Advanced Functions Class 13 Homework

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Identities and Equations (2)

1. Use an appropriate double angle formula to rewrite each expression as a single trigonometric ratio.

a. $6\sin(3x)\cos(3x)$

$= 3(2\sin 3x\cos 3x)$
$= 3\sin(6x)$

b. $1-2\cos^2(\frac{\alpha}{2})$

$= \sin^2(\frac{a}{2}) - \cos^2(\frac{a}{2})$
$= -\cos(a)$

c. $\frac{\tan\theta}{tan^2(\theta)-1}$

$= (-\frac{1}{2})(\frac{2\tan\theta}{1-\tan^2\theta})$
$= -\frac{\tan(2\theta)}{2}$

d. $\frac{\cos^2(3y)-sin^2(3y)}{\sin(3y)\cos(3y)}$

$= 2\cdot(\frac{\cos 6y}{2\sin 3y\cos 3y})$
$= 2\cdot\frac{\cos 6y}{\sin 6y}$
$= 2\cot(6y)$

2. Derive a formula for

a. $\sin(3\theta)$ in terms of $\sin(\theta)$
b. $\cos(4\theta)$ in terms of $\cos(\theta)$
c. $\cot(2\theta)$ in terms of $\cot(\theta)$

a) $\sin 3\theta$
$= -\cos(3\theta - \frac{3\pi}{2})$
$= -\cos(3(\theta - \frac{\pi}{2}))$
let $\theta - \frac{\pi}{2} = x$
$= -(\cos 2x\cos x - \sin 2x\sin x)$
$= -([2\cos^2 x - 1]\cos x - 2\sin^2 x\cos x)$
$\cos(\theta - \frac{\pi}{2}) = \sin\theta$ $= -(2\cos^3 x - \cos x - 2(1-\cos^2 x)\cos x)$
$= -(2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x)$
$= -(4\cos^3 x - 3\cos x)$
$= -4\sin^3\theta - 3\sin\theta$

b) $\cos 4\theta$
$= 2\cos^2(2\theta) - 1$
$= 2(2\cos^2\theta - 1)^2 - 1$
$= 2(4\cos^4\theta - 4\cos^2\theta + 1) - 1$
$= 8\cos^4\theta - 8\cos^2\theta - 1$

c) $\frac{1}{\tan(2\theta)} = \frac{1-\tan^2\theta}{2\tan\theta}$
$= \frac{1 - \frac{1}{\cot^2\theta}}{2\frac{1}{\cot\theta}} = \frac{\cot^2\theta - 1}{\cot^2\theta}\cdot\frac{\cot\theta}{2}$
$= \frac{\cot^2\theta - 1}{2\cot\theta}$

3. Simplify each of the following trigonometric expressions using an appropriate double angle formula, then determine the exact value of the expression.

a. $1-2\sin^2(\frac{11\pi}{8})$

$= \cos(\frac{11\pi}{4}) = -\frac{\sqrt{2}}{2}$

b. $1-2\cos^2(105°)$

$= -\cos(210)$

$= \frac{\sqrt{3}}{2}$

c. $4\sin(112.5°)\cos(112.5°)$

$= 2\sin(225)$

$= -\sqrt{2}$

d. $\frac{1-\tan^2(\frac{\pi}{12})}{\tan(\frac{\pi}{12})}$

$= 2 \cdot \frac{1-\tan^2(\frac{\pi}{12})}{2\tan(\frac{\pi}{12})}$

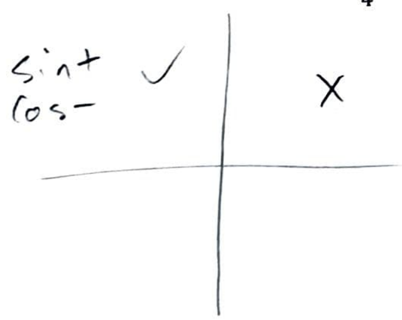
$= 2 \cdot \frac{1}{\tan(\frac{\pi}{6})}$

$= 2\sqrt{3}$

e. $\cos^2(\frac{11\pi}{12})$

$= \frac{1+\cos(\frac{11\pi}{6})}{2} = \frac{2+\sqrt{3}}{4}$

4. a. Given $\sin(\theta) = \frac{3}{4}$ where $\frac{\pi}{2} \le \theta \le \pi$, determine the exact value of $\sin(2\theta)$.



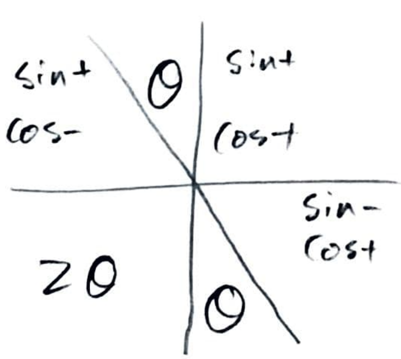
$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos\theta = -\sqrt{1-\sin^2\theta}$

$= -\frac{\sqrt{7}}{4}$

$\sin 2\theta = -\frac{3\sqrt{7}}{8}$

b. Given $\cos(2\theta) = -\frac{7}{8}$, where 2θ is an angle in standard position with a terminal arm in quadrant 3, determine the exact value of $\cos(\theta)$ and $\sin(\theta)$



