

Exercise 14.1

Q.1 In $\triangle ABC$
 $\overline{DE} \parallel \overline{BC}$

- (i) If $\overline{AD} = 1.5\text{cm}$, $\overline{BD} = 3\text{cm}$
 $\overline{AE} = 1.3\text{cm}$, then find \overline{CE}

$$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AE}}{\overline{EC}}$$

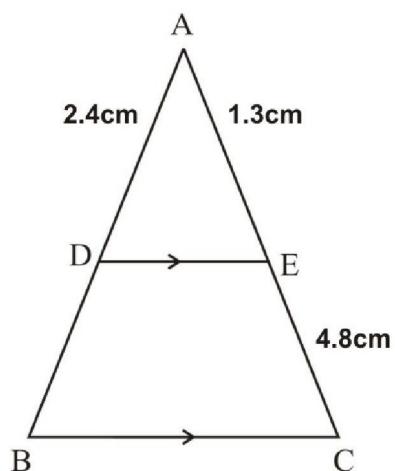
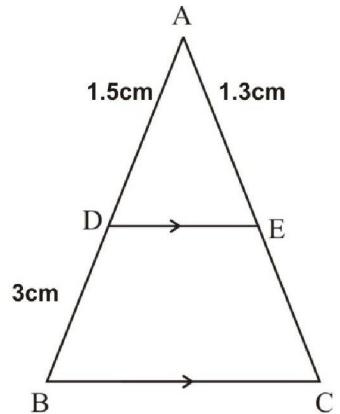
By substituting the values of \overline{AD} , \overline{BD} and \overline{AE}
So

$$\begin{aligned}\frac{1.5}{3} &= \frac{1.3}{EC} \\ EC(1.5) &= 1.3 \times 3 \\ \frac{1.3 \times 3}{1.5} &= EC \\ EC &= \frac{3.9}{1.5} \\ EC &= 2.6\text{ cm}\end{aligned}$$

- (ii) If $AD = 2.4\text{cm}$, $AE = 3.2\text{cm}$

$\overline{EC} = 4.8\text{cm}$ find AB

$$\begin{aligned}\frac{AD}{AB} &= \frac{AE}{AC} \\ \frac{AD}{AB} &= \frac{AE}{AE + EC} \\ \frac{2.4}{AB} &= \frac{3.2}{3.2 + 4.8} \\ \frac{2.4}{AB} &= \frac{3.2}{8} \\ 2.4 \times 8 &= (3.2) \overline{AB} \\ \frac{19.2}{3.2} &= \overline{AB} \\ \overline{AB} &= 6\text{cm}\end{aligned}$$



(iii) If $\frac{\overline{AD}}{\overline{BD}} = \frac{3}{5} \overline{AC} = 4.8\text{cm}$ find \overline{AE}

$$\overline{AC} = \overline{AE} + \overline{EC}$$

$$\overline{AC} = \overline{EC} + \overline{AE}$$

$$\overline{AE} = 4.8 - \overline{EC}$$

$$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AE}}{\overline{EC}}$$

$$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AC} - \overline{EC}}{\overline{EC}}$$

$$\frac{3}{5} = \frac{4.8 - \overline{EC}}{\overline{EC}}$$

$$3(\overline{EC}) = 5(4.8 - \overline{EC})$$

$$3(\overline{EC}) = 24 - 5(\overline{EC})$$

$$3(\overline{EC}) + 5(\overline{EC}) = 24$$

$$8(\overline{EC}) = 24$$

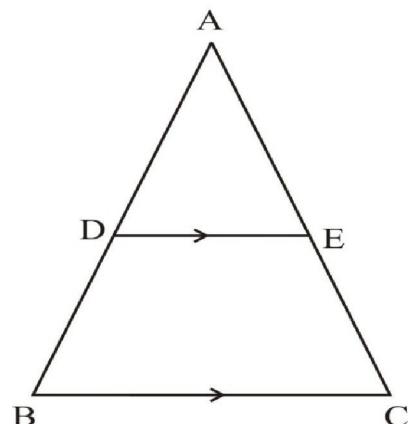
$$(\overline{EC}) = \frac{24}{8}$$

$$\overline{EC} = 3\text{cm}$$

$$\overline{AE} = \overline{AC} - \overline{EC}$$

$$= 4.8 - 3$$

$$= 1.8\text{cm}$$



(iv) If $\overline{AD} = 2.4\text{cm}$, $\overline{AE} = 3.2\text{cm}$, $\overline{DE} = 2\text{cm}$, $\overline{BC} = 5\text{cm}$. Find \overline{AB} , \overline{DB} , \overline{AC} , \overline{CE} .

