13.6: Motion in Polar Coordinates

Given coordinates $P(r, \theta)$, position, velocity, and acceleration can be represented in terms of:

- $ec{u}_r = (\cos heta) \mathbf{i} + (\sin heta) \mathbf{j}$ (unit vector in direction of $\overrightarrow{\mathit{OP}}$)
- $ec{u}_{ heta} = -(\sin heta) \mathbf{i} + (\cos heta) \mathbf{j}$ (unit vector pointing in direction of increasing heta)

$$egin{align} rac{dec{u}_r}{dt} &= heta' ec{u}_ heta \ rac{dec{u}_ heta}{dt} &= - heta' ec{u}_r \ &ec{r} &= r ec{u}_r = r \cos heta \mathbf{i} + r \sin heta \mathbf{j} \ ec{v} &= r' ec{u}_r + r heta' ec{u}_ heta \ ec{a} &= (r'' - r heta'^2) ec{u}_r + (r heta'' + 2 r' heta') ec{u}_ heta \end{aligned}$$

Cylindrical Coordinates

$$egin{aligned} ec{r} &= rec{u}_r + z\mathbf{k} \ ec{v} &= r'ec{u}_r + r heta'ec{u}_ heta + z'\mathbf{k} \ ec{a} &= (r'' - r heta'^2)ec{u}_r + (r heta'' + 2r' heta')ec{u}_ heta + z''\mathbf{k} \end{aligned}$$

15.8 Integration by Substitution > Cylindrical Coordinates for more on cylindrical coordinates.

#week3