MDM4U HW5.

P294-295:

Q5, Q6, Q11 (a, e), Q13, Q19(c),

Q20 (d, e).

P312-313: Q7, Q10;

Sol. P294-295.

Q5.

a) $_9C_0 + {_9C_1} + {_9C_2} + \cdots + {_9C_9}$

$= (1+1)^9 = 2^9 = 512$.

$\because (a+b)^n = {_nC_0}a^n + {_nC_1}a^{n-1}b + \cdots + {_nC_r}a^{n-r}b^r + \cdots + {_nC_n}b^n$

$\sum_{r=0}^{n} {_nC_r}a^{n-r}b^r$

b) $_{12}C_0 - {_{12}C_1} + {_{12}C_2} - {_{12}C_3} + \cdots - {_{12}C_{11}} + {_{12}C_{12}}$

$= (1+(-1))^{12} = 0^{12} = 0$

c) $\sum_{r=0}^{15} {_{15}C_r} = \sum_{r=0}^{15} {_{15}C_r}(1)^{15-r}(1)^r = (1+1)^{15}$

$= 2^{15} = 32768$

d) $\sum_{r=0}^{n} {}_nC_r = (1+1)^n = 2^n$

Q6. If $\sum_{r=0}^{n} {}_nC_r = 16384$

then $(1+1)^n = 16384$

$2^n = 16384$

$n = \log_2 16384 = 14$

$\because 2^{14} = 16384.$

Q11 a) $(x^2 - \frac{1}{x})^5$, $a = x^2$, $b = -\frac{1}{x} = -x^{-1}$

$n = 5.$

$= {}_5C_0 (x^2)^5 + {}_5C_1 (x^2)^4(-x^{-1}) + {}_5C_2 (x^2)^3(-x^{-1})^2 + {}_5C_3 (x^2)^2(-x^{-1})^3$

$+ {}_5C_4 (x^2)(-x^{-1})^4 + {}_5C_5 (-x^{-1})^5$

$= x^{10} - 5x^7 + 10x^4 - 10x + 5x^{-2} - x^{-5}$

e) $(\sqrt{y} - \frac{2}{\sqrt{y}})^7$

$$\left(\sqrt{y}-\frac{2}{\sqrt{y}}\right)^7 = \left(\sqrt{y}+\left(-\frac{2}{\sqrt{y}}\right)\right)^7$$

$$= {}^7C_0(\sqrt{y})^7 + {}^7C_1(\sqrt{y})^6\left(-\frac{2}{\sqrt{y}}\right) + {}^7C_2(\sqrt{y})^5\left(-\frac{2}{\sqrt{y}}\right)^2 + {}^7C_3(\sqrt{y})^4\left(-\frac{2}{\sqrt{y}}\right)^3$$

$$+ {}^7C_4(\sqrt{y})^3\left(-\frac{2}{\sqrt{y}}\right)^4 + {}^7C_5(\sqrt{y})^2\left(-\frac{2}{\sqrt{y}}\right)^5 + {}^7C_6(\sqrt{y})\left(-\frac{2}{\sqrt{y}}\right)^6 + {}^7C_7\left(-\frac{2}{\sqrt{y}}\right)^7$$

$$= y^{\frac{7}{2}} - 14y^{\frac{5}{2}} + 82y^{\frac{3}{2}} - 280y^{\frac{1}{2}} + 560y^{-\frac{1}{2}} - 672y^{-\frac{3}{2}} + 448y^{-\frac{5}{2}} - 128y^{-\frac{7}{2}}$$

Q13

a) $\left(\frac{1}{2}\right)^5 + 5\left(\frac{1}{2}\right)^5 + 10\left(\frac{1}{2}\right)^5 + 10\left(\frac{1}{2}\right)^5 + 5\left(\frac{1}{2}\right)^5 + \left(\frac{1}{2}\right)^5$.

