First Name:____________________ Last Name: __________________ Student ID: ________

Trigonometric Functions (2)

Reciprocal Functions – Investigation

$\frac{1}{\text{Sin}X} =$ $\frac{1}{\text{Cos}X} =$ $\frac{1}{\text{Tan}X} =$

RECALL: General steps for sketching **f(x) =** $\frac{1}{g(x)}$

1. Sketch the function y=g(x).

2. Identify the values of x where g(x)= 1 or g(x)= −1. At these points f(x)= g(x). That is, these points are on both f(x) and g(x). These points are called **fixed points** or **static points**.

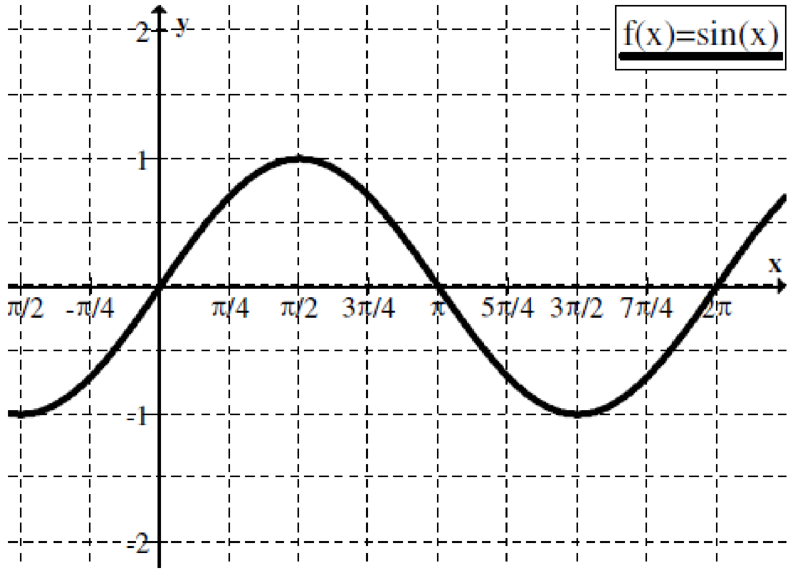
3. Identify the x-intercepts of g(x). At these points, f(x) is undefined. There will be vertical asymptotes for these values of x.

4. If required, determine what happens to the reciprocal function as x approaches the vertical asymptotes from the left and from the right.

5. If required, determine the end behaviour of f(x).

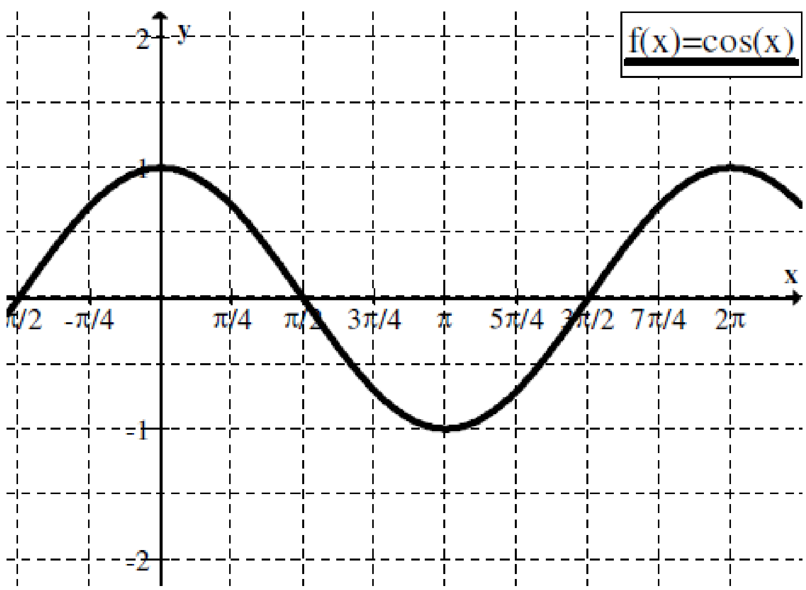
1.

