FREDERICK UNIVERSITY CYPRUS

AMEE 202
Introduction to Fluid Mechanics

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SOLUTION
Homework Assignment #3

1. A horizontal water jet with diameter d= 5 cm impacts on a vertical plate (Figure 4). After impact, the velocity normal to the plate is zero. Find the force F.

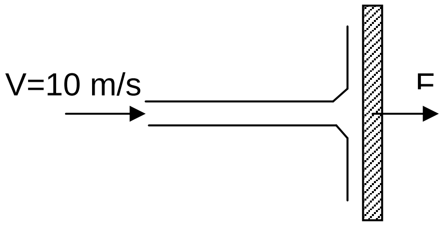


Figure 4: Impinging jet

Momentum Equation

$$\sum F_x = \dot{M}_{out_x} - \dot{M}_{in_x} = 0 - m \cdot u_{out_x} = -1000 \cdot 10 \cdot \frac{0.05^2 \pi}{4} \cdot 10 = -196.25 \text{ N}$$

This is the force acting on the fluid. The force acting on the plate is opposite $F_{R_x} = -F_x = 196.25$ N

2. To propel a light aircraft at an absolute velocity of 240 km/hr against a head wind of 48 km/hr a thrust of 10.3 kN is required. Assuming a theoretical efficiency of 90% and a constant air density of 1.2 kg/m^3 determine the diameter of ideal propeller required and the power needed to drive it. Sketch the velocity and the pressure profile along the slipstream boundary. (The efficiency is $\eta_{th} = \dfrac{u_1}{\frac{1}{2}(u_4 + u_1)}$).

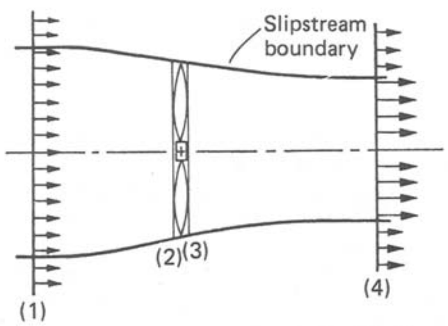


Figure: Velocity field due to a propeller (Question 2)

Consider the control volume enclosed by the slipstream boundary and planes 1 and 4. Momentum equation in the x-direction is:

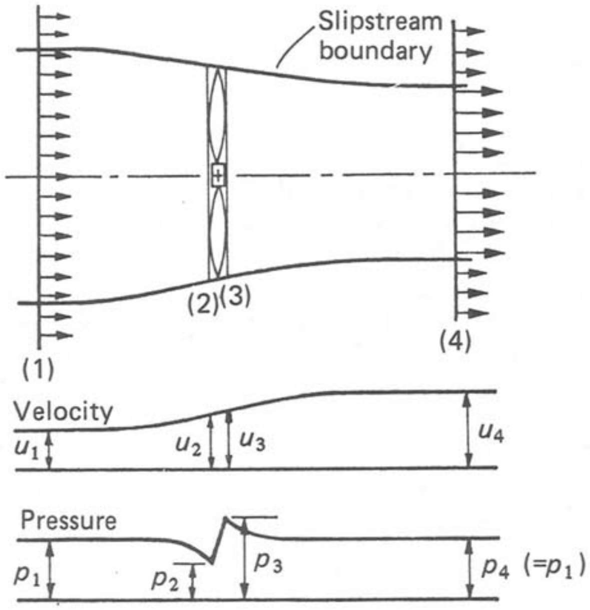
$$F_x = \dot{m}_{out} u_{\text{out}_x} - \dot{m}_{in} u_{\text{in}_x} = \dot{m}(u_4 - u_1)\,,$$

where we have employed the mass conservation equation:

$$\dot{m}_{out} = \dot{m}_{in} \Rightarrow \dot{m}_4 = \dot{m}_1 = \dot{m} = \rho u_2 A\,.$$

The pressure all around the control volume is atmospheric hence $F_x = F_{Rx}$, which is the force from the propeller to the fluid. The thrust force from the fluid to the propeller is $R_x = -F_{Rx}$. We can obtain another equation for F_{Rx}, if we take a control volume between sections 2 and 3, and assume that $u_2 \approx u_3$. The momentum equation is simply

$$F_x = 0 \Rightarrow F_{Rx} + F_{px} = 0 \Rightarrow F_{Rx} = -F_{px} = (p_3 - p_2)A\,.$$



Hence,

$$F_{Rx} = \dot{m}(u_4 - u_1) = (p_3 - p_2)A\,.$$

Employing, Bernoulli's equation between sections 1 and 2 gives:

$$p_1 + \frac{1}{2}\rho u_1^2 = p_2 + \frac{1}{2}\rho u_2^2\,.$$

Similarly, between sections 2 and 3

$$p_3 + \frac{1}{2}\rho u_3^2 = p_4 + \frac{1}{2}\rho u_4^2\,.$$

Note that, Bernoulli's equation cannot be applied between sections 2 and 3 because the flow is unsteady.

Now, $u_2 = u_3$ and also $p_1 = p_4 = \text{pressure of undisturbed fluid}$. Therefore adding the last two equations gives:

$$p_3 - p_2 = \frac{1}{2}\rho(u_4^2 - u_1^2)\,.$$

Combining this with the momentum equation gives:

$$u_2 = \frac{u_1 + u_4}{2}\,.$$

The velocity through the disc then is the arithmetic mean of the velocities upstream and downstream from it; in other words, half the change of the velocity occurs before the disc and half after it.

