Extensions for "Customer Poaching and Coupon Trading"

To check the robustness of our results, we extend our model in the following directions: (i) introducing coupon non-users; (ii) allowing continuous hassle cost of trading coupons and (iii) allowing coupons to be non-transferrable. For simplicity, we consider only offensive coupons. We find that the results are qualitatively the same as in the benchmark model.

1   Introducing coupon non-users

In our model, we have assumed that all consumers are coupon users. As we show here, our results do not change qualitatively if we introduce coupon non-users.

Assume that there is a fraction, $1 - \gamma$, of consumers who do not use coupons. They are uniformly distributed on the interval $[-L, L]$ as well, but they can have different price sensitivity than the coupon users have. Specifically, we assume that a coupon non-user located at l is indifferent between buying from either firm if and only if $l = \frac{p_2 - p_1}{\beta}$.[1] The remaining fraction γ of consumers are the same as in our benchmark model. We consider two cases: with or without mass media coupons, and find that our comparative statics results remain qualitatively the same as in the main model.[2]

First, we consider the case where firms cannot distribute mass media coupons. We look for a symmetric equilibrium ($p_2 = p_1$, $r_2 = r_1$, $\lambda_2 = \lambda_1$) and then study the comparative statics results with respect to the fraction of coupon traders α and coupon distribution cost parameter k. These results are similar to those in the main model.

With the introduction of coupon non-users, especially when they are less price sensitive than coupon users are, it is natural to consider not just poaching coupons, but mass media coupons as well.[3] For simplicity, we assume that firms can distribute mass media coupons to all consumers costlessly and we allow mass media coupons and poaching coupons to be combined. When firms send mass-media coupons, all coupon users enjoy the discount of mass media coupons while coupon non-users pay regular prices. This is equivalent to firms

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[1] We assume that $\beta \geq 1$, i.e., coupon non-users to be less price sensitive. Recall that for coupon users the marginal consumer is $l = p_2 - p_1$.

[2] Details are provided in the Appendix.

[3] If all consumers use coupons, sending mass media coupons is equivalent to a price reduction. On the other hand, if there are coupon non-users, then although mass media coupons reach all consumers, they will not be used by the non-users. Therefore, mass media coupons allow firms to charge higher prices to coupon non-users who are also less price sensitive relative to coupon users, a standard assumption in the literature, e.g., Narasimhan (1984).

charging one price for coupon users, and another price for coupon non-users. Therefore, we can treat coupon users and coupon non-users as being in separate markets. Then, the market of coupon non-users is the same as the whole market in our initial model (but of different market size) and the comparative statics results are qualitatively the same as those in Section 4.1 of the paper.

A numerical example

We normalize $L = 1$, and choose $k = 1$, $\gamma = 0.5$ (50% consumers are coupon non-users), $\alpha = 0.2$ (20% of coupon-users are coupon traders), $\beta = 2$ (coupon users are twice as price-sensitive as non-users are). Firms do not distribute mass media coupons. Then, we calculate numerical solution for the equilibrium prices and promotion intensity. The results are provided in Table 1, along with the equilibrium solutions for the benchmark model (at $L = 1$, $k = 1$ and $\alpha = 0.2$). It's easy to see that with the introduction of coupon non-users who are less price-sensitive, equilibrium prices and profits increase.

Model	λ	r	p	π
Benchmark	0.0525	0.3622	0.9743	0.9623
Coupon non-users	0.0562	0.5300	1.3100	1.2975

Table 1: Benchmark model vs. model with coupon non-users

2 Continuous hassle cost of trading coupons

In the main model, we have made a simplifying assumption about the hassle costs of trading coupons. In particular, some consumers have zero cost of trading coupons while the rest have prohibitively high costs. This assumption guarantees a clear distinction between the groups of coupon traders and non-traders. In this extension, we consider a more smooth distribution of hassle costs. Suppose that s, the hassle cost of selling coupons, is uniformly distributed on the interval $[0, S]$, and is independent of consumers' locations. In practice, the hassle cost of buying coupons is likely to be lower than that of selling coupons.[4] For simplicity, we assume that the hassle cost of buying coupons is zero.[5] This model is

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[4]To name just a few, posting and shipping the item, clearing the payment from the buyer and transacting with the auction site are additional hassle costs associated with the selling process but not the buying process.

[5]Ideally one would consider smooth distributions for hassle costs of both selling and buying coupons. The difficulty is, with hassle costs on both sides (sell/buy), we need to rely on market clearing to calculate

qualitatively different from the main model in the following sense. In the main model, all coupons that reach traders will be sold in equilibrium, since they have zero hassle cost. In the current setting, however, those who choose to sell coupons have positive hassle costs, and they have to weigh the benefit against the cost of such an action. Consequently, consumers in the middle, who do not have strong preference for either firm, are more likely to use the coupons, and forgo the hassle of the selling process.

A comparison between this and the benchmark model shows that, when S increases, hassle costs increase and fewer consumers trade coupons. This is similar in spirit to a decrease in α in our main model, and we find that the corresponding results are qualitatively the same. That is, when S decreases (or α increases), promotion frequency and depth decrease, while prices and profits increase.

<u>A numerical illustration</u>

We normalize $L = 1$ and set $S = 1$, $k = 1$. We then calculate the equilibrium. The fraction of traded coupons is about 20% under these parameter value. Based on this percentage we calculate the corresponding equilibrium of the benchmark model (with $\alpha = 0.2$) for a direct comparison. Both results are provided in Table 2.

Model	λ	r	p	π
Benchmark model	0.0525	0.3622	0.9743	0.9623
Endogenous coupon trading choices	0.0509	0.2239	0.9600	0.9528

Table 2: Benchmark model vs. model with endogenous coupon trading choices

Comparing the results from the two models,the set of parameters generate similar levels of promotion frequency (λ), price and profit, but quite different levels of promotion depth (r). The intuition is the following. In the benchmark model, the probability of trading coupons (i.e., being coupon traders) is independent of consumer's location on the interval $[-L, L]$, while in the model presented here, they are dependent. In particular, consumers in the middle are more likely to use the coupons instead of trading them. This property has two effects both contributing to a lower promotion depth. First, there will be more switchers in the middle, and smaller discounts are needed to induce them to switch. Second, and probably more important, coupons of higher face value are more likely to be sold than used, giving firms an additional incentive to lower promotion depth.

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both the prevailing coupon price and the amount of coupons traded. Moreover, the equilibrium price and quantity depend on each other, making them intractable.

3 Non-transferable coupons

In the benchmark model, we have assumed that all coupons are transferable. But why would firms allow their coupons to be transferred or traded? Often coupons have unique identifying codes, and firms can tie coupon codes to the consumers they are targeting, and refuse to honor coupons if used by others.[6] In this section, we analyze firms' choice in terms of coupon types: transferrable or non-transferrable. In particular, we introduce a stage 0 where firms simultaneously and independently decide whether they want their coupons to be transferable.[7] We assume that transferable and non-transferable coupons cost the same to distribute. Once decisions of coupon types are made, the rest of the game has similar structure as that presented in Section 3. Let (i, j) denote firms' coupon type choices, where $i, j \in \{T, NT\}$ are firm 1 and firm 2's coupon types respectively. After firms make their coupon type decisions there are four possible subgames. In the first subgame (T, T), both firms' coupons are transferable. This is the benchmark case. In the second subgame, neither firm's coupons are transferable (NT, NT). This is similar to setting $\alpha = 0$ in the benchmark model. Subgame 3 (T, NT) and 4 (NT, T) are symmetric to each other, where one firm's coupons are transferable but the other firm's are not. Due to this symmetry, we consider subgame 3 only – (T, NT).

3.1 Analysis of the (T, NT) subgame

This subgame is analyzed in two parts. In Part 1, we derive the optimal prices and promotion intensities in the (T, NT) subgame. The solution to first order conditions is the equilibrium candidate for the asymmetric game. Due to the asymmetry between firms, we can't obtain closed-form solutions any more and have to rely on numerical analysis. Then in Part 2, we show that neither firm has an incentive to unilaterally deviate. Thus, the equilibrium candidate in Part 1 is an equilibrium for (T, NT). The details of this analysis are provided in the Appendix.

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[6]This requires that firms are able to tie coupons to consumers. That is, they can identify each customer (e.g., name, address) without knowing his/her location on the interval $[-L, L]$ (preferences).

[7]While face value is printed on the coupon, certain restrictions such as transferrability may not be. Instead, there may be company policy governing the transefferability of coupons or consumers may need to find out what the exact policy is. Even if transferrability is printed on coupons, firms may still treat transferrability as a longer-term strategy and vary coupon face value more frequently than the trasfferability of coupons.

3.1.1 Comparison to the benchmark case

In the benchmark model (T,T) in Section 4, we provide a numerical example with $L = 1$, $k = \frac{1}{2}$ and $\alpha = 0.2$. Here in the (T, NT) subgame, we choose the same parameter values to facilitate comparison of the results. Firms' prices, promotion intensities and profits after a coupon type decision is made are

Coupon Type Strategies	λ	r	p	π
(T, T)	0.0987	0.3513	0.9525	0.9310
(T,NT) Firm 1	0.0583	0.2701	0.8559	0.8468
(T,NT) Firm 2	0.1831	0.4937	0.9216	0.8662
(NT,NT)	0.2056	0.4534	0.9068	0.8434

Table 3: Optimal prices, promotion strategies and profits.

Compared to the equilibrium in the benchmark model, in (T, NT), firm 1 becomes much more aggressive by increasing both promotion frequency and depth, since its coupons are non-transferable. This forces firm 2 (whose coupons are transferable) to be somewhat less aggressive. Firm 1's higher promotion intensity increases the competitive pressure, and equilibrium prices and profits go down.

3.2 The choice of coupon types

We have derived the equilibrium for the subgame (T, NT). By symmetry, we also know the equilibrium for the subgame (NT, T). The equilibria for the (T, T) and (NT, NT) subgames have been obtained earlier.[8]

Next, we analyze the couponing type decisions (stage 0 of the game). We construct a 2×2 payoff matrix. We find that in general, (T, T) is an equilibrium.[9] When k is sufficiently large (e.g., $k \geq 1$), there is another equilibrium (NT, NT), implying that firms want to mimic each other's decisions on coupon types. When (T, T) and (NT, NT) both can be supported in an equilibrium, (T, T) serves as a focal equilibrium couponing strategy since profits are higher than they would be with nontransferable coupons (NT, NT).[10]

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[8]Note that, the (NT, NT) subgame is the same as the (T, T) subgame but with $\alpha = 0$.

[9]An exception is that when both α and k are small, we have asymmetric equilibria where only one firm chooses to issue transferrable coupons. It would be interesting to analyze coupon type choices in an asymmetric firms setting. For example, one firm may have a larger loyal customer base than the other firm. Would asymmetric equilibrium in terms of coupon type choices be more likely? Which firm has more incentive to choose nontransferable coupons?

[10]In the main model, we have shown that equilibrium profits increase with α. Thus (T, T) leads to higher

A numerical example

Combining firms' profits from all subgames in Table 1 (with $L = 1$, $k = \frac{1}{2}$ and $\alpha = 0.2$), the 2×2 game of coupon type choices is presented in Table 4.

Coupon type: Firm 1 \ Firm 2	Transferable	Non-transferable
Transferable	(0.9310, 0.9310)	(0.8468, 0.8662)
Non-transferable	(0.8662, 0.8468)	(0.8434, 0.8434)

Table 4: 2×2 game of coupon type choices

For each cell, the first (second) entry is firm 1's (firm 2's) profit. We can see that (T, T) is the only equilibrium.

Appendix for the Extensions

Extension 1: Coupon non-users

A fraction, $1 - \gamma$, of consumers do not use coupons. We consider two setups: with and without mass media coupons.

Setup 1: Without mass media coupons

We divide the analysis into three parts. First, we derive the optimal prices and couponing strategies. This is the equilibrium candidate. Then, we show that neither firm has an incentive to deviate, thus confirming that the equilibrium candidate derived in Part 1 is indeed an equilibrium. Finally, we perform comparative static analysis to see how equilibrium prices and promotion strategies respond to changes in α and k. We find that, the results are qualitatively the same as those of the main model. Next, we provide the details.

Part 1: Deriving the equilibrium candidate

We first construct the profit functions with the following assumptions, which will govern the location of marginal consumers and thus the demand structure,

$$p_2 \geq p_1, \quad p_1 - r_1 < p_2 \quad \text{and } p_2 - r_2 < p_1.$$

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profits for firms than (NT, NT) does. Making coupons transferrable may also have other benefits which we do not model here. For example, verifying non-transferable coupons adds hassle costs to both the firm and customers; denying customers' rights to use coupons may upset the customers and firms may lose their business.

