

# *energyRt* model framework

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## Sets

Name	Alias	Description
comm	$c, cp, ce, cn$	commodity
region	$r, rp, re, rn$	region
year	$y, yp, ye, yn$	year
slice	$l, lp, le, ln$	time slices
sup	$u, up, ue, un$	supply
dem	$d, dp, de, dn$	demand
tech	$h, hp, he, hn$	technology
stg	$s, sp, se, sn$	storage
trade	$d, dp, de, dn$	trade between regions
expp	$x, xp, xe, xn$	export to the rest of the world (ROW)
imp	$m, mp, me, mn$	import from the ROW
group	$g, gp, ge, gn$	group of input or output commodities in technology
weather	$w, wp, we, wn$	weather

## Parameters

Name	Alias	Description
pTechOlife(tech, region)	$pTechOlife_{h,r}$	Operational life of technologies
pTechCinp2ginp(tech, comm, region, year, slice)	$pTechCinp2ginp_{h,c,r,y,l}$	Commodity input to group input
pTechGinp2use(tech, group, region, year, slice)	$pTechGinp2use_{h,g,r,y,l}$	Group input into use

pTechCinp2use(tech, comm, region, year, slice)	$pTechCinp2use_{h,c,r,y,l}$	Commodity input to use
pTechUse2cact(tech, comm, region, year, slice)	$pTechUse2cact_{h,c,r,y,l}$	Use to commodity activity
pTechCact2cout(tech, comm, region, year, slice)	$pTechCact2cout_{h,c,r,y,l}$	Commodity activity to commodity output
pTechEmisComm(tech, comm)	$pTechEmisComm_{h,c}$	Combustion factor for input commodity (from 0 to 1)
pTechAct2AInp(tech, comm, region, year, slice)	$pTechAct2AInp_{h,c,r,y,l}$	Activity to aux-commodity input
pTechCap2AInp(tech, comm, region, year, slice)	$pTechCap2AInp_{h,c,r,y,l}$	Capacity to aux-commodity input
pTechNCap2AInp(tech, comm, region, year, slice)	$pTechNCap2AInp_{h,c,r,y,l}$	New capacity to aux-commodity input
pTechCinp2AInp(tech, comm, comm, region, year, slice)	$pTechCinp2AInp_{h,c,c,r,y,l}$	Commodity input to aux-commodity input
pTechCout2AInp(tech, comm, comm, region, year, slice)	$pTechCout2AInp_{h,c,c,r,y,l}$	Commodity output to aux-commodity input
pTechAct2AOut(tech, comm, region, year, slice)	$pTechAct2AOut_{h,c,r,y,l}$	Activity to aux-commodity output
pTechCap2AOut(tech, comm, region, year, slice)	$pTechCap2AOut_{h,c,r,y,l}$	Capacity to aux-commodity output
pTechNCap2AOut(tech, comm, region, year, slice)	$pTechNCap2AOut_{h,c,r,y,l}$	New capacity to aux-commodity output
pTechCinp2AOut(tech, comm, comm, region, year, slice)	$pTechCinp2AOut_{h,c,c,r,y,l}$	Commodity to aux-commodity output
pTechCout2AOut(tech, comm, comm, region, year, slice)	$pTechCout2AOut_{h,c,c,r,y,l}$	Commodity-output to aux-commodity input
pTechFixom(tech, region, year)	$pTechFixom_{h,r,y}$	Fixed Operating and maintenance (O&M) costs (per unit of capacity)
pTechVarom(tech, region, year, slice)	$pTechVarom_{h,r,y,l}$	Variable O&M costs (per unit of activity)
pTechInvcost(tech, region, year)	$pTechInvcost_{h,r,y}$	Investment costs (per unit of capacity)
pTechEac(tech, region, year)	$pTechEac_{h,r,y}$	Equivalent annual (investment) cost
pTechShareLo(tech, comm, region, year, slice)	$pTechShareLo_{h,c,r,y,l}$	Lower bound of the share of the commodity in total group input or output
pTechShareUp(tech, comm, region, year, slice)	$pTechShareUp_{h,c,r,y,l}$	Upper bound of the share of the commodity in total group input or output
pTechAfLo(tech, region, year, slice)	$pTechAfLo_{h,r,y,l}$	Lower bound on availability factor by slices
pTechAfUp(tech, region, year, slice)	$pTechAfUp_{h,r,y,l}$	Upper bound on availability factor by slices
pTechRampUp(tech, region, year, slice)	$pTechRampUp_{h,r,y,l}$	Ramp Up on availability factor
pTechRampDown(tech, region, year, slice)	$pTechRampDown_{h,r,y,l}$	Ramp Down on availability
pTechAfsLo(tech, region, year, slice)	$pTechAfsLo_{h,r,y,l}$	Lower bound on availability factor by groups of slices
pTechAfsUp(tech, region, year, slice)	$pTechAfsUp_{h,r,y,l}$	Upper bound on availability factor by groups of slices
pTechAfcLo(tech, comm, region, year, slice)	$pTechAfcLo_{h,c,r,y,l}$	Lower bound for commodity output
pTechAfcUp(tech, comm, region, year, slice)	$pTechAfcUp_{h,c,r,y,l}$	Upper bound for commodity output
pTechStock(tech, region, year)	$pTechStock_{h,r,y}$	Technology capacity stock
pTechCap2act(tech)	$pTechCap2act_h$	Technology capacity units to activity units conversion factor

pTechCvarom(tech, comm, region, year, slice)	$pTechCvarom_{h,c,r,y,l}$	Commodity-specific variable costs (per unit of commodity input or output)
pTechAvarom(tech, comm, region, year, slice)	$pTechAvarom_{h,c,r,y,l}$	Auxiliary Commodity-specific variable costs (per unit of commodity input or output)
pDiscount(region, year)	$pDiscount_{r,y}$	Discount rate (can be region and year specific)
pDiscountFactor(region, year)	$pDiscountFactor_{r,y}$	Discount factor (cumulative)
pDiscountFactorMileStone(region, year)	$pDiscountFactorMileStone_{r,y}$	Discount factor (cumulative) sum for MileStone
pSupCost(sup, comm, region, year, slice)	$pSupCost_{u,c,r,y,l}$	Costs of supply (price per unit)
pSupAvaUp(sup, comm, region, year, slice)	$pSupAvaUp_{u,c,r,y,l}$	Upper bound for supply
pSupAvaLo(sup, comm, region, year, slice)	$pSupAvaLo_{u,c,r,y,l}$	Lower bound for supply
pSupReserveUp(sup, comm, region)	$pSupReserveUp_{u,c,r}$	Total supply reserve by region Up
pSupReserveLo(sup, comm, region)	$pSupReserveLo_{u,c,r}$	Total supply reserve by region Lo
pDemand(dem, comm, region, year, slice)	$pDemand_{d,c,r,y,l}$	Exogenous demand
pEmissionFactor(comm, comm)	$pEmissionFactor_{c,c}$	Emission factor
pDummyImportCost(comm, region, year, slice)	$pDummyImportCost_{c,r,y,l}$	Dummy costs parameters (for debugging)
pDummyExportCost(comm, region, year, slice)	$pDummyExportCost_{c,r,y,l}$	Dummy costs parameters (for debugging)
pTaxCostInp(comm, region, year, slice)	$pTaxCostInp_{c,r,y,l}$	Commodity taxes for input
pTaxCostOut(comm, region, year, slice)	$pTaxCostOut_{c,r,y,l}$	Commodity taxes for output
pTaxCostBal(comm, region, year, slice)	$pTaxCostBal_{c,r,y,l}$	Commodity taxes for balance
pSubCostInp(comm, region, year, slice)	$pSubCostInp_{c,r,y,l}$	Commodity subsidies for input
pSubCostOut(comm, region, year, slice)	$pSubCostOut_{c,r,y,l}$	Commodity subsidies for output
pSubCostBal(comm, region, year, slice)	$pSubCostBal_{c,r,y,l}$	Commodity subsidies for balance
pAggregateFactor(comm, comm)	$pAggregateFactor_{c,c}$	Aggregation factor of commodities
pPeriodLen(year)	$pPeriodLen_y$	Length of milestone-year-period
pSliceShare(slice)	$pSliceShare_l$	Share of slice
ordYear(year)	$ordYear_y$	ord year (used in GLPK-MathProg)
cardYear(year)	$cardYear_y$	card year (used in GLPK-MathProg)
pStorageInpEff(stg, comm, region, year, slice)	$pStorageInpEff_{s,c,r,y,l}$	Storage input efficiency
pStorageOutEff(stg, comm, region, year, slice)	$pStorageOutEff_{s,c,r,y,l}$	Storage output efficiency
pStorageStgEff(stg, comm, region, year, slice)	$pStorageStgEff_{s,c,r,y,l}$	Storage time-efficiency (annual)
pStorageStock(stg, region, year)	$pStorageStock_{s,r,y}$	Storage capacity stock
pStorageOlife(stg, region)	$pStorageOlife_{s,r}$	Storage operational life
pStorageCostStore(stg, region, year, slice)	$pStorageCostStore_{s,r,y,l}$	Storing costs per stored amount (annual)
pStorageCostInp(stg, region, year, slice)	$pStorageCostInp_{s,r,y,l}$	Storage input costs
pStorageCostOut(stg, region, year, slice)	$pStorageCostOut_{s,r,y,l}$	Storage output costs

