Assignment 3
Bayesian and Decision Networks

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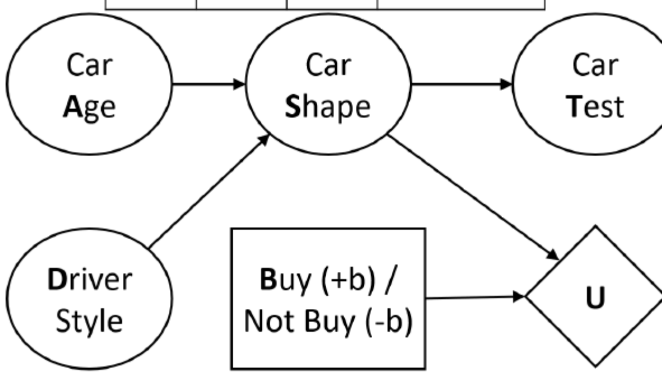
A	D	S	P(S|A, D)
+o	a	+s	0.2
+o	n	+s	0.5
+o	d	+s	0.7
-o	a	+s	0.4
-o	n	+s	0.7
-o	d	+s	0.9

A	P(A)
+o	0.6

D	P(D)
aggressive	0.2
neutral	0.5
defensive	0.3



S	T	P(T|S)
+s	+t	0.95
-s	+t	0.19

S	B	U
+g	+b	5000
+g	-b	0
-g	+b	-6000
-g	-b	0

Question 1

The joint distribution for the Bayesian Network is as follows:

$$P(D, A, S, T) = P(D) * P(A) * P(S|A, D) * P(T|S)$$

Question 2

To use variable elimination for this inference problem, we first need to find the initial factors.

P(S|T=+t) = $\sum_{d,a} P(d) * P(a) * P(S|a, d) * P(+t|S)$

Initial Factors:

- $f_1(D) = P(D)$
- $f_2(A) = P(A)$
- $f_3(S, A, D) = P(S|A, D)$

- $f_4(t, S) = P(+t|S)$

Variable Elimination:

1. Join f_1 *and* f_3

   When we do the join, we get a new factor $f_5(S, A, D)$

A	D	S	$f_5(S, A, D)$
+o	a	+s	**0.04**
+o	a	-s	**0.16**
+o	n	+s	**0.25**
+o	n	-s	**0.25**
+o	d	+s	**0.21**
+o	d	-s	**0.09**
-o	a	+s	**0.08**
-o	a	-s	**0.12**
-o	n	+s	**0.35**
-o	n	-s	**0.15**
-o	d	+s	**0.27**
-o	d	-s	**0.03**

2. Sum out D

   Getting $f_6(A, S)$

A	S	$f_6(A, S)$
+o	+s	**0.5**
+o	-s	**0.5**
-o	+s	**0.7**
-o	-s	**0.3**

3. Join f_6 *and* f_2

   Getting $f_7(A, S)$

A	S	$f_7(A, S)$
+o	+s	**0.3**
+o	-s	**0.3**
-o	+s	**0.28**
-o	-s	**0.12**

4. Sum out A

S	$f_8(S)$
+s	**0.58**
-s	**0.42**

5. Join f_4 and f_8

   Getting $f_9(S)$

S	$f_8(S)$
+s	**0.551**
-s	**0.0798**

6. Normalize to get $\mathrm{P(S|T = +t)}$

S	$\mathrm{P(S|T = +t)}$
+s	**≅0.873**
-s	**≅0.127**

Therefore, the query

$$\mathrm{P(S = +s|T = +t)} \cong 0.873$$

Question 3

$$\mathrm{P(S)} = \sum_{d,a,t} P(d) * P(a) * P(S|a,d) * P(t|S)$$

This query is very similar to the query in the previous question. If we reorder the summation, we can get:

$$\mathrm{P(S)} = \sum_{d,a} P(d) * P(a) * P(S|a,d) * \sum_{t} P(t|S)$$

By definition of the conditional probabilities, the summation of $P(t|S)$ over T equals 1. Therefore, the query simplifies to

$$\mathrm{P(S)} = \sum_{d,a} P(d) * P(a) * P(S|a,d)$$

In fact, this query is solved in the previous question until step 5**.** The distribution over S in **step 4** equals $P(S)$

