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Test 1 /47

Show your work!

Time: 75 minutes

1. Consider the following two polynomial functions (4 marks)

$P(x) = x^3 - 2x^2 + 3x - 1$ and $Q(x) = x^2 - 2x$.

Compute the required operations:

a. $P(x) + Q(x)$

$x^3 + 2x^2 + 3x - 1 + x^2 - 2x$

$= x^3 + 3x^2 + x - 1$

b. $P(x)Q(x)$

$(x^3 + 2x^2 + 3x - 1)(x^2 - 2x)$

$= x^5 + 2x^4 + 3x^3 - x^2 - 2x^4 - 4x^3 - 6x^2 + 2x$

$= x^5 - x^3 - 7x^2 + 2x$

c. Use the long division algorithm to get the quotient and the remainder for $\frac{P(x)}{Q(x)}$

$$\begin{array}{r|l} & x \\ x^2 - 2x & x^3 - 2x^2 + 3x - 1 \\ & x^3 - 2x^2 \\ \hline & 0 + 3x - 1 \end{array}$$

$P(x) = x(x^2 - 2x) + 3x - 1$

2. Use the remainder theorem to find if $2x + 1$ is a factor of $P(x) = x^3 - 2x^2 + 2x - 3$ (2 marks)

By remainder theorem:

rem $= P(r)$ where $(x-r)$ is a factor

rem $= P(-\frac{1}{2}) = -\frac{37}{8}$

3. Solve each of the following. (12 marks)

$6x^3 - 19x^2 + x + 6 = 0$

root: 3

$$\begin{array}{r|rrrr} 3 & 6 & -19 & 1 & 6 \\ & & 18 & -3 & -6 \\ \hline & 6 & -1 & -2 & 0 \end{array}$$

$= (x-3)(6x^2 - x - 2)$

$= (x-3)(2x+1)(3x-2)$

$x = 3, -\frac{1}{2}, \frac{2}{3}$

$x^4 - 12x^2 + 12 = 0$

let $w = x^2$

$w^2 - 12w + 12 = 0$

$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{12 \pm \sqrt{(12)^2 - 4(12)}}{2}$

$x^2 = 6 \pm 2\sqrt{6}$

$x^2 = 6 - 2\sqrt{6}$

$x = \pm 1.049$

$x^2 = 6 + 2\sqrt{6}$, $x = \pm 3.301$

$\frac{-3x+6}{4x-8} = -\frac{3}{4}$ $x \neq 2$

$-12x + 24 = 12x - 24$

$48 = 24x$

$x = 2$

No Solutions

$\frac{x+2}{3} + \frac{x-3}{3x-12} = \frac{1}{3x-12}$ $x \neq 4$

$\frac{x+2}{\cancel{3}} + \frac{x-3}{\cancel{3}(x-4)} = \frac{1}{\cancel{3}(x-4)}$

$(x+2)(x-4) + x - 3 = 1$

$x^2 - 4x + 2x - 8 + x - 4 = 0$

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$x = \cancel{4}, -3$

$(x-1)(2x-8)(x-3)^2 \le 0$ + − − + 0 1 3 4 $f(0) = 72$ $f(2) = -4$ $f(3.5) = -\frac{5}{8}$ $f(5) = 32$ $x \in [1,4]$	$\frac{x+3}{x^2-4} > 0$ $(x+3)(x^2-4) > 0$ $(x+3)(x+2)(x-2) > 0$ − + − + -3 -2 2 $f(-4) = -\frac{1}{12}$ $f(-2.5) = \frac{2}{9}$ $x \in (-3,-2) \cup (2, \infty)$ $f(-1) = -\frac{2}{3}$ $f(3) = \frac{6}{5}$

4. If $f(x) = mx^3 + gx^2 - x + 3$ is divided by $x + 1$, the remainder is 3. If $f(x)$ is divided by $x + 2$, the remainder is -7. What are the values of m and g? (3 marks)

by remainder theorem:

$f(-1) = 3$, $f(-2) = -7$

$m(-1)^3 + g(-1)^2 + 1 = 3$

$m(-2)^3 + g(-2)^2 + 2 = -7$ ②

$-m + g + 1 = 3$

$g = m + 2$ ①

Sub ① → ②:

$m(-2)^3 + 4(m+2) + 2 = -7$

$8m + 4m + 8 + 2 = -7$

$m = -\frac{17}{12}$, $g = \frac{7}{12}$

5. (3, 5) is a point on the graph of $f(x)$. Find the corresponding point on the graph of the function $y = 3f(-x + 1) + 2$. (2 marks)

$y = 3f(-(x-1)) + 2$

- Shift right 1   (4, 5)
- Mirror along y axis   (-4, 5)
- vertical Stretch by 3   (-4, 15)
- shift up 2   (-4, 17)

6. (a) Find a polynomial P(x) of degree five with zeros 1, -1, 2, -2, and 3 such that its graph passes through the point (0, 24). (3 marks)