pStorageFixom(stg, region, year)	$pStorageFixom_{s,r,y}$	Storage fixed O&M costs
pStorageInvcost(stg, region, year)	$pStorageInvcost_{s,r,y}$	Storage investment costs
pStorageEac(stg, region, year)	$pStorageEac_{s,r,y}$	Storage equivalent annual costs
pStorageCap2stg(stg)	$pStorageCap2stg_s$	Storage capacity units to activity units conversion factor
pStorageAfLo(stg, region, year, slice)	$pStorageAfLo_{s,r,y,l}$	Storage availability factor lower bound (minimum charge level)
pStorageAfUp(stg, region, year, slice)	$pStorageAfUp_{s,r,y,l}$	Storage availability factor upper bound (maximum charge level)
pStorageCinpUp(stg, comm, region, year, slice)	$pStorageCinpUp_{s,c,r,y,l}$	Storage input upper bound
pStorageCinpLo(stg, comm, region, year, slice)	$pStorageCinpLo_{s,c,r,y,l}$	Storage input lower bound
pStorageCoutUp(stg, comm, region, year, slice)	$pStorageCoutUp_{s,c,r,y,l}$	Storage output upper bound
pStorageCoutLo(stg, comm, region, year, slice)	$pStorageCoutLo_{s,c,r,y,l}$	Storage output lower bound
pStorageNCap2Stg(stg, comm, region, year, slice)	$pStorageNCap2Stg_{s,c,r,y,l}$	Initial storage charge level for new investment
pStorageCharge(stg, comm, region, year, slice)	$pStorageCharge_{s,c,r,y,l}$	Initial storage charge level for stock
pStorageStg2AInp(stg, comm, region, year, slice)	$pStorageStg2AInp_{s,c,r,y,l}$	Storage accumulated volume to auxiliary input
pStorageStg2AOut(stg, comm, region, year, slice)	$pStorageStg2AOut_{s,c,r,y,l}$	Storage accumulated volume output
pStorageCinp2AInp(stg, comm, region, year, slice)	$pStorageCinp2AInp_{s,c,r,y,l}$	Storage input to auxiliary input
pStorageCinp2AOut(stg, comm, region, year, slice)	$pStorageCinp2AOut_{s,c,r,y,l}$	Storage input to auxiliary output
pStorageCout2AInp(stg, comm, region, year, slice)	$pStorageCout2AInp_{s,c,r,y,l}$	Storage output to auxiliary input
pStorageCout2AOut(stg, comm, region, year, slice)	$pStorageCout2AOut_{s,c,r,y,l}$	Storage output to auxiliary output
pStorageCap2AInp(stg, comm, region, year, slice)	$pStorageCap2AInp_{s,c,r,y,l}$	Storage capacity to auxiliary input
pStorageCap2AOut(stg, comm, region, year, slice)	$pStorageCap2AOut_{s,c,r,y,l}$	Storage capacity to auxiliary output
pStorageNCap2AInp(stg, comm, region, year, slice)	$pStorageNCap2AInp_{s,c,r,y,l}$	Storage new capacity to auxiliary input
pStorageNCap2AOut(stg, comm, region, year, slice)	$pStorageNCap2AOut_{s,c,r,y,l}$	Storage new capacity to auxiliary output
pTradeIrEff(trade, region, region, year, slice)	$pTradeIrEff_{d,r,y,l}$	Inter-regional trade efficiency
pTradeIrUp(trade, region, region, year, slice)	$pTradeIrUp_{d,r,y,l}$	Upper bound on trade flow
pTradeIrLo(trade, region, region, year, slice)	$pTradeIrLo_{d,r,y,l}$	Lower bound on trade flow
pTradeIrCost(trade, region, region, year, slice)	$pTradeIrCost_{d,r,y,l}$	Costs of trade flow
pTradeIrMarkup(trade, region, region, year, slice)	$pTradeIrMarkup_{d,r,y,l}$	Markup of trade flow
pTradeIrCsrc2Ainp(trade, comm, region, region, year, slice)	$pTradeIrCsrc2Ainp_{d,c,r,y,l}$	Auxiliary input commodity in source region
pTradeIrCsrc2Aout(trade, comm, region, region, year, slice)	$pTradeIrCsrc2Aout_{d,c,r,y,l}$	Auxiliary output commodity in source region
pTradeIrCdst2Ainp(trade, comm, region, region, year, slice)	$pTradeIrCdst2Ainp_{d,c,r,y,l}$	Auxiliary input commodity in destination region
pTradeIrCdst2Aout(trade, comm, region, region, year, slice)	$pTradeIrCdst2Aout_{d,c,r,y,l}$	Auxiliary output commodity in destination region
pExportRowRes(expp)	$pExportRowRes_x$	Upper bound on accumulated export to ROW
pExportRowUp(expp, region, year, slice)	$pExportRowUp_{x,r,y,l}$	Upper bound on export to ROW

pExportRowLo(expp, region, year, slice)	$pExportRowLo_{x,r,y,l}$	Lower bound on export to ROW
pExportRowPrice(expp, region, year, slice)	$pExportRowPrice_{x,r,y,l}$	Export prices to ROW
pImportRowRes(imp)	$pImportRowRes_m$	Upper bound on accumulated import to ROW
pImportRowUp(imp, region, year, slice)	$pImportRowUp_{m,r,y,l}$	Upper bound on import from ROW
pImportRowLo(imp, region, year, slice)	$pImportRowLo_{m,r,y,l}$	Lower bound on import from ROW
pImportRowPrice(imp, region, year, slice)	$pImportRowPrice_{m,r,y,l}$	Import prices from ROW
pTradeStock(trade, year)	$pTradeStock_{d,y}$	Existing capacity
pTradeOlife(trade)	$pTradeOlife_d$	Operational life
pTradeInvcost(trade, region, year)	$pTradeInvcost_{d,r,y}$	Overnight investment costs
pTradeEac(trade, region, year)	$pTradeEac_{d,r,y}$	Equivalent annual costs
pTradeCap2Act(trade)	$pTradeCap2Act_d$	Capacity to activity factor
pWeather(weather, region, year, slice)	$pWeather_{w,r,y,l}$	weather factors
pSupWeatherUp(weather, sup)	$pSupWeatherUp_{w,u}$	weather factor for supply upper value (ava.up)
pSupWeatherLo(weather, sup)	$pSupWeatherLo_{w,u}$	weather factor for supply lower value (ava.lo)
pTechWeatherAfLo(weather, tech)	$pTechWeatherAfLo_{w,h}$	weather factor for technology availability lower value (af.lo)
pTechWeatherAfUp(weather, tech)	$pTechWeatherAfUp_{w,h}$	weather factor for technology availability upper value (af.up)
pTechWeatherAfsLo(weather, tech)	$pTechWeatherAfsLo_{w,h}$	weather factor for technology availability lower value (af.lo)
pTechWeatherAfsUp(weather, tech)	$pTechWeatherAfsUp_{w,h}$	weather factor for technology availability upper value (afs.lo)
pTechWeatherAfcLo(weather, tech, comm)	$pTechWeatherAfcLo_{w,h,c}$	weather factor for technology availability lower value (afs.lo)
pTechWeatherAfcUp(weather, tech, comm)	$pTechWeatherAfcUp_{w,h,c}$	weather factor for commodity availability upper value (afc.lo)
pStorageWeatherAfLo(weather, stg)	$pStorageWeatherAfLo_{w,s}$	weather factor for storage availability lower value (af.lo)
pStorageWeatherAfUp(weather, stg)	$pStorageWeatherAfUp_{w,s}$	weather factor for storage availability upper value (af.up)
pStorageWeatherCinpUp(weather, stg)	$pStorageWeatherCinpUp_{w,s}$	weather factor for storage commodity input upper value (cinp.up)
pStorageWeatherCinpLo(weather, stg)	$pStorageWeatherCinpLo_{w,s}$	weather factor for storage commodity input lower value (cinp.lo)
pStorageWeatherCoutUp(weather, stg)	$pStorageWeatherCoutUp_{w,s}$	weather factor for storage commodity output upper value (cout.up)
pStorageWeatherCoutLo(weather, stg)	$pStorageWeatherCoutLo_{w,s}$	weather factor for storage commodity output lower value (cout.lo)
pLECLoACT(region)	$pLECLoACT_r$	levelized costs interim parameter

