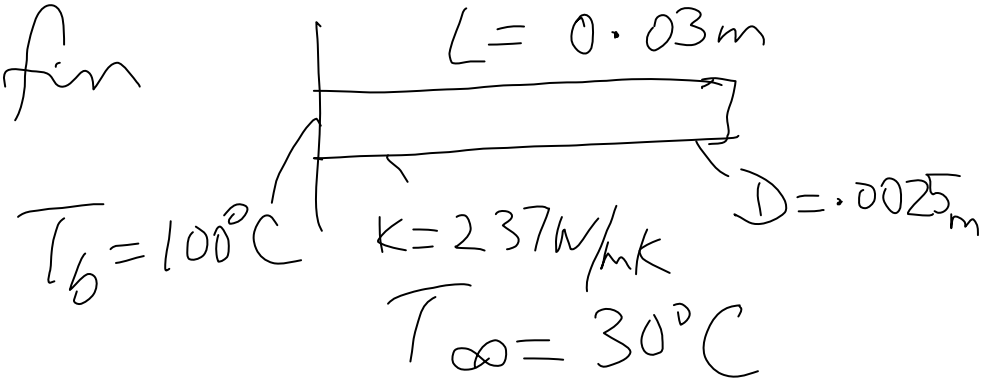
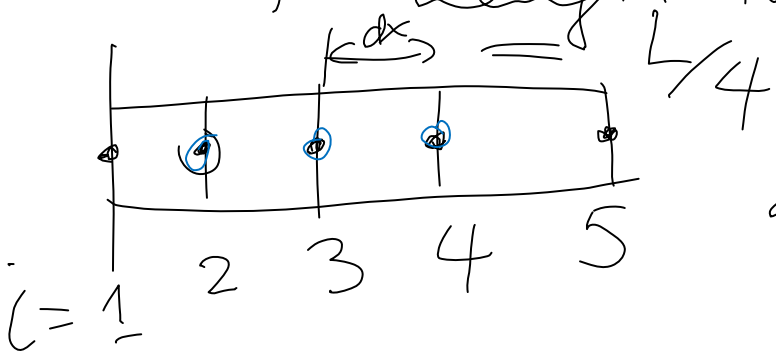
Prob 3.130

pin fin



Divide fin length into N nodes



$N = 5$

$dx = \frac{L}{N-1}$

for $i = 2:4$

$$\theta_{i-1} - (2 + m^2 dx^2)\theta_i + \theta_{i+1} = 0$$

$i = 1$

$$\theta_1 = \theta_0$$

$i = N = 5$

$$2\theta_{N-1} - (2 + m^2 dx^2)\theta_N = 0$$

or

$$2\theta_4 - (2 + m^2 dx^2)\theta_5 = 0$$

i=

1: $\theta_1 = \theta_0$

2: $\theta_1 - (2+m^2dx^2)\theta_2 + \theta_3 = 0$

3: $\theta_1 + \theta_2 - (2+m^2dx^2)\theta_3 + \theta_4 = 0$

4: $0 + 0 + \theta_3 - (2+m^2dx^2)\theta_4 + \theta_5 = 0$

5: $0 + 0 + 0 + 2\theta_4 - (2+m^2dx^2)\theta_5 = 0$

In Matrix form:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & -(2+m^2dx^2) & 1 & 0 & 0 \\ 0 & 1 & -(2+m^2dx^2) & 1 & 0 \\ 0 & 0 & 1 & -(2+m^2dx^2) & 1 \\ 0 & 0 & 0 & 2 & -(2+m^2dx^2) \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{bmatrix} = \begin{bmatrix} T_b - T_\infty \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$[M]$ $\qquad$ rhs

Solution: $\vec{\theta} = [M]^{-1} * \vec{rhs}$

Exact Solution

$$\theta_x = \theta_0 \frac{\cosh[m(L-x)]}{\cosh(mL)}$$