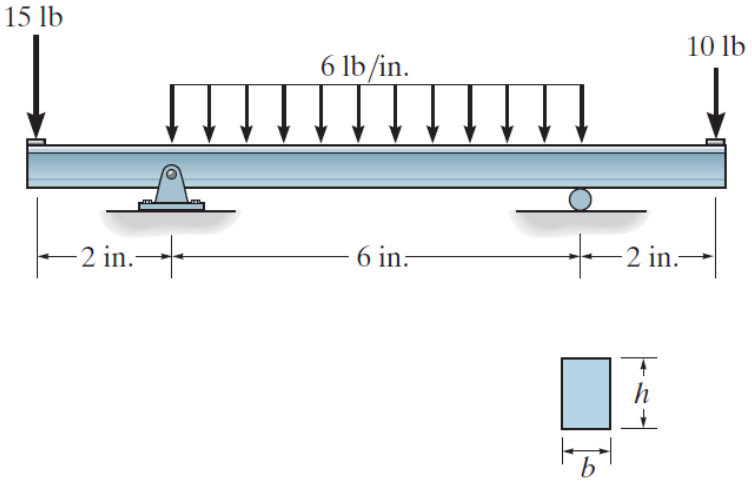
problem 11-28

*11–28. The beam is made of a ceramic material having an allowable bending stress of $\sigma_{allow} = 735$ psi and an allowable shear stress of $\tau_{allow} = 400$ psi. Determine the width b of the beam if the height $h = 2b$.



Prob. 11–28

beam

```
u = symunit;
x = sym('x');
E = sym('E');

old_assum = assumptions;
clearassum;

b = beam;
b = b.add('reaction', 'force', 'R1', 2*u.in);
b = b.add('reaction', 'force', 'R2', 8*u.in);
b = b.add('applied', 'force', -15*u.lbf, 0);
b = b.add('distributed', 'force', -6*u.lbf/u.in, [2 8]*u.in);
b = b.add('applied', 'force', -10*u.lbf, 10*u.in);
b.L = 10*u.in;
```

section properties

```
B = sym('B');
H(B) = 2*B;
b.I = B*H^3/12;
A = B*H;
```

elastic curve

```
[y(x,E,B) dy(x,E,B) m v w r] = b.elastic_curve(x, 'factor'); %#ok
y
```

y(x, E, B) =

$$\begin{cases} -\dfrac{3\ (x-2\,\text{in})\ (5\,x^2+10\,x\,\text{in}-92\,\text{in}^2)}{4\,B^4\,\text{E}}\ \text{lbf} & \text{if}\ \ x \le 2\,\text{in} \\ -\dfrac{(x-2\,\text{in})\ (x-8\,\text{in})\ (9\,x^2-100\,x\,\text{in}+320\,\text{in}^2)}{24\,B^4\,\text{E}}\ \dfrac{\text{lbf}}{\text{in}} & \text{if}\ \ x \in (2\,\text{in}, 8\,\text{in}] \\ \dfrac{(x-8\,\text{in})\ (5\,x^2-110\,x\,\text{in}+512\,\text{in}^2)}{2\,B^4\,\text{E}}\ \text{lbf} & \text{if}\ \ 8\,\text{in} < x \end{cases}$$

```
dy
```

dy(x, E, B) =

$$\begin{cases} \dfrac{3\ (112\,\text{in}^2-15\,x^2)}{4\,B^4\,\text{E}}\ \text{lbf} & \text{if}\ \ x \le 2\,\text{in} \\ -\dfrac{6\,x^3-95\,x^2\,\text{in}+488\,x\,\text{in}^2-800\,\text{in}^3}{4\,B^4\,\text{E}}\ \dfrac{\text{lbf}}{\text{in}} & \text{if}\ \ x \in (2\,\text{in}, 8\,\text{in}] \\ \dfrac{3\ (5\,x^2-100\,x\,\text{in}+464\,\text{in}^2)}{2\,B^4\,\text{E}}\ \text{lbf} & \text{if}\ \ 8\,\text{in} < x \end{cases}$$

```
m
```

m(x) =

$$\begin{cases} -15\,x\,\text{lbf} & \text{if}\ \ x \le 2\,\text{in} \\ -\dfrac{9\,x^2-95\,x\,\text{in}+244\,\text{in}^2}{3}\ \dfrac{\text{lbf}}{\text{in}} & \text{if}\ \ x \in (2\,\text{in}, 8\,\text{in}] \\ 10\ (x-10\,\text{in})\,\text{lbf} & \text{if}\ \ 8\,\text{in} < x \end{cases}$$

```
v
```

v(x) =

$$\begin{cases} -15\,\text{lbf} & \text{if}\ \ x \le 2\,\text{in} \\ -\dfrac{18\,x-95\,\text{in}}{3}\ \dfrac{\text{lbf}}{\text{in}} & \text{if}\ \ x \in (2\,\text{in}, 8\,\text{in}] \\ 10\,\text{lbf} & \text{if}\ \ 8\,\text{in} < x \end{cases}$$

```
w
```

