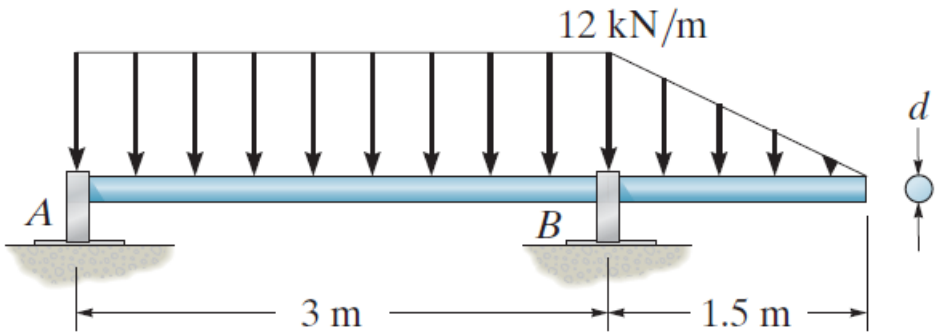
problem 6-67

6–67. The rod is supported by smooth journal bearings at A and B that only exert vertical reactions on the shaft. If $d = 90$ mm, determine the absolute maximum bending stress in the beam, and sketch the stress distribution acting over the cross section.



Probs. 6–67/68

beam

```
u = symunit;
x = sym('x');
E = sym('E');

old_assum = assumptions;
clearassum;
args = {'mode' 'factor'};
wf1(x) = -12*u.kN/u.m;
wf2 = findpoly(1, 'thru', [3*u.m -12*u.kN/u.m], [4.5*u.m 0], args{:});

b = beam; %(kN,m)
b = b.add('reaction', 'force', 'Ra', 0);
b = b.add('reaction', 'force', 'Rb', 3*u.m);
b = b.add('distributed', 'force', wf1, [0 3]*u.m);
b = b.add('distributed', 'force', wf2, [3 4.5]*u.m, [false true]);
b.L = 4.5*u.m;
```

section properties

```
D = 90*u.mm;
R = D/2;
```

```
b.I = rewrite(pi*R^4/4, u.m);
```

elastic curve

```
[y(x,E) dy(x,E) m v w r] = b.elastic_curve(x, 'factor'); %#ok
y
```

y(x, E) =

$$\begin{cases} \dfrac{1600000000\, x\, (x - 3\,\mathrm{m})\, (-2\, x^2 + 5\, x\,\mathrm{m} + 15\,\mathrm{m}^2)}{6561\,\mathrm{E}\,\pi}\ \dfrac{\mathrm{kN}}{\mathrm{m}^5} & \text{if } x \le 3\,\mathrm{m} \\ \dfrac{320000000\, (x - 3\,\mathrm{m})\, (4\, x^4 - 78\, x^3\,\mathrm{m} + 576\, x^2\,\mathrm{m}^2 - 1917\, x\,\mathrm{m}^3 + 2889\,\mathrm{m}^4)}{19683\,\mathrm{E}\,\pi}\ \dfrac{\mathrm{kN}}{\mathrm{m}^6} & \text{if } 3\,\mathrm{m} < x \end{cases}$$

```
dy
```

dy(x, E) =

$$\begin{cases} -\dfrac{1600000000\, (8\, x^3 - 33\, x^2\,\mathrm{m} + 45\,\mathrm{m}^3)}{6561\,\mathrm{E}\,\pi}\ \dfrac{\mathrm{kN}}{\mathrm{m}^5} & \text{if } x \le 3\,\mathrm{m} \\ \dfrac{3200000000\, (2\, x^4 - 36\, x^3\,\mathrm{m} + 243\, x^2\,\mathrm{m}^2 - 729\, x\,\mathrm{m}^3 + 864\,\mathrm{m}^4)}{19683\,\mathrm{E}\,\pi}\ \dfrac{\mathrm{kN}}{\mathrm{m}^6} & \text{if } 3\,\mathrm{m} < x \end{cases}$$

```
m
```

m(x) =

$$\begin{cases} -\dfrac{3\, x\, (4\, x - 11\,\mathrm{m})}{2}\ \dfrac{\mathrm{kN}}{\mathrm{m}} & \text{if } x \le 3\,\mathrm{m} \\ \dfrac{(2\, x - 9\,\mathrm{m})^3}{6}\ \dfrac{\mathrm{kN}}{\mathrm{m}^2} & \text{if } 3\,\mathrm{m} < x \end{cases}$$

```
v
```