If the undisturbed fluid be considered stationary, the propeller advances through it with velocity u_1. The product of the thrust and the velocity gives the rate at which the propeller does useful work:

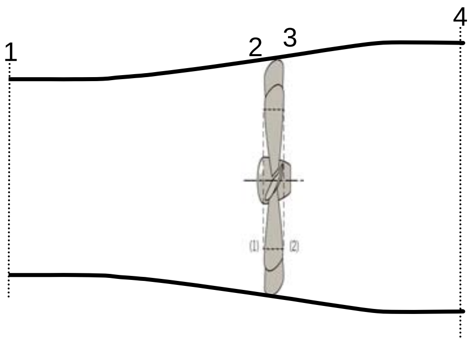
$$\text{Power Output} = F_{th}u_1 = \dot{m}(u_4 - u_1)u_1 .$$

The efficiency is the ratio of the power output to the kinetic energy given to the stream

$$\eta_{Fr} = \frac{\dot{m}(u_4 - u_1)u_1}{\dot{m}(u_4^2 - u_1^2)} = \frac{u_1}{u_1 + \frac{1}{2}(u_4 - u_1)} = \frac{u_1}{\frac{1}{2}(u_4 + u_1)}$$

$$u_1 = 288, u_2 = 320, u_4 = 352, W = F_{th}u_1 / 0.9 = 916\text{kW}, A_2 = \frac{10300}{\rho\, u_2(u_4 - u_1)} .$$

3. An ideal windmill, $12\ \text{m}$ in diameter, operates at a theoretical efficiency of 50% in a $14\ \text{m/s}$ wind. If the air density is $1.235\ \text{kg/m}^3$ determine the thrust on the windmill, the air velocity through the disc, the mean pressures immediately in front of and behind the disc, and the shaft power developed. Note: (the flow pattern of the windmill is the opposite of that of the propeller and takes energy from the fluid; the efficiency is $\eta_{th} = \frac{(u_1 + u_4)(u_1^2 - u_4^2)}{2u_1^3}$).



$$\frac{(u_4 + 14)(14^2 - u_4^2)}{2 \times 14^3} = 0.5 \Rightarrow u_4 = 8.65\ \text{m/s}$$

Momentum Equation between sections 1-4

$$\left.\begin{aligned} F_x &= \rho Q(u_4 - u_1) = \rho u_2 A(u_4 - u_1) \\ u_2 &= \frac{u_1 + u_4}{2} = 11.33\ \text{m/s (see Exercise 1)} \end{aligned}\right\} \Rightarrow F_x = \rho A \frac{(u_1 + u_4)(u_4 - u_1)}{2} = 1.235 \times \pi \frac{12^2}{4}\left(u_4^2 - u_1^2\right)$$

$$= 1.235 \times \pi \frac{12^2}{4} \frac{\left(8.65^2 - 14^2\right)}{2} = 8460\ \text{N}$$

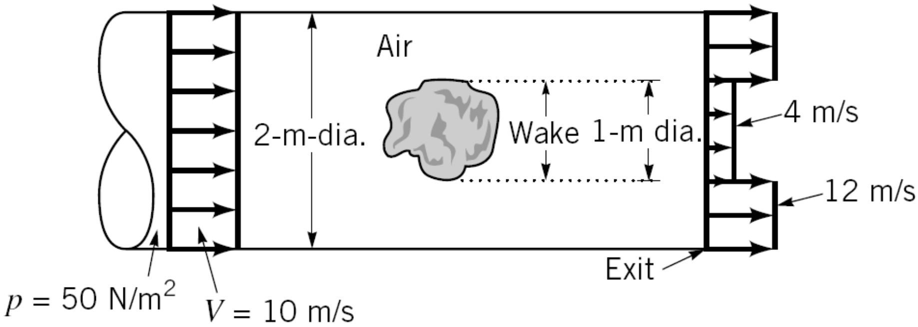
Bernoulli's equation between section 1-2

$$p_1 + \frac{1}{2}\rho u_1^2 = p_2 + \frac{1}{2}\rho u_2^2 \Rightarrow p_2 = \frac{1}{2}\rho u_2^2 - p_{atm} - \frac{1}{2}\rho u_1^2 = 41.8\ \text{Pa}$$

$$p_3 = \frac{1}{2}\rho(u_4^2 - u_1^2) + p_2 \quad \text{(see Exercise 1)} \Rightarrow p_3 = -33\ \text{Pa}$$

The shaft power is 50% of the kinetic energy of the stream. i.e. $W = 0.5\ \frac{1}{2}\ \rho A\, u_1\, u_1^2 = 95.8\ \text{kW}$

4. Air flows past an object in a pipe of $2\ \mathrm{m}$ diameter and exits as a free jet as shown in the figure. The velocity and pressure upstream are uniform at $10\ \mathrm{m/s}$ and $50\ \mathrm{N/m^2}$ respectively. At the pipe exit the velocity is non-uniform as indicated. The shear stress along the pipe wall is negligible. (a) Determine the head loss associated with a particle as it flows from the uniform velocity upstream of the object to a location in the wake at the exit plane of the pipe. (b) Determine the force that the air puts on the object.



Bernoulli's Equation

$$\frac{p_1}{\rho g}+\frac{1}{2}\frac{u_1^2}{g}=\frac{p_2}{\rho g}+\frac{1}{2}\frac{u_2^2}{g}+h_\ell \qquad \text{where } h_\ell \text{ are the losses}$$

$$\Rightarrow h_\ell=\frac{50}{1.225\times 9.81}+\frac{1}{2}\times\frac{10^2}{9.81}-\frac{1}{2}\times\frac{4^2}{9.81}=8.44\ \mathrm{m}$$

Momentum Equation

$$F_x=\dot{M}_{out_x}-\dot{M}_{in_x}=431-384.8=46.22\ \mathrm{N}$$

$$\dot{M}_{in_x}=\dot{m}_{in}\ u_{in_x}=\rho u_{in_x}Au_{in_x}=1.225\times 10^2\times\pi\times\frac{4}{4}=384.8\ \mathrm{N}$$

$$\dot{M}_{out_x}=\dot{m}_{out}\ u_{out_x}=\dot{m}_{out_{annulus}}\ u_{out_{x(annulus)}}+\dot{m}_{out_{disc}}\ u_{out_{x(disc)}}$$

