COUNTING I

Murat Osmanoglu

Counting

- Ali and Buse eat lunch together in a specific restaurant regularly.

Counting

'soup' XOR 'sandwich'

'soup' AND 'sandwich'

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Counting

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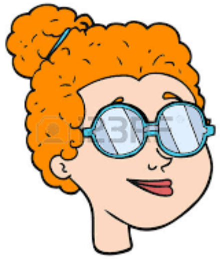

'soup' AND 'sandwich'

- Ali and Buse eat lunch together in a specific restaurant regularly.

- restaurant has 9 choices for soup
  and 16 choices for sandwich

Counting

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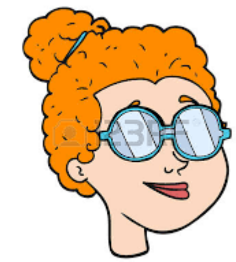

'soup' AND 'sandwich'

- Ali and Buse eat lunch together in a specific restaurant regularly.
- restaurant has 9 choices for soup and 16 choices for sandwich
- How many different meals can Ali order ?
- How many different meals can Buse order ?

Counting

- Ali can order either one soup among 9 different soups or one sandwich among 16 different sandwiches.

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- If $A_1, A_2, \ldots, A_n$ are mutually disjoint sets ($A_i \cap A_j = \emptyset$), then the number of ways of choosing a single element from A_1 or A_2 or … A_n is

$$|A_1 \cup \ldots \cup A_n| = |A_1| + \ldots + |A_n|$$

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Product Rule

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- apply the Product Rule here : 9 . 16 = 144 different choices

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- If $A_1, A_2, \ldots, A_n$ are finite sets, then the number of ways of choosing an element from $A_1, \ldots,$ an element from A_n is

$$|A_1 \times \ldots \times A_n| = |A_1| \cdot |A_2| \cdot \ldots \cdot |A_n|$$

Counting

- How many bit-strings can you create with 3-digits ?

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  101, 001, 110, . . .

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{a, b, c}

{a, b}

{c}

Counting

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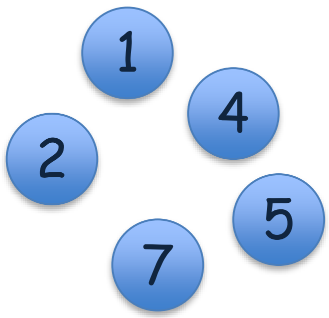
- What is the number of subsets of the set {a, b, c} ?

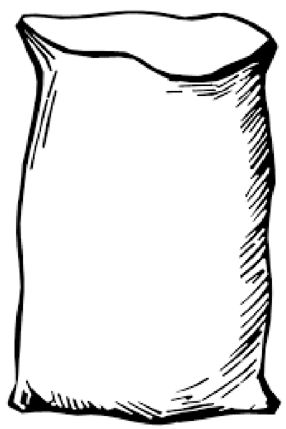
$\{a, b, c\} \longrightarrow 1\ 1\ 1$

$\{a, b\} \longrightarrow 1\ 1\ 0$

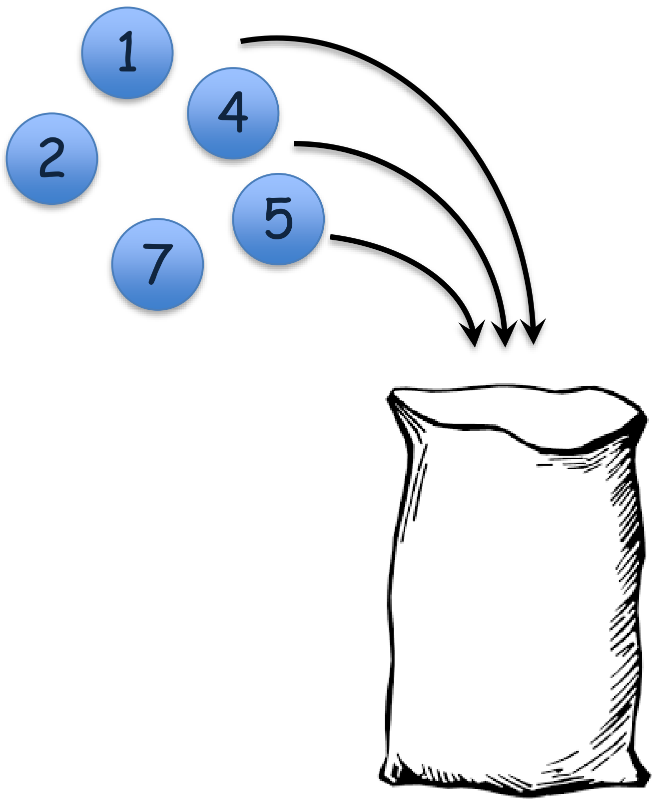
$\{c\} \longrightarrow 0\ 0\ 1$

Counting





Counting



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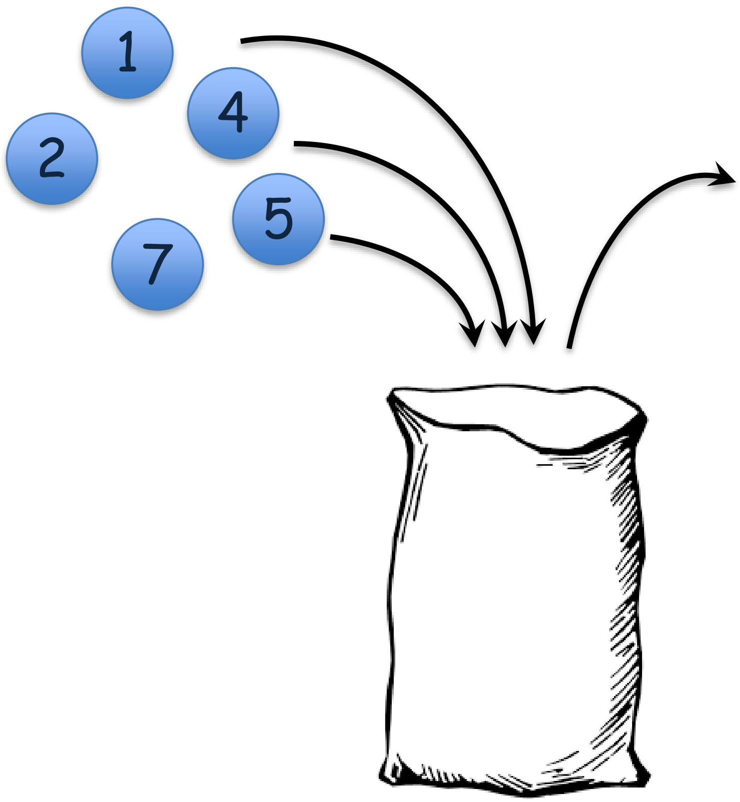
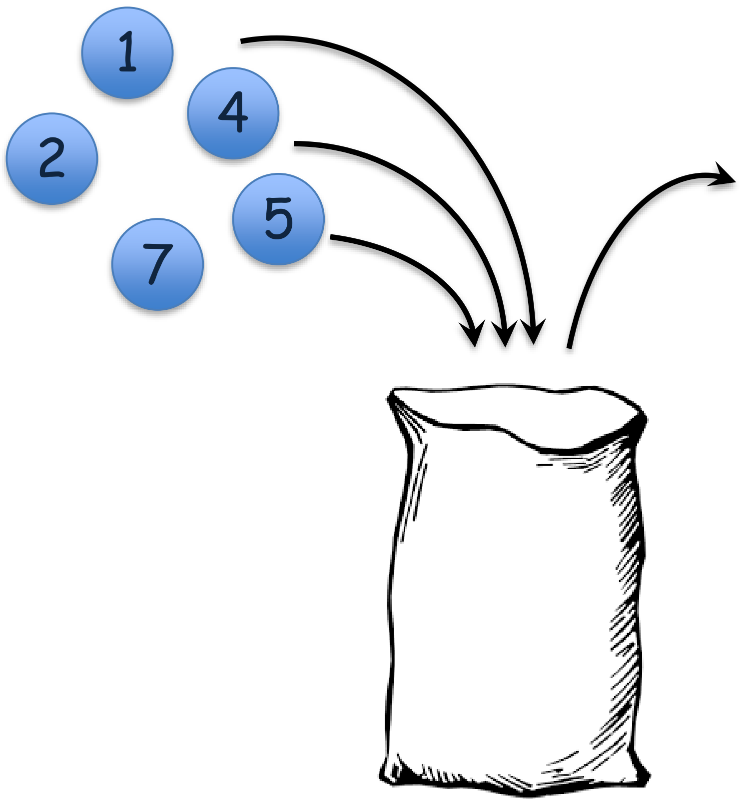


you pick one ball at a time

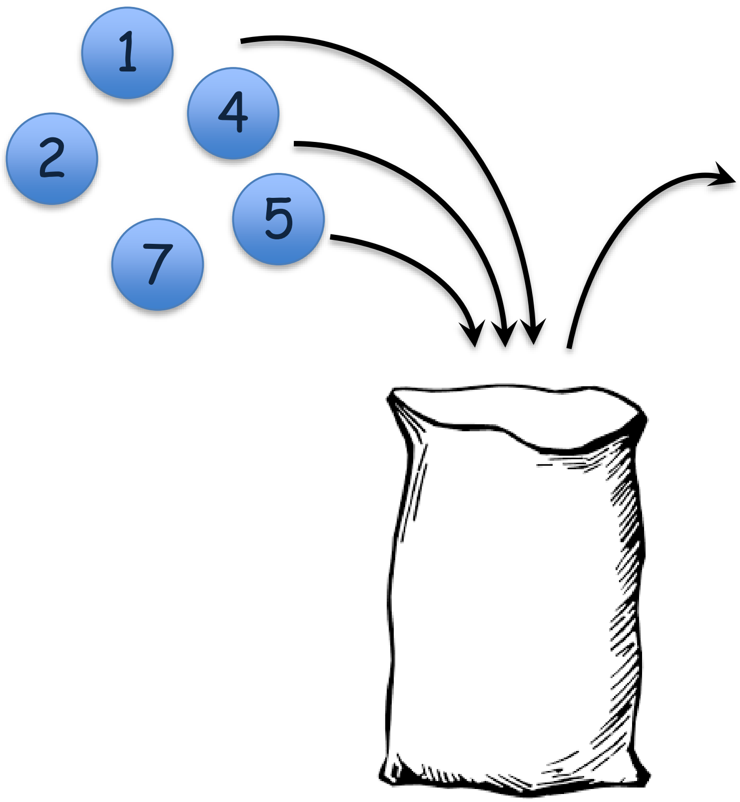
Counting



you pick one ball at a time

How many 3-digits numbers can you create with the picked numbers ?

