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For small $\theta$ in radians
$\sin(\theta) \approx \theta$

Advanced Functions Class 12 Homework

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**Identities and Equations (1)**

**1.** Use a counterexample to show that $\tan(\theta) + \sin(\theta) = \cot(\theta) + \cos(\theta)$ is not an identity.

counter: $\theta = 0$ : DNE

$\theta = 0.0001$

LHS: $2 \cdot 10^{-4}$

RHS: $10001$

**2.** Consider the trigonometric function $f(x) = \dfrac{\tan(x)+\sin(x)}{1+\cos(x)}$.

a. Identify the non-permissible values of $x$.
b. Graph $y = f(x)$ using graphing technology.
c. Use this graph to help create a possible trigonometric identity involving $f(x)$.
d. Prove your identity from part b) is true for all permissible values of the variable.

a) $1+\cos(x) = 0$ $\cos(x) = 0$

$\cos(x) = -1$

$x \neq \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

$+ 2\pi k$ where $k$ is integer

b) This looks like a tan graph

c) $\dfrac{\tan(x) + \sin(x)}{1+\cos(x)} = \tan(x)$

d)

$$LHS = \frac{\frac{\sin x}{\cos x} + \sin x}{1+\cos x}$$

$$= \frac{\sin x\left(\frac{1}{\cos x} + 1\right)}{1+\cos x}$$

$$= \frac{\sin x\left(\frac{\cancel{1+\cos x}}{\cos x}\right)}{\cancel{1+\cos x}} = \frac{\sin x}{\cos x} = \tan x = RHS$$

**3.** Prove.

a. $\csc^2(x) - \csc(x)\cot(x) = \dfrac{1}{1+\cos(x)}$

$$LHS = \frac{1}{\sin^2 x} - \frac{1}{\sin x} \cdot \frac{1}{\tan x}$$

$$= \frac{1}{\sin^2 x} - \frac{1}{\sin x} \cdot \frac{\cos x}{\sin x}$$

$$= \frac{1-\cos x}{\sin^2 x}$$

$$= \frac{1-\cos x}{1-\cos x^2}$$

$$= \frac{\cancel{1-\cos x}}{(1+\cos x)\cancel{(1-\cos x)}} = \frac{1}{1+\cos x} = RHS$$

b. $\dfrac{\sin(\theta)+1}{1-\sin(\theta)} = (\tan(\theta) + \sec(\theta))^2$

$$RHS = (\tan\theta + \sec\theta)^2$$

$$= \left(\frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)^2$$

$$= \left(\frac{\sin\theta+1}{\cos\theta}\right)^2 = \frac{(\sin\theta+1)^2}{\cos^2\theta}$$

$$= \frac{(\sin\theta+1)^2}{1-\sin^2\theta} = \frac{(\sin\theta+1)^{\cancel{2}}}{\cancel{(1+\sin\theta)}(1-\sin\theta)}$$

$$= \frac{\sin\theta+1}{1-\sin\theta} = LHS$$

c. $\dfrac{\tan(\beta)-\sin(\beta)}{\sin^3(\beta)} = \dfrac{\sec(\beta)}{1+\cos(\beta)}$

$$LHS = \frac{\frac{\sin B}{\cos B} - \sin B}{\sin^3 B}$$

$$= \frac{\cancel{\sin B}\left(\frac{1-\cos B}{\cos B}\right)}{\sin^{\cancel{3}} B}$$

$$= \frac{\left(\frac{1-\cos B}{\cos B}\right)}{\sin^2 B}$$

$$= \frac{\left(\frac{\cancel{1-\cos B}}{\cos B}\right)}{(1+\cos B)\cancel{(1-\cos B)}} = \frac{\left(\frac{1}{\cos B}\right)}{1+\cos B} = RHS$$

d. $\dfrac{\cos^3(\theta)+\sin^3(\theta)}{\sin(\theta)+\cos(\theta)} = 1 - \sin(\theta)\cos(\theta)$

let $\cos\theta = x$, $\sin\theta = y$

$$LHS = \frac{x^3+y^3}{y+x} = \frac{\cancel{(x+y)}(x^2-xy+y^2)}{\cancel{x+y}}$$

$$= x^2 - xy + y^2 = 1 - xy$$

$$= RHS$$

e. $(\csc(\theta)\sec(\theta))^2 - \dfrac{(1-\tan^2(\theta))^2}{\tan^2(\theta)} = 4$

let $\cos\theta = x$, $\sin\theta = y$

$$LHS = \left(\frac{1}{xy}\right)^2 - \frac{\left(1-\frac{y^2}{x^2}\right)^2}{\left(\frac{y}{x}\right)^2}$$

