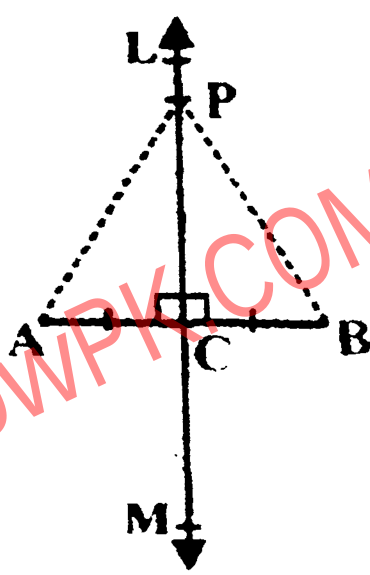
Unit 12
Line Bisectors And Angel Bisectors

THEOREM 12.1.1

Any point on the right bisector of a line segment is equidistant from its end points.



Solution:

Given:

A line $\overleftrightarrow{LM}$ intersects the line segment AB at point C such that $\overleftrightarrow{LM} \perp \overline{AB}$ and $\overline{AC} \cong \overline{BC}$.

To Prove:

$$\overline{PA} \cong \overline{PB}$$

Construction:

Take a point P on $\overleftrightarrow{LM}$. Join P to the points A and B.

Proof:

Statements	Reasons
in $\Delta ACP \leftrightarrow \Delta BCP$	
$\overline{AC} \cong \overline{BC}$	Given
$\angle ACP \cong \angle BCP$	Given $(\overline{PC} \perp \overline{AB})$
$\overline{PC} \cong \overline{PC}$	Common
$\Delta ACP \cong \Delta BCP$	S.A.S. Postulate
$\overline{PA} \cong \overline{PB}$	Corresponding sides of congruent triangles

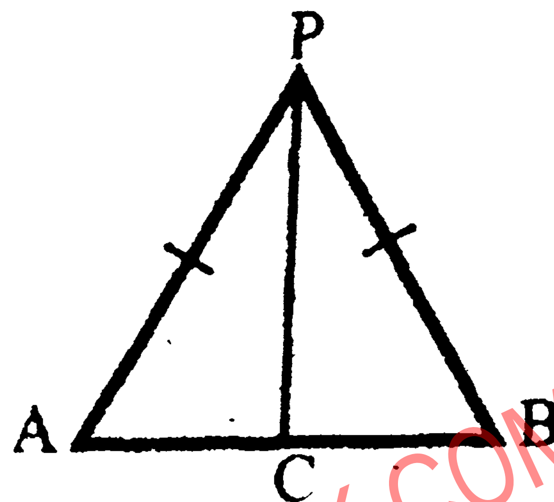
THEOREM 12.1.2

Any point equidistant from the end points of a line segment is on the right bisector of it.

Solution:

Given:

AB is a line segment. Point P is such that $\overline{PA} \cong \overline{PB}$



To Prove:

Point P is on the right bisector of $\overline{AB}$

Construction:

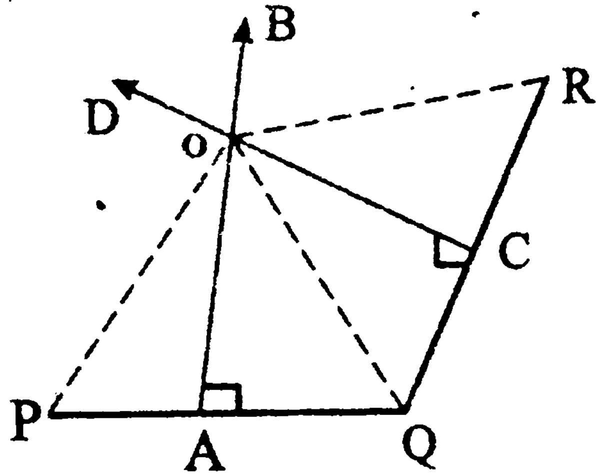
Join P to C, the midpoint of AB.

Proof:

Statements	Reasons
In $\Delta ACP \leftrightarrow \Delta BCP$	
$\overline{PA} \cong \overline{PB}$	Given
$\overline{PC} \cong \overline{PC}$	Common
$\overline{AC} \cong \overline{BC}$	Construction
$\therefore \Delta ACP \cong \Delta BCP$	S.S.S. ≅ S.S.S.
$\angle ACP \cong \angle BCP$ .... (i)	Corresponding angles of congruent triangles
But $m\angle ACP + \angle BCP = 180^0$ .... (ii)	Supplementary angles
$\therefore m\angle ACP + m\angle BCP = 90^0$	From (i) and (ii)
or $\overline{PC} \perp \overline{AB}$ .... (iii)	$m\angle ACP = 90^0$ (proved)
Also $\overline{CA} \cong \overline{AB}$ .... (iv)	Construction
$\therefore \overline{PC}$ is a right bisector of $\overline{AB}$ i.e. the point P is on the right bisector of $\overline{AB}$	from (iii) and (iv)

(ii) Take $\overline{AB}$ right bisector of $\overline{PQ}$ and $\overline{CD}$ right bisector of $\overline{QR}$. $\overline{AB}$ and $\overline{CD}$ intersect at O.

