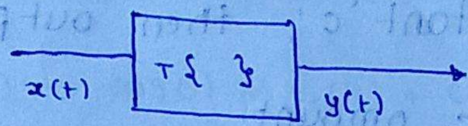
11/11/24

Unit 2 : Systems

* Continuous Time systems (CTS) is one in which continuous Time input signals are transformed into continuous time output signals.



CTS

* Discrete Time systems (DTS) is one in which discrete time input signals are transformed into discrete output signals.

→ Classification of systems :

① Linear and non-linear System :

* To check the linearity of system ...

→ A system is said to be linear if it satisfies two properties ...

(i) Additivity : It states that, if an input $x_1(t)$, produces output, $y_1(t)$ & and another input $x_2(t)$ produces output $y_2(t)$, when both the inputs acting on the system simultaneously, produces output $y_1(t) + y_2(t)$.

$$\left\{ \begin{array}{l} x_1(t) \to y_1(t) \\ x_2(t) \to y_2(t) \end{array} \right\} \to [x_1(t) + x_2(t) \to y_1(t) + y_2(t)]$$

(ii) Scaling :- It states that, if input is scaled by constant 'c', then output is also scaled by same amount.

$x(t) \to y(t)$ $\qquad$ $x[n] \to y[n]$

&

$c\,x(t) \to c\,y(t)$ $\qquad$ $c \cdot x[n] \to c \cdot y[n]$.

Q Ex : 1) $y(t) = a\,x(t)$

$x(t) = \alpha x_1(t) + \beta x_2(t)$ {let}

$y(t) = a\alpha x_1(t) + a\beta x_2(t) \to (1)$

$x_1 \to y_1(t) = a\,x_1(t)$

$\qquad y_2(t) = a\,x_2(t)$

$y(t) = \alpha y_1(t) + \beta y_2(t) \to$

So, the system is linear.

Q Ex 2) : $y(t) = a\,x(t) + b$

Let $x(t) = \alpha x_1(t) + \beta x_2(t) \to (1)$

$y(t) = a\alpha x_1(t) + a\beta x_2(t) + b$

$x_1 \to y_1(t) = a x_1(t) + b$

$\cdot y_2(t) = a x_2(t) + b$

$y(t) = \alpha y_1(t) + \beta y_2(t) \neq b$ {oka b thakkuva ayindi}

So, the system is non linear. {input = output + $\underline{\underline{b}}$}

Q Ex3: $y(t) = x(t)\cos 5t$.

A Let $x(t) = \alpha x_1(t) + \beta x_2(t)$ $\rightarrow$ ①

$\rightarrow y(t) = \cos 5t\, \alpha x_1(t) + \cos 5t\, \beta x_2(t)$

$\rightarrow y_1(t) = x_1(t)\cos 5t$ & $y_2(t) = x_2(t)\cos 5t$

$\rightarrow y(t) = \alpha y_1(t) + \beta y_2(t)$ $\rightarrow$ ②

It is linear system.

Q Ex4: $y(t) = x(t)\, x(t-4)$

A Let $x(t) = \alpha x_1(t) + \beta x_2(t)$

$\rightarrow y(t) = (\alpha x_1(t) + \beta x_2(t))(\alpha x_1(t-4) + \beta x_2(t-4))$

$\rightarrow y(t) = \alpha^2 x_1(t)\, x_1(t-4) + \alpha\beta\, x_1(t) + x_2(t-4)$

$+ \alpha\beta\, x_2(t) \cdot x_1(t-4) + \beta^2 x_2(t)\, x_2(t-4)$

$\rightarrow y_1(t) = x_1(t)\, x_1(t-4)$

$\rightarrow y_2(t) = x_2(t)\, x_2(t-4)$

$y(t) \neq y_1(t) + y_2(t)$

So, it is non linear.

Q Ex-5: $y[n] = x^2[n]$

$x[n] = \alpha x_1[n] + \beta x_2[n]$ {let}

$$y[n] = (\alpha x_1[n] + \beta x_2[n])^2$$

$$= \alpha^2 x_1^2[n] + \beta^2 x_2^2[n] + 2\alpha\beta\, x_1[n] \cdot x_2[n].$$

$$y_1[n] = x_1^2[n]$$

$$y_2[n] = x_2^2[n]$$

$\{ y[n] = \alpha^2 y_1[n] + \beta^2 y_2[n] + \ldots ? \}$

$$y[n] \neq \alpha y_1[n] + \beta y_2[n]$$

Q Ex 6 : $y(t) = |x(t)|$

Ex. $|5x(t)| = 5x(t)$

$|-5x(t)| = 5x(t)$ { -5 tho multiply chesthe +5 vas thund, }

For different types of inputs, we get same output. So It is non-linear.

Q Ex 7 : $y[n] = 2^{x[n]}$

A let $x[n] = x_1[n] + x_2[n] \rightarrow (1)$

$$y[n] = 2^{x_1[n] + x_2[n]}$$

$$= 2^{x_1[n]} \cdot 2^{x_2[n]}$$

$$y_1[n] = 2^{x_1[n]}$$

$$y_2[n] = 2^{x_2[n]}$$

$y[n] = 2^{y_1[n]} \cdot 2^{y_2[n]} \rightarrow (2)$

It is non linear.

Q Ex 8 : $y[n] = x^*[n]$

A $y[n] = \left((2+i3)\, x[n] \right)^*$ $\{$let $x[n] = (2+i3)\, x[n]\}$

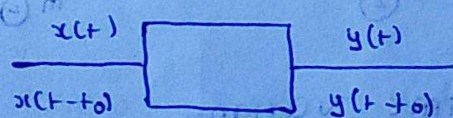
$y[n] = (2-i3)\, x^*[n]$

$\alpha x[n] \neq \alpha y[n]$

So, it is non linear.

2) Time invariant & Time variant systems:

~~If~~ a system is time invariant, if input & output characteristics doesnot change with time. The output signal is due to delayed input is equal to delayed version of output



$$y(t)\Big|_{x(t-t_0)} = y(t)\Big|_{t-t_0=t}$$

Ex ① $y(t) = x^2(t)$

$y_1(t) = x^2(t-t_0) \rightarrow ①$ $\quad x(t)$ $t-t_0)$

$y_2(t) = x^2(t-t_0) \rightarrow ②$ $\quad (t-t_0)$

$y_1(t) = y_2(t)$ is TIV

Ex ② $y(t) = t \cdot x(t)$ $\quad t \cdot x(t)$

$y_1(t) = t\, x(t-t_0) \rightarrow ①$

$y_2(t) = (t-t_0)\, x(t-t_0) \rightarrow ②$

$1 \neq 2$ Time Variant.

③ $y(t) = x(t)\cos\omega_0 t$

$y_1(t) = x(t-t_0)\cos\omega_0 t \rightarrow ①$

$y_2(t) = x(t-t_0)\cos\omega_0(t-t_0) \rightarrow ②$

$① \neq ②$

So, Time variant.

$x(at)$
$x(at - t_0)$
$x(a(t-t_0))$

④ $y(t) = x(2t)$

{ $x(t)$ ni t_0 units delay cheyalin in ①

$y_1(t) = x(2t - t_0) \rightarrow ①$

{ $x(t)$ lo, t ni t_0 units delay chayali in ②

$y_2(t) = x(2(t-t_0)) \rightarrow ②$

$① \neq ②$ So, Time variant

⑤ $y(t) = \begin{cases} x(t) & t > 0 \\ 0 & t < 0 \end{cases}$

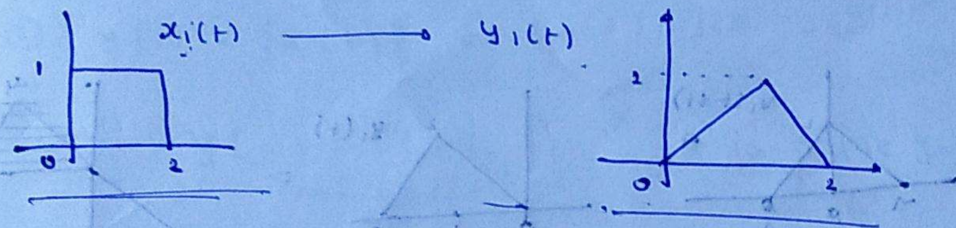
$y_1(t) = \begin{cases} x(t-t_0) & t > 0 \\ 0 & t < 0 \end{cases} \rightarrow ①$

$y_2(t) = \begin{cases} x(t-t_0) & t-t_0 > 0 \\ 0 & t-t_0 < 0 \end{cases} \rightarrow ②$

$① \neq ②$ Time variant.

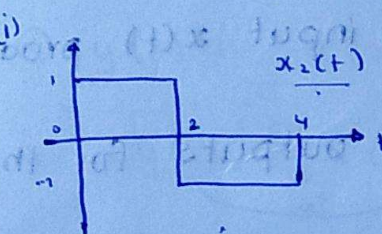
Q Consider an LTAI system whose response to the input signal is, $x_1(t) \longrightarrow y_1(t)$.

