Tree Traversal

Course Code: 00090 Course Title: Discrete Mathematics

Dept. of Computer Science
Faculty of Science and Technology

Lecturer No:	**22**	**Week No:**	**13**	**Semester:**	**Summer 21-22**
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Lecture Outline

8.3 Tree Traversal

- Preorder Traversal
- Inorder Traversal
- Postorder Traversal

Objectives and Outcomes

- Objectives: To understand the different types of tree traversal algorithms and apply them.

- Outcomes: The students are expected to be able to perform preorder, inorder and postorder tree traversal.

Tree Traversal

- **Tree traversal:**
  - A listing of the vertices of a tree
  - Is a procedure that systematically visits every vertex of an ordered rooted tree

Traversal Algorithms

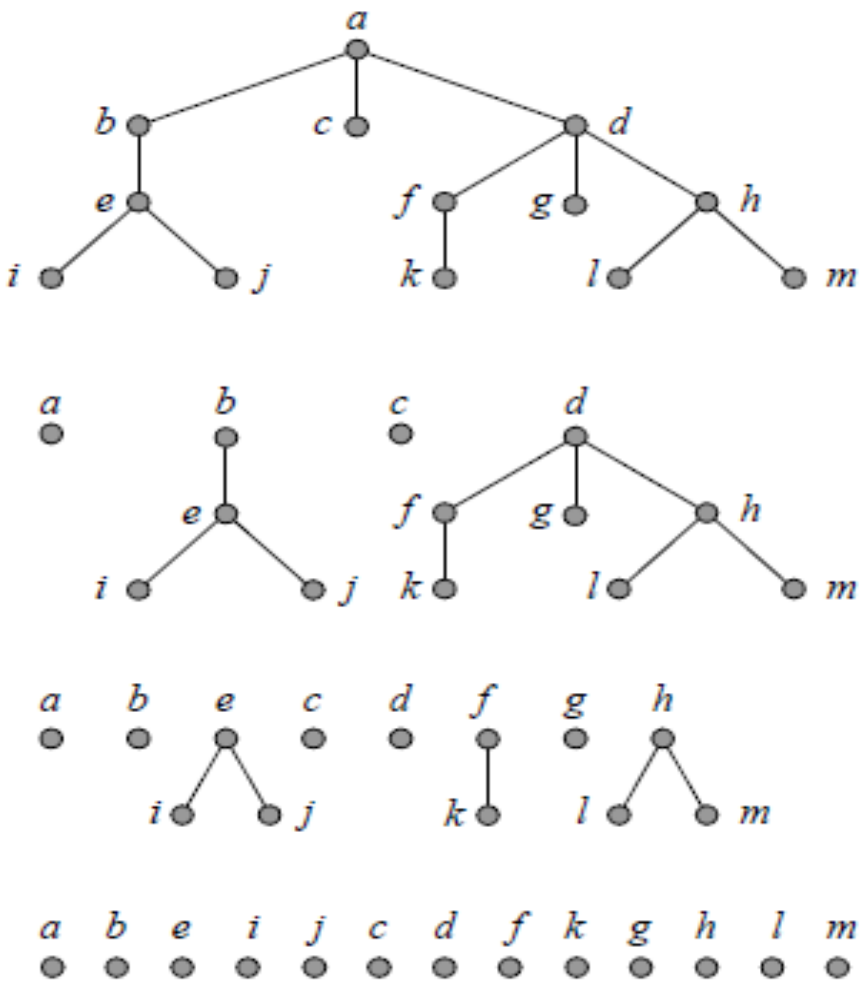
- Procedures for systematically visiting every vertex of an ordered rooted tree are called **traversal algorithms**.
- **Three most commonly used traversal algorithms:**
    1) **Preorder** traversal
    2) **Inorder** traversal
    3) **Postorder** traversal

Preorder Traversal

- **DEFINITION**: Let T be an ordered rooted tree with root r. If T consists only of r, then r is the ***preorder traversal*** of T . Otherwise, suppose that $T_1, T_2, \ldots, T_n$ are the subtrees at r from left to right in T .

  The *preorder traversal* begins by visiting *r*. It continues by traversing T_1 in preorder, then T_2 in preorder, and so on, until T_n is traversed in preorder.

Examples of Preorder Traversal



Example 2

- In which order does a preorder traversal visit the vertices in the ordered rooted tree T shown in Figure 3?

