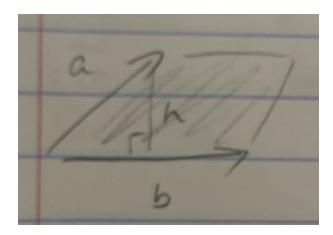
## 12.4: Cross Products

The **cross product** of  $\vec{a}$  and  $\vec{b}$  is the vector normal to  $\vec{a}$  and  $\vec{b}$  whose length is the area of the parallelogram between  $\vec{a}$  and  $\vec{b}$ .



$$ext{area} = \left\| ec{A} imes ec{B} 
ight\| = \| ec{A} \| \| ec{B} \| \sin heta$$

In full:

$$ec{A} imes ec{B} = \left( \| ec{A} \| \| ec{B} \| \sin heta 
ight) \hat{n}$$
 Unit vector  $oxdot$  to plane AB

Note that if  $ec{a} imesec{b}=0$ ,  $ec{a}$  and  $ec{b}$  are parallel.

## **Properties**

1. 
$$rec{u} imes sec{v}=(rs)(ec{u} imes ec{v})$$

2. 
$$\vec{u} imes \vec{v} = -\vec{v} imes \vec{u}$$
 (cross product is anticommutative not commutative)

з. 
$$\vec{0} imes \vec{u} = \vec{0}$$

4. 
$$ec{u} imes(ec{v}+ec{w})=(ec{u} imesec{v})+(ec{u} imesec{w})$$
 (left distributive)

5. 
$$(\vec{v}+\vec{w}) imes\vec{u}=(\vec{v} imes\vec{u})+(\vec{w} imes\vec{u})$$
 (right distributive)

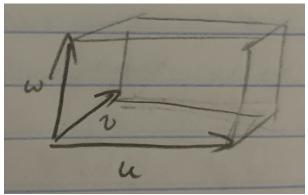
6. 
$$\vec{u} imes (\vec{v} imes \vec{w}) = (\vec{u} \cdot \vec{w}) \vec{v} - (\vec{u} \cdot \vec{v}) \vec{w}$$
 (not associative)

## **Cross product as determinant**

$$egin{aligned} ec{u} imesec{v} &= egin{aligned} \mathbf{i} & \mathbf{j} & \mathbf{k} \ u_1 & u_2 & u_3 \ v_1 & v_2 & v_3 \end{aligned} = egin{aligned} u_2 & u_3 \ v_2 & v_3 \end{aligned} \mathbf{i} - egin{aligned} u_1 & u_3 \ v_1 & v_3 \end{aligned} \mathbf{j} + egin{aligned} u_1 & u_2 \ v_1 & v_2 \end{aligned} \mathbf{k} \end{aligned}$$

## **Triple scalar product**

Used to find the volume of a parallelopiped (3D parallelogram)



$$(ec{u} imesec{v})\cdotec{w} = egin{bmatrix} dots & dots & dots \ ec{u} & ec{v} & ec{w} \ dots & dots & dots \end{bmatrix}$$

#module1 #week1