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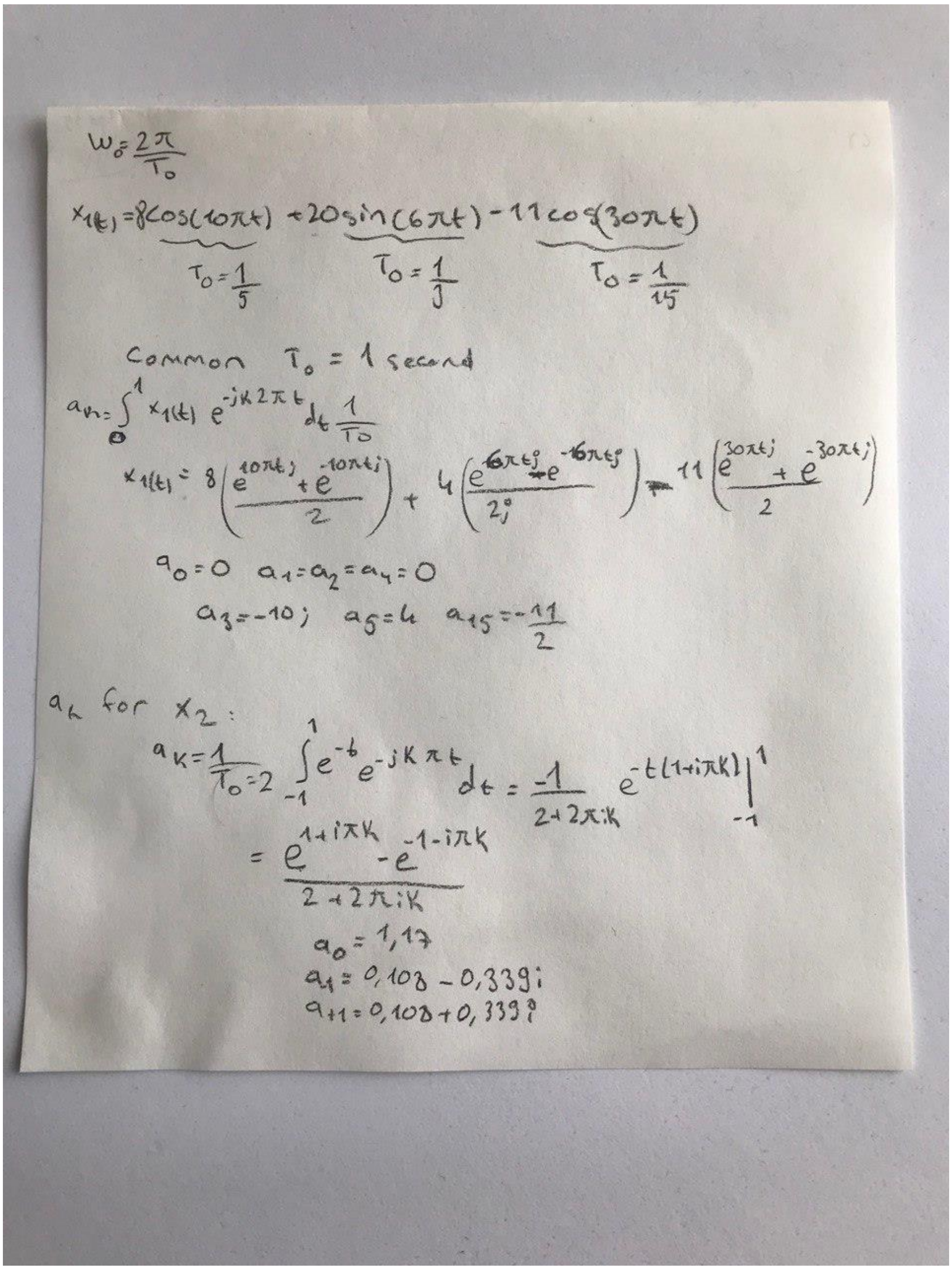
EEE-321 Lab 4

Part 1:

Part 1.1:

FSAnalysis function was written as requested. Nested loops were used when writing the FSAnalysis function. Nested loop was designed as a loop containing two for loops. While the first loop makes the calculation for each k value, the second loop makes the calculation by converting the integral in the analysis formula into a summation with a sampled version. The source code can be accessed from the Appendices section.

