Master Theorem

The Master Theorem is a technique primarily used to analyze recurrence relations. It assumes that the relation to be analyzed has the following form

$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$

where $a \geq 1, b > 1$ are constant. The solution to this recurrence has 3 general forms depending on $f(n), a$, and b.

Form 1

If $f(n) \in O(n^c)$ where $c < log_b(a)$ then $T(n) \in \Theta\left(n^{log_b(a)}\right)$

Form 2

If $\exists\ k \geq 0\ s.t.\ f(n) \in \Theta\left(n^c\ log^k(n)\right)$ where $c = log_b(a)$ then $T(n) \in \Theta\left(n^c\ log^{k+1}(n)\right)$

Form 3

If $f(n) \in \Omega\left(n^c\right)$ where $c > log_b(a)$ **and** $af\left(\frac{n}{b}\right) \leq kf(n)$ for some constant $k < 1$ and $n > n_0$ then $T(n) \in \Theta\left(f(n)\right)$

Examples

- $T(n) = 8T\left(\frac{n}{2}\right) + 1000n^2$
- $T(n) = 2T\left(\frac{n}{2}\right) + 10n$
- $T(n) = 2T\left(\frac{n}{2}\right) + n^2$