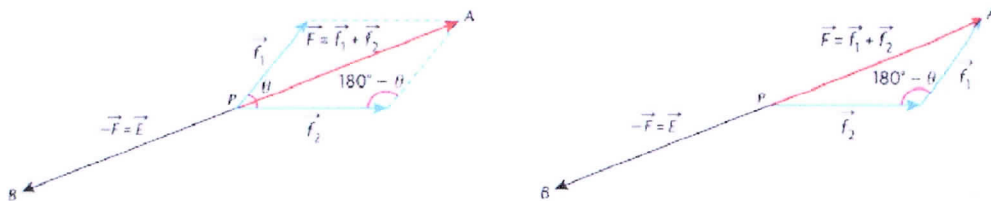


## Vectors as Forces

- When two or more forces are applied to an object, the net effect of the forces can be represented by the resultant vector determined by adding the vectors that represent each of the forces. For example,

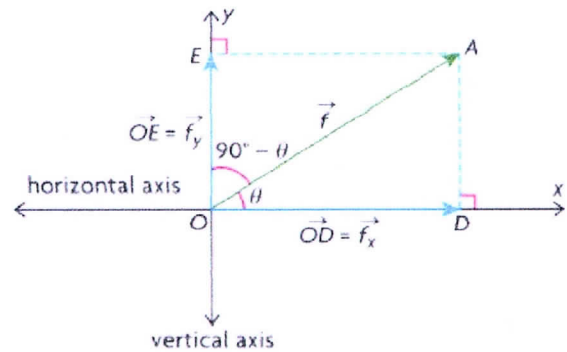
$\vec{F} = \vec{F}_1 + \vec{F}_2$  is the resultant of  $\vec{F}_1$  and  $\vec{F}_2$ .

Resultant and Equilibrant of the Force Vectors  $\vec{f}_1$  and  $\vec{f}_2$



- A system is in a state of equilibrium when the net effect of all the forces acting on an object causes no movement of the object. For example, if  $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ , then  $\vec{a}$ ,  $\vec{b}$ , and  $\vec{c}$  are in a state of equilibrium. Also note that  $-\vec{F} = -(\vec{F}_1 + \vec{F}_2)$  is the equilibrant of  $\vec{F}_1$  and  $\vec{F}_2$ .

- If the vector  $\vec{f}$  is resolved into its respective horizontal and vertical components,  $\vec{f}_x$  and  $\vec{f}_y$ , then  $|\vec{f}_x| = |\vec{f}| \cos \theta$  and  $|\vec{f}_y| = |\vec{f}| \sin \theta$ , where  $\theta$  is the angle that  $\vec{f}$  makes with the x-axis.



**Example 1** Two draft horses pull a load. The chains between the horses and the load are at an angle of  $60^\circ$  to each other. One horse pulls with a force of 230 N, the other with a force of 340 N.

- What is the resultant force on the load? In what direction is this force?
- What is the equilibrant force on the load? In what direction is this force?

**Example 2** A large promotional balloon is tethered to the top of a building by two guy wires attached at points 20 m apart. If the buoyant force on the balloon is 850 N, and the two guy wires make angles of  $58^\circ$  and  $66^\circ$  with the horizontal, find the tension in each of the wires.

**Example 3** A lawn mower is pushed with a force of 90 N directed along the handle, which makes an angle of  $36^\circ$  with the ground.

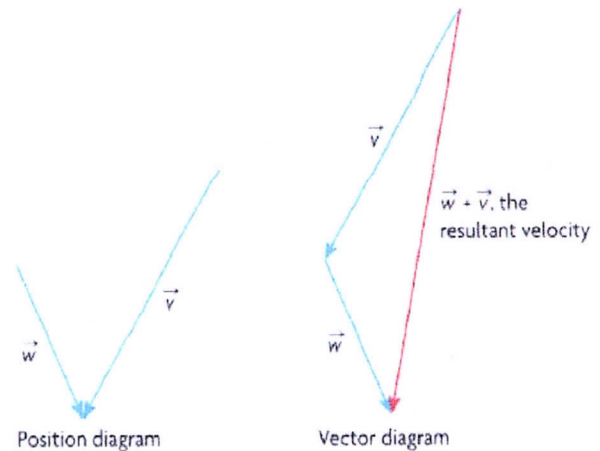
- Determine the horizontal and vertical components of the force on the mower.
- Describe the physical consequences of each component of the pushing force.

