FACULTY OF ENGINEERING AND ARCHITECTUREDEPARTMENT OF MECHATRONICS ENGINEERING

FIVE BAR MECHANISM AND FOUR BAR MECHANISM KINEMATIC SYNTHESIS

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ABSTRACT

Kinematic synthesis revolves around crafting mechanisms capable of producing predetermined motion paths or trajectories. In essence, it involves the design and configuration of systems, such as linkages and cams, to achieve desired motion patterns. This process entails determining the appropriate dimensions, shapes, and layouts of components to fulfill specific motion requirements. For instance, in industrial settings, kinematic synthesis might involve devising mechanisms to facilitate tasks like lifting and relocating objects with precision. This facet is integral to fields like robotics, automation, and machinery design.

1. AIM OF THE PROJECT

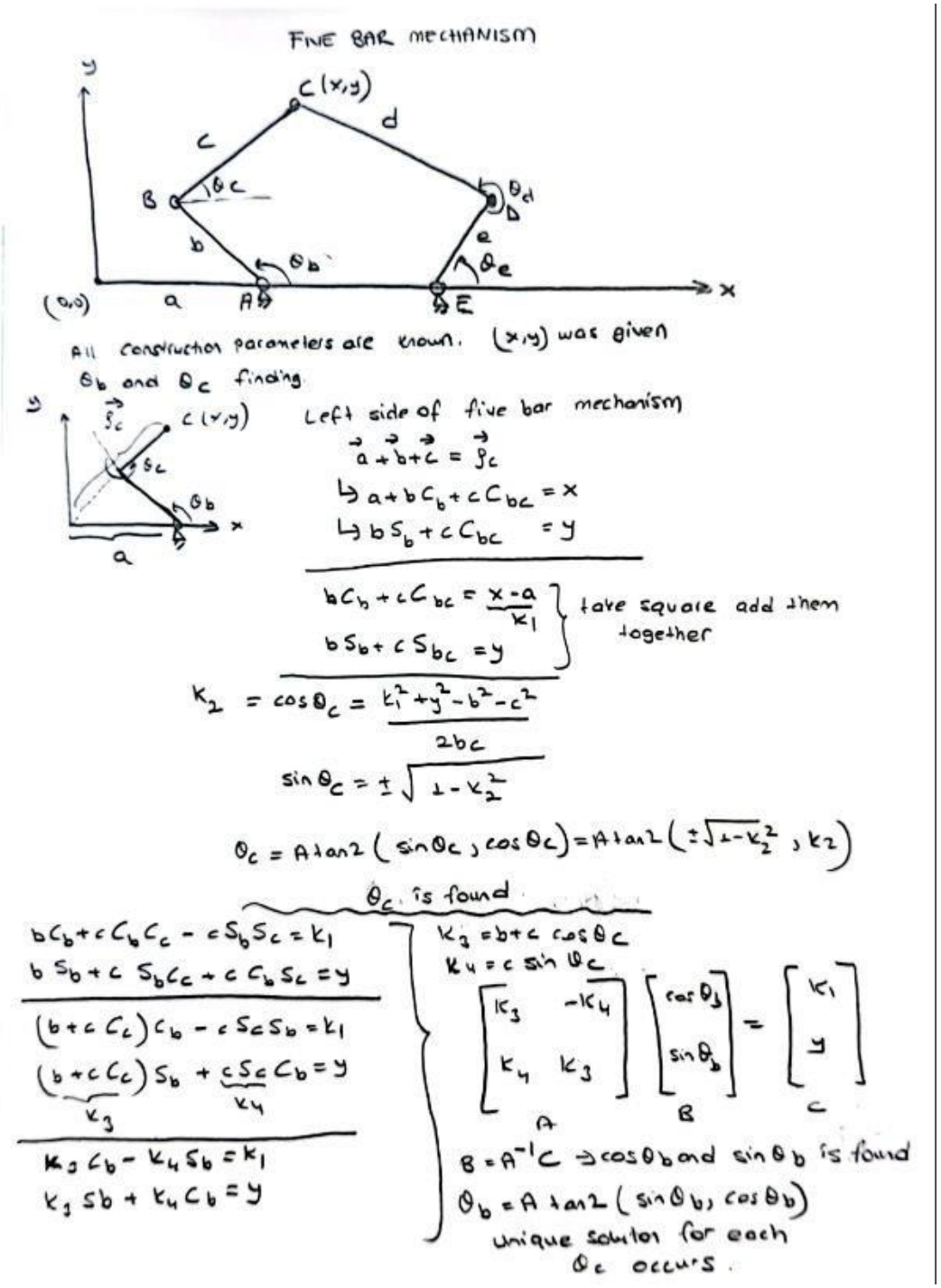
The aim of the project is to design a five-bar mechanism that follows a given function and then convert the designed five-bar mechanism into a four-bar mechanism to determine the path it follows. Subsequently, an error analysis is conducted, and the results are interpreted.

