Linear Algebra

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AGENDA

01 Connection between linear algebra and data science

02 Vector

03 Matrices

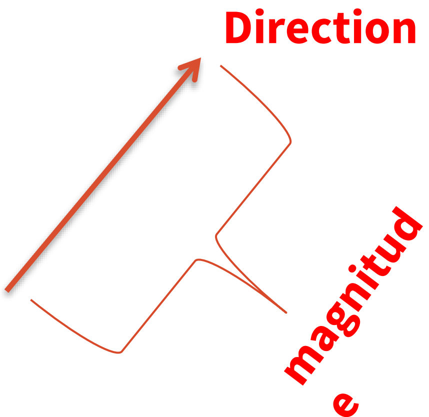
04 coding

Connection between linear algebra and data science?

Linear algebra is at the core of data science and is then largely used in the most powerful Machine Learning algorithms about recommendation engines (Netflix, Amazon…), Natural Language Processing (Alexa, Siri,...)
computer vision, etc.

– Vector :

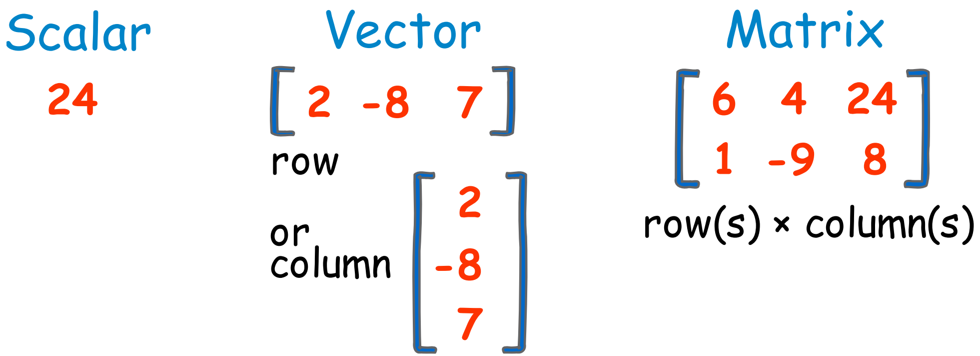
"An array of numbers, either continuous or discrete, but most machine-learning /data science problems deal with fixed-length vectors"

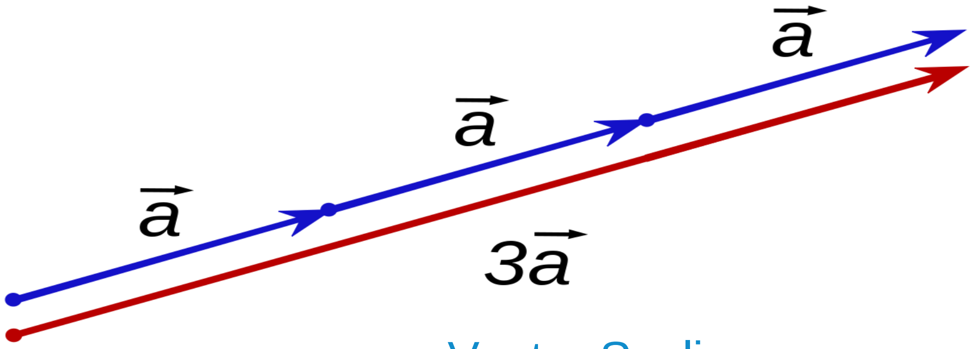


$$\begin{bmatrix} 20 \\ 10 \\ 15 \\ 22 \end{bmatrix}$$

Scalar:

one-dimensional vector is a scalar.
is a quantity that has only magnitude and no direction.
Unlike the vector that has direction
and magnitude.

