CHAPTER 01

SETS

BASIC DEFINITIONS AND FORMULAS

Concept of a set: A set is a well-defined collection of distinct objects. Each object is called an element or a member of the set. The members of the set can be anything: numbers, rivers, letters, books, fish, etc.

Notations: Sets are denoted by capital letters A, B, C …x, y, z elements of a set are denoted by small letters a, b, c….. z, y, z the number of elements in a set S are denoted by |S| or by n(S) or by O(s).

Notations for sets of numbers

i) $N = \{1, 2, 3, \ldots\}$

i.e the set of the natural numbers.

ii) $W = \{0, 1, 2, 3, \ldots\}$

i.e. the set of all non-negative integers.

iii) $Z = (\ldots -3, -2, -1, 0, 1, 2, 3, \ldots)$

i.e., the set of all integers.

iv) $Q = \{x|x = \frac{p}{q}, \text{ p and q} \in \mathbf{z}, q \neq 0\}$

i.e the set of all rational numbers.

v) $I = \{x|x \neq \frac{p}{q}, \text{ p and q} \in \mathbf{z}, q \neq 0\}$

i.e., the set of all irrational numbers.

vi) $R = \{x|x = \frac{p}{q}, \text{ p or } x \neq \frac{p}{q} \text{ and p, q} \in \mathbf{z}, q \neq 0\}$

i.e., the set of all real numbers.

Also R^+ and R^- will, respectively, denote the set of all positive and all negative real numbers.

Subsets: A set B is a subset of a set A, denoted by $B \subseteq A$, if every element of B is also an element of A, Symbolically. $B \subseteq A$.

Note: The set of all subsets of a set A is called the power set of A and is denoted by P(A).

Superset: If B is a subset of a set A, then A is called a superset of B denoted by $A \supset B$.

Proper subset : A set B is a proper subset of a set A, denoted by $B \subset A$, If B is a subset of A and if there exists at least one element in A that is not in B. Symbolically, $B \subset A$.

And there exists at least one element $\alpha \in A$ such that $\alpha \notin B$

Equal sets: Two sets A and B are equal if and only if each element of A belongs to B and vice versa

If A and B are equal then we write

$$A = B$$

Thus $A = B$ iff $A \subseteq B$ and $B \subseteq A$

Ordered pairs : Two ordered pairs (a,b) and (c, d) are equal if and only if

$$a = c, b = d$$

Thus $(a, b) \neq (b, a)$ iff $a \neq b$

The intersections of two sets: The intersection of two sets A and B denoted by $A \cap B$, is the set of all elements belonging to both the sets A and B,

$A \cap B = \{x | x \in A \text{ and } x \in B\}$

The union of two sets: The union of two sets A and B denoted by A U B, is the set of all elements belonging either to A or to B or to both A and B, i.e

$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

Equivalent sets: Two sets A and B are said to be equivalent, denoted by A ~ B, if they have same number of elements

Exhaustive sets: If A and B be subsets of a set U such that

$$A \cup B = U$$

Then the sets A and B are called exhaustive sets:

The difference of two sets: The difference of two sets A and B in the stated order denoted by A – B is the set of all elements of A that are not in B i.e.

$A - B = \{x \mid x \in A \text{ and } x \notin B\}$

Also $B - A = \{x \mid x \in B \text{ and } x \notin A\}$

Complement of a set : The complement of a set A , denoted by A` is the difference U – A where U is the universal set i.e.

$A' = U - A = \{x \mid x \in U \text{ and } x \notin A\}$

The Cartesian product of two sets : The Cartesian product of any set A with any other set B is the set of all ordered pairs (a, b), where $a \in A$ and $b \in B$. it is denoted by A × B and is read as "A cross B". Symbolically.

$A \times B \{(a, b) / a \in A, b \in B\}$

PROPERTIES OF OPERATIONS ON SETS

Clousure poerperty: Operations of union, intersection and difference on any two sets are closed in U. that is if A and B are any two sets in U, then $A \cap B$, $A \cup B$ and $A - B$ are also sets in U.

Commutative property : Operations of union and intersection of any two sets are commutative but the difference operation is not commutative in general that is

$A \cup B = B \cup A$., $A \cap B = B \cap A$ but $A - B \neq B - A$ is general

Associative laws

i) Associative property of union, If A, B and C are any three sets then $(A \cup B) \cup C = A \cup (B \cup C)$

ii) Associative property of intersection if A, B and C are any three sets, then $(A \cap B) \cap C = A \cap (B \cap C)$

Distributive laws

i) Distributive property of intersection over union

If A, B and C are any three sets

Then

a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (Left distribitivity)

b) $(B \cup C) \cap A = (B \cap A) \cup (C \cap A)$ (Right Distributivity)

Distributive property of union over intersection

If A, B and C are any three sets then

a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (Left Distributivity)

b) $(B \cap C) \cup A = (B \cup A) \cap (C \cup A)$ (Right Distributivity)

Distributive property of Cartesian product over union.

If, A, B and C are any three sets, then

a) $A \times (B \cup C) = (A \times B) \cup (A \times C)$ (Left distributivity)

b) $(B \cup C) \times A = (B \times A) \cup (C \times A)$ (Right distributivity)

Distributive property of Cartesian product over intersection

If A, B and C are any three sets then

a) $A \times (B \cap C) = (A \times B) \cap (A \times C)$ (Left Distributivity)

b) $(B \cap C) \times A = (B \times A) \cap (C \times A)$ (Right distributivity)

Distributive property of the cartesian product over complement.

a) $(A - B) \times C = (A \times C) - (B \times C)$

b) $C \times (A - B) = (C \times A) - (C \times B)$

Both equations represent different sets.

De Morgan's laws

If A, and B are any two sets, then

a) $(A \cup B)' = A' \cap B'$ b) $(A \cap B)' = A' \cup B'$

CHAPTER 02

REAL AND COMPLEX NUMBER SYSTEMS

Basic Formulas and Definitions

The System of Complex numbers: The set C in which the equation $x^2 = a$ can be solved for all $a \in R$ and is given by $C = R \times R = \{ (a, b) / a, b \in R\}$ is called the set of complex numbers and its elements (ordered pairs) are called complex numbers.