**Example 4** A 20-kg trunk is resting on a ramp inclined at an angle of  $15^\circ$ . Calculate the components of the force of gravity on the trunk that are parallel and perpendicular to the ramp. Describe the physical consequences of each. (Note that at the Earth's surface, gravity causes objects to accelerate at a rate of approximately  $9.8 \text{ m/s}^2$  as they fall.)

# Velocity

- The velocity of an object is stated relative to a frame of reference. Air speed/water speed is the speed of a plane/boat relative to a person on board. Ground speed is the speed of a plane or boat relative to a person on the ground and includes the effect of wind or current.
- When the velocity of a moving object is influenced by external forces, such as wind or the current of a river, the resultant velocity is determined by adding the vectors that represent the object in motion and the effect of the external force:

$$\vec{v}_r = \vec{v}_{object} + \vec{v}_{external\ force}$$



**Example 1** A canoeist who can paddle at a speed of 5 km/h in still water wishes to cross a river 400 m wide that has a current of 2 km/h. If he steers the canoe in a direction perpendicular to the current, determine the resultant velocity. Find the point on the opposite bank where the canoe touches.

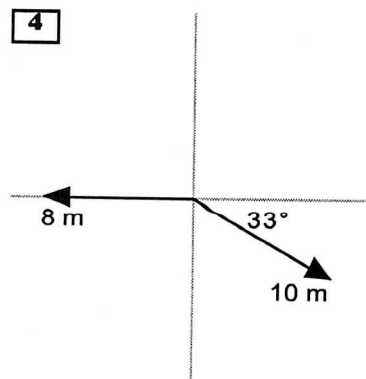
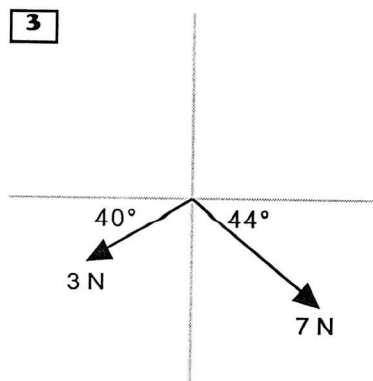
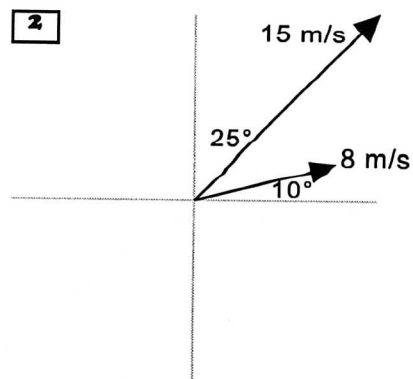
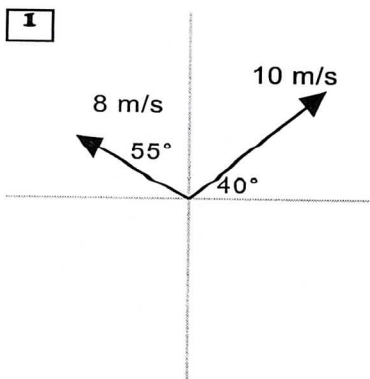
**Example 2** Suppose the canoeist of Example 1 had wished to travel straight across the river. Determine the direction in which he must head and the time it will take him to cross the river.

**Example 3** An airplane heading northwest at 500 km/h encounters a wind of 120 km/h from  $N65^\circ E$ . Determine the resultant ground velocity of the plane.

**Example 4** A pilot wishes to fly 400 km north. Due to a wind, the pilot actually flies 430 km in the direction  $N5^\circ E$ . How far, and in what direction, must the pilot now fly in order to achieve the proper displacement?

VECTORS WORKSHEETS

Adding by Vector Components



VECTORS WORKSHEETS

Adding by Vector Components