Counting

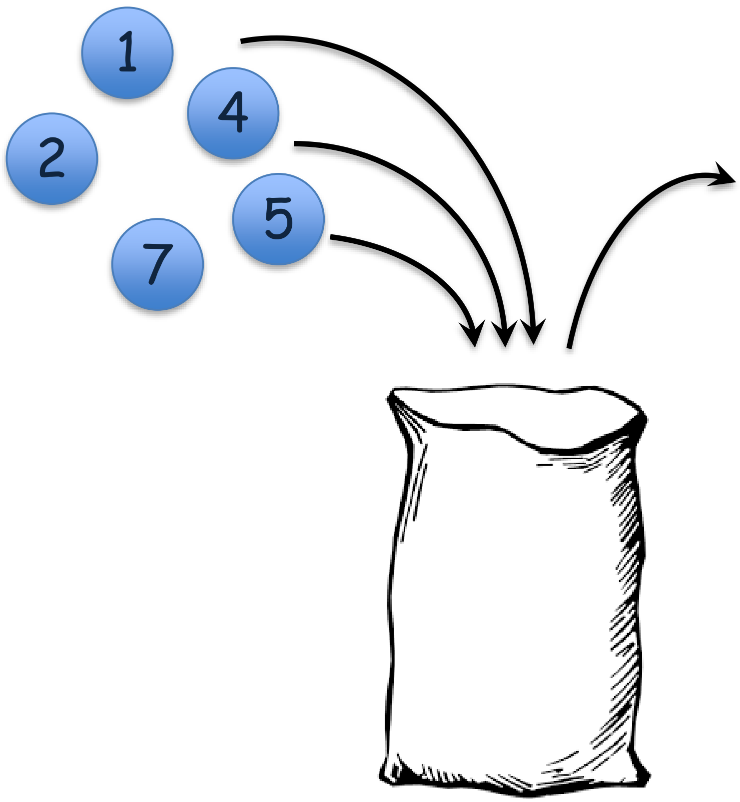


you pick one ball at a time

How many 3-digits numbers can you create with the picked numbers ?

- If you leave them to the bag after you pick

Counting



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- If you leave them to the bag after you pick
- If you keep them after you pick

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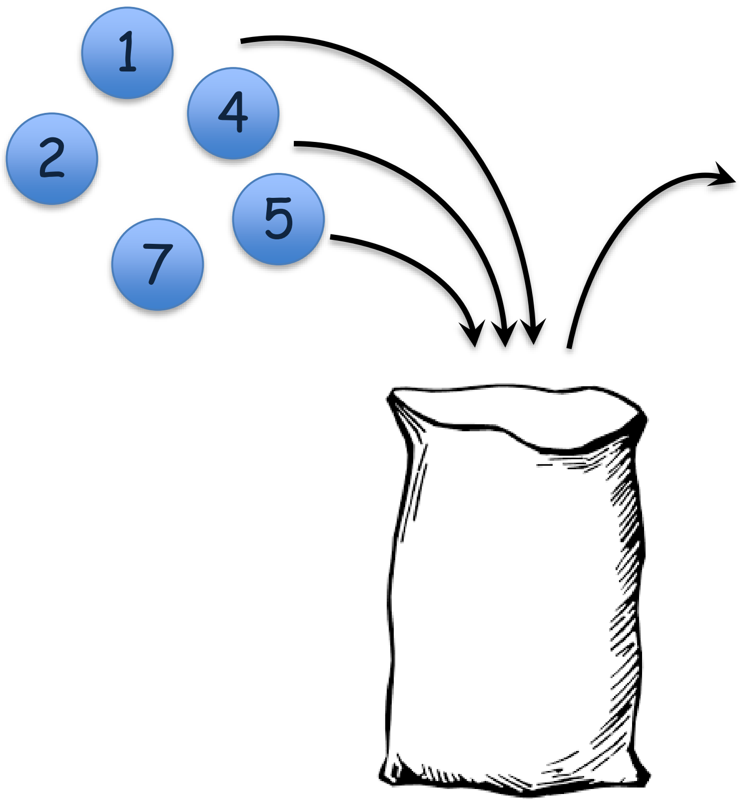

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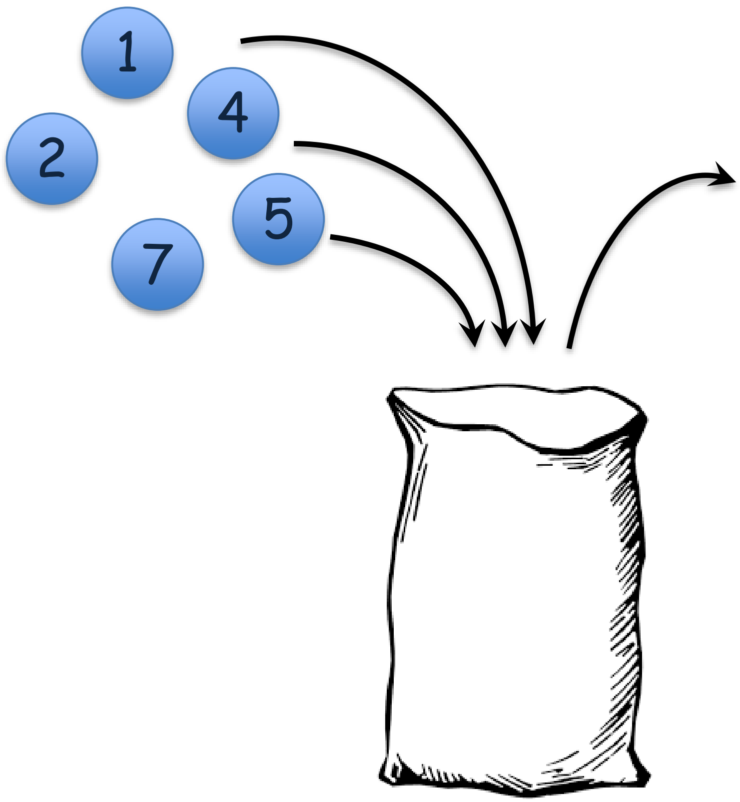
  ___ ___ ___

- If you keep them after you pick

  ___ ___ ___

Counting



you pick one ball at a time

How many 3-digits numbers can you create with the picked numbers ?

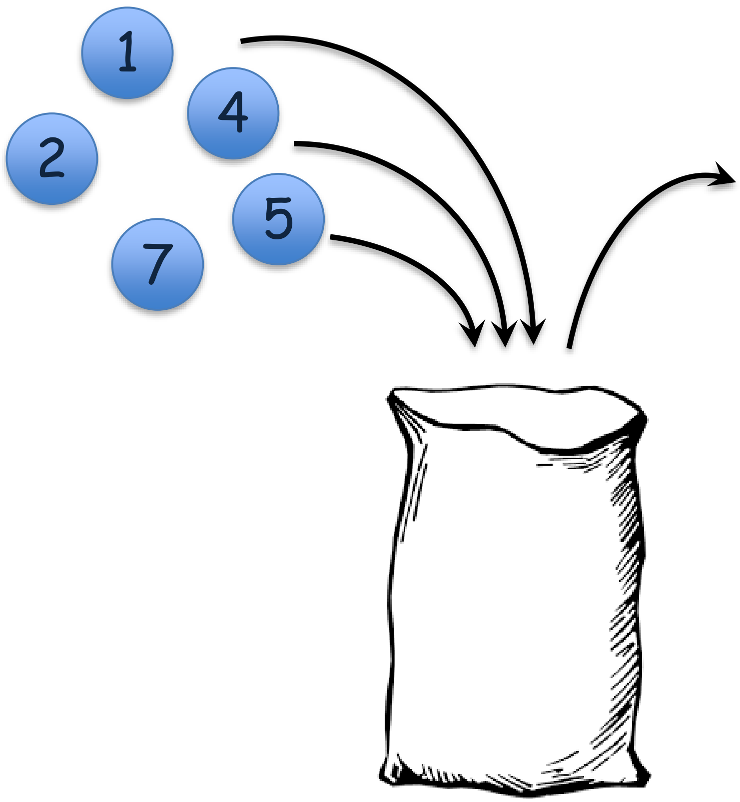
- If you leave them to the bag after you pick

$$\underline{5} \times \underline{5} \times \underline{5} = 125$$

- If you keep them after you pick

___ ___ ___

Counting



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How many 3-digits numbers can you create with the picked numbers ?

