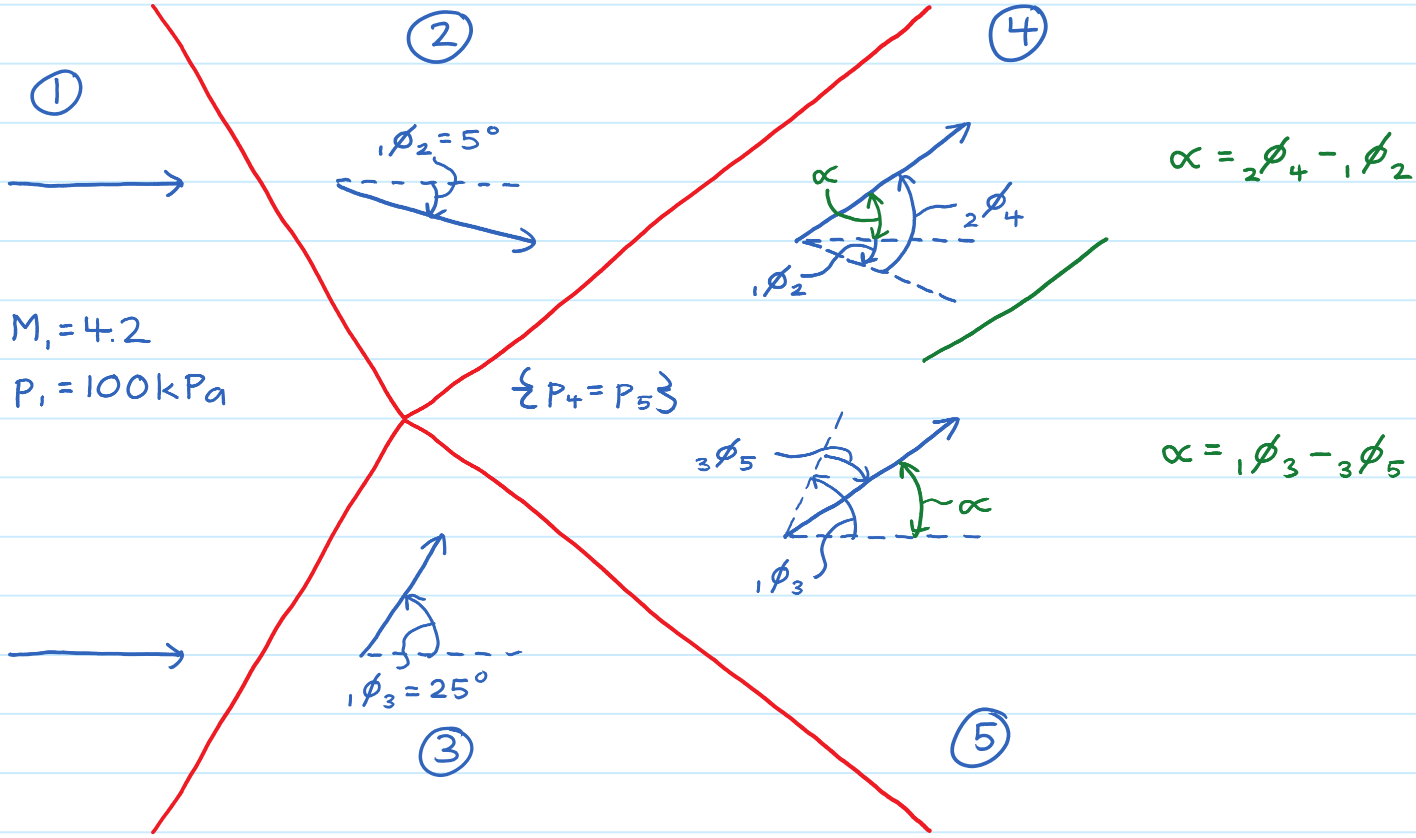
Problem 1

Thursday, April 23, 2015 8:42 PM



REGION 2:

$M_2 = 3.8119$

$\rightarrow$ $P_2/P_1 = 1.6555$

$\rightarrow$ $P_2 = P_1(P_2/P_1)$

$= (100 kPa)(1.6555)$

$P_2 = 165.5546 kPa$

REGION 3:

$M_3 = 2.2896$

$\rightarrow$ $P_3/P_1 = 7.5555$

$\longrightarrow$ $P_3 = P_1 (P_3/P_1)$

$= (100\,kPa)(7.5555)$

$P_3 = 755.5537\ kPa$

<u>SLIP LINE:</u>

① $\longrightarrow$ $$\frac{P_4}{P_1} = \frac{P_5}{P_1}$$

$$\left(\overset{F(M_2,\,{}_2\theta_4)}{\frac{P_4}{P_2}}\right)\left(\frac{P_2}{P_1}\right) = \left(\overset{F(M_3,\,{}_3\theta_5)}{\frac{P_5}{P_3}}\right)\left(\frac{P_3}{P_1}\right)$$

② $\longrightarrow$ $${}_2\overset{F(M_2,\,{}_2\theta_4)}{\phi_4} - {}_1\phi_2 = {}_3\overset{F(M_3,\,{}_3\theta_5)}{\phi_5} + {}_1\phi_3$$

soln $\longrightarrow$

$P_4 = P_5 = 1056.6000\ kPa$

${}_2\theta_4 = 38.4285°$

${}_3\theta_5 = 30.3892°$

${}_2\phi_4 = 24.3912°$

${}_3\phi_5 = 5.5909°$

$M_4 = 2.1705$

$M_5 = 2.0727$

$\alpha = 19.4091°$

See attached code + spreadsheet

Problem 2

Sunday, April 26, 2015 7:43 PM

GEOMETRY:

$\ell_1 = \ell_2 = 0.5c\sec\alpha_{w(1)}$

$= 0.5c\sec\alpha_{w(1)}$

$\ell_1 = \ell_2 = 0.1506\text{ m}$

$\ell_3 = \frac{c}{\sin\alpha_{w(5)}}\sin\alpha_{w(3)}$

$= \frac{0.3\text{ m}}{\sin(147^\circ)}\sin(30^\circ)$

$\ell_3 = 0.2754\text{ m}$

$\ell_4 = \frac{c}{\sin\alpha_{w(5)}}\sin\alpha_{w(4)}$

$= \frac{0.3\text{ m}}{\sin(147^\circ)}\sin(3^\circ)$

$\ell_4 = 0.0288\text{ m}$

UPSTREAM:

$\nu_\infty = 39.1236^\circ$

$P_\infty/P_{o\infty} = 0.0585$

REGION 1:

$\nu_1 = \nu_\infty + {}_\infty\phi_1$

$= 39.1236^\circ + 2^\circ$

$\nu_1 = 41.1236^\circ$

$M_1 = 2.5871$

$P_1/P_{o1} = 0.05113$

$P_1/P_\infty = (P_1/P_{o1})(P_{o1}/P_{o\infty})/(P_\infty/P_{o\infty})$

$= (0.05113)(1)/(0.0585)$

$P_1/P_\infty = 0.8736$

REGION 2:

$\nu_2 = \nu_1 + {}_1\phi_2$

$= 41.1236^\circ + 10^\circ$

$\nu_2 = 51.1236^\circ$

$M_2 = 3.0718$

$P_2/P_{o2} = 0.02445$

$P_2/P_\infty = (P_2/P_{o2})(P_{o2}/P_{o\infty})/(P_\infty/P_{o\infty})$

$= (0.02445)(1)/(0.0585)$

$P_2/P_\infty = 0.4178$

REGION 3:

$M_3 = 2.0859$

$P_3/P_\infty = 1.8639$

$P_{o3}/P_{o\infty} = 0.9759$

$\nu_3 = 28.7196°$

REGION 4:

$\longrightarrow$ $\nu_4 = \nu_3 + {}_3\phi_4$

$= 28.7196° + 33°$

$\nu_4 = 61.7196°$

$\longrightarrow$ $M_4 = 3.7084$

$P_4/P_{o4} = 0.0098$

$\longrightarrow$ $P_4/P_\infty = (P_4/P_{o4})(P_{o4}/P_{o3})(P_{o3}/P_{o\infty})/(P_\infty/P_{o\infty})$

$= (0.0098)(1)(0.9759)/(0.0585)$

$P_4/P_\infty = 0.1632$

COEFFICIENTS:

$\longrightarrow$ $$c_L = \frac{L}{\frac{1}{2}\gamma M_\infty^2 c}$$

$$= \frac{-(P_1/P_\infty)\ell_1\cos\alpha_1 - (P_2/P_\infty)\ell_2\cos\alpha_2 + (P_3/P_\infty)\ell_3\cos\alpha_3 + (P_4/P_\infty)\ell_4\cos\alpha_4}{\frac{1}{2}\gamma M_\infty^2 c}$$

$$= \frac{-(0.8736)(0.1506\text{ m})\cos(2°) - (0.4178)(0.1506\text{ m})\cos(12°) + (1.8639)(0.2754\text{ m})\cos(10°) + (0.1632)(0.0288\text{ m})\cos(-23°)}{\frac{1}{2}(1.4)(2.5)^2(0.3\text{ m})}$$

$c_L = 0.2414$

$\longrightarrow$ $$c_D = \frac{D}{\frac{1}{2}\gamma M_\infty^2 c}$$

$$= \frac{-(P_1/P_\infty)\ell_1\sin\alpha_1 - (P_2/P_\infty)\ell_2\sin\alpha_2 + (P_3/P_\infty)\ell_3\sin\alpha_3 + (P_4/P_\infty)\ell_4\sin\alpha_4}{\frac{1}{2}\gamma M_\infty^2 c}$$

$$= \frac{-(0.8736)(0.1506\text{ m})\sin(2°) - (0.4178)(0.1506\text{ m})\sin(12°) + (1.8639)(0.2754\text{ m})\sin(10°) + (0.1632)(0.0288\text{ m})\sin(-23°)}{\frac{1}{2}(1.4)(2.5)^2(0.3\text{ m})}$$

$c_D = 0.0531$

Problem 3

Sunday, April 26, 2015 7:43 PM

part one:

$$\rightarrow M_y = \sqrt{\frac{\gamma - 1}{2\gamma}}$$

$$= \sqrt{\frac{1.4-1}{2(1.4)}}$$

$$M_y = 0.3780$$

$$\rightarrow A_y/A_y^* = 1.6661$$

$$M_x = 1.9847$$

$$P_{oy}/P_{ox} = 0.7280$$

part two:

$$\% \text{ area increase} = \frac{A_y^* - A_x^*}{A_x^*} \times 100\%$$

$$= \left(\frac{A_y^*}{A_x^*} - 1\right) \times 100\%$$

$$= \left(\frac{1}{P_{oy}/P_{ox}} - 1\right) \times 100\%$$

$$= \left(\frac{1}{0.7280} - 1\right) \times 100\%$$

$$\% \text{ area increase} = 37.3553\%$$

Problem 4

Sunday, April 26, 2015 7:44 PM

given areas:

$\rightarrow$ $A_i = 1.0 - 1.50x_i + 1.50x_i^2$

$= 1.0 - 1.50(0) + 1.50(0)^2$

$A_i = 1.0\ m^2$

$\rightarrow$ $A_t = 1.0 - 1.50x_t + 1.50x_t^2$

$= 1.0 - 1.50(0.5m) + 1.50(0.5m)^2$

$A_t = 0.6250\ m^2$

$\rightarrow$ $A_A = 1.0 - 1.50x_A + 1.50x_A^2$

$= 1.0 - 1.50(0.9091975916\ m) + 1.50(0.9091975916\ m)^2$

$A_A = 0.8762\ m^2$

$\rightarrow$ $A_e = 1.0 - 1.50x_e + 1.50x_e^2$

$= 1.0 - 1.50(1.0m) + 1.50(1.0m)^2$

$A_e = 1.0\ m^2$

part one:

$\rightarrow$ $\frac{A_i}{A_i^*} = \frac{A_i}{A_t}$

$= \frac{1.0\ m^2}{0.6250\ m^2}$

$\frac{A_i}{A_i^*} = 1.6000$

$\rightarrow$ $M_i = 0.3970$

$P_i/P_o = 0.8971$

$P_o = P_i/(P_i/P_o)$
$= (100\ kPa)/(0.8971)$
$P_o = 111.4729\ kPa$

$$\frac{A_e}{A_e^*} = \frac{A_e}{A_t}$$
$$= \frac{1.0\ m^2}{0.6250\ m^2}$$
$$\frac{A_e}{A_e^*} = 1.6000$$

$M_e = 1.9353$
$P_e/P_o = 0.1413$

$P_e = P_o(P_e/P_o)$
$= (111.4729\ kPa)(0.1413)$
$P_e = 15.7531\ kPa$

<u>part two</u>:

$$\left(\frac{P_A}{P_{oA}}\right)\left(\frac{A_A}{A_A^*}\right) = \left(\frac{P_A}{P_{oy}}\right)\left(\frac{A_A}{A_y^*}\right)$$
$$= \left(\frac{P_A}{P_{ox}}\right)\left(\frac{A_A}{A_x^*}\right)$$
$$= \left(\frac{P_A}{P_{oi}}\right)\left(\frac{A_A}{A_t}\right)$$
$$= \left(\frac{81.24605951\ kPa}{111.4729\ kPa}\right)\left(\frac{0.8762\ m^2}{0.6250\ m^2}\right)$$
$$\left(\frac{P_A}{P_{oA}}\right)\left(\frac{A_A}{A_A^*}\right) = 1.0217$$

$M_A = 0.5500$
$P_A/P_{oA} = 0.8142$

$\longrightarrow P_{oy} = P_{oA} = P_A/(P_A/P_{oA})$

$= (81.24605951 \text{ kPa})/(0.8142)$

$P_{oy} = P_{oA} = 99.7906 \text{ kPa}$

$\longrightarrow \dfrac{P_{oy}}{P_{ox}} = \dfrac{99.7906 \text{ kPa}}{111.4729 \text{ kPa}}$

$\dfrac{P_{oy}}{P_{ox}} = 0.8952$

$M_x = 1.6000$

$\longrightarrow M_y = 0.6684$

$A_x/A_x^* = 1.2502$

$\longrightarrow A_s = A_t(A_x/A_x^*)(A_x^*/A_t)$

$= (0.6250 \text{ m}^2)(1.2502)(1)$

$A_s = 0.7814 \text{ m}^2$

$\longrightarrow A_s = 1.0 - 1.50 x_s + 1.50 x_s{}^2$

$0.7814 \text{ m}^2 = 1.0 - 1.50 x_s + 1.50 x_s{}^2$

$1.50 x_s{}^2 - 1.50 x_s + 0.2186 = 0$

$x_s = \dfrac{1.50 + \sqrt{(-1.50)^2 - 4(1.50)(0.2186)}}{2(1.50)}$

$x_s = 0.8229 \text{ m}$

$\longrightarrow \dfrac{A_e}{A_e^*} = \left(\dfrac{A_e}{A_t}\right)\left(\dfrac{A_t}{A_x^*}\right)\left(\dfrac{A_x^*}{A_y^*}\right)\left(\dfrac{A_y^*}{A_e^*}\right)$

$= \left(\dfrac{A_e}{A_t}\right)\left(\dfrac{A_t}{A_x^*}\right)\left(\dfrac{P_{oy}}{P_{ox}}\right)\left(\dfrac{A_y^*}{A_e^*}\right)$

