

4.8 Problems



Problem 4.1 Let $s \in \mathbb{C}$. Use SymPy to perform a partial fraction expansion on the following expression:

$$\frac{(s+2)(s+10)}{s^4 + 8s^3 + 117s^2 + 610s + 500}.$$

Problem 4.2 Let $x, a_1, a_2, a_3, a_4 \in \mathbb{R}$. Use SymPy to combine the cosine and sine terms that share arguments into single sinusoids with phase shifts in the following expression:

$$a_1 \sin(x) + a_2 \cos(x) + a_3 \sin(2x) + a_4 \cos(2x)$$

Problem 4.3 Let $s \in \mathbb{C}$ and $a_i, b_j \in \mathbb{R}$. Consider the following expression:

$$\frac{a_3 s^3 + a_2 s^2}{a_1 s + a_0} \cdot \frac{b_2 s^2}{b_1 s + b_0}.$$

Use SymPy to write the expression as a fraction with the numerator and denominator as polynomials in s with the terms collected such that each power of s appears only once in each polynomial.

Problem 4.4 Let $t \in \mathbb{R}$. Use SymPy to simplify the following expression:

$$\frac{\cos(t) + \sin(t)}{\cos(t) - \sin(t)} + (\cos(t) + \sin(t))^2.$$

First try using the `simplify()` function. Then try using the `trigsimp()` function.

Problem 4.5 Let $x, y, z \in \mathbb{R}$ be variables and $a, b, c \in \mathbb{R}$ be parameters. Suppose we have the following solutions for x , y , and z in terms of a , b , and c :

$$x = a^2 + b^2 \tag{4.25}$$

$$y = a^2 - b^2 \tag{4.26}$$

$$z = a^2 b^2. \tag{4.27}$$

Furthermore, suppose the numerical values of the parameters are

$$a = 1, \quad b = 2, \quad c = 3.$$

Use SymPy to substitute the solutions for x , y , and z into the following expression:

$$\frac{x+y}{x-y} + (x+y)^2 + (x-y)^2 + z^2.$$

The resulting expression should be in terms of a , b , and c only. Now substitute the numerical values of the parameters into the expression and evaluate it. The final result should be a single number.

Problem 4.6  **QR** Consider the following equation, where $x \in \mathbb{C}$ and $a, b, c \in \mathbb{R}_+$,

$$ax^2 + bx + \frac{c}{x} + b^2 = 0.$$

Use SymPy to solve for x .

Problem 4.7  **Q9** Let $w, x, y, z \in \mathbb{R}$. Consider the following system of equations:


$$8w - 6x + 5y + 4z = -20$$

$$2y - 2z = 10$$

$$2w - x + 4y + z = 0$$

$$w + 4x - 2y + 8z = 4.$$

Use SymPy to solve the system for w , x , y , and z .

Problem 4.8  **IV** Consider the truss shown in figure 4.8. Use a static analysis and the method of joints to develop a solution for the force in each member F_{AC} , F_{AD} , etc., and the reaction forces using the sign convention that tension is positive and compression is negative. The forces should be expressed in terms of the applied force f_D and the dimensions w and h only. Write a program that *solves for the forces symbolically* and answers the following questions:

- Which members are in tension?
- Which members are in compression?
- Are there any members with 0 nominal force? If so, which?
- Which member (or members) has (or have) the maximum compression?
- Which member (or members) has (or have) the maximum tension?

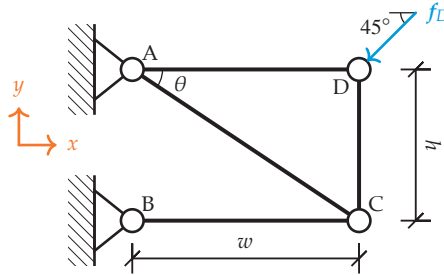



Figure 4.8. A truss with pinned joints, supported by two hinges, with an applied load f_D .

Problem 4.9  You are designing the truss structure shown in figure 4.9, which is to support the hanging of an external load $f_C = -f_C \hat{j}$, where $f_C > 0$. Your organization plans to offer customers the following options:

- Any width (i.e., $2w$)
- A selection of maximum load magnitudes $L = f_C/\alpha \in \Gamma$, where $\Gamma = \{1, 2, 4, 8, 16\}$ kN, and where α is the factor of safety

As the designer, you are to develop a design curve for the dimension h versus half-width w for each maximum load $L \in \Gamma$, under the following design constraints:

- Minimize the dimension h
- The tension in all members is no more than a given T
- The compression in all members is no more than a given C
- The magnitude of the support force at pin A is no more than a given P_A
- The magnitude of the support force at pin D is no more than a given P_D

Use a static analysis and the method of joints to develop a solution for the force in each member F_{AB} , F_{AC} , etc., and the reaction forces using the sign convention that tension is positive and compression is negative. Create a Python function that returns h as a function of w for a given set of design parameters $\{T, C, P_A, P_D, \alpha, L\}$. Use the function to create a design curve h versus $2w$ for each $L \in \Gamma$, maximum tension $T = 81$ kN, maximum compression $C = 81$ kN, maximum support A load $P_A = 50$ kN, maximum support D load $P_D = 50$ kN, and a factor of safety of $\alpha = 5$.

