Functions

Murat Osmanoglu

Functions as Relations

$$R \subseteq \underbrace{A}_{\text{domain}} \times \underbrace{B}_{\text{codomain}}$$

R(A) : the image of R, $R(A) = \{y \in B | (x, y) \in R, \exists x \in A\}$

Function is a relation that satisfies two conditions :

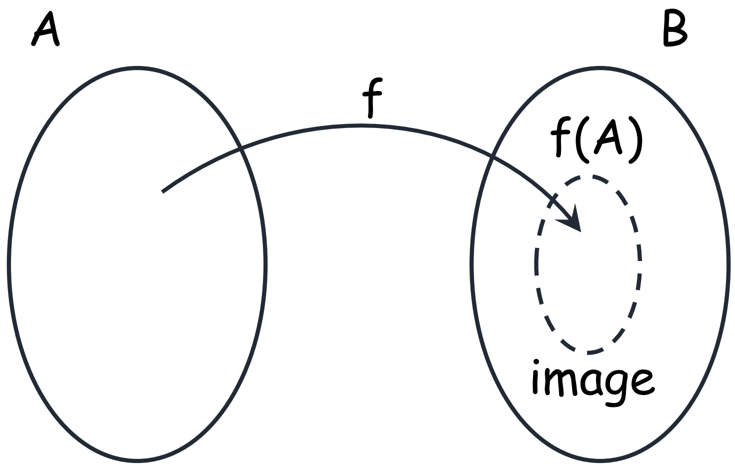
- for every element x of the domain, there is an element y in the codomain such that (x,y) is an element of the relation

  Let $R \subseteq A \times B$ be the relation, $\forall x[(x \in A) \rightarrow (\exists y \in B \;\; s.t. (x, y) \in R)]$

- for every element x of the domain, there is only one element y of the codomain such that (x,y) is an element of the relation

  Let $R \subseteq A \times B$ be the relation, $\forall x[((x, y_1) \in R \wedge (x, y_2) \in R) \rightarrow (y_1 = y_2)]$

Definition



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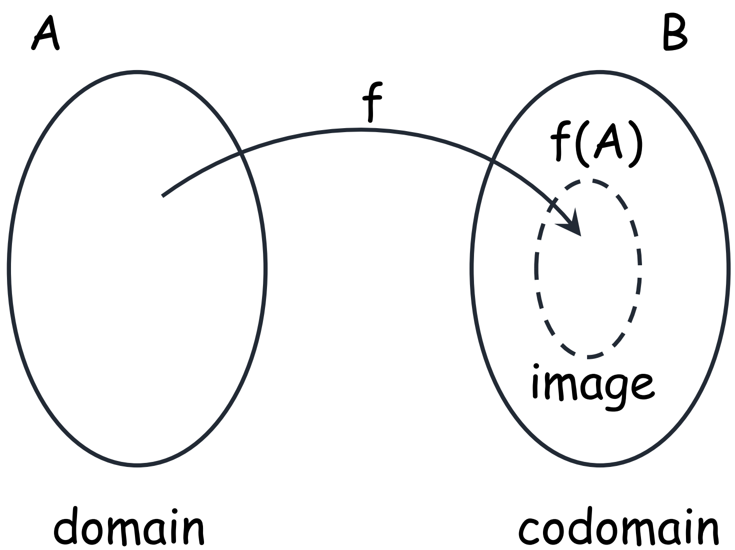

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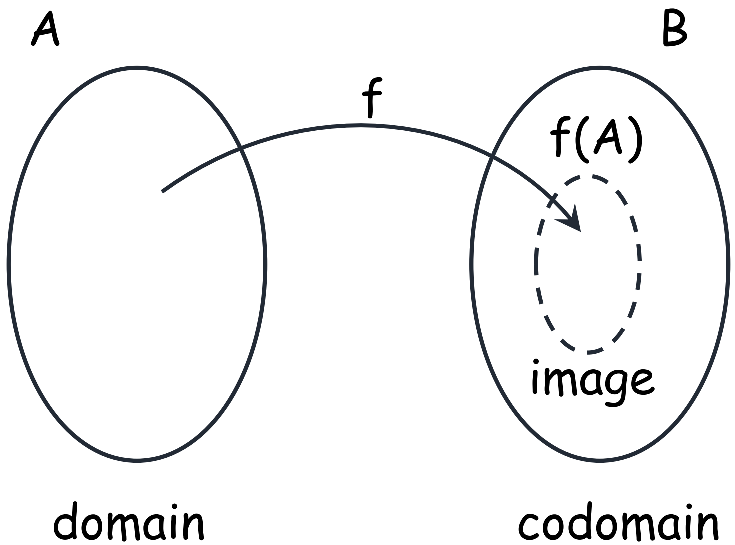

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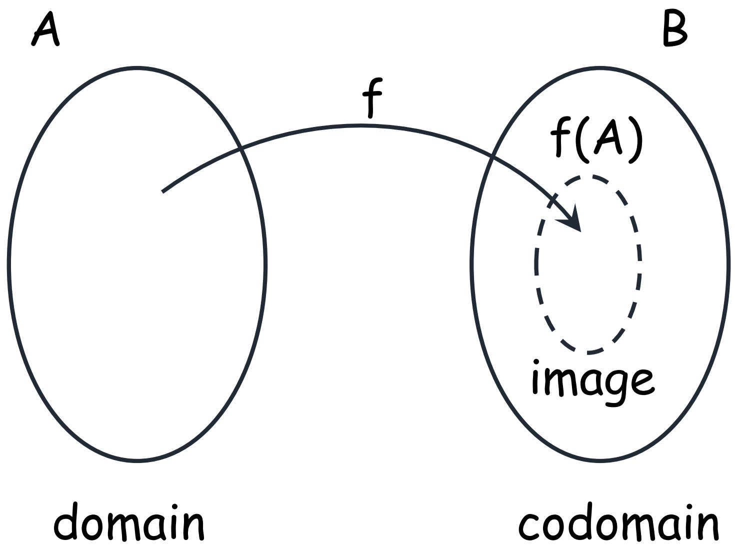

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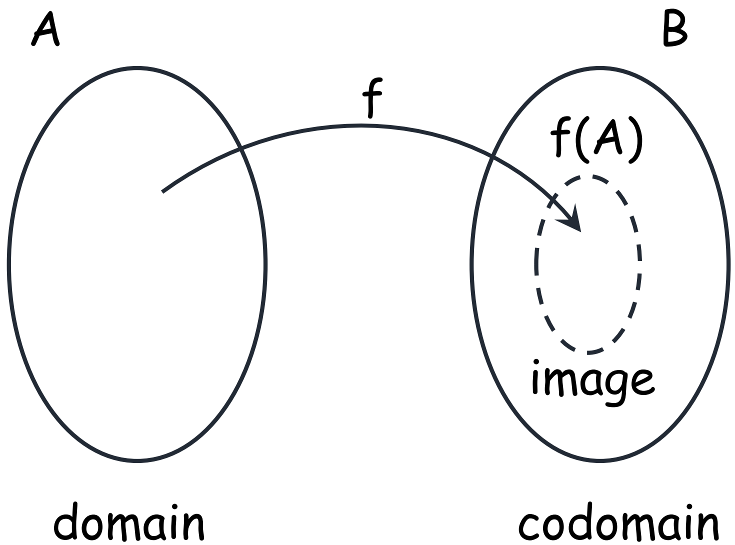

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$$\uparrow \qquad \uparrow \qquad \uparrow$$

m m m

$$m^n = |B|^{|A|} \text{ functions}$$

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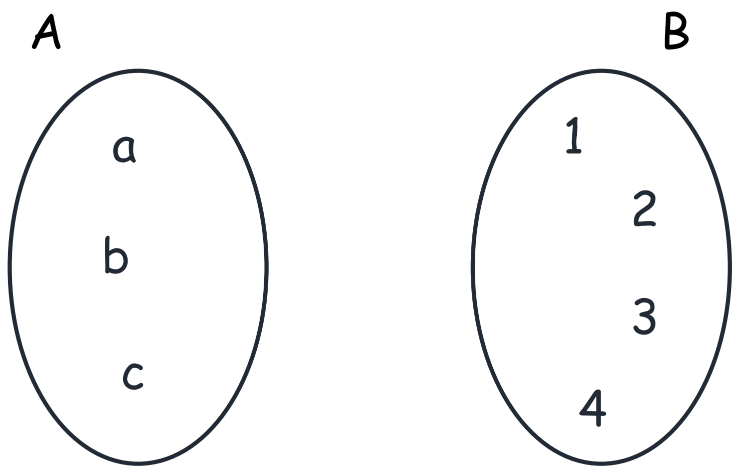

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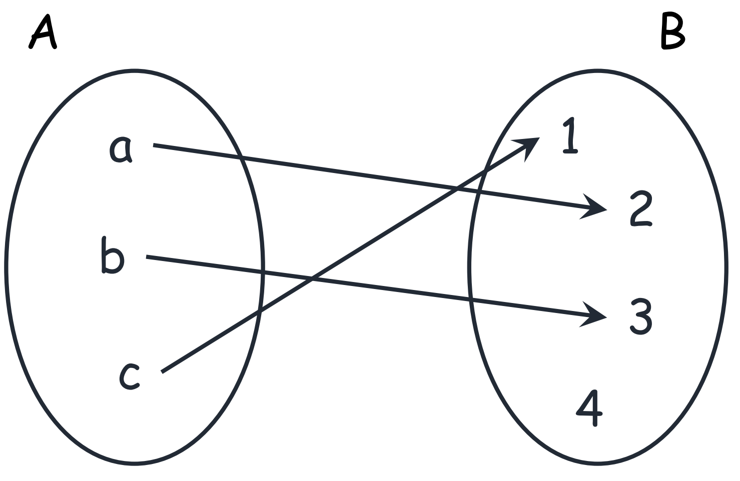

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for $x_1 = 1$ and $x_2 = -1$, $x_1 \neq x_2$ but $f(x_1) = f(x_2)$

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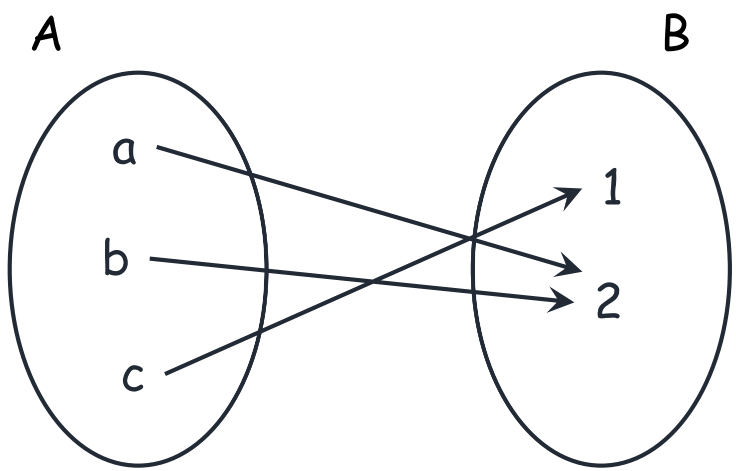

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for $5 \in \mathbb{Z}$, there is no integer $x \in \mathbb{Z}$ such that f(x) = 5.

