Unit 10: Congruent Triangle

Overview

Congruency of Triangles:

Tow triangles are said to be congruent written symbolically as $\cong$,if there exists a correspondence between them such that all the corresponding sides and angles are congruent.

i.e. if $\begin{cases} \overline{AB} \cong \overline{DE} \\ \overline{BC} \cong \overline{EF} \\ \overline{CA} \cong \overline{FD} \end{cases}$ and $\begin{cases} \angle A \cong \angle D \\ \angle B \cong \angle E \\ \angle C \cong \angle F \end{cases}$

then $\Delta ABC \cong \Delta DEF$

A.S.A postulate:

In any correspondence of two triangles, if one side and any two angles of one triangle are congruent to the corresponding side and angles of the other then the triangles are congruent this postulate is called A.S.A. postulate.

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S.S.S postulate:

In a correspondence of two triangles, if three sides of one triangle are congruent to the corresponding three sides of the other, then the two triangles are congruent this postulate is called S.S.S postulate.

H.S postulate:

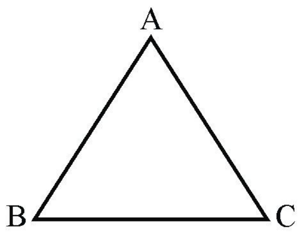
If in the correspondence of the two right-angled triangles, the hypotenuse and one side of one triangle are congruent ot the hypotenuse and the corresponding side of the other, then the triangles, are congruent this postulate is called H.S postulate.

Introduction:

Two triangles are said to be congruent if at least one $(1-1)$ correspondence can be established between them in which the angles and sides are congruent.

For example

If in the corresponding $\Delta ABC \leftrightarrow \Delta DEF$



(i) $\angle A \longleftrightarrow \angle D$ ($\angle A$ corresponds to $\angle D$)

(ii) $\angle B \longleftrightarrow \angle E$ ($\angle B$ corresponds to $\angle E$)

(iii) $\angle C \longleftrightarrow \angle F$ ($\angle C$ corresponds to $\angle F$)

(iv) $\overline{AB} \longleftrightarrow \overline{DE}$ ($\overline{AB}$ corresponds to $\overline{DE}$)

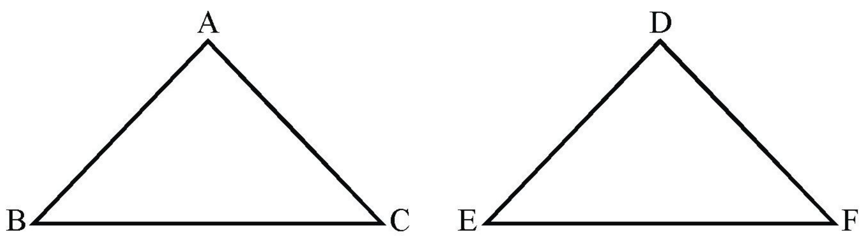
(v) $\overline{BC} \longleftrightarrow \overline{EF}$ ($\overline{BC}$ corresponds to $\overline{EF}$)

(vi) $\overline{CA} \longleftrightarrow \overline{FD}$ ($\overline{CA}$ corresponds to $\overline{FD}$)

Congruency of Triangles:

The two triangles are said to be congruent written as $\cong$ if there exists a correspondence between them such that all the corresponding sides and angles are congruent.

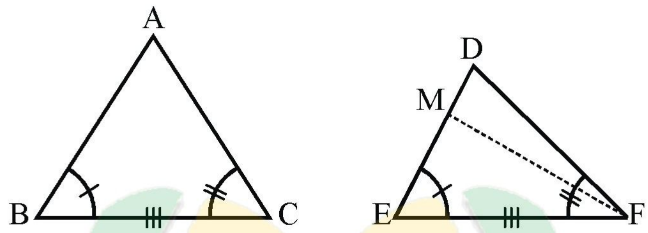
Then $\Delta ABC \cong \Delta DEF$



$$\text{If}\begin{cases}\overline{AB} \cong \overline{DE}\\ \overline{BC} \cong \overline{EF}\\ \overline{AC} \cong \overline{DF}\end{cases} \qquad \text{and} \qquad \begin{cases}\angle A \cong \angle D\\ \angle B \cong \angle E\\ \angle C \cong \angle F\end{cases}$$

Theorem 10.1.1

In any correspondence of two triangles, if one side and any two angles of one triangle are congruent to the corresponding side and angles of the other then the triangles are congruent.(A.S.A $\cong$ A.S.A.)



Given

In $\Delta ABC \leftrightarrow \Delta DEF$

$\angle B \cong \angle E$, $\overline{BC} \cong \overline{EF}$, $\angle C \cong \angle F$

To prove

$\Delta ABC \cong \Delta DEF$

Construction

Suppose $\overline{AB} \ncong \overline{DE}$. Take a point M on $\overline{DE}$ such that $\overline{AB} \cong \overline{ME}$. Join M to F

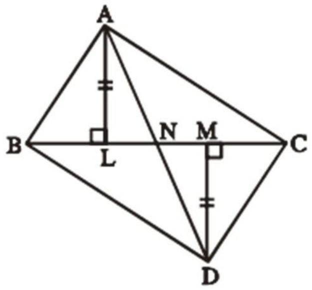
Proof

Statements	Reasons
In $\Delta ABC \leftrightarrow \Delta MEF$	
$\overline{AB} \cong \overline{ME}$ ____ (i)	Construction
$\overline{BC} \cong \overline{EF}$ ____ (ii)	Given
$\angle B \cong \angle E$ ____ (iii)	Given
$\Delta ABC \cong \Delta MEF$	S.A.S postulate
So, $\angle C \cong \angle MFE$	(Corresponding angles of congruent triangles)
But $\angle C \cong \angle DFE$	Given
$\therefore$ $\angle DFE \cong \angle MFE$	Both congruent to $\angle C$
This is possible only if D and M are the same points and $\overline{ME} \cong DE$	

So $\overline{AB} \cong \overline{DE}$ ____(iv) Thus from (ii), (iii) and (iv), we have $\Delta ABC \cong \Delta DEF$	$\overline{AB} \cong \overline{ME}$ (construction) and $\overline{ME} \cong \overline{DE}$ (proved) S.A.S postulates

Example

If ΔABC and ΔDCB are on the opposite sides of common base $\overline{BC}$ such that $\overline{AL} \perp \overline{BC}, \overline{DM} \perp \overline{BC}$ and $\overline{AL} \cong \overline{DM}$, then $\overline{BC}$ bisects $\overline{AD}$.



Given

ΔABC and ΔDCB are on the opposite sides of $\overline{BC}$ such that $\overline{AL} \perp \overline{BC}, \overline{DM} \perp \overline{BC}, \overline{AL} \cong \overline{DM}$, and $\overline{AD}$ is cut by $\overline{BC}$ at N.

To prove

$\overline{AN} \cong \overline{DN}$

Proof

Statements	Reasons
In $\Delta ALN \leftrightarrow \Delta DMN$	
$\overline{AL} \cong \overline{DM}$	Given
$\angle ALN \cong \angle \overline{DMN}$	Each angle is right angle
$\angle ALN \cong \angle \overline{DMN}$	Vertical angels
$\angle ALN \cong \angle \overline{DMN}$	SAA ≅ SAA
$\overline{AN} \cong \overline{DN}$	Corresponding sides of ≅ Δs.

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Report any mistake at freeilm786@gmail.com