$= \left(\dfrac{1.0 \text{ m}^2}{0.6250 \text{ m}^2}\right)(1)(0.8952)(1)$

$\dfrac{A_e}{A_e^*} = 1.4323$

$M_e = 0.4567$

$P_e / P_{oe} = 0.8667$

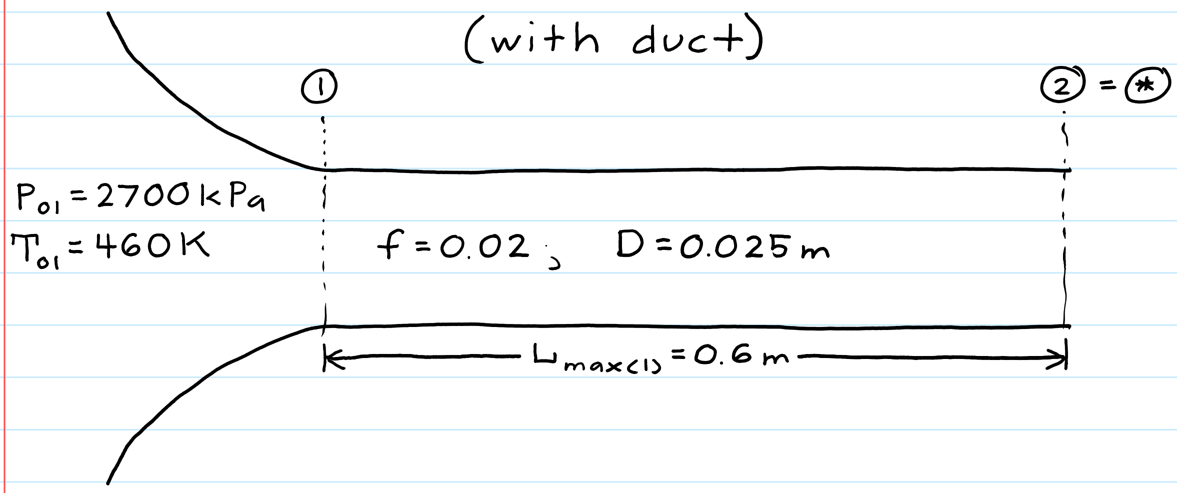
$$P_b = P_e = P_{oy} (P_e / P_{oe})(P_{oe} / P_{oy})$$

$$= (99.7906 \text{ kPa})(0.8667)(1)$$

$P_b = P_e = 86.4882 \text{ kPa}$

Problem 5

Thursday, April 23, 2015 11:15 PM



REGION 1:

→ $fL_{max(1)}/D = (0.02)(0.6\ m)/(0.025\ m)$

$fL_{max(1)}/D = 0.48$

$M_1 = 0.6028$

$P_{01}/P_0^* = 1.1849$

→ $P_1/P^* = 1.7541$

$P_1/P_{01} = 0.7823$

$T_1/T_{01} = 0.9323$

→ $P_1 = P_{01}(P_1/P_{01})$

$= (2700\ kPa)(0.7823)$

$P_1 = 2112.2156\ kPa$

→ $T_1 = T_{01}(T_1/T_{01})$

$= (460\ K)(0.9323)$

$T_1 = 428.8381\ K$

→ $\dot{m}_{max} = P_1 A_1 M_1 \sqrt{\dfrac{\gamma}{RT_1}}$

$= (2112.2156 \times 10^3\ Pa)\left[\frac{\pi}{4}(0.025\ m)^2\right](0.6028)\sqrt{\dfrac{1.4}{(287\ J/kg\text{-}K)(428.8381\ K)}}$

$\dot{m}_{max} = 2.1078\ kg/s$

$= (2112.2156\times10\ \text{Pa})[\frac{\ }{4}(0.025\ \text{m})\ ](0.6028)\sqrt{\frac{\ }{(287\ \text{J/kg-K})(428.8381\ \text{K})}}$

$\dot{m}_{max} = 2.1078\ \text{kg/s}$

REGION 2:

$P_b = P_2 = P_1/(P_1/P^*)$

$= (2112.2156\ \text{kPa})/(1.7541)$

$P_b = P_2 = 1203.7322\ \text{kPa}$

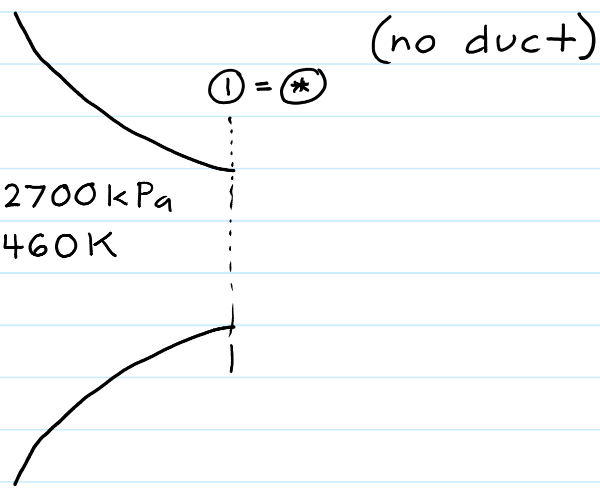
maximum mass flow is achieved when $P_b \leq 1203.7322\ \text{kPa}$

$P_{02} = P_{01}/(P_{01}/P_0^*)$

$= (2700\ \text{kPa})/(1.1849)$

$P_{02} = 2278.5798\ \text{kPa}$

$T_{02} = T_{01} = 460\ \text{K}$



$P_{01} = 2700\ \text{kPa}$

$T_{01} = 460\ \text{K}$

REGION 1:

$P_1/P_{01} = 0.5283$

$T_1/T_{01} = 0.8333$

$P_1 = P_{01}(P_1/P_{01})$

$= (2700\ \text{kPa})(0.5283)$

$P_1 = 1426.3608\ \text{kPa}$

$\longrightarrow$ $T_1 = T_{01}(T_1/T_{01})$

$= (460\,K)(0.8333)$

$T_1 = 383.3333\ K$

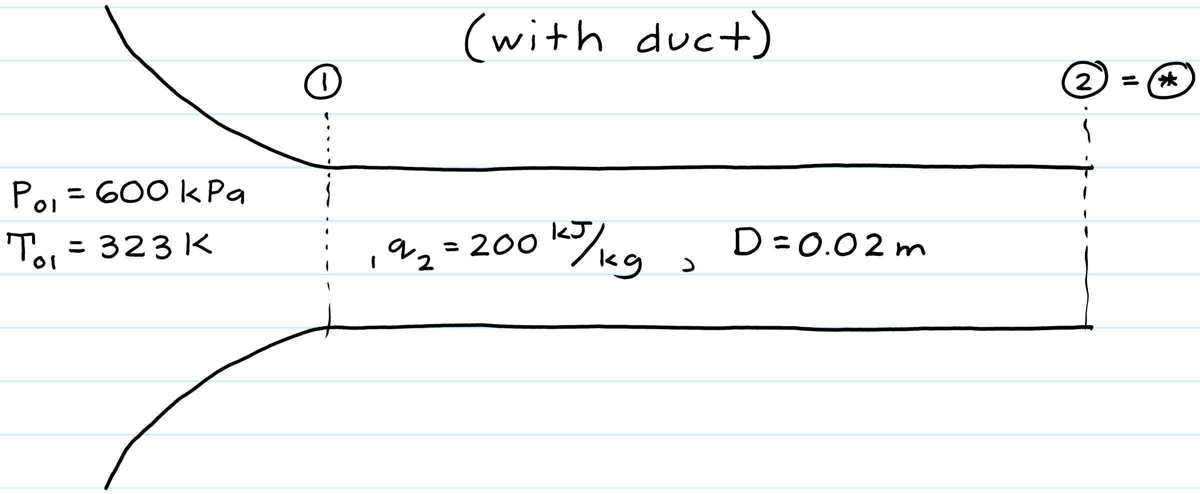
$\longrightarrow$ $\dot{m}_{max} = p_1 A_1 M_1 \sqrt{\dfrac{\gamma}{RT_1}}$

$= (1426.3608 \times 10^3\ Pa)\left[\frac{\pi}{4}(0.025\,m)^2\right](1)\sqrt{\dfrac{1.4}{(287\ J/kg\text{-}K)(383.3333\ K)}}$

