Calculus Class 2 Homework

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An introduction to calculus (2)

1. Find the following limits.

a. $\lim_{x\to5}(2x+3) = \lim_{x\to5} 2x+3 = 13$

b. $\lim_{x\to2}(-x^2+3x-2)$

$=0$

c. $\lim_{x\to1}\frac{x-1}{x+2} = 0$

d. $\lim_{x\to-1}(x^3+x^2+x+1)$

$=0$

e. $\lim_{x\to1}\frac{1-x^3}{x^2-1} = \lim_{x\to1}\frac{\cancel{(x-1)}(-x^2-x-1)}{\cancel{(x-1)}(x+1)} = -\frac{3}{2}$

f. $\lim_{x\to1}\frac{\sqrt{x}-2}{x-4} = \frac{1}{3}$

g. $\lim_{x\to1}\frac{x^3-x^2-x+1}{x^3-2x^2+x} = \frac{(x-1)(x^2-1)}{(x-1)(x^2-x)}$

$= \frac{(x+1)\cancel{(x-1)}}{x\cancel{(x-1)}} = 2$

h. $\lim_{x\to\frac{1}{2}}\frac{3-12x^2}{2x^2+x-1} = \lim_{x\to\frac{1}{2}}\frac{\cancel{(2x-1)}(-6x-3)}{\cancel{(2x-1)}(x+1)}$

$=-4$

i. $\lim_{x\to3}\frac{x^3-9x^2+27x-27}{x^3-27} = \lim_{x\to3}\frac{x^3-9x^2+27x-27}{(x-3)(x^2+3x+9)}$

$= \lim_{x\to3}\frac{\cancel{(x-3)}(x^2-6x+9)}{\cancel{(x-3)}(x^2+3x+9)} = 0$

j. $\lim_{x\to-1}\frac{\sqrt{x+9}-2\sqrt{2}}{x+1}$

$= \lim_{x\to-1}\frac{(\sqrt{x+9}-2\sqrt{2})(\sqrt{x+9}+2\sqrt{2})}{(x+1)(\sqrt{x+9}+2\sqrt{2})}$

$= \lim_{x\to-1}\frac{\cancel{x+9-8}}{\cancel{(x+1)}(\sqrt{x+9}+2\sqrt{2})} = \frac{\sqrt{2}}{8}$

k. $\lim_{x\to0}\frac{\sqrt{9-x}-\sqrt{9+x}}{x} = \lim_{x\to0}\frac{(\sqrt{9-x}-\sqrt{9+x})(\sqrt{9-x}+\sqrt{9+x})}{x(\sqrt{9-x}+\sqrt{9+x})}$

$= \lim_{x\to0}\frac{(9-x)-(9+x)}{x(\sqrt{9-x}-\sqrt{9+x})} = \frac{-2}{\sqrt{9-x}-\sqrt{9+x}} = -\frac{1}{3}$

l. $\lim_{x\to1}\frac{\sqrt{x}-1}{\sqrt[3]{x}-1}$ let $x^{\frac{1}{6}}=y$

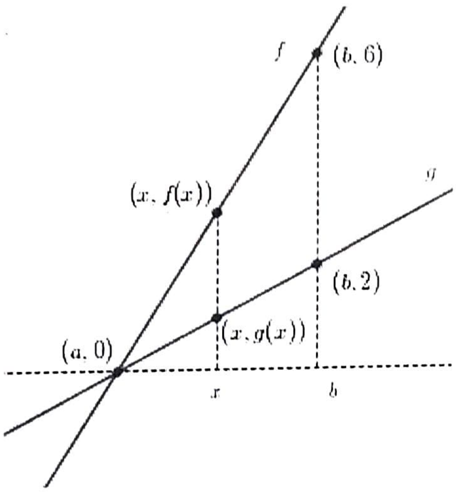
$\lim_{y\to1}\frac{y^3-1}{y^2-1} = \lim_{y\to1}\frac{\cancel{(y-1)}(y^2+y+1)}{\cancel{(y-1)}(y+1)}$

