Exercise 1.6

Q.1 Use of matrices, if possible to solve the following systems of linear equations.

(i) The matrices inversion method

(ii) The Cramer's rule

(i) $2x-2y=4$

$3x+2y=6$

By matrices inversion method

$$\begin{bmatrix}2 & -2\\3 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}4\\6\end{bmatrix}$$

Let $A=\begin{bmatrix}3 & -2\\3 & 2\end{bmatrix}X=\begin{bmatrix}x\\y\end{bmatrix}B=\begin{bmatrix}4\\6\end{bmatrix}$

$$A=\begin{bmatrix}2 & -2\\3 & 2\end{bmatrix}$$

$$|A|=\begin{vmatrix}2 & -2\\3 & 2\end{vmatrix}$$

$$|A|=(2)(2)-(-2)(3)$$

$$|A|=4+6$$

$$|A|=10$$

Then, solution is possible because A is non-singular matrix.

$$AdjA=\begin{bmatrix}2 & 2\\-3 & 2\end{bmatrix}$$

As we know that

$$AX=B$$

$$X=A^{-1}B$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{|A|}\times AdjA\times B$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{10}\begin{bmatrix}2 & 2\\-3 & 2\end{bmatrix}\begin{bmatrix}4\\6\end{bmatrix}$$

$$=\frac{1}{10}\begin{bmatrix}2\times4+2\times6\\-3\times4+2\times6\end{bmatrix}$$

$$=\frac{1}{10}\begin{bmatrix}8+12\\-12+12\end{bmatrix}$$

$$=\frac{1}{10}\begin{bmatrix}20\\0\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}\frac{20}{10}\\\frac{0}{10}\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}2\\0\end{bmatrix}$$

$$x=2, y=0$$

Solution Set = {(2, 0)}

By Cramer's rule

$$A=\begin{bmatrix}2 & -2\\3 & 2\end{bmatrix}B=\begin{bmatrix}4\\6\end{bmatrix}$$

$$|A|=\begin{vmatrix}2 & -2\\3 & 2\end{vmatrix}$$

$$=(2)(2)-(-2)(3)$$

$$=4-(-6)$$

$$=4+6$$

$$=10$$

$$|A_x|=\begin{vmatrix}4 & -2\\6 & 2\end{vmatrix}$$

$$=(4)(2)-(-2)(6)$$

$$=8+12$$

$$=20$$

$$|A_y|=\begin{vmatrix}2 & 4\\3 & 6\end{vmatrix}$$

$$=(2)(6)-(4)(3)$$

$$=12-12$$

$$=0$$

$$x=\frac{|A_x|}{|A|}$$

$$x=\frac{20}{10}$$

$$x=2$$

$$y=\frac{|A_y|}{|A|}$$

$$y=\frac{0}{10}$$
$$y=0$$

Solution Set = {(2, 0)}

(ii) $2x+y=3$

$6x+5y=1$

Matrices inversion method

$$\begin{bmatrix}2 & 1\\6 & 5\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}3\\1\end{bmatrix}$$

Let $A=\begin{bmatrix}3 & -2\\3 & 2\end{bmatrix} X=\begin{bmatrix}x\\y\end{bmatrix} B=\begin{bmatrix}4\\6\end{bmatrix}$

$$A=\begin{bmatrix}2 & 1\\6 & 5\end{bmatrix}$$
$$|A|=\begin{vmatrix}2 & 1\\6 & 5\end{vmatrix}$$
$$=(2)(5)-(1)(6)$$
$$=10-6$$
$$=4$$

Solution is possible because A is non-singular matrix.

$$AdjA=\begin{bmatrix}5 & -1\\-6 & 2\end{bmatrix}$$
$$AX=B$$
$$X=A^{-1}B$$
$$X=\frac{1}{|A|}\times AdjA\times B$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}5 & -1\\-6 & 2\end{bmatrix}\begin{bmatrix}3\\1\end{bmatrix}$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}5\times3+(-1\times1)\\-6\times3+2\times1\end{bmatrix}$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}15+(-1)\\-18+2\end{bmatrix}$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}14\\-16\end{bmatrix}$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}\frac{14}{4}\\\frac{16}{4}\end{bmatrix}$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}\frac{7}{2}\\-4\end{bmatrix}$$
$$x=\frac{7}{2}, y=-4$$

