First Name:____________________ Last Name: __________________ Student ID: ________

Trigonometric Functions (1)

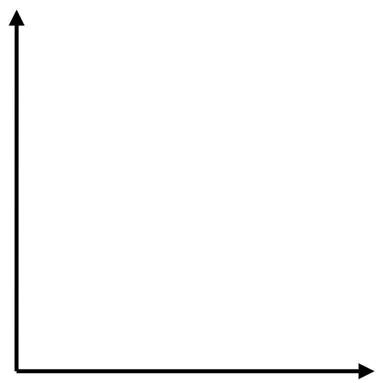
Measuring Angles

There are more ways of measuring angles:

__________________________, __________________________,

Measure each of the following angles in all three forms:

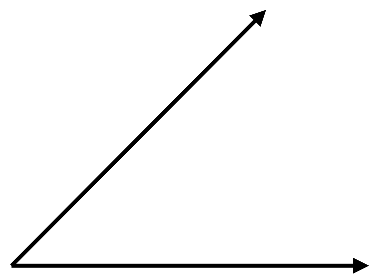
a)



D: _____________

R: _____________

b)

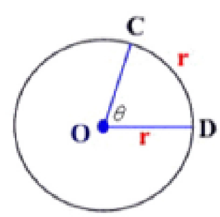


D: _____________

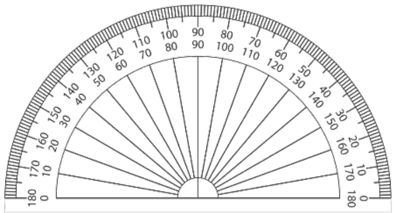
R: _____________

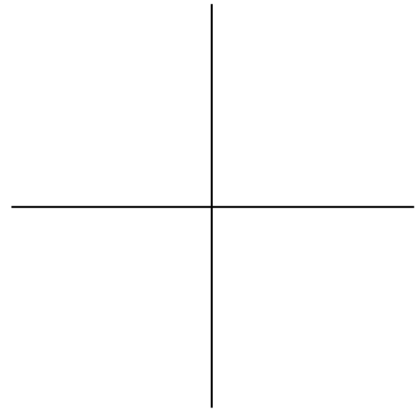
A radian is the measure of an angle θ that, when drawn as a central angle, subtends an arc whose length equals the length of the radius of the circle.

$$\theta = \frac{s}{r} = \frac{r}{r} = 1$$



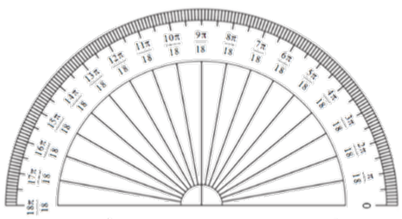
Degrees

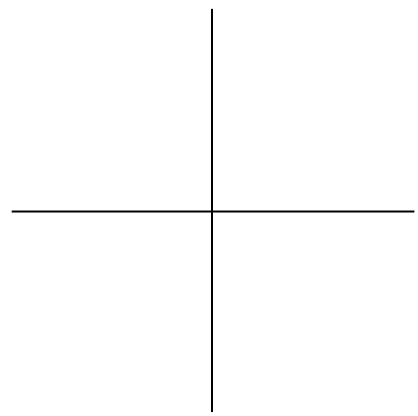




- One full turn equals ___________ degrees.
- Standard unit of measure for angles.

Radians





- One full turn equals ___________ radians.
- Mathematical unit of measure for angles.
- Measures arc length.

To change from degrees to radians, multiply by $\frac{\Pi}{180^\circ}$	To change from radians to degrees, multiply by $\frac{180^\circ}{\Pi}$

Convert each degree measure into radians and each radian measure into degrees.

