Lesson 6

1. Curve sketching

Example, sketch the graph of

$$f(x) = \log(x-1)^2 + x^2$$

Sol. $D = \{x \in \mathbb{R} \mid \text{but } x \neq 1\}$.

No symmetry.

$$\lim_{x\to 1} f(x) = \lim_{x\to 1} \log(x-1)^2 + \lim_{x\to 1} x^2$$

$$= -\infty + 1 = -\infty,$$

$\therefore$ $x=1$ is a vertical asymptote.

$$\lim_{x\to -\infty} f(x) = +\infty; \quad \lim_{x\to +\infty} f(x) = +\infty.$$

To find y-intercept. let $x=0$.

$$f(0) = \log(0-1)^2 + 0^2 = 0.$$

$\therefore$ the curve goes through $(0,0)$.

To find x-intercept(s) let $y=0$.

let $\log(x-1)^2 + x^2 = 0$

$\therefore$ ~~$x = 0$~~ or 1.19 are x-intercepts

To find increasing and decreasing intervals and local extrema.

$f'(3) = \frac{2}{2\ln 10} + 6 > 0$

$$f'(x) = \frac{2(x-1)}{(x-1)^2 \ln 10} + 2x = \frac{2}{(x-1)\ln 10} + 2x$$

$f'(-3) = \frac{2}{-4\ln 10} - 6 < 0$

let $f'(x) = 0 \Rightarrow \frac{1}{(x-1)\ln 10} = -x \Rightarrow -x(x-1)\ln 10 = 1$

$\Rightarrow -\ln 10\, x^2 + \ln 10 \cdot x - 1 = 0$

$\Rightarrow \ln 10\, x^2 - \ln 10 \cdot x + 1 = 0$

$$x = \frac{\ln 10 \pm \sqrt{(-\ln 10)^2 - 4(\ln 10)}}{2 \times \ln 10} = \cancel{\frac{\ln 10 \pm 1.14}{2\times\ln 10} = 0.58 \text{ or } 1.72}$$

< 0

no real solution

$$f''(x) = \frac{2}{\ln 10}\left(\frac{1}{x-1}\right)' + 2(x)' = -\frac{2}{\ln 10}\frac{1}{(x-1)^2} + 2$$

~~$f''(0.58) = -\frac{2}{\ln 10} \cdot \frac{1}{(0.58-1)^2} + 2 \doteq -2.92 < 0$~~

~~$\therefore f(0.58) = \log(0.58-1)^2 + (0.58)^2 \doteq -0.42$ is a local max.~~

~~$f''(1.72) = -\frac{2}{\ln 10} \cdot \frac{1}{(1.72-1)^2} + 2 \doteq 0.32 > 0$~~

~~$\therefore f(1.72) = \log(1.72-1)^2 + (1.72)^2 \approx 2.67$ is a local min~~

$$f'(x) = \frac{2}{(x-1)\ln 10} + 2x$$

$$\lim_{x \to 1^-} f'(x) = \frac{2}{0^- \ln 10} + 2 \to -\infty$$

$$\lim_{x \to 1^+} f'(x) = \frac{2}{0^+ \ln 10} + 2 \to +\infty$$

$\therefore$ $f'(x) > 0$ for all $x > 1$,

$f'(x) < 0$ for all $x < 1$.

$\therefore$ $f(x)$ has no local extrema.

but $f(x)$ increases for $x > 1$.

and decreases for $x < 1$.

$$f''(x) = -\frac{2}{\ln 10} \cdot \frac{1}{(x-1)^2} + 2$$

let $f''(x) = 0 \Rightarrow \dfrac{1}{\ln 10 (x-1)^2} = 1$

$\Rightarrow 1 = \ln 10 (x-1)^2 \Rightarrow (x-1) = \pm\sqrt{\dfrac{1}{\ln 10}}$

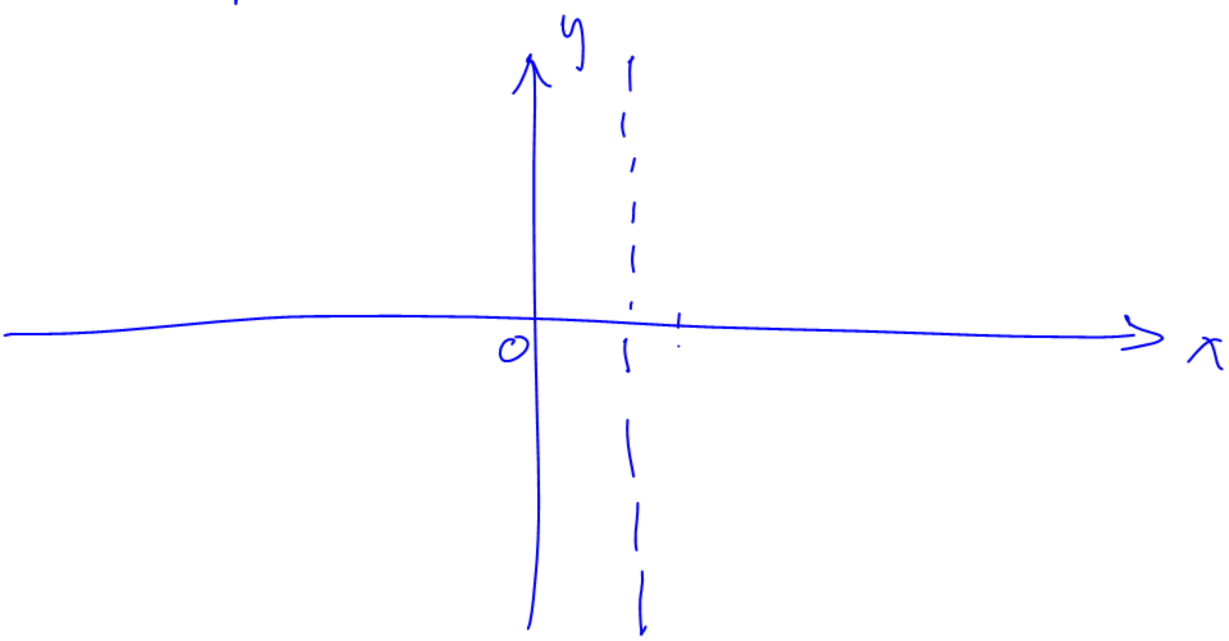
$\Rightarrow x = 1 \pm \sqrt{\dfrac{1}{\ln 10}} = 1 \pm 0{,}66$

