τ_d-tilting theory for Nakayama algebras

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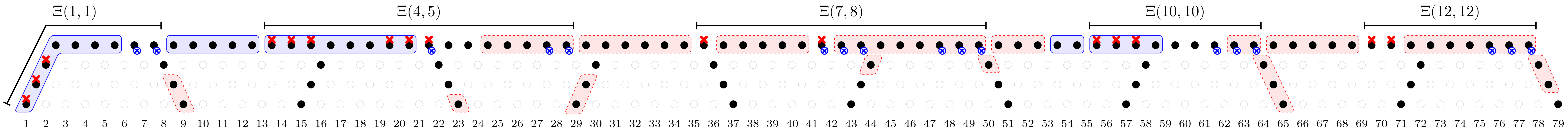

Figure 1: A τ_4-rigid pair $(\boldsymbol{M}, \boldsymbol{P})$ of $\Lambda(79, 4)$ with maximal amount of summands.

Main results

Let $\Lambda(n, l)$ be the bounded path algebra $\mathbf{k}\mathbb{A}_n/R^l$, where R is the ideal generated by arrows and

$$\mathbb{A}_n: \quad n \longrightarrow n-1 \longrightarrow \cdots \longrightarrow 3 \longrightarrow 2 \longrightarrow 1$$

Assume $\Lambda = \Lambda(n, l)$ admits a d-cluster tilting subcategory $\mathcal{C}$.

Theorem A *If $M \in \mathcal{C}$ and $P \in$ proj Λ, then the following are equivalent*

(a) $|M| + |P| \geq |N| + |P|$ for any τ_d-rigid pair (N, P),

(b) (M, P) is τ_d-rigid and $|M| + |P| = |\Lambda|$, and

(c) (M, P) is well-configured.

Theorem B

(a) If $l > 2$, then there exists a bijection between the set of paths χ in G_1 of length $p-1$ starting at an odd vertex and the set of d-torsion classes $\mathcal{U}$ in $\mathcal{C}$.

(b) If $l = 2$, then there exists a bijection between the set of paths χ in G_2 of length $p-1$ and the set of d-torsion classes $\mathcal{U}$ in $\mathcal{C}$.

Theorem C *Let $M \in \mathcal{C}$ and $P \in$ proj Λ. Then (M, P) is a τ_d-rigid pair with $|M| + |P| = |\Lambda|$ if and only if $\mathbf{P}^\bullet_{(M,P)} := P[d] \oplus \sigma_{\geq -d}\mathbf{P}^\bullet(M)$ is a silting complex in $\mathsf{K}^b(\text{proj}\,\Lambda)$.*

Constructing well-configured pairs

1. Construct local τ_d-rigid components (admissible configurations) with maximal amount of summands, see Table 1.
2. Connect the components together, using Table 2. With special considerations needed for $d = 2$.