Part 1.2:



$$w_0 = \frac{2\pi}{T_0}$$

$$x_1(t) = \underbrace{8\cos(10\pi t)}_{T_0 = \frac{1}{5}} + \underbrace{20\sin(6\pi t)}_{T_0 = \frac{1}{3}} - \underbrace{11\cos(30\pi t)}_{T_0 = \frac{1}{15}}$$

Common $T_0 = 1$ second

$$a_k = \int_0^1 x_1(t)\, e^{-jk2\pi t}\, dt\, \frac{1}{T_0}$$

$$x_1(t) = 8\left(\frac{e^{10\pi t j} + e^{-10\pi t j}}{2}\right) + 4\left(\frac{e^{6\pi t j} - e^{-6\pi t j}}{2j}\right) - 11\left(\frac{e^{30\pi t j} + e^{-30\pi t j}}{2}\right)$$

$a_0 = 0 \quad a_1 = a_2 = a_4 = 0$

$a_3 = -10j \quad a_5 = 4 \quad a_{15} = -\frac{11}{2}$

a_k for x_2:

$$a_k = \frac{1}{T_0 = 2} \int_{-1}^{1} e^{-t} e^{-jk\pi t}\, dt = \frac{-1}{2 + 2\pi i k}\, e^{-t(1+i\pi k)} \Big|_{-1}^{1}$$

$$= \frac{e^{1+i\pi k} - e^{-1-i\pi k}}{2 + 2\pi i k}$$

$a_0 = 1,17$

$a_1 = 0,108 - 0,339i$

$a_{-1} = 0,108 + 0,339i$

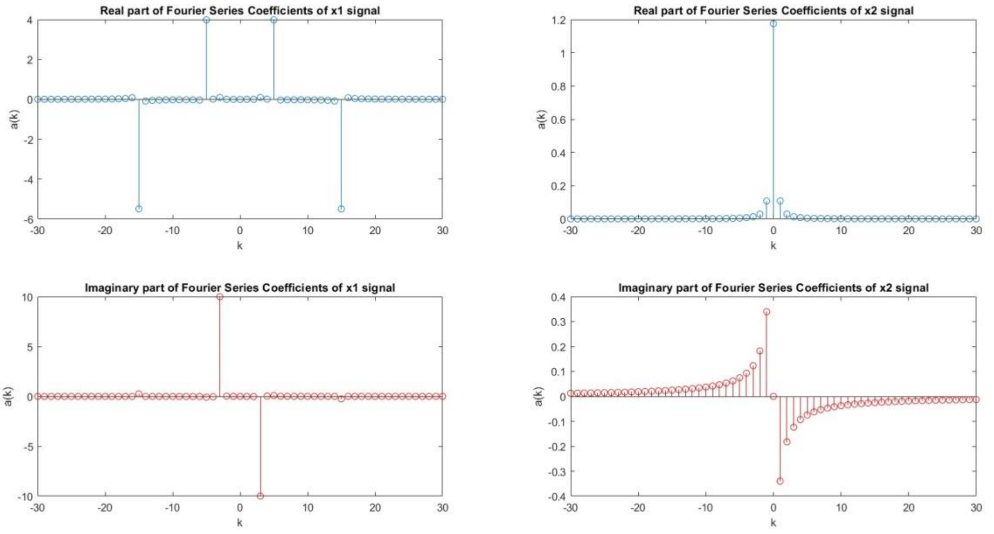


Figure 1 Graphs for Fourier Series Coefficients of x1 and x2

With Parseval's Relation we can see that the energy of a signal is independent of its domain. Thanks to our calculations, we can see that the energies of the signals in the time domain and frequency domain are the same, except for a negligible error.

```
Energy calculation result in time domain =

  292.5090


Energy calculation result in frequency domain =

  292.2164
```

Figure 2 Energies of x1 in different domain

Part 2:

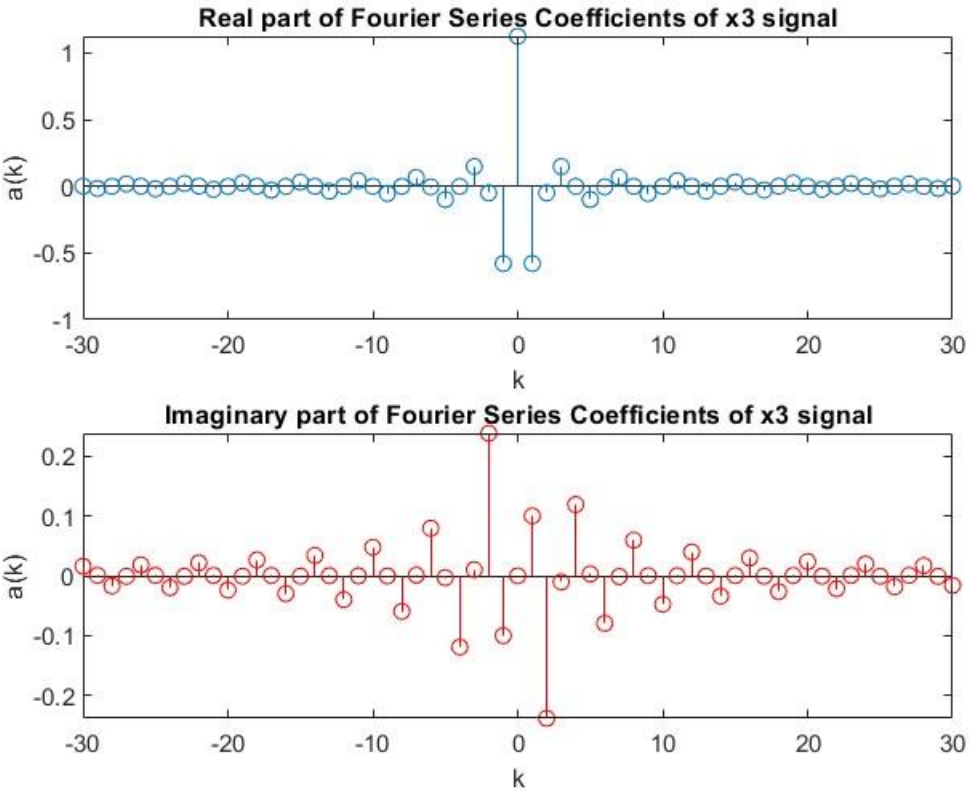


Figure 3 Graphs for Fourier Series Coefficients of x3

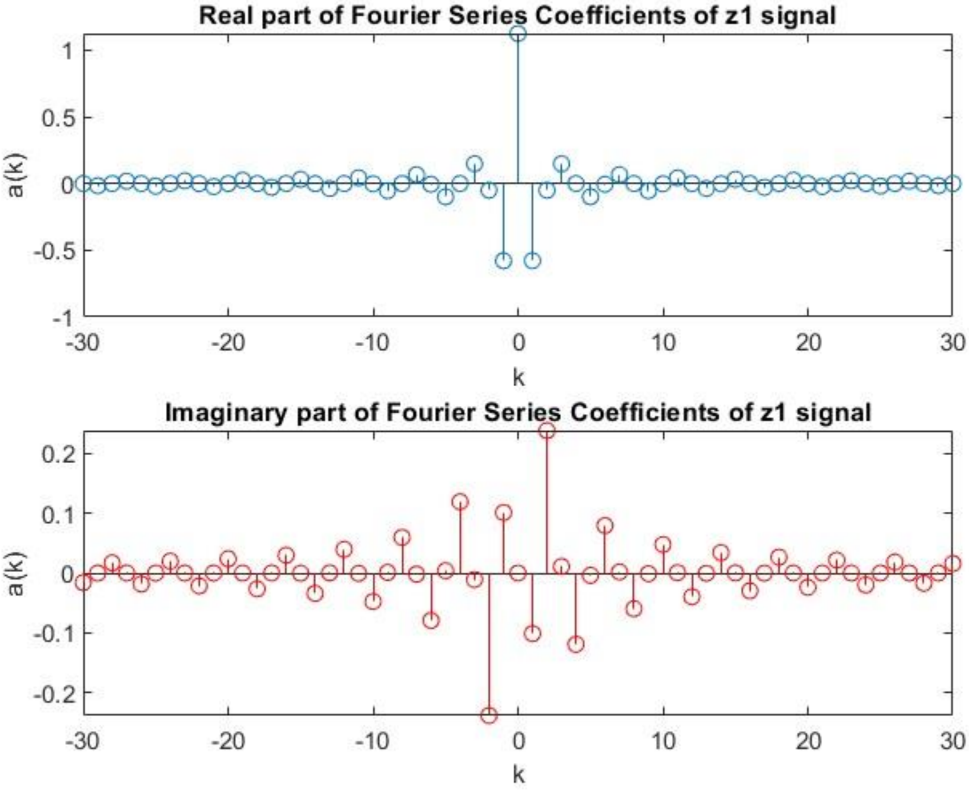


Figure 4 Graphs for Fourier Series Coefficients of z1

Z1 is the flipped version of x3(t). Therefore, as we know from conjugate symmetry property, the resulting Fourier series coefficients will be complex conjugates of the original coefficients.

