12-12-2022

2. A.C CIRCUITS

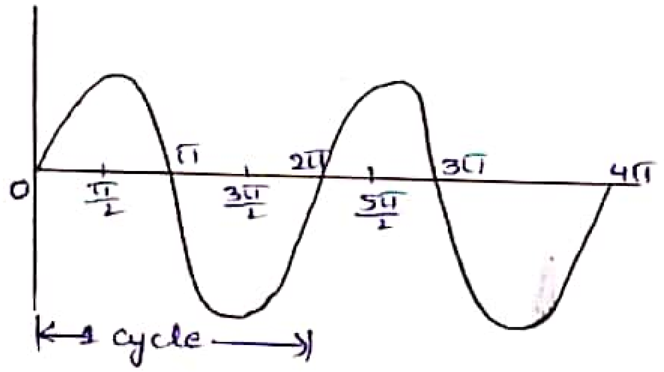
* A.C circuit :-

→ A circuit in which 'currents' & 'voltages' vary sinusoidally i.e vary with time is called Alternating current or 'A.C circuit'.

→ All A.C circuits are made up of combination of 'R', 'L', 'C'.

→ The circuit elements R, L, C are called "circuit Parameters"

* Sinusoidal signal :-



* Frequency :- No. of cycles per second.

$$\boxed{f = \frac{1}{T}} \qquad \boxed{\omega = 2\pi f = \frac{2\pi}{T}}$$

* Time period :-

→ The time taken by an alternating quantity in seconds to trace one complete cycle is called "Time period" (T).

* Peak factor :- (or) crest (or) Amplitude factor :-

$$\boxed{\text{Peak factor} = \frac{\text{Maximum value}}{\text{RMS value}}}$$

* Form factor :-

$$\boxed{\text{Form factor} = \frac{\text{RMS value}}{\text{Average value}}}$$

* RMS value :-

For sinusoidal signal, $\boxed{V_{rms} = \frac{V_m}{\sqrt{2}}}$ & $\boxed{I_{rms} = \frac{I_m}{\sqrt{2}}}$.

* Average value :-

For sinusoidal signal, $\boxed{V_{avg} = \frac{2V_m}{\pi}}$, $\boxed{I_{avg} = \frac{2I_m}{\pi}}$.

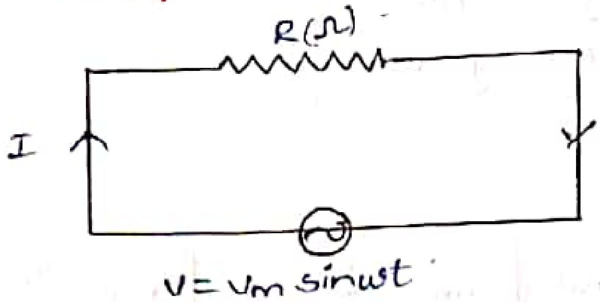
*

Circuit Element	Symbol	Resistance or Reactance	Phase of current	Phase constant	Amplitude Relation.
Resistor	R	R	In phase with V_R	0°	$V_R = I_R R$
Capacitor	C	$X_C = \frac{1}{\omega C}$	Leads V_R by 90°	90°	$V_C = I_C X_C$
Inductor	L	$X_L = \omega L$	Lags V_R by 90°	90°	$V_L = I_L X_L$

*

Element	Impedance	Admittance
R	$Z = R$	$Y = \frac{1}{R}$
L	$Z = j\omega L$	$Y = \frac{1}{j\omega L}$
C	$Z = \frac{1}{j\omega C}$	$Y = j\omega C$

* A.C through Pure Resistor:-



Here $v = V_m \sin\omega t \longrightarrow (1)$

We know $I = \frac{V}{R}$

$$\Rightarrow I = \frac{V_m \sin\omega t}{R} = \frac{V_m}{R}\sin\omega t = I_m \sin\omega t$$

$\therefore \boxed{I = I_m \sin\omega t} \longrightarrow (2)$ $\left[\because I_m = \frac{V_m}{R}\right]$

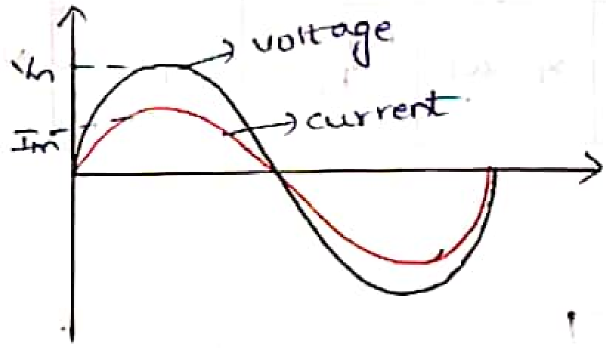
From (1) & (2)

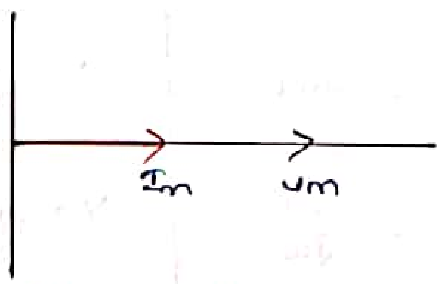
There is no phase difference

i.e, v & I are 'inphase'

$v = V_m \angle 0^\circ$ volts

$I = I_m \angle 0^\circ$ Amperes.





Phaser diagram.

Power, $P = VI$

$\Rightarrow P = V_m \sin\omega t \;\; I_m \sin\omega t$

$\Rightarrow P = V_m I_m \sin^2\omega t$.

$\Rightarrow P = V_m I_m \left(\frac{1-\cos 2\omega t}{2}\right)$.

$\Rightarrow P = \frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t$.

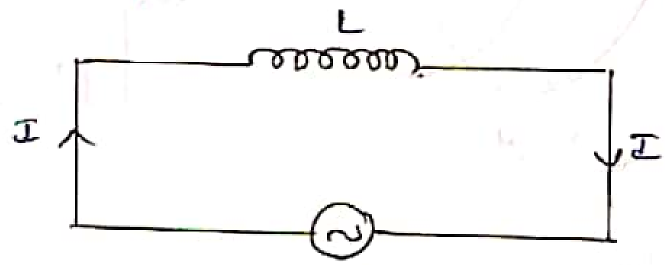
$\Rightarrow P = \frac{V_m I_m}{2} - 0$ [$\because$ In a complete cycle]

$\Rightarrow P = \frac{V_m I_m}{2}$

$\Rightarrow P = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}$

$\therefore \boxed{P = V_{rms} \cdot I_{rms}}$

* AC through Pure Inductor :-



$V_1 = V_m \sin\omega t$

Here, $V = V_m \sin\omega t \longrightarrow (1)$

We know, "$V = L\frac{dI}{dt}$"

$\Rightarrow \frac{V}{L} dt = dI$

$\Rightarrow I = \int \frac{V}{L} dt$

$\Rightarrow I = \frac{1}{L}\int V dt$

$\Rightarrow I = \frac{1}{L}\int V_m \sin\omega t \, dt$

$\Rightarrow I = \frac{V_m}{L}\left(\frac{-\cos\omega t}{\omega}\right) = \frac{V_m}{\omega L}(-\cos\omega t)$

$\Rightarrow I = \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$

$\therefore I = I_m \sin\left(\omega t - \frac{\pi}{2}\right) \longrightarrow (2)$ $\quad [\because I_m = \frac{V_m}{\omega L}]$.

$I = I_m \sin(\omega t - \frac{\pi}{2})$

Here, $I_m = \frac{V_m}{\omega L}$.

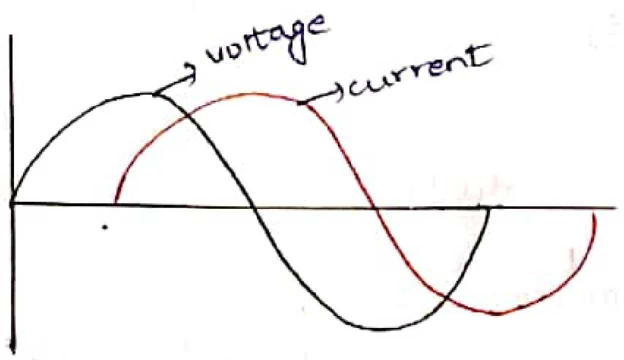
$X_L = \omega L = 2\pi f L$

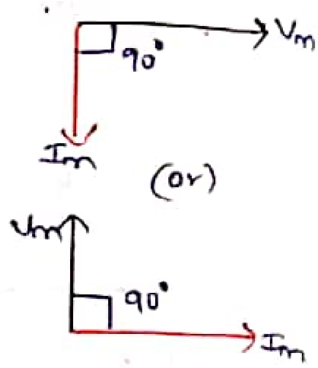
$I_m = \frac{V_m}{X_L}$. : 'X_L' is Inductive Reactance (Ω)

→ Now, $\left.\begin{array}{l} I = I_m(\sin \omega t - \frac{\pi}{2}) \\ V = V_m \sin \omega t \end{array}\right\}$ ① & ②

And $V = V_m \angle 0^\circ$

$I = I_m \angle -90^\circ$





'I' lags 'v' by '90°'

Power :-

We know, "$P = VI$".