Solution:

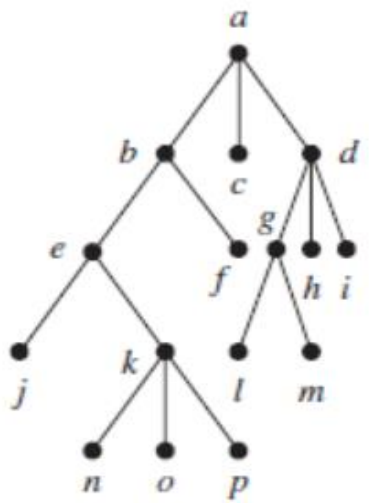
The steps of the preorder traversal of T are shown in Figure 4.

We traverse T in preorder by first listing the root *a*,

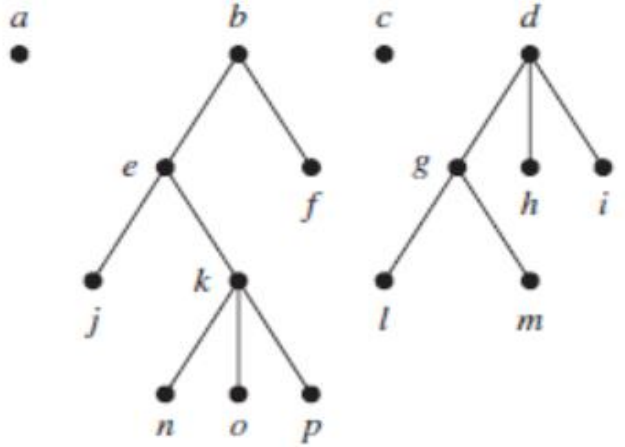
followed by the preorder list of the subtree with root *b*, the preorder list of the subtree with root *c* (which is just *c*) and the preorder list of the subtree with root *d*.

