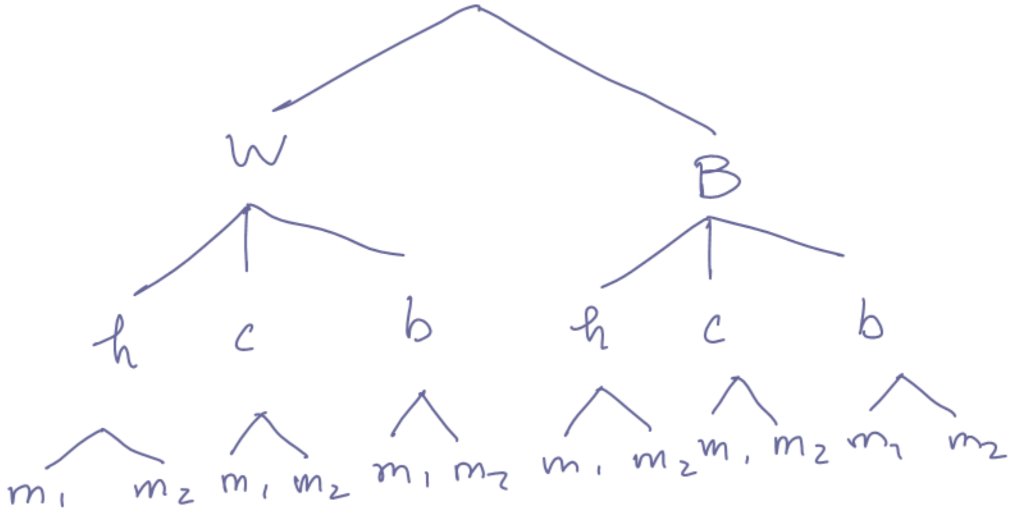
MDM4U HW1.

Page 229-231: Q1, Q2, Q3, Q8, Q12, Q19, Q22

Page 239-240: Q3, Q4, Q11, Q12, Q16, Q22, Q24

Sol. P229. Q1. using Rule of Product



3 Stages:

Stage 1 : $n_1 = 2$

Stage 2 : $n_2 = 3$

Stage 3 : $n_3 = 2$.

$\therefore$ $n_1 \times n_2 \times n_3 = 2 \times 3 \times 2 = 12$ (sandwiches)

Q2. using Rule of Sum,

Since either a sum of 4 or a sum of 11.

2 Actions:

Action 1 : $n_1 = 3$; (1,3), (3,1), (2,2)

Action 2 ; $n_2 = 2$; (5,6), (6,5)

$n_1 + n_2 = 3 + 2 = 5$ (ways)

Q3. ♠ A 2 3 - - - - - 10 | J Q K → face cards

♣ A 2 3 - - - - - 10 | J Q K

♥ A 2 3 4 5 6 7 8 9 10 | J Q K

♦ A 2 3 - - - - - 10 | J Q K

$13 \times 4 = 52$

There are four "6"s and 12 "face cards".

Using Rule of Sum.

Action 1: $n_1 = 4$.

Action 2: $n_2 = 12$,

$\therefore n_1 + n_2 = 4 + 12 = 16$ (ways)

Q8. Using Rule of Product,

5 stages, $n_1 = n_2 = n_3 = n_4 = n_5 = 6$.

$\therefore n_1 \times n_2 \times n_3 \times n_4 \times n_5 = 6^5 = 7776$ (outcomes)

P230. Q12.

Using Rule of Product.

5 Stages: $n_1 = n_2 = n_3 = n_4 = n_5 = 2$

$\therefore n_1 \times n_2 \times n_3 \times n_4 \times n_5 = 2^5 = 32$ (arrangements)

Q19. "think".

Using "indirect method" and Rule of Product.

The number of total possible outcomes is:

$${}_5P_5 = 5! = 5\times4\times3\times2\times1 = 120.$$

The number of outcomes that t and h are together.