Find response of signal due to input signal $x_1(t)$

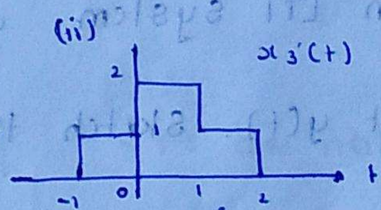


Now draw $y_2(t)$ & $y_3(t)$ for $x_2(t)$ & $x_3(t)$

(i)



(ii)

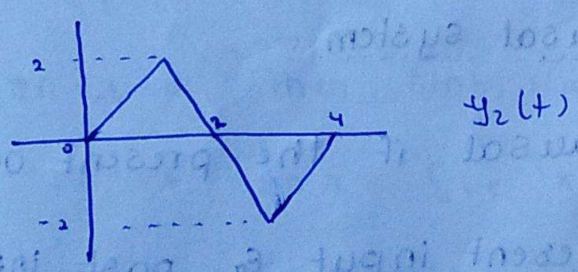


(i)

A We know $x_2(t) = x_1(t) - x_1(t-2)$

As x_1 & x_2 are linear Time invariant, the output will be $y_1(t) - y_1(t-2) = y_2(t)$

so, $y_2(t) = y_1(t) - y_1(t-2)$



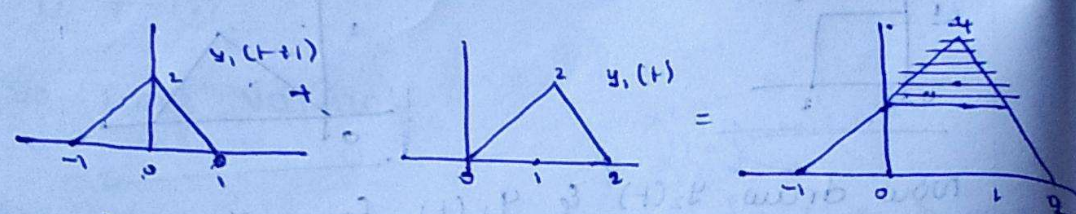
$$x_2(t) = x_1(t) - x_1(t-2) \longrightarrow \boxed{\text{LTI}}^{\,y_1(t)} \longrightarrow y_2(t) = y_1(t) - y_1(t-2)$$

(ii) $\cancel{x_3}(\cancel{t}) = \cancel{x_1}(t+\cancel{1}) + \cancel{x}(t) - x(t-1) - x(t-2)$

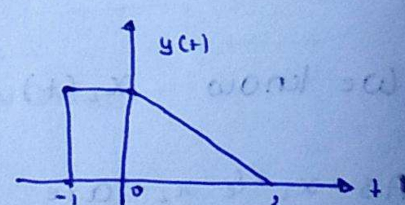
(ii) $x_3(t) = x_1(t+1) + x_1(t)$

As $x_1(t+1)$ & $x_1(t)$ is linear, the output will be $y_1(t+1)$ & $y_1(t)$

$$y_3(t) = y_1(t+1) + y_1(t)$$



Q. For an LTI system, an input $x(t)$ produces output $y(t)$. Sketch the outputs for the following in inputs.



a) $5x(t)$

b) $x(t+1) - x(t-1)$

c) $\frac{d}{dt} x(t)$.

3) Causal & non-causal system :

A system is causal if the present output depends on present input & past input but not on future values. That is known as causal system.

* non-causal systems ~~are~~ donot exist.

Ex ① $y(t) = (3t+5)\,x(t)$.

It is causal, If $t = 0$

$y(0) = (3t+5)\,x(0)$.

② $y(t) = |x(t)|$

$t=-1$, $y(-1) = |x(-1)|$ { t is same }

It is causal

③ $y(t) = |x(2t)|$

$t=1 \rightarrow y(1) = |x(2)|$ { t's are not same }

It is non-causal.

④ $y(t) = \sin\{x(t)\}$

$t=1 \rightarrow y(1) = \sin\{x(1)\}$ { t's are same }

It is causal

⑤ $y(t) = x\{\sin t\}$

If $y(t=0)$, then $\sin t = 0$

i.e $t = \pm n\pi$ { i.e $\pi, 2\pi, 3\pi, 4\pi$ }

$\pi, 2\pi, 3\pi \ldots$ are future values of t

it is non causal.

⑥ $y[n] = \sum_{k=0}^{n} x[k]$

$$y[1] = \sum_{k=0}^{1} x[k]$$

$$y[1] = \underset{\text{past input}}{x[0]} + \underset{\text{present input}}{x[1]}$$ {satisfies causal}

But $$y[-1] = \sum_{k=0}^{-1} x[k]$$

$$y[-1] = \underset{\text{future input}}{x[0]} + \underset{\text{present input}}{x[-1]}$$

It depends on future input, so

It is non causal.

(6) $$y[n] = \sum_{k=-\infty}^{n} x[k]$$

$$y[1] = \overbrace{x[-\infty] + \cdots x[-1]}^{\text{Past values}} + x(0) + x[1]$$

$$y[-1] = \underbrace{\cdots}_{\text{past values}} + x[-1]$$

It is causal.

(7) $$y[n] = \sum_{k=n-n_0}^{n+n_0} x[k]$$ {n_0 is finite}

$$y[n] = \sum_{k=n-1}^{n+1} x[k]$$ {$n_0 = 1$} let

$$y[n] = \underset{\text{Past}}{x[n-1]} + \underset{\text{present}}{x[n]} + \underset{\text{future}}{x[n+1]}$$

It depends on future values, so it is non-causal.

4) Stable & unstable system :

* A system is stable if for every bounded input, there is an Bounded output

Ex $u(t)$, $sgn(t)$, $u[n]$, $\delta[n]$

Ex ① $y(t) = x^2(t)$

A let $x(t) = u(t)$ Bounded input.

Now $x^2(t)$ is also $u(t)$ Bounded output

So, system is stable.

② $y(t) = t \cdot x(t)$.

$y(t) = t \cdot u(t) = r(t)$ {let $x(t) = u(t)$ bounded input}

{unbounded output}

So, It is unstable

③ $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

let $x(\tau) = u(\tau)$ bounded input

$\int_{-\infty}^{t} x(\tau)\, d\tau = r(\tau)$ unbounded output

So, it is unstable.

④ $y[n] = e^{x[n]}$

let $x[n]$ is finite

$y[n] = e^{finite}$ is finite

So It is stable.

⑤ $y[n] = \sum_{k=1}^{n} x[n]$

$y[2] = x[1] + x[2]$ $\{$ let $x[n] = u[n]$ $\}$

$y[2] = 2$

$y[3] = 3$

$y[4] = 4$ unbounded output.

It is unstable.

5) Static and dynamic systems:

* A system is static if input & output are observed at same instant of time., is static.

(i) $y(t) = e^{-3t} x(t)$.

$\underbrace{y(1) = e^{-3(1)} x(1)}$. same

It is static

(ii) $y(t) = g(3t+2)\, x(t)$

$y(1) = g(5)\, x(1)$ $\{$ x & y depends on same time

So it is static.

(iii) $y[n] = x[n^2]$

$\underbrace{y[2] = x[4]}_{\text{different}}$

It is dynamic.

* All static systems are causal, but all causal

systems are not static..

* Invertable & non invertable systems:

$$x(t) \rightarrow \boxed{T} \xrightarrow{y(t)} \boxed{T^{-1}} \rightarrow x(t)$$

it is invertible.

* A system is invertible if different inputs leads to different outputs. , i.e 2 different inputs for given system, should not produce same output.

1) $y(t) = x^2(t)$

Let $x(t) = 2$ $\quad x^2(t) = 4$

$$\left.\begin{array}{l} -2^2 = 4 \\ 2^2 = 4 \end{array}\right\}$$ same output so it is non invertable.

2) $y(t) = |x(t)|$

$$\left.\begin{array}{l} -2 = 2 \\ 2 = 2 \end{array}\right\}$$ same output, non invertable.

3) $y(t) = \int_{-t}^{t} x(\tau)\, d\tau$

for different inputs we get different outputs.

It is invertible

4) $y(t) = \frac{d}{dt} x(t)$

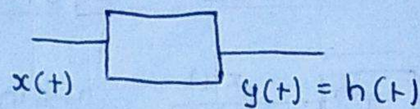
$$\left.\begin{array}{l} \frac{d}{dt}(2) = 0 \\ \frac{d}{dt}(3) = 0 \end{array}\right\}$$ same output, non invertable

5) $y[n] = x[n]\, x[n-3]$

$$\left.\begin{array}{l} \rightarrow x[n] \overset{u[n]}{=} u[n]\, u[n-3] \\ \rightarrow x[n] = -u[n] = u[n]\, u[n-3] \end{array}\right\}$$ same

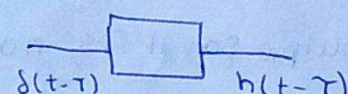
non - invertable.

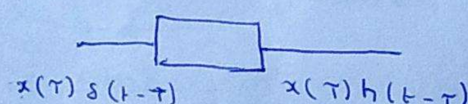
* LTI Systems : {linear time invariant systems}

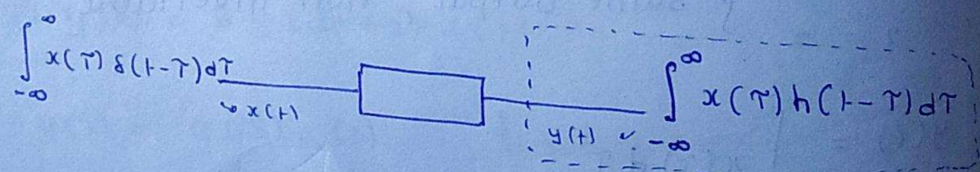


* If input is impulse, then the output is impulse response $(h(t))$ & If input is step, then the output is step response

* If The system is LTI, & the input is deleayed by 'τ' units, then the output is delayed by 'τ'.







convolution formula.

* $y(t) = x(t) * h(t)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$$

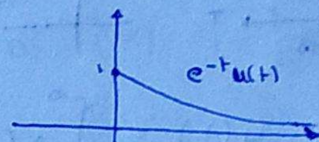
$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$$

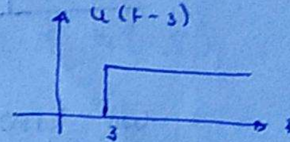
} convolution formulas.

