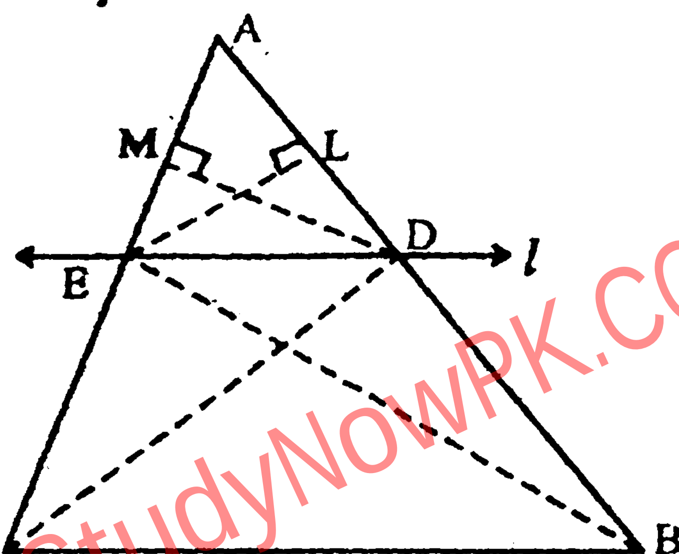
# Unit 14
# Ratio And Proportion

## THEOREM 14.1.1

**A line parallel to one side of a triangle and intersecting the other two sides divides them proportionally.**






**Given:**

In $\Delta ABC$ line $l$ is intersecting the sides $\overline{AC}$ and $\overline{AB}$ at points E and D respectively such that $\overline{ED} \| \overline{CB}$.

**To prove:**

$m\overline{AD} : m\overline{DB} = m\overline{AE} : m\overline{EC}$

**Construction:**

Join B to E and C to D and draw $\overline{DM}$ and $\overline{EL}$ perpendiculars from D and E on $\overline{AC}$ and $\overline{AB}$ to meet at the points M and L respectively.

**Proof:**

| Statements | Reasons |
|---|---|
| In triangles BED and AED, $m\overline{EL}$ is the common perpendicular | |
| $\Delta BED = \frac{1}{2} \times m\,\overline{BD} \times m\,\overline{EL}$ .....(i) | Area of a triangle = $\frac{1}{2}$ $(base \times height)$ |

| | |
|---|---|
| $\Delta AED = \frac{1}{2} \times m\overline{AD} \times m\overline{EL}$ ... (ii) | |
| $\frac{\Delta BED}{\Delta AED} = \frac{m\overline{BD}}{m\overline{AD}}$ ........... (a) | Dividing (i) by (ii) |
| Similarly | |
| $\frac{\Delta CDE}{\Delta ADE} = \frac{m\overline{EC}}{m\overline{AE}}$ ........... (b) | |
| But $\Delta BED = \Delta CDE$ | Areas of triangles with common base and same altitudes are equal. $\overline{ED} \parallel \overline{CD}$ given, so altitudes are equal |
| $\therefore$ From (a) and (b), we have | |
| $\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$ | |
| $\therefore \quad m\overline{AD} : m\overline{DB} = m\overline{AE} : m\overline{EC}$ | |



## THEOREM 14.1.2
## Converse of THEOREM 14.1.1

**If a line segment intersects the two sides of a triangle in the same ratio, then it is parallel to the third side.**

**Solution:**

**Given:**

In triangle ABC, $\overline{ED}$ intersects $\overline{AB}$ and $\overline{AC}$ such that

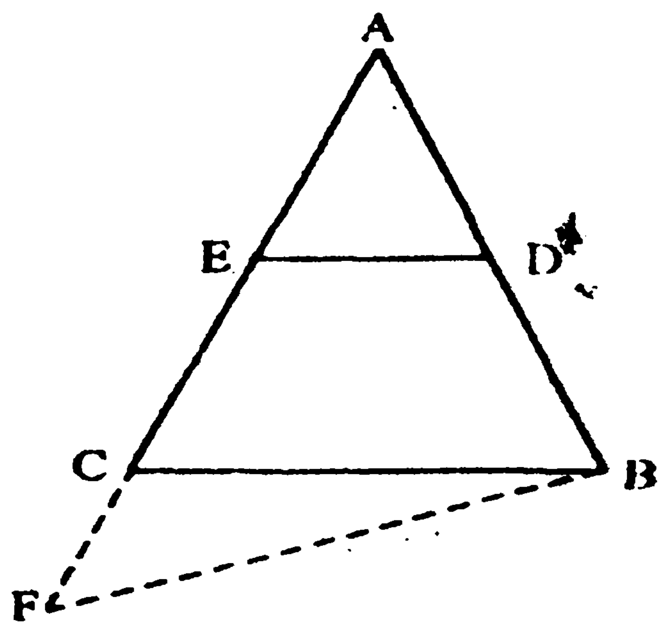
$$m\overline{AD} : m\overline{DB} : m\overline{AB} : m\overline{EC}$$

**To prove:**

$\overline{ED} \parallel \overline{CB}$

**Construction:**

If $\overline{ED} \nparallel \overline{CB}$, then draw $\overline{BF} \parallel \overline{DE}$ to meet $\overline{AC}$ produced at F.

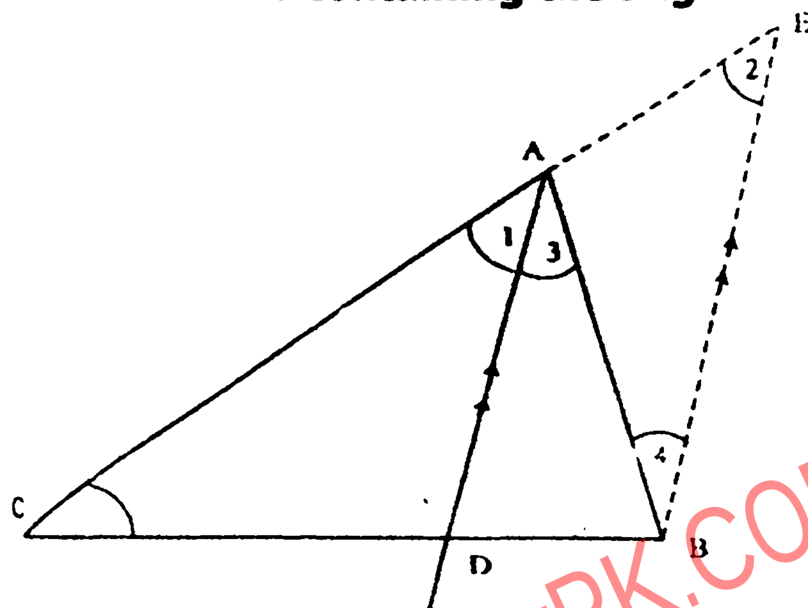



**Proof:**

| Statements | Reasons |
|---|---|
| In $\Delta ABF$ | |
| $\overline{DE} \parallel \overline{BF}$ | Construction |
| $\therefore \quad \dfrac{m\overline{AD}}{m\overline{DB}} = \dfrac{m\overline{AE}}{m\overline{EF}}$ ........ (i) | A line parallel to one side of a triangle divides the other two sides proportionally (Theorem 4) |
| But $\dfrac{m\overline{AD}}{m\overline{DB}} = \dfrac{m\overline{AE}}{m\overline{EC}}$ ..... (ii) | Given |
| $\dfrac{m\overline{AE}}{m\overline{EF}} = \dfrac{m\overline{AE}}{m\overline{EC}}$ | From (i) and (ii) |
| or $\quad m\overline{EF} = m\overline{EC}$ | |
| Which is possible only if point F is coincident with C. | Property of real numbers |
| $\therefore$ Our supposition is wrong | |
| Hence $\quad \overline{ED} \parallel \overline{CB}$ | |