1) $-\frac{4\pi}{3}$

2) $55°$

3) $135°$

4) $\frac{52\pi}{9}$

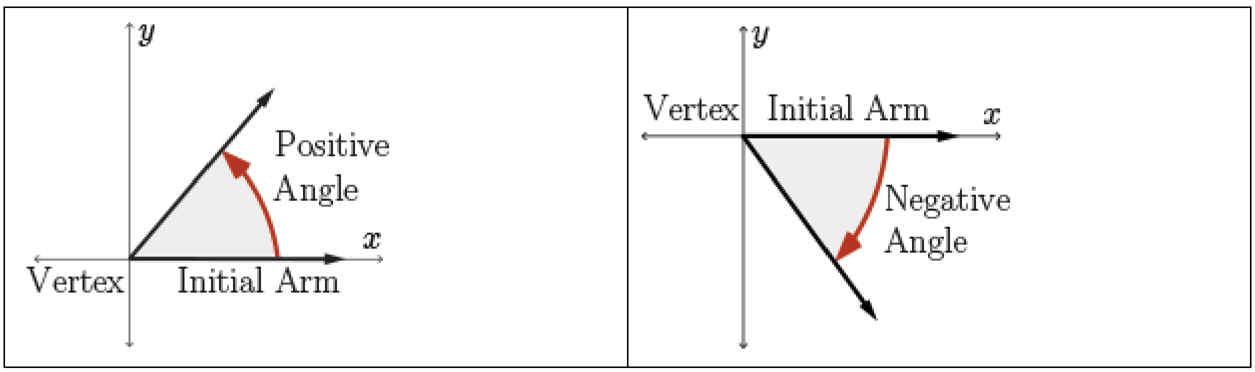
5) $\frac{23\pi}{18}$

6) $785°$

7) $-30°$

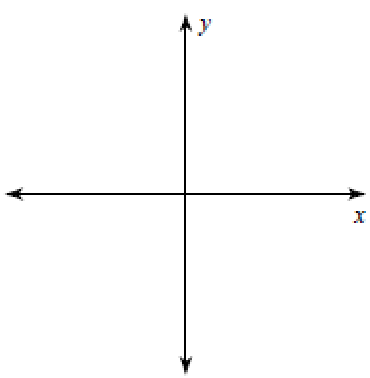
8) $-125°$

Angles in Standard Position

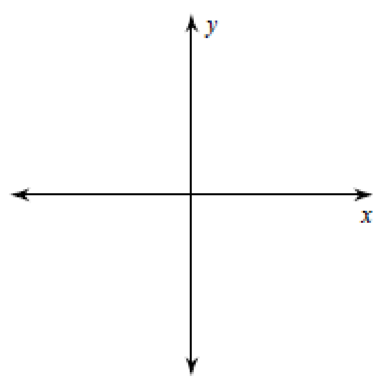


Draw an angle with the given measure in standard position.

