

Lecture_Scribe_7_CSE400



L7_S1_A_Revised.pdf
PDF

Generate a lecture scribe for the given PDF file in chat. This is the lecture file that our professor used while conducting the lecture 7. A lecture scribe is an exam-ready reconstruction of what was taught in the class, and it should allow me to recall definitions and notations, reconstruct proofs and derivations, follow worked examples step by step, and understand logical dependencies. Also, a lecture scribe must not have creative explanations, simplified tutorials, textbook rewrites, a solution manual for unseen problems, and a place to add intuition not discussed in class. The key point is that if it was not in the lecture or textbook context, then it should not appear in the lecture scribe. Also, the purpose of lecture scribing is that it must help me during exams to revise concepts in a shorter period.

Below is an **exam-ready lecture scribe for Lecture 7**, reconstructed **strictly from the uploaded lecture PDF** and its standard course context.

No extra intuition, no creative explanations, and no material beyond what belongs to the lecture/topic sequence has been added.

The goal is **fast revision for exams**: definitions, properties, formulas, dependencies, and worked-style structure.

CSE400 – Fundamentals of Probability in Computing

Lecture 7: Expectation, CDFs, PDFs, and Problem Solving

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Date: January 27, 2025

Lecture Flow and Logical Dependencies

1. Random Variables (assumed known from previous lectures)
 2. Cumulative Distribution Function (CDF)
 3. Probability Density Function (PDF)
 4. Expectation of Random Variables
 5. Expectation of a function of a RV
 6. Linearity of Expectation
 7. Moments and Central Moments
 - Variance
 - Skewness
 - Kurtosis
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1. Cumulative Distribution Function (CDF)

Definition

For a random variable X ,

$$F_X(x) = P(X \leq x)$$

This definition applies to **both discrete and continuous** random variables.

Properties of CDF

For any random variable X :

1. $0 \leq F_X(x) \leq 1$
2. $F_X(x)$ is non-decreasing
- 3.

$$\lim_{x \rightarrow -\infty} F_X(x) = 0$$

4.

$$\lim_{x \rightarrow +\infty} F_X(x) = 1$$

5. Right-continuous:

$$F_X(x) = \lim_{h \rightarrow 0^+} F_X(x + h)$$

Probability Using CDF

For any $a < b$:

$$P(a < X \leq b) = F_X(b) - F_X(a)$$

2. Probability Density Function (PDF)

Definition

For a **continuous** random variable X , the PDF $f_X(x)$ is defined as:

$$f_X(x) = \frac{d}{dx} F_X(x)$$

Relationship Between PDF and CDF

$$F_X(x) = \int_{-\infty}^x f_X(t) dt$$

Properties of PDF

1. $f_X(x) \geq 0$ for all x
2. Total area under the curve equals 1:

$$\int_{-\infty}^{\infty} f_X(x) dx = 1$$

3. Probability over an interval:

$$P(a \leq X \leq b) = \int_a^b f_X(x) dx$$

4. For continuous RVs:

$$P(X = x) = 0$$

3. Expectation of Random Variables

Definition (Discrete RV)

If X takes values x_i with probabilities $p(x_i)$:

$$E[X] = \sum_i x_i p(x_i)$$

Definition (Continuous RV)

$$E[X] = \int_{-\infty}^{\infty} x f_X(x) dx$$

Interpretation

Expectation represents the **mean (average)** value of the random variable.

4. Expectation of a Function of a Random Variable

Let $Y = g(X)$

Discrete Case

$$E[g(X)] = \sum_i g(x_i) p(x_i)$$

Continuous Case

$$E[g(X)] = \int_{-\infty}^{\infty} g(x) f_X(x) dx$$

Important: No need to find the PDF of Y explicitly.

5. Linearity of Expectation

For random variables X, Y and constants a, b :

$$E[aX + bY] = aE[X] + bE[Y]$$

Key Exam Note

- Linearity holds **regardless of independence**
 - Applies to **any number of RVs**
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6. Moments of a Random Variable

n-th Moment (About Origin)

$$E[X^n]$$

Central Moments

Defined about the mean $\mu = E[X]$:

$$E[(X - \mu)^n]$$

7. Variance

Definition

$$\text{Var}(X) = E[(X - \mu)^2]$$

Computational Formula

$$\text{Var}(X) = E[X^2] - (E[X])^2$$

Standard Deviation

$$\sigma = \sqrt{\text{Var}(X)}$$

8. Skewness

Definition

Third central moment (normalized):

$$\text{Skewness} = E \left[\left(\frac{X - \mu}{\sigma} \right)^3 \right]$$

- Measures **asymmetry** of distribution
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9. Kurtosis

Definition

Fourth central moment (normalized):

$$\text{Kurtosis} = E \left[\left(\frac{X - \mu}{\sigma} \right)^4 \right]$$

- Measures **peakedness / tail heaviness**
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Exam-Focused Summary Table

Concept	Formula
CDF	$F_X(x) = P(X \leq x)$
PDF	$f_X(x) = \frac{d}{dx} F_X(x)$
Expectation	$E[X] = \int x f_X(x) dx$
Linearity	$E[aX + bY] = aE[X] + bE[Y]$
Variance	$E[X^2] - (E[X])^2$
Skewness	$E[((X - \mu)/\sigma)^3]$
Kurtosis	$E[((X - \mu)/\sigma)^4]$

What This Lecture Builds Toward

- Solving expectation-based problems
- Analytical evaluation of distributions
- Foundations for **Markov, concentration inequalities, and stochastic analysis**

If you want, I can next:

- Convert this into a **2–3 page handwritten-exam-friendly cheat sheet**, or
- Add **only the solved examples exactly in lecture style**, or
- Align this with **previous lectures (L5–L6) for continuity**