Functions (Cont.)

Course Code: CSC 1204 Course Title: Discrete Mathematics

Dept. of Computer Science
Faculty of Science and Technology

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Lecture Outline

2.3 Functions (Cont.)

- **Inverse Functions**
- **Compositions of Functions**
- **Floor function**
- **Ceiling Function**

Objectives and Outcomes

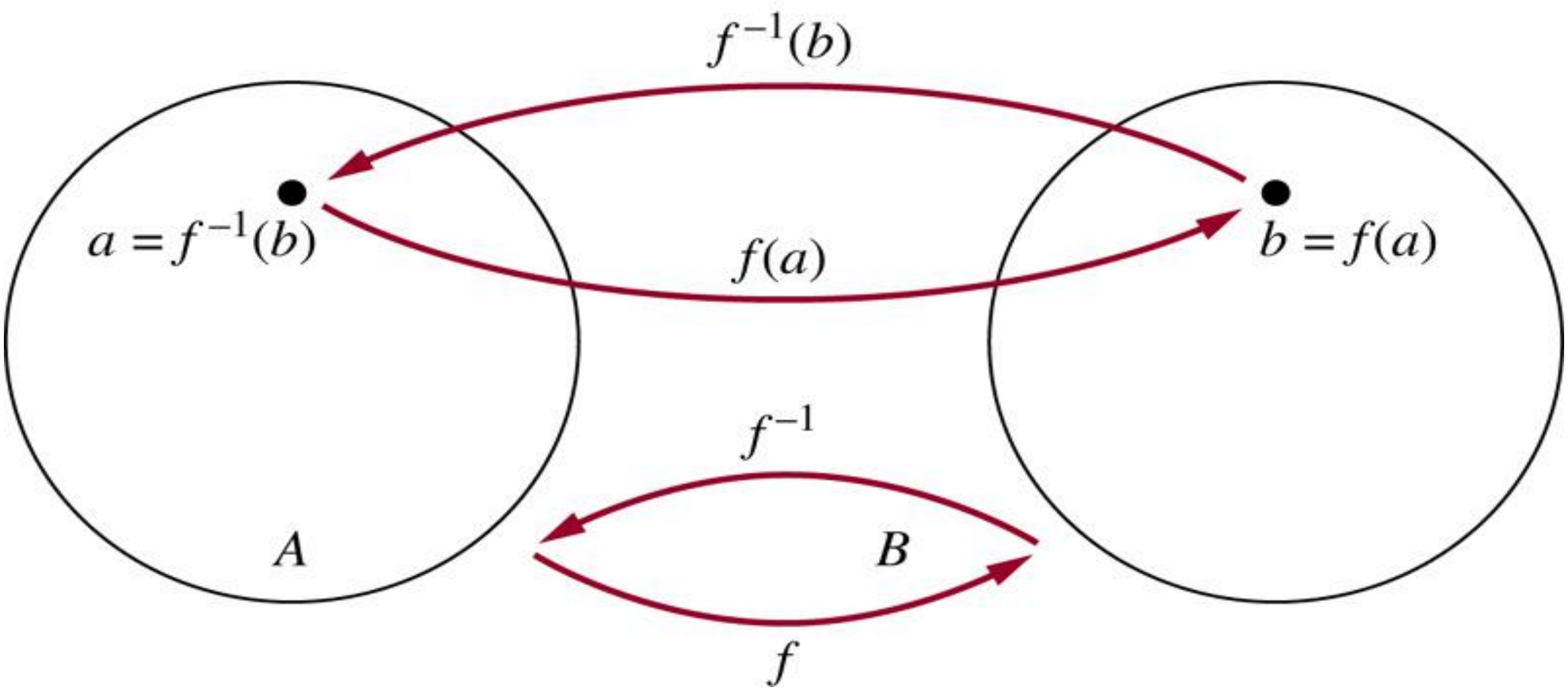
- **Objectives**: To understand Inverse Function, Compositions of Functions, Floor function, and Ceiling Function.
- **Outcomes**: Students are expected to be able to determine whether a function is invertible, be able find out the inverse of a function if the function is invertible, be able to find the composite functions of two given functions, be able to apply floor and ceiling functions.

Inverse Functions

- **Definition 9**: Let f be a one-to-one correspondence from A to B. The inverse function of f is the function that assigns to an element b belonging to B the unique element a in A such that $f(a) = b$. Hence,

  $f^{-1}(b) = a$, when $f(a) = b$

FIGURE: The Function f^{-1} is the Inverse of Function f



Inverse Functions

- A one-to-one correspondence is called **invertible** because we can define an inverse of this function.

