MDM4U HW2.

P245-246: Q5, Q6, Q10, Q11, Q15, Q18

P251-253: Q2-Q3, Q8, Q11, Q16, Q17

Sol. P245 Q5:

Using $\frac{n!}{n_1!n_2!\cdots n_k!}$, where $n_1+n_2+\cdots+n_k=n$.

$n=8$, $n_1=5$, $n_2=3$.

$\frac{8!}{5!3!}=56$ (ways)

Q6. using $1, \underline{2,2}, 3, 5, 5, \underline{6}$.

7-digit even number < 3000000

$\underline{1}$ _ _ _ _ _ $\underline{2}$: $\frac{5!}{2!}$

or $\underline{2}$ _ _ _ _ _ $\underline{2}$: $\frac{5!}{2!}$

or

$\underline{1}$ _ _ _ _ _ $\underline{6}$: $\frac{5!}{2!2!}$

or $\underline{2}$ _ _ _ _ _ $\underline{6}$: $\frac{5!}{2!}$

$3\times\frac{5!}{2!}+\frac{5!}{2!2!}$

$=210$

such 7-digit numbers.

P245 Q10.

$n=6$.

a) ${}_6P_6 = 6! = 720$ (ways)

b) $\frac{6!}{3!3!} = 20$ (ways)

c) $\frac{6!}{2!2!2!} = 90$ (ways)

P246. Q11.

$n=20$, $n_1=n_2=n_3=n_4=5$

$\therefore \frac{20!}{5!5!5!5!} = 11732745024$ (ways)

Q15.

10 students. 4 positions. 2 twins

case 1. no twins: ${}_8P_4 = 1680$

case 2. one of twins: ${}_1P_1 \times 4 \times {}_8P_3 = 1 \times 4 \times 336 = 1344$.

case 3. both twins; ${}_2P_2 \times \frac{4\times 3}{2} \times {}_8P_2 = 6 \times 56 = 336$

In total, $1680 + 1344 + 336 = 3360$ (ways)

Q18. 7 greens. 8 browns.

_ B _ B _ B _ B _ B _ B _ B _ B _

GG, G, G, G, G, G.

$$9 \times \frac{8 \times 7 \times 6 \times 5 \times 4}{5!} = \frac{{}_9P_6}{{}_5P_5} = 504$$

(ways)

P251 Q2.

using $t_{n,r} = t_{n-1,r-1} + t_{n-1,r}$

a) $t_{7,2} + t_{7,3} = t_{8,3}$

b) $t_{51,40} + t_{51,41} = t_{52,41}$

c) $t_{18,12} - t_{17,12} = t_{17,11}$

d) $t_{n,r} - t_{n-1,r} = t_{n-1,r-1}$

Q3. Sum of Row n $= 2^n$.

Q3. Sum of Row $n = 2^n$

a) Sum of Row $12 = 2^{12} = 4096$

b) Sum of Row $20 = 2^{20} = 1048576$

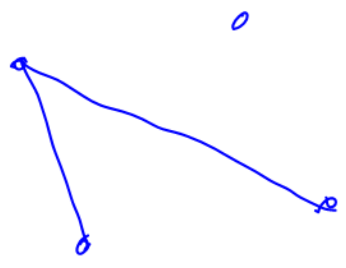
c) Sum of Row $25 = 2^{25} = 33554432$

d) Sum of Row $(n-1) = 2^{n-1}$

P252 Q8.

i) In a n-Polygon, it has n vertices, any two vertices can determine a side or a diagonal.

For example. if $n=4$,



The number of line segments determined by the n vertices is $nC_2 = \frac{nP_2}{2} = \frac{n(n-1)}{2}$

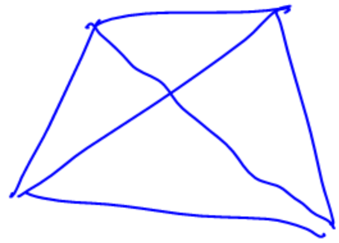
the number of diagonals is $nC_2 - nC_1 = \frac{n(n-1)}{2} - n$

$= t_{n,2} - t_{n,1}$.

i) If $n=4$.