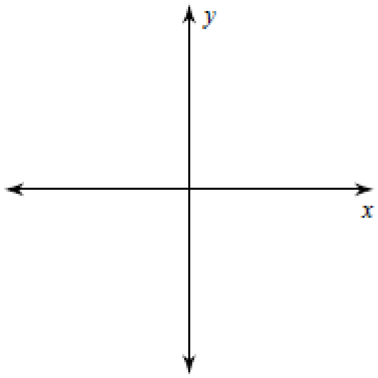
9) $100°$



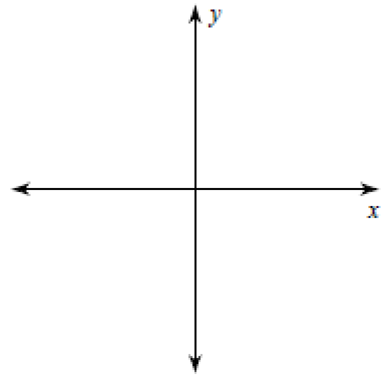
10) $-\frac{\pi}{6}$



11) $\frac{28\pi}{9}$

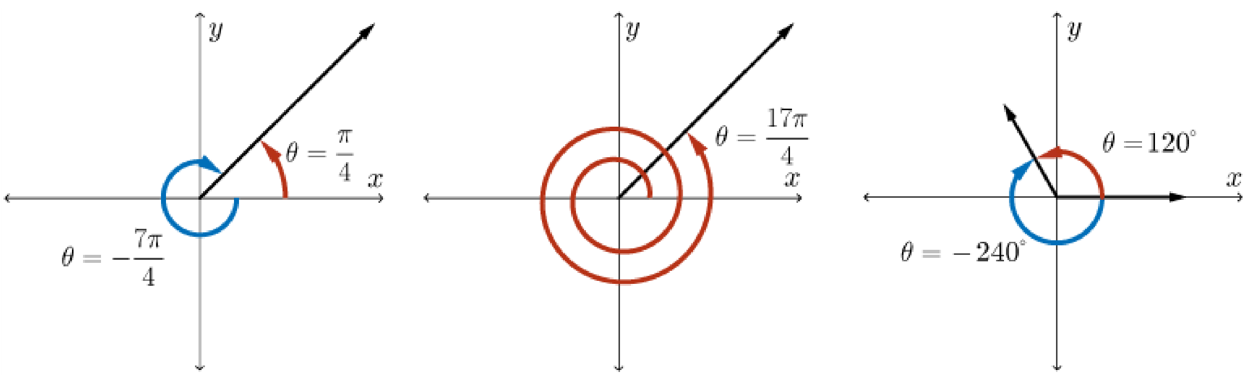


12) $-80°$



Coterminal Angles

Two angles in standard position are said to be **coterminal** if they share the same terminal arm.



Note: In general, two angles in standard position are coterminal if their difference is a non-zero integer multiple of 2π or 360^0.

Find a coterminal angle between 0 and 2π for each given angle.

13) $-\frac{11\pi}{9}$

14) $-\frac{41\pi}{36}$

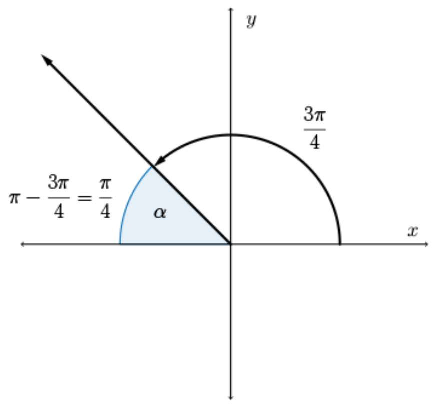
15) $\frac{9\pi}{4}$

16) $-\frac{11\pi}{36}$

17) $-\frac{17\pi}{36}$

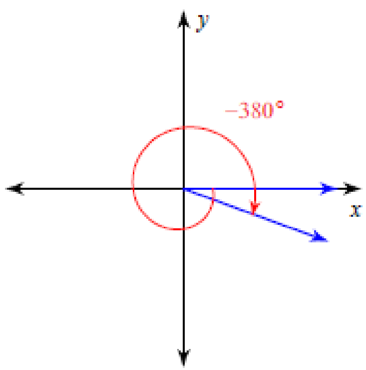
18) $\frac{37\pi}{18}$

Reference Angles

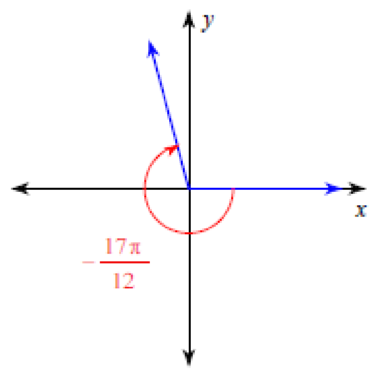
	A **reference angle** is the acute angle formed between the terminal arm of a standard position angle and the *x*-axis. A reference angle is also referred to as a related acute angle.

Find the reference angle.

