Mechanics of Materials-II

COLUMN & STRUT

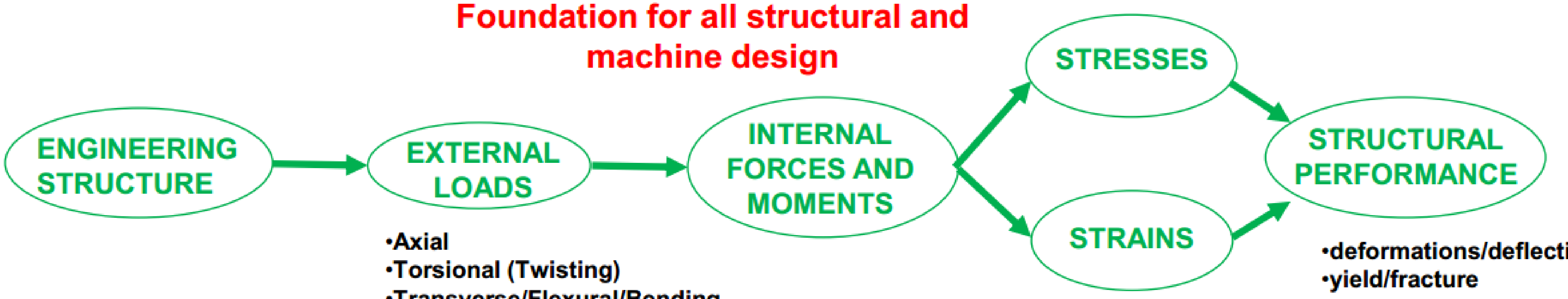
Botsa Srinivasa Rao

Assistant Professor
Dept. of Civil Engineering
RGUKT- Nuzvid

Contact No: +91-7661098698,
Email : srinivas9394258146@rguktn.ac.in

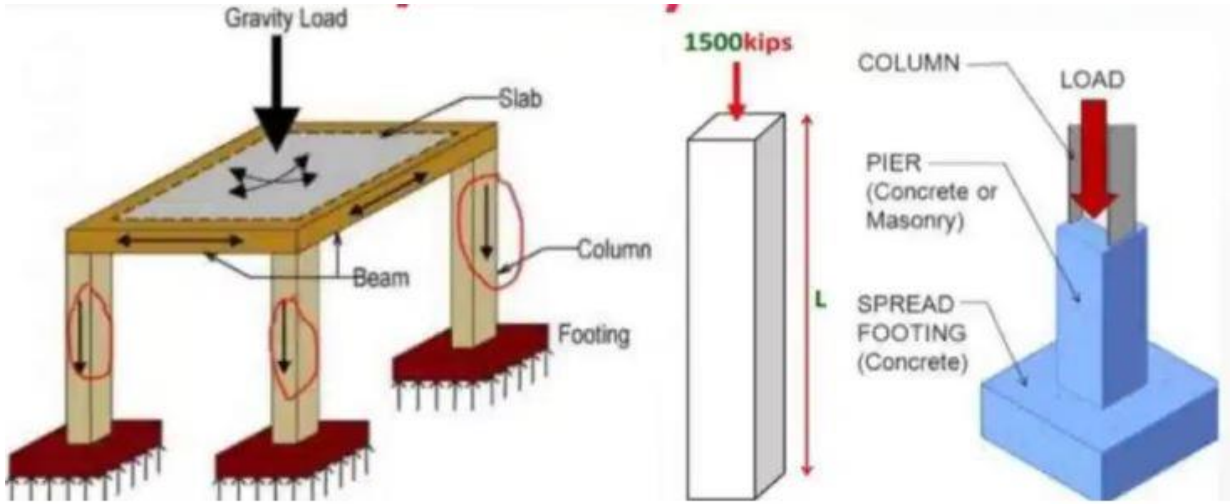
MOM (Course Outcomes)

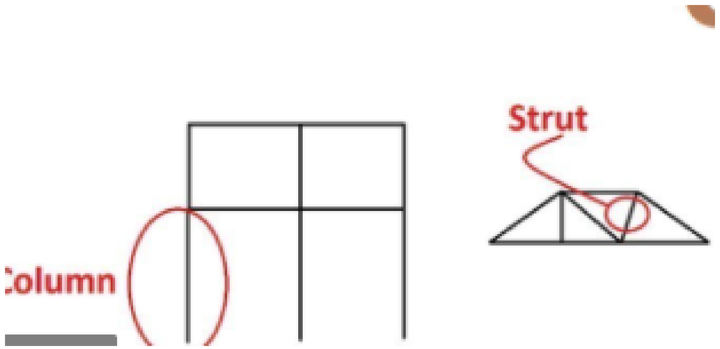
Foundation for all structural and machine design



- Axial
- Torsional (Twisting)
- Transverse/Flexural/Bending
- Pressure
- Combined Loading

- deformations/deflections
- yield/fracture
- success/failure?



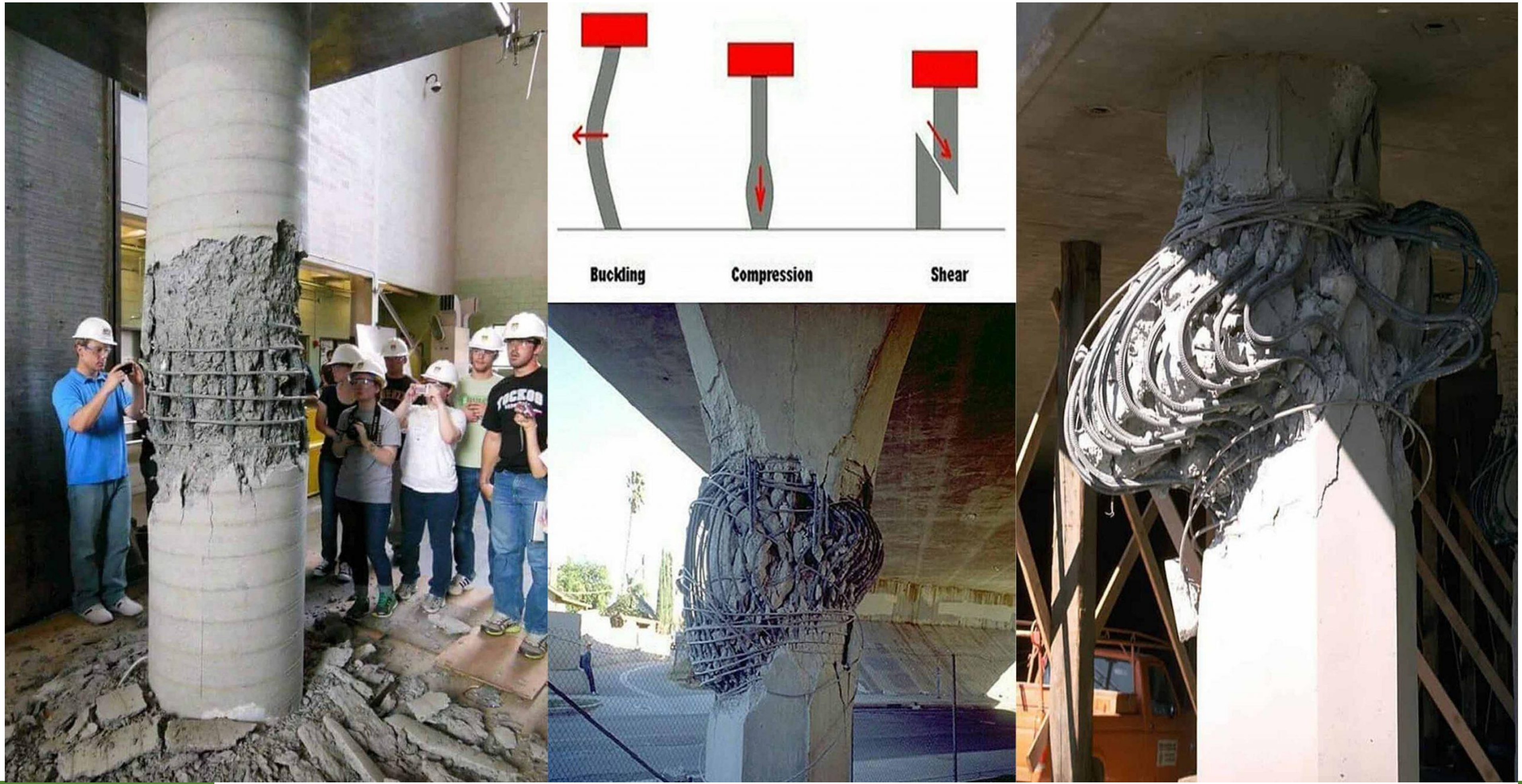


MOM (Course Outcomes)

- Any member subjected to axial compressive load is called a column or Strut.
- A vertical member subjected to axial compressive load – **COLUMN** (Eg: Pillars of a building)
- An inclined member subjected to axial compressive load - **STRUT**
- **A strut may also be a horizontal member**
- Load carrying capacity of a compression member depends not only on its **cross sectional area**, but also **on its length** and the manner in which **the ends of a column** are held.