(iii) Join O to P, Q, R.
O is the place of Children Park.



Proof:

Statements	Reasons
$\overline{OP} \cong \overline{QR} = \overline{OR}$ (i)	O is on the right bisector PQ.
$\overline{OQ} \cong \overline{OR}$ (ii)	O is on the right bisector of QR.
$\therefore$ $\overline{OP} \cong \overline{OQ} \cong \overline{OR}$	From (i) and (ii)
Hence O is equidistant from P, Q, R.	

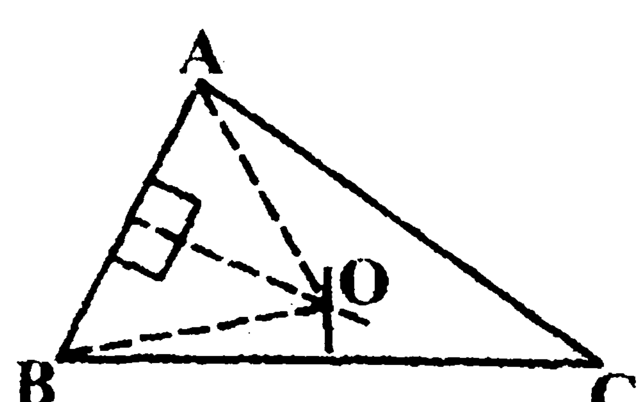
THEOREM 12.1.3

The right bisectors of the three sides of a triangle are concurrent.

Solution:

Given:
ABC is a triangle

To Prove:
The right bisectors of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ are concurrent.



Construction:

Draw the right bisectors of $\overline{AB}$ and $\overline{BC}$, which meet each other at the point O. Join O to A, B and C.

Proof:

Statements	Reasons
$\overline{OA} \cong \overline{OB}$ .... (i)	Each point on right bisector of a segment is equidistant from its end point.
$\overline{OB} \cong \overline{OC}$ .... (ii)	From (i)
$\overline{OA} \cong \overline{OC}$ .... (iii)	From (i) and (ii)
(iv) Point O is on the right bisector of $\overline{CA}$.	O is equidistant from A and C.
(v) Point O is on the right bisector of $\overline{AB}$ and $\overline{BC}$.	Construction
Thus, the right bisectors of the three sides of a triangle are concurrent.	From (iv) and (v)

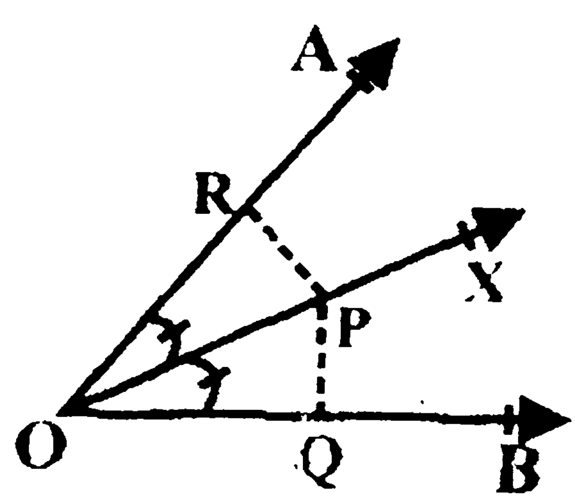
THEOREM 12.1.4

Each point on the bisector of an angle is equidistant from its arms.

Solution:

Given:

A point P is on $\overrightarrow{OX}$, the bisector of $\angle AOB$



To prove:

$\overline{PQ} \cong \overline{PR}$ i.e., P is equidistant from $\overrightarrow{OA}$ and $\overrightarrow{OB}$

Construction:

Draw $\overline{PR} \perp \overline{OA}$ and $\overline{PQ} \perp \overline{OB}$

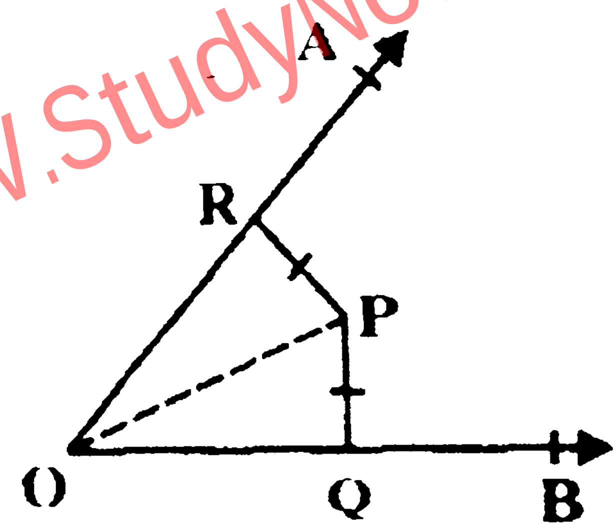
Proof:

Statements	Reasons
In $\Delta POQ \leftrightarrow \Delta POR$	
$\overline{OP} \cong \overline{OP}$	Common
$\angle PRO \cong \angle PQO$	Construction
$\angle POQ \cong \angle POR$	Given
$\therefore \Delta POQ \cong \Delta POR$	S.A.A. ≅ S.A.A.
and $\overline{PQ} \cong \overline{PR}$	Corresponding sides of congruent triangles

THEOREM 12.1.5
Converse of THEOREM 12.1.4

Any point inside an angle, equidistant from its arms is on the bisector of it.

