Exercise 1.4

Q.1 Which of the following product of matrices if conformable for multiplication?

(i) $\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \end{bmatrix}$

Yes, these matrices can be multiplied because number of columns of 1st matrix is equal to number of rows of 2nd matrix.

(ii) $\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} -2 & -1 \\ 1 & 3 \end{bmatrix}$

Yes, these matrices can be multiplied because number of columns of 1st matrix is equal to number of rows of 2nd matrix.

(iii) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$

No, these matrices cannot be multiplied because number of columns of 1st matrix is not equal to the number of rows of 2nd matrix.

(iv) $\begin{bmatrix} 1 & 2 \\ 0 & -1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$

Yes, these matrices can be multiplied because number of columns of 1st matrix is equal to number of rows of 2nd matrix.

(v) $\begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ -2 & 3 \end{bmatrix}$

Yes, these matrices can be multiplied because number of columns of 1st matrix is equal to number of rows of 2nd matrix.

Q.2 If $A=\begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$, $B=\begin{bmatrix} 6 \\ 5 \end{bmatrix}$ **find**

(i) AB

Solution: $AB=\begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 6 \\ 5 \end{bmatrix}$

$$AB=\begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

$$=\begin{bmatrix} (3\times 6)+(0\times 5) \\ (-1\times 6)+(2\times 5) \end{bmatrix}$$

$$=\begin{bmatrix} 18+0 \\ -6+10 \end{bmatrix}=\begin{bmatrix} 18 \\ 4 \end{bmatrix}$$

(ii) BA **(if possible)**

Solution:

BA is not possible became number of columns of B not equal to number of rows of A.

Q.3 Find the following products

(i) $\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}$

Solution: $\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}$

$= \begin{bmatrix}(1\times 4)+(2\times 0)\end{bmatrix}$

$= \begin{bmatrix}4+0\end{bmatrix}$

$= \begin{bmatrix}4\end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \end{bmatrix}$

Solution: $\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ -0 \end{bmatrix}$

$= \begin{bmatrix}(1\times 5)+(2\times -4)\end{bmatrix}$

$= \begin{bmatrix}5+(-8)\end{bmatrix}$

$=[5-8]$

$=[-3]$

(iii) $\begin{bmatrix} -3 & 0 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}$

Solution: $\begin{bmatrix} -3 & 0 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}$

$= \begin{bmatrix}(-3\times 4)+(0\times 0)\end{bmatrix}$

$=[-12+0]$

$=[-12]$

(iv) $\begin{bmatrix} 6 & 0 \end{bmatrix}\begin{bmatrix} 4 \\ -0 \end{bmatrix}$

Solution: $\begin{bmatrix} 6 & 0 \end{bmatrix}\begin{bmatrix} 4 \\ -0 \end{bmatrix}$

$\begin{bmatrix} 6 & +0 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}$

$=\begin{bmatrix}6\times 4+(-0)(0)\end{bmatrix}$

$=\begin{bmatrix}24-0\end{bmatrix}$

$=\begin{bmatrix}24\end{bmatrix}$

(v) $\begin{bmatrix} 1 & 2 \\ -3 & 0 \\ 6 & -1 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 0 & -4 \end{bmatrix}$

Solution: $\begin{bmatrix} 1 & 2 \\ -3 & 0 \\ 6 & -1 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 0 & -4 \end{bmatrix}$

$=\begin{bmatrix} 1\times4+2\times0 & 1\times5+2\times(-4) \\ -3\times4+0\times0 & -3(5)+0\times(-4) \\ 6(4)+(-1)(0) & 6(5)+(-1)(-4) \end{bmatrix}$

$=\begin{bmatrix} 4+0 & 5-8 \\ -12+0 & -15-0 \\ 24-0 & 30+4 \end{bmatrix}$

$=\begin{bmatrix} 4 & -3 \\ -12 & -15 \\ 24 & 34 \end{bmatrix}$

Q.4 Multiply the following matrices.

(a) $\begin{bmatrix} 2 & 3 \\ 1 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}$

Solution: $\begin{bmatrix} 2 & 3 \\ 1 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}$

$\begin{bmatrix} 2\times2+(3\times3) & (2\times-1)+(3\times0) \\ (1\times2)+(1\times3) & (1\times-1)+(1\times0) \\ (0\times2)+(-2\times3) & (0\times-1)+(-2\times0) \end{bmatrix}$

$=\begin{bmatrix} 4+9 & -2+0 \\ 2+3 & -1+0 \\ 0+-6 & 0+0 \end{bmatrix}$

$$= \begin{bmatrix} 13 & -2 \\ 5 & -1 \\ 0-6 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & -2 \\ 5 & -1 \\ -6 & 0 \end{bmatrix}$$

