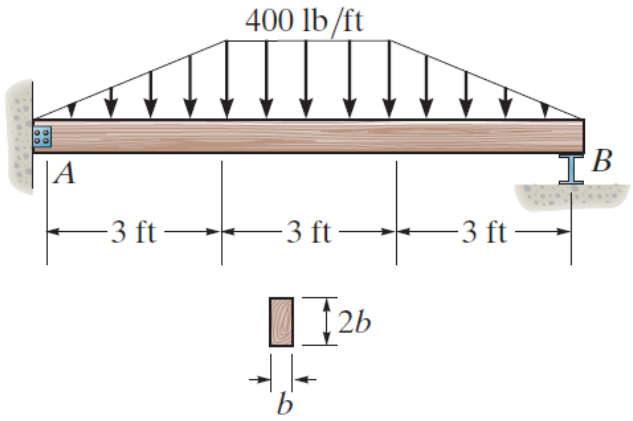
problem 6-106

6–106. The wood beam has a rectangular cross section in the proportion shown. If $b = 7.5$ in., determine the absolute maximum bending stress in the beam.



Probs. 6–105/106

beam

```
u = symunit;
x = sym('x');
E = sym('E');

old_assum = assumptions;
clearassum;
args = {'mode' 'factor'};
wf1 = findpoly(1, 'thru', [0 0], [3*u.ft -400*u.lbf/u.ft], args{:});
wf2(x) = -400*u.lbf/u.ft;
wf3 = findpoly(1, 'thru', [6*u.ft -400*u.lbf/u.ft], [9*u.ft 0], args{:});

b = beam; %(lbf,ft)
b = b.add('reaction', 'force', 'Ra', 0);
b = b.add('reaction', 'force', 'Rb', 9*u.ft);
b = b.add('distributed', 'force', wf1, [0 3]*u.ft);
b = b.add('distributed', 'force', wf2, [3 6]*u.ft, [false true]);
b = b.add('distributed', 'force', wf3, [6 9]*u.ft, [false true]);
b.L = 9*u.ft;
```

section properties

```
B = 7.5*u.in;
H = 2*B;
```

```
b.I = rewrite(B*H^3/12, u.ft);
```

elastic curve

```
[y(x,E) dy(x,E) m v w r] = b.elastic_curve(x, 'factor'); %#ok
y
```

y(x, E) =

$$\begin{cases} -\dfrac{4096\,x\,\left(x^4-180\,x^2\,\mathrm{ft}^2+8910\,\mathrm{ft}^4\right)}{375\,\mathrm{E}}\ \dfrac{\mathrm{lbf}}{\mathrm{ft}^6} & \text{if } x \le 3\,\mathrm{ft} \\ -\dfrac{4096\,\left(5\,x^4-90\,x^3\,\mathrm{ft}+90\,x^2\,\mathrm{ft}^2+2835\,x\,\mathrm{ft}^3+81\,\mathrm{ft}^4\right)}{125\,\mathrm{E}}\ \dfrac{\mathrm{lbf}}{\mathrm{ft}^5} & \text{if } x \in (3\,\mathrm{ft}, 6\,\mathrm{ft}] \\ \dfrac{4096\,\left(x-9\,\mathrm{ft}\right)\,\left(x^4-36\,x^3\,\mathrm{ft}+306\,x^2\,\mathrm{ft}^2+324\,x\,\mathrm{ft}^3+891\,\mathrm{ft}^4\right)}{375\,\mathrm{E}}\ \dfrac{\mathrm{lbf}}{\mathrm{ft}^6} & \text{if } 6\,\mathrm{ft} < x \end{cases}$$

```
dy
```

dy(x, E) =

$$\begin{cases} -\dfrac{4096\,\left(x^4-108\,x^2\,\mathrm{ft}^2+1782\,\mathrm{ft}^4\right)}{75\,\mathrm{E}}\ \dfrac{\mathrm{lbf}}{\mathrm{ft}^6} & \text{if } x \le 3\,\mathrm{ft} \\ \dfrac{4096\,\left(2\,x-9\,\mathrm{ft}\right)\,\left(-2\,x^2+18\,x\,\mathrm{ft}+63\,\mathrm{ft}^2\right)}{25\,\mathrm{E}}\ \dfrac{\mathrm{lbf}}{\mathrm{ft}^5} & \text{if } x \in (3\,\mathrm{ft}, 6\,\mathrm{ft}] \\ -\dfrac{4096\,\left(-x^4+36\,x^3\,\mathrm{ft}-378\,x^2\,\mathrm{ft}^2+972\,x\,\mathrm{ft}^3+405\,\mathrm{ft}^4\right)}{75\,\mathrm{E}}\ \dfrac{\mathrm{lbf}}{\mathrm{ft}^6} & \text{if } 6\,\mathrm{ft} < x \end{cases}$$

```
m
```

