TIME DOMAIN/RESPONSE ANALYSIS

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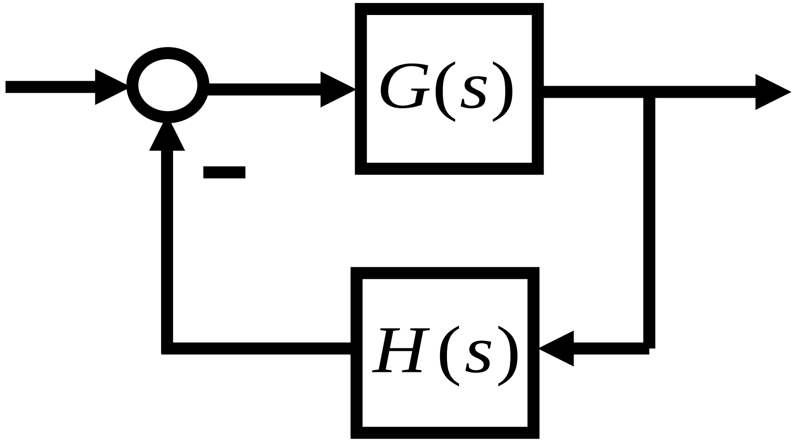
Contents:

Introduction

- **The time response of the system is the output of the closed loop system as a function of time [c(t)].**
- Can be obtained by solving differential equation governing the system or from the transfer function of the system and input to the system.



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

Introduction

- C(S) = T(S)*R(S)
- C(t) = Inverse Laplace of [C(S)]
- When the response of the system is changed from rest or equilibrium, it takes some time to settle down.
- Time response consists of two parts

1) Transient response
2) Steady state response

Transient response: Response when the input changes from one state to another.

Steady state response: Response as time, t approaches infinity.

Standard test signals

- The characteristics of actual input signals are a sudden shock, a sudden change, a constant velocity and constant acceleration.
- The dynamic behavior of a system is therefore judged and compared under application of standard test signals.

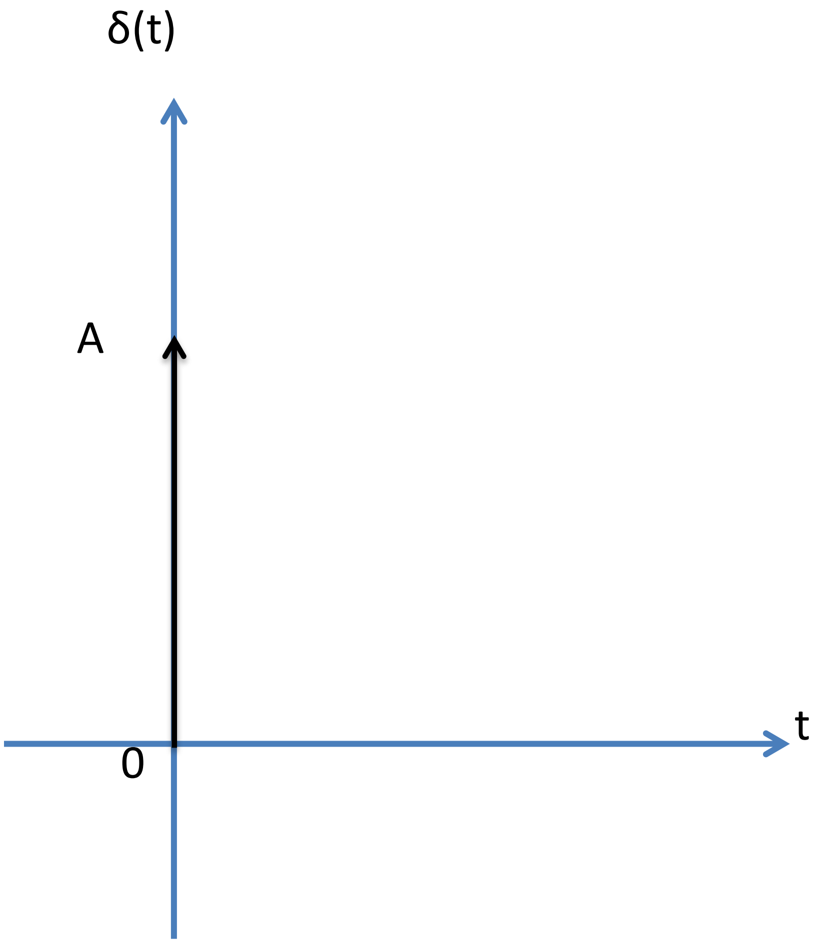
1. **Impulse signal**
2. **Step signal**
3. **Ramp signal**
4. **Parabolic signal**

Impulse signal

- The impulse signal imitate the sudden shock characteristic of actual input signal.

$$\delta(t) = \begin{cases} A & t = 0 \\ 0 & t \neq 0 \end{cases}$$

- If A=1, the impulse signal is called unit impulse signal.

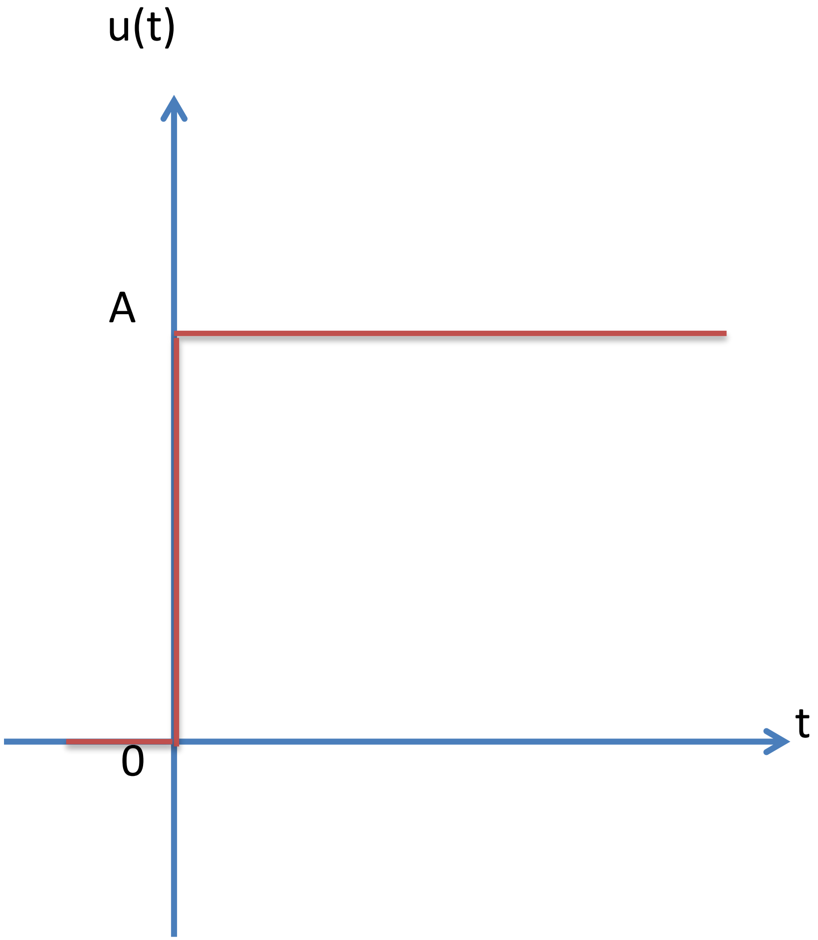


Step signal

- The step signal imitate the sudden change characteristic of actual input signal.

$$u(t) = \begin{cases} A & t \geq 0 \\ 0 & t < 0 \end{cases}$$

- If A=1, the step signal is called unit step signal

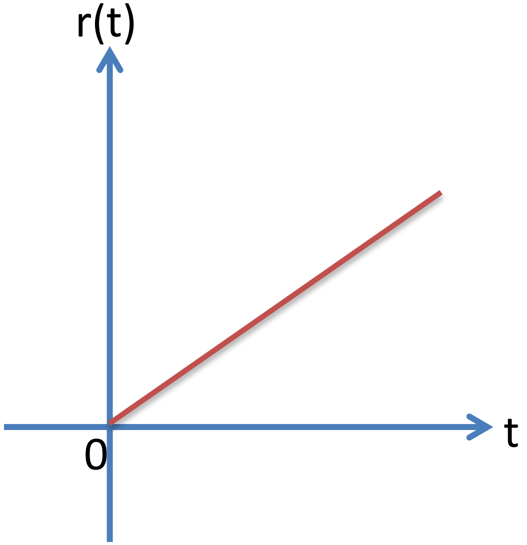


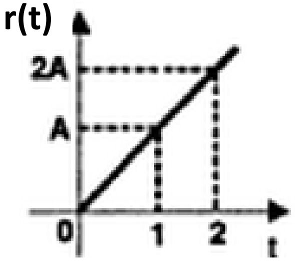
Ramp signal

- The ramp signal imitate the constant velocity characteristic of actual input signal.

