

13.6: Motion in Polar Coordinates

Given coordinates $P(r, \theta)$,

position, velocity, and acceleration can be represented in terms of:

- $\vec{u}_r = (\cos \theta)\mathbf{i} + (\sin \theta)\mathbf{j}$ (unit vector in direction of \overrightarrow{OP})
- $\vec{u}_\theta = -(\sin \theta)\mathbf{i} + (\cos \theta)\mathbf{j}$ (unit vector pointing in direction of increasing θ)

$$\frac{d\vec{u}_r}{dt} = \theta' \vec{u}_\theta$$
$$\frac{d\vec{u}_\theta}{dt} = -\theta' \vec{u}_r$$

$$\vec{r} = r\vec{u}_r = r \cos \theta \mathbf{i} + r \sin \theta \mathbf{j}$$

$$\vec{v} = r' \vec{u}_r + r\theta' \vec{u}_\theta$$

$$\vec{a} = (r'' - r\theta'^2) \vec{u}_r + (r\theta'' + 2r'\theta') \vec{u}_\theta$$

Cylindrical Coordinates

$$\vec{r} = r\vec{u}_r + z\mathbf{k}$$

$$\vec{v} = r' \vec{u}_r + r\theta' \vec{u}_\theta + z' \mathbf{k}$$

$$\vec{a} = (r'' - r\theta'^2) \vec{u}_r + (r\theta'' + 2r'\theta') \vec{u}_\theta + z'' \mathbf{k}$$

Related: [Definition of cylindrical coordinates](#)

[#module1](#) [#week3](#)