DEPARTMENT OF CIVIL ENGINEERING
RGUKT-NUZVID

Geotechnical Engineering

Permeability

Contents

Darcy’s law and Permeability

SOIL PERMEABILITY

- A soil mass is composed of small solid particles which are called the soil grains. These soil grains when depositing in a soil mass arranges themselves in a way that some amount of empty space is left between them which are called voids.
- These voids or pores are interconnected and form a highly complex network of irregular tube like structure.
- When water is subjected to a potential difference in the soil, it flows through these voids from high potential to low potential.
- The surface of the soil particles offers a resistance to the flow of water. The more irregular and narrower the voids, greater is the resistance posed to the water flowing while more open the voids, greater is the ease with which water flows through soil.

SOIL PERMEABILITY Cont..

- This property of the soil which permits the water or any liquid to flow through its voids is called permeability. It is the ease with which water can flow through the soils. A material is permeable if it contains continuous voids.
- The permeability of soils has a decisive effect on the
  - stability of foundations,
  - seepage loss through embankments of reservoirs,
  - drainage of subgrades,
  - excavation of open cuts in water bearing sand,
  - rate of flow of water into wells and many others.

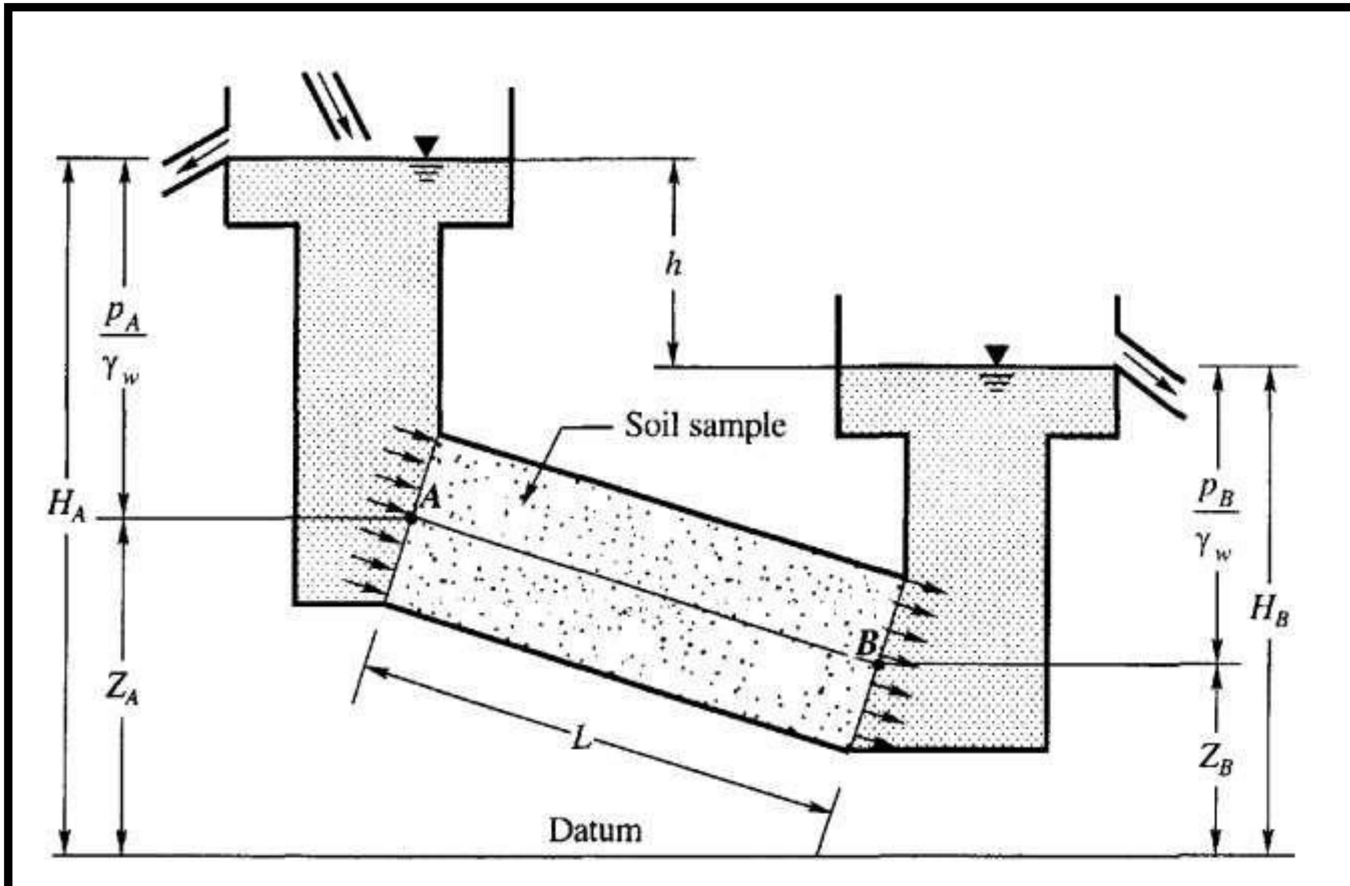
Hydraulic Gradient

- When water flows through a saturated soil mass there is certain resistance for the flow because of the presence of solid matter.
- However, the laws of fluid mechanics which are applicable for the flow of fluids through pipes are also applicable to flow of water through soils.
- As per Bernoulli's equation, the total head at any point in water under steady flow condition may be expressed as

Total head = pressure head + velocity head + elevation head

Hydraulic Gradient Cont..

- This principle can be understood with regards to the flow of water through a sample of soil of length L and cross-sectional area A.



Flow of water through a sample of soil

Hydraulic Gradient Cont..

- The heads of water at points A and B as the water flows from A to B are given as follows (with respect to a datum)
- Total head at A, $$H_A = z_A + \frac{p_A}{\gamma_w} + \frac{v_A{}^2}{2g}$$
- Total head at B, $$H_B = z_B + \frac{p_B}{\gamma_w} + \frac{v_B{}^2}{2g}$$
- As the water flows from A to B, there is an energy loss which is represented by the difference in the total heads H_A and H_B

$$H_A - H_B = \left(Z_A + \frac{p_A}{\gamma_w} + \frac{V_A^2}{2g} \right) - \left(Z_B + \frac{p_B}{\gamma_w} + \frac{V_B^2}{2g} \right) = h$$

where, z_A and z_B = datum heads, p_A and p_B = pressures, v_A and v_B = velocity, g = acceleration due to gravity, γ_w = unit weight of water, h = loss of head.

Hydraulic Gradient Cont..

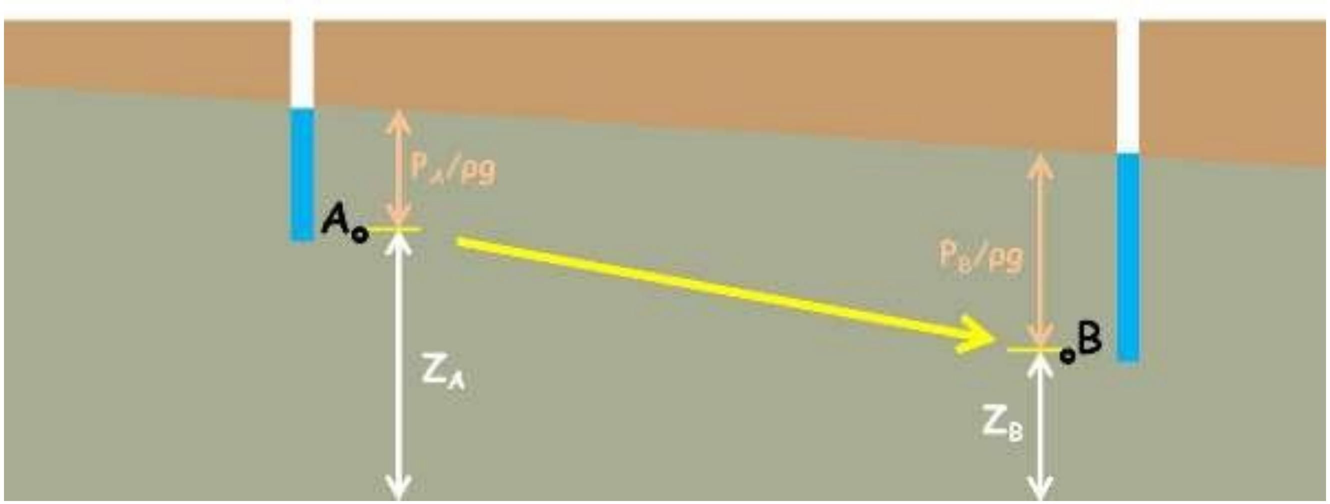
- For all practical purposes the velocity head is a small quantity and may be neglected.
- The loss of head of h units is effected as the water flows from A to B.
- The loss of head per unit length of flow may be expressed as

$$i = \frac{h}{l}$$

  where ***i*** is called the hydraulic gradient.
- This total head has the unit of height such as meters.
- When water flows through the soils its velocity is very small therefore we neglect the velocity head.

Hydraulic Gradient Cont..