Example 2

Figure 3



Preorder traversal: Visit root, visit subtrees left to right



Example 2

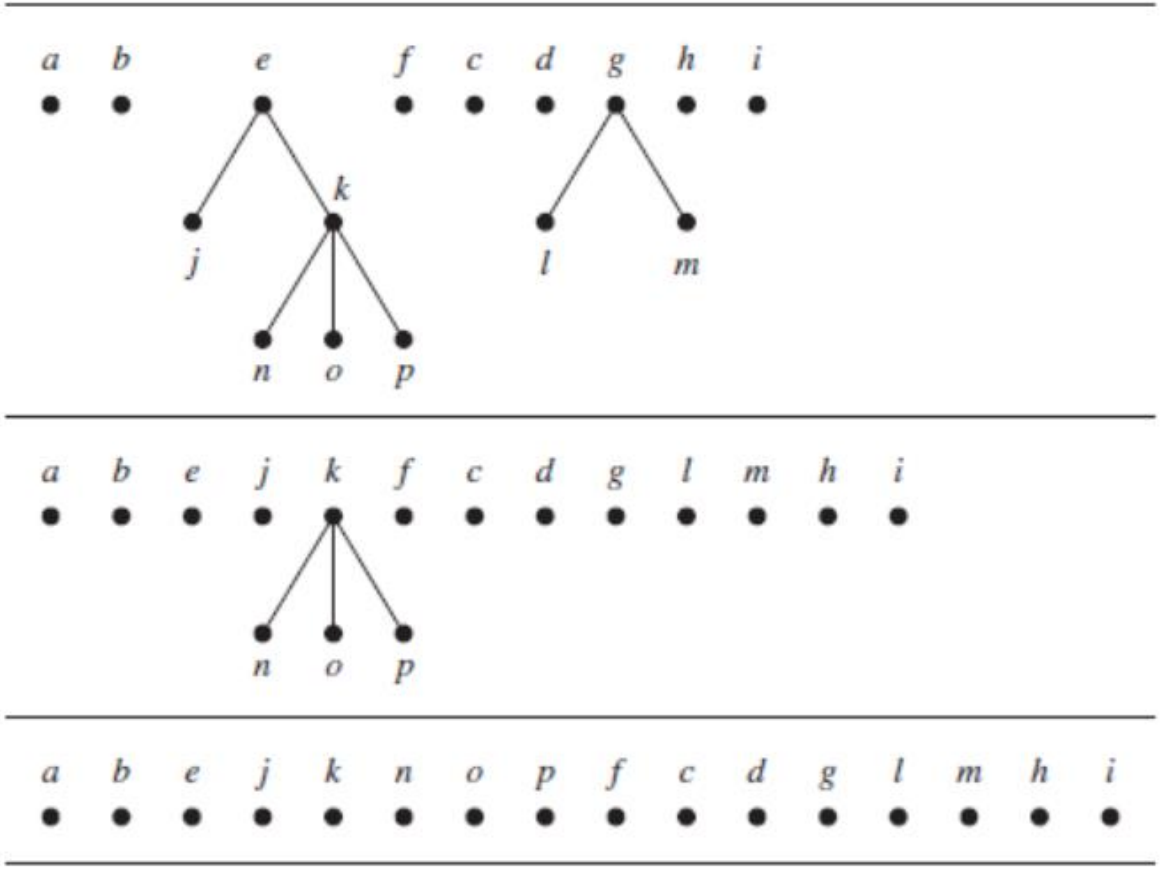
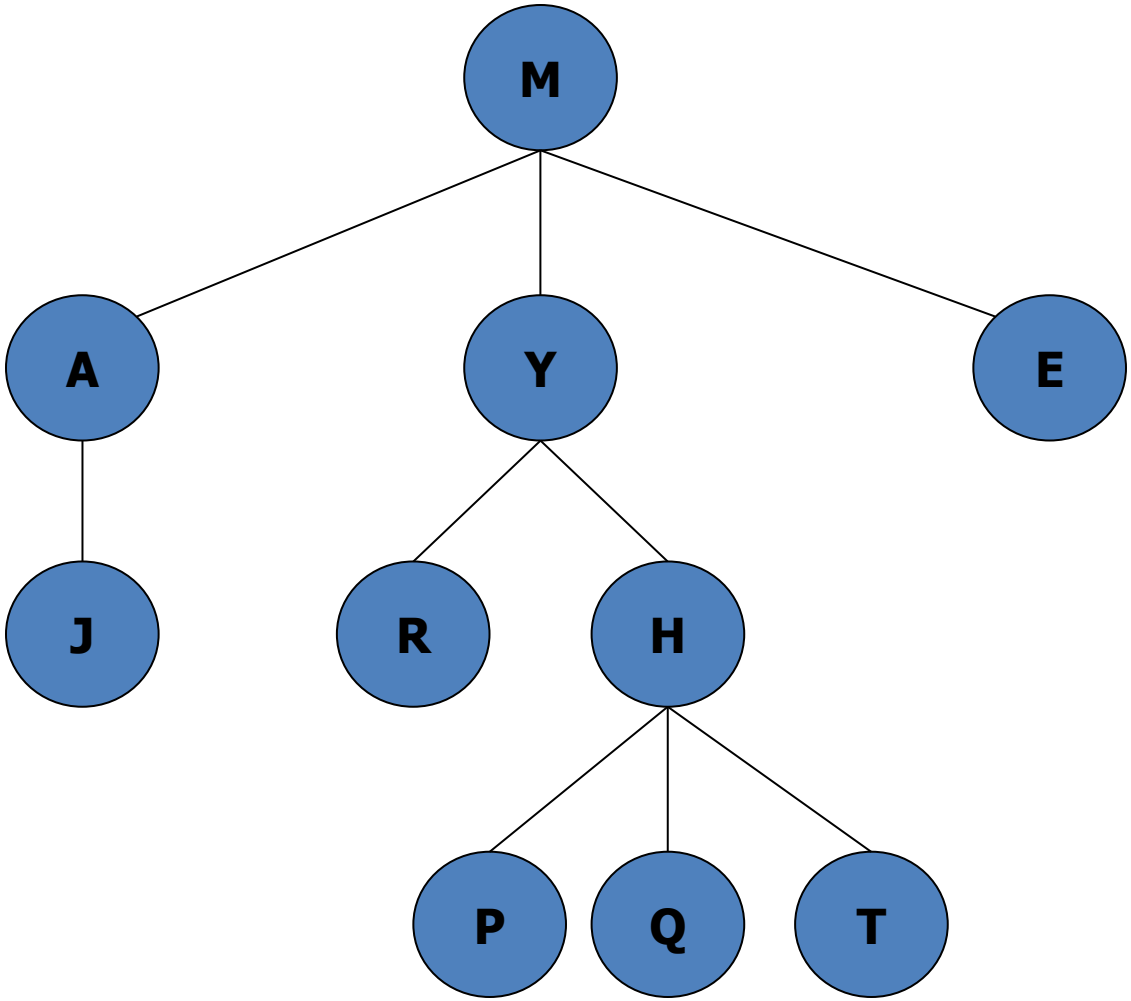
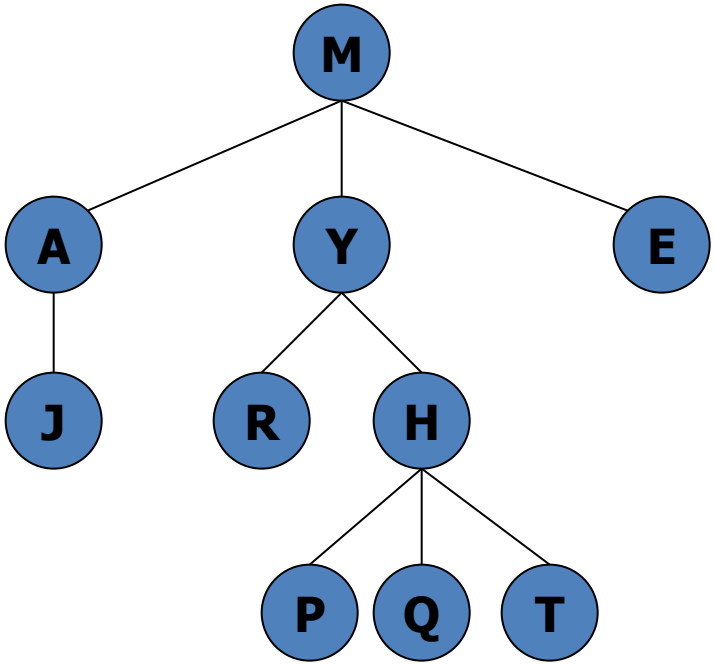