Solution:



Given:

Any point P lies inside $\angle AOB$ such that $\overline{PQ} \cong \overline{PR}$, where $\overline{PQ} \perp \overrightarrow{OB}$ and $\overline{PR} \perp \overrightarrow{OA}$

To prove:

Point P is on the bisector of $\angle AOB$.

Construction:

Join P to O.

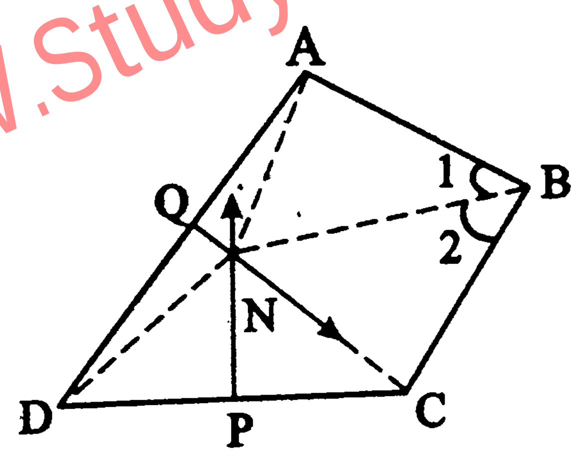
Proof:

Statements	Reasons
In $\Delta POQ \leftrightarrow \Delta POR$	
$\angle PQO \cong \angle PRO$	Given (right angles)
$\overline{PO} \cong \overline{PO}$	Common
$\overline{PQ} \cong \overline{PR}$	Given
$\therefore \quad \Delta POQ \cong \Delta POR$	H.S ≅ H.S
and $\angle POQ \cong \angle POR$ ..... (i)	Corresponding angles of congruent triangles
Hence P is on the bisector of $\angle AOB$	From (i) (proved)

EXERCISE 12.2

Q1. In a quadrilateral ABCD, $\overline{AB} \cong \overline{BC}$ and the right bisectors of $\overline{AD}$, $\overline{CD}$ meet each other at point N. Prove that $\overline{BN}$ is a bisector of $\angle ABC$.

Solution:



Given:

In the quadrilateral ABCD, $\overline{AB} \cong \overline{BC}$
$\overline{NP}$ is right bisector of $\overline{CD}$ and
$\overline{NQ}$ is right bisector of $\overline{AD}$.
They meet at N.

To Prove:

$\overline{BN}$ is a bisector of $\angle ABC$

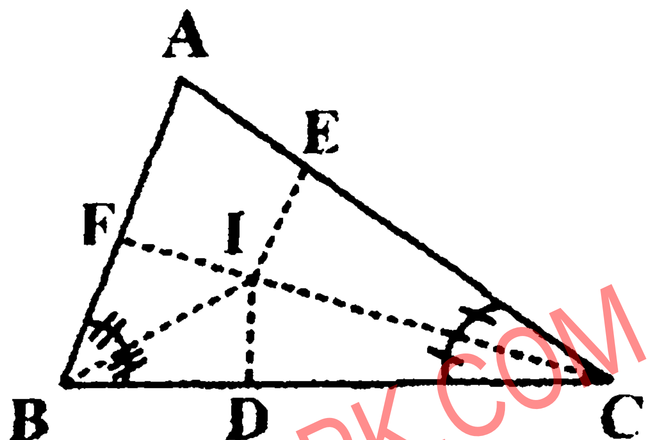
Construction:

Join N to A, B, C, D.

Hence $\overline{PO}$ is bisector of $\angle P$ or Bisector of $\angle P$ also passesthrough O.	

THEOREM 12.1.6

The bisectors of the angles of a triangle are concurrent.



Solution:

Given:

ABC is a triangle.

To prove:

The bisectors of $\angle A$, $\angle B$ and $\angle C$ are concurrent.

Construction:

Draw the bisectors or $\angle B$ and $\angle C$ which intersect at point I. From I, draw $\overline{IF} \perp \overline{AB}$, $\overline{IE} \perp \overline{CA}$ and $\overline{ID} \perp \overline{BC}$

Proof:

Statements	Reasons
$\overline{ID} \cong \overline{IF}$ Similarly,	A point on bisector of an angle is equidistant from its arms
$\overline{ID} \cong \overline{IE}$ $\overline{IE} \cong \overline{IF}$ So, the point I is on the bisector of $\angle A$ .... (i) Also the point I is on the bisectors of $\angle ABC$ and $\angle BCA$ ...... (ii)	Each is congruent to ID (proved)
Thus, the bisectors of $\angle A$, $\angle B$ and $\angle C$ are concurrent.	From (i) and (ii)

