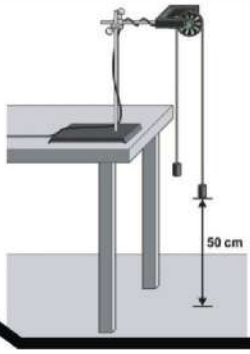

UNIT 3
DYNAMICS

LONG QUESTIONS

3.2 NEWTON'S LAWS OF MOTION

Newton's First Law of Motion

Q.1 State and Explain Newton's First law of motion (GRW 2011, 2012, 2014)

Ans: A body continues in its state of rest or of uniform motion in a straight line provided no net force acts on it.

Explanation for rest

Newton's first law of motion deals with bodies which are either at rest or moving with uniform speed in straight line. According to first law of motion, a body at rest remains at rest provided no net force act on it. This part of the law is true as we observe that objects do not move by themselves unless someone moves them.

Example

A book lying on a table remains at rest as long as no net force acts on it.

Explanation for motion

Similarly, a moving object does not stop moving by itself. A ball rolled on a rough ground stops earlier than that rolled on smooth ground. It is because rough surface offer greater friction. If there would be no force to oppose the motion of the body would never stop.

Example

When its engine of a car moving with uniform velocity is turned off it stops gradually because a net force of friction is acting in the opposite direction causes to stop it.

Law of inertia

Since Newton's first law of motion deals with the inertial property of matter, therefore, Newton's first law of motion is also known as law of inertia.

Example

Passengers standing in a bus fall forward when its driver applies brakes suddenly. It is because the upper parts of the bodies tend to continue their motion, lower parts of their bodies are in contact with the bus stop with it. Hence, they fall forward.

Newton's Second Law of Motion

Q.2 State and Explain Newton's Second law of motion (GRW 2011, LHR 2012, 2013)

Ans: "When a net force acts upon a body, it produces as acceleration in the body in the direction of force and the magnitude of acceleration is directly proportional to the net force and is inversely proportional to the mass of the body".

Mathematical Form

If the force 'F' is acting on the body of mass 'm' then we can write this in the mathematical form as,

$a \propto F$ .... (1)

$$a \propto \frac{1}{m} \quad \ldots\ldots\ldots\ldots\ldots (2)$$

From relation (1) and (2), we have

$$a \propto \frac{F}{m}$$

Changing the sign of proportionality into the sign of equality

$$a = \text{constant} \times \frac{F}{m}$$

$$a = k \times \frac{F}{m}$$

In above equation, according to international system of units if m = 1Kg, $a = 1ms^{-2}$, F = 1N then the value of the constant k will be '1'. So the equation can be written as,

$$a = 1 \times \frac{F}{m}$$

$$F = ma$$

This is the mathematical form of Newton's Second law of motion.

Unit of Force

In the System International, the unit of force is Newton, which is represented by the symbol **'N'**.

Newton

"One Newton is that force which produces an acceleration of 1 ms^{-2} in a body of mass 1 Kg". This unit of Newton can also be written as,

$$1\ N = 1\ kg \times 1\ ms^{-2}$$

$$1\ N = 1\ Kgms^{-2}$$

Q.3 Differentiate between Mass and Weight. (GRW 2011, 2012, LHR 2014, 2015)

Ans:

Mass	Weight
• Mass of a body is the quantity of matter possessed by the body.	• The weight of the body is equal to the force with which earth attracts it.
• It is a scalar quantity.	• It is a vector quantity and is toward the center of the earth.
• It is measured by physical balance.	• It is measured by spring balance.
• It remains same everywhere and does not change with change of place.	• It does not remain same at all places and varies with the value of 'g'.
• Unit of mass is kilogram (Kg).	• Unit of weight is Newton (N).
• It is a base quantity.	• It is a derived quantity.
• It can be calculated by using the formula F = ma.	• It can be calculated by using the formula w = mg.
• Mass of a body can never be zero	• Weight of body can be zero

Newton's Third Law of Motion

Q.4 State and Explain Newton's Third law of motion (LHR 2011, GRW 2013)

Ans: To every action there is always an equal but opposite reaction.

Action and Reaction

Newton's third law of motion deals with the reaction of a body when a force acts on it. Let a body A exerts a force on another body B, the body B reacts against this force and exerts a force on body A. the force exerted by body A on B is the action force whereas the force exerted by B on A is called the reaction force.

Relation between Action and Reaction

Newton has expressed action and reaction in his third law of motion. Action is always accomplished by a reaction force and the two forces must always be equal and opposite. It is to remember that "action" and "reaction" do not act on the same body but they act on two different bodies.

