

First Name: _____ Last Name: _____ Student ID: _____

Identities and Equations (2)

Recall!

Sum and Difference Formulas - Identities

$$\sin(u + v) = \sin u \cos v + \cos u \sin v$$

$$\cos(u + v) = \cos u \cos v - \sin u \sin v$$

$$\sin(u - v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

Double Angle Identities

$\sin 2A = 2 \sin A \cos A$	$\cos 2A = \cos^2 A - \sin^2 A$ $\cos 2A = 2 \cos^2 A - 1$ $\cos 2A = 1 - 2 \sin^2 A$	$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$
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The Half-Angle Formulas

$\cos \frac{\alpha}{2} =$	$\sin \frac{\alpha}{2} =$
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1. Determine the exact value of each.

a. $\sin^2(75^\circ) - \cos^2(75^\circ)$

b. $\sin(\pi/8)\cos(\pi/8)$

c. $\cos\left(\frac{5\pi}{8}\right)$

d. $\cos 22.5^\circ$

2. If $\tan(\theta) = -\frac{\sqrt{5}}{2}$ for $\frac{\pi}{2} \leq \theta \leq \pi$, determine the exact value of $\sin(2\theta)$.

3. Derive a formula for

a. $\cos(3x)$ in terms of $\cos(x)$

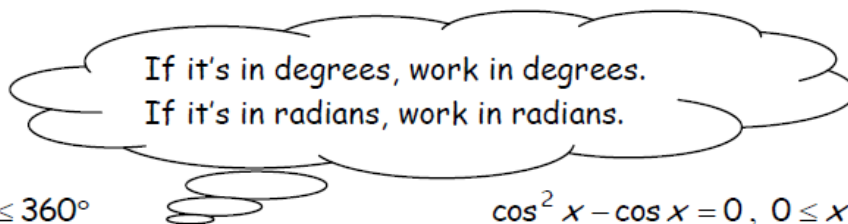
b. $\cos(4x)$ in terms of $\cos(x)$

Trigonometric Equations

Steps in solving Trigonometric Equations

- 1) Make the equation contain one trigonometric function.
- 2) Find the reference angle, r (acute angle)
- 3) Apply the "CAST" diagram to find angles from 0° to 360° / 0 to 2π .
- 4) Check the interval and write the answers using, $360^\circ n$ / $2\pi n$.

Examples



Solve:

$$2\sin\theta - 1 = 0, \quad 0 \leq \theta \leq 360^\circ$$

$$\cos^2 x - \cos x = 0, \quad 0 \leq x \leq 2\pi$$

4. Solve each of the following, where $0 \leq x \leq 2\pi$:

a. $2\sin x - 1 = 0$	b. $\sqrt{2} \sin x + 1 = 0$
c. $3\tan^2 x - 1 = 0$	d. $2\cos(3x) - 5 = 0$
e. $\sin^2 x + \cos 2x - \cos x = 0$	f. $\cot x \cos^2 x = 2\cot x$

g. $\tan^8 x - \tan^4 x = 0$	h. $2\sin^2 x + 3\cos x - 3 = 0$
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5.

a. Solve $\csc(x) + 6 = 1 - 3\csc(x)$, where $0 \leq x \leq 2\pi$, correct to two decimal places.

b. Determine the roots of $\cos(x) - \sin(x) = \cos(2x)$ where $-\pi \leq x \leq \pi$.

c. Determine the exact roots of $\sqrt{2}\sin(x)\cos(x) = \cos(x)$, where $-\pi \leq x \leq 2\pi$.

6. Consider the equation $\sin 3x = -\frac{\sqrt{3}}{2}$.

- a) Solve the equation. Present all solutions in degrees.
- b) Solve the equation. Present all solutions in radians.
- c) List all solutions between 0 and 360°.

7. Solve the following equation $\sin x + \cos x = -1$.

Practice Problems

1. Compute the exact value of $\sin 105^\circ$ using a half-angle formula.
2. Compute the exact value of each of the following.

a) $\sin 15^\circ$	c) $\sin 7.5^\circ$	e) $\tan 7.5^\circ$
b) $\cos 15^\circ$	d) $\cos 7.5^\circ$	
3. Compute the exact value of $\sin x$ and $\cos x$ if x is in the second quadrant and $\cos 2x = \frac{7}{25}$.
4. Compute the exact value of $\cos \frac{x}{2}$ and $\tan \frac{x}{2}$ if we know that $\cos x = \frac{2}{3}$.
5. Compute the exact value of $\sin \alpha$ if $\sin 2\alpha = \frac{4}{5}$.

