PHYSICAL REVIEW D VOLUME 15, NUMBER 1 1 JANUARY 1977

Energy dependence and scaling of the spin-correlation and polarization parameters in elastic proton-proton scattering*

Loyal Durand†
Theoretical Division, Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico 87545

Francis Halzen
Department of Physics, University of Wisconsin, Madison, Wisconsin 53706
(Received 10 August 1976)

We point out that the rates of decrease of the polarization and spin-correlation parameters P and C_{nn} in proton-proton scattering at momenta from 2 to 12 GeV/c are correlated in a manner such that the ratio C_{nn}/P^2 is essentially independent of energy at fixed momentum transfer. This approximate scaling law follows from a model in which the spin dependence in high-energy proton-proton scattering is associated entirely with a weak spin-orbit interaction. Various tests of the model are discussed.

In a recent letter, Miller *et al.*[1] presented measurements of the polarization and spin-correlation parameters P and C_{nn} in elastic proton-proton scattering at 2, 3, 4, and 6 GeV/c. These measurements have since been extended to 12 GeV/c.[2] Both P and C_{nn} decrease rapidly with energy over the above range, C_{nn} more rapidly than P. We wish to point out that the rates of decrease of P and C_{nn} are strongly correlated in the manner predicted by a model in which the spin dependence in high-energy proton-proton scattering arises entirely from a weak spin-orbit interaction.[3] In particular, the ratio C_{nn}/P^2 is essentially independent of the energy, and can be approximated by a universal function of the momentum transfer,

$$C_{nn}(s,t)/P^2(s,t) \approx f(t)\,. \tag{1}$$

This result, to be derived below, is not predicted by standard Regge-exchange models for proton-proton scattering. The rapid decrease of C_{nn} with increasing energy has in fact presented a serious problem for such models.[4]

We compare the scaling rule (1) with the data on P and C_{nn} in Figs. 1 and 2. Figure 1 shows that the scaling relation is well satisfied for incident momenta of 2–12 GeV/c. In Fig. 2, we have used an eyeball fit to $f(t)$, $f(t) \approx 2.3e^{0.7|t|}$, and the polarization data at each energy to predict the spin-correlation parameter C_{nn}. The predictions of a Regge-exchange model[4] fitted to the polarization data over the entire energy range and to the data on C_{nn} at 3, 4, and 6 GeV/c are included for comparison.

The consequences of the assumption that the spin dependence in high-energy proton-proton scattering arises entirely from a weak spin-orbit interaction were explored in a recent paper by the present authors.[3] This assumption can be motivated several ways. The existence of spin-orbit-type forces is certainly to be expected in a theory of spin-$\frac{1}{2}$ fermions interacting by the exchange of vector bosons (quark-gluon model). In addition, the energy dependence of a spin-orbit interaction is stronger than that of spin-spin or tensor interactions with the same intrinsic strength since $\vec{L}\cdot\vec{S}$ grows linearly with the momentum at fixed impact parameter, while the spin-spin and tensor operators are constant. We might therefore expect the spin-orbit interaction to be the most important at high energies.

The proton-proton scattering matrix can be written as an operator in the two-particle spin space,

$$\mathfrak{M} = \mathfrak{M}_0 + \mathfrak{M}_1(\vec{\sigma}_1+\vec{\sigma}_2)\cdot\hat{n} + \mathfrak{M}_2\vec{\sigma}_1\cdot\hat{n}\,\vec{\sigma}_2\cdot\hat{n} + \mathfrak{M}_3\vec{\sigma}_1\cdot\hat{q}\,\vec{\sigma}_2\cdot\hat{q} + \mathfrak{M}_4\vec{\sigma}_1\cdot\hat{l}\,\vec{\sigma}_2\cdot\hat{l}\,, \tag{2}$$

where $\vec{\sigma}_1$ and $\vec{\sigma}_2$ are the Pauli matrices for particles 1 and 2. The unit vectors $\hat{l}$, $\hat{q}$, and $\hat{n}$ are

