MATHEMATICAL MODELING OF PHYSICAL SYSTEMS

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Contents:

Introduction

- For the analysis and design of control systems, we need to formulate a mathematical description of the system.
- The process of obtaining the desired mathematical description of the system is known as “modeling”.
- The basic models of dynamic physical systems are differential equations obtained by application of the appropriate laws of nature.
- These equations may be linear or nonlinear depending on the phenomena being modeled.
- The differential equations are inconvenient for the analysis and design manipulations. So the Laplace Transformation, which converts the differential equations into algebraic equations, is used.

Introduction

- The algebraic equations may be put in transfer function form, and the system modeled graphically as a transfer function block diagram.
- Alternatively, a signal flow graph may be used.
- The physical systems are Mechanical, Electrical, Hydraulic, Thermal etc….
- Analysis of control systems

1. **The ability to predict performance**
2. **Precision of the results**
3. **Characteristic of each component**
4. **Modeling of the system**
5. **Transfer function approach and state equation approach**

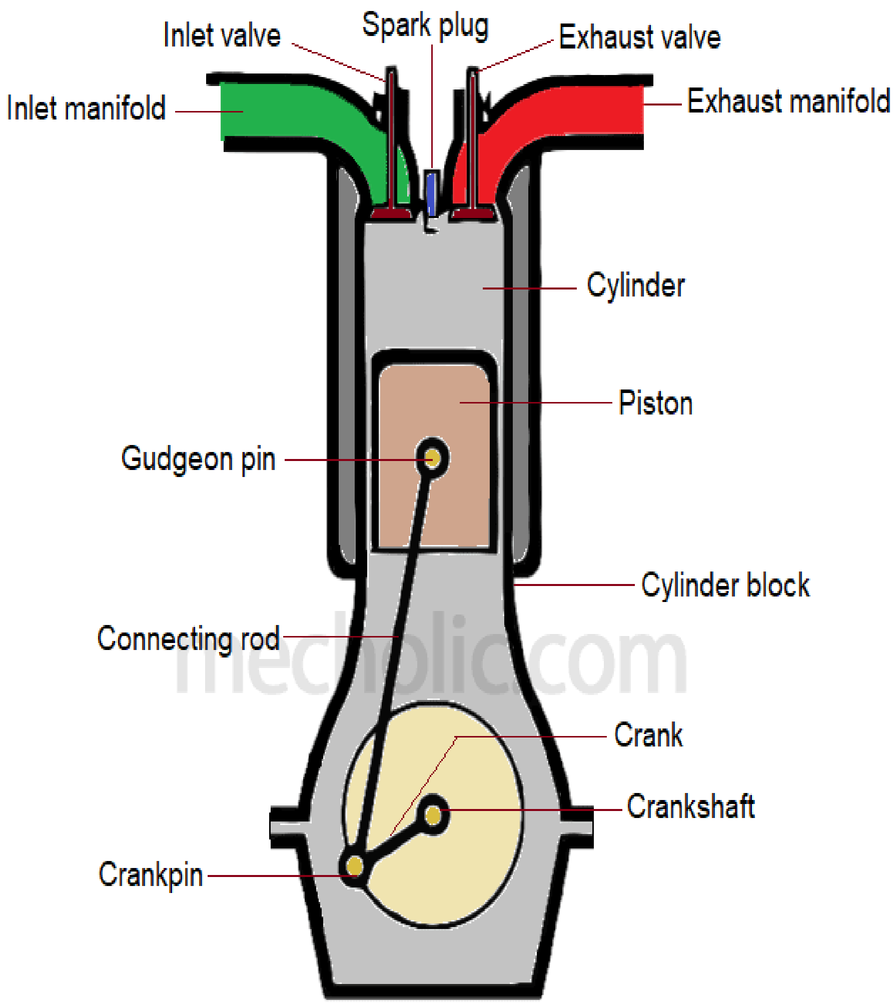
Mechanical systems:

✓ A **mechanical system** is a set of physical components that convert an input motion and force into a desired output motion and force.

1. Handle bar - Class 1 lever	12. Chain and sprocket mechanism
2. Ball bearings	13. Crank
3. Brake lever - Class 1 lever	14. Crankshaft bearing - ball bearings
4. Handle bar grips - stay in place due to friction	15. Saddle fixings - nut and bolt
5. Brakes - friction	16. Saddle and pillar stay in position due to friction
6. Brake calipers - Class 1 lever	17. Sprockets
7. Clamping bolts	18. Pedal - crank handle
8. Brake cable grip screw	19. Pedal fixing bolt and bearings
9. Wheel hub axle - nut and bolt	20. Wheel
10. Wheel hub bearing - ball bearings	21. Tyre - Friction makes the tyre grip the road surface
11. Rubber brake pads - friction	

Mechanical systems:

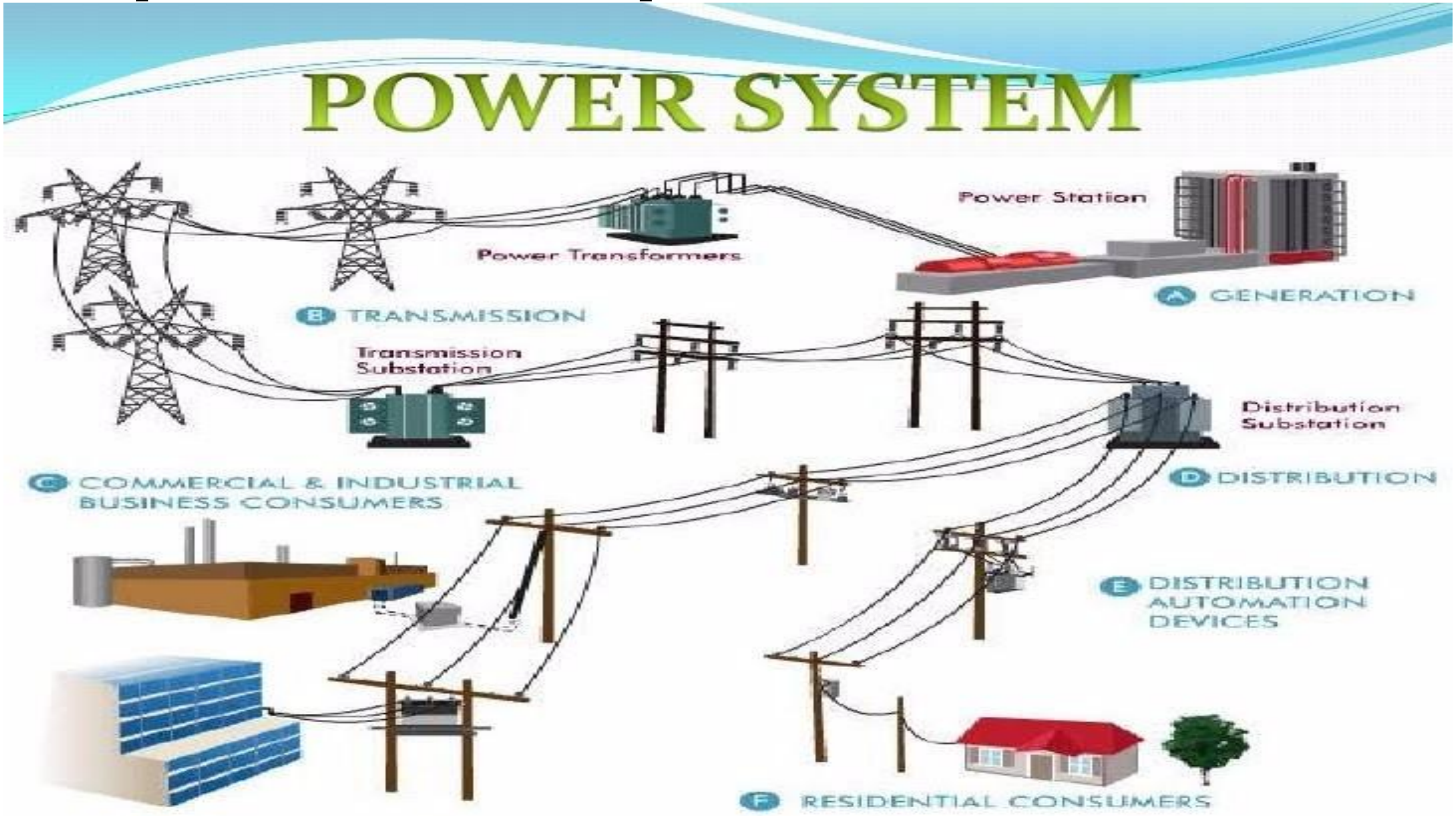
- IC engine



- Escalator

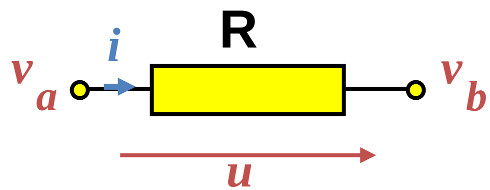
Electrical systems:

- ✓ **Electrical systems**, also called circuits or networks, are designed as combinations of mainly three fundamental components i.e. resistor, capacitor and inductor.



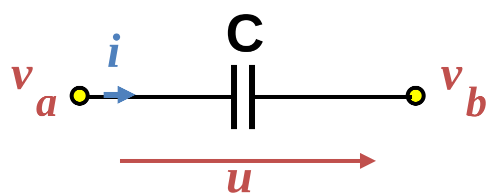
Transfer function of electrical systems
Linear Circuit Components

- Resistors



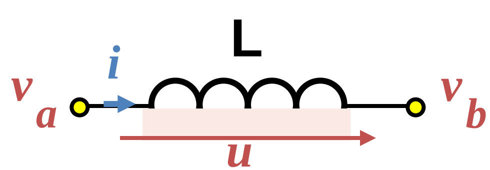
$$u = v_a - v_b$$
$$u = R \cdot i$$

- Capacitors



$$u = v_a - v_b$$
$$i = C \cdot \frac{du}{dt}$$

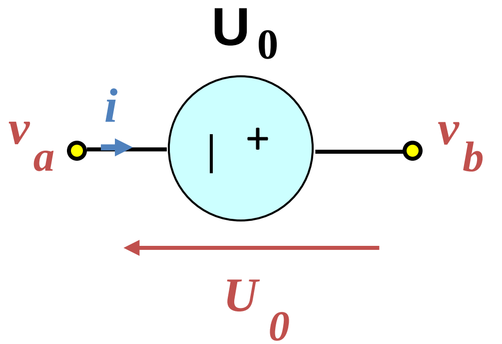
- Inductors



$$u = v_a - v_b$$
$$u = L \cdot \frac{di}{dt}$$

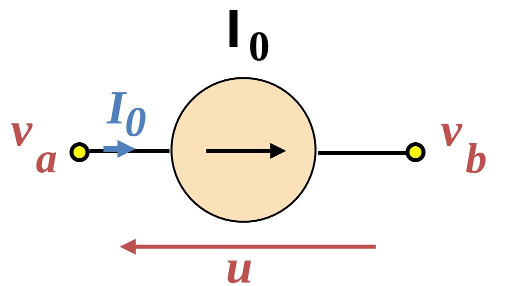
Transfer function of electrical systems
Linear Circuit Components

