∵ $x \to -2^-$ ∴ $x < -2 \Rightarrow x+2 < 0$

$\lim_{x \to a} f(x) = L$ where L is a real number constant.

$\arctan x = \tan^{-1} x$ Inverse of $\tan x$.

AP Calculus Homework One – Limit and Continuity

1.1 Definitions of Limits; 1.2 Continuity; 1.3 Limits Properties

1. Show that limits do not exist.

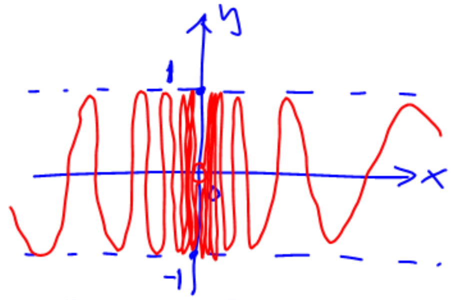
(a) $\lim_{x\to -2} \frac{x+2}{|x+2|}$

∵ $\lim_{x\to -2^-} \frac{x+2}{|x+2|} = \lim_{x\to -2^-} \frac{\cancel{x+2}}{-(\cancel{x+2})} = \lim_{x\to -2^-} (-1) = -1$;

$\lim_{x\to -2^+} \frac{x+2}{|x+2|} = \lim_{x\to -2^+} \frac{\cancel{x+2}}{\cancel{x+2}} = \lim_{x\to -2^+} (1) = 1$;

∴ $\lim_{x\to -2} \frac{x+2}{|x+2|}$ D.N.E.

(b) $\lim_{x\to 0} \sin\frac{1}{x}$



The value of $\sin\frac{1}{x}$ is oscillating between -1 and 1 as x is approaching to 0, therefore, $\lim_{x\to 0} \sin\frac{1}{x}$ D.N.E.

(c) $\lim_{x\to 0} \sqrt{3+\arctan\frac{1}{x}}$

∵ $\lim_{x\to 0^-} \sqrt{3+\arctan\frac{1}{x}} = \sqrt{3+\arctan(\frac{1}{0^-})} = \sqrt{3+\arctan(-\infty)} = \sqrt{3-\frac{\pi}{2}}$;

and $\lim_{x\to 0^+} \sqrt{3+\arctan\frac{1}{x}} = \sqrt{3+\arctan(\frac{1}{0^+})} = \sqrt{3+\arctan(+\infty)} = \sqrt{3+\frac{\pi}{2}}$; ∴ $\lim_{x\to 0} \sqrt{3+\arctan\frac{1}{x}}$ D.N.E.

2. Find limits.

(a) $\lim_{x\to 0} \frac{x^2}{2x-1} = \frac{0^2}{2(0)-1} = \frac{0}{0-1} = \frac{0}{-1} = 0$;

(b) $\lim_{x\to 2} \frac{x^3-8}{x^2-4} = \lim_{x\to 2} \frac{(\cancel{x-2})(x^2+2x+4)}{(\cancel{x-2})(x+2)} = \frac{2^2+2(2)+4}{2+2} = \frac{12}{4} = 3$;

(c) $\lim_{x\to -1} \frac{2+2/x}{x^2-4x-5} = \lim_{x\to -1} \frac{\frac{2x+2}{x}}{(x+1)(x-5)} = \lim_{x\to -1} \frac{2(\cancel{x+1})}{(\cancel{x+1})(x-5)x} = \frac{2}{(-1-5)(-1)} = +\frac{1}{3}$;

$\lim_{x\to 1} \frac{x+1}{x-1} = \frac{1+1}{1-1} = \frac{2}{0} = \pm\infty$

D.N.E.

(d) $\lim_{h\to 0} \frac{5(h-1)^2+(h-1)-4}{h} = \lim_{h\to 0} \frac{5(h^2-2h+1)+h-5}{h} = \lim_{h\to 0} \frac{5h^2-10h+\cancel{5}+h-\cancel{5}}{h}$

$= \lim_{h\to 0} \frac{\cancel{h}(5h-9)}{\cancel{h}} = 5(0)-9 = -9$;

(e) Explain, using examples, when substitution can not be used to solve a limit.

when using the Quotient Law, if $\frac{0}{0}$ occurs, i.e. $\lim_{x\to 1} \frac{x-1}{x^2-1} = \lim_{x\to 1} \frac{\cancel{x-1}}{(\cancel{x-1})(x+1)} = \frac{1}{1+1} = \frac{1}{2}$;
need to cancel out zero factor, before substitution.

