Exercise 1.5

Q.1 Find the determinant of following matrices.

(i) $A=\begin{bmatrix}-1 & 1\\ 2 & 0\end{bmatrix}$

Solution:

$A=\begin{bmatrix}-1 & 1\\ 2 & 0\end{bmatrix}$

To write the determinant form

$|A|=\begin{vmatrix}-1 & 1\\ 2 & 0\end{vmatrix}$

$= (-1)(0) - (2)(1)$
$= 0 - 2$
$= -2$

(ii) $B=\begin{bmatrix}1 & 3\\ 2 & -2\end{bmatrix}$

Solution:

$B=\begin{bmatrix}1 & 3\\ 2 & -2\end{bmatrix}$

To write in determinant form

$|B|=\begin{vmatrix}1 & 3\\ 2 & -2\end{vmatrix}$

$= (1)(-2) - (2)(3)$
$= -2-6$
$= -8$

(iii) $C=\begin{bmatrix}3 & 2\\ 3 & 2\end{bmatrix}$

Solution:

$C=\begin{bmatrix}3 & 2\\ 3 & 2\end{bmatrix}$

To write in determinant form

$|C|=\begin{vmatrix}3 & 2\\ 3 & 2\end{vmatrix}$

$= (3)(2) - (3)(2)$
$= 6-6$
$= 0$

(iv) $D=\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}$

Solution:

$D=\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}$

To write in determinant form

$|D|=\begin{vmatrix}3 & 2\\ 1 & 4\end{vmatrix}$

$= (3)(4) - (2)(1)$
$= 12-2$
$= 10$

Q.2 Find which of the following matrices are singular or non-singular?

(i) $A=\begin{bmatrix}3 & 6\\ 2 & 4\end{bmatrix}$

Solution:

$A=\begin{bmatrix}3 & 6\\ 2 & 4\end{bmatrix}$

To write in determinant form

$|A|=\begin{vmatrix}3 & 6\\ 2 & 4\end{vmatrix}$

$|A| = (3)(4)-(2)(6)$

$|A| = 12-12$

$|A| = 0$

It is a singular matrix.

(ii) $B=\begin{bmatrix}4 & 1\\ 3 & 2\end{bmatrix}$

Solution:

$B=\begin{bmatrix}4 & 1\\ 3 & 2\end{bmatrix}$

To write in determinant form

$$|B| = \begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix}$$

$$|B| = (4)\ (2) - (3)(1)$$

$$|B| = 8 - 3$$

$$|B| = 5$$

It is non-singular matrix.

(iii) $C = \begin{bmatrix} 7 & -9 \\ 3 & 5 \end{bmatrix}$

Solution:

$$C = \begin{bmatrix} 7 & -9 \\ 3 & 5 \end{bmatrix}$$

To write in determinant form

$$|C| = \begin{vmatrix} 7 & -9 \\ 3 & 5 \end{vmatrix}$$

$$|C| = (7)(5) - (3)(-9)$$

$$|C| = 35 + 27$$

$$|C| = 62$$

In not equal to zero so
It is non-singular matrix.

(iv) $D = \begin{bmatrix} 5 & -10 \\ -2 & 4 \end{bmatrix}$

Solution:

$$D = \begin{bmatrix} 5 & -10 \\ -2 & 4 \end{bmatrix}$$

To write in determinant form

$$|D| = \begin{vmatrix} 5 & -10 \\ -2 & 4 \end{vmatrix}$$

$$|D| = (5)(4) - (-2)(-10)$$

$$|D| = 20\text{-}20$$

$$|D| = 0$$

It is singular matrix.

