Exercise 2.4

Q.1 Use laws of exponents to simplify.

(i) $$\frac{(243)^{-\frac{2}{3}}(32)^{-\frac{1}{5}}}{\sqrt{(196)^{-1}}}$$

Solution: $$\frac{(243)^{-\frac{2}{3}}(32)^{-\frac{1}{5}}}{\sqrt{(196)^{-1}}}$$

$$=\frac{(243)^{-\frac{2}{3}}(32)^{-\frac{1}{5}}}{\sqrt{(196)^{-1}}}$$

$$=\frac{(3^5)^{-\frac{2}{3}}\times(2^5)^{-\frac{1}{5}}}{\sqrt{\left[(14)^2\right]^{-1}}}$$

$$=\frac{(3)^{-\frac{10}{3}}\times 2^{-1}}{\sqrt{\left[(14)^{-1}\right]^2}}$$

$$=\frac{(3)^{-\frac{10}{3}}\times 2^{-1}}{(14)^{-1}}$$

$$=\frac{14}{(3)^{\frac{10}{3}}\times 2}$$

$$=\frac{7}{3^{\frac{10}{3}}}$$

$$=\frac{7}{\sqrt[3]{3^{10}}}$$

$$=\frac{7}{\sqrt[3]{\overline{3\times3\times3}\times\overline{3\times3\times3}\times\overline{3\times3\times3}\times3}}$$

$$=\frac{7}{\sqrt[3]{3^3\times3^3\times3^3\times3}}$$

$$=\frac{7}{\sqrt[3]{3^3}\times\sqrt[3]{3^3}\times\sqrt[3]{3^3}\times\sqrt[3]{3}}$$

$$=\frac{7}{3\times3\times3\times\sqrt[3]{3}}$$

$$=\frac{7}{27\sqrt[3]{3}}$$ **Ans**

(ii) $$\left(2x^5y^{-4}\right)\left(-8x^{-3}y^2\right)$$

Solution: $$\left(2x^5y^{-4}\right)\left(-8x^{-3}y^2\right)$$

$$=-16x^{5-3}y^{-4+2}$$

$$=-16x^2y^{-2}$$

$$=\frac{-16x^2}{y^2}$$ **Ans**

(iii) $$\left[\frac{x^{-2}y^{-1}z^{-4}}{x^4y^{-3}z^{o}}\right]^{-3}$$

Solution: $$\left[\frac{x^{-2}y^{-1}z^{-4}}{x^4y^{-3}z^{o}}\right]^{-3}$$

$$=\left[x^{-2-4}y^{-1+3}z^{-4-0}\right]^{-3}$$

$$=\left(x^{-6}y^{+2}z^{-4}\right)^{-3}$$

$$=\left(x^{-6}\right)^{-3}\left(y^{2}\right)^{-3}\left(y^{-4}\right)^{-3}$$

$$=x^{18}y^{-6}z^{12}$$

$$=\frac{x^{18}z^{12}}{y^6}$$ Ans

(iv) $$\frac{(81)^n.3^5-(3)^{4n-1}(243)}{\left(9^{2n}\right)\left(3^3\right)}$$

Solution: $$\frac{(81)^n.3^5-(3)^{4n-1}(243)}{\left(9^{2n}\right)\left(3^3\right)}$$

$$=\frac{\left(3^4\right)^n.3^5-3^{4n}.3^{-1}.3^5}{\left(3^2\right)^{2n}.3^3}$$

$$=\frac{3^{4m}.3^5-3^{4n}.3^{-1+5}}{3^{4n}.3^3}$$

$$=\frac{3^{4n}.3^5-3^{4n}.3^4}{3^{4n}.3^3}$$

$$=\frac{3^{4n}.3^{4}(3-1)}{3^{4n}.3^{3}}$$

$$=3^{4n-4n}.3^{4-3}.(2)$$

$$=3^{0}.3^{1}.2$$

$$=1\times 3\times 2$$

$$=6 \ \textbf{Ans}$$

Q.2 Show that

$$\left[\frac{x^a}{x^b}\right]^{a+b}\times\left[\frac{x^b}{x^c}\right]^{b+c}\times\left[\frac{x^c}{x^a}\right]^{c+a}=1$$

Proof:

L.H.S

$$=\left[\frac{x^a}{x^b}\right]^{a+b}\times\left[\frac{x^b}{x^c}\right]^{b+c}\times\left[\frac{x^c}{x^a}\right]^{c+a}$$

$$=\left(x^{a-b}\right)^{a+b}\times\left(x^{b-c}\right)^{b+c}\times\left(x^{c-a}\right)^{c+a}$$

$$=x^{(a-b)(a+b)}\times x^{(b-c)(b+c)}\times x^{(c-a)(c+a)}$$

$$=x^{a^2-b^2}\times x^{b^2-c^2}\times x^{c^2-a^2}$$

$$=x^{\cancel{a^2}-\cancel{b^2}+\cancel{b^2}-\cancel{a^2}+\cancel{c^2}-\cancel{a^2}}$$

$$=x^{\circ}$$

$$=1$$

$$1=\text{R.H.S} \ \textbf{Ans}$$

Q.3 Simplify

(i) $$\frac{2^{\frac{1}{3}}\times(27)^{\frac{1}{3}}\times(60)^{\frac{1}{2}}}{(180)^{\frac{1}{2}}\times(4)^{-\frac{1}{3}}\times(9)^{\frac{1}{4}}}$$

