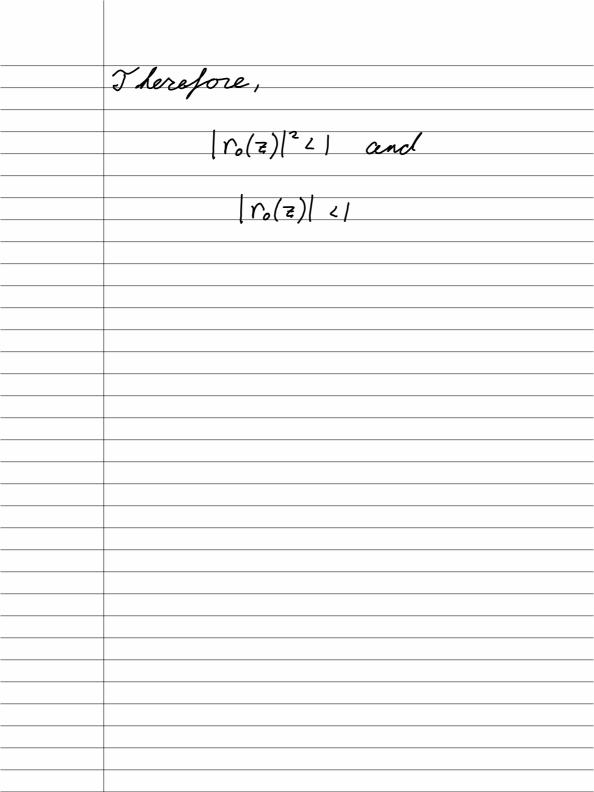
andrew Cornelio HW8 Problem ! Start with the model problem is 24. Substitute this in to the formula: $y_{n+1} = y_n + \frac{h}{2} \left[\lambda y_n + \lambda y_{n+1} \right]$ This is a difference equation that can be solved with In=r". $r^{n+1} = r^n + \frac{h}{2} \left(\lambda r^n + \lambda r^{n+1} \right)$ $r = 1 + \frac{h}{2}(\lambda + \lambda r)$ $2r - h\lambda r = 2 + h\lambda$ $r = \frac{2 + h \lambda}{2 - h \lambda}$ We have found vo. Now we must show $|r_0(\bar{z})|^2$ for all $Re(\bar{z})<0$, Let $\bar{z}=h\lambda$. Let's find $|r_0(\bar{z})|^2$: $|\Gamma_o(z)|^2 = |2+z|^2 - 4+4Re(z)+|z|^2$ $|2-z|^2 - 4-4Re(z)+|z|^2$ Since Re(2) 40, 4+4Re(z)+|z|2 < 4-4Re(z)+|z|2



Problem 3 a) The classic fourth order kunge-kulta method is yn+1 = yn + 6h (k,+2k2+2k3+ k4) $K_1 = f(t_n, y_n)$ K2 = f(tn+ =, yn+h =) $k_3 = f(t_n + \frac{h}{2}, y_n + h \frac{k_2}{2})$ K4 = f(tn + h, yn + h k3) To find the characteristic polynomial, we subsitute $\dot{y} = \lambda y$ and $y_n = r^n$: Ki= 2rn K2 = 2 (rn+h &) = 2rn+ 2h22rn K3= ス(rn+h 2)= スrn+=h22rn+ +h22rn ka = λ(rn + hk3) = λrn + hλ2rn + ± h2λ3rn + 4 h3λ4rn Substiting this back in:

= rn+ rn(th2 + th = rn + rn (h2 + = h222 + = h323 + = h424) So we have r=1+h2+ = h222+ = h323+ = h929 If we take == h?: ro(Z)=1+Z+2Z2+6Z3+24Z4 This is the 4th order expansion of ex around x=0 b) The plat is in a file named plot 26, prog. The stability threshold is ==2.7853. We can easily find the threshold of slability for the Euler method. For the Euler method, $r_0(z)=|+z|$ of Re(z)<0, then $|r_0(z)| | || when <math>z \in (-2,0)$. So the threshold of stability for $r \nmid 4$ is larger

Froblem 4 a) Solution plot is titled plot 4a.png and step size plot is titled step 4a.png.

There were 68 successful steps, 6 failed steps and 160 fn lurals. The step sizes generally increased over time. I that named plot- 45, png c) The same naming scheme applies
as 4a. There were 14/355 successful
steps, 9425 failed steps, and 904681
for evals. Step size generally decreased d) Since all the eigenvalues that we found in (b) were less than 0, the system is stiff. Since ode 45 uses runge-kutta, it has difficulty handling stiff systems. The step size of ode 45 is bounded by in step_4d.png, we see that this is a good bound.