How To Solve Double Integrals

Double integrals extend the concept of a one-dimensional integral to two dimensions. A double integral, denoted as $\iint_R f(x,y)\,dy\,dx$, measures the volume under a surface $f(x,y)$ as bounded by a region R in the xy-plane.

1 Steps to Solve a Double Integral

1. Determine and clearly define the limits of integration.
2. Evaluate the inner integral first, treating the other variable as a constant.
3. Evaluate the outer integral, which will give a number that is the solution of the double integral.

2 Example

Consider the double integral $\iint_R y\,dy\,dx$, where R is the rectangle defined by $0 \leq x \leq 2$, $0 \leq y \leq 3$.

Following the steps defined above:

1. The limits of x are 0 and 2. The limits of y are 0 and 3.
2. Begin by integrating the function $f(x,y) = y$ with respect to y:

$$\int_0^3 y\,dy = \frac{1}{2}y^2\Big|_0^3 = \frac{1}{2}(3)^2 = \frac{9}{2}$$

   This results in a function $g(x) = \frac{9}{2}$ which depends only on x, but x does not actually appear in $g(x)$.

3. Now, integrate $g(x)$ with respect to x:

$$\int_0^2 \frac{9}{2}\,dx = \frac{9}{2}x\Big|_0^2 = \frac{9}{2}(2) = 9$$

Hence, the result is 9, which represents the volume under the function y and above the rectangle R on the xy-plane.