$\Rightarrow P = V_m \sin \omega t \, (I_m \sin(\omega t - \frac{\pi}{2}))$.

$\Rightarrow P = -V_m I_m \sin \omega t \cos \omega t$

$\Rightarrow P = -\frac{V_m I_m}{2} (2 \sin \omega t \cos \omega t)$.

$\Rightarrow P = -\frac{V_m I_m}{2} (\sin 2 \omega t)$

$\therefore \boxed{P = 0}$

∴ Power dissipation in pure Inductor is '0'.

* conclusions :-

① 'I' lags 'v' by 90°.

② 'Power dissipation' in pure Inductor is zero.

③ $X_L = j\omega L = j\,2\pi f L$.

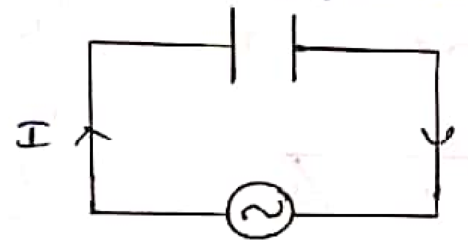
'Inductor' opposes 'A.C'

④ If frequency, 'f=0', short circuit. $[X_L=0]$

If frequency, 'f=∞', open circuit [very high]. $[X_L=\infty]$.

* A.C through pure capacitor :-

C [Farad, F)



$V = V_m \sin\omega t$.

Here, $v = V_m \sin\omega t \longrightarrow ①$

We know, $I = C \cdot \frac{dv}{dt}$.

$\Rightarrow I = C \cdot \frac{d}{dt}[V_m \sin\omega t]$.

$\Rightarrow I = CV_m\,[\cos\omega t\,(\omega)]$.

$\Rightarrow I = CV_m\omega\,[\cos\omega t]$.

$\Rightarrow I = \omega C V_m\,[\sin(\omega t + \frac{\pi}{2})]$.

$\therefore \boxed{I = I_m \sin(\omega t + \frac{\pi}{2})} \longrightarrow ②$ $[\because I_m = \omega C V_m]$.

By comparing with ohm's Law.

$I = \frac{V_m}{\frac{1}{\omega C}} \sin(\omega t + \frac{\pi}{2})$. " $I_m = \frac{V_m}{\frac{1}{\omega C}} = \frac{V_m}{X_C}$ "

Here, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$.

* conclusions :-

① 'I' leads 'v' by '90°'.

② 'v' lags 'I' by '90°'.

③ 'Power dissipation' in pure capacitor is zero.

④ $x_c = \frac{1}{j\omega c}$, $x_c = \frac{1}{j2\pi fc}$, $= \frac{-j}{2\pi fc}$

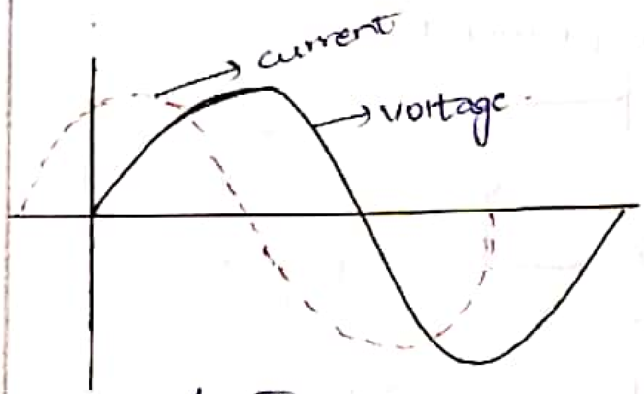
'Capacitor Blocks D.C' [$\because f = 0$ Hz].

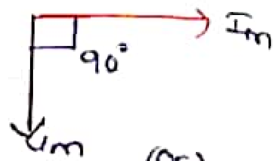
⑤ If '$f=0$', circuit is open '$x_c = \infty$' Ideal.

'$f=\infty$'. 'very high' $\Rightarrow$ short circuit $x_L = 0$.

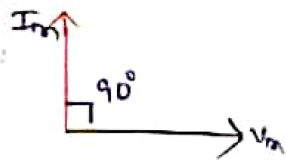
* Here $v = V_m \angle 0°$

$I = I_m \angle +90°$





(Or)



$\phi = \frac{\pi}{2}$

Power dissipation :-

We know, $P = vI$

$\Rightarrow P = V_m \sin\omega t \cdot I_m \sin(\omega t + \frac{\pi}{2})$.

$\Rightarrow P = V_m I_m \sin\omega t (\sin\omega t + \frac{\pi}{2})$

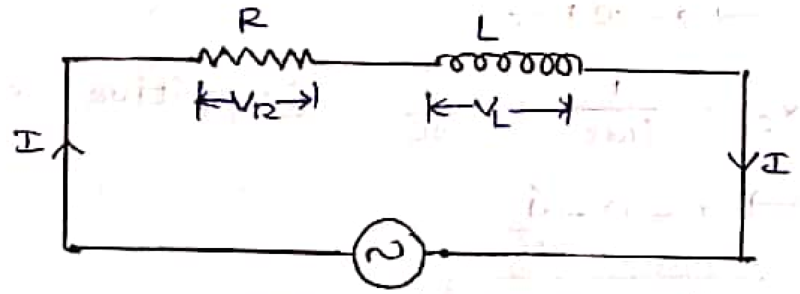
$\Rightarrow P = V_m I_m \sin\omega t \cos\omega t$

$\Rightarrow P = \frac{V_m I_m}{2} (2 \sin\omega t \cos\omega t]$

$\Rightarrow P = \frac{V_m I_m}{2} \sin 2\omega t$

$\therefore$ '$P = 0$'.

* Series RL circuit :-



$V = V_m \sin\omega t$

$I = I_m \sin(\omega t - \frac{\pi}{2})$.

→ According to KVL, '$V = V_R + V_L$'

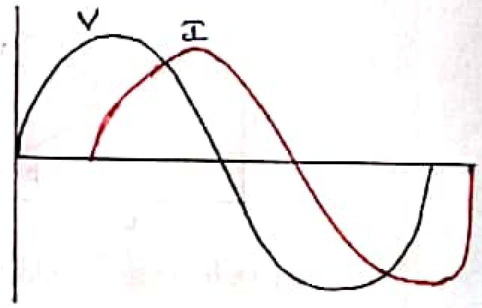
We have, $I = \frac{V}{Z}$.

$\Rightarrow Z = R + X_L$

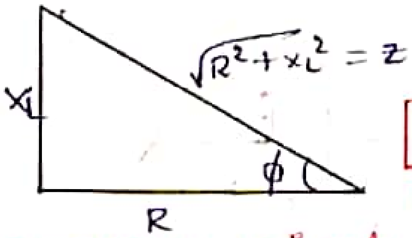
$\Rightarrow Z = R + j\omega L$

$\therefore \boxed{|Z| = \sqrt{R^2 + X_L^2}}$

$\therefore \boxed{\phi = \tan^{-1}\left(\frac{X_L}{R}\right)}$



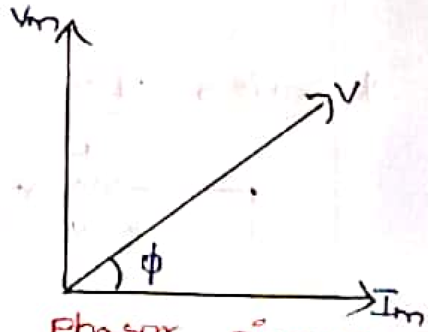
If $R = 0$, $\phi = 90^\circ$.



Impedance Triangle

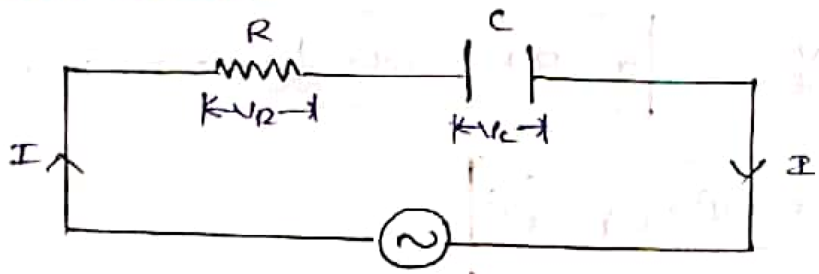
$\boxed{\cos\phi = \frac{R}{Z}}$

Power factor



Phasor Diagram.

* Series RC circuit :-



$V = V_m \sin\omega t$

$I = I_m \sin(\omega t + \frac{\pi}{2})$.

$V = V_R + V_C$.

