International Journal of Computational and Applied Mathematics
ISSN 1819-4966 Volume 5 Number 3 (2010), pp. 359–368

http://www.ripublication.com/ijcam.htm

Application of Fuzzy Ranking Method for Solving Assignment Problems with Fuzzy Costs

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Abstract

Assignment problem is a well-known topic and is used very often in solving problems of engineering and management science. In this problem c_{ij} denotes the cost for assigning the jth job to the ith person. This cost is usually deterministic in nature. In this paper c_{ij} has been considered to be triangular or trapezoidal fuzzy numbers denoted by $\tilde{c}_{ij}$ which are more realistic and general in nature. Yager's ranking method [11] has been used for ranking the fuzzy numbers. The fuzzy assignment problem has been transformed into crisp assignment problem in the LPP form and solved by using LINGO 9.0. Numerical examples show that the fuzzy ranking method offers an effective way for handling the fuzzy assignment problem.

Key words: Fuzzy Sets, Fuzzy Numbers, Fuzzy Assignment Problem, Fuzzy Ranking.

Introduction

Assignment Problem(AP) is used worldwide in solving real world problems. An assignment problem plays an important role in industry and other applications. In an assignment problem, n jobs are to be performed by n persons depending on their efficiency to do the job. In this problem c_{ij} denotes the cost of assigning the j^{th} job to the i^{th} person. We assume that one person can be assigned exactly one job; also each person can do at most one job. The problem is to find an optimal assignment so that the total cost of performing all jobs is minimum or the total profit is maximum.

In this paper we investigate a more realistic problem, namely the assignment problem with fuzzy costs or times $\tilde{c}_{ij}$. Since the objectives are to minimize the total cost or to maximize the total profit, subject to some crisp constraints, the objective function is considered also as a fuzzy number. The method is to rank the fuzzy objective values of the objective function by some ranking method for fuzzy numbers to find the best alternative. On the basis of this idea the Yager's ranking method [11] has been adopted to transform the fuzzy assignment problem to a crisp one so that the conventional solution methods may be applied to solve the AP.

The idea is to transform a problem with fuzzy parameters to a crisp version in the LPP form and to solve it by the simplex method. Other than the fuzzy assignment problem other applications of this method can be tried in project scheduling, maximal flow, transportation problem etc.

Fuzzy Assignment Problem and Fuzzy Ranking Method

In recent years, fuzzy transportation and fuzzy assignment problems have received much attention.

Lin and Wen solved the assignment problem with fuzzy interval number costs by a labeling algorithm[2]. In the paper by Sakawa et al.[7], the authors dealt with actual problems on production and work force assignment in a housing material manufacturer and a subcontract firm and formulated two kinds of two-level programming problems. Applying the interactive fuzzy programming for two-level linear and linear fractional programming problems, they derived satisfactory solutions to the problems and thereafter compared the results. They examined actual planning of the production and the work force assignment of the two firms to be implemented. Chen[8] proved some theorems and proposed a fuzzy assignment model that considers all individuals to have same skills. Wang[16] solved a similar model by graph theory. Dubois and Fortemps[3] surveys refinements of the ordering of solutions supplied by the max–min formulation, namely the discrimin partial ordering and the leximin complete preordering. They have given a general algorithm which computes all maximal solutions in the sense of these relations. Different kinds of fuzzy transportation problems are solved in the papers [6,9,12,13,14].

Dominance of fuzzy numbers can be explained by many ranking methods [1,4,10,11,15]. Of these, Yager's ranking method [11] is a robust ranking technique which satisfies the properties of compensation, linearity and additivity. We have applied Yager's ranking technique [11] in this paper.

Preliminaries on Fuzzy Numbers

Zadeh [18] in 1965 first introduced Fuzzy set as a mathematical way of representing impreciseness or vagueness in everyday life.

Fuzzy set: A fuzzy set is characterized by a membership function mapping the elements of a domain, space, or universe of discourse X to the unit interval [0, 1]. (Zadeh 1965).

