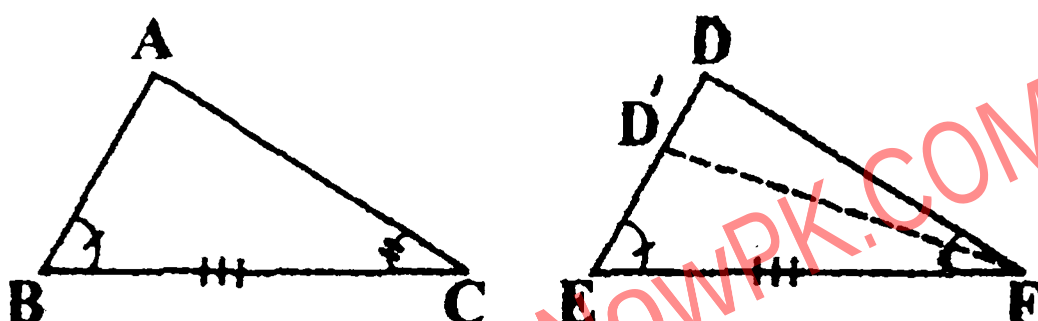
Unit 10
Congruent Triangles

THEOREM 10.1.1

If in any correspondence of two triangles, two angles and one side of a triangle are congruent to the corresponding two angles and one side of the other, the triangles are congruent. (A.S.A ≅ A.S.A)

Solution:



Given:

In $\Delta ABC \leftrightarrow \Delta DEF$

$\angle B \cong \angle E$ $\angle C \cong \angle F$ $\overline{BC} \cong \overline{EF}$

To Prove: $\Delta ABC \cong \Delta DEF$

Construction:

Suppose $\overline{AB} \ncong \overline{DE}$ and there is a point D' on $\overline{DE}$ such that $\overline{AB} \cong \overline{D'E}$. Join D' to F.

Proof:

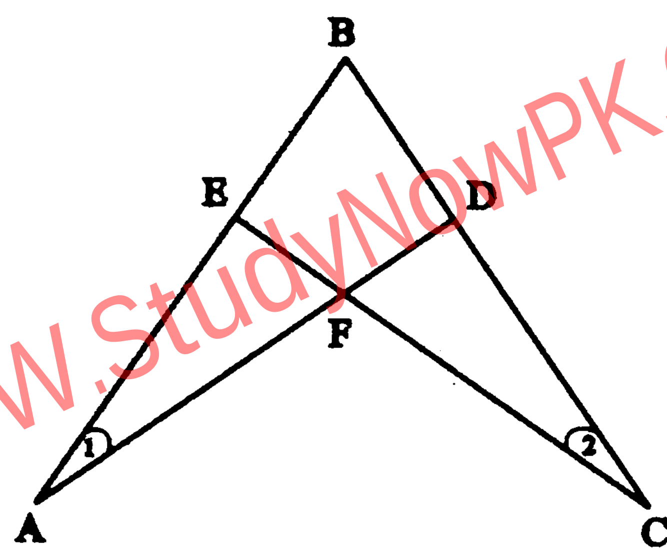
Statements	Reasons
In $\Delta ABC \leftrightarrow \Delta D'EF$	
$\overline{AB} \cong \overline{D'E}$ .... (i)	Construction / Supposition
$\overline{BC} \cong \overline{EF}$ .... (ii)	Given
$\angle B \cong \angle E$ .... (iii)	Given
$\therefore \Delta ABC \cong \Delta D'EF$	S.A.S. Postulate
So, $\angle C \cong \angle D'EF$	Corresponding angles of congruent triangles
But $\angle C \cong \angle DFE$	Given

$\therefore \quad \angle DFE \cong \angle D'FE$	Both congruent to $\angle C$
This is possible only if D and D' are the same points. So, $\overline{AB} \cong \overline{DE}$ .... (iv)	Proved that D and D' are the same points.
Thus from (ii), (iii) and (iv), we have $\Delta ABC \cong \Delta DEF$	S.A.S. postulate

EXERCISE 10.1

Q1. In the given figure, $\overline{AB} \cong \overline{CB}, \angle 1 \cong \angle 2.$
Prove that $\Delta ABD \cong \Delta CBE.$

Solution:



Given:

In the given figure $\angle 1 \cong \angle 2$ and $\overline{AB} \cong \overline{CB}$

To prove:

$\Delta ABD \cong \Delta CBE$

Proof:

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta CBE$	
$\overline{AB} \cong \overline{CB}$	Given
$\angle BAD \cong \angle BCE$	Given $\angle 1 \cong \angle 2$
$\angle ABD \cong \angle CBE$	Common
$\therefore \quad \Delta ABD \cong \Delta CBE$	S.A.A $\cong$ S.A.A

Proof:

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta ACD$	
$\overline{AD} \cong \overline{AD}$	Common
$\angle B \cong \angle C$	Given
$\angle BAD \cong \angle CAD$	Construction
$\therefore\ \Delta ABD \cong \Delta ACD$	A.A.S. ≅ A.A.S
Hence $\overline{AB} \cong \overline{AC}$	Corresponding angles of congruent triangles

EXERCISE 10.2

Q1. Prove that any two medians of an equilateral triangle are equal in measure.

