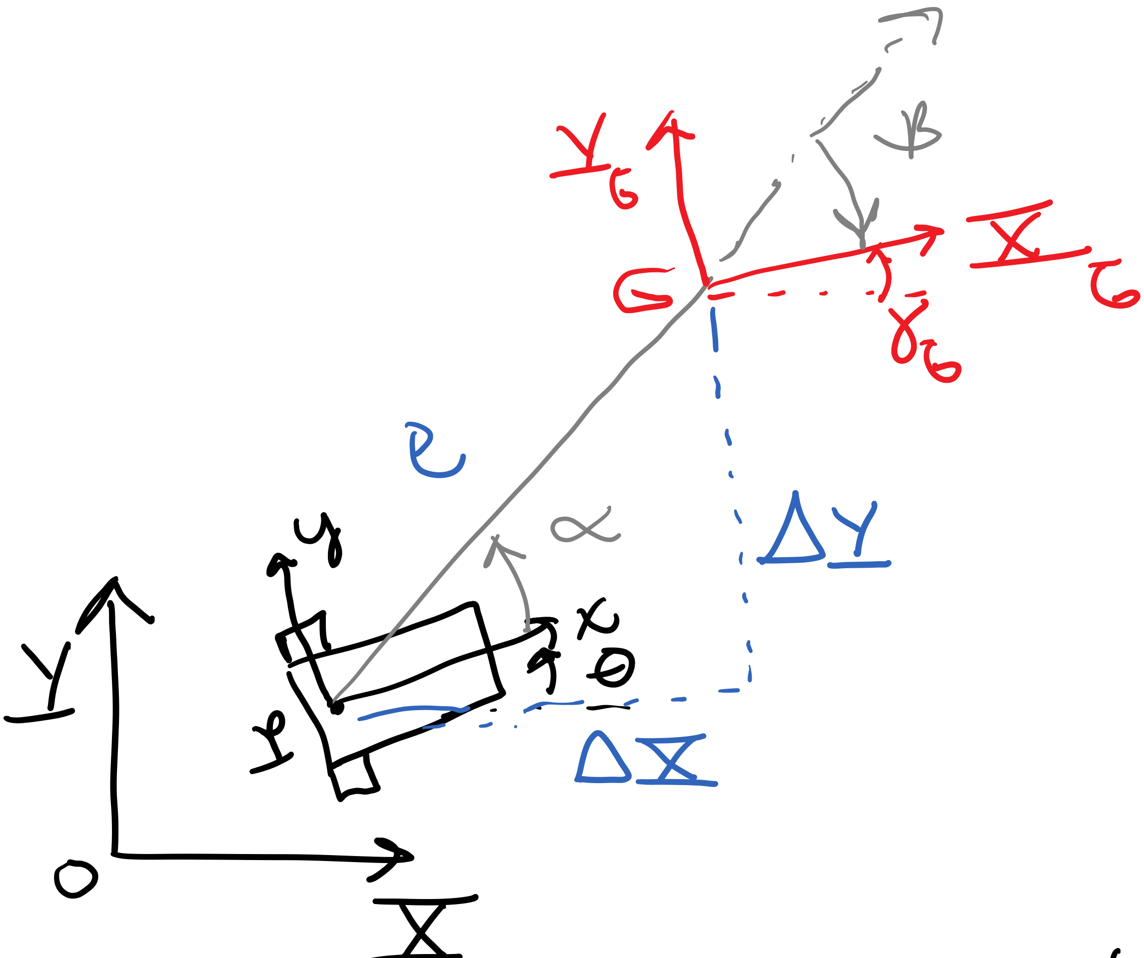
Lecture 25: Mobile Robot Control



Key question: Given a desired goal location and orientation, how do we make the robot go to the goal and reorient itself?

$\Delta X, \Delta Y$: Distance from $\underline{P}$ to G in terms of $\underline{X}$ and $\underline{Y}$ coordinate axes

e : Distance from $\underline{P}$ to G

$$e = \sqrt{\Delta X^2 + \Delta Y^2}$$

θ = angle between xyz coordinate system and $\underline{XYZ}$ coordinate system.

