Designing Agents with Task-Specific Minimal Representation

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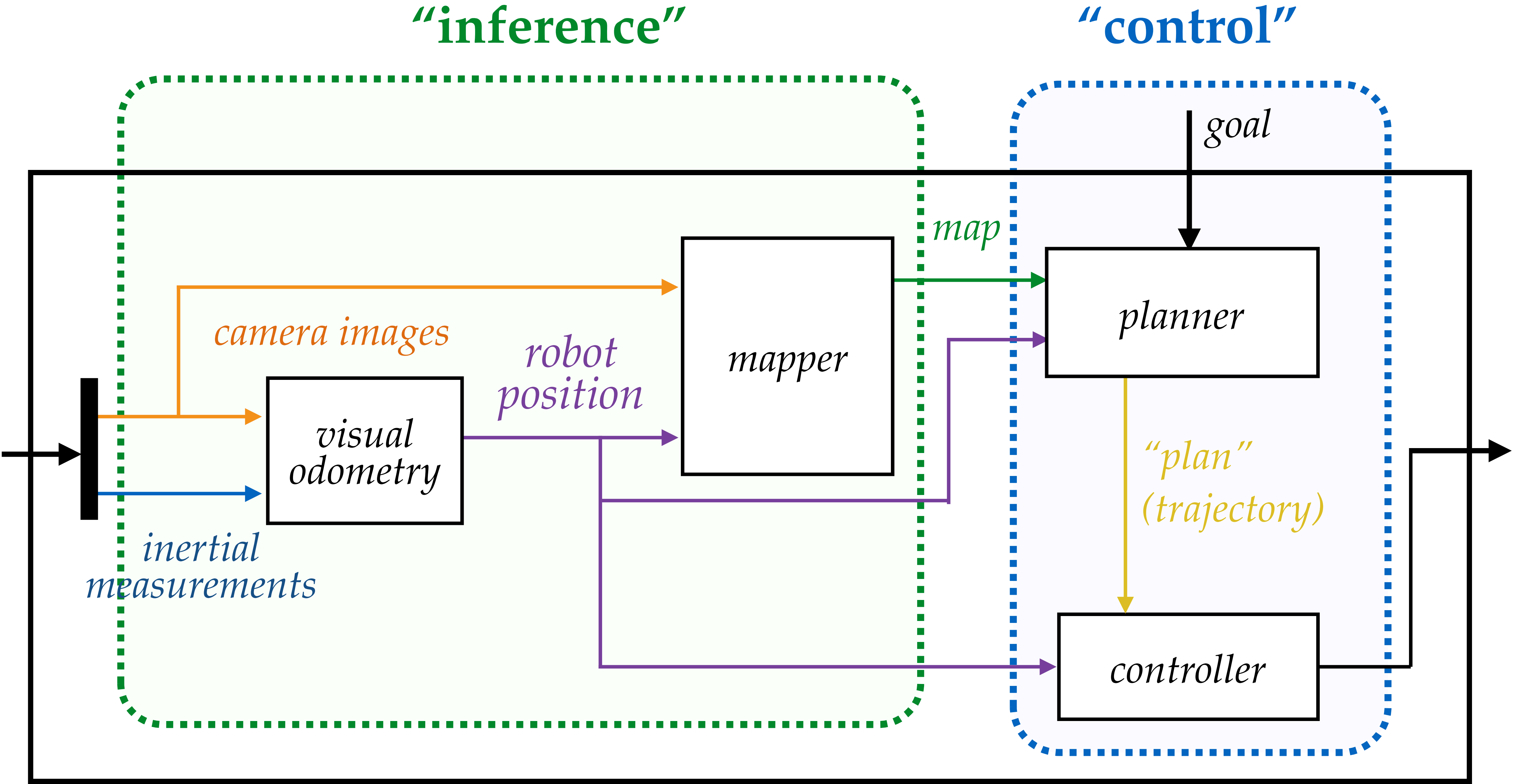
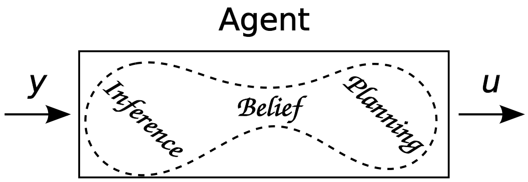