$I = \frac{V}{Z}$ $\Rightarrow Z = R + X_C$

We have, $I = \frac{V}{Z}$

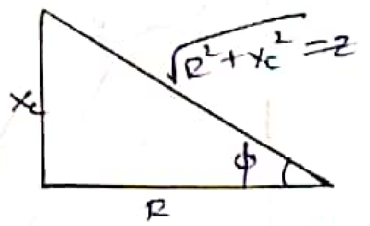
$$\Rightarrow Z = R + X_C .$$

Here, $X_C = \frac{1}{j\omega C} = \frac{-j}{\omega C}$ [Capacitive Reactance]

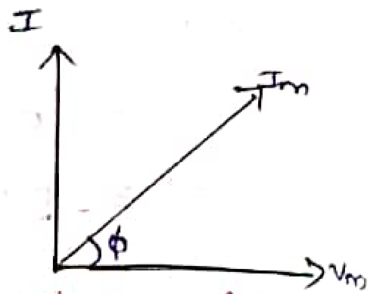
$$\Rightarrow Z = R - \frac{j}{\omega C}$$

$$\Rightarrow \boxed{|Z| = \sqrt{R^2 + X_C^2}}$$

$$\therefore \boxed{\phi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{-1}{R\omega C}\right)}$$



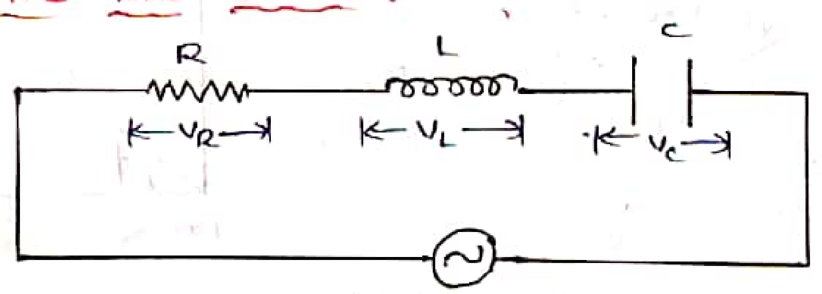
Impedance Triangle



Phasor diagram

$\boxed{\cos\phi = \frac{R}{Z}}$ Power factor

* Series RLC circuit :-



$V = V_m \sin\omega t$

Here, $V = V_R + V_L + V_C$

$I = \frac{V}{Z}$. $\boxed{Z = R + j\left(\omega L - \frac{1}{\omega C}\right) = R + j(X_L - X_C)}$

$$\therefore \boxed{|Z| = \sqrt{R^2 + (X_L - X_C)^2}}$$

$$\therefore \boxed{\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)}$$

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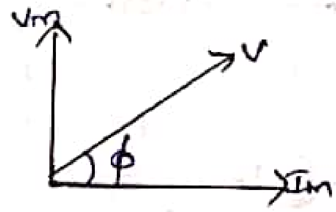
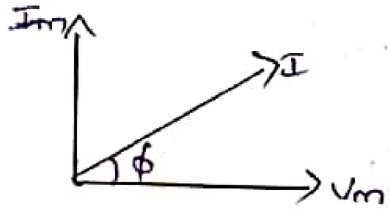
case-i :-

→ If '$X_L > X_C$' → "Inductive RLC circuit."



case-(ii) :-

→ If '$X_L < X_C$' → "capacitive RLC circuit."



$Z = \sqrt{R^2 + (X_C - X_L)^2}$

$\phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$

case-(iii) :-

→ If '$X_L = X_C$' → "Resonance condition."

$Z = R$

$\phi = 0°$

→ At "Resonance condition," circuit offers 'maximum current' and 'Impudance is minimum.'

→ This is "Pure Resistive RLC circuit."

(Inphase)

Resonance Frequency :-

$X_L = X_C$

$\Rightarrow \omega L = \frac{1}{\omega C}$

$\Rightarrow \omega^2 = \frac{1}{LC}$

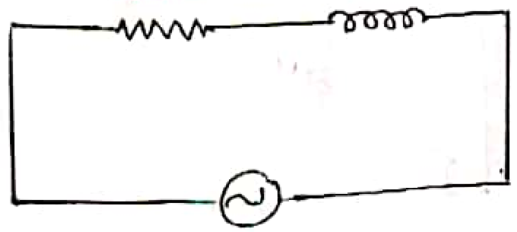
$\Rightarrow \omega = \frac{1}{\sqrt{LC}}$

$\Rightarrow 2\pi f = \frac{1}{\sqrt{LC}}$

$\therefore$ "$f = \frac{1}{2\pi\sqrt{LC}}$" is Resonance Frequency.

* Problems:

①



Find current (I) and power (P)?

A) We have, " $I = \frac{V}{Z}$ "

$Z = \sqrt{R^2 + X_L^2}$

$X_L = \omega L = 2\pi f (L)$

$= 2 \times 3.14 \times 50 \times 0.05$

$X_L = 15.7$

$\Rightarrow Z = \sqrt{R^2 + X_L^2} = \sqrt{20^2 + (15.7)^2} = 25.426$

$\therefore$ $I = \frac{V}{Z} = \frac{100}{25.426} = 3.932\ A$

Now, $\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{15.7}{20}\right) = 38.14$

$\therefore$ " $I = 3.932 \angle -38.14\ A$ "

Power $\boxed{P = |V||I| \cos\phi}$

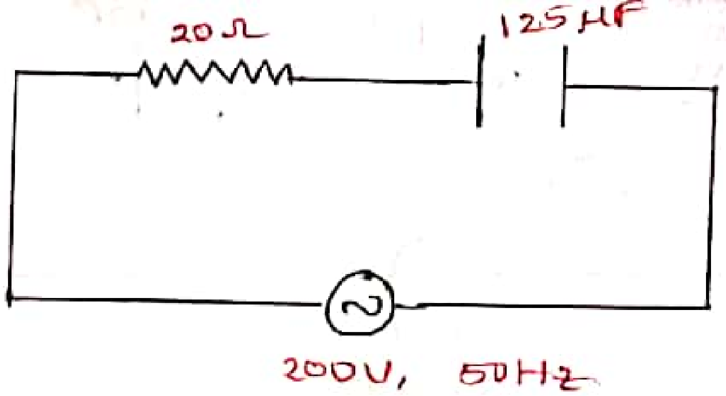
$\Rightarrow P = |100|\,|3.932| \cos(38.14)$

$\Rightarrow P = (393.2) \cos(38.14)$

$\Rightarrow P = (393.2) \times (0.786)$

$\therefore$ " $P = 309.253\ W.$ "

(8)



Find current (I) and power (P)?

A) we have, $I = \frac{V}{Z}$.

$\Rightarrow Z = \sqrt{R^2 + (X_c)^2}$

$\Rightarrow X_c = \frac{1}{\omega C} = \frac{1}{2\pi f (C)}$

$= \frac{1}{2 \times 3.14 \times 50 \times 125 \times 10^{-6}}$

$X_c = 25.478$

$\Rightarrow Z = \sqrt{20^2 + (25.478)^2} = \cancel{8525.5683} = 32.31$

$\therefore I = \frac{V}{Z} = \frac{200}{32.3} = 6.19A$

$\therefore \phi = \tan^{-1}\left(\frac{X_c}{R}\right) = \tan^{-1}\left(\frac{25.47}{20}\right) = 51.34$

$\therefore$ Power, $P = |V|\,|I| \cos\phi$

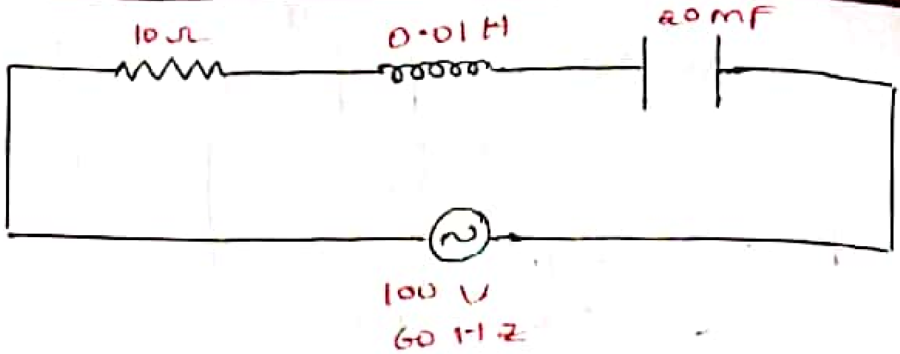
$\Rightarrow P = (200)(6.2) \cos(51.34)$

$\Rightarrow P = 1240 \times 0.624$

$\Rightarrow P = 773.76$

$\therefore$ Power $= 773.76$ watts.

(3)



Find current (I)?

A) We have, $I = \frac{V}{Z}$

$\Rightarrow Z = \sqrt{R^2 + (X_L - X_C)^2}$

$\Rightarrow X_L = \omega L = 2\pi f L = 2 \times 3.14 \times 60 \times 0.01 = 3.768$

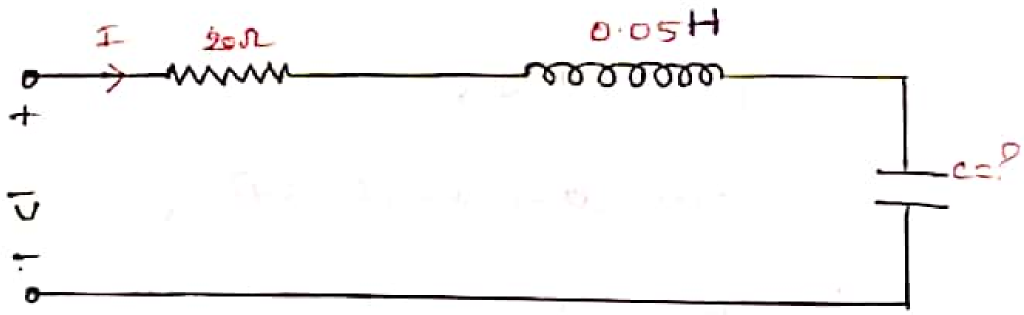
$\Rightarrow X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 60 \times 20 \times 10^{-3}} = 0.13269$

$\therefore Z = \sqrt{10^2 + (3.768 - 0.13269)^2} = 129.30 = 10.640$

$\therefore \boxed{I = \frac{V}{Z} = \frac{100}{129.30} = 0.77\ A}$

$\therefore I = \frac{V}{Z} = \frac{100}{10.640} = 9.39\ A$?

