Logic

Murat Osmanoglu

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- systematic study of the form of valid arguments

(study of the difference between valid arguments and invalid arguments)

(finding out what it is that makes an argument valid)

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1) All men are mortal
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2) John will come to the party, or Mary will come to the party
   John will not come to the party

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premises

conclusion

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most of the mathematical statements are constructed by combining one or more propositions using logical operators (connectives)

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p	~p

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p	~p
T	
F	

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Conjunction ($p \wedge q$) : "p and q" .

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- the converse of $p \to q$: $q \to p$

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p	q	$p \to q$	$q \to p$	$p \leftrightarrow q$
T	T	T	T	T
T	F	T	F	F
F	T	F	T	F
F	F	T	T	T

Logical Operators

Truth Tables

$(p \vee \sim q) \rightarrow (p \wedge q)$

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$$(p \vee \sim q) \rightarrow (p \wedge q)$$

p	q	$\sim q$	$p \wedge q$	$p \vee \sim q$	$(p \vee \sim q) \rightarrow (p \wedge q)$

Logical Operators

Truth Tables

$$(p \vee \sim q) \rightarrow (p \wedge q)$$

p	q	$\sim q$	$p \wedge q$	$p \vee \sim q$	$(p \vee \sim q) \rightarrow (p \wedge q)$
1	1				
1	0				
0	1				
0	0				

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$(p \vee \sim q) \rightarrow (p \wedge q)$

p	q	$\sim q$	$p \wedge q$	$p \vee \sim q$	$(p \vee \sim q) \rightarrow (p \wedge q)$
1	1	0			
1	0	1			
0	1	0			
0	0	1			

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$$(p \vee \sim q) \rightarrow (p \wedge q)$$

p	q	$\sim q$	$p \wedge q$	$p \vee \sim q$	$(p \vee \sim q) \rightarrow (p \wedge q)$
1	1	0	1		
1	0	1	0		
0	1	0	0		
0	0	1	0		

Logical Operators

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$$(p \lor \sim q) \rightarrow (p \land q)$$

p	q	$\sim q$	$p \land q$	$p \lor \sim q$	$(p \lor \sim q) \rightarrow (p \land q)$
1	1	0	1	1	
1	0	1	0	1	
0	1	0	0	0	
0	0	1	0	1	

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$(p \vee \sim q) \rightarrow (p \wedge q)$

p	q	$\sim q$	$p \wedge q$	$p \vee \sim q$	$(p \vee \sim q) \rightarrow (p \wedge q)$
1	1	0	1	1	1
1	0	1	0	1	0
0	1	0	0	0	1
0	0	1	0	1	0

Logical Operators

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$q \leftrightarrow (\sim p \vee \sim q)$

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p	q	~p	~q	~p∨~q	q ↔ (~p∨~q)

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p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$	$q \leftrightarrow (\sim p \vee \sim q)$
1	1				
1	0				
0	1				
0	0				

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p	q	~p	~q	$\sim p \vee \sim q$	$q \leftrightarrow (\sim p \vee \sim q)$
1	1	0	0		
1	0	0	1		
0	1	1	0		
0	0	1	1		

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p	q	~p	~q	~p∨~q	q ↔ (~p∨~q)
1	1	0	0	0	
1	0	0	1	1	
0	1	1	0	1	
0	0	1	1	1	

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$q \leftrightarrow (\sim p \vee \sim q)$

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$	$q \leftrightarrow (\sim p \vee \sim q)$
1	1	0	0	0	0
1	0	0	1	1	0
0	1	1	0	1	1
0	0	1	1	1	0

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p	q	$p \vee q$	$p \rightarrow (p \vee q)$	$\sim p$	$\sim p \wedge q$	$p \wedge (\sim p \wedge q)$
1	1					
1	0					
0	1					
0	0					

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p	q	$p \vee q$	$p \rightarrow (p \vee q)$	$\sim p$	$\sim p \wedge q$	$p \wedge (\sim p \wedge q)$
1	1	1		0	0	
1	0	1		0	0	
0	1	1		1	1	
0	0	0		1	0	

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$p \rightarrow (p \vee q)$ $\qquad$ $p \wedge (\sim p \wedge q)$

p	q	$p \vee q$	$p \rightarrow (p \vee q)$	$\sim p$	$\sim p \wedge q$	$p \wedge (\sim p \wedge q)$
1	1	1	1	0	0	
1	0	1	1	0	0	
0	1	1	1	1	1	
0	0	0	1	1	0	

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$p \rightarrow (p \vee q)$ $\qquad$ $p \wedge (\sim p \wedge q)$

p	q	$p \vee q$	$p \rightarrow (p \vee q)$	$\sim p$	$\sim p \wedge q$	$p \wedge (\sim p \wedge q)$
1	1	1	1	0	0	0
1	0	1	1	0	0	0
0	1	1	1	1	1	0
0	0	0	1	1	0	0

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Truth Tables

$p \to (p \vee q)$ $p \wedge (\sim p \wedge q)$

p	q	$p \vee q$	$p \to (p \vee q)$	$\sim p$	$\sim p \wedge q$	$p \wedge (\sim p \wedge q)$
1	1	1	1	0	0	0
1	0	1	1	0	0	0
0	1	1	1	1	1	0
0	0	0	1	1	0	0

- *A* compound proposition is called tautology if it's true for all the cases

Logical Operators

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$p \rightarrow (p \vee q)$ $\qquad\qquad$ $p \wedge (\sim p \wedge q)$

p	q	$p \vee q$	$p \rightarrow (p \vee q)$	$\sim p$	$\sim p \wedge q$	$p \wedge (\sim p \wedge q)$
1	1	1	1	0	0	0
1	0	1	1	0	0	0
0	1	1	1	1	1	0
0	0	0	1	1	0	0

- A compound proposition is called tautology if it's true for all the cases
- A compound proposition is called contradiction if it's false for all the cases

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p	q	~p	~p∨q	p → q

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1	1			
1	0			
0	1			
0	0			

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p	q	~p	~p∨q	$p \rightarrow q$
1	1	0		
1	0	0		
0	1	1		
0	0	1		

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1	1	0	1	
1	0	0	0	
0	1	1	1	
0	0	1	1	

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1	1	0	1	1
1	0	0	0	0
0	1	1	1	1
0	0	1	1	1

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p	q	$\sim p$	$\sim p \vee q$	$p \to q$
1	1	0	1	1
1	0	0	0	0
0	1	1	1	1
0	0	1	1	1

$$\sim p \vee q \equiv p \to q$$

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p	q	~p	~q	~(p ∧ q)	~p∨~q

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p	q	$\sim p$	$\sim p \vee q$	$p \rightarrow q$
1	1	0	1	1
1	0	0	0	0
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0	0	1	1	1

$$\sim p \vee q \equiv p \rightarrow q$$

p	q	$\sim p$	$\sim q$	$\sim(p \wedge q)$	$\sim p \vee \sim q$
1	1				
1	0				
0	1				
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0	0	0	1		

