Faster Nystrom with Gaussian Kernel

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If the kernel matrix happens to be Gaussian (or any kernel that random fourier feature RFF can approximate) We can speed up the Nystrom process even more. This is because we can use RFF to simulate the Integral Operator directly.

Method

1. Let the feature map be $\psi = rff$
2. Compute the integral operator

$$T_n = \frac{1}{n} \sum_{i=1}^{n} \psi(x_i) \psi(x_i)^T$$

3. Compute the eigenvalue and function of T_n

$$V(T_n) \rightarrow [\phi_1 \ \phi_2 \ \dots]$$

$$\sigma(T_n) \rightarrow n [\sigma_1 \ \sigma_2 \ \dots]$$

Note that $T_n \in \mathbb{R}^{s \times s}$ s is the width of RFF

Therefore T_n is "independent" of n.

We can use "All" the samples to get the eigenfunction.

4. Once we have the eigenfunction, the eigenvector of K

$$U = \underline{\Psi}\,\underline{\Phi}^T \Sigma \qquad \Sigma = \begin{bmatrix} 1/\sqrt{\sigma_1} & & 0 \\ & 1/\sqrt{\sigma_2} & \\ 0 & & \ddots \end{bmatrix}$$