We define equality, addition and multiplication (a, b), (c, d) as follows

(i) Equality : $(a, b) = (c, d)$ if $a = c, b = d$

ii) addition : $(a, b) + (c, d) = (a + c, b + d)$

iii) Multiplication : $(a, b) (c, d) = (ac - bd, ad + bc)$

The division of complex umbers

Let $z_1 = (a, b)$, $z_2 = (c, d)$ with $z_2 \neq (0, 0)$ be complex numbers. Then

iv) $\dfrac{z_1}{z_2} = z_1 z_2^{-1} = \left(\dfrac{ac+bd}{c^2+d^2}, \dfrac{bc-ad}{c^2+d^2}\right)$

v) $-(a, b) = (-a, -b)$ is the additive inverse of $(a, b) \in C$

vi) and $\left(\dfrac{a}{a^2+b^2}, \dfrac{b}{a^2+b^2}\right)$ is the multiplicative inverse of (a, b) where and is denoted by $(a, b)^{-1}$.

(vii) Subtraction $(a, b) - (c , d) = (a - c, b - d)$.

DEFINITIONS OF IMAGINARY NUMBERS THEIR CONJUGATES AND MODULI

The ordered pair (a, 0)in which the second member is zero has the properties of the real number a.

For example

$(a, 0) + (b, 0) = (a + b, 0)$

$(a, 0). (b, 0) = (ab, 0)$

$(a, 0) (c, d) = (ac, ad) = a(c, d). \ \forall (c, d) \in C$

Hence, the ordered pair (a, 0) is identified with $a \in R$

Imaginary Numbers

The ordered pairs (0, 1) is denoted by the letter í, read as "IOTA". Then $í^2 = í \,.\, í$

$í^2 = (0, 1) . (0, 1) \Rightarrow í^2 = (-1, 0) \in D \Rightarrow í^2 = -1 \in R$

So $í^2 = \sqrt{-1}$

"í" is called an imaginary numbers because there is no real number x satisfying the property $x^2 = -1$

The number of the form ib is called an imaginary number $\forall$ $b \in R$.

The ordered pairs (a, b) is also written in the form a + ib or a + bi, where a is called the real part and b is called the imaginary part for

$(a, b) = (a, 0) + (0 , b)$

$(a, b) = (a, 0) + (0, 1) (b, 0)$

$(a, b) = (a, 0) + í (b, 0)$

$(a, b) = a + bi$

Remarks: Operations on complex numbers can be performed by treating the complex factors as if they were real and remembering that $i^2 = -1$

Conjugate of complex numbers

$(a , b) = a - ib$ is called the complex conjugate of $(a, b) = a + ib$

Thus complex conjugate of $(a, - b)$ will be $(a,-(-b)) = (a, b)$

Hence , (a, b) and (a, –b) are complex conjugates of each other.

If $z = (a, b)$ then its conjugate $(a, -b)$ is denoted by $\bar{z}$

Modulus of a complex number

The modulus, magnitude or absolute value of a complex number $z = x + iy = (x, y)$ is a non-negative real number denoted by $|z| = |x + iy|$ and is given by $\sqrt{x^2+y^2}$.

Theorem

If z_1 and z_2 are complex numbers, then

i) $\overline{z_1+z_2} = \overline{z_1}+\overline{z_2}$

ii) $\overline{z_1.z_2} = \overline{z_1}.\overline{z_2}$

iii) $z.\overline{z} = \overline{z}.z = |z|^2$

iv) a) $z+\overline{z}$ is purely real and b) $z-\overline{z}$ is purely imaginary.

vi) $|z_1.\ z_2| = |z_1|.|z_2|$

iv) a) $z+\overline{z} = (x, y) + (x, -y) = (x + x, y - y) = (2x, 0) = 2x$, Which is purely real.

b) Similarly $z-\overline{z} = 2iy$ is purely imaginary

Note that $x = \dfrac{z+\overline{z}}{2}$ and $y = \dfrac{z-\overline{z}}{2i}$ are called conjugate coordinates.

CHAPTER 03

EQUATIONS

BASIC DEFINITIONS AND FORMULAS

SYNTHETIC DIVISION

The Steps of the method are given below.

1. Write in the first row the coefficient in the ascending powers of x in P(x).
2. Write the first coefficient in the third row below its position in the first row.
3. Write the product of factor x, the multiplier and this coefficient in the second row beneath the second coefficient in the first row and add putting the sum below them in the third row and so on.

METHOD OF COMPLETING THE SQUARES.

We know that an expression like $x^2 \pm 2ax + a^2$ can be written as $(x \pm a)^2$. Sometime a given expression contains only two terms instead of three i.e one of the three terms is missing which can be determined.

1. $\text{First term} = \dfrac{(\text{Middle Term})^2}{4\ (\text{Last Term})}$
2. $\text{Last Term} = \dfrac{(\text{Middle Term})^2}{4\ (\text{First Term})}$
3. $\text{Middle Term} = \pm\sqrt{\text{First Term} \times \text{Last term}}$

THE QUADRATIC FORMULA

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1. $(a + b)^2 = a^2 + 2ab + b^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

THE CUBE ROOTS OF UNITY

$$\left\{1, \frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}\right\} = \{1, \omega, \omega^2\}$$

PROPERTIES OF THE CUBE ROOTS OF UNITY

1. Each of the complex roots of unity is the square of the other i.e., $(\omega^2)^2 = \omega$
2. The sum of the three cube roots of unity is zero i.e., $1 + w + w^2 = 0$
3. The product of the three cube roots of unity is one i.e., $(1)(\omega)(\omega^2) = \omega^3 = 1$

SOME USEFUL FORMULAS

1. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
2. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
3. $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

EQUATIONS REDUCIBLE TO THE QUADRTIC FORM

SOME USEFUL FORMULAE

1. $(a + b)^2 = a^2 + 2ab + b^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
4. $(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$
5. $(a - b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$

NATURE OF THE ROOTS OF A QUADRATIC EQUATION

DISCRIMINANT = D = $b^2 - 4ac$

1. If D = 0 then the roots are equal
2. If D > 0 then the roots are real and unequal
3. If D < 0 then the roots are complex (imaginary) and unequal

