Exercise 4.2

Q.1 Solve

(i) If $a+b=10$ and $a-b=6$, then find the value of (a^2+b^2)

Solution:

$$2(a^2+b^2)=(a+b)^2+(a-b)^2$$
$$2(a^2+b^2)=(10)^2+(6)^2$$
$$2(a^2+b^2)=100+36$$
$$2(a^2+b^2)=136$$
$$(a^2+b^2)=\frac{\cancel{136}^{68}}{\cancel{2}}$$
$$(a^2+b^2)=68 \text{ Ans}$$

(ii) If $a+b=5, a-b=\sqrt{17}$, then find the value of ab.

Solution:

$$4ab=(a+b)^2-(a-b)^2$$
$$4ab=(5)^2-\left(\sqrt{17}\right)^2$$
$$4ab=25-17$$
$$4ab=8$$
$$ab=\frac{8}{4}$$
$$ab=2$$
$$ab=2 \text{ Ans}$$

Q.2 If $a^2+b^2+c^2=45$ and $a+b+c=-1$, then find the value of $ab+bc+ca$.

Solution: $a^2+b^2+c^2=45$

$a+b+c=-1$

$ab+bc+ca$ **=?**

We know that

$$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$$
$$(-1)^2=45+2(ab+bc+ca)$$
$$1=45+2(ab+bc+ca)$$
$$1-45=2(ab+bc+ca)$$
$$-44=2(ab+bc+ca)$$
$$\frac{\cancel{44}^{22}}{\cancel{2}}=(ab+bc+ca)$$
$$(ab+bc+ca)=-22 \text{ Ans}$$

Q.3 If $m+n+p=10$ and $mn+np+np=27$, find the value of $m^2+n^2+p^2$

Solution: $m+n+p=10$

$mn+np+np=27,$

$m^2+n^2+p^2$ **=?**

We know that

$$(m+n+p)^2=m^2+n^2+p^2+2mn+2np+2mp$$
$$(10)^2=m^2+n^2+p^2+2(mn+np+mp)$$
$$100=m^2+n^2+p^2+2(27)$$
$$100=m^2+n^2+p^2+54$$
$$100-54=m^2+n^2+p^2$$
$$m^2+n^2+p^2=46 \text{ Ans}$$

Q.4 If $x^2+y^2+z^2=78$ and $xy+yz+zx=59$, find the value of $x+y+z$.

Solution: $x^2+y^2+z^2=78$

$xy+yz+zx=59,$

$x+y+z$ **=?**

We know that

$$(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$$
$$(x+y+z)^2=78+2(xy+yz+zx)$$
$$(x+y+z)^2=78+2(59)$$
$$(x+y+z)^2=78+118$$
$$(x+y+z)^2=196$$

Taking square root at both sides

$$\sqrt{(x+y+z)^2} = \pm\sqrt{196}$$
$$x+y+z = \pm 14 \text{ Ans}$$

Q.5 **If $x+y+z=12$ and $x^2+y^2=64$, find the value of $xy+yz+zx$.**

Solution: $x+y+z=12$

$x^2+y^2=64$

$xy+yz+zx$ **=?**

We know that

$$(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$$

$$(x+y+z)^2 = x^2+y^2+z^2+2(xy+yz+zx)$$

$$(12)^2 = 64+2(xy+yz+zx)$$
$$144-64 = 2(xy+yz+zx)$$
$$80 = 2(xy+yz+zx)$$
$$\frac{^{40}\cancel{80}}{\cancel{2}} = (xy+yz+zx)$$
$$40 = xy+yz+zx$$
$$xy+yz+zx = 40 \text{ Ans}$$

Q.6 **If $x+y=7$ and $xy=12$, then find the value of x^3+y^3**

Solution: $x+y=7$

$xy=12$

x^3+y^3 **=?**

We know that

$$(x+y)^3 = x^3+y^3+3xy(x+y)$$
$$(7)^3 = x^3+y^3+3(12)(7)$$
$$343 = x^3+y^3+252$$
$$343-252 = x^3+y^3$$
$$91 = x^3+y^3$$
$$x^3+y^3 = 91 \text{ Ans}$$

Q.7 **If $3x+4y=11$ and $xy=12$, then find the value of $27x^3+64y^3$.**

Solution: $3x+4y=11$

$xy=12$

$27x^3+64y^3$ **=?**

$$(x+y)^3 = x^3+y^3+3xy(x+y)$$
$$(3x+4y)^3 = (3x)^3+(4y)^3+3(3x)(4y)(3x+4y)$$
$$(3x+4y)^3 = 27x^3+64y^3+36xy(3x+4y)$$
$$(11)^3 = 27x^3+64y^3+36(12)(11)$$
$$1331 = 27x^3+64y^3+4752$$
$$1331-4752 = 27x^3+64y^3$$
$$-3421 = 27x^3+64y^3$$
$$27x^3+64y^3 = -3421 \text{ Ans}$$