The first assumption ($p_1 \geq p_2$) is without loss of generality. The last two assumptions guarantee that coupons will be used in equilibrium. Otherwise, firms are better off not sending coupons.

Depending on whether consumers (i) are coupon users, (ii) have coupons and/or (iii) are coupon traders, we divide the consumers into the following types:

(a) Coupon users, non-traders, without coupons. The profits from this type of consumers are given by:

$$\pi_{1a} = (p_1(1-\alpha)(1-\lambda_2)L + p_1(1-\alpha)(1-\lambda_1)(p_2 - p_1))\gamma.,$$

$$\pi_{2a} = p_2(1-\alpha)(1-\lambda_1)(L - p_2 + p_1)\gamma.$$

(b) Coupon users, non-traders, with coupons. The profits generated by this type are:

$$\pi_{1b} = (p_1(1-\alpha)\lambda_2(l_{b1} + L) + (p_1 - r_1)(1-\alpha)\lambda_1 l_{b2})\gamma,$$

$$\pi_{2b} = ((p_2 - r_2)(1-\alpha)\lambda_2(0 - l_{b1}) + p_2(1-\alpha)\lambda_1(L - l_{b2}))\gamma.$$

(c) Coupon users, traders, with or without coupons. The profits generated by this type are:

$$\pi_{1c} = (p_1\alpha(l_c + L) - r_1\alpha\lambda_1 L)\gamma,$$

$$\pi_{2c} = (p_2\alpha(L - l_c) - r_2\alpha\lambda_2 L)\gamma.$$

Note that, these three types are the same as in the benchmark model. The only change we made here is to multiply the previous profits by γ, which is the fraction of coupon users.

(d) Coupon non-users. The profits made by the two firms from these types are:

$$\pi_{1d} = p_1\left(L + \frac{p_2 - p_1}{\beta}\right)(1 - \gamma),$$

$$\pi_{2d} = p_2(1 - \gamma)\left(L - \frac{p_2 - p_1}{\beta}\right).$$

The structure of profits generated from type (a) and (d) is similar but the two types differ in their price sensitivities and distributions.

Aggregating profit from each type of consumer, and subtracting the cost of distributing coupons, we can obtain firm i's overall profit,

$$\pi_i = \sum_{j=a}^{d} \pi_{ij} - k(\lambda_i L)^2, \quad i = 1, 2.$$

We look for a symmetric equilibrium to the first-order conditions. The solutions of r_1 and λ_1 as a function of p_1 are

$$r_1 = \frac{p_1 - \alpha L - \alpha p_1}{2(1-\alpha)} \quad \text{and} \tag{1}$$

$$\lambda_1 = \frac{\gamma(\alpha L - p_1 + \alpha p_1)^2}{8k(1-\alpha)L^2}. \tag{2}$$

There are three solutions of p_1 to equation $\frac{\partial \pi_1}{\partial p_1} = 0$, and we pick the real root. We then replace it into the expressions (1) and (2) to obtain r_1 and λ_1 as a function of the parameters.[11]

Part 2: Checking profits under potential deviations

In Part 1, we made the following assumptions governing the demand structure:

$$p_2 \geq p_1, \quad p_1 - r_1 < p_2 \quad \text{and } p_2 - r_2 < p_1.$$

These assumptions hold for the equilibrium solution candidate, which is symmetric. However, when a firm deviates, some of these may be violated, and we need to adjust the demand and profit functions accordingly.

Due to symmetry, we only need to check on the possibility that firm 1 will deviate from $p_1 = p_2$. Since firm 1 is the deviating firm, $p_1 - r_1 < p_2$ must hold, i.e., firm 1 must be able to poach some of firm 2's loyal customers.[12] There are two possibilities in deviation: 1) $p_1 > p_2$ and 2) $p_1 < p_2$. In both types of deviation, we fix firm 2's price, promotion frequency and promotion depth at the levels indicated in Part 1, and see whether firm 1 can improve its profit. We find that it can never do so. Therefore, no firm would have incentive to deviate.

Part 3: Comparative statics

Having shown that the equilibrium candidate in Part 1 is indeed an equilibrium, we now perform comparative static analysis similar to that in Section 4.1 of the paper. We

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[11]These expressions are too lengthy to report. Maple files for all analysis in this appendix are available upon request.

[12]When $\lambda_1 \to 0^+$, the marginal cost of distributing coupons $\frac{\partial[k(\lambda_1 L)^2]}{\partial \lambda_1} \to 0^+$. Then a deviating firm would have no incentive to distribute coupons only when its optimal prices in segment 1 and 2, if it can choose a price for each segment, are exactly the same. However, this requires the other firm to choose r and λ value well beyond the value in the equilibrium candidate (e.g. for $\lambda_2 = 1$, it requires $r_2 \approx L$, which is even above the value of p_2). This is true in all extensions which we analyze.

normalize $L = 1$, and try various value of α, k and t. We find that the comparative static results, in terms of how prices and promotion strategies vary with α or k, are qualitatively the same as those of the main model.

<u>Setup 2: With mass media coupons</u>

When firms send mass-media coupons, the coupon users enjoy the discount of mass media coupons, on top of the possible poaching coupons, while coupon non-users pay regular price. This is equivalent to firms charging one regular price for coupon users, and another regular price to coupon non-users. Then the market of coupon non-users can be treated as an independent market, separate from the market of coupon users. Consequently, the introduction of coupon non-users will not affect the equilibrium choices of λ_i, r_i and p_i qualitatively. The quantitative change is due to the change in the fraction of coupon users ($1 - \gamma$ here as opposed to 1 in the main model), which will affect the benefit of couponing, but not the cost of distributing coupons.

Extension 2: Continuous hassle cost of trading coupons

This analysis consists of three parts. In Part 1, we derive the equilibrium candidate. Then, we show that no firm wants to deviate in Part 2. In Part 3, we analyze how equilibrium prices and promotion intensity change with the coupon distribution cost parameter k and the hassle cost parameter S.

Part 1: Deriving the equilibrium candidate solution

With zero cost of buying coupons, competition among buyers would drive the market price of coupons to their face value. Therefore, we only need to calculate, under these prices, what the equilibrium supply is for each firm's coupons. The next figure shows the optimal choices of consumers who receive coupons (the potential suppliers of coupons).

