Unit-3: Forier Series

1) ~~Trig~~ There are 3 types of forier series.

* Any signal can be represented as sum of sinusoids

→ Trigonometric forier series:

→ Any signal can be represented as (or) by

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$

$$= a_0 + a_1 \cos\omega_0 t + a_2 \cos 2\omega_0 t + \dots + b_1 \sin\omega_0 t + b_2 \sin 2\omega_0 t + \dots$$

where $a_0 = \frac{1}{T}\int_0^T f(t)\,dt$

$$\left.\begin{aligned} a_n &= \frac{2}{T}\int_0^T f(t)\cos n\omega_0 t\,dt \\ b_n &= \frac{2}{T}\int_0^T f(t)\sin n\omega_0 t\,dt \end{aligned}\right\} \text{General formulas}$$

$$\left.\begin{aligned} &* \int_0^T \cos\omega_0 t\,dt = 0 \\ &* \int_0^T \sin\omega_0 t\,dt = 0 \end{aligned}\right\} \text{Reguadar formulas}$$

* $\int_0^T \cos m\omega_0 t \cdot \sin n\omega_0 t\,dt = 0$

* $\int_0^T \cos m\omega_0 t \cdot \cos n\omega_0 t\,dt = \begin{cases} \frac{T}{2}, & m=n \\ 0, & m \neq n \end{cases}$

* $\int_0^T \sin m\omega_0 t \cdot \sin n\omega_0 t\,dt = \begin{cases} T/2, & m=n \\ 0, & m \neq n \end{cases}$

Proof for general Formulas :-

1) $$\int_0^T f(t)\,dt = \int_0^T a_0\,dt + \int_0^T a_1 \cos\omega_0 t\,dt + \int_0^T a_2 \cos 2\omega_0 t\,dt \dots$$

$$+ \int_0^T b_1 \sin\omega_0 t\,dt + \int_0^T b_2 \sin 2\omega_0 t\,dt + \dots$$

$$\int_0^T f(t)\,dt = \int_0^T a_0\,dt \quad \{\because \text{from regular formulas}\}$$

$$\boxed{a_0 = \frac{1}{T}\int_0^T f(t)\,dt}$$

2) $$\int_0^T f(t) \cos n\omega_0 t = \int_0^T a_0 \cos n\omega_0 t\,dt + \int_0^T a_1 \cos\omega_0 t \cos n\omega_0 t\,dt$$

$$\dots + \int_0^T a_n \cos^2 n\omega_0 t\,dt + \int_0^T b_1 \sin\omega_0 t \cos n\omega_0 t\,dt + \dots$$

$$\int_0^T b_n \sin\omega_0 t \cdot \cos n\omega_0 t\,dt$$

$$= \int_0^T f(t)\cos n\omega_0 t\,dt = a_n \int_0^T \cos^2 n\,\omega_0 t\,dt \quad \{n=n\}$$

$$\boxed{a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_0 t \cdot dt} \quad \left\{a_n \int_0^T \frac{1+\cos 2n\omega_0 t}{2}\,dt\right\}$$

3) $$\int_0^T f(t) \sin\omega_0 t\,dt = \int_0^T a_0 \cancel{\cos n\omega_0 t} \sin n\omega_0 t\,dt +$$

$$\int_0^T a_1 \cos\omega_0 t \sin n\omega_0 t\,dt + \dots \int_0^T a_n \cos n\omega_0 t \sin n\omega_0 t$$

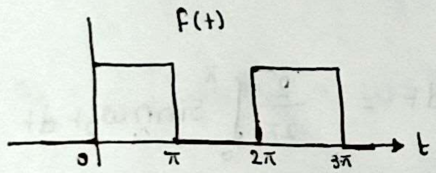
$$+ \dots \int_0^T b_1 \sin\omega_0 t \sin n\omega_0 t + \dots \int_0^T b_n \sin^2 n\omega_0 t\,dt$$

$$\int_0^T f(t)\sin n\omega_0 t \, dt = \int_0^T b_n \sin n\omega_0 t \cdot \sin n\omega_0 t \, dt$$

$$\int_0^T f(t)\sin n\omega_0 t \, dt = b_n \frac{T}{2}$$

$$\boxed{b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_0 t \, dt}$$

Q Find the trigonometric forier series of the following signal



A The following time period of signal is 2π, but, from π to 2π signal don't exist, so, we integ-rate from 0 to π

$$a_0 = \frac{1}{T}\int_0^T f(t)dt$$

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt = \frac{1}{2\pi}\int_0^{\pi} f(t)dt.$$

$$a_0 = \frac{1}{2\pi}\left[\int_0^{\pi} 1 \, dt\right]$$

$$a_0 = \frac{1}{2\pi}[\pi - 0]$$

$$a_0 = \frac{1}{2} \rightarrow (1)$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_0 t \, dt = \frac{2}{2\pi}\int_0^{\pi} \cos n\omega_0 t \, dt$$

