Lesson 9

Chapter 20: **Entropy and the second law of thermodynamics**

20-2 Entropy in the Real World: Engines

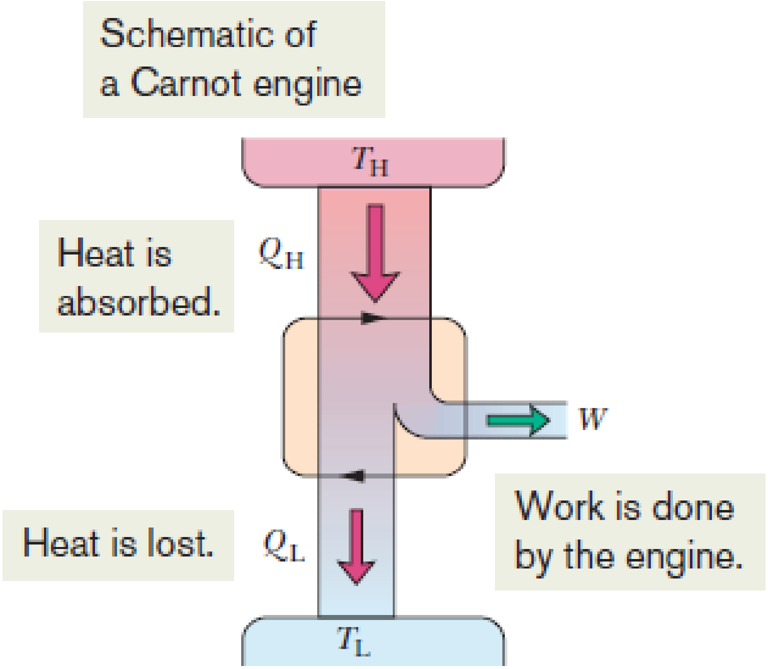
Heat Engine: A heat engine, or more simply, an engine, is a device that extracts energy from its environment in the form of heat and does useful work. At the heart of every engine is a working substance.

Carnot Engine: Although an ideal gas does not exist, any real gas approaches ideal behavior if its density is low enough. Similarly, we can study real engines by analyzing the behavior of an ideal engine.

“An ideal engine where all processes are reversible and no wasteful energy transfers occur due to, say, friction and turbulence”.

Schematic diagram of a Carnot Engine:

We shall focus on a particular ideal engine called a Carnot engine after the French scientist and engineer N. L. Sadi Carnot (pronounced "cah-no"),who first proposed the engine's concept in 1824.This ideal engine turns out to be the best (in principle) at using energy as heat to do useful work. Surprisingly, Carnot was able to analyze the performance of this engine before the first law of thermodynamics and the concept of entropy had been discovered.



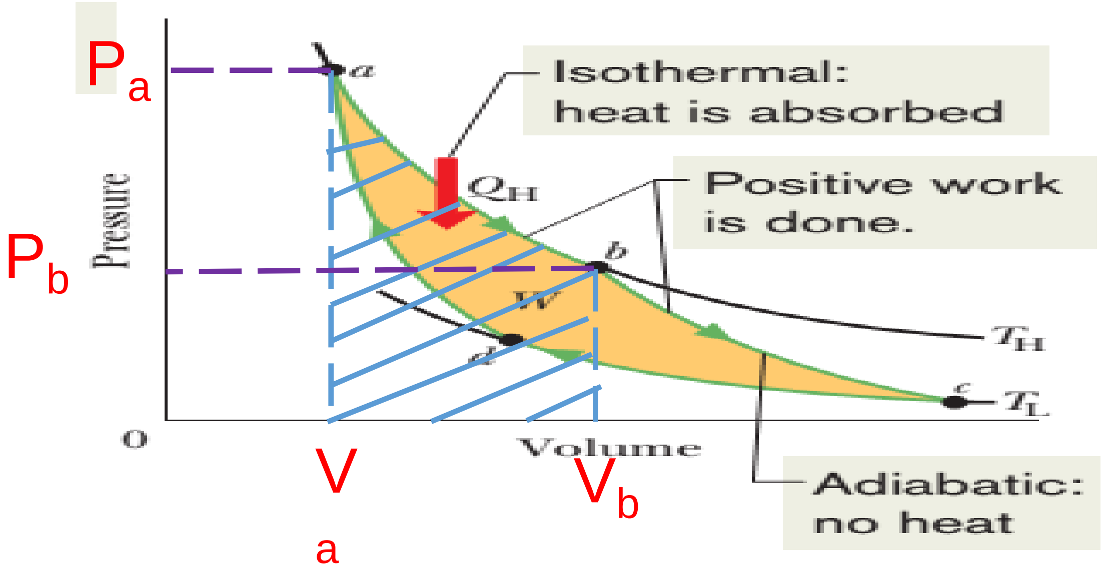
pV plot of the Carnot cycle:

