

energyRt model framework

December 1, 2022

Sets

Name	Alias	Description
weather	<i>wth1, wth1p, wth1e, wth1n</i>	weather
tech	<i>t, tp, te, tn</i>	technology
sup	<i>s1, slp, sle, sln</i>	supply
dem	<i>d, dp, de, dn</i>	demand
stg	<i>st1, st1p, st1e, st1n</i>	storage
expp	<i>e, ep, ee, en</i>	export to the rest of the world (ROW)
imp	<i>i, ip, ie, in</i>	import from the rest of the world
trade	<i>t1, t1p, t1e, t1n</i>	trade between regions
group	<i>g, gp, ge, gn</i>	group of input or output commodities in technology
comm	<i>c, cp, ce, cn</i>	commodity
region	<i>r, rp, re, rn</i>	region
year	<i>y, yp, ye, yn</i>	year
slice	<i>s, sp, se, sn</i>	time slices

Parameters

Name	Alias	Description
ordYear(year)	<i>ordYear_y</i>	ord year for GLPK-MathProg
cardYear(year)	<i>cardYear_y</i>	card year for GLPK-MathProg
pPeriodLen(year)	<i>pPeriodLen_y</i>	Length of perios for milestone year

pSliceShare(slice)	$pSliceShare_s$	Share of slice
pAggregateFactor(comm, comm)	$pAggregateFactor_{c,c}$	Aggregation factor of commodities
pTechOlife(tech, region)	$pTechOlife_{t,r}$	Operational life of technologies
pTechCinp2ginp(tech, comm, region, year, slice)	$pTechCinp2ginp_{t,c,r,y,s}$	Commodity input to group input
pTechGinp2use(tech, group, region, year, slice)	$pTechGinp2use_{t,g,r,y,s}$	Group input into use
pTechCinp2use(tech, comm, region, year, slice)	$pTechCinp2use_{t,c,r,y,s}$	Commodity input to use
pTechUse2cact(tech, comm, region, year, slice)	$pTechUse2cact_{t,c,r,y,s}$	Use to commodity activity
pTechCact2cout(tech, comm, region, year, slice)	$pTechCact2cout_{t,c,r,y,s}$	Commodity activity to commodity output
pTechEmisComm(tech, comm)	$pTechEmisComm_{t,c}$	Combustion factor for input commodity (from 0 to 1)
pTechAct2AInp(tech, comm, region, year, slice)	$pTechAct2AInp_{t,c,r,y,s}$	Activity to aux-commodity input
pTechCap2AInp(tech, comm, region, year, slice)	$pTechCap2AInp_{t,c,r,y,s}$	Capacity to aux-commodity input
pTechNCap2AInp(tech, comm, region, year, slice)	$pTechNCap2AInp_{t,c,r,y,s}$	New capacity to aux-commodity input
pTechCinp2AInp(tech, comm, comm, region, year, slice)	$pTechCinp2AInp_{t,c,c,r,y,s}$	Commodity input to aux-commodity input
pTechCout2AInp(tech, comm, comm, region, year, slice)	$pTechCout2AInp_{t,c,c,r,y,s}$	Commodity output to aux-commodity input
pTechAct2AOut(tech, comm, region, year, slice)	$pTechAct2AOut_{t,c,r,y,s}$	Activity to aux-commodity output
pTechCap2AOut(tech, comm, region, year, slice)	$pTechCap2AOut_{t,c,r,y,s}$	Capacity to aux-commodity output
pTechNCap2AOut(tech, comm, region, year, slice)	$pTechNCap2AOut_{t,c,r,y,s}$	New capacity to aux-commodity output
pTechCinp2AOut(tech, comm, comm, region, year, slice)	$pTechCinp2AOut_{t,c,c,r,y,s}$	Commodity to aux-commodity output
pTechCout2AOut(tech, comm, comm, region, year, slice)	$pTechCout2AOut_{t,c,c,r,y,s}$	Commodity-output to aux-commodity input
pTechFixom(tech, region, year)	$pTechFixom_{t,r,y}$	Fixed Operating and maintenance (O&M) costs (per unit of capacity)
pTechVarom(tech, region, year, slice)	$pTechVarom_{t,r,y,s}$	Variable O&M costs (per unit of acticity)
pTechInvcost(tech, region, year)	$pTechInvcost_{t,r,y}$	Investment costs (per unit of capacity)
pTechEac(tech, region, year)	$pTechEac_{t,r,y}$	Equivalent annual (investment) cost
pTechShareLo(tech, comm, region, year, slice)	$pTechShareLo_{t,c,r,y,s}$	Lower bound of the share of the commodity in total group input or output
pTechShareUp(tech, comm, region, year, slice)	$pTechShareUp_{t,c,r,y,s}$	Upper bound of the share of the commodity in total group input or output
pTechAfLo(tech, region, year, slice)	$pTechAfLo_{t,r,y,s}$	Lower bound on availability factor by slices
pTechAfUp(tech, region, year, slice)	$pTechAfUp_{t,r,y,s}$	Upper bound on availability factor by slices
pTechRampUp(tech, region, year, slice)	$pTechRampUp_{t,r,y,s}$	Ramp Up on availability factor
pTechRampDown(tech, region, year, slice)	$pTechRampDown_{t,r,y,s}$	Ramp Down on availability
pTechAfsLo(tech, region, year, slice)	$pTechAfsLo_{t,r,y,s}$	Lower bound on availability factor by groups of slices
pTechAfsUp(tech, region, year, slice)	$pTechAfsUp_{t,r,y,s}$	Upper bound on availability factor by groups of slices
pTechAfcLo(tech, comm, region, year, slice)	$pTechAfcLo_{t,c,r,y,s}$	Lower bound for commodity output

pTechAfcUp(tech, comm, region, year, slice)	$pTechAfcUp_{t,c,r,y,s}$	Upper bound for commodity output
pTechStock(tech, region, year)	$pTechStock_{t,r,y}$	Technology capacity stock
pTechCap2act(tech)	$pTechCap2act_t$	Technology capacity units to activity units conversion factor
pTechCvarom(tech, comm, region, year, slice)	$pTechCvarom_{t,c,r,y,s}$	Commodity-specific variable costs (per unit of commodity input or output)
pTechAvarom(tech, comm, region, year, slice)	$pTechAvarom_{t,c,r,y,s}$	Auxiliary Commodity-specific variable costs (per unit of commodity input or output)
pDiscount(region, year)	$pDiscount_{r,y}$	Discount rate (can be region and year specific)
pDiscountFactor(region, year)	$pDiscountFactor_{r,y}$	Discount factor (cumulative)
pDiscountFactorMileStone(region, year)	$pDiscountFactorMileStone_{r,y}$	Discount factor (cumulative) sum for MileStone
pSupCost(sup, comm, region, year, slice)	$pSupCost_{s1,c,r,y,s}$	Costs of supply (price per unit)
pSupAvaUp(sup, comm, region, year, slice)	$pSupAvaUp_{s1,c,r,y,s}$	Upper bound for supply
pSupAvaLo(sup, comm, region, year, slice)	$pSupAvaLo_{s1,c,r,y,s}$	Lower bound for supply
pSupReserveUp(sup, comm, region)	$pSupReserveUp_{s1,c,r}$	Total supply reserve by region Up
pSupReserveLo(sup, comm, region)	$pSupReserveLo_{s1,c,r}$	Total supply reserve by region Lo
pDemand(dem, comm, region, year, slice)	$pDemand_{d,c,r,y,s}$	Exogenous demand
pEmissionFactor(comm, comm)	$pEmissionFactor_{c,c}$	Emission factor
pDummyImportCost(comm, region, year, slice)	$pDummyImportCost_{c,r,y,s}$	Dummy costs parameters (for debugging)
pDummyExportCost(comm, region, year, slice)	$pDummyExportCost_{c,r,y,s}$	Dummy costs parameters (for debugging)
pTaxCostInp(comm, region, year, slice)	$pTaxCostInp_{c,r,y,s}$	Commodity taxes for input
pTaxCostOut(comm, region, year, slice)	$pTaxCostOut_{c,r,y,s}$	Commodity taxes for output
pTaxCostBal(comm, region, year, slice)	$pTaxCostBal_{c,r,y,s}$	Commodity taxes for balance
pSubCostInp(comm, region, year, slice)	$pSubCostInp_{c,r,y,s}$	Commodity subsidies for input
pSubCostOut(comm, region, year, slice)	$pSubCostOut_{c,r,y,s}$	Commodity subsidies for output
pSubCostBal(comm, region, year, slice)	$pSubCostBal_{c,r,y,s}$	Commodity subsidies for balance
pStorageInpEff(stg, comm, region, year, slice)	$pStorageInpEff_{st1,c,r,y,s}$	Storage input efficiency
pStorageOutEff(stg, comm, region, year, slice)	$pStorageOutEff_{st1,c,r,y,s}$	Storage output efficiency
pStorageStgEff(stg, comm, region, year, slice)	$pStorageStgEff_{st1,c,r,y,s}$	Storage time-efficiency (annual)
pStorageStock(stg, region, year)	$pStorageStock_{st1,r,y}$	Storage capacity stock
pStorageOlife(stg, region)	$pStorageOlife_{st1,r}$	Storage operational life
pStorageCostStore(stg, region, year, slice)	$pStorageCostStore_{st1,r,y,s}$	Storing costs per stored amount (annual)
pStorageCostInp(stg, region, year, slice)	$pStorageCostInp_{st1,r,y,s}$	Storage input costs
pStorageCostOut(stg, region, year, slice)	$pStorageCostOut_{st1,r,y,s}$	Storage output costs
pStorageFixom(stg, region, year)	$pStorageFixom_{st1,r,y}$	Storage fixed O&M costs