2. DESIGN PROCEDURE

2.1 FIVE BAR MECHANISM

The inverse task method was used in the design of a 5-bar mechanism. During the application of the inverse task, link dimensions were initially determined, and necessary calculations were made considering these dimensions. As a result of these calculations, 2 input and 2 output angles were obtained. One of these different solutions was selected, and a 5-bar mechanism was drawn on a graph. By means of a button, it was observed whether the mechanism followed the desired path. As a result of the process, the 5-bar mechanism successfully followed the desired path.

2.1.1 FIVE BAR MECHANISM CALCULATIONS



For the right side of mechanism replace:

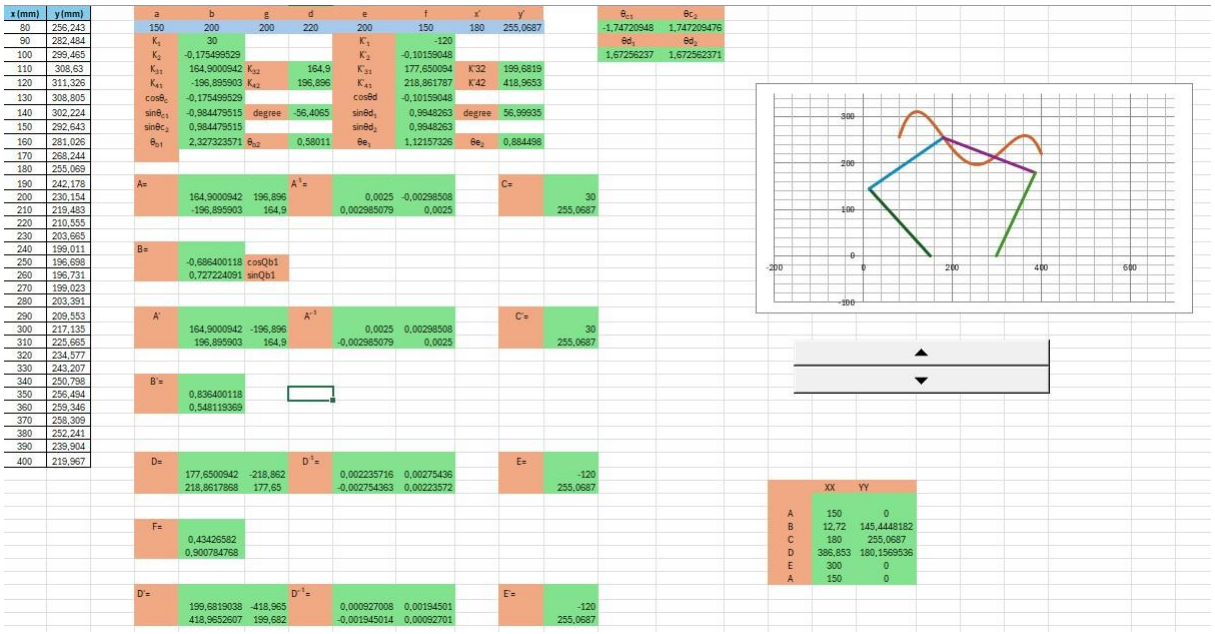
a= a+f

b=e

c=d

Qb=Qe

Qc=Qd calculations rewrited and Qe Qd degrees found. In this Project 1st solution was used.



x (mm)	y (mm)
80	256,243
90	282,484
100	299,465
110	308,63
120	311,326
130	308,805
140	302,224
150	292,643
160	281,026
170	268,244
180	255,069
190	242,178
200	230,154
210	219,483
220	210,555
230	203,665
240	199,011
250	196,698
260	196,731
270	199,023
280	203,391
290	209,553
300	217,135
310	225,665
320	234,577
330	243,207
340	250,798
350	256,484
360	259,346
370	258,309
380	252,241
390	239,904
400	219,967

a	b	g	d	e	f	x'	y'
150	200	200	220	200	150	180	255,0687
K_1	30			K'_1	-120		
K_2	-0,175499529			K'_2	-0,10159048		
K_{31}	164,9000942	K_{32}	164,9	K'_{31}	177,650094	K'32	199,6819
K_{41}	-196,895903	K_{42}	196,896	K'_{41}	218,861787	K'42	418,9653
$\cos\theta_c$	-0,175499529			cosθd	-0,10159048		
$\sin\theta_{c1}$	-0,984479515	degree	-56,4065	$\sin\theta_{d1}$	0,9948263	degree	56,99935
$\sin\theta_{c2}$	0,984479515			$\sin\theta_{d2}$	0,9948263		
θ_{b1}	2,327323571	θ_{b2}	0,58011	θ_{e1}	1,12157326	θ_{e2}	0,884498

