

Universitat Autònoma de Barcelona

RESEARCH AND INNOVATION PROJECT - I

AN AGENT BASED MODEL ANALYSIS USING SOCIAL NETWORKS
OF RELIGIOUS AFFILIATION TRANSITION

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AN AGENT BASED MODEL ANALYSIS USING SOCIAL NETWORKS OF RELIGIOUS AFFILIATION TRANSITION

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ABSTRACT

In the last decade societal transitions have become a research topic. Despite this topic is mostly related with the social sciences, due of the link with complexity theory gives it is also a nature science sense. Mathematical modeling provides a good entrance into social and physical systems by reducing complex problems to simple equations, with the support of the computers it is possible to examine long term and large scale changes in the structures and cultures of social system. We can consider the competitive dynamics of society as subgroup of this practice. In this direction when groups compete for members, the resulting dynamics of human social activity may be understandable with simple mathematical models [1]. As a competitive approach religious affiliation or un-affiliation of the societies can be modeled. [1] Such models can predict future of organized religions or coexistence phases of people in society whether they disappear or not? Five methodological different approaches can be inferred to the social dynamics which are; agent based modeling, partial differential equations, mathematical sociology, system dynamics and non-linear systems. In this project, i try to develop a simple competitive model for religious affiliation in graphs (implemented in MATLAB) which also takes into account the network structure and several other parameters. In this project the article of “A mathematical model of social group competition with application to the growth of religious non-affiliation” published by *Daniel M. Abrams and Haley A. Yapple* has been considered as a main structure. In this project a dynamic analysis and interpretation of model has been presented in the light of reference article beside from this article four different types of network were simulated in this project to represent different social structures.

Keywords: socio-physics, social networks, religion, dynamic systems

1. INTRODUCTION

In this work, I focus on social systems comprised of two mutually exclusive groups in competition for members. The tools of statistical mechanics and nonlinear dynamics have been used successfully in the past to analyze these resembling models of social phenomena ranging from language choice [2] to political party affiliation to war and peace [3].

According to last researches no religious affiliation constitute the fastest growing religious minority in many countries throughout the world. As competitive social dynamics settle down based on membership we have to as what factors determine the success of a social group and why individuals change their group membership. Especially in religion this is harder than the others. As first step modeling this problem mathematically we have to do simplifying assumptions. In this work many parameters concerned in one parameter which is most influenced one is belonging on a group in the society. In the context of these concerns a simple non-linear equation has been presented in this paper. This equation can be interpreted as a mean-field model, in which individuals react to the average affiliation of the entire population. According to the equation, a single parameter named the **“perceived utility of adhering to a religion”** determines whether the unaffiliated group will grow in a society. The model predicts that for societies in which the perceived utility of not adhering is greater than the utility of adhering, -which is mostly these societies are modern secular ones- religion will be driven toward extinction.

In second chapter I defined the equation and doing its interpretation which is presented in Yapple and Abraham’s paper. Then, the stability analysis and its interpretation of the equation performed which is practiced by me. Also in second part compatibility of the model is showed by comparing with some historical census data data of different areas. In the later of the Abraham and Yapple’s paper they made some generalizations and derive two equations for stochastic and deterministic version of main model. After that they implement a virtual experiment with two different same-type society (clustered) that linked each other. In contrary to them I have tried to simulate these model not a utopic way but more realistic for the real world societies.

In third chapter the the agent-based models and its subset named network types have been introduced and described some features of them which they are widely used for societal simulations and researches.

Next chapter the experiments, simulations and their context have been presented. The MATLAB software has been used for creating and simulations. Most of the scripts are created by me with some help from around, the some of them especially about visualization one is obtained from various platforms. Also in this chapter some results and comparison with predictions and real data which is presented in Yapple’s paper is shown. In the light of these results conclusion and comment part have been presented in this chapter as well.

2. MODEL FOR TWO-GROUP COMPETITION

Abrams et al. split an ideal society into the mutually exclusive religiously affiliated and unaffiliated, with the fraction X belonging to the unaffiliated group and Y belonging to affiliated.

According to model mechanism of switching affiliation is related with attractiveness and it increases with the number of members. [1] Additionally, attractiveness also increases with the “perceived utility”. Perceived utility notion includes lots of factor such as economic, political and security benefits derived from membership as well as spiritual mutualism with a group. Thus an equation of the dynamics of conversion can be modeled by [1]

$$\frac{dx}{dt} = yP_{yx}(x, u_x) - xP_{xy}(x, u_x) \quad (1)$$

where $P_{xy}(x, u_x)$ is the probability per unit time that an individual converts from religious to unaffiliated, x is adhering to X at time t , $0 \leq u_x \leq 1$ is the perceived utility of being unaffiliated, and $y = 1 - u_x$ is the perceived utility of being affiliated. The assumptions regarding the attractiveness of a social group imply that P_{xy} should be smooth and monotonically increasing in both arguments for that reason, authors assume that equation is symmetric under exchange of x and y ; $P_{xy}(x, u_x) = P_{yx}(1 - x, 1 - u_x)$ and this points out the idea that no one will switch to a group with no utility $P_{yx}(x, 0) = 0$ as well as no membership $P_{xy}(0, u_x) = 0$.

A functional form for the transition probabilities consistent with the minimal assumptions of the model $P_{yx}(x, u_x) = cx^a u_x$, is presented by authors where c and a are constants that scale time and determine the importance of x and u_x in attracting converts, respectively. Thus the explicit form of the equation is

$$\frac{dx}{dt} = (1 - x)cx^a u_x - xc(1 - x)^a(1 - u_x) \quad (1.2)$$

$$= cx(1 - x)[x^{a-1}u_x - (1 - x)^{a-1}(1 - u_x)]$$

2.1 Fixed Points and Stability

As stated previously that there, under the simplifying assumptions which is symmetry, monotonicity and continuity; $P_{xy}(x, u_x)$ kind of generic probability functions has at most three fixed points with changing stability.

