HACETTEPE UNIVERSITY

Department of Nuclear Engineering

NEM393 - ENGINEERING PROJECT II
Project Assignment III

Student Name : Enes Mert Ulu

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1. INTRODUCTION

In a nuclear reactor, the coolant is used to remove the waste heat from the cycle. In a typical PWR, there are three cycles which are primary, secondary, and tertiary cycles. The primary cycle is the cycle that mostly interacts with the radioactivity in the reactor core.

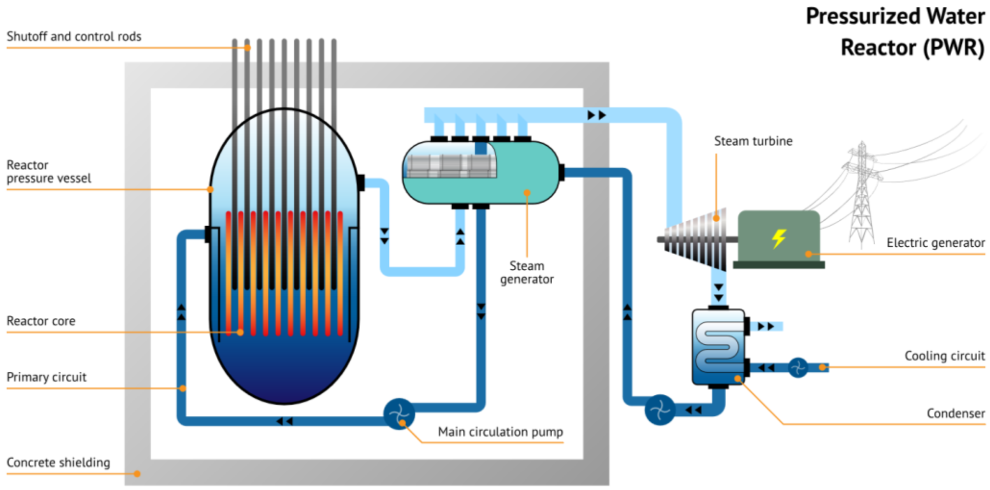


Figure 1 - PWR Nuclear Power Plant's Schematic Diagram [1]

However, if a crack occurs in the fuel rods, from this crack nuclear materials can leak to the coolant. To avoid an environmental disaster, NRC's limit of ^{131}I concentration in the water is $4x10^4\,\frac{Bq}{g}$ for PWRs.

In this study, the PWR's thermal power is $3411\ MW_{th}$ with the operation pressure of $15.5\ MPa$, and the inlet and outlet temperatures of 286℃ and 326℃, respectively. In our fuel rods, some cracks occur about 2%. From these cracks ^{131}I leaks to the coolant about $9x10^{-8}$ per cycle. In addition to these, 0.029 atoms of ^{131}I are produced from each fission.

The coolant stays in the reactor for t_1 seconds and completes the rest of the cycle in t_2 seconds. Every time that the coolant enters the reactor for a t_1 seconds, it becomes more reactive because of the leaking. So, one of the main purposes is to calculate the concentration of ^{131}I in the water that goes to the environment in the frame of NRC's limitations.

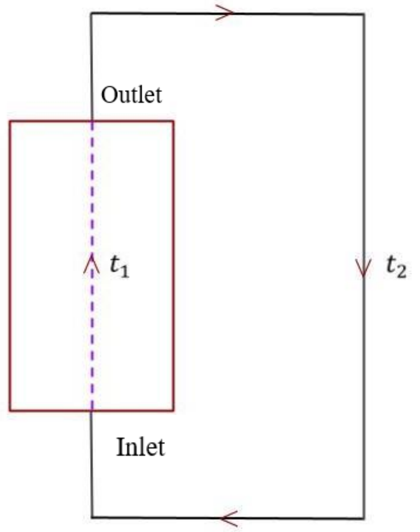


Figure 2 – The Cycle Diagram with Times

2. METHODS AND CALCULATION

Part (a)

In this section, we are asked to calculate the residence time of the coolant in the core t_1 and the circulation time t_2.