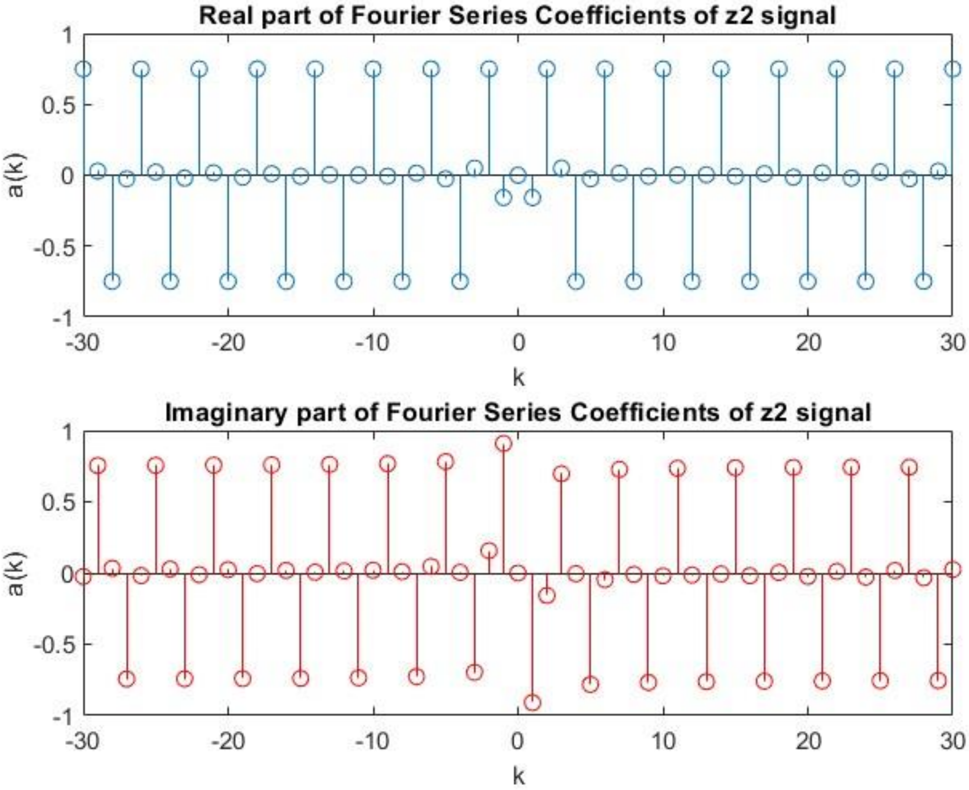


Figure 5 Graphs for Fourier Series Coefficients of z2

As described in lectures, derivatives in the time domain correspond to multiplication by jω in the frequency domain. Therefore, the Fourier series coefficients will be scaled by jω, which introduces phase shifts and changes in the magnitude of coefficients.

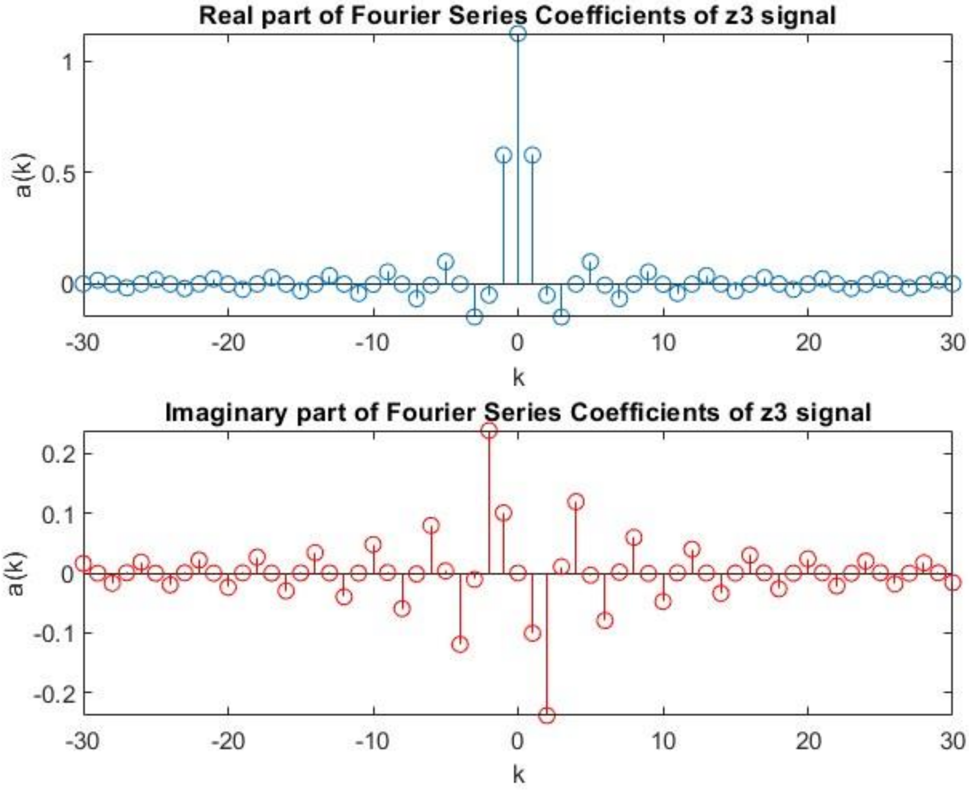


Figure 6 Graphs for Fourier Series Coefficients of z3

As described in time shift property, shifting in the time domain causes a phase shift in the frequency domain. Therefore, the resulting Fourier series coefficients will reflect this phase shift.