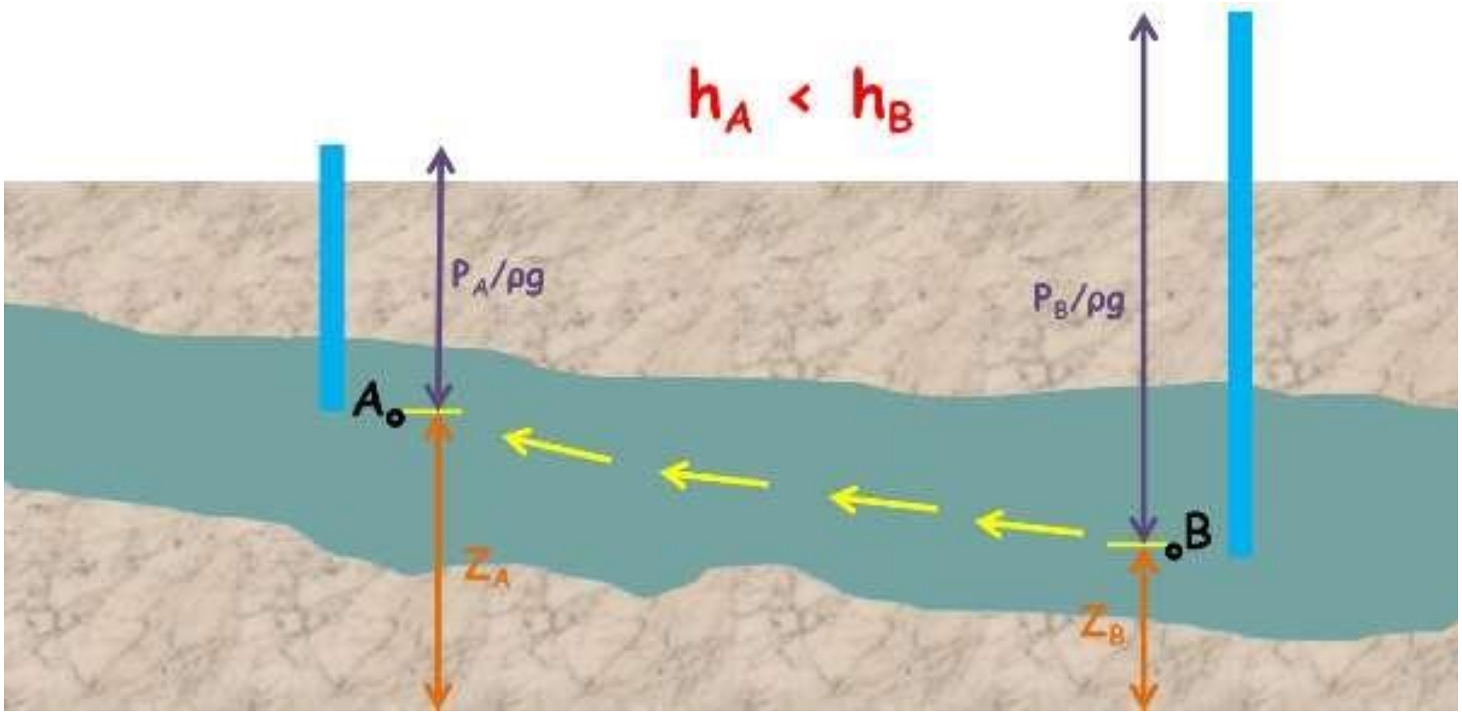
Water flows from higher energy region to lower energy region. If the hydraulic head of point A is higher than the head of B water may flow from point A to B.



Note: Water does not flow if the point A and B have the same energy or have inadequate head difference because even if A has higher elevation than point B the hydraulic gradient of both the points is same. Because hydraulic gradient which is the sum of both elevation head and pressure head, of both the points is same.

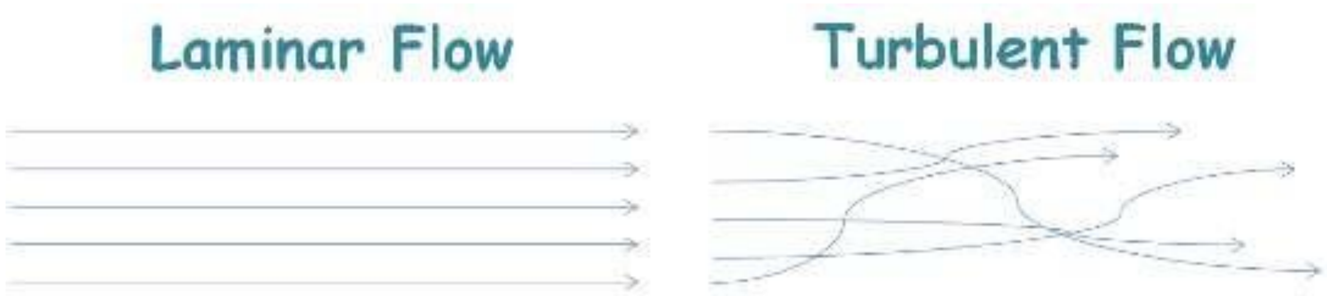
Hydraulic Gradient Cont..