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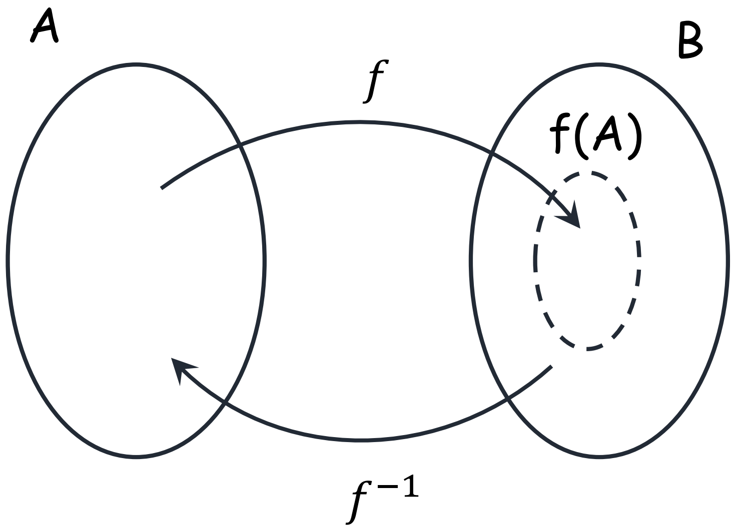
$\forall a \in A,\ \ f(a) = \mathrm{a}$, the preimage of a is itself

Inverse



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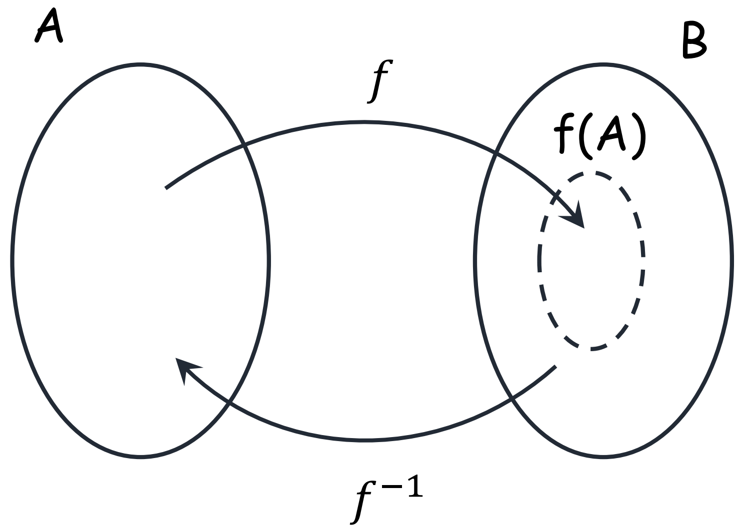

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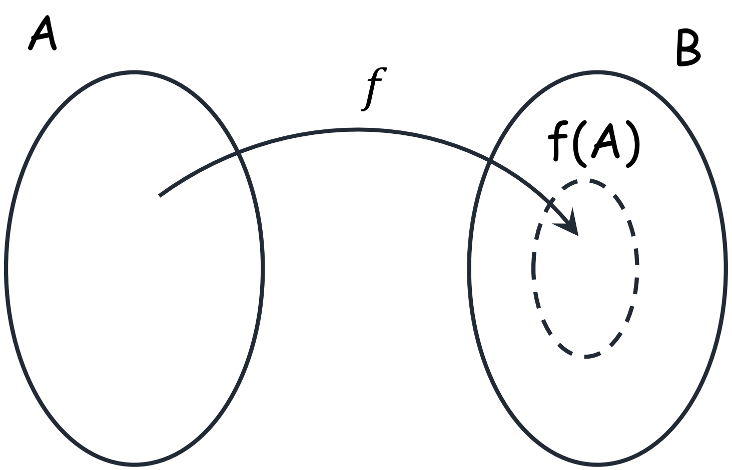

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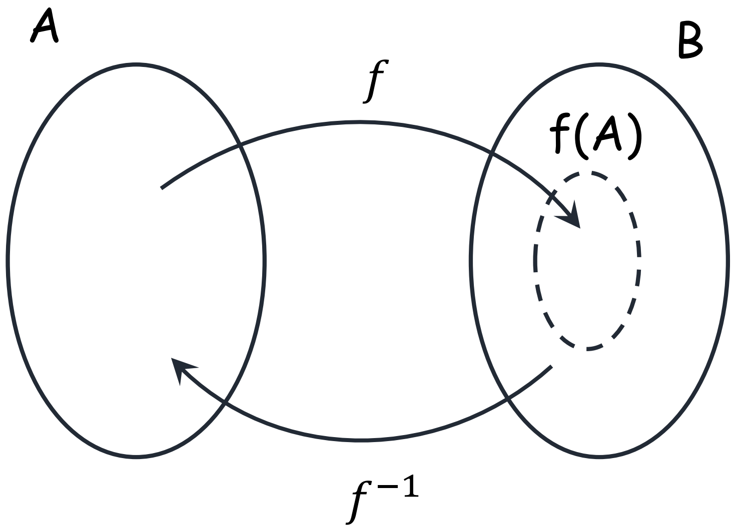

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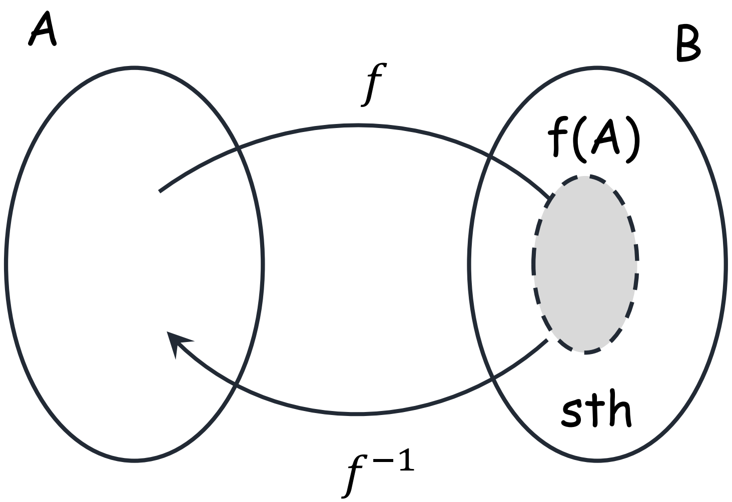

f(A)≠B

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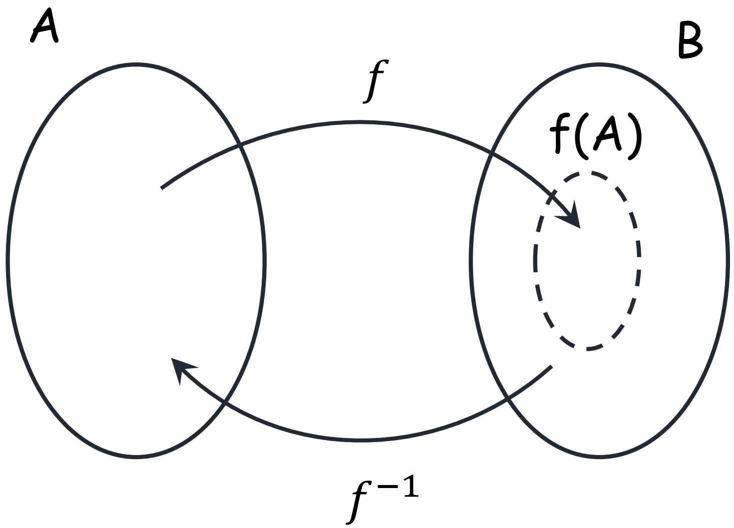

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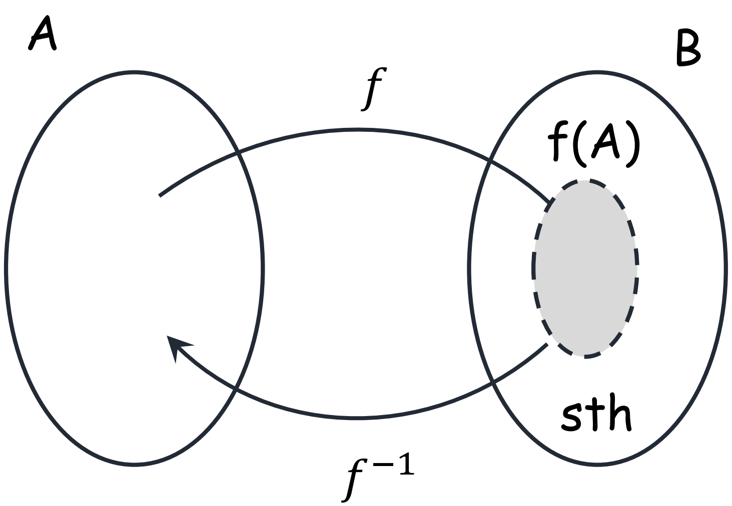

