

## 1.6 Continuity

**A Continuity**

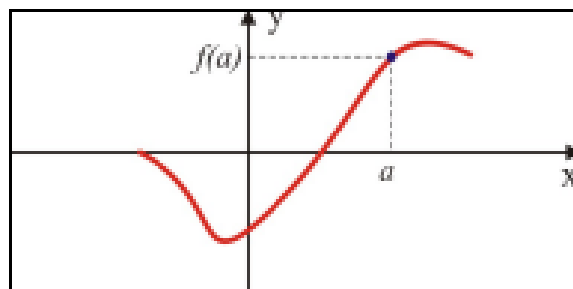
A function  $y = f(x)$  is *continuous* at a number  $a$  if

$$\lim_{x \rightarrow a} f(x) = f(a)$$

Note: A function is continuous at  $a$  if the following three conditions are met:

1.  $f(a)$  exists
2.  $\lim_{x \rightarrow a} f(x)$  exists
3.  $f(a)$  and  $\lim_{x \rightarrow a} f(x)$  are equal.

Note: A function is continuous if the graph can be drawn *without lifting the pen from paper*.

**B Discontinuity**

If  $y = f(x)$  is not continuous at  $a$  then we say:

- $y = f(x)$  is *discontinuous* at  $a$  or
- $y = f(x)$  has a *discontinuity* at  $a$

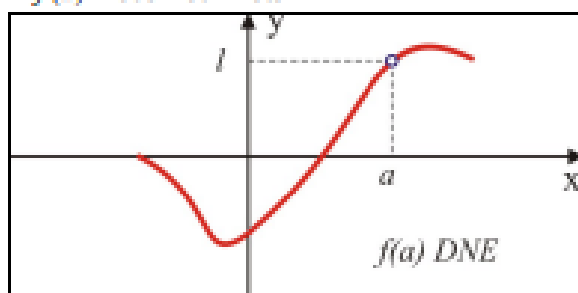
Note: There are three types of discontinuity:

- a) *removable or point discontinuity*
- b) *jump discontinuity*
- c) *infinite discontinuity (break)*

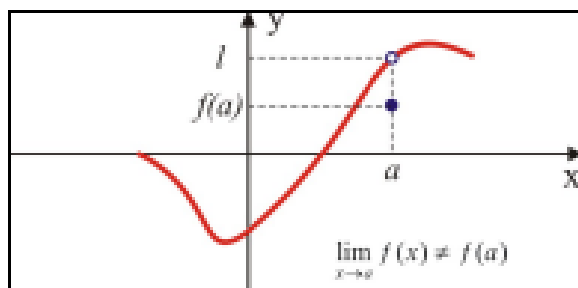
**C Removable or Point Discontinuity**

A function  $y = f(x)$  has a *removable or point discontinuity* at  $a$  if:

1.  $\lim_{x \rightarrow a} f(x)$  exists
2.  $f(a)$  Does Not Exist



or  $\lim_{x \rightarrow a} f(x) \neq f(a)$



Note: A removable or point discontinuity can be removed by redefining the function at  $a$  as

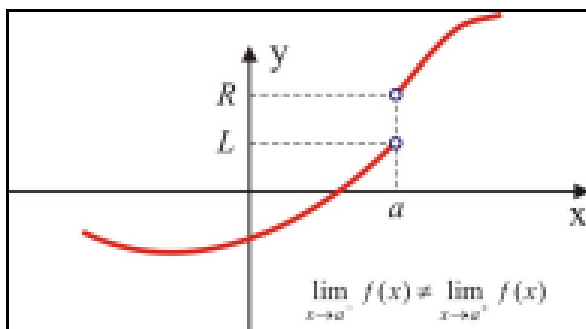
$$f(a) \stackrel{\text{def}}{=} \lim_{x \rightarrow a} f(x).$$

Ex 1. Redefine  $y = f(x) = \frac{x^2 - 4}{x - 2}$  such that  $y = f(x)$  is to be continuous everywhere (at any number). Graph the old and the new function.

