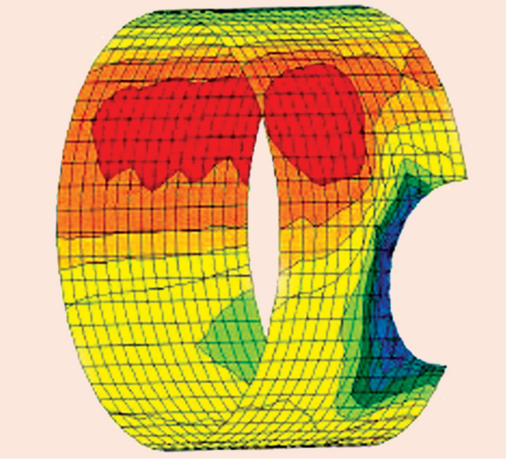
Chapter 7

Principal Stresses and Strains

Contents



7.1. Introduction

In the previous chapters, we have studied in detail, the direct tensile and compressive stress as well as simple shear. In these chapters, we have always referred the stress in a plane, which is at right angles to the line of action of the force (in case of direct tensile or compressive stress). Moreover, we have considered at a time one type of stress, acting in one direction only. But the majority of engineering, component and structures are subjected to such loading conditions (or sometimes are of such shapes) that there exists a complex state of stresses; involving direct tensile and compressive stress as well as shear stress in various directions. Now in this chapter we shall study the nature and intensity of stresses on planes, other than that, which is at right angles to the line of action of the force.

7.2. Principal Planes

It has been observed that at any point in a strained material, there are three planes, mutually perpendicular to each other, which carry direct stresses only, and no shear stress. A little consideration will show that out of these three direct stresses one will be maximum, the other minimum, and the third an intermediate between the two. These particular planes, which have no shear stress, are known as *principal planes*.

7.3. Principal Stress

The magnitude of direct stress, across a principal plane, is known as principal stress. The determination of principal planes, and then principal stress is an important factor in the design of various structures and machine components.

7.4. Methods for the Stresses on an Oblique Section of a Body

The following two methods for the determination of stresses on an oblique section of a strained body are important from the subject point of view :

1. Analytical method and **2.** Graphical method.

7.5. Analytical Method for the Stresses on an Oblique Section of a Body

Here we shall first discuss the analytical method for the determination of stresses on an oblique section in the following cases, which are important from the subject point of view :

1. A body subjected to a direct stress in one plane.
2. A body subjected to direct stresses in two mutually perpendicular directions.

7.6. Sign Conventions for Analytical Method

Though there are different sign conventions, used in different books, yet we shall adopt the following sign conventions, which are widely used and internationally recognised :

1. All the tensile stresses and strains are taken as positive, whereas all the compressive stresses and strains are taken as negative.
2. The well established principles of mechanics is used for the shear stress. The shear stress which tends to rotate the element in the clockwise direction is taken as positive, whereas that which tends to rotate in an anticlockwise direction as negative.

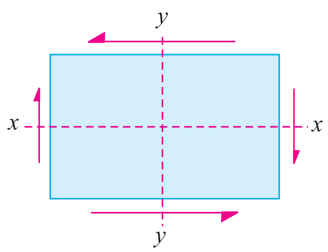


Fig. 7.1

In the element shown in Fig. 7.1, the shear stress on the vertical faces (or *x-x* axis) is taken as positive, whereas the shear stress on the horizontal faces (or *y-y* axis) is taken as negative.

7.7. Stresses on an Oblique Section of a Body Subjected to a Direct Stress in One Plane

Consider a rectangular body of uniform cross-sectional area and unit thickness subjected to a direct tensile stress along *x-x* axis as shown in Fig. 7.2 (*a*). Now let us consider an oblique section *AB* inclined with the x-x axis (*i.e.*, with the line of action of the tensile stress on which we are required to find out the stresses as shown in the figure).

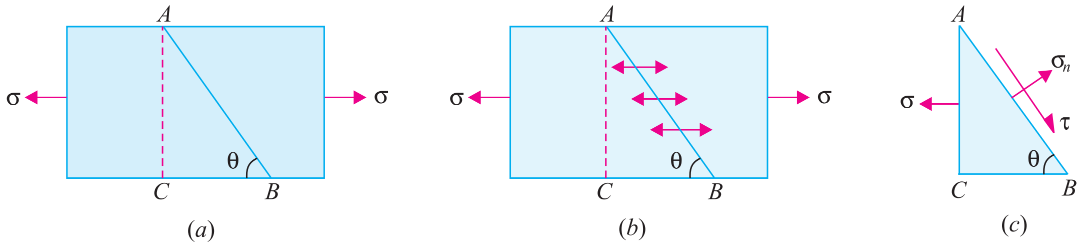


Fig. 7.2

Let σ = Tensile stress across the face AC and

θ = Angle, which the oblique section AB makes with BC *i.e.* with the x-x axis in the clockwise direction.

First of all, consider the equilibrium of an element or wedge ABC whose free body diagram is shown in fig 7.2 (*b*) and (*c*). We know that the horizontal force acting on the face AC,

$$P = \sigma \,.\, AC \;(\leftarrow)$$

Resolving the force perpendicular or normal to the section AB

$$P_n = P \sin\theta = \sigma \,.\, AC \sin\theta \qquad \text{....}(i)$$

and now resolving the force tangential to the section AB,

$$P_t = P \cos\theta = \sigma \,.\, AC \cos\theta \qquad \text{....}(ii)$$

We know that normal stress across the section AB*,

$$\sigma_n = \frac{P_n}{AB} = \frac{\sigma\, AC \sin\theta}{AB} = \frac{\sigma . AC \sin\theta}{\dfrac{AC}{\sin\theta}} = \sigma \sin^2\theta$$

$$= \frac{\sigma}{2}(1 - \cos 2\theta) = \frac{\sigma}{2} - \frac{\sigma}{2}\cos 2\theta \qquad \text{...}(iii)$$

and shear stress (*i.e.*, tangential stress) across the section AB,

$$\tau = \frac{P_t}{AB} = \frac{\sigma . AC \cos\theta}{AB} = \frac{\sigma . AC \cos\theta}{\dfrac{AC}{\sin\theta}} = \sigma \sin\theta \cos\theta$$

$$= \frac{\sigma}{2}\sin 2\theta \qquad \text{...}(iv)$$

* It can also be obtained by resolving the stress along the normal and across the section AB as shown in Fig. 7.2. (*b*).

We know that the stress across the section AB

$$= \sigma \cos\theta$$

Now resolving the stress normal to the section AB,

$$\sigma_n = \sigma \cos\theta \,.\, \cos\theta = \sigma \cos^2\theta$$

and now resolving the stress along the section AB

$$\tau = \sigma \sin\theta \,.\, \cos\theta$$

It will be interesting to know from equation (*iii*) above that the normal stress across the section *AB* will be maximum, when $\sin^2\theta = 1$ or $\sin\theta = 1$ or $\theta = 90°$. Or in other words, the face *AC* will carry the maximum direct stress. Similarly, the shear stress across the section *AB* will be maximum when $\sin 2\theta = 1$ or $2\theta = 90°$ or $270°$. Or in other words, the shear stress will be maximum on the planes inclined at 45° and 135° with the line of action of the tensile stress. Therefore maximum shear stress when θ is equal to 45°,

$$\tau_{max} = \frac{\sigma}{2}\sin 90° = \frac{\sigma}{2}\times 1 = \frac{\sigma}{2}$$

and maximum shear stress, when θ is equal to 135°,

$$\tau_{max} = -\frac{\sigma}{2}\sin 270° = -\frac{\sigma}{2}(-1) = \frac{\sigma}{2}$$

It is thus obvious that the magnitudes of maximum shear stress is half of the tensile stress. Now the resultant stress may be found out from the relation :

$$\sigma_R = \sqrt{\sigma_n^2 + \tau^2}$$

NOTE : The planes of maximum and minimum normal stresses (*i.e.* principal planes) may also be found out by equating the shear stress to zero. This happens as the normal stress is either maximum or minimum on a plane having zero shear stress. Now equating the shear stress to zero,

$$\sigma \sin\theta \cos\theta = 0$$

It will be interesting to know that in the above equation either sin θ is equal to zero or cos θ is equal to zero. We know that if sin is zero, then θ is equal to 0°. Or in other words, the plane coincides with the line of action of the tensile stress. Similarly, if cos θ is zero, then θ is equal to 90°. Or in other words, the plane is at right angles to the line of action of the tensile stress. Thus we see that there are two principal planes, at right angles to each other, one of them coincides with the line of action of the stress and the other at right angles to it.

EXAMPLE 7.1. *A wooden bar is subjected to a tensile stress of 5 MPa. What will be the values of normal and shear stresses across a section, which makes an angle of 25° with the direction of the tensile stress.*

SOLUTION. Given : Tensile stress (σ) = 5 MPa and angle made by section with the direction of the tensile stress (θ) = 25°.

Normal stress across the section

We know that normal stress across the section

$$\sigma_n = \frac{\sigma}{2} - \frac{\sigma}{2}\cos 2\theta = \frac{5}{2} - \frac{5}{2}\cos(2 \times 25°) \text{ MPa}$$
$$= 2.5 - 2.5\cos 50° = 2.5 - (2.5 \times 0.6428) \text{ MPa}$$
$$= 2.5 - 1.607 = 0.89 \text{ MPa} \quad \textbf{Ans.}$$

Shear stress across the section

We also know that shear stress across the section,

$$\tau = \frac{\sigma}{2}\sin 2\theta = \frac{\sigma}{2}\sin(2 \times 25°) = 2.5\sin 50° \text{ MPa}$$
$$= 2.5 \times 0.766 = 1.915 \text{ MPa} \quad \textbf{Ans.}$$

EXAMPLE 7.2. *Two wooden pieces 100 mm × 100 mm in cross-section are joined together along a line AB as shown in Fig. 7.3.*

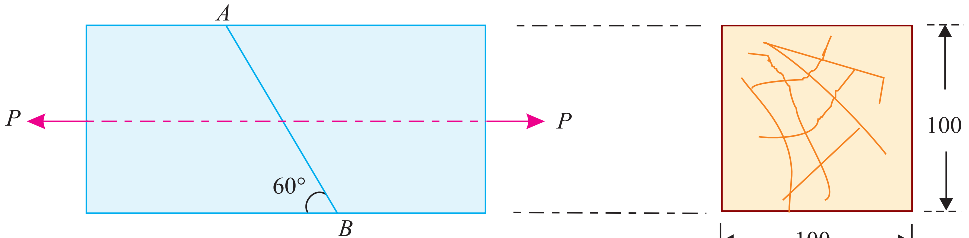


Fig. 7.3

Find the maximum force (P), which can be applied if the shear stress along the joint AB is 1.3 MPa.

SOLUTION. Given : Section = 100 mm × 100 mm ; Angle made by section with the direction of tensile stress (θ) = 60° and permissible shear stress (τ) = 1.3 MPa = 1.3 N/mm^2.

Let σ = Safe tensile stress in the member

We know that cross- sectional area of the wooden member,

$$A = 100 \times 100 = 10\,000 \text{ mm}^2$$

and shear stress (τ),

$$1.3 = \frac{\sigma}{2}\sin 2\theta = \frac{\sigma}{2}\sin(2\times 60°) = \frac{\sigma}{2}\sin 120° = \frac{\sigma}{2}\times 0.866$$

$$= 0.433\,\sigma$$

or $$\sigma = \frac{1.3}{0.433} = 3.0 \text{ N/mm}^2$$

∴ Maximum axial force, which can be applied,

$$P = \sigma.A = 3.0 \times 10\,000 = 30\,000 \text{ N} = 30 \text{ kN}$$ **Ans.**

EXAMPLE 7.3. *A tension member is formed by connecting two wooden members 200 mm × 100 mm as shown in the figure given below:*

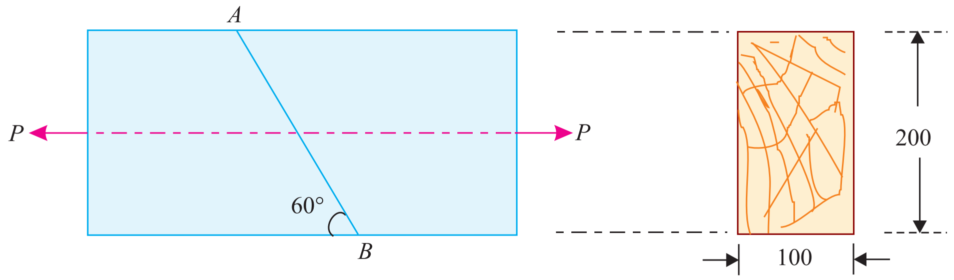


Fig. 7.4

Determine the safe value of the force (P), if permissible normal and shear stresses in the joint are 0.5 MPa and 1.25 MPa respectively.

SOLUTION. Given : Section = 200 mm × 100 mm ; Angle made by section *AB* with the direction of the tensile stress (σ) = 60° ; Permissible normal stress (σ_n) = 0.5 MPa = 0.5 N/mm^2 and permissible shear stress (τ) = 1.25 MPa = 1.25 N/mm^2.

Let σ = Safe stress in the joint in N/mm^2.

We know that cross-sectional area of the member

$$A = 200 \times 100 = 20\,000 \text{ mm}^2$$

We also know that normal stress (σ_n),

$$0.5 = \frac{\sigma}{2} - \frac{\sigma}{2}\cos 2\theta = \frac{\sigma}{2} - \frac{\sigma}{2}\cos(2 \times 60°)$$

$$= \frac{\sigma}{2} - \frac{\sigma}{2}\cos 120° = \frac{\sigma}{2} - \frac{\sigma}{2}(-0.5) = 0.75\,\sigma$$

$$\therefore \quad \sigma = \frac{0.5}{0.75} = 0.67 \text{ N/mm}^2 \quad ...(i)$$

and shear stress (τ)

$$1.25 = \frac{\sigma}{2}\sin 2\theta = \frac{\sigma}{2}\sin(2 \times 60°) = \frac{\sigma}{2}\sin 120° = \frac{\sigma}{2} \times 0.866 = 0.433\sigma$$

$$\sigma = \frac{1.25}{0.433} = 2.89 \text{ N/mm}^2 \quad ...(ii)$$

From the above two values, we find that the safe stress is least of the two values, *i.e.* 0.67 N/mm^2. Therefore safe value of the force

$$P = \sigma \,.\, A = 0.67 \times 20\,000 = 13\,400 \text{ N} = 13.4 \text{ kN} \qquad \textbf{Ans.}$$

7.8. Stresses on an Oblique Section of a Body Subjected to Direct Stresses in Two Mutually Perpendicular Directions

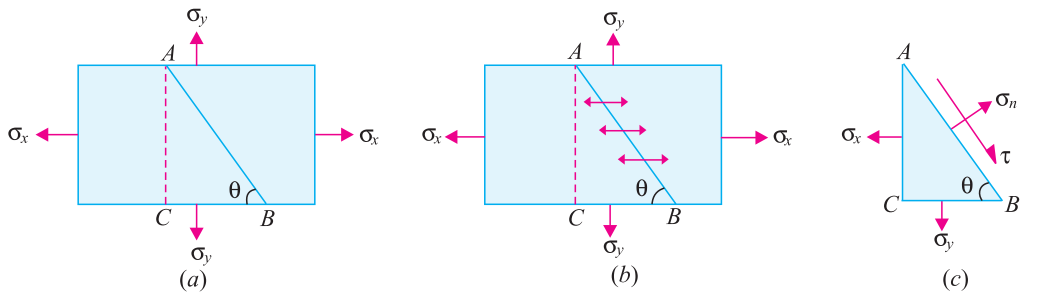


Fig. 7.5

Consider a rectangular body of uniform cross-sectional area and unit thickness subjected to direct tensile stresses in two mutually perpendicular directions along *x-x* and *y-y* axes as shown in Fig. 7.5. Now let us consider an oblique section *AB* inclined with *x-x* axis (*i.e.* with the line of action of the stress along *x-x* axis, termed as a major tensile stress on which we are required to find out the stresses as shown in the figure).

Let σ_x = Tensile stress along *x-x* axis (also termed as major tensile stress),

σ_y = Tensile stress along *y-y* axis (also termed as a minor tensile stress), and

θ = Angle which the oblique section *AB* makes with *x-x* axis in the clockwise direction.

