HW3: (Neural) Language Modeling

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March 11, 2016

Github: https://github.com/virgodi/cs287/HW3

1 Introduction

This assignment focuses on the task of language modeling, a crucial first-step for many natural language applications. In this report, we will present several count-based multinomial language models with different smoothing methods, an influential neural network based language model from the work of Bengio et al. (2003), and an extension to this language model which learns using noise contrastive estimation, as well as their implementation using Torch. We found this homework more challenging than the previous ones and encountered significant challenges that we will underline in this report.

2 Problem Description

The goal of the language models presented in this report is to learn a distributed representation for words as well as probability distribution for word sequences. Language models are usually represented as the probability of generating a new word conditioned on the preceeding words:

$$P(\boldsymbol{w}_{1:n}) = \prod_{i=1}^{n-1} P(w_{i+1}|w_i)$$

To simplify the analysis, it is common to make the assumption that a word is influenced only by the N words immediately preceeding it, which we call the context. Even with reasonably small values for N, building such models are extremely expensive computationally-wise as well as time-consuming if not ran on GPU. The joint proabibility of a sequence of 6 words taken from a vocabulary of 10 000 words could possibly imply training the model to fit up to $10^{4^6} - 1 = 10^{24} - 1$ parameters.

The objective of this homework was to predict a probability distribution over 50 words at a given place in the sentence and based on the previous words. To do so, we tried implementing N-grams models in an efficient manner. Due to computational limitations (no access to GPUs...), we faced difficulties training the Neural Network and focused our efforts on building strong count-based models to solve the problem of language modeling.

3 Model and Algorithms

We will now dive into more details of two types of models, i.e. count-based models and neural network models.

3.1 Count-based Models

The first models we implemented are based on counts of N-gram features. The main idea is here to implement n-gram multinomial estimates for $p(w_i|w_{i-n+1},\ldots,w_{i-1})$. We will use different smoothing methods and compare their performance.

Maximum Likelihood Estimation The first approach is just to compute the maximum likelihood estimation $p_{ML}(w|c)$ where c is the context (i.e. N-1 gram) and w the word predicted. This is done by simply counting the frequency of the N-gram: $(c - w)$ in the training.

$$p_{ML}(w|c) = \frac{F_{c,w}}{F_c} = \frac{F_{c,w}}{\sum_i F_{c,w_i}}$$

with $F_{c,w} = \sum_i \mathbb{1}(w_{i-n+1:i-1} = c, w_i = w)$.

We applied this method up to 5-grams. But the results are not very impressive. First, the long tail in the words distribution is not properly represented as we consider the number of occurences of such words as the ground truth. Second, this method works for one context at a time but it should be more interesting to weight and combine them.

Laplace smoohting This approach handles the first issue where we want to have a better representation for the words in the tail. Here we simply add an off-set to each count $\hat{F}_{c,w} = F_{c,w} + \alpha$.

We can use the validation set to tune the α parameter while optimizing its perplexity. We applied this pipeline on the validation set from Kaggle.

Witten-Bell smoothing Here we tackle the second issue of the MLE. The idea is to combine lower order models together to use as much information as possible. The global idea is to use recursivity:

$$p(w|c) = \lambda p_{ML}(w|c) + (1-\lambda)p(w|c')$$

with c' the lower order context (if c is the N-1 gram, c' is the N-2 gram). We stop it when c' is empty and then use the prior on the words distribution from the train (ie the frequency of the word w only).

Witten and Bell suggested a way to compute the constant: $\lambda = \frac{N1_c}{F_c + N1_c}$ with $N1_c = |\{w : F_{c,w} = 1\}|$

This method is usually called interpolation as we interpolate $p(w|c)$ with its MLE estimations at each lower order context. Another method is known as back-off where we use the lower order context only if $F_{w,c} = 0$. We applied a mixed of the two methods, ie we use interpolation and jumped directly to the lower order context if the count was 0. Also we applied an alpha smoothing for the 2-gram (ie the lowest order before the word prior). This was motivates by the need to smooth the importance of the 2-grams as the counts are higher for lower order context.

Modified Kneser-Ney smoothing This method also uses a mixture of interpolation and back-off but instead of using the MLE for the current context, it uses absolute discounting. The idea is that we would like to weigh down the count $F_{w,c}$ but differently with regards to their value. As a result, we introduce a discounting parameter which takes discrete values with regards to $F_{w,c}$.

$$p(w|c) = \frac{F_{c,w} - D(F_{c,w})}{F_c} + \gamma(F_{c,w})p(w|c)$$

with :

$$D(F_{c,w}) \begin{cases} 0, & \text{if } F_{c,w} = 0 \\ D_1, & \text{if } F_{c,w} = 1 \\ D_2, & \text{if } F_{c,w} = 2 \\ D_3, & \text{if } F_{c,w} \geq 3 \end{cases}$$

$$\gamma(F_{c,w}) = \frac{D_1 N1_c + D_2 N2_c + D_3 N3_c}{F_c}$$

with $N2_c$ and $N3_c$ similarly defined as $N1_c$.

In the original paper, they used the above definition with fixed $D1$, $D2$ and $D3$ and tuned parameters. We tried both approach and found even better results with manually tuned parameters.

3.2 Neural Networ Models

3.2.1 Regular Models

As in the neural networks build for previous homeworks, the model has for input a window of words preceding the wanted predicted word. It first convert the words in the window of size d_{win} my mapping them into a geometrical space of higher dimension d_{in} (30 in Bengio's paper). It then concatenates the words embeddings into a vector of size $d_{in} \times d_{win}$. This has for advantage of adding information about the position of the words in the window, as opposed to making a bag-of-words assumption. The higher dimensional representation of the window is then fed into a first linear model followed by a hyperbolic tangent layer to extract non- linear features. A second linear layer is then applied followed by a softmax to get a probability distribution over the vocabulary. We then train the model using a Negative Log-Likelihood criterion and stochastic gradient descent.