- Voltage sources



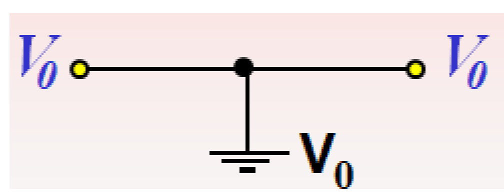
$$U_0 = v_b - v_a$$
$$U_0 = f(t)$$

- Current sources



$$u = v_b - v_a$$
$$I_0 = f(t)$$

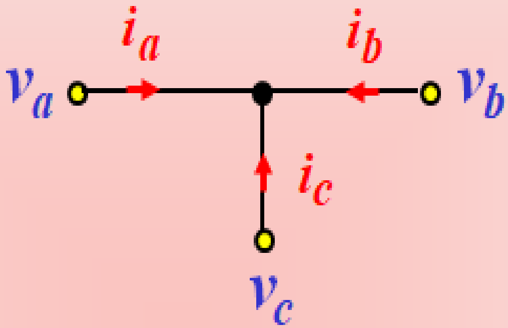
- Ground



$$V_0 = 0$$

Transfer function of electrical systems
Circuit Topology

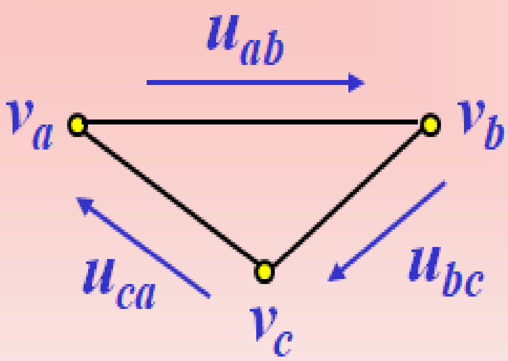
- Nodes



$$v_a = v_b = v_c$$

$$i_a + i_b + i_c = 0$$

- Meshes



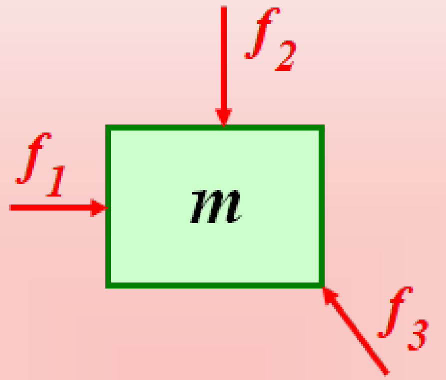
$$u_{ab} + u_{bc} + u_{ca} = 0$$

Transfer function of mechanical systems

Linear Components of Translation

- Mass

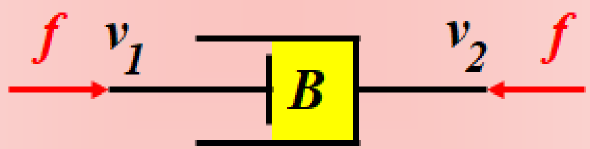
$$f(t) = Ma(t) = M\frac{dv(t)}{dt}$$



$$m\cdot a = \sum_{\forall i}(f_i)$$

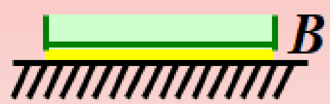
$$\frac{dv}{dt} = a$$

$$\frac{dx}{dt} = v$$



- Friction

$$f(t) = B\left[\frac{dx_1(t)}{dt} - \frac{dx_2(t)}{dt}\right]$$



$$f = B\cdot(v_1 - v_2)$$

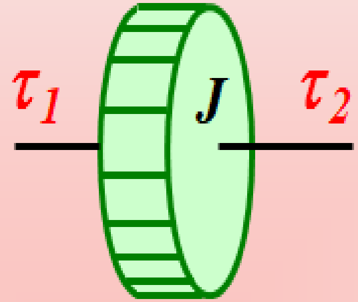
- Spring

$$f = k\cdot(x_1 - x_2)$$

Linear Components of Rotation

- Inertia

$$\tau(t) = J\frac{d^2\theta(t)}{dt^2} = J\frac{d\omega(t)}{dt}$$

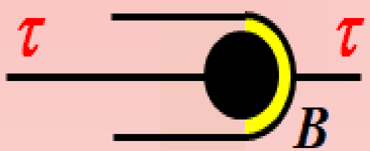


$$J\cdot\alpha = \sum_{\forall i}(\tau_i)$$

$$\frac{d\omega}{dt} = \alpha$$

$$\frac{d\theta}{dt} = \omega$$

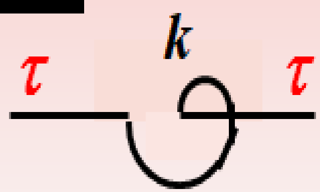
- Friction



$$\tau = B\cdot(\omega_1 - \omega_2)$$

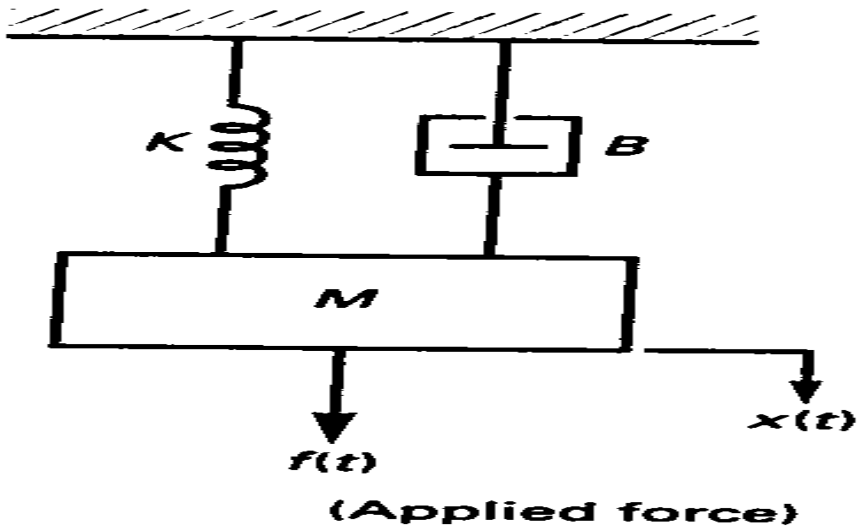
$$\tau(t) = B\left(\frac{d\theta_1(t)}{dt} - \frac{d\theta_2(t)}{dt}\right) = B[\omega_1(t) - \omega_2(t)]$$