Classification of Columns (Nature of Failure)

According to nature of failure – short, medium and long columns



Classification of Columns (Short column)

1. Short column – whose length is so related to its c/s area that failure occurs mainly **due to direct compressive stress** only and the role of bending stress is negligible

2. Medium Column - whose length is so related to its c/s area that failure occurs by a combination of direct compressive stress and bending stress

Classification of Columns (Short column)

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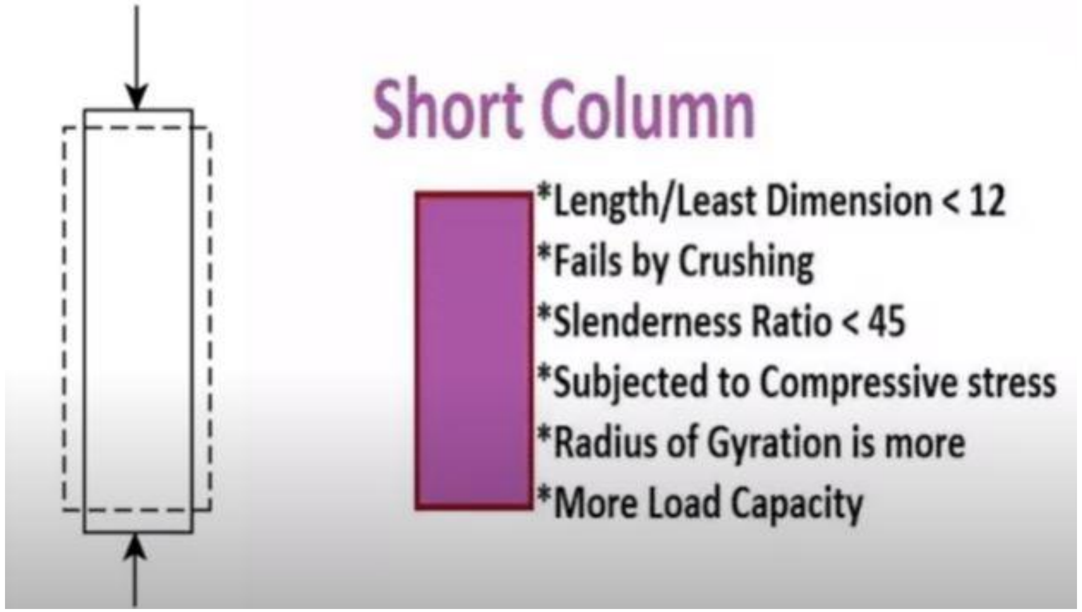
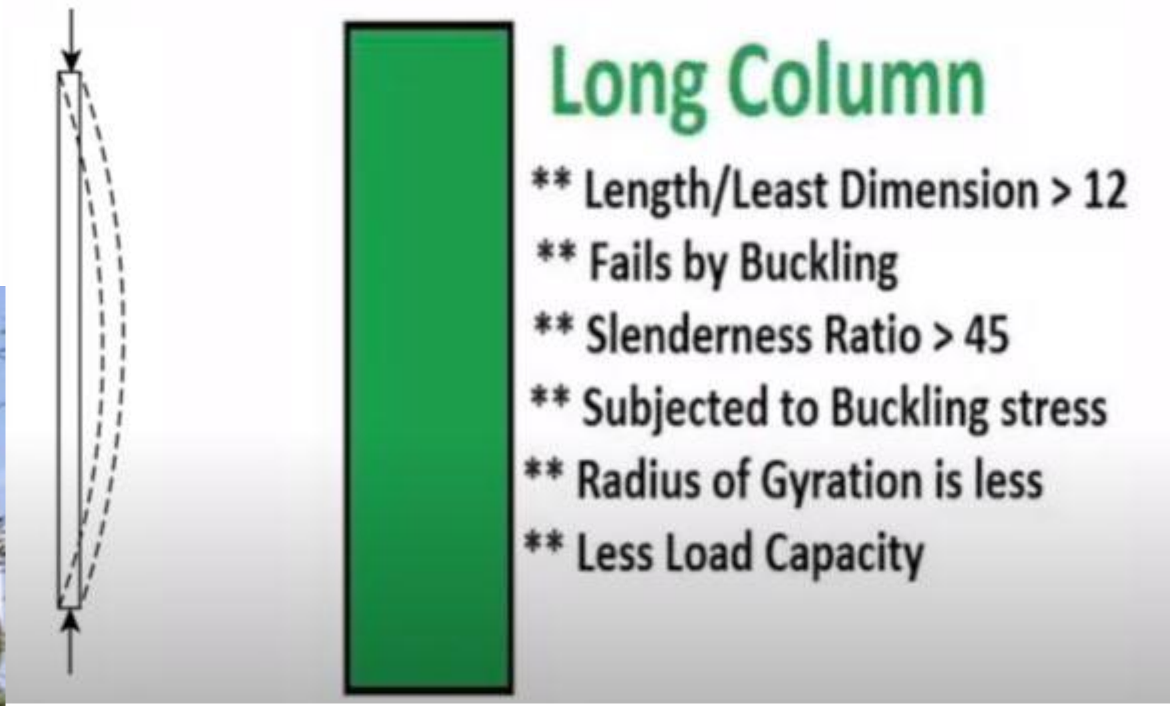
2. **Medium Column** - whose length is so related to its c/s area that failure occurs by a combination of direct compressive stress and bending stress

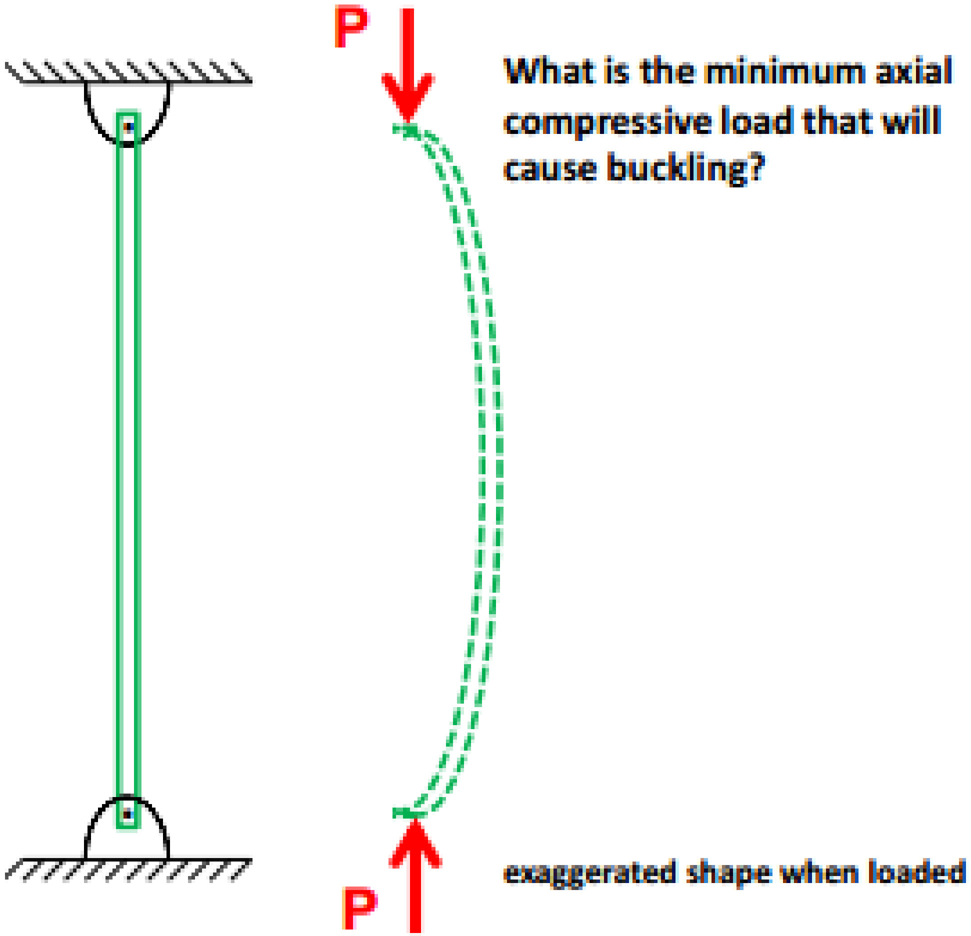


Classification of Columns

3. Long Column - whose length is so related to its c/s area that failure occurs mainly **due to bending (Buckling) stress** and the role of direct compressive stress is negligible



Classification of Columns (Long/Slender)



Classification of Columns (Short column)

Column Buckling

A simple column is a long, straight, prismatic bar subjected to compressive, axial loads

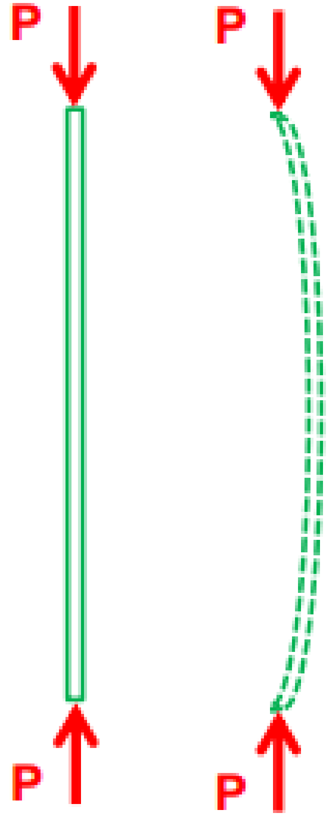
If beam remains straight, analyze using techniques from "Mechanics of Materials: Part I

Buckling occurs if the column begins to deform laterally. The deflection can become large and lead to catastrophic failure. Buckling is a large sudden deformation of a structure due to a small increase of the existing load.

Classification of Columns (Slender column)

Column Buckling

A simple column is a long, straight, prismatic bar subjected to compressive, axial loads



Buckling is when a stable equilibrium becomes unstable.

During initial compression, if a slight perturbation is laterally induced and the load is removed, the column returns to its straight configuration.

When buckling occurs, a critical value is reached at which, when perturbed laterally, the column will not return to the straight configuration.

For long slender columns, the critical buckling occurs at stress levels below the proportional limit of the material . This type of buckling is an elastic phenomenon.

Classification of Columns (Slender column)

Column Buckling

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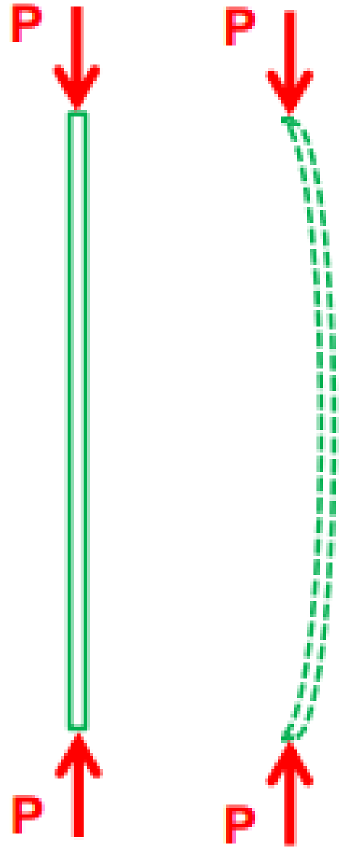

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Description	Short Column	Long Column
Length	ratio of the effective length of a column to its least lateral dimension does not exceed 12	ratio of the effective length of a column to its least lateral dimension exceeds 12
Slenderness	slenderness is less than 12	slenderness is more than 12
Radius of gyration	radius of gyration is less	higher radius of gyration
Load	load-carrying capacity of short column is more than long column.	load-carrying capacity is less compared to short column with
Strength	Stronger than long column and highly preferable.	Weaker than short column and normally not preferred.
Stress	it is subjected to compressive stress.	it is subjected to buckling stress
Failure	Mechanical failure primarily occurs due to shearing.	Long columns failed due to buckling.

