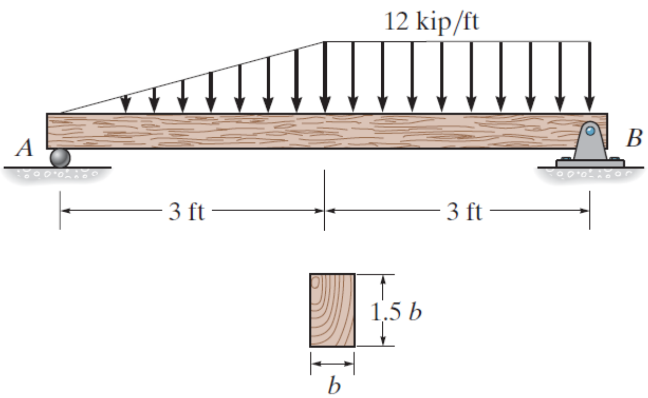
problem 11-8

***11–8.** The simply supported beam is made of timber that has an allowable bending stress of $\sigma_{allow} = 1.20$ ksi and an allowable shear stress of $\tau_{allow} = 100$ psi. Determine its smallest dimensions to the nearest $\frac{1}{8}$ in. if it is rectangular and has a height-to-width ratio of 1.5.



Prob. 11–8

beam

```
u = symunit;
x = sym('x');
E = sym('E');

old_assum = assumptions;
clearassum;
args = {'mode' 'factor'};
wf1 = findpoly(1, 'thru', [0 0], [3*u.ft -12*u.kip/u.ft], args{:});
wf2(x) = -12*u.kip/u.ft;

b = beam; %(kip,ft)
b = b.add('reaction', 'force', 'Ra', 0);
b = b.add('reaction', 'force', 'Rb', 6*u.ft);
b = b.add('distributed', 'force', wf1, [0 3]*u.ft);
b = b.add('distributed', 'force', wf2, [3 6]*u.ft, [false true]);
b.L = 6*u.ft;
```

section properties

```
B = sym('B');
H(B) = 1.5*B;
b.I = B*H^3/12;
```

```
A = B*H;
```

elastic curve

```
[y(x,E,B) dy(x,E,B) m v w r] = b.elastic_curve(x, 'factor'); %#ok
y
```

y(x, E, B) =

$$\begin{cases} -\frac{8\,x\,(2\,x^4 - 210\,x^2\,\mathrm{ft}^2 + 5049\,\mathrm{ft}^4)}{135\,B^4\,\mathrm{E}}\,\frac{\mathrm{kip}}{\mathrm{ft}^2} & \text{if } x \le 3\,\mathrm{ft} \\ \frac{8\,(x - 6\,\mathrm{ft})\,(-10\,x^3 + 70\,x^2\,\mathrm{ft} + 240\,x\,\mathrm{ft}^2 + 27\,\mathrm{ft}^3)}{45\,B^4\,\mathrm{E}}\,\frac{\mathrm{kip}}{\mathrm{ft}} & \text{if } 3\,\mathrm{ft} < x \end{cases}$$

```
dy
```

dy(x, E, B) =

$$\begin{cases} -\frac{8\,(10\,x^4 - 630\,x^2\,\mathrm{ft}^2 + 5049\,\mathrm{ft}^4)}{135\,B^4\,\mathrm{E}}\,\frac{\mathrm{kip}}{\mathrm{ft}^2} & \text{if } x \le 3\,\mathrm{ft} \\ -\frac{8\,(40\,x^3 - 390\,x^2\,\mathrm{ft} + 360\,x\,\mathrm{ft}^2 + 1413\,\mathrm{ft}^3)}{45\,B^4\,\mathrm{E}}\,\frac{\mathrm{kip}}{\mathrm{ft}} & \text{if } 3\,\mathrm{ft} < x \end{cases}$$

```
m
```

m(x) =

$$\begin{cases} \frac{x\,(63\,\mathrm{ft}^2 - 2\,x^2)}{3}\,\frac{\mathrm{kip}}{\mathrm{ft}^2} & \text{if } x \le 3\,\mathrm{ft} \\ -3\,(x - 6\,\mathrm{ft})\,(2\,x - 1\,\mathrm{ft})\,\frac{\mathrm{kip}}{\mathrm{ft}} & \text{if } 3\,\mathrm{ft} < x \end{cases}$$

```
v
```

v(x) =

$$\begin{cases} (21\,\mathrm{ft}^2 - 2\,x^2)\,\frac{\mathrm{kip}}{\mathrm{ft}^2} & \text{if } x \le 3\,\mathrm{ft} \\ -3\,(4\,x - 13\,\mathrm{ft})\,\frac{\mathrm{kip}}{\mathrm{ft}} & \text{if } 3\,\mathrm{ft} < x \end{cases}$$

```
w
```

w(x) =

$$\begin{cases} -4\,x\,\frac{\mathrm{kip}}{\mathrm{ft}^2} & \text{if } x \le 3\,\mathrm{ft} \\ -12\,\frac{\mathrm{kip}}{\mathrm{ft}} & \text{if } 3\,\mathrm{ft} < x \end{cases}$$