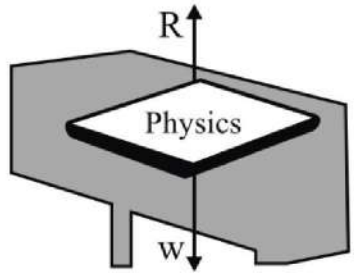

Figure 3.8: Action of the book and reaction on it.

Example 1

Consider a book lying on a table as shown in figure. The weight of the book is acting on the table in the downward direction. This is the action. The reaction of the table acts on the book in the upward direction.

Example 2

Take an air – filled balloon. When the balloon is set free, the air inside it rushes out and the balloon moves forward. In this example, the action is by the balloon that pushes the air out of it when set free. The reaction of the air which escapes out from the balloon acts on the balloon. It is due to this reaction of the escaping air that moves the balloon forward.

Example 3

A rocket moves on the same principle. When its fuel burns, hot gases escape out from its tail with a very high speed. The reaction of these gases on the rocket causes it to move opposite to the gases rushing out of its tail.

Figure 3.10: A Rocket taking off

Tension and Acceleration in a String

Q.5 Explain the tension in the string. If two bodies of masses m_1 and m_2 are hanging from the two ends of a string which is passing over a pulley, find the values of tension and acceleration in it. (LHR 2013, GRW 2015)

Ans: The force which is exerted by the string on the body when its is subjected to a pull is called the tension in the string. It is a reaction force of the weight and it is usually denoted by T. The weight acts downwards while tension T in the string is acting upwards at the block. If the object is at rest, the magnitude of tension is equal to weight.

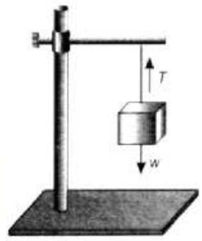

Figure 3.11: Weight of the block pulls the string downwards

Motion of Bodies connected by a string

There are two cases of motion of bodies connected by a string.

(i) When the bodies move vertically

(ii) When one body moves vertically and the other moves horizontally

Case-I

When the Bodies Move Vertically (Atwood machine)

Suppose two bodies A and B having masses m_1 and m_2 respectively are connected to two ends of an inextensible string which passes over a frictionless pulley. If m_1 is greater than m_2, then the body A will move downward and the body B will move upward. The body A being heavier must be moving downwards with some acceleration. Let this acceleration be a. At the same time, the body B attached to the other end of the string moves up with the same acceleration a. as the pulley is frictionless, hence tension will be the same throughout the string. Let the tension in the string be T.

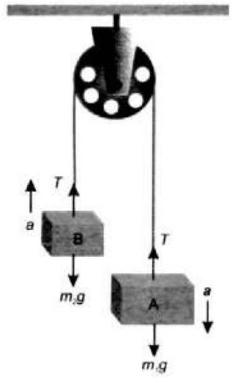

Figure 3.12: Bodies attached to the ends of a string that passes over a frictionless pulley.

Forces acting on the body A

As the body A is moving downward, the resultant force acting on it is downward due to which acceleration a is produced in it.

Net force acting on body A = $F_1 = m_1g - T$

According to Newton's second law of motion;

$$m_1g - T = m_1a \quad \ldots\ldots\ldots\ldots\ldots (1)$$

As the body B is moving upward, the resultant force acting on it is upward due to which acceleration a is produced in it.

Net force acting on body B = $T - m_1g$

According to Newton's second law of motion;

$$T - m_2 g = m_2a \quad \ldots\ldots\ldots\ldots\ldots (2)$$

Calculation of Acceleration

By adding equation (1) and equation (2), we have

$$m_1g - T + T - m_2g = m_1a + m_2a$$
$$m_1g - m_2g = m_1a + m_2a$$
$$(m_1 - m_2)g = (m_1 + m_2)a$$
$$\frac{(m_1 - m_2)g}{m_1 + m_2} = a$$

OR $$a = \frac{(m_1 - m_2)g}{m_1 + m_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Calculation of Tension

$$\frac{T - m_2g}{m_1g - T} = \frac{m_2a}{m_1a}$$
$$\frac{T - m_2g}{m_1g - T} = \frac{m_2}{m_1}$$
$$m_1\left(T - m_2g\right) = m_2\left(m_1g - T\right)$$
$$m_1T - m_1m_2g = m_1m_2g - m_2T$$
$$m_1T + m_2T = m_1m_2g + m_1m_2g$$
$$\left(m_1 + m_2\right)T = 2m_1m_2g$$
$$T = \frac{2m_1m_2g}{m_1 + m_2}$$