"inference"
"control"
goal
map
camera images
robot
position
mapper
planner
visual
odometry
"plan"
(trajectory)
inertial
measurements
controller

Separate Inference and Control

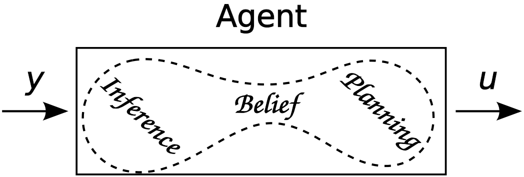
It's inefficient to do inference and control separately

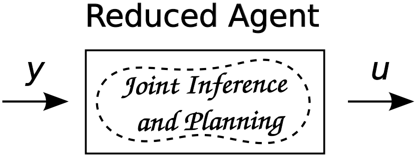


Canonical belief spaces can become a serious informational bottleneck between inference and planning modules

Joint Inference and Control

One approach is reduction





Representation Reduction

There exists a great deal of literature on the subject of representation reduction

- Continuous case: Dimensionality Reduction, Information Bottleneck...
- Discrete case: FSM Reduction

I will focus on the latter discrete case.

FSM

Definition (Finite State Machine)

A **finite state machine** is a tuple

$$\langle \Sigma, \Gamma, S, s_0, \delta, \omega \rangle \,,$$

where

- Σ is an input alphabet,
- Γ is an out alphabet,
- S is a finite set of states,
- $s_0 \in S$ is an initial state,
- $\delta : S \times \Sigma \to S$ is a state-transition function,
- $\omega : S \times \Sigma \to \Gamma$ is an output function.

ISFSM

Definition (Incompletely-Specified Finite State Machine)

An **incompletely-specified finite state machine** is a tuple

$$\langle \Sigma, \Gamma, S, s_0, \delta, \omega \rangle,$$

where

- Σ is an input alphabet,
- Γ is an out alphabet,
- S is a finite set of states,
- $s_0 \in S$ is an initial state,
- $\delta : S \times \Sigma \to S \cup \{\phi\}$ is a state-transition function,
- $\omega : S \times \Sigma \to \Gamma \cup \{\epsilon\}$ is an output function,

and ϕ and ϵ denote unspecified outputs.

Definition (Obedience)

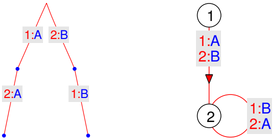
Given the decision policy $P = \langle \mathcal{U}, T : \text{Sequences}(\mathcal{Y}) \to \mathcal{U} \cup \{\epsilon\} \rangle$, we say that an ISFSM $\langle \Sigma, \Gamma, S, s_0, \delta, \omega \rangle$ **obeys** the policy P if for every finite sequence $y_1, \ldots, y_n \in \mathcal{Y}$, there exists a sequence $s_0, \ldots s_{n-1} \subseteq S$ such that

$$s_i = \delta(s_{i-1}, y_i) \text{ for all } i = 1, \ldots, n$$

and

$$T(y_1, \ldots, y_n) = \omega(s_{n-1}, y_n) \quad \text{or} \quad T(y_1, \ldots, y_n) = \epsilon.$$

Example (Obedience)



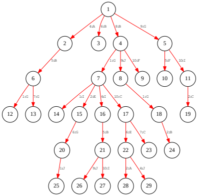
(a) Policy (b) FSM

Definition (Equivalent FSMs)

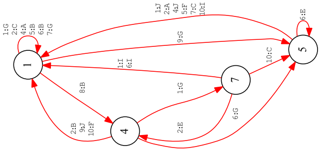
Two FSMs are equivalent if they obey all the same policies.

Equivalent FSMs may have very different forms

Example (Reduced FSMs)



(c) Canonical FSM



(d) Equivalent reduced FSM

Problem

Problem (FSM minimization)

Given a decision policy (or an ISFSM) how do we find an obedient (or equivalent) ISFSM with the smallest possible state set?

