Fundamentals of fuzzy sets and fuzzy logic

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Introduction

1. A new theory, its applications and modeling power

A new theory extending our capabilities in modeling uncertainty

Fuzzy set theory provides a major newer paradigm in modeling and reasoning with uncertainty. Though there were several forerunners in science and philosophy, in particular in the areas of multivalued logics and vague concepts, Lotfi A. Zadeh, a professor at University of California at Berkeley was the first to propose a theory of fuzzy sets and an associated logic, namely fuzzy logic (Zadeh, 1965). Essentially, a fuzzy set is a set whose members may have degrees of membership between 0 and 1, as opposed to classical sets where each element must have either 0 or 1 as the membership degree—if 0, the element is completely outside the set; if 1, the element is completely in the set. As classical logic is based on classical set theory, fuzzy logic is based on fuzzy set theory.

Major industrial application areas

The first wave: Process control

The first industrial application of fuzzy logic was in the area of fuzzy controllers. It was done by two Danish civil engineers, L.P. Holmblad and J.J. Østergaard, who around 1980 at the company F.L. Schmidt developed a fuzzy controller for cement kilns. Their results were published in 1982 (Holmblad & Østergaard, 1982). Their results were not much notice in the West, but they certainly were in Japan. The Japanese caught the idea, and applied it in an automatic-drive fuzzy control system for subway trains in Sendai City. The final product was extremely successful, and was generally praised as superior to other comparable systems based on classical control. This success encouraged a rapid increase in the Japanese's interest in fuzzy controller during the late 1980s. This led to applications in other areas, like elevator control systems and air conditioning systems. In the early 1990s, the Japanese began to apply fuzzy controller in consumer products, like camcorders, washing machines, vacuum cleaners, and cars. The Japanese success led to increased interest in Europe and the US in fuzzy controller techniques.

The second wave: information systems

The second wave of fuzzy logic systems started in Europe in the early 1990s, namely in the area of information systems, in particular in databases and information retrieval. The first fuzzy logic based search engine was developed by the author in collaboration with professor R.R. Yager, Machine Intelligence Institute, US. It was aimed for application netbased commerce systems, namely, at that time the only in the world, the French Minitel. It was first demonstrated to the public at the Joint International Conference of Artificial Intelligence in 1992 in Chambery, France. In

1999, the technique was adopted by the Danish search engine Jubii. The several ideas and results applied in the technique were published; see, for instance, (Larsen & Yager, 1993, 1997). Internet and the Web gave new interest to application of fuzzy logic technology. In the net based society we have an enormous amount of information and knowledge electronic accessible for decision makers and human in general. Much of this information is inherently uncertain—lack of precision, vague concepts, more or less reliable information, etc. On the other hand, to be useful, users must be able to utilize it, despite the uncertainties. Certainly imperfect information and knowledge cannot be ignored; for instance:

> "He said that his department…had commissioned a study…on the effect of requiring higher mileage cars, which would be smaller and could be less safe."
> (New York Times, February 22, 1991; source: E.H. Ruspini, AI Certer, SRI Internaitonal).

This is an example of information that is inherently imprecise or vague and therefore not well suited for representation and processing by classical binary logic or probabilistic based techniques. Much information is two valuable to be ignored, but it requires a human to identify it and utilize it. With this huge amount of information, we need to computer based tools to find it, evaluate it, extract the meaning, and partly utilize it for decision support. Here fuzzy logic is beginning and will in the future play a major role as tool allowing us to model and reason with the information and knowledge; in fact, fuzzy logic allows us to properly utilize the information in the uncertainty. A newer application area in this line is data mining (text mining, web mining, …) for discovery of knowledge. Hence, the second wave is, in particular due to the internet and the web, likely to be much greater then the first in the control area.

