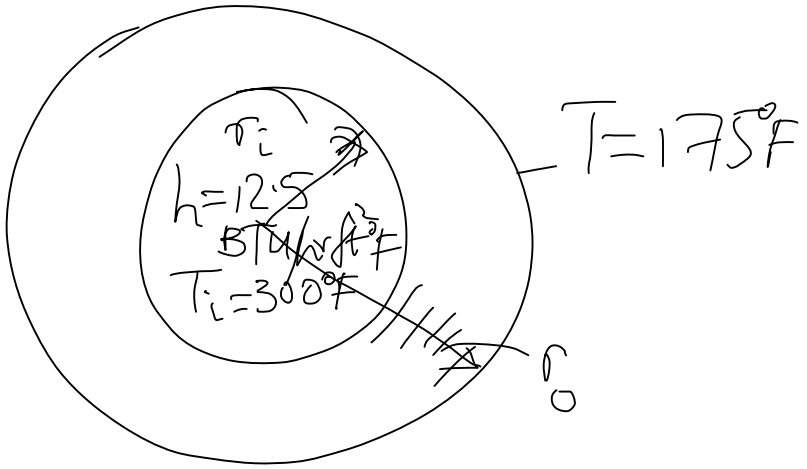
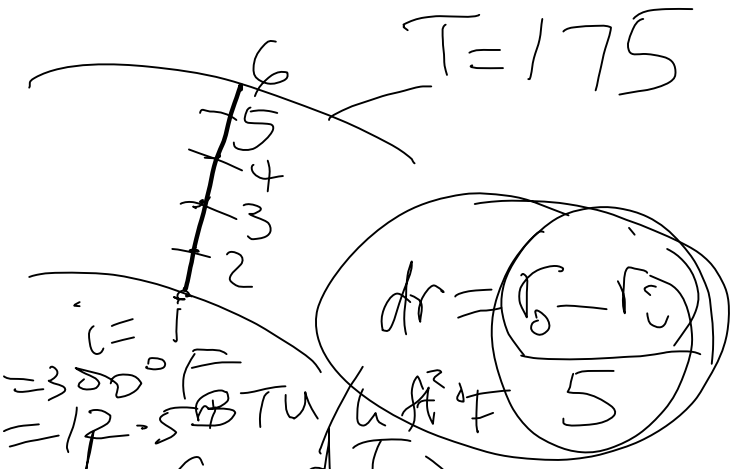
2.68E



Choose $N=6$

$\Rightarrow$ 6 nodes

5 subdivisions



Gov. Eq.

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right)=0$$

For node i

$i-1 \quad i \quad i+1$

C.D. approx:

$$\frac{r_{i+\frac{1}{2}}\left(\frac{T_{i+1}-T_i}{dr}\right)-r_{i-\frac{1}{2}}\left(\frac{T_i-T_{i-1}}{dr}\right)}{dr}=0$$

$$\frac{1}{dr^2}\left[r_{i-\frac{1}{2}}T_{i-1}-\left(r_{i-\frac{1}{2}}+r_{i+\frac{1}{2}}\right)T_i+r_{i+\frac{1}{2}}T_{i+1}\right]=0$$

For the 6 nodes:

$i=$

1 $\frac{1}{dr^2}\left[r_{\frac{1}{2}} T_0 - \left(r_{\frac{1}{2}}+r_{1\frac{1}{2}}\right) T_1 + r_{1\frac{1}{2}} T_2\right] = 0$

2 $\frac{1}{dr^2}\left[r_{1\frac{1}{2}} T_1 - \left(r_{1\frac{1}{2}}+r_{2\frac{1}{2}}\right) T_2 + r_{2\frac{1}{2}} T_3\right] = 0$

3 $\frac{1}{dr^2}\left[r_{2\frac{1}{2}} T_2 - \left(r_{2\frac{1}{2}}+r_{3\frac{1}{2}}\right) T_3 + r_{3\frac{1}{2}} T_4\right] = 0$

4 $\frac{1}{dr^2}\left[r_{3\frac{1}{2}} T_3 - \left(r_{3\frac{1}{2}}+r_{4\frac{1}{2}}\right) T_4 + r_{4\frac{1}{2}} T_5\right] = 0$

5 $\frac{1}{dr^2}\left[r_{4\frac{1}{2}} T_4 - \left(r_{4\frac{1}{2}}+r_{5\frac{1}{2}}\right) T_5 + r_{5\frac{1}{2}} T_6\right] = 0$

