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## Chapter 4: Intro to Probability Distribution, Uniform and Binomial Distribution

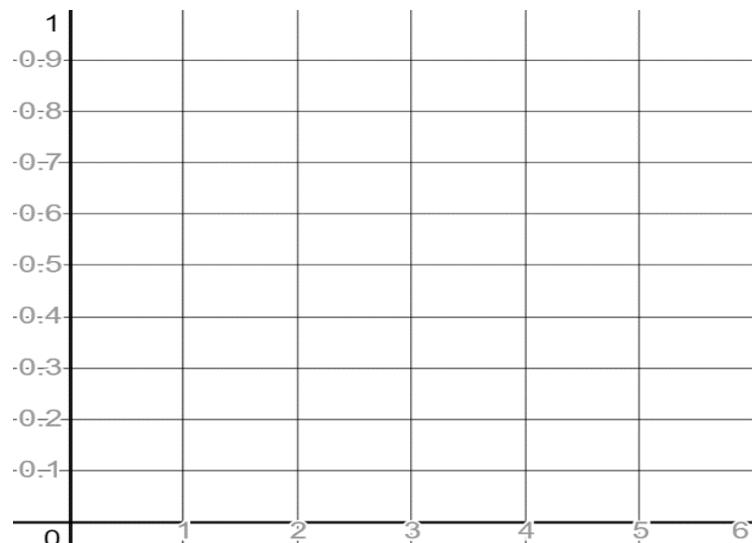
### ➤ Probability Distributions

In the previous chapter, we looked at probabilities of individual outcomes or events for an experiment. A **probability distribution** involves the probabilities for *all possible outcomes* of an experiment, often shown as a table or as a graph.

For a probability experiment, a **random variable** must be defined. What is a random variable?

**Ex. 1:** Graph the **probability distribution** for the **Random Variable,  $X$** , which is the “face-up number” displayed when rolling a standard die a single time.

| Random Variable,<br>$X = x_i$ | Probability,<br>$P(X = x_i)$ |
|-------------------------------|------------------------------|
|                               |                              |
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### ➤ uniform probability distribution

If all values in a distribution are *equally likely*, then it is called a **uniform probability distribution**.

#### Probability Function of a Discrete Uniform Distribution

$$P(X = x) = P(x) = \frac{1}{n}$$

where  $n$  is the number of possible values of the random variable  $X$

Random variables can be **discrete** or **continuous**. **Discrete variables** can only take on specific values within a given range, while **continuous variables** have an infinite number of possible values in an interval. For example, the driving time between Toronto and North Bay is found to range evenly between 195 and 240 minutes.

➤ Expected Value of a Random Variable

The **expected value (expectation)** of a random variable  $X$ ,  $E(X)$ , is the “predicted average” of all possible values of the probability experiment. The expectation is equal to the sum of the products of each value (random variable  $X = x_i$ ) with its probability,  $P(X = x_i)$ .

**Expected Value of a Random Variable X**

$$E(X) = \sum_{i=1}^n x_i P(X = x_i)$$

where  $n$  is the number of possible values of the random variable  $X$

**Ex.2:** Consider a simple game in which you draw a card from a standard deck minus the face card. When you draw

- an even club, you gain 2 points, an odd club, you lose 1 point
- an even diamond, you gain 4 points, an odd diamond, you lose 3 points
- an even spade, you gain 6 points, an odd spade, you lose 5 points
- an even heart, you gain 8 points, an odd heart, you lose 7 points

a) Graph the probability distribution of number of points scored on a single draw for this game.

b) Calculate the expected value of the number of points scored on a single draw.

c) Is this a uniform discrete distribution?

d) Is this a fair game? Why?

| Points Scored,<br>$x_i$ | Probability,<br>$P(X = x_i)$ |
|-------------------------|------------------------------|
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## ➤ Binomial Distribution

A **binomial experiment** is one that possesses the following properties:

The number of successes,  $X$ , in  $n$  trials of a binomial experiment is called a **binomial random variable**.

Probability Function of a Binomial Distribution  $X \sim B(n, p)$

$$P(X = x) = P(x) = C(n, x)p^x q^{n-x}$$

where  $n$  is the number of independent trials in the binomial experiment,

$p$  is the probability of success on any individual trial,

and  $q = 1 - p$  is the probability of failure.

The **binomial probability distribution** can be created by finding the probabilities for all values of  $x$  from \_\_\_\_\_.

**Ex.3:** A die is tossed 5 times. What is the probability of:



- d) Create a probability distribution for the number of threes in this experiment and find its expected value.

Since the probability of success for each trial is the same and each trial is independent, the **expected value (expectation)**,  $E(X)$ , for a binomial distribution is (*probability of success for each trial*)  $\times$  (*the number of trials*).

**Expectation for a Binomial Distribution  $X \sim B(n, p)$**

$$E(X) = np,$$

where  $n$  is the number of independent trials and  $p$  is the probability of success for each trial.

**Ex.4:** Hospital records show that of patients who suffer from a certain disease, 75% die as a result.

- a) What is the probability that in a sample of 20 randomly selected patients, 7 will recover?
  
  
  
  
  
  
  
  
- b) What is the expected number to recover in a sample of 20 randomly selected patients?

**Ex.5:** A manufacturer of metal pistons finds that on the average, 3% of its pistons are rejected because they are either oversize or undersize.

- a) What is the expected number of rejects in a batch of 125 pistons?
- b) What is the probability that the batch contains
  - i) no more than 5 rejects?
  - ii) at least 2 rejects?

**Ex.6:** The French mathematician Simeon-Denis Poisson (1781–1840) developed what is now known as the Poisson distribution. This distribution can be used to approximate a binomial distribution where  $p$  is very small and  $n$  is very large. It uses the formula  $P(x) = \frac{e^{-np}(np)^x}{x!}$ , where  $e$  is the irrational number 2.718 28 ... Use the Poisson distribution to approximate the following situations.

- a) A certain drug is effective in 98% of cases. If 2000 patients are selected at random, what is the probability that the drug was ineffective in exactly 10 cases?
- b) On election day, only 3% of the population voted for the Environment Party. If 1000 voters were selected at random, what is the probability that fewer than 4 of them voted for the Environment Party?