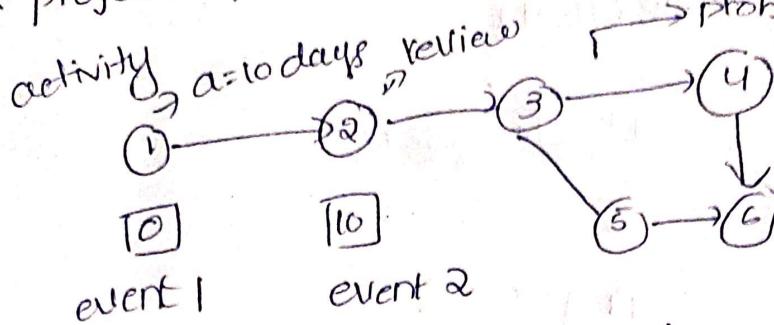


Unit 6

Unit 6

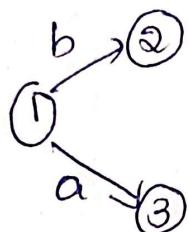
Network models (Project management)

* project → events and activities
..... → problem networks



* only one ending event and one starting event

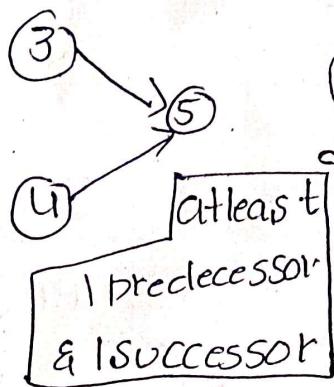
Burst event



Events

activities

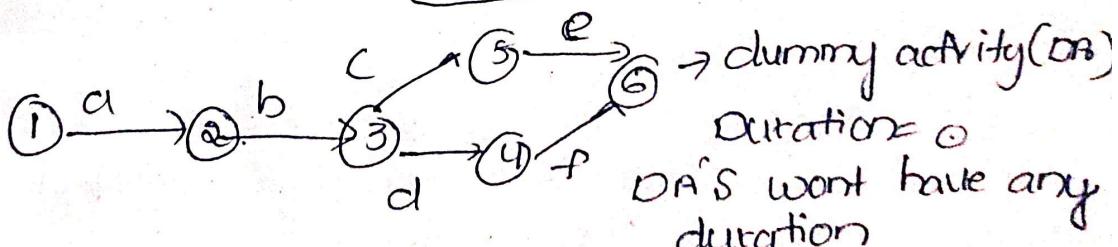
Merge event



Dual event

<u>event</u>	<u>pred</u>	<u>succ</u>
1	-	2
2	1	3
3	2	4,5
4	3	6
5	3	6
6	4,5	-

heliocentric
n - a dual event



$\leq 6 \rightarrow$ no. of DA's

* Each activity and event must have different sign

* It should only move from left to right (\rightarrow)

* we should use only one arrow, should not use multiple arrows. It should be straight

* Arrows should not intersect.

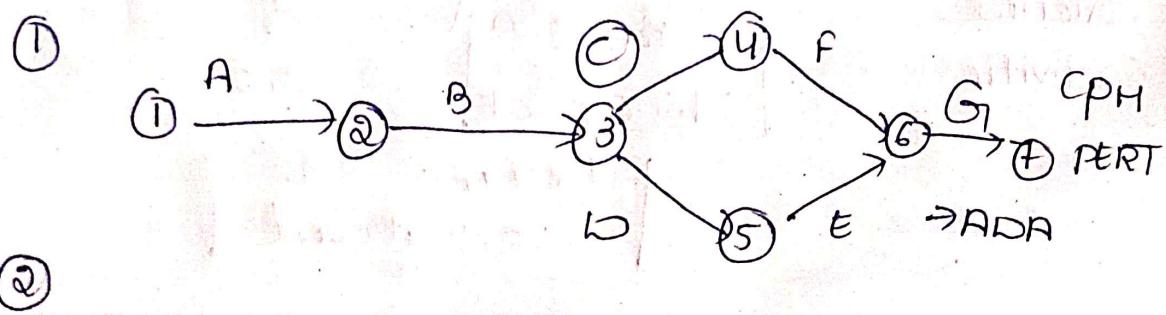
Network Diagram

- ① Activity on Arrow (AOA)
- ② Activity on Node (AON)