4. if D is a perfect square then roots are rational and unequal

TO FORM A QUADRATIC EQUATION WHEN ITS ROOTS ARE GIVEN

x^2 – (Sum of the roots) x + (product of the roots) = 0

Some useful formulae

1. $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
2. $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$
3. Additive inverse of x is $-x$
4. Multiplicative inverse of x is $\frac{1}{x}$

RELATIONS BETWEEN THE ROOTS AND THE COEFFICIENTS OF A QUADRATIC EQUATION

Let $\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$, and $\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

1. Sum of the roots $= \alpha + \beta = -\frac{b}{a} = -\frac{\text{Cofficient of } x}{\text{Cofficient of } x^2}$
2. Product of the roots $= \alpha\beta = \frac{b}{a} = \frac{\text{Constant Term}}{\text{Cofficient of } x^2}$

SOME USFUL FORMULAE

1. Perimeter of triangle = a + b + c
2. Perimeter of rectangle = 2(x + y)
3. Pythagoras theorem
   $(\text{Base})^2 + (\text{Perpendicular})^2 = (\text{Hypotenuse})^2$
4. Circumference = $2\pi r$
5. Area of circle = πr^2
6. Area of triangle = $\frac{1}{2}$ (Base)(Altitude)
7. Area of rectangle = Length × breadth
8. Area of a square = $(\text{Length})^2$

CHAPTER 04

MATRICES AND DETERMINANTS

BASIC DEFINITIONS AND FORMULAS

Matrix: A rectangular array of (real or complex) numbers of the form

$$M = \begin{bmatrix} a_{11} & a_{12} & a_m \\ a_{21} & a_{22} & a_n \\ \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{mn} \end{bmatrix}$$

Subject to certain algebraic operations, to be mentioned later is called a matrix, of order (or dimension) m × n read as "m by n" The matrix, M consists of m n number of the type.

Square matrix : A matrix in which the number of rows is equal to the number of columns i.e m =n, is called a square matrix order n. If m ≠n, the matrix is said to be rectangular matrix.

Diagonal elements: Element aíí of the square matrix (with í = j) are called diagonal elements and the line along which these elements lie is called the principal (or leading) diagonal.

Row and column matrices: Any 1 × n matrix having only one row is called a row matrix; whereas m × 1 matrix only one column is called a column matrix.

Example. $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$. $\begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$. $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ are column matrices.

Whereas [1 2], [1 2 5], [a b c] are row matrices.

Transpose Matrices: The matrix of order n × m obtained by interchanging the rows and the columns of an m × n matrix A is called the transpose of A and is denoted by A^t. read as "A transpose"

Symbolically if A = $[\,a_{íj}\,]_{(m,\,n)}$

SPECIAL TYPES OF MATRICES

Diagonal matrix: A square matrix all of whose elements are zero except those in the main diagonal is called a diagonal matrix.

e.g. $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

Unit Matrix : A diagonal matrix in which all the diagonal elements are equal to one is called a unit matrix; A unit matrix of order n Is denoted by I_n.

e.g. $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Scalar Matrix : A diagonal matrix in which all the diagonal elements are equal (same) is called a scalar matrix.

e.g. $\begin{bmatrix} b & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & b \end{bmatrix}$

Null matrix (Zero Matrix) : An m × n matrix whose elements are all zero is called the null (or zero) matrix, denoted by O_{mn} i.e $a_{ij}=0$

Symbolically $\quad O_{m,n} = [0]_{[mn]}$

e.g. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0_3$

Equal matrices: Two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ of the same order are said to be equal, denoted by A = B, if their elements in the corresponding positions are equal symbnolically

$$A = B \Leftrightarrow a_{ij} = b_{ij}$$

Addition of matrices : Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two matrices of the same order then the sum A and B denoted by A + B denoted by A + B is defined to be the matrix $C = [C_{ij}]$ where

$$C_{ij} = a_{ij} + b_{ij} \quad \text{or} \quad C = A + B$$

Subtraction of Matrices: If $A = [a_{ij}]$ and $B = [b_{ij}]$ are any two matrices of the same order then A – B is defined as

$$A - B = A + (-B) = [a_{ij} - b_{ij}]$$

Multiplication of Matrices : Let $A = [a_{ij}]$ be an m × p matrix and $B = [b_{ij}]$ an p × n matrix $C = [c_{ij}]$ where.

$$[C_{ij}] = [a_{ij}] \times [b_{ij}] \Rightarrow C = A \times B$$

PROPERTIES OF MATRIX OPERATIONS

Matrix addition and scalar multiplication: Following properties are satisfied by the matrices, A.B and C of the same order, and any two scalars k, k_2 w.r.t matrix addition and scalar multiplication.

(i) $A + B$ is also a matrix of the same order
(ii) $(A + B) + C = A + (B + C)$
(iii) $A + B = B + A$
(iv) For any matrix $A = [a_{ij}]_{(m\ n)}$ there exists a matrix of the same order called a zero matrix denoted by O, such that
$A + O = O + A = A$
(v) Where $O = [Oij]_{(m\ n)}$ is called the additive identity for the set of all $m \times n$ matrices.
v) For any matrix $A = [a_{ij}]_{(m\ n)}$ there exists a matrix B of the same order such that
$A + B = B + A = 0$
Where O is the $m \times n$ zero matrix
The matrix B is called the additive inverse of A and is denoted by $-A$
(vi) $k_1\ A$ is also a matrix of the same order
(vii) $(k_1k_2)\ A = k_1\ (k_2A)$
(viii) $(k_1 + k_2)\ A = k_1A + k_2A$ and $k_1\ (A + B) = k_1\ A + k_2\ B$
(ix) $1A = A$ and $(-1)\ A = -A$
(x) $OA = O = AO$ and $k_1O = Ok_1 = O$
Where O is the null matrix of the same order.

Properties of matrix Multiplications: If the matrices A, B, C are conformable for addition and multiplication. then

(i) $(AB)\ C = A\ (BC) = ABC$
(ii) $A\ (B + C) = AB + AC$, and $(B + C)\ A = BA + CA$
(iii) $AI = IA = A$, Where A and I are the same order
(iv) $k\ (AB) = (kA)B = A\ (kB)$. Where k is scalar

Properties of transposed Matrices: If two matrices A and B are conformable for addition and multiplication then,

(i) $(A \pm B)^t = A^t \pm B^t$
(ii) $(kA)^t = kA^t$ Where k is scalar
iii) $(AB)^t = B^tA^t$ (Reversal Law for transpose of product)
iv) $(A^t)^t = A$
Corollary 1. $(A + B + C)^t = A^t + B^t + C^t$
Corollary 2. $(ABC)^t = C^t\ B^t\ A^t$

PROPERTIES OF DETERMINANTS OF ORDER THREE

The properties given in this section are very useful in evaluating the determinants of any order.

