Calculus Class 1 Homework

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An introduction to calculus (1)

1. Determine the slope of the secant to the given curve between the specified values of *x*.

a. $y = 3^x - 4$, x = 0, x = 2

$m = \frac{5+3}{2} = 4$

b. $y = \tan(x)$, $x = \frac{\pi}{3}$, $x = \frac{\pi}{6}$

$m = \frac{\tan\frac{\pi}{3} - \tan\frac{\pi}{6}}{\frac{\pi}{3} - \frac{\pi}{6}} = \frac{\frac{2\sqrt{3}}{3}}{\frac{\pi}{6}} = \frac{4\sqrt{3}}{\pi}$

c. $y = \log_8 x$, $x = \frac{1}{2}$, x = 4

$m = \frac{\log_8(4) - \log_8(\frac{1}{2})}{4 - \frac{1}{2}} = \frac{2}{7}$

d. $y = \sin(x) - \cos(x)$, $x = \frac{\pi}{4}$, $x = \frac{3\pi}{4}$

$m = \frac{2\sqrt{2}}{\pi}$

2. Consider the function $y = f(x) = \frac{3}{2}x^2 - 2x$.

a. Find the slope of the tangent line at the generic point P(a, f(a))

b. Find the point where the tangent line is horizontal.

c. Find the point P such that $m_P = 1$.

d. Find the point P such that the tangent line at P is perpendicular to the line L: $11x - y = 3$.

a) $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \frac{(\frac{3}{2}(a+h)^2 - 2(a+h)) - (\frac{3}{2}a^2 - 2a)}{h}$

$= \lim_{h \to 0} \frac{\frac{3}{2}(a^2 + 2ah + h^2) - 2a - 2h - \frac{3}{2}a^2 + 2a}{h}$

$= \lim_{h \to 0} \frac{\cancel{h}(3a + \frac{3}{2}h - 2)}{\cancel{h}} = 3a - 2$

b) $3a - 2 = 0$

$a = \frac{2}{3}$

$f(\frac{2}{3}) = -\frac{2}{3}$

$P(\frac{2}{3}, -\frac{2}{3})$

c) $3a - 2 = 1$

$a = 1$

$f(1) = -\frac{1}{2}$

$P(1, -\frac{1}{2})$

d) $11x - y = 3$

$m = 11$

$3a - 2 = -\frac{1}{11}$

$a = \frac{7}{11}$

$f(\frac{7}{11}) = -\frac{161}{242}$

$P(\frac{7}{11}, -\frac{161}{242})$

3. The altitude of a rock climber t hours after she begins her ascent up a mountain is modelled by the equation $a(t)=-5t^2+30t$, where the altitude, $a(t)$, is measured in metres.

a. Determine the altitude of the rock climber 2 hours after she begins her climb.

40 m

b. Determine the altitude of the rock climber 3 hours after she begins her climb.

45 m

c. Determine the average rate of change of the altitude of the rock climber between 2 and 3 hours after she begins her climb.

Avg: 5 m/h

d. Determine the instantaneous rate of change of the altitude of the rock climber 3 hours after she begins her climb.

0 m/h

e. What is the significance of the instantaneous rate of change value found in part d)? Explain what this value tells us about the rock climber's travel at this point.

$$\lim_{h\to 0} \frac{45+5(3+h)^2-30(3+h)}{-h}$$

$$= \lim_{h\to 0} \frac{-45+5(9+\cancel{6h}+h^2)-\cancel{30h}}{-h}$$

$$= \lim_{h\to 0} \frac{5(h^2)}{-h} = 0$$

This point is where she reaches the top of the mountain and begins to decend after

4. The path of a robot along a track is modelled by the curve $y=2x^2+1$. As the robot moves, it passes through the point $P\,(-1,3)$. At this point, it attempts to shoot a ball at a target located at the point (1,-5). If the ball travels along the tangent line to the curve at point P, will the ball hit the target?

$$\lim_{h\to 0} \frac{2(h-1)^2\cancel{+1}-2(-1)^2\cancel{-1}}{h}$$

$$= \lim_{h\to 0} \frac{2(h-2)\cancel{(h)}}{\cancel{h}} = -4$$

$Y=-4X+b$

$3=-4(-1)+b$

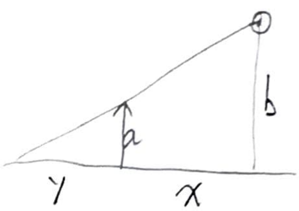
$b=-1$

$-5=-4(1)-1$

$-5=-5$ ✓

The ball will hit the target

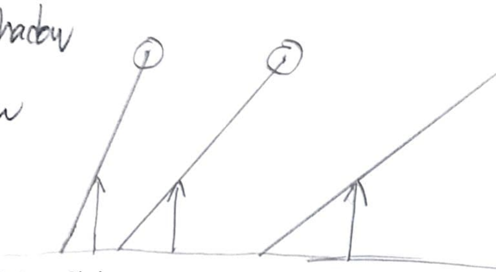
5. On an evening walk, a man passes under a streetlight and notices that the length of his shadow increases as he walks away from the base of the streetlight. If the man is 2 m tall and the streetlight is 6 m tall, determine how the length of the man's shadow is changing, in terms of the rate at which he is walking away from the streetlight. Support your answer with a diagram.