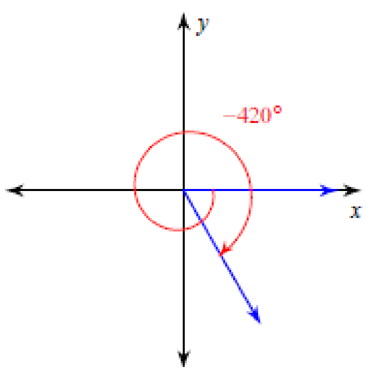
19)



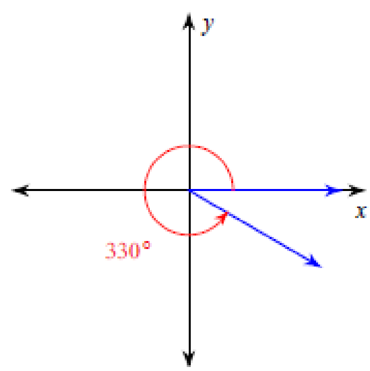
20)



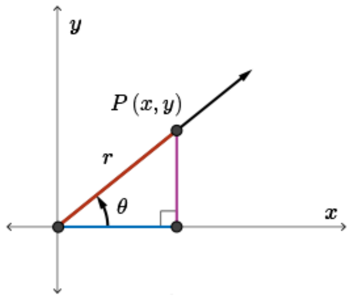
21)



22)



Relating Trigonometric Ratios and the Unit Circle

<table>
<tr>
<td>
</td>
<td>sin(θ) =

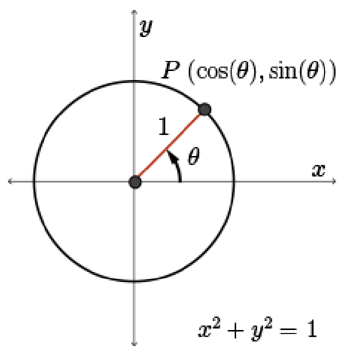
cos(θ) =

tan(θ)=</td>
</tr>
</table>

The reciprocal trigonometric ratios are defined as follows:

$\csc\theta = \frac{1}{\sin\theta}$ $\quad\quad$ $\sec\theta = \frac{1}{\cos\theta}$ $\quad\quad$ $\cot\theta = \frac{1}{\tan\theta}$

What is a unit circle?

<table>
<tr>
<td>
</td>
<td>sin(θ) =

cos(θ) =

tan(θ) =</td>
</tr>
</table>

23) Find sin(θ), cos(θ), tan(θ) if

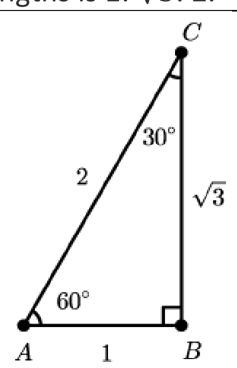
a. $\theta = \frac{\pi}{2}$ $\quad\quad$ b. $\theta = \frac{3\pi}{2}$ $\quad\quad$ c. $\theta = \pi$ $\quad\quad$ d. $\theta = 2\pi$ $\quad\quad$ e. $\theta = 0$

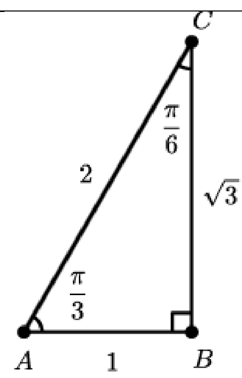
24) The point $P(-6,3)$ is on the terminal arm of an angle θ in standard position where $0 \leq \theta \leq 2\pi$. Determine the exact values of the six trigonometric ratios.

25) Point $P(-3/5,-4/5)$ is on the terminal arm of standard position angle θ. Determine the exact values of each of the six trigonometric ratios.

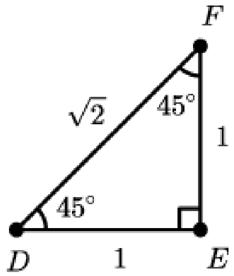
Special Triangles

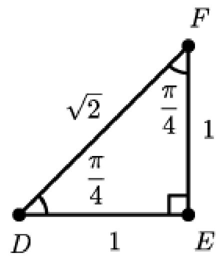
The first of the triangles is a **30^0-60^0-90^0** triangle. The ratio of the corresponding opposite side lengths is $1{:}\ \sqrt{3}{:}\ 2$.





The second triangle is a **45^0 -45^0 -90^0** triangle. The ratio of the corresponding opposite side lengths is $1{:}1{:}\sqrt{2}$.