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p	q	~p	~q	~(p ∧ q)	~p∨~q
1	1	0	0	0	0
1	0	0	1	1	1
0	1	1	0	1	1
0	0	0	1	1	1

Logical Equivalences

- If the compound propositions p and q have same truth values for all the cases, they are called logically equivalent

p	q	$\sim p$	$\sim p \vee q$	$p \rightarrow q$
1	1	0	1	1
1	0	0	0	0
0	1	1	1	1
0	0	1	1	1

$$\sim p \vee q \equiv p \rightarrow q$$

$$\sim(p \wedge q) \equiv \sim p \vee \sim q$$

p	q	$\sim p$	$\sim q$	$\sim(p \wedge q)$	$\sim p \vee \sim q$
1	1	0	0	0	0
1	0	0	1	1	1
0	1	1	0	1	1
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De Morgan's Low

$$\sim(p \vee q) \equiv \sim p \wedge \sim q$$

$$\sim(p \wedge q) \equiv \sim p \vee \sim q$$

p	q	$\sim p$	$\sim q$	$\sim(p \wedge q)$	$\sim p \vee \sim q$
1	1	0	0	0	0
1	0	0	1	1	1
0	1	1	0	1	1
0	0	0	1	1	1

Logical Equivalences

- De Morgan's Low

$\sim(p \vee q) \equiv \sim p \wedge \sim q$

$\sim(p \wedge q) \equiv \sim p \vee \sim q$

- $\sim(\sim p) \equiv p$

- $p \wedge 1 \equiv p$

  $p \vee 0 \equiv p$

- $p \wedge 0 \equiv 0$

  $p \vee 1 \equiv 1$

- $p \wedge p \equiv p$

  $p \vee p \equiv p$

- $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

  $(p \vee q) \vee r \equiv p \vee (q \vee r)$

- $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

  $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

- $p \wedge \sim p \equiv 0$

  $p \vee \sim p \equiv 1$

- $p \rightarrow q \equiv \sim p \vee q$

  $p \rightarrow q \equiv \sim q \rightarrow \sim p$

Logical Equivalences

- $\sim(p \lor (\sim p \land q) \equiv$

Logical Equivalences

- $\sim(p \vee (\sim p \wedge q) \equiv \sim p \wedge \sim(\sim p \wedge q)$

Logical Equivalences

- $\sim(p \vee (\sim p \wedge q)) \equiv \sim p \wedge \sim(\sim p \wedge q)$
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Logical Equivalences

- $\sim(p \vee (\sim p \wedge q)$ $\equiv \sim p \wedge \sim(\sim p \wedge q)$

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Logical Equivalences

- $\sim(p \vee (\sim p \wedge q)) \equiv \sim p \wedge \sim(\sim p \wedge q)$

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$$\equiv 0 \vee (\sim p \wedge \sim q)$$

Logical Equivalences

- $$\begin{aligned} \sim(p \vee (\sim p \wedge q)) &\equiv \sim p \wedge \sim(\sim p \wedge q) \\ &\equiv \sim p \wedge (p \vee \sim q) \\ &\equiv (\sim p \wedge p) \vee (\sim p \wedge \sim q) \\ &\equiv 0 \vee (\sim p \wedge \sim q) \\ &\equiv \sim p \wedge \sim q \end{aligned}$$

Logical Equivalences

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- $(p \rightarrow r) \wedge (p \rightarrow q) \equiv$

Logical Equivalences

- $\sim(p \vee (\sim p \wedge q) \equiv \sim p \wedge \sim(\sim p \wedge q)$

  $\equiv \sim p \wedge (p \vee \sim q)$

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  $\equiv 0 \vee (\sim p \wedge \sim q)$

  $\equiv \sim p \wedge \sim q$

- $(p \rightarrow r) \wedge (p \rightarrow q) \equiv (\sim p \vee r) \wedge$

Logical Equivalences

- $\sim(p \vee (\sim p \wedge q)) \equiv \sim p \wedge \sim(\sim p \wedge q)$
  $\equiv \sim p \wedge (p \vee \sim q)$
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  $\equiv 0 \vee (\sim p \wedge \sim q)$
  $\equiv \sim p \wedge \sim q$

- $(p \to r) \wedge (p \to q) \equiv (\sim p \vee r) \wedge (\sim p \vee q)$

Logical Equivalences

- $\sim(p \vee (\sim p \wedge q)) \equiv \sim p \wedge \sim(\sim p \wedge q)$
  $\equiv \sim p \wedge (p \vee \sim q)$
  $\equiv (\sim p \wedge p) \vee (\sim p \wedge \sim q)$
  $\equiv 0 \vee (\sim p \wedge \sim q)$
  $\equiv \sim p \wedge \sim q$

- $(p \rightarrow r) \wedge (p \rightarrow q) \equiv (\sim p \vee r) \wedge (\sim p \vee q)$
  $\equiv \sim p \vee (r \wedge q)$

Logical Equivalences

- $\sim(p \vee (\sim p \wedge q)) \equiv \sim p \wedge \sim(\sim p \wedge q)$
  $\equiv \sim p \wedge (p \vee \sim q)$
  $\equiv (\sim p \wedge p) \vee (\sim p \wedge \sim q)$
  $\equiv 0 \vee (\sim p \wedge \sim q)$
  $\equiv \sim p \wedge \sim q$

- $(p \rightarrow r) \wedge (p \rightarrow q) \equiv (\sim p \vee r) \wedge (\sim p \vee q)$
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  $\equiv p \rightarrow (r \wedge q)$

Predicates

Predicates

- p : '2 + 3 = 5'

  q : 'my computer is vulnerable to side channel attacks'

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- 'x + 3 = 5'

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Definition Propositions (or statements) that contains variables

Predicates

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- P(x) : 'x + 3 = 5'

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Definition Propositions (or statements) that contains variables

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Definition Propositions (or statements) that contains variables

- When a value is assigned to the variable x, then P(x) becomes a proposition and has a truth value.

