

# $\tau_d$ -tilting theory for Nakayama algebras

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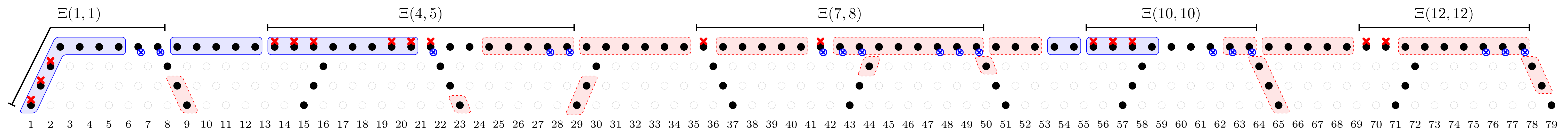


Figure 1: A  $\tau_4$ -rigid pair  $(M, P)$  of  $\Lambda(79, 4)$  with maximal amount of summands.

## Main results

Let  $\Lambda(n, l)$  be the bounded path algebra  $\mathbf{k}\Lambda_n/R^l$ , where  $R$  is the ideal generated by arrows and

$$\Lambda_n: \quad \bullet \xrightarrow{n} \bullet \xrightarrow{n-1} \cdots \xrightarrow{3} \bullet \xrightarrow{2} \bullet \xrightarrow{1}$$

Assume  $\Lambda = \Lambda(n, l)$  admits a  $d$ -cluster tilting subcategory  $\mathcal{C}$ .

**Theorem A** If  $M \in \mathcal{C}$  and  $P \in \text{proj } \Lambda$ , then the following are equivalent

- (a)  $(M, P)$  is  $\tau_d$ -rigid and  $|M| + |P| \geq |N| + |P|$  for  $(N, P)$   $\tau_d$ -rigid,
- (b)  $(M, P)$  is  $\tau_d$ -rigid and  $|M| + |P| = |\Lambda|$ , and
- (c)  $(M, P)$  is well-configured.

**Theorem B**

- (a) If  $l > 2$ , then there exists a bijection between the set of paths  $\chi$  in  $G_1$  of length  $p - 1$  starting at an odd vertex and the set of  $d$ -torsion classes  $\mathcal{U}$  in  $\mathcal{C}$ .
- (b) If  $l = 2$ , then there exists a bijection between the set of paths  $\chi$  in  $G_2$  of length  $p - 1$  and the set of  $d$ -torsion classes  $\mathcal{U}$  in  $\mathcal{C}$ .

**Theorem C** Let  $M \in \mathcal{C}$  and  $P \in \text{proj } \Lambda$ . Then  $(M, P)$  is a  $\tau_d$ -rigid pair with  $|M| + |P| = |\Lambda|$  if and only if  $\mathbf{P}_{(M,P)}^\bullet := P[d] \oplus \sigma_{\geq -d} \mathbf{P}^\bullet(M)$  is a silting complex in  $K^b(\text{proj } \Lambda)$ .

## Constructing well-configured pairs

- Construct local  $\tau_d$ -rigid components (admissible configurations) with maximal amount of summands, see Table 1.
- Connect the components together, using Table 2. With special considerations needed for  $d = 2$ .

