

## 4.8 Problems



**Problem 4.1** Let  $s \in \mathbb{C}$ . Use SymPy to perform a partial fraction expansion on the following expression:

$$\frac{(s+2)(s+10)}{s^4 + 8s^3 + 117s^2 + 610s + 500}.$$

**Problem 4.2** Let  $x, a_1, a_2, a_3, a_4 \in \mathbb{R}$ . Use SymPy to combine the cosine and sine terms that share arguments into single sinusoids with phase shifts in the following expression:

$$a_1 \sin(x) + a_2 \cos(x) + a_3 \sin(2x) + a_4 \cos(2x)$$

**Problem 4.3** Consider the following equation, where  $x \in \mathbb{C}$  and  $a, b, c \in \mathbb{R}_+$ ,

$$ax^2 + bx + \frac{c}{x} + b^2 = 0.$$

Use SymPy to solve for  $x$ .

**Problem 4.4** Let  $w, x, y, z \in \mathbb{R}$ . Consider the following system of equations:

$$8w - 6x + 5y + 4z = -20$$

$$2y - 2z = 10$$

$$2w - x + 4y + z = 0$$

$$w + 4x - 2y + 8z = 4.$$

Use SymPy to solve the system for  $w, x, y$ , and  $z$ .

**Problem 4.5** Consider the truss shown in figure 4.8. Use a static analysis and the method of joints to develop a solution for the force in each member  $F_{AC}$ ,  $F_{AD}$ , etc., and the reaction forces using the sign convention that tension is positive and compression is negative. The forces should be expressed in terms of the applied force  $f_D$  and the dimensions  $w$  and  $h$  only. Write a program that *solves for the forces symbolically* and answers the following questions:

- Which members are in tension?
- Which members are in compression?
- Are there any members with 0 nominal force? If so, which?
- Which member (or members) has (or have) the maximum compression?
- Which member (or members) has (or have) the maximum tension?

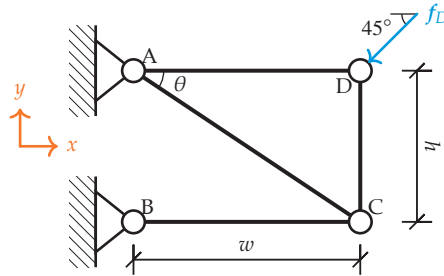



Figure 4.8. A truss with pinned joints, supported by two hinges, with an applied load  $f_D$ .

**Problem 4.6**  49 You are designing the truss structure shown in figure 4.9, which is to support the hanging of an external load  $f_C = -f_C \hat{j}$ , where  $f_C > 0$ . Your organization plans to offer customers the following options:

- Any width (i.e.,  $2w$ )
- A selection of maximum load magnitudes  $L = f_C / \alpha \in \Gamma$ , where  $\Gamma = \{1, 2, 4, 8, 16\}$  kN, and where  $\alpha$  is the factor of safety

As the designer, you are to develop a design curve for the dimension  $h$  versus half-width  $w$  for each maximum load  $L \in \Gamma$ , under the following design constraints:

- Minimize the dimension  $h$
- The tension in all members is no more than a given  $T$
- The compression in all members is no more than a given  $C$
- The magnitude of the support force at pin A is no more than a given  $P_A$
- The magnitude of the support force at pin D is no more than a given  $P_D$

Use a static analysis and the method of joints to develop a solution for the force in each member  $F_{AB}$ ,  $F_{AC}$ , etc., and the reaction forces using the sign convention that tension is positive and compression is negative. Create a Python function that returns  $h$  as a function of  $w$  for a given set of design parameters  $\{T, C, P_A, P_D, \alpha, L\}$ . Use the function to create a design curve  $h$  versus  $2w$  for each  $L \in \Gamma$ , maximum tension  $T = 81$  kN, maximum compression  $C = 81$  kN, maximum support A load  $P_A = 50$  kN, maximum support D load  $P_D = 50$  kN, and a factor of safety of  $\alpha = 5$ .

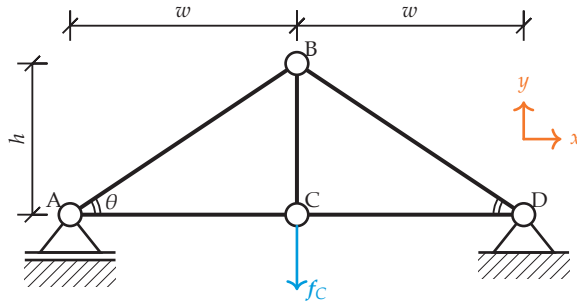



Figure 4.9. A truss with pinned joints, supported by a hinge and a floating support, with an applied load  $f_C$ .

**Problem 4.7**  Consider an LTI system modeled by the state equation of the state-space model, equation (4.24a). A **steady state** of a system is defined as the state vector  $\mathbf{x}(t)$  after the effects of initial conditions have become relatively small. For a constant input  $\mathbf{u}(t) = \bar{\mathbf{u}}$ , the constant state  $\bar{\mathbf{x}}$  toward which the system's response decays can be found by setting the time derivative vector  $\mathbf{x}'(t) = \mathbf{0}$ .

