MDM4U HW6.

P318-319: Q6, Q9, Q10, Q12, Q13;

P324-326: Q2, Q6, Q9, Q15, Q16;

Sol. Q6.

Given odds of $B = \frac{5}{4}$, so $P(B) = \frac{5}{5+4} = \frac{5}{9}$

so $P(E) = \frac{4}{9} = P(B')$

Let A be the event that Elena wins in the best of five game.

$P(A) = P(EEE) + P(EEBE) + P(EBEE) + P(BEEE)$

$+ P(BBEEE) + P(BEBEE) + P(BEEBE)$

$+ P(EBBEE) + P(EEBBE) + P(EBEBE)$

$= \left(\frac{4}{9}\right)^3 + 3\left(\frac{4}{9}\right)^3\left(\frac{5}{9}\right) + 6\left(\frac{4}{9}\right)^3\left(\frac{5}{9}\right)^2$

$= \left(\frac{4}{9}\right)^3\left[1 + 3\left(\frac{5}{9}\right) + 6\left(\frac{5}{9}\right)^2\right] = \left(\frac{4}{9}\right)^3\left(\frac{122}{27}\right) = \frac{7808}{19683}$

≈ 0.3967

$\approx 0.4.$

Q9. Given odds of T $= \frac{1}{5}$,

so $P(T) = \frac{1}{1+5} = \frac{1}{6}$;

Given odds of M $= \frac{2}{13}$,

so $P(M) = \frac{2}{2+13} = \frac{2}{15}$;

Then $P(T \cup M) = P(T) + P(M) - P(T \cap M)$

$= \frac{1}{6} + \frac{2}{15} - 0 = \frac{3}{10}$.

Hence, odds of "$T \cup M$" $= \frac{P(T \cup M)}{1 - P(T \cup M)}$

$= \frac{\frac{3}{10}}{1 - \frac{3}{10}} = \frac{3}{7}$

Q10.

♠	A	2	3	4	5	6	7	8	9	10	J	Q	K
♣	A	2	3	4	5	6	7	8	9	10	J	Q	K
♦	A	2	3	4	5	6	7	8	9	10	J	Q	K
♥	A	2	3	4	5	6	7	8	9	10	J	Q	K

a) $P(E) = \frac{n(E)}{n(S)} = \frac{{}_{12}C_1}{{}_{52}C_1} = \frac{12}{52} = \frac{3}{13}$

So odds against E $= \frac{1 - P(E)}{P(E)} = \frac{1 - \frac{3}{13}}{\frac{3}{13}} = \frac{10}{3}$

b) $P(E) = \frac{n(E)}{n(S)} = \frac{{}^{12}C_2}{{}^{52}C_2}$

$= \frac{11}{221}$

$\text{odds against } E = \frac{1-P(E)}{P(E)} = \frac{1-\frac{11}{221}}{\frac{11}{221}} = \frac{210}{11}$

Q12. Given $P(A) = 30\% = 0.3$

$P(B) = 40\% = 0.4$; $P(C) = 50\% = 0.5$

So

a) $\text{odds of } A = \frac{P(A)}{1-P(A)} = \frac{0.3}{1-0.3} = \frac{0.3}{0.7} = \frac{3}{7}$;

b) $\text{odds of } B = \frac{P(B)}{1-P(B)} = \frac{0.4}{1-0.4} = \frac{0.4}{0.6} = \frac{2}{3}$;

c) $\text{odds against } C = \frac{1-P(C)}{P(C)} = \frac{1-0.5}{0.5} = 1$

Q13. Given odds of $W = \frac{3}{1}$

so $P(W) = \frac{3}{3+1} = \frac{3}{4}$;

given odds against $L = \frac{5}{1}$, so $P(L) = \frac{1}{1+5} = \frac{1}{6}$;

given odds against $T = \frac{7}{1}$, so $P(T) = \frac{1}{1+7} = \frac{1}{8}$;

$\because P(W) + P(L) + P(T) = \frac{3}{4} + \frac{1}{6} + \frac{1}{8} = \frac{25}{24} \neq 1$, $\therefore$ odds are not true.

P324 Q2.

E is the event that at least two out of eight friends have the same birthday.

