UNIT-6. APPLICATIONS OF GRAPH THEORY

18-06-23

* 1st modules :- Euler circuits and Hamilton circuits

* Euler circuits :-

→ This module is / [Euler circuits] is developed by Leonard Euler.

→ It is a simple circuit containing all edges of the graph.

* Euler path :-

→ It is a simple path containing all the edges of the graph.

* Necessary & sufficient conditions for existence of an Euler circuit (or) Euler path :-

* case-1 :- For undirected graph.

* Theorem-1 :-

→ A graph 'G' [undirected] has an 'Euler circuit' if and only if degree of each vertex in G is an 'even' no.

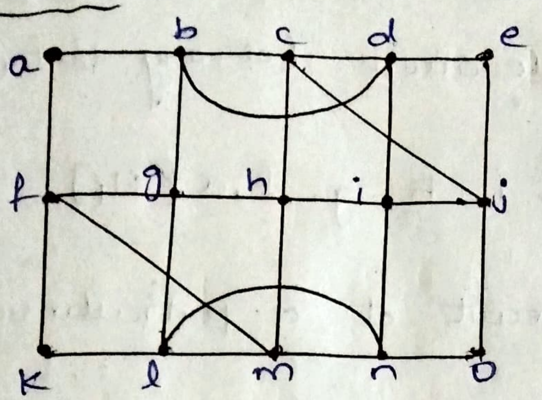
* Theorem-2 :-

→ A Graph 'G' has an 'Euler path' if and only if 'G' has exactly two vertices of 'odd degree'.

→ In this case, that Euler path from one odd degree vertex to another odd degree vertex.

* Exercise :-

1) (v)



A)

vertex	degree
a	2
b	4
c	4
d	4
e	2
f	4
g	4
h	4
i	4
j	4
k	2
l	4.
m	4
n	4
o	2

Each degree is even.

$\therefore$ By Theorem-1; G has Euler circuit.

(or)

we notice that, the degree of each vertex is even,

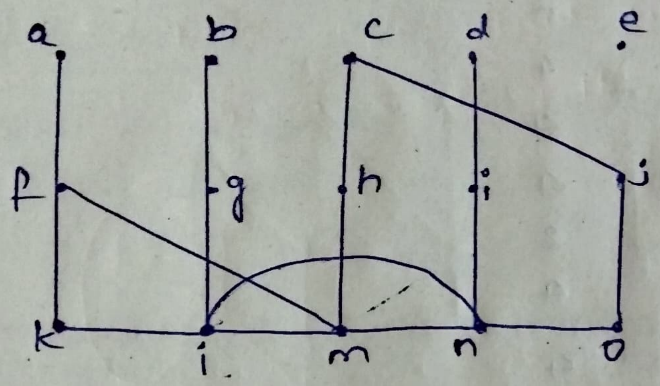
Then By Theorem-1, Graph 'G' has an Euler circuit construction of Euler circuit as follows

Step-1:- Take simple circuit [at any vertex].

a, b, c, d, e, j, i, h, g, f, a [H_1].

Step-2:- simple circuit at a particular vertex at in H_1

$H_1 = G -$ above simple circuit.



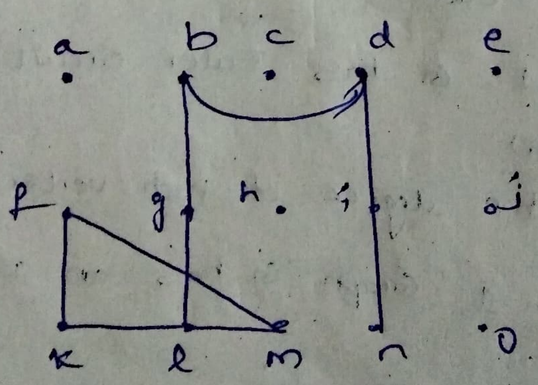
a, b, c, d, e, j, i, h, g, f.

c, j, o, n, m, h, c.

Step-3:- Join step-1 & 2. a, b, c, d, o, n, m, h, c, d, e, j, i, h, g, f, a

Step-4:- simple circuit at d in H_2

$H_2 = G -$ above simple circuit.



a, b, c, ~~d~~ j, o, n, m, h, c, d, e, j, i, h, g, f

d, i, n, l, g, b, d

step-5:- Join them. a, b, c, j, o, n, m, h, c, d, i, n, l, g, b, d, d, e, j, i, h, g, f, a

step-6:- Take simple circuit at 'f'. in H_3.

$H_3 = G -$ above simple circuit.

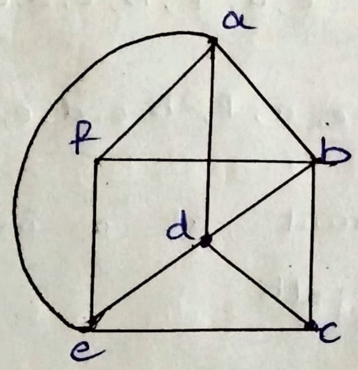
f, k, l, m, f.

step-7:- Join them.

a, b, c, j, o, n, m, h, c, d, i, n, l, g, b, d, e, j, i, h, g, f, k, l, m, f, a.

∴ This is required Euler circuit. [containing all edges & simple circuit].

(ii)



A)

vertex	degree
a	4
b	4
c	3
d	4
e	4
f	3

→ This graph contains exactly 2 - odd degrees

∴ By Theorem-2, it has Euler path.

[from c to f]

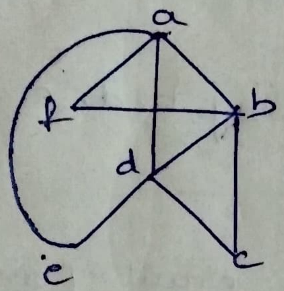
Construction of Euler path :-

step-1 :- Take simple path from 'c' to 'f'.

c, e, f

step-2 :- Take simple circuit at 'e' in H_1

H_1 = G - simple path.



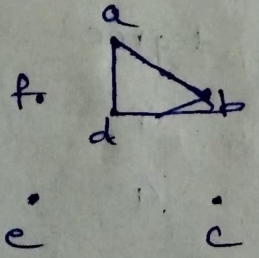
c, e, f

e, a, f, b, c, d, e.

step-3 :- Join them, c, e, a, f, b, c, d, e, f.

step-4 :- Take simple circuit at 'a' in H_2

H_2 = G - above simple path

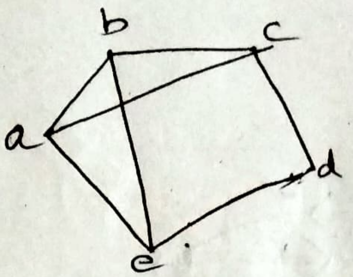


a, b, d, a

step-5 :- Join them, c, e, a, b, d, a, f, b, c, d, e, f.

