Exercise 2.6

Q.1 Identify the following statement as true or false.

(i) $\sqrt{-3}\sqrt{-3}=3$ **False**

(ii) $i^{73}=-i$ **False**

(iii) $i^{10}=-1$ **True**

(iv) Complex conjugate of $(-6i+i^2)$ is $(-1+6i)$ **True**

(v) Difference of a complex number $z=a+bi$ and its conjugate is a real number. **False**

(vi) If $(a-1)-(b+3)i=5+8i$, then $a=6$ and $b=-11$. **True**

(vii) Product of a complex number and its conjugate is always a non-negative real number. **True**

Q.2 Express the each complex number in the standard form $a+bi$, where a and b are real number.

(i) $(2+3i)+(7-2i)$

Solution:

$=2+3i+7+2i$

$=2+7+3i-2i$

$=9+i$ **Ans**

(ii) $2(5+4i)-3(7+4i)$

Solution: $2(5+4i)-3(7+4i)$

$=10+8i-21-72i$

$=10-21+8i-12i$

$=-11-4i$ **Ans**

(iii) $(-3+5i)-(4+9i)$

Solution: $(-3+5i)-(4+9i)$

$=+3-5i-4-9i$

$=3-4-5i-9i$

$=-1-14i$ **Ans**

(iv) $2i^2+6i^3+3i^{16}-6i^{19}+4i^{25}$

Solution: $2i^2+6i^3+3i^{16}-6i^{19}+4i^{25}$

$=2(-1)+$ [illegible] $i+3(i^2)^8-6(i^2)^9.i+4(i^2)^{12}.i$

$=-2+6(-1)i+3(-1)^8-6(-1).i+4(-1)^{12}.i$

$=-2-6i+3-6(-1)i+4(+1)i$

$=1-\cancel{6i}+\cancel{6i}+4i$

$=1+4i$ **Ans**

Q.3 Simplify and write your answer in the form $a+bi$

(i) $(-7+3i)(-3+2i)$

Solution: $(-7+3i)(-3+2i)$

$=-7(-3+2i)+3i(-3+2i)$

$=21-14i-9i+6i^2$

$=21-23i+6(-1)$

$=21-23i-6$

$=21-6-23i$

$=15-23i$ **Ans**

(ii) $\left(2-\sqrt{-4}\right)\left(3-\sqrt{-4}\right)$

Solution: $\left(2-\sqrt{-4}\right)\left(3-\sqrt{-4}\right)$

$$=\left(2-\sqrt{4\times-1}\right)\left(3-\sqrt{4\times-1}\right)$$
$$=\left(2-\sqrt{4i^2}\right)\left(3-\sqrt{4i^2}\right)$$
$$=\left(2-2i\right)\left(3-2i\right)$$
$$=2\left(3-2i\right)-2i\left(3-2i\right)$$
$$=6-4i-6i+4i^2$$
$$=6-10i+4\left(-1\right)$$
$$=6-10i-4$$
$$=2-10i \text{ **Ans**}$$

(iii) $\left(\sqrt{5}-3i\right)^2$

Solution: $\left(\sqrt{5}-3i\right)^2$

$$=\left(\sqrt{5}\right)^2+\left(3i\right)^2-2\left(\sqrt{5}\right)\left(3i\right)$$
$$=5+9i^2-6\sqrt{5}i$$
$$=5+9\left(-1\right)-6\sqrt{5}i$$
$$=5-9-6\sqrt{5}i$$
$$=-4-6\sqrt{5}i \text{ **Ans**}$$

(iv) $\left(2-3i\right)\left(\overline{3-2i}\right)$

Solution: $\left(2-3i\right)\left(\overline{3-2i}\right)$

$$=\left(2-3i\right)\left(3+2i\right)$$
$$=2\left(3+2i\right)-3i\left(3+2i\right)$$
$$=6+4i-9i-6i^2$$
$$=6-5i-6\left(-1\right)$$
$$=6-5i+6$$
$$=6+6-5i$$
$$=12-5i \text{ **Ans**}$$

Q.4 Simplify and write your answer in the form $a+bi$.

