Exercise 5.2

Q.1 Factorize

(i) $x^4+\frac{1}{x^4}-3$

Solution: $x^4+\frac{1}{x^4}-3$

$=\left(x^2\right)^2+\left(\frac{1}{x^2}\right)^2-3$

By adding and subtracting by 2

$=(x^2)^2+\left(\frac{1}{x^2}\right)^2+2-2-3$

$=\left[(x^2)^2+\left(\frac{1}{x^2}\right)^2-2\right]+2-3$

$=\left[(x^2)^2+\left(\frac{1}{x^2}\right)^2-2\right]-1$

$=\left(x^2-\frac{1}{x^2}\right)^2-(1)^2$

$=\left(x^2-\frac{1}{x^2}+1\right)\left(x^2-\frac{1}{x^2}-1\right)$

(ii) $3x^4+12y^4$

Solution: $3x^4+12y^4$

$=3\left(x^4+4y^4\right)$

By adding and subtracting by $2(x^2)(2y^2)$

$=3\left[\left(x^2\right)^2+\left(2y^2\right)^2+2\left(x^2\right)\left(2y^2\right)-2\left(x^2\right)\left(2y^2\right)\right]$

$=3\left[\left(x^2\right)^2+\left(2y^2\right)^2+2\left(x^2\right)\left(2y^2\right)-2\left(x^2\right)\left(2y^2\right)\right]$

$=3\left[\left(x^2+2y^2\right)^2-4x^2y^2\right]$

$=3\left[\left(x^2+2y^2\right)^2-(2xy)^2\right]$

$=3\left[\left(x^2+2y^2+2xy\right)\left(x^2+2y^2-2xy\right)\right]$

$=3\left[\left(x^2+2xy+2y^2\right)\left(x^2-2xy+2y^2\right)\right]$

(iii) $a^4+3a^2b^2+4b^4$

Solution: $a^4+3a^2b^2+4b^4$

$=\left(a^4+4b^4\right)+3a^2b^2$

$=(a^2)^2+(2b^2)^2+3a^2b^2$

By adding and subtracting by $2\left(a^2\right)\left(2b^2\right)$

$=\left(a^2\right)^2+\left(2b^2\right)^2+2\left(a^2\right)\left(2b^2\right)-2\left(a^2\right)\left(2b^2\right)+3a^2b^2$

$=\left[\left(a^2\right)^2+\left(2b^2\right)^2+2\left(a^2\right)\left(2b^2\right)\right]-2\left(a^2\right)\left(2b^2\right)+3a^2b^2$

$=(a^2+2b^2)^2-a^2b^2$

$=(a^2+2b^2)^2-(ab)^2$

$=(a^2+2b^2+ab)(a^2+2b^2-ab)$

(iv) $4x^4+81$

Solution: $4x^4+81$

$=\left(2x^2\right)^2+(9)^2$

By adding and subtracting by $2\left(2x^2\right)(9)$

$=\left[\left(2x^2\right)^2+(9)^2+2\left(2x^2\right)(9)-2\left(2x^2\right)(9)\right]$

$=\left[\left(2x^2\right)^2+(9)^2+2\left(2x^2\right)(9)\right]-2\left(2x^2\right)(9)$

$=\left(2x^2+9\right)^2-36x^2$

$=\left(2x^2+9\right)^2-(6x)^2$

$=\left(2x^2+9+6x\right)\left(2x^2+9-6x\right)$

$=\left(2x^2+6x+9\right)\left(2x^2-6x+9\right)$

(v) x^4+x^2+25

Solution: x^4+x^2+25

$=\left(x^4+25\right)+x^2$

$=\left[\left(x^2\right)^2+(5)^2\right]+x^2$

By adding and subtracting by $2\left(x^2\right)(5)$

$=\left[\left(x^2\right)^2+(5)^2+2\left(x^2\right)(5)-2\left(x^2\right)(5)\right]+x^2$

