UNIT 5
GRAVITATION

LONG QUESTIONS

5.1 THE FORCE OF GRAVITATION

Law of Gravitation

Q.1 State and explain Newton's law of gravitation. (GRW 2011, 12, 13, 15, LHR 2013)

Ans: Gravitation

In the universe, there exists a force between the bodies due to which everybody of the universe attracts every other body. This force is known as force of gravitation.

Statement

Every object in the universe attracts every other object with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers.

Explanation

Every object in this universe attracts other objects towards its centre. The attraction between two objects is called gravitation. On the basis of his observations, Newton derived a law which is called Newton's law of gravitation.

Mathematical Derivation

Consider two bodies A and B of masses m_1, and m_2, respectively. According to law of gravitation, the gravitational force of attraction F with which two mass m_1 and m_2 separated by a distance d attracts each other is given by:

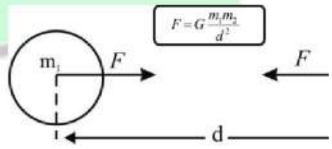

Figure 5.1: Two masses at each other with a gravitatic force of equal magnitude.

$$F \propto m_1m_2$$

$$F \propto \frac{1}{d^2}$$

OR

$$F \propto \frac{m_1m_2}{d^2}$$

$$F = \frac{Gm_1m_2}{d^2}$$

Gravitational constant

G is a constant called gravitational constant. It is called universal constant of gravitation. If $m_1 = m_2 = 1$ kg and d = 1 m, then F = G. Thus G is a force which 1 kg object exerts on another 1 kg object placed 1 m away from it. In SI units, the value of gravitational constant G is 6.67×10^{-11} Nm^2 kg^{-2}.

Dependence of Gravitational force on mass

Due to small value of G, the gravitational force of attraction between different objects around us is very small, so we do not feel it. However, if the mass of one or both the objects is very large, then we can observe the effect of gravitational force easily.

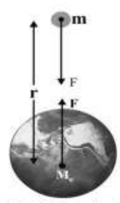

Figure 5.2: Weight of a body is due to the gravitational force between the body and the Earth.

Q.2 Explain the gravitational field? (GRW 2013)

Ans: The space around the earth in which its gravitational force acts on a body is called gravitational field. According to the Newton's law of gravitation, the gravitational force between a body of mass m and the Earth is given by,

$$F = \frac{G m M_e}{r^2}$$

Where M_e is the mass of the Earth and r is the distance of the body from the centre of the Earth.

The weight of a body is due to the gravitational force with which Earth attracts a body. Gravitational force is a non-contact force.

Example

The velocity of a body, thrown up, goes on decreasing while on returns its velocity goes on increasing. This is due to the gravitational pull of the Earth acting on the body whether the body is in contact with the Earth or not. Such a force is called the field force. It is assumed that a gravitational field exists all around the Earth. This field is directed towards the centre of the Earth. The gravitational field becomes weaker and weaker as we go farther and farther away from the Earth.

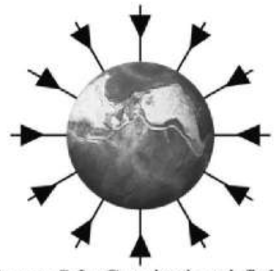

Figure 5.3: Gravitational field around the Earth is towards its centre.

Gravitational Field Strength

In the gravitational field of the Earth, the gravitational force per unit mass is called gravitational field strength of the Earth. At any place its value is equal to the value of g at that point. Near the surface of the Earth, the gravitational field strength is 10 Nkg^{-1}.

Q.3 Determine the mass of the earth by using Newton's law of gravitation.

(GRW 2014, LHR 2015)

Ans: Suppose a body of mass m is placed on the surface of the Earth. Let mass of the Earth is Me and radius of Earth be R. The distance between the body and centre of the Earth is equal to the radius of the Earth R.

According to the law of gravitation, the gravitational force F of the Earth acting on the body is given by,

$$F = \frac{G M_e m}{R^2} \quad \text{......... (1)}$$

We know that the force of gravitation with which Earth attracts the body towards its centre is equal to the weight of the body. Therefore,

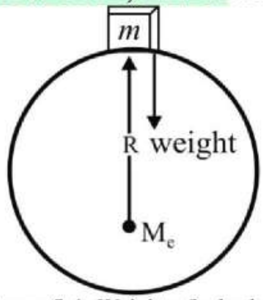

Figure 5.4: Weight of a body i Equal to the gravitational forc between the body and the Earth

Therefore, $F = w = mg$

OR $mg = \frac{G M_e m}{R^2}$

Or $g = \frac{G Me}{R^2}$ .... (2)

Or $M_e = \frac{g R^2}{G}$ ....(3)

As we know that,

$g = 10\ ms^{-2}$

$R = 6.4 \times 10^6$ m

And $G = 6.673 \times 10^{-11}\ Nm^2kg^{-2}$

By putting the value of g, R and G in equation (3), we have

$$M = \frac{gR^2}{G} = \frac{10 \times (6.4 \times 10^6)^2}{6.673 \times 10^{-11}}$$

$$M = \frac{10 \times 40.96 \times 10^{12}}{6.673 \times 10^{-11}}$$

$$M = \frac{409.6 \times 10^{12}}{6.673 \times 10^{-11}}$$

$$M = 61.4 \times 10^{23}$$

$$M = 6.14 \times 10^{24}\ \text{kg}$$

$$M = 6 \times 10^{24}\ \text{kg}$$

Hence the mass of the earth is approximately **6×10^{24} kg**.

