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A New Close Form Location Algorithm with AOA and TDOA for Mobile User

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Abstract A new closed form location algorithm for mobile user is proposed in this paper. In wireless communication systems, the time difference of arrival (TDOA) measurements can be obtained by computing the correlation function of the forward link pilot signals, the angle of arrival (AOA) measurements can be obtained by calculating the spatial spectrum for the reverse link pilot signal. With the measurements TDOA and AOA, the position of a mobile station (MS) can be determined. Firstly, the nonlinear TDOA and AOA measurement equations without noise are transformed to linear equations for the positions of MS. Secondly, the linear errors are calculated by the noisy measurement, and then a weighted least square method is used to solve the linear equations. The Cramer Rao bound (CRB) is analyzed in this paper. Finally, computer simulation is given. Numerical results demonstrate that the proposed method can attain to CRB at moderate measurement noise.

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1 Introduction

Wireless location technology has long been proposed. The research on wireless location may date back to 1960s and was first used on CAVL system—automatic vehicle localization. Nowadays, wireless location determination plays an important role in transportation, navigation, project and rescue work. Recently, due to the explosive growth of mobile communication market, wireless location service is now the most promising industry since Internet and mobile communication. Since many companies and research institutions have invested a lot of money and manpower for research and development, many positioning technologies have been proposed. As a value added service of mobile radio communications, wireless location service is playing a more and more important role in our daily life and appears to have a bright market prospect.

Ever since FCC proposed standard E-911, the research on cellular position technology has never stopped [1]. By detecting parameters such as strength of signal, angle of arrival (AOA) [2], time of arrival (TOA) or time difference of arrival (TDOA) [3–5], cellular location system is able to locate a moving station. Received signal strength (RSS) technology is based on the relationship between strength of received signal and emitter–receiver distance. In order to estimate distance between emitter and receiver, we compare the received signal strength with theoretic strength, which can be computed by known transmitted signal strength and known fading channel model. Then, we estimate moving station position by the intersection points of at least three circles. The complexity of electromagnetic environment of mobile communication, however, confine the precision of localization. Whereas the technology is still adopted where the precision of localization is not the priority because of its cheapness and feasibility. Array antenna might be adapted to measure angle of arrival (AOA) and cancel the multipath effect. Theoretically, two base stations form two lines, their intersection point can be deemed as the location of target. However, this method is not accurate when target is far away, the error of estimated angle causes severe positioning error. TOA/TDOA method has extensive application in wireless location systems based on cellular network. TOA method requires at least base stations to estimate propagation time from mobile station to base stations. Then we obtain the distance between mobile station and each base station. At last, estimated distances are used for target geolocation. This TOA method, however, also has its disadvantages—it demands that base station has a priori knowledge about the transmitting time and requires that both base station and mobile station have accurate clock. TDOA method is the improvement of TOA, it measure the time difference of arrival and use the hyperbolic location method to locate moving station. It is certain that each localization method has its own advantages and shortcomings, in practical circumstance; we usually use the joint of parameters above for localization [6].

The algorithm we proposed in this paper is a joint of TDOA and AOA method. As is well-known, the equation to measure TDOA and AOA are a non-linear equation with respect to position of target. The key to location algorithm is the processing of these non-linear equations [7–15]. In [16], Chan proposed a classical method to manage TDOA problem. When the error of TDOA is small enough, it can attain to CRLB, but when error is large, the performance of the algorithm is not guaranteed. In [6], as the same as [16], it combines TDOA and AOA, but use only one AOA information while it could have used multiple AOA information to obtain a more precise estimation. In [17], the linear equation about position vector and distance is given, and the final solution is obtained by removing the relationship between position vector and distance.

Recently, in [7], Gaber and Omar proposed a non-iterative hybrid weighted least square (HWLS) method based on TDOA and AOA estimates for wireless indoor positioning, where the initial guess of target coordinates are acquired using a WLS estimator, then the final results are derived through the equation where all elements can be presented using the initial guess. Although this method is more precise than Chan method and hybrid method, our method is better when TDOA error is large. Also, with the increase of TDOA and AOA error, the final results may not be convergent.

We proposed an algorithm that differs from [7, 16, 17] and use a linear equation with position vector only resulting from geometrical relationship. The equation is mainly based on AOA and TDOA parameters. The algorithm has less computational complexity and has better location accuracy.

The arrangement of this paper lies follows: In Sect. 2, we introduce location model of our method and some conventional location algorithms—AOA method, TDOA method, AOA and TDOA fusion method and the latest proposed WLS method. Section 3 presents the derivation of the algorithm we put forward. To compare the performance of computational complexity, we examine the amount of multiplications needed in two methods in Sect. 4. In Sect. 5, we run 10,000 Monte-Carlo simulations on HWLS method for the comparison of our method. Section 6 is the conclusion about merits and demerits of our algorithm. And the appendix is the brief derivation of CRB.

2 Location Model and Classical Location Methods

Without loss of generality, we model the problem in a 2D scenario where we can extend to 3D by analogy. As is depicted in Fig. 1, M base stations are located in a 2D plane, and AOA and TDOA measurements have already been measured. Some classical localization methods are as follows (Fig. 2).

1. AOA method

Consider M base stations in a 2D scenario. The true AOAs are given by:

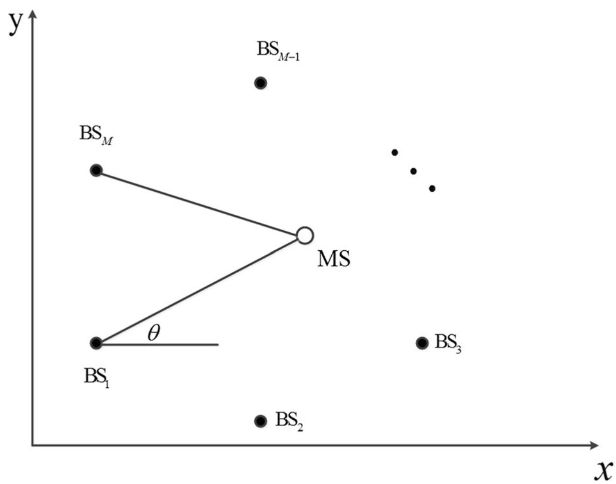
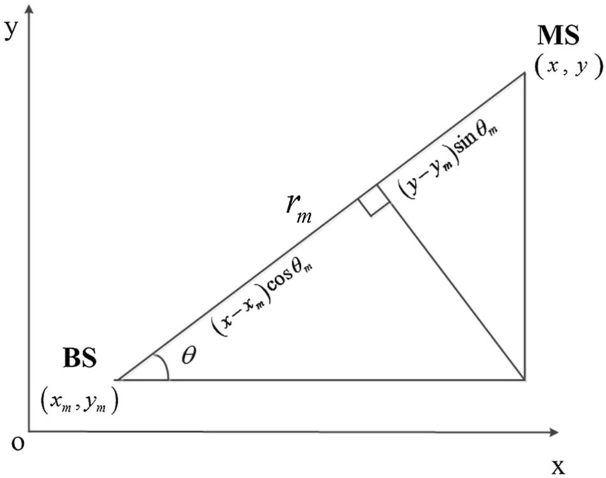