If we place the cylinder in contact with the high temperature reservoir at temperature T_H, heat Q_H is transferred to the working substance from this reservoir as the gas undergoes an isothermal expansion from volume V_a to volume V_b.

Similarly, with the working substance in contact with the low-temperature reservoir at temperature T_L, heat Q_L is transferred from the working substance to the low-temperature reservoir as the gas undergoes an isothermal compression from volume V_c to volume V_d .

Thus the heat transfers to or from the working substance only during the isothermal processes ab and cd.

Isothermal expansion at constant T_H from a to b states:



Heat Q_H is absorbed by the working substance.

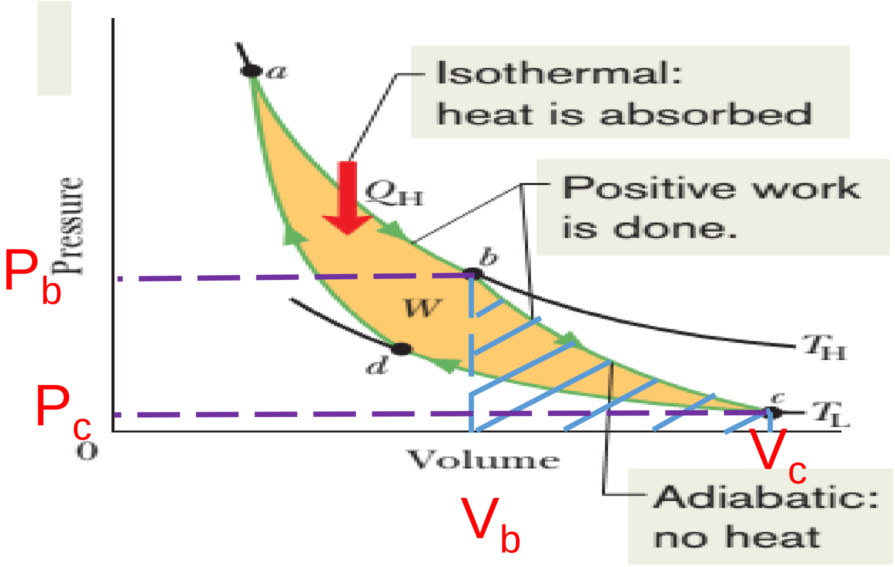
Pressure decreases from P_a to P_b

Volume increases from V_a to V_b

$$W = nRT_H \ln\frac{V_b}{V_a} \qquad [\text{positive}]$$

During the processes ab and bc, the working substance is expanding and thus doing positive work as it raises the weighted piston. This work is represented by the area under curve abc.

Adiabatic expansion from b to c states:



Heat Q is zero.

