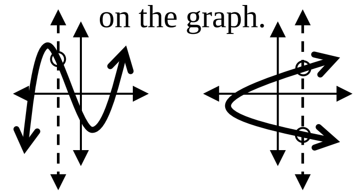
First Name:____________________ Last Name: __________________ Student ID: ________

Functions: Review Transformations and Properties

Functions Review

Relation	Function	Vertical Line Test
A set of points. *(Anything you can show on a graph)* {(6, 5), (4, 0), (-8, 5)} Relation {(3, 5), (2, -9), (3, 7)} Relation	A set of points where each x-value has only one y-value. {(6, 5), (4, 0), (-8, 5)} Function {(3, 5), (2, -9), (3, 7)} Not A Function (Two points with an x-value of 3)	If a relation is a function, a vertical line will only cross the function once at any point on the graph.  Function Not A Function

Example 1: Determine which of the following are functions:

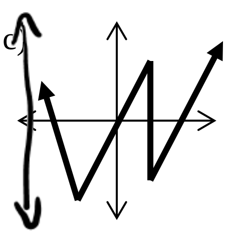
a) {(3, 4), (2, -3), (3, -1), (4, -10)}

NO

b) {(2, 6), (1, 4), (5, 6), (-10, -10)}

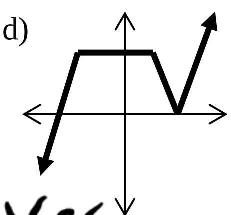
Yes

c)



No

d)



Yes

e)

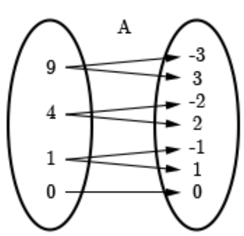
Student ID	Name
123456	John Adams
234234	Raj Sood
987654	Sara Williams

Yes

f)

Phone Number	Name
905-123-4567	John Adams
905-234-5678	Raj Sood
905-345-6789	Raj Sood

Yes



g)

No

Function Notation

	How to Write The Equation	*How to Ask a Question*
Regular Notation	y = 4x – 3	What is the value of y when x = 2?
Function Notation	$f(x) = 4x - 3$	$f(2)$

(x,y)
Cartesion

y = f(x)
x = input
y = output

x = f(y)
x = input
y = output

Independent variable: horizontal axis variable, normally x

Dependent variable: vertical axis variables, normally y

Example 2: Determine the value of each for the function $\boldsymbol{f(x) = 4x - 3}$.

a) $f(2)$

$y = 4x - 3$
$y = 8 - 3$
$y = 5$

b) $f(0)$

$y = 4 \cdot 0 - 3$
$y = -3$

c) $f(-1)$

$y = 4 \cdot (-1) - 3$
$y = -7$

d) $f(10)$

Example 3: Give $f(x) = x^2 - 1$, $g(x) = \sqrt{1-x}$, and $h(x) = \frac{1}{x-3}$, determine each of the following in its simplest form:

$g(4) = \sqrt{1-4}$
$= \sqrt{-3}$
$= i\sqrt{3}$
or $g(4)$ DNE

a) $f(-1)$ b) $g(-2)$ c) $h(2)$ d) $h(t-1)$ e) $g(1-t)$

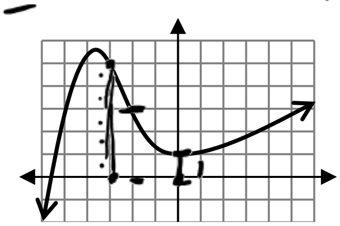
f) $h\left(\frac{1}{t}\right)$ g) $f(x) + 3$ h) $f(x+3)$ i) $3f(x)$ j) $f(\sqrt{1-x})$

k) $g(4)$ l) $h(x) + 1$

b) $g(-2) = \sqrt{1-(-2)} = \sqrt{3}$

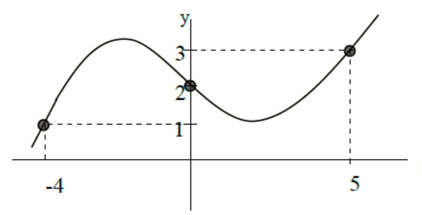
d) $h(t-1) = \frac{1}{(t-1)-3} = \frac{1}{t-4}$

$f(\sqrt{1-x}) = (\sqrt{1-x})^2 - 1$
$= 1 - x - 1 = -x$



Example 4 i. Determine the value of each for the function shown on the **graph**:

a) $f(0) = 1$ b) $f(-3) = 5$ c) $f(-2) = 3$

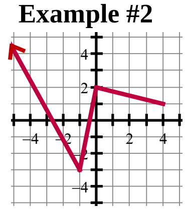
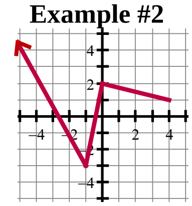


ii. From the graph of $y = f(x)$ as shown.

