MATHEMATICS HSSC–I

Time allowed: 2:35 Hours **Total Marks Sections B and C: 80**

NOTE: Attempt any ten parts from Section 'B' and any five questions from Section 'C' on the separately provided answer book. Use supplementary answer sheet i.e. Sheet–B if required. Write your answers neatly and legibly. Graph paper will be provided on request.

SECTION – B (Marks 40)

Q. 2 Attempt any TEN parts. All parts carry equal marks. **(10 x 4 = 40)**

(i) Express the complex number $1+i\sqrt{3}$ in polar form.

(ii) Show that $(A\cup B)' = A'\cap B'$ (Demorgan's Law). Where A and B are subsets of a universal set U.

(iii) If a,b are elements of a group G under the operation of multiplication. Then show that $(ab)^{-1} = b^{-1}a^{-1}$

(iv) If $A=\left[a_{ij}\right]_{3\times 3}$, and $\lambda\in\mathbb{R}$, then show that $\lambda A - A = (\lambda-1)A$

(v) Determine whether $p\rightarrow(q\rightarrow p)$ is a tautology, a contingency or an absurdity.

(vi) Discuss the nature of roots of $2x^2-5x+1=0$.

(vii) If a number exceeds its square root by 56. Find the number.

(viii) Find the $13th$ term of the sequence $x, 1, 2-x, 3-2x, \ldots$

(ix) Find the sum of n terms of the series whose n^{th} term is $3n^2+n+1$.

(x) A box contains 10 red, 30 white and 20 black marbles. A marble is drawn at random. Find the probability that it is either red or white.

(xi) If x is so small that its square and higher powers can be neglected, then show that $\dfrac{\sqrt{4+x}}{(1-x)^3}\cong 2+\dfrac{25}{4}x$

(xii) Prove that $\dfrac{\sin^2(\pi+\theta)\tan(\frac{3\pi}{2}+\theta)}{\cot^2(\frac{3\pi}{2}-\theta)\cos^2(\pi-\theta)\operatorname{cosec}(2\pi-\theta)} = \cos\theta$

(xiii) If a triangle ABC is with $a=\sqrt{3}-1$, $b=\sqrt{3}+1$ and $\gamma=60°$ then find c.

(xiv) Without using calculator or table, prove that $2\tan^{-1}\dfrac{1}{3}+\tan^{-1}\dfrac{1}{7}=\dfrac{\pi}{4}$

SECTION – C (Marks 40)

Note: Attempt any FIVE questions. All questions carry equal marks. **(5 x 8 = 40)**

Q. 3 Find the value of λ for which the system:

$$x+y+z=0$$
$$2x+y-\lambda z=0$$
$$x+2y-2z=0$$

has a non-trivial solution. Also solve the system.

Q. 4 Show that roots of $x^2+(mx+c)^2=a^2$ will be equal if $c^2=a^2(1+m^2)$

Q. 5 Sum the following series to n terms: $\dfrac{1^2}{1}+\dfrac{1^2+2^2}{2}+\dfrac{1^2+2^2+3^2}{3}+\ldots$ to n terms.

Q. 6 By the principle of mathematical induction, show that $x+y$ is a factor of $x^{2n-1}+y^{2n-1}$ $(x\neq -y)$, for all positive integer n.

Q. 7 Without using calculator / table, prove that $\sin 10°\sin 30°\sin 50°\sin 70° = \dfrac{1}{16}$

Q. 8 In a triangle ABC, with usual notations, prove that:

Area of Triangle $\Delta=\sqrt{s(s-a)(s-b)(s-c)}$ (The Hero's Formula)

Q. 9 Solve the trigonometric equation $\cos\theta+\cos 3\theta+\cos 5\theta+\cos 7\theta=0$ for its general solution.