Advanced Functions Class 4 Homework

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Polynomial Equations and Inequalities (2)

1. Factor fully.

a. x^3-x^2+x-1 $=x^2(x-1)+(x-1)$ $=(x^2+1)(x-1)$	e. $5x^3+3x^2-12x+4$ root: $\frac{2}{5}$ $=(5x-2)(x^2+x-2)$ $=(5x-2)(x+2)(x-1)$ $\frac{2}{5}$ | 5 3 -12 4 2 2 -4 5 5 -10 0
b. $2x^3+11x^2+12x-9$ root: $\frac{1}{2}$ $=(2x-1)(x^2+6x+9)$ $=(2x-1)(x+3)^2$	f. $x^3+9x^2+8x-60$ root: 2 $(x-2)(x^2+11x+30)$ $(x-2)(x+5)(x+6)$ 2 | 1 9 8 -60 2 22 60 1 11 30 0
c. x^3-7x-6 3 | 1 0 -7 -6 3 9 6 1 3 2 0 $=(x-3)(x^2+3x+2)$ $=(x-3)(x+1)(x+2)$	g. x^4-5x^2+4 let $x^2=w$ w^2-5w+4 $(w-1)(w-4)=(x^2-1)(x^2-4)$ $=(x-1)(x+1)(x-2)(x+2)$
d. $3x^3-3x^2+6x-24$ root: 2 $3(x^3-x^2+2x-8)$ $=3(x-2)(x^2+x+4)$ 2 | 1 -1 2 -8 2 2 8 1 1 4 0	h. $x^4+3x^3-38x^2+24x+64$ root: 2 $=(x-2)(x^3+5x^2-28x-32)$ $=(x-2)(x-4)(x^2+9x+8)$ $=(x-2)(x-4)(x-8)(x+1)$ 2 | 1 3 -38 24 64 2 10 -56 -64 1 5 -28 -32 0

root: 4

4 | 1 5 -28 -32
4 36 32
1 9 8 0

2. Determine the values of m and n if $3x^2-x-2$ is a factor of the polynomial $3x^4+mx^3-19x^2+nx+12$. Express the polynomial in factored form.

$a(3x^2-x-2)=f(x)$

$a(x-1)(3x+2)=$

$f(1)=0 \quad =3+m-19+n+12=m+n-4$

$m+n=4 \qquad n=4-m$

$f(-\frac{2}{3})=0 \quad =\frac{16}{27}-\frac{8m}{27}-\frac{76}{9}-\frac{2n}{3}+12$

$-\frac{8m}{27}-\frac{2n}{3}=-\frac{112}{27}$

$8m+18n=112$

$4m+9n=56$

$4m+9(4-m)=56$

$4m+36-9m=56$

$-5m=20$

$m=-4$

$n=8$

$f(x)=3x^4-4x^3-19x^2+8x+12$

$=(3x+2)(x-1)(x+2)(x-3)$

3. If $x-2$ and $x+2$ are factors of $6x^3+ax^2+bx+16$, determine the values of a and b, and any remaining factors.

$f(2)=0 = 48+4a+2b+16$ ①   ①-②: $96+4b=0$ $b=-24$

$f(-2)=0 = -48+4a-2b+16$ ②

$a=-4$

$f(x)=6x^3-4x^2-24x+16$

$$\begin{array}{r|rrrr} 2 & 6 & -4 & -24 & +16 \\ & & 12 & 16 & -16 \\ \hline & 6 & 8 & -8 & 0 \end{array} \qquad \begin{array}{r|rrr} -2 & 6 & 8 & -8 \\ & & -12 & 8 \\ \hline & 6 & -4 & 0 \end{array}$$

$=(x-2)(6x^2+8x-8)$

$=(x-2)(x+2)(6x-4)$

4. Given $f(x) = 2x^4+3x^3-5x^2+3x+2$. If k is a non-zero real root of $f(x) = 0$, show that $\frac{1}{k}$ is a also a root.

Symmetric polynomial

$f(k)=0 \quad 2(k)^4+3(k)^3-5(k)^2+3(k)+2=0$

$f(\frac{1}{k})=0 \quad 2(\frac{1}{k})^4+3(\frac{1}{k})^3-5(\frac{1}{k})^2+3(\frac{1}{k})+2=0$

$\because k\neq 0 \quad k^4 \cdot f(\frac{1}{k}) = 2+3(k)-5(k)^2+3(k)^3+2(k)^4 = f(k)=0$

$f(\frac{1}{k}) = f(k) \quad \therefore \frac{1}{k}$ is a root

5. Find all possible roots of the polynomial equation where $x \in C$. ← complex

a. $2x^3+5x^2+14x+6=0$
b. $8x^4=x$
c. $X^2(4x^2+17)=15$

a) root: $-\frac{1}{2}$

$$\begin{array}{r|rrrr} -\frac{1}{2} & 2 & 5 & 14 & 6 \\ & & -1 & -2 & -6 \\ \hline & 2 & 4 & 12 & 0 \end{array}$$