S		$f_8(S)$
+s		**0.58**
-s		**0.42**

Therefore,

$$P(S = +g) = 0.58$$

Question 4

$$EU(Buy) = P(S = +g) * U(+g, +b) + P(S = -g) * U(-g, +b)$$

$$EU(\neg Buy) = P(S = +g) * U(+g, -b) + P(S = -g) * U(-g, -b)$$

From the results of the previous question:

$$EU(Buy) = 0.58 * 5000 + 0.42 * -6000 = 380$$
$$EU(\neg Buy) = 0$$

Question 5

This is a value of information problem.

$$VPI(T) = \left[\sum_t P(t) * MEU(B|t)\right] - MEU(Buy)$$

To find this expression, we need to first find $MEU(Buy|T)$

$$EU(Buy \mid T) = P(S = +g \mid T) * U(+g, +b) + P(S = -g \mid T) * U(-g, +b)$$

$$EU(\neg Buy \mid T) = P(S = +g \mid T) * U(+g, -b) + P(S = -g \mid T) * U(-g, -b)$$

It is easily seen that $EU(\neg Buy \mid T) = 0$. For finding $EU(Buy \mid T)$, we need to do a few more calculations.

We can calculate this expected utility using the query $P(S|T = +t)$ from the second question and finding $P(S|T = -t)$.

$$P(S|T = -t) = \propto P(S, -t) = P(S) * P(-t|S)$$

From this equation, we can find the joint distribution of $P(S, -t)$ as:

S	$P(S, -t)$
+s	**0.029**
-s	**0.3402**

When we normalize this distribution:

S	$P(S \| \boldsymbol{T} = -t)$
+s	**0.079**
-s	**0.921**

From this calculation, $EU(B \mid T)$ is a distribution over $T\ and\ B$:

T	B	$U(B \mid T)$
+t	Buy	**3603**
-t	Buy	**-5131**
+t	¬Buy	**0**
-t	¬Buy	**0**

From this, we find that $MEU(B|T)$ is as follows:

For $T = +t$; optimal action is Buy and $MEU(B|T) = 3603$
For $T = -t$; optimal action is $Not\ Buy$ and $MEU(B|T) = 0$

Now that we found $U(Buy \mid T)$, we only need to find $P(T)$ and then we will be ready to find the VPI.

To find P(T), we can join the tables of $P(S, -t)$ above and $P(S, +t)$ from the second question and then marginalize over S.

S	T	P(S,T)
+s	+t	**0.551**
+s	-t	**0.029**
-s	+t	**0.0798**
-s	-t	**0.3402**

From this joint distribution:

T	$P(\boldsymbol{T})$

+t	**0.6308**
-t	**0.3692**

Using the maximum expected utilities, prior maximum expected utility, and the probabilities of T, we get:

$$VPI(T) = 0.6308 * 3603 + \big(0.3692 * (0)\big) - 380 = 1892.77$$

Therefore, the maximum amount of money to pay for tests is this.

Question 6

In rejection sampling (likelihood weighting method), we first set the weight $w = 1.0$, then start sampling in the given order. When we come across an evidence variable $E_i = e_i$, we set the weight w to $w = w * P(e_i|Parents(E_i))$

A	D	S	T	Weight
+o	d	+s	+t	$w = P(A = +o) * P(T = +t\|S = +s) = 0.6 * 0.95 = 0.57$
+o	d	+s	+t	$w = P(A = +o) * P(T = +t\|S = -s) = 0.6 * 0.19 = 0.114$

Question 7

In Gibbs sampling, we arbitrarily sample a non-evidence variable X_i using the distribution of that variable conditioned on the current values of the variables in the Markov blanket of X_i. The Markov blanket of a random variable in a BN contains the variable, its parents, children, and the parents of its children. This can be seen in the following computations:

$$P(S|A,D,T) = \frac{P(S,A,D,T)}{P(A,D,T)}$$

$$P(S|A,D,T) = \frac{P(D) * P(A) * P(S|D,A) * P(T|S)}{\sum_s P(D) * P(A) * P(s|D,A) * P(T|s)}$$

$$P(S|A,D,T) = \frac{P(S|D,A) * P(T|S)}{\sum_s P(s|D,A) * P(T|s)}$$

Given the sample $\{A = -o, D = n,\ S = +s, T = -t\}$,

$$P(S|-o,n,-t) = \frac{P(S|n,-o) * P(-t|S)}{\sum_s P(s|n,-o) * P(-t|s)}$$

First let us calculate the denominator:

$$\sum_s P(s|n,-o) * P(-t|s) = (0.7 * 0.05) + (0.3 * 0.81) = 0.278$$

The distribution in the numerator divided by 0.32 (the distribution to sample S from) becomes:

S	*Gibbs sample distribution*
+s	**0.126**
-s	**0.874**