Predicates

- $P(x)$: '$x > 3$'
- $Q(x,y)$: '$x + 3 = y$'
- $R(x,y,z)$: '$x + y = z$'

Predicates

- P(x) : '$x > 3$'
  P(4) is true

- Q(x,y) : '$x + 3 = y$'

- R(x,y,z) : '$x + y = z$'

Predicates

- $P(x)$: '$x > 3$'
  $P(4)$ is true, but $P(2)$ is false
- $Q(x,y)$: '$x + 3 = y$'
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Predicates

- $P(x)$: '$x > 3$'
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  $Q(4,7)$ is true, but $Q(4,2)$ is false
- $R(x,y,z)$: '$x + y = z$'

Predicates

- $P(x)$: '$x > 3$'
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  $Q(4,7)$ is true, but $Q(4,2)$ is false

- $R(x,y,z)$: '$x + y = z$'
  $R(2,1,3)$ is true, but $R(3,2,2)$ is false

Quantifiers

- Another way of creating a proposition from a propositional function

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Universal Quantifier

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$Q : \forall x\, P(x)$

Quantifiers

- Another way of creating a proposition from a propositional function

Universal Quantifier

$Q : \forall x\, P(x)$ If $P(x)$ is true for all x in the domain, then Q is true

Quantifiers

- Another way of creating a proposition from a propositional function

Universal Quantifier

Q : ∀x P(x) If P(x) is true for all x in the domain, then Q is true

If there is an x_0 such that P(x_0) is not true, then Q is false

Quantifiers

- Another way of creating a proposition from a propositional function

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Existential Quantifier

Quantifiers

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If P(x) is true for all x in the domain, then Q is true

If there is an x_0 such that P(x_0) is not true, then Q is false

Existential Quantifier

R : ∃x P(x)

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If P(x) is true for all x in the domain, then Q is true

If there is an x_0 such that P(x_0) is not true, then Q is false

Existential Quantifier

R : $\exists x$ P(x)

If there exists an x_0 such that P(x_0) is true, then R is true

Quantifiers

- Another way of creating a proposition from a propositional function

Universal Quantifier

Q : ∀x P(x)

If P(x) is true for all x in the domain, then Q is true

If there is an x_0 such that P(x_0) is not true, then Q is false

Existential Quantifier

R : ∃x P(x)

If there exists an x_0 such that P(x_0) is true, then R is true

If P(x) is false for all x in the domain, then R is false

Quantifiers

- P(x) : $x^2 > x$

  What is the truth value of ∀x P(x) if the domain is Z^+ ?

Quantifiers

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  What is the truth value of ∀x P(x) if the domain is Z^+ ?

  For all $x \in Z^+$ $x^2 > x$.

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  For all $x \in Z^+$ $x^2 \geq x$. So ∀x P(x) is true for Z^+ .

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- Q(x) : x = x + 1

  What is the truth value of ∃x Q(x) if the domain is R?

Quantifiers

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- Q(x) : x = x + 1

  What is the truth value of ∃x Q(x) if the domain is R?

  There is no real number x such that x = x + 1.

Quantifiers

- P(x) : $x^2 \geq x$

  What is the truth value of ∀x P(x) if the domain is Z^+ ?

  For all $x \in Z^+$ $x^2 \geq x$. So ∀x P(x) is true for Z^+ .

- Q(x) : x = x + 1

  What is the truth value of ∃x Q(x) if the domain is R?

  There is no real number x such that x = x + 1. So ∃x Q(x) is false for R.

Quantifiers

- P(x) : $x^2 + 1 < 10$, D = { 1, 2, 3}

  What is the truth value of ∀x P(x) if the domain is D?

Quantifiers

- $P(x)$: $x^2 + 1 < 10$, D = { 1, 2, 3}

  What is the truth value of $\forall x\, P(x)$ if the domain is D?

  If the domain consists of n elements,
  then $\forall x\, P(x) \equiv P(x_1) \wedge P(x_2) \wedge \ldots \wedge P(x_n)$

Quantifiers

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  What is the truth value of ∀x P(x) if the domain is D?

  If the domain consists of n elements,
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  P(1) : $2 < 10$, true

Quantifiers

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  If the domain consists of n elements,
  then ∀x P(x) $\equiv$ P(x_1) ∧ P(x_2) ∧ . . . ∧ P(x_n)

  P(1) : $2 < 10$, true
  P(2) : $5 < 10$, true

Quantifiers

- P(x) : $x^2 + 1 < 10$, D = { 1, 2, 3}

  What is the truth value of ∀x P(x) if the domain is D?

  If the domain consists of n elements,
  then ∀x P(x) ≡ P(x_1) ∧ P(x_2) ∧ . . . ∧ P(x_n)

  P(1) : $2 < 10,$ true
  P(2) : $5 < 10,$ true
  P(3) : $10 < 10,$ false

Quantifiers

- P(x) : $x^2 + 1 < 10$, D = { 1, 2, 3}

  What is the truth value of ∀x P(x) if the domain is D?

  If the domain consists of n elements,
  then ∀x P(x) ≡ P(x_1) ∧ P(x_2) ∧ . . . ∧ P(x_n)

  P(1) : $2 < 10$, true
  P(2) : $5 < 10$, true
  P(3) : $10 < 10$, false

 Since 1 ∧ 1 ∧ 0 ≡ 0, then ∀x P(x) is false for D.

Quantifiers

- Q(x) : $x^2 < 3$, D = {1, 2, 3}

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Quantifiers

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- Q(x) : $x^2 < 3$, D = {1, 2, 3}

  What is the truth value of ∃x Q(x) if the domain is D?

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  P(1) : $1 < 3$, true

Quantifiers

- Q(x) : $x^2 < 3$, D = {1, 2, 3}

  What is the truth value of ∃x Q(x) if the domain is D?

  If the domain consists of n elements,
  then ∃x P(x) ≡ P(x_1) ∨ P(x_2) ∨ . . . ∨ P(x_n)

  P(1) : $1 < 3$, true
  P(2) : $4 < 3$, false

Quantifiers

- Q(x) : $x^2 < 3$, D = {1, 2, 3}

  What is the truth value of ∃x Q(x) if the domain is D?

  If the domain consists of n elements,
  then ∃x P(x) ≡ P(x_1) ∨ P(x_2) ∨ . . . ∨ P(x_n)

  P(1) : $1 < 3$, true
  P(2) : $4 < 3$, false
  P(3) : $9 < 3$, false

Quantifiers

- Q(x) : $x^2 < 3$, D = {1, 2, 3}

  What is the truth value of ∃x Q(x) if the domain is D?

  If the domain consists of n elements,
  then ∃x P(x) ≡ P(x_1) ∨ P(x_2) ∨ . . . ∨ P(x_n)

  P(1) : $1 < 3$, true
  P(2) : $4 < 3$, false
  P(3) : $9 < 3$, false

  Since 1 ∨ 0 ∨ 0 ≡ 1, then ∃x P(x) is true for D.

Quantifiers

Negation

Quantifiers

- Every student in this class has entered the entrance exam

Negation

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- Every student in this class has entered the entrance exam

$\forall x\, P(x)$,   ' x has taken the entrance exam'

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- It's not the case that every student in this class has entered the entrance exam.

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- It's not the case that every student in this class has entered the entrance exam.

  There is a student in this class who has not taken the entrance exam.

