Textbook sections: 1.3

Importance of Classification

- Many different methods for solving DEs
- All methods for solving DEs only work for *certain* types of DEs
- Classification helps discern which methods are the best to use

Classifications

Ordinary and Partial DEs

Definition: ordinary differential equation (ODE)

Dependent variables in DE depend on only 1 independent variable.

Example

$$\frac{dy}{dt} = -k(y - T)$$

(where y is a function of t).

Definition: partial differential equation (PDE)

Dependent variable(s) depend on more than one independent variable

Example

$$\frac{\partial y}{\partial x} = k(\partial^2 y)/(\partial x^2)$$

(where y is a function of t and x).

Order

The **order** of an ODE is the highest degree derivative which appears in the equation.

$u'' + 2u' = u^3 + t^4$ is a 2nd-order ODE because the highest derivative is the 2nd.

Linear DEs vs. Non-Linear DEs

An n^{th} order **linear ODE** takes the form:

$$\sum_{k=0}^{n} a_k(t) u^{(n-k)}(t) = g(t)$$

where $a_k(t)$ and $g(t)$ are given, $u(t)$ is unknown.

In long form:

$$a_0(t)u^{(n)}(t) + a_1(t)u^{(n-1)}(t) + \ldots + a_{n-1}(t)u'(t) + a_n(t)u(t) = g(t)$$

A DE that is not of this form is **non-linear**. (The coefficients are not just functions of t.)

Linear Constant Coefficient ODEs

Linear ODE where all a_k are constants.

Homogeneous & Inhomogeneous ODEs

- If $g(t) = 0$, linear ODE is **homogeneous**.
- Otherwise, it is **inhomogeneous**.

Standard Form

The general first-order linear equation is of the form:

$$a_0 \frac{dy}{dt} + a_1 y = g(t)$$

In standard form, the dy/dt term has a coefficient of 1:

$$\frac{dy}{dt} + py = g$$

(Note when $a_0(t) \neq 0$: $p(t) = \frac{a_1(t)}{a_0(t)},\ g(t) = \frac{h(t)}{a_0(t)}$)
(In the cases where $a_0(t) = 0$, solving is very trivial, lol)