Graphical method:

1) $x(t) = e^{-t}u(t)$, $h(t) = u(t-3)$

* We know $x(t) =$





→ Step1 : Calculate the limits of $y(t)$.

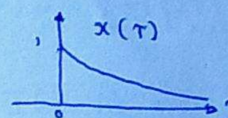
(i) sum of lower limits < t < sum of upper limits.

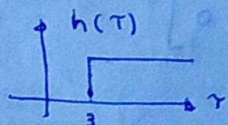
$0 + 3 < t < \infty + \infty$

$3 < t < \infty$

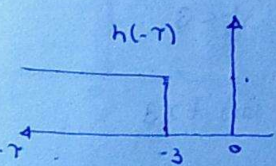
{ $y(t) = x(t) * h(t)$ convolution formula }

→ Step2 : Convert 't' domain to 'τ' domain. (t to τ)



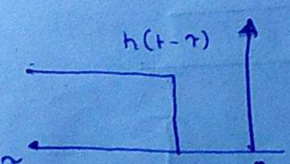


→ Step 3 : Time reversal of $h(\tau)$ i.e obtain $h(-\tau)$



This is known as folding or flipping operation.

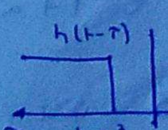
→ Step4 : Shifting 't' units of $h(-\tau)$ i.e $h(t-\tau)$

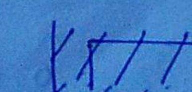


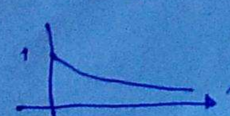
→ Step5 : multiplication.

$y(t) = \{ 0 \quad t-3 < 0$

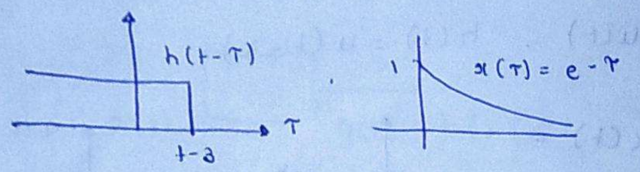
→ ①







If $t-3$ is positive.



we know $y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

Here limits are '0' to $(t-3)$

$$y(t) = \int_{0}^{t-3} e^{-\tau}(1)\, d\tau$$

$$y(t) = -\left[e^{-\tau}\right]_{0}^{t-3}$$

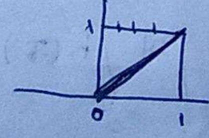
$$y(t) = -\left[e^{-t+3} - e^{0}\right]$$

$$y(t) = 1 - e^{-(t-3)} \rightarrow ②$$

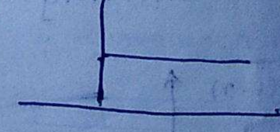
From ① & ②

$$\text{So, } y(t) = \begin{cases} 0 & t-3 < 0 \text{ (or) } t < 3 \\ 1 - e^{-(t-3)} & t-3 > 0 \text{ (or) } t > 3 \end{cases}$$

Q $x(t) =$



$h(t)$



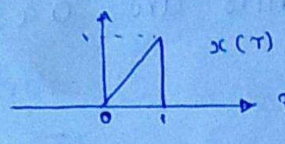
A * Calculate the limits.

Sum of Lower limits < t < sum of upper limits

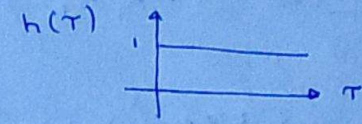
$0 + 0 < t < 1 + \infty$

$0 < t < \infty$ are the limits

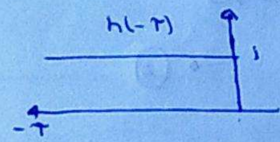
* Convert 't' into 'τ'



&

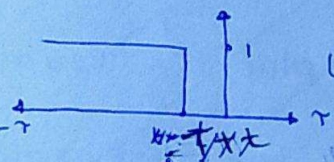


* Time reversal of $h(\tau)$ i.e obtain $h(-\tau)$



* Flipping

* Shifting i.e obtain $h(t-\tau)$.



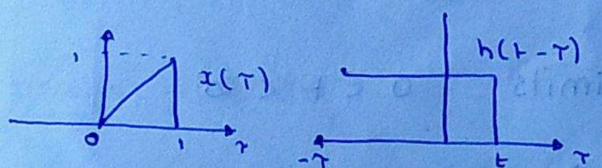
(t<0)

* Multiplication:

$0 < t < \infty$

If $t<0$, $y(t) = \{0, \quad t<0 \rightarrow ①$

If $t>0$,

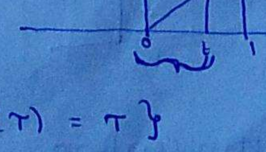


We know $y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

Case-1

* Here limits are 0 to t $\{t<1\}$ Case-i

$\int_0^t x(\tau)\, h(t-\tau)\, d\tau$

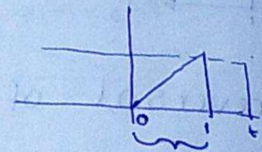


$\int_0^t \tau\,(1)\, d\tau$ $\qquad \{x(\tau) = \tau\}$

$\left(\frac{\tau^2}{2}\right)_0^t = \frac{t^2}{2} \rightarrow ②$

case-II : $t > 1$, then limits are 0 & 1

$$\int_0^1 x(\tau)\, h(t-\tau)\, d\tau$$

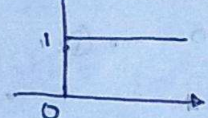


$$\int_0^1 \tau(1)\, d\tau = \left(\frac{\tau^2}{2}\right)_0^1 = \frac{1}{2} \longrightarrow ③$$

from ①, ②, & ③, we get

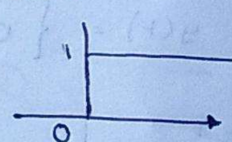
$$\boxed{y(t) = \begin{cases} 0 & ; \quad t<0 \\ t^2/2 & ; \quad t<1 \\ 1/2 & ; \quad t>1 \end{cases}}$$ //

③ $x(t)$



&

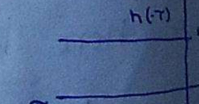
$h(t)$



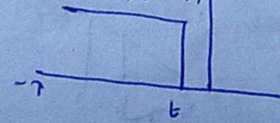
A Calculate the limits $0 < t < \infty$

Convert t into τ i.e $x(t) = x(\tau)$ & $h(t) = h(\tau)$

Time reversal of $h(\tau)$ ie obtain $h(-\tau)$



Shifting of t units. $h(t-\tau)$



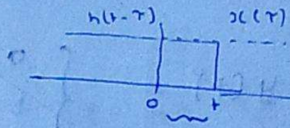
multiplication

If $t<0$, then $y(t) = 0 \quad t<0 \longrightarrow ①$

If $t>0$, then limits are 0 to t

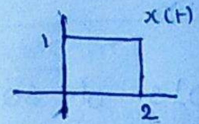
then $y(t) = \int_0^t 1(1)\,d\tau = [\tau]_0^t = t$ → ②

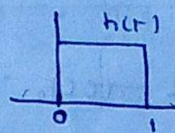
from ① & ②



$y(t) = \begin{cases} 0, & t<0 \\ t, & t>0 \end{cases}$ It is ramp signal.

---

④ 

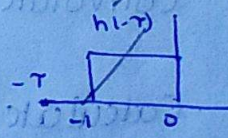


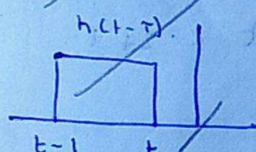
A ~~limits are $0+0 < t < 1+2$ → $0<t<3$~~

~~convert t into τ ie x(τ) & h(τ)~~

~~Time reversal of h(τ) ie obtain h(-τ)~~

~~Shifting of t units h(t-τ)~~



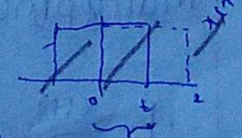


~~multiplication~~

~~If $t<0$, then $y(t)=0$ $t<0$~~ → ①

* ~~If $t>0$, & $t<2$, then limits are~~

~~0 to t.~~



→ ~~then $y(t) = \int_0^t x(\tau)\,h(t-\tau)\,d\tau$~~

~~$= \int_0^t 1(1)\,d\tau = [\tau]_0^t = t$~~ → ②

* ~~If $t>0$ & $t<2$, then limits are 0 to 2~~ & $t<3$

~~$y(t) = \int_0^2 x(\tau)\,h(t-\tau) = \int_0^2 1\,d\tau = [\tau]_0^2 = 2$~~ → ③

~~from ① ② & ③,~~

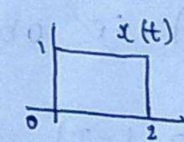
$$\cancel{y(t) = \begin{cases} 0 & ; t<0 \\ t & ; t\leq 2 \\ 2 & ; t>2 \end{cases}}$$

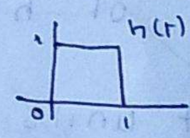
Also ~~t~~

---

* If we convolute 2 rect signals of same amplitude & same time interval, we get a triangle signal.