θ_{c1}	θ_{c2}
-1,74720948	1,747209476
θ_{d1}	θ_{d2}
1,67256237	1,672562371

	XX	YY
A	150	0
B	12,72	145,4448182
C	180	255,0687
D	386,853	180,1569536
E	300	0
A	150	0

A=				A⁻¹=			C=	
	164,9000942	196,896		0,0025	-0,00298508		30
	-196,895903	164,9		0,002985079	0,0025		255,0687

B= | | | |
|---|---|---|
| | -0,686400118 | cosQb1 |
| | 0,727224091 | sinQb1 |

A'				A'⁻¹			C'=	
	164,9000942	-196,896		0,0025	0,00298508		30
	196,895903	164,9		-0,002985079	0,0025		255,0687

B'= | |
|---|
| 0,836400118 |
| 0,548119369 |

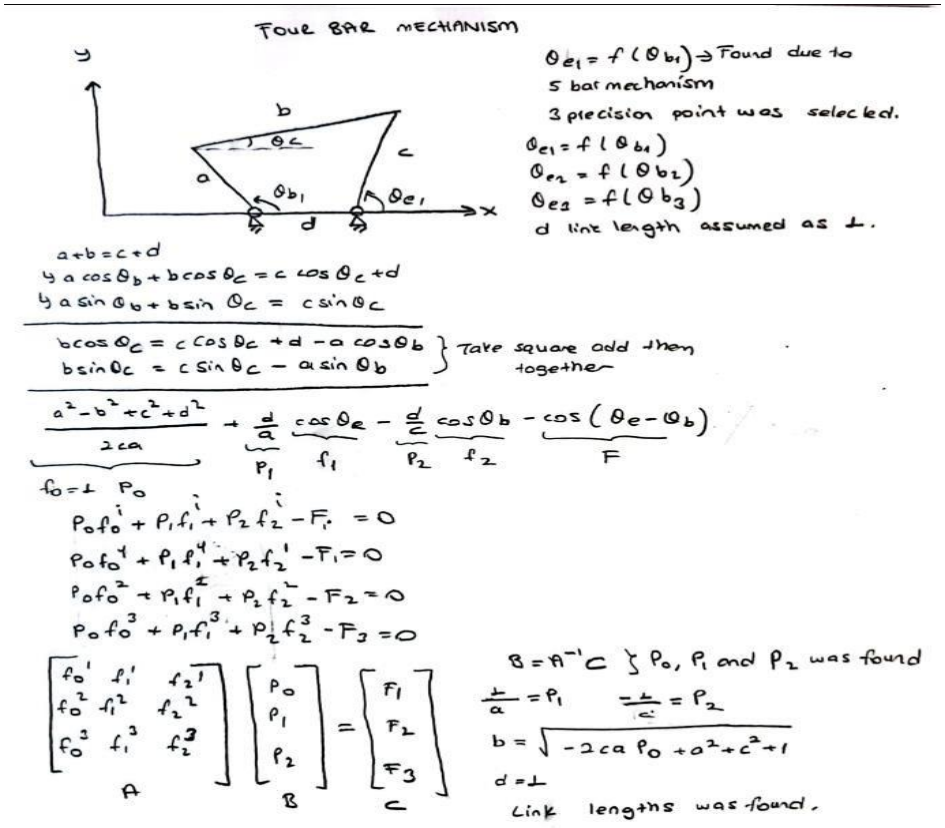
D=			D⁻¹=			E=	
	177,6500942	-218,862		0,002235716	0,00275436		-120
	218,8617868	177,65		-0,002754363	0,00223572		255,0687

F= | |
|---|
| 0,43426582 |
| 0,900784768 |

D'=			D'⁻¹=			E'=	
	199,6819038	-418,965		0,000927008	0,00194501		-120
	418,9652607	199,682		-0,001945014	0,00092701		255,0687

2.2 FOUR BAR MECHANISM

During the design stage of the 4-bar mechanism, the relationship between input and output angles was assumed to be the same as that of the 5-bar mechanism.3 precision point was selected ,based on this assumption, the link lengths of the 4-bar mechanism were calculated.



