Discrete Probability

Murat Osmanoglu

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$|E_1| = 33, |E_2| = 20$, and $|E_1 \cap E_2| = 6$; thus, $p(E_1 \cup E_2) = \frac{33+20-6}{100}$

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\begin{aligned}
|\mathrm{A} \cup \mathrm{B} \cup \mathrm{C}| &= |\mathrm{A}| + |\mathrm{B}| + |\mathrm{C}| - |\mathrm{A} \cap \mathrm{B}| - |\mathrm{A} \cap \mathrm{C}| - |\mathrm{B} \cap \mathrm{C}| + |\mathrm{A} \cap \mathrm{B} \cap \mathrm{C}| \\
&= 13 + 26 + 12 - 0 - 3 - 6 + 0 = 42
\end{aligned}
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$$p(\mathrm{A} \cup \mathrm{B} \cup \mathrm{C}) = \frac{42}{52}$$

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- $0 \leq p(x) \leq 1, \forall x \in S$

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- $\sum_{x \in S} p(x) = 1$

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$p(E) = p(1) + p(3) + p(5) = 4/7$

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p = 0.9, q = 0.1 $\binom{10}{8}(0.9)^8(0.1)^2$

Bayes' Theorem

Assume there are two boxes containing green and red balls. First one contains 2 green and 7 red balls, second one contains 4 green and 3 red balls. Assume you randomly select one box, then randomly select one ball from this box. If the ball you selected is a red ball, What is the probability that you selected that ball from the first box ?

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E : you select a red ball
F : you select the ball from the first box
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$$p(F|E) = \frac{p(F \cap E)}{p(E)} \Rightarrow p(E|F) = \frac{p(F \cap E)}{p(F)}$$

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F : you select the ball from the first box
$\bar{E}$: you select a green ball
$\bar{F}$: you select the ball from the second box

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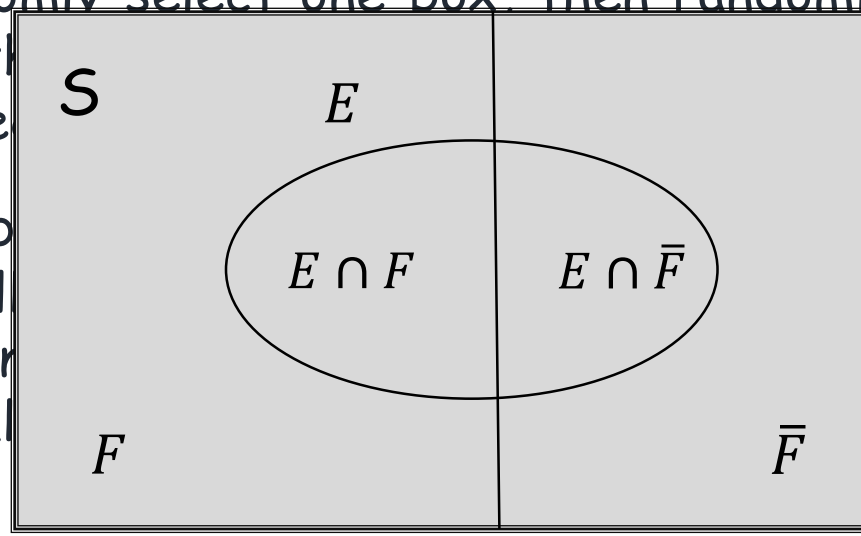

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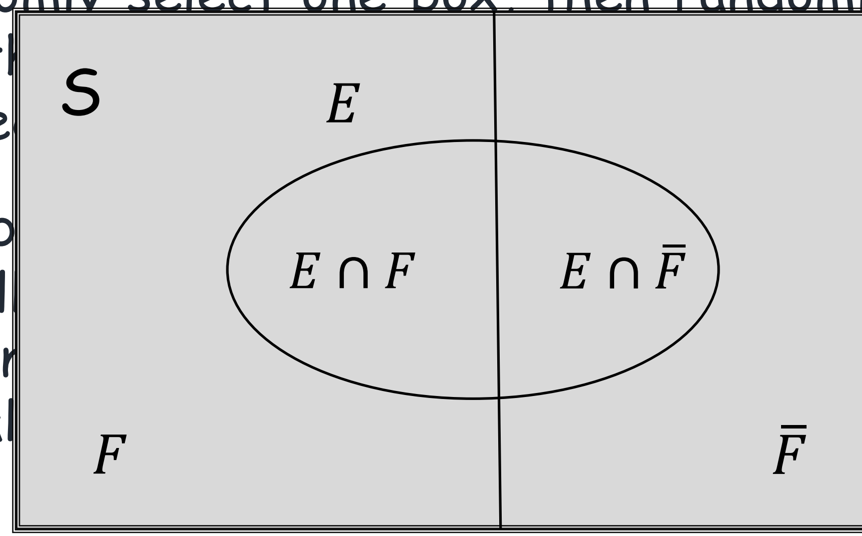