First of all, consider the equilibrium of the wedge *ABC*. We know that horizontal force acting on the face *AC* (or *x-x* axis).

$$P_x = \sigma_x \,.\, AC\ (\leftarrow)$$

and vertical force acting on the face *BC* (or *y-y* axis),

$$P_y = \sigma_y \,.\, BC\ (\downarrow)$$

Resolving the forces perpendicular or normal to the section AB,

$$P_n = P_x \sin\theta + P_y \cos\theta = \sigma_x \,.\, AC \sin\theta + \sigma_y \,.\, BC \cos\theta \qquad ...(i)$$

and now resolving the forces tangential to the section AB,

$$P_t = P_x \cos\theta - P_y \sin\theta = \sigma_x \,.\, AC \cos\theta - \sigma_y \,.\, BC \sin\theta \qquad ....(ii)$$

We know that normal stress across the section AB,

$$\sigma_n = \frac{P_n}{AB} = \frac{\sigma_x \,.\, AC \sin\theta + \sigma_y \; BC \cos\theta}{AB}$$

$$= \frac{\sigma_x \,.\, AC \sin\theta}{AB} + \frac{\sigma_y \,.\, BC \cos\theta}{AB} = \frac{\sigma_x \,.\, AC \sin\theta}{\dfrac{AC}{\sin\theta}} + \frac{\sigma_y \,.\, BC \cos\theta}{\dfrac{BC}{\cos\theta}}$$

$$= \sigma_x \sin^2\theta + \sigma_y \,.\, \cos^2\theta = \frac{\sigma_x}{2}(1 - \cos 2\theta) + \frac{\sigma_y}{2}(1 + \cos 2\theta)$$

$$= \frac{\sigma_x}{2} - \frac{\sigma_x}{2}\cos 2\theta + \frac{\sigma_y}{2} + \frac{\sigma_y}{2}\cos 2\theta$$

$$= \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta \qquad ...(iii)$$

and shear stress (*i.e.*, tangential stress) across the section AB,

$$\tau = \frac{P_t}{AB} = \frac{\sigma_x \,.\, AC \cos\theta - \sigma_y \,.\, BC \sin\theta}{AB}$$

$$= \frac{\sigma_x \,.\, AC \cos\theta}{AB} - \frac{\sigma_y \,.\, BC \sin\theta}{AB} = \frac{\sigma_x \,.\, AC \cos\theta}{\dfrac{AC}{\sin\theta}} - \frac{\sigma_y \,.\, BC \sin\theta}{\dfrac{BC}{\cos\theta}}$$

$$= \sigma_x \,.\, \sin\theta \cos\theta - \sigma_y \sin\theta \cos\theta$$

$$= (\sigma_x - \sigma_y) \sin\theta \cos\theta = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta \qquad ...(iv)$$

It will be interesting to know from equation (*iii*) the shear stress across the section AB will be maximum when $\sin 2\theta = 1$ or $2\theta = 90°$ or $\theta = 45°$. Therefore maximum shear stress,

$$\tau_{max} = \frac{\sigma_x - \sigma_y}{2}$$

Now the resultant stress may be found out from the relation :

$$\sigma_R = \sqrt{\sigma_n^2 + \tau^2}$$

EXAMPLE 7.4. *A point in a strained material is subjected to two mutually perpendicular tensile stresses of 200 MPa and 100 MPa. Determine the intensities of normal, shear and resultant stresses on a plane inclined at 30° with the axis of minor tensile stress.*

SOLUTION. Given : Tensile stress along x-x axis (σ_x) =150 MPa ; Tensile stress along y-y axis (σ_y) = 100 MPa and angle made by plane with the axis of tensile stress $\theta = 30°$

Normal stress on the inclined plane

We know that normal stress on the inclined plane,

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta$$

$$= \frac{200+100}{2} - \frac{20-100}{2} \cos (2 \times 30°) \text{ MPa}$$

$$= 150 - (50 \times 0.5) = 125 \text{ MPa} \qquad \textbf{Ans.}$$

Shear stress on the inclined plane

We know that shear stress on the inclined plane,

$$\tau = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta = \frac{200-100}{2} \times \sin (2 \times 30°) \text{ MPa}$$

$$= 50 \sin 60° = 50 \times 0.866 = 43.3 \text{ MPa} \qquad \textbf{Ans.}$$

Resultant stress on the inclined plane

We also know that resultant stress on the inclined plane,

$$\sigma_R = \sqrt{\sigma_n^2 + \tau^2} = \sqrt{(125)^2 + (43.3)^2} = 132.3 \text{ MPa} \qquad \textbf{Ans.}$$

EXAMPLE 7.5. *The stresses at point of a machine component are 150 MPa and 50 MPa both tensile. Find the intensities of normal, shear and resultant stresses on a plane inclined at an angle of 55° with the axis of major tensile stress. Also find the magnitude of the maximum shear stress in the component.*

SOLUTION. Given : Tensile stress along *x-x* axis (σ_x) = 150 MPa ; Tensile stress along *y-y* axis (σ_y) = 50 MPa and angle made by the plane with the major tensile stress (θ) = 55°.

Normal stress on the inclined plane

We know that the normal stress on the inclined plane,

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} \quad \frac{\sigma_x - \sigma_y}{2} \cos 2\theta$$

$$= \frac{150+50}{2} - \frac{150-50}{2} \cos (2 \times 55°) \text{ MPa}$$

$$= 100 - 50 \cos 110° = 100 - 50\,(-\,0.342) \text{ MPa}$$

$$= 10 + 17.1 = 117.1 \text{ MPa} \qquad \textbf{Ans.}$$

Shear stress on the inclined plane

We know that the shear stress on the inclined plane,

$$\tau = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta = \frac{150-50}{2} \times \sin (2 \times 55°) \text{ MPa}$$

$$= 50 \sin 110° = 50 \times 0.9397 = 47 \text{ MPa} \qquad \textbf{Ans.}$$

Resultant stress on the inclined plane

We know that resultant stress on the inclined plane,

$$\sigma_R = \sqrt{\sigma_n^2 + \tau^2} = \sqrt{(117.1)^2 + (47.0)^2} = 126.2 \text{ MPa} \qquad \textbf{Ans.}$$

Maximum shear stress in the component

We also know that the magnitude of the maximum shear stress in the component,

$$\tau_{max} = \pm \frac{\sigma_x - \sigma_y}{2} = \pm \frac{150-50}{2} = \pm\, 50 \text{ MPa} \qquad \textbf{Ans.}$$

EXAMPLE 7.6. *The stresses at a point in a component are 100 MPa (tensile) and 50 MPa (compressive). Determine the magnitude of the normal and shear stresses on a plane inclined at an angle of 25° with tensile stress. Also determine the direction of the resultant stress and the magnitude of the maximum intensity of shear stress.*

SOLUTION. Given : Tensile stress along x-x axis (σ_x) 100 MPa ; Compressive stress along y-y axis (σ_y) = –50 MPa (Minus sign due to compression) and angle made by the plane with tensile stress (θ) = 25°.

Normal stress on the inclined plane

We know that the normal stress on the inclined plane,

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta$$

$$= \frac{100 + (-50)}{2} - \frac{100 - (-50)}{2}\cos(2 \times 25°)\text{ MPa}$$

$$= 25 - 75\cos 50° = 25 - (75 \times 0.6428) = -23.21\text{ MPa} \qquad \textbf{Ans.}$$

Shear stress on the inclined plane

We know that the shear stress on the inclined plane,

$$\tau = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta = \frac{100 - (-50)}{2}\sin(2 \times 25°)\text{ MPa}$$

$$= 75\sin 50° = 75 \times 0.766 = 57.45\text{ MPa} \qquad \textbf{Ans.}$$

Direction of the resultant stress

Let θ = Angle, which the resultant stress makes with x-x axis.

We know that
$$\tan\theta = \frac{\tau}{\sigma_n} = \frac{57.45}{-23.21} = -2.4752 \qquad \text{or} \qquad \theta = -68° \qquad \textbf{Ans.}$$

Maximum shear stress

We also know that magnitude of the maximum shear stress,

$$\tau_{max} = \pm\frac{\sigma_x - \sigma_y}{2} = \pm\frac{100 - (-50)}{2} = \pm 75\text{ MPa} \qquad \textbf{Ans.}$$

7.9. Stresses on an Oblique Section of a Body Subjected to a Simple Shear stress

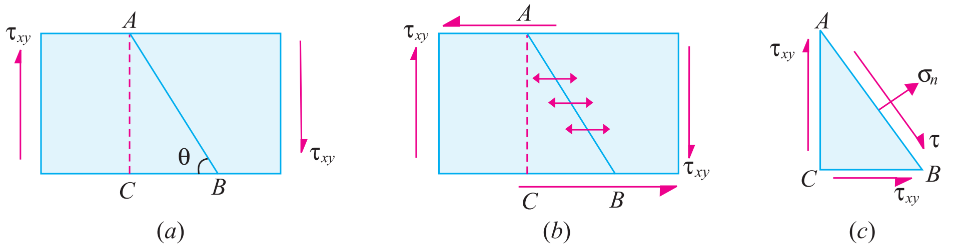


Fig. 7.6

Consider a rectangular body of uniform cross-sectional area and unit thickness subjected to a positive (*i.e.*, clockwise) shear stress along x-x axis as shown in Fig.7.6 (a). Now let us consider an oblique section AB inclined with x-x axis on which we are required to find out the stresses as shown in the figure 7.6 (b).

Let τ_{xy} = Positive (*i.e.*, clockwise) shear stress along x-x axis, and

θ = Angle , which the oblique section AB makes with x-x axis in the anticlockwise direction.

First of all, consider the equilibrium of the wedge ABC. We know that as per the principle of simple shear, the face BC, of the wedge will be subjected to an anticlockwise shear stress equal to τ_{xy} as shown in the Fig. 7.6 (b). We know that vertical force acting on the face AC,

$$P_1 = \tau_{xy} \cdot AC\ (\uparrow)$$

and horizontal force acting on the face BC,

$$P_2 = \tau_{xy} \,.\, BC\ (\rightarrow)$$

Resolving the forces perpendicular or normal to the AB,

$$P_n = P_1 \cos\theta + P_2 \sin\theta = \tau_{xy} \,.\, AC \cos\theta + \tau_{xy} \,.\, BC \sin\theta$$

and now resolving the forces tangential to the section AB,

$$P_t = P_2 \sin\theta - P_1 \cos\theta = \tau_{xy} \,.\, BC \sin\theta - \tau_{xy} \,.\, AC \cos\theta$$

We know that normal stress across the section AB,

$$\sigma_n = \frac{P_n}{AB} = \frac{\tau_{xy} \,.\, AC \cos\theta + \tau_{xy} \,.\, BC \sin\theta}{AB}$$

$$= \frac{\tau_{xy} \,.\, AC \cos\theta}{AB} + \frac{\tau_{xy} \,.\, BC \sin\theta}{AB}$$

$$= \frac{\tau_{xy} \,.\, AC \cos\theta}{\dfrac{AC}{\sin\theta}} + \frac{\tau_{xy} \,.\, BC \sin\theta}{\dfrac{BC}{\cos\theta}}$$

$$= \tau_{xy} \,.\, \sin\theta \cos\theta + \tau_{xy} \,.\, \sin\theta \cos\theta$$

$$= 2\,\tau_{xy} \,.\, \sin\theta \cos\theta = \tau_{xy} \,.\, \sin 2\theta$$

and shear stress (*i.e.* tangential stress) across the section AB

$$\tau = \frac{P_t}{AB} = \frac{\tau_{xy} \,.\, BC \sin\theta - \tau_{xy} \,.\, AC \cos\theta}{AB}$$

$$= \frac{\tau_{xy} \,.\, BC \sin\theta}{AB} - \frac{\tau_{xy} \,.\, AC \cos\theta}{AB} = \frac{\tau_{xy} \,.\, BC \sin\theta}{\dfrac{BC}{\sin\theta}} - \frac{\tau_{xy} \,.\, AC \cos\theta}{\dfrac{AC}{\cos\theta}}$$

$$= \tau_{xy} \sin^2\theta - \tau_{xy} \cos^2\theta$$

$$= \frac{\tau_{xy}}{2}\,(1 - \cos 2\theta) - \frac{\tau_{xy}}{2}\,(1 + \cos 2\theta)$$

$$= \frac{\tau_{xy}}{2} - \frac{\tau_{xy}}{2}\cos 2\theta - \frac{\tau_{xy}}{2} - \frac{\tau_{xy}}{2}\cos 2\theta$$

$$= -\,\tau_{xy} \cos 2\theta$$

...(Minus sign means that normal stress is opposite to that across AC)

Now the planes of maximum and minimum normal stresses (*i.e.*, principal planes) may be found out by equating the shear stress to zero *i.e.*

$$-\,\tau_{xy} \cos 2\theta = 0$$

The above equation is possible only if $2\theta = 90°$ or $270°$ (because $\cos 90°$ or $\cos 270° = 0$) or in other words, $\theta = 45°$ or $135°$.

7.10. Stresses on an Oblique Section of a Body Subjected to a Direct Stress in One Plane and Accompanied by a Simple Shear Stress

Consider a rectangular body of uniform cross-sectional area and unit thickness subjected to a tensile stress along *x-x* axis accompanied by a positive (*i.e.* clockwise) shear stress along *x-x* axis as shown in Fig. 7.7 (*a*). Now let us consider an oblique section AB inclined with *x-x* axis on which we are required to find out the stresses as shown in the figure.

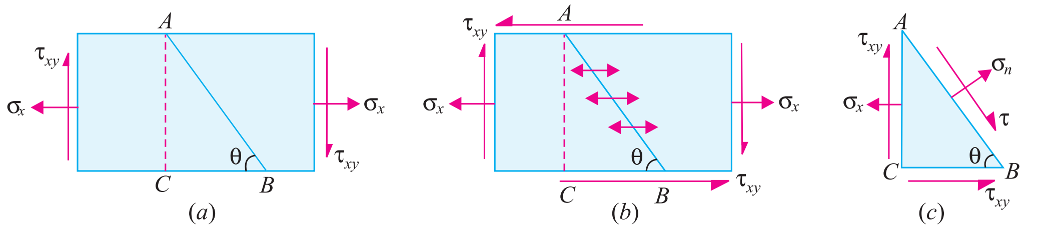


Fig. 7.7

Let σ_x = Tensile stress along *x-x* axis,

τ_{xy} = Positive (*i.e.* clockwise) shear stress along *x-x* axis, and

θ = Angle which the oblique section *AB* makes with *x-x* axis in clockwise direction.