Below is the graph of $y = \sin x$. Recalling that $\csc x = \frac{1}{\sin x}$, sketch the graph of $y = \csc x$ between $x = 0$ and $x = 2\pi$.



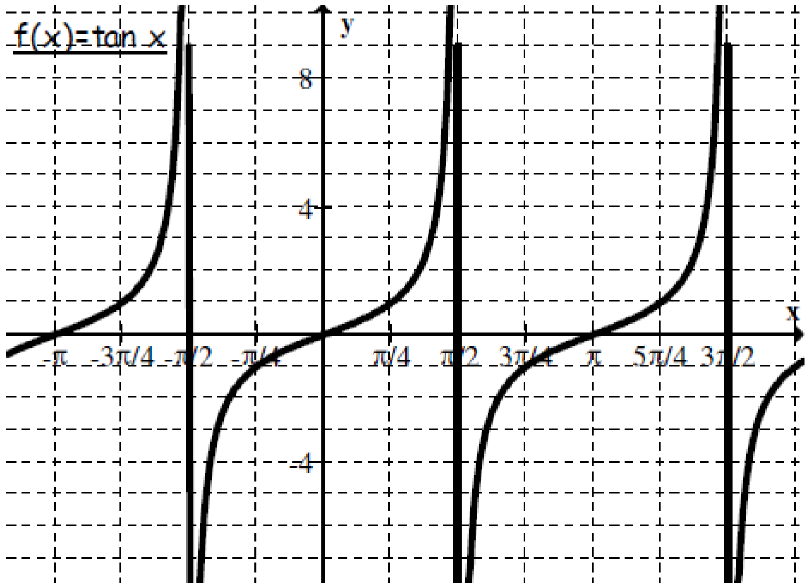
2.

Below is the graph of $y = \cos x$. Recalling that $\sec x = \frac{1}{\cos x}$, sketch the graph of $y = \sec x$ between $x = 0$ and $x = 2\pi$.



3.

Below is the graph of y = tan x. Recalling that cot x = $\frac{1}{\tan x}$, sketch the graph of y = cot x between x = 0 and x = 2 π.



Summary table

Property	y = cscx	y = secx	y = cotx
Domain			
Range			
Period			
Equations of asymptotes on the interval x ∈ [0, 2∏]			

NOTE:
The sine function and the cosine function are referred to as **sinusoidal functions**.
Their graphs have the property that they oscillate above and below a central horizontal line. For both *y* = sin(*x*) and *y* = cos(*x*), this central horizontal line or axis is *y* = 0.

RECALL:
The equation of the horizontal axis is $y = \frac{\text{maximum value} + \text{minimum value}}{2}$.
Amplitude = $\frac{\text{maximum value} - \text{minimum value}}{2}$.

Graphing Trigonometric Functions

y = af[b(x - c)] + d

GENERAL TRIGONOMETRIC FUNCTIONS				
General function	a affects vertical stretch	$b > 0$ affects horizontal stretch	c affects horizontal translation	d affects vertical translation
$y = a\sin(b(x-c)) + d$ $y = a\cos(b(x-c)) + d$	amplitude $= \lvert a \rvert$	period $= \frac{2\pi}{b}$	• $c > 0$ moves the graph right • $c < 0$ moves the graph left	• $d > 0$ moves the graph up • $d < 0$ moves the graph down • principal axis is $y = d$
$y = a\tan(b(x-c)) + d$	amplitude undefined	period $= \frac{\pi}{b}$		

Mapping Notation

$$(x, y) \rightarrow \left(\frac{1}{b}x + c,\ ay + d\right)$$

4. For each of the following equations, state the transformations, period and amplitude of the function.

Equation	**Transformations**	**Period**	**Amplitude**
$y = 3\sin[2(x + \frac{\pi}{4})] - 3$			
$y = \frac{1}{3}\tan[\frac{1}{5}(x - \frac{\pi}{3})] + 10$			
$y = \frac{1}{10}\cos(-3x - \frac{\pi}{2}) - 7$			
$y = -4\sec(\frac{1}{2}x - \pi) + 6$			

