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Polynomial Division

Numerical Long Division Review

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Polynomial Long Division

$x+5\overline{)2x^2+6x-3}$ $2x-4$ $2x^2-10x$ $-4x-3$ $-4x-20$ 17 $q(x)=2x-4$ $r(x)=17$ $(2x^2+6x-3)=(x+5)(2x-4)+17$	$3x-4\overline{)12x^3-10x^2+x-4}$

Example 1: Determine the quotient and remainder for $(-11x + 4x^3 - 9) \div (2x - 3)$, $x \neq \frac{3}{2}$

$$2x-3\overline{)4x^3+0x^2-11x-9}$$

$2x^2+3x-1$

$4x^3-6x^2$

$6x^2-11x$

$6x^2-9x$

$-2x-9$

$-2x+3$

-12

$-11x+4x^3-9$

$=$

$(2x-3)(2x^2+3x-1)-12$

Example 2: When a polynomial is divided by 2x – 5, the quotient is $2x^2 + 3x - 1$ with a remainder of -3. What is the polynomial?

$f(x) = (2x-5)(2x^2+3x-1) - 3$

$4x^3 + 6x^2 - 2x - 10x^2 - 15x + 5 - 3$

$= 4x^3 - 4x^2 - 17x + 2$

Example 3: The volume of a rectangular prism is shown by the expression $\mathbf{2x^3 + 15x^2 + 22x - 15}$. Determine the expressions for each of the dimensions of the rectangular prism if the height is **x + 3**.

$$
\begin{array}{r l}
 & 2x^2 + 9x - 5 \\
x+3 \,\big) & 2x^3 + 15x^2 + 22x - 15 \\
 & 2x^3 + 6x^2 \\
\hline
 & 9x^2 + 22x \\
 & 9x^2 + 27x \\
\hline
 & -5x - 15 \\
 & -5x - 15 \\
\hline
 & 0
\end{array}
$$

$V = lwh$

$V(x) = l(x)w(x)h(x)$

$(x+3)(2x-1)(x+5)$

$\uparrow$

h

$9x^2 + 27x$
$= (2x-1)(x+5)$

Synthetic Division

Good news ☺ There's a great short cut if the divisor is of the form x − k.

Divide $2x^2 + 5x - 8$ by $x + 3$.

$$
\begin{array}{r|rrr}
-3 & 2 & 5 & -8 \\
 & & -6 & 3 \\
\hline
 & 2 & -1 & -5
\end{array}
$$

coeff rem

$q(x) = 2x - 1$

$r = -5$

$$
\begin{array}{r l}
 & 2x - 1 \\
x+3 \,\big) & 2x^2 + 5x - 8 \\
 & 2x^2 + 6x \\
\hline
 & -x - 8 \\
 & -x - 3 \\
\hline
 & -5
\end{array}
$$

$q = 2x - 1$

$r = -5$

Now let's STOP and review.

Summary

- Long division or synthetic division can be used to determine the quotient and remainder when a polynomial (dividend) is divided by another polynomial (divisor) of the same or a lesser degree.
- To carry out the division, ensure the terms of the polynomial are in descending order of degree. Missing powers of *x* in the dividend or divisor are included using a coefficient of 0 to keep work accurate and aligned correctly.
- The division is completed when the degree of the remaining terms after subtraction is less than the degree of the divisor.
- The result of the division $P(x) \div d(x)$, where $P(x)$ is the polynomial dividend and $d(x)$ is the divisor, is given by

$$\frac{P(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$$

  Where $q(x)$ is the quotient and $r(x)$ is the remainder.
- The polynomial dividend can be expressed in terms of the quotient, divisor and remainder using

$$P(x) = d(x)q(x) + r(x)$$

To check the result of the division, multiply the quotient by the divisor and add the remainder. The result should be the dividend.

