A Hierarchical Evolutionary Algorithm with Noisy Fitness in Structural Optimization Problems

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Abstract. The authors propose a hierarchical evolutionary algorithm (HEA) to solve structural optimization problems. The HEA is composed by a lower level evolutionary algorithm (LLEA) and a higher level evolutionary algorithm (HLEA). The HEA has been applied to the design of grounding grids for electrical safety. A compact representation to describe the topology of the grounding grid is proposed. An analysis of the decision space is carried out and its restriction is obtained according to some considerations on the physical meaning of the individuals. Due to the algorithmic structure and the specific class of problems under study, the fitness function of the HLEA is noisy. A statistical approach to analyze the behavior and the reliability of the fitness function is done by applying the limit theorems of the probability theory. The comparison with the other method of grounding grid design shows the validity and the efficiency of the HEA.

1 Introduction

The grounding grids are important countermeasures to assure the safety and the reliability of the power systems and apparatus. In order to guarantee the safety level conditions, the touch voltages generated by the grounding grid in each point of the soil surface must be lower than the prearranged values fixed by Standards [1]. This requirement causes a significant increase in the cost of both conductor material and ditching. It is therefore fundamental, when a grounding grid has to be designed, to choose a criterion which guarantees both the low cost and the respect of the safety conditions.

The study of grounding grids and their design has been intensively discussed over the years and has been carried out according to different approaches. Empirical approaches have been suggested and some criteria resorting to the "compression ratio" have been given in [2]. Besides, some methods based on the genetic algorithms (GAs) have also been implemented [3], [4], [5].

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2 Description of the Hierarchical Evolutionary Algorithm

The problem of the design of a grounding grid can be formulated as follows: once leaking current, depth, volume of conductors (i.e. diameter of the cylinder conductors and number of the horizontal and vertical conductors) are prearranged, the topology of the grounding grid such that touch voltages are minimized has to be obtained.

2.1 The Lower Level Evolutionary Algorithm to Calculate U_{Tmax}

Let us consider a fixed topology of grounding grid with a section of conductors S_c which is buried at a depth d in a soil whose resistivity is ρ and leaks a fault current I_F. In order to determine the touch voltage U_T in a point P of the soil surface it is necessary to solve a system of 2nd order PDEs which in many cases cannot be solved theoretically. The Maxwell's subareas method [6] has been therefore implemented.

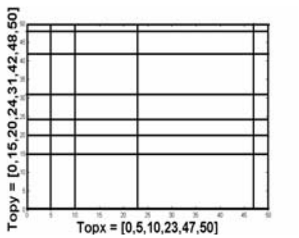

Fig. 1. Example of grounding grid (See paragraph 2.2)

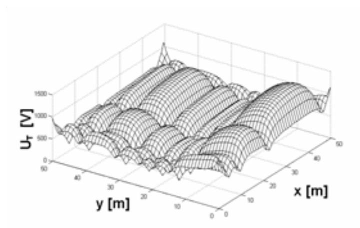

Fig. 2. Example of trend of touch voltage generated by a grounding grid

The calculation of the touch voltage by the Maxwell's method is computationally expensive and, in order to determine the maximum touch voltage U_{Tmax}, an optimization method is required. Since the trend of the touch voltage is, in general, multimodal (See Fig. 2), an approach which makes use of an exact method or a gradient-based method is not acceptable.

The LLEA here proposed is a steady-state GA [7] which works on a population of N_p points P of the soil surface initially sampled pseudo-randomly. Each point $P(x,y)$ is an individual having x and y as chromosomes. Due to the continuous trend of the touch voltage a real encoding has been chosen. In consideration of the rectangular external shape of the grounding grid and therefore of the rectangular shape of the decision space the arithmetic crossover technique [8] has been chosen. The probability of random mutation [8] has been set on 0.1. The algorithm is stopped when at least one of the two following conditions occurs: the difference $\Delta U_{T\ k}$ between the maximum touch voltage and the average value among all the touch voltages $U_{T\ k}(P)$ obtained in the k^{th} iteration is smaller than a pre-arranged value of accuracy ϵ; the number of iterations N_{iter} reaches a pre-arranged value $N_{iter\text{-}max}$.