(4)



Given $I = 5\angle 0^\circ A$, $\bar{V} = 100 + j200 V$

$\omega = 1200$ rad/s

Find 'c'?

A) Given, $R = 20 \Omega$

$L = 0.05 H$

$X_L = j\omega L$

$\therefore X_L = j(1200 \times 0.05) \Omega$

$\therefore X_L = j60 \Omega$

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$X_C = \frac{-j}{\omega C} = \frac{-j}{1200C}$.

∴ Total Impudance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= 20 + (j60 - \frac{j}{1200C})$

∴ $V = IZ$

$\Rightarrow 100 + j200 = 5\angle 0^\circ \left[20 + j60 - \frac{j}{1200C}\right]$.

$\Rightarrow 100 + j200 = 5^{+0j}\left[20 + j60 - \frac{j}{1200C}\right]$

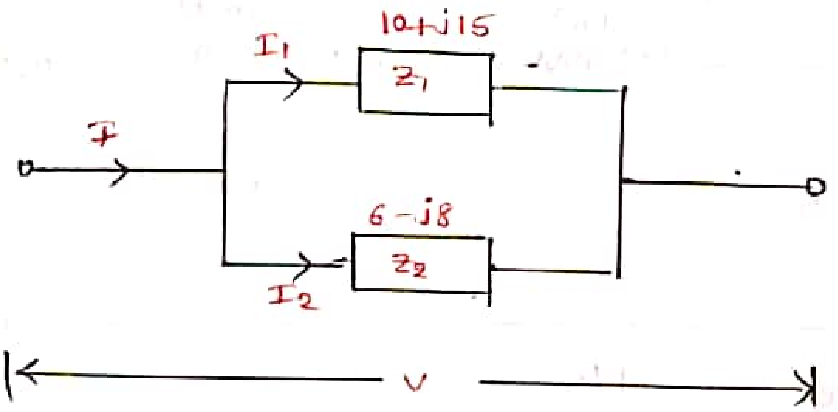
$\Rightarrow \cancel{100} + j200 = \cancel{100} + 300j - \frac{\cancel{5}j}{\cancel{1200}C}$ 240

$\Rightarrow 200j = 300j - \frac{j}{240C}$ [Comparing imaginary part]

$\Rightarrow 200 = 300 - \frac{1}{240C}$

$\Rightarrow C = 41.6\,\mu F$.

(5)



given $I = 15\angle 0^\circ$, Find I_1, I_2 & V?

A) According to current division rule

$\Rightarrow I_1 = I \times \frac{R_2}{R_1 + R_2}$

$\Rightarrow I_1 = I \times \frac{Z_2}{Z_1 + Z_2} = 15\angle 0^\circ \left[\frac{16 \mp j8}{(10+j15) + (6-j8)}\right]$

$= 15\angle 0^\circ \left[\frac{6-8j}{16+7j}\right]$

$= 15+0j \left[\frac{6-8j}{16+7j}\right]$

$= 15\left[\frac{6-8j}{16+7j}\right] = \frac{120}{61} - \frac{510}{61}j = 8.58\angle -76.7^\circ A$.

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$$I_2 = I \times \frac{Z_1}{Z_1+Z_2}$$

$$= 15\angle 0^\circ \left[\frac{10+j15}{(10+j15)+(6-j8)}\right]$$

$$= 15+0j \left[\frac{10+j15}{16+7j}\right]$$

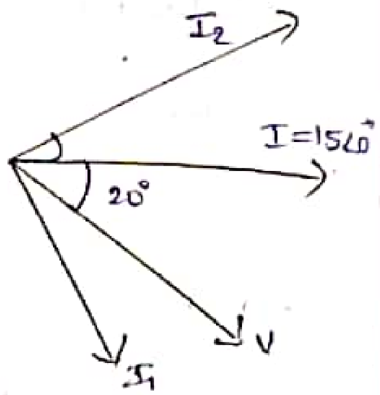
$$= 15\left[\frac{10+j15}{16+7j}\right] = \frac{795}{61} + \frac{510}{61}j = 15.49\angle 32.7^\circ A.$$

$\therefore V = IZ_1$ $[\because V = IR]$

$= (8.58\angle -76.7^\circ)(10+j15)$

$= 144.98 - 53.89j$

$V = 154.67\angle -20.39^\circ$.



(6)

given $\omega = 400$ rad/s

(a) when 'AB' opened find Z_{in}?

(b) when 'AB' short find Z_{in}?

(c) when 'AB' replaced by 10Ω. find Z_{in}?

A) $X_L = j\omega L = j(400\times 25\times 10^{-3}) = j10\ \Omega$

$$X_C = \frac{-j}{\omega C} = \frac{-j}{400\times 250\times 10^{-6}}$$

$$= \frac{-1000000j}{400\times 250} = \frac{-1000000j}{100000} = -j10\ \Omega$$

(a) when AB is opened. [25mH is dummy].

$Z_{in} = 10 + X_c$

$= 10 - j10\,\Omega$

$\therefore Z_{in} = 14.14 \angle -45^\circ \,\Omega$

(b) when AB shorted,

series :- $10\Omega + (25 \parallel 250)$

parallel :- $\frac{25 \times 250}{25+250} = \frac{X_L \times X_c}{X_L + X_c}$

$= \frac{j10 \times -j10}{j10 - j10} = \frac{j10 \times -j10}{0} = \infty$

$\therefore$ series $= 10\Omega + \infty j = 10 + \infty j$

(c) series :-
$10 + (25mH) = 10 + X_L$.

$= 10 + 10j$

$\not{=}$

parallel :- $(10+10j) \parallel (X_c)$

$= \frac{(10+10j) \times (-j10)}{(10+10j) - j10}$

$= 10 - 10j$

series :-

$\Rightarrow 10 - 10j + 10\,\Omega$

$\Rightarrow 20 - 10j$

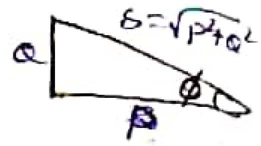
$= 22.36 \angle -26.6^\circ A$

when AB replaced by 10Ω.

* Element	Real part (P)	Imaginary Reactive part (Q)
R	$P = I_{rms} V_{rms}$	—
L	$P=0$	$Q_L = \frac{V_L}{X_L}$
C	$P=0$	$Q_C = \frac{V_C}{X_C}$

* Complex Power :-

$\boxed{S = P + jQ}$



* → This is also called 'power triangle'

→ 'cos ϕ' is 'power factor.'

→ $\boxed{I_{avg} = 0}$

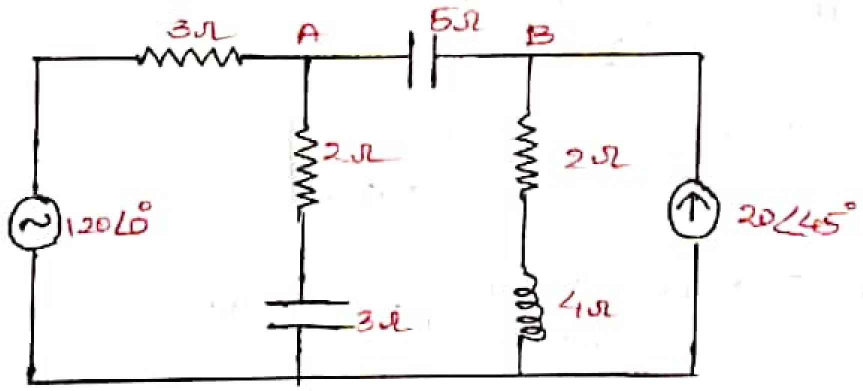
→ $\boxed{I_{rms} = \frac{I_m}{\sqrt{2}}}$