∴ This is required Euler path [from c to f].

* P-3 :-



A)

Vertex	degree
a	3
b	3
c	3
d	2
e	3

∴ This Graph has Neither Euler circuit Nor Euler path.

* Case - ii :- For Digraph

Necessary & sufficient conditions for Euler circuit or path in Digraph.

* Theorem-3 :-

→ Let 'G' be a Digraph and 'G' has an euler circuit iff the indegrees of each vertex is equal to its outdegrees.

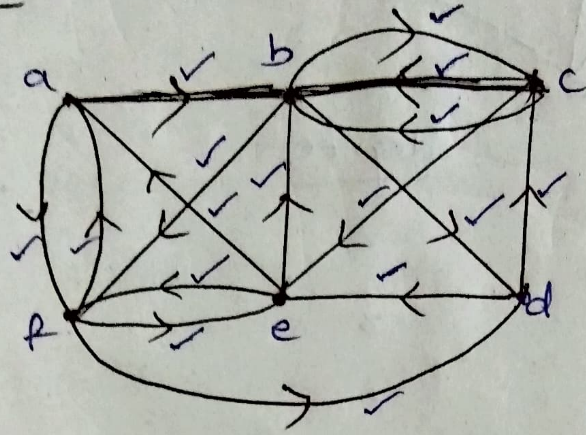
* Theorem-4 :-

→ Let 'G' be a Digraph & 'G' has Euler path iff the indegree of each vertex is equals to its outdegrees except 2 vertices say [a & b]. with Indegree of a = outdegree of a+1
Indegree of b = outdegree of b-1

→ In this case, that Euler path starts from [b to a].

* Exercise :-

(2)



A)

Vertex	Indegree	outdegree
a	2	2
b	4	3
c	2	3
d	2	2
e	3	3
f	3	3

+

−

we notice that, except vertices b & c have same In & out degrees

By Theorem - 4

This graph contains Euler path.

Also Indegree of b = out degree of b +1

Indegree of c = out degree of b-1

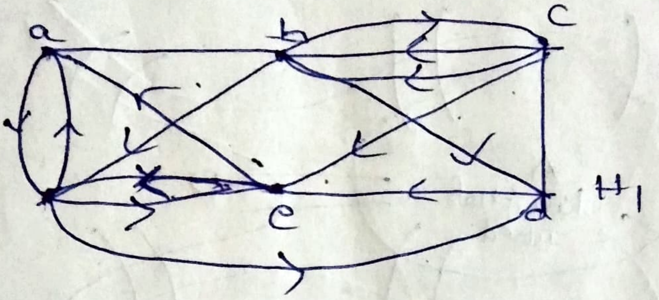
Derivation of Euler path :-

step-1 :- Take simple path from c to b.

c, e, b.

Step-2 :- Take simple circuit at 'e' in H_1

H_1 = G - above simple path.

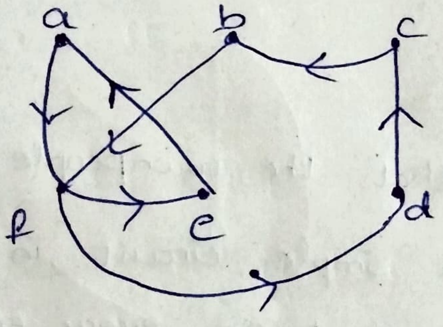


e, f, a, b, c, b, d, e.

Step-3 :- Join them, c, e, f, a, b, c, b, d, e, b.

Step-4 :- Take simple circuit at in H_2.

H_2 = G - above simple path.



e, a, f, d, c, b, f, e.

Step-5 :- Join them, 3 & 4.

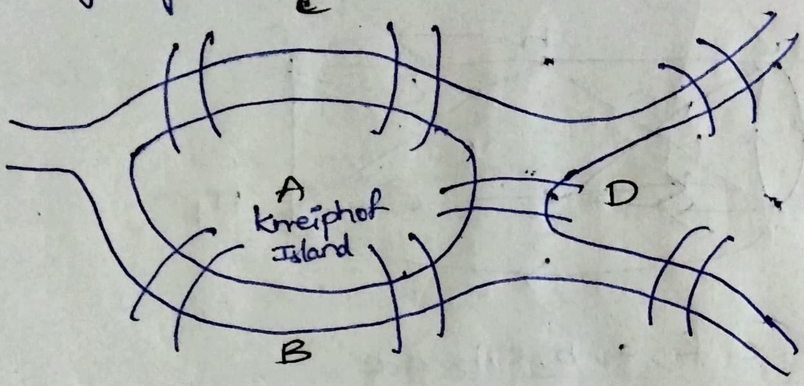
~~c, e, a, f, d.~~

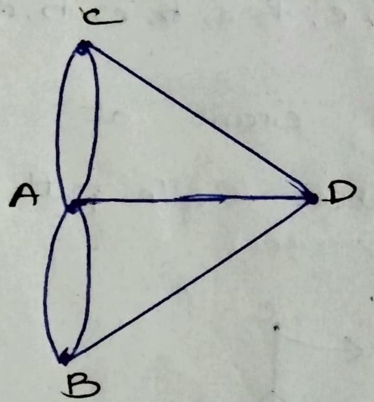
c, e, f, a, b, c, b, d, e, a, f, d, c, b, f, e, b.

clearly, it is a simple path containing all edges from c to b.

* Konigsberg seven bridges problem:-

→ Konigsberg Town.





→ The question that the townpeople asked is "Is there a simple circuit in this multigraph that contains every edge."

* Hamilton circuit:-

→ A simple circuit in 'G' that passes through every vertex of 'G' exactly once is called a Hamilton circuit."

* Note:-

→ Let 'G' be a graph with pendant vertex [degree-1] doesnot have Hamilton circuit.

* Exercise :-

3(i) Given graph doesnot contains Hamilton circuit. Because, any circuit containing all the vertices must passes through one of the vertex c, f twice. But it has Hamilton path [a, b, c, f, d, e].

(ii) This graph contains Hamiltonian/Hamilton circuit All Boundaries [a, b, c, d, e, a].

(iii) since, this graph has pendant vertex [f]
∴ It doesnot contain Hamilton circuit.

(iv) Since, this graph has pendant vertices (g, f, e)
∴ It doesnot contain Hamilton circuit.

(v) This graph doesnot contain Hamilton circuit Because, any circuit containing all vertices must passes through one of the vertex b, h, d, f twice.

(vi) This graph doesnot contain Hamilton circuit. Because, any circuit containing all vertices must passes through one of the vertices b, c twice.