- Spring



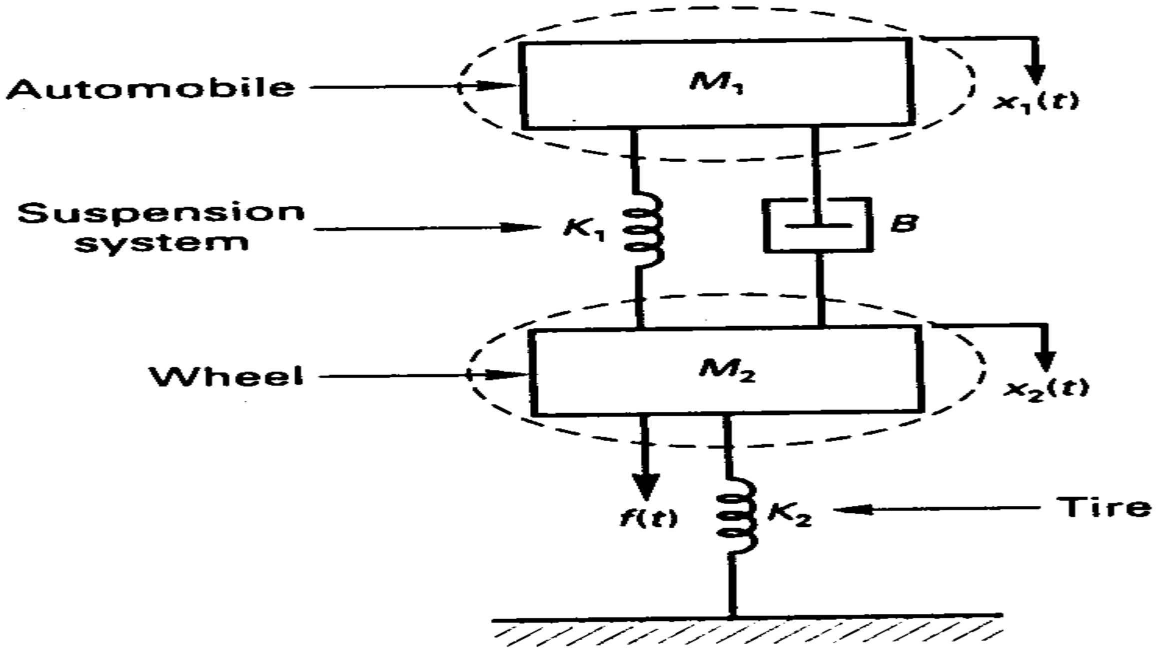
$$\tau = k\cdot(\theta_1 - \theta_2)$$

Mass-Spring-Damper system:



$$M\frac{d^2x(t)}{dt^2}+B\frac{dx(t)}{dt}+Kx(t)=f(t)$$

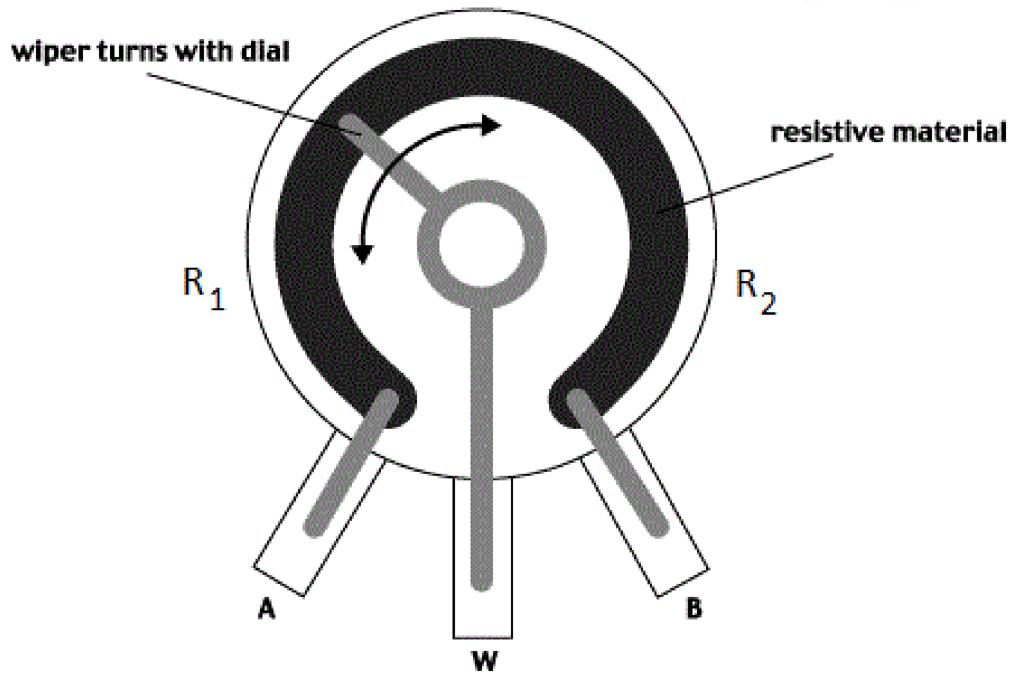
Automobile suspension system:

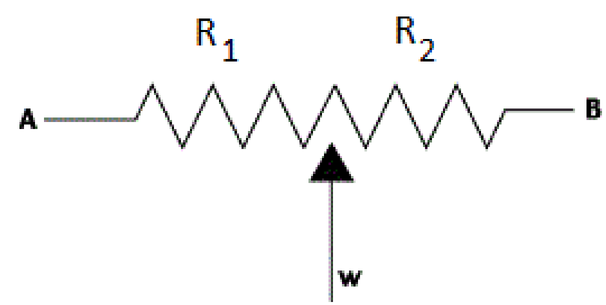


$$M_1 \frac{d^2 x_1(t)}{dt^2} = -B\left(\frac{dx_1(t)}{dt} - \frac{dx_2(t)}{dt}\right) - K_1(x_1(t) - x_2(t))$$

$$M_2 \frac{d^2 x_2(t)}{dt^2} = f(t) - B\left(\frac{dx_2(t)}{dt} - \frac{dx_1(t)}{dt}\right) - K_1(x_2(t) - x_1(t)) - K_2 x_2(t)$$