# THEOREM 14.1.3

**The internal bisector of an angle of a triangle divides the sides opposite to it in the ratio of the lengths of the sides containing the angle.**




**Given:**

In $\Delta ABC$ internal angle bisector of angle A intersects $\overline{CB}$ at the point D.

**To Prove:**

$$m\overline{BD}:m\overline{DC} = m\overline{AB}:m\overline{AC}$$

**Construction:**

Draw a line segment $\overline{BE}||\overline{DA}$ to meet $\overline{CA}$ produced, at E.

**Proof:**

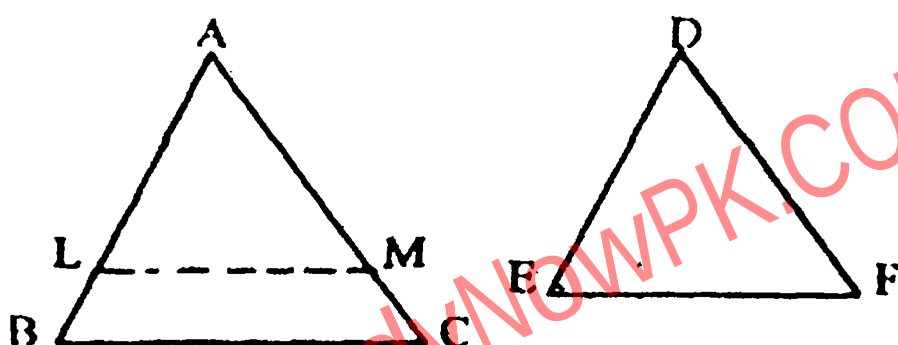
| Statements | Reasons |
|---|---|
| $\because$ $\overline{AD}||\overline{EB}$ and $\overline{EC}$ intersects there at A | Constructions |
| and E, so $m\angle 1 = m\angle 2$ | Corresponding angles |
| Again AD \|\| EB | |
| And AB intersects them | |
| So $m\angle 3 = m\angle 4$ .... (ii) | Alternate angles |
| But $m\angle 1 = m\angle 3$ | Construction (given) |
| $\therefore m\angle 2 = m\angle 4$ | |
| and $\overline{AE} \cong \overline{AB}$ | |
| Now $\overline{AD}||\overline{EB}$ | Construction |

| | |
|---|---|
| $\therefore \quad \dfrac{m\overline{BD}}{m\,\overline{DC}} = \dfrac{m\overline{EA}}{m\overline{AC}}$<br><br>or $\quad \dfrac{m\overline{BD}}{m\,\overline{DC}} = \dfrac{m\overline{EA}}{m\overline{AC}}$<br>$m\overline{BD} : m\overline{DC} = m\overline{AB} : m\overline{AC}$ | A line parallel to one side of a triangle and intersecting the other two sides divides them proportionally.<br>$\therefore m\overline{EA} = m\overline{AB}$ (Proved) |

## THEOREM 14.1.4

**If two triangles which are similar then measures of their corresponding sides are proportional.**

**Solution:**






**Given:**

In correspondence of $\Delta ABC \leftrightarrow \Delta DEF$

i.e., $\angle A \cong \angle D$ , $\angle B \cong \angle E$ and $\angle C \cong \angle F$

**To Prove:** $\dfrac{m\overline{AB}}{m\,\overline{DE}} = \dfrac{m\overline{AC}}{m\overline{DF}} = \dfrac{m\overline{BC}}{m\,\overline{EF}}$

**Construction:**

**(a)** Suppose that $m\overline{AB} > m\overline{DE}$

**(b)** $m\overline{AB} < m\overline{DE}$

On $\overline{AB}$ take a point L such that $m\overline{AL} = m\overline{DE}$

On $\overline{AC}$ take a point M such that $m\overline{AM} = m\overline{DF}$, Join L and M by the line segment $\overline{LM}$.

**Proof:**

| Statements | Reasons |
|---|---|
| In the correspondence of<br>$\Delta ALM \leftrightarrow \Delta DEF$<br>$\angle A \cong \angle D$<br>$\overline{AL} \cong \overline{DE}$ | <br><br>Given<br>Construction |

| | |
|---|---|
| $\overline{AM} \cong \overline{DF}$ | Construction |
| Thus $\Delta ALM = \Delta DEF$ | SAS Postulate |
| And $\angle L \cong \angle E$, $\angle M \cong \angle F$ | Corresponding angles of congruent triangles |
| Now $\angle E \cong \angle B$ and $\angle F = \angle C$ | Given |
| $\therefore$ $\angle L \cong \angle B$, $\angle M \cong \angle C$ | Transitivity of congruence |
| Thus $\overline{LM} \parallel \overline{BC}$. | Corresponding angles are equal. |
| Hence $\dfrac{m\overline{AL}}{m\overline{AB}} = \dfrac{m\overline{AM}}{m\overline{AC}}$ | A line parallel to one side of a triangle and intersecting the other two sides divides them proportionally. |
| or $\dfrac{m\overline{DE}}{m\overline{AB}} = \dfrac{m\overline{DF}}{m\overline{AC}}$ | $m\overline{AL} = m\overline{DE}$ (construction) $m\overline{AM} = m\overline{DF}$ |
| Similarly by intercepting segments on $\overline{BA}$ and $\overline{BC}$ we can prove that $\dfrac{m\overline{DE}}{m\overline{AB}} = \dfrac{m\overline{EF}}{m\overline{BC}}$ ..........(ii) | |
| Thus $\dfrac{m\overline{DE}}{m\overline{AB}} = \dfrac{m\overline{DF}}{m\overline{AC}} = \dfrac{m\overline{EF}}{m\overline{BC}}$ | By (i) and (ii) |
| or $\dfrac{m\overline{AB}}{m\overline{DE}} = \dfrac{m\overline{AC}}{m\overline{DF}} = \dfrac{m\overline{BC}}{m\overline{EF}}$ | By taking reciprocals. |
| **(b)** If $m\overline{AB} < m\overline{DE}$ it can similarly be proved by taking intercepts on the sides of $\Delta DEF$. | |
| If $m\overline{AB} = m\overline{DE}$ Then in the correspondence of $\Delta ABC \leftrightarrow \Delta DEF$ | |
| $\angle A \cong \angle D$ | Given |
| $\angle B \cong \angle E$ | Given |
| and $\overline{AB} \cong \overline{DE}$ | Construction |

