Exercise 5.3

Q.1 Use the remainder theorem to find the remainder when

(i) $3x^3-10x^2+13x-6$ **is divided by** $(x-2)$.

Solution:

$P(x)=3x^3-10x^2+13x-6$

Since $P(x)$ is divided by $(x-2)$.

$\therefore P(2)=R$

$R=3(2)^3-10(2)^2+13(2)-6$

$=3(2)^3-10(2)^2+13(2)-6$

$=24-40+26-6$

R=4

Hence 4 is the remainder

(ii) $4x^3-4x+3$ **is divided by** $(2x-1)$

Solution:

$P(x)=4x^3-4x+3$

Since $P(x)$ is divided by $(2x-1)$

$\therefore R=P\left(\frac{1}{2}\right)$

$=4\left[\frac{1}{2}\right]^3-\cancel{4}^2\times\frac{1}{\cancel{2}}+3$

$=\cancel{4}\times\frac{1}{\cancel{8}^2}-2+3$

$=\frac{1}{2}-2+3$

$=\frac{1-4+6}{2}=\frac{3}{2}$

$R=\frac{3}{2}$

Hence $\frac{3}{2}$ is the remainder

(iii) $6x^4+2x^3-x+2$ **is divided by** $(x+2)$ **from** $x+2=0$

Solution: Given that

$P(x)=6x^4+2x^3-x+2$

Since $P(x)$ is divided by $(x+2)$

$\therefore R=P(-2)$

$=6(-2)^4+2(-2)^3-(-2)+2$

=96-16+2+2

R=84

Hence 84 is the remainder

(iv) $(2x-1)^3+6(3+4x)^2-10$ **is divided by** $2x+1$ **from** $2x+1=0$

$x=-\frac{1}{2}$

Solution: Given that

$P(x)=(2x-1)^3+6(3+4x)^2-10$

Since $P(x)$ is divided by $2x+1$

$\therefore R=P\left(-\frac{1}{2}\right)$

$=\left[\cancel{2}\left(-\frac{1}{\cancel{2}}\right)-1\right]^3+6\left[3+\cancel{4}^2\left(\frac{-1}{\cancel{2}}\right)\right]^2-10$

$=[-1-1]^3+6[3-2]^2-10$

$=[-2]^3+6-10=-8+6-10$

$R=-12$

Hence -12 is the remainder

(v) $x^3-3x^2+4x-14$ **is divided by** $(x+2)$ **from** $x+2=0, x=-2$

Solution: Given that

$P(x)=x^3-3x^2+4x-14$

Since $P(x)$ is divided by $(x+2)$

$\therefore R=P(-2)$

$=(-2)^3-3(-2)^2+4(-2)-14$

$=-8-12-8-14$

$R=-42$

Hence -42 is the remainder

Q.2

(i) If (x+2) is a factor of $3x^2-4kx-4k^2$ then find the values of k $x+2=0$ $x=-2$

Solution: Given that

$P(x)=3x^2-4kx-4k^2$

P(-2)=$3(-2)^2-4k(-2)-4k^2$

P(-2)=$12+8k-4k^2$

If (x+2) is the factor then remainder is equal to zero

P(-2)=0

$12+8k-4k^2=0$

$4(3+2k-k^2)=0$

$-k^2+2k+3=\frac{0}{4}$

$-k^2+3k-k+3=0$

$-k(k-3)-1(k-3)=0$

$(k-3)(-k-1)=0$

$k-3=0$ $\quad -k-1=0$

$k=3$ $\quad -1=k$

$k=-1$

(ii) If (x-1) is a factor of $x^3-kx^2+11x-6$ the find the value of k from $x-1=0$ $x=1$

Solution: Given that

$P(x)=x^3-kx^2+11x-6$

P(1)=$(1)^3-k(1)^2+11(1)-6$

P(1)=$1-k+11-6$

P(1)=$6-k$

If (x-1) is the factor then remainder is equal to zero

$P(1)=0$

6-k=0

k=6

Q.3 Without long division determine whether

(i) $(x-2)$ **and** $(x-3)$ **are factor of** $P(x)=x^3-12x^2+44x-48$ **from** $x-2=0$ $x=2$

