

τ_d -tilting theory for Nakayama algebras

Endre S. Rundsveen and Laertis Vaso
arXiv:2410.19505 | ✉ endre.s.rundsveen@ntnu.no

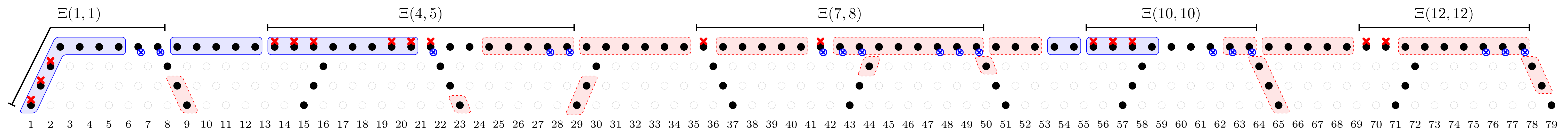


Figure 1: A τ_4 -rigid pair (M, P) of $\Lambda(79, 4)$ with maximal amount of summands.

Main results

Let $\Lambda(n, l)$ be the bounded path algebra $\mathbf{k}\Lambda_n/R^l$, where R is the ideal generated by arrows and

$$\Lambda_n: \quad \bullet \xrightarrow{n} \bullet \xrightarrow{n-1} \cdots \xrightarrow{3} \bullet \xrightarrow{2} \bullet \xrightarrow{1}$$

Assume $\Lambda = \Lambda(n, l)$ admits a d -cluster tilting subcategory \mathcal{C} .

Theorem A Let $(M, P) \in \mathcal{C} \times \text{proj } \Lambda$ be a τ_d -rigid pair, then the following are equivalent

- (a) $|M| + |P| \geq |N| + |P|$ for any τ_d -rigid pair (N, P) ,
- (b) (M, P) is τ_d -rigid and $|M| + |P| = |\Lambda|$, and
- (c) (M, P) is well-configured.

Theorem B

- (a) If $l > 2$, then there exists a bijection between the set of paths χ in G_1 of length $p - 1$ starting at an odd vertex and the set of d -torsion classes \mathcal{U} in \mathcal{C} .
- (b) If $l = 2$, then there exists a bijection between the set of paths χ in G_2 of length $p - 1$ and the set of d -torsion classes \mathcal{U} in \mathcal{C} .

Theorem C Let $M \in \mathcal{C}$ and $P \in \text{proj } \Lambda$. Then (M, P) is a τ_d -rigid pair with $|M| + |P| = |\Lambda|$ if and only if $\mathbf{P}_{(M,P)}^\bullet := P[d] \oplus \sigma_{\geq -d} \mathbf{P}^\bullet(M)$ is a silting complex in $K^b(\text{proj } \Lambda)$.

Constructing well-configured pairs

- Construct local τ_d -rigid components (admissible configurations) with maximal amount of summands, see Table 1.
- Connect the components together, using Table 2. With special considerations needed for $d = 2$.