## Variables

Name	Alias	Description
vTechInv(tech, region, year)	<b>vTechInv</b> <sub><i>h,r,y</i></sub>	Overnight investment costs
vTechEac(tech, region, year)	<b>vTechEac</b> <sub><i>h,r,y</i></sub>	Annualized investment costs
vTechOMCost(tech, region, year)	<b>vTechOMCost</b> <sub><i>h,r,y</i></sub>	Sum of all operational costs is equal vTechFixom + vTech-Varom (AVarom + CVarom + ActVarom)
vSupCost(sup, region, year)	<b>vSupCost</b> <sub><i>u,r,y</i></sub>	Supply costs
vEmsFuelTot(comm, region, year, slice)	<b>vEmsFuelTot</b> <sub><i>c,r,y,l</i></sub>	Total emissions from fuels combustion
vBalance(comm, region, year, slice)	<b>vBalance</b> <sub><i>c,r,y,l</i></sub>	Net commodity balance
vTotalCost(region, year)	<b>vTotalCost</b> <sub><i>r,y</i></sub>	Regional annual total costs
vObjective	<b>vObjective</b>	Objective costs
vTaxCost(comm, region, year)	<b>vTaxCost</b> <sub><i>c,r,y</i></sub>	Total tax levies (tax costs)
vSubsCost(comm, region, year)	<b>vSubsCost</b> <sub><i>c,r,y</i></sub>	Total subsidies (for subtraction from costs)
vAggOut(comm, region, year, slice)	<b>vAggOut</b> <sub><i>c,r,y,l</i></sub>	Aggregated commodity output
vStorageOMCost(stg, region, year)	<b>vStorageOMCost</b> <sub><i>s,r,y</i></sub>	Storage O&M costs
vTradeCost(region, year)	<b>vTradeCost</b> <sub><i>r,y</i></sub>	Total trade costs
vTradeRowCost(region, year)	<b>vTradeRowCost</b> <sub><i>r,y</i></sub>	Trade with ROW costs
vTradeIrCost(region, year)	<b>vTradeIrCost</b> <sub><i>r,y</i></sub>	Interregional trade costs

## Positive Variables

Name	Alias	Description
vTechNewCap(tech, region, year)	<b>vTechNewCap</b> <sub><i>h,r,y</i></sub>	New capacity
vTechRetiredStock(tech, region, year)	<b>vTechRetiredStock</b> <sub><i>h,r,y</i></sub>	Early retired stock
vTechRetiredNewCap(tech, region, year, year)	<b>vTechRetiredNewCap</b> <sub><i>h,r,y,y</i></sub>	Early retired new capacity
vTechCap(tech, region, year)	<b>vTechCap</b> <sub><i>h,r,y</i></sub>	Total capacity of the technology
vTechAct(tech, region, year, slice)	<b>vTechAct</b> <sub><i>h,r,y,l</i></sub>	Activity level of technology
vTechInp(tech, comm, region, year, slice)	<b>vTechInp</b> <sub><i>h,c,r,y,l</i></sub>	Input level
vTechOut(tech, comm, region, year, slice)	<b>vTechOut</b> <sub><i>h,c,r,y,l</i></sub>	Output level
vTechAInp(tech, comm, region, year, slice)	<b>vTechAInp</b> <sub><i>h,c,r,y,l</i></sub>	Auxiliary commodity input
vTechAOut(tech, comm, region, year, slice)	<b>vTechAOut</b> <sub><i>h,c,r,y,l</i></sub>	Auxiliary commodity output
vSupOut(sup, comm, region, year, slice)	<b>vSupOut</b> <sub><i>u,c,r,y,l</i></sub>	Output of supply
vSupReserve(sup, comm, region)	<b>vSupReserve</b> <sub><i>u,c,r</i></sub>	Total supply reserve

vDemInp(comm, region, year, slice)	<b>vDemInp</b> <sub>c,r,y,l</sub>	Input to demand
vOutTot(comm, region, year, slice)	<b>vOutTot</b> <sub>c,r,y,l</sub>	Total commodity output (consumption is not substracted)
vInpTot(comm, region, year, slice)	<b>vInpTot</b> <sub>c,r,y,l</sub>	Total commodity input
vInp2Lo(comm, region, year, slice, slice)	<b>vInp2Lo</b> <sub>c,r,y,l,l</sub>	Desagregation of slices for input parent to (grand)child
vOut2Lo(comm, region, year, slice, slice)	<b>vOut2Lo</b> <sub>c,r,y,l,l</sub>	Desagregation of slices for output parent to (grand)child
vSupOutTot(comm, region, year, slice)	<b>vSupOutTot</b> <sub>c,r,y,l</sub>	Total commodity supply
vTechInpTot(comm, region, year, slice)	<b>vTechInpTot</b> <sub>c,r,y,l</sub>	Total commodity input to technologies
vTechOutTot(comm, region, year, slice)	<b>vTechOutTot</b> <sub>c,r,y,l</sub>	Total commodity output from technologies
vStorageInpTot(comm, region, year, slice)	<b>vStorageInpTot</b> <sub>c,r,y,l</sub>	Total commodity input to storage
vStorageOutTot(comm, region, year, slice)	<b>vStorageOutTot</b> <sub>c,r,y,l</sub>	Total commodity output from storage
vStorageAInp(stg, comm, region, year, slice)	<b>vStorageAInp</b> <sub>s,c,r,y,l</sub>	Aux-commodity input to storage
vStorageAOut(stg, comm, region, year, slice)	<b>vStorageAOut</b> <sub>s,c,r,y,l</sub>	Aux-commodity input from storage
vDummyImport(comm, region, year, slice)	<b>vDummyImport</b> <sub>c,r,y,l</sub>	Dummy import (for debugging)
vDummyExport(comm, region, year, slice)	<b>vDummyExport</b> <sub>c,r,y,l</sub>	Dummy export (for debugging)
vStorageInp(stg, comm, region, year, slice)	<b>vStorageInp</b> <sub>s,c,r,y,l</sub>	Storage input
vStorageOut(stg, comm, region, year, slice)	<b>vStorageOut</b> <sub>s,c,r,y,l</sub>	Storage output
vStorageStore(stg, comm, region, year, slice)	<b>vStorageStore</b> <sub>s,c,r,y,l</sub>	Storage level
vStorageInv(stg, region, year)	<b>vStorageInv</b> <sub>s,r,y</sub>	Storage investments
vStorageEac(stg, region, year)	<b>vStorageEac</b> <sub>s,r,y</sub>	Storage EAC investments
vStorageCap(stg, region, year)	<b>vStorageCap</b> <sub>s,r,y</sub>	Storage capacity
vStorageNewCap(stg, region, year)	<b>vStorageNewCap</b> <sub>s,r,y</sub>	Storage new capacity
vImport(comm, region, year, slice)	<b>vImport</b> <sub>c,r,y,l</sub>	Total regional import (Ir + ROW)
vExport(comm, region, year, slice)	<b>vExport</b> <sub>c,r,y,l</sub>	Total regional export (Ir + ROW)
vTradeIr(trade, comm, region, region, year, slice)	<b>vTradeIr</b> <sub>d,c,r,r,y,l</sub>	Total physical trade flows between regions
vTradeIrAInp(trade, comm, region, year, slice)	<b>vTradeIrAInp</b> <sub>d,c,r,y,l</sub>	Trade auxiliari input
vTradeIrAInpTot(comm, region, year, slice)	<b>vTradeIrAInpTot</b> <sub>c,r,y,l</sub>	Trade total auxiliari input
vTradeIrAOut(trade, comm, region, year, slice)	<b>vTradeIrAOut</b> <sub>d,c,r,y,l</sub>	Trade auxiliari output
vTradeIrAOutTot(comm, region, year, slice)	<b>vTradeIrAOutTot</b> <sub>c,r,y,l</sub>	Trade auxiliari output total
vExportRowAccumulated(expp, comm)	<b>vExportRowAccumulated</b> <sub>x,c</sub>	Accumulated export to ROW
vExportRow(expp, comm, region, year, slice)	<b>vExportRow</b> <sub>x,c,r,y,l</sub>	Export to ROW
vImportRowAccumulated(imp, comm)	<b>vImportRowAccumulated</b> <sub>m,c</sub>	Accumulated import from ROW
vImportRow(imp, comm, region, year, slice)	<b>vImportRow</b> <sub>m,c,r,y,l</sub>	Import from ROW
vTradeCap(trade, year)	<b>vTradeCap</b> <sub>d,y</sub>	Trade capacity
vTradeInv(trade, region, year)	<b>vTradeInv</b> <sub>d,r,y</sub>	Investment in trade capacity (overnight)
vTradeEac(trade, region, year)	<b>vTradeEac</b> <sub>d,r,y</sub>	Investment in trade capacity (EAC)