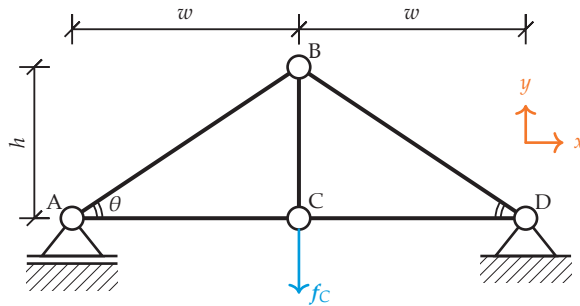



Figure 4.9. A truss with pinned joints, supported by a hinge and a floating support, with an applied load f_C .

Problem 4.10  Consider an LTI system modeled by the state equation of the state-space model, equation (4.24a). A **steady state** of a system is defined as the state vector $x(t)$ after the effects of initial conditions have become relatively small. For a constant input $u(t) = \bar{u}$, the constant state \bar{x} toward which the system's response decays can be found by setting the time derivative vector $x'(t) = 0$.

Write a Python function `steady_state()` that accepts the following arguments:

- A: A symbolic matrix representing A
- B: A symbolic matrix representing B
- `u_const`: A symbolic vector representing \bar{u}

The function should return `x_const`, a symbolic vector representing \bar{x} .

The steady-state output converges to \bar{y} the corresponding output equation of the state-space model, equation (4.24b). Write a second Python function `steady_output()` that accepts the following arguments:

- C: A symbolic matrix representing C
- D: A symbolic matrix representing D
- `u_const`: A symbolic vector representing \bar{u}
- `x_const`: A symbolic vector representing \bar{x}

This function should return `y_const`, a symbolic vector representing \bar{y} .

Apply `steady_state()` and `steady_output()` to the state-space model of the circuit shown in figure 4.10, which includes a resistor with resistance R , an inductor with inductance L , and capacitor with capacitance C . The LTI system is represented by equation (4.24) with state, input, and output vectors

$$x(t) = \begin{bmatrix} v_C(t) \\ i_L(t) \end{bmatrix}, \quad u(t) = [V_S], \quad y(t) = \begin{bmatrix} v_C(t) \\ v_L(t) \end{bmatrix}$$

and the following matrices:

$$A = \begin{bmatrix} 0 & 1/C \\ -1/L & -R/L \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1/L \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ -1 & -R \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Furthermore, let the constant input vector be

$$\bar{u} = [\bar{V}_S],$$

for constant \bar{V}_S .

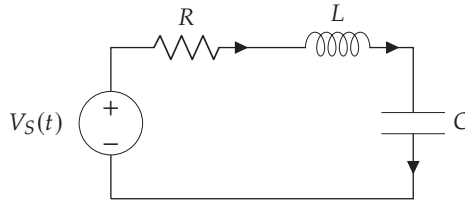



Figure 4.10. An RLC circuit with a voltage source $V_S(t)$.

Problem 4.11  Consider the electromechanical state-space model described in example 4.3. For a given set of parameters, input voltage, and initial conditions, the following vector-valued functions have been derived:

$$F = \begin{bmatrix} \int_0^t v_R(t) dt \\ \int_0^t v_L(t) dt \\ \int_0^t \Omega_B(t) dt \\ \int_0^t \Omega_J(t) dt \end{bmatrix} = \begin{bmatrix} \exp(-t) \\ \exp(-t) \\ 1 - \exp(-t) \\ 1 - \exp(-t) \end{bmatrix}, \quad G = \begin{bmatrix} \int_0^t i_R(t) dt \\ \int_0^t i_L(t) dt \\ \int_0^t T_B(t) dt \\ \int_0^t T_J(t) dt \end{bmatrix} = \begin{bmatrix} \exp(-t) \\ \exp(-t) \\ 1 - \exp(-t) \\ \exp(-t) \end{bmatrix}$$

The instantaneous power loss or stored by each element is given by the following vector of products:

$$\mathcal{P}(t) = \begin{bmatrix} v_R(t)i_R(t) \\ v_L(t)i_L(t) \\ \Omega_B(t)T_B(t) \\ \Omega_J(t)T_J(t) \end{bmatrix}.$$


The energy $\mathcal{E}(t)$ of the elements, then, is

$$\mathcal{E}(t) = \int_0^t \mathcal{P}(t) dt.$$

Write a program that satisfies the following requirements:

- It defines a function `power(F, G)` that returns the symbolic power vector $\mathcal{P}(t)$ from any inputs F and G

- b. It defines a function `energy(F, G)` that returns the symbolic energy $\mathcal{E}(t)$ from any inputs F and G (`energy()` should call `power()`)
- c. It tests the `energy()` on the specific F and G given above

Problem 4.12  For the circuit and state-space model given in problem 4.10, use SymPy to solve for $x(t)$ and $y(t)$ given the following:

- A constant input voltage $V_S(t) = \overline{V_S}$
- Initial condition $x(0) = \mathbf{0}$

Substitute the following parameters into the solution for $y(t)$ and create numerically evaluable functions of time for each variable in $y(t)$:

$$R = 50 \, \Omega, L = 10 \cdot 10^{-6} \, \text{H}, C = 1 \cdot 10^{-9} \, \text{F}, \overline{V_S} = 10 \, \text{V}.$$

Plot the outputs in $y(t)$ as functions of time, making sure to choose a range of time over which the response is best presented. *Hint:* An appropriate amount of time is on the scale of microseconds.



Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetur adipiscing elit. In hac

habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.

Fusce mauris. Vestibulum luctus nibh at lectus. Sed bibendum, nulla a faucibus semper, leo velit ultricies tellus, ac venenatis arcu wisi vel nisl. Vestibulum diam. Aliquam pellentesque, augue quis sagittis posuere, turpis lacus congue quam, in hendrerit risus eros eget felis. Maecenas eget erat in sapien mattis porttitor. Vestibulum porttitor. Nulla facilisi. Sed a turpis eu lacus commodo facilisis. Morbi fringilla, wisi in dignissim interdum, justo lectus sagittis dui, et vehicula libero dui cursus dui. Mauris tempor ligula sed lacus. Duis cursus enim ut augue. Cras ac magna. Cras nulla. Nulla egestas. Curabitur a leo. Quisque egestas wisi eget nunc. Nam feugiat lacus vel est. Curabitur consectetur.

Suspendisse vel felis. Ut lorem lorem, interdum eu, tincidunt sit amet, laoreet vitae, arcu. Aenean faucibus pede eu ante. Praesent enim elit, rutrum at, molestie non, nonummy vel, nisl. Ut lectus eros, malesuada sit amet, fermentum eu, sodales cursus, magna. Donec eu purus. Quisque vehicula, urna sed ultricies auctor, pede lorem egestas dui, et convallis elit erat sed nulla. Donec luctus. Curabitur et nunc. Aliquam dolor odio, commodo pretium, ultricies non, pharetra in, velit. Integer arcu est, nonummy in, fermentum faucibus, egestas vel, odio.

Sed commodo posuere pede. Mauris ut est. Ut quis purus. Sed ac odio. Sed vehicula hendrerit sem. Duis non odio. Morbi ut dui. Sed accumsan risus eget odio. In hac habitasse platea dictumst. Pellentesque non elit. Fusce sed justo eu urna porta tincidunt. Mauris felis odio, sollicitudin sed, volutpat a, ornare ac, erat. Morbi quis dolor. Donec pellentesque, erat ac sagittis semper, nunc dui lobortis purus, quis congue purus metus ultricies tellus. Proin et quam. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Praesent sapien turpis, fermentum vel, eleifend faucibus, vehicula eu, lacus.

Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Donec odio elit, dictum in, hendrerit sit amet, egestas sed, leo. Praesent feugiat sapien aliquet odio. Integer vitae justo. Aliquam vestibulum fringilla lorem. Sed neque lectus, consectetur at, consectetur sed, eleifend ac, lectus. Nulla facilisi. Pellentesque eget lectus. Proin eu metus. Sed porttitor. In hac habitasse platea dictumst. Suspendisse eu lectus. Ut mi mi, lacinia sit amet, placerat et, mollis vitae, dui. Sed ante tellus, tristique ut, iaculis eu, malesuada ac, dui. Mauris nibh leo, facilisis non, adipiscing quis, ultrices a, dui.

Morbi luctus, wisi viverra faucibus pretium, nibh est placerat odio, nec commodo wisi enim eget quam. Quisque libero justo, consectetur a, feugiat vitae, porttitor eu, libero. Suspendisse sed mauris vitae elit sollicitudin malesuada. Maecenas ultricies eros sit amet ante. Ut venenatis velit. Maecenas sed mi eget dui varius euismod.

Phasellus aliquet volutpat odio. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Pellentesque sit amet pede ac sem eleifend consectetur. Nullam elementum, urna vel imperdiet sodales, elit ipsum pharetra ligula, ac pretium ante justo a nulla. Curabitur tristique arcu eu metus. Vestibulum lectus. Proin mauris. Proin eu nunc eu urna hendrerit faucibus. Aliquam auctor, pede consequat laoreet varius, eros tellus scelerisque quam, pellentesque hendrerit ipsum dolor sed augue. Nulla nec lacus.

Suspendisse vitae elit. Aliquam arcu neque, ornare in, ullamcorper quis, commodo eu, libero. Fusce sagittis erat at erat tristique mollis. Maecenas sapien libero, molestie et, lobortis in, sodales eget, dui. Morbi ultrices rutrum lorem. Nam elementum ullamcorper leo. Morbi dui. Aliquam sagittis. Nunc placerat. Pellentesque tristique sodales est. Maecenas imperdiet lacinia velit. Cras non urna. Morbi eros pede, suscipit ac, varius vel, egestas non, eros. Praesent malesuada, diam id pretium elementum, eros sem dictum tortor, vel consectetur odio sem sed wisi.

5.1 Sets of Linear Algebraic Equations



Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula