Exercise 4.1

Q.1 Identify whether the following algebraic expressions are polynomials (Yes or No).

(i) $3x^2+\frac{1}{x}-5$

No (Because of $\frac{1}{x}$) Ans.

(ii) $3x^3-4x^2-x\sqrt{x}+3$

No (Because $\sqrt{x}$ or $(x)^{\frac{1}{2}}$) Ans.

(iii) $x^2-3x+\sqrt{2}$

Yes (Because no variable has power in fraction). **Ans**

(iv) $\frac{3x}{2x-1}+8$

No (Because of $\frac{1}{2x-1}$) **Ans**

Q.2 State whether each of the following expressions is a rational expression or not.

(i) $\frac{3\sqrt{x}}{3\sqrt{x}+5}$

Irrational **Ans**

(ii) $\frac{x^3-2x^3+\sqrt{3}}{2+3x-x^2}$

Rational **Ans**

(iii) $\frac{x^2+6x+9}{x^2-9}$

Rational **Ans**

(iv) $\frac{2\sqrt{x}+3}{2\sqrt{x}-3}$

Irrational **Ans**

Q.3 Reduce the following expression to the lowest form.

(i) $\frac{120x^2y^3z^5}{30x^3yz^2}$

Solution: $\frac{{}^{4}\cancel{120}x^2y^3z^5}{\cancel{30}x^3yz^2}$

$=\frac{120x^2y^3z^5}{30x^3yz^2}$

$=4x^{2-3}y^{3-1}z^{5-2}$

$=4x^{-1}y^2z^3$

$=\frac{4y^2z^3}{x}$ **Ans**

(ii) $\frac{8a(x+1)}{2(x^2-1)}$

Solution: $\frac{8a(x+1)}{2(x^2-1)}$

$=\frac{{}^{4}\cancel{8}a(x+1)}{\cancel{2}(x^2-1)}$

$=\frac{4a\cancel{(x+1)}}{(x-1)\cancel{(x+1)}}$

$=\frac{4a}{x-1}$ **Ans**

(iii) $\frac{(x+y)^2-4xy}{(x-y)^2}$

Solution: $\frac{(x+y)^2-4xy}{(x-y)^2}$

$\therefore (x+y)^2=x^2+y^2+2xy$

$\therefore (x-y)^2=x^2+y^2-2xy$

$=\frac{x^2+y^2+2xy-4xy}{x^2+y^2-2xy}$

$=\frac{x^2+y^2-2xy}{x^2+y^2-2xy}$

$$=\frac{\cancel{(x-y)^2}}{\cancel{(x-y)^2}}$$

$= 1$ **Ans**

(iv) $\dfrac{(x^3-y^3)(x^2-2xy+y^2)}{(x-y)(x^2+xy+y^2)}$

Solution: $\dfrac{(x^3-y^3)(x^2-2xy+y^2)}{(x-y)(x^2+xy+y^2)}$

$$(a^3+b^3)=(a-b)(a^2+ab+b^2)$$

$$=\frac{\cancel{(x^3-y^3)}(x^2-2xy+y^2)}{\cancel{(x^3-y^3)}}$$

$$=x^2-2xy+y^2$$

$$\therefore (x-y)^2=x^2-2xy+y^2$$

$=(x-y)^2$ **Ans**

(v) $\dfrac{(x+2)(x^2-1)}{(x+1)(x^2-4)}$

Solution: $\dfrac{(x+2)(x^2-1)}{(x+1)(x^2-4)}$

$$=\frac{(x+2)\left[(x)^2-(1)^2\right]}{(x+1)\left[(x)^2-(2)^2\right]}$$

$$=\frac{\cancel{(x+2)}(x-1)\cancel{(x+1)}}{\cancel{(x+1)}(x-2)\cancel{(x+2)}}$$

$=\dfrac{(x-1)}{(x-2)}$ **Ans**

(vi) $\dfrac{x^2-4x+4}{2x^2-8}$

Solution: $\dfrac{x^2-4x+4}{2x^2-8}$

$$\therefore (a-b)^2=a^2-2ab+b^2$$

$$\therefore a^2-b^2=(a+b)(a-b)$$