Fig. 1 Schematic diagram of AOA/TDOA location

Fig. 2 Linear expression of the AOA/TDOA equation



$$\theta_m = \arctan\frac{y - y_m}{x - x_m}, \quad m = 1\ldots M \tag{1}$$

where target is located at (x, y), (x_m, y_m) is the coordinates of m-th base station. (1) can be rewritten as:

$$(x - x_m)\sin\theta_m = (y - y_m)\cos\theta_m \tag{2}$$

Since the AOA measurements have error, we actually model AOA as: $\hat{\theta}_m = \theta_m + n_{\theta m}$, where $\mathbf{n}_\theta = [\,n_{\theta 1} \quad \cdots \quad n_{\theta M}\,]^T \sim N(\mathbf{0}, \mathbf{Q}_\theta)$. When measurement noise $n_{\theta m}$ is small, the first-order Taylor expansion is shown as follows:

$$\begin{aligned} \cos\theta_m &= \cos\hat{\theta}_m + \sin\hat{\theta}_m n_{\theta m} \\ \sin\theta_m &= \sin\hat{\theta}_m - \cos\hat{\theta}_m n_{\theta m} \end{aligned} \tag{3}$$

We substitute (3) into (2) and obtain:

$$\begin{aligned} x\sin\hat{\theta}_m - y\cos\hat{\theta}_m &= x_m\sin\hat{\theta}_m - y_m\cos\hat{\theta}_m + e_m \\ e_m &= \left[(x - x_m)\cos\hat{\theta}_m + (y - y_m)\sin\hat{\theta}_m\right] n_{\theta m} \end{aligned} \tag{4}$$

Equation (4) can also be written in the form of a vector as:

$$\mathbf{A}_\theta \mathbf{x} = \mathbf{b}_\theta + \mathbf{e}_\theta \tag{5}$$

where

$$\mathbf{A}_\theta = \begin{bmatrix} \sin\hat{\theta}_1 & -\cos\hat{\theta}_1 \\ \vdots & \vdots \\ \sin\hat{\theta}_M & -\cos\hat{\theta}_M \end{bmatrix} \tag{6}$$

$$\mathbf{b}_\theta = \begin{bmatrix} x_1 \sin\hat{\theta}_1 - y_1 \cos\hat{\theta}_1 \\ \vdots \\ x_M \sin\hat{\theta}_M - y_M \cos\hat{\theta}_M \end{bmatrix} \tag{7}$$

$$\mathbf{e}_\theta = \begin{bmatrix} \left[(x - x_1)\cos\hat{\theta}_1 + (y - y_1)\sin\hat{\theta}_1\right] n_{\theta 1} \\ \vdots \\ \left[(x - x_M)\cos\hat{\theta}_M + (y - y_1)\sin\hat{\theta}_M\right] n_{\theta M} \end{bmatrix} = \mathbf{\Lambda}\mathbf{n}_\theta \tag{8}$$

$$\mathbf{\Lambda} = diag\left\{\left[\,(x - x_1)\cos\hat{\theta}_1 + (y - y_1)\sin\hat{\theta}_1 \quad \cdots \quad (x - x_M)\cos\hat{\theta}_M + (y - y_M)\sin\hat{\theta}_M\,\right]\right\} \tag{9}$$

The statistical property of AOA measurement error vector is:

$$\begin{aligned} E(\mathbf{e}_\theta) &= \mathbf{0} \\ E\left(\mathbf{e}_\theta \mathbf{e}_\theta^T\right) &= \mathbf{Q}_1 = \mathbf{\Lambda}\mathbf{Q}_\theta \boldsymbol{\lambda}^T \end{aligned} \tag{10}$$

We get the AOA estimation based on the optimal weighted least squares:

$$\hat{\mathbf{x}}_{AOA} = \left(\mathbf{A}_\theta^T \mathbf{Q}_1^{-1} \mathbf{A}_\theta\right)^{-1} \mathbf{A}_\theta^T \mathbf{Q}_1^{-1} \mathbf{b}_\theta \tag{11}$$

2. TDOA method

We first introduce Chan algorithm [16] briefly. For the convenience, we substitute difference of distance for TDOA measurements. Assuming there are M base stations in a 2D plane. The distance between target and m-th base station is $r_m = \sqrt{(x - x_m)^2 + (y - y_m)^2}$, then we obtain m − 1 distance difference values as:

$$\hat{r}_{m1} = r_m - r_1 + n_{rm}, m = 2\ldots M \tag{12}$$

where $\mathbf{n}_r = [\,n_{r2} \quad \cdots \quad n_{rM}\,]^T \sim N(\mathbf{0}, \mathbf{Q}_r)$. (12) yields $(r_{m1} + r_1)^2 = (r_m + n_{rm})^2$, so (12) can be rewritten by:

$$\hat{r}_{m1}^2 + K_1 - K_m + 2(x_{m1}x + y_{m1}y + r_{m1}r_1) = 2r_m n_{rm} + n_{rm}^2 \tag{13}$$

where $K_m = x_m^2 + y_m^2$, $x_{m1} = x_m - x_1$, $y_{m1} = y_m - y_1$. Since $r_m \gg n_{rm}$ in fact, the second-order term of error n_{rm}^2 can be ignored, then (13) becomes:

$$\hat{r}_{m1}^2 + K_1 - K_m + 2(x_{m1}x + y_{m1}y + r_{m1}r_1) = 2r_m n_{rm} \tag{14}$$

