

Exercise 16.1

Q.1 Show that the line segment joining the midpoint of opposite sides of a parallelogram divides it into two equal parallelograms.

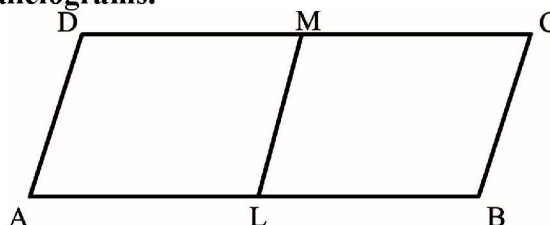
Given

ABCD is a parallelogram. L is the midpoint of \overline{AB} and M is the midpoint of \overline{DC}

To prove

Area of parallelogram ALMD = area of parallelogram LBCM.

Proof



Statements	Reasons
$\overline{AB} \parallel \overline{DC}$	Opposite sides of parallelogram ABCD.
$\overline{AL} \cong \overline{LB} \dots (i)$	L is midpoint of \overline{AB}
The parallelograms ALMD and LBCM are on equal bases and between the same parallel lines \overline{AB} and \overline{DC}	From equation (i)
Hence area of parallelogram ALMD = area of parallelogram LBCM.	They have equal areas

Q.2 In a parallelogram ABCD, $m\overline{AB} = 10\text{cm}$ the altitudes corresponding to sides AB and AD are respectively 7cm and 8cm Find \overline{AD}

$$\overline{AB} = 10 \text{ cm}$$

$$\overline{DH} = 7\text{cm}$$

$$\overline{MB} = 8\text{cm}$$

$$\overline{AD} = ?$$

Formula

Area of parallelogram = base x altitude

$$\overline{AB} \times \overline{DH} = \overline{AD} \times \overline{IB}$$

$$10 \times 7 = \overline{AD} \times 8$$

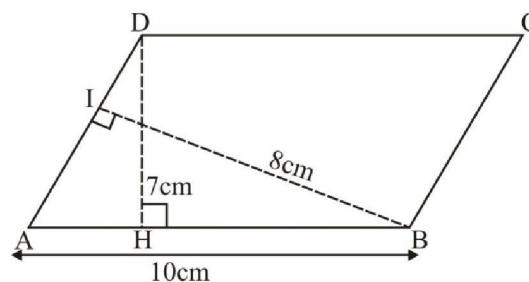
$$\frac{70}{8} = \overline{AD}$$

$$\frac{35}{4} = \overline{AD}$$

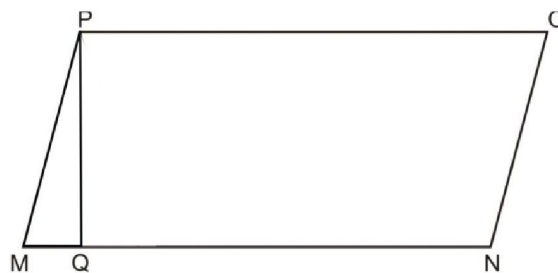
$$\overline{AD} = \frac{35}{4}$$

Or

$$\overline{AD} = 8.75\text{cm}$$



Q.3 If two parallelograms of equal areas have the same or equal bases, their altitude are equal



In parallelogram opposite side and opposite angles are Congruent.

Given

Parallelogram ABCD and parallelogram MNOP

OD is altitude of parallelogram ABCD

PQ is altitude of parallelogram MNOP

Area of ABCD $\parallel^{gm} \cong$ Area of MNOP \parallel^{gm}

To prove

$m\overline{OD} \cong m\overline{PQ}$

Proof

Statements	Reasons
Area of parallelogram ABCD =	Given
Area of parallelogram MNOP	
Area of parallelogram = base \times height	Given
$\overline{AB} \times \overline{OD} = \overline{MN} \times \overline{PQ}$	
We know that	
$\overline{AB} = \overline{MN}$	
So	
$\frac{\overline{AB}}{\overline{AB}} \times \overline{OD} = \overline{PQ}$	Proved
$\overline{OD} = \overline{PQ}$	