Let l_a be the marginal consumer who is indifferent between buying from either firm without coupon,

$$l_a = p_2 - p_1.$$

Let l_b be the marginal consumer who is indifferent between (i) buying from firm 1 without coupon and (ii) buying from 2 with a coupon,
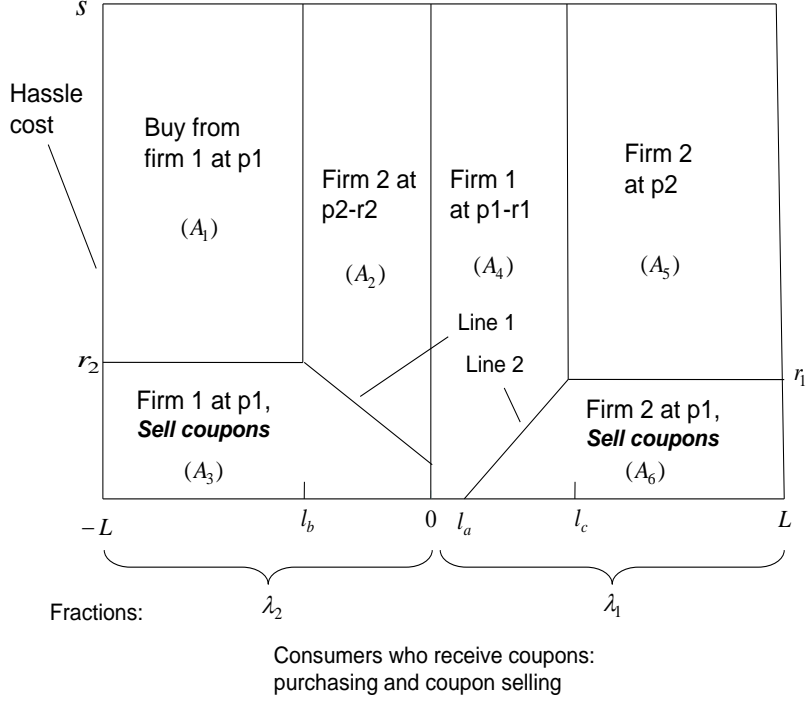
$$l_b = p_1 - (p_2 - r_2).$$

Figure 1: Endogenizing coupon trading choices

Let l_c be the marginal consumer who is indifferent between (i) buying from firm 1 with a coupon and (ii) buying from 2 without coupon,

$$l_c = (p_1 - r_1) - p_2.$$

Lines 1 and 2 in the figure define the marginal consumers who are indifferent (i) between switching and buying with coupons, and (ii) between not switching and selling coupons. For line 1 in $[l_b, 0]$, the expression is

$$l = (p_2 - r_2) - (p_1 - (r_2 - s)) \Rightarrow s = -l + (p_2 - p_1).$$

Note that, $r_1 - s$ is the gain from selling firm 1's coupon, and the slope of line 1 is -1. Similarly, line 2 in $[l_a, l_c]$ is represented by the following expression,

$$l = (p_2 - (r_1 - s)) - (p_1 - r_1) \Rightarrow s = l - (p_2 - p_1).$$

The slope of line 2 is 1. Note that, in a symmetric equilibrium, $r_1 = r_2$, $l_a = 0$. Then line 1 and line 2 both cross the point $(0, 0)$.

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Next, we will construct the profit functions. We start by calculating the sizes of areas A_1 through A_6 (Note that, they are divided by S to obtain the fraction of consumers in these areas).

$$A_1 = \frac{S - r_2}{S}(L + l_b),$$

$$A_2 = \frac{1}{2}\frac{r_2 - l_a}{S}(0 - l_b) + \frac{(S - r_2)}{S}(0 - l_b),$$

$$A_3 = L - A_1 - A_2.$$

$$A_4 = l_c - \frac{1}{2}\frac{r_1}{S}(l_c - l_a),$$

$$A_5 = \frac{S - r_1}{S}(L - l_c),$$

$$A_6 = L - A_4 - A_5.$$

From the figure, we can see that only coupons reaching consumers located in A_3 and A_6 will be traded.[13]

Let π_{1a} and π_{2a} be profits from consumers who receive coupons i.e.,

$$\pi_{1a} = p_1(A_1 + A_3)\lambda_2 + (p_1 - r_1)A_4\lambda_1,$$

$$\pi_{2a} = p_2(A_5 + A_6)\lambda_1 + (p_2 - r_2)A_2\lambda_2.$$

The losses of traded coupons are represented by,

$$loss_1 = A_6\lambda_1 r_1,$$

$$loss_2 = A_3\lambda_2 r_2.$$

Let π_{1b} and π_{2b} be profits from consumers who receive no coupon

$$\pi_{1b} = p_1 L(1 - \lambda_2) + p1 l_a(1 - \lambda_1),$$

$$\pi_{2b} = p_2(L - l_a)(1 - \lambda_1).$$

The overall profits are,

$$\pi_1 = \pi_{1a} + \pi_{1b} - loss_1 - k(\lambda_1 L)^2,$$

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[13]In equilibrium $p_1 = p_2$ thus $l_a = 0$. It can be easily seen from the figure that consumers close to 0 will switch and use the coupons instead of selling them, unless their hassle costs of selling coupons are sufficiently low. Moreover, the closer they are to the middle, the lower the threshold hassle cost needed. This is a distinct feature of the model here to differentiate it from the main model.

$$\pi_2 = \pi_{2a} + \pi_{2b} - loss_2 - k(\lambda_2 L)^2.$$

Next, we take derivatives $\frac{\partial \pi_i}{\partial p_i}$, $\frac{\partial \pi_i}{\partial r_i}$ and $\frac{\partial \pi_i}{\partial \lambda_i}$, and then impose symmetry condition

$$p_2 = p_1, r_2 = r_1, \lambda_2 = \lambda_1.$$

From $\frac{\partial \pi_1}{\partial p_1} = 0$, we can obtain

$$p_1 = \frac{r_1 \lambda_1 S - r_1^2 \lambda_1 + LS}{S(\lambda_1 + 1)}.$$

We solve for λ_1 from $\frac{\partial \pi_1}{\partial \lambda_1} = 0$. There are two solutions. We pick the one which is positive. We then use $\frac{\partial \pi_1}{\partial r_1} = 0$ to solve for r_1, and substitute it back to obtain λ_1 and p_1. The expressions are too lengthy to report, and a numerical example is provided earlier (see Table 2).