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$\sim(\forall x\, P(x)) \equiv$

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$\sim(\forall x\ P(x)) \equiv \sim(P(x_1) \wedge P(x_2) \wedge \ldots \wedge P(x_n))$

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$$\equiv \sim P(x_1) \vee \sim P(x_2) \vee \ldots \vee \sim P(x_n)$$

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$$\begin{aligned} \sim(\forall x\ P(x)) &\equiv \sim(P(x_1) \wedge P(x_2) \wedge \ldots \wedge P(x_n)) \\ &\equiv \sim P(x_1) \vee \sim P(x_2) \vee \ldots \vee \sim P(x_n) \\ &\equiv \exists x \sim P(x) \end{aligned}$$

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  $\exists x\, P(x)$, ' x has taken the entrance exam'

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  None of the students in this class has taken the entrance exam.

Quantifiers

- There is a student in this class who has taken the entrance exam.

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$$\sim(\exists x\, P(x)) \equiv \sim(P(x_1) \vee P(x_2) \vee \ldots \vee P(x_n))$$

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- There is a student in this class who has taken the entrance exam.

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  None of the students in this class has taken the entrance exam.

$$\begin{aligned} \sim(\exists x\, P(x)) &\equiv \sim(P(x_1) \vee P(x_2) \vee \ldots \vee P(x_n)) \\ &\equiv \sim P(x_1) \wedge \sim P(x_2) \wedge \ldots \wedge \sim P(x_n) \\ &\equiv \forall x \sim P(x) \end{aligned}$$

Quantifiers

- $\sim(\forall x(x^2 > x)) \equiv$

- $\sim(\exists x(x^2 = 7)) \equiv$

Quantifiers

- $\sim(\forall x(x^2 > x)) \equiv \exists x \sim(x^2 > x)$

- $\sim(\exists x(x^2 = 7)) \equiv$

Quantifiers

- $\sim(\forall x(x^2 > x)) \equiv \exists \mathbf{x} \sim(x^2 > x)$
  $\equiv \exists \mathbf{x}\ x^2 \leq x$

- $\sim(\exists x(x^2 = 7)) \equiv$

Quantifiers

- $\sim(\forall x(x^2 > x)) \equiv \exists \mathrm{x} \sim(x^2 > x)$
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- $\sim(\exists x(x^2 = 7)) \equiv \forall \mathrm{x} \sim(x^2 = 7)$

Quantifiers

- $\sim(\forall x(x^2 > x)) \equiv \exists \mathbf{x} \sim(x^2 > x)$
  $\equiv \exists \mathbf{x}\ x^2 \leq x$

- $\sim(\exists x(x^2 = 7)) \equiv \forall \mathbf{x} \sim(x^2 = 7)$
  $\equiv \forall \mathbf{x}\ x^2 \neq 7$

Quantifiers

- $\forall x \, \forall y \, ((x > 0) \land (y < 0) \rightarrow (xy < 0))$ D = R

Quantifiers

- $\forall x \, \forall y \, \overbrace{((x > 0) \land (y < 0) \to (xy < 0))}^{\text{If x is positive and y is negative, then xy is negative}}$ D = R

Quantifiers

- $$\underbrace{\forall x\, \forall y\, \overbrace{((x > 0) \land (y < 0) \rightarrow (xy < 0))}^{\text{If x is positive and y is negative, then xy is negative}}}_{\text{For every real numbers x and y, if x is positive and y is negative, then xy is negative}}$$ D = R

Quantifiers

- For every two integers, if these integers are both positive, then the sum of these integers is also positive

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$$\underbrace{\text{if } x > 0 \text{ and } y > 0\text{, then } \mathrm{x} + y > 0}_{(x > 0) \wedge (y > 0) \rightarrow (x + y > 0)}$$

Quantifiers

- For every two integers, if these integers are both positive, then the sum of these integers is also positive

- For two integers x and y, if $x > 0$ and $y > 0$, then $x + y > 0$

$$(x > 0) \land (y > 0) \rightarrow (x + y > 0)$$

$$\forall x\, \forall y\, ((x > 0) \land (y > 0) \rightarrow (x + y > 0))$$

Quantifiers

- There exist integers x and y such that $x + y = 6$

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$$\exists x\ \exists y\ (x + y = 6)$$

Quantifiers

- There exist integers x and y such that $x + y = 6$

$$\exists x\, \exists y\, (x + y = 6)$$

or

$$\exists y\, \exists x\, (x + y = 6)$$

Quantifiers

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$$\exists x \, \exists y \, (x + y = 6)$$

or

$$\exists y \, \exists x \, (x + y = 6)$$

- $\forall x \, \exists y \, (x + y = 6)$

Quantifiers

- There exist integers x and y such that $x + y = 6$

$$\exists x\ \exists y\ (x + y = 6)$$

or

$$\exists y\ \exists x\ (x + y = 6)$$

- $\forall x\ \exists y\ (x + y = 6)$

For every integer x, there exists an integer y such that $x + y = 6$

Quantifiers

- There exist integers x and y such that $x + y = 6$

$$\exists x \, \exists y \, (x + y = 6)$$

or

$$\exists y \, \exists x \, (x + y = 6)$$

- $\forall x \, \exists y \, (x + y = 6)$

For every integer x, there exists an integer y such that $x + y = 6$ (It's true)

Quantifiers

- There exist integers x and y such that $x + y = 6$

$$\exists x\ \exists y\ (x + y = 6)$$

or

$$\exists y\ \exists x\ (x + y = 6)$$

- $\forall x\ \exists y\ (x + y = 6)$

  For every integer x, there exists an integer y such that $x + y = 6$ (It's true)

- $\exists \mathrm{y}\ \forall x\ (x + y = 6)$

Quantifiers

- There exist integers x and y such that $x + y = 6$

$$\exists x \, \exists y \, (x + y = 6)$$

or

$$\exists y \, \exists x \, (x + y = 6)$$

- $\forall x \, \exists y \, (x + y = 6)$

  For every integer x, there exists an integer y such that $x + y = 6$ (It's true)

- $\exists \mathrm{y} \, \forall x \, (x + y = 6)$

  There exists an integer y so that for all integers x, $x + y = 6$

Quantifiers

- There exist integers x and y such that $x + y = 6$

$$\exists x\ \exists y\ (x + y = 6)$$

or

$$\exists y\ \exists x\ (x + y = 6)$$

- $\forall x\ \exists y\ (x + y = 6)$

  For every integer x, there exists an integer y such that $x + y = 6$ (It's true)

- $\exists \mathrm{y}\ \forall x\ (x + y = 6)$

  There exists an integer y so that for all integers x, $x + y = 6$ (It's false)

Proofs

- Valid arguments that establish the truth of mathematical statements

Proofs

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argument : sequence of sentences (propositions); premises at the beginning and conclusion at the end

Proofs

- An argument is called valid if the truthness of all its premises implies that the conlusion is true

Proofs

- An argument is called valid if the truthness of all its premises implies that the conlusion is true

- If you have a password, then you can log onto the network.