Elevation of point A is higher than that of point B. But suppose if point B has high pressure head because of any geological confined aquifer conditions then total head of point B may be higher than that of point A. In that case water will flow from point B to A even against gravity.



Laminar and Turbulent Flow

- Problems relating to the flow of fluids in general may be divided into two main classes:
  1. Those in which the flow is laminar.
  2. Those in which the flow is turbulent.
- In laminar flow the particles of water follow a defined path such that the path of one particle never intersects the path of any other particle. While turbulent flow is irregular.



Laminar and Turbulent Flow

- The fundamental laws that determine the state existing for any given case were determined by Reynolds (1883). He found the lower critical velocity is inversely proportional to the diameter of the pipe and gave the following general expression applicable for any fluid and for any system of units.

$$R_e = \frac{\gamma v d}{\mu g}$$

where,

Re = Reynolds Number taken as 2000 as the maximum value for the flow to remain always laminar,

D = diameter of pipe,

v = critical velocity below which the flow always remains laminar,

γ = unit weight of fluid at 4 °C,

μ= viscosity of fluid,

g = acceleration due to gravity.

DARCY'S LAW

- Darcy in 1856 demonstrated after doing experiments that the velocity of flow of liquid between two points in the soil is directly proportional to the hydraulic gradient applied to it.

$$v \propto i \Rightarrow v = k\, i$$

where k is termed the **hydraulic conductivity** (or coefficient of permeability) with units of velocity and i is Hydraulic gradient

From this we can write **coefficient of permeability** as

$$k = \frac{v}{i}$$

Assumptions of Darcy’s law

- To apply Darcy’s law we assume that

  1. Soil is fully saturated which means all the voids are completely filled with water.

  2. The flow is laminar inside the voids. For large soil pores of gravels and at higher hydraulic gradient, flow may become turbulent. For that Darcy’s law is invalid.

Discharge and Seepage velocities

- Discharge through soil can be written as this $Q = Av$

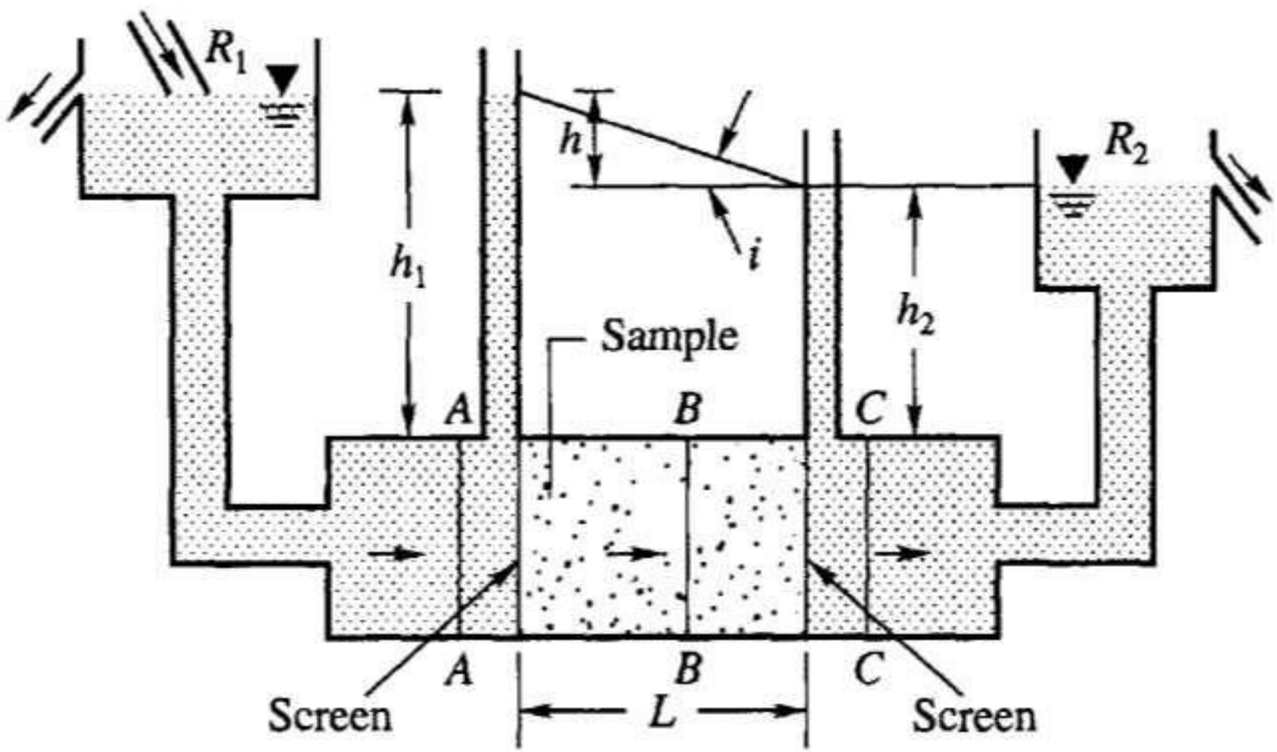
  Where A is the cross-sectional area of soil normal to the direction of flow which includes the area of the solids and the voids.
- This can be written as this :