Q.3 Find the multiplicative inverse of each

(i) $A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$

Solution:

$$A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$$

To write in determinant form

$$|A| = \begin{vmatrix} -1 & 3 \\ 2 & 0 \end{vmatrix}$$

$$|A| = (-1)(0) - (2)(3)$$

$$|A| = 0 - 6$$

$$|A| = -6 \neq 0 \text{ (Non-Singular)}$$

A^{-1} exists

To write in Adj A

$$AdjA = \begin{bmatrix} 0 & -3 \\ -2 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times AdjA$$

Putting the values

$$A^{-1} = \frac{1}{-6} \times \begin{bmatrix} 0 & -3 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} 0 \times \frac{1}{-6} & -3 \times \frac{1}{-6} \\ -2 \times \frac{1}{-6} & -1 \times \frac{1}{-6} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{0}{-6} & \frac{+3}{+6} \\ \frac{+2}{+6} & \frac{+1}{+6} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{6} \end{bmatrix}$$

(ii) $B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$

Solution:

$$B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$$

To write in determinant form

$$|B|=\begin{vmatrix}1 & 2\\ -3 & -5\end{vmatrix}$$

$$|B|=(-1)(-5)-(-3)(2)$$

$$|B|=-5+6$$

$$|B|=1\neq 0 \text{ (Non-Singular)}$$

B^{-1} exists

$$AdjB=\begin{bmatrix}-5 & -2\\ 3 & 1\end{bmatrix}$$

$$B^{-1}=\frac{1}{|B|}\times AdjB$$

Putting the values

$$B^{-1}=\frac{1}{1}\times\begin{bmatrix}-5 & -2\\ 3 & 1\end{bmatrix}=\begin{bmatrix}\frac{1}{2}\times -5 & \frac{1}{1}\times -2\\ \frac{1}{1}\times 3 & \frac{1}{1}\times 1\end{bmatrix}$$

$$=\begin{bmatrix}\frac{-5}{1} & \frac{-2}{1}\\ \frac{3}{1} & \frac{1}{1}\end{bmatrix}$$

$$=\begin{bmatrix}-5 & -2\\ 3 & 1\end{bmatrix}$$

(iii) $C=\begin{bmatrix}-2 & 6\\ 3 & -9\end{bmatrix}$

Solution:

To write in determinant form

$$|C|=\begin{vmatrix}-2 & 6\\ 3 & -9\end{vmatrix}$$

$$|C|=(-2)(-9)-(3)(6)$$

$$|C|=18-18$$

$$|C|=0 \text{ Singular}$$

C^{-1} Does not exists.

(iv) $D=\begin{bmatrix}\frac{1}{2} & \frac{3}{4}\\ 1 & 2\end{bmatrix}$

Solution:

To write in determinant form

$$D=\begin{bmatrix}\frac{1}{2} & \frac{3}{4}\\ 1 & 2\end{bmatrix}$$

$$|D|=\begin{vmatrix}\frac{1}{2} & \frac{3}{4}\\ 1 & 2\end{vmatrix}=\frac{1}{2}\times 2-\frac{3}{4}\times 1$$

$$=1-\frac{3}{4}$$

$$=\frac{4-3}{4}$$

$$|D|=\frac{1}{4}\neq 0\,(\text{Non Singular})$$

D^{-1} exists

$$AdjD=\begin{bmatrix}2 & \frac{-3}{4}\\ -1 & \frac{1}{2}\end{bmatrix}$$

$$D^{-1}=\frac{1}{|\text{D}|}\times AdjD$$

By putting the values

$$=\frac{\frac{1}{1}}{\frac{1}{4}}\begin{bmatrix}2 & -\frac{3}{4}\\ -1 & \frac{1}{2}\end{bmatrix}$$

$$=1\div\frac{1}{4}\begin{bmatrix}2 & \frac{-3}{4}\\ -1 & \frac{1}{2}\end{bmatrix}$$

$$=1\times\frac{4}{1}\begin{bmatrix}2 & \frac{-3}{2}\\ -1 & \frac{1}{2}\end{bmatrix}$$

$$=4\begin{bmatrix}2 & -\frac{3}{4}\\ -1 & \frac{1}{2}\end{bmatrix}$$

$$=\begin{bmatrix}8 & -3\\ -4 & 2\end{bmatrix}$$

Q.4 **If** $A=\begin{bmatrix}1 & 2\\ 4 & 6\end{bmatrix}$ **and** $B=\begin{bmatrix}3 & -1\\ 2 & -2\end{bmatrix}$,

