Unit 1: Matrices and Determinants

Overview

Matrix:

A rectangular array of real numbers enclosed within brackets is said to form matrix.

Rows of a Matrix:

In matrix, the entries presented in horizontal way are called rows.

i.e. $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 8 \\ 7 & 1 & 5 \end{bmatrix} \begin{array}{l} \to R_1 \\ \to R_2 \\ \to R_3 \end{array}$.

Columns of a Matrix:

In matrix, all the entries presented in vertical way are called columns of matrix.

$$\begin{array}{ccc} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 8 \\ 7 & 1 & 5 \end{bmatrix} \\ \downarrow \quad \downarrow \quad \downarrow \\ C_1 \quad C_2 \quad C_3 \end{array}.$$

Order of a Matrix:

The number of rows and columns in a matrix specifies its order. If a matrix M has m rows and n columns then M is said to be of order, $m-by-n$.

i.e. $\begin{bmatrix} 0 & 8 & 0 \\ 0 & 4 & 8 \\ 7 & 1 & 5 \end{bmatrix}$ the order matrix is $3-by-3$

Equal Matrix's:

Let A and B be two matrices. Then A is said to be equal to B, and denoted by $A=B$, if and only if;

(i) The order of A = the order of B

(ii) Their corresponding entries are equal.

i.e. $A=\begin{bmatrix} 1 & 3 \\ -4 & 2 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & 2+1 \\ -4 & 4-2 \end{bmatrix}$ are equal matrices.

Rectangular Matrix:

A matrix M is called rectangular if, the number of rows of M is not equal to the number of columns of M.

e.g., $B=\begin{bmatrix} a & b & c \\ d & e & d \end{bmatrix}$.

Square Matrix:

A matrix is called a square matrix if its number of rows is equal to its number of columns.

i.e, $A=\begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}$

Null or Zero Matrix:

A matrix M is called a null or zero matrix if each of its entries is 0.

e.g. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

Transpose of a Matrix:

A matrix obtained by interchanging the rows into columns or columns into rows of a matrix is called transpose of that matrix.

Negative of a Matrix:

Let A be matrix. Then its negative, $-A$ is obtained by changing the signs of all the entries of A,

i.e. If $A=\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$, then $-A=\begin{bmatrix} -1 & 2 \\ -3 & -4 \end{bmatrix}$

Symmetric Matrix:

A square matrix is symmetric if it is equal to its transpose i.e., matrix A is symmetric if $A^t = A$.

Skew-Symmetric Matrix:

A square matrix A is said to be skew-symmetric if $A^t = -A$.

Diagonal Matrix:

A square matrix A is called a diagonal matrix if at least any one of the entries of its diagonal is not zero and non-diagonal entries must all be zero.

i.e. $A=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

Scalar Matrix:

A diagonal matrix is called a scalar matrix, if all the diagonal entries are same and non-zero. For example $\begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$ where k is a constant $\neq 0,1$

Identity Matrix:

A diagonal matrix is called identity (unit) matrix if all diagonal entries are 1 and it is denoted by I.

e.g., $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ is a 3-by-3 identity matrix.

Addition of Matrices:

Let A and B be any two matrices with real number entries. The matrices A and B are conformable for addition, if they have the same order.

Subtraction of Matrices:

If A and B are two matrices of same order then subtraction of matrix B from matrix A is obtained by subtracting the entries of matrix B from the corresponding entries of matrix A and it is denoted by $A-B$.

Multiplication of Matrices:

Two matrices A and B conformable for multiplication, giving product AB if the number of columns of A is equal to the number of rows of B.

Determinant of a 2-by-2 Matrix:

Let $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ be a 2-by-2 square matrix. The determinant of A, denoted by **det A** or $|A|$ is defined as.

$$|A| = \det A = \det\begin{bmatrix}a & b\\ c & d\end{bmatrix} = \begin{vmatrix}a & b\\ c & d\end{vmatrix} = ad-bc = \lambda \in R$$

Singular Matrix:

A square matrix A is called singular if the determinant of A is equal to zero.

For example, $A=\begin{bmatrix}1 & 2\\ 0 & 0\end{bmatrix}$ is a singular matrix, since det $A=1\times 0-0\times 2=0$.

Non-Singular Matrix:

A square matrix A is called non-singular if the determinant of A is not equal to zero.

For example $A=\begin{bmatrix}1 & 1\\ 0 & 2\end{bmatrix}$ is non-singular, since $\det A = 1\times 2-0\times 1 = 2 \neq 0$.

Adjoint of a Matrix:

Adjoint of a square matrix $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ is obtained by interchanging the diagonal entries and changing the sign of other entries. Adjoint of matrix A is denoted as Adj A.

i.e. Adj A $=\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}$

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Report any mistake at freeilm786@gmail.com