ENDER ERKAYA
20801441

21.02.2015

EEE 473-MEDICAL IMAGING
HOMEWORK 1

1)Medical Imaging Modalities

Figure 1:X-ray image(Hand)

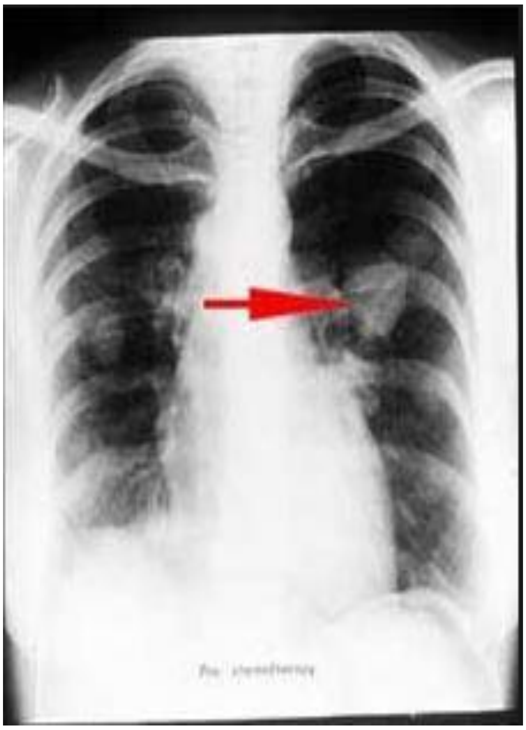


Figure 2:X-ray image(Chest)

Figure 2 shows the X-ray image of a person diagnosed with lung cancer. Figure 4 shows the CT scan of a person diagnosed with glioblastoma.

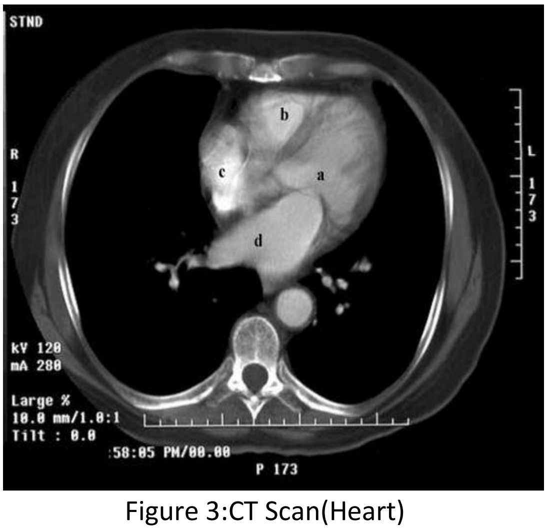


Figure 3:CT Scan(Heart)

Figure 4:Axial CT Scan(Head with Glioblastoma)

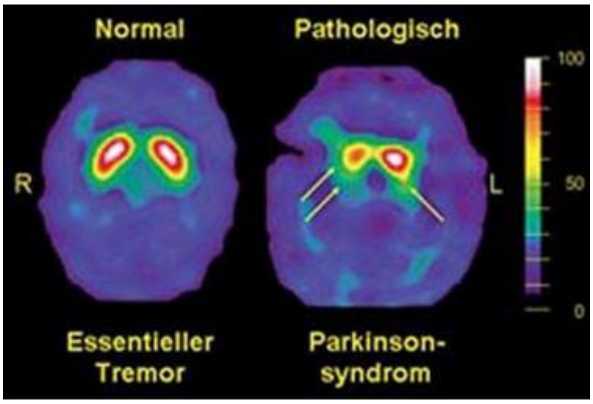


Figure 5: Pet Scan(Head)

Figure 6:Pet Scan(Chest)

Figure 5 shows the different pet scan images of a person with essential tremor and a person diagnosed with parkinson’s disease. Figure 6 shows the PET image of a person diagnosed with lymphoma.

