Advanced Functions Class 9 Homework

First Name: Adam Last Name: Chen Student ID: ________

Exponential and Logarithmic Functions (2)

1. Express each of the following in logarithmic form.

a. $27^{\frac{5}{6}} = 9\sqrt{3}$
b. $2^{-6} = 1/64$

a) $\frac{5}{6} = \log_{27}(9\sqrt{3})$

b) $-6 = \log_2(\frac{1}{64})$

2. Express each of the following in exponential form.

a. $\log_{25}(\frac{1}{625}) = -2$
b. $\log_{16}(4\sqrt{2}) = \frac{5}{8}$

a) $25^{-2} = \frac{1}{625}$

b) $16^{\frac{5}{8}} = 4\sqrt{2}$

3. Describe the transformations that would be applied to the parent function $y = \log_2(x)$ to obtain the graph of $y = f(x)$. Sketch the graph of the transformed function and state the domain, the range, and the equation of the asymptote.

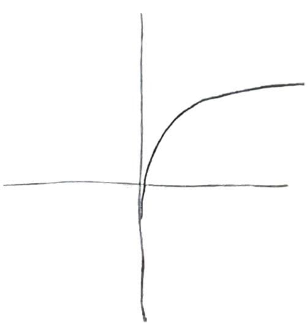
a. $f(x) = 2\log_2(x) - 3$
b. $f(x) = -3\log_2(x-2) + 1$

