First Name:_____ Last Name: _____ Student ID: _____

Polynomial Equations and Inequalities

Methods of Factoring Polynomials

- First, look for a common factor.
- Next, check to see if the polynomial can be factored by grouping.
- The factor theorem can be used to determine the first factor of the polynomial.
  If P(a) = 0, then (x−a) is a factor. Long and synthetic division can be used to find the corresponding factor.
- Polynomials of the form $ax^{2n} + bx^n + c$ are essentially quadratic in form. Factor these polynomials as you would a quadratic (it may help to make the substitution $t = x^n$ to obtain $at^2 + bt + c$).

Note: Not all polynomials can be factored using these methods.

Common Factoring

a) $30x^5 + 9x^3 + 12x^2 - 6x^4$

b) $12(10x + 7) - 3x(10x + 7)$

Grouping (Pairs)

a) $5xy - 10x - 7y + 14$

b) $x^5 + 4x^4 + 2x^3 + 8x^2 - x - 4$

Trinomials

a) $2x^2 + 15x + 7$

b) $10x^2 + x - 3$

c) $10x^2 - 80x + 150$

d) $x^2 + 7xy + 12y^2$

Difference of Squares

a) $9x^2 - 16$

b) $100x^2 - 49y^2$

c) $81x^4 - 1$

Sum and Difference of Cubes

Sum of Cubes	Difference of Cubes

a) $64x^3 - 27$

b) $1000x^3 + 343y^3$

c) $125a^3 + (a + b)^3$

Variable Replacement

Look For:

a) $3(x-3)^5 + 5x(x-3)^4$

b) $(2x+5)^7(3x-1)^4 + (2x+5)^6(3x-1)^5$

c) $6(x-5)^2 + 11(x-5) + 3$

Grouping (Trinomial)

Look For:

$x^2 + 6x + 9 - y^2$

Factoring Complex Trinomials

Look For:

a) $x^{10} - 7x^5 + 10$

b) $x^{\frac{2}{3}} - 6x^{\frac{1}{3}} - 7$

Polynomial Factoring

1. Factor $x^3 + 3x^2 - 4x - 12$.

2. Factor $6x^4 + x^3 - 46x^2 - 39x + 18$.

Solving Polynomial Equations

Example: Solve each of the following polynomial equations.

a) $2x^2 + 15x - 8 = 0$

b) $(4x - 1)(x + 2)^2(x - 9) = 0$

c) $(x+2)^3(x-6)^4 + (x+2)^2(x-6)^5 = 0$	d) $2x^3 + x^2 - 27x - 20 = x^2 + 7$

Solving Polynomial Inequalities

Example 1: Solve each inequality.

a) $(x + 1)(x - 2)^2(2x - 7) \leq 0$

Algebraic Solution

Graphical Solution

b) $x^3 + 12x^2 - 3x - 45 < 2x^2 + x - 5$

Example 2: A company manufactures and sells flashlights. For a particular model, the marketing research and financial departments estimate that at a price of $p per unit, the weekly cost (C) and revenue (R), in thousands of dollars, will be given by the equations:

$C = 7 - p$
$R = 5p - p^2$

Determine the prices at which the company will make a profit. *Round final answers to 2 decimal places, where appropriate.*

Factoring Practice

1. Factor as fully as possible:
   a) $x^4 - 13x^2 + 36$
   b) $x^8 - 32x^4 + 256$
   c) $x^{10} - 100$
   d) $x^8 - 6561$
   e) $(5 - 4x)^2 + 8(5 - 4x) - 20$
   f) $12(x + 5)^2 + 19(x + 5) + 4$
   g) $4(2x - 19)^2 - 81$
   h) $(3x + 11)^2 - (5x - 2)^2$
   i) $9x^2 - 24x + 16 - 16y^2$
   j) $x^2 + 4x + 4 - x^4$
   k) $4x(x + 9)^8 - 11(x + 9)^7$
   l) $(2x + 5)^4(10x - 1)^7 - (2x + 5)^3(10x - 1)^8$
   m) $12x(x + 6)^{-7} - 15(x + 6)^{-6}$

2. The expression $\mathbf{x^2(2x + 5) + 6x(2x + 5) + 9(2x + 5)}$ represents the volume of a square based rectangular prism. Determine the actual dimensions of the prism if the height of the prism is 21 cm.

3. The product of the ages of three friends is shown by the expression $\mathbf{x^4 - 625}$. Write an expression for the sum of their ages.

ANSWERS

1a) $(x + 2)(x - 2)(x + 3)(x - 3)$ 1b) $(x^2 + 4)^2(x + 2)^2(x - 2)^2$ 1c) $(x^5 + 10)(x^5 - 10)$
1d) $(x^4 + 81)(x^2 + 9)(x + 3)(x - 3)$ 1e) $(15 - 4x)(3 - 4x)$ 1f) $(3x + 19)(4x + 21)$
1g) $(4x - 47)(4x - 29)$ 1h) $-(8x + 9)(2x - 13)$ 1i) $(3x + 4y - 4)(3x - 4y - 4)$
1j) $-1(x - 2)(x + 1)(x^2 + x + 2)$ 1k) $(x + 9)^7(4x^2 + 36x - 11)$ 1l) $-2(2x + 5)^3(10x - 1)^7(4x - 3)$
1m) $-3(x + 6)^{-7}(x + 30)$ 2) length = 11 cm, width = 11 cm, height = 21 cm 3) $x^2 + 2x + 25$