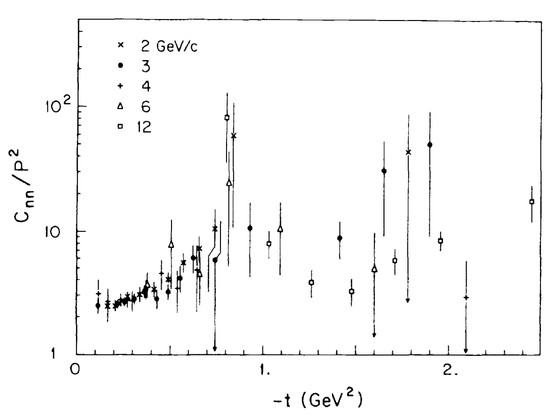

FIG. 1. The ratio C_{nn}/P^2 for the proton-proton scattering data of Refs. 1 and 2 at 2–12 GeV/c.

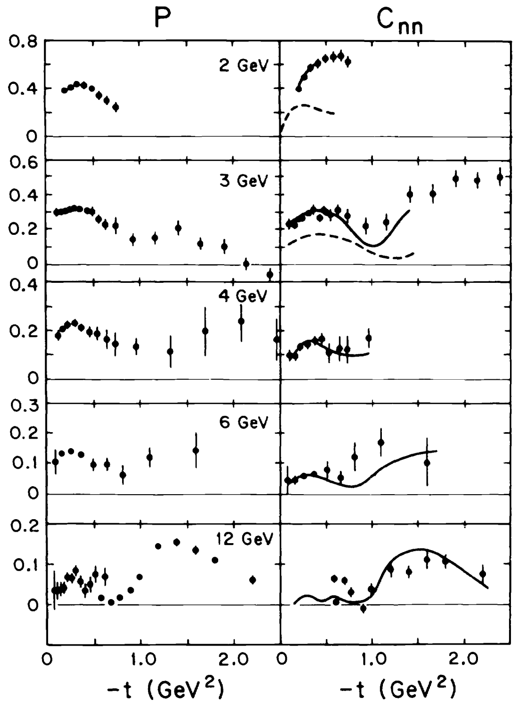

FIG. 2. Comparison of the data on C_{nn} at 2–12 GeV/c with the predictions obtained using the polarization data of Refs. 1 and 2 and the scaling law $C_{nn} = 2.3e^{0.71|t|}P^2$ (solid curves). The dashed curves give the predictions of the "super Regge" model of Ref. 4, which was fitted to the polarization data over the entire energy range and to C_{nn} at 3, 4, and 6 GeV/c.

directed along the mean direction of the incident and scattered particles, the direction of the momentum transfer, and the normal to the scattering plane.[5] The amplitude $\mathcal{M}_0$ describes the dominant spin-independent diffractive scattering. If the only spin-dependent interaction is a weak spin-orbit force with a characteristic strength χ_{LS}, $\mathcal{M}_1$ is of order χ_{LS}, $\mathcal{M}_2$ and $\mathcal{M}_3$ are of order χ_{LS}^2, and $\mathcal{M}_4$ is of order χ_{LS}^4.[6] The spin-independent amplitude $\mathcal{M}_0$ depends on χ_{LS} only in order χ_{LS}^2.

The differential-scattering cross section σ and the polarization and spin-correlation parameters are given in this model to order χ_{LS}^2 by

$$\sigma \equiv \frac{d\sigma}{d\Omega} = |\mathcal{M}_0|^2 + 2|\mathcal{M}_1|^2 ,$$

$$P\sigma = 2\,\mathrm{Re}\mathcal{M}_0^*\mathcal{M}_1 \propto \chi_{LS} , \tag{3}$$

$$C_{nn}\sigma = 2|\mathcal{M}_1|^2 + 2\,\mathrm{Re}\mathcal{M}_0^*\mathcal{M}_2 \propto \chi_{LS}^2 .$$

The approximate scaling relation between C_{nn} and P noted above follows from the observation that $\mathcal{M}_0$ varies slowly with energy for moderate values of $|t|$, and that any overall (factorizable) energy dependence of the spin-orbit interaction divides out in the ratio C_{nn}/P^2.[7] The relation may fail near zeros of $\mathcal{M}_0$ or $\mathcal{M}_1$ where small contributions not of the spin-orbit type can be important. It need not hold at all in models in which the magnitude and energy dependence of $\mathcal{M}_2$, $\mathcal{M}_3$, and $\mathcal{M}_4$ are determined primarily by spin-spin and tensor forces.