- A function is not invertible if it is not a one-to-one correspondence, because the inverse of such a function does not exist.

Example 16

- Let f be the function from {a, b, c} to {1, 2, 3} such that $f(a) = 2$, $f(b) = 3$, and $f(c) = 1$.

  Is f invertible, and if it is, what is its inverse?

- **Solution**: The function f is invertible because it is a one-to-one correspondence.
- The inverse function f^{-1} reverses the correspondence given by f. So $f^{-1}(1) = c$, $f^{-1}(2) = a$, and $f^{-1}(3) = b$

Example 17

- Let ***f*: *Z*→*Z*** be such that $f(x) = x + 1$.

  Is *f* invertible? and if it is, what is its inverse?

- ***Solution***: The function *f* is one-to-one because every element of domain has a distinct image, just 1 bigger than that element. Again, the function *f* is onto because every element of codomain has a preimage, just 1 smaller than that element. Therefore, the function *f* is a one-to-one correspondence. So, the function *f* is invertible.

- To reverse the correspondence, suppose that y is the image of x, so that y = x+1. Then x = y – 1. This means that y – 1 is the unique element of Z that is sent to y by *f*. Consequently, $f^{-1}(y) = y - 1$

Example 18

- Let f be the function from **R** to **R** with $f(x) = x^2$.
  Is f invertible?

- **Solution**: Because $f(-2) = f(2) = 4$, f is not one-to-one.
  Since f is not one-to-one, it is not one-to-one correspondence. Hence, f is **not** invertible.

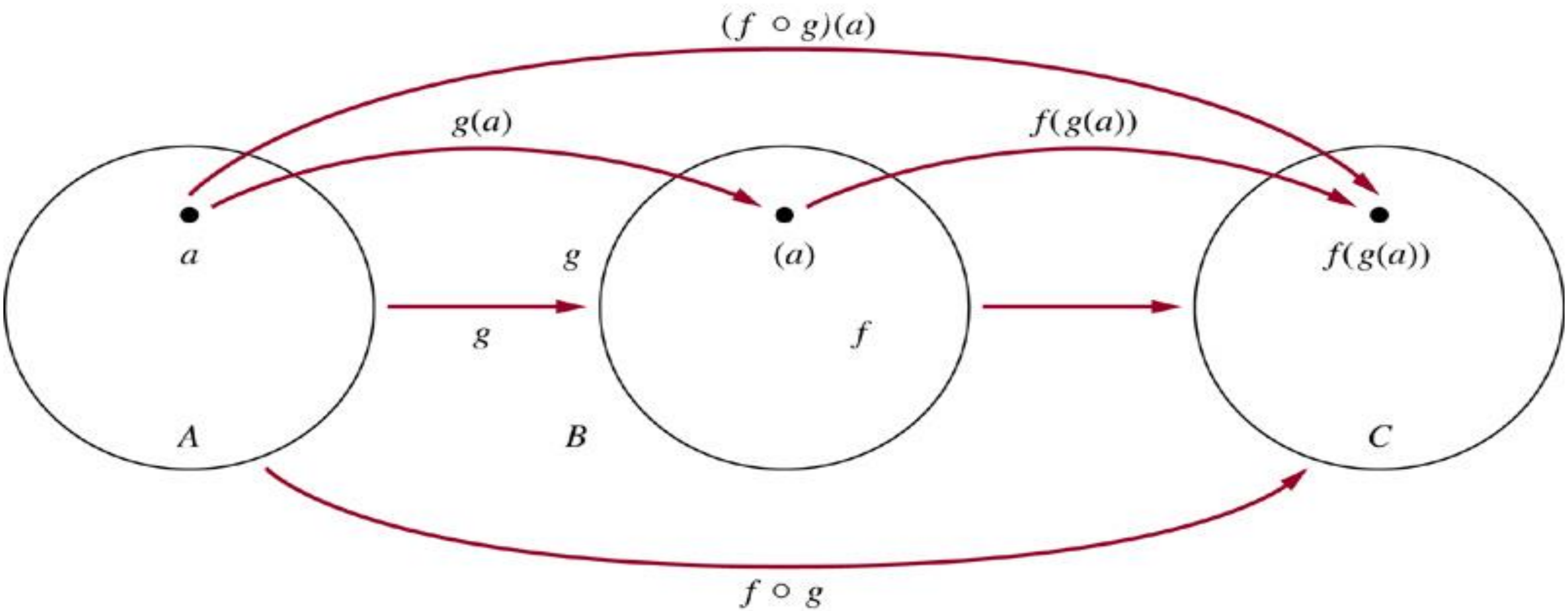
Compositions of Functions

- **Definition**: Let g be a function from the set $\boldsymbol{A}$ to the set $\boldsymbol{B}$ and let f be a function from the set $\boldsymbol{B}$ to the set $\boldsymbol{C}$.