6 $\frac{1}{dr^2}\left[r_{5\frac{1}{2}} T_5 - \left(r_{5\frac{1}{2}}+r_{6\frac{1}{2}}\right) T_6 + r_{6\frac{1}{2}} T_7\right] = 0$

---

8 values $T_0 \rightarrow T_7$

T_0 & T_7 must be eliminated using boundary conditions

Also $r_{\frac{1}{2}}$ and $r_{6\frac{1}{2}} \Rightarrow$

Set $r_{\frac{1}{2}} = r_1$

$r_{6\frac{1}{2}} = r_6$

b.c. (1) @ $r = r_1$

$$-k\frac{dT}{dr} = h(T_\infty - T)$$

Central difference

$$-k\frac{(T_2 - T_0)}{2dr} = h(T_\infty - T_1)$$

$$\therefore T_0 = T_2 + \frac{2dr\,h}{k}(T_\infty - T_1)$$

Substitute T_0 into Eq (1)

$$\frac{1}{dr^2}\left[r_1\left(T_2 - \frac{2dr\,h}{k}T_1 + \frac{2dr\,h}{k}T_\infty\right) - (r_1 + r_{1\frac{1}{2}})T_1 + r_{1\frac{1}{2}}T_2\right] = 0$$

$$\frac{1}{dr^2}\left[-(r_1 + r_{1\frac{1}{2}} + r_1\frac{2dr\,h}{k})T_1 + (r_1 + r_{1\frac{1}{2}})T_2\right] = -\frac{r_1}{dr^2}\frac{2dr\,h}{k}T_\infty$$

$r_{1\frac{1}{2}} = r_1 + \frac{dr}{2}$

@ $r = r_6$

$r_{2\frac{1}{2}} = r_1 + 1\frac{1}{2}dr$ $T_6 = \cancel{\ } 175\,^\circ F$

$r_2 = r_1 + dr$, $r_5 = r_1 + 4dr$.

$$\frac{1}{dr^2}\begin{bmatrix} -\left(2r_1+\frac{dr}{2}+r_1\frac{2drh}{k}\right) & \left(2r_1+\frac{dr}{2}\right) & 0 & 0 & 0 & 0 \\ \left(r_1+\frac{dr}{2}\right) & -\left(2r_1+2dr\right) & \left(r_1+1\tfrac{1}{2}dr\right) & 0 & 0 & 0 \\ 0 & \left(r_1+1\tfrac{1}{2}dr\right) & -\left(2r_1+4dr\right) & \left(r_1+2\tfrac{1}{2}dr\right) & 0 & 0 \\ 0 & 0 & \left(r_1+2\tfrac{1}{2}dr\right) & -\left(2r_1+6dr\right) & \left(r_1+3\tfrac{1}{2}dr\right) & 0 \\ 0 & 0 & 0 & \left(r_1+3\tfrac{1}{2}dr\right) & -\left(2r_1+8dr\right) & \left(r_1+4\tfrac{1}{2}dr\right) \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{matrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{matrix} = \begin{bmatrix} -\frac{r_1 2drh}{kdr^2}T_\infty \\ 0 \\ 0 \\ 0 \\ 0 \\ 175 \end{bmatrix}$$

$r_1 = 2'' = \frac{2}{12}$ ft

$h = 12.5$ BTU/h ft^2 °F

$K = 7.2$ BTU/h ft °F

$T_\infty = 300$°F

$$dr = \frac{r_o - r_i}{5} = \frac{(2.4-2)/12}{5} = \frac{.08}{12} \text{ ft}$$

$$\begin{bmatrix} -7661.8 & 7575.0 & 0 & 0 & 0 & 0 \\ 3825 & -7800 & 3975 & 0 & 0 & 0 \\ 0 & 3975 & -8100 & 4125 & 0 & 0 \\ 0 & 0 & 4125 & -8400 & 4275 & 0 \\ 0 & 0 & 0 & 4275 & -8700 & 4425 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{matrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{matrix} = \begin{bmatrix} -26,042 \\ 0 \\ 0 \\ 0 \\ 0 \\ 175 \end{bmatrix}$$

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 181.32 \\ 179.96 \\ 178.65 \\ 177.39 \\ 176.18 \\ 175 \end{bmatrix}$$

Cf Analytical Soln

$$\begin{bmatrix} 181.26 \\ 179.92 \\ 178.62 \\ 177.37 \\ 176.16 \\ 175 \end{bmatrix}$$