Textbook sections: 2.1, 2.2

Separable Equations (2.1)

Same as from BC.
A first-order differential equation is **separable** if it can be written as

$$\frac{dy}{dx} = f(x, y) = p(x)q(y)$$

where p, q are functions.

Integration is done to solve:

$$\begin{aligned} \frac{dy}{dx} &= p(x)q(y) \\ \frac{1}{q(y)}dy &= p(x)dx \\ \int \frac{1}{q(y)}dy &= \int p(x)dx \end{aligned}$$

Solving First-Order Linear Differential Equations (2.2)

Recall [the standard form for a first-order linear DE]:

$$y' + p(t)y = g(t)$$

We multiply each side by an integrating factor $\mu = e^{\int p(t)dt}$.
(Note that the integration constant is irrelevant for μ).

By doing so, we get

$$\mu y' + \mu p(t)y = \mu g(t).$$

The left-hand-side simplifies, leaving:

$$\frac{d}{dt}(\mu y) = \mu g.$$

Example

Solve:

$$ty' + 2y = 4t \quad (t \geq 0)$$

Note that this example is NOT separable.

Solving: $t > 0$

In standard form:

$$y' + \frac{2}{t}y = 4.$$

We find μ:

$$\mu = e^{\int \frac{2}{t} dt} = e^{2 \ln t} = t^2$$

Multiplying both sides by μ:

$$\begin{aligned} t^2 \frac{dy}{dt} + 2ty &= 4t^2 \\ \frac{d}{dt}(t^2 y) &= 4t^2 \end{aligned}$$

Then, integrate:

$$\begin{aligned} t^2 y &= \int 4t^2 dt \\ t^2 y &= \frac{4}{3}t^3 + C \\ y &= \frac{4}{3}t + \frac{C}{t^2} \end{aligned}$$

Solving: $t = 0$

The equation simplifies to...

$$\begin{aligned} (0)y' + 2y &= 4(0) \\ y &= 0 \end{aligned}$$