Lesson 9

Techniques for Integration.

1) Variable Substitution.

If $u = h(x) \Rightarrow x = h^{-1}(u)$

then $\frac{du}{dx} = h'(x) \Rightarrow du = h'(x)\,dx$

$\Rightarrow dx = \frac{1}{h'(x)}\,du = \frac{1}{h'(h^{-1}(u))}\,du$

Now, $\int f(u(x))\,dx$

usually. $\int f(x)\,dx = F(x)$, $F'(x) = f(x)$ where x is the integration variable, $f(x)$ is integrand.

For integrand is a composition function, $f(u(x))$, we need variable substitution.

Since $dx = \frac{1}{u'(x)}\,du$

so $\int f(u(x))\,dx = \int f(u) \cdot \frac{1}{u'(x)}\,du$

$= \int f(u) \frac{1}{u'(u^{-1}(u))}\,du = \int g(u)\,du = G(u) + C$

$= G(u(x)) + C$

where $G'(u) = g(u)$;

2) Integration by Parts:

By the product rule,

$$\frac{d[f(x)g(x)]}{dx} = g(x)\frac{df(x)}{dx} + f(x)\frac{dg(x)}{dx}$$

$$\Rightarrow d[f(x)g(x)] = g(x)\,df(x) + f(x)\,dg(x)$$

$$\Rightarrow \cancel{\int}\cancel{d}[f(x)g(x)] = \int g(x)\,df(x) + \int f(x)\,dg(x)$$

$$\Rightarrow f(x)g(x) = \int g(x)\,df(x) + \int f(x)\,dg(x)$$

$$\Rightarrow \int f(x)\,dg(x) = f(x)g(x) - \int g(x)\,df(x); \quad (1)$$

$$\Rightarrow \int f(x)g'(x)\,dx = f(x)g(x) - \int g(x)f'(x)\,dx, \quad (2)$$

$$\cancel{\int}\cancel{d}x = x + c$$