Nystrom Approximating Kernels

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Given a symmetric kernel matrix

$$K = \begin{bmatrix} A & B^T \\ B & C \end{bmatrix} \text{ and define } L = \begin{bmatrix} A \\ B \end{bmatrix}$$

define $\sigma(A) = \{\sigma_1, \sigma_2, \dots\}$ eigenvalues of A

define $V(A) = \{v_1, v_2, \dots\}$ eigenvectors of A

Nystrom allows us to compute

1. Entire K matrix using L
2. Eigenvectors of K using $V(A)$
3. Inverse of K

Method

1. We find $\sigma(A), V(A)$ where $A \in \mathbb{R}^{q \times q}$; $q << n$
2. The eigenfunction ϕ_i in RKHS can be obtained

$$\phi_i = \frac{1}{\sqrt{\sigma_i}} \Psi_q^T v_i \quad \text{where } \Psi_q = \begin{bmatrix} \psi(x_1) \\ \psi(x_2) \\ \vdots \\ \psi(x_q) \end{bmatrix}, \ \psi \text{ is feature map.}$$

3. Once we approximated the eigenfunction, we can compute K via

$$K = \begin{bmatrix} \phi_1(x_1) & \phi_2(x_1) & \cdots \\ \phi_1(x_2) & \phi_2(x_2) & \cdots \\ & \vdots & \end{bmatrix} \begin{bmatrix} \phi_1(x_1) & \phi_1(x_2) & \\ \phi_2(x_1) & \phi_2(x_2) & \cdots \\ \vdots & \vdots & \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{\sigma_1}} \Psi_n \Psi_q^T v_1 & \frac{1}{\sqrt{\sigma_2}} \Psi_n \Psi_q^T v_2 & \cdots \\ & & \end{bmatrix} \begin{bmatrix} \text{Transpose} \\ \\ \end{bmatrix}$$

$$= \left[\Psi_n \Psi_q^T V \Sigma \right] \left[\Sigma^T V^T \Psi_q \Psi_n^T \right]$$

$$= L V \Sigma^2 V^T L^T$$