Part 2: Checking for deviations

When constructing firms' profits, we have made the following assumptions:

$$p_2 \geq p_1, \quad p_1 - r_1 < p_2 \quad \text{and } p_2 - r_2 < p_1.$$

These assumptions need to be checked under potential deviation, and if violated, profit functions need to be adjusted. Due to symmetry, we only consider firm 1's deviations. There are several types of such deviations, depending on the demand structure.

Type 1: firm 1 deviates and $p_2 \geq p_1$, $p_2 - r_2 < p_1$

Since firm 1 is the deviating firm, it must that $p_1 - r_1 < p_2$. Thus all initial assumptions hold, we can use profit functions as in Part 1. We first solve for r_1 and λ_1 from first order conditions. We then check whether firm 1 can increases its profit by choosing $p_1 < p_2$. We find that firm 1's deviation profit increases with p_1. Thus firm 1 has no incentive to choose $p_1 < p_2$.

Type 2: firm 1 deviates and $p_2 > p_1$, $p_2 - r_2 > p_1$

In this case, firm 1 lowers p_1 to a point where firm 2's coupons are useless. We first solve for r_1 and λ_1 from first order conditions. We then find that firm 1's deviation profit increases with p_1. Thus firm 1 has no incentive to choose $p_1 < p_2$.

Type 3: firm 1 deviates and $p_1 > p_2$

In this case, $p_2 - r_2 < p_1$ must hold. Since firm 1 is the deviating firm, $p_1 - r_1 < p_2$ must also hold. Therefore, both firms' coupons can be useful. We first solve for r_1 and λ_1 from first order conditions. We then find that firm 1's deviation profit decreases with p_1. Thus firm 1 has no incentive to choose $p_1 > p_2$.

Part 3: Comparative statics when S or k changes

We normalize $L = 1$ and try various value of S (hassle cost parameter) and k (coupon distribution cost parameter). When S increases, the hassle costs of selling coupon increase. This is in the same spirit as a decrease of α in the main model. We find that when S increases, promotion frequency (λ) and promotion depth (r) increase, price and profit decrease. These results mimic qualitatively the results when α decreases in the main model. We also fix S and vary k. We find that when k increases, firms promote less frequently, but with higher coupon face value. Equilibrium price and profit increase with k just as in the main model.

Extension 3: Non-transferrable coupons

The subgame (T, NT) is analyzed in two parts. In Part 1, we derive the optimal prices and promotion intensities. This is the equilibrium candidate for the asymmetric game. In Part 2, we show that neither firm has an incentive to unilaterally deviate. Thus, the equilibrium candidate in Part 1 is an equilibrium for (T, NT).

Part 1: Deriving the equilibrium candidate in the (T, NT) subgame

We first construct the profit functions. We make the following assumptions to help us locate the position of the marginal consumers:

$$p_2 \geq p_1, \quad p_1 - r_1 < p_2 \quad \text{and } p_2 - r_2 < p_1.$$

The last two assumptions must hold in equilibrium. Otherwise, firms will be better off not to distribute coupons.

There are three types of consumers depending on whether they are coupon-traders and whether they receive coupons:

(a) non-traders without coupons. The profits generated from this type are:

$$\pi_{1a} = p_1(1 - \alpha)(1 - \lambda_2)L + p_1(1 - \alpha)(1 - \lambda_1)(p_2 - p_1),$$

$$\pi_{2a} = p_2(1 - \alpha)(1 - \lambda_1)(L - p_2 + p_1).$$

(b) non-traders with coupons. The profits from this type are:

$$\pi_{1b} = p_1(1 - \alpha)\lambda_2(l_{b1} + L) + (p_1 - r_1)(1 - \alpha)\lambda_1 l_{b2},$$

$$\pi_{2b} = (p_2 - r_2)(1 - \alpha)\lambda_2(0 - l_{b1}) + p_2(1 - \alpha)\lambda_1(L - l_{b2}).$$

Note that, these two types of consumers are the same as those in the benchmark model.

(c) traders with and without coupons. The profits from this type are:

$$\pi_{1c} = p_1\alpha\lambda_2(L + p_2 - r_2 - p_1) + p_1\alpha(1 - \lambda_2)L + p_1\alpha(p_2 - p_1) - \alpha\lambda_1 L r_1,$$

$$\pi_{2c} = (p_2 - r_2)\alpha\lambda_2(0 - (p_2 - r_2 - p_1)) + p_2\alpha(L - (p_2 - p_1)).$$

(Jie: Following paragraph needs to rewrite, will do tomorrow.) To better explain profit functions from type (c) consumers, consider first those located at $[-L, 0]$. A fraction, $\alpha\lambda_1$, of consumers receive coupons from firm 1. Since firm 1's coupons are non-transferable, these consumers will either buy from firm 1 with coupons, or buy from firm 2 and discard firm 1's coupons. For consumers in $[0, L]$, they may receive firm 2's coupons. However, firm 2's coupons are transferable, thus whether receiving them or not will not affect a trader's purchasing decision. Moreover, there is a loss of traded coupons, $\alpha\lambda_2 L r_2$ for firm 2, but none for firm 1.

Aggregating profits from each type of consumer, and subtracting the cost of coupon distribution, we obtain firm i's overall profit,

$$\pi_i = \sum_{j=a}^{c} \pi_{ij} - k(\lambda_i L)^2, \quad i = 1, 2.$$

We then use the first order conditions to solve for λ_i, r_i and p_i, $i = 1, 2$. From $\frac{\partial \pi_1}{\partial r_1} = 0$ and $\frac{\partial \pi_1}{\partial r_2} = 0$, we can obtain

$$r_1 = p_1 - \frac{p_2}{2}, \quad r_2 = \frac{-\alpha L + \alpha p_1 - 2\alpha p_2 - p_1 + 2p_2}{2(1 - \alpha)}.$$

From $\frac{\partial \pi_1}{\partial \lambda_1} = 0$ and $\frac{\partial \pi_1}{\partial \lambda_2} = 0$, we can obtain

$$\lambda_1 = \frac{p_2^2}{8kL^2}, \quad \lambda_2 = \frac{(-\alpha L - p_1 + \alpha p_1 - 2\alpha p_2 + 2p_2)^2}{8kL^2(1 - \alpha)}.$$

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From $\frac{\partial \pi_1}{\partial p_1} = 0$ and $\frac{\partial \pi_1}{\partial p_2} = 0$, we solve for p_1 and p_2. With asymmetric firms (in terms of coupon types), we can't obtain closed form solutions for p_1 and p_2, and one has to rely on numerical methods.