A fuzzy set *A* in a universe of discourse X is defined as the following set of pairs:

$A = \{(x, \mu_A(x) : x \in X\}$. Here $\mu_A: X \longrightarrow [0, 1]$ is a mapping called the membership function of the fuzzy set *A* and $\mu_A(x)$ is called the membership value or degree of membership of $x \in X$ in the fuzzy set *A*. Larger the value of $\mu_A(x)$, stronger is the grade of membership form in *A*. These membership grades are often represented by real numbers ranging from a minimum of 0 to a maximum of 1 [5,17].

Normal fuzzy set: A fuzzy set *A* of the universe of discourse X is called a normal fuzzy set implying that there exist at least one $x \in X$ such that $\mu_A(x) = 1$.

Convex fuzzy set: The fuzzy set *A* is convex if and only if, for any $x_1, x_2 \in X$, the membership function of *A* satisfies the inequality

$\mu_A\{\lambda x_1 + (1-\lambda) x_2\} \geq \min\{\mu_A(x_1), \mu_A(x_2)\}, 0 \leq \lambda \leq 1$

For a trapezoidal fuzzy number $A(x)$, it can be represented by $A(a,b,c,d;1)$ with membership function $\mu(x)$ given by

$$\mu(x) = \begin{cases} \dfrac{(x-a)}{(b-a)}, & a \leq x \leq b \\ 1, & b \leq x \leq c \\ \dfrac{(d-x)}{(d-c)}, & c \leq x \leq d \\ 0, & \text{otherwise.} \end{cases}$$

satisfying the following conditions.

(1) $\mu(x)$ is a continuous mapping from R to the closed interval [0,1]

(2) $\mu(x) = 0$ for all $x \in (-\infty, a]$;

(3) Strictly increasing and continuous on [a,b]

(4) $\mu(x) = 1$ for all $x \in [b, c]$;

(5) Strictly decreasing and continuous on [c,d]

(6) $\mu(x) = 0$ for all $x \in [d, \infty)$;

The graphic representation of a trapezoidal fuzzy number is shown in Fig. 1.

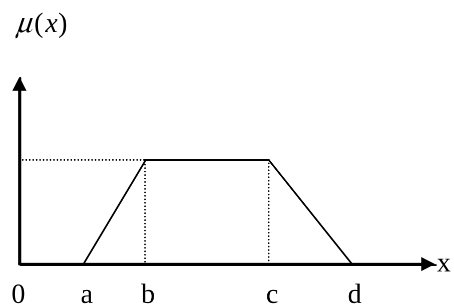

Figure 1: Graphical representation of trapezoidal fuzzy number (a, b, c, d;1).

For a triangular fuzzy number $A(x)$, it can be represented by $A(a,b,c;1)$ with membership function $\mu(x)$ given by

$$\mu(x)=\begin{cases} \dfrac{(x-a)}{(b-a)}, & a \le x \le b \\ 1 \quad , & x=b \\ \dfrac{(c-x)}{(c-b)}, & c \le x \le d \\ 0 \quad , & \text{otherwise.} \end{cases}$$

satisfying the following conditions.

(1) $\mu(x)$ is a continuous mapping from R to the closed interval [0,1]

(2) $\mu(x)=0$ for all $x \in (-\infty, a]$;

(3) Strictly increasing and continuous on [a,b]

(4) $\mu(x)=1$ at x=b;

(5) Strictly decreasing and continuous on [b,c]

(6) $\mu(x)=0$ for all $x \in [c,\infty)$;

The graphic representation of a triangular fuzzy number is shown in fig. 2.

