USING THE FFT-DDI METHOD TO MEASURE DISPLACEMENTS WITH PIEZOELECTRIC, RESISTIVE AND ICP ACCELEROMETERS

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ABSTRACT

A new frequency domain filter called FFT-DDI (Direct Digital Integration) has been recently introduced to minimize the signal shift observed during the double integration of accelerometer signals to determine displacements. This new filter significantly improves the time aliasing problems of the digital FIR filtering process, virtually solving the low-frequency displacements measurement problem. But the quality of the results obtained in practice depends on the type of accelerometer used, as shown by representative displacement measurements made using the FFT-DDI method to double integrate signals generated by piezoelectric, resistive and ICP accelerometers.

NOMENCLATURE

- t – time in seconds
- T – sample time
- $a(t)$ – acceleration signal in m/s^2
- $v(t)$ – velocity signal in m/s
- $x(t)$ – displacement signal in mm
- f – frequency of the signal in Hz
- f_s -- sample rate in Hz
- N – number of samples

1 INTRODUCTION

To certify if any structure is safe under operational conditions it is necessary, among other tasks, to identify its damaged or cracked sites, which can be localized by non-destructive techniques (NDT) such as ultrasonics, radiography, dye penetrant or magnetic particles. However, these NDT must be locally applied and, therefore, they are expensive and time consuming when used on huge structures.

On the other hand, the dynamic behavior of the structure can be used to globally verify its integrity which, at least from a theoretical point of view, can be related to the damage-induced variations on its natural frequencies, which can be easily and conveniently monitored by accelerometers. This global approach can be much more economical than the traditional NDT. However, in practice the results of this technique are still quite unreliable, mainly due to its inherent measurement errors [1]. Since the acceleration signal is very sensitive to impulses and similar noise sources, these errors can be decreased if displacements instead of acceleration are measured to monitor the dynamic structural behavior.

Moreover, important failure mechanisms, such as fatigue, fracture and plastic collapse are related to the peaks and valleys of the service loading, which frequently is impossible to be directly measured in practice. Therefore, they must be inferred from displacement and/or strain measurements, to which they are proportional in any elastic structure [2]. Since strains are a local parameter (hence heavily influenced by local structural details such as holes, notches or any other stress concentrator), displacements (which are much less sensitive to those details) reflect in a much better way the global behavior of the structure.

E.g., in [3] a technique has been proposed to detect and estimate the location and the size of cracks in concrete bridges from acceleration, velocity and displacement measurements. In [4], acceleration and displacement data were used to calculate stresses and strains in soil. In [5], the behavior of a bridge during seismic disturbances was identified from displacement measurements. In [6] the acceleration, velocity and the displacement of the soil were measured, since the peaks of these quantities were necessary for the analysis.

Therefore, displacement is a quantity that often needs to be measured in field applications, particularly when dealing with huge structures. There are many kinds of non inertial displacement sensors available [7] [8]. Among them it can be mentioned the capacitive, resistive, LVDT and eddy current.

These sensors need a fixed reference close to the point of measurement, since they are direct measurement devices. This reference may be very difficult to find in huge structures, such as stadiums and bridges. Most of the times the construction of auxiliary structures like columns or rigid beams supported directly on the soil are necessary, making the measurement expensive and time consuming.

Therefore, many others displacement measurement techniques have been developed, as optical and laser based transducers [10]. The Laser Vibrometer is a technique is in full expansion and presents excellent results in the measurement of displacements in huge structures, as pencil bridges. This method of measuring displacements in huge structures is used in [11]. In [12] it is proposed a microwave interferometer as a non contact displacement measurement to be used on large structures.

These transducers need a fixed reference too, but they have the advantage that the distance between this reference and the point of measurement can be much bigger, avoiding the columns or reference beams. But this reference can be very difficult to be achieved in many situations, such as bridges over rivers and bays.

If a seismic transducer is used, the fixed reference is not necessary. However, it is a well known fact that velocity and displacement seismic transducer can't be produced associating low frequency measurements and small size. It doesn't happen with the accelerometer, since such commercial transducers have good dynamic response (can measure a large frequency range), low price and good reliability, making the accelerometer a natural choice to measure structural vibrations [7].

The measurement of displacements using accelerometers is not an easy task. A well known method to obtain displacements from accelerometer measured data uses an analog double integrator. This method is not reliable, since these integrators have no linear phase response. Experience has shown that the analog double integration of acceleration signals introduce errors that in many cases can invalidate the analysis of the determined experimental results [13].