(vii) This graph contains Hamilton circuit
a, d, g, h, i, f, c, b, e, a.

6.2 – PLANAR GRAPHS

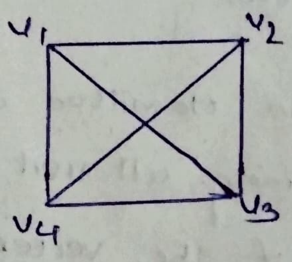
* Planar graph:-

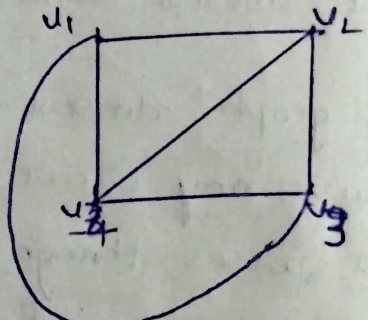
→ A graph is said to be planar if it can be drawn in the plane without any edges crossing.

→ such a drawing is caller "planar representation" of graph.

Ex:-

K4 graph

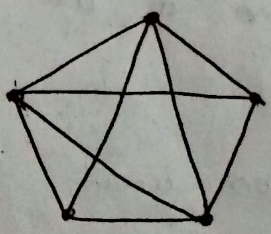




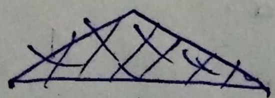
Planar graph

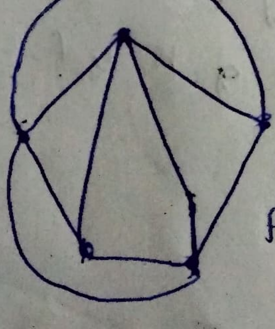
* Exercise:-

① ②



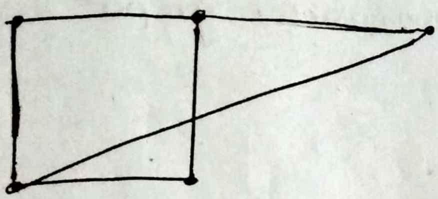
A)



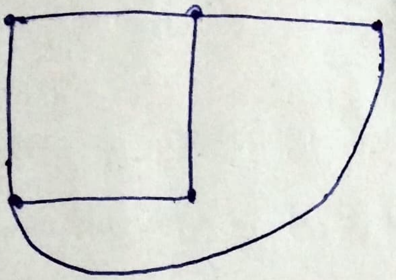


Planar graph.

Q ii

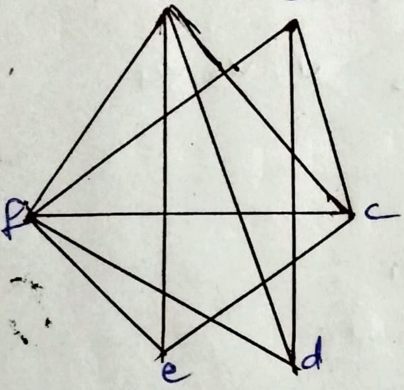


A)



planar graph.

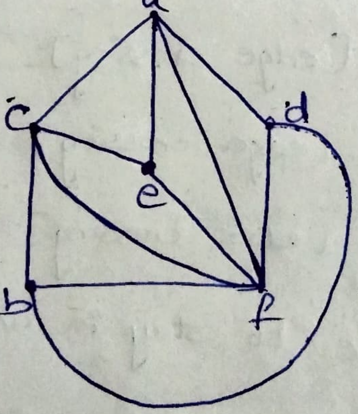
Q ii



A) $|V| = 6$, $|E|$.

circuit of length 5 [≤ 6].

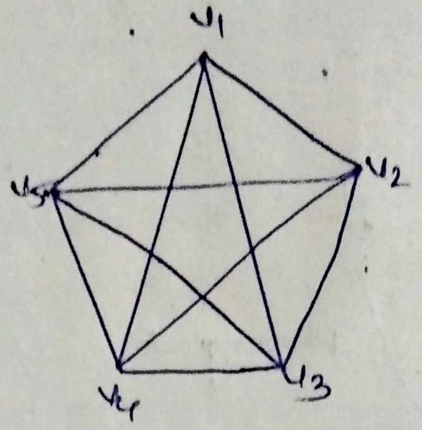
a, c, b, f, d, a



∴ clearly it is a planar graph, without crossing the edges.

* show that K_5 is Non-planar graph?

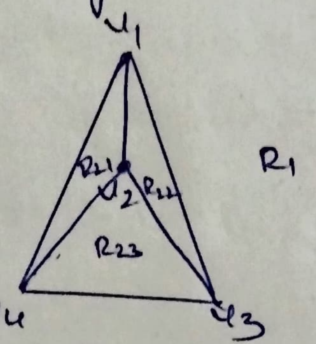
A)



$|V| = 5$

$|E| = 10$ edges.

circuit of length '3' is v_1, v_3, v_4.



v_5 has 4 possible regions, $R_1, R_{21}, R_{22}, R_{23}$

R_1 will fails [crossing].

R_{21} also fails [edge crossing].

R_{22} also fails [edge crossing]

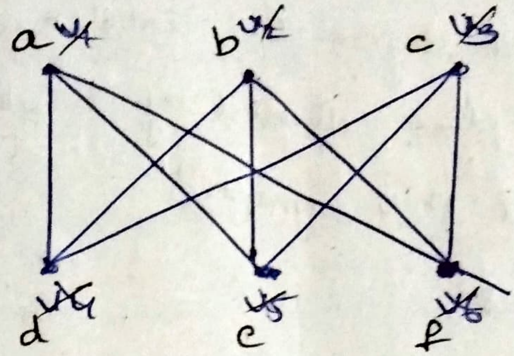
R_{23} also fails [edge crossing].

v_5 has no chance to stay in any of 4 regions

$\therefore$ k_5 is Non-planar graph.

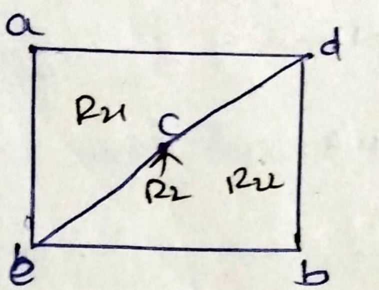
* show that $K_{3,3}$ is Non-planar graph?

A)



Take a circuit with less than no. of vertices.

Draw circuit a, d, b, e, a



If $f \in R_{21}$ $\Rightarrow$ There is edge ~~from~~ crossing.

If $f \in R_{22}$ $\Rightarrow$ There is edge crossing.