G = Goal Point

$\underline{P}$ = Point on mobile robot and the axle center

α = Angle between $\underline{r}_{\underline{P}G}$ and xyz coordinate system

γ_G = Angle between $\underline{XYZ}_G$ and $\underline{XYZ}$

β = Angle between $\underline{r}_{PG}$ and $\underline{X}\,\underline{Y}\,Z_G$ coordinate system. (opposite sign convention)

Mathematical Expressions

$$\rho = \sqrt{\Delta X^2 + \Delta Y^2}$$

$$\alpha = \operatorname{atan2}(\Delta Y, \Delta X) - \theta$$

$$\beta = -\left[(\theta + \alpha) - \gamma_G\right]$$

Control Strategies

1) "Simple but inefficient"

a) Reorient robot by turning in place so that it points towards G.

Turn in place until $\alpha = 0$

b) Drive to goal G,

drive forward until $\rho = 0$

c) Stop, turn in place until we are in desired orientation.

Turn in place until $\beta = 0$

2) Smooth Trajectory

We try to make e, α, β zero simultaneously. This is done using coordinated control of $\underset{\sim}{V_p}$ and $\underset{\sim}{\omega}$

Focus of this lecture.

Recall

$$\underset{\sim}{V}_p = V\hat{e}_x + 0\hat{e}_y + 0\hat{e}_z$$

$$\underset{\sim}{\omega} = 0\hat{e}_x + 0\hat{e}_y + \omega_z\hat{e}_z$$

$$V = (\dot{\phi}_1 + \dot{\phi}_2)\frac{r}{2}$$

$$\omega_z = (\dot{\phi}_2 - \dot{\phi}_1)\frac{r}{2L}$$

$$\begin{bmatrix} \dot{X}_p \\ \dot{Y}_p \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V \\ \omega_z \end{bmatrix}$$

If we assume $\alpha \in R_1$

$R_1 = \left(-\pi/2, \pi/2\right]$

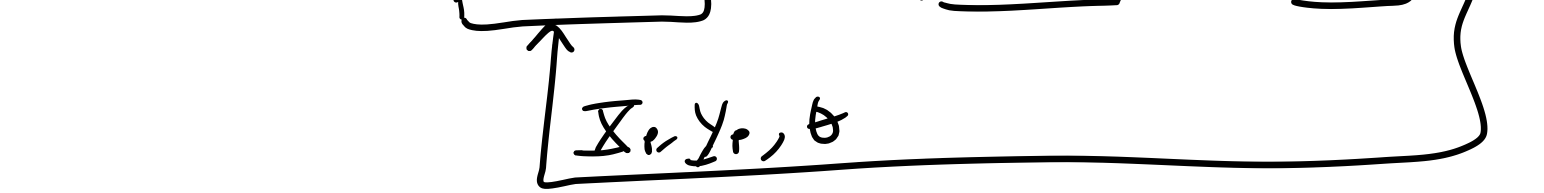
The robot is pointing towards the goal. Then we can rewrite the equations with ρ, α, β

$$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -\cos\alpha & 0 \\ \frac{\sin\alpha}{\rho} & -1 \\ -\frac{\sin\alpha}{\rho} & 0 \end{bmatrix} \begin{bmatrix} V \\ \omega_z \end{bmatrix}$$

Recall

$$\left[\begin{aligned} e &= \sqrt{\Delta X^2 + \Delta Y^2} \\ \alpha &= \operatorname{atan2}(\Delta Y, \Delta X) - \theta \\ \beta &= -(\theta + \alpha - \gamma_G) \end{aligned}\right]$$

In the feedback controls sense, e, α, β are "errors" that need to be driven to zero.



Key question: What is K?

$$\begin{bmatrix} V \\ \omega_z \end{bmatrix} = K \begin{bmatrix} \rho \\ \alpha \\ \beta \end{bmatrix}$$

K = 2x3 Gain Matrix

$$K = \begin{bmatrix} K_\rho & 0 & 0 \\ 0 & K_\alpha & K_\beta \end{bmatrix}$$

$$\boxed{\begin{aligned} V &= K_\rho \rho \\ \omega_z &= K_\alpha \alpha + K_\beta \beta \end{aligned}}$$

This structure assumes $\alpha \in R_1$

How to compute gains?

1) Locally exponentially stable
2) Robot does not reverse direction (V_ρ doesn't change sign) as it approaches goal.

$$\begin{bmatrix} k_e > 0 \\ k_\beta < 0 \\ k_\alpha + \frac{5}{3} k_\beta - \frac{2}{\pi} k_e > 0 \end{bmatrix}$$

$k_e = 3 \quad k_\beta = -1.5$

$k_\alpha = 8$

When choosing gains beware of saturation.
Motors for wheels have speed limits. This means V_p, ω_z cannot exceed certain values.

