PET 837: ENHANCED OIL RECOVERY METHODS

Methods of Predicting Waterflood Performance

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COURSE OUTLINE

- Introduction
- Predictive methods for waterflood performance
- Areal sweep effect
- Reservoir stratification
- Displacement mechanism
- Numerical method
- Empirical method
- Calculations for water flood performance
- Exercises

INTRODUCTION

- This chapter is concerned with the problem of predicting waterflood behavior.
- Given a particular waterflood prospect, we would like to predict information such as the time required for water breakthrough, oil recovery at breakthrough, water oil ratio performance after breakthrough, production-time performance, oil production-water injection performance, etc.

INTRODUCTION

- Numerous methods have been proposed to accomplish this, each differing in the manner of handling heterogeneity, areal sweep calculations, water injection performance, displacement efficiency, or many other variables which can affect waterflood performance.

Predicting Waterflood Performance

- Water flooding has been widely applied in hydrocarbon fields either to support the reservoir pressure during depletion and/or to increase hydrocarbon production as a secondary recovery process.

- The technique consists of injecting water with the purpose of maintaining pressure and/or displacing and producing hydrocarbons.

Predicting Waterflood Performance

- For purposes of description, waterflood prediction methods can be categorized into five groups.
  - Areal sweep effects
  - Reservoir stratification
  - Displacement mechanism
  - Numerical methods
  - Empirical methods.
- The most successful and most commonly used prediction methods in each of these categories will be discussed.

Areal Sweep Method

- It is shown in previous chapters that areal sweep efficiency can be correlated as a function of mobility ratio, pattern geometry, and cumulative water throughput.
- The most commonly used correlations are those developed by Caudle and coworkers.
- An application of these correlations to the prediction of waterflood behavior is illustrated by an example. See next slide

Areal Sweep Prediction Method

Example 5.1:

Consider the following data for a five-spot well pattern: Pattern area = 40 acres (20 acre well spacing); Average net pay thickness = 15 ft; Porosity = 20%; S_{oi} = 70%; S_{or}= 30%; Mobility Ratio = 2.1; Pattern Injection Rate = 200 RB/D; B_o = 1.25 RB/STB.

(a) What is the time of breakthrough and the oil recovery at breakthrough?

(b) What pattern sweep efficiency and cumulative oil recovery can be expected after injecting 0.6 pore volume of water? How long will it take?

Areal Sweep Prediction Methods

Solution: (A)

$$V_p = 7758A * h * \phi = 7758 * 40 * 15 * 0.2 = 930{,}960rb$$

$$Displaceable\ pore\ vol(V)_D = 7758A * h * \phi(S_{oi} - S_{or}) = 7758 * 40 * 15 * 0.2(0.7 - 0.3) = 372{,}384rb$$

$$E_{ABT} = 0.54602036 + \frac{0.03170817}{M} + \frac{0.30222997}{e^M} - 0.00509693M$$

$$E_{ABT} = 0.54602036 + \frac{0.03170817}{2.1} + \frac{0.30222997}{e^{2.1}} - 0.00509693 * 2.1 = \mathbf{0.6}$$

Areal Sweep Prediction Methods

$$N_{pbt} = \frac{V_D E_{ABT}}{B_o} = \frac{372{,}384 * 0.6}{1.25} = \mathbf{178{,}744}\ STB$$

$$t_{bt} = \frac{V_D E_{ABT}}{i_w} = \frac{372{,}384 * 0.6}{200} = 1117\ days\ or\ 3.1yrs$$

Solution: (B)

$$V_D = \frac{W_i}{V_p E_D} = \frac{(930{,}960)(0.6)}{(930{,}960)(0.7 - 0.3)} = 1.5$$

$$E_A = E_{ABT} + 0.2749 \ln\left(\frac{W_{inj}}{W_{iBT}}\right)$$

Areal Sweep Prediction Methods

$$E_A = 0.6 + 0.2749 ln\left(\frac{1.5}{0.6}\right) = 0.9$$

$$N_p = \frac{V_D E_A}{B_o} = \frac{372{,}384 * 0.9}{1.25} = \mathbf{268, 111} STB$$

$$t = \frac{W_i}{i_w} = \frac{(930{,}960)*(0.6)}{200} = 2793\ days\ or\ 7.65 yrs$$

Areal Sweep Prediction Methods

- Because of their many limitations, the use of areal sweep correlations, for the type of calculations illustrated by Example 5.1, should be limited to only the most cursory type of analysis.
- Among the many limitations are the following:
  - Correlations were developed using miscible fluids and, consequently, assume piston-like displacement; i.e., no oil is assumed to flow behind the front.
  - Correlations do not account for areal or vertical heterogeneities.
  - Change in mobility ratio after water breakthrough and its subsequent effect upon areal sweep efficiency are not accounted for.
  - Does not account for the effects of varying pressure which results from holding injection rate constant.

Reservoir Stratification - Dykstra-Parsons Method

- Dykstra and Parsons developed a method of predicting waterflood behavior in stratified systems which is particularly useful if a rapid approximation of waterflood recovery are needed.
- This method requires knowledge of the vertical permeability variation, V, the mobility ratio, M, the initial water saturation, S_{wi}, and fractional oil recovery at a specified water-oil ratio.
- This method is subject to several assumptions and limitations which can affect the accuracy of waterflood predictions:

Reservoir Stratification - Dykstra-Parsons Method

1. Layer-cake model with no crossflow between layers
2. Piston-like displacement with no oil production from behind the front.
3. Linear flow
4. Steady-state flow
5. Except for absolute permeability, rock and fluid properties are the same for all layers.
6. Water fillup (for gas space) occurs prior to flood response.

Reservoir Stratification - Dykstra-Parsons Method

- Dykstra and Parsons applied their mathematical model to an idealized reservoir containing 50 layers of varying permeability to prepare a correlation between permeability variation, V, and vertical coverage, C_v, for a range of values of water-oil ratio and mobility ratio.
- These correlations are presented in Figures 1 through 4 for water-oil ratios of 1, 5, 25 and 100, respectively.
- They also conducted linear waterflood tests on a large number of cores from California sands.

Reservoir Stratification - Dykstra-Parsons Method

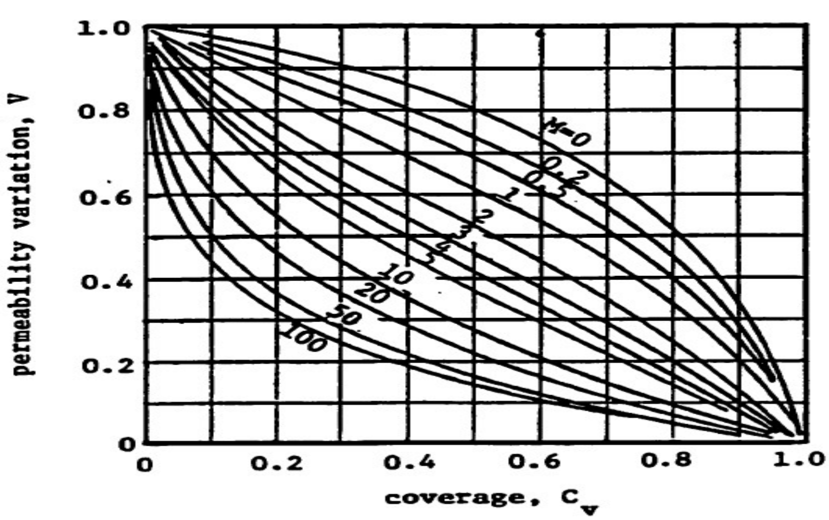


Fig. 1: Permeability variation versus vertical coverage for WOR = 1.

Reservoir Stratification - Dykstra-Parsons Method

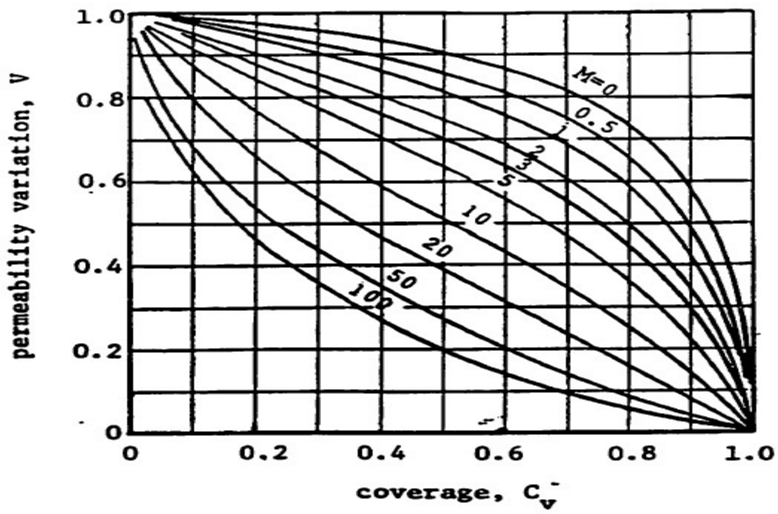


Fig. 2: Permeability variation versus vertical coverage for WOR = 5.

Reservoir Stratification - Dykstra-Parsons Method

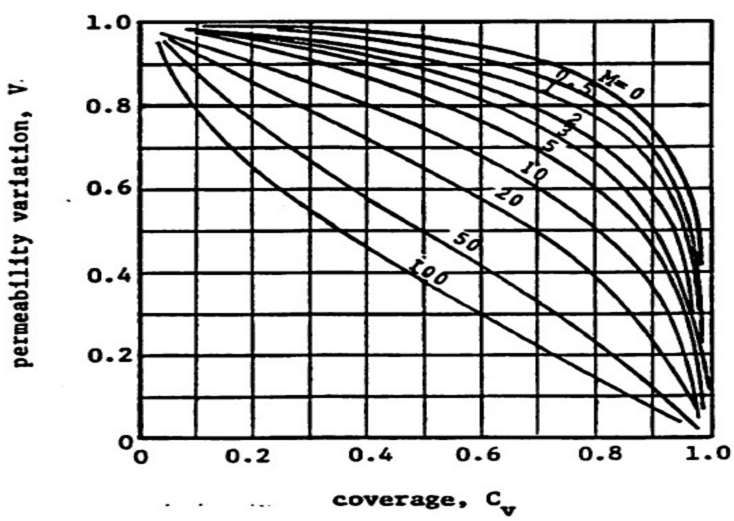


Fig. 3: Permeability variation versus vertical coverage for WOR = 25.

Reservoir Stratification - Dykstra-Parsons Method

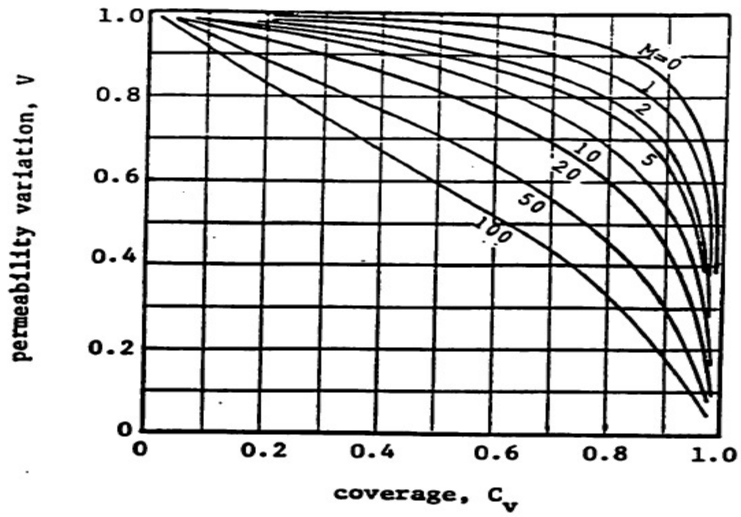


Fig. 4: Permeability variation versus vertical coverage for WOR = 100.

Reservoir Stratification - Dykstra-Parsons Method

- These cores were saturated with oil, water and gas in varying amounts and flooded to determine fractional recovery.
- The fractional recovery, E_R, was then correlated as a function of vertical coverage, water-oil ratio and initial water saturation as shown by Figure 5.
- Johnson (1956) made these correlations easier to use by combining Figures 1 to 5, thereby eliminating the variable, C_v Figure 5a

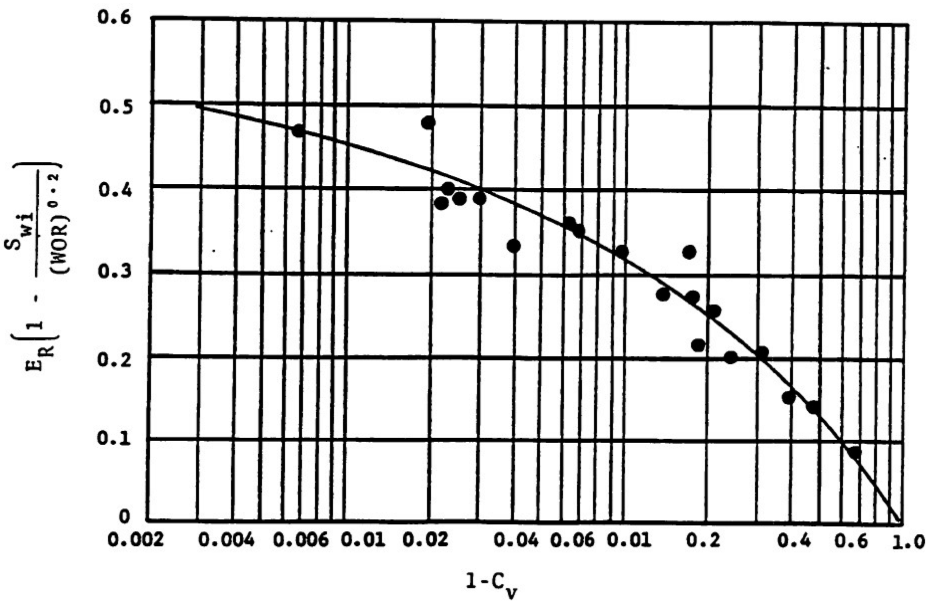


Fig. 5: Fractional recovery as a function of vertical coverage.

