Advanced Functions Class 6 Homework

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Rational Functions (2)

1. Solve each of the following equations.

a. $\frac{x^2-5x-6}{2x^2-x-3}=0$ $\quad x\neq -1, \frac{3}{2}$

$0=\frac{\cancel{(x+1)}(x-6)}{(2x-3)\cancel{(x+1)}}$

$x=6$

b. $\frac{x}{x-2}+\frac{1}{1-x}=\frac{x}{x^2-3x+2}$ $\quad x\neq 1,2$

$\frac{x(x-1)-(x-2)}{(x-2)(x-1)}=\frac{x}{(x-1)(x-2)}$

$x^2-x-x+2-x=0$

$x^2-3x+2=0$ $\quad x=1,2$

$(x-1)(x-2)=0$ $\quad$ No solutions!

c. $\frac{2x^2-x}{3}-4=\frac{3}{x}$ $\quad x\neq 0$

$2x^3-x^2-12x=9$ $\quad$ root: 3

$2x^3-x^2-12x-9=0$

$$\begin{array}{r|rrrr} 3 & 2 & -1 & -12 & -9 \\ & & 6 & 15 & 9 \\ \hline & 2 & 5 & 3 & 0 \end{array}$$

$(x-3)(2x^2+5x+3)=0$

$(x-3)(2x+3)(x+1)=0$

$x=3,-\frac{3}{2},-1$

d. $\frac{x+3}{1-\frac{3}{1-\frac{1}{x+3}}}=-\frac{x}{2}$ $\quad x\neq -3,-2,-\frac{2}{7}$

$=\frac{x+3}{1-\frac{3}{\left(\frac{x+2}{x+3}\right)}}=\frac{x+3}{1-\frac{3x+9}{x+2}}$

$=\frac{x+3}{\left(\frac{-2x-7}{x+2}\right)}=\frac{(x+3)(x+2)}{-2x-7}$

$\Rightarrow \frac{(x+2)(x+3)}{2x+7}=\frac{x}{2}$

$2(x^2+5x+6)=2x^2+7x$

$\cancel{2x^2}+10x+12=\cancel{2x^2}+7x$

$3x=-12$

$x=-4$

2. Given a rational function $f(x)=\frac{x^2-x-12}{2x^2+9x+4}$, determine all asymptotes of the function. Show, algebraically, that the graph of the function will cross the horizontal asymptote.

$f(x)=\frac{(x-4)(x+3)}{(2x+1)(x+4)}$

V. Asymptotes: $-4,-\frac{1}{2}$

H.A: $y=\frac{1}{2}$

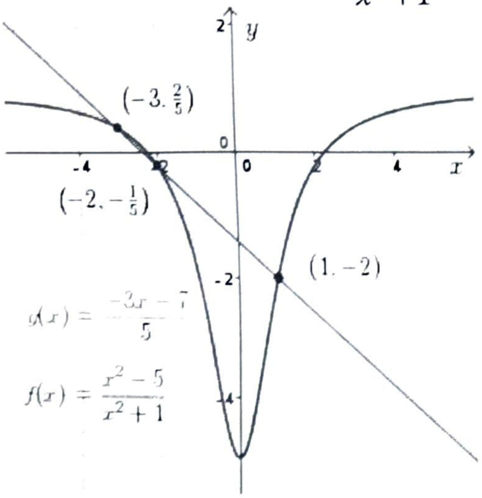
$f(-3.5)=-\frac{5}{4}$

$f(-1)=\frac{10}{3}$

$\because$ $\{-3.5,-1\}$ do not cross any vertical asymptotes,

$\therefore$ $f(x)$ crosses the horizontal asymptote $(\frac{1}{2})$

3. Given the graphs of $f(x)=\dfrac{x^2-5}{x^2+1}$ and $g(x)=\dfrac{-3x-7}{5}$, determine the solution of $\dfrac{x^2-5}{x^2+1}<\dfrac{-3x-7}{5}$



