

First Name: _____ Last Name: _____ Student ID: _____

Test 2**/48****Question 1-8 are Multiple-Choice questions (8 marks)****1.** The exact radian measure of 75° is:

- a) $\frac{5}{12}\pi$ b) $\frac{7}{12}\pi$ c) $\frac{5}{8}\pi$ d) $\frac{13}{12}\pi$

2. The equation of a cosine function with an amplitude of 3, a period of 4π , and a phase shift of $\frac{\pi}{2}$ to the left is:

- a) $y=3\cos(\frac{1}{2}x)+\frac{\pi}{2}$ b) $y=3\cos 4\pi(x + \frac{\pi}{2})$ c) $y=3\cos\frac{1}{2}(x + \frac{\pi}{2})$ d) $y=4\pi \cos 3(x + \frac{\pi}{2})$

3. The expression $\sin^2 x + \cos^2 x + \tan^2 x$ is equivalent to:

- a) 1 b) $\sec^2 x$ c) $\csc^2 x$ d) $1 + \tan x$

4. The expression $\sin\frac{\pi}{5}\cos\frac{\pi}{6} - \sin\frac{\pi}{6}\cos\frac{\pi}{5}$ is equivalent to:

- a) $\cos\frac{11}{30}\pi$ b) $\sin\frac{11}{30}\pi$ c) $\cos\frac{\pi}{30}$ d) $\sin\frac{\pi}{30}$

5. Which set of the values is the solution for $\sin x = -\frac{1}{3}$, $0 \leq x \leq 2\pi$:

- a) -0.34, 2.80 b) 5.94, 3.48 c) 1.91, 4.37 d) -0.34

6. Which set of the values is the solution for $(2\sin(x) + 1)(\cos(x) - 1) = 0$, $0 \leq x \leq 2\pi$:

- a) $\pi, \frac{7\pi}{6}, \frac{11\pi}{6}$ b) $\frac{\pi}{6}, \pi, \frac{5\pi}{6}$ c) $0, \frac{7\pi}{6}, \frac{11\pi}{6}, 2\pi$ d) $0, \frac{5\pi}{6}, \frac{7\pi}{6}, 2\pi$

7. A co-terminal angle of $\alpha = \frac{5\pi}{4}$ is:

a) $\frac{\pi}{4}$

b) π

c) $\frac{3\pi}{4}$

d) $-\frac{3\pi}{4}$

8. The exact value of $\log_{\sqrt{2}} \sqrt[5]{8}$ is:

a) $\frac{5}{8}$

b) 3

c) $\frac{6}{5}$

d) $\frac{3}{10}$

Questions 9-14 are long answer questions. Show your work to get full marks.

9. Prove the following trigonometric identities:

(9 marks)

a) $\sin x \cot^2 x + \cos x \tan^2 x = \frac{\sin^3 x + \cos^3 x}{\sin x \cos x}$

b) $\frac{\sin(a-b)}{\cos a \cos b} + \frac{\sin(b-c)}{\cos b \cos c} + \frac{\sin(c-a)}{\cos c \cos a} = 0$

c) $\frac{\cos 2x}{1 + \sin 2x} = \frac{1 - \tan x}{1 + \tan x}$

10. Write each of the following as a single logarithm: (9 marks)

i. $\frac{1}{3} \log_a x + \frac{1}{4} \log_a y - \frac{2}{5} \log_a z$ ii. $(4 \log_5 x - 2 \log_5 y) \div (3 \log_5 z)$

iii. $\frac{1}{\log_a 10} + \frac{1}{\log_b 10} + \frac{1}{\log_c 10}$

11. Radium (Ra-225) has a half-life of 15 days. For a 100 g sample, the amount of radioactive material remaining, A , after time t , is given by the equation $A(t) = 100(0.5)^{\frac{t}{15}}$, where A is measured in grams and t is measured in days. (4 marks)

a. Find the average rate of change from 3 days to 4 days, rounded to one decimal place.

b. Approximate the average rate of change at $t=3$ days, rounded to one decimal place.

12. Solve the following equations: (6 marks)

a. $5^{2x-1} = \sqrt{5}$

b. $9^x - 2(3^x) - 15 = 0$

c. $\ln(x) + \ln(x-1) = 0$

13. Evaluate the following expressions: (4 marks)

a. $27^{\log_3(90) - \log_3(18)}$

b. $4\sin^2\left(\frac{\pi}{8}\right)\cos^2\left(\frac{\pi}{8}\right)$

14. Sketch the graph of the following functions and state the domain, the range, and the equation of the asymptote (if any). (6 marks)

a. $y = \log_5(x-1) + 3$

b. $y = 2\csc\left(x + \frac{\pi}{4}\right)$

15. The function $y = \frac{1}{3}(5^{x+A}) + 4$ has a y-intercept of $\frac{301}{75}$. Determine the value of A. (2 marks)

16. Bonus Question (3 marks)

- a. Sketch the graphs of $y=\tan(x)$ and $y=\cot(x)$ for $-\frac{\pi}{2} \leq x \leq \pi$.
- b. Using the letters A, B, and C, label the three intersection points of the two functions graphed in part a).
- c. Determine the area and perimeter of $\triangle ABC$