

14.6: Tangent Planes & Differentials

Tangent Planes & Normal Lines

Recall that the [normal to a level surface](#) is ∇f .

Tangent Planes

The **tangent plane** to level surface $f(x, y, z) = c$ of a differentiable function f at point $P_0(x_0, y_0, z_0)$ where the gradient is not zero is the plane through P_0 normal to $\nabla f(x_0, y_0, z_0)$:

$$\nabla f(P_0) \cdot \overrightarrow{P_0 P} = 0$$

This can also be written in component form as:

$$f_x(P_0)(x - x_0) + f_y(P_0)(y - y_0) + f_z(P_0)(z - z_0) = 0$$

Normal Lines

The **normal line** to level surface $f(x, y, z) = c$ is the line through P_0 parallel to $\nabla f(x_0, y_0, z_0)$:

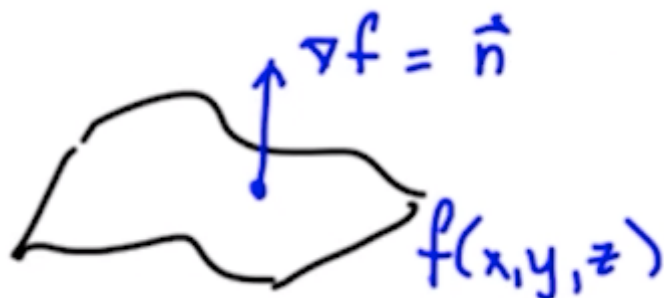
$$\vec{r}(t) = P_0 + t\nabla f(P_0)$$

This can also be written in component form as:

$$x = x_0 + f_x(P_0)t$$

$$y = y_0 + f_y(P_0)t$$

$$z = z_0 + f_z(P_0)t$$



Differentials

Linearization

The **linearization** of differentiable function $f(x, y)$ at (x_0, y_0) is:

$$L(x, y) = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

$$L(x, y) = f(x_0, y_0) + \frac{\partial f}{\partial x} \Delta x + \frac{\partial f}{\partial y} \Delta y$$

The approximation $f(x, y) \approx L(x, y)$ is called the **standard linear approximation** of f at the point.

The **total differential of f** is the resulting change from (x_0, y_0) to $(x_0 + dx, y_0 + dy)$

$$df = f_x(x_0, y_0)dx + f_y(x_0, y_0)dy$$

Error in standard linear approximation when using L to approximate f :

$$|E| \leq \frac{1}{2}M(|x - x_0| + |y - y_0|)^2$$

M represents the upper bound of the second partials on the rectangle centered at P_0 .

Extension of above formulas to more dimensions is trivial.

More Differentials

They also help in estimating change in a function in a particular direction.

To estimate the change in value of a differentiable function f when moving a small distance, ds , from point P_0 in the direction of the unit vector \hat{u} ,

$$df = f'_{\hat{u}}(P_0)ds = (\nabla f(P_0) \cdot \hat{u})ds$$

[#module2 #week5](#)