$= \left(\frac{1}{2}+\frac{1}{2}\right)^5 = 1^5 = 1.$

where n is 5, since there are 6 terms.

and ${}^5C_0\, a^5 = \left(\frac{1}{2}\right)^5$. and ${}^5C_5\, b^5 = \left(\frac{1}{2}\right)^5$.

so $a = \frac{1}{5}$, $b = \frac{1}{5}$.

b) $(0.7)^7 + 7(0.7)^6(0.3) + 21(0.7)^5(0.3)^2 + \cdots + (0.3)^7$

$= (0.7+0.3)^7 = 1^7 = 1$

where $n=7$, since there are 8 terms.

${}^7C_0\, a^7 = (0.7)^7$, ${}^7C_7\, b^7 = (0.3)^7$

so $a = 0.7$ and $b = 0.3$.

c) $7^9 - 9 \times 7^8 + 36 \times 7^7 - \cdots - 7^0$

$= (7 + (-1))^9 = 6^9 = 10077696$

where $n=9$. since there are 10 terms.

$9C_0\, a^9 = 7^9 \Rightarrow a = 7$

$9C_9\, b^9 = -7^0 = (-1)^9 7^0 \Rightarrow b = -1$

P295. Q19. c)

Find the first three tems, in the expansion of $(x^2-5)^9 (x^3+2)^6$

Sol. the general term of the expansion is:

$9C_r (x^2)^{9-r} (-5)^r \cdot 6C_t (x^3)^{6-t} (2)^t$

where $r = 0, 1, 2, \cdots, 9$; $t = 0, 1, 2, \cdots, 6$;

$= 9C_r (-5)^r\, 6C_t (2)^t\, x^{18-2r+18-3t}$

$= 9C_r (-5)^r\, 6C_t (2)^t\, x^{36-2r-3t}$

case 1: $r=0, t=0$, $9C_0 (-5)^0 6C_0 (2)^0 x^{36} = x^{36}$.

case 2: $r=1, t=0$. $9C_1 (-5)^1 6C_0 (2)^0 x^{34} = -45x^{34}$

case 3: $r=0, t=1$, $9C_0 (-5)^0 6C_1 (2^1) x^{33} = 12x^{33}$

Q20 d) Trinomial Theorem

$$(x+y+z)^n = \sum_{\substack{a+b+c=n \\ 0\le a,b,c\le n}} \frac{n!}{a!\,b!\,c!} x^a y^b z^c$$

e) $$(x+y+z)^5 = \sum_{\substack{a+b+c=n \\ 0\le a,b,c\le 5.}} \frac{5!}{a!\,b!\,c!} x^a y^b z^c$$

$$= \frac{5!}{5!\,0!\,0!} x^5 + \frac{5!}{0!\,5!\,0!} y^5 + \frac{5!}{0!\,0!\,5!} z^5 + \frac{5!}{4!\,1!\,0!} x^4 y$$

$$+ \frac{5!}{4!\,0!\,1!} x^4 z + \frac{5!}{1!\,4!\,0!} x y^4 + \frac{5!}{0!\,4!\,1!} y^4 z + \cdots$$

See Page 630 for Q20 d), e)

The Binomial Theorem could be also expressed in this format:

$$(x+y)^n = \sum_{\substack{a+b=n \\ 0\le a,b\le n.}} \frac{n!}{a!\,b!} x^a y^b = \sum_{b=0}^{n} {}_nC_b\, x^a y^b$$

P312 - 313 Q7.

a)

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

For player A.

$4+6+4=14$

chances out of 36

$\frac{14}{36} = \frac{7}{18} < \frac{1}{2}$

Therefore, player B has the advantage.

b) $4+6+6 = 16$. out of 36.

$\frac{16}{36} = \frac{4}{9} < \frac{1}{2}$.

So Player B still has the advantage.