How do we map back to wheel commands? $(\dot{\phi}_1, \dot{\phi}_2)$

$$V_p = (\dot{\phi}_1 + \dot{\phi}_2) \frac{r}{2}$$

$$\dot{\phi}_1 = V_\rho \frac{2}{r} - \dot{\phi}_2$$

$$\omega_z = (\dot{\phi}_2 - \dot{\phi}_1)\frac{r}{2L}$$

$$\frac{\omega_z 2L}{r} = \dot{\phi}_2 - \underbrace{\left(\frac{V_\rho}{2r} - \dot{\phi}_2\right)}_{\dot{\phi}_1}$$

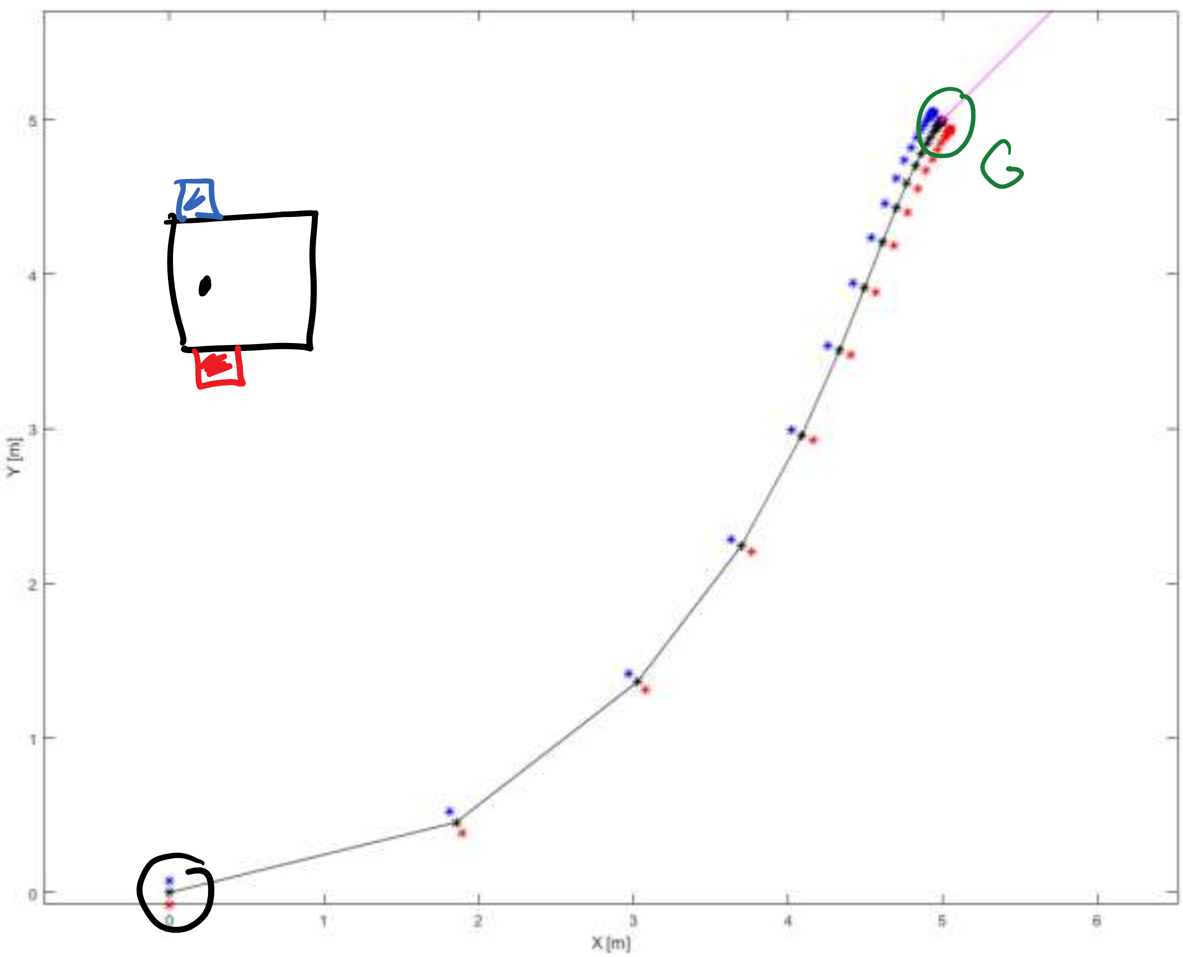
$$\frac{\omega_z 2L}{r} + \frac{V_\rho}{2r} = 2\dot{\phi}_2$$

$$\boxed{\begin{aligned} \dot{\phi}_2 &= \left[\frac{\omega_z 2L}{r} + \frac{V_\rho}{2r}\right] 1/2 \\ \dot{\phi}_1 &= \frac{V_\rho}{2r} - \frac{1}{2}\left[\frac{\omega_z 2L}{r} + \frac{V_\rho}{2r}\right] \end{aligned}}$$

