Advanced Functions Class 11 Homework

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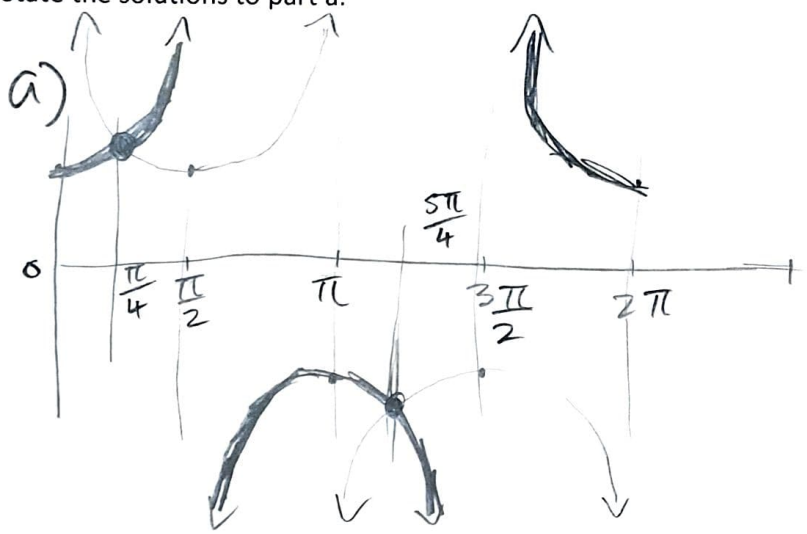
Trigonometric Functions (2)

1.

a. Using a sketch, show the solutions to $\sec(x) \geq \csc(x)$, $0 \leq x \leq 2\pi$.

b. State the solutions to part a.



— $\csc = \frac{1}{\sin}$

= $\sec = \frac{1}{\cos}$

b) $x \in [\frac{\pi}{4}, \frac{\pi}{2}) \cup (\pi, \frac{5\pi}{4}] \cup (\frac{3\pi}{2}, 2\pi)$

2. Identify the amplitude, period, phase shift, and vertical displacement for each of the following:

a. $y=6\cos[\frac{1}{2}(x-30°)]+3$

b. $y=-2+3\sin(x+\frac{\pi}{4})$

c. $y=-4\cos(2x-\frac{\pi}{3})-2$

a) amp: 6, period: $\frac{360}{(\frac{1}{2})} = 720$, vert: up 3
phase: 30 right

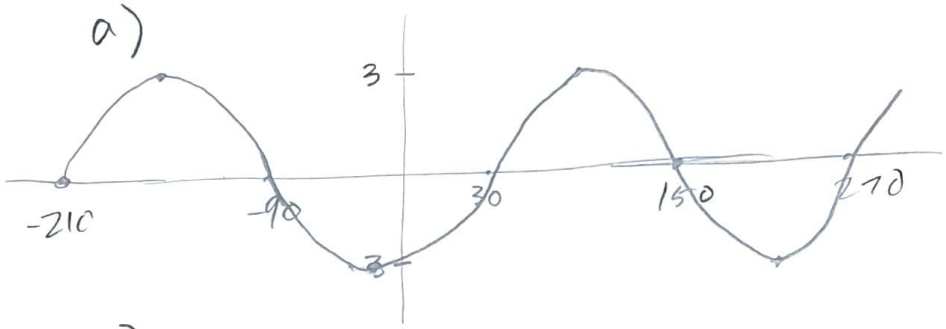
b) amp: 3 period: 2π vert: down 2
phase: $\frac{\pi}{4}$ left

c) amp: 4 period: π vert: down 2
Phase: $\frac{\pi}{6}$ right

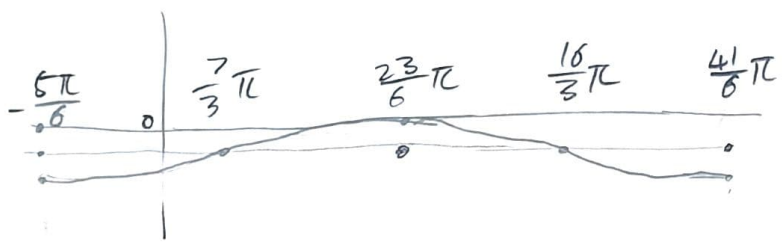
3.

a. Sketch $y= 5\sin[\frac{3}{2}(x-30°)]$, $-120° \le x \le 120°$.

b. Sketch one period of the function $f(x)= -\cos[\frac{1}{3}(x+\frac{5\pi}{6})] -2$



b) $P: 6\pi$



4.

a. Determine a sine function that is defined for all $x \ge 0$ and has its first minimum at $(\frac{\pi}{3},3)$ and its first maximum at $(\frac{4\pi}{3},9)$.

b. State an equivalent cosine function for part a.

a) Aos: $\frac{3+9}{2} = 6$ Amp: 3 Period: $(\frac{4\pi}{3} - \frac{\pi}{3}) \cdot 2 = 2\pi$

