HW 2 is way too long!!!!! SO DO NOT DO

Q3 (bottom 2 graphs)
Q4 (c,d,e,f)
Q5 (c,d,e)
Q7(c,d,e)
Q8
Q10(d,e)
Q11 (bottom 2)
Q13

} Skip

Advanced Functions Class 2 Homework

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Polynomial Functions

"limit notation"

1. Consider the following polynomial functions.

a. $y=-2x^3+4x-5$

b. $f(x)=5x^4+2x^3-4x^2+x-7$

c. $g(x)=x^5+2x^3-5x+8$

For each one, perform the following tasks.

i. Describe the end behavior of the function.

ii. Determine the maximum and minimum number of turning points.

iii. Determine the maximum and minimum number of x-intercepts.

a) i) $a<0$, odd degree

$x \to \infty,\ y \to -\infty$

$x \to -\infty,\ y \to \infty$

ii) max: 2 min: 0

iii) min: 1 max: 3

b) i) $a>0$, even degree

$x \to \infty,\ y \to \infty$

$x \to -\infty,\ y \to \infty$

ii) max: 3 min: 1

iii) min: 0 max: 4

odd degree always has root, even degree always has turning point

c) i) $a>0$, odd degree

$x \to \infty,\ y \to \infty$

$x \to -\infty,\ y \to -\infty$

ii) max: 4 min 0

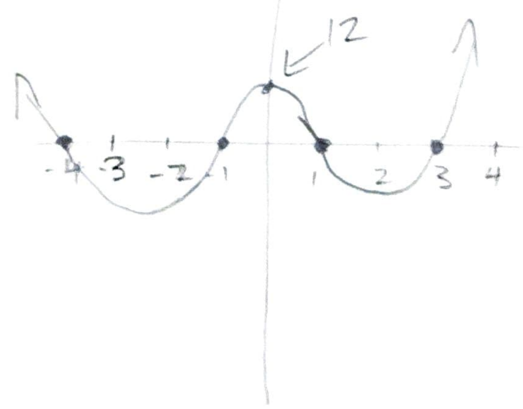
iii) max: 5 min: 1

2. Sketch a possible graph of each function by identifying the end behaviours and determining the x - and y -intercepts of the function.

a. $f(x)=(x-1)(x-3)(x+1)(x+4)$

b. $y=-2x^3-3x^2+9x$

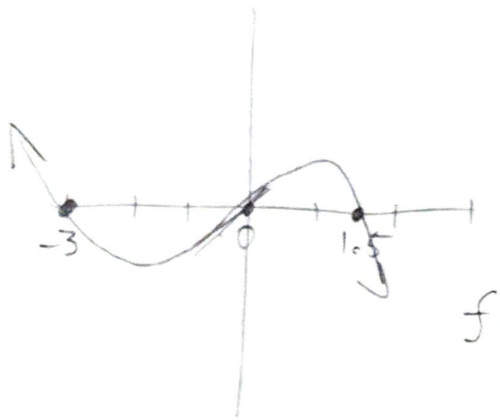
a)



b) $y=-x(2x^2+3x-9)$

$y=-x(2x-3)(x+3)$

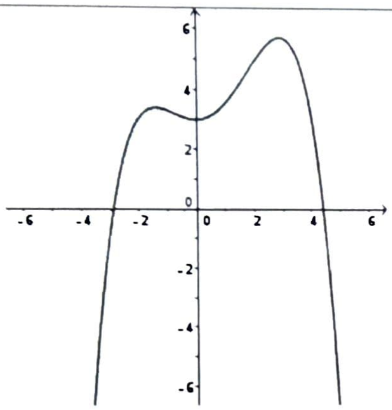
$=-2x(x-\frac{3}{2})(x+3)$



$f(4)>0$

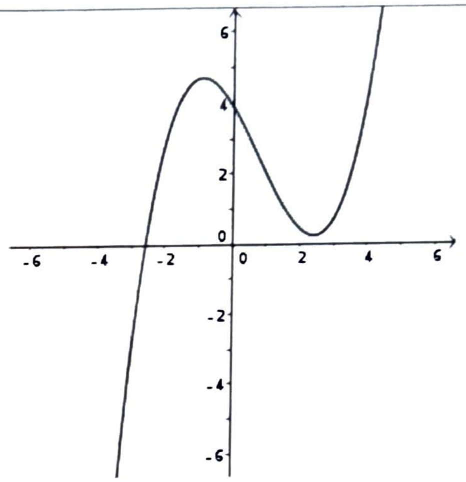
$f(2)=-10$ $f(0)=12$

3. Given the graph of the polynomial function *y=f(x)*, identify the minimum possible degree of the function and the sign of the leading coefficient.



