$f(x) = \sum_{k=0}^{n} a_k x^k$ ← k goes from 0 to n

First Name: Adam   Last Name: Chen   Student ID: ________

$f(x) = x^2 + 1 = (x-i)(x+i)$

Factored Form:

$f(x) = a_n(x-r_1) \cdot (x-r_2) \cdots (x-r_n)$ where r_i is a root i.e the zeros

Polynomial Functions

Polynomial Function

Polynomial functions are functions that have this form:

Standard form

$$\boxed{f(x) = \underline{a_n} x^n + \underline{a_{n-1}} x^{n-1} + \dots} + \underline{a_1} x + \underline{a_0}$$

- The value of **n** must be a whole number (0, 1, 2...)
- The coefficients are $a_n, a_{n-1}, \dots, a_1, a_0$. These are **real numbers**
- The degree of the polynomial function is the highest value for **n** where a_n is not equal to 0.
- a_n is called leading coeff and $a_n x^n$ is called leading term.

Example 1: Establish which of the following functions are polynomial functions. Justify your answer. If it is polynomial, please identify the degree.

a_0 = constant term

a. $f(x) = x - 3x^8$ Yes, 8

b. $y = -4x^{2016} - \sqrt[8]{5}x^2 - 7x + 2$ Yes, 2016

c. $g(x) = 8(2x+7)^3 - 3$ Yes, 3

d. $h(x) = \frac{3}{x^2}$

e. $y = 2\sqrt[8]{x} + x^{\frac{2}{5}}$

f. $x = y^4$

NO

x^{-2} isn't a whole

$\cancel{\frac{2}{5}}$

$y = \sqrt[4]{x}\,x$

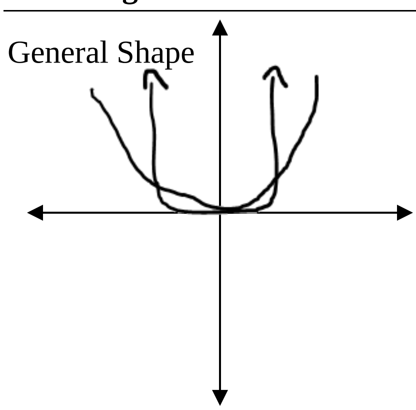
Power Functions

A **power function** is a polynomial of the form $y = ax^n$, where n is a whole number.

Standard form = sum of power functions

Even Degree Power Functions

General Shape



End Behaviour: As $x \to \infty$, $y \to \infty$

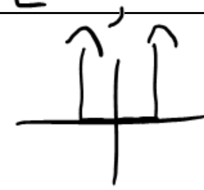
As $x \to -\infty$, $y \to \infty$

Type of Symmetry: Reflection on y axis

Domain: $x \in \mathbb{R}$

Range: $y \in [0, \infty]$

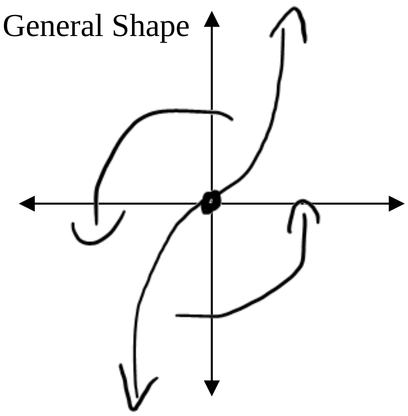
$x^{1000} \to$



As the degree of the function increases, the graph of the function It gets flatter about the origin

Odd Degree Power Functions

General Shape



End Behaviour: As $x \to \infty$, $y \to \infty$
As $x \to -\infty$, $y \to -\infty$