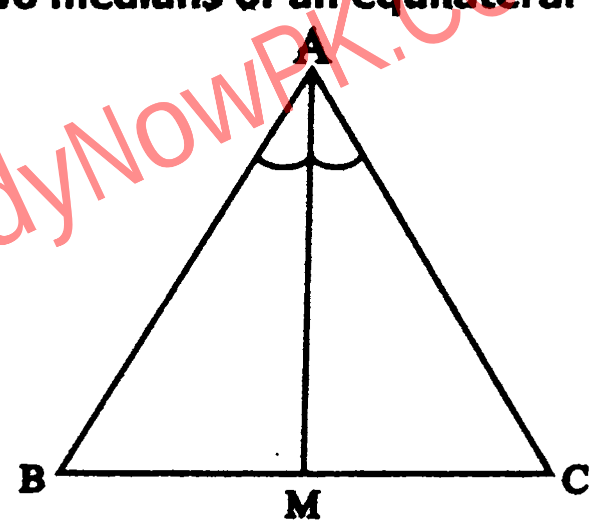
Solution:

Given:

In ΔABC, $\overline{AB} \cong \overline{AC}$ and M is mid point of BC.

To prove:

$\overline{AM}$ bisects $\angle A$ and $\overline{AM}$ is perpendicular to $\overline{BC}$.



Proof:

Statements	Reasons
In $\Delta ABM \leftrightarrow \Delta ACM$	
$\overline{AB} \cong \overline{AC}$	Given
$\overline{BM} \cong \overline{CM}$	Given M is mid point of BC.
$\overline{BM} \cong \overline{AM}$	Common
$\therefore\ \Delta ABM \cong \Delta ACM$	S.S.S. ≅ S.S.S.
So $\angle BAM \cong \angle CAM$	Corresponding sides of ≅ Δ's.
$\therefore$ AM bisects $\angle A$	
Also $\angle AMB \cong \angle AMC$	Corresponding sides of ≅ Δ's.
but $m\angle AMB \cong \angle AMC = 180^\circ$	

THEOREM 10.1.3

If in a given correspondence of two triangles, the three sides of one triangle are congruent to the corresponding three sides of the other triangle then the triangles are congruent.

Solution:

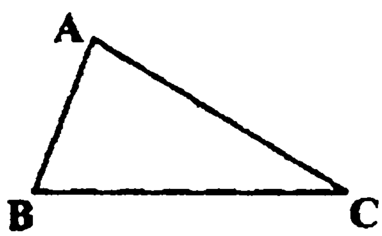


Fig. 1

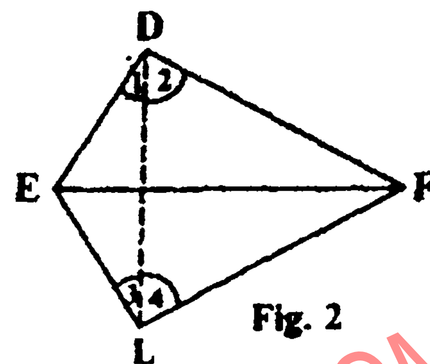


Fig. 2

Given: In $\Delta ABC \leftrightarrow \Delta DEF$

$\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$ and $\overline{CA} \cong \overline{FD}$

To Prove:

$\Delta ABC \cong \Delta DEF$

Construction:

Suppose that in ΔDEF the side $\overline{EF}$ is not smaller than any of the remaining two sides. On $\overline{EF}$ construct a ΔLEF in which, $\angle B \cong \angle FEL$ and $\overline{LE} \cong \overline{AB}$. Join D and L. In the figure, the names of some of the angles are 1, 2, 3 and 4.

Proof:

Statements	Reasons
In $\Delta ABC \leftrightarrow \Delta LEF$	
$\overline{BC} \cong \overline{EF}$	Given
$\angle B \cong \angle FEL$	Construction
$\overline{AB} \cong \overline{LE}$	Construction
$\therefore$ $\Delta ABC \cong \Delta LEF$	S.A.S postulate
and $\overline{CA} \cong \overline{FL}$ ...... (i)	Corresponding sides of congruent triangles
Also $\overline{CA} \cong \overline{FD}$ ...... (ii)	Given
$\therefore$ $\overline{FL} \cong \overline{FD}$	From (i) and (ii)

In ΔFDL	
$\angle 2 \cong \angle 4$ .... (iii)	$\overline{FL} \cong \overline{FD}$ (Proved)
Similarly $\angle 1 \cong \angle 3$ ..... (iv)	
$\therefore m\angle 2 + m\angle 1 = m\angle 4 + m\angle 3$	From (iii) and (iv)
Now, in $\Delta DEF \leftrightarrow \Delta LEF$	
$\overline{FD} \cong \overline{FL}$	Proved
And $\angle EDF \cong \angle ELF$	Proved
$\overline{DE} \cong \overline{LE}$	Each one $\cong \overline{AB}$
$\therefore \Delta DEF \cong \Delta LEF$	S.A.S. Postulate
Also $\Delta ABC \cong \Delta LEF$	Proved
Hence $\Delta ABC \cong \Delta LEF$	Each one $\cong \Delta LEF$ (Proved)

EXERCISE 10.3

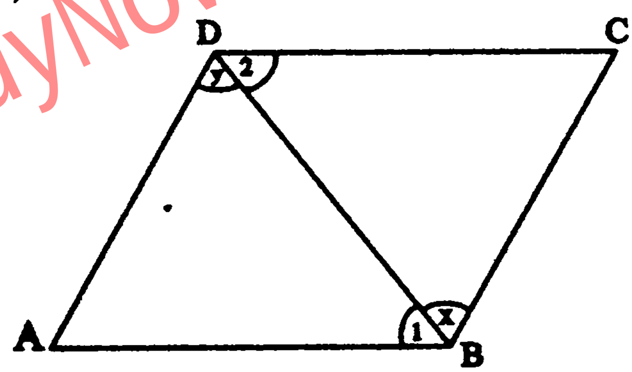
Q1. In the given figure, $\overline{AB} \cong \overline{DC}$, $\overline{AD} \cong \overline{BC}$.
Prove that $\angle A \cong \angle C$, $\angle ABC \cong \angle ADC$

