# Micro- and Macroeconomic Impacts of a Place-Based Industrial Policy

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<sup>&</sup>lt;sup>1</sup>The views expressed in this presentation are solely those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Philadelphia or the Federal Reserve System. 

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#### This Presentation

- Analyze the impacts of a new set of place-based subsidies, introduced in Turkey in 2012
  - Eligibility varies by industry
  - Generosity varies by geography
- Micro:
  - Firm-level balance sheet and subsidy take-up data to assess direct impacts
  - Production network data to measure indirect effects from subsidized firms to their customers and suppliers
- Macro:
  - Dynamic general equilibrium model with migration and trade to examine impact on regional real wage inequality
  - Measure channels through which subsidies spill over from targeted to non-targeted regions.

#### Research Questions

1. Did the program increase subsidized firms' (and industries') revenues, productivity (and, in the paper: employment, investment)?

2. Did subsidies spill over from subsidized firms to their customers and suppliers?

3. Did the program reduce regional wage inequality? In the short run? In the long run?

#### What We Find

- Did the program increase subsidized firms' (and industries') revenues, productivity (and, in the paper: employment, investment)?
   Yes: A 5 p.p. increase in the investment tax credits corresponds to a 3.2% increase in firms' TFPR.
- 2. Did subsidies spill over from subsidized firms to their customers and suppliers? Yes: Effect of having subsidized customers/suppliers is ~ one-twentieth the effect of direct subsidization.
- 3. Did the program reduce regional wage inequality? In the short run? In the long run?

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   Yes: A 5 p.p. increase in the investment tax credits corresponds to a 3.2% increase in firms' TFPR.
- 2. Did subsidies spill over from subsidized firms to their customers and suppliers? Yes: Effect of having subsidized customers/suppliers is ~ one-twentieth the effect of direct subsidization.
- Did the program reduce regional wage inequality? In the short run? In the long run?
   Only slightly. Migration and spillovers via input-output linkages mitigate relative impact on targeted regions.

#### Contribution to the Literature

- Evaluations of place-based policies: Bernini and Pellegrini (2011), Pauline, Rathelot, Sillard (2013), Busso, Gregory, Kline, (2013), Kline and Moretti (2014), Criscuolo et al. (2019)
   Our contribution: First to evaluate firm-level and aggregate impacts of Turkey's Law 2012/3305. Long-run vs. short-run and partial-equilibrium vs. general-equilibrium comparisons.
- Spillovers within production networks: Barrot and Sauvagnat (2016), Carvalho et al. (2020), Demir et al. (2020)
   Our contribution: Examine spillovers from subsidies
- Gen. eq. trade and migrations responses to policy reforms (or to other shocks): Caliendo, Dvorkin, Parro (2019), Monras (2020), Caliendo, Opromolla, Parro, Sforza (2021)
  - Our contribution: New application.

#### Outline

- 1. Institutional Background
- 2. Detecting the direct impacts of the subsidies
- 3. Identifying indirect effects
- 4. Assessing the impact on regional wage inequality

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Only firms in certain industries — mining, manufacturing, warehousing, a few others — are eligible to receive subsidies.

#### Multiple subsidy elements:

- 1. VAT and customs duties exemptions on investment machinery and equipment
- 2. support on interest rate payments (on private loans): no support in Regions 1 and 2 to 3-7 p.p.in Region 6
- 3. corporate tax credits: 15% of investment costs in Region 1 to 50% in Region 6;
- 4. support for contributions to employees' social security payments: 2 years in Region 1 up to 10 years in Region 6.

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# Targeted regions were poorer, prior to 2012



	Region						
	1	2	3	4	5	6	National
Population in 2011 (millions)	30.4	11.2	9.8	7.9	6.6	8.8	74.7
GDP Per Capita, 2011 (,000 TL)	27.36	16.54	14.95	13.38	11.23	8.30	18.95
Net Migration Rate, 2011 (%)	0.86	0.07	-0.33	-0.60	-1.09	-1.24	_
GDP Per Capita Growth Rate:	1.5	2.0	2.2	3.4	3.9	3.7	2.3
2006-2011							

# Targeted regions were a source of domestic migration



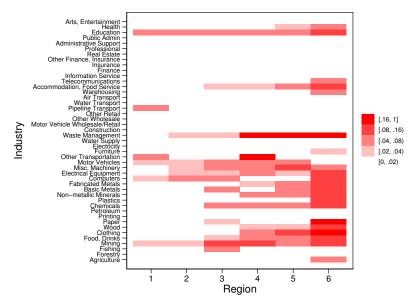
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2006-2011							