Reservoir Stratification - Dykstra-Parsons Method

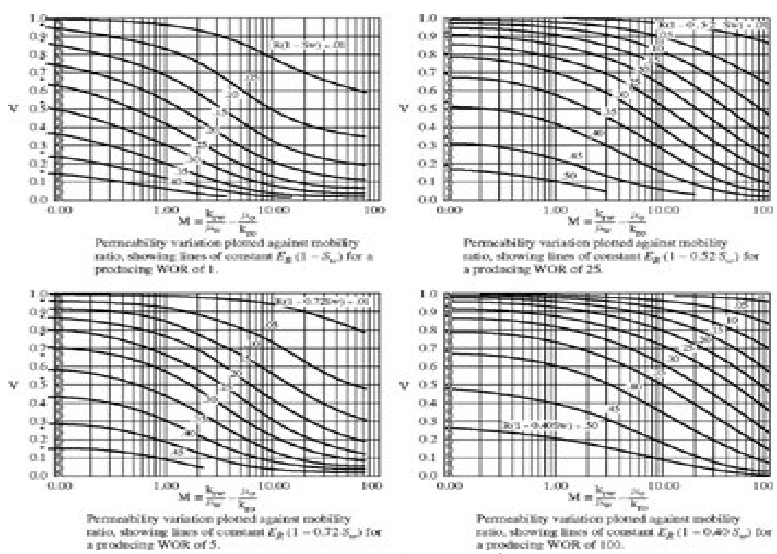


Fig. 5a: Johnson's correlations

Reservoir Stratification - Dykstra-Parsons Method

- The following procedure is utilized to predict waterflood performance if it is assumed that displaced oil is equal to the produced oil:

1. Determine the permeability variation, V.
2. Determine the mobility ratio, M.
3. Use V and M to obtain C_v at WOR values of 1, 5, 25, and 100 (see Figures 1 to 4).
4. Compute an appropriate pattern areal sweep efficiency, E_A (fraction).
5. Compute the cumulative oil production corresponding to each value of WOR; N_p.

Reservoir Stratification - Dykstra-Parsons Method

$$N_P = N * E_D * E_A * C_V$$ Eqn. 1

$$N = \frac{7758 * A * h * \emptyset * S_o}{B_o}$$ Eqn. 2

$$E_D = \frac{S_o - S_{or}}{S_o} = \frac{1 - S_{wc} - S_g - S_{or}}{1 - S_{wc} - S_g}$$ Eqn. 3

where N = oil in place at start of waterflood; C_v = coverage

E_A = Areal Sweep Efficiency; E_D = Displacement Efficiency; Swc, Sg, So and Sor = saturation of connate water, gas, oil and residual oil respectively.

Reservoir Stratification - Dykstra-Parsons Method

6. Plot N_p versus WOR. Extrapolate this curve back to a zero-WOR to obtain the recovery at break through. Based on a predetermined WOR for the economic limit, read the cumulative recovery at breakthrough from the graph. This is illustrated in Figure 6 (see next slide).

7. Compute the injected water required to fill-up the gas space (see Equation 4 in next slide).
8. Compute the injected water required to replace oil production as a function of N_p (see Equation 5 in next slide).

Reservoir Stratification - Dykstra-Parsons Method

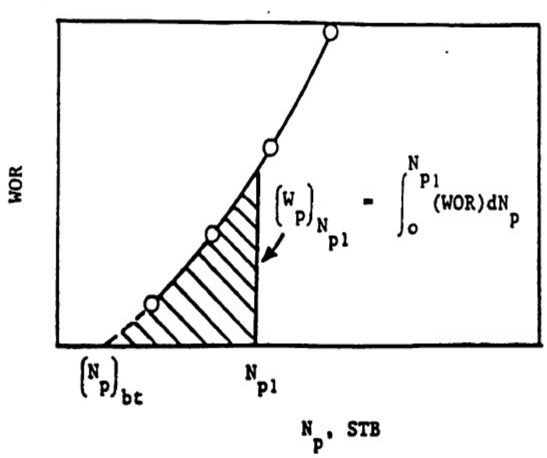


Fig. 6: WOR-N_p relationship for Dykstra-Parsons Procedure.

$$W_f = V_p(1 - S_o - S_{wi}) \qquad \text{Eqn. 4}$$

$$W_o = N_p B_o \qquad \text{Eqn. 5}$$

- Where

W_f = Injected water required to fill-up the gas space.

W_o = injected water required to replace oil production.

V_p = Pore Volume.

Reservoir Stratification - Dykstra-Parsons Method

9. Compute the injected water required to replace water production as a function of N_p:

$$WOR = \frac{q_w}{q_o} = \frac{dW_p/dt}{dN_p/dt}$$ Eqn. 6

$$W_p = \int (WOR)dN_p$$ Eqn. 7

- W_p can be computed as a function of N_p by graphically integrating the WOR-N_p curve in Figure 6 at several values of N_p.

Reservoir Stratification - Dykstra-Parsons Method

10. Compute cumulative water injected as a function of Np, and as a function of time:

$$W_i = W_f + W_o + W_p$$ Eqn. 8

$$\mathrm{t} = \frac{W_i}{i_w}$$ Eqn. 9

Reservoir Stratification - Dykstra-Parsons Method

Steps 7-10 can be summarized in tabular form as shown:

N_p	W_f	W_o	W_p	W_i	t
N_{p1}	W_f	W_{o1}	W_{p1}	W_{i1}	t_1
N_{p2}	W_f	W_{o2}	W_{p2}	W_{i2}	t_2
.	W_f	.	.	.	.
.	W_f	.	.	.	.
.	W_f	.	.	.	.
N_{pn}	W_f	W_{on}	W_{pn}	W_{in}	t_n

Example 5.2

The permeability data for an oil reservoir are presented in the Table below.

Layer	h, ft	k, mD
1	1	10.0
2	1	6.8
3	1	4.7
4	1	10.4
5	1	20.5
6	1	12.1
7	1	8.6
8	1	18.4
9	1	14.3
10	1	10.9

Additionally, the average relative permeability for this reservoir are presented in the Table below.

S_w	k_{rw}	k_{ro}
0.36	0	0.180
0.38	0.004	0.130
0.42	0.008	0.082
0.46	0.015	0.050
0.54	0.038	0.020
0.58	0.063	0.014
0.62	0.100	0.008
0.66	0.155	0.002
0.70	0.214	0

Example 5.2

Other data are:

μ_o= 2.72 cp; μ_w = 0.75 cp; Well pattern = Five-spot; Oil in place at beginning of flood = 32x10^6 STB; i_w = 15000 RB/D; B_o = 1.25 RB/STB; B_w = 1.05 RB/STB; S_{gi} = 0.0; S_{wi} = 0.36.

Use the recovery correlations of Dykstra and Parsons to determine:

A. N_p as a function of WOR

B. W_i as a function of N_p

C. N_p as a function of time

Solution

Step 1. To determine the Dykstra-Parsons permeability variation.

- This step requires that the permeabilities be rearranged in the order of decreasing permeability and that the percent greater than be computed for each value; these calculations are summarized in Table 5.1

Table 5.1:

k, mD	h, ft	h with greater k	% h greater k
20.5	1	0	0
18.4	1	1	10
14.3	1	2	20
12.1	1	3	30
10.9	1	4	40
10.4	1	5	50
10.0	1	6	60
8.6	1	7	70
6.8	1	8	80
4.7	1	9	90
	Σh = 10		

Solution

Plot the k vs %h greater k data from Table 5.1 (see Figure 7 in next slide).

Use the data from Figure 7, and compute the permeability variation

$$V = \frac{k_{50} - k_{84.1}}{k_{50}} = \frac{10.0 - 5.95}{10.0} = 0.405$$

Step 2: Calculate the mobility ratio as defined;

$$M = \frac{k_{rw}\mu_o}{k_{ro}\mu_w} \qquad \text{Eqn. 10}$$

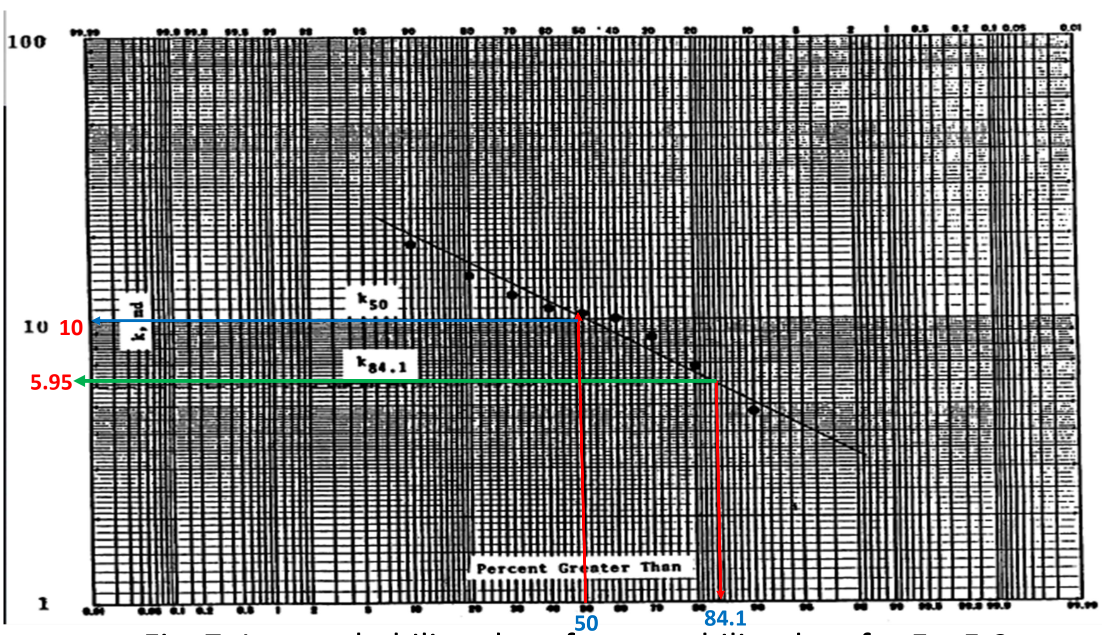


Fig. 7: Log probability plot of permeability data for Ex. 5.2

Solution

- The Dykstra-Parsons method assumes piston-like displacement; accordingly, k_{ro} is taken at the initial water saturation and k_{rw} is taken at the residual oil saturation behind the front (End point mobility ratio).

- Therefore,

$$M = \frac{(0.214)(2.72)}{(0.180)(0.75)} = 4.31$$

Solution

Step 3: Determine the Vertical sweep (coverage), C_v, from the Dykstra-Parsons charts as a function of WOR (Values of C_v on the table not found from the charts are interpolated as well as extrapolated values)

- Results are summarized in Table 5.2.

Table 5.2

WOR	**C_v**
0.1	0.24
0.2	0.29
0.5	0.40
1.0	0.51
2.0	0.63
5.0	0.79
10.0	0.88
25.0	0.94
50.0	0.96
100.0	0.98

Solution

To compute Oil recovery in STB's using the Equations below, i.e.,

$$N_P = N * E_D * E_A * C_V$$

- And $N = \frac{V_p S_o}{B_o}$

$$V_p = \frac{NB_{oi}}{1 - S_{wi}} = \frac{(32x10^6)(1.25)}{(1 - 0.36)} = 62.5x10^6 RB$$

$$N_P = (62.5x10^6)(S_o) * E_D * E_A * C_V/(B_o)$$

Since $E_D = \frac{S_o - S_{or}}{S_o}$, $N_P = (62.5x10^6)(S_o - S_{or}) * E_A * C_V/(B_o)$

Solution

Step 4: Compute an appropriate pattern areal sweep efficiency

- The areal sweep efficiency at any point in time during the flood varies from layer-to-layer; it also varies within each layer as a function of cumulative water injection.
- The basic Dykstra-Parsons calculation assumes linear flow and, accordingly, does not consider these effects.
- It will be assumed in this problem that the average areal sweep efficiency is equal to the sweep efficiency at breakthrough; this may be somewhat pessimistic but, when coupled with the optimistic Dykstra—Parson's calculations which result from assuming piston-like displacement of oil, it should give a reasonable prediction of oil recovery.

Solution

- The sweep efficiency correlations used to obtain E_{As} require a different definition of mobility ratio than used in the Dykstra-Parsons displacement calculations.

