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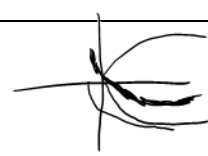
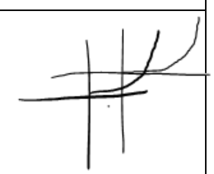
Functions: Transformations and Properties

1. For each relation given,

a. state the domain and range;

b. identify whether the relation is a function or not.

i. {(1,1),(2,1),(3,2),(4,3),(5,5),(6,8)} a) d: $\{1,2,3,4,5,6\}$ r: $\{1,2,3,5,8\}$ b) function	ii. x = -2 a) d: $\{-2\}$ r: $\mathbb{R}$ b) relation
iii. y =$(x+1)^2-2$  a) d: $\mathbb{R}$ r: $[-2, \infty)$ b) function	iv. $(x-1)^2+y^2= 9$ $y^2 = -(x-1)^2+9$ $-3 \le x-1 \le 3$ $-2 \le x \le 4$ a) d: $[-2,4]$ r: $[-3,3]$ b) relation
v. $y = -3\sqrt{x+2} + 5$  a) D: $[-2, \infty)$ R: $(-\infty, 5]$ b) function	vi. $y = 2^{x-4} + 3$  a) d: $\mathbb{R}$ r: $(3, \infty)$ b) function

2. If $f(x)=2x-3$ and $g(x)=6x^2+3x-18$, determine the value(s) of x such that $f(x)=g(x)$.

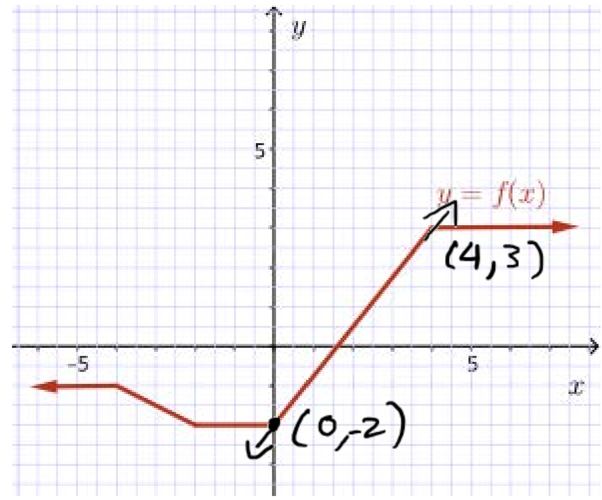
$2x-3 = 6x^2+3x-18$

$6x^2 \quad x-15=0$

$(3x+5)(2x-3)=0$

$x=-\frac{5}{3}$ or $x=\frac{3}{2}$

3. The graph of $y=f(x)$ is shown.



Determine the following:

a. the value of $f(0)$
b. the value of x such that $f(x)=0$
c. the value of $f(4)-f(-4)$
d. state the domain and range

a) $f(0)=-2$

b) $y=\frac{3+2}{4-0}x-2 \quad y=\frac{5}{4}x-2$

$0=\frac{5}{4}x-2, \quad x=\frac{8}{5}$

c) $f(4)=\frac{1}{2} \qquad f(4)-f(-4)$

$f(-4)=2 \qquad =\frac{1}{2}-2=-\frac{3}{2}$

d) D: $\mathbb{R}$

R: $[-2,3]$

4. Given $f(x)=(x-3)^2$ and $g(x)=4x+3$, determine in simplest form

a. $f(x)-g(x)$

$(x-3)^2-4x-3$
$x^2-6x+9-4x-3$
$x^2-10x+6$

b. $2f(x)$

$2(x-3)^2$
$2(x^2-6x+9)$
$2x^2-12x+18$

c. $f(x)g(x)$

$(x-3)^2(4x+3)$
$(x^2-6x+9)(4x+3)$
$4x^3+3x^2-24x^2-18x+36x+27$
$=4x^3-21x^2+18x+27$

d. $f(g(x))$

$(4x+3-3)^2$
$=(4x)^2$
$=16x^2$

e. $g(g(x))$

$4(4x+3)+3$
$16x+12+3$
$16x+15$

f. $[g(x)]^2$

$[4x+3]^2$
$=16x^2+24x+9$

5. If $f(x)=5-2x+k$ and $f(f(k))=13$, determine the value of $f(-4)$.

$f(f(k)) \Rightarrow 5-2(5-2k+k)+k=13$
$5-2(5-k)+k=13$
$5-10+2k+k=13$
$-5+3k=13$
$3k=18$
$k=6$