We can summarize the model in the following formula:

$$nnlm_1(\boldsymbol{x}) = \tanh(\boldsymbol{x}\boldsymbol{W} + \boldsymbol{b})\boldsymbol{W'} + \boldsymbol{b'}$$

where we recall that:

- $\boldsymbol{x} \in \Re^{d_{in} \cdot d_{win}}$ is the concatenated word embeddings
- $\boldsymbol{W} \in \Re^{(d_{in} \cdot d_{win}) \times d_{hid}}$, and $\boldsymbol{b} \in \Re^{d_{hid}}$
- $\boldsymbol{W'} \in \Re^{d_{hid} \times |\mathcal{V}|}$, and $\boldsymbol{b'} \in \Re^{|\mathcal{V}|}$, where $|\mathcal{V}|$ is the size of the vocabulary.

We give a diagram of the model to better illustrate it:

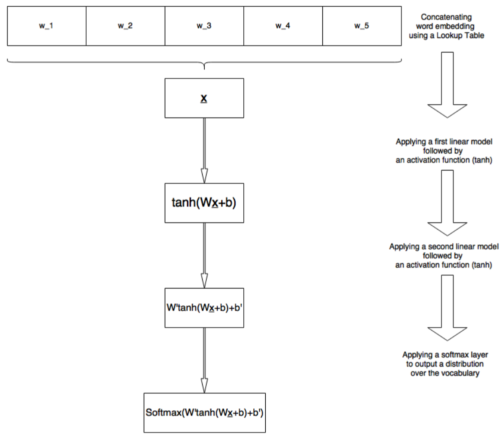


Figure 1: Neural Language Model (Bengio,2003)

We then implemented a variant of the model using a skip-layer that concatenates the output of the tanh layer again with the original embeddings. The updated formula for the model is:

$$nnlm_2(\boldsymbol{x}) = [\tanh(\boldsymbol{xW} + \boldsymbol{b}), \boldsymbol{x}]\boldsymbol{W'} + \boldsymbol{b'}$$

where this time:

- $\boldsymbol{W'} \in \Re^{(d_{hid} + d_{in} \cdot d_{win}) \times |\mathcal{V}|}$, and $\boldsymbol{b'} \in \Re^{|\mathcal{V}|}$

The updated diagram is as follows:

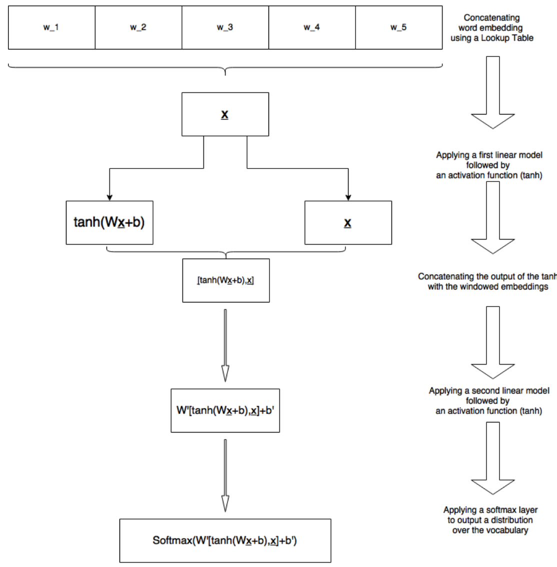


Figure 2: Skip-Layer Model

We now show the pseudo code for training these NNLMs using batch stochastic gradient descent:

```
procedure NNLM_i(win_1, ..., win_n,MaxEpoch,BatchSize,LearningRate)
    for epoch = 1,MaxEpoch do
        for batch = 1, |train|/BatchSize do
            for win in batch do
                Call NNLM_i:forward(win)
                Evalute the loss
                Evaluate derivatives of the loss
                Backprop through NNLM_i
            Update Parameters with LearningRate
```

3.2.2 Noise Contrastive Estimation

As mentioned earlier, training such model is extremely expensive in computation, especially on CPUs. The issue comes from the use of the softmax in the last layer of the model in order to obtain a distribution on a large vocabulary. In order to speed up the training time and reduce compution, we tried to implement NCE, which is a method used to fit unnomarlised method and therefore avoids using the last softmax layer.

The principle behind NCE is to sample for every context in the training data K wrong words that do not appear next in the windows using a noise distribution such a multinomial of the vocabulary simplex. We then apply a log-bilinear model. For a given context $\boldsymbol{x}$ and target y, the probability of y being a correct word for this context is given by:

$$p(D = 1|\boldsymbol{x}, \boldsymbol{y}) = \sigma(\log p(\boldsymbol{y}|D = 1, \boldsymbol{x}) - \log Kp(\boldsymbol{y}|D = 0, \boldsymbol{x}))$$

where

$$p(D = 1|\boldsymbol{x}, \boldsymbol{y}) = \sigma(logp(\boldsymbol{y}|D = 1, \boldsymbol{x}) - log(Kp(\boldsymbol{y}|D = 0, \boldsymbol{x})))$$

If the $p(y|D = 1, x)$ term still forces some normalisation in the model, thanks to the contribution of Mnih and Teh (2012), we can estimate the normalisation constant as a parameter in the model and even set to equal to 1. We can therefore replace this term by the score outpute by the linear model $z_{\boldsymbol{x}_i, w_i}$. The objective function that needs to be minimised becomes:

$$\mathcal{L}(\theta) = \sum_i \log \sigma(z_{\boldsymbol{x}_i, w_i}) - \log(Kp_{ML}(w_i))) + \sum_{k=1}^{K} \log(1 - \sigma(z_{\boldsymbol{x}_i, s_k}) - \log Kp_{ML}(s_k))$$

where p_{ML} is the pdf of the noise distribution, and s_k for $k \in \{1, ..., K\}$ are samples from the noise distribution.

4 Experiments

We now present the results of our experiments. We will first talk about the preprocessing and then continue with a comparison of the different models.

4.1 Data and Preprocessing

To complete this homework, we were given 3 datasets, one for training, one for validation and one for testing. The train set consisted of sentences with a total of over eight hundred thousands words from a vocabulary of ten thousands words. The validation set consisted of 70391 words. The particularity of the issue at hand consisted in the fact that we had to only predict a probability distribution over 50 words and not on the entire vocabulary. This is why we were provided the same validation set in the same format as for the test set. We could therefore predict 3370 words on the validation set to help us predict the 3361 words of the test set.

