Advanced Functions Class 3 Homework

First Name: Adam Last Name: Chen Student ID: _____

Polynomial Equations and Inequalities (1)

1. Express each division in terms of the quotient, divisor, and remainder.

a. $\frac{x^3+x-8x^2+37}{x+4}$, $x\neq -4$

$$\begin{array}{c|cccc} -4 & 1 & -8 & 1 & 37 \\ & & -4 & 48 & -196 \\ \hline & 1 & -12 & 49 & -159 \end{array}$$

$\therefore x^3+x-8x^2+37 = (x+4)(x^2-12x+49)-159$

b. $\frac{x^5-2x^4-7x^3+13x^2+2x-18}{x^2-2x-3}$, $x\neq -1,3$

$$\begin{array}{r} x^3 \quad -4x+5 \quad R:-3 \\ x^2-2x-3 \;\overline{)\; x^5-2x^4-7x^3+13x^2+2x-18} \\ x^5-2x^4-3x^3 \\ \hline -4x^3+13x^2+2x \\ -4x^3+8x^2+12x \\ \hline 5x^2-10x-18 \\ 5x^2-10x-15 \\ \hline -3 \end{array}$$

$\therefore x^5-2x^4-7x^3+13x^2+2x-18 = (x^2-2x-3)(x^3-4x+5)-3$

2. When a polynomial $P(x)$ is divided by $x+3$, the quotient is $3x^2-5x+4$ and the remainder is -10. Find $P(x)$ in standard form.

$P(x) = (x+3)(3x^2-5x+4)-10$

$P(x) = 3x^3-5x^2+4x+9x^2-15x+12-10$

$= 3x^3+4x^2-11x+2$

3. Find the divisor given the divident, quotient, and remainder.

a. The dividend is $3x^3-5x^2-7x-1$, the quotient is $3x^2+4x+5$, and the remainder is 14.

b. The dividend is $2x^4+11x^3+5x^2-31x+7$, the quotient is $2x^2+3x-5$, and the remainder is $-8x+2$

a) $3x^3-5x^2-7x-1 = a(3x^2+4x+5)+14$

$a = (x-3)$

$$\begin{array}{r} x-3 \\ 3x^2+4x+5 \;\overline{)\; 3x^3-5x^2-7x-15} \\ 3x^3+4x^2+5x \\ \hline -9x^2-12x-15 \\ -9x^2-12x-15 \\ \hline 0 \end{array}$$

b) $2x^4+11x^3+5x^2-31x+7 = a(2x^2+3x-5)-8x+2$

$$\begin{array}{r} x^2+4x-1 \\ 2x^2+3x-5 \;\overline{)\; 2x^4+11x^3+5x^2-23x+5} \\ 2x^4+3x^3-5x^2 \\ \hline 8x^3+10x^2-23x \\ 8x^3+12x^2-20x \\ \hline -2x^2-3x+5 \\ -2x^2-3x+5 \\ \hline 0 \end{array}$$

$a = (x^2+4x-1)$

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4. The volume of a cylinder is given by $(\pi x^3+4\pi x^2-3\pi x-18\pi)$ cm^3. If the radius of the cylinder is $(x+3)$ cm, determine the height of the cylinder in terms of x.

$V = \pi r^2 h$

$\frac{V}{\pi} = r^2 h$

$x^3+4x^2-3x-18 = (x+3)^2 h$

$= (x^2+6x+9)h$

$x^2+6x+9 \overline{)\, x^3+4x^2-3x-18}$ quotient: $x-2$

x^3+6x^2+9x

$-2x^2-12x-18$

$-2x^2-12x-18$

0

$h = x-2$

5. When $P(x)$ is divided by $(x+1)$, the remainder is 3. What is the remainder when $xP(x)$ is divided by $(x+1)$?

$P(-1)=3$

let $Q(x) = xP(x)$

$Q(x) \div (x+1)$

$= Q(-1) = xP(x)$

$= (-1)P(-1)$

$= -3$

6. Determine the value(s) of k, $k \in R$:

a. if $x-5$ is a factor of $x^3+2x^2+kx+30$

By factor theorem

$f(5)=0$

$5^3+2(5)^2+5k+30=0$

$5k=-205$

$k=-41$

b. if $2x+3$ is a factor of $2x^3+kx^2-2x+15$

$f(-\frac{3}{2})=0$

$2(-\frac{3}{2})^3+k(-\frac{3}{2})^2-2(\frac{3}{2})+15=0$

$\frac{9}{4}k = -\frac{45}{4}$

$k=-5$

7. State the equation of any cubic polynomial that has a remainder of -6 when divided by $x+3$.

$f(x) = a(x+3)-6$

$f(x) = (x+1)^2(x+3)-6$

$= (x^2+2x+1)(x+3)-6$

$= (x^3+3x^2+2x^2+6x+x+3)-6$

$= (x^3+5x^2+7x+3)-6$

$= x^3+5x^2+7x-3$

8. Find the value of a and b if x^2-5x+4 is a factor of the polynomial $2x^3+ax^2+bx-4$. Express the polynomial in factored form.

$f(x) = (x^2-5x+4)(2x-1)$

$= (x-4)(x-1)(2x-1)$

$= 2x^3 - 10x^2 + 8x - x^2 + 5x - 4$

$= 2x^3 - 11x^2 + 13x - 4 = 2x^3 + ax^2 + bx - 4$

$a = -11$, $b = 13$

9. Given the polynomial $P(x)= 4x^3+x^2-7x+3$

a. Using the rational root theorem, list the potential rational roots of $P(x)= 0$.

b. Show $P(\frac{3}{4}) = 0$. What is a linear factor of $P(x)$?

c. Determine the corresponding quadratic factor.

a) $p: \pm 1, \pm 3$

$q: \pm 1, \pm 2, \pm 4$

$\pm 1, \pm \frac{1}{2}, \pm 3, \pm \frac{1}{4}, \pm \frac{3}{2}, \pm \frac{3}{4}$

b) $P(\frac{3}{4}) = 4(\frac{3}{4})^3 + (\frac{3}{4})^2 - 7(\frac{3}{4}) + 3 = 0$, $r = (x-\frac{3}{4})$

c)

$\frac{3}{4}$	4	1	-7	3
		3	3	-3
	4	4	-4	0

$\Rightarrow 4x^2 + 4x - 4$

10. A polynomial $P(x)$ has a remainder of 3 when divided by $x-2$ and a remainder of -5 when it is divided by $x+2$. Determine the remainder when the polynomial is divided by x^2-4.

$P(2) = 3$, $P(-2) = -5$

$R(x) = 2x-1$

$P(x) = Q(x)(x^2-4) + R(x)$ $\quad R(x) = ax+b$

1) $3 = P(2) = Q(2)(2^2-4) + a(2)+b$

$3 = 2a+b$ ①

2) $-5 = P(-2) = Q(-2)((-2)^2-4) + a(2)+b$

$-5 = -2a+b$ ②

① - ②: $b = -1$

$8 = 4a$

$a = 2$