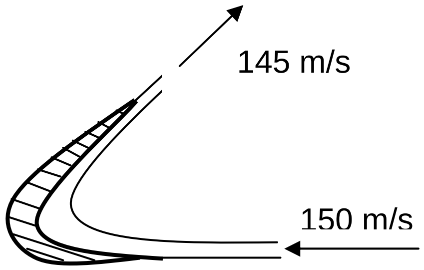
$$=\ 1.225\times 4^2\times\frac{\pi}{4}+1.225\times 4^2\times\left(\pi-\frac{\pi}{4}\right)=431\ \mathrm{N}$$

$$F_x=F_R+F_b+F_p=F_R+F_p=F_R+50\pi$$

$$\Rightarrow F_R=F_x-50\pi=46.22-157=-110.78\ \mathrm{N}$$

$$R=-F_R=110.78\ \mathrm{N}$$

5.



Steam flows smoothly onto a stationary blade and turns through 60^0. The pressure across the blade is constant and the velocity at inlet is 150 m/s. The steam leaves with a velocity 145 m/s. Determine the force (magnitude and direction) exerted on the blade for a volumetric flow rate of 0.9 m^3/s ($\rho = 10\ \text{kg/m}^3$**).**

Mass Conservation

$$\dot{m}_{in} = \dot{m}_{out} = \dot{m}$$

$$\dot{m} = \rho Q = 9\ \text{kg/s}$$

Momentum Equation

$$F_x = \dot{M}_{out_x} - \dot{M}_{in_x} = \dot{m}\left(u_{out_x} - u_{in_x}\right) = 9\left(145\cos 60 - (-150)\right)$$

$$F_y = \dot{M}_{out_y} - \dot{M}_{in_y} = \dot{m}\left(u_{out_y} - u_{in_y}\right) = 9\left(145\sin 60 - 0\right)$$

6. **A horizontal air jet having a velocity of 50 m/s and a diameter of 20 mm strikes the inside surface of a hollow hemisphere as indicated in the figure. How large is the horizontal anchoring force needed to hold the hemisphere in place? The magnitude of velocity of the air remains constant.**

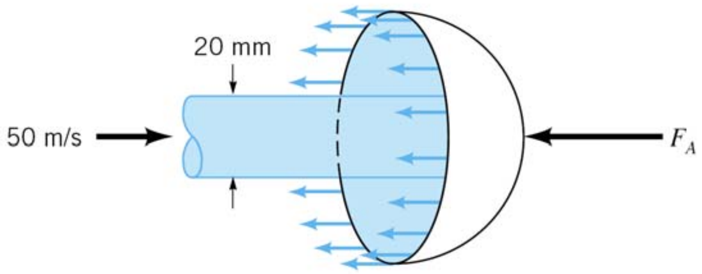
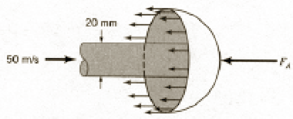


Figure : Jet striking a hollow hemisphere

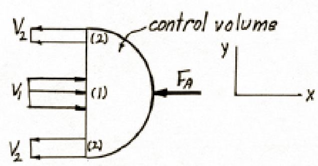
5.9R

5.9R (Linear momentum) A horizontal air jet having a velocity of 50 m/s and a diameter of 20 mm strikes the inside surface of a hollow hemisphere as indicated in Fig. P5.9R. How large is the horizontal anchoring force needed to hold the hemisphere in place? The magnitude of velocity of the air remains constant.

(ANS: 1.93 N)



■ FIGURE P5.9R



The control volume shown in the sketch is used. The x-component of the momentum equation gives

$$\int_{cs} u \rho \vec{V} \cdot \hat{n} \, dA = \sum F_x \quad \text{or}$$

$$(V_1)\rho_1(-V_1)A_1 + (-V_2)\rho_2(+V_2)A_2 = -F_A \,,$$

where for conservation of mass

$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 = \dot{m}$, the mass flowrate.

Thus,

$$F_A = \dot{m}(V_1 + V_2) = 2\dot{m}V \quad \text{since} \quad V_1 = V_2 = 50 \tfrac{m}{s}$$

Note that $V_1 = V_2$ (i.e. the speed is constant), but $\vec{V}_1 = 50\hat{i}\,\frac{m}{s} \neq \vec{V}_2 = -50\hat{i}\,\frac{m}{s}$ (i.e. the velocity changes)

With

$$\dot{m} = \rho_1 A_1 V_1 = 1.23 \tfrac{kg}{m^3}\left(\tfrac{\pi}{4}(0.020m)^2\right)\left(50\tfrac{m}{s}\right) = 0.0193 \tfrac{kg}{s}$$

we obtain

$$F_A = \left(0.0193 \tfrac{kg}{s}\right)\left(50 \tfrac{m}{s}\right)(2) = \underline{\underline{1.93\ N}}$$

7. **The figure shows an experimental flexible hose-nozzle designed to mix two liquids entering at stations 1 and 2 prior to spraying the mixture to atmosphere at station 3. The mixture proportions depend upon the gap** x **that is controlled by axial expansion or contraction of the bellows under the action of the liquid and atmospheric forces on the nozzle. Both liquids have the same density** $\rho = 1000 \text{ kg/m}^3$ **.**

   a. Determine $\dot{m}_1$ **if the stagnation pressure at station 1 is** 5 bars **and** $\dot{m}_2 = 0$ **.**

   b. f the mass flow rate at 1, $\dot{m}_1$ **, remains unaltered determine that** $\dot{m}_2 = 5.34 \text{ kg/s}$ **given that the force applied by the bellows is** 1722 N **.**

