School of Engineering and Applied Science (SEAS), Ahmedabad University

CSE 400: Fundamentals of Probability in Computing

Project Milestone 2 Submission

Topic: Multi-cycle Cyclostationary based Spectrum Sensing Algorithm for OFDM Signals with Noise Uncertainty in Cognitive Radio Networks

Group No.: 11

Domain: Wireless Communication

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1 Project System and Objective

1.1 The probabilistic problem being addressed in this project.

The project is about the studies of a smart radio receiver which must able to decide that if an OFDM signal is present or not. The receiver on the other side gets many small samples of the signal. These samples can have only noise that is H_0 or can also have OFDM signal mixed with the noise that is H_1. The receiver of this signal can not be 100 percent sure of the received signal because noise is random. So to chose which case is more likely to occur we make use of probability and statistics. This makes the work as a probabilistic binary decision problem. Here, the system will check the patterns in the received data to detect the signal.

1.2 The objective of this system.

The main objective of this system is that it should correctly detect the presence of an OFDM primary user signal. Along with this, it must ensure that the probability of false alarm is small and fixed. Also, when the exact noise power is not known even then it must work properly. Hence, the systems makes the use of a multi-cycle cyclostationary method which will improve the detection performance and will make reliable decisions based on the noisy data.

1.3 The primary sources of uncertainty involved.

The system also faces several uncertainties. Some of them are:

1. Random noise samples - Over the time, the noise changes randomly.
2. Unknown noise variance - We do not know what the exact strength of the noise is.
3. Random transmitted OFDM symbols - Variation in signal data itself.
4. Signal structure and environment effects - Due to change in the channel conditions, there might be the change in the received samples.

2 Key Random Variables and Uncertainty Modeling

2.1 Key random variables.

This system handles the signals that change randomly, so they are considered as the random variables.

1. Received signal $x[n]$

   This is the main observed data at the receiver. This can be only noise or both signal and noise. This is the reason why it is random because both signal and noise will change over time.

2. Transmitted OFDM signal $s[n]$

   These are the real transmitted signal samples which are sent by the primary user. The symbols inside the OFDM are random because digital data changes continuously.

3. Noise samples $w[n]$

   Since noise is completely random and comes from the environment or electronic devices, and it will strongly affect the detection performance.

4. Test statistic T_{mc}

   This is the detector which will calculate a specific value using the received samples. This value will be random because it will depend on the random noise and the random signal data.

2.2 Uncertainty modeling for each random variable.

- Received signal $x[n]$ - This is modeled with 2 cases.
  1. under H_0 which is only noise: $x[n] = w[n]$
  2. under H_1 which is noise + signal: $x[n] = w[n] + s[n]$
- Noise $w[n]$ - This is modeled as:
  1. Zero-mean random values.
  2. Independent and identically distributed
  3. With the help of unknown variance
- OFDM signal $s[n]$ - OFDM signal is random due to following reasons:
  1. The symbols of modulation changes.
  2. There is a repeating structure due to cyclic prefix.
- Test statistic T_{mc} - It is modeled as a random variable but we can not study distribution for this random variable. Under the case of noise-only this will follow a known probability form and this will help to compute false alarm probability.

2.3 Probabilistic assumptions made.

1. Noise samples are independent and they are distributed identically.
2. Mean of noise is assumed to be zero.
3. We have assumed that OFDM signals have the periodic patterns because of the cyclic prefix and this will be helpful for detection.

3 Probabilistic Reasoning and Dependencies

1. Independence of noise samples

   The value of one noise will not affect the another one because all the noise values will behave in the same random manner. Due to the independency the detector will be able to add many samples together safely. This will also help to make mathematical formulas for probability easier. The distribution of the test statistic will become easier to calculate.

2. Conditional Relationships

   The received signal $x[n]$ depends on the fact that which of the two hypothesis, that is H_0 or H_1 is true. This means that the detector makes the use of conditional probability to measure things such as false alarm probability $P(T_{mc} > \lambda | H_0)$. This will tell the system that how likely a wrong decision will be when there is only noise present.

3. Dependence between Variables

   Variables such as test statistics will depend on the received signals. Further, the received signals depend on both signal and noise. The detector will understand these dependencies and will use statistical reasoning to separate the useful signals from random noise.

4 Model–Implementation Alignment

4.1 Probabilistic System Modeling

The problem of spectrum sensing is modeled as a probabilistic binary hypothesis testing problem. The received signal $x[n]$ is modeled as a random process that can have either pure noise or a combination of signal and noise. Under hypothesis H_0, the received signal samples are composed of only noise ($x[n] = w[n]$), whereas under hypothesis H_1, the samples are composed of a transmitted OFDM signal and noise ($x[n] = s[n] + w[n]$).