(b) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 1 \end{bmatrix}$

Solution: $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 1 \end{bmatrix}$

$$= \begin{bmatrix} (1\times1)+(2\times3)+(3\times-1) & (1\times2)+(2\times4)+(3\times1) \\ (4\times1)+(5\times3)+(6\times-1) & (4\times2)+(5\times4)+(6\times1) \end{bmatrix}$$

$$= \begin{bmatrix} 1+6+(-3) & 2+8+3 \\ 4+15+(-6) & 8+20+6 \end{bmatrix}$$

$$= \begin{bmatrix} 7-3 & 13 \\ 19-6 & 34 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 13 \\ 13 & 34 \end{bmatrix}$$

(c) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

Solution: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

$$= \begin{bmatrix} (1\times1)+(2\times4) & (1\times2)+(2\times5) & (1\times3)+(2\times6) \\ (3\times1)+(4\times4) & (3\times2)+(4\times5) & (3\times3)+(4\times6) \\ (-1\times1)+(1\times4) & (-1\times2)+(1\times5) & (-1\times3)+(1\times6) \end{bmatrix}$$

$$= \begin{bmatrix} 1+8 & 2+10 & 3+12 \\ 3+16 & 6+20 & 9+24 \\ -1+4 & -2+5 & -3+6 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 12 & 15 \\ 19 & 26 & 33 \\ 3 & 3 & 3 \end{bmatrix}$$

(d) $\begin{bmatrix} 8 & 5 \\ 6 & 4 \end{bmatrix}\begin{bmatrix} 2 & -\frac{5}{2} \\ -4 & 4 \end{bmatrix}$

Solution: $\begin{bmatrix} 8 & 5 \\ 6 & 4 \end{bmatrix}\begin{bmatrix} 2 & -\frac{5}{2} \\ -4 & 4 \end{bmatrix}$

$$= \begin{bmatrix} (8\times2)+(5\times-4) & \left(8\times-\frac{5}{2}\right)+(5\times4) \\ (6\times2)+(4\times-4) & \left(6\times-\frac{5}{2}\right)+(4\times4) \end{bmatrix}$$

$$= \begin{bmatrix} 16+(-20) & \frac{-40}{2}+20 \\ 12+(-16) & \frac{-30}{2}+16 \end{bmatrix}$$

$$= \begin{bmatrix} 16-20 & -20+20 \\ 12-16 & -15+16 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 0 \\ -4 & 1 \end{bmatrix}$$

(e) $\begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Solution: $\begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$= \begin{bmatrix} (-1\times0)+(2\times0) & (-1\times0)+(2\times0) \\ (1\times0)+(3\times0) & (1\times0)+(3\times0) \end{bmatrix}$$

$$= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Q.5 **Let** $A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$

and $C = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$ **verify whether**

(i) $AB = BA$

Solution: $AB = BA$

L.H.S = AB

R.H.S = BA

L.H.S =AB

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times1)+(3\times-3) & (-1\times2)+(3\times-5)\\ (2\times1)+(0-3) & (2\times2)+(0-5)\end{bmatrix}$$

$$=\begin{bmatrix}-1+(-9) & -2+(-15)\\ 2+0 & 4+0\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\ 2 & 4\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}$$

$$\text{R.H.S}=\text{BA}=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}\times\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}$$

$$=\begin{bmatrix}1\times(-1)+2\times2 & 1\times3+2\times0\\ -3\times(-1)+(-5)2 & -3\times3+(-5)(0)\end{bmatrix}$$

$$=\begin{bmatrix}-1+4 & 3+0\\ 3-10 & -9-0\end{bmatrix}$$

$$=\begin{bmatrix}3 & 3\\ -7 & -9\end{bmatrix}$$

Since L.H.S $\neq$ R.H.S

L.H.S $\neq$ R.H.S

L.H.S $\neq$ R.H.S

(ii) $A(BC)=(AB)C$

Solution: $A(BC)=(AB)C$

L.H.S = A (BC)

R.H.S = (AB) C

L.H.S

L.H.S=A(BC)

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\left(\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}\right)$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}2+2 & 1+6\\ -6+(-5) & -3+(-15)\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}4 & 7\\ -6-5 & -3-15\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}4 & 7\\ -11 & -18\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times4)+(3\times-11) & (-1\times7)+(3\times-18)\\ (2\times4)+(0\times-11) & (2\times7)+(0\times-18)\end{bmatrix}$$

$$=\begin{bmatrix}-4+(-33) & -7+(-54)\\ 8+0 & 14+0\end{bmatrix}$$

$$=\begin{bmatrix}-4-33 & -7-54\\ 8 & 14\end{bmatrix}$$

$$=\begin{bmatrix}-37 & -61\\ 8 & 14\end{bmatrix}$$

R.H.S = (AB)C

$$=\left(\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}\right)\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times1)+(3\times-3) & (-1\times2)+(-3\times-5)\\ (2\times1)+(0\times-3) & (2\times2)+(0\times-5)\end{bmatrix}$$