We are given that:

$$2.5 \; x \; t_1 = t_2 \qquad eq.1$$

Firstly, we must find the mass flow rate by using eq.2 below.

$$\dot{m} = \rho_{average} \vartheta A \qquad eq.2$$

Where,

$$\rho_{average} = \frac{\rho_{inlet} + \rho_{outlet}}{2} \qquad eq.3$$

However, we must find the $\rho_{average}$ value at inlet and outlet temperatures and which are 286℃ and 326℃, respectively at 15.5 MPa. To find $\rho_{average}$ eq.4 can be used.

$$v_f = \frac{1}{\rho} \qquad eq.4$$

Where, ν_f is the specific volume of the saturated water with the unit of $\frac{m^3}{kg}$. ν_f values can be obtained from *Table A2* in the Appendix.

After finding $\rho_{average}$, we can find the coolant velocity (ϑ) by rewriting the eq.2 as below.

$$\vartheta = \frac{\dot{m}}{\rho_{average}A} \qquad eq.5$$

Then, the residence time of the coolant in the core t_1 and the circulation time t_2 can be found by using eq.6.

$$t = \frac{\chi}{\vartheta} \qquad eq.6$$

Where χ is the active fuel rod height.

Part (b)

In this part, we are asked to calculate the number of equilibriums ^{131}I atoms in the fuel region.

To find it, we must integrate eq.7 below.

$$\frac{dN}{dt} + \lambda N = R \qquad eq.7$$

To integrate eq.7, we must use integration factor method as shown below eq.8.

$$\mu = e^{\int p(x)dx} \qquad eq.8$$

By using eq.8, our integration factor can be obtained as follows:

$$\mu = e^{\lambda t} \qquad eq.9$$

Then, eq.7 must be multiplied by eq.9 as below.

$$e^{\lambda t}\left(\frac{dN}{dt} + \lambda N\right) = e^{\lambda t}R \qquad eq.10$$

As a rule of the integration factor method, eq.10 can be written as eq.11.

$$\frac{d}{dt}\left(e^{\lambda t}N\right) = e^{\lambda t}R \qquad eq.11$$

Now, we can take the integral of eq.11.

$$\int \frac{d}{dt}\left(e^{\lambda t}N\right) = \int e^{\lambda t}R \qquad eq.12$$

Then we obtain eq.13.

$$e^{\lambda t}N = \frac{R}{\lambda}e^{\lambda t} + C \qquad eq.13$$

To find C, initial condition of $N(0)$ can be used. Therefore,

$$C = N(0) - \frac{R}{\lambda} \qquad eq.14$$

By substituting eq.14 into eq.13:

$$e^{\lambda t}N = \frac{R}{\lambda}\left(1 - e^{-\lambda t}\right) + N(0) \qquad eq.15$$

To get activity equation eq.15 must be multiplied by λ.

$$\alpha(t) = \alpha(0)e^{-\lambda t} + R\left(1 - e^{-\lambda t}\right) \qquad eq.16$$

Since $\alpha(0) = 0$;

$$\alpha(t) = R\left(1 - e^{-\lambda t}\right) \qquad eq.17$$

To find $\alpha(t_1)$, eq.17 can be used as:

$$\alpha(t_1) = R\left(1 - e^{-\lambda t_1}\right) \qquad eq.18$$

For the equilibrium condition eq.19 can be written as:

$$\alpha_{eq} = \frac{R\left(1 - e^{-\lambda t_1}\right)}{\left(1 - e^{-\lambda(t_1+t_2)}\right)} \qquad eq.19$$

$where,$

$$R = (f_{leak})(Microcrack\ Ratio)\left(Number\ of\ {}^{131}I\ Atoms\ per\ Fission\right)(Fission\ Rate) \qquad eq.20$$

Part (c)

In this part, the relationship between the activity and the number of cycles that the coolant passes through the core will be calculated.

