Programming Assignment 4 Report

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1 Introduction

The purpose of this assignment is to complete programming for an ICP algorithm, building off of the programming done in the previous programming assignment. The purpose of ICP has been discussed previously. It is to register two different coordinate systems to one common one so that each tool, robot arm, or imaging system operate in the same frame.

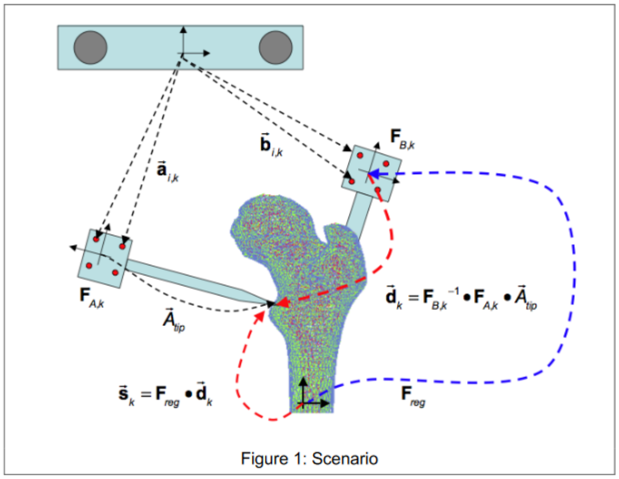

$\vec{\mathbf{b}}_{i,k}$
$\mathbf{F}_{B,k}$
$\vec{\mathbf{a}}_{i,k}$
$\mathbf{F}_{A,k}$
$\vec{A}_{tip}$
$\vec{\mathbf{d}}_k = \mathbf{F}_{B,k}^{-1} \bullet \mathbf{F}_{A,k} \bullet \vec{A}_{tip}$
$\vec{\mathbf{s}}_k = \mathbf{F}_{reg} \bullet \vec{\mathbf{d}}_k$
$\mathbf{F}_{reg}$
Figure 1: Scenario

Figure 1: Problem Description

The relevant variables from Fig. 1 are:

- $F_{A,k}$: The local coordinate frame of the calibration object A .
- $F_{B,k}$: The local coordinate frame of the calibration object B.
- $a_{i,k}$: The position of the LED marker i on the calibration object A.
- $b_{i,k}$: The position of the LED marker i on the calibration object B.
- A_{tip}: The transformation $F_{A,k}$ to the tip of the tool for calibration object A.

- B_{tip}: The transformation $F_{B,k}$ to the tip of the tool for calibration object B.
- d_k: The position of the pointer tip with respect to rigid body B.
- F_{reg}: The local coordinate frame of the EM tool (defined with respect to the EM tracker base).
- s_k: Sample point defined in the registration frame F_{reg}.
- c_k: The point on the mesh closest to the point s_k.

2 Mathematical Approach

We describe how to find matches between a test point, **a**, and a triangular mesh, as well as the general mathematic steps to set up the registration problem posed in the problem statement.

2.1 Finding c_k and d_k

This was discussed in Programming Assignment 4 Report. In sum, we find

$$d_k = F_{B,k}^{-1} F_{A,k} A_{tip}$$

For PA #3, we assumed that F_{reg} is the identity matrix. Now, we use the ICP algorithm with $F_{reg} = \mathbb{1}$ as the initial guess, and update iteratively to find the correct F_{reg} and match s_k with c_k, where:

$$s_k = F_{reg} d_k$$

and c_k is the closest point on the mesh to the point s_k. The details of the ICP algorithm are less mathematical in nature and are outlined in §Algorithmic Steps.

3 Algorithmic Steps

1. First, calculate d_k, as outlined in the mathematical approach.
2. Build a tree with the mesh provided.
3. Initialize $F_{reg} = \mathbb{1}$.
4. Initialize a maximum distance threshold for valid matches. We found that starting with $\eta_0 = 5$ works well empirically.
5. Perform ICP:
   a) Calculate $s_k = F_{reg} d_k$. Note that s_k will be updating, but d_k is constant.
   b) For some amount of max iterations:
      i. Search tree for points on mesh closest to s_k.
      ii. Find the distances between the found points c_k and given points s_k, such that $D_k = c_k - s_k$.

iii. Filter out the valid matches of c_k and s_k by only using the points which have a distance $D_k < \eta_k$.

iv. Calculate the residual errors of the valid matches.

v. Calculate some error statistics, including the maximum magnitude of the error, the standard deviation of the error, and the average error, using the residuals.

$$\sigma_n \longleftarrow \frac{\sqrt{\sum_k e_k \cdot e_k}}{NumElts(E)}$$

$$(\epsilon_{max})_n \longleftarrow \min_k \sqrt{e_k \cdot e_k}$$

$$\bar{\epsilon}_n \longleftarrow \frac{\sum_k \sqrt{e_k \cdot e_k}}{NumElts(E)}$$

vi. Check to see if the calculated F_{reg} is good enough:

A. Based on the criteria chosen of the above $\sigma_{current}$, $\epsilon_{max,current}$, $\bar{\epsilon}_{current}$, check if it is lower than some empirically determined constant. For our purposes we used the σ criterion where $\sigma_{current} < 0.01$ was sufficient.

B. Check if the error has not changed much for some number of iterations, by calculating $\gamma = \bar{\epsilon_k}/\bar{\epsilon_{k-1}}$. For our purposes we checked the past 5 ratios (γ). We also added smoothing, both to the numerator and denominator to cover for the case where the test converged within the number of ratios checked, and the error would be exactly 0, and make the ratio NaN.

C. If both of the previous two conditions are true, stop the search. If the termination criteria has not been reached, continue on.

vii. Update the points and transformation. Use point cloud registration between the current surface points s_k and matched points c_k to find F then update:

$$F_{reg} \longleftarrow FF_{reg}$$

$$s \longleftarrow F_{reg}s$$

viii. Repeat until convergence or max iterations finishes.

