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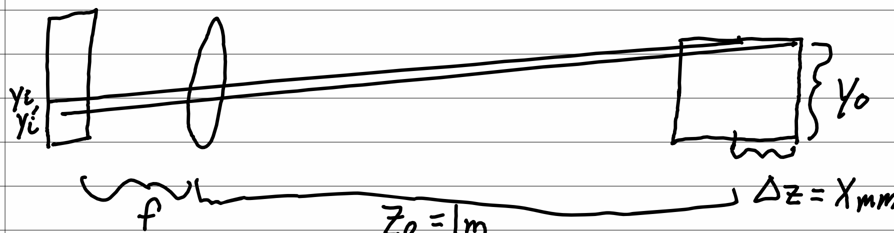
Written Assignment

1) a) i)



$$y_i = f\left(\frac{Y_o}{Z_o}\right) \qquad y_i' = f\left(\frac{Y_o}{Z_o + \Delta z}\right)$$

$$\left| \frac{y_i' - y_i}{y_i'} \right| \simeq 0.05$$

$$\left| \frac{f\left(\frac{Y_o}{Z_o}\right) - f\left(\frac{Y_o}{Z_o+\Delta z}\right)}{f\left(\frac{Y_o}{Z_o+\Delta z}\right)} \right| = 0.05$$

$$\left| \frac{\frac{1}{Z_o} - \frac{1}{Z_o+\Delta z}}{\frac{1}{Z_o+\Delta z}} \right| = \left| \frac{\Delta z \cancel{(Z_o+\Delta z)}}{Z_o \cancel{(Z_o+\Delta z)}} \right| = 0.05$$

$$\frac{\Delta z}{1m} \simeq 0.05 \qquad \Delta z = 50mm$$

$$X = 2\Delta z = 100mm$$

ii) $\frac{b}{D} = \left| \frac{y_i' - y_i}{y_i'} \right| = 0.05$

$b = 0.01\,mm$

$D = \frac{0.01}{0.05} = 0.2\,mm$

b) ii) line 1 direction: $(0,0,t)$

$(x_{vp}, y_{vp}) = \left(-1 \cdot \frac{0}{t}, -1 \cdot \frac{0}{t}\right) = (0,0)$

line 2 direction: $(0,t,t)$

$(x_{vp}, y_{vp}) = \left(-1 \cdot \frac{0}{t}, -1 \cdot \frac{t}{t}\right) = (0,-1)$

line 3 direction: $(0,-t,t)$

$(x_{vp}, y_{vp}) = \left(-1 \cdot \frac{0}{t}, -1 \cdot \frac{-t}{t}\right) = (0,1)$

iii) line 1: $(0,0,t)$

$(x_{vp}, y_{vp}) = \left(-1 \cdot \frac{0}{t}, -1 \cdot \frac{0}{t}\right) = (0,0)$

line 2 direction: $(t,0,t)$

$(x_{vp}, y_{vp}) = \left(-1 \cdot \frac{t}{t}, -1 \cdot \frac{0}{t}\right) = (-1,0)$

line 3 direction: $(-t, 0, t)$

$$(x_{vp}, y_{vp}) = \left(-1 \cdot \frac{-t}{t}, -1 \cdot \frac{0}{t}\right) = (1, 0)$$

c) We know that to find the vanishing point of a line on an image, we can translate that line so that it passes through the origin. The vanishing point is the coordinates where the translated line intersects with the image plane.

Since all the lines we are considering lie on the plane, it is sufficient to translate the entire plane so that it goes through the origin. This new plane has equation:

$$Ax + By + Cz = 0$$

Now we must find where the new plane intersects the image plane. The image plane is located 1 unit behind the origin on the z-axis, so the image plane has equation $z = -1$. Substituting in, we find the desired line:

$$Ax + By = C$$

2 Using the sum of angles formula:

$$\sin(kx+b) = \sin(kx)\cos(b) + \cos(kx)\sin(b)$$

So now we can take the fourier transform of $\sin(kx+b)$. Notice that $\sin(b)$ & $\cos(b)$ are constants w.r.t x, so we can factor them out of the fourier transform. We can then use the identities from lecture:

$$\mathcal{F}\{\sin(kx+b)\}$$

$$= \mathcal{F}\{\sin(kx)\cos(b) + \cos(kx)\sin(b)\}$$

$$= \mathcal{F}\{\sin(kx)\}\cos b + \mathcal{F}\{\cos(kx)\}\sin b$$

$$= i\pi[\delta(f+k) - \delta(f-k)]\cos b$$
$$+ \pi[\delta(f+k) + \delta(f-k)]\sin b$$

$$X = \mathrm{Im}\{\mathcal{F}\{\sin(kx+b)\}\}$$
$$= \pi[\delta(f+k) - \delta(f-k)]\cos b$$

$$Y = \mathrm{Re}\{\mathcal{F}\{\sin(kx+b)\}\}$$
$$= \pi[\delta(f+k) + \delta(f-k)]\sin b$$

$$\operatorname{atan}\left(\frac{y}{x}\bigg|_{f=-k}\right) = \operatorname{atan}\left(\frac{\sin b}{\cos b}\right) = b$$

$$\operatorname{atan}\left(\frac{y}{x}\bigg|_{f=k}\right) = \operatorname{atan}\left(-\frac{\sin b}{\cos b}\right) = -b$$