Supporting Fuzzy Rough Sets in Fuzzy Description Logics

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Introduction

- In the last years the interest in ontologies has significantly grown
- An ontology is defined as an explicit and formal specification of a shared conceptualization
- Description Logics (DLs) are a family of logics for representing structured knowledge
- They are the basis of most of the ontology languages, such as the current standard language OWL [HPS04].

- It is widely agreed that “classical” ontology languages are not appropriate to deal with *fuzzy/vague knowledge*
- With the aim of managing vagueness in ontologies, several extension of DLs have been proposed
- They may be grouped in two categories
  - Combination with fuzzy logic: *fuzzy DLs* [Str08].
    - Vagueness is quantified and expressed using a degree of membership to a vague concept
  - Combination with rough set theory [Paw82]: *rough DLs*
    - Vague concepts are approximated by means of a couple of classical sets: an upper and a lower approximation

- Fuzzy logic and rough logic are complementary formalism to manage vagueness
- Hence, it is natural to combine them by means of *fuzzy rough sets* [DP90, RK02]
- Application in, e.g., in medicine we may combine
  - Rough concepts such as “possible patient”
    - An individual affected by some of the symptoms of some disease, and hence suspected of being patient
  - With fuzzy concepts such as “high blood pressure”

Preliminaries: Mathematical Fuzzy Logic [Háj98]

- Fuzzy statements: $\phi \geqslant l$ or $\phi \leqslant u$, where $l, u \in [0,1]$ and ϕ is a statement
  - The degree of truth of ϕ is *at least* l, resp. *at most* u
- Fuzzy interpretation: $\mathcal{I} : Atoms \to [0,1]$ and is then extended inductively:

$$
\begin{array}{ll}
\mathcal{I}(\phi \wedge \psi) = \mathcal{I}(\phi) \otimes \mathcal{I}(\psi) & \mathcal{I}(\phi \vee \psi) = \mathcal{I}(\phi) \oplus \mathcal{I}(\psi), \\
\mathcal{I}(\phi \to \psi) = \mathcal{I}(\phi) \Rightarrow \mathcal{I}(\psi) & \mathcal{I}(\neg\phi) = \ominus \mathcal{I}(\phi), \\
\mathcal{I}(\exists x.\phi(x)) = \sup_{c \in \Delta^{\mathcal{I}}} \mathcal{I}(\phi(c)) & \mathcal{I}(\forall x.\phi(x)) = \inf_{c \in \Delta^{\mathcal{I}}} \mathcal{I}(\phi(c))
\end{array}
$$

$\otimes$, $\oplus$, $\Rightarrow$, and $\ominus$ are *truth combination functions*

	Łukasiewicz Logic	Gödel Logic	Product Logic	"Zadeh Logic"
$a \otimes b$	$\max(a+b-1, 0)$	$\min(a, b)$	$a \cdot b$	$\min(a, b)$
$a \oplus b$	$\min(a+b, 1)$	$\max(a, b)$	$a + b - a \cdot b$	$\max(a, b)$
$a \Rightarrow b$	$\min(1-a+b, 1)$	$\begin{cases} 1 & \text{if } a \leqslant b \\ b & \text{otherwise} \end{cases}$	$\min(1, b/a)$	$\max(1-a, b)$
$\ominus a$	$1 - a$	$\begin{cases} 1 & \text{if } a = 0 \\ 0 & \text{otherwise} \end{cases}$	$\begin{cases} 1 & \text{if } a = 0 \\ 0 & \text{otherwise} \end{cases}$	$1 - a$

- The degree of subsumption between A and B is $\inf_{x \in X} A(x) \Rightarrow B(x)$
- The inverse of R is $R^{-1} \colon Y \times X \to [0,1]$ with $R^{-1}(y,x) = R(x,y)$
- The composition of $R_1 : X \times Y \to [0,1]$ and $R_2 \colon Y \times Z \to [0,1]$ is $(R_1 \circ R_2)(x,z) = \sup_{y \in Y} R_1(x,y) \otimes R_2(y,z)$
- A fuzzy relation R is reflexive iff $\forall x \in X, R(x,x) = 1$
- R is symmetric iff $\forall x \in X, y \in Y, R(x,y) = R(y,x)$
- R is transitive iff $R(x,z) \geqslant (R \circ R)(x,z)$
- A fuzzy similarity relation is a reflexive, symmetric and transitive