A number of testable predictions of the spin-orbit-interaction model for the spin-correlation, spin-transfer, and depolarization parameters C_{ij}, K_{ij}, and D_{ij} were derived in Ref. 3.[8] If the only significant spin-dependent interaction is a weak spin-orbit interaction, K_{ij} must be equal to C_{ij} for all choices of $i,j = l,q,n$. In addition, there are no longitudinal-spin correlations, and

$$C_{ll} \approx K_{ll} \approx C_{lq} \approx K_{lq} \approx 0 . \tag{4}$$

Measurement of a nonzero value of any of these parameters or a difference between K_{ij} and C_{ij} would provide direct evidence for the presence of spin-spin or tensor-type interactions. The spin-orbit model predicts, in addition, that

$$D_{nn} \approx 1, \quad D_{ll} \approx D_{qq} \approx 1 - \tfrac{1}{2}P^2, \quad D_{lq} \approx 0 . \tag{5}$$

Significant deviations from these predictions would again provide direct evidence for non-spin-orbit interactions.

There are also a number of predictions which depend on some information about the impact-parameter profiles of the central and spin-orbit interactions. Thus, in a variety of models[3] (for example, those in which the spin-orbit interaction is somewhat peripheral), the amplitudes $\mathcal{M}_2$ and $\mathcal{M}_3$ are related to $\mathcal{M}_1$ by[9] $\mathcal{M}_2 \approx \chi_{LS}\mathcal{M}_1'$ and $\mathcal{M}_3 \approx (\chi_{LS}/q)\mathcal{M}_1$, where primes denote derivatives with respect to $q = (-t)^{1/2}$. If χ_{LS} is nearly real, as expected for a real spin-orbit interaction, these approximations imply that

$$C_{qq}\sigma \approx K_{qq}\sigma = 2\,\mathrm{Re}\mathcal{M}_0^*\mathcal{M}_3 \approx \frac{\chi_{LS}}{q} P\sigma . \tag{6}$$

For realistic models of proton-proton scattering, $\mathcal{M}_1$ is a rapidly decreasing function of q at intermediate momentum transfers, and $|\mathcal{M}_1'| \gg |\mathcal{M}_1/q|$. Hence, $|\mathcal{M}_2| \gg |\mathcal{M}_3|$, and $C_{nn} \gg C_{qq}, K_{nn} \gg K_{qq}$. The vanishing of $\mathcal{M}_4$ to order χ_{LS}^4, and the smallness of $\mathcal{M}_3$ relative to $\mathcal{M}_2$, correspond in the language of t-channel-exchange models to the suppression of unnatural-parity exchange. However, the suppression is dynamical, and need not be put in by hand.

A final approximation, valid in some (but not all) reasonable models,[3] implies that $\mathcal{M}_1 \approx \chi_{LS}\mathcal{M}_0'$ and hence that

$$P\sigma \approx \chi_{LS}\sigma' . \tag{7}$$

$$C_{nn}\sigma \approx K_{nn}\sigma \approx \chi_{LS}{}^2\sigma'' . \quad (8)$$

Relations (7) and (8) are fairly well satisfied by the data on P and C_{nn} around 6 GeV/c.[3] They appear also to be qualitatively correct at 12 GeV/c, but cannot be checked in detail because of the uncertainties involved in the calculation of the derivatives of the cross section using present data.

We conclude that present data on the spin dependence of proton-proton scattering are consistent with the assumption that the spin dependence arises almost entirely from a weak spin-orbit interaction. This assumption is directly testable. In particular, the results in Eqs. (4) and (5) depend only on the form of the spin-orbit interaction and the fact that it is weak. If our conclusions are supported by the results of future experiments, they will provide important constraints on t-channel-exchange models for proton-proton scattering.

