

First Name: _____ Last Name: _____ Student ID: _____

Exponential and Logarithmic Functions (1)

Anatomy of an Exponential Function

1. For the graph below, state the following information:

y-Intercept: _____

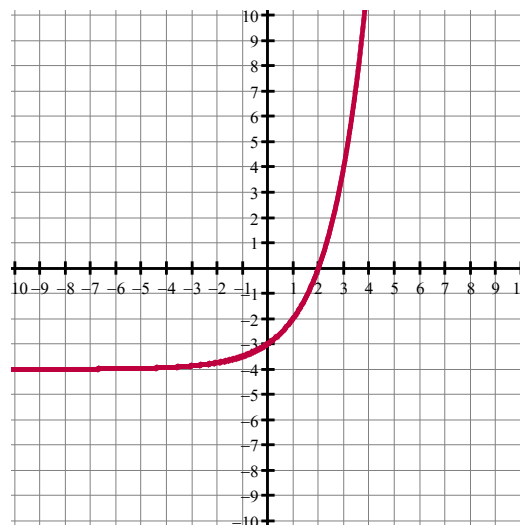
x-Intercept: _____

Horizontal Asymptote: _____

Increasing/Decreasing: _____

Domain: _____

Range: _____



2. Graph the functions $y = 2^x$ and $y = \left(\frac{1}{2}\right)^x$ by completing the tables of values below.

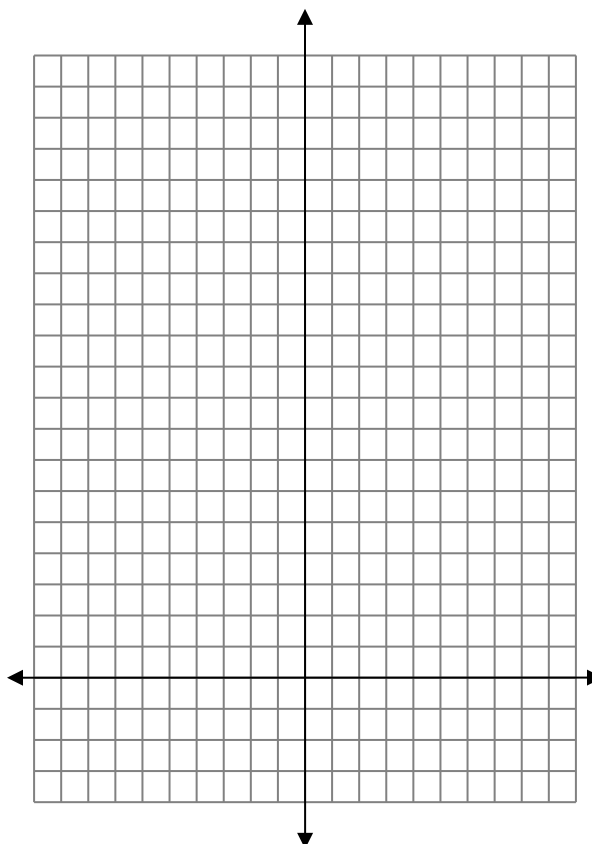
$$y = 2^x$$

X	Y
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	

$$y = \left(\frac{1}{2}\right)^x$$

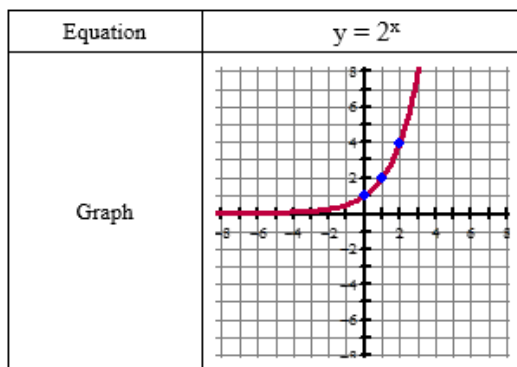
X	Y
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	

