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## **Identities and Equations (1)**

### **Objectives:**

- We will review and examine the basic fundamental relationships between trigonometric functions.
- We will identify equivalent trigonometric expressions, demonstrating equivalence algebraically and graphically.
- We will determine the non-permissible values of the variable involved in trigonometric expressions and equations.

### **Fundamental identities**

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#### **Pythagorean identities**

$$\sin^2 \theta + \cos^2 \theta = 1 \quad 1 + \cot^2 \theta = \csc^2 \theta \quad 1 + \tan^2 \theta = \sec^2 \theta$$

#### **Reciprocal identities**

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

#### **Quotient identities**

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

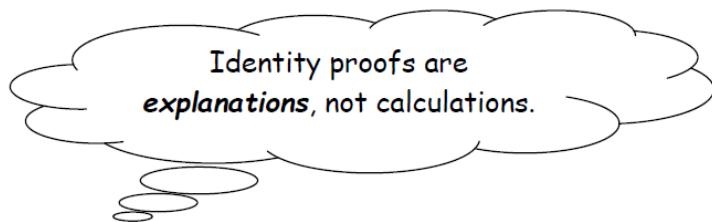
$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

## Prove It!

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### Strategies & Guidelines

- Only work with  $\sin\theta$  and  $\cos\theta$ ; does it involve a Pythagorean identity?
- Simplify anything that looks complex.
- Write your steps; only 1 *thing* changes per line.
- Make your numerators and denominators clear. Yes, sometimes it will take up 4 lines.
- Use brackets to make your explanation more readable.
- Look at the other side. That's where you are headed. Make LS = RS.



$$1. \cos x(\sec x - \cos x) = \sin^2 x$$

$$2. (1 - \sec x)(1 + \sec x) = -\sin^2 x \sec^2 x$$

$$3. \sin^3 x + \sin x \cos^2 x = \sin x$$

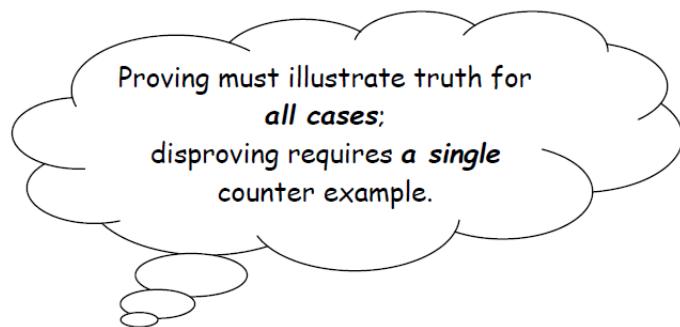
$$4. \sin^4 x - \cos^4 x = 1 - 2\cos^2 x$$

$$5. \quad 1 + \frac{1}{\sin x} = (1 + \sin x) \csc x$$

$$6. \quad (1 - \cos^2 x)(1 + \cos^2 x) = 2 \sin^2 x - \sin^4 x$$

$$7. \quad \tan x + 1 = \frac{\sin x + \cos x}{\cos x}$$

$$8. \quad \tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$$



9.

Determine whether the following statements are identities.

$$a) \quad \cot A + \cos A = \frac{\cos A(1 + \sin A)}{\sin A}$$

$$b) \quad \cot B + \cos B = \tan B + \sin B$$

### Related Angle identities

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$$\sin(\pi - x) = \underline{\hspace{5cm}}$$

$$\sin(\pi + x) = \underline{\hspace{5cm}}$$

$$\sin(2\pi - x) = \underline{\hspace{5cm}}$$

$$\sin(-x) = \underline{\hspace{5cm}}$$

$$\cos(\pi - x) = \underline{\hspace{5cm}}$$

$$\cos(\pi + x) = \underline{\hspace{5cm}}$$

$$\cos(2\pi - x) = \underline{\hspace{5cm}}$$

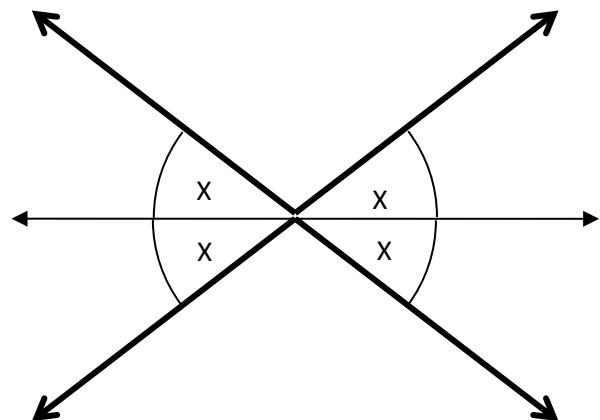
$$\cos(-x) = \underline{\hspace{5cm}}$$

$$\tan(\pi - x) = \underline{\hspace{5cm}}$$

$$\tan(\pi + x) = \underline{\hspace{5cm}}$$

$$\tan(2\pi - x) = \underline{\hspace{5cm}}$$

$$\tan(-x) = \underline{\hspace{5cm}}$$



### Use It!

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10. Prove that  $\frac{\sec x + \tan(-x)}{1 + \sin(\pi + x)} = \sec(2\pi - x)$