Equation (14) is already a linear equation controlled by location coordinates $\mathbf{x} = [x, y]^T$. In order to solve $\mathbf{x}$, define $\mathbf{z} = [\mathbf{x}^T, r_1]^T$, then M − 1 equations can be written in the form of vector as:

$$\psi = \mathbf{h} - \mathbf{G}\mathbf{z} \tag{15}$$

where

$$\mathbf{h} = \frac{1}{2}\begin{bmatrix} \hat{r}_{21}^2 + K_1 - K_2 \\ \vdots \\ \hat{r}_{m1}^2 + K_1 - K_m \end{bmatrix} \tag{16}$$

$$\mathbf{G} = -\begin{bmatrix} x_{21} & y_{21} & r_{21} \\ \vdots & \vdots & \vdots \\ x_{m1} & y_{m1} & r_{m1} \end{bmatrix} \tag{17}$$

$$\begin{aligned} \psi &= \mathbf{B}\mathbf{n}_r \\ \mathbf{B} &= diag\{r_2, r_3, \ldots r_M\} \end{aligned} \tag{18}$$

The random vector $\boldsymbol{\psi}$ is zero mean and its covariance matrix is $\boldsymbol{\psi} = \mathrm{E}\left[\psi\psi^T\right] = \mathbf{B}\mathbf{Q}_r\mathbf{B}$. Actually, $\mathbf{x}^T$ and r_1 are correlated. But we assume they are uncorrelated in order to make use of (15) to solve $\mathbf{x}^T$, then we get an optimal least squares estimator of $\mathbf{z}$ as:

$$\mathbf{z} = \left(\mathbf{G}^T\boldsymbol{\psi}^{-1}\mathbf{G}\right)^{-1}\mathbf{G}^T\boldsymbol{\psi}^{-1}\mathbf{h} \tag{19}$$

However, the estimation above is not accurate because of our assumption, the correlation between $\mathbf{x}$ and r_1 is needed, and we use another least squares estimator to obtain a more accurate solution of $\mathbf{x}$, as is shown in [18].

3. Fusion method [19]

Suppose that the positioning result of AOA method is $\hat{\mathbf{x}}_{AOA}$ and the variance of the position error is $\mathbf{Q}_{AOA}$, $\hat{\mathbf{x}}_{TDOA}$ denotes the positioning result of TDOA method and the variance of the position error is $\mathbf{Q}_{TDOA}$. According to variance fusion principle, we get:

$$\hat{\mathbf{x}}_{fusion} = \mathbf{Q}\left(\mathbf{Q}_{AOA}^{-1}\hat{\mathbf{x}}_{AOA} + \mathbf{Q}_{TDOA}^{-1}\hat{\mathbf{x}}_{TDOA}\right) \tag{20}$$

where

$$\mathbf{Q} = \left(\mathbf{Q}_{AOA}^{-1} + \mathbf{Q}_{TDOA}^{-1}\right)^{-1} \tag{21}$$

4. HWLS method

Suppose the coordinates of the base stations are known and are located at $(x_1, y_1), \ldots, (x_M, y_M)$, where M here is the number of receivers. And the target is at (x, y).

In the first step, an initial estimation of target location is computed through a W-LS estimator which can be represented as

$$\hat{\mathbf{z}} = [\hat{x}, \hat{y}]^T = \left(\mathbf{A}^T\mathbf{Q}_{TDOA}^{-1}\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{Q}_{TDOA}^{-1}\left(\mathbf{b}_1 + \mathbf{b}_2 \cdot \hat{d}_1\right) \tag{22}$$

where

$$\mathbf{A} = \begin{bmatrix} x_1 - x_2 & y_1 - y_2 \\ \vdots & \vdots \\ x_1 - x_M & y_1 - y_M \end{bmatrix}, \tag{23}$$

$$\mathbf{b}_1 = \frac{1}{2}\begin{bmatrix} x_1^2 - x_2^2 + y_1^2 - y_2^2 + d_{21}^2 \\ \vdots \\ x_1^2 - x_M^2 + y_1^2 - y_M^2 + d_{M1}^2 \end{bmatrix}, \tag{24}$$

$$\mathbf{b}_2 = [d_{21}, \ldots, d_{M1}]^T, \tag{25}$$

and $\mathbf{Q}_{TDOA}$ is the covariance matrix converted from its original TDOA form into the corresponding distance form by multiplication by c^2.

In second step, a hybrid TDOA and DOA estimator [7] is employed to solve the final result, and the estimator is shown as:

$$\hat{\mathbf{z}} = [\hat{x}, \hat{y}]^T = \hat{\mathbf{z}}_0 + \left(\mathbf{G}^T\mathbf{Q}^{-1}\mathbf{G}\right)^{-1}\mathbf{G}^T\mathbf{Q}^{-1}\mathbf{h} \tag{26}$$

where $\hat{\mathbf{z}}_0 = [\hat{x}_0, \hat{y}_0]^T$ is the initial target location estimation that has already been computed in the first step, matrix $\mathbf{G}$ and $\mathbf{h}$ are shown as:

$$\mathbf{G} = \begin{bmatrix} \frac{x_1 - \hat{x}_0}{\hat{d}_1} - \frac{x_2 - \hat{x}_0}{\hat{d}_2} & \frac{y_1 - \hat{y}_0}{\hat{d}_1} - \frac{y_2 - \hat{y}_0}{\hat{d}_2} \\ \vdots & \vdots \\ \frac{x_1 - \hat{x}_0}{\hat{d}_1} - \frac{x_M - \hat{x}_0}{\hat{d}_M} & \frac{y_1 - \hat{y}_0}{\hat{d}_1} - \frac{y_M - \hat{y}_0}{\hat{d}_M} \\ -\sin\left(\tilde{\theta}_1\right) & \cos\left(\tilde{\theta}_1\right) \\ \vdots & \vdots \\ -\sin\left(\tilde{\theta}_M\right) & \cos\left(\tilde{\theta}_M\right) \end{bmatrix} \tag{27}$$

$$\mathbf{h} = \begin{bmatrix} d_{21} - \hat{d}_{21} \\ \vdots \\ d_{M1} - \hat{d}_{M1} \\ -(x_1 - \hat{x}_0)\sin\left(\tilde{\theta}_1\right) + (y_1 - \hat{y}_0)\cos\left(\tilde{\theta}_1\right) \\ \vdots \\ -(x_M - \hat{x}_0)\sin\left(\tilde{\theta}_M\right) + (y_M - \hat{y}_0)\cos\left(\tilde{\theta}_M\right) \end{bmatrix} \tag{28}$$

where $\hat{d}_i(i = 1, \ldots, M)$ and $\hat{d}_{i1}(i = 2, \ldots, M)$ can be measured using the initial estimation $\hat{\mathbf{z}}_0 = [\hat{x}_0, \hat{y}_0]^T$ in step one.