Determine i) $f(5)$ ii) $f(0)$ iii) $f(-4)$

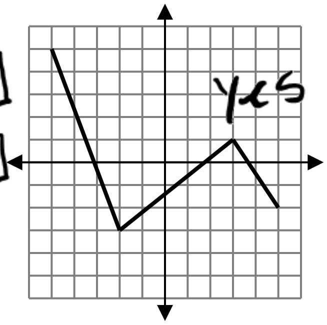
$= 3$ $= 2$ $= 1$

Domain & Range

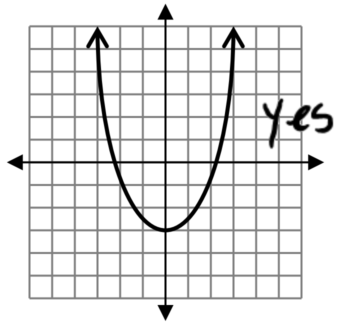
Domain	Range
The set of x-values in a relation.	The set of y-values in a relation.
From a Set of Points { xεR | x = *list all of the x-values* }	**From a Set of Points** { yεR | y = *list all of the y-values* }
From a Graph { xεR | *lowest x-value* ≤ x ≤ *highest x-value*}	**From a Graph** { yεR | *lowest y-value* ≤ y ≤ *highest y-value*}
Example #1 {(1, 5), (3, -4), (5, 5)} D = {1, 3, 5} **Example #2** 	**Example #1** {(1, 5), (3, -4), (5, 5)} R = {5, -4} R = {5, ~~-4~~, ~~5~~} **Example #2** 

The set of all real numbers such that x is an input value of the relation

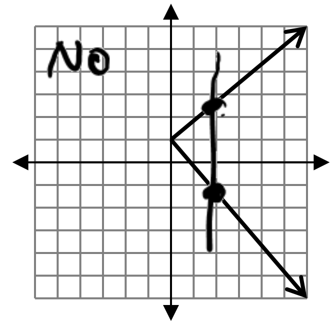
Example 5: Determine if each relation is a function. State the domain and range.

a)


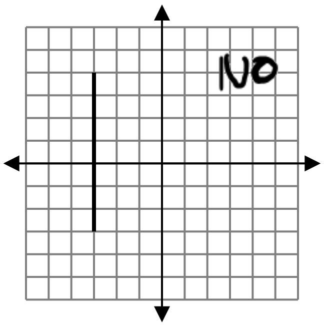
D: $[-5, 5]$
R: $[-3, 5]$
Yes

b)


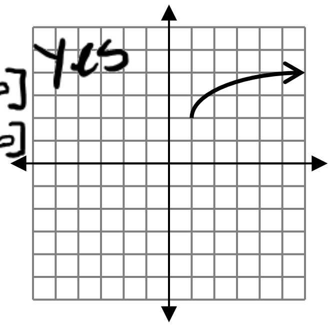
Yes

c)


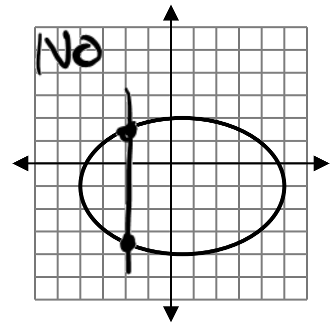
NO

d)


D: $\{-3\}$
R: $[-3, 4]$
NO

e)


Yes
D: $[1, \infty]$
R= $[2, \infty]$

f)


NO

No /0 sqrt of negative number, or log of >= 0

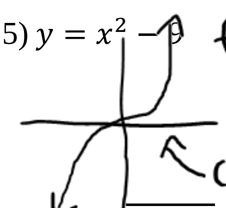
Example 6: Determine if each relation is a function. State the domain and range.

