4.8 Problems



Problem 4.1 \bigcirc 52 Let $s \in \mathbb{C}$. Use SymPy to perform a partial fraction expansion on the following expression:

$$\frac{(s+2)(s+10)}{s^4+8s^3+117s^2+610s+500}.$$

Problem 4.2 Let x, a_1 , a_2 , a_3 , $a_4 \in \mathbb{R}$. Use SymPy to combine the cosine and sine terms that share arguments into single sinusoids with phase shifts in the following expression:

$$a_1 \sin(x) + a_2 \cos(x) + a_3 \sin(2x) + a_4 \cos(2x)$$

Problem 4.3 QKR Consider the following equation, where $x \in \mathbb{C}$ and $a, b, c \in \mathbb{R}_+$,

$$ax^2 + bx + \frac{c}{x} + b^2 = 0.$$

Use SymPy to solve for x.

Problem 4.4 QG9 Let $w, x, y, z \in \mathbb{R}$. Consider the following system of equations:

$$8w - 6x + 5y + 4z = -20$$
$$2y - 2z = 10$$
$$2w - x + 4y + z = 0$$
$$w + 4x - 2y + 8z = 4.$$

Use SymPy to solve the system for w, x, y, and z.

Problem 4.5 ©IV Consider the truss shown in figure 4.8. Use a static analysis and the method of joints to develop a solution for the force in each member F_{AC} , F_{AD} , etc., and the reaction forces using the sign convention that tension is positive and compression is negative. The forces should be expressed in terms of the applied force f_D and the dimensions w and h only. Write a program that solves for the forces symbolically and answers the following questions:

- a. Which members are in tension?
- b. Which members are in compression?
- c. Are there any members with 0 nominal force? If so, which?
- d. Which member (or members) has (or have) the maximum compression?
- e. Which member (or members) has (or have) the maximum tension?

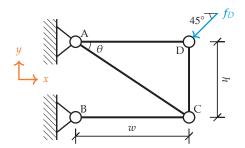


Figure 4.8. A truss with pinned joints, supported by two hinges, with an applied load f_D .

Problem 4.6 §49 You are designing the truss structure shown in figure 4.9, which is to support the hanging of an external load $f_C = -f_C \hat{j}$, where $f_C > 0$. Your organization plans to offer customers the following options:

- Any width (i.e., 2w)
- A selection of maximum load magnitudes $L = f_C/\alpha \in \Gamma$, where $\Gamma = \{1, 2, 4, 8, 16\}$ kN, and where α is the factor of safety

As the designer, you are to develop a design curve for the dimension h versus half-width w for each maximum load $L \in \Gamma$, under the following design constraints:

- Minimize the dimension h
- The tension in all members is no more than a given *T*
- The compression in all members is no more than a given *C*
- The magnitude of the support force at pin A is no more than a given P_A
- The magnitude of the support force at pin D is no more than a given P_D

Use a static analysis and the method of joints to develop a solution for the force in each member F_{AB} , F_{AC} , etc., and the reaction forces using the sign convention that tension is positive and compression is negative. Create a Python function that returns h as a function of w for a given set of design parameters $\{T, C, P_A, P_D, \alpha, L\}$. Use the function to create a design curve h versus 2w for each $L \in \Gamma$, maximum tension T = 81 kN, maximum compression C = 81 kN, maximum support A load $P_A = 50$ kN, maximum support D load $P_D = 50$ kN, and a factor of safety of $\alpha = 5$.

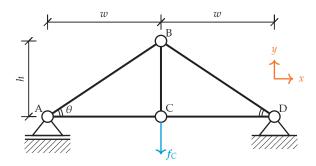


Figure 4.9. A truss with pinned joints, supported by a hinge and a floating support, with an applied load f_C .

Problem 4.7 Who Consider an LTI system modeled by the state equation of the state-space model, equation (4.24a). A **steady state** of a system is defined as the state vector x(t) after the effects of initial conditions have become relatively small. For a constant input $u(t) = \overline{u}$, the constant state \overline{x} toward which the system's response decays can be found by setting the time derivative vector $x'(t) = \mathbf{0}$.

Write a Python function steady_state() that accepts the following arguments:

- A: A symbolic matrix representing A
- B: A symbolic matrix representing B
- ullet u_const: A symbolic vector representing \overline{u}

The function should return x_const, a symbolic vector representing \overline{x} .

