15.6: Applications of Double & Triple Integrals

These are applications for double integrals and triple integrals.

Physics Definitions

(mass)

$$dm = \sigma\, dA \text{ (2 dimensions)}$$
$$dm = \rho\, dV \text{ (3 dimensions)}$$

(moment)

$$dM = r\, dm$$

(moment of inertia)

$$dI = r^2\, dm$$

Rest of these formulas can essentially be defined by these relationships.

Mass and First Moments

In three dimensions

Mass:

$$M = \iiint_D \rho\, dV$$

First moments about the coordinate planes:

$$M_{yz} = \iiint_D x\rho\, dV$$
$$M_{xz} = \iiint_D y\rho\, dV$$
$$M_{xy} = \iiint_D z\rho\, dV$$

Center of mass:

$$\bar{x} = \frac{M_{yz}}{M}$$
$$\bar{y} = \frac{M_{xz}}{M}$$
$$\bar{z} = \frac{M_{xy}}{M}$$

When density of solid object is constant ($\rho = 1$), the center of mass is called the **centroid** of the object.

In two dimensions

Mass:

$$M = \iint_D \sigma \, dA$$

First moments about the coordinate axes:

$$M_y = \iint_D x\sigma \, dA$$
$$M_x = \iint_D y\sigma \, dA$$

Center of mass:

$$\bar{x} = \frac{M_y}{M}$$
$$\bar{y} = \frac{M_x}{M}$$

Moments of Inertia

In three dimensions

$$I = \iiint r^2 \rho \, dV$$

(Around x-axis, r^2 is $(y^2 + z^2)$, etc etc)

In two dimensions

$$I = \iint r^2 \sigma \, dA$$

About origin:

$$I_O = \iint (x^2 + y^2)\sigma \, dA = I_x + I_y$$

Joint Probability Density

Joint probability density function f is a function that satisfies:

1. $f(x, y) \geq 0$
2. $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \, dx \, dy = 1$
3. $P((X, Y) \in R) = \iint_R f(x, y) \, dx \, dy$

#week8