


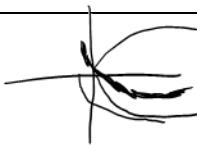

Check q 3, 6

First Name: Adam Last Name: Chen Student ID: \_\_\_\_\_**Functions: Transformations and Properties**

1. For each relation given,

a. state the domain and range;

b. identify whether the relation is a function or not.

i. $\{(1,1), (2,1), (3,2), (4,3), (5,5), (6,8)\}$ a) $d: \{1, 2, 3, 4, 5, 6\}$ $r: \{1, 2, 3, 5, 8\}$ b) function	ii. $x = -2$ a) $d: \{-2\}$ $r: \mathbb{R}$ b) relation
iii. $y = (x+1)^2 - 2$ a) $d: \mathbb{R}$ $r: [-2, \infty)$  b) function	iv. $(x-1)^2 + y^2 = 9$ $y^2 = -(x-1)^2 + 9$ a) $d: [-2, 4]$ $-3 \leq x-1 \leq 3$ $r: [-3, 3]$ $-2 \leq x \leq 4$ b) relation
v. $y = -3\sqrt{x+2} + 5$ a) $D: [-2, \infty)$ $R: (-\infty, 5]$  b) function	vi. $y = 2^{x-4} + 3$ a) $d: \mathbb{R}$ $r: (3, \infty)$  b) function

2. If  $f(x) = 2x - 3$  and  $g(x) = 6x^2 + 3x - 18$ , determine the value(s) of  $x$  such that  $f(x) = g(x)$ .

$$2x - 3 = 6x^2 + 3x - 18$$

$$6x^2 + x - 15 = 0$$

$$(3x+5)(2x-3) = 0$$

$$x = -\frac{5}{3}$$

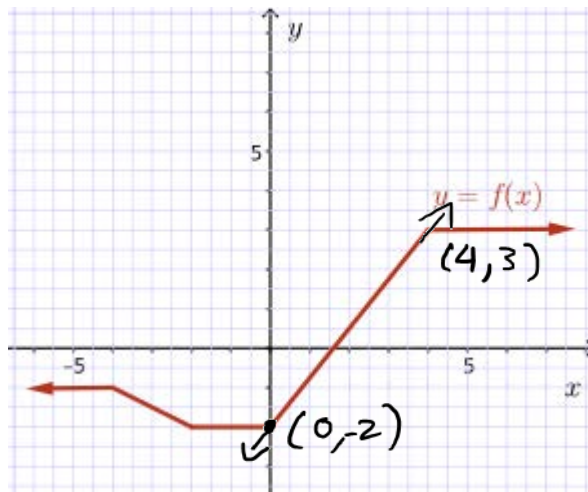
or

$$x = \frac{3}{2}$$

3. The graph of  $y=f(x)$  is shown.

Determine the following:

- the value of  $f(0)$
- the value of  $x$  such that  $f(x)=0$
- the value of  $f(4)-f(-4)$
- state the domain and range



a)  $f(0) = -2$

b)  $y = \frac{3+2}{4-0}x - 2 \quad y = \frac{5}{4}x - 2$   
 $0 = \frac{5}{4}x - 2, \quad x = \frac{8}{5}$

c)  $f(4) = \frac{1}{2} \quad f(4) - f(-4)$   
 $f(-4) = -2 \quad = \frac{1}{2} - 2 = -\frac{3}{2}$

d)  $D: \mathbb{R}$   
 $R: [-2, 3]$

4. Given  $f(x)=(x-3)^2$  and  $g(x)=4x+3$ , determine in simplest form

a.  $f(x)-g(x) \quad (x-3)^2 - 4x - 3$   
 $x^2 - 6x + 9 - 4x - 3$   
 $x^2 - 10x + 6$

b.  $2f(x) \quad 2(x-3)^2$   
 $2(x^2 - 6x + 9)$   
 $2x^2 - 12x + 18$

c.  $f(x)g(x) \quad (x-3)^2(4x+3)$   
 $(x^2 - 6x + 9)(4x+3)$   
 $4x^3 + 3x^2 - 24x^2 - 18x + 36x + 27$   
 $= 4x^3 - 21x^2 + 18x + 27$