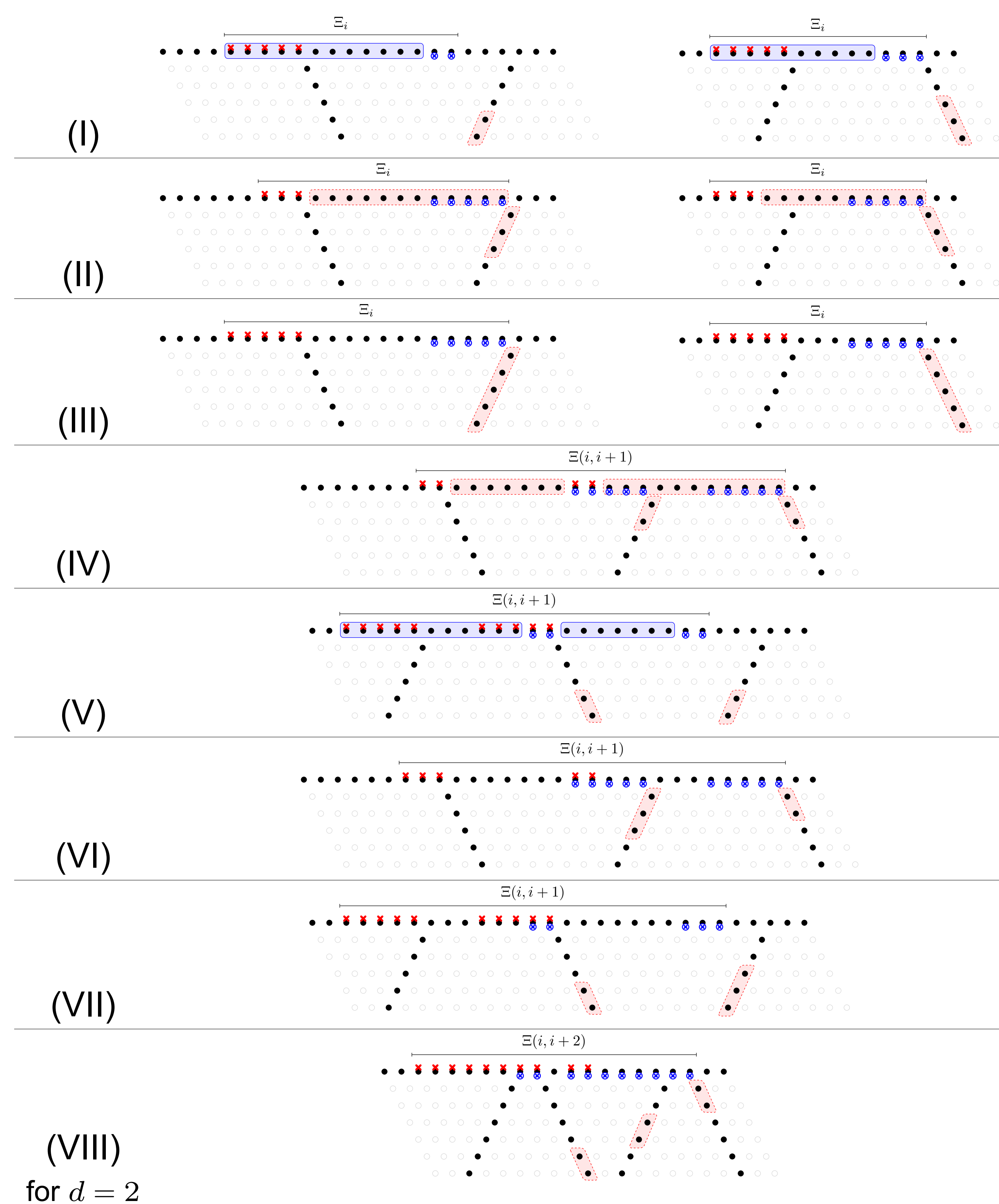


Table 1: Types of admissible configurations

$T_i \backslash T_{i+1}$	I	II	III	IV	V	VI	VII	VIII
I	S		S		S		S	S
II	A	R	A	R	A	R	A	A
III	A	R	A	R	A	R	A	A
IV	A	R	A	R	A	R	A	A
V	S		S		S		S	S
VI	A	R	A	R	A	R	A	A
VII	S		S		S		S	S
VIII	A	R	A	R	A	R	A	A

Table 2: Which components can be connected and how.

Constructing d -torsion classes

The graph G_1 is given in Figure 2 when $d > 2$, and when $d = 2$ it is given by Figure 2 with Figure 3 added.

