Properties of Limits

- If f is a polynomial function, then $\lim\limits_{x \to a} f(x) = f(a)$.
- Substituting $x = a$ into $\lim\limits_{x \to a} f(x)$ can yield the indeterminate form $\frac{0}{0}$. If this happens, you may be able to find an equivalent function that is the same as the function f for all values except at $x = a$. Then, substitution can be used to find the limit.
- To evaluate a limit algebraically, you can use the following techniques:
  - direct substitution
  - factoring
  - rationalizing
  - one-sided limits
  - change of variable

Example 1 Evaluate each limit using any appropriate technique.

a) $\lim\limits_{x \to 5} \dfrac{3x^2 - 14x - 5}{x^2 - 25}$

b) $\lim\limits_{x \to 0} \dfrac{\sqrt{x+16} - 4}{x}$

c) $\lim\limits_{x \to 3} \dfrac{\frac{1}{x} - \frac{1}{3}}{x - 3}$

d) $\lim\limits_{x \to -6} \dfrac{|x+6|}{x^2 + 2x - 24}$

e) $\lim\limits_{x \to -1} \dfrac{\sqrt[3]{x} + 1}{x + 1}$

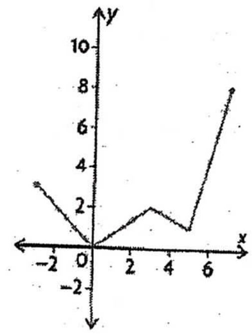
Continuity

- A function is continuous at $x = a$ if $\lim_{x \to a} f(x) = f(a)$. In other words, the values

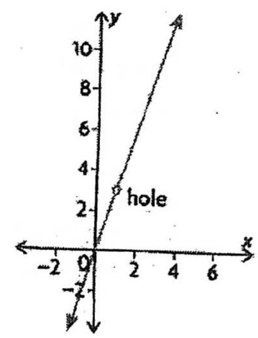
  - $\lim_{x \to a^+} f(x)$
  - $\lim_{x \to a^-} f(x)$
  - $f(a)$

  exist and are equal.

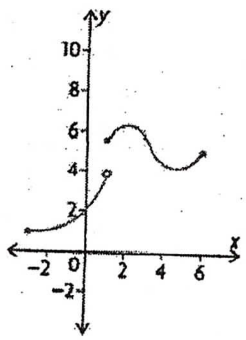
A. Continuous for all values of the domain



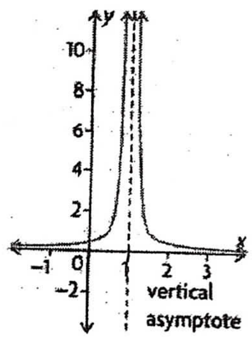
B. Discontinuous at $x = 1$ (point discontinuity)



C. Discontinuous at $x = 1$ (jump discontinuity)



D. Discontinuous at $x = 1$ (infinite discontinuity)



- A function that is not continuous has some type of break in its graph. This break is the result of a hole, jump, or vertical asymptote.

- All polynomial functions are continuous for all real numbers.
- A rational function $h(x) = \dfrac{f(x)}{g(x)}$ is continuous at $x = a$ if $g(a) \neq 0$.
- A rational function is discontinuous at the zeroes of the denominator.

Example 1 Sketch the function $f(x)=\begin{cases} x, & if\ x\leq -4 \\ 1, & if\ -4<x<2 \\ (x-4)^2-3, & if\ x>2 \end{cases}$

Determine whether the function is continuous or discontinuous.
If the function is not continuous, state the value(s) of x where there is a discontinuity.
Justify your answers.

Example 2 Determine all the values of x for which each function is continuous.

a) $f(x)=4^{-x}$

b) $g(x)=\sqrt{36-x^2}$

c) $h(x)=\dfrac{1}{\sqrt{2x+1}}$

d) $k(x)=\dfrac{x-2}{x^2+3x-10}$

Example 3 The cost of parking a car in an underground parking lot is \$3 for the first half hour (or part of a half hour), \$2.50 for the second half hour (or part), and \$2 for each additional hour (or part) up to a daily maximum of \$20.

a) Sketch a graph to represent this situation. What type of function is represented by this graph?

b) Where is the graph discontinuous? What type of discontinuity does the graph have?

Example 4 Let $f(x)=\begin{cases} a+x^2, & \text{if } x\le -3 \\ x-2a, & \text{if } x>-3 \end{cases}$

Determine the value of a that makes the function continuous.

