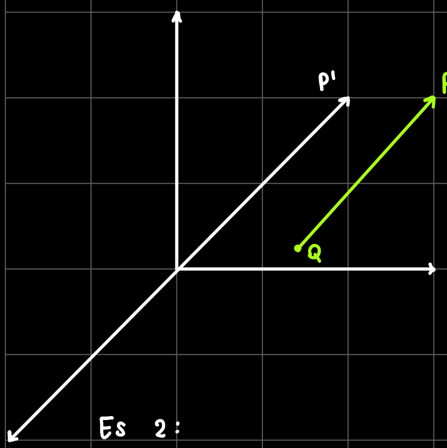
VETTORI in $\mathbb{R}^n$

1) trovare il vettore v identificato dal segmento orientato $\vec{PQ}$: NB: differenza coordinate (CODA - PUNTA)

1) $P=(1,-2,4)$; $Q=(6,1,-5)$ 1) $v \in \mathbb{R}^3$ $\vec{PQ} = (6-1, -1-(-2), -5-4) = \underbrace{(5,3,-9)}_{\text{punto il cui vettore finisce}}$

2) $P=(2,3,-6,5)$; $Q=(7,1,4,-8)$ 1) $v \in \mathbb{R}^4$ $\vec{PQ} = (7-2, 1-3, 4-(-6), -8-5) = (5,-2,10,-13)$



Es 2:

DATI: $v=(1,0,3,-2,1) \in \mathbb{R}^5$

$w=(-1,3,2,5,7) \in \mathbb{R}^5$

Determinare
1. $v+w = (0,3,5,3,8)$
2. $3v = (3,0,9,-6,3)$
3. $-w = (1,-3,-2,-5,-7)$
4. $v-w = (2,-3,1,-7,-6)$
5. $3v+w = (2,3,11,-1,10)$

Es 3:

Dati i vettori: $v=(1,0,1,0)$

$w=(1,1,1,1)$

Determinare:

1. $\|v\|, \|w\|, \|v-w\|$ $= \|v\|^2 = \langle v,v\rangle = 2 \rightarrow \|v\| = \sqrt{2}$

$= \|w\|^2 = 4 \rightarrow \|w\| = 2$

$= \|(1,0,1,0)-(1,1,1,1)\| =$

$= \|(0,-1,0,-1)\| = \sqrt{2}$

Ricorda: $\langle (a_1,a_2,a_3),(b_1,b_2,b_3)\rangle = a_1b_1 + a_2b_2 + a_3b_3$

2. $\langle v,w\rangle$ e $\langle w,v\rangle$ $= \langle (1,0,1,0),(1,1,1,1)\rangle = 2$

3. $\theta = v\hat{}w$ $\rightarrow \frac{\pi}{4}$

$= \langle v,w\rangle = \|v\| \cdot \|w\| \cdot \cos\theta$

$$\cos\theta = \frac{\langle v,w\rangle}{\|v\|\cdot\|w\|} = \frac{2}{\sqrt{2}\cdot 2} = \frac{\sqrt{2}}{2}$$

④ $P_v(w) = \dfrac{\langle w, v\rangle \cdot v}{\|v\|^2} = \dfrac{2}{2}(1,0,1,0) = (1,0,1,0)$

$P_w(v) = \langle v, w\rangle \cdot \dfrac{w}{\|w\|^2} = \dfrac{2}{4}(1,1,1,1) = \left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$

⑤ $w^{\perp}, v^{\perp}$

Sia $S = \{v, w\}$ $S^{\perp} = ?$

$w^{\perp} = \left\{ u \in \mathbb{R}^4 \;\middle|\; \langle u, w\rangle = 0 \right\}$ Sia $u = (x_1, x_2, x_3, x_4)$ $\langle u, w\rangle = \langle (x_1, x_2, x_3, x_4), (1,1,1,1)\rangle$

