

## Review Exercise 4

### Q.1 Multiple type questions?

(i) is an algebraic ...

- (a) Expression  
(c) Equation

- (b) Sentence  
(d) In-equation

(ii) The degree of polynomial  $4x^4 + 3x^2y$  is

- (a) 1  
(c) 3

- (b) 2  
(d) 4

(iii)  $a^3 + b^3$  is equal to

- (a)  $(a-b)(a^2 + ab + b^2)$   
(c)  $(a-b)(a^2 - ab + b^2)$

- (b)  $(a+b)(a^2 - ab + b^2)$   
(d)  $(a-b)(a^2 + ab + b^2)$

(iv)  $(3 + \sqrt{2})(3 - \sqrt{2})$  is equal to

- (a) 7  
(c) -1

- (b) -7  
(d) 1

(v) Conjugate of surd  $a + \sqrt{b}$  is;

- (a)  $-a + \sqrt{b}$   
(c)  $\sqrt{a} + \sqrt{b}$

- (b)  $a - \sqrt{b}$   
(d)  $\sqrt{a} - \sqrt{b}$

(vi)  $\frac{1}{a-b} - \frac{1}{a+b}$  is equal to

- (a)  $\frac{2a}{a^2 - b^2}$   
(c)  $\frac{-2a}{a^2 - b^2}$

- (b)  $\frac{2b}{a^2 - b^2}$   
(d)  $\frac{-2b}{a^2 - b^2}$

(vii)  $\frac{a^2 - b^2}{a+b}$  is equal to

- (a)  $(a-b)^2$   
(c)  $a+b$

- (b)  $(a+b)^2$   
(d)  $a-b$

(viii)  $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$  is equal to

- (a)  $a^2 + b^2$   
(c)  $a-b$

- (b)  $a^2 - b^2$   
(d)  $a+b$

ANSWER KEY

i	ii	iii	iv	v	vi	vii	viii
a	d	b	a	b	b	d	c

**Q.2 Fill in the blanks**

(i) The degree of polynomial  $x^2y^2 + 3xy + y^3$  is \_\_\_\_\_

(ii)  $x^2 - 4$  \_\_\_\_\_

(iii)  $x^3 + \frac{1}{x^3} = \left[ x + \frac{1}{x} \right] (\text{_____})$

(iv)  $2(a^2 + b^2) = (a + b)^2 + (\text{_____})^2$

(v)  $\left[ x - \frac{1}{x} \right]^2 = \text{_____}$

(vi) Order of surd  $\sqrt[3]{x}$  is \_\_\_\_\_

(vii)  $\frac{1}{2 - \sqrt{3}} = \text{_____}$

ANSWER KEY

(i) 4

(ii)  $(x - 2)(x + 2)$

(iii)  $x^2 - 1 + \frac{1}{x^2}$

(iv)  $a - b$

(v)  $x^2 + \frac{1}{x^2} - 2$

(vi) 3

(vii)  $2 + \sqrt{3}$

**Q.3** If  $x + \frac{1}{x} = 3$ , find

(i)  $x^2 + \frac{1}{x^2}$

**Solution:** Given that  $x + \frac{1}{x} = 3$

$$\therefore (a + b)^2 = a^2 + b^2 + 2ab$$

Putting the values

$$\left[ x + \frac{1}{x} \right]^2 = (x)^2 + \left( \frac{1}{x} \right)^2 + 2(x) \left( \frac{1}{x} \right)$$

$$(3)^2 = x^2 + \frac{1}{x^2} + 2$$

$$9 = x^2 + \frac{1}{x^2} + 2$$

$$9 - 2 = x^2 + \frac{1}{x^2}$$

$$x^2 + \frac{1}{x^2} = 7 \text{ Ans}$$

(ii)  $x^4 + \frac{1}{x^4}$

**Solution:** Given that  $x^2 + \frac{1}{x^2} = 7$

$$\left(x^2 + \frac{1}{x^2}\right)^2 = \left(x^2\right)^2 + \left(\frac{1}{x^2}\right)^2 + 2\left(x^2\right)\left(\frac{1}{x^2}\right)$$

$$(7)^2 = x^4 + \frac{1}{x^4} + 2$$

$$49 = x^4 + \frac{1}{x^4} + 2$$

$$x^4 + \frac{1}{x^4} = 49 - 2$$

$$x^4 + \frac{1}{x^4} = 47 \text{ Ans}$$

**Q.4** If  $x - \frac{1}{x} = 2$  find

(i)  $x^2 + \frac{1}{x^2}$

(ii)  $x^4 + \frac{1}{x^4}$

**Solution (i)**

**Given that**  $x - \frac{1}{x} = 2$

$$\therefore (a+b)^2 = a^2 + b^2 + 2ab$$

Putting the values

$$\left(x - \frac{1}{x}\right)^2 = \left(x\right)^2 + \left(\frac{1}{x}\right)^2 - 2\left(x\right)\left(\frac{1}{x}\right)$$