$\cos\theta = \pm\sqrt{\frac{1+\cos(2\theta)}{2}}$

$= \pm\frac{1}{4}$

$\sin\theta = \pm\sqrt{\frac{1-\cos(2\theta)}{2}}$

$= \pm\frac{\sqrt{15}}{4}$

If $\cos\theta = \frac{1}{4}$, $\sin\theta = -\frac{\sqrt{15}}{4}$

else if $\cos\theta = -\frac{1}{4}$, $\sin = \frac{\sqrt{15}}{4}$

Advanced Functions Class 13 Homework

5. Determine the exact value of each.

a. $\cos(22.5°)$

$= \sqrt{\frac{1+\cos 45}{2}}$

$= \sqrt{\frac{1}{2}+\frac{\sqrt{2}}{4}}$

$= \sqrt{\frac{2}{4}+\frac{\sqrt{2}}{4}}$

$= \sqrt{\frac{\sqrt{2}+2}{4}} = \frac{\sqrt{\sqrt{2}+2}}{2}$

b. $\sin(\frac{7\pi}{12})$

$= \sqrt{\frac{1-\cos(\frac{7\pi}{6})}{2}}$

$= \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}$

$= \frac{\sqrt{2+\sqrt{3}}}{2}$

c. $\tan(\frac{5\pi}{8})$

$= \frac{\sin(\frac{5\pi}{8})}{\cos(\frac{5\pi}{8})} = \frac{\sqrt{\frac{1-\cos(\frac{5\pi}{4})}{2}}}{\sqrt{\frac{1+\cos(\frac{5\pi}{4})}{2}}}$

$= -\sqrt{\frac{2+\sqrt{2}}{2-\sqrt{2}}}$

$= -\sqrt{\frac{2+\sqrt{2}}{2-\sqrt{2}} \cdot \frac{2+\sqrt{2}}{2+\sqrt{2}}}$

$= -\sqrt{\frac{(2+\sqrt{2})^2}{4-2}} = -\frac{2+\sqrt{2}}{\sqrt{2}}$

$= -\frac{\cancel{2}\sqrt{2}+\cancel{2}}{\cancel{2}}$

$= -\sqrt{2}-1$

6. Solve for x within the specified domain. Keep answers exact whenever possible.

a. $\tan(x)\cos(x) = 0$, $0 \le x \le 2\pi$

$\frac{\sin x}{\cancel{\cos x}} \cancel{\cos x} = 0 \quad \cos x \neq 0$

$\sin x = 0$

$x = 0, \pi, 2\pi$

b. $(2\cos(x)+\sqrt{3})(\csc(x)-\sqrt{2}) = 0$, $0 \le x \le 2\pi$

① $2\cos x + \sqrt{3} = 0$

$\cos x = -\frac{\sqrt{3}}{2}$

$x = \frac{5\pi}{6}, \frac{7\pi}{6}$

② $\csc x - \sqrt{2} = 0$

$\sin x = \frac{1}{\sqrt{2}}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}$

$\therefore \quad x = \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{\pi}{4}, \frac{3\pi}{4}$

c. $2\cos^2(x)+\sqrt{3}\cos(x)=0$, $-\pi \le x \le \pi$

$\cos x(2\cos x + \sqrt{3}) = 0$

$\cos x = 0$:

$x = \frac{-\pi}{2}, \frac{\pi}{2}$

$\cos x = -\frac{\sqrt{3}}{2}$:

$x = \frac{5\pi}{6}, -\frac{5\pi}{6}$

$x = -\frac{\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{6}, -\frac{5\pi}{6}$

d. $2\sin(x)\sec(x)=6\sin(x)$, $0° \le x \le 360°$

$\cancel{2}\sin x(\sec x - 3) = 0$

$\sin x = 0$:

$x = 0, 180, 360$

$\sec x = 3$

$\cos x = \frac{1}{3}$

$x = 70.53, 289.47, 0, 180, 360$

7. Determine the exact values of a and b such that the quadratic trigonometric equation $a\cos^2(x) + b\cos(x) - 3 = 0$ has the solutions $\frac{\pi}{4}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{7\pi}{4}$ in the interval $0 \le x \le 2\pi$.

① $a \cdot \frac{1}{2} + b \cdot \frac{\sqrt{2}}{2} - 3 = 0$

$a \cdot \frac{1}{2} = 3 - \frac{b\sqrt{2}}{2}$

$a = 6 - b\sqrt{2}$

② $a \cdot \frac{1}{4} - b \cdot \frac{1}{2} - 3 = 0$

$\frac{6 - b\sqrt{2}}{4} - \frac{b}{2} = 3$

$b = 3\sqrt{2} - 6$ $\quad$ $a = 6\sqrt{2}$

8. Determine roots of $\sin(x) + \cos(x) = \sqrt{\frac{3}{2}}$ for $0 \le x \le 2\pi$.

$\sin^2 x + 2\cos x \sin x + \cos^2 x = \frac{3}{2}$

$1 + \sin 2x = \frac{3}{2}$

$\sin 2x = \frac{1}{2}$

$x = \frac{\sin^{-1}\left(\frac{1}{2}\right)}{2} = \frac{1}{12}\pi, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$

$\sin\left(\frac{\pi}{12}\right) + \cos\left(\frac{\pi}{12}\right) > 0$ ✓

$\sin\left(\frac{5\pi}{12}\right) + \cos\left(\frac{5\pi}{12}\right) > 0$ ✓

$\sin\left(\frac{13\pi}{12}\right) + \cos\left(\frac{13\pi}{12}\right) < 0$ ✗ $\quad$ $x = \frac{\pi}{12}, \frac{5\pi}{12}$

$\sin\left(\frac{17\pi}{12}\right) + \cos\left(\frac{17\pi}{12}\right) < 0$ ✗