A Dimensional View towards Vagueness [1]

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We should see *vagueness* from a *dimensional* perspective. I argue that my solution is good but do *not* claim that others are wrong. Rather, my dimensional view provides a formal platform on which the disputes are possible.

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Vagueness revisited. Most verbal expressions are vague. An ordinal predicate such as "is bald" or "is a heap" leads to a paradox known as *sorites paradox*. [3] [4]

[3] Also known as paradox of heaps, paradox of baldman or *little-by-little argument*. Combining these seemingly plausible assumtoions leads to a contradiction. See:
Dominic Hyde. The Sorites Paradox. In Giuseppina Ronzitti, editor, *Vagueness: A Guide*, pages 1–17. Springer Verlag, 2011

[4] Sorites paradox matters when you analyze natural languages via logic or logical structure in natural languages.

Obviously non-bald case: A man with 2,000,000 hairs is surely *not* bald.

Obviously bald case: A man with no hair is surely bald

Tolerance Principle: Pulling a single hair does not make anyone any non-bald person into a bald one.

Unwelcome conclusion: A man with 2,000,000 hairs is bald. ■

My solution: See things dimensionally. [5] We are already familiar with the dimensional perspective. We see things not directly in a three-dimensioal structure but in a two-dimensinoal structure (as our retina does).

[5] Many solutions have been suggested. I do not argue that they are all wrong and mine is the only possible or the most plausible one. Instead, my dimensional understanding is expected to describe and embrace these different opinions.

Formalization. Technically speaking, absence and abundance of information are written in terms of *projection functions* on a dimensional structure.

Definition 1 (Dimensional structure) *Let X_i a space (set). A dimensional structure M is defined as $M = \prod_{i \in I} X_i$.*

Definition 2 (Predicates and objects) *Let $P \in PRED$ be a predicate and $o \in OBJ$ be an object. Within a dimensional structure M, a predicate P is a subset of M, written as $[\![P]\!]^M \subseteq M$. An object x is also a subset of M, $o \subseteq M$.*

Definition 3 (Projection) [6] *Let $x = \{x_0 \in X_0, x_1 \in X_1, ..., x_j \in X_j, ...\}$. Consider a dimensional structure $M^i = \prod_i X_i$. Pick an arbitrary $j \in I$. A projection $f_j : M \mapsto \Downarrow_j M$ returns $f_j(x) = \Downarrow_j x = \{x_0 \in X_0, x_1 \in X_1, ..., x_{j-1} \in X_{j-1}, x_{j+1} \in X_{j+1}, ...\}$.*

[6] There are many other possible (and expressive) projections (cf. projective geometry). This simple one is used just for the sake of explanation.

Definition 4 (Evaluation) *$M \models P(x)$ if and only if $x \subseteq [\![P]\!]$.*

Demonstration: baseball. [7]

We reach vagueness when we consider too little (*absence*) Suppose a great batter but horrible field player. If we evaluate her/him as a whole, according to our function, s/he is both good and non-good. and when we consider too much (*abundance*). Our (once fixed) evaluation is often cancelled after considering another perspective. Imagine a DH player whom you have never seen on the field. We can conclude that s/he is good without knowing his fielding ability (our function allows such) but once you start caring about tools (i.e. perspectives) ignored, your evaluation becomes more vague.

[7]

M	P1	P2	P3	P4	P5	**D1**
Fileding	+	+	+	?	-	
Hitting	+	-	?	+	-	
Running	+	+	+	?	-	
D2						

$\Downarrow_{D^*}$

$\Downarrow$ **M**	P1	P2	P3	P4	P5
In total	+	?	+	+	-

Return to Sorites. Construct $\mathbf{M} = \mathbf{D}^* \times \mathbf{D}^\#$, with $\mathbf{D}^*$ evaluating by a certain person and $\mathbf{D}^\#$ specifying the number of hair.

$\mathbf{M} =$ $\mathbf{D}^* \times \mathbf{D}^\sharp$	0	1	2	...	l	l+1	...	m	m+1	...	n	n+1	...	2,000,000	$\mathbf{D}^\#$
You	+	+	+	...	+	+	...	+	+	...	+	+	...	-	
Me	+	+	+	...	+	+	...	+	+	...	+	?	...	-	
Her	+	+	+	...	+	?	...	?	-	...	-	-	...	-	
$\mathbf{D}^*$															

$\Downarrow_{D^*}$

$\Downarrow_{\mathbf{D}^*}$ **M**	0	1	2	...	l	l+1	...	m	m+1	...	n	n+1	...	2,000,000	$\mathbf{D}^\#$
Us	+	+	+	...	+	+	...	+	?	...	?	?	...	-	

Obviously non-bald case: $M \models \neg B(x_{2,000,000})$

Obviously bald case: $M \models B(x_0)$

Tolerance Principle: There are *no* x_n and x_{n+1} such that $M \models B(x_n)$ and $M \models \neg B(x_{n+1})$

Unwelcome conclusion avoided: A person with 2,000,000 hairs does not have to be bald. □

Obvious bald/non-bald objects x_0 and $x_{2,000,000}$ are obviously so for $x_0 \subseteq [\![+]\!]^M$ and $x_2, 000, 000 \subseteq [\![-]\!]^M$. The same holds even in the more "limited" model $\Downarrow M$. [8] *Tolerance still holds.* Tolerance prohibits suddenly changing from bald to non-bald. We can construct bald x_0 and non-bald $x_{2,000,000}$ by putting $[\![?]\!]$ in between.

[8] $+$ is a shorthand for B and $-$ for $\neg B$.

Previous attempts seen dimensionally! Previous solutions are special variants of our dimensional view. For example, epistemicists [9] highlights our epistemic ignorance, corresponding to "too little to care" or "too much to care". For another instance, supervaluationists [10] would consider our ? as their "truth value gap".

[9] T Williamson. *Vagueness*. Routledge, 1994

[10] R Keefe. *Theories of Vagueness*. Cambridge University Press, 2000

References

[1] Dominic Hyde. The Sorites Paradox. In Giuseppina Ronzitti, editor, *Vagueness: A Guide*, pages 1–17. Springer Verlag, 2011.

[2] R Keefe. *Theories of Vagueness*. Cambridge University Press, 2000.

[3] T Williamson. *Vagueness*. Routledge, 1994.