2.2 The Higher Level Evolutionary Algorithm to Obtain $Grid^{opt}$

The authors propose the HLEA, a steady-state [7] GA, which uses the value of maximum touch voltage U_{Tmax} obtained by the LLEA as a fitness function to find the topology of the grounding grid $Grid^{opt}$ which generates a minimum value of maximum touch voltage. Each individual of the population is a grounding grid *Grid*(*Topx*, *Topy*). This grid (individual) has a genotype composed by two vectors of integer non-negative numbers; each of these numbers identifies a conductor of the grounding grid under study and represents the distance of this conductor from the reference axis. The origin of the reference axis is the lower left corner of the grounding grid (See Fig. 1). The two-point crossover technique [8] has been chosen. Mutation occurs, with a probability equal to 0.04, in the following way. A position of the gene to undergo the mutation is set pseudo-randomly. Another positive or negative random integer value is added to this gene. This operation is equivalent to moving a conductor of the grounding grid under examination of a small predetermined quantity. The stop criterion follows the same logic as that occurred in the LLEA.

2.3 Computational Analysis and Restriction of the Decision Space

The design problem consists in minimizing the function $U_{Tmax}(Topx, Topy)$ in the set $S = \mathbf{N}^{Nx \cdot + Ny}$ where **N** is the set of natural numbers. The number of vertical and horizontal conductors *Nx*, and *Ny* as well as the vertical and horizontal size of the grounding grid *Ly*, *Lx* are prefixed a priori. The decision space $D \subset S$ has cardinality:

$$card(D) = (Lx+1)^{Nx} (Ly+1)^{Ny}. \tag{1}$$

The position of the gene within the chromosome does not give any further information to the topology of the grounding grid. Consequently, the authors have chosen to add, at each iteration, a sorting cycle for the genes of each vector *Topx* and *Topy*. This choice decreases the cardinality of the decision space and, therefore, the computational complexity of the algorithm.

Proposition. The cardinality of the new decision space D^* is given by:

$$card(D^*) = \frac{(Nx + Lx)!}{Nx! \cdot Lx!} \cdot \frac{(Ny + Ly)!}{Ny! \cdot Ly!} \tag{2}$$

Proof: Let us consider the set of vertical conductors. Let c_l be the number of occurrences of l in the chromosome. Each individual belonging to D^* can be also expressed as a vector of occurrences $[c_0,..,c_{Lx}]$ where $c_0...+ c_{Lx} = Nx$. Applying the one-to-one correspondence of this kind to all the elements of the decision space D^* a new set M^* is generated and $card(D^*) = card(M^*)$. Let us consider a polynomial $\hat{p}(x) = (1 + x + ... + x^m)^n = \sum_{k=0}^{mn} b_k x^k$. In this case b_k is the cardinality of:

$A_k^{m,n+1} = \{[d_0 ,.. d_n] : d_i \in \{0,\dots,m\}, i = 0,\dots,n \text{ and } \sum_{i=0}^{n} d_i = k\}$. According to the above definition, $A_{Nx}^{Nx,Lx} = M^*$ and $b_{Nx} = card(A_{Nx}^{Nx,Lx}) = card(D^*)$. Performing the division of the following polynomials we obtain that $\frac{1 - x^{Nx+1}}{1 - x} = 1 + x + x^2 + ... + x^{Nx}$.

Let us consider the Lx^{th} order derivative of the geometric progression taken termwise, the common ratio $|x|$ being less than 1, $\frac{Lx!}{(1-x)^{Lx+1}} = \sum_{i=Lx}^{\infty} \frac{i!}{(i-Lx)!} x^{i-Lx}$; therefore $\frac{1}{(1-x)^{Lx+1}} = \sum_{n=0}^{\infty} \frac{(n+Lx)!}{(Lx)!n!} x^n = \sum_{n=0}^{\infty} \binom{n+Lx}{Lx} x^n$. The $card(A_{Nx}^{Nx,Lx})$ is the Nx^{th} coefficient of the classical expansion of

$p(x) = \left(\frac{1-x^{Nx+1}}{1-x} \right)^{Lx+1} = \sum_{n=0}^{\infty} \binom{n+Lx}{Lx} \cdot (\sum_{j=0}^{Lx+1} (-1)^j \binom{Lx+1}{j} x^{n+j(Nx+1)})$ that is $b_{Nx} = \binom{Nx+Lx}{Lx} = \frac{(Nx+Lx)!}{Nx!\,Lx!}$. The proof performed above is identical in the case of horizontal conductors. The sets of vertical and horizontal conductors are independent of each other, therefore the cardinality of the new decision space is given by (2)[1].