One of the authors (L.D.) would like to thank the Aspen Center for Physics for the hospitality accorded him while parts of this paper were written.

---

*Work supported in part by the University of Wisconsin Research Committee and funds granted by the Wisconsin Alumni Research Foundation, and in part by the Energy Research and Development Administration under Contract No. E(11-1)-881, COO-565.

†Permanent address: Department of Physics, University of Wisconsin, Madison, Wisconsin 53706.

[1]D. Miller, C. Wilson, R. Giese, D. Hill, K. Nield, P. Rynes, B. Sandler, and A. Yokosawa, Phys. Rev. Lett. 36, 763 (1976).

[2]K. Abe *et al*, Phys. Lett. 63B, 239 (1976).

[3]L. Durand and F. Halzen, Nucl. Phys. B104, 317 (1976).

[4]R. D. Field and P. R. Stevens, California Institute of Technology Report No. CALT-68-524, 1976 (unpublished). The curves shown in Fig. 2 correspond to the "super Regge" model. This includes pole terms and cutlike corrections for P, f, ω, ρ, A_2, π, and B exchanges, plus extra low-lying [$\alpha(0) \sim -0.5$] poles $\tilde{f}$ and $\tilde{\omega}$.

[5]The unit vectors are defined in terms of the momenta $\vec{p}$ and $\vec{p}\,'$ of the incident and scattered protons in the center-of-mass system by

$$\hat{l} = \frac{\vec{p}\,' + \vec{p}}{|\vec{p}\,' + \vec{p}|}, \quad \hat{q} = \frac{\vec{p}\,' - \vec{p}}{|\vec{p}\,' - \vec{p}|}, \quad \hat{n} = \frac{\vec{p} \times \vec{p}\,'}{|\vec{p} \times \vec{p}\,'|} .$$

[6]These relations are easily derived from the eikonal representation for the scattering matrix,

$$\mathfrak{M} = \frac{ip}{2\pi} \int (1 - e^{-\chi(\vec{b})}) e^{-i\vec{q}\cdot\vec{b}} d^2b .$$

The eikonal operator $\chi(\vec{b})$ is given in our model by the sum of a spin-independent (or central) term and a spin-orbit term,

$$\chi(\vec{b}) = \chi_c(b) - i\chi_{LS}(b)(\vec{\sigma}_1 + \vec{\sigma}_2)\cdot(\vec{b}\times\hat{l}) .$$

Expansion of the exponential in a power series in $\chi_{LS}(b)$ gives the results noted above.

[7]The energy dependence of the eikonal function $\chi_{LS}(b)$ need not be exactly factorizable for the scaling relation to hold approximately. It is sufficient for our purposes if the energy dependence of $\chi_{LS}(b)$ is not too different at different impact parameters. The quantities $|\mathfrak{M}_1|^2$ and $|\mathfrak{M}_2|^2$ will then have approximately the same energy dependence even though $\mathfrak{M}_1$ and $\mathfrak{M}_2$ may be most sensitive to somewhat different ranges of impact parameters.

[8]The spin-correlation, spin-transfer, and depolarization parameters C_{ij}, K_{ij}, and D_{ij} are defined by

$$C_{ij}\sigma = \tfrac{1}{4}\,\mathrm{Tr}\,\mathfrak{M}\sigma_{2,i}\sigma_{1,j}\mathfrak{M}^\dagger ,$$

$$K_{ij}\sigma = \tfrac{1}{4}\,\mathrm{Tr}\,\sigma_{2,i}\mathfrak{M}\sigma_{1,j}\mathfrak{M}^\dagger ,$$

$$D_{ij}\sigma = \tfrac{1}{4}\,\mathrm{Tr}\,\sigma_{1,i}\mathfrak{M}\sigma_{1,j}\mathfrak{M}^\dagger ,$$

where the trace is over the spin indices, and the differential scattering cross section is given by

$$\sigma = \tfrac{1}{4}\,\mathrm{Tr}\,\mathfrak{M}\mathfrak{M}^\dagger .$$

[9]The parameter χ_{LS} in these equations is an effective or average value of the spin-orbit eikonal function. See Ref. 3 for details.