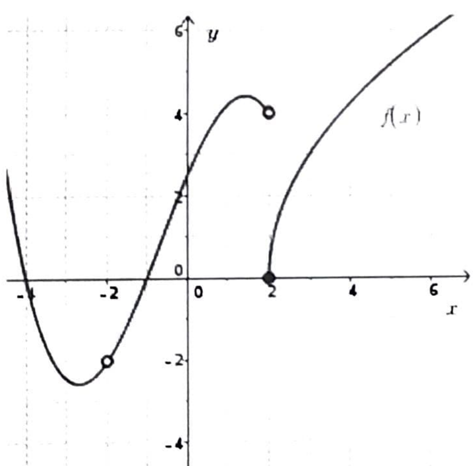
$$\frac{6}{x+y} = \frac{2}{y}$$

$y = \frac{1}{2}x$ where y = dist from lamp and x = dist of shadow

$m = \frac{1}{2}$ when he walks 1m/s, his shadow increases by $\frac{1}{2}$m/s in length



6. Given the graph of f(x), evaluate the following expressions involving f(x).



a. $\lim_{x\to -1} f(x)$ 0

b. $\lim_{x\to -2} f(x)$ −2

c. $\lim_{x\to 2^+} f(x)$ 0

d. $\lim_{x\to -2^-} f(x)$ −2

e. f(-2) DNE

f. $\lim_{x\to 2} f(x)$ DNE

g. $\lim_{x\to 2^-} f(x)$ 4

h. f(2) 0

i. $\lim_{x\to 3} f(x)$ 3

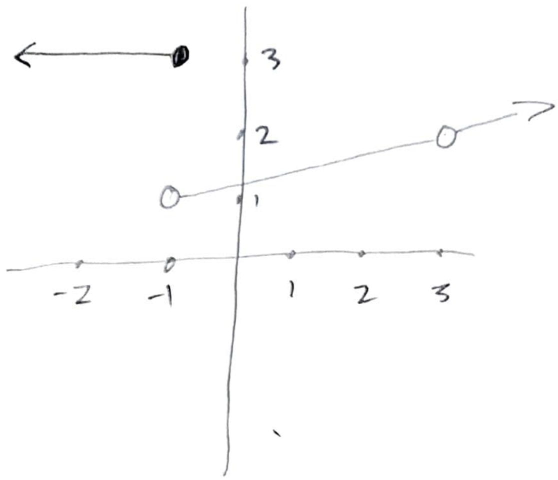
7. Sketch the graph of a function that has the following characteristics:

$\lim_{x \to -1^-} f(x) = 3$

$\lim_{x \to -1^+} f(x) = 1$

$\lim_{x \to 3} f(x) = 2$

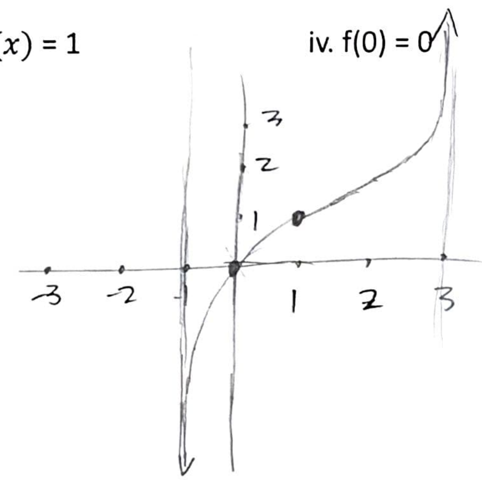
f(3) does not exist



8. Sketch the graph of a function that has the following characteristics:

i. $\lim_{x \to 3^-} f(x) \to +\infty$ ii. $\lim_{x \to -1^+} f(x) \to -\infty$

iii. $\lim_{x \to 1} f(x) = 1$ iv. f(0) = 0



9. Given that $\lim_{x \to a} f(x) = 4$ and $\lim_{x \to a} g(x) = -2$, find the following limits:

a. $\lim_{x \to a} (f(x) + g(x))$

$= \lim_{x \to a} f(x) + \lim_{x \to a} g(x) = 2$

b. $\lim_{x \to a} f(x)\, g(x)$

$= \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x) = -8$

c. $\lim_{x \to a} \frac{f(x)+2}{2-2g(x)}$

$= \dfrac{\lim_{x \to a} f(x) + 2}{-2 \cdot \lim_{x \to a} g(x) + 2} = \dfrac{6}{6} = 1$

d. $\lim_{x \to a} \frac{\sqrt{f(x)}}{g(x)}$

$= \dfrac{\sqrt{\lim_{x \to a} f(x)}}{\lim_{x \to a} g(x)} = -1$