UNIT 4

TURNING EFFECT OF FORCE

LONG QUESTIONS

4.2 ADDITION OF FORCES

Q.No.1 Which method is used for addition of forces? Explain with example.

Ans: Force is a vector quantity. It has magnitude as well direction; therefore forces are not added by ordinary arithmetic rules. They are added by a method known as head to tail rule.

Head to Tail rule

Draw the representative lines of all the vector to be added in such a way that head of first vector coincides with the tail of second vector, head of second vector coincides with the tail of third vector and so on. The line obtained by joining the tail of first vector with the head of last vector represent resultant vector.

Method

The method of addition of two vectors is given below:

- Select the frame of reference and suitable scale and draw the representative of all the vectors according to the scale; such as vector **A** and **B**.
- Take any one of the vectors as first vector e.g. vector **A**. then draw next vector **B** such that its tail coincides with the head of the first vector **A**. Similarly draw the next vector for the third force (if any) with its tail coinciding with the head of the previous vector and so on.

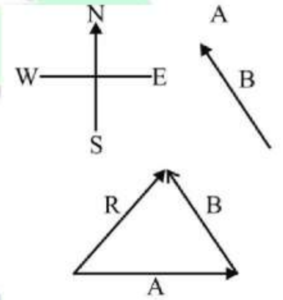


Figure 4.5: Adding vectors by head to tail rule.

- Now draw a vector **R** such that its tail is at the tail of vector **A**, the first vector, while its head is at the head of vector **B**.
- Vector R represents the resultant force completely in magnitude and direction.
- The length of the line according to scale represents the magnitude of the resultant vector.
- The direction of the resultant vector is from the tail of the first vector towards the head of the second.

4.3 RESOLUTION OF FORCES

Q.No.2 Resolve the vector into its rectangular components.

Ans: The decomposition or division of a vector into its components is called resolution of a vector.

OR

The splitting of a single vector into two mutually perpendicular components is called the resolution of that force.

The process of splitting up vectors (forces) into their component forces is called resolution of force. If a force is formed from two mutually perpendicular components then such components are called **perpendicular components**.

Determination of Rectangular components of a vector

Suppose a vector **F** acts on a body by making an angle θ with the x-axis which is represented by the vector **OA** as shown in the figure. Draw perpendicular from **A** on x-axis as **AB**. According to head to tail rule, **OA** is the resultant vector of **OB** and **BA**.

So $\quad \mathbf{OA} = \mathbf{OB} + \mathbf{BA} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$

Since the angle between **BA** and **OB** is 90^0, hence these are called the perpendicular components of the vector OA representing F.

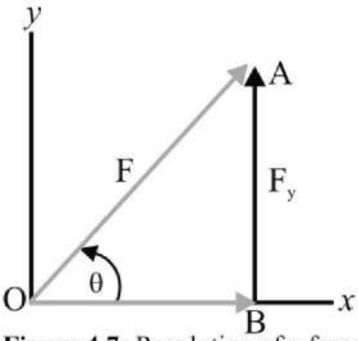


Figure 4.7: Resolution of a force

Horizontal or x-component

The component **OB** along x-axis is represented by $\mathbf{F_x}$ and is called the X-component or horizontal component of the vector **F**.

Vertical or y-component

The component **BA** is represented by $\mathbf{F_y}$ and is called the y-component or vertical component of the vector **F**.

So equation (1) can be represented by,

$$\mathbf{F} = \mathbf{F_x} + \mathbf{F_y}$$

Magnitude of Rectangular components

The magnitude of the perpendicular components F_x and F_y of forces $\mathbf{F_x}$ and $\mathbf{F_y}$ can be found by using the trigonometric ratios. In right angled triangle OAB,

$\cos\theta = \dfrac{OB}{OA}$ Or $OB = OA \cos\theta$

But $OB = F_x$ and $OA = F$

Hence

$$\boxed{\mathbf{F_x = F \cos\theta}}$$

Similarly,

$\sin\theta = \dfrac{BA}{OA}$ or $BA = OA \sin\theta$

But $BA = F_y$ and $OA = F$

Therefore,

$$\boxed{\mathbf{F_y = F \sin\theta}}$$

These two components are the two sides of the right-angled triangle where as hypotenuse represent the magnitude of the actual vector.