In order to provide one method to be applied to the double integration of commercial accelerometers measured data, Ribeiro [14] developed one integration procedure based on the use of a FIR filter in frequency domain. This method showed to be not appropriate for the measurement of transient signals, due to the occurrence of errors in the beginning of the integrated time signal, [15]. It was shown in [15] that this error was caused by time aliasing and a new digital integration technique called FFT-DDI (Fast Fourier Transform Direct Digital Integration) was introduced to overcome this problem.

The purpose of this paper is to show the application of the method developed in [15] to piezoelectric, resistive and ICP accelerometers collected results. Several representative displacement time histories were used in order to demonstrate the efficiency of the FFT-DDI in correctly double integrating the experimental data.

2 THE DOUBLE DIGITAL INTEGRATION PROBLEM

In principle, a digital integration scheme can be quite easily implemented by using the sampled signal measured by a data acquisition system as input to a numerical integration method running in some type of processor. E.g., the popular software LabView uses the Simpson formula to obtain the integral *Y(t)* of the function *x(t)*:

$$Y(i) = Y(i-1) + \frac{x(i-1) + 4 \cdot x(i) + x(i+1)}{6} \cdot \Delta t \qquad (1)$$

The digital integration main advantage is to be free from the non-linear phase response intrinsic to the analog integrator amplifiers, which in many practical cases [15] can distort the double integrated (complex) acceleration signals up to the point that the displacements simply can't be measured.

However, the digital integration has its own problems, which go beyond the obvious discretization frequency. E.g., if the structure where the displacements are going to be measured is already vibrating, one can not use the accelerometer signal to obtain the initial conditions (the displacement and the velocity at the start of the measurement) required by the integration formulas. Therefore, it is common to ***assume*** that the initial conditions are zero, a non-recommended practice which, of course, introduces a potentially severe error in the measurement.

Another problem is due to the zero shift and to the drift present in any real transducer output. Since the integration has a cumulative characteristic, any zero shift or drift will cause a time-increasing error. And this problem is magnified when the signal is double integrated.

Therefore, to safely and reliably use numerical integration methods in practice, it is necessary to remove these errors sources, which in fact can spoil measurements of dynamic displacements using accelerometers. As the main objective of this method is to measure dynamic displacements, and as these errors sources are basically caused by the signal low frequency components, they can be removed by a high-pass filter.

As the FIR filter has a linear phase response, it is generally considered [16] the best method to numerically filter a signal without distorting it. A double integration scheme can be described by the following sequence: first the acceleration signal is filtered using a high-pass FIR and then integrated, the resulting signal is filtered and integrated once more, and finally the double integrated signal is filtered again. However, the practical results obtained by this method still have some important problems.

3 THE FIR-FILTERED DIGITAL DOUBLE INTEGRATOR

Trying to solve the problem of measuring displacements from accelerometer signals, a high-pass digital FIR filter with a cut-off frequency of 0.9Hz was proposed by [14]. In this work a sample rate of 512Hz, which required 8192 data points, was chosen (because it was particularly suitable for the inertial measurement of dynamic displacements of huge structures such as stadiums or bridges, which have important spectral components in the range from 1 to 10Hz).

The error associated to the sample rate is function of the frequency of the displacement measured, the higher the frequency, the higher the error [17]. Figure 1 shows the proposed FIR filter coefficients (which has a frequency response equivalent to the classical analog double integrator of the B&K Charge Amplifier model 2635).

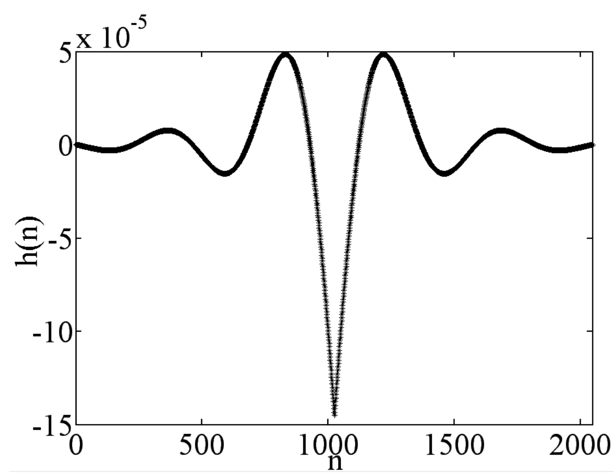

Figure 1: Impulse response of the FIR integrator.