Proofs

- An argument is called valid if the truthness of all its premises implies that the conlusion is true

- If you have a password, then you can log onto the network.

- You have a password

Proofs

- An argument is called valid if the truthness of all its premises implies that the conlusion is true

- If you have a password, then you can log onto the network.
- You have a password
- Therefore, you can log onto the network

Proofs

- An argument is called valid if the truthness of all its premises implies that the conlusion is true

- If you have a password, then you can log onto the network. $p \rightarrow q$
- You have a password p
- Therefore, you can log onto the network q

Proofs

$$\begin{array}{l} p \rightarrow q \\ p \\ \hline q \end{array}$$

Proofs

Modus Ponens

$$\begin{array}{l} p \to q \\ p \\ \hline q \end{array}$$

Proofs

Modus Ponens

$p \to q$

p

---

q

p	q	$p \to q$	$p \wedge (p \to q)$	$[p \wedge (p \to q)] \to q$
1	1	0	0	1
1	0	1	1	1
0	1	1	0	1
0	0	1	0	1

Proofs

Modus Ponens

- If $\sqrt{5} > \sqrt{3}$, then $(\sqrt{5})^2 > (\sqrt{3})^2$.

Proofs

Modus Ponens

- If $\sqrt{5} > \sqrt{3}$, then $(\sqrt{5})^2 > (\sqrt{3})^2$. $\quad$ $p \rightarrow q$

Proofs

Modus Ponens

- If $\sqrt{5} > \sqrt{3}$, then $(\sqrt{5})^2 > (\sqrt{3})^2$. p → q
- We know that $\sqrt{5} > \sqrt{3}$ p

Proofs

Modus Ponens

- If $\sqrt{5} > \sqrt{3}$, then $(\sqrt{5})^2 > (\sqrt{3})^2$. $\quad$ p → q
- We know that $\sqrt{5} > \sqrt{3}$ $\quad$ p
- So, $(\sqrt{5})^2 > (\sqrt{3})^2$

Proofs

Modus Ponens

- If $\sqrt{5} > \sqrt{3}$, then $(\sqrt{5})^2 > (\sqrt{3})^2$. $p \to q$
- We know that $\sqrt{5} > \sqrt{3}$ p
- So, $(\sqrt{5})^2 > (\sqrt{3})^2 \;\to\; 5 > 3$ q

Proofs

- To prove $\forall x\ (P(x) \to Q(x))$, show that $P(c) \to Q(c)$ is true for an arbitrary element c of the domain.

Proofs

- To prove $\forall x\ (P(x) \rightarrow Q(x))$, show that $P(c) \rightarrow Q(c)$ is true for an arbitrary element c of the domain.

1) All men are mortal

   Socrates is a man

   Socrates is mortal

Proofs

- To prove $\forall x\ (P(x) \rightarrow Q(x))$, show that $P(c) \rightarrow Q(c)$ is true for an arbitrary element c of the domain.

1) All men are mortal

   Socrates is a man

   Socrates is mortal

P(x) : x is a man

Q(x) : x is mortal

Proofs

- To prove $\forall x\, (P(x) \rightarrow Q(x))$, show that $P(c) \rightarrow Q(c)$ is true for an arbitrary element c of the domain.

1) All men are mortal	$\forall x\, (P(x) \rightarrow Q(x))$
Socrates is a man	P(Socrates)
Socrates is mortal	Q(Socrates)

P(x) : x is a man

Q(x) : x is mortal

Proofs

- To prove $\forall x\ (P(x) \rightarrow Q(x))$, show that $P(c) \rightarrow Q(c)$ is true for an arbitrary element c of the domain.
- To prove $P(c) \rightarrow Q(c)$, show that $Q(c)$ is true if $P(c)$ is true

Proofs

- To prove $\forall x\ (P(x) \rightarrow Q(x))$, show that $P(c) \rightarrow Q(c)$ is true for an arbitrary element c of the domain.
- To prove $P(c) \rightarrow Q(c)$, show that $Q(c)$ is true if $P(c)$ is true (p → q is true unless p is true but q is false)

Proofs

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Direct Proof

Proofs

- To prove $\forall x\,(P(x) \rightarrow Q(x))$, show that $P(c) \rightarrow Q(c)$ is true for an arbitrary element c of the domain.
- To prove $P(c) \rightarrow Q(c)$, show that $Q(c)$ is true if $P(c)$ is true (p → q is true unless p is true but q is false)

Direct Proof

- To prove p → q is true, first assume p is true, then show that q must also be true.

Proofs

- To prove $\forall x\ (P(x) \rightarrow Q(x))$, show that $P(c) \rightarrow Q(c)$ is true for an arbitrary element c of the domain.
- To prove $P(c) \rightarrow Q(c)$, show that $Q(c)$ is true if $P(c)$ is true (p → q is true unless p is true but q is false)

Direct Proof

- To prove p → q is true, first assume p is true, then show that q must also be true.
- Thus, if p is true, then q must also be true, so that

Proofs

- To prove $\forall x\ (P(x) \rightarrow Q(x))$, show that $P(c) \rightarrow Q(c)$ is true for an arbitrary element c of the domain.

- To prove $P(c) \rightarrow Q(c)$, show that $Q(c)$ is true if $P(c)$ is true (p → q is true unless p is true but q is false)

Direct Proof

- To prove p → q is true, first assume p is true, then show that q must also be true.

- Thus, if p is true, then q must also be true, so that the combination of p true and q false never occurs

Proofs

Direct Proof

If n is odd integer, then n^2 is odd integer.

Proofs

Direct Proof

If $\underbrace{n \text{ is odd integer}}_{p}$, then $\underbrace{n^2 \text{ is odd integer}}_{q}$.

Proofs

Direct Proof

If $\underbrace{n \text{ is odd integer}}_{p}$, then $\underbrace{n^2 \text{ is odd integer}}_{q}$.

$p \to q$

Proofs

Direct Proof

If $\underbrace{n \text{ is odd integer}}_{p}$, then $\underbrace{n^2 \text{ is odd integer}}_{q}$.

$p \rightarrow q$ assume p is true

Proofs

Direct Proof

If $\underbrace{n \text{ is odd integer}}_{p}$, then $\underbrace{n^2 \text{ is odd integer}}_{q}$.

