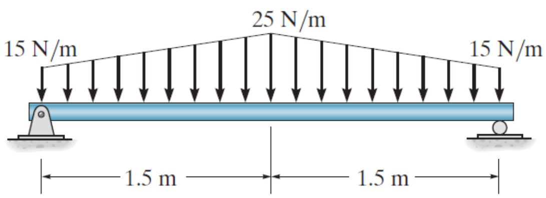
problem 11-19

11–19. The pipe has an outer diameter of 15 mm. Determine the smallest inner diameter so that it will safely support the loading shown. The allowable bending stress is $\sigma_{allow} = 167$ MPa and the allowable shear stress is $\tau_{allow} = 97$ MPa.



beam

```
u = symunit;
x = sym('x');
E = sym('E');

old_assum = assumptions;
clearassum;
args = {'mode' 'factor'};
wf1 = findpoly(1, 'thru', [0 -15*u.N/u.m], ...
                          [1.5*u.m -25*u.N/u.m], args{:});
wf2 = findpoly(1, 'thru', [1.5*u.m -25*u.N/u.m], ...
                          [3*u.m -15*u.N/u.m], args{:});

b = beam;
b = b.add('reaction', 'force', 'Ra', 0);
b = b.add('reaction', 'force', 'Rb', 3*u.m);
b = b.add('distributed', 'force', wf1, [0 1.5]*u.m);
b = b.add('distributed', 'force', wf2, [1.5 3]*u.m, [false true]);
b.L = 3*u.m;
```

section properties

```
Do = 15*u.mm;
Di = sym('Di');
Ro = Do/2;
Ri(Di) = Di/2;

yc = 4/(3*sympi)*[Ro; Ri; -Ri; -Ro];
Ac = sympi/2*[Ro^2; -Ri^2; -Ri^2; Ro^2];
Ic = (sympi/8-8/(9*sympi))*[Ro^4; -Ri^4; -Ri^4; Ro^4];
```

```
[yn Qn In] = beam.neutral_axis(yc, Ac, Ic); %#ok
b.I = rewrite(sum(In), u.m);
```

elastic curve

```
[y(x,E,Di) dy(x,E,Di) m v w r] = b.elastic_curve(x, 'factor'); %#ok
y
```

y(x, E, Di) =

$$\begin{cases} \dfrac{3200000000\, x\, (16\, x^4 + 180\, x^3\, \mathrm{m} - 1440\, x^2\, \mathrm{m}^2 + 6885\, \mathrm{m}^4)}{9\, E\, \pi\, (40000\, \mathrm{Di}^2 + 9\, \mathrm{m}^2)\, (200\, \mathrm{Di} - 3\, \mathrm{m})\, (200\, \mathrm{Di} + 3\, \mathrm{m})}\, \dfrac{\mathrm{N}}{\mathrm{m}^2} & \text{if } x \le \frac{3}{2}\, \mathrm{m} \\ -\dfrac{3200000000\, (x - 3\, \mathrm{m})\, (16\, x^4 - 372\, x^3\, \mathrm{m} + 1044\, x^2\, \mathrm{m}^2 + 2052\, x\, \mathrm{m}^3 + 81\, \mathrm{m}^4)}{9\, E\, \pi\, (40000\, \mathrm{Di}^2 + 9\, \mathrm{m}^2)\, (200\, \mathrm{Di} - 3\, \mathrm{m})\, (200\, \mathrm{Di} + 3\, \mathrm{m})}\, \dfrac{\mathrm{N}}{\mathrm{m}^2} & \text{if } \frac{3}{2}\, \mathrm{m} < x \end{cases}$$

```
dy
```

