

Exercise 6.2

Q.1 Simplify each of the following as a rational expression.

(i) $\frac{x^2 - x - 6}{x^2 - 9} + \frac{x^2 + 2x - 24}{x^2 - x - 12}$

Solution: $\frac{x^2 - x - 6}{x^2 - 9} + \frac{x^2 + 2x - 24}{x^2 - x - 12}$

$$= \frac{x^2 - x - 6}{x^2 - 9} + \frac{x^2 + 2x - 24}{x^2 - x - 12}$$

$$= \frac{x^2 - 3x - 2x - 6}{(x)^2 - (3)^2} + \frac{x^2 + 6x - 4x - 24}{x^2 - x - 12}$$

$$= \frac{x(x-3) + 2(x-3)}{(x-3)(x+3)} + \frac{x(x+6) - 4(x+6)}{x(x-4) + 3(x-4)}$$

$$= \frac{(x-3)(x+2)}{(x-3)(x+3)} + \frac{(x+6)(x-4)}{(x-4)(x+3)}$$

$$= \frac{(x+2)}{(x+3)} + \frac{(x+6)}{(x+3)}$$

$$= \frac{x+2+x+6}{x+3}$$

$$= \frac{8+2x}{x+3}$$

$$= \frac{2(x+4)}{x+3}$$

Q.2 $\left[\frac{x+1}{x-1} - \frac{x-1}{x+1} - \frac{4x}{x^2+1} \right] + \frac{4x}{x^4-1}$

Solution: $\left[\frac{x+1}{x-1} - \frac{x-1}{x+1} - \frac{4x}{x^2+1} \right] + \frac{4x}{x^4-1}$

$$= \left[\frac{x+1}{x-1} - \frac{x-1}{x+1} - \frac{4x}{x^2+1} \right] + \frac{4x}{x^4-1}$$

$$= \left[\frac{(x+1)^2 - (x-1)^2}{(x-1)(x+1)} - \frac{4x}{x^2+1} \right] + \frac{4x}{x^4-1}$$

$$= \left[\frac{x^2 + 2x + 1 - (x^2 + 1 - 2x)}{(x-1)(x+1)} - \frac{4x}{x^2+1} \right] + \frac{4x}{x^4-1}$$

$$\begin{aligned}
 &= \left[\frac{x^2 + 2x - 1 - x^2 + 2x}{x^2 - 1} - \frac{4x}{x^2 + 1} \right] + \left[\frac{4x}{x^4 - 1} \right] \\
 &= \left[\frac{4x}{x^2 - 1} - \frac{4x}{x^2 + 1} \right] + \frac{4x}{x^4 - 1} \\
 &= \left[\frac{4x(x^2 + 1) - 4x(x - 1)}{(x^2 - 1)(x^2 + 1)} \right] + \frac{4x}{x^4 - 1} \\
 &= \left[\frac{4x^3 + 4x - 4x^3 + 4x}{x^4 - 1} \right] + \frac{4x}{x^4 - 1} \\
 &= \frac{8x}{x^4 - 1} + \frac{4x}{x^4 - 1} \\
 &= \frac{8x + 4x}{x^4 - 1} \\
 &= \frac{12x}{x^4 - 1}
 \end{aligned}$$

Q.3 $\frac{1}{x^2 - 8x + 15} + \frac{1}{x^2 - 4x + 3} - \frac{2}{x^2 - 6x + 5}$

Solution: $\frac{1}{x^2 - 8x + 15} + \frac{1}{x^2 - 4x + 3} - \frac{2}{x^2 - 6x + 5}$

