# Propositional Logic (cont.)

Course Code: CSC 1204 Course Title: Discrete Mathematics


**Dept. of Computer Science**
**Faculty of Science and Technology**


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| **Lecture No:** | **2** | **Week No:** | **1** | **Semester:** | **Summer 2021-2022** |
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# Lecture Outline

## 1.1 Propositional Logic

- Logic
- Propositional Logic
- Propositions
- Propositional Variables
- Compound Propositions
- Logical Operators
- Truth Value

* We have already covered

- Truth Tables of Compound Propositions
- Conditional Statements
- Logic and Bit Operations

# Objectives and Outcomes

- **Objectives**: To understand how to construct a truth table for a compound proposition, to understand the conditional statement $p \to q$ and different equivalent expressions of $p \to q$ , to understand bit operations.

- **Outcomes**: Students are expected to be able construct a truth table for a given compound proposition, be able to explain the conditional statement $p \to q$ and it's equivalent expressions, be able to perform Bit Operations.

# Conditional Statements

- Let $p$ and $q$ be propositions.
- The *conditional statement* $p \to q$ is the proposition "if $p$, then $q$."
- The conditional statement $p \to q$ is false when p is true and q is false, and true otherwise.
- In the conditional statement $p \to q$, ***p*** *is called* ***hypothesis*** *and* ***q*** *is called* ***conclusion****.*
- This one is the English usage of "if, then" or "implies".
- The connective $\to$ is called the 'conditional connective'.
- A **conditional statement** is **also called** an **implication**.

# Truth Table for Conditional Statement



**TABLE 5 The Truth Table for the Conditional Statement $p \rightarrow q$.**

| $p$ | $q$ | $p \rightarrow q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

# Equivalent Expression of $p \rightarrow q$

In terms of words, the proposition **$p \rightarrow q$ also reads**:

(a) **if p, then q,** ***or***

**if p, q**

*(The word "**then**" is sometimes omitted in English sentences)*

(b) **p implies q**

(c) **p is a sufficient condition for q ,** ***or***

**a sufficient condition for q is p**

(d) **q is a necessary condition for p,** ***or***

**a necessary condition for p is q**

# Equivalent Expression of $p \to q$

(e) **p only if q**

(f) **q if p** , ***or***

**q, if p**

(g) **q whenever p**

(h) **q when p**

(i) **q unless $\neg$p**

# Remember!

- The hypothesis expresses a sufficient condition
- The conclusion expresses a necessary condition
- “***but***” is a logical synonym for “***and***”
- “***when***” / “***whenever***” means the same as “***if***”
- The **hypothesis** is the clause following “**if**”
- The ***conclusion*** is the clause following “***then***”
- “***only if***” clause is the **conclusion**
- **If *hypothesis*, then *conclusion***

# Example 7 (page 7)

Let p : "Maria learns discrete mathematics" and
q : "Maria will find a good job."

Express the statement $\mathbf{p \rightarrow q}$ as a statement in English.

**Solution**:

- *"**If** Maria learns discrete mathematics, **then** she will find a good job."*

There are many other ways to express this conditional statement in English.

- *"Maria will find a good job **when** she learns discrete mathematics"*
- *"**For** Maria to get a good job, **it is sufficient for her** to learn discrete mathematics".*
- *"Maria will find a good job **unless** **she does not** learn discrete mathematics."*

# Exercise 19

- Write each of these statements in the form **"if *p*, then *q*"** in English.

a) It snows **whenever** the wind blows from the northeast.

Ans. **If** the wind blows from the northeast, **then** it snows

b) The apple trees will bloom **if** it stays warm for a week.

Ans. **If** it stays warm for a week, **then** the apple trees will bloom

# Exercise 19

c) That the Pistons win the championship **implies** that they beat the Lakers.

Ans. **If** the Pistons win the championship, **then** they beat the Lakers.

d) It is **necessary** to walk 8 miles to get to the top of Long's Peak.

Ans. **If** you get to the top of Long's Peak, **then** you must have walked eight miles.

# Exercise 19

e) To get tenure as a professor, it is **sufficient** to be world-famous.

Ans. **If** you are world-famous, **then** you will get tenure as a professor.

f) **If** you drive more than 400 miles, you will need to buy gasoline.

Ans. **If** you drive more than 400 miles, **then** you will need to buy gasoline.

# Exercise 19

g) Your guarantee is good **only if** you bought your CD player less than 90 days ago.

Ans. **If** your guarantee is good, **then** you must have bought your CD player less than 90 days ago.

h) Jan will go swimming **unless** the water is too cold.

Ans. **If** the water is not too cold, **then** Jan will go swimming.

# Converse, Contrapositive, and Inverse

- We can form some new conditional statements starting with a conditional statement $p \rightarrow q$

- The **Converse of** $p \rightarrow q$ is the proposition $q \rightarrow p$
- The ***Contrapositive* of** $p \rightarrow q$ is the proposition $\neg q \rightarrow \neg p$
- The ***Inverse of*** $p \rightarrow q$ is the proposition $\neg p \rightarrow \neg q$

# Examples of Converse, Contrapositive and Inverse

- ***Converse***: $p \rightarrow q$ ==> $\boldsymbol{q \rightarrow p}$

Example: "If it is noon, then I am hungry."

**Converse**: "If I am hungry, then it is noon."

- ***Contrapositive***: $p \rightarrow q$ ==> $\boldsymbol{\neg q \rightarrow \neg p}$

Example: "If it is noon, then I am hungry."

**Contrapositive**: "If I am not hungry, then it is not noon."

- ***Inverse***: $p \rightarrow q$ ==> $\boldsymbol{\neg p \rightarrow \neg q}$

Example: "If it is noon, then I am hungry."