$P = I_{rms} V_{rms} \cos\phi$

$Q = I_{rms} V_{rms} \sin\phi$

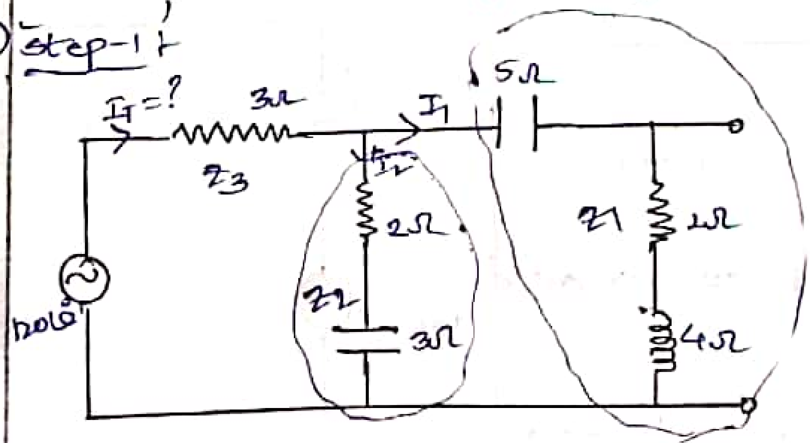
$S = I_{rms} V_{rms}$

* Problem [super position Theorem] :-



Find current passing through capacitor, $I_{CAB} = ?$

A) Step-1 :-



$Z_1 = 2+j4-j5 = (2-j1)\,\Omega$

$Z_2 = (2-3j)\,\Omega$

$\therefore Z_T = Z_3 + (Z_2 \| Z_1)$

$$Z_T = 3 + \left[\frac{(2-j)\times(2-3j)}{(2-j)+(2-3j)}\right] = 4.25 - 0.875j\,\Omega$$

Now, $I_T = \dfrac{V}{Z_T}$

$$\Rightarrow I_T = \frac{120\angle 0^\circ}{4.25-0.875j} = 27.84 + j5.90$$

$\therefore I_T = 28.46\angle 11.96^\circ$

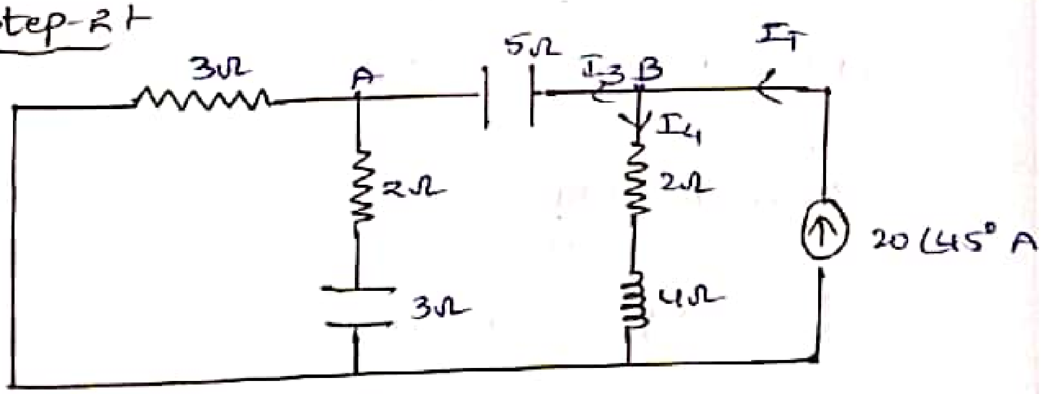
Now,

$$I_1 = I_T\left(\frac{Z_2}{Z_2+Z_1}\right)$$

$$I_1 = 28.46\angle 11.96^\circ \left[\frac{(2-3j)}{(2-j1)+(2-3j)}\right]$$

$\therefore I_1 = 18.1375 + j\,0.2075$ A [A→B].

Step-2:-



$3\Omega \| (2\Omega + 3\Omega)$

$\Rightarrow 3\Omega \| (2-3j)$

$$\Rightarrow \frac{3\times(2-3j)}{3+2-3j} = \frac{(6-9j)}{5-3j} = 1.855\angle -25.34^\circ = Z_R$$

$$\therefore I_3 = I_T\left(\frac{Z_R}{5+Z_R}\right) = 7.06 + j16.92\text{ A} \quad [B \to A]$$

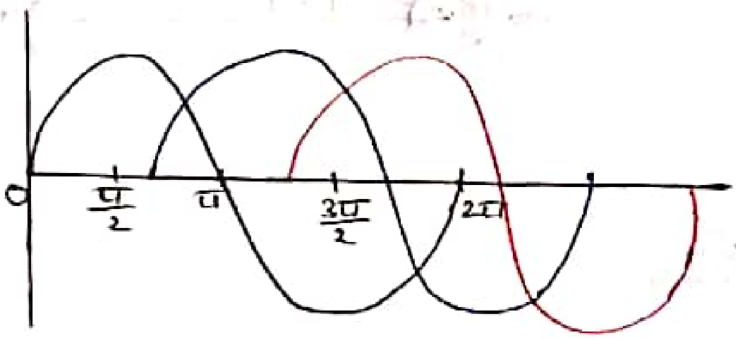
$\therefore I_{Resultant} = I_1 - I_3 =$

* star connected 3 phase Network :-

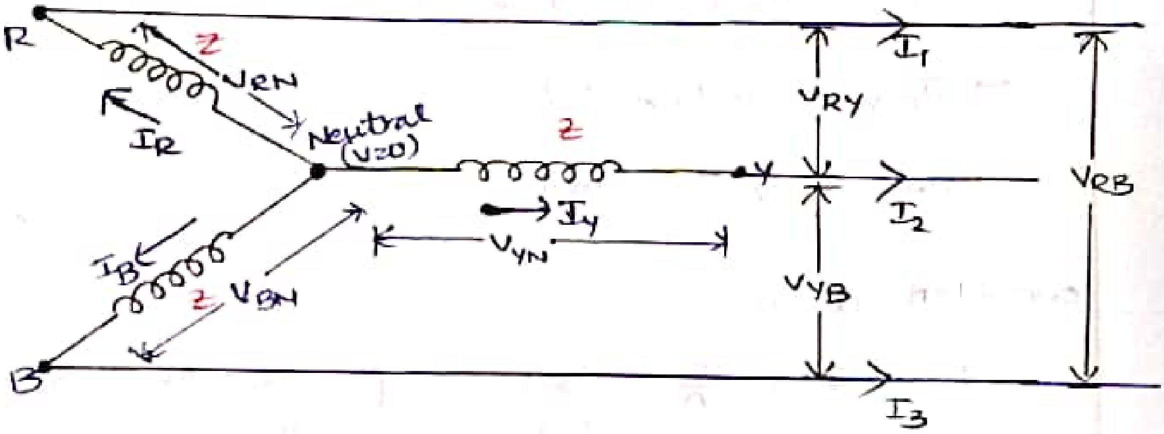
$V_1 = V_m \sin\omega t$

$V_2 = V_m \sin(\omega t + 120^\circ)$

$V_3 = V_m \sin(\omega t + 240^\circ)$.



Balanced :-



Here, 'phase currents' :- I_R, I_Y, I_B

'Line currents' :- I_1, I_2, I_3.

→ In Balanced mode,

$I_R = I_Y = I_B = I_{phase}$

$I_1 = I_2 = I_3 = I_{Line}$

$\therefore \boxed{I_{phase} = I_{Line}}$

→ Here, In Balanced Mode, $V_{RN} = V_{YN} = V_{BN} = V_{phase}$.

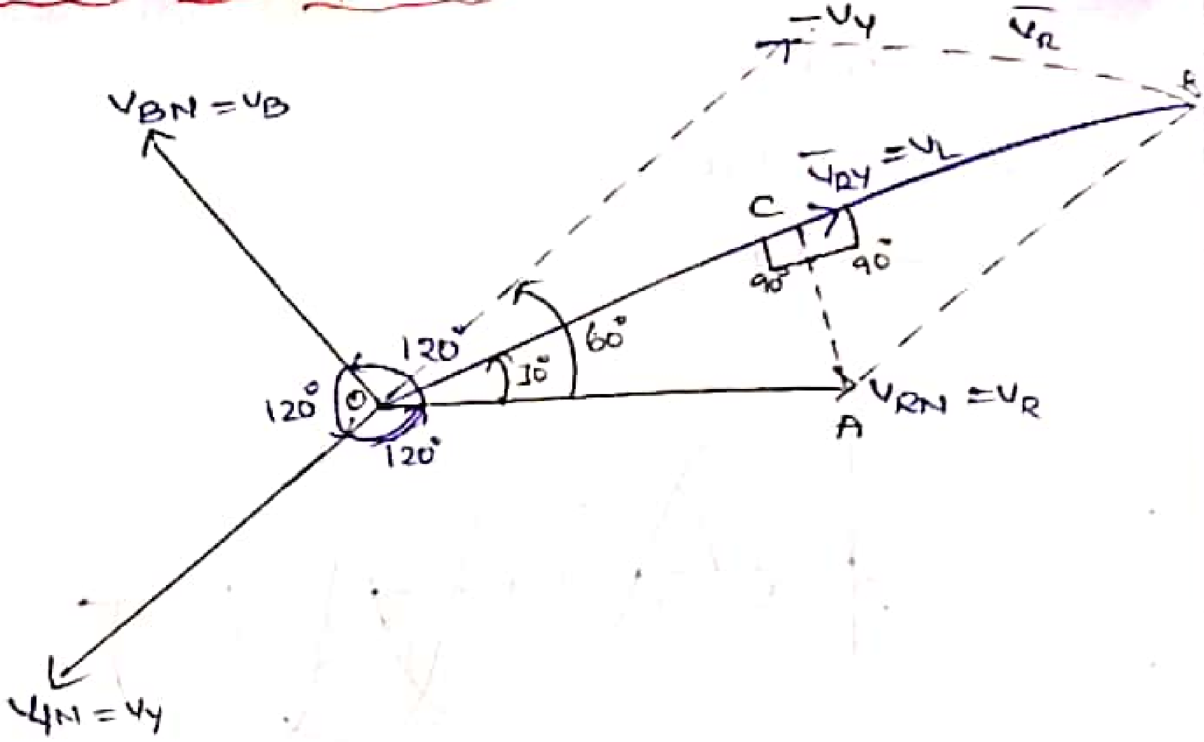
$V_{RY} = V_{YB} = V_{RB} = V_{Line}$.

