AP Calculus Homework Twelve – Applications of Definite Integral and Polar Coordinates

5.4 Polar Coordinates

1. Sketch the graphs of the polar equations.
   (a) $r^2 = 4\cos 2\theta$ (b) $r = 4\csc\theta$ (c) $r = 2^\theta,\ \theta \geq 0$

2. Find a polar equation that has the same graph as the given equation.
   (a) $x = -3$ (b) $x^2 + y^2 = 16$ (c) $x^2 - y^2 = 9$

3. Find an equation in *x* and *y* that has the same graph as the given polar equation.
   (a) $r\cos\theta = 5$ (b) $r = 6\sin\theta$ (c) $r = \tan\theta$

4. Find the slope of the tangent line to the graph of the equation at the indicated value of θ.
   (a) $r = 8\cos 3\theta,\quad \theta = \pi/4$ (b) $r\theta = 1,\quad \theta = 2\pi$

5. Sketch the graph of the equation and find the area of the region bounded by the graph.

   (a) $r = 4 + \sin\theta$ (b) $R = \{(r, \theta): 0 \le \theta \le \pi/2, 0 \le r \le e^{\theta}\}$

6. Find the area of the region bounded by one loop of the graph of the given equation.

   (a) $r^2 = 4\cos 2\theta$ (b) $r = \sin 6\theta$

7. Find the area of the region that is outside the graph of the first equation and inside the graph of the second equation: $r = 2 + 2\cos\theta, \quad r = 3$

8. Find the lengths of the following curves.

   (a) $r = e^{-\theta}; \quad 0 \le \theta \le 2\pi$ (b) $r = \cos^2(\theta/2); \quad 0 \le \theta \le \pi$