

A Dimensional View towards Vagueness ¹

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We should see *vagueness* from a *dimensional* perspective. I argue that my solution is good but do *not* claim that others are wrong. Rather, my dimensional view provides a formal platform on which the disputes are possible.

Keywords: vagueness, sorites, dimensions

VAGUENESS REVISITED. Most verbal expressions are vague. An ordinal predicate such as “is bald” or “is a heap” leads to a well known paradox called *sorites paradox*. ^{3 4}

Obviously non-bald case: A man with 200000 hairs is surely *not* bald.

Obviously bald case: A man with no hair is surely bald

Tolerance Principle: Pulling or adding a single hair does not effect baldness.

Unwelcome conclusion: A man with 2,000,000 hairs is bald. ■

MY SOLUTION: SEE THINGS DIMENSIONALLY. ⁵ We are already familiar with the dimensional perspective. We see things not directly in a three-dimensional structure but in a two-dimensional structure (as our retina does).

FORMALIZATION. Technically speaking, absence and abundance of information are written in terms of *projection functions* on a dimensional structure.

Definition 1 (Dimensional structure) Let X_i a space (set). A dimensional structure M is defined as follows:

$$M^i = \prod_{i \in I} X_i.$$

Definition 2 (Predicates and objects) Let $P \in \text{PRED}$ be a predicate and $x \in \text{OBJ}$ be an object. Within a dimensional structure M , a predicate P is a subset of M , written as $\llbracket P \rrbracket^M \subseteq M$ and an object x is a element of M , $x \in M$.

Definition 3 (Projection) Fix $j \in I$. Note $x = \{x_0 \in X_0, x_1 \in X_1, \dots, x_j \in X_j, \dots\}$. Consider a dimensional structure $M^i = \prod_i X_i$. Pick an arbitrary $j \in I$. A projection $f_j : M \rightarrow \prod_j M$ returns $f_j(x) = \prod_j x = \{x_0 \in X_0, x_1 \in X_1, \dots, x_{j-1} \in X_{j-1}, x_{j+1} \in X_{j+1}, \dots\}$.

¹ A talk for Logic and Engineering of Natural Language Semantics 15 (LENLS 15), Hiyoshi Campus of Keio University, Yokohama, Japan.

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³ Also known as paradox of heaps, paradox of baldman or *little-by-little argument*. Combining these seemingly plausible assumptions leads to a contradiction. See:

Dominic Hyde. The Sorites Paradox. In Giuseppina Ronzitti, editor, *Vagueness: A Guide*, pages 1–17. Springer Verlag, 2011

⁴ Sorites paradox matters when you analyze natural languages via logic or logical structure in natural languages.

⁵ Many solutions have been suggested. I do not argue that they are all wrong and mine is the only possible or the most plausible one. Instead, my dimensional understanding is expected to describe and embrace these different opinions.

Definition 4 (Evaluation) $M \models P(x)$ (read: “ x is P ” is true under the interpretation M) if and only if $x \subseteq ||P||$.

DEMONSTRATION: BASEBALL. We reach vagueness when we consider too little (*absence*)⁶ and when we consider too much (*abundance*).⁷

M	D ₁			
Filing	+	+	+	-
Hitting	+	-	?	-
Running	+	+	+	-
D ₂				

RETURN TO SORITES.

Obviously non-bald case: $M \models \neg B(x_{2,000,000})$

Obviously bald case: $M \models B(x_0)$

Tolerance Principle: There are no x_n and x_{n+1} such that $M \models B(x_n)$ and $M \models \neg B(x_{n+1})$

Unwelcome conclusion avoided: A man with 2,000,000 hairs does not have to be bald. ■

$M = D^* \times D^\#$	o	1	2	...	m	m+1	...	n	n+1	...	m	m+1	...	2,000,000	$D^\#$
You	+	+	+	...	+	+	...	+	+	...	+	+	...	-	
Me	+	+	+	...	+	+	...	+	+	...	+	?	...	-	
Her	+	+	+	...	+	?	...	?	-	...	-	-	...	-	
D^*															
	\Downarrow_{D^*}														
$\Downarrow_{D^*} M$	o	1	2	...	l	l+1	...	m	m+1	...	n	n+1	...	2,000,000	$D^\#$
Us	+	+	+	...	+	+	...	+	+	...	+	+	...	-	

Obvious bald/non-bald objects x_0 and $x_{2,000,000}$ are obviously so for $x_0 \subseteq ||+||^M$ and $x_{2,000,000} \subseteq ||-||^M$. The same in $\Downarrow M$.⁸ Tolerance still holds. Tolerance prohibits suddenly changing from bald to non-bald. We can construct bald x_0 and non-bald $x_{2,000,000}$ by putting *gaps* $||?||$ or *guts* $||+-||$.

PREVIOUS ATTEMPTS SEEN DIMENSIONALLY! Previous solutions are special variants of our dimensional view. For one thing, epistemicists⁹ highlights our epistemic ignorance, corresponding to “too little to care” or “too much to care”. For another thing, supervaluationists¹⁰ would consider our ? as their “truthvalue gap”.

⁶ Suppose a great batter but horrible field player. If we evaluate her/him as a whole, according to our function, s/he is both good and non-good.

⁷ Our (once fixed) evaluation is often cancelled after considering another perspective. Imagine a DH player whom you have never seen on the field. We can conclude that s/he is good without knowing his fielding ability (our function allows such) but once you start caring about tools (i.e. perspectives) ignored, your evaluation becomes more vague.

⁸ + is a shorthand for B and - for $\neg B$.

⁹ T Williamson. *Vagueness*. Routledge, 1994

¹⁰ R Keefe. *Theories of Vagueness*. Cambridge University Press, 2000

References

- [1] Dominic Hyde. The Sorites Paradox. In Giuseppina Ronzitti, editor, *Vagueness: A Guide*, pages 1–17. Springer Verlag, 2011.
- [2] R Keefe. *Theories of Vagueness*. Cambridge University Press, 2000.
- [3] T Williamson. *Vagueness*. Routledge, 1994.