FIGURE 4 The Preorder Traversal of T.

Another Example Of **Preorder** Traversal



Another Example Of **Preorder** Traversal



M A J Y R H P Q T E

Inorder Traversal

- **DEFINITION 2**: Let T be an ordered rooted tree with root r. If T consists only of *r*, then *r* is the *inorder traversal* of T. Otherwise, suppose that $T_1, T_2, \ldots, T_n$ are the subtrees at *r* from left to right.

  The *inorder traversal* begins by traversing T_1 in inorder, then visiting *r*. It continues by traversing T_2 in inorder, then T_3 in inorder,..., and finally T_n in inorder.

Inorder Traversal

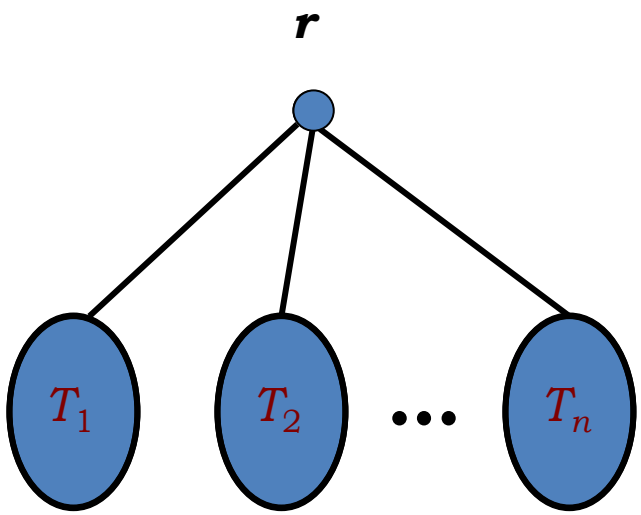
Step 1: Visit T_1 in inorder

Step 2: Visit the root r

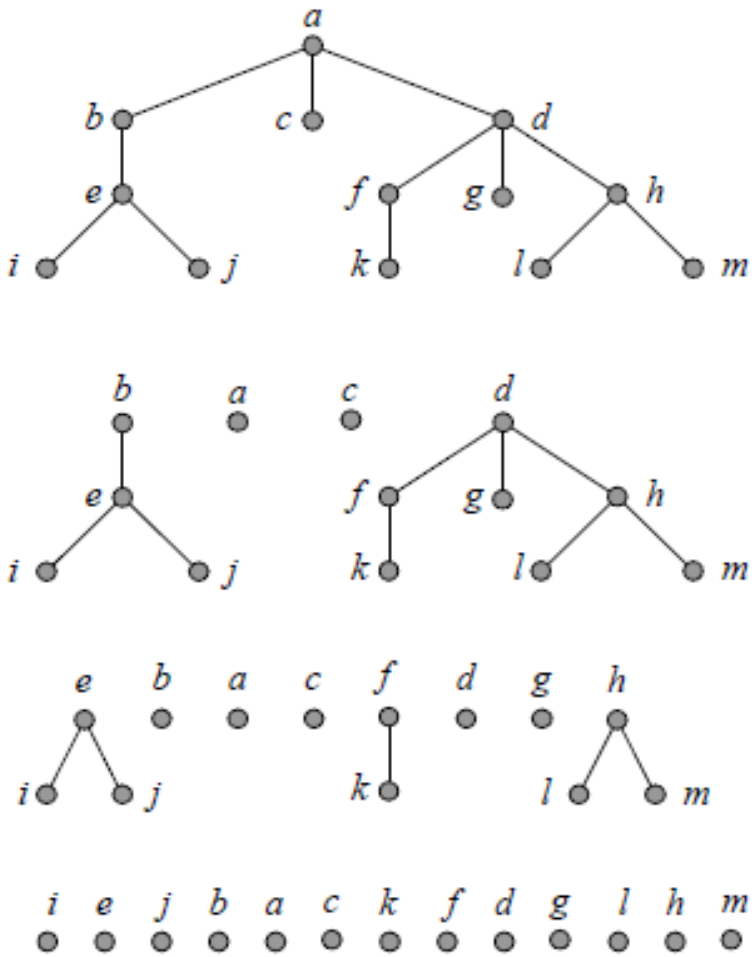