First of all, consider the equilibrium of the wedge *ABC*. We know that as per the principle of simple shear, the face *BC* of the wedge will be subjected to an anticlockwise shear stress equal to τ_{xy} as shown in Fig. 7.7 (*b*). We know that horizontal force acting on the face *AC*,

$$P_x = \sigma_x \,.\, AC\ (\leftarrow) \qquad ...(i)$$

Similarly, vertical force acting on the face *AC*,

$$P_y = \tau_{xy} \,.\, AC\ (\uparrow) \qquad ...\ (ii)$$

and horizontal force acting on the face *BC*,

$$P = \tau_{xy} \,.\, BC\ (\rightarrow) \qquad ...(iii)$$

Resolving the forces perpendicular to the section *AB*,

$$P_n = P_x \sin\theta - P_y \cos\theta - P \sin\theta$$

$$= \sigma_x \,.\, AC \sin\theta - \tau_{xy} \,.\, AC \cos\theta - \tau_{xy} \,.\, BC \sin\theta$$

and now resolving the forces tangential to the section *AB*,

$$P_t = P_x \cos\theta + P_y \sin\theta - P \cos\theta$$

$$= \sigma_x \,.\, AC \cos\theta + \tau_{xy} \,.\, AC \sin\theta - \tau_{xy} \,.\, BC \cos\theta$$

We know that normal stress across the section *AB*,

$$\sigma_n = \frac{P_n}{AB} = \frac{\sigma_x . AC \sin\theta - \tau_{xy} . AC \cos\theta - \tau_{xy} . BC \sin\theta}{AB}$$

$$= \frac{\sigma_x . AC \sin\theta}{AB} - \frac{\tau_{xy} . AC \cos\theta}{AB} - \frac{\tau_{xy} . BC \sin\theta}{AB}$$

$$= \frac{\sigma_x . AC \sin\theta}{\frac{AC}{\sin\theta}} - \frac{\tau_{xy} . AC \cos\theta}{\frac{AC}{\sin\theta}} - \frac{\tau_{xy} . BC \sin\theta}{\frac{BC}{\cos\theta}}$$

$$= \sigma_x \,.\, \sin^2\theta - \tau_{xy} \sin\theta \cos\theta - \tau_{xy} \sin\theta \cos\theta$$

$$= \frac{\sigma_x}{2}(1 - \cos 2\theta) - 2\,\tau_{xy} \sin\theta \cos\theta$$

$$= \frac{\sigma_x}{2} - \frac{\sigma_x}{2} \cos 2\theta - \tau_{xy} \sin 2\theta \qquad ...(iv)$$

and shear stress (*i.e.*, tangential stress) across the section *AB*,

$$\tau = \frac{P_t}{AB} = \frac{\sigma_x . AC \cos\theta + \tau_{xy} . AC \sin\theta - \tau_{xy} . BC \cos\theta}{AB}$$

$$= \frac{\sigma_x . AC\cos\theta}{AB} + \frac{\tau_{xy}\, AC\sin\theta}{AB} - \frac{\tau_{xy} . BC\cos\theta}{AB}$$

$$= \frac{\sigma_x . AC\cos\theta}{\frac{AC}{\sin\theta}} + \frac{\tau_{xy}\, AC\sin\theta}{\frac{AC}{\sin\theta}} - \frac{\tau_{xy} . BC\cos\theta}{\frac{BC}{\cos\theta}}$$

$$= \sigma_x \sin\theta\cos\theta + \tau_{xy}\sin^2\theta - \tau_{xy}\cos^2\theta$$

$$= \frac{\sigma_x}{2}\sin 2\theta + \frac{\tau_{xy}}{2}(1-\cos 2\theta) - \frac{\tau_{xy}}{2}(1+\cos 2\theta)$$

$$= \frac{\sigma_x}{2}\sin 2\theta + \frac{\tau_{xy}}{2} - \frac{\tau_{xy}}{2}\cos 2\theta - \frac{\tau_{xy}}{2} - \frac{\tau_{xy}}{2}\cos 2\theta$$

$$= \frac{\sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta \qquad \text{...}(v)$$

Now the planes of maximum and minimum normal stresses (*i.e.*, principal planes) may be found out by equating the shear stress to zero *i.e.*, from the above equation, we find that the shear stress on any plane is a function of σ_x, τ_{xy} and θ. A little consideration will show that the values of σ_x and τ_{xy} are constant and thus the shear stress varies with the angle θ. Now let θ_p be the value of the angle for which the shear stress is zero.

$$\therefore \quad \frac{\sigma_x}{2}\sin 2\theta_p - \tau_{xy}\cos 2\theta_p = 0 \quad \text{or} \quad \frac{\sigma_x}{2}\sin 2\theta_p = \tau_{xy}\cos 2\theta_p$$

$$\therefore \quad \tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x}$$

From the above equation we find that the following two cases satisfy this condition as shown in Fig 7.8 (*a*) and (*b*)

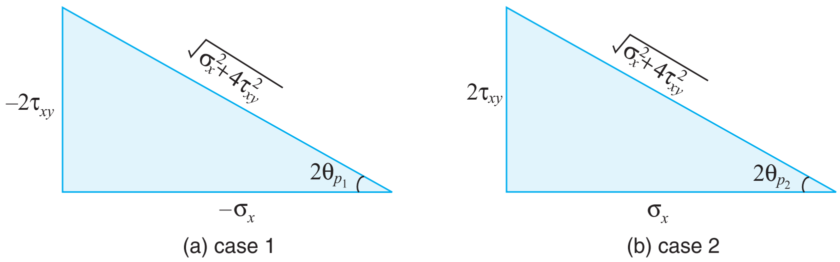


Fig. 7.8

Thus we find that these are two principal planes at right angles to each other, their inclination with *x-x* axis being θ_{p_1} and θ_{p_2}.

Now for case 1,

$$\sin 2\theta_{p_1} = \frac{-2\tau_{xy}}{\sqrt{\sigma_x^2 + 4\tau_{xy}^2}} \quad \text{and} \quad \cos 2\theta_{p_1} = \frac{-\sigma_x}{\sqrt{\sigma_x^2 + 4\tau_{xy}^2}}$$

Similarly for case 2,

$$\sin 2\theta_{p_2} = \frac{2\tau_{xy}}{\sqrt{\sigma_x^2 + 4\tau_{xy}^2}} \quad \text{and} \quad \cos 2\theta_{p_2} = \frac{\sigma_x}{\sqrt{\sigma_x^2 + 4\tau_{xy}^2}}$$

Now the values of principal stresses may be found out by substituting the above values of $2\theta_{p_1}$ and $2\theta_{p_2}$ in equation (*iv*).

Maximum principal stress, $\sigma_{p_1} = \frac{\sigma_x}{2} - \frac{\sigma_x}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$

$$= \frac{\sigma_x}{2} - \frac{\sigma_x}{2} \times \frac{-\sigma_x}{\sqrt{\sigma_x^2 + 4\tau_{xy}^2}} - \tau_{xy} \times \frac{-2\tau_{xy}}{\sqrt{\sigma_x^2 + 4\tau_{xy}^2}}$$

$$= \frac{\sigma_x}{2} + \frac{\sigma_x^2}{2\sqrt{\sigma_x^2 + 4\tau_{xy}^2}} + \frac{2\tau_{xy}^2}{\sqrt{\sigma_x^2 + 4\tau_{xy}^2}}$$

$$= \frac{\sigma_x}{2} + \frac{\sigma_x^2 + 4\tau_{xy}^2}{2\sqrt{\sigma_x^2 + 4\tau_{xy}^2}} = \frac{\sigma_x}{2} + \frac{\sqrt{\sigma_x^2 + 4\tau_{xy}^2}}{2}$$

$$= \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x^2}{2}\right) + \tau_{xy}^2}$$

Minimum principal stress, $\sigma_{p_2} = \frac{\sigma_x}{2} - \frac{\sigma_x}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$

$$= \frac{\sigma_x}{2} - \frac{\sigma_x}{2} \times \frac{\sigma_x}{\sqrt{\sigma_x^2 + 4\tau_{xy}^2}} - \tau_{xy} \times \frac{2\tau_{xy}}{\sqrt{\sigma_x^2 + 4\tau_{xy}^2}}$$

$$= \frac{\sigma_x}{2} - \frac{\sigma_x^2}{2\sqrt{\sigma_x^2 + 4\tau_{xy}^2}} - \frac{2\tau_{xy}^2}{\sqrt{\sigma_x^2 + 4\tau_{xy}^2}}$$

$$= \frac{\sigma_x}{2} - \frac{\sqrt{\sigma_x^2 + 4\tau_{xy}^2}}{2} = \frac{\sigma_x}{2} - \frac{\sigma_x^2 + 4\tau_{xy}^2}{2\sqrt{\sigma_x^2 + 4\tau_{xy}^2}}$$

$$= \frac{\sigma_x}{2} - \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

EXAMPLE 7.7. *A plane element in a body is subjected to a tensile stress of 100 MPa accompanied by a shear stress of 25 MPa. Find (i) the normal and shear stress on a plane inclined at an angle of 20° with the tensile stress and (ii) the maximum shear stress on the plane.*

SOLUTION. Given : Tensile stress along x-x axis (σ_x) = 100 MPa ; Shear stress (τ_{xy}) = 25 MPa and angle made by plane with tensile stress (θ) = 20°.

Normal and shear stresses on inclined section

We know that the normal stress on the plane,

$$\sigma_n = \frac{\sigma_x}{2} - \frac{\sigma_x}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$= \frac{100}{2} - \frac{100}{2} \cos (2 \times 20°) - 25 \sin (2 \times 20°) \text{ MPa}$$

$$= 50 - 50 \cos 40° - 25 \sin 40° \text{ MPa}$$

$$= 50 - (50 \times 0.766) - (25 \times 0.6428) \text{ MPa}$$

$$= 50 - 38.3 - 16.07 = -4.37 \text{ MPa}$$ **Ans.**

and shear stress on the plane, $\tau = \frac{\sigma_x}{2} \sin 2\theta - \tau_{xy} \cos 2\theta$

$$= \frac{100}{2} \sin (2 \times 20°) - 25 \cos (2 \times 20°) \text{ MPa}$$

$$= 50 \sin 40° - 25 \cos 40° \text{ MPa}$$

$$= (50 \times 0.6428) - (25 \times 0.766) \text{ MPa}$$

$$= 32.14 - 19.15 = 12.99 \text{ MPa}$$ **Ans.**

Maximum shear stress on the plane

We also know that maximum shear stress on the plane,

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{100}{2}\right)^2 + (25)^2} = 55.9 \text{ MPa} \qquad \textbf{Ans.}$$

Example 7.8. *An element in a strained body is subjected to a tensile stress of 150 MPa and a shear stress of 50 MPa tending to rotate the element in an anticlockwise direction. Find (i) the magnitude of the normal and shear stresses on a section inclined at 40° with the tensile stress; and (ii) the magnitude and direction of maximum shear stress that can exist on the element.*

Solution. Given : Tensile stress along horizontal *x-x* axis (σ_x) = 150 MPa ; Shear stress (τ_{xy}) – 50 MPa (Minus sign due to anticlockwise) and angle made by section with the tensile stress (θ) = 40°.

Normal and Shear stress on the inclined section

We know that magnitude of the normal stress on the section,

$$\sigma_n = \frac{\sigma_x}{2} - \frac{\sigma_x}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{150}{2} - \frac{150}{2}\cos(2 \times 40°) - (-50)\sin(2 \times 40°) \text{ MPa}$$

$$= 75 - (75 \times 0.1736) + (50 \times 0.9848) \text{ MPa}$$

$$= 75 - 13.02 + 49.24 = 111.22 \text{ MPa} \qquad \textbf{Ans.}$$

and shear stress on the section

$$\tau = \frac{\sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

$$= \frac{150}{2}\sin(2 \times 40°) - (-50)\cos(2 \times 40°) \text{ MPa}$$

$$= (75 \times 0.9848) + (50 \times 0.1736) \text{ MPa}$$

$$= 73.86 + 8.68 = 82.54 \text{ MPa} \qquad \textbf{Ans.}$$

(ii) Maximum shear stress and its direction that can exist on the element

We know that magnitude of the maximum shear stress.

$$\tau_{max} = \pm\sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} = \pm\sqrt{\left(\frac{150}{2}\right)^2 + (-50)^2} = \pm 90.14 \text{ MPa} \quad \textbf{Ans.}$$

Let θ_x = Angle which plane of maximum shear stress makes with *x-x* axis.

We know that, $$\tan 2\theta_s = \frac{\sigma_x}{2\tau_{xy}} = \frac{150}{2 \times 50} = 1.5 \qquad \text{or} \qquad 2\theta_s = 56.3°$$

$$\therefore \qquad \theta_s = 28.15° \qquad \text{or} \qquad 118.15° \qquad \textbf{Ans.}$$

Example 7.9. *An element in a strained body is subjected to a compressive stress of 200 MPa and a clockwise shear stress of 50 MPa on the same plane. Calculate the values of normal and shear stresses on a plane inclined at 35° with the compressive stress. Also calculate the value of maximum shear stress in the element.*

Solution. Given : Compressive stress along horizontal *x-x* axis (σ_x) = – 200 MPa (Minus sign due to compressive stress) ; Shear stress (τ_{xy}) = 50 MPa and angle made by the plane with the compressive stress (θ) = 35°

Normal and shear stresses across inclined section

We know that normal stress on the plane,

$$\sigma_n = \frac{\sigma_x}{2} - \frac{\sigma_x}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{-200}{2} - \frac{-200}{2}\cos(2\times 35°) - 50\sin(2\times 35°)\text{ MPa}$$

$$= -100 + (10\times 0.342) - (50\times 0.94)\text{ MPa}$$

$$= -100 + 34.2 - 46.9 = -112.9\text{ MPa} \qquad \textbf{Ans.}$$

and shear stress on the plane,

$$\tau = \frac{\sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

$$= \frac{-200}{2}\sin(2\times 35°) - 50\cos(2\times 35°)\text{ MPa}$$

$$= (-100\times 0.9397) - (50\times 0.342)\text{ MPa}$$

$$= -93.97 - 17.1 = -111.07\text{ MPa} \qquad \textbf{Ans.}$$

Maximum shear stress in the element

We also know that value of maximum shear stress in the element,

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{-200}{2}\right)^2 + (50)^2} = 111.8\text{ MPa} \qquad \textbf{Ans.}$$

7.11. Stresses on an Oblique Section of a Body Subjected to Direct Stresses in Two Mutually Perpendicular Directions Accompanied by a Simple Shear Stress

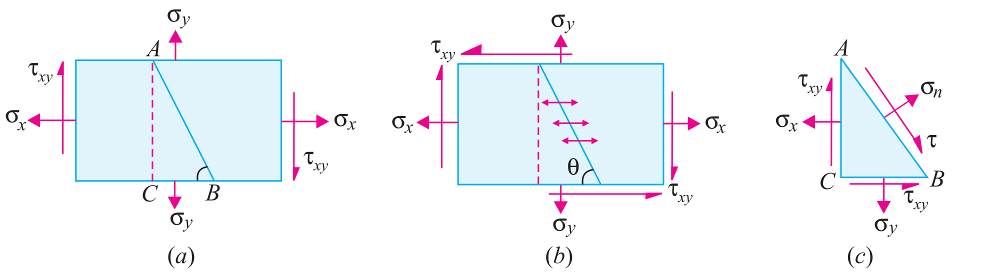


Fig. 7.9

Consider a rectangular body of uniform cross-sectional area and unit thickness subjected to tensile stresses along *x-x* and *y-y* axes and accompanied by a positive (*i.e.*, clockwise) shear stress along *x-x* axis as shown in Fig.7.9 (*b*). Now let us consider an oblique section *AB* inclined with *x-x* axis on which we are required to find out the stresses as shown in the figure.

Let σ_x = Tensile stress along *x-x* axis,

σ_y = Tensile stress along *y-y* axis,

τ_{xy} = Positive (*i.e.* clockwise) shear stress along *x-x* axis, and

θ = Angle, which the oblique section *AB* makes with *x-x* axis in an anticlockwise direction.

