

UNIVERSITY OF MARYLAND  
DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

ENEE 459C  
Computer Security  
Instructor: Charalampos Papamanthou

## Homework 3

Out: 10/17/14 Due: 10/31/14

### Instructions

1. Strictly adhere to the University of Maryland Code of Academic Integrity.
2. **Type** and submit your solutions as a pdf document at Canvas. Include your full name in the solutions document. Name the solutions document as x-hw3.pdf, where x is your last name.
3. No extensions will be given. Everything will be due at 11:59pm on 10/31/14.

### Problem 1 (20 points)

1. You are given that  $3 \cdot 5 - 2 \cdot 7 = 1$ . Use the Chinese remainder theorem to compute

$$3^{72} \mod 35.$$

2. The Blum-Micali generator outputs the next pseudorandom bit as follows: First it computes

$$x_i = g^{x_{i-1}} \mod p$$

and then it outputs the bit 1 if  $x_i < \frac{p-1}{2}$ , else it outputs the bit 0 (the procedure is repeated for  $i = 0, 1, 2, \dots$ ). Write a program that implements `get_random_bit()` using the above generator for  $p = 2147483647$  (note that in real applications,  $p$  needs to be a lot bigger) and  $g = 7$ . Use  $x_0 = 1$ . Also, write a program that implements `get_random_number_BM()` and outputs a 32-bit pseudorandom number by calling `get_random_bit()` 32 times. Note that for implementing the exponentiation above, you should use the modular power algorithm (that runs in logarithmic time) that we discussed in class.

### Problem 2 (20 points)

1. What is the difference between public-key encryption and secret-key encryption? What is a certification authority? What is a digital certificate?
2. Suppose Mallory is responsible for storing all the public keys of the students that are currently enrolled at the University of Maryland. Namely, it keeps a list of mappings of  $(d_i, pk_i)$ , where  $d_i$  is the directory identifier of student  $i$  and  $pk_i$  is the public key of student  $i$ . Suppose Bob, Alice and Eve are all students at the university. Bob wants to send a confidential message to Alice and asks Mallory for Alice's public key. However Eve is really interested in learning what Bob wants to say to Alice. Eve happens to be very good friends with Mallory. Explain how Eve could collaborate with Mallory so that Eve eventually gets to decrypt Bob's message to Alice.

3. Instead of using Mallory for managing public keys, Bob and Alice decide to setup their public keys with Verisign, Inc., a trusted certification authority. In order to arrange a meeting for the weekend, Bob exchanges with Alice brief text messages, using RSA encryption, as described in slide 7 of Lecture 9 (click [here](#) to access Lecture 9). Let  $p_B$  and  $p_A$  be the public keys of Bob and Alice respectively. A message  $m$  sent by Bob to Alice is transmitted as  $\text{Enc}_{p_A}(m)$  and the reply  $r$  from Alice to Bob is transmitted as  $\text{Enc}_{p_B}(r)$ . Although Eve cannot collaborate with Mallory anymore, she can still eavesdrop the communication and knows the following information:

- Bob and Alice are meeting either on a Saturday or on a Sunday;
- The set of 100 possible meeting places;
- Alice is going to specify the time in the format HH:MM pm or HH:MM am.
- Messages and replies are terse exchanges of the following form:

Bob: When are we meeting?

Alice: Saturday.

Bob: Where are we meeting?

Alice: Union station.

Bob: What time?

Alice: 8.30pm.

Describe how Eve can learn the meeting day, time, and place of Bob and Alice.

4. As a result of the above attack, Bob and Alice decide to modify the protocol for exchanging messages. Describe two simple modifications of the protocol that are not subject to the above attack. The first one should use random numbers and the second one should use symmetric encryption.

