Unit 16: Theorems Related With Area

Overview

Theorem 16.1.1

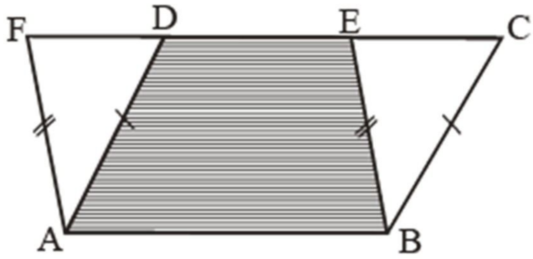
Parallelograms on the same base and between the same parallel lines (or of the same altitude) are equal in area

Given

Two parallelograms ABCD and ABEF having the same base $\overline{AB}$ between the same parallel lines $\overline{AB}$ and $\overline{DE}$

To prove

Area of parallelogram ABCD=area of parallelogram ABEF



Proof

Statements	Reasons
Area of (parallelogram ABCD) = Area of (Quad. ABED) + Area of (Δ CBE) … (1)	[Area addition axiom]
Area of (parallelogram ABEF) = Area of (Quad. ABED) + Area of (Δ DAF) … (2)	[Area addition axiom]
In Δs CBE and DAF	
$m\overline{CB} = m\overline{DA}$	[opposite sides of a Parallelogram]
$m\overline{BE} = m\overline{AF}$	[opposite sides of a Parallelogram]
$m\angle CBE = m\angle DAF$	$\left[\because \overline{BC} \parallel \overline{AD}, \overline{BE} \parallel \overline{AF}\right]$
$\Delta CBE \cong \Delta DAF$	[S.A.S Cong.axiom]
Area of (ΔCBE)= area of (ΔDAF)… (3)	[Cong. Area axiom]
Hence area of (Parallelogram ABCD) = area of (parallelogram ABEF)	From (1),(2) and (3)

Theorem 16.1.2

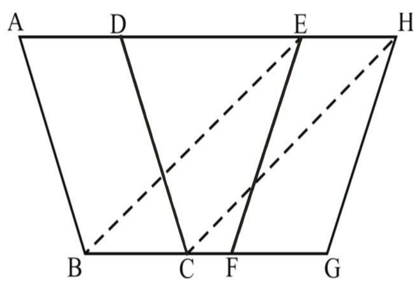
Parallelograms on equal bases and having the same (or equal) altitude area equal in area.

Given :

Parallelogram ABCD, EFGH are on equal base $\overline{BC}$, $\overline{FG}$ having equal altitudes

To prove

Area of (Parallelogram ABCD)= area of (parallelogram EFGH)



Construction

Place the parallelogram ABCD and EFGH So that their equal bases $\overline{BC}, \overline{FG}$ are in the straight line BCFG. Join $\overline{BE}$ and $\overline{CH}$

Proof

Statements	Reasons
The give 11^{mg} ABCD and EFGH are between the same parallels	
Hence ADEH is a straight line $\parallel$ to $\overline{BC}$	Their altitudes are equal (given)
$\therefore$ m$\overline{BC}$ = m$\overline{FG}$ =m$\overline{EH}$	
Now m$\overline{BC}$ =m$\overline{EH}$ and they are $\parallel$	Given
$\therefore \overline{BE}$ and $\overline{CH}$ are both equal and $\parallel$	EFGH is a parallelogram
Hence EBCH is a Parallelogram	A quadrilateral with two opposite side congruent and parallel is a parallelogram
Now $\parallel^{gm}$ ABCD = $\parallel^{gm}$ EBCH –(i)	Being on the same base $\overline{BC}$ and between the same parallels
But $\parallel^{gm}$ EBCH = $\parallel^{gm}$ EFGH – (ii)	Being on the same base $\overline{EH}$ and between the same parallels
Hence area $\parallel^{gm}$ (ABCD)= Area $\parallel^{gm}$ (EFGH)	From (i) and (ii)

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Report any mistake at freeilm786@gmail.com