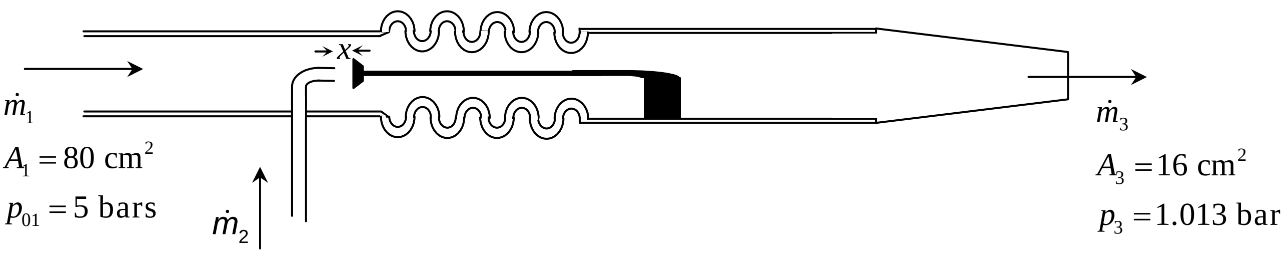


Figure : Hose-nozzle

a. Bernoulli's equation

$$p_1 + \frac{1}{2}\rho u_1^2 = p_3 + \frac{1}{2}\rho u_3^2 = 500000 \quad \text{(ignore hydrostatic pressure)}$$

$$\Rightarrow \frac{1}{2}\rho u_3^2 = 500000 - 101300 = 398700$$

$$\Rightarrow u_3 = \sqrt{\frac{2 \cdot 398700}{1000}} = \sqrt{797.4} = 28.2 \text{ m/s}$$

$$\Rightarrow \dot{m}_3 = \rho u_3 A_3 = 1000 \cdot 28.2 \cdot \frac{16}{10000} = 45.12 \text{ kg/s} = \dot{m}_1$$

$$u_1 = \frac{\dot{m}_1}{\rho A_1} = \frac{45.12 \cdot 10000}{1000 \cdot 80} = 5.64 \text{ m/s}$$

b. Momentum equation

$$\dot{m}_3 u_3 - \dot{m}_1 u_1 = -1722 + \underbrace{\left((500000 - \frac{1}{2}\rho u_1^2) - 101300 \right)}_{p_1 \text{(gage)}} A_1$$

$$\Rightarrow \frac{\dot{m}_3^2}{\rho A_3} = -1722 + \left((500000 - \frac{1}{2}\rho u_1^2) - 101300 \right) A_1 + \dot{m}_1 u_1$$

$$\Rightarrow \dot{m}_3^2 = 1594.84 \cdot 1000 \cdot 0.0016$$

$$\Rightarrow \dot{m}_2 = \dot{m}_3 - \dot{m}_1 = 50.5 - 45.16 = 5.34 \text{ kg/s}$$

8. A set of guide vanes deflects a stream of warm air onto painted parts, as illustrated in Figure 6. The guide vanes are rotated slowly to distribute the air evenly over the parts. Compute the force R_y required to hold the guide vanes in place when the angle $\theta = 45^o$ and the air is flowing at a velocity of 4 m/s. Assume that all the air that approaches a given guide vane is deflected to the angle of the vane. The air has density $\rho_{air} = 1.225 \text{ kg/m}^3$.

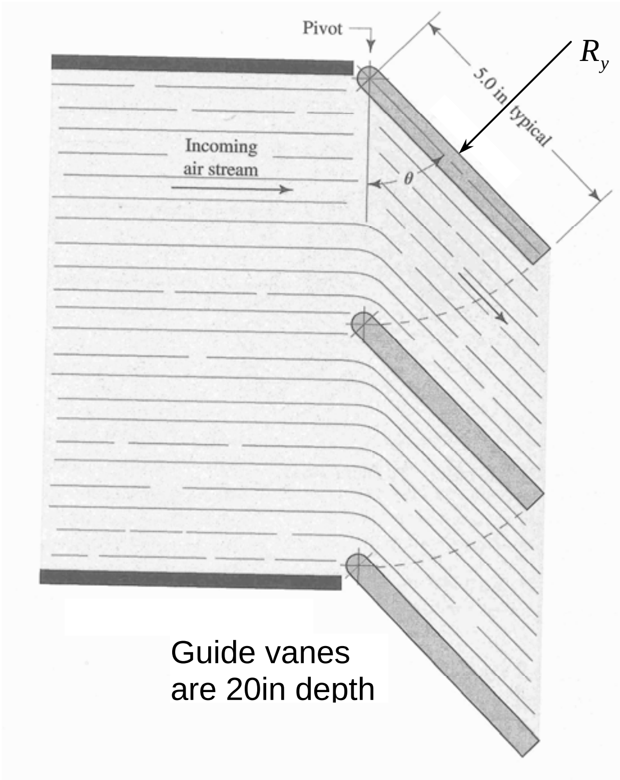


Figure: Stream of air through guide vanes

Mass Conservation

$\dot{m}_{in} = \dot{m}_{out} = \dot{m}$

$\dot{m} = \rho Q = \rho u A = 1.225 \times 4 \times (5 \times 0.0254 \times 20 \times 0.0254) = 0.316 \text{ kg/s}$