$$= \left(\frac{1}{xy} + \frac{1-\left(\frac{y}{x}\right)^2}{\left(\frac{y}{x}\right)}\right)\left(\frac{1}{xy} - \frac{1-\left(\frac{y}{x}\right)^2}{\left(\frac{y}{x}\right)}\right)$$

$$= \left(\frac{1+x^2-y^2}{xy}\right)\left(\frac{1-x^2+y^2}{xy}\right)$$

$$= \left(\frac{2x^{\cancel{2}}}{\cancel{x}y}\right)\left(\frac{2y^{\cancel{2}}}{x\cancel{y}}\right) = \left(\frac{2x}{y}\right)\left(\frac{2y}{x}\right)$$

$$= 4 = RHS$$

f. $\dfrac{\sin(x+y)}{\sin(x-y)} = \dfrac{\tan(x)+\tan(y)}{\tan(x)-\tan(y)}$

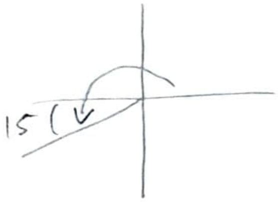
$$RHS = \frac{\frac{\sin x}{\cos x} + \frac{\sin y}{\cos y}}{\frac{\sin x}{\cos x} - \frac{\sin y}{\cos y}} = \frac{\frac{\cos y\sin x + \cos x\sin y}{\cancel{\cos x\cos y}}}{\frac{\cos y\sin x - \cos x\sin y}{\cancel{\cos x\cos y}}}$$

$$= \frac{\cos y\sin x + \cos x\sin y}{\cos y\sin x - \cos x\sin y}$$

$$= \frac{\sin(x+y)}{\sin(x-y)} = RHS$$

**4.** Determine the exact value of each trigonometric ratio. Express answers in simplest form.

a. $\sin(195^0)$

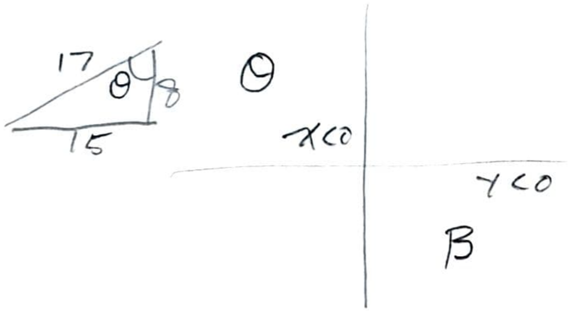



$= \sin(45-30)$
$= \sin 45 \cos 30 - \cos 45 \sin 30$
$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$
$= \frac{\sqrt{6}-\sqrt{2}}{4}$

b. $\cos(\frac{19\pi}{12})$

$\cos(\frac{19\pi}{12})$
$= -\sin(\frac{19\pi}{12} - \frac{\pi}{2})$
$= -\sin(195^0)$
$= -\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right) = \frac{\sqrt{2}-\sqrt{6}}{4}$

**5.** Given $\sin(\theta) = \frac{15}{17}$ and $\cos(\beta) = \frac{1}{3}$, where $\frac{\pi}{2} < \theta < \pi$ and $\frac{3\pi}{2} < \beta < 2\pi$, determine the exact value of $\cos(\theta + \beta)$.




1) $\sin\theta > 0$, $\cos\theta < 0$
2) $\sin\beta < 0$, $\cos\beta > 0$

1) $\cos\theta = -\sqrt{1-\sin^2\theta}$
$= -\frac{8}{17}$

2) $\sin\beta = -\sqrt{1-\cos^2\beta}$
$= -\frac{2\sqrt{2}}{3}$

$\cos(\theta+\beta) = \cos\theta\cos\beta - \sin\theta\sin\beta = \frac{30\sqrt{2}-8}{51}$

**6.** Simplify each expression to a single trigonometric ratio.

a. $\sin(3x)\cos(x) - \cos(3x)\sin(x)$
b. $\sin(\frac{\pi}{5})\sin(\frac{\pi}{3}) - \cos(\frac{\pi}{5})\cos(\frac{\pi}{3})$
c. $\frac{1-\tan(80^0)\tan(20^0)}{\tan(80^0)+\tan(20^0)}$

c) $\frac{1}{\left(\frac{\tan 80 + \tan 20}{1-\tan 80 \tan 20}\right)} = \frac{1}{\tan(100)}$
$= \cot(100)$

a) $\sin(3x-x)$
$= \sin(2x)$

b) $-(\cos\frac{\pi}{5}\cos\frac{\pi}{3} - \sin\frac{\pi}{5}\sin\frac{\pi}{3})$
$= -\cos(\frac{\pi}{5}+\frac{\pi}{3})$
$= -\cos(\frac{8\pi}{15})$