Symmetry and Degeneracy

Symmetry

There are 3 types of symmetry discussed:

- Reflective
- Periodic
- Rotational

Recall .. Vector Math

1 $u = <x(t), y(t)>$

2 $v = <x, y>$ .. a vector based on x and y

3 $P (x, y)$ .. a point at x and y

Reflective Symmetry

4 $x(t) = px + Vx (t)$

5 $y(t) = py + Vy (t)$

Periodic Symmetry

6 $x(t) = px + Vx \cos (t)$

7 $y(t) = py + Vy \sin (t)$

Rotational Symmetry

8 $x(t) = Vy (t)$

9 $y(t) = Vx (t)$

Invariance .. is when a transformation produced

10 $x2 = f(x)$

11 $d/dx = [d/dx]^{-1} f(x) \, d/dx$

12 $[d/dx]^2 =$ integrate $f(x)\, dx$

13 $H\sim [p\sim, x[2]\sim] = H\sim [p\sim, x[1]\sim]$

14 $H\sim [p\sim, x[2]\sim] \;\; \phi [a] (x[2]) = E[a] \;\; \phi [a] (x[2])$

15 $H\sim [p\sim, x\sim] \;\; \phi [a] (f(x)) = E[a] \;\; \phi [a] (f(x))$

16 $\phi [b] (x) = \phi [a] (f(x))$

17 $d/dt <Q> = 0$

Figure 10.1. The stick figure is unchanged under reflections $x \to -x$, but not under translations

Figure 10.2. The displaced sine function (dashed line) differs from the original sine function (solid line), except under translations of exactly $\Delta x = 2\pi n$.

Free Particle

18 x2 = x + a

19 d/dx2 = d/dx

20 T~ F (x) = F (x2)
21 = F (x + a)

22 T~ H~ [d/dx, x] w(x) = H~ [d/dx2, x2] w(x2)
23 = H~ [d/dx, x] w(x2)
24 H~ [d/dx, x] T~ w(x)

25 [T~, H~] = T~ H~ - H~ T~
26 = 0

20 T~ F (x) = F (x + a)
27 = F(x) + [d/dx] F a + 1/2 [d/dx]^2 F a^2 + ...
28 = Sum [n] A^n / n! [d/dx]^n F(x)
29 = exp (a d/dx) F(x)
30 = exp [i a p~/h-bar] F(x)

31 [p~, H~] = 0
H~ = p~^2 /2 m

d/dt <Q> = i / h-bar < [Q, H] >
32 d/dt <Q> = 0
33 d/dt <p> = 0

Recall .. How momentum is related to Energy
p = sqr (2 m E)

34 w(x) = a exp (i p x / h-bar) + b exp (i p x / h-bar)
35 = a exp (i sqr (2 m E) x / h-bar)
+ b exp (i sqr (2 m E) x / h-bar)

36 { ϕ [p] (x) , E [p] } for all p = [oo .. oo]
37 { 1/sqr (2 PI h-bar) exp (i p x / h-bar) , p^2/2m }

Parity

38 V(x)
39 = 0 .. for a free particle
40 = 1/2 k x^2 .. for a harmonic oscillator
.. for an infinite square well
41 = { 0 .. x < -a
42 { -V0 .. x = [a .. a]
43 { 0 .. x > a

44 x2 = -x

45 - h-bar^2 / 2m [d/dx2][d/dx2]
46 = - h-bar^2 / 2m [- d/dx][- d/dx]
47 = - h-bar^2 / 2m [d/dx][d/dx]

48 P~ F(x) = F (-x)

49 P~ H~ [dx, x] w(x)
50 = H~ [-dx, -x] w(-x)
51 = H~ [dx, x] w(-x)
52 = H~ [dx, x] P~ w(x)

53 [P~, H~] = 0
54 d/dt <P> = 0

55 P~ ϕ [b] (x) = b ϕ [b] (x)

56 P~ P~ ϕ [b] (x) = b P~ ϕ [b] (x)
57 = b^2 ϕ [b] (x)

58 P~ P~ ϕ [b] (x) = b P~ ϕ [b] (-x)
59 = ϕ [b] (x)

60 b^2 = 1
61 b = +/- 1

b = +1 .. Even Parity

62 ϕ [b] (-x) = (b) ϕ [b] (x)

