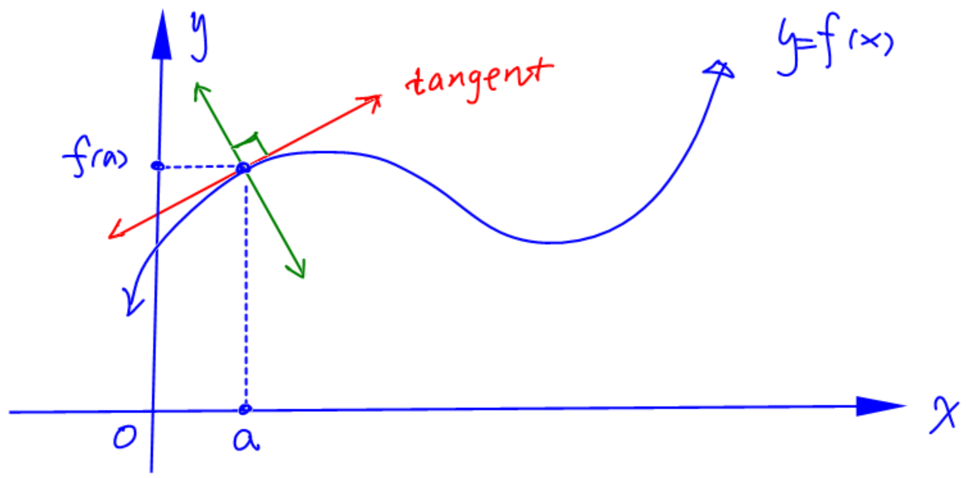
Lesson 5.

Applications of Derivatives.

For a function $y=f(x)$.

(1) $f'(a)$ is the slope of tangent to the curve of $y=f(x)$ at $x=a$.



Equation of the tangent at $x=a$:

$$y-f(a)=f'(a)(x-a).$$

Equation of the normal line at $x=a$:

$$y-f(a)=-\frac{1}{f'(a)}(x-a).$$

(2) Increasing Interval and Decreasing Interval for a function, say $y=f(x)$.

If $f'(x)>0$ for $a<x<b$,

then $f(x)$ is increasing for $a<x<b$,

or $a<x<b$ is an increasing interval for $y=f(x)$.

If $f'(x)<0$ for $a<x<b$,

then $y=f(x)$ is decreasing for $a<x<b$,

or $a<x<b$ is a decreasing interval for $y=f(x)$.

If $f'(c)=0$ or $f'(c)$ D.N.E. then

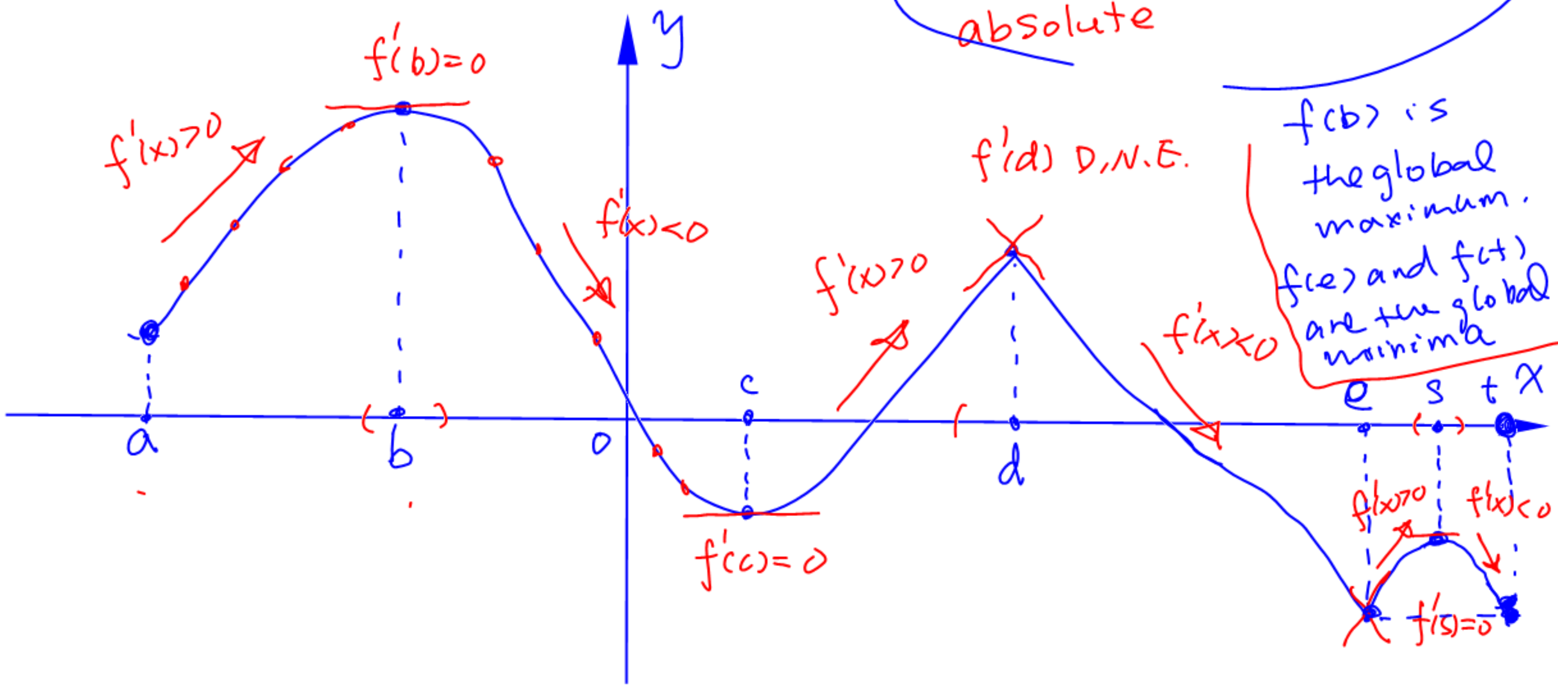
$x=c$ is a critical number of $f(x)$.