(i) $\dfrac{-2}{1+i}$

Solution: $\dfrac{-2}{1+i}$

$$=\frac{-2}{1+i}\times\frac{1-i}{1-i}$$
$$=\frac{-2\left(1-i\right)}{\left(1\right)^2-\left(i\right)^2}$$
$$=\frac{-2+2i}{1-i^2}$$
$$=\frac{-2+2i}{1-\left(-1\right)}$$
$$=\frac{-2+2i}{1+1}$$
$$=\frac{-2+2i}{2}$$
$$=-\frac{2}{2}+\frac{2i}{2}$$
$$=-1+i \text{ **Ans**}$$

(ii) $\dfrac{2+3i}{4-i}$

Solution: $\dfrac{2+3i}{4-i}$

$$=\frac{2+3i}{4-i}\times\frac{4+i}{4+i}$$
$$=\frac{\left(2+3i\right)\left(4+i\right)}{\left(4\right)^2-\left(i\right)^2}$$
$$=\frac{2\left(4+i\right)+3i\left(4+i\right)}{16-\left(-1\right)}$$
$$=\frac{8+2i+12i+3i^2}{16+1}$$
$$=\frac{8+4i+3\left(-1\right)}{17}$$
$$=\frac{8+14i-3}{17}$$
$$=\frac{8-3+14i}{17}$$
$$=\frac{5+14i}{17}$$
$$=\frac{5}{17}+\frac{14}{17}i \text{ **Ans**}$$

(iii) $\dfrac{9-7i}{3+i}$

Solution: $\dfrac{9-7i}{3+i}$

$$=\frac{9-7i}{3+i}\times\frac{3-i}{3-i}$$
$$=\frac{(9-7i)(3-i)}{(3)^2-(i)^2}$$
$$=\frac{9(3-i)-7i(3-i)}{9-(-1)}$$
$$=\frac{27-9i-21i+7i^2}{9+1}$$
$$\frac{27-30i+7(-1)}{10}$$
$$=\frac{27-30i-7}{10}$$
$$=\frac{27-7-30i}{10}$$
$$=\frac{20-30i}{10}$$
$$=\frac{20}{10}-\frac{30i}{10}$$
$$=2-3i \text{ Ans}$$

(iv) $\dfrac{2-6i}{3+i}-\dfrac{4+i}{3+i}$

Solution: $\dfrac{2-6i}{3+i}-\dfrac{4+i}{3+i}$

$$=\frac{2-6i-(4+i)}{3+i}$$
$$=\frac{2-6i-4-i}{3+i}$$
$$=\frac{2-4-6i-i}{3+i}$$
$$=\frac{-2-7i}{3+i}$$
$$=\frac{-2-7i}{3+i}\times\frac{3-i}{3-i}$$
$$=\frac{-2-7i}{3+i}\times\frac{3-i}{3-i}$$
$$=\frac{-2(3-i)-7i(3-i)}{(3)^2-(i)^2}$$
$$=\frac{-6+2i-21i+7i^2}{9-(-1)}$$
$$=\frac{-6-19i+7(-1)}{9+1}$$
$$=\frac{-6-19i-7}{10}$$
$$=\frac{-6-7-19i}{10}$$
$$=\frac{-13-19i}{10}$$
$$=\frac{-13}{10}-\frac{19i}{10} \text{ Ans}$$

(v) $\left[\dfrac{1+i}{1-i}\right]^2$

Solution: $\left[\dfrac{1+i}{1-i}\right]^2$

$$=\frac{(1+i)^2}{(1-i)^2}$$
$$=\frac{(1)^2+(i)^2+2ab}{(1)^2+(i)^2-2ab}$$
$$=\frac{(1)^2+(i)^2+2(1)(i)}{(1)^2+(i)^2-2(1)(i)}$$
$$=\frac{1+(-1)+2i}{+1+(-1)-2i}$$
$$=\frac{\cancel{1}-\cancel{1}+2i}{\cancel{1}-\cancel{1}-2i}$$
$$=\frac{\cancel{2i}}{-\cancel{2i}}=-1$$
$$=-1$$
$$=-1+0i \text{ Ans}$$