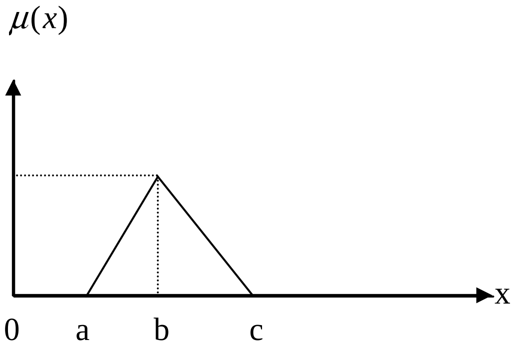

Figure 2: Graphical representation of triangular fuzzy number (a, b, c; 1).

$\alpha - cut$ of a trapezoidal fuzzy number: The $\alpha - cut$ of a fuzzy number $A(x)$ is defined as

$A(\alpha)=\{x : \mu(x) \ge \alpha\}$ $\alpha \in [0,1]$

Addition of two triangular fuzzy numbers can be performed as

$(a_1,b_1,c_1)+(a_2,b_2,c_2)=(a_1+a_2,b_1+b_2,c_1+c_2)$

Addition of two trapezoidal fuzzy numbers can be performed as

$(a_1,b_1,c_1,d_1)+(a_2,b_2,c_2,d_2)=(a_1+a_2,b_1+b_2,c_1+c_2,d_1+d_2)$

The Proposed Method

The assignment problem can be stated in the form of nxn cost matrix [cij] of real numbers as given in the following table:

Jobs

Persons

	1	2	3	---j---	N
1	c_{11}	c_{12}	c_{13}	--c_{1j}--	c_{1n}
2	c_{21}	c_{22}	c_{23}	--c_{2j}--	c_{2n}
- - i -	- - c_{i1} -	- - c_{i2} -	- - c_{i3} -	- - --c_{ij}— -	- - c_{in} -
N	c_{n1}	c_{n2}	c_{n3}	c_{n4}	c_{nn}

Mathematically assignment problem can be stated as

Minimize $z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} x_{ij}$ i=1,2,…………,n ; j= 1,2,…………,n

Subject to

$$\sum_{j=1}^{n} x_{ij} = 1, \qquad i = 1, 2, \ldots\ldots, n \tag{1}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \qquad j = 1,2,\ldots\ldots\ldots,n$$

$$x_{ij} \in \{0,1\}, \text{ where } \quad x_{ij} = \begin{cases} 1\text{ , if the i}^{\text{th}}\text{ person is assigned the j}^{\text{th}}\text{ job} \\ 0\text{ , otherwise} \end{cases}$$

is the decision variable denoting the assignment of the person I to job j. c_{ij} is the cost of assigning the j^{th} job to the i^{th} person. The objective is to minimize the total cost of assigning all the jobs to the available persons. (one job to one person).

When the costs or time $\tilde{c}_{ij}$ are fuzzy numbers, then the total cost becomes a fuzzy number.

$\tilde{z} = \sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{c}_{ij} x_{ij}$ Hence it cannot be minimized directly. For solving the problem we defuzzify the fuzzy cost coefficients into crisp ones by a fuzzy number ranking method.

Yager's ranking technique[11] is a robust ranking technique which satisfies compensation, linearity, and additivity properties and provides results which are consistent with human intuition. Given a convex fuzzy number $\tilde{c}$, the Yager's Ranking Index is defined by

$Y(\tilde{c}) = \int_0^1 0.5(c_\alpha^L + c_\alpha^U)d\alpha$, where (c_α^L, c_α^U) is the α- level cut of the fuzzy number $\tilde{c}$.

In this paper we use this method for ranking the objective values. The Yager's ranking index $Y(\tilde{c})$ gives the representative value of the fuzzy number $\tilde{c}$. It satisfies the linearity and additivity property:

If $\tilde{A} = a\tilde{B} + b\tilde{C}$ and $\tilde{D} = k\tilde{E} - t\tilde{F}$, where a, b, k, t are constants, then we have

$Y(\tilde{A}) = aY(\tilde{B}) + bY(\tilde{C})$ and $Y(\tilde{D}) = kY(\tilde{E}) - tY(\tilde{F})$. On the basis of this property the fuzzy assignment problem can be transformed into a crisp assignment problem in the LPP from. The ranking technique of Yager is :

