

Check q 3,6

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Functions: Transformations and Properties

1. For each relation given,

- a. state the domain and range;
 b. identify whether the relation is a function or not.

i. $\{(1,1), (2,1), (3,2), (4,3), (5,5), (6,8)\}$ a) $D: \{1, 2, 3, 4, 5, 6\}$ $R: \{1, 2, 3, 5, 8\}$ b) function	ii. $x = -2$ a) $D: \{-2\}$ $R: \mathbb{R}$ b) relation
iii. $y = (x+1)^2 - 2$ a) $D: \mathbb{R}$ $R: [-2, \infty)$ $\text{graph: } \begin{array}{c} \diagup \\ \diagdown \end{array}$ b) function	iv. $(x-1)^2 + y^2 = 9$ $y^2 = -(x-1)^2 + 9$ a) $D: [-2, 4]$ $R: [-3, 3]$ $-3 \leq x-1 \leq 3$ $-2 \leq x \leq 4$ b) relation
v. $y = -3\sqrt{x+2} + 5$ a) $D: [-2, \infty)$ $R: (-\infty, 5]$ $\text{graph: } \begin{array}{c} \diagup \\ \diagdown \end{array}$ b) function	vi. $y = 2^{x-4} + 3$ a) $D: \mathbb{R}$ $R: (3, \infty)$ $\text{graph: } \begin{array}{c} \diagup \\ \diagdown \end{array}$ b) function

2. If $f(x) = 2x-3$ and $g(x) = 6x^2+3x-18$, determine the value(s) of x such that $f(x)=g(x)$.

$$2x-3 = 6x^2+3x-18$$

$$6x^2+x-15=0$$

$$(3x+5)(2x-3)=0$$

$$x = -\frac{5}{3}$$

or

$$x = \frac{3}{2}$$

3. The graph of $y=f(x)$ is shown.

Determine the following:

- the value of $f(0)$
- the value of x such that $f(x)=0$
- the value of $f(4)-f(-4)$
- state the domain and range