④ Convolute

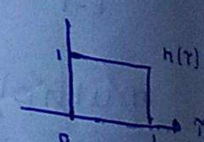




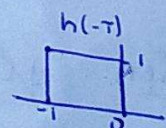
* Calculate the limits $0+0 < t < 2+1$

$\rightarrow 0 < t < 3$

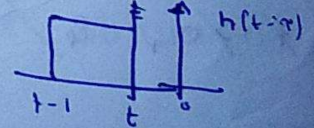
* Convert domain $t \approx \tau$ $h(t) \rightarrow h(\tau)$



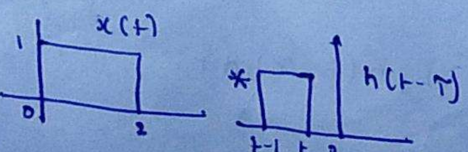
* Time reversal $h(-\tau) =$



* Time shifting $h(t-\tau) =$



* Multiplying



If $t<0$, $\rightarrow y(t) = 0 \rightarrow$ ①

If $t>0$

case i : $t<2$, so limits are 0 to t

$$\int_0^t x(t)\, h(t-\tau)\, d\tau = [\tau]_0^t = t-0 = t \rightarrow (2)$$

case-ii : $t<2$, so limits are $t-1$ to t

$$\int_t^{t-1} \tau(1)\, d\tau = [\tau]_t^{t-1} = t-1-t = -1 \rightarrow (3)$$

case-iii : $t<2$ & less than 3, so limits are

$t-1$ to 2

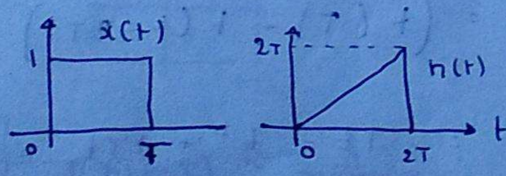
$$\int_{t-1}^{2} 1\, d\tau = [\tau]_{t-1}^{2} = 2-t+1 = 3-t \rightarrow (4)$$

If $t>0$, $y(t)=0$

signal is .. $y(t) = \begin{cases} 0 & t<0 \\ t & 0<t<1 \\ -1 & 1<t<2 \\ 3-t & 2<t<3 \\ 0 & t>0 \end{cases}$

is the resultant signal.

(5) convolute

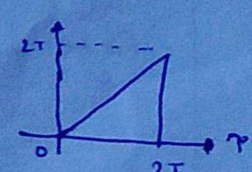


$h(t) = \begin{cases} t, & 0<t<2T \\ 0, & \text{ew} \end{cases}$

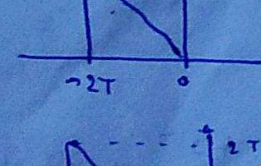
A Calculate the limits $0+0 < t < T+2T$

$0 < t < 3T$

* convert domain $t \approx \tau$, $h(t) \rightarrow h(\tau)$

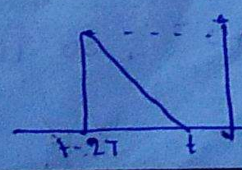


* Time reversal $h(-\tau) =$

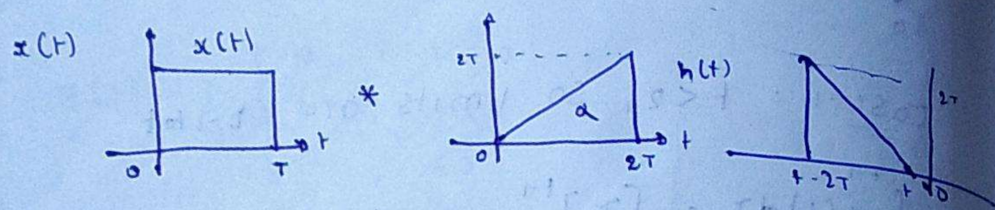


* Time shifting $h(t-\tau)$

$=$



Multiplication



If $t<0$, $y(t) = 0 \rightarrow (1)$

If $t>0$,

case-i $t<T$, so limits are 0 to t

$$\int_0^t x(t)\, h(t-\tau)\, d\tau = \int_0^t 1(t-\tau)\, d\tau = \left| t(\tau) - \frac{\tau^2}{2} \right|_0^t$$

$$\rightarrow t^2 - \frac{t^2}{2} = t^2\left(1-\frac{1}{2}\right) = \frac{t^2}{2} \rightarrow (2)$$

case-ii $t>T$ & less than $3T$ so, limits are $t-2T$ to T

$$\int_{t-2T}^{T} (t-\tau)\, d\tau = \left(t(\tau) - \frac{\tau^2}{2}\right)_{t-2T}^{T}$$

$$= \left(t(T) - t(t-2T)\right) - \left(\frac{T^2}{2} - \frac{(t-2T)^2}{2}\right)$$

$$= Tt - t^2 + 2Tt - \frac{T^2}{2} + \frac{t^2+4T^2-4Tt}{2}$$

$$= 3Tt - t^2 + \frac{t^2 + 3T^2 - 4Tt}{2}$$

$$= \frac{6Tt - 2t^2 + t^2 + 3T^2 - 4Tt}{2}$$

$$= \frac{2Tt + 3T^2 - t^2}{2}$$

$$= \frac{-1}{2}t^2 + \frac{3}{2}T^2 + tT \rightarrow (4)$$

case-i.5: $t>T$, so limits are 0 to T

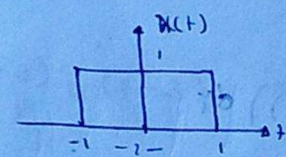
$$\left(t(\tau) - \frac{\tau^2}{2}\right)_0^T$$

$$Tt - \frac{T^2}{2} \to (3)$$

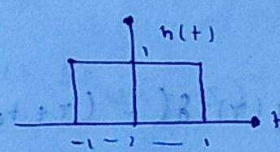
If $t > 3T$ $y(t) = 0 \to (4)$

$$y(t) = \begin{cases} 0 , & t < 0 \\ t^2/2 , & 0 < t < T \\ tT - T^2/2 & T < t < 2T \\ -\frac{1}{2}t^2 + \frac{3}{2}T^2 + tT & 2T < t < 3T \\ 0 & t > 3T \end{cases}$$

Q convolute

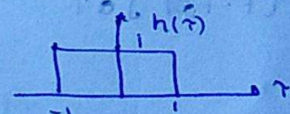


&

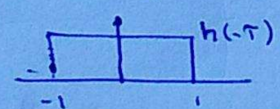


A Sum of limits $-1-1 < t < 1+1$ $= -2 < t < 2$

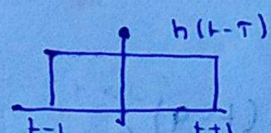
→ $h(t) = h(\tau)$



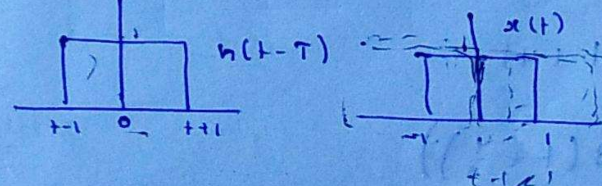
→ $h(-\tau) =$



→ $h(t-\tau) =$



Now



multiplication.

→ If $t+1 < -1$, $y(t) = 0$, → $t < -2$ → (1)

If $t > -2$, so limits are -1 to $t+1$.

$$\int_{-1}^{t+1} 1(1)\,d\tau = [\tau]_{-1}^{t+1} = t+1-(-1) = t+2 \to (2)$$

If $t > -2$, so limits are $t-1$ to 1

$$[\tau]_{t-1}^{1} = 1-t+1 = 2-t$$

$t > 2,\ y(t) = 0$

So $y(t) = \begin{cases} 0, & t < -2 \\ t+2, & -2 < t < 0 \\ 2-t, & 0 < t < 2 \\ 0, & t > 2 \end{cases}$

* Convolution property:

① $x(t) * \delta(t-t_0) = x(t-t_0)$

$$\int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$\int_{-\infty}^{\infty} x(\tau)\, \delta(-(\tau + t_0 - t))\, d\tau$$

$$\int_{-\infty}^{\infty} x(\tau)\, \delta\{\tau - (t-t_0)\}\, d\tau$$

$= x(t-t_0)$.

→ 2) Ex $x(t+5) * \delta(t-14)$

$= x((t-14)+5) = x(t-9)$ //

→ 2) $x(t) \cdot \delta(3t+4)$

$x(t)\ \delta\left(3\left(t+\frac{4}{3}\right)\right)$

$x(t) \cdot \frac{1}{3}\delta\left(t+\frac{4}{3}\right)$

$x(t) = \frac{1}{3}x\left(t+\frac{4}{3}\right)$ //

② $x_1(t) * [x_2(t) + x_3(t)]$

$= x_1(t) * x_2(t) + x_1(t) * x_2(t)$

Q $x(t) = u(t-3) - u(t-5)$

$h(t) = e^{-3t} u(t)$ so find $\frac{d}{dt} \{ x(t) * h(t) \}$

→ $\frac{d}{dt}(u(t)) = \delta(t)$, so

$\frac{d}{dt} x(t) = \delta(t-3) - \delta(t-5)$

→ $[\delta(t-3) - \delta(t-5)] * [e^{-3t} u(t)]$

$= e^{-3t} u(t) * \delta(t-3) - e^{-3t} u(t) * \delta(t-5)$

$= e^{-3(t-3)} u(t-3) - e^{-3(t-5)} u(t-5)$ //

Q $[e^{-t} u(t)] \cdot \delta(t-1)$ {general formula, not convolution}

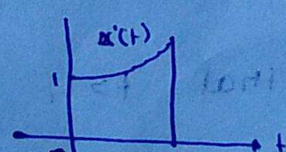
$= e^{-1} u(1) \delta(t-1)$

$= \frac{1}{e} \delta(t-1)$ //

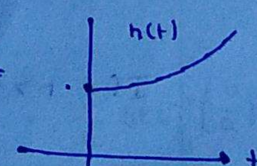
Q $x(t) = \begin{cases} 0 & t < 0 \\ e^{t} & 0 < t < 1 \\ 0 & t > 1 \end{cases}$ & $h(t) = e^{t/2} u(t)$.

convolute

A Now $x(t) =$

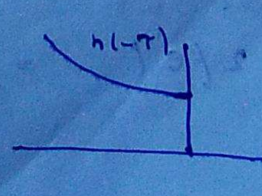


& $h(t) =$



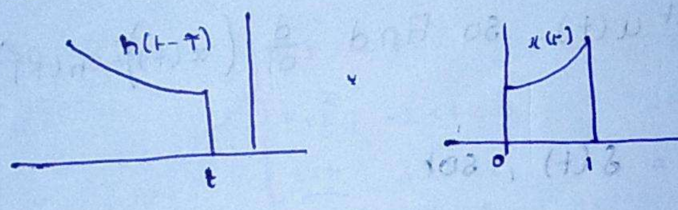
$x(t)$ limits $0 < t < \infty$

$h(t) \approx h(\tau)$



$h(\tau)$ becomes $h(-\tau)$

Time shift $h(t-\tau)$



multiplication.