Q.8 **If $x-y=4$ and $xy=21$, then find the value of x^3-y^3**

Solution: $x-y=4$

$xy=21$

x^3-y^3 **=?**

We know that

$$(x-y)^3 = x^3-y^3-3xy(x-y)$$
$$(4)^3 = x^3-y^3-3(21)(4)$$
$$64 = x^3-y^3-252$$
$$64+252 = x^3-y^3$$
$$316 = x^3-y^3$$
$$x^3-y^3 = 316 \text{ Ans}$$

Q.9 **If $5x-6y=13$ and $xy=6$, then find the value of b$125x^3-216y^3$**

Solution: $5x-6y=13$

$xy=6$

$125x^3-216y^3$ **=?**

We know that

$$(x-y)^3 = x^3-y^3-3xy(x-y)$$
$$(5x-6y)^3 = (5x)^3-(6y)^3-3(5x)(6y)(5x-6y)$$
$$(5x-6y)^3 = 125x^3-216y^3-90xy(5x-6y)$$

$$(13)^3 = 125x^3-216y^3-90(6)(13)$$
$$2197 = 125x^3-216y^3-7020$$
$$2197+7020 = 125x^3-216y^3$$

$$9217 = 125x^3 - 216y^3$$

$$125x^3 - 216y^3 = 9217 \textbf{ Ans}$$

Q.10 **If** $x + \frac{1}{x} = 3$ **then find the value of** $x^3 + \frac{1}{x^3}$

Solution: $x + \frac{1}{x} = 3$

$x^3 + \frac{1}{x^3}$**=?**

We know that

$$\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right)$$

$$(3)^3 = x^3 + \frac{1}{x^3} + 3(3)$$

$$27 = x^3 + \frac{1}{x^3} + 9$$

$$27 - 9 = x^3 + \frac{1}{x^3}$$

$$18 = x^3 + \frac{1}{x^3}$$

$$x^3 + \frac{1}{x^3} = 18 \textbf{ Ans}$$

Q.11 **If** $x - \frac{1}{x} = 7,$ **then find the value of** $x^3 - \frac{1}{x^3}$

Solution: $x - \frac{1}{x} = 7$

$x^3 - \frac{1}{x^3}$**=?**

We know that

$$\left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right)$$

$$(7)^3 = x^3 - \frac{1}{x^3} - 3(7)$$

$$343 = x^3 - \frac{1}{x^3} - 21$$

$$343 + 21 = x^3 - \frac{1}{x^3}$$

$$364 = x^3 - \frac{1}{x^3}$$

$$x^3 - \frac{1}{x^3} = 364 \textbf{ Ans}$$

Q.12 **If** $\left[3x + \frac{1}{3x}\right] = 5,$ **then find the value of** $\left[27x^3 + \frac{1}{27x^3}\right]$

Solution: $\left[3x + \frac{1}{3x}\right] = 5$

$\left[27x^3 + \frac{1}{27x^3}\right]$**=?**

We know that

$$\left(3x + \frac{1}{3x}\right)^3 = (3x)^3 + \left(\frac{1}{3x}\right)^3 + 3\cancel{(3x)}\cancel{\left(\frac{1}{3x}\right)}\left(3x + \frac{1}{3x}\right)$$

$$(5)^3 = 27x^3 + \frac{1}{27x^3} + 3\left(3x + \frac{1}{3x}\right)$$

$$125 = 27x^3 + \frac{1}{27x^3} + 3(5)$$

$$125 = 27x^3 + \frac{1}{27x^3} + 15$$

$$125 - 15 = 27x^3 + \frac{1}{27x^3}$$

$$110 = 27x^3 + \frac{1}{27x^3}$$

$$27x^3 + \frac{1}{27x^3} = 110 \textbf{ Ans}$$

Q.13 **If** $\left(5x - \frac{1}{5x}\right) = 6,$ **then find the value of** $\left(125x^3 - \frac{1}{125x^3}\right)$

