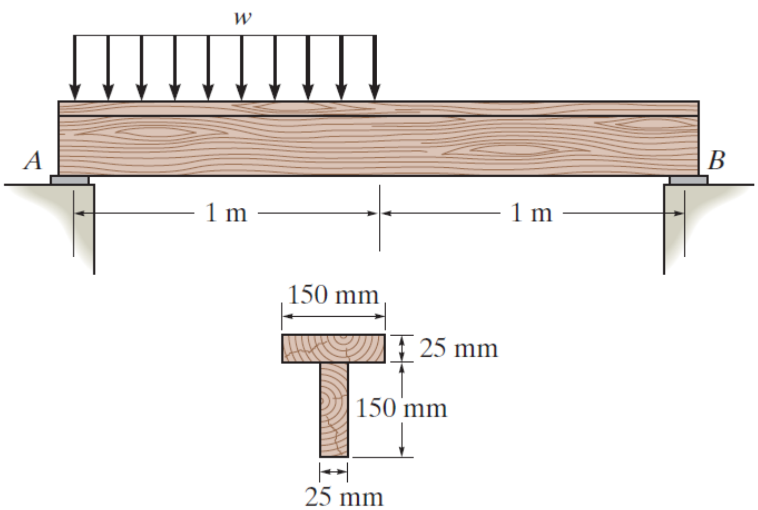
problem 11-7

11–7. If the bearing pads at A and B support only vertical forces, determine the greatest magnitude of the uniform distributed loading w that can be applied to the beam. $\sigma_{\text{allow}} = 15$ MPa, $\tau_{\text{allow}} = 1.5$ MPa.



Prob. 11–7

beam

```
u = symunit;
x = sym('x');
E = sym('E');
wo = sym('wo');

old_assum = assumptions;
clearassum;

b = beam;
b = b.add('reaction', 'force', 'Ra', 0);
b = b.add('reaction', 'force', 'Rb', 2*u.m);
b = b.add('distributed', 'force', -wo, [0 1]*u.m);
b.L = 2*u.m;
```

section properties

```
yc = [150/2; 150+25/2]*u.mm;
Ac = [25*150; 150*25]*u.mm^2;
Ic = [25*150^3; 150*25^3]*u.mm^4/12;
```

```
[yn Qn In] = beam.neutral_axis(yc, Ac, Ic); %#ok
b.I = rewrite(sum(In), u.m);
```

elastic curve

```
[y(x,E,wo) dy(x,E,wo) m v w r] = b.elastic_curve(x, 'factor'); %#ok
y
```

```
y(x, E, wo) =
```

$$\begin{cases} -\dfrac{640000\,\mathrm{wo}\,x\,(2\,x^3 - 6\,x^2\,\mathrm{m} + 9\,\mathrm{m}^3)}{663\,\mathrm{E}}\,\dfrac{1}{\mathrm{m}^4} & \text{if } x \le \mathrm{m} \\ -\dfrac{640000\,\mathrm{wo}\,(x - 2\,\mathrm{m})\,(2\,x^2 - 8\,x\,\mathrm{m} + \mathrm{m}^2)}{663\,\mathrm{E}}\,\dfrac{1}{\mathrm{m}^3} & \text{if } \mathrm{m} < x \end{cases}$$

```
dy
```

```
dy(x, E, wo) =
```

$$\begin{cases} -\dfrac{640000\,\mathrm{wo}\,(8\,x^3 - 18\,x^2\,\mathrm{m} + 9\,\mathrm{m}^3)}{663\,\mathrm{E}}\,\dfrac{1}{\mathrm{m}^4} & \text{if } x \le \mathrm{m} \\ -\dfrac{640000\,\mathrm{wo}\,(6\,x^2 - 24\,x\,\mathrm{m} + 17\,\mathrm{m}^2)}{663\,\mathrm{E}}\,\dfrac{1}{\mathrm{m}^3} & \text{if } \mathrm{m} < x \end{cases}$$

```
m
```

```
m(x) =
```

$$\begin{cases} -\dfrac{\mathrm{wo}\,x\,(2\,x - 3\,\mathrm{m})}{4} & \text{if } x \le \mathrm{m} \\ -\dfrac{\mathrm{wo}\,(x - 2\,\mathrm{m})}{4}\,\mathrm{m} & \text{if } \mathrm{m} < x \end{cases}$$

```
v
```

```
v(x) =
```

$$\begin{cases} -\dfrac{\mathrm{wo}\,(4\,x - 3\,\mathrm{m})}{4} & \text{if } x \le \mathrm{m} \\ -\dfrac{\mathrm{wo}}{4}\,\mathrm{m} & \text{if } \mathrm{m} < x \end{cases}$$

```
w
```

```
w(x) =
```

$$\begin{cases} -\mathrm{wo} & \text{if } x \le \mathrm{m} \\ 0 & \text{if } \mathrm{m} < x \end{cases}$$