If $t<0$, $y(t) = 0$ — ①

If $t>0$, so that limits are 0 to t

$$\int_0^t x(\tau)\, h(t-\tau)\, d\tau = \int_0^t e^{\tau} \cdot e^{\frac{t-\tau}{2}} u(t-\tau)\, d\tau$$

$$= \int_0^t e^{\tau} \frac{e^{t/2}}{e^{\tau/2}} u(t-\tau)\, d\tau \quad \{u(t-\tau)=1\}$$

$$= \int_0^t (e^{\tau/2})^2 \frac{e^{t/2}}{e^{\tau/2}}\, d\tau$$

$$= \int_0^t e^{\tau/2} \cdot e^{t/2}\, d\tau$$

$$= e^{t/2} \int_0^t e^{\tau/2}\, d\tau = e^{t/2} \cdot 2 \cdot \left[e^{\tau/2}\right]_0^t$$

$$= e^{t/2} \cdot 2\left[e^{t/2} - 1\right]$$

$$= e^{t/2} \cdot 2 \cdot e^{t/2}$$

— ②

$$= 2(e^{t} - e^{t/2}) \text{ ②} \quad \cdot \quad 2e^{\frac{t}{2}}(e^{t/2} - 1)$$

If $t>0$, so that $t>1$, so limits are 0 to 1

$$= e^{t/2} \cdot 2\left[e^{\tau/2}\right]_0^1 = 2 \cdot e^{t/2}\left[e^{1/2} - 1\right]$$

— ③

$$= e^{t/2} \cdot 2(e^{1/2}$$

=

$$y(t) = \begin{cases} 0 & t<0 \\ 2e^{t/2}(e^{t/2}-1) & 0<t<1 \\ 2e^{t/2}(e^{1/2}-1) & t\geq 1 \end{cases}$$

* Integration formulas:

1 $\int \frac{1}{x}\,dx = \ln|x| + c$

$\int \sin x\,dx = -\cos x + c$

$\int \cos x\,dx = \sin x + c$

$\int \sec^2 x\,dx = \tan x + c$

$\int \operatorname{cosec}^2 x\,dx = -\cot x + c$

$\int \sec x \cdot \tan x\,dx = \sec x + c$

$\int \operatorname{cosec} x \cdot \cot x\,dx = -\operatorname{cosec} x + c$

$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c$

$\int \frac{-dx}{\sqrt{1-x^2}} = \cos^{-1} x + c$

$\int \frac{dx}{1+x^2} = \tan^{-1} x + c$

$\int \frac{-dx}{1+x^2} = \cot^{-1} x + c$

$\int \frac{dx}{|x|\sqrt{x^2-1}} = \sec^{-1} x + c$

$\int (du + dv) = \int du + \int dv$

$\int e^x \cdot dx = e^x + c$

$\int a^x\,dx = \frac{a^x}{\ln|a|} + c$

Hyperbola

$\int \sinh x\,dx = \cosh x + c$

$\int \cosh x\,dx = \sinh x + c$

$\int \operatorname{sech}^2 x\,dx = \tanh x + c$

$\int \operatorname{cosech}^2 x\,dx = -\coth x + c$

$\int \operatorname{sech} x \cdot \tanh x\,dx = -\operatorname{sech} x + c$

$\int \operatorname{cosech} x \coth x\,dx = -\operatorname{cosech} x + c$

$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + c$

$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{u}{a}\right) + c$

$\int \frac{dx}{-|x|\sqrt{x^2-1}} = \operatorname{cosec}^{-1} x + c$

Inverse hyperbolic:

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \sinh^{-1}\frac{x}{a}+c$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \cosh^{-1}\frac{x}{a}+c$$

$$\int \frac{1}{a^2-x^2}\,dx = \tanh^{-1}\frac{x}{a}+c \quad |x|<a$$

$$\int \frac{1}{a^2-x^2}\,dx = \coth^{-1}\frac{x}{a}+c \quad |x|>a$$

$$\int \frac{-1}{x\sqrt{x^2-a^2}}\,dx = \operatorname{sech}^{-1}\frac{x}{a}+c$$

$$\int \frac{dx}{|x|\sqrt{x^2+a^2}} = \operatorname{cosech}^{-1}\frac{x}{a}+c$$

$$\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c$$

$$\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right|+c$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \log\left|x+\sqrt{x^2-a^2}\right|+c$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \log\left|x+\sqrt{x^2+a^2}\right|+c$$

$$\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)+c$$

$$\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\cosh^{-1}\left(\frac{x}{a}\right)+c$$

$$\int \sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2}+\frac{a^2}{2}\sinh^{-1}\left(\frac{x}{a}\right)+c$$

$$\int \tan x\,dx = -\ln|\cos x|+c \text{ (or) } \ln|\sec x|+c$$

$$\int \cot x\,dx = \ln|\sin x|+c \text{ (or) } -\ln|\operatorname{cosec} x|+c$$

$\int \sec x \, dx = \ln|\sec x + \tan x| + c$

$\int \operatorname{cosec} x \, dx = \ln|\operatorname{cosec} x + \cot x| + c = \ln|\tan \frac{x}{2}| + c$

$\int \tanh x \, dx = \ln(\cosh x) + c$

$\int \coth x \, dx = \ln|\sinh x| + c$

$\int \operatorname{sech} x \, dx = \tan^{-1}|\sinh x| + c$

$\int \operatorname{cosech} x \, dx = \ln|\tanh \frac{x}{2}| + c$

Reduction formulas :

$$\int \sin^n x \, dx = \frac{-1}{n} \sin^{n-1} x \cdot \cos x + \frac{n-1}{n} \int \sin^{n-2} x \, dx$$

$$\int \cos^n x \, dx = \frac{1}{n} \cos^{n-1} x \cdot \sin x + \frac{n-1}{n} \int \cos^{n-2} x \, dx$$

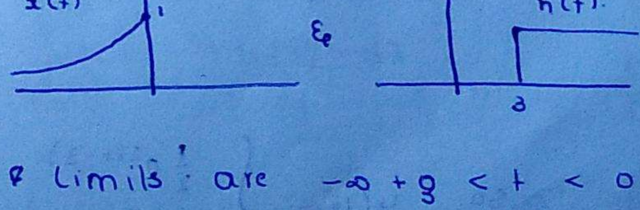
$$\int \tan^n x \, dx = \frac{1}{n-1} \tan^{n-1}(x) - \int \tan^{n-2} x \, dx$$

$$\int \operatorname{cosec}^n x \, dx = \frac{-1}{n-1} \operatorname{cosec}^{n-2} x \cdot \cot x + \frac{n-2}{n-1} \int \operatorname{cosec}^{n-2} x \, dx$$

$$\int \sec^n x \, dx = \frac{1}{n-1} \tan x \cdot \sec^{n-2} x + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx$$

$$\int \cot^n x \, dx = \frac{-1}{n-1} \cot^{n-1} x - \int \cot^{n-2} x \, dx.$$

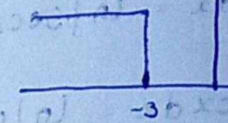
Q $x(t) = e^{2t} \cdot u(-t)$ & $h(t) = u(t-3)$ convolute.