Euler's Buckling Theory

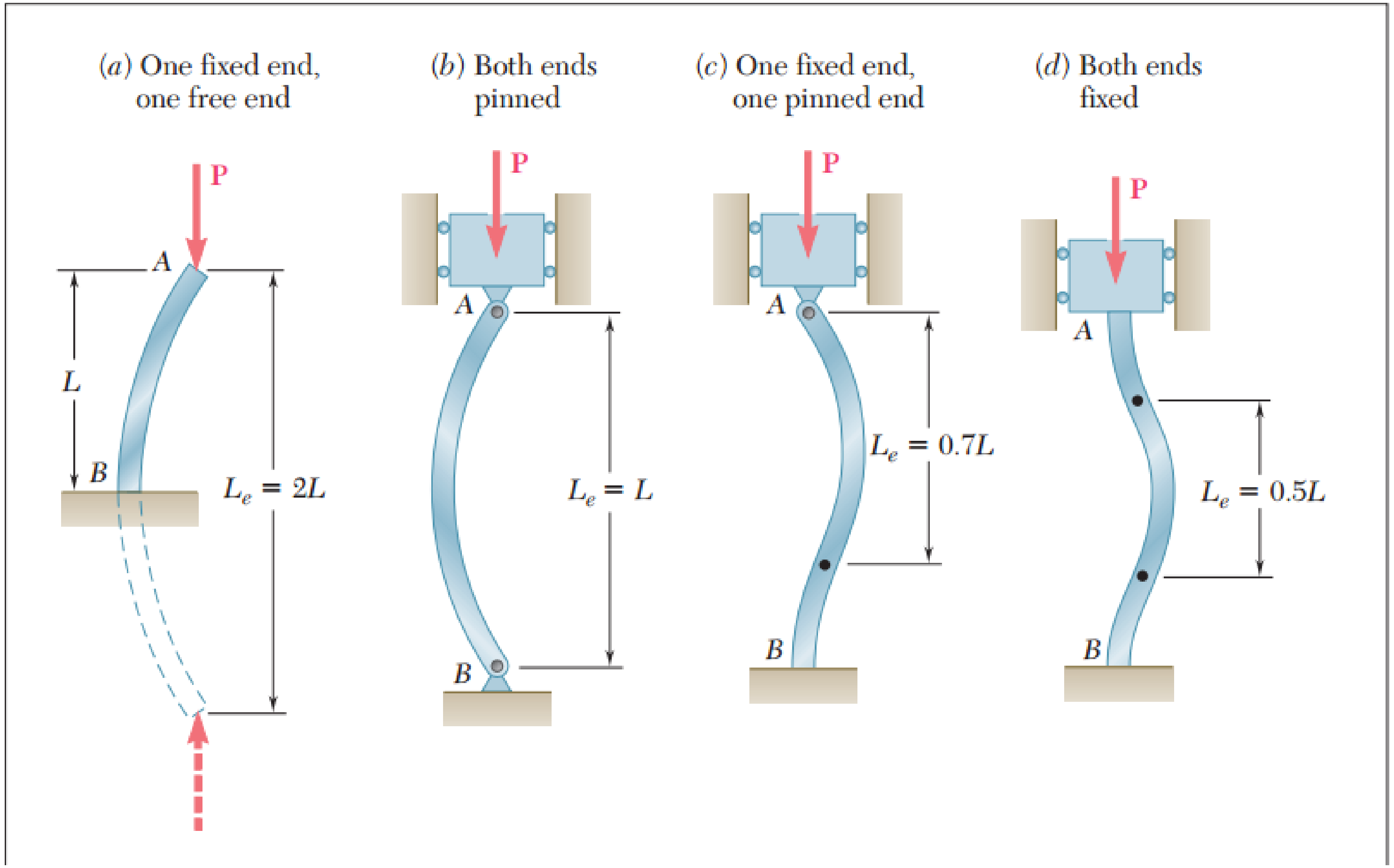
- Columns and struts which fail by buckling may be analyzed by Euler's Theory

Assumptions:

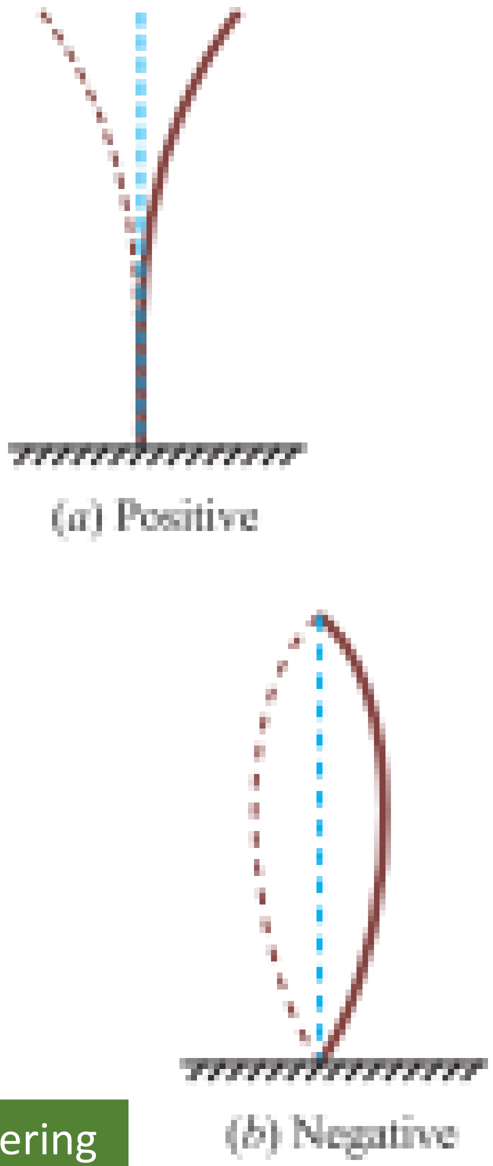
- The column is initially straight
- The cross section is uniform throughout
- The line of thrust coincides exactly with the axis of the column
- The material is homogeneous and isotropic
- The shortening of column due to axial compression is negligible

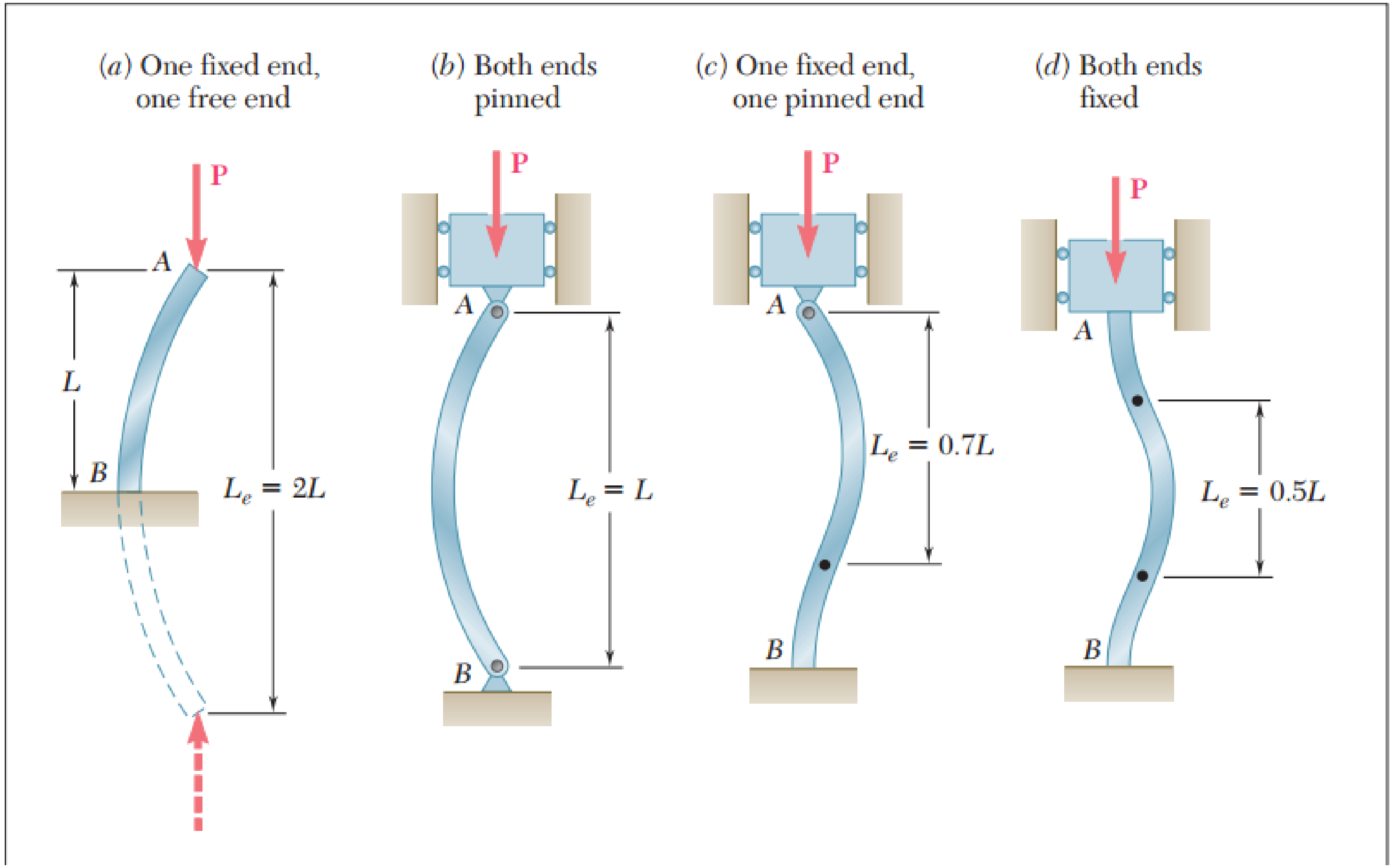
End Connections of column

- Both ends of the column is hinged
- Both ends of column is hinged
- One end is hinged and other end is Fixed
- One end is fixed and other end is free