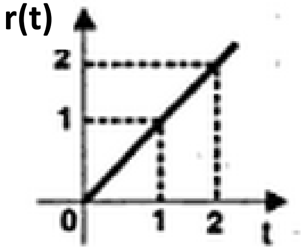
$$r(t)=\begin{cases} At & t\geq 0 \\ 0 & t<0 \end{cases}$$

- If *A=1*, the ramp signal is called unit ramp signal





ramp signal with slope A



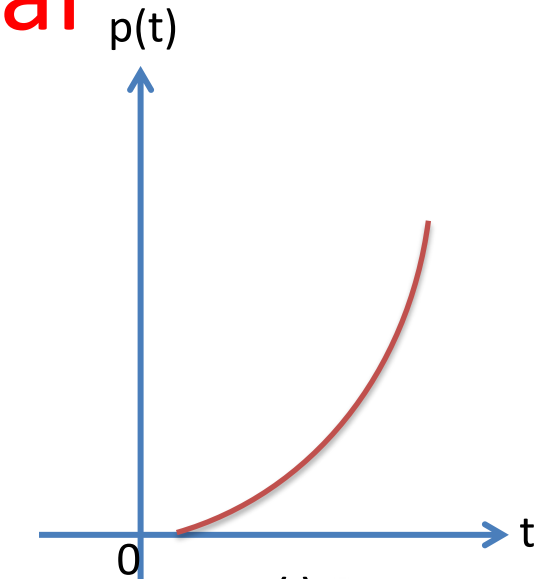
unit ramp signal

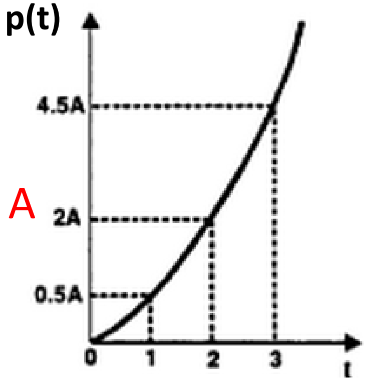
Parabolic signal

- The parabolic signal imitate the constant acceleration characteristic of actual input signal.

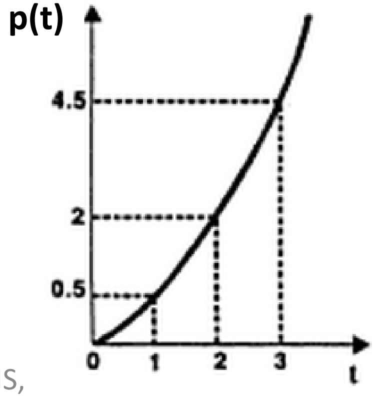
$$p(t) = \begin{cases} \frac{At^2}{2} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

- If *A=1*, the parabolic signal is called unit parabolic signal.





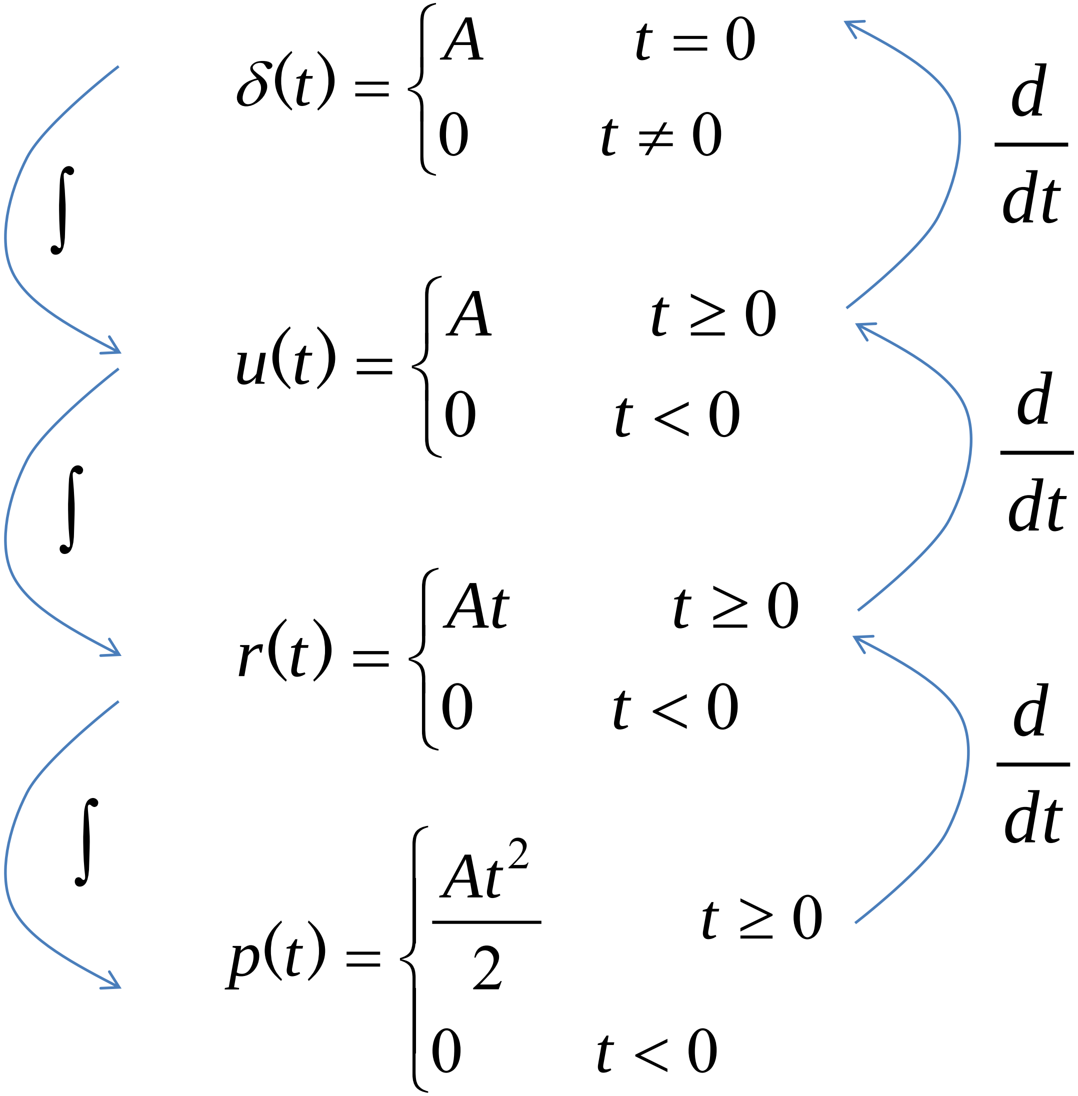
parabolic signal with slope A



Unit parabolic signal

Relation between standard Test Signals

- Impulse
- Step
- Ramp
- Parabolic



$$\delta(t)=\begin{cases}A & t=0\\ 0 & t\neq 0\end{cases}$$

$\int$ $\qquad$ $\frac{d}{dt}$

$$u(t)=\begin{cases}A & t\geq 0\\ 0 & t<0\end{cases}$$

$\int$ $\qquad$ $\frac{d}{dt}$

$$r(t)=\begin{cases}At & t\geq 0\\ 0 & t<0\end{cases}$$

$\int$ $\qquad$ $\frac{d}{dt}$

$$p(t)=\begin{cases}\frac{At^2}{2} & t\geq 0\\ 0 & t<0\end{cases}$$

Laplace Transform of Test Signals

Impulse

$$\delta(t) = \begin{cases} A & t = 0 \\ 0 & t \neq 0 \end{cases}$$

$$L\{\delta(t)\} = \delta(s) = A$$

Step

$$u(t) = \begin{cases} A & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$L\{u(t)\} = U(s) = \frac{A}{S}$$

Laplace Transform of Test Signals

Ramp

$$r(t) = \begin{cases} At & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$L\{r(t)\} = R(s) = \frac{A}{s^2}$$

Parabolic

$$p(t) = \begin{cases} \frac{At^2}{2} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$L\{p(t)\} = P(s) = \frac{A}{S^3}$$