$$q = k\,i\,A$$

$$q = kA\left(\frac{\Delta h}{L}\right)$$

- The flow of liquid takes place actually through the soil pores not through the whole cross-sectional area because primarily this cross-sectional area is composed of soil solids and very less area is available for the voids.
- But the calculations are done considering the whole soil area.

- The sample is placed in a cylindrical horizontal tube between screens. The tube is connected to two reservoirs R_1 and R_2 in which the water levels are maintained constant. The difference in head between R_1 and R_2 is h. This difference in head is responsible for the flow of water. Since Darcy's law assumes no change in the volume of voids and the soil is saturated, the quantity of flow past sections AA, BB and CC should remain the same for steady flow conditions.



Seepage velocity

- We may express the equation of continuity as follows:

$$q_{aa} = q_{bb} = q_{cc}$$

- If the soil be represented as divided into solid matter and void space, then the area available for the passage of water is only A_v. If v_s is the velocity of flow in the voids, and v, the average velocity across the section then, we have

$$A_v v_s = Av \quad \text{or} \quad v_s = \frac{A}{A_v} v$$

- Also $\frac{A}{A_v} = \frac{1}{n} = \frac{1+e}{e}, \quad v_s = \frac{v}{n} = \left(\frac{1+e}{e}\right) v$
- Since A is always greater than A_v , v_s is always greater than v
- Here, v_s is called the seepage velocity and v the discharge velocity.

Methods of determination of hydraulic conductivity of soils

Methods:

- Methods that are in common use for determining the coefficient of permeability k can be classified under laboratory and field methods.
- Laboratory methods:
  1. Constant head permeability method
  2. Falling head permeability method
- Field methods:
  1. Pumping tests
  2. Bore hole tests
- Indirect Method:

  Empirical correlations

- The various types of apparatus which are used in soil laboratories for determining the permeability of soils are called permeameters.
- The apparatus used for the constant head permeability test is called a constant head permeameter and the one used for the falling head test is a falling head permeameter.
- The soil samples used in laboratory methods are either undisturbed or disturbed.
- Since it is not possible to obtain undisturbed samples of cohesionless soils, laboratory tests are always conducted on samples which are reconstructed to the same density as they exist in nature which is not reliable and Direct field testing is required.
- Since this method is quite costly, it is generally carried out in connection with major projects such as foundation investigation for dams and large bridges or building foundation jobs where lowering of the water table is involved.