reactions

```
Ra = r.Ra %#ok
```

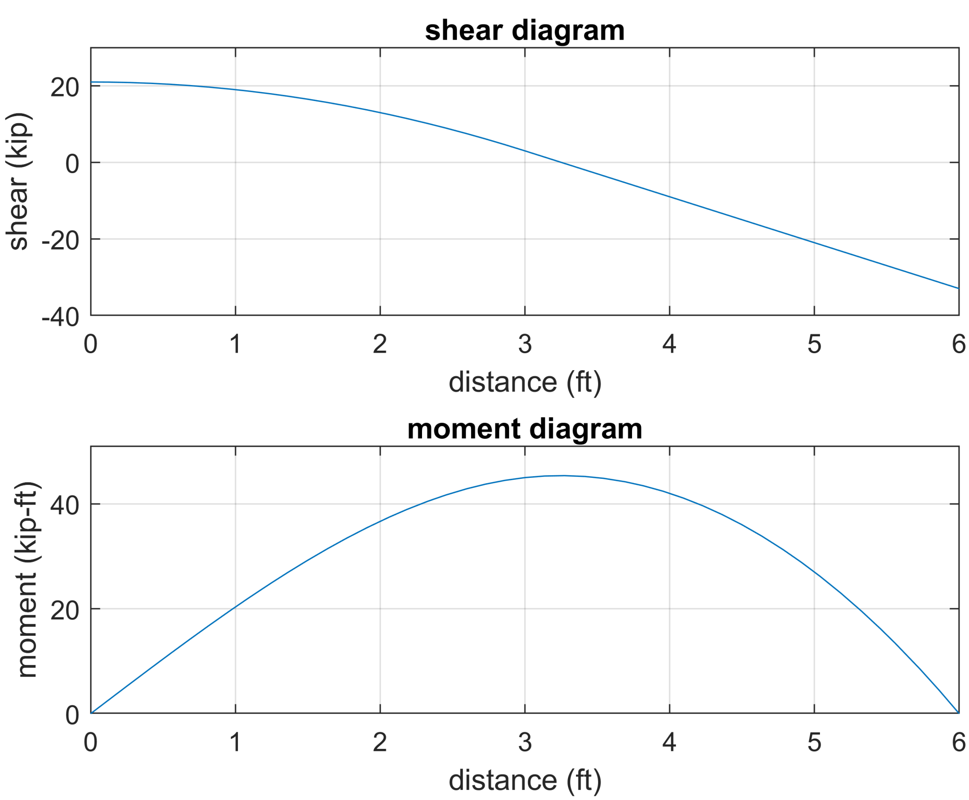
Ra = 21 kip

```
Rb = r.Rb %#ok
```

Rb = 33 kip

shear and moment diagram

```
beam.shear_moment(m, v, [0 6], {'kip' 'ft'});
subplot(2,1,1);
axis([0 6 -40 30]);
subplot(2,1,2);
axis([0 6 0 51]);
```



maximum loads

```
assume(0 < x & x < b.L & in(x, 'real'));
xmax = solve(v == 0, x);
M_val = m(xmax);
```

```
M_max = vpa(M_val, 4) %#ok
```

M_max = $45.38\ \mathrm{ft\ kip}$

```
V_max = v(b.L)
```

V_max = $-33\ \mathrm{kip}$

```
M_max = M_val;
```

maximum stresses

```
C = H/2;
sigma_max = rewrite(M_max, u.kip*u.in)*C/b.I
```

sigma_max(B) =

$$\frac{1452}{B^3}\ \mathrm{in\ kip}$$

```
tau_max = 3*abs(V_max)/(2*A)
```

tau_max(B) =

$$\frac{33}{B^2}\ \mathrm{kip}$$

minimum beam dimension

```
sigma_allow = 1.20*u.ksi;
tau_allow = 100*u.psi;

assume(B > 0 & in(B, 'real'));
clear B_min;

B_min.bend = solve(sigma_max == rewrite(sigma_allow, u.kip/u.in^2));
B_min.bend = simplify(B_min.bend);
B_min_bend = vpa(B_min.bend, 4) %#ok
```

B_min_bend = $10.66\ \mathrm{in}$

```
B_min.shear = solve(tau_max == rewrite(tau_allow, u.kip/u.in^2));
B_min.shear = simplify(B_min.shear);
B_min_shear = vpa(B_min.shear, 4) %#ok
```

B_min_shear = $18.17\ \mathrm{in}$

```
B_min_vals = [B_min.bend B_min.shear];
loc = sigma_max(B_min_vals) <= sigma_allow & ...
```

```
      tau_max(B_min_vals) <= tau_allow;
B_min.limit = B_min_vals(isAlways(loc));
B_min_limit = vpa(B_min.limit, 4) %#ok
```

B_min_limit = $18.17\ \mathrm{in}$

clean up

```
setassum(old_assum, 'clear');
clear args old_assum Ra Rb M_val;
clear B_min_bend B_min_shear B_min_vals loc B_min_limit;
```