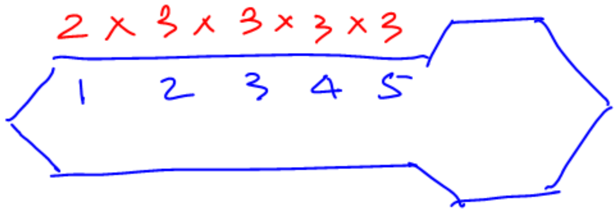
th or ht: $\quad 2\times {}_4P_4 = 2\times 4! = 48.$

Then the number of ways required is

$$120 - 48 = 72 \text{ (ways)}$$

Q22. 10 types

Using Rule of Product.



$$n_1\times n_2\times n_3\times n_4\times n_5$$
$$= 2\times3\times3\times3\times3 = 162 \text{ for each type}$$

Since 10 types. so

$$10\times162 = 1620 \text{ Keys}$$

P239-240. Q3. Express in terms of $_nP_r$

a) $6 \times 5 \times 4 = {}_6P_3$

b) $9 \times 8 \times 7 \times 6 = {}_9P_4$

c) $20 \times 19 \times 18 \times 17 = {}_{20}P_4$

d) $101 \times 100 \times 99 \times 98 \times 97 = {}_{101}P_5$

e) $76 \times 75 \times 74 \times 73 \times 72 \times 71 \times 70 = {}_{76}P_7$

Q4.

a) $P(10,4) = 10 \times 9 \times 8 \times 7 = 5040$

b) $P(16,4) = 16 \times 15 \times 14 \times 13 = 43680$

c) ${}_5P_2 = 5 \times 4 = 20$

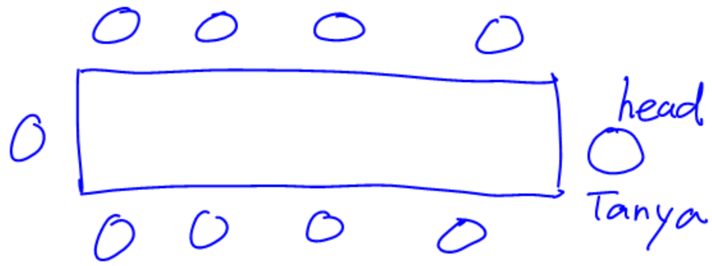
d) ${}_9P_4 = 9 \times 8 \times 7 \times 6 = 3024$

e) $7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$

Q11. 12 members. two captains together.

$2 \times {}_{11}P_{11} = 2 \times 11! = 79833600$ (ways)

Q12. 10 people, Tanya sit at the head of table, Henry and Wilson or Henry and Nancy do not sit next each other, but Wilson, Henry and Nancy sit together is OK.



Using "indirect method".

the number of total sitting arrangements:

$$_9P_9 = 9! = 362880,$$

Henry and Wilson: $2 \times 8! = 80640$

Henry and Nancy: $2 \times 8! = 80640$;

So the number of sitting arrangements required

is $362880 - 2 \times 80640 = 201600$ (ways)

Q16.

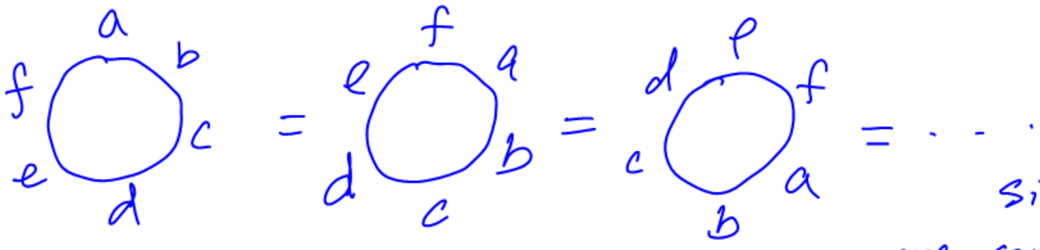
a) Try. $10 \times 9 \times 8 \times 7 \times 6 \times 5 = {}_{10}P_6 = 151200$ > 60000

So, try ${}_{10}P_5 = 30240 < 60000$.

Therefore, the code should be 6-digit long.

b). $3! \times 7! = 30240$ (codes)

Q22.



six cases are considered as the same.

$$\frac{6!}{6} = 5! = 120 \text{ (ways)}$$

Generally, the round permutation or circular permutation of n items is

$$\frac{n!}{n} = (n-1)!$$

Q24.

A. B C

$$10 \times 9 \times 8 \times \underline{3 \times 2 \times 7 \times 6} \times \underline{1 \times 1 \times 5 \times 4} = 10!$$

$$= 3628800 \text{ (ways)}$$