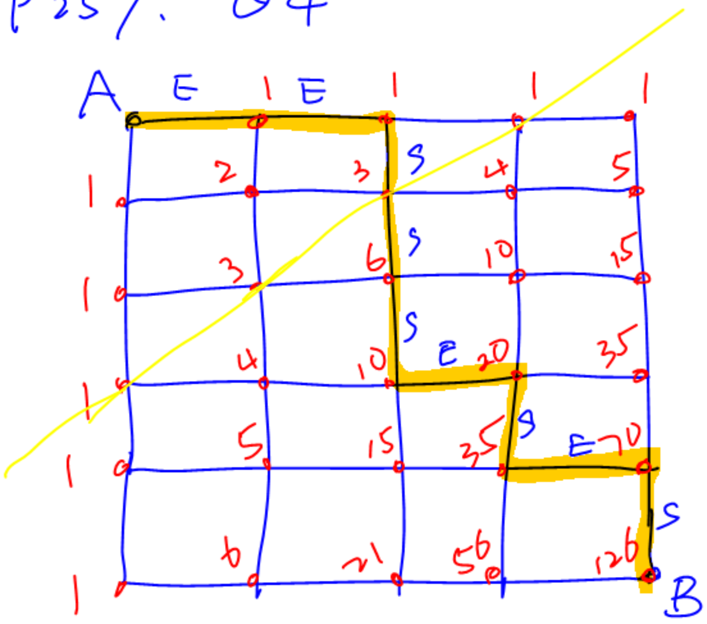
MDM4U HW3

P257-259: Q4, Q7, Q13, Q15.

P271-272: Q5, Q7, Q9, Q10.

Sol. P257. Q4

a).



Method 1.

using Pascal's method.

There are 126 routes from A to B if moving East or South.

Method 2.

using permutation with identical items.

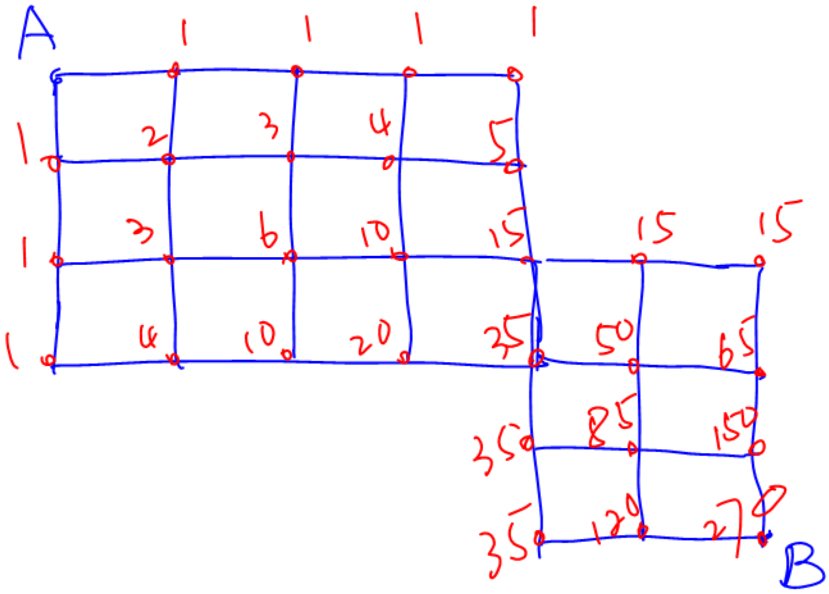
A route consists of 4 Es and 5 Ss.

i.e. EESSSESES

$$\frac{(4+5)!}{4!\,5!} = \frac{9!}{4!5!} = {}_9C_4 = {}_9C_5$$

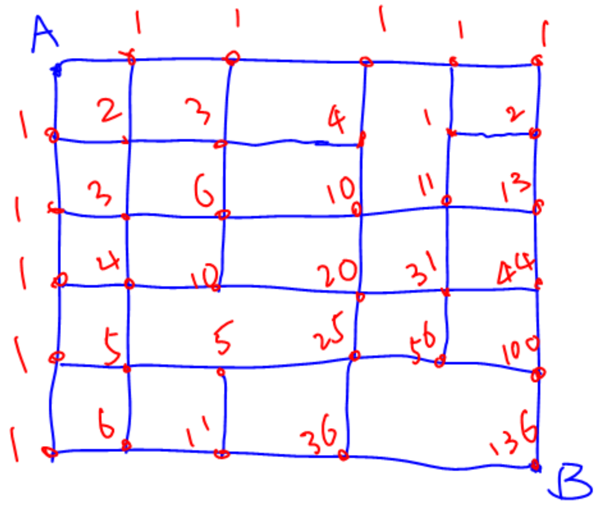
$$= \frac{{}_9P_4}{4!} = 126.$$

b).



