

AP Calculus Homework Nine – Antiderivatives and the Definite Integral

4.4 Definition of Definite Integral and Properties of Definite Integral

4.4 The Mean Value Theorem for Definite Integral; 4.5 The Fundamental Theorem of Calculus

1. Evaluate the definite integrals.

(a) $\int_1^4 (x^2 - 4x - 3) dx$

(b) $\int_1^2 [5/(8x^6)] dx$

(c) $\int_{-3}^6 |x - 4| dx$

(d) $\int_{-\pi/6}^{\pi/6} (x + \sin 5x) dx$

(e) $\int_0^1 \frac{1}{(3-2v)^2} dv$

(f) $\int_1^4 \frac{1}{\sqrt{x}(\sqrt{x}+1)^3} dx$

2. Let f be continuous on $[-a, a]$. if f is an even function, show that $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$ and interpret this result geometrically. Verify the result for the special case $f(x) = \cos x$.

3. Evaluate the definite integrals.

$$(a) \int_{-1}^4 (x^2 - x - 1) dx$$

$$(b) \int_0^3 \frac{dt}{\sqrt{4-t}}$$

$$(c) \int_0^1 (2t - 1)^3 dt$$

$$(d) \int_4^9 \frac{2+x}{2\sqrt{x}} dx$$

4. Evaluate the definite integrals.

$$(a) \int_0^1 e^{-x} dx$$

$$(b) \int_0^1 xe^{x^2} dx$$

$$(c) \int_0^{\pi/6} \frac{\cos \theta}{1 + 2 \sin \theta} d\theta$$

$$(d) \int_{\sqrt{2}}^2 \frac{u}{u^2 - 1} du$$

5. Evaluate the definite integrals.

$$(a) \int_{\sqrt{2}}^2 \frac{u}{(u^2 - 1)^2} du$$

$$(b) \int_0^{\pi/4} \cos^2 \theta d\theta$$

(c) $\int_{\pi/12}^{\pi/4} \frac{\cos 2x dx}{\sin^2 2x}$

(d) $\int_0^{\pi/2} \sin^2 \frac{x}{2} dx$

6. Evaluate the definite integrals with substitutions.

(a) $\int_1^2 \frac{\sqrt{4-x^2}}{x} dx$, let $x = 2 \sin \theta$

(b) $\int_1^{\sqrt{3}} \sqrt{1+x^2} dx$, let $x = \tan \theta$

(c) $\pi \int_0^{2a} y^2 dx$, let $y = 2a \cos^2 \theta$ and $x = 2a \tan \theta$, where $0 \leq \theta \leq \pi$

7. In each sub-question, find numbers that satisfy the conclusion of the Mean Value Theorem for Definite Integral.

(a) $\int_0^3 3x^2 dx = 27$

(b) $\int_0^a \sqrt{a^2 - x^2} dx = (\pi a^2)/4$, $a > 0$