Exercise 6.1

Q.1 Find the H.C.F of the following expressions.

(i) $39x^7y^3z$ **and** $91x^5y^6z^7$

Solution:

$39x^7y^3z = 3\times 13\times x.x.x.x.x.x.x.y.y.y.z$

$91x^5y^6z^7 = 7\times 13\times x.x.x.x.x.y.y.y.y.y.y.z.z.z.z.z.z.z$

$\text{H.C.F} = 13\times x.x.x.x.x.y.y.y.z$

$\text{H.C.F} = 13x^5y^2z$

(ii) $102xy^2z, 85x^2yz$ **and** $187xyz^2$

Solution:

$102xy^2z = 2\times 3\times 17\times x.y.y.z$

$85x^2yz = 5\times 17\times x.x.y.z$

$187xyz^2 = 11\times 17\times x.y.z.z$

$\text{H.C.F} = 17xyz$

Q.2 Find the H.C.F of the following expression by factorization.

(i) $x^2+5x+6, x^2-4x-12$

Solution: $x^2+5x+6, x^2-4x-12$

Factorizing x^2+5x+6

$= x^2+3x+2x+6$

$= x(x+3)+2(x+3)$

$= (x+3)(x+2)$

Factorizing $x^2-4x-12$

$= x^2-6x+2x-12$

$= x(x-6)+2(x-6)$

$= (x-6)(x+2)$

So,

$\text{H.C.F} = (x+2)$

(ii) $x^2-27, x^2+6x-27, 2x^2-18$

Solution: $x^2-27, x^2+6x-27, 2x^2-18$

Factorizing x^3-27

$=(x)^3-(3)^3$

$=(x-3)\left[(x)^2+(x)(3)+(3)^2\right]$

$=(x-3)(x^2+3x+9)$

Factorizing $x^2+6x-27$

$=x^2+9x-3x-27$

$=x(x+9)-3(x+9)$

$=(x+9)(x-3)$

Factorizing $2x^2-18$

$=2(x^2-9)$

$=2\left[(x)^2-(3)^2\right]$

$=2(x-3)(x+3)$

So,

H.C.F $=(x-3)$

(iii) $x^3-2x^2+x, x^2+2x-3, x^2+3x-4$

Factorizing x^3-2x^2+x

$=x(x^2-2x+1)$

$x(x^2-x-x+1)$

$=x\left[x(x-1)-1(x-1)\right]$

$=x(x-1)(x-1)$

Factorizing x^2+2x-3

$=x^2+3x-x-3$

$=x(x+3)-1(x+3)$

$=(x+3)(x-1)$

Factorizing x^2+3x-4

$=x^2+4x-x-4$

$=x(x+4)-1(x+4)$

$=(x+4)(x-1)$

So,

H.C.F $=(x-1)$

(iv) $18(x^3-9x^2+8x), 24(x^2+3x+2)$

Solution: $18(x^3-9x^2+8x), 24(x^2+3x+2)$

Factorizing $18(x^3-9x^2+8x)$

$=6\times3\times x(x^2-9x+8)$

$= 6\times 3\times x\left(x^2-8x-x+8\right)$

$= 6\times 3\times x\left[x(x-8)-1(x-8)\right]$

$= 6\times 3\times x(x-8)(x-1)$

Factorizing $24\left(x^2+3x+2\right)$

$= 6\times 4\left(x^2-3x+2\right)$

$= 6\times 4\left(x^2-2x-x+2\right)$

$= 6\times 4\left[x(x-2)-1(x-2)\right]$

$= 6\times 4(x-2)(x-1)$

So,

H.C.F $= 6(x-1)$

(v) $36\left(3x^4+5x^2-2x^2\right), 54\left(27x^4-x\right)$

Factorizing $36\left(3x^4+6x^3-2x^2\right)$

$= 3\times 3\times 2\times 2\times x^2\left(3x^2+5x-2\right)$

$= 3\times 3\times 2\times 2\times x^2\left(3x^2+6x-x-2\right)$

$= 3\times 3\times 2\times 2\times x^2\left[3x(x+2)-1(x+2)\right]$

$= 3\times 3\times 2\times 2\times x^2(x+2)(3x-1)$

Factorizing $54\left(27x^4-x\right)$

$= 3\times 3\times 3\times 2\times x\left(27x^3-1\right)$

$= 3\times 3\times 3\times 2\times x\left[(3x)^3-(1)\right]$

$= 3\times 3\times 3\times 2\times x(3x-1)\left[(3x)^2+(3x)(1)+(1)^2\right]$

$= 3\times 3\times 3\times 2\times x(3x-1)\left(9x^2+3x+1\right)$

So,

H.C.F $= 3\times 3\times 2\times x(3x-1)$

$= 18x(3x-1)$

Q.3 Find the H.C.F of the following by division method.