or $(c, f(c))$ is a critical point of $f(x)$.

Most Likely, $f(c)$ is a local maximum or local minimum. (or just local extremum)

For example,

global / absolute max. or min.



$a<x<b \cup c<x<d \cup e<x<s$ are increasing intervals

or $x \in (a,b) \cup (c,d) \cup (e,s)$

$x \in (b,c) \cup (d,e) \cup (s,t)$ are decreasing intervals

(3) Second Derivatives and Concavity.

Points of Inflection. (P.O.I.s)

If $f''(x) > 0$ for $a < x < b$,

then $y = f(x)$ is concave upward for $a < x < b$.
(C.U.)

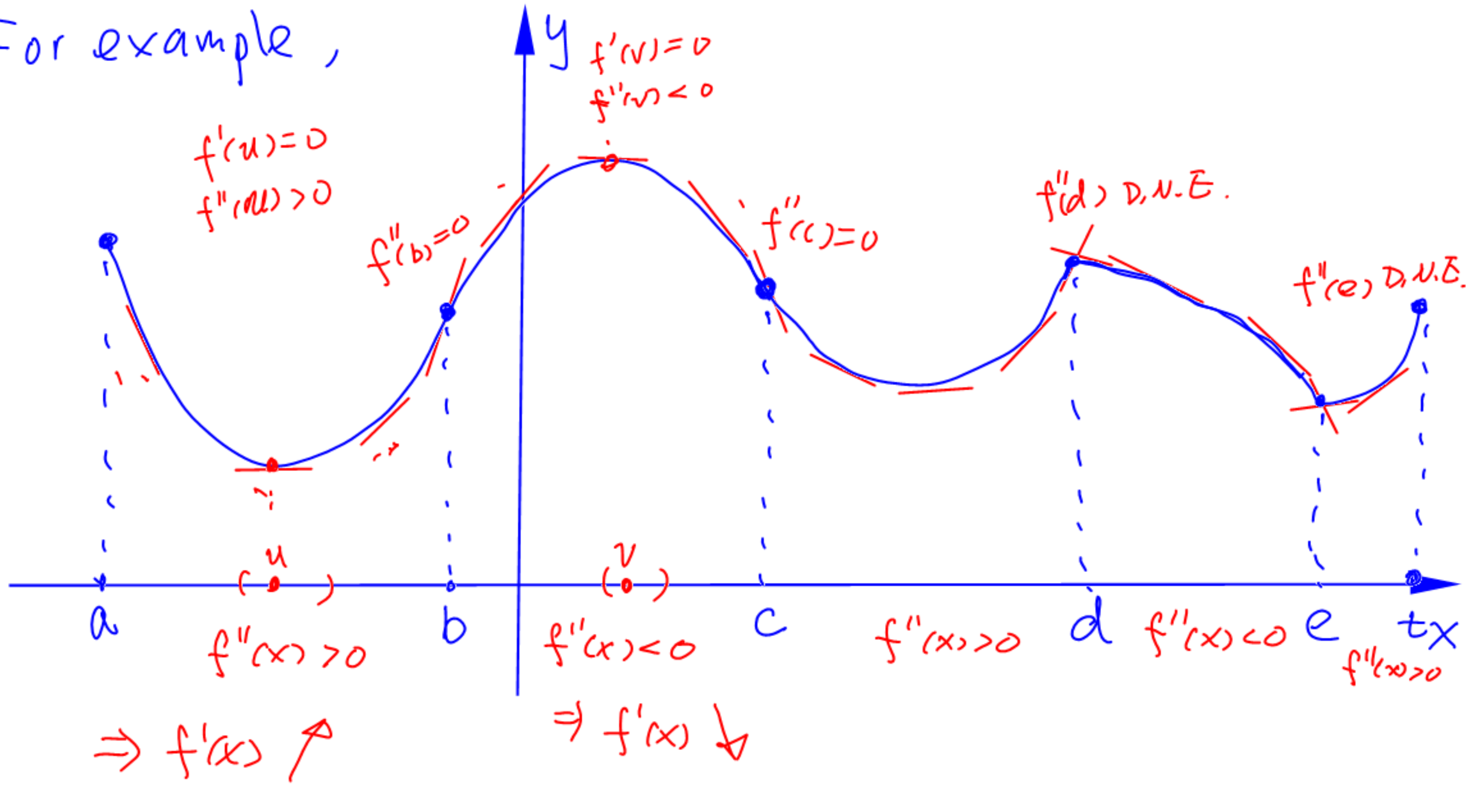
If $f''(x) < 0$ for $a < x < b$.