① Activity Predecessor

A	-
B	A
C	B
D	B
E	D
F	C
G	E, F

Network diagram



* We can draw in different structures ↓ don't have

EST, LFT can't
found them

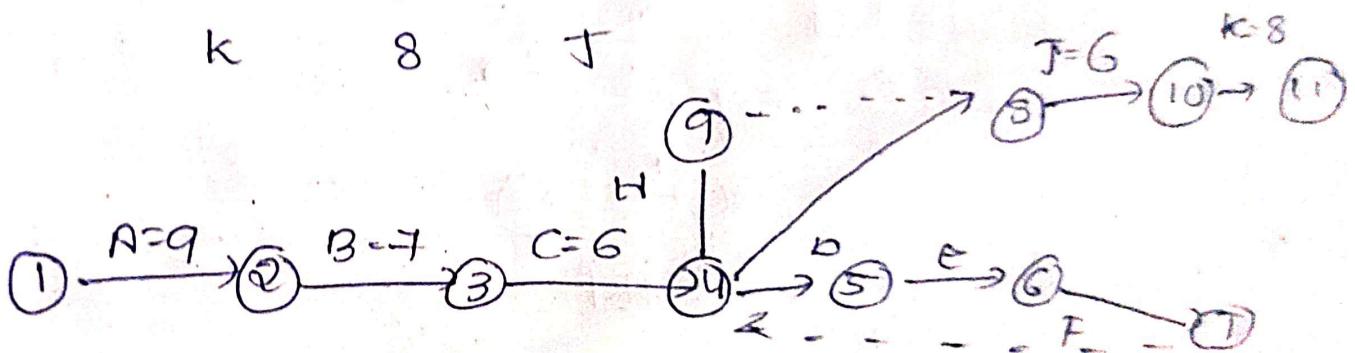
(P) Activity Duration Predessor

A	9	-
B	7	A
C	6	B
D	4	C
E	2	D
F	3	E
G	2	E } Burst

H	1	C
I	5	C,F } Burst

Merge: ↙ I ↘ J G, H, I

K 8 J



events = 11

Activities = 11

Dummy Activities = 2

* only one starting & one ending event

Critical Path method (CPM) (or) Deterministic model

(P) Activity duration (weeks)

A(1-2) 7

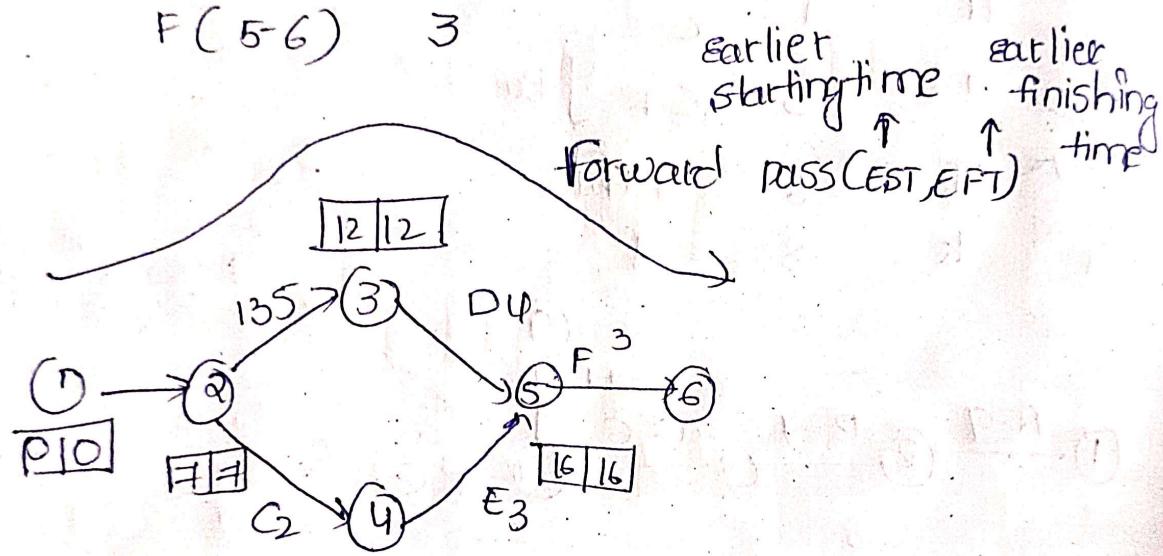
B(2-3) 5

C(2-4) 2

D(3-5) 4

E(4-5) 3

F(5-6) 3



Backward pass (LST, LFT)