pStorageInvcost(stg, region, year)	$pStorageInvcost_{st1,r,y}$	Storage investment costs
pStorageEac(stg, region, year)	$pStorageEac_{st1,r,y}$	Storage equivalent annual costs
pStorageCap2stg(stg)	$pStorageCap2stg_{st1}$	Storage capacity units to activity units conversion factor
pStorageAfLo(stg, region, year, slice)	$pStorageAfLo_{st1,r,y,s}$	Storage availability factor lower bound (minimum charge level)
pStorageAfUp(stg, region, year, slice)	$pStorageAfUp_{st1,r,y,s}$	Storage availability factor upper bound (maximum charge level)
pStorageCinpUp(stg, comm, region, year, slice)	$pStorageCinpUp_{st1,c,r,y,s}$	Storage input upper bound
pStorageCinpLo(stg, comm, region, year, slice)	$pStorageCinpLo_{st1,c,r,y,s}$	Storage input lower bound
pStorageCoutUp(stg, comm, region, year, slice)	$pStorageCoutUp_{st1,c,r,y,s}$	Storage output upper bound
pStorageCoutLo(stg, comm, region, year, slice)	$pStorageCoutLo_{st1,c,r,y,s}$	Storage output lower bound
pStorageNCap2Stg(stg, comm, region, year, slice)	$pStorageNCap2Stg_{st1,c,r,y,s}$	Initial storage charge level for new investment
pStorageCharge(stg, comm, region, year, slice)	$pStorageCharge_{st1,c,r,y,s}$	Initial storage charge level for stock
pStorageStg2AInp(stg, comm, region, year, slice)	$pStorageStg2AInp_{st1,c,r,y,s}$	Storage accumulated volume to auxiliary input
pStorageStg2AOut(stg, comm, region, year, slice)	$pStorageStg2AOut_{st1,c,r,y,s}$	Storage accumulated volume output
pStorageCinp2AInp(stg, comm, region, year, slice)	$pStorageCinp2AInp_{st1,c,r,y,s}$	Storage input to auxiliary input
pStorageCinp2AOut(stg, comm, region, year, slice)	$pStorageCinp2AOut_{st1,c,r,y,s}$	Storage input to auxiliary output
pStorageCout2AInp(stg, comm, region, year, slice)	$pStorageCout2AInp_{st1,c,r,y,s}$	Storage output to auxiliary input
pStorageCout2AOut(stg, comm, region, year, slice)	$pStorageCout2AOut_{st1,c,r,y,s}$	Storage output to auxiliary output
pStorageCap2AInp(stg, comm, region, year, slice)	$pStorageCap2AInp_{st1,c,r,y,s}$	Storage capacity to auxiliary input
pStorageCap2AOut(stg, comm, region, year, slice)	$pStorageCap2AOut_{st1,c,r,y,s}$	Storage capacity to auxiliary output
pStorageNCap2AInp(stg, comm, region, year, slice)	$pStorageNCap2AInp_{st1,c,r,y,s}$	Storage new capacity to auxiliary input
pStorageNCap2AOut(stg, comm, region, year, slice)	$pStorageNCap2AOut_{st1,c,r,y,s}$	Storage new capacity to auxiliary output
pTradeIrEff(trade, region, region, year, slice)	$pTradeIrEff_{t1,r,r,y,s}$	Inter-regional trade efficiency
pTradeIrUp(trade, region, region, year, slice)	$pTradeIrUp_{t1,r,r,y,s}$	Upper bound on trade flow
pTradeIrLo(trade, region, region, year, slice)	$pTradeIrLo_{t1,r,r,y,s}$	Lower bound on trade flow
pTradeIrCost(trade, region, region, year, slice)	$pTradeIrCost_{t1,r,r,y,s}$	Costs of trade flow
pTradeIrMarkup(trade, region, region, year, slice)	$pTradeIrMarkup_{t1,r,r,y,s}$	Markup of trade flow
pTradeIrCsrc2Ainp(trade, comm, region, region, year, slice)	$pTradeIrCsrc2Ainp_{t1,c,r,r,y,s}$	Auxiliary input commodity in source region
pTradeIrCsrc2Aout(trade, comm, region, region, year, slice)	$pTradeIrCsrc2Aout_{t1,c,r,r,y,s}$	Auxiliary output commodity in source region
pTradeIrCdst2Ainp(trade, comm, region, region, year, slice)	$pTradeIrCdst2Ainp_{t1,c,r,r,y,s}$	Auxiliary input commodity in destination region
pTradeIrCdst2Aout(trade, comm, region, region, year, slice)	$pTradeIrCdst2Aout_{t1,c,r,r,y,s}$	Auxiliary output commodity in destination region
pExportRowRes(expp)	$pExportRowRes_e$	Upper bound on accumulated export to ROW
pExportRowUp(expp, region, year, slice)	$pExportRowUp_{e,r,y,s}$	Upper bound on export to ROW
pExportRowLo(expp, region, year, slice)	$pExportRowLo_{e,r,y,s}$	Lower bound on export to ROW