(i) $x^3+3x^2-16x+12,\ x^3+x^2-10x+8$

Solution: $x^3+3x^2-16x+12,\ x^3+x^2-10x+8$

$$x^3+x^2-10x+8\ \overline{)\,x^3+3x^2-16x+12\,}\quad (1)$$

$$\underline{\pm x^3 \pm x^2 \mp 10x \pm 8}$$

$$2x^2-6x+4$$

$$2\left(x^2-3x+2\right)$$

$$x^2-3x+2\ \overline{)\,\cancel{x^3}+x^2-10x+8\,}\quad (x+4)$$

$$\underline{\cancel{-x^3} \mp 3x^2 \pm 2x}$$

$$4x^2-12x+8$$

$$\underline{\pm 4x^2 \mp 12x \pm 8}$$

$$\times$$

H.C.F = $\left(x^2-3x+2\right)$

(ii) $x^4+x^3-2x^2+x-3,\ 5x^3+3x^2-17x+6$

Solution: $x^4+x^3-2x^2+x-3,\ 5x^3+3x^2-17x+6$

$$5x^3+3x^2-17x+6\ \overline{)\,x^4+x^3-2x^2+x-3\,}\quad (x+2)$$

$$\underline{\times 5}$$

$$5x^4+5x^3-10x^2+5x-15$$

$$\underline{\pm 5x^4 \pm 3x^3 \mp 17x^2 \pm 6x}$$

$$2x^3+7x^2-x-15$$

$$\underline{\times 5}$$

$$10x^3+35x^2-5x-75$$

$$\underline{\pm 10x^3 \pm 6x^2 \mp 34x \pm 12}$$

$$29x^2+29x-87$$

$$29(x^2+x-3)$$

$$x^2+x-3\ \overline{)\,5x^3+3x^2-17x+6\,}\quad (5x-2)$$

$$\pm 5x^3 \pm 5x^2 \mp 15x$$

$$-2x^2-2x+6$$

$$\underline{\mp 2x^2 \mp 2x \pm 6}$$

$$\times$$

H.C.F $=\left(x^2+x-3\right)$

(iii) $2x^5-4x^4-6x, x^5+x^4-3x^3-3x^2$

$$2x^5-4x^4-6x \,\overline{\big)\, x^5+x^4-3x^3-3x^2}\quad 1$$

$$\times 2$$

$$2x^5+2x^4-6x^3-6x^2$$

$$\underline{-2x^5 \mp 4x^4 \qquad \mp 6x}$$

$$6x^4-6x^3-6x^2+6x$$

$$6\left(x^4-x^3-x^2+x\right)$$

$$x^4-x^3-x^2+x \,\overline{\big)\, 2x^5-4x^2-6x}\quad 2x-2$$

$$\underline{\pm 2x^5 \pm 2x^2 \qquad \mp 2x^4 \mp 2x^3}$$

$$-2x^4+2x^3-2x^2-6x$$

$$\underline{\mp 2x^4 \pm 2x^3 \pm 2x^2 \mp 2x}$$

$$-4x^2-4x$$

$$-4\left(x^2+x\right)$$

$$x^2+x \,\overline{\big)\, x^4-x^3-x^2+x}\quad x^2-2x+1$$

$$\underline{-x^4 \pm x^3}$$

$$-2x^3-x^2+x$$

$$\underline{\mp 2x^3 \mp 2x^2}$$

$$x^2+x$$

$$\underline{\pm x^2 \pm x}$$

$$\times$$

H.C.F $=x^2+x$

Q.4 Find the L.C.M of the following expressions.

(i) $39x^7y^3z$ **and** $91x^5y^6z^7$

Solution:

$39x^7y^3z = 3\times 13\times x.x.x.x.x.x.x.y.y.y.z$

$91x^5y^6z^7 = 7\times 13\times x.x.x.x.x.y.y.y.y.y.y.z.z.z.z.z.z.z$

Common$=13x^5y^8z$

Uncommon$=3\times 7\times x^2y^3z^6$

$= 21x^2y^3z^6$

L.C.M= common factors$\times$uncommon factors

$=13x^5y^3z\times 21x^2y^3z^6$

$273x^7y^6z^7$

(ii) $102xy^2z, 85x^2yz$ **and** $187xyz^2$

Solution:

$102xy^2z = 2\times3\times17.x.y.y.z$

$85x^2yz = 5\times17\times x.x.y.z$

$187xyz^2 = 11\times17.x.y.z.z$

Common=$17xyz$

Uncommon= $2\times3\times5\times11.xyz$

=$330xyz$

L.C.M=common x uncommon

$= 17xyz\times330xyz$

$= 5610x^2y^2z^2$

Q.5 Find the L.C.M of the following by factorizing.

