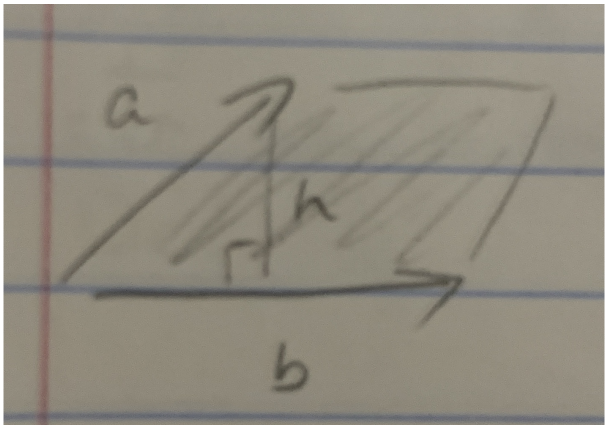
12.4: Cross Products

Area of parallelogram:



$$\|\vec{b}\|\underbrace{\|\vec{a}\|\sin\theta}_{h}$$

Cross product: vector $\perp$ to $\vec{a}$ and $\vec{b}$ whose length is this area

$$\left\|\vec{A}\times\vec{B}\right\| = \|\vec{A}\|\|\vec{B}\|\sin\theta$$

$$\vec{A}\times\vec{B} = \left(\|\vec{A}\|\|\vec{B}\|\sin\theta\right)\underbrace{\hat{n}}_{\text{Unit vector }\perp\text{ to plane AB}}$$

$\vec{a}$ and $\vec{b}$ are parallel if $\vec{a}\times\vec{b}=0$

Properties

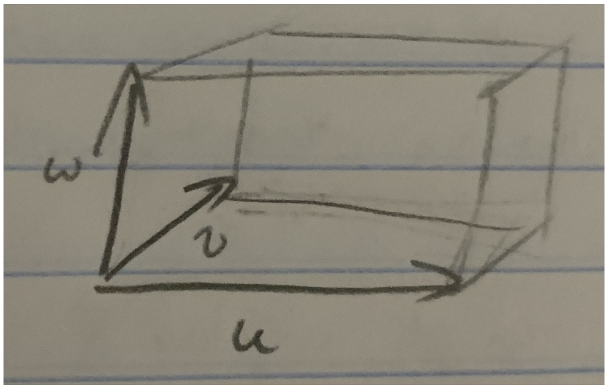
1. $r\vec{u}\times s\vec{v} = (rs)(\vec{u}\times\vec{v})$
2. $\vec{u}\times\vec{v} = -\vec{v}\times\vec{u}$ (*cross product is <u>anticommutative</u> not commutative*)
3. $\vec{0}\times\vec{u} = \vec{0}$
4. $\vec{u}\times(\vec{v}+\vec{w}) = (\vec{u}\times\vec{v}) + (\vec{u}\times\vec{w})$ (left distributive)
5. $(\vec{v}+\vec{w})\times\vec{u} = (\vec{v}\times\vec{u}) + (\vec{w}\times\vec{u})$ (right distributive)
6. $\vec{u}\times(\vec{v}\times\vec{w}) = (\vec{u}\cdot\vec{w})\vec{v} - (\vec{u}\cdot\vec{v})\vec{w}$ (**not** associative)

Cross product as determinant

$$\vec{u} \times \vec{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix} \mathbf{k}$$

Triple scalar product

Used to find the area of a parallelopiped (3D parallelogram)



$$(\vec{u} \times \vec{v}) \cdot \vec{w} = \begin{vmatrix} \vdots & \vdots & \vdots \\ \vec{u} & \vec{v} & \vec{w} \\ \vdots & \vdots & \vdots \end{vmatrix}$$

#week1