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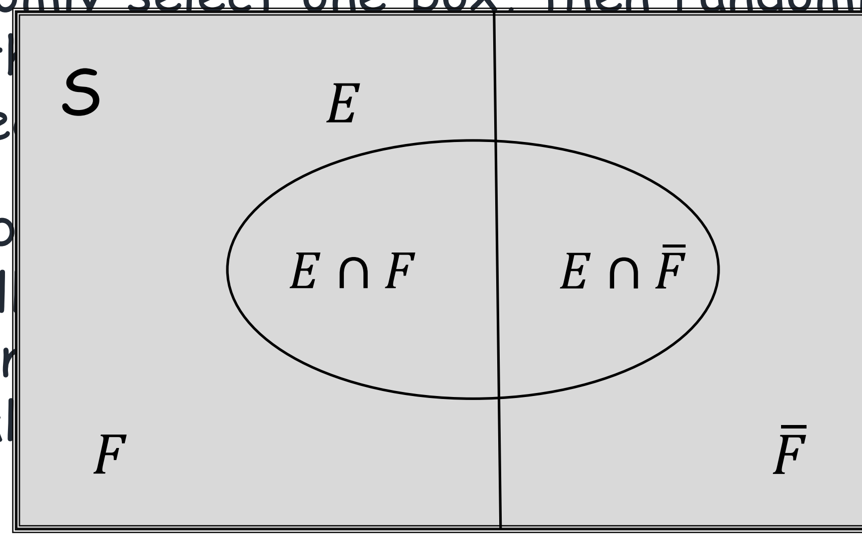

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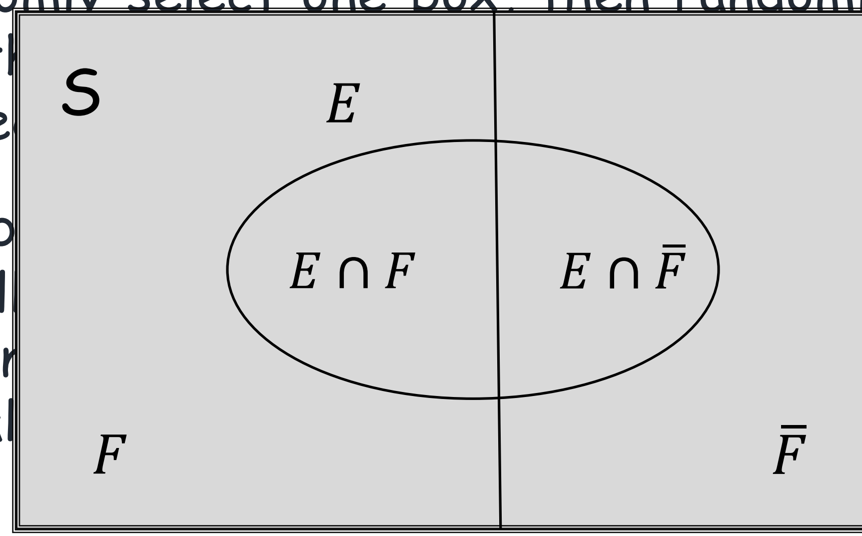

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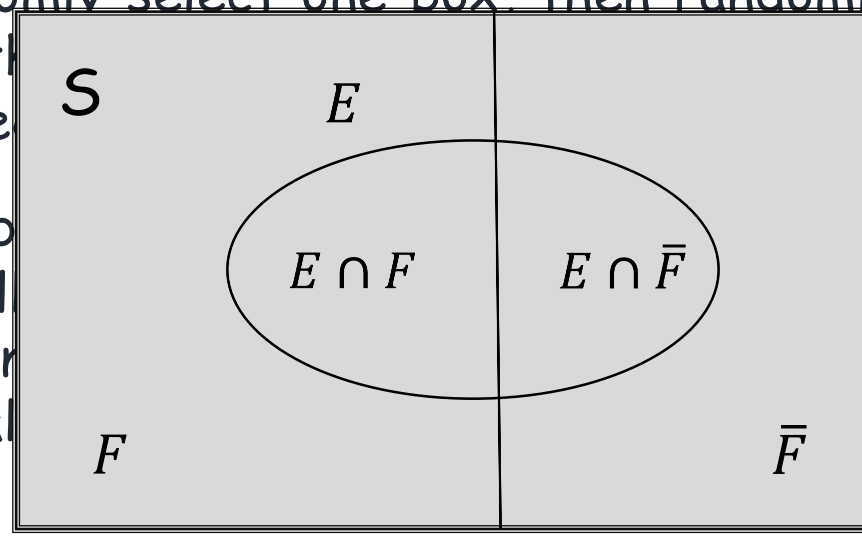

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Bayesian Spam Filters

Suppose that the word 'rolex' appears in 125 of 1000 messages which were identified as spam, and in 5 of 1000 messages which were identified as non-spam. Assume that it is equally likely that an incoming message is spam or non-spam. Estimate the probability that an incoming message containing 'rolex' is spam. If our threshold for rejecting a message as spam is 0.9, will such messages be rejected or not ?

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$p(F|E) = \frac{125}{1000} = 0.125$ and $p\left(F|\bar{E}\right) = \frac{5}{1000} = 0.005$

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Random Variables

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$P(X = 0) = 1/8$, $P(X = 1) = 3/8$, $P(X = 2) = 3/8$, $P(X = 3) = 1/8$

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Expected Value

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The deviation of X at s in S : X(s) – E(X), the difference between the value of X and the mean of X

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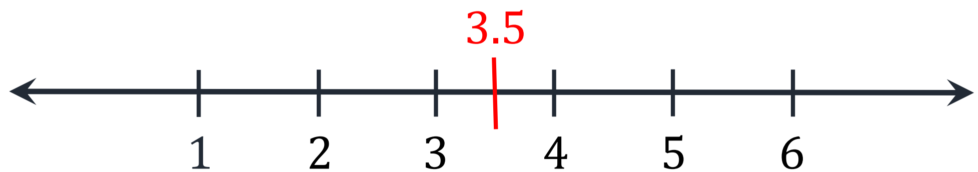

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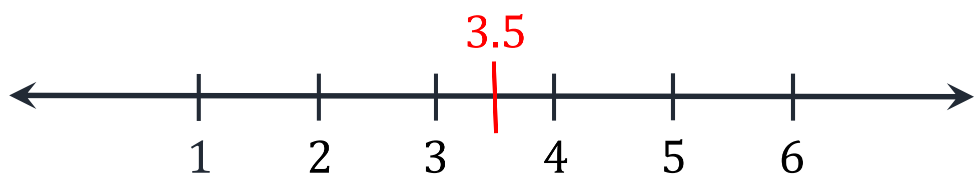

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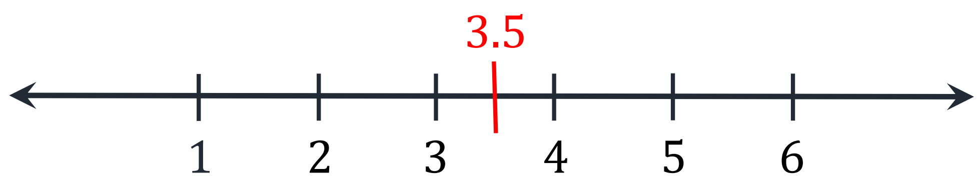
$$E(X) = \frac{1}{6}.1 + \frac{1}{6}.2 + \frac{1}{6}.3 + \frac{1}{6}.4 + \frac{1}{6}.5 + \frac{1}{6}.6 = 3.5$$



- A fair coin is flipped 3 times. Let X be the random variable that assigns the number of the heads to each otucome in the sample space. What is the expected value of X ?

TTT, TTH, THT, HTT, THH, HTH, HHT, HHH

Expected Value

- Let X be the number that comes up when a fair die is rolled. What is the expected value of X ?