$=\frac{3}{2}$

2. If $\lim_{x\to1} f(x) = -2$ and $\lim_{x\to1} g(x) = 3$, then what is the value of $\lim_{x\to1} \frac{[f(x)]^3+[g(x)]^2}{5-2g(x)}$?

$$= \frac{\left[\lim_{x\to1} f(x)\right]^3 + \left[\lim_{x\to1} g(x)\right]^2}{\lim_{x\to1} 5 - 2\lim_{x\to1} g(x)} = -1$$

3. Let a<b be real numbers. Consider two linear functions as shown in the graph.



Evaluate $\lim_{x\to a} \frac{f(x)}{g(x)}$.

$f(x)$: $m = \frac{6}{b-a}$ $\quad g(x)$: $m = \frac{2}{b-a}$

$f(x) = \frac{6}{b-a}(x-a) \quad g(x) = \frac{2}{b-a}(x-a)$

$$\lim_{x\to a} \frac{\frac{6}{\cancel{b-a}}\cancel{(x-a)}}{\frac{2}{\cancel{b-a}}\cancel{(x-a)}} = 3$$

4. Consider the piecewise function f(x) defined below, where A is a constant.

$$f(x) = \begin{cases} A^2x - 3A & if\ x \geq 1 \\ -2 & if\ x < 1 \end{cases}$$

Determine all values of A so that $\lim_{x\to1} f(x)$ exist.

$\lim_{x\to1^-} f(x) = -2$

$\lim_{x\to1^+} A^2x - 3A = A^2 - 3A$

$A^2 - 3A = -2$

$A^2 - 3A + 2 = 0$

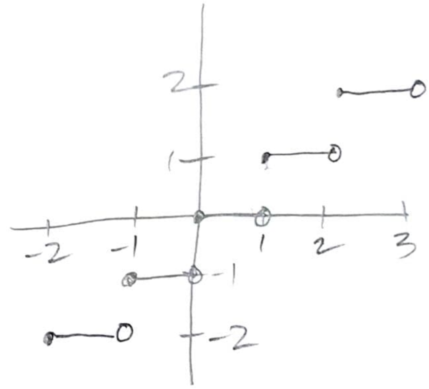
$(A-1)(A-2) = 0$

$A = 1$ or 2

5. The greatest integer function(or the step/floor function) is defined as $f(x)=[x]=n$, where n is an integer such that

$$n \le x < n+1 .$$

a. Sketch the graph of $f(x)=[x]$.



b. For what values of p do the following one sided limits exist?

i. $\lim_{x\to p^-} f(x)$

$P \in \mathbb{R}$

ii. $\lim_{x\to p^+} f(x)$

c. For what values of p do the the right and left hand limits exist, but $\lim_{x\to p^-} f(x) \neq \lim_{x\to p^+} f(x)$?

$P \in \mathbb{Z}$

d. For what values of p does $\lim_{x\to p} f(x)$ exist?

$x \in \mathbb{R} / \{\mathbb{Z}\}$

6. a) $\lim_{x\to -\pi^-} (\sin x - 1) = -1$

$\lim_{x\to -\pi^+} (x^2 - \pi^2) = 0$

discontinous at $x = -\pi$

$\lim_{x\to 0^-} (x^2 - \pi^2) = -\pi^2$

$\lim_{x\to 0^+} (-\pi^2 + x) = -\pi^2$

continous at $x = 0$

6. Analyse the continuity of the functions:

a. $f(x) = \begin{cases} \sin(x) - 1, & x < -\pi \\ x^2 - \pi^2, & -\pi \le x \le 0 \\ -\pi^2 + x, & x > 0 \end{cases}$

b. $f(x) = \begin{cases} \frac{|x|}{x} & if\ x \neq 0 \\ a & if\ x = 0 \end{cases}$ where a is a real number.

b)

$\lim_{x\to 0^-} \frac{-x}{x} = -1$

$\lim_{x\to 0^+} \frac{x}{x} = 1$

discontinous at 0

7. Give an example of functions $f(x)$ and $g(x)$ such that $\lim_{x\to1}(f(x)+g(x))$ exists but $\lim_{x\to1} f(x)$ and $\lim_{x\to1} g(x)$ do not exist.