It can be calculated by using eq.21 [2] for n cycle as below.

$$\alpha_n = \frac{R\left(1 - e^{-\lambda t_1}\right)\left(1 - e^{-n\lambda(t_1+t_2)}\right)}{1 - e^{-\lambda(t_1+t_2)}} \qquad eq.21$$

To obverse it more accurately, this part will be evaluated on Python for 10^{10} cycles.

Part (d)

In this part, the equilibrium activity per gram of water will be calculated and discussed. It can be calculated by using eq.22 below.

$$\alpha_{Dose} = \alpha_{limit} m_{coolant} \qquad eq.22$$

If $\alpha_{Dose} > \alpha_{limit}$, it is not suitable for the NRC's regulations.
If $\alpha_{Dose} < \alpha_{limit}$, it is suitable for the NRC's regulations.

3. RESULTS

Part (a)

From the *Table A2* in the Appendix, v_f value at 286℃ is obtained by interpolation as:

$$v_f = (0.2)(0.001366 - 0.001348) + 0.001348$$

$$v_f = 0.001352\ \frac{m^3}{kg}$$

Therefore, by using eq.4

$$\rho_{inlet} = \frac{1}{0.001352\ \frac{m^3}{kg}} = 739.864\ \frac{kg}{m^3}$$

From the *Table A2* in the Appendix, v_f value at 326℃ is obtained by interpolation as:

$$v_f = (0.2)(0.001561 - 0.001528) + 0.001528$$

$$v_f = 0.001535 \ \frac{m^3}{kg}$$

Therefore, by using eq.4

$$\rho_{outlet} = \frac{1}{0.001535 \frac{m^3}{kg}} = 651.636 \ \frac{kg}{m^3}$$

Then, $\rho_{average}$ can be found by using eq.2 as:

$$\rho_{average} = \frac{739.864 \ \frac{kg}{m^3} + 651.636 \ \frac{kg}{m^3}}{2} = 695.750 \ \frac{kg}{m^3}$$

Now, we can find the coolant velocity (ϑ) by using eq.5 as:

$$\vartheta = \frac{1.74x10^4 \frac{kg}{s}}{\left(695.750 \frac{kg}{m^3}\right)(5.31 \ m^2)} = 4.710 \ \frac{m}{s}$$

Since χ is given as $3.660 \ m$ in *Table A1* in the Appendix, by using eq.6 the residence time of the coolant in the core t_1 can be obtained as:

$$t_1 = \frac{3.660 \ m}{4.710 \frac{m}{s}} = 0.7771 \ s$$

By using eq.1 t_2 can be obtained as:

$$t_2 = 2.5 \ x \ t_1 = 1.9428 \ s$$

Part (b)

In Part (a), we found $t_1 = 0.7771 \ s$ by using this value, we will calculate the equilibrium activity at t_1.

Firstly, by using eq.20, R value must be found.

$$R = (9x10^{-8})(0.02)\left(0.029 \frac{atoms}{fission}\right)\left(1.065x10^{20} \frac{fission}{s}\right) = 5.56x10^9 \frac{atoms}{s}$$

Then λ can be calculated as:

$$\lambda = \frac{\ln 2}{692{,}988.48\ s} = 1x10^{-6}\ s^{-1}$$

After that, equilibrium activity can be calculated by using eq.19.

$$\alpha_{eq} = \frac{\left(5.56x10^{9}\,\frac{atoms}{s}\right)(7.771x10^{-7})}{2.720x10^{-6}}$$