Solution: $\left(5x - \frac{1}{5x}\right) = 6$

$\left(125x^3 - \frac{1}{125x^3}\right)$**=?**

We know that

$$\left(5x-\frac{1}{5x}\right)^3=(5x)^3-\left(\frac{1}{5x}\right)^3-3\cancel{(5x)}\left(\frac{1}{\cancel{5x}}\right)\left(5x-\frac{1}{5x}\right)$$

$$(6)^3=125x^3-\frac{1}{125x^3}-3(6)$$

$$216=125x^3-\frac{1}{125x^3}-18$$

$$216+18=125x^3-\frac{1}{125x^3}$$

$$234=125x^3-\frac{1}{125x^3}$$

$$125x^3-\frac{1}{125x^3}=234 \text{ Ans}$$

Q.14 Factorize

(i) x^3-y^3-x+y

Solution: x^3-y^3-x+y

$$=(x)^3-(y)^3-1(x-y)$$

$$=(x-y)\left(x^2+xy+y^2\right)-1(x-y)$$

$$=(x-y)\left(x^2+xy+y^2-1\right) \text{ Ans}$$

(ii) $8x^3-\frac{1}{27y^3}$

Solution: $8x^3-\frac{1}{27y^3}$

$$=(2x)^3-\left(\frac{1}{3y}\right)^3$$

$$=\left[2x-\frac{1}{3y}\right]\left[(2x)^2+(2x)\left(\frac{1}{3y}\right)+\left(\frac{1}{3y}\right)^2\right]$$

$$=\left(2x-\frac{1}{3y}\right)\left(4x^2+\frac{2x}{3y}+\frac{1}{9y^2}\right) \text{ Ans}$$

Q.15 Find the products, using formula.

(i) $\left(x^2+y^2\right)\left(x^4-x^2y^2+y^4\right)$

Solution: $\left(x^2+y^2\right)\left(x^4-x^2y^2+y^4\right)$

$$=\left(x^2+y^2\right)\left[\left(x^2\right)^2-\left(x^2\right)\left(y^2\right)+\left(y^2\right)^2\right]$$

$$\left[\left(x^2\right)^3+\left(y^2\right)^3\right]$$

$$=x^6+y^6 \text{ Ans}$$

(ii) $\left(x^3-y^3\right)\left(x^6+x^3y^3+y^6\right)$

Solution: $\left(x^3-y^3\right)\left(x^6+x^3y^3+y^6\right)$

$$\left(x^3-y^3\right)\left[\left(x^3\right)^2+\left(x^3\right)\left(y^3\right)+\left(y^3\right)^2\right]$$

$$=\left(x^3\right)^3-\left(y^3\right)^3$$

$$=x^9-y^9 \text{ Ans}$$

(iii) $(x-y)(x+y)\left(x^2+y^2\right)\left(x^2+xy+y^2\right)$

$\left(x^2+xy+y^2\right)\left(x^4-x^2y^2+y^4\right)$

Solution:

$(x-y)(x+y)\left(x^2+y^2\right)\left(x^2+xy+y^2\right)$

$\left(x^2+xy+y^2\right)\left(x^4-x^2y^2+y^4\right)$

$$=\left[(x-y)\left(x^2+xy+y^2\right)\right]\left[(x+y)\left(x^2-xy+y^2\right)\right]$$

$$\left[\left(x^2+y^2\right)\left(x^4-x^2y^2+y^4\right)\right]$$

$$=\left[\left(x^3-y^3\right)\left(x^3+y^3\right)\right]\left[\left(x^2\right)^3+\left(y^2\right)^3\right]$$

$$=\left[\left(x^3\right)^2-\left(y^3\right)^2\right]\left[\left(x^6+y^6\right)\right]$$

$$=\left[\left(x^6-y^6\right)\left(x^6+y^6\right)\right]$$

$$=\left[\left(x^6\right)^2-\left(y^6\right)^2\right]$$

$$=x^{12}-y^{12} \text{ Ans}$$

(iv) $\left(2x^2-1\right)\left(2x^2+1\right)\left(4x^4+2x^2+1\right)\left(4x^4-2x^2+1\right)$

Solution:

$$\left(2x^2-1\right)\left(2x^2+1\right)\left(4x^4+2x^2+1\right)\left(4x^4-2x^2+1\right)$$
$$=\left[\left(2x^2-1\right)\left(4x^4+2x^2+1\right)\right]\left[\left(2x^2+1\right)\left(4x^4-2x^2+1\right)\right]$$
$$=\left[\left(2x^2\right)^3-(1)^3\right]\left[\left(2x^2\right)^3+(1)^3\right]$$
$$=\left(8x^6-1\right)\left(8x^6+1\right)$$
$$=\left(8x^6\right)^2-(1)^2$$
$$=64x^{12}-1 \quad \textbf{Ans}$$

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