Mathematical Induction

Murat Osmanoglu

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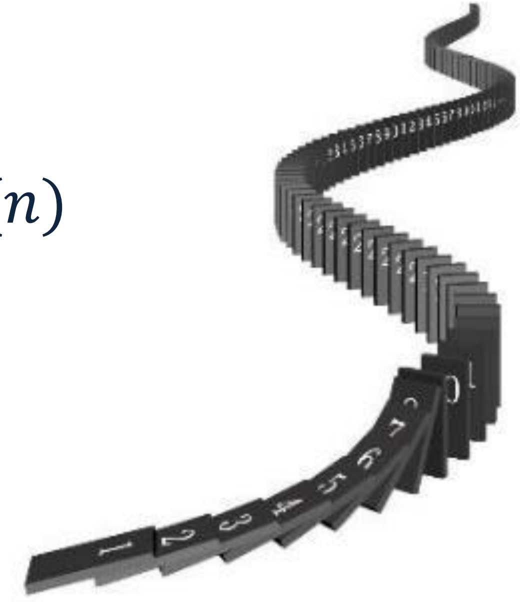

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<u>Basic Step</u> P(0) : $a = \frac{ar^{0+1}-a}{r-1} = \frac{a(r-1)}{r-1} = a$

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since $n \geq 3$, $2k - 6 \geq 0$

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$$H_j = 1 + \frac{1}{2} + \ldots + \frac{1}{j}$$

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$$\rightarrow [H_1 + \ldots + H_{k+1} = (k+2)H_{k+1} - (k+1)\,]$$

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$k + 1 = (a - 5).3 + 15 + 1$
$k + 1 = (a - 5).3 + 2.8$

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Proofs

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    for all $k \in Z^{+}$ (Inductive Step)

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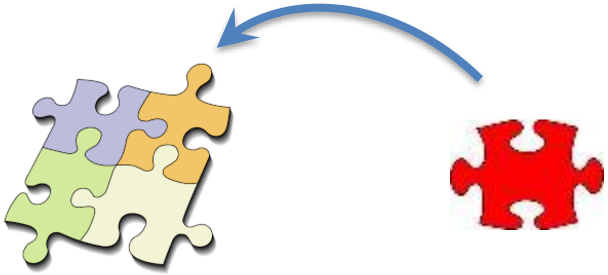

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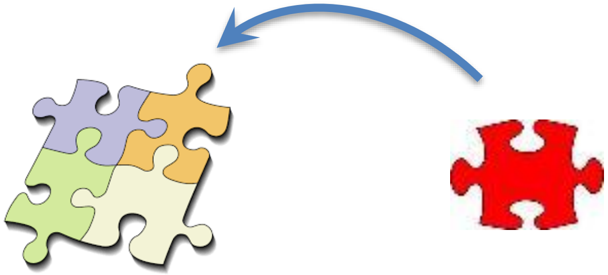

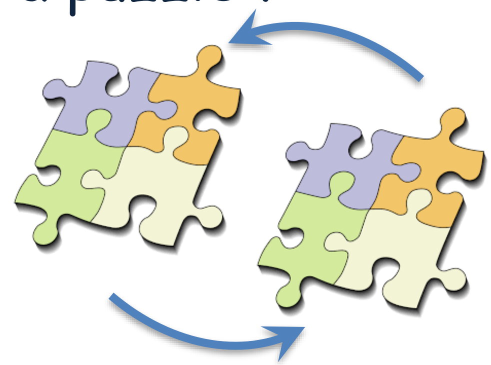

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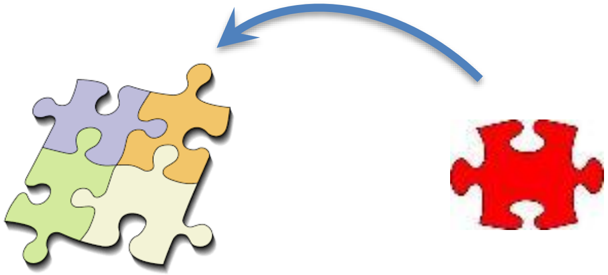

