MATHEMATICS HSSC-I

Time allowed: 2:35 Hours   Total Marks Section B & C: 80

SECTION - B (Marks 40)

Q.2 Attempt any TEN parts. All parts carry equal marks. (10×4=40)

(i) If $z_1 = 2 + i$, $z_2 = 3 + 2i$, $z_3 = 1 + 3i$ then express $\frac{\bar{z}_1 \cdot \bar{z}_2}{z_3}$ in the form $a + ib$

(ii) Prove that
$p \vee (\sim p \wedge \sim q) \vee (p \wedge q) = p \vee (\sim p \wedge \sim q)$

(iii) Show that:

$$\begin{vmatrix} x & 1 & 1 & 1 \\ 1 & x & 1 & 1 \\ 1 & 1 & x & 1 \\ 1 & 1 & 1 & x \end{vmatrix} = (x+3)(x-1)^3$$

(iv) Solve the Equation $4.2^{2x+1} - 9.2^x + 1 = 0$

(v) Resolve into partial fractions $\frac{2x+1}{(x+3)(x-1)(x+2)^2}$

(vi) Find the Sum to n^{th} term of series:
$r + (1+k)r^2 + (1+k+k^2)r^3 + \ldots\ldots n$

(vii) Find the numbers greater than 23000 that can be formed from the digits 1, 2, 3, 5, 6 without repeating any digit.

(viii) if x is so small that its square and higher powers maybe neglected then show that

$$\frac{(1+x)^{\frac{1}{2}}(4-3x)^{\frac{3}{2}}}{(4+5x)^{\frac{1}{3}}} \approx 4\left(1 - \frac{5x}{6}\right)$$

(ix) Find correct to the nearest centimeter, the distance at which a coin of diameter '1' cm should be held so as to conceal the full moon whose diameter subtends an angle of $31'$ at the eye of the observer on the earth.

(x) Prove that $\sqrt{\frac{1+\sin\alpha}{1-\sin\alpha}} = \frac{\sin\frac{\alpha}{2} + \cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2} - \cos\frac{\alpha}{2}}$

(xi) Draw the graph of y= cos x from 0 to 2π

(xii) By using usual notation, prove that:
$r_1 = \frac{\Delta}{s-a}$

(xiii) Show that $\cos^{-1}(-x) = \pi - \cos^{-1} x$

(xiv) Find the solution set of
$\sin 3x + \cos 2x + \sin x = 0$

SECTION – C (Marks 40)

Note: Attempt any FIVE questions. All questions carry equal marks. (5×8=40)

Q3. Use matrices to solve the system of equations:

$$\begin{aligned} 2x_1 + x_2 + 3x_3 &= 3 \\ x_1 + x_2 - 2x_3 &= 0 \\ -3x_1 - x_2 + 2x_3 &= -4 \end{aligned}$$

Q4. Solve the system of equation
$X^2 - Y^2 = 5, 4X^2 - 3xy = 18$

Q5. If the numbers $\frac{1}{2}, \frac{4}{21}$ and $\frac{1}{36}$ are subtracted from the three consecutive terms of a G.P., the resulting numbers in H.P. Find the numbers of their product is $\frac{1}{27}$.

Q.6 Identify the following series and find its sum.

$$1 - \frac{1}{2}\left(\frac{1}{2}\right) + \frac{1.3}{2.4}\left(\frac{1}{2}\right)^2 + \frac{1.3.5}{2.4.6}\left(\frac{1}{2}\right)^3 + \ldots\ldots\ldots, \infty$$

Q.7 Find $\sin(\alpha + \beta)$ and $\cos(\alpha + \beta)$ if $\tan\alpha = -\frac{15}{8}$, $\sin\beta = -\frac{7}{25}$ and neither the terminal side of the angle of measure α nor that of β is in the IV quadrant.

Q8. Prove the following:

$$\cos^{-1}\frac{63}{65} + 2\tan^{-1}\frac{1}{5} = \sin^{-1}\frac{3}{5}$$

Q9. Show that the set consisting of elements of the form $a + \sqrt{3}b$ $(a, b \text{ being rational})$, is an Abelian group w.r.t. addition.