dy(x, E, Di) =

$$\begin{cases} -\dfrac{16000000000\, (2\, x - 3\, \mathrm{m})\, (-8\, x^3 - 84\, x^2\, \mathrm{m} + 306\, x\, \mathrm{m}^2 + 459\, \mathrm{m}^3)}{9\, E\, \pi\, (40000\, \mathrm{Di}^2 + 9\, \mathrm{m}^2)\, (200\, \mathrm{Di} - 3\, \mathrm{m})\, (200\, \mathrm{Di} + 3\, \mathrm{m})}\, \dfrac{\mathrm{N}}{\mathrm{m}^2} & \text{if } x \le \frac{3}{2}\, \mathrm{m} \\ -\dfrac{16000000000\, (2\, x - 3\, \mathrm{m})\, (8\, x^3 - 156\, x^2\, \mathrm{m} + 414\, x\, \mathrm{m}^2 + 405\, \mathrm{m}^3)}{9\, E\, \pi\, (40000\, \mathrm{Di}^2 + 9\, \mathrm{m}^2)\, (200\, \mathrm{Di} - 3\, \mathrm{m})\, (200\, \mathrm{Di} + 3\, \mathrm{m})}\, \dfrac{\mathrm{N}}{\mathrm{m}^2} & \text{if } \frac{3}{2}\, \mathrm{m} < x \end{cases}$$

```
m
```

m(x) =

$$\begin{cases} -\dfrac{5\, x\, (4\, x^2 + 27\, x\, \mathrm{m} - 108\, \mathrm{m}^2)}{18}\, \dfrac{\mathrm{N}}{\mathrm{m}^2} & \text{if } x \le \frac{3}{2}\, \mathrm{m} \\ \dfrac{5\, (x - 3\, \mathrm{m})\, (4\, x^2 - 51\, x\, \mathrm{m} + 9\, \mathrm{m}^2)}{18}\, \dfrac{\mathrm{N}}{\mathrm{m}^2} & \text{if } \frac{3}{2}\, \mathrm{m} < x \end{cases}$$

```
v
```

v(x) =

$$\begin{cases} -\dfrac{5\, (x + 6\, \mathrm{m})\, (2\, x - 3\, \mathrm{m})}{3}\, \dfrac{\mathrm{N}}{\mathrm{m}^2} & \text{if } x \le \frac{3}{2}\, \mathrm{m} \\ \dfrac{5\, (x - 9\, \mathrm{m})\, (2\, x - 3\, \mathrm{m})}{3}\, \dfrac{\mathrm{N}}{\mathrm{m}^2} & \text{if } \frac{3}{2}\, \mathrm{m} < x \end{cases}$$

```
w
```

w(x) =

$$\begin{cases} -\frac{5\,(4\,x+9\,\mathrm{m})}{3}\ \frac{\mathrm{N}}{\mathrm{m}^2} & \text{if } x \le \frac{3}{2}\,\mathrm{m} \\ \frac{5\,(4\,x-21\,\mathrm{m})}{3}\ \frac{\mathrm{N}}{\mathrm{m}^2} & \text{if } \frac{3}{2}\,\mathrm{m} < x \end{cases}$$

reactions

```
Ra = r.Ra %#ok
```

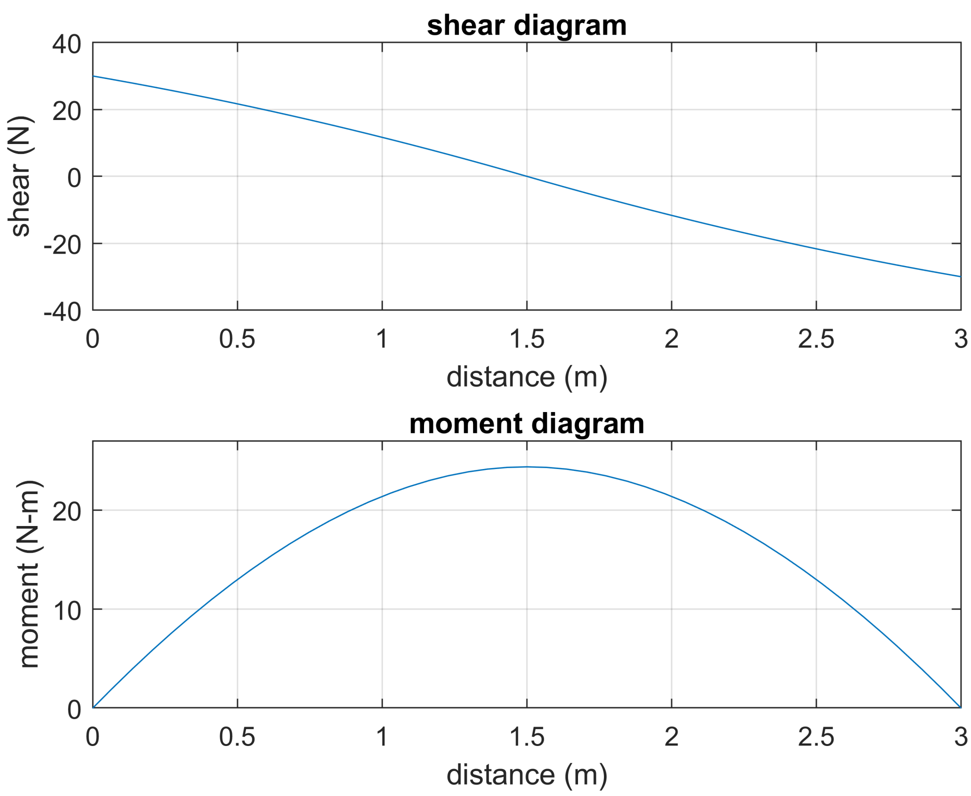
Ra = $30\ \mathrm{N}$

```
Rb = r.Rb %#ok
```