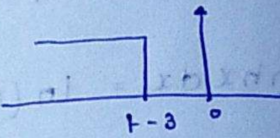


& Limits are $-\infty + 3 < t < 0 + \infty$

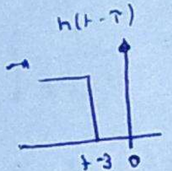
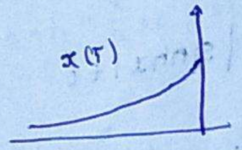
Limits are $-\infty < t < \infty$

change the domain. $h(t) = h(\tau)$

time reversal $\rightarrow$ $h(-\tau)$ 

time shift $\rightarrow$ $h(t-\tau)$ 

Now multiplication

 & 

If $t-3$ is less than 0 . . limits are $-\infty$ to $t-3$

$$\rightarrow \int_{-\infty}^{t-3} x(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{t-3} e^{2\tau} u(-\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{t-3} e^{2\tau} (1)(1)\, d\tau = \left.\frac{e^{2\tau}}{2}\right|_{-\infty}^{t-3} \rightarrow \frac{1}{2}\left[e^{2(t-3)} - e^{-\infty}\right]$$

$$= \frac{1}{2}\left[e^{2(t-3)} - \frac{1}{e^{\infty}}\right] \qquad \left\{ \frac{1}{e^{\infty}} = 0 \right\}$$

$$= \frac{1}{2} e^{2(t-3)}.$$

If $t-3 > 0$, so limits are $-\infty$ to 0

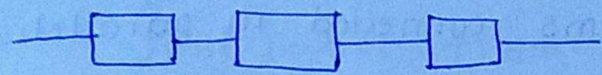
$$\rightarrow \int_{-\infty}^{0} e^{2\tau} (1)(1)\, d\tau = \left.\frac{e^{2\tau}}{2}\right|_{-\infty}^{0}$$

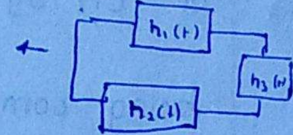
$$\frac{1}{2}\left[e^{0} - 0\right]$$

$$= \frac{1}{2}$$

$$\text{so } y(t) = \begin{cases} \frac{1}{2} e^{2(t-3)} & \boxed{-\infty < t-3 < 0} \quad -\infty < t < 0 \\ \frac{1}{2} & 0 < t < \infty \quad \boxed{0 < t-3 < \infty} \end{cases}$$

* Interconnection of systems

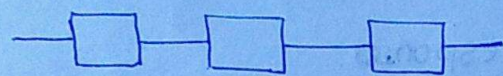




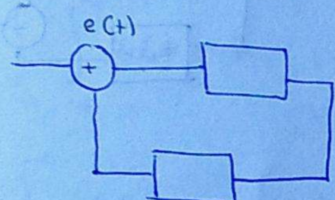
(or)

$h_1(t) * h_2(t) * h_3(t)$

→ Series connection of systems:



→ Feed back.



* Properties of continuous time LTI ~~systems~~ convolution.

→ Commutative property: It states that response of an LTI system is not effected if the position of input and impulse response are interchanged.

→ $x(t) * h(t) = h(t) * x(t)$.

Proof: $\displaystyle\int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$ $\quad \{ \text{let } (t-\tau) = \lambda \}$

$\{ -d\tau = d\lambda \}$

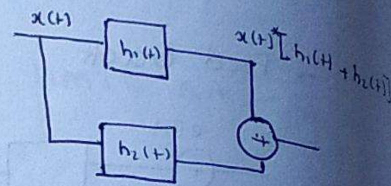
$\rightarrow \displaystyle\int_{+\infty}^{-\infty} x(t-\lambda)\, h(\lambda)\, -d\lambda$ $\quad \left\{ \begin{array}{l} \text{If } \tau = \infty,\ \lambda = -\infty \\ \text{If } \tau = -\infty,\ \lambda = \infty \end{array} \right\}$

$= \displaystyle\int_{-\infty}^{\infty} x(t-\lambda)\, h(\lambda)\, d\lambda$

$= \displaystyle\int_{-\infty}^{\infty} h(\lambda)\, x(t-\lambda)\, d\lambda = h(t) * x(t)$

* Distributive property: It states that any number of LTI systems connected in parallel can all being combined into a single system with an impulse response which is equal to sum of all individual responses.

$x(t) * [h_1(t) + h_2(t)]$



$x(t) * h_1(t) + x(t) * h_2(t)$

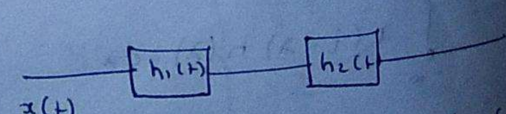
$\rightarrow \quad x(t) * [h_1(t) + h_2(t)]$

$$= \int_{-\infty}^{\infty} x(\tau)\,[h_1(t-\tau) + h_2(t-\tau)]\,d\tau$$

$$= \int_{-\infty}^{\infty} x(\tau)\,h_1(t-\tau)\,d\tau + \int_{-\infty}^{\infty} x(\tau)\,h_2(t-\tau)\,d\tau$$

$= x(t) * h_1(t) + x(t) * h_2(t)$..

* Associative property: It states that any number of LTI systems connected in cascade (series) can all be combined in single system, whose impulse response is equal to convolution of individual impulse responses



$(x(t) * h_1(t)) * h_2(t) = x(t) * (h_1(t) * h_2(t))$

$\rightarrow x(t) * (h_1(t) * h_2(t))$

$\rightarrow x(t) * \int_{-\infty}^{\infty} h_1(\tau) \cdot h_2(t-\tau)\, d\tau$

$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x(k)\, h_1(\tau) \cdot h_2(t-\tau-k)\, dk\, d\tau$

let $\tau + k = l$

$\tau = l - k$

$d\tau = dl$

$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x(k)\, h_1(l-k)\, h_2(t-l)\, dk\, dl$

$= \int_{-\infty}^{\infty} (x(l) * h_1(l))\, h_2(t-l)\, dl$

$= (x(t) * h_1(t)) * h_2(t)$ //

* Time shifting property :- If $x(t) * h(t) = y(t)$,

then $x(t) * h(t-t_0) = y(t-t_0)$

$x(t-t_0) * h(t) = y(t-t_0)$

$x(t-t_1) * h(t-t_2) = y(t-t_1-t_2)$

$\rightarrow x(t) * \delta(t-t_0) = x(t-t_0)$

$\rightarrow x(t-t_1) * \delta(t-t_2) = x(t-t_1-t_2)$

Proof : $x(t-t_1) * h(t-t_2)$

$= \int_{-\infty}^{\infty} x(\tau - t_1)\, h(t-t_2-\tau)\, d\tau$

let $\tau - t_1 = k \rightarrow \tau = k + t_1$ & $d\tau = dk$.

$$= \int_{-\infty}^{\infty} x(k)\, h(-k-t_1+t-t_2)\, dk$$

$$= \int_{-\infty}^{\infty} x(k)\, h(t-t_1-t_2-k)\, dk$$

$$= y(t-t_1-t_2)$$

* Differentiation property: If $x(t) * h(t) = y(t)$

then $\frac{d}{dt} x(t) * h(t) = \frac{d}{dt} y(t)$

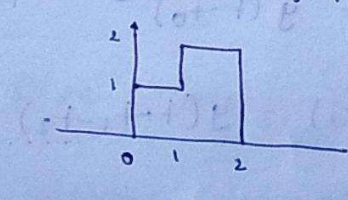
and $x(t) * \frac{d}{dt} h(t) = \frac{d}{dt} y(t)$

Proof: $y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

$$\frac{d(y(t))}{dt} = \int_{-\infty}^{\infty} x(\tau) \frac{d}{dt} h(t-\tau)\, d\tau$$

$$= x(t) * \frac{d}{dt} h(t)$$

Q $x(t) = \delta(t) - 2\delta(t-1) + \delta(t-2)$

$h(t) =$ 

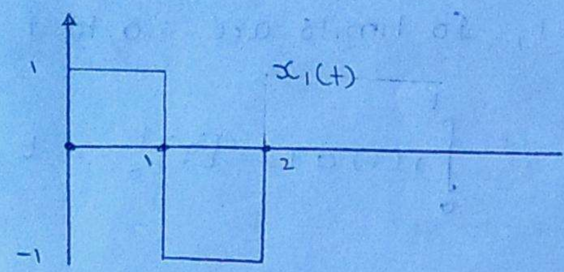
Sketch the graph of $x(t) * h(t)$

→ $x(t) = \delta(t) - 2\delta(t-1) + \delta(t-2)$

$x(t) = \frac{d}{dt} u(t) - 2\frac{d}{dt}(u(t-1)) + \frac{d}{dt}(u(t-2))$

$x(t) = \frac{d}{dt}\left[u(t) - 2u(t-1) + u(t-2)\right]$

let $x_1(t) = u(t) - 2u(t-1) + u(t-2)$



Now $x(t) * h(t) =$

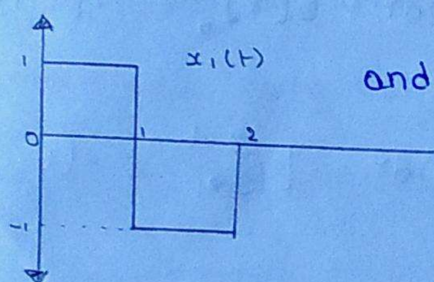
$$\frac{d}{dt} x_1(t) * h(t)$$

$$= \frac{d}{dt} y(t).$$

→ we have $x_1(t) * h(t) = y(t)$.

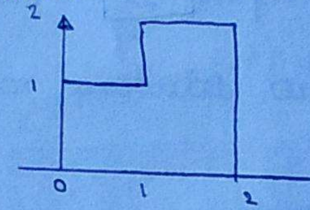
then we get the value $x(t) * h(t) = \frac{d}{dt} y(t)$.

$x_1(t)$



and

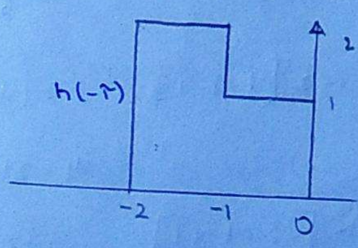
$h(t)$



1) Limits $0+0 < t < 2+2 \rightarrow 0 < t < 4$.