$\omega_0 = \frac{2\pi}{t}$ Here $t = 2\pi$, so $\omega = 1$

$$a_n = \frac{1}{\pi}\int_0^{\pi} \cos nt \, dt$$

$$a_n = \frac{1}{\pi}\left[\frac{1}{n}\sin nt\right]_0^{\pi}$$

$$a_n = \frac{1}{\pi}\left[\frac{1}{n}\left[\sin n\pi - \sin n(0)\right]\right] \quad \{\sin n\pi = 0\}$$

$$a_n = 0 \rightarrow (2)$$

$$b_n = \frac{2}{T}\int_0^{T} f(t)\sin n\omega_0 t \, dt = \frac{2}{2\pi}\int_0^{\pi} \sin n\omega_0 t \, dt$$

$$b_n = \frac{1}{\pi}\left[\frac{-1}{n}\cos nt\right]_0^{\pi}$$

$$b_n = \frac{1}{\pi}\left[\frac{-1}{n}\cos n\pi + \frac{1}{n}\cos 0\right]$$

$$b_n = \frac{1}{\pi}\left[\frac{-1}{n}(1) + \frac{1}{n}\right] \quad \{\text{If } n \text{ is even}\}$$

$$b_n = 0 \rightarrow 3(a)$$

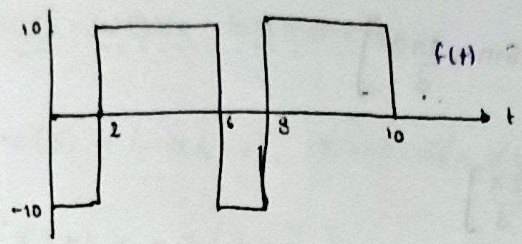
$$b_n = \frac{1}{\pi}\left[\frac{-1}{n}(-1) + \frac{1}{n}\right] \quad (n \text{ is even})$$

$$b_n = \frac{2}{n\pi} \rightarrow 3(b)$$

~~The series is~~ ..

$$\cancel{f(t) = \sum_{n=1}^{\infty} a \cos n\omega_0 t}$$

Q Find the Trigonometric forier series of the following signal.



A The time period of the signal is .. 6.

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt$$

$$a_0 = \frac{1}{6}\left[\int_0^2 -10\,dt + \int_2^6 10\,dt\right]$$

$$a_0 = \frac{1}{6}\left[-10t\Big|_0^2 + 10t\Big|_2^6\right]$$

$$a_0 = \frac{1}{6}\left[-10(2) + [10(6) - 10(2)]\right]$$

$$a_0 = \frac{1}{6}\left[-20 + [60 - 20]\right]$$

$$a_0 = \frac{20}{6} = \frac{10}{3}$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_0 t\,dt \qquad \left\{\omega_0 = \frac{2\pi}{t} = \frac{2\pi}{6} = \frac{\pi}{3}\right\}$$

$$a_n = \frac{2}{6}\int_0^6 f(t)\cos n\frac{\pi}{3}t\,dt$$

$$a_n = \frac{1}{3}\left[-10\int_0^2 \cos n\frac{\pi}{3}t\,dt + 10\int_2^6 \cos n\frac{\pi}{3}t\,dt\right]$$

$$a_n = \frac{1}{3}\left[-10\frac{1}{n\pi/3}\sin n\frac{\pi}{3}t\Big|_0^2 + 10\frac{1}{n\pi/3}\sin n\frac{\pi}{3}t\Big|_2^6\right]$$

$$a_n = \frac{1}{3}\left[\frac{-30}{n\pi}\sin n\frac{\pi}{3}t\Big|_0^2 + \frac{30}{n\pi}\sin n\frac{\pi}{3}t\Big|_2^6\right]$$

$$a_n = \frac{1}{3}\left[\frac{-30}{n\pi}\left[\sin n\frac{2\pi}{3} - \sin 0\right] + \frac{30}{n\pi}\left(\sin n2\pi + \sin n\frac{2\pi}{3}\right)\right]$$

$$a_n = \frac{1}{3}\left[\frac{-30\times 2}{n\pi}\left(\sin n\frac{2\pi}{3}\right)\right]$$

$$a_n = \frac{-60}{3n\pi}\left[\sin n\frac{2\pi}{3}\right]$$

$$a_n = \frac{-20}{n\pi}\left[\sin n\frac{2\pi}{3}\right] \longrightarrow (2)$$

$$b_n = \left[\frac{2}{6}\left[\int_0^2 -10\sin n\frac{\pi}{3}t\,dt + \int_2^6 10\sin n\frac{\pi}{3}t\,dt\right]\right.$$

$$b_n = \frac{1}{3}\left[\int_0^2 -10\sin n\frac{\pi}{3}t\,dt + \int_2^6 10\sin n\frac{\pi}{3}t\,dt\right]$$

$$b_n = \frac{1}{3}\left[10\left[\frac{3}{n\pi}\cos n\frac{\pi}{3}t\Big|_0^2\right] - 10\left[\frac{3}{n\pi}\cos n\frac{\pi}{3}t\Big|_2^6\right]\right]$$