$$\frac{\overline{AD}}{\overline{AB}} = \frac{\overline{AE}}{\overline{AC}} = \frac{\overline{DE}}{\overline{BC}}$$

$$\frac{2.4}{AB} = \frac{3.2}{AC} = \frac{2}{5}$$

$$\frac{2.4}{AB} = \frac{2}{5}$$

$$(2.4)5 = 2(AB)$$

$$\frac{12.0}{2} = AB$$

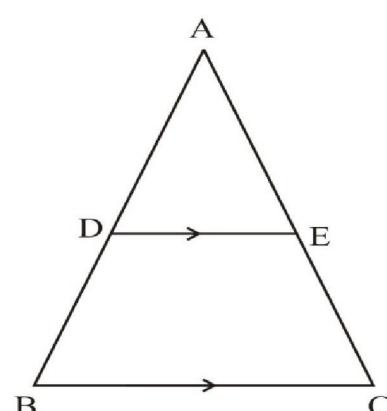
$$\overline{AB} = 6\text{cm}$$

$$\frac{3.2}{AC} = \frac{2}{5}$$

$$16.0 = 2(AC)$$

$$\frac{16}{2} = AC$$

$$\overline{AC} = 8\text{cm}$$



$$\overline{DB} = \overline{AB} - \overline{AD}$$

$$\overline{DB} = 6 - 2.4$$

$$\overline{BD} = 3.6 \text{ cm}$$

$$\frac{\overline{AD}}{\overline{AB}} = \frac{\overline{AE}}{\overline{AC}}$$

$$\frac{2.4}{6} = \frac{\overline{AE}}{8}$$

$$\overline{AE} = \frac{2.4}{6} \times 8$$

$$\overline{AE} = \frac{19.2}{6}$$

$$\overline{AE} = 3.2 \text{ cm}$$

$$\overline{CE} = \overline{AC} - \overline{AE}$$

$$\overline{CE} = 8 - 3.2$$

$$\overline{CE} = 4.8 \text{ cm}$$

If $\overline{AD} = 4x - 3$ $\overline{AE} = 8x - 7$

$\overline{BD} = 3x - 1$ and $\overline{CE} = 5x - 3$ Find the value of x

$$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AE}}{\overline{EC}}$$

By putting the value of \overline{AD} , \overline{AE} , \overline{BD} and \overline{CE}

$$\frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$$

By cross multiplying

$$(4x-3)(5x-3) = (8x-7)(3x-1)$$

$$20x^2 - 12x - 15x + 9 = 24x^2 - 8x - 21x + 7$$

$$20x^2 - 27x + 9 = 24x^2 - 29x + 7$$

$$0 = 24x^2 - 20x^2 - 29x + 27x + 7 - 9$$

$$4x^2 - 2x - 2 = 0$$

$$2(2x^2 - x - 1) = 0$$

$$2x^2 - 2x + 1x - 1 = \frac{0}{2}$$

$$2x(x-1) + 1(x-1) = 0$$

$$(x-1)(2x+1) = 0$$

$$x - 1 = 0$$

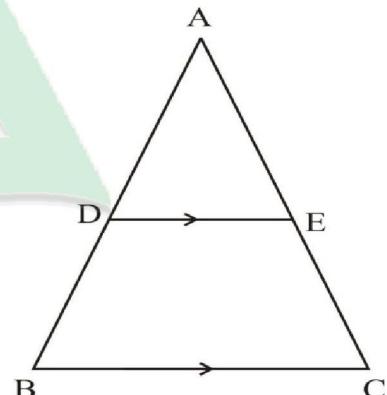
$$2x + 1 = 0$$

$$x = 1$$

$$2x = -1$$

$$x = -\frac{1}{2}$$

Distance is not taken in negative it is always in positive so the value of $x = 1$.



- Q.2 In $\triangle ABC$ is an isosceles triangle $\angle A$ is vertex angle and \overline{DE} intersects the sides \overline{AB} and \overline{AC} as shown in the figure so that**

$$m\overline{AD} : m\overline{DB} = m\overline{AE} : m\overline{EC}$$

Prove that $\triangle ADE$ is also an isosceles triangle.