4 Software Overview / Program Structure

Description of files in the programs directory. Subsections represent directories. Detailed explanations of what every function or subroutine does can be found inline in comments of code, including input and output arguments.

PA_4

- `main.m`: Function that executes the algorithmic steps in §Algorithmic Steps for a single text file. Edit the inputs near the top to change the files used. `problemNum` for PA #4 will always be 4, `sampleReadingLetter` corresponds to the different scenarios posed by the text files, and `fileType` can be either "debug" or "unknown." The output will be saved in the **output** folder with the designated filename.

- `init.m`: This file adds all files and files in sub-directories to the path. It is run at the beginning of main to initialize the paths needed.

4.1 programs

- `generate_outputs.m`: runs the main file multiple times for all text files given.

- `ICP.m`: Performs the ICP algorithm outlined in §Algorithmic Steps.

- `CalcStats.m`: Calculates the standard deviation, maximum error, and average error given a residual matrix.

- `TerminationTest.m`: Checks if the algorithm has converged or not based on convergence criterion outlined in §Algorithmic Steps.

4.1.1 test_data

Folder of data used for unit testing.

- `generate_cube.m`: makes a cube in 3D space ranging from (+/-1,+/-1,+/-1) and saves it to cube.mat

- `generate_rectangular_prism_mat.m`: makes a rectangular box in 3D space ranging from (+/-1,+/-2,+/-3) and saves it to rect_prism.mat

- `cube.mat`: resulting file from `generate_cube.m`, containing vertices and the corresponding corner indices.

- `rectangular_prism_mat.mat`: resulting file from `generate_rectangular_prism_mat.m`, containing vertices and the corresponding corner indices.

4.1.2 unit_testing

Files here are for unit testing different functions. We will go into more depth in the testing section.

- `ICP_Box_Unit_Testing.m`: Tests that the ICP algorithm can correctly reconstruct three transformations on a box mesh: rotation, translation, and rotation + translation

- `ICP_Mesh_Unit_Testing.m`: Tests that the ICP algorithm can correctly reconstruct three transformations on the `Problem4MeshFile.sur` mesh: rotation, translation, and rotation + translation

4.1.3 util

Files here are general usage functions used throughout `main.m` as well as the unit tests, all from previous programming assignments.

- **kinematic**
  - `homoify.m`: Takes in a homogeneous vector and makes it non-homogenous (removes the 1). Alternatively, takes in a non-homogeneous vector, and makes it homogeneous.
  - `randomSE3.m`: Generates a random valid rotation and translation and returns the transform.
  - `rot_rpy.m`: Given an input vector q = [roll, pitch, yaw], rotates about the correct axes and returns the resultant rotation matrix.
  - `RX.m`: Given a scalar angle, returns the rotation about the x-axis by this angle.
  - `RY.m`: Given a scalar angle, returns the rotation about the y-axis by this angle.
  - `RZ.m`: Given a scalar angle, returns the rotation about the z-axis by this angle
  - `SE3_INV.m`: Uses SE3 properties to quickly calculate the inverse of a given transformation.
  - `SE3.m`: Takes in a rotation and translation, and quickly appends them to the right format
  - `randomSE3.m`: Generates a random valid member of SE3.
- **registrations**: These are the registration/calibration functions from PA # 1
  - `Point_Cloud_Registration.m`: Takes in two sets of points (A, B, with points in row format), finds the transformation from one set of points to the other by performing point cloud calibration/registration.
- **read**: These are the files read in the text data given into a desired format.
  - `read_ProblemX_BodyY.m`: Reads a `ProblemX_BodyY.txt` file and parses the data
  - `read_SampleReadings.m`: Reads a `SampleReadings.txt` file and parse the data.
  - `read_ProblemXMesh.m`: Reads a `ProblemXMesh.txt` file and parses the data.
  - **tree_search**
    - `Project_On_Segment.m`: This function takes a point c and projects it on the line formed by the two points p and q, and returns the projection.
    - `Find_Closest_Point_Triangle.m`: This function takes a matrix of triangle vertices, and a point a, and calculates the closest point on the triangle to the point.

    - `Find_Closest_Point_Mesh.m`: This function implements brute force search for closest point on mesh via a linear search that keeps track of the closest point and minimum distance.
- **data_structures** Folder for classes used for different search methods.
    - `CovTreeNode.m`: Covariance tree node class. Implements the functions for the search as well as for tree building.
    - `Triangle.m`: Triangle class. Implements functions to find the closest point on a triangle to a test point, and to create and enlarge the bounding boxes of a triangle.

5 Verification

5.1 Unknown Data Results

This is the output files of all the unknown data. See the appendix for the output data for the debugging data.