Solution Set = $\{(\frac{7}{2},-4)\}$

By Cramer's Rule

$A=\begin{bmatrix}2 & 1\\6 & 5\end{bmatrix} B=\begin{bmatrix}3\\1\end{bmatrix}$

$$|A|=\begin{vmatrix}2 & 1\\6 & 5\end{vmatrix}$$
$$=(2)(5)-(1)(6)$$
$$=10-6$$
$$=4$$

Solution is possible because A is non-singular matrix.

$$|A_x|=\begin{vmatrix}3 & 1\\1 & 5\end{vmatrix}$$
$$=(3)(5)-(1)(1)$$
$$=15-1$$
$$=14$$
$$|A_y|=\begin{vmatrix}2 & 3\\6 & 1\end{vmatrix}$$
$$=(2)(1)-(3)(6)$$
$$=2-18$$
$$=-16$$
$$x=\frac{|A_x|}{|A|}$$
$$x=\frac{14}{4}$$
$$x=\frac{7}{2}$$
$$y=\frac{|A_y|}{|A|}$$
$$y=\frac{16}{4}$$
$$y=-4$$

Solution Set $=\left\{\left(\frac{7}{2},-4\right)\right\}$

(iii) $4x+2y=8$

$3x-y=-1$

By Matrices Inversion Method

$$\begin{bmatrix}4 & 2\\3 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}8\\-1\end{bmatrix}$$

Let $A=\begin{bmatrix}3 & -2\\3 & 2\end{bmatrix} X=\begin{bmatrix}x\\y\end{bmatrix} B=\begin{bmatrix}4\\6\end{bmatrix}$

$$|A|=\begin{vmatrix}4 & 2\\3 & -1\end{vmatrix}$$

$$=(4)(-1)-(2)(3)$$

$$=-4-6$$

$$=-10$$

Solution is possible because A is non singular matrix.

$$AdjA=\begin{bmatrix}-1 & -2\\-3 & 4\end{bmatrix}$$

As we know that

$$AX=B$$

$$X=A^{-1}B$$

$$X=A^{-1}B$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{|A|}\times AdjA\times B$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{-10}\begin{bmatrix}-1 & -2\\-3 & 4\end{bmatrix}\begin{bmatrix}8\\-1\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{-10}\begin{bmatrix}-8+2\\-24+(-4)\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{-10}\begin{bmatrix}-6\\-28\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}\frac{-6}{-10}\\\frac{-28}{-10}\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}\frac{3}{5}\\\frac{14}{5}\end{bmatrix}$$

$$x=\frac{3}{5}, y=\frac{14}{5}$$

Solution Set $=\left\{\left(\frac{3}{5},\frac{14}{5}\right)\right\}$

By Cramer's rule

$$A=\begin{bmatrix}4 & 2\\3 & -1\end{bmatrix} B=\begin{bmatrix}8\\-1\end{bmatrix}$$

$$|A|=\begin{vmatrix}4 & 2\\3 & -1\end{vmatrix}$$

$$=(4)(-1)-(2)(3)$$

$$=-4-6$$

$$=-10$$

Solution is possible because A is non singular matrix.

$$|A_x|=\begin{vmatrix}8 & 2\\-1 & -1\end{vmatrix}$$

$$=(8)(-1)-(2)(-1)$$

$$=-8-(-2)$$

$$=-6$$

$$x=\frac{|A_x|}{|A|}$$

$$x=\frac{6}{10}$$

$$x=\frac{3}{5}$$

$$|A_y|=\begin{vmatrix}4 & 8\\3 & -1\end{vmatrix}$$

$$=(4)(-1)-(8)(3)$$

$$=-4-24$$

$$=-28$$

$$y=\frac{|A_y|}{|A|}$$

$$y=\frac{28}{10}$$

$$y=\frac{14}{5}$$

Solution Set $=\left\{\left(\frac{3}{5},\frac{14}{5}\right)\right\}$

(iv) $3x$-$2y$=-6

$5x$-$2y$=-10

By Matrices Inversion Method

$$\begin{bmatrix}3 & -2\\5 & -2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-6\\-10\end{bmatrix}$$