$= x_1 + x_2 + x_3 + x_4 = 0$

Es: $(-1, 0, 0, 1)$

Isolo $x_1 = -x_2 - x_3 - x_4$

$x_2 = t \in \mathbb{R}$

$x_3 = t' \in \mathbb{R}$

$x_4 = t'' \in \mathbb{R}$

$x_1 = -t - t' - t''$

$w^{\perp} = \left\{ (-t, -t', -t'', t, t', t'') \;\middle|\; t, t', t'' \in \mathbb{R} \right\}$

Esercizio: Individuiamo le coppie di vettori ortogonali:

$v_1 = (1,-1,0,4)$ $v_3 = (0,-4,3,0)$

$v_2 = (0,3,4,0)$ $v_4 = (7,0,0,0)$

$v_5 = (2,2,1,0)$

$\langle v_1, v_2\rangle = 3 \rightarrow v_1 \not\perp v_2 \neq 0$ $\langle v_2, v_3\rangle = 0$ $\langle v_3, v_4\rangle = 0$

$\langle v_1, v_3\rangle = 4 \rightarrow \neq 0$ $\langle v_2, v_4\rangle = 0$ $\langle v_3, v_5\rangle = -5 \neq 0$

$\langle v_1, v_4\rangle = 7 \rightarrow \neq 0$ $\langle v_2, v_5\rangle = 10 \neq 0$ NO $\langle v_4, v_5\rangle = 14 \neq 0$

$\langle v_1, v_5\rangle = 0 \rightarrow v_1 \perp v_5$

ESE: Sono $v, w \in \mathbb{R}^n$, $w \neq 0 \rightarrow$ vettore nullo

Dimostrare che

NB: $\langle a, b+c \rangle = \langle a, b \rangle + \langle a, c \rangle$

$\langle ka, b \rangle = k \langle a, b \rangle \quad k \in \mathbb{R}$

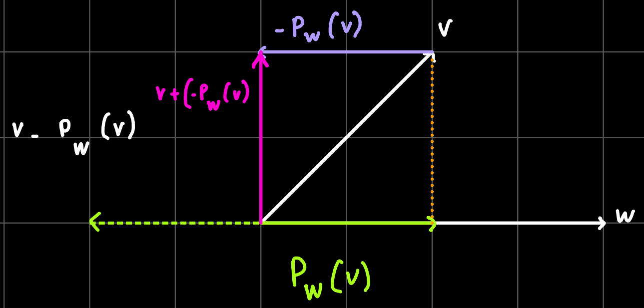
$P_w(v)$ e $v - P_w(v)$ sono ortogonali.

DIM: Calcolo $\langle P_w(v), v - P_w(v) \rangle = \langle \langle v, w \rangle \frac{w}{\|w\|^2}, v - \langle v, w \rangle \frac{w}{\|w\|^2} \rangle$

$$= \langle \langle v, w \rangle \frac{w}{\|w\|^2}, v \rangle + \langle \langle v, w \rangle \frac{w}{\|w\|^2}, -\langle v, w \rangle \frac{w}{\|w\|^2} \rangle$$

$$= \frac{\langle v, w \rangle}{\|w\|^2} \langle w, v \rangle - \frac{\langle v, w \rangle}{\|w\|^2} \frac{\langle v, w \rangle}{\|w^2\|} \langle w, w \rangle$$

$$= \frac{\langle v, w \rangle^2}{\|w\|^2} - \frac{\langle v, w \rangle^2}{\|w\|^{\cancel{4}}} \cancel{\|w\|^2} = 0$$



Es: $v = (-1, 1, 3)$ $\quad w = (2, 1, 5)$ $\quad$ 1) Determinare un vettore ortogonale a v e a w.

2) L'area del parallelogramma di lati v e w.

1) $v \times w = \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix} \times \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} = \left((1 \cdot 5 - 3 \cdot 1), -(-1 \cdot 5 - 2 \cdot 3), -1 \cdot 1 - 2 \cdot 1 \right)$

$$= \begin{pmatrix} 2 \\ 11 \\ -3 \end{pmatrix}$$

2) $\|v \times w\| = \sqrt{2^2 + 11^2 + -3^2} = \sqrt{134}$

$\|v \times w\| = \|v\| \cdot \|w\| \cdot \sin \theta$