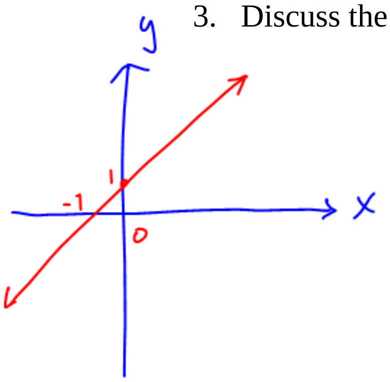
when evaluating the limit of roots, i.e. $\lim_{x\to 2} \sqrt{1-x} \neq \sqrt{\lim_{x\to 2}(1-x)} = \sqrt{1-2} = \sqrt{-1}$. D.N.E.
No complex number is acceptable.

$\lim_{x \to c} f(x) = f(c)$

$f(x) = x+1$, for $x \in R$

$2 \times 2 \times 2 \times 2 \times 2 = 2^5$

3. Discuss the continuity and sketch the graph of $f(x) = \begin{cases} \dfrac{x^2 + x}{x}, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$. $= \begin{cases} x+1, & \text{if } x \neq 0, \\ 1, & \text{if } x = 0. \end{cases}$



$\because f(0) = 1$, and

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (x+1) = 0^- + 1 = 1$;

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (x+1) = 0^+ + 1 = 1$;

$\therefore \lim_{x \to 0} f(x) = 1 = f(0)$

Therefore $f(x)$ is continuous at $x=0$, as well as all other real numbers of x.

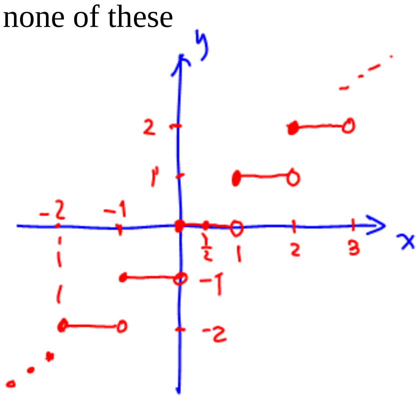
4. If $[x]$ is the greatest integer not greater than x, then $\lim_{x \to \frac{1}{2}} [x]$ is

(A) 1/2 (B) 1 (C) nonexistent (D) 0 (E) none of these

$[\frac{1}{2}] = 0$.

$\because \lim_{x \to \frac{1}{2}} [x] = 0$

$\therefore$ (D)



5. Find a value of k such that $f(x)$ is continuous at $x = 0$.

$$f(x) = \begin{cases} \dfrac{x^2 - x}{2x}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases} = \begin{cases} \dfrac{x-1}{2}, & \text{if } x \neq 0; \\ k, & \text{if } x = 0; \end{cases}$$

$\because \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{x-1}{2} = -1/2$;

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{x-1}{2} = -\frac{1}{2}$;

$\therefore \lim_{x \to 0} f(x) = -\frac{1}{2}$; $\therefore k = -\frac{1}{2}$.

Since $\lim_{x \to 0} f(x) = f(0) = -\frac{1}{2}$; then $f(x)$ is continuous at $x=0$.

6. The function $s(x)$ is defined as follows. Find a value of k such that $s(x)$ is continuous for all x.

$$s(x) = \begin{cases} 4x - 11, & \text{if } x < 3 \\ kx^2, & \text{if } x \geq 3 \end{cases}$$

$\because \lim_{x \to 3^-} S(x) = \lim_{x \to 3^-} (4x - 11) = 4(3) - 11 = 1$;

$\because \lim_{x \to 3^+} S(x) = \lim_{x \to 3^+} (kx^2) = k(3)^2 = 9k$;

and $f(3) = k(3)^2 = 9k$, too;

$\therefore$ let $9k = 1 \Rightarrow k = \frac{1}{9}$.

7. Discuss the continuity of the graph of $y = \dfrac{x^2 - 9}{3x - 9}$, indicating type of discontinuity if there is one.

Obviously, $f(3)$ is undefined, and $\lim_{x \to 3} \frac{x^2-9}{3x-9} = \lim_{x \to 3} \frac{\cancel{(x-3)}(x+3)}{3\cancel{(x-3)}} = \lim_{x \to 3} \frac{x+3}{3} = \frac{3+3}{3} = 2$;

Therefore, $f(x) = \frac{x^2-9}{3x-9}$ has a removable discontinuity at $x = 3$. (a hole)

and $f(x)$ is continuous elsewhere for other $x \in R$.