If $f \in R_1$ $\Rightarrow$ There is edge crossing.

$\therefore$ The given graph is not planar graph.

* Euler's Formula:

→ ~~Every~~ conn't

→ Let 'G' be a (~~Connected~~ simple) planar graph with 'E' edges & 'V' vertices then "$v - e + r = 2$."

where r = No. of regions

2] (a) Suppose that a connected planar graph has 8 vertices, each of degree 3. Into how many regions is the plane divided by a planar representation of this graph?

A) Given that $v=8$,

Degree of each vertex $=3$.

By Handshaking theorem,

$$2e = 8 \times 3$$

$$\Rightarrow e = \frac{24}{2} = 12$$

By Euler's Formula,

$$v - e + r = 2$$

$$\Rightarrow r = 2 + e - v$$

$$\therefore r = 2 + 12 - 8 = \underline{6}$$

(b) Suppose that a connected planar graph has 30 edges. If a planar representation of this graph divides the plane into 20 regions, how many vertices does this graph have?

A) Given $e = 30$

$r = 20$

$$\therefore v - e + r = 2$$

$$v = 2 + e - r$$

$$= 2 + 30 - 20$$

$\therefore v = 12$ vertices.

* Corollary-1:-

→ Let 'G' be a connected simple graph with 'e' edges & 'v' vertices. [$v \geq 3$].

→ If 'G' is planar then '$e \leq 3v-6$'.

→ If '$e \nleq 3v-6$' then 'G' is Not planar.

Ex:- K_5

$|v|=5, \ |e|=10$

$\Rightarrow e \leq 3(5)-6$

$\Rightarrow e \leq 9$

$\Rightarrow 10 \nleq 9$

$\therefore K_5$ is Not planar.

* Problem:-

Let 'G' be a connected simple graph with 15 vertices & 40 edges. Is 'G' planar?

A) Given $e=40, \quad v=15$

$\therefore e \leq 3(15)-6$

$\Rightarrow 40 \leq 45-6$

$\Rightarrow 40 \nleq 39$

$\therefore$ Not planar.

* Corollary-2:-

→ If G is a connected planar simple graph with no loops then 'G' has a vertex of degree not exceding 5.

Ex:- K_7, K_8, K_9 are not planar.

* Corollary-3 :-

→ Let 'G' be a connected simple graph with 'v' vertices & 'e' edges. 'G' has no circuits of length 3. If 'G' is 'planar' then '$e \leq 2v-4$'

→ If '$e \nleq 2v-4$' then 'G' is 'Not planar'.

Ex :- $K_{3,3}$

By corollary-1 $\Rightarrow 9 \leq 3(6)-6$

$\Rightarrow 9 \leq 12$ ✓

By corollary-3 $\Rightarrow 9 \leq 2(6)-4$

$\Rightarrow 9 \leq 8$ ✗

∴ $K_{3,3}$ is not planar.

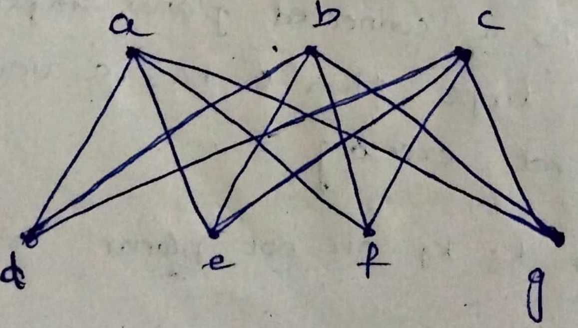
* Crossing Number :-

→ The 'crossing number' of a simple graph is minimum no. of crossings that can occur when graph is drawn in the plane where no 3 arcs representing the edges are permitted to cross the same point.

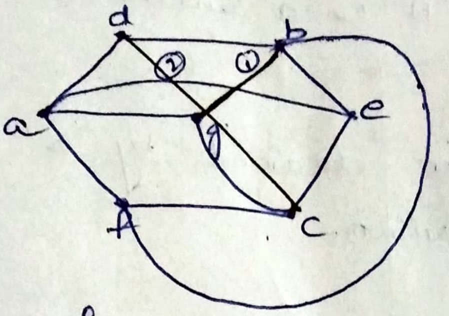
* Exercise :-

4] ⓒ $K_{3,4}$.

A)



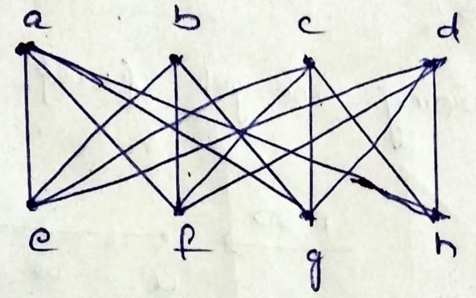
a, d, b, e, c, f, a.



Min no. of edge crossings = 2.

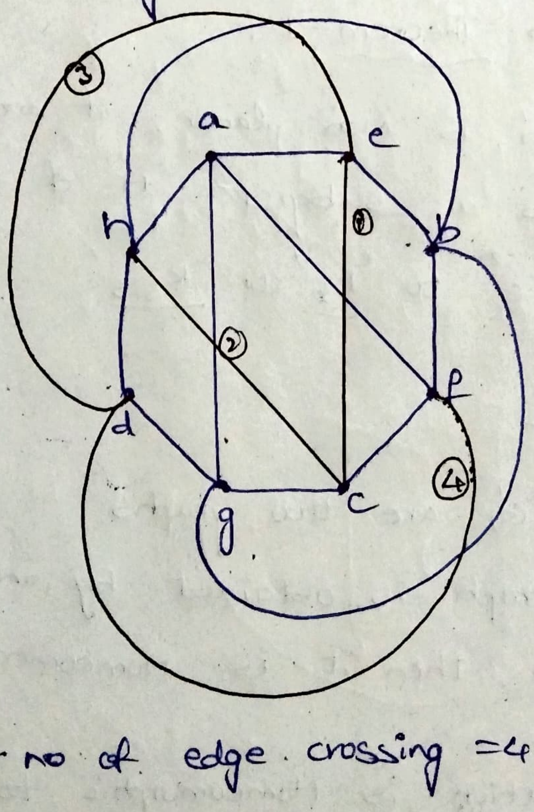
(d) $K_{4,4}$

A)



consider a ~~path~~ circuit a, e, b, f, c, g, d, h

is of length - 8.