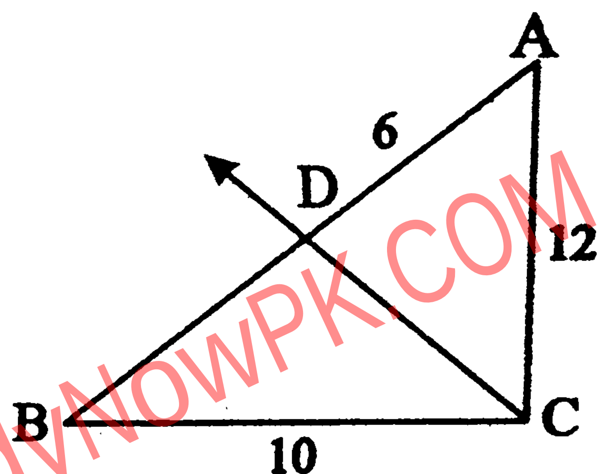
| | |
|---|---|
| so $\Delta ABC \cong \Delta DEF$ | ASA ≅ ASA |
| Thus $\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{AC}}{m\overline{DF}} = \frac{m\overline{BC}}{m\overline{EF}} = 1$ | $\overline{AC} \cong \overline{DF}$ |
| Thus result is true for all cases | $\overline{BC} \cong \overline{EF}$ |

## EXERCISE 14.2

**Q1. In $\Delta ABC$ as shown in the figure, $\overrightarrow{CD}$ bisects $\angle C$ and meets $\overline{AB}$ at D. $m\overline{BD}$ is equal to**

**(a) 5 (b) 16 (c) 10 (d) 18**

**Solution:**




In $\Delta ABC$, $\overline{CD}$ bisect $\angle C$ meets $\overline{AB}$ at D.

As $\overline{CD}$ is the internal bisector of $\angle C$

So $\frac{m\overline{BD}}{m\overline{DA}} = \frac{m\overline{BC}}{m\overline{CA}}$

$\frac{m\overline{BD}}{6} = \frac{10}{12}$

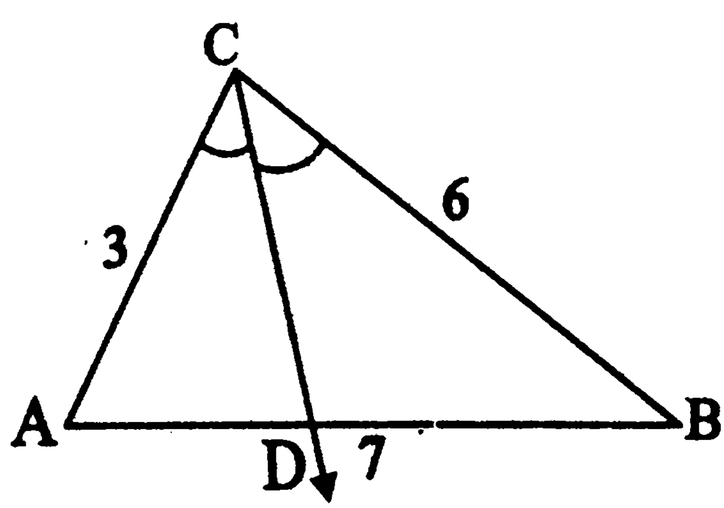
$m\overline{BD} = 6 \times \frac{10}{12} = 5$

So the correct answer is (a).



**Q2. In $\Delta ABC$ shown in the figure, $\overrightarrow{CD}$ bisects $\angle C$. If $m\overline{AC} = 3$, $m\overline{CB} = 6$ and $m\overline{AB} = 7$, then find $m\overline{AD}$ and $m\overline{DB}$.**

**Solution:**




In $\Delta ABC$, $m\overline{AC} = 3$

$m\overline{CB} = 6$, $m\overline{AB} = 7$

Let $m\overline{AD} = x$

then $m\overline{DB} = 7 - x$

As $\overrightarrow{CD}$ is internal bisector of $\angle C$

So $\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AC}}{m\overline{CB}}$

$\frac{x}{7-x} = \frac{3}{6}$

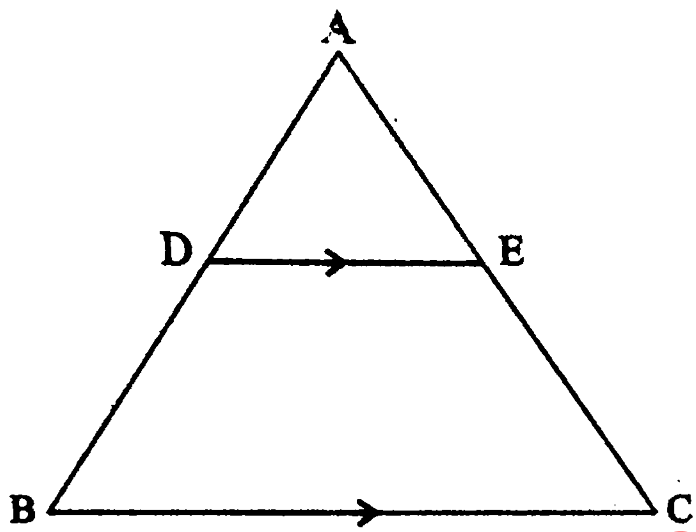
$6x = 21 - 3x$

$9x = 21$

# EXERCISE 14.1

**Q1. In $\Delta ABC$, $\overline{DE} \parallel \overline{BC}$.**

**Solution:**




**(i)** $\overline{AD} = 1.5$ cm, $\overline{BD} = 3$ cm, $\overline{AE} = 1.3$ cm, $\overline{CE} = ?$

In $\Delta ABC$, $\quad \overline{DE} \parallel \overline{BC}$

$$\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$$

$$\therefore \quad \frac{1.5}{3} = \frac{1.3}{m\overline{EC}}$$

$$\Rightarrow \quad 1.5(m\overline{EC}) = (1.3)3$$

$$m\overline{EC} = \frac{1.3 \times 1.3}{1.5} = \frac{13 \times 3 \times 10}{10 \times 15}$$

$$= \frac{13 \times 3}{15} = \frac{13}{5} = 2.6 \text{ cm}$$

**(ii)** $\overline{AD} = 2.4$ cm, $\overline{AE} = 3.2$ cm, $\overline{EC} = 4.8$ cm, $\overline{AB} = ?$

In $\Delta ABC$, $\quad \overline{DE} \parallel \overline{BC}$

$$\text{As} \quad \frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$$

$$\frac{2.4}{m\overline{DB}} = \frac{3.2}{4.8}$$

$$3.2(m\overline{DB}) = (2.4)(4.8)$$

$$m\overline{DB} = \frac{(2.4)(4.8)}{1.3} = \frac{24 \times 10 \times 48}{10 \times 32 \times 10}$$