It turns out,

- It is easy for CSFSM (polynomial in $|S|$).
- It is hard (NP-hard) for ISFSM.

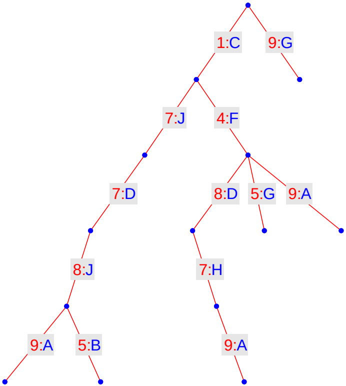
Previous Work

Previous work: Heuristics for ISFSMs

- C. P. Pfleeger. "State reduction in incompletely specified finite-state machines". In: *IEEE Trans. Comput* 22.12 (1973), pp. 1099 –110
- A. Grasselli and F. Luccio. "A method for minimizing the number of internal states in incompletely-specified sequential networks". In: *IEEE Trans. Comput* 13.3 (1965), pp. 350 –359
- J. Pena and A. Oliveira. "A new algorithm for the reduction of incompletely specified finite state machines". In: *IEEE/ACM International Conference on Computer-Aided Design* (1998), pp. 4482 –489
- S. Goren and F. Ferguson. "On state reduction of incompletely specified finite state machines". In: *Computers and Electrical Engineering* 33.1 (2007), pp. 58 –69

Bit-at-a-time (Censi)

Proposed by Andrea Censi, MIT-LIDS: Greedily separate ambiguous contexts along decision tree.



(a) Decision Tree

Bit-at-a-time (Censi)

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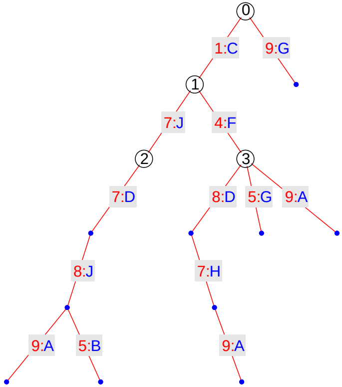

(a) Decision Tree

Bit-at-a-time (Censi)

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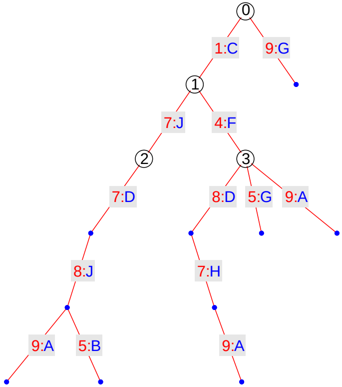
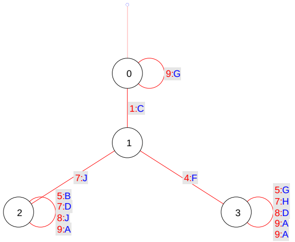