Let $A=\begin{bmatrix}3 & -2\\3 & 2\end{bmatrix} X=\begin{bmatrix}x\\y\end{bmatrix} B=\begin{bmatrix}4\\6\end{bmatrix}$

$$|A|=\begin{vmatrix}3 & -2\\5 & -2\end{vmatrix}$$
$$=(3)(-2)-(-2)(5)$$
$$=-6-(-10)$$
$$=-6+10$$
$$=4$$
Solution is possible because A is non singular matrix.
$$AdjA=\begin{bmatrix}-2 & 2\\-5 & 3\end{bmatrix}$$
$$X=A^{-1}B$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{|A|}\times AdjA\times B$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}-2 & 2\\-5 & 3\end{bmatrix}\begin{bmatrix}-6\\-10\end{bmatrix}$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}-2\times-6+2\times-10\\-5\times-63\times-10\end{bmatrix}$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}12+(-20)\\30+(-30)\end{bmatrix}$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}12-20\\30-30\end{bmatrix}$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}-8\\0\end{bmatrix}$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}\frac{-8}{4}\\\frac{0}{4}\end{bmatrix}$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-2\\0\end{bmatrix}$$
$x=-2$, $y=0$

Solution Set $=\{(-2,0)\}$

By Cramer's rule
$$A=\begin{bmatrix}3 & -2\\5 & -2\end{bmatrix}, B=\begin{bmatrix}-6\\-10\end{bmatrix}$$
$$|A|=\begin{vmatrix}3 & -2\\5 & -2\end{vmatrix}$$
$$=(3)(-2)-(-2)(5)$$
$$=-6-(-10)$$
$$=-6+10$$
$$=4$$
Solution is possible because A is non singular matrix.
$$|A_x|=\begin{vmatrix}-6 & -2\\-10 & -2\end{vmatrix}$$
$$=(-6)(-2)-(-2)(-10)$$
$$=+12-(+20)$$
$$=12-20$$
$$=-8$$
$$|A_y|=\begin{vmatrix}3 & -6\\5 & -10\end{vmatrix}$$
$$=(3)(-10)-(-6)(5)$$
$$=-30-(-30)$$
$$=-30+30$$
$$=0$$
$$x=\frac{|A_x|}{|A|}$$
$$x=\frac{-8}{4}$$
$$x=-2$$
$$y=\frac{|A_y|}{|A|}$$
$$=\frac{0}{4}$$
$$y=0$$
Solution Set $=\{(-2,0)\}$

(v) $3x-2y=4$
$-6x+4y=7$

By Matrices Inversion Method
$$\begin{bmatrix}3 & -2\\-6 & 4\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}4\\7\end{bmatrix}$$
$$A=\begin{bmatrix}3 & -2\\3 & 2\end{bmatrix} X=\begin{bmatrix}x\\y\end{bmatrix} B=\begin{bmatrix}4\\6\end{bmatrix}$$
$$|A|=\begin{vmatrix}3 & -2\\-6 & 4\end{vmatrix}$$
$$=(3)(4)-(-2)(-6)$$
$$=12-(+12)$$
$$=12-12$$
$$=0$$
Solution is not possible because A is singular matrix.

(vi) $4x+y=9$

$-3x-y=-5$

By Matrices Inversion Method

$$\begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

Let $A=\begin{bmatrix} 3 & -2 \\ 3 & 2 \end{bmatrix} X=\begin{bmatrix} x \\ y \end{bmatrix} B=\begin{bmatrix} 4 \\ 6 \end{bmatrix}$

$$|A|=\begin{vmatrix} 4 & 1 \\ -3 & -1 \end{vmatrix}$$

$=(4)(-1)-(-1)(-3)$

$=-4+3$

$=-1$

Solution is possible because $|A|$ is non singular

$$AdjA=\begin{vmatrix} -1 & -1 \\ 3 & 4 \end{vmatrix}$$

As we know that

$X=A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\frac{1}{|A|}\times AdjA\times B$$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\frac{1}{-1}\begin{bmatrix} -1 & -1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\frac{1}{-1}\begin{bmatrix} -9+5 \\ 27+(-20) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\frac{1}{-1}\begin{bmatrix} -4 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} \frac{-4}{-1} \\ \frac{7}{-1} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 4 \\ -7 \end{bmatrix}$$