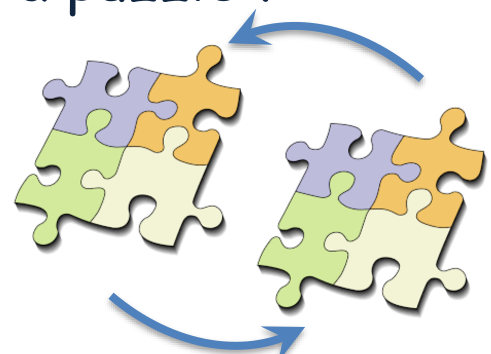

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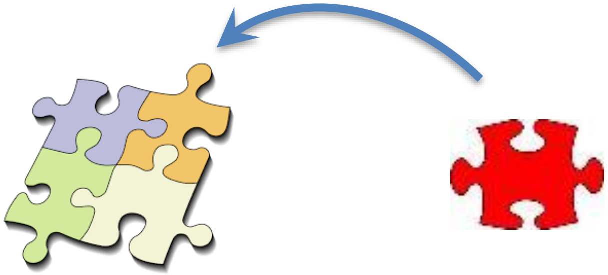

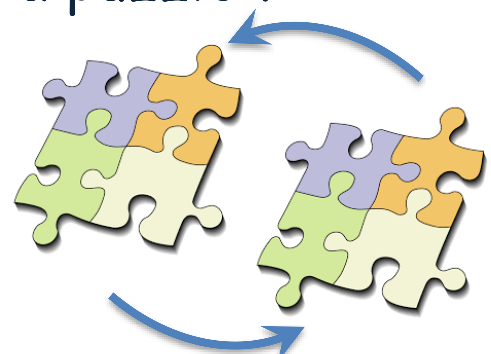

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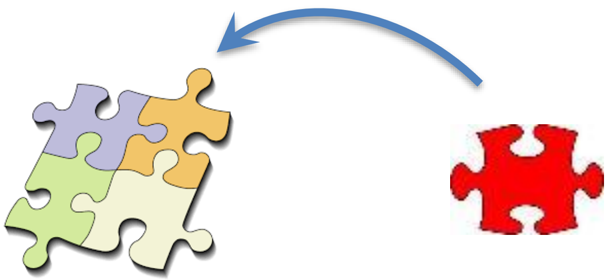

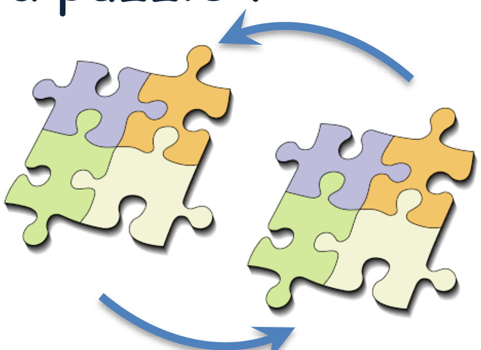

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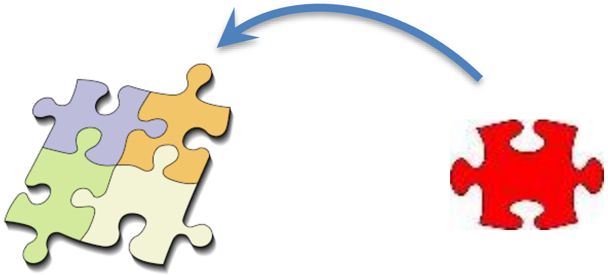

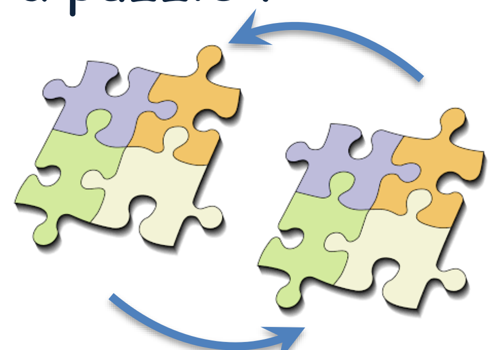

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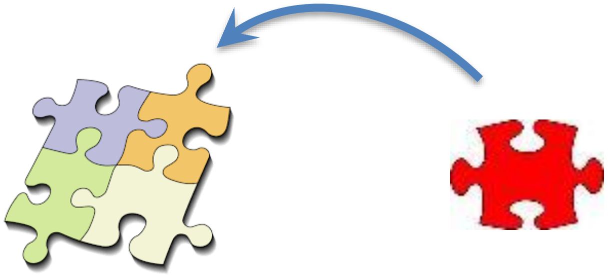

