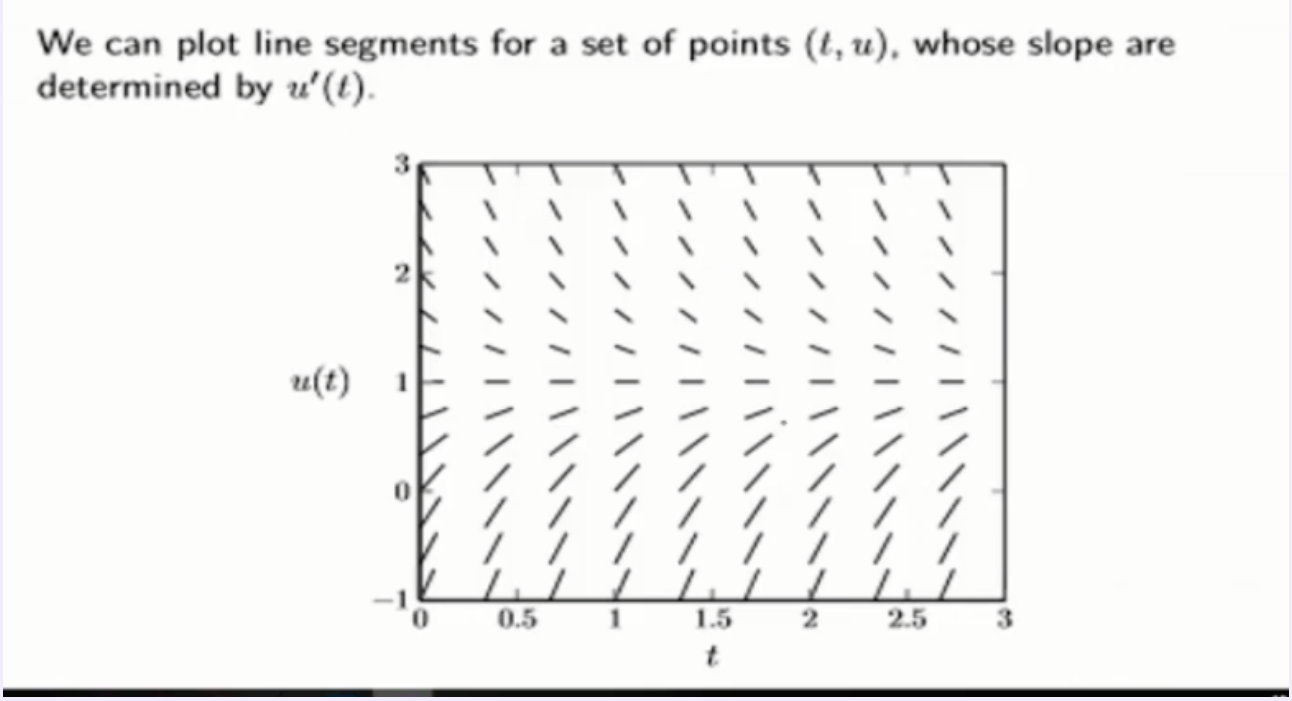
Textbook sections: 1.2
This section explores *phase lines* and *direction fields*.

Direction Fields

Exactly the same as direction fields from BC.

Example

Here is the direction field for $\frac{du}{dt} = 2 - 2u$.



Autonomous DEs

Autonomous differential equations are of the form

$$\frac{dy}{dt} = f(y).$$

Very easy to find the equilibrium points of.

Equilibrium Points

An **equilibrium solution** (or **critical points**, **fixed points**, **stationary points**, **steady-state points**) for a DE in $y(t)$ satisfies $y = \text{constant}$.

- i.e. $\frac{dy}{dt} = 0$.
- Used in optimization problems :D

Classifications for Equilibria in Autonomous Equations

classification	intepretation
asymptotically stable	solution curves close to & on either side of y_1 converge as $t \to \infty$
unstable	solution curves close to & on either side of y_1, diverge as $t \to \infty$
semi-stable	solution curves close to & on one side converges as $t \to \infty$ and away on the other side

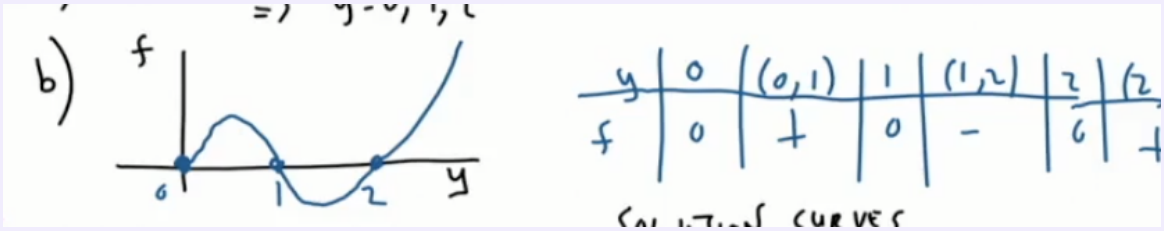
See also: week 3 equilibrium points and week 10 equilibrium points.

Phase Lines

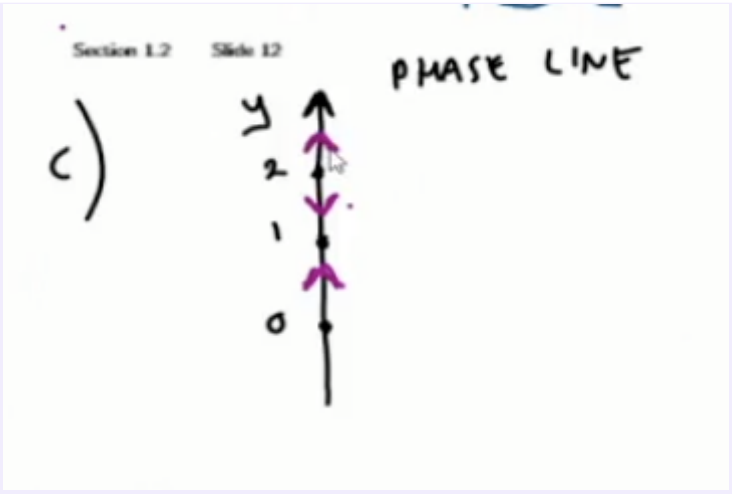
Phase line (or **phase portrait**) is a number line that indicates where solutions tend towards.

Example

Suppose $\frac{dy}{dt} = \overbrace{y(y-1)(y-2)}^{f(y)}$, $y(0) \geq 0$, $t \geq 0$.

1. The equilibrium points: $y = 0, 1, 2$
2. Sketch $f(y)$ vs y.
   - 

3. Use (1) and (2) to sketch phase line.

- 

4. Use (1), (2), (3) to sketch a few solution curves (aka **integral curves**) for the DE.
   - At $y = 0, 1, 2$, $\frac{dy}{dt}$ is 0, so it should be horizontal at those points. $\frac{dy}{dt}$ on $u \in (0, 1)$ is positive, so y is increasing. etc.
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