Tasks

Definition (Task)

A **task** on the **state-space** $\mathcal{X}$ is a *partial ordering* $\overset{T}{\preccurlyeq}$ *on* ProbMeas($\mathcal{X}$)

- We can restrict this to $\mathcal{P}(\mathcal{X})$ (possibilistic) or $\mathcal{X}$ (deterministic).
- Given a state-value function $E : \mathcal{X} \to \mathbb{R}$, and a partial ordering $\overset{T}{\preccurlyeq}_{\mathbb{R}}$ on ProbMeas($\mathbb{R}$), we can derive an ordering

$$\mu_1 \overset{T}{\preccurlyeq} \mu_2 \iff (\mu_1 \circ E) \overset{T}{\preccurlyeq}_{\mathbb{R}} (\mu_2 \circ E)$$

for $\mu_1, \mu_2 \in \text{ProbMeas}(\mathcal{X})$. We would usually expect $\overset{T}{\preccurlyeq}_{\mathbb{R}}$ to respect stochastic dominance, i.e.

$$\mu_1 \leq \mu_2 \implies \mu_1 \overset{T}{\preccurlyeq}_{\mathbb{R}} \mu_2$$

Sensor

Definition (Sensor)

A **sensor** on $\mathcal{X}$ is a function $S : \text{Sequences}(\mathcal{X}) \to \text{ProbMeas}(\mathcal{X})$.
A sensor is **memoryless** if it is a function of the last entry of its sequence.

In the following, we will only consider memoryless sensors, although most of the definitions can be extended to general sensors.

Definition (Sensor Consistency)

A sensor is **consistent with** the task $\overset{T}{\preccurlyeq}_{\mathcal{X}}$ when

$$S(x_1) \overset{T}{\preccurlyeq} S(x_2) \implies \delta_{x_1} \overset{T}{\preccurlyeq} \delta_{x_2} \qquad \text{for all } x_1, x_2 \in \mathcal{X},$$

where $\delta_x(A) := \mathbb{1}(x \in A)$ for $x \in \mathcal{X}$ and $A \subseteq \mathcal{X}$.

Sensors, continued

Definition (Sensor Refinement)

A sensor S_1 **is refined by** a sensor S_2 with respect to $\overset{T}{\preccurlyeq}$ when

$$S_1(x_1) \overset{T}{\preccurlyeq} S_2(x_2) \implies S_2(x_1) \overset{T}{\preccurlyeq} S_2(x_2) \qquad \text{for all } x_1, x_2 \in \mathcal{X}.$$

A **perfect** sensor S with respect to $\overset{T}{\preccurlyeq}$ is one such that

$$\delta_{x_1} \overset{T}{\preccurlyeq} \delta_{x_2} \implies S(x_1) \overset{T}{\preccurlyeq} S(x_2) \qquad \text{for all } x_1, x_2 \in \mathcal{X},$$

i.e. it satisfies the converse of the consistency condition.