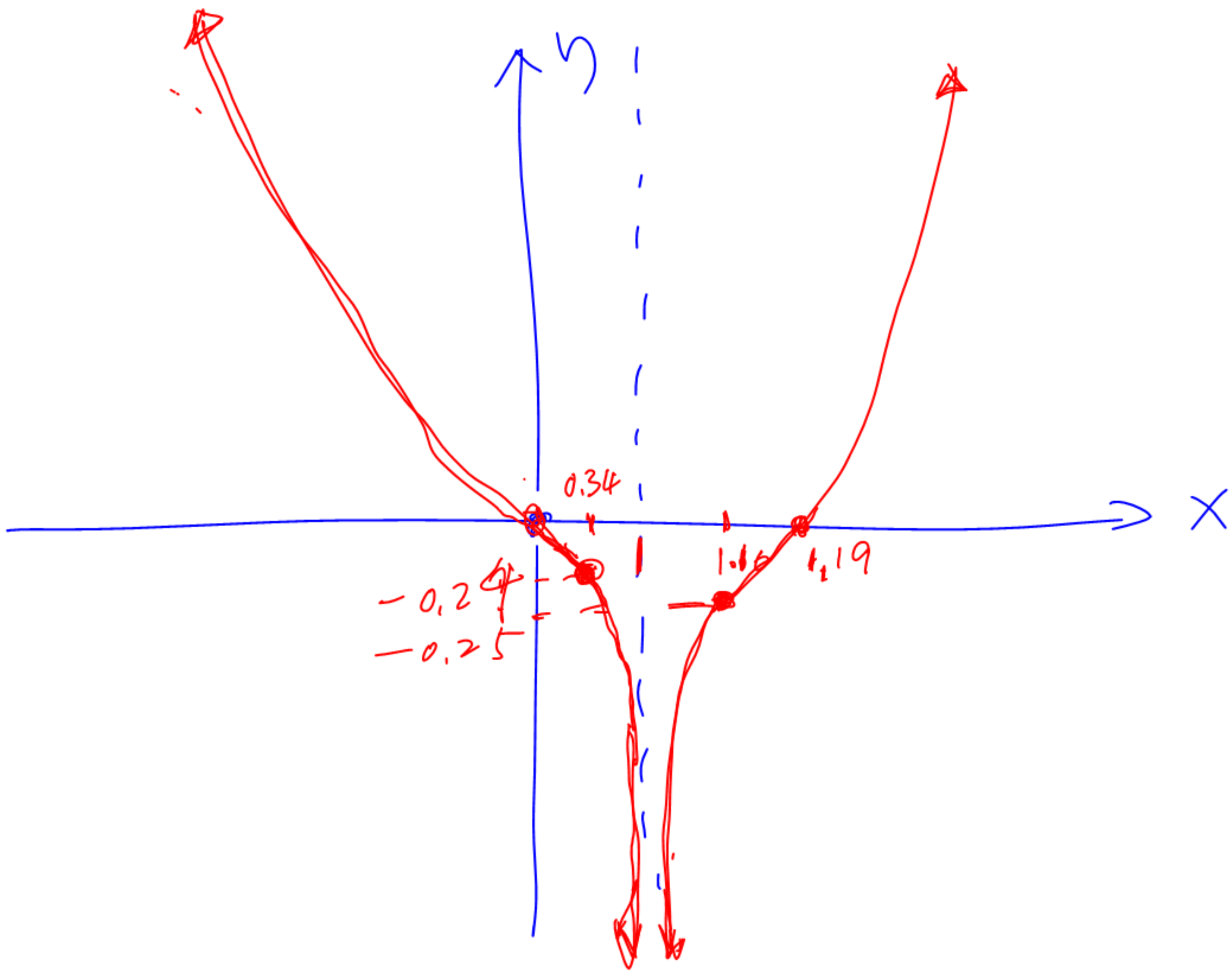
$\therefore \quad x \doteq 1 \pm 0.66$

$x \approx 0.34$ or 1.66

$$f''(x) = -\frac{2}{\ln 10} \cdot \frac{1}{(x-1)^2} + 2$$

Interval	$(-\infty, 0.34)$	$(0.34, 1)$	$(1, 1.66)$	$(1.66, \infty)$
test value	0	0.5	1.2	2
$f''(x) = -\frac{2}{\ln 10 (x-1)^2} + 2$	+	−	−	+
$f(x)$	∪	∩	∩	∪





$$f(0.34)=\log(0.34-1)^2+(0.34)^2\approx-0.24$$

$$f(1.16)=\log(1.16-1)^2+(1.16)^2\approx-0.25$$