$x=4, y=-7$

Solution Set$=\{(4,-7)\}$

By Cramer's rule

$A=\begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix}, B=\begin{bmatrix} 9 \\ -5 \end{bmatrix}$

$$|A|=\begin{vmatrix} 4 & 1 \\ -3 & -1 \end{vmatrix}$$

$=(4)(-1)-(1)(-3)$

$=-4-(-3)$

$=-4+3$

$=-1$

$$|A_x|=\begin{vmatrix} 9 & 1 \\ -5 & -1 \end{vmatrix}$$

$=(9)(-1)-(1)(-3)$

$=-9-(-5)$

$=-9+5$

$=-4$

$$x=\frac{|A_x|}{|A|}$$

$$=\frac{-4}{-1}$$

$x=4$

$$|A_y|=\begin{bmatrix} 4 & 9 \\ -3 & -5 \end{bmatrix}$$

$=(4)(-5)-(9)(-3)$

$=-20-(-27)$

$=--20+27$

$=7$

$$y=\frac{|A_y|}{|A|}$$

$$=\frac{7}{-1}$$

$y=-7$

Solution Set$=\{(4_7,-7)\}$

(vii) $2x-2y=4$

$-5x-2y=-10$

By Matrices Inversion Method

$$\begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

Let $A=\begin{bmatrix} 3 & -2 \\ 3 & 2 \end{bmatrix} X=\begin{bmatrix} x \\ y \end{bmatrix} B=\begin{bmatrix} 4 \\ 6 \end{bmatrix}$

$$|A|=\begin{vmatrix} 2 & -2 \\ -5 & -2 \end{vmatrix}$$

$=(2)(-2)-(-2)(-5)$

$=-4-(+10)$

$=-4-10$

$=-14$

Solution is possible

$$AdjA=\begin{bmatrix}-2 & 2\\ 5 & 2\end{bmatrix}$$

As we know that

$$X=A^{-1}B$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{|A|}\times AdjA\times B$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{-14}\begin{bmatrix}-2 & 2\\ 5 & 2\end{bmatrix}\begin{bmatrix}4\\ -10\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{-14}\begin{bmatrix}-8+(-20)\\ 20+(-20)\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{-14}\begin{bmatrix}-8-20\\ 20-20\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{-14}\begin{bmatrix}-28\\ 0\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}\frac{-28}{-14}\\ \frac{0}{14}\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}2\\ 0\end{bmatrix}$$

$x=2, y=0$

Solution Set $=\{(2,0)\}$

By Cramer's rule

$$A=\begin{bmatrix}2 & -2\\ -5 & -2\end{bmatrix} B=\begin{bmatrix}4\\ -10\end{bmatrix}$$

$$|A|=\begin{vmatrix}2 & -2\\ -5 & -2\end{vmatrix}$$

$$=(2)(-2)-(-2)(-5)$$
$$=-4-(+10)$$
$$=-4-10$$
$$=-14$$

Set is possible

$$|A_x|=\begin{bmatrix}4 & -2\\ -10 & -2\end{bmatrix}$$

$$=(4)(-2)-(-2)(-10)$$
$$=-8-(+20)$$
$$=-8-20$$
$$=-28$$

$$|A_y|=\begin{bmatrix}2 & 4\\ -5 & -10\end{bmatrix}$$

$$=(2)(-10)-(4)(-5)$$
$$=-20-(-20)$$
$$=-20+20$$
$$=0$$

$$x=\frac{|A_x|}{|A|}$$

$$x=\frac{-28}{-14}$$

$x=2$

Solution Set $=\{(2,0)\}$

(viii) $3x-4y=4$

$x+2y=8$

By Matrices Inversion Method

$$\begin{bmatrix}3 & -4\\ 1 & 2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}4\\ 8\end{bmatrix}$$

Let $A=\begin{bmatrix}3 & -2\\ 3 & 2\end{bmatrix} X=\begin{bmatrix}x\\ y\end{bmatrix} B=\begin{bmatrix}4\\ 6\end{bmatrix}$

$$|A|=\begin{vmatrix}3 & -4\\ 1 & 2\end{vmatrix}$$

$$=(3)(2)-(-4)(1)$$
$$=6-(-4)$$
$$|A|=6+4$$
$$=10$$

Solution is possible because A is non singular matrix.

