Impact of Input Data Preparation on Multi-Criteria Decision Analysis Results

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1 Multi-Criteria Decision Analysis (MCDA) methods

1.1 The ARAS method

The following steps of the ARAS method (Additive Ratio Assessment) are described based on [4]. Like other MCDA methods, this method requires a decision matrix with performance values of m alternatives regarding n evaluation criteria.

Step 1. Normalize the decision matrix with the Sum normalization method using Equation (1) for profit criteria and Equation (2) for cost criteria

$$r_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}} \tag{1}$$

$$r_{ij} = \frac{\frac{1}{x_{ij}}}{\sum_{i=1}^{m} \frac{1}{x_{ij}}} \tag{2}$$

where $X = [x_{ij}]_{m \times n}$ denotes the decision matrix including performance values of m alternatives regarding n evaluation criteria. The default normalization method for ARAS is sum normalization.

Step 2. Calculate the weighted normalized decision matrix $D = [d_{ij}]_{m \times n}$ with Equation (3)

$$d_{ij} = r_{ij} w_j \tag{3}$$

where w_j represents jth criteria weight values.

Step 3. Calculate the optimality function S_i for each ith alternative using Equation (4).

$$S_i = \sum_{j=1}^{n} d_{ij} \tag{4}$$

Step 4. Calculate the utility value U_i for each ith alternative as Equation (5) shows

$$U_i = S_i/S_o \tag{5}$$

where S_o represents the optimality function value for the optimal alternative. U_i values are between 0 and 1. The best-ranked alternative is the option with the highest U_i value. Thus, alternatives are ranked descendingly according to U_i values to build the ARAS ranking.

1.2 The CoCoSo method

The CoCoSo method is detailed based on [10].

Step 1. Normalization of decision matrix using Minimum-Maximum normalization method using Equation (6) for profit criteria and Equation (7) for cost criteria.

$$r_{ij} = \frac{x_{ij} - min_j(x_{ij})}{max_j(x_{ij}) - min_j(x_{ij})} \tag{6}$$

$$r_{ij} = \frac{max_j(x_{ij}) - x_{ij}}{max_j(x_{ij}) - min_j(x_{ij})} \tag{7}$$

where j represent criteria ($j = 1, 2, \ldots, n$) and i denotes alternatives ($i = 1, 2, \ldots, m$).

Step 2. The total of the weighted comparability sequence and the whole of the power weight of comparability sequences for each alternative sum of the weighted comparability sequence and also an amount of the power weight of comparability sequences for each alternative as S_i using Equation (8) and P_i by Equation (9)

$$S_i = \sum_{j=1}^{n} (w_j r_{ij}) \tag{8}$$

$$P_i = \sum_{j=1}^{n} (r_{ij})^{w_j} \tag{9}$$

where w_j denotes criteria weights.

Step 3. Relative weights of the alternatives using the following aggregation strategies are computed. In this step, three appraisal score strategies are used to generate relative weights of other options, which are derived using following Equations (10), (11) and (12).

$$k_{ia} = \frac{P_i + S_i}{\sum_{i=1}^{m} P_i + S_i} \tag{10}$$

$$k_{ib} = \frac{S_i}{\min_i S_i} + \frac{P_i}{\min_i P_i} \tag{11}$$

$$k_{ic} = \frac{\lambda(S_i) + (1-\lambda)(P_i)}{(\lambda \underset{i}{max} S_i + (1-\lambda)\underset{i}{max} P_i)};\ 0 \leq \lambda \leq 1 \tag{12}$$

where λ in Equation (12) can be set by decision-makers and is usually equal to 0.5

Step 4. The final ranking of the alternatives is determined based on k_i, values obtained using Equation (13). The best evaluated alternative has the highest k_i value.

$$k_i = (k_{ia}k_{ib}k_{ic})^{\frac{1}{3}} + \frac{1}{3}(k_{ia} + k_{ib} + k_{ic}) \tag{13}$$

1.3 The TOPSIS Method

The following stages of the TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) method are presented in the following steps [3].

Step 1. Normalization of decision matrix. This procedure is performed in original TOPSIS algorithm with vector normalization technique, as demonstrated in Equation 14 for profit r_{ij}^{+} and cost r_{ij}^{-} criteria,

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}},\ r_{ij} = 1 - \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \tag{14}$$

where $X = [x_{ij}]_{m \times n}$ denotes the decision matrix including performance values collected for m alternatives considering n criteria of assessment.

