Applications of Trees

Course Code: 00090 Course Title: Discrete Mathematics

Dept. of Computer Science
Faculty of Science and Technology

Lecturer No:	**22**	**Week No:**	**12**	**Semester:**	**Summer 21-22**
Lecturer:	*Md. Mahmudur Rahman (mahmudur@aiub.edu)*				

Lecture Outline

9.2 Applications of Trees

- Binary Search Tree (BST)

Objectives and Outcomes

- Objectives: To understand various applications of trees, to understand binary search tree (BST) and the algorithm for constructing binary search tree.

- Outcomes: The students are expected to be able to explain applications of trees; be able to construct a BST from a given list of items.

Common Uses of Trees

- Manipulate hierarchical data
- Make information easy to search
- Manipulate sorted lists of data
- As a workflow for compositing digital images for visual effects
- Routing Algorithms

Applications of Trees

- Three problems can be studied using trees –

1) How should items in a list be stored so that an item can be easily located?
2) What series of decisions should be made to find an object with a certain property in a collection of objects of a certain type?
3) How should a set of characters be efficiently coded by bit strings?

Applications of Trees

- Binary search tree
  - A simple data structure for sorted lists
- Decision tree
  - A rooted tree where each **vertex** represents a possible outcome of a **decision** and the **leaves** represent the possible **solutions**
  - Minimum comparisons in sorting algorithms
- Prefix code
  - A code that has the property that the code of a character is never a prefix of the code of another character
  - Huffman coding

Binary Search Tree (BST)

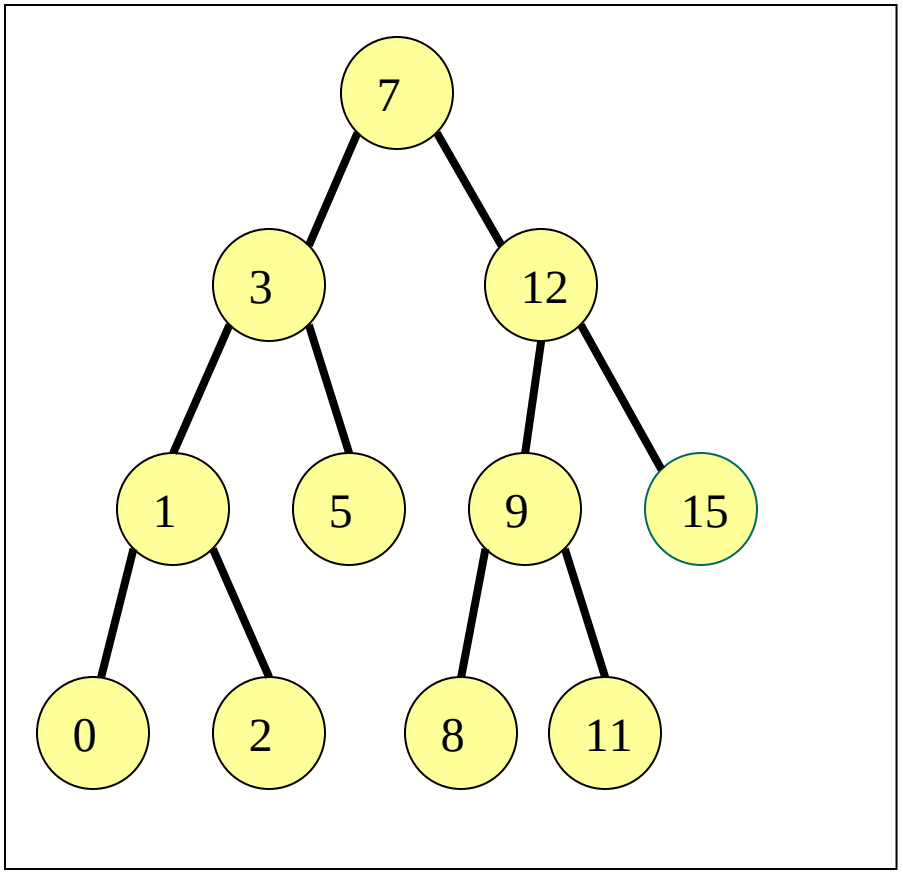
- A binary tree in which each child of a vertex is designated as a right or left child.
- No two vertex has more than one right child or left child, and each vertex is labeled with a key, which is one of the items.
- Vertices are assigned keys so that the key of a vertex is both larger than the keys of all vertices in its left subtree and smaller than the keys of all vertices in its right subtree.