$$= \frac{36}{10} = 3.6 \text{ cm}$$

$$m\overline{AB} = m\overline{AD} + m\overline{DB} = 2.4 + 3.6$$

$$= 6.0 \text{ cm}$$

**(iii)** $\dfrac{\overline{AD}}{\overline{DB}} = \dfrac{3}{5}, \overline{AC} = 4.8, \overline{AE} = ?$

In $\Delta ABC$, $\overline{DE} \parallel \overline{BC}$

As
$$\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$$
$$\frac{3}{5} = \frac{\overline{AE}}{\overline{EC}}$$
$$\frac{3}{5} + 1 = \frac{\overline{AE}}{\overline{EC}} + 1$$
$$\frac{8}{5} = \frac{\overline{AE}+\overline{EC}}{\overline{EC}} = \frac{\overline{AC}}{\overline{EC}}$$
$$\frac{8}{5} = \frac{4.8}{\overline{EC}}$$
$$\Rightarrow \quad 8\overline{EC} = 4.8 \times 5 = 24$$
$$\overline{EC} = \frac{24}{8} = 3$$
$$m\overline{AE} = m\overline{AC} - m\overline{EC} = 4.8 - 3 = 1.8 \text{ cm}$$

**(iv)** $\overline{AD} = 2.4$ **cm,** $\overline{AE} = 3.2$ **cm,** $\overline{DE} = 2$ **cm,** $\overline{BC} = 5$ **cm find** $\overline{AB}, \overline{DB}, \overline{AC}, \overline{CE}$**.**

In $\Delta ABC$, $\overline{DE} \parallel \overline{BC}$
$$\frac{\overline{AD}}{\overline{AB}} = \frac{\overline{AE}}{\overline{AC}} = \frac{\overline{DE}}{\overline{BC}}$$
$$\frac{2.4}{\overline{AB}} = \frac{3.2}{\overline{AC}} = \frac{2}{5}$$
$$2(\overline{AB}) = 5(2.4) = 12$$
$$\overline{AB} = \frac{12}{2} = 6 \text{ cm}$$
$$2(\overline{AC}) = (3.2)5 = 16$$
$$\overline{AC} = \frac{16}{2} = 8 \text{ cm}$$
$$\overline{DE} = \overline{AB} - \overline{AD}$$
$$= 6 - 2.4 = 3.6 \text{ cm}$$
$$\overline{CE} = \overline{AC} - \overline{AE}$$
$$= 8 - 3.2 = 4.8 \text{ cm}$$

**(v)** $\overline{AD} = 4x - 3, \overline{AE} = 8x - 7, \overline{BD} = 3x - 1, \overline{CE} = 5x - 3,$ **find** $x$**.**

In $\Delta ABC$, $\overline{DE} \parallel \overline{BC}$
$$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AE}}{\overline{EC}}$$
$$\Rightarrow \quad \frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$$
$$(4x-3)(5x-3) = (8x-7)(3x-1)$$
$$\Rightarrow \quad 20x^2 - 12x - 15x + 9 = 24x^2 - 8x - 21x + 7$$

or $20x^2 - 24x^2 - 27x + 29x + 9 - 7 = 0$

$-4x^2 + 2x + 2 = 0$

or $2x^2 - x - 1 = 0$

$2x^2 - 2x + x - 1 = 0$

$2x(x-1) + (x-1) = 0$

$(x-1)(2x+1) = 0$

$x = 1, -\frac{1}{2}$

For $x = -\frac{1}{2}$ sides become negative.

So $x = 1$

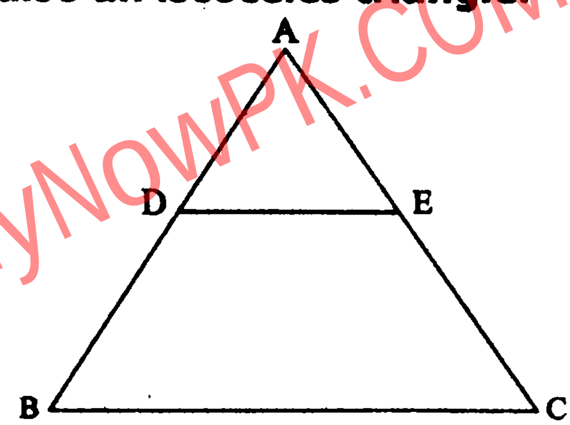
**Q2. If $\Delta ABC$ is an isosceles triangle, $\angle A$ is vertex angle and $\overline{DE}$ intersects the sides $\overline{AB}$ and $\overline{AC}$ as shown in the figure so that**

**$m\overline{AD} : m\overline{DB} = m\overline{AE} : m\overline{EC}$**

**Prove that $\Delta ADE$ is also an isosceles triangle.**



**Solution:**




**Given:**

In $\Delta ABC$, $\overline{AB} \cong \overline{AC}$

and $\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$

**To prove:**

$\Delta ADE$ is isosceles.

**Proof:**

| Statements | Reasons |
|---|---|
| $\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$ | Given |
| $\frac{m\overline{AD}}{m\overline{DB}} + 1 = \frac{m\overline{AE}}{m\overline{EC}} + 1$ | |
| Or $\frac{m\overline{AD}+m\overline{DB}}{m\overline{DB}} = \frac{m\overline{AE}+m\overline{EC}}{m\overline{EC}}$ | |
| i.e. $\frac{m\overline{AB}}{m\overline{DB}} = \frac{m\overline{AC}}{m\overline{EC}}$ | Given $\overline{AB} \cong \overline{AC}$ |
| $\Rightarrow m\overline{DB} = m\overline{EC}$ | From figure |
| $m\overline{AB} - m\overline{DB} = m\overline{AC} - m\overline{EC}$ | |
| $m\overline{AD} = m\overline{AE}$ | |
| or $\overline{AD} \cong \overline{AE}$ | |
| $\therefore \Delta ADE$ is isosceles. | |

**Q3. In an equilateral triangle ABC shown in the figure, $m\overline{AE} : m\overline{AC} = m\overline{AD} : m\overline{AB}$. Find all the three angles of Δ ADE and name it also.**

**Solution:**

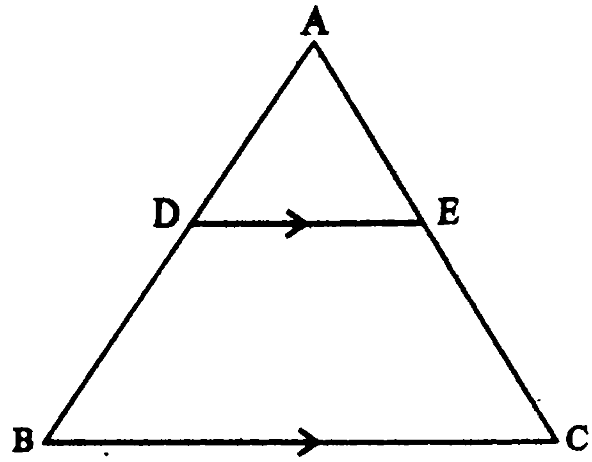
**Given:**

ABC is an equilateral triangle

$$\frac{m\overline{AE}}{m\overline{AC}} = \frac{m\overline{AD}}{m\overline{AB}}$$

**To prove:**

All angles of ΔADE.