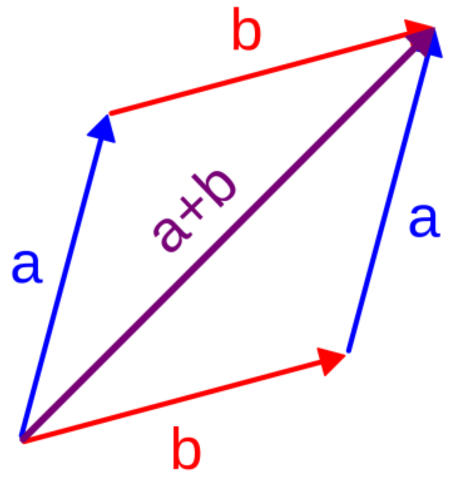


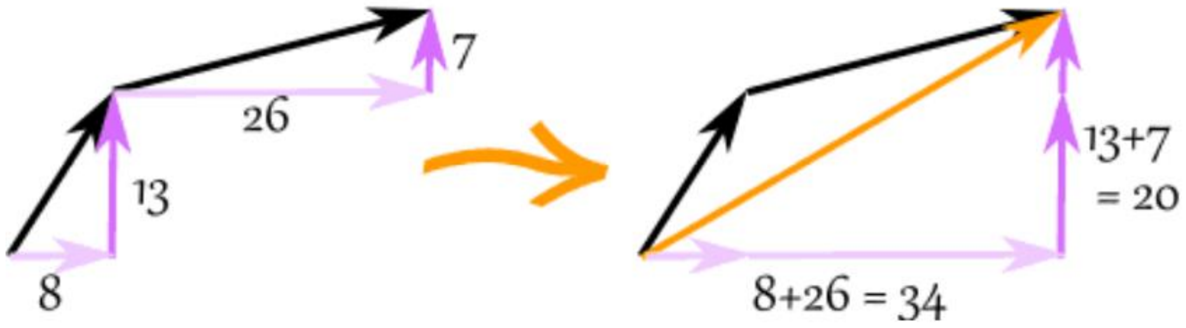


Vector Scaling

Vector Addition

$$\begin{bmatrix} 8 \\ 13 \end{bmatrix} + \begin{bmatrix} 26 \\ 7 \end{bmatrix} = \begin{bmatrix} 34 \\ 20 \end{bmatrix}$$

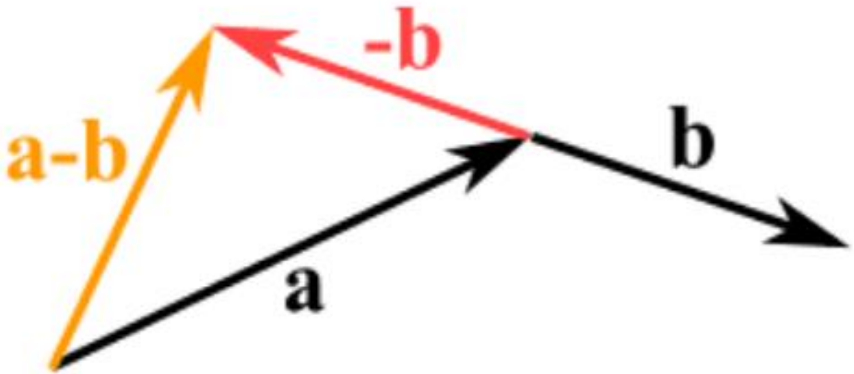




Vector Subtraction

first we reverse the direction of the vector we want to subtract, then add them as usual

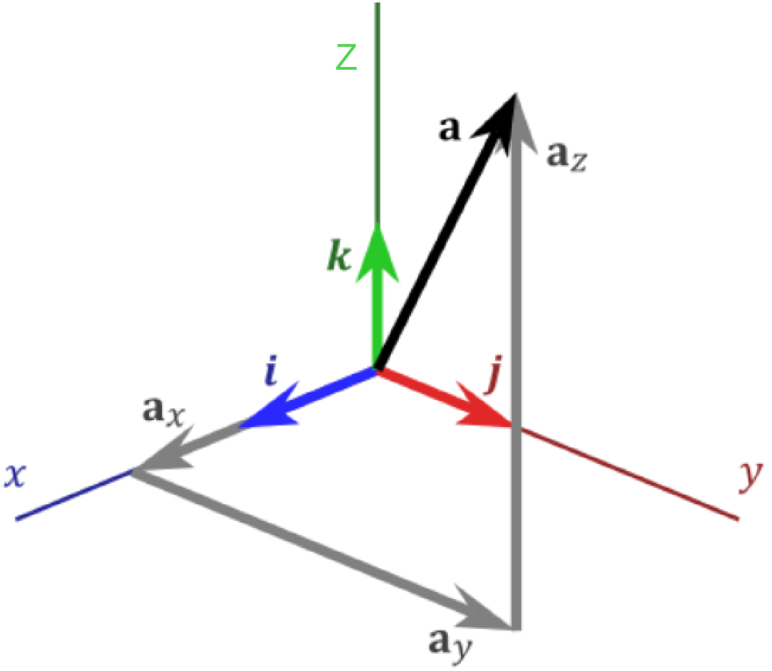
$$\begin{bmatrix} 25 \\ 9 \end{bmatrix} - \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 22 \\ 8 \end{bmatrix}$$