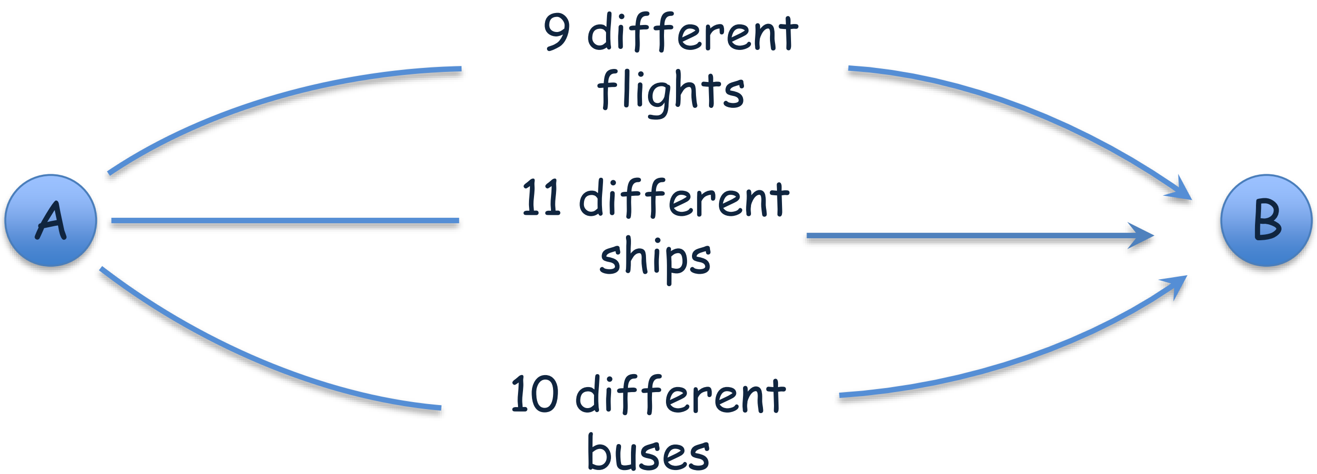
- If you leave them to the bag after you pick

$$\underline{5} \times \underline{5} \times \underline{5} = 125$$

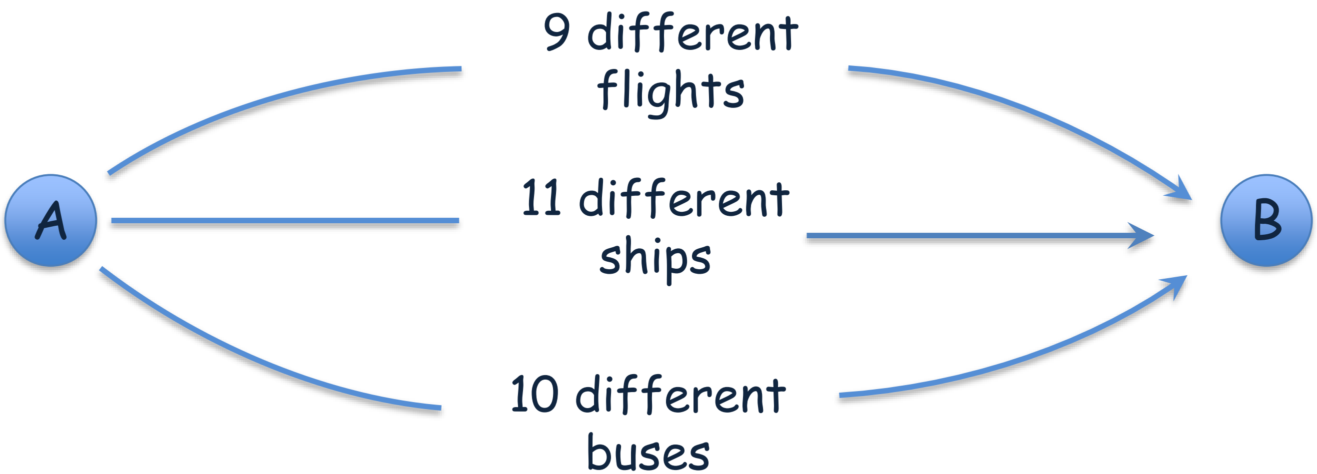
- If you keep them after you pick

$$\underline{5} \times \underline{4} \times \underline{3} = 60$$

Counting

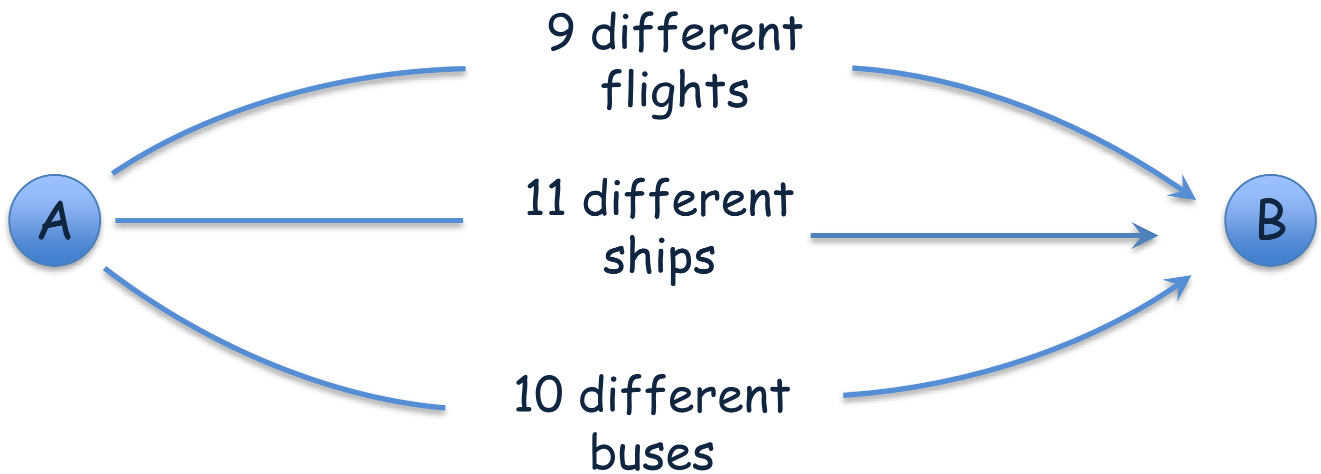

9 different flights
11 different ships
10 different buses
A
B

Counting



In how many different ways can you go from the city A to the city B ?

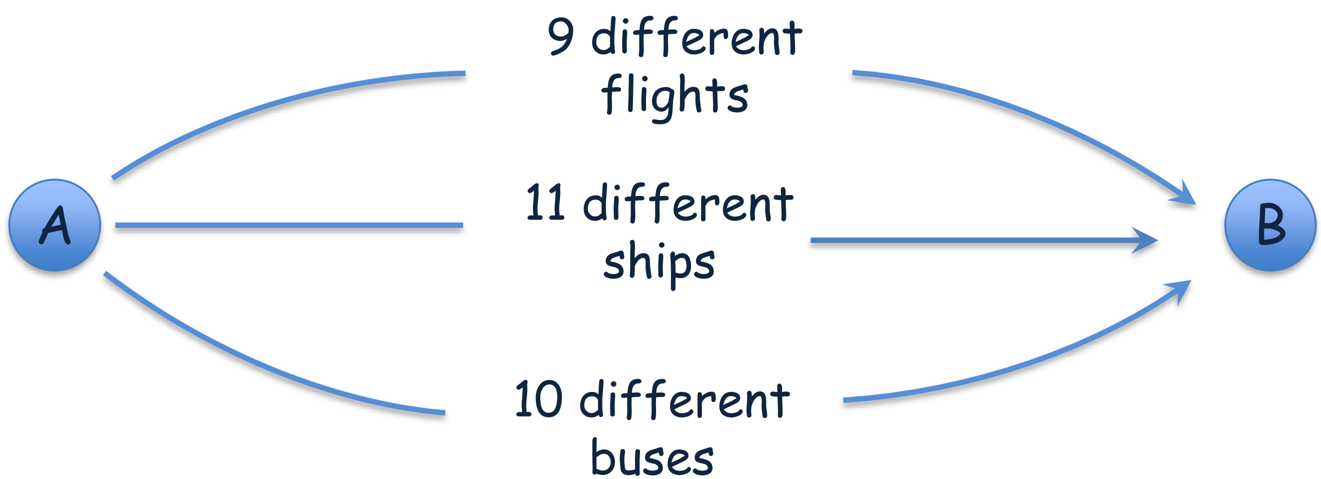
Counting



In how many different ways can you go from the city A to the city B ?

$$9 + 11 + 10 = 30$$

Counting



In how many different ways can you go from the city A to the city B ?

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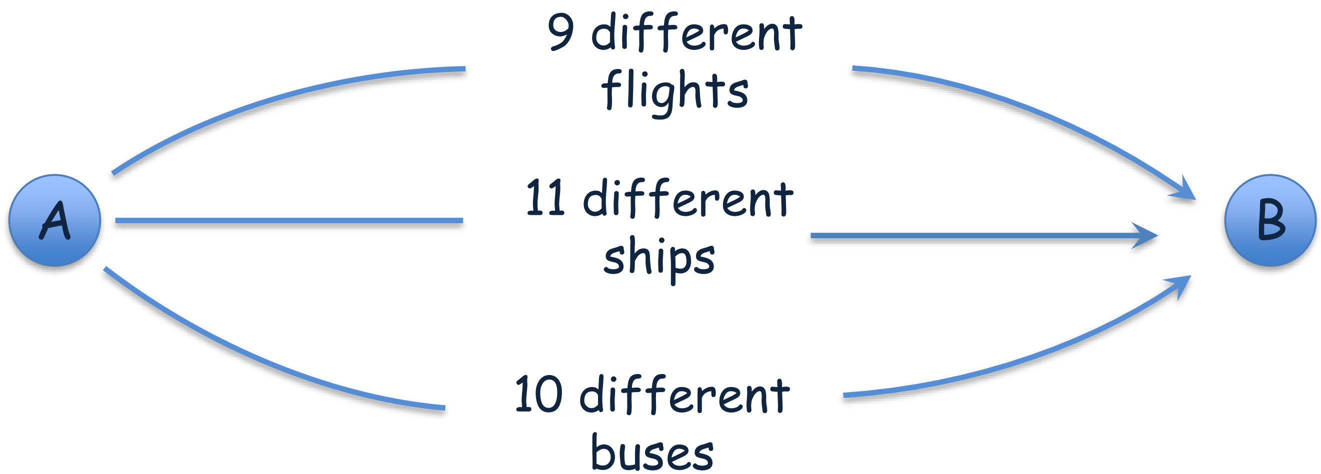
In how many different ways can you go to B and come back to A ?

Counting



In how many different ways can you go from the city A to the city B ?

$$9 + 11 + 10 = 30$$

In how many different ways can you go to B and come back to A ?

$$30 \times 30 = 900$$

Counting

- You prepare a meal for your friends

Counting

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- There are 5 kinds of bagels, 7 kinds of sandwiches, 6 drinks (hot coffee, hot tea, iced tea, cola, orange juice, apple juice)

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- You prepare a meal for your friends
- There are 5 kinds of bagels, 7 kinds of sandwiches, 6 drinks (hot coffee, hot tea, iced tea, cola, orange juice, apple juice)
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(bagel, hot) or (sand, cold)

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How many different meals can you prepare?

(bagel, hot) or (sand, cold)

$$5 \times 2 \quad + \quad 7 \times 4 \quad = \quad 38$$

Counting

- How many different licence plates can be created if a plate contains of 2 digits indicating the city the car registered followed by 3 uppercase Turkish letters followed by 3 digits ?

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___ ___ ___ ___ ___ ___ ___

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$$\underbrace{\_\_\_\_}_{\text{indicating the city}} \quad \underbrace{\_\_\_\ \ \_\_\_\ \ \_\_\_}_{\text{Turkish letters}} \quad \underbrace{\_\_\_\ \ \_\_\_\ \ \_\_\_}_{\text{numbers}}$$

Counting

- How many different licence plates can be created if a plate contains of 2 digits indicating the city the car registered followed by 3 uppercase Turkish letters followed by 3 digits ?

$$\underbrace{\underline{\;81\;}}_{\text{indicating the city}} \times \underbrace{\underline{\quad}\;\underline{\quad}\;\underline{\quad}}_{\text{Turkish letters}}\;\underbrace{\underline{\quad}\;\underline{\quad}\;\underline{\quad}}_{\text{numbers}}$$

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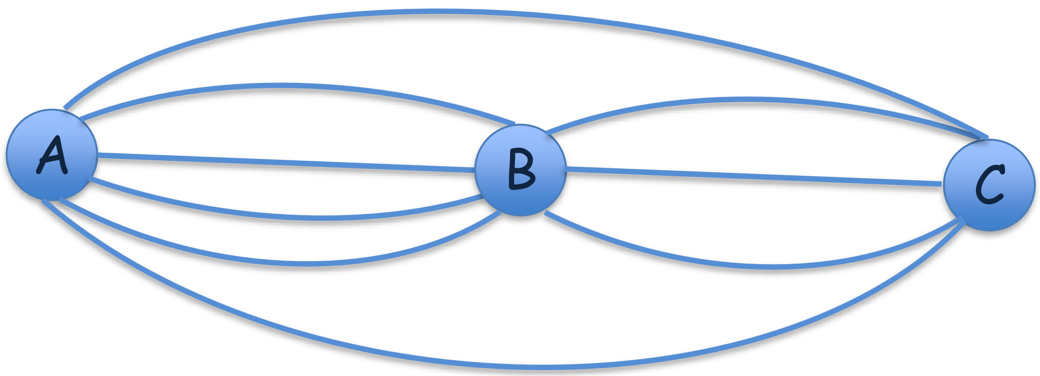
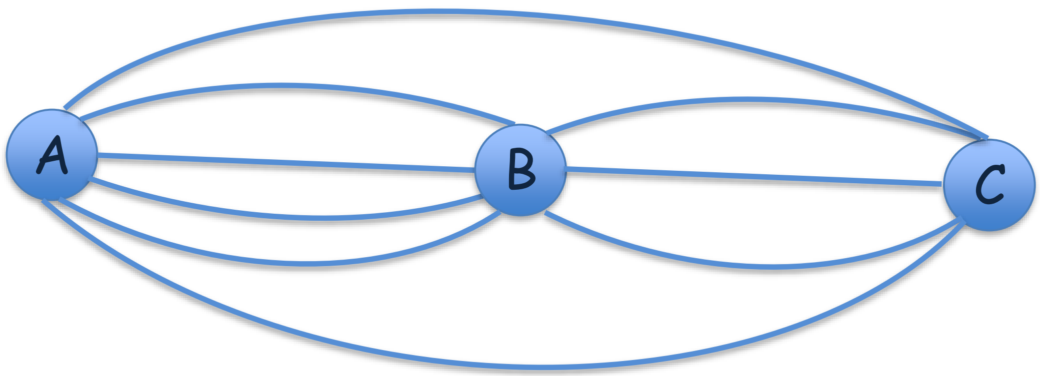
Counting

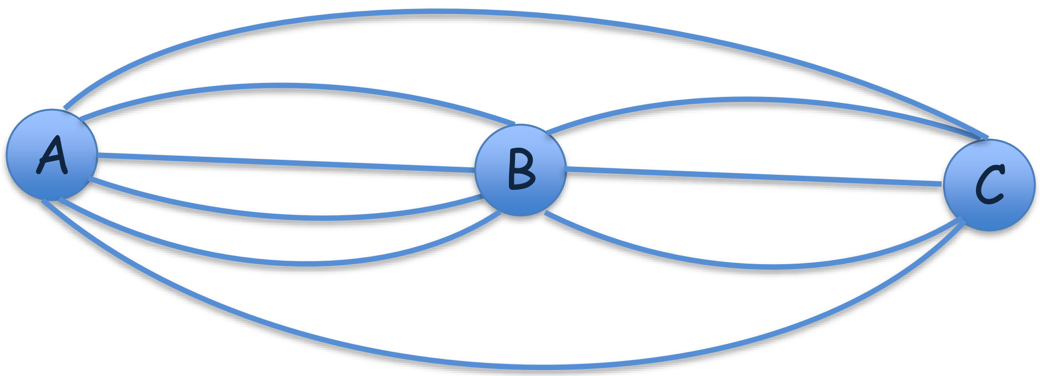


Counting



In how many different ways can you go from the city A to the city C ?

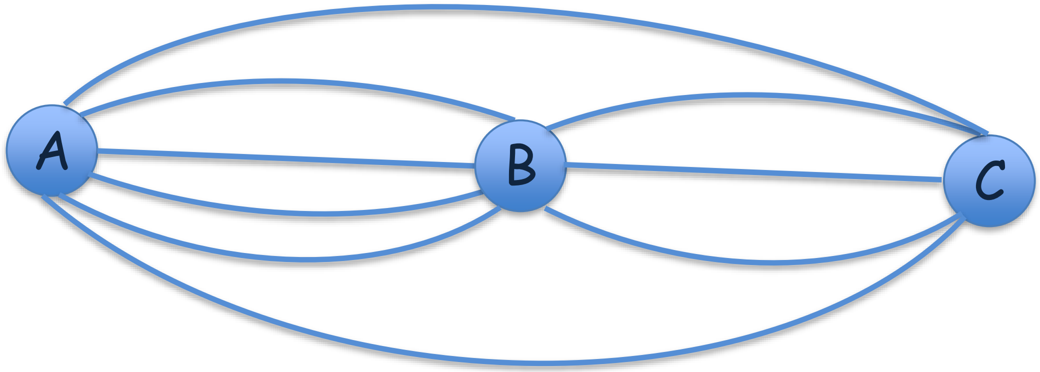
Counting



In how many different ways can you go from the city A to the city C ?

$$4 \times 3 + 2 = 14$$

Counting

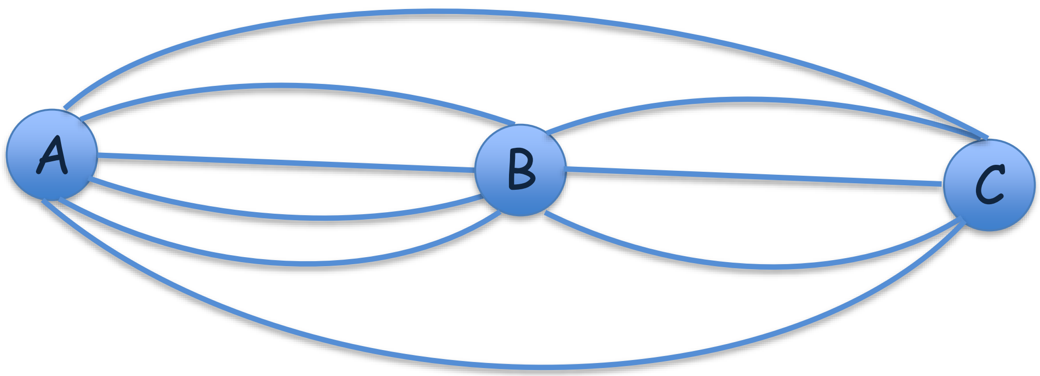


In how many different ways can you go from the city *A* to the city *C* ?

$$4 \times 3 + 2 = 14$$

In how many different ways can you go to *C* and come back to *A* ?

