

Q3 (bottom 2 graphs)
Q4 (c,d,e,f)
Q5 (c,d,e)
Q7(c,d,e)
Q8
Q10(d,e)
Q11 (bottom 2)
Q13

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First Name: _____ Last Name: _____ Student ID: _____

Polynomial Functions

1. Consider the following polynomial functions.

- a. $y = -2x^3 + 4x - 5$
- b. $f(x) = 5x^4 + 2x^3 - 4x^2 + x - 7$
- c. $g(x) = x^5 + 2x^3 - 5x + 8$

For each one, perform the following tasks.

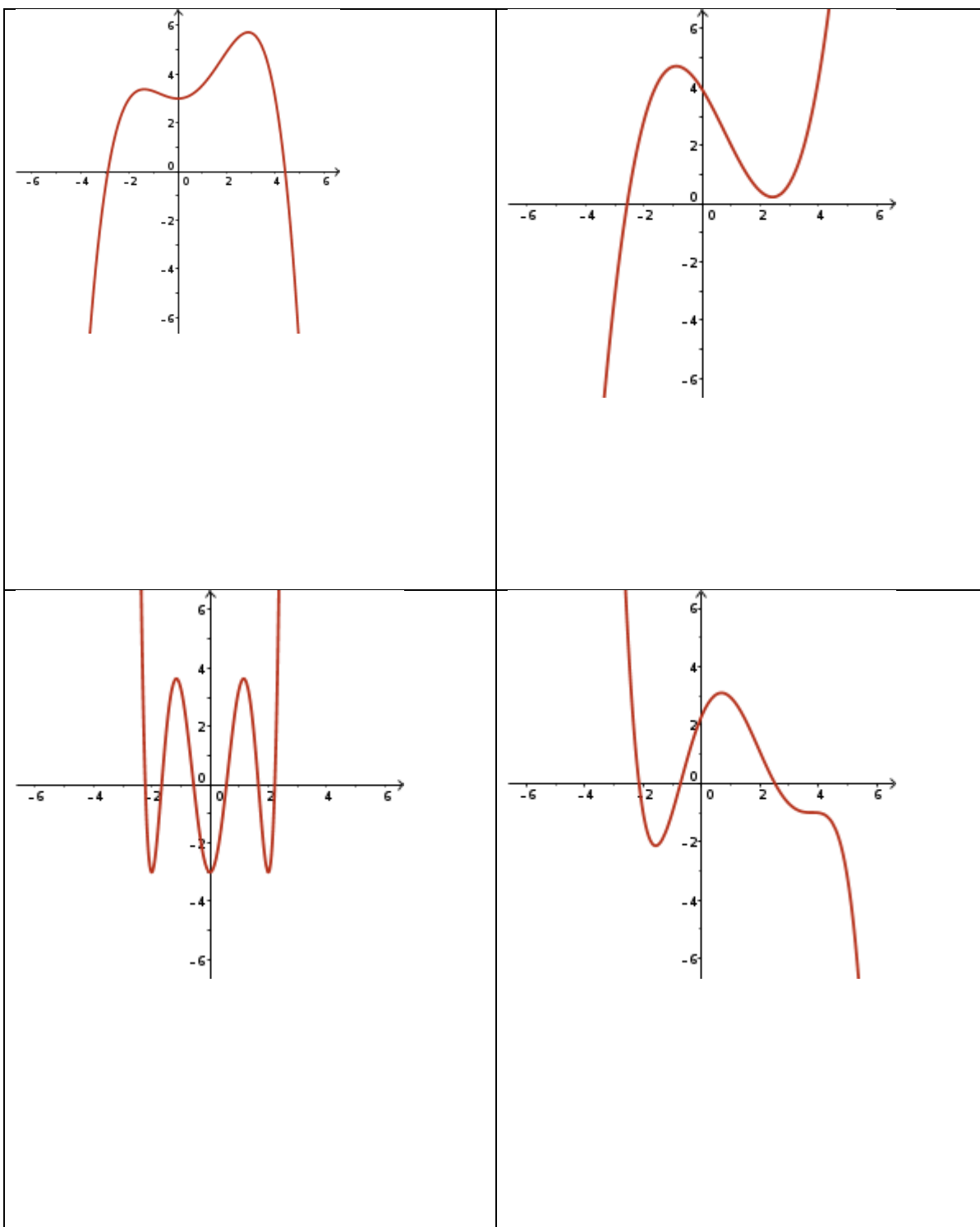
- i. Describe the end behavior of the function.
- ii. Determine the maximum and minimum number of turning points.
- iii. Determine the maximum and minimum number of x-intercepts.