Solution:

Given:

In the figure

$\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$



To prove:

$\angle A \cong \angle C$

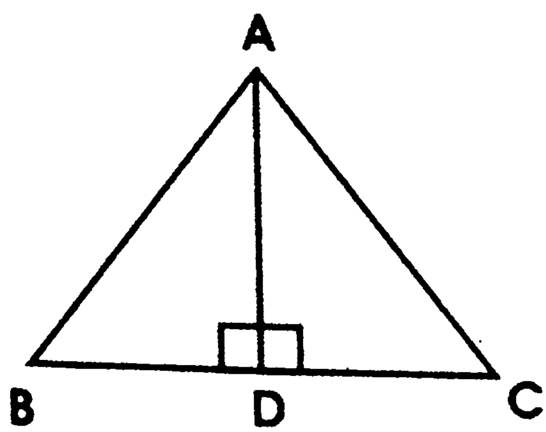
$\angle ABC \cong \angle ADC$

Construction:

Join B to D.

Proof:

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta CDB$	
$\overline{AB} \cong \overline{DC}$	Given
$\overline{AD} \cong \overline{CD}$	Given
$\overline{BD} \cong \overline{DB}$	Common
$\therefore \Delta ABC \cong \Delta CDB$	S.S.S. $\cong$ S.S.S.
$\therefore \angle A \cong \angle C$	Corresponding sides of $\cong$ Δs.



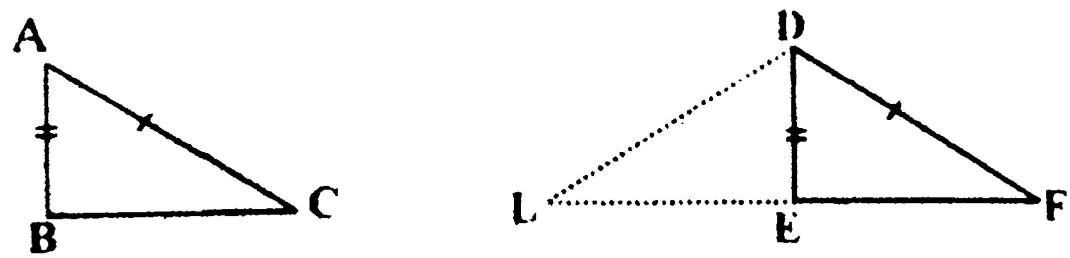
Proof:

Statement	Reasons
In the correspondence of $\triangle ABD \leftrightarrow \triangle ACD$	
$\overline{AB} \cong \overline{AC}$	Given
$\overline{AD} \cong \overline{AD}$	Common
$\overline{BD} \cong \overline{DC}$	Given
$\therefore \triangle ABD \cong \triangle ACD$	S.S.S. postulate
Thus $\angle BAD \cong \angle CAD$	Corresponding angles of congruent triangle
$m\angle ADB + m\angle ADC = 180^0$	Supplementary angles
$m\angle ADC + m\angle ADC = 180^0$	
$\Rightarrow 2\, m\angle ADC = 180^0$	As $m\angle ADC = m\angle ADB$
$\Rightarrow m\angle ADC = 90^0$	
Hence $\overline{AD} \perp \overline{BC}$	

THEOREM 10.1.4

If in the correspondence of the two right-angled triangles, the hypotenuse and one side of one triangle are congruent to the hypotenuse and the corresponding side of the other, then the triangles is congruent.

Solution:



Given: In $\Delta ABC \leftrightarrow \Delta DEF$

$\angle B \cong \angle E$ (Right angles) ; $\overline{CA} \cong \overline{FD}$

$\overline{AB} \cong \overline{DE}$

To Prove:

$\Delta ABC \cong \Delta DEF$

Construction:

Produce $\overline{EF}$ to point L such that $\overline{EL} \cong \overline{BC}$ and join points D and L.

Proof:

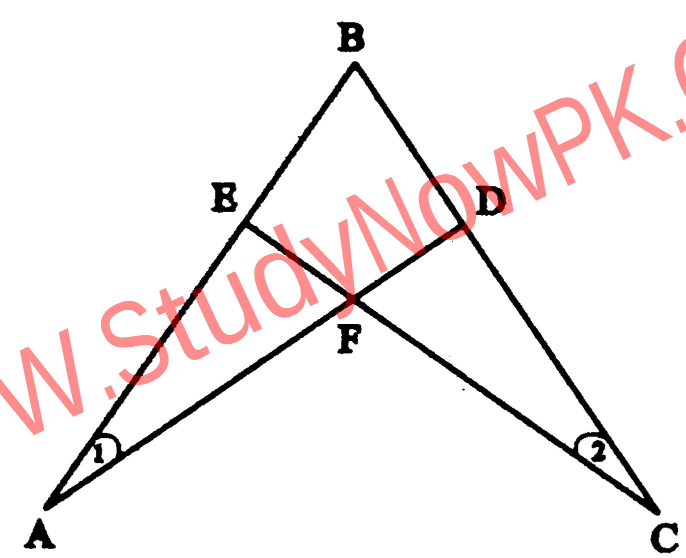
Statements	Reasons
$m\angle DEF + m\angle DEL = 180^0$. (i)	Supplementary angles
Now $m\angle DEF = 90^0$ ...... (ii)	Given
$\therefore$ $m\angle DEL = 90^0$	From (i) and (ii)
In $\Delta ABC \leftrightarrow \Delta DEL$	
$\overline{BC} \cong \overline{EL}$	Construction
$\angle ABC \cong DEL$	Each equal to 90^0
$\overline{AB} \cong \overline{DE}$	Given
$\therefore$ $\Delta ABC \cong \Delta DEL$	S.A.S. postulate
And $\angle C \cong \angle L$	Corresponding angles of congruent triangles
$\overline{CA} \cong \overline{LD}$	Corresponding sides of congruent triangles
But $\overline{CA} \cong \overline{FD}$	Given
$\therefore$ $\overline{LD} \cong \overline{FD}$	each is congruent to $\overline{CA}$
In ΔDLF	
$\angle F \cong \angle L$	$\overline{FD} \cong \overline{LD}$ (proved)
But $\angle C \cong \angle L$	Proved
$\angle C \cong \angle F$	Each is congruent to $\angle L$
In $\Delta ABC \leftrightarrow \Delta DEF$	
$\overline{AB} \cong \overline{DE}$	Given
$\angle ABC \cong \angle DEF$	Given
$\therefore$ $\Delta ABC \cong \Delta DEF$	S.A.A. $\cong$ S.A.A

