

## Quiz 5 for Linear Algebra

Name :

True/False. Answer only

1. Let  $A$  be an  $n \times n$  matrix with orthonormal columns. If  $\{v_1, \dots, v_n\}$  is an orthonormal basis for  $\mathbb{R}^n$ , so is  $\{Av_1, \dots, Av_n\}$ .

2. An equation  $Ax = b$  has a solution if and only if  $b$  is orthogonal to all solutions of  $A^T x = 0$ .

3. Given the data set  $\{(0,1), (1,1), (2,0), (3,2), (4,2)\}$ , find the best-fitting quadratic polynomial  $y = at^2 + bt + c$ .

4. Find a  $QR$  factorization of  $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ . i.e. express  $A$  as  $QR$ , where  $Q$  is a matrix with orthonormal columns and  $R$  is an upper-triangular invertible matrix.

5. Let  $u$  be a unit vector in  $R^n$  and let  $A = I - 2uu^T$ . Show that any real eigenvalue of  $A$  must be  $\pm 1$ .