

4.8 Problems



Problem 4.1 Let $s \in \mathbb{C}$. Use SymPy to perform a partial fraction expansion on the following expression:

$$\frac{(s+2)(s+10)}{s^4 + 8s^3 + 117s^2 + 610s + 500}.$$

Problem 4.2 Let $x, a_1, a_2, a_3, a_4 \in \mathbb{R}$. Use SymPy to combine the cosine and sine terms that share arguments into single sinusoids with phase shifts in the following expression:

$$a_1 \sin(x) + a_2 \cos(x) + a_3 \sin(2x) + a_4 \cos(2x)$$

Problem 4.3 Consider the following equation, where $x \in \mathbb{C}$ and $a, b, c \in \mathbb{R}_+$,

$$ax^2 + bx + \frac{c}{x} + b^2 = 0.$$

Use SymPy to solve for x .

Problem 4.4 Let $w, x, y, z \in \mathbb{R}$. Consider the following system of equations:

$$8w - 6x + 5y + 4z = -20$$

$$2y - 2z = 10$$

$$2w - x + 4y + z = 0$$

$$w + 4x - 2y + 8z = 4.$$

Use SymPy to solve the system for w, x, y , and z .

Problem 4.5 Consider the truss shown in figure 4.8. Use a static analysis and the method of joints to develop a solution for the force in each member F_{AC} , F_{AD} , etc., and the reaction forces using the sign convention that tension is positive and compression is negative. The forces should be expressed in terms of the applied force f_D and the dimensions w and h only. Write a program that *solves for the forces symbolically* and answers the following questions:

- Which members are in tension?
- Which members are in compression?
- Are there any members with 0 nominal force? If so, which?
- Which member (or members) has (or have) the maximum compression?
- Which member (or members) has (or have) the maximum tension?

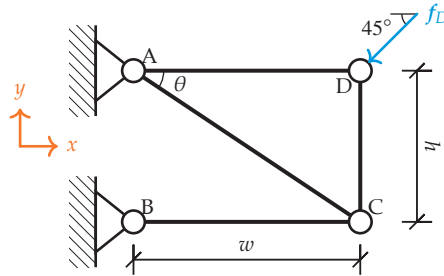



Figure 4.8. A truss with pinned joints, supported by two hinges, with an applied load f_D .

Problem 4.6  49 You are designing the truss structure shown in figure 4.9, which is to support the hanging of an external load $f_C = -f_C \hat{j}$, where $f_C > 0$. Your organization plans to offer customers the following options:

- Any width (i.e., $2w$)
- A selection of maximum load magnitudes $L = f_C / \alpha \in \Gamma$, where $\Gamma = \{1, 2, 4, 8, 16\}$ kN, and where α is the factor of safety

As the designer, you are to develop a design curve for the dimension h versus half-width w for each maximum load $L \in \Gamma$, under the following design constraints:

- Minimize the dimension h
- The tension in all members is no more than a given T
- The compression in all members is no more than a given C
- The magnitude of the support force at pin A is no more than a given P_A
- The magnitude of the support force at pin D is no more than a given P_D

Use a static analysis and the method of joints to develop a solution for the force in each member F_{AB} , F_{AC} , etc., and the reaction forces using the sign convention that tension is positive and compression is negative. Create a Python function that returns h as a function of w for a given set of design parameters $\{T, C, P_A, P_D, \alpha, L\}$. Use the function to create a design curve h versus $2w$ for each $L \in \Gamma$, maximum tension $T = 81$ kN, maximum compression $C = 81$ kN, maximum support A load $P_A = 50$ kN, maximum support D load $P_D = 50$ kN, and a factor of safety of $\alpha = 5$.

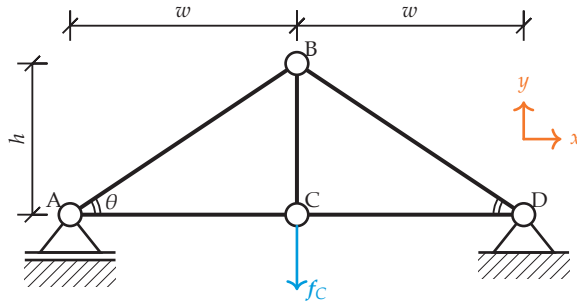



Figure 4.9. A truss with pinned joints, supported by a hinge and a floating support, with an applied load f_C .

Problem 4.7  Consider an LTI system modeled by the state equation of the state-space model, equation (4.24a). A **steady state** of a system is defined as the state vector $\mathbf{x}(t)$ after the effects of initial conditions have become relatively small. For a constant input $\mathbf{u}(t) = \bar{\mathbf{u}}$, the constant state $\bar{\mathbf{x}}$ toward which the system's response decays can be found by setting the time derivative vector $\mathbf{x}'(t) = \mathbf{0}$.