$$(2)^2 = x^2 + \frac{1}{x^2} - 2$$

$$4 + 2 = x^2 + \frac{1}{x^2}$$

$$x^2 + \frac{1}{x^2} = 6 \text{ Ans}$$

**Solution (ii)**

Given that  $x^2 + \frac{1}{x^2} = 6$

$$\left(x^2 + \frac{1}{x^2}\right)^2 = x^4 + \frac{1}{x^4} + 2\left(x^2\right)\left(\frac{1}{x^2}\right)$$

$$(6)^2 = x^4 + \frac{1}{x^4} + 2$$

$$x^4 + \frac{1}{x^4} = 36 - 2$$

$$x^4 + \frac{1}{x^4} = 34 \text{ Ans}$$

**Q.5** Find the value of  $x^3 + y^3$  and  $xy$  if  $x + y = 5$  and  $x - y = 3$ .

**Solution:** Given that  $x + y = 5$

$$x - y = 3$$

As we know that

$$\therefore (x+y)^2 - (x-y)^2 = 4xy$$

Putting the values

$$4xy = (5)^2 - (3)^2$$

$$4xy = 25 - 9$$

$$4xy = 16$$

$$xy = \frac{16}{4}$$

$$xy = 4 \text{ Ans}$$

As we know that

$$(x+y)^3 = x^3 + y^3 + 3xy(x+y)$$

Putting the values

$$(5)^3 = x^3 + y^3 + 3 \times 4 \times 5$$

$$125 = x^3 + y^3 + 60$$

$$x^3 + y^3 = 125 - 60$$

$$x^3 + y^3 = 65$$

$$x^3 + y^3 = 65 \text{ Ans}$$

**Q.6** If  $P = 2 + \sqrt{3}$ , find

(i)  $P + \frac{1}{P}$

**Solution:** Given that  $P = 2 + \sqrt{3}$

$$\frac{1}{P} = \frac{1}{2 + \sqrt{3}}$$

$$\frac{1}{P} = \frac{1}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}}$$

$$\frac{1}{P} = \frac{2 - \sqrt{3}}{(2)^2 - (\sqrt{3})^2}$$

$$\frac{1}{P} = \frac{2 - \sqrt{3}}{4 - 3}$$

$$\frac{1}{P} = \frac{2 - \sqrt{3}}{1}$$

$$\frac{1}{P} = 2 - \sqrt{3}$$

$$P + \frac{1}{P} = 2 + \cancel{\sqrt{3}} + 2 - \cancel{\sqrt{3}}$$

$$P + \frac{1}{P} = 4 \text{ Ans}$$

(ii)  $P - \frac{1}{P}$

As we know that

$$\frac{1}{P} = 2 - \sqrt{3} \text{ and}$$

$$P = 2 + \sqrt{3}$$

$$P - \frac{1}{P} = 2 + \sqrt{3} - (2 - \sqrt{3})$$

$$= \cancel{2} + \sqrt{3} - \cancel{2} + \sqrt{3}$$

$$= 2\sqrt{3} \text{ Ans}$$

(iii)  $P^2 + \frac{1}{P^2}$

**Solution:** Given that  $P + \frac{1}{P} = 4$

$$\therefore (a + b)^2 = a^2 + b^2 + 2ab$$

$$\left(P + \frac{1}{P}\right)^2 = (P)^2 + \left(\frac{1}{P}\right)^2 + 2(\cancel{P})\left(\frac{1}{\cancel{P}}\right)$$

$$(4)^2 = P^2 + \frac{1}{P^2} + 2$$

$$16 - 2 = P^2 + \frac{1}{P^2}$$

$$P^2 + \frac{1}{P^2} = 14 \text{ Ans}$$

(iv)  $P^2 - \frac{1}{P^2}$

**Solution:**

$$P^2 - \frac{1}{P^2} = \left(P + \frac{1}{P}\right)\left(P - \frac{1}{P}\right)$$

$$P^2 - \frac{1}{P^2} = (4)(2\sqrt{3})$$

$$= 8\sqrt{3} \text{ Ans}$$

**Q.7** If  $q = \sqrt{5} + 2$  find.

(i)  $q + \frac{1}{q}$

**Solution:** Given that  $q = \sqrt{5} + 2$

$$\frac{1}{q} = \frac{1}{\sqrt{5} + 2} \times \frac{\sqrt{5} - 2}{\sqrt{5} - 2}$$

$$= \frac{\sqrt{5} - 2}{(\sqrt{5})^2 - (2)^2}$$

$$= \frac{\sqrt{5} - 2}{5 - 4}$$

$$= \sqrt{5} - 2$$

$$q + \frac{1}{q} = \sqrt{5} + \cancel{2} + \sqrt{5} - \cancel{2}$$

$$q + \frac{1}{q} = 2\sqrt{5} \text{ Ans}$$

(ii)  $q - \frac{1}{q}$

**Solution:** Given that  $q = \sqrt{5} + 2$

$$\frac{1}{q} = \sqrt{5} - 2$$

$$\begin{aligned} q - \frac{1}{q} &= \sqrt{5} + 2 - (\sqrt{5} - 2) \\ &= \cancel{\sqrt{5}} + 2 - \cancel{\sqrt{5}} + 2 \\ q - \frac{1}{q} &= 4 \text{ Ans} \end{aligned}$$