The combined covariance matrix of TDOA and DOA is defined as:

$$\mathbf{Q} = \begin{bmatrix} \mathbf{Q}_{TDOA} & 0 \\ 0 & \mathbf{Q}_{DOA} \end{bmatrix} \tag{29}$$

where $\mathbf{Q}_{DOA} = diag\{\sigma_1^2\hat{d}_1^2, \ldots, \sigma_M^2\hat{d}_M^2\}$. And $\sigma_i^2(i = 1, \ldots, M)$ are DOA variance.

3 Proposed Approach

In Chan method [16], the authors bring in unknown r_1 in the first least squares estimator and assume $\mathbf{x}$ and r_1 are uncorrelated. For a more precise estimation, another least squares estimator is needed and the computational complexity is high. If we have AOA measurements of target already, we do not have to use r_1 and solve least squares solution twice.

According to geometry relationship, we have:

$$r_m \cos\theta_m = x - x_m \tag{30}$$

$$r_m \sin\theta_m = y - y_m \tag{31}$$

Combine Eq. (30) and (31), yields:

$$r_m = (x - x_m)\cos\theta_m + (y - y_m) sin\theta_m \tag{32}$$

Note that:

$$(x - x_m)\sin\theta_m = (y - y_m)\cos\theta_m \tag{33}$$

As a matter of fact, we are not able to measure the θ_m and r_m precisely, so we rewrite the equations that measure angle and distance as:

$$\hat{\theta}_m = \theta_m + n_{\theta m} \tag{34}$$

$$\hat{r}_{m1} = r_m - r_1 + n_{rm} \tag{35}$$

where $\mathbf{n}_\theta, \mathbf{n}_r$ are not correlated. Substituting (34) into (32) and using the first order Taylor series, we get:

$$x\left(\cos\hat{\theta}_m - \cos\hat{\theta}_1\right) + y\left(\sin\hat{\theta}_m - \sin\hat{\theta}_1\right) = x_m\cos\hat{\theta}_m - x_1\cos\hat{\theta}_1 + y_m\sin\hat{\theta}_m - y_1\sin\hat{\theta}_1 + \hat{r}_{m1} + \varepsilon_m \tag{36}$$

where

$$\varepsilon_m = \left[(x_m - x)\sin\hat{\theta}_m + (y - y_m)\cos\hat{\theta}_m\right]n_{\theta m} - \left[(x_1 - x)\sin\hat{\theta}_1 + (y - y_1)\cos\hat{\theta}_1\right]n_{\theta 1} - n_{rm} \tag{37}$$

Then we get another linear equation about target location $\mathbf{x} = [x, y]^T$, and the equation do not have unknown term r_1. So, we need only one least squares estimator and the computation load is only the half. For a more precise estimation, we rewrite (36) in the form of vector and combine linear Eq. (5):

$$\mathbf{Ax} = \mathbf{b} + \mathbf{e} \tag{38}$$

where:

$$\mathbf{A} = \begin{bmatrix} \sin\hat{\theta}_1 & -\cos\hat{\theta}_1 \\ \vdots & \vdots \\ \sin\hat{\theta}_M & -\cos\hat{\theta}_M \\ \cos\hat{\theta}_2 - \cos\hat{\theta}_1 & \sin\hat{\theta}_2 - \sin\hat{\theta}_1 \\ \vdots & \vdots \\ \cos\hat{\theta}_M - \cos\hat{\theta}_1 & \sin\hat{\theta}_M - \sin\hat{\theta}_1 \end{bmatrix} \tag{39}$$

$$\mathbf{b} = \begin{bmatrix} x_1 \sin\hat{\theta}_1 - y_1 \cos\hat{\theta}_1 \\ \vdots \\ x_M \sin\hat{\theta}_M - y_M \cos\hat{\theta}_M \\ x_2 \cos\hat{\theta}_2 - x_1 \cos\hat{\theta}_1 + y_2 \sin\hat{\theta}_2 - y_1 \sin\hat{\theta}_1 + r_{21} \\ \vdots \\ x_M \cos\hat{\theta}_M - x_1 \cos\hat{\theta}_1 + y_M \sin\hat{\theta}_M - y_1 \sin\hat{\theta}_1 + r_{M1} \end{bmatrix} \tag{40}$$

$$\mathbf{e} = \begin{bmatrix} \left[(x - x_1)\cos\hat{\theta}_1 + (y - y_1)\sin\hat{\theta}_1\right] n_{\theta 1} \\ \vdots \\ \left[(x - x_M)\cos\hat{\theta}_M + (y - y_1)\sin\hat{\theta}_M\right] n_{\theta M} \\ \left[(x_2 - x)\sin\hat{\theta}_2 + (y - y_2)\cos\hat{\theta}_2\right] n_{\theta 2} - \left[(x_1 - x)\sin\hat{\theta}_1 + (y - y_1)\cos\hat{\theta}_1\right] n_{\theta 1} - n_{r2} \\ \vdots \\ \left[(x_M - x)\sin\hat{\theta}_M + (y - y_M)\cos\hat{\theta}_M\right] n_{\theta M} - \left[(x_1 - x)\sin\hat{\theta}_1 + (y - y_1)\cos\hat{\theta}_1\right] n_{\theta 1} - n_{rM} \end{bmatrix}$$
$$= \begin{bmatrix} \mathbf{\Lambda} \mathbf{n}_\theta \\ \mathbf{C}\mathbf{n}_\theta - \mathbf{n}_r \end{bmatrix} \tag{41}$$

where

$$\mathbf{\Lambda} = diag\left\{\left[\, (x - x_1)\cos\hat{\theta}_1 + (y - y_1)\sin\hat{\theta}_1 \quad \cdots \quad (x - x_M)\cos\hat{\theta}_M + (y - y_M)\sin\hat{\theta}_M \,\right]\right\} \tag{42}$$

$$\mathbf{C} = [\,\mathbf{a} \quad \mathbf{\Sigma}\,] \tag{43}$$

$$\mathbf{a} = -\left[(x_1 - x)\sin\hat{\theta}_1 + (y_1 - y)\cos\hat{\theta}_1\right] \mathbf{1}_{M-1} \tag{44}$$

$$\mathbf{\Sigma} = diag\left\{\left[\, (x_2 - x)\sin\hat{\theta}_2 + (y - y_2)\cos\hat{\theta}_2 \quad \cdots \quad (x_M - x)\sin\hat{\theta}_M + (y - y_M)\cos\hat{\theta}_M \,\right]\right\} \tag{45}$$

$$\mathbf{n}_\theta = [n_{\theta 2}, \ldots, n_{\theta M}]^T \tag{46}$$

$$\mathbf{n}_r = [n_{r2}, \ldots, n_{rM}]^T \tag{47}$$

where $\mathbf{1}_{M-1}$ in Eq. (44) is a $(M-1) \times 1$ vector which all elements are 1.