$$\begin{aligned}
 &= \frac{1}{x^2 - 8x + 15} + \frac{1}{x^2 - 4x + 3} - \frac{2}{x^2 - 6x + 5} \\
 &= \frac{1}{x^2 - 3x - 5x + 15} + \frac{1}{x^2 - 3x - 1x + 3} - \frac{2}{x^2 - 5x - x + 5} \\
 &= \frac{1}{x(x - 3) - 5(x - 3)} + \frac{1}{x(x - 3) - 1(x - 3)} - \frac{2}{x(x - 5) - 1(x - 5)} \\
 &= \frac{1}{(x - 3)(x - 5)} + \frac{1}{(x - 3)(x - 1)} - \frac{2}{(x - 5)(x - 1)} \\
 &= \frac{(x - 1) + (x - 5) - 2(x - 3)}{(x - 3)(x - 5)(x - 1)} \\
 &= \frac{x - x + x - 5 - 2x + 6}{(x - 3)(x - 5)(x - 1)} \\
 &= \frac{0}{(x - 3)(x - 5)(x - 1)} \\
 &= 0
 \end{aligned}$$

Q.4 $\frac{(x+2)(x+3)}{x^2-9} + \frac{(x+2)(2x^2-32)}{(x-4)(x^2-x-6)}$

Solution:
$$\begin{aligned} & \frac{(x+2)(x+3)}{x^2-9} + \frac{(x+2)(2x^2-32)}{(x-4)(x^2-x-6)} \\ &= \frac{(x+2)(x+3)}{x^2-9} + \frac{(x+2)(2x^2-32)}{(x-4)(x^2-x-6)} \\ &= \frac{(x+2)(x+3)}{(x-3)^2} + \frac{(x+2)[2(x^2-16)]}{(x-4)(x^2-3x+2x-6)} \\ &= \frac{(x+2)(x+3)}{(x-3)(x+3)} + \frac{(x+2)[2(x^2-(4)^2)]}{(x-4)[x(x-3)+2(x-3)]} \\ &= \frac{(x+2)}{(x-3)} + \frac{(x+2)[2(x+4)(x-4)]}{(x-4)(x-3)(x+2)} \\ &= \frac{(x+2)}{(x-3)} + \frac{2(x+4)}{(x-3)} \\ &= \frac{x+2}{x-3} + \frac{2x+8}{x-3} \\ &= \frac{x+2+2x+8}{x-3} \\ &= \frac{3x+10}{x-3} \end{aligned}$$

Q.5 $= \frac{x+3}{2x^2+9x+9} + \frac{1}{2(2x-3)} - \frac{4x}{4x^2-9}$

Solution:
$$\begin{aligned} & \frac{x+3}{2x^2+9x+9} + \frac{1}{2(2x-3)} - \frac{4x}{4x^2-9} \\ &= \frac{x+3}{2x^2+9x+9} + \frac{1}{2(2x-3)} - \frac{4x}{4x^2-9} \\ &= \frac{x+3}{2x^2+6x+3x+9} + \frac{1}{2(2x-3)} - \frac{4x}{(2x)^2-(3)^2} \\ &= \frac{x+3}{2x(x+3)+3(x+3)} + \frac{1}{2(2x-3)} - \frac{4x}{(2x-3)(2x+3)} \\ &= \frac{(x+3)}{(x+3)(2x+3)} + \frac{1}{2(2x-3)} - \frac{4x}{(2x-3)(2x+3)} \\ &= \frac{1}{2x+3} + \frac{1}{2(2x-3)} - \frac{4x}{(2x-3)(2x+3)} \\ &= \frac{2(2x-3)+(2x+3)-4x \times 2}{2(2x-3)(2x+3)} \\ &= \frac{4x-6+2x+3-8x}{2(2x-3)(2x+3)} \end{aligned}$$

$$\begin{aligned}
 &= \frac{-2x-3}{2(2x-3)(2x+3)} \\
 &= \frac{-1(2x+3)}{2(2x-3)(2x+3)} \\
 &= \frac{-1}{2(2x-3)}
 \end{aligned}$$