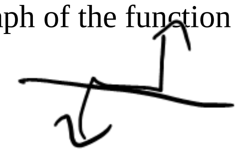
Type of Symmetry: Rotation

Domain: $\mathbb{R}$

Range: $\mathbb{R}$

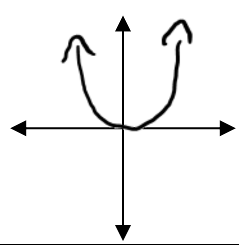
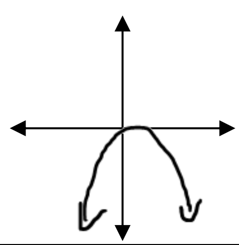
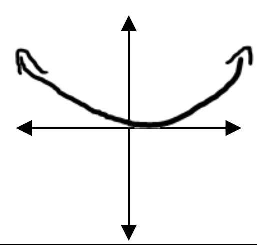
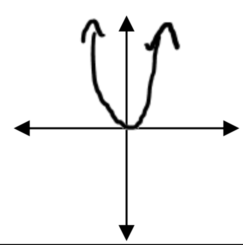
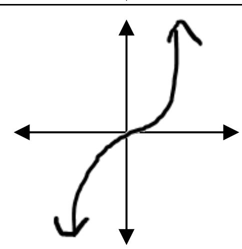
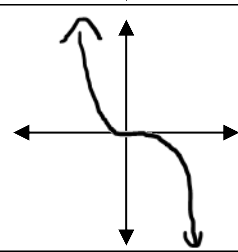
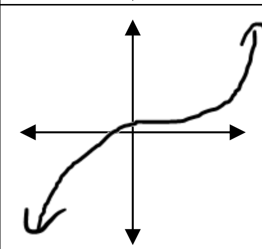
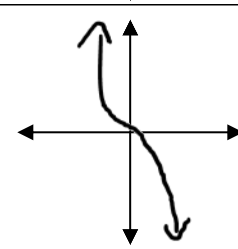
As the degree of the function increases, the graph of the function It gets flatter about the origin

x^{1001}

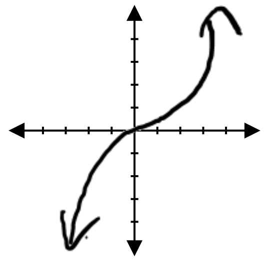


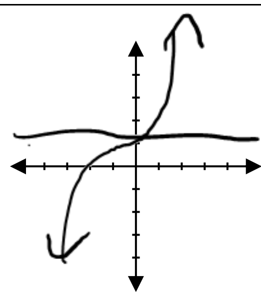
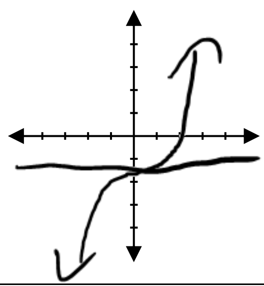
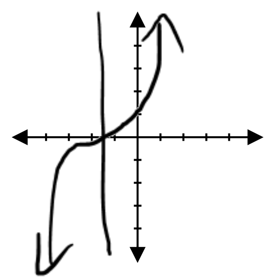
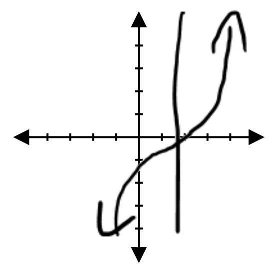
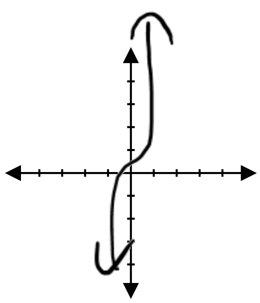
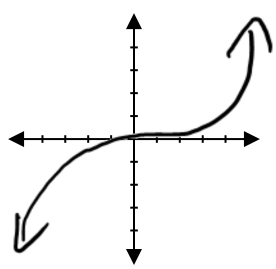
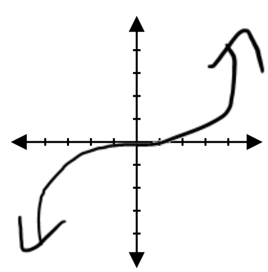
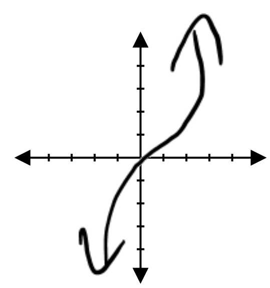
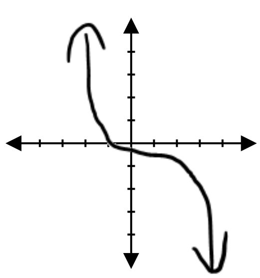
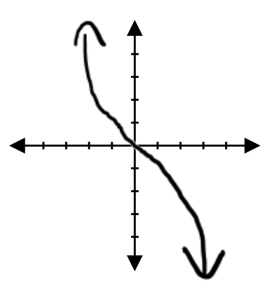
Leading Coefficient

