CAPS

Mind the Gap!

Mathematics
Study Guide

Grade 12

basic education

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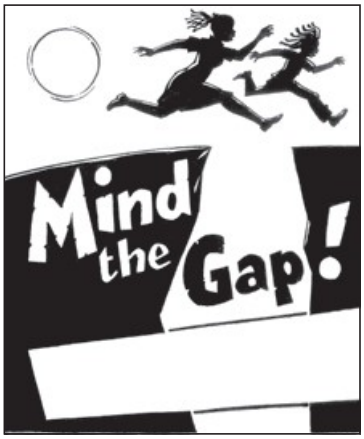

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Curriculum and Assessment Policy Statement (CAPS) Grade 12

Mind the Gap study guide for Mathematics

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The first edition, published in 2012, for the Revised National Curriculum Statement (RNCS) Grade 12 Mind the Gap study guides for Accounting, Economics, Geography and Life Sciences; the second edition, published in 2014 , aligned these titles to the Curriculum and Assessment Policy Statement (CAPS) and added more titles to the series in 2015, including the CAPS Grade 12 **Mind the Gap study guide for Mathematics.**
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Ministerial foreword

Matsie Angelina Motshekga, MP
Minister of Basic Education

The Department of Basic Education (DBE) has pleasure in releasing the second edition of the *Mind the Gap* study guides for Grade 12 learners. These study guides continue the innovative and committed attempt by the DBE to improve the academic performance of Grade 12 candidates in the National Senior Certificate (NSC) examination.

The study guides have been written by teams of exerts comprising teachers, examiners, moderators, subject advisors and coordinators. Research, which began in 2012, has shown that the *Mind the Gap* series has, without doubt, had a positive impact on grades. It is my fervent wish that the *Mind the Gap* study guides take us all closer to ensuring that no learner is left behind, especially as we celebrate 20 years of democracy.

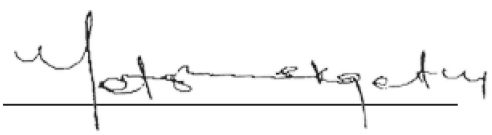
The second edition of *Mind the Gap* is aligned to the 2014 Curriculum and Assessment Policy Statement (CAPS). This means that the writers have considered the National Policy pertaining to the programme, promotion requirements and protocols for assessment of the National Curriculum Statement for Grade 12 in 2014.

The CAPS aligned *Mind the Gap* study guides take their brief in part from the 2013 National Diagnostic report on learner performance and draw on the Grade 12 Examination Guidelines. Each of the *Mind the Gap* study guides defines key terminology and offers simple explanations and examples of the types of questions learners can expect to be asked in an exam. Marking memoranda are included to assist learners to build their understanding. Learners are also referred to specific questions from past national exam papers and examination memos that are available on the Department's website – www.education.gov.za.

The CAPS editions include Accounting, Economics, Geography, Life Sciences, Mathematics, Mathematical Literacy and Physical Sciences Part 1: Physics and Part 2: Chemistry. The series is produced in both English and Afrikaans. There are also nine English First Additional Language (EFAL) study guides. These include EFAL Paper 1 (Language in Context); EFAL Paper 3 (Writing) and a guide for each of the Grade 12 prescribed literature set works included in Paper 2. These are Short Stories, Poetry, *To Kill a Mockingbird, A Grain of Wheat, Lord of the Flies, Nothing but the Truth* and *Romeo and Juliet*. (Please remember when preparing for EFAL Paper 2 that you need only study the set works you did in your EFAL class at school.)

The study guides have been designed to assist those learners who have been underperforming due to a lack of exposure to the content requirements of the curriculum and aim to mind-the-gap between failing and passing, by bridging the gap in learners' understanding of commonly tested concepts, thus helping candidates to pass.

All that is now required is for our Grade 12 learners to put in the hours required to prepare for the examinations. Learners, make us proud – study hard. We wish each and every one of you good luck for your Grade 12 examinations.



Matsie Angelina Motshekga, MP
Minister of Basic Education
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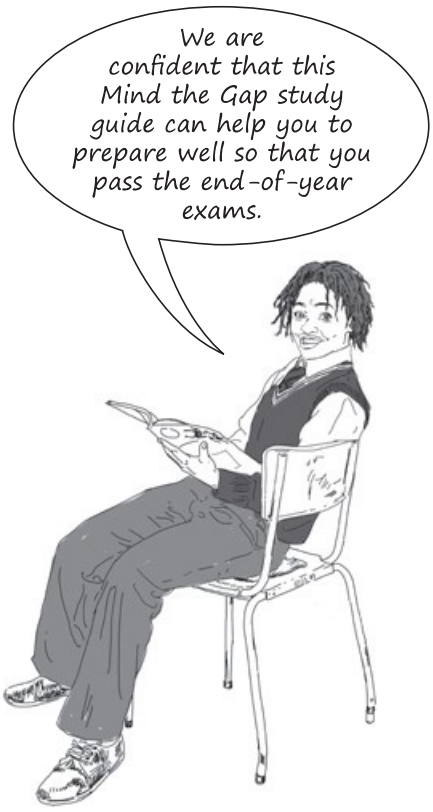
Dear Grade 12 learner



This *Mind the Gap* study guide helps you to prepare for the end-of-year CAPS Grade 12 exam.

The study guide does NOT cover the entire curriculum, but it does focus on core content of each knowledge area and points out where you can earn easy marks.

You must work your way through this study guide to improve your understanding, identify your areas of weakness and correct your own mistakes.

To ensure a good pass, you should also cover the remaining sections of the curriculum using other textbooks and your class notes.

Overview of the Grade 12 exam

The following topics make up each of the TWO exam papers that you write at the end of the year:

Paper	Topics	Duration	Total	Date	Marking
1	Patterns and sequences Finance, growth and decay Functions and graphs Algebra, equations and inequalities Differential Calculus Probability	3 hours	150	October/ November	Externally
2	Euclidean Geometry Analytical Geometry Statistics and regression Trigonometry	3 hours	150	October/ November	Externally

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Cognitive level	Description of skills to be demonstrated	Weighting	Approximate number of marks in a 150-mark paper
Knowledge	• Recall • Identification of correct formula on the information sheet (no changing of the subject) • Use of mathematical facts • Appropriate use of mathematical vocabulary • Algorithms • Estimation and appropriate rounding of numbers	20%	30 marks
Routine Procedures	• Proofs of prescribed theorems and derivation of formulae • Perform well-known procedures • Simple applications and calculations which might involve few steps • Derivation from given information may be • involved • Identification and use (after changing the • subject) of correct formula • Generally similar to those encountered in class	35%	52–53 marks
Complex Procedures	• Problems involve complex calculations and/or • higher order reasoning • There is often not an obvious route to the • solution • Problems need not be based on a real world • context • Could involve making significant connections • between different representations • Require conceptual understanding • Learners are expected to solve problems by • integrating different topics.	30%	45 marks
Problem Solving	• Non-routine problems (which are not necessarily difficult) • Problems are mainly unfamiliar • Higher order reasoning and processes are involved • Might require the ability to break the problem down into its constituent parts • Interpreting and extrapolating from solutions obtained by solving problems based in unfamiliar contexts.	15%	22–23 marks