- In order to obtain E_{As} at breakthrough from Figure 8 (see next slide), the mobility ratio is computed according to the Equation below:

$$M = \frac{(k_{rw})_{\bar{S}_{wbt}} * \mu_o}{(k_{ro})_{S_{wi}} * \mu_w} \qquad \text{Eqn. 11}$$

Solution

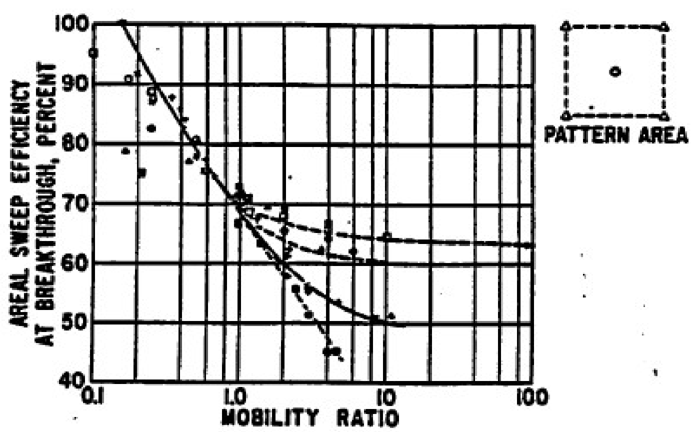


Fig. 8: Areal sweep efficiency at breakthrough, developed five-spot pattern

Solution

- Figure 9 (see next slide) presents the fractional flow curve for this reservoir; it is determined from this graph that $\bar{S}_{wbt}$ = 0.548, and it is found by linear interpolation from the relative permeability data that the corresponding value of k_{rw} is 0.043. Thus,

$$M = \frac{0.043 * 2.72}{0.180 * 0.75} = 0.87$$

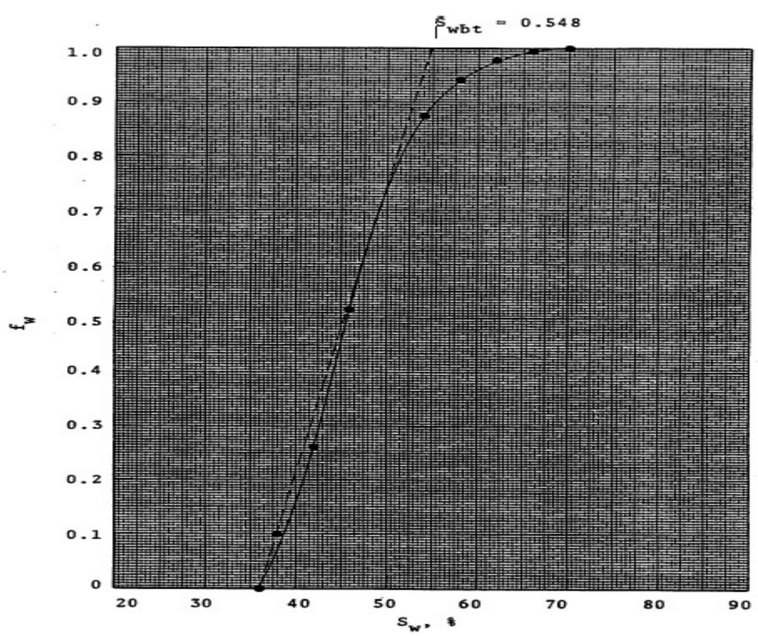


Fig. 9: Fractional flow curve for Example 5.1

Solution

- From Figure 8, E_{As} = 70%, and from relative permeability table, S_{or} = 30% and S_o = 64%

Step 5: Compute the cumulative oil production corresponding to each value of WOR; N_p. Compute cumulative oil recovery for the various C_v values as a function of water-oil ratio. See table 5.2 in next slide

A. Calculating Cumulative Oil Produced as function of WOR.

$$N_P = (62.5x10^6)(0.64 - 0.3) * 0.7 * C_V/(1.25)$$

$$N_P = 11.9x10^6 C_V$$

Step 6: Plot WOR vs N_p (see Figure 10 , two (2) slide down)

$$N_{P,@WOR=0.1} = 11.9 \times 10^6 * 0.24 = 2.85x10^6$$

Solution

Table 5.2

WOR	C_v	N_p, STBx10^6
0.1	.24	2.85
0.2	.29	3.45
0.5	.40	4.76
1.0	.51	6.07
2.0	.63	7.59
5.0	.79	9.40
10.0	.88	10.47
25.0	.94	11.19
50.0	.96	11.42
100.0	.98	11.66

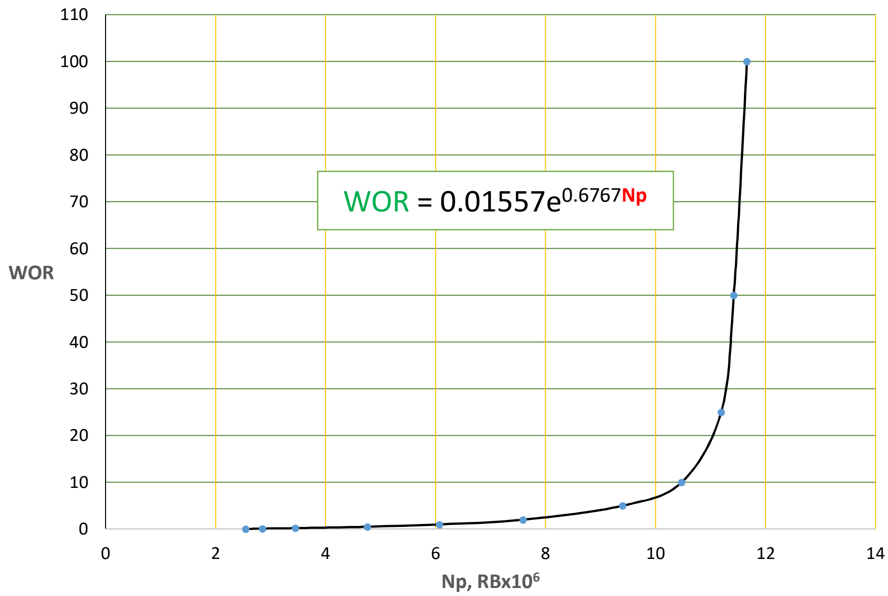


Fig. 10: Cumulative oil recovery versus water oil-ratio for Example 5.1.

Solution

Step 7: Compute the injected water required to fill-up the gas space

B: The cumulative water injected at any time is computed according to the relationship:

$$W_i = W_f + W_o + W_p$$

Where; $W_f = V_p(1 - S_o - S_{wi})$ Eqn. 12

$$W_f = 62.5x10^6(1 - 0.64 - 0.36) = 0$$

Step 8: Compute the injected water required to replace oil production as a function of N_p

- The water required to replace the produced oil, W_o, is computed according to Equation 5, i.e.,

$$W_o = N_p B_o$$

$$W_o = 2.85\text{x}10^6 * 1.25 = 3.56x10^6 \text{ bbl}$$

Solution

Step 9: Compute the injected water required to replace water production as a function of N_p:

- The water required to replace produced water is defined by Equation 7 and is obtained by graphically integrating the area under the WOR-N_p curve in Figure 10.

$$W_p = \int_{N_{p(n-1)}}^{N_{p(n)}} (WOR)dN_p$$

From Figure 10, at WOR = 0, N_p(at breakthrough) = 2.55x10^6 RB
The relationship between WOR and N_p is given as

$$WOR = ae^{bN_p}$$

Where a and b are constant. From Figure 10, **a = 0.01557** and **b = 0.6767**.

Solution

- Since a and b are known, the equation for WOR becomes

$$WOR = 0.01557e^{0.6767N_p}$$

- Then the integral function becomes

$$W_p = \int_{N_{p(n-1)}}^{N_{p(n)}} 0.01557e^{0.6767N_p}dN_p$$

$$W_p = 0.01557\int_{N_{p(n-1)}}^{N_{p(n)}} e^{0.6767N_p}dN_p$$

Solution

- Cumulative water produced Wp is then computed as

$$W_p = \frac{0.01557}{0.6767} e^{0.6767 N_p} \Bigg|_{N_{p,n-1}}^{N_{p,n}}$$

$$W_p = 0.02301\left(e^{0.6767 N_{p,n}} - e^{0.6767\ {}_{p,n-1}}\right)$$

$$W_{p,@WOR=0.1} = 0.02301\left(e^{0.6767(2.85)} - e^{0.6767(2.55)}\right)$$
$$= 0.0298 \approx \mathbf{0.03} bbl \times 10^6$$

$$W_{p,@WOR=0.2} = 0.02301\{\left(e^{0.6767(2.85)} - e^{0.6767(2.55)}\right) + \left(e^{0.6767(3.45)} - e^{0.6767(2.85)}\right)\}$$
$$= 0.108 \approx \mathbf{0.11} bbl \times 10^6$$

Solution

$$W_{p,@WOR=0.5} = 0.02301\{\left(e^{0.6767(2.85)} - e^{0.6767(2.55)}\right) + \left(e^{0.6767(3.45)} - e^{0.6767(2.85)}\right)$$
$$+\left(e^{0.6767(4.76)} - e^{0.6767(3.45)}\right)\}$$
$$= 0.447 \approx \mathbf{0.}45 bbl \times 10^6$$

- The water required to fill up gas space is zero in this project since there is no initial gas saturation. These calculations are summarized in the Table (see next slide).

Step 10: Compute cumulative water injected as a function of Np, and as a function of time:

Solution

WOR	N_p, STBx10^6	W_0, bblx10^6	W_p, bblx10^6	W_i, bblx10^6
0	N_p at Breakthrough 2.55	-	-	-
0.1	2.85	3.56	0.03	3.59
0.2	3.45	4.31	0.11	4.42
0.5	4.76	5.95	0.45	6.40
1.0	6.07	7.58	1.27	8.85
2.0	7.59	9.49	3.78	13.27
5.0	9.40	11.75	13.19	24.94
10.0	10.47	13.09	27.34	40.43
25.0	11.19	13.99	44.59	58.58
50.0	11.42	14.27	52.12	66.39
100.0	11.66	14.58	61.34	75.92

Solution

C: Waterflood performance can be put on a time basis using the Equation below, i.e..

$$t = \frac{W_i}{i_w} = \frac{W_1 = 3.59x10^6}{15000} = 239\ days$$

$$t = \frac{W_2 = 4.42x10^6}{15000} = 295\ days$$

- These calculations are summarized in the Table on the side.

WOR	N_p, STBx10^6	t, days
0.1	2.85	239
0.2	3.45	295
0.5	4.76	427
1.0	6.07	590
2.0	7.59	885
5.0	9.40	1663
10.0	10.47	2695
25.0	11.19	3905
50.0	11.42	4426
100.0	11.66	5061

Displacement Mechanism Prediction Methods

- The methods considered thus far have assumed piston-like displacement behind the water front.
- However, it is generally recognized that a saturation gradient does exist behind the front and that oil production can be expected after water breakthrough from the swept area.
- The following methods account for the mechanism of displacement in predicting waterflood behavior.

Displacement Mechanism - Buckley-Leverett Method

- This method serves as the basis for describing the mechanism of immiscible fluid displacement in a waterflood.
- It has been shown that this method can be extended to describe the saturation behavior in radial systems and in five-spot systems.
- In other extensions to be described, we will see that the method can also be applied to multilayered systems.

Displacement Mechanism - Roberts Method

- Roberts suggested that the performance of each layer in a layered system could be computed using Buckley-Leverett theory, with the injection into each layer being proportional to the capacity of the layer.
- Assumptions and limitations are:
  - All assumptions involved in the Buckley-Leverett Method apply to each layer.
  - Layer-cake model with no crossflow.
  - Injection into each layer is proportional to the fractional capacity of the layer.
  - Constant injection rate.

Displacement Mechanism - Roberts Method

- The following procedure for applying the Roberts Method is suggested by Langnes, et al.

1. Construct a fractional flow curve and determine the average water saturation behind the front.
2. Draw several tangents to the fractional flow curve at S_w values greater than the breakthrough saturation. Determine $\bar{S}_w$ and $\grave{f}_w = df_w/dS_w$ corresponding to these values. Plot f_w' versus S_w and construct a smooth curve through the points.
3. Define the layers within the reservoir and determine the average permeability, porosity, and thickness for each layer.
4. Compute the capacity, kh, and fraction of total capacity, ΔC , for each layer.

Displacement Mechanism - Roberts Method

5. Compute the injection rate into each layer.

$$i_{wj} = (i_{wt})(\Delta C) \qquad \text{Eqn. 12}$$

6. Calculate the cumulative water injection, W_{ij}, into each layer to reach each S_w point chosen in (2).

$$W_{ij} = \frac{7758 A_j h_j \emptyset_j}{f'_w} \qquad \text{Eqn. 13}$$

7. Calculate q_{oj} and q_{wj} for each layer at each S_w point. Before breakthrough in a given bed,

$$q_{oj} = \frac{i_{wj}}{B_o}, \qquad q_{wj} = 0 \qquad \text{Eqn. 14}$$

Displacement Mechanism - Roberts Method

- After breakthrough in a bed

$$q_{oj} = \frac{i_{wj}}{B_o}(1 - f_w), \qquad q_{wj} = i_{wj} * f_w \qquad \text{Eqn. 15}$$

8. Calculate the recovery at breakthrough, $\left(N_{pj}\right)_{bt}$ and the time to breakthrough, $\left(t_j\right)_{bt}$ for each layer

$$\left(N_{pj}\right)_{bt} = 7758 A_j h_j \emptyset_j \left(\frac{S_{wbt} - S_{wi}}{B_o}\right) \qquad \text{Eqn. 16}$$

$$\left(t_j\right)_{bt} = \frac{\left(W_{ij}\right)_{bt}}{k_j} \qquad \text{Eqn. 17}$$

Displacement Mechanism - Roberts Method

9. Calculate the recovery, N_{pj}, and the time, t_j , to each point.

10. Plot the oil production rate for each layer as a function of time. Use this plot to construct a graph of total oil production rate versus time.