Part 2: Checking for deviations

In deriving the profit functions, we have made the following assumptions:

$$p_2 \geq p_1, \quad p_1 - r_1 < p_2, \quad p_2 - r_2 < p_1.$$

It can be shown that, these assumptions hold for the equilibrium solution candidate. However, when a firm deviates, these assumptions may be violated. When that happens, we need to adjust the demand function to derive the deviation profit.

Since firms are asymmetric, we have to check deviation for each firm. There are a total of 6 types of deviations, depending on which firm deviates and the ensuing demand structure. Note that, the demand structure only depends on p_i and r_i, not on λ_i. There are a total of six types of deviations:

- Type 1: firm 1 deviates without violating the demand structure

- Type 2: firm 1 deviates, $p_1 < p_2$, $p_1 < p_2 - r_2$

- Type 3: firm 1 deviates, $p_1 > p_2$, $p_1 < p_2 - r_2$

- Type 4: firm 2 deviates without violating the demand structure

- Type 5: firm 2 deviates, $p_2 < p_1$, $p_2 > p_1 - r_1$

- Type 6: firm 2 deviates, $p_2 < p_1$, $p_2 < p_1 - r_1$.

In the first three types of deviation, firm 1 is the deviating firm, and $p_1 - r_1 < p_2$ must hold, i.e., firm 1 must be able to poach some of firm 2's loyal customers.[14] In the last three types of deviations, firm 2 is the deviating firm, and $p_2 - r_2 < p_1$ must hold. We find that firms have no incentive to deviate, for each of the six types of deviations.[15] Therefore, what we derived in Part 1 is an equilibrium for the subgame (T, NT). $\blacksquare$

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[14]When $\lambda_1 \to 0^+$, the marginal cost of distributing coupons $\frac{\partial [k(\lambda_1 L)^2]}{\partial \lambda_1} \to 0^+$. Then a deviating firm would have no incentive to distribute coupons only when its optimal prices in segment 1 and 2, if it can choose a price for each segment, are exactly the same. However, this requires the other firm to choose r and λ value well beyond the value in the equilibrium candidate (e.g. for $\lambda_2 = 1$, it requires $r_2 \approx L$, which is even above the value of p_2).

[15]A Maple program with detailed analysis is available upon request.

Defensive coupon

We allow firms to send defensive coupons alone or together with offensive coupons. And we show that in equilibrium, firms will not do so. Let (λ_{11}, r_{11}) denote firm 1's defensive couponing intensity in segment 1, where firm 1's loyal customers are located. Similarly, let (λ_{12}, r_{12}) denote firm 1's offensive couponing intensity in segment 2. Note that, if $\lambda_{12} > 0$ and $\lambda_{11} = 0$, then firm 1 is sending offensive coupons only. Firm 2's offensive couponing intensity is given by (λ_{21}, r_{21}) and its defensive couponing intensity is given by (λ_{22}, r_{22}). When consumers do not use coupons, they will pay either p_1 or p_2, depending on which firm they buy from. Consumers located at segment $j = 1, 2$ who have coupons, will pay $p_1 - r_{1j}$ if they buy from firm 1, or pay $p_2 - r_{2j}$ if they buy from firm 2. Next, we introduce the cost of distributing coupons takes the form of $k(\lambda_{ij}L)^2$ for firm i's promotion effort at segment j. If firm i sends out both offensive and defensive coupons, then its total promotion cost is $k(\lambda_{i1}L)^2 + k(\lambda_{i2}L)^2$.

Consumers can be segmented into the following groups, depending on whether they are coupon traders and whether they receive coupons.

Type (a): Non-traders with neither firm's coupon;

Type (b): Non-traders with only offensive coupons;

Type (c): Traders with or without coupons.

Type (d): Non-traders with only defensive coupons;

Type (e): Non-traders with both firms' coupons;

The first three types of consumers have been analyzed in the paper, we now provide analysis for type (d) and (e) consumers.

Type (d): Non-traders with defensive coupons only

The fourth type of consumers are non-traders who receive defensive coupons only. That is, non-traders on $[-L, 0]$ who receive firm 1's coupons and those on $[0, L]$ who receive firm 2's coupons. Their densities are $(1 - \alpha)(1 - \lambda_{11})\lambda_{21}$ and $(1 - \alpha)\lambda_{12}(1 - \lambda_{22})$ respectively.

Let's start with consumers on the interval $[-L, 0]$. These consumers prefer firm 1's products and we assumed that $p_2 \geq p_1$. Moreover, they receive coupons from their preferred firm but not the other firm. Thus, they will all buy from firm 1 and pay $p_1 - r_{11}$. Next, we consider consumers on $[0, L]$. All these consumers prefer firm 2's product. Although $p_2 \geq p_1$, they receive firm 2's coupon. Let l_d denote the marginal customer who is indifferent between buying from firm 2 with coupon and buying from firm 1 without coupon. Then,
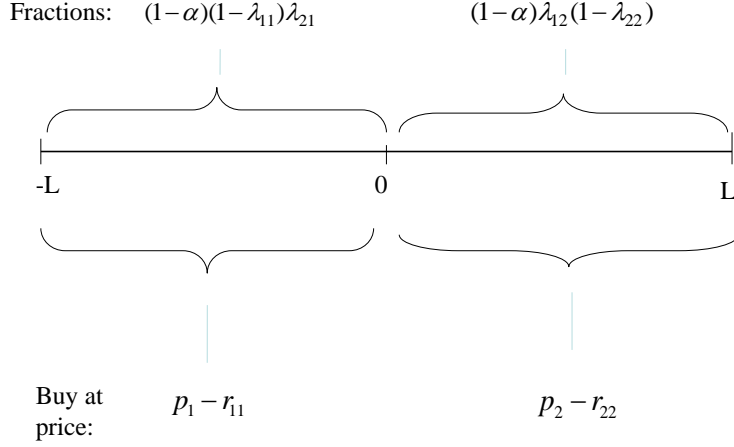
Figure 1: Type (d): Non-traders with defensive coupons only