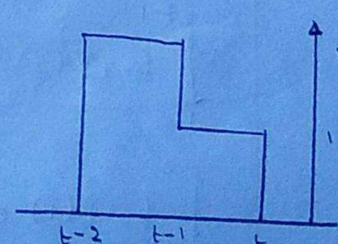
2) $h(t) \approx h(\tau)$

3) $h(\tau)$ to be converted as $h(-\tau)$



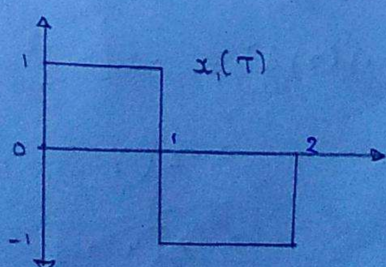
Time shifting

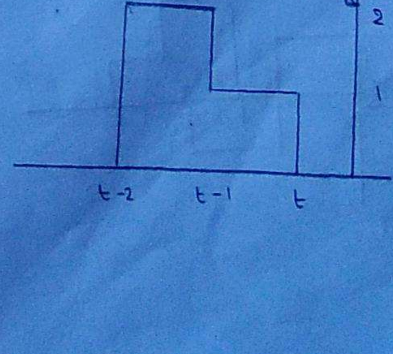
$h(t-\tau) \rightarrow$



Multiplication

$x_1(\tau) \cdot h(t-\tau)$.

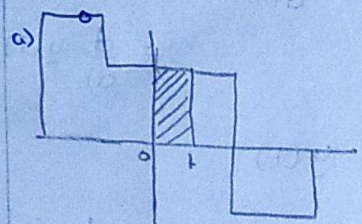




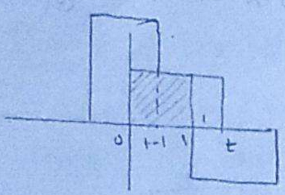
If $t<0$, $y(t) = 0 \rightarrow$ 1

If $t>0$, and $t<1$, So limits are - 0 to t

$$\int_0^t x(\tau)\, h(t-\tau)\, d\tau = \int_0^t 1(1)\, d\tau = [\tau]_0^t = t \rightarrow (1)$$

a) 

* If $t>0$, & $t>1$, So limits are 0 to 1.

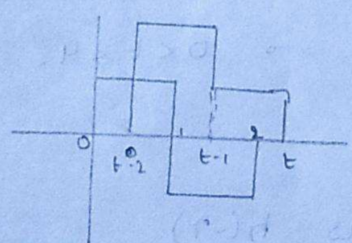


$$\int_{t-1}^{1} 1(1)\, d\tau = [\tau]_{t-1}^{1} = 1-t+1 = 2-t \rightarrow (2)$$

$$\int_0^{t-1} 2(1)\, d\tau = 2[\tau]_0^{t-1} = 2[t-1] = 2t-2 \rightarrow (3)$$

$$\int_1^t (-1)(1)\, d\tau = -[\tau]_1^t = -(t-1) = 1-t \rightarrow (4)$$

If $t>0$ & $t>1$, $t>2$



$$\int_{t-2}^{1} 1(2)\, d\tau = 2[\tau]_{t-2}^{1} = 2[1-t+2] = 2[3-t] = 6-2t \rightarrow (5)$$

$$\int_1^{t-1} (-1)(2)\, d\tau = -2[\tau]_1^{t-1} = -2[t-1-1] = -2[t-2] = 4-2t \rightarrow (6)$$

$$\int_{t-1}^{2} (-1)(1)\, d\tau = -1[\tau]_{t-1}^{2} = -[2-t+1] = -[3-t) = t-3 \rightarrow (7)$$

$$\int_{t-2}^{2} (-1)(2)\, d\tau = -2[\tau]_{t-2}^{2}$$

$$= -2[2-t+2]$$

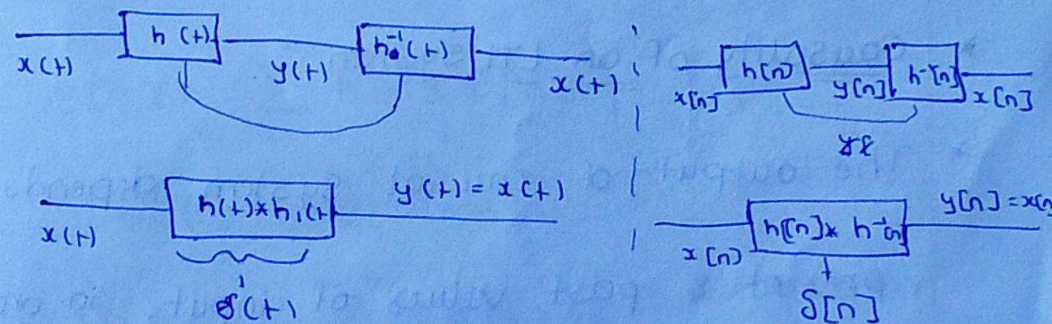
$$= -2[4-t]$$

$$= 2t-8 \rightarrow (8)$$

$$y(t) = \begin{cases} t & 0 < t < 0.5 \\ 2-t & 0.5 < t < 1 \\ 2t-2 & 1 < t < 1.5 \\ 1-t & 1.5 < t < 2 \\ 6-2t & 2 < t < 2.5 \\ t-3 & 2.5 < t < 3 \\ 2t-8 & 3 < t < 3.5 \\ 0 & ew \end{cases}$$

17) * **Invertibility of LTI system**

A system is invertable when connected in series with the original system produces an output equal to the input.



$$x(t) * \delta(t) = x(t)$$

$$h(t) * h_1(t) = \delta(t)$$

$$h[n] * h^{-1}[n] = \delta[n]$$

* A system is memory less, if its output at any time depends on input at that time. The only way that can be true for an LTI system is

$$h[n] = 0 \quad : n \neq 0$$

$$y[n] = \sum_{n=-\infty}^{\infty} x[n]\, h[n-k]$$

$$y[2] = \sum_{n=-\infty}^{\infty} x[2]\, h[2-k]$$

$$= \dots + \underbrace{x[-1]\,h[3] + x[0]\,h[2] + x[1]\,h[1]}\ + x[2]h[0] + \dots$$

not required as output don't depend on same time of input

$= \boxed{h[n] = 0, \quad n \neq 0}$ $\{ h[n] = k \cdot \delta[n] \}$

$= y[n] = x[n] * k\,\delta[n]$

$= \boxed{y[n] = k \cdot x[n]}$

write in continuous.

* Causality of an LTI system:

* The output of causal system depends on present & past values of input. in order for an LTI system to be causal $y[n]$ must not depend on $x(k)$ for $k > n$.

So, this is to be true, all of the coefficients $h[n-k]$ that multiply values of $x[k]$ for $k > n$ must be '0'.

$y[n] = x[n] * h[n]$ $\quad \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$

$$y[2] = \sum_{k=-\infty}^{\infty} x[k]\, h[2-k]$$

$\ldots + x[-2]h[4] + x[-1]h[3] + x[0]h[2] + x[1]h[1]$

$+ x[2]h[0] + x[3]h[-1]$

$$y[n] = \sum_{k=-\infty}^{n} x[k]\, h[n-k]$$

$$y(t) = x(t) * h(t) \; ; \; \int_{-\infty}^{\infty} x(t)\, h(T-t)$$

* Stability for LTI systems:-

* System is stable if every bounded input produces bounded output. Consider an input $x[n]$ that is bounded in magnitude.

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

$$|x[n]| = B \quad \forall n$$

$$|y[n]| = \sum_{k=-\infty}^{\infty} |h[k]|\, B$$

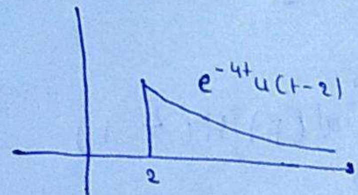
$$= B \sum_{k=-\infty}^{\infty} |h(k)| < \infty$$

$$\boxed{\left|\sum_{k=-\infty}^{\infty} h(k)\right| < \infty}$$ condition.

Property	CT LTI	DT LTI
Stability	$h(t) = 0,\ t<0$	$h[n] = 0,\ n<0$
Inequality	$\sum_{k=-\infty}^{\infty} \|h(k)\| < \infty$	$\int_{-\infty}^{\infty} h(t)dt < \infty$
memoryless	$h(t) * h_1(t) = \delta(t)$	$h[n] * h_1[n] = \delta[n]$
/Static	$h(t) = 0\ \ t \neq 0$ $= k\,\delta(t)$	$h[n] = 0,\ n \neq 0$ $= k\,\delta[n]$

Q. Find whether the following systems are causal. $h(t) = e^{-4t}u(t-2)$

A.

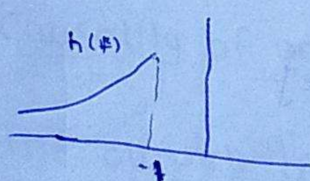


causal.
stable

2) $h(t) = e^{-6|t|}$

non-causal
stable.

3) $h(t) = e^{3t}u(-1-t)$

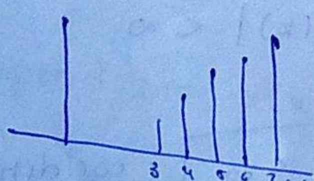


non-causal
stable

4) $h[n] = 5^n u[n-3]$

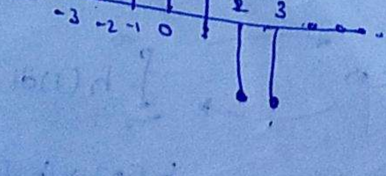
5) $h[n] = u[n+3] - 2u[n-2] + u[n-4]$.