Q.6 $A - \frac{1}{A}$, Where $A = \frac{a+1}{a-1}$

Solution: $A - \frac{1}{A}$, Where $A = \frac{a+1}{a-1}$

$$= A - \frac{1}{A} = ?$$

Put the value of A

$$\begin{aligned}
 &= \frac{a+1}{a-1} - \frac{a-1}{a+1} \\
 &= \frac{(a+1)^2 - (a-1)^2}{(a-1)(a+1)} \\
 &= \frac{a^2 + 2a + 1 - (a^2 - 2a + 1)}{a^2 - 1} \\
 &= \frac{a^2 + 2a + 1 - a^2 + 2a - 1}{a^2 - 1} \\
 &= \frac{4a}{a^2 - 1}
 \end{aligned}$$

Q.7 $\left[\frac{x-1}{x-2} + \frac{2}{2-x} \right] - \left[\frac{x+1}{x+2} + \frac{4}{4-x^2} \right]$

Solution: $\left[\frac{x-1}{x-2} + \frac{2}{2-x} \right] - \left[\frac{x+1}{x+2} + \frac{4}{4-x^2} \right]$

$$= \left[\frac{x-1}{x-2} + \frac{2}{2-x} \right] - \left[\frac{x+1}{x+2} + \frac{4}{4-x^2} \right]$$

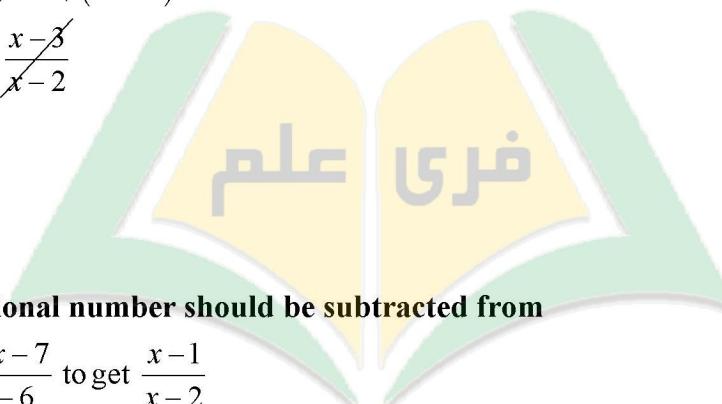
$$= \left[\frac{x-1}{x-2} + \frac{2}{-x+2} \right] - \left[\frac{x+1}{x+2} + \frac{4}{-x^2+4} \right]$$

$$= \left[\frac{x-1}{x-2} - \frac{2}{x-2} \right] - \left[\frac{x+1}{x+2} + \frac{4}{-(x^2-4)} \right]$$

$$= \left[\frac{x-1}{x-2} - \frac{2}{x-2} \right] - \left[\frac{x+1}{x+2} - \frac{4}{x^2-4} \right]$$

$$\begin{aligned}
 &= \left[\frac{x-1-2}{x-2} \right] - \left[\frac{x+1}{x+2} - \frac{4}{(x+2)(x-2)} \right] \\
 &= \frac{(x-3)}{(x-2)} - \frac{(x+1)(x-2)-4}{(x+2)(x-2)} \\
 &= \frac{x-3}{x-2} - \frac{x^2-x-2-4}{(x+2)(x-2)} \\
 &= \frac{x-3}{x-2} - \frac{x^2-x-6}{(x-2)(x+2)} \\
 &= \frac{x-3}{x-2} - \frac{x^2-3x+2x-6}{(x-2)(x+2)} \\
 &= \frac{x-3}{x-2} - \frac{x(x-3)2(x-3)}{(x-2)(x+2)} \\
 &= \frac{x-3}{x-2} - \frac{(x-3)(x+2)}{(x-2)(x+2)} \\
 &= \frac{x-3}{x-2} - \frac{x-3}{x-2}
 \end{aligned}$$