$$AdjA=\begin{bmatrix}2 & 4\\ -1 & 3\end{bmatrix}$$

As we know that

$$AX=B$$
$$X=A^{-1}B$$
$$X=\frac{1}{|A|}\times AdjA\times B$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{10}\begin{bmatrix}2 & 4\\ -1 & 3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{10}\begin{bmatrix}2\times4+4\times8\\ -1\times+3\times8\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{10}\begin{bmatrix}8+32\\ -4+24\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{10}\begin{bmatrix}40\\ 20\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}\frac{40}{10}\\ \frac{20}{10}\end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$x=4, y=2$

Solution Set $=\{(4,2)\}$

By Cramer's rule

$$A=\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} B=\begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

$$|A|=\begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix}$$

$=(3)(2)-(-4)(1)$

$=6-(-4)$

$=6+4$

$=10$

Solution is possible

$$|A_x|=\begin{bmatrix} 4 & -4 \\ 8 & 2 \end{bmatrix}$$

$$|A_x|=\begin{vmatrix} 4 & -4 \\ 8 & 2 \end{vmatrix}$$

$=(4)(2)-(-4)(8)$

$=8-(-32)$

$=8+32$

$=40$

$$|A_y|=\begin{bmatrix} 3 & 4 \\ 1 & 8 \end{bmatrix}$$

$$|A_y|=\begin{vmatrix} 3 & 4 \\ 1 & 8 \end{vmatrix}$$

$=(3)(8)-(4)(1)$

$=24-4$

$=20$

$$x=\frac{|A_x|}{|A|}$$

$$x=\frac{40}{10}$$

$x=4$

$$y=\frac{|A_y|}{|A|}$$

$$y=\frac{20}{10}$$

$y=2$

Solution Set $=\{(4,2)\}$

Q.2 The length of a rectangle is 4 times it width. The perimeter of the rectangle is 150cm. Find the dimensions of the rectangle.

Solution:

Let width of rectangle $=x$

Length of rectangle $=y$

According to 1st condition

$y=4x$

$-4x+y=0 \qquad \rightarrow ...(i)$

According to 2nd condition

2(length + Width)=Perimeter

$2(y+x)=150$

$$y+x=\frac{\cancel{150}^{75}}{\cancel{2}}$$

$x+y=75 \qquad \rightarrow ...(ii)$

$-4x+y=0$

$x+y=75$

Changing into matrix form

$$\begin{bmatrix} -4 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 0 \\ 75 \end{bmatrix}$$

$X=A^{-1}B$

(By matrix inversion method)

Let $A=\begin{bmatrix} -4 & 1 \\ 1 & 1 \end{bmatrix} X=\begin{bmatrix} x \\ y \end{bmatrix} B=\begin{bmatrix} 0 \\ 75 \end{bmatrix}$

$$|A|=\begin{vmatrix} -4 & 1 \\ 1 & 1 \end{vmatrix}$$

$=(-4)(1)-(1)(1)$

$=-4-1$

$=-5$

$$AdjA=\begin{bmatrix} 1 & -1 \\ -1 & -4 \end{bmatrix}$$

As we know that

$X=A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\frac{1}{|A|}\times AdjA\times B$$

$$=-\frac{1}{5}\begin{bmatrix} 1 & -1 \\ -1 & -4 \end{bmatrix}\begin{bmatrix} 0 \\ 75 \end{bmatrix}$$

$$=-\frac{1}{5}\begin{bmatrix} 0-75 \\ 0-300 \end{bmatrix}$$

$$=\frac{1}{-5}\begin{bmatrix} -75 \\ -300 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{-75}{-5} \\ \frac{-300}{-5} \end{bmatrix}$$

$$= \begin{bmatrix} 15 \\ 60 \end{bmatrix}$$

$x = 15, y = 60$

Width of rectangle = x = 15cm

Length of rectangle = y = 60cm

By Cramer's rule

$$A = \begin{bmatrix} -4 & 1 \\ 1 & 1 \end{bmatrix} B = \begin{bmatrix} 0 \\ 75 \end{bmatrix}$$