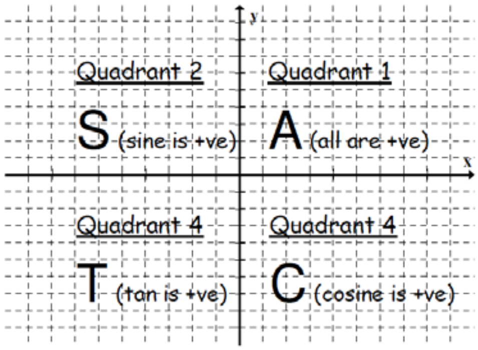
Using radian measure, the first triangle is a $\pi/6$ - $\pi/3$ - $\pi/2$ triangle and the second triangle is a $\pi/4$ - $\pi/4$ - $\pi/2$ triangle.

Determine the exact values of the six trigonometric ratios for each of the angles $\frac{\pi}{6}$, $\frac{\pi}{4}$, and $\frac{\pi}{3}$.

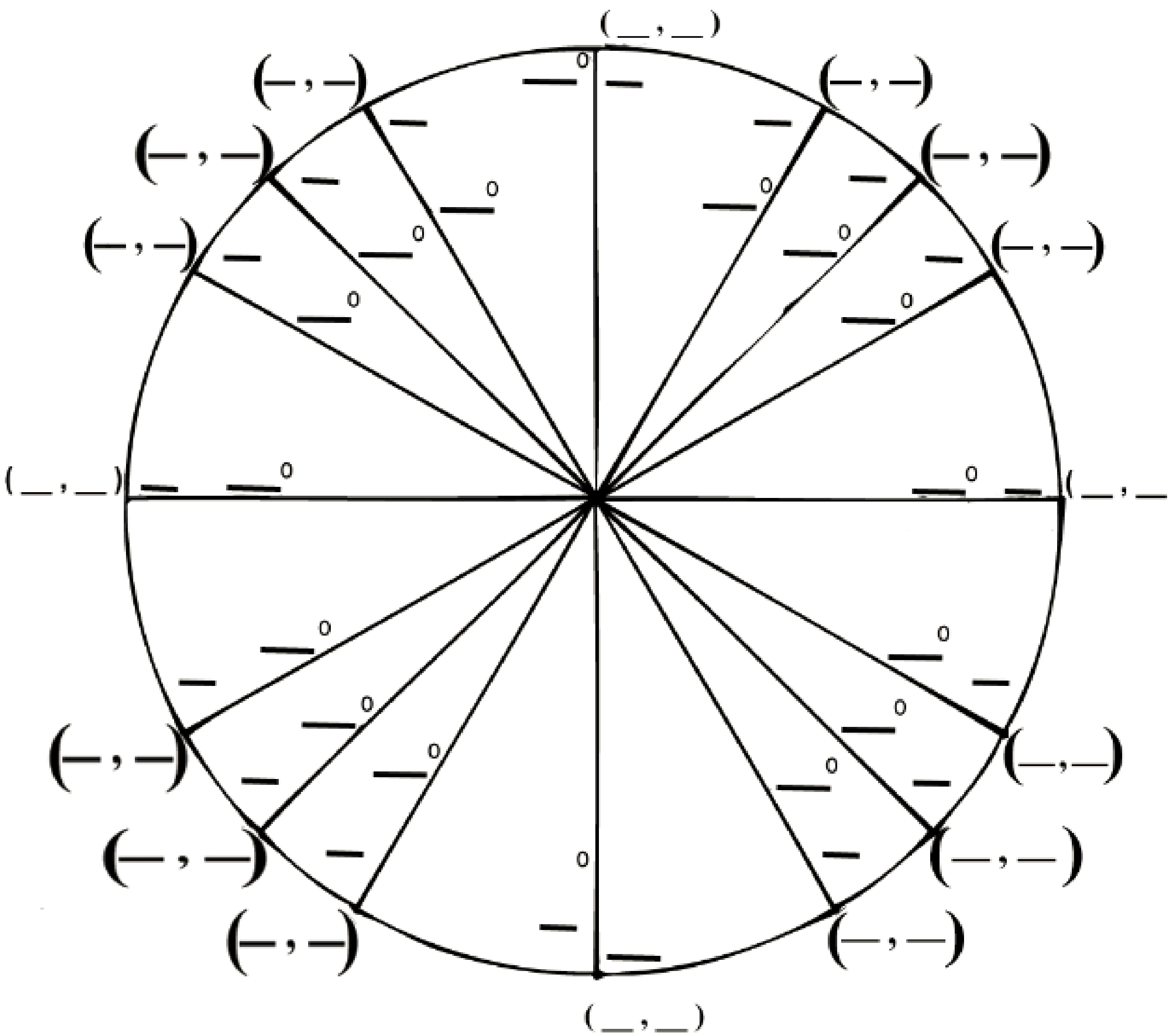
Angle(θ)	sin(θ)	tan(θ)	tan(θ)
$\frac{\pi}{6}$ or 30°			
$\frac{\pi}{3}$ or 60°			
$\frac{\pi}{4}$ or 45°			