In the properties given below R_1, R_2 and R_3 represent first second and third rows; C_1 $C_2$and C_3 represent first, second and third columns respectively. Also |A| represents

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Property 1. The value of a determinant is unaltered by changing its rows and column i.e for any matrix A

$$|A| = |A^t|$$

Note that the expansion of $|A^t|$ is simply the expansion of |A| by its first column.

Property 2. The interchange of any two rows, or of any two columns, of a matrix A changes the sign of its determinant without altering its numerical value.

Property 3. If two rows of a matrix A are identical then

$$|A| = 0$$

Property 4. If all the elements of a row of a square matrix A are zero then

$$|A| = 0$$

Property 5. If every element in a row of a matrix A is multiplied by the same number k, then |A| gets multiplied by k.

Property 6. if every element of a row of a matrix A be expressed as the sum of two terms then |A| can be expressed as the sum of determinants of two matrices differing in the elements of that row but with remaining rows as the same as those of |A|.

Property 7. If the elements of one row of a matrix A are k times the elements of its another row, then |A| = 0

Property 8. If to each element of a row of a matrix A is added. A constant multiple of the corresponding elements of another row then the value of |A| is unaltered.

CHAPTER 05

GROUPS

BASIC DEFINITIONS AND FORMULAS

BINARY OPERATIONS : A binary operation on a non –empty set S. is a function $S \times S \longrightarrow S$, Thus binary operation is a rule which associates with each ordered pair (a, b); a and b $\in$ s a unique element a b of s.

MULTIPLICATION (OR COMPOSION) TABLE

$S = \{1, -1, i, -i\}$

•	1	–1	i	$-i$
1	1	–1	i	$-i$
–1	–1	1	$-i$	i
i	i	$-i$	–1	1
$-i$	$-i$	i	1	–1

$S = \{1, \omega, \omega^2\}$

x	1	ω	ω^2
1	1	ω	ω^2
ω	ω	ω^2	1
ω^2	ω^2	1	ω

PROPERTIES OF BINARY OPERATIONS:-

1) **COMMUTATIVE BINARY OPERATIONS**: A binary operation * on a set S is said to be commutative if

   a * b = b * a, $\forall$ a, b $\in$ S

2) **ASSOCIATIVE BINARY OPERATIONS**: a binary operation * on a set S is said to be associative if (a * b) * c = a * (b * c), $\forall$ a, b, c, $\in$ S

3) **IDENTITY ELEMENT**: Let s be a set with a binary operation *. an element e $\in$ S. is said to be an identity of s w.r.t *- if

   a * e = e * a = a, $\forall$ a $\in$ S

4) **INVERSE ELEMENTS:** Let S be a set with a binary operation * having an identity element e. An element b $\in$ S is said to be an inverse of a $\in$ S w.r.t * if a * b = b * a = e

GROUPOIDS: A groupoid (S, *) is an ordered pair consisting of a non-empty set S and a binary operation * defined on S.

SEMI GROUPS/ ASSOCIATIVE GROUPOIDS: A groupoid (S, *) is called a semi group if * is associative in S.

ABELIAN SEMI GROUOPS: A semi group (S, *) is called an abelian semi group if * is commutative in S.

GROUPS: An ordered pair (G, *) of a non-empty set G and a binary operation * is said to be a group if the following properties are satisfied.

1. * is associative in G
2. There exists on identity element e $\in$ G w.r.t.*

   a * e = e * a = a, $\forall$ a $\in$ G
3) For every element a' $\in$ G. There exists an element a' $\in$ G called the inverse of a such that a' * a = a * a' = e

ABELIAN GROUPS: A group (G, *) is said to be an abelian group if * is commutative on G i.e if a * b = b * a, $\forall$ a, b $\in$ G.

CHAPTER 06

SEQUENCES AND SERIES

BASIC DEFINITION AND FORMULAE

ARITHMETIC SEQUENCE OR ARITHMETIC PROGRESSION (A.P): A sequence in which each term is formed by adding a fixed number to the one preceding it, is called are arithmetic sequence or arithmetic progression. (A. P)

STANDARD FORM OF AN A.P.: If the first term of an A.P is "a" and the common difference "d" then by definition.

T_1 = the first term = a or $a + (1 - 1)$ d;
T_2 = the second term = a + d or $a + (2 - 1)$ d;
T_3 = the third term = a + 2d or $a + (3 - 1)$ d;
T_4 = the fourth term = a + 3d or $a + (4 - 1)$ d;

Hence by similarly, we deduce that

T_n = the n the term = $a + (n - 1)$ d, . . .(1)

Which is the formula for finding the nth term of an arithmetic sequence.

Whose first term is "a" and common difference "d".

T_n is called the general term of the sequence

a, a + d, a + 2d, a + 3d, . . . , a + (n – 1)d is known as the standard form of an A.P.

ARITHMETIC SERIES: The sum of the terms of an arithmetic sequence is called an arithmetic series In general, an arithmetic series of n terms with "a" as its first term and "d" as its common difference is.

$a + (a + d) + (a + 2d) + \cdots + \{a + (n - 1) d\}$

$S_n = \frac{n}{2}\{2a + (n - 1) d\} \Rightarrow S_n = \frac{n}{2}(a + \ell)\ \ T_n = \ell = a + (n - 1) d$

ARITHMETIC MEANS

1) A Single Arithmetic mean between a and b

$$A.M = \frac{a+b}{2}$$

2) n Arithmetic means between a and b, $(n > 1)$

$A_1 = a + d \Rightarrow A_2 = a + 2d \Rightarrow A_3 = a + 3d \Rightarrow A_4 = a + 4d$

$A_n = a + nd$

Where $d = \frac{b-a}{n+1}$

a = First term ⇒ b = Second term
n = Number of A.M's ⇒ d = Common difference

3) Let three numbers in A.P be (a – d), a (a + d)
4) Let four numbrs in A.P be (a – 3d), (a – d) (a + d) (a + 3d) -
5) Let five numbers in A.P be (a–2d), (a–d), a, (a + d), (a + 2d)

GEOMETRIC SEQUENCE OR GEOMETRIC PROGRESSION (G . P) : A sequence in which the first term is a non-zero number and each of the terms is formed by multiplying the previous one by a certain non-zero fixed number is called a geometric sequence or a geometric progression (G. P).