(vi) $\dfrac{1}{(2+3i)(1-i)}$

Solution: $\dfrac{1}{(2+3i)(1-i)}$

$$=\frac{1}{2(1-i)+3i(1-i)}$$

$$=\frac{1}{2-2i+3i-3i^2}$$
$$=\frac{1}{2+i-3(-1)}$$
$$=\frac{1}{2+i+3}$$
$$=\frac{1}{2+3+i}$$
$$=\frac{1}{5+i}$$
$$=\frac{1}{5+i}\times\frac{5-i}{5-i}$$
$$=\frac{1(5-i)}{(5)^2-(i)^2}$$
$$=\frac{5-i}{25-(-1)}$$
$$=\frac{5-i}{25+1}$$
$$=\frac{5-i}{26}$$
$$=\frac{5}{26}-\frac{1i}{26}\ \textbf{Ans}$$

Q.5 **Calculate** $(a)\bar{z}\,(b)\,z+\bar{z}\,(c)\,z-\bar{z}\,(d)\,z\bar{z}$ **for each of the following.**

(i) $z=-i$

Solution: $z=-i$

(a) $\bar{z}=+i$

(b) $z+\bar{z}=-i+i$
$=0$

(c) $z-\bar{z}=(-i)-(i)$
$=-2i$

(d) $z\bar{z}=(-i)(i)$
$=-i^2$
$=-(-1)$
$=1$ **Ans**

(ii) $z=2+i$

Solution: $z=2+i$

$z+2i$

(a) $\bar{z}=2-i$

(b) $z+\bar{z}=(2+i)+(2-i)$
$=2+\cancel{i}+2-\cancel{i}$
$=2+2$
$=4$

(c) $z-\bar{z}=(2+i)-(2-i)$
$=\cancel{2}+i-\cancel{2}+i$
$=i+i$
$=2i$

(d) $z\bar{z}=(2+i)(2-i)$
$=(2)^2-(i)^2$
$=4-i^2$
$=4-(-1)$
$=4+1$
$=5$ **Ans**

(iii) $z=\frac{1+i}{1-i}$

Solution: $z=\frac{1+i}{1-i}$

$$z=\frac{1+i}{1-i}\times\frac{1+i}{1+i}$$
$$=\frac{1(1+i)+i(1+i)}{(1-i)(1+i)}$$
$$=\frac{1+i+i+(-1)}{(1)^2-(i)^2}$$
$$=\frac{1+2i+(-1)}{1-(-1)}$$
$$=\frac{\cancel{1}+2i-\cancel{1}}{1+1}$$
$$=\frac{\cancel{2}i}{\cancel{2}}$$
$=i$

$z=i$

(a) $\bar{z}=-i$

(b) $z+\bar{z}=i+(-i)$
$=\cancel{i}\cancel{-i}$

$= 0$

(c) $z - \bar{z} = i - (-i)$

$= i + i$

$= 2i$

(d) $z\bar{z} = (i)(-i)$

$= -i^2$

$= -(-1)$

$= +1$ **Ans**

(iv) $z = \dfrac{4-3i}{2+4i}$

Solution: $z = \dfrac{4-3i}{2+4i}$

$z = \dfrac{4-3i}{2+4i} \times \dfrac{2-4i}{2-4i}$

$= \dfrac{4(2-4i) - 3i(2-4i)}{(2+4i)(2-4i)}$

$= \dfrac{8 - 16i - 6i + 12i^2}{(2)^2 - (4i)^2}$

$= \dfrac{8 - 22i + 12(-1)}{4 - 16i^2}$

$= \dfrac{8 - 22i - 12}{4 - 16(-1)}$

$= \dfrac{8 - 12 - 22i}{4 + 16}$

$= \dfrac{-4 - 22i}{20}$

$= \dfrac{-4}{20} - \dfrac{22}{20}i$

$= -\dfrac{1}{5} - \dfrac{11}{10}i$

(a) $\bar{z} = \dfrac{-1}{5} + \dfrac{11}{10}i$

(b)

