ELEC531/ELEC431 Handout #5
Adaptive Signal Processing Sunday, November 28, 2021

Homework Set #4
Due: Saturday, December 11, 2021.

Instructions:

For the question below, prepare Matlab/Python/Julia programs and generate corresponding reports as in previous homeworks.

In your reports, for each part of the question generate a different section. Make sure that you put introductory, descriptive and explanatory statements as well as equations formatted in Latex (if needed) for each section, such that it is organized as a full report.

Note that for this homework, you will need to write a separate Matlab/Python/Julia function for part a).

1. *Adaptive Channel Equalization*

Consider the following linear equalization setup show in Figure1.

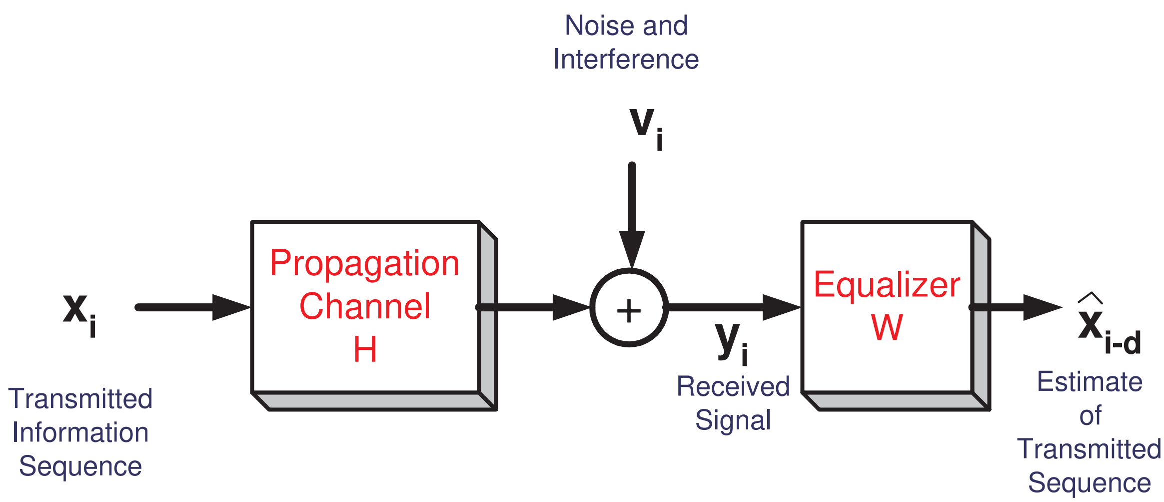

Figure 1: Linear FIR equalization setup.

In this setup,

- x_i is the zero mean i.i.d. transmission sequence with variance 1.
- v_i is the zero mean i.i.d. noise sequence with variance 0.0001.
- y_i is the observations at the receiver.
- d is the equalization delay.
- W is the FIR equalizer.

The goal is to estimate the delayed version x_{i-d} of the transmission sequence from the observations at the receiver y_i.

(a) Write a function to calculate Minimum Mean Square Error (MMSE) FIR equalizer coefficients based on the optimal stochastic filtering formulation expression in class. Before coding, first derive expressions for

$$\mathbf{R}_{x_{i-d}\mathbf{y}_i}, \text{ and } \mathbf{R}_{\mathbf{y}_i}, \tag{1}$$

in terms of channel impulse response vector $\mathbf{h} = \begin{bmatrix} h_0 & h_1 & \ldots & h_{L-1} \end{bmatrix}$, equalizer length N, noise variance σ_v^2. Based on these derivations write your Matlab function. Below is the prototype for the function:

```
function [w, mmse]=findmmsefireq(h,r,N,d)
% MATLAB function that calculates the MMSE FIR equalizer
%  h : Impulse response of the channel to be equalized
%  r : Variance of the iid noise
%  N : Length of the FIR equalizer
%  d : Equalization delay
%  w : Equalizer coefficients as output
% mmse: Minimum Mean Square Error as output
% This function assumes that transmission sequence has variance 1.
```

Your report should include the latex formatted expressions for $\mathbf{R}_{x_{i-d}\mathbf{y}_i}, \mathbf{R}_{\mathbf{y}_i}$.

(b) Given $h = \begin{bmatrix} 1 & 1.5 \end{bmatrix}$ and the length of the equalizer 10, calculate MMSE for $d = 0$ to 8 and plot it as a function of delay.

(c) For the same channel,

- Generate i.i.d x_is with values -1 and 1,
- Pass it through the channel,
- Add i.i.d. Gaussian noise with variance 0.0001 to the channel output.

For $d = 8$ and $\mu = 0.03$ perform LMS for 500 samples and obtain square error convergence curve as a function of time. Repeat this for 1000 times to average the square error curves.

(d) Repeat part (c) for $\mu = 0.02$. Plot MMSE convergence curves for part (c) and part (d) on the same graph (plot 10*log10(mmse)). On the same plot draw a line indicating the optimal MMSE level. What are the excess MMSE and convergence times in each case.

(e) Find the convergence curve for $\mu = 0.5$. What is your observation? Explain the reason for this behavior based on the statistics of the observation signal y_i. Test to find the μ value which is the border value for convergence.

(f) Apply RLS algorithm to obtain adaptive filter coefficients. Choose λ very close to 1.