63 ϕ [+] (-x) = (+) ϕ [+] (x)

b = -1 .. Odd Parity

64 ϕ [b] (-x) = (b) ϕ [b] (x)

65 ϕ [-] (-x) = (-) ϕ [-] (x)

66 a+ (-x -dx) = -a+ (x, dx)

Recall .. the General Form for the Harmonic Oscillator

67 ϕ [n] (x) = 1/ sqr (n!) (a+)^n ϕ [0] (x)

68 P~ ϕ [n] (x) = ϕ [n] (-x)

69 = (-1)^n ϕ [n] (x)

70 P~ exp (i p x / h-bar) = exp (-i p x / h-bar)

71 != +/- exp (i p x / h-bar)

34 w(x) = a exp (i p x / h-bar) + b exp (i p x / h-bar)

72 ϕ [E] (x) = a exp (i p x / h-bar) + b exp (i p x / h-bar)

Recall .. How momentum is related to Energy

p = sqr (2 m E)

a = b .. Even Parity .. Energy Eigen State

73 ϕ [E+] (x) = A cos (px / h-bar)

a = -b .. Odd Parity .. Energy Eigen State

74 ϕ [E-] (x) = B sin (px / h-bar)

75 { ϕ [E+] (x), ϕ [E-] (x)}

76 P~ p~ f(x) - (-p~) f(-x)

77 = - p~ P~ f(x)

78 != p~ P~ F(x)

79 [P~, p~] = 0

Particle In the Square

Assume a cartesian grid of (L x L), where 0 is centered at (L/2). Point P (x,y) can be reflected around the the equation (L / 2), or the line (x = y).

80 x2 = L - x1
81 y2 = L - y1

83 x2 = y
84 y2 = x

85 H~ = -h-bar^2 / 2m ([d/dx]^2 + [d/dy]^2) + V (x, y)

86 V(x,y) = v(x) + v(y)

87 v(x) =
88 { 0 .. x = [0 .. L]
89 {00 .. otherwise

90 H~ = Hx~ + Hy~

91 Hx~ = -h-bar^2 / 2m [d/dx]^2 + v (x)
92 Hy~ = -h-bar^2 / 2m [d/dy]^2 + v (y)

93 ϕ (x, y) = w (x) w (y)

Recall .. TISE
H~ ϕ = E ϕ

94 < ϕ | H | w > = ϕ (y) Hx~ w (x) + w (x) Hy~ ϕ (y)
= ϕ (y) Hx~ w (x) + w (x) Hy~ ϕ (y)

95 H~ ϕ (x) w (x) = E ϕ1 (y) w (x)
H~ ϕ w (x) / w (x) = E ϕ1 (y) w (x) / w (x)

Note .. ϕ1 (y) w (x) will only work when they are orthogonal

96 Hx~ w (x) / w(x) = Hy~ w(x) / w(y) = E
97 Hx~ w [n] (x) / w(x) = E[n] w [n] (x)

98 Hx~ w [m] (x) / w(x) = E[m] w [m] (x)

99 w [n] (x) = sqr (2 / L) sin (n PI x / L)
100 E[n] = n^2 h-bar^2 PI^2 / (2 m L^2)

101 w [m] (x) = sqr (2 / L) sin (m PI x / L)
102 E[m] = m^2 h-bar^2 PI^2 / (2 m L^2)

103 ϕ[n, m] (x, y) = (2/L) sin (n PI x / L) sin (m PI x / L)
104 E [n, m] = (n^2 + m^2) h-bar^2 PI^2 / (2 m L^2)