$z + \bar{z} = \left(-\dfrac{1}{5} - \dfrac{11}{10}i\right) + \left(-\dfrac{1}{5} + \dfrac{11}{10}i\right)$

$= -\dfrac{1}{5} - \cancel{\dfrac{11}{10}i} - \dfrac{1}{5} + \cancel{\dfrac{11}{10}i}$

$= -\dfrac{1}{5} - \dfrac{1}{5}$

$= \dfrac{-1-1}{5}$

$= -\dfrac{2}{5}$

(c)

$z - \bar{z} = \left(-\dfrac{1}{5} - \dfrac{11}{10}i\right) - \left(-\dfrac{1}{5} + \dfrac{11}{10}i\right)$

$= -\cancel{\dfrac{1}{5}} - \dfrac{11}{10}i + \cancel{\dfrac{1}{5}} - \dfrac{11}{10}i$

$= -\dfrac{11}{10}i - \dfrac{11}{10}i = \dfrac{-11i - 11i}{10}$

$= -\dfrac{\cancel{22}^{11}i}{\cancel{10}^{5}}$

$= -\dfrac{11}{5}i$

(d) $z\bar{z} = \left(-\dfrac{1}{5} - \dfrac{11}{10}i\right)\left(-\dfrac{1}{5} + \dfrac{11}{10}i\right)$

$= \left(-\dfrac{1}{5}\right)^2 - \left(\dfrac{11}{10}i\right)^2$

$= \dfrac{1}{25} - \dfrac{121}{100}i^2$

$= \dfrac{1}{25} - \dfrac{121}{100}(-1)$

$= \dfrac{1}{25} + \dfrac{121}{100}$

$= \dfrac{4 + 121}{100}$

$= \dfrac{125}{100}$

$= \dfrac{5}{4}$ **Ans**

Q.6 **If $z = 2 + 3i$ and show that.**

(i) $\overline{z + w} = \bar{z} + \bar{w}$

Solution: $\overline{z + w} = \bar{z} + \bar{w}$

$z + w = 2 + 3i + 5 - 4i$

$= 2 + 5 + 3i - 4i$

$= 7 - i$

L.H.S $= \overline{z + w}$

$= \overline{7 - i}$

$= 7 + i$ … (i)

R. H. S $= \bar{z} + \bar{w}$

$= \left(\overline{2 + 3i}\right) + \left(\overline{5 - 4i}\right)$

$= 2 - 3i + 5 + 4i$

$= 2 + 5 - 3i + 4i$

$= 7 + i$ ... (ii)

From (i) and (ii) we get

L.H.S=R.H.S

$\overline{z+w} = \bar{z} + \bar{w}$

Hence proved

(ii) $\overline{z-w} = \bar{z} - \bar{w}$

Solution: $\overline{z-w} = \bar{z} - \bar{w}$

$z - w = (2+3i) - (5-4i)$

$= 2 + 3i - 5 + 4i$

$= 2 - 5 + 3i + 4i$

$= -3 + 7i$

L.H.S $= \overline{z-w}$

$= \overline{-3+7i}$

$= -3 - 7i$ ...(i)

