AP Calculus Homework Six – Applications of Differential Calculus

3.4 Curve Sketching; 3.5 Optimization Problems; 3.6 Local Linear Approximations

1. Find the best approximation, in cubic inches, to the increase in volume of a sphere when the radius is increased from 3 to 3.1 inches.

2. If the side e of a square is increased by 1%, find the increase of the area in terms of e.

3. Sketch a curve for which both $f'(x)$ and $f''(x)$ are negative.

4. Sketch a curve for which $f'(x)$ is negative but $f''(x)$ is positive.

5. What is the area of the largest rectangle that can be drawn with one side along the x-axis and two vertices on the curve $y = e^{-x^2}$?

6. Find the point(s) on the curve $x^2 - y^2 = 4$ closest to the point (6, 0).

7. The sum of the squares of two positive numbers is 200; what is their minimum product?

8. What is the best linear approximation for $f(x) = \tan x$ near $x = \pi/4$?

9. If $f(6) = 30$ and $f'(x) = x^2/(x + 3)$, what is the estimate of $f(6.02)$, using the local linearization?

10. Find the tangent line approximation for $f(x) = \sqrt{x^2 + 16}$ near $x = -3$.