Lesson No. 02

NUMBER SYSTEMS

Binary to Decimal conversion

Most real world quantities are represented in Decimal Number System. Digital Systems on the other hand are based on the Binary Number System. Therefore, when converting from the Digital Domain to the real-world, Binary numbers have to be represented in terms of their Decimal equivalents.

The method used to convert from Binary to Decimal is the Sum-of-Weights method. The Sum-of-Weights method has been used to represent the Caveman numbers ∆↑, ∆Ω↑∑ and the Binary numbers 10011 and 1011.101 in the first lecture.

1. Sum-of-Weights Method

Sum-of-weights as the name indicates sums the weights of the Binary Digits (bits) of a Binary Number which is to be represented in Decimal. The Sum-of-Weights method can be used to convert a Binary number of any magnitude to its equivalent Decimal representation.

In the Sum-of-Weights method an extended expression is written in terms of the Binary Base Number 2 and the weights of the Binary number to be converted. The weights correspond to each of the binary bits which are multiplied by the corresponding binary value. Binary bits having the value 0 do not contribute any value towards the final sum expression.

The Binary number 10110_2 is therefore written in the form of an expression having weights 2^0, 2^1, 2^2, 2^3 and 2^4 corresponding to the bits 0, 1, 1, 0 and 1 respectively. Weights 2^0 and 2^3 do not contribute in the final sum as the binary bits corresponding to these weights have the value 0.

$$
\begin{aligned}
10110_2 &= 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \\
&= 16 + 0 + 4 + 2 + 0 \\
&= 22
\end{aligned}
$$

2. Sum-of-non-zero terms

In the Sum-of-Weights method, the Binary bits 0 do not contribute towards the final sum representing the decimal equivalent. Secondly, the weight of each binary bit increases by a factor of 2 starting with a weight of 1 for the least significant bit. For example, the Binary number 10110_2 has weights $2^0=1$, $2^1=2$, $2^2=4$, $2^3=8$ and $2^4=16$ corresponding to the bits 0, 1, 1, 0 and 1 respectively.

The Sum-of-non-zero terms method is a quicker method to determine decimal equivalents of binary numbers without resorting to writing an expression. In the Sum-of-non-zero terms method the weights of non-zero binary bits are summed, as the weights of zero binary bits do not contribute towards the final sum representing the decimal equivalent.

The weights of binary bits starting from the right most least significant bit is 1, The next significant bit on the left has the weight 2, followed by 4, 8, 16, 32 etc. corresponding to higher significant bits. In binary number system the weights of successive bits increase by an order of 2 towards the left side and decrease by an order of 2 towards the right side. Thus a binary number can be quickly converted into its decimal equivalent by adding weights of non-zero terms which increase by a factor of 2. Binary numbers having an integer and a fraction part can similarly be converted into their decimal equivalents by applying the same method.

A quicker method is to add the weights of non-zero terms. Thus for the numbers

- $10011_2 = 16 + 2 + 1 = 19$
- $1011.101_2 = 8 + 2 + 1 + ½ + 1/8 = 11 + 5/8 = 11.625$

Decimal to Binary conversion

Conversion from Decimal to Binary number system is also essential to represent real-world quantities in terms of Binary values. The Sum-of-weights and repeated division by 2 methods are used to convert a Decimal number to equivalent Binary.

1. Sum-of-Weights

The Sum-of-weights method used to convert Binary numbers into their Decimal equivalent is based on adding binary weights of the binary number bits. Converting back from the decimal number to the original Binary number requires finding the highest weight included in the sum representing the decimal equivalent. A Binary 1 is marked to represent the bit which contributed its weight in the Sum representing the decimal equivalent. The weight is subtracted from the sum decimal equivalent. The next highest weight included in the sum term is found. A binary 1 is marked to represent the bit which contributed its weight in the sum term and the weight is subtracted from the sum term. This process is repeated until the sum term becomes equal to zero. The binary 1s and 0s represent the binary bits that contributed their weight and bits that did not contribute any weight respectively.

The process of determining Binary equivalent of a Decimal number 392 and 411 is illustrated in a tabular form. Table 2.1.

Sum Term	Highest Weight	Binary Number	Sum Term = Sum Term – Highest Weight
411	256	100000000	155
155	128	110000000	27
27	16	110010000	11
11	8	110011000	3
3	2	110011010	1
1	1	110011011	0

Table 2.1a Converting Decimal to Binary using Sum-of-Weights Method

Sum Term	Highest Weight	Binary Number	Sum Term = Sum Term – Highest Weight
392	256	100000000	136
136	128	110000000	8
8	8	110001000	0

Table 2.1b Converting Decimal to Binary using Sum-of-Weights Method

The Sum of weights method requires mental arithmetic and is a quick way of converting small decimal numbers into binary. With practice large Decimal numbers can be converted into Binary equivalents.

2. Repeated Division-by-2

Repeated Division-by-2 method allows decimal numbers of any magnitude to be converted into binary. In this method the Decimal number to be converted into its Binary equivalent is repeatedly divided by 2. The divisor is selected as 2 because the decimal number is being converted into Binary a Base-2 Number system. Repeated division method can be used to convert decimal number into any Number system by repeated division by the Base-Number. For example, the decimal number can be converted into the Caveman Number system by repeatedly dividing by 5, the Base number of the Caveman Number System. The Repeated Division method will be used in latter lectures to convert decimal into Hexadecimal and Octal Number Systems.

In the Repeated-Division method the Decimal number to be converted is divided by the Base Number, in this particular case 2. A quotient value and a remainder value is generated, both values are noted done. The remainder value in all subsequent divisions would be either a 0 or a 1. The quotient value obtained as a result of division by 2 is divided again by 2. The new quotient and remainder values are again noted down. In each step of the repeated division method the remainder values are noted down and the quotient values are repeatedly divided by the base number. The process of repeated division stops when the quotient value becomes zero. The remainders that have been noted in consecutive steps are written out to indicate the Binary equivalent of the Original Decimal Number.

Number	Quotient after division	Remainder after division
392	196	0
196	98	0
98	49	0
49	24	1
24	12	0
12	6	0
6	3	0
3	1	1
1	0	1

Table 2.2 Converting Decimal to Binary using Repeated Division by 2 Method

The process of determining the Binary equivalent of a Decimal number 392 is illustrated in a tabular form. Table 2.2. Reading the numbers in the Remainder column from bottom to top 110001000 gives the binary equivalent of the decimal number 392