$$=\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-1+(-9) & -2+(-15)\\ 2+0 & -4+0\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\ 2 & 4\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}(-10\times2)+(-17\times1) & (-10\times1)+(-17\times3)\\ (2\times2)+(4\times1) & (2\times1)(4\times3)\end{bmatrix}$$

$$=\begin{bmatrix}-20+(-17) & -10+(-51)\\ 4+4 & 2+12\end{bmatrix}$$

$$=\begin{bmatrix}-20-17 & -10-51\\ 8 & 14\end{bmatrix}$$

$$=\begin{bmatrix}-37 & -61\\ 8 & 14\end{bmatrix}$$

Since

L.H.S = R.H.S $\Rightarrow$ A (BC) = (AB)C

Hence proved

(iii) $A(B+C)=AB+AC$

Solution: $A(B+C)=AB+AC$

L.H.S = A (B+C)

R.H.S = AB+AC

L.H.S

LHS=A (B+C)

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\left(\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}+\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}\right)$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1+2 & 2+1\\ -3+1 & -5+3\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}3 & 3\\ -2 & -2\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times3)+(3\times-2) & (-1\times3)+(3\times-2)\\ (2\times3)+(0\times-2) & (2\times3)+(0\times-2)\end{bmatrix}$$

$$=\begin{bmatrix}-3+(-6) & -3+(-6)\\ 6+0 & 6+0\end{bmatrix}$$

$$=\begin{bmatrix}-3-6 & -3-6\\ 6 & 6\end{bmatrix}$$

$$=\begin{bmatrix}-9 & -9\\ 6 & 6\end{bmatrix}$$

R.H.S=AB+AC

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}+\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times1)+(3\times-3) & (-1\times2)+(3\times-5)\\ (2\times1)+(0\times-3) & (2\times+2)+(0\times-5)\end{bmatrix}+\begin{bmatrix}(-1\times2)+(3\times1) & (-1\times1)+(3\times3)\\ (2\times2)+(0\times1) & (2\times1)+(0\times3)\end{bmatrix}$$

$$=\begin{bmatrix}-1+(-3) & -2+(-15)\\ 2+0 & +4+0\end{bmatrix}+\begin{bmatrix}-2+3 & -1+9\\ 4+0 & 2+0\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\ 2 & +4\end{bmatrix}+\begin{bmatrix}1 & 8\\ 4 & 2\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\ 2 & +4\end{bmatrix}+\begin{bmatrix}1 & 8\\ 4 & 2\end{bmatrix}$$

$$=\begin{bmatrix}-10+1 & -17+8\\ 2+4 & +4+2\end{bmatrix}$$

$$=\begin{bmatrix}-9 & -9\\ 6 & 6\end{bmatrix}$$

Since LHS=RHS

A (B+C) = AB+AC

Hence proved

(iv) A(B–C) = AB–AC

Solution: A (B–C) = AB–AC

L.H.S = A (B–C)

R.H.S = AB–AC

L.H.S=A(B–C)

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\left(\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}-\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}\right)$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1-2 & 2-1\\ -3-1 & -5-3\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}-1 & 1\\ -4 & -8\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times-1)+(3\times-4) & (-1\times1)+(3\times-8)\\ (2\times-1)+(0l-4) & (2\times1)+(0\times-8)\end{bmatrix}$$

$$=\begin{bmatrix}+1+(-12) & -1+(-24)\\ -2+0 & 2+0\end{bmatrix}$$

$$=\begin{bmatrix}1-12 & -1-24\\ -2 & 2\end{bmatrix}$$

$$=\begin{bmatrix}-11 & -25\\ -2 & 2\end{bmatrix}$$

R.H.S=AB–AC

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}-\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times1)+(3\times-3) & (-1\times2)+3\times-5)\\ (2\times1)+(0\times-3) & (2\times2)+(0\times-5)\end{bmatrix}$$

$$-\begin{bmatrix}(-1\times2)+(3\times1) & (-1\times1)+(3\times3)\\ (2\times2)+(0\times1) & (2\times1)+(0\times3)\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times1)-(3\times3) & (-1\times2)+(3\times-5)\\ (2\times1)+(0\times-3) & (2\times2)+(0\times-5)\end{bmatrix}$$

$$-\begin{bmatrix}(-1\times2)+(3\times1) & (-1\times1)+(3\times3)\\ (2\times2)+(0\times1) & (2\times1)+(0\times3)\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\ 2+0 & 4+0\end{bmatrix}-\begin{bmatrix}-2+3 & -1+9\\ 4+0 & 2+0\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}-\begin{bmatrix}1 & 8\\ 4 & 2\end{bmatrix}$$

$$=\begin{bmatrix}-10-1 & -17-8\\ 2-4 & 4-2\end{bmatrix}$$

$$=\begin{bmatrix}-11 & -25\\ -2 & 2\end{bmatrix}$$

Since L.H.S = R.H.S

A (B-C) =AB-AC, Hence **proved.**

Q.6 For the matrices $A=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}$,

$B=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$ **and** $C=\begin{bmatrix}-2 & 6\\ 3 & -9\end{bmatrix}$