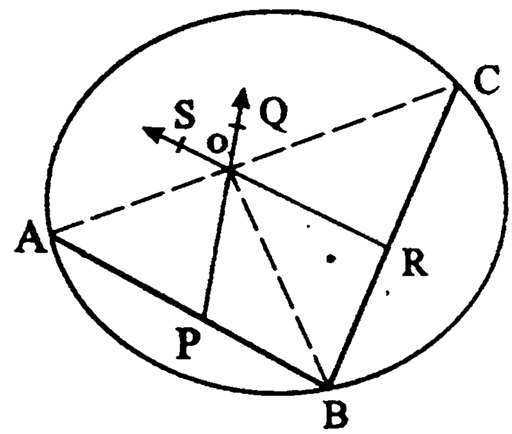
EXERCISE 12.1

Q1. Prove that the centre of a circle is on the right bisectors of each of its chords.

Solution:

Given:

A, B, C are three non-collinear points.



Required:

To find the centre of the circle passing through A, B, C

Construction:

(i) Join B to A,C

(ii) Take $\overrightarrow{PQ}$ right bisector of $\overline{AB}$ and $\overrightarrow{RS}$ right bisector of $\overline{BC}$. They intersect at O.

(iii) Join O to A, B, C

O is the centre of the circle.

Proof:

Statements	Reasons
$In\ \overline{OA} \cong \overline{OB}$ (i)	O is on right bisector of AB
$\overline{OB} \cong \overline{OC}$ (ii)	O is on right bisector of BC
$\therefore \overline{OA} \cong \overline{OB} \cong \overline{OC}$ (iii)	From (i), (ii)
Hence O is equidistant from A, B, C.	
Therefore O is the required centre of the circle.	

Q2. Where will be the centre of a circle passing through three non-collinear points?

Solution:

Given:

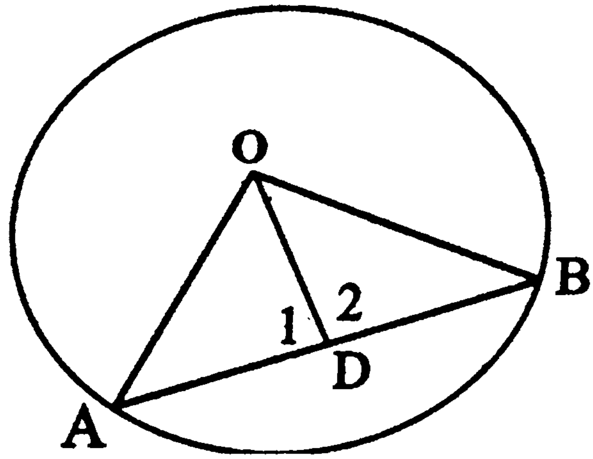
O is the centre of a circle. $\overline{AB}$ is any chord of the circle.

To Prove:

O is right bisector of $\overline{AB}$.

Construction:

Take mid point D of AB and join D to O.



Proof:

Statements	Reasons
In $\Delta AOD \leftrightarrow \Delta BOD$	
$\overline{(OA)} \cong \overline{(OB)}$	Radii of same circle
$\overline{(OD)} \cong \overline{(OD)}$	Common
$\overline{(AD)} \cong \overline{(BD)}$	Construction
$\therefore\ \Delta AOD \cong \Delta BOD$	S.S.S.≅ S.S.S.
But $m\angle 1 \cong m\angle 2 = 180^0$	Supplementary angles
$\therefore\ m\angle 1 + m\angle 2 = 180^o$	From (i)
$2m\angle 1 = 180^o$	
$m\angle 1 = 90^o$	
$\therefore$ DO is riht bisector of AB. i.e. O is on the right bisector of AB.	

Q3. Three villages P, Q and R are not on the same line. The people of these villages want to make a Children Park at such a place which is equidistant from these three villages. After fixing the place of Children Park, prove that the park is equidistant from three villages.

Solution:

Given:

P, Q, R are three villages on the same straight line

To prove:

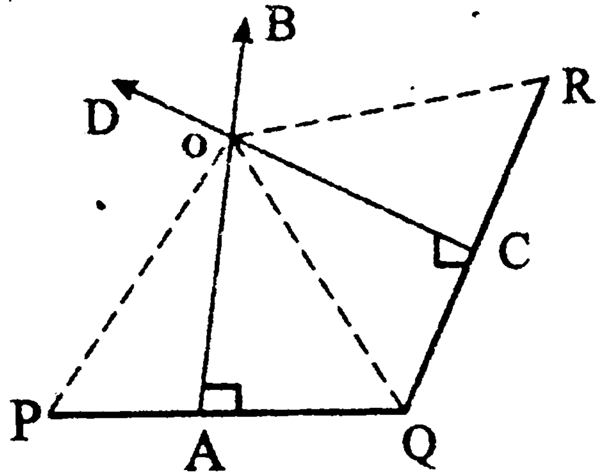
To find the point equidistant from P, Q, R.

Construction:

(i) Join Q to P and R.