then $y = f(x)$ is concave downward for $a < x < b$.
(C.D.)

If $f''(p) = 0$ or $f''(p)$ D.N.E. and $f''(x)$ changes sign on the two sides of $x = p$.

then $(p, f(p))$ is a P.O.I.,

For example,

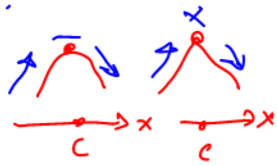


Two Testing Rules for Local Extrema:

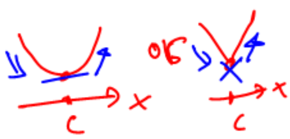
① The First Derivative Test.

If $f'(c)=0$ or D.N.E. and

i) if $f'(x)$ changes sign from $+$ to $-$ at $x=c$,
then $f(c)$ is a local max.



ii) if $f'(x)$ changes sign from $-$ to $+$ at $x=c$,
then $f(c)$ is a local min.



iii) if $f'(x)$ does not change sign at $x=c$.
then $f(c)$ is neither local max. nor local min.

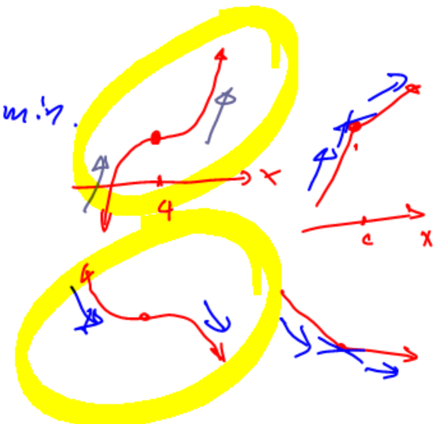
$f(x) = (x-4)^3$

$f'(x) = 3(x-4)^2$

$f'(4) = 3(4-4)^2 = 0$

$f''(x) = 6(x-4)$

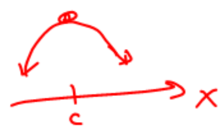
$f''(4) = 6(4-4) = 0$



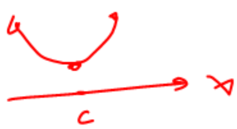
② The Second Derivative Test

If $f'(c)=0$ and

(i) if $f''(c)<0$, then $f(c)$ is a local max.



(ii) if $f''(c)>0$, then $f(c)$ is a local min.



(iii) if $f''(c)=0$ or D.N.E.
then no conclusion about $f(c)$.
we need to use the first derivative test.