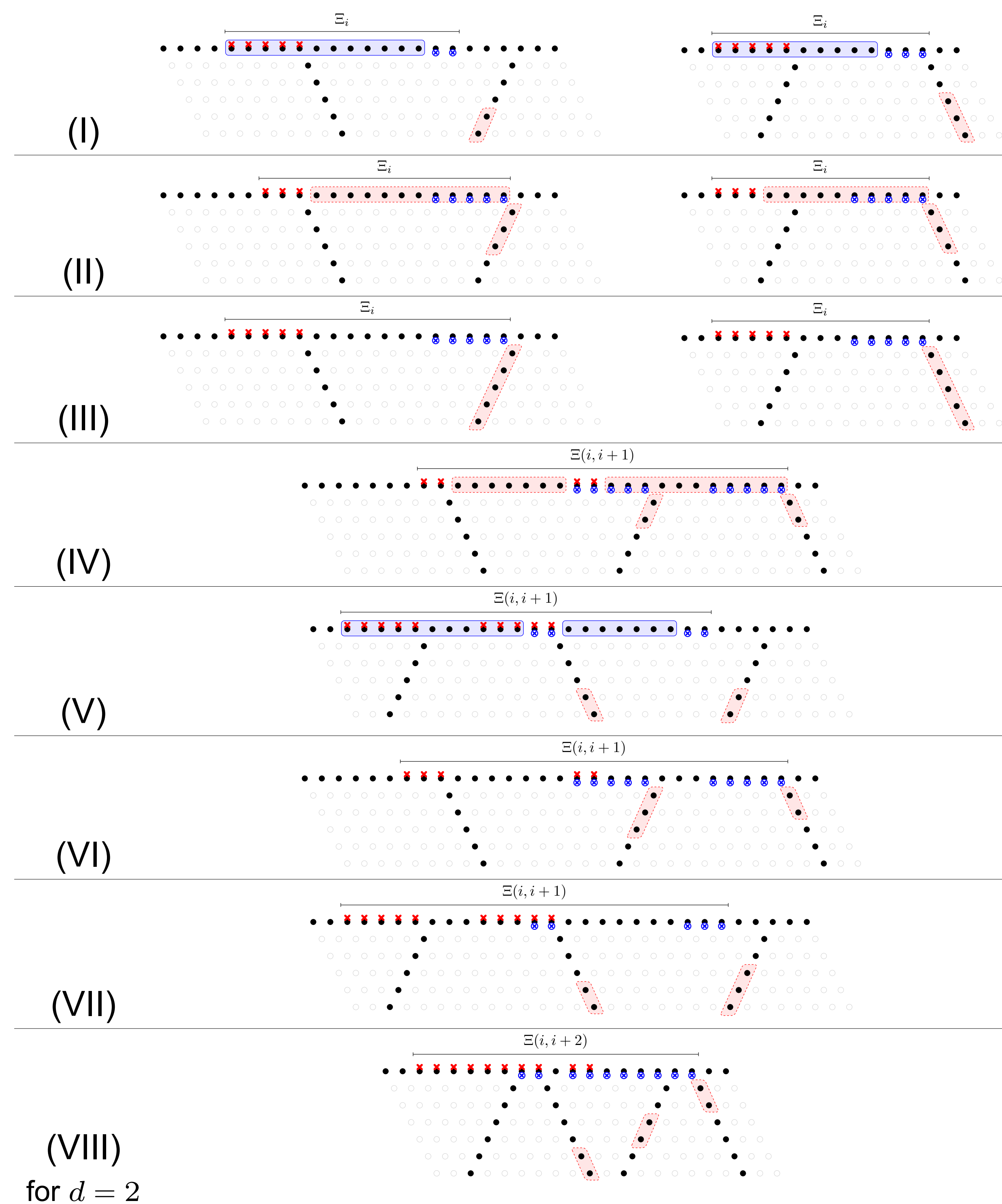


Table 1: Types of admissible configurations

$T_i \backslash T_{i+1}$	I	II	III	IV	V	VI	VII	VIII
I	S		S		S		S	S
II	A	R	A	R	A	R	A	A
III	A	R	A	R	A	R	A	A
IV	A	R	A	R	A	R	A	A
V	S		S		S		S	S
VI	A	R	A	R	A	R	A	A
VII	S		S		S		S	S
VIII	A	R	A	R	A	R	A	A

Table 2: Which components can be connected and how.

## Constructing $d$ -torsion classes

The graph  $G_1$  is given in Figure 2 when  $d > 2$ , and when  $d = 2$  it is given by Figure 2 with Figure 3 added.