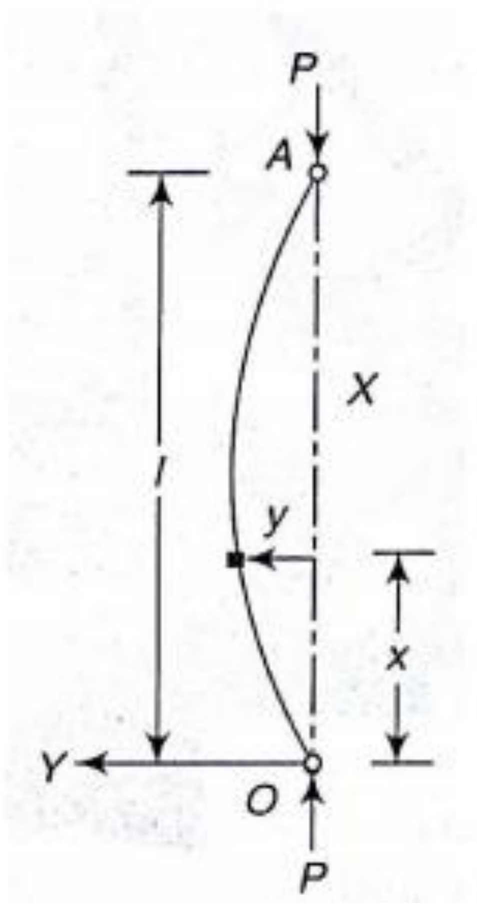


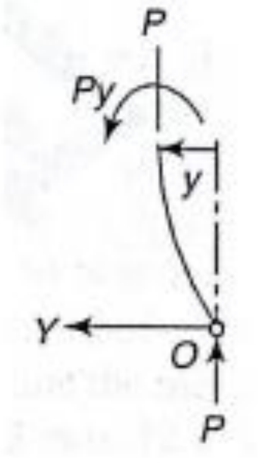
End Connections of column (Moment Sign)

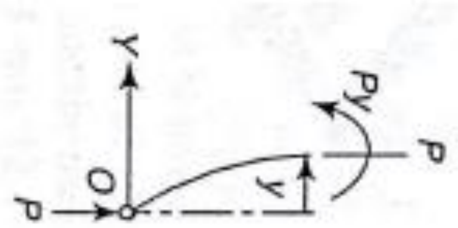




1.Both ends of the column is hinged







$$EI\frac{d^2y}{dx^2} = M = -Py$$

1.Both ends of the column is hinged

$$EI\frac{d^2y}{dx^2} = M = -Py$$

The equation can be written as $\frac{d^2y}{dx^2} + \alpha^2 y = 0$ where $\alpha^2 = \frac{P}{EI}$

The solution is $y = A\sin\alpha x + B\cos\alpha x$

At $x = 0, y = 0, \therefore B = 0$

at $x = l, y = 0$ and thus $A\sin\alpha l = 0$

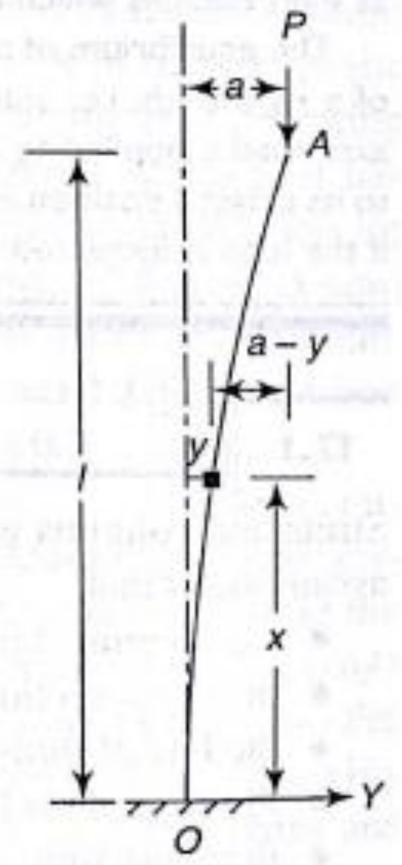
If $A = 0$, y is zero for all values of load and there is no bending.

$\therefore \quad \sin\alpha l = 0 \quad$ or $\quad \alpha l = \pi \qquad$ (considering the least value)

or $\quad \alpha = \pi / l$

$\therefore \quad$ Euler crippling load, $P_e = \alpha^2 EI = \frac{\pi^2 EI}{l^2}$

2.One end is fixed other is free



$$EI\frac{d^2y}{dx^2} = M = P(a-y) = Pa - Py$$

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$$\frac{d^2y}{dx^2} + \alpha^2 y = \frac{P\cdot a}{EI} \quad \text{where} \quad \alpha^2 = \frac{P}{EI}$$

The solution is $y = A\sin\alpha x + B\cos\alpha x + \dfrac{P.a}{EI\alpha^2}$

$$= A\sin\alpha x + B\cos\alpha x + a$$

$$x = 0,\ y = 0,\ \therefore B = -a;$$

$$x = 0,\ \frac{dy}{dx} = 0$$

or $\quad A\alpha\cos\alpha x - B\alpha\sin\alpha x = 0 \quad$ or $\quad A = 0$

$$y = -a\cos\alpha x + a = a(1-\cos\alpha x)$$

2.One end is fixed other is free

$$EI\frac{d^2y}{dx^2} = M = P(a-y) = Pa - Py$$

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$x = 0, y = 0, \therefore B = -a;$

$x = 0, \dfrac{dy}{dx} = 0$

or $A\alpha\cos\alpha x - B\alpha\sin\alpha x = 0$ or $A = 0$

$$y = -a\cos\alpha x + a = a(1-\cos\alpha x)$$

At $x = l, y = a, \therefore a = a(1-\cos\alpha l)$

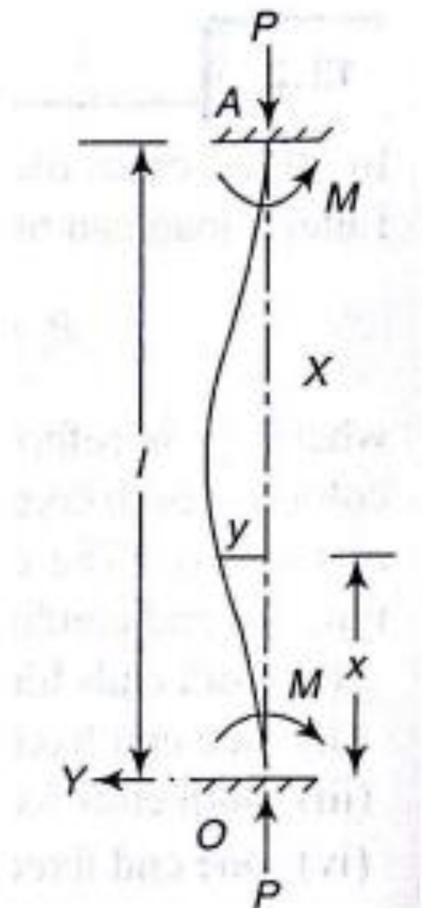
or $\cos\alpha l = 0$ or $\alpha l = \dfrac{\pi}{2}$ (considering the least value)

$$\alpha = \pi/2l$$

$\therefore$ Euler crippling load, $P_e = \alpha^2 EI = \dfrac{\pi^2 EI}{4l^2}$

$$EI\frac{d^2y}{dx^2} = M = P(a-y) = Pa - Py$$

3. Both Ends Fixed



$$EI\frac{d^2y}{dx^2} = -Py + M$$

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$$\frac{d^2y}{dx^2} + \alpha^2 y = \frac{M}{EI} \quad \text{where} \quad \alpha^2 = \frac{P}{EI}$$

The solution is $y = A\sin\alpha x + B\cos\alpha x + \frac{M}{EI\alpha^2} = A\sin\alpha x + B\cos\alpha x + \frac{M}{P}$

$$x = 0, y = 0, \therefore\ B = -\frac{M}{P};$$

$$x = 0, \frac{dy}{dx} = 0$$

or $\quad A\alpha\cos\alpha x - B\alpha\sin\alpha x = 0 \quad$ or $\quad A = 0$

$$\therefore\ y = -\frac{M}{P}\cos\alpha x + \frac{M}{P} = \frac{M}{P}(1 - \cos\alpha x)$$

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$$x = 0, y = 0, \therefore \ B = -\frac{M}{P};$$

$$x = 0, \frac{dy}{dx} = 0$$

$$\text{or} \quad A\alpha\cos\alpha x - B\alpha\sin\alpha x = 0 \quad \text{or} \quad A = 0$$

$$\therefore \ y = -\frac{M}{P}\cos\alpha x + \frac{M}{P} = \frac{M}{P}(1-\cos\alpha x)$$

At $x = l, y = 0, \therefore \ 0 = \frac{M}{P}(1-\cos\alpha l)$ or $\cos\alpha l = 1$

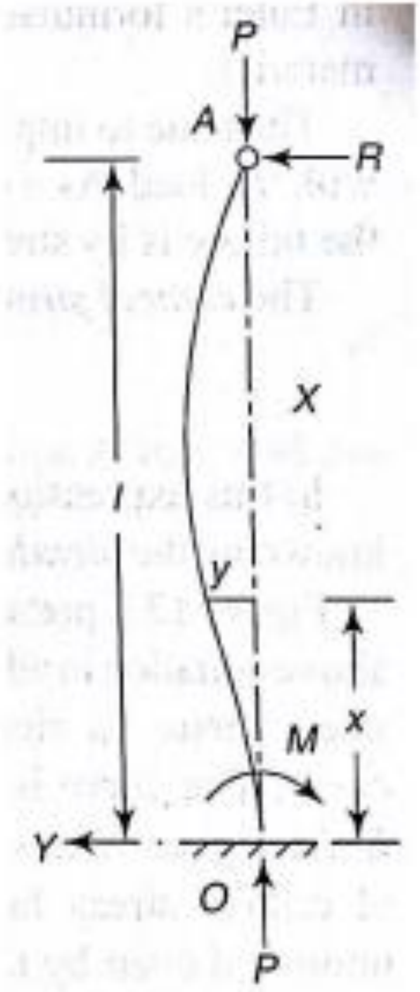
or $\alpha l = 2\pi$ (considering the least value)

or $\alpha = 2\pi/l$

$$\therefore \text{ Euler crippling load, } P_e = \alpha^2 EI = \frac{4\pi^2 EI}{l^2}$$

$$EI\frac{d^2y}{dx^2} = -Py + M$$

4.One end is fixed other is hinged



$$EI\frac{d^2y}{dx^2} = -Py + R(l-x)$$

$$\frac{d^2y}{dx^2} + \alpha^2 y = \frac{R(l-x)}{EI} \quad \text{where} \quad \alpha^2 = \frac{P}{EI}$$