$$b_n = \frac{1}{3}\left[\frac{30}{n\pi}\left[\cos\frac{n2\pi}{3} - 1\right] - \frac{30}{n\pi}\left[\cos 2n\pi - \cos\frac{n2\pi}{3}\right]\right]$$

$$b_n = \frac{1}{3}\left[\frac{-30}{n\pi}\left[1-\cos\frac{2n\pi}{3}\right] - \frac{30}{n\pi}\left[1-\cos\frac{2n\pi}{3}\right]\right]$$

$$b_n = \frac{1}{3}\left[\frac{-60}{n\pi}\left[1-\cos\frac{2n\pi}{3}\right]\right]$$

$$b_n = \frac{-20}{n\pi}\left[1-\cos\frac{2n\pi}{3}\right]$$

$$b_n = \frac{20}{n\pi}\left[\cos\frac{2n\pi}{3} - 1\right]_{4}$$

* Even Symmetry:

$x(t) = x(-t)$, then $\rightarrow a_n$ exist and $b_n = 0$

* Odd symmetry:

$x(t) = -x(-t)$, then $\rightarrow a_n = 0$ & b_n exist.

* Half-wave symmetry :-

~~$x(t) = -x(-t)$, then $a_n \neq 0$ &~~

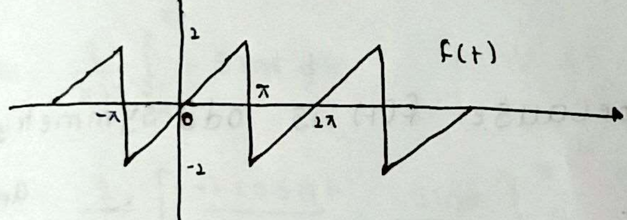
$x(t) = -x\left(t - \frac{T}{2}\right)$

a_n for n = odd exist

b_n for n = odd exist

i.e a_2 or a_4 or ... & b_2 or b_4 or ... are zeroes.

Q calculate furier series of following signal.



It is odd symmetry , i.e $f(t) = -f(-t)$.

calculate slope from $-\pi$ to π

$\{ y - y_0 = m(x - x_0) \}$ slope of $(0,0)$ & $(\pi, 2)$

$$m = \frac{2-0}{\pi - 0} = \frac{2}{\pi}$$

$\rightarrow f(t) \pm 2 = \frac{2}{\pi}(t \pm \pi)$

$\rightarrow f(t) \pm 2 = \frac{2t}{\pi} \pm 2$

$\rightarrow f(t) = \frac{2t}{\pi}$

$\rightarrow a_0 = \frac{1}{T}\int_0^T f(t) \cdot dt$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t)\, dt$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{2t}{\pi}\, dt$$

$$= \frac{1}{\pi^2} \int_{-\pi}^{\pi} t\, dt$$

$$= \frac{1}{\pi^2} \left. \frac{t^2}{2} \right|_{-\pi}^{\pi}$$

$$= \frac{1}{2\pi^2} \left[\pi^2 - (-\pi)^2 \right]$$

$$= \frac{1}{2\pi^2} [0] = 0$$

$a_0 = 0$, [because $f(t)$ is odd symmetry, then $a_n = 0$]

$$a_n = \frac{2}{T} \int f(t) \cos n\omega_0 t\, dt \qquad \omega_0 = \frac{2\pi}{t} = (t = 2\pi)$$

$$a_n = \frac{2}{2\pi} \int_{-\pi}^{\pi} \frac{2t}{\pi} \cos n\omega_0 t\, dt \qquad \omega_0 = 1$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} \frac{2t}{\pi} \cos nt\, dt$$

$$a_n = \frac{2}{\pi^2} \int_{-\pi}^{\pi} t \cos nt\, dt$$

$$a_n = \frac{2}{\pi^2} \left[t \int \cos nt\, dt - \int 1 \int \cos nt\, dt \right]$$

$$a_n = \frac{2}{\pi^2} \left[t \left(\frac{\sin nt}{n} \right) - \int \frac{\sin nt}{n} \right]_{-\pi}^{\pi}$$

$$a_n = \frac{2}{\pi^2} \left[\frac{t \sin nt}{n} + \frac{\cos nt}{n^2} \right]_{-\pi}^{\pi}$$

$$a_n = \frac{2}{\pi^2}\left[\frac{nt\sin nt + \cos nt}{n^2}\right]_{-\pi}^{\pi}$$

$$a_n = \frac{2}{\pi^2 n^2}\left[nt\sin nt + \cos nt\right]_{-\pi}^{\pi}$$

$$a_n = \frac{2}{\pi^2 n^2}\left[(\pi n\sin n\pi + \cos n\pi) - (-\pi n\sin n\pi + \cos n\pi)\right]$$

$$a_n = \frac{2}{\pi^2 n^2}\left[\cos n\pi - \cos n\pi\right] \qquad \{\sin n\pi = 0\}$$