Momentum Equation

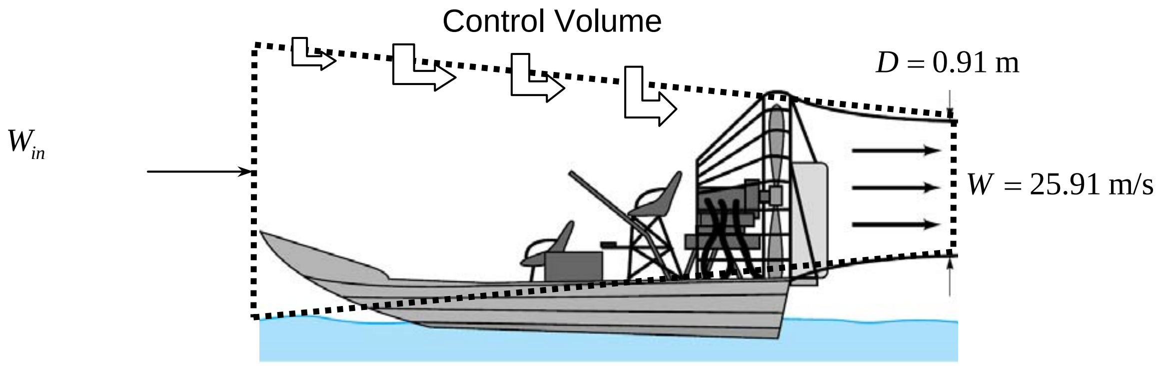
$F_y = \dot{M}_{out_y} - \dot{M}_{in_y} = \dot{m}\left(u_{out_y} - u_{in_y}\right) = 0.316\left(0 - 4\sin 45\right) = -0.894 \text{ N}$

F_y is the force acting on the fluid. On the guide vane it will be equal and opposite

$R = \text{-}F_y = 0.894 \text{ N}$

(Pressure is assumed uniform)

9. The propeller on a swamp boat produces a jet of air having a diameter of $D = 0.91\ \text{m}$ as illustrated in the figure. The ambient air density is $\rho = 1.178\ \text{kg/m}^3$ and the axial velocity of the flow is $25.91\ \text{m/s}$ relative to the boat. What propulsive forces are produced by the propeller when the boat is stationary and when the boat moves forward with a constant velocity of $6.1\ \text{m/s}$?



Choose a control volume such that the left side is far enough from the propeller such that the relative velocity is zero. Then, the air mass flux in would correspond to the mass flux from the sides. The total mass flux in must be equal to the total mass flux out:

a. When the boat is stationary

$$\dot{m}_{out} = \dot{m}_{in} = \dot{m} = \rho\ u_{out} A_{out} = 1.178 \cdot 25.91 \cdot \pi \frac{0.91^2}{4} = 19.851\ \text{kg/s}$$

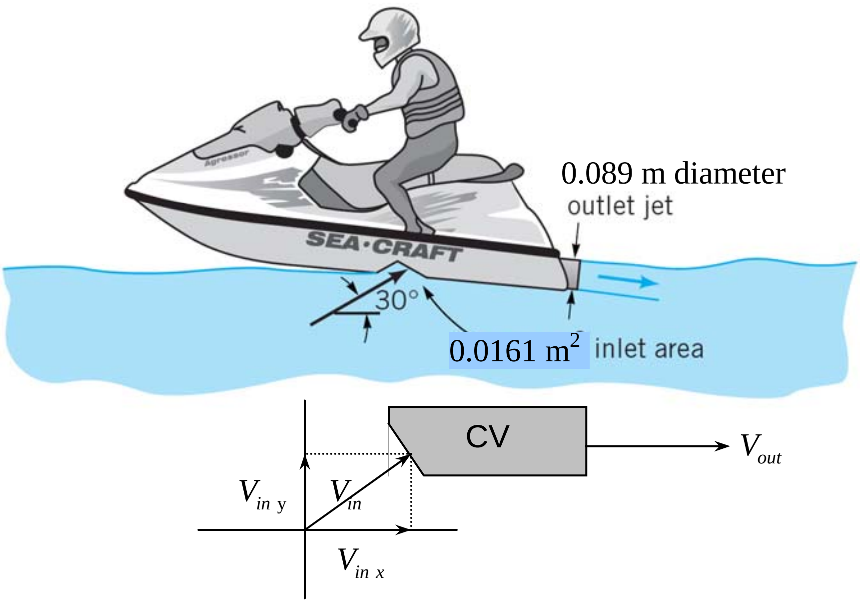
$$F_x = \dot{M}_{out_x} - \dot{M}_{in_x} = \dot{m}\ (u_{out_x} - \overbrace{u_{in_x}}^{\approx 0}) = 19.8511 * 25.91 = 514.34\ \text{N}$$

a. When the boat is moving

Use relative velocities $\dot{m}_{out} = \dot{m}_{in} = \dot{m} = \rho\ W_{out} A_{out} = 1.178 \cdot 25.91 \cdot \pi \dfrac{0.91^2}{4} = 19.851\ \text{kg/s}$

$$F_x = \dot{M}_{out_x} - \dot{M}_{in_x} = \dot{m}\ (W_{out_x} - \overbrace{W_{in_x}}^{=6.1}) = 19.8511 * (25.91 - 6.1) = 393.25\ \text{N}$$

10. The thrust developed to propel the jet ski shown in the figure is a result of water pumped through the vehicle and exiting as a high-speed water jet. For the conditions shown in the figure, what flowrate is needed to produce a 1334 N thrust? Assume the inlet and outlet jets of water are free jets.



Mass Conservation

$$\dot{m}_{in} = \dot{m}_{out} \;\Rightarrow\; \rho W_{in} A_{in} = \rho W_{out} A_{out} \;\Rightarrow\; W_{in} = \frac{W_{out} A_{out}}{A_{in}}$$

Momentum Conservation

$$F_x = \dot{m}(W_{out\ x} - W_{in\ x}) = \dot{m}\left(W_{out} - W_{in}\cos(30)\right) = \dot{m}\left(W_{out} - \frac{W_{out} A_{out}}{A_{in}}\cos(30)\right)$$

