Discrete Structures Homework

(1) I assume you mean third of spade. (Maça 3).

$$\frac{51}{52} \cdot \frac{50}{51} \cdot \frac{49}{50} \cdot \frac{48}{49} \cdot \frac{47}{48} \cdot \frac{1}{47}$$

$\frac{51}{52} \cdot \frac{50}{51} \cdot \frac{49}{50} \cdot \frac{48}{49} \cdot \frac{47}{48}$ → we calculate the possibility of the card is not the third of spade.

$\frac{1}{47}$ → In sixth draw, we calculate the possibility of that the card we draw is third of spade.

Answer is $1/52$ //

(2) A: The people who diseased.

B: The people who got positive

$$P(A|B) = \frac{P(A) \cdot P(B|A)}{P(B)}$$ } Bayes Theorem

$P(A) = \frac{1}{2}$

$P(B) = \frac{1}{2} \cdot 0,96 + \frac{1}{2} \cdot 0,8$

$P(B|A) = 0,96$

$$P(A|B) = \frac{\frac{1}{2} \cdot (0,96)}{\frac{1}{2} \cdot (1,04)}$$

$$= \frac{96}{104} //$$

(3) $a_n = 7 a_{n-1} - 12 \cdot a_{n-2}$ where $n \geq 2$, $a_0 = 5$ $a_1 = 18$

$a_n - 7a_{n-1} + 12 a_{n-2} = 0$ (homogen, second order)

$r^2 - 7r + 12 = 0$ (characteristic equation)

$(r-4)\cdot(r-3)$

$r_1 = 4$ $r_2 = 3$ (characteristic roots)

$a_n = c_1 \cdot (4)^n + c_2 \cdot (3)^n$

$a_0 = c_1 + c_2 = 5 \quad /-3$

$a_1 = 4c_1 + 3c_2 = 18$

$c_1 = 3$ $c_2 = 2$

$a_n = 3 \cdot 4^n + 2 \cdot 3^n$ //

(4) $A = \{ x \in \mathbb{Z} \mid 1 \leq x \leq 500 \text{ and } x \text{ is divisible by } 3 \}$

$B = \{ x \in \mathbb{Z} \mid 1 \leq x \leq 500 \text{ and } x \text{ is divisible by } 5 \}$

$C = \{ x \in \mathbb{Z} \mid 1 \leq x \leq 500 \text{ and } x \text{ is divisible by } 8 \}$

$|\bar{A} \cap \bar{B} \cap \bar{C}| = ? = |\overline{A \cup B \cup C}|$

$= |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

$= 166 + 100 + 62 - 33 - 20 - 12 + 4$

$A \cap B = \{ x \in \mathbb{Z}^+ \mid x \leq 500 \text{ is divisible by } 15 \}$

$A \cap C = \{ x \in \mathbb{Z}^+ \mid x \leq 500 \text{ is divisible by } 24 \}$

$B \cap C = \{ x \in \mathbb{Z}^+ \mid x \leq 500 \text{ is divisible by } 40 \}$

$A \cap B \cap C = \{ x \in \mathbb{Z}^+ \mid x \leq 500 \text{ is divisible by } 120 \}$

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