Binary Search Trees (BST)

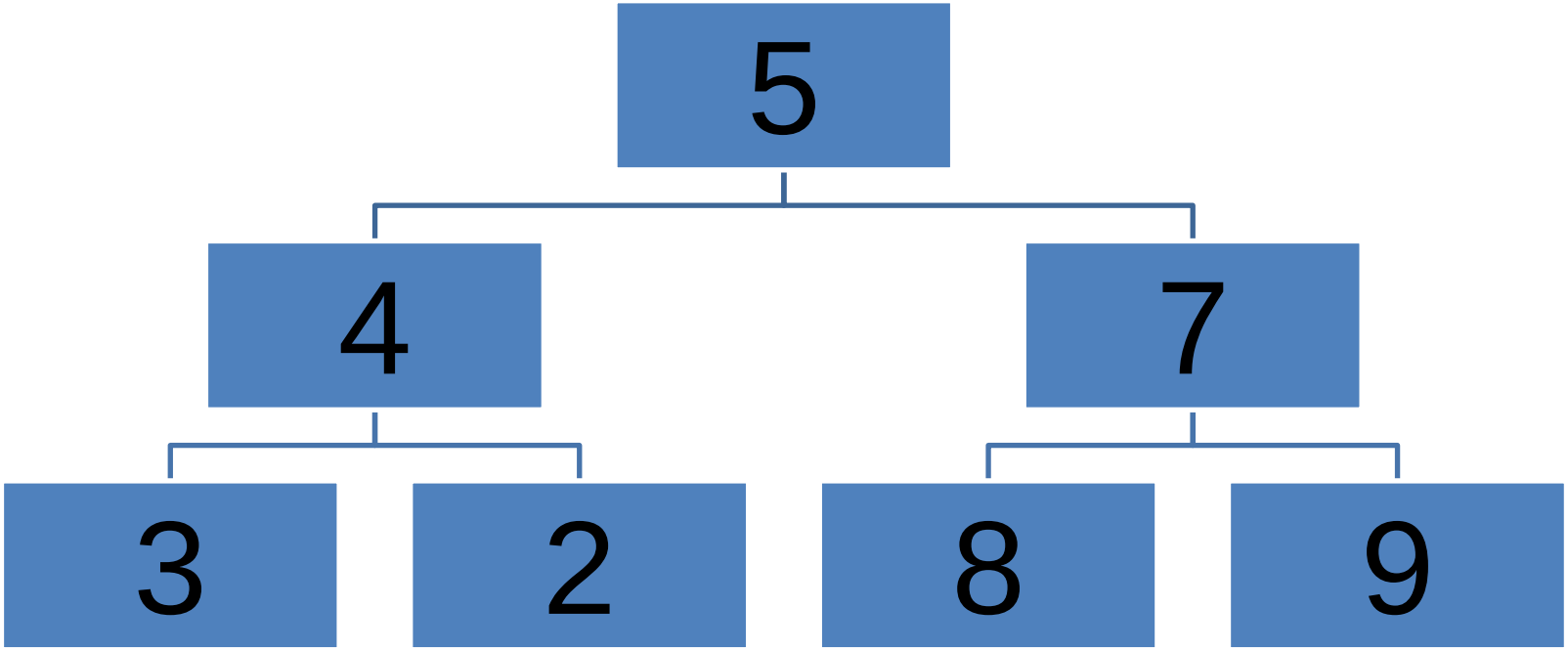
- Binary Search Tree (BST): A binary tree in which the vertices are labeled with items so that a label of a vertex is greater than the labels of all vertices in the left subtree of this vertex and is less than the labels of all vertices in the right subtree of this vertex.

Binary Search Trees (BST): Example

- Items are stored at individual tree nodes.
- We arrange for the tree to always obey this invariant:
- For every item *x*,
  - Every node in *x*'s left subtree is less than *x*.
  - Every node in *x*'s right subtree is greater than *x*.



Is this a BST?



No, this is NOT a BST. Why?

