AP Calculus Homework Eight – Antiderivatives and the Definite Integral

4.1 Antiderivatives; 4.2 Area

1. Find the most general antiderivatives of the functions.

(a) $f(x) = 6/\sqrt[3]{x} - \sqrt[3]{x}/6 + 7$

(b) $f(x) = (4 + 3x^2 \cos 4x)/x^2$

(c) $f(x) = 2\cos 3x - 3\sin 2x$

(d) $f(x) = \sin(4x)/\cos(2x)$

2. Solve the differential equations subject to the given boundary conditions.

(a) $f'''(x) = 6x, \quad f''(0) = 2, \quad f'(0) = -1, \quad f(0) = 4$

(b) $f''(x) = 4\sin 2x + 16\cos 4x, \quad f'(0) = 1, \quad f(0) = 6$

3. Evaluate the integrals without using your calculator.

(a) $\int (2 - 3x)^5 \, dx$

(b) $\int \dfrac{1 - 3y}{\sqrt{2y - 3y^2}} \, dy$

4. Evaluate the integrals without using your calculator.

(a) $\int \frac{x\,dx}{1+4x^2}$

(b) $\int \frac{dx}{1+4x^2}$

(c) $\int \frac{x\,dx}{\left(1+4x^2\right)^2}$

(d) $\int \frac{x\,dx}{\sqrt{1+4x^2}}$

5. Evaluate the integrals without using your calculator.

(a) $\int \sin\theta\cos\theta\; d\theta$

(b) $\int \frac{\sin\sqrt{x}\; dx}{\sqrt{x}}$

(c) $\int \cos^2 2x\; dx$

(d) $\int \sin 2\theta\; d\theta$

6. Evaluate the integrals without using your calculator.

(a) $\int \frac{\sin 2x\,dx}{\sqrt{1+\cos^2 x}}$

(b) $\int \sec^{3/2} x \tan x\,dx$

(c) $\int \tan\theta \, d\theta$

(d) $\int \frac{dx}{\sin^2 2x}$

7. Evaluate the integrals without using your calculator.

(a) $\int \frac{\tan^{-1} y}{1+y^2} \, dy$

(b) $\int \sin 2\theta \cos\theta \, d\theta$

(c) $\int \cot 2u \, du$

(d) $\int e^{2\theta} \sin e^{2\theta} \, d\theta$

8. Evaluate the integrals without using your calculator.

(a) $\int x^2 e^x dx$

(b) $\int \frac{e^x + e^{-x}}{e^x - e^{-x}} \, dx$

(c) $\int \frac{\ln v dv}{v}$

(d) $\int x^3 \ln x \, dx$

9. Evaluate the integrals without using your calculator.

(a) $\int \ln x^3 \, dx$

(b) $\int \frac{\ln y}{y^2} \, dy$

(c) $\int \frac{dx}{x^2 + 2x + 2}$

(d) $\int u \sec^2 u \, du$

10. Evaluate the integrals without using your calculator.

(a) $\int \frac{2x - 1}{\sqrt{4x - 4x^2}} \, dx$

(b) $\int \frac{dx}{1 - e^x}$

(c) $\int e^{2 \ln u} \, du$

(d) $\int (\tan \theta - 1)^2 \, d\theta$

11. A projectile is fired vertically upward from ground level with a velocity of 500 m/s. If air resistance is neglected, find its distance s(t) above ground at time t. What is its maximum height?

12. Suppose $f(x) = x^3$ and P is the partition of [-2, 4] into the four subintervals determined by $x_0 = -2$, $x_1 = 0$, $x_2 = 1$, $x_3 = 3$ and $x_4 = 4$. Find the Riemann sun R_p of $f(x)$ if w_i is the right-hand endpoint of the interval $[x_{i-1}, x_i]$.

13. Find the area under the graph of $f(x)$ from a to b using inscribed rectangles. In each case sketch the graph and typical rectangles, labeling the drawing.

    (a) $f(x) = x^2; \quad a = 0, b = 5$

14. Find the area under the graph of $f(x)$ from a to b using circumscribed rectangles. In each case sketch the graph and typical rectangles, labeling the drawing.

    (a) $f(x) = x^3 + 1; \quad a = 1, \ b = 2$