Most of the preprocessing was about building the N-grams for the different sets. We included in the preprocess.py file different functions to evaluate the windows as well as counting the different occurences of each N-grams. For instance, looking at 6-grams gave:

- 772 670 unique 6-grams on the training set,
- 887 522 6-grams in total,
- 70 391 6-grams on the validation set,
- 3 370 words to predict on the validation set,
- and 3 361 words of the test set

4.2 Evaluation

To evaluate the models, we will use the perplexity measure. For a set of m N-grams, $\boldsymbol{w}_1, ..., \boldsymbol{w}_m$, it is defined to be:

$$P(w_1, ..., w_m) = \exp\left(-\frac{1}{m}\sum_{i=1}^{m}\log P(w_N^i | w_{N-1}^i, ..., w_1^i)\right)$$

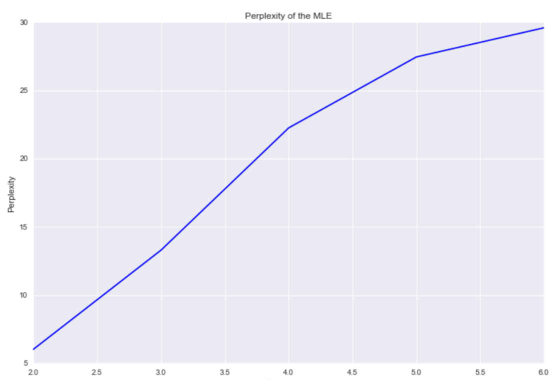
In other words, the perplexity translates how likely is the predicted word given the previous N-1 words. In this report, we will evaluated perplexity both on the entire vocabulary but also on the reduced 50 words to predict from. Values between these two ”different” perplexities wil range from 3 to 1000.

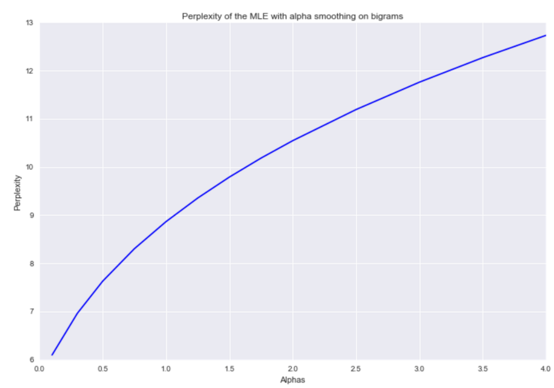
4.3 Count-based Models

We first present the results of the different count-based models:

Maximum Likelihood Estimation For this first model, the only hyperparameter is the number of N-grams in the feature. We applied this method on different N-gram models and provides the results. The computation time remains very low for each model; the only issue as we increase the size of the N-gram is the memory footprint.

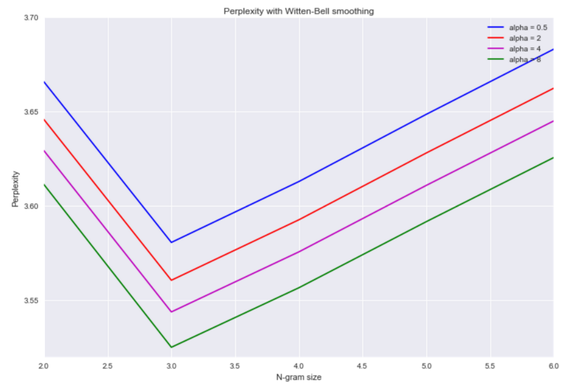
Laplace Smoothing We studied the impact of the alpha parameter and the best value with regard to the perlexity on the Kaggle development set.





Witten-Bell Smoothing For the Witten-Bell Smoothing, we benchmarked both the alpha parameter and the context from which to start the interpolation.

First, we applied a different normalization when we were using our model for the Kaggle dataset. For each entry in this dataset, we build a language model on a really small subset of the training dictionary (50 out of 10 001 words). As a result, only a small fraction of all the N-grams can be considered and we decided to compute the factor $N1$ and F_c only among the output vocabulary (50 words). This change lead to better results, we directly applied it on the modified Kneser-Ney.

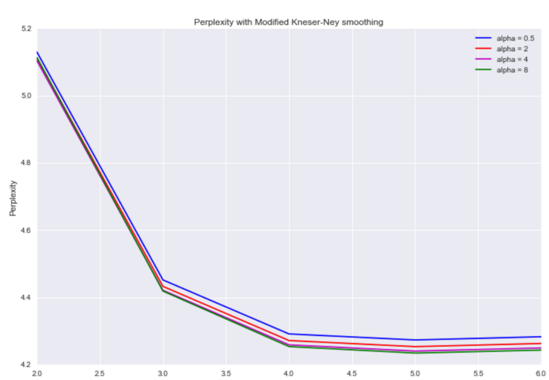


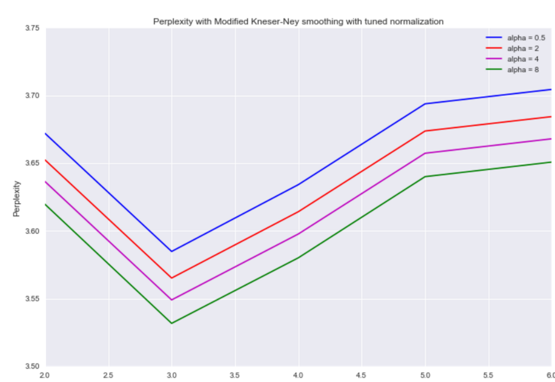
This plots shows that the Witten-Bell smoothing does not manage to successfully balance the different orders of the models when they are too many. The best perplexity is always reached for the tri-grams. This may be cause by the fact that with large context (N¿2), the probability may be disturbed by a lower order model very present even though the rest of the context is not (see the example of SAN FRANCISCO in the paper).

Moreover, we see that the perplexity gets smaller as α increases. This is because we may want a strong smoothing of the lower order model (here we use alpha only on the lower order model computed). The relative differences are bigger in the lower order models as the contexts are there more frequent (because smaller) so large smoothing may decrease their influence and favorise the long tail.

Modified Kneser-Ney This model introduces different hyperparameters. The absolute discounting D induces 2 kinds of hyperparameters: the number of values it takes and the values. We first applied those suggested in the paper but also decided to tune them manually as the paper provides better results in that case. We also observed this result.

We plot the results with a model with fixed parameters D as suggested in the paper. As with the Witten-Bell smoothing, we evaluated the models against different starting contexts. In that case, the best results were reached for the 3-grams.