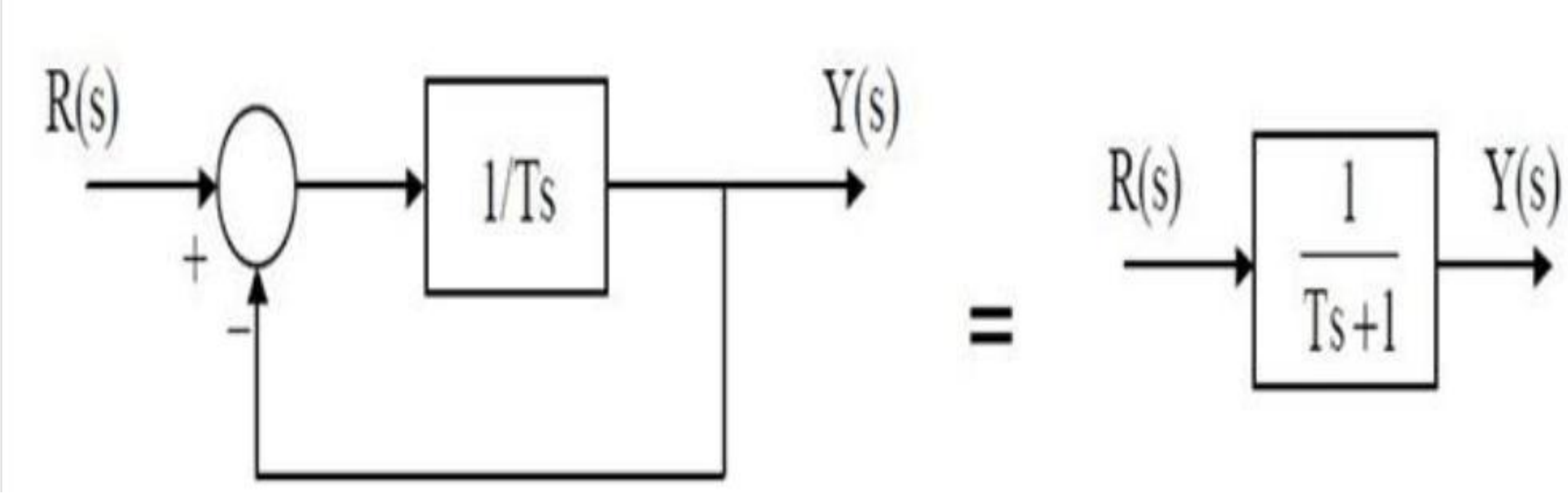
Time Response of Control Systems

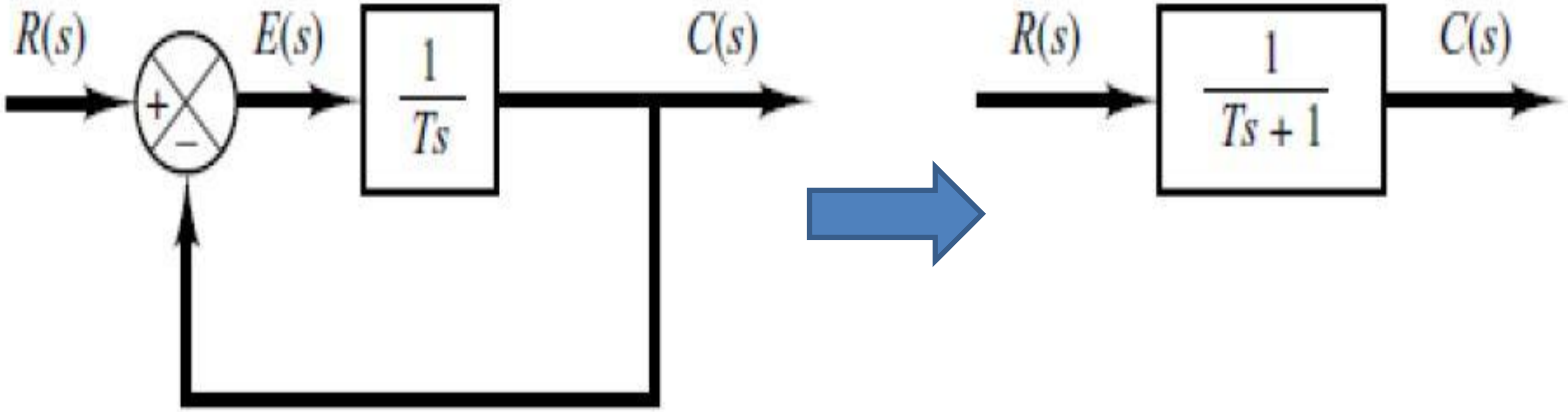
- ✓How quickly a system responds.
- ✓Transient response is dependent upon the system poles only and not on the type of input.
- ✓It is therefore sufficient to analyze the transient response using a step input.
- ✓The steady-state response depends on system dynamics and the input quantity.
- ✓It is then examined using different test signals by final value theorem.
- ✓Any time function can be expressed in terms of linear combinations of test signals and hence output of the system can be obtained by superposition principle.
- ✓Convolution integral can also be used to determine the response of a linear system for any given input, if the response is known for a step or an impulse input.

Time response of first order system

- ✓ The system whose input-output equation is a first order differential equation or a system whose dynamic behavior is described by a first order differential equation.
- ✓ The order of the differential equation is the highest degree of derivative present in an equation.
- ✓ First order system contains only one energy storing element. Usually a capacitor or combination of two capacitors is used for this purpose.
- ✓ Most of the practical models are first order systems.
- ✓ If a system with higher order has a dominant first order mode it can be considered as a first order system.

Time response of first order system





Time response of first order system

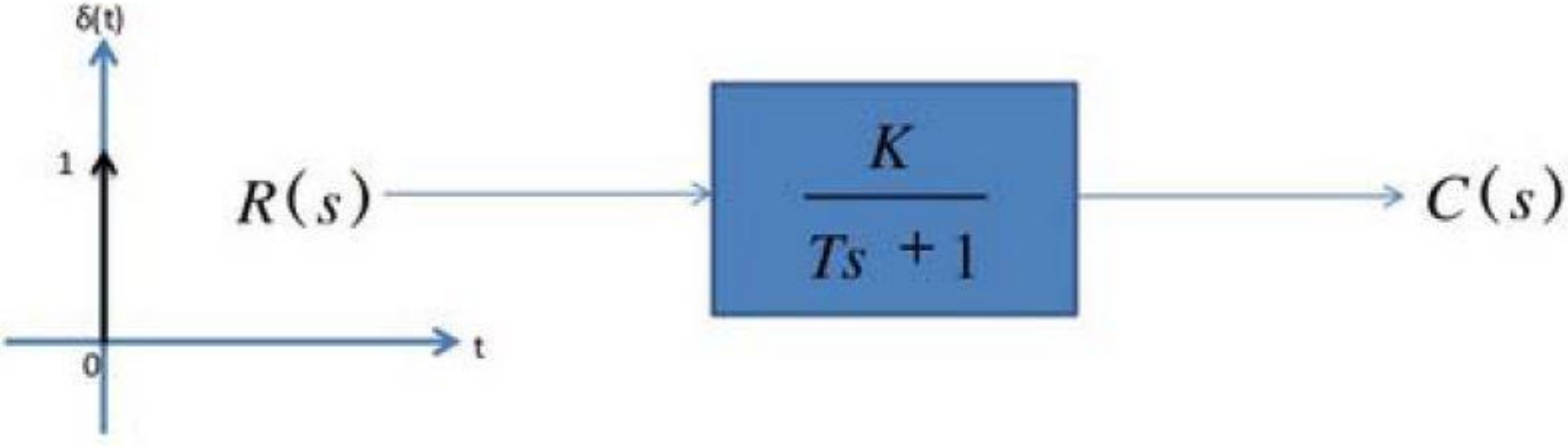
$$\frac{C(s)}{R(s)} = K\frac{1}{Ts+1}$$

- ✓ K= Gain (ratio between the input signal and steady state value of the output)
- ✓ T= time constant of the system (measure of how quickly a 1st order system responds)
- ✓ The time constant for a first order system is given by :

  T=RC (for a system with capacitors)

  T=L/R (for a system with inductors)