The Greatest Integer Function

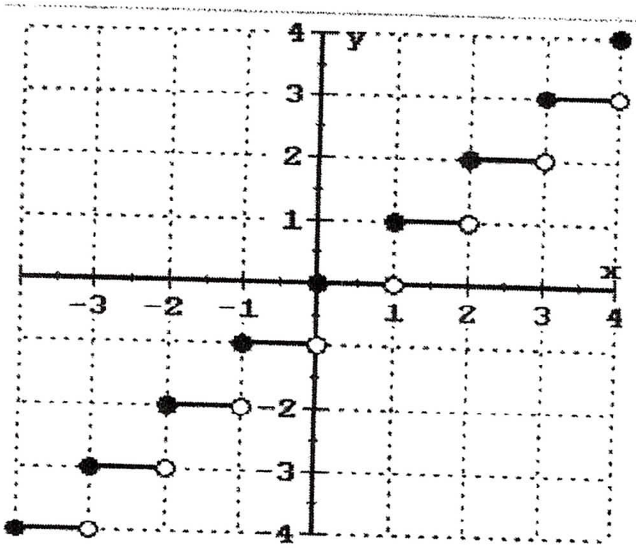
$y = [x]$

Definition: For each real number x, the greatest integer function is defined to be the largest integer that is less than or equal to x.

For example,

$[1.9] = 1$ $\quad [2] = 2$ $\quad [4.999] = 4$

$[4.0001] = 4$ $\quad [0.95] = 0$ $\quad [-2.5] = -3$



Find the following limits:

a) $\lim_{x\to 1^-}[x] =$ b) $\lim_{x\to 1^+}[x] =$ c) $\lim_{x\to 1}[x] =$

d) $\lim_{x\to -2^-}[x] =$ e) $\lim_{x\to -2^+}[x] =$ f) $\lim_{x\to -2}[x] =$

g) $\lim_{x\to 2.5^-}[x] =$ h) $\lim_{x\to 2.5^+}[x] =$ i) $\lim_{x\to 2.5}[x] =$

j) $\lim_{x\to n^-}[x] =$ k) $\lim_{x\to n^+}[x] =$ l) $\lim_{x\to n}[x] =$

Limits as $x \to \pm\infty$

Find the limits.

1. $\lim\limits_{x\to\infty} \frac{x}{2x+1}$

2. $\lim\limits_{x\to\infty} \frac{x^2-3x+1}{3x^2-2x}$

3. $\lim\limits_{x\to-\infty} \frac{x}{\sqrt{7x^2+1}}$

4. $\lim\limits_{x\to\infty} \frac{3x+2}{x-1}$

5. $\lim\limits_{x\to-\infty} \frac{2x^2-x+5}{5x^2+6x-1}$

6. $\lim\limits_{x\to\infty} \frac{2x+7}{x^2-x}$

Supplemental Exercises

Find the following limits:

1. $\lim_{x \to 3} x^2 + 2x - 7 =$

2. $\lim_{x \to 5} \dfrac{x^2 - 2x - 15}{x - 5} =$

3. $\lim_{x \to 1} \dfrac{4x^4 - 5x^2 + 1}{x^2 + 2x - 3} =$

4. $\lim_{x \to 2} \dfrac{(2x+1)^2 - 25}{x - 2} =$

5. $\lim_{x \to 1} \dfrac{\dfrac{2x}{x+1} - 1}{x - 1} =$

6. $\lim_{x \to -2} \dfrac{x^4 - 2x^2 - 8}{x^2 - x - 6} =$

7. $\lim_{x \to 0} \dfrac{x^2 + 7x + 6}{x + 3} =$

8. $\lim_{x \to 2} \dfrac{x^3 + x^2 - 4x - 4}{x^2 + x - 6} =$

9. $\lim_{x \to -1} \dfrac{\dfrac{1}{x} + 1}{x + 1} =$

10. $\lim_{x \to 0} \dfrac{(x+1)^2 - 1}{x} =$

11. $\lim_{x \to -3} \dfrac{2x^2 + 2x - 12}{x^2 + 4x + 3} =$

12. $\lim_{x \to 2} \dfrac{(3x-2)^2 - (x+2)^2}{x - 2} =$

13. $\lim_{x \to 2} \dfrac{\dfrac{2}{x^2} - \dfrac{1}{2}}{x - 2} =$

14. $\lim_{x \to 1} \dfrac{x - 1}{\sqrt{x} - 1} =$

15. $\lim_{x \to -2} \dfrac{\dfrac{x}{x+4} + 1}{x + 2} =$

16. $\lim_{x \to 3} \dfrac{x^2 - 2x - 3}{x + 5} =$

ANSWERS

1.	8	5.	1/2	9.	−1	13.	−1/2
2.	8	6.	24/5	10.	2	14.	2
3.	3/2	7.	2	11.	5	15.	1
4.	20	8.	12/5	12.	16	16.	0