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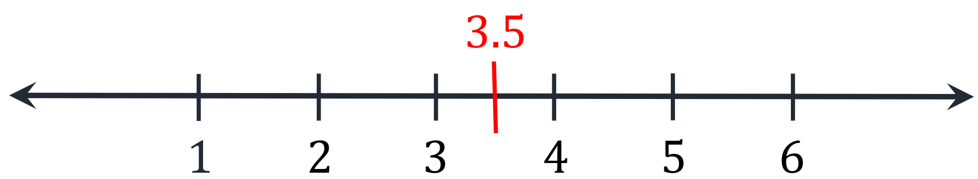

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TTT, TTH, THT, HTT, THH, HTH, HHT, HHH

X(TTT) = 0, X(THT) = 1, X(HHT) = 2, X(HHH) = 3,

Expected Value

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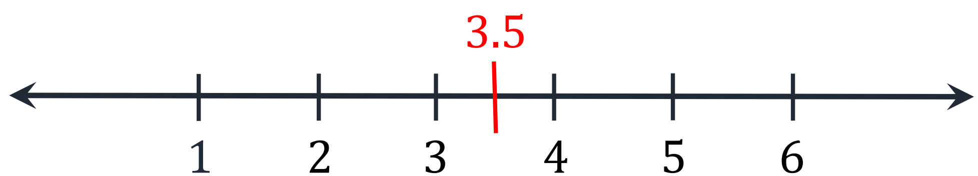

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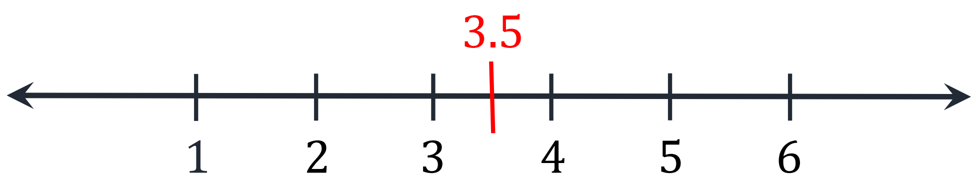
TTT, TTH, THT, HTT, THH, HTH, HHT, HHH
X(TTT) = 0, X(THT) = 1, X(HHT) = 2, X(HHH) = 3,

$$E(X) = \frac{1}{8}.0 + \frac{3}{8}.1 + \frac{3}{8}.2 + \frac{1}{8}.3$$

Expected Value

- Let X be the number that comes up when a fair die is rolled. What is the expected value of X ?

$$E(X) = \frac{1}{6}.1 + \frac{1}{6}.2 + \frac{1}{6}.3 + \frac{1}{6}.4 + \frac{1}{6}.5 + \frac{1}{6}.6 = 3.5$$



- A fair coin is flipped 3 times. Let X be the random variable that assigns the number of the heads to each otucome in the sample space. What is the expected value of X ?

TTT, TTH, THT, HTT, THH, HTH, HHT, HHH
X(TTT) = 0, X(THT) = 1, X(HHT) = 2, X(HHH) = 3,

$$E(X) = \frac{1}{8}.0 + \frac{3}{8}.1 + \frac{3}{8}.2 + \frac{1}{8}.3 = 1.5$$

Expected Value

- What is the expected value of the sum of the numbers that appear when a pair of dice is rolled ?

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$$E(X) =$$

Expected Value

- What is the expected value of the sum of the numbers that appear when a pair of dice is rolled ?

$$E(X) = \frac{1}{36}.2 + \frac{2}{36}.3 + \frac{3}{36}.4 + \frac{4}{36}.5 + \frac{5}{36}.6 + \frac{6}{36}.7$$

$$= \frac{5}{36}.8 + \frac{4}{36}.9 + \frac{3}{36}.10 + \frac{2}{36}.11 + \frac{1}{36}.12$$

Expected Value

- What is the expected value of the sum of the numbers that appear when a pair of dice is rolled ?

$$E(X) = \frac{1}{36}.2 + \frac{2}{36}.3 + \frac{3}{36}.4 + \frac{4}{36}.5 + \frac{5}{36}.6 + \frac{6}{36}.7$$

$$= \frac{5}{36}.8 + \frac{4}{36}.9 + \frac{3}{36}.10 + \frac{2}{36}.11 + \frac{1}{36}.12 = 7$$

Expected Value

- Suppose that the probability that a coin comes up tails is p This coins is flipped repeatedly until it comes up tails. What is the expected number of flips ?

