Key Equations

Matrix Operations

Transpose
$A^t_{m \times n} = B_{n \times m}$, where $b_{ij} = a_{ji}$

Scalar Multiplication
$kA = C$, where $c_{ij} = ka_{ij}$

Addition
$A + D = E$, where $e_{ij} = a_{ij} + d_{ij}$

Multiplication
$A_{m \times n} F_{n \times p} = G_{m \times p}$, where $g_{ij} = \sum_{k=1}^{n} a_{ik} f_{kj}$

Inverse
For $H = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $H^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ if $ad \neq bc$

Statistics of One Variable

	Population	Sample	Weighted Mean
Mean:	$\mu = \frac{\sum x}{N}$	$\bar{x} = \frac{\sum x}{n}$	$\bar{x}_w = \frac{\sum w_i x_i}{\sum w_i}$
Variance:	$\sigma^2 = \frac{\sum (x-\mu)^2}{N}$	$s^2 = \frac{\sum (x-\bar{x})^2}{n-1}$	
Standard Deviation:	$\sigma = \sqrt{\frac{\sum (x-\mu)^2}{N}}$	$s = \sqrt{\frac{\sum (x-\bar{x})^2}{n-1}}$ $= \sqrt{\frac{n(\sum x^2) - (\sum x)^2}{n(n-1)}} = \sqrt{\frac{(\sum x^2) - n(\bar{x})^2}{n-1}}$ $\cancel{= \sqrt{\frac{n\sum x^2 - (\sum x)^2}{n(n-1)}}}$	
Z-score:	$z = \frac{x - \mu}{\sigma}$	$z = \frac{x - \bar{x}}{s}$	
Grouped Data:	$\mu \doteq \frac{\sum f_i m_i}{\sum f_i}$	$\bar{x} \doteq \frac{\sum f_i m_i}{\sum f_i}$, where m_i is midpoint of ith interval	
	$\sigma \doteq \sqrt{\frac{\sum f_i (m_i - \mu)^2}{N}}$	$s \doteq \sqrt{\frac{\sum f_i (m_i - \bar{x})^2}{n-1}}$	

Statistics of Two Variables

Correlation Coefficient

$$r = \frac{s_{XY}}{s_X s_Y}$$

$$= \frac{n\sum xy - \sum x \sum y}{\sqrt{n\sum x^2 - (\sum x)^2}\sqrt{n\sum y^2 - (\sum y)^2}}$$

Least Squares Line of Best Fit

$y = ax + b$, where $a = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$ and $b = \bar{y} - a\bar{x}$

Coefficient of Determination

$$r^2 = \frac{\sum (y_{est} - \bar{y})^2}{\sum (y - \bar{y})^2}$$

Permutations and Organized Counting

Factorial: $n! = n \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1$

Permutations

r objects from n different objects: ${}_nP_r = \frac{n!}{(n-r)!}$

n objects with some alike: $\frac{n!}{a!b!c!\ldots}$

Combinations and the Binomial Theorem

Combinations

r items chosen from n different items: ${}_nC_r = \frac{n!}{(n-r)!r!}$

at least one item chosen from n distinct items: $2^n - 1$

at least one item chosen from several different sets of identical items: $(p+1)(q+1)(r+1) \ldots -1$

Pascal's Formula: ${}_nC_r = {}_{n-1}C_{r-1} + {}_{n-1}C_r$

Binomial Theorem: $(a+b)^n = \sum_{r=0}^{n} {}_nC_r a^{n-r} b^r$

Introduction to Probability

Equally Likely Outcomes: $P(A) = \frac{n(A)}{n(S)}$

Complement of A: $P(A') = 1 - P(A)$

Odds: odds in favour of $A = \frac{P(A)}{P(A')}$

If odds in favour of $A = \frac{h}{k}$, $P(A) = \frac{h}{h+k}$

Conditional Probability: $P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$

Independent Events: $P(A \text{ and } B) = P(A) \times P(B)$

Dependent Events: $P(A \text{ and } B) = P(A) \times P(B \mid A)$

Mutually Exclusive Events: $P(A \text{ or } B) = P(A) + P(B)$

Non-Mutually Exclusive Events: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Markov Steady State: $S^{(n)} = S^{(n)}P$

Discrete Probability Distributions

Expectation: $E(x) = \sum_{i=1}^{n} x_i P(x_i)$

Discrete Uniform Distribution: $P(x) = \frac{1}{n}$

Binomial Distribution: $P(x) = {}_nC_x p^x q^{n-x}$ $\quad E(x) = np$

Geometric Distribution: $P(x) = q^x p$ $\quad E(x) = \frac{q}{p}$

Hypergeometric Distribution: $\cancel{P(x) = \frac{{}_aC_x \times {}_{n-a}C_{r-x}}{{}_nC_r}}$ $\quad \cancel{E(x) = \frac{ra}{n}}$ $\quad P(x) = \frac{C(K,x) \times C(N-K,n-x)}{C(N,n)}$

$E(X) = n\left(\frac{K}{N}\right)$

Continuous Probability Distributions

Exponential Distribution: $y = ke^{-kx}$, where $k = \frac{1}{\mu}$

Normal Distribution: $y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

Normal Approximation to Binomial Distribution: $\mu = np$ and $\sigma = \sqrt{npq}$ if $np > 5$ and $nq > 5$

Distribution of Sample Means: $\mu_{\bar{x}} = \mu$ and $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

Confidence Intervals: $\bar{x} - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$ $\quad \hat{p} - z_{\frac{\alpha}{2}} \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}} < p < \hat{p} + z_{\frac{\alpha}{2}} \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}$