DBSCAN

Theme

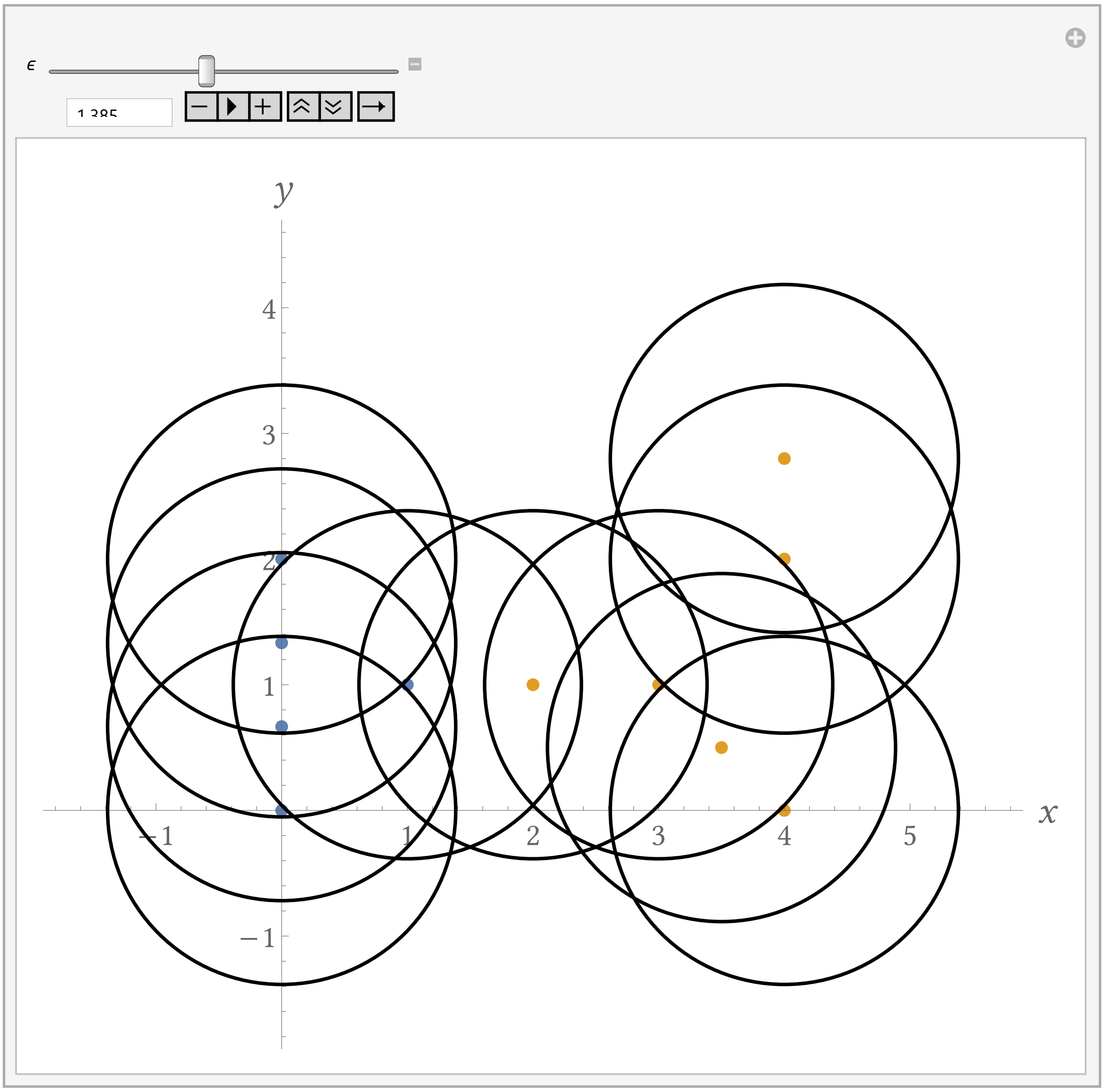
Data

In[13]:=
```
data = {
   {1, 1}, {0, 0}, {0, 2}, {0, 2/3}, {0, 4/3},
   {2, 1},
   {4, 0}, {4, 2}, {3, 1}, {3.5, 0.5}, {4, 2.8}
  };
```