The solution is $y = A\sin\alpha x + B\cos\alpha x + \dfrac{R(l-x)}{EI\alpha^2}$

$$= A\sin\alpha x + B\cos\alpha x + \frac{R}{P}(l-x)$$

At $x = 0, y = 0, \therefore\ B = -\dfrac{Rl}{P}$;

At $x = 0, \dfrac{dy}{dx} = 0$

or $A\alpha\cos\alpha x - B\alpha\sin\alpha x - \dfrac{R}{P} = 0$ or $A = \dfrac{R}{P\alpha}$

$$EI\frac{d^2y}{dx^2} = -Py + R(l-x)$$

4.One end is fixed other is hinged

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$$EI\frac{d^2y}{dx^2} = -Py + R(l-x)$$

$$\therefore \quad y = \frac{R}{P\alpha}\sin\alpha x - \frac{Rl}{P}\cos\alpha x + \frac{R}{P}(l-x)$$

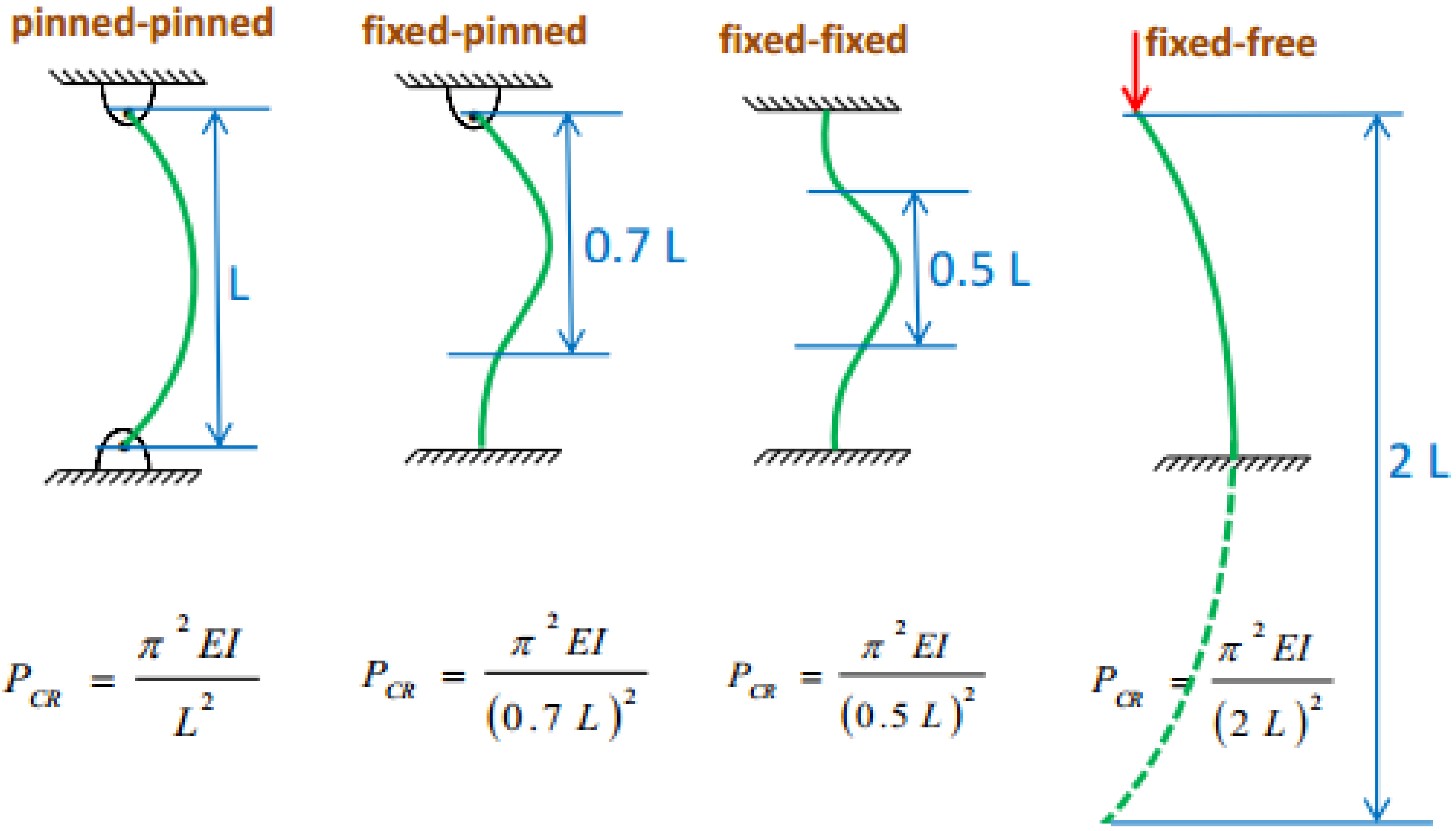
At $x = l, y = 0, \therefore 0 = \dfrac{R}{P\alpha}\sin\alpha l - \dfrac{Rl}{P}\cos\alpha l$

or $\tan\alpha l = \alpha l$

$\alpha l = 4.49$ rad (considering the least value)

$\alpha = 4.49 / l$

Euler crippling load, $P_e = \alpha^2 EI = \dfrac{4.49^2 EI}{l^2} = \dfrac{20.2EI}{l^2} \approx \dfrac{2\pi^2 EI}{l^2}$



$$P_{CR} = \frac{\pi^2 EI}{L^2} \qquad P_{CR} = \frac{\pi^2 EI}{(0.7L)^2} \qquad P_{CR} = \frac{\pi^2 EI}{(0.5L)^2} \qquad P_{CR} = \frac{\pi^2 EI}{(2L)^2}$$

$$P_{CR} = \frac{\pi^2 EI}{(L_{EFFECTIVE})^2}$$

where $L_{EFFECTIVE}$ **is the distance between two successive inflection points or points of zero moment (must be modified for actual end conditions)**

Tables are using DRCS & DSS

Table.Effective length of compression member

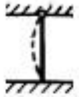
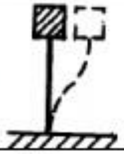
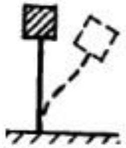
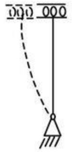
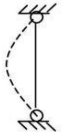
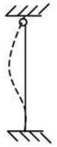
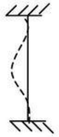
Sl. No.	Degree of End Restraint of Compression Members	Figure	Theo. Value of Effective Length	Reco. Value of Effective Length
1	Effectively held in position and restrained against rotation in both ends		0.50 l	0.65l
2	Effectively held in position at both ends, restrained against rotation at one end		0.70 l	0.80l
3	Effectively held in position at both ends, but not restrained against rotation		1.0 l	1.0l
4	Effectively held in position and restrained against rotation at one end, and at the other restrained against rotation but not held in position		1.0 l	1.20l
5	Effectively held in position and restrained against rotation in one end, and at the other partially restrained against rotation but not held in position		-	1.5l
6	Effectively held in position at one end but not restrained against rotation, and at the other end restrained against rotation but not held in position		2.0 l	2.0l
7	Effectively held in position and restrained against rotation at one end but not held in position nor restrained against rotation at the other end		2.0 l	2.0l

TABLE 4: EFFECTIVE LENGTH OF PRISMATIC COMPRESSION MEMBERS

Boundary Conditions				Schematic representation	Effective Length
At one end		At the other end			
Translation	Rotation	Translation	Rotation		
Restrained	Restrained	Free	Free		2.0L
Free	Restrained	Restrained	Free		
Restrained	Free	Restrained	Free		1.0L
Restrained	Restrained	Free	Restrained		1.2L
Restrained	Restrained	Restrained	Free		0.8L
Restrained	Restrained	Restrained	Restrained		0.65 L

Note – L is the unsupported length of the compression member (7.2.1).

Euler's Critical Stress

- Critical stress (σ_c) – average stress over the cross section

$$\sigma_c = \frac{P_e}{A} = \frac{\pi^2 EI}{Al_e^2}$$

$$= \frac{\pi^2 EAk^2}{Al_e^2}$$

$$\sigma_c = \frac{\pi^2 E}{(l_e/k)^2}$$

- *l*/k is known as **Slenderness Ratio**

Limitations of Euler's Formula

- Assumption- Struts are initially perfectly straight and the load is exactly axial
- There is always some eccentricity and initial curvature present
- In practice a strut suffers a deflection before the crippling load

Both ends linged	L = 1	Constant = 1
Both ends fixed	$L = \frac{l}{2}$	Constant = $\frac{1}{2}$
One end fixed and other end hinged	$L = \frac{l}{\sqrt{2}}$	Constant = $\frac{1}{\sqrt{2}}$
One end fixed and other end free	$L = 2l$	Constant = 2

Rankine's Formula (or) Rankine - Gorden Formula

- Euler's formula is applicable to long columns only for which l/k ratio is larger than a particular value.
- Also doesn't take in to account the direct compressive stress.
- Thus for columns of medium length it doesn't provide accurate results.
- Rankine's forwarded an empirical relation

$$\frac{1}{P} = \frac{1}{P_c} + \frac{1}{P_e}$$

where P = Rankine's crippling load

P_c = ultimate load for a strut = $\sigma_u \cdot A$, constant for a material

P_e = Eulerial load for a strut = $\pi^2 EI/l^2$

Rankine's Formula (or) Rankine - Gorden Formula

$$\frac{1}{P} = \frac{1}{P_c} + \frac{1}{P_e}$$

where P = Rankine's crippling load

P_c = ultimate load for a strut = $\sigma_u \cdot A$, constant for a material

P_e = Eulerial load for a strut = $\pi^2 EI/l^2$

- For short columns, P_e is very large and therefore $1/P_e$ is small in comparison to $1/P_c$. Thus the crippling load P is practically equal to P_c
- For long columns, P_e is very small and therefore $1/P_e$ is quite large in comparison to $1/P_c$. Thus the crippling load P is practically equal to P_e

Rankine’s Formula (or) Rankine - Gorden Formula

$$\frac{1}{P} = \frac{1}{P_c} + \frac{1}{P_e}$$

$$\frac{1}{P} = \frac{P_e + P_c}{P_c P_e}$$

$$P = \frac{P_c P_e}{P_e + P_c} = \frac{P_c}{1 + P_c / P_e} = \frac{\sigma_c \cdot A}{1 + \frac{\sigma_c A \cdot l^2}{\pi^2 EI}}$$

$$= \frac{\sigma_c \cdot A}{1 + \frac{\sigma_c \cdot A \cdot l^2}{\pi^2 EAk^2}}$$

$$P = \frac{\sigma_c \cdot A}{1 + a\left(\frac{l}{k}\right)^2}$$

where σ_c is the crushing stress

a is the Rankine’s constant $(\sigma_c / \pi^2 E)$

Rankine’s formula for columns with other end conditions

$$P = \frac{\sigma_c \cdot A}{1 + a\left(\frac{l_e}{k}\right)^2}$$

A Factor of Safety may be considered for the value of σ_c in the above formula

Problems on Euler Formula

1. A mild steel tube 4m long, 3 cm internal diameter and 4 mm thick is used as a strut with both ends hinged. Find the collapsing load, what will be the crippling load if
   i. Both ends are built in
   ii. One end is built-in and one end is free?

