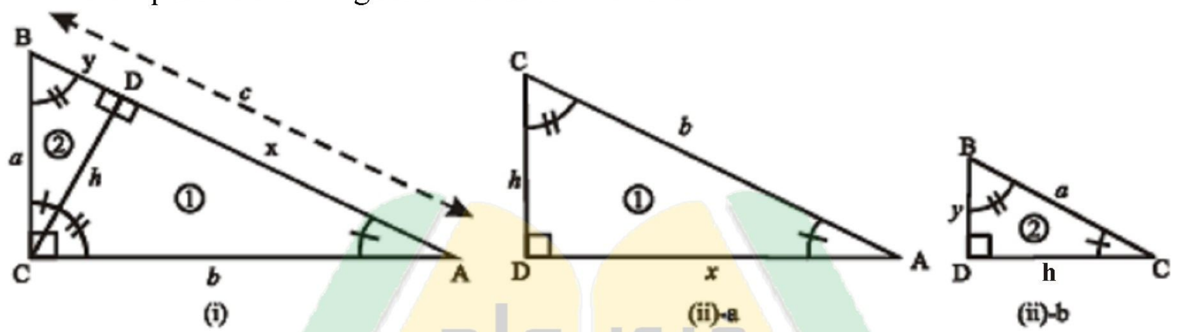
Unit 15: Pythagoras Theorem

Overview

Theorem 15.1.1

In a right angled triangle, the square of the length of hypotenuse is equal to the sum of the squares of the lengths of the other two sides



Given

Δ ACB is a right angled triangle in which $m\angle C = 90°$ and $m\overline{BC} = a$, $m\overline{AC} = b$ and $m\overline{AB} = c$

To prove

$c^2 = a^2 + b^2$

Construction

Draw $\overline{CD}$ perpendicular from C on $\overline{AB}$

Let $m\overline{CD} = h$, $m\overline{AD} = x$ and $m\overline{BD} = y$. Line segment CD splits ΔABC into two Δs ADC and BDC which are separately shown in the figures (ii) –a and (ii) –b respectively

Proof (using similar Δs)

Statements	Reasons
In Δ ADC $\leftrightarrow$ Δ ACB	Refer to figure (ii)-a and (i)
$\angle A \cong \angle A$	Common – Self Congruent
$\angle ADC \cong \angle ACB$	Construction- given each angle = 90°
$\angle C \cong \angle B$	$\angle C$ and $\angle B$ complements of $\angle A$

$\therefore \Delta ADC \sim \Delta ACB$	Congruency of three angles
$\therefore \frac{x}{b} = \frac{b}{c}$	(Measures of corresponding sides of similar triangles are proportional)
or $x = \frac{b^2}{c}$ ________ (i)	
Again in $\Delta BDC \leftrightarrow \Delta BCA$	Refer to figure (ii)-b and (i)
$\angle B \cong \angle B$	Common – self Congruent
$\angle BDC \cong \angle BCA$	Construction – given each angle = 90°
$\angle C \cong \angle A$	$\angle C$ and $\angle A$ complements of $\angle B$
$\therefore \Delta BDC \sim \Delta BCA$	Congruency of three angles
$\therefore \frac{y}{a} = \frac{a}{c}$	(Corresponding sides of similar triangles are proportional)
or $y = \frac{a^2}{c}$ ________ (ii)	
But $y + x = c$	Supposition
$\therefore \frac{a^2}{c} + \frac{b^2}{c} = c$	By (i) and (ii)
or $a^2 + b^2 = c^2$	Multiplying both side by c
i-e $c^2 = a^2 + b^2$	

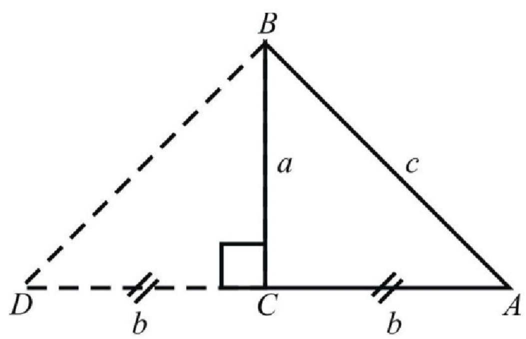
Theorem 15.1.2 Converse of Pythagoras Theorem 15.1.1

If the Square of one side of a triangle is equal to the sum of the square of the other two sides then the triangle is a right angled triangle

Given

In a ΔABC , $m\overline{AB} = c, m\overline{BC} = a, m\overline{AC} = b$

Such that $a^2 + b^2 = c^2$



To prove

ΔACB is a right angled triangle

Construction

Draw $\overline{CD}$ perpendicular to $\overline{BC}$ Such that

$\overline{CD} \cong \overline{CA}$. Join the points B and D

Proof

Statements	Reasons
ΔDCB is a right angled triangle	Construction
$\therefore \left(m\overline{BD}\right)^2 = a^2 + b^2$	Pythagoras theorem
But $a^2 + b^2 = c^2$	Given
$\therefore \left(m\overline{BD}\right)^2 = c^2$	
or $m\overline{BD} = c$	Taking Square root on both sides
Now in $\Delta DCB \leftrightarrow \Delta ACB$	
$\overline{CD} \cong \overline{CA}$	Construction
$\overline{BC} \cong \overline{BC}$	Common
$\overline{DB} \cong \overline{AB}$	Each side = c
$\therefore \Delta\ DCB \cong \Delta\ ACB$	S.S.S ≅ S.S.S
$\therefore \Delta\ DCB \cong \angle ACB$	(Corresponding angles of congruent triangle)
But $m\angle\ DCB = 90°$	Construction
$\therefore m\angle\ ACB = 90°$	
Hence the Δ ACB is a Right angled triangle	

Last Updated: September 2020

Report any mistake at freeilm786@gmail.com