For function $f(x) = ax^n$ $0 < |a| < 1$

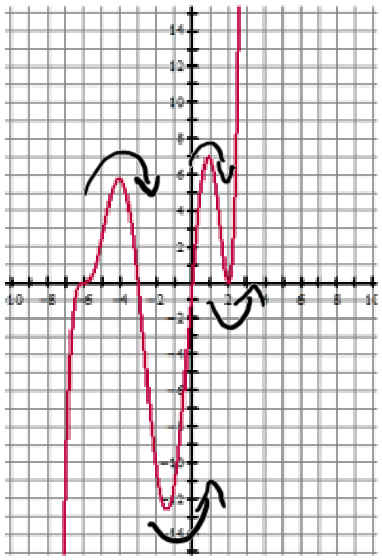
Value of a	**a = 1**	**a < 0**	**0 < a < 1 or 0 > a > -1**	**a > 1 or a < -1**
Effect $x^n \to ax^n$	x^n	vertical reflection	(compression (vertical))	stretch (vertical)
Even Degree Graph				
Odd Degree Graph				

Power Functions & Transformations Investigation

Function	$y = x^3$
Graph	

Function	$y = x^3 + 2$	$y = x^3 - 2$	$y = (x + 2)^3$	$y = (x - 2)^3$
Graph				
Function	$y = 2x^3$	$y = \frac{1}{2}x^3$	$y = (2x)^3$	$y = (\frac{1}{2}x)^3$
Graph				
Function	$y = -x^3$	$y = (-x)^3$		
Graph				

Graphing Polynomial Functions Investigation



Determine the number of x-intercept and local max/min points for the following function.

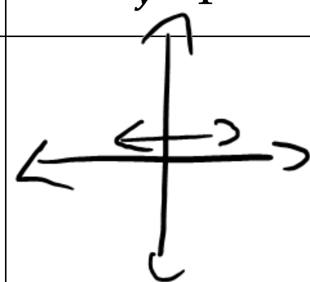
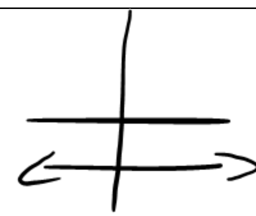
Number of x-intercepts	4
Number of Local Max/Min Points "Turning points"	4

Given the degree of the polynomial, what is the most zeros and max/min points the graph can have?

Constant Functions

a) How many x-intercepts can a Constant Function have?

b) What is the maximum number of max/min points that a Constant Function can have?

Function	$y = 1$	$y = -7$
Graph		
# of x-intercepts	0	0
# of Local Max/Min Points	0	0

y≠0 strict max

Linear Function

Function	$y = x$	$y = -2x + 1$	$y = 4x - 8$
Factored	y=x	y=-2x+1	y= 4x-8
Graph			

# of X-Intercept	1	1	1
# of Local Max/Min Points	0	0	0

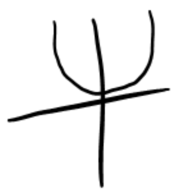
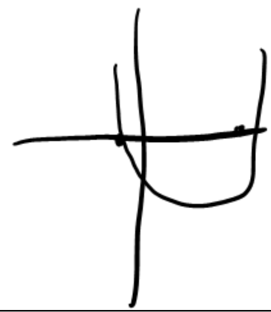
a) How many x-intercepts can a Linear Function have?