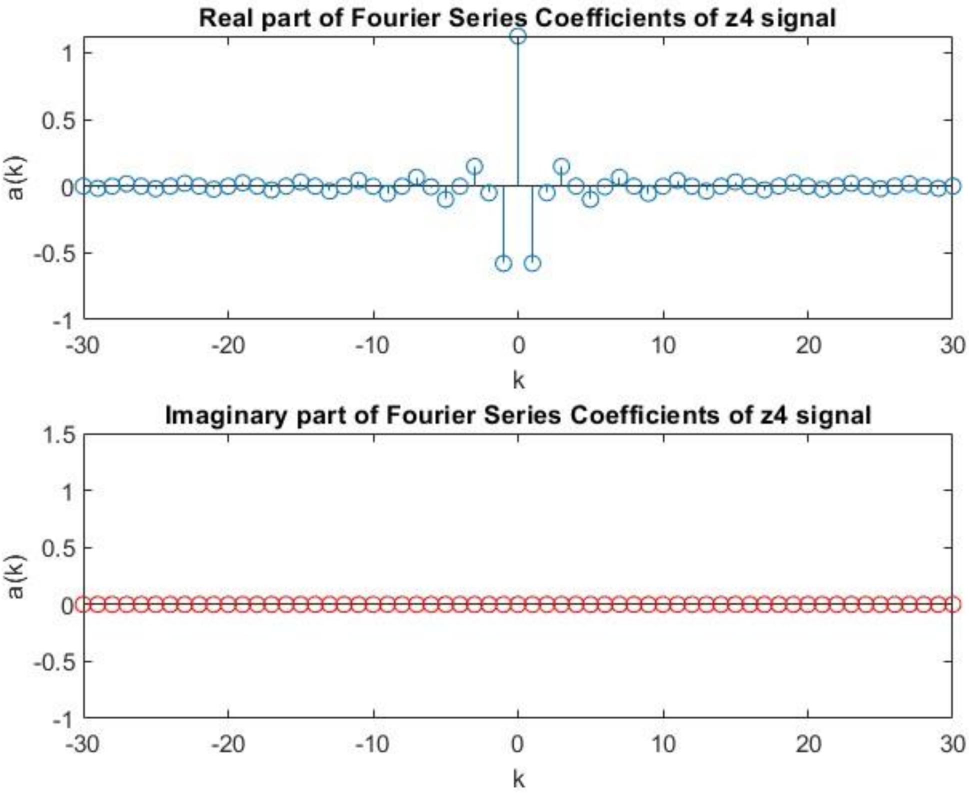


Figure 7 Graphs for Fourier Series Coefficients of z4

In this signal, only the even components of the x3 signal are taken. The result is that, as learned in properties, properties are also expected to be even.