For our case; the fixed points x^* can be written as solutions to the equation Eq. (1), equal zero. Hence for a given value of u_x ;

$$0 = (1 - x)P_{yx}(x, u_x) - xP_{xy}(1 - x, 1 - u_x)$$

Or,

$$0 = cx(1 - x)[x^{a-1}u_x - (1 - x)^{a-1}(1 - u_x)]$$

According to these equations $x^*=0$, $x^*=1$ is fixed points and there is also third fixed points which its value and stability are changing with a and u_x . For an important notation it we have to note that; if there is a single intermediate fixed point, in order for other fixed points to appear, the continuous curve connecting $(x; u_x) = (0; 0)$ to $(x; u_x) = (1; 1)$ would have to have zero slope at some value of u_x . Thus the condition for a single intermediate fixed point is that $\frac{dx}{du_x} > 0$ for all u_x (stable), or vice versa for unstable. This equation provides that condition. One can see [1] for proof and further explanation.

For stability analysis,

$$0 = (1 - x)P_{yx}(x, u_x) - xP_{xy}(1 - x, 1 - u_x)$$

We found in previous step the fixed points. For stability analysis At first glance one can take the symmetry limiting properties along with the monotonicity $P_{yx}(x, 0) = 0$ and $P_{xy}(0, u_x) = 0$ shows that; when $u_x = 0$ the fixed point $x^*=0$ is stable and similarly when $u_x = 1$ fixed point $x^*=1$ is stable and the other not. On the other hand, since we have another fixed point and it has an altering stability as mentioned above the we must examine stability analysis of fixed points.

For the fixed point $x^*=0$ and note that $x^*=1$ will have the same result. Let's set $x=\delta$ a small perturbation from the $x=0$ point;

$$\begin{aligned} \frac{d\delta}{dt} &= (1 - \delta)P_{yx}(\delta; u) - \delta P_{xy}(1 - \delta; 1 - u) \\ &\approx P_{yx}(0; u) + \delta[P'_{xy}(0; u) - P_{xy}(0; u) - P_{xy}(1; 1 - u)] \\ &= -\delta[P_{xy}(1; 1 - u) - P'_{xy}(0; u)] \end{aligned}$$

After neglecting the higher order terms we can understand if $P_{xy}(1; 1 - u) > P'_{xy}(0; u)$, and since this probability function is $P_{yx}(x, u_x) = cx^a u_x$ explicitly, that the fixed point $x^*=0$ stable if $a > 1$ and unstable for $a < 0$. (See fig. 2.1) This conditions exactly valid for the other fixed point $x^*=1$. (See fig. 2.1)

For probability equation critique value is $a=1$. In that situation if we perform again for $x^*=0$ and substitute x^* and $a=1$ into the equation (1.2) we can see that there are only two fixed points opposite stability which has the condition that the stability depends the sign of $u_x - \frac{1}{2}$ (See fig. 2.2)

However, the stability of the intermediate fixed point is fully determined since the stabilities of the two endpoints $x^*=0$ and $x^*=1$ are known. Because it is a one-dimensional flow, the intermediate fixed point must be stable when the endpoints are unstable, and vice-versa when the endpoints are stable. (See Fig 2.3)

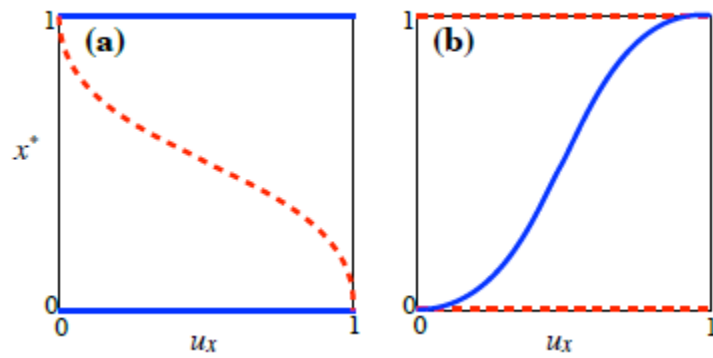


Figure 2.1. Typical fixed points for Eq. (1). Here (a) $a = 3$ and (b) $a = 1/2$. Red dashed lines indicate unstable branches, blue solid lines indicate stable branches of fixed points.

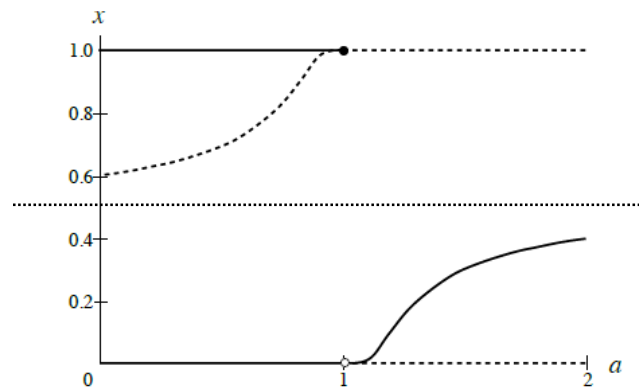


Figure 2.2 Fixed points of Eq. (2.1) and their stability versus a , shown for $u_x = 0.6$. Solid lines and filled circle indicate stability, dashed lines and open circle indicate instability.

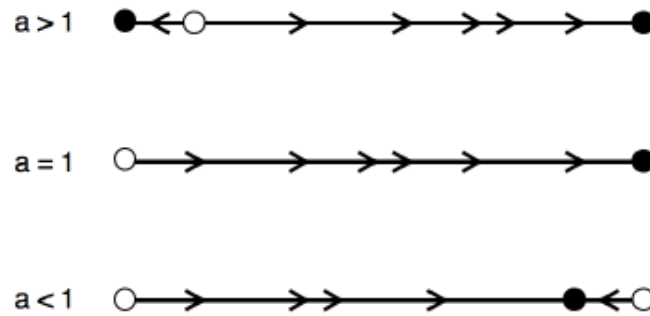


Figure 2.3. The flow in x for various values of the constant a .

