Highway Patrol Officer Scheduling Using an Optimization-based Scheduling Model

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Abstract

***Abstract*—In light of the need to cut personnel cost and appease officer complaints about shift schedules, the National Highway Police Bureau needs to evaluate the efficiency of its manually designed shift schedules. This research applies integer linear programming and develops a scheduling model to help the NHPB solve the problem. The model is combined with Monte Carlo simulation method for data generation and performs the ANOVA test and paired t-test to verify feasibility of the model. A case study is conducted on one highway platoon. The results indicate that the model provides an alternative scheduling model to cut cost and help settle complaint problems.**

Keywords-component; scheduling models, manpower, integer linear program, statistical test

I. INTRODUCTION

In recent years, the central government in Taiwan has ordered all government departments to streamline their operations. As a result, the National Highway Police Bureau (NHPB) has been trying to revamp its patrol officer scheduling procedures in order to cut back personnel budgets without jeopardizing operational effectiveness. Currently the shift schedule is designed by hand on a daily basis to meet varying manpower demands. Arranging the shift schedule normally takes up fifty minutes of the scheduler's time. Not only is the manual method time-consuming and therefore unable to be modified easily, it also relies on the experiences of the scheduler, both of which make it very difficult to determine how to improve each day's schedule and to quantify the impact of schedule changes on productivity. Hence is the need to develop a computerized model to assist in determining the best shift schedule under stochastic daily demand.

The NHPB is currently divided into nine brigades with three platoons each, working rotating eight-hour shifts in a five-day workweek. The shift schedule is designed manually by the platoon lieutenant. Each shift, rather than of eight consecutive hours, is divided into two 'semi-shifts' of four hours, with an eight-hour break in between. During the break, officers must stay in the platoon station in case there are emergencies that require extra patrol support. The eight-hour break mandated by the Taiwan Police Regulations and Procedures Manual was first introduced in the 60's to cope with in-station manpower shortage. However, along the years, there have been complaints from officers because (1) this scheduling arrangement takes up 16 hours of their day, and (2) overtime pay policy only covers a maximum of four hours, leaving the other four hours worked unpaid. In light of decreased officer morale, the platoon lieutenant is inclined to change the mandatory break policy but lacks persuasive analysis as to how long the ideal break should be (or if the break policy should be abolished) to convince the National Police Agency (NPA) of a policy change. Therefore, the second goal of this research is to develop a computer modeling system that can determine whether a policy change can contribute to deployment efficiency, and consequently a reduction in personnel cost.

Our proposed model uses integer linear programming combined with Monte Carlo simulation method. Our primary goal is to develop a scheduling model that can generate an optimal schedule under stochastic demands. Such an optimal schedule must not only assign officers to daily duties but also maximize the number of officers in station. The model can be a useful tool for all the platoon lieutenants to arrange their shift schedules according to their manpower forecasts. In addition to the primary goal, the proposed model also allows the scheduler to change the break length to evaluate alternative schedules for the same demand scenario. This feature is of great importance for it serves as the analytical evidence for the NPA to consider changing the mandatory break policy. Furthermore, to demonstrate feasibility, the model was tested with the 2006 data from one of the twenty-seven platoons, Taian Platoon, and the results of the case study were discussed.

The rest of this paper is organized as follows. In section 2 we review past research on scheduling problems. Section 3 introduces the proposed model. In section 4 we conduct a case study and discuss the results. Finally, we present our conclusions.

II. BACKGROUND RESEARCH

Mathematical solutions to personnel scheduling problems were first proposed by Dantzig [1]. Since then, researchers have worked on models to generate optimal solutions [2]. Since Beasley and Cao [3] categorized personnel scheduling problems into three groups, other researchers have delved into finding the solution to each group. The three groups include airline crew scheduling ([4],[5],[6],[7];), mass transit crew scheduling ([8],[9];), and generic crew scheduling ([10]; [11], [12], [13];). We found that the problems dealt with in

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the above literature are different from the one in this study in that the patrol officers are scheduled based on stochastic demands

In addition to the categorization of Beasley and Cao, Morris [14] further classified personnel scheduling problems into three types: shift scheduling, day-off scheduling, and tour scheduling problems. Each type has been well discussed. Some researchers focusing on shift scheduling problems include Lau [15] and Aykin [16]; on day-off scheduling, Narasimhan [17]; on tour scheduling, Bechtold et al. [18], and Brusco and Johns [19]. Although the problem faced in this research belongs to the first type, the models presented in the aforesaid studies can not be directly applied to or modified for this research because there is not any long break of hours included in any of the models.