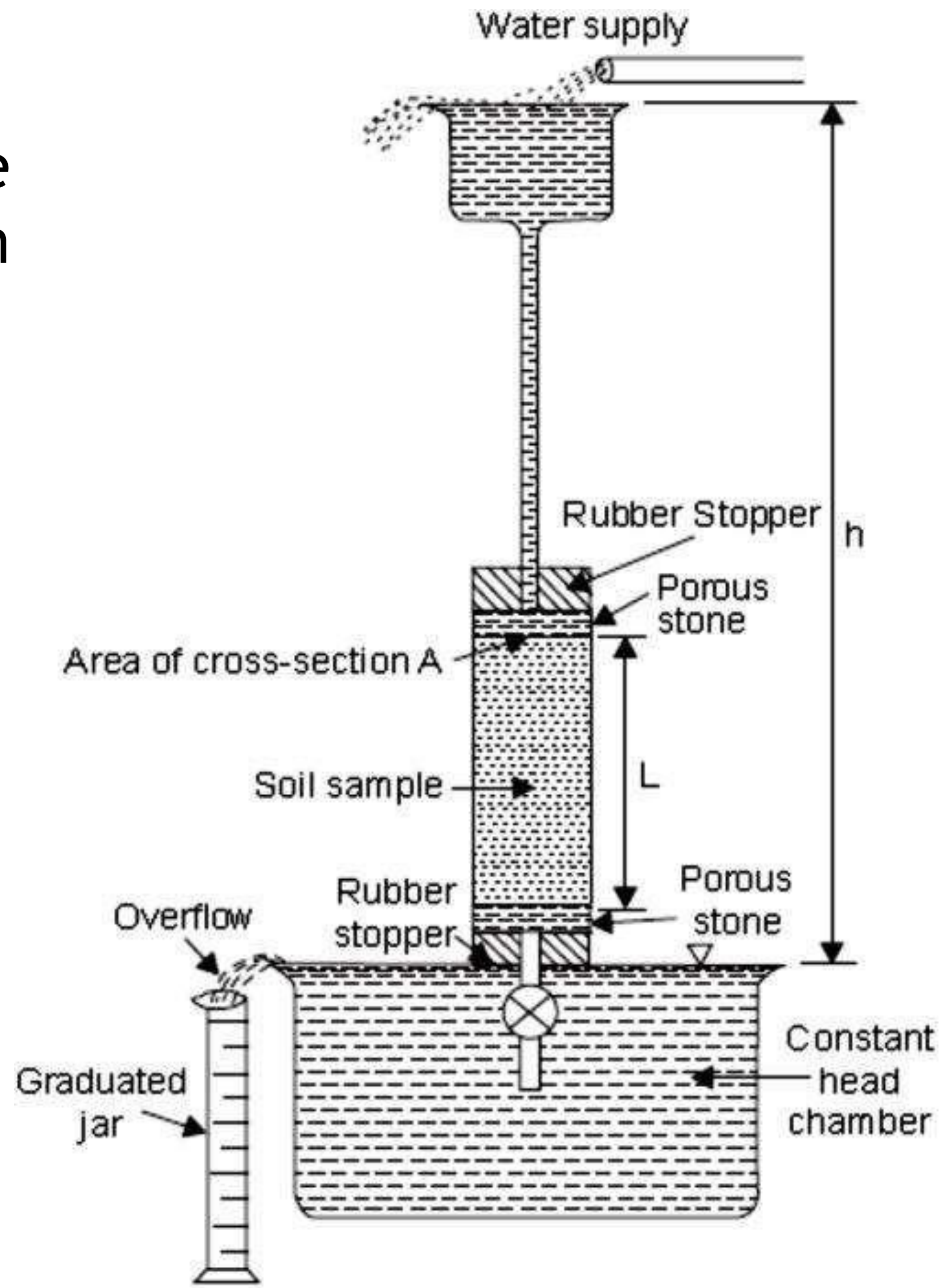
CONSTANT HEAD PERMEABILITY TEST

- Constant head method is suitable for coarse grained soils which are relatively more pervious because of their larger voids.
- A constant head permeameter consists of a vertical tube containing a soil sample which is reconstructed or undisturbed as the case may be.
- The diameter and height of the tube can be of any convenient dimensions.
- The head and tail water levels are kept constant by overflows.
- The sample of length L and cross-sectional area A is subjected to a head h which is constant during the progress of a test.
- A test is performed by allowing water to flow through the sample and measuring the quantity of discharge Q in time t.

- The value of k can be computed directly from Darcy's law expressed as:

$$Q = k\frac{h}{L}At$$

$$k = \frac{QL}{hAt}$$



FALLING HEAD PERMEABILITY TEST

- The soil sample is kept in a vertical cylinder of cross-sectional area ‘A’.
- A transparent stand pipe of cross sectional area, ‘a’, is attached to the test cylinder.
- The test cylinder is kept in a container filled with water, the level of which is kept constant by overflows.
- Before the commencement of the test the soil sample is saturated by allowing the water to flow continuously through the sample from the stand pipe.
- After saturation is complete, the stand pipe is filled with water up to a height of h_o and a stop watch is started.
- Let the initial time be t_o. The time t_1 when the water level drops from h_o to h_1 is noted.
- The hydraulic conductivity k can be determined on the basis of the drop in head (h_o – h_1) and the elapsed time (t_1 - t_o) required for the drop.

- Let h be the head of water at any time t. Let the head drop by an amount dh in time dt.
- The quantity of water flowing through the sample in time dt from Darcy's law is