2.2 Numerical Solution and Validation Of Model

D.Abraham and H.Yaple in their article compare their model with the collected historical census data which used validating this model originated in census surveys from a range of countries worldwide. A total of 85 data sets had 5 or more independent data points. These came from various regions of 9 different countries: Australia, Austria, Canada, the Czech Republic, Finland, Ireland, the Netherlands, New Zealand, and Switzerland. (See Fig. 2.4)

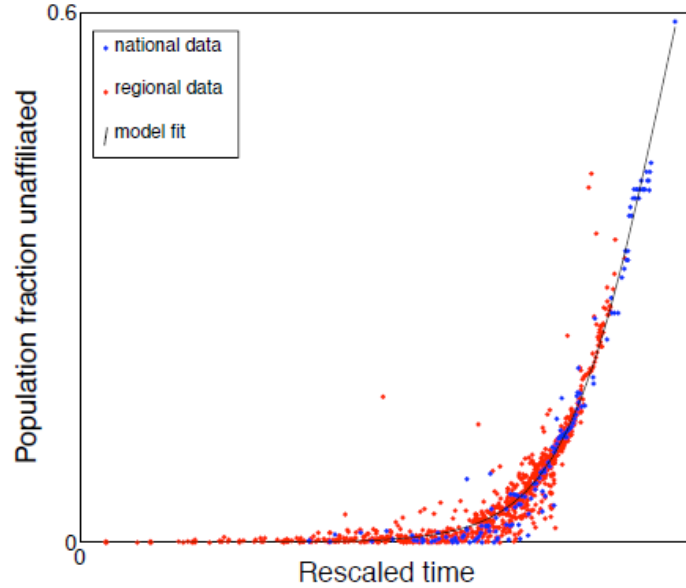


Figure 2.4: All data on changes in religious affiliation with time (85 data sets). Time has been rescaled. Red dots correspond to regions within countries, while blue dots correspond to entire countries. Black line indicates model prediction for $u_x = 0.65$.

For the fits discussed in this paper, $a \sim 1$ is chosen both for simplicity and because of the broad minimum visible around $a \sim 1$. The parameter c , which simply sets a time scale, was approximately 0.2. One can ask why the minimum error obtained for $a=1$ or very close. This can be understood analytically for $a = 1$, in which case we have;

$$\frac{dx}{dt} = cx(1-x)(2u_x - 1)$$

logistic growth. An analysis of the fixed points of this equation tells us that religion will disappear if its perceived utility is less than that of non affiliation, regardless of how large a fraction initially adheres to a religion. However, if a is less than but close to one, a small social group coexist with a large social group. Additionally logistic growth is a competitive hypothesis for collected data as well. (See Figure 2.4). This result is expected. At least this model (1.1) is a very particular case of growth law.

There is also one important point that has to be explain; one of the key parameters of the model is u_x perceived utility which is assumed constant in the model. It has been speculated that for most of human history, the perceived utility of religion was high. Religiously non-affiliated people persisted but in small numbers. With the birth of modern secular societies, the perceived utility of adherence to religion versus non-affiliation has changed significantly in numerous countries. For that reason we can implement and make experiments with higher perceived utility rates which the datas also confirm that opinion.

One can find numerical solution of the equation of model (2.1) in the MATLAB m-files named *affiliation.m* and *affiliation_ode.m* . Thanks to MATLAB ordainay differantal equation solver ODE45 (Runge-Kutta Method 4th and 5th order) these kind of simple nonlinear equations.

For the validation of our program we can solve it for the same inputs with the model fit (Fig. 2.4) which the parameters are $c=0.2$, $a \sim 1$ or $a=1$ and the perceived utility $u_x=0.65$. (See Figure 2.5)

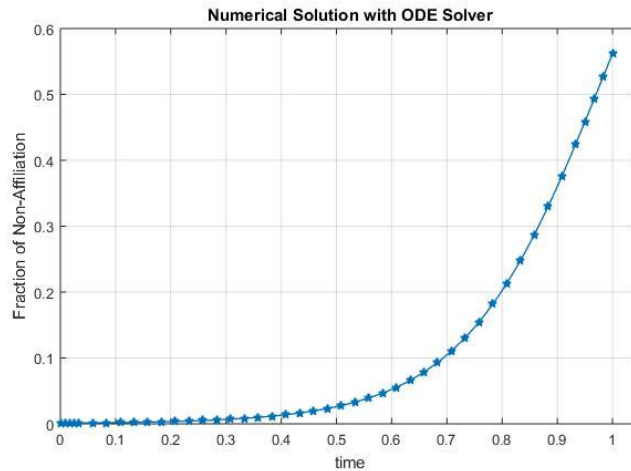
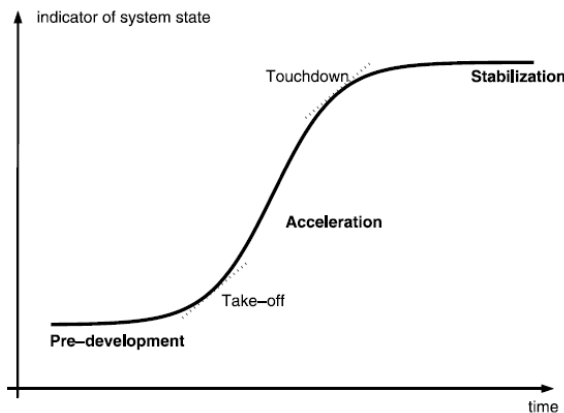


Figure 2.5

If we compare the my numerical solution is with the model-fit solution we can see that numerical solution nearly fits. With this solutions and the comparison we obtain the characteristic curve of the societal transition. As mentioned above one of the particular case of growth law. As societal analysis you can see that the interpretation of this curve (See Fig. 2.6) and have an opinion where we are.



3. IMPLEMENTATION OF MODEL

3.1 Agent Based Modelling

Agent-based modelling and simulation (ABMS) is a relatively new approach to modelling complex systems composed of interacting, autonomous ‘agents’. Agents have behaviours, often described by simple rules, and interactions with other agents, which in turn influence their behaviours. By modelling agents individually, the full effects of the diversity that exists among agents in their attributes and behaviours can be observed as it gives rise to the behaviour of the system as a whole.[4]

Minimal continuous dynamical systems lend themselves well to rigorous analysis, but important details may be left out for simplicity. For instance, the model that D.Abrams and H.Yaple et al. presented in their paper gives result for homogen and all-to-all societies. Thus they ignore heterogeneity in religious affiliation utility and social desire for similarly affiliated friends.