The best perplexity was reached with a manually tuned D (on the Kaggle development set with the perplexity). The perplexity reached is 3.476.

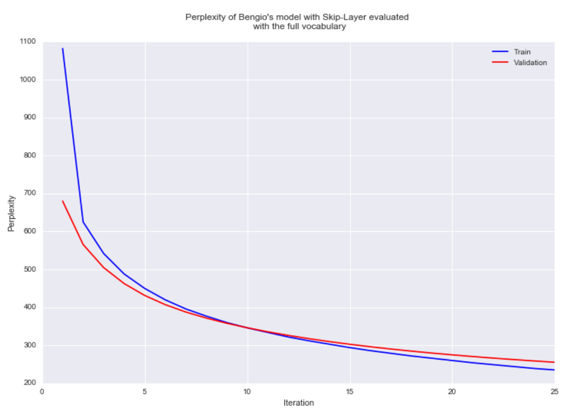
4.4 Neural Models

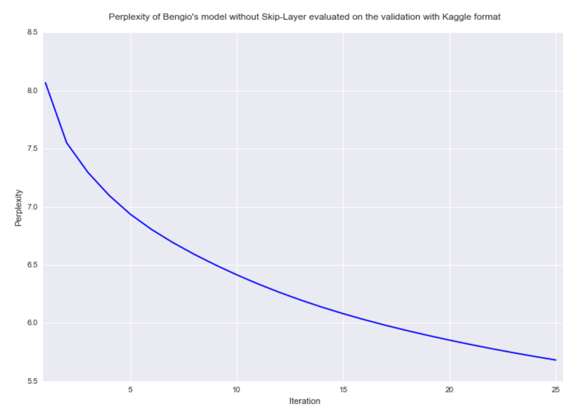
The main issue we faced with Bengio's model was training time. Even we managed to have a working model with a smaller vocabulary, we struggled at first to get a code that ran fast enough to experiment extensively with different paremetrisations. Our original code ran one epoch in about one hour for the paremetrisation. While trying to code the NCE approximation, we nevertheless managed to cut the training time to about 14-15 minutes.

We started to train the more simple neural network i.e. the one without the skip-layer, with the parametrisation suggested by Bengio:

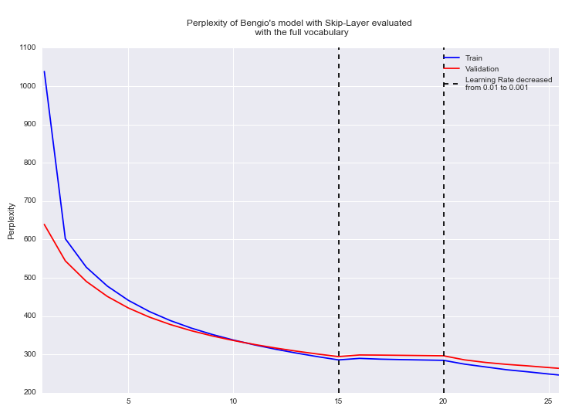
- Window size: $d_{win} = 5$
- Dimension of the embeddings: $d_{in} = 30$
- Hidden dimension: $d_{hid} = 100$

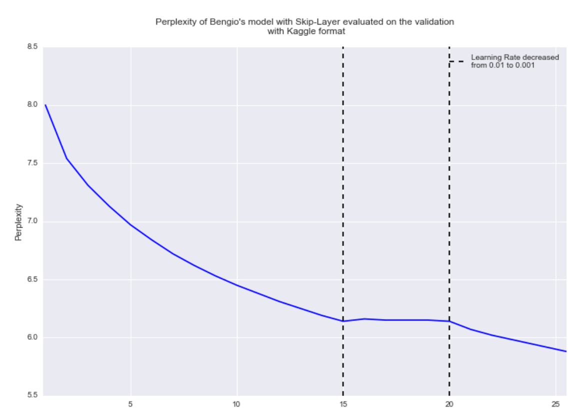
We summarize the results in the graphs below:





Before making a submission on kaggle, we decided to compare the encouraging results with the Skip-Layer model. We ran the experiment 5 epochs at a time to give us control on the learning rate. We thought that after the 15 epochs, the model was close to convergence, and decided to decrease the learning rate from 0.01 to 0.001. We obtain the following results:





As one can see, changing the learning rate negatively impacted the results. It plateaued but perplexity started decreasing again as soon as we re-up the learning rate. It also unclear how much improvement the skip-layer model brings compared to the original Bengio model, and this even without the change in learning rate. Based on these observation, we decided to submit to Kaggle the results of the simple model. We obtained:

$$Perp_{nnlm}^{test} = 5.47$$

Nevertheless, a final observation on the training of these NNLMs is that the models haven't seem to converge fully after 25 epochs. Running the algorithm for 5-10 extra epochs would most probably yielded better results and helped us reach the level of the count-base models.

4.5 NCE

We unfortunately did not succeed to implement a valid version of the Noise Contrastive Estimation. We did not managed to have a speed improvement and even worse observed that the perplexity on the validation was increasing instead of decreasing.

4.6 Mixtures of models

In order to increase our score, we decided to combine the differente approaches by averaging the results over the distributions outputted by various models.

We managed to reach our best perplexity with a weigthed mean of the Witten-Bell smoothing, the modified Kneser-Kney with tuned parameters and local normalization and the Neural Network.

Formally, the output is built as follows:

$$p(w|c) = \frac{2p_{WB}(w|c) + p_{mKN}(w|c) + p_{NN}(w|c)}{5}$$

We reached on the Kaggle development set: $perp_{Kaggle} = 3.36$
We provide here a summary of our models on the Kaggle development set:

Table 1: Final Performances

Model	Perplexity (dev Kaggle)
MLE with Laplace smoothing	6.01
Witten Bell (trigram)	3.52
Modified Kneser Kney	3.47
Neural Network	5.68
Neural Network with skip Layer	5.66
Ensemble (WB + mKN + NNsl)	**3.36**

5 Conclusion

In this homework we successfully build very efficient count-based Language models that yield great results. On the other hand, we were surprised to see that these models were beating in performance the model presented by Bengio which is much more elaborated. Nevertheless, we obtained the best results by taking a mixture of the two different types of results. These could be explained by the fact that the stochastic gradient descent may converge to different local minimums.