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Q.8 What rational number should be subtracted from

$$= \frac{2x^2 + 2x - 7}{x^2 + x - 6} \text{ to get } \frac{x-1}{x-2}$$

Solution: let required rational number be $P(x)$

According to condition

$$\begin{aligned}
 &\frac{2x^2 + 2x - 7}{x^2 + x - 6} - P(x) = \frac{x-1}{x-2} \\
 &P(x) = \frac{2x^2 + 2x - 7}{x^2 + x - 6} - \frac{x-1}{x-2} \\
 &= \frac{2x^2 + 2x - 7}{x^2 + 3x - 2x - 6} - \frac{x-1}{x-2} \\
 &= \frac{2x^2 + 2x - 7}{x(x+3) - 2(x+3)} - \frac{x-1}{x-2} \\
 &= \frac{2x^2 + 2x - 7}{(x+3)(x-2)} - \frac{x-1}{x-2} \\
 &= \frac{2x^2 + 2x - 7 - (x-1)(x+3)}{(x+3)(x-2)} \\
 &= \frac{2x^2 + 2x - 7 - (x^2 + 2x - 3)}{(x+3)(x-2)}
 \end{aligned}$$

$$= \frac{2x^2 + 2x - 7 - x^2 - 2x + 3}{(x+3)(x-2)}$$

$$= \frac{x^2 - 4}{(x+3)(x-2)}$$

$$= \frac{x^2 - 2^2}{(x+3)(x-2)}$$

$$= \frac{(x+2)(\cancel{x-2})}{(x+3)(\cancel{x-2})}$$

$$= \frac{x+2}{x+3}$$

Q.9 $= \frac{x^2 + x - 6}{x^2 - x - 6} \times \frac{x^2 - 4}{x^2 - 9}$

Solution: $= \frac{x^2 + x - 6}{x^2 - x - 6} \times \frac{x^2 - 4}{x^2 - 9}$

$$= \frac{x^2 + x - 6}{x^2 - x - 6} \times \frac{x^2 - 4}{x^2 - 9}$$

$$= \frac{x^2 + 3x - 2x - 6}{x^2 - 3x + 2x - 6} \times \frac{x^2 - 2^2}{x^2 - 3^2}$$

$$= \frac{x(x+3) - 2(x+3)}{2(x-3) + 2(x-3)} \times \frac{(x-2)(x+3)}{(x-3)(x+3)}$$

$$= \frac{(\cancel{x+3})(x-2)}{(\cancel{x-3})(\cancel{x+2})} \times \frac{(x-2)(\cancel{x+2})}{(\cancel{x-3})(\cancel{x+3})}$$

$$= \frac{(x-2)^2}{(x-3)^2}$$

Q.10 $\frac{x^3 - 8}{x^2 - 4} \times \frac{x^2 + 6x + 8}{x^2 - 2x + 1}$

Solution: $\frac{x^3 - 8}{x^2 - 4} \times \frac{x^2 + 6x + 8}{x^2 - 2x + 1}$

$$= \frac{x^3 - 8}{x^2 - 4} \times \frac{x^2 + 6x + 8}{x^2 - 2x + 1}$$

$$\begin{aligned}
 &= \frac{(x)^3 - (2)^3}{(x^2) - (2)^2} \times \frac{x^2 + 4x + 2x + 8}{x^2 - x - x + 1} \\
 &= \frac{(x-2)(x^2 + 2x + 4)}{(x-2)(x+2)} \times \frac{x(x+4) + 2(x+4)}{x(x-1) - 1(x-1)} \\
 &= \frac{x^2 + 2x + 4}{(x+2)} \times \frac{(x+4)(x+2)}{(x-1)(x-1)} \\
 &= \frac{(x^2 + 2x + 4)(x+4)}{(x-1)^2}
 \end{aligned}$$

Q.11 $\frac{x^4 - 8x}{2x^2 + 5x - 3} \times \frac{2x-1}{x^2 + 2x + 4} \times \frac{x+3}{x^2 - 2x}$