This method of integration is called FIR-DDI (Finite Impulse Response Direct Digital Integration), and it can integrate an acceleration signal without introducing the distortions caused by the non-linear phase response of the analog integrators. However, this FIR-DDI introduces a delay of 2 seconds in the signal, which causes potentially severe errors in the transient part of the measured displacement signals, as illustrated in the following examples.

Figure 2 shows the simulation of the double integration of a damped sinusoidal acceleration signal using this FIR-DDI method. Figure 3 shows the same simulation, but with the delay of 2 seconds removed to compare the results.

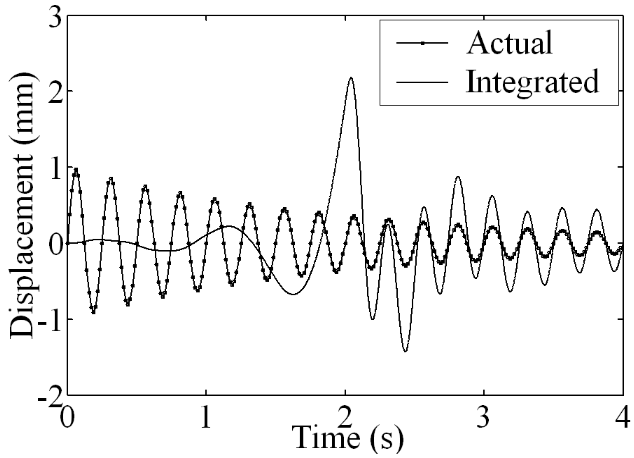

Figure 2: Application of the FIR-DDI to a damped sinusoid.

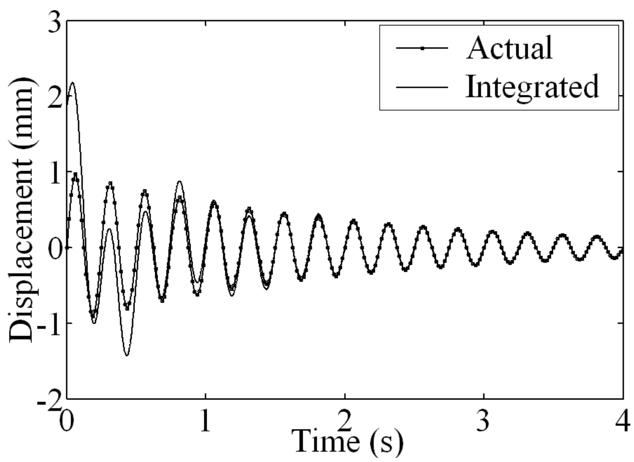

Figure 3: Application of the FIR-DDI to a damped sinusoid, removing the delay.

It can be shown that, in addition to this delay, this FIR filter has a transient response that introduces an error in the beginning of the measurements [18][19][20]. Figures 4 and 5 show some representative displacement measurements. They have been made by double integrating acceleration signals using this FIR-DDI method. The error due to the transient response can be verified.

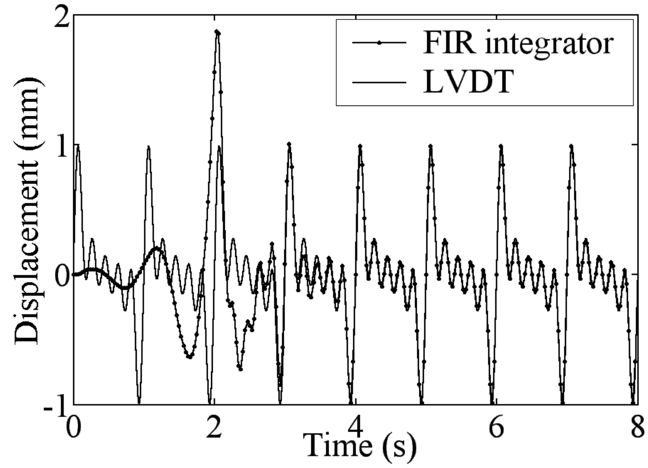

Figure 4: Application of the FIR-DDI integrator to an acceleration signal compared to an LVDT signal.