Sign: −

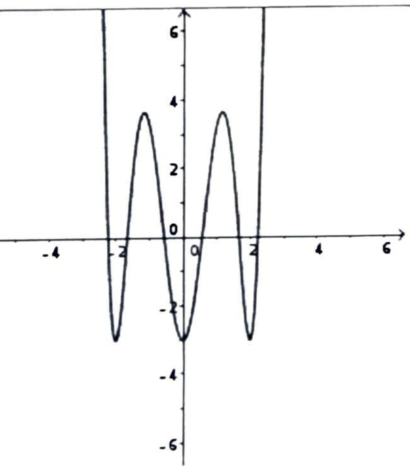
min: 4



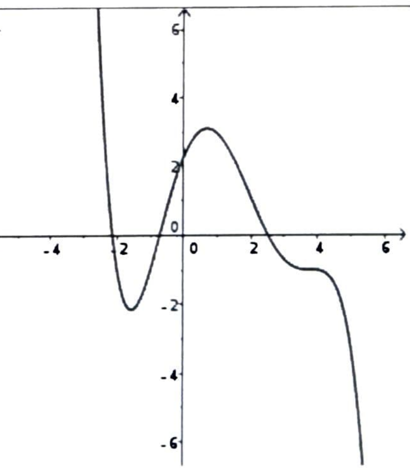
Sign: +

min: 3

X



X



4. Sketch a graph of a polynomial function that satisfies each set of conditions.

a. Degree three, two distinct x -intercepts, two turning points, and end behavior such that $y \rightarrow \infty$ as $x \rightarrow -\infty$ and $y \rightarrow -\infty$ as $x \rightarrow \infty$

b. Degree four, two distinct x -intercepts, three turning points, and end behavior such that $y \rightarrow \infty$ as $x \rightarrow \pm\infty$

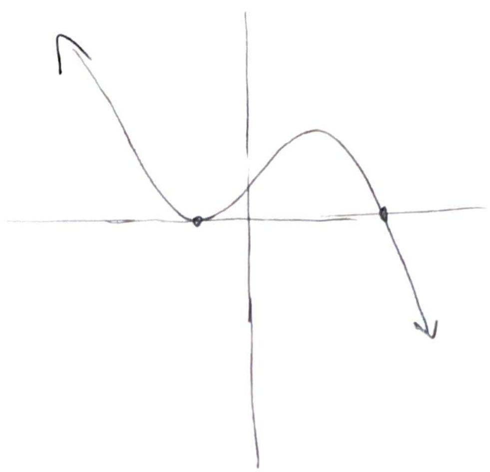
~~c.~~ Degree four, negative leading coefficient, three distinct x -intercepts, three turning points

~~d.~~ Degree three, positive leading coefficient, one x -intercept, two turning points

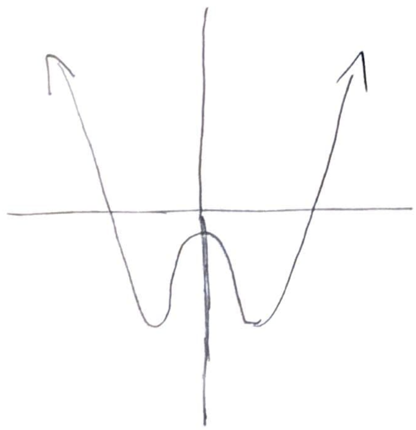
~~e.~~ Degree five, negative leading coefficient, two distinct x -intercepts, two turning points

~~f.~~ Degree five, positive leading coefficient, one x -intercept, four turning points

a)



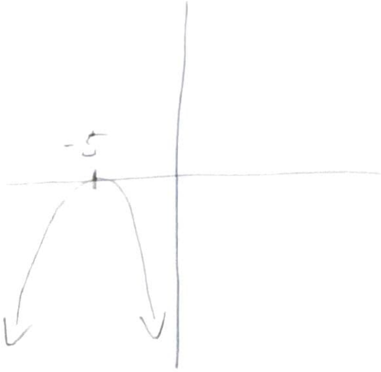
b)



