First Name:____________________ Last Name: __________________ Student ID: ________

Exponential and Logarithmic Functions (2)

A logarithm with a base of a positive number *b* is defined to be:

$$y = \log_b(x) \Leftrightarrow x = b^y$$

A logarithmic function is the inverse of an exponential function. The answer to $\log_b(x)$ gives you the exponent that **b** needs to be raised to in order to get an answer of **x**.

1. Express in logarithmic form:

a) $2^4 = 16$

b) $6^2 = 36$

c) $3^{-1} = \frac{1}{3}$

d) $x^y = z$

2. Express in exponential form:

a) $\log_5 25 = 2$

b) $\log_2 \frac{1}{8} = -3$

c) $\log_{16} 4 = \frac{1}{2}$

d) $\log_x y = z$

3. Determine the inverse of the following functions:

a) $f(x) = 3^{x+4} - 1$

b) $g(x) = -3 \cdot 2^{1-x} + 4$

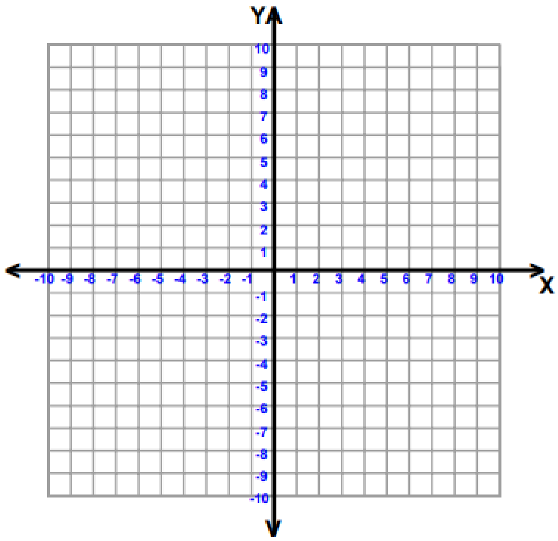
c) $h(x) = \log_3(x+2)$

d) $k(x) = -2\log(x-4) - 5$

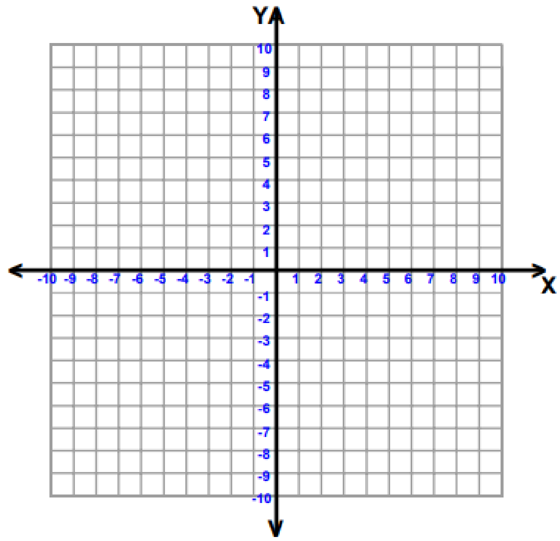
Graphing Logarithms

4. Give the domain and range of each function, then graph.

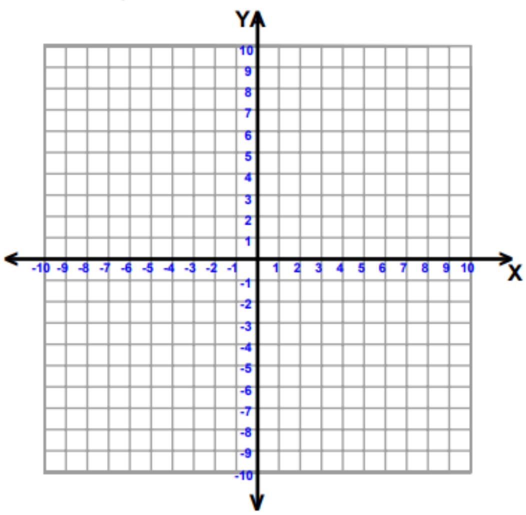
1) $y = \log(x - 5) - 3$



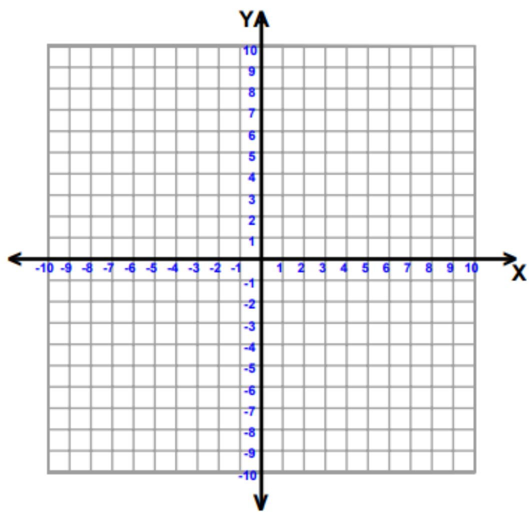
2) $y = \log_9(x - 2) - 3$



3) $y = \log_5(x + 3) - 5$



4) $y = \log(x - 4) - 2$



5. For the function $g(x) = 2\log_3\left[\frac{1}{2}(x+2)\right]+1$:

a) State the transformations on $\log_3 x$ that produced g(x).

b) State the domain and range for g(x).

