Advanced Functions Class 7 Homework

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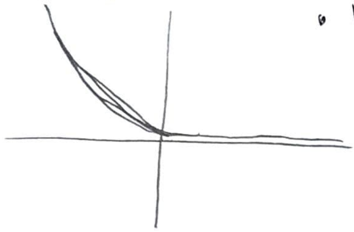
Exponential and Logarithmic Functions (1)

1. Describe the transformations that can be applied to the function $y=2^x$ to obtain the graph of each functions. Rewrite the equation if necessary. Sketch the graph of the functions.

a. $y=\frac{1}{3}(2^{-x-2})$

$y=\frac{1}{3}(2^{-(x+2)})$

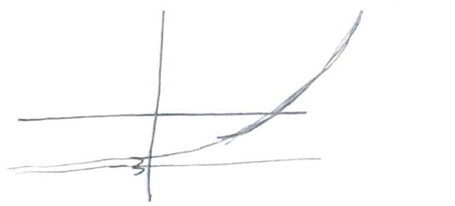
- h shift left by 2
- Mirror across y
- v compress by 3



b. $y=4^{\frac{1}{2}x}-3$

$y=2^{x}-3$

- v - shift down by 3



2. A mapping defined by $(x,y)\rightarrow(-x+3, \frac{1}{2}y-2)$ is applied to each point (x,y) on the graph of the function $y = 4^x$ to obtain the graph of $y = f(x)$.

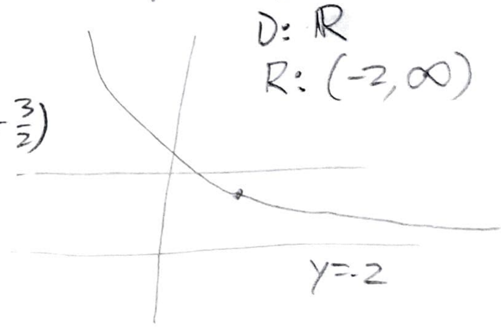
a. Sketch the graph of $y = f(x)$ and identify its domain and range.
b. State the equation of $y = f(x)$.
c. What mapping must be applied to the points on $y = 2^x$ to obtain the same graph as $y = f(x)$?

a) $a f(b(x-c))+d \Rightarrow (x,y)\rightarrow(\frac{1}{b}x+c, ay+d)$

$(x,y) \rightarrow (-x+3, \frac{1}{2}y-2)$

D: $\mathbb{R}$

R: $(-2,\infty)$



b) $f(x)=\frac{1}{2}4^{-(x-3)}-2$ $(0,1)\rightarrow(3,-\frac{3}{2})$

c) $f(x)=\frac{1}{2}2^{-2(x-3)}-2$

$(x,y) \rightarrow (-\frac{1}{2}x+3, \frac{1}{2}y-2)$

3. Solve for x.

a. $\sqrt{8^{x+1}} = \frac{1}{32}$

$(8^{x+1})^{\frac{1}{2}} = \frac{1}{2}$

$8^{\frac{1}{2}(x+1)} = 2^{-5}$

$2^{\frac{3}{2}(x+1)} = 2^{-5}$

$\frac{3}{2}(x+1) = -5$

$(x+1) = -\frac{10}{3}$

$x = -\frac{13}{3}$

b. $4^{3-5x} = 1$

$x^0 = 1$

$3-5x=0$

$5x=3$

$x=\frac{3}{5}$

c. $27^{x^2} = 3(9^{-x})$

$3^{3x^2} = 3^{-2x+1}$

$3x^2 = -2x+1$

$3x^2+2x-1=0$

$(3x-1)(x+1)=0$

$x=\frac{1}{3}, -1$

d. $\left(\frac{1}{4^x}\right)^{x-4} = \frac{16^{x-3}}{2^x}$

$(4^x)^{-(x-4)} = \frac{2^{4(x-3)}}{2^x}$

$4^{-x^2+4x} = 2^{3x-12}$

$2^{-2x^2+8x} = 2^{3x-12}$

$-2x^2+8x = 3x-12$

$-2x^2+5x+12=0$

$2x^2-5x-12=0$

$(2x+3)(x-4)=0$

$x=-\frac{3}{2}, 4$

$x=-1, 1$

e. $5^x(25)^{\frac{1}{x^2}} = 125$

$5^x \cdot 5^{\frac{2}{x^2}} = 5^3$

$5^{\frac{2}{x^2}+x} = 5^3$

$\frac{2}{x^2}+x = 3$

$2+x^3 = x^2$

$x^3-x^2+2=0$

$$\begin{array}{r|rrrr} -1 & 1 & -1 & 0 & 2 \\ & & -1 & 2 & -2 \\ \hline & 1 & -2 & 2 & 0 \end{array}$$