Min. no of edge crossing = 4

$\therefore$ crossing = 4

* Elementary subdivision :- [ESD]

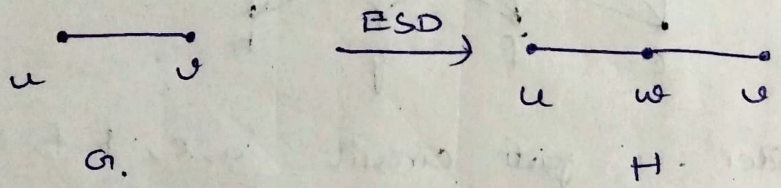
→ If a Graph 'H' is a subdivision of another Graph 'G'.

If 'H' can be obtained by 'G' by applying elementary subdivision.

$G \longrightarrow H$

→ Delete an edge {u, v} in G and add a new vertex w(say) (which is not in G) together with the edges.

→ After joining we get {u, w}, {w, v}



→ ESD doesnot effects planarity.

* kuratowski's Theorem-1 :-

→ A Graph 'G' is Non planar, if and only if there exists a subgraph 'H' of 'G' is "Homeomorphic" to 'K_5' (or) '$K_{3,3}$'.

Homeomorphic :-

→ If 'G_1' and 'G_2' are two graphs,

"If one graph is obtained by another graph using ESD then it is Homeomorphic"

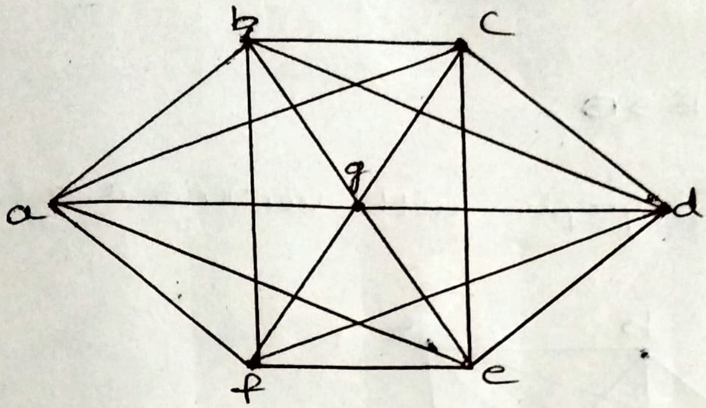
→ Any graph which is Homeomorphic to planar graph is also planar.

* **Kuratowski's Theorem-2 :**

→ A Graph 'G' is planar iff 'G' doesnot contain any subgraph Homeomorphic to one of the Kuratowski's 'K_5' or '$K_{3,3}$'

* **Exercise :**

3(b)



A) Let us assume given graph is 'G'.

Here $|V| = 7$

Degree of each vertex except 'g' is 5.

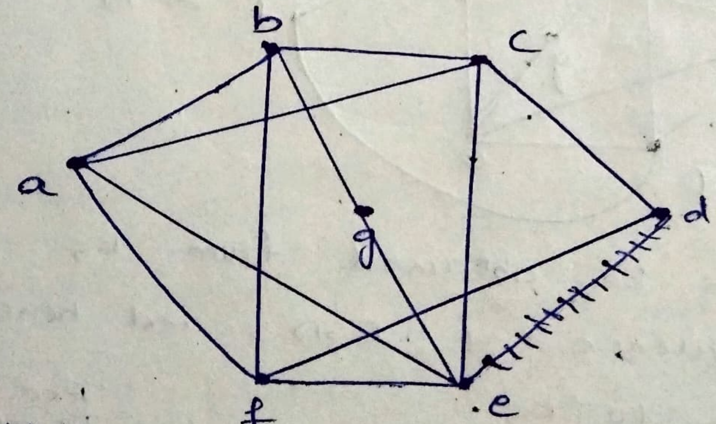
g - degree = 6

By Handshaking Theorem, $\therefore 6 \times 5 + 6 = 2e$

$36 = 2e$

$e = 18$.

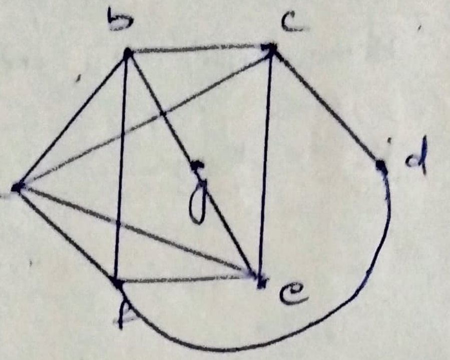
The subgraph can be obtained by 'G' by deleting the following edges.



$18 - 6 = 12$ edges.

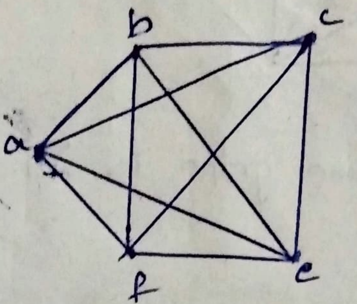
Another graph .H.

Another form of graph H.



~~By using ESD~~,

consider K_5 graph with vertices a, b, c, e, f

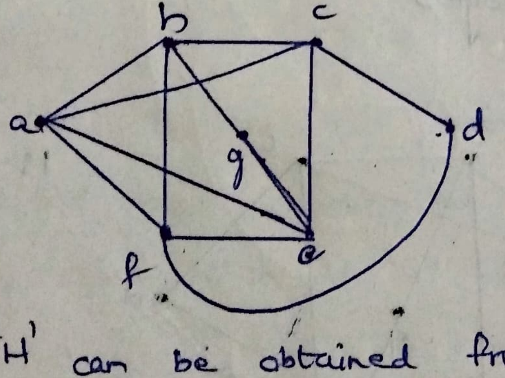


K_5.

(i) Delete an edge {b, e} and add new vertex 'g' together with {b, g}, {g, e}.

(ii) Delete an edge {c, f} and add new vertex 'd' together with {c, d}, {d, f}.

By applying (i) & (ii)



← H

Thus 'H' can be obtained from K_5 by applying sequence of ESD's and hence 'H' is Homeomorphic to K_5.

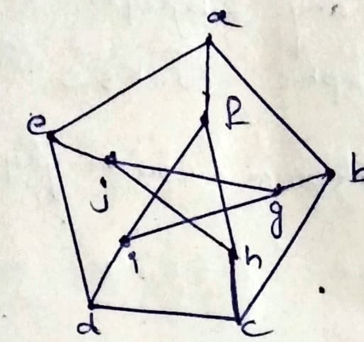
Thus there is a subgraph 'H' of 'G'.

Then By Kurat Theorem-I, 'G' is Non planar.

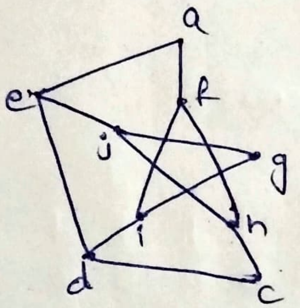
* Example-10 :- Petersen Graph.

