

AP Calculus Homework One – Limit and Continuity

1.1 Definitions of Limits; 1.2 Continuity; 1.3 Limits Properties

1. Show that limits do not exist.

(a) $\lim_{x \rightarrow -2} \frac{x+2}{|x+2|}$

(b) $\lim_{x \rightarrow 0} \sin \frac{1}{x}$

(c) $\lim_{x \rightarrow 0} \sqrt{3 + \arctan \frac{1}{x}}$

2. Find limits.

(a) $\lim_{x \rightarrow 0} \frac{x^2}{2x-1}$

(b) $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 4}$

(c) $\lim_{x \rightarrow -1} \frac{2+2/x}{x^2 - 4x - 5}$

(d) $\lim_{h \rightarrow 0} \frac{5(h-1)^2 + (h-1) - 4}{h}$

(e) Explain, using examples, when substitution can not be used to solve a limit.

3. Discuss the continuity and sketch the graph of $f(x) = \begin{cases} \frac{x^2 + x}{x}, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$.

4. If $[x]$ is the greatest integer not greater than x , then $\lim_{x \rightarrow \frac{1}{2}} [x]$ is
 (A) 1/2 (B) 1 (C) nonexistent (D) 0 (E) none of these

5. Find a value of k such that $f(x)$ is continuous at $x = 0$.

$$f(x) = \begin{cases} \frac{x^2 - x}{2x}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$$

6. The function $s(x)$ is defined as follows. Find a value of k such that $s(x)$ is continuous for all x .

$$s(x) = \begin{cases} 4x - 11, & \text{if } x < 3 \\ kx^2, & \text{if } x \geq 3 \end{cases}$$

7. Discuss the continuity of the graph of $y = \frac{x^2 - 9}{3x - 9}$, indicating type of discontinuity if there is one.