First of all, consider the equilibrium of the wedge *ABC*. We know that as per the principle of simple shear, the face *BC* of the wedge will be subjected to an anticlockwise shear stress equal to τ_{xy}

as shown in Fig. 7.9 (*b*). We know that horizontal force acting on the face *AC*,

$$P_1 = \sigma_x \,.\, AC\ (\leftarrow) \qquad \ldots (i)$$

and vertical force acting on the face *AC*,

$$P_2 = \tau_{xy} \,.\, AC\ (\uparrow) \qquad \ldots (ii)$$

Similarly, vertical force acting on the face *BC*,

$$P_3 = \sigma_y \,.\, BC\ (\downarrow) \qquad \ldots (iii)$$

and horizontal force on the face *BC*,

$$P_4 = \tau_{xy} \,.\, BC\ (\rightarrow) \qquad \ldots (iv)$$

Now resolving the forces perpendicular to the section *AB*,

$$P_n = P_1 \sin\theta - P_2 \cos\theta + P_3 \cos\theta - P_4 \sin\theta$$
$$= \sigma_x \,.\, AC \sin\theta - \tau_{xy} \,.\, AC \cos\theta + \sigma_y \,.\, BC \cos\theta - \tau_{xy} \,.\, BC \sin\theta$$

and now resolving the forces tangential to *AB*,

$$P_t = P_1 \cos\theta + P_2 \sin\theta - P_3 \sin\theta - P_4 \cos\theta$$
$$= \sigma_x \,.\, AC \cos\theta + \tau_{xy} \,.\, AC \sin\theta - \sigma_y \,.\, BC \sin\theta - \tau_{xy} \,.\, BC \cos\theta$$

Normal Stress (across the inclined section *AB*)

$$\sigma_n = \frac{P_n}{AB} = \frac{\sigma_x . AC \sin\theta - \tau_{xy} . AC \cos\theta + \sigma_y . BC \cos\theta - \tau_{xy} . BC \sin\theta}{AB}$$

$$= \frac{\sigma_x . AC \sin\theta}{AB} - \frac{\tau_{xy} . AC \cos\theta}{AB} + \frac{\sigma_y . BC \cos\theta}{AB} - \frac{\tau_{xy} . BC \sin\theta}{AB}$$

$$= \frac{\sigma_x . AC \sin\theta}{\dfrac{AC}{\sin\theta}} - \frac{\tau_{xy} . AC \cos\theta}{\dfrac{AC}{\sin\theta}} + \frac{\sigma_y . BC \cos\theta}{\dfrac{BC}{\cos\theta}} - \frac{\tau_{xy} . BC \sin\theta}{\dfrac{BC}{\cos\theta}}$$

$$= \sigma_x \,.\, \sin^2\theta - \tau_{xy} \sin\theta \cos\theta + \sigma_y \,.\, \cos^2\theta - \tau_{xy} \,.\, \sin\theta \cos\theta$$

$$= \frac{\sigma_x}{2}(1 - \cos 2\theta) + \frac{\sigma_y}{2}(1 + \cos 2\theta) - 2\,\tau_{xy} \,.\, \sin\theta\cos\theta$$

$$= \frac{\sigma_x}{2} - \frac{\sigma_x}{2}\cos 2\theta + \frac{\sigma_y}{2} + \frac{\sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

or

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta \qquad \ldots (v)$$

Shear Stress or Tangential Stress (across inclined the section *AB*)

$$\tau = \frac{P_t}{AB} = \frac{\sigma_x . AC \cos\theta + \tau_{xy} . AC \sin\theta - \sigma_y . BC \sin\theta - \tau_{xy}\ BC \cos\theta}{AB}$$

$$= \frac{\sigma_x . AC \cos\theta}{AB} + \frac{\tau_{xy} . AC \sin\theta}{AB} - \frac{\sigma_y . BC \sin\theta}{AB} - \frac{\tau_{xy}\ BC \cos\theta}{AB}$$

$$= \frac{\sigma_x . AC \cos\theta}{\dfrac{AC}{\sin\theta}} + \frac{\tau_{xy} . AC \sin\theta}{\dfrac{AC}{\sin\theta}} - \frac{\sigma_y . BC \sin\theta}{\dfrac{BC}{\cos\theta}} - \frac{\tau_{xy} . BC \cos\theta}{\dfrac{BC}{\cos\theta}}$$

$$= \sigma_x \sin\theta\cos\theta + \tau_{xy}\sin^2\theta - \sigma_y \sin\theta\cos\theta - \tau_{xy}\cos^2\theta$$

$$= (\sigma_x - \sigma_y)\sin\theta\cos\theta + \frac{\tau_{xy}}{2}(1 - \cos 2\theta) - \frac{\tau_{xy}}{2}(1 + \cos 2\theta)$$

or

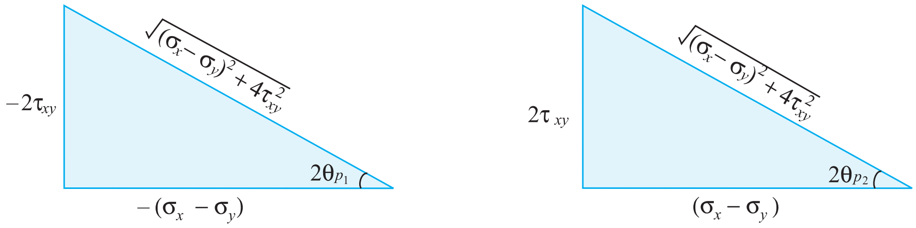
$$\tau = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta - \tau_{xy}\cos 2\theta \qquad \ldots (vi)$$

Now the planes of maximum and minimum normal stresses (*i.e.* principal planes) may be found out by equating the shear stress to zero. From the above equations, we find that the shear stress to any plane is a function of σ_y, σ_x, τ_{xy} and θ. A little consideration will show that the values of σ_y, σ_x and τ_{xy} are constant and thus the shear stress varies in the angle θ. Now let θ_p be the value of the angle for which the shear stress is zero.

$$\therefore \quad \frac{\sigma_x - \sigma_y}{2} \sin 2\theta_p - \tau_{xy} \cos 2\theta_p = 0$$

$$\text{or} \quad \frac{\sigma_x - \sigma_y}{2} \sin 2\theta_p = \tau_{xy} \cos 2\theta_p \quad \text{or} \quad \tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

From the above equation, we find that the following two cases satisfy this condition as shown in Fig 7.10 (*a*) and (*b*).



(*a*) Case 1 (*b*) Case 2

Fig. 7.10

Thus we find that there are two principal planes, at right angles to each other, their inclinations with *x-x* axis being θ_{p_1} and θ_{p_2}.

Now for case 1,

$$\sin 2\theta_{p_1} = \frac{-2\tau_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}} \quad \text{and} \quad \cos 2\theta_{p_1} = \frac{-(\sigma_x - \sigma_y)}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}$$

Similarly for case 2,

$$\sin 2\theta_{p_2} = \frac{2\tau_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}} \quad \text{and} \quad \cos 2\theta_{p_2} = \frac{(\sigma_x - \sigma_y)}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}$$

Now the values of principal stresses may be found out by substituting the above values of $2\theta_{p_1}$ and $2\theta_{p_2}$ in equation (*v*).

Maximum Principal Stress,

$$\sigma_{p_1} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$= \frac{\sigma_x + \sigma_y}{2} - \left(\frac{\sigma_x - \sigma_y}{2} \times \frac{-(\sigma_x - \sigma_y)}{\sqrt{(\sigma_x - \sigma_y^2) + 4\tau_{xy}^2}}\right) - \left(\tau_{xy} \times \frac{-2\tau_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}\right)$$

$$= \frac{\sigma_x + \sigma_y}{2} + \frac{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}{2\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2}$$

$$\text{or} \quad \boxed{\sigma_{p_1} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

Minimum Principal Stress

$$\sigma_{p2} = \frac{\sigma_x + \sigma_y}{2} - \frac{(\sigma_x - \sigma_y)}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{\sigma_x + \sigma_y}{2} - \left(\frac{\sigma_x - \sigma_y}{2} \times \frac{(\sigma_x - \sigma_y)}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}\right) - \left(\tau_{xy} \times \frac{2\tau_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}\right)$$

$$= \frac{\sigma_x + \sigma_y}{2} - \frac{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}{2\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}} = \frac{\sigma_x - \sigma_y}{2} - \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2}$$

or

$$\sigma_{p_2} = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

EXAMPLE 7.10. *A point is subjected to a tensile stress of 250 MPa in the horizontal direction and another tensile stress of 100 MPa in the vertical direction. The point is also subjected to a simple shear stress of 25 MPa, such that when it is associated with the major tensile stress, it tends to rotate the element in the clockwise direction. What is the magnitude of the normal and shear stresses on a section inclined at an angle of 20° with the major tensile stress?*

SOLUTION. Given : Tensile stress in horizontal *x-x* direction (σ_x) = 250 MPa ; Tensile stress in vertical *y-y* direction (σ_y) = 100 MPa ; Shear stress (τ_{xy}) = 25 MPa and angle made by section with the major tensile stress (θ) = 20°.

Magnitude of normal stress

We know that magnitude of normal stress,

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{250 + 100}{2} - \frac{250 - 100}{2}\cos(2 \times 20°) - 25\sin(2 \times 20°)$$

$$= 175 - 75\cos 40° - 25\sin 40° \text{ MPa}$$

$$= 175 - (75 \times 0.766) - (25 \times 0.6428) \text{ MPa}$$

$$= 175 - 57.45 - 16.07 = 101.48 \text{ MPa}$$ **Ans.**

Magnitude of shear stress

We also know that magnitude of shear stress,

$$\tau = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

$$= \frac{250 - 100}{2}\sin(2 \times 20°) - 25\cos(2 \times 20°)$$

$$= 75\sin 40° - 25\cos 40° \text{ MPa}$$

$$= (75 \times 0.6428) - (25 \times 0.766) \text{ MPa}$$

$$= 48.21 - 19.15 = 29.06 \text{ MPa}$$ **Ans.**

EXAMPLE 7.11. *A plane element in a boiler is subjected to tensile stresses of 400 MPa on one plane and 150 MPa on the other at right angles to the former. Each of the above stresses is accompanied by a shear stress of 100 MPa such that when associated with the minor tensile stress tends to rotate the element in anticlockwise direction. Find*

(*a*) *Principal stresses and their directions.*

(*b*) *Maximum shearing stresses and the directions of the plane on which they act.*

SOLUTION. Given : Tensile stress along *x-x* axis (σ_x) = 400 MPa ; Tensile stress along *y-y* axis (σ_y) = 150 MPa and shear stress (τ_{xy}) = – 100 MPa (Minus sign due to anticlockwise on *x-x* direction).

(a) Principal stresses and their directions

We know that maximum principal stress,

$$\sigma_{max} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{400 + 150}{2} + \sqrt{\left(\frac{400 - 150}{2}\right)^2 + (-100)^2} \text{ MPa}$$

$$= 275 + 160.1 = 435.1 \text{ MPa} \qquad \textbf{Ans.}$$

and minimum principal stress,

$$\sigma_{min} = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{400 + 150}{2} - \sqrt{\left(\frac{400 - 150}{2}\right)^2 + (-100)^2} \text{ MPa}$$

$$= 275 - 160.1 = 114.9 \text{ MPa} \qquad \textbf{Ans.}$$

Let θ_p = Angle which plane of principal stress makes with *x-x* axis.

We know that, $$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2 \times 100}{400 - 150} = 0.8 \quad \text{or} \quad 2\theta_p = 38.66°$$

$$\therefore \quad \theta_p = 19.33° \quad \text{or} \quad 109.33° \qquad \textbf{Ans.}$$

(b) Maximum shearing stresses and their directions

We also know that maximum shearing stress

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{400 - 150}{2}\right)^2 + (-100)^2}$$

$$= 160.1 \text{ MPa} \qquad \textbf{Ans.}$$

Let θ_s = Angle which plane of maximum shearing stress makes with *x-x* axis.

We know that, $$\tan 2\theta_s = \frac{\sigma_x - \sigma_y}{2\tau_{xy}} = \frac{400 - 150}{2 \times 100} = 1.25 \quad \text{or} \quad 2\theta_s = 51.34°$$

$$\theta_s = 25.67° \quad \text{or} \quad 115.67° \qquad \textbf{Ans.}$$

EXAMPLE 7.12. *A point in a strained material is subjected to the stresses as shown in Fig. 7.11.*

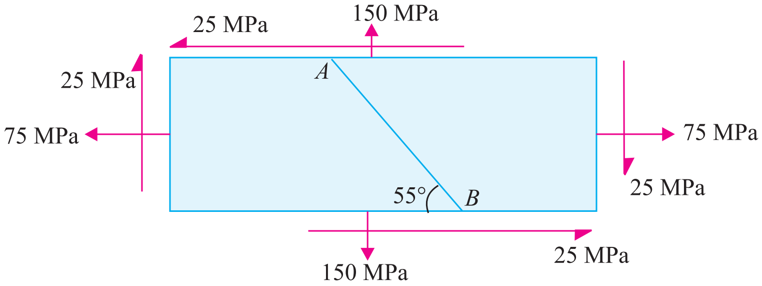


Fig. 7.11

Find graphically, or otherwise, the normal and shear stresses on the section AB.

SOLUTION. Given : Tensile stress along horizontal *x-x* axis (σ_x) = 75 MPa ; Tensile stress along vertical *y-y* axis (σ_y) = 150 MPa ; Shear stress (τ_{xy}) = 25 MPa and angle made by section with the horizontal direction (θ) = 55°.

Normal stress on the section AB

We know that normal stress on the section *AB*,

$$\sigma_n = \frac{\sigma_x - \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$= \frac{75+150}{2} - \frac{75-150}{2} \cos (2 \times 55°) - 25 \sin (2 \times 55°) \text{ MPa}$$

$$= 112.5 + 37.5 \cos 110° - 25 \sin 110° \text{ MPa}$$

$$= 11.25 + 37.5 \times (-0.342) - (25 \times 0.9397) \text{ MPa}$$

$$= 112.5 - 12.83 - 23.49 = 76.18 \text{ MPa} \qquad \textbf{Ans.}$$

Shear stress on the section AB

We also know that shear stress on the section *AB*.

$$\tau = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta - \tau_{xy} \cos 2\theta$$

$$= \frac{75-150}{2} \sin (2 \times 55°) - 25 \cos (2 \times 55°) \text{ MPa}$$

$$= -37.5 \sin 110° - 25 \cos 110° \text{ MPa}$$

$$= -37.5 \times 0.9397 - 25 \times (-0.342) \text{ MPa}$$

$$= -35.24 + 8.55 = -26.69 \text{ MPa} \qquad \textbf{Ans.}$$

EXAMPLE 7.13. *A plane element of a body is subjected to a compressive stress of 300 MPa in x-x direction and a tensile stress of 200 MPa in the y-y direction. Each of the above stresses is subjected to a shear stress of 100 MPa such that when it is associated with the compressive stress, it tends to rotate the element in an anticlockwise direction. Find graphically, or otherwise, the normal and shear stresses on a plane inclined at an angle of 30° with the x-x axis.*

SOLUTION. Given : Compressive stress in *x-x* direction (σ_x) = – 300 MPa (Minus sign due to compressive stress) ; Tensile stress in *y-y* direction (σ_y) = 200 MPa ; Shear stress (τ_{xy}) = – 100 MPa (Minus sign due to anticlockwise direction along the compressive stress *i.e.*, σ_x) and angle made by section with the *x-x* axis (θ) = 30°.

Normal stress on the plane

We know that normal stress on the plane,

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$= \frac{-300+200}{2} - \frac{-300-200}{2} \cos (2 \times 30°) - [-100 \sin (2 \times 30°]$$

$$= -50 - (-250 \cos 60°) + 100 \sin 60° \text{ MPa}$$

$$= -50 + (250 \times 0.5) + (10 \times 0.866) \text{ MPa}$$

$$= -50 + 125 + 86.6 = 161.6 \text{ MPa} \qquad \textbf{Ans.}$$

Shear stress on the plane

We also know that shear stress on the plane.

$$\tau = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta - \tau_{xy} \cos 2\theta$$

$$= \frac{-300-200}{2} \sin (2 \times 30°) - [-100 \cos (2 \times 30°)] \text{ MPa}$$
$$= -250 \sin 60° + 100 \cos 60° \text{ MPa}$$
$$= -250 \times 0.866 + 100 \times 0.5 \text{ MPa}$$
$$= -216.5 + 50 = -166.5 \text{ MPa} \qquad \textbf{Ans.}$$

EXAMPLE 7.14. *A machine component is subjected to the stresses as shown in the figure given below :*

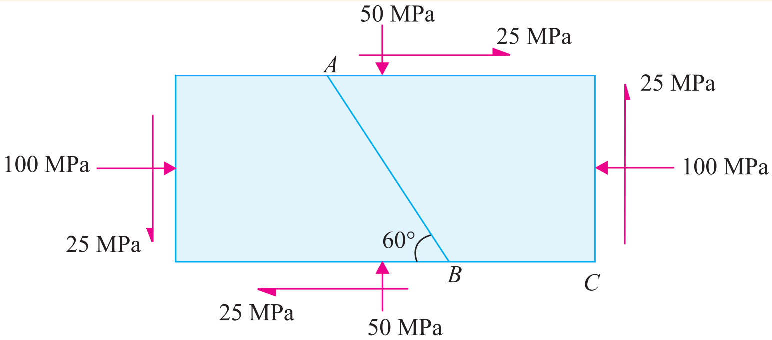


Fig. 7.12

Find the normal and shearing stresses on the section AB inclined at an angle of 60° with x-x axis. Also find the resultant stress on the section.

SOLUTION. Given : Compressive stress along horizontal *x-x* axis (σ_x) = – 100 MPa (Minus sign due to compressive stress) ; Compressive stress along vertical *y-y* axis (σ_y) = – 50 MPa (Minus sign due to compressive stress) ; Shear stress (τ_{xy}) = – 25 MPa (Minus sign due to anticlockwise on *x-x* axis) and angle made by section *AB* with *x-x* axis (θ) = 60°.