To solve the linear Eq. (38), the least square solution can be obtained:

$$\mathbf{x}_{LS} = \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{b} \tag{48}$$

To promote to performance of the solution of the linear Eq. (38), we using the weighted least square method to solve the equation, firstly, the statistical property of error vector $\mathbf{e}$ is shown as follows:

$$E(\mathbf{e}) = \mathbf{0}_{2M-1} \tag{49}$$

Table 1 The calculation procedure for proposed location algorithm

Step 1	Using Eq. (39), (40) to obtain the matrix $\mathbf{A}$ and vector $\mathbf{b}$
Step 2	Using Eq. (48) to obtain the least square solution $\mathbf{x}_{LS}$
Step 3	Using $\mathbf{x}_{LS}$, Eqs. (42), (43), (44), (45) and (50) to compute weighted matrix $\mathbf{Q}$
Step 4	Using Eq. (51) to obtain the weighted least square solution $\mathbf{x}_{WLS}$

$$E\left(\mathbf{e}\mathbf{e}^T\right) = \mathbf{Q} = \begin{bmatrix} \boldsymbol{\Lambda}\mathbf{Q}_\theta\boldsymbol{\Lambda}^T & \boldsymbol{\lambda}\mathbf{Q}_\theta\mathbf{C}^T \\ \mathbf{C}\mathbf{Q}_\theta\boldsymbol{\Lambda}^T & \mathbf{C}\mathbf{Q}_\theta\mathbf{C}^T + \mathbf{Q}_r \end{bmatrix} \tag{50}$$

where $\mathbf{0}_{2M-1}$ is a $(2M-1)\times 1$ vector which all elements are zero. The optimal weighed least square solution of target coordinates $\mathbf{x}$ is represented as:

$$\mathbf{x}_{WLS} = \left(\mathbf{A}^T\mathbf{Q}^{-1}\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{Q}^{-1}\mathbf{b} \tag{51}$$

Notice that weighted matrix $\mathbf{Q}$ depends on unknown vector $\mathbf{x}$, we set $\mathbf{Q}$ as unit matrix firstly, then use the initial solution to solve $\mathbf{Q}$, thus we get the final location solution through (51) (Table 1).

4 Computational Complexity

In this section, we will calculate the computation complexity of HWLS method and our method. We test only the multiplications, and we do not care the computation load of matrix elements.

1. HWLS Method [7]

In the first step, the matrices $\mathbf{A}$, $\mathbf{b}_1$, $\mathbf{b}_2$ and $\mathbf{Q}_{TDOA}^{-1}$ are computed. Since $\mathbf{A}$ is a $(M-1)\times 2$ matrix, $\mathbf{b}_1$, $\mathbf{b}_2$ are the size of $(M-1)\times 1$, and $\mathbf{Q}_{TDOA}^{-1}$ is $(M-1)$ order matrix, which need $(M-1)^3$ multiplications to calculate them. Thus the $\left(\mathbf{A}^T\mathbf{Q}_{TDOA}^{-1}\mathbf{A}\right)^{-1}$ require $(M-1)^3 + 2(M-1)^2 + 4(M-1) + 2^3$ multiplications. So, the step one need totally $(M-1)^3 + 4(M-1)^2 + 11(M-1) + 8$ multiplications.

While in the second step, as is the same as the first step, needs $(2M-1)^3 + 4(2M-1)^2 + 10(2M-1) + 8$ multiplications.

Total multiplications of HWLS method is:

$$\begin{aligned}&(M-1)^3 + 4(M-1)^2 + 11(M-1) + (2M-1)^3 + 4(2M-1)^2 + 10(2M-1) + 16 \\ &\quad = 9M^3 + 5M^2 + 16M + 1\end{aligned}$$

2. Proposed method

In our method, we first calculate $\mathbf{x}_{LS}$ to acquire an initial guess, where $\mathbf{x}_{LS} = \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{b}$. Since $\mathbf{A}$ and $\mathbf{b}$ are not special matrices, we need $10(2M-1)+8$ multiplications to calculate them.

Then, we use $\mathbf{x}_{LS}$ to construct weighted matrix $\mathbf{Q}$. Since $\mathbf{\Lambda}$, $\mathbf{Q}_\theta$ are diagonal matrices, $\mathbf{C}$ is the union of a vector and a diagonal matrix, we need $M^2 + 7M - 5$ multiplications to generate $\mathbf{Q}$.

At last, a weighted least squares estimator is employed, as the same as WLS method, $(2M-1)^3 + 4(2M-1)^2 + 10(2M-1) + 8$ multiplications are required.

Total multiplications of our method is:

$$(2M-1)^3 + 4(2M-1)^2 + 10(2M-1) + M^2 + 7M + 3 = 8M^3 + 5M^2 + 17M + 6$$

Compare the multiplications required of both two methods, we find that the computational complexity of our method is slightly smaller than HWLS method.

5 Simulation Results

A. Performance of several specified target locations with different receivers' location and different measurement errors.

Simulation context: TDOA and AOA measurement errors are mutually independent zero mean Gaussian white noise. Their variance are $\sigma_t = 10/c\text{m}$, $\sigma_\theta = 1°$, c is the speed of light. Assuming M base stations and their distributional radius $R = 3\text{km}$, locating at $\mathbf{x}_{rm} = R[\cos(2\pi(m-1)/M), \sin(2\pi(m-1)/M)], \quad m = 1, 2\ldots M$, we simulate under the circumstances of $M = 4, 5, 6$ and simulation results are shown in Figs. 3, 4 and 5, respectively.

