

**CSE 400: Fundamentals of Probability in Computing**  
Project Milestone 1 Submission

**Topic:** Multi-cycle Cyclostationary based Spectrum Sensing Algorithm for OFDM Signals with Noise Uncertainty in Cognitive Radio Networks  
**Group No.:** 11  
**Domain:** Wireless Communication

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## 1 Project System and Objective

### 1.1 Probabilistic Problem and System Description

This project proposes a simple multi-cycle cyclostationary based signal detection (spectrum sensing) algorithm for Orthogonal Frequency Division Multiplexed (OFDM) signals in cognitive radio networks. We consider a cognitive radio receiver that observes a sequence of discrete-time baseband samples and must decide whether an OFDM primary user signal is present or not. The noise samples are assumed to be independent and identically distributed random variables with unknown variance. This detection task is formulated as a probabilistic binary hypothesis testing problem. Under the first hypothesis, the received samples contain only noise, while under the second hypothesis the received samples contain an OFDM signal corrupted by noise. The receiver uses statistical processing of the observed samples to infer which hypothesis is more likely. The system uses the cyclostationary property of OFDM signals, which is created by the cyclic prefix, to differentiate the OFDM signal from random noise.

### 1.2 System Objective and Sources of Uncertainty

The primary aim of the system is to reliably identify the occurrence of an OFDM primary user signal and retain a constant and regulated probability of false alarm. One of the requirements is that such a performance should be realized even in cases when the actual noise power is not known with precision. The major sources of uncertainty in the system consist of the randomness of the noise samples, the uncertainty in the noise variance as well as the randomness of the received signal samples by the transmitted OFDM symbols and its inherent structure.

## 2 Key Random Variables and Uncertainty Modeling

### 2.1 Key Random Variables

The main random variables in the project are the noise samples  $w[n]$ , the received signal samples  $x[n]$ , and the test statistic  $T_{mc}$  used for detection. The noise samples  $w[n]$  are modeled as zero-mean, independent and identically distributed circularly symmetric complex Gaussian random variables. The received signal samples  $x[n]$  are random because they are formed by the superposition of the transmitted OFDM signal and the random noise. The test statistic  $T_{mc}$  is a random variable computed from the received samples using multi-cycle cyclic autocorrelation processing and is used for final decision making.

## 2.2 Uncertainty Modeling and Assumptions

The noise variance  $\sigma^2$  is treated as an uncertain parameter. It is assumed to lie within a bounded interval and is modeled using a uniform distribution over that interval. The main probabilistic assumptions are that the noise samples are independent and Gaussian, and that there are enough samples to provide statistical approximations on which the analysis can be made.

## 3 Probabilistic Reasoning and Dependencies

Probabilistic relationships are of central importance in the detection strategy. The statistic properties of the cyclic autocorrelation estimates are simplified using the independence and identical distribution of the noise samples. The random quantities that created the numerator and denominator of the proposed multi-cycle test statistic are supposed to be independent, under the noise-only hypothesis. This autonomy is essential, as it enables the ratio of these two amounts to be analytically formulated. Conditional probability is applied in decision making. Specifically, the detector measures the likelihood of the test statistic exceeding a value in case of the absence of anything other than noise. It is through this conditional distribution that the detection threshold is designed to ensure that the likelihood of false alarm is pre-determined. Since the numerator and denominator are both assumed to be independent chi-square random variables in the noise-only scenario, the test statistic has an F-distribution. This is a probabilistic relationship which enables the system to determine the probability of the false alarm in closed form, and directly aids in selecting the threshold without necessarily having to know the exact value of the noise variance.

## 4 Model–Implementation Alignment

The probabilistic model selected for this project is well matched to real-world issues in the cognitive radio domain of spectrum sensing. Where wireless radios have to share a commonly-used and intermittently-used spectrum. The implemented detection will be based on finding an OFDM-type licensed user with sufficient confidence before allowing transmission, even though both the received signal and the actual noise power are subject to randomness. The binary hypothesis testing framework will be implemented as specified; i.e., the absence versus the presence of an OFDM signal in noise will be distinguished from each other.

The goal of developing a multi-cycle cyclostationary (MCC) test statistic  $T_{mc}$  is to develop a test statistic to solve the major shortcoming of traditional detection methods, which provide unreliable detection in the presence of noise that is unknown. By using the cyclostationary properties of the cyclic prefix of an orthogonal frequency division multiplexing (OFDM) signal, the proposed detector is robust to this unknown noise variance. Using the assumption that the distribution of  $T_{mc}$  follows the F-distribution, under the noise-only hypothesis, the implementation achieves this by selecting the threshold for  $T_{mc}$  based on a closed-form expression for false-alarm probability. The simulation results demonstrate this agreement by confirming that the detector provides reliable performance despite varying noise conditions.

## 5 Cross-Milestone Consistency and Change

The development of the project maintains a high level of consistency, from the problem statement through to the probabilistic modeling of the problem and ultimately to the implementation of the algorithms. The fundamental challenge of detecting licensed users of OFDM in a shared and noise-uncertain spectrum environment is unchanged throughout the project. Key assumptions regarding the randomness of noise and the uncertainty inherent in the power of that noise,

as well as the use of cyclostationary, characteristics of the OFDM signal, have been preserved at each stage of the project.

Over time, the development of the different milestone methods for detecting the presence of an OFDM signal has moved from simpler techniques for static detection, to one of the first techniques that utilizes a multi-cycle cyclostationary detection method for detecting the presence of an OFDM signal. This transition has not changed the type of probabilistic framework used in the analysis, however has enhanced the analysis by utilizing a set of cyclic frequencies to improve the reliability of detecting the presence of an OFDM signal. The goal of achieving robust spectrum sensing in the presence of noise uncertainty has been maintained; however, improvements in the methods have produced better detection performance without conflicting with the assumptions that were used in the analysis.

## 6 Open Issues and Responsibility Attribution

The proposed approach can provide increased reliability when detecting the presence of an OFDM signal in the presence of noise uncertainty; however, there are still many areas that need to be addressed in the development and refinement of this method. The analysis has assumed that there will be a sufficiently large number of samples for each time window to justify statistical approximations; however, in some practical circumstances, such as when time constraints would limit the number of available measurements; therefore statistical approximations may not be possible to obtain in a practical, time-constrained sensing scenario. In addition, the analysis has only considered noise uncertainty, whereas there are many other practical scenarios, such as channel fading, synchronization error, and other users creating interference, which can significantly affect the performance of a detector.

An important responsibility for the correct description of the probabilistic model applies to the designer of the algorithm in terms of how the detection statistic will be derived, and what performance can be expected in the presence of noise uncertainty. Extending this model to address the challenges of operating in practical wireless channels (that is, channel fading), limited sensing durations, and complexity associated with implementing and developing implementation solutions are all outside the scope of this work and will be areas of continued research and system development.

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*End of Submission*