$$=\frac{(x)^2-2(x)(2)+(2)^2}{2(x^2-4)}$$

$$=\frac{(x-2)^2}{2\left[(x)^2-(2)^2\right]}$$

$$=\frac{(x-2)^2}{2(x+2)(x-2)}$$

$$=\frac{(x-2)\cancel{(x-2)}}{2(x+2)\cancel{(x-2)}}$$

$=\dfrac{x-2}{2(x+2)}$ **Ans**

(vii) $\dfrac{64x^5-64x}{(8x^2+8)(2x+2)}$

Solution: $\dfrac{64x^5-64x}{(8x^2+8)(2x+2)}$

$$=\frac{64x(x^4-1)}{8(x^2+1).2(x+1)}$$

$$=\frac{64\left[(x^2)^2-(1)^2\right]}{16(x^2+1)(x+1)}$$

$$=\frac{{}^{4}\cancel{64}(x^2-1)\cancel{(x^2+1)}}{\cancel{16}\cancel{(x^2+1)}(x+1)}$$

$$=\frac{4x(x-1)\cancel{(x+1)}}{\cancel{(x+1)}}$$

$=4x(x-1)$ **Ans**

(viii) $\dfrac{9x^2-(x^2-4)^2}{4+3x-x^2}$

Solution: $\dfrac{9x^2-(x^2-4)^2}{4+3x-x^2}$

$$=\frac{(3x)^2-(x^2-4)^2}{4+3x-x^2}$$

$$=\frac{(3x+x^2-4)(3x-x^2+4)}{4+3x-x^2}$$

$$=\frac{(x^2+3x-4)\cancel{(-x^2+3x+4)}}{\cancel{(-x^2+3x+4)}}$$

$$=x^2+3x-4 \textbf{ Ans}$$

Q.4 Evaluate

(a) $\dfrac{x^3y-2z}{xz}$ **for**

(i) $x=3, y=-1, z=-2$

(ii) $x=-1, y=-9, z=4$

Solution for 1st part

When $x=3,\ y=-1,\ z=-2$

$$\frac{x^3y-2z}{xz}=$$

$$=\frac{(3)^3(-1)-2(-2)}{(3)(-2)}$$

$$=\frac{27(-1)+4}{-6}$$

$$=\frac{-27+4}{-6}$$

$$=\frac{-23}{-6}$$

$$=\frac{23}{6} \textbf{ Ans}$$

Solution for 2nd Part.

When $x=-1,\ y=-9,\ z=4$

$$\frac{x^3y-2z}{xz}=$$

$$=\frac{(-1)^3(-9)-2(4)}{(-1)(4)}$$

$$=\frac{-1(-9)-8}{-4}$$

$$=\frac{9-8}{-4}$$

$$=\frac{1}{-4}$$

$$=-\frac{1}{4} \textbf{ Ans}$$

(b) $\dfrac{x^2y^2-5z^4}{xyz}$ for $x=4, y=-2$ and $z=-1$

Solution: $\dfrac{x^2y^3-5z^4}{xyz}$

$$=\frac{(4)^2(-2)^3-5(-1)^4}{(4)(-2)(-1)}$$

$$=\frac{16(-8)-5(1)}{8}$$

$$=\frac{16(-8)-5(1)}{8}$$

$$=\frac{-128-5}{8}$$

$$=-\frac{133}{8}$$

$$=-16\frac{5}{8} \textbf{ Ans}$$

Q.5 Perform the indicated operation and simplify.

(i) $\dfrac{15}{2x-3y}-\dfrac{4}{3y-2x}$

Solution: $\dfrac{15}{2x-3y}-\dfrac{4}{3y-2x}$

$$=\frac{15}{2x-3y}-\frac{4}{-2x+3y}$$

$$=\frac{15}{2x-3y}-\frac{4}{-(2x-3y)}$$

$$=\frac{15}{2x-3y}+\frac{4}{2x-3y}$$

$$=\frac{19}{2x-3y} \textbf{ Ans}$$

(ii) $\dfrac{1+2x}{1-2x}-\dfrac{1-2x}{1+2x}$

Solution: $\dfrac{1+2x}{1-2x}-\dfrac{1-2x}{1+2x}$

$$=\frac{(1+2x)^2-(1-2x)^2}{(1-2x)(1+2x)}$$

$$=\frac{(1)^2+(2x)^2+2(2x)(1)-\left[(1)^2+(2x)^2-2(2x)(1)\right]}{(1)^2-(2x)^2}$$