STANDARD FORM OF G.P: If the first term of a G.P is "a" and the common ratio "r" then by definition.

T_1 = the first term = a or $a\,r^{1-1}$
T_2 = the second term = ar or $a\,r^{2-1}$
T_3 = the third term = ar^2 or ar^{3-1}
T_4 = the fourth term = ar^3 or ar^{4-1}

And so on

Hence, by similarity, we deduce that

T_n = the nth term = $ar^{n-1} \longrightarrow (1)$

Which is the formula for finding the nth term of a geometric sequence whose first term is "a" and common ratio r.

T_n is called general term of the sequence.

The sequence

$a,\ ar,\ ar^2,\ ar^3,\ \cdots\ ar^{n-1} \longrightarrow (1)$

is known as standard form of G.P.

GEOMETRIC SERIES: The sum of the terms of a geometric sequence is called geometric series. In general a geometric series with a as its first term and "r" as its common ratio is

$$a + ar + ar^2 + ar^3 + \cdots + ar^{n-1} \longrightarrow (1)$$

If S_n denotes the sum to n term of the series, We have

$$S_n = a + ar + ar^2 + \cdots + ar^{n-1}$$

Multiply each side by r we have

$$rS_n = ar + ar^2 + ar^3 + \cdots ar^n \longrightarrow (2)$$

Hence from (1) and (2) by subtraction

$$S_n - rS_n = a - ar^n$$

Or $S_n(1-r) = a(1-r^n)$

If $r \neq 1$ dividing by $(1 - r)$, We get

$$S_n = \frac{a(1-r^n)}{1-r},\ r < 1 \rightarrow (3) \text{ Or } S_n = \frac{a(r^n-1)}{r-1},\ r > 1 \rightarrow (4)$$

Formula (3) and (4) are used respectively

When $r < 1$ and $r > 1$

If ℓ depends the last term i.e

If $\ell = T_n = ar^{n-1}$

Then we have.

INFINITE GEOMETRIC SERIES

1) Infinite Geometric progession (G.P) $a, ar, ar^2, ar^3, \cdots$
2) Infinite Geometric Series (G.S) $a + ar + ar^2 + ar^3 + \cdots$
3) sum of the infinite Geometric series (s)

$$S = \frac{a}{1-r}$$

Where a = first term and r = common ratio.

GEOMETRIC MEANS TO INSERT

1) A single Geometric mean between a and b is G.M = $\pm\sqrt{a \times b}$
2) "n" Geometric means between a and b are

$$G_1 = ar = a\left(\frac{b}{a}\right)^{\frac{1}{n+1}} \Rightarrow G_2 = ar^2 = a\left(\frac{b}{a}\right)^{\frac{2}{n+1}}$$

$$G_3 = ar^3 = a\left(\frac{b}{a}\right)^{\frac{3}{n+1}} \Rightarrow G_n = ar^n = a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$$

Where

a = first term, b = last term and r = common ratio = $\left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

3) Let the three numbers in G.P be $\frac{a}{r}, a, ar$
4) Let the four number in G.P be $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$
5) Let the five numbers in G.P be $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

HARMONIC SEQUENCE OR HARMONIC PROGESSION (H.P): A sequence is said to be a harmonic sequence or a harmonic progression (H.P) if the reciprocals of its terms are in arithmetic progression.

$a, (a + d), (a + 2d) \cdots \{a + (n - 1) d\}$ are in A.P

$\frac{1}{a}, \frac{1}{(a+d)}, \frac{1}{(a+2d)}, \cdots \frac{1}{\{a+(n-1)d\}}$ are in H.P.

GENERAL TERM OF AN H.P

$$T_n = \frac{ab}{b+(n-1)(a-b)}$$

Where a = First term

b = second term

n = number of terms

T_n = nth term = last term

AN IMPORTANT THEOREM

If $T_p = x$, $T_q = y$ and $T_r = z$ then

$$\begin{vmatrix} \frac{1}{x} & \frac{1}{y} & \frac{1}{z} \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} = 0$$

HARMONIC MEANS

1) A single harmonic mean between a and b is

$$H.M = \frac{2ab}{a+b}$$

2) n Harmonic mean between a and b are H_1, H_2, H_3 & H_4.

$$H_1 = \frac{(n+1)ab}{a+nb} \qquad H_3 = \frac{(n+1)ab}{3a+(n-2)b}$$

$$H_2 = \frac{(n+1)ab}{2a+(n-1)b} \qquad H_4 = \frac{(n+1)ab}{4a+(n-3)b}$$

CHAPTER 07

PERMUTATIONS COMBINATIONS AND INTRODUCTION TO PROBABILITY

BASIC DEFINITION AND FORMULAE

1) If A and B are two sets (overlapping) then
O (AUB) = O (A) + O(B) – O(A∩B).

2) If A and B are disjoint sets then 0(AUB) = 0(A) + 0(B).

3) $0 (A \times B) = 0(A). 0(B)$

COUNTING

A process of determining the number of elements contained in a set is called counting the symbol 0(A) is used to denote the number of elements in the set A if A = {1, 2, 3, 4} then 0(A) = 4

FACTORIAL NOTATION

The product of first n natural numbers is denoted by n! and is read as factorial n.

