



Ministry of Education, Culture & Higher Education

National Examinations and Certifications Office

Form Four National Examinations

JUNE 2024

SUBJECT: Math

TIME: 2 HOURS

INSTRUCTIONS: Answer all questions in the ANSWER BOOKLET

PART ONE: MULTIPLE CHOICE QUESTIONS (20 × 2 = 40 MARKS).

1. The algebraic expression for half of  $x$  is

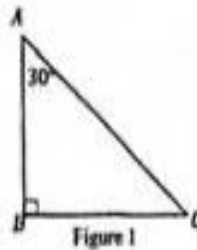
- a)  $x + 2$       b)  $x - 2$       c)  $2x$       d)  $\frac{x}{2}$

2.  $\cot x \cdot \sin 2x \cdot \sec^2 x =$

- a) 0      b) 1      c) 2      d) 4

3. The value of  $\widehat{C}$  in the figure below (figure 1) is

- a)  $50^\circ$       b)  $40^\circ$       c)  $60^\circ$       d)  $70^\circ$



4. The expression  $\sqrt[3]{16} \times \sqrt[3]{4}$  is equal

- a) 16      b) 4      c) 3      d) 12

5. The number 20.003 has \_\_\_\_\_ significant digits

- a) 2      b) 3      c) 4      d) 5

6. A group of 250 students took a test. If 76% of them passed the test. How many students passed the exam?

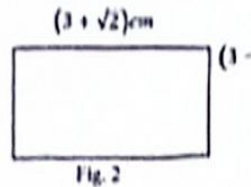
- a) 100      b) 190      c) 200      d) 60

7. If  $A = \begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}$ , then  $\det(A) =$

- a) 1      b) 9      c) -1      d) 0

8. Find the area of the rectangle below (Fig. 2)

- (a)  $9 \text{ cm}^2$   
 (b)  $2 \text{ cm}^2$   
 (c)  $7 \text{ cm}^2$   
 (d)  $5 \text{ cm}^2$



9. The factorization of  $a^3 - 8$  is:

- (a)  $(a + 2)(a^2 - 2a + 4)$   
 (b)  $(a - 2)(a^2 - 2a + 4)$   
 (c)  $(a + 2)(a^3 + 4a + 4)$   
 (d)  $(a - 2)(a^3 + 2a + 4)$

10. The mean of four numbers is 7, three of the numbers are 4, 6 and 10. The fourth number is:

- a) 10                      b) 4                      c) 6                      d) 8

11.  $\int_1^2 x dx =$

- a)  $\ln x + c$                       b)  $2 \ln x + c$                       c)  $\ln 2x + c$                       d)  $\ln \frac{x}{2} + c$

12. If  $z = 3 + 4i$  then the magnitude of  $z$ ,  $|z| =$

- a) 3                      b) 4                      c) 5                      d) 7

13. The  $\lim_{x \rightarrow -1} x^3 + x^2 + x + 1 =$

- a) 0                      b) 1                      c) 2                      d) 3

14. The derivative of  $y = x \cos x$  is

- a)  $y = \cos x - x \sin x$                       b)  $y = x \cos x$   
 c)  $y = x \sin x$                       d)  $y = \cos x + x \sin x$

15. The 6th term of a geometric series is 486 and the 7th term is 1458. Find the common ratio.

- a) 1                      b) 2                      c) 3                      d) 4

16. The value of  $\sin(\pi - x)$  is:

- a)  $\cos x$                       b)  $\sin x$                       c)  $\sec x$                       d)  $\tan x$

17.  $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} =$

- a)  $f(x)$                       b)  $f'(x)$                       c)  $f(a)$                       d)  $f'(a)$

18. From a class of 25 students 4 are selected to work on a special assignment. In how many different ways can the 4 students be selected?