If $Y(\tilde{U}) \le Y(\tilde{V})$, then $\tilde{U} \le \tilde{V}$ i. e. $\min\{\{\tilde{U}, \tilde{V}\} = \tilde{U}$

For the assignment problem (1), with fuzzy objective function min $\tilde{z}$ = $\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{c}_{ij}x_{ij}$ we apply Yager's ranking method [11] (using the linearity and additive property) to get the minimum objective value $\tilde{z}^*$ from the formulation

$$Y(\tilde{z}^*) = \text{minimize z} = \sum_{i=1}^{n}\sum_{j=1}^{n} Y(\tilde{c}_{ij})x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = 1 , \qquad i = 1, 2, \ldots\ldots\ldots, n \tag{2}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \qquad j = 1,2,\ldots\ldots\ldots\ldots,n$$

$$x_{ij} \in \{0,1\}, \text{ where } \quad x_{ij} = \begin{cases} 1 \text{ , if the } i^{th} \text{ person is assigned the } j^{th} \text{ job} \\ 0 \text{ , otherwise} \end{cases}$$

is the decision variable denoting the assignment of the person i to job j. $\tilde{c}_{ij}$ is the cost of assigning the j^{th} job to the i^{th} person. The objective is to minimize the total cost of assigning all the jobs to the available persons. (one job to one person).

Since $Y(\tilde{c}_{ij})$ are crisp values, this problem (2) is obviously the crisp assignment problem of the form (1) which can be solved by the conventional methods, namely the Hungarian Method or the Simplex method to solve the LPP form of the problem. Once the optimal solution x^* of Model (2) is found, the optimal fuzzy objective value $\tilde{z}^*$ of the original problem can be calculated as

$$\tilde{z}^* = \sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{c}_{ij}x_{ij}^* .$$

Numerical Example

Exampl1: Let us consider a Fuzzy Assignment Problem with rows representing 4 persons A,B, C, D and columns representing the 4 jobs Job1, Job2, Job3 and Job4. The cost matrix $[\tilde{c}_{ij}]$ is given whose elements are trapezoidal fuzzy numbers. The problem is to find the optimal assignment so that the total cost of job assignment becomes minimum

$$[\tilde{c}_{ij}] = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} (3,5,6,7) & (5,8,11,12) & (9,10,11,15) & (5,8,10,11) \\ (7,8,10,11) & (3,5,6,7) & (6,8,10,12) & (5,8,9,10) \\ (2,4,5,6) & (5,7,10,11) & (8,11,13,15) & (4,6,7,10) \\ (6,8,10,12) & (2,5,6,7) & (5,7,10,11) & (2,4,5,7) \end{pmatrix} \end{array}$$

Solution: In conformation to Model (2) the fuzzy assignment problem can be formulated in the following mathematical programming form

$$\begin{aligned} &\text{Min}[\, Y(3,5,6,7)\,x_{11} + Y(5,8,11,12)\,x_{12} + Y(9,10,11,15)\,x_{13} + Y(5,8,10,11)\,x_{14} \\ &+ Y(7,8,10,11)\,x_{21} + Y(3,5,6,7)\,x_{22} + Y(6,8,10,12)\,x_{23} + Y(5,8,9,10)\,x_{24} \\ &+ Y(2,4,5,6)\,x_{31} + Y(5,7,10,11)\,x_{32} + Y(8,11,13,15)\,x_{33} + Y(4,6,7,10)\,x_{34} \\ &+ Y(6,8,10,12)\,x_{41} + Y(2,5,6,7)\,x_{42} + Y(5,7,10,11)\,x_{43} + Y(2,4,5,7)\,x_{44}\,] \end{aligned}$$

$$\begin{aligned} \text{s.t.}\quad & x_{11} + x_{12} + x_{13} + x_{14} = 1, && x_{11} + x_{21} + x_{31} + x_{41} = 1, \\ & x_{21} + x_{22} + x_{23} + x_{24} = 1, && x_{12} + x_{22} + x_{32} + x_{42} = 1, \\ & x_{31} + x_{32} + x_{33} + x_{34} = 1, && x_{13} + x_{23} + x_{33} + x_{43} = 1, \\ & x_{41} + x_{42} + x_{43} + x_{44} = 1, && x_{14} + x_{24} + x_{34} + x_{44} = 1, \\ & x_{ij} \in \{0,1\} \end{aligned} \tag{3}$$