Impulse response of first order system

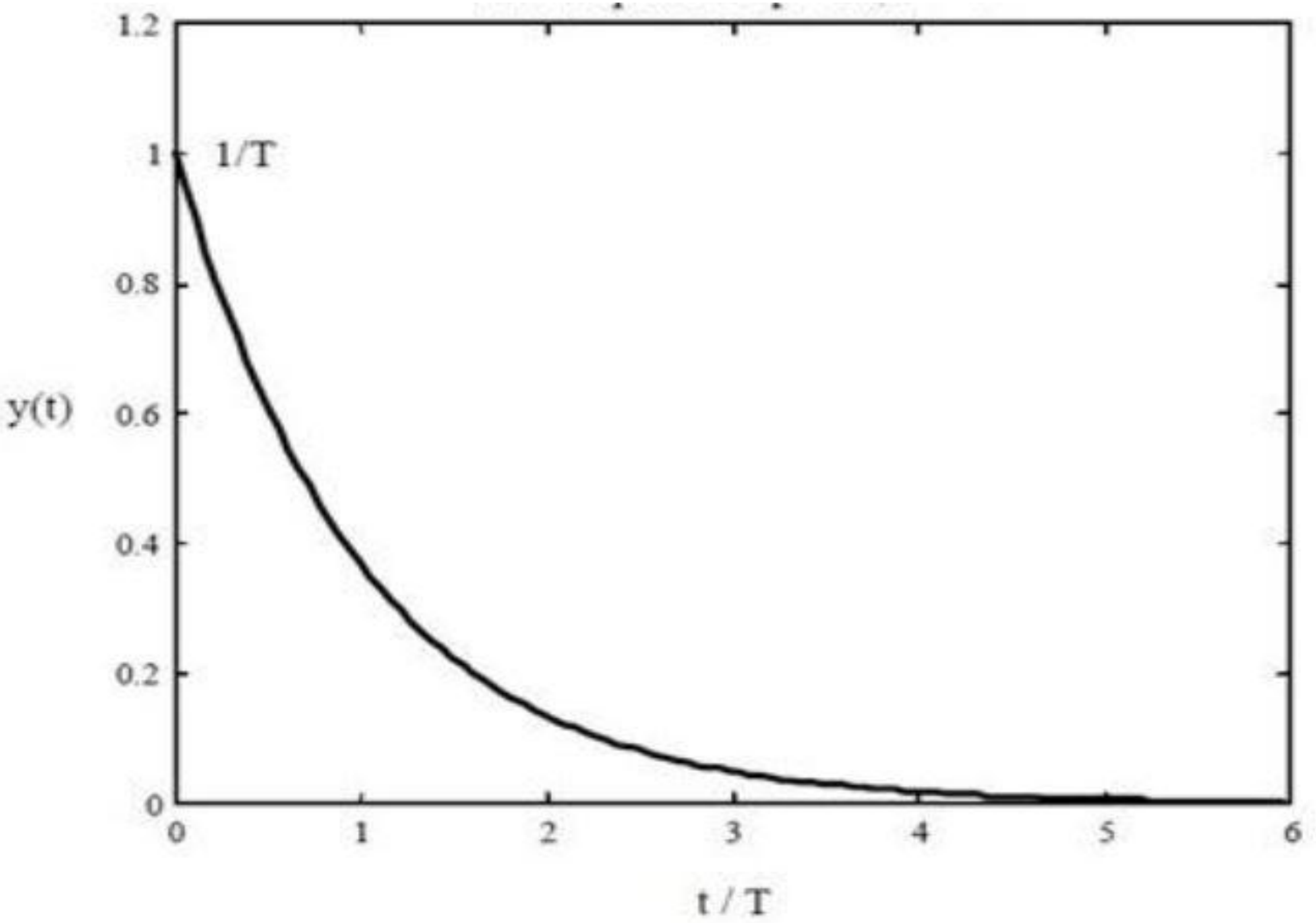


$$R(s) = \delta(s) = 1$$

$$C(s) = \frac{K}{Ts+1}$$

$$c(t) = \frac{K}{T}e^{-\frac{t}{T}}$$

Impulse response of first order system



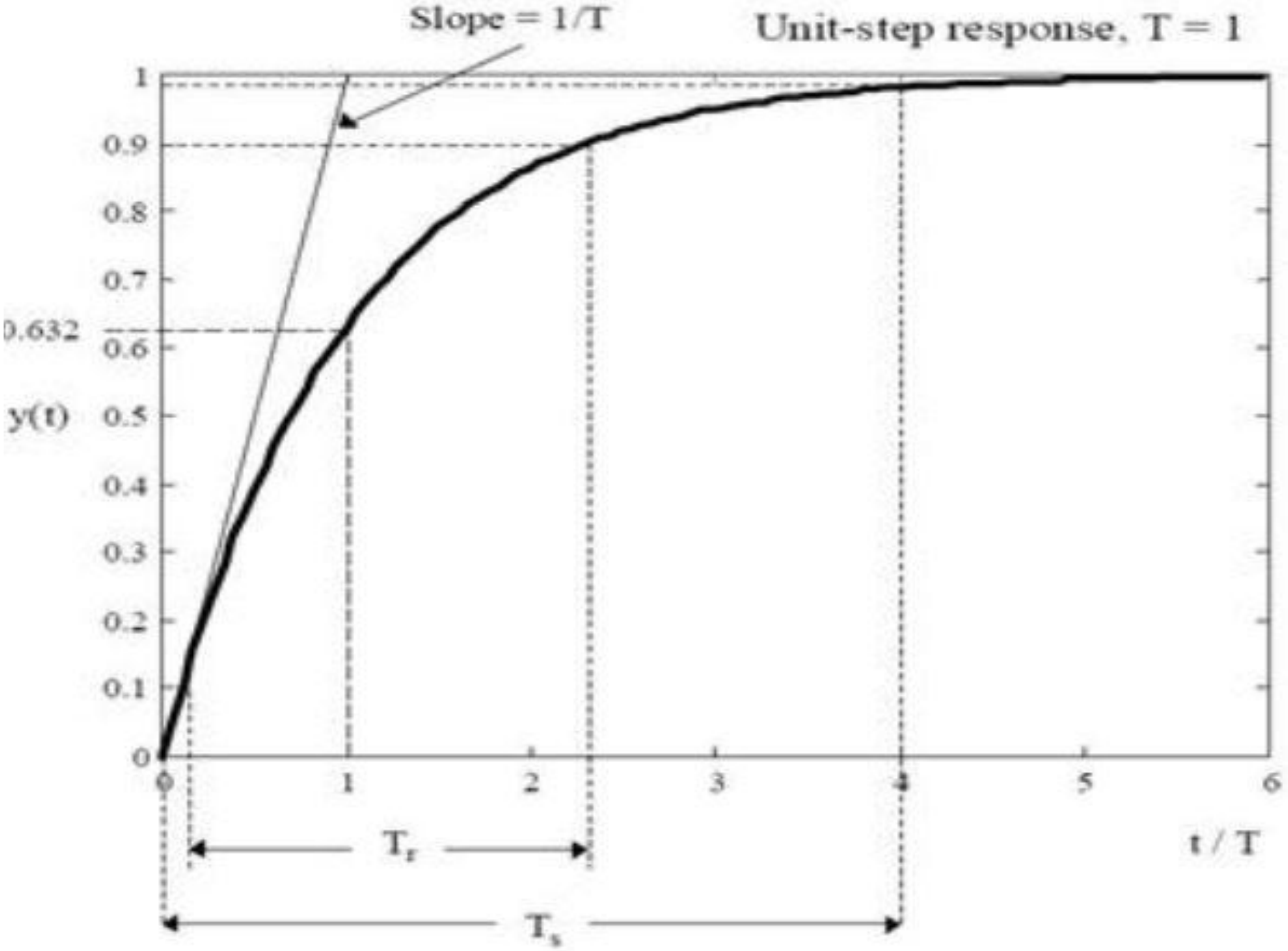
Step response of first order system

$$R(s) = \frac{1}{s}$$

$$C(s) = \frac{K}{Ts + 1}\frac{1}{s}$$

$$c(t) = Ku(t) - e^{-\frac{t}{T}}$$

Step response of first order system



Step response of first order system

- ✓ The time constant is indicative of speed of response or how fast the response is approaching the final value.
- ✓ Speed of response is inversely proportional to time constant.
- ✓ Another important characteristic is the error between desired value and actual value under steady state conditions, known as steady state error (E(s), e_{ss}).
- ✓ For step, ess = 0
- ✓ Yields the desired information about the speed of transient response.