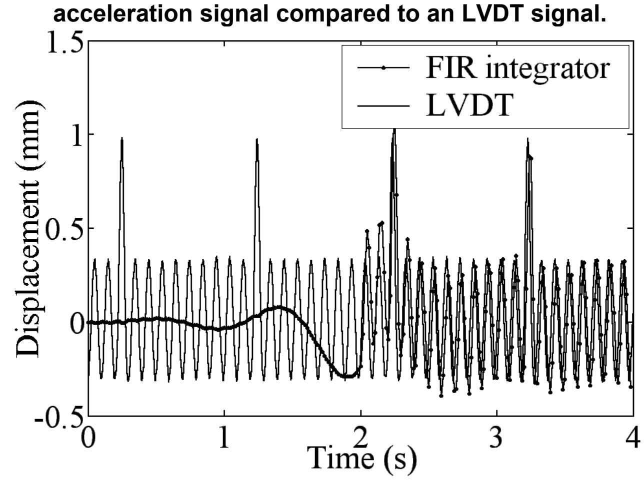

Figure 5: Application of the FIR-DDI to an impulsive signal compared to an LVDT signal.

These results show that this method, which is often cited as the solution for the double integration of acceleration signal in order to obtain displacements [18], is not suitable for low frequency displacement measurements.

4 THE FFT-DDI METHOD

To solve the FIR-DDI transient problem, the new FFT-DDI method has been proposed [22] to digitally double integrate low-frequency acceleration signals. Its idea is to filter in the frequency domain, and can be briefly described using the following example. A sinusoidal acceleration of 4.125Hz contaminated by a zero shift is sampled at a rate of 512Hz, as shown in Equation 2.

$$a_M(t) = -(8.25\cdot\pi)^2 \cdot [sin(8.25\cdot\pi\cdot t) + 0.1] \quad (2)$$

It is desired to remove only the shift from the signal in order to obtain the desired acceleration shown in Equation 3.

$$a_D(t) = -(8.25\cdot\pi)^2 \cdot sin(8.25\cdot\pi\cdot t) \quad (3)$$

If the measured acceleration has 2048 points sampled at 512Hz, it will not have an integer number of periods sampled. Therefore, it is not possible to obtain this desired acceleration by subtracting the measured acceleration and its mean value.

Figure 6 shows the real part of the FFT of $a_M(t)$ (measured acceleration) and $a_D(t)$ (desired acceleration). It can be verified that the difference between them is in the component at the frequency of 0Hz.

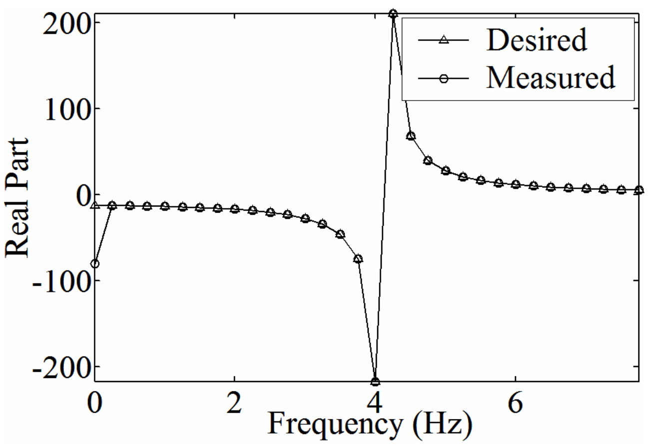

Figure 6: Real part of the FFT of the desired $a_D(t)$ and the measured $a_M(t)$ accelerations.

Figure 7 shows the imaginary part of the FFT of the cited accelerations. Since it is a real signal, there is no difference between them, and their value at 0Hz is zero. The graphs of the real part of the FFT of the desired acceleration have one characteristic, the value at frequencies below 0.75Hz are almost equal. Since the problem is to calculate the shift that has to be removed from the signal, this characteristic can be used as an estimator.

The filtering in the frequency domain can then be made using the real part of the FFT of the measured signal. The value of the real part at 0Hz, 0.25Hz and 0.5 can be made equal to the value at 0.75Hz. It is evident that this is not true, but since the required signals are above 1Hz, this is a good estimator of the value at low frequencies.

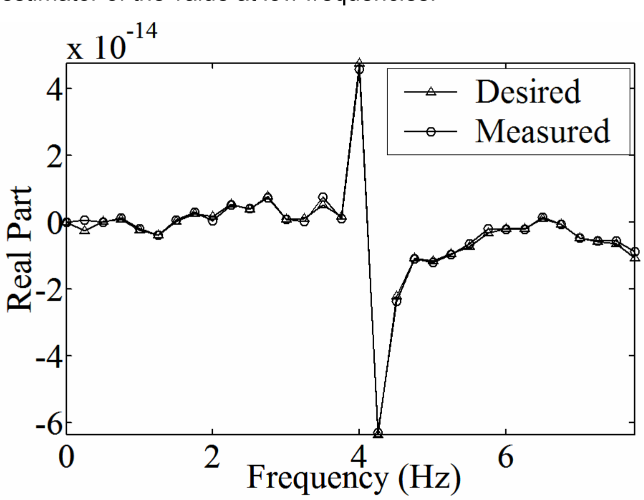

Figure 7: Imaginary part of the FFT of the desired $a_D(t)$, and of the measured $a_M(t)$ accelerations.

