Unit 12: Line Bisectors and Angle Bisectors

Overview

Right Bisector of a line segment:

Right bisection of a line segment means to draw a perpendicular at the mid-point of line segment.

Bisector of an angle:

Bisection of an angle means to draw a ray to divide the given angle into two equal parts.

Theorem 12.1.1

Statement:

Any point on the right bisector of a line segment is equidistant from its end points.

Given

A line $\overleftrightarrow{LM}$ intersects the line segment AB at the point C Such that $\overleftrightarrow{LM} \perp \overline{AB}$ and $\overline{AC} \cong \overline{BC}$. P is a point on $\overleftrightarrow{LM}$
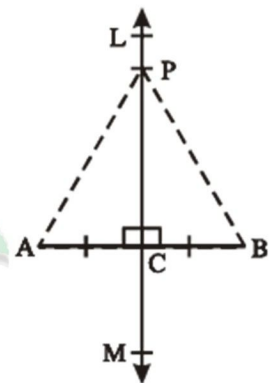
To prove

$\overline{PA} \cong \overline{PB}$

Construction

Join P to the point A and B

Proof

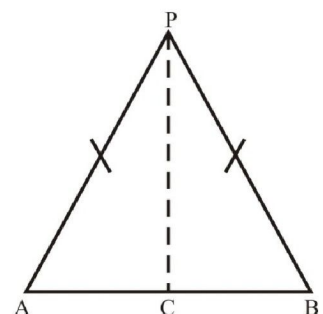| Statements	Reasons
In $\triangle ACP \leftrightarrow \triangle BCP$	
$\overline{AC} \cong \overline{BC}$	Given
$\angle ACP \cong \angle BCP$	Given $\overline{PC} \perp \overline{AB}$, so that each $\angle$ at $C = 90^o$
$\overline{PC} \cong \overline{PC}$	Common
$\therefore \triangle ACP \cong \triangle BCP$	S.A.S Postulate
Hence $\overline{PA} \cong \overline{PB}$	(Corresponding sides of congruent triangles)

Theorem 12.1.2

$\{$Converse of Theorem 12.1.1$\}$

Any point equidistant from the end points of a line segment is on the right bisector of it.

Given

$\overline{AB}$ is a line segment. Point P is such that $\overline{PA} \cong \overline{PB}$

To prove

The point P is the on the right bisector of $\overline{AB}$

Construction

Join P to C, the midpoint of $\overline{AB}$

Proof

Statements	Reasons
In $\triangle ACP \leftrightarrow \triangle BCP$	
$\overline{PA} \cong \overline{PB}$	Given
$\overline{PC} \cong \overline{PC}$	Common
$\overline{AC} \cong \overline{BC}$	Construction
$\therefore \triangle ACP \cong \triangle BCP$	$S.S.S \cong S.S.S$
$\angle ACP \cong \angle BCP$ _____ (i)	Corresponding angles of congruent triangles
But $m\angle ACP + m\angle BCP = 180^o$ ___ (ii)	Supplementary angles
$\therefore m\angle ACP = m\angle BCP = 90^o$	From (i) and (ii)
i.e $\overline{PC} \perp \overline{AB}$ _____ (iii)	$m\angle ACP = 90^o$ (Proved)
Also $\overline{CA} \cong \overline{CB}$ _____ (iv)	**Construction**
$\therefore \overline{PC}$ is a right bisector of $\overline{AB}$	from (iii) and (iv)
i.e. the point P is on the right bisector of $\overline{AB}$	
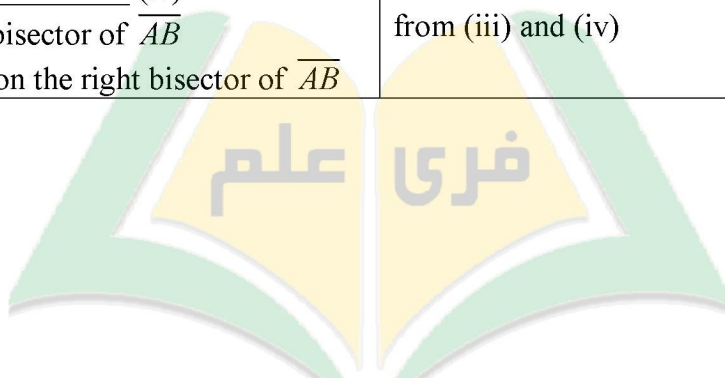
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