Binary Search Trees (BST)

- Binary search tree construction **Algorithm**
  - Start with a tree containing just one vertex (the root).
  - Let *v* = the root.
  - To add a new item *a*, do the following until stop:

If *a*< *v*:

  - If *v* has a left child, then *v* = left child of *v*(move to the left)
  - Else add a new left child to *v* with this item as its key.
  - Stop.

If *a*> *v*:

  - If *v* has a right child, then *v*= right child of *v*(move to the right).
  - Else add a new right child to *v* with this item as its key.
  - Stop.

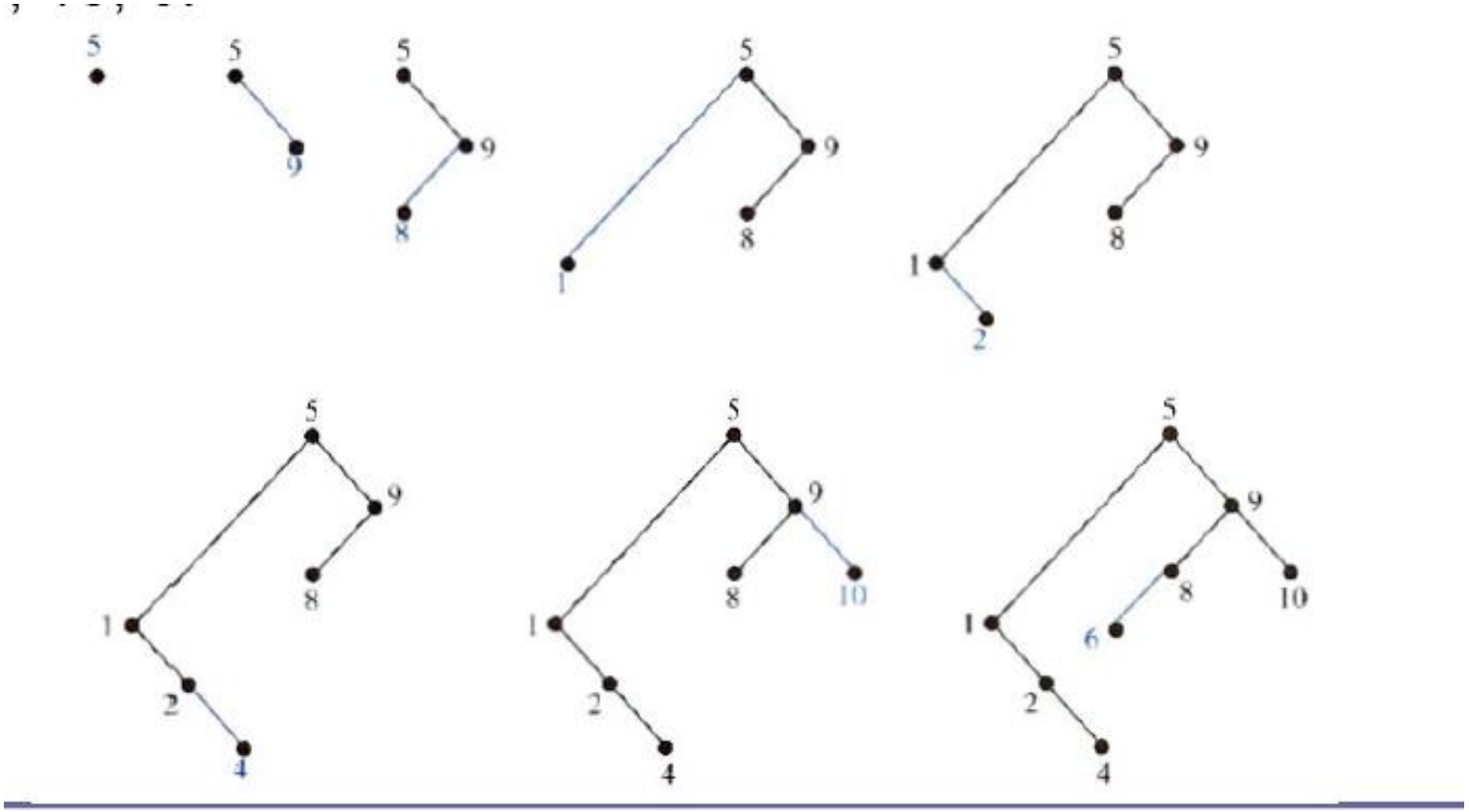
Construction of BST

- Example: Construct/Form a BST using the numbers below.