Yes 1) $y = (x+5)^2$
D= $\mathbb{R}$
R= $[0, \infty]$

2) $y = -(7-x)^2$
Function
D: $\mathbb{R}$
R: $[-\infty, 0]$

3) $y = -(2x+3)^2$
Func
D: $\mathbb{R}$
R: $[-\infty, 0]$

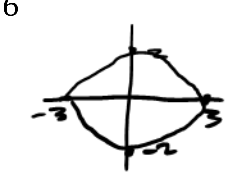
4) $y = 2x^2 - 4x + 7$

5) $y = x^2 - 9$
function


cubic

6) $y = 2x^3 + 16$
D= $\mathbb{R}$
R = $\mathbb{R}$

7) $x^2 + y^2 = 49$
NO
D= $[-7, 7]$
R= $[-7, 7]$

8) $4x^2 + 9y^2 = 36$
Not function
D= $[-3, 3]$
R= $[-2, 2]$


9) $y = \sqrt{4 - x^2}$
D= $[-2, 2]$
R= $[0, 2]$
$4 - x^2 \geq 0$
$4 \geq x^2$

10) $y = \sqrt{9x^2 - 4}$

11) $y = -\sqrt{x^2 + 9}$

12) $y = -\sqrt{16 - x^2}$

13) $y = \frac{2}{\sqrt{x}} + 1$

14) $y = \frac{-4}{\sqrt{x-1}} + 2$
Yes
D= $(1, \infty)$
R= $(-\infty, 2)$

15) $y = x^3 - x$

16) $y = \frac{-2x^3}{5}$

17) $y = \frac{1}{\sqrt{x-1}}$

18) $y^2 = \frac{1}{x+3}$

19) $y = 2|x+3| - 7$

20) $y = -3|x-2| + 4$

21) $y = 2^x - 1$

Transformations

Transformation is when you move a function without changing its shape

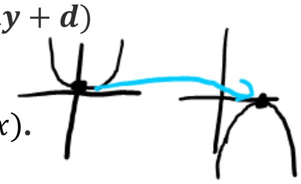
- **Shifting**
  - Given a function $y = f(x)$ and a constant c > 0
    - $y = f(x) + c$ shifts the graph up c units (add c to y-values)
    - $y = f(x) - c$ shifts the graph down c units (subtract c from y- values)
    - $y = f(x + c)$ shifts the graph left c units (subtract c from x-values)
    - $y = f(x - c)$ shifts the graph right c units (add c to x-values)
- **Stretching & compressing**
  - Given a function $y = f(x)$ and a constant c > 1
    - $y = c \cdot f(x)$ vertical stretch by a factor of c (multiply y-values by c)
    - $y = \frac{1}{c} \cdot f(x)$ vertical compress by a factor of $\frac{1}{c}$ (divide y-values by c)
    - $y = f(c \cdot x)$ horizontal compress by a factor of $\frac{1}{c}$ (divide x-values by c)
    - $y = f(\frac{1}{c} \cdot x)$ horizontal stretch by a factor of c (multiply x-values by c)
- **Reflecting**
  - Given a function $y = f(x)$
    - $y = -f(x)$ reflects graph about the x-axis (multiply all y values by -1)
    - $y = f(-x)$ reflects graph about the y-axis (multiply all x values by -1)

(x, y) is on the graph y = f(x)

Mapping Notation $\quad y = af(b(x-c)) + d \quad (x, y) \rightarrow (\frac{1}{b}x + c, ay + d)$

$x_1 \to x_2 \quad y_1 \to y_2$

tracks a single point



Example 7: Describe each transformation that must be applied to the function $y = f(x)$.

a) $y = 2f(x-5)$

- Vertical strech by 2
- Horizontal translation right by 5

b) $y = f(x+1) - 4$

- horizontal translation left by 1
- Vertical translation down by 4

c) $y = -f(3x) + 1$

- Horizontal stretch by 1/3
- reflection in x axis
- vertical shift up by 1

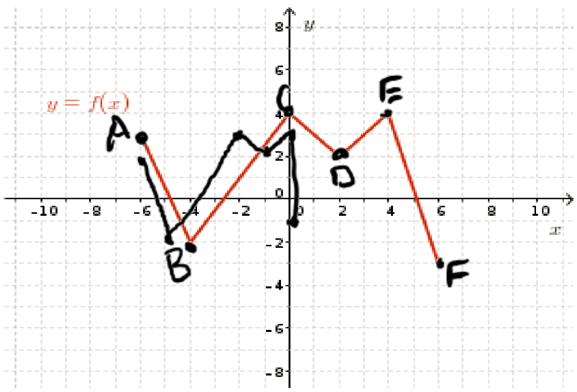
Example 8: Given the graph of the function $y = f(x)$, draw the graphs of the following transformed function $y = \frac{1}{2}f(2(x+3))$

0
↓
d

$y = a f(\boxed{b}(x - c)) + d$

$(x, y) \to (\frac{1}{b}x + c, ay + d)$

$\to (\frac{x}{2} - 3, \frac{y}{2})$

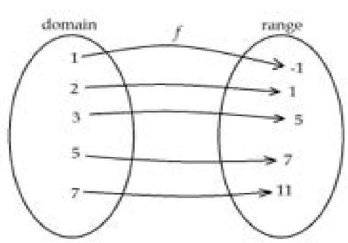


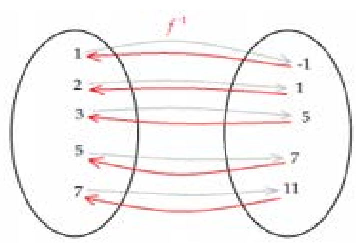
All these transformations PRESERVE straight lines and circles: Meaning if you keep track of where the endpoints of a line segment get mapped, those points become the new endpoints. i.e we only need to track where the endpoints get mapped.