E' is the event that no friends have the same birthday.

$$P(E') = \frac{n(E')}{n(S)} = \frac{{}_{365}P_8}{365^8} \approx 0.9257$$

$$\therefore P(E) = 1 - P(E') \approx 1 - 0.9257 = 0.0743.$$

Q6. 3 black, 5 blue, 8 white

$$3+5+8=16.$$

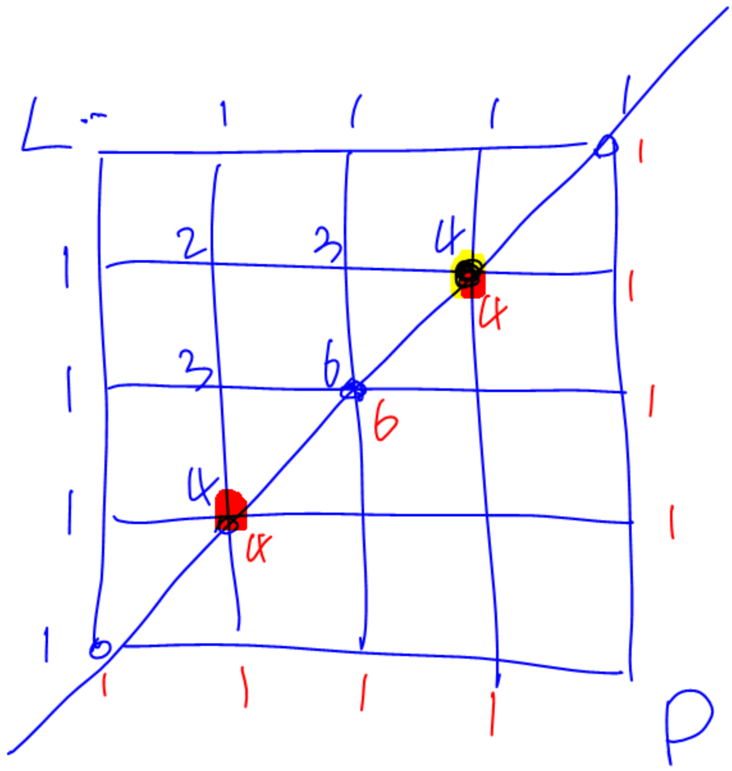
$$P(E) = \frac{n(E)}{n(S)} = \frac{{}_8C_2 \cdot {}_8C_0}{{}_{16}C_2} = \frac{7}{30}$$

Q9. 2 goalies; 6 defenders; 8 wingers; 4 centres; $2+6+8+4=20$.

a) $$P(E) = \frac{n(E)}{n(S)} = \frac{{}_8C_4 \cdot {}_{12}C_0}{{}_{20}C_4} = \frac{14}{969} \approx 0.014$$

b) $$P(E) = \frac{n(E)}{n(S)} = \frac{{}_{14}C_4 \cdot {}_6C_0}{{}_{20}C_4} = \frac{1001}{4845} \approx 0.207$$

Q15



$$P(E) = \frac{n(E)}{n(S)} = \frac{1^2+4^2+6^2+4^2+1^2}{16 \times 16} = \frac{70}{256} = \frac{35}{128}$$

Q16.

E is the event that the committee is comprised of nurses only.

Given $P(E') \geq 90\%$

let K be the size of the committee.

where $K = 1, 2, 3, \cdots, 8$;

Find the minimum value of K so that

$P(E') \geq 90\%$.

Then $1 - P(E) \geq 90\%$

$$0.1 = 1 - 0.9 \geq P(E)$$

8 N, 2 D.

$$P(E) = \frac{n(E)}{n(S)} \leq 0.1$$

$$P(E) = \frac{{}_8C_K \cdot {}_2C_0}{{}_{10}C_K} = \frac{{}_8C_K}{{}_{10}C_K} \leq 0.1$$

If $K=1$, $P(E) = \frac{{}_8C_1}{{}_{10}C_1} = \frac{8}{10} = 0.8 \nleq 0.1$

If $K=2$, $P(E) = \frac{{}_8C_2}{{}_{10}C_2} = \frac{28}{45} \approx 0.62 \nless 0.1$

$\vdots$

If $K=5$, $P(E) = \frac{{}_8C_5}{{}_{10}C_5} = \frac{2}{9} \approx 0.\bar{2} \nless 0.1$

If $K=6$, $P(E) = \frac{{}_8C_6}{{}_{10}C_6} = \frac{2}{15} \approx 0.1\bar{3} \nless 0.1$

If $K=7$. $P(E) = \frac{{}_8C_7}{{}_{10}C_7} = \frac{8}{120} \approx 0.0\bar{6} < 0.1$

So the minimum size of the committee required is 7.