Homework #4 – Solutions

2.43

2.43 For the inclined-tube manometer of Fig. P2.43 the pressure in pipe A is 0.6 psi. The fluid in both pipes A and B is water, and the gage fluid in the manometer has a specific gravity of 2.6. What is the pressure in pipe B corresponding to the differential reading shown?

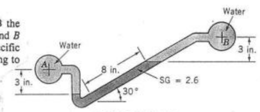


FIGURE P2.43

$$p_A + \gamma_{H_2O}\left(\tfrac{3}{12}\text{ ft}\right) - \gamma_{gf}\left(\tfrac{8}{12}\text{ ft}\right)\sin 30^\circ - \gamma_{H_2O}\left(\tfrac{3}{12}\text{ ft}\right) = p_B$$

(where γ_{gf} is the specific weight of the gage fluid)

Thus,

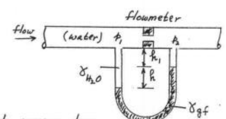
$$p_B = p_A - \gamma_{gf}\left(\tfrac{8}{12}\text{ ft}\right)\sin 30^\circ$$

$$= \left(0.6\,\frac{\text{lb}}{\text{in.}^2}\right)\left(144\,\frac{\text{in.}^2}{\text{ft}^2}\right) - (2.6)\left(62.4\,\frac{\text{lb}}{\text{ft}^3}\right)\left(\tfrac{8}{12}\text{ ft}\right)(0.5) = 32.3\,\frac{\text{lb}}{\text{ft}^2}$$

$$= 32.3\ \text{lb/ft}^2 / 144\ \text{in.}^2/\text{ft}^2 = \underline{\underline{0.224\ \text{psi}}}$$

2.44

2.44 A flowrate measuring device is installed in a horizontal pipe through which water is flowing. A U-tube manometer is connected to the pipe through pressure taps located 3 in. on either side of the device. The gage fluid in the manometer has a specific weight of 112 lb/ft³. Determine the differential reading of the manometer corresponding to a pressure drop between the taps of 0.5 lb/in.².



Let p_1 and p_2 be pressures at pressure taps. Write manometer equation between p_1 and p_2. Thus,

$$p_1 + \gamma_{H_2O}(h_1 + h) - \gamma_{gf}\,h - \gamma_{H_2O}\,h_1 = p_2$$

so that

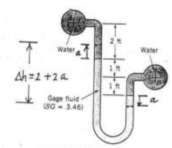
$$h = \frac{p_1 - p_2}{\gamma_{gf} - \gamma_{H_2O}} = \frac{\left(0.5\,\frac{\text{lb}}{\text{in.}^2}\right)\left(144\,\frac{\text{in.}^2}{\text{ft}^2}\right)}{112\,\frac{\text{lb}}{\text{ft}^3} - 62.4\,\frac{\text{lb}}{\text{ft}^3}}$$

$$= \underline{1.45\ \text{ft}}$$

21 bi

2.61

2.61 The manometer fluid in the manometer of Fig. P2.61 has a specific gravity of 3.46. Pipes A and B both contain water. If the pressure in pipe A is decreased by 1.3 psi and the pressure in pipe B increases by 0.9 psi, determine the new differential reading of the manometer.



■ FIGURE P2.61

For the initial configuration:

$$p_A + \gamma_{H_2O}(2) + \gamma_{gf}(2) - \gamma_{H_2O}(1) = p_B \qquad (1)$$

where all lengths are in ft. When p_A decreases to p_A' and p_B increases to p_B' the heights of the fluid columns change as shown on figure. For the final configuration:

$$p_A' + \gamma_{H_2O}(2-a) + \gamma_{gf}(2+2a) - \gamma_{H_2O}(1+a) = p_B' \qquad (2)$$

Subtract Eq.(2) from Eq.(1) to obtain

$$p_A - p_A' + \gamma_{H_2O}(a) - \gamma_{gf}(2a) + \gamma_{H_2O}(a) = p_B - p_B'$$

or

$$a = \frac{(p_B - p_B') - (p_A - p_A')}{2(\gamma_{H_2O} - \gamma_{gf})}$$

Since, $p_A - p_A' = 1.3$ psi, $p_B - p_B' = -0.9$ psi, and $\gamma_{gf} = 3.46\,\gamma_{H_2O}$

