AP Calculus In-Class Eight – Antiderivatives and the Definite Integral

4.1 Antiderivatives; 4.2 Area

1. Find the most general antiderivatives of the functions.

(a) $f(x) = 4x^2 - 8x + 1$

(b) $f(x) = 4/x^7 - 7/x^4 + x$

(c) $f(x) = (8x - 5)/\sqrt[5]{x}$

(d) $f(x) = (x^3 + 3x^2 - 9x - 2)/(x - 2)$

2. Solve the differential equations subject to the given boundary conditions.

(a) $f'(x) = 9x^2 + x - 8, \quad f(-1) = 1$

(b) $f'''(x) = (\pi^3/2)\sin(\pi x/4), \quad f''(0) = 4 - 2\pi^2, \quad f'(0) = 0, \quad f(0) = 32$

3. Evaluate the integrals without using your calculator.

(a) $\int (3x^2 - 2x + 3)\,dx$

(b) $\int \left(x - \frac{1}{2x}\right)^2 dx$

(c) $\int \sqrt{4-2t}\ dt$

(d) $\int \frac{dx}{3(2x-1)^2}$

(e) $\int \frac{2du}{1+3u}$

(f) $\int \frac{t}{\sqrt{2t^2-1}}\ dt$

(g) $\int \cos 3x dx$

4. Evaluate the integrals without using your calculator.

(a) $\int \frac{dy}{\sqrt{4-y^2}}$

(b) $\int \frac{ydy}{\sqrt{4-y^2}}$

(c) $\int \frac{2x+1}{2x} dx$

(d) $\int \frac{(x-2)^3}{x^2} dx$

(e) $\int \left(\sqrt{t} - \frac{1}{\sqrt{t}}\right)^2 dt$

5. Evaluate the integrals without using your calculator.

(a) $\int \left(4x^{1/3} - 5x^{3/2} - x^{-1/2}\right) dx$

(b) $\int \left(\frac{x^3 - x - 1}{x^2} \right) dx$

(c) $\int \frac{dy}{\sqrt{y}\left(1 - \sqrt{y}\right)}$

(d) $\int \frac{u du}{\sqrt{4 - 9u^2}}$

(e) $\int t \cos(2t)^2 \, dt$

6. Evaluate the integrals without using your calculator.

(a) $\int x \cos x \, dx$

(b) $\int \frac{du}{\cos^2 3u}$

(c) $\int \frac{\cos x dx}{\sqrt{1 + \sin x}}$

(d) $\int \frac{\cos(\theta - 1)}{\sin^2(\theta - 1)} \, d\theta$

(e) $\int \sec(t/2) dt$

7. Evaluate the integrals without using your calculator.

(a) $\int \frac{\sin 2t}{1-\cos 2t}\,dt$

(b) $\int \frac{e^x}{e^x-1}\,dx$

(c) $\int \frac{(x-1)dx}{x(x-2)}$

(d) $\int xe^{x^2}dx$

(e) $\int \cos\theta\, e^{\sin\theta}\,d\theta$

8. Evaluate the integrals without using your calculator.

(a) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

(b) $\int xe^{-x}dx$

(c) $\int \frac{e^x}{1+e^{2x}}\,dx$

(d) $\int \frac{\ln\sqrt{x}\,dx}{x}$

(e) $\int \ln\eta\,d\eta$

9. Evaluate the integrals without using your calculator.

(a) $\int \frac{dv}{v \ln v}$

(b) $\int \frac{y-1}{y+1}\, dy$

(c) $\int \sqrt{x}\left(\sqrt{x}-1\right)dx$

(d) $\int e^{\theta} \cos\theta \, d\theta$

(e) $\int \frac{(1-\ln t)^2}{t}\, dt$

10. Evaluate the integrals without using your calculator.

(a) $\int \frac{2x+1}{4+x^2}\, dx$

(b) $\int \frac{1-x}{\sqrt{1-x^2}}\, dx$

(c) $\int \frac{e^{2x}}{1+e^{x}}\, dx$

(d) $\int \frac{\cos\theta}{1+\sin^2\theta}\, d\theta$

(e) $\int \arctan x \, dx$

(f) $\int \frac{(2-y)^2}{4\sqrt{y}} \, dy$

(g) $\int \frac{dy}{y(1+\ln y^2)}$

(h) $\int \frac{d\theta}{1+\sin\theta}$

11. An object is dropped from a height of 300 m. Neglecting air resistance, find the distance it falls in t s. What is its velocity at the end of 3 s? When will it strike the ground?

12. If an automobile starts from rest, what constant acceleration will enable it travel 150 m in 10 s?

13. The volume V of as a balloon is changing with respect to time t at a rate given by $dV/dt = 3\sqrt{t} + t/4$ m^3/s. if, at t = 4s, the volume is 6 m^3, express V as a function of t.

14. If $f(x) = 8 - x^2/2$, find the Riemann sum R_p of $f(x)$ where P is the partition of [0, 6] into the six equal subintervals determined by $x_0 = 0$ and $x_6 = 6$, and w_i is the midpoint of the interval $[x_{i-1}, x_i]$.

15. Find the area under the graph of $f(x)$ from a to b using inscribed rectangles. In each case sketch the graph and typical rectangles, labeling the drawing.

    (a) $f(x) = 2x + 3; \quad a = 0, \ b = 4$

    (b) $f(x) = 3x^2 + 5; \quad a = 1, \ b = 4$

16. Find the area under the graph of $f(x)$ from a to b using circumscribed rectangles. In each case sketch the graph and typical rectangles, labeling the drawing.

    (a) $f(x) = 4x^2 + 3x + 2; \quad a = 1, b = 5$

    (b) $f(x) = x^3$; $a = -2, b = 6$