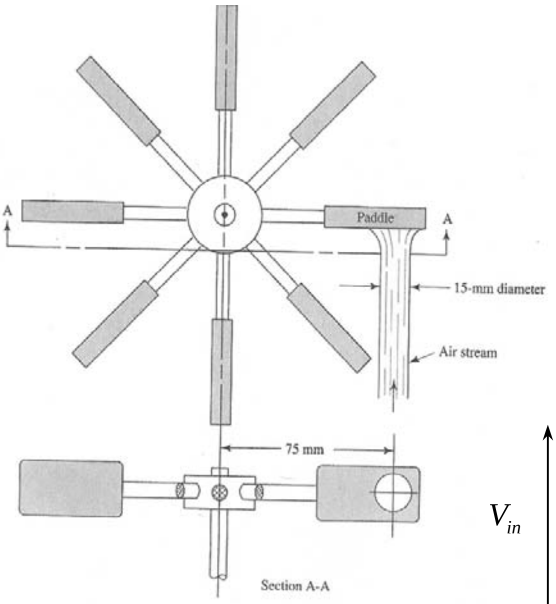
Note: $W_{\text{out}\,x} = W_{\text{out}}$ and $A_{out} = \pi D^2/4 = 0.0062\ \text{m}^2$

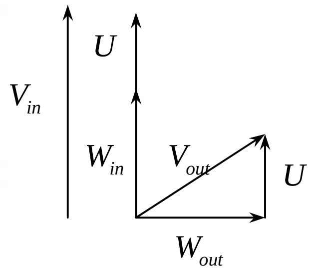
$$F_x = \rho W_{out} A_{out}\left(W_{out} - \frac{W_{out} A_{out}}{A_{in}}\cos(30)\right) \;\Rightarrow\; W_{out}^2 = \frac{F_x}{\rho A_{out}}\left(1 - \frac{A_{out}\cos(30)}{A_{in}}\right)^{(-1)}$$

$$\Rightarrow W_{out} = \sqrt{\frac{F_x}{\rho A_{out}} \Big/ \left(1 - \frac{A_{out}\cos(30)}{A_{in}}\right)} \;\Rightarrow\; \dot{m} = \rho A_{out}\sqrt{\frac{F_x}{\rho A_{out}} \Big/ \left(1 - \frac{A_{out}\cos(30)}{A_{in}}\right)}$$

$$\dot{m} = \sqrt{\rho A_{out} F_x \Big/ \left(1 - \frac{A_{out}\cos(30)}{A_{in}}\right)} = \sqrt{1000 * 0.0062 * 1334 \Big/ \left(1 - \frac{0.0062\cos(30)}{0.0161}\right)} = 111\ \text{kg/s}$$

11. Shown in the figure is a small decorative wheel fitted with flat paddles so the wheel turns about its axis when acted on by a blown stream of air. Assuming that all the air in a $15-\mathrm{mm}$ **diameter stream moving at** $0.35\ \mathrm{m/s}$ **strikes one paddle and is deflected by it at right angles, compute the force exerted on the wheel initially when it is stationary. The air has density** $1.20\ \mathrm{kg/m^3}$**. Compute the force exerted on the paddle when the wheel rotates at** $40\ \mathrm{rpm}$**. Estimate the power and torque and plot it in terms of the paddle speed.**





If the wheel is stationary

$$F_y = \dot{m}\ (V_{out\ y} - V_{\text{in}\ y}) = \rho\ V_{\text{in}\ y}\ A_{in}(V_{out\ y} - V_{\text{in}\ y})$$

$$= 1.2 * 0.35 * \left(\frac{\pi * 0.015^2}{4}\right)(0 - 0.35) = -2.6 * 10^{-5}\,\mathrm{N}$$

If the wheel is moving

$$F_y = \dot{m}\ (V_{out\ y} - V_{\text{in}\ y}) = \rho\ V_{\text{in}\ y}\ A_{in}(V_{out\ y} - V_{\text{in}\ y})$$

In order to estimate V_{out} we assume that there are no mechanical losses hence $W_{in} = W_{out}$.

From the velocity diagram we can deduce

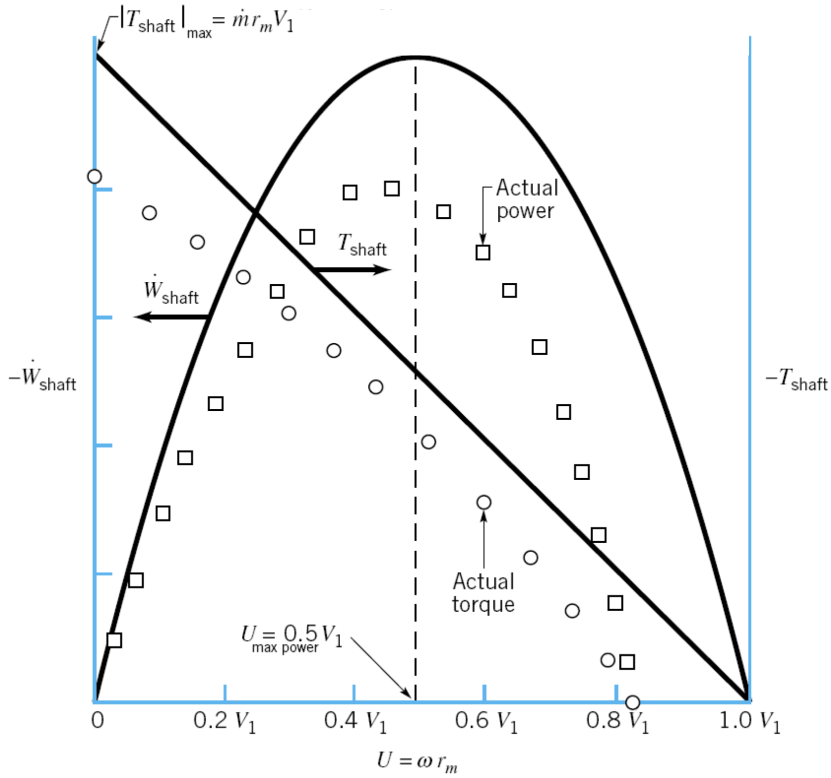
$$V_{out\ y} = U = \omega r = \frac{40 \cdot 2 \cdot \pi}{60} \cdot 0.075 = 0.314\ \mathrm{m/s}$$