# Targeted regions had faster pre-policy growth



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# Subsidization varies by geography and industry



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#### Data

#### Main Components

- Firm balance sheet data: 2006-2018
- Firm-to-firm production network: 2006-2018
- Subsidization take-up: 2012-2018
- ▶ Linked employer-employee data: 2012-2018: Used to compute migration rates

#### Caveats

- Only covers firms and employees in the formal economy
  - Use estimates of formality by region and by industry when calibrating our aggregate model
- Firm-level balance sheet data links industries provinces to that of the headquarter firm
  - ► For multi-establishment firms, we can observe employment by establishment & where subsidy took place
  - ▶ Industry-level exercises records subsidization at the proper industry and province

# Our empirical setup to detect direct effects

$$y_{pnt} = \beta_{pn} + \beta_{nt} + \beta_1 S_{pnt} + \varepsilon_{pnt}$$

ightharpoonup p=province; n=industry; t=year;  $y_{pnt}$ =activity measure;  $S_{pnt}$ =subsidy measure

- 1. Subsidies were targeted towards already-fast-growing regions: Pre-trends?
- 2. Not all eligible firms received subsidies; measurement error in subsidies received

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- 2. Not all eligible firms received subsidies; measurement error in subsidies received Instrument received subsidies with statutory eligibility/generosity; e.g.,
  - 0.50 for a region 6 firm in an eligible industry post 2012;
  - 0.15 for a region 1 firm in an eligible industry post 2012;
  - 0 for a firm in an ineligible industry or before 2012

# Impact of Subsidies On Industry Revenues

$$y_{pnt} = \beta_{pn} + \beta_{nt} + \beta_1 S_{pnt} + \varepsilon_{pnt}$$

Dependent Variable	Revenues		
	(1)	(2)	
Investment Tax Credit Rate	3.534***	8.934***	
investment Tax Credit Nate	(1.015)	(1.610)	
First Stage			
Statutory rate on investment	0.061***	0.075***	
tax credits	(0.004)	(0.007)	
Year FEs	Yes	No	
Year $ imes$ Industry FEs	No	Yes	
N	221,790	221,366	

▶ 5 p.p. ↑ in investment tax credit subsidies received ⇔ 17.7% to 44.7% higher revenues.

# Impact of Subsidies On Industry Revenues and Firm Count

$$y_{pnt} = \beta_{pn} + \beta_{nt} + \beta_1 S_{pnt} + \varepsilon_{pnt}$$

Dependent Variable	Revenues			Firm Count	
	(1)	(2)	_	(3)	(4)
Investment Tay Condit Date	3.534***	8.934***	_	0.945*	4.003***
nvestment Tax Credit Rate (1.015)	(1.610)		(0.567)	(0.766)	
First Stage					
Statutory rate on investment	0.061***	0.075***	_	0.061***	0.075***
tax credits	(0.004)	(0.007)		(0.004)	(0.007)
Year FEs	Yes	No		Yes	No
Year $ imes$ Industry FEs	No	Yes		No	Yes
N	221,790	221,366		221,790	221,366

➤ 5 p.p. ↑ in investment tax credit subsidies received  $\iff$  4.7% to 20.0% higher firm count.

## Impact of Subsidies On Firm Revenues

$$y_{ft} = \beta_{nt} + \beta_f + \beta_1 S_{ft} + \varepsilon_{ft}$$

Dependent Variable	Revenues		
	(1)	(2)	
Investment Tax Credit Rate	2.607***	3.194***	
investment Tax Credit Nate	(0.467)	(0.559)	
First Stage			
Statutory rate on investment	0.142***	0.136***	
tax credits	(0.010)	(0.019)	
Year FEs	Yes	No	
Year $ imes$ Industry FEs	No	Yes	
N	870,557	870,557	

▶ 5 p.p. ↑ in investment tax credit subsidies received ⇔ 13.0% to 16.0% higher revenues.

## Impact of Subsidies On Firm Revenues and TFP

$$y_{ft} = \beta_{nt} + \beta_f + \beta_1 S_{ft} + \varepsilon_{ft}$$

Dependent Variable	Reve	enues	TI	FP
	(1)	(2)	(3)	(4)
Investment Tax Credit Rate	2.607***	3.194***	0.989***	0.649***
	(0.467)	(0.559)	(0.153)	(0.220)
First Stage				
Statutory rate on investment	0.142***	0.136***	0.143***	0.139***
tax credits	(0.010)	(0.019)	(0.010)	(0.019)
Year FEs	Yes	No	Yes	No
Year $ imes$ Industry FEs	No	Yes	No	Yes
N	870,557	870,557	815,855	815,377

▶ 5 p.p.  $\uparrow$  in investment tax credit subsidies received  $\iff$  3.2% higher TFPR.