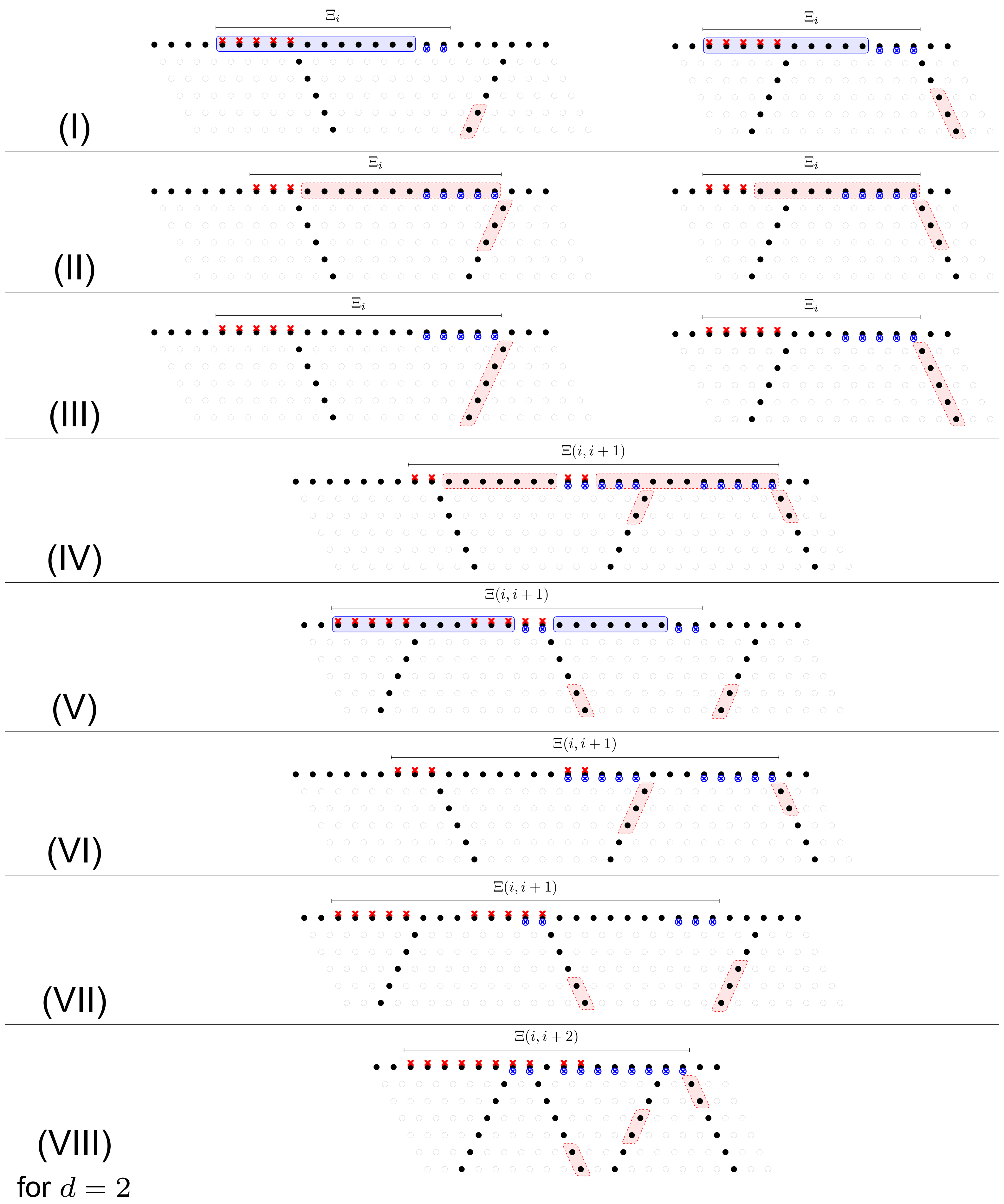

Table 1: Types of admissible configurations

T_i \ T_{i+1}	I	II	III	IV	V	VI R	VI SR	VII	VIII
I	S	■	S	■	S	■	S	S	S
II	A	R	A	R	A	R	A	A	A
III	A	R	A	R	A	R	A	A	A
IV	A	R	A	R	A	R	A	A	A
V	S	■	S	■	S	■	S	S	S
VI	A	R	A	R	A	R	A	A	A
VII S	S	■	S	■	S	■	S	S	S
VII SR	A	R	A	R	A	R	A	A	A
VIII	A	R	A	R	A	R	A	A	A

Table 2: Which components can be connected and how.

Constructing d-torsion classes

The graph G_1 is given in Figure 2 when $d > 2$, and when $d = 2$ it is given by Figure 2 with Figure 3 added.