Hence

$$F_y = \rho\ V_{\text{in}}\ A_{in}(U - V_{in}) = 1.2 \cdot 0.35 \cdot 0.000177 \cdot (0.314 - 0.35) = -2.67 \cdot 10^{-6}\,N$$

$$\text{Torque}(T) = F \cdot r = r\rho\ V_{\text{in}}\ A_{in}(U - V_{in})$$

$$\text{Power}(\dot{W}) = T \cdot \omega = \omega r \rho\ V_{\text{in}}\ A_{in}(U - V_{in}) = \dot{m}U(U - V_{in})$$



$|T_{shaft}|_{max} = \dot{m} r_m V_1$
Actual power
T_{shaft}
$\dot{W}_{shaft}$
$-\dot{W}_{shaft}$
$-T_{shaft}$
Actual torque
$U_{max\ power} = 0.5 V_1$
0
$0.2\ V_1$
$0.4\ V_1$
$0.6\ V_1$
$0.8\ V_1$
$1.0\ V_1$
$U = \omega r_m$

12. **A Pelton wheel vane directs a horizontal, circular cross-sectional jet of water symmetrically as indicated in Figure 1. The jet leaves the nozzle with a velocity of $30\ \mathrm{m/s}$. Determine the x-direction component of anchoring force required to:**
    a. **Hold the vane stationary.**
    b. **Confine the speed of the vane to a value of $3\ \mathrm{m/s}$ to the right. The fluid speed magnitude remains constant along the vane surface.**

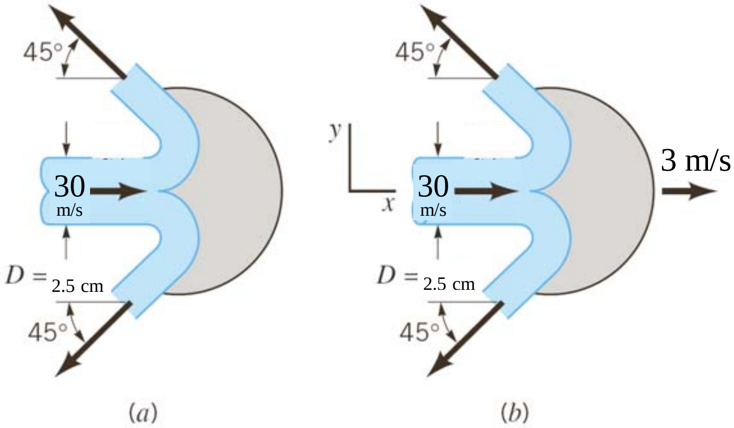


Figure: Jet impinging on a vane

Mass Conservation

$\dot{m}_{in} = \dot{m}_{out} = \dot{m}$

Momentum Equation

a. $F_x = \dot{M}_{out_x} - \dot{M}_{in_x} = \dot{m}\left(u_{out_x} - u_{in_x}\right) = \rho u A\left(-30\sin 45 - 30\right) =$

$= -1000 \times 30 \times \dfrac{\pi \times 0.025^2}{4}\left(30\sin 45 + 30\right) = -805\ \mathrm{N}$

Body forces $F_b = 0$

Pressure forces $F_p = 0$ (uniform pressure)

b. $W_{in} = V - U_{\mathrm{cv}} = 30 - 3 - 27$

$F_x = \dot{M}_{out_x} - \dot{M}_{in_x} = \dot{m}\left(W_{out_x} - W_{in_x}\right) = \rho W_{in} A\left(W_{out}\sin 45 - W_{in_x}\right) =$

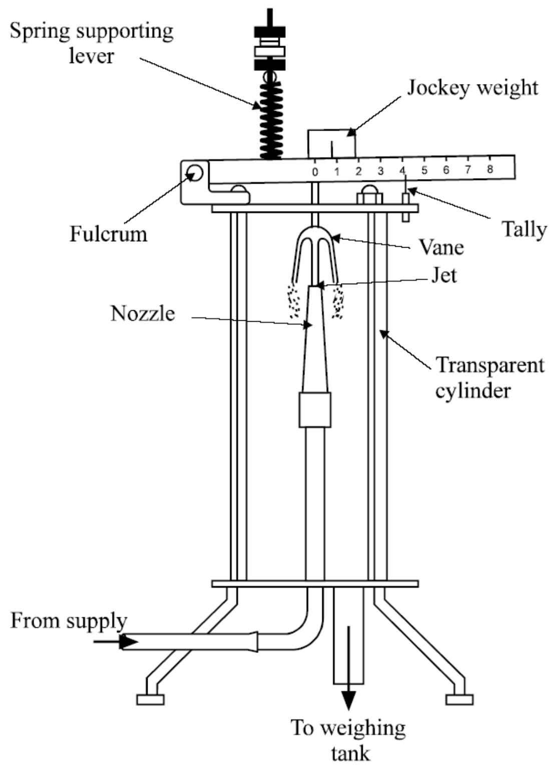
$= -1000 \times 27 \times \dfrac{\pi \times 0.025^2}{4}\left(27\sin 45 + 27\right) = -652\ \mathrm{N}$

13. In an experiment to determine the force due to the impact of a jet on a vane (see figure) the following measurements were taken:

