Nystrom Approximate Eigenvectors

Tuesday, August 30, 2022 6:55 PM

Once we can approximate eigenfunctions with a subset of sample, this allows us to approximate the eigenvector of the whole matrix with A, L, and V(A).

The key proposition is

$$u_i = \frac{1}{\sqrt{\sigma_i}} \underline{\Psi}_n \phi_i$$

where u_i is the ith eigenvector of K

and $$\underline{\Psi}_n = \begin{bmatrix} \psi(x_1) \\ \vdots \\ \psi(x_n) \end{bmatrix}$$

Previously we know that the eigenfunction approximate is

$$\phi_i = \frac{1}{\sqrt{\sigma_i}} \underline{\Psi}_q^T v_i$$

Therefore we have

$$u_i = \frac{1}{\sqrt{\sigma_i}} \underline{\Psi}_n \phi_i \implies u_i = \frac{1}{\sqrt{\sigma_i}} \underline{\Psi}_n \frac{1}{\sqrt{\sigma_i}} \underline{\Psi}_q^T v_i$$

$$\Rightarrow \quad u_i = \frac{1}{\sigma_i} \underline{\Psi}_n \underline{\Psi}_q^T v_i$$

$$= \frac{1}{\sigma_i} L v_i$$

$$\Rightarrow \begin{bmatrix} u_1 & u_2 & \cdots \end{bmatrix} = L \begin{bmatrix} v_1 & v_2 & \cdots \end{bmatrix} \begin{bmatrix} \frac{1}{\sigma_1} & & 0 \\ & \frac{1}{\sigma_2} & \\ 0 & & \ddots \end{bmatrix}$$

$$U = L V \Sigma$$