~~K5~~

A) Petersen Graph :-

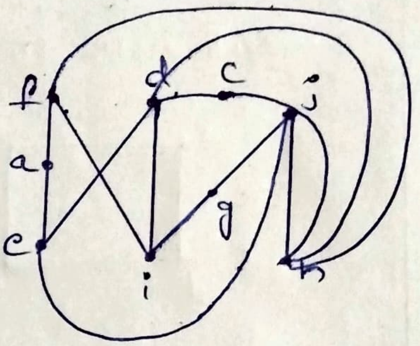


Delete vertex-b :-



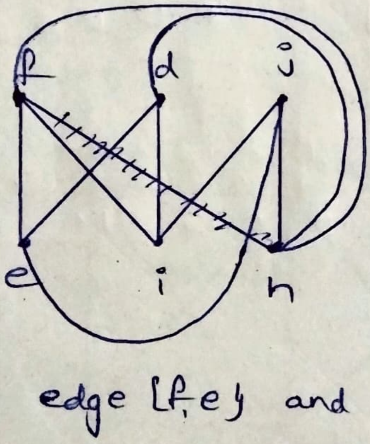
H-graph

Another form of H :-



H-graph

Consider $K_{3,3}$ graph with ~~R~~ d. vertex sets $\{f, d, j\}$ and $\{e, i, h\}$.



(i) Delete edge $\{f, e\}$ and add new vertex 'a'.

(ii) Delete edge $\{i, j\}$ and add new vertex 'g'.

(iii) Delete edge $\{d, h\}$ and add new vertex 'c'.

By applying above, we will get h-graph. & It is Homeomorphic.

6.3 - GRAPH COLORING.

* Graph coloring:-

→ It is coloring of a simple graph.

→ No two adjacent sides have same color.

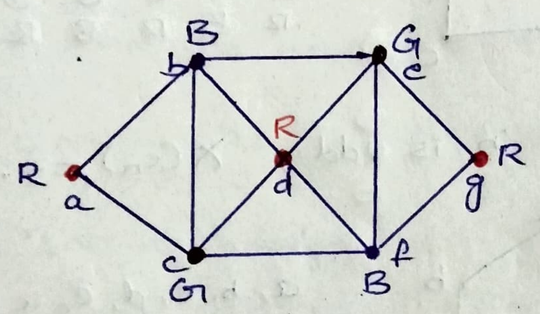
* chromatic number:-

→ The least no. of colors required to a graph coloring is called chromatic Number.

→ It is denoted by 'chi' [X] - Ki

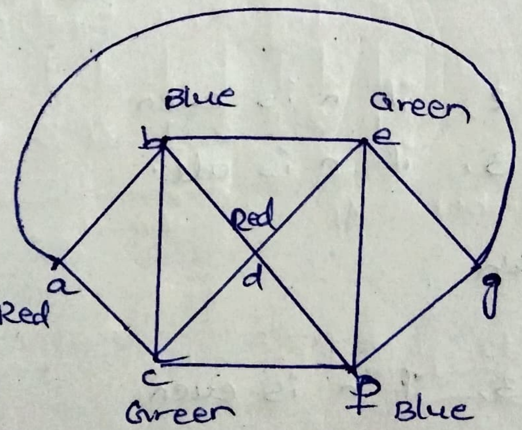
* Ex-1:-

A) ①



$X = 3$

②



$X = 4$

* chromatic number for different graphs :-

1. For planar graphs, chromatic no is not greater than '4'.

2. For Non-planar graphs, χ is large number

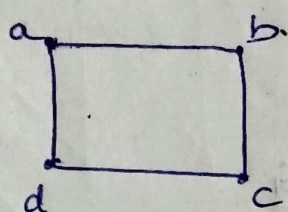
3. For complete graph, '$\chi(K_n) = n$'.

4. For Bipartite graph, '$\chi(K_{m,n}) = 2$'

4. For cycle, (C_n)

case-i :- If 'n' is even.

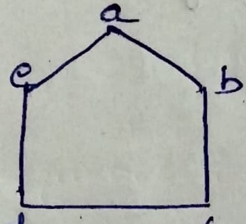
→ If $n=4$, C_4, $\chi(C_n) = 2$.



a, B, c, d, a
↓ ↓ ↓ ↓ ↓
R B R B R

case-ii :- If 'n' is odd, $\chi(C_n) = 3$.

Ex:-



a, b, c, d, e, a
↓ ↓ ↓ ↓ ↓ ↓
R B R B G R

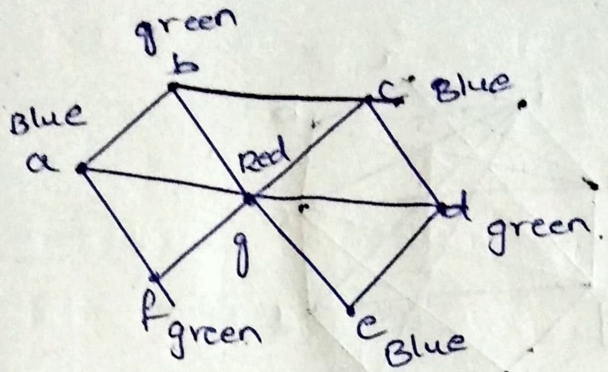
$\therefore \chi(C_n) = 2$, if n is even
$\quad\quad\quad\;\; 3$, if n is odd.

5. For wheel, W_n

Ex:-

$$\chi(W_n) = \begin{cases} 3, & \text{if } n \text{ is even.} \\ 4, & \text{if } n \text{ is odd.} \end{cases}$$

* P-2 :-

A) 

$\chi(G) = 3.$

* P-3 :-

A) For an isolated node, $\chi = 1.$

* Welsh-Powell algorithm :-

step-1 :- Find the degree of each vertex in 'G'.

step-2 :- Arrange the vertices by using degree sequence

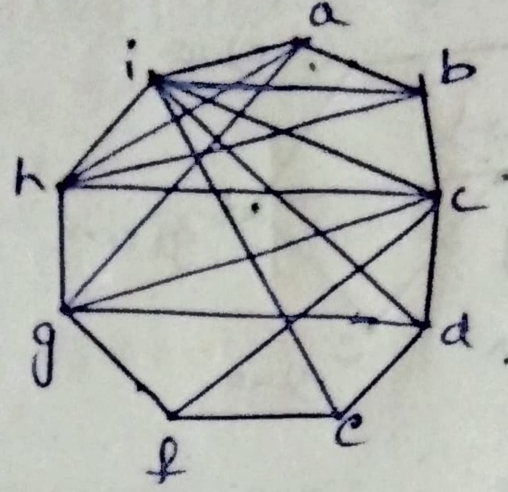
step-3 :- Let us assign color-1 to the first vertex in the vertices list. and move forward through the list until we get a non-adjacent ~~step-4~~ :- vertex to a vertex having color-1 and color it. Keep on moving through the list find the Non-adjacent vertex to vertices having color-1 and assign color as color-1.

step-4 :- Repeat step-3 for non-coloured vertices in the list. using color-2.

step-5 :- Repeat step-3 & 4 using colors-3, 4, ---

step-6 :- The maximum no occurred in the colored row becomes the chromatic no. of given graph.