Given:

$\triangle ABC$ is an isosceles triangle, $\angle A$ is vertex and \overline{DE} intersects the sides \overline{AB} and \overline{AC} .

$$\frac{m\overline{AD}}{m\overline{BD}} = \frac{m\overline{AE}}{m\overline{EC}}$$

To Prove

$$m\overline{AD} = m\overline{AE}$$

Proof

$$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AE}}{\overline{EC}}$$

$$\text{Or } \frac{\overline{BD}}{\overline{AD}} = \frac{\overline{EC}}{\overline{AE}}$$

$$\text{Or } \frac{\overline{AD} + \overline{BD}}{\overline{AD}} = \frac{\overline{AE} + \overline{EC}}{\overline{EC}}$$

As we know

$$\overline{AB} = \overline{AD} + \overline{BD}$$

$$\overline{AC} = \overline{AE} + \overline{EC}$$

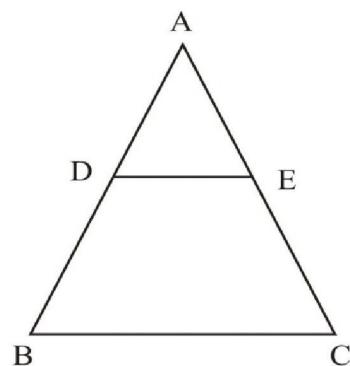
$$\frac{\overline{AB}}{\overline{AD}} = \frac{\overline{AC}}{\overline{AE}}$$

From this

$$\frac{\overline{AB}}{\overline{AD}} = \frac{\overline{AC}}{\overline{AE}}$$

$$\overline{AD} = \overline{AE}$$

$$\overline{AB} = \overline{AC} \text{ (Given)}$$



- Q.3 In an equilateral triangle ABC shown in the figure $m\overline{AE}:m\overline{AC}=m\overline{AD}:m\overline{AB}$ find all the three angles of $\triangle ADE$ and name it also.**

Given

$\triangle ABC$ is equilateral triangle

To prove

To find the angles of $\triangle ADE$

Solution:

$$\frac{m\overline{AE}}{m\overline{AC}} = \frac{m\overline{AD}}{m\overline{AB}}$$

All angles are equal as it is an equilateral triangle which are equal to 60° each

$$\angle A = \angle B = \angle C$$

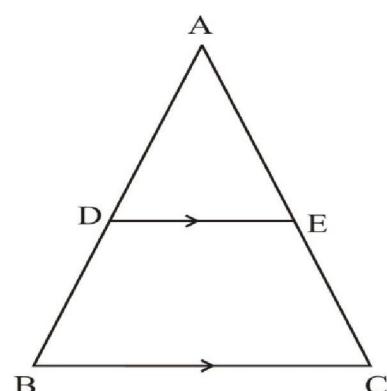
$$m\overline{BC} \parallel m\overline{DE}$$

$$\angle ADE = \angle ABC = 60^\circ$$

$$\angle AED = \angle ACB = 60^\circ$$

$$\angle A = 60^\circ$$

$\triangle ADE$ is an equilateral triangle



Q.4 Prove that line segment drawn through the midpoint of one side of a triangle and parallel to another side bisects the third side

Given

$$\overline{AD} = \overline{BD}$$

$$\overline{DE} \parallel \overline{BC}$$

To Prove

$$\overline{AE} = \overline{EC}$$

In $\triangle ABC$

$$\overline{DE} \parallel \overline{BC}$$

In theorem it is already discussed that

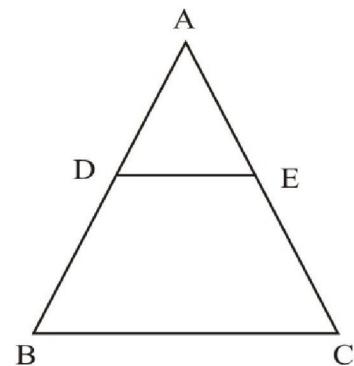
$$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AE}}{\overline{EC}}$$

As we know $\overline{AD} = \overline{BD}$ or $\overline{BD} = \overline{AD}$

$$\frac{\overline{AD}}{\overline{AD}} = \frac{\overline{AE}}{\overline{EC}}$$

$$1 = \frac{\overline{AE}}{\overline{EC}}$$

$$\overline{EC} = \overline{AE}$$



Q.5 Prove that the line segment joining the midpoint of any two sides of a triangle is parallel to the third side