(ii) Take $\overline{AB}$ right bisector of $\overline{PQ}$ and $\overline{CD}$ right bisector of $\overline{QR}$. $\overline{AB}$ and $\overline{CD}$ intersect at O.

(iii) Join O to P, Q, R.
O is the place of Children Park.



Proof:

Statements	Reasons
$\overline{OP} \cong \overline{QR} = \overline{OR}$ (i)	O is on the right bisector PQ.
$\overline{OQ} \cong \overline{OR}$ (ii)	O is on the right bisector of QR.
$\therefore$ $\overline{OP} \cong \overline{OQ} \cong \overline{OR}$	From (i) and (ii)
Hence O is equidistant from P, Q, R.	

THEOREM 12.1.3

The right bisectors of the three sides of a triangle are concurrent.

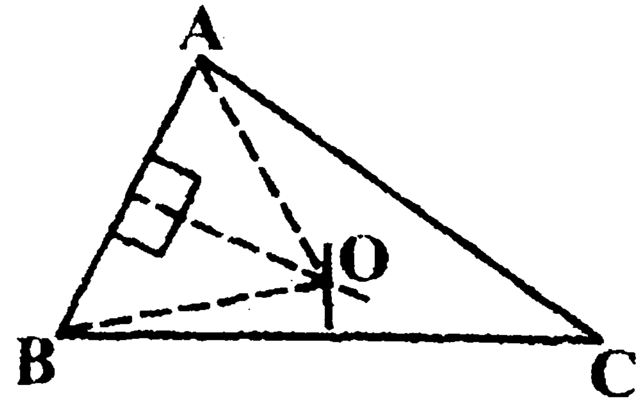
Solution:

Given:

ABC is a triangle

To Prove:

The right bisectors of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ are concurrent.



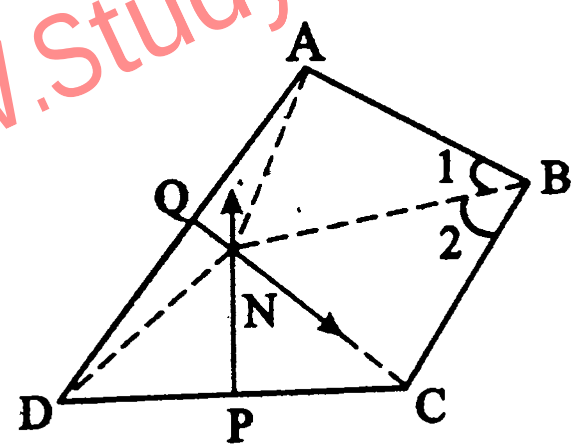
Proof:

Statements	Reasons
In $\Delta POQ \leftrightarrow \Delta POR$	
$\angle PQO \cong \angle PRO$	Given (right angles)
$\overline{PO} \cong \overline{PO}$	Common
$\overline{PQ} \cong \overline{PR}$	Given
$\therefore \Delta POQ \cong \Delta POR$	H.S ≅ H.S
and $\angle POQ \cong \angle POR$ ..... (i)	Corresponding angles of congruent triangles
Hence P is on the bisector of $\angle AOB$	From (i) (proved)

EXERCISE 12.2

Q1. In a quadrilateral ABCD, $\overline{AB} \cong \overline{BC}$ and the right bisectors of $\overline{AD}$, $\overline{CD}$ meet each other at point N. Prove that $\overline{BN}$ is a bisector of $\angle ABC$.

Solution:



Given:

In the quadrilateral ABCD, $\overline{AB} \cong \overline{BC}$
$\overline{NP}$ is right bisector of $\overline{CD}$ and
$\overline{NQ}$ is right bisector of $\overline{AD}$.
They meet at N.

To Prove:

$\overline{BN}$ is a bisector of $\angle ABC$

Construction:

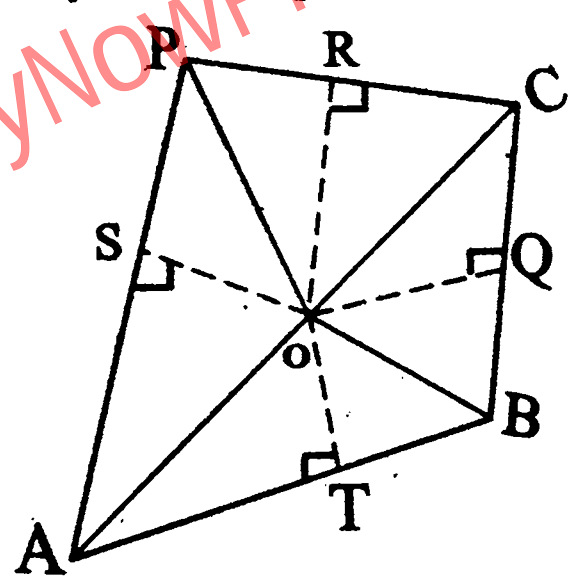
Join N to A, B, C, D.