Now we calculate Y(3,5,6,7) by applying the Yager's Ranking Method. The membership function of the trapezoidal number (3,5,6,7) is

$$\mu(x) = \begin{cases} \dfrac{(x-3)}{2}, & 3 \le x \le 5 \\ 1, & 5 \le x \le 6 \\ \dfrac{(7-x)}{1}, & 6 \le x \le 7 \\ 0, & \text{otherwise.} \end{cases}$$

The $\alpha - cut$ of the fuzzy number (3,5,6,7) is $(c_\alpha^L, c_\alpha^U) = (2\alpha + 3, 7 - \alpha)$ for which

$$Y(\tilde{c}_{11}) = Y(3,5,6,7) = \int_0^1 0.5(c_\alpha^L + c_\alpha^U)\,d\alpha = \int_0^1 0.5(2\alpha + 3 + 7 - \alpha)\,d\alpha = \int_0^1 0.5(\alpha + 10)\,d\alpha = 5.25$$

Proceeding similarly, the Yager's ranking indices for the fuzzy costs $\tilde{c}_{ij}$ are calculated as:

$Y(\tilde{c}_{12}) = 9$, $Y(\tilde{c}_{13}) = 11.25$, $Y(\tilde{c}_{14}) = 8.5$, $Y(\tilde{c}_{21}) = 9$, $Y(\tilde{c}_{22}) = 5.25$, $Y(\tilde{c}_{23}) = 9$, $Y(\tilde{c}_{24}) = 8$, $Y(\tilde{c}_{31}) = 4.25$, $Y(\tilde{c}_{32}) = 8.25$, $Y(\tilde{c}_{33}) = 11.75$, $Y(\tilde{c}_{34}) = 6.75$, $Y(\tilde{c}_{41}) = 9$, $Y(\tilde{c}_{42}) = 5$, $Y(\tilde{c}_{43}) = 8.25$, $Y(\tilde{c}_{44}) = 4.5$.

We replace these values for their corresponding $\tilde{c}_{ij}$ in (3), which results in a conventional assignment problem in the LPP form. We solve it by using LINGO 9.0 to get the following optimal solution

$x_{13}^* = x_{22}^* = x_{31}^* = x_{44}^* = 1,$

$x_{11}^* = x_{12}^* = x_{14}^* = x_{21}^* = x_{23}^* = x_{24}^* = x_{32}^* = x_{33}^* = x_{34}^* = x_{41}^* = x_{42}^* = x_{43}^* = 0,$

with the optimal objective value $Y(\tilde{z}^*) = 25.25$ which represents the optimal total cost. In other words the optimal assignment is

$A \longrightarrow 3$, $B \longrightarrow 2$, $C \longrightarrow 1$, $D \longrightarrow 4$.

The fuzzy optimal total cost is calculated as

$$\tilde{c}_{13} + \tilde{c}_{22} + \tilde{c}_{31} + \tilde{c}_{44} = (9,10,11,15) + (3,5,6,7) + (2,4,5,6) + (2,4,5,7)$$
$$= (16, 23, 27, 35).$$

Also we find that $Y(\tilde{z}^*) = Y(16,23,27,35) = 25.25$

Example2: Let us consider a fuzzy Assignment Problem with rows representing 3 persons A,B, C and columns representing the 3 jobs Job1, Job2 and Job3. The cost matrix $[\tilde{c}_{ij}]$ is given whose elements are triangular or trapezoidal fuzzy numbers. The problem is to find the optimal assignment so that the total cost of job assignment becomes minimum.

