First Name:____________________ Last Name: __________________ Student ID: ________

Operations on Functions

Sum, Difference, Product, and Quotient

If f and g are two functions, then the **sum** $f + g$, the **difference** $f - g$, the **product** fg, and the **quotient** f/g are defined as follows:

$$(f+g)(x) = f(x) + g(x), \qquad (1)$$
$$(f-g)(x) = f(x) - g(x), \qquad (2)$$
$$(fg)(x) = f(x)g(x), \qquad (3)$$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \text{ provided } g(x) \neq 0. \qquad (4)$$

Example 1

Let $f(x) = \sqrt{4-x}$ and $g(x) = \sqrt{3+x}$. Find the functions $f + g$, $f - g$, fg, and f/g, and find their domains.

Example 2

Let $f(x) = 2x - 1$ and $g(x) = x^2 + x - 2$. Find each of the following functions:

a. $(f+g)(x)$ **b.** $(f-g)(x)$ **c.** $(fg)(x)$ **d.** $\left(\frac{f}{g}\right)(x)$.

Determine the domain for each function.

Study Tip

If the function $\frac{f}{g}$ can be simplified, determine the domain *before* simplifying.

Example 3

Two functions, f and g, are defined using the tables below.

x	-2	-1	0	1	2	3
f(x)	3	2	3	2	3	2

x	-2	-1	0	1	2	3
g(x)	3	2	1	0	-1	-2

Evaluate the following:

a. $(f+g)(2) =$

b. $(g-f)(-2) =$

c. $(f/g)(1) =$

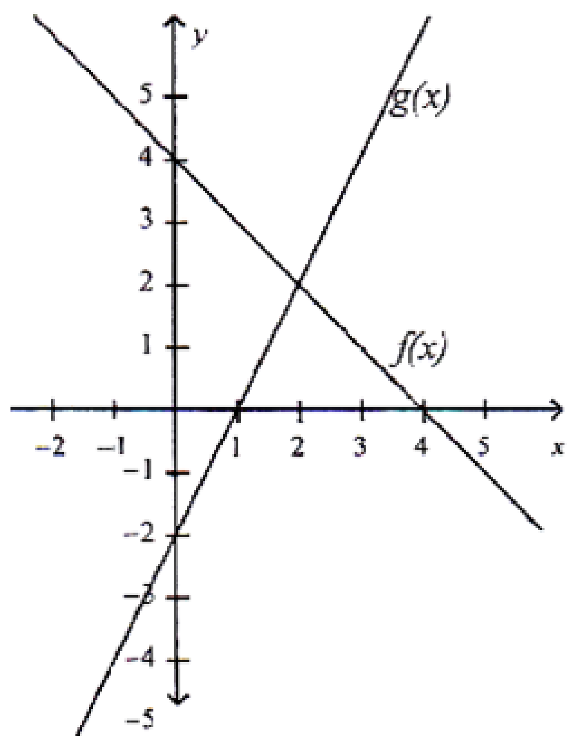
Example 4

a. what is $(g-f)(2)$?

b. what is $(g \times f)(4)$?

c. what is $(\frac{g}{f})(1)$?

d. what is $(\frac{f}{g})(1)$?



Example 5

Graph the function $h\,(x) = (f + g)\,(x)$ for the domain $[-3, 3]$, given the functions $f\,(x) = -x - 3$ and $g\,(x) = 2^x$.

Example 6

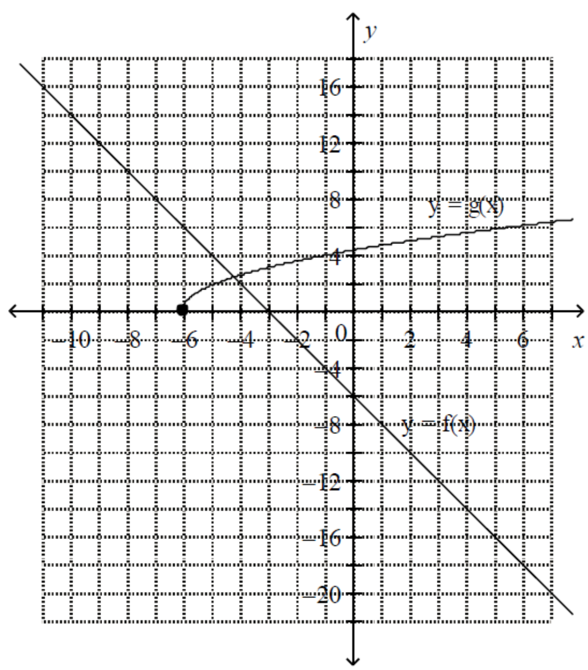
Use the graphs of $y = f(x)$ and $y = g(x)$.

a) State the domain and range of $y = f(x)$.

b) State the domain and range of $y = g(x)$.

c) Sketch the graph of $y = f(x) \cdot g(x)$. (Make a table of values)

d) What is the domain of $y = f(x) \cdot g(x)$? How is it related to the domain of $y = f(x)$ and $y = g(x)$?