Proof:

Statements	Reasons
$\overline{ND} \cong \overline{NC}$ (i)	N is on right bisector of $\overline{DC}$
$\overline{ND} \cong \overline{NA}$ (ii)	N is on right bisector of $\overline{AC}$
$\overline{NA} \cong \overline{NC}$ (iii)	From (i), (ii)
In $\Delta BNA \leftrightarrow \Delta BNC$	
$\overline{NA} \cong \overline{NC}$	From (iii)
$\overline{AB} \cong \overline{CD}$	Given
$\overline{BN} \cong \overline{BN}$	Common
$\therefore$ $\Delta BNA \leftrightarrow \Delta BNC$	S.S.S.≅S.S.S.
Hence $\angle 1 \cong \angle 2$	Corresponding angles of congruent triangles.
Hence $\overline{BN}$ is bisector of $\angle ABC$.	

Q2. The bisectors of $\angle A, B$ and $\angle C$ of a quadrilateral ABCP meet each other at point O, prove that the bisector of $\angle P$ will also pass through the point O.

Solution:



Given:

ABCP is a quadrilateral.
$\overline{Ao}, \overline{BO}, \overline{CO}$ are bisector of $\angle A, \angle B, \angle C$, respectively.
P is joined to O.

To prove:

PO is bisector of $\angle P$

Construction:

From O draw $\overline{OT} \perp \overline{AB}$ $\overline{OQ} \perp \overline{BC}, \overline{OR} \perp \overline{PC}$ and $\overline{OS} \perp \overline{AP}$ respectively.

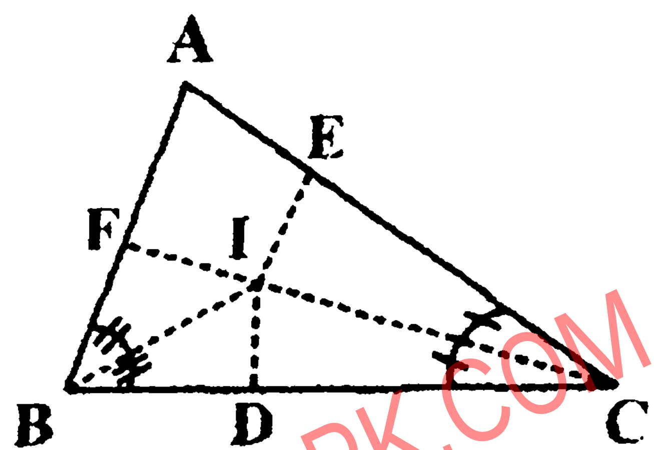
Proof:

Statements	Reasons
$\overline{OS} \cong \overline{OT}$ (i)	AO is bisector of $\angle$A
$\overline{OT} \cong \overline{OQ}$ (ii)	BO is bisector of $\angle B$
$\overline{OQ} \cong \overline{OR}$ (iii)	CO is bisector of $\angle C$
$\therefore$ $\overline{OS} \cong \overline{OR}$	From (i), (ii), (iii)
$\therefore$ O is on bisector of $\angle P$.	

Hence $\overline{PO}$ is bisector of $\angle P$ or Bisector of $\angle P$ also passesthrough O.	

THEOREM 12.1.6

The bisectors of the angles of a triangle are concurrent.



Solution:

Given:

ABC is a triangle.

To prove:

The bisectors of $\angle A$, $\angle B$ and $\angle C$ are concurrent.

Construction:

Draw the bisectors or $\angle B$ and $\angle C$ which intersect at point I. From I, draw $\overline{IF} \perp \overline{AB}$, $\overline{IE} \perp \overline{CA}$ and $\overline{ID} \perp \overline{BC}$

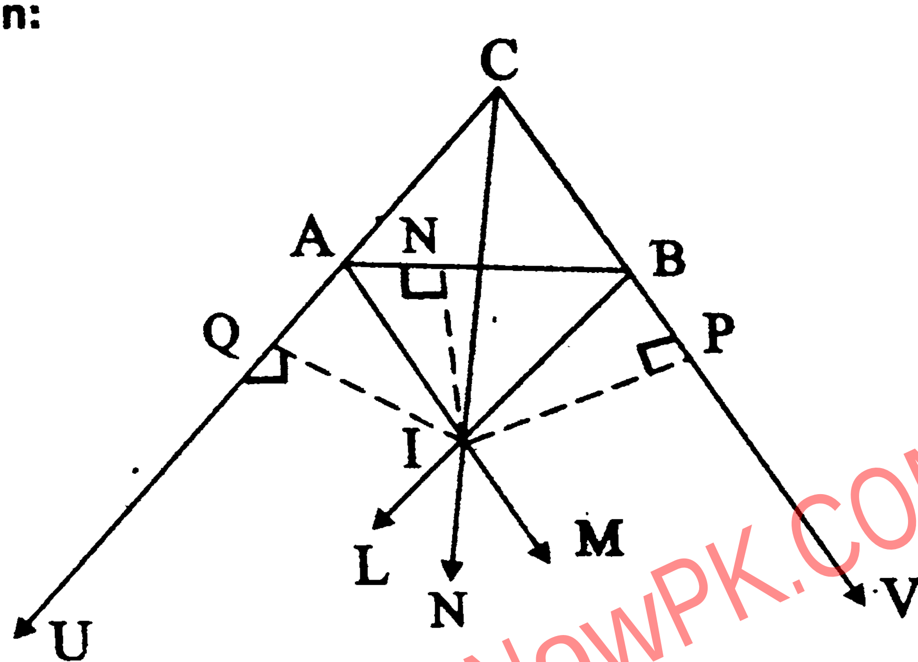
Proof:

Statements	Reasons
$\overline{ID} \cong \overline{IF}$ Similarly,	A point on bisector of an angle is equidistant from its arms
$\overline{ID} \cong \overline{IE}$ $\overline{IE} \cong \overline{IF}$	Each is congruent to ID (proved)
So, the point I is on the bisector of $\angle A$ .... (i) Also the point I is on the bisectors of $\angle ABC$ and $\angle BCA$ ...... (ii)	
Thus, the bisectors of $\angle A$, $\angle B$ and $\angle C$ are concurrent.	From (i) and (ii)

EXERCISE 12.3

Q1. Prove that the bisectors of the angles of base of an isosceles triangle intersect each other on its altitude.

Solution:



Given:

In ΔABC, sides $\overline{CA}$ and $\overline{CB}$ are produced.

$\overrightarrow{BL}$ is bisector of $\angle ABV$.

$\overline{AM}$ is bisector of $\angle BAU$.

$\overline{BL}$ and $\overline{AM}$ is intersect at I.

C is joined to I,

To Prove:

CI is bisector of $\angle C$

Construction:

Draw $IP \perp CV, IQ \perp CU$ and $\overline{IN} \perp \overrightarrow{AB}$.

Proof:

Statements	Reasons
$\overline{IN} \cong \overline{IP}$ (i)	$\overline{BI}$ is bisector of $\angle ABV$
$\overline{IN} \cong \overline{IQ}$ (ii)	$\overline{AI}$ is a bisector of $\angle BAU$
$\overline{IP} \cong \overline{IQ}$	From (i) and (ii)
Now $\overline{IP}$ and $\overline{IQ}$ are perpendicular to $\overline{CB}$ and $\overline{CA}$ produced CI is bisector of angles $\angle C$.	

REVIEW EXERCISE 12

Q1. Which of the following are true and which are false?

(i) Bisection means to divide into two parts.

(ii) Right bisection of line segment means to draw perpendicular which passes through the mid-point of line segment.

(iii) Any point on the right bisector of a line segment is not equidistant from its end points.

(iv) Any point equidistant from the end points of a line segment is on the right bisector of it.

(v) The right bisector of the sides of a triangle is not concurrent.

(vi) The bisectors of the angles of a triangle are concurrent.

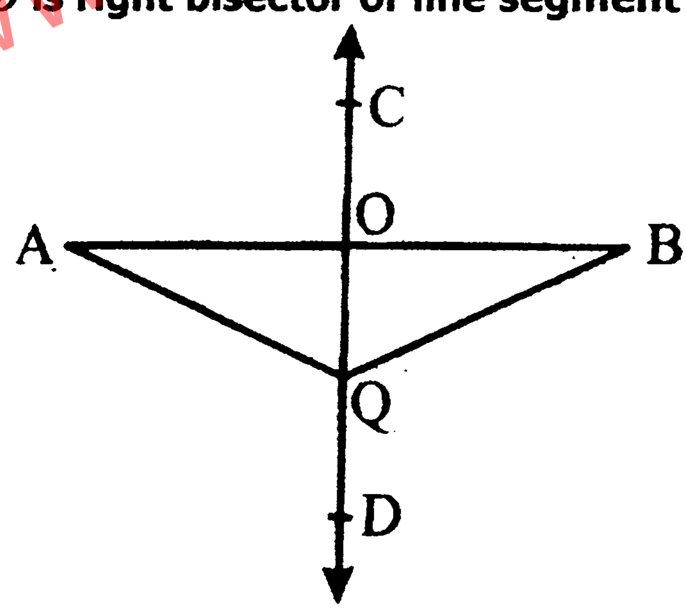
(vii) Any point on the bisector of an angle is not equidistant from its arm.

(viii) Any point inside an angle, equidistant from its arms, is on the bisector of it.

Answers:

(i) T	(ii) T	(iii) F	(iv) T
(v) F	(vi) T	(vii) F	(viii) T

Q2. If $\overleftrightarrow{CD}$ is right bisector of line segment $\overline{AB}$, then



(i) $m\overline{OA} = \ldots\ldots\ldots$ (ii) $m\overline{AQ} = \ldots\ldots\ldots$

Answers:

(i) $m\overline{OB}$	(ii) $m\overline{BQ}$

Q3. Define the following.

