Exercise 9.2

Q.1 Show whether the points with vertices (5,-2),(5,4) and (-4,1) are the vertices of equilateral triangle or an isosceles triangle

$P(5,-2), Q(5,4), R(-4,1)$

Solution:

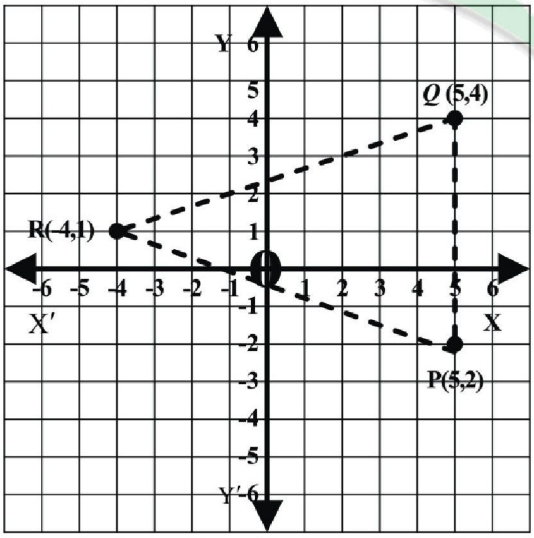
We know that the distance formula is

$$=\sqrt{|x_2-x_1|^2+|y_2-y_1|^2}$$

We have $P(5,-2), Q(5,4)$

$$|PQ|=\sqrt{|5-5|^2+|4-(-2)|^2}$$

$$|PQ|=\sqrt{(0)^2+(4+2)^2}$$



$$|PQ|=\sqrt{(6)^2}$$

$$|PQ|=6$$

$Q(5,4), R(-4,1)$

$$|QR|=\sqrt{|-4-5|^2+|1-4|^2}$$

$$|QR|=\sqrt{(-9)^2+(-3)^2}$$

$$|QR|=\sqrt{81+9}$$

$$|QR|=\sqrt{90}$$

$$|QR|=\sqrt{9\times 10}=3\sqrt{10}$$

$R(-4,1), P(5,-2)$

$$|RP|=\sqrt{|5-(-4)|^2+|-2-1|^2}$$

$$|RP|=\sqrt{(5+4)^2+(-3)^2}$$

$$|RP|=\sqrt{(9)^2+9}$$

$$|RP|=\sqrt{81+9}$$

$$|RP|=\sqrt{90}$$

$$|RP|=\sqrt{9\times 10}=3\sqrt{10}$$

$$|QR|=|PR|$$

Two lengths of triangle are equal

So it is a isosceles triangle

Q.2 Show whether or not the points with vertices $(-1,1),(2,-2)$ and $(-4,1)$ form a Square

Solution:

$P(-1,1)Q(5,4)R(2,-2)S(-4,1)$

Distance $=\sqrt{|x_2-x_1|^2+|y_2-y_1|^2}$

$$|PQ|=\sqrt{\left|5-(-1)^2\right|+|4-1|^2}$$

$$|PQ|=\sqrt{|5+1|^2+|3|^2}$$

$$|PQ|=\sqrt{6^2+9}$$

$$|PQ|=\sqrt{36+9}$$

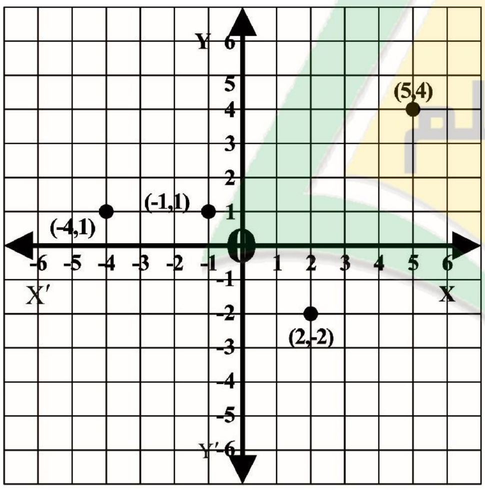
$$|PQ|=\sqrt{45}$$

$$|PQ|=\sqrt{9\times 5}$$

$$|PQ|=3\sqrt{5}$$

$$|QR|=\sqrt{|2-5|^2+|-2-4|^2}$$

$$|Q\,R| = \sqrt{(-3)^2 + (6)^2}$$
$$|Q\,R| = \sqrt{9+36}$$
$$|Q\,R| = \sqrt{45}$$
$$|Q\,R| = \sqrt{9\times 5}$$
$$|Q\,R| = 3\sqrt{5}$$
$$|R\,S| = \sqrt{|-4-2|^2 + |1-(-2)|^2}$$
$$|R\,S| = \sqrt{(-6)^2 + (1+2)^2} = \sqrt{36+(3)^2}$$
$$|R\,S| = \sqrt{36+9}$$
$$|R\,S| = \sqrt{45}$$
$$|R\,S| = \sqrt{9\times 5}$$
$$|R\,S| = 3\sqrt{5}$$



$$|S\,P| = \sqrt{|-4-(-1)|^2 + |1-1|^2}$$
$$|S\,P| = \sqrt{(-4+1)^2 + (0)^2}$$
$$|S\,P| = \sqrt{(-3)^2}$$
$$|S\,P| = \sqrt{9}$$
$$|S\,P| = 3$$

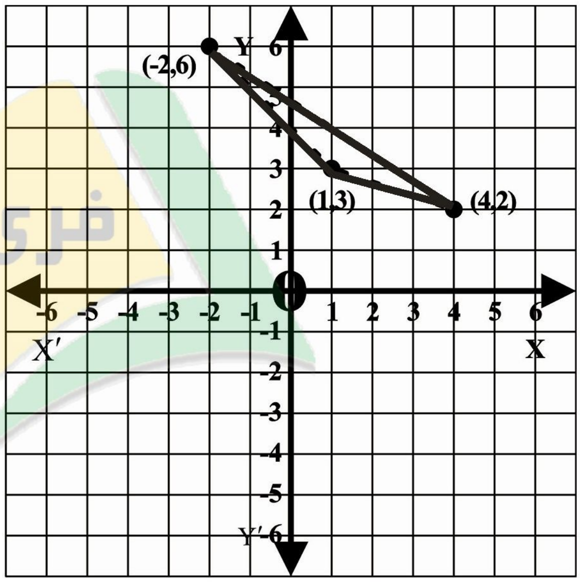
If all the length are same then it will be a Square all the length are not equal so it is not square.

$$|P\,Q| = |Q\,R| = |R\,S| \neq |S\,P|$$

Q.3 **Show whether or not the points with coordinates $(1,3),(4,2)$ and $(-2,6)$ are vertices of a right triangle?**

Solution:

$$A(1,3), B(4,2), C(-2,6)$$
$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$
$$|A\,B| = \sqrt{|4-1|^2 + |2-3|^2}$$
$$|A\,B| = \sqrt{(3)^2 + (-1)^2}$$
$$|A\,B| = \sqrt{9+1}$$
$$|A\,B| = \sqrt{10}$$



$$|B\,C| = \sqrt{|-2-4|^2 + |6-2|^2}$$
$$|B\,C| = \sqrt{(-6)^2 + (4)^2}$$
$$|B\,C| = \sqrt{36+16}$$
$$|B\,C| = \sqrt{52}$$
$$|C\,A| = \sqrt{|-2-1|^2 + |6-3|^2} = \sqrt{(-3)^2 + (3)^2}$$
$$|C\,A| = \sqrt{9+9}$$
$$|C\,A| = \sqrt{18}$$

By Pythagoras theorem

$$(\text{Hyp})^2 = (\text{Base})^2 + (\text{Perp})^2$$
$$\left(\sqrt{52}\right)^2 = \left(\sqrt{18}\right)^2 + \left(\sqrt{10}\right)^2$$

$52 = 18 + 10$

$52 = 28$

Since $52 \neq 28$

So it not right angle triangle.

