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Chapter 3: Dependent and Independent Events, Mutually Exclusive Events
Applying Matrices to Probability Problems

There are times when we may deal with probabilities involving **two or more events**. For example, flipping a coin and then rolling a die is an example of two separate events, known as **compound events**.

- Independent Events

In some situation involving compound events, the occurrence of one event, *A*, has *no effect* on the occurrence of another event, *B*. In such cases, events *A* and *B* are **independent**.

Ex.1: A coin is tossed 3 times and turns up heads each time.

a) What is the probability that the fourth toss will be heads?
b) Find the probability of tossing four heads in a row.

Ex.2: A coin is flipped while a six-sided die is rolled. What is the probability of flipping tails *and* rolling a 3 in a single trial?

The Product Rule for Independent Events

A compound probability asks us to find the likelihood that **event A *and* event B** will occur.

In general, the compound probability of two independent can be calculated as follows:

$$P(\boldsymbol{A\ and\ B}) = P(\boldsymbol{A}) \times P(\boldsymbol{B})$$

- Dependent Events

Dependent events are events whose outcomes are affected by each other. For example, if two cards are drawn from a deck without replacement, the outcome of the second event depends on the outcome of the first event which is the first card drawn.

Ex.4: Suppose we have a standard deck of 52 cards. What is the probability of selecting two diamonds: a) with replacement? b) without replacement?

The Product Rule for Dependent Events

The probability that both A and B will occur equals the probability of A times the probability of B given that A has occurred:

$$P(\boldsymbol{A} \text{ and } \boldsymbol{B}) = P(\boldsymbol{A}) \times P(\boldsymbol{B} \text{ given } \boldsymbol{A}) \text{ or } P(\boldsymbol{A} \cap \boldsymbol{B}) = P(\boldsymbol{A}) \times P(\boldsymbol{B}|\boldsymbol{A})$$

Ex.5: Cards are drawn from a standard deck of 52 cards without replacement. Calculate the probability of obtaining:

a) a king, then another king
b) a club, then a heart
c) a black card, then a heart, then a diamond

Ex.6: A jar contains blue and purple marbles. Two marbles are chosen without replacement. The probability of selecting a purple marble and then a blue marble is $\frac{16}{69}$, and the probability of selecting a purple marble on the first draw is $\frac{4}{11}$.

a) What is the probability of selecting a blue marble on the second draw, given that the first marble drawn was purple?

b) What is the probability of selecting a blue marble on the second draw, given that the first marble drawn was blue?

Ex.7: At an athletic event, athletes are tested for steroids using two different tests. The first test has a 93% probability of giving accurate results, while the second test is accurate 87% of the time. If both tests are used on an athlete who has steroids in his system, what is the probability that:

a) neither test shows that steroids are present?

b) both tests show that steroids are present?

c) at least one of the tests shows that steroids are present?

Ex.8: A hockey team has jerseys in three different colors. There are 4 green, 6 white and 5 orange jerseys in the bag. Todd and Blake are given a jersey at random. What is the probability that both jerseys are the same color?

When we looked at two events $\boldsymbol{A}$ and $\boldsymbol{B}$ that had some "overlap" (**intersection**), we noted that we could count the total number of **A or B** (**union**) using $n(\boldsymbol{A} \cup \boldsymbol{B}) = n(\boldsymbol{A}) + n(\boldsymbol{B}) - n(\boldsymbol{A} \cap \boldsymbol{B})$. This helped us avoid double counting the number of outcomes where **A** ***and*** **B** could occur together.

- **Mutually Exclusive Events vs. Non-Mutually Exclusive Events**

Mutually exclusive events are events that **cannot occur simultaneously**. For example, getting a 2 and getting a 1 on a single roll of die. **Non-mutually exclusive events** are events that **may occur simultaneously**. For example, when we were looking for the number of ways to select a queen *or* a heart from a standard deck of cards.

Ex.9: Classify the events in each experiment as being either mutually exclusive or non-mutually exclusive.

a) The experiment is rolling a single die once.
Event A: rolling an even number, Event B: rolling a prime number.

b) The experiment is playing a game of hockey.
Event A: your team scores a goal, Event B: your team wins the game.

c) The experiment is selecting a gift.
Event A: the gift is edible, Event B: the gift is an ukulele.

The Addition Rule for Mutually Exclusive Events

$$P(\boldsymbol{A}\ or\ \boldsymbol{B}) = P(\boldsymbol{A} \cup \boldsymbol{B}) = P(\boldsymbol{A}) + P(\boldsymbol{B})$$

The Addition Rule for Non-Mutually Exclusive Events

$$P(\boldsymbol{A}\ or\ \boldsymbol{B}) = P(\boldsymbol{A} \cup \boldsymbol{B}) = P(\boldsymbol{A}) + P(\boldsymbol{B}) - P(\boldsymbol{A} \cap \boldsymbol{B})$$

Ex.10: If two standard dice are rolled,

a) what is the probability of rolling doubles *or* a total of 11?
b) what is the probability of rolling doubles or a total of 10?

Ex.11: The probability that Dana will make the hockey team is $\frac{2}{3}$. The probability that she will make the swimming team is $\frac{3}{4}$. If the probability of Dana making both teams is $\frac{1}{2}$, determine the probability that she will not make neither team.

Ex.12: In an animal-behaviour study, hamsters were tested with a number of intelligence tasks, as shown in the table below.

Number of Tests	Number of Hamsters
0	10
1	6
2	4
3	3
4 or more	5

If a hamster is randomly chosen from this study group, what is the likelihood that the hamster has participated in

a) exactly three tests?
b) fewer than two tests?
c) either one or two tests?
d) no tests or more than three tests?

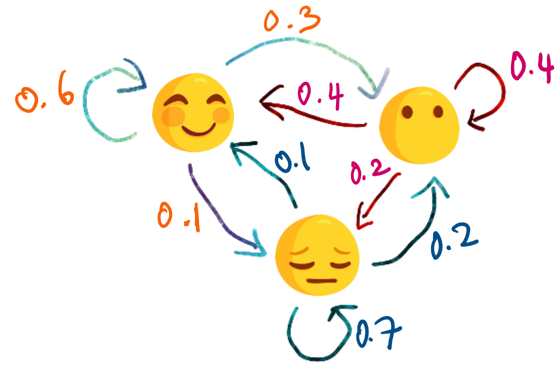
➢ Applying Matrices to Probability Problems

Dependent probabilities can be calculated using Markov Chains, a powerful probability model that can be applied to study stock market trends, consumer habits, etc.

Ex.13: Rami after reflecting on her past experience concluded that the states of her mood (cheerful, sad, so-so) switches according to the following diagram:

a) Suppose Rami is cheerful today, what is the probability that she is
   i) cheerful ii) sad iii) so-so
   the next day?

b) What about 3 days from today?



- initial probability vector:

- transition matrix:

Regular Markov chains always achieve a steady state. A Markov chain is regular if the **transition matrix P** or some power of **P** has no zero entries. A regular Markov chain will reach the same steady state regardless of the **initial probability vector**.

c) Determine the long-term probability of each of her mood states.