$f(x)=5-2x+6$
$=11-2x$
$f(-4)=11-2(-4)$
$=11+8$
$=19$

$9x^2 - 6x + 5$

$(-3x+1)(-3x+5)$

6. If $g(x)=1-3x$ and $f(g(x))=9x^2-6x+5$, determine the value of $f(5)$.

$g(x) = 1-3x$

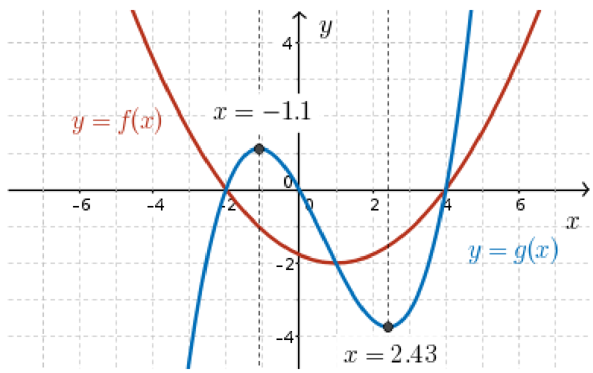
$g(x) = 5: \quad 5 = 1-3x$

$4 = -3x \quad x = \frac{4}{-3}$

$f(\frac{4}{-3}) = 9(\frac{4}{-3})^2 - 6(-\frac{4}{3}) + 5$

$= 29$

7. Given the graphs of $y=f(x)$ and $y=g(x)$ as shown in the graph below



identify the following.

a. where $f(x)=g(x)$
b. the interval(s) where $g(x)<0$
c. the interval(s) where $f(x)\geq g(x)$
d. the interval(s) where both functions are decreasing.
e. the local maxima and minima for both functions

a) Points where $f(x) = g(x)$
{(-2, 0), (1, -2), (4, 0)}

b) $(-\infty, -2) \cup (0, 4)$

c) $(-\infty, -2] \cup [1, 4]$

d) $(-1.1, 1)$

e) f(x) - Local minima: {(1,-2)}
g(x) - Local minima: {(2.43,-3.6)}
Local maxima: {(-1.1, 1)}

8. Solve. Write your answers using interval notation.

a. $-3 < \frac{2x+5}{3} \le 5$

$\frac{2x+5}{3} \le 5$

$-3 < \frac{2x+5}{3}$

$2x+5 \le 15$

$-9 < 2x+5$

$2x \le 10$

$-14 < 2x$

$x \le 5$

$-7 < x$

$x: (-7, 5]$

b. $\frac{3}{x-2} > 1$

$x-2>0$

$3 > x-2$

$x-2<0 \to x<2$

$3<x-2 \quad 5<x$ X

$(2, 5)$

c. $\frac{3x-6}{4x-8} \le 1$

$\frac{3\cancel{(x-2)}}{4\cancel{(x-2)}} \le 1 \quad \frac{3}{4} \le 1$ ✓

$x-2 \ne 0$

$x \ne 2$

$(-\infty, 2) \cup (2, \infty)$

d. $x(x+3)(x-4) > 0$

roots: $\{-3, 0, 4\}$

Test by plugging in

$-4: <0$

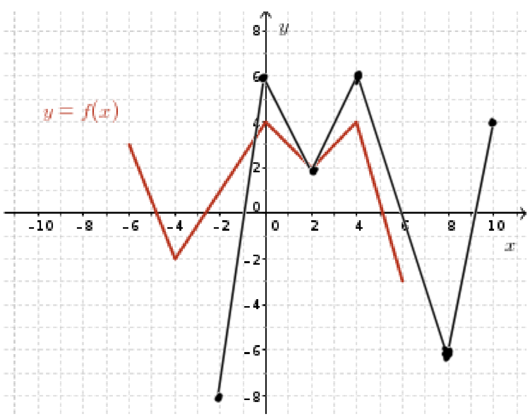
$-2: >0$

$1: <0$

$5: >0$

$(-3, 0) \cup (4, \infty)$

9. Given the graph of the function $y=f(x)$, draw the graphs of the following transformed function $y= 2f(-(x-4))-2$:



$(x, y) \to (-x+4, 2y-2)$

10. The function $f(x)$ satisfies the equation $f(x)=f(x-1)+f(x+1)$ for all values of x. Define $f(1)=1$ and $f(3)=3$; then, $f(2)=1+3=4$. Determine the value of $f(1867)$.

$f(1) = f(0) + f(2)$

$f(1) = f(0) + 4 = 1$

$f(0) = -3$

$f(3) = 4 + f(4) = 3$

$f(4) = 3 - 4 = -1$

$|-3, 1, 4, 3, -1, -4, |-3, 1, 4 \ldots$

$\;\;0 \;\; 1 \;\; 2 \;\; 3 \;\; 4 \;\;\; 5 \;\;\;\; 6 \;\; 7 \;\; 8$

$f(n) = f(n-1) - f(n-2)$

$f(1867) = f(1867 \bmod 6) = f(1) = 1$