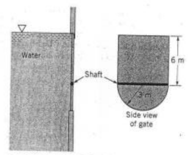
$$a = \frac{\left(-0.9\,\frac{\text{lb}}{\text{in.}^2}\right)\left(144\,\frac{\text{in.}^2}{\text{ft}^2}\right) - \left(1.3\,\frac{\text{lb}}{\text{in.}^2}\right)\left(144\,\frac{\text{in.}^2}{\text{ft}^2}\right)}{2\left(62.4\,\frac{\text{lb}}{\text{ft}^3}\right)(1 - 3.46)} = 1.03\ \text{ft}$$

and therefore

$$\Delta h = 2\text{ ft} + 2a = 2\text{ ft} + 2(1.03\text{ ft}) = \underline{\underline{4.06\ \text{ft}}}$$

2.85

2.85 A gate having the shape shown in Fig. P2.85 is located in the vertical side of an open tank containing water. The gate is mounted on a horizontal shaft. (a) When the water level is at the top of the gate, determine the magnitude of the fluid force on the rectangular portion of the gate above the shaft and the magnitude of the fluid force on the semicircular portion of the gate below the shaft. (b) For this same fluid depth determine the moment of the force acting on the semicircular portion of the gate with respect to an axis which coincides with the shaft.



■ FIGURE P2.85

(a) For rectangular portion,

$(F_R)_r = \gamma h_c A$ where $h_c = 3\,m$

So that

$(F_R)_r = (9800\,\frac{N}{m^3})(3\,m)(6\,m \times 6\,m) = \underline{1060\,kN}$

For semi-circular portion,

$(F_R)_{sc} = \gamma h_c A$ where $h_c = 6\,m + \frac{4R}{3\pi}$ (See Fig. 2.13)

$= 6\,m + \frac{4(3\,m)}{3\pi} = 7.27\,m$

so that

$(F_R)_{sc} = (9800\,\frac{N}{m^3})(7.27\,m)(\frac{\pi}{2}(3\,m)^2) = \underline{1010\,kN}$

(b) For semi-circular portion

$y_R = \frac{I_{xc}}{y_c A} + y_c = \frac{0.1098 R^4}{(7.27\,m)(\frac{\pi}{2})R^2} + 7.27\,m$

$= \frac{0.1098\,(3\,m)^4}{(7.27\,m)(\frac{\pi}{2})(3\,m)^2} + 7.27\,m = 7.36\,m$

Thus, moment with respect to shaft, M,

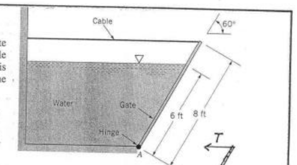
$M = (F_R)_{sc} \times (7.36\,m - 6.00\,m)$

$= (1010 \times 10^3\,N)(1.36\,m)$

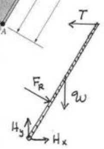
$= \underline{1.37 \times 10^6\,N \cdot m}$

2.87

2.87 A homogeneous, 4-ft-wide, 8-ft-long rectangular gate weighing 800 lb is held in place by a horizontal flexible cable as shown in Fig. P2.87. Water acts against the gate which is hinged at point A. Friction in the hinge is negligible. Determine the tension in the cable.



■ FIGURE P2.87



$F_R = \gamma h_c A$ where $h_c = (\frac{6\,ft}{2})\sin 60°$

Thus,

$F_R = (62.4\,\frac{lb}{ft^3})(\frac{6\,ft}{2})(\sin 60°)(6\,ft \times 4\,ft)$

$= 3890\,lb$

To locate F_R,

$y_R = \frac{I_{xc}}{y_c A} + y_c$ where $y_c = 3\,ft$

so that

$y_R = \frac{\frac{1}{12}(4\,ft)(6\,ft)^3}{(3\,ft)(6\,ft \times 4\,ft)} + 3\,ft = 4.0\,ft$

For equilibrium,

$\Sigma M_H = 0$

and

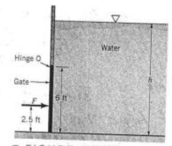
$T\,(8\,ft)(\sin 60°) = w\,(4\,ft)(\cos 60°) + F_R\,(2\,ft)$

