First Name:_____ Last Name: _____ Student ID: _____

Rational Functions (2)

Objective: Solve rational equations and inequalities.

1. Solve each of the following equations:

$\frac{3x+5}{x+1} = 2$	$\frac{2x}{x+3} = \frac{1}{x-1}$
$\frac{2x+1}{x^2+4x} + \frac{4x}{x^2-3x} = \frac{x-2}{x^2+x-12}$	$\frac{2}{x^2-1} = \frac{x}{x+1}$

An **extraneous solution** is a solution of an equation derived from an original equation that is not a solution of the original equation. When you solve a rational equation, it is possible to get extraneous solutions. These values should be eliminated from the solution set. **Always check** your solutions by substituting them into the original equation.

2. Determine the point(s) of intersection of the functions $f(x)=\frac{2}{x^2-1}$ and $g(x)=\frac{x}{x+1}$ and illustrate the situation graphically.

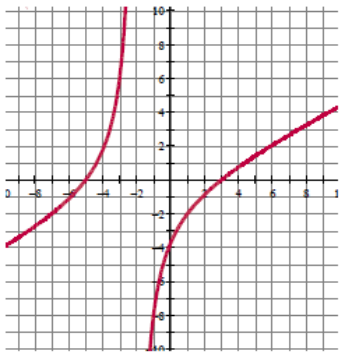
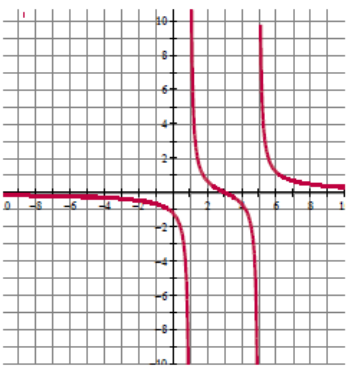
3. Solve the following equations

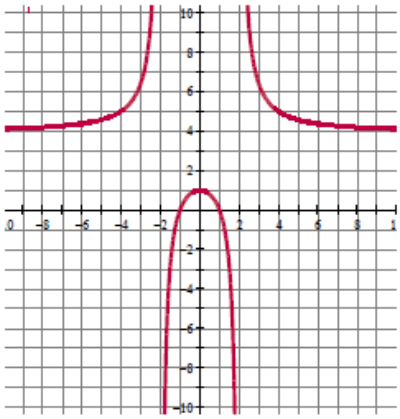
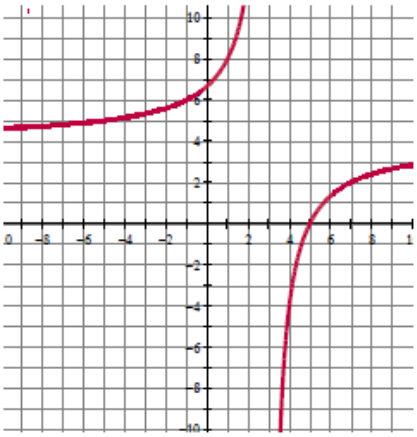
a. $\frac{2}{x-3}+\frac{5}{x-2}=\frac{7x-19}{x^2-5x+6}$

b. $\frac{2}{x-1}=\frac{1}{5x-5}$

4. Pedro left home at noon and cycled 72 km to his family cottage. His sister, Alexandra, left home on her bike at 1 PM and arrived at the cottage 12 minutes after Pedro. If she cycles, on average, 3 km/h faster than Pedro, how long did it take Pedro to make the trip, and what was his average speed?

5. Highlight the sections of the graph that meet the following inequalities. State the regions of the graph that meet the criteria.

f(x) > 0	f(x) < 0
	

$f(x) \geq 0$	$f(x) \leq 0$
	

6. Solve the following rational inequalities.

a. $\frac{3}{x-2} > \frac{1}{x+1}$

Algebraic Method	Graphical Method

b. $\frac{x+1}{x-1} \geq 1$ (Chose the method)

c. $\frac{2}{x^2+2x-3} < \frac{x}{x^2+3x-4}$ (Chose the method)

KEY CONCEPTS:

- **Solving a rational equation** means *finding the value(s) of x* so that **f(x) = 0**
- You can solve rational equations/inequalities algebraically or graphically
- **Solving an inequality** means *finding all the possible values of the variable (x)* that satisfy the inequality.
- To solve the inequality algebraically, **rearrange** the inequality so that one side is zero. Then find the common denominator, and simplify and *factor the numerator*. Make **a table** to **find the sign of each factor** and the **signs of the entire factors** on the intervals created by the zeros of both numerator and denominator.