reactions

```
Ra = r.Ra %#ok
```

Ra =

$$\frac{3\,\mathrm{wo}}{4}\ \mathrm{m}$$

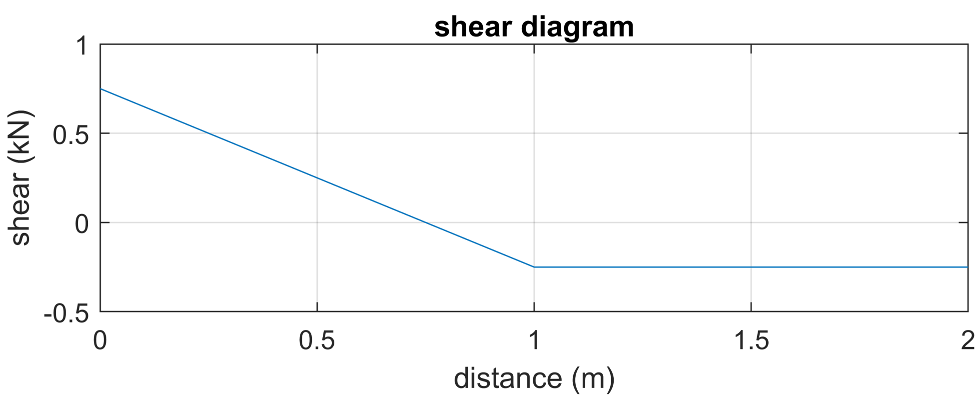
```
Rb = r.Rb %#ok
```

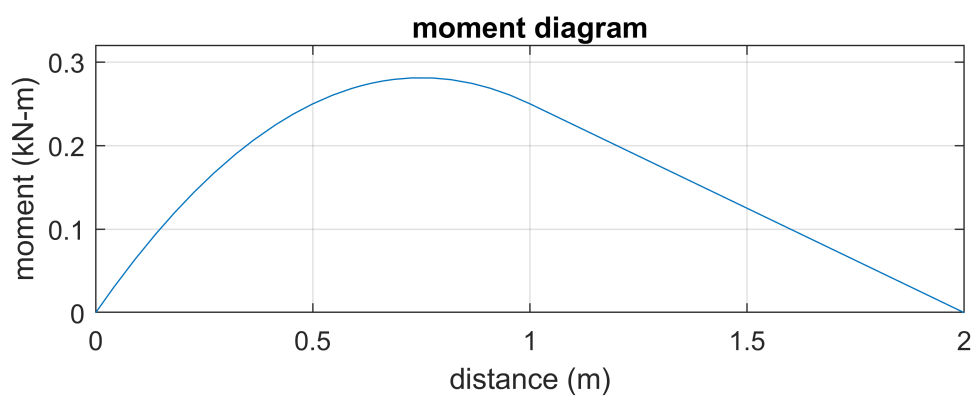
Rb =

$$\frac{\mathrm{wo}}{4}\ \mathrm{m}$$

shear and moment diagram

```
beam.shear_moment(m, v, [0 2], {'kN' 'm'}, wo, 1);
subplot(2,1,1);
axis([0 2 -0.5 1]);
subplot(2,1,2);
axis([0 2 0 0.32]);
```





maximum loads

```
assume(0 < x & x < b.L & in(x, 'real'));
xmax = solve(v == 0, x);
M_max(wo) = m(xmax)
```

M_max(wo) =

$$\frac{9\,\mathrm{wo}}{32}\ \mathrm{m}^2$$

```
V_max(wo) = v(0)
```

V_max(wo) =

$$\frac{3\,\mathrm{wo}}{4}\ \mathrm{m}$$

maximum stresses

```
C = symmax([yn (150+25)*u.mm-yn]);
b.I = rewrite(b.I, u.mm);
sigma_max = rewrite(M_max*C/b.I, u.m)
```

sigma_max(wo) =

$$\frac{342000\,\mathrm{wo}}{221}\ \frac{1}{\mathrm{m}}$$

```
Q_max = (yn/2)*(25*u.mm*yn);
t_min = 25*u.mm;
tau_max = rewrite(V_max*Q_max/(b.I*t_min), u.m)
```

tau_max(wo) =

$$\frac{54150\,\mathrm{wo}}{221}\ \frac{1}{\mathrm{m}}$$

maximum distributed force

```
sigma_allow = 15*u.MPa;
tau_allow = 1.5*u.MPa;

assume(wo > 0 & in(wo, 'real'));
clear wo_max;

wo_max.bend = solve(sigma_max == rewrite(sigma_allow, u.kN/u.m^2));
wo_max.bend = simplify(wo_max.bend);
wo_max_bend = vpa(wo_max.bend, 3) %#ok
```

```
wo_max_bend =
```

$9.69 \frac{\mathrm{kN}}{\mathrm{m}}$

```
wo_max.shear = solve(tau_max == rewrite(tau_allow, u.kN/u.m^2));
wo_max.shear = simplify(wo_max.shear);
wo_max_shear = vpa(wo_max.shear, 3) %#ok
```

```
wo_max_shear =
```

$6.12 \frac{\mathrm{kN}}{\mathrm{m}}$

```
wo_max_vals = [wo_max.bend wo_max.shear];
loc = sigma_max(wo_max_vals) <= sigma_allow & ...
      tau_max(wo_max_vals) <= tau_allow;
wo_max.limit = wo_max_vals(isAlways(loc));
wo_max_limit = vpa(wo_max.limit, 3) %#ok
```

```
wo_max_limit =
```

$6.12 \frac{\mathrm{kN}}{\mathrm{m}}$

clean up

```
setassum(old_assum, 'clear');
clear old_assum Ra Rb;
clear wo_max_bend wo_max_shear wo_max_vals loc wo_max_limit;
```