5. Sketch a possible graph of a polynomial function that satisfies the following conditions.

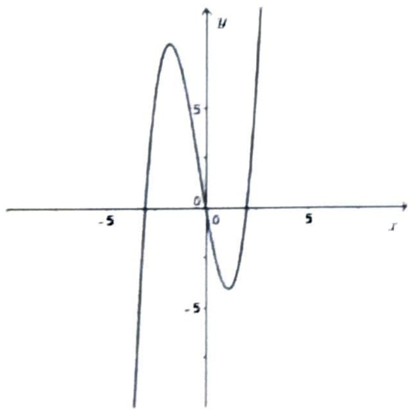
a. A quadratic function with a negative leading coefficient and a zero at $x=-5$ of multiplicity 2.
b. A 5th degree function with a positive leading coefficient, a zero at the origin of order 2, and a zero at $x=3$ of order 3.
c. A quartic function with a positive leading coefficient and two real zeros, $x=0$ and $x=3$ of order 2.
d. A cubic function with a negative leading coefficient and only one zero at $x=4$ and two non-real zeros.
e. A quintic function with a positive leading coefficient, a zero at $x=-2$, and a second zero at $x=1$ of multiplicity 4.

a)

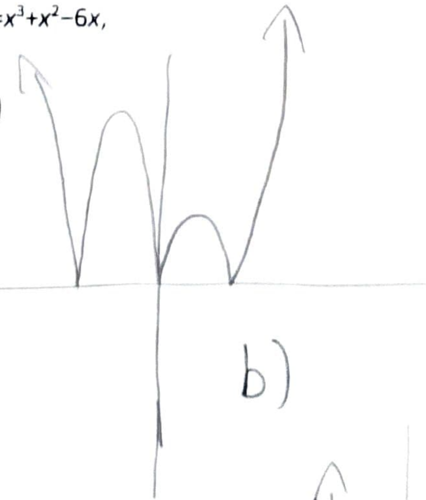


b)

6. Given the graph of the polynomial function $f(x)=x^3+x^2-6x$,



a)



b)

Sketch the graph of

a. $y=|f(x)|$

b. $y=f(|x|)$

7. Sketch a possible graph for each of the following functions.

a. $y=-x(x+2)\,(2x-5)$

b. $f(x)=2(x-2)^2(x+3)^2$

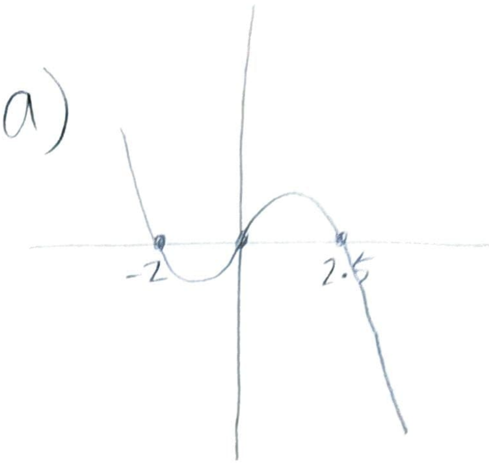
~~c.~~ $g(x)=-0.5(x-3)\,(x+1)^3$

~~d.~~ $y=2x^2(x-4)^3$

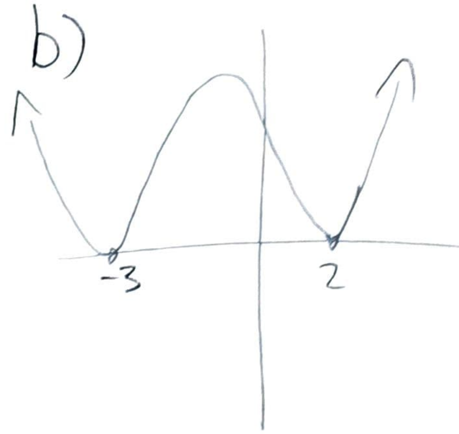
~~e.~~ $f(x)=-x(2x+3)\,(x-2)^2$

Solution:

a)



b)



~~8.~~ Sketch a possible graph for each function.