pExportRowPrice(expp, region, year, slice)	$pExportRowPrice_{e,r,y,s}$	Export prices to ROW
pImportRowRes(imp)	$pImportRowRes_i$	Upper bound on accumulated import to ROW
pImportRowUp(imp, region, year, slice)	$pImportRowUp_{i,r,y,s}$	Upper bound on import from ROW
pImportRowLo(imp, region, year, slice)	$pImportRowLo_{i,r,y,s}$	Lower bound on import from ROW
pImportRowPrice(imp, region, year, slice)	$pImportRowPrice_{i,r,y,s}$	Import prices from ROW
pTradeStock(trade, year)	$pTradeStock_{t1,y}$	Existing capacity
pTradeOlife(trade)	$pTradeOlife_{t1}$	Operational life
pTradeInvcost(trade, region, year)	$pTradeInvcost_{t1,r,y}$	Overnight investment costs
pTradeEac(trade, region, year)	$pTradeEac_{t1,r,y}$	Equivalent annual costs
pTradeCap2Act(trade)	$pTradeCap2Act_{t1}$	Capacity to activity factor
pWeather(weather, region, year, slice)	$pWeather_{wth1,r,y,s}$	weather factors
pSupWeatherUp(weather, sup)	$pSupWeatherUp_{wth1,s1}$	weather factor for supply upper value (ava.up)
pSupWeatherLo(weather, sup)	$pSupWeatherLo_{wth1,s1}$	weather factor for supply lower value (ava.lo)
pTechWeatherAfLo(weather, tech)	$pTechWeatherAfLo_{wth1,t}$	weather factor for technology availability lower value (af.lo)
pTechWeatherAfUp(weather, tech)	$pTechWeatherAfUp_{wth1,t}$	weather factor for technology availability upper value (af.up)
pTechWeatherAfsLo(weather, tech)	$pTechWeatherAfsLo_{wth1,t}$	weather factor for technology availability lower value (af.lo)
pTechWeatherAfsUp(weather, tech)	$pTechWeatherAfsUp_{wth1,t}$	weather factor for technology availability upper value (afs.lo)
pTechWeatherAfcLo(weather, tech, comm)	$pTechWeatherAfcLo_{wth1,t,c}$	weather factor for technology availability lower value (afs.lo)
pTechWeatherAfcUp(weather, tech, comm)	$pTechWeatherAfcUp_{wth1,t,c}$	weather factor for commodity availability upper value (afc.lo)
pStorageWeatherAfLo(weather, stg)	$pStorageWeatherAfLo_{wth1,st1}$	weather factor for storage availability lower value (af.lo)
pStorageWeatherAfUp(weather, stg)	$pStorageWeatherAfUp_{wth1,st1}$	weather factor for storage availability upper value (af.up)
pStorageWeatherCinpUp(weather, stg)	$pStorageWeatherCinpUp_{wth1,st1}$	weather factor for storage commodity input upper value (cinp.up)
pStorageWeatherCinpLo(weather, stg)	$pStorageWeatherCinpLo_{wth1,st1}$	weather factor for storage commodity input lower value (cinp.lo)
pStorageWeatherCoutUp(weather, stg)	$pStorageWeatherCoutUp_{wth1,st1}$	weather factor for storage commodity output upper value (cout.up)
pStorageWeatherCoutLo(weather, stg)	$pStorageWeatherCoutLo_{wth1,st1}$	weather factor for storage commodity output lower value (cout.lo)
pLECLoACT(region)	$pLECLoACT_r$	levelized costs interim parameter

Variables

Name	Alias	Description
vTechInv(tech, region, year)	vTechInv _{<i>t,r,y</i>}	Overnight investment costs
vTechEac(tech, region, year)	vTechEac _{<i>t,r,y</i>}	Annualized investment costs
vTechOMCost(tech, region, year)	vTechOMCost _{<i>t,r,y</i>}	Sum of all operational costs is equal vTechFixom + vTechVarom (AVarom + CVarom + ActVarom)
vSupCost(sup, region, year)	vSupCost _{<i>s1,r,y</i>}	Supply costs
vEmsFuelTot(comm, region, year, slice)	vEmsFuelTot _{<i>c,r,y,s</i>}	Total emissions from fuels combustion
vBalance(comm, region, year, slice)	vBalance _{<i>c,r,y,s</i>}	Net commodity balance
vTotalCost(region, year)	vTotalCost _{<i>r,y</i>}	Regional annual total costs
vObjective	vObjective	Objective costs
vTaxCost(comm, region, year)	vTaxCost _{<i>c,r,y</i>}	Total tax levies (tax costs)
vSubsCost(comm, region, year)	vSubsCost _{<i>c,r,y</i>}	Total subsidies (for subtraction from costs)
vAggOut(comm, region, year, slice)	vAggOut _{<i>c,r,y,s</i>}	Aggregated commodity output
vStorageOMCost(stg, region, year)	vStorageOMCost _{<i>st1,r,y</i>}	Storage O&M costs
vTradeCost(region, year)	vTradeCost _{<i>r,y</i>}	Total trade costs
vTradeRowCost(region, year)	vTradeRowCost _{<i>r,y</i>}	Trade with ROW costs
vTradeIrCost(region, year)	vTradeIrCost _{<i>r,y</i>}	Interregional trade costs

Positive Variables

Name	Alias	Description
vTechNewCap(tech, region, year)	vTechNewCap _{<i>t,r,y</i>}	New capacity
vTechRetiredStock(tech, region, year)	vTechRetiredStock _{<i>t,r,y</i>}	Early retired stock
vTechRetiredNewCap(tech, region, year, year)	vTechRetiredNewCap _{<i>t,r,y,y</i>}	Early retired new capacity
vTechCap(tech, region, year)	vTechCap _{<i>t,r,y</i>}	Total capacity of the technology
vTechAct(tech, region, year, slice)	vTechAct _{<i>t,r,y,s</i>}	Activity level of technology
vTechInp(tech, comm, region, year, slice)	vTechInp _{<i>t,c,r,y,s</i>}	Input level
vTechOut(tech, comm, region, year, slice)	vTechOut _{<i>t,c,r,y,s</i>}	Output level
vTechAInp(tech, comm, region, year, slice)	vTechAInp _{<i>t,c,r,y,s</i>}	Auxiliary commodity input
vTechAOut(tech, comm, region, year, slice)	vTechAOut _{<i>t,c,r,y,s</i>}	Auxiliary commodity output
vSupOut(sup, comm, region, year, slice)	vSupOut _{<i>s1,c,r,y,s</i>}	Output of supply
vSupReserve(sup, comm, region)	vSupReserve _{<i>s1,c,r</i>}	Total supply reserve

vDemInp(comm, region, year, slice)	vDemInp _{c,r,y,s}	Input to demand
vOutTot(comm, region, year, slice)	vOutTot _{c,r,y,s}	Total commodity output (consumption is not substracted)
vInpTot(comm, region, year, slice)	vInpTot _{c,r,y,s}	Total commodity input
vInp2Lo(comm, region, year, slice, slice)	vInp2Lo _{c,r,y,s,s}	Desagregation of slices for input parent to (grand)child
vOut2Lo(comm, region, year, slice, slice)	vOut2Lo _{c,r,y,s,s}	Desagregation of slices for output parent to (grand)child
vSupOutTot(comm, region, year, slice)	vSupOutTot _{c,r,y,s}	Total commodity supply
vTechInpTot(comm, region, year, slice)	vTechInpTot _{c,r,y,s}	Total commodity input to technologies
vTechOutTot(comm, region, year, slice)	vTechOutTot _{c,r,y,s}	Total commodity output from technologies
vStorageInpTot(comm, region, year, slice)	vStorageInpTot _{c,r,y,s}	Total commodity input to storage
vStorageOutTot(comm, region, year, slice)	vStorageOutTot _{c,r,y,s}	Total commodity output from storage
vStorageAInp(stg, comm, region, year, slice)	vStorageAInp _{st1,c,r,y,s}	Aux-commodity input to storage
vStorageAOut(stg, comm, region, year, slice)	vStorageAOut _{st1,c,r,y,s}	Aux-commodity input from storage
vDummyImport(comm, region, year, slice)	vDummyImport _{c,r,y,s}	Dummy import (for debugging)
vDummyExport(comm, region, year, slice)	vDummyExport _{c,r,y,s}	Dummy export (for debugging)
vStorageInp(stg, comm, region, year, slice)	vStorageInp _{st1,c,r,y,s}	Storage input
vStorageOut(stg, comm, region, year, slice)	vStorageOut _{st1,c,r,y,s}	Storage output
vStorageStore(stg, comm, region, year, slice)	vStorageStore _{st1,c,r,y,s}	Storage level
vStorageInv(stg, region, year)	vStorageInv _{st1,r,y}	Storage investments
vStorageEac(stg, region, year)	vStorageEac _{st1,r,y}	Storage EAC investments
vStorageCap(stg, region, year)	vStorageCap _{st1,r,y}	Storage capacity
vStorageNewCap(stg, region, year)	vStorageNewCap _{st1,r,y}	Storage new capacity
vImport(comm, region, year, slice)	vImport _{c,r,y,s}	Total regional import (Ir + ROW)
vExport(comm, region, year, slice)	vExport _{c,r,y,s}	Total regional export (Ir + ROW)
vTradeIr(trade, comm, region, region, year, slice)	vTradeIr _{t1,c,r,r,y,s}	Total physical trade flows between regions
vTradeIrAInp(trade, comm, region, year, slice)	vTradeIrAInp _{t1,c,r,y,s}	Trade auxiliari input
vTradeIrAInpTot(comm, region, year, slice)	vTradeIrAInpTot _{c,r,y,s}	Trade total auxiliari input
vTradeIrAOut(trade, comm, region, year, slice)	vTradeIrAOut _{t1,c,r,y,s}	Trade auxiliari output
vTradeIrAOutTot(comm, region, year, slice)	vTradeIrAOutTot _{c,r,y,s}	Trade auxiliari output total
vExportRowAccumulated(expp, comm)	vExportRowAccumulated _{e,c}	Accumulated export to ROW
vExportRow(expp, comm, region, year, slice)	vExportRow _{e,c,r,y,s}	Export to ROW
vImportRowAccumulated(imp, comm)	vImportRowAccumulated _{i,c}	Accumulated import from ROW
vImportRow(imp, comm, region, year, slice)	vImportRow _{i,c,r,y,s}	Import from ROW
vTradeCap(trade, year)	vTradeCap _{t1,y}	Trade capacity
vTradeInv(trade, region, year)	vTradeInv _{t1,r,y}	Investment in trade capacity (overnight)
vTradeEac(trade, region, year)	vTradeEac _{t1,r,y}	Investment in trade capacity (EAC)

vTradeNewCap(trade, year)	vTradeNewCap _{t1,y}	New trade capacity
vTotalUserCosts(region, year)	vTotalUserCosts _{r,y}	Total additional costs (set by user)

