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Test 2

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Question 1-8 are Multiple-Choice questions **(8 marks)**

1. The exact radian measure of 75^0 is:

a) $\frac{5}{12}\pi$ b) $\frac{7}{12}\pi$ c) $\frac{5}{8}\pi$ d) $\frac{13}{12}\pi$

2. The equation of a cosine function with an amplitude of 3, a period of 4π, and a phase shift of $\frac{\pi}{2}$ to the left is:

a) $y=3\cos(\frac{1}{2}x)+\frac{\pi}{2}$ b) $y=3\cos 4\pi(x+\frac{\pi}{2})$ c) $y=3\cos\frac{1}{2}(x+\frac{\pi}{2})$ d) $y=4\pi\cos 3(x+\frac{\pi}{2})$

3. The expression $\sin^2 x + \cos^2 x + \tan^2 x$ is equivalent to:

a) 1 b) $\sec^2 x$ c) $\csc^2 x$ d) $1 + \tan x$

4. The expression $\sin\frac{\pi}{5}\cos\frac{\pi}{6} - \sin\frac{\pi}{6}\cos\frac{\pi}{5}$ is equivalent to:

a) $\cos\frac{11}{30}\pi$ b) $\sin\frac{11}{30}\pi$ c) $\cos\frac{\pi}{30}$ d) $\sin\frac{\pi}{30}$

5. Which set of the values is the solution for $\sin x = -\frac{1}{3}, 0 \leq x \leq 2\pi$:

a) -0.34, 2.80 b) 5.94, 3.48 c) 1.91, 4.37 d) -0.34

6. Which set of the values is the solution for $(2\sin(x)+1)(\cos(x)-1) = 0, 0 \leq x \leq 2\pi$:

a) $\pi, \frac{7\pi}{6}, \frac{11\pi}{6}$ b) a) $\frac{\pi}{6}, \pi, \frac{5\pi}{6}$ c) $0, \frac{7\pi}{6}, \frac{11\pi}{6}, 2\pi$ d) $0, \frac{5\pi}{6}, \frac{7\pi}{6}, 2\pi$

7. A co-terminal angle of $\alpha = \frac{5\pi}{4}$ is:

a) $\frac{\pi}{4}$ b) π c) $\frac{3\pi}{4}$ d) $-\frac{3\pi}{4}$

8. The exact value of $\log_{\sqrt{2}} \sqrt[5]{8}$ is:

a) $\frac{5}{8}$ b) 3 c) $\frac{6}{5}$ d) $\frac{3}{10}$

Questions 9-14 are long answer questions. Show your work to get full marks.

9. Prove the following trigonometric identities: (9 marks)

a) $\sin x \cot^2 x + \cos x \tan^2 x = \dfrac{\sin^3 x + \cos^3 x}{\sin x \cos x}$

$LHS = \cancel{\sin x}\dfrac{\cos^2 x}{\sin^{\cancel{2}} x} + \cancel{\cos x}\dfrac{\sin^2 x}{\cos^{\cancel{2}} x}$

$= \dfrac{\cos^2 x}{\sin x} + \dfrac{\sin^2 x}{\cos x}$

$= \dfrac{\cos^3 x + \sin^3 x}{\sin x \cos x}$

$= RHS$

b) $\dfrac{\sin(a-b)}{\cos a \cos b} + \dfrac{\sin(b-c)}{\cos b \cos c} + \dfrac{\sin(c-a)}{\cos c \cos a} = 0$

let $a, b, c = \sin(a, b, c)$

let $x, y, z = \cos(a, b, c)$

$LHS = \dfrac{ay - xb}{xy} + \dfrac{bz - cy}{yz} + \dfrac{cx - az}{zx}$

$= \dfrac{\cancel{ayz} - \cancel{xbz} + \cancel{bzx} - \cancel{cyx} + \cancel{cyx} - \cancel{ayz}}{xyz}$

$= \dfrac{0}{xyz} = 0 = RHS$

c) $\dfrac{\cos 2x}{1 + \sin 2x} = \dfrac{1 - \tan x}{1 + \tan x}$

$RHS = \dfrac{1 - \frac{\sin x}{\cos x}}{1 + \frac{\sin x}{\cos x}}$

$= \sin x + \cos x -$

10. Write each of the following as a single logarithm: (9 marks)

i. $\frac{1}{3}\log_a x + \frac{1}{4}\log_a y - \frac{2}{5}\log_a z$

$= 20\log_a x + 15\log_a y - 30\log_a z$

$= \log_a(x^{20}y^{15}) - 30\log_a z$

$= \log_a\left(\frac{x^{20}y^{15}}{z^{30}}\right)$

ii. $(4\log_5 x - 2\log_5 y) \div (3\log_5 z)$

$= \frac{\log_5\left(\frac{x^4}{y^2}\right)}{\log_5(z^3)}$

$= \log_{(z^3)}\left(\frac{x^4}{y^2}\right)$

iii. $\frac{1}{\log_a 10} + \frac{1}{\log_b 10} + \frac{1}{\log_c 10}$

$= \frac{\log_a a}{\log_a 10} + \frac{\log_b b}{\log_b 10} + \frac{\log_c c}{\log_c 10} = \log_{10}(a) + \log_{10}(b) + \log_{10}(c)$

$= \log_{10}(abc)$

11. Radium (Ra-225) has a half-life of 15 days. For a 100 g sample, the amount of radioactive material remaining, *A*, after time *t*, is given by the equation $A(t) = 100(0.5)^{\frac{t}{15}}$, where *A* is measured in grams and *t* is measured in days. (4 marks)

a. Find the average rate of change from 3 days to 4 days, rounded to one decimal place.