↓ ↗ late
later finishing time
standing time

$$* \text{ EFT} = \text{EST} + D$$

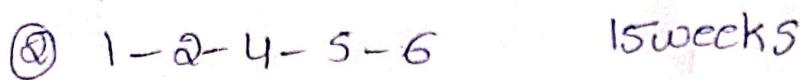
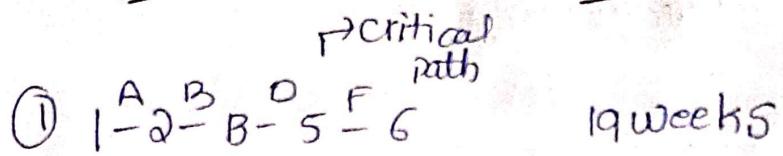
$$* \text{ LST} = \text{LFT} - D$$

$$* \text{ Float} = \text{LFT} - \text{EFT} \text{ (or)} \text{ LST} - \text{EST}$$

↓
float = 0 \rightarrow no duration \rightarrow critical activity (or)
job (or) task

\rightarrow no freedom to reschedule

↓
highest duration

Paths

① EST for ② = EST of ① + Activity Duration

② EST for ⑤ = $\max \left\{ \begin{array}{l} \text{EST of } ③ + 'D' \text{ duration} \\ \text{EST of } ④ + 'E' \text{ duration} \end{array} \right\}$

③ LFT of ⑤ = LFT of ⑥ - 'F' duration

④ LFF of ② = $\min \left\{ \begin{array}{l} \text{LFT of } ③ - 'B' \text{ duration} \\ \text{LFT of } ⑨ - 'C' \text{ duration} \end{array} \right\}$

For starting path $EST = LFT$

ending path. $EST = LFT$

$LST - EST$
 (or)
 $LFT - EFT$

<u>Activity</u>	<u>Duration</u>	<u>EST</u>	<u>EFT</u>	<u>LST</u>	<u>LFT</u>	<u>float</u>
A(1-2)	7	0	7	0	7	A 0
B(2-3)	5	7	12	7	12	B 0
C(3-4)	2	7	9	11	13	4
D(3-5)	4	12	16	12	16	D 0
E(4-5)	3	9	12	13	16	4
F(5-6)	3	16	19	16	19	F 0

For A, B, D, F float = 0

↳ No freedom

C, E float = 4

have '4' freedom

(P) Activity Duration (days)

A(1-2) 5

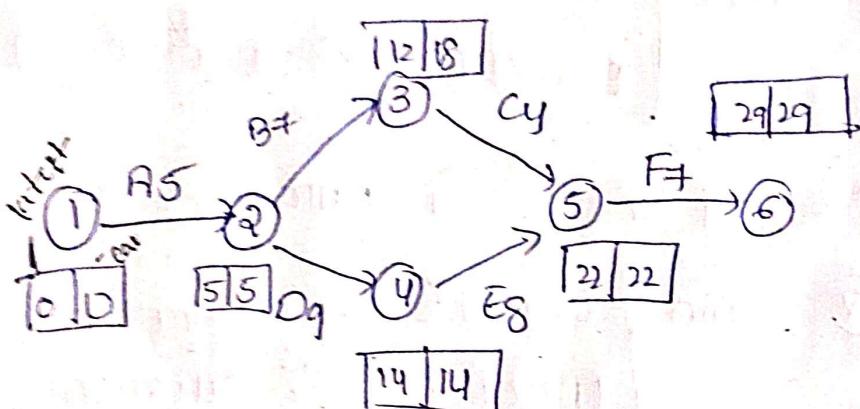
B(2-3) 7

C(3-5) 4

D(2-4) 9

E(4-5) 8

F(5-6) 4



Paths

Duration

① 1 → A → B → C → F → 6 23 days

② 1 → A → D → E → F → 6 27 days

Activity Duration EST EF LST LF Float

A(1-2) 5 0 5 0 5 0 A

B(2-3) 7 5 12 11 18 6

C(3-5) 4 12 16 18 22 6

D(2-4) 9 5 14 5 14 0 D

E(4-5) 8 14 22 14 22 0 E

F(5-6) 4 22 29 22 29 0 F

A, D, E, F → Float = 0, B, C → Float > 0

CBA

PERT

① Deterministic

probimodel

slack

② Float

$$= LFT - EFT$$

Continuous

$$LST - EST$$

$$= LFT - EFT$$

(CV)