1, ∞ if $y=0$

b) What is the maximum number of max/min points that a Linear Function can have?

0

Quadratic Functions

Function	$y = x^2$	$y = x^2 - 4x - 5$	$y = -x^2 - 6x - 11$
Factored	$y=x^2$	$y=(x+1)(x-5)$	None
Graph			
# of X-Intercept	1	2	0
# of Local Max/Min Points	1	1	1

a) How many x-intercepts can a Quadratic Function have?

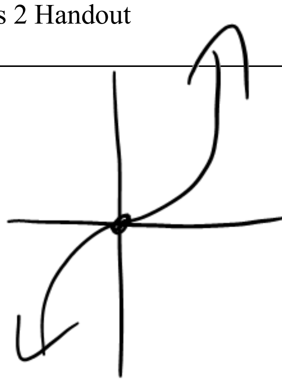
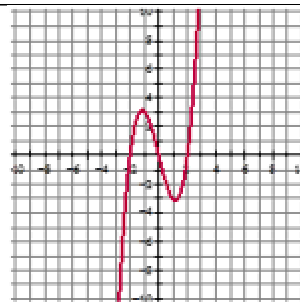
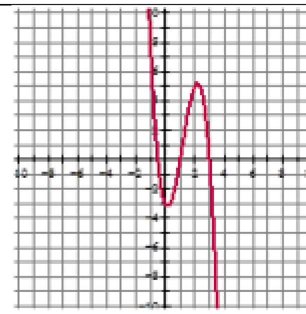
≤ 2

b) What is the maximum number of max/min points that a Quadratic Function can have?

≤ 1

Cubic Functions

Function	$y = x^3$	$y = x^3 - 4x$	$y = -2x^3 + 7x^2 - 2x - 3$
Factored	$y=x^3$	$y=x(x-2)(x+2)$	$y = -1(2x + 1)(x - 1)(x - 3)$

Graph			
# of X-Intercept	1	3	3
# of Local Max/Min Points	0	2	2

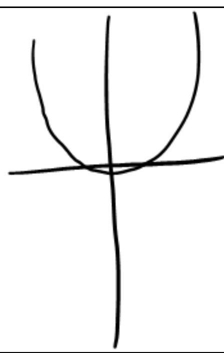
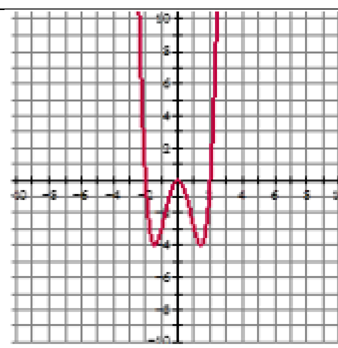
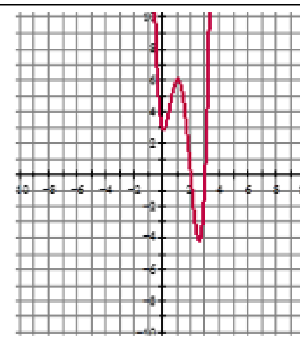
a) How many x-intercepts can a Cubic Function have?

≤ 3

b) What is the maximum number of max/min points that a Cubic Function can have?