Figure 7:Ultrasound(Liver)

Figure 8:Ultrasound(Kidney)

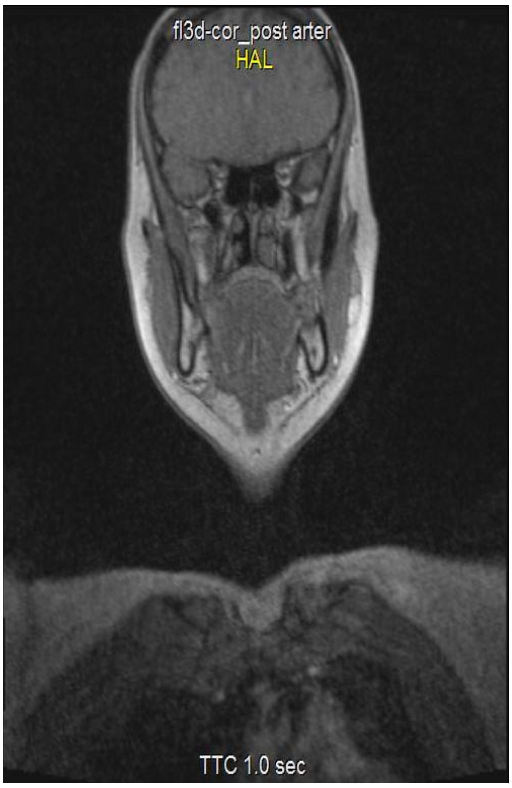


Figure 9:MRI (Skull&Spinal Bulb)

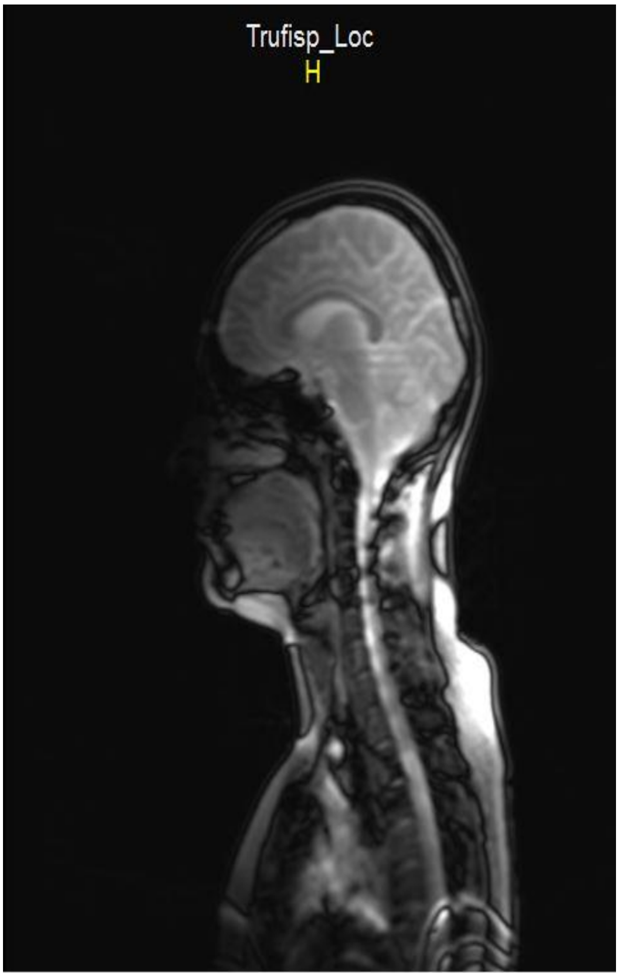


Figure 10:MRI (Skull&Cervical Spine&Corpus Callosum)

Figure 9 is coronal MRI image of my sister’s head. Figure 10 is sagittal MRI image of my sister’s cervical spine and head. It also explicitly shows Corpus Callosum.

2)

Linearity:

$$\mathcal{S}\left[\sum_{k=1}^{K} w_k f_k(x,y)\right] = \sum_{k=1}^{K} w_k \mathcal{S}\left[f_k(x,y)\right]$$

Shift Invariance:

$$g(x-x_0, y-y_0) = \mathcal{S}\left[f(x-x_0, y-y_0)\right]$$

a) $g(x,y) = f(x,y) \cdot f(x-x_0, y)$

i) $$\mathcal{S}\left[\sum_k w_k \cdot f_k(x,y)\right] = \sum_k w_k \cdot f_k(x,y) \cdot \sum_k w_k \cdot f_k(x-x_0, y)$$

$$\neq \sum w_k \underbrace{f_k(x,y)\, f_k(x-x_0,y)}_{g_k(x,y)}$$

$\downarrow$

NOT LINEAR

ii) $$\mathcal{S}\left[f(x-X, y-Y)\right] = f(x-X, y-Y) \cdot f(x-X-x_0, y-Y)$$

$$= g(x-X, y-Y)$$

$\downarrow$

SHIFT INVARIANT

b) $g(x,y) = \int_{-\infty}^{\infty} f(x,\eta)\, d\eta$

i) $$\mathcal{S}\left[\sum_k w_k f_k(x,y)\right] = \int_{-\infty}^{\infty} \sum_k w_k f_k(x,\eta)\, d\eta$$

$$= \sum_k w_k \underbrace{\int_{-\infty}^{\infty} f_k(x,\eta)\, d\eta}_{g_k(x,y)} = \sum_k w_k \cdot g_k(x,y)$$