Determination of a Force from Its Perpendicular Components

Q.No.3 Find the magnitude and direction of a vector whose rectangular components are given.

Ans: If we have the perpendicular components of any vector then we can find the magnitude and direction of the resultant vector. It is reverse of resolving the vector.

As we know that x-component $\mathbf{F_x}$ of the force **F** is **F cos θ** and the y-component $\mathbf{F_y}$ is **F sin θ**. These two perpendicular components are represented by lines OP and PR respectively as shown in the figure.

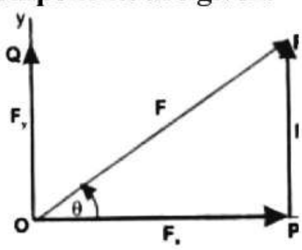


Figure 4.8: Determination of a fo
by its perpendicular components

According to head to tail rule:

$$\vec{OR} = \vec{OP} + \vec{PR}$$

Thus $\vec{OR}$ will completely represent the force $\vec{F}$ where x and y-components are $\vec{Fx}$ and $\vec{Fy}$ respectively.

$$\vec{F} = \vec{Fx} + \vec{Fy}$$

Magnitude of actual vector

The magnitude of the force F can be determined using the right angled triangle OPR as,

$$(OR)^2 = (OP)^2 + (PR)^2$$

$$F^2 = F_x^{\ 2} + F_y^{\ 2}$$

Hence $$F = \sqrt{F_x^{\ 2} + F_y^{\ 2}}$$

Direction of actual vector

Direction of the force F with x-axis is given by,

$$\tan\theta = \frac{PR}{OP} = \frac{F_y}{F_x}$$

So $$\theta = \tan^{-1}\frac{F_y}{F_x}$$

The value of the angle can be determined by using trigonometric tables or calculator.

4.4 TORQUE OR MOMENT OF A FORCE

Q.No.4 Explain torque or moment of a force. **(GRW 2011, LHR 2012)**

Ans: Definition

"The rotational effect of a force is measured by a quantity, known as torque".

Dependence of Torque

Rotation (torque) produced in a body depends on the following two factors:

(i) Force.

(ii) Moment arm

Force

Greater is the force; greater is the moment of the force (torque).

Example

While riding the bicycle, if you press the pedal hard with your feet, its wheels start rotating fast and the speed of the bicycle increases. Similarly if you press the pedal softly, the wheel will rotate slowly and the speed of the bicycle will be less.

Line of action of Force

The line along which a force acts is called the line of action of force. In figure the line BC is the line of action of force.

Moment arm **(LHR 2015)**

The perpendicular distance between the line of action of the force and the axis of rotation, is known as moment arm.

Longer is the moment arm greater is the moment of force.

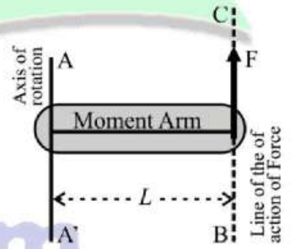


Figure 4.12: Factors affecting the moment of a force

Example

Mechanics loosen or tighten the nut or a bolt with the help of a spanner. A spanner having long arm helps him to do it with greater ease then the one having short arm. It is because the turning effect of the force is different in the two cases. The moment produced by the same force but using a spanner of short arm

Mathematical Form

Torque depends upon the force F and the moment arm r. torque is determined by the product of force F and its moment arm L. So we can write,

Torque = Moment arm x Force

$$\boldsymbol{\tau = F \times L}$$

Unit

In the system international, the unit of torque is Newton meter (Nm). A torque of 1 N m is caused by a force of 1 N acting perpendicular to the moment arm of 1m long.