Solution: $$\frac{2^{\frac{1}{3}}\times(27)^{\frac{1}{3}}\times(60)^{\frac{1}{2}}}{(180)^{\frac{1}{2}}\times(4)^{-\frac{1}{3}}\times(9)^{\frac{1}{4}}}$$

$$=\frac{2^{\frac{1}{3}}\times\left(3^{\cancel{3}}\right)^{\frac{1}{\cancel{3}}}\times(2\times2\times3\times5)^{\frac{1}{2}}}{(2\times2\times3\times3\times5)^{\frac{1}{2}}\times\left(2^{2}\right)^{-\frac{1}{3}}\times\left(3^{\cancel{2}}\right)^{\frac{1}{\cancel{2}}}}$$

$$=\frac{2^{\frac{1}{3}}\times3\left(2^{2}\right)^{\frac{1}{2}}\times3^{\frac{1}{2}}\times5^{\frac{1}{2}}}{\left(2^{2}\right)^{\frac{1}{2}}\times\left(3^{2}\right)^{\frac{1}{2}}\times(5)^{\frac{1}{2}}\times2^{-\frac{2}{3}}\times3^{\frac{1}{2}}}$$

$$=\frac{2^{\frac{1}{3}}\times3\times2\times3^{\frac{1}{2}}\times5^{\frac{1}{2}}}{2\times3\times5^{\frac{1}{2}}\times2^{-\frac{2}{3}}\times3^{\frac{1}{2}}}$$

$$=2^{\frac{1}{3}}\times2^{+1}\times2^{-1}\times2^{\frac{+2}{3}}\times3^{1}\times3^{\frac{1}{2}}\times3^{-1}\times3^{\frac{-1}{2}}\times5^{\frac{1}{2}}\times5^{\frac{-1}{2}}$$

$$=2^{\frac{1}{3}\cancel{+1}\cancel{-1}+\frac{2}{3}}\times3^{\cancel{1}+\cancel{\frac{1}{2}}-\cancel{1}-\cancel{\frac{1}{2}}}\times5^{\cancel{\frac{1}{2}}\cancel{\frac{-1}{2}}}$$

$$=2^{\frac{1}{3}+\frac{2}{3}}\times3^{0}\times5^{0}$$

$$=2^{\frac{1}{3}+\frac{2}{3}}\times1\times1$$

$$=2^{\frac{\cancel{3}}{\cancel{3}}}$$

$$=2 \ \textbf{Ans}$$

(ii) $$\sqrt{\frac{(216)^{\frac{2}{3}}\times(25)^{\frac{1}{2}}}{(0.04)^{-\frac{1}{2}}}}$$

Solution: $$\sqrt{\frac{(216)^{\frac{2}{3}}\times(25)^{\frac{1}{2}}}{(0.04)^{-\frac{1}{2}}}}$$

$$=\sqrt{\frac{\left(6^{3}\right)^{\frac{2}{3}}\times\left(5^{2}\right)^{\frac{1}{2}}}{\left(\frac{4}{100}\right)^{-\frac{1}{2}}}}$$

$$=\sqrt{\frac{6^{2}\times5}{\left(\frac{\overset{25}{\cancel{100}}}{\cancel{4}}\right)^{\frac{1}{2}}}}$$

$$=\sqrt{\frac{6^{2}\times5}{(25)^{\frac{1}{2}}}}$$

$$=\sqrt{\frac{6^{2}\times5}{\left(5^{\cancel{2}}\right)^{\frac{1}{\cancel{2}}}}}$$

$$=\sqrt{\frac{6^{2}\times5}{5}}$$

$$=\sqrt{6^{2}\times5^{+1}\times5^{-1}}$$

$$=\sqrt{6^{2}\times5^{\cancel{+1}\cancel{-1}}}$$

$$=\sqrt{6^{2}\times5^{0}}$$

$$=\sqrt{6^{2}\times1}$$

$$=\sqrt{6^{2}}$$

$$=6 \ \textbf{Ans}$$

(iii) $5^{2^3} \div \left(5^2\right)^3$

Solution: $5^{2^3} \div \left(5^2\right)^3$

$$= 5^8 \div 5^6$$
$$= 5^{8-6}$$
$$= 5^2$$
$$= 25 \text{ Ans}$$

(iv) $\left(x^3\right)^2 \div x^{3^2}, x \neq 0$

Solution: $\left(x^3\right)^2 \div x^{3^2}, x \neq 0$

$$= x^6 \div x^9$$
$$= x^{6-9}$$
$$= x^{-3}$$
$$= \frac{1}{x^3} \textbf{ Ans}$$

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Report any mistake at freeilm786@gmail.com