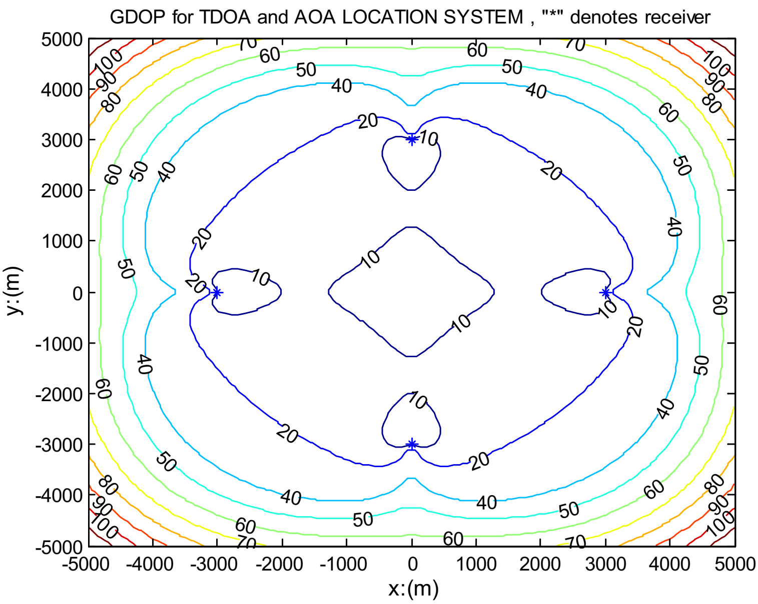


Fig. 3 GDOP for the TDOA/AOA location, four BSs, $\sigma_d = 10\text{m}$, $\sigma_\theta = 1°$

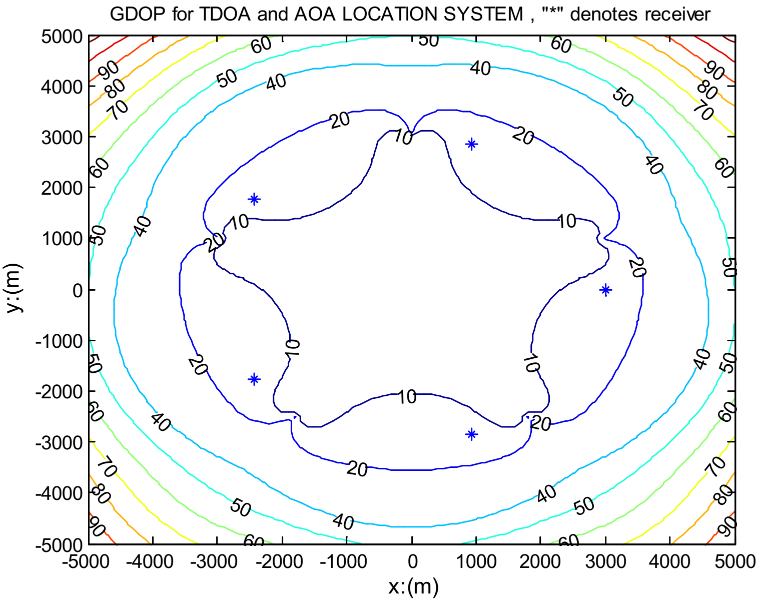


Fig. 4 GDOP for the TDOA/AOA location, five BSs, $\sigma_d = 10\text{m}$, $\sigma_\theta = 1^\circ$

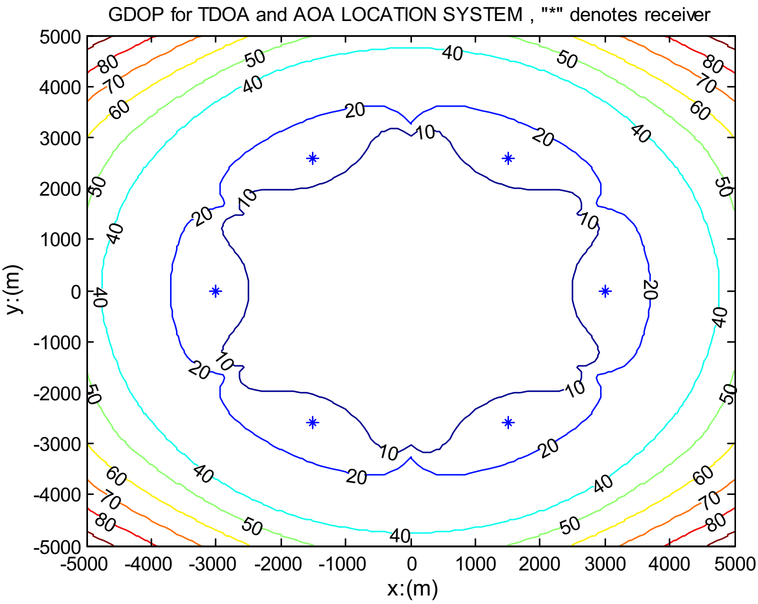


Fig. 5 GDOP for the TDOA/AOA location, six BSs, $\sigma_d = 10\text{m}$, $\sigma_\theta = 1^\circ$

B. The performance of our algorithm and the WLS method and their comparison with CRB.

We assume target at (1, 1 km), and place receivers in the corners of a regular hexagon whose radius is $R=$ 3 km to simulate a scenario of cellular network. Receivers positions are at $\mathbf{x}_{rm} = R[\cos(2\pi(m-1)/6), \sin(2\pi(m-1)/6)], \quad m = 1, 2\ldots6$. Measurements are AOA and TDOA (time difference of arrival with respect to the first receiver) information. Measurement errors are zero mean Gaussian white noise and are uncorrelated. $RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\mathbf{x}-\hat{\mathbf{x}}_i)^T(\mathbf{x}-\hat{\mathbf{x}}_i)}$. HWLS method is based on [7].

When AOA error is fixed, we compare the performance of these two location methods. We set AOA error $\sigma_\theta = 0.5^\circ, 1^\circ, 1.5^\circ, 2^\circ$, after 10,000 Monte Carlo simulations, we get Figs. 6, 7, 8 and 9.

C. When TDOA error is fixed, we test the influence of these two methods through different AOA errors. We set TDOA error as $\sigma_d = 50, 100, 200$ m, we run Monte Carlo simulations 10,000 times and obtain Figs. 10, 11 and 12.