**Proof:**

| Statements | Reasons |
|---|---|
| $\frac{m\overline{AC}}{m\overline{AF}} = \frac{m\overline{AB}}{m\overline{AD}}$ | Given |
| $\frac{m\overline{AC}}{m\overline{AE}} - 1 = \frac{m\overline{AB}}{m\overline{AD}} - 1$ | |
| $\frac{m\overline{AC}-m\overline{AE}}{m\overline{AE}} = \frac{m\overline{AB}-m\overline{AD}}{m\overline{AD}}$ | |
| $\frac{m\overline{EC}}{m\overline{AE}} = \frac{m\overline{DB}}{m\overline{AD}}$ | |
| or $\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$ | Given |
| $\therefore \overline{DE} \parallel \overline{BC}$ ......... (i) | |
| $m\angle A = m\angle B = m\angle C = 60°$ | From (i) |
| $\angle AED \cong \angle ACD \cong 60°$ | |
| ∴ Each angle of Δ ADE has measure of 60° | |
| So Δ ADE is equiangular or equilateral | |

**Q4. Prove that the line segment drawn through the mid-point of one side of a triangle and parallel to another side bisects the third side.**

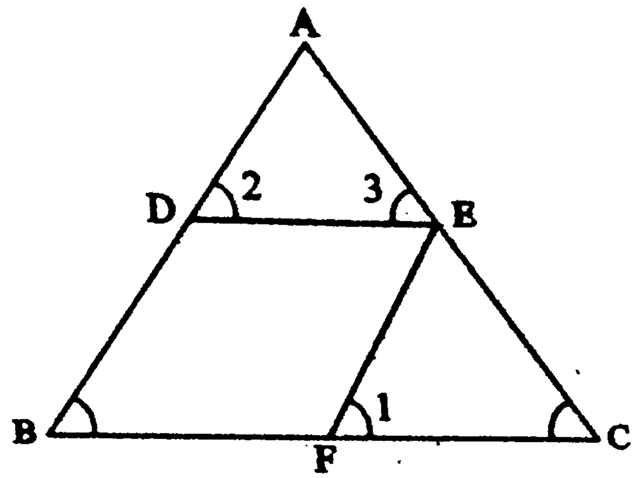
**Solution:**

**Given:**

In ΔABC, D is mid-point of AB. $\overline{DE} \parallel \overline{BC}$

**To prove:**

$\overline{EA} \cong \overline{EC}$



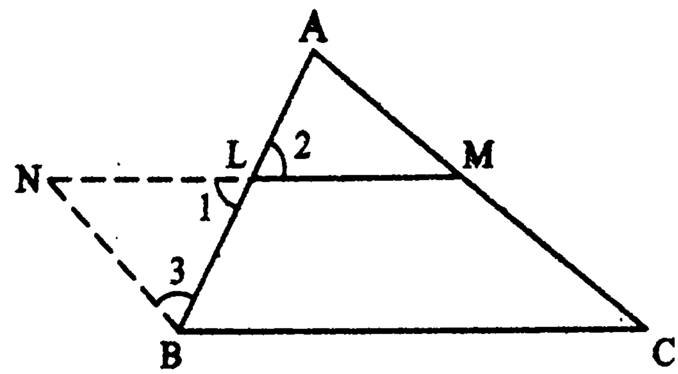

**Construction:**

Take $\overline{EF} \parallel \overline{AB}$

**Proof:**

| Statements | Reasons |
|---|---|
| $\overline{DE} \parallel \overline{BF}$ | Given |
| $\overline{EF} \parallel \overline{BD}$ | Construction |
| $\therefore$ DBEF is a parallelogram | |
| $\overline{EF} \cong \overline{DB}$ (i) | Opposite sides |
| $\overline{AD} \cong \overline{DB}$ (ii) | Given |
| $\therefore \overline{EF} \cong \overline{AD}$ (iii) | From (i), (ii) |
| $\angle 1 \cong \angle B$ | Corresponding angles |
| And $\angle 2 \cong \angle B$ | |
| $\therefore \angle 1 \cong \angle 2$ (iv) | |
| In correspondence | |
| $\Delta ADE \longleftrightarrow \Delta EFC$ | |
| $\angle 2 \cong \angle 1$ | From (iv) |
| $\angle 3 \cong \angle C$ | Corresponding angles |
| $\overline{AD} \cong \overline{EF}$ | From (ii) |
| Hence $\Delta ADE \cong \Delta EFC$ | A.A.S $\cong$ A.A.S |
| $\therefore \overline{EA} \cong \overline{EC}$ | Corresponding sides of congruent angles. |

**Q5. Prove that the line segment joining the mid-points of any two sides of a triangle is parallel to the third side.**




**Solution:**

**Given:**

In $\Delta$ ABC, the mid-points of $\overline{AB}$ and $\overline{AC}$ are L and M respectively.

**To prove:**

$\overline{LM} \parallel \overline{BC}$

**Construction:**

Join M to L and produce $\overline{ML}$ to N such that $\overline{ML} \cong \overline{LN}$. Join N to B and in the figure, name the angles as $\angle 1$, $\angle 2$ and $\angle 3$.

**Proof:**

| Statements | Reasons |
|---|---|
| In $\Delta BLN \longleftrightarrow \Delta ALM$ | |
| $\overline{BL} \cong \overline{AL}$ | Given |
| $\angle 1 \cong \angle 2$ | Vertical angles |
| $\overline{NL} \cong \overline{ML}$ | Construction |
| $\therefore \Delta BLN \cong \Delta ALM$ | S.A.S. postulate |
| And $\angle A \cong \angle 3$ ......... (i) | |
| $\overline{NB} \cong \overline{AM}$ ......... (ii) | Corresponding angles of congruent triangles |
| $\overline{NB} \parallel \overline{AM}$ | Corresponding angles of congruent triangles |
| Thus $\overline{NB} \parallel \overline{MC}$ ......... (iii) | |
| $\overline{MC} \cong \overline{AM}$ ......... (iv) | (M is mid-point of $\overline{AC}$) |
| $\overline{NB} \cong \overline{AM}$ ......... (v) | Given |
| $\therefore$ BCMN is a parallelogram | from (ii) and (iv) |
| $\overline{BC} \parallel \overline{LM}$ | From (i) and (v) |
| or $\overline{BC} \parallel \overline{NL}$ (vi) | Opposite sides of a parallelogram (BCMN) |

| | |
|---|---|
| so $\Delta ABC \cong \Delta DEF$ | ASA ≅ ASA |
| Thus $\dfrac{m\overline{AB}}{m\overline{DE}} = \dfrac{m\overline{AC}}{m\overline{DF}} = \dfrac{m\overline{BC}}{m\overline{EF}} = 1$ | $\overline{AC} \cong \overline{DF}$ |
| Thus result is true for all cases | $\overline{BC} \cong \overline{EF}$ |