How fuzzy sets extends our modeling capabilities

By fuzzy set theory we can provide exact representations of concepts and relations that are vague, that is, with no sharp yes-no borderline between cases covered, and cases not covered, by the concept or relation. This allows us to represent, for instance, that a document deals with a topic T1 to some degree (between 0 and 1), that a user is interested in a topic T2 to some degree, and that a topic T3 implies a topic T4 to some degree. By fuzzy set theory and fuzzy logic, we can not only represent such knowledge, but also utilize it to its full extent, taking the kind and the form of the uncertainties into account. This does not mean than fuzzy logic renders classical logic and probability theory obsolete. On the contrary, though fuzzy sets and fuzzy logic extend membership degrees and truth values from 0 and 1 to the real interval from 0 to 1, the definition of the fuzzy logic formalism still rely on the classical logic. Further more, we apply statistics based on probability theory in fuzzy data mining of knowledge — the main difference being that probabilities now are associated to fuzzy sets. Another advantage of fuzzy logic is that it allows fast processing of large bodies of complex knowledge, since processing is performed by numerical computations and not symbolic unification as in, e.g., logic programming formalisms. As opposed to neural nets, fuzzy logic has the advantage that it supports explicit representation of knowledge, like in symbolic formalisms, allowing us to combine knowledge in a controlled way.

2. Outline of the course

The direction taken in the course

By knowledge modeling in the framework of fuzzy sets we introduce a new powerful basis for development of advanced information systems. It should be mentioned, that though lots of research has been done if fuzzy logic theory that has been much further developed since Zadeh's seminal paper from 1965, works on fuzzy logic software and knowledge engineering, and efficient algorithms for fuzzy knowledge processing, have been very limited; the latter in particular due to the fact that the well known classical algorithms assume classical binary logic. I hope with this course in fuzzy logic (that had also been referred to as Fuzzy Logic Information Technology, Fuzzy Systems, and Fuzzy Logic Engineering), to contribute to an improvement of this situation.

With respect to application frameworks, we shall in particular consider information access in the broad, including database querying, information retrieval, and object recognition—that essentially are solved through some variant of classificatory problem solving—and, further, data mining where "hidden" knowledge is retrieved from the information base, as essentially represents some form of inductive problem solving.

Though the focus in this course is fuzzy logic, we shall cover central aspects of information retrieval, database querying, and search engines, as well as knowledge representation and algorithms, as related to the engineering part. As opposed to tradition approaches in teaching these topics (courses and text books), we adopt fuzzy logic as the basic logical framework, and emphasizes algorithms for fuzzy knowledge processing.

Main plan for the course

The main structure (the red thread) in the course is intended as follows. The lectures 1–2 provide a general introduction with an outline of fundamentals of fuzzy sets and fuzzy logic. Lecture 3 covers the triangular norm aggregation operators, providing fuzzy set intersection and union operators. The lectures 4–7, we cover averaging aggregation operators, that is, the mean function in fuzzy logic. We present two families of such operators, namely OWA operators and quasi-arithmetic mean operators, and cover central aspects such as andness/orness and importance weighting. Fuzzy number arithmetic is covered in lecture 8. Lecture 9 covers fuzzy relations, with focus on binary fuzzy relations, and operations on such. Lecture 10 covers fuzzy semantic-term nets. Lecture 11 covers a number of advanced topics for flexible, intelligent information access in the framework of fuzzy logic, such as fuzzy query answering and fuzzy databases. Lecture 12 introduces a model for text mining in the framework of fuzzy logic and probability theory. Possibility theory that is built on fuzzy set theory is introduced in Lecture 13. In lecture 14 is introduced a model for situation recognition in the framework of fuzzy logic and possibility theory. Finally, lecture 15 introduces to fuzzy clustering and present the fuzzy c-means algorithm.

A note on project work in relation to the course

Since semester project work is related to the course, it is encouraged to start early think about a project theme. Start in particular early, even before a concrete project idea has been identified, to implement some of the fuzzy logic operations for a tool box. This will both give a good understanding of the operators, and be a help for the chosen project that may be aimed for developing an application system. As the operators are developed and included in the software library, be sure that they are sufficiently generic, and are well documented, and, of course, proven or tested as being correct.