Q.4 Use distance formula to prove whether or not the points $(1,1),(-2,-8)$ and $(4,10)$ lie on a straight line?

Solution:

$A(1,1), B(-2,-8), C(4,10)$

$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|AB| = \sqrt{|-2-1|^2 + |-8-1|^2}$

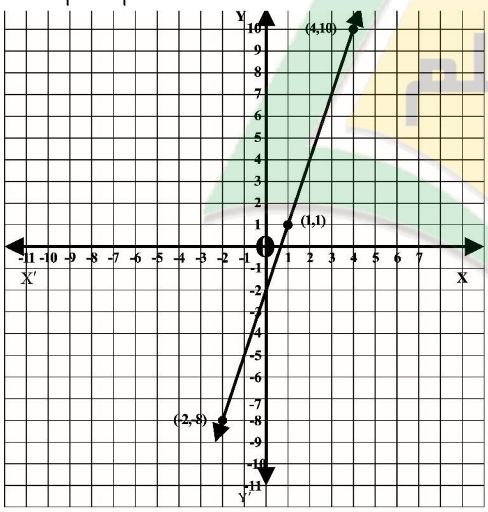
$|AB| = \sqrt{(-3)^2 + (-9)^2}$

$|AB| = \sqrt{9+81}$

$|AB| = \sqrt{90}$

$|AB| = \sqrt{9 \times 10}$

$|AB| = 3\sqrt{10}$



$|BC| = \sqrt{|4-(-2)|^2 + |10-(-8)|^2}$

$|BC| = \sqrt{(4+2)^2 + (10+8)^2}$

$|BC| = \sqrt{(6)^2 + (18)^2}$

$|BC| = \sqrt{36+324}$

$|BC| = \sqrt{360}$

$|BC| = \sqrt{36 \times 10}$

$|BC| = 6\sqrt{10}$

$|AC| = \sqrt{|4-1|^2 + |10-1|^2}$

$|AC| = \sqrt{(3)^2 + (9)^2}$

$|AC| = \sqrt{9+81}$

$|AC| = \sqrt{90}$

$|AC| = \sqrt{9 \times 10}$

$|AC| = 3\sqrt{10}$

$|AC| + |AB| = |BC|$

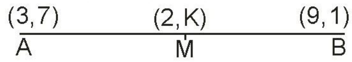
$3\sqrt{10} + 3\sqrt{10} = 6\sqrt{10}$

$6\sqrt{10} = 6\sqrt{10}$

It means that they lie on same line so they are collinear.

Q.5 Find K given that point (2, K) is equidistance from (3, 7) and (9,1)

Solution: $M(2,K), A(3,7)$ and $B(9,1)$



$|\overline{AM}| = |\overline{BM}|$

$\sqrt{|2-3|^2 + |K-7|^2} = \sqrt{|9-2|^2 + |1-K|^2}$

$\sqrt{(-1)^2 + (K-7)^2} = \sqrt{(7)^2 + (1-K)^2}$

Taking square on both Side

$\left(\sqrt{1+K^2+49-14K}\right)^2 = \left(\sqrt{49+1+K^2-2K}\right)^2$

$K^2 - 14K + 50 = 50 + K^2 - 2K$

$\cancel{K^2} - 14K \cancel{+50} \cancel{-50} \cancel{-K^2} + 2K = 0$

$-12K = 0$

$K = \dfrac{0}{-12}$

$K = 0$

Q.6 Use distance formula to verify that the points $A(0,7), B(3,-5), C(-2,15)$ are Collinear.