11. Repeat step 10 for the water production rate.

12. Use the total oil and water production rates to construct a plot of WOR versus time.

Displacement Mechanism - Roberts Method

13. Plot cumulative oil recovery from each layer as a function of time and use this plot to construct a graph of total recovery versus time.

14. Based on estimated expenses, decide on an appropriate WOR cutoff and from the data in Step 12, estimate the life of the project.

15. Use the WOR-time cutoff to determine the projects ultimate recovery from data in Step 13.

Displacement Mechanism - Craig-Geffen-Morse Method

- This is one of the most thorough and most practical prediction methods available for five-spot systems.
- The technique is also applicable to other patterns if certain required experimental correlations are available.
- The method utilizes a modified Welge equation to consider the displacement mechanism in the swept area.
- Variations in injectivity for constant pressure water injection are accounted for using the experimental correlations of Caudle and Witte, and the effects of increases in areal sweep efficiency beyond break through are included on the basis of experimental correlations presented by Craig, Geffen, and Morse (CGM).

Displacement Mechanism - Craig-Geffen-Morse Method

- The Craig-Geffen-Morse (CGM) method of waterflood prediction is a steady state technique which combines areal sweep effects, displacement mechanism, stratification, and variable injectivity to predict waterflood performance in a five-spot pattern.
- The method is valid with or without free gas initially present, provided there is no trapped gas behind the front.
- The calculations can be adopted for use in other pattern floods but do not account for edge or bottomwater influx.

Displacement Mechanism - Craig-Geffen-Morse Method

- The method assumes 100 percent vertical sweep efficiency within each layer of the stratified reservoir.

- Experimentally derived correlations are used to determine areal sweep efficiency at breakthrough and after breakthrough.
- Calculations are made in four stages:

Displacement Mechanism - Craig-Geffen-Morse Method

- ***Stage 1*** - This stage begins with the start of water injection and ends when oil banks formed around adjacent injectors meet. This meeting of oil banks is termed interference. Stage 1 will not occur unless free gas is present at the start of the flood. Oil production during this time period is simply a continuation of previously existing primary production. No secondary oil is recovered during this part of the flood.

- ***Stage 2*** - This period extends from interference until all pre-existing gas space is filled by injected water. Only primary oil production occurs during this stage.

Displacement Mechanism - Craig-Geffen-Morse Method

- ***Stage 3*** - This period extends from gas fillup to water breakthrough at producing wells. Oil production caused by the waterflood begins at the start of Stage 3. Furthermore, oil production during this stage is a combination of incremental waterflood recovery and a continuation of primary recovery. Water production begins at the end of Stage 3.

- ***Stage 4*** - This stage extends from water breakthrough to the economic limit.

Displacement Mechanism - Craig-Geffen-Morse Method

- Stages 1, 2, and 3 are illustrated in Figure 11.

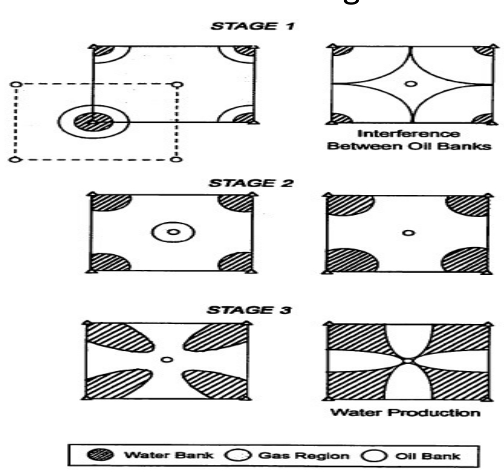


Figure 11

Displacement Mechanism - Craig-Geffen-Morse Method

- Calculation will be shown first on how waterflood predictions are made for a five-spot pattern reservoir with only one layer.
- Extended calculations for multi-layered five-spot reservoirs will be presented in a subsequent section.

Displacement Mechanism - Craig-Geffen-Morse Method

Initial Calculations - Single Layer

- Before considering the detailed procedures necessary to predict flood performance during each of the four stages, it is convenient to present the following calculations.

A. Calculate pattern pore volume, V_p.

B. Calculate stock tank oil-in-place at the beginning of the waterflood, N.

C. Calculate mobility ratio, M, prior to water breakthrough using Equation 10 and fractional flow data.

Displacement Mechanism - Craig-Geffen-Morse Method

D. Determine sweep efficiency at water breakthrough, using the mobility ratio from Step C and the correlation shown in Figure below.

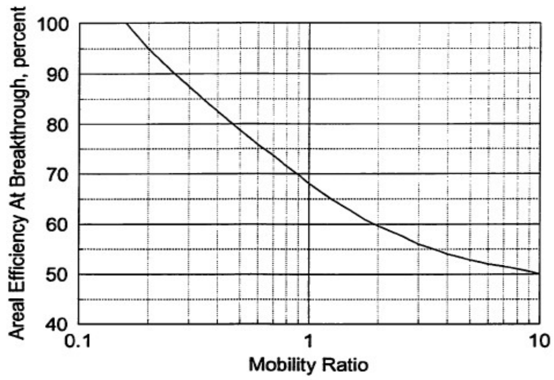


Fig. 12: areal sweep efficiency at breakthrough (developed five-spot pattern)

Displacement Mechanism - Craig-Geffen-Morse Method

E. Determine the maximum value of gas saturation, S_g^*, for which the Craig-Geffen-Morse method is valid.

$$S_g^* = C(S_o - \bar{S}_{obt}) \quad \text{Eqn. 18}$$

where:

C = coefficient from Figure E.7, SPE Monograph III.

$\bar{S}_{obt}$ = average oil saturation in swept portion of reservoir at time of water breakthrough, fraction.

- If $S_g > S^*_g$, Craig indicates, without justification, that this prediction method will yield higher WOR's and oil recovery values at any injected values than will actually occur in the field.

Displacement Mechanism - Craig-Geffen-Morse Method

F. Calculate cumulative water injected at the time of interference.

$$W_{ii} = \frac{\pi r_{ei}^2 h \emptyset S_g}{5.615} \qquad \text{Eqn. 19}$$

where:

- W_{ii} = cumulative water injected at interference, bbls
- r_{ei} = half the distance between adjacent injectors, feet.

G. Calculate cumulative water injected at gas fillup (W_{if}).

$$W_{if} = V_p S_g \qquad \text{Eqn. 20}$$

Displacement Mechanism - Craig-Geffen-Morse Method

H. Calculate cumulative water injected at the time of water breakthrough.

$$W_{ibt} = V_p E_{Abt}(\bar{S}_{wbt} - S_{wc}) \qquad \text{Eqn. 21}$$

- where:

W_{ibt} = cumulative water injected at breakthrough, bbls

- $\bar{S}_{wbt}$ = average water saturation in swept region at breakthrough, fraction
- S_{wc} = connate water saturation at start of flood, fraction
- W_{if} = cumulative water injected at gas fillup, bbls
- S_g = gas saturation at start of flood, fraction

Displacement Mechanism - Craig-Geffen-Morse Method

STAGE 1; PERFORMANCE PRIOR TO INTERFERENCE

- It is assumed during this period that the water and oil banks are radial in shape and that Darcy's radial flow equation can be used to predict water injection into the reservoir.

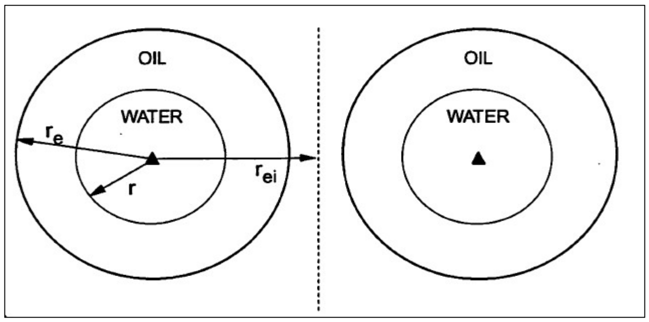


Fig. 13:Radial water and oil banks associated with injection wells during stage 1

Displacement Mechanism - Craig-Geffen-Morse Method

STAGE 1; PERFORMANCE PRIOR TO INTERFERENCE

- For a constant pressure differential (Δp), the water injection rate prior to interference will be:

$$i_w = \frac{0.00708 h k_o \Delta P}{\frac{\mu_w}{k_{rw}} ln \frac{r}{r'_w} + \frac{\mu_o}{k_{ro}} ln \frac{r_e}{r}} \qquad \text{Eqn. 22}$$

- where:

i_w = water injection rate, bbls/day;

h= net pay, feet;

K = base permeability used to define relative permeability, mD [usually the effective permeability to oil at irreducible water $(k_o)_{S_{wir}}$, mD;

Displacement Mechanism - Craig-Geffen-Morse Method

STAGE 1; PERFORMANCE PRIOR TO INTERFERENCE

k_{ro} = relative permeability to oil in oil bank at S_{wc};

k_{rw} = relative permeability to water in water bank at $\bar{S}_{wbt}$

r'_w = effective wellbore radius, ft = $r_w e^{-Si}$;

r = radius of water bank, ft;

r_w = wellbore radius, ft;

r_e = radius of oil bank, ft;

S_i = Skin factor at injection well, dimensionless;

ΔP = applied pressure differential;

μ_o = oil viscosity, cp and μ_w = water viscosity, cp.

Displacement Mechanism - Craig-Geffen-Morse Method

- The radii of the water and oil banks required by the equation of iw depends upon the cumulative water injection W_i.

$$r_e = \left[\frac{5.615W_i}{\pi h \emptyset S_g}\right]^{1/2} \qquad \text{Eqn. 23}$$

$$r = r_e\left[\frac{S_g}{\bar{S}_{wbt} - S_{wc}}\right]^{1/2} \qquad \text{Eqn. 24}$$

Summary – Stage 1 Calculations

1. Select values of W_i from zero to W_{ii}. There are no rigorous rules for making this selection; generally ten intervals of equal ΔW_i increment will be adequate.
2. Compute r_e for each value of W_i using Equation 23
3. Compute r for each value of W_i using Equation 24
4. Compute i_w for each value of W_i using Equation 25
5. Compute the average water injection rate for each increment of water injection.

$$\left[(i_w)_{avg}\right]_n = \frac{(i_w)_n + (i_w)_{n-1}}{2} \qquad \text{Eqn. 25}$$

Summary – Stage 1 Calculations

6. Compute the time required for each increment of water injection.

$$(\Delta t)_n = \frac{(W_i)_n + (W_i)_{n-1}}{\left[(i_w)_{avg}\right]_n} \qquad \text{Eqn. 26}$$

7. Compute cumulative time for each value of W_i.

$$t_n = \sum (\Delta t)_n \qquad \text{Eqn. 27}$$

STAGE 2: PERFORMANCE FROM INTERFERENCE TO FILLUP

- At the time of interference, the shape of the oil bank and water bank is radial; however, from interference to fillup, the shape of the oil bank must continuously change as the remaining gas space within the five-spot pattern is filled.
- Because of the changing geometry of the banks during this period, it is not possible to write a simple equation to predict water injection behavior.
- Fortunately, the length of stage 2 is generally short compared to other stages.

STAGE 2: PERFORMANCE FROM INTERFERENCE TO FILLUP

- Accordingly, we are going to compute the water injection rates at the end of Stage 1 and the beginning of Stage 3 and assume that i_w changes linearly between these two values.
- Therefore, the time differential between interference and fillup is:

$$(\Delta t) = \frac{(W_{if}) - (W_{ii})}{0.5(i_{wi} + i_{wf})} \qquad \text{Eqn. 28}$$

- Values of W_{if} and W_{ii} are known from initial calculations. The water injection rate at interference, i_w, corresponds to the rate at the end of Stage 1.

STAGE 2: PERFORMANCE FROM INTERFERENCE TO FILLUP

- The injection rate at fillup, i_{wf}, as well as injection rates from fillup to water breakthrough, and to the end of the economic life of the project, can be calculated as:

$$i_w = \gamma i_{base} \qquad \text{Eqn. 29}$$

- Where:

γ = conductance ratio

i_{base}= base water injection rate, bbls

$$i_{base} = \frac{0.003541(k_o)_{S_{wir}} h\Delta p}{\mu_o\left(ln\frac{d}{r_w} - 0.619 + 0.5s_p + 0.5s_i\right)} \qquad \text{Eqn. 30}$$

STAGE 2: PERFORMANCE FROM INTERFERENCE TO FILLUP

Where:

i_{base}= base water injection rate (steady-state water injection rate in an oil-field, five-spot pattern with a unit mobility ratio), bbls/day.

d = diagonal distance between adjacent injection and production wells, ft

$(k_o)_{S_{wir}}$= effective permeability to oil at immobile connate water saturation, md,

s_p and s_i = skin factor at production and injection well respectively.

Δp = bottom-hole pressure difference between the injection and production well after fillup, psi.

STAGE 2: PERFORMANCE FROM INTERFERENCE TO FILLUP

- The conductance ratio, γ, is an experimentally determined factor based on the work of Claudle and Witte which, when used in Equation 29, gives the correct injection rate.

- The conductance ratio is presented in Figure 14 (see next page) as a function of mobility ratio, M, and areal sweep efficiency, E_A.

- Note in Figure 14 that for M = 1.0, γ = 1.0 and i_w is a constant.
- For M > 1.0, γ and i_w increase with increasing sweep efficiency.
- When M < 1.0, γ and i_w decrease with increasing sweep efficiency.