Step 3: Visit T_2 in inorder

⋮

Step n+1: Visit T_n in inorder



Examples of Inorder Traversal



Example Inorder Traversal

- **EXAMPLE 3:** In which order does an inorder traversal visit the vertices of the ordered rooted tree T in Figure 3?

- **Solution**: The steps of the inorder traversal of the ordered rooted tree T are shown in Figure 6. The inorder traversal begins with an inorder traversal of the subtree with root *b*, the root *a*, the inorder listing of the subtree with root *c*, which is just *c*, and the inorder listing of the subtree with root *d*.

Example of Inorder Traversal

Figure 3

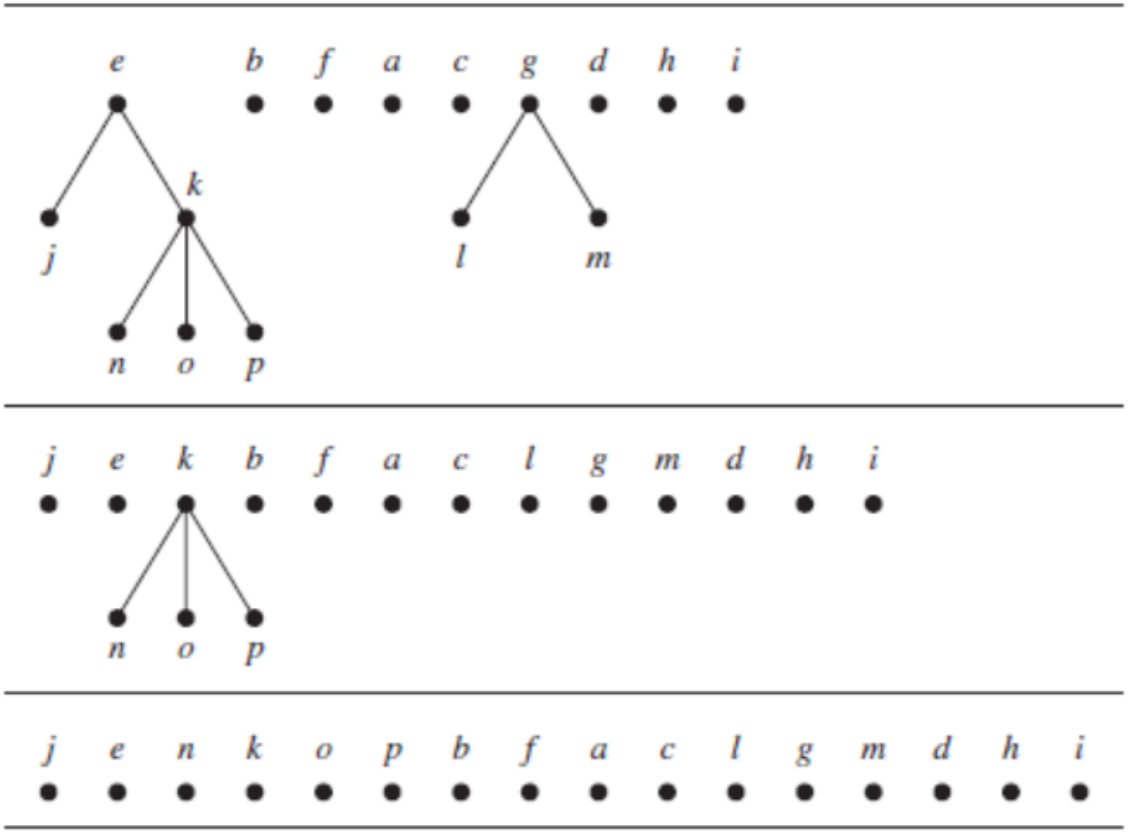


FIGURE 6 The Inorder Traversal of T.

Postorder Traversal

- **DEFINITION 3:** Let T be an ordered rooted tree with root r. If T consists only of *r*, then *r* is the postorder traversal of T . Otherwise, suppose that $T_1, T_2, \ldots, T_n$ are the subtrees at *r* from left to right.

  The postorder traversal begins by traversing T_1 in postorder, then T_2 in postorder, . . . , then T_n in postorder, and ends by visiting *r*.

Postorder Traversal

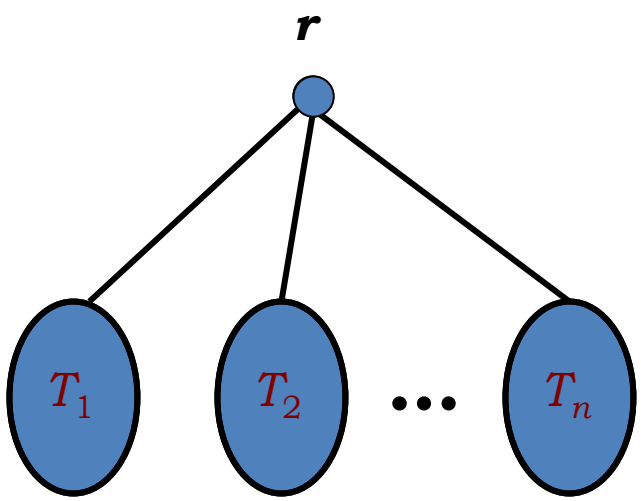
Step 1: Visit T_1 in postorder

Step 2: Visit T_2 in postorder

⋮

Step n: Visit T_n in postorder

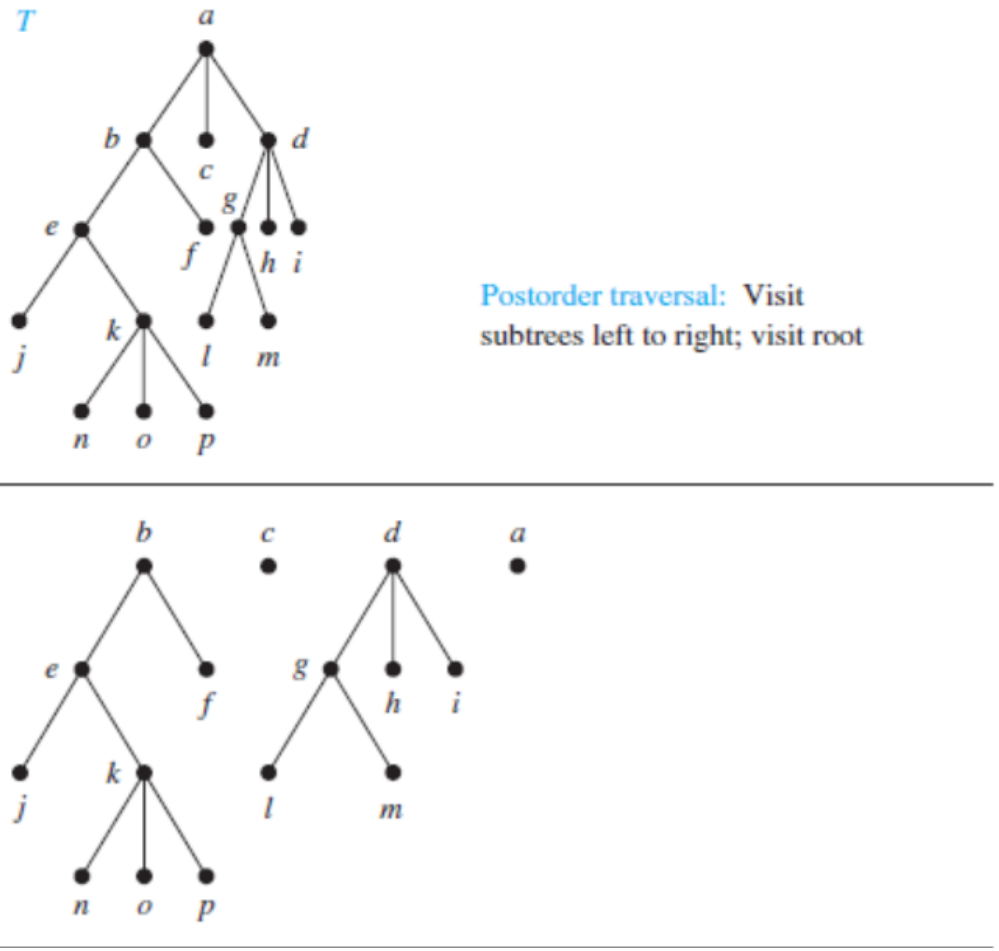
Step n+1: Visit r



Example 4

- EXAMPLE 4 : In which order does a postorder traversal visit the vertices of the ordered rooted tree T shown in Figure 3?
- **Solution:** The steps of the postorder traversal of the ordered rooted tree T are shown in Figure 8. The postorder traversal begins with the postorder traversal of the subtree with root *b*, the postorder traversal of the subtree with root *c*, which is just *c*, the postorder traversal of the subtree with root *d*, followed by the root *a*.

Figure 3


T
a
b
c
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o
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Postorder traversal: Visit
subtrees left to right; visit root
b
c
d
a
e
f
g
h
i
j
k
l
m
n
o
p