vTradeNewCap(trade, year)	<b>vTradeNewCap</b> <sub>d,y</sub>	New trade capacity
vTotalUserCosts(region, year)	<b>vTotalUserCosts</b> <sub>r,y</sub>	Total additional costs (set by user)

## Equations

### Technology

#### Activity Input & Output

Technology input to output

*eqTechSng2Sng*<sub>h,c,r,y,l</sub>

$$\mathbf{vTechInp}_{h,c,r,y,l} * pTechCinp2use_{h,c,r,y,l} = \frac{\mathbf{vTechOut}_{h,cp,r,y,l}}{pTechUse2cact_{h,cp,r,y,l} * pTechCact2cout_{h,cp,r,y,l}} \quad (1)$$

Technology group input to output

*eqTechGrp2Sng*<sub>h,r,g,cp,y,l</sub>

$$pTechGinp2use_{h,g,r,y,l} * \sum_c (\mathbf{vTechInp}_{h,c,r,y,l} * pTechCinp2ginp_{h,c,r,y,l}) = \frac{\mathbf{vTechOut}_{h,cp,r,y,l}}{pTechUse2cact_{h,cp,r,y,l} * pTechCact2cout_{h,cp,r,y,l}} \quad (2)$$

Technology input to group output

*eqTechSng2Grp*<sub>h,r,c,gp,y,l</sub>

$$\mathbf{vTechInp}_{h,c,r,y,l} * pTechCinp2use_{h,c,r,y,l} = \sum_{cp} \left( \frac{\mathbf{vTechOut}_{h,cp,r,y,l}}{pTechUse2cact_{h,cp,r,y,l} * pTechCact2cout_{h,cp,r,y,l}} \right) \quad (3)$$

Technology group input to group output

*eqTechGrp2Grp*<sub>h,r,g,gp,y,l</sub>

$$pTechGinp2use_{h,g,r,y,l} * \sum_c (\mathbf{vTechInp}_{h,c,r,y,l} * pTechCinp2ginp_{h,c,r,y,l}) = \sum_{cp} \left( \frac{\mathbf{vTechOut}_{h,cp,r,y,l}}{pTechUse2cact_{h,cp,r,y,l} * pTechCact2cout_{h,cp,r,y,l}} \right) \quad (4)$$

#### Shares for grouped commodities

Technology lower bound on input share

*eqTechShareInpLo*<sub>h,r,g,c,y,l</sub>



$$\mathbf{vTechInp}_{h,c,r,y,l} \geq pTechShareLo_{h,c,r,y,l} * \sum_{cp} (\mathbf{vTechInp}_{h,cp,r,y,l}) \quad (5)$$

Technology upper bound on input share  
 $eqTechShareInpUp_{h,r,g,c,y,l}$

$$\mathbf{vTechInp}_{h,c,r,y,l} \leq pTechShareUp_{h,c,r,y,l} * \sum_{cp} (\mathbf{vTechInp}_{h,cp,r,y,l}) \quad (6)$$

Technology lower bound on output share  
 $eqTechShareOutLo_{h,r,g,c,y,l}$

$$\mathbf{vTechOut}_{h,c,r,y,l} \geq pTechShareLo_{h,c,r,y,l} * \sum_{cp} (\mathbf{vTechOut}_{h,cp,r,y,l}) \quad (7)$$

Technology upper bound on output share  
 $eqTechShareOutUp_{h,r,g,c,y,l}$

$$\mathbf{vTechOut}_{h,c,r,y,l} \leq pTechShareUp_{h,c,r,y,l} * \sum_{cp} (\mathbf{vTechOut}_{h,cp,r,y,l}) \quad (8)$$

### Auxiliary input & output

Technology auxiliary commodity input  
 $eqTechAInp_{h,c,r,y,l}$

$$\begin{aligned} \mathbf{vTechAInp}_{h,c,r,y,l} &= (\mathbf{vTechAct}_{h,r,y,l} * pTechAct2AInp_{h,c,r,y,l}) \\ &+ (\mathbf{vTechCap}_{h,r,y} * pTechCap2AInp_{h,c,r,y,l}) \\ &+ (\mathbf{vTechNewCap}_{h,r,y} * pTechNCap2AInp_{h,c,r,y,l}) + \sum_{cp} (pTechCinp2AInp_{h,c,cp,r,y,l} * \mathbf{vTechInp}_{h,cp,r,y,l}) \\ &+ \sum_{cp} (pTechCout2AInp_{h,c,cp,r,y,l} * \mathbf{vTechOut}_{h,cp,r,y,l}) \end{aligned} \quad (9)$$

Technology auxiliary commodity output  
 $eqTechAOut_{h,c,r,y,l}$

$$\begin{aligned}
\mathbf{vTechAOut}_{h,c,r,y,l} &= (\mathbf{vTechAct}_{h,r,y,l} * pTechAct2AOut_{h,c,r,y,l}) \\
&+ (\mathbf{vTechCap}_{h,r,y} * pTechCap2AOut_{h,c,r,y,l}) \\
&+ (\mathbf{vTechNewCap}_{h,r,y} * pTechNCap2AOut_{h,c,r,y,l}) + \sum_{cp} (pTechCinp2AOut_{h,c,cp,r,y,l} * \mathbf{vTechInp}_{h,cp,r,y,l}) \\
&+ \sum_{cp} (pTechCout2AOut_{h,c,cp,r,y,l} * \mathbf{vTechOut}_{h,cp,r,y,l})
\end{aligned} \tag{10}$$

### Availability

Technology availability factor lower bound

$eqTechAfLo_{h,r,y,l}$

$$pTechAfLo_{h,r,y,l} * pTechCap2act_h * \mathbf{vTechCap}_{h,r,y} * pSliceShare_l * \prod_w (pTechWeatherAfLo_{w,h} * pWeather_{w,r,y,l}) \leq \mathbf{vTechAct}_{h,r,y,l} \tag{11}$$

Technology availability factor upper bound

$eqTechAfUp_{h,r,y,l}$

$$\mathbf{vTechAct}_{h,r,y,l} \leq pTechAfUp_{h,r,y,l} * pTechCap2act_h * \mathbf{vTechCap}_{h,r,y} * pSliceShare_l * \prod_w (pTechWeatherAfUp_{w,h} * pWeather_{w,r,y,l}) \tag{12}$$