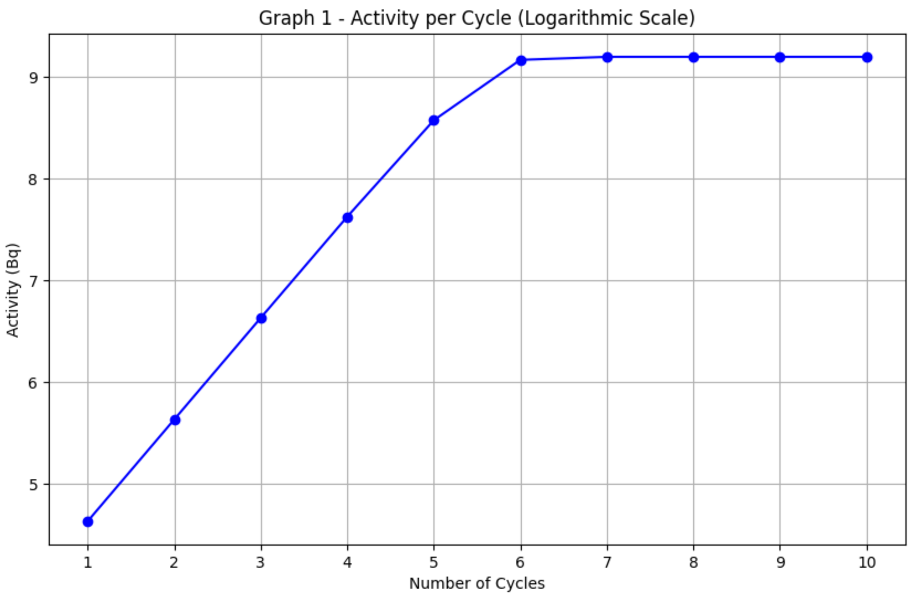
$$\alpha_{eq} = 1.589x10^{9}\ \frac{atoms}{s}$$

Part (c)

By solving eq.21 on Python, corresponding values are obtained, and they are given in Table 1 and Graph 1 below.

Table 1 – Activity per Cycle

Number of Cycle	Activity (Bq)
10	42864.65
10^2	428594.07
10^3	4280699.53
10^4	42287550.17
10^5	375303176.16
10^6	1472179550.16
10^7	1576014706.0
10^8	1576014706.0
10^9	1576014706.0
10^{10}	1576014706.0



As shown in Graph 1, as the number of cycles increases, the activity starts to converge a value and stays constant. This value is α_{eq}.

Part (d)

In this part, NRC's limitation versus our data will be compared by using eq.22.

$$\alpha_{limit} = 4x10^4 \frac{Bq}{g}$$

$$\alpha_{Dose} = \frac{1.589x10^9\ Bq}{2.5x10^8\ g} = 6.356\ \frac{Bq}{g}$$

$$\because \alpha_{limit} > \alpha_{Dose}$$

Therefore, our ^{131}I concentration in the water is acceptable. Because it is lower than NRC's limits. This amount of concentration might be safe for the environment and humanity.

4. CONCLUSION

To sum up, in a fuel rod, there might be some cracks. These cracks might come from the fabrication and also these might occur inside of the reactor, since it works under high pressure and temperature. From these cracks radioactive materials can leak to the environment via coolant water in an accident situation. Most of the countries has nuclear regulatory agencies, these agencies put limits to these leaked radioactive materials to protect environment and humanity.

In our study, the equilibrium concentration of ^{131}I increases for every additional cycle at the beginning. After that, since number of cycles increases rapidly, it converges to a value, that α_{eq}, equilibrium activity. So, we can understand that the equilibrium concentration of ^{131}I has a limit value, it does not go to the infinity. In addition to this, our radioactivity level in the coolant is lower than the NRC's limit, in other words our radioactive waste level is acceptable. However, these radioactive materials must be more solid to protect the world.

5. REFERENCES

[1] (n.d.). *Pressurized Water Reactor (PWR)*. Energy Encyclopedia. Retrieved December 7, 2023, from https://www.energyencyclopedia.com/en/nuclear-energy/the-nuclear-reactors/pressurized-water-reactor-pwr

6. APPENDIX

i. Python Code: NEM393-Project-III[Appendix i - Python Code].py

ii. Python Code Output: NEM393-Project-III[Appendix ii - Code Output].txt

iii.

Table A1 - Reference design parameters for the PWR

Thermal Power	3411 MW_{th}
Operating Pressure	15.5 MPa
Inlet Temperature	286 ^oC
Outlet Temperature	326 ^oC
Number of Fuel Rods	50,952
Core Flow Rate	17.4 Mg/s
Active Fuel Rod Height	3.66 m
Core Wide Coolant Area	5.31 m^2
Total Weight of Coolant in Primary Side	2.5x105 kg

iv.