Write a Python function `steady_state()` that accepts the following arguments:

- A: A symbolic matrix representing  $A$
- B: A symbolic matrix representing  $B$
- $\mathbf{u\_const}$ : A symbolic vector representing  $\bar{\mathbf{u}}$

The function should return  $\mathbf{x\_const}$ , a symbolic vector representing  $\bar{\mathbf{x}}$ .

The steady-state output converges to  $\bar{\mathbf{y}}$  the corresponding output equation of the state-space model, equation (4.24b). Write a second Python function `steady_output()` that accepts the following arguments:

- C: A symbolic matrix representing  $C$
- D: A symbolic matrix representing  $D$
- $\mathbf{u\_const}$ : A symbolic vector representing  $\bar{\mathbf{u}}$
- $\mathbf{x\_const}$ : A symbolic vector representing  $\bar{\mathbf{x}}$

This function should return  $\mathbf{y\_const}$ , a symbolic vector representing  $\bar{\mathbf{y}}$ .

Apply `steady_state()` and `steady_output()` to the state-space model of the circuit shown in figure 4.10, which includes a resistor with resistance  $R$ , an inductor with inductance  $L$ , and capacitor with capacitance  $C$ . The LTI system is represented by equation (4.24) with state, input, and output vectors

$$\mathbf{x}(t) = \begin{bmatrix} v_C(t) \\ i_L(t) \end{bmatrix}, \quad \mathbf{u}(t) = [V_S], \quad \mathbf{y}(t) = \begin{bmatrix} v_C(t) \\ v_L(t) \end{bmatrix}$$

and the following matrices:

$$A = \begin{bmatrix} 0 & 1/C \\ -1/L & -R/L \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1/L \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ -1 & -R \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Furthermore, let the constant input vector be

$$\bar{u} = [\bar{V}_S],$$

for constant  $\bar{V}_S$ .

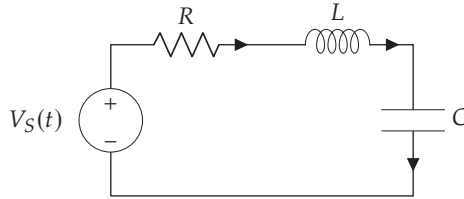



Figure 4.10. An RLC circuit with a voltage source  $V_S(t)$ .

**Problem 4.8**  Consider the electromechanical state-space model described in example 4.3. For a given set of parameters, input voltage, and initial conditions, the following vector-valued functions have been derived:

$$F = \begin{bmatrix} \int_0^t v_R(t) dt \\ \int_0^t v_L(t) dt \\ \int_0^t \Omega_B(t) dt \\ \int_0^t \Omega_J(t) dt \end{bmatrix} = \begin{bmatrix} \exp(-t) \\ \exp(-t) \\ 1 - \exp(-t) \\ 1 - \exp(-t) \end{bmatrix}, \quad G = \begin{bmatrix} \int_0^t i_R(t) dt \\ \int_0^t i_L(t) dt \\ \int_0^t T_B(t) dt \\ \int_0^t T_J(t) dt \end{bmatrix} = \begin{bmatrix} \exp(-t) \\ \exp(-t) \\ 1 - \exp(-t) \\ \exp(-t) \end{bmatrix}$$

The instantaneous power loss or stored by each element is given by the following vector of products:

$$\mathcal{P}(t) = \begin{bmatrix} v_R(t)i_R(t) \\ v_L(t)i_L(t) \\ \Omega_B(t)T_B(t) \\ \Omega_J(t)T_J(t) \end{bmatrix}.$$


The energy  $\mathcal{E}(t)$  of the elements, then, is

$$\mathcal{E}(t) = \int_0^t \mathcal{P}(t) dt.$$

Write a program that satisfies the following requirements:

- It defines a function `power(F, G)` that returns the symbolic power vector  $\mathcal{P}(t)$  from any inputs  $F$  and  $G$

- b. It defines a function `energy(F, G)` that returns the symbolic energy  $\mathcal{E}(t)$  from any inputs  $F$  and  $G$  (`energy()` should call `power()`)
- c. It tests the `energy()` on the specific  $F$  and  $G$  given above

**Problem 4.9**  For the circuit and state-space model given in problem 4.7, use SymPy to solve for  $x(t)$  and  $y(t)$  given the following:

- A constant input voltage  $V_S(t) = \overline{V_S}$
- Initial condition  $x(0) = \mathbf{0}$

Substitute the following parameters into the solution for  $y(t)$  and create numerically evaluable functions of time for each variable in  $y(t)$ :

$$R = 50 \, \Omega, L = 10 \cdot 10^{-6} \, \text{H}, C = 1 \cdot 10^{-9} \, \text{F}, \overline{V_S} = 10 \, \text{V}.$$

Plot the outputs in  $y(t)$  as functions of time, making sure to choose a range of time over which the response is best presented. *Hint:* An appropriate amount of time is on the scale of microseconds.



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## 5.1 Sets of Linear Algebraic Equations



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