5 9 8 1 2 4 6 10

Construction of BST

5 9 8 1 2 4 6 10



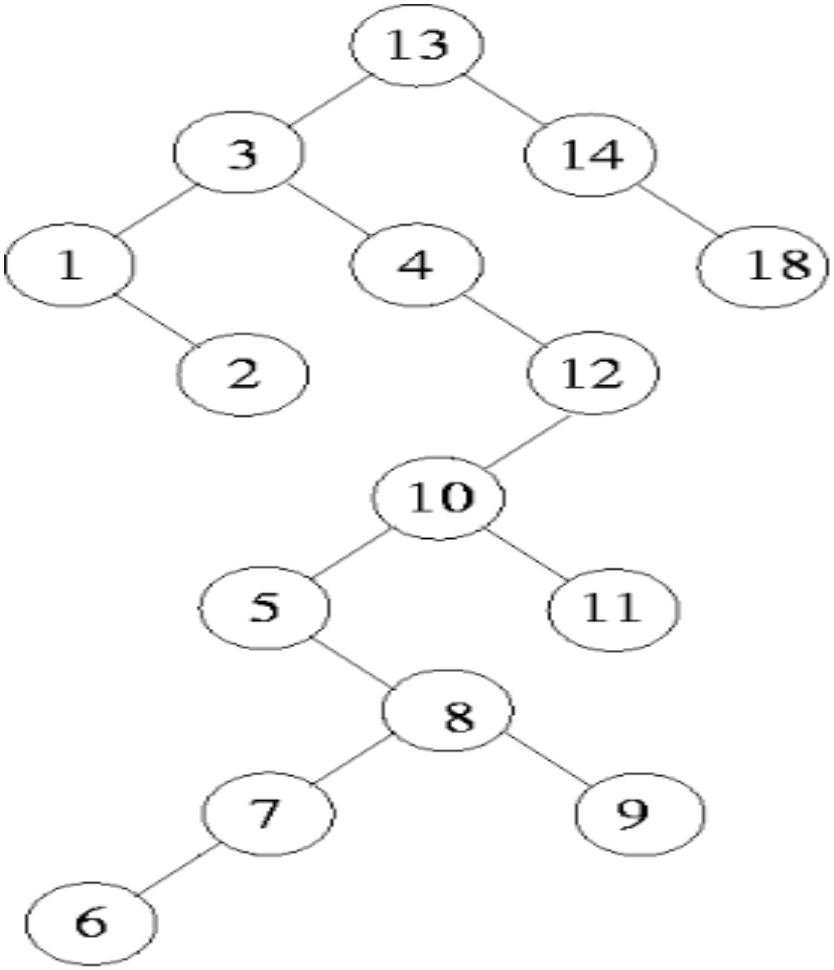
Class Work

- Construct/Form a BST using the numbers below.

  13, 3, 4, 12, 14, 10, 5, 1, 8, 2, 7, 9, 11, 6, 18

13, 3, 4, 12, 14, 10, 5, 1, 8, 2, 7, 9, 11, 6, 18

Solution



Example 1

- **EXAMPLE 1:** Form a binary search tree for the words mathematics, physics, geography, zoology, meteorology, geology, psychology, and chemistry (using alphabetical order).

Solution of Example 1

Solution: Figure 1 displays the steps used to construct this binary search tree. The word mathematics is the key of the root. Because physics comes after mathematics (in alphabetical order), add a right child of the root with key physics. Because geography comes before mathematics, add a left child of the root with key geography. Next, add a right child of the vertex with key physics, and assign it the key zoology, because zoology comes after mathematics and after physics. Similarly, add a left child of the vertex with key physics and assign this new vertex the key meteorology. Add a right child of the vertex with key geography and assign this new vertex the key geology. Add a left child of the vertex with key zoology and assign it the key psychology. Add a left child of the vertex with key geography and assign it the key chemistry. (The reader should work through all the comparisons needed at each step.)