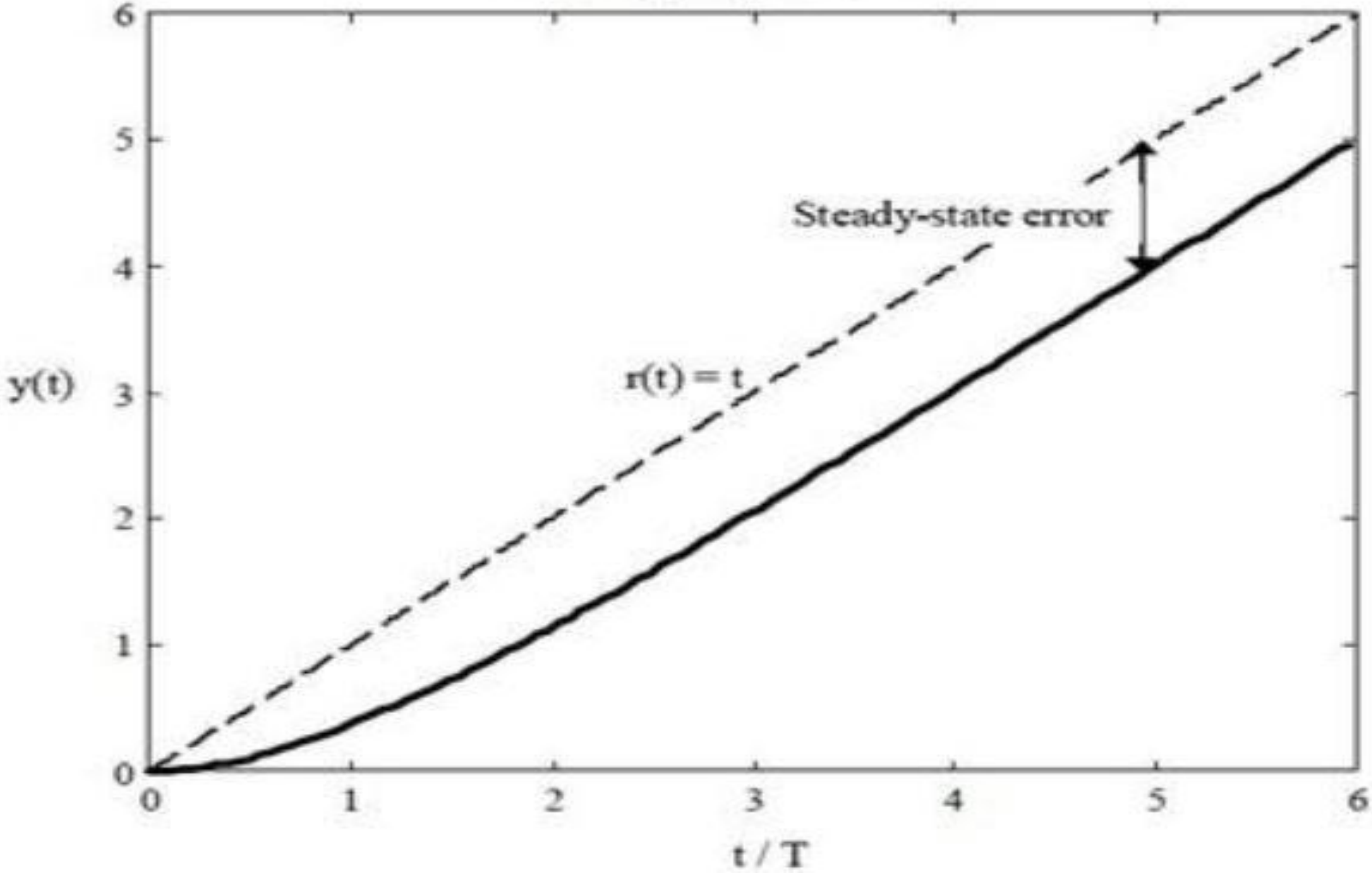
Ramp response of first order system

$$R(s) = \frac{1}{s^2}$$

$$C(s) = \frac{K}{Ts + 1}\frac{1}{s^2}$$

$$c(t) = Kt - T + Te^{-\frac{t}{T}}$$

Ramp response of first order system



Ramp response of first order system

- ✓ $e_{ss} = T$
- ✓ The 1st order system can not track the ramp input without a finite steady state error.
- ✓ Do not give any information regarding speed of response.

Parabolic response of first order system

$$R(s) = \frac{1}{S^3}$$

$$C(S) = \frac{1}{S^{3}\,(Ts+1)}$$

$$C(t) = T^2 - Tt + \frac{t^2}{2} - T^2 e^{-t/T}$$

$$e(t) = -T^2 + Tt + T^2 e^{-t/T}$$

Parabolic response of first order system

- ✓ $e_{ss} = \infty$
- ✓ A first order system has infinite steady state error for a parabolic input.
- ✓ Do not give any information regarding speed of response.
- ➢ **From the above discussions it is clear that, it is sufficient to study the behavior of any system to a unit step input for understanding transient response and use ramp and parabolic inputs for understanding the steady state behavior of the system.**

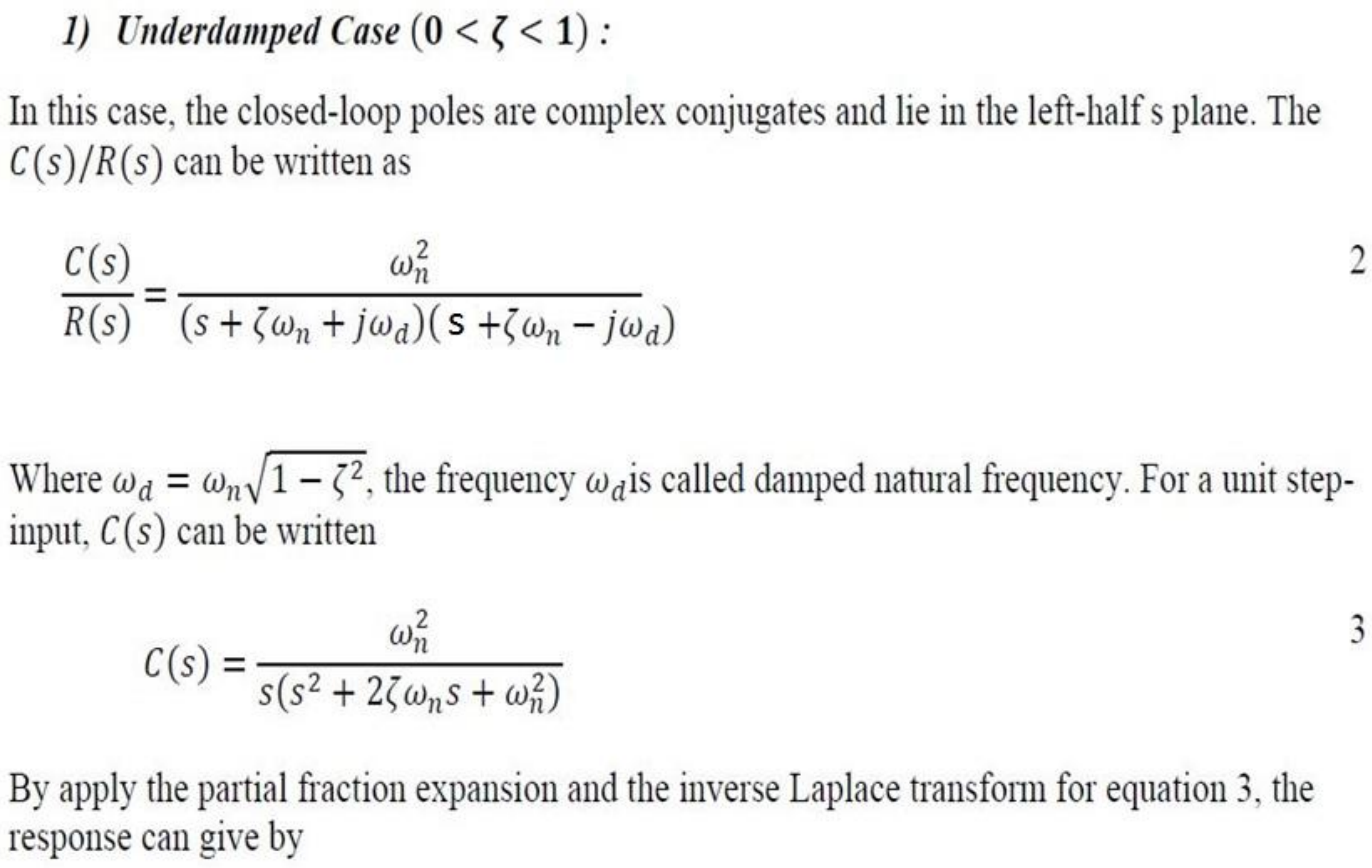
Step response of second order system

- ✓ Systems described by the second order differential equations.
- ✓ The closed loop transfer function C(S)/R(S) is given by the equation

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad 1$$

- ✓ This form is called as the standard form of second order system.
- ✓ ζ = Damping ratio (under damped, critical, over damped)
- ✓ ω_n = Natural frequency

Step response of second order system



1) Underdamped Case $(0 < \zeta < 1)$:

In this case, the closed-loop poles are complex conjugates and lie in the left-half s plane. The $C(s)/R(s)$ can be written as

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{(s + \zeta\omega_n + j\omega_d)(s + \zeta\omega_n - j\omega_d)} \qquad 2$$

Where $\omega_d = \omega_n\sqrt{1-\zeta^2}$, the frequency ω_d is called damped natural frequency. For a unit step-input, $C(s)$ can be written

$$C(s) = \frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)} \qquad 3$$

By apply the partial fraction expansion and the inverse Laplace transform for equation 3, the response can give by