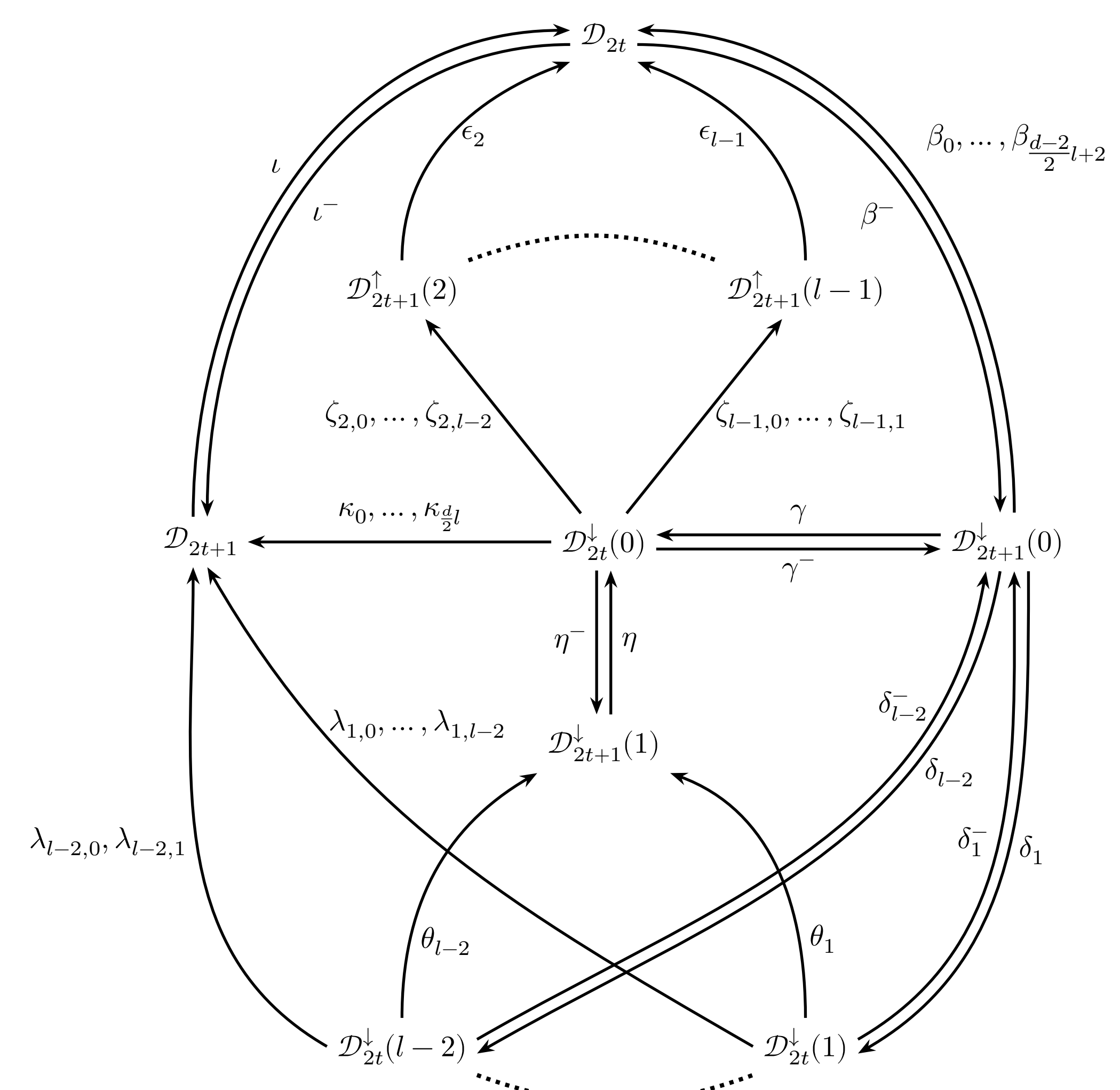


Figure 2: Construction graph for d -torsion classes.

