Impact of Input Data Preparation on Multi-Criteria Decision Analysis Results

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1 MCDA methods

1.1 The ARAS method

The following steps of the ARAS method (Additive Ratio Assessment) are described based on [2]. Like other MCDA methods, this method requires a decision matrix with performance values of m alternatives regarding n evaluation criteria.

Step 1. Normalize the decision matrix with the Sum normalization method using Equation (1) for profit criteria and Equation (2) for cost criteria

$$r_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}} \tag{1}$$

$$r_{ij} = \frac{\frac{1}{x_{ij}}}{\sum_{i=1}^{m} \frac{1}{x_{ij}}} \tag{2}$$

where $X = [x_{ij}]_{m \times n}$ denotes the decision matrix including performance values of m alternatives regarding n evaluation criteria. The default normalization method for ARAS is sum normalization.

Step 2. Calculate the weighted normalized decision matrix $D = [d_{ij}]_{m \times n}$ with Equation (3)

$$d_{ij} = r_{ij} w_j \tag{3}$$

where w_j represents jth criteria weight values.

Step 3. Calculate the optimality function S_i for each ith alternative using Equation (4).

$$S_i = \sum_{j=1}^{n} d_{ij} \tag{4}$$

Step 4. Calculate the utility value U_i for each ith alternative as Equation (5) shows

$$U_i = S_i/S_o \tag{5}$$

where S_o represents the optimality function value for the optimal alternative. U_i values are between 0 and 1. The best-ranked alternative is the option with the highest U_i value. Thus, alternatives are ranked descendingly according to U_i values to build the ARAS ranking.

1.2 The CoCoSo method

The CoCoSo method is detailed based on [3].

Step 1. Normalization of decision matrix using Minimum-Maximum normalization method using Equation (6) for profit criteria and Equation (7) for cost criteria.

$$r_{ij} = \frac{x_{ij} - min_j(x_{ij})}{max_j(x_{ij}) - min_j(x_{ij})} \tag{6}$$

$$r_{ij} = \frac{max_j(x_{ij}) - x_{ij}}{max_j(x_{ij}) - min_j(x_{ij})} \tag{7}$$

where j represent criteria ($j = 1, 2, \ldots, n$) and i denotes alternatives ($i = 1, 2, \ldots, m$).

Step 2. The total of the weighted comparability sequence and the whole of the power weight of comparability sequences for each alternative sum of the weighted comparability sequence and also an amount of the power weight of comparability sequences for each alternative as S_i using Equation (8) and P_i by Equation (9)

$$S_i = \sum_{j=1}^{n} (w_j r_{ij}) \tag{8}$$

$$P_i = \sum_{j=1}^{n} (r_{ij})^{w_j} \tag{9}$$

where w_j denotes criteria weights.

Step 3. Relative weights of the alternatives using the following aggregation strategies are computed. In this step, three appraisal score strategies are used to generate relative weights of other options, which are derived using following Equations (10), (11) and (12).

$$k_{ia} = \frac{P_i + S_i}{\sum_{i=1}^{m} P_i + S_i} \tag{10}$$

$$k_{ib} = \frac{S_i}{\underset{i}{min} S_i} + \frac{P_i}{\underset{i}{min} P_i} \tag{11}$$

$$k_{ic} = \frac{\lambda(S_i) + (1-\lambda)(P_i)}{(\lambda \max_i S_i + (1-\lambda) \max_i P_i)};\ 0 \leq \lambda \leq 1 \tag{12}$$

where λ in Equation (12) can be set by decision-makers and is usually equal to 0.5

Step 4. The final ranking of the alternatives is determined based on k_i, values obtained using Equation (13). The best evaluated alternative has the highest k_i value.

$$k_i = (k_{ia} k_{ib} k_{ic})^{\frac{1}{3}} + \frac{1}{3}(k_{ia} + k_{ib} + k_{ic}) \tag{13}$$

1.3 The TOPSIS Method

The following stages of the TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) method are presented in the following steps [1].

Step 1. Normalization of decision matrix. This procedure is performed in original TOPSIS algorithm with vector normalization technique, as demonstrated in Equation 14 for profit r_{ij}^{+} and cost r_{ij}^{-} criteria,

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}},\ r_{ij} = 1 - \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \tag{14}$$

where $X = [x_{ij}]_{m \times n}$ denotes the decision matrix including performance values collected for m alternatives considering n criteria of assessment.

Step 2. Calculate the weighted normalized decision matrix according to Equation (15),

$$v_{ij} = w_j r_{ij} \tag{15}$$

where w_j represents j-th criteria weight values. Weights are determined with the selected weighting method.

Step 3. Calculate the Positive Ideal Solution v_j^{+} (PIS) and Negative Ideal Solution v_j^{-} (NIS) following Equation (16). PIS denotes a vector with the maximum values of the weighted normalized decision matrix, and NIS includes its minimal values. Splitting criteria into profit and cost types is unnecessary because the decision matrix was normalized in the previous step.

$$v_j^{+} = \{v_1^{+}, v_2^{+}, \ldots, v_n^{+}\} = \{max_j(v_{ij})\},\ v_j^{-} = \{v_1^{-}, v_2^{-}, \ldots, v_n^{-}\} = \{min_j(v_{ij})\} \tag{16}$$

Step 4. Compute the distance from PIS D_i^{+} and NIS D_i^{-} for each considered alternative. The default metric for distance determination in the TOPSIS method is Euclidean distance.

$$D_i^{+} = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^{+})^2},\ D_i^{-} = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^{-})^2} \tag{17}$$

Step 5. Calculate the TOPSIS preference values for each alternative for period p as Equation (18) demonstrates. The C_i takes values from 0 to 1. The highest C_i value represents best scored alternative.

$$C_i = \frac{D_i^-}{D_i^- + D_i^+} \tag{18}$$

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References

1. Boumaiza, A., Sanfilippo, A., Mohandes, N.: Modeling multi-criteria decision analysis in residential PV adoption. Energy Strategy Reviews **39**, 100789 (2022). `https://doi.org/https://doi.org/10.1016/j.esr.2021.100789`
2. Goswami, S., Mitra, S.: Selecting the best mobile model by applying AHP-COPRAS and AHP-ARAS decision making methodology. International Journal of Data and Network Science **4**(1), 27–42 (2020). `https://doi.org/http://dx.doi.org/10.5267/j.ijdns.2019.8.004`
3. Yazdani, M., Zarate, P., Kazimieras Zavadskas, E., Turskis, Z.: A combined compromise solution (CoCoSo) method for multi-criteria decision-making problems. Management Decision **57**(9), 2501–2519 (2019). `https://doi.org/https://doi.org/10.1108/MD-05-2017-0458`