Write a Python function `steady_state()` that accepts the following arguments:

- A: A symbolic matrix representing A
- B: A symbolic matrix representing B
- $\mathbf{u_const}$: A symbolic vector representing $\bar{\mathbf{u}}$

The function should return $\mathbf{x_const}$, a symbolic vector representing $\bar{\mathbf{x}}$.

The steady-state output converges to $\bar{\mathbf{y}}$ the corresponding output equation of the state-space model, equation (4.24b). Write a second Python function `steady_output()` that accepts the following arguments:

- C: A symbolic matrix representing C
- D: A symbolic matrix representing D
- $\mathbf{u_const}$: A symbolic vector representing $\bar{\mathbf{u}}$
- $\mathbf{x_const}$: A symbolic vector representing $\bar{\mathbf{x}}$

This function should return $\mathbf{y_const}$, a symbolic vector representing $\bar{\mathbf{y}}$.

Apply `steady_state()` and `steady_output()` to the state-space model of the circuit shown in figure 4.10, which includes a resistor with resistance R , an inductor with inductance L , and capacitor with capacitance C . The LTI system is represented by equation (4.24) with state, input, and output vectors

$$\mathbf{x}(t) = \begin{bmatrix} v_C(t) \\ i_L(t) \end{bmatrix}, \quad \mathbf{u}(t) = [V_S], \quad \mathbf{y}(t) = \begin{bmatrix} v_C(t) \\ v_L(t) \end{bmatrix}$$

and the following matrices:

$$A = \begin{bmatrix} 0 & 1/C \\ -1/L & -R/L \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1/L \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ -1 & -R \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Furthermore, let the constant input vector be

$$\bar{u} = [\bar{V}_S],$$

for constant \bar{V}_S .

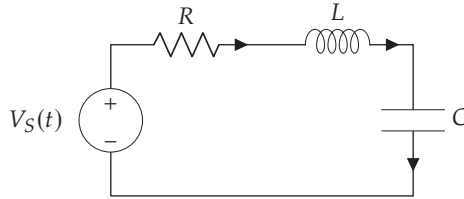



Figure 4.10. An RLC circuit with a voltage source $V_S(t)$.

Problem 4.8  Consider the electromechanical state-space model described in example 4.3. For a given set of parameters, input voltage, and initial conditions, the following vector-valued functions have been derived:

$$F = \begin{bmatrix} \int_0^t v_R(t) dt \\ \int_0^t v_L(t) dt \\ \int_0^t \Omega_B(t) dt \\ \int_0^t \Omega_J(t) dt \end{bmatrix} = \begin{bmatrix} \exp(-t) \\ \exp(-t) \\ 1 - \exp(-t) \\ 1 - \exp(-t) \end{bmatrix}, \quad G = \begin{bmatrix} \int_0^t i_R(t) dt \\ \int_0^t i_L(t) dt \\ \int_0^t T_B(t) dt \\ \int_0^t T_J(t) dt \end{bmatrix} = \begin{bmatrix} \exp(-t) \\ \exp(-t) \\ 1 - \exp(-t) \\ \exp(-t) \end{bmatrix}$$

The instantaneous power loss or stored by each element is given by the following vector of products:

$$\mathcal{P}(t) = \begin{bmatrix} v_R(t)i_R(t) \\ v_L(t)i_L(t) \\ \Omega_B(t)T_B(t) \\ \Omega_J(t)T_J(t) \end{bmatrix}.$$


The energy $\mathcal{E}(t)$ of the elements, then, is

$$\mathcal{E}(t) = \int_0^t \mathcal{P}(t) dt.$$

Write a program that satisfies the following requirements:

- It defines a function `power(F, G)` that returns the symbolic power vector $\mathcal{P}(t)$ from any inputs F and G

- b. It defines a function `energy(F, G)` that returns the symbolic energy $\mathcal{E}(t)$ from any inputs F and G (`energy()` should call `power()`)
- c. It tests the `energy()` on the specific F and G given above

Problem 4.9  For the circuit and state-space model given in problem 4.7, use SymPy to solve for $x(t)$ and $y(t)$ given the following:

- A constant input voltage $V_S(t) = \overline{V_S}$
- Initial condition $x(0) = \mathbf{0}$

Substitute the following parameters into the solution for $y(t)$ and create numerically evaluable functions of time for each variable in $y(t)$:

$$R = 50 \, \Omega, L = 10 \cdot 10^{-6} \, \text{H}, C = 1 \cdot 10^{-9} \, \text{F}, \overline{V_S} = 10 \, \text{V}.$$

Plot the outputs in $y(t)$ as functions of time, making sure to choose a range of time over which the response is best presented. *Hint:* An appropriate amount of time is on the scale of microseconds.



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5.1 Sets of Linear Algebraic Equations



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