Sign conventions

Under the action of the torque if the rotation produced is anticlockwise, the torque is considered to be positive. If the rotation produced is clockwise, then the torque is taken as negative.

4.6 CENTRE OF MASS

Q.No.5 What is Centre of Mass? Explain its effect on rotation.

Centre of mass of a system is such a point where an applied force causes the system to move without rotation.

Explanation

It is observed that the centre of mass of a system as if its entire mass is confined that point. A force applied at such a point in the body does not produce any torque in it i.e. the body moves in the direction of net force F without rotation.

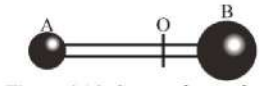


Figure 4.16: Centre of mas of two unequal masses.

Example

Consider a system of two particles A and B connected by a light rigid rod as shown in fig. let O is the point anywhere between A and B such that the force F is applied at point O as shown in fig. if the system moves in the direction of force F without rotation, then point O is the centre of mass of the system.

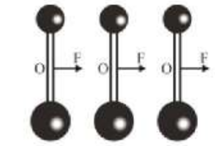


Figure 4.17: A force applied at COM moves the system without rotation.

System move without rotation if the force acts elsewhere on it.

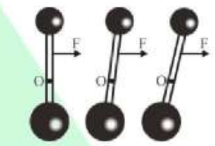


Figure 4.18: The system moves as well as rotates when a force is applied away from COM.

(i) Let the force be applied near the lighter particle as shown in fig. the system will move as well as rotate.

(ii) Let the force be applied near the heavier near the heavier particle as shown in fig. in this case, also the system moves as well as rotate.

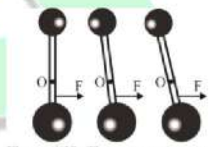


Figure 4.19: The system moves as well as rotates when a force is applied away from COM.

4.6 CENTRE OF GRAVITY

Q.No.6 What is meant by centre of gravity of a body? Explain an experiment to find the centre of gravity of a plate of uniform thickness. (GRW 2014, LHR 2014)

Ans: A point in a body where the weight of the body appears to act vertically downward is called the centre of gravity.

The centre of gravity can exist inside a body or outside the body. Position of the centre of gravity depends upon the shape of the body.

Method

A body is made up of a large number of particles as shown in figure. Earth attracts each of these particles vertically downwards towards its centre. The pull of the Earth acting on a particle is equal to its weight. These forces acting on the particles of a body are almost parallel. The resultant of all these parallel force is a single force equal to the weight of the body. A point where this resultant force acts vertically towards the centre of the Earth is called the centre of gravity G.

Centre of Gravity of Some Symmetrical Objects

The centre of gravity of objects which have symmetrical shapes can be found from their geometry.

Examples

- The centre of gravity of a uniform rod lies at a point where it is balanced. The balance point is its middle point G.
- The centre of gravity of a uniform square or a rectangular sheet is the point of intersection of its diagonals
- The centre of gravity of a uniform circular disc is its centre
- The centre of gravity of a solid sphere or hollow sphere is the centre of the spheres
- The centre of gravity of uniform triangular sheet is the point of intersection of its medians.
- The centre of gravity of a uniform circular ring is the centre of the ring
- The centre of gravity of a uniform solid or hollow cylinder is the middle point on its axis

Centre of Gravity of an Irregular Shaped Thin Lamina

A simple method to find the centre of gravity of a body is by the use of plumb line. A plumb line consists of small metal bob (lead or brass) supported by a string. When the bob is supported is suspended freely by the string, it rests along the vertical direction due to its weight acting vertically downward as shown in figure. In this state, centre of gravity of the bob is exactly below its point of suspension.