In[59]:=
```
padding = 1.9;
plotRange =
  {MinMax[data[[ ;; , 1]]] + {-padding, padding}, MinMax[data[[ ;; , 2]]] + {-padding, padding}};
createClusterPlot[ϵ_, nMin_] := ListPlot[
  FindClusters[data, Method → {"DBSCAN", "NeighborhoodRadius" → ϵ, "NeighborsNumber" → nMin}],
  PlotTheme → "myTheme",
  PlotRange → plotRange,
  AspectRatio → Automatic,
  AxesLabel → {it["x"], it["y"]},
  Epilog → {
    Opacity[0],
    EdgeForm[Thick],
    Table[Disk[p, ϵ], {p, data}]
   }
 ]
```

In[69]:=
```
Manipulate[
 createClusterPlot[ϵ, 4]
 , {ϵ, 0.1, 3}]
```

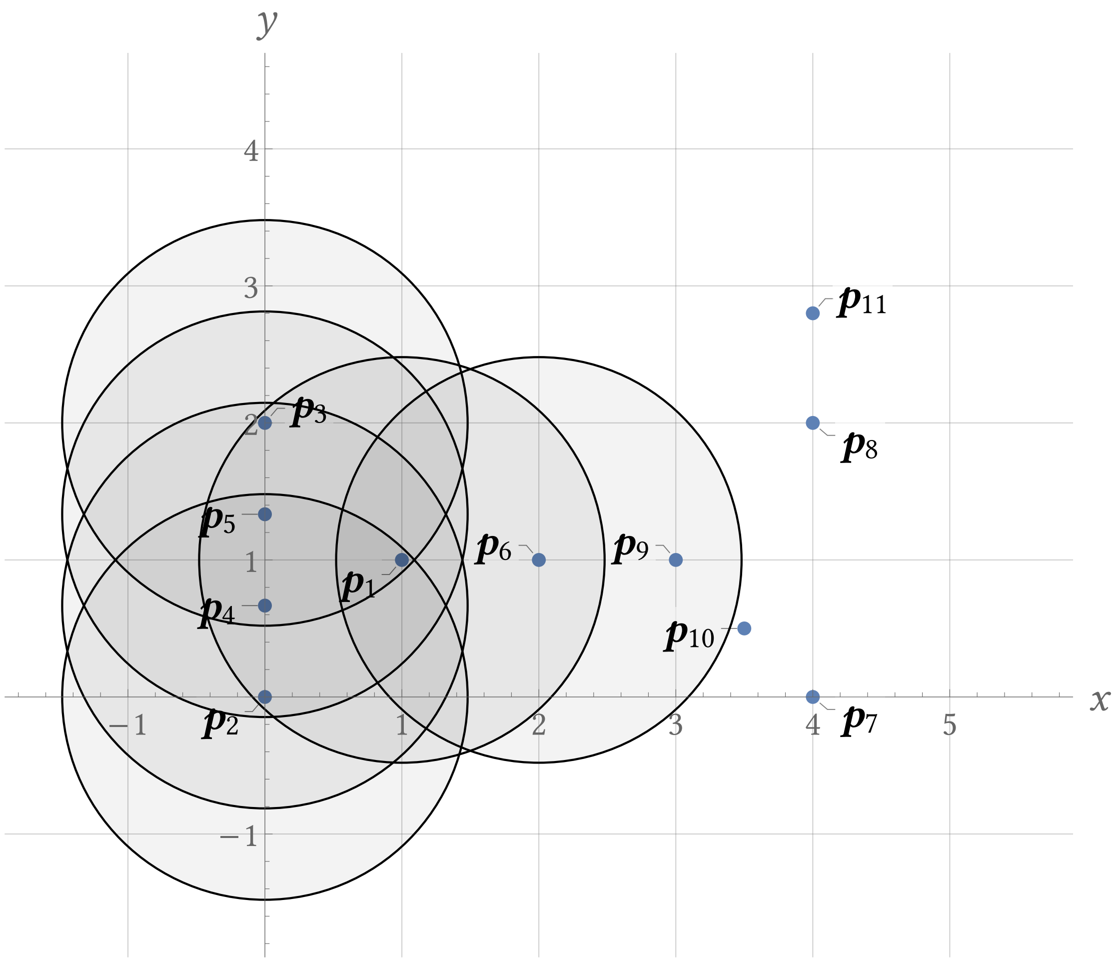
Out[69]=



Example

In[20]:=
```
plotExample = Show[
  ListPlot[MapThread[(Callout[#1, Subscript[bi["p"], #2]] &), {data, Range[Length[data]]}],
   PlotRange → {MinMax[data[[ ;; , 1]]] + {-padding, padding},
     MinMax[data[[ ;; , 2]]] + {-padding, padding}},
   AspectRatio → Automatic,
   GridLines → Automatic,
   PlotTheme → "myTheme",
   AxesLabel → {it["x"], it["y"]}
  ],
  Graphics[{
    Opacity[0.05],
    EdgeForm[Thickness[0.002]],
    Table[Disk[p, 1.48], {p, data[[1 ;; 6, ;;]]}]
   }]
 ]
```

