

Exercise 5.4

Q.1 $x^3 - 2x^2 - x + 2$

Solution: Given that

$$P(x) = x^3 - 2x^2 - x + 2$$

$P=2$ and possible factor of 2 are $\pm 1, \pm 2$.

Here $q=1$ and possible factor of 1 are ± 1 .

So possible factor of $P(x)$ will be form $\frac{P}{q} = \pm 1, \pm 2$

$$P(x) = x^3 - 2x^2 - x + 2$$

Put $x=1$

$$P(1) = (1)^3 - 2(1)^2 - 1 + 2 = 1 - 2 - 1 + 2 = 0 \text{ Remainder is equal to zero so } (x-1) \text{ is factor}$$

Put $x=-1$

$$P(-1) = (-1)^3 - 2(-1)^2 - (-1) + 2 = -1 - 2 + 1 + 2 = 0 \text{ Remainder is equal to zero so } (x+1) \text{ is factor}$$

Put $x=2$

$$P(2) = (2)^3 - 2(2)^2 - (2) + 2 = 8 - 8 - 2 + 2 = 0 \text{ Remainder is equal to zero so } (x-2) \text{ is factor}$$

$$x^3 - 2x^2 - x + 2 = (x-1)(x+1)(x-2)$$

Q.2 $x^3 - x^2 - 22x + 40$

Solution: Given that

$$P(x) = x^3 - x^2 - 22x + 40$$

$P=40$

possible factor of 40 = $\pm 1, \pm 2, \pm 4, \pm 5, \pm 8, \pm 10, \pm 20, \pm 40$

Here $q=1$ and possible factor of 1 are ± 1

So possible factor of $P(x)$ will be from

$$\frac{P}{q} = \pm 1, \pm 2, \pm 4, \pm 5, \pm 8, \pm 10, \pm 20, \pm 40$$

$$P(x) = x^3 - x^2 - 22x + 40$$

Put $x = 2$

$$P(2) = (2)^3 - (2)^2 - 22(2) + 40$$

$$= 8 - 4 - 44 + 40 = 0$$

Remainder is equal to zero so $(x-2)$ is a factor

Put $x=4$

$$P(4) = (4)^3 - (4)^2 - 22(4) + 40$$

$$= 64 - 16 - 88 + 40 = 0$$

Remainder is not equal to zero so $(x-4)$ is a factor

Put $x=-5$

$$\begin{aligned} P(-5) &= (-5)^3 - (-5)^2 - 22(-5) + 40 \\ &= -125 - 25 + 110 + 40 \\ &= -150 + 150 \\ &= 0 \end{aligned}$$

Remainder is equal to zero so $(x+5)$ is a factor

$$\text{Hence } x^3 - x^2 - 22x + 40 = (x-2)(x-4)(x+5)$$

Q.3 $x^3 - 6x^2 + 3x + 10$

Solution: Given that

$$P(x) = x^3 - 6x^2 + 3x + 10$$

$$P=10$$

So possible factor of 10 are $\pm 1, \pm 2, \pm 5, \pm 10$

Here $q=1$ So, possible factor of 1 are ± 1 .

So possible of factor of $P(x)$ can be found from $\frac{P}{q} = \pm 1, \pm 2, \pm 5, \pm 10$

$$P(x) = x^3 - 6x^2 + 3x + 10$$

$$\text{Put } x = -1$$

$$P(-1) = (-1)^3 - 6(-1)^2 + 3(-1) + 10 = -1 - 6 - 3 + 10 = 0$$

Remainder is equal to zero so $(x+1)$ is a factor

$$\text{Put } x = 2$$

$$P(2) = (2)^3 - 6(2)^2 + 3(2) + 10 = 8 - 24 + 6 + 10 = 0$$

Remainder is equal to zero so $(x-2)$ is a factor

$$\text{Put } x = 5$$

$$P(5) = (5)^3 - 6(5)^2 + 3(5) + 10 = 125 - 150 + 15 + 10 = 0 \text{ Remainder is equal to zero so } (x-5) \text{ is a factor}$$

$$\text{Hence } x^3 - 6x^2 + 3x + 10 = (x+1)(x-2)(x-5)$$

Q.4 $x^3 + x^2 - 10x + 8$

Solution: Given that

$$P(x) = x^3 + x^2 - 10x + 8$$

$$P=8 \text{ So possible factor of 8}$$

are $\pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 7, \pm 8$.