$$|A| = \begin{vmatrix} -4 & 1 \\ 1 & 1 \end{vmatrix}$$

$$= (-4)(1) - (1)(1)$$

$$= -4 - 1$$

$$= -5$$

$$|A_x| = \begin{vmatrix} 0 & 1 \\ 75 & 1 \end{vmatrix}$$

$$= (0)(1) - (1)(75)$$

$$= 0 - 75$$

$$= -75$$

$$|A_y| = \begin{vmatrix} -4 & 0 \\ 1 & 75 \end{vmatrix}$$

$$= (-4)(75) - (0)(1)$$

$$= 0 - 300$$

$$= -300$$

$$x = \frac{|A_x|}{|A|}$$

$$= \frac{-75}{-5}$$

$$x = 15$$

$$y = \frac{|A_y|}{|A|}$$

$$= \frac{-300}{-5}$$

$$y = 60$$

Then

Width of rectangle = x = 15 cm

Length of rectangle = y = 60 cm

Q.3 Two sides of a rectangle differ by 3.5cm. Find the dimension of the rectangle if its perimeter is 67cm.

Solution:

Suppose Width of rectangle = x

Length of rectangle = y

According to 1st condition

$$y - x = 3.5$$

$$-x + y = 3.5 \quad \rightarrow \ldots(\text{i})$$

According to 2nd condition

$$2(L + B) = P$$

$$2(y + x) = 67$$

$$x + y = \frac{67}{2}$$

$$x + y = 33.5 \quad \rightarrow (\text{ii})$$

Changing into matrix form

$$\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3.5 \\ 33.5 \end{bmatrix}$$

(By matrix inversion method)

Let $A = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} X = \begin{bmatrix} x \\ y \end{bmatrix} B = \begin{bmatrix} 3.5 \\ 33.5 \end{bmatrix}$

$$|A| = \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix}$$

$$= (-1)(1) - (1)(1)$$

$$= -1 - 1$$

$$= -2$$

$$AdjA = \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}$$

As we know that

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|A|} \times AdjA \times B$$

$$= \frac{1}{-2} \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 3.5 \\ +33.5 \end{bmatrix}$$

$$= \frac{1}{-2} \begin{bmatrix} 1\times 3.5 & 1\times 33.5 \\ -1\times 3.5 & 1\times 33.5 \end{bmatrix}$$

$$= \frac{1}{-2} \begin{bmatrix} 3.5(-33.5) \\ -3.5(-33.5) \end{bmatrix}$$

$$= \frac{1}{-2} \begin{bmatrix} -30 \\ -37 \end{bmatrix}$$

$$=\begin{bmatrix}\frac{30}{2}\\ \frac{37}{2}\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}15\\ \frac{37}{2}\end{bmatrix}$$

$$x=15, y=\frac{37}{2}=18.5$$

By Cramer's rule

$$A=\begin{bmatrix}-1 & 1\\ 1 & 1\end{bmatrix} B=\begin{bmatrix}3.5\\ 33.5\end{bmatrix}$$

$$|A|=\begin{vmatrix}-1 & 1\\ 1 & 1\end{vmatrix}$$

$$=(-1)(1)-(1)(1)$$
$$=-1-1$$
$$=-2$$

$$|A_x|=\begin{vmatrix}3.5 & 1\\ 33.5 & 1\end{vmatrix}$$

$$=(3.5)(1)-(1)(33.5)$$
$$=3.5-33.5$$
$$=-30$$

$$|A_y|=\begin{vmatrix}-1 & 3.5\\ 1 & 33.5\end{vmatrix}$$

$$=(-1)(33.5)-(3.5)(1)$$
$$=-33.5-3.5$$
$$=-37$$

$$x=\frac{|A_x|}{|A|}$$
$$x=\frac{-30}{-2}$$
$$x=15$$
$$y=\frac{|A_y|}{|A|}$$
$$y=\frac{-37}{-2}$$
$$y=\frac{37}{2}=18.5$$

Width of rectangle =x=15cm
Length of rectangle =y=18.5cm

Q.4 The third angle of an isosceles Δ is 16° less than the sum of two equal angles. Find three angles of the triangle.