### Correlated Angle identities

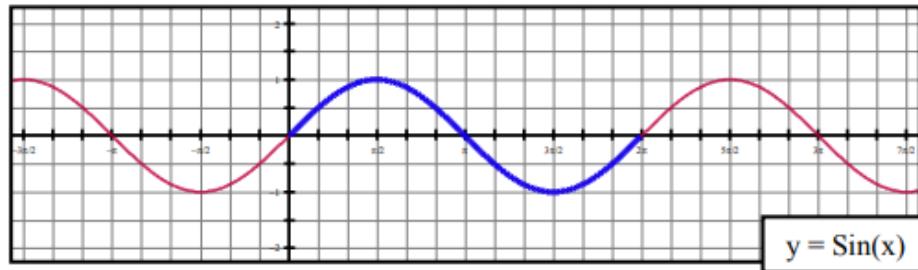
Determine the equation of the Sine graph shown below as a function of Cosine.

$$\sin x = \underline{\hspace{10cm}}$$

$$\sin x = \underline{\hspace{10cm}}$$

$$\sin x = \underline{\hspace{10cm}}$$

$$\sin x = \underline{\hspace{10cm}}$$



$$\cos\left(\frac{\pi}{2} - x\right) = \underline{\hspace{10cm}}$$

$$\cos\left(\frac{3\pi}{2} - x\right) = \underline{\hspace{10cm}}$$

$$\cos\left(\frac{\pi}{2} + x\right) = \underline{\hspace{10cm}}$$

$$\cos\left(\frac{3\pi}{2} + x\right) = \underline{\hspace{10cm}}$$

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\sin\left(\frac{3\pi}{2} - x\right) = -\cos x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x$$

$$\tan\left(\frac{3\pi}{2} - x\right) = \cot x$$

$$\sin\left(\frac{\pi}{2} + x\right) = \cos x$$

$$\sin\left(\frac{3\pi}{2} + x\right) = -\cos x$$

$$\tan\left(\frac{\pi}{2} + x\right) = -\cot x$$

$$\tan\left(\frac{3\pi}{2} + x\right) = -\cot x$$

### Use It!

11. Prove that  $\tan x + \tan\left(\frac{\pi}{2} - x\right) = -\csc x \sec(\pi + x)$

### **Compound Angle Formulas**

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$$\cos(x + y) = \underline{\hspace{10cm}}$$
      
$$\cos(x - y) = \underline{\hspace{10cm}}$$

$$\sin(x + y) = \underline{\hspace{10cm}}$$
      
$$\sin(x - y) = \underline{\hspace{10cm}}$$

$$\tan(x + y) = \underline{\hspace{10cm}}$$
      
$$\tan(x - y) = \underline{\hspace{10cm}}$$

12. Using the Compound Angle Formulas, determine the formula for each of the following:

a.  $\sin 2x$

b.  $\cos 2x$

c.  $\tan 2x$

### **Double Angle Formulas**

$\sin 2x =$	$\cos 2x =$	$\tan 2x =$
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### **Practice**

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13. Determine the exact value of each of the following:

a)  $\sin \frac{7\pi}{12}$

b)  $\cos \frac{5\pi}{12} \cos \frac{7\pi}{12} - \sin \frac{5\pi}{12} \sin \frac{7\pi}{12}$

14. Prove the following identities:

a.  $2\sin x \sin y = \cos(x - y) - \cos(x + y)$

b.  $1 + (\tan x)(\cot y) = \frac{\sin(x + y)}{\cos x \sin y}$

c.  $\cos(2x + 3y) \cos(2x - 3y) - \sin(2x + 3y) \sin(2x - 3y) = \cos 4x$

d.  $\sin(x + y) \sin(x - y) = \cos^2 y - \cos^2 x$

e.  $\sin(x + y) \sin(x - y) = \cos^2 y - \cos^2 x$

f.  $\tan\left(x + \frac{\pi}{4}\right) = \frac{\cos x + \sin x}{\cos x - \sin x}$

**Extra practice**

$$1. \sin(x + \pi/4) - \cos(x - \pi/4) = 0$$

$$2. \sin(x + \pi/6) - \cos(\pi/3 - x) = 0$$

$$3. \frac{\sin(x + \pi/4) - \cos(x + \pi/4)}{\sin(x + \pi/4) + \cos(x + \pi/4)} = \tan x$$

$$4. \frac{\tan(x - y) + \tan y}{1 - \tan(x - y) \tan y} = \tan x$$

$$5. \tan x \tan(\pi/4 - x) = \frac{1 - \tan x}{1 + \cot x}$$

$$6. \frac{\tan x + \tan y}{\tan(x + y)} + \frac{\tan x - \tan y}{\tan(x - y)} = 2$$

$$7. \frac{\sin(x - y)}{\sin x \sin y} + \frac{\sin(y - z)}{\sin y \sin z} + \frac{\sin(z - x)}{\sin z \sin x} = 0$$