$\downarrow$ LINEAR

ii) $$\mathcal{S}\left[f(x-X, y-Y)\right] = \int_{-\infty}^{\infty} f(x-X, \eta)\, d\eta = g(x-X, y-Y)$$

$\downarrow$

SHIFT INVARIANT

3) $f(x,y) = x + y^2$

a) $f(x,y) \cdot \delta(x-1, y-2) = f(x,y)\Big|_{(1,2)} \cdot \delta(x-1, y-2) = 5 \cdot \delta(x-1, y-2)$

b) $$f(x,y) * \delta(x-1,y-2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\xi,\eta)\, \delta(x-\xi-1, y-\eta-2)\, d\xi\, d\eta$$

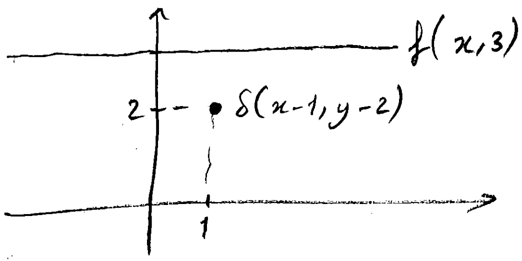
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x-\xi, y-\eta)\, \delta(\xi-1, \eta-2)\, d\xi\, d\eta$$

$$= f(x-1, y-2) \underbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \delta(\xi-1, \eta-2)\, d\xi\, d\eta}_{1}$$

$$= f(x-1, y-2)$$

c) $$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \delta(x-1, y-2) \cdot f(x,3)\, dx\, dy \qquad f(x,3) = x+9$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} 0 \cdot dx\, dy = \underline{0}$$



d) $\delta(x-1, y-2) * f(x+1, y+2) = f(x+1, y+2) * \delta(x-1, y-2)$

$= f(x-1+1, y-2+2) = f(x,y)$ (using the property found in part (b))

4)

b) * $\text{comb}(x,y) \xrightarrow{F_{2D}} \text{comb}(u,v)$

$$\delta_s(x,y;\Delta x,\Delta y) = \frac{1}{\Delta x \Delta y}\,\text{comb}\left(\frac{x}{\Delta x},\frac{y}{\Delta y}\right)$$

$\downarrow F_{2D}$ $\quad\quad$ $\downarrow F_{2D}$ $\longrightarrow$ using scaling property of FT

$$F_{2D}(\delta_s)(u,v) = \frac{1}{\cancel{\Delta x \Delta y}} \cdot \cancel{\Delta x . \Delta y} . \text{comb}(u.\Delta x, v.\Delta y)$$

$$F_{2D}(\delta_s)(u,v) = \text{comb}(u.\Delta x, v.\Delta y)$$

---

c) $s(x,y) = \sin(2\pi(u_0x + v_0y))$

$$S(u,v) = \iint_{-\infty}^{\infty} \frac{1}{2} e^{-j\pi/2}\left[e^{+j2\pi(u_0x+v_0y)} - e^{-j2\pi(u_0x+v_0y)}\right] \ldots$$

$$\ldots \cdot e^{-j2\pi(ux+vy)}\, dx\,dy$$

$$S(u,v) = \frac{1}{2} e^{-j\pi/2} \iint_{-\infty}^{\infty} e^{j2\pi[(u_0-u)x+(v_0-v)y]} - e^{-j2\pi[(u_0+u)x+(v_0+v)y]}\, dx\,dy$$

$$= \frac{1}{2} e^{-j\pi/2}\left[\int_{-\infty}^{\infty} e^{j2\pi(u_0-u)x}dx \int_{-\infty}^{\infty} e^{j2\pi(v_0-v)y}dy + \ldots\right.$$

$$\left.\ldots - \int_{-\infty}^{\infty} e^{-j2\pi(u_0+u)x}dx \int_{-\infty}^{\infty} e^{-j2\pi(u_0+v)y}dy\right]$$

$$= \frac{1}{2} e^{-j\pi/2}\left[\delta(u-u_0, v-v_0) - \delta(u+u_0, v+v_0)\right]$$

$$= \frac{1}{2} e^{-j\pi/2}\delta(u-u_0, v-v_0) + \frac{1}{2} e^{+j\pi/2}\delta(u+u_0, v+v_0)$$