### Problem 3 (20 points)

In class we talked about some basic number theory and its significance for various cryptographic algorithms such as RSA and ElGamal. Answer the following questions:

1. Suppose  $n = 43434563$ . How many elements in  $\mathbb{Z}_n - \{0\}$  have a multiplicative inverse modulo  $n$ ? Name such an element (i.e., one that has a multiplicative inverse modulo  $n$ ). You might want to visit [this webpage](#) to answer this question.
2. Consider an RSA key set with  $p = 11$ ,  $q = 29$ ,  $n = pq = 319$ , and  $e = 3$ . What value of  $d$  should be used in the secret key? What is the encryption of message  $M = 100$ ? Why should  $e$  always be an odd number (to answer this question, assume  $p$  and  $q$  are always greater than 2)? You might want to visit [this webpage](#) to answer this question.
3. Prove that *all* elements in  $\mathbb{Z}_n - \{0\}$  have a multiplicative inverse modulo  $n$  **if and only if**  $n$  is a prime. Note that you need to prove both directions.
4. Let now  $n$  be a positive integer and let  $\mathbb{Z}_n^* = \{x \in [0, n] : \text{GCD}(x, n) = 1\}$ . Let also  $k < n$  and  $a \in \mathbb{Z}_n^*$ . What is the multiplicative inverse of  $a^k \in \mathbb{Z}_n^*$  when  $n$  is a prime? What is the multiplicative inverse of  $a^k \in \mathbb{Z}_n^*$  in case that  $n$  is not a prime (and has  $p$  and  $q$  as prime factors)? You do not have to use the extended Euclidean algorithm to answer this question.
5. Prove that for all integers  $a$ ,  $k$  and  $n$ , it is  $\text{GCD}(a, n) = \text{GCD}(a + kn, n)$  (Hint: Use the same technique we used in class to prove that  $\text{GCD}(a, b) = \text{GCD}(b, a \bmod b)$ ).
6. Prove that for all integers  $a$ ,  $b$  and primes  $p$  it is  $(a + b)^p = a^p + b^p \bmod p$  (Hint: Use Fermat's little theorem).

- Recall the extended Euclidean algorithm: Given two integers  $a$  and  $b$ , it outputs two other integers  $x$  and  $y$  such that  $xa + yb = \text{GCD}(a, b)$ . Devise an algorithm that uses the extended Euclidean algorithm as a black box, such that, on input 3 integers  $a$ ,  $b$  and  $c$ , it outputs three integers  $u$ ,  $v$  and  $z$  such that  $ua + vb + zc = \text{GCD}(a, b, c)$  (Hint: Use the fact that  $\text{GCD}(a, b, c) = \text{GCD}(\text{GCD}(a, b), c)$ ).

**Problem 4** (20 points)

In this exercise, you will obtain a personal certificate and send signed emails. This task can be performed under Windows or Linux with no restrictions or differences. This task requires the configuration of a full email client (or more formally a mail user agent (MUA)). There exist multiple MUAs for Linux, Windows and Mac OS X. We will be using the open-source, multi-platform email client Mozilla Thunderbird.

- Configure Thunderbird to access your university email account by specifying the following parameters:

E-mail address: <your address>@umd.edu  
 Incoming mail server (IMAPS): imaps.umd.edu  
 Outgoing mail server (SMTPS): smtps.umd.edu  
 Account name: user login

Verify that you have correctly configured your MUA by sending an email to yourself and then receiving it.

**What is the difference between the protocols IMAP and IMAPS, and SMTP and SMTPS?**

- Obtain a free client certificate from the following provider <http://www.cacert.org/> for your school email address. Please note that the procedure may vary depending on your browser and OS. Use the information provided on the web site of CACert to generate the certificate.

Thanks to a special code contained in the html page, the web site instructs the browser to generate a key pair. The private key of the user remains on the local disk. The public key is then sent to the authority that generates the certificate. Finally, the CA will send you an e-mail informing you when your certificate is ready.

