# Sets

Murat Osmanoglu

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- These objects are called elements (or members) of the set.
- $x \in A$ , x is an element of the set A
- $x \notin A$ , x is not an element of the set A

```
• Z = \{2, -22, 12, 0, 43, -1287, ...\}

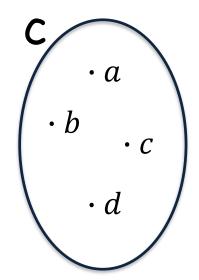
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- Venn Diagram



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- The universal set, denoted by U, contains all possible elements under the consideration
- The empty set, denoted by Ø, has no element

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$$\leftrightarrow \exists x \sim [x \in A \to x \in B]$$

### **Definitions**

$$A \subseteq B \leftrightarrow \forall x [x \in A \rightarrow x \in B]$$

$$\mathbf{A} \not\subseteq \mathbf{B} \leftrightarrow \sim \forall x [x \in A \to x \in B] 
\leftrightarrow \exists x \sim [x \in A \to x \in B] 
\leftrightarrow \exists x \sim [\sim x \in A \lor x \in B]$$

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$$\Leftrightarrow \exists x [x \in A \land \sim x \in B] 
\leftrightarrow \exists x [x \in A \land x \notin B]$$

 A set A is a subset of a set B if and only if every element of A is also an element of B.

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•  $\emptyset \subseteq A$  and  $A \subseteq A$ .

$$A \subseteq B \leftrightarrow \forall x [x \in A \rightarrow x \in B]$$

- $\emptyset \subseteq A$  and  $A \subseteq A$ .
- A = B if and only if  $A \subseteq B$  and  $B \subseteq A$

•  $A = \{x | x = 4k + 1 \text{ for some } k \in Z\}$ ,  $B = \{x | x = 4k - 3 \text{ for some } k \in Z\}$ Show that whether the sets A and B are equal or not.

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Thus, A=B.

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The power set of a given set is the set of all possible subsets.

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• If |S|=n, then  $|P(S)|=2^n$ 

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• The complement of A, denoted by  $\overline{A}$ , contains elements that are in U but not in A.

$$\overline{A} = \{ x \in U | x \notin A \}$$

•  $A \cup \emptyset = A$  $A \cap U = A$ 

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• 
$$A \cup \emptyset = A$$
  $p \lor 0 \equiv p$   $A \cap U = A$   $p \land 1 \equiv p$ 

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 $A \cap \emptyset = \emptyset$   $p \land 0 \equiv 0$ 

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$$A \cap \overline{A} = \emptyset$$
  
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$$p \land \sim p \equiv 0$$
$$p \lor \sim p \equiv 1$$

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  $p \lor p \equiv p$   $A \cap A = A$   $p \land p \equiv p$ 

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$$\overline{(\overline{A})} = A$$
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IAUBI = IAI + IBI - IA∩BI

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$$|A \times B| = |A| \cdot |B|$$

• The Cartesian products of the sets  $A_1, A_2, ..., A_n$  is the set of ordered n-tuples  $(a_1, a_2, ..., a_n)$  where  $a_1 \in A_1, a_2 \in A_2, ..., a_n \in A_n$ .

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$$A_1 x ... x A_n = \{(a_1, a_2, ..., a_n) | a_i \in A_i, i = 1... n\}$$