# 13.1, 13.2: Vector Functions

A **vector function** is a function that takes one or more variables and returns a vector.

- Given real valued functions $f_1, f_2, f_3$,
  - $\vec{f}(t) = f_1(t)\mathbf{i} + f_2(t)\mathbf{j} + f_3(t)\mathbf{k}$
- $f_1, f_2, f_3$ are the **components** of $\vec{f}$
- Parametrization should be obvious

# Limits of Vector Functions

Let $\vec{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$,

$$\lim_{t \to t_0} \vec{r}(t) = \vec{L}$$

if for every $\epsilon > 0$, there exists corresponding $\delta > 0$ such that for all $t \in D$,

$$\|\vec{r}(t) - \vec{L}\| < \epsilon \text{ whenever } 0 < \|t - t_0\| < \delta$$

To evaluate a limit, find the limit of each component.

$\vec{f}$ is **continuous** at point $t = t_0$ if $\lim_{t \to t_0} \vec{f}(t) = \vec{f}(t_0)$
($\vec{f}$ is continuous if all points are continuous)

## Limit rules

Given $\vec{f}(t) \to \vec{L}, \vec{g}(t) \to \vec{M}, u(t) \to U$, then:

1. $\vec{f}(t) + \vec{g}(t) \to \vec{L} + \vec{M}$
2. $\alpha\vec{f}(t) \to \alpha\vec{L}$
3. $u(t)\vec{f}(t) \to U\vec{L}$
4. $\vec{f}(t) \cdot \vec{g}(t) \to \vec{L} \cdot \vec{M}$
5. $\vec{f}(t) \times \vec{g}(t) \to \vec{L} \times \vec{M}$

# Derivatives of Vector Functions

$$\vec{r}'(t) = \lim_{\Delta t \to 0} \frac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta t}$$

If limit exists, $\vec{r}$ is **differentiable** at t.
($\vec{r}$ is differentiable if all points are differentiable)

$\vec{r}'$ is the same as the sum of the derivatives of each component

## Derivative rules

1. $(\vec{f} + \vec{g})'(t) = \vec{f}'(t) + \vec{g}'(t)$
2. $(\alpha\vec{f})'(t) = \alpha\vec{f}'(t)$
3. $(\vec{f} \cdot \vec{g})'(t) = \vec{f}'(t) \cdot \vec{g}(t) + \vec{f}(t) \cdot \vec{g}'(t)$ (product rule, applies for *dot product*, *cross product*, *scalar multiplication*)
4. $(\vec{f} \circ u)'(t) = \vec{f}'(u(t))u'(t)$ (chain rule)

## Tangent Lines, Velocity, Acceleration

$\vec{r}(t)$ is **smooth** if $\frac{dr}{dt}$ is continuous & never zero

The **tangent line** to smooth curve $\vec{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$ at $t = t_0$ is the line that passes $\vec{r}(t_0)$ & is parallel to $\vec{r}'(t_0)$.

Let $\vec{r}$ be position. Then:

| Quantity | Identity |
|---|---|
| velocity ($\vec{v}$) | $\frac{d\vec{r}}{dt}$ |
| acceleration ($\vec{a}$) | $\frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$ |
| speed | $\lVert\vec{v}\rVert$ |
| direction of motion / direction of $\vec{v}$ / unit tangent vector | $\frac{\vec{v}}{\lVert\vec{v}\rVert}$ |

# Integrals of Vector Functions

$\vec{R}(t)$ is an antiderivative of $\vec{r}(t)$ on interval $I$ if $\frac{d\vec{R}}{dt} = \vec{r}$ at each point in $I$.

$$\int \vec{r}(t)\,dt = \vec{R}(t) + \vec{C}$$

To calculate integrals, componentize $\vec{r}$ and solve for each component.

## Integral rules

1. $\int_a^b (\vec{f} + \vec{g})(t)\,dt = \int_a^b \vec{f}(t)\,dt + \int_a^b \vec{g}(t)\,dt$
2. $\int_a^b (\alpha \vec{f})(t)\,dt = \alpha \int_a^b \vec{f}(t)\,dt$
3. $\int_a^b (\vec{c}\vec{f})(t)\,dt = \vec{c} \int_a^b \vec{f}(t)\,dt$
4. $\left\| \int_a^b (\vec{f})(t)\,dt \right\| \leq \int_a^b \left\| \vec{f}(t) \right\| dt$

# Projectile Motion

bro use kinematics

#module1 #week2