Listing 1: Program Generated PA4-G-Unknown-SampleReadingsTest.txt

```
200, pa4-G-Output.txt
  -42.89   -16.65   -27.30        -42.89   -16.65   -27.30      0.002
   17.27    19.15   -30.41         17.27    19.15   -30.41      0.005
  -44.05    -8.49   -26.39        -44.04    -8.49   -26.38      0.007
   38.23    -0.35    -4.81         38.23    -0.35    -4.81      0.000
  -36.50    -3.73   -41.89        -36.49    -3.73   -41.89      0.001
   10.92    24.87    15.04         10.92    24.87    15.04      0.007
   35.23    -4.39   -10.09         35.23    -4.39   -10.09      0.000
   22.28    14.71    38.43         22.28    14.71    38.43      0.001
   -0.78    -6.13    56.36         -0.78    -6.12    56.36      0.008
   17.40    -4.36    45.03         17.41    -4.37    45.03      0.002
   23.18     2.86    43.09         23.18     2.86    43.09      0.000
   27.38    22.97    -7.35         27.38    22.97    -7.35      0.004
   20.45    22.38   -26.42         20.45    22.39   -26.42      0.008
  -38.76   -19.87   -16.10        -38.76   -19.87   -16.10      0.002
   18.47    26.59     2.20         18.47    26.59     2.20      0.005
  -24.02   -18.49    -6.40        -24.02   -18.49    -6.40      0.001
   -4.63    13.56    52.66         -4.63    13.56    52.66      0.005
   27.67    22.29     6.51         27.67    22.29     6.51      0.002
  -42.09   -11.51   -17.57        -42.09   -11.51   -17.57      0.000
  -15.50    13.44    21.61        -15.50    13.45    21.61      0.004
   -7.05     6.06    48.14         -7.05     6.06    48.14      0.004
   15.36    -0.04   -21.56         15.36    -0.04   -21.56      0.000
   -6.05   -23.47   -15.21         -6.05   -23.47   -15.21      0.003
    1.12    14.21    -1.78          1.11    14.22    -1.78      0.008
   -2.72    -6.04    36.03         -2.72    -6.05    36.03      0.003
   20.78    25.09     7.25        20.78    25.09     7.25       0.002
   34.06    -5.38     5.61        34.06    -5.38     5.61       0.001
   26.95     1.18    25.83        26.95     1.18    25.83       0.004
  -22.66   -20.41    -6.86       -22.66   -20.41    -6.86       0.003
```

Listing 2: Program Generated PA4-H-Unknown-SampleReadingsTest.txt

```
200, pa4-H-Output.txt
    5.95    -9.62    16.37         5.95    -9.61    16.37       0.003
   -9.18     1.27    23.07        -9.18     1.27    23.07       0.002
   18.52   -10.77     4.67        18.52   -10.77     4.67       0.006
   -8.41   -26.19   -16.80        -8.40   -26.20   -16.80       0.006
   -5.63   -23.54   -36.43        -5.63   -23.54   -36.43       0.003
  -18.98   -13.21   -49.02       -18.98   -13.21   -49.02       0.002
   -5.33   -22.27   -13.99        -5.33   -22.27   -13.99       0.004
   16.73     7.64   -23.27        16.72     7.64   -23.27       0.003
  -22.60     5.27   -42.09       -22.60     5.27   -42.09       0.002
    9.32   -15.48   -14.09         9.32   -15.47   -14.09       0.009
    7.79   -16.08    -7.78         7.79   -16.07    -7.78       0.010
    5.76    17.60    -4.38         5.75    17.60    -4.38       0.001
   -2.85    -5.15    44.83        -2.86    -5.15    44.83       0.004
  -27.62     9.83   -32.78       -27.62     9.83   -32.78       0.000
  -36.34   -26.49   -27.45       -36.34   -26.48   -27.45       0.008
  -17.69    10.54   -35.43       -17.69    10.54   -35.43       0.005
  -30.15   -14.94   -46.96       -30.15   -14.94   -46.97       0.005
   -8.83     8.21   -24.59        -8.82     8.21   -24.59       0.002
  -41.88   -17.18   -21.04       -41.88   -17.18   -21.03       0.001
   13.23    25.37    13.23        13.23    25.39    13.23       0.016
   -1.14     3.07   -24.27        -1.14     3.07   -24.27       0.001
   18.81    -6.22    21.68        18.81    -6.22    21.68       0.001
   -5.44    -1.48    48.67        -5.43    -1.48    48.67       0.006
   11.56   -16.08    -2.58        11.56   -16.08    -2.58       0.001
   15.47    24.36   -18.28        15.47    24.37   -18.28       0.006
  -20.59   -27.04   -11.42       -20.59   -27.04   -11.41       0.009
  -24.87     8.36   -37.75       -24.87     8.36   -37.75       0.001
   17.70    -7.43   -22.64        17.70    -7.43   -22.64       0.004
  -39.47   -15.85   -37.82       -39.47   -15.85   -37.82       0.000
```

Listing 3: Program Generated PA4-J-Unknown-SampleReadingsTest.txt

```
200, pa4-J-Output.txt
  -12.08     5.34     7.98       -12.07     5.34     7.98       0.007
   -1.64    -9.67   -36.34        -1.62    -9.67   -36.35       0.017
    4.90    -7.65    55.95         4.92    -7.42    55.95       0.239
  -31.39   -11.08    -6.71       -31.43   -11.08    -6.66       0.068
   21.10    20.63    25.64        21.13    20.68    25.65       0.059
  -31.91     8.61   -29.96       -31.99     8.74   -29.98       0.154
```

```
   21.47     2.52    52.49        21.52     2.50    52.50      0.058
   -1.58    18.45    45.40        -1.58    18.47    45.41      0.022
    3.72    14.00    -5.76         3.77    13.94    -5.70      0.098
    7.49    21.62    -4.98         7.52    21.59    -4.96      0.043
  -37.48     0.85   -16.01       -37.48     0.85   -16.01      0.000
   14.01    21.23   -29.35        14.00    21.24   -29.36      0.015
   -8.61   -28.24   -30.48        -8.58   -28.29   -30.49      0.065
  -19.11   -19.00   -47.25       -19.10   -19.01   -47.28      0.033
   39.38     7.73     4.40        39.39     7.73     4.40      0.017
  -10.40   -28.29   -17.94       -10.42   -28.26   -17.95      0.043
  -34.91     2.72   -38.16       -34.94     2.76   -38.19      0.065
  -18.83     5.65   -13.67       -18.86     5.71   -13.61      0.090
  -14.53    21.88    27.80       -14.51    21.86    27.79      0.027
  -27.43    11.21   -24.41       -27.42    11.18   -24.41      0.030
  -11.39   -20.01   -44.84       -11.39   -19.86   -44.86      0.150
    2.23    -6.62    51.17         2.21    -6.83    51.17      0.210
    9.43    -4.08   -22.08         9.43    -4.08   -22.07      0.009
    6.20    20.36    45.62         6.20    20.36    45.62      0.001
  -15.92    15.58    30.24       -16.00    15.57    30.27      0.082
  -21.89     5.00   -42.31       -21.89     5.00   -42.32      0.004
   10.75    26.61     7.03        10.79    26.43     7.02      0.188
    5.68    18.96    -3.29         5.64    18.97    -3.31      0.042
  -10.74   -26.50   -15.17       -10.76   -26.46   -15.22      0.063
```