$\therefore \quad \angle DFE \cong \angle D'FE$ This is possible only if D and D' are the same points. So, $\overline{AB} \cong \overline{DE}$ .... (iv)	Both congruent to $\angle C$ Proved that D and D' are the same points.
Thus from (ii), (iii) and (iv), we have $\Delta ABC \cong \Delta DEF$	S.A.S. postulate

EXERCISE 10.1

Q1. In the given figure, $\overline{AB} \cong \overline{CB}, \angle 1 \cong \angle 2.$
Prove that $\Delta ABD \cong \Delta CBE.$

Solution:



Given:

In the given figure $\angle 1 \cong \angle 2$ and $\overline{AB} \cong \overline{CB}$

To prove:

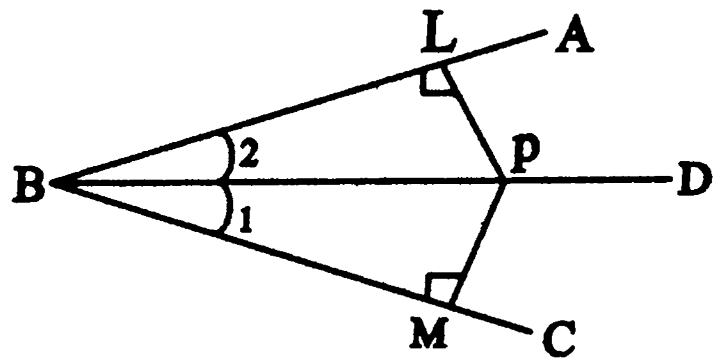
$\Delta ABD \cong \Delta CBE$

Proof:

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta CBE$	
$\overline{AB} \cong \overline{CB}$	Given
$\angle BAD \cong \angle BCE$	Given $\angle 1 \cong \angle 2$
$\angle ABD \cong \angle CBE$	Common
$\therefore \quad \Delta ABD \cong \Delta CBE$	S.A.A $\cong$ S.A.A

Q2. **From a point on the bisector of an angle, perpendiculars are drawn to the arms of the angle. Prove that these perpendiculars are equal in measure.**

Solution:



Given:

$\overline{BD}$ is bisector of $\angle ABC$. P is point on $\overline{BD}$ and $\overline{PL}$ and $\overline{PM}$ are perpendicular to $\overline{AB}$ and $\overline{CB}$ respectively.

To prove:

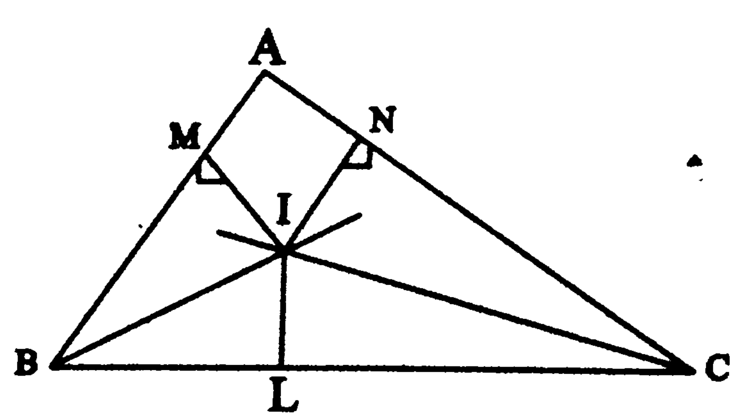
$$\overline{PL} \cong \overline{PM}$$

Proof:

Statements	Reasons
In $\Delta BLP \leftrightarrow \Delta BMP$	
$\overline{BP} \cong \overline{BP}$	Common
$\angle BLP \cong \angle BMP$	Each right angle (given)
$\angle LBP \cong \angle MBP$	Given $\overline{BD}$ is bisector of angle B
$\therefore \quad \Delta BLP \cong \Delta BMP$	S. A. A $\cong$ S. A. A.
So $\quad \overline{PL} \cong \overline{PM}$	Corresponding sides of $\cong \Delta$'s.

Q3. **In a triangle ABC, the bisectors of $\angle B$ and $\angle C$ meet in a point I. Prove that I is equidistant from the three sides of ΔABC.**

Solution:



Given:

In ΔABD the bisector of $\angle B$ and $\angle C$ meet at I, IL, IM and IN are perpendiculars to the three sides of ΔABD.