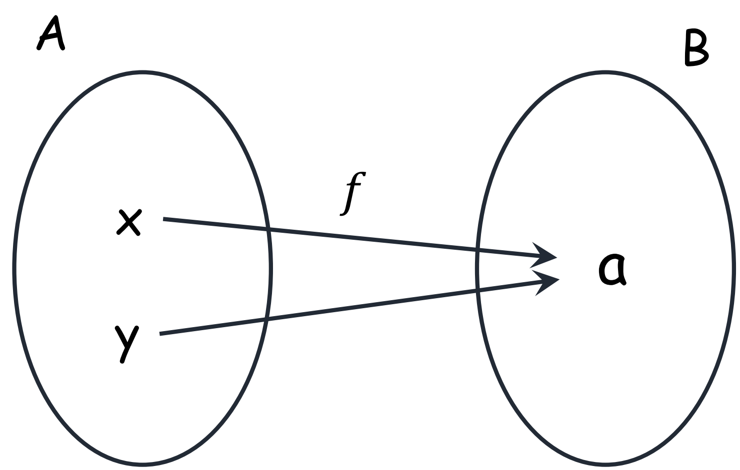

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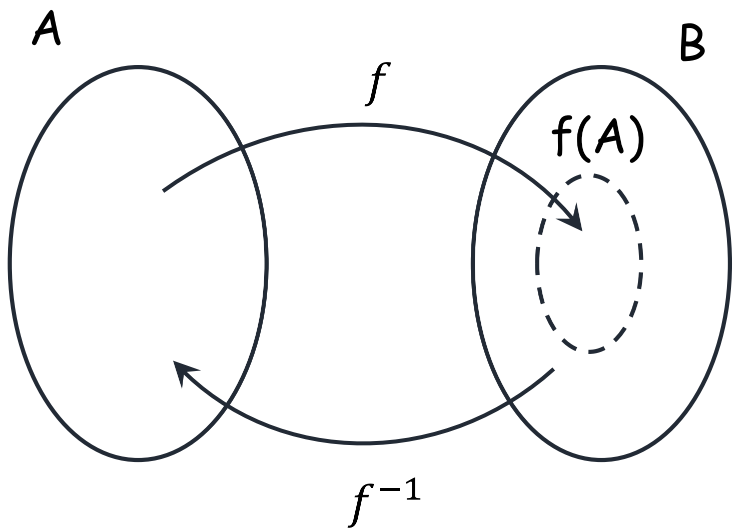

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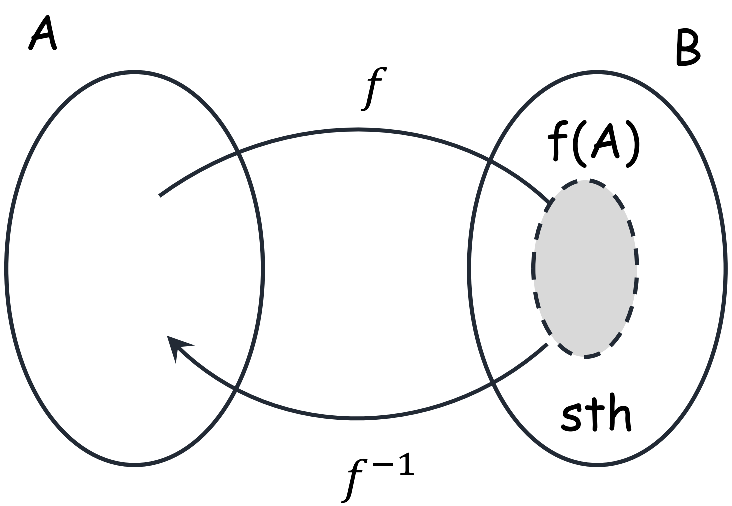

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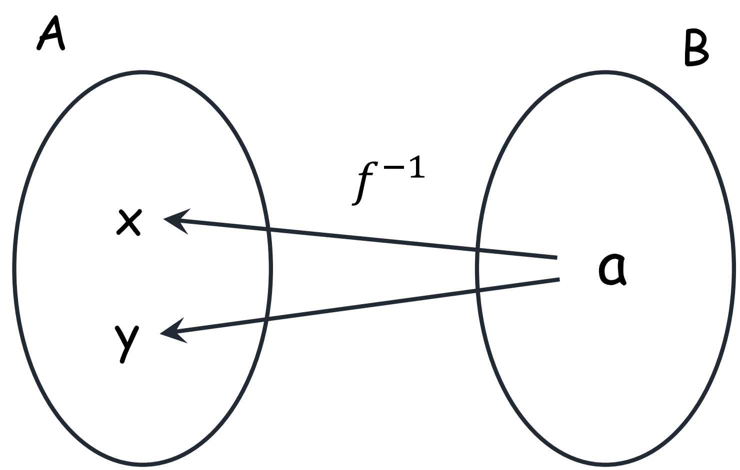

$$f(x) = f(y) = a$$

$f^{-1}(a) = x$ and $f^{-1}(a) = y$

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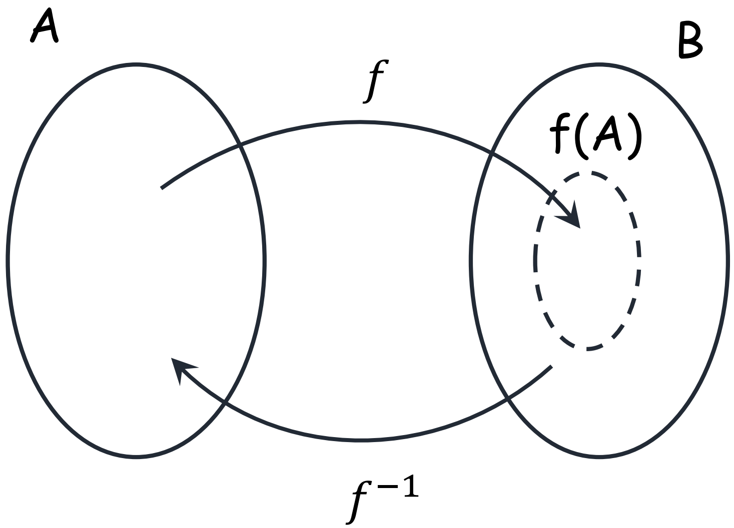

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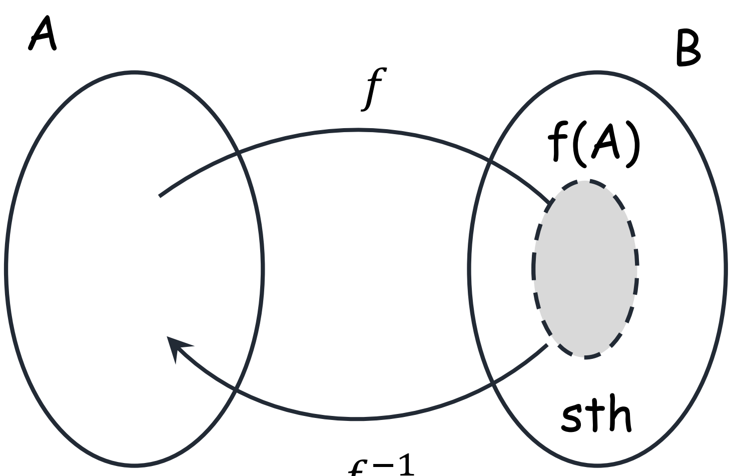

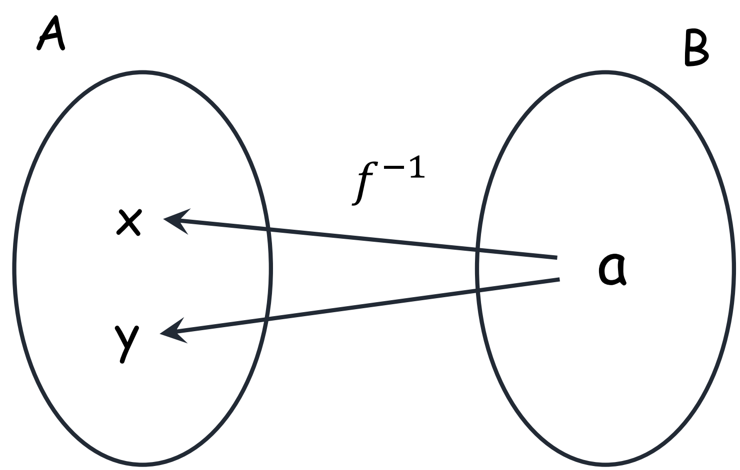

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$$\to x_1 = x_2 \text{ (one-to-one)}$$

$$\forall y \in \mathbb{Z}, f(x) = \mathrm{y} \leftrightarrow x + 1 = y$$
$$\leftrightarrow x = y - 1 \in \mathbb{Z} \text{ (onto)}$$

Inverse

- If a function both one-to-one and onto, it is called bijection.

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$$f^{-1}(x) = x - 1$$

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$$\forall y \in \mathbb{Z}, \exists x \in \mathbb{Z}\ f(x) = y \leftrightarrow 2x + 1 = y$$

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$$\forall y \in \mathbb{Z}, \exists x \in \mathbb{Z}\ f(x) = y \leftrightarrow 2x + 1 = y$$
$$\leftrightarrow x = \frac{y-1}{2}$$

but for some $y \in \mathbb{Z}$, $x = \frac{y-1}{2} \notin \mathbb{Z}$ (not onto)

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- $f: \mathbb{Z} \to \mathbb{N}$, defined as $f(x) = \begin{cases} 2x-1 & if \;\; x > 0 \\ -2x & if \;\; x \leq 0 \end{cases}$, f is invertible ?