$l_d = (p_2 - r_{22}) - p_1$. Depending on the sign of l_d, there are two cases. In the first case, $l_d \leq 0$, i.e., all consumers on $[0, L]$ buy from firm 2. This case is depicted in Figure 1. Firms' profits are,

$$\pi_{1d} = (p_1 - r_{11})(1 - \alpha)\lambda_{11}(1 - \lambda_{21})L,$$

$$\pi_{2d} = (p_2 - r_{22})(1 - \alpha)(1 - \lambda_{12})\lambda_{22}L.$$

In the other case, $l_d > 0$. Then consumers on $[0, l_d)$ buy from firm 1 while those in $[l_d, L]$ buy from firm 2. Firms' profits become

$$\pi_{1d} = (p_1 - r_{11})(1 - \alpha)\lambda_{11}(1 - \lambda_{21})L + p_1(1 - \alpha)\lambda_{22}(1 - \lambda_{12})l_d$$

$$\pi_{2d} = (p_2 - r_{22})(1 - \alpha)(1 - \lambda_{12})\lambda_{22}(L - l_d)$$

Type (e): Non-traders with both firms' coupons

Non-traders with both firms' coupons are depicted in Figure 2. Their densities are $(1 - \alpha)\lambda_{11}\lambda_{21}$ on $[-L, 0]$ and $(1 - \alpha)\lambda_{12}\lambda_{22}$ on $[0, L]$ respectively.

Let l_{e1} and l_{e2} denote the marginal consumer in segment 1 and 2 respectively. The left marginal consumer, located at l_{e1}, is indifferent between buying from firm 1 with a coupon (thus paying $p_1 - r_{11}$) and buying from firm 2 also with a coupon (thus paying $p_2 - r_{21}$). Similarly, the right marginal consumer (located at l_{e2}) is indifferent between buying from
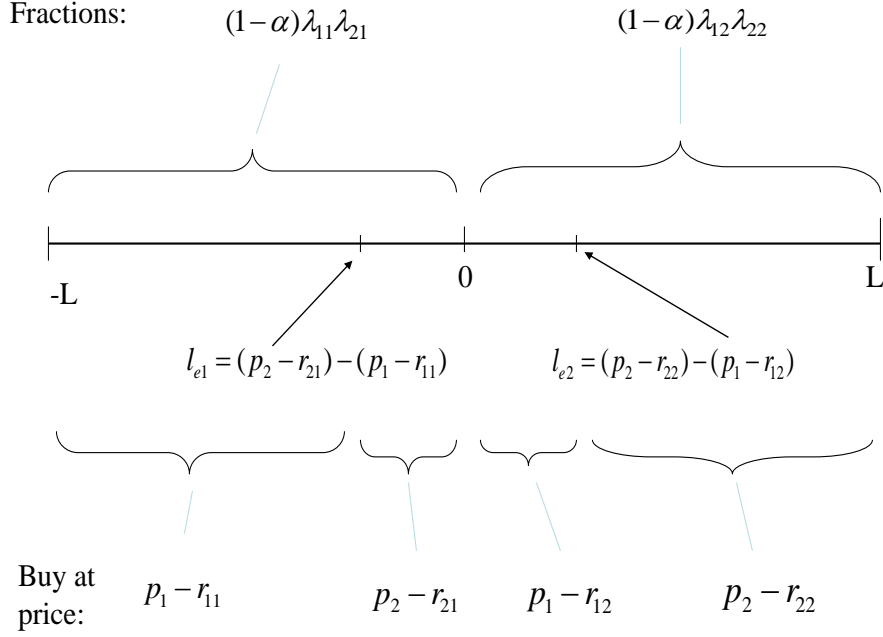
Figure 2: Type (d): Non-traders with both firms' coupons

firm 1 at a price of $p_1 - r_{12}$ and buying from firm 2 at a price of $p_2 - r_{22}$. The exact locations of these two marginal consumers are

$$l_{d1} = (p_2 - r_{21}) - (p_1 - r_{11}), \quad l_{d2} = (p_2 - r_{22}) - (p_1 - r_{12}).$$

Consumers located in the interval $[-L, l_{d1}]$ receive coupons from both firms, and the face value of firm 2's coupon is not enough to compensate for their strong preferences for firm 1's product. As a result, they will use firm 1's coupons and buy from firm 1 at $p_1 - r_{11}$. Since they are non-traders, they will not sell firm 2's coupons. However, for consumers located in $(l_{e1}, 0]$, they only have a weak preference for firm 1's product. With firm 2's coupons, they will choose to buy from firm 2 and pay $p_2 - r_{21}$. Similarly, consumers located in $[0, l_{e2})$ will buy from firm 1 at $p_1 - r_{12}$, and consumers in $[l_{e2}, L]$ will buy from firm 2 at $p_2 - r_{22}$. Consequently, firms' profits are

$$\pi_{1e} = (p_1 - r_{11})(1 - \alpha)\lambda_{11}\lambda_{21}(L + l_{e1}) + (p_1 - r_{12})(1 - \alpha)\lambda_{12}\lambda_{22}l_{e2}$$

$$\pi_{2e} = (p_2 - r_{21})(1 - \alpha)\lambda_{11}\lambda_{21}(0 - l_{e1}) + (p_2 - r_{22})(1 - \alpha)\lambda_{12}\lambda_{22}(L - l_{e2})$$

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Next, we show that firms will not distribute defensive coupon in equilibrium. First, in Lemma 1, we show that a firm has no incentive to distribute both offensive and defensive coupons, whether in equilibrium or off the equilibrium path. Second, in Lemma 2, we prove that, firms will not distribute defensive coupons alone in any pure strategy equilibrium.

Lemma 1. *When coupon distribution is costly ($k > 0$), firms have no incentive to distribute both offensive and defensive coupons, whether in equilibrium or off the equilibrium path.*

Proof. See Appendix. ∎

Lemma 2. *There is no pure strategy equilibrium where firms distribute defensive coupons only.*

Proof. See Appendix. ∎

Proof of Lemma 1. Without loss of generality due to symmetry, we only show that firm 1 has no incentive to distribute both offensive and defensive coupons. Suppose not, fix firm 2's price and promotion strategies, and let p_1, λ_{11}, λ_{12}, r_{11} and r_{12} denote firm 1's best response to firm 2's strategy. We call this the *initial strategy*. Since firm 1 distributes both offensive and defensive coupons, thus $\lambda_{1j} > 0$ and $r_{1j} > 0$, $j = 1, 2$.