$y = 3\sin(k+x)+6$ $\Rightarrow$ $y = 3\sin(x - \frac{5}{6}\pi)+6$

$3 = 3\sin(\frac{\pi}{3}+k)+6$

$-3 = 3\sin(\frac{\pi}{3}+k)$

$\sin(\frac{\pi}{3}+k) = -1$

$\frac{\pi}{3}+k = -\frac{1}{2}\pi$

$k = -\frac{5}{6}\pi$

b) $y = 3\cos(x - \frac{4}{3}\pi)+6$

5.

a. Determine a sinusoidal function $f(x)$ that

- has a maximum of 100;
- has a minimum of 20;
- a period of 30;
- has the point (15,60) on its curve; and
- for $x \geq 0$, reaches its first maximum before its first minimum.

a) amp: 40 AoE: 60

$y = 40\sin(12(x+k)) + 60 \Rightarrow y = 40\sin(12(x-15)) + 60$

$60 = 40\sin(180 - 12k) + 60$

$\sin(180 - 12k) = 0$ $\quad k = -15$

$y = -40\sin(12[x+15]) + 60$

b. Use your function from part a) to determine the first value of x, $x \geq 0$ such that $f(x) = 80$

$80 = -40\sin(12[x-15]) + 60$

$\sin(12[x-15]) = -\frac{1}{2}$ $\quad x = \frac{5}{2}$

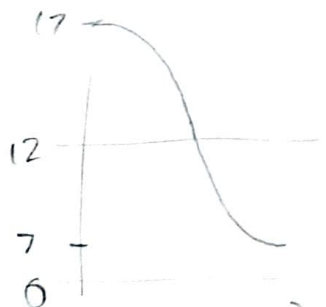
$12[x+5] = -30$

$x+5 = -\frac{5}{2}$

6. The minimum depth, d (in metres), of water in a harbour, t hours after midnight, can be approximated by the function $d(t) = 5\cos(0.5t)+12$, where $0 \leq t \leq 24$.

a. Determine the maximum and minimum depths of water in the harbour.
b. Determine the period of the depth function.
c. What is the depth of water, to the nearest tenth of a metre, at 2:00 AM?
d. A ship, which requires a minimum depth of 8.5 metres, is docked at midnight. By what time, to the nearest minute, must it leave in order to prevent being grounded?
e. What is the next time, to the nearest minute, that the ship can return to the harbour?

$a = 5, b = \frac{1}{2}$ $\quad c = 0, d = 12$



a) max $= d + a = 12 + 5 = 17$ m

min $= d - a = 12 - 5 = 7$ m

Center $= d = 12$ m

b) $P = \frac{2\pi}{|b|} = \frac{2\pi}{(\frac{1}{2})} = 4\pi$

c) $t = 2$: $d(2) = 5\cos(1) + 12 \doteq 14.70$ m

d) $8.5 = 5\cos(0.5t) + 12$

$-\frac{7}{10} = \cos(0.5t)$

$t = 4.69$

4:41 AM

e) $P - 4.69 = 7.87$

7:52 AM

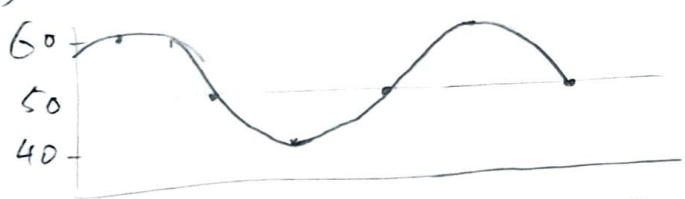
7. A weight attached to the end of a long spring hanging above the ground is bouncing up and down. As it bounces, its distance from the floor varies sinusoidal with time (assume no friction is present in the spring). A stopwatch is used to measure its height above the floor as a function of time. When the stopwatch reads 0.3 s, the weight first reaches a high point 60 cm above the floor. The next low point, at 40 cm above the floor, occurs at 1.8 s.

a. Draw a sketch to illustrate d, the spring's distance from the floor in centimetres, over the interval $0 \le t \le 6$, where t is in seconds.
b. Determine a function, $d(t)$, that describes the spring's distance from the floor as a function of time.
c. What is the distance from the floor (to the nearest millimetre) when the stopwatch reads 8.1 s?
d. At what time is the weight 45 cm above the floor for the first time?

a: 10 center: 50 P: $(1.8-0.3)\cdot 2 = 3$

$y = 10\cos\left(\frac{2}{3}\pi(x-0.3)\right)+50$

a)



b) $f(x) = 10\cos\left(\frac{2}{3}\pi(x-0.3)\right)+50$

c) $f(8.1) = 41.9$ cm

d) $45 = 10\cos\left(\frac{2}{3}\pi(x-0.3)\right)+50$

$\cos\left(\frac{2}{3}\pi(x-0.3)\right) = -\frac{1}{2}$

$x = 1.3$