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Rational Functions (1)

1. State the domain of each function, then determine the equation of any vertical asymptotes and/or coordinates of any holes in the graph of the function.

a. f(x) = $\frac{2x}{x-3}$

b. f(x) = $\frac{2x^2+x}{x^2-5x+6}$

c. f(x) = $\frac{3x^2-21x}{6x^2-39x-21}$

d. f(x) = $\frac{x^3+x}{6x^3+x^2-x}$

a) D: $\mathbb{R} \setminus \{3\}$

V.A: $x=3$

b) D: $\mathbb{R} \setminus \{2,3\}$

$x^2-5x+6 \neq 0$

$(x-2)(x-3)$

V.A: $x=2$, $x=3$

c) $\frac{3x(x-7)}{3(2x^2-13x-7)}$ D: $\mathbb{R} \setminus \{-\frac{1}{2}, 7\}$

$= \frac{3x(x-7)}{3(2x+1)(x-7)}$ VA: $x=-\frac{1}{2}$ hole: $(x=7, y=\frac{7}{15})$

d) $\frac{x(x^2+1)}{x(6x^2+x-1)}$ D: $\mathbb{R} \setminus \{-\frac{1}{2}, \frac{1}{3}, 0\}$ VA: $x=-\frac{1}{2}$, $x=\frac{1}{3}$

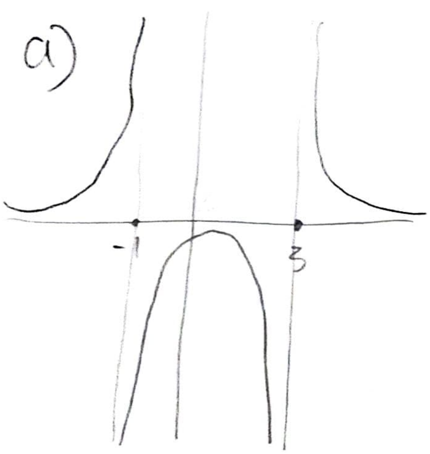
$= \frac{x(x^2+1)}{x(2x+1)(3x-1)}$ hole: $(x=0, y=-1)$

2. Determine, with support, an equation for a rational function of the form y=$\frac{g(x)}{h(x)}$ that satisfies the given conditions.

a. Vertical asymptotes of x=−1 and x=3

b. A hole at (1/3,−2) and a vertical asymptote of x=1

a)



$y = \frac{1}{(x+1)(x-3)}$

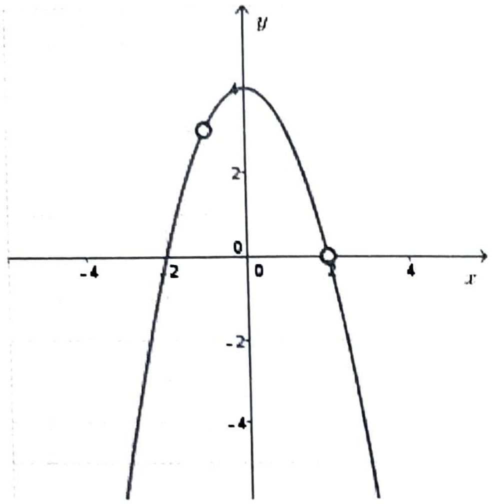
b)

$y = \frac{(x-\frac{1}{3})k}{(x-\frac{1}{3})(x-1)}$

$k = \frac{4}{3}$

$y = \frac{4}{3} \cdot \frac{(x-\frac{1}{3})}{(x-\frac{1}{3})(x-1)}$

3. Determine an equation for the rational function shown in the graph below.



$Y = a(x-0)^2+4$

$= ax^2-4$

$3 = a(-1)^2+4$

$a = -1$

$Y = -x^2+4$

$y = \frac{(4-x^2)(x+1)(x-2)}{(x+1)(x-2}$

4. a. Under what conditions does a rational function have an oblique asymptote?

b. Explain how to determine the equation of the oblique asymptote of a rational function that satisfies the conditions in part a).

c. Which of these functions has an oblique asymptote? Determine the equation of the oblique asymptote, if it exists.