then

Then verify that

(i) A(AdjA)=(AdjA)A=(detA)I

Solution: A(AdjA)=(AdjA)A=(detA)I

$$A=\begin{bmatrix}1 & 2\\ 4 & 6\end{bmatrix}$$

$$\text{AdjA}=\begin{bmatrix}6 & -2\\ -4 & 1\end{bmatrix}$$

$$\text{detA}=\begin{bmatrix}1 & 2\\ 4 & 6\end{bmatrix}$$

$$=1\times 6 \quad 2\times 4$$

$$=6-8$$

$$=-2$$

$$A(AdjA)=\begin{bmatrix}1 & 2\\ 4 & 6\end{bmatrix}\begin{bmatrix}6 & -2\\ -4 & 1\end{bmatrix}$$

$$=\begin{bmatrix}6-8 & (-2)+2\\ 24-4 & -8+6\end{bmatrix}$$

$$\text{A (Adj A)}=\begin{bmatrix}-2 & 0\\ 0 & -2\end{bmatrix} \text{\_\_\_\_\_\_\_\_ (i)}$$

$$(AdjA)A=\begin{bmatrix}6 & -2\\ -4 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\ 4 & 6\end{bmatrix}$$

$$(AdjA)A=\begin{bmatrix}(6)\times 12\times 4 & (6)\times 2+(-2)\times 6\\ (-4)\times 1+(1)(4) & (-4)(2)+(1)(6)\end{bmatrix}$$

$$=\begin{bmatrix}6-8 & 12-12\\ -4+4 & -8+6\end{bmatrix}$$

$$\text{(AdjA)A}=\begin{bmatrix}-2 & 0\\ 0 & -2\end{bmatrix} \text{\_\_\_\_\_\_\_\_ (ii)}$$

$$(\det A)\text{I=-2}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}-2\times 1 & 0\times 2\\ -2\times 0 & 1\times -2\end{bmatrix}$$

$$\text{(detA)I}=\begin{bmatrix}-2 & 0\\ 0 & -2\end{bmatrix} \text{\_\_\_\_\_\_\_\_ (iii)}$$

Hence proved

From eq (i), (ii) and (iii)

A(AdjA)=(AdjA)A=(detA)I

(ii) $BB^{-1} = I = B^{-1}B$

Solution: $BB^{-1} = I = B^{-1}B$

To write in determinant form

$$|B|=\begin{vmatrix}3 & -1\\ 2 & -2\end{vmatrix}$$

$$=-6-(-2)$$

$$=-6+2$$

$$=-4\neq 0 \text{ (None singular)}$$

$=B^{-1}$ exists.

To write in AdjB

$$AdjB=\begin{bmatrix}-2 & 1\\ -2 & 3\end{bmatrix}$$

$$B^{-1}\frac{1}{|B|}Adj$$

$$B^{-1}=\frac{1}{-4}\begin{bmatrix}-2 & 1\\ -2 & 3\end{bmatrix}$$

New

$$BB^{-1}=\begin{bmatrix}3 & -1\\ 2 & -2\end{bmatrix}\times\frac{1}{-4}\begin{bmatrix}-2 & 1\\ -2 & 3\end{bmatrix}$$

$$=\frac{1}{-4}\begin{bmatrix}3 & -1\\ 2 & -2\end{bmatrix}\begin{bmatrix}-2 & 1\\ -2 & 3\end{bmatrix}$$

$$=-\frac{1}{4}\begin{bmatrix}-6+2 & 3-3\\ -4+4 & 2-6\end{bmatrix}$$

$$=\frac{1}{-4}\begin{bmatrix}-4 & 0\\ 0 & -4\end{bmatrix}$$

$$=\begin{bmatrix}\frac{4}{4} & 0\\ 0 & \frac{4}{4}\end{bmatrix}$$

$$BB^{-1}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$$

and

$$B^{-1}B=\frac{1}{-4}\begin{bmatrix}-2 & 1\\ -2 & 3\end{bmatrix}\begin{bmatrix}3 & -1\\ 2 & -2\end{bmatrix}$$

$$=\frac{1}{-4}\begin{bmatrix}-6+2 & 2-2\\ -6+6 & 2-6\end{bmatrix}$$

$$=\frac{1}{-4}\begin{bmatrix}-4 & 0\\ 0 & -4\end{bmatrix}$$

$$=\begin{bmatrix} \frac{-4}{-4} & 0 \\ 0 & \frac{-4}{-4} \end{bmatrix}$$

$$=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$B^{-1}B$ =I

From (i) and (ii)

BB^{-1}=I=$B^{-1}B$

Hence proved

Q.5 Determine whether the given matrices are multiplicative inverses of each other.