Solution:

$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|AB| = \sqrt{|3-0|^2 + |-5-7|^2}$

$|AB| = \sqrt{(3)^2 + (-12)^2}$

$|AB| = \sqrt{9+144}$

$|AB| = \sqrt{153}$

$|AB| = \sqrt{9 \times 17}$

$|A\ B| = 3\sqrt{17}$

$|B\ C| = \sqrt{|-2-3|^2 + |15-(-5)|^2}$

$|B\ C| = \sqrt{(-5)^2 + (15+5)^2}$

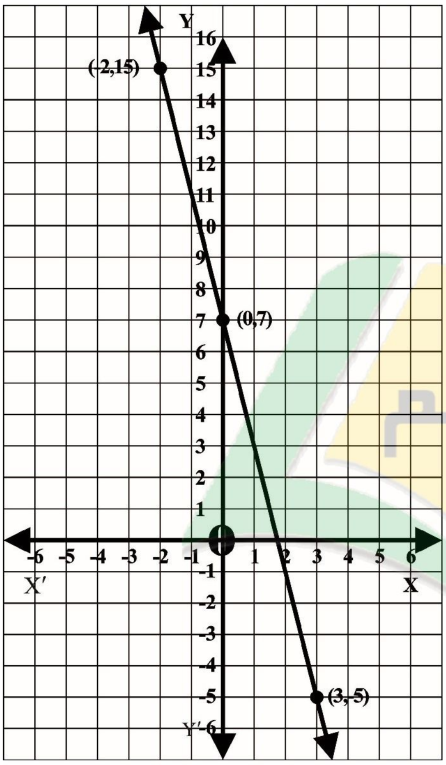
$|B\ C| = \sqrt{25+(20)^2}$

$|B\ C| = \sqrt{25+400}$

$|B\ C| = \sqrt{425}$

$|B\ C| = \sqrt{25\times 17}$

$|B\ C| = 5\sqrt{17}$



$|A\ C| = \sqrt{|-2-0|^2 + |15-7|^2}$

$|A\ C| = \sqrt{(-2)^2 + (8)^2}$

$|A\ C| = \sqrt{4+64}$

$|A\ C| = \sqrt{68}$

$|A\ C| = \sqrt{4\times 17}$

$|A\ C| = 2\sqrt{17}$

$|A\ B| + |A\ C| = |B\ C|$

$3\sqrt{17} + 2\sqrt{17} = 5\sqrt{17}$

$5\sqrt{17} = 5\sqrt{17}$

L.H.S = R.H.S So

They lie on same line and they are collinear.

Q.7 Verify whether or not the points $O(0,0)\ A(\sqrt{3},1), B(\sqrt{3},-1)$ are the vertices of an equilateral triangle

Solution:

$d = \sqrt{|x_2-x_1|^2 + |y_2-y_1|^2}$

$|O\ A| = \sqrt{|\sqrt{3}-0|^2 + |0-1|^2}$

$|O\ A| = \sqrt{(\sqrt{3})^2 + (-1)^2}$

$|O\ A| = \sqrt{3+1}$

$|O\ A| = \sqrt{4}$

$|O\ A| = 2$

$|O\ B| = \sqrt{|\sqrt{3}-0|^2 + |-1-0|^2}$

$|O\ B| = \sqrt{(\sqrt{3})^2 + (-1)^2}$

$|O\ B| = \sqrt{3+1}$

$|O\ B| = \sqrt{4}$

$|O\ B| = 2$

$|A\ B| = \sqrt{|\cancel{\sqrt{3}} - \cancel{\sqrt{3}}|^2 + |-1-1|^2}$

$|A\ B| = \sqrt{0+(-2)^2}$

$|A\ B| = \sqrt{4}$

$|A\ B| = 2$

All the sides are same in length so it is equilateral triangle

Q.8 Show that the points $A(-6,-5), B(5,-5), C(5,-8)$ and $D(-6,-8)$ are the vertices of a rectangle find the length of its diagonals are equal

Solution:

$d = \sqrt{|x_2-x_1|^2 + |y_2-y_1|^2}$

$A(-6,-5), B(5,-5)$

$$|AB| = \sqrt{|5-(-6)|^2 + |-5-(-5)|^2}$$

$$|AB| = \sqrt{(5+6)^2 + (-5+5)^2}$$

$$|AB| = \sqrt{(11)^2 + (0)^2} = \sqrt{121}$$

$$|AB| = 11$$

$$(5,-5), C(5,-8)$$

$$|BC| = \sqrt{|5-5|^2 + |-8-(-5)^2|}$$

$$|BC| = \sqrt{(0)^2 + (-8+5)^2} = \sqrt{(-3)^2}$$

$$|BC| = \sqrt{(-3)^2} = \sqrt{9}$$

$$|BC| = 3$$

$$C(5,-8), D(-6,-8)$$

$$|DC| = \sqrt{|-6-5|^2 + |-8-(-8)|^2}$$

$$|DC| = \sqrt{(-11)^2 + (-8+8)^2}$$

$$|DC| = \sqrt{121+0} = \sqrt{121}$$

$$|DC| = 11$$

$$D(-6,-8), A(-6,-5)$$

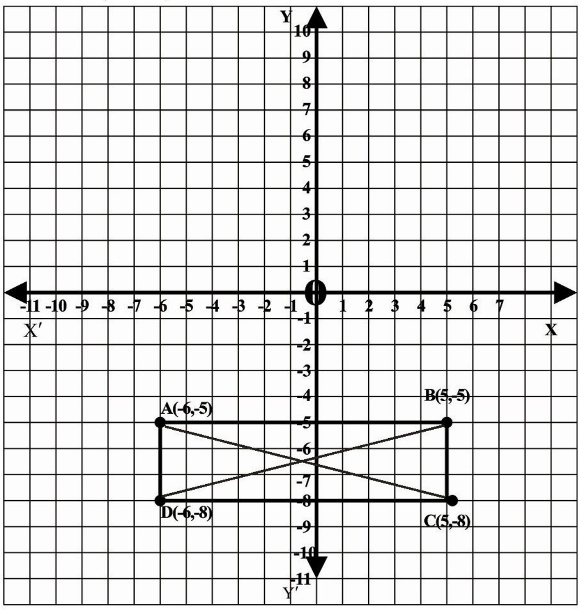
$$|DA| = \sqrt{|-6-(-6)|^2 + |-5-(-8)|^2}$$

$$|DA| = \sqrt{(-6+6)^2 + (-5+8)^2}$$

$$|DA| = \sqrt{(0)^2 + (3)^2} = \sqrt{0+9}$$

$$|DA| = \sqrt{9}$$

$$|DA| = 3$$



Diagonal distance of

$|AC|$ or $|BD|$

$$A(-6,-5), C(5,-8)$$

$$|AC| = \sqrt{|5-(-6)|^2 + |-8-(-5)|^2}$$

$$|AC| = \sqrt{(5+6)^2 + (-8+5)^2}$$

$$|AC| = \sqrt{(11)^2 + (-3)^2}$$

$$|AC| = \sqrt{121+9}$$

$$|AC| = \sqrt{130}$$

$$B(5,-5), D(-6,-8)$$

$$|BD| = \sqrt{|-6-5|^2 + |-8-(-5)|^2}$$

$$|BD| = \sqrt{(-11)^2 + (-8+5)^2}$$

$$|BD| = \sqrt{121 + (-3)^2} = \sqrt{121+9}$$

$$|BD| = \sqrt{130}$$

By Pythagoras theorem

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

$$(AC)^2 = (DC)^2 (AD)^2$$

$$\left(\sqrt{130}\right)^2 = (11)^2 + (3)^2$$

130 = 121 + 9

130 = 130

L.H.S = R.H.S

Length of both diagonal are same it is a rectangle

Q.9 Show that the point $M(-1, 4)$, $N(-5, 3)$, $P(1,-3)$ and $Q(5,-2)$ are vertices of a parallelogram.

