

Review Exercise 1

Q.1 Select the correct answer in each of the following.

(i) The order of matrix $\begin{bmatrix} 2 & 1 \end{bmatrix}$ is....

- (a) 2-by-1
(c) 1-by-1

- (b) 1-by-2
(d) 2-by-2

(ii) $\begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix}$ is called ...matrix.

- (a) Zero
(c) Scalar

- (b) Unit
(d) Singular

(iii) Which is order of a square matrix?

- (a) 2-by-2
(c) 2-by-1

- (b) 1-by-2
(d) 3-by-2

(iv) Order of transpose of $\begin{bmatrix} 2 & 1 \\ 0 & 1 \\ 3 & 2 \end{bmatrix}$ is...

- (a) 3-by-2
(c) 1-by-3

- (b) 2-by-3
(d) 3-by-1

(v) Adjoint of $\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$ is...

- (a) $\begin{bmatrix} -1 & -2 \\ 0 & 1 \end{bmatrix}$
(c) $\begin{bmatrix} -1 & 2 \\ 0 & -1 \end{bmatrix}$

- (b) $\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$
(d) $\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}$

(vi) Product of $\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ is...

- (a) $[2x + y]$
(c) $[2x - y]$

- (b) $[x - 2y]$
(d) $[x + 2y]$

(vii) If $\begin{bmatrix} 2 & 6 \\ 3 & x \end{bmatrix} = 0$, then x is equal to...

- (a) 9
(c) 6

- (b) -6
(d) -9

(viii) If $X + \begin{bmatrix} -1 & -2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then X is equal to...

- (a) $\begin{bmatrix} 2 & 2 \\ 2 & 0 \end{bmatrix}$

- (b) $\begin{bmatrix} 0 & 2 \\ 2 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$

ANSWER KEY

i	ii	iii	iv	v	vi	vii	viii
b	c	a	b	a	c	a	d

Q.2 Complete the following:

(i) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is called ... matrix.

(ii) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is called ... matrix.

(iii) Additive inverse of $\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$ is....

(iv) In matrix multiplication, in general, $AB \dots BA$.

(v) Matrix $A+B$ may be found if order of A and B is...

(vi) A matrix is called ... matrix if number of rows and columns are equal.

ANSWER KEY

i	ii	iii	iv	v	vi
Null	Unit	$\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$	\neq	Same	Square

Q.3 If $\begin{bmatrix} a+3 & 4 \\ 6 & b-1 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$, then find a and b .

Solution: $\begin{bmatrix} a+3 & 4 \\ 6 & b-1 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$
 $a+3 = -3$ $b-1 = 2$
 $a = -3-3$ $b = 2+1$
 $a = -6$ $b = 3$ **Ans**

Q.4 If $A = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}$, then find the following.

(i) $2A+3B$

(ii) $-3A+2B$

(iii) $-3(A+2B)$

(iv) $\frac{2}{3}(2A-3B)$

Solution: (i)

$$\begin{aligned}
 2A+3B &= 2\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} + 3\begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix} \\
 &= \begin{bmatrix} 4 & 6 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 15 & -12 \\ -6 & -3 \end{bmatrix} \\
 &= \begin{bmatrix} 4+15 & 6-12 \\ 2-6 & 0-3 \end{bmatrix} \\
 &= \begin{bmatrix} 19 & -6 \\ -4 & -3 \end{bmatrix} \text{ **Ans** }
 \end{aligned}$$

Solution: (ii)

$$\begin{aligned}
 -3A+2B &= -3\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} + 2\begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix} \\
 &= \begin{bmatrix} -6 & -9 \\ -3 & 0 \end{bmatrix} + \begin{bmatrix} 10 & -8 \\ -4 & -2 \end{bmatrix} \\
 &= \begin{bmatrix} -6+10 & -9-8 \\ -3-4 & 0-2 \end{bmatrix}
 \end{aligned}$$

$$= \begin{bmatrix} 4 & -17 \\ -7 & -2 \end{bmatrix} \text{ Ans}$$

Solution: (iii)

$$\begin{aligned} -3(A+2B) &= -3 \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} + 2 \begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix} \\ &= -3 \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 10 & -8 \\ -4 & -2 \end{bmatrix} \\ &= -3 \begin{bmatrix} 2+10 & 3-8 \\ 1-4 & 0-2 \end{bmatrix} \\ &= \begin{bmatrix} 12 & -5 \\ -3 & -2 \end{bmatrix} \\ &= \begin{bmatrix} -36 & 15 \\ 9 & 6 \end{bmatrix} \text{ Ans} \end{aligned}$$