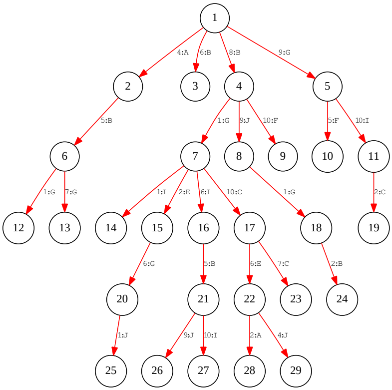
(a) Decision Tree



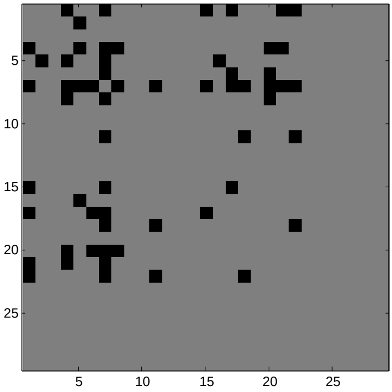
(b) Reduced FSM

Greedy Clique Covering

Greedily combine compatible states



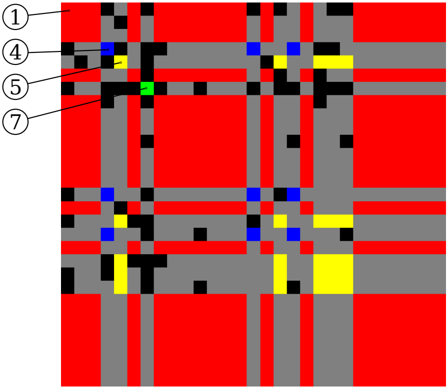
(c) Decision Tree



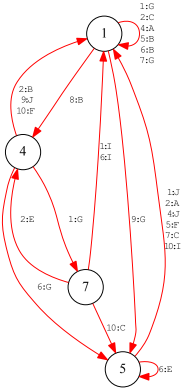
(d) Compatibility Matrix

Greedy Clique Covering

Greedily combine compatible states



(e) Greedy Clique Covering



(f) Reduced Rep

Alberto and Simão, '09

A. Simão A. Alberto. "Minimization of Incompletely Specified Finite State Machines Based on Distinction Graphs". In: *Latin American Test Workshop* (2009), pp. 1 –6

Method

1. Construct equivalence graph
2. Select maximal anticlique in equivalence graph
3. Merge remaining states with members of the anticlique graph

1. Equivalence Graph

Construct an equivalence graph using Hopcroft's algorithm[1] (its complement, the distinction graph, is shown).

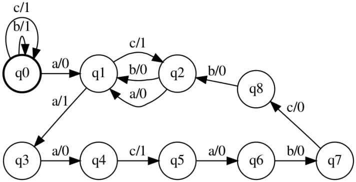


Figure 1: An incompletely-specified FSM

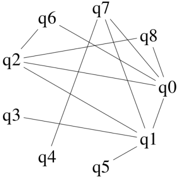


Figure 2: Distinction graph for the FSM in Figure 1

[1] J. Hopcroft. "An $n \log n$ algorithm for minimizing states in a finite automaton". In: *Theory of machines and computations* (1971), pp. 198 –196.

2. Maximal Anticlique

The next step requires that we find a maximal anticlique of the equivalence graph.

- Cliques on the equivalence graph identify sets of states that can be collapsed into a single state. The minimal clique-covering, that is the smallest collection of disjoint cliques that covers the equivalence graph, correponds to a minimal reduction of the FSM.
- Finding a maximum anticlique is also NP-hard (same paper), but this algorithm only *approximates* a minimal reduction

3. Iteratitve Merge

Next, states outside the anticlique selected, following a simple heuristic, and merged with compatible states on the anticlique (there must exist at least one).

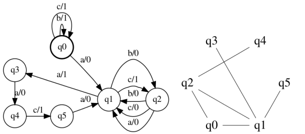


Figure 3: Reduced FSM after first iteration and its distinction graph

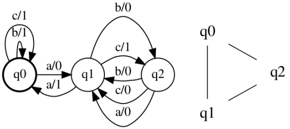


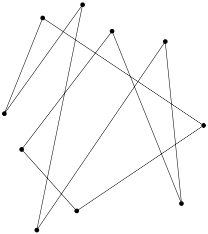
Figure 4: Reduced FSM after second iteration and its distinction graph

Limitations

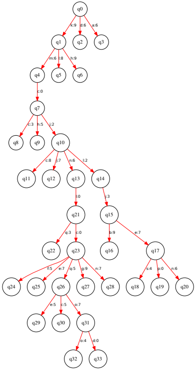
- Finding the maximal anticlique is still an NP-hard problem.
- Can do very badly on certain cases

Example (Failure Case)

