AP Calculus In-Class Seven – Applications of Differential Calculus

3.7 Motion along a Curve: Velocity and Acceleration; 3.8 Related Rates; 3.9 Slope of a Polar Curve

1. If the position of a particle moving along a horizontal line is given by $s = t^4 - 6t^3 + 12t^2 + 3$.

   $2t$
   t

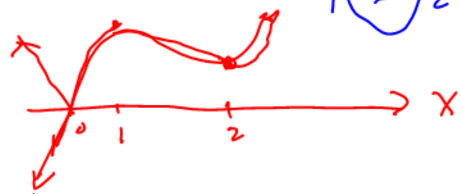
   a) When is the particle at rest?
   b) Find the values of t for which the velocity is increasing.
   c) Find the values of t for which the speed is increasing.

   $v(1) = 4 - 18 + 24$
   $v(2) = 32 -$

a) $v(t) = s'(t) = (t^4 - 6t^3 + 12t^2 + 3)' = 4t^3 - 18t^2 + 24t$
$= 2t(2t^2 - 9t + 12)$. let $v(t) = 0 \Rightarrow t = 0$. $\therefore$ the particle rests at $t=0$.

b) $a(t) = v'(t) = 12t^2 - 36t + 24 = 12(t^2 - 3t + 2) = 12(t-1)(t-2)$

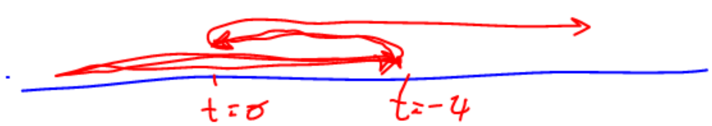
so $v(t)$ increases for $t<1$ or $t>2$.



c) speed $= |v(t)| = 2|t||2t^2 - 9t + 12|$
is increasing for $t > 0$.

2. If a particle moves along a line according to the law $s = t^5 + 5t^4$, what is the number of times it reverses its direction.

$v(t) = s'(t) = 5t^4 + 20t^3 = 5t^3(t+4)$



3. A circular conical reservoir, vertex down, has depth 20 ft and radius of the top 10 ft. Water is leaking out so that the surface is falling at the rate of 0.5 ft/hr. What is the rate, in cubic feet per hour, at which the water is leaving the reservoir when the water is 8 ft deep?

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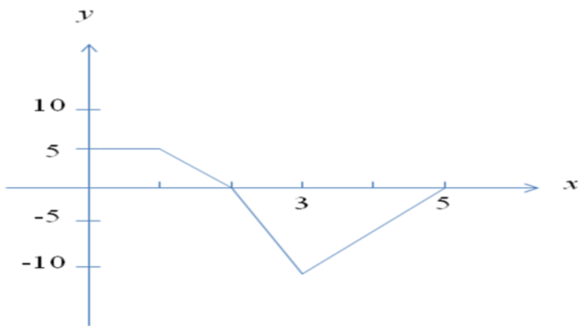
Use the graph shown for Question 4 – 10. It shows the velocity of an object moving along a straight line during the time $0 \le t \le 5$.



4. Find the value of t when the object attains its maximum speed.

5. What is the time interval during which the speed of the object is increasing?

6. What is the time interval during which the acceleration of the object is positive?

7. How many times on $0 < t < 5$ is the object's acceleration undefined?

8. What is the object's acceleration (in cm/sec^2) during $2 < t < 3$?

9. Find the value of t when the object is furthest to the right.

10. What is the object's average acceleration (in cm/sec^2) for the time interval $0 \le t \le 3$?

11. The table shows the velocity at various times of an object moving along a line. What is the estimate of its acceleration (in cm/sec^2) at $t = 1$.

t (sec)	1.0	1.5	2.0	2.5
v (cm/sec)	12.2	13.0	13.4	13.7

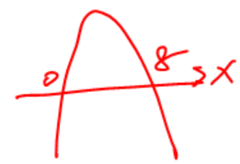
$\vec{R}(t) = [x(t), y(t)] = [2t, 4t - t^2]$

For Questions 12 and 15, the motion of a particle in a plane is given by the pair of equations $x = 2t$ and $y = 4t - t^2$.

$\therefore t = \frac{x}{2}$

12. What is the name of the curve for the particle to move along?

$y = 4\left(\frac{x}{2}\right) - \left(\frac{x}{2}\right)^2 = 2x - \frac{1}{4}x^2 = -\frac{1}{4}x(x-8)$.



a parabola

13. Find the speed of the particle at any time t.

$\vec{v}(t) = \vec{R}'(t) = [2(t)', 4(t)' - (t^2)'] = [2, 4-2t]$

speed $= |\vec{v}(t)| = \sqrt{2^2 + (4-2t)^2} = \sqrt{4 + 16 - 16t + 4t^2} = 2\sqrt{t^2 - 4t + 5}$

$= 2\sqrt{(t-2)^2 + 1}$

14. What is the minimum speed of the particle?

minimum speed occurs at $t = 2$, the minimum speed is 2.

15. Find the acceleration of particle at any time t.

$\vec{a}(t) = \vec{v}'(t) = [0, -2]$

16. A particle is moving on the curve of $y = 2x - \ln x$ so that $\frac{dx}{dt} = -2$ at all times t. Find $\frac{dy}{dt}$ at the point (1, 2).

17. A vertical circular cylinder has radius r ft and height h ft. If the height and radius both increase at the constant rate of 2 ft/sec, then what is the rate at which the lateral surface area increases?

18. Find the slope of the line tangent to the polar curve $r = \theta^{-0.5}$ at $\theta = \pi/4$.

19. Determine the equations of all lines that are tangent to the polar curve $r = 3 - 3\cos\theta$ and horizontal.

20. Find the points on the polar curve $r = 1 - 2\cos\theta$ where the tangent line is horizontal and those where it is vertical.