Fig. 6 Performance comparison using TDOA and AOA measurements, $\sigma_\theta = 0.5^\circ$

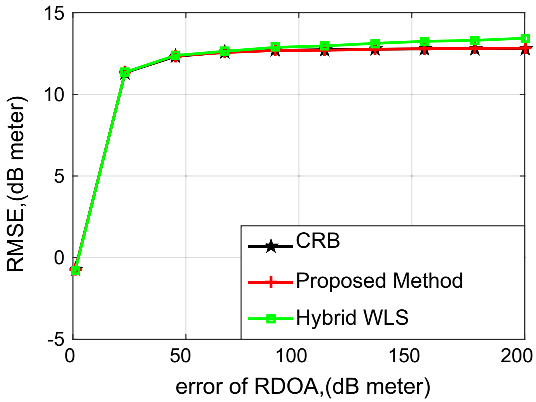
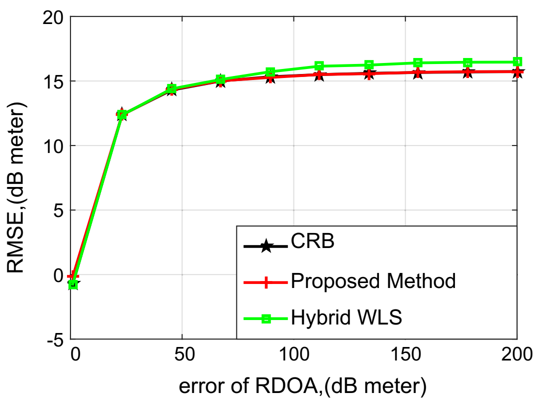


Fig. 7 Performance comparison using TDOA and AOA measurements, $\sigma_\theta = 1^\circ$



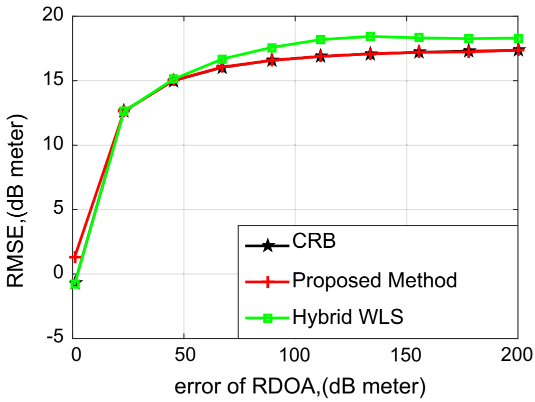


Fig. 8 Performance comparison using TDOA and AOA measurements, $\sigma_\theta = 1.5°$

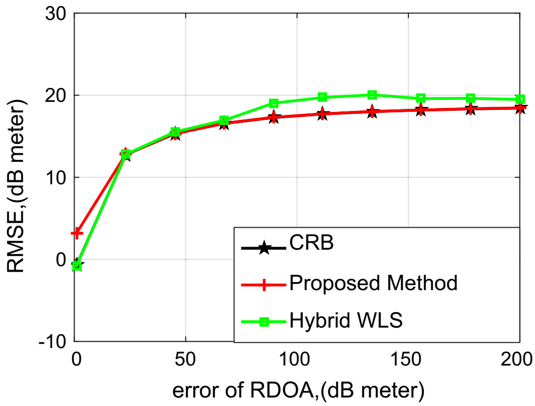


Fig. 9 Performance comparison using TDOA and AOA measurements, $\sigma_\theta = 2°$

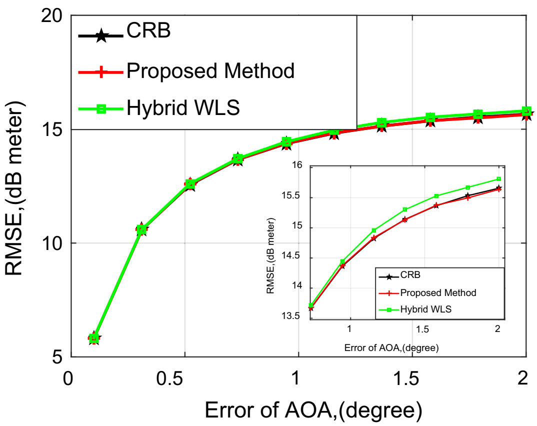


Fig. 10 Performance comparison using TDOA and AOA measurements, $\sigma_d = 50$m

Concluding from simulation figures above, we find that the algorithm we proposed can attain to CRB (the derivation of CRB is presented in appendix) and outperforms HWLS method. Both two methods work well when AOA and TDOA errors are small.

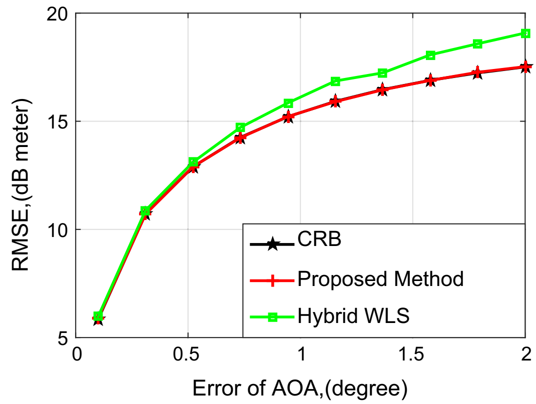


Fig. 11 Performance comparison using TDOA and AOA measurements, $\sigma_d = 100$m

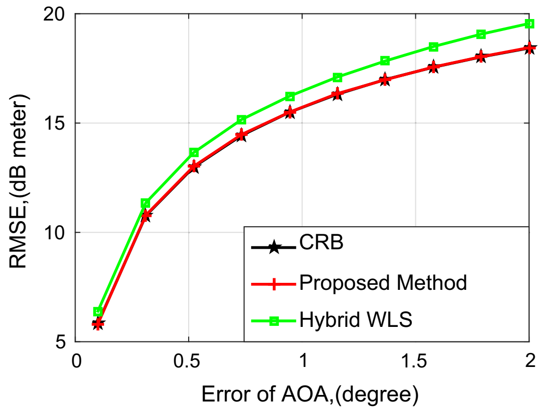


Fig. 12 Performance comparison using TDOA and AOA measurements, $\sigma_d = 200$m

As the growing of measurement error, WLS method start to deviate CRB. As we can see from Fig. 10, when $\sigma_d = 50$m, as $\sigma_\theta < 0.7^\circ$, two methods show good performance on target localization, but as the error of AOA increase, the proposed method can be better than HWLS. In Fig. 12, when $\sigma_d = 200$ m, HWLS method perform worse than our method. That's because the HWLS method rely on the precision of initial guess in the first step, which is based only on TDOA information. While in our method, we unite both AOA and TDOA information to locate target. Actually, we use AOA information twice. When we use target location to linearize TDOA equation, we derive a target location estimator that unite AOA, TDOA information and target coordinates together based on geometry relationship in Fig. 2. That is why the proposed method weaken the influence of the measurement error. Moreover, we use more AOA information (we use it six times as in our simulation) to reduce the reliance on the precision of AOA measurements, as can be proved by the simulation results – the proposed method can always attain to CRB.