2. Sketch a possible graph of each function by identifying the end behaviours and determining the x - and y -intercepts of the function.

a. $f(x) = (x-1)(x-3)(x+1)(x+4)$

b. $y = -2x^3 - 3x^2 + 9x$

3. Given the graph of the polynomial function $y=f(x)$, identify the minimum possible degree of the function and the sign of the leading coefficient.



4. Sketch a graph of a polynomial function that satisfies each set of conditions.

a. Degree three, two distinct x -intercepts, two turning points, and end behavior such that $y \rightarrow \infty$ as $x \rightarrow -\infty$ and $y \rightarrow -\infty$ as $x \rightarrow \infty$

b. Degree four, two distinct x -intercepts, three turning points, and end behavior such that $y \rightarrow \infty$ as $x \rightarrow \pm\infty$

c. Degree four, negative leading coefficient, three distinct x -intercepts, three turning points

d. Degree three, positive leading coefficient, one x -intercept, two turning points

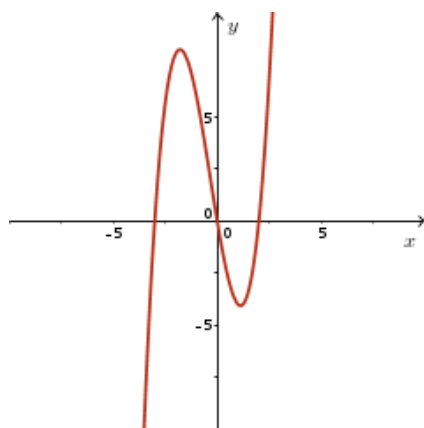
e. Degree five, negative leading coefficient, two distinct x -intercepts, two turning points

f. Degree five, positive leading coefficient, one x -intercept, four turning points

5. Sketch a possible graph of a polynomial function that satisfies the following conditions.

- A quadratic function with a negative leading coefficient and a zero at $x=-5$ of multiplicity 2.
- A 5th degree function with a positive leading coefficient, a zero at the origin of order 2, and a zero at $x=3$ of order 3.
- A quartic function with a positive leading coefficient and two real zeros, $x=0$ and $x=3$ of order 2.
- A cubic function with a negative leading coefficient and only one zero at $x=4$ and two non-real zeros.
- A quintic function with a positive leading coefficient, a zero at $x=-2$, and a second zero at $x=1$ of multiplicity 4.

6. Given the graph of the polynomial function $f(x)=x^3+x^2-6x$,



Sketch the graph of

a. $y=|f(x)|$

b. $y=f(|x|)$

Advanced Functions Class 2 Homework

7. Sketch a possible graph for each of the following functions.

a. $y = -x(x+2)(2x-5)$

b. $f(x) = 2(x-2)^2(x+3)^2$

c. $g(x) = -0.5(x-3)(x+1)^3$

d. $y = 2x^2(x-4)^3$

e. $f(x) = -x(2x+3)(x-2)^2$

Solution:

8. Sketch a possible graph for each function.

a. $f(x) = -2x^3 + 8x$

b. $f(x) = -x^4 - 5x^3 - 6x^2$

c. $f(x) = x^4 - 2x^2 + 1$

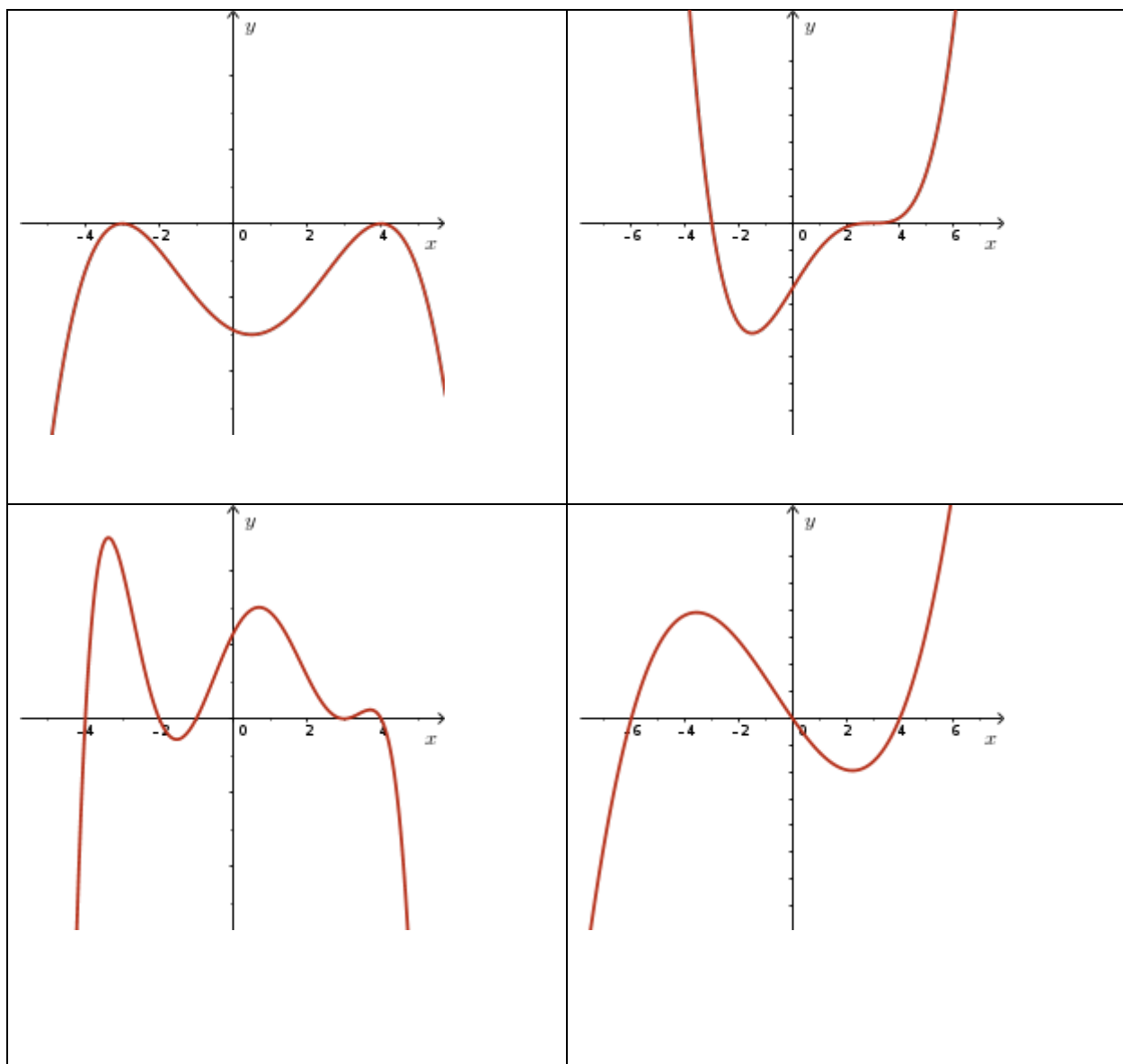
9. A family of quintic functions has a zero at $x=-3$ and turning points tangent to the x -axis at $x=1$ and 4 .

- a. State the general equation of the family.
- b. State the equations of two members of the family that have end behaviour $y \rightarrow \infty$ as $x \rightarrow -\infty$ and $y \rightarrow -\infty$ as $x \rightarrow \infty$.

10. State the equation of the family of polynomial functions satisfying the following conditions:

- a) A cubic with zeros $x=-3$, $x=-\frac{1}{2}$, and $x=\frac{5}{3}$.
- b) A sixth degree function with zeros $x=-2$ (order 2), $x=1$ (order 1), and $x=5$ (order 3).
- c) A quartic that passes through the origin and has a point of inflection at $(\frac{2}{3}, 0)$.
- d) Cubic function, x -intercept at $x=-4$, a turning point at $(1, 0)$, and $f(x) \rightarrow -\infty$ as $x \rightarrow \infty$.
- e) A quartic function with zeros at $x=\pm\sqrt{5}$ and $x=-1\pm\sqrt{2}$.

11. Given the graph of $y=f(x)$, determine a general equation for a family of polynomials with the same end behaviour and zeros of $f(x)$ (note: all zeros are integer in value).



12. State the equation of the quartic function with zeros $x = -\frac{1}{2}$ and 5 (both of multiplicity 1) and $x = 2$ (multiplicity 2), having a y-intercept of 4.

13. Find the general equation of the family of

- a) quadratic functions with zeros $-3-\sqrt{5}$ and $-3+\sqrt{5}$.
- b) cubic functions with zeros $0, 1-2\sqrt{3}$ and $1+2\sqrt{3}$.
- c) quartic functions with zeros $-2, 1$ and $\pm 3i$.
- d) quartic functions with zeros $3\pm\sqrt{2}$ and $-4\pm i\sqrt{3}$

14. Determine the equation of the quartic function with rational coefficients, zeros $4-\sqrt{2}$ and $-3+\sqrt{6}$, and a y-intercept of -21 .

15. The function $f(x) = \frac{1}{4}(x-2)^2(x+2)^2$ has a turning point at $(0, 4)$. Determine

- a) The intervals where $f(x)$ is positive and negative.
- b) The intervals where $f(x)$ is increasing and decreasing