- a) 6375600                      b) 12650                      c) 13800                      d) 303600

19. If  $P(A) = 0.53$ ,  $P(B) = 0.36$ , and  $P(A \cup B) = 0.74$ , then  $P(A \cap B) =$

- a) 0.47                      b) 0.64                      c) 0.15                      d) 0.26

20. The value of  $\frac{1}{i^2}$  is equal to

- a)  $i$                       b)  $-i$                       c) 1                      d)  $-1$

Part Two: Match Column A with the appropriate mathematical expression in column B (10 Marks)

COLUMN A	ANSWER	COLUMN B
1. If $f(s) = s^2 + 3s - 2$ , then $f(-2) =$	6	3
2. The integral of constant $k$ with respect to $x$ is	3	-2
3. $2(i)^{58} =$	10	4
4. $P(A')$ can not be higher than	1	-4
5. $6 \cos\left(\frac{\pi}{2} - x\right) \sec\left(\frac{\pi}{2} - x\right) =$	4	1
6. The number of rows of $\begin{pmatrix} 1 & 2 & 0 & 0 \\ 1 & 1 & 3 & 0 \\ 0 & 2 & 1 & 4 \end{pmatrix}$ is	9	15
7. There are 16 students in a class. If the ratio of boys to girls is 3:5. Then the number of girls in the class is	8	0
8. $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x}$	2	$kx + c$
9. The magnitude of the vector $\begin{pmatrix} 9 \\ 12 \end{pmatrix}$ is	5	6
10. The distance between $(-1, 3)$ and $(3, 3)$ is	7	10

## Trigonometry

1.  $\cot x = \tan 65^\circ$  (4 marks)

OR

$$\begin{aligned}
 \cot x &= \tan 65^\circ \\
 \tan(90^\circ - \theta) &= \tan 65^\circ \quad (1 \text{ mark}) \\
 90^\circ - \theta &= 65^\circ \quad (1 \text{ mark}) \\
 -\theta &= 65^\circ - 90^\circ \quad (1 \text{ mark}) \\
 -\theta &= -25^\circ \\
 \theta &= 25^\circ \quad (1 \text{ mark})
 \end{aligned}$$

$$\begin{aligned}
 \cot x &= \tan 65^\circ \\
 \text{Let } \cot x &= \frac{1}{\tan x} \quad (1 \text{ mark}) \\
 \text{Substitute} \\
 \frac{1}{\tan x} &= \tan 65^\circ \quad (2 \text{ marks}) \\
 x &= 65^\circ \quad (1 \text{ mark})
 \end{aligned}$$

2. Show that  $\cos 3x = 4\cos^3 x - 3\cos x$  (5 marks)

$$\cos 3x = \cos (2x + x) \quad (1 \text{ mark})$$

$$\cos (2x + x) = \cos 2x \cos x - \sin 2x \sin x \quad (1 \text{ mark})$$

$$= (2\cos^2 x - 1)\cos x - (2\sin x \cos x)\sin x$$

$$= 2\cos^3 x - \cos x - (2\sin^2 x \cos x) \quad (1 \text{ mark})$$

$$= 2\cos^3 x - \cos x - (2(1 - \cos^2 x)\cos x) \quad (1 \text{ mark})$$

$$= 2\cos^3 x - \cos x - (2(\cos x - \cos^3 x))$$

$$= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x \quad (1 \text{ mark})$$

$$= 4\cos^3 x - 3\cos x$$

### Complex Number

1. Use De Moivre's theorem to calculate the value of  $\frac{1}{(1-i)^6}$  [4 Marks]

$$R = \sqrt{1^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2} \quad (1 \text{ mark})$$

$$\theta = \tan^{-1}\left(\frac{-1}{1}\right) = -45^\circ \text{ which lies in Q IV} \quad (1 \text{ mark})$$

$$\therefore \theta = 0^\circ - 45^\circ \quad \text{or} \quad \theta = 360^\circ - 45^\circ$$

$$\theta = -45^\circ \quad \theta = 315^\circ$$

$$\frac{1}{(1-i)^6} = (1-i)^{-6} \quad \text{or} \quad \frac{1}{(1-i)^6} = (1-i)^{-6}$$