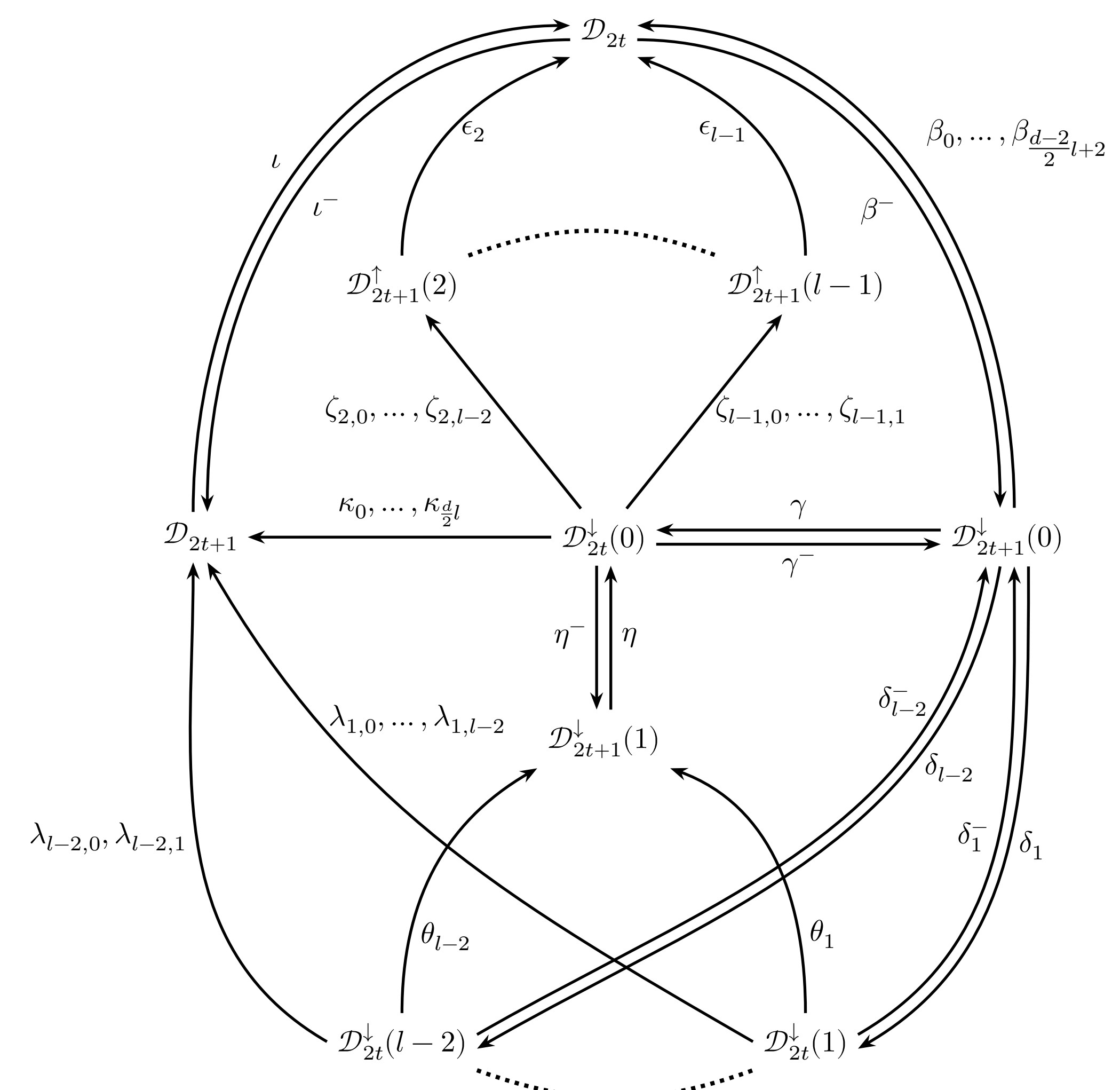


Figure 2: Construction graph for  $d$ -torsion classes.