Given

$\triangle ABC$ the midpoint of \overline{AB} and \overline{AC} are L and M respectively

To Prove

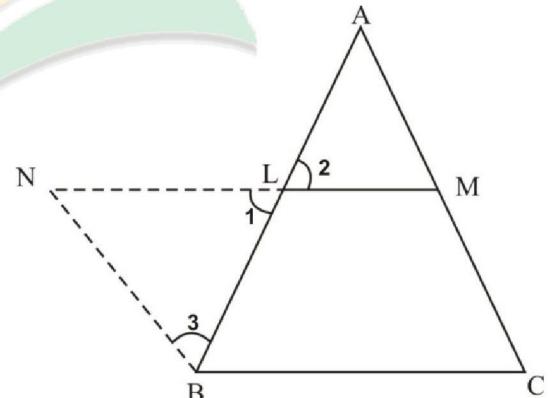
$$\overline{LM} \parallel \overline{BC} \text{ and } m\overline{LM} = \frac{1}{2}\overline{BC}$$

Construction

Join M to L and produce \overline{ML} to N such that $\overline{ML} \cong \overline{LN}$

Join N to B and in the figure name the angles $\angle 1$, $\angle 2$, and $\angle 3$

Proof



Statements	Reasons
$\Delta BLN \leftrightarrow \Delta ALM$	
$\overline{BL} \cong \overline{AL}$	Given
$\angle 2 = \angle 1$ or $\angle 1 = \angle 2$	Vertical angles
$\overline{NL} \cong \overline{ML}$	Construction
$\therefore \Delta BLN \cong \Delta ALM$	Corresponding angle of congruent triangles Given
$\therefore \angle A = \angle 3$	
And $\overline{NB} \cong \overline{AM}$	
$\overline{NB} \parallel \overline{AM}$	

$\overline{ML} = \overline{AM}$	Given
$\overline{NB} \cong \overline{ML}$	
$\overline{BC} \overline{MN}$ is parallelogram	
$\therefore \overline{BC} \parallel \overline{LM}$ or $\overline{BC} \parallel \overline{NL}$	
$\overline{BC} \cong \overline{NM}$	(Opposite side of parallelogram BCMN)
$m\overline{LM} = \frac{1}{2} m\overline{NM}$	(Opposite side of parallelogram)
Hence $m\overline{LM} = \frac{1}{2} m\overline{BC}$	

Theorem 14.1.3

The internal bisector of an angle of a triangle divides the sides opposite to it in the ratio of the lengths of the sides containing the angle.

Given

In $\triangle ABC$ internal angle bisector of $\angle A$ meets \overline{CB} at the points D.

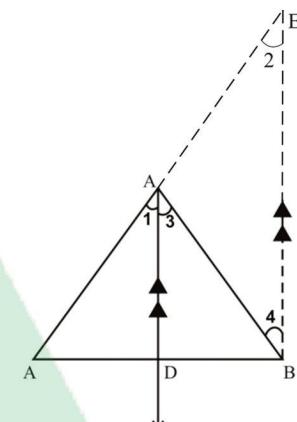
To prove

$$m\overline{BD}:m\overline{DC}=m\overline{AB}:m\overline{AC}$$

Construction

Draw a line segment $\overline{BE} \parallel \overline{DA}$ to meet \overline{CA} Produced at E