R.H.S $= \bar{z} - \bar{w}$

$= \left(\overline{2+3i}\right) - \left(\overline{5-4i}\right)$

$= 2 + 3i - (5+4i)$

$= 2 - 3i - 5 - 4i$

$= -3 - 7i$

From (i) and (ii) we get

L.H.S=R.H.S

$\overline{z-w} = \overline{z-w}$

Hence proved

(iii) $\overline{zw} = \bar{z}\,\bar{w}$

Solutions: $\overline{zw} = \bar{z}\,\bar{w}$

$zw = (2+3i)(5+4i)$

$= 2(5-4i) + 3i(5-4i)$

$= 10 - 8i + 15i - 12i^2$

$= 10 + 7i - 12(-1)$

$= 10 + 7i + 12$

$= 22 + 7i$

L.H.S $= \overline{zw}$

$= \overline{22+7i}$

$= 22 - 7i$

R.H.S $= \overline{zw}$

$= \left(\overline{2+3i}\right)\left(\overline{5-4i}\right)$

$= (2-3i)(5+4i)$

$= 2(5+4i) - 3i(5+4i)$

$= 10 + 8i - 15i - 12i^2$

$= 10 - 7i - 12(-1)$

$= 10 - 7i + 12$

$= 22 - 7i$

From (i) and (ii) we get

L.H.S=R.H.S

$\overline{zw} = \bar{z}\bar{w}$

Hence proved

(iv) $\overline{\left[\dfrac{z}{w}\right]} = \dfrac{\bar{z}}{\bar{w}}$, where $\neq 0$

Solutions: $\overline{\left[\dfrac{z}{w}\right]} = \dfrac{\bar{z}}{\bar{w}}$

$\dfrac{z}{w} = \dfrac{2+3i}{5-4i} \times \dfrac{5+4i}{5+4i}$

$= \dfrac{2(5+4i) + 3i(5+4i)}{(5-4i)(5+4i)}$

$= \dfrac{10 + 8i + 15i + 12i^2}{(5)^2 - (4i)^2}$

$= \dfrac{10 + 23i + 12(-1)}{25 - 16i^2}$

$= \dfrac{10 + 23i - 12}{25 - (6(-1))}$

$= \dfrac{10 + 23i - 12}{25 + 16}$

$= \dfrac{-2 + 23i}{41}$

L.H.S$= \overline{\left(\dfrac{z}{w}\right)}$

$= \overline{\left(\dfrac{-2+23i}{41}\right)}$

$= \dfrac{-2}{41} - \dfrac{23}{41}i$ ... (i)