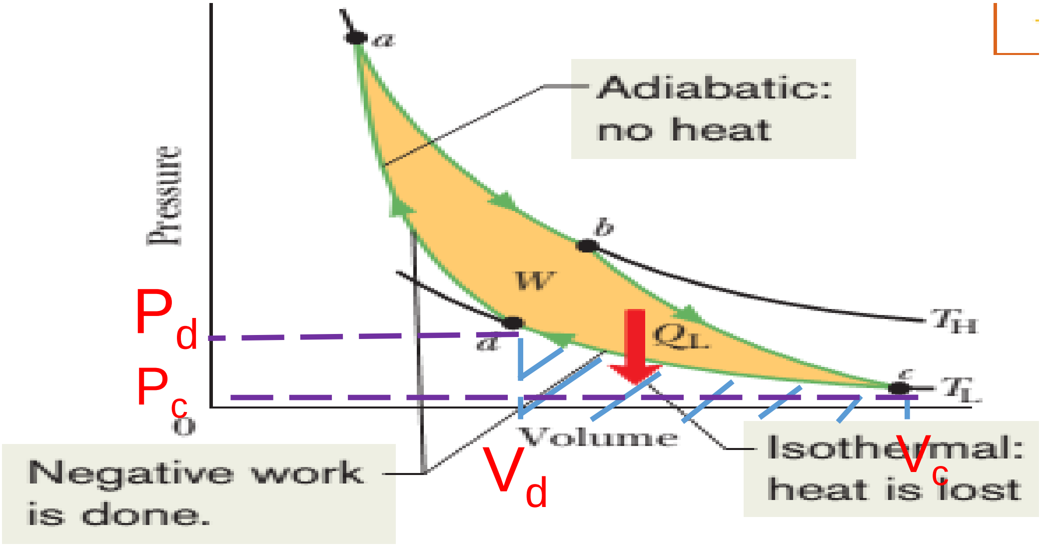
Pressure decreases from P_b to P_c

Volume increases from V_b to V_c

$$W = \frac{P_bV_b - P_cV_c}{\gamma - 1}$$

Isothermal compression at constant T_L from c to d states:

Heat Q_L is released.



Pressure increases from P_c to P_d

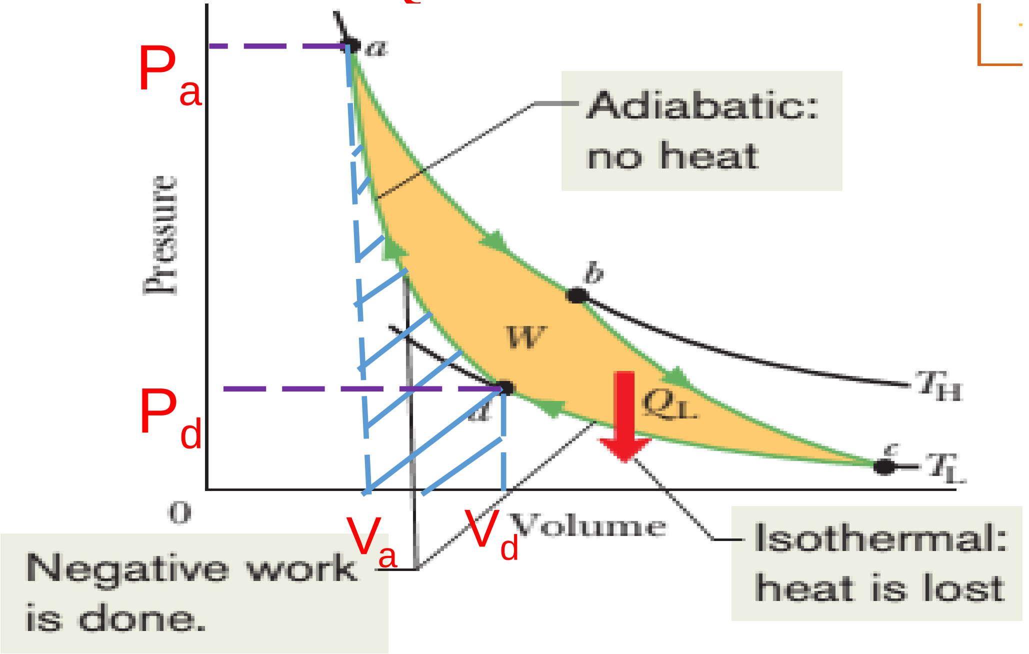
Volume decreases from V_c to V_d

$$W = nRT_L \ln \frac{V_d}{V_c} \qquad \text{[negative]}$$

During the processes cd and da, the working substance is being compressed, which means that it is doing negative work on its environment. This work is represented by the area under curve cda.

Adiabatic compression from d to a states:

Heat Q is zero.



