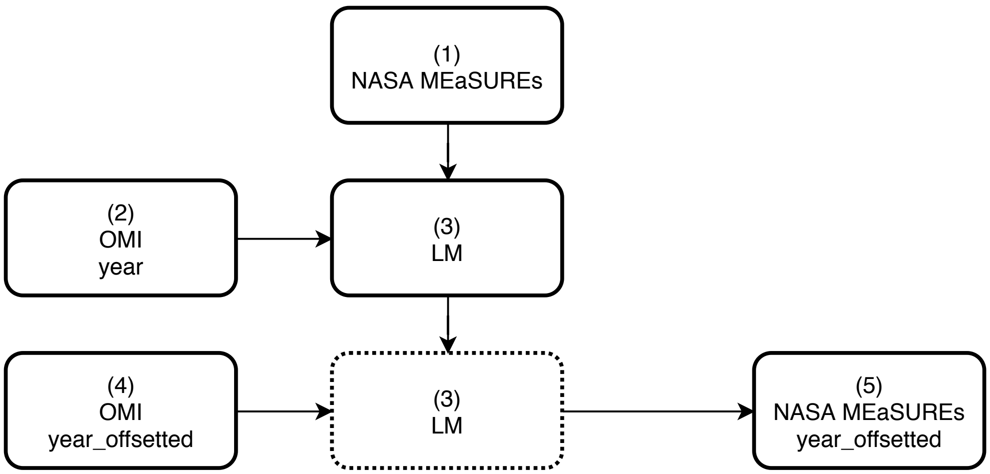

The uncertainty in (5) is the combination of both the model uncertainty (3) and the NASA MEaSUREs (1) one.

$$Y_i = a + BX_i + (m_i + r_i)$$

where:
- Yi is emission at year i
- Xi is set of predictors (e.g. concentrations at various radii)
- m_i "measurement" error from NASA
- r_i residuals from linear model

Error from model

To estimate the error & confidence interval **for each prediction**,
we directly ask lm predict method to give us a **confidence -or- prediction interval**

```
predicted, predicted_lower, predicted_upper <- predict(m, data, interval="confidence", level=0.05)
```

-or-

```
predicted, predicted_lower, predicted_upper <- predict(m, data, interval="confidence", level=0.05)
```

That's where lies a key uncertainty

Read more here:
https://rpubs.com/aaronsc32/

This gives us a 95% confidence interval. We retrofit an equivalent σ using:

σ_{model} (source, year) = [predicted_upper(source, year) - predicted_lower(source, year)] / 2 / 1.96

/regression-confidence-prediction-intervals

Error from NASA

In the prediction, a "year" corresponds to Sep (year -1) -> Aug (year).
The attached NASA uncertainty for a given source in a given "offsetted year" is approximated by:

$$\sigma_{NASA}\text{(source, year)} = 4/12^{*}\ \sigma_{NASA}\text{(source, year-1)} + 8/12^{*}\ \sigma_{NASA}\text{(source, year)]}$$

For 2020, we assume σ_{NASA} (source, 2020) = σ_{NASA} (source, 2019)

Combining both -> Error on the sum of emissions

For each prediction (i.e. for each source and year), we now have both σ_{model} (source, year) and σ_{NASA} (source, year).
We assume the errors are independent, and therefore use Var(X+Y) = Var(X) + Var(Y) to derive:

$$\sigma_{TOTAL}\text{(sector, year)} = \text{sqrt[sum for all sector sources(}\ \sigma_{NASA}\text{(source, year)}^2 + \sigma_{model}\text{(source, year)}^2\text{)]}$$

We then compute the 95% confidence interval using:

$$\text{prediction\_lower(sector, year)} = \text{prediction(source, year)} - 1.96 * \sigma_{TOTAL}\text{(sector, year)}$$

$$\text{prediction\_upper(sector, year)} = \text{prediction(source, year)} + 1.96 * \sigma_{TOTAL}\text{(sector, year)}$$

Y-o-y uncertainty

The uncertainty of the y-o-y ratio can be derived from uncertainties in year y and year y-1, using for instance the Fieller method (https://zenodo.org/record/820551/files/review_CI_ratio.pdf)