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  $p(T) = p$,

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  T, HT, HHT, HHHT, HHHHT

  $p(T) = p$, $p(HT) =(1-p)p$,

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  T, HT, HHT, HHHT, HHHHT

  p(T) = p, p(HT) =(1-p)p, p(HHT) = (1-p)(1-p)p, . . .

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  T, HT, HHT, HHHT, HHHHT

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  Let X be the random variable that is the number of flips for an outcome

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  $p(T) = p$, $p(HT) = (1-p)p$, $p(HHT) = (1-p)(1-p)p$, . . .

  Let X be the random variable that is the number of flips for an outcome

  $X(T) = 1$, $X(HT) = 2$, $X(HHT) = 3$, . . .

Expected Value

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  T, HT, HHT, HHHT, HHHHT

  p(T) = p, p(HT) =(1-p)p, p(HHT) = (1-p)(1-p)p, . . .

  Let X be the random variable that is the number of flips for an outcome

  X(T) = 1, X(HT) = 2, X(HHT) = 3, . . .

$$E(X) = X(T).p(T) + X(HT).p(HT) + X(HHT)p(HHT) + \ldots$$

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Let X be the random variable that is the number of flips for an outcome

X(T) = 1, X(HT) = 2, X(HHT) = 3, . . .

$$E(X) = X(T).p(T) + X(HT).p(HT) + X(HHT)p(HHT) + \ldots$$
$$E(X) = 1.p + 2.(1-p)p + 3.(1-p)^2p + \ldots$$

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T, HT, HHT, HHHT, HHHHT

p(T) = p, p(HT) =(1-p)p, p(HHT) = (1-p)(1-p)p, . . .

Let X be the random variable that is the number of flips for an outcome

X(T) = 1, X(HT) = 2, X(HHT) = 3, . . .

$$E(X) = X(T).p(T) + X(HT).p(HT) + X(HHT)p(HHT) + \ldots$$
$$E(X) = 1.p + 2.(1-p)p + 3.(1-p)^2p + \ldots$$
$$E(X) = p[1.(1-p)^0 + 2.(1-p)^1 + 3.(1-p)^2p + \ldots]$$

Expected Value

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T, HT, HHT, HHHT, HHHHT

p(T) = p, p(HT) =(1-p)p, p(HHT) = (1-p)(1-p)p, . . .

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$$E(X) = 1.p + 2.(1-p)p + 3.(1-p)^2p + \ldots$$
$$E(X) = p[1.(1-p)^0 + 2.(1-p)^1 + 3.(1-p)^2p + \ldots]$$
$$E(X) = p\sum_{i=1}^{\infty} i.(1-p)^{i-1}$$

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T, HT, HHT, HHHT, HHHHT

p(T) = p, p(HT) =(1-p)p, p(HHT) = (1-p)(1-p)p, . . .

Let X be the random variable that is the number of flips for an outcome

X(T) = 1, X(HT) = 2, X(HHT) = 3, . . .

$$E(X) = X(T).p(T) + X(HT).p(HT) + X(HHT)p(HHT) + \ldots$$
$$E(X) = 1.p + 2.(1-p)p + 3.(1-p)^2 p + \ldots$$
$$E(X) = p[1.(1-p)^0 + 2.(1-p)^1 + 3.(1-p)^2 p + \ldots]$$
$$E(X) = p\sum_{i=1}^{\infty} i.(1-p)^{i-1} = p.\frac{1}{p^2} = \frac{1}{p}$$

Expected Value

- Suppose that a casino offers a game for a single player at which a fair cion is tossed. The initial stake is 2 dollars, and it is doubled every time heads appears. The game ends when the first tails appears. What would be the fair price to pay the casino in order to enter the game ?

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  T, HT, HHT, HHHT, HHHHT, . . .

  $p(T) = \frac{1}{2}, p(HT) = \frac{1}{2}.\frac{1}{2}\ , p(HHT) = \frac{1}{2}.\frac{1}{2}.\,\frac{1}{2}, \ldots$

Expected Value

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  T, HT, HHT, HHHT, HHHHT, . . .

  $p(T) = \frac{1}{2}$, $p(HT) = \frac{1}{2} . \frac{1}{2}$, $p(HHT) = \frac{1}{2} . \frac{1}{2} . \frac{1}{2}$, . . .

  Let X be the random variable defined as the amount of Money the player wins

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T, HT, HHT, HHHT, HHHHT, . . .

p(T) = $\frac{1}{2}$, p(HT) = $\frac{1}{2}.\frac{1}{2}$, p(HHT) = $\frac{1}{2}.\frac{1}{2}.\frac{1}{2}$, . . .

Let X be the random variable defined as the amount of Money the player wins
X(T) = 2, X(HT) = 2.2, X(HHT) = 2.2.2, . . .

Expected Value

- Suppose that a casino offers a game for a single player at which a fair cion is tossed. The initial stake is 2 dollars, and it is doubled every time heads appears. The game ends when the first tails appears. What would be the fair price to pay the casino in order to enter the game ?

T, HT, HHT, HHHT, HHHHT, . . .

p(T) = $\frac{1}{2}$, p(HT) = $\frac{1}{2}.\frac{1}{2}$, p(HHT) = $\frac{1}{2}.\frac{1}{2}.\frac{1}{2}$, . . .

Let X be the random variable defined as the amount of Money the player wins

X(T) = 2, X(HT) = 2.2, X(HHT) = 2.2.2, . . .

$$E(X) = X(T).p(T) + X(HT).p(HT) + X(HHT)p(HHT) + \ldots$$

Expected Value

- Suppose that a casino offers a game for a single player at which a fair cion is tossed. The initial stake is 2 dollars, and it is doubled every time heads appears. The game ends when the first tails appears. What would be the fair price to pay the casino in order to enter the game ?

T, HT, HHT, HHHT, HHHHT, . . .

p(T) = $\frac{1}{2}$, p(HT) = $\frac{1}{2}.\frac{1}{2}$, p(HHT) = $\frac{1}{2}.\frac{1}{2}.\frac{1}{2}$, . . .

Let X be the random variable defined as the amount of Money the player wins
X(T) = 2, X(HT) = 2.2, X(HHT) = 2.2.2, . . .

$$E(X) = X(T).p(T) + X(HT).p(HT) + X(HHT)p(HHT) + \ldots$$
$$E(X) = 2.\frac{1}{2} + 2.2.\frac{1}{2}.\frac{1}{2} + 2.2.2.\frac{1}{2}.\frac{1}{2}.\frac{1}{2} + \ldots$$

Expected Value

- Suppose that a casino offers a game for a single player at which a fair cion is tossed. The initial stake is 2 dollars, and it is doubled every time heads appears. The game ends when the first tails appears. What would be the fair price to pay the casino in order to enter the game ?

T, HT, HHT, HHHT, HHHHT, . . .

p(T) = $\frac{1}{2}$, p(HT) = $\frac{1}{2}.\frac{1}{2}$, p(HHT) = $\frac{1}{2}.\frac{1}{2}.\frac{1}{2}$, . . .

Let X be the random variable defined as the amount of Money the player wins

X(T) = 2, X(HT) = 2.2, X(HHT) = 2.2.2, . . .

$$E(X) = X(T).p(T) + X(HT).p(HT) + X(HHT)p(HHT) + \ldots$$
$$E(X) = 2.\frac{1}{2} + 2.2.\frac{1}{2}.\frac{1}{2} + 2.2.2.\frac{1}{2}.\frac{1}{2}.\frac{1}{2} + \ldots$$
$$E(X) = 1 + 1 + 1 + \ldots$$

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p(T) = $\frac{1}{2}$, p(HT) = $\frac{1}{2}.\frac{1}{2}$, p(HHT) = $\frac{1}{2}.\frac{1}{2}.\frac{1}{2}$, . . .

Let X be the random variable defined as the amount of Money the player wins

X(T) = 2, X(HT) = 2.2, X(HHT) = 2.2.2, . . .

$$E(X) = X(T).p(T) + X(HT).p(HT) + X(HHT)p(HHT) + \ldots$$

$$E(X) = 2.\frac{1}{2} + 2.2.\frac{1}{2}.\frac{1}{2} + 2.2.2.\frac{1}{2}.\frac{1}{2}.\frac{1}{2} + \ldots$$

$$E(X) = 1 + 1 + 1 + \ldots$$

St. Petersburg Paradox