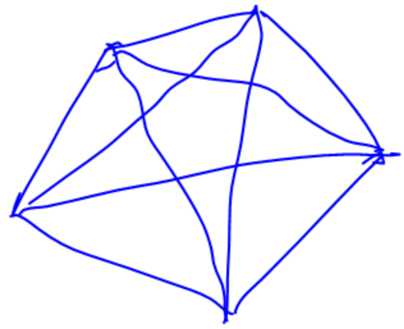
$$4C_2 - 4C_1 = \frac{4\times 3}{2} - 4 = 6 - 4 = 2.$$

$(t_{4,2} - t_{4,1})$



ii) If $n=5$,

$$5C_2 - 5C_1 = t_{5,2} - t_{5,1}$$
$$= 10 - 5 = 5$$



iii) If $n=6$

$$6C_2 - 6C_1 = t_{6,2} - t_{6,1}$$
$$= 15 - 6 = 9$$

b) If $n=7$.

$$7C_2 - 7C_1 = t_{7,2} - t_{7,1} = 21 - 7 = 14.$$

If $n=8$

$$8C_2 - 8C_1 = t_{8,2} - t_{8,1} = 28 - 8 = 20$$

Q11. a)

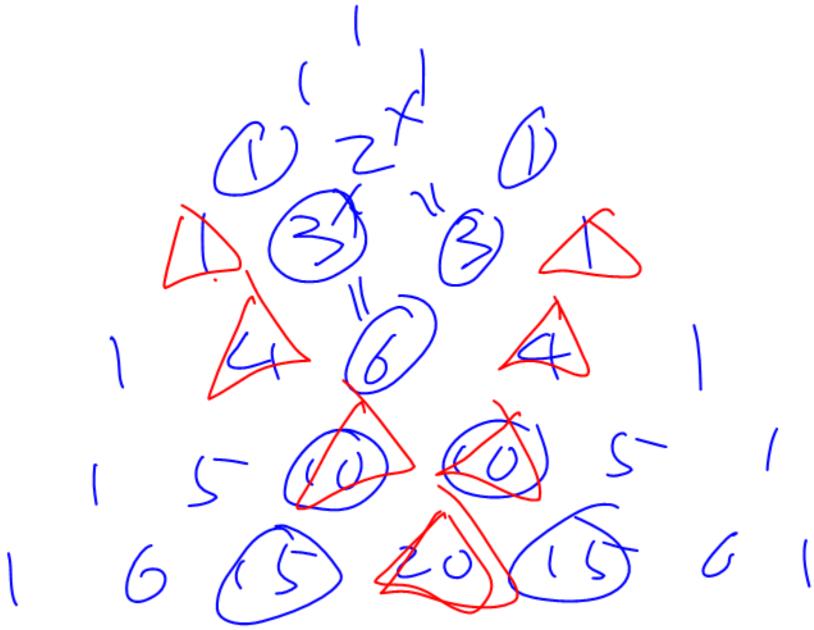
If $\{b_n\}$ is the sequence of tetrahedral numbers.

then $b_n = t_{n+2,\, n-1}$

$b_1 = t_{3,0}$

$b_2 = t_{4,1}$

$b_3 = t_{5,2}$



b) $b_{12} = t_{14,11} = {}_{14}C_{11} = 364$

Q16.

a) $n=1$,  : $3\times1 = 3$

$n=2$,  : $3\times1+3\times2 = 9$

$n=3$,  : $3\times1+3\times2+3\times3 = 18$

⋮

n, $3\times1+3\times2+\cdots+3\times n$

$= 3(1+2+\cdots+n) = 3\cdot\frac{n(n+1)}{2}$

$= 3\cdot {}_{n+1}C_2$

$= 3\cdot t_{n+1,2}$

b) if $n=10$,

$3\cdot t_{11,2} = 3\cdot\frac{10\times11}{2} = 165$;

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