Solution: Given that

$P(x)=x^3-12x^2+44x-48$

If (x-2) is the factor then remainder is equal to zero

$P(2)=(2)^3-12(2)^2+44(2)-48=8-48+88-48=0$

Hence $x-2$ is a factor of $P(x)$

For $x-3$

$R=P(3)$

$=(3)^3-12(3)^2+44(3)-48$

$=(3)^3-12(3)^2+44(3)-48$

=27-108+132-48

=159-156

R=3

3 is remainder hence $x-3$ is not factor of $P(x)$

P(3) is not equal to zero then x-3 is not factor of $P(x)=x3-12x^2+44x-48$

(ii) $(x-2),(x+3)$ **and** $(x-4)$ **are factor of** $q(x)=x^3+2x^2-5x-6$ **from** $x-2=0$**,** $x=2$

Solution: Given that

$q(x)=x^3+2x^2-5x-6$

For (x-2), putt x-2=0

$x=2$

$R=q(2)$

$=(2)^3+2(2)^2-5(2)-6$

$R=8+8-10-6$

$R=16-16$

$R=0$

Hence $x-2$ is factor of $q(x)=x^3+2x^2-5x-6$

For (x+3), putt x+3=0

x=-3

$R=q(-3)$

$= (-3)^3 + 2(-3)^2 - 5(-3) - 6$

$= -27 + 18 + 15 - 6$

$R = 0$

Hence x-2 is factor of $q(x) = x^3 + 2x^2 - 5x - 6$

For x-4, x-4=0

x=4

$R = q(4)$

$= (4)^3 + 2(4)^2 - 5(4) - 6$

$= 64 + 32 - 20 - 6$

R=70

Hence x-4 is not a factor of $q(x) = x^3 + 2x^2 - 5x - 6$

Q.4 For what value of *m* is the polynomial P(*x*) =4x^3-7x^2+6x-3*m* exactly divisible by *x*+2?

Solution:

$P(x) = 4x^3 - 7x^2 + 6x - 3m$

From x+2=0, x=-2

P(-2)=4(-2)^3-7(-2)^2 $+6(-2)-3m$

P(-2)=-32-28-12-3m=-72-3m

If (x+2) is the factor then remainder is equal to zero

$P(-2) = 0$

$-72 - 3m = 0$

$-72 = 3m$

$m = -\frac{72}{3}$

$m = -24$

Q.5 Determine the value of *k* if $P(x) = kx^3 + 4x^2 + 3x - 4$ and $q(x) = x^3 - 4x + k$ leaves the same remainder when divided by $(x-3)$.

Solution:

$q(x) = x^3 - 4x + k$

from x-3=0 x=3

$R_1 = q(3)$

$= (3)^3 - 4(3) + k$

$= 27 - 12 + k$

$= 15 + k$

$R_1 = 15 + k$ ....(i)

$R_2 = P(3)$

$= k(3)^3 + 4(3)^2 + 3(3) - 4$

$= 27k + 36 + 9 - 4$

$R_2 = 27k + 41$ ....(ii)

Since it leaves the same remainder.

Hence $R_1 = R_2$

$15 + k = 27k + 41$

$15 - 41 = 27k - k$

$-26 = 26k$

$k = \frac{-26}{26}$

$k = -1$

Q.6 The remainder after dividing the polynomial $P(x) = x^3 + ax^2 + 7$ by $(x+1)$ is 2*b* calculate the value of a and *b* if this expression leaves a remainder of $(b+5)$ on being dividing by $(x-2)$

Solution:

Let

$P(x) = x^3 + ax^2 + 7$

Since $P(x)$ is divided by (x+1)

Put $x + 1 = 0$ $x = -1$

R=P(-1)

$= (-1)^3 + a(-1)^2 + 7$

$= -1 + a + 7$

$R = a + 6$

According to first condition remainder is 2b

$2b = a + 6$ ...(i)

Since $P(x)$ is divided by (x-2)

Put $x - 2 = 0$

$x = 2$

P(2)=(2)^3+a(2)^2+7

=8+4a+7

R=15+4a

According to second condition remainder is (b+5)

15+4a=b+5

4a-b=5-15

4a-b=-10 ...(ii)

Solving equations (i) and (ii)
From equation (ii) b=10+4a putting the value of be in equation (i)
a+6=2(10+4a)
a=20+8a-6
-8a+a=14
-7a=14
$a = \frac{14}{-7}$
a=-2
Putting the value of a in equation (ii)
$4a - b = -10$
$4(-2) - b = -10$
$-8 - b = -10$
$-8 + 10 = b$
$2 = b$
$b = 2$

Q.7 The polynomial $x^3 + lx^2 + mx + 24$ has a factor $(x+4)$ and it leaves a remainder of 36 when divided by $(x-2)$
Find the values of *l* and *m*.

