Homework 5-ENEE351

Problem 1

By using Fast Fourier Transform (FFT) algorithm, evaluate the polynomial $A(x) = 3x^4 + 2x^3 - 2x^2 - x$ at the complex 6th roots of unity. Show at least two levels of recursion.

Problem 2

Let $A(x) = x^2 + x + 2$ and $B(x) = 2x + 3$. In this question, we will compute the polynomial $C(x) = A(x) \cdot B(x)$ by using the FFT algorithm.

(a) What is the minimum number of points we need to use? Explain.

(b) Evaluate $A(x)$ at the complex 4th roots of unity. Show at least two levels of recursion.

(c) Evaluate $B(x)$ at the complex 4th roots of unity. Show at least two levels of recursion.

(d) Compute $C(x)$ at the complex 4th roots of unity.

(e) Find the coefficients of $C(x)$.

Problem 3

Let a and b be two n-bit nonnegative integers, and let

$$(a_{n-1}, a_{n-2}, \cdots, a_1, a_0) \qquad \text{and} \qquad (b_{n-1}, b_{n-2}, \cdots, b_1, b_0)$$

be their respective binary representations. Define integer $c = a \cdot b$. In this question we are interested in algorithm for computing c.

(a) Define polynomials

$$A(x) = a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0$$

and

$$B(x) = b_{n-1}x^{n-1} + b_{n-2}x^{n-2} + \cdots + b_1 x + b_0 \, .$$

Show that $a = A(2)$ and $b = B(2)$.

(b) Define polynomial $C(x) = A(x) \cdot B(x)$. (Note that the polynomial $C(x)$ can be computer from $A(x)$ and $B(x)$ using FFT algorithm.) Is $C(2) = c$? Prove or disprove.

(c) What is the degree m of $C(x)$?

(d) Observe that the coefficients of $C(x)$ are not neccessarily all 0 and 1. Propose an algorithm that computes the binary representation of c, namely

$$(c_m, c_{m-1}, \cdots, c_1, c_0) \, ,$$

from the coefficients of $C(x)$ using only $O(n)$ arithmetic operations over integers with $O(\log n)$ bits.