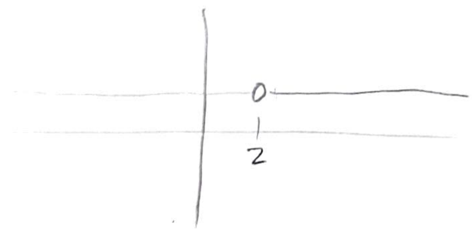
8. Give an example of function $f(x)$ such that $\lim_{x\to3} f^2(x)$ exists but $\lim_{x\to3} f(x)$ do not exist.

$$f(x) = \begin{cases} 1, & \text{if } x<3 \\ -1, & \text{if } x\geq 3 \end{cases}$$

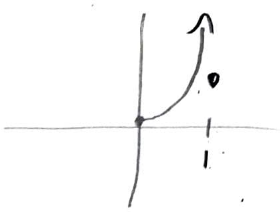
9. For each of the following, sketch the graph of a function $f(x)$ that satisfies the given description.

a. f is continuous for all $x\neq2$, and has a removable discontinuity at $x=2$.

$$f(x)=\frac{x-2}{x-2}$$



b. The domain of f is $\{x|0\leq x\leq1, x\in R\}$, f is continuous from the right at $x=0$, continuous on $0<x<1$, and has an infinite discontinuity at $x=1$.



10. Find the value of a for which the following limit statement is true:

$$\lim_{x\to1}\frac{\sqrt{a-x}-\sqrt{8+x}}{x-1} = -2$$

$$\lim_{x\to1}\frac{(\sqrt{a-x}-\sqrt{8+x})(\sqrt{a-x}+\sqrt{8+x})}{(x-1)(\sqrt{a-x}+\sqrt{8+x})} = \lim_{x\to1}\frac{(a-8)-2x}{(x-1)(\sqrt{a-x}+\sqrt{8+x})} = -2$$

$$\lim_{x\to1}\frac{-2\left(x-\frac{1}{2}(a-8)\right)}{(x-1)(\sqrt{a-x}+\sqrt{8+x})} = -2 \qquad \frac{1}{2}(a-8)=1 \quad a=10$$

$$\lim_{x\to1}\frac{-2}{6} \neq -2$$

No Solutions

11. Evaluate the following limits, using an answer of $+\infty$ or $-\infty$ whenever appropriate.

a. $\lim_{x\to\infty} \frac{-14x+37}{7x-3}$

b. $\lim_{x\to-\infty} \frac{100-3x^2+7x^5-6x^7}{2x^7-1}$

c. $\lim_{x\to-\infty} \left(x - \frac{x^2-4x+1}{x-3}\right)$

d. $\lim_{x\to-\infty} \frac{\sqrt{x^2+2}}{x+1}$

a)

$$\lim_{x\to\infty} \frac{\cancel{x}(-14+\cancel{\frac{37}{x}})}{\cancel{x}(7-\cancel{\frac{3}{x}})}$$

$$= -2$$

b)

$$\lim_{x\to-\infty} \frac{x^7\left(\frac{100}{x^7}-\frac{3}{x^5}+\frac{7}{x^2}-6\right)}{x^7\left(2-\frac{1}{x^7}\right)}$$

$$= -\frac{6}{2} = -3$$

c)

$$\lim_{x\to-\infty} \frac{x(x-3)-(x^2-4x+1)}{x-3}$$

$$= \lim_{x\to-\infty} \frac{\cancel{x^2}-3x-\cancel{x^2}+4x-1}{x-3}$$

$$= \lim_{x\to-\infty} \frac{x-1}{x-3}$$

$$= 1$$

d)

$$\lim_{x\to-\infty} \frac{\sqrt{x^2\left(1+\frac{2}{x^2}\right)}}{x+1}$$

$$= \lim_{x\to-\infty} \frac{\cancel{-x}\sqrt{1+\frac{2}{x^2}}}{\cancel{x}\left(1+\frac{1}{x}\right)}$$

$$= -1$$