Normal stress on the section AB

We know that normal stress on the section *AB*,

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$
$$= \frac{-100+(-50)}{2} - \frac{-100-(-50)}{2} \cos (2 \times 60°) - [-25 \sin (2 \times 60°)]$$
$$= -75 + 25 \cos 120° + 25 \sin 120° \text{ MPa}$$
$$= -75 + [25 \times (-0.5)] + (25 \times 0.866) \text{ MPa}$$
$$= -75 - 12.5 + 21.65 = -65.85 \text{ MPa} \qquad \textbf{Ans.}$$

Shearing stress on the section AB

We know that shearing stress on the section *AB*,

$$\tau = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta - \tau_{xy} \cos 2\theta$$
$$= \frac{-100-(-50)}{2} \sin (2 \times 60°) - [-25 \cos (2 \times 60°)]$$
$$= -25 \sin 120° + 25 \cos 120° = -25 \times 0.866 + [25 \times (-0.5)] \text{ MPa}$$
$$= -21.65 - 12.5 = -34.15 \text{ MPa} \qquad \textbf{Ans.}$$

Resultant stress on the section AB

We also know that resultant stress on the section *AB*,

$$\sigma_R = \sqrt{\sigma_n^2 + \tau^2} = \sqrt{(-65.85)^2 + (-34.15)^2} = 74.2 \text{ MPa} \qquad \textbf{Ans.}$$

EXERCISE 7.1

1. A bar is subjected to a tensile stress of 100 MPa, Determine the normal and tangential stresses on a plane making an angle of 30° with the direction of the tensile stress.
(**Ans.** 75 MPa ; 43.3 MPa)
2. A point in a strained material is subjected to a tensile stress of 50 MPa. Find the normal and shear stress at an angle of 50° with the direction of the stress. (**Ans.** 29.34 MPa ; 24.62 MPa)
3. At a point in a strained material, the principal stresses are 100 MPa and 50 MPa both tensile. Find the normal and shear stresses at a section inclined at 30° with the axis of the major principal stress. (**Ans.** 87.5 MPa ; 21.65 MPa)
4. A point in a strained material is subjected to a tensile stress of 120 MPa and a clockwise shear stress of 40 MPa. What are the values of normal and shear stresses on a plane inclined at 45° with the normal to the tensile stress. (**Ans.** 20 MPa ; 60 MPa)
5. The principal stresses or a point in the section of a member are 50 MPa or 20 MPa both tensile. If there is a clockwise shear stress of 30 MPa, find the normal and shear stresses on a section inclined at an angle of 15° with the normal to the major tensile stress.
(**Ans.** 32.99 MPa ; 33.48 MPa)

7.12. Graphical Method for the Stresses on an Oblique Section of a Body

In the previous articles, we have been discussing the analytical method for the determination of normal, shear and resultant stresses across a section. But we shall now discuss a graphical method for this purpose. This is done by drawing a Mohr's Circle of Stresses. The construction of Mohr's Circle of Stresses as well as determination of normal, shear and resultant stresses is very easier than the analytical method. Moreover, there is a little chance of committing any error in this method. In the following pages, we shall draw the Mohr's Circle of Stresses for the following cases :

1. A body subjected to a direct stress in one plane.
2. A body subjected to direct stresses in two mutually perpendicular directions.
3. A body subjected to a simple shear stress.
4. A body subjected to a direct stress in one plane accompanied by a simple shear stress.
5. A body subjected to direct stresses in two mutually perpendicular directions accompanied by a simple shear stress.

7.13. Sign Conventions for Graphical Method

Though there are different sign conventions used in different books for graphical method also, yet we shall adopt the following sign conventions, which are widely used and internationally recognised :

1. The angle is taken with reference to the *X-X* axis. All the angles traced in the anticlockwise direction to the *X-X* axis are taken as negative, whereas those in the clockwise direction as positive as shown in Fig. 7.13 (*a*). The value of angle θ, until and unless mentioned is taken as positive and drawn clockwise.
2. The measurements above *X-X* axis and to the right of *Y-Y* axis are taken as positive, whereas those below *X-X* axis and to the left of *Y-Y* axis as negative as shown in Fig 7.13 (*b*) and (*c*).
3. Sometimes there is a slight variation in the results obtained by analytical method and graphical method. The values obtained by graphical method are taken to be correct if they agree upto the first decimal point with values obtained by analytical method, *e.g.*, 8.66 (Analytical) = 8.7 (Graphical), similarly 4.32 (Analytical) = 4.3 (Graphical)

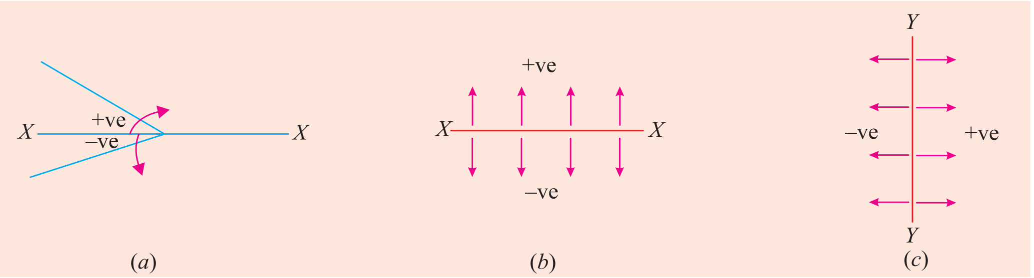


Fig. 7.13

7.14. Mohr's Circle for Stresses on an Oblique Section of a Body Subjected to a Direct Stress in One Plane

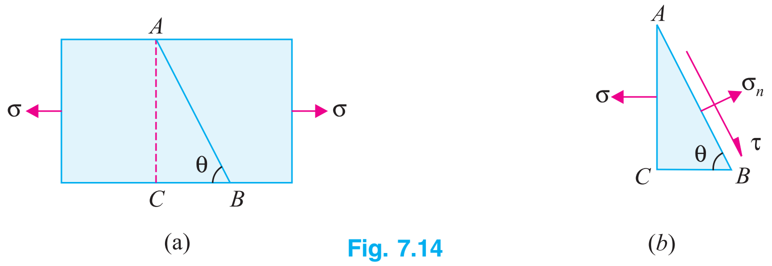


Fig. 7.14

Consider a rectangular body of uniform cross-sectional area and unit thickness subjected to a direct tensile stress along $X–X$ axis as shown in Fig 7.14 (*a*) and (*b*). Now let us consider an oblique section AB inclined with $X–X$ axis, on which we are required to find out the stresses as shown in the figure.

Let σ = Tensile stress, in x-x direction and

θ = Angle which the oblique section AB makes with the x-x axis in clockwise direction.

First of all, consider the equilibrium of the wedge ABC. Now draw the Mohr's* Circle of Stresses as shown in Fig.7.15 and as discussed below :

1. First of all, take some suitable point O and through it draw a horizontal line XOX.
2. Cut off OJ equal to the tensile stress (σ) to some suitable scale and towards right (because σ is tensile). Bisect OJ at C. Now the point O represents the stress system on plane BC and the point J represents the stress system on plane AC.
3. Now with C as centre and radius equal to CO and or CJ draw a circle. It is known as Mohr's Circle for Stresses.

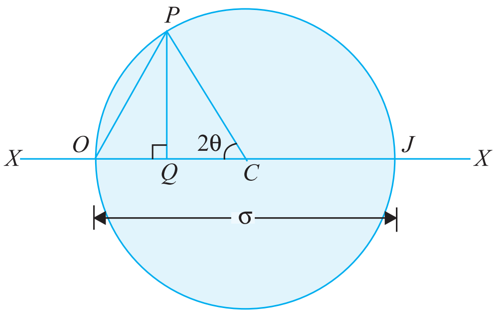


Fig. 7.15

* The diagram was first presented by German Scientist Otto Mohr in 1982.

4. Now through C draw a line CP making an angle of 2θ with CO in the clockwise direction meeting the circle at P. The point P represents the section AB.
5. Through P, draw PQ perpendicular to OX. Join OP.
6. Now OQ, QP and OP will give the normal stress, shear stress and resultant stress respectively to the scale. And the angle POJ is called the angle of obliquity (θ).

Proof

From the geometry of the Mohr's Circle of Stresses, we find that,

$$OC = CJ = CP = \sigma/2 \qquad \text{... (Radius of the circle)}$$

$\therefore$ Normal Stress.

$$\sigma_n = OQ = OC - QC = \left(\frac{\sigma}{2}\right) - \left(\frac{\sigma}{2}\right)\cos 2\theta \qquad \text{...(Same as in Art. 7.7)}$$

and shear stress

$$\tau = QP = CP \sin 2\theta = \frac{\sigma}{2}\sin 2\theta \qquad \text{...(Same as in Art. 7.7)}$$

We also find that maximum shear stress will be equal to the radius of the Mohr's Circle of Stresses *i.e.*, $\frac{\sigma}{2}$. It will happen when 2θ is equal to 90° or 270° *i.e.*, θ is equal to 45° or 135°.

However when $\theta = 45°$ then the shear stress is equal to $\frac{\sigma}{2}$.

And when $\theta = 135°$ then the shear stress is equal to $-\frac{\sigma}{2}$.

7.15. Mohr's Circle for Stresses on an Oblique Section of a Body Subjected to Direct Stresses in Two Mutually Perpendicular Directions

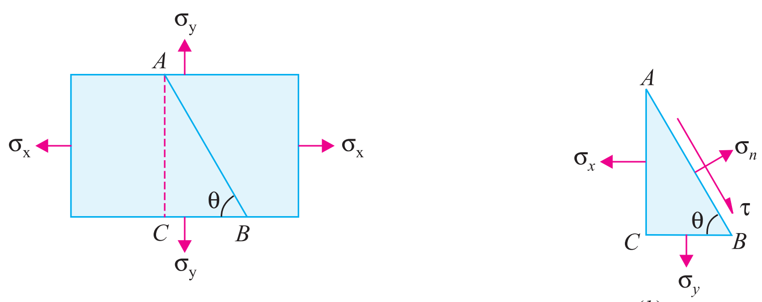


Fig. 7.16

Consider a rectangular body of uniform cross-sectional area and unit thickness subjected to direct tensile stresses in two mutually perpendicular directions along *x-x* and *y-y* axis as shown in Fig 7.16 (*a*) and (*b*). Now let us consider an oblique section AB inclined with *x-x* axis on which we are required to find out the stresses as shown in the figure.

Let σ_x = Tensile stress in *x-x* direction (also termed as major tensile stress),

σ_y = Tensile stress in *y-y* direction (also termed as minor tensile stress). and

θ = Angle which the oblique section AB makes with *x-x* axis in clockwise direction.

First of all consider the equilibrium of the wedge ABC. Now draw the Mohr's Circle of Stresses as shown in Fig. 7.17 and as discussed below :

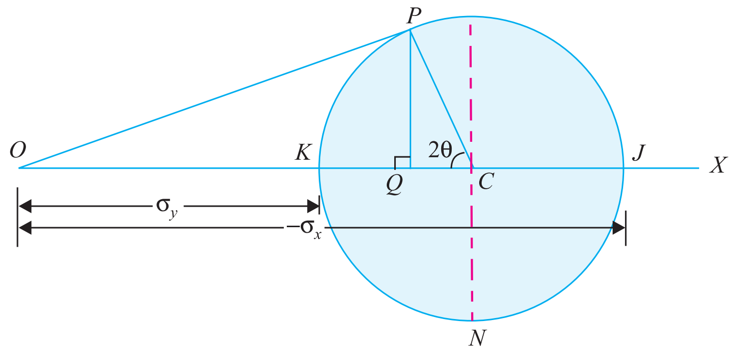


Fig. 7.17

1. First of all, take some suitable point *O* and draw a horizontal line *OX*.
2. Cut off *OJ* and *OK* equal to the tensile stresses σ_x and σ_y to some suitable scale towards right (because both the stresses are tensile). The point *J* represents the stress system on plane *AC* and the point *K* represents the stress system on plane *BC*. Bisect *JK* at *C*.
3. Now with *C* as centre and radius equal to *CJ* or *CJ* draw a circle. It is known as Mohr's Circle of Stresses.
4. Now through *C*, draw a line *CP* making an angle of 2θ with *CK* in clockwise direction meeting the circle at *P*. The point *P* represents the stress systems on the section *AB*.
5. Through *P*, draw *PQ* perpendicular to the line *OX*. Join *OP*.
6. Now *OQ*, *QP* and *OP* will give the normal stress, shear stress and resultant stress respectively to the scale. Similarly *CM* or *CN* will give the maximum shear stress to the scale. The angle *POC* is called the angle of obliquity.

Proof

From the geometry of the Mohr's Circle of Stresses, we find that

$$KC = CJ = CP = \frac{\sigma_x - \sigma_y}{2}$$

or

$$OC = OK + KC = \sigma_y + \frac{\sigma_x - \sigma_y}{2} = \frac{2\sigma_y + \sigma_x - \sigma_y}{2} = \frac{\sigma_x + \sigma_y}{2}$$

∴ Normal stress,

$$\sigma_n = OQ = OC - CQ = \frac{\sigma_x - \sigma_y}{2} - CP\cos 2\theta$$

$$= \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta \qquad \text{...(Same as Art. 7.8)}$$

and shear stress,

$$\tau = QP = CP\sin 2\theta$$

$$= \frac{\sigma_x + \sigma_y}{2}\sin 2\theta \qquad \text{...(Same as Art. 7.8)}$$

We also find that the maximum shear stress will be equal to the radius of the Mohr's Circle of Stresses. *i.e.*, $\frac{\sigma_x - \sigma_y}{2}$. It will happen when 2θ is equal to 90° or 270° *i.e.*, when θ is equal to 45° or 135°.

However when $\theta = 45°$ then the shear stress is equal to $\dfrac{\sigma_x - \sigma_y}{2}$

And when $\theta = 135°$ then the shear stress will be equal to $\dfrac{-(\sigma_x - \sigma_y)}{2}$ or $\dfrac{\sigma_y - \sigma_x}{2}$.

EXAMPLE 7.15. *The stresses at a point of a machine component are 150 MPa and 50 MPa both tensile. Find the intensities of normal, shear and resultant stresses on a plane inclined at an angle of 55° with the axis of major tensile stress.*

Also find the magnitude of the maximum shear stresses in the component.

***SOLUTION.** Given : Tensile stress along horizontal *x-x* axis (σ_x) = 150 MPa ; Tensile stress along vertical *y-y* axis (σ_y) = 50 MPa and angle made by the plane with the axis of major tensile stress (θ) = 55°.

The given stresses on the planes *AC* and *BC* in the machine component are shown in Fig. 7.18 (*a*). Now draw the Mohr's Circle of Stresses as shown in Fig. 7.18 (*b*) and as discussed below :

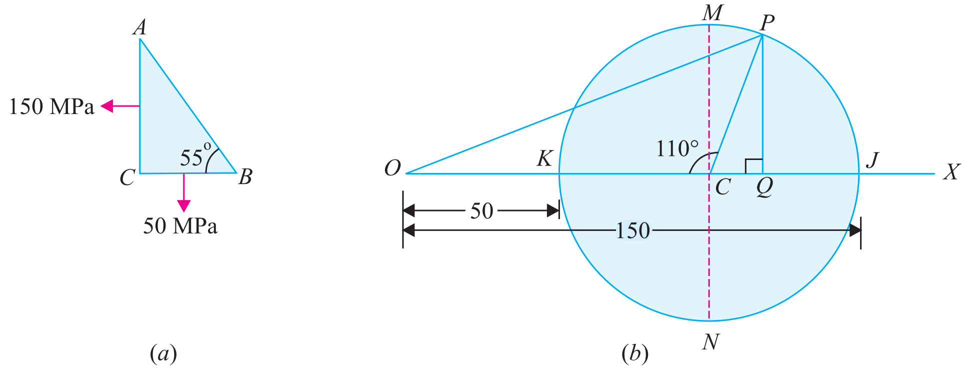


Fig. 7.18

1. First of all, take some suitable point *O* and draw a horizontal line *OX*.
2. Cut off *OJ* and *OK* equal to the tensile stresses σ_x and σ_y respectively (*i.e.* 150 MPa and 50 MPa) to some suitable scale towards right. The point *J* represents the stress system on the plane *AC* and the point *K* represents the stress system on the plane *BC*. Bisect *KJ* at *C*.
3. Now with *C* as centre and radius equal to *CJ* or *CK* draw the Mohr's Circle of Stresses.
4. Now through *C* draw two lines *CM* and *CN* at right angles to the line *OX* meeting the circle at *M* and *N*. Also through *C* draw a line *CP* making an angle of 2 × 55° = 110° with *CK* in clockwise direction meeting the circle at *P*. The point *P* represents the stress system on the plane *AB*.
5. Through *P*, draw *PQ* perpendicular to the line *OX*. Join *OP*.