But $\boxed{V_{ph} \neq V_L}$

If N is connected to ground,

$V_R = V_Y = V_B = V_{ph}$.

* Derivation for V_{ph} & V_L :-



Now. $V_{RY} = V_R - V_Y$

$$\Rightarrow \bar{V}_{RY} = \bar{V}_R + (-\bar{V}_Y) \longrightarrow (1)$$

$$\Rightarrow \bar{V}_{RB} = \bar{V}_R + (-\bar{V}_B) \longrightarrow (2)$$

$$\Rightarrow \bar{V}_{YB} = \bar{V}_Y + (-\bar{V}_B) \longrightarrow (3)$$

consider, ΔOCA

$$\Rightarrow \cos 30° = \frac{OC}{OA} = \frac{\left(\frac{V_L}{2}\right)}{V_{ph}}$$

$\left[\because V_R = V_{phase} \text{ \& } V_L = V_{RY}\right]$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{\left(\frac{V_L}{2}\right)}{V_{ph}}$$

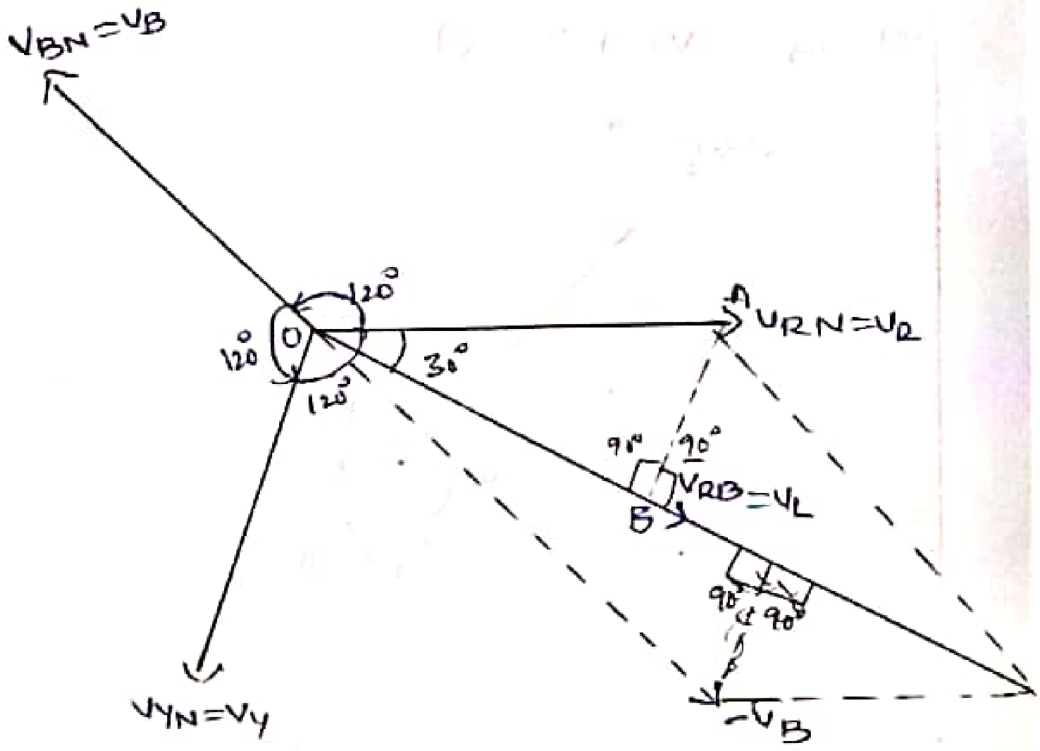
$$\Rightarrow \frac{\sqrt{3}}{\cancel{2}} = \frac{1}{\cancel{2}}\left(\frac{V_L}{V_{ph}}\right)$$

$$\Rightarrow \sqrt{3} = \frac{V_L}{V_{ph}}$$

$$\therefore \boxed{V_L = \sqrt{3}\, V_{ph}} \quad \text{(or)} \quad \boxed{V_{ph} = \frac{V_L}{\sqrt{3}}}$$

Now, $\bar{V}_{RB} = \bar{V}_R - \bar{V}_B$

$\Rightarrow \bar{V}_{RB} = \bar{V}_R + (-\bar{V}_B)$



Consider, ΔOAB

$\Rightarrow \cos\theta = \frac{Adj}{Hyp}$

$\Rightarrow \cos 30^\circ = \frac{OB}{OA}$

$\Rightarrow \cos 30^\circ = \frac{\frac{V_{RB}}{2}}{V_R}$ $\quad \left[\because \frac{V_{RB}}{2} = \frac{V_L}{2}\right]$

$\left[\because V_R = V_{ph}\right]$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{\frac{V_L}{2}}{V_{ph}}$

$\Rightarrow \frac{\sqrt{3}}{\cancel{2}} = \frac{1}{\cancel{2}} \frac{V_L}{V_{ph}}$

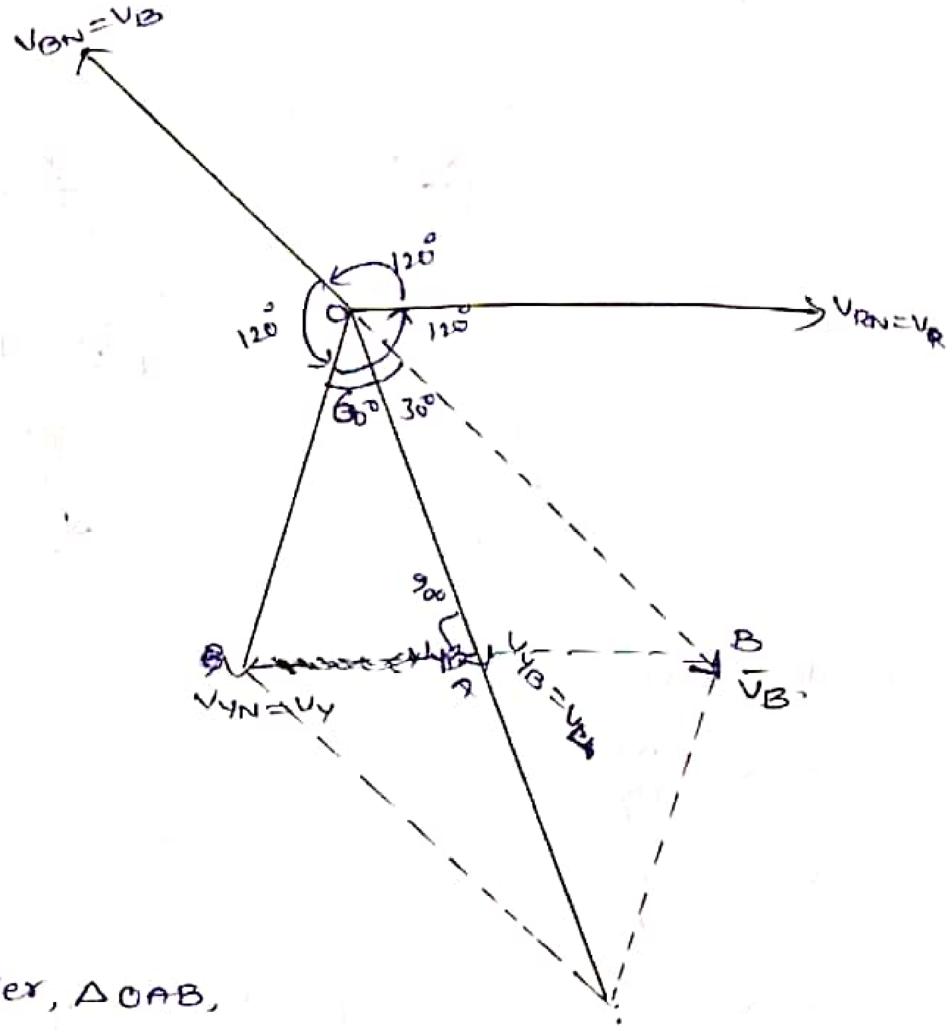
$\Rightarrow \sqrt{3} = \frac{V_L}{V_{ph}}$

$\therefore \boxed{V_L = \sqrt{3}\, V_{ph}}$ or) $\boxed{V_{ph} = \frac{V_L}{\sqrt{3}}}$

