13.6: Motion in Polar Coordinates

Given coordinates $P(r, \theta)$,
position, velocity, and acceleration can be represented in terms of:

- $\vec{u}_r = (\cos\theta)\mathbf{i} + (\sin\theta)\mathbf{j}$ (unit vector in direction of $\overrightarrow{OP}$)
- $\vec{u}_\theta = -(\sin\theta)\mathbf{i} + (\cos\theta)\mathbf{j}$ (unit vector pointing in direction of increasing θ)

$$\frac{d\vec{u}_r}{dt} = \theta'\vec{u}_\theta$$
$$\frac{d\vec{u}_\theta}{dt} = -\theta'\vec{u}_r$$

$$\vec{r} = r\vec{u}_r = r\cos\theta\mathbf{i} + r\sin\theta\mathbf{j}$$
$$\vec{v} = r'\vec{u}_r + r\theta'\vec{u}_\theta$$
$$\vec{a} = (r'' - r\theta'^2)\vec{u}_r + (r\theta'' + 2r'\theta')\vec{u}_\theta$$

Cylindrical Coordinates

$$\vec{r} = r\vec{u}_r + z\mathbf{k}$$
$$\vec{v} = r'\vec{u}_r + r\theta'\vec{u}_\theta + z'\mathbf{k}$$
$$\vec{a} = (r'' - r\theta'^2)\vec{u}_r + (r\theta'' + 2r'\theta')\vec{u}_\theta + z''\mathbf{k}$$

15.8 Integration by Substitution > Cylindrical Coordinates for more on cylindrical coordinates.

#week3