Consider the case when the distinction graph looks like



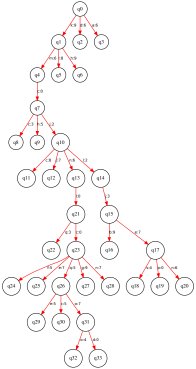
Poisson Random Tree

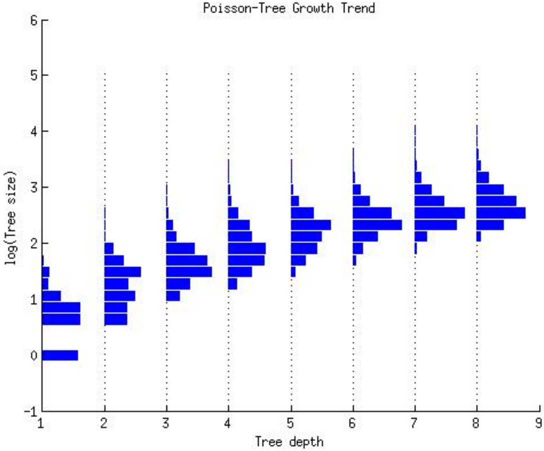


Generated by recursively adding $X \sim \text{Poisson}(\lambda)$ children to each new node. Result is conditioned on process not terminating before depth H.

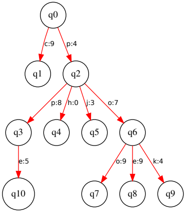
Models a birth/death process where individuals continuously produce offspring at a rate of λ per lifetime.

