Task 1. *(*) Let $p_1, p_2, \dots, p_n$ be a finite sequence of natural numbers, $0 \le p_1 \le p_2 \le \dots \le p_n$. Let $p'_k = \sum_{i=1}^{k} p_i$. Prove that a tournament with outdegree $p_1, p_2, \dots, p_n$ exists if and only if $p'_k \ge \binom{k}{2}$ for $1 \le k \le n$ and $p'_n = \binom{n}{2}$.*

Task 2. *Let T be a tournament where everyone loses at least one match ($\delta^-(T) > 0$).*

(i) Prove that given king x, there exists another king $y \in V(T)$ such that $y \to x$.

(ii) Prove (using (i)) that T has at least 3 kings.

Task 3. *Check whether the following structures are posets. If so, find all minimal elements ans all maximal ones, and the minimum element and the maximum one (provided they exists). Check whether an infinite chain and antichain exists.*

(i) $(\mathcal{P}(\mathbb{N}), \subseteq)$, where $\mathcal{P}$ denotes the power set: $\mathcal{P}(\mathbb{N}) = \{A \subseteq \mathbb{N}\}$.

(ii) $(\mathcal{P}(\mathbb{N}), \supseteq)$

(iii) $(\mathbb{N}^+, |)$, where $|$ is a divisibility relation, i.e. $a|b$ means that a divides b.

(iv) $(\mathbb{N} \times \mathbb{N}, \preceq)$, where $(a_1, a_2) \preceq (b_1, b_2)$ iff $(a_1 \le b_1) \wedge (a_2 \le b_2)$.

Task 4. *Draw Hasse diagrams, find all minimal and maximal elements as well as the minimum and the maximum one (provided they exists) and determine the length ℓ and the width w of the following posets. Show that they are covered by w chains and by ℓ antichains, with respective values of w and ℓ.*

(i) $(\mathcal{P}(\{1,2,3,4\}) \setminus \{\emptyset\}, \subseteq)$

(ii) $(\{3,4,\dots,16\}, |)$

(iii) $(\{1,2,3,4\} \times \{1,2,3,4\}, \preceq)$, where $(a_1, a_2) \preceq (b_1, b_2)$ wtw. $(a_1 \le b_1) \wedge (a_2 \le b_2)$.

Task 5. *(*) Prove* Mirski's Theorem*: The length of a finite poset $(X, \preceq)$ is equal to the minimum number of antichains that cover all elements of X.*

Task 6. *Prove that every finite partial order has an linear extension.*