26) A unit circle is shown with *OP* on the terminal arm of a $\pi/3$ radian standard position angle. Determine the coordinates of *P*.

Determining the Sign of a Trigonometric Ratio



Finding More Points on the Unit Circle



27) If $\cos(\theta) = -\frac{1}{4}$, determine the possible values of θ such that $-180^0 \le \theta \le 180^0$.

28) Determine the exact value of the following:

a. $\cos(300^0)$

b. $\tan(-\ 855^0)$

c. $\sec(\frac{17\pi}{3})$

d. $\tan(2018\pi)$

e. $\csc(\frac{17\pi}{2})$

f. $\cot(-\frac{25\pi}{4})$

29) If $\tan(\theta)= -\sqrt{3}$, determine the possible values of θ such that $0 \le \theta \le 2\pi$.

30) If $\sin(\theta) = -\frac{\sqrt{2}}{2}$, determine the possible values of θ such that $-\pi \le \theta \le 3\pi$.

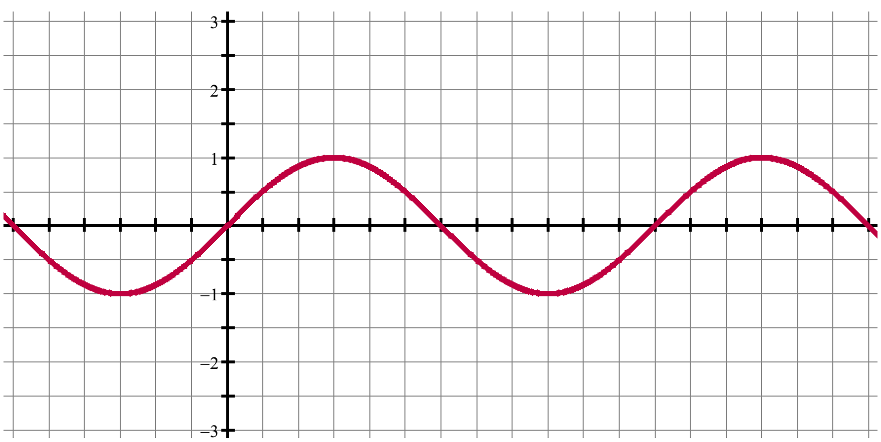
31) Determine the central angle subtended by an arc of length 10 cm with a radius of 4 cm.

32) For $\tan\theta = -\frac{5}{24}$, where $0^0 \leq \theta \leq 360^0$ determine the trig ratios in each possible quadrant.

Graphing $\sin x$, $\cos x$, and $\tan x$

For each of the following trigonometric graphs, identify the function, mark on the scale, and highlight one cycle of the graph. State the amplitude, period, max/min values, domain, range, and end behaviours.

