Computer Vision Fall 2024
Early Assignment

Instructor: Saining Xie

Sept 5, 2024

Submission Instructions

You must submit solutions to both the theory and coding portion of this homework to be eligible for full credit on this assignment.

Please navigate to the "Assignments" page of the course website in order to download or copy the coding portion of the assignment.You are strongly encouraged to typeset your answers to the theory questions below using LaTeX, via the course homework template. You must submit your answers to the coding problems by filling out the provided iPython notebook. We encourage you to use Google Colab to write and test your code.

This problem set is due on Sept 12, 2024, 6:00 PM. When you have completed both portions of the homework, submit them **on the course Grade-scope as two separate files, with the coding portion in `.ipynb` format** by the due date. **No other forms of submissions will be accepted. Late submissions will also not be accepted.**

You may not discuss the questions in this problem set with other students. If you employ LLMs for help with your assignment, make sure to explicitly credit it. Include the prompts, the responses, and any adjustments.

Theory Questions

Probability and Calculus

Question 1 (10 points)

Two players take turns trying to kick a ball into a net while playing soccer. Player 1 succeeds at kicking the ball into the net with probability $\frac{1}{5}$, while Player 2 succeeds with the probability $\frac{1}{4}$. Whoever succeeds first, wins the game. Assuming that Player 1 takes the first shot, what is the probability that Player 1 wins the game? Show all steps of computation.

Question 2 (10 points)

You know that 1% of the population has COVID. You also know that 90% of the people who have COVID get a positive test result, while 10% of people who do not have COVID also test positive. What is the probability that you have COVID, given that you tested positive?

Question 3 (10 points)

Let the function $f(x)$ be defined as below.

$$f(x) = \begin{cases} 0 & \text{for } x < 0 \\ \frac{1}{(1+x)} & \text{otherwise.} \end{cases} \tag{1}$$

Is $f(x)$ a valid probability-density function (PDF)? If yes, then prove that it is a PDF. If not, then prove that it is not a PDF.

Question 4 (10 points)

Assume that X and Y are two independent, identically distributed (i.i.d.) random variables with probability density function $f(x)$, such that

$$f(x) = \begin{cases} 2x & \text{if} \qquad 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

What is the value of $\mathbb{P}(X + Y \le 1)$?

Question 5 (10 points)

Let X be a uniformly distributed random variable over the open interval $(0, 1)$, $X \sim Unif(0, 1)$. Let $Y = e^X$, so that Y is the random variable that results from exponentiating X. Compute the expectation of Y. Show your work.

Question 6 (10 points)

The number of errors associated with a computer program follows a Poisson distribution with mean $\lambda = 5$. Suppose that we have 125 program submissions and let $X_i \in \{X_1, \ldots, X_{125}\}$ be an i.i.d. random variable denoting the number of errors associated with each submission i. Compute the probability that the average number of errors in this sample, $\overline{X} = \frac{1}{125}\sum_{i=1}^{125} X_i$, is strictly less than 5.5. Show your work.

Question 7 (10 points)

Let $X_n = f(W_n, X_{n-1})$ represent the value of a recurrence defined over $n \in \{1, \ldots, p\}$ for some unknown function $f(\cdot)$. Consider E given by $E = ||c - X_p||^2$, where c is an unknown constant.

Compute the value of gradient $\frac{\partial E}{\partial X_0}$. Write your answer in terms of f, c, and $W_i \in \{W_1, \ldots, W_p\}$.

Linear Algebra

Question 8 (10 points)

Let $\mathbf{A}$ denote a matrix,

$$\mathbf{A} = \begin{bmatrix} 2 & 6 & 7 \\ 3 & 1 & 2 \\ 5 & 3 & 4 \end{bmatrix}$$

and let $\mathbf{x}$ denote a column vector,

$$\mathbf{x} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}.$$

Let $\mathbf{A}^T$ and $\mathbf{x}^T$ denote the transposes of $\mathbf{A}$ and $\mathbf{x}$, respectively. Compute the matrix-vector products $\mathbf{A}\mathbf{x}$, $\mathbf{A}^T\mathbf{x}$, and $\mathbf{x}^T\mathbf{A}$.

Question 9 (10 points)

Determine whether the following matrices are invertible. If so, compute their inverses. Show all steps of computation.

(a)

$$\begin{bmatrix} 6 & 2 & 3 \\ 3 & 1 & 1 \\ 10 & 3 & 4 \end{bmatrix} \tag{3}$$

(b)

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 2 \\ 1 & 4 & 5 \end{bmatrix} \tag{4}$$

Question 10 (10 points)

What is an eigenvalue of a matrix? What is an eigenvector of a matrix? Describe one method (any method) you could use to compute both of them. Use the method you described in order to compute the eigenvalues of the matrix below. Show all steps of computation.

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & 0 \\ -2 & 2 & 1 \end{bmatrix} \tag{5}$$