---

d) $c(x,y) = \cos(2\pi(u_0x + v_0y)) = \frac{1}{2}\left[e^{j(2\pi(u_0x+v_0y))} - e^{-j2\pi(u_0x+v_0y)}\right]$

The difference from part c is the scaling factor and the sign of the part 2 integral. Hence,

$$C(u,v) = \frac{1}{2}\left[\delta(u-u_0, v-v_0) + \delta(u+u_0, v+v_0)\right]$$

4)

e) $f(x,y) = \frac{1}{2\pi\sigma^2} \exp\{-(x^2+y^2)/2\sigma^2\}$

$f(x,y)$ is separable such that

$$f(x,y) = \frac{1}{2\pi\sigma^2} \exp\{-x^2/2\sigma^2\} \cdot \exp\{-y^2/2\sigma^2\}$$

$$\Big\downarrow F_{2D} \qquad\qquad * \Big\downarrow F_{1D} \qquad\qquad * \searrow F_{1D}$$

$$F(u,v) = \frac{1}{2\pi\sigma^2} \cdot \sigma \cdot \sqrt{2\pi} \cdot \exp\{-\pi\sigma^2 2\pi u^2\} \cdot \sigma\sqrt{2\pi} \exp\{-\pi\sigma^2 2\pi v^2\}$$

$$= \frac{1}{\cancel{2\pi\sigma^2}} \cancel{\sigma^2 2\pi} \exp\{-2\pi^2\sigma^2(u^2+v^2)\}$$

$$= \exp\{-2\pi^2\sigma^2(u^2+v^2)\}$$

In calculation of F_{1D} of exponential signals scaling property of Fourier transform is used. $\mathcal{F}$: $\{e^{-\pi x^2}\} \rightarrow \{e^{-\pi u^2}\}$

5) $$F(q) = 2\pi \int_0^\infty f(r) . J_0(2\pi q r)\, r\, dr$$

$$F(q) = 2\pi \int_0^1 1 . J_0(2\pi q r)\, r\, dr$$

$$* \quad \frac{d}{dx}\left(x^m J_m(x)\right) = x^m J_{m-1}(x) \text{ [1]}$$

Using the identity above

$$\left[x . J_1(x)\right]' = x . J_0(x)$$

$$\left[ax . J_1(ax)\right]' = a^2 x\, J_0(ax)$$

$$F(q) = 2\pi . \frac{1}{(2\pi q)^2}\; 2\pi q r\, J_1(2\pi q r)\Big|_0^1$$

$$= \frac{1}{\cancel{2\pi} q^2}\; \cancel{2\pi} q\, J_1(2\pi q) = \frac{J_1(2\pi q)}{q}$$

[1] = Wolfram Mathworld. 'Bessel Function of First Kind', Eq. 59
< mathworld.wolfram.com/BesselFunctionoftheFirstKind.html >

6)MATLAB Question

a) s(x,y) with u0=5 and v0=1

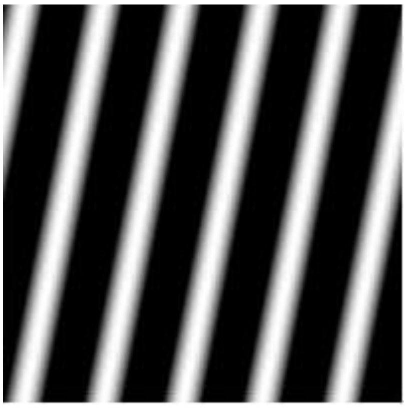


Figure 11:Gray-scale image of function a

Matlab Code:

```
clear all
u0=5;
v0=1;
[x,y] = meshgrid(0:5e-3:1,0:5e-3:1);
f=sin(2*pi*(u0*x+v0*y));
imshow(f)
```

b) s(x,y) with u0=3 and v0=3

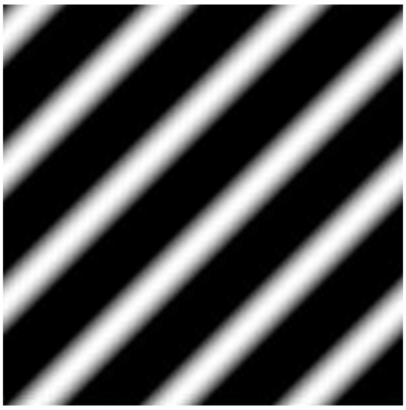


Figure 12:Gray-scale image of function b

Matlab Code:

```
clear all
u0=3;
v0=3;
[x,y] = meshgrid(0:5e-3:1,0:5e-3:1);
f=sin(2*pi*(u0*x+v0*y));
imshow(f)
```