Out[20]=



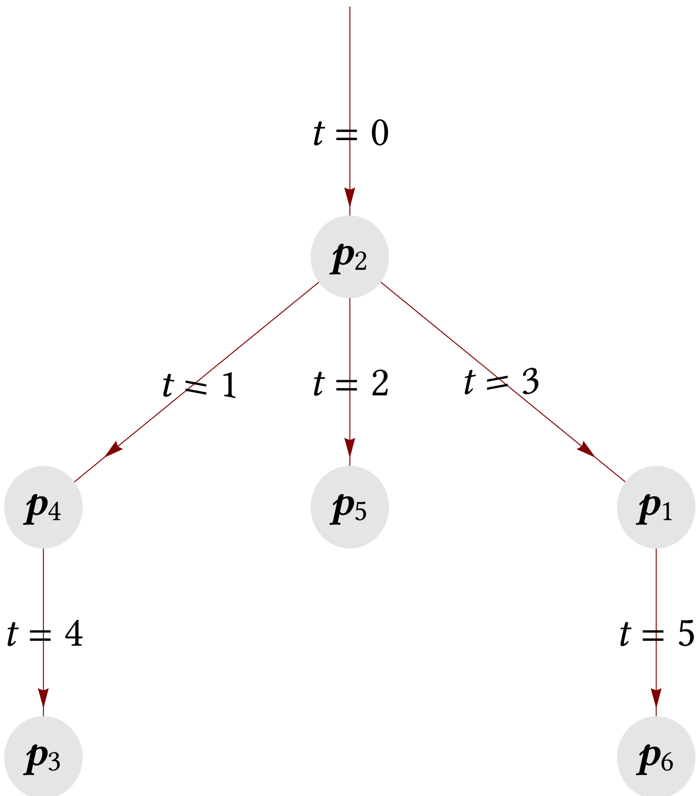
In[53]:=

```
vertexStyle[expr_, pos_] := Module[{},
  If[expr == "",
   Return[{}]
  ];

  {Text[
    Framed[expr,
     Background → GrayLevel[0.9],
     RoundingRadius → 50,
     FrameMargins → {{10, 10}, {12, 6}},
     FrameStyle → Thickness[0.01]
    ], pos]}
 ]
labels = (Subscript[bi["p"], #] &) /@ Range[Length[data]];

createTreePlot[edges_] := Module[{t, fEdges},
  t = 0;
  fEdges = {{"" → labels[[edges[[1, 1]]]], Row[{it["t"], " = " <> ToString[t]}]}};
  Do[
   t++;
   AppendTo[fEdges,
    {labels[[e[[1]]]] → labels[[e[[2]]]], Row[{it["t"], " = " <> ToString[t]}]}];
   , {e, edges}];

  TreePlot[fEdges, Top, "",
   VertexLabeling → True,
   DirectedEdges → True,
   VertexRenderingFunction → (vertexStyle[#2, #1] &),
   PlotStyle → Directive[FontFamily → "Libertinus Serif", FontSize → 22],
   PlotRangePadding → None,
   ImageSize → Automatic
  ]
 ]
```

In[56]:=

```
plotTree = createTreePlot[{2 → 4, 2 → 5, 2 → 1, 4 → 3, 1 → 6}]
```

Out[56]=

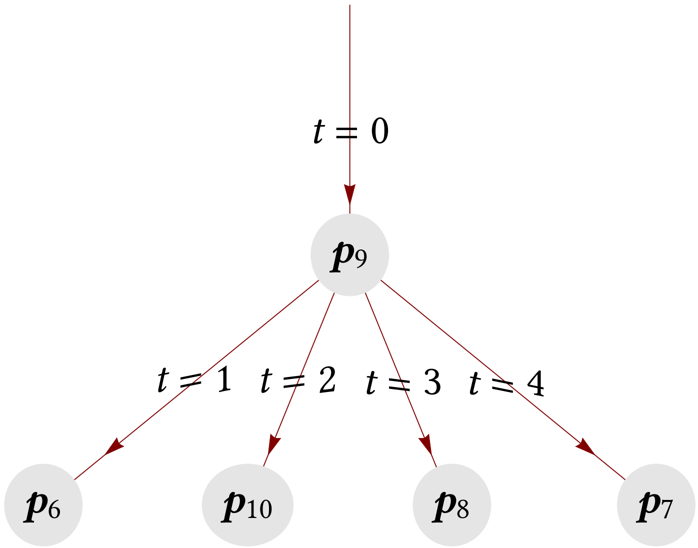


Part 1: Remaining Algorithm

From the point p_9 we reach 4 other points. However, each of these neighbours has not enough points in its own neighbourhood so that the tree does not grow further.