6 References

- Bengio, Y., Ducharme, R., Vincent, P., and Janvin, C. (2003). A neural probabilistic language model. *Journal of Machine Learning Research*, 3:11371155.
- Mnih, A. and Kavukcuoglu, K. (2013). Learning word embeddings efficiently with noise-contrastive estimation. *In Advances in Neural Information Processing Systems*, pages 22652273.
- Chen, S. and Goodman, J. (1998). An Empirical Study of Smoothing Techniques for Language. *Technical Report TR-10-98*, Computer Science Group, Harvard University.

Appendices

Preprocessing:

```python
#!/usr/bin/env python

"""Language modeling preprocessing
"""

import numpy as np
import h5py
import argparse
import sys
import re
import codecs
from collections import Counter

# Your preprocessing, features construction, and word2vec code.


def get_words2index(filename):
    '''
    Loading the tags to index mapping
    '''
    words2index = {}
    with open(filename) as f:
        for line in f:
            (val, key, num) = line.split()
            words2index[key] = int(val)
    return words2index


def get_index2words(filename):
    '''
    Loading the tags to index mapping
    '''
    index2words = {}
    with open(filename) as f:
        for line in f:
            (val, key, num) = line.split()
            index2words[int(val)] = key
    return index2words

index2words = get_index2words('data/words.dict')
index2words1000 = get_index2words('data/words.1000.dict')
```

```


def valid_test_Ngram(filepath, words2index, N, test=False):
    results = []
    if test == False:
        with open(filepath) as f:
            i = 1
            for line in f:
                lsplit = line.split()
                if lsplit[0] == 'Q':
                    topredict = np.array([words2index[x] for x in lsplit[1:]])
                if lsplit[0] == 'C':
                    l = np.append(
                        np.repeat(words2index['<s>'], N-1), [words2index[x] for x in lsplit[1:-1]])
                    lastNgram = l[-N+1:]
                    results.append((lastNgram, topredict))
    else:
        with open(filepath) as f:
            i = 1
            for line in f:
                lsplit = line.split()
                if lsplit[0] == 'Q':
                    topredict = np.array([words2index[x] for x in lsplit[1:]])
                if lsplit[0] == 'C':
                    l = np.append(
                        np.repeat(words2index['<s>'], N-1), [words2index[x] for x in lsplit[1:-1]])
                    lastNgram = l[-N+1:]
                    results.append((lastNgram, topredict))
    return results


def train_get_ngram(filename, words2index, N):
    '''
    Generating N-grams
    '''
    results = []
    with open(filename) as f:
        for line in f:
            lsplit = [words2index[x] for x in line.split()]
            l = np.append(np.repeat(words2index['<s>'], N-1), lsplit)

```

```
            for i in range(len(lsplit)):
                g = l[i:N-1+i]
                v = lsplit[i]
                results.append((g, v))
        results.append((l[-N+1:], words2index['</s>']))
    return results

def tomatrix(results, train=True, count = True):

    N = len(results[0][0])+1

    if train:
        if count:
            tuplelist = []
            for i in range(len(results)):
                tuplelist.append(
                    tuple(list(np.append(results[i][0], results[i][1]))))
            Count = Counter(tuplelist).most_common()
            tooutput = np.empty((len(Count), N+1))

            for i in range(len(Count)):
                tooutput[i, :] = np.append(np.array(Count[i][0]), Count[i][1])

            return tooutput.astype(int)

        else:
            tooutput_ = np.empty((len(results),N))
            for i in range(len(results)):
                tooutput_[i,:] = np.append(results[i][0],results[i][1])
            return tooutput_

    else:
        tooutput = np.empty((len(results), 50+N-1))

        for i in range(len(results)):
            tooutput[i, :] = np.hstack((results[i][1], results[i][0]))

        return tooutput.astype(int)

def validation_kaggle(filepath):
    it = 0
    results = []
    with open(filepath) as f:
```

```
        for line in f:
            if it == 0 :
                it+=1
            else:
                lsplit = line.split(',')
                l = [int(x.rstrip()) for x in lsplit[1:]]
                results.append(l)
    return np.array(results)


def get_prior(filepath, words2index):
    '''
    Case N=1: ie prior on the word distribution from the train text
    '''
    counter = Counter()
    with open(filepath) as f:
        lines = f.readlines()
        for line in lines:
            # Adding the end of line prediction
            lsplit = line.split() + ['</s>']
            counter.update(lsplit)
    # Build count matrix: (N_words, 2), col 1: word index, col2: word cout
    count_matrix = np.zeros((len(counter), 2), dtype=int)

    for i,t in enumerate(counter.iteritems()):
        k, v = t
        count_matrix[i, 0] = words2index[k]
        count_matrix[i, 1] = v
    return count_matrix


FILE_PATHS = ("data/train.txt",
                          "data/train.1000.txt",
             "data/valid.txt",
             "data/valid_blanks.txt",
             "data/test_blanks.txt",
             "data/words.dict",
             "data/words.1000.dict",
             "data/valid_kaggle.txt")
args = {}


def main(arguments):
    global args
```

```python
    parser = argparse.ArgumentParser(
        description=__doc__,
        formatter_class=argparse.RawDescriptionHelpFormatter)
    parser.add_argument('--N', default=3, type=int, help='Ngram size')
    args = parser.parse_args(arguments)
    N = args.N
    train, train1000, valid_txt, valid, test, word_dict, word_dict_1000, kaggle = FILE_PATHS

    words2index = get_words2index(word_dict)
    words2index1000 = get_words2index(word_dict_1000)
    index2words = get_index2words(word_dict)

    train_list = train_get_ngram(train, words2index, N)
    train_matrix_count = tomatrix(train_list)
    train_matrix = tomatrix(train_list,True,False)

    train_list_1000 = train_get_ngram(train1000, words2index1000, N)
    train_matrix_1000_count = tomatrix(train_list_1000,True,False)
    train_matrix_1000 = tomatrix(train_list_1000)

    valid_txt_list = train_get_ngram(valid_txt, words2index, N)
    valid_txt_matrix= tomatrix(valid_txt_list,True,False)

    valid_list = valid_test_Ngram(valid, words2index, N)
    valid_matrix = tomatrix(valid_list, False)

    test_list = valid_test_Ngram(test, words2index, N, True)
    test_matrix = tomatrix(test_list, False)

    valid_kaggle = validation_kaggle(kaggle)

    filename = str(N) + '-grams.hdf5'
    with h5py.File(filename, "w") as f:

        f['train'] = train_matrix_count
        f['train_1000_nocounts'] = train_matrix_1000
        f['train_1000'] = train_matrix_1000_count
        f['train_nocounts'] = train_matrix
        f['valid_txt'] = valid_txt_matrix
        f['valid'] = valid_matrix
        f['valid_output'] = valid_kaggle
        f['test'] = test_matrix
        f['nwords'] = np.array([np.max(index2words.keys())])
```

```python
if __name__ == '__main__':
    sys.exit(main(sys.argv[1:]))
```