Solution:
Let
$P(x) = x^3 + lx^2 + mx + 24$
From $x + 4 = 0$ $x = -4$
$P(-4) = (-4)^3 + l(-4)^2 + m(-4) + 24$
$P(-4) = -64 + 16l - 4m + 24$
$P(-4) = 16l - 4m - 40$
According to condition (*x*+4) is the factor then
$16l - 4m - 40 = 0$
$4[4l - m - 10] = 0$
$4l - m - 10 = 0$ (i)
from $x - 2 = 0$ $x = 2$
Now $P(2) = (2)^3 + l(2)^2 + m(2) + 24$
$P(2) = 8 + 4l + 2m + 24$
$P(2) = 4l + 2m + 32$
According the condition
$4l + 2m + 32 = 36$
$4l + 2m = 36 - 32$
$4l + 2m = 4$
$4l + 2m - 4 = 0$ (ii)
Subtracting (i) from (ii)
$\cancel{4l} + 2m - 4 = 0$
$\underline{\pm \cancel{4l} \mp m \mp 10 = 0}$
$3m + 6 = 0$
$3m + 6 = 0$
$3m = -6$
$m = \frac{-\cancel{6}2}{\cancel{3}}$
$m = -2$
Putting the value of m is equation (i)
$4l - (-2) - 10 = 0$
$4l + 2 - 10 = 0$
$4l - 8 = 0$
$4l = 8$
$l = \frac{2\cancel{8}}{\cancel{4}}$
$l = 2$

Q.8 The expression $lx^3 + mx^2 - 7$ leaves remainder of -3 and 12 when divided by $(x-1)$ and $(x+2)$ respectively. Calculate the value of *l* and *m*.

Solution:
$P(x) = lx^3 + mx^2 - 7$
from *x*-1=0 *x*=1
$P(1) = l(1)^3 + m(1)^2 - 4$
$P(1) = l + m - 4$
According to conditions *l*+*m*-4=-3
$l + m = 4 - 3$
$l = 1 - m$ (i)
From $x + 2 = 0$ $x = -2$
$P(-2) = l(-2)^3 + m(-2)^2 - 4$
$P(-2) = -8l + 4m - 4$
According to condition
$-8l + 4m - 4 = 12$
Putting the value of *l* in the equation

$-8[1-m]+4m=16$

$-8+8m+4m=16$

$12m=16+8$

$12m=24$

$m=\frac{\cancel{24}^{2}}{\cancel{12}}$

$m=2$

Putting the value of m is equation (i)

$l=1-2$

$l=-1$

$m=2$

$l=-1$

Q.9 The expression $ax^3-9x^2+bx+3a$ is exactly divisible by x^2-5x+6. Find the value of a and b.

Solution: Given that

$P(x)=ax^3-9x^2+bx+3a$

$x^2-5x+6 \quad =x^2-2x-3x+6$

$=x[x-2]-3[x-2]$

$=[x-2][x-3]$

$(x-2)(x-3)$ is divides the expression $ax^3-9x^2+bx+3a$ from $x-2=0$, $x=2$

$P(2)=a(2)^3-9(2)^2+b(2)+3a$

$P(2)=8a-36+2b+3a$

$P(2)=11a+2b-36$

According to condition (x-2) is the factor so

11a+2b-36=0 (i)

From x-3=0,x=3

$P(3)=a(3)^3-9(3)^2+b(3)+3a$

$P(3)=27a-81+3b+3a$

$P(3)=30a+3b-81$

According to condition (x-3) is the factor so

$30a+3b-81=0$ (ii)

$3(10a+b-27)=0$

$10a+b-27=\frac{0}{3}$

$b=27-10a$ (iii)

Putting the value of b in equation (i)

$11a+2[27-10a)-36=0$

$11a+54-20a-36=0$

$-9a+18=0$

$+18=9a$

$a=\frac{+\cancel{18}\,2}{\cancel{9}}$

$a=+2$

Putting the value of a in equation (iii)

$b=27-10(+2)$

$b=27-20$

$b=7$

$a=2$

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Report any mistake at freeilm786@gmail.com