Research focused on police scheduling problem first began in 1967 [20], followed by Felkenes and Whisenand [21], Heller et al [22], and Comrie and Kings [23]. Taylor and Huxley [24] gave an excellent summary on the research efforts and valuable bibliographic resources up until 1989. As they pointed out, neither the hypercube model ([25] [26]) nor Rand's patrol car allocation model ([27] [28]) provided solutions to the patrol officer scheduling problem. Taylor and Huxley used "integer-heuristic" search approach and developed the Police Patrol Scheduling System (PPSS), which provided the San Francisco Police Department with mathematical evidence of the benefits of deploying a mix of one- and two-officer patrol cars and switching from a 5/8 to 4/10 workweek schedule. We have surveyed other studies in the past two decades and found that in the U.S. shifts of eight consecutive hours are most prevalent, while shifts of longer hours (10 or 12) have been adopted by larger agencies [29]. A few studies by the police departments in both the U.S. and the U.K. have focused on the feasibility of a twelve-hour shift ([30]; [31]; [32]).

Our review of the past research concluded that the models developed in the U.K. and the U.S. can not be adopted in Taiwan due to the absence of the mandatory eight-hour break period. In addition, these models were based on two to five shift start times, whereas the start times in Taiwan are twelve. Therefore, a new model that includes these two factors is needed.

III. MODEL FORMULATION

Although the need for patrol officers is present throughout the day, the number of officers required often varies by hour by day. As mentioned earlier, the current shift schedule is designed daily to ensure that the hourly officer requirements for the day are met. The hourly requirements are determined based on historical data collected by the NHPB. As explained earlier, the goals of the proposed model are to schedule officers to meet the hourly officer requirements, and to evaluate alternative schedules should there be a change to the break policy. Other constraints included in the model are as follows:

- The model must be able to generate a schedule within considerably less time than the current manual model of 50 minutes.
- A day is divided into 12 timeslots, the first timeslot starting midnight. There are twelve shift start times.
- The shift start time for officers who start their workweek must not be earlier than 8 am, and the shift end time for officers who finish their workweek must not be later than 8 pm.
- The schedule must not interfere with the current off-duty schedules. All officers work 5 consecutive days with the following two days off. Days off are rotated by month. For example, an officer with the weekend off this month will have Friday and Saturday off the next month.
- Each day the officers on duty are divided into five squads of approximately equal number of officers. According to the existing schedules, each squad is of 5 to 8 officers, which are set to be the lower and upper bound for squad size.

To meet the hourly officer requirements, the model generates various schedules, each with a sum of the total officers scheduled, which is referred to as the 'objective' in this research. As the requirements are only forecasts of police manpower demand, the optimal schedule will be the one with the smallest objective, meaning that not only are daily duties covered, the platoon also has spare manpower in station to cope with emergencies.

A. Basic model

The symbol notations used in the model formulation are as follows:

N	:	the set of timeslots. A day is divided into 12 timeslots, starting at midnight; $N \equiv \{1, 2, 3, \ldots\ldots 12\}$
C	:	the set of squads on duty. $C \equiv \{1, 2, 3, 4, 5\}$.
i	:	index for the i^{th} squad; $i \in C$.
j	:	the j^{th} shift start time $j \in N$.
k	:	the k^{th} timeslot $k \in N$
Y_{ij}	:	Y_{ij} = 0 or 1, 1 meaning to arrange j^{th} shift for the i^{th} squad.
x_{ijk}	:	The number of officers from the i^{th} squad scheduled for the j^{th} shift in the k^{th} timeslot
B	:	A very large value for ease of modeling.
P	:	The length of break; $P \equiv \{0, 2, 4, 6, 8, 10\}$.
d_K	:	Officer requirements for the k^{th} timeslot.
l	:	lower bound for the squad size, l=5
u	:	upper bound for the squad size, u=8