Points: $A(-6,3) \to A'(-6,\frac{3}{2})$ $F(6,-3)$
$B(-4,2) \to B'(-5,-1)$ $F'($
$C(0,4) \to C'(-3,2)$
$D(2,2) \to D'(-2,1)$
$E(4,4) \to E'($

Inverse of a Function

Consider the function f shown on the picture below. The inverse relation, denoted by f^{-1} is obtained by reversing the assignments defined by f.





Mapping representation

Note: $f^{-1}(x) \neq \frac{1}{f(x)}$

inverse ≠ reciprocal

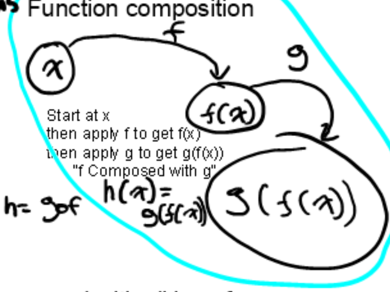
Function maps input -> output
inverse maps input <- output
(new out) (new in)

Example 9: Given $g(x) = \sqrt{x+2}$

a. Draw the graph of $y = g(x)$ and $y = g^{-1}(x)$.
b. Determine the equation of $g^{-1}(x)$.

$\sin^{-1}(\theta)$ vs $\sin^{2}(\theta)$
inverse exponent

A function of functions



x and y get switched

$y = f(x)$
$x = f^{-1}(y)$

$a + b = 0$ $b = -a$ "opposite"
$ab = 1$ $b = \frac{1}{a}$ "reciprocal"
$\{f \circ g = id, \; g \circ f = id\}$ $g = f^{-1}$ "inverse"
$a + b = b + a$ $f \circ g \neq g \circ f$

Function Composition of "f composed with g" is gof

id(x) = x for all x (the do nothing function)

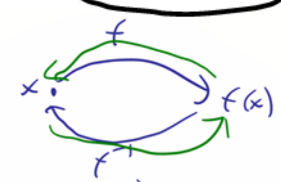
(foid)(x) = f(id(x)) = f(x)

foid = f
idof = f

Definition: The "identity function", written id(x)

$(f \circ g)(x) = f(g(x))$

Inverse



So $f^{-1}(f(x)) = x$
$f(f^{-1}(x)) = x$

$a, b \in \mathbb{R}, a < b$ $[a,b] = \{x \in \mathbb{R} \mid a \leq x \leq b\}$

square: closed (endpoints included)

Interval Notation

Round: open interval endpoints not included $(a,b) = \{x \in \mathbb{R} \mid a < x < b\}$

Example 10: Complete the chart below.

Interval Notation	Inequality Notation	English Sentence
(-∞, -2)	$x < -2$	The set of all real numbers less than -2.
[1, 10]	$1 \leq x \leq 10$	The set of all real numbers greater than or equal to one and less than or equal to 10
[-2,2] U [6, ∞) union	$-2 \leq x \leq 2$ or $x \geq 6$	The set of all real numbers less than or equal to 6 or greater than 8.

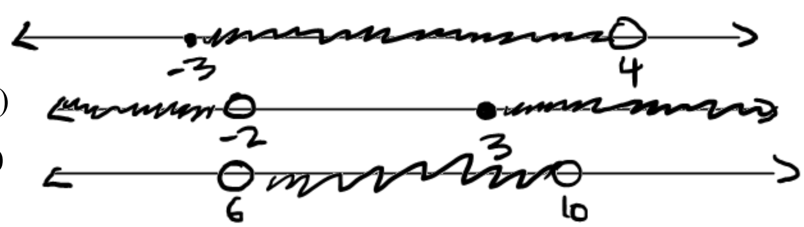
Example 11: Use a number line to graph the intervals below.

• inclusive
O exclusive

a. [-3,4)

b. (-∞, -2) U [3, ∞)

c. x ≥ 6 and x ≤ 10



Example 12: Given the graph of $y = f(x)$ shown, identify the following:

a. The domain of the graph
b. The range of the graph
c. The increasing and decreasing intervals
d. The intervals where $f(x) = 0$