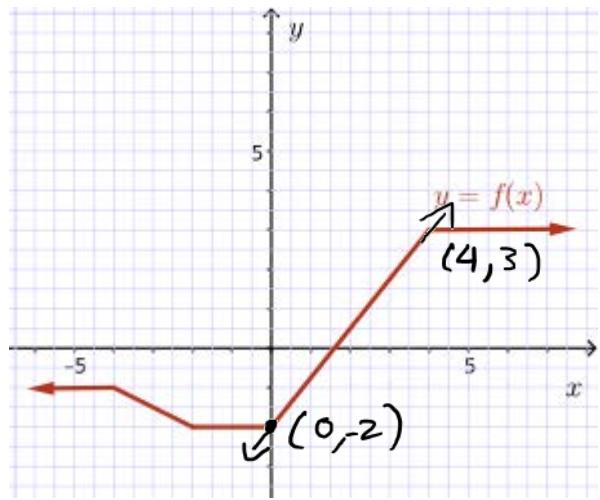
$$a) f(0) = 2$$

$$b) y = \frac{3+2}{4}x - 2 \quad y = \frac{5}{4}x - 2$$

$$0 = \frac{5}{4}x - 2, \quad x = \frac{8}{5}$$

$$c) f(4) = \frac{1}{2} \quad f(4) - f(-4)$$

$$f(-4) = 2 \quad = \frac{1}{2} - 2 = -\frac{3}{2}$$



$$d) D: \mathbb{R}$$

$$R: [-2, 3]$$

4. Given $f(x)=(x-3)^2$ and $g(x)=4x+3$, determine in simplest form

$$a. f(x)-g(x) \quad (x-3)^2 - 4x - 3$$

$$\quad x^2 - 6x + 9 - 4x - 3$$

$$\quad x^2 - 10x + 6$$

$$d. f(g(x)) \quad (4x+3-3)^2$$

$$= (4x)^2$$

$$= 16x^2$$

$$b. 2f(x) \quad 2(x-3)^2$$

$$2(x^2-6x+9)$$

$$2x^2-12x+18$$

$$e. g(g(x)) \quad 4(4x+3)+3$$

$$16x+12+3$$

$$16x+15$$

$$c. f(x)g(x) \quad (x-3)^2(4x+3)$$

$$(x^2-6x+9)(4x+3)$$

$$4x^3 + 3x^2 - 24x^2 - 18x + 36x + 27$$

$$= 4x^3 - 21x^2 + 18x + 27$$

$$f. [g(x)]^2 \quad [4x+3]^2$$

$$= 16x^2 + 24x + 9$$

5. If $f(x)=5-2x+k$ and $f(f(k))=13$, determine the value of $f(-4)$.

$$f(f(k)) \Rightarrow 5 - 2(5 - 2k + k) + k = 13$$

$$5 - 2(5 - k) + k = 13$$

$$5 - 10 + 2k + k = 13$$

$$-5 + 3k = 13$$

$$3k = 18$$

$$k = 6$$

$$f(x) = 5 - 2x + 6$$

$$= 11 - 2x$$

$$f(-4) = 11 - 2(-4)$$

$$= 11 + 8$$

$$= 19$$

$$\begin{aligned} & 9x^2 - 6x + 5 \\ & (-3x+1)(3x+5) \end{aligned}$$

6. If $g(x)=1-3x$ and $f(g(x))=9x^2-6x+5$, determine the value of $f(5)$.

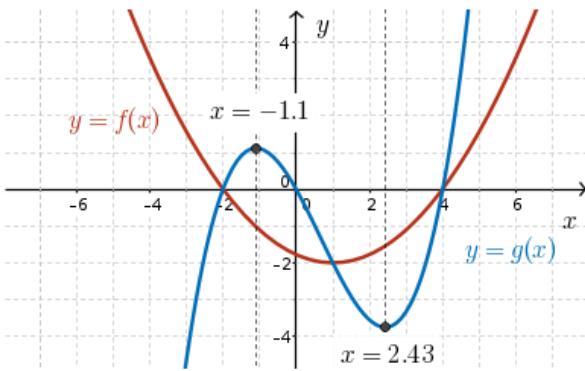
$$g(x) = 1-3x$$

$$g(x) = 5 : 5 = 1-3x$$

$$4 = -3x \quad x = \frac{4}{3}$$

$$\begin{aligned} f\left(\frac{4}{3}\right) &= 9\left(\frac{4}{3}\right)^2 - 6\left(-\frac{4}{3}\right) + 5 \\ &= 29 \end{aligned}$$

7. Given the graphs of $y=f(x)$ and $y=g(x)$ as shown in the graph below



identify the following.

- where $f(x)=g(x)$
- the interval(s) where $g(x)<0$
- the interval(s) where $f(x)\geq g(x)$
- the interval(s) where both functions are decreasing.
- the local maxima and minima for both functions

a) Points where
 $f(x) = g(x)$
 $\{(-2, 0), (1, -2), (4, 0)\}$

- b) $(-\infty, -2) \cup (0, 4)$
c) $(-\infty, -2] \cup [1, 4]$
d) $(-1.1, 1)$

e) $f(x)$ - Local minima: $\{(1, -2)\}$
 $g(x)$ - Local minima: $\{(2.43, -3.6)\}$
Local maxima: $\{(-1.1, 1)\}$

8. Solve. Write your answers using interval notation.

$$\begin{aligned} \text{a. } -3 < \frac{2x+5}{3} \leq 5 \\ -9 < 2x+5 &\leq 15 \\ -14 < 2x &\leq 10 \\ -7 < x &\leq 5 \end{aligned}$$

$$\text{c. } \frac{3x-6}{4x-8} \leq 1$$

$z(x)$

$$x: [-7, 5]$$

$$\text{Roots: } \{3, 0, 4\}$$

$$\text{b. } \frac{3}{x-2} > 1$$

$$\begin{aligned} x-2 > 0 \\ 3 > x-2 \end{aligned}$$

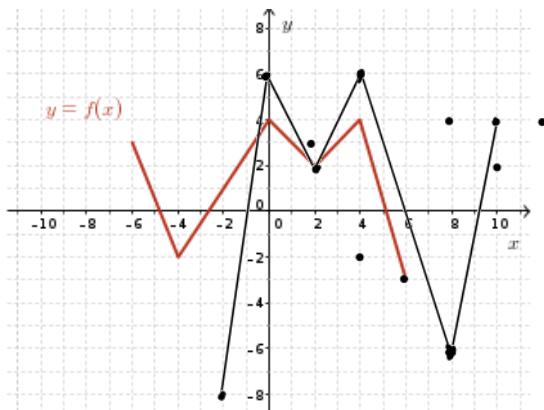
$$\begin{aligned} x-2 < 0 \rightarrow x < 2 \\ 3 < x-2 \quad 5 < x \end{aligned}$$

$$(2, 5)$$

$$\begin{aligned} \text{Test by plugging} \\ \text{in} \\ -4: &< 0 \\ -2: &> 0 \\ 1: &= 0 \\ 5: &> 0 \end{aligned}$$

$$(-3, 0) \cup (4, \infty)$$

9. Given the graph of the function $y=f(x)$, draw the graphs of the following transformed function $y=2f(-(x-4))-2$:



- Shift 4 right
- Reflect on y axis
- horizontal stretch by 2
- Shift 2 down

$$(x, y) \rightarrow (-x+4, 2y-2)$$

10. The function $f(x)$ satisfies the equation $f(x)=f(x-1)+f(x+1)$ for all values of x .

Define $f(1)=1$ and $f(3)=3$; then, $f(2)=1+3=4$. Determine the value of $f(1867)$.

$$f(1) = f(0) + f(2)$$

$$f(1) = f(0) + 4 = 1$$

$$f(0) = -3$$

$$f(3) = 4 + f(4) = 3$$

$$f(4) = 3 - 4 = -1$$

$$\begin{aligned} &|-3, 1, 4, 3, -1, -4, | -3, 1, 4 \dots f(n) = f(n-1) - f(n-2) \\ &0, 2, 3, 4, 5, 6, 7, 8 \\ f(1867) = f(1867 \bmod_4 6) &= f(1) = 1 \end{aligned}$$