Equations

Technology

Activity Input & Output

Technology input to output
*eqTechSng2Sng*_{t,r,c,cp,y,s}

$$\mathbf{vTechInp}_{t,c,r,y,s} * pTechCinp2use_{t,c,r,y,s} = \frac{\mathbf{vTechOut}_{t,cp,r,y,s}}{pTechUse2cact_{t,cp,r,y,s} * pTechCact2cout_{t,cp,r,y,s}} \quad (1)$$

Technology group input to output
*eqTechGrp2Sng*_{t,r,g,cp,y,s}

$$pTechGinp2use_{t,g,r,y,s} * \sum_c (\mathbf{vTechInp}_{t,c,r,y,s} * pTechCinp2ginp_{t,c,r,y,s}) = \frac{\mathbf{vTechOut}_{t,cp,r,y,s}}{pTechUse2cact_{t,cp,r,y,s} * pTechCact2cout_{t,cp,r,y,s}} \quad (2)$$

Technology input to group output
*eqTechSng2Grp*_{t,r,c,gp,y,s}

$$\mathbf{vTechInp}_{t,c,r,y,s} * pTechCinp2use_{t,c,r,y,s} = \sum_{cp} \left(\frac{\mathbf{vTechOut}_{t,cp,r,y,s}}{pTechUse2cact_{t,cp,r,y,s} * pTechCact2cout_{t,cp,r,y,s}} \right) \quad (3)$$

Technology group input to group output
*eqTechGrp2Grp*_{t,r,g,gp,y,s}

$$pTechGinp2use_{t,g,r,y,s} * \sum_c (\mathbf{vTechInp}_{t,c,r,y,s} * pTechCinp2ginp_{t,c,r,y,s}) = \sum_{cp} \left(\frac{\mathbf{vTechOut}_{t,cp,r,y,s}}{pTechUse2cact_{t,cp,r,y,s} * pTechCact2cout_{t,cp,r,y,s}} \right) \quad (4)$$

Shares for grouped commodities

Technology lower bound on input share
*eqTechShareInpLo*_{t,r,g,c,y,s}

$$\mathbf{vTechInp}_{t,c,r,y,s} \geq pTechShareLo_{t,c,r,y,s} * \sum_{cp} (\mathbf{vTechInp}_{t,cp,r,y,s}) \quad (5)$$

Technology upper bound on input share
 $eqTechShareInpUp_{t,r,g,c,y,s}$

$$\mathbf{vTechInp}_{t,c,r,y,s} \leq pTechShareUp_{t,c,r,y,s} * \sum_{cp} (\mathbf{vTechInp}_{t,cp,r,y,s}) \quad (6)$$

Technology lower bound on output share
 $eqTechShareOutLo_{t,r,g,c,y,s}$

$$\mathbf{vTechOut}_{t,c,r,y,s} \geq pTechShareLo_{t,c,r,y,s} * \sum_{cp} (\mathbf{vTechOut}_{t,cp,r,y,s}) \quad (7)$$

Technology upper bound on output share
 $eqTechShareOutUp_{t,r,g,c,y,s}$

$$\mathbf{vTechOut}_{t,c,r,y,s} \leq pTechShareUp_{t,c,r,y,s} * \sum_{cp} (\mathbf{vTechOut}_{t,cp,r,y,s}) \quad (8)$$

Auxiliary input & output

Technology auxiliary commodity input
 $eqTechAInp_{t,c,r,y,s}$

$$\begin{aligned} \mathbf{vTechAInp}_{t,c,r,y,s} &= (\mathbf{vTechAct}_{t,r,y,s} * pTechAct2AInp_{t,c,r,y,s}) \\ &+ (\mathbf{vTechCap}_{t,r,y} * pTechCap2AInp_{t,c,r,y,s}) \\ &+ (\mathbf{vTechNewCap}_{t,r,y} * pTechNCap2AInp_{t,c,r,y,s}) + \sum_{cp} (pTechCinp2AInp_{t,c,cp,r,y,s} * \mathbf{vTechInp}_{t,cp,r,y,s}) \\ &+ \sum_{cp} (pTechCout2AInp_{t,c,cp,r,y,s} * \mathbf{vTechOut}_{t,cp,r,y,s}) \end{aligned} \quad (9)$$

Technology auxiliary commodity output
 $eqTechAOut_{t,c,r,y,s}$

$$\begin{aligned}
\mathbf{vTechAOut}_{t,c,r,y,s} &= (\mathbf{vTechAct}_{t,r,y,s} * pTechAct2AOut_{t,c,r,y,s}) \\
&+ (\mathbf{vTechCap}_{t,r,y} * pTechCap2AOut_{t,c,r,y,s}) \\
&+ (\mathbf{vTechNewCap}_{t,r,y} * pTechNCap2AOut_{t,c,r,y,s}) + \sum_{cp} (pTechCinp2AOut_{t,c,cp,r,y,s} * \mathbf{vTechInp}_{t,cp,r,y,s}) \\
&+ \sum_{cp} (pTechCout2AOut_{t,c,cp,r,y,s} * \mathbf{vTechOut}_{t,cp,r,y,s})
\end{aligned} \tag{10}$$

Availability

Technology availability factor lower bound

$eqTechAfLo_{t,r,y,s}$

$$pTechAfLo_{t,r,y,s} * pTechCap2act_t * \mathbf{vTechCap}_{t,r,y} * pSliceShare_s * \prod_{wth1} (pTechWeatherAfLo_{wth1,t} * pWeather_{wth1,r,y,s}) \leq \mathbf{vTechAct}_{t,r,y,s} \tag{11}$$

Technology availability factor upper bound

$eqTechAfUp_{t,r,y,s}$

$$\mathbf{vTechAct}_{t,r,y,s} \leq pTechAfUp_{t,r,y,s} * pTechCap2act_t * \mathbf{vTechCap}_{t,r,y} * pSliceShare_s * \prod_{wth1} (pTechWeatherAfUp_{wth1,t} * pWeather_{wth1,r,y,s}) \tag{12}$$

Technology availability factor for sum of slices lower bound

$eqTechAfsLo_{t,r,y,s}$

$$pTechAfsLo_{t,r,y,s} * pTechCap2act_t * \mathbf{vTechCap}_{t,r,y} * pSliceShare_s * \prod_{wth1} (pTechWeatherAfsLo_{wth1,t} * pWeather_{wth1,r,y,s}) \leq \sum_{sp} (\mathbf{vTechAct}_{t,r,y,sp}) \tag{13}$$

Technology availability factor for sum of slices upper bound

$eqTechAfsUp_{t,r,y,s}$

$$\sum_{sp} (\mathbf{vTechAct}_{t,r,y,sp}) \leq pTechAfsUp_{t,r,y,s} * pTechCap2act_t * \mathbf{vTechCap}_{t,r,y} * pSliceShare_s * \prod_{wth1} (pTechWeatherAfsUp_{wth1,t} * pWeather_{wth1,r,y,s}) \tag{14}$$