Listing 4: Program Generated PA4-K-Unknown-SampleReadingsTest.txt

```
200, pa4-K-Output.txt
    2.03    19.36     4.14         2.05    19.35     4.14      0.015
  -22.00   -18.23    -6.39       -22.00   -18.23    -6.36      0.028
   -1.46   -20.38   -33.53        -1.47   -20.37   -33.52      0.018
  -31.34     0.97   -42.30       -31.36     1.00   -42.35      0.058
   11.03    22.26    40.58        11.08    21.91    40.50      0.354
   18.87     7.27   -26.48        18.85     7.25   -26.50      0.029
   17.85    -5.08    36.27        17.74    -4.84    36.26      0.259
   19.69    21.25   -27.80        19.71    21.26   -27.84      0.043
   23.71    -3.49    24.72        23.73    -3.54    24.73      0.060
  -21.25   -32.63   -29.77       -21.25   -32.70   -29.77      0.065
  -12.49   -14.65    -3.66       -12.51   -14.72    -3.58      0.117
  -29.39   -29.58   -34.49       -29.41   -29.63   -34.51      0.065
  -26.47   -24.53   -43.11       -26.47   -24.50   -43.06      0.052
  -36.92   -25.98   -27.33       -36.89   -25.95   -27.32      0.033
  -34.32     7.66   -25.32       -34.31     7.64   -25.32      0.017
  -37.36     0.66   -15.63       -37.37     0.67   -15.63      0.012
   22.36    13.69    42.68        22.28    13.64    42.66      0.097
  -22.38     4.97   -13.83       -22.39     4.99   -13.80      0.031
   23.75    -1.79    30.95        23.70    -1.74    30.94      0.066
```

−33.23	−12.32	−7.35	−33.24	−12.32	−7.34	0.013
−5.74	−7.96	19.35	−5.69	−7.91	19.34	0.064
−39.62	−21.23	−18.88	−39.60	−21.22	−18.89	0.031
−30.95	−27.81	−16.35	−30.94	−27.78	−16.38	0.043
10.73	22.53	30.93	10.73	22.48	30.93	0.049
−34.01	4.24	−37.40	−34.00	4.22	−37.38	0.032
31.80	9.97	15.32	31.80	9.97	15.32	0.000
35.35	−4.63	−14.03	35.24	−4.58	−14.04	0.113
30.10	13.70	15.74	30.05	13.67	15.73	0.053
−22.87	−20.78	−46.49	−22.86	−20.77	−46.48	0.016

5.2 Testing

5.2.1 `ICP_Box_Unit_Testing.m`:

This unit testing file does multiple tests. A rectangular prism with vertices (±1, ±2, ±3) was generated. Then, a known F_{reg} was made and applied to every vertex. The algorithm was then tested with this dummy example. To check that the algorithm performed correctly, we compared the unmapped points to the original mesh and made sure that the error was small ($\epsilon < 1e - 13$). We did this because transformations are surjective, meaning that the mapping is not unique. The tests done were incremental. It was first tested with a rotation about the Z-axis only. Then, it was tested with a small random translation and identity rotation. Finally, it was tested with a completely random rotation and translation. The points were correctly recovered each time. Note that we had to choose a transformation such that it was close to the initial guess. If it was too far, the algorithm would not converge. This is normal as ICP is subject to local minima.

5.2.2 `ICP_Mesh_Unit_Testing.m`:

This unit test uses the performs the same tests as the previous file except that instead of using a box mesh, we use the provided mesh `Problem4MeshFile.sur`. We sample n random triangles on the mesh and then generate random barycentric coordinate vectors to sample a random point on each triangle. Although we ran the same tests as in the previous file, all of while passed on the box, we ran into some issues with the mesh. If the mesh is too complex and not enough points are sampled, the ICP algorithm can easily find a registration that is a local minimum rather than the desired transformation. Also, we realized that the transformation we apply to the sampled points cannot be too great otherwise, the algorithm won't be able to correctly register the sampled points to the mesh. In practice, this can be overcome by providing an initial registration that is good. We were able to overcome these issues by sampling a lot of points and using small test transformations.

5.2.3 util:

All of the functions in the util folder have been tested before in previous programming assignments. They are verified to work correctly. Please refer to those reports for the respective unit tests.

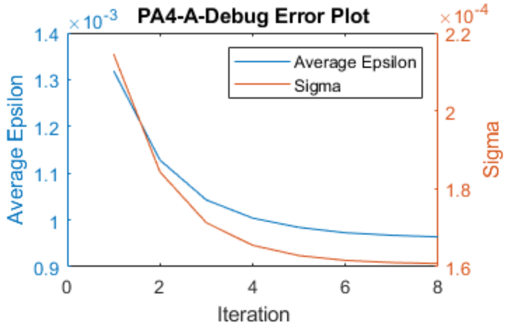
6 Discussion

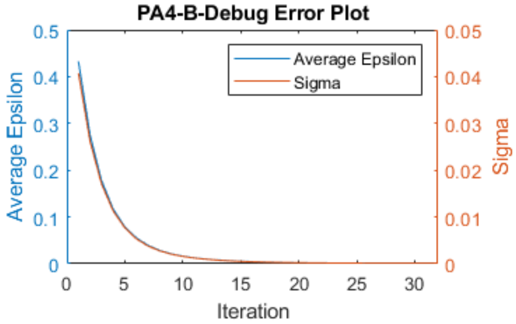
The norm of the error between the matched closest point and the given point is shown in the last column on Table 1. We can see that it is relatively small for every dataset. That, combined with the unit testing we did, confirms that the algorithm works correctly. As noted before, we were able to test the limits of ICP algorithm in the mesh unit test file. We realized that if we do not provide enough sample points or use too large of a transformation on a complicated mesh structure (without a good initial registration), we will not be able to perform a good ICP registration. However, when we compare the results of the ICP algorithm on the sample data to the example output files provided, we see that for each case, the error norm between our results and the example results is on a similar scale, although there are some minute differences due to some randomization.