Practice Problems - Answers

1. $\frac{\sqrt{2+\sqrt{3}}}{2}$
2. a) $\frac{\sqrt{6}-\sqrt{2}}{4}$ b) $\frac{\sqrt{6}+\sqrt{2}}{4}$ c) $\frac{1}{2}\sqrt{\frac{4-\sqrt{6}-\sqrt{2}}{2}}$ d) $\frac{1}{2}\sqrt{\frac{4+\sqrt{6}+\sqrt{2}}{2}}$
 e) $\frac{\sqrt{6}-\sqrt{2}}{4+\sqrt{6}+\sqrt{2}} = -2 + \sqrt{6} + \sqrt{2} - \sqrt{3}$
3. $\sin x = \frac{3}{5}$ $\cos x = -\frac{4}{5}$
4. $\cos \frac{x}{2} = \pm \frac{\sqrt{30}}{6}$ and $\tan \frac{x}{2} = \pm \frac{\sqrt{5}}{5}$
5. $\pm \frac{2\sqrt{5}}{5}, \pm \frac{1\sqrt{5}}{5}$

Practice Problems

1. Solve : $0^\circ \leq \theta \leq 360^\circ$

- a) $2 \sin \theta = \sqrt{3}$ b) $2 \sin \theta = -1$ c) $2 \cos \theta - 1 = 0$ d) $2 \sin \theta - 3 = 0$
 e) $4 \sin^2 \theta - 3 = 0$ f) $3 \tan^2 \theta = 1$ g) $\tan^2 \theta + \tan \theta = 0$ h) $2 \cos^3 \theta - \cos \theta = 0$
 i) $\cos \theta - \sec \theta = 0$ j) $\sec^2 \theta - \tan^2 \theta = 0$ k) $\cos^2 \theta - \sin^2 \theta = \frac{1}{2}$ l) $\tan^2 \theta + \cot^2 \theta = 2$

2. Solve for angles between 0 and 2π .

- a) $2 \sin^2 x - 5 \sin x + 2 = 0$ b) $2 \cos^2 x - \cos x - 1 = 0$ c) $2 \sin x \cos x + \sin x = 0$
 d) $2 \sin^2 x - \sin x - 1 = 0$ e) $6 \cos^2 x + 5 \cos x - 4 = 0$ f) $1 - \sin x = 2 \sin^2 x$
 g) $2 \sin^2 x + 5 \sin x - 3 = 0$ h) $4 \cos^2 x - 1 = 0$ i) $4 \sin^2 x - 1 = 0$
 j) $5 \sin \theta - 3 = 1$ k) $8 \cos \theta + 2 = -5$ l) $3 \tan x + 5 = 6$
 m) $4 \sin x - 1 = 2$ n) $1 - 6 \cos x = 3$ o) $\sin 2\theta = 1$
 p) $\cos 2\theta = 0.5$ q) $\tan^2 2\theta = 1$ r) $\sin\left(\frac{\theta}{2}\right) = \frac{\sqrt{3}}{2}$
 s) $\cos^2\left(\frac{\theta}{2}\right) = 0.5$

Answers

1. a) $60^\circ, 120^\circ$ b) $210^\circ, 330^\circ$ c) $60^\circ, 300^\circ$ d) no solutions
 e) $60^\circ, 120^\circ, 240^\circ, 300^\circ$ f) $30^\circ, 210^\circ, 150^\circ, 330^\circ$ g) $0, 180^\circ, 135^\circ, 315^\circ, 360^\circ$
 h) $45^\circ, 90^\circ, 135^\circ, 225^\circ, 270^\circ, 315^\circ$ i) $0, 180^\circ, 360^\circ$ j) no solution
 k) $30^\circ, 150^\circ, 210^\circ, 330^\circ$ l) $45^\circ, 135^\circ, 225^\circ, 315^\circ$
2. a) $\frac{\pi}{6}, \frac{5\pi}{6}$ b) $0, \frac{5\pi}{6}, \frac{7\pi}{6}, 2\pi$ c) $0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, 2\pi$ d) $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$
 e) $\frac{\pi}{3}, \frac{5\pi}{3}$ f) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$ g) $\frac{\pi}{6}, \frac{5\pi}{6}$ h) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$
 i) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ j) $\sin^{-1}\left(\frac{4}{5}\right), \pi - \sin^{-1}\left(\frac{4}{5}\right)$ k) $\cos^{-1}\left(\frac{-7}{8}\right), 2\pi - \cos^{-1}\left(\frac{-7}{8}\right)$
 l) $\tan^{-1}\left(\frac{1}{3}\right), \pi + \tan^{-1}\left(\frac{1}{3}\right)$ m) $\sin^{-1}\left(\frac{3}{4}\right), \pi - \sin^{-1}\left(\frac{3}{4}\right)$ n) $\cos^{-1}\left(\frac{-1}{3}\right), 2\pi - \cos^{-1}\left(\frac{-1}{3}\right)$
 o) $\frac{\pi}{4}, \frac{5\pi}{4}$ p) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ q) $\frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$
 r) $\frac{2\pi}{3}, \frac{4\pi}{3}$ s) $\frac{\pi}{2}, \frac{3\pi}{2}$