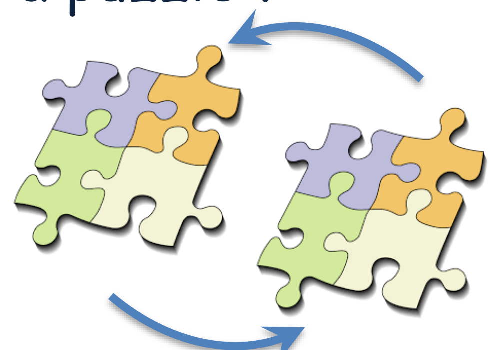

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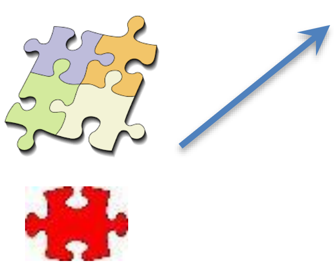

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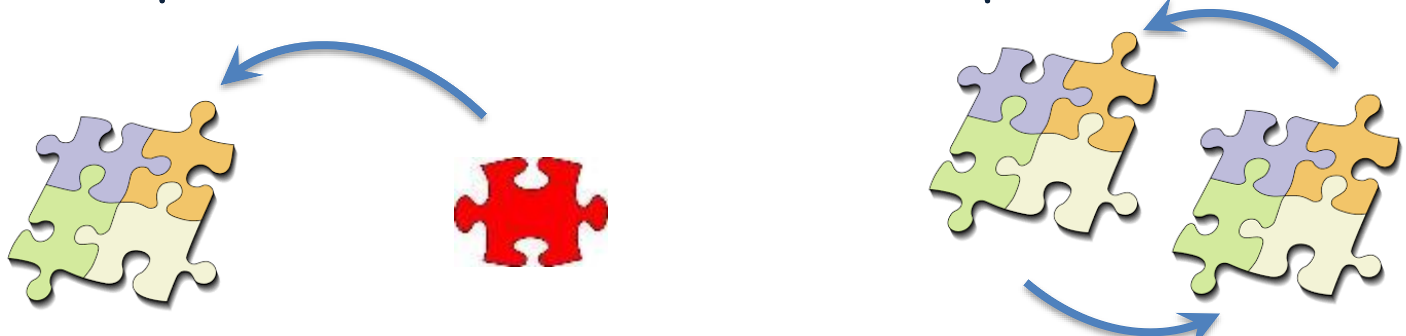

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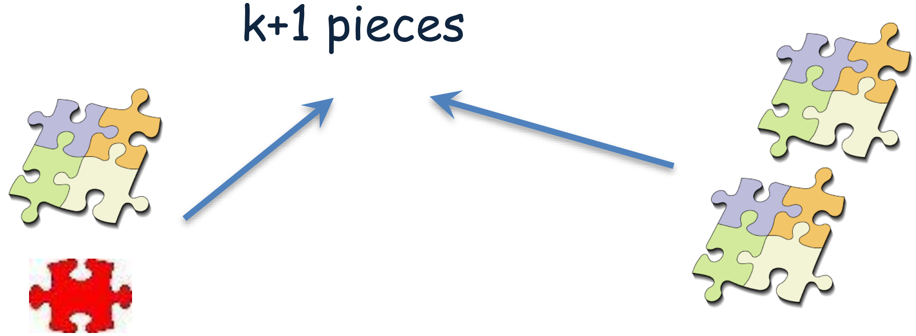

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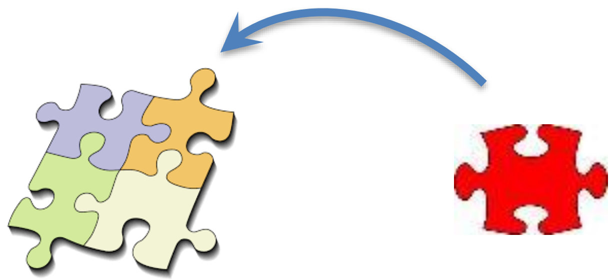

