Midterm (Solution)

1. (20p) Construct the truth table for the proposition $(p \to q) \wedge (\neg p \to r)$

Solution :

p	q	r	$\neg p$	$p \to q$	$\neg p \to r$	$(p \to q) \wedge (\neg p \to r)$
1	*1*	*1*	*0*	*1*	*1*	*1*
1	*1*	*0*	*0*	*1*	*1*	*1*
1	*0*	*1*	*0*	*0*	*1*	*0*
1	*0*	*0*	*0*	*0*	*1*	*0*
0	*1*	*1*	*1*	*1*	*1*	*1*
0	*1*	*0*	*1*	*1*	*0*	*0*
0	*0*	*1*	*1*	*1*	*1*	*1*
0	*0*	*0*	*1*	*1*	*0*	*0*

2. (20p) Prove that if $3n^2 + 2n + 3$ is even integer, then n is odd integer.

Solution :
(Proof by Contrapositive)
$p \to q \equiv \neg q \to \neg p$

If n *is not odd integer, then* $3n^2 + 2n + 3$ *is not even integer*

assume n is not odd integer (even integer),
$n = 2k, \exists k \in Z$, *then* $3n^2 + 2n + 3 = 12k^2 + 4k + 3$
$3n^2 + 2n + 3 = 2(6k^2 + 2k + 1) + 1$
$3n^2 + 2n + 3 = 2m + 1, \exists m \in Z,$
then $3n^2 + 2n + 3$ *is not even integer*

(Proof by Contradiction)
assume $\neg(p \to q) \equiv \neg(\neg p \vee q) \equiv p \wedge \neg q$
assume $3n^2 + 2n + 3$ *is even integer and* n *is even integer,*
$n = 2k, \exists k \in Z$, *then* $3n^2 + 2n + 3 = 12k^2 + 4k + 3$
$3n^2 + 2n + 3 = 2(6k^2 + 2k + 1) + 1$
$3n^2 + 2n + 3 = 2m + 1, \exists m \in Z$
then $3n^2 + 2n + 3$ *is odd integer* $(\neg p)$*, it is a contradiction. Thus,* $(p \to q)$ *must be true.*

3. (20p) What value is returned by the following algorithm? What is its basic operation? How many times is the basic operation executed? Give the worst-case running time of the algorithm using Big Oh notation.

Cardano (n)
input *: a positive integer n*
r ← 0
for i = 1 to 2n
    for j = 1 to n
        for k = 1 to 3n

r ← r + 5
return r

Solution :

basic operation(s) : r ← r + 5 (addition and assignment, can be considered as just 1 operation)
T(n) : $\sum_{i=1}^{2n}\sum_{j=1}^{n}\sum_{k=1}^{3n} 1 = 6n^3 = O(n^3)$
returned value : 5T(n)

4. (20p) Given the sets $A = \{a, b, c\}$ and $B = \{1, 2, 3, 4, 5, 6\}$,

a) How many one-to-one functions are there from A to B?
a) How many onto functions are there from A to B?

Solution :

a) P(6, 3) = 6.5.4 = 120
b) since $|A| < |B|$*, onto functions cannot be defined*

5. (20p) In how many ways can 4 men and 4 women be seated if no two men and no two women sit next to each other? (linear)

Solution :

M F M F M F M F or F M F M F M F M (linear, not circular), 2.4!.4! = 1152