Minimize

$$z = \sum_{i \in C} \sum_{j \in N} \sum_{k \in N} x_{ijk} \qquad (1)$$

Subject to

$$\sum_{i\in C}\sum_{j\in N} x_{ijk} \geq d_k \qquad (2)$$

$$\sum_{i\in C}\sum_{J\in N} x_{ijk} \leq BY_{ij} \qquad (3)$$

$$Y_{ij} = \begin{cases} 1, & x_{ijk} = x_{ij(k+1)} = x_{ij(k+2+p)} = x_{ij(k+3+p)} \\ 0, & \text{otherwise} \end{cases}$$

$$j = k, \forall i \in C, \forall j \in N, \forall k \in N \qquad (4)$$

$$l \leq \sum_{j\in C}\sum_{k\in C} x_{ijk} \leq u \qquad \forall i \in C, \forall j \in N \qquad (5)$$

$$x_{ijk} \geq 0 \;\; \& \;\; x_{ijk} \in I \qquad \forall i \in C, \forall j \in N \qquad (6)$$

The objective function, Equation (1), is to minimize the objective (sum of total officers scheduled). Constraint (2) ensures that the hourly requirements are met in each timeslot. Equations (3) and (4) determine how many officers from which squad should be scheduled. Once the first semi-shift is scheduled, the second semi-shift will be automatically determined according to the break length. Constraint (5) ensures that each squad is of approximately equal size, while Equation (6) ensures that the variables are non-negative and integral.

B. Model Application

Because the proposed model generates schedules against historical data, we adopt Monte Carlo model for data simulation should there be not enough computerized data available. The following steps explain the computation procedure:

- Step 1: Determine P value for the scheduling model that conforms to the break policy.
- Step 2: Input available sets of historical data. One data set (one demand scenario) includes the hourly officer requirements of one given day. The model then generates M sets of simulated data.
- Step 3: Set m=1; m records how many sets of simulated data have been used by the model.
- Step 4: Run all the data sets on the model to get the objective of each demand scenario. The calculation continues until m= M.
- Step 5: Record the objective of all the demand scenarios for evaluation.
- Step 6: Reset P value and repeat step 1 to 6 for an alternative scheduling model.
- Step 7: Evaluate the credibility of the simulated data by performing the ANOVA test and the paired t-test. Then compare the different models to determine the best scheduling model.

IV. Case Study

To test the applicability of the model, we contacted the NPA and obtained the 2006 data of Taian Platoon, the one with the most complete daily data, for our case study. We were not able to obtain data of 2007 onwards due to confidentiality issue. We used Excel, coupled with its built-in features (random number generator and mathematical programming solver) and Visual Basic. The tests were performed on a Pentium 4 – 2GHz with 1Gb RAM computer with Microsoft Windows XP. We used the available data to construct the model and then proposed alternative solutions. Finally, we analyzed the solutions.

A. Data analysis

According to the data of 2006, there are fifty-eight officers in Taian Platoon. There are on average sixteen officers off duty on any given day, so the number of officers available for assignments is forty-two, which is the maximum value for the objective (Max-objective). If a schedule has an objective over the Max-objective, it is regarded as a failed schedule.

The duties for the highway patrol officers include patrol duty, toll stations inspection, and charge of quarters (CQ). The data of 2006 shows, as in Table 1, that 75 percent of shift time is spent on patrol duty, followed by 15 percent on CQ and 10 percent on inspection. The number of officers required by hour ranges from 8 (7.41) to 12 (11.74) and the average number of officers needed for a day is 26. Also, about 9 officers (8.43) are needed for a timeslot of two hours.

