Advanced Functions Class 14 Homework

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Rates of Change

1. Given the function $f(x)=2x^3+3x^2-2x$:

a. Find the average rate of change of the function $f(x)$ from $x=1$ to $x=2$.

b. Find the average rate of change of the function $f(x)$ from $x=1$ to $x=1.1$.

c. Find the average rate of change of the function $f(x)$ from $x=1$ to $x=1.01$.

d. Find the average rate of change of the function $f(x)$ from $x=1$ to $x=1.001$.

e. Using the results from parts b) to d), approximate the instantaneous rate of change of $f(x)$ at $x=1$

a) $\frac{f(2)-f(1)}{2-1} = 21$ b) $\frac{f(1)-f(1.1)}{1-1.1} = 10.92$ c) 10.1 d) 10.0

e) approx. 10

2. An object is fired upward into the air from a platform. The object's height above the ground is modeled by $h(t)=-5t^2+50t+1$ where h is the height in metres, and t is the time in seconds since the object was launched.

a. Determine the average velocity between 4 seconds and 4.1 seconds. That is, determine the average rate of change of height between 4 seconds and 4.1 seconds. Include units.

$\frac{h(4)-h(4.1)}{4-4.1} = 9.5\ m/s$

b. Determine the average velocity between 4 seconds and 4.01 seconds. Include units.

$\frac{h(4)-h(4.01)}{4-4.01} = 9.95\ m/s$

c. Explain how you could approximate the instantaneous rate of change at $x=a$.

You can plug in $\dfrac{h(a)-h(a+k)}{a-(a+k)}$ where k is a very small number

d. Approximate the instantaneous rate of change of the height of the object rounded to three decimal places, or, equivalently, the instantaneous velocity at which the object is moving, 4 seconds after it is launched.

$$\frac{h(4)-h(4.000000001)}{4-4.000000001} \doteq 10.000 \text{ m/s}$$

3. The depth of water, D, in metres, at the end of a pier in Vacation Village, varies with the tides throughout the day and can be modeled by the equation $D(t)=1.5\cos[0.575(t-3.5)]+3.8$, where t is the time of day, measured in hours past 12 am.

a. Find the average rate of change of D from 2 hours to 2.5 hours.

b. Approximate the instantaneous rate of change at $t=2$ hours.

a) $\dfrac{D(2)-D(2.5)}{2-2.5} = 0.565954$ m/h

b) $\dfrac{D(2)-D(2.000000001)}{2-2.000000001} \doteq 0$ m/h

4. Given the function, $f(x)=-x^2-6x+7$:

a. Find the slope of the tangent at $x=-3$.

b. Find the equation of the tangent to the function $f(x)=-x^2-6x+7$ at $x=-3$.

a) $\lim_{h \to 0} \frac{f(h-3)-f(-3)}{h} = \frac{-(h-3)^2-6(h-3)+7-(-(-3)^2-6(-3)+7)}{h}$

$= \frac{-(h^2-6h+9)-6h+25-(-9+18+7)}{h} = \frac{-h^2+\cancel{6h}-9-\cancel{6h}+25-16}{h}$

$= \frac{-h^2}{h} = -h = 0 \qquad m=0$

b) $y=0+b \quad \Rightarrow \quad y=16 \qquad$ or $\qquad y=f(a)+m(x-a)$
$= 16$

5. For the function $f(x)=\frac{1}{x^2}$, find the slope of the tangent at (1,1).

$\lim_{h \to 0} \frac{f(h+1)-f(1)}{h} = \frac{\frac{1}{(h+1)^2}-\frac{1}{1}}{h} = \frac{1-(h+1)^2}{(h+1)^2} \cdot \frac{1}{h}$

$1-(h+1)^2 = (1-(h+1))(1+h+1)$
$= (-h)(h+2)$

$= \frac{-\cancel{(h)}(h+2)}{\cancel{h}(h+1)^2} = -\frac{h+2}{(h+1)^2} = -\frac{0+2}{(0+1)^2} = -2$