## EXERCISE 14.2

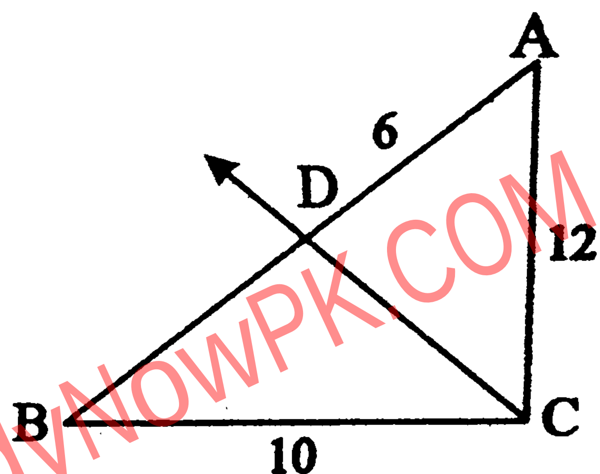
**Q1. In $\Delta ABC$ as shown in the figure, $\overrightarrow{CD}$ bisects $\angle C$ and meets $\overline{AB}$ at D. $m\overline{BD}$ is equal to**

**(a) 5 (b) 16 (c) 10 (d) 18**

**Solution:**

In $\Delta ABC$, $\overline{CD}$ bisect $\angle C$ meets $\overline{AB}$ at D.

As $\overline{CD}$ is the internal bisector of $\angle C$




So $\dfrac{m\overline{BD}}{m\overline{DA}} = \dfrac{m\overline{BC}}{m\overline{CA}}$

$\dfrac{m\overline{BD}}{6} = \dfrac{10}{12}$

$m\overline{BD} = 6 \times \dfrac{10}{12} = 5$

So the correct answer is (a).



**Q2. In $\Delta ABC$ shown in the figure, $\overrightarrow{CD}$ bisects $\angle C$. If $m\overline{AC} = 3$, $m\overline{CB} = 6$ and $m\overline{AB} = 7$, then find $m\overline{AD}$ and $m\overline{DB}$.**

**Solution:**

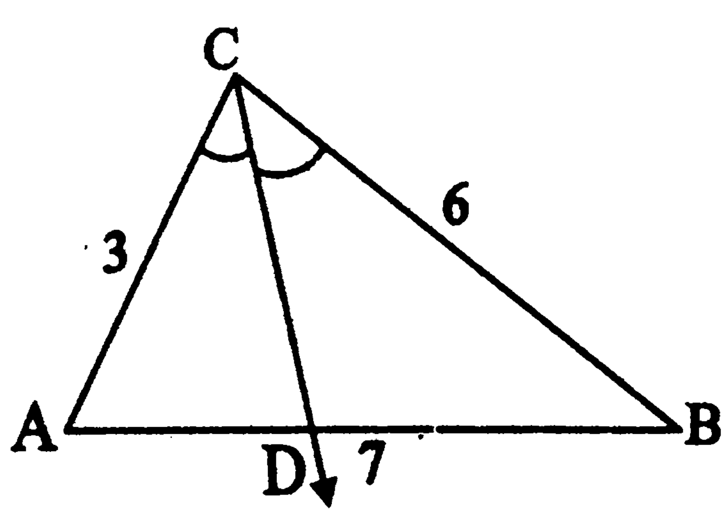
In $\Delta ABC$, $m\overline{AC} = 3$

$m\overline{CB} = 6$, $m\overline{AB} = 7$

Let $m\overline{AD} = x$

then $m\overline{DB} = 7 - x$

As $\overrightarrow{CD}$ is internal bisector of $\angle C$




So $\dfrac{m\overline{AD}}{m\overline{DB}} = \dfrac{m\overline{AC}}{m\overline{CB}}$

$\dfrac{x}{7-x} = \dfrac{3}{6}$

$6x = 21 - 3x$

$9x = 21$

$x = \frac{21}{9}$

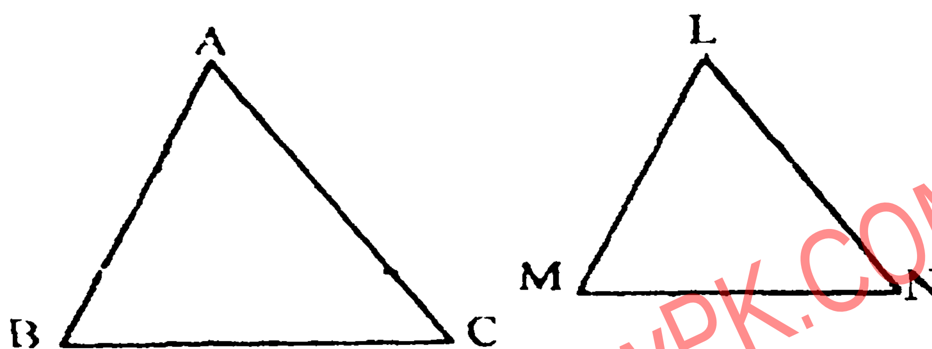
$m\overline{AD} = \frac{21}{9} = \frac{7}{3}$

$m\overline{DB} = m\overline{AB} - m\overline{AD}$

$m\overline{DB} = 7 - \frac{21}{9} = \frac{63-21}{9} = \frac{42}{9} = \frac{14}{3}$

**Q3. Show that in any corresponding of two triangles, if two angles of one triangle are congruent to the corresponding angles of the other, then the triangles are similar.**

**Solution:**




Let the two $\Delta$s be ABC and LMN

It is given that

$m\angle A = m\angle L$

$m\angle B = m\angle M$

As sum of the angles of a triangle is 180°

$m\angle A + m\angle B + m\angle C = 180°$

$m\angle L + m\angle M + m\angle C = m\angle L + m\angle M + m\angle N$

$m\angle L + m\angle M + m\angle C = m\angle L + m\angle M + m\angle N$

$m\angle C = m\angle N$

$\therefore$ The two triangles ABC and LMN are similar.