$=\left[\left(x^2\right)^2+(5)^2+2\left(x^2\right)(5)\right]-2\left(x^2\right)(5)+x^2$

$=\left(x^2+5\right)^2-10x^2+x^2$

$=(x^2+5)^2-9x^2$

$= (x^2+5)^2 - (3x)^2$
$= \left(x^2+5+3x\right)\left(x^2+5-3x\right)$
$= (x^2+3x+5)(x^2-3x+5)$

(vi) x^4+4x^2+16

Solution: x^4+4x^2+16
$= (x^2)^2+16+4x^2$
$= (x^2)^2+(4)^2+4x^2$

By adding and subtracting by $2\left(x^2\right)(4)$

$= \left(x^2\right)^2+(4)^2+2\left(x^2\right)(4)-2\left(x^2\right)(4)+4x^2$
$= (x^2)^2+(4)^2+2(x^2)(4)-2(x^2)(4)+4x^2$
$= \left(x^2+4\right)^2-8x^2+4x^2$
$= (x^2+4)^2-4x^2$
$= (x^2+4)^2-(2x)^2$
$= \left(x^2+4+2x\right)\left(x^2+4-2x\right)$
$= \left(x^2+2x+4\right)\left(x^2-2x+4\right)$

Q.2 Factorize

(i) $x^2+14x+48$

Solution: $x^2+14x+48$
$= x^2+8x+6x+48$
$= x(x+8)+6(x+8)$
$= (x+8)(x+6)$

(ii) $x^2-21x+108$

Solution: $x^2-21x+108$
$= x^2-12x-9x+108$
$= x(x-12)-9(x-12)$
$= (x-9)(x-12)$

(iii) $x^2-11x-42$

Solution: $x^2-11x-42$
$= x^2-14x+3x-42$
$= x(x-14)+3(x-14)$
$= (x+3)(x-14)$

(iv) $x^2+x-132$

Solution: $x^2+x-132$
$= x^2+12x-11x-132$
$= x(x+12)-11(x+12)$
$= (x-11)(x+12)$

Q.3 Factorize

(i) $4x^2+12x+5$

Solution: $4x^2+12x+5$
$= 4x^2+2x+10x+5$
$= 2x(2x+1)+5(2x+1)$
$= (2x+5)(2x+1)$

(ii) $30x^2+7x-15$

Solution: $30x^2+7x-15$
$= 30x^2+25x-18x-15$
$= 5x(6x+5)-3(6x+5)$
$= (5x-3)(6x+5)$

(iii) $24x^2-65x+21$

Solution: $24x^2-65x+21$
$= 24x^2-56x-9x+21$
$= 8x(3x-7)-3(3x-7)$
$= (8x-3)(3x-7)$

(iv) $5x^2-16x-21$

Solution: $5x^2-16x-21$
$= 5x^2+5x-21x-21$
$= 5x\,(x+1)-21(x+1)$
$= (5x-21)(x+1)$

(v) $4x^2-17xy+4y^2$

Solution: $4x^2-17xy+4y^2$
$= 4x^2-16xy-xy+4y^2$
$= 4x(x-4y)-y(x-4y)$
$= (4x-y)(x-4y)$

(vi) $3x^2-38xy-13y^2$

Solution: $3x^2-38xy-13y^2$
$= 3x^2-39xy+xy-13y^2$
$= 3x(x-13y)+y(x-13y)$
$= (3x+y)(x-13y)$

(vii) $5x^2+33xy-14y^2$

Solution: $5x^2+33xy-14y^2$

$= 5x^2 + 35xy - 2xy - 14y^2$
$= 5x(x+7y) - 2y(x+7y)$
$= (5x-2y)(x+7y)$

(viii) $\left(5x-\frac{1}{x}\right)^2 + 4\left(5x-\frac{1}{x}\right) + 4, x \neq 0$

Solution: $\left(5x-\frac{1}{x}\right)^2 + 4\left(5x-\frac{1}{x}\right) + 4, x \neq 0$