$\dot{m}_{max} = 2.4977\ kg/s$

Problem 6

Friday, April 24, 2015 1:20 AM



REGION 1:

$$\rightarrow \quad {}_1q_2 = c_p T_{01}\left(\frac{T_{02}/T_0^*}{T_{01}/T_0^*} - 1\right)$$

$$200\ \text{kJ/kg} = (1.0045\ \text{kJ/kg-K})(323\ \text{K})\left(\frac{1}{T_{01}/T_0^*} - 1\right)$$

$$T_{01}/T_0^* = 0.6187$$

$M_1 = 0.4529$

$P_{01}/P_0^* = 1.1338$

$\rightarrow \quad P_1/P^* = 1.8645$

$P_1/P_{01} = 0.8687$

$T_1/T_{01} = 0.9697$

$$\rightarrow \quad P_1 = P_{01}(P_1/P_{01})$$
$$= (600\ \text{kPa})(0.8687)$$
$$P_1 = 521.2394\ \text{kPa}$$

$$\rightarrow \quad T_1 = T_{01}(T_1/T_{01})$$
$$= (323\ \text{K})(0.9697)$$
$$T_1 = 310.2711\ \text{K}$$

$$\rightarrow \quad \dot{m}_{max} = p_1 A_1 M_1 \sqrt{\frac{\gamma}{RT_1}}$$

$$= (521.2394 \times 10^3\ Pa)\left[\frac{\pi}{4}(0.02\,m)^2\right](0.4529)\sqrt{\frac{1.4}{(287\ J/kg\text{-}K)(310.2711\ K)}}$$

$$\dot{m}_{max} = 0.2941\ kg/s$$

REGION 2:

$$\rightarrow \quad P_b = P_2 = P_1/(P_1/P^*)$$

$$= (521.2394\ kPa)/(1.8645)$$

$$P_b = P_2 = 279.5526\ kPa$$

maximum mass flow is achieved when $P_b \leq 279.5526\ kPa$

$$\rightarrow \quad P_{o2} = P_{o1}/(P_{o1}/P_o^*)$$

$$= (600\ kPa)/(1.1338)$$

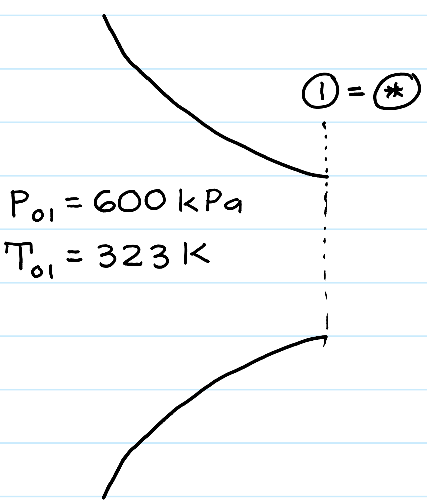
$$P_{o2} = 529.1732\ kPa$$

$$\rightarrow \quad T_{o2} = T_{o1}/(T_{o1}/T_o^*)$$

$$= (323\ K)/(0.6187)$$

$$T_{o2} = 522.1040\ K$$

(no duct)



REGION 1:

$$P_1/P_{01} = 0.5283$$
$$T_1/T_{01} = 0.8333$$

$$P_1 = P_{01}(P_1/P_{01})$$
$$= (600\,kPa)(0.5283)$$
$$P_1 = 316.9691\,kPa$$

$$T_1 = T_{01}(T_1/T_{01})$$
$$= (323\,K)(0.8333)$$
$$T_1 = 269.1667\,K$$

$$\dot{m}_{max} = P_1 A_1 M_1 \sqrt{\frac{\gamma}{RT_1}}$$
$$= (316.9691 \times 10^3\,Pa)\left[\frac{\pi}{4}(0.02\,m)^2\right](1)\sqrt{\frac{1.4}{(287\,J/kg\text{-}K)(269.1667\,K)}}$$
$$\dot{m}_{max} = 0.4239\,kg/s$$