   By measurement, we find that the normal stress (σ_n) = *OQ* = 117.1 MPa ; Shear stress (τ) = *QP* = 47.0 MPa ; Resultant stress (σ_R) = *OP* = 126.2 MPa and maximum shear stress (τ_{max}) = *CM* = ± 50 MPa **Ans.**

EXAMPLE 7.16. *The stresses at a point in a component are 100 MPa (tensile) and 50 MPa (compressive). Determine the magnitude of the normal and shear stresses on a plane inclined at an angle of 25° with tensile stress. Also determine the direction of the resultant stress and the magnitude of the maximum intensity of shear stress.*

* We have already solved this question analytically, as example 7.5.

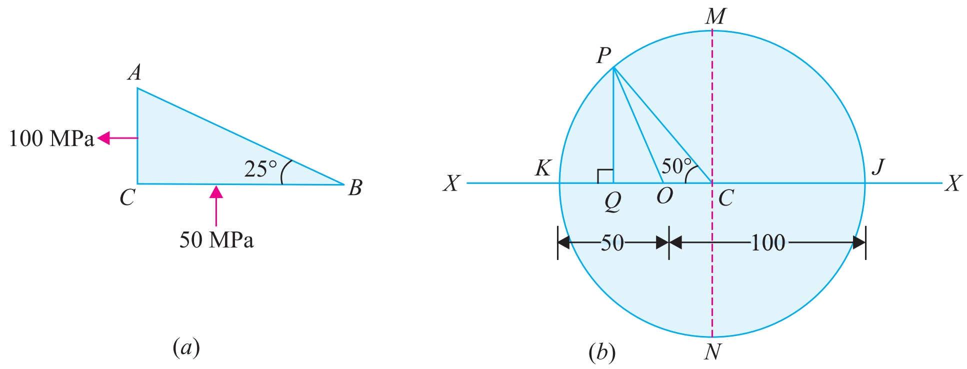


Fig. 7.19

***Solution.** Given : Tensile stress along horizontal *x-x* axis (σ_x) = 100 MPa ; Compressive stress along vertical *y-y* axis (σ_y) = – 50 MPa (Minus sign due to compressive) and angle made by plane with tensile stress (θ) = 25°.

The given stresses on the planes *AC* and *BC* of the component are shown in Fig 7.19 (*a*). Now draw the Mohr's Circle of Stresses as shown in Fig. 7.19 (*b*) and as discussed below :

1. First of all, take some suitable point *O* and through it draw a horizontal line *XOX*.
2. Cut off *OJ* and *OK* equal to the stresses and respectively (*i.e.*, 100 MPa and – 50 MPa) to some suitable scale such that *J* is towards right (because of tensile stress) and *B* is towards left (because of compressive stress). The point *J* represents the stress system on the plane *AC* and the point *K* represents the stress systems on the plane *BC*. Bisect *KJ* at *C*.
3. Now with *C* as centre and radius equal to *CJ* or *CK* draw the Mohr's Circle of Stresses.
4. Now through *C*, draw two lines *CM* and *CN* at right angles to the line *OX* meeting the circle at *M* and *N*. Also through *C*, draw a line *CP* making an angle of 2 × 25° = 50° with *CK* in clockwise direction meeting the circle at *P*. The point *P* represents the stress system on the plane *AB*.
5. Through *P*, draw *PQ* perpendicular to the line *OX*. Join *OP*.

By measurement, we find that the normal stress (σ_n) = – 23.2 MPa ; Shear stress (τ) = *PQ* = 57.45 MPa; Direction of the resultant stress ∠*POQ* = 68.1° and maximum shear stress (τ_{max}) = *CM* = *CN* = ± 75 MPa **Ans.**

7.16. Mohr's Circle for Stresses on an Oblique Section of a Body Subjected to a Direct Stresses in One Plane Accompanied by a Simple Shear Stress

Consider a rectangular body of uniform cross-sectional area and unit thickness subjected to a direct tensile stress along *X-X* axis accompanied by a positive (*i.e.* clockwise) shear stress along *X-X* axis as shown in Fig 7.20 (*a*) and (*b*). Now let us consider an oblique section *AB* inclined with *x-x* axis on which we are required to find out the stresses as shown in the figure 7.20.

* We have already solved this question analytically, as example 7.6.

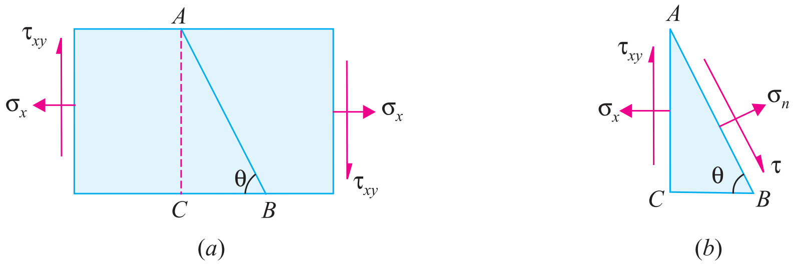


Fig. 7.20

Let

σ_x = Tensile stress in *x-x* direction,

τ_{xy} = Positive (*i.e.*, clockwise) shear stress along *x-x* axis, and

θ = Angle which oblique section *AB* makes with *x-x* axis in clock wise direction.

First of all consider the equilibrium of the wedge *ABC*. **We know that as per the principle of simple shear the face *BC* of the wedge will also be subjected to an anticlockwise shear stress.** Now draw the Mohr's Circle of Stresses as shown in Fig.7.21 and as discussed below :

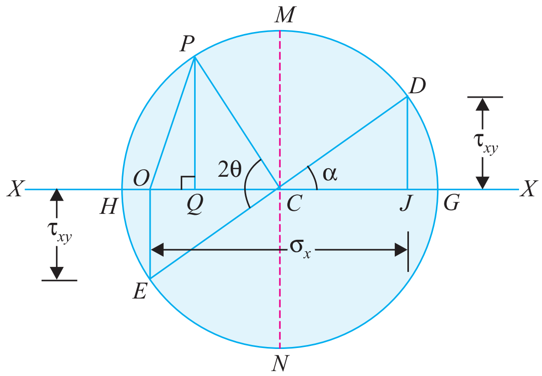


Fig. 7.21

1. First of all, take some suitable point *O* and through it draw a horizontal line *XOX*.
2. Cut off *OJ* equal to the tensile stress σ_x to some suitable scale and towards right (because σ_x is tensile).
3. Now erect a perpendicular at *J* above the line *X-X* (because τ_{xy} is positive along *x-x* axis) and cut off *JD* equal to the shear stress τ_{xy} to the scale. The point *D* represents the stress system on plane *AC*. Similarly, erect a perpendicular below the line *x-x* (**because τ_{xy} is negative along *y-y* axis**) and cut off *OE* equal to the shear stress τ_{xy} to the scale. The point *E* represents the stress system on plane *BC*. Join *DE* and bisect it at *C*.
4. Now with *C* as centre and radius equal to *CD* or *CE* draw a circle. It is known as Mohr's Circle of Stresses.
5. Now through *C*, draw a line *CP* making an angle 2θ with *CE* in clockwise direction meeting the circle at *P*. The point *P* represents the stress system on the section *AB*.
6. Through *P*, draw *PQ* perpendicular to the line *OX*. Join *OP*.
7. Now *OQ*, *QP* and *OP* will give the normal, shear and resultant stresses to the scale. And the angle *POC* is called the angle of obliquity.

Proof

From the geometry of the Mohr's Circle of Stresses, we find that

$$OC = \frac{\sigma_x}{2}$$

and radius of the circle,

$$R = EC = CD = CP = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

Now in the right angled triangle DCJ,

$$\sin\alpha = \frac{DJ}{CD} = \frac{\tau_{xy}}{R} \qquad \text{and} \qquad \cos\alpha = \frac{JC}{CD} = \frac{\sigma_x}{2} \times \frac{1}{R} = \frac{\sigma_x}{2R}$$

and similarly in right angled triangle CPQ,

$$\angle PCQ = (2\theta - \alpha)$$

$$\therefore \quad CQ = CP\cos(2\theta - \alpha) = R[\cos(2\theta - \alpha)]$$

$$= R[\cos\alpha\cos 2\theta + \sin\alpha\sin 2\theta]$$

$$= R\cos\alpha\cos 2\theta + R\sin\alpha\sin 2\theta$$

$$= R \times \frac{\sigma_x}{2R}\cos 2\theta + R \times \frac{\tau_{xy}}{R}\sin 2\theta$$

$$= \frac{\sigma_x}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

We know that normal stress across the section AB,

$$\sigma_n = OQ = OC - CQ = \frac{\sigma_x}{2} - \left(\frac{\sigma_x}{2}\cos 2\theta + \tau_{xy}\sin 2\theta\right)$$

$$= \frac{\sigma_x}{2} - \frac{\sigma_x}{2}\cos 2\theta - \tau_{xy}\sin 2\theta \qquad \text{...(Same as in Art. 7.10)}$$

and shear stress, $\tau = QP = CP\sin(2\theta - \alpha) = R\sin(2\theta - \alpha)$

$$= R(\cos\alpha\sin 2\theta - \sin\alpha\cos 2\theta)$$

$$= R\cos\alpha\sin 2\theta - R\sin\alpha\cos 2\theta$$

$$= R \times \frac{\sigma_x}{2R}\sin 2\theta - R \times \frac{\tau_{xy}}{2}\cos 2\theta$$

$$= \frac{\sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta \qquad \text{...(Same as in Art. 7.10)}$$

We also know that maximum stress,

$$\sigma_{max} = OG = OC + CG = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

and minimum stress

$$\sigma_{min} = OH = OC - CH = \frac{\sigma_x}{2} - \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

We also find that the maximum shear stress will be equal to the radius of the Mohr's circle of stresses *i.e.*, $\sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$. It will happen when $(2\theta - \alpha)$ is equal to 90° or 270°.

However when $(2\theta - \alpha)$ is equal to 90° then the shear stress is equal to $+\sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$.

And when $(2\theta - \alpha) = 270°$ then the shear stress is equal to $-\sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$.

EXAMPLE 7.17. *A plane element in a body is subjected to a tensile stress of 100 MPa accompanied by a clockwise shear stress of 25 MPa. Find (i) the normal and shear stress on a plane inclined at an angle of 20° with the tensile stress ; and (ii) the maximum shear stress on the plane.*

***SOLUTION.** Given : Tensile stress along horizontal *x-x* axis (σ_x) = 100 MPa ; Shear stress (τ_{xy}) = 25 MPa and angle made by plane with tensile stress (θ) = 20°.

The given stresses on the element and a complimentary shear stress on the *BC* plane are shown in Fig. 7.22 (*a*). Now draw the Mohr's Circle of Stresses as shown in Fig 7.22 (*b*) and as discussed below :

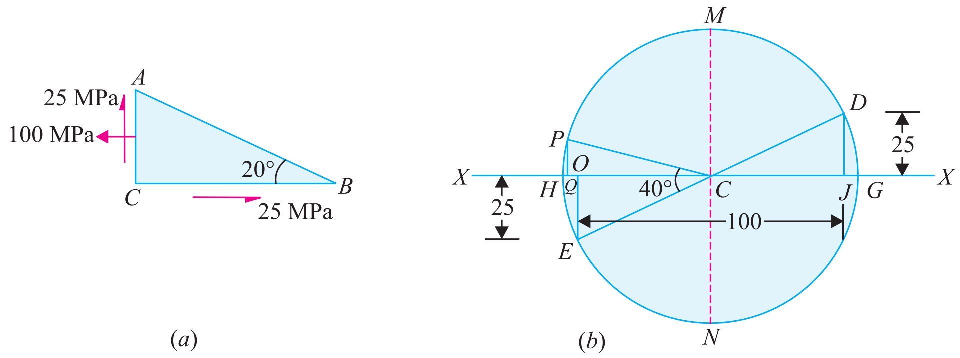


Fig. 7.22

1. First of all, take some suitable point *O*, and through it draw a horizontal line *XOX*.
2. Cut off *OJ* equal to the tensile stress on the plane *AC* (*i.e.,* 100 MPa) to some suitable scale towards right.
3. Now erect a perpendicular at *J* above the line *X-X* and cut off *JD* equal to the positive shear stress on the plane *BC* (*i.e.,* 25 MPa) to the scale. The point *D* represents the stress system on the plane *AC*. Similarly erect a perpendicular at *O* below the line *X-X* and cut off *OE* equal to the negative shear stress on the plane *BC* (*i.e.*, 25 MPa) to the scale. The point *E* represents the stress system on the plane *BC*. Join *DE* and bisect it at *C*.
4. Now with *C* as centre and radius equal to *CD* or *CE* draw the Mohr's Circle of Stresses.
5. Now through *C*, draw two lines *CM* and *CN* at right angle to the line *OX* meeting the circle at *M* and *N*. Also through *C*, draw a line *CP* making an angle of 2 × 20° = 40° with *CE* in clockwise direction meeting the circle at *P*. The point *P* represents the stress system on the section *AB*.
6. Through *P*, draw *PQ* perpendicular to the line *OX*.

   By measurement, we find that the normal stress (σ_n) = *OQ* = 4.4 MPa (compression) ; Shear stress (τ) = *QP* = 13.0 MPa and maximum shear stress (τ_{max}) = *CM* = 55.9 MPa **Ans.**

EXAMPLE 7.18. *An element in a strained body is subjected to a tensile stress of 150 MPa and a shear stress of 50 MPa tending to rotate the element in an anticlockwise direction. Find (i) the magnitude of the normal and shear stresses on a section inclined at 40° with the tensile stress ; and (ii) the magnitude and direction of maximum shear stress that can exist on the element.*

* We have already solved this question analytically, as example 7.7.

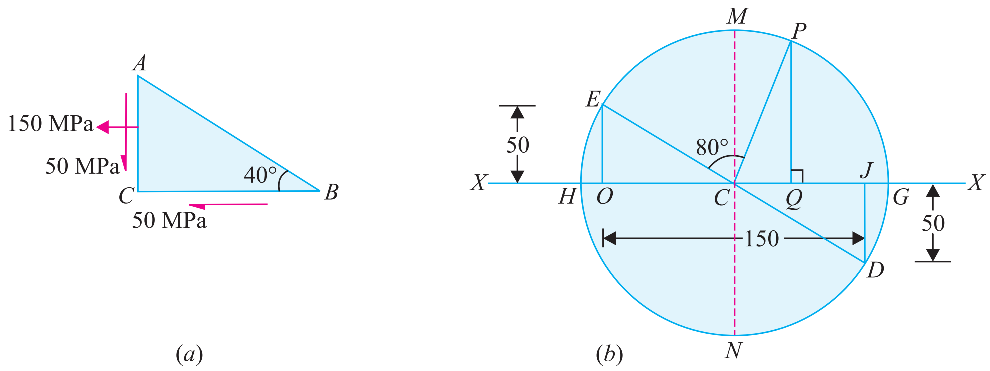


Fig. 7.23

***Solution.** Given : Tensile stress along horizontal *x-x* axis $(\sigma_x) = 150$ MPa ; Shear stress $(\tau_{xy}) = -50$ MPa (Minus sign due to anticlockwise) and angle made by section with the tensile stress $(\theta) = 40°$.

The given stresses on the plane *AB* of the element and a complimentary shear stress on the plane *BC* are shown in Fig 7.23 (*a*). Now draw the Mohr's Circle of Stresses as shown in Fig. 7.23 (*b*) and as discussed below :

1. First of all, take some suitable point *O*, and through it draw a horizontal line *XOX*.
2. Cut off *OJ* equal to the tensile stress on the plane *AC* (*i.e.*, 150 MPa) to some suitable scale towards right.
3. Now erect a perpendicular at *J* below the line *X-X* and cut off *JD* equal to the negative shear stress on the plane *AC* (*i.e.*, 50 MPa) to the scale. The point *D* represents the stress system on the plane *AC*. Similarly, erect a perpendicular at *O* above the line *X-X* and cut off *OE* equal to the positive shear stress on the plane *BC* (*i.e.*, 50 MPa) to the scale. The point *E* represents the stress system on the plane *BC*. Join *DE* and bisect it at *C*.
4. Now with *C* as centre and radius equal to *CD* or *CE* draw the Mohr's Circle of Stresses meeting the line *X-X* at *G* and *H*.
5. Through *C*, draw two lines *CM* and *CN* at right angles to the line *X-X* meeting the circle at *M* and *N*. Also through *C*, draw a line *CP* making an angle of $2 \times 40° = 80°$ with *CE* in clockwise direction meeting the circle at *P*. The point *P* represents the stress system on the section *AB*.
6. Through *P*, draw *PQ* perpendicular to the line *OX*.

