

AP Calculus In-Class Ten – Applications of Definite Integral and Polar Coordinates

5.1 Area and Solids of Revolution

In Questions 1 - 11, evaluate the area of the region whose boundaries are given.

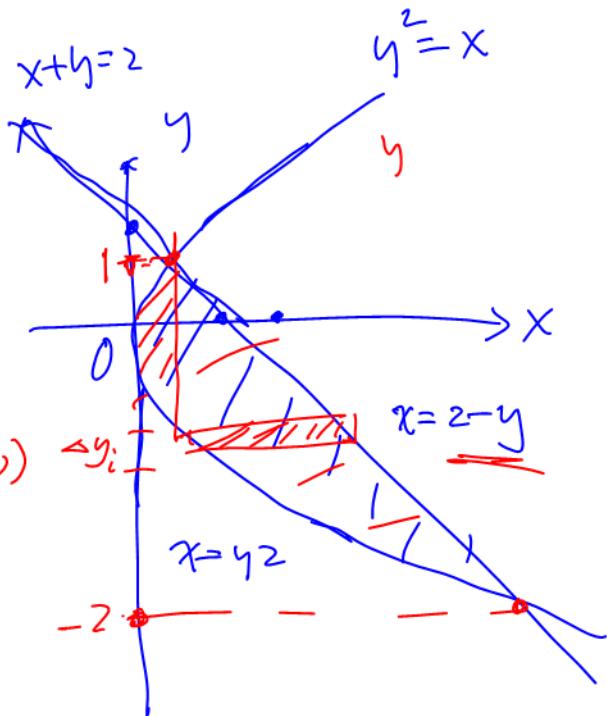
1. The curve of $y = x^2$, $y = 0$, $x = -1$, and $x = 2$.

$$\begin{aligned}y^2 &= 2 - y \\y^2 + y - 2 &= 0 \\(y+2)(y-1) &= 0\end{aligned}$$

2. The parabola of $y^2 = x$ and the line $x + y = 2$.

$$\begin{aligned}A &= \int_{-2}^1 [f(y) - g(y)] dy = \int_{-2}^1 [2-y - y^2] dy \\&= \left[2y - \frac{y^2}{2} - \frac{y^3}{3} \right]_{-2}^1 = 2(1-(-2)) - \frac{1}{2}(1^2 - (-2)^2) - \frac{1}{3}(1^3 - (-2)^3) \\&= 6 + \frac{3}{2} - \frac{3}{3} = 3 + \frac{3}{2} = 4\frac{1}{2}\end{aligned}$$

3. The curve of $y = 2/x$ and $x + y = 3$.



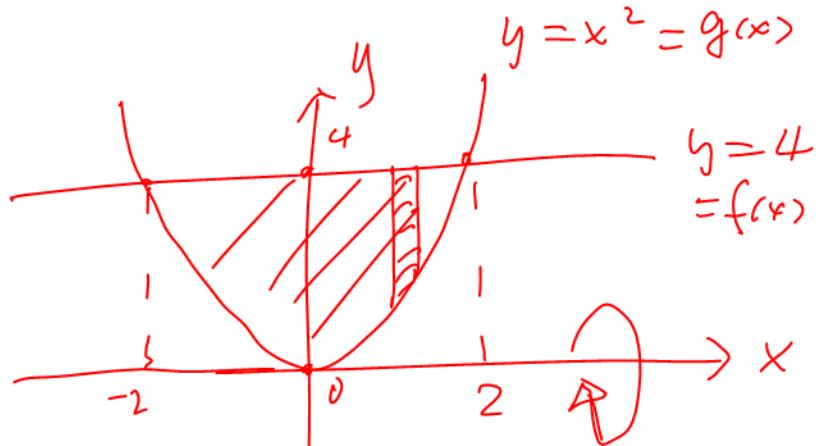
4. In the first quadrant, bounded below by the x -axis and above by the curves of $y = \sin x$ and $y = \cos x$.

5. The curve of $y = \cot x$, the line $x = \pi/4$, and the x -axis.

6. The curve of $y = x^3 - 2x^2 - 3x$ and the x -axis.
7. Find the total area bounded by the cubic $x = y^3 - y$ and the line $x = 3y$.
8. What is the area enclosed by the ellipse with parametric equations
 $x = 2 \cos \theta$ and $y = 3 \sin \theta$?
9. Find the area enclosed by one arch of the cycloid with parametric equations
 $x = \theta - \sin \theta$ and $y = 1 - \cos \theta$.
10. Write a definite integral to represent the area enclosed by the curve $y^2 = x(1 - x)$.
11. Find the area bounded by the curve $y = x^3$, $y = c^3$, and the y -axis.

In Questions 12 - 15, the region whose boundaries are given is rotated about the line indicated. Calculate the volume of the solid generated.

12. The first quadrant region bounded by $y = x^2$, the y -axis, and $y = 4$; about the y -axis.



13. $y = x^2$ and $y = 4$; about the x -axis.

$$\begin{aligned}
 V &= \pi \int_{-2}^2 ([f(x)]^2 - [g(x)]^2) dx \\
 &= \pi \int_{-2}^2 (4^2 - (x^2)^2) dx = 2\pi \int_0^2 (16 - x^4) dx = 2\pi \left[16x - \frac{1}{5}x^5 \right]_0^2 \\
 &= 2\pi \left[16(2) - \frac{1}{5}(2)^5 \right] = 2\pi \left[32 - \frac{1}{5}(32) \right] = 64\pi \left(\frac{4}{5} \right) \\
 &= \frac{256\pi}{5}
 \end{aligned}$$

14. The first-quadrant region under $y = e^{-x}$; about the x -axis.

15. A trapezoid with vertices at $(2, 0)$, $(2, 2)$, $(4, 0)$, and $(4, 4)$; about the x -axis.

In Questions 15 - 18, evaluate the area, if it exists, of the region described.

16. In the first quadrant under the curve of $y = xe^{-x^2}$.

17. In the first quadrant above $y = 1$, between by the y -axis and the curve $xy = 1$.

18. Between the curve $y = 4/(1 + x^2)$ and the x -axis.

19. Above the x -axis, between the curve $y = \frac{4}{\sqrt{1-x^2}}$ and its asymptotes.

In Questions 20 - 21, write a definite integral that gives the area of the region whose boundaries are given.

20. The area bounded by the parabola $y = 2 - x^2$ and the line $y = x - 4$.

21. The area enclosed by the hypocycloid with parametric equations $x = \cos^3 t$ and $y = \sin^3 t$.

22. Suppose the following is a table of ordinates for $y = f(x)$, given that f is continuous on $[1, 5]$:

x	1	2	3	4	5
y	1.62	4.15	7.5	9.0	12.13

If a trapezoid sum is used, when $n = 4$, then find the area under the curve, from $x = 1$ to $x = 5$, rounding up (or down) the answer to two decimal places.

23. What is the exact total area bounded by the curve $y = x^3 - 4x^2 + 3x$ and the x -axis?

24. If the curve of $f(x)$ and $g(x)$ intersect for $x = a$ and $x = b$ and if $f(x) > g(x) > 0$ for all x on (a, b) , then what is the volume obtained when the region bounded by the curves is rotated about the x -axis?