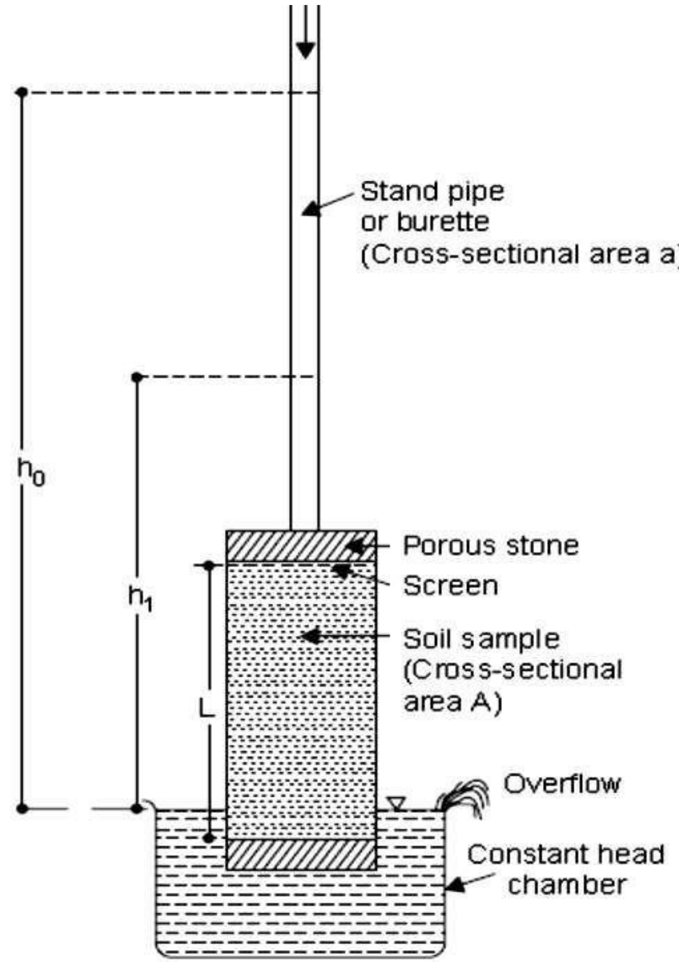
$$dQ = kiA\,dt = k\frac{h}{L}A\,dt$$

- The quantity of discharge dQ can be expressed as dQ = -a dh
- Since the head decreases as time increases, dh is a negative quantity.

$$-a\,dh = k\frac{h}{L}A\,dt$$

- The discharge *Q in time (t^ - fQ) can be obtained by integrating the above equation.*
- Therefore, the equation can be rearranged and integrated as follows:

$$-a\int_{h_0}^{h_1}\frac{dh}{h} = \frac{kA}{L}\int dt \quad \text{or} \quad a\log_e\frac{h_0}{h_1} = \frac{kA}{L}(t_1 - t_0)$$



- The general expression for k is

$$k = \frac{aL}{A(t_1 - t_0)} \log_e \frac{h_0}{h_1} \quad \text{or} \quad k = \frac{2.3aL}{A(t_1 - t_0)} \log_{10} \frac{h_0}{h_1}$$

- The setup is generally used for comparatively fine materials such as fine sand and silt where the time required for the drop in head from h_o to h_1 is neither unduly too long nor too short for accurate recordings.
- If the time is too long evaporation of water from the surface of the water might take place and also temperature variations might affect the volume of the sample. These would introduce serious errors in the results. The set up is suitable for soils having permeabilities ranging from 10^{-3} to 10^{-6} cm per sec.
- According to Indian Standards we report the permeability values at 27°C temperature. So if we measure the temperature of the water as T°C then using this formula we can calculate the permeability at 27°C.

$$k_{27} = k_T \frac{\mu_T}{\mu_{27}}$$

- Here k_{27} and k_T are the permeability of the soil at 27°C and at test temperature T°C respectively. μ_{27} and μ_T are the coefficient of viscosity at temperature 27°C and test temperature T°C respectively.

Example: In a constant head permeameter test, the following observations were taken.
Distance between piezometer tappings = 100 mm
Difference of water levels in piezometers = 60 mm
Diameter of the test sample = 100 mm
Quantity of water collected = 350 ml
Duration of the test = 270 seconds
Determine the coefficient of permeability of the soil.

Solution. From Eq. 8.5, $k = \frac{qL}{Ah}$

In this case, $q = 350/270 = 1.296$ ml/sec

Therefore, $k = \frac{1.296 \times 10.0}{(\pi/4) \times (10)^2 \times 6.0} = 0.0275$ cm/sec.

Example: A sand sample of 35cm^2 cross sectional area and 20 cm long was tested in a constant head permeameter. Under a head of 60 cm, the discharge was 120 ml in 6min. The dry weight of sand used for the test was 1120 g, and G_s = 2.68. Determine (a) the hydraulic conductivity in cm/sec, (b) the discharge velocity, and (c) the seepage velocity.