$$=\frac{1+4x^2+4x-\left[1+4x^2-4x\right]}{1-4x^2}$$

$$=\frac{1+4x^2+4x-1-4x^2+4x}{1-4x^2}$$

$$=\frac{4x+4x}{1-4x^2}$$

$$=\frac{8x}{1-4x^2}\ \textbf{Ans}$$

(iii) $\dfrac{x^2-25}{x^2-36}-\dfrac{x+5}{x+6}$

Solution: $\dfrac{x^2-25}{x^2-36}-\dfrac{x+5}{x+6}$

$$=\frac{(x)^2-(5)^2}{(x)^2-(6)^2}-\frac{x+5}{x+6}$$

$$=\frac{(x+5)(x-5)}{(x+6)(x-6)}-\frac{x+5}{x+6}$$

$$=\frac{(x+5)(x-5)-(x-6)(x+5)}{(x+6)(x-6)}$$

$$=\frac{(x+5)\left[(x-5)-(x-6)\right]}{x^2-6^2}$$

$$=\frac{(x+5)(x-5-x+6)}{x^2-36}$$

$$=\frac{(x+5)(1)}{x^2-36}$$

$$=\frac{x+5}{x^2-36}\ \textbf{Ans}$$

(iv) $\dfrac{x}{x-y}-\dfrac{y}{x+y}-\dfrac{2xy}{x^2-y^2}$

Solution: $\dfrac{x}{x-y}-\dfrac{y}{x+y}-\dfrac{2xy}{x^2-y^2}$

$$=\frac{x(x+y)-y(x-y)}{(x-y)(x+y)}-\frac{2xy}{x^2-y^2}$$

$$=\frac{x^2+\cancel{xy}-\cancel{xy}+y^2}{(x)^2-(y)^2}-\frac{2xy}{x^2-y^2}$$

$$=\frac{x^2+y^2}{x^2-y^2}-\frac{2xy}{x^2-y^2}$$

$$=\frac{x^2+y^2-2xy}{x^2-y^2}$$

$$=\frac{(x-y)^2}{x^2-y^2}$$

$$=\frac{(x-y)\cancel{(x-y)}}{(x+y)\cancel{(x-y)}}$$

$$=\frac{x-y}{x+y}\ \textbf{Ans}$$

(v) $\dfrac{x-2}{x^2+6x+9}-\dfrac{x+2}{2x^2-18}$

Solution: $\dfrac{x-2}{x^2+6x+9}-\dfrac{x+2}{2x^2-18}$

$$=\frac{x-2}{(x)^2+2(3)(x)+3^2}-\frac{x+2}{2(x^2-9)}$$

$$=\frac{x-2}{(x+3)^2}-\frac{x+2}{2\left[(x)^2-(3)^2\right]}$$

$$=\frac{x-2}{(x+3)^2}-\frac{x+2}{2(x-3)(x+3)}$$

$$=\frac{x-2}{(x+3)(x+3)}-\frac{x+2}{2(x+3)(x-3)}$$

$$=\frac{2(x-3)(x-2)-(x+3)(x+2)}{2(x+3)(x+3)(x-3)}$$

$$=\frac{2(x^2-2x-3x+6)-(x^2+2x+3x+6)}{2(x+3)(x+3)(x-3)}$$

$$=\frac{2(x^2-5x+6)-(x^2+5x+6)}{2(x+3)(x+3)(x-3)}$$

$$=\frac{2x^2-10x+12-x^2-5x-6}{2(x+3)^2(x-3)}$$

$$=\frac{x^2-15x+6}{2(x+3)^2(x-3)}\ \textbf{Ans}$$

(vi) $\dfrac{1}{x-1}-\dfrac{1}{x+1}-\dfrac{2}{x^2+1}-\dfrac{4}{x^4-1}$

Solution: $\dfrac{1}{x-1}-\dfrac{1}{x+1}-\dfrac{2}{x^2+1}-\dfrac{4}{x^4-1}$

$$=\frac{(x+1)-(x-1)}{(x-1)(x+1)}-\frac{2}{x^2+1}-\frac{4}{x^4-1}$$

$$=\frac{\cancel{x}+1-\cancel{x}+1}{x^2-1}-\frac{2}{x^2+1}-\frac{4}{x^4-1}$$

$$=\frac{2}{x^2-1}-\frac{2}{x^2+1}-\frac{4}{x^4-1}$$

$$=\frac{2(x^2+1)-2(x^2-1)}{(x^2-1)(x^2+1)}-\frac{4}{x^4-1}$$