Technology availability factor for sum of slices lower bound

$eqTechAfsLo_{h,r,y,l}$

$$pTechAfsLo_{h,r,y,l} * pTechCap2act_h * \mathbf{vTechCap}_{h,r,y} * pSliceShare_l * \prod_w (pTechWeatherAfsLo_{w,h} * pWeather_{w,r,y,l}) \leq \sum_{lp} (\mathbf{vTechAct}_{h,r,y,lp}) \tag{13}$$

Technology availability factor for sum of slices upper bound

$eqTechAfsUp_{h,r,y,l}$

$$\sum_{lp} (\mathbf{vTechAct}_{h,r,y,lp}) \leq pTechAfsUp_{h,r,y,l} * pTechCap2act_h * \mathbf{vTechCap}_{h,r,y} * pSliceShare_l * \prod_w (pTechWeatherAfsUp_{w,h} * pWeather_{w,r,y,l}) \tag{14}$$

Technology ramp up

$eqTechRampUp_{h,r,y,l}$

$$\frac{\mathbf{vTechAct}_{h,r,y,l}}{pSliceShare_l} - \sum_{lp} \left( \frac{\mathbf{vTechAct}_{h,r,y,lp}}{pSliceShare_{lp}} \right) \leq \frac{pSliceShare_l * 365 * 24 * pTechCap2act_h * \mathbf{vTechCap}_{h,r,y}}{pTechRampUp_{h,r,y,l}} \tag{15}$$

Technology ramp down

$eqTechRampDown_{h,r,y,l}$

$$\sum_{lp} \left( \frac{\mathbf{vTechAct}_{h,r,y,lp}}{pSliceShare_{lp}} \right) - \frac{\mathbf{vTechAct}_{h,r,y,l}}{pSliceShare_l} \leq \frac{pSliceShare_l * 365 * 24 * pTechCap2act_h * \mathbf{vTechCap}_{h,r,y}}{pTechRampDown_{h,r,y,l}} \quad (16)$$

**Connect activity with output**

Technology activity to commodity output

$eqTechActSng_{h,c,r,y,l}$

$$\mathbf{vTechAct}_{h,r,y,l} = \frac{\mathbf{vTechOut}_{h,c,r,y,l}}{pTechCact2cout_{h,c,r,y,l}} \quad (17)$$

Technology activity to group output

$eqTechActGrp_{h,g,r,y,l}$

$$\mathbf{vTechAct}_{h,r,y,l} = \sum_c \left( \frac{\mathbf{vTechOut}_{h,c,r,y,l}}{pTechCact2cout_{h,c,r,y,l}} \right) \quad (18)$$

**Availability commodity factor**

Technology commodity availability factor lower bound

$eqTechAfcOutLo_{h,r,c,y,l}$

$$\begin{aligned} & pTechCact2cout_{h,c,r,y,l} * pTechAfcLo_{h,c,r,y,l} * pTechCap2act_h * \mathbf{vTechCap}_{h,r,y} * pSliceShare_l * \prod_w (pTechWeatherAfcLo_{w,h,c} * pWeather_{w,r,y,l}) \\ & \leq \mathbf{vTechOut}_{h,c,r,y,l} \end{aligned} \quad (19)$$

Technology commodity availability factor upper bound

$eqTechAfcOutUp_{h,r,c,y,l}$

$$\mathbf{vTechOut}_{h,c,r,y,l} \leq pTechCact2cout_{h,c,r,y,l} * pTechAfcUp_{h,c,r,y,l} * pTechCap2act_h * \mathbf{vTechCap}_{h,r,y} * \prod_w (pTechWeatherAfcUp_{w,h,c} * pWeather_{w,r,y,l}) \quad (20)$$

Technology commodity availability factor lower bound

$eqTechAfcInpLo_{h,r,c,y,l}$

$$pTechAfcLo_{h,c,r,y,l} * pTechCap2act_h * \mathbf{vTechCap}_{h,r,y} * pSliceShare_l * \prod_w (pTechWeatherAfcLo_{w,h,c} * pWeather_{w,r,y,l}) \leq \mathbf{vTechInp}_{h,c,r,y,l} \quad (21)$$

Technology commodity availability factor upper bound

$eqTechAfcInpUp_{h,r,c,y,l}$

$$\mathbf{vTechInp}_{h,c,r,y,l} \leq pTechAfcUp_{h,c,r,y,l} * pTechCap2act_h * \mathbf{vTechCap}_{h,r,y} * pSliceShare_l * \prod_w (pTechWeatherAfcUp_{w,h,c} * pWeather_{w,r,y,l}) \quad (22)$$

### Capacity and costs equations

Technology capacity

$eqTechCap_{h,r,y}$

$$\mathbf{vTechCap}_{h,r,y} = pTechStock_{h,r,y} - \mathbf{vTechRetiredStock}_{h,r,y} + \sum_{yp} \left( pPeriodLen_{yp} * \left( \mathbf{vTechNewCap}_{h,r,yp} - \sum_{ye} (\mathbf{vTechRetiredNewCap}_{h,r,yp,ye}) \right) \right) \quad (23)$$

Retirement of new capacity

$eqTechRetiredNewCap_{h,r,y}$

$$\sum_{yp} (\mathbf{vTechRetiredNewCap}_{h,r,y,yp}) \leq \mathbf{vTechNewCap}_{h,r,y} \quad (24)$$

Retirement of stock

$eqTechRetiredStock_{h,r,y}$

$$\mathbf{vTechRetiredStock}_{h,r,y} \leq pTechStock_{h,r,y} \quad (25)$$

Technology Equivalent Annual Cost (EAC)

$eqTechEac_{h,r,y}$

$$\mathbf{vTechEac}_{h,r,y} = \sum_{yp} \left( pTechEac_{h,r,yp} * pPeriodLen_{yp} * \left( \mathbf{vTechNewCap}_{h,r,yp} - \sum_{ye} (\mathbf{vTechRetiredNewCap}_{h,r,yp,ye}) \right) \right) \quad (26)$$

Technology overnight investment costs

$eqTechInv_{h,r,y}$

$$\mathbf{vTechInv}_{h,r,y} = pTechInv_{cost_{h,r,y}} * \mathbf{vTechNewCap}_{h,r,y} \quad (27)$$

Technology O&M costs

$eqTechOMCost_{h,r,y}$

$$\begin{aligned}
\mathbf{vTechOMCost}_{h,r,y} &= pTechFixom_{h,r,y} * \mathbf{vTechCap}_{h,r,y} + \sum_l (pTechVarom_{h,r,y,l} * \mathbf{vTechAct}_{h,r,y,l}) \\
&+ \sum_l \left( \sum_c (pTechCvarom_{h,c,r,y,l} * \mathbf{vTechInp}_{h,c,r,y,l}) \right) + \sum_l \left( \sum_c (pTechCvarom_{h,c,r,y,l} * \mathbf{vTechOut}_{h,c,r,y,l}) \right) \\
&+ \sum_l \left( \sum_c (pTechAvarom_{h,c,r,y,l} * \mathbf{vTechAOut}_{h,c,r,y,l}) \right) + \sum_l \left( \sum_c (pTechAvarom_{h,c,r,y,l} * \mathbf{vTechAInp}_{h,c,r,y,l}) \right)
\end{aligned} \tag{28}$$

## Supply

Supply availability upper bound

$eqSupAvaUp_{u,c,r,y,l}$

$$\mathbf{vSupOut}_{u,c,r,y,l} \leq pSupAvaUp_{u,c,r,y,l} * \prod_w (pSupWeatherUp_{w,u} * pWeather_{w,r,y,l}) \tag{29}$$

Supply availability lower bound

$eqSupAvaLo_{u,c,r,y,l}$

$$\mathbf{vSupOut}_{u,c,r,y,l} \geq pSupAvaLo_{u,c,r,y,l} * \prod_w (pSupWeatherLo_{w,u} * pWeather_{w,r,y,l}) \tag{30}$$

Total supply of each commodity

$eqSupTotal_{u,c,r}$

$$\mathbf{vSupReserve}_{u,c,r} = \sum_{y,l} (pPeriodLen_y * \mathbf{vSupOut}_{u,c,r,y,l}) \tag{31}$$

