14.7: Extreme Values

Local Extrema

Let $f(x, y)$ be defined on a region R containing point (a, b). Then:

1. $f(a, b)$ is a **local maximum** of f if $f(a, b) \geq f(x, y)$ for all domain points (x, y) in an open disk around (a, b).
2. $f(a, b)$ is a **local minimum** of f if $f(a, b) \leq f(x, y)$ for all domain points (x, y) in an open disk around (a, b).

First Derivative Test

If $f(x, y)$ has a local extremum at interior point (a, b), then $\nabla f(a, b) = \vec{0}$ (all the partial derivatives are 0).
Critical points: The points where $\nabla f = \vec{0}$.
Saddle point: Critical point that isn't a local extremum (some points around greater, some are less)

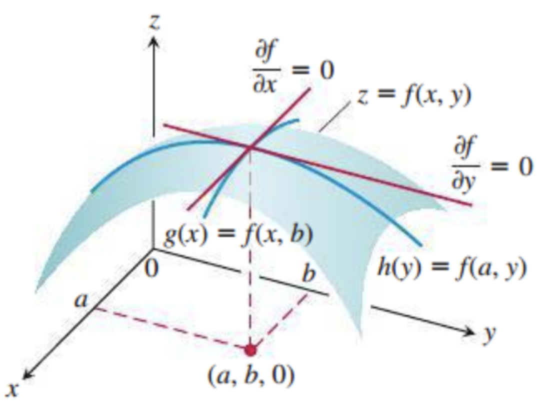

FIGURE 14.44 If a local maximum of f occurs at $x = a, y = b$, then the first partial derivatives $f_x(a, b)$ and $f_y(a, b)$ are both zero.

Second Partials Test

This is analogous to the 2nd derivative test from single-variable calculus.

- Used to determine if a critical point is a saddle point or a local min or max

$$
\begin{aligned}
A &= f_{xx}(x_0, y_0) \\
B &= f_{xy}(x_0, y_0) \\
C &= f_{yy}(x_0, y_0) \\
D &= \begin{vmatrix} A & B \\ B & C \end{vmatrix} = AC - B^2
\end{aligned}
$$

1. If $D < 0$, (x_0, y_0) is a saddle point.
2. If $D > 0$ and $A > 0$, local minimum.
3. If $D > 0$ and $A < 0$, local maximum.
4. If $D = 0$, test is inconclusive.

Why?
If $D > 0$, the surface is curving in the same way in all directions. Then, you can check A or C for if it is concave up ($A > 0$) or down ($A < 0$).

If $D < 0$, the surface is not curving in the same way in all directions. It must be a saddle point then.

Absolute Extrema

Absolute maximum: Greatest value $f(x, y)$ for all $(x, y) \in D$
Absolute maximum: Smallest value $f(x, y)$ for all $(x, y) \in D$

Process for finding absolute extrema:

1. Find candidates:
    - Interior points in D that are critical points
    - Boundary points in D that are possible candidates (critical points and end points)
2. Evaluate f at candidates.
3. Find lowest and highest of those.

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