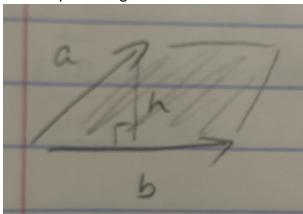
## 12.4: Cross Products

Area of parallelogram:



$$\|\vec{b}\|\underbrace{\|\vec{a}\|\sin heta}_{h}$$

**Cross product**: vector  $\perp$  to  $\vec{a}$  and  $\vec{b}$  whose length is this area

$$egin{aligned} \left\| ec{A} imes ec{B} 
ight\| &= \|ec{A}\| \|ec{B}\| \sin heta \ ec{A} imes ec{B} &= \left( \|ec{A}\| \|ec{B}\| \sin heta 
ight) \hat{n} \end{aligned}$$
 Unit vector  $ec{A}$  to plane AB

 $ec{a}$  and  $ec{b}$  are parallel if  $ec{a} imes ec{b} = 0$ 

## **Properties**

1. 
$$rec{u} imes sec{v}=(rs)(ec{u} imesec{v})$$

2. 
$$ec{u} imes ec{v} = -ec{v} imes ec{u}$$
 (cross product is anticommutative not commutative)

з. 
$$ec{0} imesec{u}=ec{0}$$

4. 
$$ec{u} imes(ec{v}+ec{w})=(ec{u} imesec{v})+(ec{u} imesec{w})$$
 (left distributive)

5. 
$$(ec{v}+ec{w}) imesec{u}=(ec{v} imesec{u})+(ec{w} imesec{u})$$
 (right distributive)

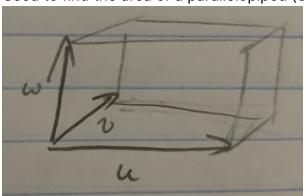
6. 
$$\vec{u} imes (\vec{v} imes \vec{w}) = (\vec{u} \cdot \vec{w}) \vec{v} - (\vec{u} \cdot \vec{v}) \vec{w}$$
 (not associative)

## **Cross product as determinant**

$$ec{u} imesec{v}=egin{array}{ccc} \mathbf{i} & \mathbf{j} & \mathbf{k} \ u_1 & u_2 & u_3 \ v_1 & v_2 & v_3 \ \end{array} = egin{array}{ccc} u_2 & u_3 \ v_2 & v_3 \ \end{array} egin{array}{ccc} \mathbf{i} -egin{array}{ccc} u_1 & u_3 \ v_1 & v_3 \ \end{array} egin{array}{ccc} \mathbf{j} +egin{array}{ccc} u_1 & u_2 \ v_1 & v_2 \ \end{array} egin{array}{cccc} \mathbf{k} \end{array}$$

## **Triple scalar product**

Used to find the area of a parallelopiped (3D parallelogram)



$$(ec{u} imesec{v})\cdotec{w} = egin{bmatrix} dots & dots & dots \ ec{u} & ec{v} & ec{w} \ dots & dots & dots \ dots & dots & dots \ \end{pmatrix}$$