Experiment

Take an irregular piece of cardboard. Make holes A, B and C as shown in the figure near its edge. Fix a nail on a wall. Support the cardboard on the nail through one of the holes (let it be A), so that the cardboard can swing freely about A. the cardboard will come to rest with its centre of gravity just vertically below the nail. Vertical line from A can be located using a plumb line hung from the nail. Mark the line on the cardboard behind the plumb line. Repeat it by supporting the cardboard from hole B. The line from B will intersect at a point G. Similarly, draw another line from the hole C. Note that this line also passes through G. it will be found that all the vertical lines from holes A, B, and C have a common point G. this common point G is the centre of gravity of the cardboard.

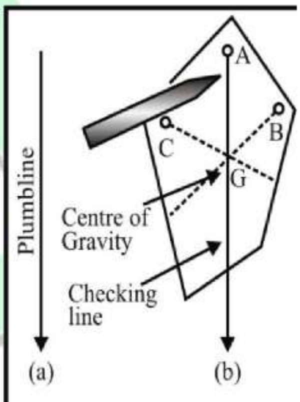


Figure 4.23 **(a)** Plumbline **(b)** Loca the centre of gravity of a piece of cardboard by using plumbline.

4.7 COUPLE

Q.No.7 Define Couple. Give examples and find torque produced by couple.

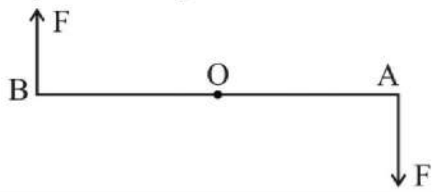
A couple is formed by two unlike parallel forces of the same magnitude but not along the same line.

Examples

- While turning a car, the forces applied on the steering wheel by hands provide the necessary couple.
- While opening or closing a water tap,
- While locking or opening the stopper of a bottle or a jar.

Explanation

A double arm spanner is used to open a nut. Equal forces each of magnitude F are applied on ends A and B of a spanner in opposite direction as shown in figure. These forces form a couple that turns the spanner about a point O. the torques produced by both forces of the couple have same direction. The total torque produced by the couple will be,



Total torque of the couple = F x OA + F x OB

= F (OA + OB)

Torque of the couple = F x AB

The above equation shows that torque produced by the couple of forces F and F separated by distance AB. The toque of a couple is given by the product of one of the two forces and perpendicular distance between them.

Couple arm

The perpendicular distance "r" between the two forces of the couple is called the couple arm.

4.8 EQUILIBRIUM

Q.No.8 What is equilibrium? State and explain the conditions of equilibrium.

Ans: Equilibrium

A body is said to be in equilibrium if no net force acts on it.

First Condition of Equilibrium

A body will be in equilibrium if the resultant of all the forces acting on it is zero. This is first condition of equilibrium.

Explanation

Let n number of forces F_1, F_2, F_3, ………, F_n are acting on a body such that

$F_1 + F_2 + F_3 + \ldots\ldots + F_n$

$\sum F = 0$ ……………. (1)

The symbol $\sum$ is a Greek letter called sigma used for summation. The first condition of equilibrium can also be stated in terms of x and y-component of the forces on the body as:

$F_{1x} + F_{2x} + F_{3x} + \ldots\ldots + F_{nx} = 0$

And $F_{1y} + F_{2y} + F_{3y} + \ldots\ldots + F_{ny} = 0$

OR $\sum F_x = 0$ ………… (2)

$\sum F_y = 0$ ………… (3)

Examples

Examples of bodies satisfying the first condition of equilibrium are given below:

- A book lying on a table or a picture hanging on a wall are at rest
- A paratrooper coming down with terminal velocity (constant velocity)

Figure 4.27: A wall hanging is in equilibrium

Linear acceleration

When the 1st condition of equilibrium is satisfied, no linear acceleration is produced in the body.

Second Condition of Equilibrium

If a number of forces act on a body so that the total sum of the torques of these forces is zero, the body will be in equilibrium.