The filtered acceleration $a_f(t)$ obtained by this method is then integrated to obtain the velocity. This calculated velocity will have a shift due to the unknown initial condition that can be removed using the same method. The filtered velocity is then integrated again to obtain the displacement, that is finally filtered once more to remove its zero shift.

This method has been implemented in virtual instruments (VI) written in Labview. Figure 8 shows the **FFT filter.vi** module, where the FFT filtering is made. Figure 9 shows the VI called **Double int.vi** where the digital double integration is applied to the filtered signal.

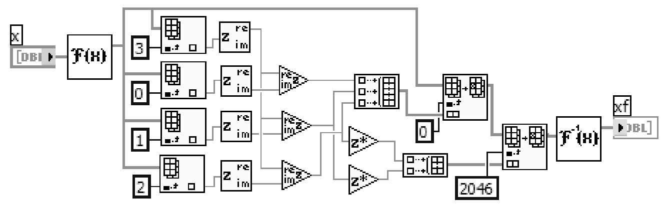

Figure 8: The VI called FFT filter.vi used to filter the signal removing the drift present in the measurement.

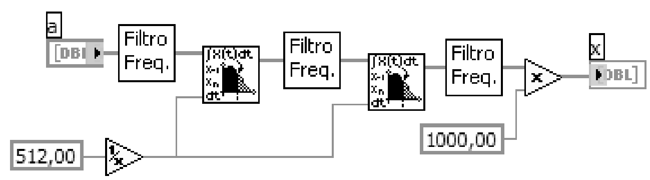

Figure 9: The VI called Double int.vi that does the digital double integration of the signal.

Since this method filters in the frequency domain using the signal FFT, it has been called the FFT-DDI (Fast Fourier Transform Direct Digital Integration) method. In [22], an equivalent method is applied, which is based in the curve-fit of the Power Spectrum of the displacement. The method is equivalent because it is concerned in estimating the values of the low frequency components of the displacement, since they are an important source of error.

5 EXPERIMENTAL RESULTS

The FFT-DDI method was used to measure displacements generated by an Instron servo-hydraulic shaker equipped with an LVDT, which was used as the reference transducer. An accelerometer and its signal conditioner and a computer with a DAQ-1200 running under LabView, both from National Instruments, complemented the experimental setup. The FFT-DDI algorithm in Labview was used to obtain displacements.

The acceleration was measured using three different types of accelerometer: a piezoelectric model 4370 with a charge amplifier model 2635 both from B&K, an ICP model 752-100 with the amplifier model 2792B from Endevco, and a resistive model AS-50HB with the amplifier model DPM-6H from Kyowa. The ICP accelerometer had to be used in conjunction with a low-pass analog filter, to remove the high DC component present in its output.

Figures 10 to 21 compare the results obtained with the three different accelerometers. In Figures 10, 11 and 12, 9 cycles of a quasi-sinusoidal signal were followed by an overload cycle, simulating an impulsive perturbation. In figures 13, 14 and 15, the displacement signal was composed by 1Hz, 3Hz and 9Hz components. In Figures 16, 17 and 18 the spectral components of the displacement signal were 1Hz, 2Hz, 3Hz, 4Hz and 5Hz. These four signals can be considered as representative of the type of displacement signals obtained in the field when monitoring huge structures, which were the main motivation for this study.

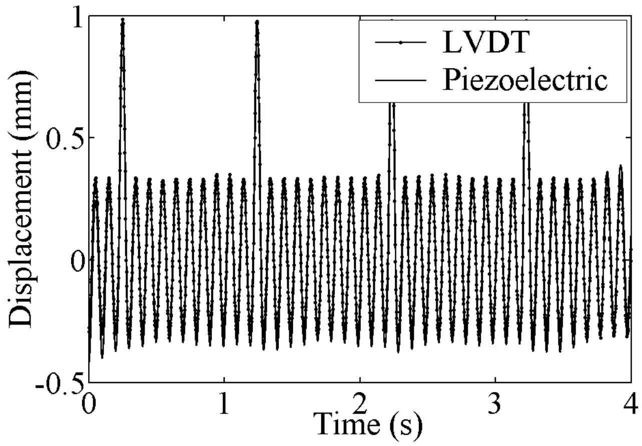

Figure 10: "Impulsive" signal measured using the piezoelectric accelerometer.

