14.10: Partial Derivatives w/ Constraints

Previous partial derivatives assumed all variables were independent, but what if some of the variables have known relationships?

The notation $\left(\frac{\partial w}{\partial y}\right)_{z,t}$ represents the partial derivative of w with respect to y, given that z and t are independent of y.

To evaluate a partial derivative with constraints:

1. Decide which variables are dependent & independent
2. Eliminate the other dependent variables
3. Differentiate and solve

Example

If $w = x^2 + y - z + \sin(t)$ and $x + y = t$, find $\left(\frac{\partial w}{\partial y}\right)_{z,t}$

Method 1: Eliminating other independent variables first

Since $x + y = t$, substitute $x = t - y$ into w, resulting in:

$$w = (t-y)^2 + y - z + \sin(t)$$

All of the variables are now independent, so just compute the partial.

$$\frac{\partial w}{\partial y} = -2(t-y) + 1$$

Method 2: Using chain rule

We know x is dependent on y, so $\frac{\partial x}{\partial y}$ must be non-zero.

$$\frac{\partial w}{\partial y} = 2x\frac{\partial x}{\partial y} + 1$$

Then, since $x = t - y$,

$$\frac{\partial x}{\partial y} = -1$$

Substitute $\frac{\partial x}{\partial y}$ and x, you get the answer from before.

#module2 #week6