Now, equation-3

$\Rightarrow \bar{V}_{YB} = \bar{V}_Y - \bar{V}_B$

$\Rightarrow \bar{V}_{YB} = \bar{V}_Y + (-\bar{V}_B)$



consider, ΔOAB,

$$\Rightarrow \cos 30^\circ = \frac{OA}{OB}$$

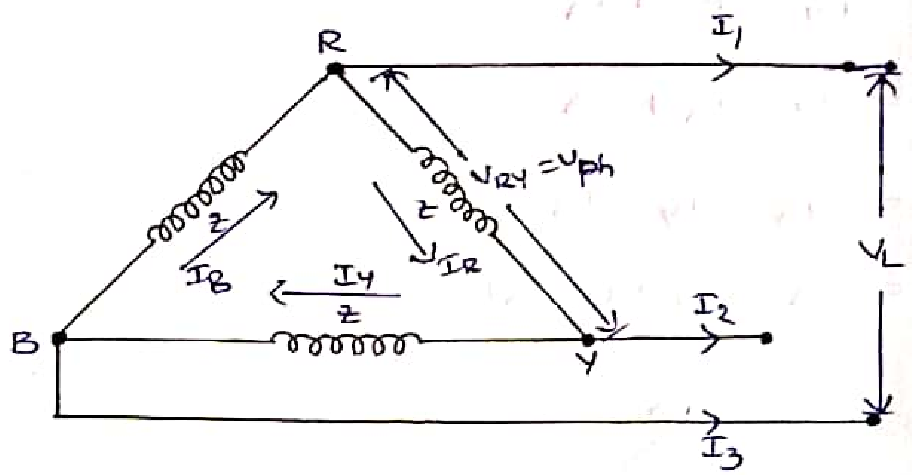
$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{\frac{V_Y}{2}}{V_B}$$

$$\Rightarrow \frac{\sqrt{3}}{\cancel{2}} = \cancel{\frac{1}{2}} \frac{V_L}{V_{ph}} \qquad \left[\because \begin{array}{l} V_Y = V_L \\ V_B = V_{ph} \end{array}\right]$$

$$\Rightarrow \sqrt{3} = \frac{V_L}{V_{ph}}$$

$\therefore$ $\boxed{V_L = \sqrt{3}\, V_{ph}}$ (or) $\boxed{V_{ph} = \frac{V_L}{\sqrt{3}}}$

* Delta connected 3phase Networks:-



→ In Delta connected Network, Line current is not equal to phase current

Here, 'I_R', 'I_Y', 'I_B' are phase currents

'I_1', 'I_2', 'I_3' are Line currents.

$\therefore$ $\boxed{I_{ph} \neq I_L}$

→ Here, $\boxed{V_{ph} = V_L}$

→ Apply KCL at R :-

$I_B = I_1 + I_R$

$I_1 = I_B - I_R$

$I_1 = I_B + (-I_R)$

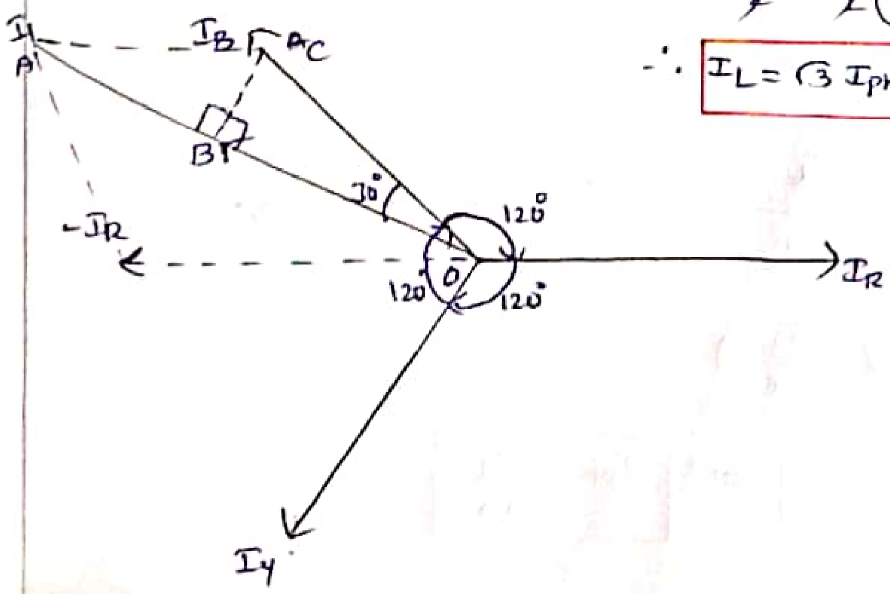
consider ΔOBC

$\Rightarrow \cos 30^\circ = \dfrac{OB}{OC}$

$\Rightarrow \cos 30^\circ = \dfrac{\frac{I_L}{2}}{I_{ph}}$ $\left[\because OB = I_1 = \frac{I_L}{2},\ OC = I_B = I_{ph}\right]$

$\Rightarrow \dfrac{\sqrt{3}}{\cancel{2}} = \dfrac{1}{\cancel{2}}\left(\dfrac{I_1}{I_{ph}}\right)$

$\therefore$ $\boxed{I_L = \sqrt{3}\, I_{ph}}$ (or) $\boxed{I_{ph} = \dfrac{I_L}{\sqrt{3}}}$



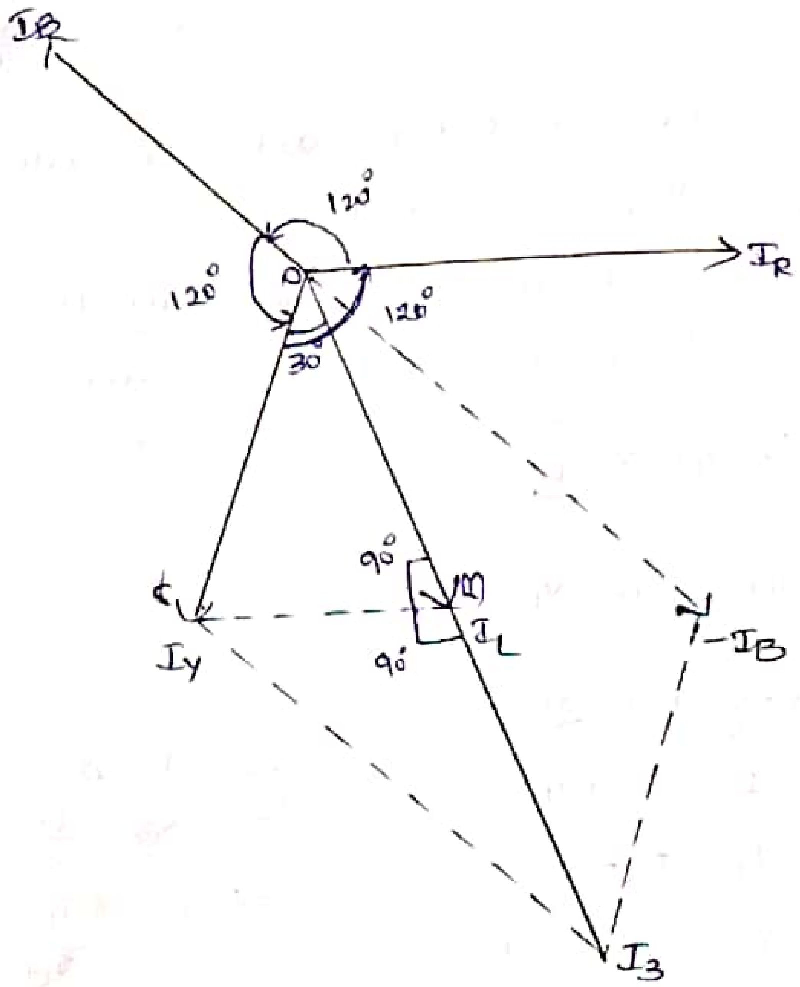
Applying KCL at B :-

$I_Y = I_3 + I_B$

$\Rightarrow \bar{I}_Y = \bar{I}_3 + \bar{I}_B$

$\Rightarrow \bar{I}_3 = \bar{I}_Y - \bar{I}_B$

$\Rightarrow \bar{I}_3 = \bar{I}_Y + (-\bar{I}_B)$



consider Δ OAB.

$\Rightarrow \cos 30^\circ = \frac{OB}{OA}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{\frac{I_L}{2}}{I_{ph}}$ $\left[\because I_Y = I_{ph} \ \& \ \frac{I_3}{2} = \frac{I_L}{2} \right]$.