   By measurement, we find that the Normal stress $(\sigma_n) = OQ = 112.2$ MPa ; Shear stress $(\tau) = QP = 82.5$ MPa and maximum shear stress, that can exist on element $(\tau_{max}) = \pm\, CM = CN = 90.14$ MPa **Ans.**

Example 7.19. *An element in a strained body is subjected to a compressive stress of 200 MPa and a clockwise shear stress of 50 MPa on the same plane. Calculate the values of normal and shear stresses on a plane inclined at 35° with the compressive stress. Also calculate the value of maximum shear stress in the element.*

****Solution.** Given : Compressive stress along horizontal *x-x* axis $(\sigma_x) = -200$ MPa (Minus sign due to compressive stress) ; Shear stress $(\tau_{xy}) = 50$ MPa ; and angle made by plane with the compressive stress $(\theta) = 35°$.

* We have already solved this question analytically, as example 7.8.
** We have already solved this question analytically, as example 7.9.

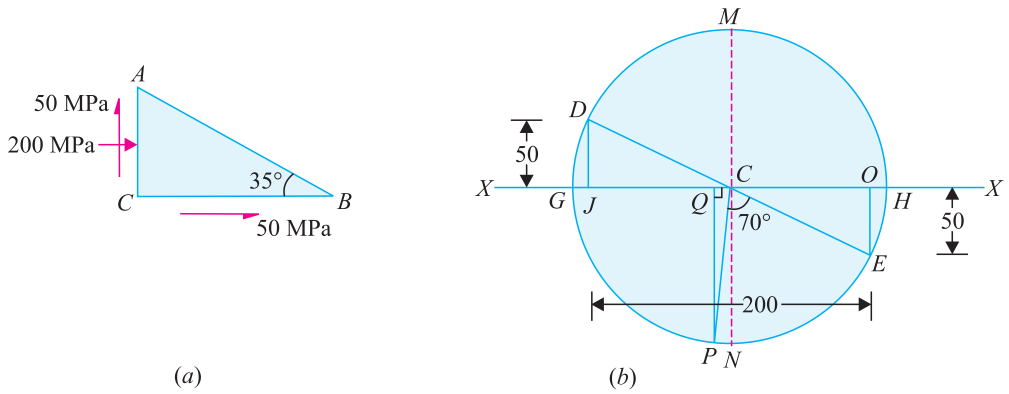


Fig. 7.24

The given stresses on the plane *AC* of the element and a complimentary shear stress on the plane *BC* are shown in Fig. 7.24 (*a*). Now draw the Mohr's Circle of Stresses as shown in Fig.7.24 (*b*) and as discussed below :

1. First of all, take some suitable point *O*, and through it draw a horizontal line *XOX*.
2. Cut off *OJ* equal to the compressive stress on the plane *AC* (*i.e.*, 200 MPa) to some suitable scale towards left .
3. Now erect a perpendicular at *J* above the line *X-X* and cut off *JD* equal to the positive shear stress on the plane *AC* (*i.e.*, 50 MPa) to the scale. The point *D* represents the stress system on the plane *AC*. Similarly, erect a perpendicular at *O* below the line *X-X* and cut off *OE* equal to the negative shear stress on the plane *BC* (*i.e.,* 50 MPa) to the scale. The point *E* represents the stress system on the plane *BC*. Join *DE* and bisect it at *C*.
4. Now with *C* as centre and radius equal to *CD* or *CE* draw the Mohr's Circle of Stresses. Meeting the line *X-X* at *G* and *H*.
5. Through *C,* draw two lines *CM* and *CN* at right angles to the line *X-X* meeting the circle at *M* and *N*. Also through *C* draw a line *CP* making an angle of 2 × 35° = 70° with *CE* in clockwise direction meeting the circle at *P*. The point *P* represents the stress system on the plane *AB*.
6. Through *P*, draw *PQ* perpendicular to the line *OX*.

   By measurement, we find that the Normal stress (σ_n) = *OQ* = – 112.8 MPa ; Shear stress (τ) = *QP* = – 111.1 MPa and maximum shear stress in the element (t_{max}) = ± *CM* = *CN* = 112.1 MPa **Ans.**

7.17. Mohr's Circle for Stresses on an Oblique Section of a Body Subjected to Direct Stresses in Two Mutually Perpendicular Directions Accompanied by a Simple Shear Stress

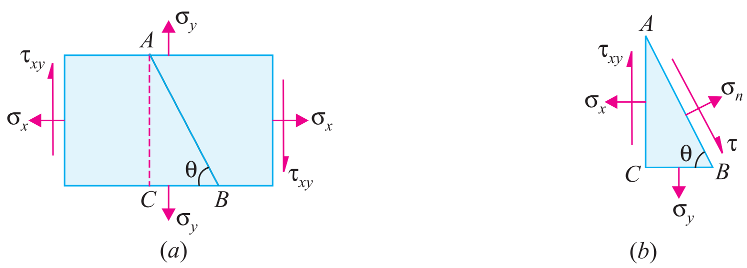


Fig. 7.25

Consider a rectangular body of uniform cross-sectional area and unit thickness subjected to direct tensile stresses in two mutually perpendicular directions along *X-X* and *Y-Y* axes accompanied by a positive (*i.e.*, clockwise) shear stress along *X-X* axis as shown in Fig. 7.25 (*a*) and (*b*). Now let us consider an oblique section *AB* inclined with *X-X* axis on which we are required to find out the stresses as shown in the figure.

Let σ_x = Tensile stress in *X-X* direction,

σ_y = Tensile stress in *Y-Y* direction,

τ_{xy} = Positive (*i.e.*, clockwise) shear stress along *X-X* axis, and

θ = Angle which the oblique section *AB* makes with *X-X* axis in clockwise direction.

First of all, consider the equilibrium of the wedge *ABC*. We know that as per the principle of simple shear, the face *BC* of the wedge will be subjected to an anticlockwise shear stress equal to τ_{xy} as shown in Fig. 7.25 (*b*). Now draw the Mohr's Circle of Stresses as shown in Fig. 7.26 and as discussed below :

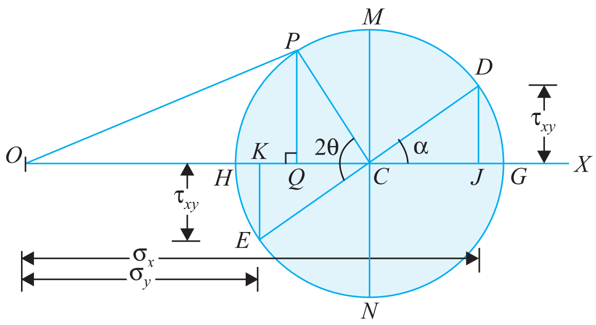


Fig. 7.26

1. First of all, take some suitable point *O* and through it draw a horizontal line *OX*.
2. Cut off *OJ* and *OK* equal to the tensile stresses σ_x and σ_y respectively to some suitable scale and towards right (because both the stresses are tensile).
3. Now erect a perpendicular at *J* above the line *X-X* (because τ_{xy} is positive along *X-X* axis) and cut off *JD* equal to the shear stress τ_{xy} to the scale. The point *D* represents the stress system on plane *AC*. Similarly, erect perpendicular below the line *X-X* (because τ_{xy} is negative along *Y-Y* axis) and cut off *KE* equal to the shear stress τ_{xy} to the scale. The point *E* represents the plane *BC*. Join *DE* and bisect it at *C*.
4. Now with *C* as centre and radius equal to *CD* or *CE* draw a circle. It is known as Mohr's Circle of Stresses.
5. Now through *C*, draw a line *CP* making an angle 2θ with *CE* in clockwise direction meeting the circle at *P*. The point *P* represents the stress system on section AB.
6. Through *P*, draw *PQ* perpendicular to the line *OX*. Join *OP*.
7. Now *OQ*, *QP* and *OP* will give the normal stress, shear stress and resultant stress respectively to the scale. Similarly *OG* and *OH* will give the maximum and minimum principal shear stresses to the scale. The angle *POC* is called the angle of obliquity.

Proof

From the geometry of the Mohr's Circle of Stresses, we find that

$$OC = \frac{\sigma_x + \sigma_y}{2}$$

and radius of the circle

$$R = EC = CD = CP = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Now in the right angled triangle *DCJ*

$$\sin\alpha = \frac{JD}{DC} = \frac{\tau_{xy}}{R} \quad \text{and} \quad \cos\alpha = \frac{JD}{DC} = \frac{\sigma_x - \sigma_y}{2} \times \frac{1}{R} = \frac{\sigma_x - \sigma_y}{2R}$$

Similarly in right angled triangle *CPQ*

$$\therefore \quad \angle PCQ = (2\theta - \alpha)$$

$$\begin{aligned} CQ &= CP \cos 2\theta - \alpha \\ &= R\,[\cos(2\theta - \alpha)] \\ &= R\,[\cos\alpha \cos 2\theta + \sin\alpha \sin 2\theta] \\ &= R\cos\alpha \cos 2\theta + R\sin\alpha \sin 2\theta \\ &= R \times \frac{\sigma_x - \sigma_y}{2R} \cos 2\theta + R \times \frac{\tau_{xy}}{R} \sin 2\theta \\ &= \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \end{aligned}$$

Normal Stress (across the inclined section *AB*)

$$\sigma_n = OQ = OC - CQ$$

or $$\sigma_n = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$ ...(Same as in Art. 7.11)

Shear Stress or Tangential Stress (across the inclined section *AB*)

$$\begin{aligned} \tau &= QP = CP \sin[(2\theta - \alpha)] = R\sin(2\theta - \alpha) \\ &= R(\cos\alpha \sin 2\theta - \sin\alpha \cos 2\theta) \\ &= R\cos\alpha \sin 2\theta - R\sin\alpha \cos 2\theta \\ &= R \times \frac{\sigma_x - \sigma_y}{2R} \sin 2\theta - R \times \frac{\tau_{xy}}{R} \cos 2\theta \end{aligned}$$

or $$\tau = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta - \tau_{xy} \cos 2\theta$$ ...(Same as in Art. 7.11)

Maximum Principal Stress

$$\sigma_{max} = OG = OC + CG = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Minimum Principal Stress

$$\sigma_{min} = OH = OC - CH = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

We also find the maximum shear stress will be equal to the radius of the Mohr's circle of Stresses.

i.e., $\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$. It will happen when $(2\theta - \alpha)$ is equal to 90° or 270°.

However when $(2\theta - \alpha) = 90°$ then the shear stress is equal to $+\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$.

And when $(2\theta - \alpha) = 270°$ then the shear stress is equal to $-\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$.

EXAMPLE 7.20. *A point is subjected to a tensile stress of 250 MPa in the horizontal direction and another tensile stress of 100 MPa in the vertical direction. The point is also subjected to a simple shear stress of 25 MPa, such that when it is associated with the major tensile stress, it tends to rotate the element in the clockwise direction. What is the magnitude of the normal and shear stresses inclined on a section at an angle of 20° with the major tensile stress ?*

***SOLUTION.** Given : Tensile stress in horizontal direction (σ_x) = 250 MPa ; Tensile stress in vertical direction (σ_y) = 100 MPa ; Shear stress (τ) = 25 MPa and angle made by section with major tensile stress (θ) = 20°.

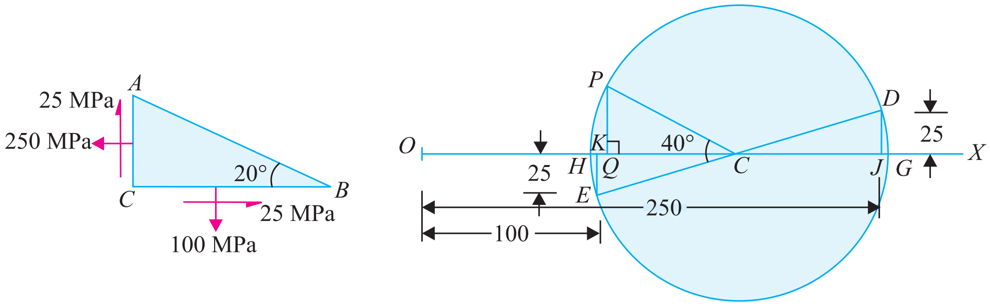


Fig. 7.27

The given stresses on the face *AC* of the point alongwith a tensile stress on the plane *BC* and a complimentary shear stress on the plane *BC* are shown in Fig 7.27 (*a*). Now draw the Mohr's Circle of Stresses as shown in Fig. 7.27 (*b*) and as discussed below :

1. First of all, take some suitable point *O*, and through it draw a horizontal line *OX*.
2. Cut off *OJ* and *OK* equal to the tensile stresses σ_x and σ_y respectively (*i.e.*, 250 MPa and 100 MPa) to some suitable scale towards right.
3. Now erect a perpendicular at *J* above the line *OX* and cut off *JD* equal to the positive shear stress on the plane *AC* (*i.e.,* 25 MPa) to the scale. The point *D* represents the stress system on the plane *AC*. Similarly, erect a perpendicular at *K* below the *OX* and cut off *KE* equal to the negative shear stress on the plane *BC* (*i.e.*, 25 MPa) to the scale. The point *E* represents the stress system on the plane *BC*. Join *DE* and bisect it at *C*.
4. Now with *C* as centre and radius equal to *CD* or *CE* draw the Mohr's Circle of Stresses.
5. Now through *C* draw a line *CP* making an angle of 2 × 20° = 440° with *CE* in clockwise direction meeting the circle at *P*. The point *P* represents the stress system on the section to *AB*.
6. Through *P*, draw *PQ* perpendicular to the line *OX*.

   By measurement, we find that the normal stress, (σ_x) = *OQ* = 101.5 MPa and shear stress τ = *QP* = 29.0 MPa **Ans.**

EXAMPLE 7.21. *A plane element in a boiler is subjected to tensile stresses of 400 MPa on one plane and 150 MPa on the other at right angle to the former. Each of the above stresses is accompanied by a shear stress of 100 MPa such that when associated with the major tensile stress tends to rotate the element in an anticlockwise direction. Find (a) Principal stresses and their directions. (b) Maximum shearing stresses and directions of the plane on which they act.*

* We have already solved this question analytically, as example 7.10.

***Solution.** Given : Tensile stress along horizontal *x-x* axis (σ_x) = 400 MPa ; Tensile stress along vertical *y-y* axis (σ_y) = 150 MPa and Shear stress (τ_{xy}) = – 100 MPa (Minus sign due to anticlockwise on *x-x* axis).

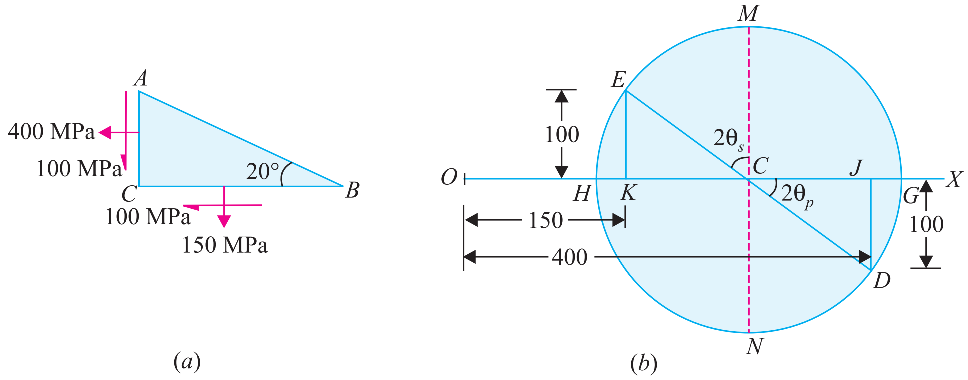


Fig. 7.28

The given stresses on the plane *AC* and *BC* of the element along with a complimentary shear stress on the plane *BC* are shown in Fig. 7.28 (*a*). Now Draw the Mohr's Circle of Stresses as shown in Fig 7.28 (*b*) and as discussed below :

1. First of all, take some suitable point *O*, and draw a horizontal line *OX*.
2. Cut off *OJ* and *OK* equal to the tensile stresses σ_x and σ_y respetively (*i.e*,. 400 MPa and 150 MPa) to some suitable scale towards right.
3. Now erect a perpendicular at *J* below the line *OX* and cut off *JD* equal to the negative shear stress on the plane *AC* (*i.e.*, 100 MPa) to the scale. The point *D* represents the stress systems on the plane *AC*. Similarly, erect a perpendicular at *K* above the line *OX* and cut off *KE* equal to the positive shear stress on the plane *BC* (*i.e.*, 100 MPa) to the scale. The point *E* represents the stress system on the plane *BC*. Join *DE* and bisect it at *C*.
4. Now with *C* as centre and radius equal to *CD* or *CE* draw the Mohr's Circle of Stresses meeting the line *OX* at *G* and *H*.
5. Through *C* draw two lines *CM* and *CN* at right angles to the line *OX* meeting the circle at *M* and *N*.