The above arrangement is also known as Atwood machine. It can also used to find the acceleration due to gravity by equation (3)

$$g = \frac{m_1 + m_2}{m_1 - m_2}a$$

Case-II

When One Body Moves Vertically and the Other Moves Horizontally

Two bodies A and B having masses m_1 and m_2 respectively are connected to an inextensible string which passes over the pulley as shown in figure. The body A moves vertically downward with an acceleration a. The body B moves on the horizontal smooth surface towards the pulley with the same acceleration a. As the pulley is frictionless, hence tension T will be the same throughout the string.

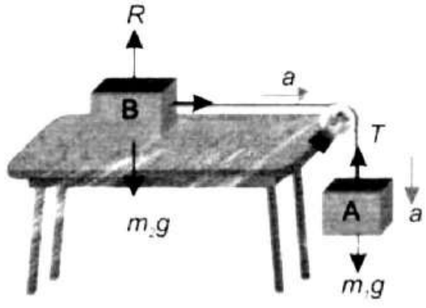

Figure 3.13: Motion of masses attached to a string that passes over a frictionless pulley.

Forces acting on the body A

As the body A is moving downward, therefore, weight m_1g is greater than the tension T in the string.

Net force acting on body A = $m_1g - T$

According to Newton's second law of motion;

$$m_1g - T = m_1a \quad \ldots\ldots\ldots\ldots\ldots (1)$$

Forces acting on the body B

Now consider the motion of the body B. Three forces are acting on it.

(i) Its weight $w_2 = m_2g$ of the body B acting downward

(ii) The upward reaction R on the horizontal surface acting on the body B in the upward direction.

(iii) Tension T on the string pulling the body in the horizontal direction over the smooth surface.

As the body B is not moving vertically, therefore, vertical forces cancel each other and their resultant is zero. The only remaining force T due to which the body B is moving in the horizontal direction with acceleration 'a'.

Hence according to Newton's second law of motion,

$$T = m_2a \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Calculation of Acceleration

Adding eqution (1) and (2)

$$m_1g - T + T = m_1a + m_2a$$

$$m_1g = (m_1 + m_2)a$$

$$a = \frac{m_1g}{m_1 + m_2}$$

Calculation of Tension

In order to find the value of T, put the value of a in equation (2), we have

$$T = m_2 \times \frac{m_1g}{m_1 + m_2}$$

$$T = \frac{m_1m_2g}{m_1 + m_2} \quad \ldots\ldots\ldots\ldots\ldots \quad (4)$$

Force and the Momentum

Q.6 How you can prove that rate of change in momentum of a body is equal to the applied force? Or Derive the relation between momentum and force. (LHR 2015)

Ans: When a force acts on a body, it produces an acceleration in the body and will be equal to the rate of change of momentum of the body.

Suppose a force 'F' acts on a body of mass 'm' moving with initial velocity 'v_i' which produces an acceleration a in it. This changes the velocity of body to 'v_f' after time t. If P_i and P_f be the initial momentum and final momentum of the body related to initial and final velocities, Then,

Momentum of the body having velocity $v_i = P_i = mv_i$
Momentum of the body having velocity $v_f = P_f = mv_f$
Change in momentum = final momentum – initial momentum
$= P_f - P_i = mv_f - mv_i = m(v_f - v_i)$

Rate of change in momentum $= \frac{P_f - P_i}{t} = \frac{mv_f - mv_i}{t}$

Rate of change in momentum $= m\frac{v_f - v_i}{t}$

Since $\frac{(v_f - v_i)}{t}$ is the rate of change of velocity equal to acceleration produced by the force F.

Rate of change of momentum = ma

According to Newton's second law of motion,

$$F = ma$$

$\therefore$Rate of change of momentum = F

Rate of change of momentum of a body is equal to the applied force on it and the direction of change of in momentum is in the direction of the force.

Hence when a force acts on a body, it produces an acceleration in the body and will be equal to the rate of change of momentum of the body.

This is statement of Newton's second law of motion in terms of momentum.

Law of Conservation of Momentum

Q.7 State and explain Law of conservation of Momentum. (GRW 2013, LHR 2014)

Ans: The momentum of an isolated system of two or more than two interacting bodies remains constant.

An isolated system is a group of interacting bodies on which no external force is acting. If no unbalanced or net force acts on a system then its momentum remains constant.