Solution:

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$M(-1,4), N(-5,3)$$

$$|MN| = \sqrt{|-5-(-1)|^2 + |3-4|^2}$$

$$|MN| = \sqrt{(-5+1)^2 + (-1)^2}$$

$$|MN| = \sqrt{(-4)^2 + 1} = \sqrt{16+1}$$

$$|MN| = \sqrt{17}$$

$N(-5,3), P(1,-3)$

$|NP| = \sqrt{|1-(-5)|^2 + |-3-3|^2}$

$|NP| = \sqrt{(1+5)^2 + (-6)^2}$

$|NP| = \sqrt{(6)^2 + (6)^2} = \sqrt{36+36}$

$|NP| = \sqrt{72}$

$|NP| = \sqrt{36\times 2}$

$|NP| = 6\sqrt{2}$

$P(1,-3), Q(5,-2)$

$|PQ| = \sqrt{|5-1|^2 + |-2-(-3)|^2}$

$|PQ| = \sqrt{(4)^2 + (-2+3)^2}$

$|PQ| = \sqrt{16+(1)^2}$

$|PQ| = \sqrt{16+1}$

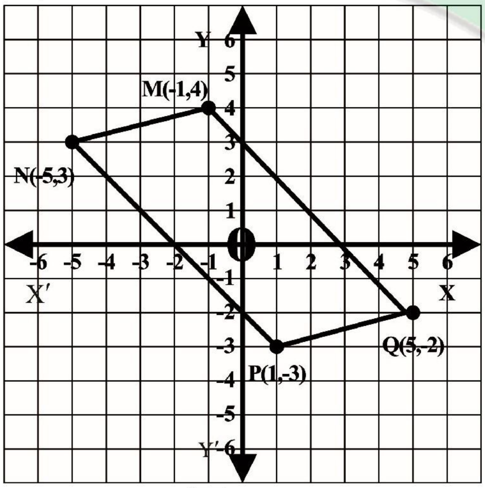
$|PQ| = \sqrt{17}$

$M(-1,4), Q(5,-2)$

$|MQ| = \sqrt{|5-(-1)|^2 + |-2-4|^2}$

$|MQ| = \sqrt{(5+1)^2 + (-6)^2} = \sqrt{(6)^2 + 36}$

$|MQ| = \sqrt{36+36}$



$|MQ| = \sqrt{72}$

$|MQ| = \sqrt{36\times 2}$

$|MQ| = 6\sqrt{2}$

Anyone diagonals distance

$N(-5,3), Q(5,-2)$

$|NQ| = \sqrt{|5-(-5)|^2 + |-2-3|^2}$

$|NQ| = \sqrt{(5+5)^2 + (-5)^2}$

$|NQ| = \sqrt{(10)^2 + (25)}$

$|NQ| = \sqrt{100+25} = \sqrt{125}$

By Pythagoras theorem

$(QM)^2 + (MN)^2 = (QN)^2$

$(6\sqrt{2})^2 + (\sqrt{17})^2 = (\sqrt{125})^2$

$36\times 2 + 17 = 125$

72 + 17 = 125

89 = 125

They are not equal so it is not right angle triangle

But

$|MN| = |PQ| \text{and} |NP| = |MQ|$

Opposite side are equal so it is a Parallelogram.

Q.10 Find the length of the diameter of the circle having centre at $C(-3,6)$ and passing through P (1 , 3)

Solution:

CP is the radius of a circle

So

$|CP| = \sqrt{|-3-1|^2 + |6-3|^2}$

$|CP| = \sqrt{(-4)^2 + (3)^2}$

$|CP| = \sqrt{16+9}$

$|CP| = \sqrt{25}$

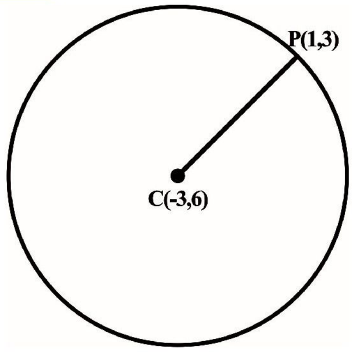
$|CP| = 5$

Diameter = 2 radius

Diameter = 2(CP)