$\Rightarrow \frac{\sqrt{3}}{\cancel{2}} = \frac{1}{\cancel{2}} \left(\frac{I_L}{I_{ph}} \right)$

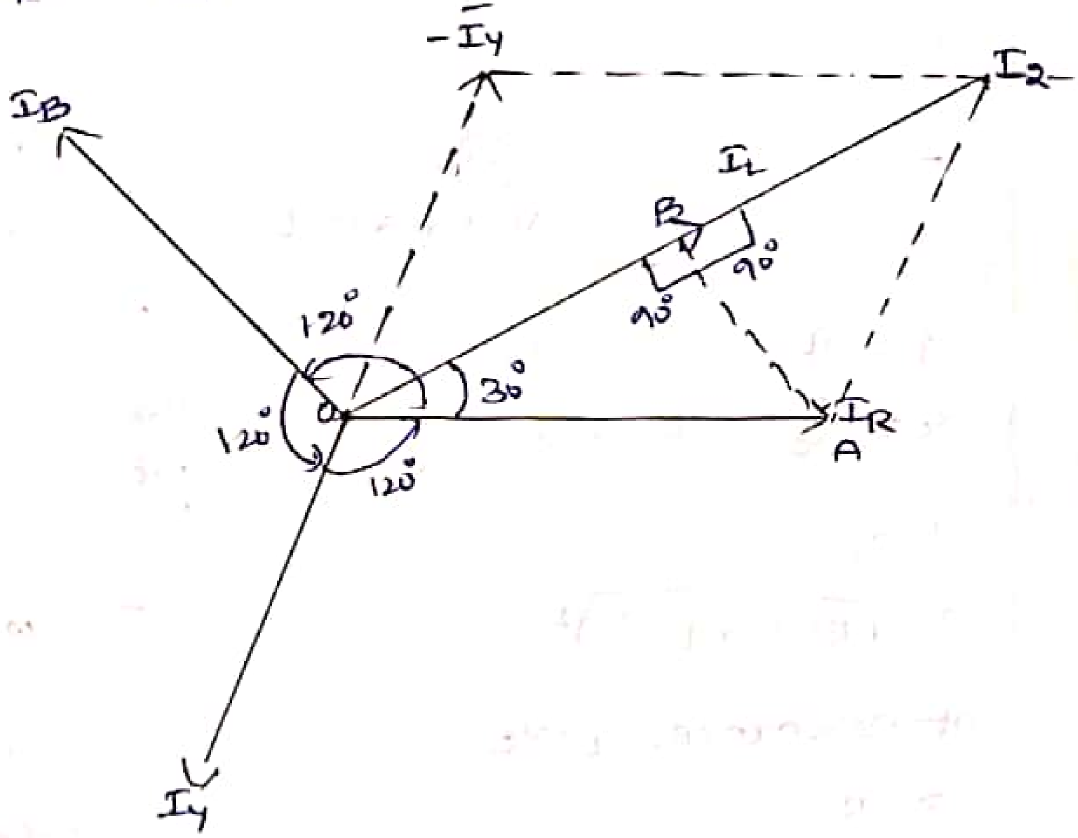
$\Rightarrow \boxed{I_L = \sqrt{3}\, I_{ph}}$ (or) $\boxed{I_{ph} = \frac{I_L}{\sqrt{3}}}$

Applying KCL at y:

$\cdot\ I_R = I_2 + I_y$

$\Rightarrow I_2 = I_R - I_y$

$\Rightarrow I_2 = \bar{I}_R + (-\bar{I}_y)$



Consider Δ OAB,

$$\Rightarrow \cos 30^\circ = \frac{OB}{OA}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{\frac{I_L}{2}}{I_R}$$

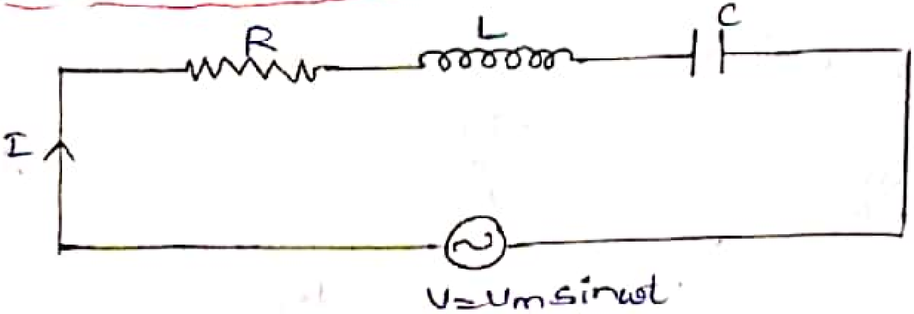
$$\Rightarrow \frac{\sqrt{3}}{\cancel{2}} = \frac{1}{\cancel{2}}\left[\frac{I_L}{I_{ph}}\right] \qquad [\because I_R = I_{ph}]$$

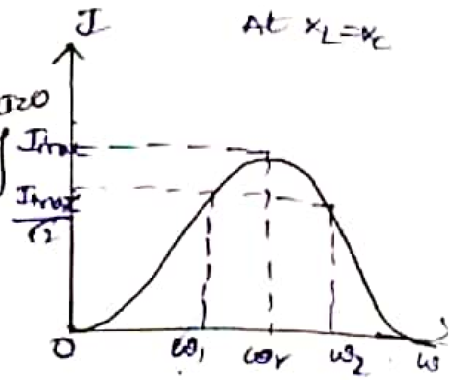
$$\Rightarrow \sqrt{3} = \frac{I_L}{I_{ph}}$$

$$\therefore I_L = \sqrt{3}\, I_{ph} \quad (\text{or}) \quad I_{ph} = \frac{I_L}{\sqrt{3}}$$

29-12-

* RLC - Series circuit :-





$X_L = \omega L$: if $\omega = 0$, $X_L = 0$, $X_C = \infty$ } $I = 0$

$X_C = \frac{1}{\omega C}$: if $\omega = \infty$, $X_C = 0$, $X_L = \infty$

At $X_L = X_C$

$$Z = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$$

At Resonance, $X_L = X_C$

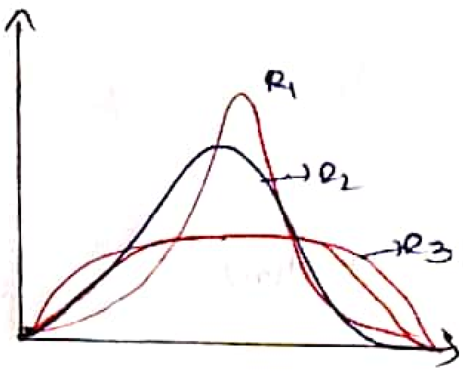
$Z = R$

purely resistive.

$$\boxed{I_{max} = \frac{V}{R}}$$

ω_1 = lower cutoff frequency

ω_2 = higher cutoff freq



$R_1 < R_2 < R_3$.

(1) Quality factor (Q) :-

$$\boxed{Q = 2\pi \frac{\text{Energy stored}}{\text{Energy dissipated}}}$$

(2) Bandwidth (BW) :-

→ The difference between Half power frequencies is called Bandwidth.

$\boxed{BW = \omega_2 - \omega_1}$ [in the above graph].

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}.$$

$$I_1 = \frac{V}{\sqrt{R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2}} = \frac{I_{max}}{\sqrt{2}} = \frac{\frac{V}{R}}{\sqrt{2}} = \frac{V}{\sqrt{2}R}. \quad \left[\because I_{max} = \frac{V}{R}\right]$$

$$\Rightarrow I_1^2 = \frac{V^2}{R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2} = \frac{V^2}{2R^2}$$

$$= R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2 = 2R^2$$

$$\left(\omega_1 L - \frac{1}{\omega C}\right)^2 = 2R^2 - R^2$$

$$\left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2 = R^2$$

$$\therefore \boxed{\omega_1 L - \frac{1}{\omega_1 C} = \pm R}$$

Similarly, $\boxed{\omega_2 L - \frac{1}{\omega_2 C} = \pm R.}$

Now, $\omega_1 < \omega_r < \omega_2$.

$$\Rightarrow \omega_1 L - \frac{1}{\omega_1 C} = -R$$

$$\Rightarrow \omega_1^2 LC - 1 = -R\omega_1 C$$

$$\Rightarrow \omega_1^2 LC + RC\omega_1 = 1$$

$$\Rightarrow \omega_1^2 LC + RC\omega_1 - 1 = 0$$

$$\Rightarrow \frac{\omega_1^2 LC}{LC} + \frac{RC\omega_1}{LC} - \frac{1}{LC} = 0$$

$$\Rightarrow \omega_1^2 + \frac{R}{L}\omega_1 - \frac{1}{LC} = 0$$

$$\Rightarrow \omega_1 = \frac{-R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4(1)\left(-\frac{1}{LC}\right)}$$

$$= \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}}{2(1)}$$

$$\omega_1 = +\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}.$$

Similarly, $\omega_2 = +\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$

$\therefore$ Bandwidth, $BW = \omega_2 - \omega_1 = \frac{2R}{2L} = \frac{R}{L}$ $\quad \therefore \boxed{BW = \omega_2 - \omega_1 = \frac{R}{L}.}$

Quality factor, $Q = \cancel{2\pi} \cdot \frac{I^2 X_L}{I^2 R} = \cancel{2\pi} \frac{I^2 X_C}{I^2 R}$.

$\Rightarrow Q = \cancel{2\pi} \cdot \frac{X_L}{R} = \cancel{2\pi} \frac{X_C}{R}$.

$\Rightarrow Q = \cancel{2\pi} \frac{\omega_0 L}{R} = \cancel{2\pi} \frac{1}{\omega_0 R C}$, $\omega_0 = \frac{1}{\sqrt{LC}}$.

$\Rightarrow Q = \frac{1}{\sqrt{LC}} \cdot \frac{L}{R} = \frac{1}{R} \sqrt{\frac{L}{C}}$; $\boxed{Q = \frac{1}{R} \sqrt{\frac{L}{C}}}$

Also,

$\boxed{Q = \frac{\omega_r}{BW}}$ $\Rightarrow$ $\boxed{Q = \frac{\omega_0}{\omega_2 - \omega_1}}$

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