$$a_n = 0$$

* $$b_n = \frac{2}{T}\int_{-\pi}^{\pi} f(t)\sin n\omega_0 t\, dt$$

$$b_n = \frac{2}{\pi^2}\int_{-\pi}^{\pi} t\sin nt\, dt$$

$$b_n = \frac{2}{\pi^2}\left[\frac{-t\cos nt}{n} + \frac{\sin nt}{n^2}\right]_{-\pi}^{\pi}$$

$$b_n = \frac{2}{\pi^2}\left[\frac{\sin nt - nt\cos nt}{n^2}\right]_{-\pi}^{\pi}$$

$$b_n = \frac{2}{n^2\pi^2}\left[(\sin n\pi - \pi n\cos n\pi) - (\sin(-n)\pi - (-\pi n\cos(-n)\pi))\right]$$

$$b_n = \frac{2}{n^2\pi^2}\left[\sin n\pi - \pi n\cos n\pi - (-\sin n\pi + \pi n\cos n\pi)\right]$$

$$b_n = \frac{2}{n^2\pi^2}\left[0 - \pi n\cos n\pi + \sin n\pi - \pi n\cos n\pi\right]$$

$$b_n = \frac{2}{n^2\pi^2}\left[-2\pi n\cos n\pi\right]$$

$$b_n = \frac{-4\pi n\cos n\pi}{n^2\pi^2} = \boxed{\frac{-4\cos n\pi}{\pi n}}$$

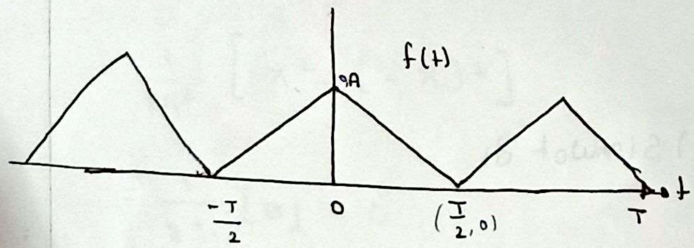
If n is even, $\cos n\pi = 1$

then $\boxed{b_n = \frac{-4}{n\pi}}$

If n is odd $\boxed{b_n = \frac{4}{n\pi}}$

$$b_n = \begin{cases} \frac{-4}{n\pi} & n \text{ is even} \\ \frac{4}{n\pi} & n \text{ is odd} \end{cases}$$

* Write the forier series of the following signal.



A The given signal is ... even symmetry,

so, $b_n = 0$

ø

* Complex exponential forier Series :

we know

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t \rightarrow (1)$$

we also know, $e^{j\theta} = \cos\theta + j\sin\theta$

$$e^{-j\theta} = \cos\theta - j\sin\theta$$

then $\cos\theta = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

Similarly $\cos(n\omega_0 t) = \dfrac{e^{jn\omega_0 t} + e^{-j(n\omega_0 t)}}{2}$

and $\sin(n\omega_0 t) = \dfrac{e^{jn\omega_0 t} - e^{-j(n\omega_0 t)}}{2j}$

Substitute in (1)

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \left[\frac{e^{jn\omega_0 t} + e^{-jn\omega_0 t}}{2}\right] + \sum_{n=1}^{\infty} b_n \left[\frac{e^{jn\omega_0 t} - e^{-jn\omega_0 t}}{2j}\right]$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left(\frac{a_n}{2} + \frac{b_n}{2j}\right) e^{jn\omega_0 t} + \sum_{n=1}^{\infty} \left(\frac{a_n}{2} - \frac{b_n}{2j}\right) e^{-jn\omega_0 t}$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left(\frac{a_n}{2} - \frac{jb_n}{2}\right) e^{jn\omega_0 t} + \sum_{n=1}^{\infty} \left(\frac{a_n}{2} + \frac{jb_n}{2}\right) e^{-jn\omega_0 t}$$

$\{-j = \frac{1}{j}\}$

let $\dfrac{a_n - jb_n}{2} = C_n$ & $\dfrac{a_n + jb_n}{2} = C_{-n}$

$$f(t) = a_0 + \sum_{n=1}^{\infty} C_n e^{jn\omega_0 t} + \sum_{n=1}^{\infty} C_{-n} e^{-jn\omega_0 t}$$

$\{a_0 = 0\}$ $\{c_1, c_2, c_3 \ldots$ are positives$\}$ $\{c_{-1}, c_{-2}, c_{-3}, \ldots$ are negatives$\}$

$$\boxed{f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}}$$

then $\boxed{c_n = \frac{1}{T}\int_0^T f(t)\, e^{-jn\omega_0 t}\, dt}$

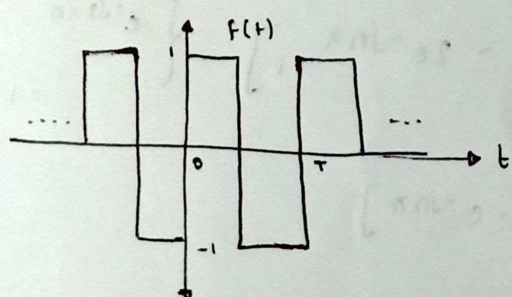
$$\boxed{c_0 = \frac{1}{T}\int_0^T f(t)\, dt}$$

also, $\boxed{c_0 = a_0}$

$$\boxed{c_n = \frac{a_n - jb_n}{2}}$$

$$\boxed{c_n = \frac{a_n + jb_n}{2j}}$$

Q Find the complex exponential forier series. representation of given signal.