$= \left(5x-\frac{1}{x}\right)^2 + 2\left(5x-\frac{1}{x}\right)(2) + (2)^2$

$= \left(5x-\frac{1}{x}+2\right)^2$

$= \left(5x-\frac{1}{x}+2\right)\left(5x-\frac{1}{x}+2\right)$

Q.4

(i) $(x^2+5x+4)(x^2+5x+6)-3$

Solution: $(x^2+5x+4)(x^2+5x+6)-3$

Suppose that

$x^2+5x = y$

So,

$(x^2+5x+4)(x^2+5x+6)-3$
$= (y+4)(y+6)-3$
$= [y(y+6)+4(y+6)-3$
$= (y^2+6y+4y+24)-3$
$= (y^2+10y+24)-3$
$= y^2+10y+24-3$
$= y^2+10y+21$
$= y^2+7y+3y+21$
$= y(y+7)+3(y+7)$
$= (y+3)(y+7)$

We know that $y = x^2+5x$

$= (x^2+5x+3)(x^2+5x+7)$

(ii) $(x^2-4x)(x^2-4x-1)-20$

Solution: $(x^2-4x)(x^2-4x-1)-20$

Suppose that

$x^2-4x = y$

So,

$= (y)(y-1)-20$
$= (y^2-y)-20$
$= y^2-y-20$
$= y^2-5y+4y-20$
$= y(y-5)+4(y-5)$
$= (y+4)(y-5)$

We know that $a = x^2-4x$

$= (x^2-4x+4)(x^2-4x-5)$

$= \left[(x)^2-2(x)(2)+(2)^2\right]\left[x^2-5x+x-5\right]$

$= (x-2)^2\left[x(x-5)+1(x-5)\right]$

$= (x-2)^2(x-5)(x+1)$

$= (x-5)(x+1)(x-2)^2$

(iii) $(x+2)(x+3)(x+4)(x+5)-15$

Solution: $(x+2)(x+3)(x+4)(x+5)-15$

$= [(x+2)(x+5)][(x+3)(x+4)]-15$
$= [x(x+5)+2(x+5)][x(x+4)+3(x+4)]-15$
$= [x^2+5x+2x+10][x^2+4x+3x+12]-15$
$= (x^2+7x+10)(x^2+7x+12)-15$

Suppose that

$x^2+7x = y$

So,

$(x^2+7x+10)(x^2+7x+12)-15$
$= (y+10)(y+12)-15$
$= [y(y+12)+10(y+12)]-15$
$= (y^2+12y+10y+120)-15$
$= (y^2+22y+120)-15$
$= y^2+22y+120-15$
$= y^2+22y+105$
$= y^2+15y+7y+105$
$= y(y+15)+7(y+15)$
$= y(y+15)+7(y+15)$
$= (y+7)(y+15)$