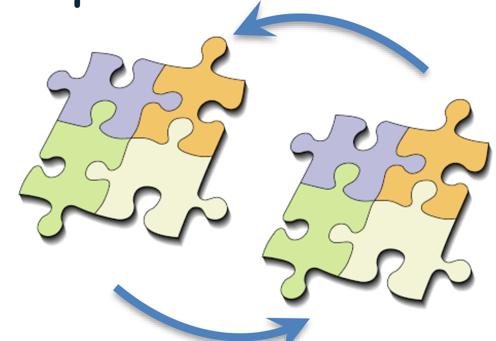

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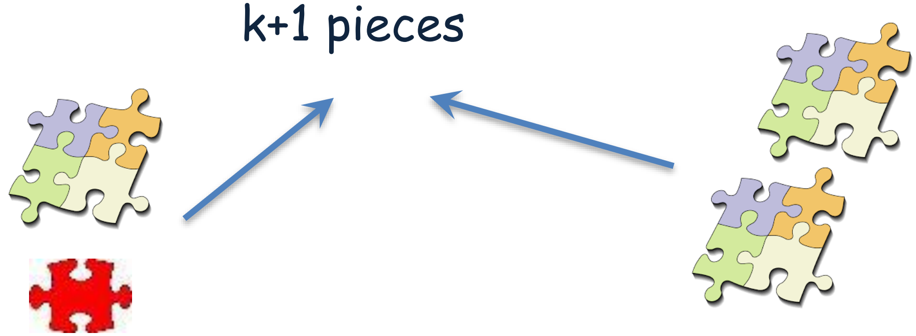

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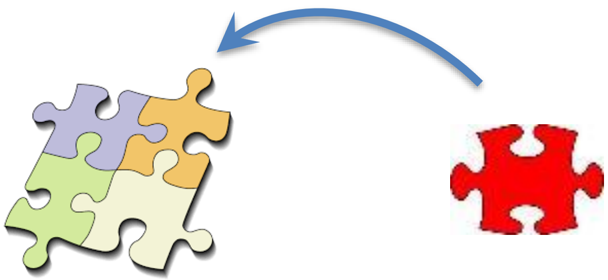

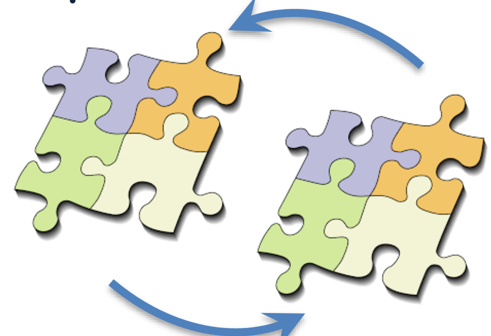

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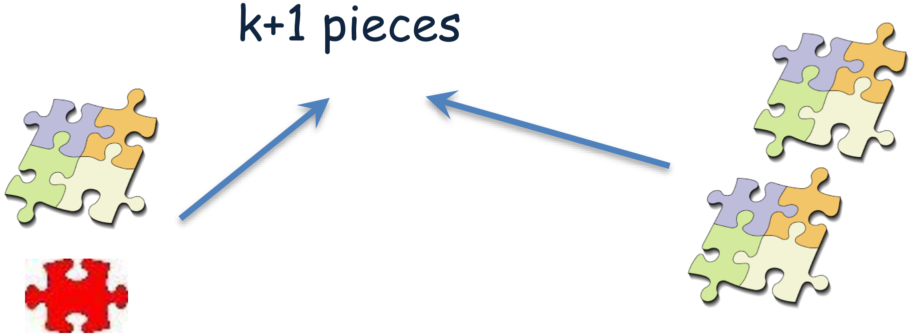

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$\alpha = \frac{1+\sqrt{5}}{2}$ is a solution of the equation $\alpha^2 - \alpha - 1 = 0$. Thus, $\alpha^2 = \alpha + 1$

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$\alpha = \frac{1+\sqrt{5}}{2}$ is a solution of the equation $\alpha^2 - \alpha - 1 = 0$. Thus, $\alpha^2 = \alpha + 1$

Strong Induction

- Prove that for every integer $n \geq 3$, $\mathrm{F}(n) > \alpha^{n-2}$ where $\alpha = (1+\sqrt{5})/2$

Fibonacci sequence : $\mathrm{F(1) = 1, F(2) = 1,}$ and $\mathrm{F(n) = F(n-1) + F(n-2)}$

Basic Step P(3) : $\mathrm{F}(3) = 2 > \alpha^{3-2} = (1+\sqrt{5})/2$

Inductive Step $[P(1) \wedge \ \dots \wedge P(k)] \rightarrow P(k+1)$

assume that P(i) is true for all i such that $2 \leq i \leq k$, i.e $\mathrm{F}(i) > \alpha^{i-2}$

for $P(k+1)$: $\mathrm{F}(k+1) = \mathrm{F}(k) + \mathrm{F}(k-1) > \alpha^{i-2} + \alpha^{i-3}$

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