To prove:

$$\overline{IL} \cong \overline{IM} \cong \overline{IN}$$

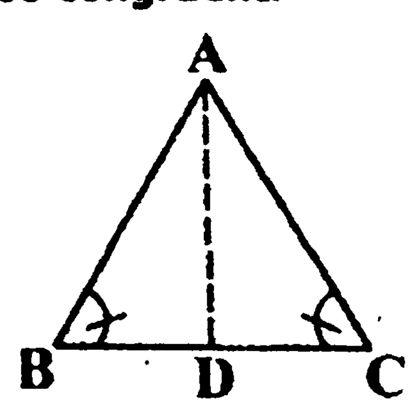
Proof:

Statements	Reasons
In $\Delta ILB \leftrightarrow \Delta IMB$	
$\overline{BI} \cong \overline{BI}$	Common
$\angle IBL \cong \angle IBM$	Given BI is bisector of $\angle B$
$\angle ILB \cong \angle IMB$	Given each $\angle$ is right angles.
$\Delta ILB \cong \Delta IMB$	S. A. A $\cong$ S. A. A.
$\overline{IL} \cong \overline{IM}$ (i)	Corresponding sides of $\cong$ Δs.
Similarly	
$\Delta IAC \cong \Delta INC$	
So $\overline{IL} \cong \overline{IN}$ (ii)	
From (i)and (ii)	Corresponding sides of $\cong$ Δs.
$\overline{IL} \cong \overline{IM} \cong \overline{IN}$	
$\therefore$ I is equidistant from the three sides of ΔABC.	

THEOREM 10.1.2

If two angles of a triangle are congruent, then the sides opposite to them are also congruent.

Solution:



Given:

In ΔABC,

$\angle B \cong \angle C$

To Prove:

$$\overline{AB} \cong \overline{AC}$$

Construction:

Draw the bisector of $\angle A$, to meet $\overline{BC}$ at point D.

In ΔFDL	
$\angle 2 \cong \angle 4$ .... (iii)	$\overline{FL} \cong \overline{FD}$ (Proved)
Similarly $\angle 1 \cong \angle 3$ ..... (iv)	
$\therefore m\angle 2 + m\angle 1 = m\angle 4 + m\angle 3$	From (iii) and (iv)
Now, in $\Delta DEF \leftrightarrow \Delta LEF$	
$\overline{FD} \cong \overline{FL}$	Proved
And $\angle EDF \cong \angle ELF$	Proved
$\overline{DE} \cong \overline{LE}$	Each one $\cong \overline{AB}$
$\therefore \Delta DEF \cong \Delta LEF$	S.A.S. Postulate
Also $\Delta ABC \cong \Delta LEF$	Proved
Hence $\Delta ABC \cong \Delta LEF$	Each one $\cong \Delta LEF$ (Proved)

EXERCISE 10.3

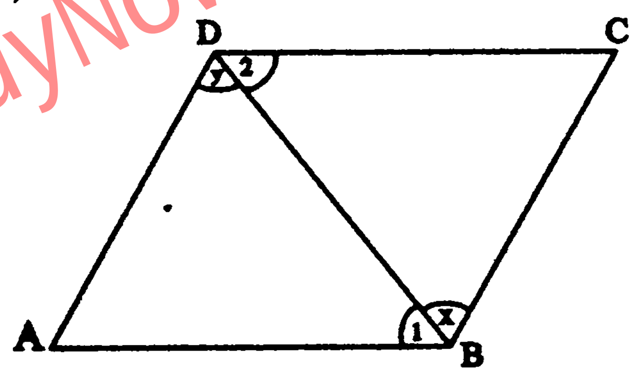
Q1. In the given figure, $\overline{AB} \cong \overline{DC}$, $\overline{AD} \cong \overline{BC}$.
Prove that $\angle A \cong \angle C$, $\angle ABC \cong \angle ADC$

Solution:

Given:

In the figure

$\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$



To prove:

$\angle A \cong \angle C$

$\angle ABC \cong \angle ADC$

Construction:

Join B to D.

Proof:

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta CDB$	
$\overline{AB} \cong \overline{DC}$	Given
$\overline{AD} \cong \overline{CD}$	Given
$\overline{BD} \cong \overline{DB}$	Common
$\therefore \Delta ABC \cong \Delta CDB$	S.S.S. $\cong$ S.S.S.
$\therefore \angle A \cong \angle C$	Corresponding sides of $\cong \Delta$s.

$\left.\begin{array}{c}\hat{1} \cong \hat{2} \\ \text{and} \\ \angle x \cong \angle y\end{array}\right\}$	Corresponding sides of $\cong \Delta$s.
$\therefore$ by adding above equations $\hat{1} + \hat{x} = \hat{2} + \hat{y}$	Addition of angles
or $\angle ABC \cong \angle ADC$	

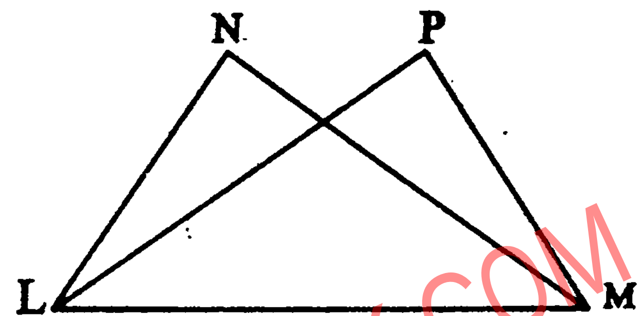
Q2. In the figure, $\overline{LN} \cong \overline{MP}, \overline{MN} \cong \overline{LP}$.
Prove that $\angle N \cong \angle P, \angle NML \cong \angle PLM$.