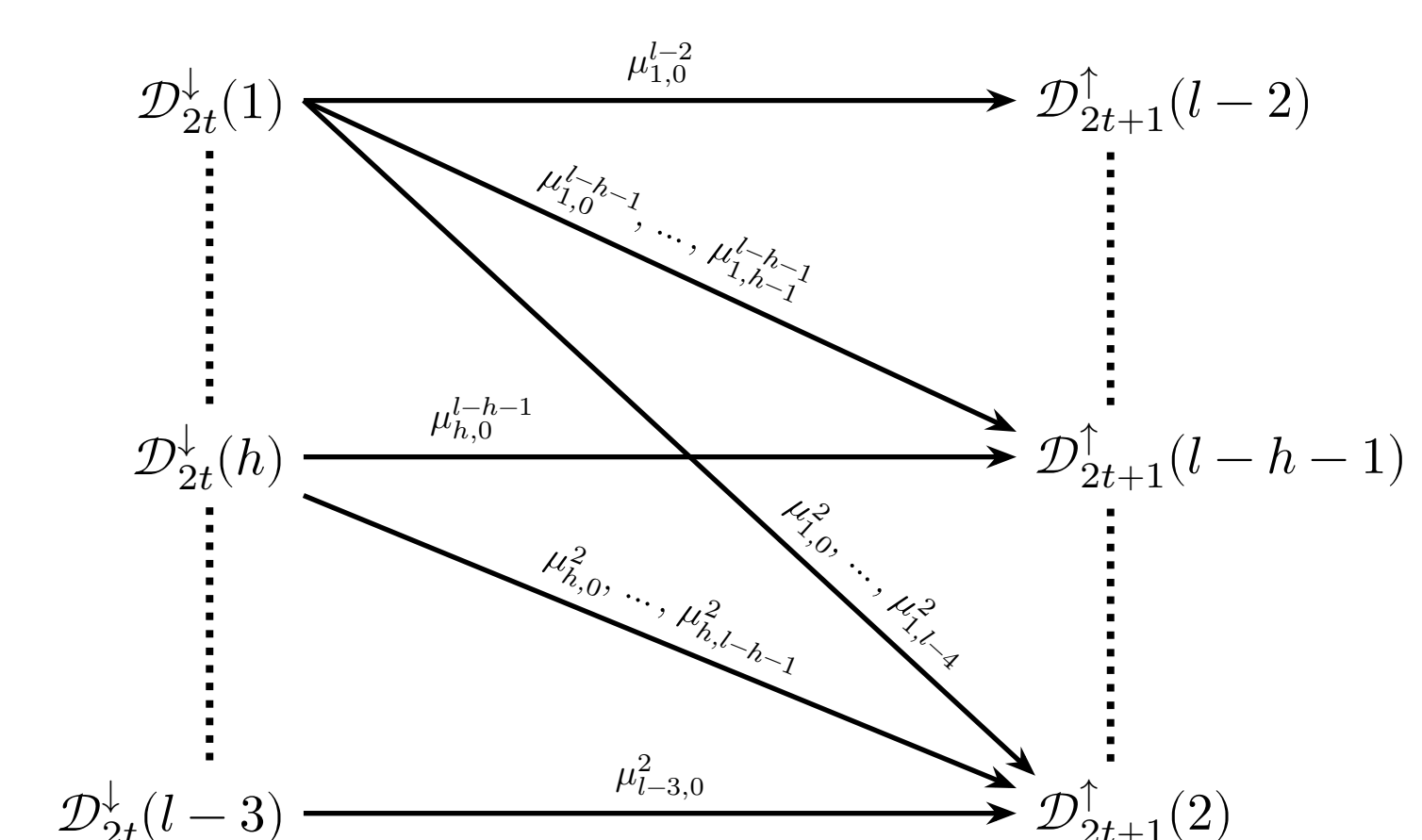


Figure 3: Add-on when  $d = 2$ .

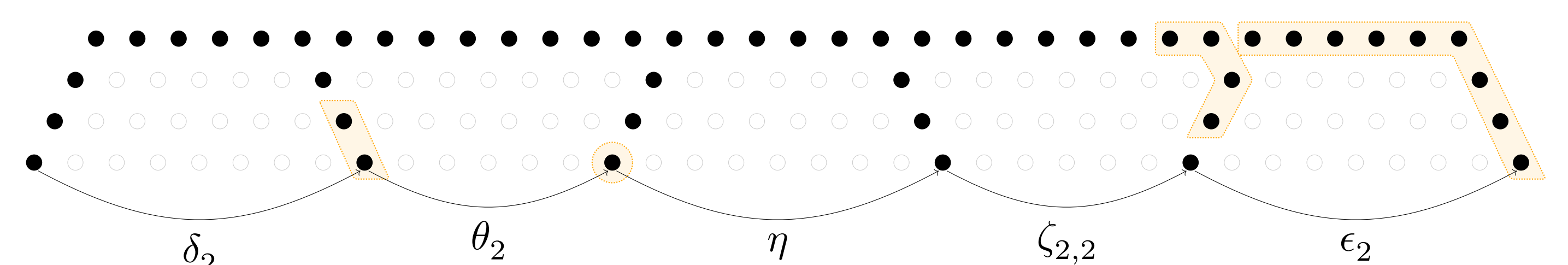


Figure 4: A 4-torsion class of  $\Lambda(37, 4)$ .



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