1



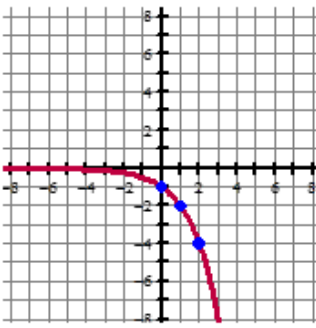
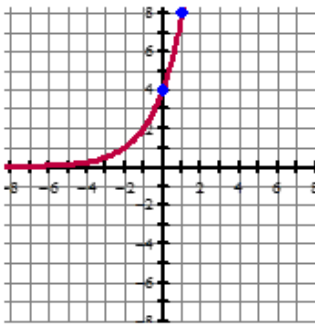
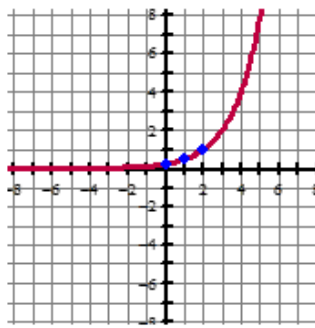
Graphing Exponential Functions Investigation

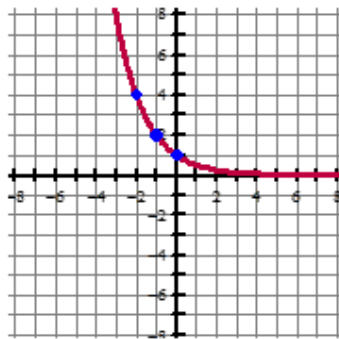
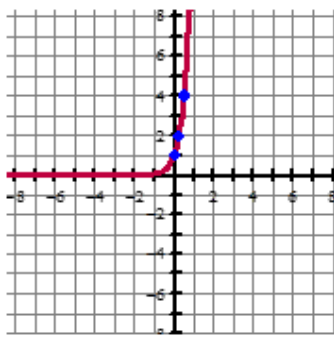
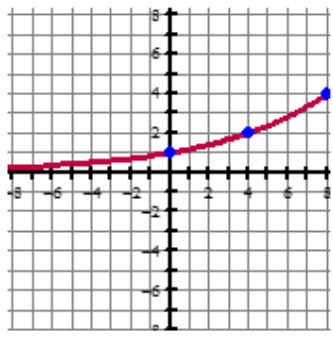
For each of the following graphs, state the horizontal asymptote, whether it is increasing or decreasing, and how it compares to the function $y = 2^x$.



Equation	$y = 2^x + 3$	$y = 2^x - 3$
Graph		
How Does Graph Compare to $y = 2^x$		

Equation	$y = 2^{(x+3)}$	$y = 2^{(x-3)}$
Graph		
How Does Graph Compare to $y = 2^x$		

Equation	$y = -(2^x)$	$y = 3(2^x)$	$y = \frac{1}{4}(2^x)$
Graph			
How Does Graph Compare to $y = 2^x$			

Equation	$y = 2^{(-x)}$	$y = 2^{(4x)}$	$y = 2^{\left(\frac{1}{4}x\right)}$
Graph			
How Does Graph Compare to $y = 2^x$			

Make a Prediction

How do each of the following letters effect the function $y = 2^x$.

$$y = a[2^{b(x+h)}] + k$$

3. State the effects that the following transformations have had on the base function.

a) $y = -5 \left[8^{[4(x-7)]} \right] + 2$

b) $y = -\frac{1}{5} \left[\frac{1}{7} \left[-\frac{1}{2}^{(x+6)} \right] \right] - 9$

4. Write an equation of the base function $y = 3^x$ that has undergone the following transformations.

a) Up 18

Horizontal Reflection (in y-axis)

Vertical Compression of 3

b) Left 17

Vertical Stretch of 10

Horizontal Compression of 8

5. Determine the horizontal asymptote of each of the following functions.

a) $y = 8 \left[\frac{1}{2} \left[-\frac{1}{4}^{(x-3)} \right] \right] - 10$

b) $y = -\frac{1}{4} \left[6^{[9(x+1)]} \right] + 2$

6. Determine the y-intercept of each of the following functions.

a) $y = 5 \left[\frac{1}{2} \left[\frac{1}{3}^{(x-6)} \right] \right] + 1$

b) $y = -\frac{1}{9} \left[3^{[2(x+2)]} \right] + 5$

Solving Exponential Equations

Strategy 1 – Powers of the Same Base

Solve each of the following:

a) $5^{3x-2} = 625$

b) $16^{x+5} = 64^{x-2}$

c) $\frac{5^{2x+7} \cdot 25^{6x}}{125^{x+1}} = 625^{x-4}$

d) $6480(6^{9x-2}) - 2 = 3$

Strategy 2 – Variable Replacement

a) $3^{x+1} + 3^{x+2} = 108$

b) $2^{x+3} + 2^{x+2} - 3 = 0$

c) $4^{2x} + 4^x - 2 = 0$

d) $5^{2x} - 26(5^x) + 25 = 0$

Try yourself!