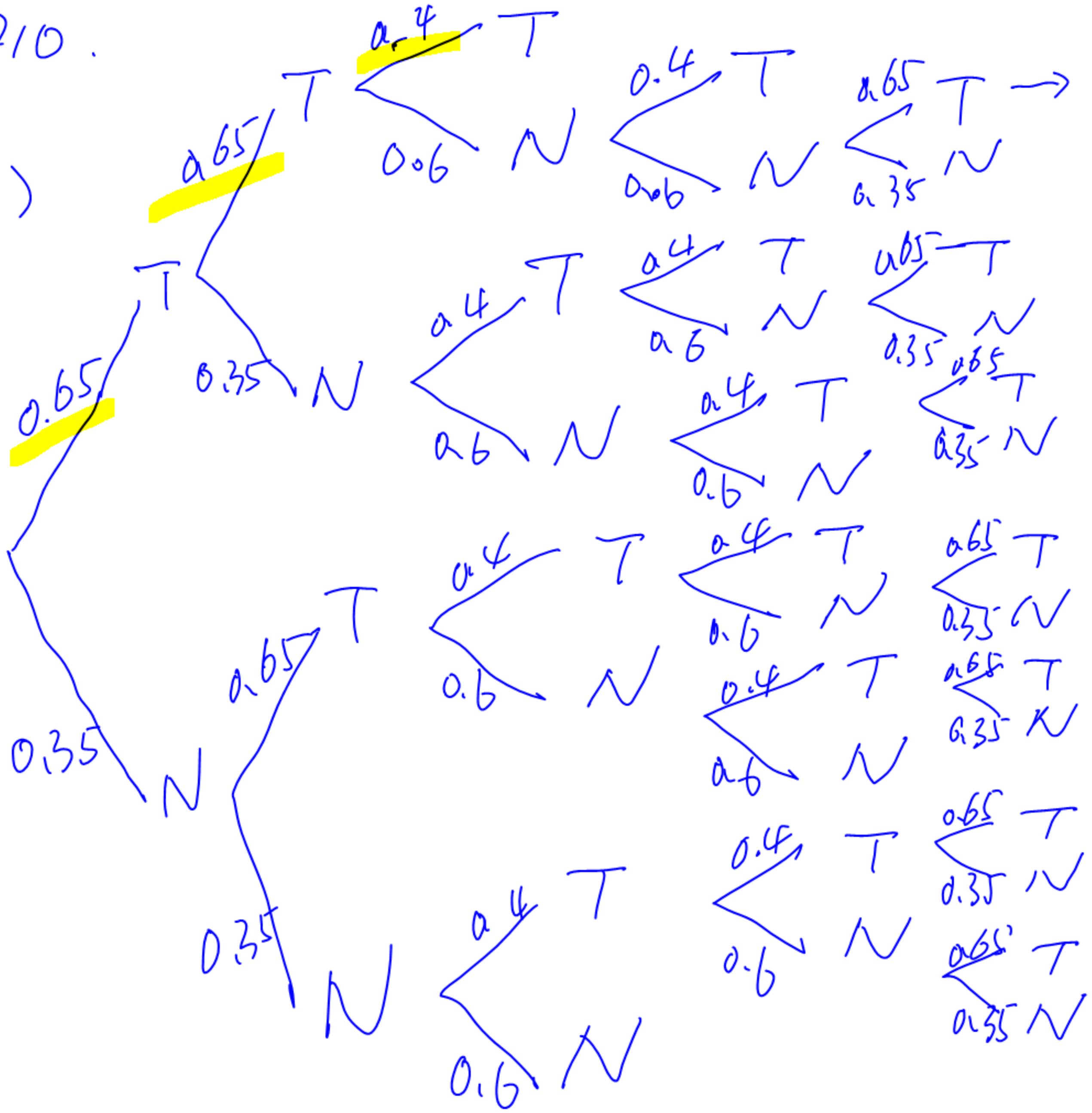
c) player A wins if rolling an even sum: 2, 4, 6, 8, 10 or 12.

that is 18 out of 36.

or $\frac{18}{36} = \frac{1}{2}$.

Q10.

a)



b) $P(TTT) = 0.65 \times 0.65 \times 0.4 = 0.169$

c) $P(TTNT) = 0.65 \times 0.65 \times 0.6 \times 0.4 = 0.1014$

⋮ ⋮

$P(NNN) = 0.35 \times 0.35 \times 0.6 = 0.0735$