AP Calculus In-Class Nine – Antiderivatives and the Definite Integral

4.4 Definition of Definite Integral and Properties of Definite Integral

4.4 The Mean Value Theorem for Definite Integral; 4.5 The Fundamental Theorem of Calculus

1. Evaluate the definite integrals.

(a) $\int_{7}^{12} dz$

(b) $\int_{-8}^{8} \left(\sqrt[3]{s^2} + 2 \right) ds$

(c) $\int_{5}^{5} \sqrt[3]{x^2 + \sqrt{x^5 + 1}} \, dx$

(d) $\int_{0}^{-1} \frac{x^3 + 8}{x+2} \, dx$

(e) $\int_{\pi/2}^{\pi} \cos(x/3) dx$

2. If g is differentiable and f is continuous for all x, prove that
$$D_x \int_{a}^{g(x)} f(t)dt = f(g(x))g'(x).$$

3. Use Q2 to find $D_x \int_{-3}^{x^2 - x + 1} \sqrt[3]{t^4 + 1} \, dt$.

4. Evaluate the integrals.

(a) $\int (3x - 2)^{10} \, dx$

(b) $\int \sqrt{t^3 - 1} \, t^2 \, dt$

(c) $\displaystyle\int \frac{x-2}{\left(x^2-4x+3\right)^3}\,dx$

(d) $\displaystyle\int \frac{\left(\sqrt{u}+3\right)^4}{\sqrt{u}}\,du$

(e) $\displaystyle\int \sin(1+6x)\,dx$

(f) $\displaystyle\int \cos 3x \sqrt[3]{\sin 3x}\,dx$

(g) $\displaystyle\int (2+5\cos x)^3 \sin x\,dx$

(h) $\displaystyle\int_0^1 \frac{1}{(3-2v)^2}\,dv$

(i) $\displaystyle\int_1^4 \frac{1}{\sqrt{x}\left(\sqrt{x}+1\right)^3}\,dx$

5. Find

(a) $\displaystyle D_x\int \frac{1}{\sqrt{x^3+x+5}}\,dx$

(b) $\displaystyle\int_0^3 D_x\sqrt{x^2+16}\,dx$

(c) $\displaystyle D_x\int_0^1 \sqrt{x^2+4}\;x\,dx$

6. Let f be continuous on $[-a, a]$. if f is an odd function, show that $\int_{-a}^{a} f(x)dx = 0$ and interpret this result geometrically. Verify the result for the special case $f(x) = \sin x$.

7. Evaluate the definite integrals.

(a) $\int_{1}^{2} \frac{3x-1}{3x} dx$

(b) $\int_{-1}^{0} \sqrt{3u + 4}\, du$

(c) $\int_{2}^{3} \frac{dy}{2y - 3}$

(d) $\int_{0}^{\sqrt{3}} \frac{x}{\sqrt{4 - x^2}} dx$

(e) $\int_{-3}^{3} \frac{dx}{9 + x^2}$

8. Evaluate the definite integrals.

(a) $\int_{0}^{\pi/4} \sin 2\theta\, d\theta$

(b) $\int_{1}^{2} \frac{dz}{3 - z}$

(c) $\int_{0}^{\pi} \cos^2 \theta \sin \theta\, d\theta$

(d) $\int_{1}^{e} \frac{\ln x}{x} dx$

(e) $\int_{0}^{1} xe^x dx$

9. Evaluate the definite integrals.

(a) $\int_0^1 \dfrac{1+e^{-x}}{e^{-x}}dx$

(b) $\int_0^1 \dfrac{e^x}{1+e^x}dx$

(c) $\int_{-1}^3 \left| x \right| dx$

(d) $\int_{-3}^2 \left| x+1 \right| dx$

(e) $\int_0^{\pi/4} \sqrt{1-\cos 2\alpha}\,d\alpha$

10. Evaluate the definite integrals with substitutions.

(a) $\int_3^8 \dfrac{dx}{x\sqrt{x+1}}$, let $u = \sqrt{x+1}$

(b) $\int_2^4 xy\,dx$, let $x = 4\cos\theta$ and $y = 3\sin\theta$

11. In each sub-question, find numbers that satisfy the conclusion of the Mean Value Theorem for Definite Integral.

(a) $\int_{-2}^{18} 1/(x-8)\,dx = 4$

(b) $\int_{-1}^8 3\sqrt{x+1}\,dx = 54$

12. If $f(x) = x$ and $0 < a < b$, find (without integrating) a number z in (a, b) such that $\int_a^b f(x)dx = f(z)(b-a)$.