c) f(x,y) with sigma=10

Figure 13:Grey-scale function c

Matlab Code:

```
clear all
u0=3;
v0=3;
[x,y] = meshgrid(0:5e-3:1,0:5e-3:1);
f=(1/(2*pi*100))*exp(-(x.*x+y.*y)/200);
imshow(f)
```

d) f(x,y) with sigma=1

Figure 14:Function d

Matlab Code:

```
clear all
u0=3;
v0=3;
[x,y] = meshgrid(0:5e-3:1,0:5e-3:1);
f=(1/(2*pi))*exp(-(x.*x+y.*y)/2);
imshow(f)
```

e) rect(r)

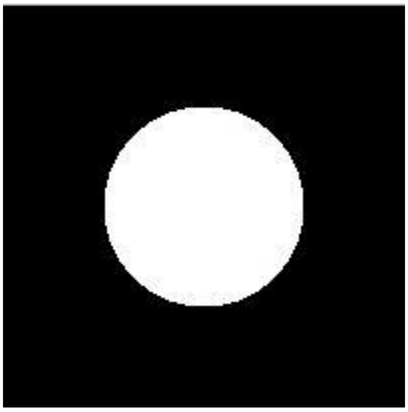


Figure 15:rect(r)

Matlab Code:

```
clear all
[x,y] = meshgrid(-2:2e-2:2,-2:2e-2:2);
r=sqrt(x.*x+y.*y);
f=zeros(length(r(:,1)),length(r(:,1)));
for i=1:length(r(:,1))
    for j=1:length(r(:,1))
        if r(i,j)<1
            f(i,j)=1;
        else
            f(i,j)=0;
        end
    end
end
imshow(f)
```

f) jinc(r)

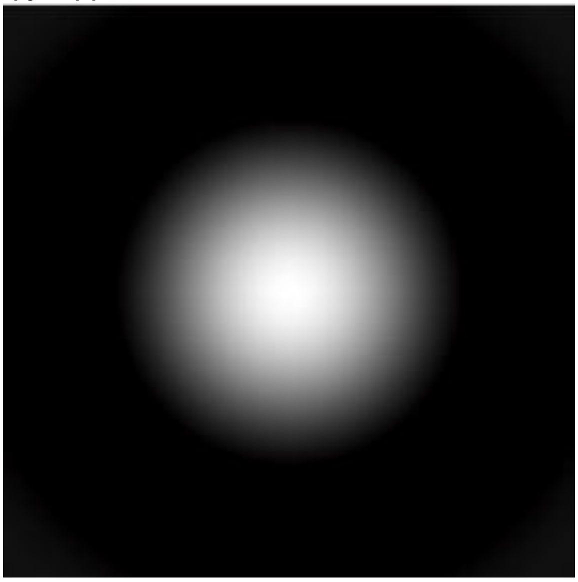


Figure 16:jinc(r)

Matlab Code:

```
clear all
[x,y] = meshgrid(-1:7e-3:1,-1:7e-3:1);
r=sqrt(x.*x+y.*y);
imshow(jinc(r))

function y=jinc(r)
y=real(2*besselj(1,2*pi*r)./ (2*pi*r));
y(isnan(y))=1;
end
```

g) rect(x,y)

Figure 17:rect(x,y)

Matlab Code:

```
[x,y] = meshgrid(-2:2e-2:2,-2:2e-2:2);
f=rectangularPulse(x).*rectangularPulse(y);
imshow(f)
```

h)sinc(x,y)

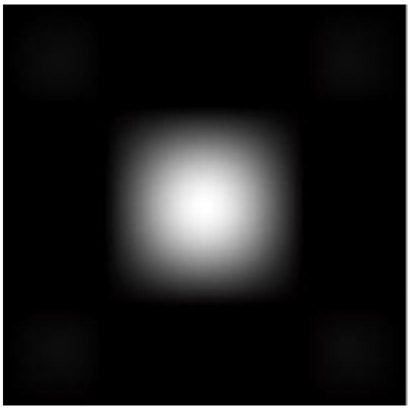


Figure 18:sinc(x,y)

Matlab Code:

```
[x,y] = meshgrid(-2:2e-2:2,-2:2e-2:2);
f=sinc(x).*sinc(y);
imshow(f)
```