Solution:

Given:

In the figure

$\overline{LN} \cong \overline{MP}$ and $\overline{LP} \cong \overline{MN}$



To prove:

$\angle N \cong \angle P$ and

$\angle NML \cong \angle PLM$

Proof:

Statements	Reasons
In $\Delta LMN \leftrightarrow \Delta MLP$	
$\overline{LN} \cong \overline{MP}$	Given
$\overline{LP} \cong \overline{MN}$	Given
$\overline{LM} \cong \overline{ML}$	Common
$\therefore \Delta LMN \cong \Delta MLP$	S.S.S. $\cong$ S.S.S
$\therefore \angle N \cong \angle P$	Corresponding sides of $\cong \Delta$'s.
$\angle NML \cong \angle PLM$	Corresponding sides of $\cong \Delta$'s.

Q3. Prove that the median bisecting the base of an isosceles triangle bisects the vertex angle and it is perpendicular to the base.

Solution:

Given:

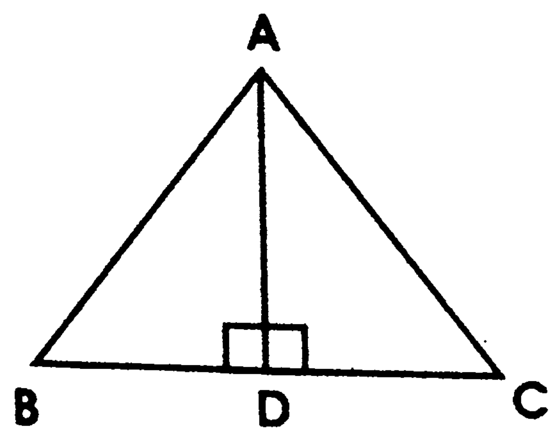
An isosceles triangle ABC with base $\overline{BC}$ and $\overline{AD}$ bisects at point D.

i.e. $\overline{BD} \cong \overline{DC}$ and $\overline{AB} \cong \overline{AC}$

To prove:

$\angle BAD \cong \angle CAD$

and $\overline{AD} \perp \overline{BC}$



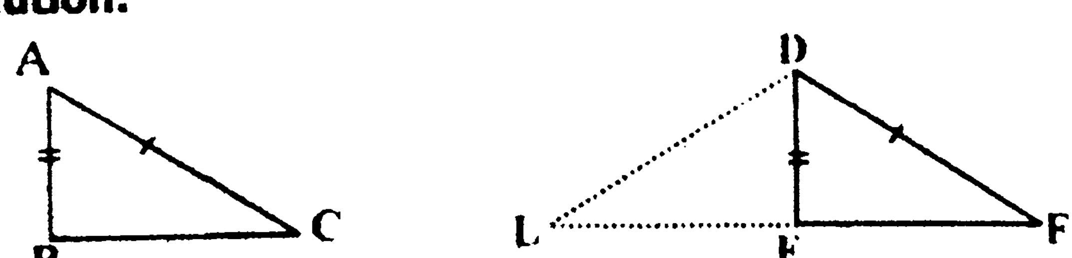
Proof:

Statement	Reasons
In the correspondence of $\Delta ABD \leftrightarrow \Delta ACD$	
$\overline{AB} \cong \overline{AC}$	Given
$\overline{AD} \cong \overline{AD}$	Common
$\overline{BD} \cong \overline{DC}$	Given
$\therefore \Delta ABD \cong \Delta ACD$	S.S.S. postulate
Thus $\angle BAD \cong \angle CAD$	Corresponding angles of congruent triangle
$m\angle ADB + m\angle ADC = 180^0$	Supplementary angles
$m\angle ADC + m\angle ADC = 180^0$	
$\Rightarrow 2\, m\angle ADC = 180^0$	As $m\angle ADC = m\angle ADB$
$\Rightarrow m\angle ADC = 90^0$	
Hence $\overline{AD} \perp \overline{BC}$	

THEOREM 10.1.4

If in the correspondence of the two right-angled triangles, the hypotenuse and one side of one triangle are congruent to the hypotenuse and the corresponding side of the other, then the triangles is congruent.

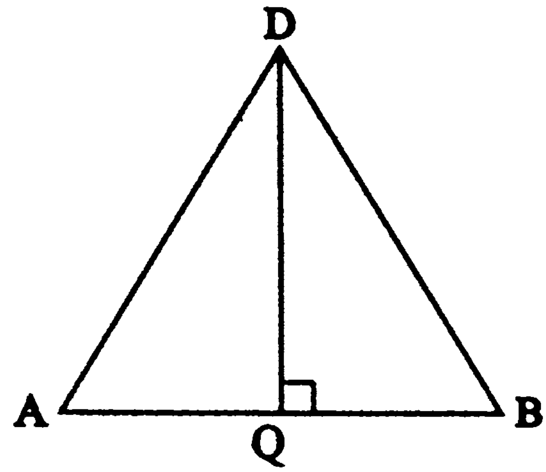
Solution:



EXERCISE 10.4

Q1. In Δ PAB of figure, $\overline{PQ} \perp \overline{AB}$, and $\overline{PA} \cong \overline{PB}$, Prove that $\overline{AQ} \cong \overline{BQ}$, and $\angle APQ \cong \angle BPQ$.

Solution:



Given:

In ΔPAB,

$\overline{PQ} \perp \overline{AB}$, and $\overline{PA} \cong \overline{PB}$

To prove:

$\overline{AQ} \cong \overline{BQ}$

and $\angle APQ \cong \angle BPQ$

Proof:

Statements	Reasons
In $\Delta APQ \leftrightarrow \Delta BPQ$	
$\overline{PA} \cong \overline{PB}$	Given
$\angle AQP \cong \angle BQP$	Given $PQ \perp AB$
$\overline{PQ} \cong \overline{PQ}$	Common
$\therefore \quad \Delta APQ \cong \Delta BPQ$	H.S. $\cong$ H.S.
So $\quad \overline{AQ} \cong \overline{BQ}$	Corresponding sides of $\cong \Delta$s.
and $\quad \angle APQ \cong \angle BPQ$	Corresponding sides of $\cong \Delta$s.