$0! = 1, \quad 1! = 1$

$2! = 2 \times 1 = 2$

$3! = 3 \times 2 \times 1 = 6$

$4! = 4 \times 3 \times 2 \times 1 = 24$

PERMUTATIONS

The number of the different arrangement that can be form with r – object from a group of n–distinct objects is denoted by,

$$^nP_r = \frac{n!}{(n-r)!}$$

$r = 0, \ ^nP_o = 1$

$r = 1, \ ^nP_1 = n$

$r = n, \ ^nP_n = n!$

COMBINATIONS:-

Combinations means all possible selections consisting of r-different things, selected from the n-given things

Formula

$$^nC_r = \binom{n}{r} = \frac{n!}{(n-r)!\,r!}$$

$$\binom{n}{0} = 1, \quad \binom{n}{n} = 1$$

$$\binom{n}{1} = n, \quad \binom{n}{n-1} = n$$

GROUPS PERMUTATIONS: The number of groups permutations of n-objects of which n_1 are alike, n_2 are alike, n_3 are alike - - - n_k are alike is given by.

$$P = \binom{n}{n_1 n_2 n_3 - - - n_k} = \frac{n!}{n_1! n_2! n_3! - - - nk!}$$

Note:-

$n = n_1 + n_2 + - - - nk$

CIRCULAR PERMUTATIONS

1) The number of permutations of n-objects arranged in circle is $(n - 1)!$

2) Necklace or Garland permutations = $\frac{1}{2}(n - 1)!$

Playing cards = 52
Red cards = 26
Black cards = 26
Spade (black) = 13
Diamond (red) = 13
Heart (red) = 13
Club (black) = 13
Face Cards
King = 4 (2 black + 2 red)
Queen = 4 (2 black + 2 red)
Ace = 4(2 black + 2 red)
Jack = 4 (2 black + 2 red)

DIVISION INTO SECTIONS OR PARCELS

Theorem 1: The number of ways of partitioning a set consisting of (r + s) elements into Pairs of two disjoint subsets such that one subset consists of r elements and the other of S elements is.

$$\frac{(r+s)!}{r!s!}$$

Theorem 2 : Let A_1, A_2, A_3 be matually disjoint sets each consisting of r_1, r_2, r_3 elements respectively then the total number of mutually disjoint subsets of $A_1 \cup A_2 \cup A_3$ each consisting of r_1, r_2, r_3 elements is $\frac{(r_1+r_2+r_3)!}{r_1!\, r_2! r_3!}$

Theorem 3: If the sets A_1, A_2. A_3 consist of an equal numbers of elements i.e if $r_1 = r_2 = r_3 = r$ say then the required total number of mutually disjoint subsets of $A_1 \cup A_2 \cup A_3$ is $\frac{1}{n!}\frac{(nr)!}{(r!)^n}$

SAMPLE SPACE: A set of all sample points or out comes of an experiment is called the sample space. It is denoted by S.

EVENT: Any subset of sample space is called an event it is denoted by A or B or C.

THEOREM (PROBABILITY MEASURE): If in a random experiment the sample space S consists on n distinct outcomes or elements which are all equally likely and if A is an event of the corresponding sample space consisting of exactly m elements, then the probability of event A is given by $P(A) = \frac{O(A)}{O(S)} = \frac{m}{n}$

ADDITION LAW OF PROBABILITY

Theorem.1:

Let $A, B \subseteq S$ then (Total probability)
$P(A \cup B) + P(A \cap B) = P(A) + P(B)$

Theorem.2

If A and B are any two events then
$P[(A \cup B)]' = 1 - P(A \cup B)$

Theorem.3

If A and B be mutually exclusive events i.e if $A, B \subseteq S$ and $A \cap B = \phi$, then $P(A \cap B) = P(\phi) = 0$ so that $P(A \cup B) = P(A) + P(B)$

CHAPTER 08

MATHEMATICAL INDUCTION AND BINOMIAL THEOREM

BASIC DEFINITION AND FORMULAE

Description

1) Induction is the technique by which we verify the given proposition for n. where n is the integer.

2) This technique has following steps.

STEP NO.1: Denote the proposition by p(n) verify for n = 1. this step is called condition I.

STEP NO.2: Assume that p(n) is true for n = k verify the proposition for n = k + 1, this step is called condition II.

$$1 + 3 + 5 + \text{----------} + (2n-1) = n^2$$

L.H.S = Series, R.H.S = sum

Basic formulas

Sum of integral powers of natural numbers.

1) $1 + 2 + 3 + \text{- - -} n = \Sigma n = \dfrac{n(n+1)}{2}$

2) $1^2 + 2^2 + 3^2 + \text{- - -} + n^2 = \Sigma n^2 = \dfrac{n(n+1)(2n+1)}{6}$

3) $1^3 + 2^3 + 3^3 + \text{- - -} + n^3 = \Sigma n^3 = \left[\dfrac{n(n+1)}{2}\right]^2$

The Binomial Theorem for Positive Power

1) $(a + b)^1 = a + b$
2) $(a + b)^2 = a^2 + 2ab + b^2$
3) $(a + 3)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
4) $(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$
5) $(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \text{- -} + b^n$
6) $(a + b)^n = a^n + {}^nC_1\, a^{n-1}b + {}^nC_2\, a^{n-2}\, b^2 + {}^nC_3\, a^{n-3}\, b^3 + \text{- - -} + b^n$
7) $(a + b)^n = a^n + na^{n-1}b + \dfrac{n(n-1)}{2!}a^{n-2}b^2 + \dfrac{n(n-1)(n-2)}{3!}a^{n-3}b^3 + \text{- - -} + b^n$
8) The general term of $(a + b)^n$

$$T_{r+1} = \binom{n}{r} a^{n-r}\, b^r = {}^nC_r\, a^{n-r}\, b^r$$

9) Binomial Coefficient of First term = 1
10) Binomial Coefficient of Second term = n
11) Binomial Coefficient of Third term = $\dfrac{n(n-1)}{2!}$
12) Binomial Coefficient of 4^{th} term = $\dfrac{n(n-1)(n-2)}{3!}$
13) The Middle Terms
   i) If n is even then position of middle term = $T\left(\dfrac{n+2}{2}\right)$
   ii) If n is odd then position of middle terms $= T\left(\dfrac{n+1}{2}\right)$ and $T\left(\dfrac{n+3}{2}\right)$

The Binomial theorem for any index.