$\sum \tau = 0$ ……………. (4)

This is called the 2nd condition of equilibrium. If these two conditions are satisfied, the body is completely in equilibrium.

Explanation

Consider a body pulled by two forces F_1 and F_2 as shown in figure. The two forces are equal but opposite to each other. Both are acting along the same line, hence their resultant will be zero. According to first condition of equilibrium, the body will be in equilibrium. Now shift the location of the forces as shown in the figure. In this situation, the body is not in equilibrium although the first condition of equilibrium is still satisfied. It is because the body has the tendency to rotate. This situation demands another condition for equilibrium in addition to first condition of equilibrium.

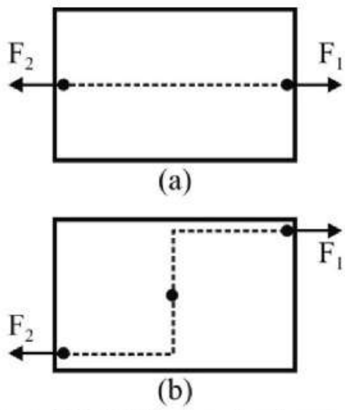


Figure 4.30: (a) Two equal and opposite forces acting along the same lines **(b)** Two equal and opposite forces acting along different lines.

Rotational acceleration

When the 2nd condition of equilibrium is satisfied, then no rotational acceleration is produced in the body.

States of Equilibrium

Q.No.9 Define and explain the three states of equilibrium.

Ans: There are three states of equilibrium:

(i) Stable equilibrium
(ii) Unstable equilibrium
(iii) Neutral equilibrium

Stable equilibrium

A body is said to in stable equilibrium if after a slight tilt it returns to its previous position. When body is in stable equilibrium, its centre of gravity is at the lowest position. When it is tilted, its centre of gravity rises. It returns to its stable state by lowering its centre of gravity. A body remains in stable equilibrium as long as the centre of gravity acts through the base of the body.

Example

Consider a block as shown in figure. When the block is tilted, its centre of gravity G rises. If the vertical line through G passes through its base in the tilted position, the block returns to its previous position. If the vertical line through G gets out its base, it does not return to its previous position.

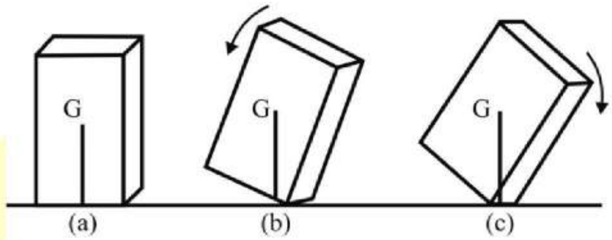


Figure 4.34 (a) Block in stable equilibrium **(b)** Slightly tilted block is returning to its previous position. **(c)** A more tilted block topples over its base and does not return to its previous position.

It topples over its base and moves to new stable equilibrium position. That is why a vehicle made heavy at its bottom to keep its centre of gravity as low as possible. A lower centre of gravity keeps it stable. Moreover, the base of the vehicle is made wide so that the vertical lien passing through the centre of gravity should not get out of its base during a turn.

More Examples

Table, chair, box and brick lying on a floor.

Unstable equilibrium

If a body does not returns to its previous position when sets after a slightest tilt is said to in unstable equilibrium.

The centre of gravity of the body is at its highest point in the state of unstable equilibrium. As the body topples over about its base, its centre of gravity moves towards its lower position and does not return to its previous position.

Example

A pencil is made to stand in equilibrium on its tip. When you leave it, the pencil topples over about its tip and falls down. The body may be made to stay only for a moment. Thus a body is unable to keep itself in the state of unstable equilibrium.

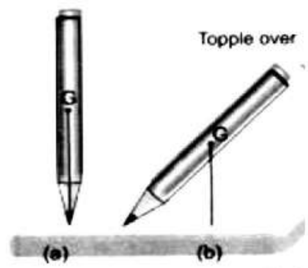


Figure 4.36: Unstable equilibri (a) pencil just balanced at its with centre of gravity G at highest position. **(b)** Pencil topp over caused by the torque of weight acting at G.