a. $f(x)=-2x^3+8x$
b. $f(x)=-x^4-5x^3-6x^2$
c. $f(x)=x^4-2x^2+1$

9. A family of quintic functions has a zero at $x=-3$ and turning points tangent to the x -axis at $x=1$ and 4.

a. State the general equation of the family.
b. State the equations of two members of the family that have end behaviour $y \to \infty$ as $x \to -\infty$ and $y \to -\infty$ as $x \to \infty$.

a) $f(x) = a(x+3)(x-1)^2(x-4)^2 \quad a \neq 0$

b) $y \to \infty$ as $x \to -\infty$: $f(x) \in a < 0$
$y \to -\infty$ as $x \to \infty$: $f(x) \in a < 0$

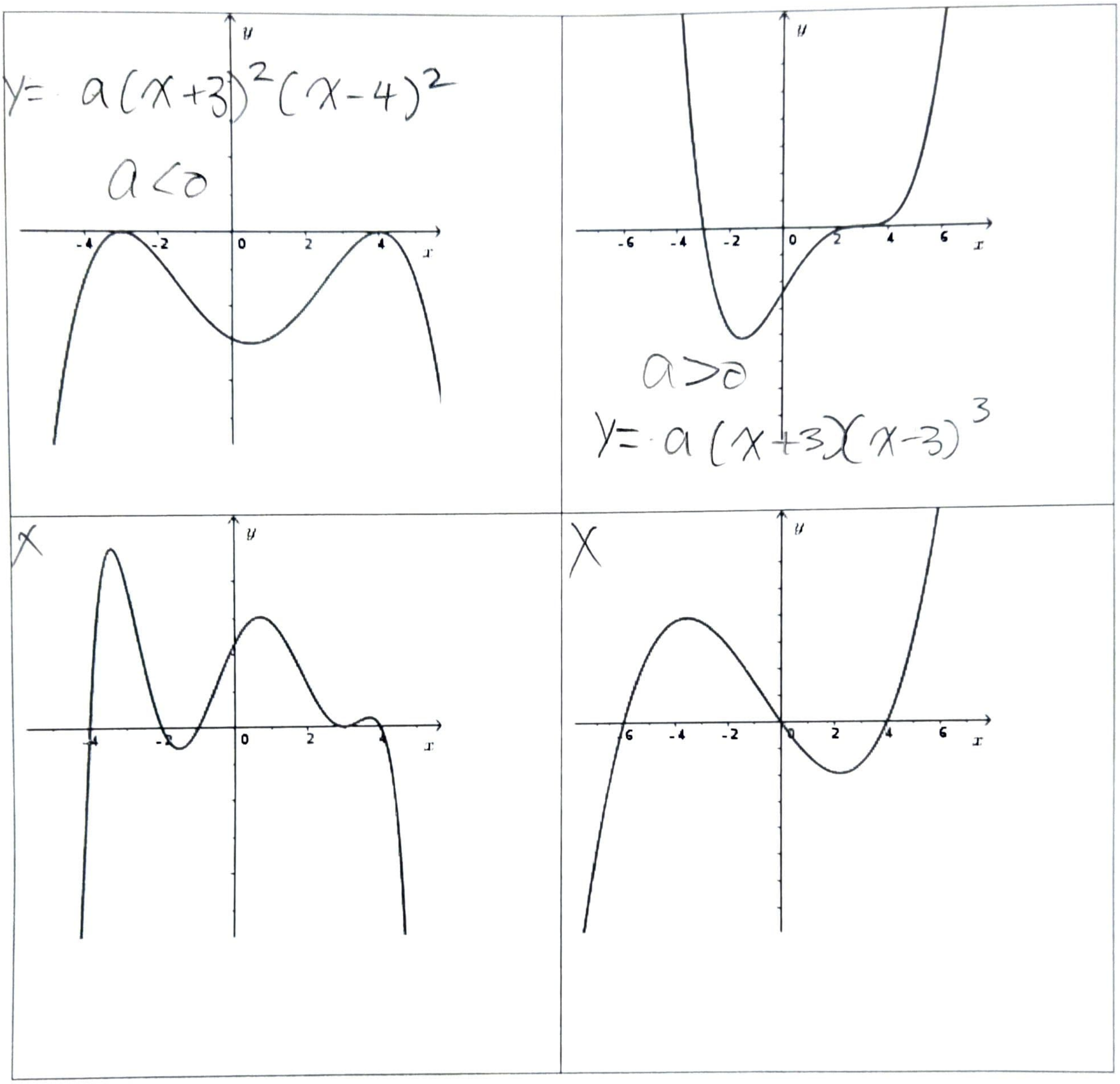
10. State the equation of the family of polynomial functions satisfying the following conditions:

a) A cubic with zeros $x=-3$, $x=-\frac{1}{2}$, and $x=\frac{5}{3}$.
b) A sixth degree function with zeros $x=-2$ (order 2), $x=1$ (order 1), and $x=5$ (order 3).
c) A quartic that passes through the origin and has a point of inflection at $(\frac{2}{3},0)$.
d) Cubic function, x -intercept at $x=-4$, a turning point at $(1,0)$, and $f(x) \to -\infty$ as $x \to \infty$.
e) A quartic function with zeros at $x=\pm\sqrt{5}$ and $x=-1\pm\sqrt{2}$.

a) $f(x) = a(x+3)(x+\frac{1}{2})(x-\frac{5}{3})$
b) $f(x) = a(x+2)^2(x-1)(x-5)^3$
c)

$f(x) = a(x-\frac{3}{2})^3 x$

11. Given the graph of $y=f(x)$, determine a general equation for a family of polynomials with the same end behaviour and zeros of $f(x)$ (note: all zeros are integer in value).



$y = a(x+3)^2(x-4)^2$

$a<0$

$a>0$

$y = a(x+3)(x-3)^3$

X

X

12. State the equation of the quartic function with zeros $x=-\frac{1}{2}$ and 5 (both of multiplicity 1) and $x=2$ (multiplicity 2), having a y-intercept of 4.

$h(x)=(x+\frac{1}{2})(x-5)(x-2)^2 \cdot k$

$h(0)=4 \Rightarrow (0+\frac{1}{2})(0-5)(0-2)^2 \cdot k=4$

$k=-\frac{2}{5}$

$f(x)=-\frac{2}{5}(x+\frac{1}{2})(x-5)(x-2)^2$

13. Find the general equation of the family of

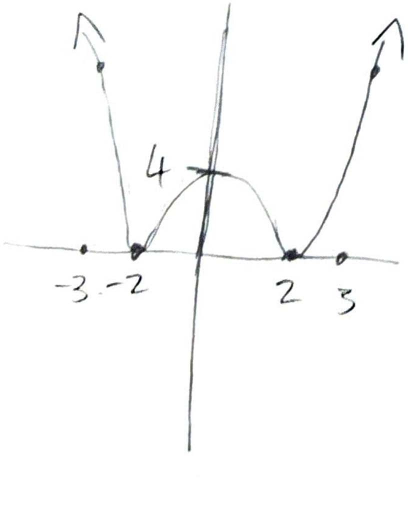
a) quadratic functions with zeros $-3-\sqrt{5}$ and $-3+\sqrt{5}$.
b) cubic functions with zeros $0, 1-2\sqrt{3}$ and $1+2\sqrt{3}$.
c) quartic functions with zeros $-2, 1$ and $\pm 3i$.
d) quartic functions with zeros $3\pm\sqrt{2}$ and $-4\pm i\sqrt{3}$

14. Determine the equation of the quartic function with rational coefficients, zeros $4-\sqrt{2}$ and $-3+\sqrt{6}$, and a y-intercept of -21.

$$h(x) = a(x-4+\sqrt{2})(x-4-\sqrt{2})(x+3-\sqrt{6})(x+3+\sqrt{6})$$
$$= a((x-4)^2-2)((x+3)^2-6)$$
$$h(0) = -21 \Rightarrow a = -\frac{1}{2}$$
$$f(x) = -\frac{1}{2}([x-4]^2-2)([x+3]^2-6)$$

15. The function $f(x)=\frac{1}{4}(x-2)^2(x+2)^2$ has a turning point at $(0,4)$. Determine

a) The intervals where $f(x)$ is positive and negative.
b) The intervals where $f(x)$ is increasing and decreasing



Negative: $x \in \emptyset$

a) Positive:
$x \in (-\infty,-2) \cup (-2,2) \cup (2,\infty)$

b) increasing:
$x \in (-2,0) \cup (2,\infty)$
decreasing
$x \in (-\infty,-2) \cup (0,2)$

f(x)	Y
-3	$\frac{25}{4}$
-2	0
0	4
2	0
3	$\frac{25}{4}$