**Textbook sections**: 2.5

# Concavity of Autonomous Equations

Recall Autonomous DEs.

Note that

$$\frac{d^2y}{dt^2} = \frac{df}{dy}\frac{dy}{dt}$$

- Concave up if df/dy, dy/dt same signs
- Concave down if df/dy, dy/dt different signs

**Example**

Population obeys logistic equation:

$$\frac{dy}{dt} = ry\left(\frac{1-y}{K}\right)$$

1. Sketch $f$ vs $y$, identify and classify the equilibrium points of $y$.
2. For $y \in \mathbb{R}$, determine whether y is concave up or concave down.
3. Use the information in parts (a), (b) to sketch integral curves of the DE.

*Work is on paper.*

**Example**

A population obeys the DE $y' = a - y^2$ for a parameter $a$.

1. Determine the equilibrium points for any $a \in \mathbb{R}$. There are 3 cases.
2. Sketch the phase lines for each case and classify the critical points.
3. For the case when $a > 0$, sketch a few solution curves.
4. Sketch the location of the critical point as a function of $a$ in an $ay$-plane. This is known as a **bifurcation** diagram.

*Work is on paper.*