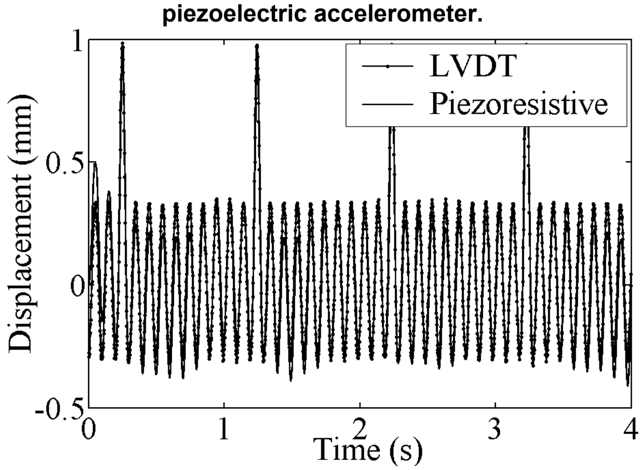

Figure 11: "Impulsive" signal measured using the resistive accelerometer.

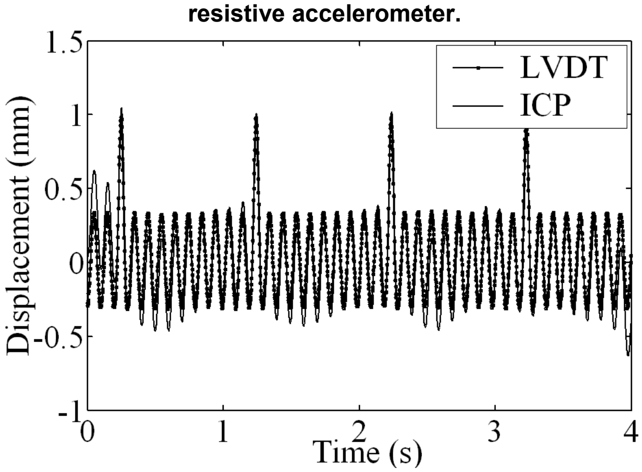

Figure 12: "Impulsive" signal measured using the ICP accelerometer.

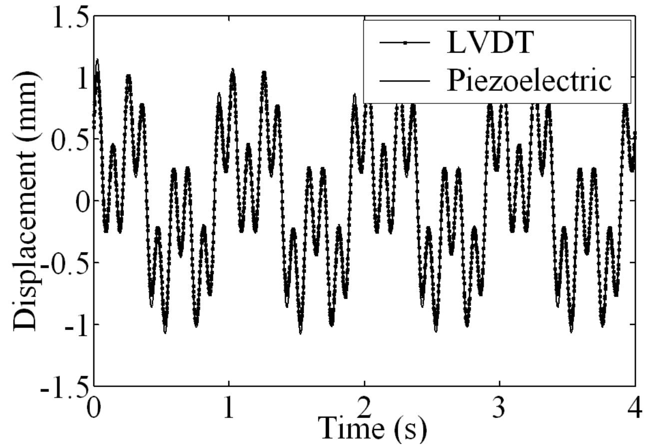

Figure 13: Signal with frequencies of 1Hz, 3Hz and 9Hz, measured using the piezoelectric accelerometer.

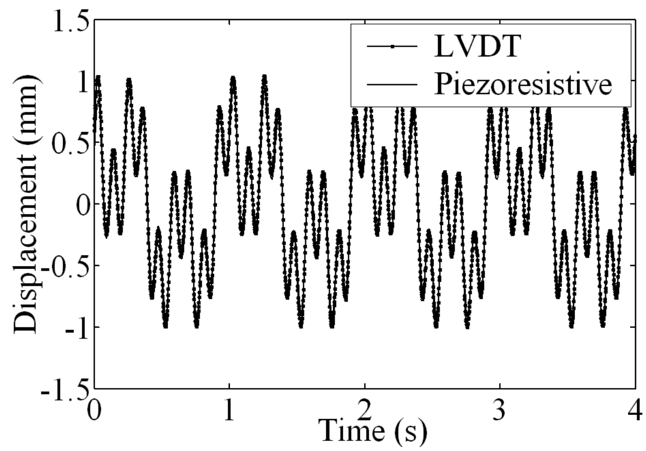

Figure 14: Signal with frequencies of 1Hz, 3Hz and 9Hz, measured using the resistive accelerometer.