Counting



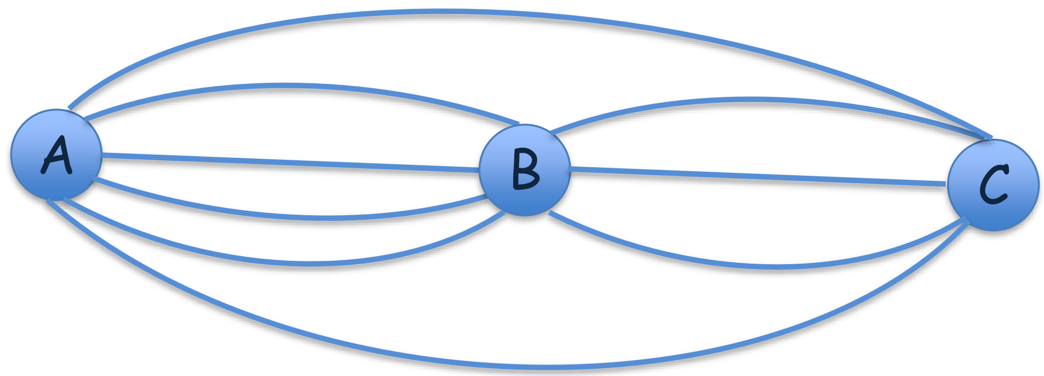
In how many different ways can you go from the city *A* to the city *C* ?

$$4 \times 3 + 2 = 14$$

In how many different ways can you go to *C* and come back to *A* ?

$$14 \times 14 = 196$$

Counting



In how many different ways can you go from the city *A* to the city *C* ?

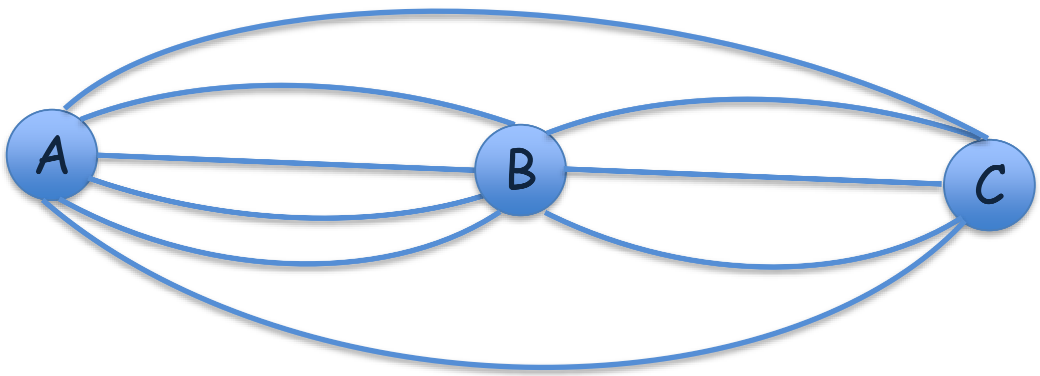
$$4 \times 3 + 2 = 14$$

In how many different ways can you go to *C* and come back to *A* ?

$$14 \times 14 = 196$$

In how many different ways can you go to *C* and come back to *A* so that you can use same route to come back?

Counting



In how many different ways can you go from the city A to the city C ?

$$4 \times 3 + 2 = 14$$

In how many different ways can you go to C and come back to A ?

$$14 \times 14 = 196$$

In how many different ways can you go to C and come back to A so that you can use same route to come back?

$$14 \times 13 = 182$$

Permutation

Assume there are 10 students. We choose five of them, and make them sit together to get a picture

How many such arrangements can we make?

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How many such arrangements can we make?

- assign them numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

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Assume there are 10 students. We choose five of them, and make them sit together to get a picture

How many such arrangements can we make?

- assign them numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- possible arrangements 13429, 60938, 19082, . . .

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$$\underline{10} \times \underline{9} \times \underline{8} \times \underline{7} \times \underline{6} = 30240$$

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$$\underline{10} \times \underline{9} \times \underline{8} \times \underline{7} \times \underline{6} = 30240$$

How many different arrangements can we make for all students ?

Permutation

Assume there are 10 students. We choose five of them, and make them sit together to get a picture

How many such arrangements can we make?

- assign them numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- possible arrangements 13429, 60938, 19082, . . .

$$\underline{10}\times\underline{9}\times\underline{8}\times\underline{7}\times\underline{6} = 30240$$

How many different arrangements can we make for all students ?

$$\underline{10}\times\underline{9}\times\underline{8}\times\underline{7}\times \ldots \times\underline{1} = 3628800$$

Permutation

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How many such arrangements can we make?

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$$10 \times 9 \times 8 \times 7 \times 6$$

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How many such arrangements can we make?

- assign them numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- possible arrangements 13429, 60938, 19082, . . .

$$10 \times 9 \times 8 \times 7 \times 6 \; \frac{\times 5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1}$$

Permutation

Assume there are 10 students. We choose five of them, and make them sit together to get a picture

How many such arrangements can we make?

- assign them numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- possible arrangements 13429, 60938, 19082, . . .

$$10 \times 9 \times 8 \times 7 \times 6 \, \frac{\times 5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1} = \frac{10!}{5!}$$

Permutation

Assume there are 10 students. We choose five of them, and make them sit together to get a picture

How many such arrangements can we make?

- assign them numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- possible arrangements 13429, 60938, 19082, . . .

$$10 \times 9 \times 8 \times 7 \times 6 \; \frac{\times 5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1} = \frac{10!}{5!}$$

- the number of different permutation of size 5 for 10 objects

Permutation

Assume there are 10 students. We choose five of them, and make them sit together to get a picture

How many such arrangements can we make?

- assign them numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- possible arrangements 13429, 60938, 19082, . . .

$$10 \times 9 \times 8 \times 7 \times 6 \; \frac{\times 5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1} = \frac{10!}{5!}$$

- the number of different permutation of size 5 for 10 objects
- the number of different permutation of size r for n objects

Permutation

Assume there are 10 students. We choose five of them, and make them sit together to get a picture

How many such arrangements can we make?

- assign them numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- possible arrangements 13429, 60938, 19082, . . .

$$10 \times 9 \times 8 \times 7 \times 6 \, \frac{\times 5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1} = \frac{10!}{5!}$$

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$$P(n,r) = \frac{n!}{(n-r)!}$$

Permutation

- Using the letters of the word 'COMPUTER' how many different words can you create ?

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$$P(8, 5) = 8! / 5! = 336$$

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$$8 \times 8 \times 8 \times 8 \times 8 = 32768$$

Permutation

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Permutation

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$$4!$$

Permutation

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$\cancel{4!}$

Permutation

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$\cancel{4!}$

BALL, BLAL, BLLA, ABLL, ALBL, ALLB
LBAL, LBLA, LABL, LALB, LLAB, LLBA

Permutation

- Using the letters of the word 'BALL', how many different words can you create?

$\cancel{4!}$ 12

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5! / 3! = 20

- pretend they are different A's
- fix other letters and reorder A's

Permutation

- There are 5 people : A, B, C, D, E
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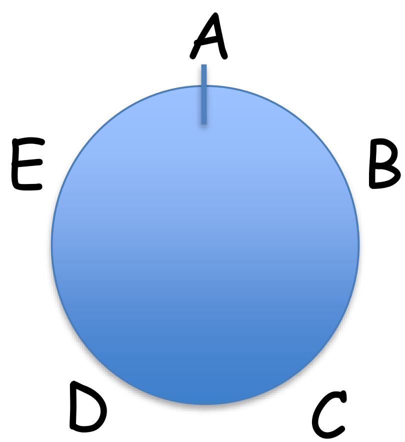

A B C D E

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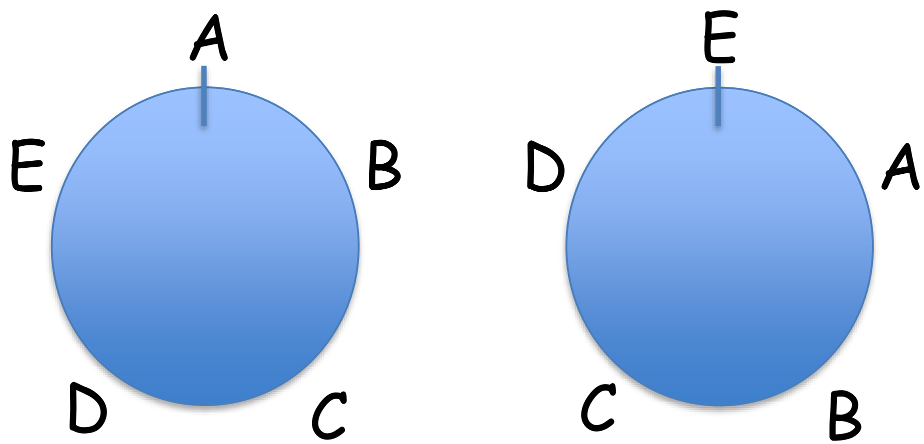

A B C D E
E A B C D

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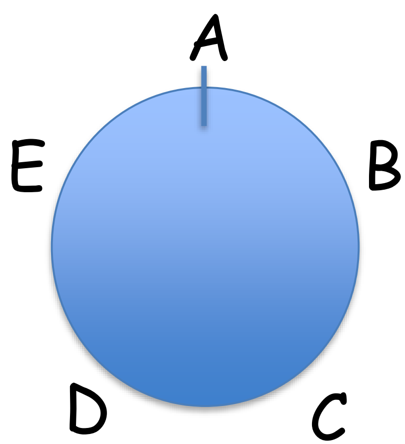