Total reserve upper value

$eqSupReserveUp_{u,c,r}$

$$pSupReserveUp_{u,c,r} \geq \mathbf{vSupReserve}_{u,c,r} \tag{32}$$

Total reserve lower value

$eqSupReserveLo_{u,c,r}$

$$\mathbf{vSupReserve}_{u,c,r} \geq pSupReserveLo_{u,c,r} \tag{33}$$

Total supply costs

$eqSupCost_{u,r,y}$

$$\mathbf{vSupCost}_{u,r,y} = \sum_{c,l} (pSupCost_{u,c,r,y,l} * \mathbf{vSupOut}_{u,c,r,y,l}) \tag{34}$$

## Demand

Demand equation

$eqDemInp_{c,r,y,l}$

$$\mathbf{vDemInp}_{c,r,y,l} = \sum_d (pDemand_{d,c,r,y,l}) \quad (35)$$

## Emission & Aggregating commodity equation

Aggregating commodity output

$eqAggOut_{c,r,y,l}$

$$\mathbf{vAggOut}_{c,r,y,l} = \sum_{cp} \left( pAggregateFactor_{c,cp} * \sum_{lp} (\mathbf{vOutTot}_{cp,r,y,lp}) \right) \quad (36)$$

Emissions from commodity consumption (i.e. fuels combustion)

$eqEmsFuelTot_{c,r,y,l}$

$$\mathbf{vEmsFuelTot}_{c,r,y,l} = \sum_{cp} \left( pEmissionFactor_{c,cp} * \sum_h \left( pTechEmisComm_{h,cp} * \sum_{lp} (\mathbf{vTechInp}_{h,cp,r,y,lp}) \right) \right) \quad (37)$$

## Storage

Input & Output

Storage level

$eqStorageAInp_{s,c,r,y,l}$

$$\begin{aligned} \mathbf{vStorageAInp}_{s,c,r,y,l} = & \sum_{cp} (pStorageStg2AInp_{s,c,r,y,l} * \mathbf{vStorageStore}_{s,cp,r,y,l}) + \sum_{cp} (pStorageCinp2AInp_{s,c,r,y,l} * \mathbf{vStorageInp}_{s,cp,r,y,l}) \\ & + \sum_{cp} (pStorageCout2AInp_{s,c,r,y,l} * \mathbf{vStorageOut}_{s,cp,r,y,l}) + \sum_{cp} (pStorageCap2AInp_{s,c,r,y,l} * \mathbf{vStorageCap}_{s,r,y}) \\ & + \sum_{cp} (pStorageNCap2AInp_{s,c,r,y,l} * \mathbf{vStorageNewCap}_{s,r,y}) \end{aligned} \quad (38)$$

Storage availability factor lower

$eqStorageAOut_{s,c,r,y,l}$

$$\begin{aligned}
\mathbf{vStorageAOut}_{s,c,r,y,l} = & \sum_{cp} (pStorageStg2AOut_{s,c,r,y,l} * \mathbf{vStorageStore}_{s,cp,r,y,l}) + \sum_{cp} (pStorageCinp2AOut_{s,c,r,y,l} * \mathbf{vStorageInp}_{s,cp,r,y,l}) \\
& + \sum_{cp} (pStorageCout2AOut_{s,c,r,y,l} * \mathbf{vStorageOut}_{s,cp,r,y,l}) + \sum_{cp} (pStorageCap2AOut_{s,c,r,y,l} * \mathbf{vStorageCap}_{s,r,y}) \\
& + \sum_{cp} (pStorageNCap2AOut_{s,c,r,y,l} * \mathbf{vStorageNewCap}_{s,r,y})
\end{aligned} \tag{39}$$

Storage availability factor upper  
 $eqStorageStore_{s,c,r,y,l}$

$$\begin{aligned}
\mathbf{vStorageStore}_{s,c,r,y,l} = & pStorageCharge_{s,c,r,y,l} + (pStorageNCap2Stg_{s,c,r,y,l} * \mathbf{vStorageNewCap}_{s,r,y}) + \sum_{lp} (pStorageInpEff_{s,c,r,y,lp} * \mathbf{vStorageInp}_{s,c,r,y,lp}) \\
& + \sum_{lp} \left( \left( pStorageStgEff_{s,c,r,y,l}^{pSliceShare_l} \right) * \mathbf{vStorageStore}_{s,c,r,y,lp} \right) - \sum_{lp} \left( \frac{\mathbf{vStorageOut}_{s,c,r,y,lp}}{pStorageOutEff_{s,c,r,y,lp}} \right)
\end{aligned} \tag{40}$$

Storage output vs level  
 $eqStorageAfLo_{s,c,r,y,l}$

$$\mathbf{vStorageStore}_{s,c,r,y,l} \geq pStorageAfLo_{s,r,y,l} * pStorageCap2stg_s * \mathbf{vStorageCap}_{s,r,y} * \prod_w (pStorageWeatherAfLo_{w,s} * pWeather_{w,r,y,l}) \tag{41}$$

Storage aux-commodity input  
 $eqStorageAfUp_{s,c,r,y,l}$

$$\mathbf{vStorageStore}_{s,c,r,y,l} \leq pStorageAfUp_{s,r,y,l} * pStorageCap2stg_s * \mathbf{vStorageCap}_{s,r,y} * \prod_w (pStorageWeatherAfUp_{w,s} * pWeather_{w,r,y,l}) \tag{42}$$

Storage aux-commodity output  
 $eqStorageClean_{s,c,r,y,l}$

$$\frac{\mathbf{vStorageOut}_{s,c,r,y,l}}{pStorageOutEff_{s,c,r,y,l}} \leq \mathbf{vStorageStore}_{s,c,r,y,l} \tag{43}$$

Storage input upper constraint  
 $eqStorageInpUp_{s,c,r,y,l}$

$$\mathbf{vStorageInp}_{s,c,r,y,l} \leq pStorageCap2stg_s * \mathbf{vStorageCap}_{s,r,y} * pStorageCinpUp_{s,c,r,y,l} * pSliceShare_l * \prod_w (pStorageWeatherCinpUp_{w,s} * pWeather_{w,r,y,l}) \quad (44)$$

Storage input lower constraint  
 $eqStorageInpLo_{s,c,r,y,l}$

$$\mathbf{vStorageInp}_{s,c,r,y,l} \geq pStorageCap2stg_s * \mathbf{vStorageCap}_{s,r,y} * pStorageCinpLo_{s,c,r,y,l} * pSliceShare_l * \prod_w (pStorageWeatherCinpLo_{w,s} * pWeather_{w,r,y,l}) \quad (45)$$

Storage output upper constraint  
 $eqStorageOutUp_{s,c,r,y,l}$

$$\mathbf{vStorageOut}_{s,c,r,y,l} \leq pStorageCap2stg_s * \mathbf{vStorageCap}_{s,r,y} * pStorageCoutUp_{s,c,r,y,l} * pSliceShare_l * \prod_w (pStorageWeatherCoutUp_{w,s} * pWeather_{w,r,y,l}) \quad (46)$$

Storage output lower constraint  
 $eqStorageOutLo_{s,c,r,y,l}$

$$\mathbf{vStorageOut}_{s,c,r,y,l} \geq pStorageCap2stg_s * \mathbf{vStorageCap}_{s,r,y} * pStorageCoutLo_{s,c,r,y,l} * pSliceShare_l * \prod_w (pStorageWeatherCoutLo_{w,s} * pWeather_{w,r,y,l}) \quad (47)$$

### Capacity and costs for storage

Storage capacity  
 $eqStorageCap_{s,r,y}$

$$\mathbf{vStorageCap}_{s,r,y} = pStorageStock_{s,r,y} + \sum_{yp} (pPeriodLen_{yp} * \mathbf{vStorageNewCap}_{s,r,yp}) \quad (48)$$

Storage overnight investment costs  
 $eqStorageInv_{s,r,y}$

$$\mathbf{vStorageInv}_{s,r,y} = pStorageInvcost_{s,r,y} * \mathbf{vStorageNewCap}_{s,r,y} \quad (49)$$

Storage equivalent annual cost  
 $eqStorageEac_{s,r,y}$



$$\mathbf{vStorageEac}_{s,r,y} = \sum_{yp} (pStorageEac_{s,r,yp} * pPeriodLen_{yp} * \mathbf{vStorageNewCap}_{s,r,yp}) \quad (50)$$

