MLE for Bernoulli

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Given a Bernoulli distribution:

$$p(x) = \theta^{x}(1-\theta)^{1-x} \qquad x = \{0, 1\}$$

Given N samples

$$p(\underline{X}) = \prod_{i=1}^{N} p(x_i) = \prod_{i=1}^{N} \theta^{x_i}(1-\theta)^{1-x_i}$$

$$= \theta^{\sum_{i=1}^{N} x_i}(1-\theta)^{\sum_{i=1}^{N}(1-x_i)}$$

To perform maximum likelihood, we take the derivative with respect to the "hyperparameter", θ and set it to 0.

$$\frac{\partial}{\partial\theta} \ln p(\underline{X}) = \frac{\partial}{\partial\theta}\left[\sum_{i=1}^{N} x_i \ln\theta + \sum_{i=1}^{N}(1-x_i)\ln(1-\theta)\right]$$

$$= \frac{1}{\theta}\sum_{i=1}^{N} x_i - \frac{1}{1-\theta}\sum_{i=1}^{N}(1-x_i)$$

$$\frac{1}{\theta}\sum_{i=1}^{N} x_i = \frac{1}{(1-\theta)}\sum_{i=1}^{N}(1-x_i)$$

$$\frac{1-\theta}{\theta} = \frac{\sum_{i=1}^{N}(1-x_i)}{\sum_{i=1}^{N} x_i} = \frac{N - \sum_{i=1}^{N} x_i}{\sum_{i=1}^{N} x_i}$$

$$\frac{1}{\theta} - 1 = \frac{N - \sum_{i=1}^{N} x_i}{\sum_{i=1}^{N} x_i}$$

$$\frac{1}{\theta} = \frac{\sum_{i=1}^{N} x_i}{\sum_{i=1}^{N} x_i} - \frac{N - \sum_{i=1}^{N} x_i}{\sum_{i=1}^{N} x_i}$$

$$\frac{1}{\theta} = \frac{N}{\sum_{i=1}^{N} x_i}$$

$$\theta = \sum_{i=1}^{N} x_i / N$$

Since $\sum_{i=1}^{N} x_i$ is the count of x_i happening, this creates a histogram.