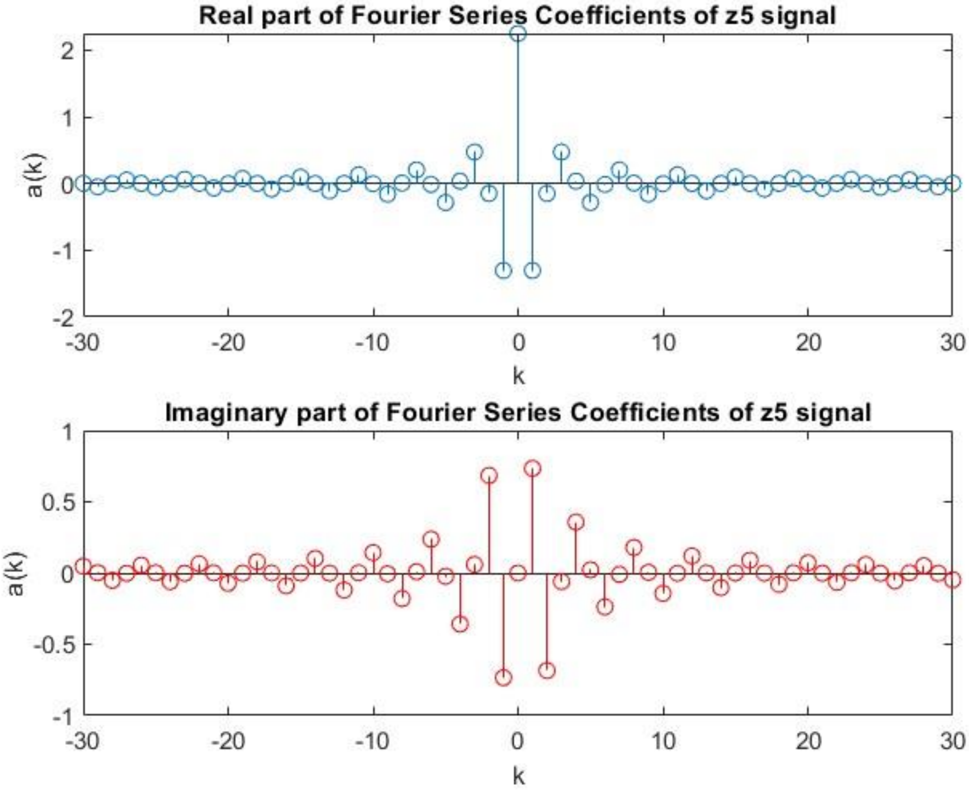


Figure 8 Graphs for Fourier Series Coefficients of z5

As described in multiplication property, squares the signal (x3(t) x3(t)), which causes convolution in the frequency domain. Therefore, squaring the signal in the time domain doubles the Fourier series coefficients comparing to the x3(t)'s Fourier series coefficients.

Part 3:

Part 3.1:

$$\mathcal{F}[y(t)] = b_K$$

$$\mathcal{F}[f(t)] = a_K$$

$$M \frac{\partial^2 y(t)}{\partial t^2} + c \frac{\partial y(t)}{\partial t} + K y(t) = f(t)$$

from linearity

$$-K^2 \omega_0^2 M b_K + c b_K j K \omega_0 + \mathcal{K} b_K = a_K$$

$$b_K = \frac{a_K}{-K^2 \omega_0^2 M + j K \omega_0 c + \mathcal{K}}$$

$$Y(j\omega) = H(j\omega) \cdot X(j\omega)$$

$$H(j\omega_0 K) = \frac{Y(j\omega_0 K)}{X(j\omega_0 K)} = \frac{b_K}{a_K} = \frac{1}{-K^2 \omega_0^2 M + j K \omega_0 c + \mathcal{K}}$$

$$-K^2 \omega_0^2 M + j \omega_0 K c + \mathcal{K} = 0$$

Part 3.2:

from definition:

$$y'[n] = x[n] - x[n-1]$$

$$y''[n] = x[n] - 2x[n-1] + x[n-2]$$

$$M(y[n] - 2[n-1] + y[n-2]) + C(y[n] - y[n-1]) + K\,y[n] = x[n]$$

$$\Rightarrow \quad y[n] = \frac{x[n] - M\,y[n-2] + (2M+C)\,y[n-1]}{M+C+K}$$

In order to find the response of an LTI system in the time domain according to any x(t) signal, we need to convolve with h(t). However, according to the properties shown and proven in the course, it is sufficient to multiply H(jw) and X(jw) to find the response of any system in the frequency domain. So Y(jw) = H(jw) X(jw). Thanks to these features, we can write the system with linear differential operators. These features were used effectively in Part 3.

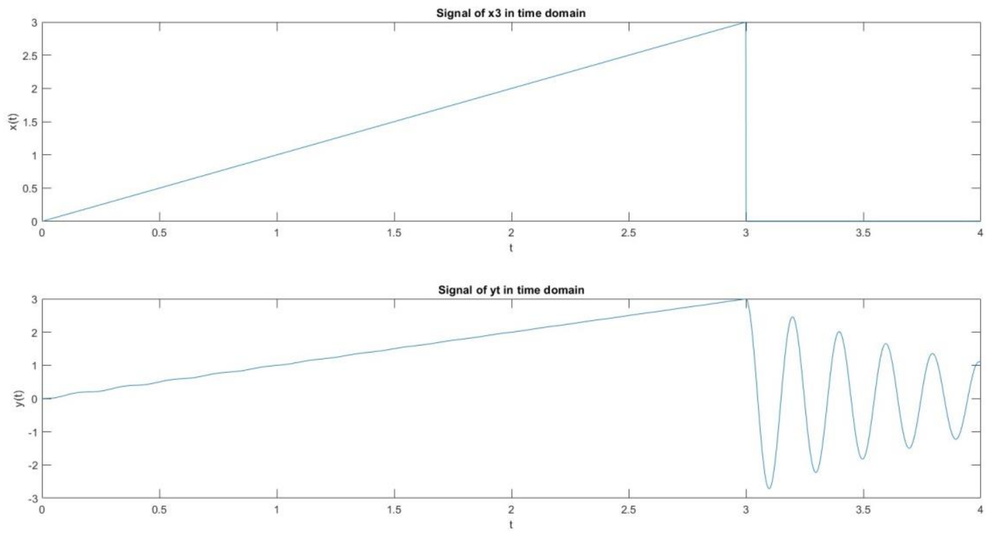


Figure 9 Graphs for x(t) and y(t)