Table A2 - Thermodynamic Properties of Water, Saturated Water *

		Specific Volume, m^3/kg			Internal Energy, kJ/kg		
Temp. (°C)	**Press. (kPa)**	**Sat. Liquid** v_f	**Evap.** v_{fg}	**Sat. Vapor** v_g	**Sat. Liquid** u_f	**Evap.** u_{fg}	**Sat. Vapor** u_g
0.01	0.6113	0.001000	206.131	206.132	0	2375.33	2375.33
5	0.8721	0.001000	147.117	147.118	20.97	2361.27	2382.24
10	1.2276	0.001000	106.376	106.377	41.99	2347.16	2389.15
15	1.705	0.001001	77.924	77.925	62.98	2333.06	2396.04
20	2.339	0.001002	57.7887	57.7897	83.94	2318.98	2402.91
25	3.169	0.001003	43.3583	43.3593	104.86	2304.90	2409.76
30	4.246	0.001004	32.8922	32.8932	125.77	2290.81	2416.58
35	5.628	0.001006	25.2148	25.2158	146.65	2276.71	2423.36
40	7.384	0.001008	19.5219	19.5229	167.53	2262.57	2430.11
45	9.593	0.001010	15.2571	15.2581	188.41	2248.40	2436.81
50	12.350	0.001012	12.0308	12.0318	209.30	2234.17	2443.47
55	15.758	0.001015	9.56734	9.56835	230.19	2219.89	2450.08
60	19.941	0.001017	7.66969	7.67071	251.09	2205.54	2456.63
65	25.03	0.001020	6.19554	6.19656	272.00	2191.12	2463.12
70	31.19	0.001023	5.04114	5.04217	292.93	2176.62	2469.55
75	38.58	0.001026	4.13021	4.13123	313.87	2162.03	2475.91
80	47.39	0.001029	3.40612	3.40715	334.84	2147.36	2482.19
85	57.83	0.001032	2.82654	2.82757	355.82	2132.58	2488.40
90	70.14	0.001036	2.35953	2.36056	376.82	2117.70	2494.52
95	84.55	0.001040	1.98082	1.98186	397.86	2102.70	2500.56
100	101.3	0.001044	1.67185	1.67290	418.91	2087.58	2506.50
105	120.8	0.001047	1.41831	1.41936	440.00	2072.34	2512.34
110	143.3	0.001052	1.20909	1.21014	461.12	2056.96	2518.09
115	169.1	0.001056	1.03552	1.03658	482.28	2041.44	2523.72
120	198.5	0.001060	0.89080	0.89186	503.48	2025.76	2529.24
125	232.1	0.001065	0.76953	0.77059	524.72	2009.91	2534.63
130	270.1	0.001070	0.66744	0.66850	546.00	1993.90	2539.90
135	313.0	0.001075	0.58110	0.58217	567.34	1977.69	2545.03
140	361.3	0.001080	0.50777	0.50885	588.72	1961.30	2550.02
145	415.4	0.001085	0.44524	0.44632	610.16	1944.69	2554.86
150	475.9	0.001090	0.39169	0.39278	631.66	1927.87	2559.54
155	543.1	0.001096	0.34566	0.34676	653.23	1910.82	2564.04
160	617.8	0.001102	0.30596	0.30706	674.85	1893.52	2568.37
165	700.5	0.001108	0.27158	0.27269	696.55	1875.97	2572.51
170	791.7	0.001114	0.24171	0.24283	718.31	1858.14	2576.46
175	892.0	0.001121	0.21568	0.21680	740.16	1840.03	2580.19
180	1002.2	0.001127	0.19292	0.19405	762.08	1821.62	2583.70
185	1122.7	0.001134	0.17295	0.17409	784.08	1802.90	2586.98
190	1254.4	0.001141	0.15539	0.15654	806.17	1783.84	2590.01

(*) Source: Borgnakke, C., & Sonntag, R. E. (2012). *Fundamentals of Thermodynamics* (8th ed., p. 776). Wiley.