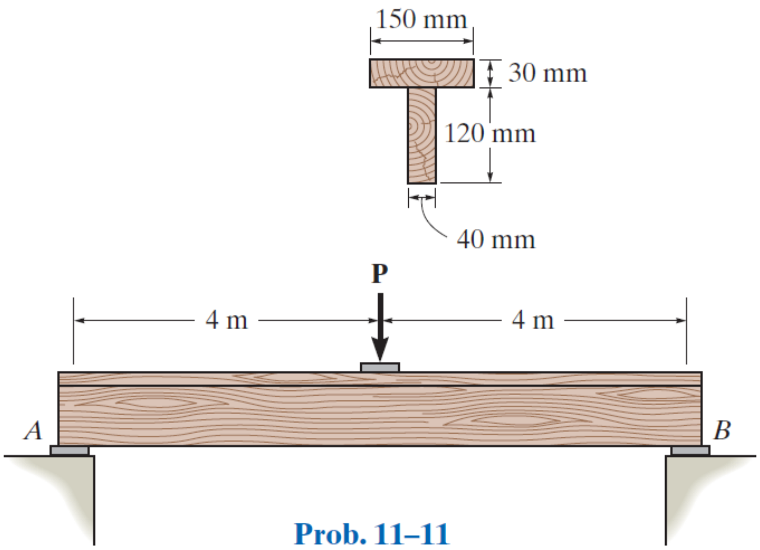
problem 11-11

11–11. The timber beam is to be loaded as shown. If the ends support only vertical forces, determine the greatest magnitude of **P** that can be applied. $\sigma_{\text{allow}} = 25$ MPa, $\tau_{\text{allow}} = 700$ kPa.



Prob. 11–11

beam

```
u = symunit;
x = sym('x');
E = sym('E');
P = sym('P');

old_assum = assumptions;
clearassum;

b = beam;
b = b.add('reaction', 'force', 'Ra', 0);
b = b.add('reaction', 'force', 'Rb', 8*u.m);
b = b.add('applied', 'force', -P, 4*u.m);
b.L = 8*u.m;
```

section properties

```
yc = [120/2; 120+30/2]*u.mm;
Ac = [40*120; 150*30]*u.mm^2;
Ic = [40*120^3; 150*30^3]*u.mm^4/12;

[yn Qn In] = beam.neutral_axis(yc, Ac, Ic); %#ok
b.I = rewrite(sum(In), u.m);
```

elastic curve

```
[y(x,E,P) dy(x,E,P) m v w r] = b.elastic_curve(x, 'factor'); %#ok
y
```

y(x, E, P) =

$$\begin{cases} -\frac{3100000000\,P\,x\,(48\,\mathrm{m}^2 - x^2)}{712827\,\mathrm{E}}\,\frac{1}{\mathrm{m}^4} & \text{if } x \le 4\,\mathrm{m} \\ -\frac{3100000000\,P\,(x - 8\,\mathrm{m})\,(x^2 - 16\,x\,\mathrm{m} + 16\,\mathrm{m}^2)}{712827\,\mathrm{E}}\,\frac{1}{\mathrm{m}^4} & \text{if } 4\,\mathrm{m} < x \end{cases}$$

```
dy
```

dy(x, E, P) =

$$\begin{cases} \frac{3100000000\,P\,(x - 4\,\mathrm{m})\,(x + 4\,\mathrm{m})}{237609\,\mathrm{E}}\,\frac{1}{\mathrm{m}^4} & \text{if } x \le 4\,\mathrm{m} \\ -\frac{3100000000\,P\,(x - 4\,\mathrm{m})\,(x - 12\,\mathrm{m})}{237609\,\mathrm{E}}\,\frac{1}{\mathrm{m}^4} & \text{if } 4\,\mathrm{m} < x \end{cases}$$

```
m
```

m(x) =

$$\begin{cases} \frac{P\,x}{2} & \text{if } x \le 4\,\mathrm{m} \\ -\frac{P\,(x - 8\,\mathrm{m})}{2} & \text{if } 4\,\mathrm{m} < x \end{cases}$$

```
v
```

v(x) =

$$\begin{cases} \frac{P}{2} & \text{if } x \le 4\,\mathrm{m} \\ -\frac{P}{2} & \text{if } 4\,\mathrm{m} < x \end{cases}$$

```
w
```