Theorem 16.1.3

Triangle on the same base and of the same (i.e...equal) altitudes are equal in area

Given

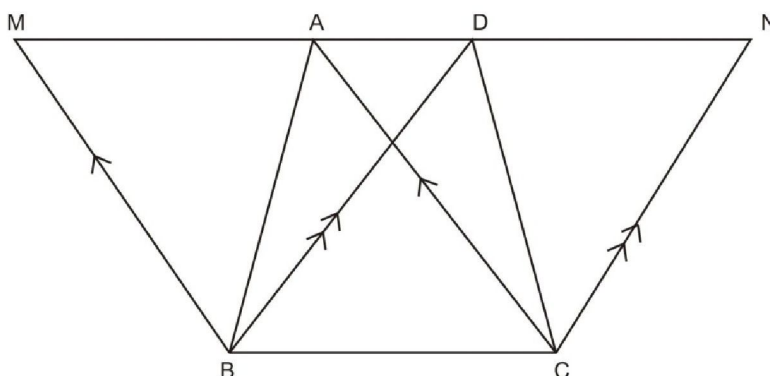
Δ 's ABC, DBC on the

Same base \overline{BC} and

having equal altitudes

To prove

Area of (Δ ABC) = area of (Δ DBC)



Construction:

Draw $\overline{BM} \parallel$ to \overline{CA} , $\overline{CN} \parallel$ to \overline{BD} meeting \overline{AD} produced in M.N.

Proof

Statements	Reasons
$\triangle ABC$ and $\triangle DBC$ are between the same \parallel^s	Their altitudes are equal
Hence \overline{MADN} is parallel to \overline{BC}	
$\therefore \text{Area} \parallel^{\text{gm}} (\text{BCAM}) = \text{Area} \parallel^{\text{gm}} (\text{BCND})$	These \parallel^{gm} are on the same base \overline{BC} and between the same \parallel^s
But $\triangle ABC = \frac{1}{2} \parallel^{\text{gm}} (\text{BCAM})$ ----- (ii)	
And $\triangle DBC = \frac{1}{2} \parallel^{\text{gm}} (\text{BCND})$ ----- (iii)	Each diagonal of a \parallel^{gm}
Hence $\text{area} (\triangle ABC) = \text{Area} (\triangle DBC)$	Bisects it into two congruent triangles From (i) (ii) and (iii)

Theorem 16.1.4

Triangles on equal bases and of equal altitudes are equal in area.

Given

$\triangle s$ ABC, DEF on equal bases \overline{BC} , \overline{EF} and having altitudes equal

To prove

$\text{Area} (\triangle ABC) = \text{Area} (\triangle DEF)$

Construction:

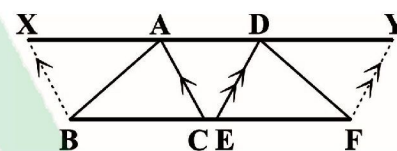
Place the $\triangle s$ ABC and DEF so that their equal bases \overline{BC} and \overline{EF} are in the same

straight line BCEF and their vertices on the same side of it .Draw $\overline{BX} \parallel \overline{CA}$ and $\overline{FY} \parallel \overline{ED}$

meeting \overline{AD} produced in X, Y respectively

Proof

Statements	Reasons
$\triangle ABC$, $\triangle DEF$ are between the same parallels	Their altitudes are equal (given)



$\therefore XADY$ is \parallel^{gm} to $BCEF$	
$\therefore \text{area } \parallel^{\text{gm}} (BCAX) = \text{Area } \parallel^{\text{gm}} (EFYD) \text{----(i)}$	These \parallel^{gm} are on equal bases and between the same parallels
But $\Delta ABC = \frac{1}{2} \parallel^{\text{gm}} (BCAX) \text{----(ii)}$	Diagonal of a \parallel^{gm} bisect it
And area of $\Delta DEF = \frac{1}{2} \text{ area of } \parallel^{\text{gm}} (EFYD) \text{--- (iii)}$	
$\therefore \text{area } (\Delta ABC) = \text{area } (\Delta DEF)$	From (i),(ii)and(iii)