Storage total costs

$eqStorageCost_{s,r,y}$

$$\mathbf{vStorageOMCost}_{s,r,y} = pStorageFixom_{s,r,y} * \mathbf{vStorageCap}_{s,r,y} \quad (51)$$

$$+ \sum_c \left( \sum_l (pStorageCostInp_{s,r,y,l} * \mathbf{vStorageInp}_{s,c,r,y,l}) + \sum_l (pStorageCostOut_{s,r,y,l} * \mathbf{vStorageOut}_{s,c,r,y,l}) + \sum_l (pStorageCostStore_{s,r,y,l} * \mathbf{vStorageStore}_{s,c,r,y,l}) \right)$$

## Interregional and ROW Trade equations

Flow

Import equation

$eqImport_{c,dst,y,l}$

$$\mathbf{vImport}_{c,dst,y,l} = \sum_{lp} \left( \sum_d \left( \sum_{src} (pTradeIrEff_{d,src,dst,y,lp} * \mathbf{vTradeIr}_{d,c,src,dst,y,lp}) \right) \right) + \sum_{lp} \left( \sum_m (\mathbf{vImportRow}_{m,c,dst,y,lp}) \right) \quad (52)$$

Export equation

$eqExport_{c,src,y,l}$

$$\mathbf{vExport}_{c,src,y,l} = \sum_{lp} \left( \sum_d \left( \sum_{dst} (\mathbf{vTradeIr}_{d,c,src,dst,y,lp}) \right) \right) + \sum_{lp} \left( \sum_x (\mathbf{vExportRow}_{x,c,src,y,lp}) \right) \quad (53)$$

Trade upper bound

$eqTradeFlowUp_{d,c,src,dst,y,l}$

$$\mathbf{vTradeIr}_{d,c,src,dst,y,l} \leq pTradeIrUp_{d,src,dst,y,l} \quad (54)$$

Trade lower bound

$eqTradeFlowLo_{d,c,src,dst,y,l}$

$$\mathbf{vTradeIr}_{d,c,src,dst,y,l} \geq pTradeIrLo_{d,src,dst,y,l} \quad (55)$$

Total trade costs

$eqCostTrade_{r,y}$

$$\mathbf{vTradeCost}_{r,y} = \mathbf{vTradeRowCost}_{r,y} + \mathbf{vTradeIrCost}_{r,y} \quad (56)$$

Costs of trade with the Rest of the World (ROW)

$eqCostRowTrade_{r,y}$

$$\mathbf{vTradeRowCost}_{r,y} = \sum_{m,c,l} (pImportRowPrice_{m,r,y,l} * \mathbf{vImportRow}_{m,c,r,y,l}) - \sum_{x,c,l} (pExportRowPrice_{x,r,y,l} * \mathbf{vExportRow}_{x,c,r,y,l}) \quad (57)$$

Costs of import

$eqCostIrTrade_{r,y}$

$$\begin{aligned} \mathbf{vTradeIrCost}_{r,y} = & \sum_d (\mathbf{vTradeEac}_{d,r,y}) + \sum_{d,src} \left( \sum_c \left( \sum_l (((pTradeIrCost_{d,src,r,y,l} + pTradeIrMarkup_{d,src,r,y,l}) * \mathbf{vTradeIr}_{d,c,src,r,y,l})) \right) \right) \\ & - \sum_{d,dst} \left( \sum_c \left( \sum_l ((pTradeIrMarkup_{d,r,dst,y,l} * \mathbf{vTradeIr}_{d,c,r,dst,y,l})) \right) \right) \end{aligned} \quad (58)$$

Export to ROW upper constraint

$eqExportRowUp_{x,c,r,y,l}$

$$\mathbf{vExportRow}_{x,c,r,y,l} \leq pExportRowUp_{x,r,y,l} \quad (59)$$

Export to ROW lower constraint

$eqExportRowLo_{x,c,r,y,l}$

$$\mathbf{vExportRow}_{x,c,r,y,l} \geq pExportRowLo_{x,r,y,l} \quad (60)$$

Cumulative export to ROW

$eqExportRowCumulative_{x,c}$

$$\mathbf{vExportRowAccumulated}_{x,c} = \sum_{r,y,l} (pPeriodLen_y * \mathbf{vExportRow}_{x,c,r,y,l}) \quad (61)$$

Cumulative export to ROW upper constraint

$eqExportRowResUp_{x,c}$

$$\mathbf{vExportRowAccumulated}_{x,c} \leq pExportRowRes_x \quad (62)$$

Import from ROW upper constraint

$eqImportRowUp_{m,c,r,y,l}$

$$\mathbf{vImportRow}_{m,c,r,y,l} \leq pImportRowUp_{m,r,y,l} \quad (63)$$

Import of ROW lower constraint

$eqImportRowLo_{m,c,r,y,l}$

$$\mathbf{vImportRow}_{m,c,r,y,l} \geq pImportRowLo_{m,r,y,l} \quad (64)$$

Cumulative import from ROW

$eqImportRowAccumulated_{m,c}$

$$\mathbf{vImportRowAccumulated}_{m,c} = \sum_{r,y,l} (pPeriodLen_y * \mathbf{vImportRow}_{m,c,r,y,l}) \quad (65)$$

Cumulative import from ROW upper constraint

$eqImportRowResUp_{m,c}$

$$\mathbf{vImportRowAccumulated}_{m,c} \leq pImportRowRes_m \quad (66)$$

Trade capacity

$eqTradeCapFlow_{d,c,y,l}$

$$pSliceShare_l * pTradeCap2Act_d * \mathbf{vTradeCap}_{d,y} \geq \sum_{src,dst} (\mathbf{vTradeIr}_{d,c,src,dst,y,l}) \quad (67)$$

Trade overnight investment costs

$eqTradeCap_{d,y}$

$$\mathbf{vTradeCap}_{d,y} = pTradeStock_{d,y} + \sum_{yp} (pPeriodLen_{yp} * \mathbf{vTradeNewCap}_{d,yp}) \quad (68)$$

Trade equivalent annual costs

$eqTradeInv_{d,r,y}$

$$\mathbf{vTradeInv}_{d,r,y} = pTradeInv_{cost_{d,r,y}} * \mathbf{vTradeNewCap}_{d,y} \quad (69)$$

Trade capacity to activity

$eqTradeEac_{d,r,y}$

$$\mathbf{vTradeEac}_{d,r,y} = \sum_{yp} (pTradeEac_{d,r,yp} * pPeriodLen_{yp} * \mathbf{vTradeNewCap}_{d,yp}) \quad (70)$$

### Trade IR capacity equations

#### Auxiliary input & output equations

Trade auxiliary commodity input

$$eqTradeIrAInp_{d,c,r,y,l}$$

$$\mathbf{vTradeIrAInp}_{d,c,r,y,l} = \sum_{dst} \left( pTradeIrCsrc2Ainp_{d,c,r,dst,y,l} * \sum_{cp} (\mathbf{vTradeIr}_{d,cp,r,dst,y,l}) \right) + \sum_{src} \left( pTradeIrCdst2Ainp_{d,c,src,r,y,l} * \sum_{cp} (\mathbf{vTradeIr}_{d,cp,src,r,y,l}) \right) \quad (71)$$

Trade auxiliary commodity output

$$eqTradeIrAOut_{d,c,r,y,l}$$

$$\mathbf{vTradeIrAOut}_{d,c,r,y,l} = \sum_{dst} \left( pTradeIrCsrc2Aout_{d,c,r,dst,y,l} * \sum_{cp} (\mathbf{vTradeIr}_{d,cp,r,dst,y,l}) \right) + \sum_{src} \left( pTradeIrCdst2Aout_{d,c,src,r,y,l} * \sum_{cp} (\mathbf{vTradeIr}_{d,cp,src,r,y,l}) \right) \quad (72)$$

Trade auxiliary commodity input

$$eqTradeIrAInpTot_{c,r,y,l}$$

$$\mathbf{vTradeIrAInpTot}_{c,r,y,l} = \sum_{d,lp} (\mathbf{vTradeIrAInp}_{d,c,r,y,lp}) \quad (73)$$