6 Conclusion

In this paper, we proposed a new 2D target positioning method using AOA/TDOA information in mobile communication systems. We deduced the linear equation by means of AOA/TDOA and target location and then unite AOA information to obtain finally linear equation. At last, we solve a closed-form solution of target information using weighted least squares method. Simulation shows that our algorithm can attain to CRB and has good performance when AOA and TDOA parameters have large measurement error while HWLS method deviate CRB. In our paper, we do not bring in visual range and solve directly, which reduce the computation cost.

Appendix

The derivation of CRB:

We present the PDF based on (34), (35) as:

$$p(\mathbf{x}|\boldsymbol{\theta},\mathbf{r}) = \frac{1}{(2\pi)^{\frac{2M-1}{2}}|\mathbf{Q}_\theta|^{\frac{1}{2}}|\mathbf{Q}_r|^{\frac{1}{2}}}\exp\left\{-\frac{\left(\hat{\theta}-\boldsymbol{\theta}\right)^T\mathbf{Q}_\theta^{-1}\left(\hat{\theta}-\boldsymbol{\theta}\right)}{2}\right\}\exp\left\{-\frac{(\hat{\mathbf{r}}-\mathbf{r})^T\mathbf{Q}_r^{-1}(\hat{\mathbf{r}}-\mathbf{r})}{2}\right\} \tag{52}$$

The likelihood function is as follows:

$$L = \ln\{p(\mathbf{x}|\boldsymbol{\theta},\mathbf{r})\} = c - \frac{\left(\hat{\theta}-\boldsymbol{\theta}\right)^T\mathbf{Q}_\theta^{-1}\left(\hat{\theta}-\boldsymbol{\theta}\right)}{2} - \frac{(\hat{\mathbf{r}}-\mathbf{r})^T\mathbf{Q}_r^{-1}(\hat{\mathbf{r}}-\mathbf{r})}{2} \tag{53}$$

where $c = -\frac{2M-1}{2}\ln(2\pi) + \frac{1}{2}\ln|\mathbf{Q}_\theta| + \frac{1}{2}\ln|\mathbf{Q}_r|$ is the constant which is independent of $\mathbf{x}$.

FISHER information matrix is given by:

$$\mathbf{J} = E\left\{\frac{\partial L}{\partial \mathbf{x}}\frac{\partial L}{\partial \mathbf{x}^T}\right\} = \frac{\partial \boldsymbol{\theta}^T}{\partial \mathbf{x}}\mathbf{Q}_\theta^{-1}\frac{\partial \boldsymbol{\theta}}{\partial \mathbf{x}^T} + \frac{\partial \mathbf{r}^T}{\partial \mathbf{x}}\mathbf{Q}_r^{-1}\frac{\partial \boldsymbol{\theta}}{\partial \mathbf{r}^T} \tag{54}$$

The CRB of the problem is:

$$CRB\_\mathbf{x} = \mathbf{J}^{-1} \tag{55}$$

where

$$\frac{\partial \boldsymbol{\theta}^T}{\partial \mathbf{x}} = \begin{bmatrix} \frac{\partial \theta_1}{\partial x} & \frac{\partial \theta_1}{\partial y} \\ \vdots & \\ \frac{\partial \theta_M}{\partial x} & \frac{\partial \theta_M}{\partial y} \end{bmatrix} \qquad \frac{\partial \mathbf{r}^T}{\partial \mathbf{x}} = \begin{bmatrix} \frac{\partial r_{21}}{\partial x} & \frac{\partial r_{21}}{\partial y} \\ \vdots & \\ \frac{\partial r_{M1}}{\partial x} & \frac{\partial r_{M1}}{\partial y} \end{bmatrix} \tag{56}$$

$$\frac{\partial \theta_m}{\partial x} = \frac{-(y-y_m)}{(x-x_m)^2+(y-y_m)^2} = \frac{-\sin\theta_m}{r_m} \tag{57}$$

$$\frac{\partial\theta_m}{\partial y}=\frac{x-x_m}{(x-x_m)^2+(y-y_m)^2}=\frac{\cos\theta_m}{r_m} \tag{58}$$

$$\frac{\partial r_m}{\partial x}=\frac{x-x_m}{\sqrt{(x-x_m)^2+(y-y_m)^2}}=\cos\theta_m \tag{59}$$

$$\frac{\partial r_m}{\partial y}=\frac{y-y_m}{\sqrt{(x-x_m)^2+(y-y_m)^2}}=\sin\theta_m \tag{60}$$

If the measurement errors are identity, that is:

$$\mathbf{Q}_\theta=\sigma_\theta^2\mathbf{I}_M \quad \mathbf{Q}_r=\sigma_r^2\mathbf{H}_{M-1} \tag{61}$$

where

$$\mathbf{H}_{M-1}=\mathbf{1}\mathbf{1}^T+\mathbf{I}_{M-1} \tag{62}$$

According to matrix inverse theory, we have:

$$\mathbf{H}_{M-1}^{-1}=-\frac{1}{M}\mathbf{1}\mathbf{1}^T+\mathbf{I}_{M-1} \tag{63}$$

Yields:

$$\mathbf{J}=\begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22}\end{bmatrix} \tag{64}$$

where

$$J_{11}=\sigma_\theta^{-2}\sum_{m=1}^{M}\frac{\sin^2\theta_m}{r_m^2}+\sigma_r^{-2}\frac{M-1}{M}\sum_{m=2}^{M}(\cos\theta_m-\cos\theta_1)^2 \tag{65}$$

$$J_{12}=J_{21}=-\sigma_\theta^{-2}\sum_{m=1}^{M}\frac{\sin\theta_m\cos\theta_m}{r_m^2}+\sigma_r^{-2}\frac{M-1}{M}\sum_{m=2}^{M}(\cos\theta_m-\cos\theta_1)(\sin\theta_m-\sin\theta_1) \tag{66}$$

$$J_{22}=\sigma_\theta^{-2}\sum_{m=1}^{M}\frac{\cos^2\theta_m}{r_m^2}+\sigma_r^{-2}\frac{M-1}{M}\sum_{m=2}^{M}(\sin\theta_m-\sin\theta_1)^2 \tag{67}$$

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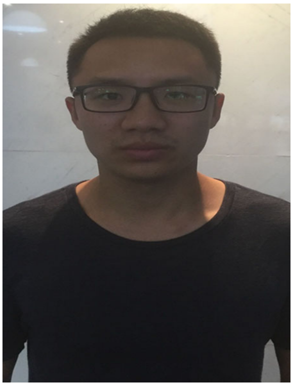

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