Solution: (iv) $\frac{2}{3}(2A-3B)$

$$\begin{aligned} &= \frac{2}{3} \left(2 \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} - 3 \begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix} \right) \\ &= \frac{2}{3} \left(\begin{bmatrix} 4 & 6 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 15 & -12 \\ -6 & -3 \end{bmatrix} \right) \\ &= \frac{2}{3} \begin{bmatrix} 4-15 & 6-(-12) \\ 2-(-6) & 0-(-3) \end{bmatrix} \\ &= \frac{2}{3} \begin{bmatrix} -11 & 6+12 \\ 2+6 & 0+3 \end{bmatrix} \\ &= \frac{2}{3} \begin{bmatrix} -11 & 18 \\ 8 & 3 \end{bmatrix} \\ &= \begin{bmatrix} -11 \times \frac{2}{3} & 18 \times \frac{2}{3} \\ 8 \times \frac{2}{3} & 3 \times \frac{2}{3} \end{bmatrix} \\ &= \begin{bmatrix} \frac{-22}{3} & 12 \\ \frac{16}{3} & 2 \end{bmatrix} \text{ Ans} \end{aligned}$$

Q.5 Find the value of X, if

$$\begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix} + X = \begin{bmatrix} 4 & -2 \\ -1 & -2 \end{bmatrix}.$$

Solution: Given that

$$\begin{aligned} \begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix} + X &= \begin{bmatrix} 4 & -2 \\ -1 & -2 \end{bmatrix} \\ X &= \begin{bmatrix} 4 & -2 \\ -1 & -2 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix} \\ &= \begin{bmatrix} 4-2 & -2-1 \\ -1-3 & -2-(-3) \end{bmatrix} \\ &= \begin{bmatrix} 2 & -3 \\ -4 & -2+3 \end{bmatrix} \\ X &= \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix} \text{ Ans} \end{aligned}$$

Q.6 If $A = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}, B = \begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix},$

then prove that

- (i) $AB \neq BA$
(ii) $A(BC) = (AB)C$

Solution: Given that

$$A = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}, B = \begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix}$$

(i) $AB \neq BA$

$$\begin{aligned} \text{L.H.S} = AB &= \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix} \\ &= \begin{bmatrix} 0 \times (-3) + 1 \times 5 & 0 \times 4 + 1 \times (-2) \\ 2 \times (-3) + (-3) \times 5 & 2 \times 4 + (-3) \times (-2) \end{bmatrix} \\ &= \begin{bmatrix} 0+5 & 0-2 \\ -6-15 & 8+6 \end{bmatrix} \\ &= \begin{bmatrix} 5 & -2 \\ -21 & 14 \end{bmatrix} \rightarrow (i) \end{aligned}$$

$$\begin{aligned} \text{R.H.S} = BA &= \begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} \\ &= \begin{bmatrix} -3(0) + 4(2) & -3(1) + 4(-3) \\ 5(0) + (-2)(2) & 5(1) + (-2)(-3) \end{bmatrix} \\ &= \begin{bmatrix} 0+8 & -3-12 \\ 0-4 & 5+6 \end{bmatrix} \\ &= \begin{bmatrix} 8 & -15 \\ -4 & 11 \end{bmatrix} \rightarrow (ii) \end{aligned}$$

From (i) and (ii), we get

$$L.H.S \neq R.H.S$$

$$AB \neq BA$$

Hence proved

$$(ii) A(BC) = (AB)C$$

Solution:

We cannot solve because matrix C is not given.

Q.7 If $A = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 4 \\ -3 & -5 \end{bmatrix}$,

then verify that

$$(i) (AB)^t = B^t A^t$$

$$(ii) (AB)^{-1} = B^{-1} A^{-1}$$

Solution: Given that

$$A = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 4 \\ -3 & -5 \end{bmatrix}$$

$$(i) (AB)^t = B^t A^t$$

$$\begin{aligned} AB &= \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ -3 & -5 \end{bmatrix} \\ &= \begin{bmatrix} 3(2) + 2(-3) & 3(4) + 2(-5) \\ 1(2) + (-1)(-3) & 1(4) + (-1)(-5) \end{bmatrix} \\ &= \begin{bmatrix} 6-6 & 12-10 \\ 2+3 & 4+5 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 2 \\ 5 & 9 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} L.H.S &= (AB)^t = \begin{bmatrix} 0 & 2 \\ 5 & 9 \end{bmatrix}^t \\ &= \begin{bmatrix} 0 & 5 \\ 2 & 9 \end{bmatrix} \rightarrow (i) \end{aligned}$$