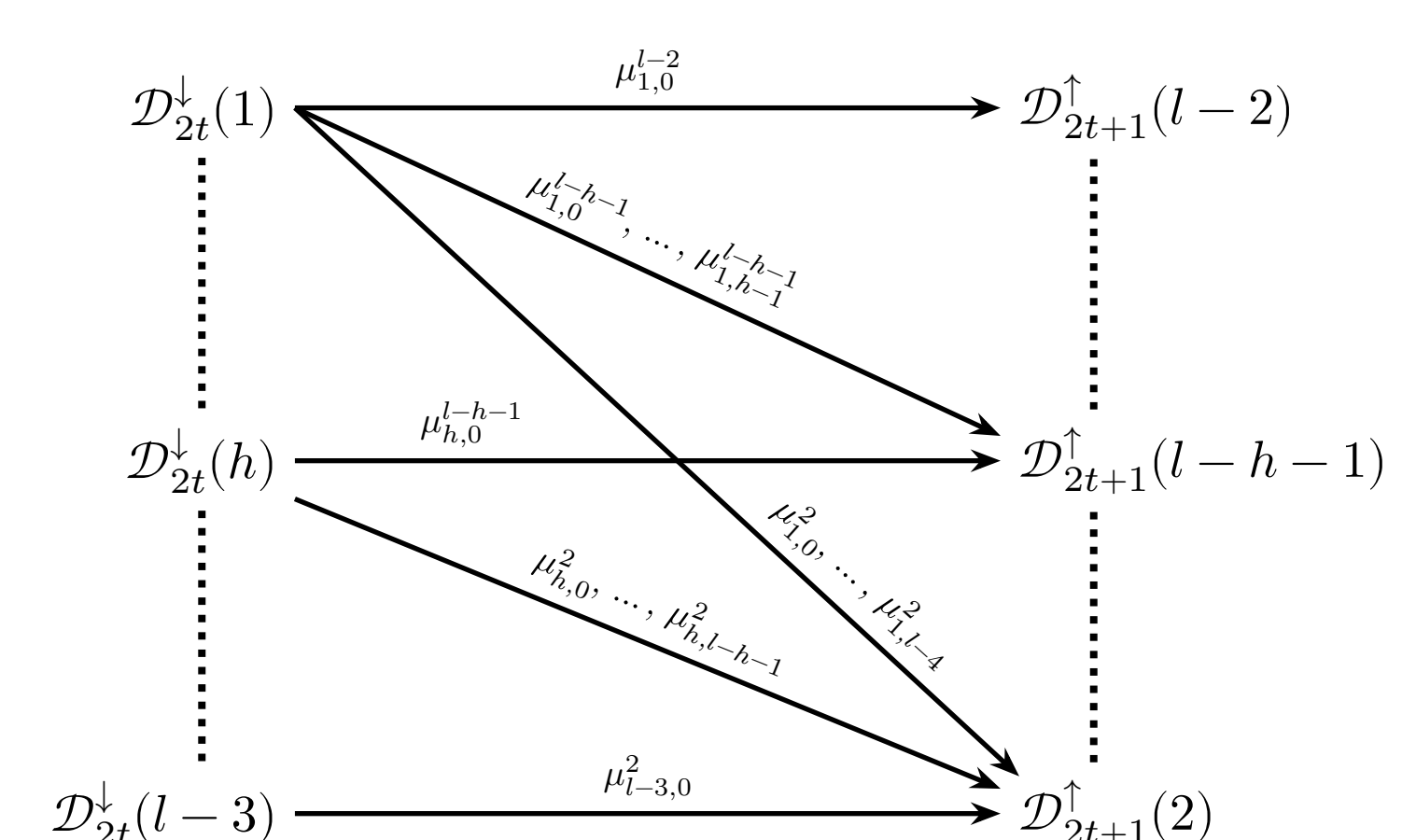


Figure 3: Add-on when $d = 2$.

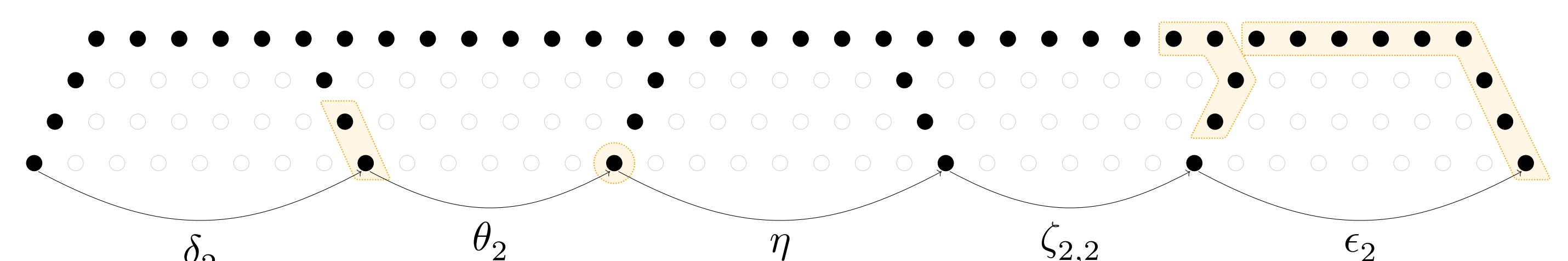


Figure 4: A 4-torsion class of $\Lambda(37, 4)$.



PDF of poster
+ Extra resources
<https://endresr.github.io/poster/>