By applying the Monte Carlo model, we generated simulated data using a discrete random genenaror based on the probability distribution of manpower in 2006. According to the law of large numbers, the Monte Carlo model will display $1/\sqrt{n}$ convergence—i.e. quadrupling the number of sample points will halve the error. This shows that the number of simulations (n) has a strong influence on the test results. As the available historical data is only about one year's time in this case, we set the M value at 1000 in order to simulate data sets for three year's time.

In order to varify the similarity between the simulated data and the real data in each timeslot, the Chi-Square test of goodness of fit was applied. Accordingly, the numbers for annual manpower requirements are regarded as expected (theoretical) ones (E-value) and the lated (simulated) manpower requirements are regarded as observed numbers (O-value). Equation (7) carries out the Chi-Square test and the results of the test are shown in Table 2. With a significance level below 0.05, all results are accepted.

B. The results of the scheduling models

As our secondary goal is to determine whether a change to the break policy can result in improvement in productivity, with the 1,000 simulated data sets, we generated six scheduling models, each with a different break length, from 0-hour break to 10-hour break. To compare the five alternative models (P=0, 2, 4, 6, or 10) with the current model (P=8), three indexes are used: the mean, standard deviation of the objective, and discrepancy between the objective and Max-objective. These indexes are shown in Table 3.

Table 1 Manpower distribution by every two hours for Taian Platoon in 2006

Duties / Manpower (man-hour) / Hour(timeslot)	Patrol duty	Toll station in spection	CQ	Sum	Mean (per day)	Standard deviation
00:00-02:00	2115	49	337	2501	7.42	0.75
02:00-04:00	2236	49	337	2622	7.78	1.12
04:00-06:00	1426	49	337	1812	5.38	0.58
06:00-08:00	1910	49	337	2296	6.81	1.18
08:00-10:00	3138	11	337	3486	10.34	2.24
10:00-12:00	3498	13	337	3848	11.42	2.01
12:00-14:00	3899	17	337	4253	12.62	2.57
14:00-16:00	4063	26	337	4426	13.13	2.93
16:00-18:00	2549	288	337	3174	9.42	2.29
18:00-20:00	2196	289	337	2822	8.37	1.57
20:00-22:00	2069	681	337	3087	9.16	1.47
22:00-24:00	1622	683	337	2642	7.84	0.79
Total	30719	2415	4044	36967	109.96	3.14
Average	2560	201	337	3081	9.14	1.05

$$\chi^2 = \sum_{i=1} \frac{(o_i - e_i)^2}{e_i} \quad \cdots\cdots\cdots\cdots \quad (7)$$

Oi = number of simulated manpower demand
Ei = number of actual manpower

Table 2 Results for the goodness of fit from the Chi-Square test for every time slot

Item / Hour	$\sum (O_i-E_i)^2/E_i$	Degree of freedom	Critical value	Result of the test
00:00-02:00	6.161	4	9.49	accepted
02:00-04:00	3.453	4	9.49	accepted
04:00-06:00	6.392	4	9.49	accepted
06:00-08:00	5.433	5	11.07	accepted
08:00-10:00	9.780	13	22.36	accepted
10:00-12:00	8.578	14	23.68	accepted
12:00-14:00	17.892	13	23.36	accepted
14:00-16:00	18.905	17	27.59	accepted
16:00-18:00	16.351	14	23.68	accepted
18:00-20:00	8.317	9	16.92	accepted
20:00-22:00	3.246	10	18.31	accepted
22:00-24:00	3.957	5	11.07	accepted

Table 3. Results for the mean, standard deviation, discrepancy and maximum officer requirement (in persons)

alternative models	Objective			Discrepancy (the objective and officer requirement)			The number of failed schedules
	Mean	standard deviation	Max	Max	Min	Avg.	
officer requirement	27.53	1.51	32.5	-	-	-	-
0-hr break	34.90	3.00	45	17.5	1.75	7.37	16
2-hr break	32.45	2.41	45	30.5	0.75	4.92	1
4-hr break	30.26	1.95	38	8.25	0	2.73	0
6-hr break	31.23	2.67	45	13.5	0	3.70	1
8-hr break (current)	31.14	2.85	47	15.5	0	3.61	1
10-hr break	48.15	9.68	90	61.25	4	20.62	681