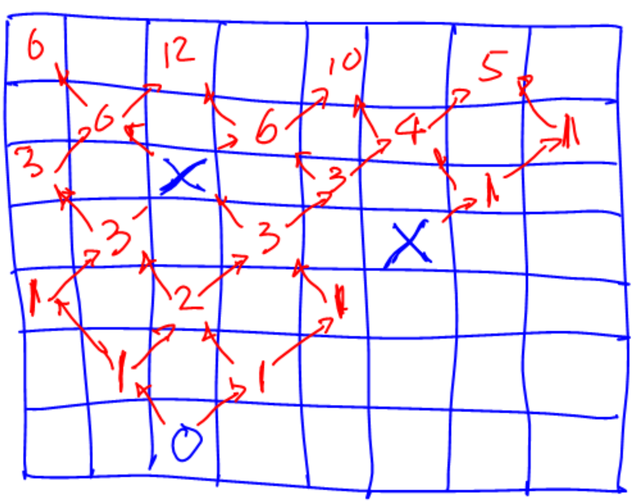
There are 270 routes from A to B, if moving East or South only.

c)



There are 136 routes from A to B, if moving East or South only.

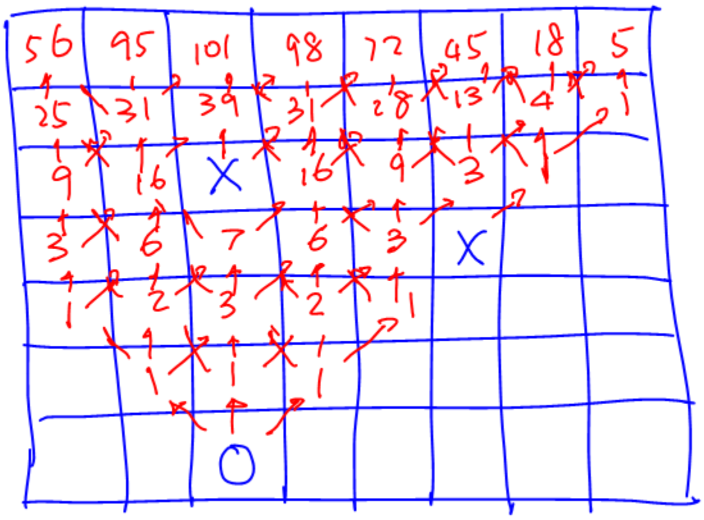
Q7.



a).

$6+12+10+5 = 33$

so 33 paths to the top.



$56+95+101+98+72+45+18+5 = 490$ paths from bottom to top.

P258 Q13.

Express $t_{n,r}$ in terms of $t_{n-3,i}$.

Sol. $t_{n,r} = t_{n-1,r-1} + t_{n-1,r}$

$= t_{n-2,r-2} + t_{n-2,r-1} + t_{n-2,r-1} + t_{n-2,r}$

$= t_{n-2,r-2} + 2t_{n-2,r-1} + t_{n-2,r}$

$= t_{n-3,r-3} + t_{n-3,r-2} + 2(t_{n-3,r-2} + t_{n-3,r-1})$

$+ t_{n-3,r-1} + t_{n-3,r}$

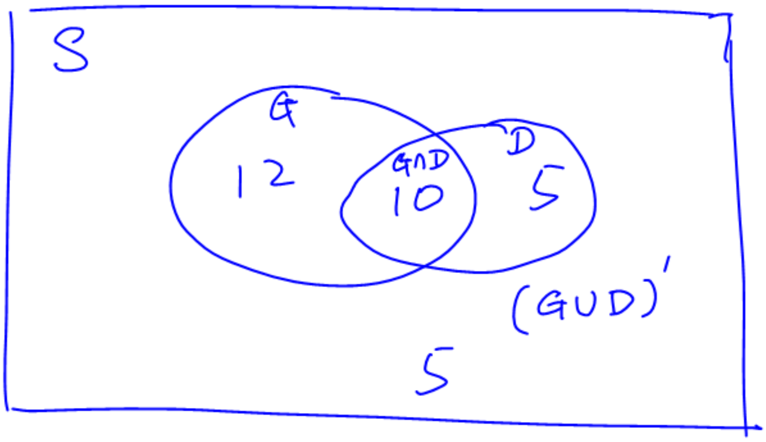
$= t_{n-3,r-3} + 3t_{n-3,r-2} + 3t_{n-3,r-1} + t_{n-3,r}$

P259 Q15.