$p \rightarrow q$ assume p is true

$$n = 2k + 1, \exists k \in Z$$

Proofs

Direct Proof

If $\underbrace{n \text{ is odd integer}}_{p}$, then $\underbrace{n^2 \text{ is odd integer}}_{q}$.

p → q assume p is true

$$n = 2k + 1, \exists k \in Z$$
$$n^2 = (2k + 1)^2$$

Proofs

Direct Proof

$\underbrace{\text{If } n \text{ is odd integer}}_{p}$, then $\underbrace{n^2 \text{ is odd integer}}_{q}$.

$p \rightarrow q$ assume p is true

$$n = 2k + 1, \exists k \in Z$$
$$n^2 = (2k + 1)^2$$
$$n^2 = 4k^2 + 2k + 1$$

Proofs

Direct Proof

If $\underbrace{n \text{ is odd integer}}_{p}$, then $\underbrace{n^2 \text{ is odd integer}}_{q}$.

$p \rightarrow q$ assume p is true

$$n = 2k + 1, \exists k \in Z$$
$$n^2 = (2k + 1)^2$$
$$n^2 = 4k^2 + 2k + 1$$
$$n^2 = 2(2k^2 + k) + 1$$

Proofs

Direct Proof

If n is odd integer, then n^2 is odd integer.

$$\underbrace{\text{If } n \text{ is odd integer}}_{p}, \text{ then } \underbrace{n^2 \text{ is odd integer}}_{q}.$$

p → q assume p is true

$$n = 2k + 1, \exists k \in Z$$
$$n^2 = (2k + 1)^2$$
$$n^2 = 4k^2 + 2k + 1$$
$$n^2 = 2(2k^2 + k) + 1$$
$$n^2 = 2m + 1, \exists m \in Z$$

Proofs

Direct Proof

If $\underbrace{n \text{ is odd integer}}_{p}$, then $\underbrace{n^2 \text{ is odd integer}}_{q}$.

$p \rightarrow q$

assume p is true

$$n = 2k + 1, \exists k \in Z$$
$$n^2 = (2k + 1)^2$$
$$n^2 = 4k^2 + 2k + 1$$
$$n^2 = 2(2k^2 + k) + 1$$
$$n^2 = 2m + 1, \exists m \in Z$$

q is also true

Proofs

Direct Proof

If a and b are odd integers, then $a + b$ is even integer.

Proofs

Direct Proof

If $\underbrace{a \text{ and } b \text{ are odd integers}}_{p}$, then $\underbrace{a + b \text{ is even integer}}_{q}$.

Proofs

Direct Proof

If $\underbrace{a \text{ and } b \text{ are odd integers}}_{p}$, then $\underbrace{a+b \text{ is even integer}}_{q}$.

$p \rightarrow q$ assume p is true

Proofs

Direct Proof

If $\underbrace{a \text{ and } b \text{ are odd integers,}}_{p}$ then $\underbrace{a+b \text{ is even integer.}}_{q}$

$p \rightarrow q$ assume p is true

$a = 2x + 1$ and $b = 2y + 1$ $\exists x, y \in Z$

Proofs

Direct Proof

$$\text{If } \underbrace{a \text{ and } b \text{ are odd integers,}}_{p} \text{ then } \underbrace{a + b \text{ is even integer.}}_{q}$$

$p \rightarrow q$

assume p is true

$a = 2x + 1$ and $b = 2y + 1 \; \exists x, y \in Z$

$a + b = 2x + 1 + 2y + 1$

Proofs

Direct Proof

If $\underbrace{a \text{ and } b \text{ are odd integers}}_{p}$, then $\underbrace{a + b \text{ is even integer}}_{q}$.

$p \rightarrow q$ assume p is true

$a = 2x + 1$ and $b = 2y + 1$ $\exists x, y \in Z$

$a + b = 2x + 1 + 2y + 1$

$a + b = 2x + 2y + 2$

Proofs

Direct Proof

If $\underbrace{a \text{ and } b \text{ are odd integers,}}_{p}$ then $\underbrace{a + b \text{ is even integer.}}_{q}$

$p \rightarrow q$ assume p is true

$a = 2x + 1$ and $b = 2y + 1 \; \exists x, y \in Z$

$a + b = 2x + 1 + 2y + 1$

$a + b = 2x + 2y + 2$

$a + b = 2(x + y + 1)$

Proofs

Direct Proof

If $\underbrace{a \text{ and } b \text{ are odd integers,}}_{p}$ then $\underbrace{a + b \text{ is even integer.}}_{q}$

$p \rightarrow q$ assume p is true

$$a = 2x + 1 \text{ and } b = 2y + 1 \ \exists x, y \in Z$$
$$a + b = 2x + 1 + 2y + 1$$
$$a + b = 2x + 2y + 2$$
$$a + b = 2(x + y + 1)$$
$$a + b = 2m, \exists m \in Z$$

Proofs

Direct Proof

If $\underbrace{a \text{ and } b \text{ are odd integers,}}_{p}$ then $\underbrace{a + b \text{ is even integer.}}_{q}$

$p \rightarrow q$ assume p is true

$$a = 2x + 1 \text{ and } b = 2y + 1 \ \exists x, y \in Z$$
$$a + b = 2x + 1 + 2y + 1$$
$$a + b = 2x + 2y + 2$$
$$a + b = 2(x + y + 1)$$
$$a + b = 2m, \exists m \in Z$$

q is also true

Proofs

Direct Proof

If m and n are perfect squares, then $m.n$ is also a perfect square.

Proofs

Direct Proof

If $\underbrace{m \text{ and } n \text{ are perfect squares}}_{p}$, then $\underbrace{m.n \text{ is also a perfect square.}}_{q}$

Proofs

Direct Proof

If $\underbrace{m \text{ and } n \text{ are perfect squares}}_{p}$, then $\underbrace{m.n \text{ is also a perfect square}}_{q}$.

$p \rightarrow q$

Proofs

Direct Proof

If $\underbrace{m \text{ and } n \text{ are perfect squares}}_{p}$, then $\underbrace{m.n \text{ is also a perfect square}}_{q}$.

$p \rightarrow q$ assume p is true

Proofs

Direct Proof

If $\underbrace{m \text{ and } n \text{ are perfect squares}}_{p}$, then $\underbrace{m.n \text{ is also a perfect square.}}_{q}$

$p \rightarrow q$ assume p is true

$m = x^2$ and $n = y^2$, $\exists x, y \in Z$

Proofs

Direct Proof

If $\underbrace{m \text{ and } n \text{ are perfect squares}}_{p}$, then $\underbrace{m.n \text{ is also a perfect square.}}_{q}$

p → q   assume p is true

$m = x^2$ and $n = y^2$, $\exists x, y \in Z$

$m.n = x^2y^2$

Proofs

Direct Proof

If $\underbrace{m \text{ and } n \text{ are perfect squares}}_{p}$, then $\underbrace{m.n \text{ is also a perfect square}}_{q}$.