Verify that

(i) $(AB)^t = B^t A^t$

Solution: $(AB)^t = B^t A^t$

L.H.S $= (AB)^t$

R.H.S $= B^t A^t$

$$(AB)=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times1)+(3\times-3) & (-1\times2)+(3\times-5)\\ (+2\times1)+(0\times-3) & (2\times2)+(0\times-5)\end{bmatrix}$$

$$=\begin{bmatrix}-1+(-9) & -2+(-15)\\ 2+0 & 4+0\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\ 2 & 4\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}$$

LHS$= (AB)^t$

$$=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}^t$$

$$=\begin{bmatrix}-10 & 2\\ -17 & 4\end{bmatrix}$$

$$B^t=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}^t$$

$$B^t=\begin{bmatrix}1 & -3\\ 2 & -5\end{bmatrix}$$

$$A^t=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}$$

$$A^t=\begin{bmatrix}-1 & 2\\ 3 & 0\end{bmatrix}$$

L.H.S$=B^tA^t$

$$=\begin{bmatrix}1 & -3\\ 2 & -5\end{bmatrix}\times\begin{bmatrix}-1 & 2\\ 3 & 0\end{bmatrix}$$

$$=\begin{bmatrix}(1\times-1)+(-3\times3) & (1\times2)+(-3\times0)\\ (2\times-1)+(-5\times3) & (2\times2)+(-5\times0)\end{bmatrix}$$

$$=\begin{bmatrix}-1+(-9) & 2+0\\ -2+(-15) & 4+0\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & 2\\ -2-15 & 4\end{bmatrix}$$

$$=\begin{bmatrix}-10 & 2\\ -17 & 4\end{bmatrix}$$

Since L.H.S = R.H.S

$(AB)^t = B^tA^t$

Hence proved

L.H.S = R.H.S

(ii) $\mathbf{(BC)^t= C^tB^t}$

Solution: $(BC)^t= C^tB^t$

L.H.S $= (BC)^t$

R.H.S $= C^t B^t$

To solve L.H.S

$$BC=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}\times\begin{bmatrix}-2 & 6\\ 3 & -9\end{bmatrix}$$

$$=\begin{bmatrix}(1\times-2)+(2\times3) & (1\times6)+(2\,\text{x}\times-9)\\ (-3\times-2)+(-5\times3) & (-3\times6)+(-5\times-9)\end{bmatrix}$$

$$=\begin{bmatrix}-2+6 & 6+(-18)\\ 6+(-15) & -18+45\end{bmatrix}$$

$$=\begin{bmatrix}4 & 6-18\\ 6-15 & 27\end{bmatrix}$$

$$=\begin{bmatrix}4 & -12\\ -9 & 27\end{bmatrix}$$

Taking transpose of BC:-

$$(BC)^t=\begin{bmatrix}4 & -12\\ -9 & 27\end{bmatrix}^t$$

$$\text{LHS} = (B\,C)^t = \begin{bmatrix}4 & -9\\ -12 & 27\end{bmatrix}$$

To solve R.H.S =

Taking transpose of matrix C

$$C^t=\begin{bmatrix}-2 & 3\\ 6 & -9\end{bmatrix}$$

Taking transpose of matrix B

$$B^t=\begin{bmatrix}1 & -3\\ 2 & -5\end{bmatrix}$$

Now, multiplying matrices, $B^t C^t$

$$\text{R.H.S} = C^t B^t=\begin{bmatrix}-2 & 3\\ 6 & -9\end{bmatrix}\times\begin{bmatrix}1 & -3\\ 2 & -5\end{bmatrix}$$

$$=\begin{bmatrix}(-2\times1)+(3\times2) & (-2\times-3)+(3\times-5)\\ (6\times1)+(-9\times2) & (6\times-3)+(-9\times-5)\end{bmatrix}$$

$$=\begin{bmatrix} -2+6 & 6+(-15) \\ 6+(-18) & -18+45 \end{bmatrix}$$

$$=\begin{bmatrix} 4 & 6-15 \\ 6-18 & 27 \end{bmatrix}$$

$$=\begin{bmatrix} 4 & -9 \\ -12 & 27 \end{bmatrix}$$

Hence proved
L.H.S = R.H.S

Last Updated: September 2020

Report any mistake at freeilm786@gmail.com