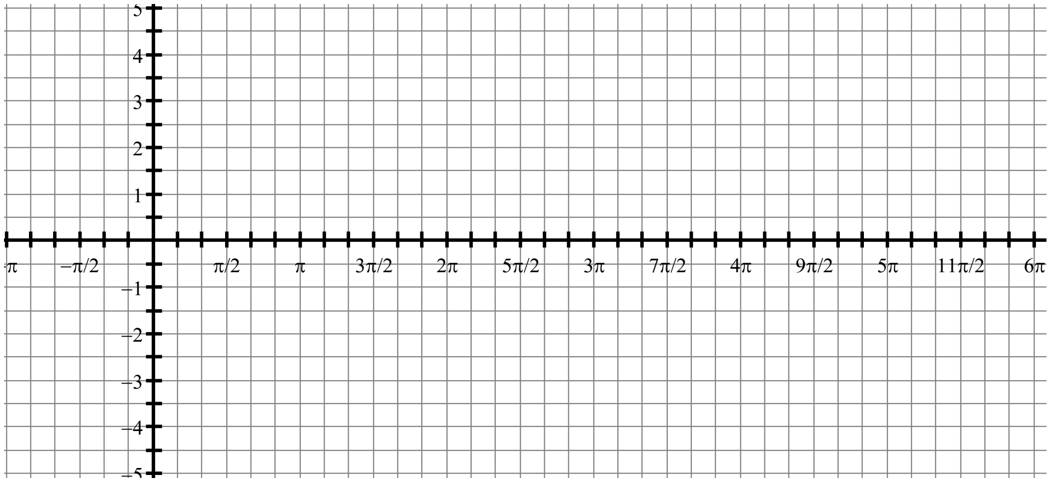
5. Write the equation of **y = cos(x)** if it has undergone the following transformations:

a) Up 6
Left 2π
Vertical Reflection
Vertical Compression of 8
Horizontal Stretch of 6

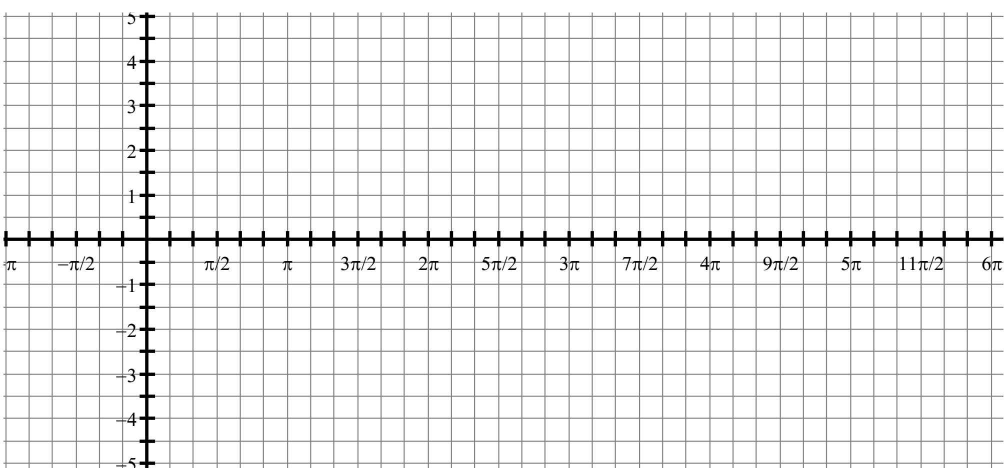
b) Down 3
Right π
Vertical Stretch of 6
Horizontal Compression of 5
Horizontal Reflection

6. Graph one cycle of each of the following functions.

$y = 3\sin[3\,(x - \frac{\pi}{2})] - 1$



$y = -\cos[\frac{1}{2}(x + \frac{5\pi}{6})] + 3$



7. The piston in an engine moves up and down along a crankshaft in the middle. The height of the piston over time is shown by the graph below.

a) How long does it take for the piston to move up and down once?

b) What is the maximum height that the piston reaches?