Technology ramp up

$eqTechRampUp_{t,r,y,s}$

$$\frac{\mathbf{vTechAct}_{t,r,y,s}}{pSliceShare_s} - \sum_{sp} \left(\frac{\mathbf{vTechAct}_{t,r,y,sp}}{pSliceShare_{sp}} \right) \leq \frac{pSliceShare_s * 365 * 24 * pTechCap2act_t * \mathbf{vTechCap}_{t,r,y}}{pTechRampUp_{t,r,y,s}} \tag{15}$$

Technology ramp down

$eqTechRampDown_{t,r,y,s}$

$$\sum_{sp} \left(\frac{\mathbf{vTechAct}_{t,r,y,sp}}{pSliceShare_{sp}} \right) - \frac{\mathbf{vTechAct}_{t,r,y,s}}{pSliceShare_s} \leq \frac{pSliceShare_s * 365 * 24 * pTechCap2act_t * \mathbf{vTechCap}_{t,r,y}}{pTechRampDown_{t,r,y,s}} \quad (16)$$

Connect activity with output

Technology activity to commodity output

$eqTechActSng_{t,c,r,y,s}$

$$\mathbf{vTechAct}_{t,r,y,s} = \frac{\mathbf{vTechOut}_{t,c,r,y,s}}{pTechCact2cout_{t,c,r,y,s}} \quad (17)$$

Technology activity to group output

$eqTechActGrp_{t,g,r,y,s}$

$$\mathbf{vTechAct}_{t,r,y,s} = \sum_c \left(\frac{\mathbf{vTechOut}_{t,c,r,y,s}}{pTechCact2cout_{t,c,r,y,s}} \right) \quad (18)$$

Availability commodity factor

Technology commodity availability factor lower bound

$eqTechAfcOutLo_{t,r,c,y,s}$

$$pTechCact2cout_{t,c,r,y,s} * pTechAfcLo_{t,c,r,y,s} * pTechCap2act_t * \mathbf{vTechCap}_{t,r,y} * pSliceShare_s * \prod_{wth1} (pTechWeatherAfcLo_{wth1,t,c} * pWeather_{wth1,r,y,s}) \leq \mathbf{vTechOut}_{t,c,r,y,s} \quad (19)$$

Technology commodity availability factor upper bound

$eqTechAfcOutUp_{t,r,c,y,s}$

$$\mathbf{vTechOut}_{t,c,r,y,s} \leq pTechCact2cout_{t,c,r,y,s} * pTechAfcUp_{t,c,r,y,s} * pTechCap2act_t * \mathbf{vTechCap}_{t,r,y} * \prod_{wth1} (pTechWeatherAfcUp_{wth1,t,c} * pWeather_{wth1,r,y,s}) \quad (20)$$

Technology commodity availability factor lower bound

$eqTechAfcInpLo_{t,r,c,y,s}$

$$pTechAfcLo_{t,c,r,y,s} * pTechCap2act_t * \mathbf{vTechCap}_{t,r,y} * pSliceShare_s * \prod_{wth1} (pTechWeatherAfcLo_{wth1,t,c} * pWeather_{wth1,r,y,s}) \leq \mathbf{vTechInp}_{t,c,r,y,s} \quad (21)$$

Technology commodity availability factor upper bound

$eqTechAfcInpUp_{t,r,c,y,s}$

$$\mathbf{vTechInp}_{t,c,r,y,s} \leq pTechAfcUp_{t,c,r,y,s} * pTechCap2act_t * \mathbf{vTechCap}_{t,r,y} * pSliceShare_s * \prod_{wth1} (pTechWeatherAfcUp_{wth1,t,c} * pWeather_{wth1,r,y,s}) \quad (22)$$

Capacity and costs equations

Technology capacity

$eqTechCap_{t,r,y}$

$$\mathbf{vTechCap}_{t,r,y} = pTechStock_{t,r,y} - \mathbf{vTechRetiredStock}_{t,r,y} + \sum_{yp} \left(pPeriodLen_{yp} * \left(\mathbf{vTechNewCap}_{t,r,yp} - \sum_{ye} (\mathbf{vTechRetiredNewCap}_{t,r,yp,ye}) \right) \right) \quad (23)$$

Retirement of new capacity

$eqTechRetiredNewCap_{t,r,y}$

$$\sum_{yp} (\mathbf{vTechRetiredNewCap}_{t,r,y,yp}) \leq \mathbf{vTechNewCap}_{t,r,y} \quad (24)$$

Retirement of stock

$eqTechRetiredStock_{t,r,y}$

$$\mathbf{vTechRetiredStock}_{t,r,y} \leq pTechStock_{t,r,y} \quad (25)$$

Technology Equivalent Annual Cost (EAC)

$eqTechEac_{t,r,y}$

$$\mathbf{vTechEac}_{t,r,y} = \sum_{yp} \left(pTechEac_{t,r,yp} * pPeriodLen_{yp} * \left(\mathbf{vTechNewCap}_{t,r,yp} - \sum_{ye} (\mathbf{vTechRetiredNewCap}_{t,r,yp,ye}) \right) \right) \quad (26)$$

Technology overnight investment costs

$eqTechInv_{t,r,y}$

$$\mathbf{vTechInv}_{t,r,y} = pTechInvcost_{t,r,y} * \mathbf{vTechNewCap}_{t,r,y} \quad (27)$$

Technology O&M costs

$eqTechOMCost_{t,r,y}$

$$\begin{aligned} \mathbf{vTechOMCost}_{t,r,y} = & pTechFixom_{t,r,y} * \mathbf{vTechCap}_{t,r,y} + \sum_s (pTechVarom_{t,r,y,s} * \mathbf{vTechAct}_{t,r,y,s}) \\ & + \sum_s \left(\sum_c (pTechCvarom_{t,c,r,y,s} * \mathbf{vTechInp}_{t,c,r,y,s}) \right) + \sum_s \left(\sum_c (pTechCvarom_{t,c,r,y,s} * \mathbf{vTechOut}_{t,c,r,y,s}) \right) \\ & + \sum_s \left(\sum_c (pTechAvarom_{t,c,r,y,s} * \mathbf{vTechAOut}_{t,c,r,y,s}) \right) + \sum_s \left(\sum_c (pTechAvarom_{t,c,r,y,s} * \mathbf{vTechAInp}_{t,c,r,y,s}) \right) \end{aligned} \quad (28)$$

Supply

Supply availability upper bound

$eqSupAvaUp_{s1,c,r,y,s}$

$$\mathbf{vSupOut}_{s1,c,r,y,s} \leq pSupAvaUp_{s1,c,r,y,s} * \prod_{wth1} (pSupWeatherUp_{wth1,s1} * pWeather_{wth1,r,y,s}) \quad (29)$$

Supply availability lower bound

$eqSupAvaLo_{s1,c,r,y,s}$

$$\mathbf{vSupOut}_{s1,c,r,y,s} \geq pSupAvaLo_{s1,c,r,y,s} * \prod_{wth1} (pSupWeatherLo_{wth1,s1} * pWeather_{wth1,r,y,s}) \quad (30)$$

Total supply of each commodity

$eqSupTotal_{s1,c,r}$

$$\mathbf{vSupReserve}_{s1,c,r} = \sum_{y,s} (pPeriodLen_y * \mathbf{vSupOut}_{s1,c,r,y,s}) \quad (31)$$

Total reserve upper value

$eqSupReserveUp_{s1,c,r}$

$$pSupReserveUp_{s1,c,r} \geq \mathbf{vSupReserve}_{s1,c,r} \quad (32)$$

Total reserve lower value

$eqSupReserveLo_{s1,c,r}$

$$\mathbf{vSupReserve}_{s1,c,r} \geq pSupReserveLo_{s1,c,r} \quad (33)$$

Total supply costs

$eqSupCost_{s1,r,y}$

$$\mathbf{vSupCost}_{s1,r,y} = \sum_{c,s} (pSupCost_{s1,c,r,y,s} * \mathbf{vSupOut}_{s1,c,r,y,s}) \quad (34)$$

