Unit 11: Parallelograms and Triangles

Overview

Parallelogram:

If two opposite sides of a quadrilateral are congruent and parallel, it is a parallelogram.

Medians

A line segment joining a vertex of a triangle to the mid-point of the opposite side is called median of the triangle.

Trisection

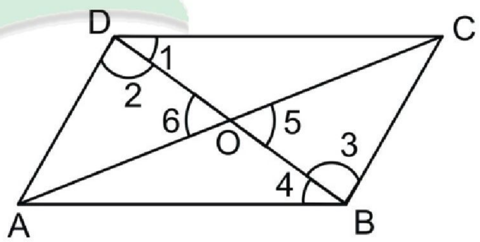
The process to divide a line segment into three equal parts.

Theorem 11.11

In a parallelograms
(i) Opposite sides are congruent
(ii) Opposite angles are congruent
(iii)The diagonals bisect each other

Given

In a quadrilateral $ABCD$, $\overline{AB} \parallel \overline{DC}, \overline{BC} \parallel \overline{AD}$ and the diagonals $\overline{AC}, \overline{BD}$ meet each other at point O.



To Prove

(i) $\overline{AB} \cong \overline{DC}, \overline{AD} \cong \overline{BC}$

(ii) $\angle ADC \cong \angle ABC,\ \angle BAD \cong \angle BCD$

(iii) $\overline{OA} \cong \overline{OC},\ \overline{OB} \cong \overline{OD}$

Construction

In the figure as shown, we label the angles as $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$, $\angle 5$ and $\angle 6$.

Proof

Statements	Reasons
(i) In $\Delta ABD \leftrightarrow \Delta CDB$	
$\angle 4 \cong \angle 1$	Alternate angles
$\overline{BD} \cong \overline{BD}$	Common
$\angle 2 \cong \angle 3$	Alternate angles
$\therefore \Delta ABD \cong \Delta CDB$	$A.S.A \cong A.S.A$
$So, \overline{AB}, \overline{DC}, \overline{AD} \cong \overline{BC}$	(Corresponding sides of congruent triangles)
and $\angle A \cong \angle C$	(Corresponding angles of congruent triangles)
(ii) Since	
and $\angle 1 \cong \angle 4$................(a)	Proved

$\angle 2 \cong \angle 3 ....(b)$	Proved
$\therefore m\angle 1 + m\angle 2 = m\angle 4 + m\angle 3$ or $m\angle ADC = m\angle ABC$ or $\angle ADC \cong \angle ABC$	From (a) and (b)
and $\angle BAD \cong m\angle BCD$	Proved in (i)
(iii) In $\Delta BOC \leftrightarrow \Delta DOA$	
$\overline{BC} \cong \overline{AD}$	Proved in (i)
$\angle 5 \cong \angle 6$	Vertical angles
$\angle 3 \cong \angle 2$	Proved
$\because \Delta BOC \cong \Delta DOA$	$(A.A.S \cong A.A.S)$
Hence $\overline{OC} \cong \overline{OA}, \overline{OB} \cong \overline{OD}$	(Corresponding sides of congruent triangles)

Example

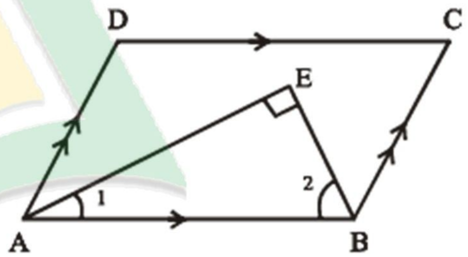
The bisectors of two angles on the same side of a parallelogram cut each other at right angles.

Given

A parallelogram ABCD, in which

$\overline{AB} \parallel \overline{DC}, \overline{AD} \parallel \overline{BC}$

The bisectors of EA and EB cut each other at E.



To Prove

$m\angle E = 90°$

Construction:

Name the angles $\angle 1$ and $\angle 2$ as shown in the figure.

Proof

Statements	Reasons
$m\angle 1 + m\angle 2$ $= \frac{1}{2}(m\angle BAD + m\angle ABC)$	$m\angle 1 = \frac{1}{2}m\angle BAD$, $m\angle 2 = \frac{1}{2}m\angle ABC$
$= \frac{1}{2}(180°)$ $= 90°$	int. angles on the same side of $\overline{AB}$ which cuts $\parallel$ segments $\overline{AD}$ and $\overline{BC}$ are supplementary.
Hence in $\Delta ABE, m\angle E = 90°$	$m\angle 1 + m\angle 2 = 90°$ (proved)

Last Updated: September 2020

Report any mistake at freeilm786@gmail.com