Step 2. Calculate the weighted normalized decision matrix according to Equation (15),

$$v_{ij} = w_j r_{ij} \tag{15}$$

where w_j represents j-th criteria weight values. Weights are determined with the selected weighting method.

Step 3. Calculate the Positive Ideal Solution v_j^{+} (PIS) and Negative Ideal Solution v_j^{-} (NIS) following Equation (16). PIS denotes a vector with the maximum values of the weighted normalized decision matrix, and NIS includes its minimal values. Splitting criteria into profit and cost types is unnecessary because the decision matrix was normalized in the previous step.

$$v_j^{+} = \{v_1^{+}, v_2^{+}, \ldots, v_n^{+}\} = \{max_j(v_{ij})\},\ v_j^{-} = \{v_1^{-}, v_2^{-}, \ldots, v_n^{-}\} = \{min_j(v_{ij})\} \tag{16}$$

Step 4. Compute the distance from PIS D_i^{+} and NIS D_i^{-} for each considered alternative. The default metric for distance determination in the TOPSIS method is Euclidean distance.

$$D_i^{+} = \sqrt{\sum_{j=1}^{n}(v_{ij} - v_j^{+})^2},\ D_i^{-} = \sqrt{\sum_{j=1}^{n}(v_{ij} - v_j^{-})^2} \tag{17}$$

Step 5. Calculate the TOPSIS preference values for each alternative for period p as Equation (18) demonstrates. The C_i takes values from 0 to 1. The highest C_i value represents best scored alternative.

$$C_i = \frac{D_i^-}{D_i^- + D_i^+} \tag{18}$$

2 Normalization methods

2.1 Minimum-Maximum Normalization

. When using the minimum-maximum normalization, the normalized values r_{ij}^+ for profit criteria and r_{ij}^- for cost criteria are calculated by the Equation (19). This method is applied by default for CoCoSo [10] and MABAC [8].

$$r_{ij}^+ = \frac{x_{ij} - min_j(x_{ij})}{max_j(x_{ij}) - min_j(x_{ij})},\ r_{ij}^- = \frac{max_j(x_{ij}) - x_{ij}}{max_j(x_{ij}) - min_j(x_{ij})} \tag{19}$$

This method has the advantage that the measurement scale is precisely between 0 and 1 for each attribute. Therefore, this procedure is also adequate for data including negative or zero values.

2.2 Maximum Normalization

. The maximum normalization method is a technique in which only the maximum value for each criterion is used to obtain normalized values r_{ij}^+ for profit criteria and r_{ij}^- for cost criteria as shown in Equation (20).

$$r_{ij}^+ = \frac{x_{ij}}{max_j(x_{ij})},\ r_{ij}^- = 1 - \frac{x_{ij}}{max_j(x_{ij})} \tag{20}$$

2.3 Sum Normalization

. In the sum method, all values in the evaluated set are summed. This normalization method is applied by default for COPRAS and ARAS [5]. Normalized values r_{ij}^+ for profit criteria and r_{ij}^- for cost criteria are received using Equation 21.

$$r_{ij}^+ = \frac{x_{ij}}{\sum_{i=1}^m x_{ij}},\ r_{ij}^- = \frac{\frac{1}{x_{ij}}}{\sum_{i=1}^m \frac{1}{x_{ij}}} \tag{21}$$

2.4 Linear Normalization

. Normalized values for profit criteria r_{ij}^+ and for cost criteria r_{ij}^- are obtained using linear normalization as Equation (22) shows. It is the default normalization method for CODAS [1] and WASPAS [9].

$$r_{ij}^+ = \frac{x_{ij}}{max_j(x_{ij})},\ r_{ij}^- = \frac{min_j(x_{ij})}{x_{ij}} \tag{22}$$

2.5 Vector Normalization

. In the vector method, the square root of all values is computed. Formulas for achieving normalized values r_{ij}^{+} for profit criteria and r_{ij}^{-} for cost criteria are presented in Equation (23). This normalization technique is the default for TOPSIS [2], MOORA (Multi-Objective Optimization on the basis of Ratio Analysis) [6], and MULTIMOORA methods [7].

$$r_{ij}^{+} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^{2}}},\ r_{ij}^{-} = 1 - \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^{2}}} \tag{23}$$

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