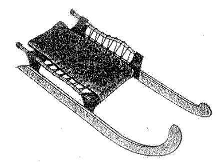
Homework #2 – Solutions

1.77

1.77 The sled shown in Fig. P1.77 slides along on a thin horizontal layer of water between the ice and the runners. The horizontal force that the water puts on the runners is equal to 1.2 lb when the sled's speed is 50 ft/s. The total area of both runners in contact with the water is 0.08 ft², and the viscosity of the water is 3.5×10^{-5} lb s/ft². Determine the thickness of the water layer under the runners. Assume a linear velocity distribution in the water layer.



■ FIGURE P1.77

F (force) $= \tau A$

$\tau = \mu \frac{dv}{dy} = \mu \frac{V}{d}$ where d = thickness of water layer

Thus,

$$F = \mu \frac{V}{d} A$$

and

$$d = \frac{\mu V A}{F} = \frac{\left(3.5 \times 10^{-5} \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}\right)\left(50 \frac{\text{ft}}{\text{s}}\right)\left(0.08\ \text{ft}^2\right)}{1.2\ \text{lb}}$$

$$= \underline{\underline{11.7 \times 10^{-4}\ \text{ft}}}$$

1.78

1.78 A 25-mm-diameter shaft is pulled through a cylindrical bearing as shown in Fig. P1.78 The lubricant that fills the 0.3-mm gap between the shaft and bearing is an oil having a kinematic viscosity of $8.0 \times 10^{-4}\ m^2/s$ and a specific gravity of 0.91. Determine the force P required to pull the shaft at a velocity of 3 m/s. Assume the velocity distribution in the gap is linear.

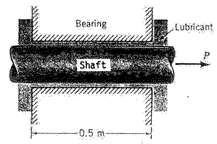
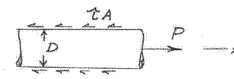


FIGURE P1.78



$\sum F_x = 0$

Thus, $P = \tau A$

where $A = \pi D \times (\text{shaft length in bearing}) = \pi D \ell$

and $\tau = \mu \dfrac{(\text{velocity of shaft})}{(\text{gap width})} = \mu \dfrac{V}{b}$

so that

$$P = \left(\mu \frac{V}{b}\right)(\pi D \ell)$$

Since $\mu = \nu \rho = \nu (SG)(\rho_{H_2O @ 4°C})$,

$$P = \frac{(8.0 \times 10^{-4} \frac{m^2}{s})(0.91 \times 10^3 \frac{kg}{m^3})(3 \frac{m}{s})(\pi)(0.025 m)(0.5 m)}{(0.0003 m)}$$

$$= \underline{\underline{286\ N}}$$

1.81

1.81 A layer of water flows down an inclined fixed surface with the velocity profile shown in Fig. P1.81. Determine the magnitude and direction of the shearing stress that the water exerts on the fixed surface for $U = 2$ m/s and $h = 0.1$ m.

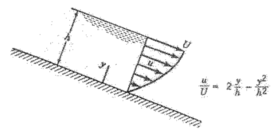


FIGURE P1.81

$$\tau = \mu \frac{du}{dy}$$

$$\frac{du}{dy} = U\left(\frac{2}{h} - \frac{y^2}{h^2}\right)$$

Thus, at the fixed surface $(y=0)$

$$\left(\frac{du}{dy}\right)_{y=0} = \frac{2U}{h}$$

so that

$$\tau = \mu\left(\frac{2U}{h}\right) = \left(1.12\times10^{-3}\,\frac{N\cdot s}{m^2}\right)(2)\,\frac{\left(2\,\frac{m}{s}\right)}{(0.1\,m)}$$

$$= 4.48\times10^{-2}\,\frac{N}{m^2} \quad \text{acting in direction of flow}$$

1.84

1.84 A new computer drive is proposed to have a disc, as shown in Fig. P1.84. The disc is to rotate at 10,000 rpm, and the reader head is to be positioned 0.0005 in. above the surface of the disc. Estimate the shearing force on the reader head as result of the air between the disc and the head.

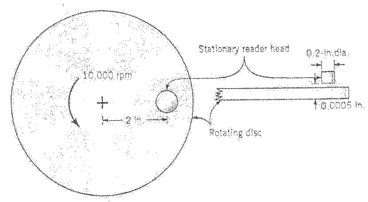


FIGURE P1.84

F = shear force on head $= \tau A$, where, if the velocity profile in the gap between the disc and head is linear and uniform across the head, then

$$\tau = \mu \frac{du}{dy} = \mu \frac{U}{b}, \text{ where}$$

$$U = \omega R = 10{,}000 \frac{rev}{min}\left(\frac{1\,min}{60\,s}\right)\left(\frac{2\pi\, rad}{rev}\right)\left(\frac{2}{12}\,ft\right) = 175\,\frac{ft}{s}$$

Thus,

$$\tau = \left(3.74 \times 10^{-7}\,\frac{lb \cdot s}{ft^2}\right)\frac{175\,\frac{ft}{s}}{\left(\frac{0.005}{12}\,ft\right)} = 1.57\,\frac{lb}{ft^2}$$

so that

$$F = \tau A = \left(1.57\,\frac{lb}{ft^2}\right)\frac{\pi}{4}\left(\frac{0.2}{12}\,ft\right)^2 = \underline{\underline{3.43 \times 10^{-4}\,lb}}$$

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