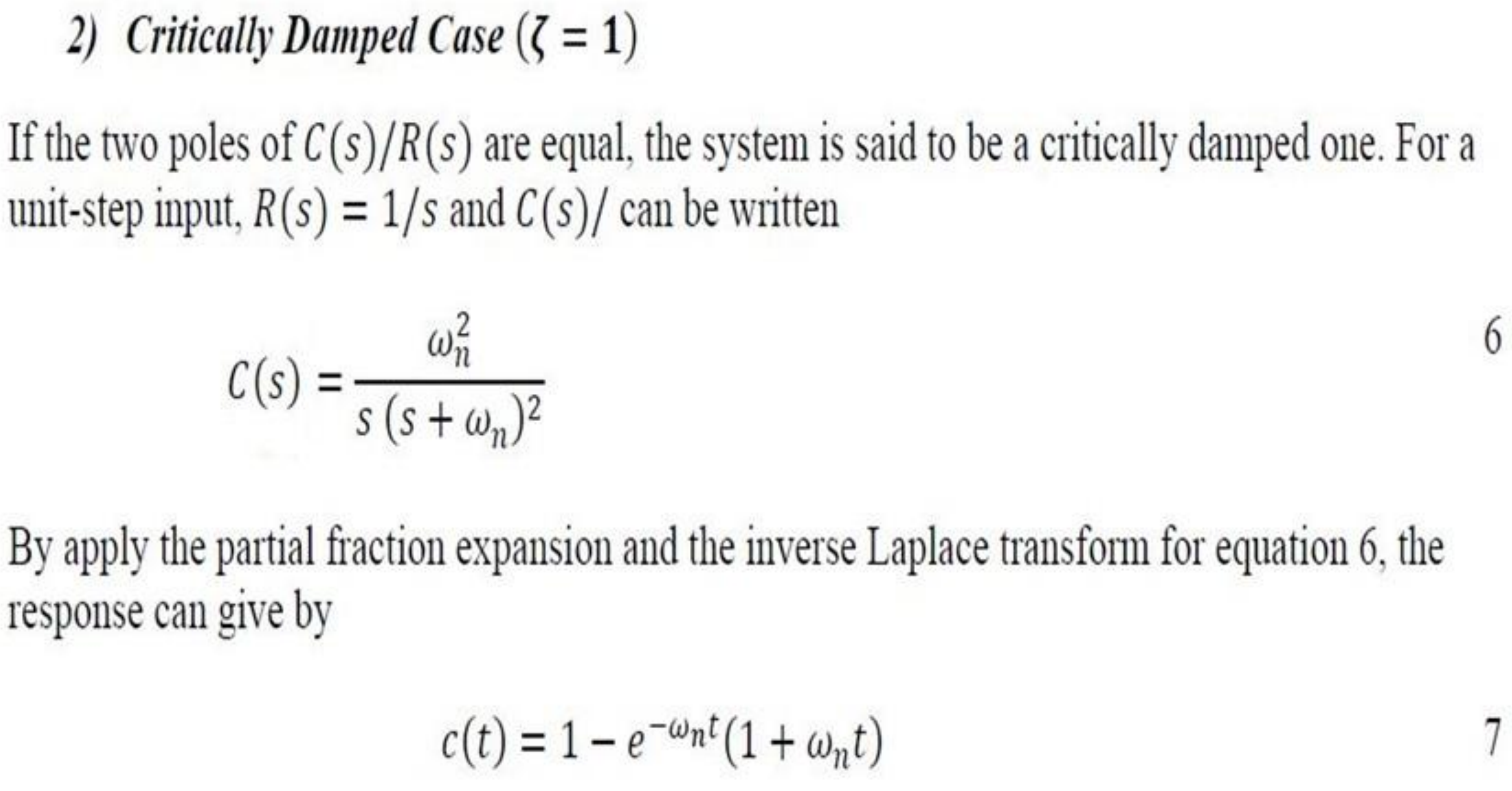
Step response of second order system

$$c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin\left(\omega_d t + \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)\right) \qquad 4$$

If the damping ratio ζ is equal to zero, the response becomes undamped and oscillations continue indefinitely. The response $c(t)$ for the zero damping case may be obtained by substituting $\zeta = 0$ in Equation 4, yielding

$$c(t) = 1 - \cos \omega_n t \qquad 5$$

Step response of second order system



2) *Critically Damped Case* $(\zeta = 1)$

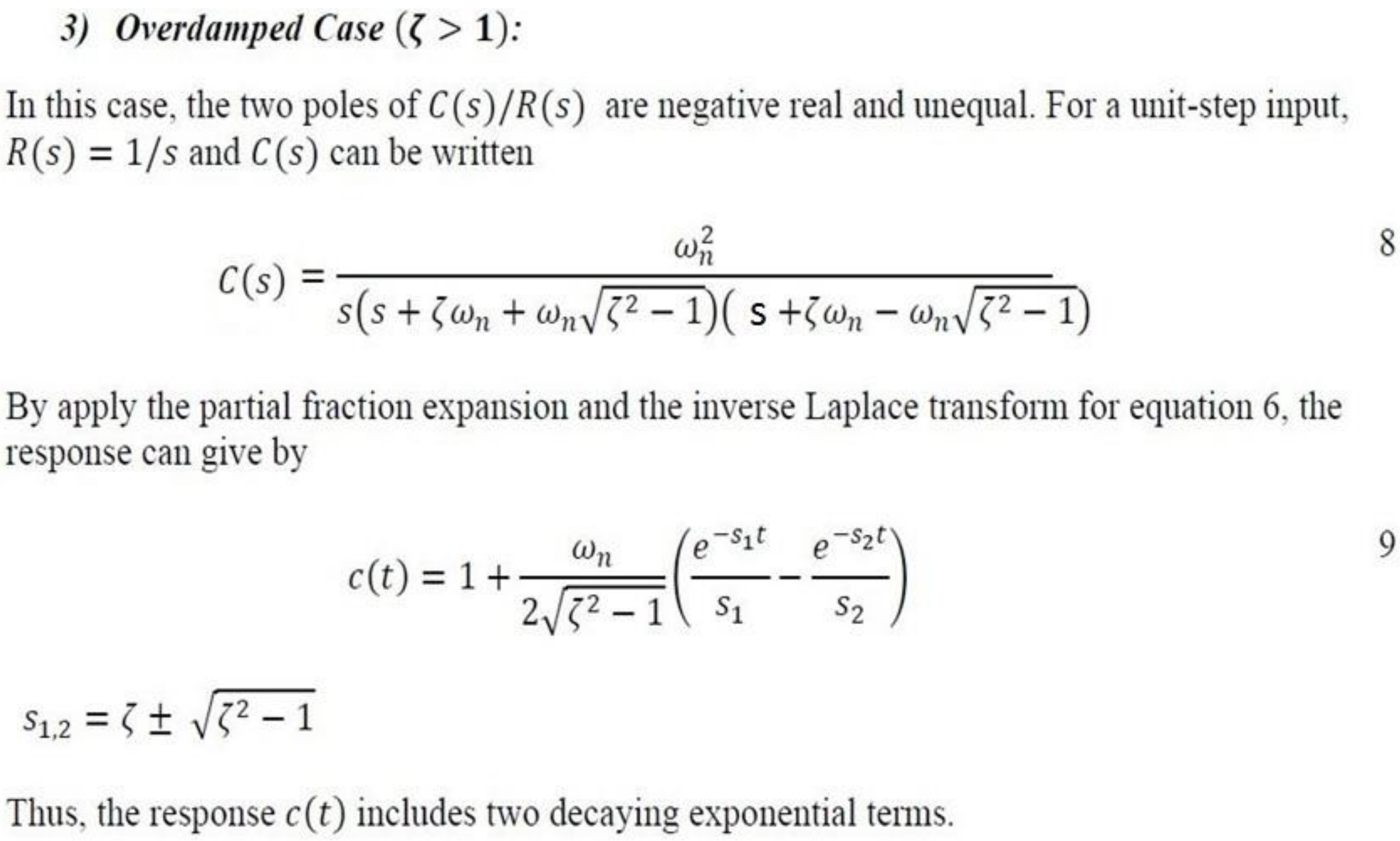
If the two poles of $C(s)/R(s)$ are equal, the system is said to be a critically damped one. For a unit-step input, $R(s) = 1/s$ and $C(s)/$ can be written

$$C(s) = \frac{\omega_n^2}{s\,(s + \omega_n)^2} \qquad 6$$

By apply the partial fraction expansion and the inverse Laplace transform for equation 6, the response can give by

$$c(t) = 1 - e^{-\omega_n t}(1 + \omega_n t) \qquad 7$$

Step response of second order system



3) Overdamped Case ($\zeta > 1$)***:***

In this case, the two poles of $C(s)/R(s)$ are negative real and unequal. For a unit-step input, $R(s) = 1/s$ and $C(s)$ can be written

$$C(s) = \frac{\omega_n^2}{s\left(s + \zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1}\right)\left(s + \zeta\omega_n - \omega_n\sqrt{\zeta^2 - 1}\right)} \qquad 8$$

By apply the partial fraction expansion and the inverse Laplace transform for equation 6, the response can give by

$$c(t) = 1 + \frac{\omega_n}{2\sqrt{\zeta^2 - 1}}\left(\frac{e^{-s_1 t}}{s_1} - \frac{e^{-s_2 t}}{s_2}\right) \qquad 9$$

$$s_{1,2} = \zeta \pm \sqrt{\zeta^2 - 1}$$

Thus, the response $c(t)$ includes two decaying exponential terms.

Step response of second order system

A family of unit-step response curves $c(t)$ with various values of ζ is shown in Figure1 , where the abscissa is the dimensionless variable $\omega_n t$.