Function: **y =** ___________________



Amplitude: __________

Period: __________

Maximum: __________

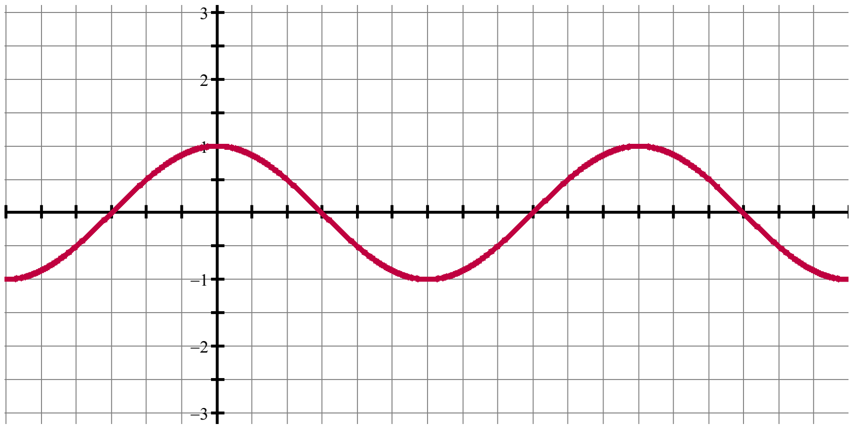
Minimum: __________

Domain: _____________________________

End Behaviour:

Range: _____________________________

Function: **y =** ___________________



Amplitude: __________

Period: __________

Maximum: __________

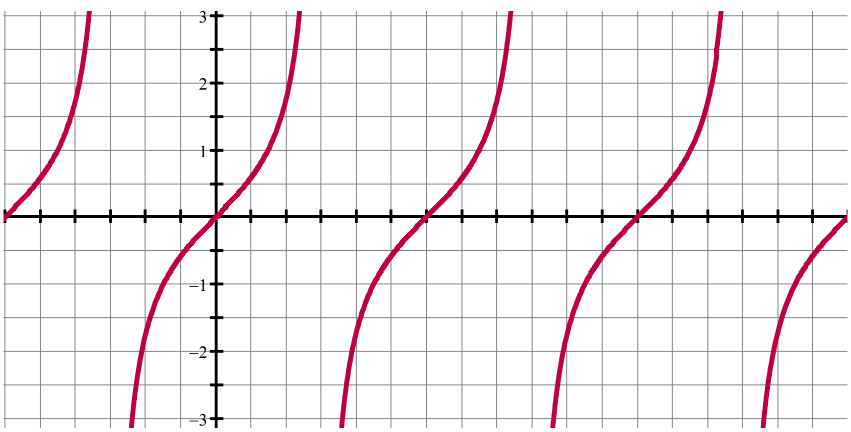
Minimum: __________

Domain: _____________________________

End Behaviour:

Range: _____________________________

Function: **y =** ___________________



Amplitude: __________

Period: __________

Maximum: __________

Minimum: __________

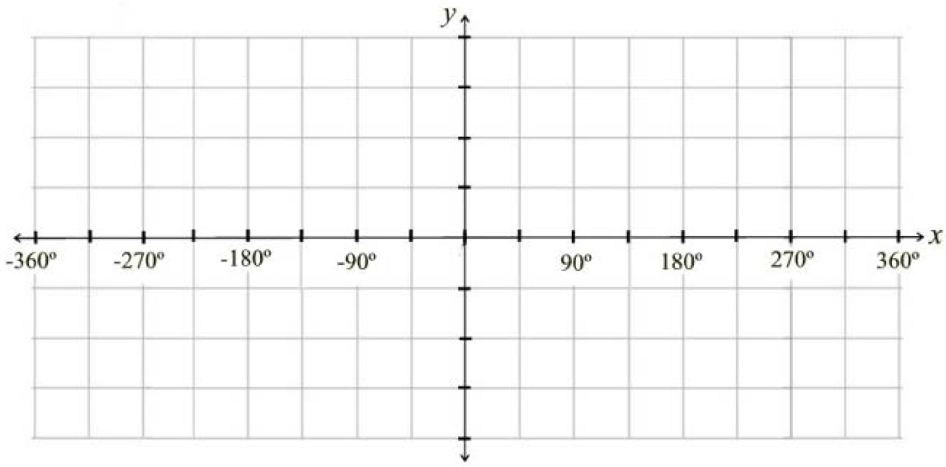
Domain: _____________________________

End Behaviour:

Range: _____________________________

Warm UP!