Count-Based Models:

```lua
--------------------------------
---- helper
--------------------------------

-- Loading train of the gram_size N
function get_train(N)
    local filename = N .. '-grams.hdf5'
    --print(filename)
    myFile = hdf5.open(filename,'r')
    train = myFile:all()['train']
    myFile:close()
    return train
end

function perplexity(distribution, true_words)
    -- exp of the average of the cross entropy of the true word for each line
    -- true words (N_words to predict, one hot true value among 50)
    local perp = 0
    local N = true_words:size(1)
    for i = 1,N do
        mm,aa = true_words[i]:max(1)
        perp = perp + math.log(distribution[{i, aa[1]}])
    end
    perp = math.exp(- perp/N)
    return perp
end


function build_context_count(count_tensor)
    local indexes
    local indexN
    -- Ngram count (depend on w and context)
    -- {'index1-...-indexN-1': {'indexN' : count}}
    local F_c_w = {}
    -- F_c dict (independent of w, only context based)
    -- {index1-...-indexN-1 : count all words in c}
    local F_c = {}
    -- N_c dict (independent of w, only context based)
    -- {index1-...-indexN-1 : count unique type of words in c}
    local N_c = {}

```

```lua
    local N = count_tensor:size(1)
    local M = count_tensor:size(2)

    for i=1, N do
        indexN = count_tensor[{i,M-1}]

        -- build the key index1-...-indexN-1
        indexes = tostring(count_tensor[{i,1}])
        for j=2, M - 2 do
            indexes = indexes .. '-' .. tostring(count_tensor[{i,j}])
        end

        -- Filling F_c_w
        if F_c_w[indexes] == nil then
            F_c_w[indexes] = {[indexN] = count_tensor[{i, M}]}
        else
            F_c_w[indexes][indexN] = count_tensor[{i, M}]
        end

        -- Updating F_c and F_c
        if F_c[indexes] == nil then
            F_c[indexes] = count_tensor[{i, M}]
            N_c[indexes] = 1
        else
            F_c[indexes] = count_tensor[{i, M}] + F_c[indexes]
            N_c[indexes] = 1 + N_c[indexes]
        end
    end

    return F_c_w, F_c, N_c
end

--------------------
---- Maximum Likekihood Estimation
--------------------

function compute_mle_line(N, entry, F_c_w, alpha)
    -- Compute the maximum likelihood estimation with alpha smoothing on the
    -- input in entry,
    --
    -- Return vector (50) predicting the distribution from entry
    -- N represent the Ngram size used in the prediction so context is N-1 gram
    local prediction = torch.zeros(50)
```

```
    local indexN

    -- context (at least with one element)
    local indexes = tostring(entry[{1, entry:size(2)}])
    for j=entry:size(2) - 1, entry:size(2) - 1 - (N-3), -1 do
        indexes = tostring(entry[{1, j}]) .. '-' .. indexes
    end
    -- check if context is unseen, otherwise go to next context
    if F_c_w[indexes] == nil then
        --print('unseen context')
        prediction:fill(alpha)
    else
        -- Compute MLE for each word
        for j=1,50 do
            indexN = entry[{1, j}]
            if F_c_w[indexes][indexN] ~= nil then
                prediction[j] = F_c_w[indexes][indexN] + alpha
            else
                --print('unseen word')
                prediction[j] = alpha
            end
        end
    end

    return prediction:div(prediction:sum())
end

-- Prediction with the MLE (with Laplace smoothing, no back-off and interpolation)

function mle_proba(N, data, alpha)
    -- Output format: distribution predicted for each N word along the
    -- 50 possibilities
    local N_data = data:size(1)

    -- Train model
    local train = get_train(N)
    local F_c_w = build_context_count(train)

    -- Prediction
    local distribution = torch.zeros(N_data, 50)
    for i=1, N_data do
        distribution:narrow(1, i, 1):copy(compute_mle_line(N, data:narrow(1,i,1), F_c_w, alpha))
    end
```

```
    return distribution
end

--------------------
---- Witten-Bell
--------------------

function compute_wb_line(N, entry, F_c_w_table, alpha)
    -- Compute the interpolated Witten-Bell model where we jump tp lower
    -- order models if the context count is 0 or all the words counts in that
    -- context is 0 also.
    --
    -- Return vector (50) predicting the distribution from entry
    -- N represent the Ngram size used in the prediction so context is N-1 gram
    -- alpha is only used for the MLE without any context
    --
    -- NB: the normalization is done based on the words contained in the first 50
    -- columns of the entry as we are building a distribution on a sub sample of a
    -- dictionnary (so we are using the count only of these words to normalize).
    -- Hence the variable denom and N_c_local
    local prediction = torch.zeros(50)
    local indexN
    local indexes
    local denom
    local N_c_local

    -- case where computation only on the prior
    if N == 1 then
        for j=1,50 do
            indexN = entry[{1, j}]
            -- Corner case when prediction on words not on the dict (case for <s>)
            if F_c_w_table[1][tostring(indexN)] == nil then
                prediction[j] = 0
            else
                prediction[j] = F_c_w_table[1][tostring(indexN)][indexN] + alpha
            end
```

```
        end
        -- Normalizing
        return prediction:div(prediction:sum(1)[1])
    else
        -- Compute the MLE for current N
        -- context (at least with one element)
        indexes = tostring(entry[{1, entry:size(2)}])
        for j=entry:size(2) - 1, entry:size(2) - 1 - (N-3), -1 do
            indexes = tostring(entry[{1, j}]) .. '-' .. indexes
        end

        -- check if context is unseen, otherwise go to next context
        if F_c_w_table[N][indexes] == nil then
            --print('unseen context')
            return compute_wb_line(N-1, entry, F_c_w_table, alpha)
        end

        -- local variable initialization
        denom = 0
        N_c_local = 0
        -- Compute MLE for each word
        for j=1,50 do
            indexN = entry[{1, j}]
            if F_c_w_table[N][indexes][indexN] ~= nil then
                prediction[j] = F_c_w_table[N][indexes][indexN]
                denom = denom + F_c_w_table[N][indexes][indexN] + 1
                N_c_local = N_c_local + 1
            end
        end

        -- Check that MLE predicted at least one words, otherwise go to next context
        if prediction:sum(1)[1] == 0 then
            --print('unseen words')
            return compute_wb_line(N-1, entry, F_c_w_table, alpha)
        end

        -- Combining with next context
        prediction:add(compute_wb_line(N-1, entry, F_c_w_table, alpha):mul(N_c_local)):div(denom)
        return prediction
    end
end

-- Witten Bell: new version, computation done at once line by line
```

```
--
-- p_wb(w|c) = (F_c_w + N_c_. * p_wb(w|c'))/(N_c_. + F_c_.)
function distribution_proba_WB(N, data, alpha)
    local N_data = data:size(1)
    local M = data:size(2)

    -- Building the count matrix for each ngram size lower than N.
    local F_c_w_table = {}
    for i=1,N do
        train = get_train(i)
        F_c_w_table[i] = build_context_count(train)
    end

    -- Vector initialisation
    local distribution = torch.zeros(N_data, 50)
    for i=1,N_data do
        -- Compute witten bell for the whole line i
        distribution:narrow(1, i, 1):copy(compute_wb_line(N, data:narrow(1,i,1), F_c_w_table, alpha))
    end
    return distribution
end


------------------------
---- Modified Kneser Ney
------------------------