(i) $x^2-25x+100$ **and** x^2-x-20

Solution: $x^2-25x+100$ **and** x^2-x-20

Factorizing $x^2-25x+100$

$= x^2-20x-5x+100$

$= x(x-20)-5(x-20)$

$=(x-20)(x-5)$

Factorizing x^2-x-20

$= x^2-5x+4x-20$

$= x(x-5)+4(x-5)$

$=(x-5)(x+4)$

So,

L.C.M = $(x-5)(x+4)(x-20)$

(ii) $x^2+4x+4, x^2-4, 2x^2+x-6$

Solution: $x^2+4x+4, x^2-4, 2x^2+x-6$

Factorizing x^2+4x+4

$= x^2+2x+2x+4$

$= x(x+2)+2(x+2)$

$=(x+2)(x+2)$

Factorizing x^2-4

$=(x)^2-(2)^2$

$=(x-2)(x+2)$

Factorizing $2x^2+x-6$

$= 2x^2+4x-3-6$

$= 2x(x+2)-3(x+2)$

$=(x+2)(2x-3)$

So,

L.C.M $=(x+2)(x+2)(x-2)(2x-3)$

$=(x+2)^2(x-2)(2x-3)$

(iii) $2(x^4-y^4), 3(x^3+2x^2y-xy^2-2y^3)$

Factorizing $2(x^4-y^4)$

$=2\left[(x^2)^2-(y^2)^2\right]$

$=2(x^2+y^2)(x^2-y^2)$

$=2(x^2+y^2)(x+y)(x-y)$

Factorizing $3(x^3+2x^2y-xy^2-2y^3)$

$=3\left[x^2(x+2y)-y^2(x+2y)\right]$

$=3(x+2y)(x^2-y^2)$

$=3(x+2y)(x+y)(x-y)$

So,

L.C.M $=(x+y)(x-y)(x^2+y^2)(x+2y)\times 2\times 3$

$=6(x+y)(x-y)(x^2+y^2)(x+2y)$

$=6(x+2y)(x^4-y^4)$

(iv) $4(x^4-1), 6(x^3-x^2-x+1)$

Solution: $4(x^4-1), 6(x^3-x^2-x+1)$

Factorizing $4(x^4-1)$

$= \;=2\times 2\left[(x^2)^2-(1)^2\right]$

$=2\times 2(x^2+1)(x^2-1)$

$=2\times 2(x^2+1)(x+1)(x-1)$

$=6(x^3-x^2-x+1)$

$=2\times 3\left[x^2(x-1)-1(x-1)\right]$

$=2\times 3\left[(x-1)(x^2\text{-}1)\right]$

$=2\times 3(x-1)(x-1)(x+1)$

L.C.M$=2\times 2\times 3(x-1)(x+1)(x-1)(x^2+1)$

$=12(x-1)^2(x+1)(x^2+1)$

$=12(x-1)(x^4-1)$

Q.6 For what value of k is $(x+4)$ the H.C.F of $x^2+x-(2k+2)$ and $2x^2+kx-12$?

Solution:

$P(x)=x^2+x-(2k+2)$

Since $x+4$ is H.C.F of $P(x)$ and $q(x)$

$\therefore x+4$ is a factor of $P(x)$

Hence $P(-4)=0$

$x^2+x-(2k+2)=0$

By putting the value of x

$(-4)^2+(-4)-(2k+2)=0$

$16-4-2k-2=0$

$-2k+10=0$

$2k=10$

$k=\frac{\cancel{10}^{5}}{\cancel{2}}$

$k=5$

Q.7 If $(x+3)(x-2)$ is the H.C.F of $P(x)=(x+3)(2x^2-3x+k)$ and $q(x)=(x-2)(3x^2+7x-l)$ the find k and l