* P-6:

A) 

Vertex	Degree
a	4
b	4
c	6
d	4
e	3
f	3
g	5
h	5
i	6

Degree sequence :- 6, 6, 5,5, 4,4,4, 3, 3.

$$\begin{array}{ccccccccc} c, & i, & g, & h, & a, & b, & d, & e, & f \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 2 & 2 & 3 & 1 & 4 & 3 & 1 & 3 \end{array}$$

∴ chromatic Number = 4 color - 1, 2, 3, 4

6.4 - TREES

* Trees:-

→ A 'Tree' is an 'undirected connected graph' without circuits (or) simple circuits.

* Trivial Tree and Non-Trivial Tree:-

→ A Tree which contains only one vertex is called 'Trivial Tree' otherwise 'Non-Trivial Tree'

* Theorem-1:-

→ Any undirected connected graph is tree iff there is an unique simple path b/w any two different vertices.

* Theorem-2:-

→ Let 'G' be a undirected graph with 'n' vertices then the following statements are equivalent.

S_1

S_2

S_3

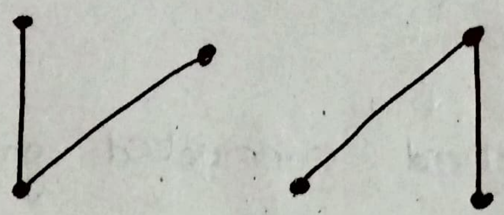
$$\left.\begin{array}{l} S_1 \Rightarrow S_2 \\ S_2 \Rightarrow S_3 \\ S_3 \Rightarrow S_1 \end{array}\right\} \text{Equivalent.}$$

1. G is a tree.
2. G is connected with n-1 edges.
3. G is circuitless with n-1 edges.
4. G is minimally connected graph.

* Exercise:

1) Which are trees?

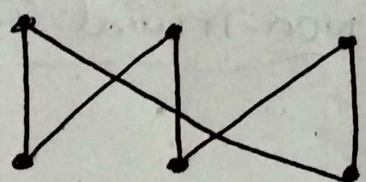
(b)



A) It is disconnected graph

∴ Not a Tree.

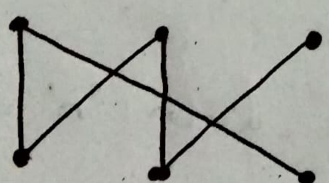
(f)



A) It is a connected graph. But it contains circuit

∴ Not a tree.

(h)

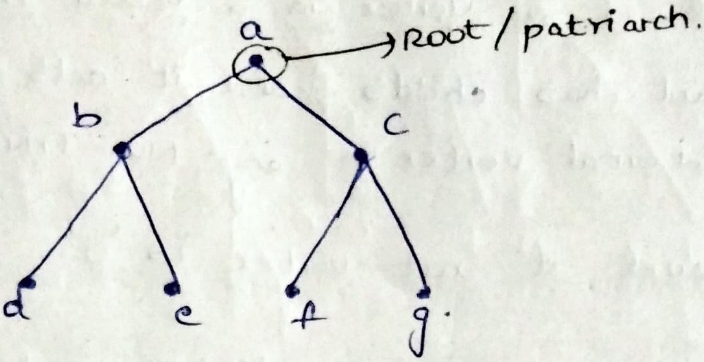


A) It is a connected graph. & No circuits are there.

∴ It is a Tree

* Rooted Tree :-

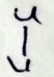
→ A rooted tree in which one vertex is designated as the root and every edge is directed away from the root



Terminology :-

(i) * Parent :-

→ Let 'T' is a rooted tree.

T 

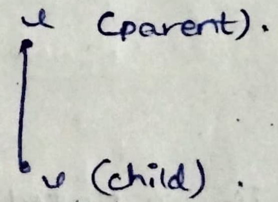
Then the parent of v is u.

(u, v) 

① Parent vertex is always unique.

(ii) * child :-

→ If 'u' is parent of 'v' then 'v' is child of 'u'.



(iii) * Internal vertex :-

→ It is one of the vertex in Rooted Tree that have children.

(iv) Leaf:-

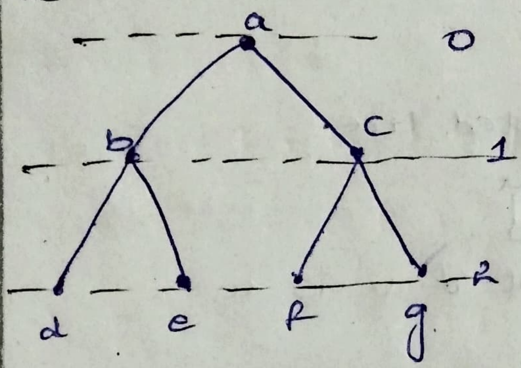
→ If a vertex has no child then it is a 'Leaf'.

(v) Level of a vertex in a rooted Tree:-

→ Root has childs and it acts as a Internal vertex in any Non-Trivial Tree.

→ Level of root vertex is '0'.

Ex:-



Vertex	Level
a	0
b	1
c	1
d	2
e	2
f	2
g	2

(vi) Height of a rooted tree:-

→ The Maximum level number is called Height

(or)

The longest path length from root to any vertex is called 'Height'.

(vii) Ancestors:-

→ Let 'T' is a rooted tree then v (other than root) vertex.

→ By Finding all the path b/w root and v & excluding them the remaining vertices b/w them are called 'Ancestors'.

→ Root is Ancestor to any vertex.

(viii) Descendents:-

→ Let 'v' be a vertex in rooted Tree 'T'

→ Then descendents of 'v' are those vertices that have 'v' as an ancestor.

(ix) subtree:-

→ If 'a' is a vertex in tree, subtree with 'a' as its root is the 'subgraph' consisting of 'a', its descendents & all edges incident with them.