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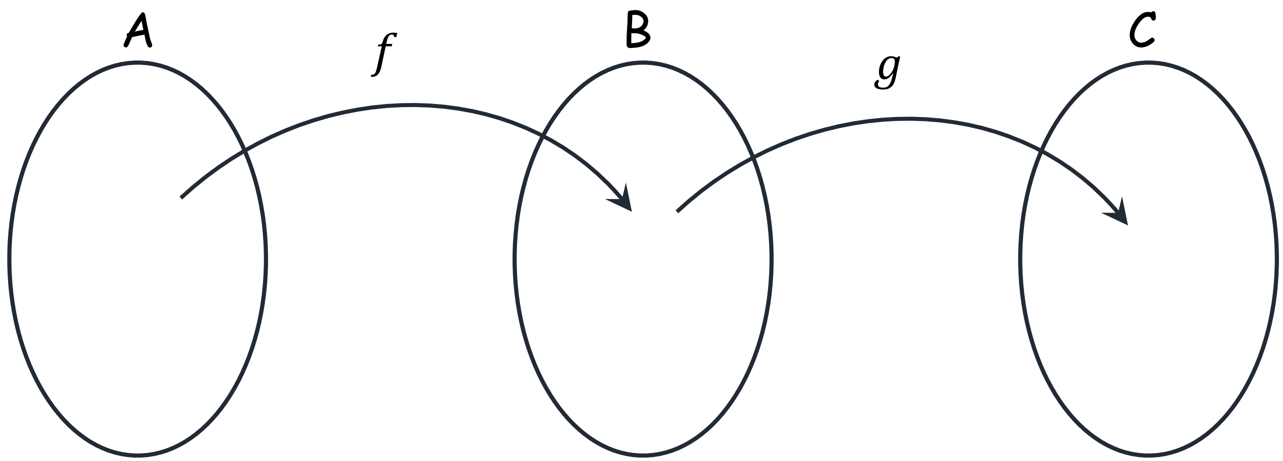
(onto)

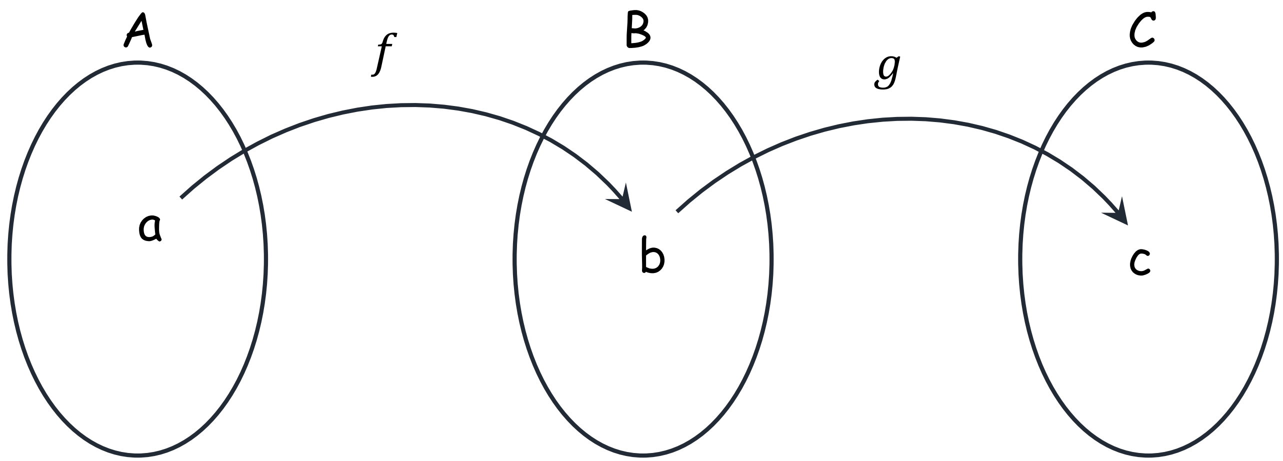
Composition



$f: A \to B$ and $g: B \to C$

$g \circ f: A \to C$

Composition

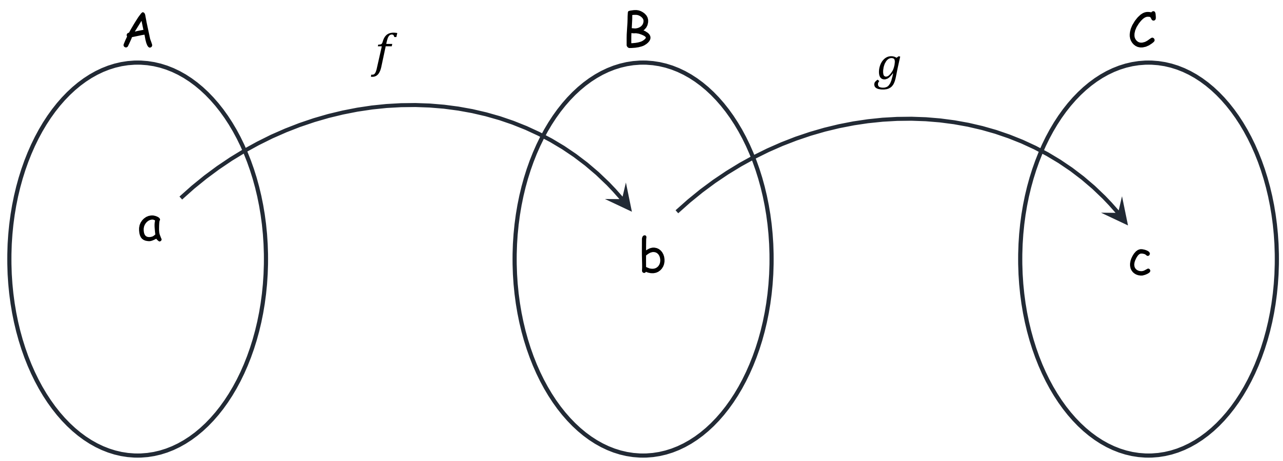


$f: A \to B$ and $g: B \to C$

$g \circ f: A \to C$

$f(a) = b$ and $g(b) = c$

Composition



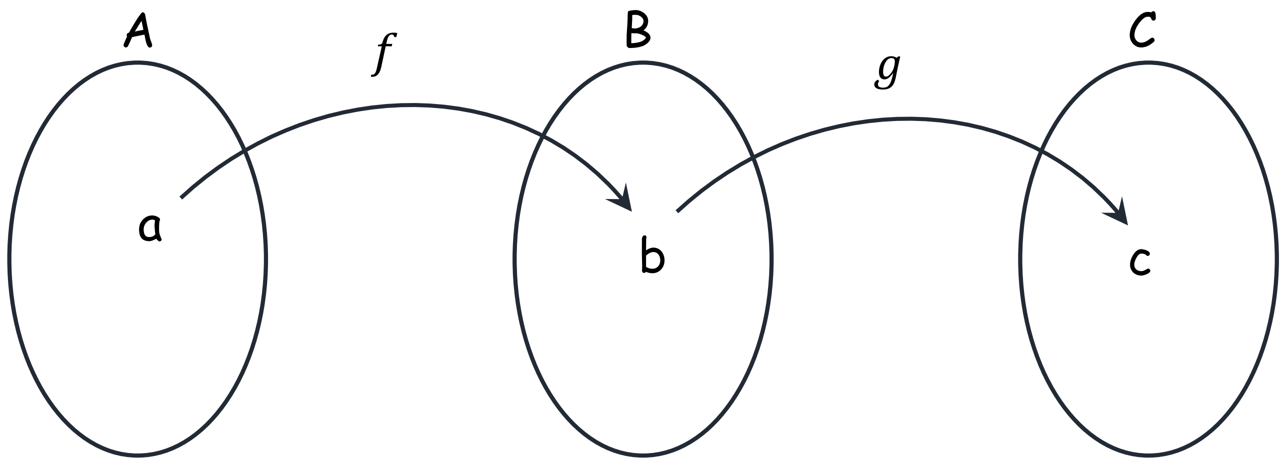
$f: A \to B$ and $g: B \to C$

$g \circ f: A \to C$

$f(a) = b$ and $\mathrm{g}(b) = c$

$g \circ f(a)$

Composition



$f: A \to B$ and $g: B \to C$

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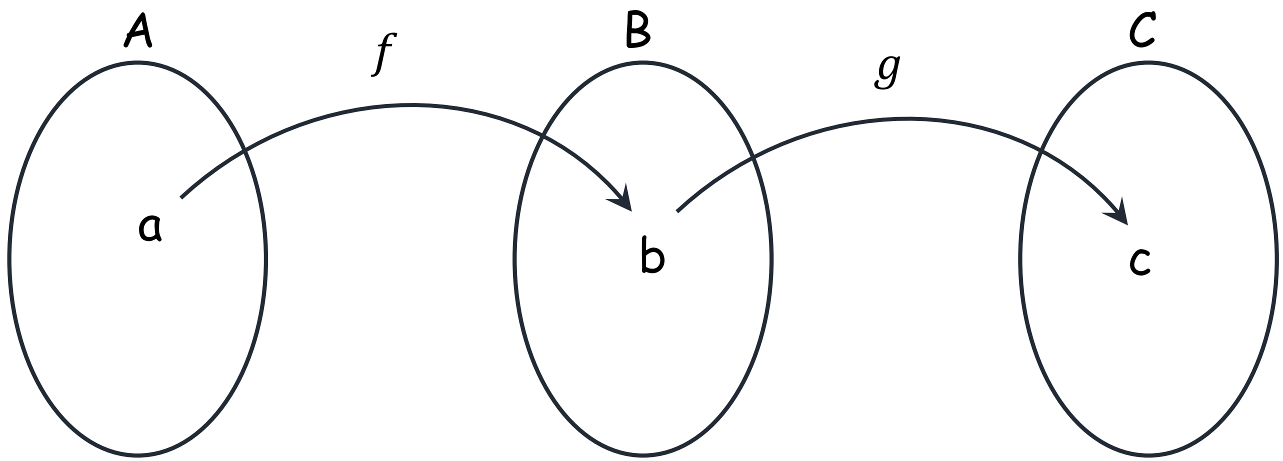
$f(a) = b$ and $\mathrm{g}(b) = c$