STAGE 2: PERFORMANCE FROM INTERFERENCE TO FILLUP

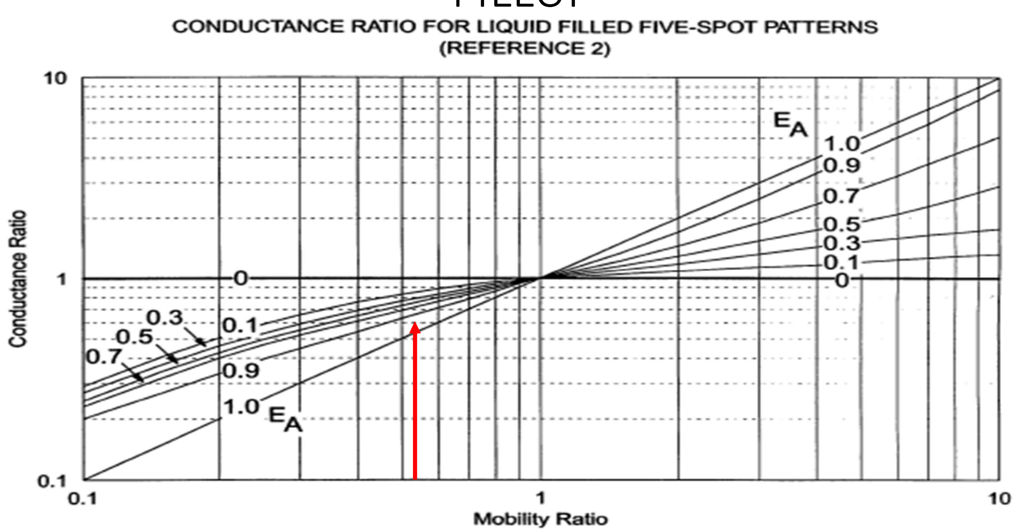


Fig. 14: Conductance Ratio for Liquid Filled Five-Spot Patterns

STAGE 2: PERFORMANCE FROM INTERFERENCE TO FILLUP

- The areal sweep efficiency required by Figure 14 can be computed as:

$$E_A = \frac{W_{if}}{V_p(\bar{S}_{wbt} - S_{wc})}$$

Eqn. 31

Summary – Stage 2 Calculations

1. Obtain values of W_{if} and W_{ii} from initial calculations.
2. Obtain value of i_w from Stage 1 calculations where $W_i = W_{ii}$.
3. Compute E_A at fillup using Equation 31.
4. Obtain the mobility ratio, M, from Step C of initial calculations.
5. Determine γ at fillup from Figure 14.
6. Compute i_{base} using Equation 30.
7. Compute water injection rate at fillup, i_{wf}, using Equation 29.
8. Compute time interval required for Stage 2 using Equation 28.

STAGE 3: PERFORMANCE FROM FILLUP TO BREAKTHROUGH

- The end of the gas fillup period marks the beginning of secondary oil production.
- It is assumed that, on a reservoir volume basis, the total oil producing rate during this stage is equal to the water injection rate.
- The water injection can be determined using Equation 29. Thus the oil producing rate in STB/D is:

$$q_o = \frac{i_w}{B_o}$$

Eqn. 32

STAGE 3: PERFORMANCE FROM FILLUP TO BREAKTHROUGH

- Cumulative oil production, N_p, since the beginning of Stage (fillup) can be computed in terms of cumulative water injected during Stage 3 as:

$$N_p = \frac{(W_i) - \left(W_{if}\right)}{B_o}$$

Eqn. 33

Summary – Stage 3 Calculations

1. Select values of W_i from W_{if} to W_{ibt} using a convenient interval.
2. Determine E_A for each value of W_i using Equation 31.
3. Determine γ for each value of W_i using Figure 14.
4. Compute i_w using Equation 29.
5. Compute average value of i_w for each interval.
6. Compute incremental and cumulative times associated with each interval.
7. Compute q_o using Equation 32.
8. Compute cumulative oil recovery using Equation 33.

STAGE 4: PERFORMANCE AFTER WATER BREAKTHROUGH

- This stage, which marks the beginning of water production, is characterized by an increasing mobility ratio, increasing areal sweep efficiency, increasing water-oil ratio, and decreasing oil producing rate.

- The producing water-oil ratio is governed by the amount of oil and water flowing from the previously swept region of the reservoir plus the amount of oil displaced as the swept area increases.

STAGE 4: PERFORMANCE AFTER WATER BREAKTHROUGH

- Oil and water production from the previously swept region is governed by fractional flow data and can be predicted using previously developed methods.

- Oil displaced from the newly swept portion of the reservoir is assumed to be that displaced by the water saturation immediately behind the stabilized zone, S_{wsz} (practically, S_{wsz} is equal to S_{wf}, the saturation at the leading edge of the front).

STAGE 4: PERFORMANCE AFTER WATER BREAKTHROUGH

- Considering a given time interval, incremental oil produced from the previously unswept portion of the reservoir, ΔN_{pu}, will depend upon the incremental increase in areal sweep efficiency, ΔE_A, the change in water saturation in the newly swept area (S_{wsz} – S_{wc}), and the pore volume, V_p.

$$\Delta N_{pu} = \Delta E_A (S_{wsz} - S_{wc}) V_p \quad \text{Eqn. 34}$$

- The term $\Delta E_A/(\Delta W_i/W_{ibt})$ is a convenient term to include in these calculations.

STAGE 4: PERFORMANCE AFTER WATER BREAKTHROUGH

- Accordingly, multiplying both sides of Equation 34 by this term results in:

$$\Delta N_{pu} = \frac{\Delta E_A}{\Delta W_i / W_{ibt}} (S_{wsz} - S_{wc}) V_p \left(\frac{\Delta W_i}{W_{ibt}} \right)$$ Eqn. 35

$$\Delta N_{pu} = \lambda (S_{wsz} - S_{wc}) V_p \left(\frac{\Delta W_i}{W_{ibt}} \right)$$ Eqn. 36

- Where:

$$\lambda = \frac{\Delta E_A}{\Delta W_i / W_{ibt}}$$ Eqn. 37

STAGE 4: PERFORMANCE AFTER WATER BREAKTHROUGH

- These calculations can be put on the basis of one total injection (or production, since injection and production rate are assumed equal at reservoir conditions) by setting ΔW_i = 1.0. Thus,

$$\Delta N_{pu} = \frac{\lambda V_p (S_{wsz} - S_{wc})}{W_{ibt}}$$ Eqn. 38

- The water injected at breakthrough is:

$$W_{ibt} = V_p E_{Abt} (\bar{S}_{wbt} - S_{wc})$$ Eqn. 39

$$\Delta N_{pu} = \lambda \frac{(S_{wsz} - S_{wc})}{E_{Abt} (\bar{S}_{wbt} - S_{wc})}$$ Eqn. 40

STAGE 4: PERFORMANCE AFTER WATER BREAKTHROUGH

- The oil produced from the previously unswept area, ΔN_{pu}, during the time that ΔW_i barrels of water are injected can be predicted Equation 40 provided that λ is known.

- However, notice that λ depends upon the increase in areal sweep efficiency, ΔE_A, which occurs as a result of injecting ΔW_i barrels of water.

- Craig, Geffen and Morse found experimentally that E_A increases linearly with the logarithm of W_i/W_{ibt}.

STAGE 4: PERFORMANCE AFTER WATER BREAKTHROUGH

- This relationship is shown graphically by Figure X1, and can be expressed as:

$$\boxed{E_A = 0.2749 ln(W_i/W_{ibt}) + E_{Abt}}$$ Eqn. 41

It follows that:

$$\frac{\Delta E_A}{\Delta W_i} \cong \frac{dE_A}{dW_i} = \frac{0.2749}{W_i}$$ Eqn. 42

- And:

$$\boxed{\lambda = 0.2749 \left(\frac{W_i}{W_{ibt}}\right)^{-1}}$$ Eqn. 43

STAGE 4: PERFORMANCE AFTER WATER BREAKTHROUGH

- Thus

$$\Delta N_{pu} = 0.2749 \frac{S_{wsz} - S_{wc}}{E_{Abt}(\bar{S}_{wbt} - S_{wc})} \left(\frac{W_i}{W_{ibt}}\right)^{-1}$$ Eqn. 43

- Incremental oil from the previously swept region, ΔN_{ps}, based upon one barrel of total production is:

$$\Delta N_{ps} = f_{02}(1 - \Delta N_{pu})$$ Eqn. 44

- Where:

f_{o2} = fraction of oil in the producing stream = 1 – f_{w2}

f_{w2} = fraction of water in the producing stream.

STAGE 4: PERFORMANCE AFTER WATER BREAKTHROUGH

- Since ΔN_{pu} is known (Equation 44), it is obvious from Equation 45 that ΔN_{ps} can be determined if f_{o2} can be defined.

- The big question at this point is – how can f_{o2} (or f_{w2}) be determined at any time after breakthrough?

- It will be recall from frontal advance theory that f_{w2} can be determined from the fractional flow curve (Figure 15) if S_{w2}, the saturation at the producing well, is known.

STAGE 4: PERFORMANCE AFTER WATER BREAKTHROUGH

- Unfortunately we do not know S_{w2}; however, we do know that S_{w2} is the tangent point on the fractional flow curve defined by a tangent line of slope.

$$\boxed{\left(\frac{df_w}{dS_w}\right)_{S_{w2}} = \frac{1}{(Q_i)_{S_{w2}}}}$$

Eqn. 45

- Where:

Q_i = pore volumes of water injected at the time in question

STAGE 4: PERFORMANCE AFTER WATER BREAKTHROUGH

- If Q_i were known, it would be possible to compute the slope of the tangent line using Equation 45. S_{w2} and f_{w2} could then be determined from the fractional flow curve (see Figure 15 in next slide).
- When expressed in water-contacted pore volumes, this becomes:

$$\boxed{Q_{ibt} = \frac{W_{ibt}}{E_{Abt}V_p} = \bar{S}_{wbt} - S_{wc}}$$

Eqn. 46

STAGE 4: PERFORMANCE AFTER WATER BREAKTHROUGH

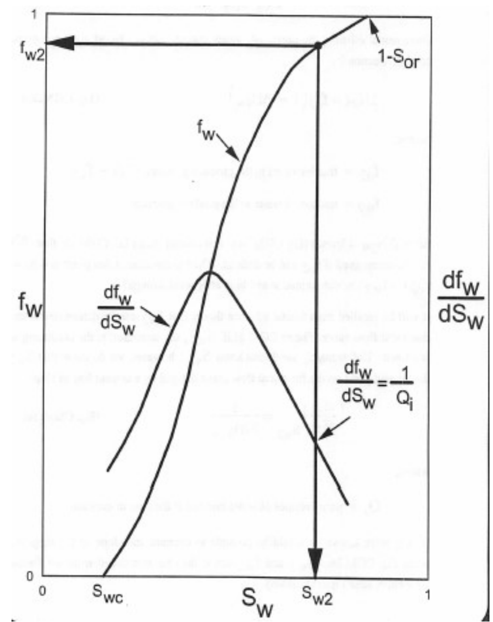


Fig. 15: Fractional flow curve demonstrating use of Q_i to determine f_{w2}.

STAGE 4: PERFORMANCE AFTER WATER BREAKTHROUGH

- The cumulative water injected at any time beyond breakthrough is equal to the water injected at breakthrough plus the additional water injected beyond breakthrough.

$$W_i = W_{ibt} + (\Delta W)_{beyond\ breakthrough} \qquad \text{Eqn. 47}$$

- Expressed in terms of water-contacted pore volumes:

$$Q_i = Q_{ibt} + (Q)_{beyond\ breakthrough} \qquad \text{Eqn. 48}$$

- The sweep efficiency after breakthrough increases as W_i increases.

STAGE 4: PERFORMANCE AFTER WATER BREAKTHROUGH

$$\frac{Q_i}{Q_{ibt}} = 1.0 + E_{Abt} \int_{1.0}^{W_i/W_{ibt}} \frac{d(W_i/W_{ibt})}{E_{Abt} + 0.2749 ln(W_i/W_{ibt})}$$ Eqn. 49

- A tabular solution of Equation 49 is present in the tables in the next slides.
- Once Q_i/Q_{ibt} is determined from this table, Q_i can be calculated and used along with the fractional flow curve to define f_{w2}.
- Finally, $f_{o2} = 1.0 - f_{w2}$ and ΔN_{ps} can be computed using Equation 44.