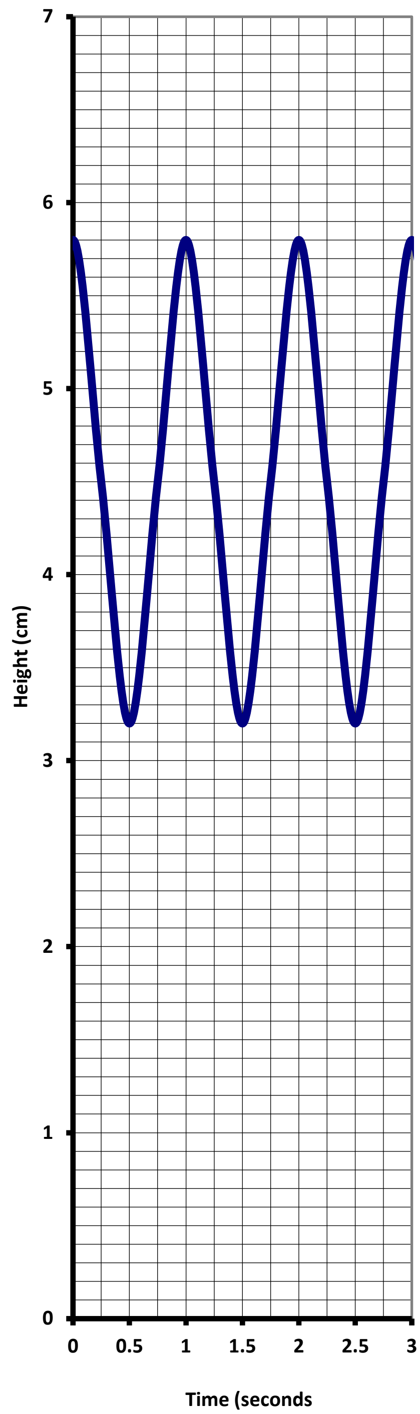
c) What is the lowest height that the piston reaches?

d) How high is the crankshaft?

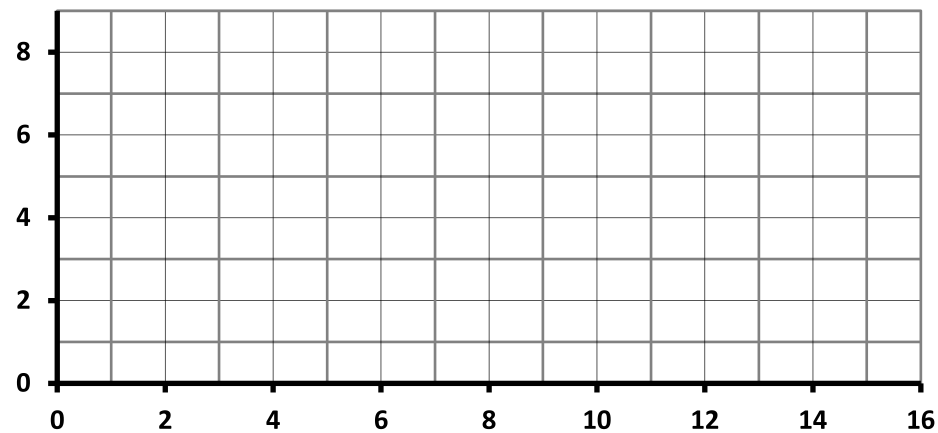
e) Determine the equation of the function.

f) What would you expect to happen to the graph if you revved the engine?

f) What would you expect to happen to the graph if the crankshaft was lower?



8. The water level in ocean harbour is 5 m during low tide and 8 m during high tide. If takes 8 hours to complete one full tide cycle.



a) Sketch two tide cycles starting at high tide.