Example

Consider the example of an air-filled balloon. In this case, balloon and the air inside it form a system. Before releasing the balloon, the system was at rest and hence the initial momentum of the system was zero. As soon as the balloon is set free, air escapes out of it with some velocity. The air coming out of it possesses momentum. To conserve momentum, balloon moves in the direction opposite to the air coming out of it.

Mathematical Explanation

Consider an isolated system of two spheres of masses m_1 and m_2 as shown figure. They are moving in a straight line with initial velocities u_1 and u_2 respectively, such that u_1 is greater than u_2. Sphere of mass m_1 approaches the sphere of mass m_2 as they move.

Initial momentum of mass $m_1 = m_1u_1$
Initial momentum of mass $m_2 = m_2u_2$
Total momentum of the system before collision $= m_1u_1 + m_2u_2$

After sometime mass m_1 hits m_2 with some force. According to Newton's third law of motion, m_2 exerts an equal and opposite reaction force on m_1. Let their velocities become v_1 and v_2 respectively after collision.

Final momentum of mass $m_1 = m_1v_1$
Final momentum of mass $m_2 = m_2v_2$
Total momentum of the system after collision $= m_1v_1 + m_2v_2$
Total momentum of system before collision = total momentum of system after collision

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

The above equation that the momentum of the isolated system before and after collision remains same which is the law of conservation of momentum.

Application of Law of Conservation of Momentum

This law is applicable universally i.e. true not only for bigger bodies but also for atoms and molecules.

Example

Consider a system of gun and a bullet. Before firing, the velocity of the bullet as well as that of gun was zero. Therefore, the total momentum of both the objects was also zero. We can write it as,

Total momentum of gun and bullet before firing = 0

When the gun is fired, bullet shoots out of the gun and acquire momentum. To conserve momentum the gun recoils backward. Now according to the law of conservation of momentum, the total momentum of the gun and bullet will also be zero after the gun is fired. Let m be the mass of the bullet and v be its velocity on firing the gun; M be the mass of the gun and V be the velocity with which it recoils. Thus the total momentum of the gun is fired will be:

The momentum of the gun and bullet after the gun is fired = M V + m v

According to the law of conservation of momentum

Total momentum before firing = Total momentum after firing

$$MV + mv = 0$$

OR $$MV = -mv$$

Hence $$V = -\frac{m}{M}v$$

The above equation gives the velocity V of the gun. Here negative sign indicates that velocity gun is opposite to the velocity of bullet. That is why the shoulder pressed hard during firing. Since mass of the gun is much larger than the bullet, therefore, the recoil is much smaller than the velocity of the bullet.

Application in Rocket or Jet engine

Rocket or Jet engine also works on this same principle. In both of them, gases are produced at a high temperature due to the burning of fuel. These gases rush out with large momentum. Therefore the rockets or jet engines gain an equal and opposite momentum. This enables them to move with very high velocities.

3.3 FRICTION

Q.8 Define friction. Explain cause of friction and derive its mathematical formula.

Ans: The force which opposes the motion of moving objects is called friction.

Cause of friction

No surface is perfectly smooth. A surface that appears smooth has pits and bumps that can be seen under microscope. A magnified view of a surface in contact shows the gaps and contacts between them. The contact points between the two surfaces form a sort of cold welds. These cold welds resist the surfaces from sliding over each other. Adding weight over the upper block increases the force pressing the surfaces together which increases the resistance. Thus greater is the pressing force greater will be the friction between sliding surfaces.

Mathematical Derivation

Friction is equal to the applied force that tends to move a body at rest. This friction at rest is called the static friction. It increases with the applied force. Friction can also be increased to a certain maximum value. It does not increase beyond this. This maximum value of friction is known as force of limiting friction (Fs). It depends on the normal reaction (pressing force) between the two surfaces in contact. The ratio between the force of limiting friction Fs and the normal reaction R is constant. This constant is called the coefficient of friction and is represented by μ.

Thus $$\mu = \frac{Fs}{R}$$

Or $$Fs = \mu R$$

If m is the mass of the block, then for horizontal surface;

$$R = mg$$

Hence $$Fs = \mu mg$$

Friction is desirable

Friction is needed to walk on the ground. It is risky to run on wet floor with shoes that have smooth soles. Athletes use special shoes that have extraordinary ground grip. Such shoes prevent them from slipping while running fast. To stop bicycle we apply brakes. The rubber pads pressed against the rims provide friction. It is the friction that stops the bicycle.

Rolling Friction

Q.9 Explain the rolling friction.

