

*teacher  
note*

First Name: AdamLast Name: ChenStudent # Q13

HW 2 is way too long!!!! SO DO NOT DO:

Q3 (bottom 2 graphs)

Q4 (c,d,e,f)

Q5 (c,d,e)

Q7(c,d,e)

Q8

Q10(d,e)

Q11 (bottom 2)

Q13

} Skip

### Polynomial Functions

1. Consider the following polynomial functions.

- a.  $y = -2x^3 + 4x - 5$
- b.  $f(x) = 5x^4 + 2x^3 - 4x^2 + x - 7$
- c.  $g(x) = x^5 + 2x^3 - 5x + 8$

For each one, perform the following tasks.

- i. Describe the end behavior of the function.
- ii. Determine the maximum and minimum number of turning points.
- iii. Determine the maximum and minimum number of x-intercepts.

a) i) as  $x \rightarrow \infty, y \rightarrow -\infty$   
          as  $x \rightarrow -\infty, y \rightarrow -\infty$   
     ii) 1, because it's a parabola  
     iii) max: 2, min: 0

b) i) as  $x \rightarrow \infty, y \rightarrow \infty$   
          as  $x \rightarrow -\infty, y \rightarrow \infty$

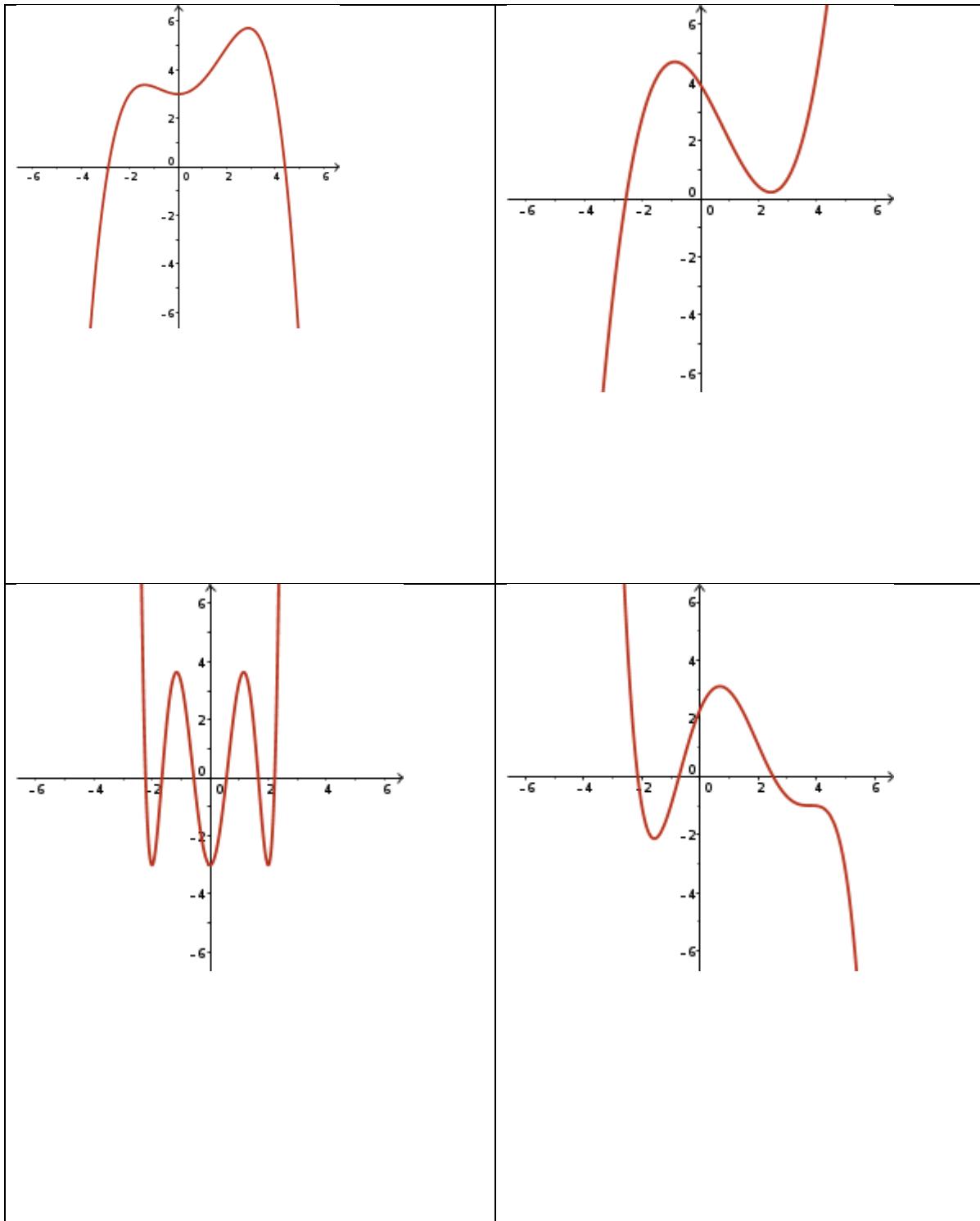
2. Sketch a possible graph of each function by identifying the end behaviours and determining the x- and y-intercepts of the function.

a.  $f(x) = (x-1)(x-3)(x+1)(x+4)$

b.  $y = -2x^3 - 3x^2 + 9x$

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3. Given the graph of the polynomial function  $y=f(x)$ , identify the minimum possible degree of the function and the sign of the leading coefficient.