1) $(1 + x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \dfrac{n(n-1)(n-2)}{3!}x^3 + \text{- - -}$
2) $(1 + x)^{-1} = 1 - x + x^2 - x^3 + \text{- - -}$
3) $(1 - x)^{-1} = 1 + x + x^2 + x^3 + \text{- - -}$
4) $(1 + x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \text{- - -}$
5) $(1 - x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \text{- - -}$
6) The general term of $(1 + x)^n$ if n is any index

$$Tr + 1 = \frac{[n(n-1)(n-2)---(n-r+1)]x^r}{r!}$$

APPROXIMATIONS

1) $(1 + x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \dfrac{n(n-1)(n-2)}{3!}x^3 + \text{---}$
2) First Approximation, we may omit the terms containing squares and higher powers of x.
   $(1 + x)^n = 1 + nx$
3) Second Approximation, we may omit the terms containing cubes and higher powers of x.

$$(1 + x)^n = 1 + nx + \frac{n(n-1)x^2}{2!}$$

CHAPTER 09

FUNDAMENTALS OF TRIGONOMETRY

BASIC FORMULAS AND DIENINITIONS

RELATION BETWEEN RADIAN AND DEGREE MEASURE: In any circle of radius r units, a central angle of one radian intercepts an arc of length r units. Thus an arc length is directly proportional to the measure of its central angle, for the arc length r is equal to $\frac{1}{2\pi r}$ or $\frac{1}{2\pi}$ of the circumference.

Therefore an angle of π radians will intercept an arc of length πr or half of the circumference. So,

π radians = 180 degrees

i.e., 1 radian = $\frac{180}{\pi}$ degrees = 57.3°

and 1 degree = $\frac{\pi}{180}$ radians = .01745 radian

The following table shows some common angles measured both in degrees and radians.

Degrees	0	30	45	60	90	120	135	150	180
Radians	0	$\frac{\pi}{6}$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$	$\frac{\pi}{2}$	$\frac{2}{3}\pi$	$\frac{3}{4}\pi$	$\frac{5}{6}\pi$	π

RELATION BETWEEN ARC-LENGTH, RADUS AND CENTRAL ANGLE. :Consider an arc of length s of a circle with radus of length r measured in the same units. Let θ be the number of radians in the angle AOB subtended by the arc at the centre of the circle. Since the length of an arc in a circle is directly proportional to the measure of its central angle.

we have

$$\frac{s}{2\pi r} = \frac{\theta}{2\pi}$$

So, $S = r\theta$

Note: Here r and s are measured in the same units and θ is essentially the number of radians in the central angle.

SIGNS OF THE TRIGNOMETRIC FUNCTIONS IN THE FOUR QUADRANTS : Let r be a general angle in the standard position such that p(r) = (x, y) then

i) If ρ (r) is in the first quadrant, then $x > 0$, $y > 0$ so that cos r > 0 and sin r > 0 Hence tan r cosec r,

Sec r and cot r are all positive.

ii) If ρ (r) is in the second quadrant we have $x < 0$ and $y > 0$. so sin r and cosec r are both positive while tan r, cot r. cos r and sec r are negative.

iii) If ρ (r) is in the third quadrant, then $x < 0$ and $y < 0$. so that tan r and cot r are positive and others are negative.

iv) if ρ (r) is in the fourth quadrant. Then $x > 0$ and $y < 0$, so we have cos r and sec r positive and the others are negative .

Figure below gives the signs of all the trigonometric functions in the four quadrants.

If the value of one trigonometric function is knows the values of the other trigonometric functions can be found.

II Sin θ, Cosec θ = +ve All others = –ve	I All + ve
III tan θ, Cot θ = + ve	IV Cos θ, Sec θ = + ve

all others = –ve	All others = –ve

By definition of Radian function

Let $P(\theta) = (x, y)$ in a unit circle

Where $\cos\theta = x \Rightarrow \sin\theta = y \Rightarrow \tan\theta = \frac{y}{x}$

$\Rightarrow \cot\theta = \frac{x}{y} \Rightarrow \sec\theta = \frac{1}{x} \Rightarrow \text{Cosec} = \frac{1}{y}$

And $x^2 + y^2 = 1$

Table

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
Sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
Cosec θ	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
Sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
Cot θ	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

CHAPTER 10

TRIGONOMETRIC IDENTITIES

BASIC FORMULAS AND DIENINITIONS

1) $\cos^2\theta + \sin^2\theta = 1$
2) $1 + \tan^2\theta = \sec^2\theta$
3) $1 + \cot^2\theta = \text{Cosec}^2\theta$
4) $\tan\theta = \frac{\sin\theta}{\cos\theta}$
5) $\cot\theta = \frac{\cos\theta}{\sin\theta}$
6) $\sin\theta = \frac{1}{\text{Cosec}\theta}$, $\text{Cosec}\theta = \frac{1}{\sin\theta}$
7) $\cos\theta = \frac{1}{\sec\theta}$, $\sec\theta = \frac{1}{\cos\theta}$
8) $\tan\theta = \frac{1}{\cot\theta}$, $\cot\theta = \frac{1}{\tan\theta}$
9) $a^2 - b^2 = (a + b)(a - b)$
10) $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$
11) Distance Formula

    $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
12) Mid-point formula

    $M(x, y) = M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
13) Isosceles triangle has two sides are equal.
14) Equilateral triangle has three sides are equal.
15) Pythagoras theorem,

    $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

Sum and difference formulas

16) $\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$

17) $\sin(\alpha - \beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta$

18) $\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$

19) $\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$

20) $\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

21) $\tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

22) $\sin 2\theta = 2\sin\theta\cos\theta$

23) $\begin{bmatrix} \cos 2\theta = 2\cos^2\theta - 1 \\ \cos 2\theta = 1 - 2\sin^2\theta \\ \cos 2\theta = \cos^2\theta - \sin^2\theta \end{bmatrix}$

24) $\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

25) $\sin\theta = 2\sin\left(\dfrac{\theta}{2}\right) \cos\left(\dfrac{\theta}{2}\right)$

26) $\begin{bmatrix} \cos\theta = \cos^2\left(\frac{\theta}{2}\right) - \sin^2\left(\frac{\theta}{2}\right) \\ \cos\theta = 2\cos^2\left(\frac{\theta}{2}\right) - 1 \\ \cos\theta = 1 - \sin^2\left(\frac{\theta}{2}\right) \end{bmatrix}$

27) $\tan\theta = \dfrac{2\tan\left(\frac{\theta}{2}\right)}{1 - \tan^2\left(\frac{\theta}{2}\right)}$