$$z^n = r^n(\cos n\theta + i\sin n\theta)$$

$$z^n = r^n(\cos n\theta + i\sin n\theta)$$

$$z^6 = (\sqrt{2})^{-6}(\cos -6(-45^\circ) + i\sin -6(-45^\circ)) \quad (1 \text{ mark}) = (\sqrt{2})^{-6}(\cos 315^\circ + i\sin 315^\circ) \quad (1 \text{ mark})$$

$$z^{-6} = (\sqrt{2})^{-6}(\cos(270) + i\sin(270)) = (\sqrt{2})^{-6}(\cos -6(315^\circ) + i\sin 6(315^\circ))$$

$$z^{-6} = \frac{1}{(\sqrt{2})^6}(\cos(270) + i\sin(270)) = \frac{1}{(\sqrt{2})^6}(\cos(-1890^\circ) + i\sin(-1890^\circ))$$

$$= \frac{1}{8}(\cos(270) + i\sin(270)) \quad (1 \text{ mark}) = \frac{1}{8}(0 + i(-1)) = -\frac{1}{8}i \quad (1 \text{ mark})$$

$$= \frac{1}{8}(0 + i(-1)) = -\frac{1}{8}i$$

2. Show that

[5 Marks]

$$\left[ \frac{1+i}{1-i} \right]^4 + \left[ \frac{1-i}{1+i} \right]^4 = 2$$

$$\left[ \frac{(1+i)(1+i)}{(1-i)(1+i)} \right]^4 + \left[ \frac{(1-i)}{(1+i)} \times \frac{(1-i)}{(1-i)} \right]^4 \quad (1 \text{ mark})$$

$$\left[ \frac{(1+i+i+i^2)}{(1-i^2)} \right]^4 + \left[ \frac{(1-i-i+i^2)}{(1-i^2)} \right]^4 \quad (1 \text{ mark})$$

$$\left[ \frac{(1+2i-1)}{(1+1)} \right]^4 + \left[ \frac{(1-2i-1)}{(1+1)} \right]^4 \quad (1 \text{ mark})$$

$$\left[ \frac{2i}{2} \right]^4 + \left[ \frac{-2i}{2} \right]^4 = 2 \quad (1 \text{ mark})$$

$$\left[ \frac{16}{16} \right] + \left[ \frac{16}{16} \right] = 2 \quad (1 \text{ mark})$$

$$1 + 1 = 2$$

### Statistics and probability

1. Given that the row data 4,6,8,8,18,9,9,18,12,18

Find

- a) Mean (3 Marks)
- b) Mode (3 Marks)
- c) Median (3 Marks)
- d) Range (3 Marks)

Ans :

$$a) \text{ Mean} = \frac{4+6+8+8+9+9+12+18+18+18}{10} = \frac{110}{10} = 11$$

$$b) \text{ Mode} = 18$$

$$c) \text{ Median} = \frac{9+9}{2} = 9$$

$$d) \text{ Range} = 18 - 4 = 14$$

2. Find the value of N that satisfies  $n_{C_2} = 15$  (5 Marks)

Ans

$$n_{C_2} = 15$$

$$\begin{aligned} n_{C_2} &= \frac{n!}{r!(n-r)!} \\ \frac{n(n-1)(n-2)!}{2!(n-2)!} &= 15 \quad (1 \text{ Mark}) \\ \frac{n(n-1)(n-2)!}{2 \times 1(n-2)!} &= 15 \end{aligned}$$

$$\frac{n(n-1)}{2} = 15 \quad (1 \text{ Mark})$$

$$n(n-1) = 30 \quad (1 \text{ Mark})$$

$$n^2 - n - 30 = 0 \quad (2 \text{ Marks})$$

NB by solving this equation can use one of the 3 methods

- 1. Factorization
- 2. Quadratic formula and by completing the square

by factorization

$$= (n-6)(n+5) = 0$$

$$\therefore n = 6 \text{ or } n = -5 \text{ so we will take } +6$$



### Calculus

1. Find  $\lim_{x \rightarrow 4} \frac{x^2-16}{x-4}$  (3 Marks)

Ans

$$\lim_{x \rightarrow 4} \frac{x^2-16}{x-4} = \lim_{x \rightarrow 4} \frac{(x+4)(x-4)}{(x-4)} \quad (1 \text{ Mark})$$