$T = \frac{(800\,lb)(4\,ft)(\cos 60°) + (3890\,lb)(2\,ft)}{(8\,ft)(\sin 60°)}$

$= \underline{1350\,lb}$

2.88

2.88 A rectangular gate 6 ft tall and 5 ft wide in the side of an open tank is held in place by the force F as indicated in Fig. P2.88. The weight of the gate is negligible, and the hinge at O is frictionless. **(a)** Determine the water depth, h, if the resultant hydrostatic force of the water acts 2.5 ft above the bottom of the gate, i.e., it is collinear with the applied force F. **(b)** For the depth of part (a), determine the magnitude of the resultant hydrostatic force. **(c)** Determine the force that the hinge puts on the gate under the above conditions.



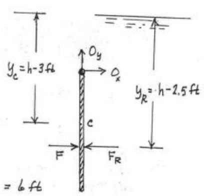
■ FIGURE P2.88

(a) $$y_R - y_c = \frac{I_{xc}}{y_c A}$$

$$(h - 2.5\,ft) - (h - 3.0\,ft) = \frac{\frac{1}{12}(5\,ft)(6\,ft)^3}{(h - 3\,ft)(6\,ft \times 5\,ft)}$$

$$0.5\,ft = \frac{3\,ft^2}{h - 3\,ft}$$

$$\underline{\underline{h = 9.00\,ft}}$$



(b) $F_R = \gamma h_c A$ where $h_c = h - 3\,ft = 6\,ft$

$$= (62.4\,\tfrac{lb}{ft^3})(6\,ft)(6\,ft \times 5\,ft)$$

$$= \underline{\underline{11,200\,lb}}$$

(c) For equilibrium,

$$\sum M_o = 0$$

So that

$$F\,(3.5\,ft) = F_R\,(3.5\,ft)$$

$$F = F_R$$

$$\sum F_x = 0$$

$$O_x + F - F_R = 0$$

$$\underline{\underline{O_x = 0}}$$

$$\sum F_y = 0 \qquad \underline{\underline{O_y = 0}}$$

2.95

2.95 A gate having the cross section shown in Fig. P2.95 closes an opening 5 ft wide and 4 ft high in a water reservoir. The gate weighs 500 lb and its center of gravity is 1 ft to the left of AC and 2 ft above BC. Determine the horizontal reaction that is developed on the gate at C.

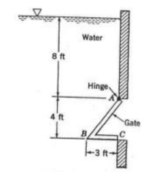


FIGURE P2.95

$F_1 = \gamma h_{c1} A_1$ where $h_{c1} = 8\,ft + 2\,ft$

Thus,

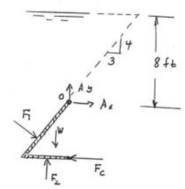
$$F_1 = (62.4\,\tfrac{lb}{ft^3})(10\,ft)(5\,ft \times 5\,ft)$$

$$= 15,600\,lb$$

To locate F_1,

$$y_1 = \frac{I_{xc}}{y_{c1} A_1} + y_{c1}$$

where $y_{c1} = \dfrac{8\,ft}{\frac{4}{5}} + 2.5\,ft = 12.5\,ft$



So that

$$y_1 = \frac{\frac{1}{12}(5\,ft)(5\,ft)^3}{(12.5\,ft)(5\,ft \times 5\,ft)} + 12.5\,ft = 12.67\,ft$$

Also,

$F_2 = p_2 A_2$ where $p_2 = \gamma_{H_2O}\,(8\,ft + 4\,ft)$

so that

$$F_2 = \gamma_{H_2O}\,(12\,ft)(A_2) = (62.4\,\tfrac{lb}{ft^3})(12\,ft)(3\,ft \times 5\,ft) = 11,230\,lb$$

For equilibrium,

$$\sum M_o = 0$$

and

$$F_1\left(y_1 - \frac{8\,ft}{\frac{4}{5}}\right) + W\,(1\,ft) - F_2\left(\tfrac{1}{2}\right)(3\,ft) - F_c\,(4\,ft)$$

so that

$$F_c = \frac{(15,600\,lb)(12.67\,ft - 10\,ft) + (500\,lb)(1\,ft) - (11,230\,lb)(\frac{3}{2}\,ft)}{4\,ft} = \underline{\underline{6330\,lb}}$$