Agent based modeling is a natural way to test the robustness of the continuous model results under assumptions of heterogeneity adaptation. There is different agent interaction topologies that have been performing for years. In my case “network” will be used which is widely using for social simulation. (See Fig. 3.1)

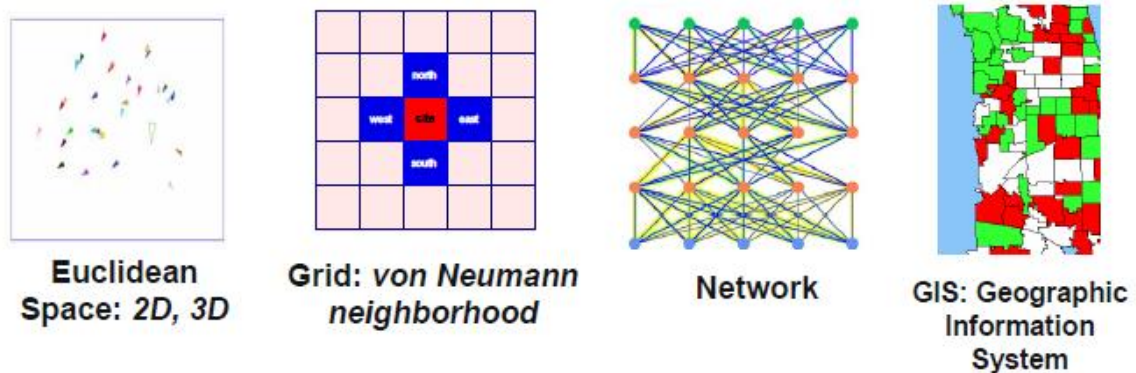


Figure 3.1 Different ABM Topologies

3.2 Networks

A social network is a social structure made up of a set of social actors (such as individuals or organizations), sets of ties, and other social interactions between actors. In the context of network theory, a complex network is a graph (network) with non-trivial topological features that do not occur in simple networks such as lattices or random graphs but often occur in graphs modelling of real systems. **The study of complex networks is a young and active area of scientific research inspired largely by the empirical study of real-world networks such as computer networks, technological networks, and social networks.** [6]

For the purposes of our simulations, i used four network structures which are *random graph*, *small world network* , *scale-free network* and last one *all-to-all network*. These structures can be characterized by properties such as average shortest path lengths, clustering coefficients, and the degree of connectivity.

3.2.1 Random Graph

Random graphs can be used as a first approximation for networks about whose structure is known nothing except for the number of vertices and edges. Such graphs which have been first extensively studied by Erdős and Renyi can be generated in two slightly different ways. First one is the graph is defined by the total number of nodes, and by the probability of any two nodes to be connected, the other one is number of edges in the graph such that every arrangement of edges is equally probable. For our simulation first one is adopted. Generally random graphs have a short average path length. They typically have a small clustering coefficient. Thus, all connections are random.

In my simulation file one can find the MATLAB file script named *create_graph_rnd.m* for generate random graph created by me. Basically, the the algorithm of script is generating random graphs with a given number of vertices N and a given mean degree d . The $(N*d/2)$ edges are placed between randomly chosen pairs (j, k) of vertices and creates a sparse matrix. Then it calls a function `[]` which provides a visualization of graph. (See Figure 3.2). For more aesthetic visualization i used software named *Gephi* for all graphs. (See Figure 3.2)

The other graph that we are going to simulate in this work is all-to-all coupling graph. This approach is the least realistic model among the others but for exact comparison with the Abrams and Yapple model which we studied previous chapter given equation (1) implies all-to-all connections. The all-to-all graph is one of the subset of random graph algorithm. If one choose the parameters which number of vertices as N and the degree of vertices as $(N-1)$ the algorithm will generate all-to-all coupling graph. (See Figure 3.3)

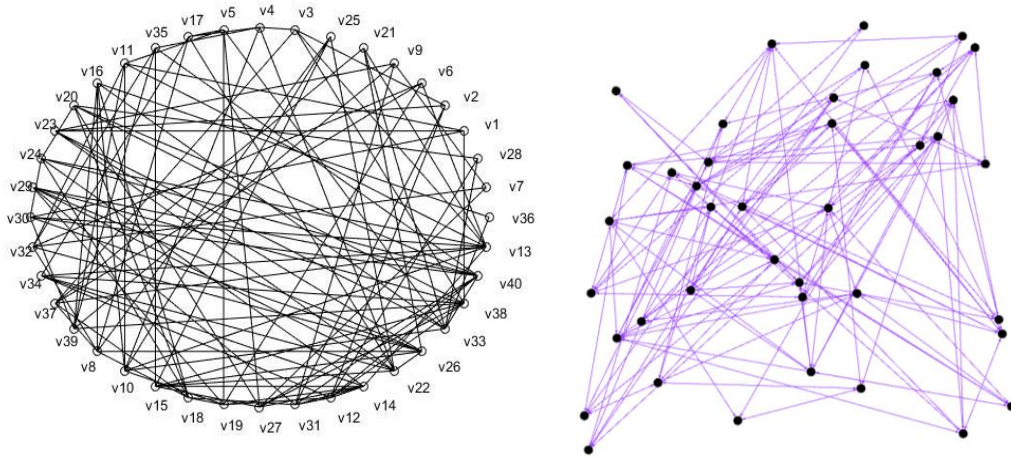


Figure 3.2 For $N=40$, $d=6$ parameters generated Random Graph.
(Left) Plot of MATLAB file.
(Right) Exprot from Gephi

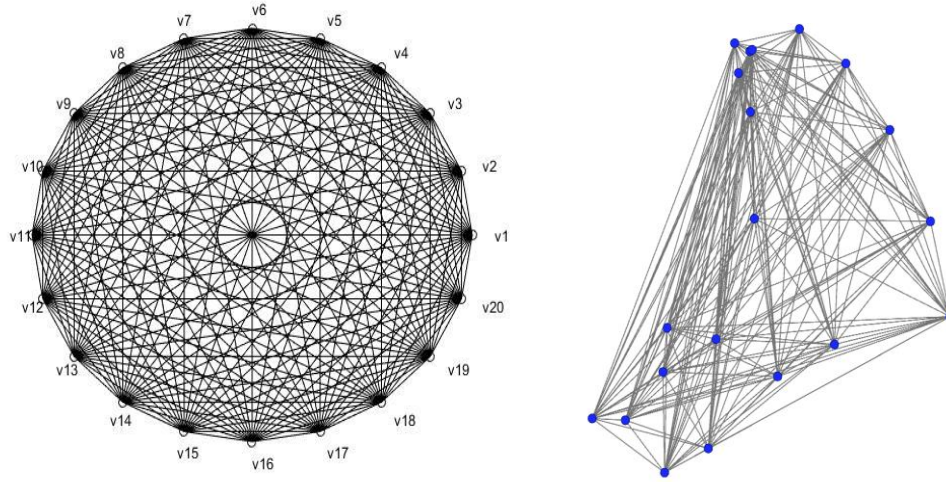


Figure 3.2 For $N=20$, parameter generated All-to-all Random Graph.
 (Left) Plot of MATLAB file.
 (Right) Export from GEPH