$$=\frac{\cancel{2x^2}+2-\cancel{2x^2}+2}{(x^2)^2-(1)^2}-\frac{4}{x^4-1}$$

$$=\frac{4}{x^4-1}-\frac{4}{x^4-1}$$

$$=\frac{4-4}{x^4-1}$$

$$=\frac{0}{x^4-1}$$

$= 0$ **Ans**

Q.6 Perform the indicated operation and simplify.

(i) $(x^2-49).\dfrac{5x+2}{x+7}$

Solution: $(x^2-49).\dfrac{5x+2}{x+7}$

$$=\left[(x)^2-(7)^2\right].\frac{5x+2}{x+7}$$

$$=(x+7)(x-7)\frac{(5x+2)}{(x+7)}$$

$$=(x-7)(5x+2)\ \textbf{Ans}$$

(ii) $\dfrac{4x-12}{x^2-9}\div\dfrac{18-2x^2}{x^2+6x+9}$

Solution: $\dfrac{4x-12}{x^2-9}\div\dfrac{18-2x^2}{x^2+2(x)(3)+(3)^2}$

$$=\frac{4(x-3)}{(x^2)-(3)^2}\div\frac{2(9-x^2)}{(x+3)^2}$$

$$=\frac{4\cancel{(x-3)}}{\cancel{(x-3)}(x+3)}\times\frac{(x+3)^2}{2(9-x^2)}$$

$$=\frac{4}{x+3}\times\frac{(x+3)^2}{2(3+x)(3-x)}$$

$$=\frac{{}^2\cancel{4}x\,\cancel{(x+3)^2}}{\cancel{2}\,\cancel{(x+3)^2}(3-x)}$$

$$=\frac{2}{3-x}\ \textbf{Ans}$$

(iii) $\dfrac{x^6-y^6}{x^2-y^2}\div(x^4+x^2y^2+y^4)$

Solution: $\dfrac{x^6-y^6}{x^2-y^2}\div(x^4+x^2y^2+y^4)$

$$=\frac{(x^2)^3-(y^2)^3}{x^2-y^2}\div(x^4+x^2y^2+y^4)$$

$$=\frac{\cancel{(x^2-y^2)}\left[(x^2)^2+x^2y^2+(y^2)^2\right]}{\cancel{(x^2-y^2)}}\div(x^4+x^2y^2+y^4)$$

$$=\cancel{(x^4+x^2y^2+y^4)}\times\frac{1}{\cancel{(x^4+x^2y^2+y^4)}}$$

$= 1$ **Ans**

(iv) $\dfrac{x^2-1}{x^2+2x+1}.\dfrac{x+5}{1-x}$

Solution: $\dfrac{x^2-1}{x^2+2x+1}.\dfrac{x+5}{1-x}$

$$=\frac{(x+1)(x-1)}{(x^2+2(x)(1)+(1)^2)}\times\frac{x+5}{-(x-1)}$$

$$=\frac{(x+1)\cancel{(x-1)}}{(x+1)^2}\times\frac{(x+5)}{-\cancel{(x-1)}}$$

$$= -\frac{\cancel{(x+1)}(x+5)}{\cancel{(x+1)}(x+1)}$$

$$= -\frac{(x+5)}{x+1} \textbf{ Ans}$$

(v) $\frac{x^2+xy}{y(x+y)} \cdot \frac{x^2+xy}{y(x+y)} \div \frac{x^2-x}{xy-2y}$

Solution: $\frac{x^2+xy}{y(x+y)} \cdot \frac{x^2+xy}{y(x+y)} \div \frac{x^2-x}{xy-2y}$

$$= \frac{x\cancel{(x+y)}}{y\cancel{(x+y)}} \cdot \frac{x\cancel{(x+y)}}{y\cancel{(x+y)}} \div \frac{x(x-1)}{y(x-2)}$$

$$= \frac{x.\cancel{x}}{y.\cancel{y}} \times \frac{\cancel{y}(x-2)}{\cancel{x}(x-1)}$$

$$= \frac{x(x-2)}{y(x-1)} \textbf{ Ans}$$

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