  The composition of functions f and g, denoted by $\boldsymbol{f \circ g}$, is defined by $(f \circ g)(a) = f(g(a))$

- **Note:** $\boldsymbol{f \circ g}$ and $\boldsymbol{g \circ f}$ are not equal
- **Note:** The composition $\boldsymbol{f \circ g}$ can NOT be defined ***unless*** the range of g is a subset of the domain of f

FIGURE : The Composition of the Functions *f* and *g*



Modified Example 21

@ p. 141(6^{th} ed.) @ p.149 (7^{th} ed.)

- Let *f* and *g* be the functions from the set of integers to the set of integers defined by $f(x) = 2x + 3$ and $g(x) = 3x + 2$.

  Find the composite functions of *f* and *g.*

Solution:

- The composition of the functions *f* and *g* is

  $(f \circ g)(x) = f(g(x)) = f(3x+2) = 2(3x+2) + 3 = 6x + 7$

- The composition of the functions *g* and *f is*

  $(g \circ f)(x) = g(f(x)) = g(2x+3) = 3(2x+3) + 2 = 6x + 11$

- Practice @ Home: Example 20 (p.149)

Some Important Functions

- Two important functions in discrete mathematics. These functions are often used when objects are counted. They play an important role in the analysis of the number of steps used by procedures to solve problems of a particular size.
  - **Floor function**
  - **Ceiling Function**
- The **floor** and **ceiling** functions **map** the **real numbers** onto the **integers** (**R**→**Z**).

Floor Function

- The floor function assigns to the real number *x* the largest integer that is less than or equal to *x*.
  - Let *x* be a real number. The floor function rounds *x* down to the closest integer less than or equal to *x*.

- The value of the floor function at *x* is denoted by $\lfloor x \rfloor$

- **Examples:** $\lfloor 2.3 \rfloor = 2$, $\lfloor 2 \rfloor = 2$, $\lfloor 0.5 \rfloor = 0$, $\lfloor -3.5 \rfloor = -4$

Ceiling Function

- The ceiling function assigns to the real number *x* the smallest integer that is greater than or equal to *x*.

- The value of the ceiling function at x is denoted by $\lceil x \rceil$

- **Examples:** $\lceil 2.3 \rceil = 3, \lceil 2 \rceil = 2, \lceil 0.5 \rceil = 1, \lceil -3.5 \rceil = -3$

Examples of Ceiling Function

- How many **bytes** are required to encode 600 kilobits of data? [***Note***: Each byte is made up of 8 bits]
- Answer: $\lceil (600X1000)/8 \rceil$ = 75000 bytes

- How many bytes are required to encode 1001 bits of data?
- Answer: $\lceil 1001/8 \rceil$ = 126 bytes

Example of Floor Function

- In asynchronous transfer mode (ATM), data are organized into cells of 53 bytes. **How many ATM cells** can be transmitted in 1 minute over a connection that transmits data at the rate of 500 kilobits per second?
- **Solution**: In 1 minute this connection can transmit 500.1000.60 bits = 30,000,000 bits

  Each ATM cell is 53 bytes long, which means that it is

  53.8 = 424 bits long

  Number of ATM cells that can be transmitted is

  $= \lfloor 30{,}000{,}000/424 \rfloor = 70{,}754$

Practice @ Home

- Relevant odd-numbered exercises from text book

Books

1. *Discrete Mathematics and its applications with combinatorics and graph theory (7th edition)* by Kenneth H. Rosen [Indian Adaptation by KAMALA KRITHIVASAN], published by McGraw-Hill

References

1. Discrete Mathematics, *Richard Johnsonbaugh*, Pearson education, Inc.
2. Discrete Mathematical Structures, *Bernard Kolman*, *Robert C. Busby*, *Sharon Ross,* Prentice-Hall, Inc.
3. *SCHAUM'S outlines Discrete Mathematics(2nd edition)*, by *Seymour Lipschutz*, *Marc Lipson*