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#### Indirect Effects

In our macro-model calibration: key object of interest is *direct* productivity impact of subsidization on productivity

Subsidies in one firm potentially spill over...

- ... to customers or suppliers: Let  $s_{f o \vartheta}^{ ext{upstream}}$  and  $s_{\vartheta o f}^{ ext{downstream}}$  denote share of f's suppliers or customers who are subsidized
- ightharpoonup ... to local wages: let  $w_{npt}$  denote average wage in year t, in industry n, and province p

$$y_{ft} = \beta_f + \beta_{nt} + \beta_1 S_{ft} + \beta_2 \cdot w_{npt} + \beta_{\text{up}} s_{\vartheta \to ft}^{\text{upstream}} + \beta_{\text{down}} s_{f \to \vartheta, t}^{\text{downstream}} + \varepsilon_{ft}$$

### Impact of Subsidies On Revenues and TFP

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Dependent Variable	Reve	enues	TI	FP
	(1)	(2)	(3)	(4)
Investment Tax Credit Rate	2.235***	2.488***	1.054***	0.668***
Received	(0.370)	(0.646)	(0.222)	(0.190)
Weight of subsidized firms in total	0.067***	0.025**	-0.003	-0.013*
sales	(0.014)	(0.012)	(800.0)	(0.007)
Weight of subsidized firms in total	0.065***	0.071***	0.035**	0.025**
purchases	(0.013)	(0.012)	(0.014)	(0.012)
Landeile come in land labor marks	0.049***	0.035***	-0.016**	-0.008
Log daily wage in local labor market	(0.009)	(0.009)	(0.006)	(0.005)
Instrument for $S_{ft}$ ?	Yes	Yes	Yes	Yes
Year FEs	Yes	No	Yes	No
Year ×Industry FEs	No	Yes	No	Yes
N	785,579	785,220	735,915	735,531

▶ 5 p.p. ↑ in investment tax credit subsidies received ⇔ 3.2% higher TEPR > < ○

### Impact of Subsidies On Revenues and TFP

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5 p.p.  $\uparrow$  counterparties' subsidization  $\iff$  0.5% $\uparrow$  revenues  $\downarrow$  1 =  $\downarrow$ 0.00



### Impact of Subsidies On Revenues and TFP

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▶ 5 p.p.  $\uparrow$  counterparties' subsidization  $\iff$  0.5% $\uparrow$  revenues, 0.1% $\uparrow$  TFPR=

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- 1. Institutional Background
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# We apply a dynamic g.e. model with trade and migration to understand the subsidy policy's impact on regional inequality

We apply the model of Caliendo, Dvorkin, Parro (2019) See the equations

- Households
  - Consume output specific to their region and industry.
  - ► Face dynamic migration decision on where to work in the future
    - Depends on expectations over future real wages, time-invariant migration costs, i.i.d. taste shocks
- Landlords
  - Rent out structures they own to intermediate goods firms. Consume.

# We apply a dynamic g.e. model with trade and migration to understand the subsidy policy's impact on regional inequality

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- ► Intermediate goods firms
  - Operate with CRS production function: labor, structures, material inputs.
  - ► Time-varying total factor productivity
  - Sell output to "final goods producers"
- ► Final goods firms
  - Bundle different varieties with a CES production function.
  - ▶ Source inputs from intermediate goods firms. The share of varieties sourced from a given region depends on suppliers' marginal cost, iceberg trade costs
  - Output is bundled, sold to households for consumption and intermediate goods producers as material inputs

## Key spatial spillovers in the model

- ► Input-output linkages
  - Subsidy lowers marginal costs downstream of subsidized firms, increases labor demand upstream
- ▶ Domestic migration
  - ▶ In-migration to subsidized areas reduces real wages in subsidized region-industries
- Capital rents
  - Increases in rental income of structures in subsidized areas benefit landowners throughout the country

# The subsidy plan had a modest impact on real wage inequality

Object of interest: What is the effect of the subsidy policy's on real wages (and employment) in each region-industry pair?

- Consider counterfactual equilibrium: suppose total factor productivity was lower (especially in subsidized region-industries) absent the subsidy policy
  - We estimated: 1 p.p. increase in investment tax credits  $\rightarrow$ 0.6% increase in TFP.
  - ► Combine with info on investment tax credits received by industry×region×year.

