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Spectrum Sensing in Cognitive Radio Using Cyclostationary Features

CSE400 - Section 1
Group 11

Project Overview

Problem Statement

- Detect presence of primary OFDM user
- Work under unknown noise variance
- Ensure controlled false alarm probability

Core Idea of Paper

- Use cyclostationary structure of OFDM
- Combine multiple cyclic frequencies
- Derive analytical false alarm expression

Milestone-2 Focus

- Identify and explain 4 core mathematical models
- Analyze random variables and distributions
- Understand F-distribution derivation

Problem Framing: Spectrum Sensing Under Uncertainty

Primary Goal

➔ Detect presence of OFDM primary user

Mathematical Formulation

➔ Binary hypothesis testing

➔ $H_0 : x[n] = w[n]$

➔ $H_1 : x[n] = s[n] + w[n]$

Core Challenge

➔ Noise is random

➔ Noise variance is unknown

➔ Finite samples only

❖ w[n] noise & s[n] OFDM signal

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Why a Probabilistic Detection Framework is Required?

Observation Under Uncertainty

- ➔ x[n] is random
- ➔ w[n] is random
- ➔ Noise variance σ^2 is unknown

Decision is Not Deterministic

- ➔ Cannot be 100% certain
- ➔ Risk of false alarm and missed detection

Therefore: Probabilistic Modeling Required

- ➔ Model distributions of test statistic
- ➔ Derive probability of false alarm
- ➔ Select threshold analytically

Motivation for Structural Detection

- ➔ Energy detector depends on σ^2
- ➔ Cyclostationary model reduces
- ➔ dependence

Mathematical Model 1: Signal Observation Model

Binary Hypothesis Testing Model

- $H_0 : x[n] = w[n]$
- $H_1 : x[n] = s[n] + w[n]$

Statistical Assumptions

- $w[n] \sim \text{Gaussian}(0, \sigma^2)$
- Samples independent
- σ^2 unknown

Random Variables Defined

- $w[n] \rightarrow$ Additive noise
- $s[n] \rightarrow$ OFDM signal
- $x[n] \rightarrow$ Observed sample

Purpose of This Model

- Formalizes detection problem
- Defines randomness source
- Foundation for all later models

Mathematical Model 2: Noise Statistical Model

Noise Statistical Characterization

- $w[n] \sim \mathcal{N}(0, \sigma^2)$

Random Variables Defined

- $w[n]$ → additive noise sample
- σ^2 → noise variance (unknown parameter)

Statistical Assumptions

- Zero mean
- Independent across time
- Gaussian distribution
- σ^2 not perfectly known

Purpose of This Model

- Quantify randomness in received samples
- Identify main source of uncertainty
- Explain why energy-based detection becomes unreliable

Mathematical Model 3: Cyclostationary Signal Model

Cyclostationary Characterization of OFDM Signal

Time-varying autocorrelation:

- ➔ $Ry(t, \tau) = E[y(t)\, y(t + \tau)]$

Cyclic autocorrelation at frequency α:

- ➔ $Ry^\alpha(\tau)$

Random Variables Defined

- ➔ $y(t)$ → received signal process
- ➔ $Ry(t, \tau)$ → time-varying autocorrelation
- ➔ $Ry^\alpha(\tau)$ → cyclic autocorrelation component

Statistical Assumptions

- ➔ OFDM contains cyclic prefix
- ➔ Signal exhibits periodic statistical behavior
- ➔ Noise has zero cyclic autocorrelation for $\alpha \neq 0$

Purpose of This Model

- ➔ Capture structural property of OFDM
- ➔ Differentiate signal from noise using periodicity
- ➔ Provide feature independent of energy magnitude

Mathematical Model 4: Multi-Cycle Detection & Distribution Model

Multi-Cycle Test Statistic

- T(mc) = (Sum of cyclic feature energies) / (Reference energy)

Random Variables Defined

- R̂x^α(τ) → estimated cyclic autocorrelation
- T(mc) → test statistic
- λ → detection threshold

Statistical Assumptions

- Multiple cyclic frequencies combined
- Numerator and denominator independent under H_0
- Large sample approximation
- Under H_0: T(mc) follows F-distribution
- Pf = 1 / (λ + 1)

Purpose of This Model

- Construct robust detection statistic
- Cancel unknown noise variance
- Derive analytical threshold
- Control probability of false alarm

Decision Logic and Randomness Flow

Final Decision Rule

- If $T(mc) \geq \lambda \rightarrow$ Decide H_1
- Else $\rightarrow$ Decide H_0

Flow of Randomness

- $w[n] \rightarrow x[n] \rightarrow \tilde{R}\_x^\alpha(\tau) \rightarrow T\_mc \rightarrow$ Decision

Core Challenge

- Probability of Detection (Pd)
- Probability of False Alarm (Pf)

Model–Implementation Alignment

How Theory Connects to Practice

- Observation model → Sample collection
- Cyclostationary model → Feature extraction
- Detection model → Statistic computation
- Distribution model → Threshold selection

Simulation Validation

- Matches analytical Pf
- Works across OFDM modulations
- Robust under noise uncertainty

Limitations and Milestone-2 Contribution

Current Assumptions

- ➔ Independent Gaussian noise
- ➔ Large sample approximation
- ➔ Ideal synchronization

Future Extensions

- ➔ Channel fading models
- ➔ Small-sample distribution analysis
- ➔ Hardware imperfections

Milestone-2 Contribution

- ➔ Clear identification of 4 core probabilistic models
- ➔ Structured random variable analysis
- ➔ Analytical understanding of F-distribution result
- ➔ Stronger theoretical clarity than Milestone 1

Thank you!