-- Version tailored for modified Kneser-Ney:
-- Modif: now we enable a local computation of D
-- (that will be based on the sub vocabulary used in the validation and tesst)

function build_context_count_split(count_tensor, K)
    -- count_tensor in format (N_words, N + 1):
    -- col1, ..., colN = indexes for the Ngram, colN+1 = N_gram count
    -- K: number of count separate cases (need K > 1, usually K = 3)
    --
    -- Ngram count (depend on w and context)
    -- {'index1-...-indexN-1': {'indexN' : count}}
    local F_c_w = {}
    -- n_table: stores the total number of N_grams ending with indexN
    -- with exact number of occurences stored in their key k:
    -- {k : {'indexN': # N_grams ending with indexN with exactly k occurences}}
```

```lua
    local n_table = {}
    for j=1,K+1 do
        n_table[j] = {}
    end

    local N = count_tensor:size(1)
    local M = count_tensor:size(2)

    for i=1, N do
        local indexN = count_tensor[{i,M-1}]

        -- build the key index1-...-indexN-1
        indexes = tostring(count_tensor[{i,1}])
        for j=2, M - 2 do
            indexes = indexes .. '-' .. tostring(count_tensor[{i,j}])
        end

        -- Filling F_c_w
        if F_c_w[indexes] == nil then
            F_c_w[indexes] = {[indexN] = count_tensor[{i, M}]}
        else
            F_c_w[indexes][indexN] = count_tensor[{i, M}]
        end

        -- Building the key to update the corresponding part of n_table
        if count_tensor[{i, M}] > K then
            key_N_c = K
        else
            key_N_c = count_tensor[{i, M}]
        end

        -- Updating n_table
        if count_tensor[{i, M}] <= K + 1 then
            if n_table[count_tensor[{i, M}]][indexN] == nil then
                n_table[count_tensor[{i, M}]][indexN] = 1
            else
                n_table[count_tensor[{i, M}]][indexN] = n_table[
                    count_tensor[{i, M}]][indexN] + 1
            end
        end
    end

    return F_c_w, n_table
end
```

```
-- V2: with local normalization on the validation sub vocabulary

function compute_mkn_line(N, entry, F_c_w_table, n_table, alpha, K, D)
    -- Compute the Modified Kneser Ney model where we jump to lower
    -- order models if the context count is 0 or all the words counts in that
    -- context is 0 also.
    --
    -- Return vector (50) predicting the distribution from entry
    -- N represent the Ngram size used in the prediction so context is N-1 gram
    -- alpha is only used for the MLE without any context
    local prediction = torch.zeros(50)
    local indexN
    local F_local
    local N_c_local = {}
    for k=1,K do
        N_c_local[k] = 0
    end
    local n_table_local = {}
    for k=1,K+1 do
        n_table_local[k] = 0
    end

    -- case where computation only on the prior
    if N == 1 then
        for j=1,50 do
            indexN = entry[{1, j}]
            -- Corner case when prediction on words not on the dict (case for <s>)
            if F_c_w_table[1][tostring(indexN)] == nil then
                prediction[j] = 0
            else
                prediction[j] = F_c_w_table[1][tostring(indexN)][indexN] + alpha
            end
        end
        -- Normalizing
        return prediction:div(prediction:sum(1)[1])
    else
        -- Compute the MLE for current N
        -- context (at least with one element)
        local indexes = tostring(entry[{1, entry:size(2)}])
        for j=entry:size(2) - 1, entry:size(2) - 1 - (N-3), -1 do
            indexes = tostring(entry[{1, j}]) .. '-' .. indexes
```

```
        end
        -- check if context is unseen, otherwise go to next context
        if F_c_w_table[N][indexes] == nil then
            --print('unseen context')
            return compute_mkn_line(N-1, entry, F_c_w_table, n_table, alpha, K, D)
        end

        -- Building local n_table
        for j=1,50 do
            indexN = entry[{1, j}]
            -- Updating local n_table
            for k=1,K+1 do
                -- Possible Case where there is no Ngrams ending with indexN with count of K
                if n_table[N][k][indexN] ~= nil then
                    n_table_local[k] = n_table_local[k] + n_table[N][k][indexN]
                end
            end
        end

        -- Check no 0 in n_table_local
        for k=1,K+1 do
            if n_table_local[k] == 0 then
                print('0 count in n_table_local for ', indexN, k, N)
                n_table_local[k] = 1
            end
        end

        -- Building D (needed to compute prediction rows)
        -- Computing local D

        if D == nil then
            local Y = n_table_local[1]/(n_table_local[1] + 2*n_table_local[2])
            D = {}
            for k=1,K do
                D[k] = k - (1 + k)*Y*n_table_local[1 + k]/n_table_local[k]
            end
        end

        F_local = 0
        -- Compute curent order level with modified absolute discouting
```

```
            for each word
        for j=1,50 do
            indexN = entry[{1, j}]
            -- case word seen
            if F_c_w_table[N][indexes][indexN] ~= nil then
                -- Building the key for the different case of absolute discounting
                if F_c_w_table[N][indexes][indexN] > K then
                    key_N_c = K
                else
                    key_N_c = F_c_w_table[N][indexes][indexN]
                end
                prediction[j] = F_c_w_table[N][indexes][indexN] - D[key_N_c]
                F_local = F_local + F_c_w_table[N][indexes][indexN]
                N_c_local[key_N_c] = N_c_local[key_N_c] + 1
            end
        end

        -- Check that MLE predicted at least one words, otherwise go to next context
        if prediction:sum(1)[1] == 0 then
            --print('unseen words')
            return compute_mkn_line(N-1, entry, F_c_w_table, n_table, alpha, K, D)
        end

        -- Computing factor of lower order model (no denominator because we normalize afterwards)
        local gamma = 0
        for k=1,K do
            if N_c_local[k] ~= nil then
                gamma = gamma + D[k]*N_c_local[k]
            end
        end
        if gamma < 0 then
            --print('gamma error')
            return compute_mkn_line(N-1, entry, F_c_w_table, n_table, alpha, K, D)
        end
        -- Combining with next context
        prediction:add(compute_mkn_line(N-1, entry, F_c_w_table, n_table, alpha, K, D):mul(gamma)):div(F_local)
        -- Normalization
        -- TODO: why??? We normalize at the end
```

```
        -- prediction:div(prediction:sum(1)[1])
        return prediction
    end
end

-- Modified Kneser Ney: computation done at once line by line
--
-- p_wb(w|c) = (F_c_w + N_c_. * p_wb(w|c'))/(N_c_. + F_c_.)
function distribution_proba_mKN(N, data, alpha, K, D)
    local N_data = data:size(1)
    local M = data:size(2)

    -- Building the count matrix for each ngram size lower than N.
    local F_c_w_table = {}
    local n_table = {}
    for i=1,N do
        train = get_train(i)
        F_c_w_table[i], n_table[i] = build_context_count_split2(train, K)
    end

    -- Vector initialisation
    local distribution = torch.zeros(N_data, 50)
    for i=1,N_data do
        -- Compute witten bell for the whole line i
        distribution:narrow(1, i, 1):copy(compute_mkn_line2(N, data:narrow(1,i,1), F_c_w_table, n_table, alpha, K, D))
    end
    --distribution:cdiv(distribution:sum(2):expand(distribution:size(1), distribution:size(2)))
    return distribution
end
```