Ans: Wheel as greatest invention

The most important invention in the history of mankind was a wheel. The first thing about a wheel is that it rolls as it moves rather than to slide. This greater reduces the friction.

Less friction in Rolling Friction

When axle of a wheel is pushed, the force of friction between the wheel and the ground at the point of contact provides the reaction force. The reaction force acts at the contact points of the wheel in a direction opposite to the direction to the applied force. The wheel rolls without rupturing the cold welds. That is why this rolling friction is extremely small than sliding friction. The fact that rolling friction is less than sliding friction is applied in ball bearing to reduce losses due to friction.

Necessary Road Grip

The wheel would not roll on pushing it if there would be no friction between the wheel and the ground. Thus, friction is desirable for wheels to roll over a surface. It is dangerous to drive on a wet road because the friction between the road and the tyres is very small. This increases the chance of slipping the tyres from the road. The threading of tyres is designed to increase friction. Thus, threading improves road grip and make it safer to drive even on wet road.

Sliding Friction in Brakes

A cyclist applies brakes to stop his/her bicycle. As soon as brakes are applied, the wheels stop rolling and begin to slide over the road. Since sliding friction is much greater than rolling friction, the cycle stops very quickly.

Braking and Skidding

Q.10 Explain the roll of friction in Braking and explain the Skidding.

Ans: The wheels of a moving vehicle have velocity components:

(i) Motion of wheel along the road

(ii) Rotation of wheels about their axis

To move a vehicle on the road as well as to stop a moving vehicle requires friction between its tyres and the road.

Example

If the road is slippery or the tyres are worn out then the tyres instead of rolling, slip over the road. The vehicle will not move if the wheels start slipping at the same point on the slippery road. Thus for the wheels to roll, the force of friction (gripping force) between the tyres and the road must be enough that prevents them from slipping.

Similarly to stop a car quickly, a large force of friction between the tyres and the road is needed. But there is a limit to this force of friction that tyres can provide.

Skidding

If the brakes are applied too strongly, the wheels of the car will lock up (stop turning) and the car will skid due to its large momentum. It will lose its directional control that may result in an accident. In order to reduce the chance of skidding, it is advisable not to apply brakes too hard that lock up their rolling motion especially at high speeds. Moreover, it is unsafe to drive a vehicle with worn out tyres.

3.4 UNIFORM CIRCULAR MOTION

Centripetal Force

Q.11 Define centripetal force and centripetal acceleration and derive the mathematical relation for centripetal force and acceleration.

Ans: A force that keeps a body to move in a circle is known as centripetal force.

Explanation

Consider a body tied at the end of a string moving with uniform speed in a circular path. A body has the tendency to move in a straight line due to inertia. The string to which body is tied keeps it to move in a circle by pulling the body towards the center of the circle. The string pulls the body perpendicular to its motion. The pulling force continuously changes the direction of motion and remains towards the center of the circle. This center seeking force is called the centripetal force. It keeps the body to move in a circle. Centripetal force always acts perpendicular to the motion of the body.

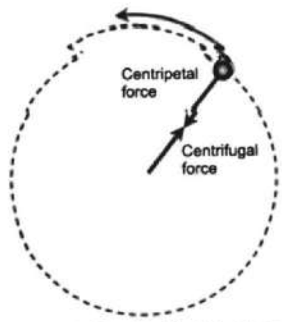

Figure 3.28: Centripetal force acting on the stone and the centrifugal force acting on the

Examples

(i) A stone is tied to one end of a string rotating in a circle. The tension in the string provides the required centripetal force. It keeps the stone to remain in the circle. If the string is not strong enough to provide the necessary tension, it breaks and the stone moves away along the tangent to the circle.

(ii) The moon revolves around the Earth. The gravitational force of the Earth provides required centripetal force.

Mathematical Formula

If an object of mass m is moving with velocity v in a circle of radius r, the magnitude of centripetal force F_c acting on it can be found by using the following equation.

$$F_c = \frac{mv^2}{r}$$

Centripetal acceleration

The acceleration produced by the centripetal force which is always directed towards the center of the circle is known as centripetal acceleration. It is represented by a_c.

According to Newton's second law of motion, the direction of this acceleration is along the direction of the centripetal force F_c, i.e., perpendicular to the velocity and directed towards the centre of the circle.

$$F_c = ma_c$$

So, $$ma_c = \frac{mv^2}{r}$$

So, $$a_c = \frac{v^2}{r}$$

Dependence

The above equation shows that centripetal force of the body moving in a circular path depends upon:

- Mass m of the body
- Square of its velocity