4A



~~non~~ causal
non stable

5A



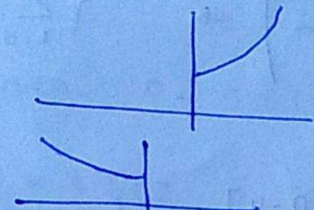
non-causal
stable

Q. Given $h(t) = e^{\alpha t}u(t) + e^{\beta t}u(-t)$ for what values of α, β 0, system is stable.

A. $\alpha \leq 0$ & $\beta \geq 0$

$e^{\alpha t} u(t)$

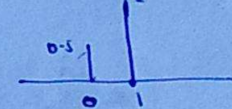
$e^{\beta t} u(t)$



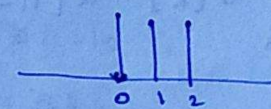
So { α = negative either, & β becomes positive, condition will be satisfied }

Q. consider an LTI system, convolute the following

$x[n] = \{0.5, 2\}$ (arrow under 0.5)



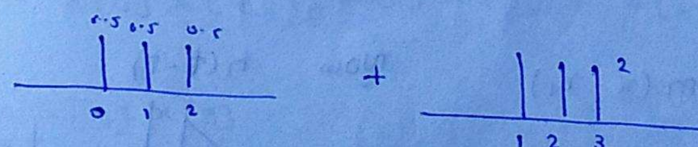
$h[n] = \{1, 1, 1\}$ (arrow under the first 1)



A. $y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$

$= x(0)\, h(n) + x[1]\, h[n-1]$

$= 0.5 h[n] + 2h[n-1]$



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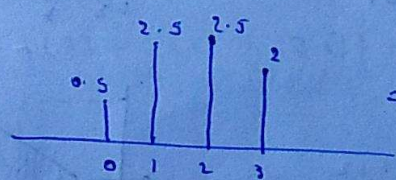


$= y[n]$

Q. $y[n] = a^n u[n] * b^n u[n]$

$\rightarrow \sum_{k=-\infty}^{\infty} x[n] \cdot h[n-k] = \sum_{k=-\infty}^{\infty} a^k u[k] \cdot b^{n-k} u[n-k].$

$= \sum_{k=-\infty}^{\infty} a^k u[k] \cdot b^{n-k} u[n-k]$ $\{k > 0$ & ~~n-k~~ $k < n\}$

$= \sum_{k=-\infty}^{\infty} a^k \cdot b^{n-k} = \sum_{k=-\infty}^{\infty} a^k \cdot b^n \cdot b^{-k}$

$$b^n \sum_{k=0}^{n} \left(\frac{a}{b}\right)^k$$

$$= b^n \left(\frac{1-(a/b)^{n+1}}{1-(a/b)} \right) \Rightarrow \left\{ \sum_{k=0}^{n} a^n = \frac{1-a^{n+1}}{1-a} \right\}$$

Q $y[n] = u[n] * u[n-4]$, find $y[5]$.

A $$\sum_{k=-\infty}^{\infty} u[k] \cdot u[n-k-4]$$

$$\sum_{k=-\infty}^{\infty} u[k] \cdot u[n-(k+4)]$$ $\{ k > 0$ & $n-(k+4) > 0$

$k+4 < n$

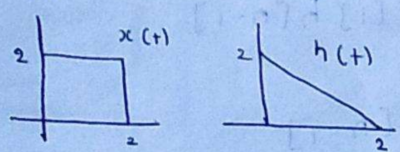
$k < n-4$

$$\sum_{k=0}^{n-4} u[k] \cdot u[n-(k+4)]$$ Here $n=5$

$$\sum_{k=0}^{1} u[k] \cdot u[1-k] = u[0] \cdot u[1-0] + u[1] \cdot u[1]$$

$$= 1 + 1 = \boxed{2}.$$

Q



, claclulate $y(t)$ at $t = 3$

$y - y_1 = m(x - x_1)$

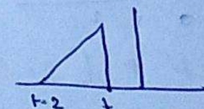
$h(t) - 2 = -1(t-0)$

$h(t) = -t + 2$

$h(t-\tau) = -(t-\tau) + 2$

$= (-t + \tau + 2)$

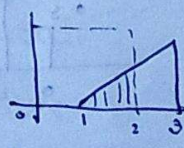
Now $h(t-\tau)$



$t = 3$

$t-2 = 1$

$t = 3$



$$\int_1^2 2(-3 + \tau + 2) \, d\tau$$ gives the value.

$$= \int_1^2 -6 + 2\tau + 4 \, d\tau$$

$$= -6\tau + 2\frac{\tau^2}{2} + 4\tau \Big|_1^2$$

$$= -6\tau + \tau^2 + 4\tau \Big|_1^2$$

$$= [-6(2) + 2^2 + 4(2)] - [-6(1) + 1^2 + 4(1)]$$

$= -12 + 4 + 8 - [-6 + 1 + 4]$

$= 1$

Q $x[n] = n$, when $0 \le n \le 10$

$y[n] = x[n] * x[n]$, $y[4] = ?$

$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, x[n-k]$

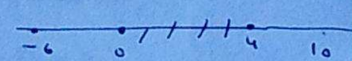
$\{ 0 \le k \le 10$ & $0 \le n-k \le 10 \}$

$0 \le k \le 10$ & $n \ge n-k \ge n-10 \}$ $n = 4$

$4 \ge 4-k \ge -6$

$y[n]$

$= \sum_{k=-\infty}^{\infty} x[k]\, x[4-k]$



Limits are 0 to 4

$= \sum_{k=0}^{4} x[k]\, x[4-k]$

$= x[0]\,x[4] + x[1]\,x[3] + x[2]\,x[2] + x[3]\,(x[1])$

$= x[4]\,x[0]$

$\rightarrow 0(4) + 1(3) + 2(2) + 3(1) + 4(0)$

$= 10$

$x[n] = \{1, 2, 3, 4\}$

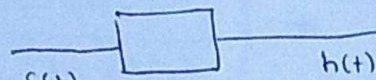
$h[n] = \{4, 5, 6\}$

$= x[n] * y[n] =$

	1	2	3	4
4	4	8	12	6
5	5	10	15	20
6	6	12	18	24

$\{4, 13, 28, 43, 38, 24\}$

Step Response:



$\rightarrow \quad u(t) = \int_{-\infty}^{t} \delta(t)\, dt$

$$\boxed{\begin{aligned} &\rightarrow \quad s(t) = \int_{-\infty}^{t} h(t) \cdot dt \\ &\rightarrow \quad = \; h(t) = \frac{d}{dt} s(t) \end{aligned}}$$

Q If the unit step response of system is $(1-e^{-\alpha t})u(t)$. Find unit impulse response

A Given $s(t) = (1-e^{-\alpha t})u(t)$ {asked for $h(t)$.

$h(t) = \frac{d}{dt}(1-e^{-\alpha t})u(t)$

$h(t) = (1-e^{-\alpha t}) \frac{d}{dt} u(t) + u(t) \frac{d}{dt}(1-e^{-\alpha t})$

$h(t) = (1-e^{-\alpha t})\, \delta(t) + u(t) \cdot \alpha e^{-\alpha t}$

$h(t) = (1-e^{0 t})\, \delta(t) + u(t) \cdot \alpha e^{-\alpha t}$

$\boxed{h(t) = u(t) \cdot \alpha e^{-\alpha t}}$ $\left\{ x(t)\, \delta(t-t_0) = x(t_0) \cdot \delta(t-t_0) \right.$

Q Find the impulse response system, if step response is .. $s(t) = \cos \omega_0 t \cdot u(t)$.

A $\frac{d}{dt} h(t) = \frac{d}{dt} \cos\omega_0 t \, u(t)$

$h(t) = \cos\omega_0 t \, \delta(t) - u(t) \sin\omega_0 t \cdot \omega$

$h(t) = \cos\omega_0 (0) \cdot \delta(t) - u(t)\, \omega_0 \sin\omega_0 t$

$h(t) = \delta(t) - u(t)\, \omega_0 \sin\omega_0 t \quad \{ x(t)\, \delta(t-t_0) = x(t_0) \cdot \delta(t-t_0) \}$

Q Find the step response of system if $h[n] = (0.5)^n u[n]$

A $s[n] = \sum_{k=-\infty}^{n} h[n]$

$= \sum_{k=-\infty}^{n} (0.5)^n u[n] \qquad (n > 0)$

$= \sum_{k=0}^{n} (0.5)^n \cancel{k}\, u[n]$

$= (0.5)^0 (1) + (0.5)^1 + (0.5)^2 (1) + \cdots (0.5)^n (1).$

$= 1 + 0.5 + (0.5)^2 + (0.5)^3 + \cdots + (0.5)^n$

(or) $= \sum_{k=0}^{n} a^n = \frac{1 - a^{n+1}}{1-a} = \frac{1 - (0.5)^{n+1}}{0.5}$ //.

Solution to constant coefficient D.E :-

Q $\frac{dy(t)}{dt} + y(t) = x(t) = \begin{cases} e^{3t} & t > 0 \\ 0 & t < 0 \end{cases}$ Solve after definition

$\checkmark\ \cancel{m} \ne 1 \rightarrow m + 1 = 0$ Where y is the state variable

$\rightarrow m = -1$ $x(t)$ is

The ~~so~~ complete solution is of this form i.e

$y(t) = y_c(t) + y_p(t)$

where $y_c(t)$ = zero input response or natural response

and $y_p(t)$ = zero state response or forced response

$$m + 1 = 0$$

$$m = -1$$

$$\Rightarrow y_c(t) = c \cdot e^{-t}$$

$$\Rightarrow y_p(t) = \frac{1}{D+1} e^{3x} \qquad t > 0$$

$$= \frac{1}{3+1} e^{3x}$$

$$= \frac{1}{4} e^{3x}$$

$$y(t) = c \cdot e^{-t} + \frac{1}{4} e^{3x} \qquad \text{Given } y(0) = 0$$

then $c + \frac{1}{4} = 0$

$c = \frac{-1}{4}$

$$y(t) = \frac{1}{4}\left(e^{3x} - e^{-t}\right)$$ //