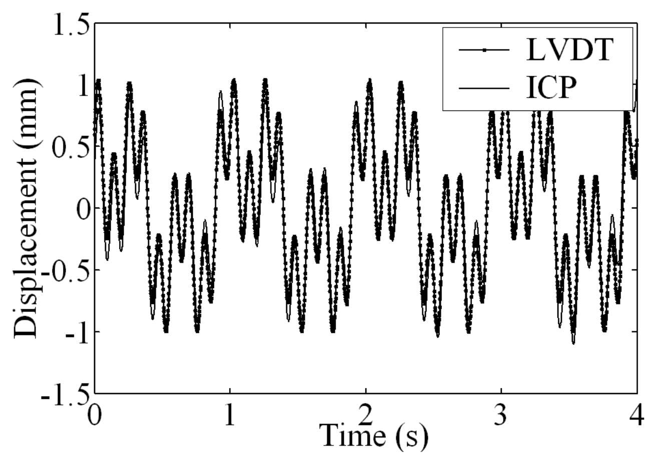

Figure 15: Signal with frequencies of 1Hz, 3Hz and 9Hz, measured using the ICP accelerometer.

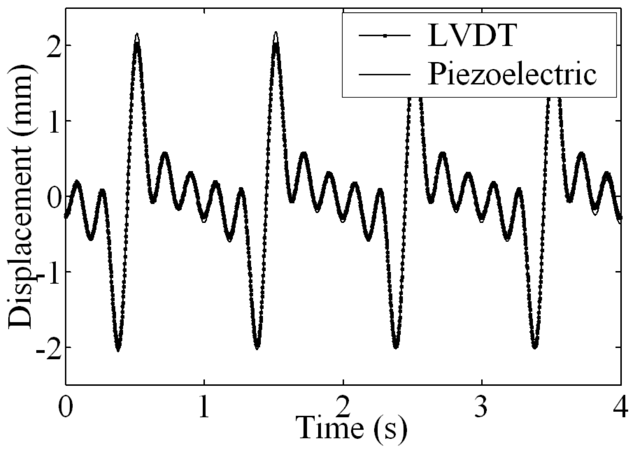

Figure 16: Signal with frequencies of 1Hz, 2Hz, 3Hz, 4Hz and 5Hz, measured using the piezoelectric accelerometer.

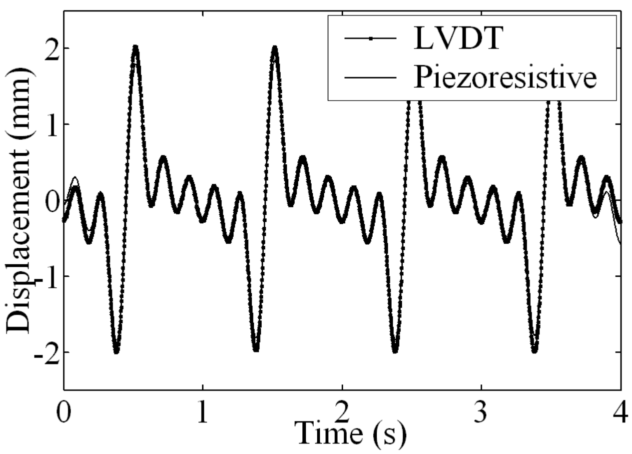

Figure 17: Signal with frequencies of 1Hz, 2Hz, 3Hz, 4Hz and 5Hz, measured using the resistive accelerometer.

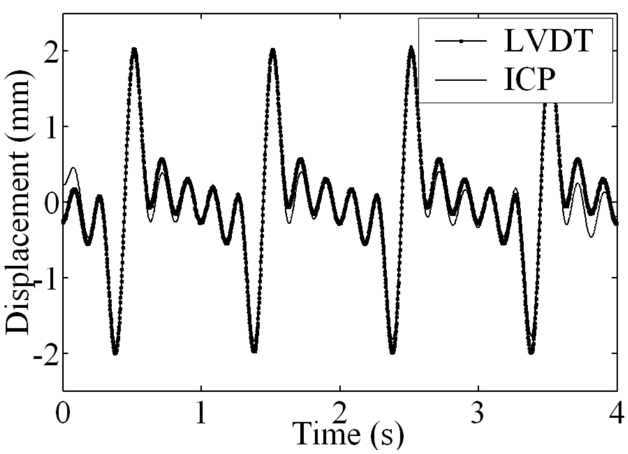

Figure 18: Signal with frequencies of 1Hz, 2Hz, 3Hz, 4Hz and 5Hz, measured using the ICP accelerometer.

