Matrix Chain Rule Derivatives

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Given a multivariate function

$$f(x_1, x_2, x_3, x_4) = x_1 x_2 + x_3 + x_4^2$$

with
$x_1 = t$, $x_2 = 3t+1$, $x_3 = e^t$, $x_4 = t^2$

Let $X = (x_1, x_2, x_3, x_4)$ together, therefore

$$f(x_1, x_2, x_3, x_4) = f(X)$$

The derivative is

$$\frac{df(x_1, x_2, x_3, x_4)}{dt} = \left\langle \begin{bmatrix} df/dx_1 \\ df/dx_2 \\ df/dx_3 \\ df/dx_4 \end{bmatrix}, \begin{bmatrix} dx_1/dt \\ dx_2/dt \\ dx_3/dt \\ dx_4/dt \end{bmatrix} \right\rangle$$

$$= \sum_{i=1}^{4} \frac{df}{dx_i} \frac{dx_i}{dt}$$

We can also separate the equation based on addition.

$$f(x_1, x_2, x_3, x_4) = \underbrace{x_1 x_2}_{f_1(x_1, x_2)} + \underbrace{x_3}_{f_2(x_3)} + \underbrace{x_4^2}_{f_3(x_4)}$$

$$\frac{df}{dt} = \begin{bmatrix} \frac{df}{dx_1} & \frac{df}{dx_2} \end{bmatrix} \begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} + \frac{df}{dx_3}\frac{dx_3}{dt} + \frac{df}{dx_4}\frac{dx_4}{dt}$$