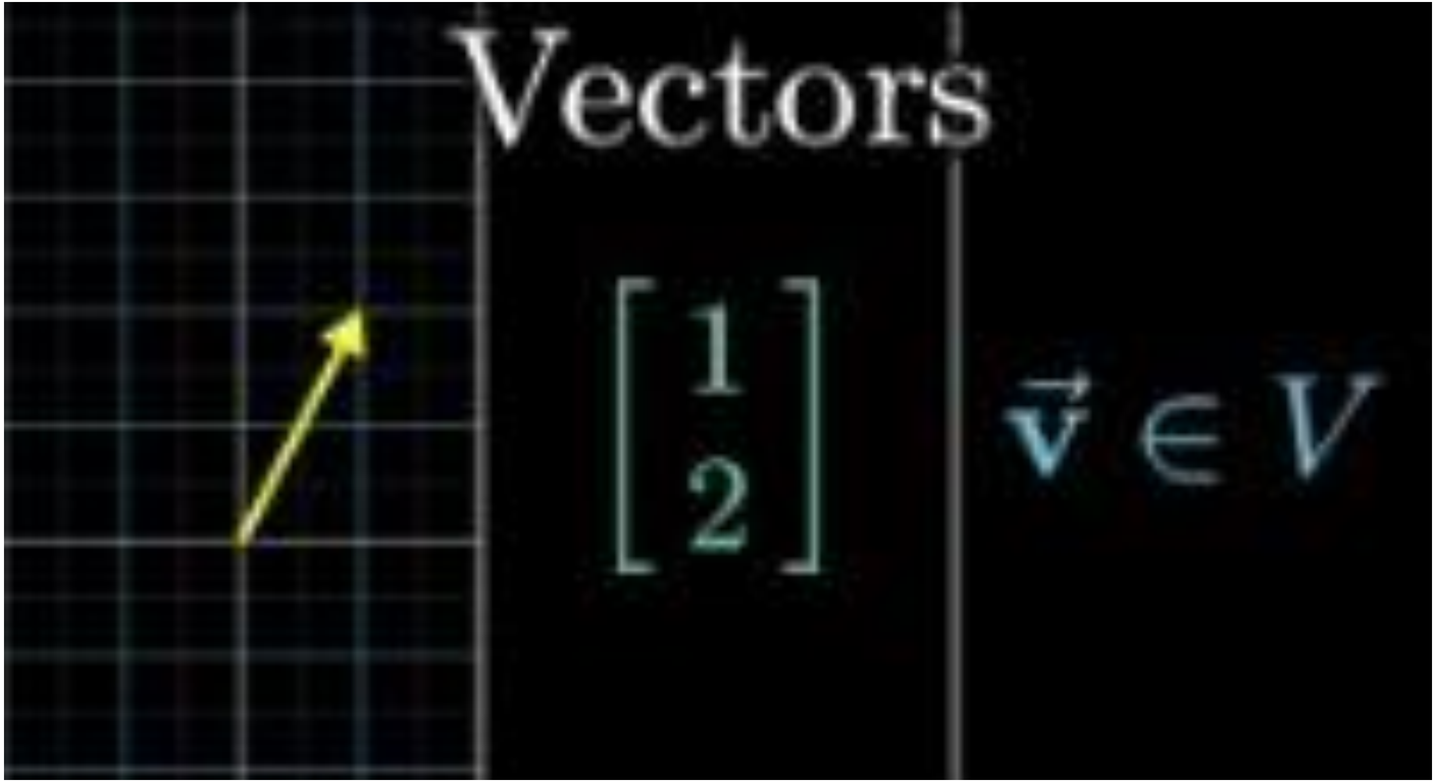


Vector Addition & Subtraction (3D)