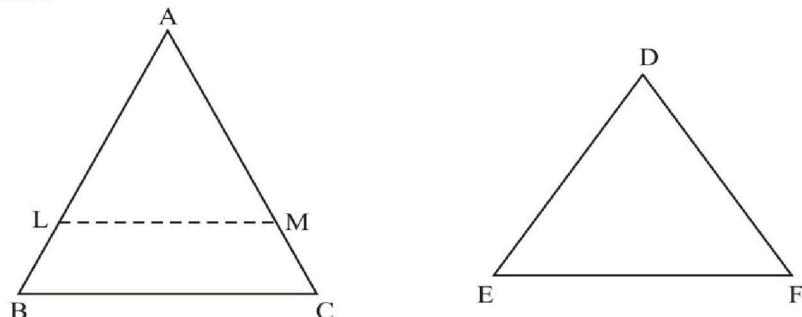
Proof



Statements	Reasons
$\because \overline{AD} \parallel \overline{EB}$ and \overline{EC} intersect them	Construction
$m\angle 1 = m\angle 2 \dots \dots \dots \text{(i)}$	Corresponding angles
Again $\overline{AD} \parallel \overline{EB}$ and \overline{AB} intersects them	
$\therefore m\angle 3 = m\angle 4 \dots \dots \dots \text{(ii)}$	Alternate angles
But $m\angle 1 = m\angle 3$	Given
$\therefore m\angle 2 = m\angle 4$	From (i) and (ii)
And $\overline{AB} \cong \overline{AE}$ or $\overline{AE} \cong \overline{AB}$	In a Δ , the sides opposite to congruent angles are also congruent
Now $\overline{AD} \parallel \overline{EB}$	Construction
$\therefore \frac{m\overline{BD}}{m\overline{DC}} = \frac{m\overline{EA}}{m\overline{AC}}$	A line parallel to one side of a triangle and intersecting the other two sides divides them proportionally.
or $\frac{m\overline{BD}}{m\overline{DC}} = \frac{m\overline{AB}}{m\overline{AC}}$	
Thus $m\overline{BD}:m\overline{DC}=m\overline{AB}:m\overline{AC}$	$m\overline{EA} = m\overline{AB}$ (proved)

Theorem 14.1.4

If two triangles are similar, then the measures of their corresponding sides are proportional



Given

$$\triangle ABC \sim \triangle DEF$$

i.e. $\angle A \cong \angle D$, $\angle B \cong \angle E$ and $\angle C \cong \angle F$

To Prove

$$\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{AC}}{m\overline{DF}} = \frac{m\overline{BC}}{m\overline{EF}}$$

Construction

(I) Suppose that $m\overline{AB} > m\overline{DE}$

(II) $m\overline{AB} \leq m\overline{DE}$

On \overline{AB} take a point L such that $m\overline{AL} = m\overline{DE}$

On \overline{AC} take a point M such that $m\overline{AM} = m\overline{DF}$

Join L and M by the line segment LM

Proof

Statements	Reasons
In $\triangle ALM \leftrightarrow \triangle DEF$	
$\angle A \cong \angle D$	Given
$\overline{AL} \cong \overline{DE}$	Construction
$\overline{AM} \cong \overline{DF}$	Construction
Thus $\triangle ALM \cong \triangle DEF$	S.A.S Postulate
And $\angle L \cong \angle E$, $\angle M \cong \angle F$	(Corresponding angles of congruent triangles)
Now $\angle E \cong \angle B$ and $\angle F \cong \angle C$	Given
$\therefore \angle L \cong \angle B$, $\angle M \cong \angle C$	Transitivity of congruence
Thus $\overline{LM} \parallel \overline{BC}$	Corresponding angles are equal
Hence $\frac{m\overline{AL}}{m\overline{AB}} = \frac{m\overline{AM}}{m\overline{AC}}$	A line parallel to one side of a triangle and intersecting the other two sides divides them proportionally.
Or $\frac{m\overline{DE}}{m\overline{AB}} = \frac{m\overline{DF}}{m\overline{AC}}$(i)	$m\overline{AL} = m\overline{DE}$ and $m\overline{AM} = m\overline{DF}$ (Construction)
Similarly by intercepting segments on \overline{BA} and \overline{BC} , we can prove that	
$\frac{m\overline{DE}}{m\overline{AB}} = \frac{m\overline{EF}}{m\overline{BC}}$(ii)	

$$\text{Thus } \frac{m\overline{DE}}{m\overline{AB}} = \frac{m\overline{DF}}{m\overline{AC}} = \frac{m\overline{EF}}{m\overline{BC}}$$

$$\text{Or } \frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{AC}}{m\overline{DF}} = \frac{m\overline{BC}}{m\overline{EF}}$$

If $m\overline{AB} = m\overline{DE}$

Then in $\Delta ABC \leftrightarrow \Delta DEF$

(II) If $m\overline{AB} < m\overline{DE}$, it can similarly be proved by taking intercepts on the sides of ΔDEF

$$\angle A \cong \angle D$$

$$\angle B \cong \angle E$$

$$\text{And } \overline{AB} \cong \overline{DE}$$

So $\Delta ABC \cong \Delta DEF$

$$\text{Thus } \frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{AC}}{m\overline{DF}} = \frac{m\overline{BC}}{m\overline{EF}} = 1$$

Hence the result is true for all the cases.

By (i) and (ii)

By taking reciprocals

A.S.A \cong A.S.A

$\overline{AC} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$



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Report any mistake at freeilm786@gmail.com