$g \circ f(a) = g\big(f(a)\big)$

Composition



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$f(a) = b$ and $\mathrm{g}(b) = c$

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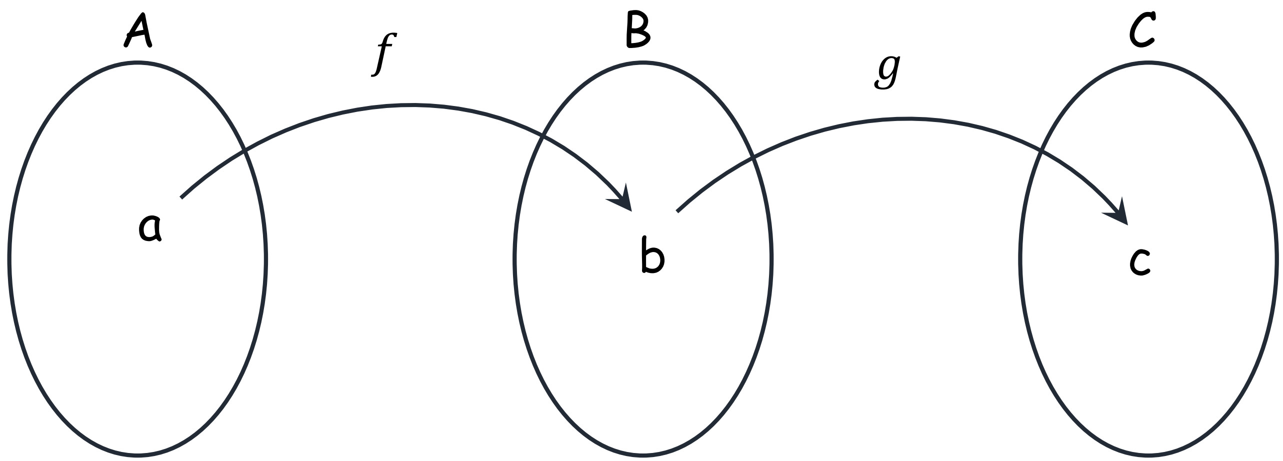
Composition



$f: A \to B$ and $g: B \to C$

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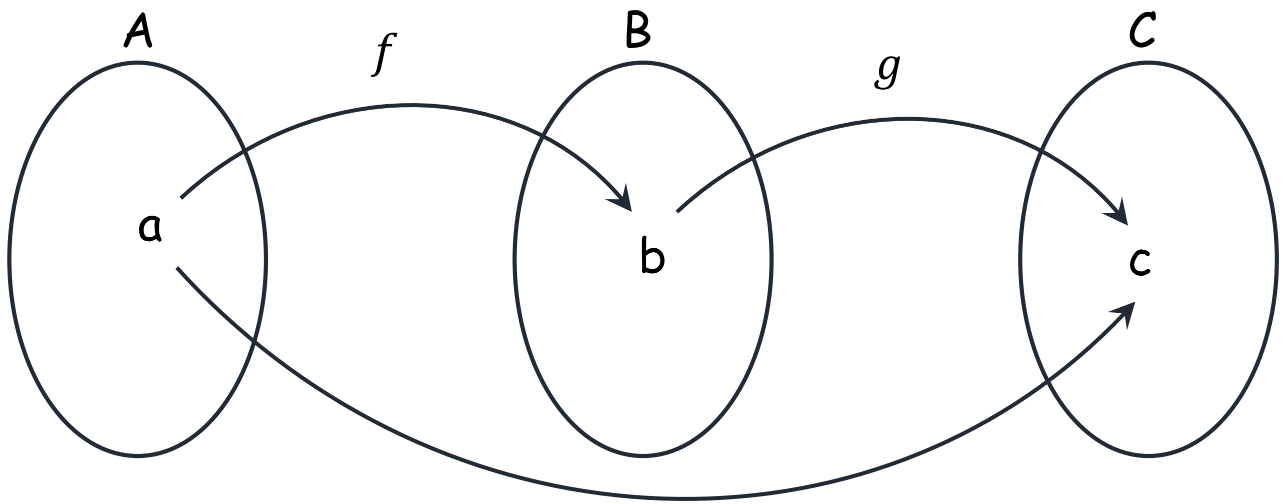
Composition



$g \circ f$

$f: A \to B$ and $g: B \to C$

$g \circ f: A \to C$

$f(a) = b$ and $\mathrm{g}(b) = c$

$g \circ f(a) = g\big(f(a)\big) = g(b) = c$

Composition

- $f, g: \mathbb{Z} \to \mathbb{Z}$,
  $f(x) = 3x + 1$ and $\mathrm{g}(x) = 2x - 1$

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  $f \circ g(x)$

Composition

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$$1 \ne 1 + 1$$

Sequences

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- $a_n = \frac{1}{3^n+2}$ $a_0 = \frac{1}{2}, a_1 = \frac{1}{5},\ a_2 = \frac{1}{11}, \ldots$

Sequences

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initial term

common ratio

general term

Sequences

Geometric Sequence :

$$\underbrace{a}_{\text{initial term}},\ \underbrace{ar}_{\text{common ratio}},\ ar^2, \ldots, \underbrace{ar^n}_{\text{general term}}, \ldots$$

$$a_n = (-1)^n$$

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initial term   common ratio   general term

$$a_n = (-1)^n \qquad a_n = 2.3^n$$

$$1, -1, 1, -1, \ldots \qquad 2, 2.3, 2.9, 2.27, \ldots$$

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initial term

common ratio

general term

$$a_n = (-1)^n \qquad a_n = 2.3^n \qquad a_n = 3.(1/2)^n$$

$$1, -1, 1, -1, \ldots \qquad 2, 2.3, 2.9, 2.27, \ldots \qquad 3, 3/2, 3/4, 3/8, \ldots$$

Sequences

Arithmetic Sequence :

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$$a, a + d, a + 2d, \ldots, a + n.d, \ldots$$

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$$1, 2, 3, 4, \dots$$

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$1, 2, 3, 4, \ldots$

$a_n = 2 - 4n$

$2, -2, -6, -10, \ldots$

$a_n = -1 + 8n$

$-1, 7, 15, 23, \ldots$

Summations

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$$rS_n = S_n + ar^{n+1} - a \;\rightarrow\; S_n = \frac{ar^{n+1} - a}{r - 1}$$

Recurrence Relations

- sometimes the elements of the sequence are defined recursively in terms of previous and the initial elements of the sequence

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$$a_0 = 1,\, a_1 = 5,\, a_2 = 13,\, a_3 = 29,\, a_4 = ?$$

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a_1 &= 2a_0 + 3 = 5 \\
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\end{aligned}$$

Definition : an equation that express the general term of the sequence in terms of previous terms. A sequence is called a solution of a recurrence relation if its terms satisfy the recurrence relation.

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$$
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a_3 &= 225 = 3.(3.(3.5)) \\
&\vdots
\end{aligned}
$$

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  $a_2 = A.7^2 \rightarrow 98 = A.49 \rightarrow A = 2$

Recurrence Relations

- $a_{n+1} = 3a_n, \;\; a_0 = 5$

$a_1 = 15 = 3.5$
$a_2 = 75 = 3.(3.5)$
$a_3 = 225 = 3.(3.(3.5))$
$\vdots$
$a_n = 3^n 5$; the unique solution of the given recurrence relation

- $a_{n+1} = d.a_n, \;\; a_0 = A$ where d is constant

  the solution of the recurrence relation will be $a_n = A.d^n$

- solve the recurrence relation $a_{n+1} = 7.a_n$ where $n \geq 1$ and $a_2 = 98$

  $a_2 = A.7^2 \rightarrow 98 = A.49 \rightarrow A = 2$

  the solution is $a_n = 2.7^n$

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1 + 3	1 + 2 + 1
2 + 2	2 + 1 + 1
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1 + 1 + 2	1 + 1 + 1 + 1

- $a_4 = 2.a_3$,

Recurrence Relations

- 3 can be written as a sum of positive integers in 4 different ways:

3
1 + 2
2 + 1
1 + 1 + 1

2
1 + 1

- In how many different ways can n be written as a sum of positive integers ?

4
1 + 3
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first order linear homogeneous recurrence relation

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The solutions for the characteristic equation are called characteristic roots; r_1 and r_2

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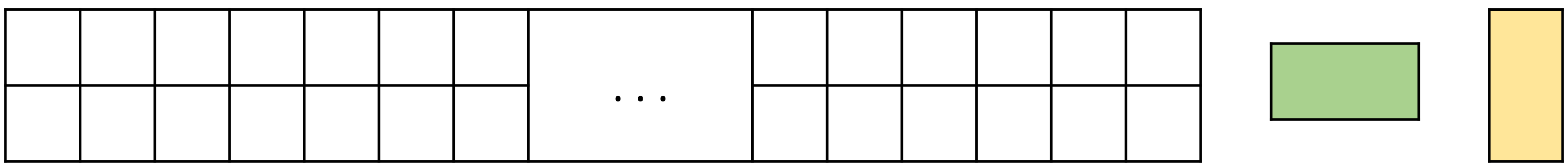

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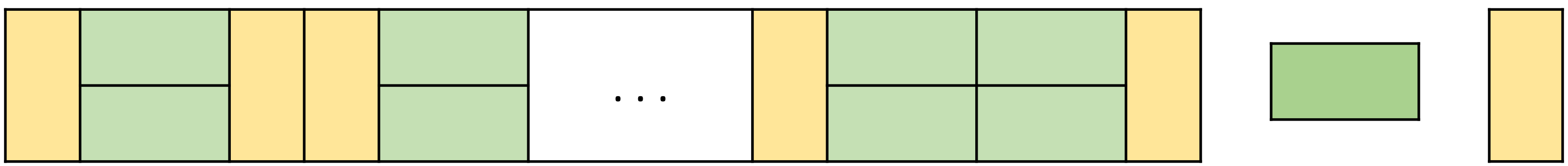

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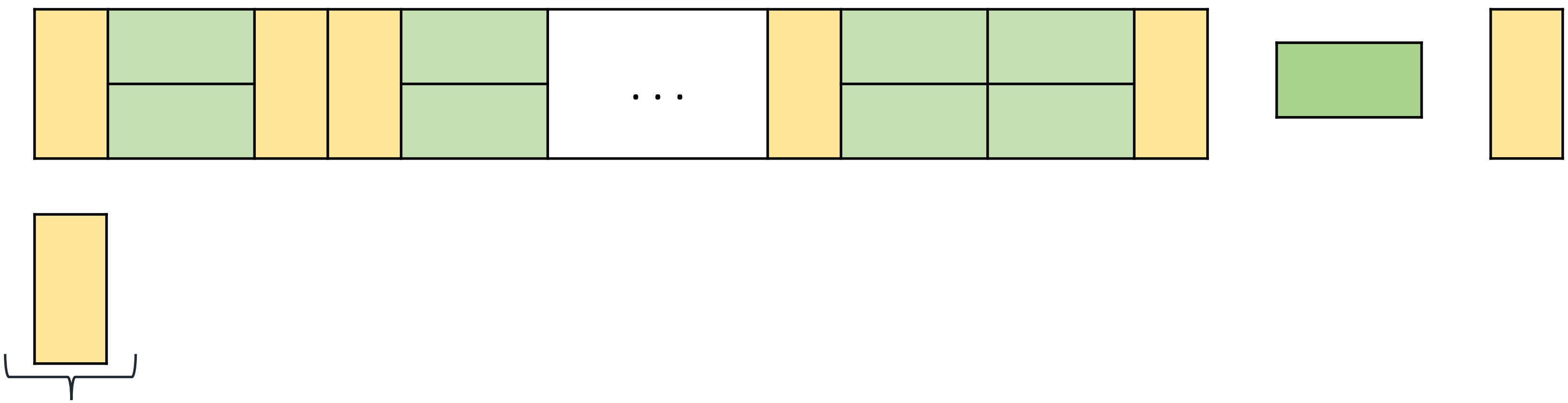

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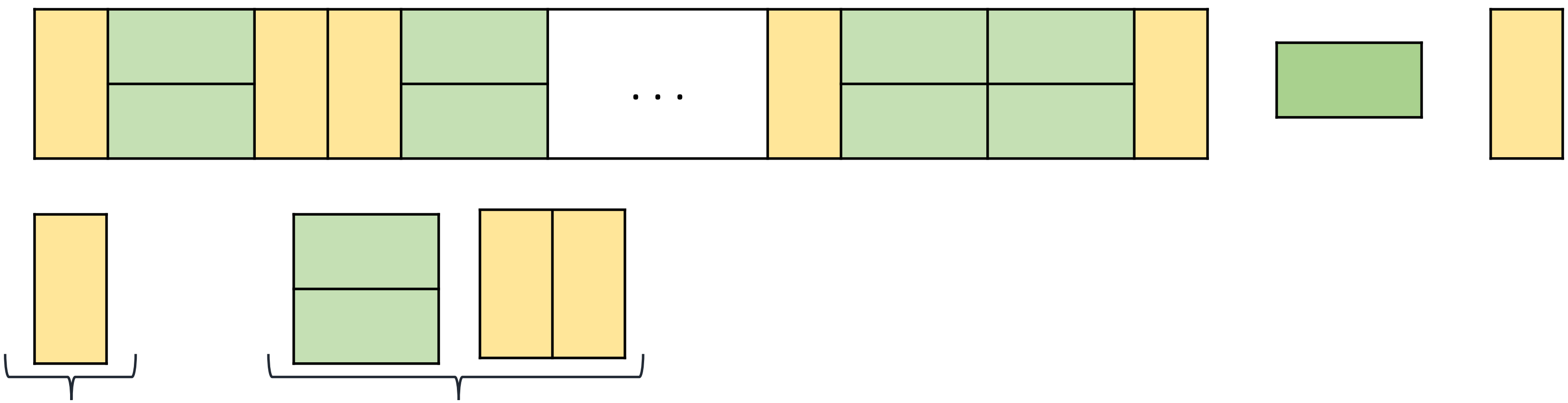

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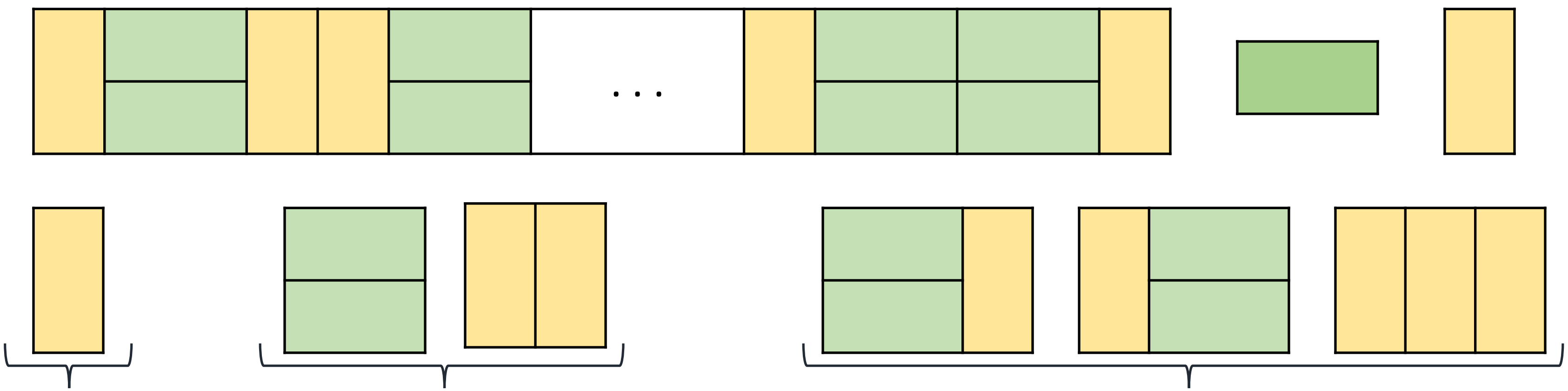

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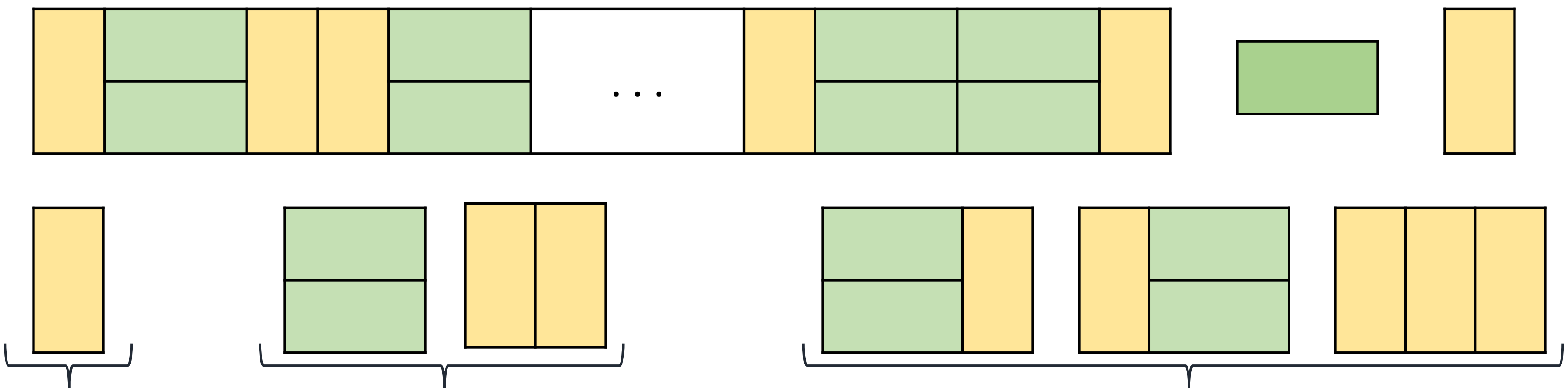

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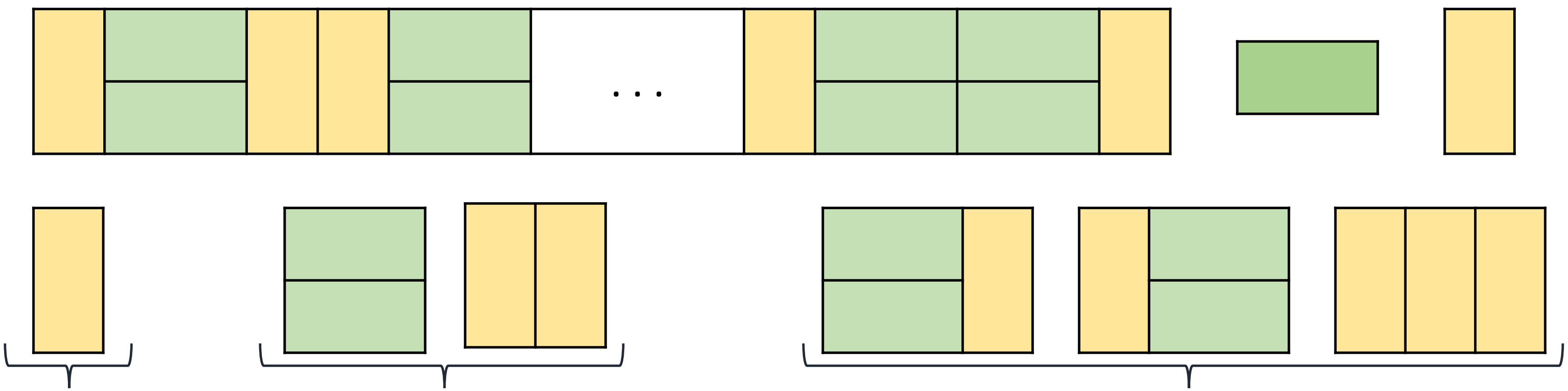

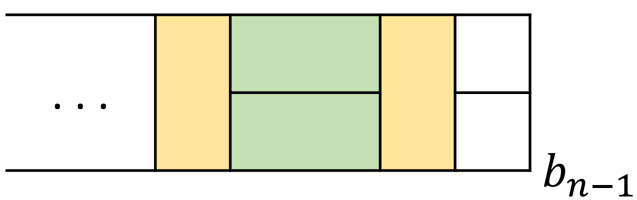

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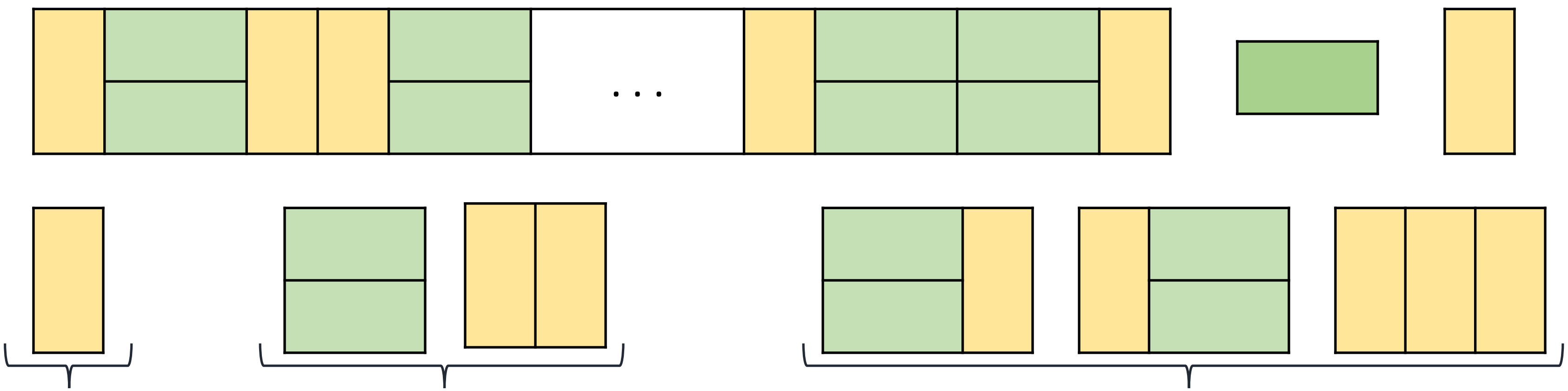

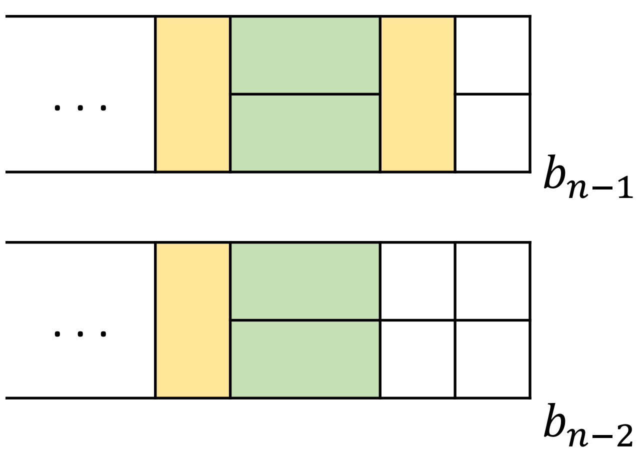

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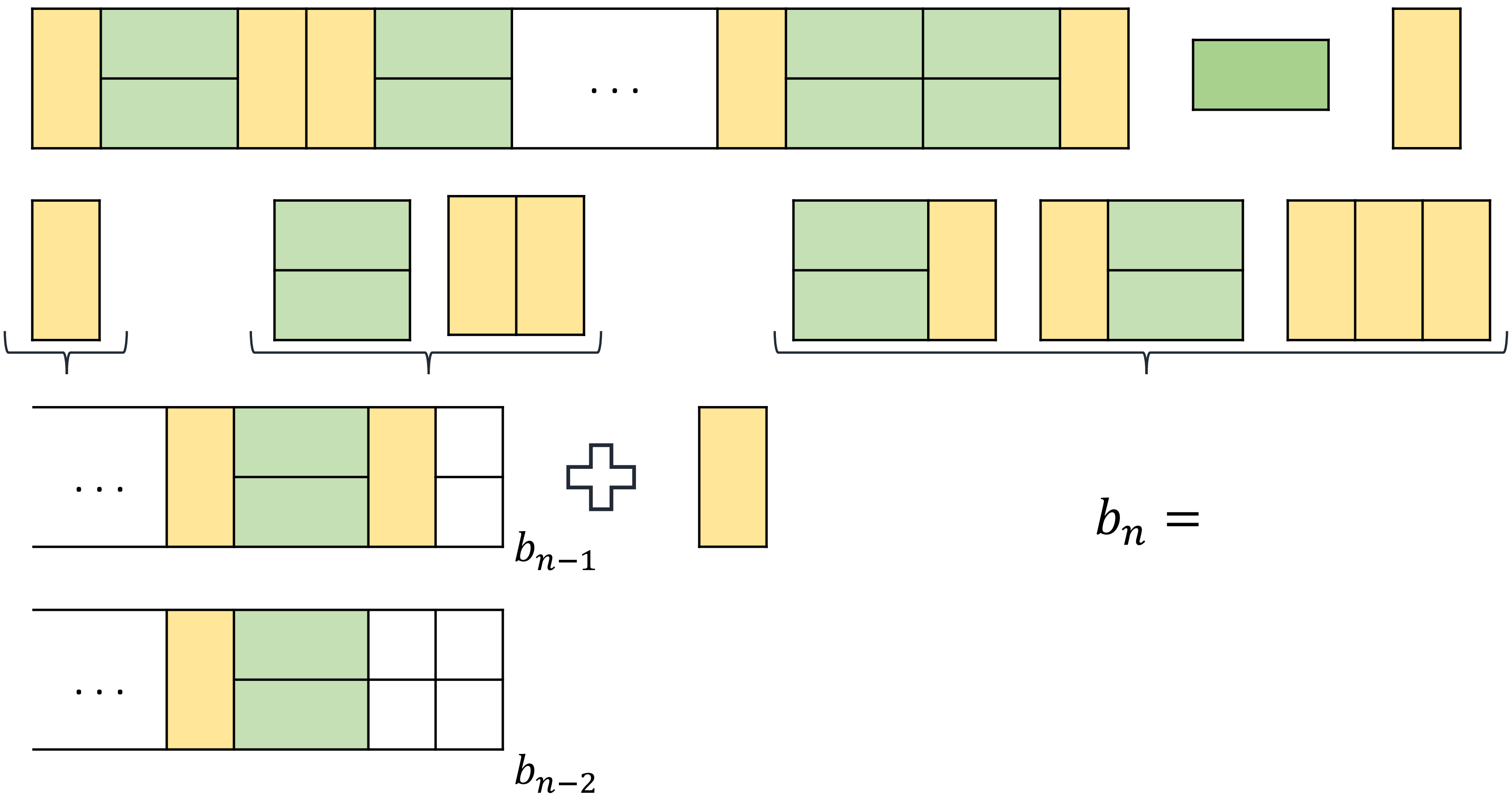