Trade auxiliary commodity output

$$eqTradeIrAOutTot_{c,r,y,l}$$

$$\mathbf{vTradeIrAOutTot}_{c,r,y,l} = \sum_{d,lp} (\mathbf{vTradeIrAOut}_{d,c,r,y,lp}) \quad (74)$$

#### Balance equations & dummy import & export

PRODUCTION<sub>i</sub> = CONSUMPTION commodity balance

$$eqBalLo_{c,r,y,l}$$

$$\mathbf{vBalance}_{c,r,y,l} \geq 0 \quad (75)$$

PRODUCTION<sub>j</sub> = CONSUMPTION commodity balance

$$eqBalUp_{c,r,y,l}$$

$$\mathbf{vBalance}_{c,r,y,l} \leq 0 \quad (76)$$

PRODUCTION == CONSUMPTION commodity balance  
 $eqBalFx_{c,r,y,l}$

$$\mathbf{vBalance}_{c,r,y,l} = 0 \quad (77)$$

Commodity balance  
 $eqBal_{c,r,y,l}$

$$\mathbf{vBalance}_{c,r,y,l} = \mathbf{vOutTot}_{c,r,y,l} - \mathbf{vInpTot}_{c,r,y,l} \quad (78)$$

Total commodity output  
 $eqOutTot_{c,r,y,l}$

$$\begin{aligned} \mathbf{vOutTot}_{c,r,y,l} = & \mathbf{vDummyImport}_{c,r,y,l} + \mathbf{vSupOutTot}_{c,r,y,l} + \mathbf{vEmsFuelTot}_{c,r,y,l} + \mathbf{vAggOut}_{c,r,y,l} + \mathbf{vTechOutTot}_{c,r,y,l} \\ & + \mathbf{vStorageOutTot}_{c,r,y,l} + \mathbf{vImport}_{c,r,y,l} + \mathbf{vTradeIrAOutTot}_{c,r,y,l} + \sum_{lp} (\mathbf{vOut2Lo}_{c,r,y,lp,l}) \end{aligned} \quad (79)$$

Total commodity input  
 $eqOut2Lo_{c,r,y,l}$

$$\begin{aligned} \sum_{lp} (\mathbf{vOut2Lo}_{c,r,y,l,lp}) = & \mathbf{vSupOutTot}_{c,r,y,l} + \mathbf{vEmsFuelTot}_{c,r,y,l} + \mathbf{vAggOut}_{c,r,y,l} + \mathbf{vTechOutTot}_{c,r,y,l} \\ & + \mathbf{vStorageOutTot}_{c,r,y,l} + \mathbf{vImport}_{c,r,y,l} + \mathbf{vTradeIrAOutTot}_{c,r,y,l} \end{aligned} \quad (80)$$

From commodity slice to lo level  
 $eqInpTot_{c,r,y,l}$

$$\begin{aligned} \mathbf{vInpTot}_{c,r,y,l} = & \mathbf{vDemInp}_{c,r,y,l} + \mathbf{vDummyExport}_{c,r,y,l} + \mathbf{vTechInpTot}_{c,r,y,l} + \mathbf{vStorageInpTot}_{c,r,y,l} \\ & + \mathbf{vExport}_{c,r,y,l} + \mathbf{vTradeIrAInpTot}_{c,r,y,l} + \sum_{lp} (\mathbf{vInp2Lo}_{c,r,y,lp,l}) \end{aligned} \quad (81)$$

From commodity slice to lo level  
 $eqInp2Lo_{c,r,y,l}$

$$\sum_{lp} (\mathbf{vInp2Lo}_{c,r,y,l,lp}) = \mathbf{vTechInpTot}_{c,r,y,l} + \mathbf{vStorageInpTot}_{c,r,y,l} + \mathbf{vExport}_{c,r,y,l} + \mathbf{vTradeIrAInpTot}_{c,r,y,l} \quad (82)$$

Supply total output

*eqSupOutTot*<sub>c,r,y,l</sub>

$$\mathbf{vSupOutTot}_{c,r,y,l} = \sum_u \left( \sum_{lp} (\mathbf{vSupOut}_{u,c,r,y,lp}) \right) \quad (83)$$

Technology total input

*eqTechInpTot*<sub>c,r,y,l</sub>

$$\mathbf{vTechInpTot}_{c,r,y,l} = \sum_h \left( \sum_{lp} (\mathbf{vTechInp}_{h,c,r,y,lp}) \right) + \sum_h \left( \sum_{lp} (\mathbf{vTechAInp}_{h,c,r,y,lp}) \right) \quad (84)$$

Technology total output

*eqTechOutTot*<sub>c,r,y,l</sub>

$$\mathbf{vTechOutTot}_{c,r,y,l} = \sum_h \left( \sum_{lp} (\mathbf{vTechOut}_{h,c,r,y,lp}) \right) + \sum_h \left( \sum_{lp} (\mathbf{vTechAOut}_{h,c,r,y,lp}) \right) \quad (85)$$

Storage total input

*eqStorageInpTot*<sub>c,r,y,l</sub>

$$\mathbf{vStorageInpTot}_{c,r,y,l} = \sum_s (\mathbf{vStorageInp}_{s,c,r,y,l}) + \sum_s (\mathbf{vStorageAInp}_{s,c,r,y,l}) \quad (86)$$

Storage total output

*eqStorageOutTot*<sub>c,r,y,l</sub>

$$\mathbf{vStorageOutTot}_{c,r,y,l} = \sum_s (\mathbf{vStorageOut}_{s,c,r,y,l}) + \sum_s (\mathbf{vStorageAOut}_{s,c,r,y,l}) \quad (87)$$

## Objective and aggregated costs equations

Total costs

*eqCost*<sub>r,y</sub>

$$\begin{aligned} \mathbf{vTotalCost}_{r,y} = & \sum_h (\mathbf{vTechEac}_{h,r,y}) + \sum_h (\mathbf{vTechOMCost}_{h,r,y}) + \sum_u (\mathbf{vSupCost}_{u,r,y}) + \sum_{c,l} (pDummyImportCost_{c,r,y,l} * \mathbf{vDummyImport}_{c,r,y,l}) \\ & + \sum_{c,l} (pDummyExportCost_{c,r,y,l} * \mathbf{vDummyExport}_{c,r,y,l}) + \sum_c (\mathbf{vTaxCost}_{c,r,y}) - \sum_c (\mathbf{vSubsCost}_{c,r,y}) \\ & + \sum_s (\mathbf{vStorageOMCost}_{s,r,y}) + \sum_s (\mathbf{vStorageEac}_{s,r,y}) + \mathbf{vTradeCost}_{r,y} + \mathbf{vTotalUserCosts}_{r,y} \end{aligned} \quad (88)$$

Commodity taxes

*eqTaxCost<sub>c,r,y</sub>*

$$\mathbf{vTaxCost}_{c,r,y} = \sum_l (pTaxCostOut_{c,r,y,l} * \mathbf{vOutTot}_{c,r,y,l}) + \sum_l (pTaxCostInp_{c,r,y,l} * \mathbf{vInpTot}_{c,r,y,l}) + \sum_l (pTaxCostBal_{c,r,y,l} * \mathbf{vBalance}_{c,r,y,l}) \quad (89)$$

Commodity subsidy

*eqSubsCost<sub>c,r,y</sub>*

$$\mathbf{vSubsCost}_{c,r,y} = \sum_l (pSubCostOut_{c,r,y,l} * \mathbf{vOutTot}_{c,r,y,l}) + \sum_l (pSubCostInp_{c,r,y,l} * \mathbf{vInpTot}_{c,r,y,l}) + \sum_l (pSubCostBal_{c,r,y,l} * \mathbf{vBalance}_{c,r,y,l}) \quad (90)$$

Objective equation

*eqObjective*

$$\mathbf{vObjective} = \sum_{r,y} (\mathbf{vTotalCost}_{r,y} * pDiscountFactorMileStone_{r,y}) \quad (91)$$

**LEC equation**

levelized costs (auxiliary equation)

*eqLECActivity<sub>h,r,y</sub>*

$$\sum_l (\mathbf{vTechAct}_{h,r,y,l}) \geq pLECLoACT_r \quad (92)$$