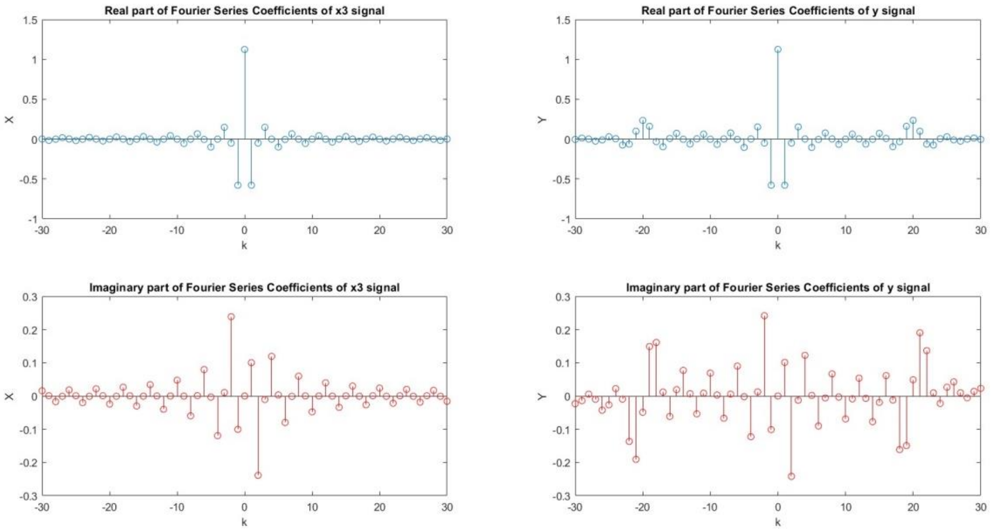


Figure 10 Graphs of Fourier Series Coefficients of X and Y

Appendices:

Part 1.1:

```
function [fsCoeffs] = FSAnalysis(x, k)
    akbeforedividingt = zeros(2*k + 1, 1);
    N = length(x);
    for i = -k:k %we are finding the coefficients for each k
        for d = 0: N-1  %we are taking the integral
            akbeforedividingt(i+k+1) = akbeforedividingt(i+k+1)+((x(d+1))*exp((-
1i)*pi*2*i*d/N));
        end
    end
    fsCoeffs = akbeforedividingt /N; %it comes from analysis equation. we are
dividing by T0
end
```

Part 1.2:

```
k=30;
Ts = 0.001
timevectorforfirst = 0:Ts:1
timevectorforsecond = -1:Ts:1
first =  8 * cos(10 * pi * timevectorforfirst) + 20 * sin(6 * pi *
timevectorforfirst) - 11 * cos(30 * pi * timevectorforfirst);
second = exp(-timevectorforsecond)
firstresponse = FSAnalysis(first,k);
secondresponse = FSAnalysis(second,k);


figure;
subplot(2,2,1)
stem( -k:k,real(firstresponse) );
```

```
xlabel('k');
ylabel('a(k)');
title('Real part of Fourier Series Coefficients of x1 signal');

subplot(2,2,2)
stem( -k:k,real(secondresponse) );
xlabel('k');
ylabel('a(k)');
title('Real part of Fourier Series Coefficients of x2 signal');

subplot(2,2,3)
stem( -k:k,imag(firstresponse), 'r' );
xlabel('k');
ylabel('a(k)');
title('Imaginary part of Fourier Series Coefficients of x1 signal');

subplot(2,2,4)
stem( -k:k,imag(secondresponse), 'r' );
xlabel('k');
ylabel('a(k)');
title('Imaginary part of Fourier Series Coefficients of x2 signal');

parintdmforfirst = Ts * sum(abs(first).^2);
parinfdmforfirst = sum(abs(firstresponse).^2);
display(parintdmforfirst, 'Energy calculation result in time domain');
display(parinfdmforfirst, 'Energy calculation result in frequency domain');
```