TABLE CGM-3

Values of Q_i/Q_{ibt} for various Values of Breakthrough Areal Sweep Efficiency

	E_{Abt}, percent									
W_i/W_{ibt}	50	51	52	53	54	55	56	57	58	59
1.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.2	1.190	1.191	1.191	1.191	1.191	1.191	1.191	1.191	1.192	1.192
1.4	1.365	1.366	1.366	1.367	1.368	1.368	1.369	1.369	1.370	1.370
1.6	1.529	1.530	1.531	1.532	1.533	1.535	1.536	1.536	1.537	1.538
1.8	1.684	1.686	1.688	1.689	1.691	1.693	1.694	1.696	1.697	1.699
2.0	1.832	1.834	1.837	1.839	1.842	1.844	1.846	1.849	1.851	1.853
2.2	1.974	1.977	1.981	1.984	1.987	1.990	1.993	1.996	1.999	2.001
2.4	2.111	2.115	2.119	2.124	2.127	2.131	2.135	2.139	2.142	2.146
2.6	2.244	2.249	2.254	2.259	2.264	2.268	2.273	2.277	2.282	2.286
2.8	2.373	2.379	2.385	2.391	2.397	2.402	2.407	2.413	2.418	2.422
3.0	2.500	2.507	2.513	2.520	2.526	2.533	2.539	2.545	2.551	2.556
3.2	2.623	2.631	2.639	2.646	2.653	2.660	2.667	2.674	2.681	2.687
3.4	2.744	2.752	2.761	2.770	2.778	2.786	2.793	2.801	2.808	2.816
3.6	2.862	2.872	2.881	2.891	2.900	2.909	2.917	2.926	2.934	2.942
3.8	2.978	2.989	3.000	3.010	3.020	3.030	3.039	3.048	3.057	3.066
4.0	3.093	3.105	3.116	3.127	3.138	3.149	3.159	3.169	3.179	3.189
4.2	3.205	3.218	3.231	3.243	3.254	3.266	3.277	3.288	3.299	3.309
4.4	3.316	3.330	3.343	3.357	3.369	3.382	3.394	3.406	3.417	3.428
4.6	3.426	3.441	3.455	3.469	3.483	3.496	3.509	3.521	3.534	3.546
4.8	3.534	3.550	3.565	3.580	3.594	3.609	3.622	3.636	3.649	
5.0	3.641	3.657	3.674	3.689	3.705	3.720	3.735			
5.2	3.746	3.764	3.781	3.798	3.814	3.830				
5.4	3.851	3.869	3.887	3.905	3.922					
5.6	3.954	3.973	3.993	4.011						
5.8	4.056	4.077	4.097							
6.0	4.157	4.179								
6.2	4.257									
Values of W_i/W_{ibt} at which E_A = 100 percent										
	6.164	5.944	5.732	5.527	5.330	5.139	4.956	4.779	4.608	4.443

TABLE CGM-3 (continued)

Values of Q_i/Q_{ibt} for various Values of Breakthrough Areal Sweep Efficiency

	E_{Abt}, percent									
W_i/W_{ibt}	60	61	62	63	64	65	66	67	68	69
1.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.2	1.192	1.192	1.192	1.192	1.192	1.192	1.193	1.193	1.193	1.193
1.4	1.371	1.371	1.371	1.372	1.372	1.373	1.373	1.373	1.374	1.374
1.6	1.539	1.540	1.541	1.542	1.543	1.543	1.544	1.545	1.546	1.546
1.8	1.700	1.702	1.703	1.704	1.706	1.707	1.708	1.709	1.710	1.711
2.0	1.855	1.857	1.859	1.861	1.862	1.864	1.866	1.868	1.869	1.871
2.2	2.044	2.007	2.009	2.012	2.014	2.016	2.019	2.021	2.023	2.025
2.4	2.149	2.152	2.155	2.158	2.161	2.164	2.167	2.170	2.173	2.175
2.6	2.290	2.294	2.298	2.301	2.305	2.308	2.312	2.315	2.319	2.322
2.8	2.427	2.432	2.436	2.441	2.445	2.449	2.453	2.457	2.461	2.465
3.0	2.562	2.567	2.572	2.577	2.582	2.587	2.592	2.597	2.601	2.606
3.2	2.693	2.700	2.705	2.711	2.717	2.723	2.728	2.733	2.738	2.744
3.4	2.823	2.830	2.836	2.843	2.849	2.855	2.862	2.867	2.873	
3.6	2.950	2.957	2.965	2.972	2.979	2.986	2.993			
3.8	3.075	3.083	3.091	3.099	3.107					
4.0	3.198	3.207	3.216	3.225						
4.2	3.319	3.329								
4.4	3.439									
Values of W_i/W_{ibt} at which E_A = 100 percent										
	4.285	4.132	3.984	3.842	3.704	3.572	3.444	3.321	3.203	3.088

TABLE CGM-3 (continued)
Values of Q_i/Q_{ibt} for various Values of Breakthrough Areal Sweep Efficiency

	E_{Abt}, percent									
W_i/W_{ibt}	70	71	72	73	74	75	76	77	78	79
1.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.2	1.193	1.193	1.193	1.193	1.193	1.193	1.193	1.194	1.194	1.194
1.4	1.374	1.375	1.375	1.375	1.376	1.376	1.376	1.377	1.377	1.377
1.6	1.547	1.548	1.548	1.549	1.550	1.550	1.551	1.551	1.552	1.552
1.8	1.713	1.714	1.715	1.716	1.717	1.718	1.719	1.720	1.720	1.721
2.0	1.872	1.874	1.875	1.877	1.878	1.880	1.881	1.882	1.884	1.885
2.2	2.027	2.029	2.031	2.033	2.035	2.037	2.039	2.040	2.042	2.044
2.4	2.178	2.180	2.183	2.185	2.188	2.190	2.192	2.195	2.197	
2.6	2.325	2.328	2.331	2.334	2.337	2.340				
2.8	2.469	2.473	2.476	2.480						
3.0	2.610	2.614								
Values of W_i/W_{ibt} at which E_A = 100 percent										
	2.978	2.872	2.769	2.670	2.575	2.483	2.394	2.309	2.226	2.147

TABLE CGM-3 (continued)
Values of Q_i/Q_{ibt} for various Values of Breakthrough Areal Sweep Efficiency

	E_{Abt}, percent									
W_i/W_{ibt}	90	91	92	93	94	95	96	97	98	99
1.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.2	1.194	1.195	1.195	1.195	1.195	1.195	1.195	1.195	1.195	1.195
1.4	1.380	1.380	1.380	1.380	1.381					
1.6	1.558									
Values of W_i/W_{ibt} at which E_A = 100 percent										
	1.439	1.387	1.338	1.290	1.244	1.199	1.157	1.115	1.075	1.037

TABLE CGM-3 (continued)
Values of Q_i/Q_{ibt} for various Values of Breakthrough Areal Sweep Efficiency

	E_{Abt}, percent									
W_i/W_{ibt}	80	81	82	83	84	85	86	87	88	89
1.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.2	1.194	1.194	1.194	1.194	1.194	1.194	1.194	1.194	1.194	1.194
1.4	1.377	1.378	1.378	1.378	1.378	1.379	1.379	1.379	1.379	1.379
1.6	1.553	1.553	1.554	1.555	1.555	1.555	1.556	1.556	1.557	1.557
1.8	1.722	1.723	1.724	1.725	1.725	1.726	1.727	1.728		
2.0	1.886	1.887	1.888	1.890						
2.2	2.045									
Values of W_i/W_{ibt} at which E_A = 100 percent										
	2.070	1.996	1.925	1.856	1.790	1.726	1.664	1.605	1.547	1.492

STAGE 4: PERFORMANCE AFTER WATER BREAKTHROUGH

- The incremental water produced on a one barrel basis, ΔW_{ps}, is computed as:

$$\Delta W_{ps} = 1 - \left(\Delta N_{ps} + \Delta N_{pu}\right) \quad \text{Eqn. 50}$$

- The water-oil ratio at reservoir pressure, WOR_p, is defined as:

$$WOR_p = \frac{1 - \Delta N_{ps} - \Delta N_{pu}}{\Delta N_{ps} + \Delta N_{pu}} \quad \text{Eqn. 51}$$

- The water-oil ratio at surface condition is

$$WOR_s = WOR_p \frac{B_o}{B_w} \quad \text{Eqn. 52}$$

STAGE 4: PERFORMANCE AFTER WATER BREAKTHROUGH

- The oil producing rate in STB/D can be determined as:

$$q_o = \frac{i_w\left(\Delta N_{ps} + \Delta N_{pu}\right)}{B_o}$$ Eqn. 53

- The water producing rate in STB/D is:

$$q_w = \frac{i_w\left(1 - \Delta N_{ps} - \Delta N_{pu}\right)}{B_w}$$ Eqn. 54

- Cumulative oil production, N_p, in STB is

$$N_p = \frac{V_p\left[E_A(\bar{S}_w - S_{wc}) - S_g\right]}{B_o}$$ Eqn. 55

STAGE 4: PERFORMANCE AFTER WATER BREAKTHROUGH

- Where:

$\bar{S}_w$= average water saturation in the reservoir at the time of interest.

$$\bar{S}_{w2} = S_{w2} + Q_i f_{o2}$$ Eqn. 56

An alternate means of computing cumulative oil recovery since the start of waterflooding is with the aid of the following Equations:

STAGE 4: PERFORMANCE AFTER WATER BREAKTHROUGH

- Incremental waterflood oil production during Stage 4, ΔN_p, is:

$$\Delta N_p = \left[\frac{(q_o)_N + (q_o)_{N-1}}{2}\right](\Delta t)$$

Eqn. 57

- And cumulative oil production since the start of waterflooding, N_p, at any time step, during Stage 4 is:

$$N_p = \left(N_p\right)_{end\ of\ stage\ 3} + \left(\sum \Delta N_p\right)_{Stage\ 4}$$

Eqn. 58

STAGE 4: PERFORMANCE AFTER WATER BREAKTHROUGH

- The cumulative water produced, W_p, in STB can be computed as:

$$W_p = \frac{W_i - N_p B_o - V_p S_g}{B_w}$$

Eqn. 59

Summary – Stage 4 Calculations

1. Select values of Wi from W_{ibt} to the economic limit and express as a ratio of W_i/W_{ibt}.
2. Compute E_A using Equation 41 for each value of W_i.
3. Determine values of Q_i/Q_{ibt} from Table CGM-3, and compute:

$$Q_i = Q_{ibt}(Q_i/Q_{ibt})$$

$$Q_i = (\bar{S}_{wbt} - S_{wc})(Q_i/Q_{ibt})$$

4. Compute the slope of the fractional flow curve, df_w/dS_w, using Equation 45.
5. Use the slope from step 4 and the fractional flow curve to determine S_{w2}.

Summary – Stage 4 Calculations

6. Using S_{w2}, determine f_{w2} from the fractional flow curve; then, $f_{o2} = 1.0 – f_{w2}$.
7. Compute $\bar{S}_w$ using Equation 56.
8. Compute λ using Equation 43.
9. Compute ΔN_{pu} using Equation 40.
10. Compute ΔN_{ps} using Equation 44.
11. Compute WORs using Equation 52
12. Compute N_p using Equation 55.

Summary – Stage 4 Calculations

13. Determine the mobility ratio, M, according to the relationship:

$$M = \frac{(k_{rw})_{\bar{S}_{w2}} * \mu_o}{(k_{ro})_{S_{wi}} * \mu_w}$$ Eqn. 60

14. Determine γ from Figure 14.
15. Compute i_w using Equation 29.
16. Compute incremental and cumulative times associated with each interval.
17. Compute q_o using Equation 53 and q_w using Equation 54.
18. Compute W_p using Equation 59.

Example 5.3

Use the Craig-Geffen-Morse method to calculate the performance of the five-spot pattern waterflood described below.

Pattern area, A = 40 acres; Oil formation volume factor, B_o = 1.056 RB/STB; Water formation volume factor, B_w = 1.0 RB/STB; Oil viscosity, μ_o = 0.853 cp; Water viscosity, μ_w = 0.375 cp; Injection pressure = 3200 psig; Average reservoir pressure at start of waterflood = 100 psig; Producing well pressure after the end of Stage 2 = 100 psig; Wellbore radius, r_w = 0.5ft; injection well skin factor = 0, production well skin factor = 0; porosity, Ø = 0.16;

Example 5.3

Permeability, $(k_o)_{Swir}$ = 20md; Formation thickness, h = 1.5ft; Oil saturation at beginning of flood, S_o = 0.70; Gas saturation at beginning of flood, S_g = 0.10; Water saturation at beginning of flood, S_{wc} = 0.20; Distance between injector and producer = 954 ft; Oil production rate at beginning of flood = 1.0 BOPD.

Relative permeability data for the reservoir and calculations to determine the fractional flow curve are given in the Table 1 (see next slide); and the fractional flow curve is presented in the figure (see two slide away). The derivative of the fractional flow curve is presented in another figure (see three slide away).