More Examples

- A stick standing vertically on the tip of a finger.
- A cone standing on the tip of a finger.

Neutral equilibrium

If a body remains in its new position when disturbed from its previous position, it is said to be in a state of neutral equilibrium.

Example

A ball lying on a horizontal surface is shown in figure. Roll the ball over the surface and leave it after displacing from the previous position. It remains in its new position and does not return to its previous position.

In neutral equilibrium, the centre of gravity of a body remains at the same height, irrespective of its new position.

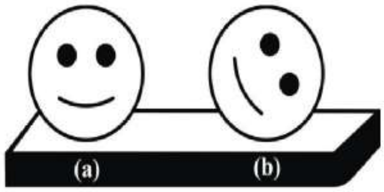


Figure 4.37: Neutral equilibrium **(a)** a ball is placed on a horizontal surface **(b)** the ball remains in its new displaced position.

More Examples

A pencil, a sphere, and cylinder, a roller, an egg lying horizontally on a flat surface.

4.9 STABILITY AND POSITION OF CENTRE OF MASS

Q.No.10 How Stability and Position of centre of mass are related to each other?

Ans: As we have learnt that position of centre of mass of an object plays an important role in their stability. To make them stable, their centre of mass must be kept as low as possible. It is due to the reason, racing cars are made heavy at the bottom and their height is kept to be minimum.

Examples

Here are few examples in which lowering of centre of mass makes the objects stable. These objects return to their stable states when disturbed. In each case centre of mass is vertically below their point of support. This makes their equilibrium stable.

- Circus artists such as tight rope walker use long poles to lower their centre of mass. In this way they are prevented from topple over.
- Figure shows a sewing needle fixed in a cork. The cork is balanced on the tip of the needle by hanging forks. The forks lower the centre of mass of the system.

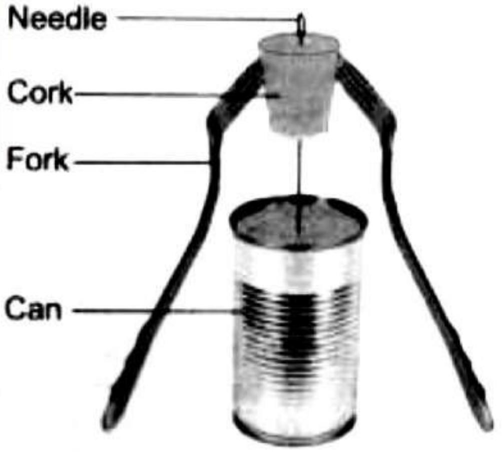


Figure 4.38: A needle is made to balance at its tip.

- Figure shows a perched parrot which is made heavy at its tail. Figure shows a toy that keeps itself upright when tilted. It has heavy semi spherical base. When it is tilted, its centre of mass rises. It returns to the upright position at which its centre of mass is at the lowest.

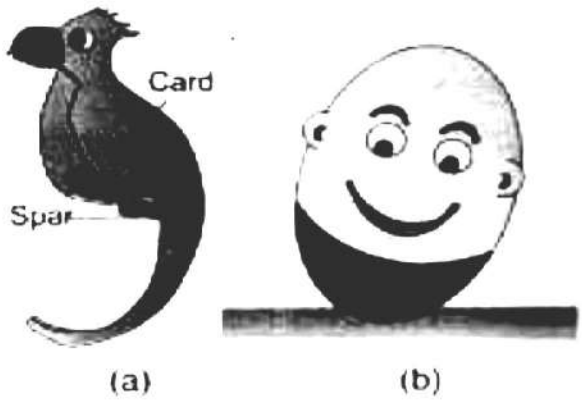


Figure 4.39 **(a)** A perched parrot
(b) A self righting toy