$x^2-2x+2=0$

$(x-1)(x-1)=0$

f. $5(25)^x - 26(5^x) + 5 = 0$ let $w=5^x$

$5(5^x)^2 - 26(5^x) + 5 = 0$

$5w^2-26w+5=0$

$(5w-1)(w-5)=0$

$w=5, \frac{1}{5}$

$x=1,-1$

g. $4^x + 5(2^x) + 6 = 0$ let $2^x = w$

$(2^x)^2+5(2^x)+6=0$

$w^2+5w+6=0$

$(w+2)(w+3)=0$

No real solutions

h. $3^x - 6(\sqrt{3})^x - 27 = 0$ let $w=(\sqrt{3})^x$

$w^2-6w-27=0$

$(w-9)(\cancel{w+3})=0$

$\sqrt{3}^x = 9$

$3^{\frac{x}{2}} = 3^2$

$\frac{x}{2}=2$

$x=4$

4. Cameron would like to invest \$1000 for the next three and a half years. He is considering two different investment alternatives:

- Option 1: 3.2% per annum, compounded quarterly
- Option 2: 2.7% per annum, compounded monthly

14 Q
42 m

Determine the amount of interest earned with each option.

Option 1:

$$Y = 1000\left(1+\frac{0.032}{4}\right)^{14}$$
$$= \$1118.01$$

interest: \$118.01

Option 2:

$$Y = 1000\left(1+\frac{0.027}{12}\right)^{42}$$
$$= 1098.99$$

interest: \$98.99

5. Strontium-90, ^{90}Sr, has a half life of 29 years.

a. If 42.5 grams remain after 50 years, what is the initial mass of ^{90}Sr, to the nearest gram?
b. How long will it take for 180 grams of the substance to decay to 11.25 grams?

a) $42.5 = x\left(\frac{1}{2}\right)^{\frac{50}{29}} \Rightarrow x = 140.49$

b) $11.25 = 180\left(\frac{1}{2}\right)^{\frac{x}{29}}$

$\frac{1}{16} = \left(\frac{1}{2}\right)^{\frac{x}{29}}$

$\left(\frac{1}{2}\right)^{4} = \left(\frac{1}{2}\right)^{\frac{x}{29}}$

$\frac{x}{29} = 4$

$x = 116$ Years

6. Solve the following system of equations.

$5^{2x+y} = 625$ $\qquad 5^{2x+y} = 5^{4}$

$25^{x+2y} = \frac{1}{25}$ $\qquad 5^{2x+4y} = 5^{-2}$

$2x+y = 4$ ②

$2x+4y = -2$ ①

① - ②:

$3y = -6$

$y = -2$

$x = 3$

7. Determine all values of *k* for which the equation

$k(2^x) + 2^{-x} = 3$ has a single root.

let $t = 2^x$

$t > 0$

$kt + \frac{1}{t} - 3 = 0$

$kt^2 - 3t + 1 = 0$

when $k = 0$:

$-3t + 1 = 0$

$t = \frac{1}{3} > 0$

when $k \neq 0$

look for $r_1 > 0$, $r_2 < 0$

$b^2 - 4ac$

$kt^2 - 2t + 1 = k(t - r_1)(t - r_2)$

$kt^2 - 3t + 1 = k(t^2 - (r_1 + r_2) + r_1 r_2)$

$\underset{>0}{\underline{1}} = k \underset{>0}{\underline{r_1}} \underset{<0}{\underline{r_2}}$

$k < 0$ since $k = \frac{1}{r_1 r_2}$

$9 - 4k > 0$ $k < \frac{9}{4}$

$\therefore$ $k < 0$ gives a single root

$9 - 4k = 0$ $k = \frac{9}{4}$

$\frac{9}{4}t^2 - 3t + 1 = 0 \Rightarrow (3t - 2)^2$

$t = \frac{2}{3} > 0$

$\therefore$ $k = \frac{9}{4}$ also gives a single (repeated) root

Successful values:

$k < 0$, $k = \frac{9}{4}$, $k = 0$

$\therefore k \in (-\infty, 0] \cup \{\frac{9}{4}\}$