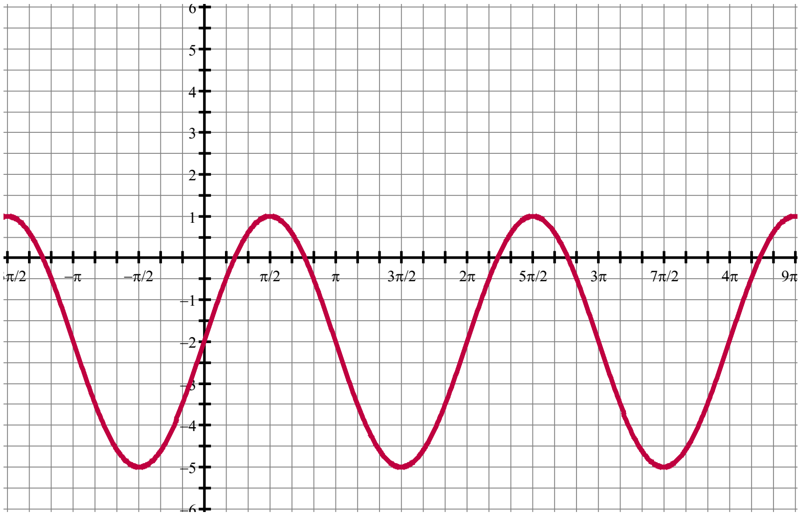
b) Determine an equation for the tide function.

c) If low tide occurs at 8:00 AM, at what time would you expect it to be high tide?

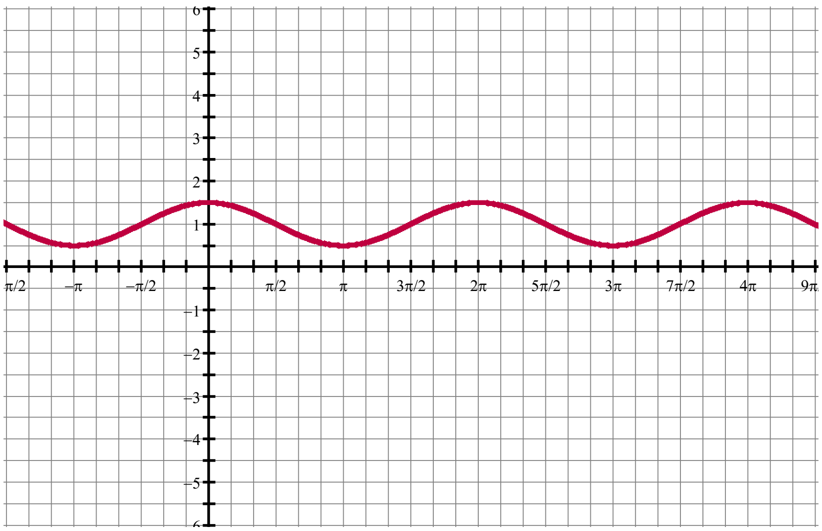
d) If low tide occurs at 8:00 AM, what would you expect the height of the water to be at 6:00 AM the next day?

9. Find the equation of the following function as both a SINE function and a COSINE function.

a)

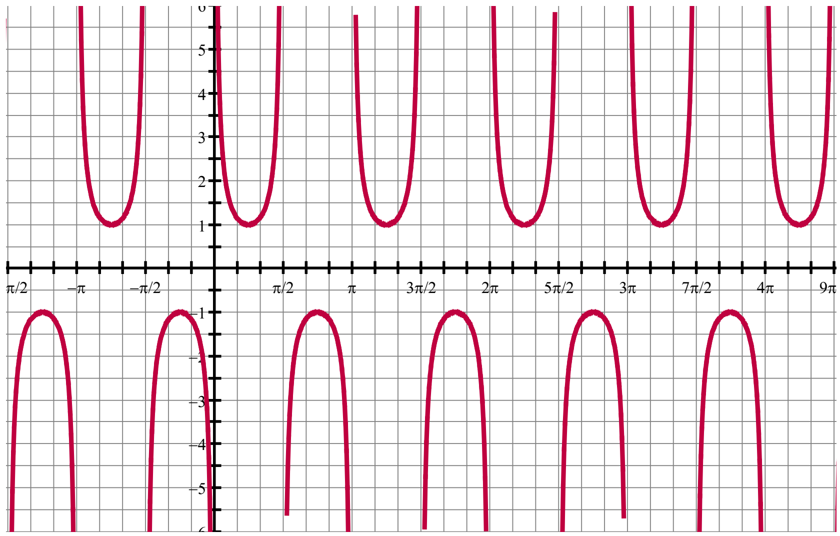


b)