Solution: $(x-2)$ $(x+3)$ will divide $P(x)=(x+3)$ $(2x^2-3x+K)$

$(x-2)$ $(x+3)$ will divide $P(x)=(x+3)$ $(2x^2-3x+K)$

$x-2=0$

$x=2$

$P(2)=(2+3)(2(2)^2-3(2)+K)$

$P(2)=(5)(2\times 4-6+K)$

$P(2)=5(8-6+K)$

$P(2)=5(2+K)$

Remainder is equal to zero

$5(2+K)=0$

$2+K=\frac{0}{5}$

$2+K=0$

$K=-2$

$q(x)=(x-2)(3x^2+7x-1)$

$(x-2)(x+3)$ will be divide $q\ (x)=(x-2)(3x^2+7x-l)$

$x+3=0$

$x=-3$

$q(-3)=(-3-2)\left[3(-3)^2+7(-3)-l\right]$

$q(-3)=(-5)\left[3(9)-21-l\right]$

$q(-3)=(-5)[27-21-l]$

$q(-3)=(-5)(6-l)$

Remainder is equal to zero

$-5(6-l)=0$

$6-l=0$

$l=6$

Q.8 **The L.C.M and H.C.F of two polynomials** $P(x)$ **and** $q(x)$ **are** $2(x^4-1)$ **and** $(x+1)(x^2+1)$ **respectively. If** $P(x)=x^3+x^2+x+1$, **find** $q(x)$

Solution: $\therefore P(x)\times q(x)=\text{L.C.M}\times\text{H.C.F}$

$\therefore P(x)\times q(x)=\text{L.C.M}\times\text{H.C.F}$

$$q(x)=\frac{L.C.M\times H.C.F}{P(x)}$$

By putting the values

$$q(x)=\frac{2(x^4-1)(x+1)(x^2+1)}{x^3+x^2+x+1}$$

$$q(x)=\frac{2(x^4-1)(x+1)(x^2+1)}{x^2(x+1)+1(x+1)}$$

$$q(x)=\frac{2(x^4-1)\cancel{(x+1)}\cancel{(x^2+1)}}{\cancel{(x+1)}\cancel{(x^2+1)}}$$

$$q(x)=2(x^4-1)$$

Q.9 **Let** $p(x)=10(x^2-9)(x^2-3x+2)$ **and** $q(x)=10x(x+3)(x-1)^2$ **. if the H.C.F of** $p(x),\mathrm{q}(x)$ **is** $10(x+3)(x-1)$**, Find their L.C.M**

Solutions: $p(x)\times q(x)=:L.C.M\times H.C.F$

$p(x)\times q(x)=:L.C.M\times H.C.F$

$$L.C.M=\frac{p(x)\times q(x)}{H.C.F}$$

By putting the values

$$L.C.M=\frac{\cancel{10}(x^2-9)(x^2-3x+2)\times 10x\cancel{(x+3)}(x-1)^{\cancel{2}}}{\cancel{10}\cancel{(x+3)}\cancel{(x-1)}}$$

$$L.C.M=10x(x^2-9)(x^2-3x+2)(x-1)$$

Q.10 **Let the product of L.C.M and H.C.F of two polynomial be** $(x+3)^2(x-2)(x+5)$**. If one polynomial is** $(x+3)(x-2)$ **and the second polynomial is** $x^2+kx+15$, **find the value of k.**

Solution: $p(x)\times q(x) = L.C.M \times H.C.F$

$p(x)\times q(x) = L.C.M \times H.C.F$

By putting the values

$(x+3)(x-2)(x^2+kx+15) = (x+3)^2(x-2)(x+5)$

$x^2+kx+15 = \dfrac{(x+3)^{\cancel{2}}\cancel{(x-2)}(x+5)}{\cancel{(x+3)}\cancel{(x-2)}}$

$x^2+kx+15 = (x+3)(x+5)$

$x^2+kx+15 = x^2+8x+15$

$kx = \cancel{x^2}+8x+\cancel{15}-\cancel{x^2}-\cancel{15}$

$kx = 8x$

$k = \dfrac{8\cancel{x}}{\cancel{x}}$

$k = 8$

Q.11 **Waqas wishes to distribute 128 bananas and also 176 apples equally among a certain number of children. Find the highest number of children who can get fruit in this way.**

Solution

$$\begin{array}{r|l} 128 & 176 \;(1) \\ & \underline{128} \\ 48 & 128 \;(2) \\ & \underline{-96} \\ 32 & 48 \\ & \underline{-32} \\ 16 & 32 \;(2) \\ & \underline{-32} \\ & 0 \end{array}$$

Highest no. of children =16

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Report any mistake at freeilm786@gmail.com