Example 4

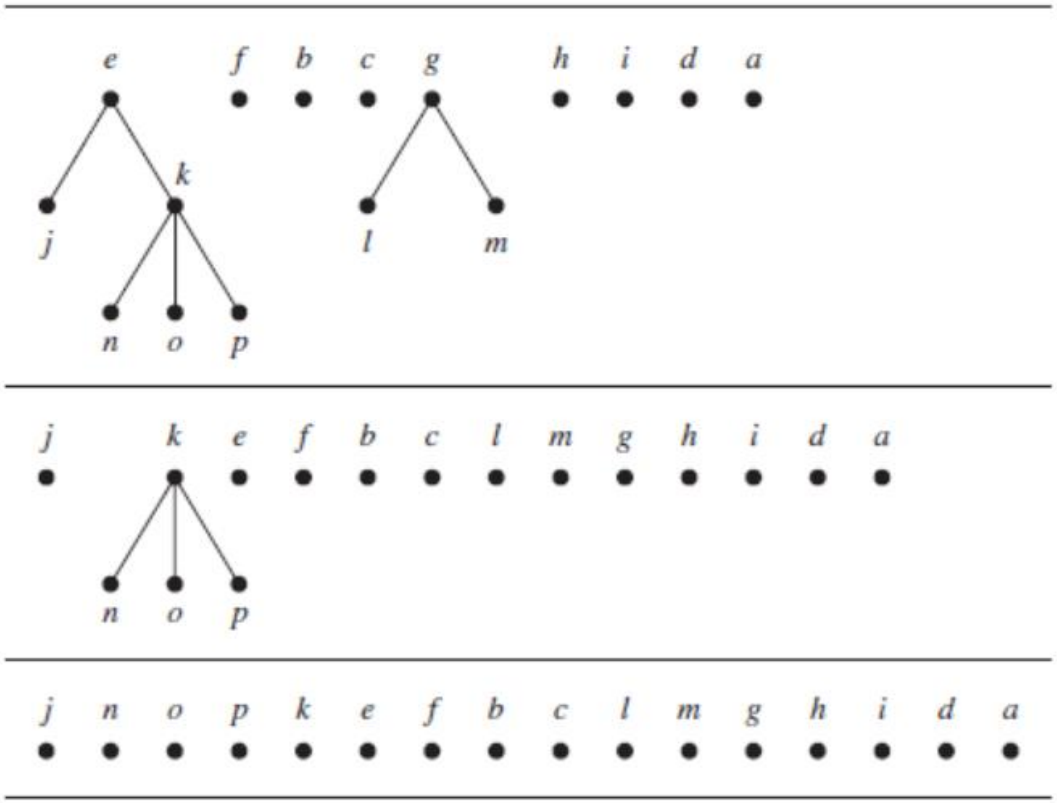
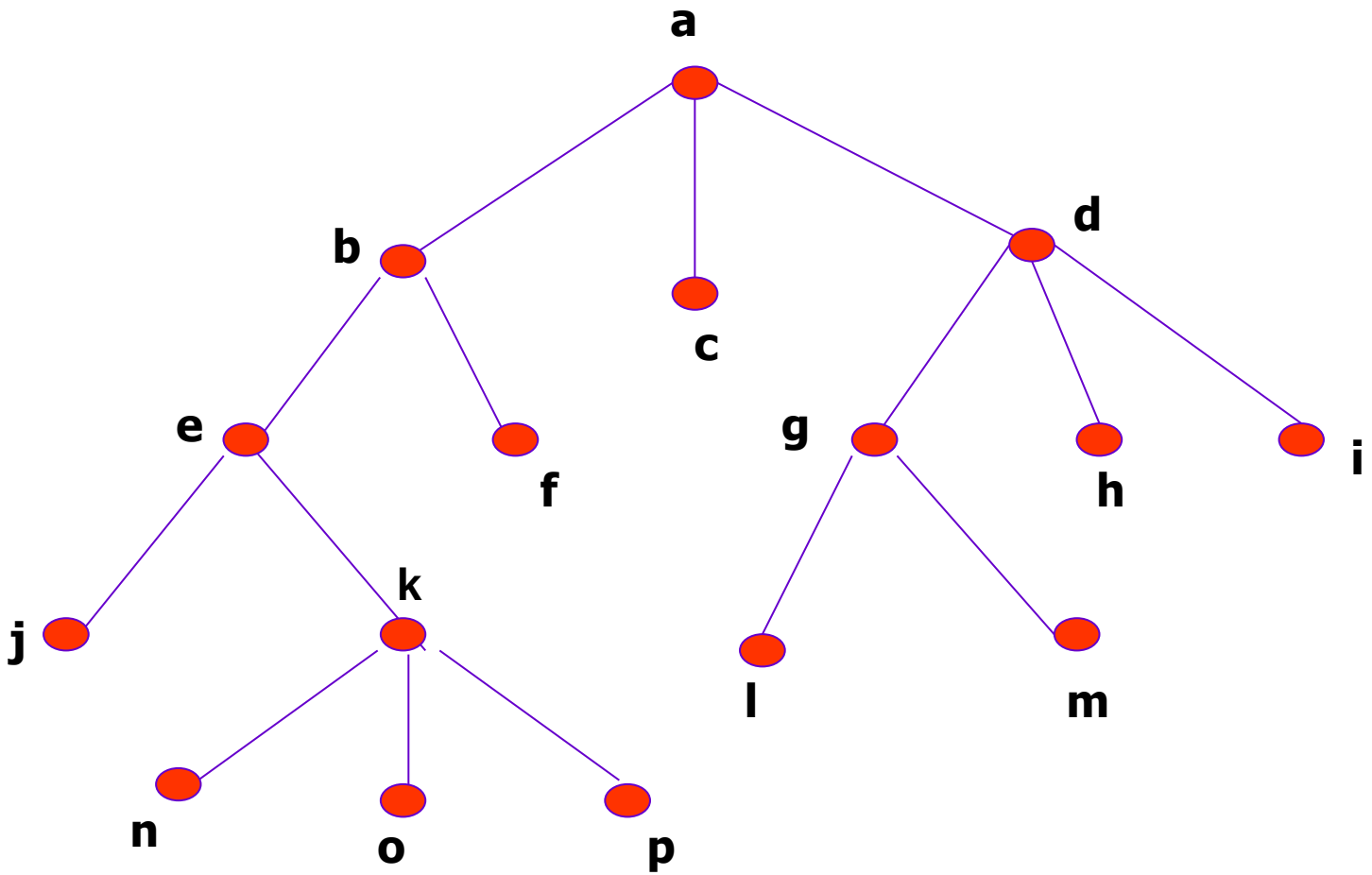


FIGURE 8 The Postorder Traversal of T.

Class Work

Determine the order in which a (a) **Preorder**, (b) **Inorder**, and (c) **Postorder** traversal visits the vertices of the ordered rooted tree below.



Answers

a) **Preorder**: *a b e j k n o p f c d g l m h i*

b) **Inorder**: *j e n k o p b f a c l g m d h i*

c) **Postorder**: *j n o p k e f b c l m g h i d a*

Books

- Rosen, K. H., & Krithivasan, K. (2012). Discrete mathematics and its applications: with combinatorics and graph theory. Tata McGraw-Hill Education. (7th Edition)

References

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2. Discrete Mathematical Structures, *Bernard Kolman*, *Robert C. Busby*, *Sharon Ross,* Prentice-Hall, Inc.
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- University of Hawaii
http://courses.ics.hawaii.edu/ReviewICS241/morea/trees/TreeTraversal-QA.pdf
- Florida State University
http://www.cs.fsu.edu/~lacher/lectures/Output/trees_intro/script.html