$$\begin{aligned} A^t &= \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}^t \\ &= \begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix} \end{aligned}$$

$$B^t = \begin{bmatrix} 2 & 4 \\ -3 & -5 \end{bmatrix}^t = \begin{bmatrix} 2 & -3 \\ 4 & -5 \end{bmatrix}$$

$$R.H.S = B^t A^t = \begin{bmatrix} 2 & -3 \\ 4 & -5 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}$$

$$\begin{aligned} &= \begin{bmatrix} 2 \times 3 + (-3) \times 2 & 2(1) + (-3)(-1) \\ 4 \times 3 + (-5) \times 2 & 4(1) + (-5)(-1) \end{bmatrix} \\ &= \begin{bmatrix} 6-6 & 2+3 \\ 12-10 & 4+5 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 5 \\ 2 & 9 \end{bmatrix} \rightarrow (ii) \end{aligned}$$

From equal (i) and (ii) we get

$$L.H.S = R.H.S$$

$$(AB)^t = B^t A^t$$

Hence proved

$$(ii) (AB)^{-1} = B^{-1} A^{-1}$$

$$\begin{aligned} |AB| &= \begin{vmatrix} 0 & 2 \\ 5 & 9 \end{vmatrix} \\ &= 0 \times 9 - 2 \times 5 \\ &= 0 - 10 \\ &= -10 \text{ (Non singular)} \end{aligned}$$

Inverse exists

$$Adj(AB) = \begin{bmatrix} 9 & -2 \\ -5 & 0 \end{bmatrix}$$

$$L.H.S = (AB)^{-1} = \frac{1}{|AB|} Adj(AB)$$

$$\begin{aligned} &= \frac{1}{-10} \begin{bmatrix} 9 & -2 \\ -5 & 0 \end{bmatrix} \\ &= \begin{bmatrix} \frac{9}{-10} & \frac{-2}{-10} \\ \frac{-5}{-10} & \frac{0}{-10} \end{bmatrix} \\ &= \begin{bmatrix} \frac{-9}{10} & \frac{1}{5} \\ \frac{1}{2} & 0 \end{bmatrix} \rightarrow (i) \end{aligned}$$

$$B = \begin{bmatrix} 2 & 4 \\ -3 & -5 \end{bmatrix}$$

$$\begin{aligned} |B| &= \begin{vmatrix} 2 & 4 \\ -3 & -5 \end{vmatrix} \\ &= 2(-5) - 4 \times (-3) \\ &= -10 + 12 \\ &= 2 \text{ (non singular)} \\ \therefore B^{-1} \text{ exists} \end{aligned}$$

$$\text{Adj}B = \begin{bmatrix} -5 & -4 \\ 3 & 2 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|} \text{Adj}B$$

$$= \frac{1}{2} \begin{bmatrix} -5 & -4 \\ 3 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & 2 \\ 1 & -1 \end{vmatrix}$$

$$= 3(-1) - 2 \times 1$$

$$= -3 - 2$$

$$= -5 \text{ (non singular)}$$

$$\therefore A^{-1} \text{ exists}$$

$$\text{Adj}A = \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{Adj}A$$

$$= \frac{1}{-5} \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$$

$$\text{R.H.S} = B^{-1}A^{-1}$$

$$= \left(\frac{1}{2} \begin{bmatrix} -5 & -4 \\ 3 & 2 \end{bmatrix} \right) \times \left(\frac{1}{-5} \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix} \right)$$

$$= \frac{1}{2} \left(-\frac{1}{5} \right) \begin{bmatrix} -5 & -4 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$$

$$= -\frac{1}{10} \begin{bmatrix} -5(-1) + (-4)(-1) & -5(-2) + (-4)(3) \\ 3(-1) + 2(-1) & 3(-2) + 2(3) \end{bmatrix}$$

$$= -\frac{1}{10} \begin{bmatrix} 5+4 & 10-12 \\ -3-2 & -6+6 \end{bmatrix}$$

$$= -\frac{1}{10} \begin{bmatrix} 9 & -2 \\ -5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{9}{-10} & \frac{-2}{-10} \\ \frac{-5}{-10} & \frac{0}{-10} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-9}{10} & \frac{1}{5} \\ \frac{1}{2} & 0 \end{bmatrix}$$

→ (ii)

From equation (i) and (ii) we get

$$\text{L.H.S} = \text{R.H.S}$$

$$(AB)^{-1} = B^{-1}A^{-1}$$

Hence proved

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Report any mistake at freeilm786@gmail.com