Exercise 13.1

Q.1 Two sides of a triangle measure 10cm and 15 cm which of the following measure is possible for the third side?

(a) 5*cm*
(b) 20 *cm*
(c) 25 *cm*
(d) 30 *cm*

Solution

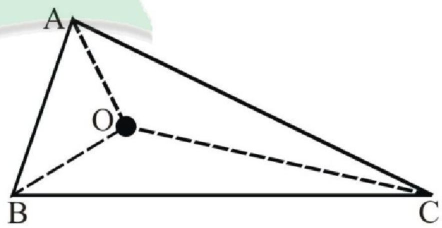
Lengths of two sides are 15 and 10 cm.
So, sum of two lengths of triangle = 10 + 15 = 25 m
10 + 15 > 20
10 + 20 > 15
15 + 20 > 10
$\therefore$ 20 cm is possible for third side
Or
Sum of length of two sides is always greater than the third sides of a triangle.
Given

Q.2 Point O is interior of ΔABC
Show that

$$m\overline{OA}+m\overline{OB}+m\overline{OC}>\frac{1}{2}\left(m\overline{AB}+m\overline{BC}+m\overline{CA}\right)$$

Given
Point O is interior of ΔABC
To prove:

$$m\overrightarrow{OA}+m\overline{OB}+m\overline{OC}<\frac{1}{2}\left(m\overline{AB}+m\overline{BC}+m\overline{AC}\right)$$



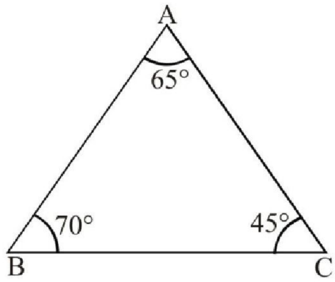
Construction
Join O with A, B and C.
So that we get three triangle ΔOAB, ΔOBC and ΔOAC
Proof

Statements	Reasons
In ΔOAB $m\overline{OA}+\overline{OB} > m\overline{AB}$ _____(i)	In any triangle the sum of length of two sides is greater then the third sides.
In ΔOAC $m\overline{OC}+m\overline{OA} > m\overline{AC}$ _____(ii)	As in (i)
In ΔOBC $mOB+\overline{OC} > m\overline{BC}$ _____(iii)	As in (i)
Adding equation i, ii and iii $\overline{OA}+\overline{OC}+\overline{OA}+\overline{OB}+\overline{OB}+\overline{OC}>\overline{AC}+\overline{AB}+\overline{BC}$ $2\overline{OA}+2\overline{OC}+2\overline{OB}>\overline{AB}+\overline{BC}+\overline{CA}$ $2(OA+OC+OB)>\overline{AB}+\overline{BC}+\overline{CA}$	

$\frac{\cancel{2}(OA+OC+OB)}{\cancel{2}} > \frac{\overline{AB}+\overline{BC}+\overline{CA}}{2}$ $(OA+OC+OB) > \frac{1}{2}(\overline{AB}+\overline{BC}+\overline{CA})$	Dividing both sides by 2

Q.3 In the ΔABC m∠B = 70° and m∠C = 45° which of the sides of the triangle is longest and which is the shortest.

Solution



Sum of three angle in a triangle is 180°

$\angle A + \angle B + \angle C = 180$

$\angle A + 70 + 45 = 180$

$\angle A + 115 = 180$

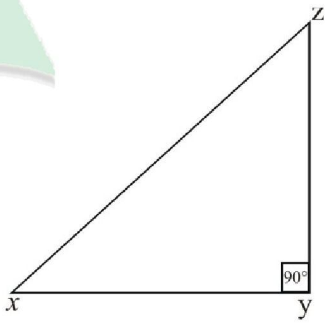
$\angle A = 180 - 115$

$\angle A = 65^o$

Sides of the triangle depend upon the angles largest angle has largest opposite side and smallest angle has smallest opposite side here $\angle B$ is largest so, $\overline{AC}$ is largest $\angle C$ is smallest, so $\overline{AB}$ is smallest side.

Q.4 Prove that in a right-angled triangle, the hypotenuse is longer than each of the other two sides.

Solution



Sum of three angles in a triangle is equal to 180°. So in a triangle one angle will be equal to 90° and rest of two angles are acute angle (less than 90°)

$\therefore$ $m\angle y = 90$

And $m\angle x + m\angle z = 90$

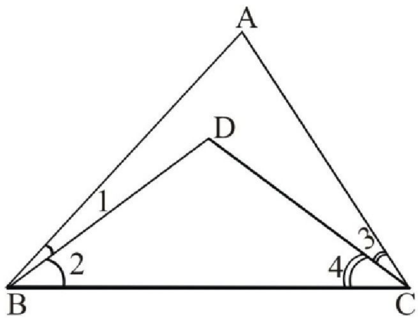
So $m\angle x$ and $m\angle z$ are acute angle

$\therefore$ Opposite to $m\angle y = 90^o$ is hypotenuse

It is largest side.

Q.5 In the triangular figure $\overline{AB} > \overline{AC}$. $\overline{BD}$ and $\overline{CD}$ are the bisectors of ∠B and ∠C respectively prove that $\overline{BD} > \overline{DC}$

Given



InΔABC

$\overline{AB} > \overline{AC}$

$\overline{BD}$ and $\overline{CD}$ are the bisectors of $\angle B$ and $\angle C$

To prove

$\overline{BD} > \overline{CD}$

Construction

Label the angles $\angle 1, \angle 2, \angle 3$ and $\angle 4$

Proof

Statements	Reasons
In ΔABC	
$\overline{AB} > \overline{AC}$	Given
$\overline{BD}$ is the bisector of $\angle B$	
$\frac{1}{2}m\angle ACB > \frac{1}{2}m\angle ABC$	
$m\angle ABC$	
$m\angle 2 < m\angle 4$	
$\overline{CD}$ is the bisector of $\angle C$	Given
In ΔBCD	
$\overline{BD} > \overline{DC}$	Side opposite to greater angle is greater

Theorem 13.1.4

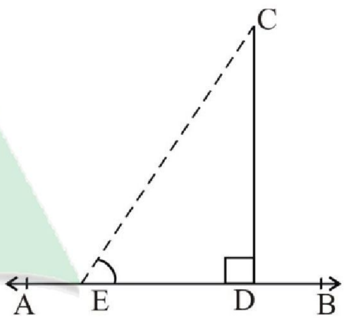
From a point, out side a line, the perpendicular is the shortest distance from the point to the line.

Given:

A line AB and a point C

(Not lying on $\overleftrightarrow{AB}$)and a point D on $\overleftrightarrow{AB}$ such that

$\overline{CD} \perp \overleftrightarrow{AB}$



To prove

m$\overline{CD}$ is the shortest distance from the point C to $\overleftrightarrow{AB}$

Construction

Take a point E on $\overleftrightarrow{AB}$ .Join C and E to form a ΔCDE

Proof

Statements	Reasons
In ΔCDE	
m$\angle$CDB > m$\angle$CED	(An exterior angle of a triangle is greater than non adjacent interior angle)
But m$\angle$CDB = m$\angle$CDE	Supplement of right angle
$\therefore$ m$\angle$CDE > m$\angle$CED	
Or m$\angle$CED < m$\angle$CDE	
Or m$\overline{CD}$ < m$\overline{CE}$	Side opposite to greater angle is greater.
But E is any point on $\overleftrightarrow{AB}$	
Hence m$\overline{CD}$ is the shortest distance from C to $\overleftrightarrow{AB}$	

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Report any mistake at freeilm786@gmail.com