On the same grid, sketch two cycles of *y* = sin *x* and *y* = cos *x*.



Rewrite *y* = sin *x* as a cosine function:

Rewrite *y* = cos *x* as a cosine function:

Summary

	$y = \sin(x)$	$y = \cos(x)$	$y = \tan(x)$
Domain	$\{x \mid x \in \mathbb{R}\}$	$\{x \mid x \in \mathbb{R}\}$	$\left\{x \mid x \neq \frac{\pi}{2} + n\pi, n \in \mathbb{Z}, x \in \mathbb{R}\right\}$
Range	$\{y \mid -1 \leq y \leq 1, y \in \mathbb{R}\}$	$\{y \mid -1 \leq y \leq 1, y \in \mathbb{R}\}$	$\{y \mid y \in \mathbb{R}\}$
Maximum	$y = 1$	$y = 1$	none
Minimum	$y = -1$	$y = -1$	none
Period	2π	2π	π
Amplitude	1	1	not defined
Vertical Asymptotes	none	none	$x = \frac{\pi}{2} + n\pi, n \in \mathbb{Z}$
y-intercept	0	1	0
x-intercepts	$x = n\pi, \in \mathbb{Z}$	$x = \frac{\pi}{2} + n\pi, n \in \mathbb{Z}$	$x = n\pi, n \in \mathbb{Z}$

Extra practice

1. Given $\cos\left(\frac{7\pi}{12}\right)$, determine an equivalent expression in terms of the related acute angle.
2. Given $\tan\left(\frac{17\pi}{15}\right)$, determine an equivalent expression in terms of the related acute angle.
3. Given $\tan\left(\frac{4\pi}{5}\right)$, determine an equivalent expression in terms of the related acute angle.
4. Given $\cos\left(\frac{-5\pi}{9}\right)$, determine an equivalent expression in terms of the related acute angle.
5. Given $\cos\left(\frac{7\pi}{6}\right)$, determine an equivalent expression in terms of the related acute angle.
6. Given $\cos\left(\frac{3\pi}{5}\right)$, determine an equivalent expression in terms of the related acute angle.
7. Given $\sin\left(\frac{-7\pi}{5}\right)$, determine an equivalent expression in terms of the related acute angle.
8. Given $\sin\left(\frac{19\pi}{15}\right)$, determine an equivalent expression in terms of the related acute angle.
9. Given $\cos\left(\frac{-11\pi}{12}\right)$, determine an equivalent expression in terms of the related acute angle.
10. Given $\sin\left(\frac{17\pi}{9}\right)$, determine an equivalent expression in terms of the related acute angle.

Answers

1. $\cos\left(\frac{7\pi}{12}\right)=-\cos\left(\frac{5\pi}{12}\right);$	6. $\cos\left(\frac{7\pi}{6}\right)=-\cos\left(\frac{\pi}{6}\right);$
2. $\tan\left(\frac{17\pi}{15}\right)=\tan\left(\frac{2\pi}{15}\right)$	7. $\sin\left(\frac{-7\pi}{5}\right)=\sin\left(\frac{2\pi}{5}\right);$
3. $\tan\left(\frac{4\pi}{5}\right)=-\tan\left(\frac{\pi}{5}\right);$	8. $\sin\left(\frac{19\pi}{15}\right)=-\sin\left(\frac{4\pi}{15}\right)$
4. $\cos\left(\frac{-5\pi}{9}\right)=-\cos\left(\frac{4\pi}{9}\right);$	9. $\cos\left(\frac{-11\pi}{12}\right)=-\cos\left(\frac{\pi}{12}\right);$
5. $\cos\left(\frac{7\pi}{6}\right)=-\cos\left(\frac{\pi}{6}\right);$	10. $\sin\left(\frac{17\pi}{9}\right)=-\sin\left(\frac{\pi}{9}\right)$