Example: Potentiometer





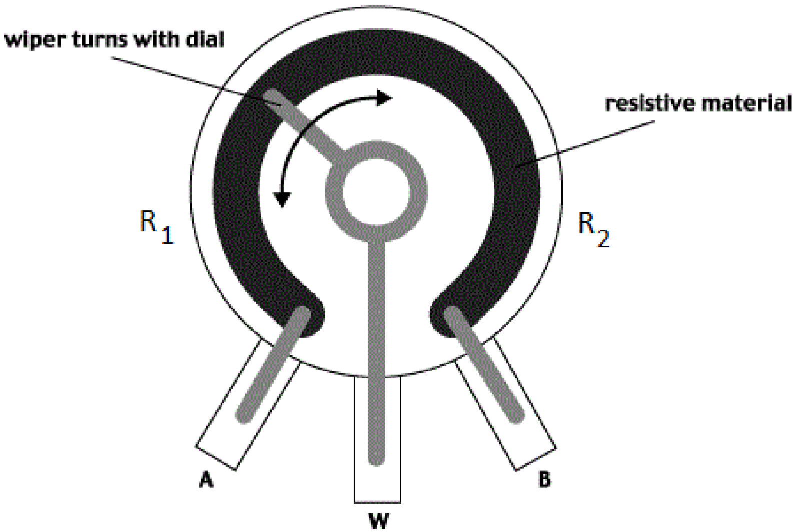
Example: Potentiometer

- The resistance between the wiper (slider) and "A" is labeled R_1, the resistance between the wiper and "B" is labeled R_2.

- The total resistance between "A" and "B" is constant, $R_1+R_2=R_{tot}$.

- If the potentiometer is turned to the extreme counter clockwise position such that the wiper is touching "A" we will call this $\theta=0$; in this position $R_1=0$ and $R_2=R_{tot}$.

- If the wiper is in the extreme clockwise position such that it is touching "B" we will call this $\theta=\theta_{max}$; in this position $R_1=R_{tot}$ and $R_2=0$.

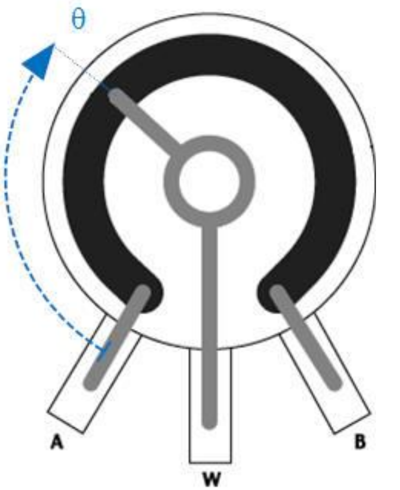


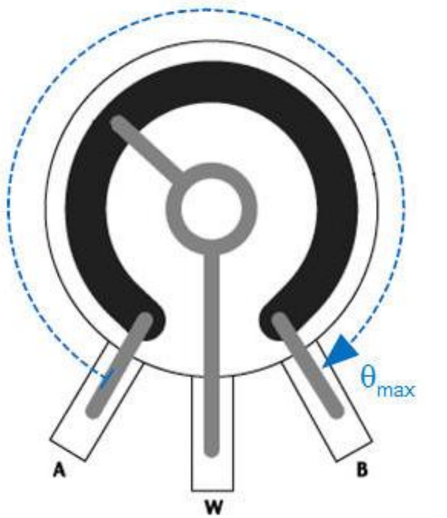
Example: Potentiometer

✓R_1 and R_2 vary linearly with θ between the two extremes:

$$R_1 = \frac{\theta}{\theta_{\max}} R_{tot}$$

$$R_2 = \frac{\theta_{\max} - \theta}{\theta_{\max}} R_{tot}$$





Example: Potentiometer

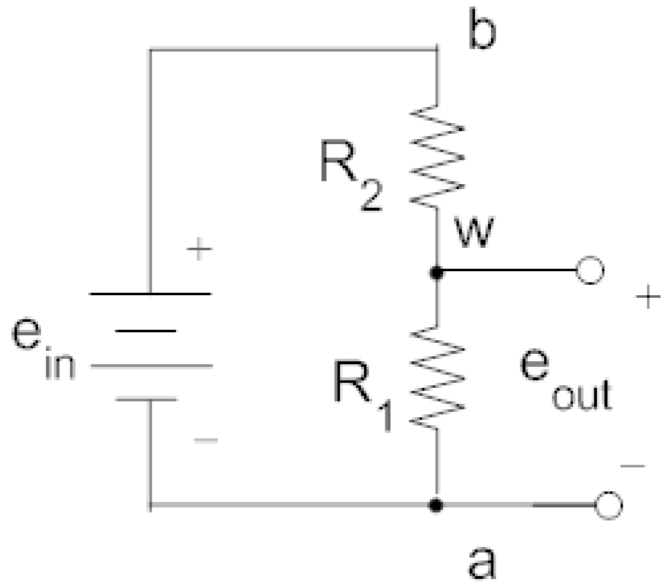
- ✓ Potentiometer can be used to sense angular position, consider the circuit of figure-1.