Problems on Euler Formula

$\therefore$ External diameter of the tube, $D = d + 2t$
$= 3+2(0.4)$
$= 3.8$ cm.

Assuming E for steel = 2×10^6 Kg/cm^2

M.O.I of the column section,

$$I = \frac{\pi}{64}\left[D^4 - d^4\right]$$

$$= \frac{\pi}{64}\left[(3.8)^4 - (3)^2\right]$$

$I = 6.26$ cm ^{4}

$\therefore = l = 400$ cm

$\therefore$ Euler's crippling load $\Rightarrow$ $P_{cr} = \frac{\pi^2 EI}{L^2}$

$$= \frac{\pi^2 \times 2\times10^6 \times 6.26}{(400)^2}$$

$P_{cr} = 772.30 Kg.$

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

$$= \frac{\pi^2 \times 2\times10^6 \times 6.26}{(200)^2}$$

P_{cr} = 3089.19 Kg.

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

$$= \frac{\pi^2 \times 2\times10^6 \times 6.26}{(800)^2}$$

P_{cr} = 193.07 Kg.

Problems on Euler Formula

1. A column having a T section with a flange 120mm X 16 mm and web 150 mm X 16 mm is 3 m long. Assuming the column to be hinged at both ends, Find the crippling load by using Euler's theory formula. E=2x10^6 kg/cm2

Problems on Euler Formula

$$\therefore \bar{Y} = \frac{12 \times 1.6 \times \frac{1.6}{2} + 15 \times 1.6\left(1.6 + \frac{15}{2}\right)}{12 \times 1.6 + 15 \times 1.6}$$

$$\bar{Y} = 5.41\,cm$$

Distance of C.G from bottom fibre = (15+1.6) - 5.41
= 11.19cm

Now M.O.I of the whole section about X-X axis.

$$I_{XX} = \left[\frac{12 \times (1.6)^3}{12} + (12 \times 1.6)\left(5.41 - \frac{1.6}{2}\right)^2\right] + \left[\frac{1.6 \times (15)^3}{12} + (1.6\right.$$

$$I_{XX} = 1188.92\,cm^4$$

M.I of the whole section about Y-Y axis

$$I_{yy} = \frac{1.6 \times (12)^3}{12} + \frac{15 \times (106)^3}{12} = 235.52\,cm^4$$

$$\therefore I_{min} = 235.52\,cm^4$$

$\therefore$ Euler's Crippling load,

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

$$= \frac{\pi^2 \times 2 \times 10^6 \times 235.52}{(300)^2} \quad ; \qquad P_{cr} = 51655.32\,Kg.$$

Problems on Euler Formula

A steel bar of solid circular cross-section is 50 mm in diameter. The bar is pinned at both ends and subjected to axial compression. If the limit of proportionality of the material is 210 MPa and E = 200 GPa, determine the m minimum length to which Euler's formula is valid. Also determine the value of Euler's buckling load if the column has this minimum length.

∴ Euler's buckling load,

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

$$\frac{P_{cr}}{A} = \frac{\pi^2 E}{(L/K)^2}$$

For Euler's formula to be valid, value of its minimum effective length L r out by equating the buckling stress to f

$$\frac{\pi^2 E}{\left(\frac{L}{K}\right)^2} = 210$$

$$L^2 = \frac{\pi^2 E \times k^2}{210} \qquad L^2 = \frac{\pi^2 \times 2 \times 10^5 \times 156.25}{210}$$

L = 1211.89 mm = 1212 mm = 1.212 m

∴ The required minimum actual length l = L = 1.212 m

For this value of minimum length,

$$\text{Euler's buckling load} = \frac{\pi^2 EI}{L^2}$$

$$= \frac{\pi^2 \times 2 \times 10^5 \times 306.75 \times 10^3}{(1212)^2}$$

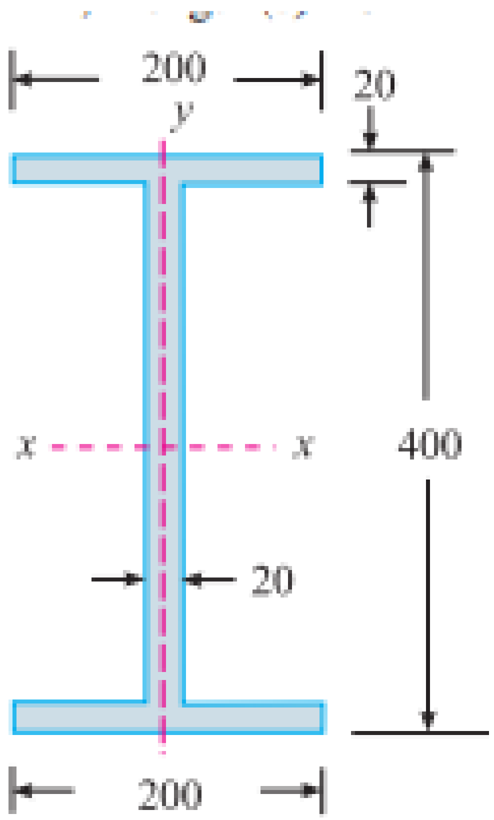
= 412254 N = 412.254 KN

Result:

Minimum actual length l = L = 1.212 m
Euler's buckling Load =412.254 KN

Problems on Euler Formula

An I Section joist 400 mmx200 mmx 20 mm and 6 m long is used as strut with both ends fixed Crippling load for the column? E=200 GPa



From the geometry of the figure, we find that inner depth,

$$d = 400 - (2 \times 20) = 360 \text{ mm}$$

and inner width, $$b = 200 - 20 = 180 \text{ mm}$$

We know that moment of inertia of the joist section about *X-X* axis,

$$I_{XX} = \frac{1}{12}[BD^2 - ba^3]$$

$$= \frac{1}{12}[200 \times (400)^3 - 180 \times (360)^3] \text{mm}^4$$

$$= 366.8 \times 10^6 \text{ mm}^4 \qquad ...(i)$$

Similarly, $$I_{YY} = \left[2 \times \frac{2 \times (200)^3}{12}\right] + \frac{360 \times (20)^3}{12}$$

$$= 2.91 \times 10^6 \text{ mm}^4$$

$$L_e = \frac{l}{2} = \frac{(6 \times 10^3)}{2} = 3 \times 10^3 \text{ mm}$$

ıd for the column,

$$P_E = \frac{\pi^2 EI}{L_e^{\,2}} = \frac{\pi^2 \times (200 \times 10^3) \times (2.91 \times 10^6)}{(3 \times 10^3)^2} =$$

$$= 638.2 \text{ kN} \qquad \textbf{Ans.}$$

Rankine’s Formula (or) Rankine's - Gorden Formula

$$\frac{1}{P} = \frac{1}{P_c} + \frac{1}{P_e}$$

where P = Rankine’s crippling load

P_c = ultimate load for a strut = $\sigma_u \cdot A$, con

P_e = Eulerial load for a strut = $\pi^2 EI/l^2$

- For short columns, P_e is very large and therefore $1/P_e$ is small ir load P is practically equal to P_c
- For long columns, P_e is very small and therefore $1/P_e$ is quite crippling load P is practically equal to P_e

$$\frac{1}{P} = \frac{1}{P_c} + \frac{1}{P_e}$$

$$\frac{1}{P} = \frac{P_e + P_c}{P_c P_e}$$

$$P = \frac{P_c P_e}{P_e + P_c} = \frac{P_c}{1 + P_c / P_e} = \frac{\sigma_c \cdot A}{1 + \frac{\sigma_c A \cdot l^2}{\pi^2 EI}}$$

$$= \frac{\sigma_c \cdot A}{1 + \frac{\sigma_c \cdot A \cdot l^2}{\pi^2 EAk^2}}$$

$$P = \frac{\sigma_c \cdot A}{1 + a\left(\frac{l}{k}\right)^2}$$

where σ_c is the crushing stress

a is the Rankine’s constant $(\sigma_c/\pi^2 E)$

Rankine's Formula (or) Rankine - Gorden Formula

$$\frac{1}{P} = \frac{1}{P_c} + \frac{1}{P_e}$$

$$\frac{1}{P} = \frac{P_e + P_c}{P_c P_e}$$

$$P = \frac{P_c P_e}{P_e + P_c} = \frac{P_c}{1 + P_c / P_e} = \frac{\sigma_c \cdot A}{1 + \frac{\sigma_c A \cdot l^2}{\pi^2 EI}}$$

$$= \frac{\sigma_c \cdot A}{1 + \frac{\sigma_c \cdot A \cdot l^2}{\pi^2 EAk^2}}$$

$$P = \frac{\sigma_c \cdot A}{1 + a\left(\frac{l}{k}\right)^2}$$

where σ_c is the crushing stress

a is the Rankine's constant $(\sigma_c/\pi^2 E)$

Material	f_c N/mm^2	$\alpha = \frac{f_c}{\pi^2 E}$
Wrought iron	250	$\frac{1}{9000}$
Cast iron	550	$\frac{1}{1600}$
Mild steel	320	$\frac{1}{7500}$
Timber	50	$\frac{1}{750}$

Rankine's Formula -Problems

A rolled steel joist ISMB 300 is to be used a column of 3 meters length with both ends fixed. Find the safe axial load on the column. Take factor of safety 3, $f_c = 320$ N/mm^2 and $\alpha = \frac{1}{7500}$. Properties of the column section.