A $c_n = \frac{1}{T}\int_0^T f(t)\, e^{-jn\omega_0 t}\, dt$

$$c_n = \frac{1}{T}\left[\int_0^{T/2} e^{-jn\omega_0 t}\, dt + \int_{T/2}^{T} -e^{-jn\omega_0 t}\, dt\right]$$

$$c_n = \frac{1}{T}\left[\int_0^{T/2} e^{-jn\omega_0 t}\, dt - \int_{T/2}^{T} e^{-jn\omega_0 t}\, dt\right]$$

$\omega_0 = \frac{2\pi}{T}$

$$c_n = \frac{1}{T}\left[\int_0^{T/2} e^{-jn\omega_0 t}\, dt - \int_{T/2}^{T} e^{-jn\omega_0 t}\, dt\right]$$

put $-jn\omega_0 t = u$

$$dt = \frac{du}{-jn\omega_0}$$

$$c_n = \frac{1}{T}\left[\frac{1}{-jn\omega_0} e^{-jn\omega_0 t}\Big|_0^{T/2} + \frac{1}{jn\omega_0} e^{-jn\omega_0 t}\Big|_{T/2}^{T}\right]$$

$$c_n = \frac{1}{Tjn\omega_0}\left[\left(e^{-jn\omega_0 T} - e^{-jn\omega_0 \frac{T}{2}}\right) - \left(e^{-jn\omega_0 \frac{T}{2}} - 1\right)\right]$$

$$c_n = \frac{1}{Tjn\omega_0}\left[e^{-jn\omega_0 T} - 2e^{-jn\omega_0 \frac{T}{2}} + 1\right]$$

$$\left\{\omega_0 = \frac{2\pi}{T}\right\}$$

$$c_n = \frac{1}{Tjn\omega_0}\left[e^{-j2\pi n} - 2e^{-j\frac{2\pi n}{2}} + 1\right]$$

$$c_n = \frac{1}{Tjn\omega_0}\left[e^{-j2\pi n} - 2e^{-j\pi n} + 1\right]$$

$$c_n = \frac{\cancel{T}}{\cancel{T}jn2\pi}\left[1 - 2e^{-jn\pi} + 1\right] \quad \left\{e^{-j2\pi n} = \cos 2n\pi - j\sin 2n\pi = 1\right\}$$

$$c_n = \frac{2}{2jn\pi}\left[1 - e^{-jn\pi}\right]$$

$$c_n = \frac{2}{j2\pi n}\left[1 - (\cos \pi n - j\sin n\pi)\right]$$

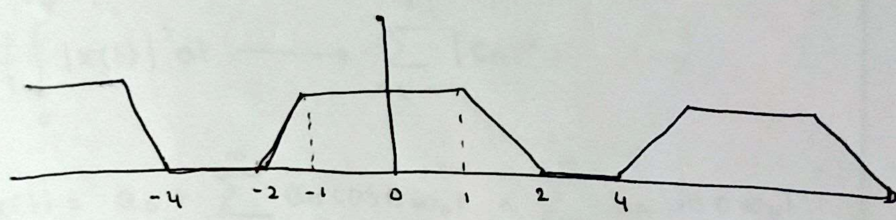
$$c_n = \frac{2}{j2\pi n}\left[1 - \cos n\pi\right]$$

$$c_n = \frac{1}{j\pi n}\left[1 - \cos n\pi\right]$$

n is even, $c_n = 0$

n is odd, $c_n = \dfrac{2}{j\pi n}$

Q Find complex exponential forier series for the following signal.



Parsevel's theorem: [power theorem]:

$$x(t) \xrightarrow{F.S} C_n$$

$$\frac{1}{T}\int_0^T |x(t)|^2 dt \longrightarrow \sum_{n=-\infty}^{\infty} |C_n|^2$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$

power of dc $= a_0^2$

power of $a_n \cos n\omega_0 t = \left(\frac{a_n}{\sqrt{2}}\right)^2 = \frac{a_n^2}{2}$

power of $b_n \sin n\omega_0 t = \frac{b_n^2}{2}$

$$C_n = \frac{a_n - jb_n}{2} \qquad |C_n| = \frac{\sqrt{a_n^2 + b_n^2}}{2}$$

$$C_{-n} = \frac{a_n + jb_n}{2} \qquad |C_n| = \frac{\sqrt{a_n^2 + b_n^2}}{2}$$

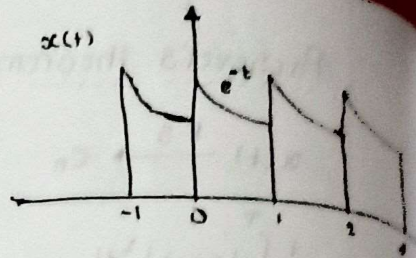
$$|C_n|^2 + |C_{-n}|^2 = \frac{a_n^2 + b_n^2}{4} + \frac{a_n^2 + b_n^2}{4} = \frac{a_n^2 + b_n^2}{2}$$