105 { ϕ[n, m] (x, y), E [n, m] }

106 E [n m] = E [m n], n != m

107 Rx~ f (x, y) = f (L - x, y)
108 Ry~ f (x, y) = f (x, L - y)

109 Rx~ Ry~ f (x,y) = Ry~ Rx~ f (x,y)
110 = f (L - x, L - y)
111 = [Rx, Ry]
112 = 0

113 sin (n PI (L - x) / L) =
114   { 1 .. n .. odd
115   { -1 .. n even

116 I~ f(x, y) = f (y, x)
117 I~ Rx~ f (x,y) = f (y, L - x)
118 Rx~ I~ f (x,y) = f (L - y, x)

119 I~ I~ ϕ [b] (x, y)
120 = I~ ϕ [b] (y, x)
121 = ϕ [b] (x, y)

122 I~ I~ ϕ [b] (x, y)
123 = B I~ ϕ [b] (x, y)
124 = B^2 ϕ [b] (x, y)

125 B^2 = 1

126 B = +/- 1

ϕ [n, m] (x , y) and ϕ [m, n] (x , y) have the same energy

127 ϕ (x, y) = a ϕ [n m] (x, y) + b ϕ [m n] (x, y)

ϕ (x , y) = ϕ (y, x)

128 = a sin (n PI x / L) sin (m PI x / L)

+ b sin (n PI x / L) sin (m PI x / L)

129 ϕ [nm+] = 1/sqr(2) (ϕ [nm] (x, y) + ϕ [nm] (y, x))

130 ϕ [nm-] = 1/sqr(2) (ϕ [nm] (x, y) - ϕ [nm] (y, x))

131 ϕ [nm+] = ϕ [nn] (x, y)

The Quantum Corral

132 V(x) =
133 { 0 .. r < R
134 { oo .. r > R

135 r2 = r1
136 θ2 = θ + δ2 θ

137 x = r cos θ
138 y = r sin θ

139 ∇^2 = [d/dx]^2 + [d/dy]^2
140 = [d/dr]^2 + 1/r [d/dr] + 1/r^2 [d/dθ]^2

141 H~ = -h-bar^2 / 2m ∇^2 + V(r)
142 = -h-bar^2 / 2m ([d/dr]^2 + 1/r [d/dr] + 1/r^2 [d/dθ]^2) + V(r)

143 R [δ θ]~ f (r, θ)
144 = f (r, θ + δ θ)
144 = f (r, θ) + [d/dθ] f (r, θ) δ θ + 1/2 [d/dθ]^2 f (r, θ) ...
145 = exp (δ θ [d/dθ]) f (r, θ)

146 R [δ θ]~ = exp (δ θ [d/dθ])

147 [R [δ θ], H] = 0
147 [[d/dθ], H] = 0

148 d/dθ = [d/dθ] x [d/dx] + [d/dθ] y [d/dy]
149 = -r sin θ [d/dx] + r cos θ [d/dy]
150 = x [d/dy] - y [d/dx]
151 = i / h-bar (x py~ - y px~)

Lz = xp[y] - yp[x]
Lz~ = x~p[y]~ - y~p[x]~

152 R [δ θ] = exp (i δ θ Lz / h-bar)
153 [Lz, H] = 0
154 [d/dt] <Lz> = 0

155 Lz = -i h-bar [d/dθ]

156 H~ = -h-bar^2 / 2m ([d/dr]^2 + 1/r [d/dr] + 1/r^2 [d/dθ]^2)
+ V(r)

157 Lz ϕ (r, θ)
158   = a ϕ (r, θ)
159 = -i h-bar [d/dθ] ϕ (r, θ)

160 ϕ (r, θ) = w(r) exp (i a θ / h-bar)
ϕ [n] (r, θ) = w [n] (r) exp (i n θ)

161 ϕ (r, θ + 2 PI) = ϕ (r, θ)

162 a = n h-bar, n = integers

163 H~
164 = -h-bar^2 / 2m ([d/dr]^2 + 1/r [d/dr] + 1/r^2 [d/dθ]^2) ϕ [n]
+ V(r) ϕ [n]
165 = E [n] ϕ [n]

166 Lz ϕ [n] = n h-bar ϕ [n]

167 H~
168 = -h-bar^2 / 2m ([d/dr]^2 + 1/r [d/dr] + 1/r^2 [d/dθ]^2) ϕ [n](r)
+ V(r) ϕ [n] (r)
= [-h-bar^2 / 2m ([d/dr]^2 + 1/r [d/dr] + 1/r^2 [d/dθ]^2)
+ V(r)] ϕ [n] (r)
169 = E [n] ϕ [n] (r)