Solution of Example 1

mathematics, physics, geography, zoology, meteorology, geology, psychology, and chemistry

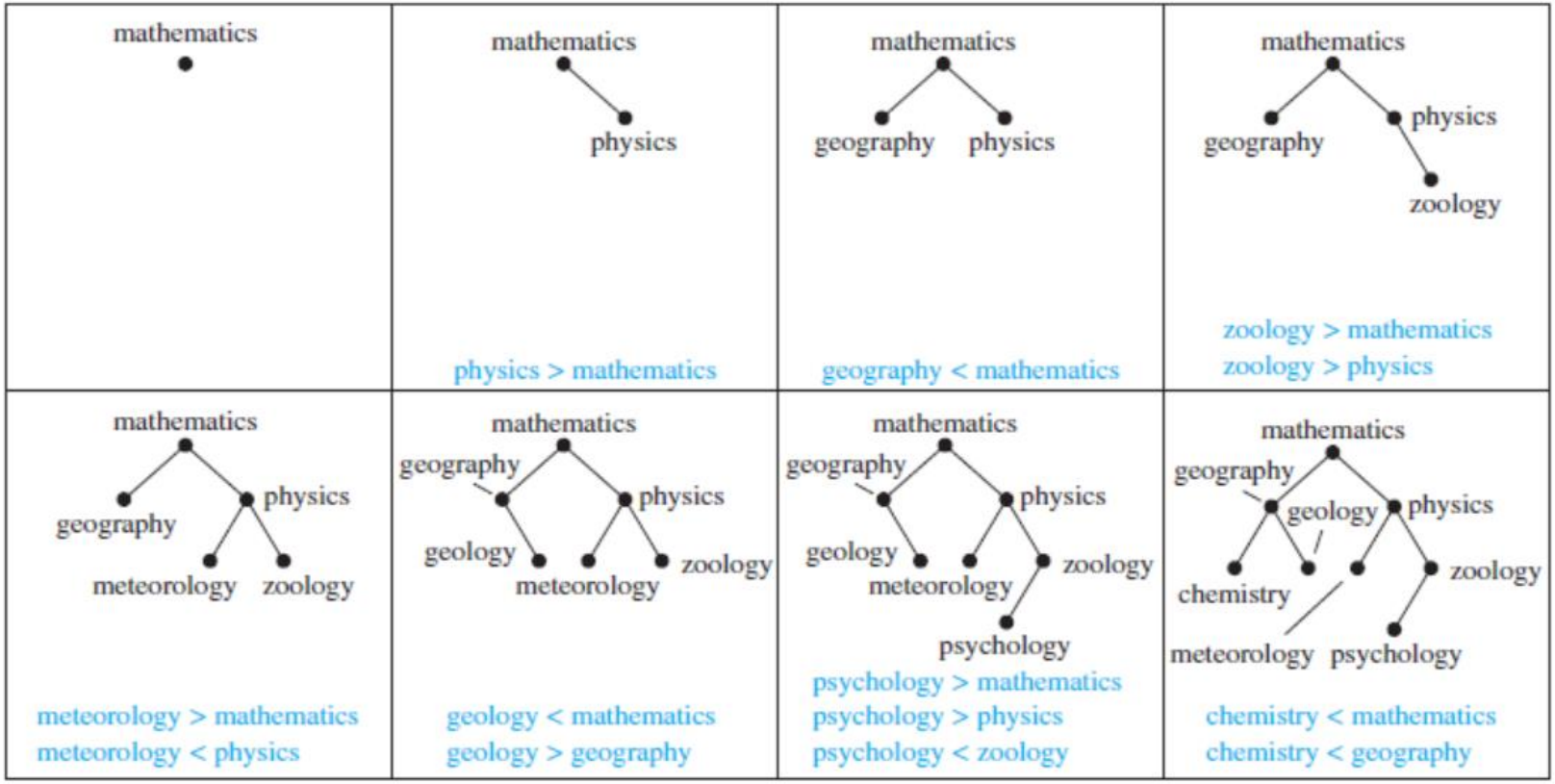


FIGURE 1 **Constructing a Binary Search Tree.**

Class work

- Exercise 1: Build a binary search tree for the words ***banana***, ***peach***, ***apple***, ***pear***, ***coconut***, ***mango***, and ***papaya*** using alphabetical order.

- Exercise 5: Using alphabetical order, construct a binary search tree for the words in the sentence

  "The quick brown fox jumps over the lazy dog."

Books

- Rosen, K. H., & Krithivasan, K. (2012). Discrete mathematics and its applications: with combinatorics and graph theory. Tata McGraw-Hill Education. (7th Edition)

References

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