m(x) =

$$\begin{cases} \dfrac{200\,x\,\left(54\,\mathrm{ft}^2-x^2\right)}{9}\ \dfrac{\mathrm{lbf}}{\mathrm{ft}^2} & \text{if } x \le 3\,\mathrm{ft} \\ -200\,\left(x^2-9\,x\,\mathrm{ft}+3\,\mathrm{ft}^2\right)\ \dfrac{\mathrm{lbf}}{\mathrm{ft}} & \text{if } x \in (3\,\mathrm{ft}, 6\,\mathrm{ft}] \\ \dfrac{200\,\left(x-9\,\mathrm{ft}\right)\,\left(x^2-18\,x\,\mathrm{ft}+27\,\mathrm{ft}^2\right)}{9}\ \dfrac{\mathrm{lbf}}{\mathrm{ft}^2} & \text{if } 6\,\mathrm{ft} < x \end{cases}$$

```
v
```

v(x) =

$$\begin{cases} \dfrac{200\,\left(18\,\mathrm{ft}^2-x^2\right)}{3}\ \dfrac{\mathrm{lbf}}{\mathrm{ft}^2} & \text{if } x \le 3\,\mathrm{ft} \\ -200\,\left(2\,x-9\,\mathrm{ft}\right)\ \dfrac{\mathrm{lbf}}{\mathrm{ft}} & \text{if } x \in (3\,\mathrm{ft}, 6\,\mathrm{ft}] \\ \dfrac{200\,\left(x^2-18\,x\,\mathrm{ft}+63\,\mathrm{ft}^2\right)}{3}\ \dfrac{\mathrm{lbf}}{\mathrm{ft}^2} & \text{if } 6\,\mathrm{ft} < x \end{cases}$$

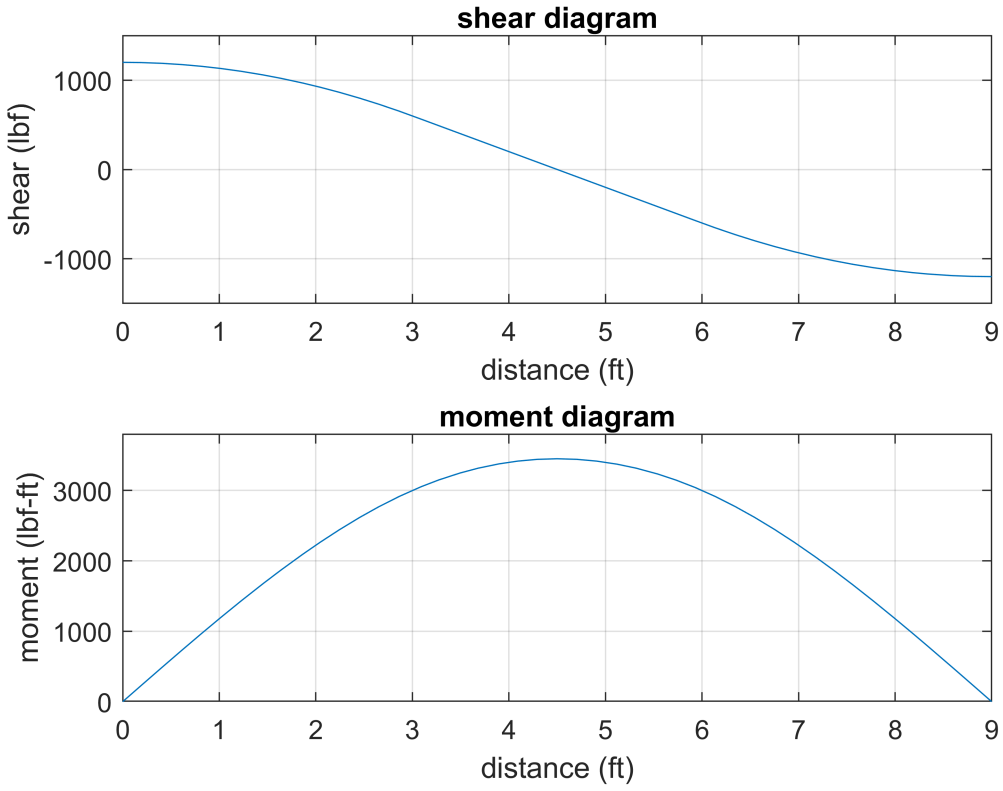
```
w
```

```
w(x) =
```

$$w(x) = \begin{cases} -\frac{400\,x}{3}\ \frac{\text{lbf}}{\text{ft}^2} & \text{if } x \le 3\ \text{ft} \\ -400\ \frac{\text{lbf}}{\text{ft}} & \text{if } x \in (3\ \text{ft}, 6\ \text{ft}] \\ \frac{400\ (x - 9\ \text{ft})}{3}\ \frac{\text{lbf}}{\text{ft}^2} & \text{if } 6\ \text{ft} < x \end{cases}$$

shear and bending moment diagrams

```
beam.shear_moment(m, v, [0 9], {'lbf' 'ft'});
subplot(2,1,1);
axis([0 9 -1500 1500]);
subplot(2,1,2);
axis([0 9 0 3800]);
```



maximum bending moment

```
assume(0 < x & x < b.L & in(x, 'real'));
xmax = solve(v == 0, x);
```

```
M_max = m(xmax)
```

M_max = 3450 ft lbf

maximum bending stress

```
C = H/2;
b.I = rewrite(b.I, u.in);
sigma_val = rewrite(M_max*C/b.I, u.psi);
sigma_max = vpa(sigma_val) %#ok
```

sigma_max = 147.2 psi

```
sigma_max = sigma_val;
```

clean up

```
setassum(old_assum, 'clear');
clear args old_assum sigma_val;
```