3.2.2 Scale Free Networks

A scale-free network is a network whose degree distribution follows a power law thus, there are few nodes that are highly connected, and more nodes that are moderately or mildly connected. This means that highly connected vertices are more likely to be linked with newly added ones.

The method for creating such graphs proposed by Barabasi and Albert [1999] is based on preferential attachment. Compared to random graphs, they have a smaller average shortest path and includes more reasonable assumption for social network context. Therefore even a scale-free graph cannot be the ultimate model of such networks.

In my simulation file one can find the MATLAB file script named `create_graph_sf.m` for generate scale-free networks which is created by me. It follows the algorithm described by Barabasi and Albert [1999]. The input parameters are again the number of vertices N and the mean degree d . Basically the algorithm contains these steps; first a subgraph is created and store the vertices already placed. Then in a new loop each new vertex i is connected to $d/2$ of the already placed (old) ones by help of a auxiliary vector. In last step created adjacency matrix converting to a sparse matrix. (See Figure 3.3)

3.2.3 Small World Networks

Small world graphs have characteristics that lie in between random graphs and highly clustered graphs they have a high clustering coefficient similar to the scale-free, but also have a small average shortest path similar to the random graph. Many real-world networks have been observed to have a small world structure. Also do not exhibit power law. Their name suggests that their diameter grows slowly with increasing size of the graph.

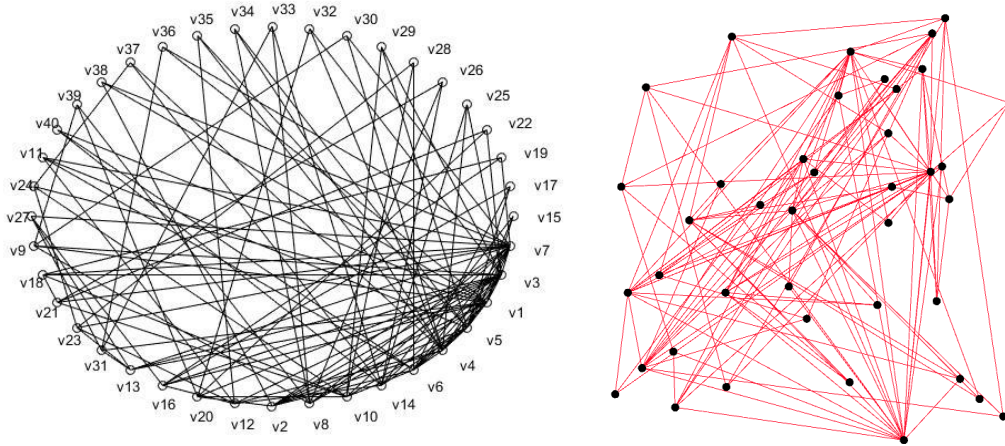


Figure 3.3 For $N=40$, $d=4$ generated Scale Free Graph.
 (Left) Plot of MATLAB file.
 (Right) Export from GEPH

They can be created in a way described by Watts and Strogatz [1998] by starting with some regular graph and randomly “rewiring” a given fraction p of edges. The limiting cases $p = 0$ and $p = 1$ correspond to completely regular and completely random graphs, respectively. For some intermediate values of p , the resulting graph has certain properties of a random graph (slowly growing diameter) and other properties resembling a regular graph (high clustering).

In my simulation file one can find the MATLAB file script named *create_graph_sw.m* for generate small world networks. While creating this file i inspired MIT Network Toolbox. The algorithm of creating this network is basicly, first we first calculate the number of edges corresponding to a mean degree d' given by the natural number. Then, each vertex is linked with the $\lfloor d'/2 \rfloor$ following neighbours in the network. Then in a loop over all edges, each edge is reconnected with a given probability p . One end of this edge is kept at its original position while the other one is placed at a randomly chosen vertex. Finally, some edges are added or removed to match the required value of mean degree d if it is not a natural number.(See Figure 3.4)

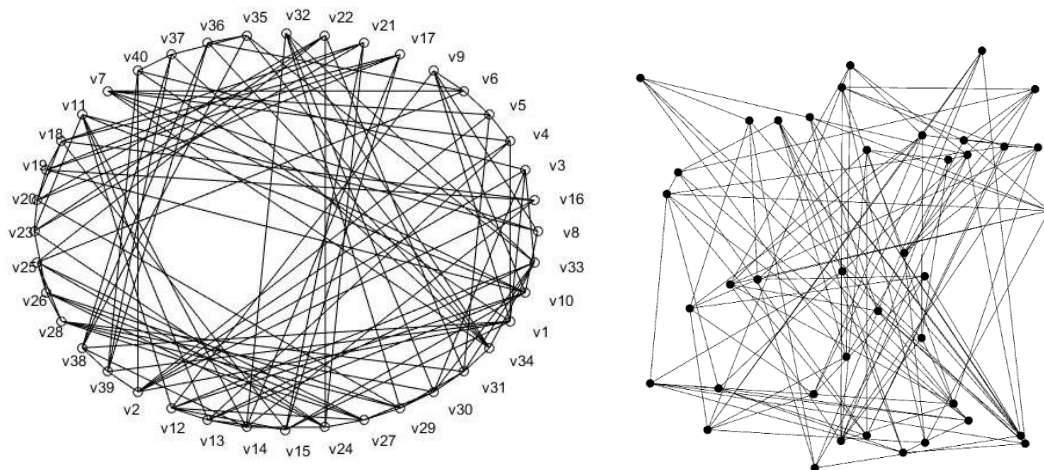


Figure 3.3 For $N=40$, $d=6$ generated Small World Graph.