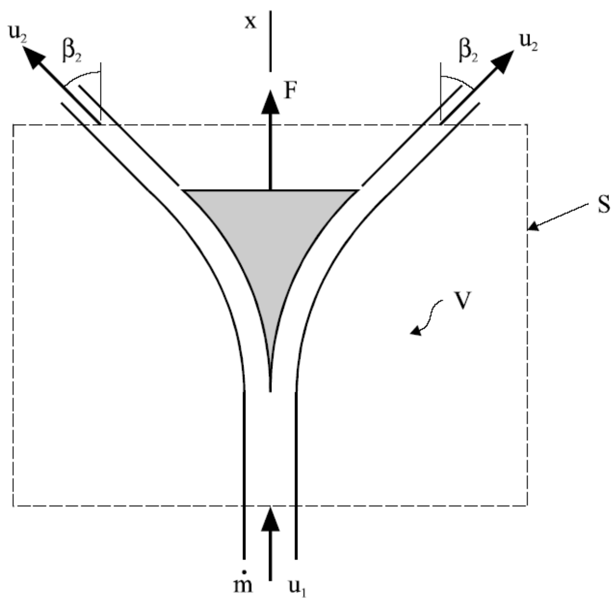
a. The volume of water gathered was $5\ \text{lts}$ **in** $40\ \text{secs}$

b. The diameter of the nozzle is $10\ \text{mm}$

c. The vane used was a hemispherical cup

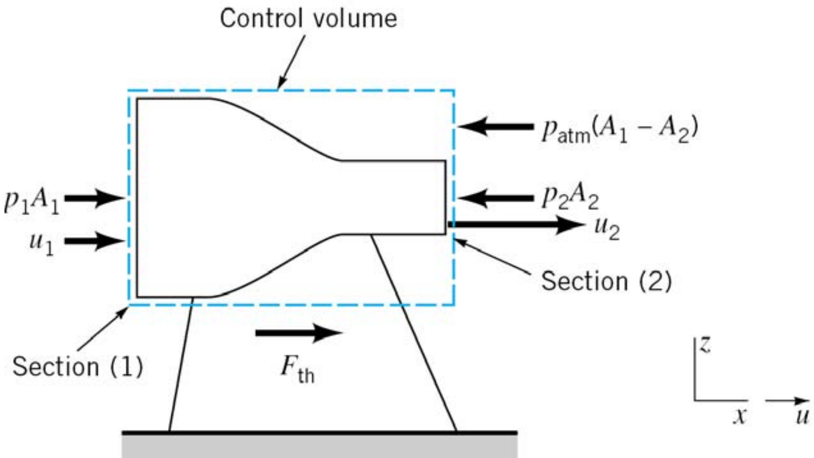


i. Determine the force on the vane by considering the following control volume.



ii. What mass is required to be hanged on the lever to restore the lever to its original position. The distance from the vane to the pivot is $150\ \text{mm}$ **and from the hanging weight to the pivot is** $210\ \text{mm}$**.**

14. A static thrust stand as sketched in the Figure is to be designed for testing a jet engine. The following conditions are known for a typical test: Intake air velocity $= 200$ m/s **; exhaust gas velocity** $= 500$ m/s **; intake cross sectional area** $= 1\ \text{m}^2$ **; intake static pressure** $= -22.5$ kPa=78.5 kPa (abs); **intake static temperature** $= 268$ K **; exhaust static pressure** $= 0$ kPa=101 kPa (abs)**. Estimate the thrust.**



Note: Intake air density can be calculated using the ideal gas law

$$\rho_1 = \frac{p_1}{R\ T_1} = \frac{78500}{286.9 \times 268} = 1.02\ \text{kg/m}^3$$

The cylindrical control volume outlined with a dashed line in Fig. E5.15 is selected. The external forces acting in the axial direction are also shown. Application of the momentum equation (Eq. 5.22) to the contents of this control volume yields

$$\int_{cs} u\rho \mathbf{V} \cdot \hat{\mathbf{n}}\, dA = p_1A_1 + F_{th} - p_2A_2 - p_{atm}(A_1 - A_2) \qquad (1)$$

where the pressures are absolute. Thus, for one-dimensional flow, Eq. 1 becomes

$$(+u_1)(-\dot{m}_1) + (+u_2)(+\dot{m}_2) = (p_1 - p_{atm})A_1 - (p_2 - p_{atm})A_2 + F_{th} \qquad (2)$$

The positive direction is to the right. The conservation of mass equation (Eq. 5.12) leads to

$$\dot{m} = \dot{m}_1 = \rho_1A_1u_1 = \dot{m}_2 = \rho_2A_2u_2 \qquad (3)$$

Combining Eqs. 2 and 3 and using gage pressure we obtain

$$\dot{m}(u_2 - u_1) = p_1A_1 - p_2A_2 + F_{th} \qquad (4)$$

Solving Eq. 4 for the thrust force, F_{th}, we obtain

$$F_{th} = -p_1A_1 + p_2A_2 + \dot{m}(u_2 - u_1) \qquad (5)$$

We need to determine the mass flowrate, $\dot{m}$, to calculate F_{th}, and to calculate $\dot{m} = \rho_1A_1u_1$, we need ρ_1. From the ideal gas equation of state

$$\rho_1 = \frac{p_1}{RT_1} = \frac{(78.5\ \text{kPa})(1000\ \text{Pa/kPa})[1(\text{N/m}^2)/\text{Pa}]}{(286.9\ \text{J/kg}\cdot\text{K})(268\ \text{K})(1\ \text{N}\cdot\text{m/J})} = 1.02\ \text{kg/m}^3$$

Thus,

$$\dot{m} = \rho_1 A_1 u_1 = (1.02\ \text{kg/m}^3)(1\ \text{m}^2)(200\ \text{m/s}) = 204\ \text{kg/s} \qquad \textbf{(6)}$$

Finally, combining Eqs. 5 and 6 and substituting given data with $p_2 = 0$, we obtain

$$F_{\text{th}} = -(1\ \text{m}^2)(-22.5\ \text{kPa})(1000\ \text{Pa/kPa})[1(\text{N/m}^2)/\text{Pa}]$$
$$+ (204\ \text{kg/s})(500\ \text{m/s} - 200\ \text{m/s})[1\ \text{N}/(\text{kg} \cdot \text{m/s}^2)]$$

and

$$F_{\text{th}} = 22{,}500\ \text{N} + 61{,}200\ \text{N} = 83{,}700\ \text{N} \qquad \textbf{(Ans)}$$

The force of the thrust stand on the engine is directed toward the right. Conversely, the engine pushes to the left on the thrust stand (or aircraft).