Matrix Multiplication

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Basics

- Matrix is a rectangular array of numbers, symbols, or expressions, arranged in rows and columns.

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

Matrix Multiplication

- Given matrix A (a x b) and B (b x c), C = AB and size of C is a x c.

$$\mathbf{C}[i,j] = \sum_{k=1}^{b} \mathbf{A}[i,k]\,\mathbf{B}[k,j] \quad (1 \le i \le a, 1 \le j \le c)$$

- ((AB)C) = (A(BC))
- Plain method O(N^3)
- Currently lower bound O(N^2.36) (Coppersmith Winograd algorithm)

Fast Matrix Power

- When computing a^t, we can first get a^[t/2] and then get a^t .
- For Matrix power, same $a^t = \begin{cases} \left(a^{\lfloor \frac{t}{2} \rfloor}\right)^2 \\ \left(a^{\lfloor \frac{t}{2} \rfloor}\right)^2 \cdot a \end{cases}$
- O(N^3 lg t)