3.3 Simulation

As opposed to simple models which describe the competitive dynamics by differential equations for the unknown numbers of individuals in each state as functions of time, i use a graph-based model. Each individual is represented by a node in a graph whose weighted edges correspond to social relations between different individuals. The weights can then be thought of as some quantifiers of their intensities for example expressing the amount of time that each pair of individuals spends together.

As mentioned above, the graphs were represented by their $N \times N$ adjacency matrices. States of the N individuals were stored in an N -dimensional vector with each element having either the value 0 corresponding to the affiliated state, 1 for the non-affiliation. Also i create some additional states which; after once an individual get unaffiliated its **willingness** to spread the idea to their connections. Newcomers always more enthusiastic to tell their friends. After becoming state 1 individual some time later becomes 1, 2, . . . , L which representing the L willingness state. These states affect and increase probability of influence per individual but getting lower as L and time increase. After some point the state will be again 1 of the individuals.

The following function *social_step* takes as its parameters the state vector *pre_states*, the adjacency matrix *graph* which is calling the *creating_graph* functions, a vector *willingness* containing the coefficients values of the different stages of the of the individuals willingness and finally $P_{yx}(x, u_x)$ as k parameter. Basically this step contains creating the probabilities matrix and multiplying with the willingness state vector and the $P_{yx}(x, u_x)$ vector (k) and stored in *prob* vector. Because of the perceived utility is increasing with the membership $P_{yx}(x, u_x)$ is continuously calculating and storing in a k vector in main script. Also and $P_{xy}(1 - x, 1 - u_x)$ vector is calculated as *prob1*.

Afterwards, non-affiliated individuals are evolved deterministically either to the next stage of their willingness or the last stage again 1. Then an competitive step starts between two group, after the states are change and stored in *new_states* as update.

The whole simulation was then simulated using the script *social_main* whose parameters are the vector *initstates* of initial states, the adjacency matrix *graph*, one can choose the network type. the willingness vector *willingness*. In the main loop of the script $P_{yx}(x, u_x)$ the perceived utility calculating each step proportional to fraction of X and repeatedly calls the function *social_step* keeps updating the state vector *states*. The function returns an 'n x 2' matrix and finally result of each step vector *states* counted by the function *social_run* and stores in the *history*. Then history plotted.

4. SIMULATION RESULTS

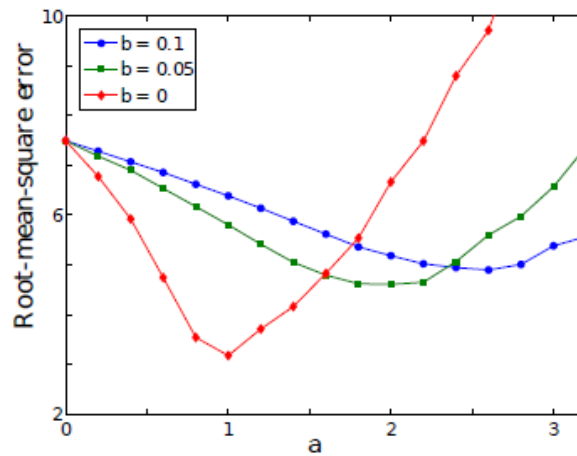
My model allows us to change several free parameters of the system and study the influence of these changes on the system dynamics. For a given type of network these parameters are: the size N of the graph, the mean degree d of its vertices, the willingness vector and for defining the probability of getting non-affiliating $P_{yx}(x, u_x)$ the perceived utility u_x , c time scale parameter and a . As a first step simulations has run for parameters that Abraham and Yapple used and validated with the real datasets. After that we will make different experiments.

These parameters are:

- Graph Size: $N = 1000$
- Mean Degree: $d = 10.0$
- Willingness Vector : $[0.000, 0.24, 0.08]$
- Perceived Utility: $= 0.65$
- $a = 1$ and $c=0.2$ (See Figure 4.1)
- Realization = 50

The graph size $N = 1000$ is small enough to allow reasonably fast simulations while at the same time reliable results. Normally the mean degree $d = 6.0$ is based on my rough estimate of the number of strong social ties that an average person might have but this time i want to estimate $d=10$ in order to add internet effect. Later we will discuss and make comparison different degree values.

Finally, remarking that the initial state vector, indicating which individuals are exist at the beginning, could be also regarded as a free parameter of the system. However, for the sake of simplicity and i assume the initial value as %0.01 of the population. For parameters c it doesnt make difference only arranges the time scale contrarily a is important one because it effects exponentially. As mentioned before minimum error rate is at $a=1$ (See Figure 4.1)



Summed root-mean-square error over all data sets versus parameter a .
The best fitt value of b appears to be near $b = 0$ and $a = 1$.

As mentioned previous chapters in theoretical work of Abraham and Yapple after they present the equation of competitive dynamics they compare with real data of contries. Then they offered two version of the system one of “discrete stochastic system” and the other is “continuous deterministic system.” Then they create two all-to-all network which separete at beginning and they did small perturbations between two cliques they compare with the original model. That model offers an highly idealized (all-to-all assumption) system because of that i try to see spreading the this behaviour in societies which perceived utility higher than 0.5 . I simulated serially the model four different network which is random , scale-free, small world and finally all-to-all network. (for compare with the article)

4.1 Simulation Results with all Networks

In this chapter, all the simulation results have presented here, with the corresponding inputs above. The graphs representing all the realizations with each networks in addition compare with the analytical solution. One can see the bold line on the graph as the solution of differential equation of competitive dynamics. (Eq.1)

Next goal was to look at the time dependence of the numbers of none adherents, for that reason i simulated 50 simulations of each network (See Figures 4.1,4.2,4.3) Although these figures that they are not very suitable for a closer analysis of this difference. Despite being quite illustrative and they possibly helpful for a crude assessment of basic features of the transition, it is rather difficult to extract any quantitative results from them. Since graphs are creating mostly randomly, for in each simulation they are slightly different results. Most of inconsistency is in random graphs then scale-free and small-world following. Due to small world networks created in two stage they are more controllable and consistent. In Figure 4.4 totally same conditions simulated with the Abrahams and Yaple work and compared their solution. Beside that Since the standard deviations are relatively small one can analyze the characteristic of networks and with smaller realization. Thus we can show the different parameters effect.