Q2. In the figure, $m\angle C = m\angle D = 90^\circ$ and $\overline{BC} \cong \overline{AD}$. Prove that $\overline{AC} \cong \overline{BD}$, and $\angle BAC \cong \angle ABD$.

Solution:

Given:

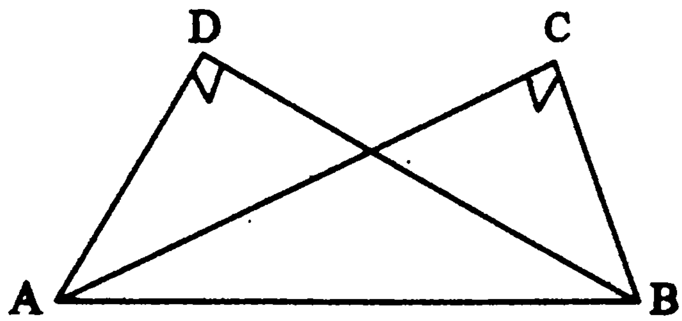
In the figure,

$m\angle C = m\angle D = 90^\circ$

and $\overline{BC} \cong \overline{AD}$

To prove:

$\angle ABC \cong \angle ABD$

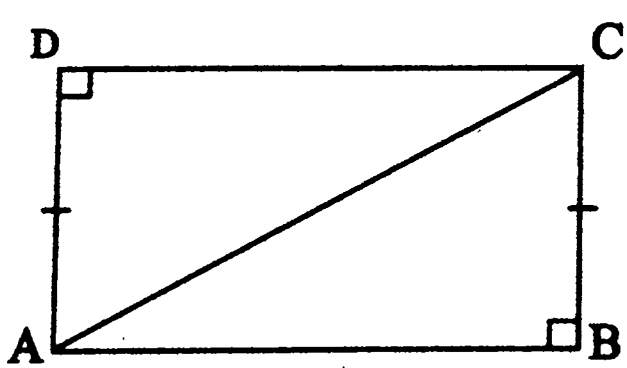


Proof:

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta BAC$	
$\hat{D} \cong \hat{C}$	Given
$\overline{AD} \cong \overline{BC}$	Given
$\overline{AB} \cong \overline{BA}$	Common
$\therefore$ $\Delta ABD \cong \Delta BAC$	H. S. $\cong$ H. S.
So $\overline{AC} \cong \overline{BD}$	Corresponding sides of $\cong$ Δ's.
and $\angle BAC \cong \angle ABD$	Corresponding sides of $\cong$ Δ's.

Q3. In the figure, $m\angle B = m\angle D = 90°$ and $\overline{AD} \cong \overline{BC}$. Prove that ABCD is a rectangle.

Solution:



Given:

In the figure,

$m\angle B = m\angle D = 90°$ and $\overline{AD} \cong \overline{BC}$

To prove:

ABCD is a rectangle.

Construction:

Join A to C.

Proof:

Statements	Reasons
In $\Delta ABC \leftrightarrow \Delta CDA$	
$\hat{B} \cong \hat{D}$	Given each angle = 90°
$\overline{AC} \cong \overline{CA}$	Common
$\overline{BC} \cong \overline{DA}$	Given
$\therefore$ $\Delta ABC \cong \Delta CDA$	H.S. $\cong$ H.S.
$\therefore$ $\overline{AB} \cong \overline{CD}$	Corresponding sides of $\cong$ Δ's.
and $\angle ACB \cong \angle CAD$	Corresponding sides of $\cong$ Δ's.
Hence ABCD is a rectangle	

REVIEW EXERCISE 10

Q1. Which of the following are true and which are false?

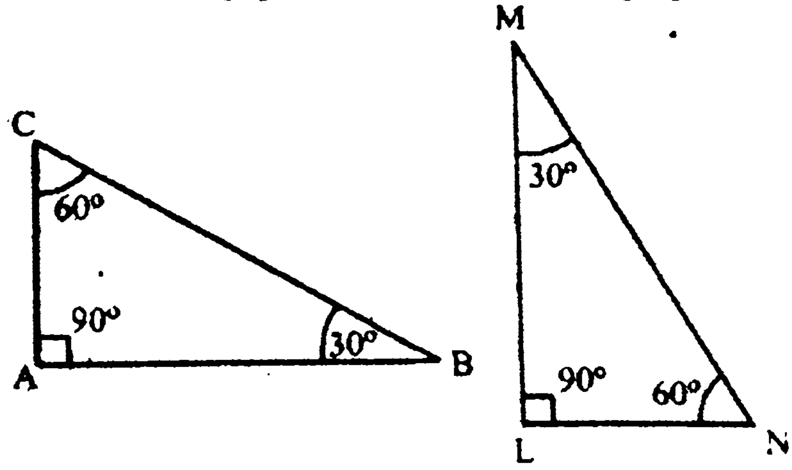
(i) A ray has two end points.
(ii) In a triangle, there can be only right angle.
(iii) Three points are said to be collinear if they lie on same line.
(iv) Two parallel lines intersect only at a point.
(v) Two lines can intersect only at one point.
(vi) A triangle of congruent sides has non-congruent angles.