Area = 5626 mm^2, $I_{XX} = 8.603 \times 10^7$ mm^4

$I_{yy} = 4.539 \times 10^7$ mm^4

Rankine's Formula -Problems

$\therefore$ Effective length, $L = \frac{l}{2} = \frac{3000}{2} = 1500mm$

Since I_{yy} is less then I_{xx}, $\therefore$ The column section,

$$I = I_{\min} = I_{yy} = 4.539 \times 10^7 \, mm^4$$

$\therefore$ Least radius of gyration of the column section,

$$K = \sqrt{\frac{I}{A}} = \sqrt{\frac{4.539 \times 10^7}{5626}} = 89.82mm$$

Crippling load as given by Rakine's formula,

$$P_{cr} = \frac{f_c \times A}{1 + \alpha\left(\frac{L}{K}\right)^2} = \frac{320 \times 5626}{1 + \frac{1}{7500}\left(\frac{1500}{89.82}\right)^2}$$

P_{cr} = 1343522.38 N

P_{cr} = 1343522.38 N

Allowing factor of safety 3,

$$\text{Safe load} = \frac{\textit{Crippling Load}}{\textit{Factor of safety}}$$

$$= \frac{1343522.38}{3} = 447840.79N$$

Result:

i. Crippling Load (P_{cr}) = 1343522.38 N
ii. Safe load =447840.79N

Rankine's Formula -Problems

A built up column consisting of rolled steel beam ISWB 300 with two plates 200 mm x 10 mm connected at the top and bottom flanges. Calculate the safe load the column carry, if the length is 3m and both ends are fixed. Take factor of safety 3 $f_c = 320$ N/mm² and $\alpha = \frac{1}{7500}$

Take properties of joist: A = 6133 mm²

$I_{xx} = 9821.6 \times 10^4$ mm⁴ ; $I_{yy} = 990.1 \times 10^4$ mm⁴

Rankine's Formula -Problems

7500

Sectional area of the built up column,

$$A = 6133 + 2(200 \times 10) = 10133 mm^2$$

Moment of inertia of the built up column section abut xx axis,

$$I_{XX} = 9821.6 \times 10^4 + 2\left[\frac{200 \times 10^3}{12} + (200 \times 10)(155)^2\right]$$

$= 1.94 \times 10^8 \text{ mm}^4$

Moment of inertia of the built up column section abut YY axis,

$$I_{YY} = 990.1 \times 10^4 + 2\left(\frac{10 \times 200^3}{12}\right)$$

$= 0.23 \times 10^8 \text{ mm}^4$

Since I_{yy} is less than I_{xx} , The column will tend to buckle about Y-Y axis.

Least moment of inertia of the column section,

$$I = I_{min} = I_{YY} = 0.23 \times 10^8 mm^4$$

The column is fixed at both ends.

$\therefore$ Effective length,

$$L = \frac{l}{2} = \frac{3000}{2} = 1500 mm$$

$\therefore$ Least radius of gyration o the column section,

$$K = \sqrt{\frac{J}{A}} = \sqrt{\frac{0.23 \times 10^8}{10133}} = 47.64 mm$$

Crippling load as given by Rankine's formula,

$$P_{cr} = \frac{f_c \times A}{1 + \alpha\left(\frac{L}{K}\right)^2} = \frac{320 \times 10133}{1 + \frac{1}{7500}\left(\frac{1500}{47.64}\right)^2}$$

$= 2864023.3$ N

$$\text{Safe load} = \frac{\text{Crippling load}}{\text{Factor of safety}} = \frac{2864023.3}{3} = 954674.43 N$$

Result:

i. Crippling load = 2864023.3 N
ii. Safe load = 954674.43 N

Long & Short Columns Subjected Eccentric Loading

i. Rankine's formula:

Consider a short column subjected to an eccentric load P with an eccentricity e form axis.

Maximum stress = Direct Stress + Bending stress

$$f_c = \frac{P}{A} + \frac{M}{Z} \qquad Z = \frac{I}{y}$$

$$= \frac{P}{A} + \frac{p.e.y_c}{Ak^2} \qquad I = Ak^2$$

$$k = \sqrt{\frac{I}{A}}$$

Long & Short Columns Subjected Eccentric Loading

$$f_c = \frac{P}{A}\left(1 + \frac{ey_c}{k^2}\right)$$

$\therefore$

$$\boxed{\text{Eccentric load, } P = \frac{f_c \times A}{1 + \frac{ey_c}{k^2}}}$$

Where $\left(1 + \frac{ey_c}{k^2}\right)$ is the reduction factor for eccentricity of loading.

For long column, loaded with axial loading, the crippling load,

$$P = \frac{f_c \times A}{1 + \alpha\left(\frac{L}{K}\right)^2}$$

Where $\left(1 + \alpha\left(\frac{L}{K}\right)^2\right)$ is the reduction factor for buckling of long column.

Hence for a long column loaded with eccentric loading, the safe load,

$$\boxed{P = \frac{f_c \times A}{\left(1 + \frac{ey_c}{K^2}\right)\left[1 + \alpha\left(\frac{L}{K}\right)^2\right]}}$$

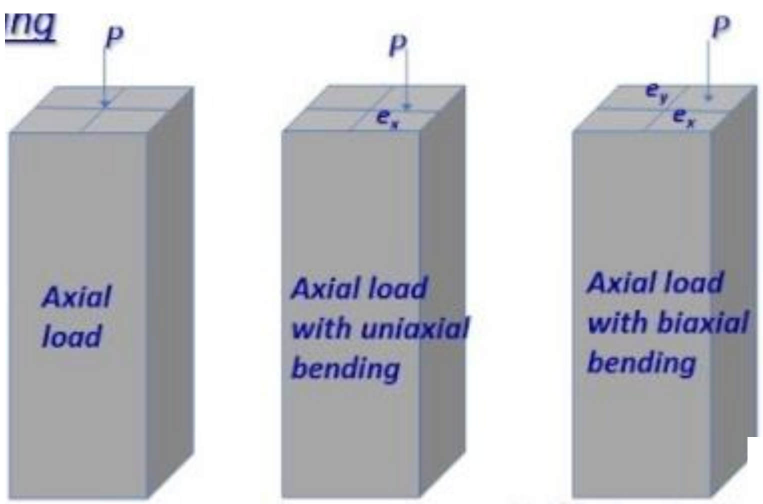
ii. Euler's formula

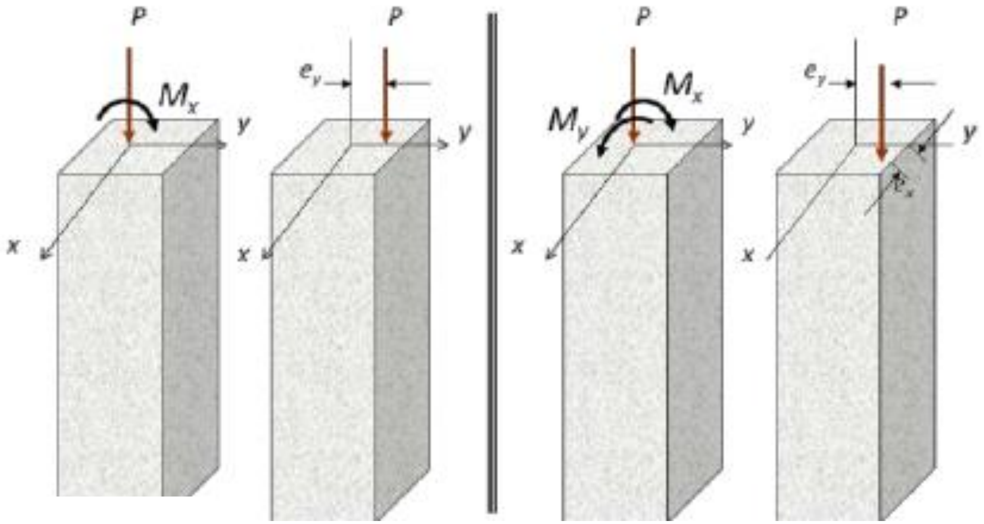
Maximum stress n the column = Direct stress + Bending stress

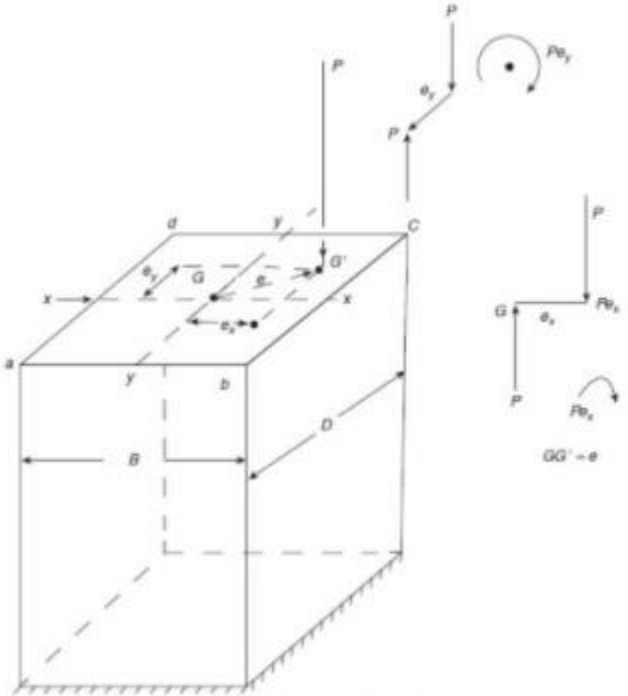
$$= \frac{P}{A} + \frac{P \times e \sec\sqrt{P/EI}\,\frac{l}{2}}{Z}$$

Hence, the maximum stress induced in the column having both ends hinged and an eccentricity of e is $\frac{P}{A} + \frac{Pe}{Z}\sec\left(\sqrt{P/EI}\,\frac{l}{2}\right)$