$$LST - EST$$

Event-based

③ Activity based

* probability of completing the project in D

days

$$\tau = \frac{D - t_e}{s + D}$$

$$t_e = \text{tot} u(\ln) - 1 \cdot p$$

6

$Z = 0.00 \quad Z = 1.11$

Z	0.00	0.01	0.02	0.03	0.04
0.00	0.500				
0.01					
0.02					
⋮					
1.00					0.8413
1.10					
1.12					

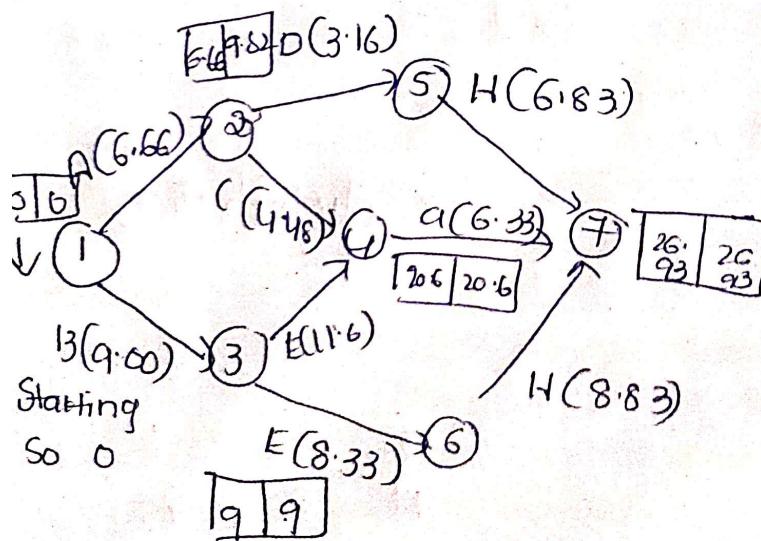
$\rightarrow Z$ distribution

71	1	1
13	80	95
2501.	501.	7501.

PERT

<u>Activity</u>	<u>\hat{t}_o</u>	<u>\hat{t}_e</u>	<u>\hat{t}_p</u>	<u>\hat{t}_c</u>
A(1→2)	3	2	9	$(3+4(1)+9)/6 = 6.66$
B(1→3)	5	9	13	$(5+36+13)/6 = 9$
C(2→4)	2	5	2	4.48
D(2→5)	1	3	6	3.16
E(3→4)	8	12	14	
F(3→6)	6	9	8	11.6
G(4→7)	3	6	11	8.33
H(5→7)	4	2	9	6.33
I(6→7)	6	9	11	6.83
				8.83

Probability in 30 days Probability in estimated time
 → D value Prob = 50%.



<u>Paths</u>	<u>Duration (days)</u>
① 1-2-5-7	16.66
② 1-2-4-7	7.4
③ 1-3-4-7	26.98
④ 1-3-6-7	26.16

B, E, F, I → slack = 0
 ↓
 critical activities

	<u>EST</u>	<u>EFT</u>	<u>LST</u>	<u>LFT</u>	<u>Slack</u>
A(1-2)	0	6.66	9.46	16.12	9.46
B(1-3)	0	9.00	0	9.00	0 - B
C(2-4)	6.66	11.14	16.12	20.6	9.46
D(2-5)	6.66	9.82	16.94	20.1	10.28
E(3-4)	9	20.6	9.0	20.6	0 - E
F(3-6)	9	17.33	9.44	18.1	0.77
G(4-7)	20.6	26.93	20.6	26.93	0 - G
H(5-7)	9.8	16.65	20.1	26.93	10.28
I(6-7)	17.33	26.16	18.1	26.93	0.77

