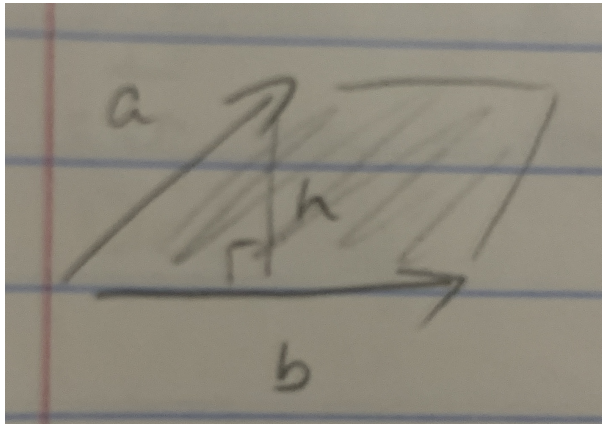


12.4: Cross Products

Area of parallelogram:



$$\|\vec{b}\| \underbrace{\|\vec{a}\| \sin \theta}_h$$

Cross product: vector \perp to \vec{a} and \vec{b} whose length is this area

$$\|\vec{A} \times \vec{B}\| = \|\vec{A}\| \|\vec{B}\| \sin \theta$$

$$\vec{A} \times \vec{B} = \left(\|\vec{A}\| \|\vec{B}\| \sin \theta \right) \underbrace{\hat{n}}_{\text{Unit vector } \perp \text{ to plane AB}}$$

\vec{a} and \vec{b} are parallel if $\vec{a} \times \vec{b} = 0$

Properties

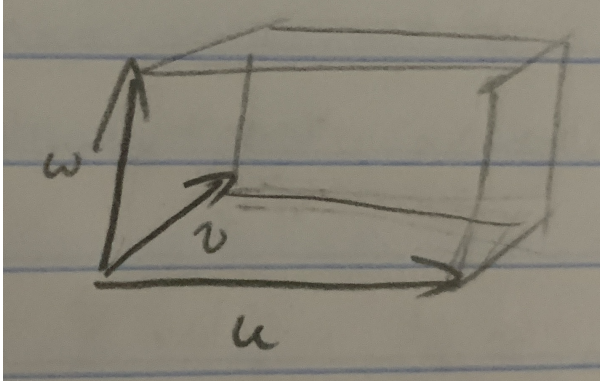
1. $r\vec{u} \times s\vec{v} = (rs)(\vec{u} \times \vec{v})$
2. $\vec{u} \times \vec{v} = -\vec{v} \times \vec{u}$ (cross product is anticommutative not commutative)
3. $\vec{0} \times \vec{u} = \vec{0}$
4. $\vec{u} \times (\vec{v} + \vec{w}) = (\vec{u} \times \vec{v}) + (\vec{u} \times \vec{w})$ (left distributive)
5. $(\vec{v} + \vec{w}) \times \vec{u} = (\vec{v} \times \vec{u}) + (\vec{w} \times \vec{u})$ (right distributive)
6. $\vec{u} \times (\vec{v} \times \vec{w}) = (\vec{u} \cdot \vec{w})\vec{v} - (\vec{u} \cdot \vec{v})\vec{w}$ (**not** associative)

Cross product as determinant

$$\vec{u} \times \vec{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix} \mathbf{k}$$

Triple scalar product

Used to find the area of a parallelepiped (3D parallelogram)



$$(\vec{u} \times \vec{v}) \cdot \vec{w} = \begin{vmatrix} \vdots & \vdots & \vdots \\ \vec{u} & \vec{v} & \vec{w} \\ \vdots & \vdots & \vdots \end{vmatrix}$$