1.3 Permutations with Identical Items.

P242 Ex. 1.

DOLE , DOLL , LOLL.

Permutation of the letters of DOLE :

$4! = 4P_4 = 4 \times 3 \times 2 \times 1 = 24$ (ways)

Permutation of the letters of DOLL:

Since no arrangements are needed for two identical Ls.

So $\frac{4!}{2!} = \frac{24}{2} = 12$ (ways)

Permutation of the letters of LOLL:

Since no arrangements are needed for three identical Ls.

So $\frac{4!}{3!} = \frac{24}{6} = 4$ (ways).

LOLL, OLLL, LLOL, LLLO

Generally, if n items consist of k kinds,

and there are n_1 items of the 1^{st} kind,

n_2 items of the 2^{nd} kind,

$\vdots \qquad \vdots$

n_k items of the k^{th} kind,

where $n_1 + n_2 + \cdots + n_k = n$.

Then, the total permutation of the n items

is $\dfrac{n!}{n_1!\, n_2! \cdots n_k!}$.

P243. Ex. 3.

bookkeeper.

$n = 10$; $k = 6$;

$n_1 = 1$, $n_2 = 2$, $n_3 = 2$, $n_4 = 3$, $n_5 = 1$, $n_6 = 1$

The permutation of the letters of "bookkeeper" is:

$$\frac{10!}{1!\,2!\,2!\,3!\,1!\,1!} = 151200 \text{ (ways)}$$

1.4 Pascal's Triangle.

1	--- Row 0	--- $t_{0,0}$
1 1	----- Row 1	--- $t_{1,0}$ $t_{1,1}$
1 2 1	---- Row 2	--- $t_{2,0}$ $t_{2,1}$ $t_{2,2}$
1 3 3 1	---- Row 3	--- $t_{3,0}$ $t_{3,1}$ $t_{3,2}$ $t_{3,3}$
1 4 6 4 1	---- Row 4	--- $t_{4,0}$ $t_{4,1}$ $t_{4,2}$ $t_{4,3}$ $t_{4,4}$
$\vdots$	$\vdots$	$\vdots$

Generally, in Row n (the $n+1$ th row).

There are $n+1$ numbers:

$t_{n,0}, t_{n,1}, t_{n,2}, \cdots, \underline{t_{n,r}}, \cdots, t_{n,n}$.

Properties of Pascal's Triangle.

1) Recursive Relation (or Pascal's formula)

$$t_{n,r} = t_{n-1,r-1} + t_{n-1,r}$$

where $n = 0, 1, 2, 3, \cdots$;
$r = 1, 2, 3, \cdots, n-1$;

$t_{n,0} = t_{n,n} = 1$;

i.e. $t_{12,5} = t_{11,4} + t_{11,5}$

2) Symmetric Relation:

$$t_{n,r} = t_{n,n-r}; \quad \text{where } r = 0,1,2,\cdots,n; \quad n = 0,1,2,3,\cdots;$$

i.e. $t_{7,2} = t_{7,7-2} = t_{7,5}$

3) Each number is a Combination

$$t_{n,r} = nCr$$

Therefore, $nCr = nC_{n-r}$ as well.

i.e. $5C_2 = 5C_{5-2} = 5C_3$

$$5C_2 = \frac{5P_2}{2!} = \frac{5\times4}{2} = 10.$$

$$5C_3 = \frac{5P_3}{3!} = \frac{5\times4\times3}{3\times2\times1} = 10.$$

4) Sum of Row n is 2^n.

$$t_{n,0} + t_{n,1} + t_{n,2} + \cdots + t_{n,n} = 2^n;$$

so $nC_0 + nC_1 + nC_2 + \cdots + nC_n = 2^n;$

If $n=0$, $t_{0,0} = 1 = 2^0$

If $n=1$, $t_{1,0} + t_{1,1} = 1+1 = 2 = 2^1$

If $n=2$, $t_{2,0} + t_{2,1} + t_{2,2} = 1+2+1 = 4 = 2^2$

If $n=3$, $t_{3,0} + t_{3,1} + t_{3,2} + t_{3,3} = 1+3+3+1 = 8 = 2^3$

$\vdots$ $\vdots$

5) Containing Triangular Number Sequence.

Let $\{a_n\}$ be the triangular number sequence,

then $a_1 = 1 \qquad\qquad = t_{2,2}$

$a_2 = 1+2 = 3 \qquad = t_{3,2}$

$a_3 = 1+2+3 = 6 \qquad = t_{4,2}$

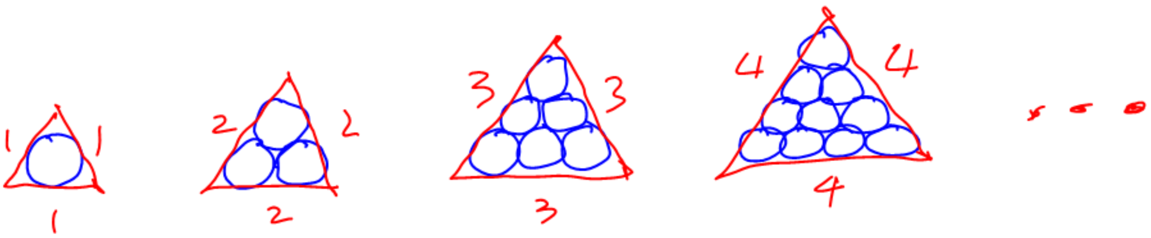
$a_4 = 1+2+3+4 = 10 \quad = t_{5,2}$

$\vdots$

$$a_n = 1+2+3+\cdots+n = \frac{n(1+n)}{2} = t_{n+1,2}$$

$\vdots \qquad\qquad\qquad \vdots$

i.e. $a_{10} = 1+2+3+\cdots+10 = \dfrac{10(1+10)}{2} = 5\times 11 = 55$



1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1