$x \in (-\infty,-2] \cup (-3,1)$

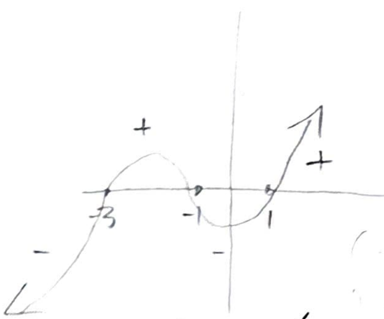
4. Solve each inequality algebraically. State the solution using interval notation, where $x\in$ R.

a. $\dfrac{3x+4}{2x-1}>0$

$(3x+4)(2x-1) > 0$

$+ \quad -\tfrac{4}{3} \quad - \quad \tfrac{1}{2} \quad +$

$x \in (-\infty,-\tfrac{4}{3}) \cup (\tfrac{1}{2},\infty)$



b. $\dfrac{3-x}{2x+2}>\dfrac{x}{2}$ $\quad x \neq -1$

$\dfrac{3-x}{2x+2} - \dfrac{x(x+1)}{2(x+1)} > 0$

$\dfrac{3-x-x^2-x}{2x+2} > 0$

$\dfrac{(x+3)(x-1)}{(x+1)} < 0$

$0 \le \dfrac{4x-5}{x-3}$

$- \quad \tfrac{5}{4} \quad - \quad 3 \quad +$

c. $\dfrac{3}{x-2}-\dfrac{x-3}{x+1}>\dfrac{x}{x-2}$

$\dfrac{3-x}{x-2} - \dfrac{x-3}{x+1} > 0$

$\dfrac{(3-x)(x+1)-(x-3)(x-2)}{(x-2)(x+1)} > 0$

$\dfrac{(x-3)(x+1)+(x-3)(x-2)}{(x-2)(x+1)} > 0$

$\dfrac{(x-3)(2x-1)}{(x-2)(x+1)} < 0$

$+ \quad -1 \quad - \quad \tfrac{1}{2} \quad + \quad 2 \quad - \quad 3 \quad +$

$x \in (-1,\tfrac{1}{2}) \cup (2,3)$

d. $\left|\dfrac{x+4}{x-3}\right| \le 3$

$-3 \le \dfrac{x+4}{x-3}$, $\qquad \dfrac{x+4}{x-3} \le 3$

$0 \le \dfrac{x+4}{x-3}+3$ $\qquad \dfrac{x+4}{x-3}-3 \le 0$

$0 \le \dfrac{x+4+3(x-3)}{x-3}$ $\qquad \dfrac{x+4-3(x-3)}{x-3} \le 0$

$\dfrac{-2x-5}{x-3} \le 0$

$- \quad -\tfrac{5}{2} \quad + \quad 3 \quad -$

$x \in [3,\infty) \cup (-\infty,-\tfrac{5}{2}]$

5. a. If $T = x^2 + \frac{1}{x^2}$, determine the values of b and c so that $x^6 + \frac{1}{x^6} = T^3 + bT + c$ for all non-zero real numbers x.

b. If x is a real number satisfying $x^3 + 1/x^3 = 2\sqrt{5}$, determine the exact value of $x^2 + 1/x^2$.

a)

$$T^3 = \left(x^2 + \frac{1}{x^2}\right)^3 = x^6 + 3x^2 + \frac{3}{x^2} + \frac{1}{x^6}$$

$$bT = bx^2 + \frac{b}{x^2}$$

$$c = c$$

$$\cancel{x^6} + \cancel{\frac{1}{x^6}} = \cancel{x^6} + (3+b)x^2 + \frac{(3+b)}{x^2} + \cancel{\frac{1}{x^6}} + c$$

This is true for all x

$3 + b = 0$, $b = -3$, $c = 0$

b)