Solution: $\frac{x^4 - 8x}{2x^2 + 5x - 3} \times \frac{2x-1}{x^2 + 2x + 4} \times \frac{x+3}{x^2 - 2x}$

$$= \frac{x^4 - 8x}{2x^2 + 5x - 3} \times \frac{2x-1}{x^2 + 2x + 4} \times \frac{x+3}{x^2 - 2x}$$

$$= \frac{x(x^3 - 8)}{2x^2 + 6x - x - 3} \times \frac{2x-1}{x^2 + 2x + 4} \times \frac{x+3}{x(x-2)}$$

$$= \frac{x[(x)^3 - (2)^3]}{2x(x+3) - 1(x+3)} \times \frac{2x-1}{x^2 + 2x + 4} \times \frac{x+3}{x(x-2)}$$

$$= \frac{x(x-2)(x^2 + 2x + 4)}{(2x-1)(x+3)} \times \frac{2x-1}{x^2 + 2x + 4} \times \frac{x+3}{x(x-2)}$$

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Q.12 $\frac{2y^2 + 7y - 4}{3y^2 - 13y + 4} \div \frac{4y^2 - 1}{6y^2 + y - 1}$

Solution: $\frac{2y^2 + 7y - 4}{3y^2 - 13y + 4} \div \frac{4y^2 - 1}{6y^2 + y - 1}$

$$\begin{aligned}
 &= \frac{2y^2 + 7y - 4}{3y^2 - 13y + 4} \div \frac{4y^2 - 1}{6y^2 + y - 1} \\
 &= \frac{2y^2 + 8y - 1y - 4}{3y^2 - 12y - y + 4} \div \frac{(2y)^2 - (1)^2}{6y^2 + 3y - 2y - 1} \\
 &= \frac{2y(y+4) - 1(y+4)}{3y(y-4) - 1(y-4)} \div \frac{(2y-1)(2y+1)}{3y(2y+1) - 1(2y+1)} \\
 &= \frac{(y+4)(2y-1)}{(3y-1)(y-4)} \div \frac{(2y-1)(2y+1)}{(3y-1)(2y+1)} \\
 &= \frac{(y+4)(2y-1)}{(3y-1)(y-4)} \times \frac{(3y-1)}{(2y+1)} \\
 &= \frac{y+4}{y-4}
 \end{aligned}$$

Q.13 $\left[\frac{x^2 + y^2}{x^2 - y^2} - \frac{x^2 - y^2}{x^2 + y^2} \right] \div \left[\frac{x+y}{x-y} - \frac{x-y}{x+y} \right]$

Solution: $\left[\frac{x^2 + y^2}{x^2 - y^2} - \frac{x^2 - y^2}{x^2 + y^2} \right] \div \left[\frac{x+y}{x-y} - \frac{x-y}{x+y} \right]$

$$\begin{aligned}
 &= \left[\frac{x^2 + y^2}{x^2 - y^2} - \frac{x^2 - y^2}{x^2 + y^2} \right] \div \left[\frac{x+y}{x-y} - \frac{x-y}{x+y} \right] \\
 &= \left[\frac{(x^2 + y^2)^2 - (x^2 - y^2)^2}{(x^2 - y^2)(x^2 + y^2)} \right] \div \left[\frac{(x+y)^2 - (x-y)^2}{(x-y)(x+y)} \right] \\
 &= \left[\frac{(x^4 + 2x^2y^2 + y^4) - (x^4 - 2x^2y^2 + y^4)}{(x^2 - y^2)(x^2 + y^2)} \right] \div \left[\frac{(x^2 + 2xy + y^2) - (x^2 - 2xy + y^2)}{x^2 - y^2} \right] \\
 &= \left[\frac{4x^2y^2}{(x^2 - y^2)(x^2 + y^2)} \right] \div \left[\frac{4xy}{x^2 - y^2} \right] \\
 &= \frac{4x^2y^2}{(x^2 - y^2)(x^2 + y^2)} \times \frac{x^2 - y^2}{4xy} \\
 &= \frac{4\cancel{xy} \cdot \cancel{xy}}{\cancel{(x^2 - y^2)}(x^2 + y^2)} \times \frac{\cancel{x^2 - y^2}}{\cancel{4xy}}
 \end{aligned}$$

$$= \frac{xy}{x^2 + y^2} \text{ Ans}$$

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