w(x) = 0

reactions

```
Ra = r.Ra %#ok
```

Ra =

$$\frac{P}{2}$$

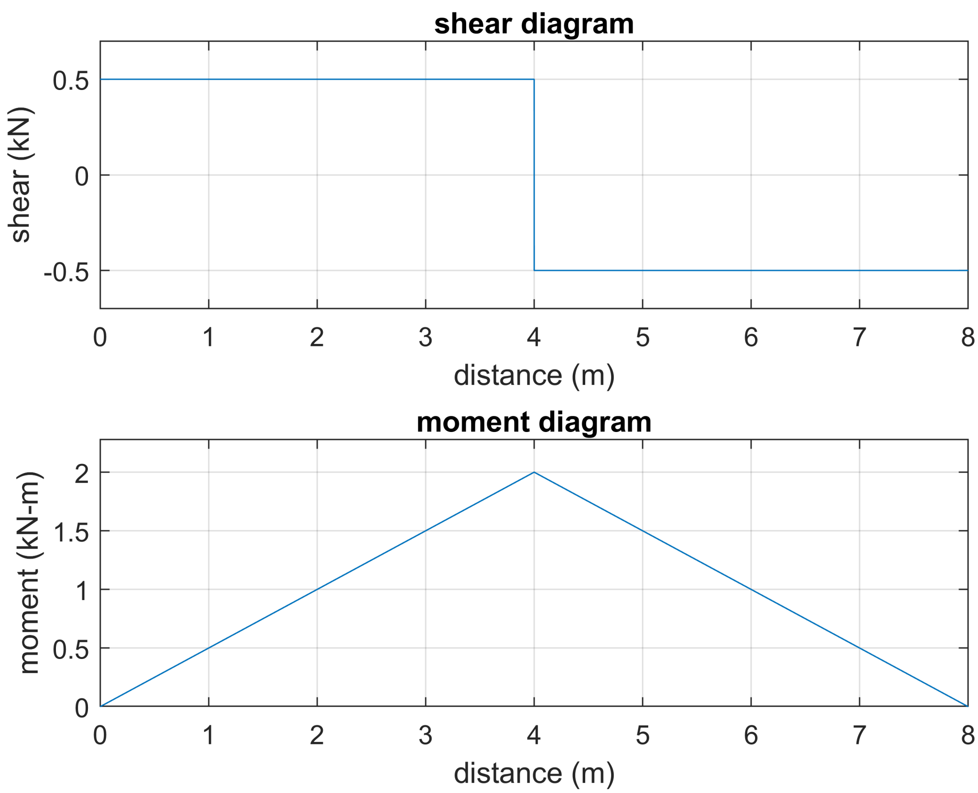
```
Rb = r.Rb %#ok
```

Rb =

$$\frac{P}{2}$$

shear and moment diagram

```
beam.shear_moment(m, v, [0 8], {'kN' 'm'}, P, 1);
subplot(2,1,1);
axis([0 8 -0.7 0.7]);
subplot(2,1,2);
axis([0 8 0 2.28]);
```



maximum loads

```
M_max(P) = m(4*u.m)
```

M_max(P) = $2\,P\,\mathrm{m}$

```
V_max(P) = v(0)
```

V_max(P) =

$$\frac{P}{2}$$

maximum stresses

```
C = symmax([yn (120+30)*u.mm-yn]);
b.I = rewrite(b.I, u.mm);
sigma_max = rewrite(M_max, u.mm)*C/b.I
```

sigma_max(P) =

$$\frac{796\,P}{79203}\,\frac{1}{\mathrm{mm}^2}$$

```
Q_max = (yn/2)*(40*u.mm*yn);
t_min = 40*u.mm;
tau_max = V_max*Q_max/(b.I*t_min)
```

tau_max(P) =

$$\frac{39601\,P}{327372400}\,\frac{1}{\mathrm{mm}^2}$$

maximum applied force

```
sigma_allow = 25*u.MPa;
tau_allow = 700*u.kPa;

assume(P > 0 & in(P, 'real'));
clear P_max;

P_max.bend = solve(sigma_max == rewrite(sigma_allow, u.kN/u.mm^2));
P_max.bend = simplify(P_max.bend);
P_max_bend = vpa(P_max.bend, 3) %#ok
```

P_max_bend = $2.49\ \mathrm{kN}$

```
P_max.shear = solve(tau_max == rewrite(tau_allow, u.kN/u.mm^2));
P_max.shear = simplify(P_max.shear);
P_max_shear = vpa(P_max.shear, 3) %#ok
```

P_max_shear = $5.79\ \mathrm{kN}$

```
P_max_vals = [P_max.bend P_max.shear];
loc = sigma_max(P_max_vals) <= sigma_allow & ...
      tau_max(P_max_vals) <= tau_allow;
```

```
P_max.limit = P_max_vals(isAlways(loc));
P_max_limit = vpa(P_max.limit, 3) %#ok
```

P_max_limit = $2.49\ \mathrm{kN}$

clean up

```
setassum(old_assum, 'clear');
clear old_assum Ra Rb;
clear P_max_bend P_max_shear P_max_vals loc P_max_limit;
```