**Q4. If line segments AB and CD are intersecting at point X and $\frac{m\overline{AX}}{m\overline{XB}} = \frac{m\overline{CX}}{m\overline{XD}}$**

**Solution:**

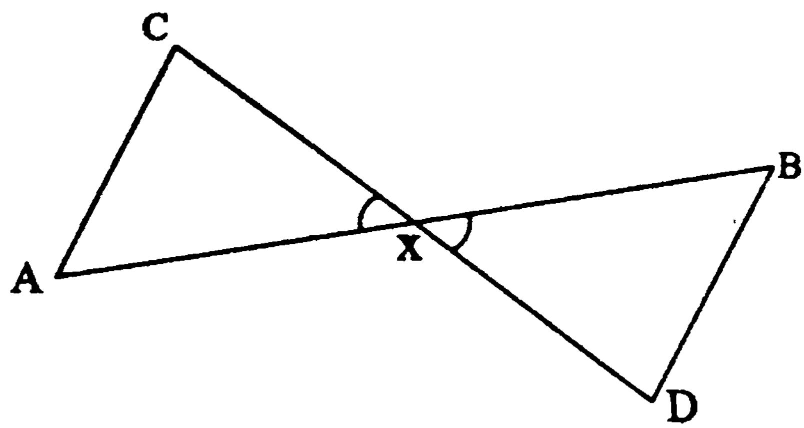
**Given:**

Line segment $\overline{AB}$ and $\overline{CD}$ intersect at X and

$\frac{m\overline{AX}}{m\overline{XB}} = \frac{m\overline{CX}}{m\overline{XD}}$

**To prove:**

$\Delta AXC$ and $\Delta BXD$ are similar.




**Proof:**

| Statements | Reasons |
|---|---|
| $\frac{m\overline{AX}}{m\overline{XB}} = \frac{m\overline{CX}}{m\overline{XD}}$ | Given |
| So $\overline{AC} \parallel \overline{BD}$ | |
| In $\Delta$s AXC and BXD | |
| $m\angle AXC = m\angle BXD$ | Vertical angles |
| $m\angle A = m\angle B$ | Alternate angles |
| $m\angle C = m\angle D$ | Alternate angles |
| So the triangles are similar. | |



## REVIEW EXERCISE 14

**Q1. Which of the following are true and which are false?**

**(i)** Congruent triangles are of same size and shape.
**(ii)** Similar triangles are of same shape but different sizes.
**(iii)** Symbol used for congruent is' ~'.
**(iv)** Symbol used for similarity is '≅'.
**(v)** Congruent triangles are similar.
**(vi)** Similar triangles are congruent.
**(vii)** A line segment has only one mid-point.
**(viii)** One and only one line can be drawn through two points.
**(ix)** Proportion is non-equality of two ratios.
**(x)** Ratio has no unit.

**Answers:**

| | | | | |
|---|---|---|---|---|
| (i) T | (ii) T | (iii) F | (iv) F | (v) T |
| (vi) F | (vii) T | (viii) T | (ix) F | (x) T |

**Q2. Define the following:**

**(i) Ratio** **(ii) Proportion**

**(iii) Congruent Triangle** **(iv) Similar Triangles**

**Solution:**

**(i) Ratio**

The ratio of two quantities a and b of same kind is denoted as a : b and is defined as:

$$a : b = a \div b$$

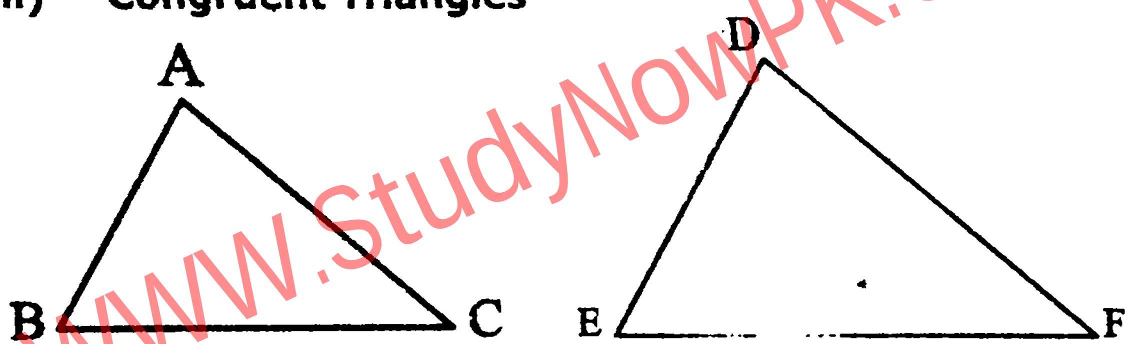
The ratio a : b $= \frac{a}{b}$ is the comparison of two like quantities 'a' and 'b' are called terms of 'a' ratio 'b'. Terms must be expressed in the same units.

**(ii) Proportion**

The statement of equality of two ratios is called proportion.

i.e. if a : b = c : d then a, b, c and d are said to be in proportion.

**(iii) Congruent Triangles**






Two triangles are said to be congruent written symbolically as ≅, if there exists a correspondence between them such that all the corresponding sides and angles are congruent.

If $\begin{cases} \overline{AB} \cong \overline{DE} \\ \overline{BC} \cong \overline{EF} \\ \overline{CA} \cong \overline{FD} \end{cases}$ and $\begin{matrix} \angle A \cong \angle D \\ \angle B \cong \angle E \\ \angle C \cong \angle F \end{matrix}$

Then $\Delta ABC \cong \Delta DEF$

**(iv) Similar Triangles**

If in $\Delta ABC \longleftrightarrow \Delta DEF$

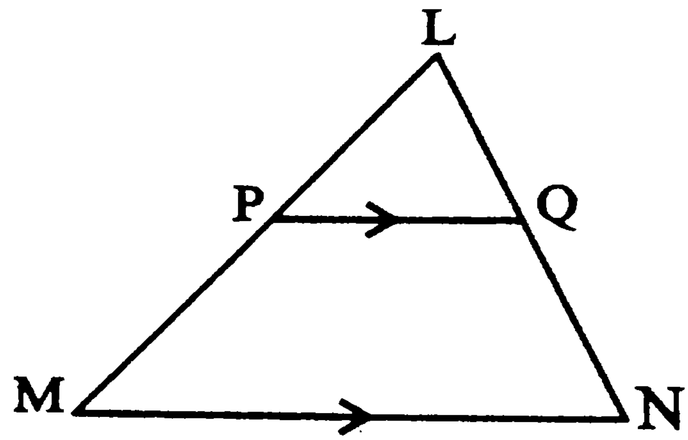
$\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$

And $\frac{\overline{AB}}{\overline{DE}} = \frac{\overline{BC}}{\overline{EF}} = \frac{\overline{CA}}{\overline{FD}}$

Then $\Delta ABC$ and $\Delta DEF$ are called a similar triangle which is symbolically written as $\Delta ABC \sim \Delta DEF$.