The Rules for Logarithms

*For all rules, we will assume that **a, b, A, B,** and **C** are positive numbers.*

Definition of a logarithm:

$$\log_b(x) = y \iff b^y = x$$

Useful properties of logarithms:

$$\log_a(A \cdot B) = \log_a A + \log_a B \qquad \log_a\left(\frac{B}{C}\right) = \log_a B - \log_a C$$

$$\log_a(A^n) = n \cdot \log_a A$$

Cancelation Rules:

$$\log_a(a^n) = n \qquad a^{\log_a n} = n$$

Change of Base Formula:

$$\log_a(C) = \frac{\log_b(C)}{\log_b(a)}$$

6. Evaluate each of the following. Do NOT use your calculator.

a) $\log_5 125$

b) $\log_2 128$

c) $\log_3 81$

d) $\log_8 1$

e) $\log_2\left(\frac{1}{32}\right)$

f) $\log_{10}(-100)$

g) $\log_8 \sqrt[5]{8}$

h) $\log_2\left(\sqrt[7]{2}\right)^5$

i) $\log_7\left(\sqrt[3]{\frac{1}{49}}\right)^4$

7. Using the change of base formula, evaluate:

$\log_2 8 \log_8 16 \log_{16} 32 \log_{32} 64 \log_{64} 128$

8. Evaluate each of the following;

$2\log_5 10 - \log_5 4$	$\log_2 56 - \log_4 49$	$25^{\left(-\log_5 \sqrt{2}\right)}$
$10^{\log_{100} 9}$	$\log 5 + \log 20$	$\log_6 108 + \log_6 60 - \log_6 5$

9. Prove that the following statements are true.

a) $\dfrac{1}{\log_5 a} + \dfrac{1}{\log_3 a} = \dfrac{1}{\log_{15} a}$

b) $(\log_a b)(\log_b a) = 1$

c) $\dfrac{2}{\log_8 a} - \dfrac{4}{\log_2 a} = \dfrac{1}{\log_4 a}$

Solving Exponential & Logarithmic Equations

1. To solve an exponential equation, first isolate the exponential expression, then **take the logarithm of both sides of the equation** and solve for the variable.
2. To solve a logarithmic equation, first isolate the logarithmic expression, then **exponentiate both sides of the equation** and solve for the variable.

10. Solve the exponential equations.

a. $2^x = 7$

b. $4^{x-3} = 9$

c. $2e^x = 10$

11. Solve the logarithmic equations.

$2\log_4 x = 5$	$3\log x = 6$
$20\ln 0.2x = 30$	$\log_3 2x - \log_3(x-3) = 1$
$\log_2 x + \log_2(x-1) = 1$	$(\log_5 x)^2 - 3\log_5 x + 2 = 0$

Note: You should *always* check your solution in the original equation.

12. A sample of 500 cells in a medical research lab doubles every 20 min. The formula for the number of cells at time t, where t is measured in minutes is $N(t) = 500 \cdot (2)^{\frac{t}{20}}$. How long will it take for the population to reach 18 000? Answer correct to 2 decimal places.

13. A sample of radioactive iodine-131 atoms has a half-life of about 8 days. Suppose that 1 000 000 iodine-131 atoms are initially present. The formula for the number of atoms at time *t*, where *t* represents number of days is given by $N(t) = 1000000\left(\frac{1}{2}\right)^{\frac{t}{8}}$. How long will it take for the sample to reach 180 000 atoms? Answer correct to 2 decimal places.

Extra practice

1. Evaluate:

a) $\log_{10} 1000$ b) $\log_4 1$ c) $\log_3 27$

d) $\log_2 \frac{1}{4}$ e) $\log_a a^x$

2. Solve for x.

a) $\log_4 x = 2$ b) $\log_{\frac{1}{3}} x = 4$ c) $\log_{10}(2x+1) = 2$

d) $\log_2 64 = x$ e) $\log_b 81 = 4$

3. a) Use log laws to solve $\log_3 x = \log_3 7 + \log_3 3$.

b) Without tables, simplify $2\log_{10} 5 + \log_{10} 8 - \log_{10} 2$.

c) If $\log_{10} 8 = x$ and $\log_{10} 3 = y$, express the following in terms of x and y only:

i. $\log_{10} 24$ ii. $\log_{10} \frac{9}{8}$ iii. $\log_{10} 720$

Solutions:

1 a) 3 b) 0 c) 3 d) -2 e) x

2 a) 16 b) $\frac{1}{81}$ c) 49.5 or $\frac{99}{2}$ d) 6 e) 3

3a) 21 b) 2 c) i. $x+y$ ii. $2y-x$ iii. $2y+x+1$