Textbook sections: 1.1

Differential Equations

- A **differential equation** (DE) is an equation involving a function and its derivatives
- A **solution** is a differentiable function that satisfies the DE on some interval.
- We can substitute to see if the function satisfies DE.

Example: Newton's Law of Cooling

The rate of change of the temperature of an object is negatively proportional to the difference between the temperature of the object and the temperature of the environment the object is in (the ambient temperature).

Let temperature at time t be $T(t)$, and ambient temperature be T_{amb}. Then:

$$\frac{dT}{dt} = -c(T(t) - T_{amb}), \quad k > 0$$

This is an example of a **differential equation**, T is the **unknown**, c and T_{amb} are **parameters** of the system.

Example: Verifying Solutions to DE

Verify that $u(t) = Ce^{-kt} + T$, $C \in \mathbb{R}$, is a solution to:

$$\frac{du}{dt} = -k(u - T)$$

Solution:

$$\begin{aligned} u(t) &= Ce^{-kt} + T \\ \frac{du}{dt} &= -kCe^{-kt} \\ \frac{du}{dt} &= -k(u - T) \end{aligned}$$

Dynamical System

Newton's law is an example of an equation describing a **dynamical system**.
A dynamical system is composed of:

- A system:
  - We are observing a *phenomenon* which behaves according to a set of laws
  - Phenomenon may be mechanical, biological, social, etc.
- Dynamic:
  - The system involves time

Our task (in this course) to **predict** and/or **characterize** the long-term behavior of the dynamical system.