Here $q=1$ So possible factor can be found from $\frac{P}{q} = \pm 1, \pm 2, \pm 4, \pm 8$

$$P(x) = x^3 + x^2 - 10x + 8$$

$$\text{Put } x = 1$$

$$P(1) = (1)^3 + (1)^2 - 10(1) + 8 = 1 + 1 - 10 + 8 = 0$$

Remainder is equal to zero so $(x-1)$ is a factor

$$\text{Put } x = 2$$

$$P(2) = 2^3 + 2^2 - 10(2) + 8$$

$$= 8 + 4 - 20 + 8$$

$$= 20 - 20$$

$$= 0$$

Remainder is equal to zero so $(x-2)$ is a factor

Put $x = -4$

$$\begin{aligned} P(-4) &= (-4)^3 + (-4)^2 - 10(-4) + 8 \\ &= -64 + 16 + 40 + 8 \\ &= -64 + 64 \\ &= 0 \end{aligned}$$

Remainder is equal to zero so $(x+4)$ is a factor

$$\text{Hence } x^3 + x^2 - 10x + 8 = (x-1)(x-2)(x+4)$$

Q.5 $x^3 - 2x^2 - 5x + 6$

Solution: Given that

$$P(x) = x^3 - 2x^2 - 5x + 6$$

$P = 6$ So factors of 6 are $\pm 1, \pm 2, \pm 3, \pm 6$

Here $q=1$ So factors of 1 are ± 1 .

So possible factors of $P(x)$ can be found from $\frac{P}{q} = \pm 1, \pm 2, \pm 3, \pm 6$

$$P(x) = x^3 - 2x^2 - 5x + 6$$

Put $x=1$

$$\begin{aligned} P(1) &= (1)^3 - 2(1)^2 - 5(1) + 6 \\ &= 1 - 2 - 5 + 6 \\ &= -7 + 7 \\ &= 0 \end{aligned}$$

Remainder is equal to zero so $(x-1)$ is a factor

Put $x=-2$

$$\begin{aligned} P(-2) &= (-2)^3 - 2(-2)^2 - 5(-2) + 6 \\ &= -8 - 8 + 10 + 6 \\ &= -16 + 16 \\ &= 0 \end{aligned}$$

Remainder is equal to zero so $(x+2)$ is a factor

Put $x=3$

$$\begin{aligned} P(3) &= (3)^3 - 2(3)^2 - 5(3) + 6 \\ &= 27 - 6 - 15 + 6 \\ &= 27 - 27 \\ &= 0 \end{aligned}$$

Remainder is equal to zero so $(x-3)$ is a factor

$$\text{Hence } x^3 - 2x^2 - 5x + 6 = (x-1)(x+2)(x-3)$$

Q.6 $x^3 + 5x^2 - 2x - 24$

Solution: Given that

$$P(x) = x^3 + 5x^2 - 2x - 24$$

$P = -24$ So possible factors of 24 are $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

Here $q=1$. So possible factors of 1 are ± 1 .

So possible factors of $P(x)$ will be found from

$$\frac{P}{q} = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$$

$$P(x) = x^3 + 5x^2 - 2x - 24$$

Put $x=2$

$$P(2) = (2)^3 + 5(2)^2 - 2(2) - 24$$

$$= 8 + 20 - 4 - 24$$

$$= 28 - 28$$

$$= 0$$

Remainder is equal to zero so $(x-2)$ is a factor

Put $x=-3$

$$P(-3) = (-3)^3 + 5(-3)^2 - 2(-3) - 24$$

$$= -27 + 45 + 6 - 24$$

$$= -51 + 51$$

$$= 0$$

Remainder is equal to zero so $(x+3)$ is a factor

Put $x=-4$

$$P(-4) = (-4)^3 + 5(-4)^2 - 2(-4) - 24$$

$$= -64 + 80 + 8 - 24$$

$$= -88 + 88$$

$$= 0$$

Remainder is equal to zero so $(x+4)$ is a factor

$$\text{Hence } x^3 + 5x^2 - 2x - 24 = (x-2)(x+3)(x+4)$$

$$\text{Q.7 } 3x^3 - x^2 - 12x + 4$$

Solution: Given that

$$P(x) = 3x^3 - x^2 - 12x + 4$$

$P=4$ So possible factors of 4 are $\pm 1, \pm 2, \pm 4$.