R.H.S $= \dfrac{\bar{z}}{\bar{w}}$

$$=\frac{\overline{(2+3i)}}{\overline{(5-4i)}}$$

$$=\frac{2-3i}{5+4i}$$

$$=\frac{2-3i}{5+4i}\times\frac{5-4i}{5-4i}$$

$$=\frac{2(5-4i)-3i(5-4i)}{(5+4i)(5-4i)}$$

$$=\frac{10-8i-15i+12i^2}{(5)^2-(4i)^2}$$

$$=\frac{10-23i+12(-1)}{25-16i^2}$$

$$=\frac{10-23i+12(-1)}{25-16(-1)}$$

$$=\frac{10-23i-12}{25+16}$$

$$=\frac{-2-23i}{41}$$

$$=\frac{-2}{41}-\frac{23}{41}i \qquad \ldots \text{(ii)}$$

From (i) and (ii) we get

L.H.S=R.H.S

Hence Proved

$$\overline{\left[\frac{z}{w}\right]}=\frac{\overline{z}}{\overline{w}}$$

(v) $\frac{1}{2}(z+\overline{z})$ **is the real part of** z**.**

Solution: $\frac{1}{2}(z+\overline{z})$

$$=\frac{1}{2}\left[(2+3i)+\overline{(2+3i)}\right]$$

$$=\frac{1}{2}\left[(2+3i)+(2-3i)\right]$$

$$=\frac{1}{2}\left[2+\cancel{3i}+2-\cancel{3i}\right]$$

$$=\frac{1}{2}[2+2]$$

$$=\frac{1}{\cancel{2}}\left[\cancel{4}^{2}\right]$$

$$=2=\operatorname{Re}(z)$$

$\frac{1}{2}(z+\overline{z})$ is the real part of z**. Ans**

(vi) $\frac{1}{2}(z-\overline{z})$ **is the imaginary part of** z**.**

Solution: $\frac{1}{2}(z-\overline{z})$

$$\frac{1}{2}(z-\overline{z})=$$

$$=\frac{1}{2}\left[(2+3i)-\overline{(2+3i)}\right]$$

$$=\frac{1}{2}\left[(2+3i)-(2-3i)\right]$$

$$=\frac{1}{2}\left[\cancel{2}+3i-\cancel{2}+3i\right]$$

$$=\frac{1}{\cancel{2}}\left[{}^{3}\cancel{6}i\right]$$

$$=3i$$

$$=\text{Imaginary}(z)$$

$\frac{1}{2}(z-\overline{z})$ is the imaginary part of z. **Ans**

Q.7 Solve the following equations for real x **and** y**.**

(i) $(2-3i)(x+yi)=4+i$

Solution: $(2-3i)(x+yi)=4+i$

$$x+yi=\frac{4+i}{2-3i}$$

$$x+yi=\frac{4+i}{2-3i}\times\frac{2+3i}{2+3i}$$

$$=\frac{4(2+3i)+i(2+3i)}{(2-3i)(2+3i)}$$

$$=\frac{8+12i+2i+3i^2}{(2)^2-(3i)^2}$$

$$=\frac{8+14i+3(-1)}{4-9i^2}$$

$=\dfrac{8+14i-3}{4-9(-1)}$

$=\dfrac{8-3+14i}{4+9}$

$=\dfrac{5+14i}{13}$

$x+yi=\dfrac{5}{13}+\dfrac{14}{13}i$

$x=\dfrac{5}{13}, y=\dfrac{14}{13}$ **Ans**

(ii) $(3-2i)(x+yi)=2(x-2yi)+2i-1$

Solution:

$(3-2i)(x+yi)=2(x-2yi)+2i-1$

$3(x+yi)-2i(x+yi)=2x-4yi+2i-1$

$3x+3yi-2xi-2yi^2=(2x-1)+i(2-4y)$

$3x+(3x-2x)i-2y(-1)=(2x-1)+i(2-4y)$

$3x+(3y-2x)i+2y=(2x-1)+i(2-4)$

$(3x+2y)+(3y-2x)i=(2x-1)+(2-4y)i$

Comparing the real and imaginary parts.

$3x+2y=2x-1$,

$3y-2x=2-4y$

$3x-2x+2y=-1$,

$3y-2x=2-4y$

$x+2y=-1$,

$-2x+3y+4y=2$

$-2x+7y=2$

$x+2y=-1$ ________ (i)

$-2x+7y=2$ ________ (ii)

Multiply equation (i) with (2)

$2(x+2y)=-1\times 2$

$2x+4y=-2$ ________ (iii)

$\cancel{2x}+4y=\cancel{-2}$

$\cancel{-2x}+7y=\cancel{2}$

$11y=0$

$y=\dfrac{0}{11}$

$y=0$

Putting $y=0$ in equation (i)

$x+2y=-1$

$x+2(0)=-1$

$x+0=-1$

$x=-1+0$

$x=-1$ **Ans**

(iii) $(3+4i)^2-2(x-yi)=x+yi$

Solution: $(3+4i)^2-2(x-yi)=x+yi$

$(3+4i)^2-2(x-yi)=x+yi$

$9+24i+16i^2-2x+2yi=x+yi$

$9+24i+16(-1)-2x+2yi=x+yi$

$9+24i-16-2x+2yi=x+yi$

$9+24i-16-2x=x+2yi-yi=0$

$9+24i-16-3x+yi=0$

$-3x+yi=-9-24i+16$

$-3x+yi=16-9-24i$

$$-3x + yi = 7 - 24i$$

Comparing the real and imaginary parts.

$$-3x = 7$$

$$y = -24$$

$$x = \frac{-7}{3}$$

$$y = -24$$ **Ans**

Last Updated: September 2020

Report any mistake at freeilm786@gmail.com