$$\text{power of } x(t) = a_0^2 + \sum_{n=1}^{\infty}\left(\frac{a_n^2 + b_n^2}{2}\right)$$

$$= C_0^2 + \sum_{n=1}^{\infty} |C_n|^2 + |C_{-n}|^2$$

$$= \sum_{n=-\infty}^{\infty} |C_n|^2$$

1) Find the e.Forier series of the following signal $x(t)$: Find the power in DC and first 2 Harmonic components.



$$c_n = \frac{1}{T}\int_0^T x(t)\, e^{-jn\omega_0 t}\, dt$$

$$c_n = \frac{1}{T}\int_0^T e^{-t}\, e^{-jn\omega_0 t}\, dt$$

$$c_n = \frac{1}{T}\int_0^T e^{-jn\omega_0 t - t}\, dt$$

$$c_n = \frac{1}{T}\int_0^T e^{-t(jn\omega_0 + 1)}\, dt$$

$$c_n = \frac{1}{T}\left[\frac{-e^{-t(jn\omega_0+1)}}{jn\omega_0 + 1}\right]_0^T$$

$$c_n = \frac{1}{T}\left[\frac{-e^{-T(jn\omega_0+1)}}{jn\omega_0+1} + 1\right]$$

$$c_n = \frac{1 - e^{-T(jn\omega_0+1)}}{T\,(jn\omega_0+1)}$$

$$c_n = \frac{1 - e^{-T(jn\omega_0+1)}}{jn\omega_0 T + T}$$

$$c_n = \frac{1 - e^{-1}}{1 + j2\pi n}$$

Power: $P = \sum_{n=-\infty}^{\infty} |c_n|^2$

$$= \sum_{n=-2}^{2} |c_n|^2 = \sum_{n=-2}^{2} \left|\frac{1-e^{-1}}{1+j2n\pi}\right|^2$$

$$= \left|\frac{1-e^{-1}}{1-4j\pi}\right|^2 + \left|\frac{1-e^{-1}}{1-2j\pi}\right|^2 + \left|\frac{1-e^{-1}}{1}\right|^2 + \left|\frac{1+e^{-1}}{1+j2\pi}\right|^2 + \left|\frac{1+e^{-1}}{1+j4\pi}\right|^2$$

modulus

$$= (1-e^{-1})^2\left[\frac{2}{(1+(4\pi)^2)} + \frac{2}{1+(2\pi)^2} + 1\right]$$

$= 0.424$ watt $(e = 2.71)$ $(\pi = 3.14)$

The exponential forier series representation of continuous time periodic signal $x(t)$ is defined as

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

The following information is given about $x(t)$ and a_k

1) The $x(t)$ is real and even having a fundamental time period of 6.

2) The average value of $x(t)$ is 2.

3) $a_k \neq k$, when it is from 0 to 3 and 0 when $k >$, i.e

$$a_k = \begin{cases} k & 1 \leq k \leq 3 \\ 0 & k > 0 \end{cases}$$

* Find the average power of the signal $x(t)$

Q The forier series of a real periodic signal with fundamental frequency f_0 is given by

$$g_p(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi f_0 n t}$$

It is given that $C_3 = 3 + j5$ find C_{-3}

A $C_{-3} = 3 - j5$

Q Let $x(t)$ be a periodic signal with period T and forier series coefficients be C_n. Let $y(t) = x(t - t_0) + x(t + t_0)$. The forier series coefficient of y_0 is d_n, $d_n = 0$ $\forall$ odd $\rightarrow n$. Find all t_0 terms

A Given $y(t) = x(t - t_0) + x(t + t_0)$

By using property of forier series

$$\int_0^T [x(t-t_0) + x(t+t_0)] e^{-jn\omega_0 t} dt$$

$$\int_0^T x(t-t_0) e^{-jn\omega_0 t} dt + \int_0^T x(t+t_0) e^{-jn\omega_0 t} dt$$

$$= (e^{-jn\omega_0 t_0} + e^{jn\omega_0 t_0}) C_n$$

$$= (e^{-jn\omega_0 t_0} + e^{jn\omega_0 t_0}) \int_0^T x(t-t_0) e^{-jn\omega_0 (t-t_0)} dt$$

$\rightarrow x(t) \rightarrow C_n$

$x(t+t_0) + x(t-t_0) \rightarrow e^{-jn\omega_0 t_0} c_n + e^{jn\omega_0 t_0} c_n$

$= c_n \left[e^{jn\omega_0 t_0} + e^{-jn\omega_0 t_0} \right]$

$= c_n \left[\cos n\omega_0 t_0 + j\cancel{\sin n\omega_0 t_0} + \cos n\omega_0 t_0 - j\cancel{\sin n\omega_0 t_0} \right]$