3 Statistical Analysis of the Noisy Fitness and Numerical Results

As previously described the HLEA makes use of the fitness given by the LLEA. Since the latter is a GA, it gives a value of touch voltage that is not deterministic but it takes its own value according to a stochastic process. Therefore the HLEA works with a noisy fitness.

Results shown in Table 1 and Table 2 refer to a grounding grid whose genotype is *Grid* ([0 , 5, 20, 25, 35, 40, 50], [0, 10, 15, 30, 36, 40, 50]) laying in a domain 50m x 50m. The values for the parameters characterizing the problem are the following:

d=0.5m $\quad$ S_c=50 mm^2 $\quad$ ρ=100 Ωm $\quad$ I_F=400 A $\quad$ $N_{iter\text{-}max}$=20 $\quad$ N_{exp} =500

For each population size N_{pop}, a set of values of U_{Tmax} has been obtained. Among these values, the maximum $U_{Tmax}{}^{max}$, the minimum $U_{Tmax}{}^{min}$ and the width of the interval $W = U_{Tmax}{}^{max} - U_{Tmax}{}^{min}$ have been determined. Hence, fixing a percent accuracy $acc\%$ and therefore the interval V=[(100-$acc\%$)/100 $U_{Tmax}{}^{max}$, $U_{Tmax}{}^{max}$], the probability q^* that a value given by the LLEA falls within the interval V has been approximated, according to the law of the big numbers, by $q = N_V/N_{exp}$ where N_V is the number of simulations so that the value falls within V and N_{exp} is the total number of simulations (experiments)-see Table 1.

The accuracy of the approximation of q^* has been verified by the central limit theorem (see Table 2). The probability P that the approximated probability q does not deviate from the real value q^* more than a value δ is expressed by:

$$P\{|q-q^*| \le \delta\} \approx 2\Phi\left(\delta\sqrt{N_{\exp}/q(1-q)}\right)-1 \quad \text{where} \quad \Phi(x) = 1/\sqrt{2\pi}\int_{-\infty}^{x} e^{-\frac{y^2}{2}}dy$$

[1] The problem studied and then the results got are similar to a classic problem of quantum mechanics (Bose-Einstein Statistics).

Table 1. Estimation of the noise and calculation time in the LLEA

N_{pop}	U_{Tmax}^{max} [V]	U_{Tmax}^{min} [V]	W [V]	Approximated probabilities q that U_{Tmax} determined by the LLEA falls within the interval V (for several values of $acc\%$)						Mean time[2] t [s]
				1%	2%	3%	5%	7%	9%	
40	97.241	83.562	13.679	0.72	0.74	0.75	0.76	0.77	0.99	4.82
70	97.241	88.874	8.387	0.83	0.84	0.85	0.86	0.88	1	11.03
100	97.241	88.874	8.387	0.94	0.97	0.97	0.97	0.97	1	32.64

Table 2. Confidence intervals δ for approximation performed with confidence level P=0.995

N_{pop}	Confidence intervals δ related to the probabilities q for several population sizes N_{pop} and values of accuracy $acc\%$					
	1%	2%	3%	5%	7%	9%
40	0.083	0.080	0.079	0.079	0.077	0.018
70	0.069	0.067	0.066	0.064	0.060	0
100	0.044	0.031	0.031	0.031	0.031	0

Table 3. Comparison between the HEA and other designing methods

Method	Objective	Assumptions	Decision Space
[2]	None	Exponential regularity	Continuous
[4]	Volume	Symmetry, particular proportions	Continuous
[3]	Cost	Symmetry, conductors on the perimeter	$2^{\frac{Lx}{2}+\frac{Ly}{2}+2}$
[5]	Touch voltage	None	$(Lx+1)^{Nx}(Ly+1)^{Ny}$
HEA	Touch voltage	None	$\frac{(Nx+Lx)!}{Nx!\cdot Lx!}\cdot\frac{(Ny+Ly)!}{Ny!\cdot Ly!}$