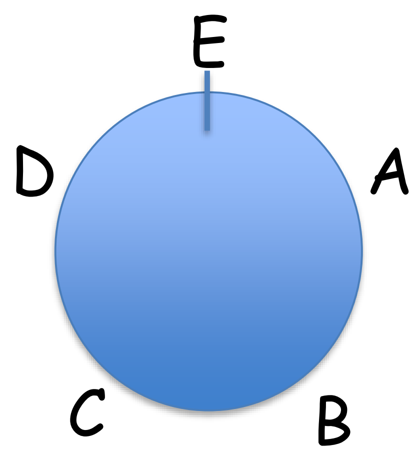

A B C D E
E A B C D
D E A B C
C D E A B
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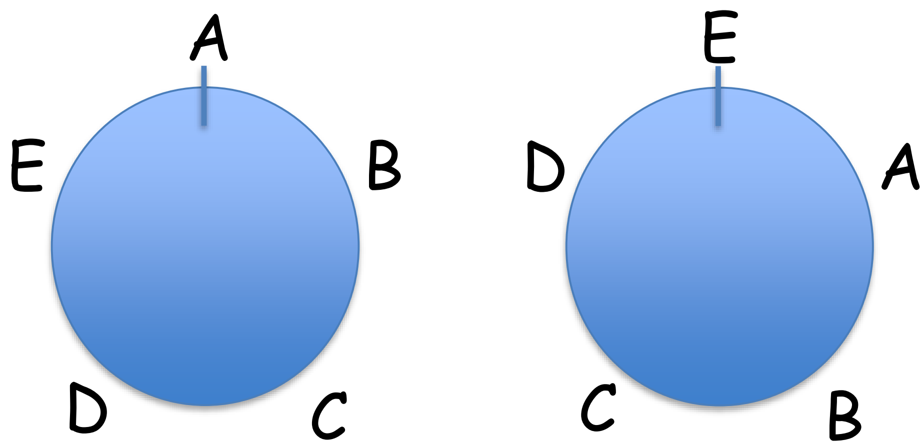

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For each circular arrangement, there are 5 linear arrangements

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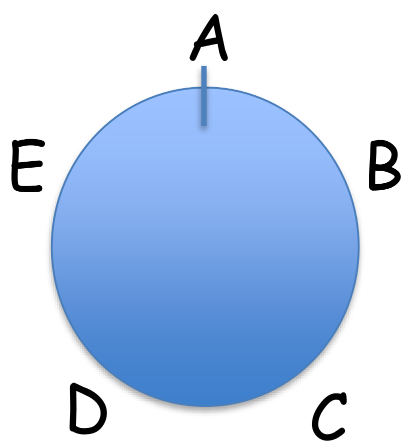

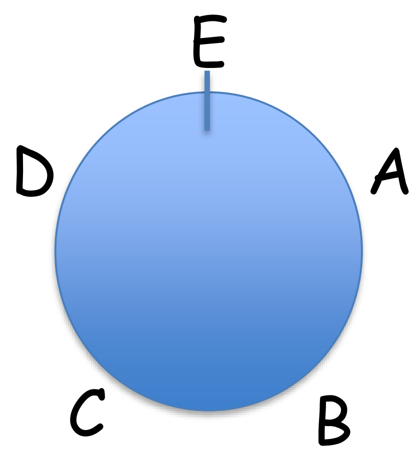

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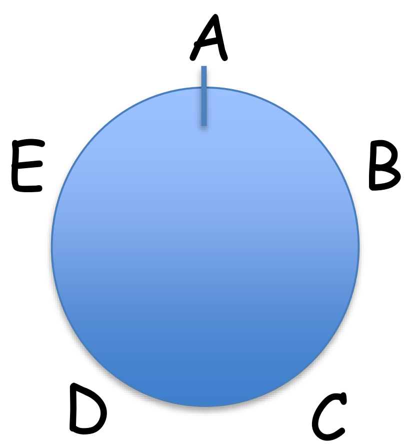

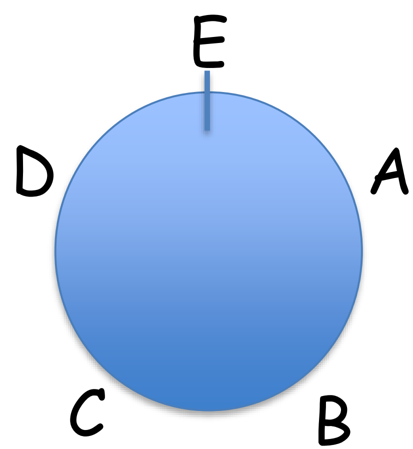

A B C D E
E A B C D
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For each circular arrangement, there are 5 linear arrangements

5 x (# of circular) = (# of linear)

(# of circular) = 5! / 5 = 24

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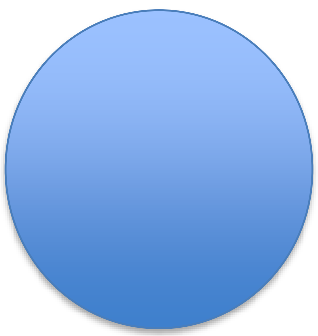

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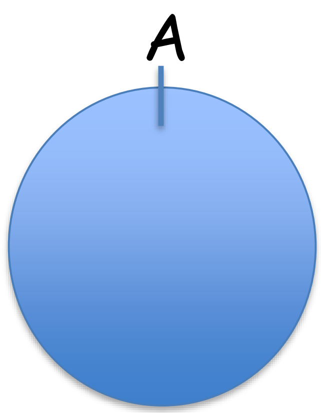

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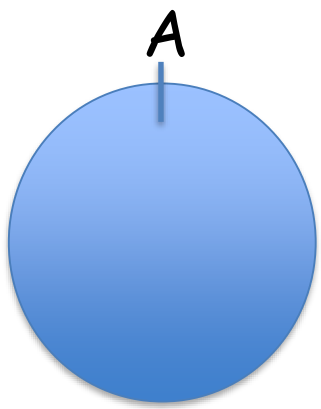

- fix one of them
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- 4!

Permutation

- You invite 2 couples for the dinner (3 couples at total)

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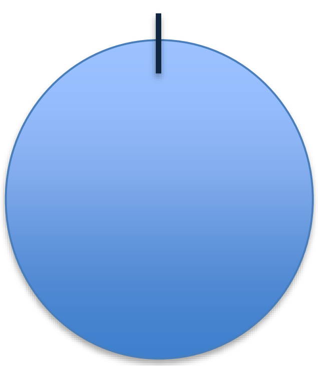

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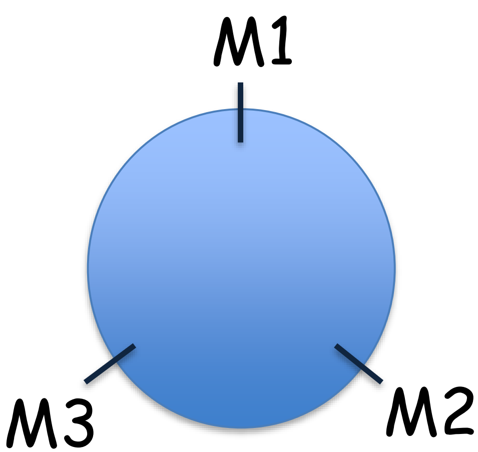

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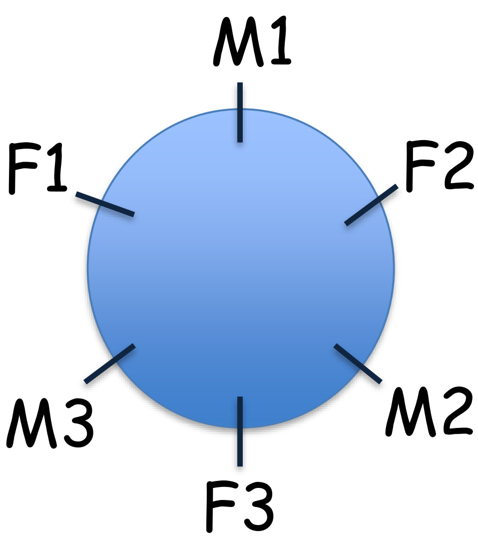

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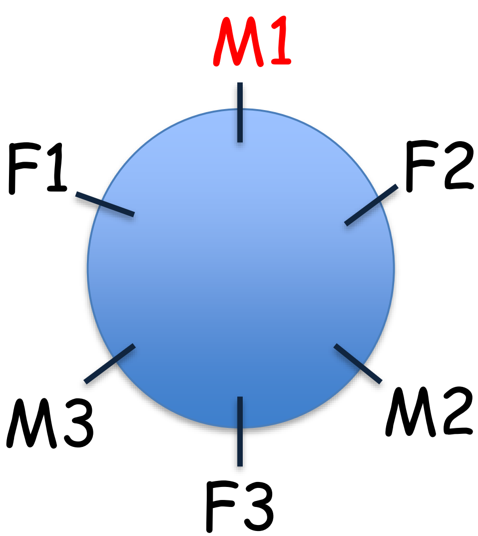

- $3 \times 2 \times 2 \times 1 \times 1 = 12$

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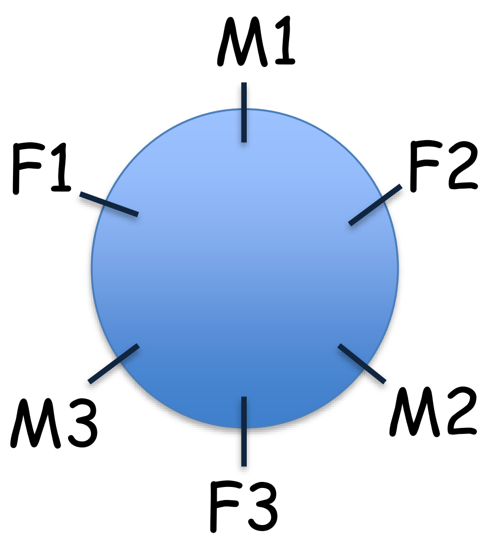