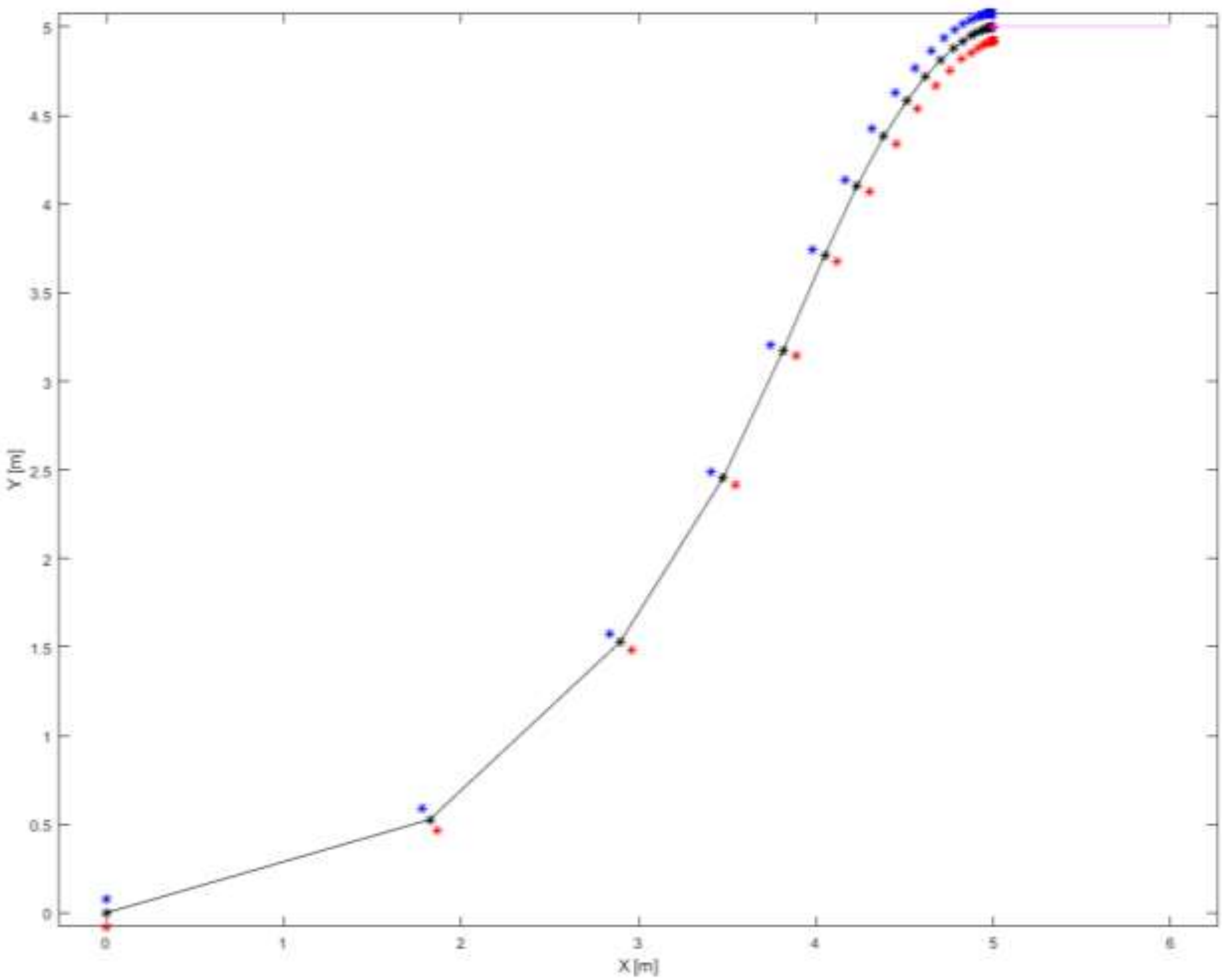
Find Error (ρ, α, β)

Compute ω_z, V_ρ using K matrix

Find $\dot{\phi}_1, \dot{\phi}_2$



Y [m]
X [m]
G



Y [m]
X [m]
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
6