By measurement, we find that maximum principal stress (σ_{max}) = *OG* = 435.0 MPa ; Minimum principal stress (σ_{min}) = *OH* = 115.0 MPa ; By measurement ∠*JCD* therefore angle which the plane of principal stress makes with *x-x* axis (θ_p) = $\dfrac{\angle JCD}{2} = \dfrac{38.66°}{2}$ = 19.33° ; Maximum shearing stress (τ_{max}) = *CM* = 160.0 MPa ; By measurement ∠ *MCE* = $2\theta_s$ = 51.34°, therefore angle which the plane of maximum shearing stress makes with *x-x* axis (θ_s) = $\dfrac{51.34°}{2}$ = 25.7° **Ans.**

* We have already solved this question analytically, as example 7.11.

Example 7.22. *A point in a strained material is subjected to the stresses as shown in Fig. 7.29. Find graphically, or otherwise, the normal and shear stresses on the section AB.*

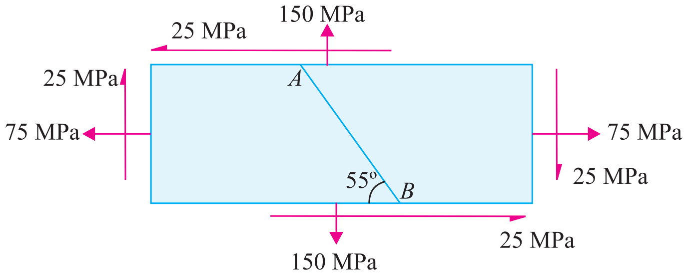


Fig. 7.29

***Solution.** Given : Tensile stress along horizontal *x-x* axis (σ_x) = 75 MPa ; Tensile stress along vertical *y-y* axis (σ_y) = 150 MPa ; Shear stress (τ_{xy}) = 25 MPa and angle made by section with horizontal tensile stress in clockwise direction (θ) = 55°.

The given stresses on the planes *AC* and *BC* are shown in Fig.7.30 (*a*). Now draw the Mohr's Circle of Stresses as shown in Fig. 7.30 (*b*) and as discussed below :

1. First of all, take some suitable point *O*, and draw a horizontal line *OX*.
2. Cut off *OJ* and *OK* equal to the tensile stresses σ_x and σ_y respectively (*i.e.*,75 MPa and 150 MPa) to some suitable scale towards right.

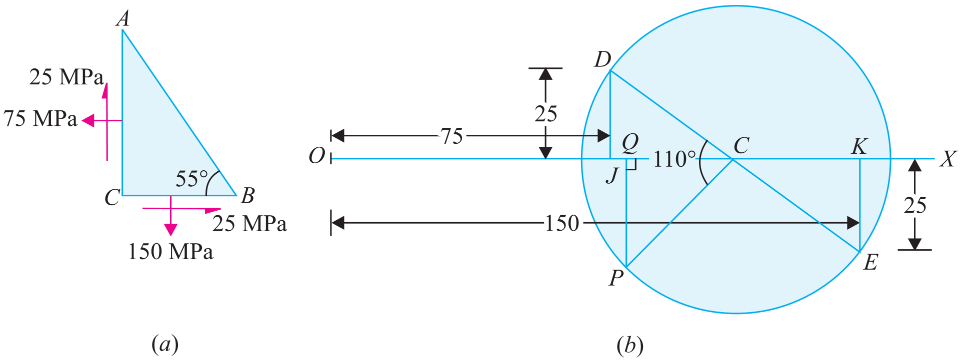


Fig. 7.30

3. Now erect a perpendicular at J above the line *OX* and cut off *JD* equal to the positive shear stress on the plane *AC* (*i.e.,* 25 MPa) to the scale. The point *D* represents the stress system on the plane *AC*. Similarly, erect a perpendicular at *K* below the line *OX* and cut off *KE* equal to the negative shear stress on the plane *BC* (*i.e.*, 25 MPa) to the scale. The point *E* represents the stress system on the plane *BC*. Join *DE* and bisect it at *C*.
4. Now with *C* as centre and radius equal to *CD* or *CE* draw the Mohr's Circle of Stresses.
5. Now through *C* draw a line *CP* making an angle of 2 × 55° = 110° with *CD* in an anticlockwise direction meeting the circle at *P*. The point *P* represents the stress system on the section *AB*.

   By measurement, we find that the normal stress (σ_n) = *OQ* = 76.1 MPa and shear stress (τ) = *PQ* = – 26.7 MPa. **Ans.**

* We have already solved this question analytically, as example 7.12.

EXAMPLE 7.23. *A plane element of a body is subjected to a compressive stress of 300 MPa in x-x direction and a tensile stress of 200 MPa in the y-y direction. Each of the above stresses is subjected to a shear stress of 100 MPa such that when it is associated with the compressive stress, it tends to rotate the element in an anticlockwise direction.*

Find graphically, or otherwise, the normal and shear stresses on a plane inclined at an angle of 30° with the x-x axis.

***SOLUTION.** Given : Compressive stress in *x-x* direction (σ_x) = – 300 MPa (Minus sign due to compressive). Tensile stress in *y-y* direction (σ_y) = 200 MPa ; Shear stress (τ_{xy}) = 100 MPa (Minus sign due to anticlockwise direction along the compressive stress *i.e.*, σ_x) and angle of plane with *x-x* axis (θ) = 30°.

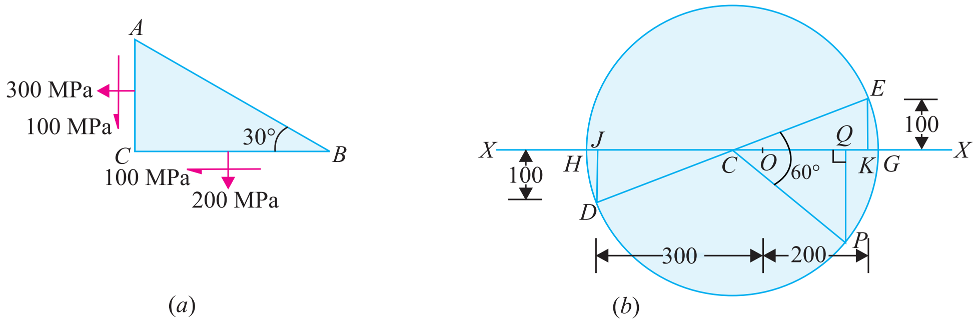


Fig. 7.31

The given stresses on the plane *AC* of the element alongwith a tensile stress on the plane *BC* and a complimentary shear stress on the plane *BC* are shown in Fig. 7.31 (*a*). Now draw the Mohr's Circle of Stresses as shown in Fig. 7.31 (*b*) and as discussed below :

1. First of all, take some suitable point *O*, and through it draw horizontal line *XOX*.
2. Cut off *OJ* and *OK* equal to the stresses σ_x and σ_y respectively (*i.e.*, – 300 MPa and 200 MPa) to some suitable scale such that *J* is towards left (because of compressive) and *K* is towards right (because of tensile).
3. Now erect a perpendicular at *J* below the line *XOX* and cut off *JD* equal to the negative shear stress on the plane *AC* (*i.e.*, 100 MPa) to the scale. The point *D* represents the stress system on the plane *AC*. Similarly, erect a perpendicular at *K* above the line *XOX* and cut off *KE* equal to the positive shear stress on the plane *BC* (*i.e.*, 100 MPa) to the scale. The point *E* represents the stress system on the plane *BC*. Join *DE* and bisect it at *C*.
4. Now with *C* as centre and radius equal to *CD* or *CE* draw the Mohr's Circle of Stresses.
5. Now through *C* draw a line *CP* making an angle of 2 × 30° = 60° with *CE* in clockwise direction meeting the circle at *P*. The point *P* represents the stress system on plane *AB*.
6. Through, *P*, draw *PQ* perpendicular to the line *OX*.

   By measurement, we find that the normal stress (σ_n) = *OQ* = 161.6 MPa ; and shear stress (τ) = *QP* = – 166.5 MPa **Ans.**

EXAMPLE 7.24. *A machine component is subjected to the stresses as shown in Fig. 7.32.*

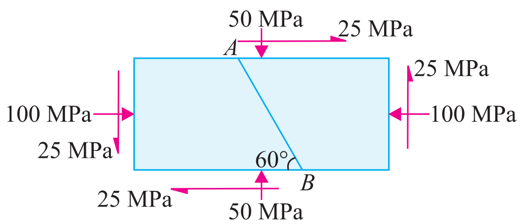


Fig. 7.32

* We have already solved this question analitically, as example 7.13.

Find the normal and shearing stresses on the section AB inclined at an angle of 60° with x-x axis. Also find the resultant stress on the section.

***Solution.** Given : Compressive stress along horizontal x-x axis (σ_x) = – 100 MPa (Minus sign due to compressive) ; Compressive stress along vertical y-y axis (σ_y) = – 50 MPa (Minus sign due to compressive) ; Shear stress (τ_{xy}) = – 25 MPa (Minus sign due to anticlockwise on x-x axis and angle between section and horizontal x-x axis (θ) = 60°.

The given stresses on the planes *AC* and *BC* are shown in Fig. 7.33 (*a*). Now draw the Mohr's Circle of Stresses as shown in Fig. 7.33 (*b*) and as discussed below :

1. First of all, take some suitable point *O* and through it draw a horizontal line, such that *X* is towards left. (because both the stress are compressive)
2. Cut off *OJ* and *OK* equal to the compressive stresses σ_x and σ_y respectively (*i.e.,* –100 MPa and –50 MPa) to some suitable scale towards left.

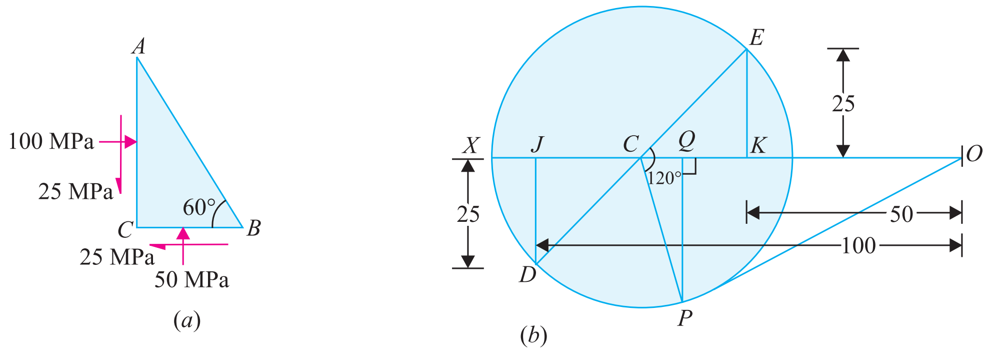


Fig. 7.33

3. Now erect a perpendicular at *J* below the line *XO* and cut off *JD* equal to the negative shear stress on the plane *AC* (*i.e.*, 25 MPa) to the scale. The point *D* represents the stress system on the plane *AC*. Similarly, erect a perpendicular at *K* above the line *XO* and cut off *KE* equal to the positive shear stress on the plane *BC* (*i.e.*, 25 MPa) to the scale. The point *E* represents the stress system on the plane *BC*. Join *DE* and bisect it at *C*.
4. Now with *C* as centre and radius equal to *CD* or *CE* draw the Mohr's Circle of Stresses.
5. Now through *C*, draw a line *CP* making an angle of 2 × 60° = 120° with *CE* in clockwise direction meeting the circle at *P*. The point *P* represents the stress system on the section *AB*.
6. Through *P*, draw *PQ* perpendicular to the line *XO*. Join *OP*.

   By measurement, we find that the normal stress (σ_n) = *OQ* = – 65.8 MPa ; Shear stress (τ) = QP = – 34.1 MPa and resultant stress (σ_R) = *OP* = 74 MPa **Ans.**

EXERCISE 7.2

1. At a point in a strained material, the principal stresses are 100 MPa and 50 MPa both tensile. Find the normal and shear stresses at a section inclined at 60° with the axis of the major principal stress. (**Ans.** 87.5 MPa ; 21.65 MPa)
2. A point in a strained material is subjected to a tensile stress of 120 MPa and a clockwise shear stress of 40 MPa. What are the values of normal and shear stresses on a plane inclined at 25° with the normal to the tensile stress. (**Ans.** 20 MPa ; 60 MPa)

* We have already solved this question analytically, as example 7.14.

3. The principal stresses at a point in the section of a member are 50 MPa and 20 MPa both tensile. If there is a clockwise shear stress of 30 MPa, find graphically the normal and shear stresses on a section inclined at an angle of 15° with the normal to the major tensile stress.
(**Ans.** 32.99 MPa ; 33.48 MPa)

4. A point is subjected to tensile stresses of 200 MPa and 150 MPa on two mutually perpendicular planes and an anticlockwise shear stress of 30 MPa. Determine by any method the values of normal and shear stresses on a plane inclined at 60° with the minor tensile stress.
(**Ans.** 188.48 MPa ; 36.65 MPa)

5. At a point in a stressed element, the normal stresses in two mutually perpendicular directions are 45 MPa and 25 MPa both tensile. The complimentary shear stress is these directions is 15 MPa. By using Mohr's circle method, or otherwise, determine the maximum and minimum principal stresses. (**Ans.** 188.48 MPa ; 36.65 MPa)

QUESTIONS

1. Define principal planes and principal stresses and explain their uses.
2. Derive an expression for the stresses on an oblique section of a rectangular body, when it is subjected to (*a*) a direct stress in one plane only and (*b*) direct stresses in two mutually perpendicular directions.
3. Obtain an expression for the major and minor principal stresses on a plane, when the body is subjected to direct stresses in two mutually perpendicular directions accompanied by a shear stress.
4. How will you find out graphically the resultant stress on an oblique section when the body is subjected to direct stresses in two mutually perpendicular directions?

OBJECTIVE TYPE QUESTIONS

1. When a body is subjected to a direct tensile stress (σ) in one plane, then normal stress on an oblique section of body inclined at an angle to the normal of the section is equal to

(*a*) $\sigma \sin \theta$ (*b*) $\sigma \cos \theta$ (*c*) $\sigma \sin^2 \theta$ (*d*) $\sigma \cos^2 \theta$

2. When a body is subjected to a direct tensile stress (σ) in one plane, then the tangential stress on an oblique section of the body inclined at an angle (θ) to normal of the section is equal to

(*a*) $p \sin 2\theta$ (*b*) $p \cos 2\theta$ (*c*) $\frac{P}{2}\sin 2\theta$ (*d*) $\frac{P}{2}\cos 2\theta$

3. When a body is subjected to a direct tensile stress (σ) in one plane and accompanied by a single shear stress (τ), the maximum normal stress is

(*a*) $\frac{\sigma}{2}+\frac{1}{2}\sqrt{\sigma^2+4\tau^2}$ (*b*) $\frac{\sigma}{2}-\frac{1}{2}\sqrt{\sigma^2+4\tau^2}$

(*c*) $\frac{\sigma}{2}+\sqrt{\sigma^2-4\tau^2}$ (*d*) $\frac{\sigma}{2}-\frac{1}{2}\sqrt{\sigma^2-4\tau^2}$

4. When a body is subjected to the mutually perpendicular stress (σ_x and σ_y) then the centre of the Mohr's circle from y-axis is taken as

(*a*) $\frac{\sigma_x+\sigma_y}{2}$ (*b*) $\frac{\sigma_x-\sigma_y}{2}$ (*c*) $\frac{\sigma_x-\sigma_y}{2}+\tau_{xy}$ (*d*) $\frac{\sigma_x-\sigma_y}{2}-\tau_{xy}$

ANSWERS

1. (*d*) **2.** (*c*) **3.** (*a*) **4.** (*b*)