i. $y=\frac{x^2}{x+3}$

ii. $y=\frac{3x}{x^2+1}$

iii. $y=\frac{x^2+4x+5}{x^2-4}$

iv. $y=\frac{2x^2-3x+5}{x-4}$

v. $y=\frac{x^3-1}{x^2-1}$

a) a rational function is obique when the degree of the denominator is 1 less than the numerator

b) The equation can be found with division

i) $x+3 \overline{\smash{)}\, x^2+0}$ with quotient $x-3$

x^2+3x

$-3x+0$

$-3x-9$

9

Asymptote: $y=x-3$

iv) $x-4 \overline{\smash{)}\, 2x^2-3x+5}$ with quotient $2x+5$

$2x^2-8x$

$5x+5$

$5x-20$

25

Asymptote: $y=2x+5$

v) $x^3-1 = (\cancel{x+1})(x^2+x+1)$

$x^2-1 = (x+1)(\cancel{x+1})$

$$\begin{array}{r|rrr} -1 & 1 & 1 & 1 \\ & & -1 & 0 \\ \hline & 1 & 0 & 1 \end{array}$$

$Y=x$

5. By identifying asymptotes and intercepts, match the equation of each function to the most appropriate graph. Justify your choice.

a. $y=\dfrac{1}{x^2+x-6}$

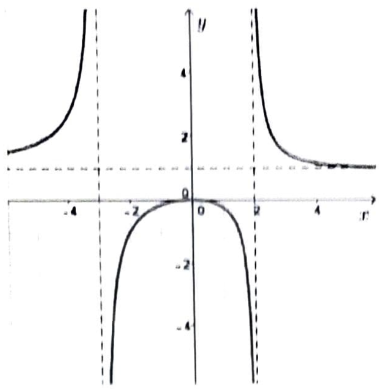
b. $y=\dfrac{x}{x^2+x-6}$

c. $y=\dfrac{x^2}{x^2+x-6}$

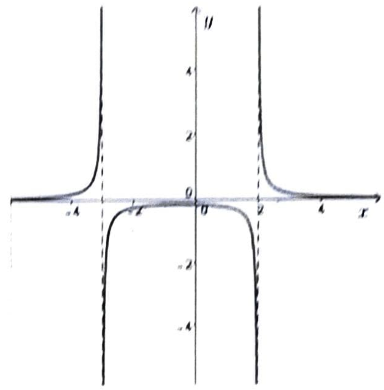
a) $\dfrac{1}{(x-2)(x+3)} \Rightarrow$ ii) No x int

b) $\dfrac{x}{(x-2)(x+3)} \Rightarrow$ iii) x int at 0

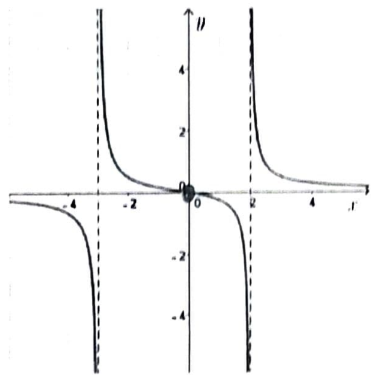
i.



ii.



iii.



c) $\dfrac{f(x)}{g(x)}$ has H.A at $y=0$ when $\deg(g)>\deg(f)$

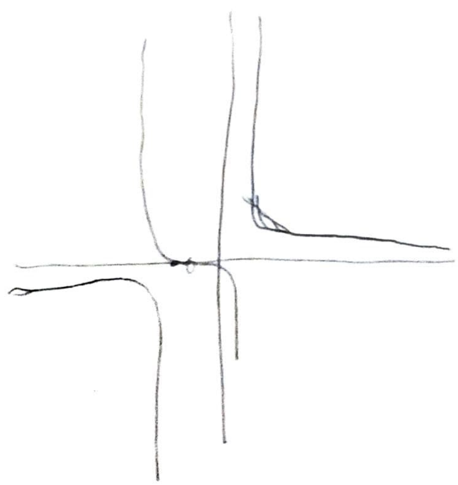
$(\Rightarrow$ i)

6. Complete the comparison table below. Use this information, along with additional points, to sketch the graph of each function.

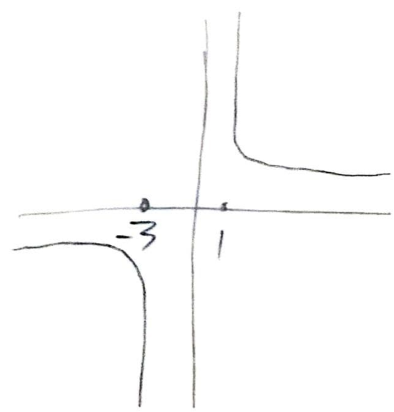
Function	a. $y=\dfrac{x+1}{x^2+2x-3}$ $\quad \dfrac{x+1}{(x+3)(x-1)}$	b. $y=\dfrac{x-1}{x^2+2x-3}$ $\quad \dfrac{x-1}{(x+3)(x-1)}$
Domain	$x\in\mathbb{R}\setminus\{-3,1\}$	$x\in\mathbb{R}\setminus\{-3,1\}$