Solution:

Let each equal angles are x and third angle is y

According to condition $y=2x-16$

$2x-y=16$ (i)

As we know that

$x+x+y=180$

$2x+y=180$ (ii)

$2x-y=16$

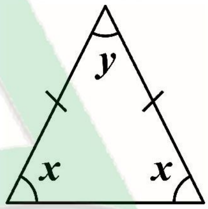
$2x+y=180$

Changing into matrix form

$$\begin{bmatrix}2 & -1\\ 2 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}16\\ 180\end{bmatrix}$$

$X=A^{-1}B$

Let $A=\begin{bmatrix}2 & -1\\ 2 & 1\end{bmatrix} X=\begin{bmatrix}x\\ y\end{bmatrix} B=\begin{bmatrix}16\\ 180\end{bmatrix}$



$X=A^{-1}B$

Where

$$A=\begin{bmatrix}2 & -1\\ 2 & 1\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}, B=\begin{bmatrix}16\\ 180\end{bmatrix}$$

$$|A|=\begin{vmatrix}2 & -1\\ 2 & 1\end{vmatrix}$$

$$=2\times1-(-1)\times2$$
$$=2+2$$

$=4\neq0$ (None singular)

A^{-1} exist

$$AdjA=\begin{bmatrix}1 & 1\\ -2 & 2\end{bmatrix}$$

$$X=A^{-1}B$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}1 & 1\\ -2 & 2\end{bmatrix}\begin{bmatrix}16\\ 180\end{bmatrix}$$

$$=\frac{1}{4}\begin{bmatrix}1\times16+1\times180\\ -2\times16+2\times180\end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}16+180\\-32+360\end{bmatrix}$$

$$=\begin{bmatrix}196\\328\end{bmatrix}$$

$$=\begin{bmatrix}\frac{196}{4}\\\frac{328}{4}\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}49\\82\end{bmatrix}$$

$x = 49$

$y = 82$

Cramer Rule

$$A=\begin{bmatrix}2 & -1\\2 & 1\end{bmatrix} B=\begin{bmatrix}16\\180\end{bmatrix}$$

$$|A|=\begin{vmatrix}2 & -1\\2 & 1\end{vmatrix}$$

$=(2)(1)-(-1)(2)$

$=2-(-2)$

$=2+2$

$=4$

$$|A_x|=\begin{bmatrix}16 & -1\\180 & 1\end{bmatrix}$$

$=(16)(1)-(-1)(180)$

$=16+180$

$=196$

$$|A_y|=\begin{bmatrix}2 & 16\\2 & 180\end{bmatrix}$$

$=(2)(180)-(16)(2)$

$=360-32$

$=328$

$$x=\frac{|A_x|}{|A|}$$

$$x=\frac{196}{4}$$

$x = 49$

$$y=\frac{|A_y|}{|A|}$$

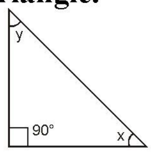
$$y=\frac{328}{4}$$

$y=82$

1^{st} angle = x = 49° Ans
2^{nd} angle= x = 49° Ans
3^{rd} angle =y= 82° Ans

Q.5 One acute angle of a right triangle is 12° more than twice the other acute angle. Find the acute angles of the right triangle.

Solution:



Let one acute angle =x
And other acute angle =y
According to 1^{st} condition

$x=2y+12$

$x-2y=12 \qquad \rightarrow(\text{i})$

As we know

$x+y=90 \qquad \rightarrow(\text{ii})$

By matrices inversion method
Changing into matrix form

$$\begin{bmatrix}1 & -2\\1 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}12\\90\end{bmatrix}$$

$X = A^{-1}B$

Let $A=\begin{bmatrix}1 & -2\\1 & 1\end{bmatrix} X=\begin{bmatrix}x\\y\end{bmatrix} B=\begin{bmatrix}12\\90\end{bmatrix}$

$$|A|=\begin{vmatrix}1 & -2\\1 & 1\end{vmatrix}$$

$=(1)(1)-(-2)(1)$

$=1-(-2)$

$=3$ (Non singular)