$$\begin{bmatrix}3\\7\\4\end{bmatrix} + \begin{bmatrix}2\\9\\11\end{bmatrix} = \begin{bmatrix}5\\16\\15\end{bmatrix}$$

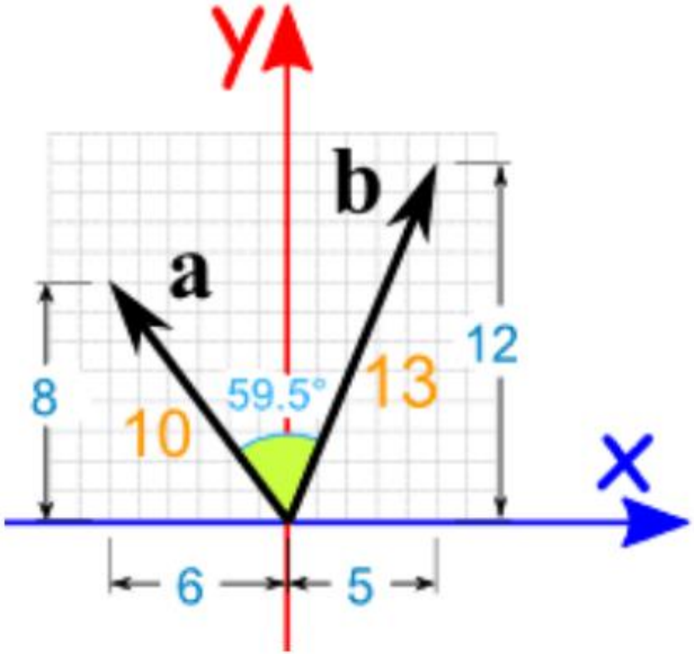
$$\begin{bmatrix}3\\7\\4\end{bmatrix} - \begin{bmatrix}2\\9\\11\end{bmatrix} = \begin{bmatrix}1\\-2\\-7\end{bmatrix}$$




Vectors
$\begin{bmatrix}1\\2\end{bmatrix}$
$\vec{\mathbf{v}} \in V$

Vector Dot/Inner Product

- ***The result is Scalar***
- $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| \times |\mathbf{b}| \times \cos(\theta)$
- $\mathbf{a} \cdot \mathbf{b} = 10 \times 13 \times \cos(59.5°)$
- $\mathbf{a} \cdot \mathbf{b} = 10 \times 13 \times 0.5075...$
- $\mathbf{a} \cdot \mathbf{b} = 65.98... = 66$ (rounded)
- $\mathbf{a} \cdot \mathbf{b} = a_x \times b_x + a_y \times b_y$
- $\mathbf{a} \cdot \mathbf{b} = -6 \times 5 + 8 \times 12$
- $\mathbf{a} \cdot \mathbf{b} = -30 + 96$
- $\mathbf{a} \cdot \mathbf{b} = 66$



$$v_1.v_2 = v_1^T v_2 = v_2^T v_1 = v_{11}v_{21} + v_{12}v_{22} + \ldots + v_{1n}v_{2n} = \sum_{k=1}^{n} v_{1k}v_{2k}$$

Matrices

A matrix is a multi-dimensional array that has a fixed number of rows and columns and contains a number at the intersection of each row and column. A matrix is usually delimited by square brackets.

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

A rectangular array of numbers of the form

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

Is called an $m \times n$ matrix, where m and n represents the number of rows and columns, respectively. A can be written as $A = [a_{ij}]$, where a_{ij} is the (i^{th}, j^{th}) element of matrix A. The element of a matrix can be real or complex numbers.