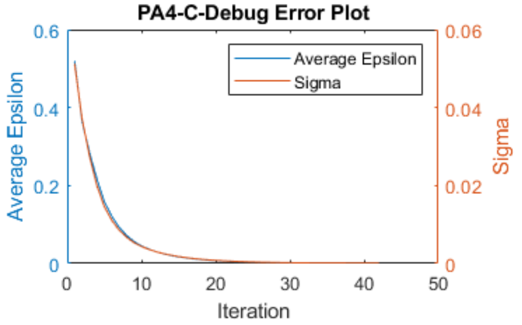
In figure 2 we have added the error plots of the ICP algorithm on the sample data. We plot the average error and the standard deviation of the error of the matches until convergence. We can see that some sample problems converge much faster than others. For example, sample A converges in 8 iterations, where as sample D takes around 50. Also, some plot exhibit a 'step' where the error does not improve for a few iterations before decreasing again. This shows that the termination test allows the algorithm to explore a little when progress starts to stall before it ends.

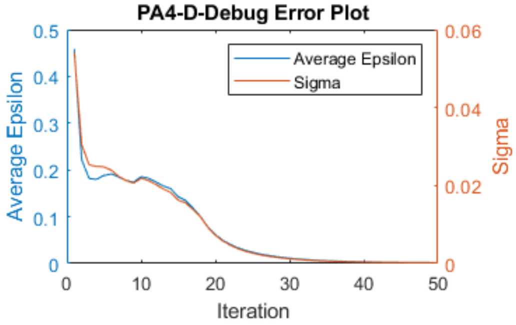
			Absolute Average Difference Between "Answer" File and Tree Search Output						
File Type	Scenario	Stat.	c_x	c_y	c_z	d_x	d_y	d_z	norm
Debug	A	Avg.	0.0017	0.0047	0.0003	0.0012	0.0025	0.0004	0.0020
		Std.	0.0038	0.0050	0.0016	0.0033	0.0044	0.0020	0.0013
Debug	B	Avg.	0.0014	0.0053	0.0002	0.0010	0.0028	0.0007	0.0022
		Std.	0.0035	0.0050	0.0014	0.0030	0.0045	0.0026	0.0018
Debug	C	Avg.	0.0019	0.0028	0.0010	0.0014	0.0020	0.0008	0.0022
		Std.	0.0039	0.0046	0.0030	0.0035	0.0040	0.0026	0.0017
Debug	D	Avg.	0.0046	0.0056	0.0015	0.0036	0.0038	0.0014	0.0040
		Std.	0.0054	0.0061	0.0036	0.0051	0.0054	0.0035	0.0034
Debug	E	Avg.	0.0482	0.0639	0.0580	0.0505	0.0535	0.0515	0.0293
		Std.	0.0278	0.0417	0.0254	0.0289	0.0409	0.0307	0.0252
Debug	F	Avg.	0.0176	0.0208	0.0119	0.0171	0.0167	0.0109	0.0117
		Std.	0.0124	0.0155	0.0090	0.0128	0.0141	0.0092	0.0092
Unknown	G	Avg.	0.0032	0.0041	0.0021	0.0026	0.0027	0.0020	0.0034
		Std.	0.0047	0.0055	0.0041	0.0044	0.0048	0.0040	0.0030
Unknown	H	Avg.	0.0035	0.0050	0.0020	0.0028	0.0038	0.0018	0.0036
		Std.	0.0053	0.0056	0.0040	0.0048	0.0053	0.0039	0.0030
Unknown	J	Avg.	0.0126	0.0107	0.0143	0.0103	0.0089	0.0136	0.0098
		Std.	0.0100	0.0081	0.0102	0.0094	0.0075	0.0098	0.0068
Unknown	K	Avg.	0.0297	0.0204	0.0156	0.0253	0.0162	0.0146	0.0169
		Std.	0.0222	0.0140	0.0116	0.0197	0.0138	0.0118	0.0141

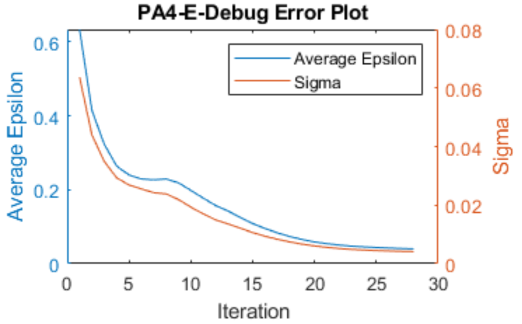
Table 1: Average Difference Between "Answer" File and ICP Algorithm Output