170 ([d/dr]^2 + 1/r [d/dr] + (2 m E / h-bar^2 - n^2 / r^2)) ϕ [n](r)
= 0

V (r) = ∞
ϕ (r) = 0, r >> R

171 k^2 = 2 m E / h-bar^2
172 p = k r
173 r = p / k
174 = p / sqr (2 m E) / h-bar

([d/dp]^2 + 1/p [d/dp] + (1 - n^2 / p^2)) ϕ [n] (r) = 0

175 w [n] (r) = J [n] (kr)
176 E = (1)^2 h-bar^2 k^2 / 2 m, n = 1

170 ([d/dr]^2 + 1/r [d/dr] + (2 m E / h-bar^2 - n^2 / r^2)) w (r) = 0
175 ([d/dp]^2 + 1/p [d/dp] + (1 - n^2 / p^2)) w (r) = 0

176 ϕ [n] (R, θ) = 0
177 w [n] (R, θ) = 0
178 J [n] (k R) = 0
179 J [n] (x [n, j]) = 0

180 { x [n1], x [n2], x [n3], .. x [nn] }
181 x[0j] = 2.405, 5.520, 8.654, ...
182 x[1j] = 3.832, 7.016, 10.173, ...

183 k R = x [n j]
184 E [n j] = h-bar^2 k^2 / 2 m

185 { ϕ [n j], E [n j], Lz }
186 ϕ [n, j] = J [n] (x [n j] r / R) exp (i n θ)
187 E [n, j] = h-bar^2 x[n j]^2 / (2 m R^2)
188 Lz = n h-bar

P[θ]~ f (r , θ) = f (r , −θ)

Landau Levels

189 Force = q E - q/c v x B

190 Force = mass * acceleration

191 h = 1/2m (p - q/c A) · (p - q/c A) + q/c ϕ

192 E = -1/c [d/dt] A - ∇ ϕ
193 B = ∇ x A

194 A -> A - ∇ x
195 B = dx Ay - dy Ax

196 Ax = 0
197 Ay = Bx

198 H~
199 = -h-bar^2 / 2m (px^2 + py^2 - 2 q/c Bx py + q^2/c^2 B^2 x^2)

200 [H, py] = 0

201 H~ w [kn] (x, y) = E [kn] ϕ [kn] (x, y)
202 py~ w [kn] (x, y) = (h-bar k) ϕ [kn] (x, y)

203 k = [oo .. oo]
204 ϕ [kn] (x, y) = 1 / sqr (2 PI h-bar) exp (i k y) ϕ [nk] (x)

205 Xk = kc / qB
x = Xk

Recall .. the Spring Constant (kx)
kx = q^2 B^2 / m c^2
213 w = q B / m c

206 = -h-bar^2 / 2m (px^2 + k^2 - 2 q/c Bxk + q^2/c^2 B^2 x^2)
207 = 1 / 2m (px^2 + q^2/c^2 B^2 (x - xk)^2) ϕ [nk] (x)
208 = E [n k] ϕ [k n] (x)

209 ϕ [kn] (x)
210 = ϕ [n] (x - xk)
 = 1 / sqr (2 PI h-bar) exp (i k y) ϕ [n] (x - xk)

211 E [kn] = E [n]
212 = h-bar w (n + 1/2)

214 [H, y] = i h-bar / m (q/c Bx -py)
215 [H, px] = i h-bar / m (q^2/c^2 B^2x - q/c B py)
216 = q/ c B [H, y]

217 Q~ = px~ - q/c B y~

218 [H, Q] = 0

219 ϕ [n] (x, y) = sqr (2 / Ly) sin (ky) ϕ [n] (x - xk)
220 k = PI j / Ly
221 j = [1 .. max]
222 max = (1 / PI) (q / c) B x Area

Xk-max = k-max c / q B
 = PI j-max c / q B Ly
 = Lx

j-max = q B Area / PI c

Area = Lx × Ly

Complete Set of Observables

223 { exp (i sqr (2 m E) x / h-bar), exp (-i sqr (2 m E) x / h-bar) }
224 E = [0 .. oo]

225 { cos (sqr (2 m E) x / h-bar), sin (sqr (2 m E) x / h-bar) }
226 E = [0 .. oo]