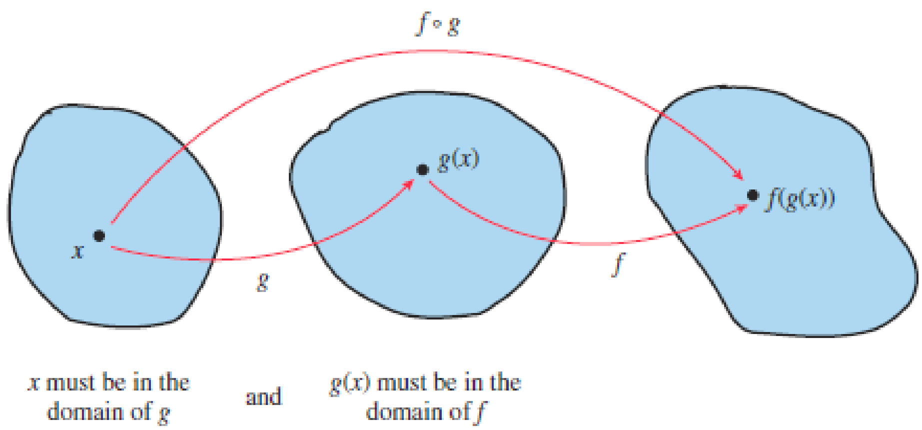
Composition of Functions

If f and g are two functions, then the **composition** of f and g, denoted by $f \circ g$, is the function defined by

$$(f \circ g)(x) = f(g(x)). \tag{5}$$

The **composition** of g and f, denoted by $g \circ f$, is the function defined by

$$(g \circ f)(x) = g(f(x)). \tag{6}$$



Example 7

Find the functions and their domains given $f(x) = x + \frac{1}{x}$, $g(x) = \frac{x+17}{x+2}$.

(a) $f \circ g$

(b) $g \circ f$

(c) $f \circ f$

(d) $g \circ g$

Example 8

Two functions, f and g, are defined using the tables below.

x	-2	-1	0	1	2	3
f(x)	3	2	3	2	3	2

x	-2	-1	0	1	2	3
g(x)	3	2	1	0	-1	-2

Evaluate the following.

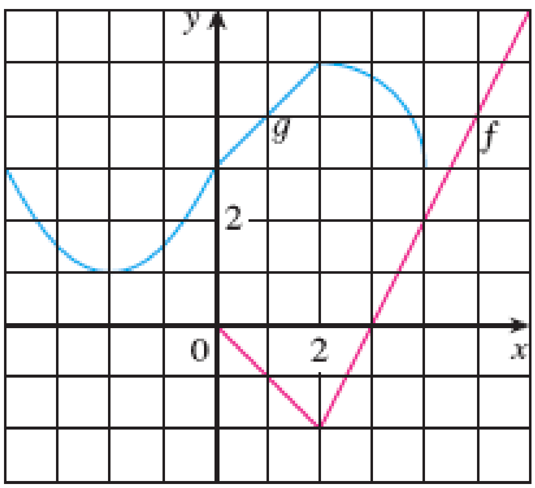
a. $(f \circ g)(1) =$

b. $(f \circ g)(3) =$

c. $(g \circ f)(0) =$

Example 9

Use the given graphs of *f* and *g* to evaluate each expression, or if the expression is undefined, enter UNDEFINED.



(a) $f(g(2))$

(b) $g(f(0))$

(c) $(f \circ g)(0)$

(d) $(g \circ f)(6)$

(e) $(g \circ g)(-2)$

(f) $(f \circ f)(4)$

Example 10

Express each function as a composition of two or more functions. Verify your choice of functions.

a. $f(x) = 2^{3x-1}$

b. $g(x) = \frac{3}{2\sin(x) - 1}$

Example 11

Express the function in the form $f \circ g \circ h$. (Use non-identity functions for f, g, and h.)

$H(x) = 3\sqrt{7 + |x|}$

Example 12

Let $f(x) = x^2 - 1$ and let $g(x) = \sqrt{x}$. Find the domains of the composite functions

(a) $g \circ f$ **(b)** $f \circ g$.

Example 13

If you invest x dollars at 5% interest compounded annually, then the amount $A(x)$ of the investment after one year is $A(x) = 1.05x$.

(a) Find $A \circ A \circ A$.

(b) What does this composition represent?

(c) Find a formula for the composition of n copies of A.

Bonus Questions – Extra Practice

1.

Three on a Match Match each function f with a function g and a domain D so that $(f \circ g)(x) = x^2$ with domain D.

f	g	D
e^x	$\sqrt{2-x}$	$x \neq 0$
$(x^2+2)^2$	$x+1$	$x \neq 1$
$(x^2-2)^2$	$2\ln x$	$(0, \infty)$
$\dfrac{1}{(x-1)^2}$	$\dfrac{1}{x-1}$	$[2, \infty)$
x^2-2x+1	$\sqrt{x-2}$	$(-\infty, 2]$
$\left(\dfrac{x+1}{x}\right)^2$	$\dfrac{x+1}{x}$	$(-\infty, \infty)$

2.

Identifying Identities An *identity* for a function operation is a function that combines with a given function f to return the same function f. Find the identity functions for the following operations:

(a) Function addition. That is, find a function g such that $(f+g)(x) = (g+f)(x) = f(x)$.

(b) Function multiplication. That is, find a function g such that $(fg)(x) = (gf)(x) = f(x)$.

(c) Function composition. That is, find a function g such that $(f \circ g)(x) = (g \circ f)(x) = f(x)$.