≤ 2

Quartic Functions

Function	$y = x^4$	$y = x^4 - 4x^2$	$y = 2.5x^4 - 12.5x^3 + 15.5x^2 - 2.5x + 3$
Factored			$y = 0.5(x-2)(x-3)(5x^2+1)$
Graph			
# of X-Intercept	1	3	2
# of Local Max/Min Points	1	3	3

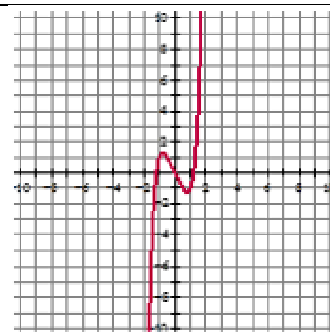
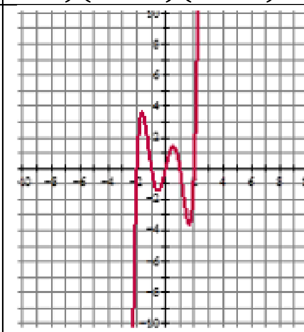
a) How many x-intercepts can a Quartic Function have?

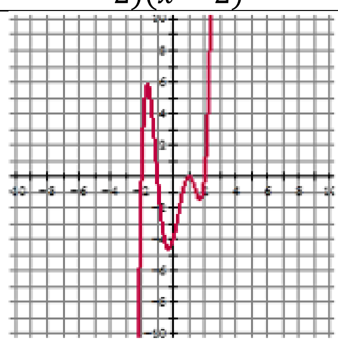
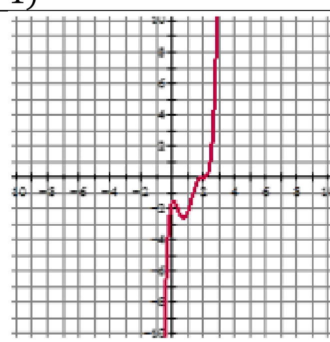
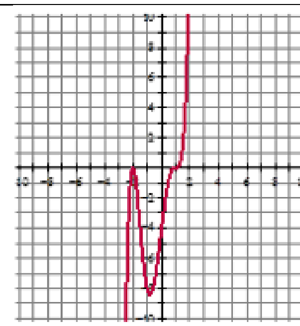
≤ 4

b) What is the maximum number of max/min points that a Quartic Function can have?

≤ 3

Quintic Functions

Function	$y = x^5$	$y = x^5 - 2x$	$y = 5x^5 - 5x^3 + 4x$
Factored			$y = x(x+1)(x-1)(x+2)(x-2)$
Graph			
# of X-Intercept			
# of Local Max/Min Points			

Function	$y = x^5 - x^4 - 5x^3 + 5x^2 + 4x - 4$	$y = 2x^5 - 12x^4 + 24.2x^3 - 17.2x^2 + 2.4x - 1.6$	$y = x^5 + x^4 - 5x^3 - x^2 + 8x - 4$
Factored	$y = (x+1)(x-1)^2(x+2)(x-2)$	$y = 0.2(x-2)^3(10x^2+1)$	$y = (x+2)^2(x-1)^3$
Graph			
# of X-Intercept			
# of Local Max/Min Points			

a) How many x-intercepts can a Quintic Function have?

≤ 5

b) What is the maximum number of max/min points that a Quintic Function can have?

≤ 4

We did not prove these answers, we just observed
the patterns from the examples

How does the maximum number of local max/min points compare to the degree of the function?

One less than the degree

Predict the degree of a function that has 7 local max/min points.

degree >= 8

How does the number of x-intercepts that a function can have compare to the degree of the function?

Degree >= # intercepts
r is rootL (x-r) is a factor

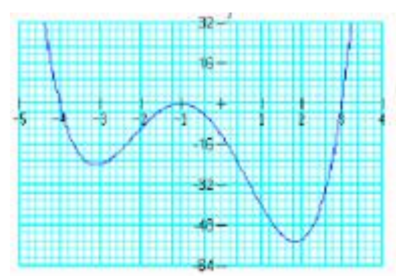
Predict the degree of a function that has 11 x-intercepts.

degree >= 11

What relationship do you notice between the factored form of the equation and the graph of the function?