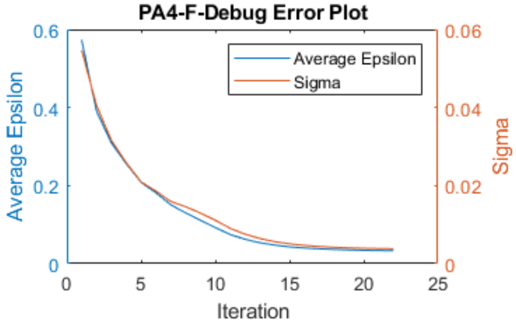

PA4-A-Debug Error Plot

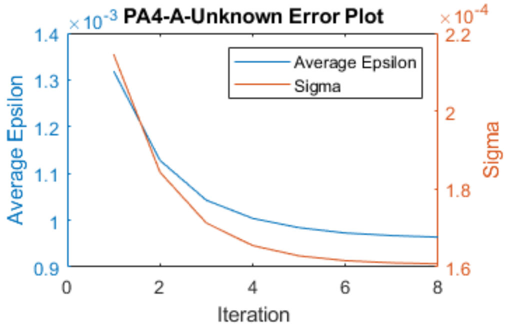

PA4-B-Debug Error Plot

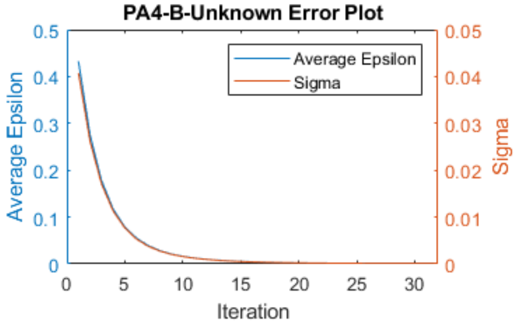

PA4-C-Debug Error Plot

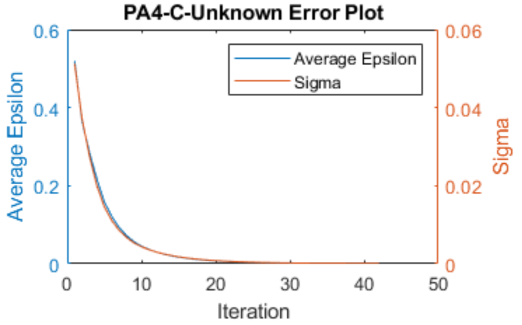

PA4-D-Debug Error Plot

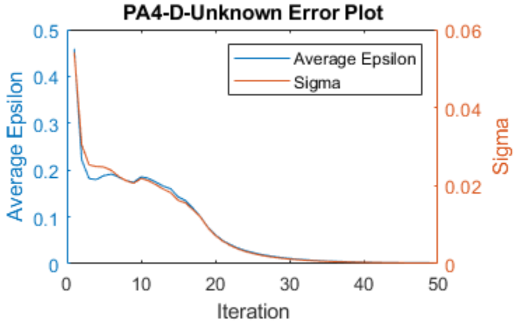

PA4-E-Debug Error Plot


PA4-F-Debug Error Plot


PA4-A-Unknown Error Plot


PA4-B-Unknown Error Plot


PA4-C-Unknown Error Plot


PA4-D-Unknown Error Plot

7 Contributions

Jessica: Worked on termination criteria in main as well as unit tests. Worked on the report.
Andrew: Worked on the main ICP algorithm. Contributed to unit tests. Worked on the report.

8 Citation

Taylor, Russel. "Registration Part I." (2020). pgs 52-59

9 Appendix

9.1 Debugging Data Results

Listing 5: Program Generated PA4-A-Debug-SampleReadingsTest.txt

```
75, pa4-A-Output.txt
   -35.05     3.35    -37.36         -35.05      3.35    -37.36     0.001
   -19.41   -18.10     -6.44         -19.41    -18.10     -6.44     0.001
   -27.75   -18.69     -7.20         -27.75    -18.69     -7.20     0.000
    25.93     3.16     31.18          25.93      3.16     31.18     0.003
    -3.65    -4.54    -38.69          -3.65     -4.54    -38.69     0.002
    20.64     0.06     51.19          20.63      0.06     51.19     0.003
    15.19    19.22     55.62          15.19     19.22     55.62     0.002
    -9.74   -19.57     -8.48          -9.74    -19.56     -8.48     0.002
    -6.80     3.50     51.88          -6.80      3.50     51.88     0.001
    16.49    18.24     55.47          16.50     18.25     55.47     0.004
     2.60    -5.38    -25.63           2.60     -5.38    -25.63     0.001
    -1.54     8.43    -15.92          -1.54      8.44    -15.92     0.002
    -4.05    -5.71    -40.18          -4.06     -5.71    -40.17     0.005
    22.98     4.22     45.05          22.98      4.22     45.05     0.001
   -11.29    -4.25    -46.23         -11.29     -4.25    -46.23     0.000
   -38.93     0.54    -34.86         -38.93      0.54    -34.86     0.000
    14.80    21.88     38.50          14.80     21.87     38.50     0.004
    14.95   -17.37     -9.22          14.95    -17.36     -9.22     0.003
    13.50    22.08    -27.23          13.50     22.08    -27.23     0.002
    29.33    12.99     17.69          29.33     12.99     17.69     0.003
     3.11   -12.19      5.67           3.11    -12.18      5.67     0.002
    17.01   -10.87    -19.81          17.01    -10.87    -19.81     0.000
    11.33    -6.36     50.64          11.33     -6.36     50.64     0.000
    -1.47    -5.17    -30.03          -1.47     -5.17    -30.03     0.000
     8.07    22.98     27.97           8.07     22.98     27.97     0.000
    15.39    24.84     14.12          15.39     24.84     14.12     0.001
   -23.20   -24.42     -9.22         -23.20    -24.42     -9.22     0.004
    34.99    -5.05    -14.38          34.99     -5.06    -14.38     0.002
   -24.88     3.35    -13.52         -24.88      3.35    -13.53     0.002
```