Demand

Demand equation

$eqDemInp_{c,r,y,s}$

$$\mathbf{vDemInp}_{c,r,y,s} = \sum_d (pDemand_{d,c,r,y,s}) \quad (35)$$

Emission & Aggregating commodity equation

Aggregating commodity output

$eqAggOut_{c,r,y,s}$

$$\mathbf{vAggOut}_{c,r,y,s} = \sum_{cp} \left(pAggregateFactor_{c,cp} * \sum_{sp} (\mathbf{vOutTot}_{cp,r,y,sp}) \right) \quad (36)$$

Emissions from commodity consumption (i.e. fuels combustion)

$eqEmsFuelTot_{c,r,y,s}$

$$\mathbf{vEmsFuelTot}_{c,r,y,s} = \sum_{cp} \left(pEmissionFactor_{c,cp} * \sum_t \left(pTechEmisComm_{t,cp} * \sum_{sp} (\mathbf{vTechInp}_{t,cp,r,y,sp}) \right) \right) \quad (37)$$

Storage

Input & Output

Storage level

$eqStorageAInp_{st1,c,r,y,s}$

$$\begin{aligned} \mathbf{vStorageAInp}_{st1,c,r,y,s} = & \sum_{cp} (pStorageStg2AInp_{st1,c,r,y,s} * \mathbf{vStorageStore}_{st1,cp,r,y,s}) + \sum_{cp} (pStorageCinp2AInp_{st1,c,r,y,s} * \mathbf{vStorageInp}_{st1,cp,r,y,s}) \\ & + \sum_{cp} (pStorageCout2AInp_{st1,c,r,y,s} * \mathbf{vStorageOut}_{st1,cp,r,y,s}) + \sum_{cp} (pStorageCap2AInp_{st1,c,r,y,s} * \mathbf{vStorageCap}_{st1,r,y}) \\ & + \sum_{cp} (pStorageNCap2AInp_{st1,c,r,y,s} * \mathbf{vStorageNewCap}_{st1,r,y}) \end{aligned} \quad (38)$$

Storage availability factor lower

$eqStorageAOut_{st1,c,r,y,s}$

$$\begin{aligned}
\mathbf{vStorageAOut}_{st1,c,r,y,s} = & \sum_{cp} (pStorageStg2AOut_{st1,c,r,y,s} * \mathbf{vStorageStore}_{st1,cp,r,y,s}) + \sum_{cp} (pStorageCinp2AOut_{st1,c,r,y,s} * \mathbf{vStorageInp}_{st1,cp,r,y,s}) \\
& + \sum_{cp} (pStorageCout2AOut_{st1,c,r,y,s} * \mathbf{vStorageOut}_{st1,cp,r,y,s}) + \sum_{cp} (pStorageCap2AOut_{st1,c,r,y,s} * \mathbf{vStorageCap}_{st1,r,y}) \\
& + \sum_{cp} (pStorageNCap2AOut_{st1,c,r,y,s} * \mathbf{vStorageNewCap}_{st1,r,y})
\end{aligned} \quad (39)$$

Storage availability factor upper
 $eqStorageStore_{st1,c,r,y,s}$

$$\begin{aligned}
\mathbf{vStorageStore}_{st1,c,r,y,s} = & pStorageCharge_{st1,c,r,y,s} + (pStorageNCap2Stg_{st1,c,r,y,s} * \mathbf{vStorageNewCap}_{st1,r,y}) \\
& + \sum_{sp} (pStorageInpEff_{st1,c,r,y,sp} * \mathbf{vStorageInp}_{st1,c,r,y,sp}) \\
& + \sum_{sp} \left(\left(pStorageStgEff_{st1,c,r,y,s}^{pSliceShare_s} * \mathbf{vStorageStore}_{st1,c,r,y,sp} \right) - \sum_{sp} \left(\frac{\mathbf{vStorageOut}_{st1,c,r,y,sp}}{pStorageOutEff_{st1,c,r,y,sp}} \right) \right)
\end{aligned} \quad (40)$$

Storage output vs level
 $eqStorageAfLo_{st1,c,r,y,s}$

$$\mathbf{vStorageStore}_{st1,c,r,y,s} \geq pStorageAfLo_{st1,r,y,s} * pStorageCap2stg_{st1} * \mathbf{vStorageCap}_{st1,r,y} * \prod_{wth1} (pStorageWeatherAfLo_{wth1,st1} * pWeather_{wth1,r,y,s}) \quad (41)$$

Storage aux-commodity input
 $eqStorageAfUp_{st1,c,r,y,s}$

$$\mathbf{vStorageStore}_{st1,c,r,y,s} \leq pStorageAfUp_{st1,r,y,s} * pStorageCap2stg_{st1} * \mathbf{vStorageCap}_{st1,r,y} * \prod_{wth1} (pStorageWeatherAfUp_{wth1,st1} * pWeather_{wth1,r,y,s}) \quad (42)$$

Storage aux-commodity output
 $eqStorageClean_{st1,c,r,y,s}$

$$\frac{\mathbf{vStorageOut}_{st1,c,r,y,s}}{pStorageOutEff_{st1,c,r,y,s}} \leq \mathbf{vStorageStore}_{st1,c,r,y,s} \quad (43)$$

Storage input upper constraint
 $eqStorageInpUp_{st1,c,r,y,s}$

$$\begin{aligned} \mathbf{vStorageInp}_{st1,c,r,y,s} &\leq pStorageCap2stg_{st1} * \mathbf{vStorageCap}_{st1,r,y} * pStorageCinpUp_{st1,c,r,y,s} \\ &\quad * pSliceShare_s * \prod_{wth1} (pStorageWeatherCinpUp_{wth1,st1} * pWeather_{wth1,r,y,s}) \end{aligned} \quad (44)$$

Storage input lower constraint
 $eqStorageInpLo_{st1,c,r,y,s}$

$$\begin{aligned} \mathbf{vStorageInp}_{st1,c,r,y,s} &\geq pStorageCap2stg_{st1} * \mathbf{vStorageCap}_{st1,r,y} * pStorageCinpLo_{st1,c,r,y,s} \\ &\quad * pSliceShare_s * \prod_{wth1} (pStorageWeatherCinpLo_{wth1,st1} * pWeather_{wth1,r,y,s}) \end{aligned} \quad (45)$$

Storage output upper constraint
 $eqStorageOutUp_{st1,c,r,y,s}$

$$\begin{aligned} \mathbf{vStorageOut}_{st1,c,r,y,s} &\leq pStorageCap2stg_{st1} * \mathbf{vStorageCap}_{st1,r,y} * pStorageCoutUp_{st1,c,r,y,s} \\ &\quad * pSliceShare_s * \prod_{wth1} (pStorageWeatherCoutUp_{wth1,st1} * pWeather_{wth1,r,y,s}) \end{aligned} \quad (46)$$

Storage output lower constraint
 $eqStorageOutLo_{st1,c,r,y,s}$

$$\begin{aligned} \mathbf{vStorageOut}_{st1,c,r,y,s} &\geq pStorageCap2stg_{st1} * \mathbf{vStorageCap}_{st1,r,y} * pStorageCoutLo_{st1,c,r,y,s} \\ &\quad * pSliceShare_s * \prod_{wth1} (pStorageWeatherCoutLo_{wth1,st1} * pWeather_{wth1,r,y,s}) \end{aligned} \quad (47)$$

Capacity and costs for storage

Storage capacity
 $eqStorageCap_{st1,r,y}$

$$\mathbf{vStorageCap}_{st1,r,y} = pStorageStock_{st1,r,y} + \sum_{yp} (pPeriodLen_{yp} * \mathbf{vStorageNewCap}_{st1,r,yp}) \quad (48)$$

Storage overnight investment costs
 $eqStorageInv_{st1,r,y}$

$$\mathbf{vStorageInv}_{st1,r,y} = pStorageInvcost_{st1,r,y} * \mathbf{vStorageNewCap}_{st1,r,y} \quad (49)$$

Storage equivalent annual cost

$eqStorageEac_{st1,r,y}$

$$\mathbf{vStorageEac}_{st1,r,y} = \sum_{yp} (pStorageEac_{st1,r,yp} * pPeriodLen_{yp} * \mathbf{vStorageNewCap}_{st1,r,yp}) \quad (50)$$

