First Name:____________________ Last Name: __________________ Student ID: ________

Test 1 /47

Show your work!

Time: 75 minutes

1. Consider the following two polynomial functions (4 marks)

$P(x) = x^3 - 2x^2 + 3x - 1$ and $Q(x) = x^2 - 2x$.

Compute the required operations:

a. $P(x) + Q(x)$

b. $P(x)Q(x)$

c. Use the long division algorithm to get the quotient and the remainder for $\frac{P(x)}{Q(x)}$

2. Use the remainder theorem to find if $2x + 1$ is a factor of $P(x) = x^3 - 2x^2 + 2x - 3$ (2 marks)

3. Solve each of the following. (12 marks)

$6x^3 - 19x^2 + x + 6 = 0$	$x^4 - 12x^2 + 12 = 0$
$\frac{-3x+6}{4x-8} = -\frac{3}{4}$	$\frac{x+2}{3} + \frac{x-3}{3x-12} = \frac{1}{3x-12}$

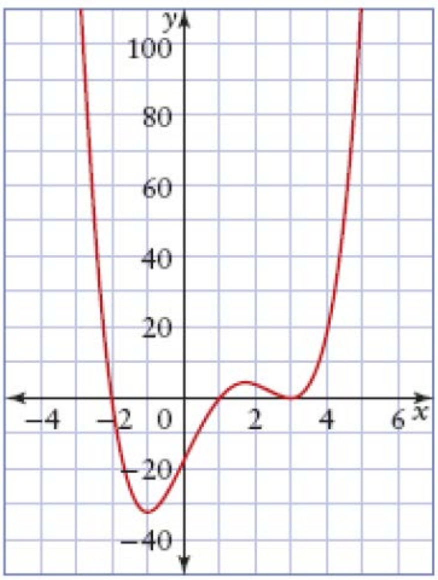
(x-1)(2x-8)$(x-3)^2 \leq 0$	$\frac{x+3}{x^2-4} > 0$

4. If f(x) = $mx^3 + gx^2 - x + 3$ is divided by x + 1, the remainder is 3. If f(x) is divided by x + 2, the remainder is -7. What are the values of m and g? (3 marks)

5. (3, 5) is a point on the graph of $f(x)$. Find the corresponding point on the graph of the function $y = 3f(-x + 1) + 2$. (2 marks)

6. (a) Find a polynomial P(x) of degree five with zeros 1, -1, 2, -2, and 3 such that its graph passes through the point (0, 24). (3 marks)

(b) Find a possible equation for the graph bellow. (3 marks)



(c) Find a cubic polynomial function f(x) that has two zeros at 1 and 2, and also f(0) = 1

and f(-1) = 4. (3 marks)

(8 marks)

7. Sketch the graph of the following functions. Specify the x- and y-intercepts and the asymptotes.

a. h(x) = $\frac{x^2-1}{x+2}$

b. h(x) = $\frac{x^4-2x^3-3x^2+4x+4}{x^2-x-2}$

(4 marks)

8. Sketch a possible graph of a polynomial function that satisfies the following conditions:

a. A quintic function with a positive leading coefficient, a zero at $x=-2$, and a second zero at $x=1$ of multiplicity 4.

b. Degree five, positive leading coefficient, one x -intercept, four turning points

(3 marks)

9. Find constants a and b that guarantee that the graph of the function defined by

$f(x) = \frac{ax^2+6}{16-bx^2}$ will have vertical asymptotes of $x = \frac{2}{3}$ and $x = -\frac{2}{3}$, and a horizontal asymptote of

$y = -2$.

Bonus question (3 marks)

1. An *Indicator function*, *f*(*x*), is a function that takes the value 1 if some condition on *x* is true, and the value 0 if the condition is not true. For integers *x*, define the indicator functions:

f(x) = $f(x) = \begin{cases} 1, & if\ x\ is\ divisible\ by\ 2 \\ 0, & if\ x\ is\ not\ divisible\ by\ 2 \end{cases}$

and

g(x) = $g(x) = \begin{cases} 1, & if\ x\ is\ divisible\ by\ 3 \\ 0, & if\ x\ is\ not\ divisible\ by\ 3 \end{cases}$.

a. Find *f*(2), *f*(9), *f*(2017), *g*(2), *g*(9), *g*(2017).
b. Let *h*(*x*) = *f*(*x*)*g*(*x*). Determine whether *h*(*x*) is an indicator function, and, if so, describe in words the condition it indicates.
c. Let *k*(*x*) = $\left[\frac{f(x)+g(x)+1}{2}\right]$. Determine whether *k*(*x*) is an indicator function, and, if so, describe in words the condition it indicates.(where $[x]$ represents the integer part of x)

2. Graph the following relations $|x-y| \leq 2$. (3 marks)