16.4: Green's Theorem

Circulation density of a vector field $\vec{F} = M\mathbf{i} + N\mathbf{j}$ at point (x, y) is the scalar expression:

$$\text{curl } \vec{F} \cdot \mathbf{k} = \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}$$

Divergence (flux density) of vector field $\vec{F} = M\mathbf{i} + N\mathbf{j}$ at (x, y) is:

$$\text{div } \vec{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}$$

See ∇, div, and curl for a full definition of div and curl.

Circulation-Curl or Tangential Form

Let C be piecewise smooth, simple closed curve enclosing region R in the plane.
Let $\vec{F} = M\mathbf{i} + N\mathbf{j}$ be a vector field with M and N have continuous 1st partial derivatives in open region containing R.

Then:
The **counterclockwise circulation** of $\vec{F}$ around C equals the double integral of $\text{curl } \vec{F} \cdot \mathbf{k}$ over R.

$$\oint_C \vec{F} \cdot \vec{T} \, ds = \oint_C M \, dx + N \, dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx \, dy$$

Flux-Divergence or Normal Form

Let C be piecewise smooth, simple closed curve enclosing region R in the plane.
Let $\vec{F} = M\mathbf{i} + N\mathbf{j}$ be a vector field with M and N have continuous 1st partial derivatives in open region containing R.

Then:
The **outward flux** of $\vec{F}$ around C equals the double integral of $\text{div } \vec{F}$ over R.

$$\oint_C \vec{F} \cdot \vec{n} \, ds = \oint_C M \, dy - N \, dx = \iint_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} \right) dx \, dy$$

Area

Green's Theorem can be used to write area in terms of a line integral.

$$\begin{aligned} A_R &= \iint_R dy \, dx \\ &= \iint_R \left(\frac{1}{2} + \frac{1}{2} \right) dy \, dx \\ &= \frac{1}{2} \oint x \, dy - y \, dx \end{aligned}$$

#module4 #week12