$p \rightarrow q$ assume p is true

$m = x^2$ and $n = y^2$, $\exists x, y \in Z$

$m.n = x^2y^2$

$m.n = (x.y)^2$

Proofs

Direct Proof

If $\underbrace{m \text{ and } n \text{ are perfect squares}}_{p}$, then $\underbrace{m.n \text{ is also a perfect square}}_{q}$.

$p \rightarrow q$ assume p is true

$m = x^2$ and $n = y^2$, $\exists x, y \in Z$
$m.n = x^2y^2$
$m.n = (x.y)^2$
$m.n = k^2$, $\exists k \in Z$

Proofs

Direct Proof

If m and n are perfect squares, then $m.n$ is also a perfect square.

$$\underbrace{\text{If } m \text{ and } n \text{ are perfect squares}}_{p}, \text{ then } \underbrace{m.n \text{ is also a perfect square}}_{q}.$$

$p \rightarrow q$ assume p is true

$m = x^2$ and $n = y^2$, $\exists x, y \in Z$

$$m.n = x^2 y^2$$

$$m.n = (x.y)^2$$

$$m.n = k^2, \exists k \in Z$$

q is also true

Proofs

Proof by Contraposition

Proofs

Proof by Contraposition

- Instead of proving $p \rightarrow q$, prove logically equivalent proposition $\sim q \rightarrow \sim p$

Proofs

Proof by Contraposition

- Instead of proving $p \rightarrow q$, prove logically equivalent proposition $\sim q \rightarrow \sim p$ --WHY?

Proofs

Proof by Contraposition

If $3n + 2$ is an odd integer, then n is odd integer

Proofs

Proof by Contraposition

If $\underbrace{3n + 2 \text{ is an odd integer}}_{p}$, then $\underbrace{n \text{ is odd integer}}_{q}$

Proofs

Proof by Contraposition

If $\underbrace{3n + 2 \text{ is an odd integer}}_{p}$, then $\underbrace{n \text{ is odd integer}}_{q}$

$p \rightarrow q$ assume p is true

Proofs

Proof by Contraposition

If $\underbrace{3n + 2 \text{ is an odd integer}}_{p}$, then $\underbrace{n \text{ is odd integer}}_{q}$

$p \rightarrow q$ assume p is true

$$3n + 2 = 2k + 1, \exists k \in Z$$

Proofs

Proof by Contraposition

If $\underbrace{3n + 2 \text{ is an odd integer}}_{p}$, then $\underbrace{n \text{ is odd integer}}_{q}$

$p \rightarrow q$ assume p is true

$$3n + 2 = 2k + 1, \exists k \in Z$$
$$3n = 2k - 1$$

Proofs

Proof by Contraposition

If $\underbrace{3n + 2 \text{ is an odd integer}}_{p}$, then $\underbrace{n \text{ is odd integer}}_{q}$

$p \rightarrow q$ assume p is true

$$3n + 2 = 2k + 1, \exists k \in Z$$
$$3n = 2k - 1$$
$$n = \frac{2k-1}{3}$$

Proofs

Proof by Contraposition

$p \to q \equiv \sim q \to \sim p$

If $3n + 2$ is an odd integer, then n is odd integer

Proofs

Proof by Contraposition $p \to q \equiv \sim q \to \sim p$

If $3n + 2$ is an odd integer, then n is odd integer

If n is not odd integer, then $3n + 2$ is not odd integer

Proofs

Proof by Contraposition

p → q ≡ ~q → ~p

If $3n + 2$ is an odd integer, then n is odd integer

If $\underbrace{\text{n is not odd integer}}_{\sim q}$, then $\underbrace{3n + 2 \text{ is not odd integer}}_{\sim p}$

Proofs

Proof by Contraposition $p \rightarrow q \equiv \sim q \rightarrow \sim p$

If $3n + 2$ is an odd integer, then n is odd integer

If $\underbrace{\text{n is not odd integer}}_{\sim q}$, then $\underbrace{3n + 2 \text{ is not odd integer}}_{\sim p}$

assume ~q is true

Proofs

Proof by Contraposition   p → q ≡ ~q → ~p

If $3n + 2$ is an odd integer, then n is odd integer

If $\underbrace{\text{n is not odd integer}}_{\sim q}$, then $\underbrace{3n + 2 \text{ is not odd integer}}_{\sim p}$

assume ~q is true $n = 2k, \exists k \in Z$

Proofs

Proof by Contraposition $\quad p \to q \equiv {\sim}q \to {\sim}p$

If $3n + 2$ is an odd integer, then n is odd integer

If $\underbrace{\text{n is not odd integer}}_{\sim q}$, then $\underbrace{3n + 2 \text{ is not odd integer}}_{\sim p}$

assume ~q is true

$$n = 2k, \exists k \in Z$$
$$3n + 2 = 6k + 8$$

Proofs

Proof by Contraposition

$p \rightarrow q \equiv \sim q \rightarrow \sim p$

If $3n+2$ is an odd integer, then n is odd integer

If $\underbrace{\text{n is not odd integer}}_{\sim q}$, then $\underbrace{3n+2 \text{ is not odd integer}}_{\sim p}$

assume ~q is true

$$n = 2k, \exists k \in Z$$
$$3n+2 = 6k+8$$
$$3n+2 = 2(3k+4)$$

Proofs

Proof by Contraposition

$p \rightarrow q \equiv \sim q \rightarrow \sim p$

If $3n + 2$ is an odd integer, then n is odd integer

If $\underbrace{\text{n is not odd integer}}_{\sim q}$, then $\underbrace{3n + 2 \text{ is not odd integer}}_{\sim p}$

assume ~q is true

$$n = 2k, \exists k \in Z$$
$$3n + 2 = 6k + 8$$
$$3n + 2 = 2(3k + 4)$$
$$3n + 2 = 2m, \exists m \in Z$$

Proofs

Proof by Contraposition $p \rightarrow q \equiv \sim q \rightarrow \sim p$

If $3n + 2$ is an odd integer, then n is odd integer

If $\underbrace{\text{n is not odd integer}}_{\sim q}$, then $\underbrace{3n + 2 \text{ is not odd integer}}_{\sim p}$

assume ~q is true

$n = 2k, \exists k \in Z$

$3n + 2 = 6k + 8$

$3n + 2 = 2(3k + 4)$

$3n + 2 = 2m, \exists m \in Z$

~p is also true

Proofs

Proof by Contraposition

p → q ≡ ~q → ~p

Prove that for all real numbers x and y, if $x + y \geq 100$, then $x \geq 50$ or $y \geq 50$.

Proofs

Proof by Contraposition

$p \to q \equiv {\sim}q \to {\sim}p$

Prove that for all real numbers x and y, if $x + y \geq 100$, then $x \geq 50$ or $y \geq 50$.