$$[\tilde{c}_{ij}] = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{pmatrix} (2,4,6) & (7,8,10,12) & (4,5,6,8) \\ (8,10,12,13) & (10,13,15) & (5,7,10) \\ (2,3,5,6) & (6,7,9,10) & (9,11,12) \end{pmatrix} \end{array}$$

Solution: In conformation to Model (2) the fuzzy assignment problem can be formulated in the following mathematical programming form

Min[$Y(2,4,6)x_{11} + Y(7,8,10,12)x_{12} + Y(4,5,6,8)x_{13} + Y(8,10,12,13)x_{21} + Y(10,13,15)x_{22} + Y(5,7,10)x_{23} + Y(2,3,5,6)x_{31} + Y(6,7,9,10)x_{32} + Y(9,11,12)x_{33}$]

Subject to

$x_{11} + x_{12} + x_{13} = 1$, $\quad x_{11} + x_{21} + x_{31} = 1$,

$x_{21} + x_{22} + x_{23} = 1$, $x_{12} + x_{22} + x_{32} = 1$,

$x_{31} + x_{32} + x_{33} = 1$, $x_{13} + x_{23} + x_{33} = 1$,

$x_{ij} \in \{0,1\}$

Using Yager's Ranking Method, the above problem can be transformed into the crisp version of the assignment problem in the LPP form:

Minimize z = $Y(\tilde{z}^*)$ = Min [$4x_{11} + 9.25x_{12} + 5.25x_{13} + 10.75x_{21} + 12.75x_{22} + 7.25x_{23} + 4x_{31} + 8x_{32} + 10.75x_{33}$]

Subject to

$x_{11} + x_{12} + x_{13} = 1$, $x_{11} + x_{21} + x_{31} = 1$,

$x_{21} + x_{22} + x_{23} = 1$, $x_{12} + x_{22} + x_{32} = 1$,

$x_{31} + x_{32} + x_{33} = 1$, $x_{13} + x_{23} + x_{33} = 1$,

$x_{ij} \in \{0,1\}$

This problem is solved by LINGO 9.0. The optimal solution is

$x_{11}^* = x_{23}^* = x_{32}^* = 1$,

$x_{12}^* = x_{13}^* = x_{21}^* = x_{22}^* = x_{31}^* = x_{33}^* = 0$,

with the optimal objective value $Y(\tilde{z}^*)$ = 19.25 which represents the optimal total cost. In other words the optimal assignment is

A $\longrightarrow$ 1 , B $\longrightarrow$ 3 , C $\longrightarrow$ 2.

The fuzzy optimal total cost is calculated as

$\tilde{c}_{11} + \tilde{c}_{23} + \tilde{c}_{32}$ = (2,4,6) + (5,7,10) + (6,7,9,10)

= (2,4,4,6) + (5,7,7,10) + (6,7,9,10)

= (13, 18, 20, 26).

Also we find that $Y(\tilde{z}^*) = Y(13,18,20,26) = 19.25$

In the above examples it has been shown that the total optimal cost obtained by our method remains same as that obtained by defuzzifying the total fuzzy optimal cost by applying the Yager's ranking method.[11].

Conclusions

In this paper, the assignment costs are considered as imprecise numbers described by fuzzy numbers which are more realistic and general in nature. Moreover, the fuzzy assignment problem has been transformed into crisp assignment problem using Yager's ranking indices [11]. Numerical examples show that by this method we can have the optimal assignment as well as the crisp and fuzzy optimal total cost. By using Yager's [11] ranking indices we have shown that the total cost obtained is optimal. Moreover, one can conclude that the solution of fuzzy problems (for more realistic sense) can be obtained by Yager's ranking method effectively. This technique can also be tried in solving other types of problems like, project schedules, transportation problems and network flow problems.

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