

Each MCDA method requires a decision matrix demonstrated by Equation (1)

$$X = [x_{ij}]_{m \times n} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \quad (1)$$

where m denotes the number of alternatives, and n represents the number of criteria.

1. The TOPSIS Method

The following stages of the TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) method are provided below.

Step 1. Normalization of the decision matrix. The Minimum-Maximum normalization method or another normalization method can be used for performing the normalization procedure. In Minimum-Maximum normalization r_{ij} normalized values are obtained applying Equation (2) for profit criteria and (3) for cost criteria.

$$r_{ij} = \frac{x_{ij} - \min_j(x_{ij})}{\max_j(x_{ij}) - \min_j(x_{ij})} \quad (2)$$

$$r_{ij} = \frac{\max_j(x_{ij}) - x_{ij}}{\max_j(x_{ij}) - \min_j(x_{ij})} \quad (3)$$

Step 2. Calculation of weighted normalized decision matrix using Equation (4).

$$v_{ij} = w_j r_{ij} \quad (4)$$

Step 3. Determination of Positive Ideal Solution with Equation (5) and Negative Ideal Solution with Equation (6). PIS contains the maximums of the weighted normalized decision matrix, while NIS contains its minimums. Due to the previous normalization application, there is no need to divide the criteria into profit and cost in this step.

$$v_j^+ = \{v_1^+, v_2^+, \dots, v_n^+\} = \{\max_j(v_{ij})\} \quad (5)$$

$$v_j^- = \{v_1^-, v_2^-, \dots, v_n^-\} = \{\min_j(v_{ij})\} \quad (6)$$

Step 4. Computation of distance from PIS (7) and NIS (8) for each alternative. The default metric for distance computation in TOPSIS algorithm is Euclidean distance.

$$D_i^+ = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^+)^2} \quad (7)$$

$$D_i^- = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^-)^2} \quad (8)$$

Step 5. Computation of the score for each considered alternative according to Equation (9). The C_i value is always between 0 to 1, and the alternative that has the highest C_i value is the best. It implies that for TOPSIS, the ranking is created by descending sorting of alternatives by preference value.

$$C_i = \frac{D_i^-}{D_i^- + D_i^+} \quad (9)$$

2. The VIKOR Method

The subsequent steps of the VIKOR (VIseKriterijumska Optimizacija Kompromisno Resenje) method are given below.

Step 1. Determination of the best f_j^* and the worst f_j^- values for each criteria functions. For profit criteria Equation (10) is applied and for cost criteria Equation (11) is employed.

$$f_j^* = \max_i f_{ij}, \quad f_j^- = \min_i f_{ij} \quad (10)$$

$$f_j^* = \min_i f_{ij}, \quad f_j^- = \max_i f_{ij} \quad (11)$$

Step 2. Computation of the S_i and R_i values using Equations (12) and (13).

$$S_i = \sum_{j=1}^n w_j (f_j^* - f_{ij}) / (f_j^* - f_j^-) \quad (12)$$

$$R_i = \max_j [w_j (f_j^* - f_{ij}) / (f_j^* - f_j^-)] \quad (13)$$

Step 3. Calculation of the Q_i values with Equation (14)

$$Q_i = v(S_i - S^*) / (S^- - S^*) + (1 - v)(R_i - R^*) / (R^- - R^*) \quad (14)$$

where

$$S^* = \min_i S_i, \quad S^- = \max_i S_i$$

$$R^* = \min_i R_i, \quad R^- = \max_i R_i$$

v denotes the weight assigned for the strategy of "most criteria". For computations in this paper $v = 0.5$ was chosen.

Step 4. The rankings of alternatives are constructed by sorting S , R , and Q values in ascending order. The result is three ranked lists.

Step 5. A compromise solution is proposed considering the conditions of good advantage and acceptable stability within the three vectors obtained in the previous step. The best alternative is the one with the lowest value and the leading position in the ranking Q .

3. The AHP method

Step 1. Normalization of the decision matrix (Min-Max normalization).

Step 2. Calculate weighted normalized decision matrix.

Step 3. Aggregate score values by summing rows of weighted normalized decision matrix for each alternative.