$$3(16^x) + 2(81^x) = 5(36^x)$$

Hint: Divide both sides by $81^x > 0$

$$4^{1-\frac{1}{x}} - 5 \cdot 6^{-\frac{1}{x}} = 9^{1-\frac{1}{x}}$$

$$\text{Hint: } 4 \cdot 4^{-\frac{1}{x}} - 5 \cdot 6^{-\frac{1}{x}} = 9 \cdot 9^{-\frac{1}{x}}$$

Divide both sides by $9^{-\frac{1}{x}} > 0$

Applications of Exponents - Growth & Decay

The model for **Exponential Growth** or **Decay** is $A(t) = A_0(r)^{t/p}$

where, $A(t)$ = amount as a function of time (final amount)

A_0 = initial amount (@ time 0)

r = the ratio/multiplier of growth or decay

t = time (must be the same unit of time as the period)

p = period of time for the growth or decay to occur

The **half-life** of a radioactive substance is the time required for a sample of the material to decay to half of its initial amount.

7. The value of an antique vase increases by 10% every 7 years. If the vase was purchased for \$200, approximately how long will it take until it is worth \$1000?

Extra Practice

1. Solve.

- a) $2^x = 16$
- c) $2^x = 128$
- e) $4^y = 256$
- g) $(-3)^x = -27$
- i) $(-5)^a = 25$
- k) $-2^x = -16$
- m) $-5^x = -625$
- o) $(-1)^m = -1$

- b) $3^x = 27$
- d) $5^x = 125$
- f) $729 = 9^z$
- h) $(-2)^x = -32$
- j) $81 = (-3)^x$
- l) $-4^y = -64$
- n) $(-1)^x = 1$

2. Solve.

- a) $7^{w-2} = 49$
- c) $2^{1-x} = 128$
- e) $5^{3x-1} = 25$
- g) $4^{x-1} = 1$
- i) $(-1)^{2x} = 1$

- b) $3^{x+4} = 27$
- d) $4^{3k} = 64$
- f) $-81 = -3^{2x+8}$
- h) $3^{2-2x} = 1$

3. Solve and check.

- a) $6^{x+3} = 6^{2x}$
- c) $3^{2y+3} = 3^{y+5}$
- e) $7^{5d-1} = 7^{2d+5}$

- b) $2^{x+3} = 2^{2x-1}$
- d) $2^{4x-7} = 2^{2x+1}$
- f) $3^{b-5} = 3^{2b-3}$

4. Solve.

- a) $16^{2x} = 8^{3x}$
- c) $27^{x-1} = 9^{2x}$
- e) $16^{2p+1} = 8^{3p+1}$

- b) $4^t = 8^{t+1}$
- d) $25^{2-c} = 125^{2c-4}$
- f) $(-8)^{1-2x} = (-32)^{1-x}$

5. Solve and check.

- a) $2^{x+5} = 4^{x+2}$
- c) $9^{2q-6} = 3^{q+6}$
- e) $27^{y-1} = 9^{2y-4}$

- b) $2^x = 4^{x-1}$
- d) $4^x = 8^{x+1}$
- f) $8^{x+3} = 16^{2x+1}$

6. Solve and check.

- a) $5^{4-x} = \frac{1}{5}$
- c) $6^{3x-7} = \frac{1}{6}$
- e) $5^{2n+1} = \frac{1}{125}$

- b) $10^{y-2} = \frac{1}{10\,000}$
- d) $3^{3x-1} = \frac{1}{81}$
- f) $\frac{1}{256} = 2^{2-5w}$

7. Solve and check.

- a) $4^x = 8$
- c) $(-8)^y = -2$
- e) $2^{9x} = \frac{1}{8}$
- g) $2^x = 16^4$
- i) $9^{2s+1} = 27$
- b) $64^z = 16$
- d) $9^{-x} = 3$
- f) $9^{6x} = \frac{1}{27}$
- h) $2^{-2g} = 32$