EXERCISE

- Calculate time, cumulative water injected, and cumulative produced fluids at:

A. Inference
B. Fillup
C. Water breakthrough
D. Economic limit (q_o = 5.0STB/D)

Table E.1: Relative permeability Data

Problem CGM:1 - Table 1: Relative permeability data and fractional flow calculations

S_w	k_{ro}	k_{rw}	$\frac{k_{ro}}{k_{rw}} \frac{\mu_w}{\mu_o}$	f_w
0.20	1.0000	0.0000	-	0.000
0.35	0.4120	0.0678	2.6714	0.272
0.40	0.2720	0.1040	1.1498	0.465
0.45	0.1770	0.1300	0.5986	0.626
0.50	0.1090	0.1630	0.2940	0.773
0.55	0.0627	0.2030	0.1358	0.880
0.60	0.0317	0.2540	0.0549	0.948
0.65	0.0111	0.3180	0.0153	0.985
0.70	0.0000	0.3970	0.0000	1.000

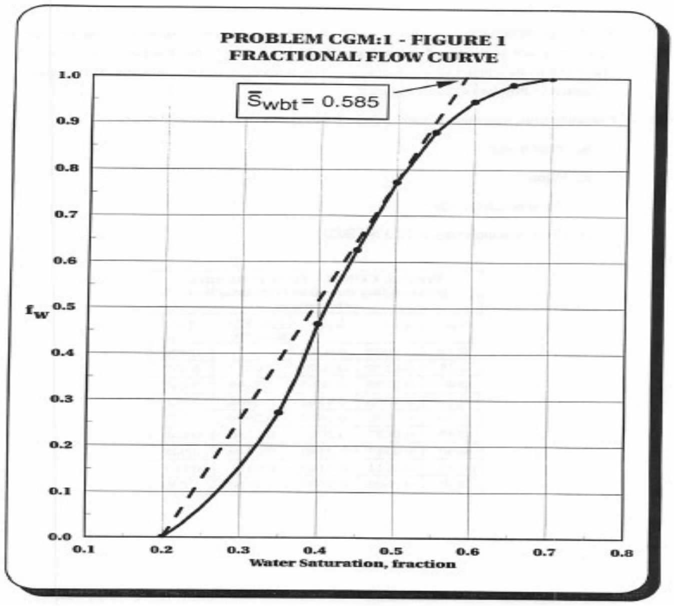


Fig. E.1: The f_w curve for Example 5.3

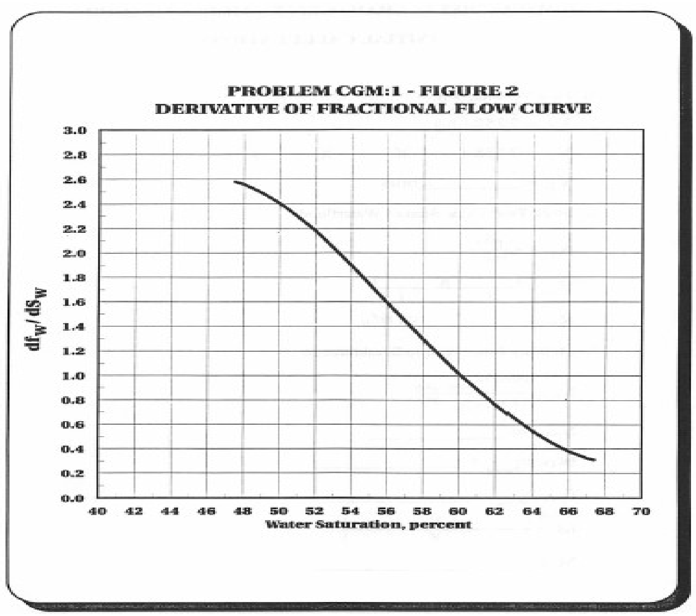


Fig. E.2: Derivative Curve for Example 5.3.

Solution

1. Pore Volume, Vp

$$V_p = 7758 * A * h * \emptyset = 7758 * 40 * 1.5 * 0.16$$
$$= 74476.8\, rb = 0.745x106\, rb$$

2. Stock tank Oil at Start of Waterflood

$$N_o = \frac{V_p S_o}{B_o} = \frac{0.745 \times 10^6 * (0.7)}{1.056} = 0.4937 \times 10^6 stb$$

Solution

3. Mobility ratio prior to Breakthrough

$$M = \frac{(k_{rw})_{\bar{S}_{wbt}}}{(k_{ro})_{S_{wc}}} \frac{\mu_o}{\mu_w} = \frac{0.2387}{1.000} \frac{0.853}{0.375} = 0.2387 * 2.2747 = \mathbf{0.54}$$

4. Sweep Efficiency at Breakthrough. From Figure 8 at Mobility of 0.54.

$$E_{Abt} = 0.78 = 78\%$$

Solution

5. Compute S_g^*

$$S_g^* = C(S_o - \bar{S}_{obt}) = C\{S_o - (1 - \bar{S}_{wbt})\}$$

$$Where, C = 0.62 + \frac{0.46836}{M} = 0.62 + \frac{0.46836}{0.54} = \mathbf{1.49}$$

$$Therefore\ S_g^* = 1.49\{0.70 - (1 - 0.585)\} = 0.4247 \approx 0.42$$

- S_g = 0.1 and $S_g^* = 0.42$. Since S_g^* is greater than S_g, CRAIG-GEFFEN-MORSE METHOD is Valid.

Solution

6. Water injection at time interference, W_{ii}.

$$r_{ei} = \sqrt{\frac{A * 43560}{\pi}} = \sqrt{\frac{40 * 43560}{3.142}} = 745ft$$

$$W_{ii} = \frac{\pi r_{ei}^2 h \emptyset S_g}{5.615} = \frac{3.142 * (745)^2 * 1.5 * 0.16 * 0.1}{5.615}$$

$$W_{ii} = 7454\ bbls$$

Solution

7. Cumulative Water injection at Gas Fillup, W_{if}.

$$W_{if} = V_p S_g = (0.745 \times 10^6) * (0.1) = \mathbf{74500}\ bbls$$

8. Water injected at Time of Water Breakthrough, W_{ibt}

$$W_{ibt} = V_p E_{Abt}(\bar{S}_{wbt} - S_{wc})$$

$$W_{ibt} = (0.745 \times 10^6) * 0.78 * (0.585 - 0.20)$$

$$W_{ibt} = 223724\ bbls$$

Solution

- The summary of stage 1 is summarized in Table 1

- Where

$$A_w = \frac{\mu_w}{k_{rw}} ln \frac{r}{r'_w}$$

$$A_o = \frac{\mu_o}{k_{ro}} ln \frac{r_e}{r}$$

$$r'_w = r_w e^{-s_i} = 0.5 * e^{-0} = 0.5 * 1 = 0.5$$

Solution

Table 1: Summary of Stage 1 Calculate

W_i,bbls	r_e^2	r_e, ft	r, ft	A_w	A_o	A_w+A_o	I_w, bbl/D	$(i_w)_{avg}$	Δt	t, Days
750	55846	236	120	8.62	0.57	9.19	73.95		10.14	10.14
1500	111692	334	170	9.16	0.57	9.74	69.82	71.88	10.43	20.58
2250	167539	409	209	9.48	0.57	10.05	67.60	68.71	10.92	31.49
3000	223385	473	241	9.70	0.57	10.28	66.12	66.86	11.22	42.71
3750	279231	528	269	9.88	0.57	10.46	65.01	65.56	11.44	54.15
4500	335077	579	295	10.02	0.57	10.60	64.13	64.57	11.62	65.76
5250	390923	625	319	10.14	0.57	10.72	63.41	63.77	11.76	77.52
6000	446770	668	341	10.25	0.57	10.82	62.79	63.10	11.89	89.41
6750	502616	709	361	10.34	0.57	10.92	62.26	62.53	11.99	101.40
7454	555037	745	380	10.42	0.57	10.99	61.82	62.04	11.35	**112.75**

Solution

STAGE 2: Performance from Interference to Fillup

1. Water injection at time interference, W_{ii}.

$$W_{ii} = 7454\ bbls$$

2. Cumulative Water injection at Gas Fillup, W_{if}

$$W_{if} = \mathbf{74500}\ bbls$$

3. Areal Sweep Efficiency at Fillup (E_A)

$$\boxed{E_A = \frac{W_{if}}{V_p(\bar{S}_{wbt} - S_{wc})} = \frac{74500}{745000(0.585 - 0.2)} = \mathbf{0.26}}$$

Solution

STAGE 2: Performance from Interference to Fillup

4. Mobility ratio = 0.54.
5. Determine Conductance ratio, γ from Figure 14. **γ = 0.82**
6. Compute i_{base} using Equation 30.

$$i_{base} = \frac{0.003541(k_o)_{S_{wir}} h\Delta p}{\mu_o\left(ln\frac{d}{r_w} - 0.619 + 0.5s_p + 0.5s_i\right)} = \frac{0.003541(20)*(1.5)*(3200)}{0.853\left(ln\frac{954}{0.5} - 0.619 + 0.5(0) + 0.5(0)\right)}$$

$$= 57.5\ bbl/day$$

Solution

STAGE 2: Performance from Interference to Fillup

7. Compute water injection rate at fillup, i_{wf}, using Equation 29.

$$i_{wf} = \gamma * i_{base} = 0.82 * 57.5 = 47\ bbls/day$$

Read the values of i_{wi} at $w_i = w_{ii}$, and time to interference t_{ii} from Table 1.

i_{wi} = ***61.82*** bbls/day and t_{ii} = ***112.75*** days.

Solution

STAGE 2: Performance from Interference to Fillup

8. Compute time interval required for Stage 2, Δt

$$(\Delta t) = \frac{(W_{if}) - (W_{ii})}{0.5(i_{wi} + i_{wf})} = \frac{74500 - 7454}{0.5(62 + 47)} = 1230\ days$$

9. Cumulative Time to Fillup = Δt + t_{ii} = 1230 + 112.75
= ***1342.75*** days.

Solution

STAGE 3: Performance from Fillup to Water Breakthrough

1. Water injected at Time of Water Breakthrough, W_{ibt}

$$W_{ibt} = 223724\ bbls$$

2. Assume several values of cumulative water injected W_{inj} between W_{if} and Wibt and calculate the areal sweep efficiency at each W_{inj} from Equation 31.

$$E_A = \frac{W_{if}}{V_p(\bar{S}_{wbt} - S_{wc})}$$

Solution

STAGE 3: Performance from Fillup to Water Breakthrough

3. Determine the conductance ratio γ for each assumed value of Winj from Figure 14.
4. Calculate the water injection rate at each W_{inj} by applying Equation 29.

$$i_w = \gamma i_{base}$$

5. Calculate Oil flowrate Q_o during this stage from:

$$Q_o = \frac{i_w}{B_o}$$

Solution

STAGE 3: Performance from Fillup to Water Breakthrough

6. Calculate cumulating oil production N_p from the following expression:

$$N_P = \frac{W_{inj} - W_{if}}{B_o}$$

Time to Fillup, t_f = ***1342.75*** days

Solution

Table 2: Performance from Fillup to Water Breakthrough Calculation

	(1) W_i ,bbls	(2) E_A	(3) γ from Figure 14	(4) i_w	(5) $(i_w)_{avg}$	(6) Δt	(7) t, days	(8) q_o, STB/d	(9) W_i - W_{if}	(10) N_p, STB
W_{if} =	74500	0.26	0.82	47.15			1342.75	44.65	0	0
	100000	0.35	0.79	45.43	46.29	550.90	1893.65	43.02	25500	24148
	125500	0.44	0.77	44.28	44.85	568.56	2462.22	41.93	51000	48295
	151000	0.53	0.75	43.13	43.70	583.52	3045.74	40.84	76500	72443
	176500	0.62	0.73	41.98	42.55	599.29	3645.04	39.75	102000	96591
	202000	0.70	0.71	40.83	41.40	615.94	4260.98	38.66	127500	120739
W_{ibt} =	223724	0.78	0.69	39.68	40.25	539.73	**4800.70**	37.57	149224	**141311**

Solution

STAGE 3: Performance from Fillup to Water Breakthrough

- From the calculation summarized in Table 2, it indicate that the time to breakthrough will occur after 4800.70 days from the start of flood with cumulative oil produced of 141,311 STB.

Solution

STAGE 4: Performance from Water Breakthrough to End of the Project

1. Assume several values for the ratio W_{inj}/W_{iBT} that correspond to the values from 1.0, 1.2, 1.4 …… etc.

2. Calculate the cumulative water injected for each assumed ratio of (W_{inj}/W_{iBT}) from:

$$W_{inj} = \left(\frac{W_{inj}}{W_{iBT}}\right) W_{iBT}$$

Solution

STAGE 4: Performance from Water Breakthrough to End of the Project

3. Calculate the areal sweep efficiency at each assumed (W_{inj}/W_{iBT}) using the Equation.

$$E_A = E_{ABT} + 0.633 log\left(\frac{W_{inj}}{W_{iBT}}\right)$$

4. Read the ratio (Q_i/Q_{iBT}) that corresponds to each value of (W_{inj}/W_{iBT}) with E_A at W_{iBT} from Table CGM-3. Note for the values of (W_{inj}/W_{iBT}) above 2.4, the ratio (Q_i/Q_{iBT}) is obtain by extrapolation.

Solution

STAGE 4: Performance from Water Breakthrough to End of the Project

5. Determine the total pore volumes of water injected by multiplying each ratio of (Q_i/Q_{iBT}) by Q_{iBT}, or:

$$Q_i = \left(\frac{Q_i}{Q_{iBT}}\right) Q_{iBT}$$

6. From the definition of Q_i, as expressed by Equation 45;

$$\left(\frac{df_w}{dS_w}\right)_{S_{w2}} = \frac{1}{(Q_i)_{S_{w2}}}$$

Solution

STAGE 4: Performance from Water Breakthrough to End of the Project.