Vertical Asymptote(s) and/or Points of Discontinuity	$x=3$, $x=1$ VA	$x=3$, $x=1$ VA Hole
Horizontal Asymptote(s) deg: $\frac{1}{2}$ $1<2$	$y=0$	$y=0$
x-intercepts	$x=-1$	none
y-intercepts $f(0)$	$(0, -\frac{1}{3})$	$(0, \frac{1}{3})$
Symmetry (Even/Odd)	none	none

a)



b)



7. For the following functions

- Identify all asymptotes, points of discontinuity and intercepts.
- Discuss the behaviour of the graph of the function near its asymptotes.
- Based on your findings from parts a) and b), along with additional points, sketch a graph of the function.

a. $g(x) = \frac{x^2+3x-8}{x+2}$:

$$\begin{array}{r} x+1 \\ x+2 \,\overline{)\, x^2+3x-8} \\ x^2+2x \\ \hline x-8 \\ x+2 \\ \hline -10 \end{array} \Rightarrow g(x) = (x+1) - \frac{10}{x+2}$$

V.A: $x = -2$

oblique: $y = x+1$

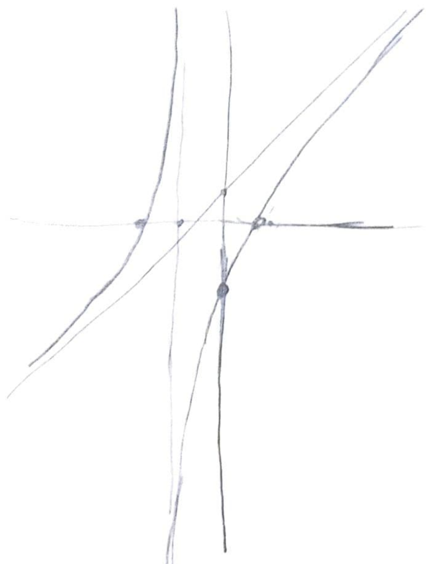
x-int: $-\frac{3}{2} \pm \frac{\sqrt{41}}{2}$

y-int: $g(0) = -4$

$x \to \infty,\ y = x+1$

$x \to -\infty,\ y = x+1$

$x \to -2^-,\ y \to \infty$ $\quad x \to -2^+,\ y \to -\infty$



b) $y = \frac{x^3-1}{x^2+2x}$

$y = \frac{(x-1)(x^2+x+1)}{x(x-2)}$

$$\begin{array}{r} x-2 \\ x^2+2x \,\overline{)\, x^3+0x^2+0x-1} \\ x^3+2x^2 \\ \hline -2x^2+0x \\ -2x^2-4x \\ \hline 4x-1 \end{array}$$

V.A: $x = 0,\ x = 2$

oblique: $y = x-2$

x-int: 1 $\quad y = (x-2) + \frac{4x-1}{x^2+2x}$

y-int: $y(0) = $ undefined

$x \to \pm\infty,\ y = x-2$

$x \to -2^-,\ y \to -\infty$

$x \to -2^+,\ y \to +\infty$

$x \to 0^-,\ y \to +\infty$

$x \to 0^+,\ y \to -\infty$

8. The function $f(x) = \frac{2x^2+ax+b}{5x^2-26x+b}$, where a and b are real numbers, has a point of discontinuity (hole) when $x=6$.

a. Determine the values of a and b.

b. Determine the location of the hole, the x- and y-intercepts, and the equations of the asymptotes of the function.

a)

$2\cdot 6^2 + a\cdot 6 + b = 0$ ①

$5\cdot 6^2 - 26\cdot 6 + b = 0$ ②

② − ①: $3\cdot 6^2 - 26\cdot 6 - a\cdot 6 = 0$

$-48 - 6a = 0$

$6a = -48$

$a = -8$

$b = -24$

b) $f(x) = \frac{2x^2-8x-24}{5x^2-26x-24}$

$= \frac{2(x+2)(x-6)}{(5x+4)(x-6)}$

hole: $(6, y) \Rightarrow (6, \frac{8}{17})$

$f(6) = \frac{8}{17}$

x-int: $-\frac{4}{5}$

V.A: $x = -\frac{4}{5}$

H.A: $y = \frac{2}{5}$