v(x) =

$$\begin{cases} -\dfrac{3\, (8\, x - 11\,\mathrm{m})}{2}\ \dfrac{\mathrm{kN}}{\mathrm{m}} & \text{if } x \le 3\,\mathrm{m} \\ (2\, x - 9\,\mathrm{m})^2\ \dfrac{\mathrm{kN}}{\mathrm{m}^2} & \text{if } 3\,\mathrm{m} < x \end{cases}$$

```
w
```

w(x) =

$$\begin{cases} -12\ \dfrac{\mathrm{kN}}{\mathrm{m}} & \text{if } x \le 3\,\mathrm{m} \\ 4\, (2\, x - 9\,\mathrm{m})\ \dfrac{\mathrm{kN}}{\mathrm{m}^2} & \text{if } 3\,\mathrm{m} < x \end{cases}$$

reactions

```
Ra = vpa(r.Ra) %#ok
```

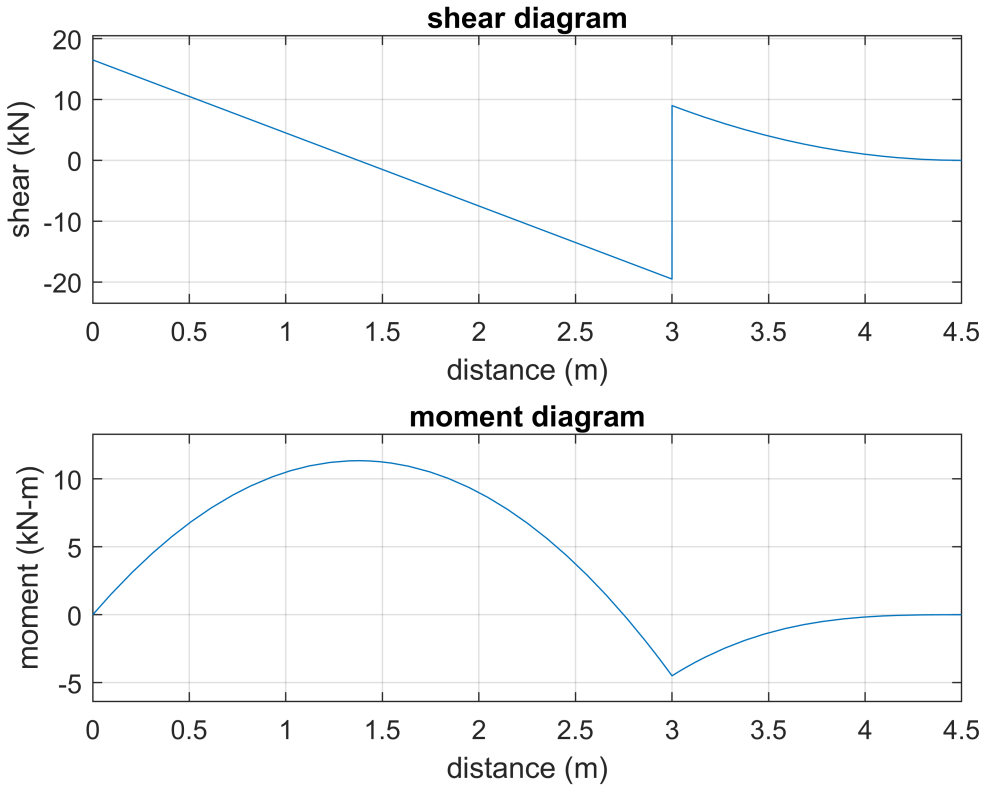
Ra = 16.5 kN

```
Rb = vpa(r.Rb) %#ok
```

Rb = 28.5 kN

shear and moment diagram

```
beam.shear_moment(m, v, [0 4.5], {'kN' 'm'});
subplot(2,1,1);
axis([0 4.5 -23.5 20.5]);
subplot(2,1,2);
axis([0 4.5 -6.4 13.3]);
```



maximum moment

```
assume(0 < x & x < b.L & in(x, 'real'));
xmax = solve(v == 0, x);
M_val = m(xmax);
```

```
M_max = vpa(M_val, 4) %#ok
```

M_max = 11.34 kN m

```
M_max = M_val;
```

maximum bending stress

```
C = R;
b.I = rewrite(b.I, u.mm);
sigma_val = rewrite(M_max*C/b.I, u.MPa);
sigma_max = vpa(sigma_val, 5) %#ok
```

sigma_max = 158.5 MPa

```
sigma_max = sigma_val;
```

clean up

```
setassum(old_assum, 'clear');
clear args old_assum Ra Rb M_val sigma_val;
```