Textbook sections: 6.3, 6.4

Eigenvalue Review

- **Algebraic multiplicity**, number of eigenvalue repeats in characteristic polynomial
- **Geometric multiplicity**, number of linearly independent eigenvectors

$$0 < g_n \leq a_n \leq n$$

Homo Systems

Extends [Solutions to Systems of 2 FOLDEs]().

Given FOLDE system:

$$\vec{x}' = P\vec{x}, \quad P \in \mathbb{R}^{n \times n}$$

We know solutions are of the form $\vec{x}(t) = e^{\lambda t}\vec{v}$.

Eigenvalues have 3 cases:

	Section	Eigenvalues	P
1	6.3	All $\mathbb{R}$	not defective
2	6.4	All $\mathbb{C}$	not defective
3	6.7	All $\mathbb{C}$	defective

6.7 is out of the scope of this course.
A defective matrix is a matrix where $\sum g_n \neq n$.

- i.e. number of eigenvectors < n, non-diagonalizable.

Real, Non-Defective Solutions (6.3)

Since matrix is not defective, it is diagonalizable.

Given linearly independent eigenpairs $(\lambda_1, \vec{v}_1), \ldots, (\lambda_n, \vec{v}_n)$ for matrix A, then for

$$\vec{x}' = A\vec{x},$$

the fundamental set of solutions is

$$\left\{e^{\lambda_1 t}\vec{v}_1, \ldots, e^{\lambda_n t}\vec{v}_n\right\}$$

and the general solution is a linear combination of these:

$$\vec{x} = \sum_{i=1}^{n} c_i e^{\lambda_i t} \vec{v}_t$$

Example

Determine general solution to

$$\vec{x}' = P\vec{x}, \quad P = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \\ 1 & -1 & 2 \end{bmatrix}.$$

The eigenvalues of P are 0 and 1.

$\lambda = 0$

$$P - 0I = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \\ 1 & -1 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Then, $\vec{v} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$.

$\lambda = 1$

$$P - I = \begin{bmatrix} -1 & 1 & -1 \\ 1 & -1 & 1 \\ 1 & -1 & 1 \end{bmatrix} \sim \begin{bmatrix} -1 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Then $\vec{v} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

Complex, Non-Defective Solutions (6.4)

For each complex pair, convert into real values by the formula from the complex eigenvalues section.