Factored forms tells us the x intercepts and their
order (By looking at the exponent)

Characteristics of Polynomial Functions



<u>Local Minimum and Maximum Points:</u>

Let's look at the graph of the polynomial function defined by $y = x^4 + 3x^3 - 9x^2 - 23x - 12$.

In general, polynomial function graphs consist of a smooth line with a series of hills and valleys. The hills and valleys are called

Turning points. Each turning point corresponds to a Local min

or a Local Max

The maximum possible number of local min/max points is 1 less than the degree of the polynomial.

Example: The polynomial above $y = x^4 + 3x^3 - 9x^2 - 23x - 12$ has degree 4 and has 2 local minimums and 1 local maximum, for a total of 3 turning points.

Even degree poly's have an odd number of turning points
odd degree poly's habe an even number of turning points

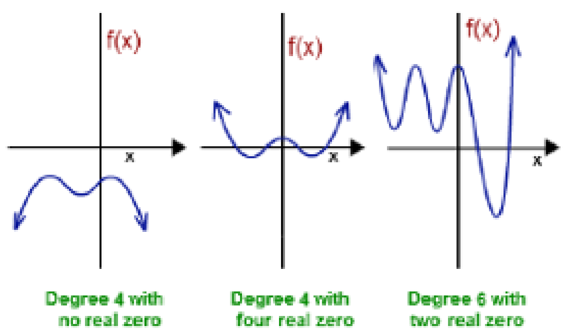
Zero (or x-intercepts) of polynomial functions

A **zero** of a polynomial function is an x value for which $y = 0$. At these x values, its graph **cuts or touches** the x axis.

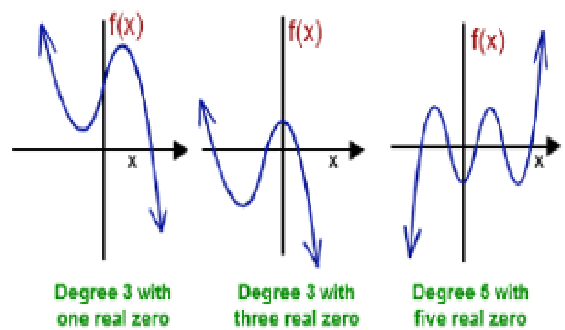
- **The maximum number of zeros of <u>any</u> polynomial is ______________ its degree.**

Example: The polynomial function $y = x^4 + 3x^3 - 9x^2 - 23x - 12$, graphed above, has ___3___ zeros at $x =$ ________________________. This polynomial ______________ the x-axis at -4 and 3, but only ______________ the x-axis at $x = -1$.

- **Even-degree polynomials may not have zeros.**



- **Odd-degree polynomials must have at least one zero.**



Key Features of Graphs of Polynomial Functions with Odd Degree

- Odd-degree polynomial have at least one zero, up to a maximum of n x-intercepts, where n is the degree of the function.
- The domain is $\{x \in R\}$ and the range is $\{y \in R\}$.
- They have no maximum point and no minimum point.
- They <u>may</u> have point symmetry.

Positive Leading Coefficient

- Graph extends from quadrant 3 to quadrant 1.
  OR "as $x \longrightarrow -\infty$, $y \longrightarrow -\infty$" and "as $x \longrightarrow \infty$, $y \longrightarrow \infty$"

Negative Leading Coefficient

- Graph extends from quadrant 2 to quadrant 4.
  OR "as $x \longrightarrow -\infty$, $y \longrightarrow \infty$" and "as $x \longrightarrow \infty$, $y \longrightarrow -\infty$"

Key Features of Graphs of Polynomial Functions with Even Degree

- Even-degree polynomial have no zeros, up to a maximum of n x-intercepts, where n is the degree of the function.
- The domain is $\{x \in R\}$.
- They <u>may</u> have a line of symmetry.