8. Solve and check.

- a) $9^{x+1} = 27^{2x}$
- c) $36^{t-2} = 216^{-2t}$
- e) $25^{1-3x} = 125^{-x}$
- b) $16^y = 64^{2y-1}$
- d) $8^{2x-1} = 16^{x-1}$
- f) $16^{3+k} = 32^{1-2k}$

9. Solve and check.

- a) $5 = 25^{\frac{x}{2}}$
- c) $9^{\frac{y}{5}} = 27$
- e) $4^{\frac{x}{4}} = \frac{1}{8}$
- b) $8 = 2^{\frac{x}{3}}$
- d) $\frac{1}{2} = 2^{\frac{a}{3}}$
- f) $\left(\frac{3}{2}\right)^{\frac{m}{2}} = \frac{4}{9}$

10. Solve.

- a) $3(5^{x+1}) = 15$
- b) $2(3^{y-2}) = 18$
- c) $5(4^x) = 10$
- d) $2(4^{v+1}) = 1$
- e) $2 = 6(3^{4f-2})$
- f) $27(3^{3x+1}) = 3$

11. Solve and check.

- a) $2^{x+2} - 2^x = 48$
- b) $4^{x+3} + 4^x = 260$
- c) $2^{a+5} + 2^a = 1056$
- d) $6^{x+1} + 6^{x+2} = 7$
- e) $3^{x+3} - 3^{x+1} = 648$
- f) $10^{z+4} + 10^{z+3} = 11$
- g) $2^{x+2} - 2^{x+5} = -7$
- h) $3^{m+1} + 3^{m+2} - 972 = 0$
- i) $5^{n+2} - 5^{n+3} = -2500$

19. Solve.

a) $\frac{27^x}{9^{2x-1}} = 3^{x+4}$ b) $27^x(9^{2x-1}) = 3^{x+4}$ c) $27^{x+1} = \left(\frac{1}{9}\right)^{2x-5}$

20. Solve.

a) $2^{x^2+2x} = 2^{x+6}$ b) $3^{x^2-2x} = 3^{x-2}$ c) $2^{2x^2-3x} = 2^{x^2-2x+12}$

22. Solve and check.

a) $\frac{2^{2x+1}}{2^{x-3}} = 4$ b) $\frac{9^{x+4}}{27^{x-1}} = 81$ c) $\frac{8^{x+2}}{4^{x+3}} = 16^{x-3}$

Section 1.3, pp. 23–25

1. a) 4 b) 3 c) 7 d) 3 e) 4 f) 3 g) 3 h) 5 i) 2 j) 4 k) 4 l) 3 m) 4
 n) x any even integer o) m any odd integer 2. a) 4 b) -1 c) -6
 d) 1 e) 1 f) -2 g) 1 h) 1 i) all values of x 3. a) 3 b) 4 c) 2 d) 4
 e) 2 f) -2 4. a) 0 b) -3 c) -3 d) 2 e) 1 f) -2 5. a) 1 b) 2 c) 6
 d) -3 e) 5 f) 1 6. a) 5 b) -2 c) 2 d) -1 e) -2 f) 2 7. a) $\frac{3}{2}$
 b) $\frac{2}{3}$ c) $\frac{1}{3}$ d) $-\frac{1}{2}$ e) $-\frac{1}{3}$ f) $-\frac{1}{4}$ g) 16 h) $-\frac{5}{2}$ i) $\frac{1}{4}$ 8. a) $\frac{1}{2}$ b) $\frac{3}{4}$
 c) $\frac{1}{2}$ d) $-\frac{1}{2}$ e) $\frac{2}{3}$ f) $-\frac{1}{2}$ 9. a) 1 b) 9 c) $\frac{15}{2}$ d) -3 e) -6 f) -4
 10. a) 0 b) 4 c) $\frac{1}{2}$ d) $-\frac{3}{2}$ e) $\frac{1}{4}$ f) -1 11. a) 4 b) 1 c) 5 d) -1 e) 3
 f) -3 g) -2 h) 4 i) 2 12. The equation is true for all values of
 b) 84 years c) 140 years 15. a) $\frac{1}{8}$ b) 26 days 16. a) 2 m
 b) 11% 17. a) 5 h b) 20.4 years c) 30 s 18. 59.6 h 19. a) -1
 b) 1 c) 1 20. a) 2, -3 b) 1, 2 c) 4, -3 21. 16 days 22. a) -2
 b) 7 c) 4 23. $x = -17, y = 2$