(iii)  $q^2 + \frac{1}{q^2}$

**Solution:** Given that  $q - \frac{1}{q} = 4$

Squaring both sides

$$\left(q - \frac{1}{q}\right)^2 = (4)^2$$

$$q^2 + \frac{1}{q^2} - 2 = 16$$

$$q^2 + \frac{1}{q^2} = 16 + 2$$

$$q^2 + \frac{1}{q^2} = 18 \text{ Ans}$$

(iv)  $q^2 - \frac{1}{q^2}$

**Solution:** Given that  $q + \frac{1}{q} = 2\sqrt{5}$

$$q - \frac{1}{q} = 4$$

By using formula

$$\begin{aligned} q^2 - \frac{1}{q^2} &= \left(q + \frac{1}{q}\right)\left(q - \frac{1}{q}\right) \\ &= (2\sqrt{5})(4) \\ &= 8\sqrt{5} \text{ Ans} \end{aligned}$$

### Q.8 Simplify

(i)  $\frac{\sqrt{a^2+2} + \sqrt{a^2-2}}{\sqrt{a^2+2} - \sqrt{a^2-2}}$

**Solution:**

$$\begin{aligned} &= \frac{\sqrt{a^2+2} + \sqrt{a^2-2}}{\sqrt{a^2+2} - \sqrt{a^2-2}} \times \frac{\sqrt{a^2+2} + \sqrt{a^2-2}}{\sqrt{a^2+2} + \sqrt{a^2-2}} \\ &= \frac{(\sqrt{a^2+2} + \sqrt{a^2-2})^2}{(\sqrt{a^2+2})^2 - (\sqrt{a^2-2})^2} \\ &= \frac{(\sqrt{a^2+2})^2 + (\sqrt{a^2-2})^2 + 2(\sqrt{a^2+2})(\sqrt{a^2-2})}{a^2+2 - a^2+2} \\ &= \frac{a^2 + 2 + a^2 - 2 + 2(\sqrt{a^4 - 2a^2 + 2a^2 - 4})}{4} \end{aligned}$$

$$\begin{aligned} &= \frac{2a^2 + 2\sqrt{a^4 - 4}}{4} \\ &= \frac{2(a^2 + \sqrt{a^4 - 4})}{4} \\ &= \frac{a^2 + \sqrt{a^4 - 4}}{2} \text{ Ans} \end{aligned}$$

(ii)  $\frac{1}{a - \sqrt{a^2 - x^2}} - \frac{1}{a + \sqrt{a^2 - x^2}}$

$$\begin{aligned} &= \left( \frac{1}{a - \sqrt{a^2 - x^2}} \times \frac{a + \sqrt{a^2 - x^2}}{a + \sqrt{a^2 - x^2}} \right) \\ &\quad - \left( \frac{(1)}{(a + \sqrt{a^2 - x^2})} \frac{(a - \sqrt{a^2 - x^2})}{(a - \sqrt{a^2 - x^2})} \right) \\ &= \left( \frac{a + \sqrt{a^2 - x^2}}{(a)^2 - (\sqrt{a^2 - x^2})^2} \right) - \left( \frac{a - \sqrt{a^2 - x^2}}{(a)^2 - (\sqrt{a^2 - x^2})^2} \right) \\ &= \left( \frac{a + \sqrt{a^2 - x^2}}{a - (a^2 - x^2)} \right) - \left( \frac{a - \sqrt{a^2 - x^2}}{a - (a^2 - x^2)} \right) \\ &= \left( \frac{a + \sqrt{a^2 - x^2}}{a^2 - a^2 + x^2} \right) - \left( \frac{a - \sqrt{a^2 - x^2}}{\cancel{a} - \cancel{a} + x^2} \right) \end{aligned}$$

$$\begin{aligned}
 &= \left( \frac{a + \sqrt{a^2 - x^2}}{x^2} \right) - \left( \frac{a - \sqrt{a^2 - x^2}}{x^2} \right) \\
 &= \frac{\cancel{a} + \sqrt{a^2 - x^2} - \cancel{a} + \sqrt{a^2 - x^2}}{x^2} \\
 &= \frac{2\sqrt{a^2 - x^2}}{x^2} \text{ Ans}
 \end{aligned}$$

**Last Updated: September 2020**

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