7. Read the value of S_{w2}, that is, water saturation at the producing well, that corresponds to each slope from the plot of $\left(\frac{df_w}{dS_w}\right)_{S_{w2}}$ versus S_{w2} (see Figure E.2)
8. Calculate the reservoir water cut at the producing well f_{w2} for each S_{w2} from Figure E.1. or f_{o2} = 1-f_{w2}
9. Determine the average water saturation in the swept area $\bar{S}_{w2}$ using Equation 56. $\bar{S}_{w2} = S_{w2} + Q_i f_{o2}$

Solution

STAGE 4: Performance from Water Breakthrough to End of the Project

10. Calculate the incremental oil produced from the newly swept zone which expressed as

$$(\Delta N_P)_{New} = E\lambda = \Delta N_{pu}$$

$$E = \frac{S_{wf} - S_{wc}}{E_{ABT}(\bar{S}_{wBT} - S_{wc})} = \frac{0.5 - 0.2}{0.78(0.585 - 0.2)}$$

E = 0.999 ≈ 1.00

Solution

STAGE 4: Performance from Water Breakthrough to End of the Project

$$\lambda = 0.2749\left(\frac{W_{inj}}{W_{ibt}}\right)^{-1}$$

11. Incremental oil from the previously swept region, ΔN_{ps}, based upon one barrel of total production is:

$$\Delta N_{ps} = f_{02}(1 - \Delta N_{pu})$$

Solution

STAGE 4: Performance from Water Breakthrough to End of the Project

12.The water-oil ratio at reservoir pressure, WOR_p, is defined as:

$$WOR_p = \frac{1 - \Delta N_{ps} - \Delta N_{pu}}{\Delta N_{ps} + \Delta N_{pu}}$$

13.The water-oil ratio at surface condition is

$$WOR_s = WOR_p \frac{B_o}{B_w}$$

Solution

STAGE 4: Performance from Water Breakthrough to End of the Project

14.Compute the Cumulative oil production, N_p, in STB

$$N_p = \frac{V_p\left[E * (\bar{S}_{w2} - S_{wc}) - S_g\right]}{B_o}$$

15.Calculate krw @ $\bar{S}_{w2}$ and determine the mobility ratio M after breakthrough from

$$M = \frac{(k_{rw})_{\bar{S}_{w2}}}{(k_{ro})_{S_{wc}}} \frac{\mu_o}{\mu_w}$$

Solution

STAGE 4: Performance from Water Breakthrough to End of the Project

16. Calculate the conductance ratio γ from Figure 14.

17. Determine the water injection rate i_w from Equation 29.

$$i_w = \gamma * i_{base}$$

18. Calculate the oil and water production rate

$$q_o = \frac{i_w(\Delta N_{ps} + \Delta N_{pu})}{B_o}$$

$$q_w = \frac{i_w(1 - \Delta N_{ps} - \Delta N_{pu})}{B_w} = q_o * WOR_s$$

Solution

STAGE 4: Performance from Water Breakthrough to End of the Project

16. Calculate cumulative water Produced, W_P.

$$W_P = \frac{W_{inj} - N_P B_o - (V_P * S_g)}{B_w}$$

Table 3: Summary of Stage 4 Calculation

	(1) W_i ,bbls	(2) W_i/W_{ibt}	(3) E_A	(4) Q_i/Q_{ibt}	(5) Q_i = (4)*Q_{ibt}	df_w/ds_w = 1.0/Q_i	(7) S_{w2}	(8) f_{w2}	(9) f_{o2}	(10) $S_{w2avg.}$
W_{ibt} =	223724	1.0	0.78	1.000	0.385	2.60	0.477	0.72	0.28	0.585
	268469	1.2	0.83	1.194	0.460	2.18	0.52	0.8	0.2	0.612
	313214	1.4	0.87	1.377	0.530	1.89	0.533	0.84	0.16	0.618
	357958	1.6	0.91	1.552	0.598	1.67	0.545	0.88	0.12	0.617
	402703	1.8	0.94	1.720	0.662	1.51	0.568	0.9	0.1	0.634
	447448	2.0	0.97	1.884	0.725	1.38	0.573	0.92	0.08	0.631
	559310	2.5	1.00	2.042	0.786	1.27	0.584	0.93	0.07	0.639
	671172	3.0	1.00	2.197	0.846	1.18	0.59	0.935	0.065	0.645
	894896	4.0	1.00	2.349	0.904	1.11	0.592	0.94	0.06	0.646
	1118620	5.0	1.00	2.500	0.963	1.04	0.599	0.95	0.05	0.647
	1342344	6.0	1.00	2.642	1.017	0.98	0.604	0.955	0.045	0.650
	1789792	8.0	1.00	3.332	1.283	0.78	0.607	0.958	0.042	0.661
W_i@EL=	**2237240**	10.0	1.00	4.577	1.762	0.57	0.609	0.961	0.039	0.678
	3355860	15.0	1.00	6.223	2.396	0.42	0.612	0.962	0.038	0.703

Table 3: Summary of Stage 4 Calculation Contd.

(11) λ	(12) ΔN_{pu}	(13) ΔN_{ps}	(14) WOR_p	(15) WOR_s	(16) N_p ,STB	(17) k_{rw}@S_{w2avg}	(18) M	(19) γ	(20) i_w, bbl/D
0.275	0.2746	0.2031	1.0932	1.1545	141311	0.440	1.001	1.00	57.50
0.229	0.2289	0.1542	1.6104	1.7006	170700	0.505	1.149	1.15	66.13
0.196	0.1962	0.1286	2.0791	2.1955	186637	0.526	1.196	1.19	68.43
0.172	0.1716	0.0994	2.6894	2.8400	196739	0.521	1.185	1.18	67.85
0.153	0.1526	0.0847	3.2139	3.3938	217894	0.556	1.265	1.26	72.45
0.137	0.1373	0.0690	3.8467	4.0621	224581	0.551	1.253	1.25	71.88
0.110	0.1099	0.0623	4.8085	5.0778	249061	0.570	1.297	1.30	74.75
0.092	0.0915	0.0590	5.6405	5.9563	243381	0.578	1.315	1.32	75.90
0.069	0.0687	0.0559	7.0297	7.4234	244285	0.589	1.340	1.34	77.05
0.055	0.0549	0.0473	8.7868	9.2788	244894	0.596	1.356	1.36	78.20
0.046	0.0458	0.0429	10.2725	10.8478	246762	0.608	1.383	1.38	79.35
0.034	0.0343	0.0406	12.3536	13.0454	254597	0.618	1.406	1.41	81.08
0.027	0.0275	0.0379	14.2925	15.0929	266481	0.626	1.424	1.42	81.65
0.018	0.0183	0.0373	16.9815	17.9325	284343	0.632	1.438	1.44	82.80

Table 3: Summary of Stage 4 Calculation

(21) $(i_w)_{avg}$	(22) Δw_{inj}	(23) Δt	(24) t, days	(25) q_o, STB/D	(26) q_w, STB/D	(27) W_p, bbls
			4801	26	30	0
61.81	44744.8	724	5525	24	41	13710
67.28	89489.6	1330	6855	21	46	41624
68.14	134234	1970	8825	17	49	75702
70.15	178979	2551	11376	16	55	98107
72.16	223724	3100	14476	14	57	135791
73.31	335586	4577	19054	12	62	221801
75.33	447448	5940	24994	11	64	339662
76.48	671172	8776	33771	9	67	562431
77.63	894896	11528	45299	8	70	785512
78.78	1118620	14200	59499	7	72	1007263
80.21	1566068	19524	79023	6	75	1446438
81.36	2013516	24747	103771	**5 (Economic Limit)**	**76**	**1881336**
82.23	3132136	38092	141863	4	78	2981093

Solution

Note:

- Column 4 (Q_i/Q_{iBT}) of Table 3 is obtain from Table CGM-3. The values are obtain by reading the corresponding value of E_A = 0.78 and the various value of W_i/W_{iBT}.
- Column 7 (S_{w2}) of Table 3 is obtain from Figure E.2.
- Column 8 (f_{w2}) of Table 3 is obtain from Figure E.1.
- Column 17 ($k_{rw}@\bar{S}_{w2}$) of Table 3 is computed from Table E.1.
- Column 19 (γ) of Table 3 is read from Figure 14.
- W_i@EL = Water injection of Economic Limit flowrate of 5 STB/D

Displacement Mechanism - Higgins-Leighton Method

- This method basically applies the displacement theory of Buckley and Leverett to any flooding pattern for which the iso-potential and flow streamlines are available. It is more complicated to use than previously discussed methods and requires the use of a computer.

- To apply the method, the reservoir is divided into flow channels based on flow streamlines as determined from potentiometric model studies, or other methods.

Displacement Mechanism - Higgins-Leighton Method

- Each stream channel is subdivided into equal volume cells and assuming unidirectional flow, a Buckley-Leverett type material balance on each cell yields the rate of water accumulation and oil displacement from which saturation gradients can be determined.

- From individually calculated flow resistances for each cell, and the total pressure drop between wells, instantaneous oil and water flow rates can be computed.

Displacement Mechanism - Higgins-Leighton Method

- Data required for the method are relative permeabilities, viscosities, absolute permeability, layer thicknesses, applied differential pressure, and the iso-potential and streamline configuration for the particular well pattern studied.

- A major limitation of the method is its dependence on the resistance factors (shape factor) which must be known for each cell to properly account for sweep variations induced by the different cell geometries.
- These resistance factors have been presented in the literature for many commonly used flooding patterns.

Displacement Mechanism - Higgins-Leighton Method

- A major assumption in setting up the cell models is that stream channels determined using unit mobility can be applied to any system.

- This method has given very good agreement in matching experimental and field waterflood results.

Numerical Models

- A complete solution to the multiphase, multidimensional partial differential equations which govern fluid flow in a porous and permeable media is probably the best prediction model that we can use.

- Such a model can account for directional variation in fluid and rock properties, layering effects, crossflow, gravity, capillary pressure, irregular boundaries, individual well behavior, etc.

Numerical Models

- The effects of varying injection patterns, well locations, injection and producing rates, plus many other factors, can be studied which were not possible using previously discussed models.

- In general, mathematical models are very expensive to develop and run. Furthermore, extensive amounts of data are generally required to take advantage of the flexibility and accuracy afforded by these models. Many studies simply do not justify the use of such a model.

Empirical Models

- Several models are available which attempt to relate waterflood recovery to pertinent project variables based on the past performance of waterfloods.
- Although these models can generally give answers that are reasonably correct, they should only be used to make a cursory analysis of a project.
- They should certainly not be used as the basis for the final design of a waterflood.

EXERCISE

QUESTION 1:

Solve Example 5.2 but define the mobility ratio in Step 2 as the **Shock Front Mobility Ratio.** Make different performance plots from the results to show the difference in the prediction outcomes

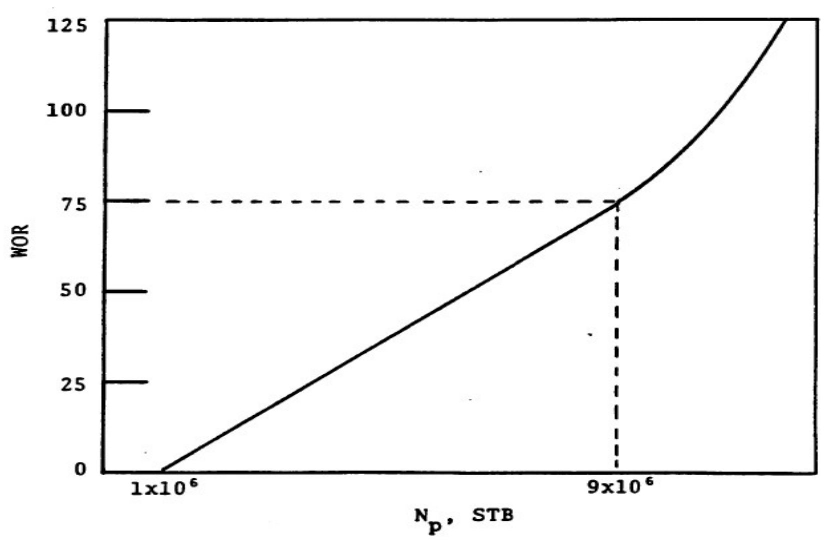
EXERCISE

QUESTION 2:

Shown (next slide) is a plot of producing (surface) water-oil ratio versus cumulative oil production obtained from Dykstra-Parsons calculations for an oil reservoir. Other reservoir data are: B_o = 1.30 RB/STB; B_w = 1.0 RB/STB; i_w = 50,000 RB/D; S_{gi} = 12%; V_p = 30.0×10^6 RB.

a. Calculate the cumulative water production to be expected while producing 9×10^6 STB's of oil.
b. How long will it take for this production to occur?
c. How long will it take for water breakthrough to occur?

EXERCISE



EXERCISE

Question 3

An oil reservoir is under consideration for further development by water injection.

The relative permeability data are given below:

S_w	0.10	0.20	0.30	0.40	0.70	0.85
k_{rw}	0.00	0.00	0.02	0.05	0.35	0.60
k_{ro}	1.00	0.93	0.60	0.35	0.05	0.00

- Calculate and plot the reservoir performance during the following stages:
- Start—interference
- Interference—fill-up
- Fill-up—breakthrough
- After breakthrough

EXERCISE

Question 3

Additional data are given below:

Flood pattern	= Five-spot
Absolute permeability	= 70 md
Thickness	= 20 ft
Porosity	= 15%
S_{gi}	= 15%
S_{wi}	= 20%
μ_o	= 3.1 cp
μ_w	= 1.0 cp
B_o	= 1.25 bbl/STB
B_w	= 1.01 bbl/STB
Pattern area	= 40 acres
r_w	= 1.0 ft
$\Delta p = (P_{inj} - P_{wf})$	= 1000 psi (constant)

EXERCISE

Question 4

The following core analysis is available on a reservoir that is being considered for a waterflooding project:

Sample	h, ft	k, md
1	2	14
2	2	39
3	1	108
4	2	77
5	2	28
6	1	212
7	1	151
8	3	10
9	2	20
10	3	55

EXERCISE

Question 3

Other data:

i_w = 1000 bbl/day; μ_o = 9.0 cp; μ_w = 0.95 cp; Mobility ratio = 4.73

N_s = 6 MMSTB; B_o = 1.02 bbl/STB; B_w = 1.00 bbl/STB; S_{wi} = 0.2; S_{oi} = 0.8.

Using the simplified Dykstra–Parsons method, determine the following recovery parameters as a function of time:

a) Q_o
b) Q_w
c) WOR
d) N_p
e) W_p

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