(i) $\begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix}$ and $\begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$

Solution: $\begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix}$ and $\begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$

$$\begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix}\begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$$

$$=\begin{bmatrix} 21+(-20) & -15+15 \\ 28+(-28) & -20+21 \end{bmatrix}$$

$$=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The given matrices are multiplicative inverse of each other.

(ii) $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $\begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$

Solution: $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $\begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$$

$$=\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$$

$$=\begin{bmatrix} -3+4 & 2+(-2) \\ -6+6 & 4+(-3) \end{bmatrix}$$

$$=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Given matrices are multiplicative inverse of each other

Q.6

(i) $(AB)^{-1} = B^{-1}A^{-1}$

Solution: $(AB)^{-1} = B^{-1}A^{-1}$

$$A=\begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}, B=\begin{bmatrix} -4 & -2 \\ 1 & -1 \end{bmatrix}$$

$$AB=\begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -4 & -2 \\ 1 & -1 \end{bmatrix}$$

$$=\begin{bmatrix} 4\times(-4)+0(1) & 4\times(-2)+0(-1) \\ -1\times(-4)+2(1) & -1\times(-2)+2(-1) \end{bmatrix}$$

$$=\begin{bmatrix} -16+0 & -8+0 \\ 4+2 & 2+(-2) \end{bmatrix}$$

$$=\begin{bmatrix} -16 & -8 \\ 6 & 0 \end{bmatrix}$$

To write in determinant form

$$|AB|=\begin{vmatrix} -16 & -8 \\ 6 & 0 \end{vmatrix}$$

$$|AB|=0-(-48)$$

$$|AB|=48$$

To write in Adj (AB)

$$\text{Adj (AB)}=\begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix}$$

$$(AB)^{-1}=\frac{1}{|AB|}\times AdjAB$$

$$=\frac{1}{48}\times\begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix}$$

$$=\begin{bmatrix} \frac{0}{48} & \frac{8}{48} \\ \frac{-6}{48} & \frac{-16}{48} \end{bmatrix}$$

$$=\begin{bmatrix} 0 & \frac{1}{6} \\ -\frac{1}{8} & -\frac{1}{3} \end{bmatrix}$$

To solve R.H. S

To write in determinant form

$$|B|=\begin{vmatrix} -4 & -2 \\ 1 & -1 \end{vmatrix}$$

$|B| = 4-(-2)$

$|B| = 4+2$

$|B| = 6$

To write in Adj B

$$AdjB = \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|} \times AdjB$$

By putting value

$$B^{-1} = \frac{1}{6} \times \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix}$$

To write in determinant form

$$|A| = \begin{vmatrix} 4 & 0 \\ -1 & 2 \end{vmatrix}$$

$|A| = 8-(-0)$

$|A| = 8$

To write in Adj A

$$AdjA = \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times AdjA$$

$$= \frac{1}{8} \times \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

To solve R.H.S

$$B^{-1}A^{-1} = \frac{1}{6}\begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix} \times \frac{1}{8}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$= \frac{1}{6} \times \frac{1}{8}\begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$= \frac{1}{48}\begin{bmatrix} -2+2 & 0+8 \\ -2-4 & 0-16 \end{bmatrix}$$

$$= \frac{1}{48}\begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{0}{48} & \frac{8}{48} \\ \frac{-6}{48} & \frac{-16}{48} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{1}{6} \\ -\frac{1}{8} & -\frac{1}{3} \end{bmatrix}$$

$$= \begin{bmatrix} 0 \times \frac{1}{48} & 8 \times \frac{1}{48} \\ -6 \times \frac{1}{48} & -16 \times \frac{1}{48} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{1}{6} \\ -\frac{1}{8} & -\frac{1}{3} \end{bmatrix}$$

Hence proved

L.H.S = R.H.S

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Report any mistake at freeilm786@gmail.com