**Describe the procedure you have followed to generate your certificate. Why do we have to download and install the certificate of the Certificate Authority (CA) before installing our own certificate? Which is the precise identity of the CA? The certificate is valid from which date to which date? In which field of the certificate you find your e-mail? Your public key? The CA?**

Thunderbird has its own certificate store and does not share it with other applications. In order to be able to send signed and/or encrypted emails, you have to import it to Thunderbird (and potentially export it first from the browser) by using a file in the format pkcs12.

Import certificate to Thunderbird:

Tools / Options / Privacy / Security /  
 Show certificates / Authorities / Import

**Why do we need to install the certificates in the MUA and the web browser separately? Is there a system repository of certificates? Is it used by all browsers and all MUAs?**

- Send a signed email to your TA through Thunderbird containing your responses to all bolded questions.

**Problem 5** (20 points)

In this programming assignment, you will hack into a server running at IP 129.2.212.154 by figuring out the private key of a user! To prove to us that your attack succeeded, you are going to submit the contents of the file `/home/chell/cake.txt` that is stored at the server, as well as leave your own message in a file inside the same directory. We now give some guidelines:

**SSH public-key authentication.** SSH (secure shell) is a protocol that is used to establish secure connection to a server. To establish an SSH connection, a user typically generates a public/private key pair by executing command `ssh-keygen`, and then adds the produced public key to `.ssh/authorized_keys`. Then the user can use his private key to log into the server with SSH. You can read more about SSH [here](#).

**The bug.** `openssl` is a cryptographic library that is used by `ssh-keygen`. In 2006, Debian, a popular Linux distribution, introduced a bug into `openssl` by commenting out code dealing with getting entropy for the pseudorandom number generator. You can read more details about the bug [here](#). The effect of the change was that the only source of entropy for `openssl` was now the process's PID, which can be any value from 1 to `/proc/sys/kernel/pid_max` (usually 32768).

This meant that, if someone was using the buggy version of `openssl` to get randomness and generate key pairs, instead of generating one of the possible  $\sim 2^{2048}$  key pairs, he would be generating only one of the 32768 key pairs.

**How to hack it.** You know that the user `chell@129.2.212.154` is using a key pair generated with a vulnerable version of `openssl`. Your goal is to log into his account and read the file `/home/hackme/file.txt`.

To achieve that, write a program to generate all 32768 key pairs and try logging into the server with them until you find the correct one. To generate the vulnerable keys, you can download from [here](#) an `.iso` image of an old version of Ubuntu that uses the vulnerable version of `openssl` and use virtualization software (e.g., Virtual Box, visit webpage [here](#)) to boot the `.iso`. Then, in the virtual machine, run `ssh-keygen` with PID from 1 to 32768 to generate all the possible keys.

The following code will run `ssh-keygen` with a PID of 1, and will try to log into the server with the generated key. First, since it is hard to make a program run with a chosen PID, we instead use `LD_PRELOAD` to intercept calls to `getpid()`, so that `ssh-keygen` eventually runs with our chosen PID. Save the following code to a file named `fakepid.c`:

```
#include <stdlib.h>
int getpid() {
    /* instead of returning the actual PID, return whatever
     * number is in the environment variable FAKEPID */
    return atoi(getenv("FAKEPID"));
}
```

Now, compile `fakepid.c` as a shared library, so that we can use `LD_PRELOAD` to tell the linker to link `ssh-keygen` with our version of `getpid()` instead of the normal one:

```
$ gcc -shared -fPIC fakepid.c -o fakepid.so
```

Now, run `ssh-keygen`, using `LD_PRELOAD` and `fakepid.so` to execute `ssh-keygen` with the PID we want (in Linux, '`ENVIRONMENT_VARIABLE=value` command' runs `command` with the environment variable set to `value`):

```
$ FAKEPID=1 LD_PRELOAD="./fakepid.so" ssh-keygen -N "" -f key_for_pid_1
```

Finally, try to log into the server using the key you just generated:

```
$ ssh -i key_for_pid_1 hackme@107.20.43.191
```

Submit the contents of the file `/home/chell/cake.txt`, along with any code you wrote for the problem, and leave a message containing at least your name.