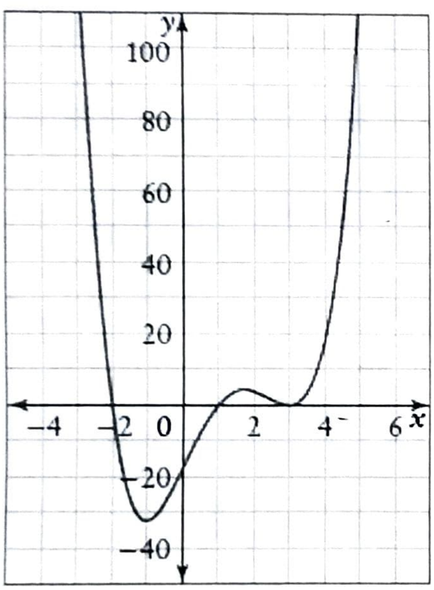
$P(x) = a(x-1)(x+1)(x-2)(x+2)(x-3)$

$24 = a(0-1)(0+1)(0-2)(0+2)(0-3)$

$a = -2$

$P(x) = -2(x-1)(x+1)(x-2)(x+2)(x-3)$

(b) Find a possible equation for the graph bellow. (3 marks)



$f(x) = a(x+2)(x-1)(x-3)^2$

(c) Find a cubic polynomial function f(x) that has two zeros at 1 and 2, and also f(0) = 1 and f(-1) = 4. (3 marks)

$f(x) = a(x-1)(x-2)(x-k)$

$1 = a(-1)(-2)(-k)$

$4 = a(-2)(-3)(-1-k)$

$f(x) = -\frac{1}{6}(x-1)(x-2)(x-3)$

$\frac{1}{2} = -ak$ ①

$\frac{2}{3} = a(-1-k)$

$\frac{2}{3} = -a - ak$ ②

$k = 3$

Sub ① → ②

$\frac{2}{3} = -a + \frac{1}{2}$

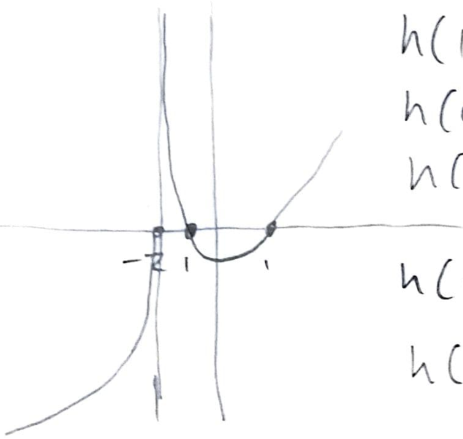
$a = -\frac{1}{6}$

(8 marks)

7. Sketch the graph of the following functions. Specify the x- and y-intercepts and the asymptotes.

a. $h(x) = \dfrac{x^2-1}{x+2} = \dfrac{(x-1)(x+1)}{x+2}$

b. $h(x) = \dfrac{x^4 - 2x^3 - 3x^2 + 4x + 4}{x^2 - x - 2}$



$h(10) = 8.25$

$h(0) = -\frac{1}{2}$

$h(-1.5) = \frac{5}{2}$

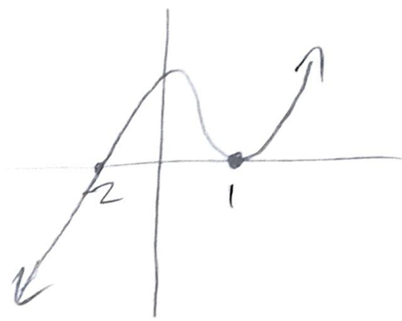
$h(-3) = -8$

$h(-10) = -12.3$

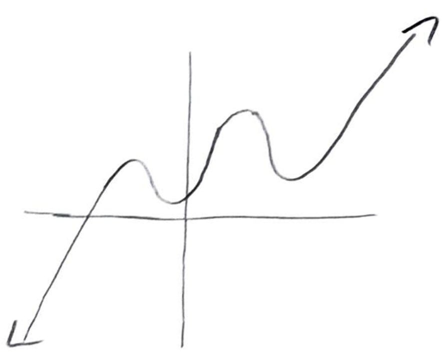
(4 marks)

8. Sketch a possible graph of a polynomial function that satisfies the following conditions:

a. A quintic function with a positive leading coefficient, a zero at $x=-2$, and a second zero at $x=1$ of multiplicity 4.



b. Degree five, positive leading coefficient, one x-intercept, four turning points



(3 marks)

9. Find constants a and b that guarantee that the graph of the function defined by

$f(x) = \frac{ax^2+6}{16-bx^2}$ will have vertical asymptotes of $x = \frac{2}{3}$ and $x = -\frac{2}{3}$, and a horizontal asymptote of

$y = -2$.

Bonus question (3 marks)

1. An *Indicator function*, *f*(*x*), is a function that takes the value 1 if some condition on *x* is true, and the value 0 if the condition is not true. For integers *x*, define the indicator functions:

f(x) = $f(x) = \begin{cases} 1, & if\ x\ is\ divisible\ by\ 2 \\ 0, & if\ x\ is\ not\ divisible\ by\ 2 \end{cases}$

and

g(x) = $g(x) = \begin{cases} 1, & if\ x\ is\ divisible\ by\ 3 \\ 0, & if\ x\ is\ not\ divisible\ by\ 3 \end{cases}$.

a. Find *f*(2), *f*(9), *f*(2017), *g*(2), *g*(9), *g*(2017).
b. Let *h*(*x*) = *f*(*x*)*g*(*x*). Determine whether *h*(*x*) is an indicator function, and, if so, describe in words the condition it indicates.
c. Let *k*(*x*) = $\left[\frac{f(x)+g(x)+1}{2}\right]$. Determine whether *k*(*x*) is an indicator function, and, if so, describe in words the condition it indicates.(where $[x]$ represents the integer part of x)

2. Graph the following relations $|x - y| \leq 2$. (3 marks)

$x - y \leq 2$ $x - y \geq 2$