**D Jump Discontinuity**

A function  $y = f(x)$  has a *jump discontinuity* at  $a$  if the left-side and the right-side limits exist but they are not equal:

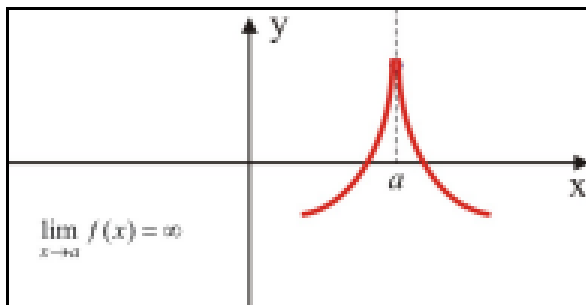
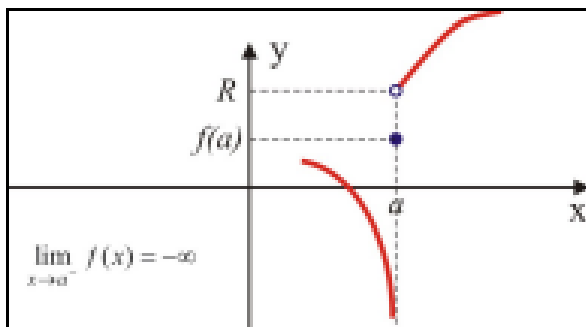
$$\lim_{x \rightarrow a^-} f(x) \neq \lim_{x \rightarrow a^+} f(x)$$



Ex 2. Analyze the continuity of the function  $y = f(x) = \frac{|x-3|}{x-3}$  at  $x = 3$ . Graph the function to illustrate the situation.

**E Infinite Discontinuity**

A function  $y = f(x)$  has an *infinite discontinuity* at  $a$  if at least one of the left-side or the right-side limits is *unbounded* (approaches to  $\infty$  or  $-\infty$ ).



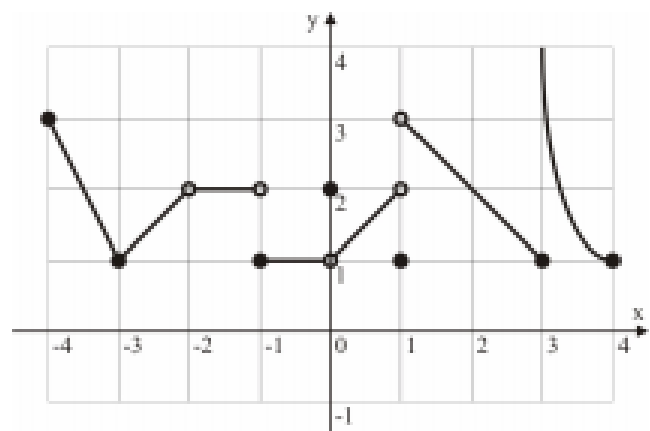
To write  $\lim_{x \rightarrow a} f(x) = \infty$  is better (more information is included) than to write  $\lim_{x \rightarrow a} f(x)$  DNE.

Ex 3. Analyze the continuity of the function  $f(x) = \frac{1}{x}$  at  $x = 0$ .

Ex 4. The function  $y = f(x)$  is represented graphically in the figure on the right side.

Analyze the continuity of this function at:

- a)  $x = -3$
- b)  $x = -2$
- c)  $x = -1$
- d)  $x = 0$
- e)  $x = 3$



### F Elementary Functions

Elementary functions (polynomial, power, rational, trigonometric, exponential, and logarithmic) are *continuous* over their domain.

Ex 5. Analyze the continuity of the function:

$$f(x) = \begin{cases} x, & x < 0 \\ x^2, & 0 \leq x \leq 1 \\ x^3 + 1, & x > 1 \end{cases}$$

Ex 6. For what value of the constant  $c$  is the function

$$f(x) = \begin{cases} x + c, & x < 2 \\ cx^2 + 1, & x \geq 2 \end{cases}$$

continuous at any number (everywhere)?