Unit 14: Ratio and Proportion

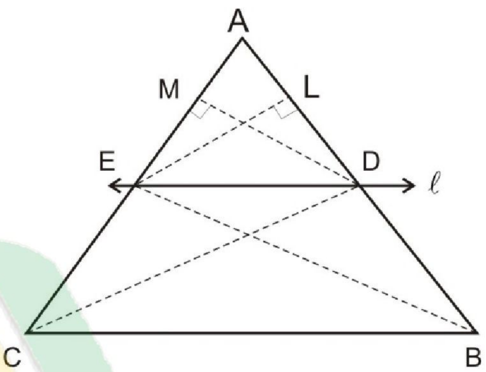
Overview

Theorem 14.1.1

A line parallel to one side of a triangle and intersecting the other two sides divides them proportionally.

Given:

In ΔABC, the line ℓ is intersecting the sides $\overline{AC}$ and $\overline{AB}$ at points E and D respectively such that $\overline{ED} \parallel \overline{CB}$



To Prove

$m\overline{AD} : \overline{DB} = m\overline{AE} : m\overline{EC}$

Construction:

Join B to E and C to D .From D draw $\overline{DM} \perp \overline{AC}$ and from E draw $\overline{EL} \perp \overline{AB}$

Proof

Statements	Reasons
In triangles BED and AED, EL is the common perpendicular	
$\therefore$ Area of $\Delta BED = \frac{1}{2} \times m\overline{BD} \times m\overline{EL}$.......(i)	Area of a $\Delta = \frac{1}{2}$(base)(height)
and Area of $\Delta AED = \frac{1}{2} \times m\overline{AD} \times m\overline{EL}$.......(ii)	
Thus Area of $\frac{\Delta BED}{\Delta AED} = \frac{m\overline{DB}}{m\overline{AD}}$........(iii)	Dividing (i) by (ii)
Similarly $\frac{\text{Area of } \Delta CDE}{\text{Area of } \Delta ADE} = \frac{m\overline{EC}}{m\overline{AE}}$........(iv)	
But $\Delta BED \cong \Delta CDE$	(Areas of triangles with common base and same altitudes are equal. Given that $\overline{ED} \parallel \overline{CB}$, so altitudes are equal).
$\therefore$ From (iii) and (iv) We have $\frac{m\overline{DB}}{m\overline{AD}} = \frac{m\overline{EC}}{m\overline{AE}}$ or	
$\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}}$	Taking reciprocal of both sides.
Hence $m\overline{AD} : m\overline{DB} = m\overline{AE} : m\overline{EC}$	

Theorem: 14.1.2 Converse of Theorem 14.1.1

If a line segment intersects the two sides of a triangle in the same ratio, then it is parallel to the third side.

Given

In ΔABC, $\overline{ED}$ intersect $\overline{AB}$ and $\overline{AC}$ such that

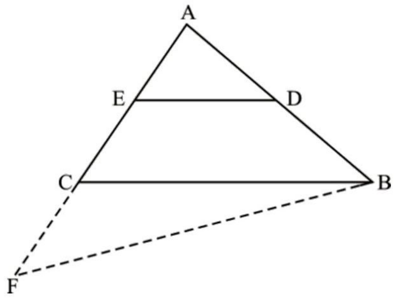
$m\overline{AD}:\overline{DB}=m\overline{AE}:m\overline{EC}$

To Prove

$\overline{ED}\parallel\overline{CB}$

Construction

If $\overline{ED}\nparallel\overline{CB}$ then draw $\overline{BF}\parallel\overline{DE}$ to meet $\overline{AC}$ Produced at F



Proof

Statements	Reasons
In ΔABF	
$\overline{DE}\parallel\overline{BF}$	Construction
$\therefore \dfrac{m\overline{AD}}{m\overline{DB}}=\dfrac{m\overline{AE}}{m\overline{EF}}$ ....(i)	(A line parallel to one side of a triangle divides the other two sides proportionally Theorem 14.1.1)
But $\dfrac{m\overline{AD}}{m\overline{DB}}=\dfrac{m\overline{AE}}{m\overline{EC}}$ ....(ii)	Given
$\therefore \dfrac{m\overline{AE}}{m\overline{EF}}=\dfrac{m\overline{AE}}{m\overline{EC}}$	From (i) and (ii)
or $m\overline{EF}=m\overline{EC}$,	
This is possible only if point F is coincident with C.	(Property of real numbers)
$\therefore$ Our supposition is wrong	
Hence $\overline{ED}\parallel\overline{CB}$	

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Report any mistake at freeilm786@gmail.com