B, E, G \rightarrow slack = 0

No freedom
to reschedule

A, C, D, F, H, I, J

have freedom to
reschedule

$$\text{Variance } \left(\frac{tp - po}{6} \right)^2$$

$$B = (8/6)^2 = 1.77$$

$$E = (6/6)^2 = 1$$

$$G = (8/6)^2 = 1.77$$

total variance 4.54

$$STD = \sqrt{4.54} = 2.13$$

$$\text{Probability} = \frac{D - L_c}{std} = \frac{30 - 26.93}{2.13} = 1.44$$

z-table
 $0.425 + 0.5$
 $0.925 \cdot 1$

Crashing of a project :- To reduce cost & reduce time

* Normal → time
→ cost

* Crash → time
cost

$$\begin{aligned} * \text{slope} &= \frac{\text{Crash cost} - \text{Normal cost}}{\text{Normal time} - \text{crash time}} \\ &\downarrow \\ \text{if is same} & \end{aligned}$$

$$\begin{aligned} * \text{crash limit} &= \text{normal time} - \text{crash time} \\ &\downarrow \\ &\text{it will be changing} \end{aligned}$$

In problem they will mention changes for every iteration

* Indirect cost = Rs 20

* Total cost = Direct cost + Indirect cost

Iteration	1-2-4-5-6	1-2 A	Slope	Crash limit
	2-4 C	50	9 → can't crash	
	4-5 E	60	3 → crash	
	5-6 F	65	2	
		100	1	

we will select

with less slope

① Total cost = previous cost + slope - Indirect cost

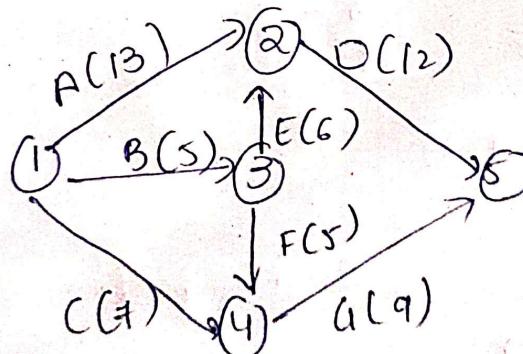
⇒ 2500

→ we have to stop
iteration

2423	It must be
2470	less than
2460	previous
2465	X

① <u>Activity</u>	Normal <u>time</u>	weeks Normal <u>cost</u>	(days)		weeks crash <u>time</u>	crash <u>cost</u>	<u>slope</u>
			Normal	crash			
A	1-2	13	400	9	900	50	
B	1-3	5	400	4	460	60	
C	1-4	4	600	4	810	70	
D	2-5	12	800	11	865	65	
E	3-2	6	900	4	1130	115	
F	3-4	5	1000	3	1150	90	
G	4-5	9	1500	6	1800	100	
			<u>5900</u>				

* Indirect cost is Rs. 160 per week
 Crash it by 1 week each time. Find the project cost & time after crashing



$$\begin{aligned}
 1-2-5 &= 25 \\
 1-3-2-5 &= 23 \\
 1-3-4-5 &= 19 \\
 1-4-5 &= 16
 \end{aligned}$$

$$\text{Indirect cost} = 25 \times 160 = \text{Rs. } 4000$$

Total cost = Direct + Indirect cost

$$5900 + 4000$$

$$\text{Rs. } 9900$$

Iteration

1-2-5 A(1-2)

Slope

50

Crash limit

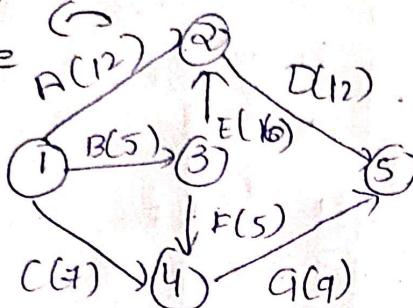
(NT - CT)

D(2-5)

65

4 (13.9 : 4)

Crashed
decrease



Critical path

$$1-2-5 = 24$$

$$1-3-2-5 = 23$$

$$1-3-4-5 = 19$$

$$1-4-5 = 16$$

$$\begin{aligned} \text{Total cost} &= 9900 + 50 - 160 \\ &= 9790 \end{aligned}$$

Iteration 2

1-2-5 A(1-2)

Slope

50

Crash limit

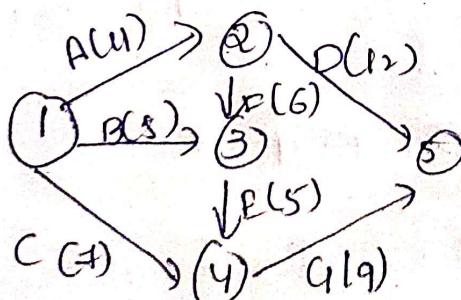
D(2-5)

65

3

1

$$\begin{aligned} \text{Total cost} &= 9790 + 50 - 160 \\ &= 9686 \end{aligned}$$