Positive Leading Coefficient

- Graph extends from quadrant 2 to quadrant 1.
  OR "as $x \longrightarrow -\infty$, $y \longrightarrow \infty$" and "as $x \longrightarrow \infty$, $y \longrightarrow \infty$"
- The range is $\{y \in R, y \geq a\}$, where a is the minimum value of the function.
- It will have at least one minimum point.

Negative Leading Coefficient

- Graph extends from quadrant 4 to quadrant 4.
  OR "as $x \longrightarrow -\infty$, $y \longrightarrow -\infty$" and "as $x \longrightarrow \infty$, $y \longrightarrow -\infty$"
- The range is $\{y \in R, y \leq a\}$, where a is the maximum value of the function.
- It will have at least one maximum point.

Example 1: Sketch a possible graph for each of the following functions.

a. $y = -x(x+2)(2x-5)$

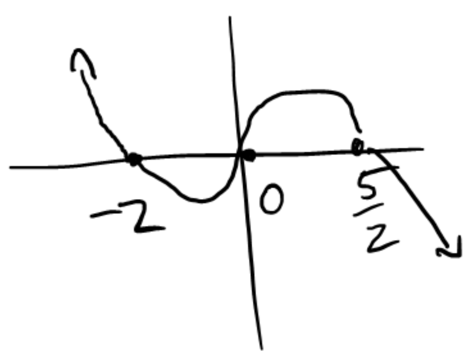
b. $f(x) = 2(x-2)^2(x+3)^2$

c. $g(x) = -0.5(x-3)(x+1)^3$

d. $y = 2x^2(x-4)^3$

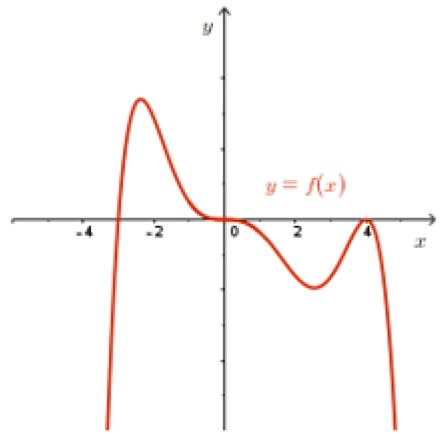
e. $f(x) = -x(2x+3)(x-2)^2$

1a



degree: 3
leading: -2 Lo

Example 2: Determine a possible equation for the polynomial function $y = f(x)$ shown below.



Example 3

a. State the equation of a cubic function that has exactly two x-intercepts.

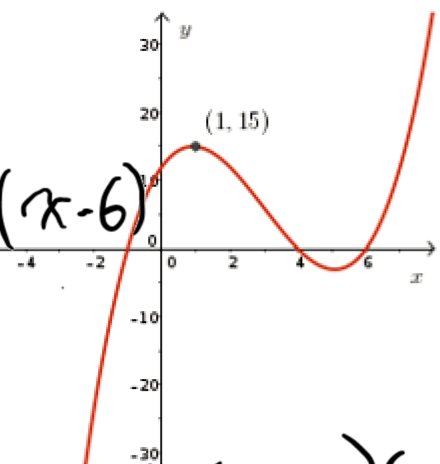
b. State the equation of a quartic function that has exactly two x-intercepts.

c. State the equation of a quintic function that has exactly two x-intercepts.

Example 4: Determine the equation given the graph of the polynomial function $y = g(x)$ with integer zeros.

Looks like a cubic

roots at -1, 4 and 6



So, $f(x) = a(x+1)(x-4)(x-6)$

$f(1) = 15$

$15 = a(2)(-3)(-5)$

$15 = a30$

$a = \frac{1}{2}$

$f(x) = \frac{1}{2}(x+1)(x-4)(x-6)$