<u>NNLM:</u>

```
function build_model(dwin, nwords, hid1, hid2)
    -- Model with skip layer from Bengio, standards parameters
    -- should be:
    -- dwin = 5
    -- hid1 = 30
    -- hid2 = 100

    -- To store the whole model
    dnnlm = nn.Sequential()

    -- Layer to embedd (and put the words along the window into one vector)
```

```
    LT = nn.Sequential()
    LT_ = nn.LookupTable(nwords,hid1)
    LT:add(LT_)
    LT:add(nn.View(-1, hid1*dwin))

    dnnlm:add(LT)

    concat = nn.ConcatTable()

    lin_tanh = nn.Sequential()
    lin_tanh:add(nn.Linear(hid1*dwin,hid2))
    lin_tanh:add(nn.Tanh())

    id = nn.Identity()

    concat:add(lin_tanh)
    concat:add(id)

    dnnlm:add(concat)
    dnnlm:add(nn.JoinTable(2))
    dnnlm:add(nn.Linear(hid1*dwin + hid2, nwords))
    dnnlm:add(nn.LogSoftMax())

    -- Loss
    criterion = nn.ClassNLLCriterion()

    return dnnlm, criterion
end


function train_model(train_input, dnnlm, criterion, dwin, nwords, eta,
   nEpochs, batchSize)
    -- Train the model with a mini batch SGD
    -- standard parameters are
    -- nEpochs = 1
    -- batchSize = 32
    -- eta = 0.01

    -- To store the loss
    av_L = 0

    -- Memory allocation
    inputs_batch = torch.DoubleTensor(batchSize,dwin)
    targets_batch = torch.DoubleTensor(batchSize)
    outputs = torch.DoubleTensor(batchSize, nwords)
```

```
    df_do = torch.DoubleTensor(batchSize, nwords)

    for i = 1, nEpochs do
        -- timing the epoch
        timer = torch.Timer()
        av_L = 0

        -- max renorm of the lookup table
        dnnlm:get(1):get(1).weight:renorm(2,1,1)

        -- mini batch loop
        for t = 1, train_input:size(1), batchSize do
            -- Mini batch data
            current_batch_size = math.min(batchSize,train_input:size(1)-t)
            inputs_batch:narrow(1,1,current_batch_size):copy(train_input:narrow(1,t,current_batch_size))
            targets_batch:narrow(1,1,current_batch_size):copy(train_output:narrow(1,t,current_batch_size))

            -- reset gradients
            dnnlm:zeroGradParameters()
            --gradParameters:zero()

            -- Forward pass (selection of inputs_batch in case the batch is not full, ie last batch)
            outputs:narrow(1,1,current_batch_size):copy(dnnlm:forward(inputs_batch:narrow(1,1,current_batch_size)))

            -- Average loss computation
            f = criterion:forward(outputs:narrow(1,1,current_batch_size), targets_batch:narrow(1,1,current_batch_size))
            av_L = av_L +f

            -- Backward pass
            df_do:narrow(1,1,current_batch_size):copy(criterion:backward(outputs:narrow(1,1,current_batch_size), targets_batch:narrow(1,1,current_batch_size)))
            dnnlm:backward(inputs_batch:narrow(1,1,current_batch_size), df_do:narrow(1,1,current_batch_size))
            dnnlm:updateParameters(eta)

        end

        print('Epoch '..i..': '..timer:time().real)
```

```
            print('Average Loss: '..av_L/math.floor(train_input:size(1)/batchSize))

        end

        return dnnlm

end
```