- $\mathcal{I} \models \phi \geqslant l$ iff $\mathcal{I}(\phi) \geqslant l$. $\mathcal{I} \models \phi \leqslant u$ iff $\mathcal{I}(\phi) \leqslant u$
- The notions of satisfiability and logical consequence are defined in the standard way
- $\phi \geqslant l$ is a tight logical consequence of a set of fuzzy statements $\mathcal{K}$ iff $l = \sup\{r \mid \mathcal{K} \models \phi \geqslant r\}$

Preliminaries: Rough Set and Fuzzy Rough Set Theory [Paw82, DP90]

- Key idea: approximation of a vague concept by means of a pair a concepts
  - a sub-concept or lower approximation
    - describing the sets of elements which definitely belong to the vague set
  - a super-concept or upper approximation
    - describing the sets of elements which possibly belong to the vague set

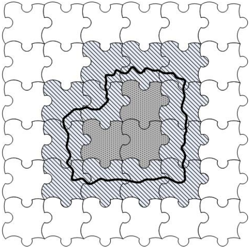

Figure: Vague concept (bold line), upper approximation (striped line) and lower approximation (dotted line)

- Approximation is based on an equivalence relation between elements of the domain
- Crisp Case: given an equivalence relation R
  - Upper approximation of set S:

$$\overline{S} = \{x \mid \exists y.(x, y) \in R \wedge y \in S\}$$

  - Lower approximation of set S:

$$\underline{S} = \{x \mid \forall y.(x, y) \in R \rightarrow y \in S\}$$

- Fuzzy Rough Sets: given fuzzy similarity relation R, t-norm $\otimes$ and an implication function $\Rightarrow$
  - Upper approximation of a fuzzy set S: for all $x \in X$,

$$\overline{S}(x) = \sup_{y \in \Delta^{\mathcal{I}}} \{R(x, y) \otimes S(y)\}$$

  - Lower approximation is defined as: for all $x \in X$,

$$\underline{S}(x) = \inf_{y \in \Delta^{\mathcal{I}}} \{R(x, y) \Rightarrow S(y)\}$$

Fuzzy Rough DLs

- We extend Fuzzy Description Logics
- Fuzzy Concepts may be Upper and Lover Approximated

Description Logics (DLs)

- The logics behind OWL-DL and OWL-Lite, `http://dl.kr.org/`.
- Concept/Class: names are equivalent to unary predicates
  - In general, concepts equiv to formulae with one free variable
- Role or attribute: names are equivalent to binary predicates
  - In general, roles equiv to formulae with two free variables
- Taxonomy: Concept and role hierarchies can be expressed
- Individual: names are equivalent to constants
- Operators: restricted so that:
  - Language is decidable and, if possible, of low complexity
  - No need for explicit use of variables
    - Restricted form of $\exists$ and $\forall$
  - Features such as counting can be succinctly expressed

The Crisp DL Family

- A given DL is defined by set of concept and role forming operators
- Basic language: $\mathcal{ALC}$($\mathcal{A}$ttributive $\mathcal{L}$anguage with $\mathcal{C}$omplement)

Syntax				Semantics	Example
C, D	$\rightarrow$	$\top$	|	$\top(x)$	
		$\bot$	|	$\bot(x)$	
		A	|	$A(x)$	$Human$
		$C \sqcap D$	|	$C(x) \wedge D(x)$	$Human \sqcap Male$
		$C \sqcup D$	|	$C(x) \vee D(x)$	$Nice \sqcup Rich$
		$\neg C$	|	$\neg C(x)$	$\neg Meat$
		$\exists R.C$	|	$\exists y.R(x,y) \wedge C(y)$	$\exists has\_child.Blond$
		$\forall R.C$		$\forall y.R(x,y) \rightarrow C(y)$	$\forall has\_child.Human$
$C \sqsubseteq D$				$\forall x.C(x) \rightarrow D(x)$	$Happy\_Father \sqsubseteq Man \sqcap \exists has\_child.Female$
$a{:}C$				$C(a)$	$John{:}Happy\_Father$