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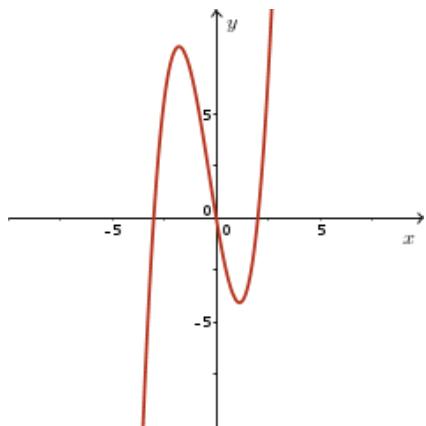
- 4.** Sketch a graph of a polynomial function that satisfies each set of conditions.
- a. Degree three, two distinct  $x$ -intercepts, two turning points, and end behavior such that  $y \rightarrow \infty$  as  $x \rightarrow -\infty$  and  $y \rightarrow -\infty$  as  $x \rightarrow \infty$
  - b. Degree four, two distinct  $x$ -intercepts, three turning points, and end behavior such that  $y \rightarrow \infty$  as  $x \rightarrow \pm\infty$
  - c. Degree four, negative leading coefficient, three distinct  $x$ -intercepts, three turning points
  - d. Degree three, positive leading coefficient, one  $x$ -intercept, two turning points
  - e. Degree five, negative leading coefficient, two distinct  $x$ -intercepts, two turning points
  - f. Degree five, positive leading coefficient, one  $x$ -intercept, four turning points

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5. Sketch a possible graph of a polynomial function that satisfies the following conditions.

- a. A quadratic function with a negative leading coefficient and a zero at  $x=-5$  of multiplicity 2.
- b. A 5th degree function with a positive leading coefficient, a zero at the origin of order 2, and a zero at  $x=3$  of order 3.
- c. A quartic function with a positive leading coefficient and two real zeros,  $x=0$  and  $x=3$  of order 2.
- d. A cubic function with a negative leading coefficient and only one zero at  $x=4$  and two non-real zeros.
- e. A quintic function with a positive leading coefficient, a zero at  $x=-2$ , and a second zero at  $x=1$  of multiplicity 4.

6. Given the graph of the polynomial function  $f(x)=x^3+x^2-6x$ ,



Sketch the graph of

a.  $y=|f(x)|$

b.  $y=f(|x|)$

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7. Sketch a possible graph for each of the following functions.

a.  $y = -x(x+2)(2x-5)$

b.  $f(x) = 2(x-2)^2(x+3)^2$

c.  $g(x) = -0.5(x-3)(x+1)^3$

d.  $y = 2x^2(x-4)^3$

e.  $f(x) = -x(2x+3)(x-2)^2$

Solution:

8. Sketch a possible graph for each function.

a.  $f(x) = -2x^3 + 8x$

b.  $f(x) = -x^4 - 5x^3 - 6x^2$

c.  $f(x) = x^4 - 2x^2 + 1$

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**9.** A family of quintic functions has a zero at  $x=-3$  and turning points tangent to the  $x$ -axis at  $x=1$  and  $4$ .