Here $q=3$ So possible factors of 3 are $\pm 1, \pm 3$.

So possible factors of $P(x)$ can be found from

$$\frac{P}{q} = \pm 1, \pm 2, \pm 4, \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}$$

Put $x=2$

$$P(2) = 3(2)^3 - (2)^2 - 12(2) + 4$$

$$= -24 - 4 + 24 + 4$$

$$= 28 - 28$$

$$= 0$$

Remainder is equal to zero so $(x-2)$ is a factor

Put $x=-2$

$$P(-2) = 3(-2)^3 - (-2)^2 - 12(-2) + 4$$

$$= -24 - 4 + 24 + 4$$

$$= 28 - 28$$

$$= 0$$

Remainder is equal to zero so $(x+2)$ is a factor

Put $x = \frac{1}{3}$

$$P\left(\frac{1}{3}\right) = 3\left(\frac{1}{3}\right)^3 - \left(\frac{1}{3}\right)^2 - 12\left(\frac{1}{3}\right) + 4$$

$$= \cancel{2} \left(\frac{1}{\cancel{2} \cancel{9}} \right) - \frac{1}{9} - \cancel{4} + \cancel{4}$$

$$P\left(\frac{1}{3}\right) = \frac{1}{9} - \frac{1}{9}$$

$$= 0$$

$$x = \frac{1}{3} \Rightarrow \begin{matrix} 3x = 1 \\ 3x - 1 \end{matrix}$$

Remainder is equal to zero so $(3x-1)$ is a factor

$$\text{Hence } 3x^3 - x^2 - 10x + 4 = (x-2)(x+2)(3x-1)$$

Q.8 $2x^3 + x^2 - 2x - 1$

Solution: Given that

$$P(x) = 2x^3 + x^2 - 2x - 1$$

$P = -1$ So possible factors of -1 are $\pm 1, \pm 2$.

Here $q=1$. So possible factors of $P(x)$ will be found from $\frac{P}{q}$

$$\frac{P}{q} = \pm 1, \pm 2, \pm \frac{1}{2}$$

$$P(x) = 2x^3 + x^2 - 2x - 1$$

Put $x=1$

$$P(1) = 2(1)^3 + (1)^2 - 2(1) - 1$$

$$= 2 + 1 - 2 - 1$$

$$= 3 - 3$$

$$= 0$$

Remainder is equal to zero $(x-1)$ is a factor

Put $x=-1$

$$P(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$$

$$= 2 + 1 - 2 - 1$$

$$= 3 - 3$$

$$= 0$$

Remainder is equal to zero $(x+1)$ is a factor

$$\text{Put } x = \frac{-1}{2}$$

$$P\left(\frac{-1}{2}\right) = 2\left[\frac{-1}{2}\right]^3 + \left[\frac{-1}{2}\right]^2 - \cancel{2}\left[\frac{-1}{\cancel{2}}\right] - 1$$

$$P\left(\frac{-1}{2}\right) = 2\left[\frac{-1}{\cancel{2}4}\right] + \frac{1}{4} + \cancel{1} - \cancel{1}$$

$$P\left(\frac{-1}{2}\right) = -\frac{1}{4} + \frac{1}{4}$$
$$= 0$$

$$x = -\frac{1}{2}$$

$$2x = -1$$

$$2x + 1 = 0$$

Remainder is equal to zero so $(2x+1)$ is a factor

$$\text{Hence } 2x^3 + x^2 - 2x - 1 = (x-1)(x+1)(2x+1)$$

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Report any mistake at freeilm786@gmail.com