Answers:

(i) F	(ii) T	(ii) T	(iv) F	(v) T	(vi) F

Q2. If $\Delta ABC \cong \Delta LMN$, then

(i) $m\angle M = \cdots$ **(ii)** $m\angle N = \cdots$ **(iii)** $m\angle A = \cdots$

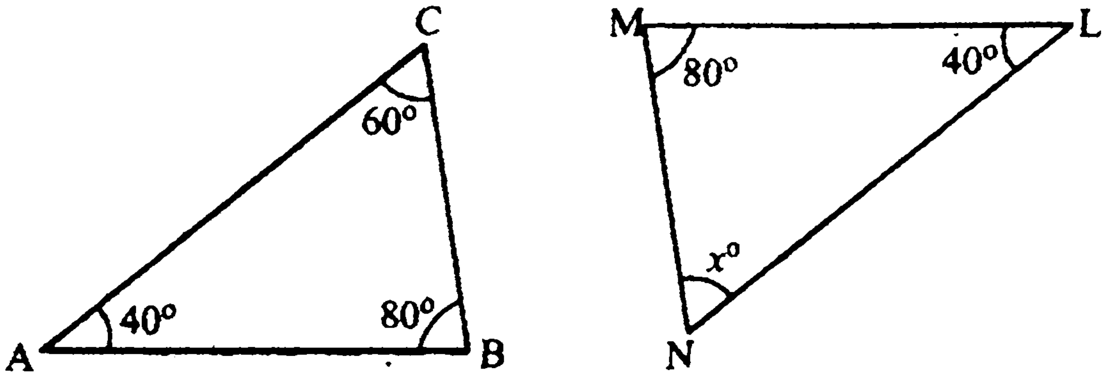


Solution:

(i) $m\angle B$	(ii) $m\angle C$	(iii) $m\angle L$

Q3. If $\Delta ABC = \Delta LMN$, then find the unknown x.

Solution:



Given that $\Delta ABC = \Delta LMN$

$\therefore$ $\angle C \cong \angle M$

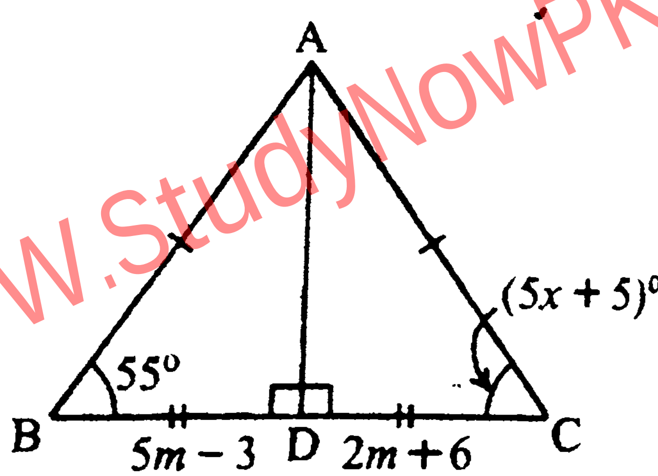
or $m\angle C \cong m\angle M$

$\Rightarrow$ $60^\circ = x^\circ$

$\Rightarrow$ $x = 60^0$

Q4. Find the value unknowns for the given congruent triangles.

Solution:



$\Delta ADB \cong \Delta ADC$

$\overline{BD} \cong \overline{CD}$

Corresponding sides of $\cong \Delta's$.

$\Rightarrow$ $m\overline{BD} \cong m\overline{CD}$

$\Rightarrow$ $5m - 3 = 2m + 6$

or $5m - 2m = 6 + 3$

$3m = 9$

$\angle B \cong \angle C$

Corresponding sides of $\cong \Delta's$.

$\Rightarrow$ $m\angle B \cong m\angle C$

$55^\circ = (5x + 5)^\circ$

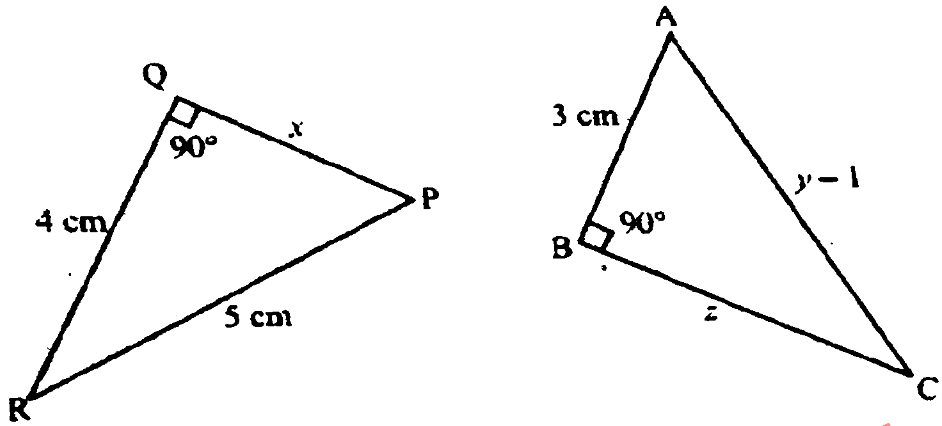
$\Longrightarrow \quad 55 = 5x + 5$

or $\quad 5x = 55 - 5 = 50$

$\Longrightarrow \quad x = 10^0$

Q5. If PQR ≅ ABC, then find the unknowns.

Solution:



$\Delta PQR \cong \Delta ABC$

$\therefore \quad \overline{BD} \cong \overline{CD}$

Corresponding sides of ≅ Δ's.

$\Longrightarrow \quad$ **x = 3 cm**

$\overline{PR} \cong \overline{AC}$

Corresponding sides of ≅ Δ's.

$\Longrightarrow \quad 5 = y - 1$

$y = 5 + 1$

$\Longrightarrow \quad$ **y = 6**

Also $\quad \overline{QR} \cong \overline{BC}$

Corresponding sides of ≅ Δ's.

$m\overline{QR} \cong m\overline{BC}$

$4\text{ cm} = z$

or $\quad$ **z = 4 cm**