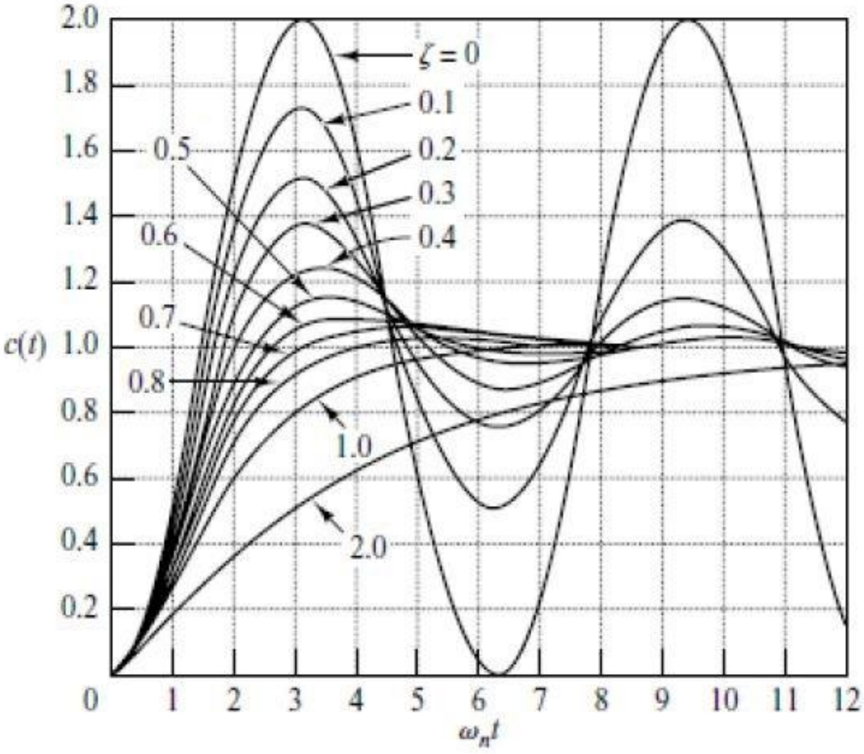


Figure 1: Unit step response curves of the system

Time response specifications

- The performance characteristics of a control system are specified in terms of the transient response to a unit step input.
- The transient response of a system to a unit step input depends on the initial conditions.
- It is a common practice to use the standard initial condition that the system is at rest initially with the output and all times derivatives thereof zero. Then the response characteristics of many systems can be easily compared.
- The transient response of a practical control system often exhibits damped oscillations before reaching steady state.

Time response specifications

- In specifying the transient-response characteristics of a control system to a unit-step input, it is common to specify the following:

1. Delay time, t_d
2. Rise time , t_r
3. Peak time, t_p
4. Maximum overshoot, M_p
5. Settling time, t_s

Time response specifications

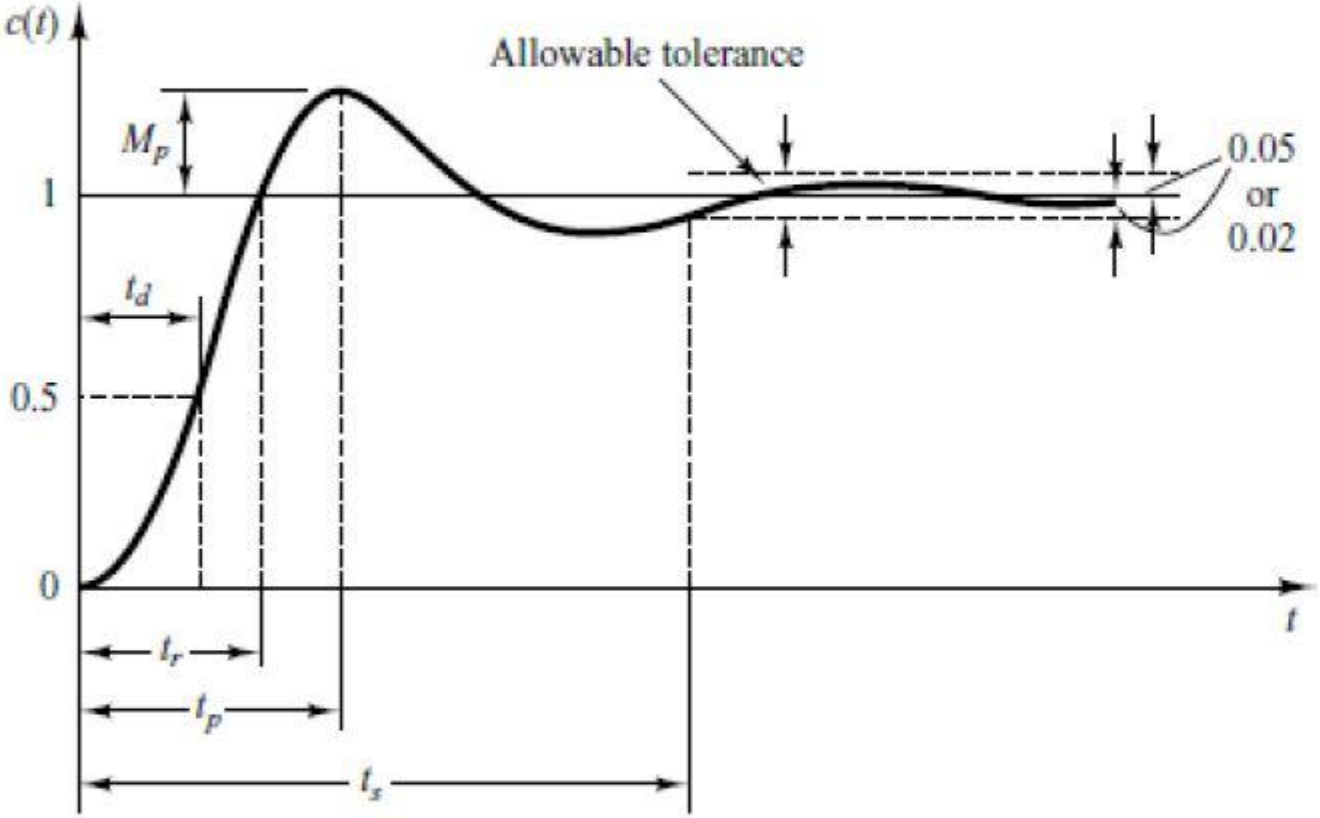
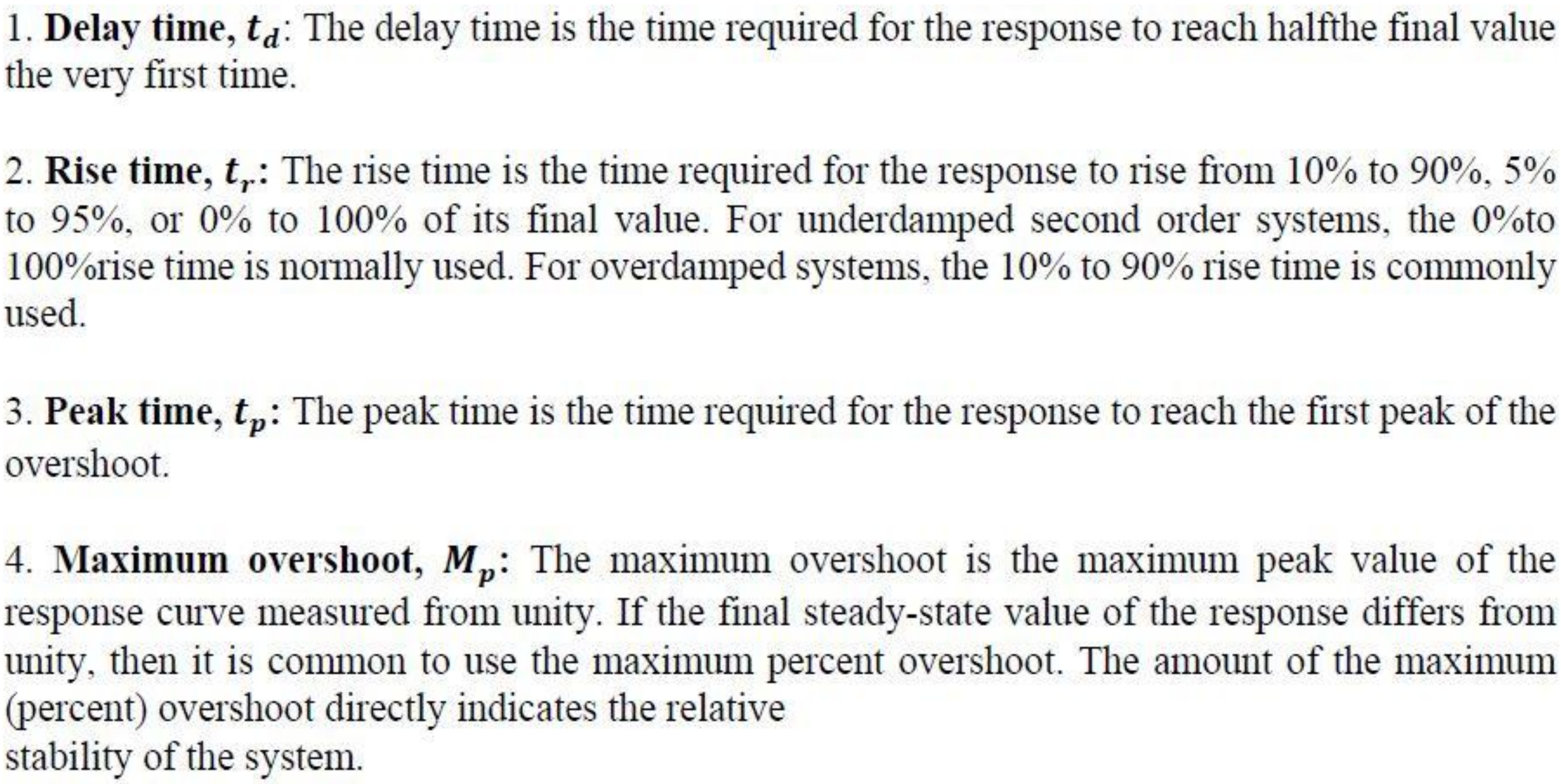