Matrix A can be considered a matrix that contains n number of column vectors $\in \mathbb{R}^m$ stacked side-by-side. We represent the matrix as $A_{m \times n} \in \mathbb{R}^{m \times n}$.

Addition of Two Matrices:

The addition of two matrices A and B implies their element-wise addition. We can only add two matrices, provided their dimensions match.

- Example

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \text{ then } A + B = \begin{bmatrix} 1+5 & 2+6 \\ 3+7 & 4+8 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$$

Subtraction of Two Matrices:

The subtraction of two matrices A and B implies their element-wise subtraction.

We can only subtract two matrices provided their dimensions match.

- Example

$$A=\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix} B=\begin{bmatrix}5 & 6\\7 & 8\end{bmatrix} \text{ then } A-B=\begin{bmatrix}1-5 & 2-6\\3-7 & 4-8\end{bmatrix}=\begin{bmatrix}-4 & -4\\-4 & -4\end{bmatrix}$$

Transpose of a Matrix:

- The transpose of a matrix $A \in R^{m \times n}$ is generally represented by $A^T \in R^{n \times m}$ and is obtained by transposing the column vectors as row vectors.

$$a'_{ji} = a_{ij} \quad \forall\, i \in \{1,2,..m\}, \forall\, j \in \{1,2,..n\}$$

$$where\ a'_{ji} \in A^T\ and\ a_{ij} \in A$$

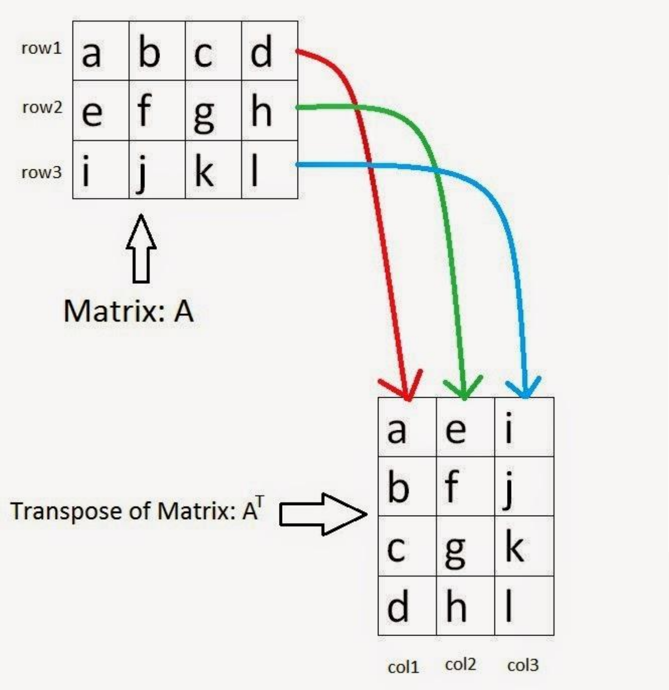
Transpose of a Matrix

$$\begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$$

Input Matrix — Transpose Matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

Example of transpose of a matrix



$$\begin{bmatrix} 6 & 4 & 24 \\ 1 & -9 & 8 \end{bmatrix}^T = \begin{bmatrix} 6 & 1 \\ 4 & -9 \\ 24 & 8 \end{bmatrix}$$

Product of Two Matrices

- For two matrices $A \in R^{m\times n}$ and $B \in R^{p\times q}$ to be multipliable, n should be equal to p. The resulting matrix is $A \in R^{m\times n}$. The elements of C can be expressed as $C \in R^{m\times q}$

$$c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj} \qquad \forall i \in \{1,2,..m\}, \forall j \in \{1,2,..q\}$$