#### Impact of subsidy on Region 6 relative to Region 1 real wages

- ▶ In 2017: 0.5 percentage points (1.6% increase in Region 6 vs. 1.1% in Region 1)
- ► In 2022: 0.3 p.p.
- ► In 2027: 0.2 p.p.
- ► In 2032: 0.1 p.p.

#### We consider three additional calibrations of our model

- 1. "No migration": Utility cost of migrating across subsidy regions is infinite; households may switch industries within regions
- "No migration, autarky": Also, the iceberg cost across subsidy regions is infinite.
- 3. "No migration, autarky, no structures": Also, the structures share in value added also equals 0.

# Spillovers due to migration and input-output linkages blunt the policy's impact on real wage inequality

	2017	2022	2027	2032
Benchmark	0.5 p.p.	0.3 p.p.	0.2 p.p.	0.1 p.p.
No Migration	1.7 p.p.	2.0 p.p.	2.0 p.p.	2.0 p.p.
No Migration, Autarky	5.8 p.p.	4.2 p.p.	4.1 p.p.	4.1 p.p.
No Migration, Autarky, No Structures	3.8 p.p.	4.2 p.p.	4.1 p.p.	4.1 p.p.

#### Conclusion

#### Results

- ► Micro: 2012 subsidy program had substantial impact on treated firms' sales, TFP (in the paper: investment, employment).
- Macro: 2012 subsidy program had modest impact on regional real wage inequality.

#### Implications for the place-based policy literature:

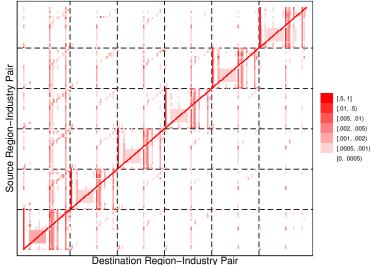
- ► Migration responds slowly to real-wage differentials ⇒Short- and long-run impacts; partial and general equilibrium subsidy impacts differ considerably.
- ▶ Spillovers need not be restricted to nearby geographic areas.

#### Open questions:

➤ To what extent did the policy boost nation-wide investment? Was the policy cost effective?

# Flows of Individuals Across Region-Industry Pairs





# Trade Flows Across Region-Industry Pairs





# We apply a dynamic g.e. model with trade and migration to understand aggregate effects

We apply the model of Caliendo, Dvorkin, Parro (2019) Go back

- Households
  - Consume output specific to their region and industry.
  - ► Face dynamic migration decision on where to work in the future
    - Depends on expectations over future real wages, time-invariant migration costs, i.i.d. taste shocks
  - Lifetime utility

$$U_t^{nj} = \sum_{k=1}^J \alpha^k \log \left( c_t^{nj,k} \right) + \max_{\{i,k\}} \beta \mathbb{E} \left[ U_{t+1}^{ik} - \tau^{nj,ik} + \nu \epsilon_t^{ik} \right]$$

Migration probabilities

$$\mu_t^{nj,ik} = \frac{\exp\left(\beta \mathbb{E}\left[U_{t+1}^{ik}\right] - \tau^{nj,ik}\right)^{1/\nu}}{\sum_{m=1}^{N} \sum_{j=0}^{J} \exp\left(\beta \mathbb{E}\left[U_{t+1}^{mh}\right] - \tau^{nj,mh}\right)^{1/\nu}}$$

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- ► Intermediate goods firms
  - ▶ Operate with CRS production function: labor, structures, material inputs

$$q_t^{nj} = z^{nj} \left( A_t^{nj} \left( h_t^{nj} \right)^{\xi^n} \left( I_t^{nj} \right)^{1-\xi^n} \right)^{\gamma^{nj}} \prod_{k=1}^J \left( M_t^{nj,nk} \right)^{\gamma^{nj,nk}}$$

► Marginal cost:

$$x_t^{nj} = B^{nj} \frac{\left[ \left( r^{nj} \right)^{\xi^n} \left( w^{nj} \right)^{1-\xi^n} \right]^{\gamma^{nj}}}{z^{nj} \left( A_t^{nj} \right)^{\gamma^{nj}}} \prod_{k=1}^J \left( P_t^{nk} \right)^{\gamma^{nj,nk}}$$

- ► Final goods firms
  - ▶ Bundle different varieties with a CES production function
  - ▶ Source from a given supplier with probability proportional to:

$$\pi_t^{nj,ij} = \frac{\left(x_t^{ij} \cdot \kappa_{nj,ij}\right)^{-\theta^j}}{\sum_{m=1}^{N} \left(x_t^{mj} \cdot \kappa_{nj,mj}\right)^{-\theta^j}}.$$