Link length of ‘f’ five bar mechanism must be equal to link length of ‘d’ for four bar mechanism. First step d link length was assumed to 1 but five bar mechanism ‘f’ link length is 150 mm so, four bar mechanism link lengths must be scale by multiplying each link to 150.

	PRECİSİON POİNTS	
	Qb1	Qe1
PP1	2,681999507	1,60753741
PP2	2,232612218	0,85224385
PP3	1,308795868	0,11685332

A MATRIX		
1	-0,036732818	0,89623305
1	0,658295725	0,61455038
1	0,993180416	-0,2590133

A-1 MARRIX		
1,522690293	-1,717183508	1,19449321
-1,703449301	2,252730563	-0,5492813
-0,653025225	2,008331045	-1,3553058

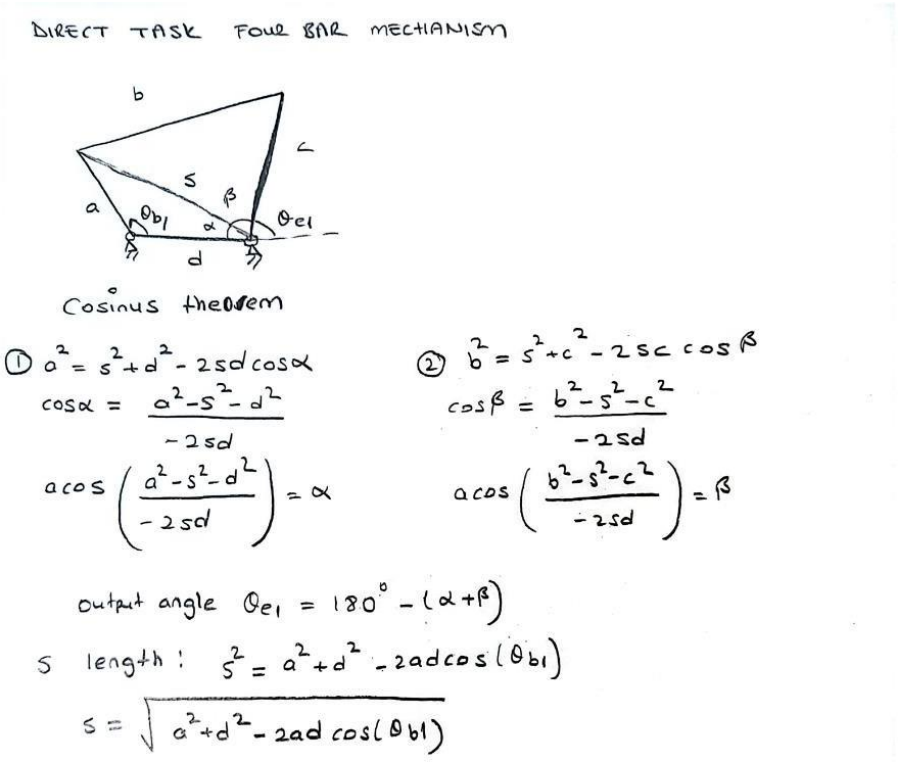
C MATRIX
0,476205309
0,189279135
0,369855771

A-1*C=B MATRIX	
P0	0,84188
P1	-0,58795
P2	-0,43211

FOUR BAR LINK LENGTHS		
		scale
a	-1,70082	-255,123061
b	1,61897	242,8459216
c	-2,31424	-347,136567
d	1	150

