EEE 473/573 – Spring 2015
Homework #4

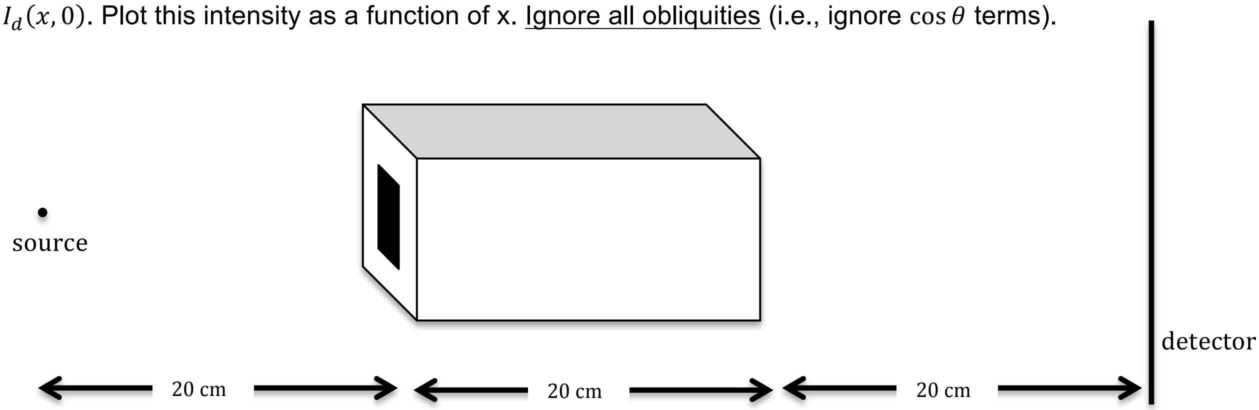
Upload your homework solution (as a PDF) file on Moodle under "HW#4 Upload"
by 11:00PM on Sunday, April 12th. <u>No late submissions allowed</u>.

1) [20 points] Answer the following questions. Simplify your answers as much as possible.

- **a)** [4 points] Calculate the 2D convolution $\mathrm{rect}\left(\frac{x}{4}, y\right) * \delta(x-1, y-2)$.
- **b)** [4 points] Calculate the 2D convolution $\mathrm{rect}\left(\frac{x}{4}, y\right) * \delta(y-2)$.
- **c)** [4 points] Calculate the 2D convolution $\cos(2\pi x + 4\pi y) * \mathrm{sinc}(2x, 3y)$.
- **d)** [4 points] Calculate the 2D convolution $\cos(2\pi u_0 x + 2\pi v_0 y) * \exp(-x^2 - y^2)$.
- **e)** [4 points] What is the 2D Fourier transform of the following function?

  $f(x,y) = f(r) = \mathrm{rect}\left(\frac{r-a}{b}\right), \text{where a} > \text{b}$.

2) [20 points] A <u>hollow</u> prism (i.e., with a hole at its center) has length L = 20 cm, outer width 8cm X 8cm, inner width 2cm X 2cm, and a constant linear attenuation coefficient of $\mu_0 = 0.05\ \mathrm{cm}^{-1}$. This prism is imaged with a point source x-ray imaging system, as shown below. Formulate the intensity on the detector along the x-axis, $I_d(x, 0)$. Plot this intensity as a function of x. <u>Ignore all obliquities</u> (i.e., ignore $\cos\theta$ terms).



3) [20 points] A 2D function $f(x,y)$ (or $f(r,\emptyset)$) produces 1D projections given by

$$g(l,\theta) = 4\, sinc(2l)\cos(4\pi l)$$

- **a)** [10 points] Determine the 2D function $f(x,y)$ (or $f(r,\emptyset)$).
- **b)** [10 points] If the CT image reconstruction is performed with a filtered backprojection system using a modified filter $|\rho|\mathrm{rect}\left(\frac{\rho}{2\rho_0}\right)$, determine the resultant reconstructed image as a function of ρ_0. Simplify your answer as much as possible.

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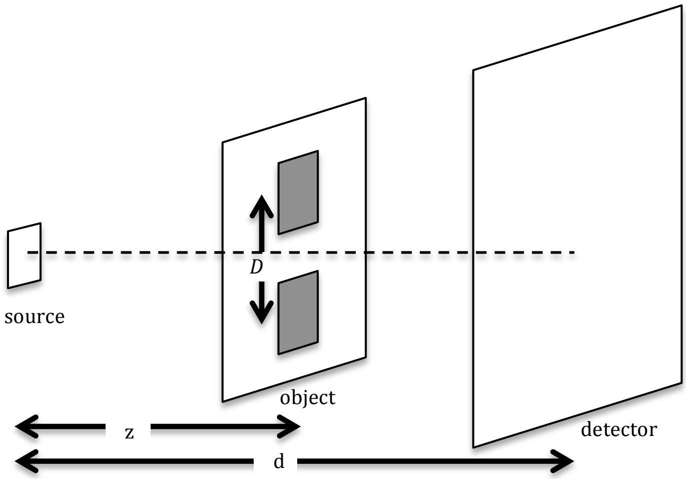
4) [20 points] A square source of size *L* by *L* is used to image a planar object that contains two square holes (lesions), each size *W* by *W*. The rest of the planar object has zero transmittivity. The centers of the two holes are separated by a distance *D* along the x-direction. The exact depth of the planar object is not known, except that it is between $z = d/2$ and $z = 2d/3$.



- **a)** [8 points] Find the largest source size, *L*, that ensures that the two lesions remain fully resolved, i.e., they remain not touching in the image, for all z within the range specified.
- **b)** [4 points] What is the largest value of *L* if $D = 9W/4$?
- **c)** [8 points] Using the value from part (b), find the value(s) of z (within the range specified) that maximize the image intensity at the center of the lesions.

<u>Ignore all obliquities</u> (i.e., ignore $\cos\theta$ terms).
Hint: There is no need to fully calculate $I_d(x, y)$ for this question.

5) [20 points] **MATLAB Question:** As we covered in Chapter 6, the process of taking the projections of a 2D function is also called "Radon Transform", and the inverse process of reconstructing the images from projections is called the "Inverse Radon Transform". These transforms are available in MATLAB as built-in functions "*radon*" and "*iradon*".

Download the MATLAB data file "*projections.mat*" provided on Moodle. This file contains 4 projection sets of the same 2D object, where $glt_i(l_i, theta_i)$ is the projections taken at angles $theta_i$. These projections were computed using "*radon*" function of MATLAB.

- **a)** Display the sinograms and the reconstructed images (using "iradon") for all 4 projection sets.
- **b)** Some of the reconstructed images will have clearly visible artifacts on them. What kind of an artifact is this? Why is this artifact not present in some of the images? How can we avoid (or at least reduce) this artifact?
- **c)** For the projection set $glt_3(l_3, theta_3)$, reconstruct the image using three different filters:

  1) Default filter in "*iradon*" (cropped ramp filter, i.e., ramp filter multiplied with a rect function),

  2) Hamming windowed filter,

  3) No filter (this would be direct backprojection reconstruction, without any filtering).

  These are options available in "*iradon*" function, so type "*help iradon*" to see how you can use these filters. Comment on the differences that you see in the reconstructed images. Which filter provides the best image?
- **d)** Repeat part (c) for the projection set $glt_4(l_4, theta_4)$. Comment on the differences that you see in the images. Which filter provides the best image?