Induction

Induction is a technique to prove statements. It is typically performed on the set of natural numbers, but it can be generalized to use on any well-ordered set (e.g. $\mathbb{N} \cup \{-1\}$). The two most commonly used forms of induction are called **Weak Induction** and **Strong Induction**, although both methods are equivalent. In both forms we show that a situation is true in an initial (base) case, and then show that it holds for all subsequent cases.

Weak Induction

This is the basic form of induction, and consists of two steps:

1. **Base Case:** Prove that the statement hold for the first member in your well-ordered set
2. **Inductive Step:** Prove that, given the statement holds for an arbitrary index n in your set, show that it holds for $n+1$

The statement that you assume to be true during the inductive step is referred to as the hypothesis.

Strong Induction

This is a less commonly used version of induction. It is called "Strong" because it involves making a stronger hypothesis. Whereas in the case of weak induction, when we were trying to show that the hypothesis is true in the case of $n+1$, we assumed that the hypothesis is true for the case of n. In strong induction, we assume that the hypothesis is true for all $i \leq n$

False Counter Example

All horses are the same color (from wikipedia). Proof:

1. Basis: If there is only one horse, there is only one color.
2. Induction step: Induction step: Assume as induction hypothesis that within any set of n horses, there is only one color. Now look at any set of n + 1 horses. Number them: 1, 2, 3, ..., n, n + 1. Consider the sets 1, 2, 3, ..., n and 2, 3, 4, ..., n + 1. Each is a set of only n horses, therefore within each there is only one color. But the two sets overlap, so there must be only one color among all n + 1 horses.

Where is the error?

Examples

Prove the following:

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
- $\sum_{i=2}^{2*n} i(1-(i \mod 2)) = n(n+1)$
- $\sum_{i=1}^{n} 2i - 1 = n^2$
- $\forall n \geq 1, 8^n - 3^n \mod 5 = 0$
- $n! > 2^n \ \forall \ n \geq 4$
- Let $T_1 = T_2 = T_3 = 1$ and $T_n = T_{n-1} + T_{n-2} + T_{n-3}$ for $n \geq 4$. Prove $T_n < 2^n \ \forall \ n > 0$