AP Calculus Homework Two – Limit and Continuity
1.4 Other Basic Limits; 1.5 Asymptotes

1. Use the Sandwich Theorem and the fact that $\lim_{x \to 0} (\,|x|+1\,) = 1$ to prove that $\lim_{x \to 0} (\,x^2 + 1\,) = 1$.

2. Find limits.

(a) $\lim_{x \to -\infty} \dfrac{5x^3 + 27}{20x^2 + 10x + 9}$

(b) $\lim_{x \to \infty} \dfrac{2^{-x}}{2^x}$

(c) $\lim_{x \to 0} \dfrac{4x^2 + 3x \sin x}{x^2}$

(d) $\lim_{t \to 0} \dfrac{1 - \cos t}{t^{2/3}}$

(e) $\lim_{x \to +\infty} \left(1 + \dfrac{2}{x}\right)^x$

(f) $\displaystyle\lim_{x\to+\infty} \frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x+1}}$

(g) $\displaystyle\lim_{x\to+\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}} - \sqrt{x} \right)$

3. Find a value of k such that $g(x)$ is continuous at $x = 0$.
$$g(x) = \begin{cases} \ln(x+k), & \text{if } 0 < x < 3 \\ \cos(kx), & \text{if } x \le 0 \end{cases}$$

4. Find all asymptotes for the graph of $f(x) = \dfrac{2x^2 + 4}{2 + 7x - 4x^2}$.

5. Find all vertical and horizontal asymptotes for the graph of $h(x) = \dfrac{e^{-x}}{x}$.

6. For what vales of k will $\displaystyle\lim_{x\to 3} \frac{x-3}{x^2 - 6x + k}$ exist?

7. Show that $f(x) = \dfrac{x^2 - 5}{x+1}$ has a root between $x = 2$ and $x = 3$.