

## AP Calculus Homework Nine – Antiderivatives and the Definite Integral

4.4 Definition of Definite Integral and Properties of Definite Integral

4.4 The Mean Value Theorem for Definite Integral; 4.5 The Fundamental Theorem of Calculus

1. Evaluate the definite integrals.

(a)  $\int_1^4 (x^2 - 4x - 3)dx$

(b)  $\int_1^2 [5/(8x^6)]dx$

(c)  $\int_{-3}^6 |x - 4|dx$

(d)  $\int_{-\pi/6}^{\pi/6} (x + \sin 5x)dx$

(e)  $\int_0^1 \frac{1}{(3 - 2v)^2} dv$

(f)  $\int_1^4 \frac{1}{\sqrt{x}(\sqrt{x} + 1)^3} dx$

2. Let  $f$  be continuous on  $[-a, a]$ . if  $f$  is an even function, show that  $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$  and interpret this result geometrically. Verify the result for the special case  $f(x) = \cos x$ .

3. Evaluate the definite integrals.

(a)  $\int_{-1}^4 (x^2 - x - 1) dx$

(b)  $\int_0^3 \frac{dt}{\sqrt{4-t}}$

(c)  $\int_0^1 (2t-1)^3 dt$

(d)  $\int_4^9 \frac{2+x}{2\sqrt{x}} dx$

4. Evaluate the definite integrals.

(a)  $\int_0^1 e^{-x} dx$

(b)  $\int_0^1 x e^{x^2} dx$

(c)  $\int_0^{\pi/6} \frac{\cos \theta}{1+2\sin \theta} d\theta$

(d)  $\int_{\sqrt{2}}^2 \frac{u}{u^2-1} du$

5. Evaluate the definite integrals.

(a)  $\int_{\sqrt{2}}^2 \frac{u}{(u^2-1)^2} du$

(b)  $\int_0^{\pi/4} \cos^2 \theta d\theta$

$$(c) \int_{\pi/12}^{\pi/4} \frac{\cos 2x dx}{\sin^2 2x}$$

$$(d) \int_0^{\pi/2} \sin^2 \frac{x}{2} dx$$

6. Evaluate the definite integrals with substitutions.

$$(a) \int_1^2 \frac{\sqrt{4-x^2}}{x} dx, \text{ let } x = 2 \sin \theta$$

$$(b) \int_1^{\sqrt{3}} \sqrt{1+x^2} dx, \text{ let } x = \tan \theta$$

$$(c) \pi \int_0^{2a} y^2 dx, \text{ let } y = 2a \cos^2 \theta \text{ and } x = 2a \tan \theta, \text{ where } 0 \leq \theta \leq \pi$$

7. In each sub-question, find numbers that satisfy the conclusion of the Mean Value Theorem for Definite Integral.

$$(a) \int_0^3 3x^2 dx = 27$$

$$(b) \int_0^a \sqrt{a^2 - x^2} dx = (\pi a^2)/4, \quad a > 0$$