Part 2:

```
Ts = 0.001;
k = 30
t = 0:Ts:4;
stepfunc = ones(size(t));
stepfuncminus3=zeros(size(t));
stepfuncminus3(3001:end)=1;

thirdfunc = (t.* stepfunc) - ((t-3) .* stepfuncminus3) - (3*stepfuncminus3)

responseforthird = FSAnalysis(thirdfunc, k);

minusofthirdfunc = thirdfunc(end:-1:1);
mel1 = FSAnalysis(minusofthirdfunc, k);

derofthirdfunc = diff(thirdfunc)
derofthirdfunc = derofthirdfunc / Ts
mel2 = FSAnalysis(derofthirdfunc, k);

sh2ofthirdfunc = zeros(size(thirdfunc));
sh2ofthirdfunc(1:2001) = thirdfunc(2001:end);
sh2ofthirdfunc(2002:end) = thirdfunc(1:2000);
mel3 = FSAnalysis(sh2ofthirdfunc,k);

ciftpartofthirdfunc = (thirdfunc + minusofthirdfunc)/2;
mel4 = FSAnalysis(ciftpartofthirdfunc,k);

sarofthirdfunc = thirdfunc .^ 2;
mel5 = FSAnalysis(sarofthirdfunc,k);
```

```
figure;
subplot(2,1,1)
stem( -k:k,real(responseforthird) );
xlabel('k');
ylabel('a(k)');
title('Real part of Fourier Series Coefficients of x3 signal');

subplot(2,1,2)
stem( -k:k,imag(responseforthird), 'r' );
xlabel('k');
ylabel('a(k)');
title('Imaginary part of Fourier Series Coefficients of x3 signal');

figure;
subplot(2,1,1)
stem( -k:k,real(mel1) );
xlabel('k');
ylabel('a(k)');
title('Real part of Fourier Series Coefficients of z1 signal');

subplot(2,1,2)
stem( -k:k,imag(mel1), 'r' );
xlabel('k');
ylabel('a(k)');
title('Imaginary part of Fourier Series Coefficients of z1 signal');

figure;
subplot(2,1,1)
stem( -k:k,real(mel2) );
xlabel('k');
ylabel('a(k)');
title('Real part of Fourier Series Coefficients of z2 signal');

subplot(2,1,2)
stem( -k:k,imag(mel2), 'r' );
xlabel('k');
ylabel('a(k)');
title('Imaginary part of Fourier Series Coefficients of z2 signal');

figure;
subplot(2,1,1)
stem( -k:k,real(mel3) );
xlabel('k');
ylabel('a(k)');
title('Real part of Fourier Series Coefficients of z3 signal');

subplot(2,1,2)
stem( -k:k,imag(mel3), 'r' );
xlabel('k');
ylabel('a(k)');
title('Imaginary part of Fourier Series Coefficients of z3 signal');

figure;

subplot(2,1,1)
ylim([-1, 1.5]);
stem( -k:k,real(mel4) );
xlabel('k');
```

```
ylabel('a(k)');
title('Real part of Fourier Series Coefficients of z4 signal');

subplot(2,1,2)
stem( -k:k,imag(mel4), 'r' );
ylim([-1, 1.5]);
xlabel('k');
ylabel('a(k)');
title('Imaginary part of Fourier Series Coefficients of z4 signal');

figure;
subplot(2,1,1)
stem( -k:k,real(mel5) );
xlabel('k');
ylabel('a(k)');
title('Real part of Fourier Series Coefficients of z5 signal');

subplot(2,1,2)
stem( -k:k,imag(mel5), 'r' );
xlabel('k');
ylabel('a(k)');
title('Imaginary part of Fourier Series Coefficients of z5 signal');
```

Part 3.2:

```
Ts = 0.001;
k = 30;
t = 0:Ts:4;
stepfunc = ones(size(t));
stepfuncminus3=zeros(size(t));
stepfuncminus3(3001:end)=1;
thirdfunc = (t.* stepfunc) - ((t-3) .* stepfuncminus3) - (3*stepfuncminus3);
c_katsayi = 0.1;
degisik = 0.1;
m_katsayi = 100;
yintimedomain = zeros(1,length(thirdfunc));

for i = 3:length(thirdfunc)
    yintimedomain(i)= (thirdfunc(i) + (-m_katsayi)*yintimedomain(i-2) + (c_katsayi
+ 2*m_katsayi)*yintimedomain(i-1))/(m_katsayi+c_katsayi+degisik);
end
yintimedomain = yintimedomain ./ 10;
yinfreqdomain = FSAnalysis(yintimedomain,k);
thirdinfreqdomain = FSAnalysis(thirdfunc,k);

figure;
subplot(2,1,1);
plot(t,thirdfunc);
ylabel('x(t)');
xlabel('t');
title('Signal of x3 in time domain');
```

```
subplot(2,1,2);
plot(t,yintimedomain);
ylabel('y(t)');
xlabel('t');
title('Signal of yt in time domain');

figure;
subplot(2,2,1)
stem( -k:k,real(thirdinfreqdomain) );
xlabel('k');
ylabel('X');
title('Real part of Fourier Series Coefficients of x3 signal');

subplot(2,2,2)
stem( -k:k,real(yinfreqdomain) );
xlabel('k');
ylabel('Y');
title('Real part of Fourier Series Coefficients of y signal');

subplot(2,2,3)
stem( -k:k,imag(thirdinfreqdomain), 'r' );
xlabel('k');
ylabel('X');
title('Imaginary part of Fourier Series Coefficients of x3 signal');

subplot(2,2,4)
stem( -k:k,imag(yinfreqdomain), 'r' );
xlabel('k');
ylabel('Y');
title('Imaginary part of Fourier Series Coefficients of y signal');
```