Homework #3

You write your solutions on paper by yourself, scan (or photo capture through a mobile application such as CamScanner) and submit them as a single .pdf file. Your solutions have to be handwritten. **Solutions must be submitted electronically before 23.59 on December 25**. No credit will be given to solutions obtained verbatim from the Internet or other sources.

1. Let R be the relation on the set of integers defined as $\forall a, b \in \mathbb{Z}, (a, b) \in R$ if $a.b < 0$. Determine which properties (reflexive, symmetric, antisymmetric, transitive) the relation satisfies. Justify your answer.

2. A relation R defined on a set A is called irreflexive if for all $a \in A$, $(a, a) \notin R$.

a) Give an example of a relation R on Z where R is irreflexive and transitive but not symmetric.
b) Let R be a nonempty relation on a set A. Prove that if R satisfies any two of the following properties -irreflexive, symmetric, and transitive- then it cannot satisfy the third.
c) If $|A| = n \geq 1$, how many irreflexive relations can be defined on A?

3. Let R be the relation defined on $A = Z \times Z$ in the following way :

$$\big((x_1, y_1), (x_2, y_2)\big) \in R \Leftrightarrow x_1.y_1 = x_2.y_2$$

Determine whether the relation R is an equivalence relation on A or not.

4. Let p and q be two distinct primes.

a) Draw the Hasse diagram of all positive divisors ofp^3q^6.
b) How many edges are there in the Hasse diagram of all positive divisors of $p^m q^n$ where m and n are positive integers ?