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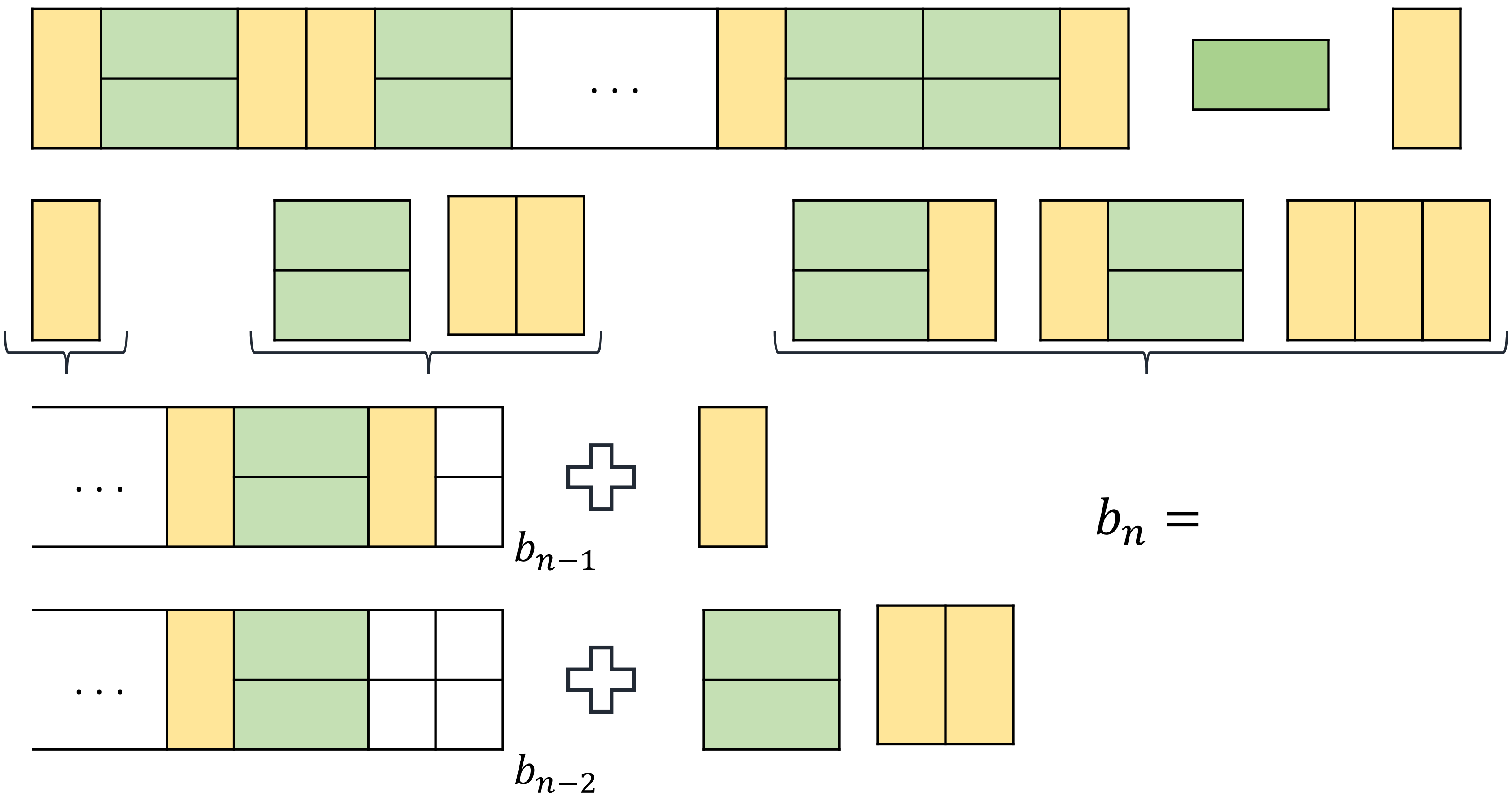

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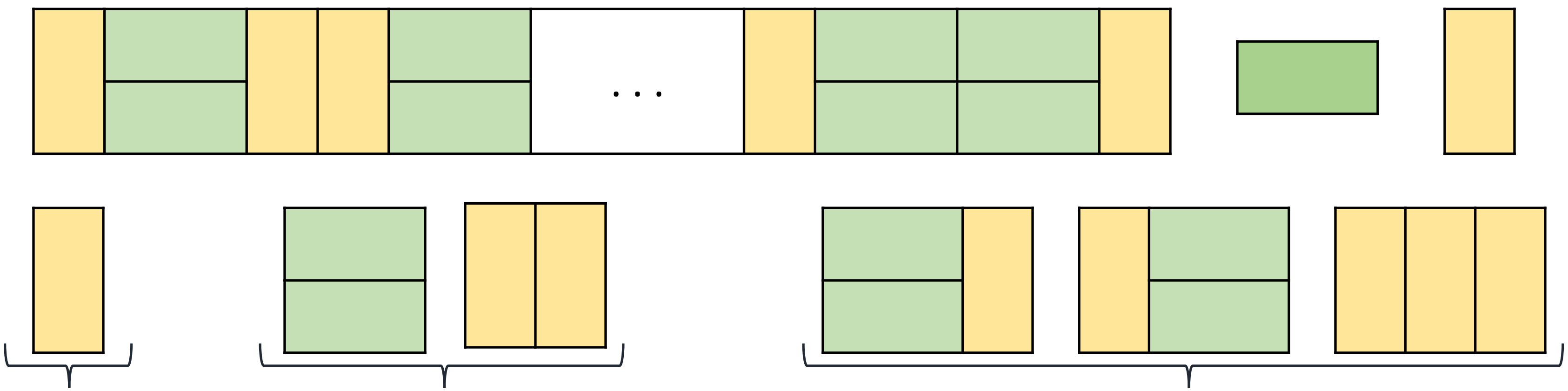

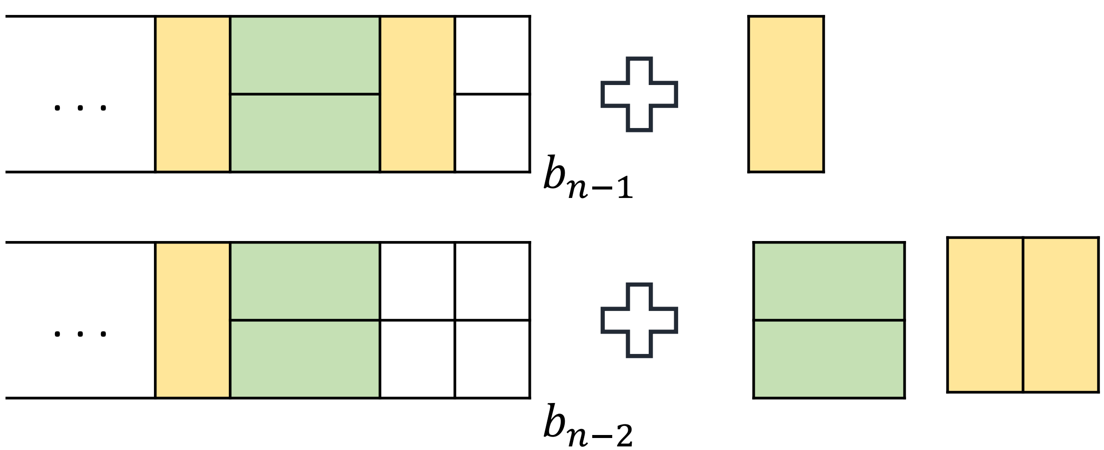

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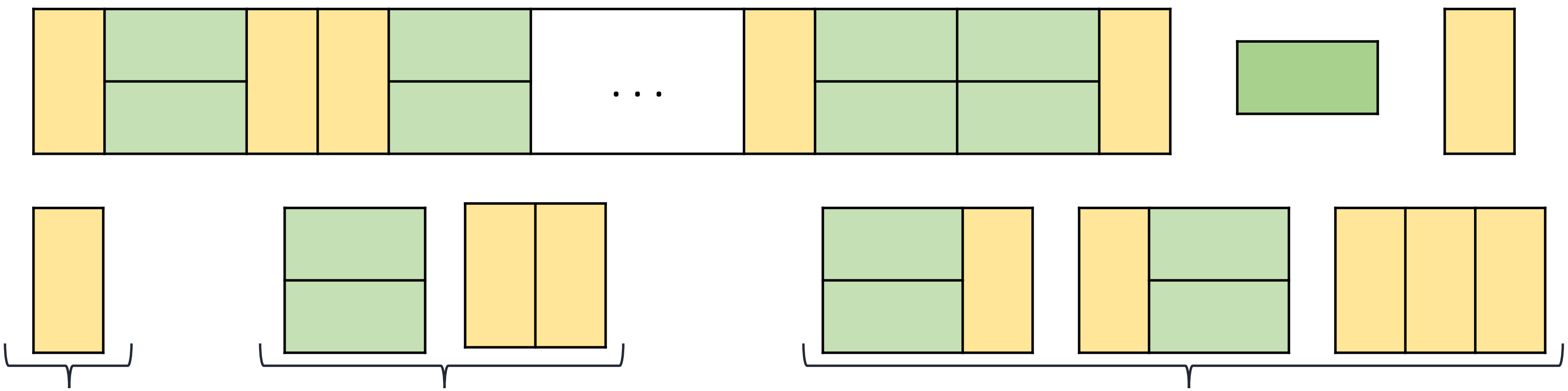

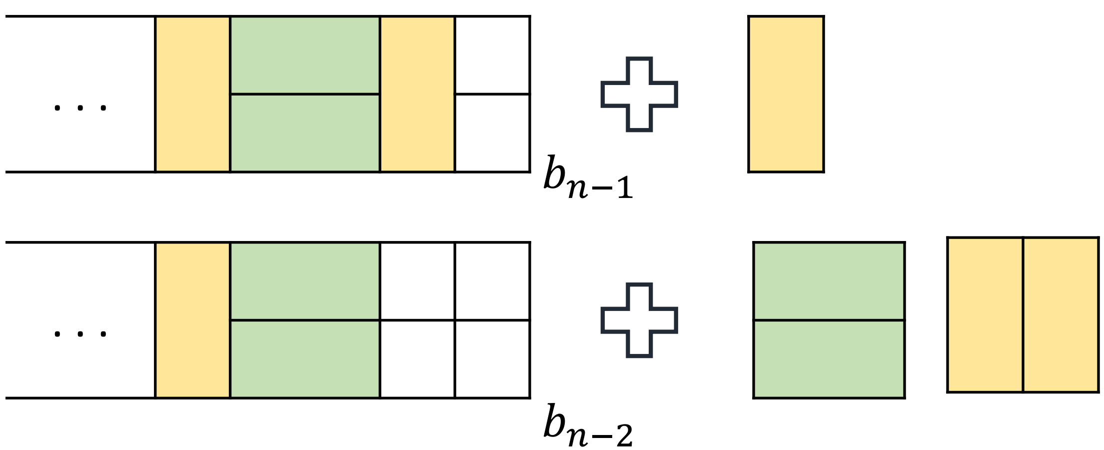

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