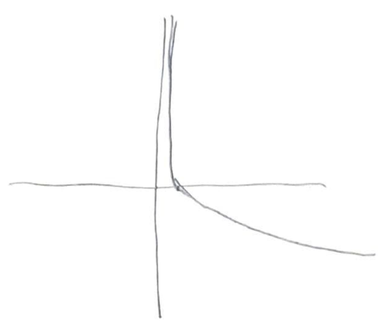
a) vertical stretch by 2
Shift 3 down

Domain: $(0, \infty)$
Range: $\mathbb{R}$
Asymptote: $x=0$

b) Shift right 2
reflect across x
vertical stretch by 3
Shift up 1

Domain: $(2, \infty)$
Range: $\mathbb{R}$
Asymptote: $x=2$





4. For the function, $y = g(x)$, determine the equation of $y = g^{-1}(x)$.

$g(x) = -\log_3(\frac{1}{2}x - 3) + 4$

$y = -\log_3(\frac{1}{2}x - 3) + 4$

$-y + 4 = \log_3(\frac{1}{2}x - 3)$

$3^{4-y} + 3 = \frac{1}{2}x$

$x = 2\,3^{4-y} + 6$

$g^{-1}(x) = 2\;3^{4-x} + 6$

5. Simplify, then evaluate without using a calculator.

a. $\log_6(18) + \log_6(2)$

$= \log_6(36)$

$= \log_6(6^2)$

$= 2$

b. $\log_2(56) - \log_2(7)$

$= \log_2(\frac{56}{7})$

$= \log_2(8)$

$= \log_2(2^3)$

$= 3$

c. $\log(4) - 4\log(2) - \log(25)$

$= \log(\frac{4}{2^4}) - \log(25)$

$= \log(\frac{1}{4} \div 25)$

$= \log(\frac{1}{100})$

$= -2$

d. $5^{2\log_5 6}$

$5^{\log_5(36)}$

$= 36$

e. $3\log_5(10) - \log_5(40)$

$= \log_5(\frac{10^3}{40})$

$= \log_5(25)$

$= \log_5(5^2)$

$= 2$

f. $\log_4(24) - 2\log_4(3) + 0.5\log_4(144)$

$= \log_4(\frac{24}{3^2}) + 0.5\log_4(144)$

$= \log_4(\frac{8}{3}) + 0.5\log_4(144)$

$= \log_4(\frac{96}{3})$

$= \log_4(32) = \log_4(2^5)$

$= 5 \cdot \frac{1}{2} = \frac{5}{2}$

g. $(\log_3(216)) / (\log_3(6))$

$= \log_6(216)$

$= \log_6(6^3)$

$= 3$

h. $9^{\log_3(12) - \log_3(3)}$

$= 9^{(\log_3(4))}$

$= 3^{2\log_3(4)}$

$= \cancel{3}^{\cancel{\log_3}(16)}$

$= 16$

6. Identify the restrictions on x. Convert each equation to the equivalent exponential form and solve for x.

a. $\log_8(\frac{1}{16\sqrt{2}}) = x$

$x = -\frac{3}{2}$

b. $\log_3(4x+6) = 2$ $\quad x > -\frac{6}{4}$

$\frac{\log(4x+6)}{\log(3)} = 2$

$\log(4x+6) = 2\log(3) = \log(9)$

$4x+6=9 \quad x = \frac{3}{4}$

c. $\log(3x-2) = 1$ $\quad x > \frac{2}{3}$

$3x-2 = 10$

$x = 4$

d. $\log_x(16) = -2$ $\quad x > 0$

$16 = x^{-2}$

$x = \frac{1}{4}$

7. Solve. Check for extraneous roots.

a. $\log_5(2x-7) = \log_5(x) - \log_5(4)$ $\quad x>0,\ x>\frac{7}{2}$

$\log_5(8x-28) = \log_5(x)$

$8x-28 = x$

$7x = 28$

$x = 4$

b. $\log(x) + \log(x-4) = \log(12)$

$\log(\frac{x^2-4x}{12}) = 0$ $\quad x > 4$

$x^2-4x = 12$

$x^2-4x-12=0$

$(x+2)(x-6)=0$

$x = \cancel{x},\ 6$

c. $2\log_2(x) = \log_2(3) + \log_2(12)$ $\quad x > 0$

$\log_2(x^2 \div 36) = 0$

$x^2 = 36$

$x = \pm 6$

$x = 6$

d. $2(5^{6x}) - 9(5^{4x}) + 10(5^{2x}) - 3 = 0$

Let $5^{2x} = w$ $\quad$ root: 3

$2w^3 - 9w^2 + 10w - 3 = 0$

$$\begin{array}{c|cccc} 3 & 2 & -9 & 10 & -3 \\ & & 6 & -9 & 3 \\ \hline & 2 & -3 & 1 & \end{array}$$

$2w^2-3w+1=0$

$(w-3)(w-1)(2w-1)=0$

$x = \frac{\log_5(3)}{2},\ 0,\ \frac{\log_5(\frac{1}{2})}{2}$

e. $\log_3(x) + \log_2(x) = 5$

$\frac{\log(x)}{\log(3)} + \frac{\log(x)}{\log(2)} = 5$

$\log(x)\left(\frac{1}{\log(3)} + \frac{1}{\log(2)}\right) = 5$

$x = 10^{\left(\frac{5}{\frac{1}{\log(3)} + \frac{1}{\log(2)}}\right)}$

f. $\log(\log x) = 0$

$\log(x) = 1$

$x = 10^1 = 10$

8. A $1000 investment earns interest at a rate of 4.2% per annum, compounded monthly. Another investment of $1600earns interest at a rate of 3.6% per annum, compounded semi-annually. How long, if ever, will it take for the lower initial investment to be worth more than the higher one?

$$y = 1000\left(1+\frac{0.042}{12}\right)^{12x}$$

$$y = 1600\left(1+\frac{0.036}{2}\right)^{2x}$$

$$1000\left(1+\frac{0.042}{12}\right)^{12x} = 1600\left(1+\frac{0.036}{2}\right)^{2x}$$

$$1000\left(\frac{2007}{2000}\right)^{12x} = 1600\left(\frac{509}{500}\right)^{2x} \qquad a^x = b^{\log_b(a^x)}$$

let $\frac{2007}{2000} = a$, $\frac{509}{500} = b$

$$1000a^{12x} = 1600b^{2x}$$

$$b^{12x\log_b(a)} = \frac{8}{5}b^{2x}$$

$$b^{x(12\log_b a - 2)} = \frac{8}{5}$$

$$\log_b\left(\frac{8}{5}\right) = x(12\log_b a - 2)$$

$$x = \frac{\log_b\left(\frac{8}{5}\right)}{12\log_b(a) - 2} \qquad \text{Sub in } a, b$$

$x = 75.23$ years