$= c_n \left[2\cos n \frac{2\pi}{T} t_0 \right]$

for odd , 0

$2 c_n \cos n\omega_0 t_0 = 0 \quad \{n=1\}$

$\cos\omega_0 t_0 = \cos 90$

$\frac{2\pi}{T} t_0 = \frac{\pi}{2}$

$t_0 = \frac{T}{2\pi} \times \frac{\pi}{2}$

$\boxed{t_0 = \frac{T}{4}}$

Q Find the exponential forier series coefficients for the periodic signal $x(t)$, where

$x(t) = 2 + \cos\left(\frac{2\pi}{3}t\right) + 4\sin\left(\frac{5\pi}{3}t\right)$

$= a_0 + \cos\left[2\left(\frac{\pi}{3}t\right)\right] + 4\sin\left[5\left(\frac{\pi}{3}t\right)\right] \left\{\omega_0 = \frac{\pi}{3}\right\}$

$= a_0 + a_2 + 4b_5$

$a_0 = c_0 = 2$

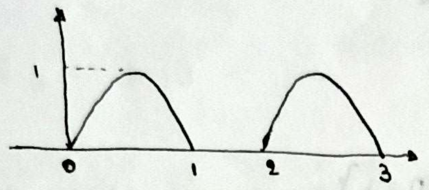
$c_2 = \frac{a_n - jb_n}{2} = \frac{1}{2}$

$c_{-2} = \frac{a_2 + jb_n}{2} = \frac{1}{2}$

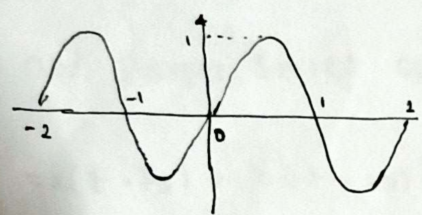
$c_5 = \frac{a_5 - jb_5}{2} = \frac{-j(4)}{2} = -2j$

$c_{-5} = \frac{a_5 + jb_5}{2} = 2j$

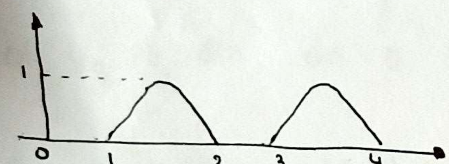
Q The forier Series coefficient of the signal $x(t)$ shown in fig R. $C_0 = \frac{1}{\pi}$, $C_1 = -j0.25$, $C_n = \frac{1}{\pi(1-n^2)}$ when n is even. find Forier series coefficuents of $y(t)$, $f(t)$ and $g(t)$



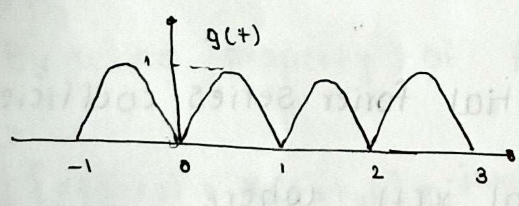
$x(t)$



$y(t)$ $f(t)$



$y(t)$



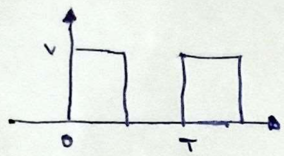
Dirichlet conditions [converge of F.S]

1) Input signal must be absolutely integrable even the range of time period

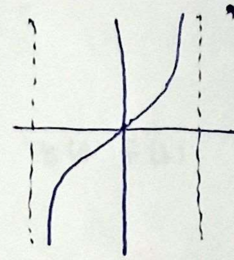
$$\sum \int_0^T |x(t)| \, dt < \infty$$

we cannot calculate F.S for tant.

Ex.

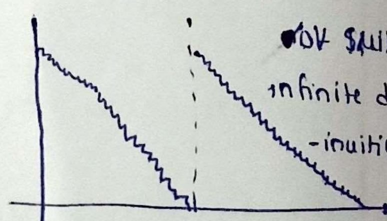


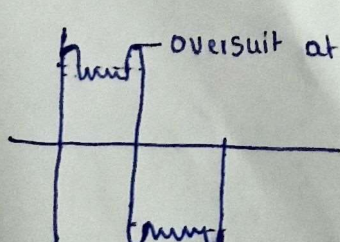
convergence



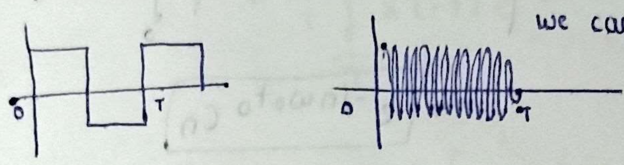
2) The signal must have finite number of discontinuity over the range of time period.

* At a discontinuity the F.S representation of a function will be oversuit 9% its value. It never dissapear even in the limit of infinite no. of terms, this is called Gibb's phenomenon.