Storage total costs

$eqStorageCost_{st1,r,y}$

$$\mathbf{vStorageOMCost}_{st1,r,y} = pStorageFixom_{st1,r,y} * \mathbf{vStorageCap}_{st1,r,y} \quad (51)$$

$$+ \sum_c \left(\sum_s (pStorageCostInp_{st1,r,y,s} * \mathbf{vStorageInp}_{st1,c,r,y,s}) + \sum_s (pStorageCostOut_{st1,r,y,s} * \mathbf{vStorageOut}_{st1,c,r,y,s}) + \sum_s (pStorageCostStore_{st1,r,y,s} * \mathbf{vStorageStore}_{st1,c,r,y,s}) \right)$$

Interregional and ROW Trade equations

Flow

Import equation

$eqImport_{c,dst,y,s}$

$$\mathbf{vImport}_{c,dst,y,s} = \sum_{sp} \left(\sum_{t1} \left(\sum_{src} (pTradeIrEff_{t1,src,dst,y,sp} * \mathbf{vTradeIr}_{t1,c,src,dst,y,sp}) \right) \right) + \sum_{sp} \left(\sum_i (\mathbf{vImportRow}_{i,c,dst,y,sp}) \right) \quad (52)$$

Export equation

$eqExport_{c,src,y,s}$

$$\mathbf{vExport}_{c,src,y,s} = \sum_{sp} \left(\sum_{t1} \left(\sum_{dst} (\mathbf{vTradeIr}_{t1,c,src,dst,y,sp}) \right) \right) + \sum_{sp} \left(\sum_e (\mathbf{vExportRow}_{e,c,src,y,sp}) \right) \quad (53)$$

Trade upper bound

$eqTradeFlowUp_{t1,c,src,dst,y,s}$

$$\mathbf{vTradeIr}_{t1,c,src,dst,y,s} \leq pTradeIrUp_{t1,src,dst,y,s} \quad (54)$$

Trade lower bound

$eqTradeFlowLo_{t1,c,src,dst,y,s}$

$$\mathbf{vTradeIr}_{t1,c,src,dst,y,s} \geq pTradeIrLo_{t1,src,dst,y,s} \quad (55)$$

Total trade costs

$eqCostTrade_{r,y}$

$$\mathbf{vTradeCost}_{r,y} = \mathbf{vTradeRowCost}_{r,y} + \mathbf{vTradeIrCost}_{r,y} \quad (56)$$

Costs of trade with the Rest of the World (ROW)

$eqCostRowTrade_{r,y}$

$$\mathbf{vTradeRowCost}_{r,y} = \sum_{i,c,s} (pImportRowPrice_{i,r,y,s} * \mathbf{vImportRow}_{i,c,r,y,s}) - \sum_{e,c,s} (pExportRowPrice_{e,r,y,s} * \mathbf{vExportRow}_{e,c,r,y,s}) \quad (57)$$

Costs of import

$eqCostIrTrade_{r,y}$

$$\begin{aligned} \mathbf{vTradeIrCost}_{r,y} = & \sum_{t1} (\mathbf{vTradeEac}_{t1,r,y}) + \sum_{t1,src} \left(\sum_c \left(\sum_s (((pTradeIrCost_{t1,src,r,y,s} + pTradeIrMarkup_{t1,src,r,y,s}) * \mathbf{vTradeIr}_{t1,c,src,r,y,s})) \right) \right) \\ & - \sum_{t1,dst} \left(\sum_c \left(\sum_s ((pTradeIrMarkup_{t1,r,dst,y,s} * \mathbf{vTradeIr}_{t1,c,r,dst,y,s})) \right) \right) \end{aligned} \quad (58)$$

Export to ROW upper constraint

$eqExportRowUp_{e,c,r,y,s}$

$$\mathbf{vExportRow}_{e,c,r,y,s} \leq pExportRowUp_{e,r,y,s} \quad (59)$$

Export to ROW lower constraint

$eqExportRowLo_{e,c,r,y,s}$

$$\mathbf{vExportRow}_{e,c,r,y,s} \geq pExportRowLo_{e,r,y,s} \quad (60)$$

Cumulative export to ROW

$eqExportRowCumulative_{e,c}$

$$\mathbf{vExportRowAccumulated}_{e,c} = \sum_{r,y,s} (pPeriodLen_y * \mathbf{vExportRow}_{e,c,r,y,s}) \quad (61)$$

Cumulative export to ROW upper constraint

$eqExportRowResUp_{e,c}$

$$\mathbf{vExportRowAccumulated}_{e,c} \leq pExportRowRes_e \quad (62)$$

Import from ROW upper constraint

*eqImportRowUp*_{*i,c,r,y,s*}

$$\mathbf{vImportRow}_{i,c,r,y,s} \leq pImportRowUp_{i,r,y,s} \quad (63)$$

Import of ROW lower constraint

*eqImportRowLo*_{*i,c,r,y,s*}

$$\mathbf{vImportRow}_{i,c,r,y,s} \geq pImportRowLo_{i,r,y,s} \quad (64)$$

Cumulative import from ROW

*eqImportRowAccumulated*_{*i,c*}

$$\mathbf{vImportRowAccumulated}_{i,c} = \sum_{r,y,s} (pPeriodLen_y * \mathbf{vImportRow}_{i,c,r,y,s}) \quad (65)$$

Cumulative import from ROW upper constraint

*eqImportRowResUp*_{*i,c*}

$$\mathbf{vImportRowAccumulated}_{i,c} \leq pImportRowRes_i \quad (66)$$

Trade capacity

*eqTradeCapFlow*_{*t1,c,y,s*}

$$pSliceShare_s * pTradeCap2Act_{t1} * \mathbf{vTradeCap}_{t1,y} \geq \sum_{src,dst} (\mathbf{vTradeIr}_{t1,c,src,dst,y,s}) \quad (67)$$

Trade overnight investment costs

*eqTradeCap*_{*t1,y*}

$$\mathbf{vTradeCap}_{t1,y} = pTradeStock_{t1,y} + \sum_{yp} (pPeriodLen_{yp} * \mathbf{vTradeNewCap}_{t1,yp}) \quad (68)$$

Trade equivalent annual costs

*eqTradeInv*_{*t1,r,y*}

$$\mathbf{vTradeInv}_{t1,r,y} = pTradeInv_{cost,t1,r,y} * \mathbf{vTradeNewCap}_{t1,y} \quad (69)$$

Trade capacity to activity

*eqTradeEac*_{*t1,r,y*}

$$\mathbf{vTradeEac}_{t1,r,y} = \sum_{yp} (pTradeEac_{t1,r,yp} * pPeriodLen_{yp} * \mathbf{vTradeNewCap}_{t1,yp}) \quad (70)$$

Trade IR capacity equations

Auxiliary input & output equations

Trade auxiliary commodity input

$$eqTradeIrAInp_{t1,c,r,y,s}$$

$$\begin{aligned} & \mathbf{vTradeIrAInp}_{t1,c,r,y,s} \\ &= \sum_{dst} \left(pTradeIrCsrc2Ainp_{t1,c,r,dst,y,s} * \sum_{cp} (\mathbf{vTradeIr}_{t1,cp,r,dst,y,s}) \right) + \sum_{src} \left(pTradeIrCdst2Ainp_{t1,c,src,r,y,s} * \sum_{cp} (\mathbf{vTradeIr}_{t1,cp,src,r,y,s}) \right) \end{aligned} \quad (71)$$

Trade auxiliary commodity output

$$eqTradeIrAOut_{t1,c,r,y,s}$$

$$\mathbf{vTradeIrAOut}_{t1,c,r,y,s} = \sum_{dst} \left(pTradeIrCsrc2Aout_{t1,c,r,dst,y,s} * \sum_{cp} (\mathbf{vTradeIr}_{t1,cp,r,dst,y,s}) \right) + \sum_{src} \left(pTradeIrCdst2Aout_{t1,c,src,r,y,s} * \sum_{cp} (\mathbf{vTradeIr}_{t1,cp,src,r,y,s}) \right) \quad (72)$$

Trade auxiliary commodity input

$$eqTradeIrAInpTot_{c,r,y,s}$$

$$\mathbf{vTradeIrAInpTot}_{c,r,y,s} = \sum_{t1,sp} (\mathbf{vTradeIrAInp}_{t1,c,r,y,sp}) \quad (73)$$