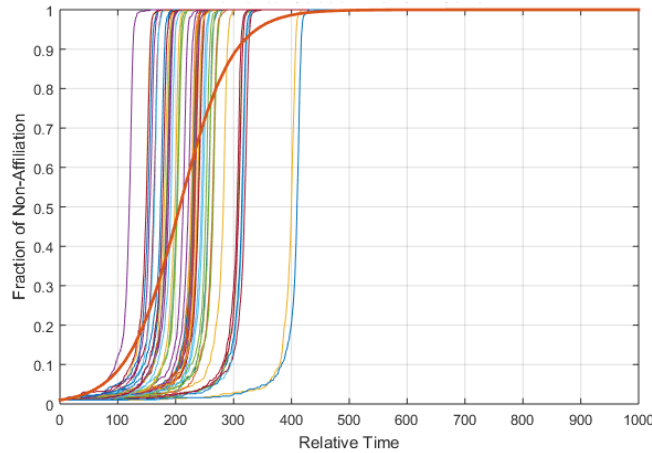


Figure 4.1 Simulation Results of Random Network given parameters
red bold line is analytical solution of Eq1

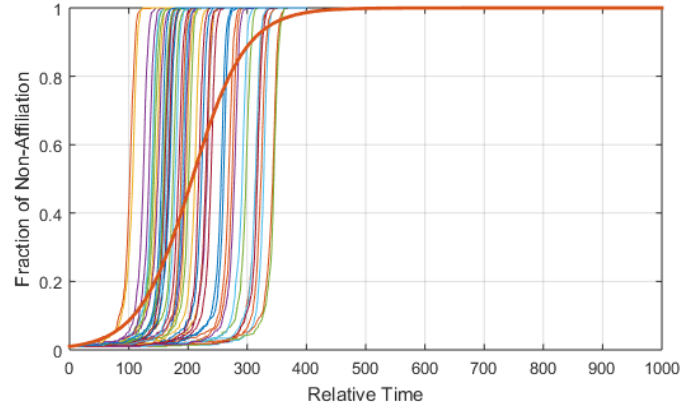


Figure 4.2 Simulation results of scale-free network given parameters
red bold line is analytical solution of Eq1

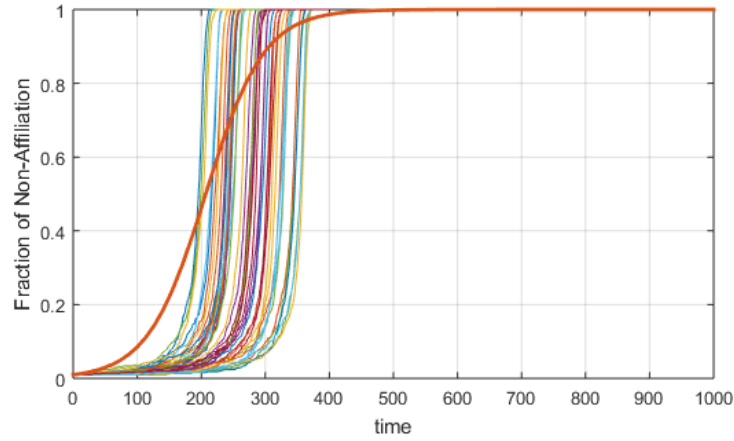


Figure 4.3 Simulation results of small-world network given parameters
red bold line is analytical solution of Eq1

As we can see from graphs above the societal transition is really speeded up after some point. As we can observe from the figures in real world assumption thus in a society this acceleration critical value might be reached later than the ideal predict (analytical solution). But as we assume the probability of transition is increasing with the membership vastness of the non-affiliation in real world models, spreading speed of the behaviour is higher than idealized one. If we examine the slope of the lines we can see the speed of transition, of course in here time scale can be illusive. But if we consider the real data fitness graphs which is showed in second chapter (Figure 2.4) the slope of acceleration is nearly close each other. In this study the assumption of “willingness” parameter has increased the speed transition. The willingness is representing two things which one of is due to high communication devices (social media, TV etc) the impact of the people to each other is quite high recently. The other one is having a new ideology thrills the people and makes them tell to the friends and their social interacts. In simulations there is three stage of non-affiliated people if they are new and have the idea long time (2nd stage), to effect the probability matrix more than first and third stages.

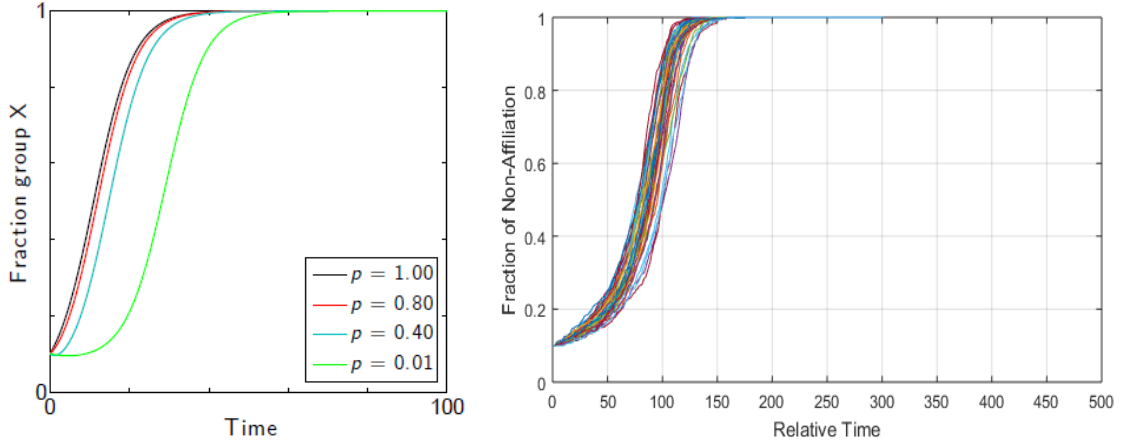


Figure 4.4 Simulation results of all-to all network given parameters
red bold line is analytical solution of Eq1

As mentioned before the Abrahams and Yapple study they created and two cliques which connect all agent which each other in the same cluster. Then they put some links between each other. This represents a two polarized group in a society. Despite the remarkable results have been observed from this experiment it would be good if one simulate with the same parameters in mixed society. The real world assumptions has been presented in Fig. 4.1-4.2-4.3. If we assume mixed agents but all to all society. The same parameters with the article ($N:500$, $u_x=0.6$, $X_0=0.1$) We nearly have same behaviour only slightly time differences which is make sense because of the mixed society.