Rb = $30\ \mathrm{N}$

shear and moment diagram

```
beam.shear_moment(m, v, [0 3], {'N' 'm'});
subplot(2,1,1);
axis([0 3 -40 40]);
subplot(2,1,2);
axis([0 3 0 27]);
```



maximum loads

```
M_val = m(1.5*u.m);
M_max = vpa(M_val, 4) %#ok
```

M_max = $24.38 \mathrm{~N~m}$

```
V_max = v(0)
```

V_max = $30 \mathrm{~N}$

```
M_max = M_val;
```

maximum stresses

```
C = Ro;
b.I = simplify(rewrite(b.I, u.mm));
sigma_max = simplify(rewrite(M_max, u.N*u.mm)*C/b.I)
```

sigma_max(Di) =

$$\frac{11700000}{\pi\ (50625\ \mathrm{mm}^4 - \mathrm{Di}^4)}\ \mathrm{N\ mm}^2$$

```
Q_max(Di) = simplify(sum(index(Qn, 1:2)));
t_min(Di) = Do-Di;
tau_max = simplify(V_max*Q_max/(b.I*t_min))
```

tau_max(Di) =

$$\frac{160\ (\mathrm{Di}^2 + 15\ \mathrm{Di}\ \mathrm{mm} + 225\ \mathrm{mm}^2)}{\pi\ (50625\ \mathrm{mm}^4 - \mathrm{Di}^4)}\ \mathrm{N}$$

minimum inner diameter

```
sigma_allow = 167*u.MPa;
tau_allow = 97*u.MPa;

assume(0 < Di & Di < Do & in(Di, 'real'));
clear Di_min;

Di_min.bend = solve(sigma_max == rewrite(sigma_allow, u.N/u.mm^2));
Di_min.bend = simplify(Di_min.bend);
Di_min_bend = vpa(Di_min.bend, 4) %#ok
```

Di_min_bend = $12.97 \mathrm{~mm}$

```
Di_min.shear = solve(tau_max == rewrite(tau_allow, u.N/u.mm^2));
```

```
Di_min.shear = simplify(separateUnits(Di_min.shear))*u.mm;
Di_min_shear = vpa(Di_min.shear, 4) %#ok
```

Di_min_shear = 14.97 mm

```
Di_min_vals = [Di_min.bend Di_min.shear];
loc = sigma_max(Di_min_vals) <= sigma_allow & ...
      tau_max(Di_min_vals) <= tau_allow;
Di_min.limit = Di_min_vals(isAlways(loc));
Di_min_limit = vpa(Di_min.limit, 4) %#ok
```

Di_min_limit = 12.97 mm

clean up

```
setassum(old_assum, 'clear');
clear args old_assum Ra Rb M_val;
clear Di_min_bend Di_min_shear Di_min_vals loc Di_min_limit;
```