Listing 6: Program Generated PA4-B-Debug-SampleReadingsTest.txt

```
200, pa4-B-Output.txt
    16.10    10.95    -26.48          16.11     10.95    -26.48     0.003
    19.35    15.21    -31.23          19.35     15.21    -31.23     0.002
    14.02    18.73     62.19          14.02     18.73     62.19     0.001
   -12.49     9.74     24.79         -12.49      9.74     24.79     0.003
    39.78     2.33     -8.33          39.79      2.33     -8.33     0.002
   -27.61   -31.18    -32.51         -27.62    -31.18    -32.51     0.002
   12.68    23.21    26.38        12.68    23.21    26.38     0.001
   14.01   -10.63   -19.98        14.01   -10.63   -19.98     0.003
   16.40    24.96   -16.50        16.40    24.95   -16.50     0.009
   -3.83    -3.61    46.44        -3.83    -3.61    46.44     0.002
   15.69    23.18    22.79        15.69    23.18    22.79     0.001
   21.47    15.71    40.72        21.47    15.71    40.72     0.001
   11.99    21.85    43.44        11.99    21.86    43.44     0.006
   -5.72     1.79   -33.25        -5.71     1.79   -33.25     0.003
   -6.52     5.38   -27.67        -6.52     5.38   -27.67     0.000
   21.80     8.85    55.94        21.80     8.85    55.94     0.002
  -40.60   -19.55   -30.94       -40.60   -19.55   -30.94     0.002
   32.55    -6.64     3.94        32.55    -6.65     3.94     0.007
   -1.23    -6.05    39.92        -1.23    -6.05    39.92     0.003
   22.71    -5.85    16.67        22.71    -5.85    16.67     0.001
   -1.07    16.86    54.59        -1.07    16.87    54.59     0.003
   38.50     0.03     5.23        38.50     0.03     5.23     0.001
  -11.15   -29.25   -20.23       -11.15   -29.25   -20.23     0.004
   34.35    16.31   -10.82        34.35    16.31   -10.82     0.000
   33.13    12.45   -21.86        33.13    12.45   -21.86     0.002
  -14.73    22.11    25.36       -14.73    22.11    25.36     0.002
   39.32     9.04     0.68        39.33     9.04     0.68     0.005
   14.26    -5.93    41.22        14.26    -5.93    41.22     0.000
    1.53    23.64    14.58         1.53    23.64    14.58     0.001
```

Listing 7: Program Generated PA4-C-Debug-SampleReadingsTest.txt

```
200, pa4-C-Output.txt
   27.50    22.98    -2.23        27.50    22.98    -2.23     0.000
   11.06    -8.65    15.15        11.06    -8.65    15.15     0.001
   -1.84   -22.73   -24.22        -1.83   -22.73   -24.22     0.002
   22.50     7.92    50.64        22.50     7.92    50.65     0.005
    4.81   -11.44    11.45         4.81   -11.44    11.45     0.002
  -21.35    12.12   -29.46       -21.35    12.12   -29.46     0.002
  -22.69    -1.14   -47.13       -22.69    -1.14   -47.13     0.004
   24.19   -14.98   -13.80        24.18   -14.97   -13.80     0.001
   -7.75     9.94     4.32        -7.75     9.94     4.32     0.003
  -22.25     9.78   -36.52       -22.25     9.78   -36.51     0.000
  -21.88     1.06   -45.60       -21.88     1.07   -45.60     0.002
  -10.01    16.81    38.08       -10.01    16.81    38.08     0.002
   25.34     7.01    34.06        25.34     7.01    34.06     0.002
  -28.02   -24.97   -10.48       -28.02   -24.97   -10.48     0.000
   22.57     0.81    41.09        22.57     0.81    41.09     0.001
  -29.06     8.48   -17.93       -29.06     8.48   -17.93     0.002
   22.62     4.89    48.05        22.62     4.88    48.05     0.001
   21.82    15.28    39.93        21.82    15.28    39.93     0.001
  -32.99    -2.99   -44.00       -32.99    -2.99   -44.00     0.002
  -0.24   -10.95    12.17       -0.24   -10.95    12.17      0.002
  33.89    11.26   -21.01       33.89    11.26   -21.01      0.000
  19.12    -1.00    56.75       19.12    -1.00    56.75      0.002
  20.63   -12.13    -0.15       20.63   -12.13    -0.15      0.005
 -24.84     4.06   -13.76      -24.84     4.06   -13.76      0.001
  -3.20    -3.46    63.22       -3.20    -3.46    63.22      0.001
 -22.35    -4.36   -47.59      -22.35    -4.36   -47.60      0.002
  18.60    -5.40    27.77       18.60    -5.40    27.77      0.003
 -37.85   -23.35   -33.31      -37.85   -23.35   -33.31      0.001
  28.99     6.76   -29.23       28.98     6.76   -29.23      0.004
```

Listing 8: Program Generated PA4-D-Debug-SampleReadingsTest.txt

```
200, pa4-D-Output.txt
 -14.83   -27.05   -40.33      -14.83   -27.05   -40.33      0.001
  30.44    12.40    15.90       30.44    12.40    15.90      0.001
   6.74    21.21    38.49        6.74    21.22    38.49      0.002
  -6.28     3.04   -31.73       -6.28     3.04   -31.73      0.005
 -12.26    10.70    18.29      -12.25    10.70    18.29      0.009
 -20.50    -2.26   -47.31      -20.50    -2.26   -47.31      0.003
   1.94    18.66    52.38        1.95    18.64    52.38      0.016
  -3.77    -4.88    38.65       -3.76    -4.87    38.65      0.013
 -29.71   -24.34   -42.05      -29.71   -24.34   -42.05      0.000
  30.36   -10.53   -14.54       30.36   -10.53   -14.54      0.001
 -14.46     0.04   -45.58      -14.46     0.04   -45.58      0.002
  16.30    -6.49    22.89       16.30    -6.49    22.89      0.000
   0.57   -11.05    11.70        0.57   -11.04    11.70      0.000
  34.23    10.51    11.37       34.23    10.51    11.37      0.001
  -8.11     8.34    35.93       -8.11     8.34    35.93      0.003
  -1.95    23.74    29.43       -1.95    23.73    29.42      0.003
  -5.56    13.92     8.12       -5.56    13.92     8.12      0.002
  -0.03   -12.83     1.45       -0.03   -12.83     1.45      0.005
  -1.46    -5.83    44.01       -1.46    -5.84    44.01      0.010
  16.96   -16.47    -3.85       16.96   -16.47    -3.85      0.003
   4.78   -15.07    -3.82        4.78   -15.06    -3.82      0.010
  34.52    -4.97     5.54       34.51    -4.97     5.54      0.007
  -6.60   -11.49     3.94       -6.60   -11.49     3.94      0.006
  -3.71    -4.90    38.76       -3.71    -4.90    38.76      0.003
  17.93   -11.66     3.44       17.93   -11.66     3.44      0.005
  33.89    11.32   -20.97       33.89    11.32   -20.97      0.001
  22.31    14.62    38.60       22.31    14.63    38.60      0.007
  20.11   -11.01     2.33       20.11   -11.00     2.33      0.007
  -0.11   -17.77   -30.76       -0.10   -17.77   -30.76      0.003
```