Next, we will rank r_{11} and r_{12}. Intuitively, firms are more aggressive and charge lower prices in the other firm's turf, due to best-response asymmetry (One firm's weak market is the other firm's strong market). This implies $r_{12} > r_{11}$. We will show that, firm 1 can improve its profit by playing the following strategy instead

$$p_1' = p_1 - r_{11}, \lambda_{12}' = \lambda_{12}, r_{12}' = r_{12} - r_{11}, \lambda_{11}' = r_{11}' = 0.$$

We call this strategy the *alternative strategy*, where firm 1 distributes offensive coupons only with a price cut. As we prove next, in comparison to the initial strategy, under the alternative strategy, firm 1 (i) earns weakly higher profit from the non-traders; (ii) earns weakly higher profit from the traders, and (iii) saves on coupon distribution cost. The result is quite intuitive and we present the proof for the sake of completeness.

Higher profit from non-traders

We first calculate firm 1's profit from non-traders under the initial strategy. Start with consumers in segment 1, i.e., those located in $[-L, 0]$. Let $\pi_{11}(p)$ denote firm 1's expected profit in this segment when its effective price is p. Firm 1's profit in segment 1 is

$$(1 - \lambda_{11})\pi_{11}(p_1) + \lambda_{11}\pi_{11}(p_1 - r_{11}).$$

Similarly firm 1's profit in segment 2 is

$$(1 - \lambda_{12})\pi_{12}(p_1) + \lambda_{12}\pi_{12}(p_1 - r_{12}).$$

Note that, it must be that $\pi_{11}(p_1) < \pi_{11}(p_1 - r_{11})$ and $\pi_{12}(p_1) < \pi_{12}(p_1 - r_{12})$. Otherwise, firm 1 would be better off not distributing coupons. Moreover, since the initial strategy is a best-response to firm 2's strategy, $p_1 - r_{12}$ maximizes firm 1's profits. This implies that

$$\pi_{12}(p_1 - r_{12}) > \pi_{12}(p_1 - r_{11}) > \pi_{11}(p_1),$$

since $p_1 - r_{12} < p_1 - r_{11} < p_1$, and $\pi_{12}(p)$ is concave in p (demand is linear in p), and $p_1 - r_{12}$ is a maximum.

Firm 1's profit from non-traders under the initial strategy (distributing both types of coupons) is

$$\pi_1 = (1 - \lambda_{11})\pi_{11}(p_1) + \lambda_{11}\pi_{11}(p_1 - r_{11}) + (1 - \lambda_{12})\pi_{12}(p_1) + \lambda_{12}\pi_{12}(p_1 - r_{12}).$$

Similarly, we can show that firm 1's profit from non-traders under the alternative strategy (with offensive coupons only) is

$$\pi_1' = (1 - \lambda_{12})\pi_{12}(p_1) + \lambda_{12}\pi_{12}(p_1 - r_{12}) + \pi_{11}(p_1 - r_{11}).$$

We have shown that $\pi_{12}(p_1 - r_{12}) > \pi_{12}(p_1)$ and $\pi_{11}(p_1 - r_{11}) > \pi_{11}(p_1)$. Thus

$$\pi_1' \geq \pi_1,$$

and firm 1 earns higher profit from the non-traders under the alternative strategy. The inequality is strict unless $\lambda_{11} = \lambda_{12} = 0$.

Higher profit from traders

Next, we compare firm 1's profit from traders under the initial strategy with that under the alternative strategy. In both segments, firm 1's final prices after coupons are distributed are the same under either strategy, while the regular price is lower under the alternative strategy. For traders who receive firm 1's coupons, a lower regular price means that they are more likely to buy from firm 1, instead of buying from firm 2 and selling firm 1's coupons. Similarly, for traders not receiving firm 1's coupons, lower regular price improves firm 1's profits as well. Moreover, when coupons are traded, a lower coupon face value

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implies lower loss for firm 1. Therefore, firm 1's profit from traders is higher under the alternative strategy.

<u>Saving on coupon distribution cost</u>

The coupon distribution cost is $k(\lambda_{11}L)^2 + k(\lambda_{11}L)^2$ under the initial strategy, while it's only $k(\lambda_{11}L)^2$ under the alternative strategy. Whenever $k > 0$, coupon distribution cost is strictly lower under the alternative strategy.

To summarize, firm 1 earns higher profits from traders and non-traders and saves on coupon distribution costs under the alternative strategy. Thus the initial strategy, under which firm 1 distributes both offensive and defensive coupons, cannot be a best-response to firm 2's strategy. Therefore, it will never distribute both types of coupons in equilibrium or off the equilibrium path(in deviation). ∎

Proof of Lemma 2. To show that both firms distributing defensive coupons alone is not an equilibrium, we show that if firm 2 sends defensive coupons only, then firm 1's optimal choice cannot be defensive coupons only.

We start by deriving firm 1's best response in each segment, assuming that, *hypothetically*, it can choose an individual price for each segment. Recall that $[-L, 0]$ and $(0, L]$ are segment 1 and 2 respectively. Let p_{ij} and π_{ij}, $i = 1, 2$, $j = 1, 2$ denote firm i's price and profit from segment j. Then

$$\pi_{21} = p_{21}[0 - (p_{21} - p_{11})], \quad \pi_{22} = p_{22}[L - (p_{22} - p_{12})].$$

The first-order conditions are

$$\frac{\partial \pi_{21}}{\partial p_{21}} = -2p_{21} + p_{11} = 0 \Rightarrow p_{21} = \frac{1}{2}p_{11},$$

$$\frac{\partial \pi_{22}}{\partial p_{22}} = L - 2p_{22} + p_{12} = 0 \Rightarrow p_{22} = \frac{1}{2}L + \frac{1}{2}p_{12}.$$

For firm 2 to have an incentive to send defensive coupons, it must be that $p_{22} < p_{21}$ which is equivalent to $p_{11} - p_{12} > L$ based on the above expressions, an impossible case when firm 1 sends only defensive coupons. The reason is the following. The effective prices for consumers who do not receive its coupons, are the same, $p_{11} = p_{12}$. For those who do receive firm 1's defensive coupons (in segment 2), we have $p_{11} < p_{12}$. In either case, it is impossible to have $p_{11} - p_{12} > L$. Therefore, whenever firm 1 sends defensive coupons only, firm 2's best response is to choose $p_{21} < p_{22}$ in this hypothetical case. This suggests that firm 2 actually has incentive to send offensive coupons as a best response to firm 1's defensive coupons. ∎