Q.4 Explain the variation of 'g' with altitude. **(GRW 2015)**

Ans: As we know that

$$g = \frac{G\,Me}{R^2}$$

The above equation show that the acceleration due to gravity depends on the radius of Earth at its surface. The value of g is inversely proportional to the square of the radius of the Earth. It does not remain constant. It decreases with altitude. Altitude is the height of an object or place above sea level. The value of g is greater at sea level than at the hills.

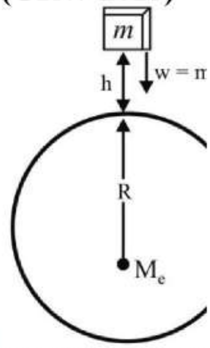

Figure 5.5: Weight of decreases as its height in from the surface of the E

Mathematical Form

Suppose a body of mass m at an altitude h. The distance of the body from the centre of the Earth is R+h. By using above equation, we have

$$g_h = \frac{G\,M}{(R + h)^2}$$

According to the above equation, we come to know that at a height equal to one Earth radius above the surface of the Earth, g becomes one fourth of its value on the Earth. Similarly, at a distance of two Earth's radius above the Earth's surface, the value of g becomes one ninth of its value on the Earth.

Q.5 What are artificial satellites? Define orbital velocities and what do you know about communication satellites? **(LHR 2013)**

Ans: An object that revolves around a planet is called a satellite.

Orbital Velocity

It is the velocity of the satellite with which it moves around the earth at specific height.

Natural satellite of Earth

The moon revolves around the Earth so moon is the natural satellite of Earth.

Artificial satellites

Scientists have sent many objects into space. Some of these revolve around the Earth. These are called artificial satellites.

Most of the artificial satellites orbiting around the Earth are used for communication purposes. Artificial satellites carry instruments or passengers to perform experiments in the space.

Large numbers of artificial satellites have been launched in different orbits around the Earth. They take different time to complete their one revolution around the Earth depending upon their distance h from the Earth.

Communication Satellites

Communication satellites take 24 hours to complete their one revolution around the Earth. As Earth also completes one rotation about its axis in 24 hours, hence, these communication satellites appear to be stationary with respect to Earth. It is due to this reason that the orbit of such satellites is called geostationary orbit. Dish antennas sending and receiving the signals from them have fixed direction depending upon their location on the Earth.

Q.6 Explain the motion of an artificial satellite and derive the formula for orbital velocity of an artificial satellite. **(LHR 2013, 2014, 2015, GRW 2014)**

Ans: A satellite requires centripetal force that keeps it to move around the Earth. The gravitational force of attraction between the satellite and the Earth provides the necessary centripetal force.

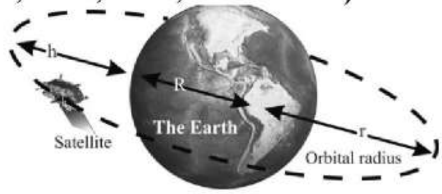

Figure 5.6: A satellite is orbiting around the Earth at a height ***h*** above the surface of the Earth.

Mathematical Derivation

Suppose a satellite of mass m is revolving around the Earth at a height 'h' in an orbit of radius r_o with orbital velocity v_o. The necessary centripetal force F_c required to keep the satellite moving is given by,

$$F_c = \frac{mv_o^2}{r_o} \text{ .... (1)}$$

This centripetal force is provided to the satellite by the gravitational force of attraction between the Earth and satellite and is equal to the weight of the satellite w (mg_h). thus

$$F_c = w = mg_h \text{ .... (2)}$$

By comparing equation (1) and equation (2), we get

Or $$mg_h = \frac{mv_o^2}{r_o}$$

Or $$v_o^2 = g_h r_o$$

Or $$v_o = \sqrt{g_h r_o}$$

As $$r_o = R + h$$

So $$v_o = \sqrt{g_h (R + h)} \text{ .... (3)}$$

This equation represents the orbital velocity, which a satellite must possess when launched in an orbit of radius $r_o = R + h$ around the Earth. An approximation can be made for a satellite revolving close to the Earth such that $R >> h$.

$R + h \approx R$

And $g_h \approx g$

So $v_o = \sqrt{g\ R}$

A Satellite revolving around very close to the Earth has speed nearly 8 kms^{-1} or 29000 kmh^{-1}.