- **Example**

The matrix multiplication of the two matrices A, $B \in R^{2\times 2}$ can be computed as seen here:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$$c_{11} = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 7 \end{bmatrix} = 1\times 5 + 2\times 7 = 19 \quad c_{12} = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 6 \\ 8 \end{bmatrix} = 1\times 6 + 2\times 8 = 22$$

$$c_{21} = \begin{bmatrix} 3 & 4 \end{bmatrix} \begin{bmatrix} 5 \\ 7 \end{bmatrix} = 3\times 5 + 4\times 7 = 43 \quad c_{22} = \begin{bmatrix} 3 & 4 \end{bmatrix} \begin{bmatrix} 6 \\ 8 \end{bmatrix} = 3\times 6 + 4\times 8 = 50$$

$$C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

Matrix-Vector Product

- To multiply a matrix A by a vector $\boldsymbol{x}$, the number of columns in A must equal the number of rows in $\boldsymbol{x}$.
- So, if A is an $m \times n$ matrix, then the product $A\boldsymbol{x}$ is defined for $n \times 1$ column vectors $\boldsymbol{x}$. If we let $A\boldsymbol{x} = b$, then b is an $m \times 1$ column vector. The number of rows of A determines the number of rows in the product b.
- Performing the multiplication can be explained in two different ways:
  1. Multiplication a row at a time.
  2. Combination of columns.

1. **Multiplication a row at a time**: Taking the dot product of each row of A with the vector $\boldsymbol{x}$:

$$A\boldsymbol{x} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} (a_{11}, a_{12}, \dots, a_{1n}) \cdot (x_1, x_2, \dots, x_n) \\ (a_{21}, a_{22}, \dots, a_{2n}) \cdot (x_1, x_2, \dots, x_n) \\ \vdots \\ (a_{m1}, a_{m2}, \dots, a_{mn}) \cdot (x_1, x_2, \dots, x_n) \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

2. Combination of columns: The matrix A acts on $\boldsymbol{x}$ vector . The output $A\boldsymbol{x}$ is a **combination** b of **the columns** of A :

$$A\boldsymbol{x} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = x_1 \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix} + x_2 \begin{bmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{bmatrix} + \cdots + x_n \begin{bmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

$$A\boldsymbol{x} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \end{bmatrix}$$

Example

- The transpose of the product of two matrices A and B is the product of the transposes of matrices A and B in the reverse order; i.e $(AB)^T = B^T A^T$
- For example, if we have two matrices $A = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$, then

$$(AB) = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix} \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 95 & 132 \\ 301 & 400 \end{bmatrix} \text{ and hence } (AB)^T = \begin{bmatrix} 95 & 301 \\ 132 & 400 \end{bmatrix}$$

Now, $A^T = \begin{bmatrix} 19 & 43 \\ 22 & 50 \end{bmatrix}$ and $B^T = \begin{bmatrix} 5 & 7 \\ 6 & 8 \end{bmatrix}$

$$B^T A^T = \begin{bmatrix} 5 & 7 \\ 6 & 8 \end{bmatrix} \begin{bmatrix} 19 & 43 \\ 22 & 50 \end{bmatrix} = \begin{bmatrix} 95 & 301 \\ 132 & 400 \end{bmatrix}$$

Basic Matrix Operation

- Matrix multiplication is **ASSOCIATIVE**:

$$\mathbf{A}(\mathbf{BC}) = (\mathbf{AB})\mathbf{C}$$

- Matrix operations are **DISTRIBUTIVE**:

$$\mathbf{A}(\mathbf{B}+\mathbf{C}) = \mathbf{AB}+\mathbf{AC}$$

$$(\mathbf{B}+\mathbf{C})\mathbf{D} = \mathbf{BD}+\mathbf{CD}$$

$$\boxed{(AB)^T = B^T A^T}$$

- Matrix multiplication is **NOT COMMUTATIVE**:

(usually)

$$AB \neq BA$$

THE TASK

- **Find C = AB then get C transpose:**

$$a)\ A = \begin{bmatrix} 3 & 1 & 0 \\ 2 & 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ 2 & 5 \\ -1 & 3 \end{bmatrix}$$

$$b)\ A = \begin{bmatrix} 1 & 0 & 3 \\ 4 & 5 & -1 \\ 0 & 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

[MA214: Linear Algebra]

[khan Academy]

[MIT 18.065]

[3Blue1Brown]

THANKS
Any questions?

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