Listing 9: Program Generated PA4-E-Debug-SampleReadingsTest.txt

```
200, pa4-E-Output.txt
```

```
   -9.88    22.45    18.29         -9.85    22.38    18.34       0.083
  -11.84     2.93    14.73        -11.87     2.93    14.74       0.030
  -10.09    24.49    28.28        -10.13    24.60    28.31       0.120
    0.04   -19.16   -27.59          0.09   -19.18   -27.60       0.055
   28.72    -9.50     0.98         28.71    -9.47     0.97       0.039
  -39.78   -22.48   -28.27        -39.73   -22.45   -28.27       0.055
   20.02   -11.92   -20.81         20.11   -11.85   -20.73       0.136
    0.94    19.98     6.24          1.01    19.91     6.28       0.102
   12.00    22.06    35.91         12.01    22.02    35.91       0.048
   22.14    10.64    50.54         22.16    10.65    50.54       0.023
   32.84     9.66   -24.73         32.70     9.62   -24.67       0.157
  -16.62   -31.63   -32.12        -16.61   -31.66   -32.13       0.029
    6.13    18.14    -4.73          6.09    18.15    -4.75       0.047
   -3.59   -20.14   -11.65         -3.63   -20.07   -11.70       0.090
    2.52    19.82    44.34          2.52    19.84    44.34       0.022
   -2.76    19.70    11.15         -2.79    19.73    11.13       0.040
   -3.48    19.43    11.48         -3.43    19.39    11.52       0.071
    2.45    20.82     5.14          2.48    20.79     5.17       0.049
   14.02    13.41   -17.13         14.12    13.47   -17.12       0.116
   -6.84     5.86    56.87         -7.12     5.87    56.88       0.285
  -16.19    -9.80    -0.75        -16.15    -9.77    -0.79       0.063
  -32.22   -23.60   -41.17        -32.17   -23.53   -41.10       0.109
   16.63    -6.14    25.22         16.64    -6.22    25.23       0.081
  -13.81   -11.24    -0.82        -13.87   -11.33    -0.75       0.130
  -10.30    16.76    15.24        -10.27    16.75    15.27       0.045
   -5.77   -24.65   -34.87         -5.79   -24.64   -34.86       0.019
   -2.01    15.37    60.04         -2.14    15.55    60.07       0.216
    6.19    20.00    -3.92          6.22    19.99    -3.91       0.028
   33.78    17.11     4.47         33.78    17.11     4.47       0.000
```

Listing 10: Program Generated PA4-F-Debug-SampleReadingsTest.txt

```
200, pa4-F-Output.txt
   -1.93   -11.55   -37.40         -1.90   -11.54   -37.41       0.036
   -8.72   -24.88   -14.73         -8.70   -24.91   -14.71       0.035
   -5.09    12.46     5.06         -5.10    12.47     5.06       0.008
   -8.80     8.41    32.55         -8.88     8.36    32.58       0.105
   29.15   -11.66   -16.40         29.09   -11.56   -16.38       0.116
   -5.86     2.11   -32.59         -5.89     2.10   -32.59       0.030
  -30.65     6.76   -36.91        -30.63     6.74   -36.89       0.039
   35.10     1.90   -20.63         35.13     1.90   -20.64       0.031
  -29.72   -30.41   -31.92        -29.71   -30.38   -31.91       0.030
  -12.19   -27.45   -15.78        -12.19   -27.52   -15.73       0.087
  -23.25     3.76   -43.56        -23.25     3.78   -43.59       0.034
   -6.96     3.37    52.39         -6.84     3.38    52.40       0.127
   18.75    21.19    30.25         18.78    21.23    30.27       0.047
```

−13.13	−12.08	−47.67	−13.12	−12.08	−47.68	0.016
−9.35	24.67	29.29	−9.35	24.56	29.25	0.116
−22.98	−8.14	−48.90	−22.98	−8.12	−48.94	0.041
−14.43	−26.99	−13.57	−14.44	−26.97	−13.58	0.017
−5.18	9.26	−5.24	−5.18	9.25	−5.24	0.008
6.89	−7.36	56.62	6.88	−7.29	56.62	0.070
−15.06	−15.48	−47.87	−15.06	−15.46	−47.83	0.049
9.92	24.10	−5.54	9.93	24.09	−5.53	0.025
20.74	16.21	42.23	20.83	16.26	42.25	0.103
11.52	−2.71	63.25	11.52	−2.71	63.23	0.023
−7.26	−8.82	−44.74	−7.32	−8.83	−44.68	0.084
2.89	8.40	−15.48	2.89	8.39	−15.48	0.011
−3.14	−21.27	−15.31	−3.13	−21.29	−15.30	0.025
−7.81	1.93	28.73	−7.91	1.91	28.75	0.109
−4.81	11.23	0.76	−4.77	11.18	0.78	0.063
−7.52	15.61	12.83	−7.59	15.67	12.76	0.121