In[57]:=
```
createTreePlot[{9 → 6, 9 → 10, 9 → 8, 9 → 7}]
```

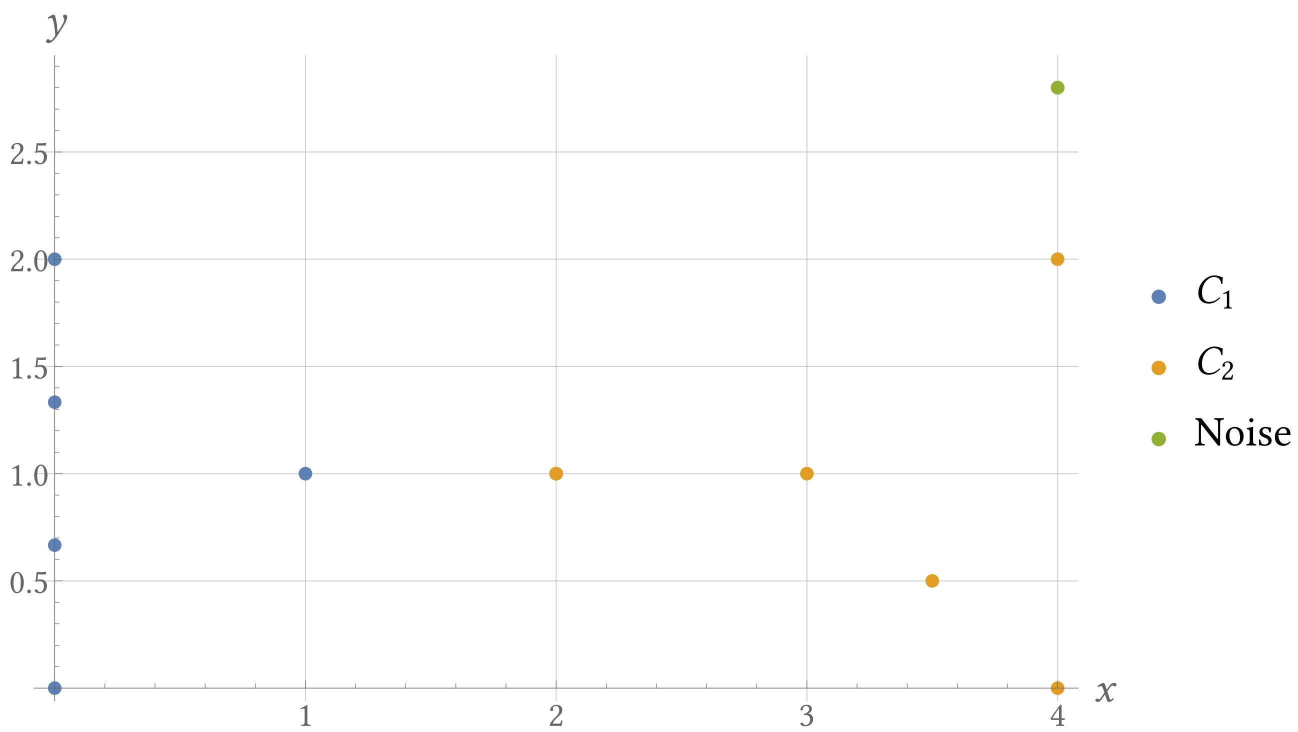
Out[57]=



So, we have $C_2 = \{p_9, p_6, p_{10}, p_8, p_7\}$ and the point p_{11} is a noise point.