w(x) =

$$\begin{cases} 0 & \text{if } x < 2\,\text{in} \\ -6\,\frac{\text{lbf}}{\text{in}} & \text{if } x \in [2\,\text{in}, 8\,\text{in}] \\ 0 & \text{if } 8\,\text{in} < x \end{cases}$$

reactions

```
R1 = vpa(r.R1, 4) %#ok
```

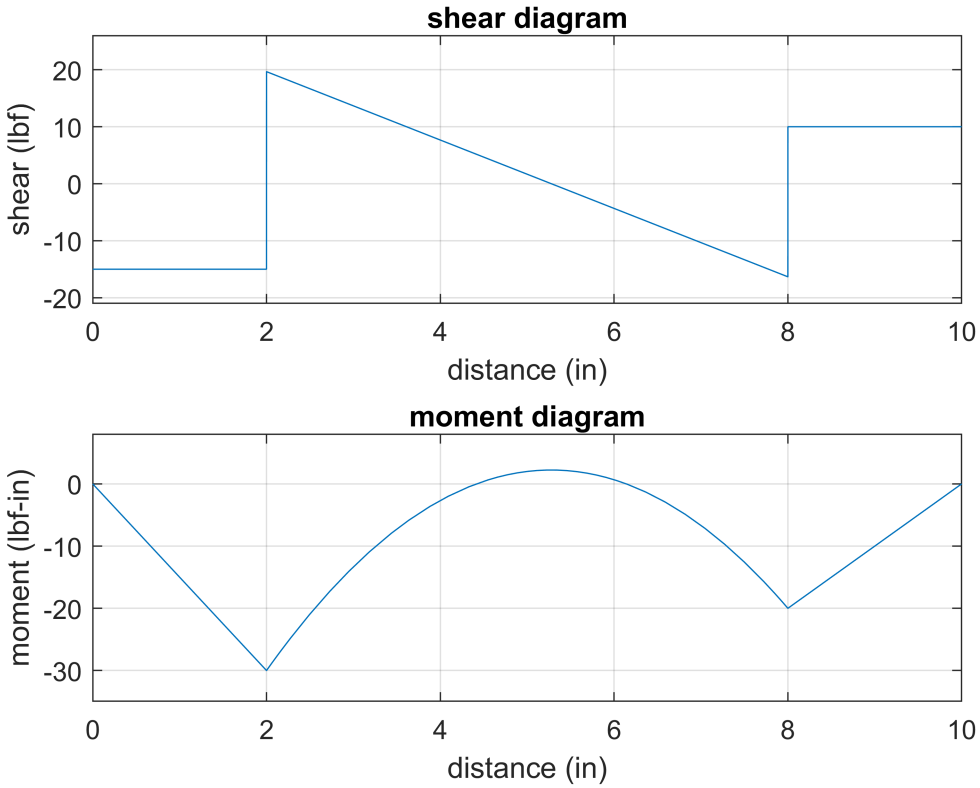
R1 = 34.67 lbf

```
R2 = vpa(r.R2, 4) %#ok
```

R2 = 26.33 lbf

shear and moment diagram

```
beam.shear_moment(m, v, [0 10], {'lbf', 'in'});
subplot(2,1,1);
axis([0 10 -21 26]);
subplot(2,1,2);
axis([0 10 -35 8]);
```



maximum loads

```
M_max = m(2*u.in)
```

M_max = $-30\ \text{in lbf}$

```
V_val = subs(expression(v,2), x, 2*u.in);
V_max = vpa(V_val, 4) %#ok
```

V_max = $19.67\ \text{lbf}$

```
V_max = V_val;
```

maximum stresses

```
C = H/2;
sigma_max = abs(M_max)*C/b.I
```

sigma_max(B) =

$$\frac{45}{B^3}\ \text{in lbf}$$

```
tau_max = 3*V_max/(2*A)
```

tau_max(B) =

$$\frac{59}{4\,B^2}\ \text{lbf}$$

minimum inner diameter

```
sigma_allow = 735*u.psi;
tau_allow = 400*u.psi;

assume(B > 0 & in(B, 'real'));
clear B_min;

B_min.bend = solve(sigma_max == rewrite(sigma_allow, u.lbf/u.in^2));
B_min.bend = simplify(B_min.bend);
B_min_bend = vpa(B_min.bend, 2) %#ok
```

B_min_bend = $0.39\ \text{in}$

```
B_min.shear = solve(tau_max == rewrite(tau_allow, u.lbf/u.in^2));
B_min.shear = simplify(B_min.shear);
B_min_shear = vpa(B_min.shear, 2) %#ok
```

B_min_shear = 0.19 in

```
B_min_vals = [B_min.bend B_min.shear];
loc = sigma_max(B_min_vals) <= sigma_allow & ...
      tau_max(B_min_vals) <= tau_allow;
B_min.limit = B_min_vals(isAlways(loc));
B_min_limit = vpa(B_min.limit, 2) %#ok
```

B_min_limit = 0.39 in

clean up

```
setassum(old_assum, 'clear');
clear old_assum R1 R2 V_val;
clear B_min_bend B_min_shear B_min_vals loc B_min_limit;
```