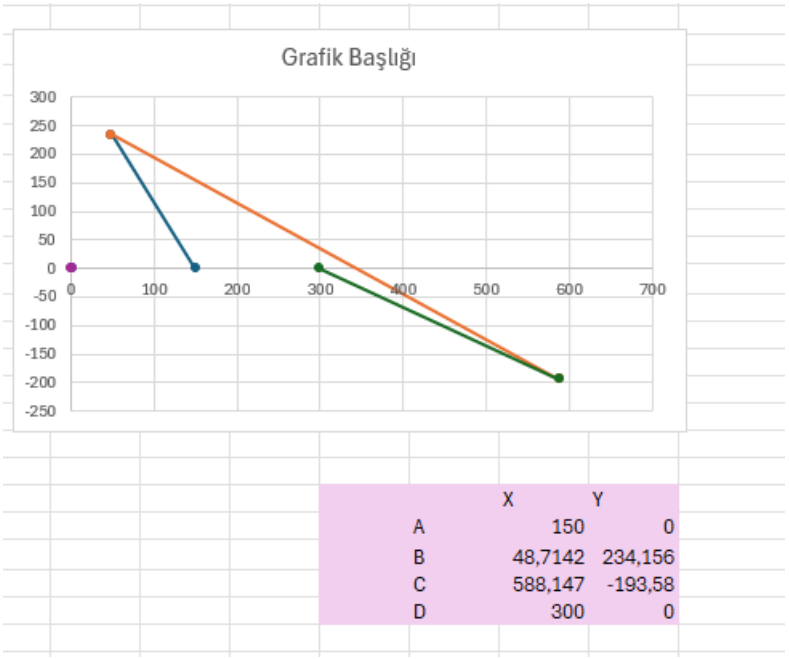
2.2.1 FOUR BAR MECHANISM DIRECT TASK

By performing the direct task, the relationship between the input and output angles of the known 4-bar mechanism is obtained. Thus, the design of the 4-bar mechanism is completed.



input degree								
derece Qb1	radyan Qb1	S	degree α	radyan α	degree β	radyan β	degree Qe1	radyan Qe1
153,6672524	2,681999507	137,81459	124,799	2,17814	147,307	2,57099	-92,105	-1,6075
153,0942946	2,671999507	139,05319	123,875	2,16203	146,891	2,56372	-90,766	-1,5842
152,5213368	2,661999507	140,30518	122,964	2,14612	146,485	2,55665	-89,449	-1,5612
151,948379	2,651999507	141,57009	122,063	2,1304	146,091	2,54977	-88,154	-1,5386
151,3754212	2,641999507	142,84743	121,173	2,11487	145,707	2,54306	-86,88	-1,5163
150,8024634	2,631999507	144,13676	120,294	2,09953	145,333	2,53654	-85,627	-1,4945
150,2295056	2,621999507	145,43763	119,425	2,08437	144,97	2,5302	-84,395	-1,473
149,6565478	2,611999507	146,7496	118,567	2,06939	144,616	2,52402	-83,183	-1,4518
149,08359	2,601999507	148,07225	117,719	2,05459	144,272	2,51802	-81,991	-1,431
148,5106323	2,591999507	149,40517	116,881	2,03997	143,938	2,51219	-80,819	-1,4106
147,9376745	2,581999507	150,74795	116,053	2,02551	143,613	2,50652	-79,666	-1,3904
147,3647167	2,571999507	152,1002	115,235	2,01122	143,298	2,50101	-78,532	-1,3706
146,7917589	2,561999507	153,46154	114,425	1,9971	142,991	2,49567	-77,417	-1,3512
146,2188011	2,551999507	154,83158	113,626	1,98314	142,694	2,49047	-76,319	-1,332
145,6458433	2,541999507	156,20997	112,835	1,96934	142,405	2,48543	-75,239	-1,3132
145,0728855	2,531999507	157,59635	112,053	1,95569	142,124	2,48054	-74,177	-1,2946
144,4999277	2,521999507	158,99038	111,279	1,94219	141,853	2,47579	-73,132	-1,2764
143,9269699	2,511999507	160,39171	110,514	1,92884	141,589	2,47119	-72,103	-1,2584
143,3540121	2,501999507	161,80002	109,758	1,91563	141,333	2,46673	-71,091	-1,2408
142,7810543	2,491999507	163,215	109,009	1,90257	141,085	2,4624	-70,094	-1,2234
142,2080965	2,481999507	164,63632	108,269	1,88964	140,845	2,45821	-69,114	-1,2063
141,6351387	2,471999507	166,06369	107,536	1,87685	140,612	2,45415	-68,148	-1,1894
141,0621809	2,461999507	167,49681	106,811	1,8642	140,387	2,45021	-67,198	-1,1728
140,4892231	2,451999507	168,93539	106,093	1,85167	140,169	2,4464	-66,262	-1,1565

(Relationship between input and output angle)



	X	Y
A	150	0
B	48,7142	234,156
C	588,147	-193,58
D	300	0

(Four bar mechanism)

2.3 ERROR CALCULATION

The polynomial approximation method was chosen for error calculation.

$$f(c,v) - g(v) = e$$

$$P_0 f_0 + P_1 f_1 + P_2 f_2 - F = \text{error}$$

error = 0 @ Precision point

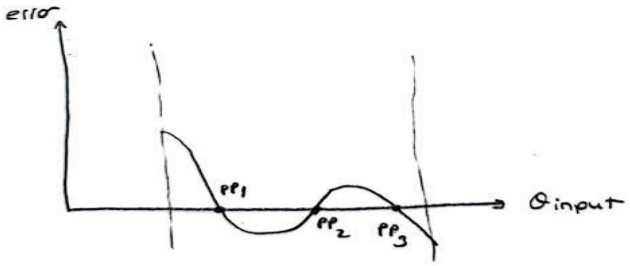
P_0, P_1, P_2 unknowns (3 eqn 3 precision points)

$$P_0 f_0^1 + P_1 f_1^1 + P_2 f_2^1 - F_1 = 0$$

$$P_0 f_0^2 + P_1 f_1^2 + P_2 f_2^2 - F_2 = 0$$

$$P_0 f_0^3 + P_1 f_1^3 + P_2 f_2^3 - F_3 = 0$$

$$\begin{bmatrix} f_0 & f_1 & f_2 \\ f_0 & f_1 & f_2 \\ f_0 & f_1 & f_2 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}$$