Critical path

$$1-2-5 = 23 \checkmark$$

$$1-3-2-5 = 23 \checkmark$$

$$1-3-4-5 = 19$$

$$1-4-5 = 16 \checkmark$$

Iteration 3

① 1-2-5

A(1-2)

Slope

50

α

to avoid
simultaneously
creating

D(2-5)

65*

2

we take
'65' as

② 1-3-2-5

B(1-3)

E(3-2)

D(2-5)

60

115

1

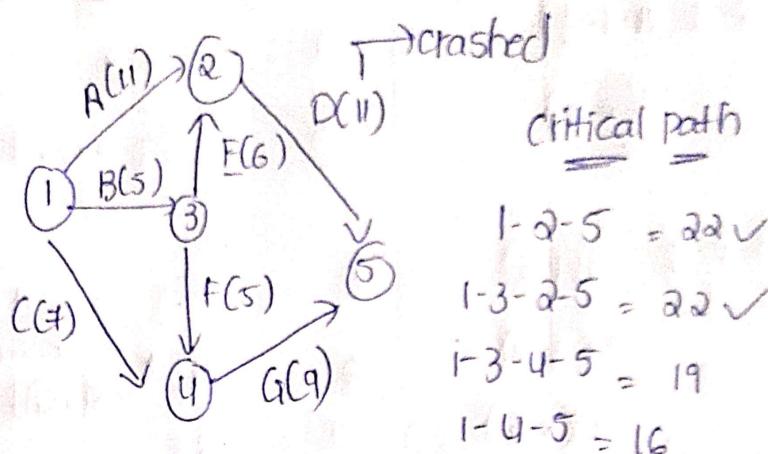
slope

65*

2

1

$$\text{Total cost} = 9680 + 65 - 160 = \text{Rs } 9585$$



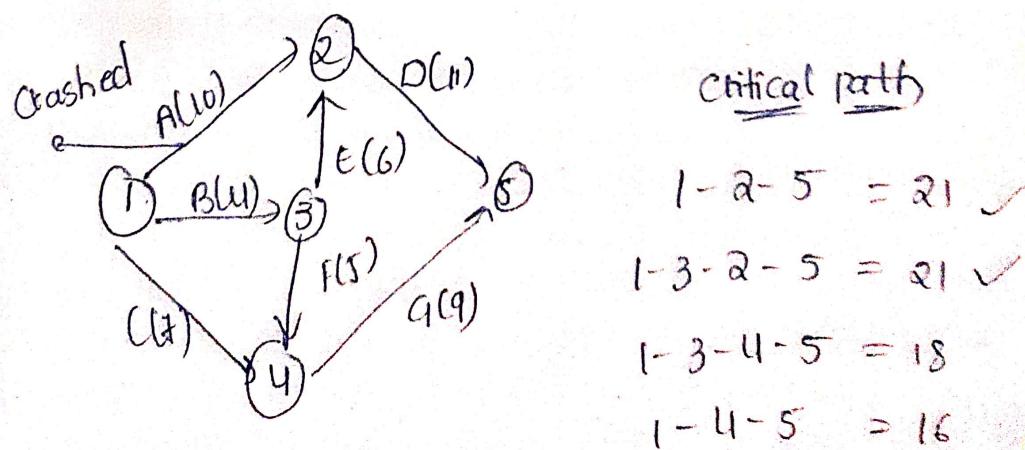
Iteration 4

		slope	CL
①	1-2-5 A(1-2)	50*	2
	D(2-5)	65	0

②	1-3-2-5 B(1-3)	60*	1
	E(3-2)	115	2
	D(2-5)	65	0

By crashing 'A', 'B' is crashed simultaneously

$$\begin{aligned} \text{Total cost} &= 9585 + 50 + 60 - 160 \\ &= \text{Rs } 9535 \end{aligned}$$



Iteration - 5

	<u>CL</u>	Slope
I-2-5 A(1-2)	1	50*
D(2-5)	0	65 → can't crash
I-3-2-5 B(1-3)	0	60 → can't crash
E(3-2)	2	115* → because CL > 0 → simultaneous crash
D(2-5)	0	65

Total cost = 9535 + 50 + 115 - 160
= Rs. 9540/- → it increases when compared to previous

So, here iteration stops

* project cost = Rs. 9535

* Project time = 21 weeks (10+11+21)