$m = \frac{A(3) - A(4)}{3-4} = -3.9$ g/day

b. Approximate the ~~average~~ instant rate of change at t=3 days, rounded to one decimal place.

$m = \frac{A(3) - A(3.00001)}{3 - 3.00001} = -4.0$

12. Solve the following equations: (6 marks)

a. $5^{2x-1} = \sqrt{5}$

$5^{2x-1} = 5^{\frac{1}{2}}$

$2x-1 = \frac{1}{2}$

$2x = \frac{3}{2}$

$x = \frac{3}{4}$

b. $9^x - 2(3^x) - 15 = 0$

let $3^x = a$

$(3^x)^2 - 2(3^x) - 15 = 0$

$a^2 - 2a - 15 = 0$

$(a-5)(a+3) = 0$

$x = \log_3(5)$ or $\cancel{\log_3(-3)}$

c. $\ln(x) + \ln(x-1) = 0$

$\log_e x + \log_e(x-1) = 0$

$\log_e(x^2 - x) = 0$

$x^2 - x = 1$

$x^2 - x - 1 = 0$

$x = \frac{1 \pm \sqrt{1-4(1)(-1)}}{2}$

$x = \frac{1}{2} + \frac{\sqrt{5}}{2}, \frac{1}{2} - \frac{\sqrt{5}}{2}$

Test x: $\frac{1}{2} + \frac{\sqrt{5}}{2}$ ✓

$\frac{1}{2} - \frac{\sqrt{5}}{2}$ ×

13. Evaluate the following expressions: (4 marks)

a. a. $27^{\log_3(90) - \log_3(18)}$

$= 27^{\log_3(5)}$

$= 3^{3\log_3(5)}$

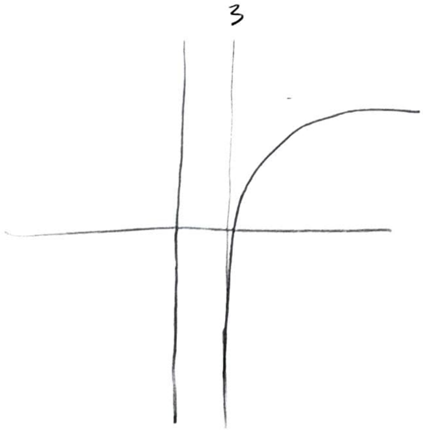
$= 3^{\log_3(5^3)}$

$= 5^3 = 125$

b. $4\sin^2(\frac{\pi}{8})\cos^2(\frac{\pi}{8})$

14. Sketch the graph of the following functions and state the domain, the range, and the equation of the asymptote (if any). (6 marks)

a. $y = \log_5(x-1) + 3$



Asymptote: $x = 3$

D: $x \in (3, \infty)$

R: $y \in (-\infty, \infty)$

b. $y = 2\csc(x + \frac{\pi}{4})$

$y = 2\frac{1}{\sin(x+\frac{\pi}{4})}$

asymptotes: $x = -\frac{\pi}{4} + k\pi$

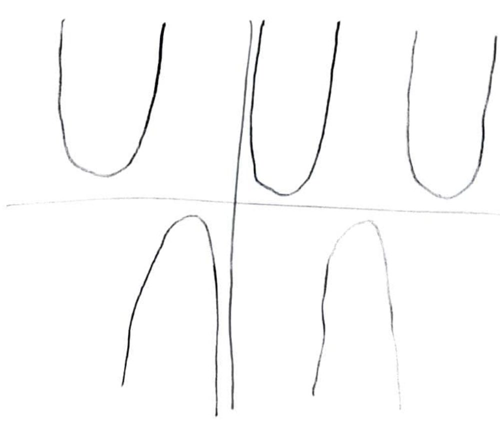
$\sin(x + \frac{\pi}{4}) = 0$

$x + \frac{\pi}{4} = 0$

$x = -\frac{\pi}{4}$

$x \neq -\frac{\pi}{4} + k\pi$

where k is integer



D: $x \in R \setminus \{-\frac{\pi}{4} + k\pi\}$

R: $y \in (-\infty, -2) \cup (2, \infty)$

15. The function $y=\frac{1}{3}\left(5^{x+A}\right)+4$ has a y-intercept of $\frac{301}{75}$. Determine the value of **A**. (2 marks)

$$\frac{301}{75}=\frac{1}{3}\left(5^{A}\right)+4$$

$$\frac{1}{75}=\frac{1}{3}\left(5^{A}\right)$$

$$\frac{1}{25}=5^{A}$$

$$A=\log_5\left(\frac{1}{25}\right)$$

$$=-2$$

16. Bonus Question (3 marks)

a. Sketch the graphs of $y=\tan(x)$ and $y=\cot(x)$ for $-\frac{\pi}{2}\le x\le \pi$.

b. Using the letters A, B, and C, label the three intersection points of the two functions graphed in part a).

c. Determine the area and perimeter of $\triangle ABC$