Toy Example

$$
\begin{array}{rcl}
Sex & = & Male \sqcup Female \\
Male \sqcap Female & \sqsubseteq & \bot \\
Person & \sqsubseteq & Human \sqcap \exists hasSex.Sex \\
MalePerson & \sqsubseteq & Person \sqcap \exists hasSex.Male
\end{array}
$$

$$umberto : Person \sqcap \exists hasSex.\neg Female$$

$$KB \models umberto : MalePerson$$

Note on DL Naming

$\mathcal{AL}$: $C, D \longrightarrow \top \mid \bot \mid A \mid C \sqcap D \mid \neg A \mid \exists R.\top \mid \forall R.C$

$\mathcal{C}$: Concept negation, $\neg C$. Thus, $\mathcal{ALC} = \mathcal{AL} + \mathcal{C}$

$\mathcal{S}$: Used for $\mathcal{ALC}$ with transitive roles $\mathcal{R}_+$

$\mathcal{U}$: Concept disjunction, $C_1 \sqcup C_2$

$\mathcal{E}$: Existential quantification, $\exists R.C$

$\mathcal{H}$: Role inclusion axioms, $R_1 \sqsubseteq R_2$, *e.g.*, $\mathit{is\_component\_of} \sqsubseteq \mathit{is\_part\_of}$

$\mathcal{N}$: Number restrictions, $(\geqslant n\ R)$ and $(\leqslant n\ R)$, *e.g.*, $(\geqslant 3\ \mathit{has\_Child})$ (has at least 3 children)

$\mathcal{Q}$: Qualified number restrictions, $(\geqslant n\ R.C)$ and $(\leqslant n\ R.C)$, *e.g.*, $(\leqslant 2\ \mathit{has\_Child}.\mathit{Adult})$ (has at most 2 adult children)

$\mathcal{O}$: Nominals (singleton class), $\{a\}$, *e.g.*, $\exists \mathit{has\_child}.\{\mathit{mary}\}$.
Note: $a{:}C$ equiv to $\{a\} \sqsubseteq C$ and $(a, b){:}R$ equiv to $\{a\} \sqsubseteq \exists R.\{b\}$

$\mathcal{I}$: Inverse role, R^-, *e.g.*, $\mathit{isPartOf} = \mathit{hasPart}^-$

$\mathcal{F}$: Functional role, f, *e.g.*, $\mathit{functional}(\mathit{hasAge})$

$\mathcal{R}_+$: transitive role, *e.g.*, $\mathit{transitive}(\mathit{isPartOf})$

$\mathcal{R}$: role inclusions with composition, $R_1 \circ R_2 \sqsubseteq S$, *e.g.*, $\mathit{isPartOf} \circ \mathit{isPartOf} \sqsubseteq \mathit{isPartOf}$

For instance,

$$\begin{array}{lcll}
\mathcal{SHIF} & = & \mathcal{S}+\mathcal{H}+\mathcal{I}+\mathcal{F} = \mathcal{ALCR_+HIF} & \text{OWL-Lite} \\
\mathcal{SHOIN} & = & \mathcal{S}+\mathcal{H}+\mathcal{O}+\mathcal{I}+\mathcal{N} = \mathcal{ALCR_+HOIN} & \text{OWL-DL} \\
\mathcal{SROIQ} & = & \mathcal{S}+\mathcal{R}+\mathcal{O}+\mathcal{I}+\mathcal{Q} = \mathcal{ALCR_+ROIN} & \text{OWL 2}
\end{array}$$

Concrete Domains

- Concrete domains: reals, integers, strings, ...

  $(tim, 14) : hasAge$
  $(sf, \text{``}SoftComputing\text{''}) : hasAcronym$
  $(source1, \text{``}ComputerScience\text{''}) : isAbout$
  $(service2, \text{``}InformationRetrievalTool''\,) : Matches$
  $Minor = Person \sqcap \exists hasAge. \leqslant_{18}$

- Semantics: a clean separation between “object” classes and concrete domains
  - $D = \langle \Delta_\beta, \Phi_\beta \rangle$
  - Δ_β is an interpretation domain
  - Φ_β is the set of concrete domain predicates d with a predefined arity n and fixed interpretation $d^\beta \subseteq \Delta_\beta^n$
  - Concrete properties: $R^\mathcal{I} \subseteq \Delta^\mathcal{I} \times \Delta_D$
- Notation: (D). E.g., $\mathcal{ALC}(D)$ is $\mathcal{ALC}$ + concrete domains

Fuzzy DLs Basics

The semantics is an immediate consequence of applying mathematical fuzzy logic to the First-Order-Logic translation of DLs expressions