Project proposals are available on the net.

References

Zadeh, L.A. (1965): Fuzzy sets, *Information and Control* 8(3):338–353.

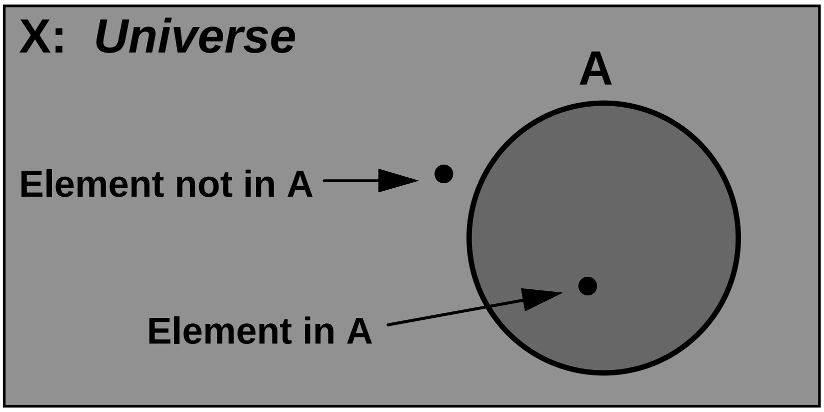
L.P. Holmblad and J.J. Østergaard (1982): Control of a cement kiln by fuzzy logic. In M.M. Gupta and E. Sanchez, Eds., *Fuzzy Information and Decision Processes*, North-Holland, New York, pp. 389–399.

Larsen, H.L., and Yager, R.R.: The use of fuzzy relational thesauri for classificatory problem solving in information retrieval and expert systems. *IEEE J. on System, Man, and Cybernetics* **23**(1):31–41, 1993.

Larsen, H.L., and Yager, R.R.: Query Fuzzification for Internet Information retrieval. In Dubois, D., Prade H., and Yager, R.R., Eds., *Fuzzy Set methods in Information Engineering: A Guided Tour of Applications*, John Wiley & Sons, pp. 291–310, 1997.

Fuzzy logic and fuzzy sets

Classical set theory

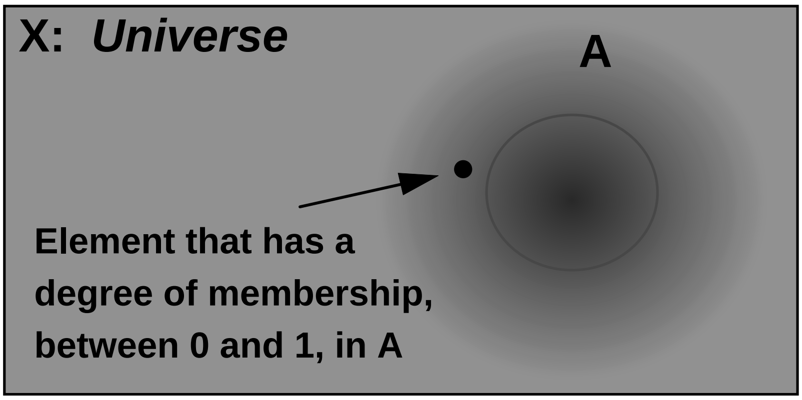


The set (concept, predicate) A is characterized by a membership function m: X $\rightarrow \{0, 1\}$

$$\mu_A(x) = \begin{cases} 1 & x \in A \\ 0 & x \notin A \end{cases}$$

Fuzzy logic & fuzzy sets

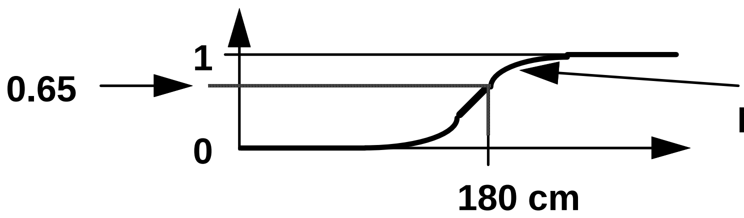
Fuzzy set theory



- A has no sharp border line. It is a fuzzy subset of X, characterized by a membership function μ: X → [0, 1]