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Combinations

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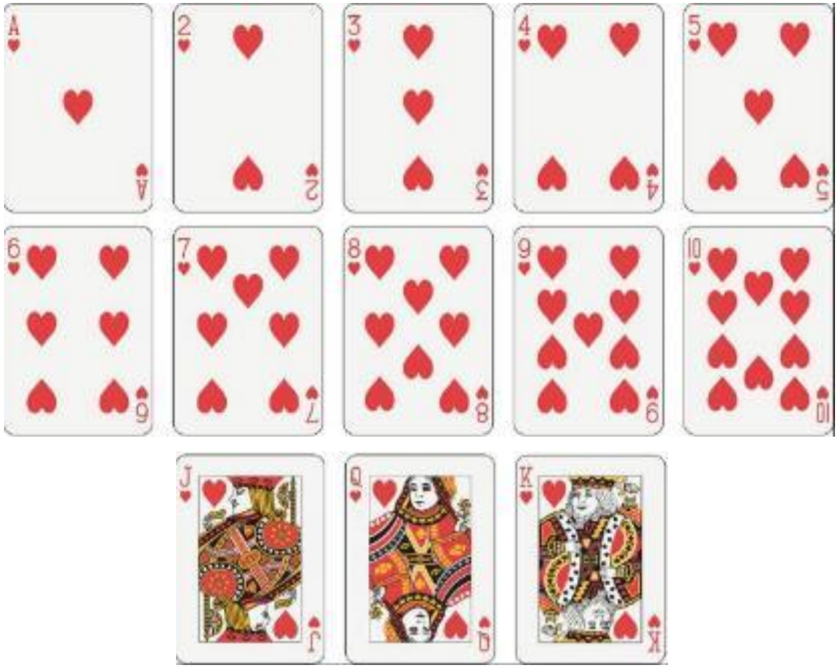

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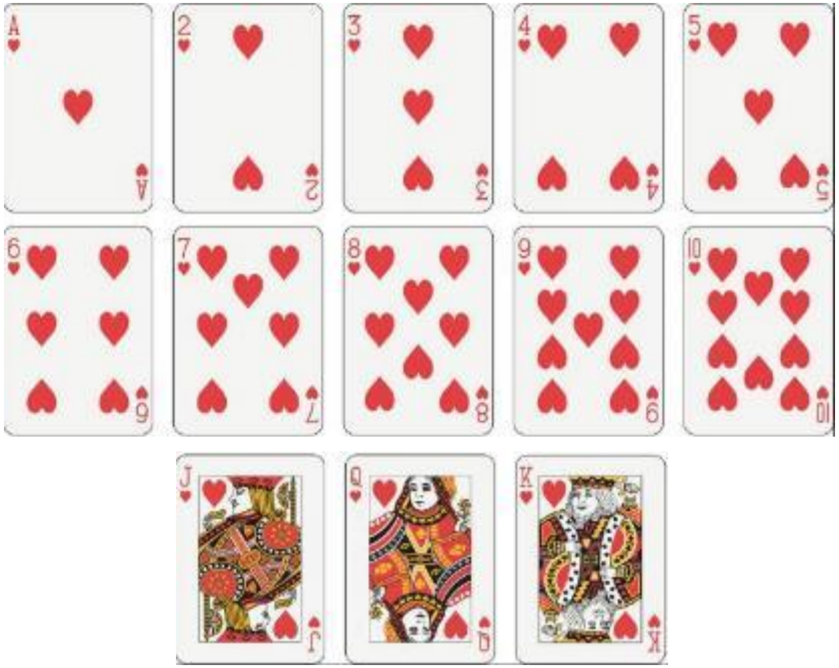

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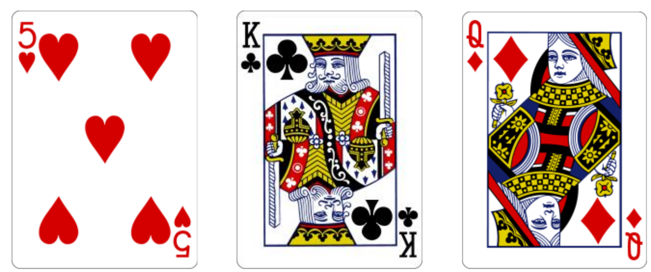

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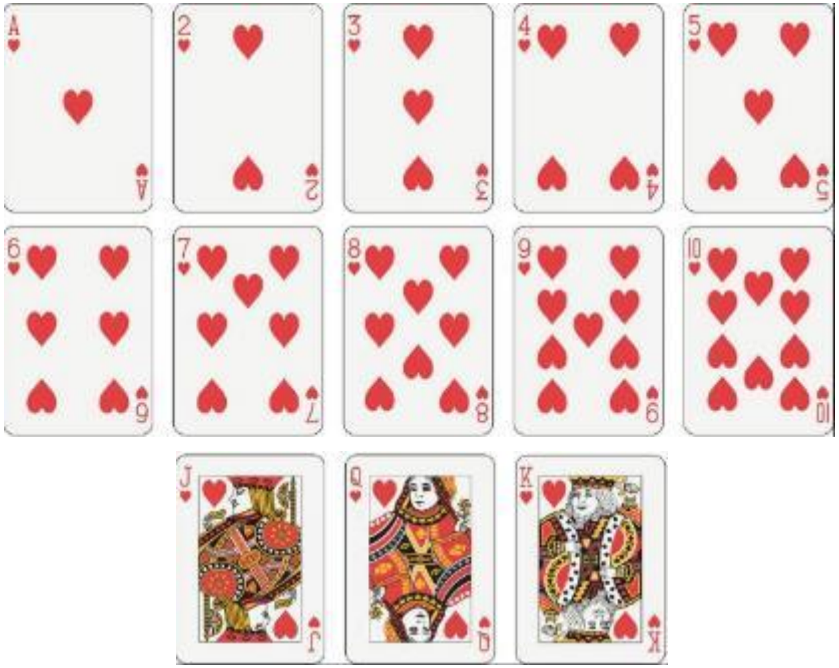

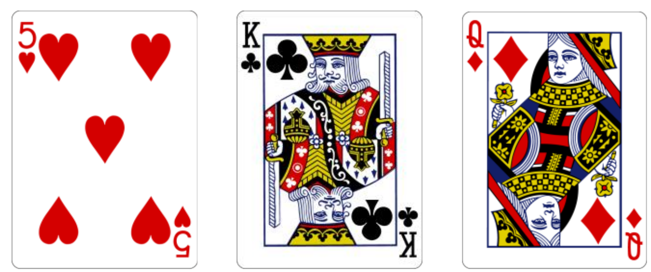

- Assume you play a game such that a player holds 3 cards.

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$$P(52,3) = 52! \,/\, (52-3)!$$

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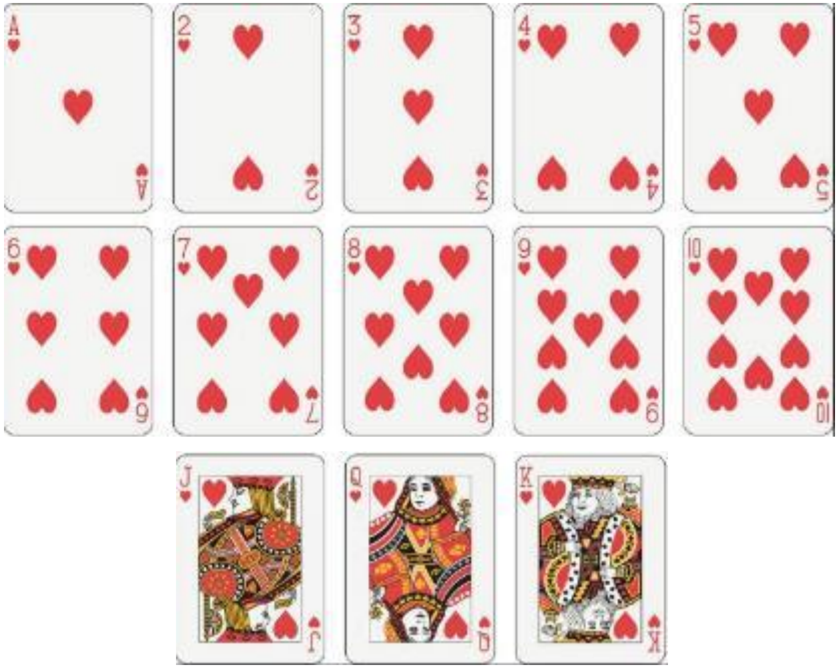