$\therefore A^{-1}$ exists

$$AdjA \quad =\begin{bmatrix}1 & 2\\-1 & 1\end{bmatrix}$$

As we know that

$X = A^{-1}B$ or

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{|A|}\times AdjA\times B$$

$$=\frac{1}{3}\begin{bmatrix}1 & 2\\-1 & 1\end{bmatrix}\begin{bmatrix}12\\90\end{bmatrix}$$

$$=\frac{1}{3}\begin{bmatrix}12+180\\-12+90\end{bmatrix}$$

$$=\frac{1}{3}\begin{bmatrix}192\\78\end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{192}{3} \\ \frac{78}{3} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 64 \\ 26 \end{bmatrix}$$

$x = 64, y = 26$

Then

1^{st} angle = x = 64°

2^{nd} angle = y = 26°

By Cramer's rule

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix} B = \begin{bmatrix} 12 \\ 90 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -2 \\ 1 & 1 \end{vmatrix}$$

$= (1)(1) - (-2)(1)$

$= 1 - (-2)$

$= 1 + 2$

$= 3$

$$|A_x| = \begin{vmatrix} 12 & -2 \\ 90 & 1 \end{vmatrix}$$

$= (12)(1) - (-2)(90)$

$= 12 + 180$

$= 192$

$$|A_y| = \begin{vmatrix} 1 & 12 \\ 1 & 90 \end{vmatrix}$$

$= (90) - (12)$

$= 90 - 12$

$= 78$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{192}{3}$$

$x = 64$

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{78}{3}$$

$y = 26$

1^{st} angle = x = 64°

2^{nd} angle = y = 26°

Q.6 Two cars that are 600 km apart are moving towards each other. Their speeds differ by 6 km per hour and the cars are 123 km apart after $4\frac{1}{2}$ hours. Find the speed of each car.

Solution:

Suppose speed of 1^{st} car = x

Suppose speed of 2^{nd} car = y

According to 1^{st} condition

$x - y = 6 \quad \rightarrow (i)$

According to 2^{nd} condition

Total distance = 600 km

Left distance = 123 km

Covered distance = total distance-left distance

Covered distance = 600-123

= 477 km

Total time = $4\frac{1}{2}$ hours =or $\frac{9}{2}$ hours

$$\text{Total Speed} = \frac{\text{Total Distance Covered}}{\text{Total Time Taken}}$$

$$x + y = \frac{477}{\frac{9}{2}} = 477 \div \frac{9}{2} = 477 \times \frac{2}{9}$$

$$x + y = \frac{\cancel{477}^{53} \times 2}{\cancel{9}}$$

$x + y = 106 \quad \rightarrow (ii)$

x - y = 6

x + y = 106

By matrices inversion method

Changing into matrix form

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 106 \end{bmatrix}$$

$X = A^{-1}B$, where

Let $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 6 \\ 106 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}$$

$$= (1)(1) - (-1)(1)$$

$$= 1 - (-1)$$

$$= 1 + 1$$

$$= 2$$

$$AdjA = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|A|} \times AdjA \times B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 106 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 6+106. \\ -6+106 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 112 \\ 100 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{112}{2} \\ \frac{100}{2} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 56 \\ 50 \end{bmatrix}$$

$$x = 56, y = 50$$

Speed of 1st car = x= 56km/h

Speed of 2nd car = y= 50km/h

By Cramer's rule

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} B = \begin{bmatrix} 6 \\ 106 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}$$

$$= (1)(1) - (-1)(1)$$

$$= 1 - (-1)$$

$$= 1 + 1$$

$$= 2$$

$$|A_x| = \begin{vmatrix} 6 & -1 \\ 106 & 1 \end{vmatrix}$$

$$= (6)(1) - (-1)(106)$$

$$= 6 - (-106)$$

$$= 6 + 106$$

$$= 112$$

$$|A_y| = \begin{vmatrix} 1 & 6 \\ 1 & 106 \end{vmatrix}$$

$$= (106)(1) - (6)(1)$$

$$= 106 - 6$$

$$= 100$$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{112}{2}$$

$$x = 56$$

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{100}{2}$$

$$y = 50$$

Then

Speed of 1st car = x = 56km/h

Speed of 2nd car = y = 50km/h

Last Updated: September 2020

Report any mistake at freeilm786@gmail.com