4.2 Dependencies to Parameters

In this chapter some example results are presented for changing the behaviour. The one of the most important parameter of model is perceived utility which is means conformity rate of being any membership of a group. As mention before in modern secular societies the non affiliation perceived utility is about 0.7 according to Abrahams and Yapple. As you can see from Figure 4.5 the higher rate(0.75) is quickly reach the state and the other(0.55) is more late. As analyzed in previous chapter if perceived utility below 0.5 the non-affiliation could not increase.

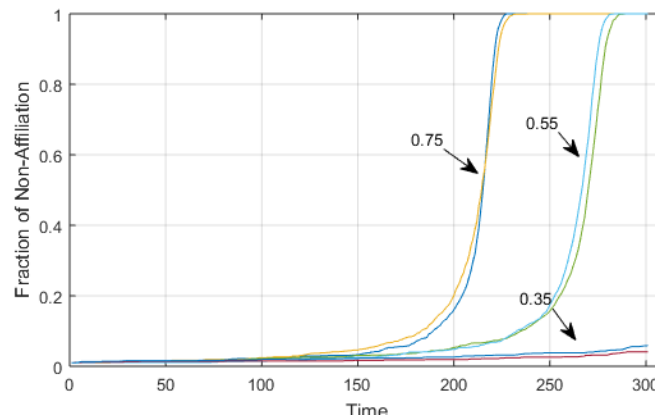


Figure 4.5 Dependencies of Perceived Utility U_x

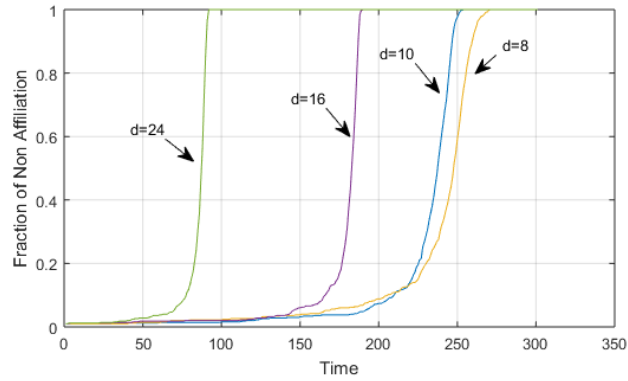


Figure 4.6 Dependencies of Mean Node Degree (d)

In figure 4.6 examined the mean degree dependencies of the system with small-world network. As you can see in more “connect” societies the ideas spread more quickly than the other. In same way the size of a network would effect the behaviour but not much as mean degree. The quantity of concatchness more important to its wideness. So in this context not only near physical social contacts (friends , family ,school) but also communication tools such as internet etc. is playing important role how to people know each other. So in this model i introduced the willingness parameter as mentioned before. The willingness vector also speed ups the non-affiliation process. Willingness vector includes two coefficent which changes according to the agents state, it effect the probabily of their concact by adding these coefficent to the agent’s probability. Thus this increases the transition speed. This parameters contains effect of social media and willingness of a person to spread the ideology that adopted newly.

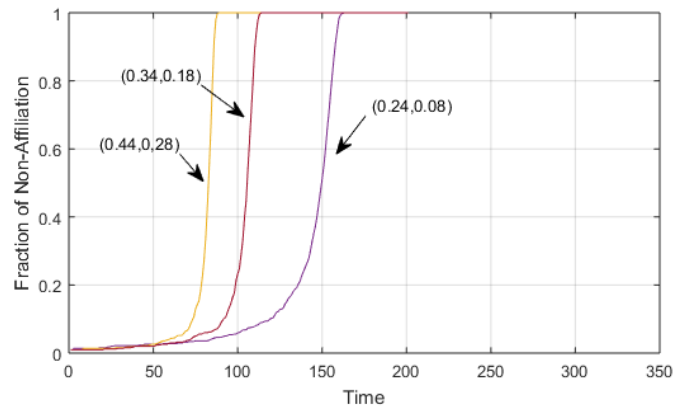


Figure 4.6 Dependencies of Mean Node Degree (d)

5. Outlook and Discussion

Abraham and Yapple have developed a general framework for modeling competition between social groups and analyzed the behavior of the model. They found that a particular case of the solution fits census data on competition between religious and irreligious segments of modern secular societies in 85 regions around the world. The model indicates that in these societies the perceived utility of religious non-affiliation is greater than that of adhering to a religion, and therefore predicts continued growth of non-affiliation, tending toward the disappearance of religion. They showed that this all-to-all model fits the real world behaviour.

I have used MATLAB to implement a simple agent base model of competitive dynamics in networks described by graphs. The four types of networks used in this project are: random networks, scale-free networks (generated according to Barabási and Albert [1999]), small-world networks (described by Watts and Strogatz [1998],) and all-to-all random networks. Our results show that although the studied dependences are usually qualitatively similar for all four types of networks, the ability of the small-world networks seems slightly reliable and consistent in comparison with the other networks. Addition to an regular model, in this model the parameter of “willingness” is studied. It was include the effects of the social media and friendship of the agents.

This agent based model of religious affiliation replicates the continuous dynamical systems model on a network under the specified conditions . Specifically, the agent based model predicts logarithmic growth of the unaffiliated group and the eventual extinction of the affiliated group and the eventual extinction of the affiliated group. Its sure that it is impossible to select which of the models best describes religious shift or what the eventual outcome will be. The dynamics change area by area all over the world. For further works it will be more realistic to create more accurate networks. There are number of network type which have different algorithms also combining the basic ones. There are social media networks projects improving everyday. Beside that, more realistic parameterization could be study in this further works.

6. References

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