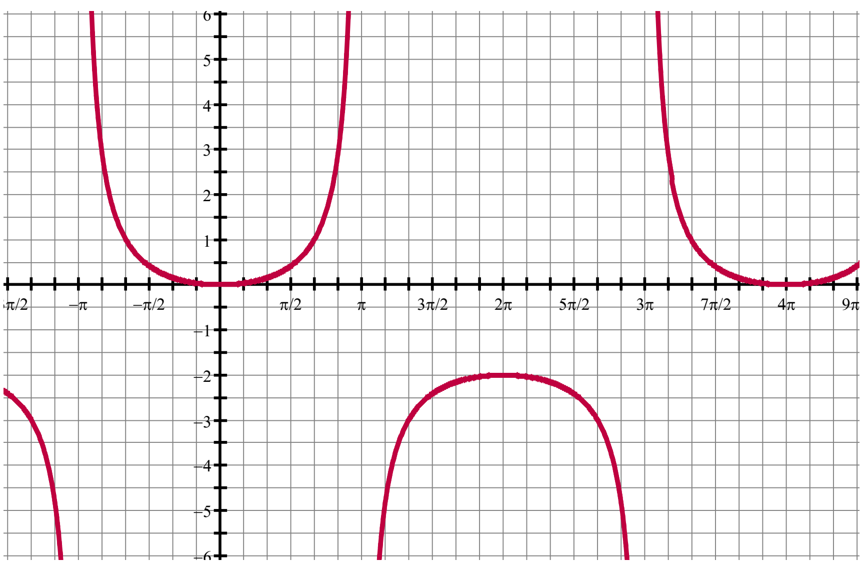


10. Determine the equation for each of the following graphs:

a)



b)



Practice Problems

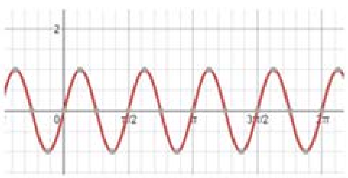
Use what you know about the graphs of the sine and cosine function to match the equations to the graphs

1) $y = \sin 4x$ 2) $y = \sin x + 4$ 3) $y = \sin\left(x + \frac{\pi}{4}\right)$ 4) $y = \cos\left(x + \frac{\pi}{4}\right)$

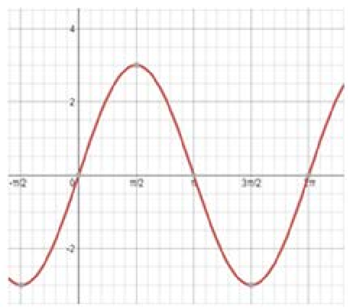
5) $y = \cos\left(x - \frac{\pi}{4}\right)$ 6) $y = -2 + \sin x$ 7) $y = -2 + \cos x$ 8) $y = 3\sin x$

9) $y = 2\cos x$ 10) $y = \cos 2x$ 11) $y = 2\sin 2x$ 12) $y = 3\sin x + 1$

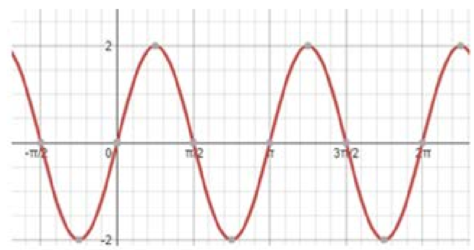
A)



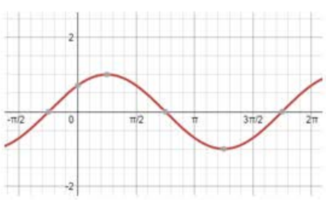
B)



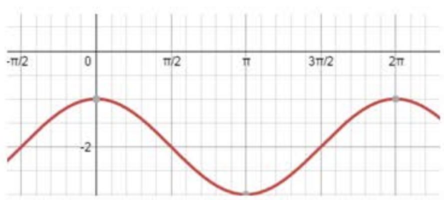
C)