The steady-state output converges to \overline{y} the corresponding output equation of the state-space model, equation (4.24b). Write a second Python function steady_output() that accepts the following arguments:

- C: A symbolic matrix representing *C*
- $\bullet\,$ D: A symbolic matrix representing D
- u_const: A symbolic vector representing \overline{u}
- x_const: A symbolic vector representing \overline{x}

This function should return y_const, a symbolic vector representing \overline{y} .

Apply steady_state() and steady_output() to the state-space model of the circuit shown in figure 4.10, which includes a resistor with resistance R, an inductor with inductance L, and capacitor with capacitance C. The LTI system is represented by equation (4.24) with state, input, and output vectors

$$x(t) = \begin{bmatrix} v_C(t) \\ i_L(t) \end{bmatrix}$$
, $u(t) = \begin{bmatrix} V_S \end{bmatrix}$, $y(t) = \begin{bmatrix} v_C(t) \\ v_L(t) \end{bmatrix}$

and the following matrices:

$$A = \begin{bmatrix} 0 & 1/C \\ -1/L & -R/L \end{bmatrix}, \ B = \begin{bmatrix} 0 \\ 1/L \end{bmatrix}, \ C = \begin{bmatrix} 1 & 0 \\ -1 & -R \end{bmatrix}, \ D = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Furthermore, let the constant input vector be

$$\overline{u} = \left[\overline{V_S} \right]$$
,

for constant $\overline{V_S}$.

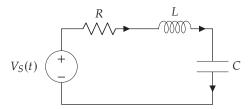


Figure 4.10. An RLC circuit with a voltage source $V_S(t)$.

Problem 4.8 ©8U Consider the electromechanical state-space model described in example 4.3. For a given set of parameters, input voltage, and initial conditions, the following vector-valued functions have been derived:

$$F = \begin{bmatrix} \int_0^t v_R(t) \ dt \\ \int_0^t v_L(t) \ dt \\ \int_0^t \Omega_B(t) \ dt \end{bmatrix} = \begin{bmatrix} \exp(-t) \\ \exp(-t) \\ 1 - \exp(-t) \\ 1 - \exp(-t) \end{bmatrix}, \quad G = \begin{bmatrix} \int_0^t i_R(t) \ dt \\ \int_0^t i_L(t) \ dt \\ \int_0^t T_B(t) \ dt \\ \int_0^t T_I(t) \ dt \end{bmatrix} = \begin{bmatrix} \exp(-t) \\ \exp(-t) \\ 1 - \exp(-t) \\ \exp(-t) \end{bmatrix}$$

The instantaneous power lossed or stored by each element is given by the following vector of products:

$$\mathcal{P}(t) = \begin{bmatrix} v_R(t)i_R(t) \\ v_L(t)i_L(t) \\ \Omega_B(t)T_B(t) \\ \Omega_J(t)T_J(t) \end{bmatrix}.$$

The energy $\mathcal{E}(t)$ of the elements, then, is

$$\mathcal{E}(t) = \int_0^t \mathcal{P}(t) dt.$$

Write a program that satisfies the following requirements:

a. It defines a function power (F, G) that returns the symbolic power vector $\mathcal{P}(t)$ from any inputs F and G

b. It defines a function energy (F, G) that returns the symbolic energy $\mathcal{E}(t)$ from any inputs F and G (energy () should call power ())

c. It tests the energy () on the specific F and G given above

Problem 4.9 For the circuit and state-space model given in problem 4.7, use SymPy to solve for x(t) and y(t) given the following:

- A constant input voltage $V_S(t) = \overline{V_S}$
- Initial condition x(0) = 0

Substitute the following parameters into the solution for y(t) and create numerically evaluable functions of time for each variable in y(t):

$$R = 50 \Omega$$
, $L = 10 \cdot 10^{-6} \text{ H}$, $C = 1 \cdot 10^{-9} \text{ F}$, $\overline{V_S} = 10 \text{ V}$.

Plot the outputs in y(t) as functions of time, making sure to choose a range of time over which the response is best presented. *Hint*: An appropriate amount of time is on the scale of microseconds.

Numerical Analysis and Design Techniques [Outlined]



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5.1 Sets of Linear Algebraic Equations



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