Polynomial approximation

PRECİSİON POİNTS

	Qb1	Qe1
PP1	2,682	-1,60754
POINT1	2,472	-1,18941
PP2	2,23261	-0,85152
POINT 2	1,72199	-0,38527
PP3	1,3088	-0,11876

A MATRIX

1	-0,03673	0,89623
1	0,65884	0,61455
1	0,99296	-0,25901

A-1 MARRIX

1,52063	-1,71447	1,19383
-1,70114	2,24968	-0,54854
-0,65064	2,00517	-1,35454

C MATRIX

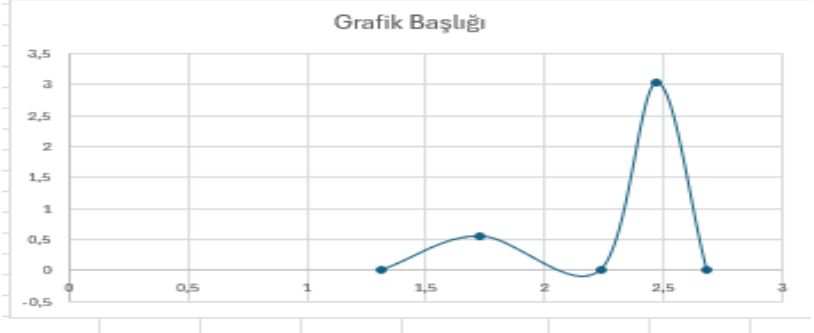
-0,41036
-0,99835
0,14275

A-1*C=B MATRIX

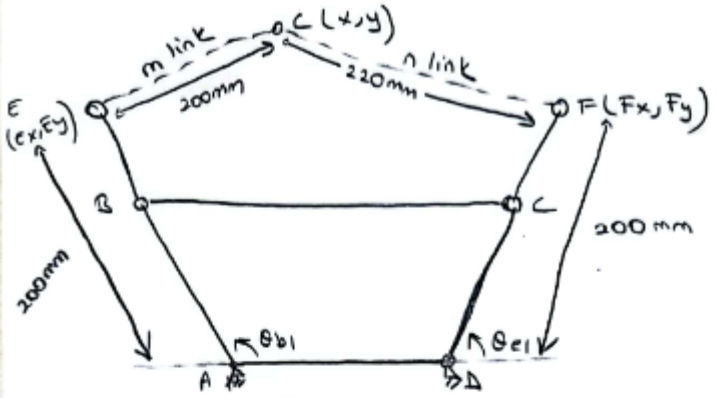
P0	1,25804
P1	-1,62618
P2	-1,92823

error at pp1	0
error at point 1	3,03255
error at pp2	0
error at point 2	0,55259
error at pp3	0

ERROR GRAPH



FINDING THE PATH WHICH FOUR BAR MECHANISM'S TOP POINT FOLLOWS.



$* \; (x-E_x)^2 + (y-E_y)^2 = m^2 \qquad m = 200\,mm$

$\text{Eqn}(1) \Rightarrow \underbrace{200^2 - E_x^2 - E_y^2}_{K_1} = x^2 + y^2 - 2(E_x \cdot x + E_y \cdot y)$

$* \; (x-F_x)^2 + (y-F_y)^2 = n^2 \qquad n = 220\,mm$

$\text{Eqn}(2) \rightarrow \underbrace{220^2 - F_x^2 - F_y^2}_{K_2} = x^2 + y^2 - 2(F_x \cdot x + F_y \cdot y)$

$\text{Eqn}(2) - \text{Eqn}(1) \Rightarrow \frac{K_2 - K_1}{2} = x\underbrace{(-F_x + E_y)}_{j} + y\underbrace{(-F_y + E_y)}_{z}$

$\frac{K_2 - K_1}{2} = jx + zy$

$y = \frac{\frac{K_2 - K_1}{2} - jx}{z}$ (substitute y eqn to eqn(1) then find the x value)

$\underbrace{220^2 - F_x - F_y}_{K_2} = x^2 + \left(\frac{\left(\frac{K_2 - K_1}{2}\right) - jx}{z} \right)^2 - 2(F_x \cdot x + F_y \cdot y)$

NOTE: $L = \frac{K_2 - K_1}{2}$

$0 = x^2 - K_2 + \frac{L^2 + j^2x^2 - 2Ljx}{z^2} - 2F_x \cdot x - 2F_y \cdot \frac{L - jx}{z}$

$\Rightarrow \underbrace{x^2(z^2 + j^2)}_{a} + x\underbrace{(-2Lj - 2F_x z^2 - 2F_y j z)}_{b} - \underbrace{K_2 z^2 - 2F_y L z}_{c}$

$\Rightarrow ax^2 + bx + c = 0$ (Find the roots of the eqn)

$x_1 = \frac{-b \pm \sqrt{\Delta}}{2a}$ (x founded) $\qquad y = \frac{\frac{K_2 - K_1}{2} - jx_1}{z}$ (y founded)

∇ Repeat this calculations for every position then, find the path function

							L			coefficients of the equation				The path which is four bar mechanism follow			
Ex	Ey	M	K1	Fx	Fy	K2	(K2-K1)/2	J=-Fx+Ey	Z=-Fy+Ey	a	b	c	delta	x1	x2	y1	y2
-29,247	-199,87	200	-801,39	291,919	-219,85	-85151	-42175	-491,78	19,9865	242251	-5E+07	-336624126	49452941,6	197,089	-7,0505	2739,36	-2283,7
-28,35	-199,98	200	398,301	297,059	-219,98	-88235	-44317	-497,04	19,9982	247450	-5E+07	-354630853	52147238	203,703	-7,0354	2846,85	-2390,9
-29,247	-199,87	200	400,696	291,919	-219,85	-85151	-42776	-491,78	19,9865	242251	-5E+07	-341906178	50054893,6	199,552	-7,0727	2769,88	-2314,3
-26,505	-199,9	200	330,492	307,088	-219,89	-94253	-47292	-506,98	19,9896	257433	-5E+07	-378072985	56230029,2	211,482	-6,9445	2997,87	-2541,9
-29,247	-199,87	200	400,696	291,919	-219,85	-85151	-42776	-491,78	19,9865	242251	-5E+07	-341906178	50054893,6	199,552	-7,0727	2769,88	-2314,3
-24,589	-199,42	200	186,024	316,775	-219,36	-100065	-50125	-516,19	19,9418	266852	-6E+07	-398745738	60164472,9	218,625	-6,8348	3145,52	-2690,5
-29,247	-199,87	200	400,696	291,919	-219,85	-85151	-42776	-491,78	19,9865	242251	-5E+07	-341906178	50054893,6	199,552	-7,0727	2769,88	-2314,3
-22,603	-198,59	200	-26,328	326,113	-218,44	-105668	-52821	-524,7	19,8586	275703	-6E+07	-416601567	63939164,5	225,203	-6,7097	3290,42	-2837,1
-29,247	-199,87	200	400,696	291,919	-219,85	-85151	-42776	-491,78	19,9865	242251	-5E+07	-341906178	50054893,6	199,552	-7,0727	2769,88	-2314,3
-20,547	-197,44	200	-298,02	335,101	-217,18	-111060	-55381	-532,54	19,7438	283987	-6E+07	-431655215	67546341,5	231,278	-6,5721	3433,14	-2982,3
-29,247	-199,87	200	400,696	291,919	-219,85	-85151	-42776	-491,78	19,9865	242251	-5E+07	-341906178	50054893,6	199,552	-7,0727	2769,88	-2314,3
-18,424	-196,01	200	-620,83	343,74	-215,61	-116244	-57811	-539,75	19,6007	291711	-7E+07	-443971813	70981386	236,903	-6,4244	3574,17	-3126,4
-29,247	-199,87	200	400,696	291,919	-219,85	-85151	-42776	-491,78	19,9865	242251	-5E+07	-341906178	50054893,6	199,552	-7,0727	2769,88	-2314,3
-16,233	-194,33	200	-987,01	352,033	-213,76	-121220	-60116	-546,36	19,4326	298885	-7E+07	-453655818	74242347,5	242,129	-6,2687	3714,01	-3269,8
-29,247	-199,87	200	400,696	291,919	-219,85	-85151	-42776	-491,78	19,9865	242251	-5E+07	-341906178	50054893,6	199,552	-7,0727	2769,88	-2314,3
-13,976	-192,42	200	-1389,3	359,987	-211,66	-125992	-62301	-552,41	19,2422	305525	-7E+07	-460841079	77329494,9	246,997	-6,1068	3853,08	-3413,1
-29,247	-199,87	200	400,696	291,919	-219,85	-85151	-42776	-491,78	19,9865	242251	-5E+07	-341906178	50054893,6	199,552	-7,0727	2769,88	-2314,3
-11,653	-190,32	200	-1820,9	367,608	-209,35	-130565	-64372	-557,93	19,0322	311648	-8E+07	-465682179	80244911,1	251,546	-5,9403	3991,81	-3556,4
-29,247	-199,87	200	400,696	291,919	-219,85	-85151	-42776	-491,78	19,9865	242251	-5E+07	-341906178	50054893,6	199,552	-7,0727	2769,88	-2314,3
-9,266	-188,05	200	-2275,5	374,904	-206,86	-134943	-66334	-562,96	18,8051	317272	-8E+07	-468347073	82992129,1	255,81	-5,7706	4130,58	-3700,2
-29,247	-199,87	200	400,696	291,919	-219,85	-85151	-42776	-491,78	19,9865	242251	-5E+07	-341906178	50054893,6	199,552	-7,0727	2769,88	-2314,3
-6,8149	-185,63	200	-2747,6	381,886	-204,19	-139132	-68192	-567,52	18,563	322419	-8E+07	-469010987	85575814,9	259,819	-5,5987	4269,77	-3844,7
-29,247	-199,87	200	400,696	291,919	-219,85	-85151	-42776	-491,78	19,9865	242251	-5E+07	-341906178	50054893,6	199,552	-7,0727	2769,88	-2314,3
-4,301	-183,08	200	-3231,8	388,563	-201,39	-143138	-69953	-571,64	18,3079	327110	-8E+07	-467851473	88001492,2	263,602	-5,4258	4409,73	-3990,3
-29,247	-199,87	200	400,696	291,919	-219,85	-85151	-42776	-491,78	19,9865	242251	-5E+07	-341906178	50054893,6	199,552	-7,0727	2769,88	-2314,3
-1,7254	-180,42	200	-3723,6	394,946	-198,46	-146967	-71622	-575,36	18,0416	331366	-9E+07	-465044516	90275309,4	267,181	-5,2527	4550,81	-4137,3
-29,247	-199,87	200	400,696	291,919	-219,85	-85151	-42776	-491,78	19,9865	242251	-5E+07	-341906178	50054893,6	199,552	-7,0727	2769,88	-2314,3
0,91087	-177,66	200	-4218,6	401,045	-195,42	-150627	-73204	-578,7	17,7657	335211	-9E+07	-460761536	92403842,3	270,579	-5,08	4693,33	-4286
-29,247	-199,87	200	400,696	291,919	-219,85	-85151	-42776	-491,78	19,9865	242251	-5E+07	-341906178	50054893,6	199,552	-7,0727	2769,88	-2314,3
3,60677	-174,82	200	-4713,1	406,871	-192,3	-154123	-74705	-581,69	17,4816	338666	-9E+07	-455167181	94393932,1	273,814	-4,9084	4837,62	-4436,7
-29,247	-199,87	200	400,696	291,919	-219,85	-85151	-42776	-491,78	19,9865	242251	-5E+07	-341906178	50054893,6	199,552	-7,0727	2769,88	-2314,3
6,36123	-171,91	200	-5203,7	412,436	-189,1	-157462	-76129	-584,34	17,1908	341753	-9E+07	-448417759	96252551,4	276,905	-4,7385	4984,01	-4589,6
-29,247	-199,87	200	400,696	291,919	-219,85	-85151	-42776	-491,78	19,9865	242251	-5E+07	-341906178	50054893,6	199,552	-7,0727	2769,88	-2314,3
9,17315	-168,94	200	-5687,3	417,75	-185,84	-160650	-77481	-586,69	16,8942	344492	-9E+07	-440660210	97986696,7	279,867	-4,5706	5132,8	-4745
-29,247	-199,87	200	400,696	291,919	-219,85	-85151	-42776	-491,78	19,9865	242251	-5E+07	-341906178	50054893,6	199,552	-7,0727	2769,88	-2314,3

3. RESULTS

In the project, the path that the mechanism needed to follow was a function in the x and y planes. This means that a 2-degree-of-freedom mechanism can follow this path with zero error. We proved this with the synthesis of the 5-bar mechanism. However, when we converted our 5-bar mechanism to a 4-bar mechanism, the degrees of freedom reduced to 1, causing the mechanism to fail to follow the required path and errors to occur. In the final step of the project, this error was calculated to reach a conclusion.