Remainder Theorem

Determine the remainder when $6x^2 + 5x - 2$ is divided by $x - 3$.

$$\begin{array}{r|rrr} 3 & 6 & 5 & -2 \\ & & 18 & 69 \\ \hline & 6 & 23 & 67 \end{array}$$

$q(x) = 6x + 23$

$r = 67$

$f(3) = 6(3)^2 + 5(3) - 2 = 54 + 15 - 2$
$= 67$

Remainder Theorem

When a polynomial P(x) is divided by a linear factor (x-k), the remainder is given by P(k)

Proof: $P(x) =$

Example 1: Find the remainder when:

a. $(6x^3 - 2x + 3) \div (x - 1)$

$f(1) = 6 - 2 + 3$
$= 7$

$r = 7$

b. $(8x^2 - 6x + 2) \div (2x - 3)$

$f(\frac{3}{2}) = \frac{11}{2}$

$2(x - \frac{3}{2})$

$r = 2 \cdot \frac{11}{2}$

$r = 11$

Example 2: When $3x^3 + 4x^2 - kx + 2$ is divided by $x + 3$, the remainder is -1. Find k.

$f(x) =$
$f(-3) = -1 = 3(-3)^3 + 4(-3)^2 - k(-3) + 2$
$k = 14$

Factor Theorem

Factor $x^2 + 9x + 20$.

$(x+5)(x+4)$

Determine the remainder for $(x^2 + 9x + 20) \div (x + 5)$

$f(-5) = r$
$25 - 45 + 20 = r$
$r = 0$

Factor Theorem
(x-k) is a factor of p(x) <-> p(k) = r = 0

Example 1: Determine whether $x - 3$ a factor of each of the following:

a. $x^3 - 2x^2 + x - 12$

$f(3) = 0$

$\to f(3) = 0$

$\therefore x - 3$ is a factor

b. $2x^4 - 5x - 10$

$f(3) = 137$
$137 \neq 0$
$\therefore x - 3$ is not a factor

Example 2: Determine the value of k if x + 1 is a factor of the function $f(x) = x^5 - 3x^4 + kx^2 + 13$.

$f(-1)=0$

solve for k

Polynomial Factoring

Expand the expression **(2x + 1)(x – 4)(x + 3)**

Using the expanded expression and the factor theorem, verify that **2x + 1**, **x – 4** and **x + 3** are factors.

Brainstorm a strategy that you could follow to find the factors of a polynomial expression.

Unfortunately, there is no general formula that lets us plug in a polynomial and output the roots/factors

1) Guess and check for roots to identify linear factors
2) Synthetic Division
3) Repeat with your quotient which is degree 1 less than above

Hope this works until we get down to a quadratic because we have the quadratic equation

find 1 root and you can get all the other roots

1. Factor $x^3 + 3x^2 - 4x - 12$.

2. Factor $6x^4 + x^3 - 46x^2 - 39x + 18$.

More practice. **Have fun!**

Use polynomal division to simplify each of the following quotients.

a) $\dfrac{x^4+3x^3-x^2-x+6}{x+3}$ b) $\dfrac{2x^4-5x^3+2x^2+5x-10}{x-2}$ c) $\dfrac{7x^4-10x^3+3x^2+3x-3}{x-1}$

d) $\dfrac{2x^4+8x^3-5x^2-4x+2}{x^2+4x-2}$ e) $\dfrac{3x^4-x^3+8x^2+5x+3}{x^2-x+3}$ f) $\dfrac{3x^4+9x^3-5x^2-6x+2}{3x^2-2}$

g) $\dfrac{x^3-2x^2-4}{x-2}$ h) $\dfrac{x^3-4x^2+9}{x-3}$ i) $\dfrac{x^4-13x-42}{x^2-x-6}$

Answers

a) x^3-x+2 b) $2x^3-x^2+5$ c) $7x^3-3x^2+3$

d) $2x^2-1$ e) $3x^2+2x+1$ f) x^2+3x-1

g) x^2+2x+2 h) x^2-x-3 i) x^2+x+7