4.2 Random Variables and Distributions in Implementation

The process directly implements the probabilistic model. The noise samples $w[n]$ are modeled as independent and identically distributed Gaussian random variables with zero mean and unknown variance. The OFDM signal $s[n]$ is modeled as a random process because of the time-varying modulation symbols, but it is cyclostationary because of the cyclic prefix. The test statistic derived from the cyclic autocorrelation is also a random variable. The F-distribution is the distribution of the test statistic under the noise-only hypothesis, which enables the computation of the probability of false alarm.

4.3 Practical Implementation Using Finite Samples

In theoretical modeling, the cyclic autocorrelation is based on infinite observations. In reality, only finite data are available. Hence, the implementation is based on the estimated cyclic autocorrelation values calculated from the observed data. The additional randomness introduced by the estimated values makes the test statistic stochastic. This illustrates how theoretical probabilistic models are developed into practical algorithms.

4.4 Planned Simulation and Validation

To make the probabilistic model consistent with implementation, simulations will be carried out. Random noise samples will be created using Gaussian distributions, and the simulated OFDM signals will be added to test the detection performance. The test statistic will be calculated over a number of trials to see the distribution of the test statistic. Basic plots of the probability density function and cumulative distribution function of the test statistic will be created to confirm the theoretical assumptions.

5 Cross-Milestone Consistency and Change

5.1 Aspects That Remain Consistent

The basic probabilistic model of the project is still consistent with the previous milestones. The system is still modeled as a binary hypothesis test. The assumptions about independent noise samples, random OFDM signal structure, and probabilistic decision-making are still the same. The application of cyclostationary features for signal detection is still the main approach.

5.2 Improvements in the Current Milestone

In this milestone, the probabilistic modeling has moved towards a more concrete and mathematically defined framework. The major random variables in the system have been identified, and their probabilistic characteristics have been defined. The distribution of the test statistic under the noise-only hypothesis has been understood, allowing for the analytical calculation of the false alarm probability. Furthermore, the group has started the process of simulation validation by generating random variables and analyzing their statistical properties through PDF and CDF graphs.

5.3 Aspects Requiring Further Refinement

Although some progress has been made, there are still several aspects of the model that need to be further refined. The noise model currently in place is based on independence and Gaussian distribution, which may not be entirely accurate in modeling the wireless channel. The impact of channel fading, interference, and synchronization errors has not yet been taken into account in the probabilistic model. The impact of finite sample size on estimation and detection also needs to be explored.

5.4 Expected Evolution in Future Milestones

In the future milestones, the probabilistic model could be improved by considering more realistic channel conditions and correlated noise characteristics. Further simulations will be performed to analyze the detection results under different signal-to-noise ratios and modulation schemes. Such improvements will enhance the credibility of the probabilistic model and help to make the theoretical assumptions more realistic.

6 Open Issues and Responsibility Attribution

6.1 Open Probabilistic Issues

However, there are some probabilistic issues that have not been resolved at this point. The model developed here considers that the noise samples are independent and follow a Gaussian distribution. However, in real-world wireless networks, there could be some correlated interference and fading. The distribution of the test statistic for the signal present hypothesis has not been completely specified.

6.2 Modeling and Simulation Uncertainties

The current implementation of the project is based on theoretical assumptions that may not be entirely true in real-world applications. The finite sample size may impact the estimation of the cyclic autocorrelation and the distribution of the test statistic. The effect of the variation of the signal-to-noise ratio on the performance of the detection also needs to be explored further through simulations. The generation of the PDF and CDF graphs of the test statistic for various scenarios will aid in the understanding of the aforementioned uncertainties.

6.3 Practical System Limitations

The current model is based on ideal OFDM signals and perfect synchronization. However, in practical systems, there can be timing errors, channel fading, and multipath effects, which may introduce additional randomness that is not yet accounted for in the model. This will have to be incorporated into the probabilistic model in future milestones.

6.4 Responsibility Attribution

To tackle the remaining issues, the tasks will be assigned to the group members. One member will concentrate on enhancing the probabilistic and mathematical modeling of the system. Another member will take care of the simulation work, such as the creation of random variables and the plotting of the PDF/CDF graphs. The third member will analyze more research papers to add realistic channel and noise models. The fourth member will take care of the consistency of the documentation and will make sure that the probabilistic assumptions are consistent.

End of Submission