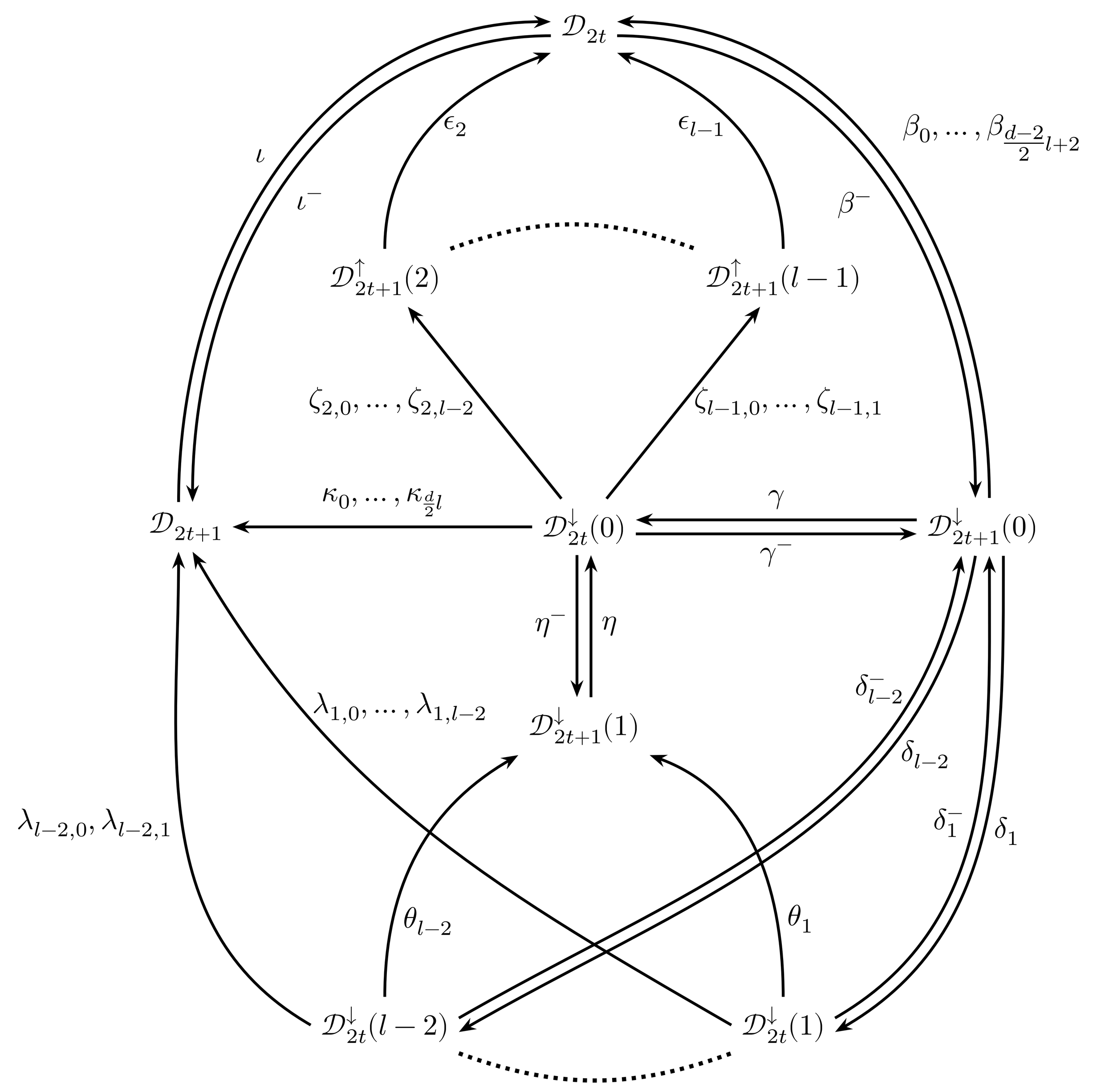

Figure 2: Construction graph for d-torsion classes.

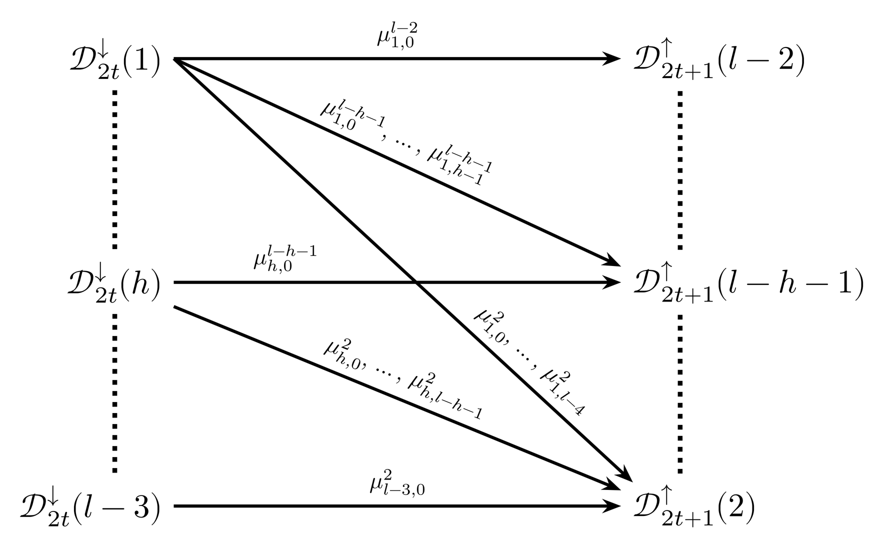

Figure 3: Add-on when $d = 2$.

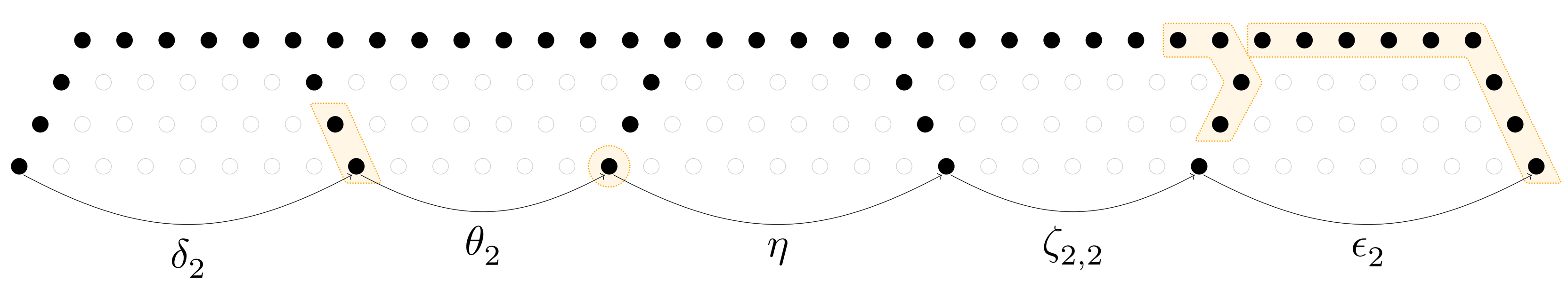

Figure 4: A 4-torsion class of $\Lambda(37, 4)$.