— “What is Spatial Logic?”

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In a nutshell...

This introductory chapter of van Benthem and Bezhanishvili (2007) overview *spatial logic*, containing its description and historcial contexts. It has mattered to spatial logic (as other logics) to take a *balance* between *expressive power* and *computational complexity*.

Keywords: spatial logic, Hilbert, Tarski,

Definiton and analogy.

Definition of spatial logic. “By a *spatial logic*, we understand any formal language interpreted over a class of structures featurung geometrical entities and relations, broadly construed.” (p.1) Note that the authors do *limit* it to a certain kind of ‘spaces’. Any ‘space’, say topological, affine, metric, or more specific ones like Euclidean three dimensional space are all what spatial logics matter.

Pararell between space and time. It is helpful to see the parellels between space (spatial logic) and time (temporal logic), the more established area. Temporal logic has its appeal in its good balance between expressive power and computational complexity. (p. 2) So expected is spatial logic.

Historical Contexts: Hilbert and Tarski.

Historical contexts: Hilbert. Euclidean classical geometry was a target of Hilbert’s mathematical analysis. Hilbert relied on (abstract and lightly mathematicalized but still) idiomatic German for its language.

Historical contexts: Tarski. Spaces needed to wait Tarski 1959 for its being completely formalized. The development of formal logic and model-theoretic semantics made possible to “prove the precise inferential and expressive resources of geometry” (p. 2). The aim behind his 1959 paper was to search consequences of restricing the syntax (particularly, of a first-order logic). Sacrificing its expressive power, Tarski found that the theory of elementary geometry is *decidable*, meaning that a mechanical method exists to determine whether a given sentence is true under the intended interpretation. Note that the second-order theory required for all Hilbert’s axioms is undecidable.

Tarski’s impact. “Tarski’s discovery illustrates the most distinctive feature of logic in the wake of the model-theoretic revolution of the previous century: its fundamentally linguistic orientation.” (p. 3) “On this view, spatial logic, as defined above, becomes the study of the relationship between geometrical structures and the spatial languages which describe them.” (p.3)

Elementary geometry aximatized In elementary geometry, objects can be defined by a fixed number of points in points in the Euclidean plane. But we want more: polygons, arbitrary connected regions and what Tarski’s Geometry of Solids dealed. Tarski’s own system uses the second-order syntax and the object variables are assigned to not point but certain regions called solids (more specifically, the regular closed subsets of R^3). The result is remarkable. The resulting theory (i) can be completely axiomatized in the second-order language and (ii) is categorical. [1]

More coarse: binding topology Tarski and McKinsey 1944 found that “topology has small decidable fragments” once interpretating topological interior as a modal operator.

Three princpal directions of spatial logic

1. What geometrical entities do you use for interpreatation?

[1] A theory is categorical when any model of the teory is isomorphic to the normal interpretation on the reals.

2. What do you assign to the non-logical primitives? Like $||p||$.

3. Which syntax?

Classification tradition of geometries, connected. Spatial logic clasifies geometrical languages with respect to their spatial primitives. This shares the same idea with the long-standing tradition of classification of geometries (e.g. Klein's Erlanger Programm). However, the difference is "the logical approach opens up many new possibilities in this regard, such as, for instance, a new sort of invariance between topological patterns, much coarser even than topological homeomorphism, viz. modal bisimulation." (p. 5)

Four salient issues of spatial logic

1. How can we characterize its valid formulas?

2. What is its expressive power?

3. What is its computational complexity?

4. Whata laternative interpretations does it have?

Recent trends: from computer science.

Qualitative spatial reasoning.

Spatial database.

Image processing.

To a misunderstanding.

A possible misunderstanding.

A reply (with an aid of historical backgrounds)

Construction of this volume.

A. Tarskian spirit: first-order languages.

B. Modal logics for topology: Framgement of first order logic.

C. Space combined with others: time.

D. Further mathematical and computational advance.

E. A coda: metaphysics.

What does *not* this book talk about

Comments by me

References

van Benthem, J. and Bezhanishvili, G. (2007). Modal Logics of Space. In Aiello, M., Pratt-Hartmann, I., and van Benthem, J., editors, *Handbook of Spatial Logics*, chapter 5, pages 217 – 298. Springer, Dordrecht, The Netherlands.