Poisson Random Tree

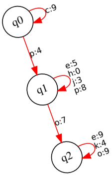




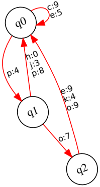
Reduction Examples



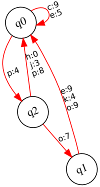
(g) Original



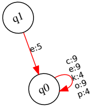
(h) Censi



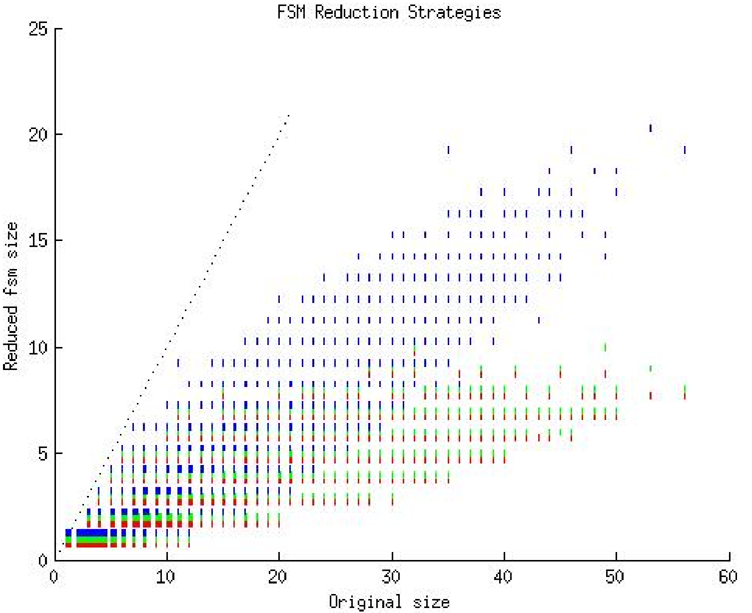
(i) Josh

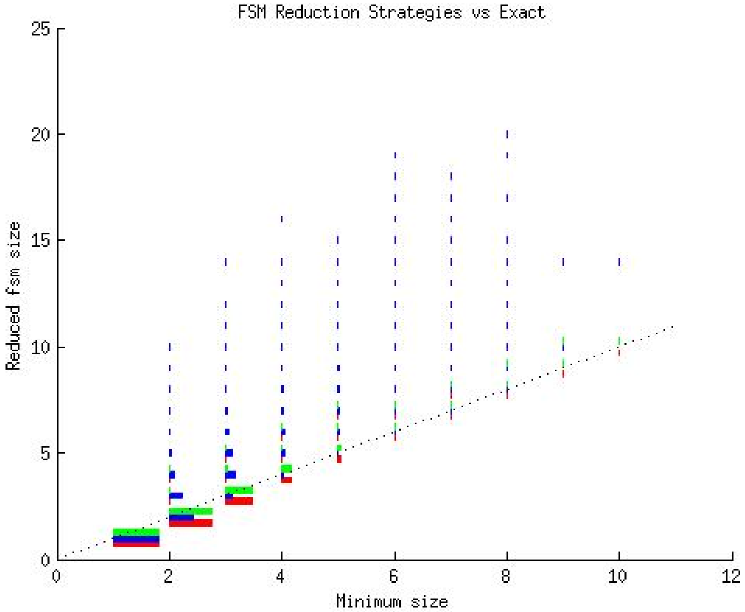


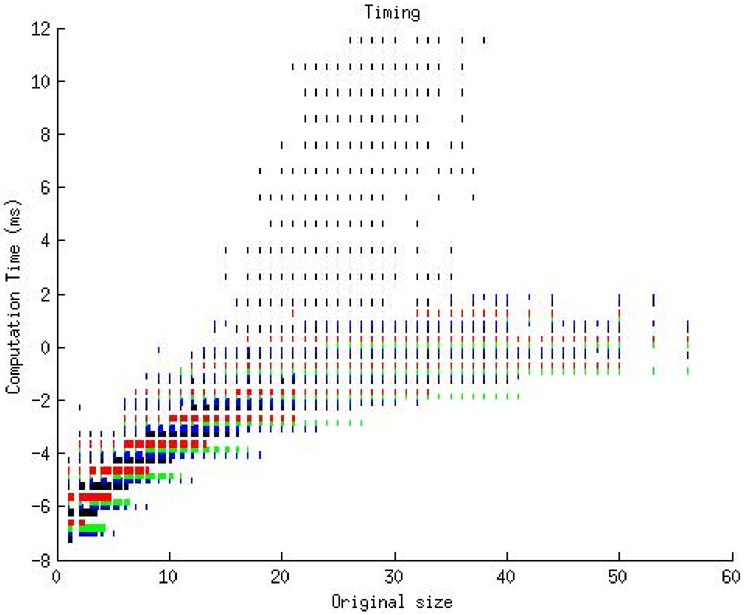
(j) Alberto



(k) Exact

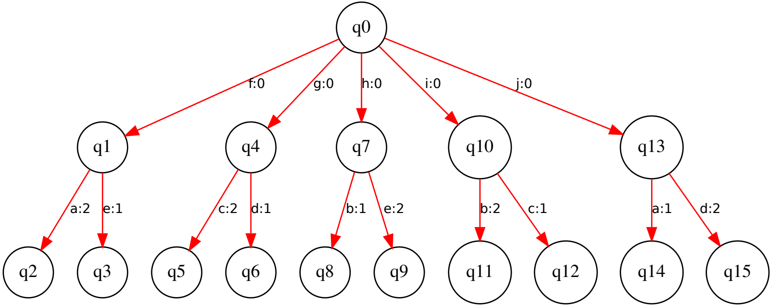

FSM Reduction Strategies
Reduced fsm size
Original size
0
5
10
15
20
25
0
10
20
30
40
50
60


FSM Reduction Strategies vs Exact
Reduced fsm size
Minimum size
0
5
10
15
20
25
0
2
4
6
8
10
12