Interpretation:

$$\begin{array}{lcl} \mathcal{I} & = & \Delta^{\mathcal{I}} \\ C^{\mathcal{I}} & : & \Delta^{\mathcal{I}} \to [0,1] \\ R^{\mathcal{I}} & : & \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \to [0,1] \end{array} \qquad \begin{array}{lcl} \otimes & = & \text{t-norm} \\ \oplus & = & \text{s-norm} \\ \ominus & = & \text{negation} \\ \Rightarrow & = & \text{implication} \end{array}$$

Concepts:

Syntax		Semantics		
$C, D \longrightarrow$	$\top \mid$	$\top^{\mathcal{I}}(x)$	$=$	1
	$\bot \mid$	$\bot^{\mathcal{I}}(x)$	$=$	0
	$A \mid$	$A^{\mathcal{I}}(x)$	$\in$	$[0,1]$
	$C \sqcap D \mid$	$(C_1 \sqcap C_2)^{\mathcal{I}}(x)$	$=$	$C_1{}^{\mathcal{I}}(x) \otimes C_2{}^{\mathcal{I}}(x)$
	$C \sqcup D \mid$	$(C_1 \sqcup C_2)^{\mathcal{I}}(x)$	$=$	$C_1{}^{\mathcal{I}}(x) \oplus C_2{}^{\mathcal{I}}(x)$
	$\neg C \mid$	$(\neg C)^{\mathcal{I}}(x)$	$=$	$\ominus C^{\mathcal{I}}(x)$
	$\exists R.C \mid$	$(\exists R.C)^{\mathcal{I}}(x)$	$=$	$\sup_{y \in \Delta^{\mathcal{I}}} R^{\mathcal{I}}(x,y) \otimes C^{\mathcal{I}}(y)$
	$\forall R.C$	$(\forall R.C)^{\mathcal{I}}(u)$	$=$	$\inf_{y \in \Delta^{\mathcal{I}}} R^{\mathcal{I}}(x,y) \Rightarrow C^{\mathcal{I}}(y)\}$

Assertions: $\langle a{:}C, r\rangle$, $\mathcal{I} \models \langle a{:}C, r\rangle$ iff $C^{\mathcal{I}}(a^{\mathcal{I}}) \geqslant r$ (similarly for roles)

- individual a is instance of concept C at least to degree r, $r \in [0,1] \cap \mathbb{Q}$

Inclusion axioms: $\langle C \sqsubseteq D, r\rangle$,

- $\mathcal{I} \models \langle C \sqsubseteq D, r\rangle$ iff $\inf_{x \in \Delta^{\mathcal{I}}} C^{\mathcal{I}}(x) \Rightarrow D^{\mathcal{I}}(x) \geqslant r$

Fuzzy DL: Specific Constructs

- Concrete data types
  - *e.g.*, $Sedan \sqcap (\geqslant price\ 22.000)$
- Fuzzy constraints
  - numerical features may be constrained by so-called fuzzy membership functions

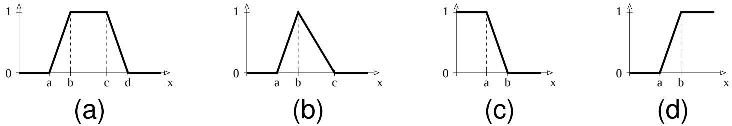

Figure: (a) Trapezoidal function $trz(a, b, c, d)$, (b) triangular function $tri(a, b, c)$, (c) left shoulder function $ls(a, b)$, and (d) right shoulder function $rs(a, b)$.

  - For instance, $(\exists price.ls(22000, 26000))$ dictates that given a price it returns the degree to which the constraint is satisfied

Definition (Specific Concept Expressions)