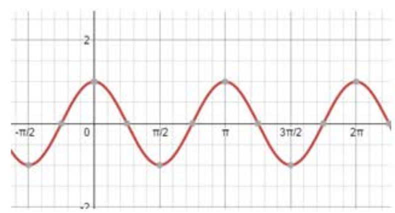
D)



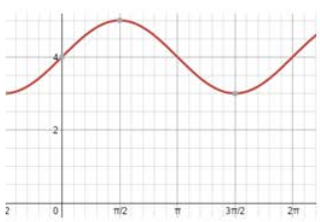
E)



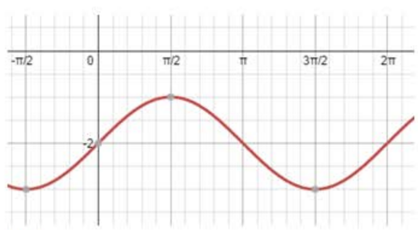
F)



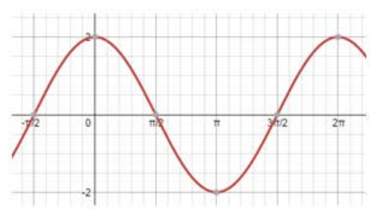
G)



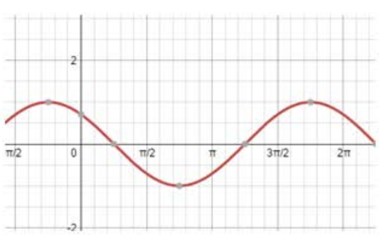
H)



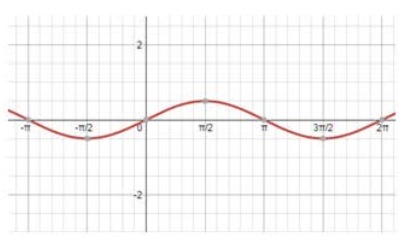
I)



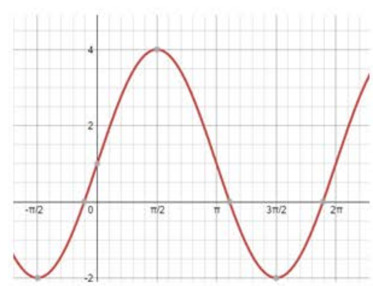
J)



K)



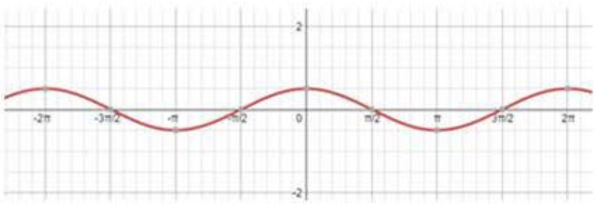
L)



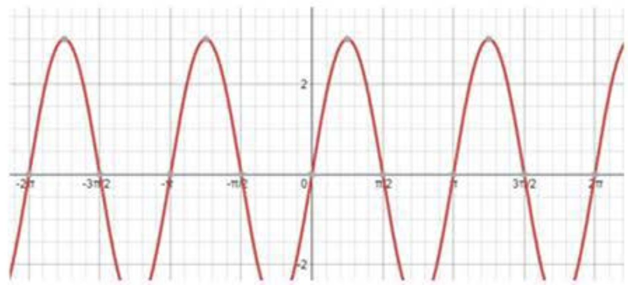
Determine the amplitude and period for each function

13) $y = \sin 4x$ 14) $y = 3\cos(-2x)$ 15) $y = 4\cos x$ 16) $y = 3\sin 2x$

17)



18)



Solutions

Match sine and cosine function to their graph

1) A 2) G 3) K 4) J 5) D 6) H 7) E 8) B 9) I 10) F 11) C 12) L

13) Amplitude: 1

Period: $\frac{\pi}{2}$

14) Amplitude: 3

Period: π

15) Amplitude: 4

Period: 2π

16) Amplitude: 3

Period: π

17) Amplitude: 0.5

Period: 2π

18) Amplitude: 3

Period: π