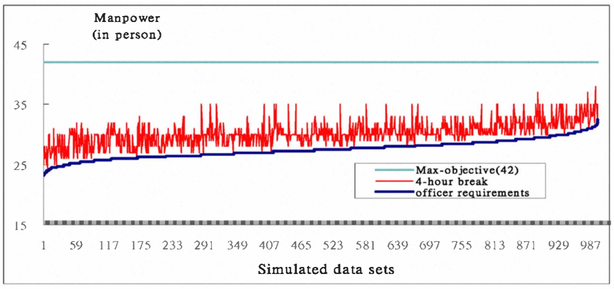

Figure 1 Distribution of the objective for the 4-hr break schedule

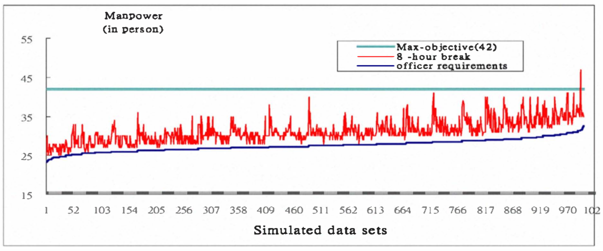

Figure 2 Distribution of the objective for the 8-hr break schedule

We also graphed the distribution of the objective of 1000 demand scenarios for the six schedules with different break lengths. We found that the 4-hr break model had the smallest fluctuation range, with the objective mostly closest to the officer requirements in different scenarios as shown Fig 1. The worst solution is the 10-hr break model, with the objective often exceeding the Max-objective, which resulted in failed schedules. The distribution of the objective for the 6-hr break model is similar to that for the 8-hr model (as shown Fig 2), and as for the 0-hr break model, the fluctuation range gets wider and the objective deviates further away from the officer requirements.

Table 3 shows large discrepancies between the mean objective of each model and the average number of officers needed for a day (27.53). While the 4-hr-break schedule requires 2.73 more officers, the 10-hr-break schedule needs almost 80% more manpower. The maximum and standard deviation of the objective all demonstrate that the 4-hr-break model is the optimal solution, followed by either 6-or 8-hr-break schedule, with the 10-hr-break model being the worst solution, which also produces 681 failed schedules.

To further evaluate the feasibility of these scheduling models, we conducted two other tests: the ANOVA test and the paired t-test.

First, ANOVA (Analysis of Variance) was used to test the equality of the means at one time. The hypotheses are shown in Equation (8). We set the significance level at =0.05. The results were that the F value (2164.53) was over the critical value (2.22), which provided evidence to reject the null hypothesis. The 6 models are not all equal.

$$\begin{cases} H_0: \mu_1 = \mu_2 = \cdots = \mu_k \\ H_1: \mu_1 \neq \mu_2, or \cdots, \mu_k \end{cases} \quad \cdots\cdots\cdots\cdots \quad (8)$$

Second, the paired t-test can be used to compare two means when there are two samples in which observations in one sample can be paired with observations in the other sample. The null hypothesis is that the differences between the two observations is 0（D=0）. We set the significance level at 0.05. The hypotheses are shown in Equation (9). The results showed that every t value was above the critical value, as shown in Table 4. We also changed the null hypothesis of this test to $D \leq 0$. This allowed us to determine the best scheduling model, which has the lowest mean objective value.

$$\begin{cases} H_0 : D = 0 (\text{the difference between the two observations is } 0) \\ H_1 : D \neq 0 (\text{the difference is not } 0) \end{cases} \quad \dots (9)$$

Table 4 Results of **the** Paired t-test:

		0-hr break	2-hr break	4-hr break	6-hr break	8-hr break	10-hr break
0-hr break	t-value	-	25.879	51.488	36.927	39.091	-43.471
	p-value	-	0.000	0.000	0.000	0.000	0.000
2-hr break	t-value	-	-	37.920	17.445	17.956	-53.493
	p-value	-	-	0.000	0.000	0.000	0.000
4-hr break	t-value	-	-	-	-19.706	-14.212	-58.767
	p-value	-	-	-	0.000	0.000	0.000
6-hr break	t-value	-	-	-	-	2.174	-54.086
	p-value	-	-	-	-	0.030	0.000
8-hr break	t-value	-	-	-	-	-	-54.086
	p-value	-	-	-	-	-	0.000
10-hr break	t-value	-	-	-	-	-	-
	p-value	-	-	-	-	-	-

These results of the Paired t-test are sorted in decreasing order as in Equation (10), with the 4-hr-break schedule being the optimal solution because it has the lowest mean objective.

4-hr break <8-hr break（current）<2-hr break <0-hr break < 10-hr break （10）

It is worth noting that although the historical data was the daily records of 8-hr-break shift schedules, the 8-hour-break model does not produce the best objective. There are two possible explanations for this: (1) the model is to produce integer objective to meet the hourly demand, meaning the number of officers scheduled is either equal to or larger than the number of officer required; (2) in real life, in cases of increased sick leave or natural disasters, the platoon lieutenant may have to shorten the break in order to cope with manpower shortage.

C. Analysis of results

In this study, the time taken to run one model was 7860 seconds, which means it takes an average of 7.9 second to generate one schedule. The efficiency of scheduling model exceeded that of the manual method by almost four hundred times. For ease of calculation, we set the time to generate one schedule at 5 minutes. The adoption of the proposed model will save the platoon lieutenant approximately 16,425 productive hours per year.

Furthermore, the study found that the 4-hour-break model is the optimal one because it requires the least number of officers (30.26) to perform the same daily duties, and generates no failed schedules. The current 8-hour-break model, while being the second best model for its second lowest discrepancy value (3.61), has one failed schedule. From the viewpoint of cutting cost, the 4-hr-break model requires 1 officer less (3.61-2.73) per day. At US $20 per hour, that is a savings of US$ 584,000 per year.

Another benefit of switching from the 8-hour-break model to the 4-hr-break model is that it resolves the two complaints from officers, as mentioned in the introduction. Not only do officers have more personal time, the break period will be fully covered by overtime pay policy.

V. CONCLUSIONS

In this research, we develop a scheduling model that can assist the NHPB in arranging the daily shift schedule and evaluating the need to change the current break policy. The model also factors in other constraints, such as shift times, off-duty schedules, and squad size, to ensure that the model can be immediately adopted with no effect on those schedule policies other than the break policy.

The model is formulated by integer linear programming. Monte Carlo simulation technique is employed to resolve the problem of data shortage, and the Chi-Square test was performed to demonstrate the quality of the simulated data.

A case study was conducted using the 2006 data of Taian Platoon, which was obtained from the NHPB. Based on the one year's data, 1000 sets of simulated data were generated and used to produce six scheduling models for the evaluation of a break policy change.

From the case study several points can be concluded: (1) the model can be very useful for the scheduler as it dramatically decrease the scheduling time from 50 minutes to less than 5 minutes, which equals to 16,425 hours per year saved for the scheduler; (2) the current 8-hr break model is viable because it has a better objective than that of the 0-hr, 6-hr, or 10-hr break model. Under the current model, officers are scheduled relevantly effectively to meet the daily stochastic demand; (3) a change to the break policy should be considered because not only does it save approximately US$ 584,000 per year on personnel cost, it can boost officer morale; (4) regardless of break policy, the model can help the scheduler generate a schedule fast enough in cases of national emergencies.

For other police departments, the model can be suitably modified to reflect scheduling preferences such as squad number and size, and the number of start times. This model can also serve as the base upon which a more complicated model that includes off-duty officer scheduling and patrol cars scheduling can be built. This could be a direction for future research on police scheduling and deployment in Taiwan.

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