

## AP Calculus Homework One – Limit and Continuity

1.1 Definitions of Limits; 1.2 Continuity; 1.3 Limits Properties

1. Show that limits do not exist.

(a)  $\lim_{x \rightarrow -2} \frac{x+2}{|x+2|}$

(b)  $\lim_{x \rightarrow 0} \sin \frac{1}{x}$

(c)  $\lim_{x \rightarrow 0} \sqrt{3 + \arctan \frac{1}{x}}$

2. Find limits.

(a)  $\lim_{x \rightarrow 0} \frac{x^2}{2x-1}$

(b)  $\lim_{x \rightarrow 2} \frac{x^3-8}{x^2-4}$

(c)  $\lim_{x \rightarrow -1} \frac{2+2/x}{x^2-4x-5}$

(d)  $\lim_{h \rightarrow 0} \frac{5(h-1)^2 + (h-1) - 4}{h}$

(e) Explain, using examples, when substitution can not be used to solve a limit.

3. Discuss the continuity and sketch the graph of  $f(x) = \begin{cases} \frac{x^2 + x}{x}, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$ .
4. If  $[x]$  is the greatest integer not greater than  $x$ , then  $\lim_{x \rightarrow \frac{1}{2}} [x]$  is  
(A)  $1/2$                       (B)  $1$                       (C) nonexistent                      (D)  $0$                       (E) none of these
5. Find a value of  $k$  such that  $f(x)$  is continuous at  $x = 0$ .  
$$f(x) = \begin{cases} \frac{x^2 - x}{2x}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$$
6. The function  $s(x)$  is defined as follows. Find a value of  $k$  such that  $s(x)$  is continuous for all  $x$ .  
$$s(x) = \begin{cases} 4x - 11, & \text{if } x < 3 \\ kx^2, & \text{if } x \geq 3 \end{cases}$$
7. Discuss the continuity of the graph of  $y = \frac{x^2 - 9}{3x - 9}$ , indicating type of discontinuity if there is one.