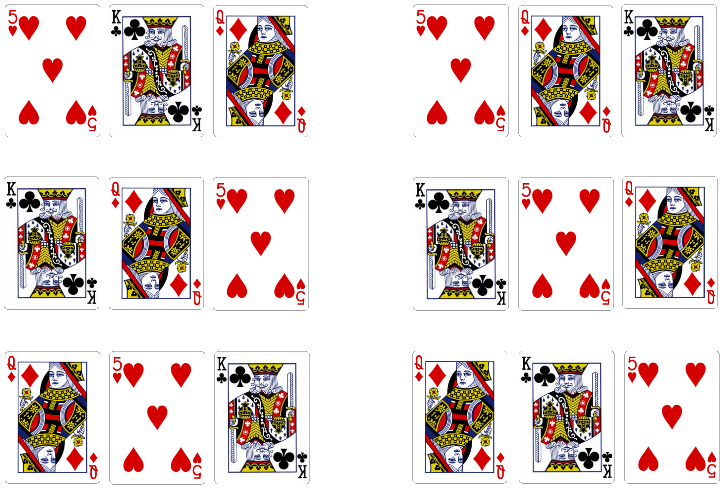

- Assume you play a game such that a player holds 3 cards.
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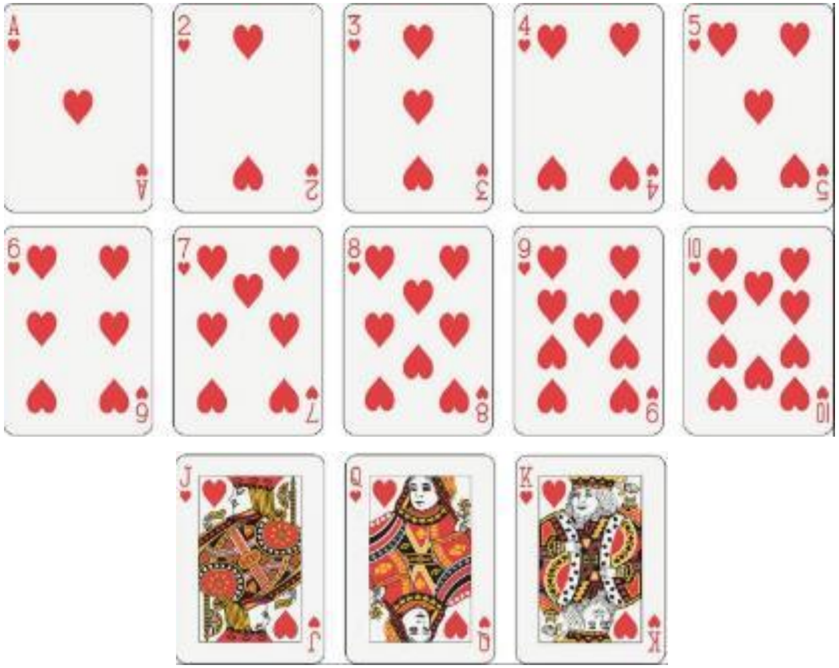

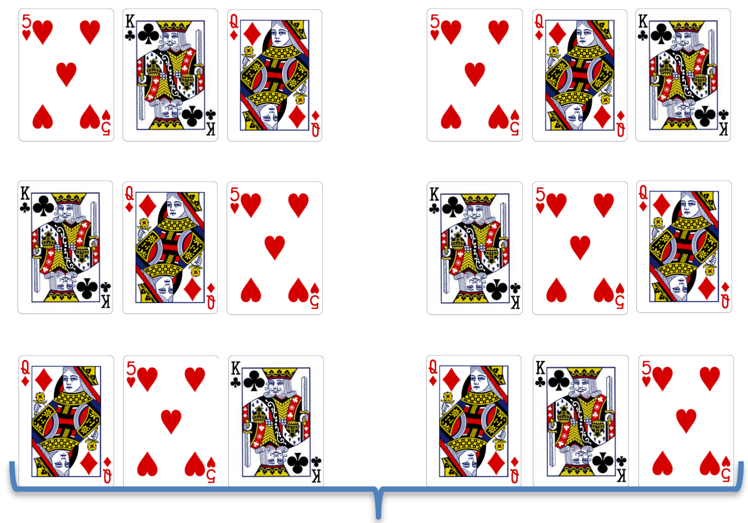

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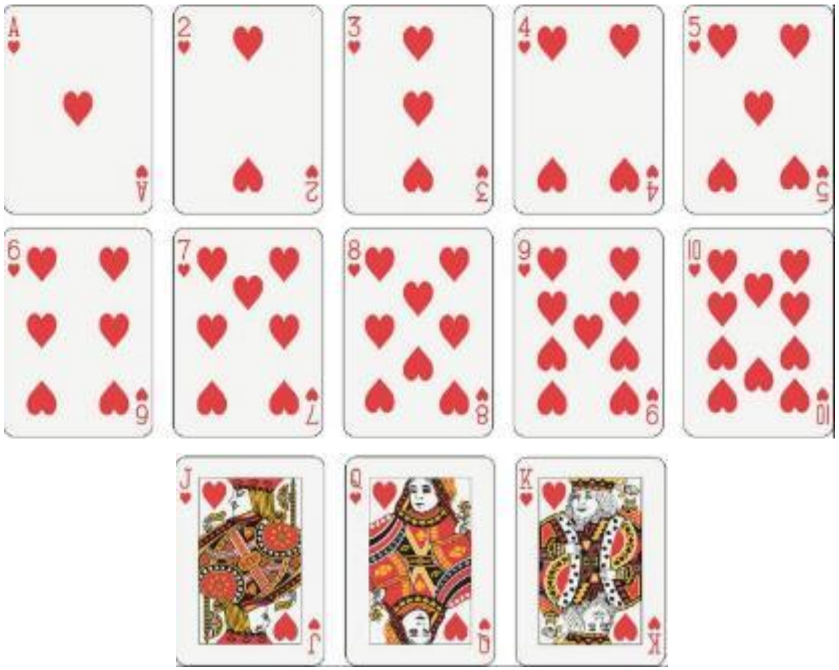

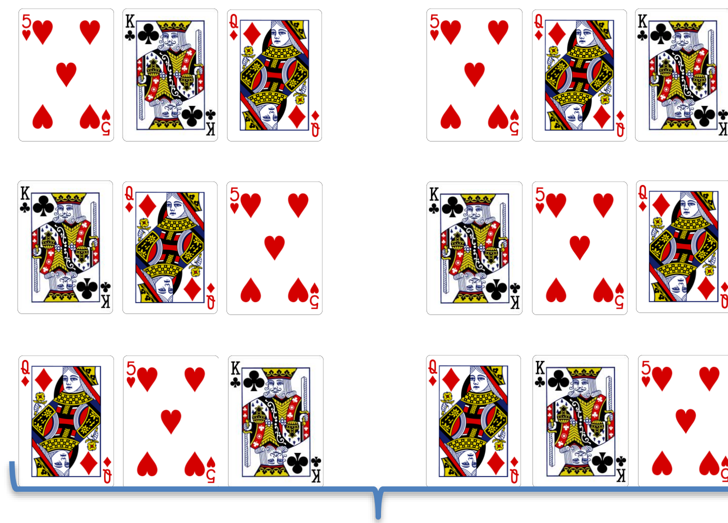

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$$52! / [(52-3)! \,.\, 3!]$$

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$$\binom{5}{4} \text{x} \binom{5}{3} = \frac{5!}{1!.4!} \text{x} \frac{5!}{2!.3!} = 50$$

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How many of them contains no club ?

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$$\binom{30}{6}\binom{24}{6}\binom{25}{6}\binom{19}{6}$$

- Suppose you are playing a game with five cards.

How many different hands can you have ? $\binom{52}{5}$

How many of them contains no club ? $\binom{39}{5}$

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$\Sigma = \{0, 1\}$.

Let's use this alphabet to create three digits encoding:

000, 010, 111, 011, . . .

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Assume $x = x_1 x_2 \ldots x_n$, the weight of a given encoding is

$$w(x) = x_1 + x_2 + \ldots + x_n$$

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$$\binom{8}{0} + \binom{8}{2} + \ldots + \binom{8}{8}$$

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- How many of them contains exactly three 1's ?

Assume $x = x_1x_2...x_n$, the weight of a given encoding is

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Σ = {0, 1, 2}.
Let's use this alphabet to create three digits encoding:

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$$3 \times 3 \times \ldots \times 3 = 3^8$$

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Let x and y be variables and n be non-negative integer, then

$$(x+y)^n$$

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- What is the coefficient of x^5y^2 in the expansion of $(x+y)^7$?

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$$\binom{n}{i} x^{n-i} y^i \Rightarrow \mathrm{n} = 25 \text{ and } i = 15 \Rightarrow \binom{25}{15} (2x)^{10} (-3y)^{15}$$

Binomial Theorem

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$$\Rightarrow \binom{25}{15} 2^{10} (-3)^{15} x^{10} y^{15}$$

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- What is the coefficient of ${x_1}^{n_1} {x_2}^{n_2} \ldots {x_k}^{n_k}$ in the expansion of $(x_1 + x_2 + \cdots + x_k)^n$?

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$$\binom{n}{n_1}$$

Binomial Theorem

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$$\binom{n}{n_1} \cdot \binom{n - n_1}{n_2}$$

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Binomial Theorem

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- What is the coefficient of $x^3 y^2 z^2$ in the expansion of $(x+y+z)^7$?

$$\frac{7!}{3!\,2!\,2!} = 210$$

Binomial Theorem

- Prove that $\sum_{i=0}^{n} \binom{n}{i} = 2^n$

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Suppose there is a set $A=\{1, 2, . . ., n\}$.

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Suppose there is a set A={1, 2, . . ., n}. Let's write the elements of the power set P(A):

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{1}, {2}, {3}, . . ., {n}

{1, 2}, {1, 3}, {1, 4}, . . ., {n-1, n}

Binomial Theorem

- Prove that $\sum_{i=0}^{n} \binom{n}{i} = 2^n$

Suppose there is a set A={1, 2, . . ., n}. Let's write the elements of the power set P(A):

∅

{1}, {2}, {3}, . . ., {n}

{1, 2}, {1, 3}, {1, 4}, . . ., {n-1, n}

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$$\begin{tikzcd}[column sep=small]
 & & \binom{0}{0} \arrow[dl, dash] \arrow[dr, dash] & \\
 & \binom{1}{0} \arrow[dl, dash] & & \binom{1}{1} \\
\binom{2}{0} & & &
\end{tikzcd}$$

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1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

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$$= \binom{2}{2} + \binom{2}{1} + \binom{3}{1} + \ldots + \binom{n-1}{1} + \binom{n}{1}$$

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