Trade auxiliary commodity output

$$eqTradeIrAOutTot_{c,r,y,s}$$

$$\mathbf{vTradeIrAOutTot}_{c,r,y,s} = \sum_{t1,sp} (\mathbf{vTradeIrAOut}_{t1,c,r,y,sp}) \quad (74)$$

Balance equations & dummy import & export

PRODUCTION _i= CONSUMPTION commodity balance

$$eqBalLo_{c,r,y,s}$$

$$\mathbf{vBalance}_{c,r,y,s} \geq 0 \quad (75)$$

PRODUCTION _j= CONSUMPTION commodity balance

$$eqBalUp_{c,r,y,s}$$

$$\mathbf{vBalance}_{c,r,y,s} \leq 0 \quad (76)$$

PRODUCTION == CONSUMPTION commodity balance

$eqBalFx_{c,r,y,s}$

$$\mathbf{vBalance}_{c,r,y,s} = 0 \quad (77)$$

Commodity balance

$eqBal_{c,r,y,s}$

$$\mathbf{vBalance}_{c,r,y,s} = \mathbf{vOutTot}_{c,r,y,s} - \mathbf{vInpTot}_{c,r,y,s} \quad (78)$$

Total commodity output

$eqOutTot_{c,r,y,s}$

$$\begin{aligned} \mathbf{vOutTot}_{c,r,y,s} = & \mathbf{vDummyImport}_{c,r,y,s} + \mathbf{vSupOutTot}_{c,r,y,s} + \mathbf{vEmsFuelTot}_{c,r,y,s} + \mathbf{vAggOut}_{c,r,y,s} + \mathbf{vTechOutTot}_{c,r,y,s} \\ & + \mathbf{vStorageOutTot}_{c,r,y,s} + \mathbf{vImport}_{c,r,y,s} + \mathbf{vTradeIrAOutTot}_{c,r,y,s} + \sum_{sp} (\mathbf{vOut2Lo}_{c,r,y,sp,s}) \end{aligned} \quad (79)$$

Total commodity input

$eqOut2Lo_{c,r,y,s}$

$$\begin{aligned} \sum_{sp} (\mathbf{vOut2Lo}_{c,r,y,sp,s}) = & \mathbf{vSupOutTot}_{c,r,y,s} + \mathbf{vEmsFuelTot}_{c,r,y,s} + \mathbf{vAggOut}_{c,r,y,s} + \mathbf{vTechOutTot}_{c,r,y,s} \\ & + \mathbf{vStorageOutTot}_{c,r,y,s} + \mathbf{vImport}_{c,r,y,s} + \mathbf{vTradeIrAOutTot}_{c,r,y,s} \end{aligned} \quad (80)$$

From commodity slice to lo level

$eqInpTot_{c,r,y,s}$

$$\begin{aligned} \mathbf{vInpTot}_{c,r,y,s} = & \mathbf{vDemInp}_{c,r,y,s} + \mathbf{vDummyExport}_{c,r,y,s} + \mathbf{vTechInpTot}_{c,r,y,s} + \mathbf{vStorageInpTot}_{c,r,y,s} \\ & + \mathbf{vExport}_{c,r,y,s} + \mathbf{vTradeIrAInpTot}_{c,r,y,s} + \sum_{sp} (\mathbf{vInp2Lo}_{c,r,y,sp,s}) \end{aligned} \quad (81)$$

From commodity slice to lo level

$eqInp2Lo_{c,r,y,s}$

$$\sum_{sp} (\mathbf{vInp2Lo}_{c,r,y,sp,s}) = \mathbf{vTechInpTot}_{c,r,y,s} + \mathbf{vStorageInpTot}_{c,r,y,s} + \mathbf{vExport}_{c,r,y,s} + \mathbf{vTradeIrAInpTot}_{c,r,y,s} \quad (82)$$

Supply total output

$eqSupOutTot_{c,r,y,s}$

$$\mathbf{vSupOutTot}_{c,r,y,s} = \sum_{s1} \left(\sum_{sp} (\mathbf{vSupOut}_{s1,c,r,y,sp}) \right) \quad (83)$$

Technology total input
 $eqTechInpTot_{c,r,y,s}$

$$\mathbf{vTechInpTot}_{c,r,y,s} = \sum_t \left(\sum_{sp} (\mathbf{vTechInp}_{t,c,r,y,sp}) \right) + \sum_t \left(\sum_{sp} (\mathbf{vTechAInp}_{t,c,r,y,sp}) \right) \quad (84)$$

Technology total output
 $eqTechOutTot_{c,r,y,s}$

$$\mathbf{vTechOutTot}_{c,r,y,s} = \sum_t \left(\sum_{sp} (\mathbf{vTechOut}_{t,c,r,y,sp}) \right) + \sum_t \left(\sum_{sp} (\mathbf{vTechAOut}_{t,c,r,y,sp}) \right) \quad (85)$$

Storage total input
 $eqStorageInpTot_{c,r,y,s}$

$$\mathbf{vStorageInpTot}_{c,r,y,s} = \sum_{st1} (\mathbf{vStorageInp}_{st1,c,r,y,s}) + \sum_{st1} (\mathbf{vStorageAInp}_{st1,c,r,y,s}) \quad (86)$$

Storage total output
 $eqStorageOutTot_{c,r,y,s}$

$$\mathbf{vStorageOutTot}_{c,r,y,s} = \sum_{st1} (\mathbf{vStorageOut}_{st1,c,r,y,s}) + \sum_{st1} (\mathbf{vStorageAOut}_{st1,c,r,y,s}) \quad (87)$$

Objective and aggregated costs equations

Total costs
 $eqCost_{r,y}$

$$\begin{aligned} \mathbf{vTotalCost}_{r,y} = & \sum_t (\mathbf{vTechEac}_{t,r,y}) + \sum_t (\mathbf{vTechOMCost}_{t,r,y}) + \sum_{s1} (\mathbf{vSupCost}_{s1,r,y}) + \sum_{c,s} (pDummyImportCost_{c,r,y,s} * \mathbf{vDummyImport}_{c,r,y,s}) \\ & + \sum_{c,s} (pDummyExportCost_{c,r,y,s} * \mathbf{vDummyExport}_{c,r,y,s}) + \sum_c (\mathbf{vTaxCost}_{c,r,y}) - \sum_c (\mathbf{vSubsCost}_{c,r,y}) \\ & + \sum_{st1} (\mathbf{vStorageOMCost}_{st1,r,y}) + \sum_{st1} (\mathbf{vStorageEac}_{st1,r,y}) + \mathbf{vTradeCost}_{r,y} + \mathbf{vTotalUserCosts}_{r,y} \end{aligned} \quad (88)$$

Commodity taxes

eqTaxCost_{c,r,y}

$$\mathbf{vTaxCost}_{c,r,y} = \sum_s (pTaxCostOut_{c,r,y,s} * \mathbf{vOutTot}_{c,r,y,s}) + \sum_s (pTaxCostInp_{c,r,y,s} * \mathbf{vInpTot}_{c,r,y,s}) + \sum_s (pTaxCostBal_{c,r,y,s} * \mathbf{vBalance}_{c,r,y,s}) \quad (89)$$

Commodity subsidy

eqSubsCost_{c,r,y}

$$\mathbf{vSubsCost}_{c,r,y} = \sum_s (pSubCostOut_{c,r,y,s} * \mathbf{vOutTot}_{c,r,y,s}) + \sum_s (pSubCostInp_{c,r,y,s} * \mathbf{vInpTot}_{c,r,y,s}) + \sum_s (pSubCostBal_{c,r,y,s} * \mathbf{vBalance}_{c,r,y,s}) \quad (90)$$

Objective equation

eqObjective

$$\mathbf{vObjective} = \sum_{r,y} (\mathbf{vTotalCost}_{r,y} * pDiscountFactorMileStone_{r,y}) \quad (91)$$

LEC equation

levelized costs (auxiliary equation)

eqLECActivity_{t,r,y}

$$\sum_s (\mathbf{vTechAct}_{t,r,y,s}) \geq pLECLoACT_r \quad (92)$$