As for $\mathcal{SHIF}$

+

$$\begin{array}{lcl} C & \to & DR \quad \text{(datatype restriction)} \\ DR & \to & (\geqslant\ t\ val) \mid (\leqslant\ t\ val) \mid (=\ t\ val) \end{array}$$

e.g. $Sedan \sqcap (\leqslant\ price\ 26.000)$

+

$$\begin{array}{lcl} C & \to & \forall t.d \mid \exists t.d \quad \text{(fuzzy constraints)} \\ d & \to & ls(a,b) \mid rs(a,b) \mid tri(a,b,c) \mid trz(a,b,c,d) \end{array}$$

e.g. $Car \sqcap (\exists price.ls(22000, 26000))$

+

$$\begin{array}{lcl} C & \to & TC \quad \text{(threshold concept)} \\ TC & \to & C[\geqslant\ n] \mid C[\leqslant\ n] \end{array}$$

e.g. $(Sedan \sqcap Cheap \sqcap (\leqslant\ price\ 30.000))[\geqslant\ 0.8]$

+

$$\begin{array}{lcl} C & \to & WC \quad \text{(weighted sum concept)} \\ WC & \to & (w_1 \cdot C_1 + w_2 \cdot C_2 + \ldots + w_k \cdot C_k) \end{array}$$

where $\sum_{i=1}^{k} w_i = 1$. E.g., $0.2 \cdot (\leqslant\ price\ 30.000) + 0.8 \cdot (\exists hasColor.Red)$

+

$$C \quad \to \quad mod(C) \quad \text{(modified concept)}$$

where *mod* is a linear hedge. E.g., $SportCar \sqsubseteq Car \sqcap \exists \text{hasSpeed}.very(High)$

Definition (Specific Concept Expressions)

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$$\begin{array}{lcl} C & \rightarrow & DR \quad (\text{datatype restriction}) \\ DR & \rightarrow & (\geqslant t\ val) \mid (\leqslant t\ val) \mid (= t\ val) \end{array}$$

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C \to mod(C) \quad (\text{modified concept})
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where *mod* is a linear hedge. E.g., $SportCar \sqsubseteq Car \sqcap \exists \mathrm{hasSpeed}.very(High)$

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$$C \rightarrow mod(C) \quad \text{(modified concept)}$$

where *mod* is a linear hedge. E.g., $SportCar \sqsubseteq Car \sqcap \exists \text{hasSpeed}.very(High)$

Definition (Rough Fuzzy Concept Expressions)

+

$$C \quad \rightarrow \quad \begin{array}{ll} \overline{C}^{i} & \text{(upper approximation)} \\ \underline{C}_{i} & \text{(lower approximation)} \end{array}$$

where R_i ($i = 1, ..., m$) are m fuzzy similarity relations.

SARS Example [JWDT09]

SARS (Severe Acute Respiratory Syndrome)

- Is a respiratory disease in humans, caused by SARS coronavirus
- Definition of SARS cannot be expressed precisely
- Mainly two kinds of diagnostic criteria for SARS
  - Suspected diagnostic criteria
    - patients who accord with suspected diagnostic criteria may have SARS (but, not necessarily)
    - these patients may be defined as the upper approximation concept of SARS, *i.e.*, $\overline{SARS}$
  - Clinically diagnosed criteria
    - patients who accord with clinically diagnosed criteria necessarily have SARS
    - these patients may be defined as the lower approximation concept of SARS, *i.e.*, $\underline{SARS}$
- Example of suspected diagnostic criteria rule

$$\overline{Close\_Contact}^1 \sqsubseteq \overline{SDC}^2$$

where *SDC* stands for

"The patient has had close contact with SARS patients or similar cases in recent two weeks, or there is accurate evidence of SARS cases that have infected this patient"

Reasoning

- Recall that
  - Fuzzy Rough Sets: given fuzzy similarity relation R, t-norm $\otimes$ and an implication function $\Rightarrow$
    - Upper approximation of a fuzzy set S: for all $x \in X$,

$$\overline{S}(x) = \sup_{y \in \Delta^{\mathcal{I}}} \{R(x, y) \otimes S(y)\}$$

    - Lower approximation is defined as: for all $x \in X$,

$$\underline{S}(x) = \inf_{y \in \Delta^{\mathcal{I}}} \{R(x, y) \Rightarrow S(y)\}$$

- We consider the transformation:

$$\begin{aligned} \overline{C}^{i} &\mapsto \exists R_i.C && (1) \\ \underline{C}_{i} &\mapsto \forall R_i.C && (2) \end{aligned}$$

  where R_i is reflexive, symmetric and transitive
- Thus, reasoning with fuzzy similarity relations is sufficient
  - Has been implemented into the fuzzyDL system [BS08] (see `http://www.straccia.info`)

Conclusions

- We have shown that Fuzzy Rough Sets/Concepts may smoothly be incorporated into Fuzzy Description Logics

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