We know that $y = x^2+7x$

$= (x^2+7x+7)(x^2+7x+15)$

(iv) $(x+4)(x-5)(x+6)(x-7)-504$

Solution: $(x+4)(x-5)(x+6)(x-7)-504$

$=[(x+4)(x-5)][(x+6)(x-7)]-504$

$=[x(x-5)+4(x-5)][x(x-7)+6(x-7)]-504$

$=(x^2-5x+4x-20)(x^2-7x+6x-42)-504$

$=(x^2-x-20)(x^2-x-42)-504$

Suppose that

$x^2-x=y$

So,

$=(y-20)(y-42)-504$

$=[y(y-42)-20(y-42)]-504$

$=(y^2-42y-20y+840)-504$

$=y^2-62y+840-504$

$=y^2-62y+336$

$=y^2-56y-6y+336$

$=y(y-56)-6(y-56)$

$=(y-6)(y-56)$

We know that $a=x^2-x$

$=(x^2-x-6)(x^2-x-56)$

$=(x^2-3x+2x-6)(x^2-8x+7x-56)$

$=[x(x-3)+2(x-3)][x(x-8)+7(x-8)]$

$=(x+2)(x-3)(x+7)(x-8)$

(v) $(x+1)(x+2)(x+3)(x+6)-3x^2$

Solution: $(x+1)(x+2)(x+3)(x+6)-3x^2$

$=[(x+1)(x+6)][(x+2)(x+3)]-3x^2$

$=[x(x+6)+1(x+6)][x(x+3)+2(x+3)]-3x^2$

$=(x^2+6x+x+6)(x^2+3x+2x+6)-3x^2$

$=(x^2+6+7x)(x^2+6+5x)-3x^2$

Suppose that

$x^2+6=y$

So,

$=(y+7x)(y+5x)-3x^2$

$=[y(y+5x)+7x(y+5x)]-3x^2$

$=(y^2+5xy+7xy+35x^2-3x^2)$

$=y^2+12xy+32x^2$

$=y^2+8xy+4xy+32x^2$

$=y(y+8x)+4x(y+8x)$

$=(y+4x)(y+8)$

We know that $y=x^2+6$

$=(x^2+6+4x)(x^2+6+8x)$

$=(x^2+4x+6)(x^2+8x+6)$

Q.5

(i) $x^3+48x-12x^2-64$

Solution: $x^3+48x-12x^2-64$

$=x^3-12x^2+48x-64$

$a^3-3a^2b+3ab^2-b^3=(a-b)^3$

$=(x)^3-3(x)^2(4)+3(x)(4)^2-(4)^3$

$=(x-4)^3$

(ii) $8x^3+60x^2+150x+125$

Solution: $8x^3+60x^2+150x+125$

$a^3+3a^2b+3ab^2+b^3=(a+b)^3$

$=(2x)^3+3(2x)^2(5)+3(2x)(5)^2+(5)^3$

$=(2x+5)^3$

(iii) $x^3-18x^2+108x-216$

Solution: $x^3-18x^2+108x-216$

$a^3-3a^2b+3ab^2-b^3=(a-b)^3$

$=(x)^3-3(x)^2(6)+3(x)(6)^2-(6)^3$

$=(x-6)^3$

(iv) $8x^3-125y^3-60x^2y+150xy^2$

Solution: $8x^3-125y^3-60x^2y+150xy^2$

$=8x^3-60x^2y+150xy^2-125y^3$

$a^3-3a^2b+3ab^2-b^3=(a-b)^3$

$=(2x)^3-3(2x)^2(5y)+3(2x)(5y)^2-(5y)^3$

$=(2x-5y)^3$

Q.6

(i) $27+8x^3$

Solution: $27+8x^3$

$=(3)^3+(2x)^3$

$=(3+2x)[(3)^2-(3)(2x)+(2x)^2]$

$=(3+2x)(9-6x+4x^2)$

(ii) $125x^3-216y^3$

Solution: $125x^3-216y^3$

$=(5x)^3-(6y)^3$

$(a-b)(a^2+ab+b^2)=a^3-b^3$

$=(5x-6y)[(5x)^2+(5x)(6y)+(6y)^2]$

$= (5x-6y)(25x^2+30xy+36y^2)$

(iii) $64x^3+27y^3$

Solution: $64x^3+27y^3$

$= (4x)^3+(3y)^3$

$(a+b)(a^2+ab+b^2) = a^3+b^3$

$= (4x+3y)\left[(4x)^2-(4x)(3y)+(3y)^2\right]$

$= (4x+3y)(16x^2-12xy+9y^2)$

(iv) $(2x)^3+(5y)^3$

Solution: $(2x)^3+(5y)^3$

$(a-b)(a^2+ab+b^2) = a^3-b^3$

$= (2x+5y)\left[(2x)^2-(2x)(5y)+(5y)^2\right]$

$= (2x+5y)(4x^2-10xy+25y^2)$

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Report any mistake at freeilm786@gmail.com