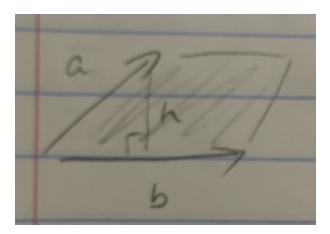
12.4: Cross Products

The **cross product** of \vec{a} and \vec{b} is the vector normal to \vec{a} and \vec{b} whose length is the area of the parallelogram between \vec{a} and \vec{b} .



$$ext{area} = \left\| ec{A} imes ec{B}
ight\| = \| ec{A} \| \| ec{B} \| \sin heta$$

In full:

$$ec{A} imes ec{B} = \Big(\| ec{A} \| \| ec{B} \| \sin heta \Big) \hat{\underline{n}}$$
Unit vector $oldsymbol{\perp}$ to plane AB

Note that if $\vec{a} \times \vec{b} = 0$, \vec{a} and \vec{b} are parallel.

Properties

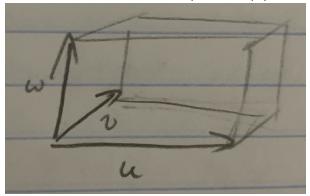
- 1. $rec{u} imes sec{v}=(rs)(ec{u} imes ec{v})$
- 2. $\vec{u} imes \vec{v} = -\vec{v} imes \vec{u}$ (cross product is <u>anticommutative</u> not commutative)
- з. $ec{0} imesec{u}=ec{0}$
- 4. $ec{u} imes(ec{v}+ec{w})=(ec{u} imesec{v})+(ec{u} imesec{w})$ (left distributive)
- 5. $(ec{v}+ec{w}) imesec{u}=(ec{v} imesec{u})+(ec{w} imesec{u})$ (right distributive)
- 6. $\vec{u} imes (\vec{v} imes \vec{w}) = (\vec{u} \cdot \vec{w}) \vec{v} (\vec{u} \cdot \vec{v}) \vec{w}$ (not associative)

Cross product as determinant

$$ec{u} imesec{v}=egin{array}{ccc} \mathbf{i} & \mathbf{j} & \mathbf{k} \ u_1 & u_2 & u_3 \ v_1 & v_2 & v_3 \ \end{array} = egin{array}{ccc} u_2 & u_3 \ v_2 & v_3 \ \end{array} egin{array}{ccc} \mathbf{i} -egin{array}{ccc} u_1 & u_3 \ v_1 & v_3 \ \end{array} egin{array}{ccc} \mathbf{j} +egin{array}{ccc} u_1 & u_2 \ v_1 & v_2 \ \end{array} egin{array}{cccc} \mathbf{k} \end{array}$$

Triple scalar product

Used to find the area of a parallelopiped (3D parallelogram)



$$(ec{u} imesec{v})\cdotec{w} = egin{vmatrix} dots & dots & dots \ ec{u} & ec{v} & ec{w} \ dots & dots & dots \end{bmatrix}$$

#module1 #week1