* Note:-

① Notation,

n = No. of ~~edges~~ vertices.

i = No. of internal vertices.

l = No. of leaves

e = no. of edges

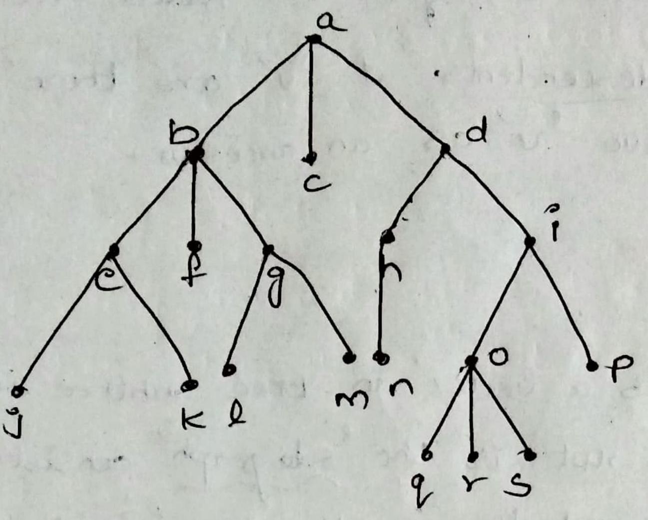
h = height of rooted Tree.

② The no. of edges (e) = The no. of children in a rooted tree.

③ The no. of vertices (n) = $i+l$ ∴ $\boxed{n=i+l}$

$\boxed{i=n-l}$ (or) $\boxed{l=n-i}$

3) Exercises



A) ⓐ root is 'a'

ⓑ Internal - b, d, e, g, h, i, o

ⓒ leaves - c, f, j, k, l, m, n, p, q, r, s.

ⓓ children of 'o' - q, r, s.

ⓔ parent of 'h' - d

ⓕ siblings of 'l' - m

ⓖ Ancestors of 'm' - a, b, g.

ⓗ Descendants of 'b' - e, f, g, j, k, l, m.

④ write down the Level.

A)

Vertex	Level	Vertex	Level
a	0	j	3
b	1	k	3
c	1	l	3
d	1	m	3
e	2	n	3
f	2	o	3
g	2	p	3
h	2	q	4
i	2	r	4
		s	4

(or)

Vertex	Level
a	0
b, c, d	1
e, f, g, h, i	2
j, k, l, m, n, o, p	3
q, r, s	4

* **m-ary Tree :-**

→ A rooted tree is said to be m-ary tree if every internal vertex contains no more than 'm' children.

* **Full m-ary rooted Tree :-**

→ If every internal vertex has exactly 'm' children then it is "Full m-ary rooted tree".

* **Binary Tree :-**

→ If Internal vertex has 2 children then it is Binary Tree.

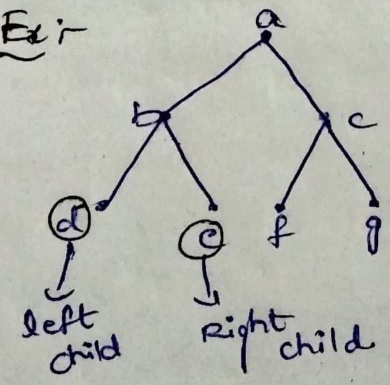
* **complete m-ary Rooted Tree :-**

→ A full m-ary tree, where every leaf is at the same level.

* **ordered Rooted Tree :-**

→ The children of every rooted Tree is in order, in the ordered Rooted Tree.

Ex :-



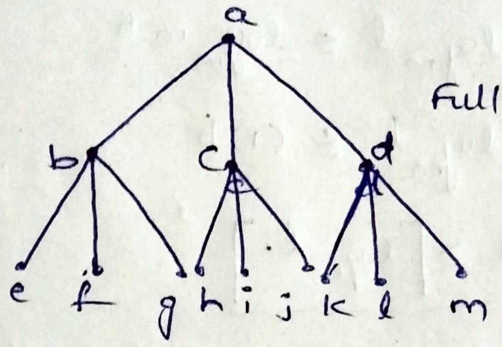
* **Left sub tree :-**

→ Let 'T' be an ordered rooted Binary Tree then Left sub Tree is a subtree and rooted at Left child.

* Right Subtree :-

→ Let 'T' be an ordered Binary Tree then Right subtree is a subtree which is rooted at Right child.

* Example :-



Full 3-ary Rooted Tree.

* Balanced Tree :-

→ If the height is 'h' (or) h−1 then it is Balanced Tree.

* Theorem-3 :-

→ Let 'T' be a Full m-ary tree with 'i' internal vertices.

① m → fixed

② n
③ i
④ l

} Parameters.

(i) No. of edges, $e = mi$

(ii) No. of vertices, $n = mi + 1$

~~(iii) N~~

* Theorem-4 :-

→ Let 'T' be a full m-ary tree. [fixed constant]

(i) If 'i' is known, $n = mi + 1$ & $\boxed{l = n - i = mi + 1 - i = (m-1)i + 1}$

(ii) If 'n' is known, $i = \frac{n-1}{m}$ & $\boxed{l = n - i = \frac{(m-1)n+1}{n}}$

(iii) If 'l' is known, $n = \frac{ml-1}{m-1}$ & $\boxed{i = n - l = l-1/m-1}$

* Theorem-5:-

→ The no. of leaves for a m-ary rooted tree of height 'h' atmost 'm^h' i.e, '$l \le m^h$'.

* Theorem-6:-

→ Applying logarithm to $l \le m^h$.

$\Rightarrow \log_m l \le \log_m m^h$.

$\Rightarrow \log_m l \le (1) h$

$\therefore$ '$h \ge \lceil \log_m l \rceil$'

→ If given tree is a full m-ary balanced rooted tree then '$h = \lceil \log_m l \rceil$'.

* Exercise:-

③ Is the rooted tree in Exercise 2 a full m-ary tree for some positive integer m?

A) There is no 'm' value exists, so it is not possible to be a full m-ary tree.

⑤ How many edges does a tree with 10,000 vertices have?

A) No. of edges = vertices − 1 = 10,000 − 1 = 9999.

③ How many vertices does a full 5-ary tree with 100 internal vertices have?

A) $n = mi + 1$

$= 5(100) + 1 = 501$ vertices.

8. How many leaves does a full 3-ary tree with 100 vertices have?

A) $l = \frac{(m-1)n+1}{m} = \frac{(3-1)\,100+1}{3} = \frac{201}{3} = 67$ leaves

9. Given, $m = 5$

$i = 10{,}000$

The no. of people receives the letter $= n-1$

[∵ root doesnot receive]

$= mi$

$= 5(10{,}000)$

$= 50{,}000.$

The no. of leaves, $l = (m-1)i+1$

$= (5-1)\,10{,}000+1$

$= 40{,}001$ received letter but not send out.