In[79]:=
```
ListPlot[{
   data[[{2, 4, 5, 1, 3, 6}]],
   data[[{9, 6, 10, 8, 7}]],
   data[[{11}]]
  },
  PlotLegends → PointLegend[97, {"C1", "C2", "Noise"}, LegendMarkerSize → 15],
  PlotTheme → "myTheme",
  GridLines → Automatic,
  AxesLabel → {it["x"], it["y"]}
]
```

Out[79]=



Part 2: Ambiguity

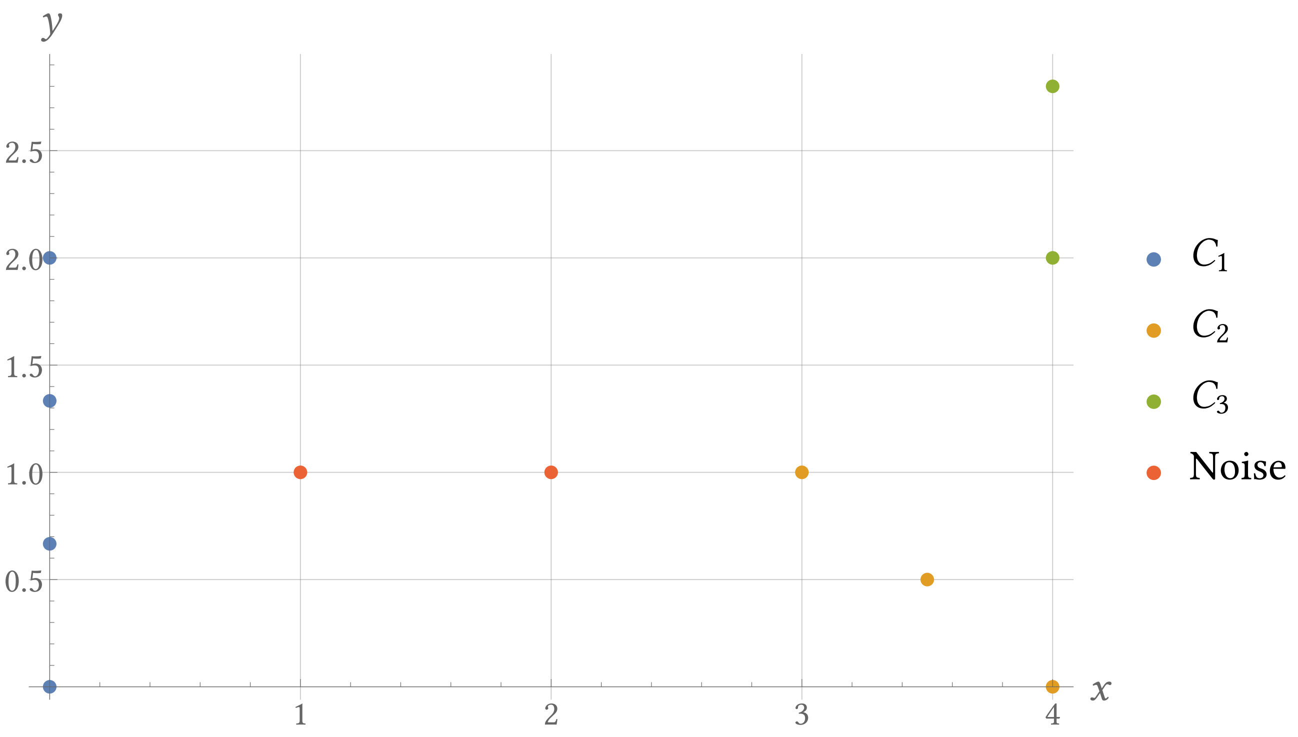
The point p_6 may be added to the first or the second cluster depending on the ordering of the seed points. This is possible for every border point. In the end, we may get different results when applying the DBSCAN algorithm multiple times.

Part 3: New Parameters

With the new parameters, we get four clusters and two noise points. It is to note that both parameters have a high influence on the clustering result.

In[81]:=
```
ListPlot[{
  data[[{2, 4, 5, 3}]],
  data[[{9, 10, 7}]],
  data[[{8, 11}]],
  data[[{1, 6}]]
 },
 PlotLegends → PointLegend[97, {"C₁", "C₂", "C₃", "Noise"}, LegendMarkerSize → 15],
 PlotTheme → "myTheme",
 GridLines → Automatic,
 AxesLabel → {it["x"], it["y"]}
]
```

Out[81]=



Part 4: Comparison

- Both algorithms are non-deterministic. However, the results differ probably more with *k*-means due to the random initialization of the first cluster centroids. In DBSCAN, only the border points may vary.
- *k*-means is centroid- and DBSCAN density-based. Also, *k*-means minimizes the variance criterion.
- Only one iteration is required for DBSCAN (then all points must have a label or are considered noise) whereas *k*-means requires usually multiple iterations.

- Only DBSCAN can detect noise points. If *k*-means should be applied on a noisy dataset, the fuzzy version may be appropriate (cf. future exercise).

Part 5: Smiley Face dataset

k-means fails on this dataset since it does not have the usual blob-like structure. What is more, a centroid does not make much sense for this kind of cluster (especially the outer ring).