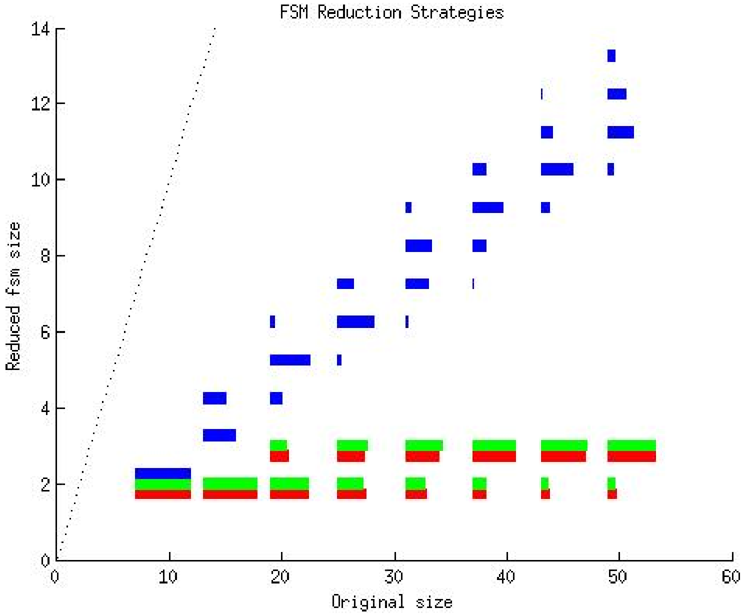


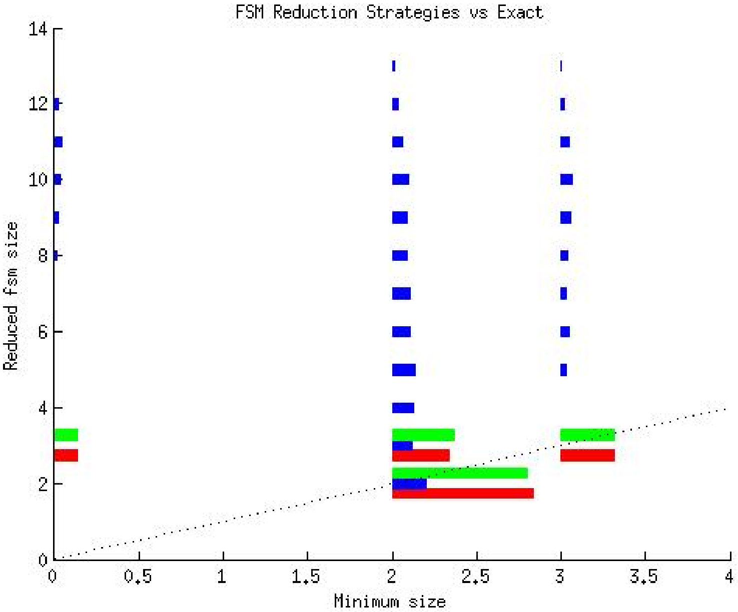
Timing
Computation Time (ms)
Original size
12
10
8
6
4
2
0
-2
-4
-6
-8
0
10
20
30
40
50
60

Pathological Tree

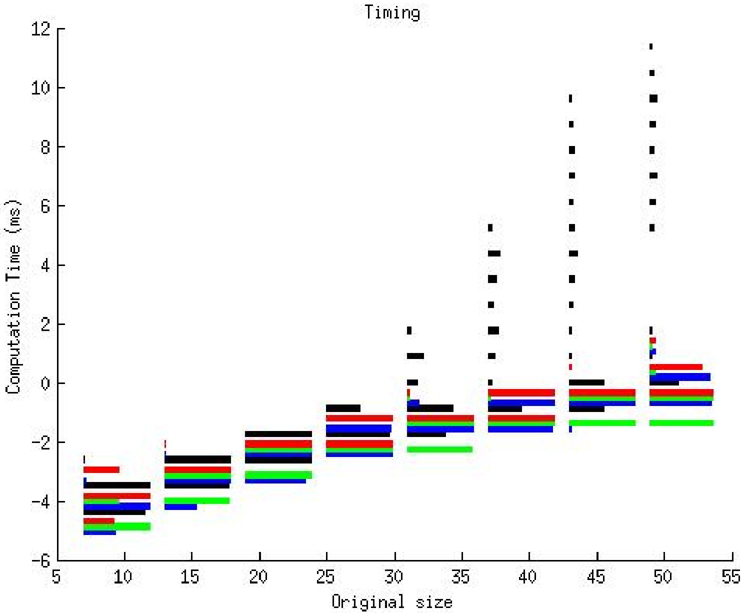


Each of the states at depth 1 is incompatible with exactly two others. This creates a distinction graph consisting of disjoint rings.


FSM Reduction Strategies
Reduced fsm size
Original size
0
2
4
6
8
10
12
14
0
10
20
30
40
50
60



FSM Reduction Strategies vs Exact
Reduced fsm size
Minimum size
0
2
4
6
8
10
12
14
0
0.5
1
1.5
2
2.5
3
3.5
4


Timing
Computation Time (ms)
Original size
12
10
8
6
4
2
0
-2
-4
-6
5
10
15
20
25
30
35
40
45
50
55