As the results in Table 1 show, if the size of the population of the LLEA is small, the fitness of the HLEA is quickly calculable but it is noisier; on the contrary, if the size is large the fitness is less noisy but each single evaluation requires a longer time. On the other hand, we should consider that the functions whose values take a large variability demand a large population size in order to find the global optimum [10]. In other words, if the population size of the LLEA is small, the HLEA, due to the noisy fitness, requires a large population size; if population size of the HLEA is small, a low-noisy fitness is required and therefore a large population of the LLEA. According to the obtained results, the authors propose for 50m<Lx,Ly<1000m the population sizes $N_{pop\,L}$ =70 and $N_{pop\,H}$ =50 for LLEA and HLEA, respectively.

[2] The calculation times refer to a PC with a frequency of 3 GHz and 512 Mb RAM.

Table 3 shows the comparison between the HEA and the other designing method found in literature.

Table 4. Numerical results obtained by different designing methods

Method	*Topx Topy*	U_{Tmax} [V]	Decision Space
[2]	[0,9.88,22.89,40,57.11,70.11,80] [0,13.05,30,46.95,60]	905.3762	Continuous
[3]	[0,6,22,40,58,74,80][0,10,30,50,60]	818.2894	4.7224×10^{21}
[5]	[0,7,22,40,58,73,80][0,10,30,50,60]	801.5457	1.9322×10^{22}
HEA	[0,7,22,40,58,73,80][0,10,30,50,60]	801.5457	4.8265×10^{16}

Table 4 shows the results obtained for the following set of parameters [3]:

D=0.5m S_c=69 mm^2 ρ=100 Ωm I_F=5 kA Lx=80m Ly=60m

4 Conclusions

The HEA performed better results in terms of efficiency and computational complexity compared to the other methods found in literature. Besides, the HEA is a completely general and automatic method which does not require any assumption on the topology of the grid. The HEA works in a decision space much smaller than the other designing methods thanks to the compact representation and the sorting cycle implemented. Though the decision space is small it contains not only all the representations of grounding grids considered in [3] but also non-symmetrical solutions that can be optimal in the case of non-constant resistivity. The statistical analysis carried out proves the reliability of the algorithm notwithstanding the noisy fitness. Moreover, the HEA is an extremely flexible method that can be applied in several other structural optimization problems.

References

1. IEEE Standard 80 – 2000: IEEE Guide for Safety in AC Substation Grounding. (2000)
2. Sun, W., He, J., Gao, Y., Zeng, R., Wu, W., Su, Q.: Optimal Design Analysis of Grounding Grids for Substations built in non-uniform soil. Proc. of Powercon. Intern. Conference on Power System Technology, Vol. 3. (2000) 1455-1460
3. Otero, A. F., Cidras, J., Garrido, C.: Genetic Algorithm Based Method for Grounding Grids Design. Proc. of the IEEE Inter. Conference on Evolutionary Computation. World Congress of Computational Intelligence. (1998) 120-123
4. Costa, M. C., Filho, M. L. P., Marechal, Y., Coulomb, J.-L., Cardoso, J. R.: Optimization of Grounding Grids by Response Surface and Genetic Algorithms. IEEE Transactions on Magnetics, Nol. 39, No 3. (2003) 1301-1304
5. Neri, F.: A New Evolutionary Method for Designing Grounding Grids by Touch Voltage Control. Proc. of IEEE Intern. Symposium on Industrial Electronics, (2004) 1501-1505

6. Sylos Labini, M., Covitti, A., Delvecchio, G., Marzano, C.: A Study for Optimizing the Number of Subareas in the Maxwell's Method. IEEE Trans. on Magnetics, Vol. 39, No. 3. (2003) 1159-1162
7. Eshelman, L. J.: The CHC adaptative search algorithm: How to have safe search when engaging in nontraditional genetic recombination. In FOGA-1 (1991) 265-283
8. Michalewicz, Z.: Genetic Algorithms + Data Structures = Evolution Programs. 3rd edn. Springer-Verlag, Berlin Heidelberg New York (1996)
9. Goldberg, D. E., Deb, K., Clark, J. H.: Genetic algorithms, noise, and sizing of populations. Complex Systems 6(4). (1992) 333-362