$=(x+\frac{1}{2})(2x^2+4x+12)$

$=(2x+1)(x^2+2x+6)$

$x^2+2x=-6$

$x^2+2x+1=-5$

$(x+1)^2=-5$

$x+1=\pm 5i$

roots: $-\frac{1}{2}, -1\pm 5i$

b) $8x^4-x=0$

$8x(x^3-\frac{1}{8})=0$

$8x(x-\frac{1}{2})(x^2+\frac{x}{2}+\frac{1}{4})=0$

$x=0, \frac{1}{2}, \frac{-1\pm\sqrt{3}}{4}$

c) $4x^4+17x^2-15$ let $w=x^2$

$(4w-3)(w+5) \to x=\pm\frac{\sqrt{3}}{2}$

or $\pm\sqrt{5}$

6. Sketch a possible graph for each polynomial function, using the intercepts and end behavior of the function.

a. $y=2x^3-12x^2+18x$
b. $y=-x^3+4x^2+x-4$
c. $y=x^4-8x^2+16$

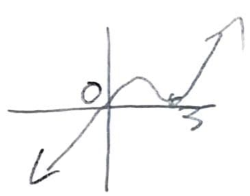
a) $2x(x-3)^2$

b) $-(x-4)(x-1)(x+1)$

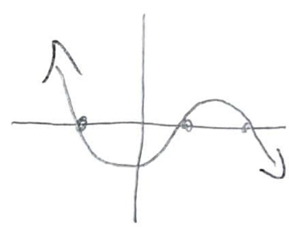
c) let $w=x^2$

$w^2-8w+16$
$=(w-4)^2$
$=(x^2-4)^2=(x+2)^2(x-2)^2$

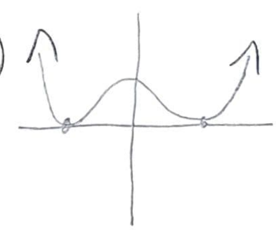
a



b)



c)



7. Explain why

a. $15x^5+4x^4+9x^2+7x+380=0$ has at least one real root.
b. $5x^6+3x^4+8x^2+120=0$ has no real roots.

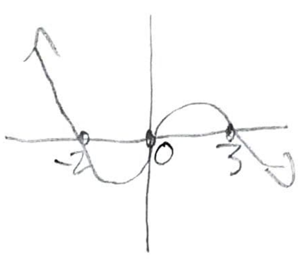
a) it has an odd degree

b) all terms have even deg, and + . Thus they are greater than 0. The graph is also shifted up by 120

8. Solve each of the following polynomial inequalities using a graphical approach, $x \in R$.

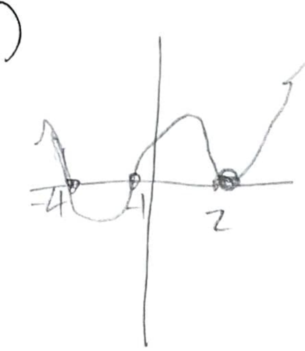
a. $-2x(x+2)(x-3)<0$
b. $(x+4)(x+1)(x-2)^2 \le 0$

a)



$x \in (-2,0) \cup (3,\infty)$

b)



$x \in [-4,-1] \cup \{2\}$

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9. Solve each of the following polynomial inequalities using an interval sign table, $x \in R$

a. $2(x+3)(x-1)(x-5) \le 0$

b. $-3(x+4)(x-3)^3 > 0$

a) <-3 <1 <5 >5

$-$ $+$ $-$ $+$

$x \in (-\infty, -3) \cup [1, 5]$

b) <-4 <3 >3

$-$ $+$ $-$

$x \in (-4, 3)$

10. Suppose $P(x)$ is a quadratic whose coefficients are all odd integers. Prove that $P(x)=0$ has no rational roots.

$P(x) = ax^2 + bx + c$, a, b, c are odd

let $\frac{m}{n}$ be a rational root

$m, n \in Z$, $n \neq 0$

$0 = P(\frac{m}{n}) = a(\frac{m}{n})^2 + b(\frac{m}{n}) + c$

$P(\frac{m}{n}) \cdot n^2 = am^2 + bmn + cn^2$

By rational root theorem $n | a$ and $m | c$

$\because$ a, c are odd and all divisors and factors of odd #'s are odd

$\therefore$ m, n are odd

$\underbrace{0}_{\text{even}} = \underbrace{am^2}_{\text{odd}} + \underbrace{bmn}_{\text{odd}} + \underbrace{cn^2}_{\text{odd}}$

The terms sum to an odd number, but 0 is even

$\therefore$ $P(x)=0$ has no rational roots