∞ μ(x) is the element x's degree of membership in A

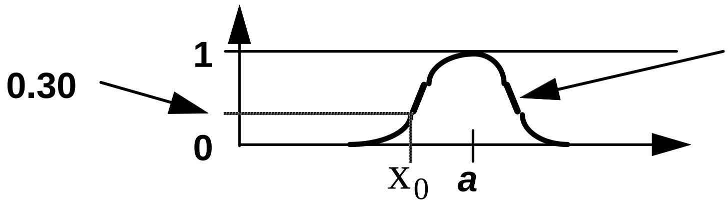
Example 1: A = tall (i.e. tall person heights)



Membership function for A

$$\mu_A(180 \text{ cm}) = 0.65$$

Example 2: A = about the value *a*



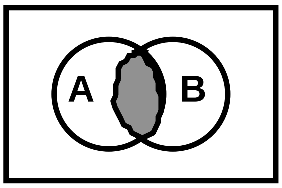
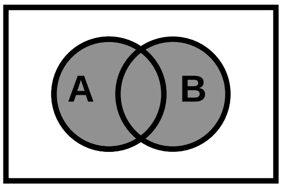
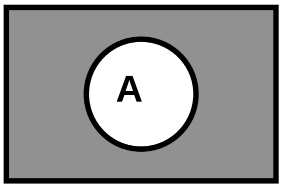
Membership function for A

$$\mu_A(x_0) = 0.30$$

Fuzzy logic & fuzzy sets

Fuzzy logic

Correspondance between set theory and logic in the fuzzy case like in the classical case

	Intersection	**Union**	**Complement**
	$A \cap B$	$A \cup B$	$\bar{A}$
			
	$\mu_{A\cap B}(x) =$	$\mu_{A\cup B}(x) =$	$\mu_{\bar{A}}(x) =$
classical	$\begin{cases} 1 & x \in A \cap B \\ 0 & x \notin A \cap B \end{cases}$	$\begin{cases} 1 & x \in A \cup B \\ 0 & x \notin A \cup B \end{cases}$	$\begin{cases} 1 & x \notin A \\ 0 & x \in A \end{cases}$
fuzzy	$\min(\mu_A(x), \mu_B(x))$	$\max(\mu_A(x), \mu_B(x))$	$1 - \mu_A(x)$
	AND	**OR**	**NOT**

Fuzzy logic
Kinds of operators

AND (intersection) operators

t-norms: $f:[0,1]\times[0,1]\rightarrow[0,1]$

Examples: min: f(a,b) = min(a,b)

prod: f(a,b) = a·b

OR (union) operators

t-conorms: $g:[0,1]\times[0,1]\rightarrow[0,1]$

Examples: max: g(a,b) = max(a,b)

alg.sum: g(a,b) = a+b - a·b

Between AND and OR

Averaging operators: $h:[0,1]^n\rightarrow[0,1]$

Examples:

arithmetic mean: $h(a_1,\ldots,a_n)=\frac{1}{n}\sum_{i=1}^{n}a_i$

We have for any set of operators f, g, and h:

$$\forall a,b\in[0,1]: f(a,b)\le h(a,b)\le g(a,b)$$

In general, since f , g are associative and commutative:

$$\forall a_1,\ldots,a_n\in[0,1]: f(a_1,\ldots,a_n)\le h(a_1,\ldots,a_n)\le g(a_1,\ldots,a_n)$$