- ✓ Using the voltage divider principle we can write:

$$e_{out} = \frac{R_1}{R_1 + R_2} e_{in} = \frac{R_1}{R_{tot}} e_{in}$$

$$e_{out} = \frac{\theta}{\theta_{\max}} e_{in}$$

Figure-1

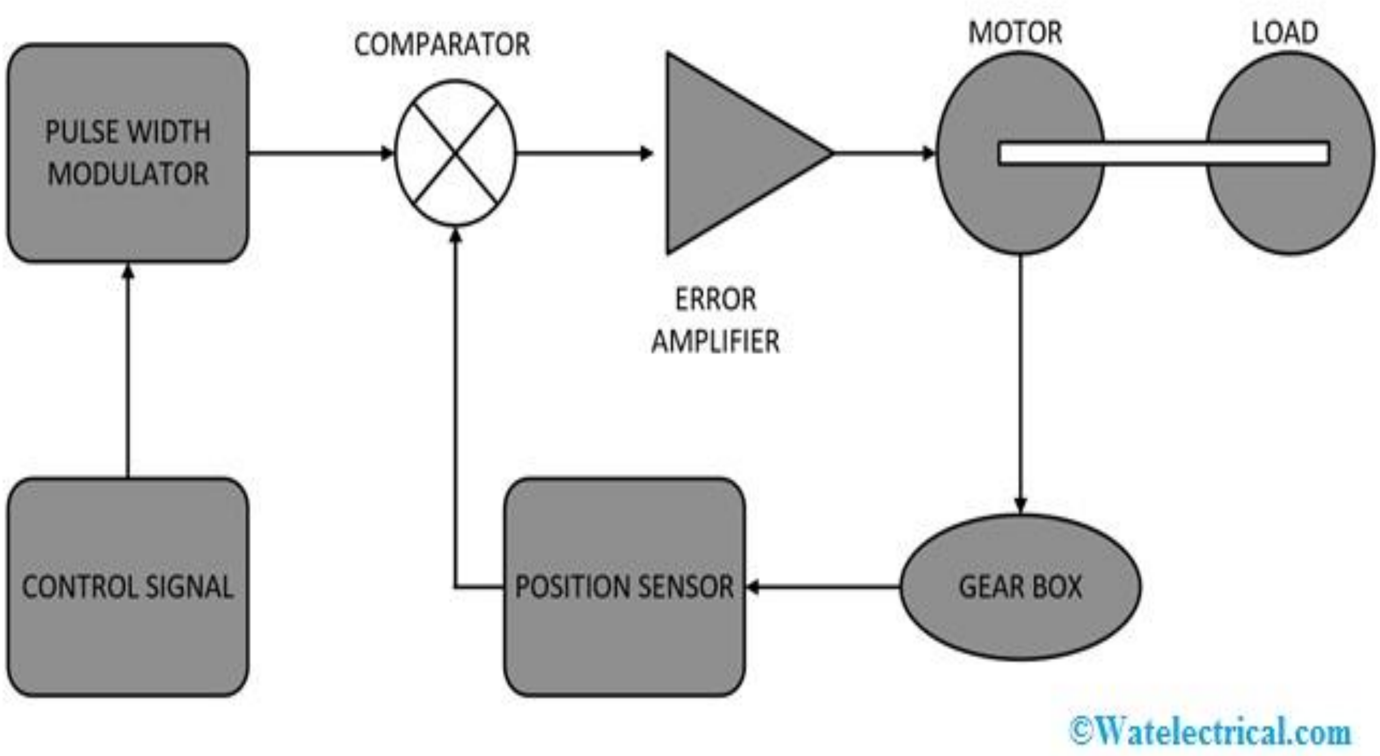


Servo Drives:

- ✓ Designed to convert electrical power into precision controlled motion
    1. Controlled torque(torque servo)
    2. Controlled speed(velocity servo)
    3. Controlled position(position servo)
- ✓ Requires at least three elements
    1) The motor
    2) Feedback of some sort
    3) An amplifier
- ✓ Industrial motion control
- ✓ Factory automation, robotics, machine tool and packaging etc.....

Servo Drives:

✓ Can be Either rotary or linear and Either DC or AC



Servomotor:

- ✓ The servomotor includes the motor that drives the load and a position detection component, such as an encoder.
- ✓ The most common type is brushless motor.
- ✓ The rotor has a powerful permanent magnet.
- ✓ The stator is composed of multiple conductor coils and the rotor spins when the coils are powered in the specified order.
- ✓ The movement of the rotor is determined by the stator's frequency, phase, polarity and current when the correct current is applied to the stator coils at the appropriate time.

Servomotor: **Encoder**

- ✓ Encoder allows high-speed and high-precision control according to the given position and speed commands.
- ✓ Generate speed and position feedback.
- ✓ Many cases built into the servomotor or attached to the servomotor.

Stepper motor:

A stepper motor is a special electrical machine which rotates in discrete angular steps in response to a programmed sequence of input electrical pulses.

- Operate on discrete control pulses.
- Rotate in discrete steps.

Construction:

- ✓ The stator has excitation windings. The excitation voltage to the coils is D.C and the number of phases indicate the number of windings.
- ✓ The rotor is of salient structure without any windings, and it may or may not have permanent magnets
- ✓ Normally of two types

  1) Permanent magnet

  2) Variable reluctance

Stepper motor: Working Principle

A magnetic interaction takes place between the rotor and the stator, which make rotor move.

The stator pole pitch,

$$\theta_s = \frac{360^0}{Number\ of\ stator\ pole\ teeth} = \frac{360^0}{\mathbf{Ns}}$$

The rotor pole pitch,

$$\theta_r = \frac{360^0}{Number\ of\ rotor\ pole\ teeth} = \frac{360^0}{\mathbf{Nr}}$$