Half Angle Formulas

28) $\sin\left(\dfrac{\theta}{2}\right) = \pm\sqrt{\dfrac{1 - \cos\theta}{2}}$

29) $\cos\left(\dfrac{\theta}{2}\right) = \pm\sqrt{\dfrac{1 + \cos\theta}{2}}$

30) $\tan\left(\dfrac{\theta}{2}\right) = \pm\sqrt{\dfrac{1 - \cos\theta}{1 + \cos\theta}}$

Product to Sum formulas

31) $\sin\alpha\cos\beta = \dfrac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$

32) $\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$

33) $\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$

34) $\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$

Sum to Product Formulas

35) $\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$

36) $\sin u - \sin v = 2\cos\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$

37) $\cos u + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$

38) $\cos U - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$

39) $\tan\left(\frac{\theta}{2}\right) = \frac{1-\cos\theta}{\sin\theta}$

40) $\tan\left(\frac{\theta}{2}\right) = \frac{\sin\theta}{1+\cos\theta}$

CHAPTER 12

SOLUTIONS OF TRIANGLES

BASIC FORMULAS AND DIENINITIONS

Trigonometric Ratios

$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$ $\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}$

$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$

Law of Sines

$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$ OR $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$

Laws of Cosines

i) $\cos\alpha = \frac{b^2+c^2-a^2}{2bc}$ ii) $\cos\beta = \frac{c^2+a^2-b^2}{2ca}$

iii) $\cos\gamma = \frac{a^2+b^2-c^2}{2ab}$

OR

i) $a^2 = b^2 + c^2 - 2bc\cos\alpha$ ii) $b^2 = a^2 + c^2 - 2ac\cos\beta$

iii) $c^2 = a^2 + b^2 - 2ab\cos\gamma$

LAWS OF TANGENTS:-

1) $\frac{a-b}{a+b} = \frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(\frac{\alpha+\beta}{2}\right)}$ 2) $\frac{b-c}{b+c} = \frac{\tan\left(\frac{\beta-\gamma}{2}\right)}{\tan\left(\frac{\beta+\gamma}{2}\right)}$

3) $\frac{c-a}{c+a} = \frac{\tan\left(\frac{\gamma-\alpha}{2}\right)}{\tan\left(\frac{\gamma+\alpha}{2}\right)}$

Theorem:- Sum of the measures of three angles of a triangle is equal to 180°.

OR $\alpha + \beta + \gamma = 180^\circ$.

Area of a triangle

Case I: When the measures of two sides and the measure of the included angle are given.

$\Delta = \frac{1}{2}bc\sin\alpha$ $\Delta = \frac{1}{2}ac\sin\beta$ $\Delta = \frac{1}{2}ab\sin\gamma$

Case II: When the measures of two angles and the measure of one side are known.

1) $\Delta = \frac{1}{2}a^2 \frac{\sin\beta\sin\gamma}{\sin\alpha}$ 2) $\Delta = \frac{1}{2}b^2 \frac{\sin\alpha\sin\gamma}{\sin\beta}$

3) $\Delta = \frac{1}{2}c^2 \frac{\sin\alpha\sin\beta}{\sin\gamma}$

Case III: When the measures of three sides are known

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

Where s = semi – Perimeter

$$S = \frac{a+b+c}{2}$$

HALF ANGLE FORMULAE IN TERMS OF a, b, c & s.

1) $\sin\left(\frac{\alpha}{2}\right) = \sqrt{\frac{(s-b)(s-c)}{bc}}$ 2) $\sin\left(\frac{\beta}{2}\right) = \sqrt{\frac{(s-a)(s-c)}{ac}}$

3) $\sin\left(\frac{\gamma}{2}\right) = \sqrt{\frac{(s-a)(s-b)}{ab}}$ 4) $\cos\left(\frac{\alpha}{2}\right) = \sqrt{\frac{s(s-a)}{bc}}$

5) $\cos\left(\frac{\beta}{2}\right) = \sqrt{\frac{s(s-b)}{ac}}$ 6) $\cos\left(\frac{\gamma}{2}\right) = \sqrt{\frac{s(s-c)}{ab}}$

7) $\tan\left(\frac{\alpha}{2}\right) = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$ 8) $\tan\left(\frac{\beta}{2}\right) = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$

9) $\tan\left(\frac{\gamma}{2}\right) = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$ 10) $\tan\left(\frac{\alpha}{2}\right) = \frac{r}{s-a}$

11) $\tan\left(\frac{\beta}{2}\right) = \frac{r}{s-b}$ 12) $\tan\left(\frac{\gamma}{2}\right) = \frac{r}{s-c}$

13) $r^2 = \frac{(s-a)(s-b)(s-c)}{s}$

CIRCUM – RADIUS = R

$$R = \frac{abc}{4\Delta}$$

IN RADIUS = r

$$r = \frac{\Delta}{s}$$

ESCRIBED – RADIUS

1) $r_1 = \frac{\Delta}{s-a}$ 2) $r_2 = \frac{\Delta}{s-b}$

3) $r_3 = \frac{\Delta}{s-c}$

CHAPTER 13

INVERSE TRIGONOMETRIC FUNCTIONS AND TRIGONOMETRIC EQUATIONS

BASIC FORMULAS AND DIENINITIONS

1) $\sin^{-1}\theta + \cos^{-1}\theta = \frac{\pi}{2}, -1 \le \theta \le 1$

2) $\sec^{-1}\theta + \operatorname{cosec}^{-1}\theta = \frac{\pi}{2}, \theta \le -1 \text{ or } \theta \ge 1$

3) $\tan^{-1}\theta + \cot^{-1}\theta = \frac{\pi}{2}, -\infty < \theta < \infty$

4) $\tan^{-1}A + \tan^{-1}B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$

5) $\tan^{-1}A - \tan^{-1}B = \tan^{-1}\left(\frac{A-B}{1+AB}\right)$

6) $f = \{(x, y) / (x, y) \in R \times R\}$
7) $f^{-1} = \{(y, x) / (y, x) \in R \times R\}$
8) The domain f becomes the range of f^{-1}
9) The range of f becomes the domain of f^{-1}
10)

function	Domain	Range

arc Sin	$-1 \leq \sin\theta \leq 1$	$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$
arc Cos	$-1 \leq \cos\theta \leq 1$	$0 \leq \theta \leq \pi$
arc tan	R	$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$