Textbook sections: 7.3, 7.4

Logistic Growth

Logistic growth can be represented with

$$\frac{dx}{dt} = x(\varepsilon - \sigma x),$$

where:

- ε is a growth rate
- $\frac{\sigma}{\varepsilon}$ is a saturation level
- $(\varepsilon - \sigma x)$ represents the environmental capacity for a species (such as food supply)
- $x(t)$ represents the population of **one species** at time t.

Competing Species (7.3)

Suppose there are two species that do not interact directly but compete for the same food supply.

We could use two logistic equations...

$$\begin{aligned} \frac{dx}{dt} &= x(\varepsilon_1 - \sigma_1 x) \\ \frac{dy}{dt} &= y(\varepsilon_2 - \sigma_2 y), \end{aligned}$$

but this fails to consider the same, limited food source.

Thus, we can include a term to associate the two...

$$\begin{aligned} \frac{dx}{dt} &= x(\varepsilon_1 - \sigma_1 x - \alpha_1 y) \\ \frac{dy}{dt} &= y(\varepsilon_2 - \sigma_2 y - \alpha_2 x). \end{aligned}$$

Predator-Prey Model (7.4)

Suppose that we have two species that interact directly with each other, with the assumptions...

- $x(t)$ and $y(t)$ are their population
- y preys on x
- without prey, y dies out ($y' = -Cy, C > 0$)

- without predators, x grows exponentially ($x' = Ax, A > 0$)
- the number of encounters is proportional to xy

The **Lotka-Volterra** equations:

$$\frac{dx}{dt} = Ax - \alpha xy$$
$$\frac{dy}{dt} = -Cx + \gamma xy$$