7. An open top box is to be made with the following conditions:

- The length of the base is 10 cm longer then the width of the base.
- The height of the box must be greater than the width.
- The volume must be 375 cm^3.

Determine specific restrictions, if any, on the width, length, and height of the box.

Extra Practice

Solve each equation or inequality. Check your solutions.

1. $\frac{12}{x} + \frac{3}{4} = \frac{3}{2}$ 16

2. $\frac{x}{x-1} - 1 = \frac{x}{2}$ −1, 2

3. $\frac{p+10}{p^2-2} = \frac{4}{p}$ $-\frac{2}{3}$, 4

4. $\frac{s}{s+2} + s = \frac{5s+8}{s+2}$ 4

5. $\frac{5}{y-5} = \frac{y}{y-5} - 1$ all reals except 5

6. $\frac{1}{3x-2} + \frac{5}{x} = 0$ $\frac{5}{8}$

7. $\frac{5}{t} < \frac{9}{2t+1}$ $t < -5$ or $-\frac{1}{2} < t < 0$

8. $\frac{1}{2h} + \frac{5}{h} = \frac{3}{h-1}$ $\frac{11}{5}$

9. $\frac{4}{w-2} = \frac{-1}{w+3}$ −2

10. $5 - \frac{3}{a} < \frac{7}{a}$ $0 < a < 2$

11. $\frac{4}{5x} + \frac{1}{10} < \frac{3}{2x}$ $0 < x < 7$

12. $8 + \frac{3}{y} > \frac{19}{y}$ $y < 0$ or $y > 2$

13. $\frac{4}{p} + \frac{1}{3p} < \frac{1}{5}$ $p < 0$ or $p > \frac{65}{3}$

14. $\frac{6}{x-1} = \frac{4}{x-2} + \frac{2}{x+1}$ ∅

15. $g + \frac{g}{g-2} = \frac{2}{g-2}$ −1

16. $b + \frac{2b}{b-1} = 1 - \frac{b-3}{b-1}$ −2

17. $2 = \frac{x+2}{x-3} + \frac{x-2}{x-6}$ $\frac{14}{3}$

18. $5 - \frac{3d+2}{d-1} = \frac{2d-4}{d+2}$ 6

19. $\frac{1}{n+2} + \frac{1}{n-2} = \frac{3}{n^2-4}$ $\frac{3}{2}$

20. $\frac{c+1}{c-3} = 4 - \frac{12}{c^2-2c-3}$ $-\frac{5}{3}$, 5

21. $\frac{3}{k-3} + \frac{4}{k-4} = \frac{25}{k^2-7k+12}$ 7

22. $\frac{4v}{v-1} - \frac{5v}{v-2} = \frac{2}{v^2-3v+2}$ −1, −2

23. $\frac{y}{y+2} + \frac{7}{y-5} = \frac{14}{y^2-3y-10}$ 0

24. $\frac{x^2+4}{x^2-4} + \frac{x}{2-x} = \frac{2}{x+2}$ ∅

25. $\frac{r}{r+4} + \frac{4}{r-4} = \frac{r^2+16}{r^2-16}$
all reals except −4 and 4

26. $3 = \frac{6a-1}{2a+7} + \frac{22}{a+5}$ −2

27. **BASKETBALL** Kiana has made 9 of 19 free throws so far this season. Her goal is to make 60% of her free throws. If Kiana makes her next x free throws in a row, the function $f(x) = \frac{9+x}{19+x}$ represents Kiana's new ratio of free throws made. How many successful free throws in a row will raise Kiana's percent made to 60%? 6

28. **OPTICS** The lens equation $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ relates the distance p of an object from a lens, the distance q of the image of the object from the lens, and the focal length f of the lens. What is the distance of an object from a lens if the image of the object is 5 centimeters from the lens and the focal length of the lens is 4 centimeters? 20 cm