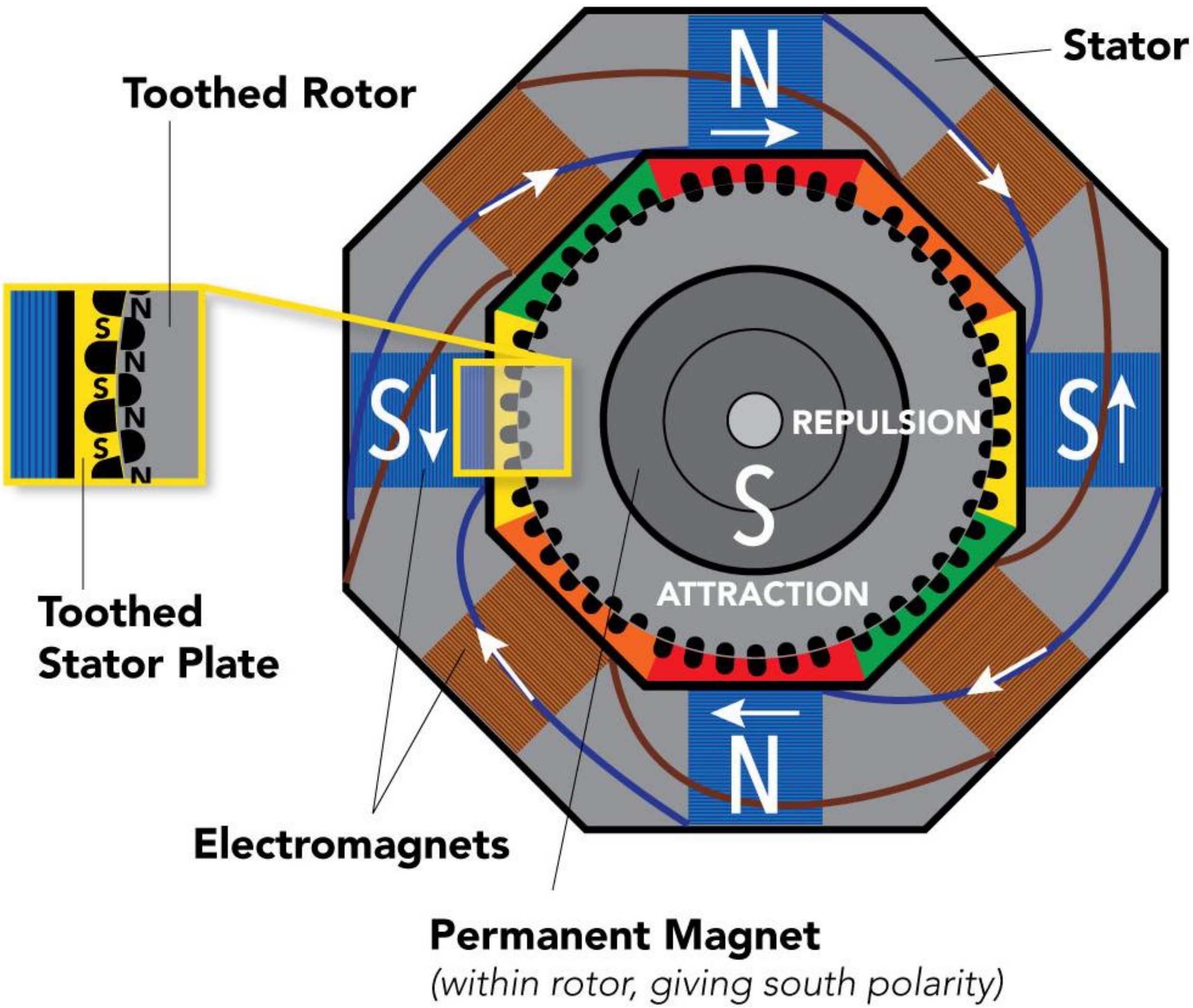
The full step angle,

$$\theta_{fs} = \theta_s \sim \theta_r$$

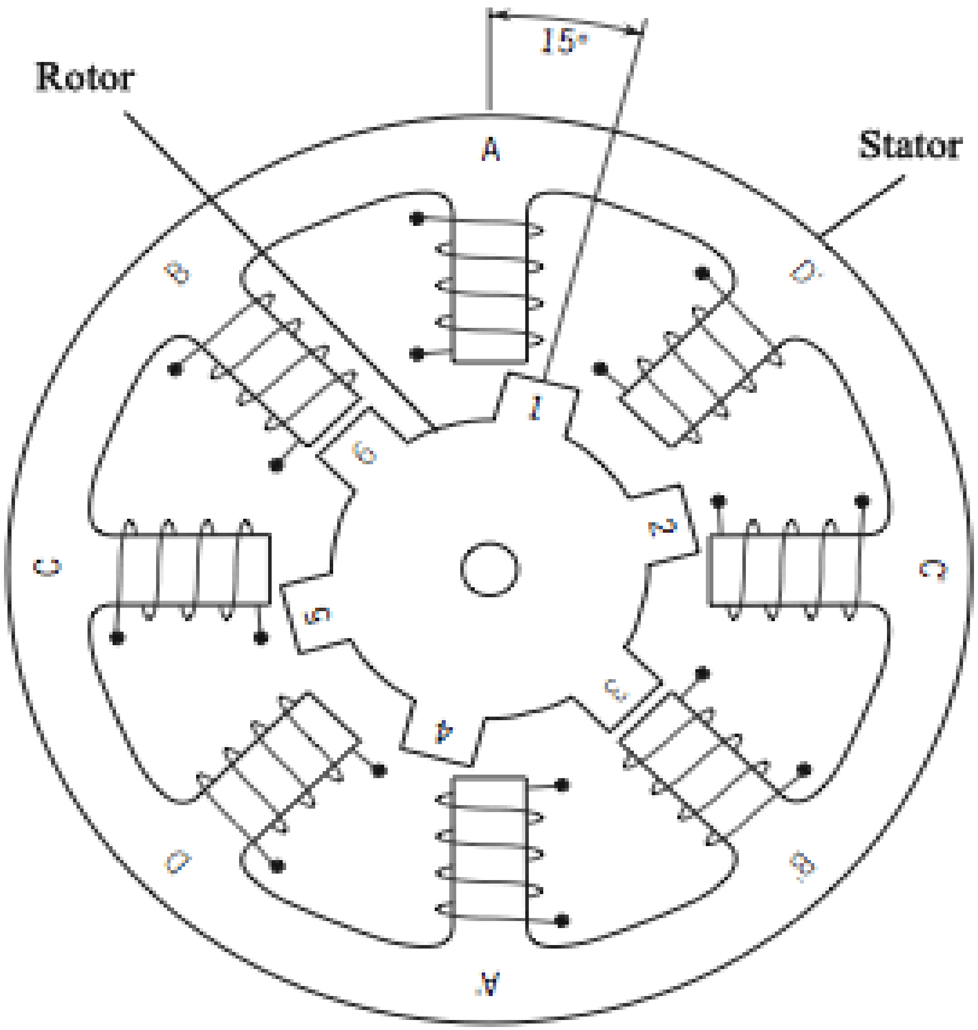
The half step angle,

$$\theta_{hs} = (\theta_s \sim \theta_r)/2$$

Stepper motor:



Stepper motor:



The stator pole pitch,

$$\theta_s = \frac{360^0}{\text{Ns}} = \frac{360}{8} = 45$$

The rotor pole pitch,

$$\theta_r = \frac{360^0}{\text{Nr}} = \frac{360}{6} = 60$$

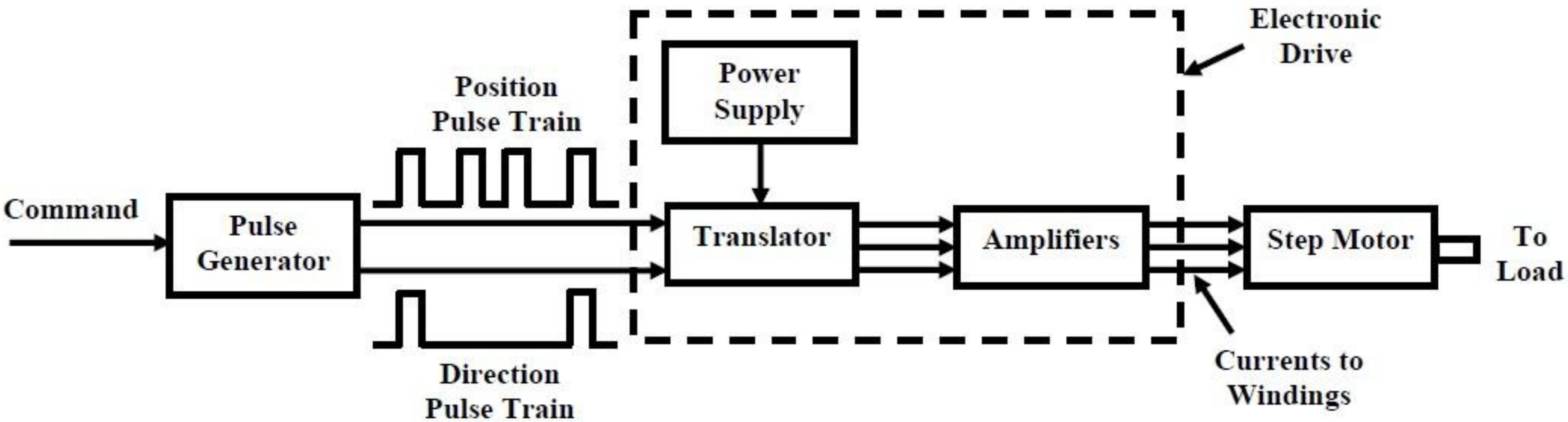
The full step angle,

$$\theta_{fs} = \theta_s \sim \theta_r = 60\text{-}45 = 15$$

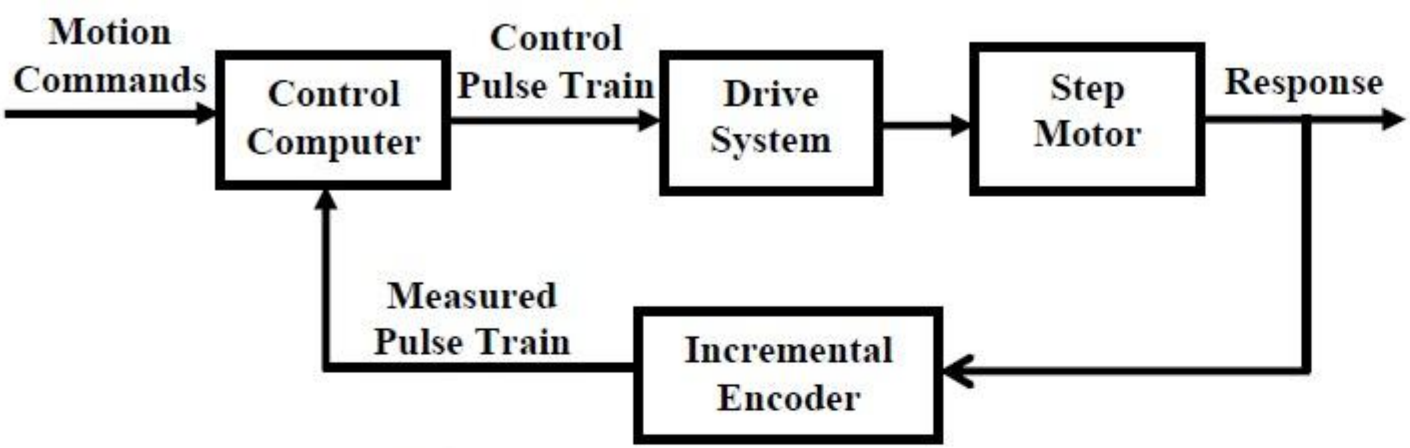
The half step angle,

$$\theta_{hs} = (\theta_s \sim \theta_r)/2 = 15/2\ 7.5$$

Stepper motor:



Open-loop control of a step motor



Feedback control of a step motor

Stepper motor:

Application:

- ✓ Application of stepper motor in diverse areas ranging from a small wrist watch to artificial satellites.
- ✓ Widely employed in industrial control, specifically for CNC machines.
- ✓ Power range 1W to 2.5KW
- ✓ Torque range 1μN to 40 Nm