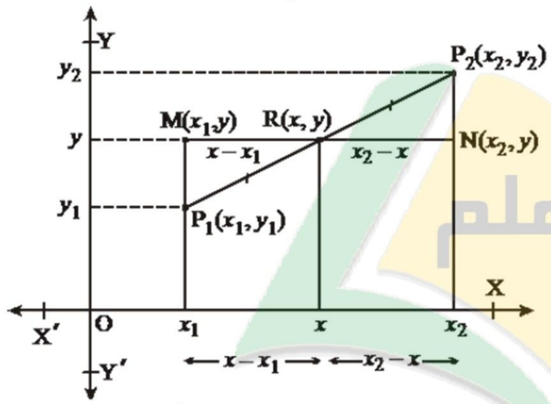
Diameter = 2(5)

Diameter = 10



Recognition of the midpoint formula for any two points in the plane

Let $P_1(x, y)$ and $P_2(x_2, y_2)$ be any two points in the plane and $R(x, y)$ be midpoint of point P_1 and P_2 on the line segment P_1P_2 as shown in the figure.



If the line segment MN, parallel to x-axis has its midpoint $R(x, y)$,

then, $x_2 - x = x - x_1$

$x_2 + x_1 = x + x$

$2x = x_1 + x_2 \Rightarrow x = \frac{x_1 + x_2}{2}$

Similarly, $\mathrm{y} = \frac{\mathrm{y}_1 + \mathrm{y}_2}{2}$

Thus the point R (x, y)

$= R\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ is the midpoint of the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$

Verification of the midpoint formula

$$|P_1R| = \sqrt{\left(\frac{x_1 + x_2}{2} - x_1\right)^2 + \left(\frac{y_1 + y_2}{2} - y_1\right)^2}$$

$$|P_1R| = \sqrt{\left(\frac{x_1 + x_2 - 2x_1}{2}\right)^2 + \left(\frac{y_1 + y_2 - 2y_1}{2}\right)^2}$$

$$|P_1R| = \sqrt{\left(\frac{x_2 - x_1}{2}\right)^2 + \left(\frac{y_2 - y_1}{2}\right)^2}$$

$$|P_1R| = \sqrt{\frac{(x_2 - x_1)^2}{4} + \frac{(y_2 - y_1)^2}{4}}$$

$$|P_1R| = \sqrt{\frac{(x_2 - x_1)^2 + (y_2 - y_1)^2}{4}}$$

$$|P_1R| = \sqrt{\frac{(x_2 - x_1)^2 + (y_2 - y_1)^2}{2}}$$

OR

$$|P_1R| = \frac{1}{2}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \frac{1}{2}|P_1P_2|$$

and $|P_2R| = \sqrt{\left(\frac{x_1 + x_2}{2} - x_2\right)^2 + \left(\frac{y_1 + y_2}{2} - y_2\right)^2}$

$$|P_2R| = \sqrt{\left(\frac{x_1 + x_2 + 2x_2}{2}\right)^2 + \left(\frac{y_1 + y_2 - 2y_2}{2}\right)^2}$$

$$|P_2R| = \sqrt{\left(\frac{x_1 - x_2}{2}\right)^2 + \left(\frac{y_1 - y_2}{2}\right)^2}$$

$$|P_2R| = \sqrt{\frac{(x_1 - x_2)^2}{4} + \frac{(y_1 - y_2)^2}{4}}$$

$$|P_2R| = \sqrt{\frac{(x_1 - x_2)^2 + (y_1 - y_2)^2}{4}}$$

$$|P_2R| = \frac{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}{2}$$

OR

$$|P_2R| = \frac{1}{2}\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$\Rightarrow |P_2R| = |P_1R| = \frac{1}{2}|P_1R_2|$$

Thus it verifies that

$R\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ is the midpoint of the line segment P_1RP_2 which lies on the line segment since $|P_1R| + |P_2R| = |P_1P_2|$

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Report any mistake at freeilm786@gmail.com