If $\underbrace{x < 50 \text{ and } y < 50}_{\sim q}$, then $\underbrace{x + y < 100}_{\sim p}$

Proofs

Proof by Contraposition

$p \rightarrow q \equiv {\sim}q \rightarrow {\sim}p$

Prove that for all real numbers x and y, if $x + y \geq 100$, then $x \geq 50$ or $y \geq 50$.

If $\underbrace{x < 50 \text{ and } y < 50}_{\sim q}$, then $\underbrace{x + y < 100}_{\sim p}$

assume ~q is true

Proofs

Proof by Contraposition $p \to q \equiv {\sim}q \to {\sim}p$

Prove that for all real numbers x and y, if $x + y \geq 100$, then $x \geq 50$ or $y \geq 50$.

If $\underbrace{x < 50 \text{ and } y < 50}_{\sim q}$, then $\underbrace{x + y < 100}_{\sim p}$

assume ~q is true $x < 50$ and $y < 50$

Proofs

Proof by Contraposition $p \to q \equiv {\sim}q \to {\sim}p$

Prove that for all real numbers x and y, if $x + y \geq 100$, then $x \geq 50$ or $y \geq 50$.

If $\underbrace{x < 50 \text{ and } y < 50}_{\sim q}$, then $\underbrace{x + y < 100}_{\sim p}$

assume ~q is true $x < 50$ and $y < 50$

$x + y < 100$

Proofs

Proof by Contraposition

$p \to q \equiv {\sim}q \to {\sim}p$

Prove that for all real numbers x and y, if $x + y \geq 100$, then $x \geq 50$ or $y \geq 50$.

If $\underbrace{x < 50 \text{ and } y < 50}_{\sim q}$, then $\underbrace{x + y < 100}_{\sim p}$

assume ~q is true

$x < 50$ and $y < 50$

$x + y < 100$

~p is also true

Proofs

Proof by Contradiction

Proofs

Proof by Contradiction

- To prove that 'p is true', find a contradiction q such that $\sim p \rightarrow q$ is true.

Proofs

Proof by Contradiction

- To prove that 'p is true', find a contradiction q such that $\sim p \rightarrow q$ is true.

$$\sim p \rightarrow q$$

Proofs

Proof by Contradiction

- To prove that 'p is true', find a contradiction q such that $\sim p \rightarrow q$ is true.

$$\sim p \rightarrow q$$

$$\rightarrow F \equiv T$$

Proofs

Proof by Contradiction

- To prove that 'p is true', find a contradiction q such that $\sim p \rightarrow q$ is true.

$$\sim p \rightarrow q$$

$$F \rightarrow F \equiv T$$

Proofs

Proof by Contradiction

- To prove that 'p is true', find a contradiction q such that $\sim p \rightarrow q$ is true.

$$\sim p \rightarrow q$$

$$F \rightarrow F \equiv T$$

$$q \equiv r \wedge \sim r$$

Proofs

Proof by Contradiction

- To prove that 'p is true', find a contradiction q such that $\sim p \rightarrow q$ is true.

$$\sim p \rightarrow q$$

$$F \rightarrow F \equiv T$$

$$q \equiv r \wedge \sim r \equiv 0$$

Proofs

Proof by Contradiction

- To prove that 'p is true', find a contradiction q such that $\sim p \rightarrow q$ is true.

$$\sim p \rightarrow q$$

$$F \rightarrow F \equiv T$$

- assuming '~p is true' leads us a contradiction

$$q \equiv r \wedge \sim r \equiv 0$$

Proofs

Proof by Contradiction

- Prove that the sum of an irrational number and rational number is irrational.

Proofs

Proof by Contradiction

- Prove that the sum of an irrational number and rational number is irrational.

Assume that the sum of an irrational number x and a rational number y is rational.

Proofs

Proof by Contradiction

- Prove that the sum of an irrational number and rational number is irrational.

Assume that the sum of an irrational number x and a rational number y is rational. ($\sim p$ is true)

Proofs

Proof by Contradiction

- Prove that the sum of an irrational number and rational number is irrational.

Assume that the sum of an irrational number x and a rational number y is rational. (~p is true)

$y = \frac{a}{b}$ and $x + y = \frac{c}{d}$, $\exists\ a, b, c, d \in Z$

Proofs

Proof by Contradiction

- Prove that the sum of an irrational number and rational number is irrational.

Assume that the sum of an irrational number x and a rational number y is rational. (~p is true)

$y = \frac{a}{b}$ and $x + y = \frac{c}{d}$, $\exists\ a, b, c, d \in Z$

There is no integers e,f such that $x = \frac{e}{f}$

Proofs

Proof by Contradiction

- Prove that the sum of an irrational number and rational number is irrational.

Assume that the sum of an irrational number x and a rational number y is rational. (~p is true)

$y = \frac{a}{b}$ and $x + y = \frac{c}{d}$, $\exists\ a, b, c, d \in Z$

There is no integers e,f such that $x = \frac{e}{f}$ (the proposition r)

Proofs

Proof by Contradiction

- Prove that the sum of an irrational number and rational number is irrational.

Assume that the sum of an irrational number x and a rational number y is rational. (~p is true)

$y = \frac{a}{b}$ and $x + y = \frac{c}{d}$, $\exists\ a, b, c, d \in Z$

There is no integers e,f such that $x = \frac{e}{f}$ (the proposition r)

$$x + y = \frac{c}{d}$$

Proofs

Proof by Contradiction

- Prove that the sum of an irrational number and rational number is irrational.

Assume that the sum of an irrational number x and a rational number y is rational. (~p is true)

$y = \frac{a}{b}$ and $x + y = \frac{c}{d}$, $\exists\ a, b, c, d \in Z$

There is no integers e,f such that $x = \frac{e}{f}$ (the proposition r)

$x + y = \frac{c}{d} \quad \rightarrow \quad x + \frac{a}{b} = \frac{c}{d}$

Proofs

Proof by Contradiction

- Prove that the sum of an irrational number and rational number is irrational.

Assume that the sum of an irrational number x and a rational number y is rational. (~p is true)

$y = \frac{a}{b}$ and $x + y = \frac{c}{d}$, $\exists\ a, b, c, d \in Z$
There is no integers e,f such that $x = \frac{e}{f}$ (the proposition r)

$x + y = \frac{c}{d} \quad \rightarrow \quad x + \frac{a}{b} = \frac{c}{d} \quad \rightarrow \quad x = \frac{c}{d} - \frac{a}{b}$

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~p → (r ∧ ~r) : assuming '~p is true' leads us a contradiction.

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- Prove that if $3n + 2$ is an odd integer, then n is odd integer

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$3n + 2$ is an even integer. (Contradiction!)

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$5n + 4 = 2(5k + 4) + 1$

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$q \rightarrow p$ (proof by contraposition)

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