$\dfrac{(n+m)!}{n!\,m!}$ is the number of routes to move n blocks north and m block west.

P271. Q5.

Find $n[(G\cup D)']$.



Given $n(S)=32$.

$n(G)=22$

$n(D)=15$

$n(G\cap D)=10$.

Sol. $n(G\cup D)=n(G)+n(D)-n(G\cap D)$

$=22+15-10$

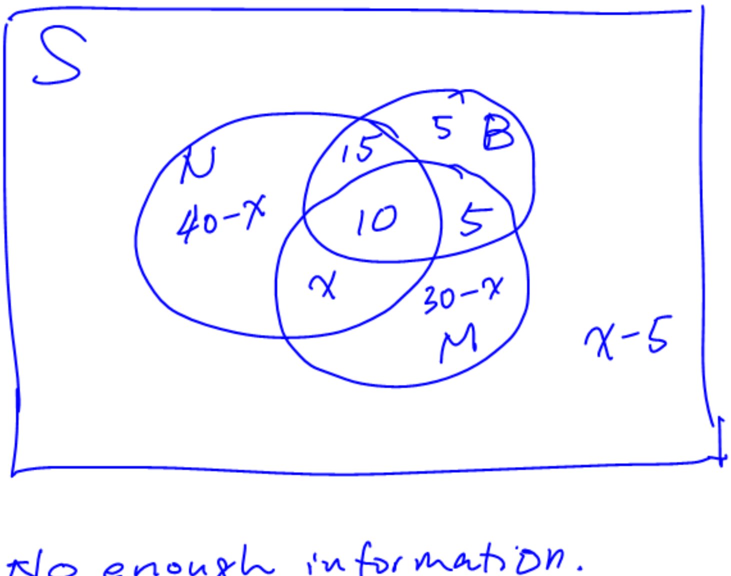
$=27$.

$n[(G\cup D)']=n(S)-n(G\cup D)$

$=32-27=5$

There are 5 employees speak nether German nor Dutch.

Q7.



$100-(105-x)$

$=x-5$

a) Max. of $n(N\cap M)$

$=10+x=10+30$

$=40$

Since $30-x\geq 0$

or $x\leq 30$.

b) No enough information.

If we knew the number of people who read newspapers only or who read magagines only, we would be able to determine the number of people who read both newspapers and magazines.

Q9. 12 forwards, 8 defences. 2 goalies.

a) $12+8+2=22$

b) $12+8=20$

c) $12+8+2-3=19$

d) $12+8-2=18$

Q10. Principle of Inclusion and Exclusion

a) for 3 sets:

$$n(A\cup B\cup C)=n(A)+n(B)+n(C)-n(A\cap B)-n(A\cap C)$$
$$-n(B\cap C)+n(A\cap B\cap C)$$

b) for 4 sets:

$$n(A\cup B\cup C\cup D)=n(A)+n(B)+n(C)+n(D)$$
$$-n(A\cap B)-n(A\cap C)-n(A\cap D)-n(B\cap C)-n(B\cap D)-n(C\cap D)$$
$$+n(A\cap B\cap C)+n(A\cap B\cap D)+n(A\cap C\cap D)+n(B\cap C\cap D)-n(A\cap B\cap C\cap D)$$

c) for n sets:

$$n(A_1\cup A_2\cup\cdots\cup A_n)=n(A_1)+n(A_2)+\cdots+n(A_n)$$
$$-n(A_1\cap A_2)-n(A_1\cap A_3)-\cdots-n(A_1\cap A_n)-n(A_2\cap A_n)-\cdots-n(A_{n-1}\cap A_n)$$
$$+n(A_1\cap A_2\cap A_3)+n(A_1\cap A_2\cap A_4)+\cdots+n(A_{n-2}\cap A_{n-1}\cap A_n)+\cdots+(-1)^{n-1}n(A_1\cap A_2\cap\cdots A_n)$$