Homework #1

You write your solutions on paper by yourself, scan (or photo capture through a mobile application such as CamScanner) and submit them as a single .pdf file. Your solutions have to be handwritten. **Solutions must be submitted electronically before 23.59 on November 13.** No credit will be given to solutions obtained verbatim from the Internet or other sources.

1. Prove that there is no integer a such that $a \equiv 2 \ (mod \ 6)$ and $a \equiv 7 \ (mod \ 9)$. (Use proof by contradiction)

2. Prove that if $2n^2 + 3n$ is even integer, then n is even integer.

3. Find the number of permutations of the letters a b c d e … x y z (26 letters) in which none of the patterns 'spin' or 'net' occurs.

4. How many ways are there to distribute 30 balls to 4 people Hasan, Mehmet, Ayla, and Büşra if Ayla and Büşra together get no more than 20 balls and Mehmet gets at least 7?

5. Let $A = \{a, b, c\}$, $B = \{1, 2, …, n, n + 1\}$, and $S = \{ f : A \to B | f(a) < f(c) \ and \ f(b) < f(c)\}$.
   a) For $n \geq i \geq 1$, let $X = \{ f : A \to B | f \in S \ and \ f(c) = 5\}$. What is $|X|$?
   b) Let $Y = \{ f : A \to B | f \in S \ and \ f(a) = f(b)\}$. What is $|Y|$?