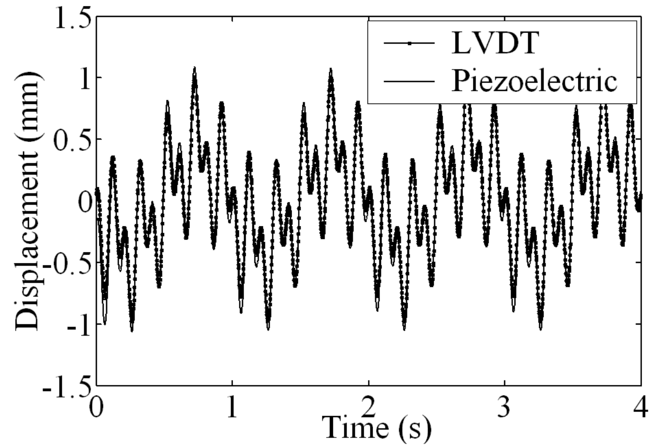

Figure 19: Signal with frequencies of 1Hz, 5Hz, 10Hz measured using the piezoelectric accelerometer.

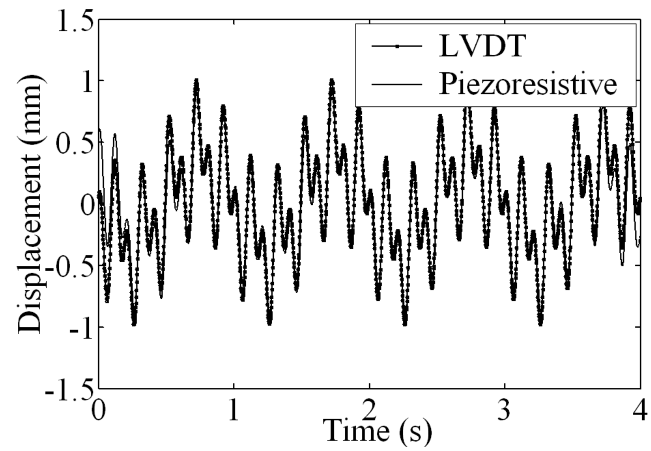

Figure 20: Signal with frequencies of 1Hz, 5Hz, 10Hz measured using the resistive accelerometer.

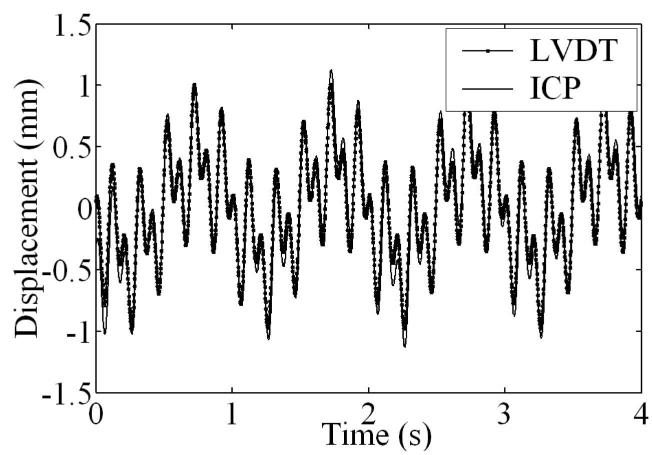

Figure 21: Signal with frequencies of 1Hz, 5Hz, 10Hz using the ICP accelerometer.

6 CONCLUSIONS

The FFT-DDI method has shown excellent results for the measurement of displacement signals with a piezoelectric accelerometer and a charge amplifier system. No transient or any other disturbances were noted in the signals, which were virtually identical to the LVDT output used as reference for these tests. Indeed this method can be recommended as an economical solution for the inertial measurements of low frequency vibrations in huge structures.

However, as shown in the Figures above, the results were not so good when resistive and ICP accelerometers were used in the measurements. The reason for this behavior is still not clear. But, despite this problem, the displacements measured with these transducers and the FFT-DDI method were much better than those obtained by the FIR-DDI or the analog double integration, confirming the recommendation stated above.

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