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L11_S1_A.pdf

PDF

Generate a lecture scribe for the given PDF file in chat. This is the lecture file that our professor used while conducting the lecture 11. A lecture scribe is an exam-ready reconstruction of what was taught in the class, and it should allow me to recall definitions and notations, reconstruct proofs and derivations, follow worked examples step by step, and understand logical dependencies. Also, a lecture scribe must not have creative explanations, simplified tutorials, textbook rewrites, a solution manual for unseen problems, and a place to add intuition not discussed in class. The key point is that if it was not in the lecture or textbook context, then it should not appear in the lecture scribe. Also, the purpose of lecture scribing is that it must help me during exams to revise concepts in a shorter period. Hence, for the given lecture pdf, make the lecture scribe that also explains from where the particular formula gets derived.



Here is the **exam-ready Lecture Scribe — Lecture 11 (Transformation of Random Variables)** reconstructed strictly from the provided lecture slides, with derivation flow exactly following the class material.

CSE400 — Lecture 11 Scribe

Transformation of Random Variables

(Exam-Focused Reconstruction)

Source: L11_S1_A

1. Problem Setting and Notation

Given:

- A continuous random variable X with known PDF $f_X(x)$.
- A new random variable defined by transformation

$$Y = g(X)$$

Goal:

- Find $F_Y(y)$ and $f_Y(y)$.

As shown on **page 1**, the lecture begins with:

- PDF/CDF of X known a priori.
 - Define new RV through mapping $Y = g(X)$.
 - Need derived distribution of Y . $\square_{\text{L11_S1_A}}$
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2. Outline of Lecture

From **page 2**, three main components were taught:

1. Transformation of random variables.
2. Function of two random variables.
3. Example: $Z = X + Y$. $\square_{\text{L11_S1_A}}$

Logical dependency:

Single RV Transformation \rightarrow Joint RV Transformation \rightarrow Example

3. Transformation of One Random Variable

3.1 Assumption

Transformation:

$$Y = g(X)$$

Case shown in lecture:

- $g(\cdot)$ monotonic (increasing or decreasing).
- Inverse exists:

$$X = g^{-1}(Y)$$

(See sketches on page 3.) □ L11_S1_A

3.2 Step-1: CDF Method (Derivation Origin)

Definition:

$$F_Y(y) = P(Y \leq y)$$

Substitute transformation:

$$F_Y(y) = P(g(X) \leq y)$$

If g is increasing:

$$F_Y(y) = P(X \leq g^{-1}(y))$$

Hence:

$$F_Y(y) = F_X(g^{-1}(y))$$

This formula is obtained directly from rewriting the probability event using the inverse mapping, as shown in **page 3 derivation flow**. □ L11_S1_A

3.3 Step-2: PDF from CDF

Differentiate:

$$f_Y(y) = \frac{d}{dy} F_Y(y)$$

Substitute previous expression:

$$f_Y(y) = \frac{d}{dy} F_X(g^{-1}(y))$$

Using chain rule:

$$f_Y(y) = f_X(g^{-1}(y)) \cdot \frac{d}{dy} g^{-1}(y)$$

Since

$$\frac{d}{dy}g^{-1}(y) = \frac{dx}{dy}$$

Final result written in lecture:

$$f_Y(y) = f_X(x) \left| \frac{dx}{dy} \right|_{x=g^{-1}(y)}$$

This is obtained by:

- CDF definition
- Differentiation
- Chain rule

(as shown on page 3 right side). □ L11_S1_A

3.4 Decreasing Transformation Case

From page 4:

If $g(\cdot)$ is decreasing:

$$F_Y(y) = P(X \geq g^{-1}(y)) = 1 - F_X(g^{-1}(y))$$

Differentiate:

$$f_Y(y) = -f_X(x) \frac{dx}{dy}$$

Taking magnitude gives same unified formula:

$$f_Y(y) = f_X(x) \left| \frac{dx}{dy} \right|$$

Hence absolute value appears due to sign change during differentiation. □ L11_S1_A

4. Worked Example (Single Variable Transformation)

From page 5.

Given:

$$X \sim U(-1, 1)$$

$$f_X(x) = \begin{cases} \frac{1}{2}, & -1 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Transformation:

$$Y = \sin\left(\frac{\pi X}{2}\right)$$

4.1 Inverse Mapping

Solve for x :

$$x = \frac{2}{\pi} \sin^{-1}(y)$$

4.2 Jacobian Term (Derivative Origin)

Differentiate inverse:

$$\frac{dx}{dy} = \frac{2}{\pi} \frac{1}{\sqrt{1-y^2}}$$

This derivative appears from differentiating the inverse function used in the general formula.

4.3 PDF of Y

Apply transformation rule:

$$f_Y(y) = f_X(x) \left| \frac{dx}{dy} \right|$$

Substitute $f_X(x) = \frac{1}{2}$:

$$f_Y(y) = \frac{1}{2} \cdot \frac{2}{\pi} \frac{1}{\sqrt{1-y^2}}$$

$$f_Y(y) = \frac{1}{\pi\sqrt{1-y^2}}, \quad -1 < y < 1$$

Limits obtained from mapping:

$$x = -1 \Rightarrow y = -1, \quad x = 1 \Rightarrow y = 1$$

(as annotated on page 5). □ L11_S1_A

5. Function of Two Random Variables

From page 6:

Define:

$$Z = X + Y$$

Tasks discussed:

1. Find $f_Z(z)$.
2. Case when X, Y independent.
3. Example: $X, Y \sim N(0, 1) \Rightarrow Z \sim N(0, 2)$.
4. Case: exponential RVs.

These were presented as derivation setup problems, not solved fully in slide text.

□ L11_S1_A

6. Derivation of Distribution of $Z = X + Y$

From page 7 graphical derivation.

6.1 Start from CDF Definition

$$F_Z(z) = P(Z \leq z) = P(X + Y \leq z)$$

6.2 Convert to Double Integral

$$F_Z(z) = \iint_{x+y \leq z} f_{XY}(x, y) dx dy$$

Region shown in shaded diagram:

- Boundary line $x + y = z$.
 - Integration limits depend on geometry drawn in slide.
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6.3 Change Order of Integration

Lecture shows equivalent expression:

$$F_Z(z) = \int_{x=-\infty}^{\infty} \int_{y=-\infty}^{z-x} f_{XY}(x, y) dy dx$$

This step comes from describing the shaded triangular region shown in the diagram on page 7.  L11_S1_A

7. Logical Flow Summary (Exam Structure)

Lecture progression:

1. Start from CDF definition of transformed RV.
 2. Use inverse mapping.
 3. Differentiate to obtain PDF.
 4. Extend idea to joint RVs using probability region.
 5. Apply to example $Z = X + Y$.
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8. Final Key Formulas (As Derived in Lecture)

Single RV Transformation

$$F_Y(y) = F_X(g^{-1}(y))$$

$$f_Y(y) = f_X(g^{-1}(y)) \left| \frac{dx}{dy} \right|$$

Sum of Two RVs

$$F_Z(z) = \iint_{x+y \leq z} f_{XY}(x, y) dx dy$$

End of Lecture 11 Scribe

Three helpful follow-up questions:

1. Do you want Lecture 11 rewritten into **compact exam revision sheets (1-page formula summary)**?
2. Should I generate the **full convolution derivation** for $Z = X + Y$ exactly matching CSE400 notation?
3. Do you want a **step-by-step comparison** between CDF method vs Jacobian method as used in this lecture?



Sources



Do you like this personality?