Pressure increases from P_d to P_a

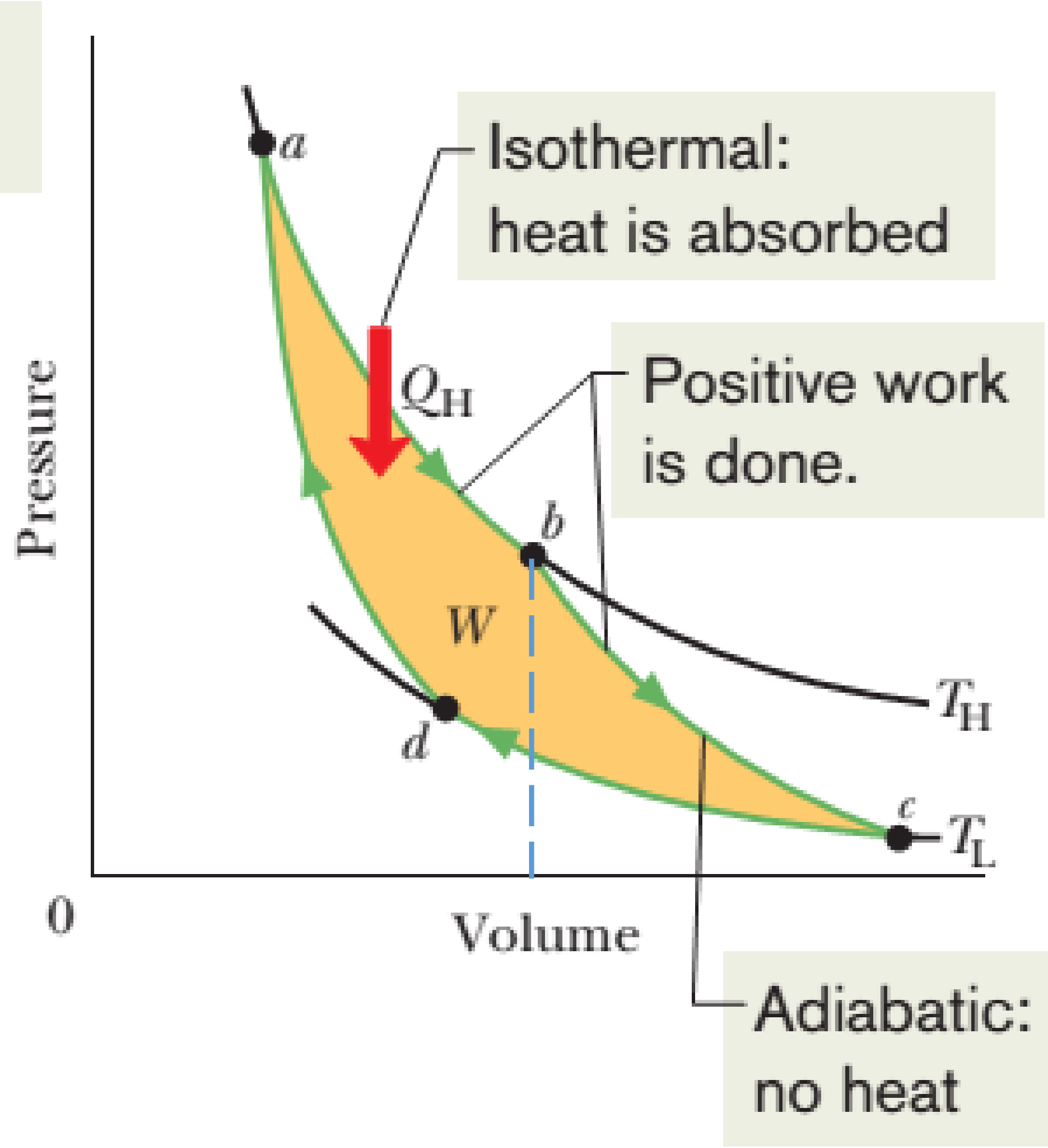
Volume decreases from V_d to V_a

$$W = \frac{P_d V_d - P_a V_a}{\gamma - 1} \qquad \text{[negative]}$$

Carnot cycle:

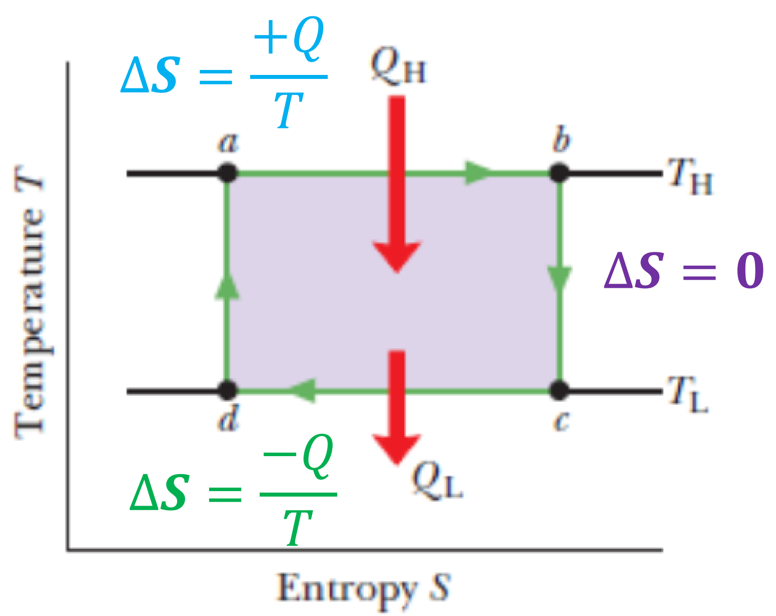
W = abcd area inside the curve [positive]

The net work per cycle is the difference between these two areas and is a positive quantity equal to the area enclosed by cycle abcda (yellow area).



Carnot cycle on a temperature–entropy diagram

In the temperature–entropy $(T-S)$ diagram the lettered points $a, b, c,$ and d there correspond to the lettered points in the $p-V$ diagram for the Carnot cycle.



The two horizontal lines in Fig. correspond to the two isothermal processes of the cycle. Process ab is the isothermal expansion and process bc is the isothermal compression of the cycle.

As the working substance (reversibly) absorbs energy Q_H as heat at constant temperature T_H during the expansion, its entropy increases.

$$\Delta S = \frac{+Q}{T}$$

Similarly, during the isothermal compression cd, the working substance (reversibly) loses energy Q_L as heat at constant temperature T_L, and its entropy decreases.

$$\Delta S = \frac{-Q}{T}$$

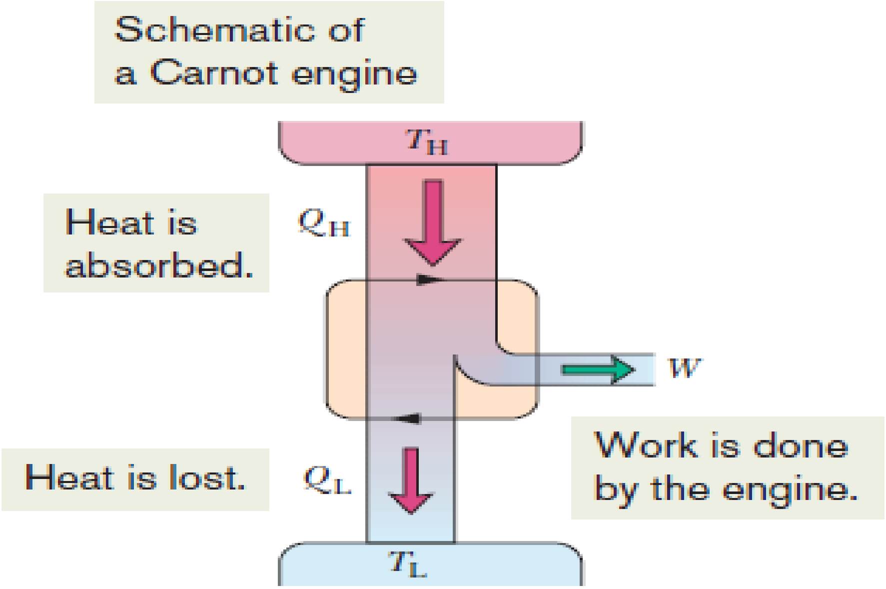
The two vertical lines correspond to the two adiabatic processes of the Carnot cycle. Because no energy is transferred as heat during the two processes (Q = 0), the entropy of the working substance is constant during them.

$$\Delta S = 0$$

The Work: In a Carnot engine, the working substance completes reversible cycles.

For a complete cycle of the working substance, the net internal energy change, $\Delta E_{int} = 0$

In each cycle of a Carnot engine, the heat Q_H is transferred to the working substance from the high temperature reservoir T_H and the heat Q_L is transferred from the working substance to the low temperature reservoir T_L.