Figure 2: Step response specification

Time response specifications

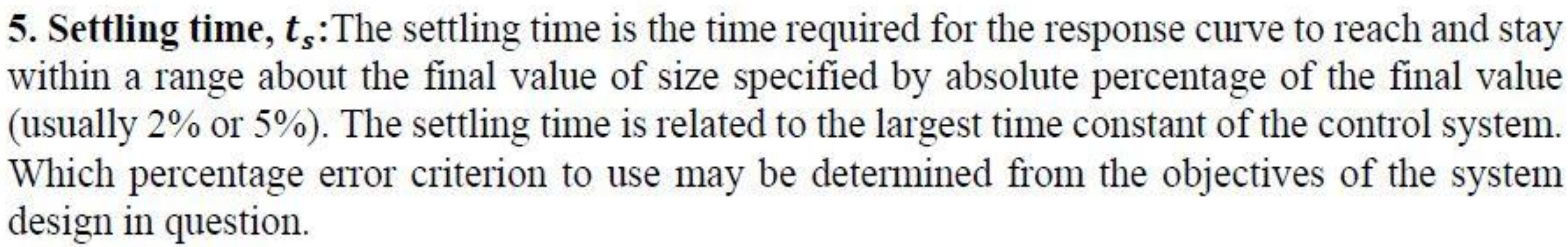


1. **Delay time, t_d**: The delay time is the time required for the response to reach halfthe final value the very first time.

2. **Rise time, t_r:** The rise time is the time required for the response to rise from 10% to 90%, 5% to 95%, or 0% to 100% of its final value. For underdamped second order systems, the 0%to 100%rise time is normally used. For overdamped systems, the 10% to 90% rise time is commonly used.

3. **Peak time, t_p:** The peak time is the time required for the response to reach the first peak of the overshoot.

4. **Maximum overshoot, M_p:** The maximum overshoot is the maximum peak value of the response curve measured from unity. If the final steady-state value of the response differs from unity, then it is common to use the maximum percent overshoot. The amount of the maximum (percent) overshoot directly indicates the relative
stability of the system.



5. Settling time, t_s: The settling time is the time required for the response curve to reach and stay within a range about the final value of size specified by absolute percentage of the final value (usually 2% or 5%). The settling time is related to the largest time constant of the control system. Which percentage error criterion to use may be determined from the objectives of the system design in question.

Time response specifications

1. Rise time , t_r

$$t_r = \frac{\pi - \beta}{\omega_d}$$

where angle β is defined in figure 3. Clearly, for a small value of t_r , ω_d must be large.

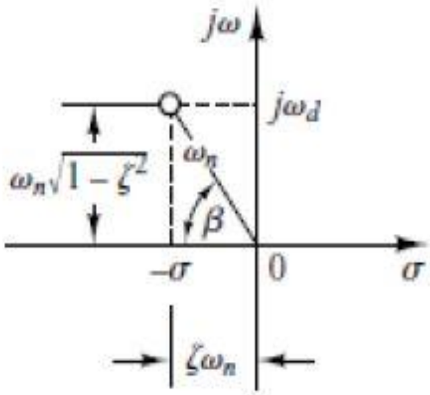


Figure 3

2. Peak time, t_p

Since the peak time corresponds to the first peak overshoot,

$$t_p = \frac{\pi}{\omega_d}$$

The peak time $t_p$corresponds to one-half cycle of the frequency of damped oscillation.

Time response specifications

3. **Maximum overshoot, M_p**

Assuming that the final value of the output is unity

$$M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$$

If the final value $c(\infty)$ of the output is not unity, then we need to use the following equation:

$$M_p = \frac{c(t_p) - c(\infty)}{c(\infty)}$$

4. **Settling time, t_s**

For convenience in comparing the responses of systems, we commonly define the settling time , t_s to be

$t_s = \frac{4}{\zeta\omega_n}$	(2% criterion)
$t_s = \frac{3}{\zeta\omega_n}$	(5% criterion)

Example-1 :

$$T(s) = \frac{16}{s^2+4s+16}$$

Standard form $= T(s) = \dfrac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

$$\omega_n^2 = 16 \Rightarrow \omega_n^2 = 4^2 \Rightarrow \omega_n = 4$$

$$2\zeta\omega_n = 4 \Rightarrow \zeta = \frac{4}{2\omega_n} = \frac{\cancel{4}}{2\times\cancel{4}} = 0.5$$

$$t_s = \frac{4}{\zeta\omega_n} = \frac{4}{0.5\times4} = 2$$

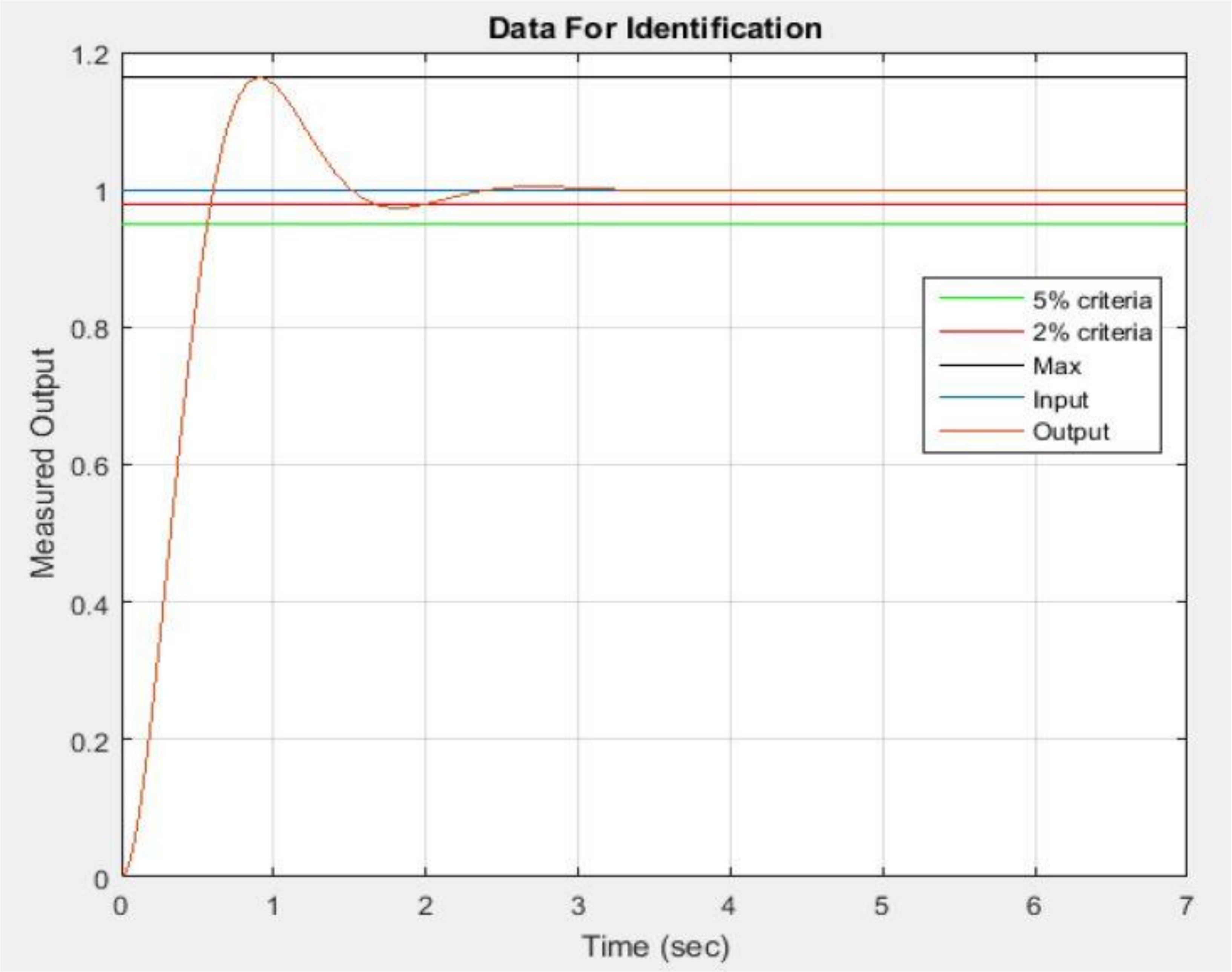
$$M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} = 0.163$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 3.464$$

$$\beta = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta} = 60^\circ = 1.0472.$$

$$t_r = \frac{\pi-\beta}{\omega_d} = 0.6046$$

$$t_p = \frac{\pi}{\omega_d} = 0.9069.$$


Data For Identification
Measured Output
Time (sec)
0
0.2
0.4
0.6
0.8
1
1.2
0
1
2
3
4
5
6
7
5% criteria
2% criteria
Max
Input
Output

Steady state errors:

- ✓ The steady state error is one of the important design specifications for a control system.
- ✓ The steady state output of any system should be close to desired output as possible.
- ✓ The steady state error reflects the accuracy of the system.