$$= \lim_{x \rightarrow 4} (x + 4) \quad (1 \text{ Mark})$$

$$= 4 + 4$$

$$= 8 \quad (1 \text{ Mark})$$

2. Find the first and second derivative of (2 Marks)

$$y = x^5 - 2x^2 + 3x - 1$$

$$\text{Ans } \frac{dy}{dx} = 5x^4 - 4x + 3 \quad (1 \text{ Mark})$$

$$\frac{d^2y}{dx^2} = 20x^3 - 4 \quad (1 \text{ Mark})$$

3. If  $y = \sin 5x - \cos 3x$ , find  $\frac{dy}{dx}$  ? (2 Marks)

Ans we use chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$y = \sin 5x - \cos 3x \quad \text{let } u = 5x \quad \text{and} \quad u = 3x$$
$$\frac{dy}{du} = \cos u = \cos 5x \quad \text{and} \quad \frac{dy}{du} = -\sin u = \sin 3x$$

$$\frac{du}{dx} = 5 \quad \text{and} \quad \frac{du}{dx} = 3 \quad (1 \text{ Mark})$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 5\cos 5x + 3\sin 3x \quad (1 \text{ Mark})$$

4. Find the value  $\int_{-1}^2 (3x^2 + 2x + 1)dx$  (4 Marks)

Ans

$$\int_{-1}^2 (3x^2 + 2x + 1)dx =$$

$$= \left( \frac{3x^3}{3} + \frac{2x^2}{2} + x + c \right) \quad (1 \text{ Mark})$$

$$= \left( \frac{3(2)^3}{3} + \frac{2(2)^2}{2} + (2) + c \right) - \left( \frac{3(-1)^3}{3} + \frac{2(-1)^2}{2} + (-1) + c \right) \quad (1 \text{ Mark})$$

$$= \left( \frac{3(8)}{3} + \frac{2(4)}{2} + 2 + c \right) - (-1 + 1 - 1 + c)$$

$$= \left( \frac{24}{3} + \frac{8}{2} + 2 + c \right) - (-1 + c) \quad (1 \text{ Mark})$$

$$= (8 + 4 + 2 + c) - (-1 + c) = 14$$

$$= (14 + 1)$$

$$= 15 \quad (1 \text{ Mark})$$

OR

$$\int_{-1}^2 (3x^2 + 2x + 1)dx =$$

$$= \left( \frac{3x^3}{3} + \frac{2x^2}{2} + x + c \right) \quad (1 \text{ Mark})$$

$$= (x^3 + x^2 + x + c)$$

$$= ((2)^3 + (2)^2 + 2 + c) - ((-1)^3 + (-1)^2 + (-1) + c) \quad (1 \text{ Mark})$$

$$= (8 + 4 + 2 + c) - (-1 + 1 - 1 + c) = 14 \quad (1 \text{ Mark})$$

$$= (14 + 1)$$

$$= 15 \quad (1 \text{ Mark})$$



5. Evaluate  $\int \frac{1-\cos^2 x}{\cos^2 x} dx$  (4 Marks)

$$\int \frac{1-\cos^2 x}{\cos^2 x} dx = \int \left( \frac{1}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x} \right) dx \quad (1 \text{ Mark}) \quad \text{or} \quad \int \frac{1-\cos^2 x}{\cos^2 x} dx = \int \frac{\sin^2 x}{\cos^2 x} dx$$

$$= \int (\sec^2 x - 1) dx \quad (1 \text{ Mark}) \quad = \int \tan^2 x dx$$

$$= \int \sec^2 x dx - \int dx \quad (1 \text{ Mark}) \quad = \int (\sec^2 x - 1) dx$$

$$= \tan x - x + c \quad (1 \text{ Mark}) \quad = \tan x - x + c$$