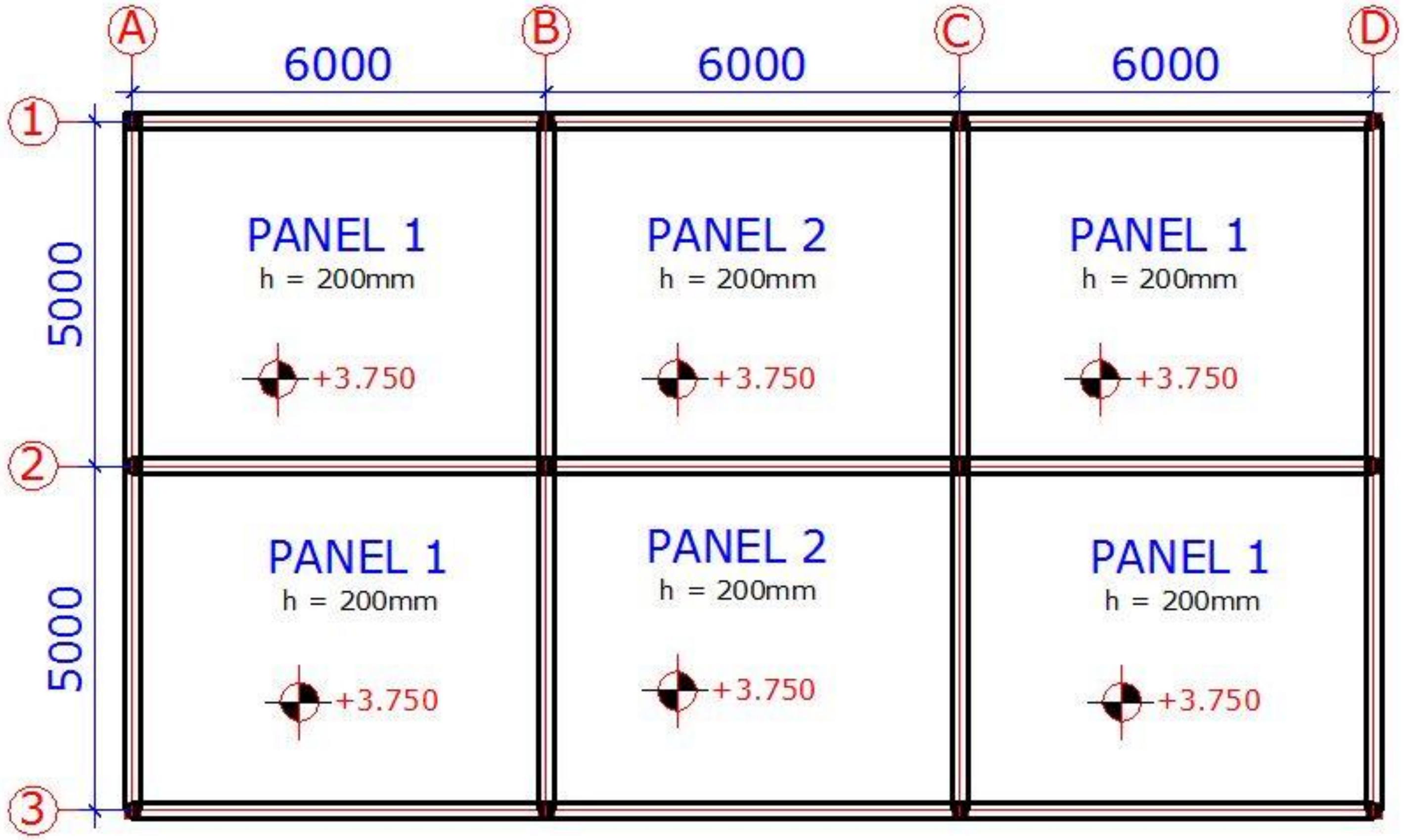
Eccentric Loaded column







Eccentric Loaded column



CORE (or) KERNEL OF A SECTION

- When load acts in such a way on region around the CG of the section So that in that region stress everywhere is compressive and NO TENSION developed any where.

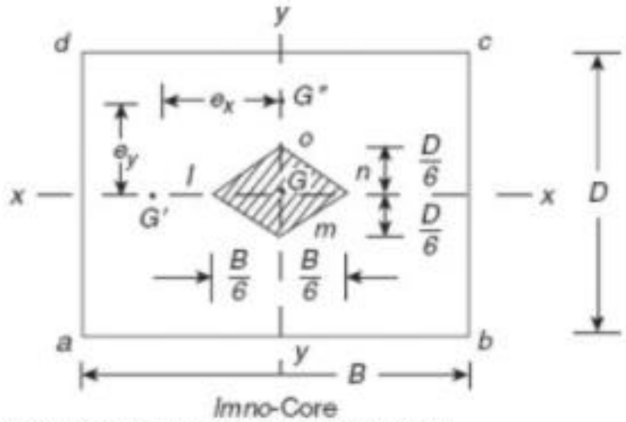


Figure 10.6 Core of rectangular section

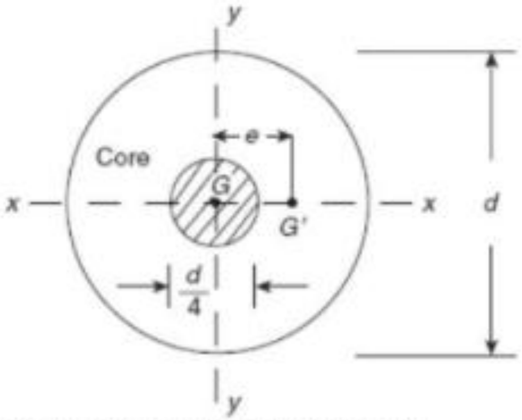


Figure 10.7 Core of a circular section

Core of the rectangular section = Area of the shaded portion

$$= 2 \times \frac{1}{2} \times \frac{b}{3} \times \frac{d}{6}$$

$$= \frac{bd}{18}$$

Core of the circular section = Area of the shaded portion

$$= \pi (D/8)^2$$

$$= \frac{\pi D^2}{64}$$

CORE (or) KERNEL OF A SECTION

- When load acts in such a way on region around the CG of the section So that in that region stress everywhere is compressive and NO TENSION developed any where.

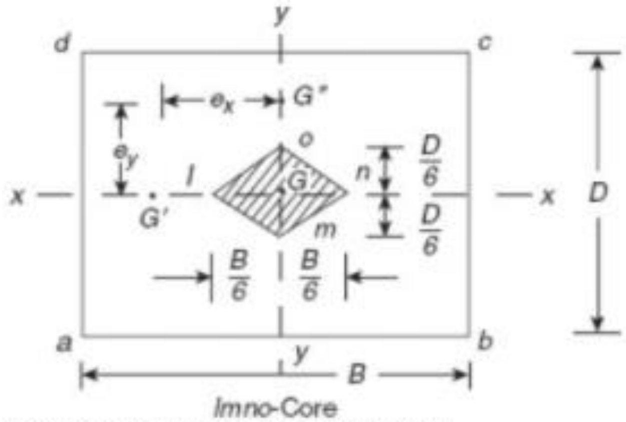


Figure 10.6 Core of rectangular section

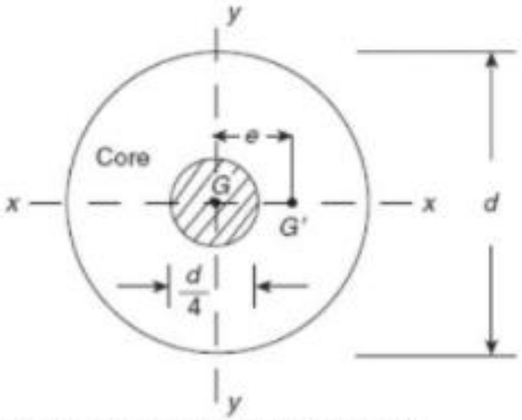


Figure 10.7 Core of a circular section

Core of the rectangular section = Area of the shaded portion

$$= 2 \times \frac{1}{2} \times \frac{b}{3} \times \frac{d}{6}$$

$$= \frac{bd}{18}$$

Core of the circular section = Area of the shaded portion

$$= \pi (D/8)^2$$

$$= \frac{\pi D^2}{64}$$

Eccentric loaded column Problems

A column of circular section has 150 mm dia and 3m length. Both ends of the column are fixed. The column carries a load of 100 KN at an eccentricity of 15 mm from the geometrical axis of the column. Find the maximum compressive stress in the column section. Find also the maximum permissible eccentricity to avoid tension in the column section. $E = 1 \times 10^5$ N/mm^2

Eccentric loaded column Problems

A column of circular section has 150 mm dia and 3m length. Both ends of the column are fixed. The column carries a load of 100 KN at an eccentricity of 15 mm from the geometrical axis of the column. Find the maximum compressive stress in the column section. Find also the maximum permissible eccentricity to avoid tension in the column section. $E = 1 \times 10^5$ N/mm^2

Area of the column section $A = \frac{\pi \times D^2}{4}$

$$= \frac{\pi}{4}(150)^2$$

$$= 17671 \text{ mm}^2$$

Moment of inertia of the column section N.A.,

$$I = \frac{\pi}{64} \times D^4 = \frac{\pi}{64} \times (150)^4$$

$$= 24.85 \times 10^6 \text{ mm}^4$$

Section modulus,

$$Z = \frac{I}{y} = \frac{I}{D/2}$$

$$= \frac{24.85 \times 10^6}{\frac{150}{2}} = 331339 mm^3$$

Both the ends of the column 2 are fixed.

Effective length of the column, $L = \frac{l}{2} = \frac{3000}{2} = 1500mm$

Now, the angle

$$\sqrt{P/EI} \times \frac{L}{2} = \sqrt{\frac{100 \times 10^3}{1 \times 10^5 \times 24.85 \times 10^6}} \times \frac{1500}{2}$$

$$= 0.1504 \text{ rad} = 8.61^\circ$$

Maximum compressive stress,

$$= \frac{P}{A} + \frac{P \times e}{Z}\left(\sec\sqrt{P/EI}\,\frac{L}{2}\right)$$

$$= \frac{100 \times 10^3}{17671} + \frac{100 \times 10^3 \times 15 \times \sec 8.61^\circ}{331339}$$

$$= 10.22 \text{ N/mm}^2$$

To avoid tension we know,

$$\frac{P}{A} = \frac{M}{Z}$$

$$\Rightarrow \quad \frac{P}{A} = \frac{p \times e \times \sec .8.61^\circ}{Z}$$

$$\frac{100 \times 10^3}{17671} = \frac{100 \times 10^3 \times e \times \sec .8.61^\circ}{331339}$$

$$e = 18.50 \text{ mm}$$

Result:

i. Maximum compressive stress $= 10.22$ N/mm^2
ii. Maximum eccentricity $= 18.50$ mm

Column & Strut

Queries?