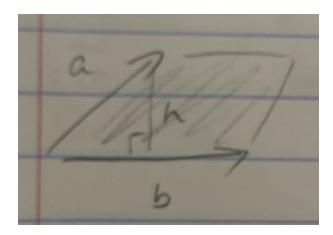
12.4: Cross Products

The **cross product** of \vec{a} and \vec{b} is the vector normal to \vec{a} and \vec{b} whose length is the area of the parallelogram between \vec{a} and \vec{b} .



$$ext{area} = \left\| ec{A} imes ec{B}
ight\| = \| ec{A} \| \| ec{B} \| \sin heta$$

In full:

$$ec{A} imes ec{B} = \left(\| ec{A} \| \| ec{B} \| \sin heta
ight) \hat{n}$$
 Unit vector $oxdot$ to plane AB

Note that if $ec{a} imesec{b}=0$, $ec{a}$ and $ec{b}$ are parallel.

Properties

1.
$$rec{u} imes sec{v}=(rs)(ec{u} imes ec{v})$$

2.
$$\vec{u} imes \vec{v} = -\vec{v} imes \vec{u}$$
 (cross product is anticommutative not commutative)

з.
$$\vec{0} imes \vec{u} = \vec{0}$$

4.
$$ec{u} imes(ec{v}+ec{w})=(ec{u} imesec{v})+(ec{u} imesec{w})$$
 (left distributive)

5.
$$(\vec{v}+\vec{w}) imes\vec{u}=(\vec{v} imes\vec{u})+(\vec{w} imes\vec{u})$$
 (right distributive)

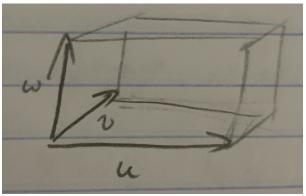
6.
$$\vec{u} imes (\vec{v} imes \vec{w}) = (\vec{u} \cdot \vec{w}) \vec{v} - (\vec{u} \cdot \vec{v}) \vec{w}$$
 (not associative)

Cross product as determinant

$$egin{aligned} ec{u} imesec{v} &= egin{aligned} \mathbf{i} & \mathbf{j} & \mathbf{k} \ u_1 & u_2 & u_3 \ v_1 & v_2 & v_3 \end{aligned} = egin{aligned} u_2 & u_3 \ v_2 & v_3 \end{aligned} \mathbf{i} - egin{aligned} u_1 & u_3 \ v_1 & v_3 \end{aligned} \mathbf{j} + egin{aligned} u_1 & u_2 \ v_1 & v_2 \end{aligned} \mathbf{k} \end{aligned}$$

Triple scalar product

Used to find the volume of a parallelopiped (3D parallelogram)



$$(ec{u} imesec{v})\cdotec{w} = egin{bmatrix} dots & dots & dots \ ec{u} & ec{v} & ec{w} \ dots & dots & dots \end{bmatrix}$$

Why?

The cross product creates a vector in a direction perpendicular to \vec{u} and \vec{v} , with a magnitude of the area between \vec{u} and \vec{v} .

The volume of a parallelopiped is V = lwh, where h is the height perpendicular to the area lw.

h would be $\vec{w}\cos\theta$ (where θ is the angle between $\vec{u}\times\vec{v}$ and \vec{w}). Hence, $V=(\vec{u}\times\vec{v})\cdot\vec{w}$.

#module1 #week1