**Q3. In $\Delta LMN$ shown in the figure, $\overline{MN} \parallel \overline{PQ}$**

**Solution:**




(i) **$m\overline{LM} = 5$ cm, $m\overline{LP} = 2.5$ cm,**
**$m\overline{LQ} = 2.3$ cm, $m\overline{LN} = ?$**

$\overline{PQ} \parallel \overline{MN}$

$$\frac{m\overline{LP}}{m\overline{LM}} = \frac{m\overline{LQ}}{m\overline{LN}}$$

$$\frac{2.5}{5} = \frac{2.3}{m\overline{LN}}$$

$$2.5(m\overline{LN}) = (2.3) \times = 11.5$$

$$m\overline{LN} = \frac{11.5}{2.5} = \frac{11.5}{25} = \frac{23}{5} = 4.6 \text{ cm}$$

(ii) **$m\overline{LM} = 6$ cm, $m\overline{LQ} = 2.5$ cm, $m\overline{QN} = 5$ cm,**
**$m\overline{LP} = ?$**

$m\overline{LN} = m\overline{LQ} + m\overline{QN} = 2.5 \text{ cm} + 5 \text{ cm} = 7.5$ cm

As $\overline{PQ} \parallel \overline{MN}$

$$\frac{m\overline{LP}}{m\overline{LM}} = \frac{m\overline{LQ}}{m\overline{LN}}$$

$$\frac{m\overline{LP}}{6} = \frac{2.5}{7.5}$$

$$m\overline{LP} = \frac{2.5}{7.5} \times 6 = \frac{25}{75} \times 6 = 2 \text{ cm}$$

**Q4. In the shown figure, let $m\overline{PA} = 8x - 7$, $m\overline{PB} = 4x - 3$, $m\overline{AQ} = 5x - 3$, $m\overline{BR} = 3x - 1$. Find the value of x if $\overline{AB} \parallel \overline{QR}$.**

**Solution:**

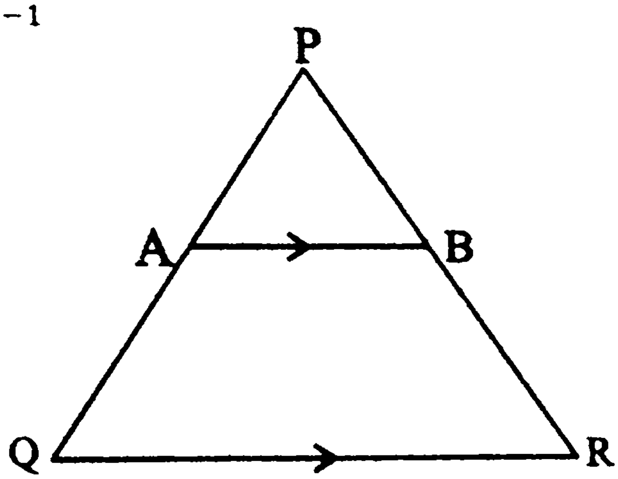
$m\overline{PA} = 8x - 7$, $m\overline{PB} = 4x - 3$

$m\overline{AQ} = 5x - 3$, $m\overline{BR} = 3x - 1$

As $\overline{AB} \parallel \overline{QR}$

$$\frac{m\overline{PA}}{m\overline{AQ}} = \frac{m\overline{PB}}{m\overline{BR}}$$

$$\frac{8x-7}{5x-3} = \frac{4x-3}{3x-1}$$




$(8x - 7)(3x - 1) = (4x - 3)(5x - 3)$

$24x^2 - 8x - 21x + 7 = 20x^2 - 12x - 15x + 9$

$24x^2 - 20x^2 - 29x + 27x + 7 = 9$

$4x^2 - 2x + 7 = 9$

$4x^2 - 2x - 2 = 0$

$2x^2 - x - 1 = 0$

$2x^2 - 2x + x - 1$

$2x(x - 1) + (x - 1) = 0$

$(x - 1)(2x + 1) = 0$

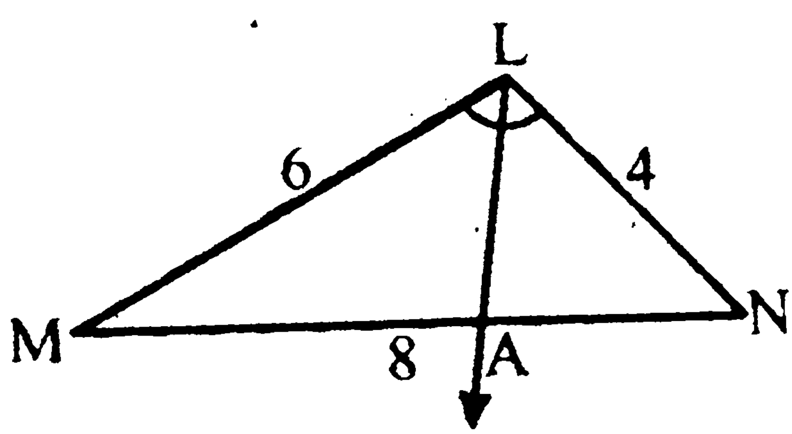
$x = 1, -\frac{1}{2}$

**$x = 1$ is the required value.**



**Q5. In $\Delta$ LMN show in the figure, $\overrightarrow{LA}$ bisect $\angle L$. If $m\overline{LN} = 4$ $m\overline{LM} = 6$, $m\overline{MN} = 8$, then find $m\overline{MA}$ and $m\overline{AN}$.**

**Solution:**




$m\overline{LN} = 4$, $m\overline{LN} = 6$, $m\overline{MN} = 8$

$\overrightarrow{LA}$ is bisector of $\angle L$

$$\frac{m\overline{MA}}{m\overline{NA}} = \frac{m\overline{LM}}{m\overline{LN}} = \frac{6}{4}$$

i.e. $\quad m\overline{MA} : m\overline{NA} = 6 : 4$

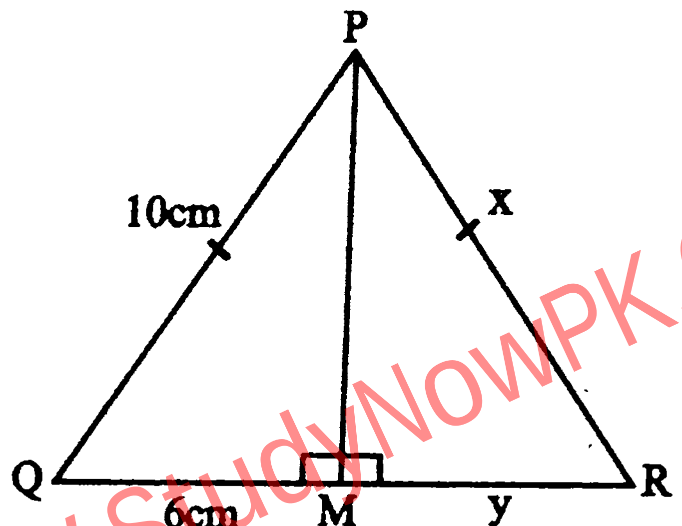
but $\quad m\overline{MN} = m\overline{MA} + m\overline{AN} = 8$

$$m\overline{MA} = \frac{6}{10} \times 8 = \frac{48}{10} = \mathbf{4.8}$$

and $\quad m\overline{AN} = \frac{4}{10} \times 8 = \frac{32}{10} = \mathbf{3.2}$

**Q6. In isosceles $\Delta PQR$ shown in the figure, find the value of x and y.**

**Solution:**




$\overline{PQ} \cong \overline{PR}$

$\Rightarrow$ $\quad$ **x = 10 cm**

$PM \perp QR$ where PQR is an isosceles triangle

$\therefore$ $\quad m\overline{MQ} = m\overline{MR}$

$\Rightarrow$ $\quad$ **y = 6 cm**