d.  $f(g(x)) \quad (4x+3-3)^2$   
 $= (4x)^2$   
 $= 16x^2$

e.  $g(g(x)) \quad 4(4x+3)+3$   
 $16x+12+3$   
 $16x+15$

f.  $[g(x)]^2 \quad [4x+3]^2$   
 $= 16x^2 + 24x + 9$

5. If  $f(x)=5-2x+k$  and  $f(f(k))=13$ , determine the value of  $f(-4)$ .

$f(f(k)) \Rightarrow 5 - 2(5 - 2k + k) + k = 13$

$5 - 2(5 - k) + k = 13$

$5 - 10 + 2k + k = 13$

$-5 + 3k = 13$

$3k = 18$

$k = 6$

$f(x) = 5 - 2x + 6$   
 $= 11 - 2x$

$f(-4) = 11 - 2(-4)$   
 $= 11 + 8$   
 $= 19$

$$9x^2 - 6x + 5$$

$$(-3x+1)(-3x+5)$$

6. If  $g(x)=1-3x$  and  $f(g(x))=9x^2-6x+5$ , determine the value of  $f(5)$ .

$$g(x) = 1 - 3x$$

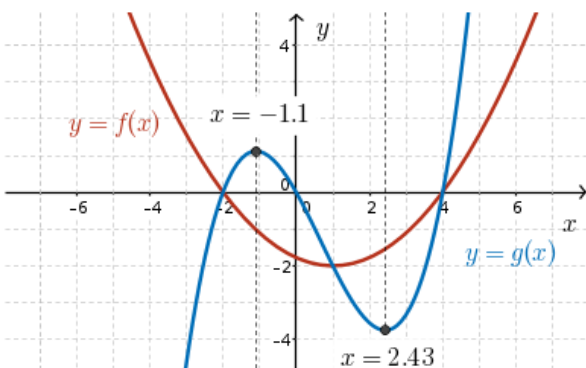
$$g(x) = 5 : 5 = 1 - 3x$$

$$4 = -3x \quad x = \frac{4}{-3}$$

$$f\left(\frac{4}{-3}\right) = 9\left(\frac{4}{-3}\right)^2 - 6\left(-\frac{4}{3}\right) + 5$$

$$= 29$$

7. Given the graphs of  $y=f(x)$  and  $y=g(x)$  as shown in the graph below



a) Points where  $f(x) = g(x)$

$$\{(-2, 0), (1, -2), (4, 0)\}$$

b)  $(-\infty, -2) \cup (0, 4)$

c)  $(-\infty, -2] \cup [1, 4]$

d)  $(-1.1, 1)$

e)

$$f(x) - \text{Local minima: } \{(1, -2)\}$$

$$g(x) - \text{Local minima: } \{(2.43, -3.6)\}$$

$$\text{Local maxima: } \{(-1.1, 1)\}$$

identify the following.

- where  $f(x)=g(x)$
- the interval(s) where  $g(x)<0$
- the interval(s) where  $f(x)\geq g(x)$
- the interval(s) where both functions are decreasing.
- the local maxima and minima for both functions

8. Solve. Write your answers using interval notation.

a.  $-3 < \frac{2x+5}{3} \leq 5$

$$\begin{aligned} -3 &< \frac{2x+5}{3} \leq 5 \\ -9 &< 2x+5 \leq 15 \\ -14 &< 2x \leq 10 \\ -7 &< x \leq 5 \end{aligned}$$

c.  $\frac{3x-6}{4x-8} \leq 1$

$2(x)$

b.  $\frac{3}{x-2} > 1$

$$\begin{aligned} x-2 &> 0 \\ 3 &> x-2 \end{aligned}$$

$$\begin{aligned} x-2 &< 0 \rightarrow x < 2 \\ 3 &< x-2 \rightarrow 5 < x \end{aligned}$$

$\rightarrow (2, 5)$

roots:  $\{-3, 0, 4\}$

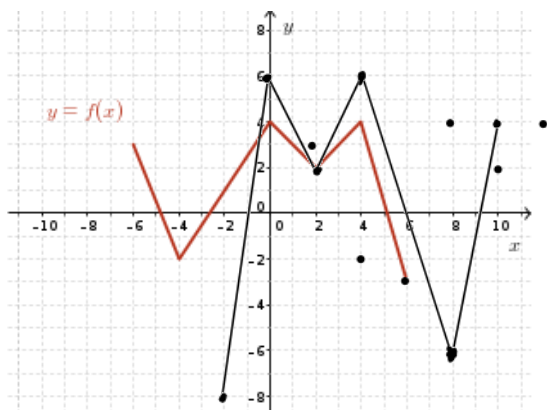
d.  $x(x+3)(x-4) > 0$

$$\begin{aligned} -4 &: < 0 \\ -2 &: > 0 \\ 1 &: < 0 \\ 5 &: > 0 \end{aligned}$$

Test by plugging in

$(-3, 0) \cup (4, \infty)$

9. Given the graph of the function  $y=f(x)$ , draw the graphs of the following transformed function  $y=2f(-(x-4))-2$ :



- Shift 4 right
- Reflect on y axis
- horizontal stretch by 2
- shift 2 down

$(x, y) \rightarrow (-x+4, 2y-2)$

10. The function  $f(x)$  satisfies the equation  $f(x)=f(x-1)+f(x+1)$  for all values of  $x$ . Define  $f(1)=1$  and  $f(3)=3$ ; then,  $f(2)=1+3=4$ . Determine the value of  $f(1867)$ .

$f(1) = f(0) + f(2)$

$f(1) = f(0) + 4 = 1$

$f(0) = -3$

$f(3) = 4 + f(4) = 3$

$f(4) = 3 - 4 = -1$

$-3, 1, 4, 3, -1, -4, -3, 1, 4, \dots$   
 $0, 1, 2, 3, 4, 5, 6, 7, 8$   
 $f(1867) = f(1867 \bmod 6) = f(1) = 1$