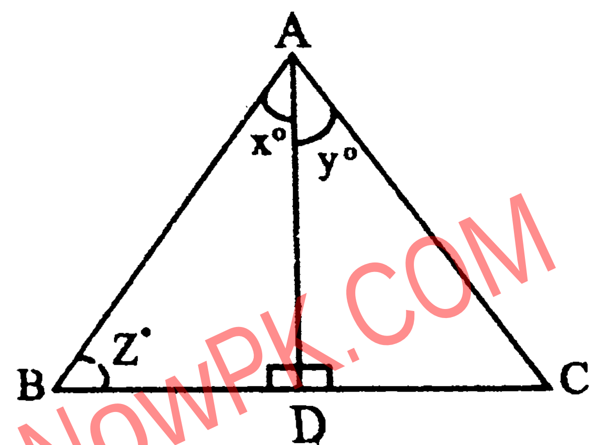
(i) Bisector of a line segment:

A line passing through the midpoint of a segment is called the bisector of line segment.

(ii) Bisector of an angle:

A ray that bisects an angle is called bisector of the angle.

Q4. The given triangle ABC is equilateral triangle and $\overline{AD}$ is bisector of angle A, then find the values of unknown x^0, y^0 and z^0.



Solution:

ΔABC is equilateral

$m\angle A = m\angle B = m\angle C = 60^o$

$\therefore \quad z^o = 60^o$

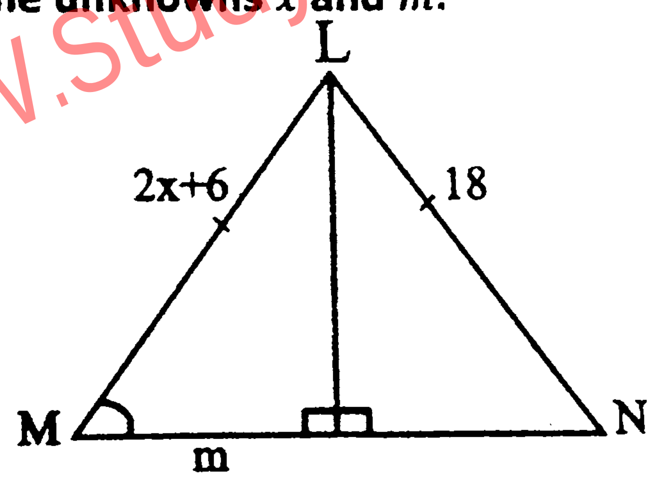
$\overline{AD}$ is bisector of $\angle A$

$x^o = y^o = \frac{1}{2} m\angle A$

$= \frac{1}{2}(60^o) = 30^o$

$\therefore \quad x^0 = y^0 = 30^0$

Q5. In the given congruent triangles LMO and LNO, find the unknowns x and m.



Solution:

Corresponding sides of congruent triangles ΔLMO and ΔLNO.

$\overline{LM} \cong \overline{LN}$

$\therefore \quad 2x + 6 = 18$

$\Rightarrow \quad 2x = 18 - 6 = 12$

$x = \frac{12}{6} = 6$

- Given that $m\,\overline{ON} = 12$

  Since given triangles are congruent therefore

  $m\,\overline{OM} = m\,\overline{ON} = 12$

  $m\,\overline{OM} = m = 12$

Q6. $\overline{CD}$ is the right bisector of the line segment AB.

(i) If $m\,\overline{AB} = 6\ cm$, then find the $m\,\overline{AL}$ and $m\,\overline{LB}$

(ii) If $m\,\overline{BD} = 4\ cm$, then find the $m\,\overline{AD}$

Solution:

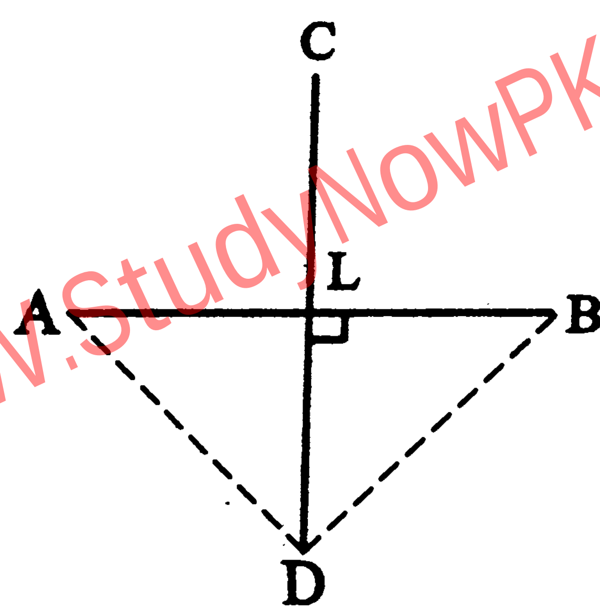
CD is right bisector

$\therefore \quad \overline{AL} \cong \overline{BL}$

$\therefore \quad m\overline{AL} = m\overline{BL}$

$$= \frac{1}{2}(m\overline{AB}) = \frac{1}{2}(6\ cm) = 3cm$$

$m\overline{AL} = m\overline{BL} = 3\ cm$



In $\quad \Delta ALD \leftrightarrow \Delta BLD$

$\overline{AL} \cong \overline{BL}$

$\angle ALD \cong \angle BLD$

and $\quad DL \cong DL$

$\therefore \quad \Delta ALD \cong \Delta BLD$

So $\quad m\overline{AD} \cong m\overline{BD} = 4cm$

$m\overline{AD} = 4cm$