**Inverse**: "If it is not noon, then I am not hungry."

# Bi-Conditional

- Let *p* and *q* be propositions.
- The *bi-conditional statement* $p \leftrightarrow q$ is the proposition **"*p* if and only if *q*."**
- **The bi-conditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise.**
- Bi-conditional statements are also called "*bi-implications*"

- Question : Which operator is the opposite of $\leftrightarrow$?
- ***Answer:*** $\leftrightarrow$ has exactly the opposite truth table as $\oplus$

# Truth Table for Bi-conditional $p \leftrightarrow q$

**TABLE 6** The Truth Table for the Biconditional $p \leftrightarrow q$.

| $p$ | $q$ | $p \leftrightarrow q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

# Example of Bi-conditional statement

- **Example 10 ( Page 9):** Let *p* be the statement " You can take the flight" and *q* be the statement " You buy a ticket". What is the statement for $\boldsymbol{p \leftrightarrow q}$ **?**

- **Solution:**

"You can take the flight if and only if you buy a ticket"

# How to **Construct** a **Truth Table** for a Compound Proposition?

- At first look at the number of propositions (e.g. *p*, *q*, *r* ) in the given compound proposition.
- There will be **$2^n$ number of rows** in the truth table, where *n* is the number of propositions in the compound proposition.
- Draw the table. In the first row, write down the name of propositions (e.g. *p*, *q*, *r* ) starting from left/first column.
- Construct the truth table step by step.

## **Example**: Construct a truth table for

## $(p \oplus q) \oplus r$

| p | q | r | $p \oplus q$ | $(p \oplus q) \oplus r$ |
|---|---|---|---|---|
| T | T | T | F | T |
| T | T | F | F | F |
| T | F | T | T | F |
| T | F | F | T | T |
| F | T | T | T | F |
| F | T | F | T | T |
| F | F | T | F | T |
| F | F | F | F | F |

# Precedence of Logical Operators

- Negation operator is applied before all other logical operators
- Conjunction operator takes precedence over disjunction operator
- Conditional and bi-conditional operators have lower precedence
- Parentheses are used whenever necessary

# Precedence of Logical Operators



**TABLE 8** Precedence of Logical Operators.

| *Operator* | *Precedence* |
|---|---|
| $\neg$ | 1 |
| $\wedge$<br>$\vee$ | 2<br>3 |
| $\rightarrow$<br>$\leftrightarrow$ | 4<br>5 |

# Logic and Bit Operations

- **bit** ==> ***b***inary dig***it***
- Boolean variable: either **true** or **false**
  - Can be represented by a bit
- **Bit String**: A *bit string* is a sequence of zero or more bits. The *length* of the string is the number of bits in the string.
- Example 20(p.15): 101010011 is a bit string of length *nine*

# Bit Operations

**Computer bit operations** correspond to the **logical connectives**. By replacing **true** by a **one (1)** and **false** by a **zero (0)** in the truth tables for the operators $\wedge$, $\vee$, and $\oplus$, the tables shown in Table 9 for the corresponding bit operations are obtained. We will also use the notation ***OR***, ***AND***, *and* ***XOR*** *for the operators* $\vee$, $\wedge$, *and* $\oplus$ *respectively, as is done in various programming languages.*

# Table for Bit Operations



**TABLE 9 Table for the Bit Operators *OR*, *AND*, and *XOR*.**

| $x$ | $y$ | $x \vee y$ | $x \wedge y$ | $x \oplus y$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 0 |

# Bit string and bit operation

**DEFINITION 7** A *bit string* is a sequence of zero or more bits. The *length* of this string is the number of bits in the string.

**EXAMPLE 12** 101010011 is a bit string of length nine. ◀

We can extend bit operations to bit strings. We define the **bitwise** ***OR***, **bitwise** ***AND***, and **bitwise** ***XOR*** of two strings of the same length to be the strings that have as their bits the *OR*, *AND*, and *XOR* of the corresponding bits in the two strings, respectively. We use the symbols $\vee$, $\wedge$, and $\oplus$ to represent the bitwise *OR*, bitwise *AND*, and bitwise *XOR* operations, respectively. We illustrate bitwise operations on bit strings with Example 13.

**EXAMPLE 13** Find the bitwise *OR*, bitwise *AND*, and bitwise *XOR* of the bit strings 01 1011 0110 and 11 0001 1101. (Here, and throughout this book, bit strings will be split into blocks of four bits to make them easier to read.)

*Solution:* The bitwise *OR*, bitwise *AND*, and bitwise *XOR* of these strings are obtained by taking the *OR*, *AND*, and *XOR* of the corresponding bits, respectively. This gives us

$$
\begin{array}{ll}
\phantom{0}01\ 1011\ 0110 & \\
\underline{11\ 0001\ 1101} & \\
11\ 1011\ 1111 & \text{bitwise } OR \\
01\ 0001\ 0100 & \text{bitwise } AND \\
10\ 1010\ 1011 & \text{bitwise } XOR
\end{array}
$$

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# Books

- *Discrete Mathematics and its applications with combinatorics and graph theory (7$^{th}$ edition)* by Kenneth H. Rosen [Indian Adaptation by KAMALA KRITHIVASAN], published by McGraw-Hill

# References


1. Discrete Mathematics, *Richard Johnsonbaugh*, Pearson education, Inc.
2. Discrete Mathematical Structures, *Bernard Kolman*, *Robert C. Busby*, *Sharon Ross,* Prentice-Hall, Inc.
3. *SCHAUM'S outlines Discrete Mathematics(2nd edition)*, by *Seymour Lipschutz*, *Marc Lipson*