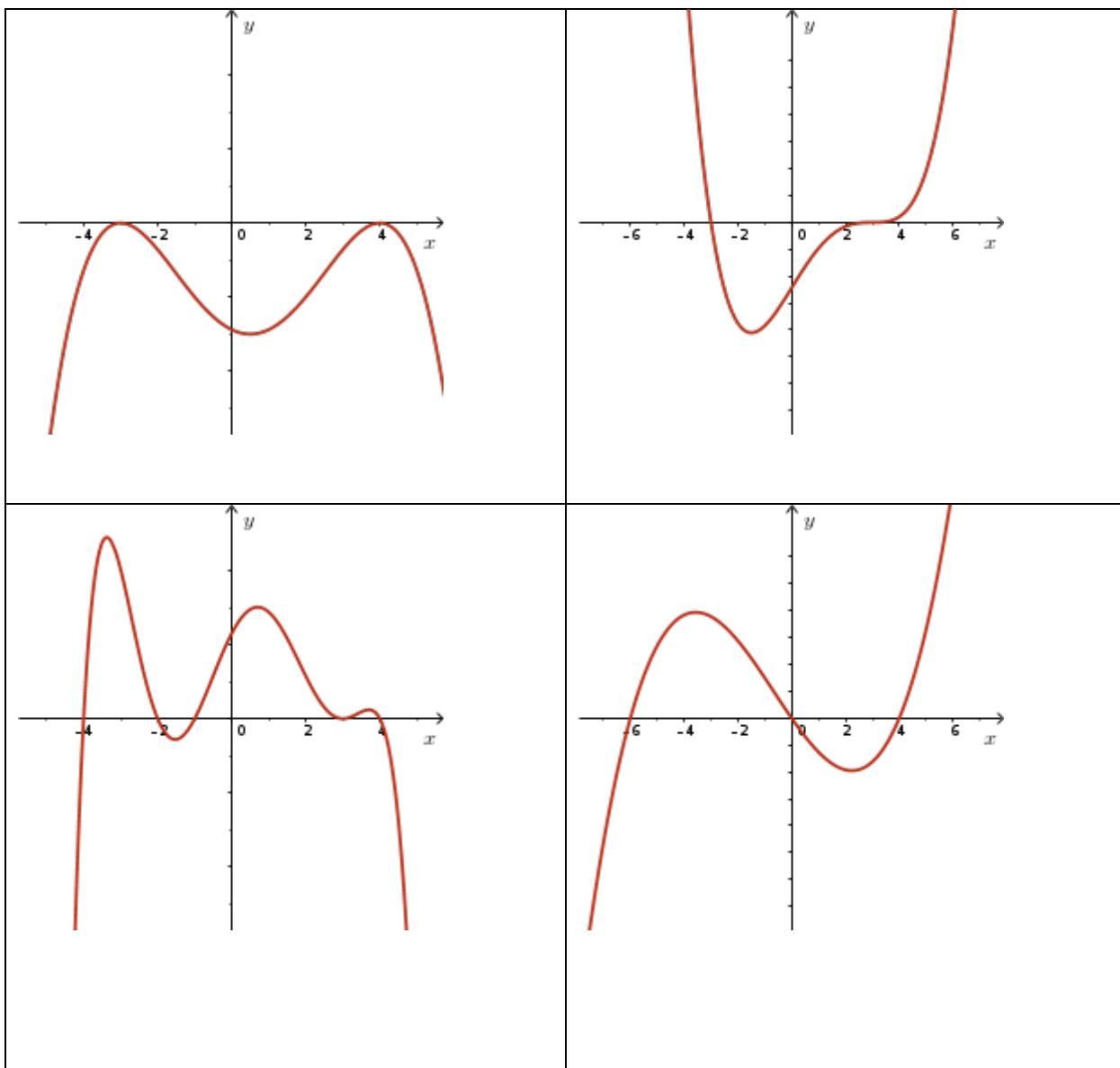
- a. State the general equation of the family.
- b. State the equations of two members of the family that have end behaviour  $y \rightarrow \infty$  as  $x \rightarrow -\infty$  and  $y \rightarrow -\infty$  as  $x \rightarrow \infty$ .

**10.** State the equation of the family of polynomial functions satisfying the following conditions:

- a) A cubic with zeros  $x=-3$ ,  $x=-\frac{1}{2}$ , and  $x=\frac{5}{3}$ .
- b) A sixth degree function with zeros  $x=-2$  (order 2),  $x=1$  (order 1), and  $x=5$  (order 3).
- c) A quartic that passes through the origin and has a point of inflection at  $(\frac{2}{3}, 0)$ .
- d) Cubic function,  $x$ -intercept at  $x=-4$ , a turning point at  $(1, 0)$ , and  $f(x) \rightarrow -\infty$  as  $x \rightarrow \infty$ .
- e) A quartic function with zeros at  $x=\pm\sqrt{5}$  and  $x=-1\pm\sqrt{2}$ .

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- 11.** Given the graph of  $y=f(x)$ , determine a general equation for a family of polynomials with the same end behaviour and zeros of  $f(x)$  (note: all zeros are integer in value).



- 12.** State the equation of the quartic function with zeros  $x=-\frac{1}{2}$  and 5 (both of multiplicity 1) and  $x=2$  (multiplicity 2), having a  $y$ -intercept of 4.

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**13.** Find the general equation of the family of

- a) quadratic functions with zeros  $-3-\sqrt{5}$  and  $-3+\sqrt{5}$ .
- b) cubic functions with zeros  $0, 1-2\sqrt{3}$  and  $1+2\sqrt{3}$ .
- c) quartic functions with zeros  $-2, 1$  and  $\pm 3i$ .
- d) quartic functions with zeros  $3\pm\sqrt{2}$  and  $-4\pm i\sqrt{3}$

**14.** Determine the equation of the quartic function with rational coefficients, zeros  $4-\sqrt{2}$  and  $-3+\sqrt{6}$ , and a  $y$ -intercept of  $-21$ .

**15.** The function  $f(x)=\frac{1}{4}(x-2)^2(x+2)^2$  has a turning point at  $(0,4)$ . Determine

- a) The intervals where  $f(x)$  is positive and negative.
- b) The intervals where  $f(x)$  is increasing and decreasing