So, the net heat transfer per cycle,

$$Q = |Q_H| - |Q_L|$$

The first law of thermodynamics for the Carnot cycle,

$$\Delta E_{int} = Q - W$$

$$0 = Q - W$$

$$W = Q$$

$$W = |Q_H| - |Q_L|$$

This is the net work done by a Carnot engine during a cycle.

Entropy Changes:

There are two isothermal processes in each cycle of a Carnot engine.

During the isothermal expansion, the working substance absorbs heat $|Q_H|$ at temperature T_H.

The increase in entropy, $\Delta S_H = \frac{+|Q_H|}{T_H}$

Again during the isothermal compression, the working substance releases heat $|Q_H|$ at constant temperature T_L.

The decrease in entropy, $\Delta S_L = \frac{-|Q_L|}{T_L}$

Thus the net entropy change per cycle,

$$\Delta S = \Delta S_H + \Delta S_L$$

$$\Delta S = \frac{+|Q_H|}{T_H} + \frac{-|Q_L|}{T_L}$$

For a complete cycle, $\Delta S = 0$

$$0 = \frac{+|Q_H|}{T_H} + \frac{-|Q_L|}{T_L}$$

$$\frac{|Q_H|}{T_H} = \frac{|Q_L|}{T_L} \qquad \frac{|Q_H|}{|Q_L|} = \frac{T_H}{T_L}$$

Efficiency of a Carnot Engine:

Thermal efficiency of any engine is defined as,

$$\varepsilon = \frac{\text{energy we get}}{\text{energy we provide}} = \frac{|W|}{|Q_H|}$$

$$\varepsilon = \frac{|Q_H| - |Q_L|}{|Q_H|}$$

$$\varepsilon = 1 - \frac{|Q_L|}{|Q_H|} \quad \text{[any engine]}$$

$$\varepsilon = 1 - \frac{T_L}{T_H} \quad \text{[Carnot engine]} \qquad [T_L < T_H]$$

Because $T_L < T_H$, the efficiency of Carnot engine is less than unity or less than 100%. Thus only a part of the extracted heat is available to do work and the rest is delivered to the low temperature reservoir.

23. A Carnot engine whose low-temperature reservoir is at 17 ^{0}C has an efficiency of 40%. By how much should the temperature of the high-temperature reservoir be increased to increase the efficiency to 50%?

Solution:

Given,

$$T_L = 17^0C = 290\ K$$

$$Initial\ efficiency, \varepsilon_c = 40\%$$

$$Final\ efficiency, \varepsilon'_c = 50\%$$

$$\Delta T_H = ?$$

For the initial state,

$$\varepsilon_c = 1 - \frac{T_L}{T_H}$$

$$\Rightarrow 40\% = 1 - \frac{T_L}{T_H}$$

$$\Rightarrow \frac{T_L}{T_H} = 1 - 0.40$$

$$\therefore T_H = 483.33\ K$$

For the final state,

$$\varepsilon'_c = 1 - \frac{T_L}{T_H}$$

$$\Rightarrow 50\% = 1 - \frac{T_L}{T'_H}$$

$$\Rightarrow \frac{T_L}{T'_H} = 1 - 0.50$$

$$\therefore T'_H = 580\ K$$

So the increased temperature of the high temperature reservoir,

$$\Delta T_H = T'_H - T_H$$

$$= (580 - 483.33)\ K$$

$$= 96.67\ K$$

24. A Carnot engine absorbs 52 kJ as heat and exhausts 36 kJ as heat in each cycle. Calculate (a) the engine's efficiency and (b) the work done per cycle in kilojoules.