8) Signals must have finite no. of maxima and minima over the range of time period.



we cannot find F.S

* Properties of Forier series

1) Linearity:
$$x(t) \xrightarrow{F.S} C_n$$
$$y(t) \xrightarrow{F.S} D_n$$

then $\alpha x(t) + \beta (y(t)) \xrightarrow{F.S} \alpha C_n + \beta D_n$

Do the proof for it.

2) Time Shifting:

$x(t) \rightarrow C_n$

$x(t-t_0) \rightarrow e^{-jn\omega_0 t_0} C_n$

Proof: $C_n = \frac{1}{T}\int_0^T x(t) e^{-jn\omega_0 t} dt$

$= \frac{1}{T}\int_0^T x(t-t_0) e^{-jn\omega_0 t} dt$

let $t - t_0 = \tau$

$t = \tau + t_0$

$dt = d\tau$

$$= \frac{1}{T}\int_0^T x(\tau)\, e^{-jn\omega_0(\tau+t_0)}\, d\tau$$

$$= e^{-jn\omega_0 t_0}\, \frac{1}{T}\int_0^T x(\tau)\, e^{-jn\omega_0 \tau}\, d\tau$$

$$\therefore = \boxed{e^{-jn\omega_0 t_0}\, C_n}$$

3) Frequency shifting :

$$x(t) \xrightarrow{F.S} C_n$$

$$e^{jm\omega_0 t}\, x(t) \longrightarrow C_{n-m}$$

Proof : $C_n = \frac{1}{T}\int_0^T x(t)\, e^{-jn\omega_0 t}\, dt$

$$= \frac{1}{T}\int_0^T e^{jm\omega_0 t}\, x(t)\, e^{-jn\omega_0 t}\, dt$$

$$= \frac{1}{T}\int_0^T e^{-j(n-m)\omega_0 t}\, x(t)\, dt$$

$$= C_{n-m}$$

4) Differentiation property :

$$x(t) \longrightarrow C_n$$

$$\frac{d}{dt}x(t) \longrightarrow (jn\omega_0)\, C_n$$

Proof : $\frac{d}{dt}x(t) = \sum_{n=-\infty}^{\infty} C_n \frac{d}{dt} e^{jn\omega_0 t}$

$$= \sum_{n=-\infty}^{\infty} (jn\omega_0)\, C_n\, e^{jn\omega_0 t} = (jn\omega_0)\, C_n$$

5) Time Reversal : $x(t) \xrightarrow{F.S} C_n$

$$x(-t) \longrightarrow C_{-n}$$

Proof → $C_n = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-jn\omega_0 t}\, dt$

$$= \frac{1}{T}\int_{-T/2}^{T/2} x(-t)\, e^{-jn\omega_0 t}\, dt$$

let $-t = T$

$-d(t) = dT$

$dt = -dT$

$$= \frac{1}{T}\int_{+T/2}^{-T/2} x(T)\, e^{jn\omega_0 T}\, (-dT)$$

$$= \frac{1}{T}\int_{-T/2}^{T/2} x(T)\, e^{jn\omega_0 T}\, dT$$

$$= \frac{1}{T}\int_{-T/2}^{T/2} x(T)\, e^{-j(-n)\omega_0 T}\, dT = C_{-n}$$

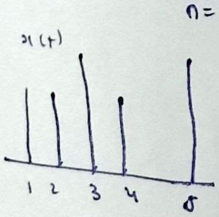
6) Time Scaling :-

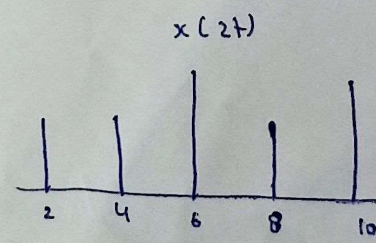
$x(t) \xrightarrow{FS} C_n$

$x(at) \longrightarrow C_n$

Proof $x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$

$$x(at) = \sum_{n=-\infty}^{\infty} C_n e^{j(na)\omega_0 t}$$





No change in F.S coefficients. But the representation is different due to change in fundamental frequency.

7) Conjugate property:

$$x(t) \xrightarrow{F.S} C_n$$

$$x^*(t) \longrightarrow C^*_{-n}$$

$$C_n = \frac{1}{T}\int_0^T x(t)\, e^{-jn\omega_0 t}\, dt .$$

$$C^*_{-n} = \frac{1}{T}\int_0^T x^*(t)\, e^{+jn\omega_0 t}\, dt$$

$$= \frac{1}{T}\int_0^T x^*(t)\, e^{-jn\omega_0 t}\, dt .$$

$$x^*(t) \longrightarrow C_n^*$$

8) Multiplication property

$$x(t) \longrightarrow C_n \quad \& \quad y(t) \longrightarrow D_n$$

$$x(t)\cdot y(t) \longrightarrow C_n * D_n$$

proove it

9) Integration property :

$$x(t) \xrightarrow{FS} C_n$$

$$\int_{-\infty}^{t} x(\tau)\, d\tau \longrightarrow \frac{C_n}{jn\omega_0}$$

Proove it