Problem 1

Assuming 4 spheres (with errors) do share exatly 1 common point (or 1 small common area). Namely, there is no such terrible cases like the Figure 1 in 3D:

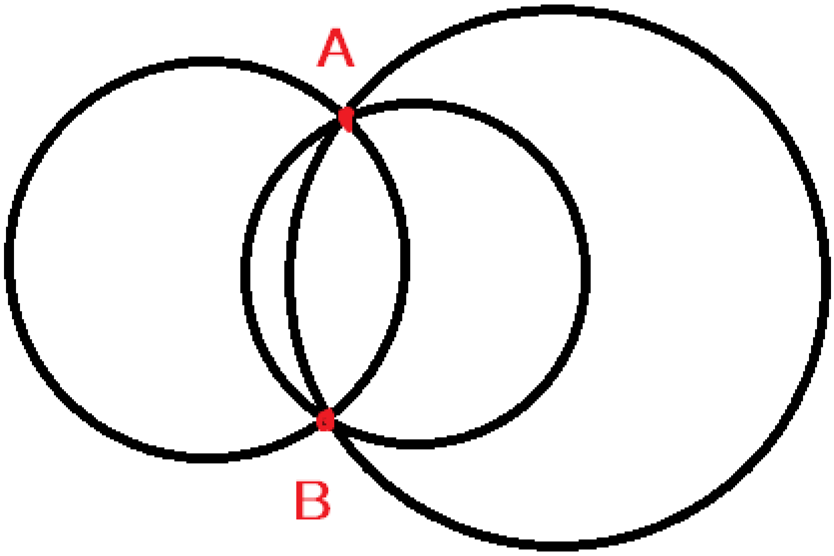

Figure 1. a terrible case that there are 2 possible locations in 2D.

There are 3 cases:

Case A: All these 4 centers are collinear.

By the geometric symmetry, the location must be also collinear with these centers, or there exist a whole circle that the location can be.

Obviously, in this case, all 4 spheres are tangent to each other. Therefore, we can pick up any 2 spheres, and use them to compute the location, which is the tangent point.

Say we have $(x - c_1)^2 = r_1^2$, $(x - c_2)^2 = r_2^2$,where $r_1 \leq r_2$. The tangent point is $c_2 + r_2 \times \frac{c1-c2}{||c1-c2||}$

Case B: All these 4 centers are coplanar but not collinear.

By the geometric symmetry, the location must be also coplanar with these centers, or there exist at least 2 possible location.

Since these 4 centers are not collinear, there must exist a choice to pick up 3 centers, such that $(c_1 - c_2), (c_1 - c_3)$ are linearly independent. Namely, the intersection circle C_{12} between Sphere S_1, S_2, and the intersection circle C_{13} are not coplanar. Because $c_1 - c_2$ is the normal vector of the plane P_{12} where C_{12} is, and $c_1 - c_3$ is the one of P_{13}.

Also, we have a plane P_0 whose normal vector is $(c_1 - c_2) \times (c_1 - c_3)$, namely, this coplanar plane.

Given the assumption, the intersection of these 3 plane is the location.

For P_0, we know c_1 is on it. we can compute the equation of P_0.

Say $d_{12} = ||c_1 - c_2||$. Considering P_{12}, we can show $c_1 + k \times (c_2 - c_1)$ is on P_{12}, where $k = \frac{d^2 - r_2^2 + r_1^2}{2d}$.

Similarly, we can also compute a point on P_{13}.

Now, we have 3 linearly-indepent linear equations, say $AX = b$, and hence the location $X = A^{-1}b$.

Case C: these 4 centers are not coplanar.

Therefore, there must exists 3 different P_1, P_2, P_3 where the intersection circle is. We can just compute each of them by computing the normal vector and a point on it.

Now this problem also is reduced to $AX = b$. The location $X = A^{-1}b$.

Exceptions

Notice that all these solutions are based on the assumption. Therefore, once we computed the location, we can examinate it using thses 4 distances. Once it does not according to any distances, the only case is there are more than 1 possible locations (or these 4 spheres have no common point), which is not the case we are going to solve.