Solution:

Given,

$$|Q_H| = 52\ KJ = 52 \times 10^3\ J$$

$$|Q_L| = 36\ KJ = 36 \times 10^3\ J$$

$$(a)\varepsilon_c = ?$$

We know,

$$\varepsilon_c = \left(1 - \frac{|Q_L|}{|Q_H|}\right) \times 100\%$$

$$= \left(1 - \frac{36 \times 10^3}{52 \times 10^3}\right) \times 100\%$$

$$= 30.77\ \%$$

$$(b)W = ?$$

We know

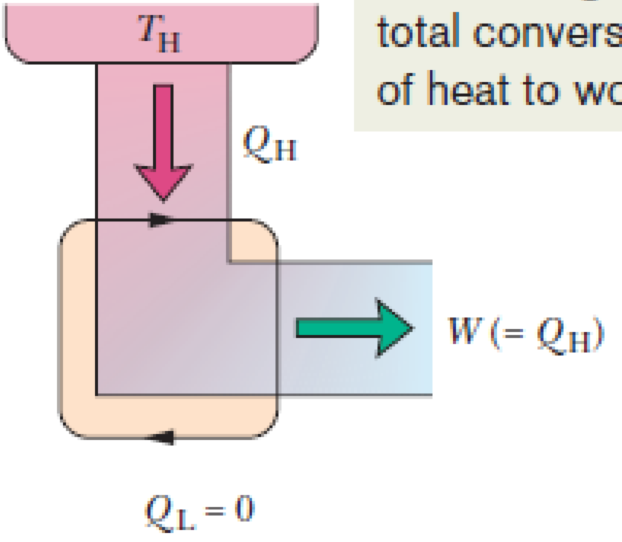
$$W = |Q_H| - |Q_L|$$

$$= 52\ KJ - 36\ KJ$$

$$= 16\ KJ$$

20-2 No Perfect Engines:

Perfect engine: total conversion of heat to work



Inventors continually try to improve engine efficiency by reducing the energy Q_L that is "thrown away" during each cycle. The inventor's dream is to produce the perfect engine, diagrammed in Fig, in which Q_L is reduced to zero and Q_H is converted completely into work.

Also, a perfect engine is only a dream: impossible requirements

$$\varepsilon = 1 - \frac{T_L}{T_H} = = 1 - \frac{0}{\infty} = 1 = 100\%$$

Instead, experience gives the following alternative version of the second law of thermodynamics, which says in short, there are no perfect engines:

"No series of processes is possible whose sole result is the transfer of energy as heat from a thermal reservoir and the complete conversion of this energy to work".

25. A Carnot engine has an efficiency of 22.0%. It operates between constant-temperature reservoirs differing in temperature by 75.0 C^0. What is the temperature of the (a) lower-temperature and (b) higher-temperature reservoir?

Given,

Efficiency, $\varepsilon_c = 22.0\% = 0.22$

Difference in temperature, $T_H - T_L = 75^0 C = 75\ K$

$$(T_H + 273) - (T_L + 273) = 75K$$

$(a)\ T_L = ?$

We know,

$$\varepsilon_c = 1 - \frac{T_L}{T_H}$$

$$\Rightarrow 22\% = 1 - \frac{T_L}{75 + T_L} \quad [as \quad T_H = 75 + T_L]$$

$$\Rightarrow 0.22 = \frac{75 + T_L - T_L}{75 + T_L}$$

$$\Rightarrow 0.22 = \frac{75}{75 + T_L}$$

$$\Rightarrow 0.22(75 + T_L) = 75$$

$$\therefore T_L = 266\ \mathrm{K}$$

$(b)\ T_H = ?$

We have,

$$T_H - T_L = 75\ K$$

$$\therefore T_H = 341\ \mathrm{K}$$

27. A Carnot engine operates between 235 ^{0}C and 115 ^{0}C, absorbing 6.30x10^4 J per cycle at the higher temperature. (a) What is the efficiency of the engine? (b) How much work per cycle is this engine capable of performing?

Given,

$$T_H = 235^0C = 508\ K$$

$$T_L = 115^0C = 388\ K$$

$$Q_H = 6.3 \times 10^4\ J$$

$$(a)\ \varepsilon_c = ?$$

We know,

$$\varepsilon_c = \left(1 - \frac{T_L}{T_H}\right) \times 100\%$$

$$= \left(1 - \frac{388}{508}\right) \times 100\%$$

$$= 23.62\%$$

(b) *The work done per cycle,*

$$W = \varepsilon_c\,|Q_H| \qquad \left[\varepsilon_c = \frac{W}{|Q_H|}\right]$$

$$= 23.62\% \times (6.3 \times 10^4)$$

$$= 1.48 \times 10^4\ J$$