Solution: Q = 120 ml, t = 6 min, A = 35cm^2, L = 20 cm, and h = 60 cm

$$k = \frac{120 \times 20}{60 \times 35 \times 6 \times 60} = 3.174 \times 10^{-3} \text{ cm/sec}$$

Discharge velocity, $v = ki = 3.174 \times 10^{-3} \times \frac{60}{20} = 9.52 \times 10^{-3}$ cm/sec

Seepage velocity v_s

$$\gamma_d = \frac{W_s}{V} = \frac{1120}{35 \times 20} = 1.6 \text{ g/cm}^3$$

From Eq. (3.18a), $\gamma_d = \frac{\gamma_w G_s}{1+e}$ or $e = \frac{G_s}{\gamma_d} - 1$ since $\gamma_w = 1$ g/cm^3

Substituting, $e = \frac{2.68}{1.6} - 1 = 0.675$

$$n = \frac{e}{1+e} = \frac{0.675}{1+0.675} = 0.403$$

Now, $v_s = \frac{v}{n} = \frac{9.52 \times 10^{-3}}{0.403} = 2.36 \times 10^{-2}$ cm/sec

Example: The falling-head permeability test was conducted on a soil sample of 4 cm diameter and 18 cm length. The head fell from 1.0 in to 0.40 m in 20 minutes. If the cross- sectional area of the stand pipe was 1 cm^2, determine the coefficient of permeability.

Solution. From Eq. 8.6,

$$k = \frac{aL}{At} \log_e (h_1/h_2)$$

$$= \frac{1.0 \times 18.0}{(\pi/4) \times (4.0)^2 \times 20 \times 60} \log_e (1.0/0.40)$$

$$= 1.09 \times 10^{-3} \text{ cm/sec.}$$

Example: A soil has the coefficient of permeability of 4.75 x 10^{-2} mm/sec at 30°C. Determine its value at 27°C. Take the coefficient of viscosity at 30°C and 27°C as 8.0 milli poise and 8.5 milli poise, respectively.

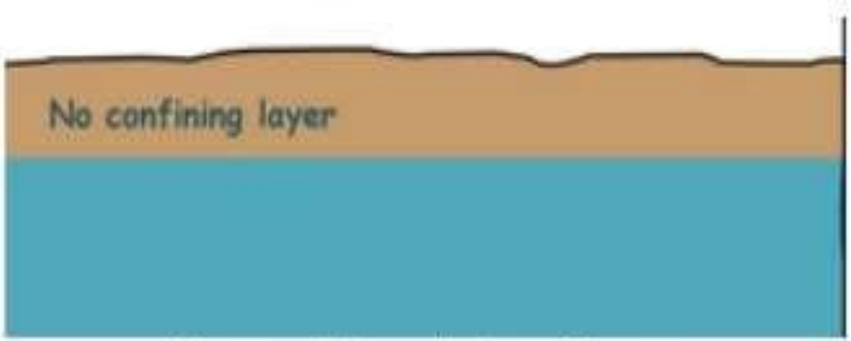
$$k_{27} = k_t \frac{\mu_t}{\mu_{27}}$$

$$= 4.75 \times 10^{-2} \times 8.0/8.5 = 4.48 \times 10^{2} \text{ mm/sec.}$$

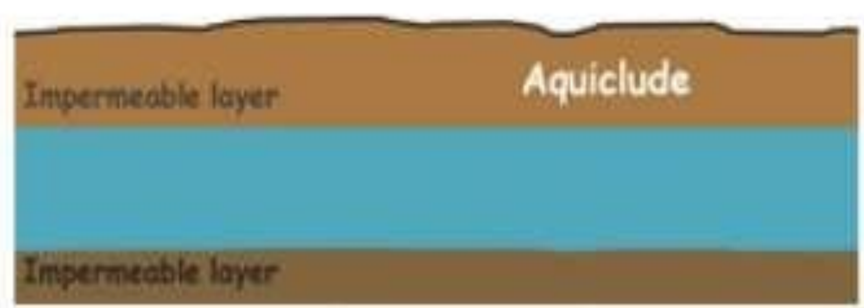
Direct determination of k of soils in place by **Pumping test**

- The most reliable information concerning the permeability of a **deposit of coarse grained material below the water table** can usually be obtained by conducting pumping tests in the field.
- Although such tests have their most extensive application in connection with dam foundations, they may also prove advisable on large bridge or building foundation jobs where the water table must be lowered.
- The arrangement consists of a **test well** and a series of **observation wells**. The test well is sunk through the permeable stratum up to the impermeable layer.

- A well sunk into a water bearing stratum, termed an aquifer, and tapping free flowing ground water having a free **ground water table under atmospheric pressure**, is termed a **gravity or unconfined well.**
- A well sunk into an aquifer where the ground water flow is confined between two impermeable soil layers, and is under pressure greater than atmospheric, is termed as **artesian or confined well.**
- Observation wells are drilled at various distances from the test or pumping well along two straight lines, one oriented approximately in the direction of ground water flow and the other at right angles to it.
- A minimum of two observation wells and their distances from the test well are needed.

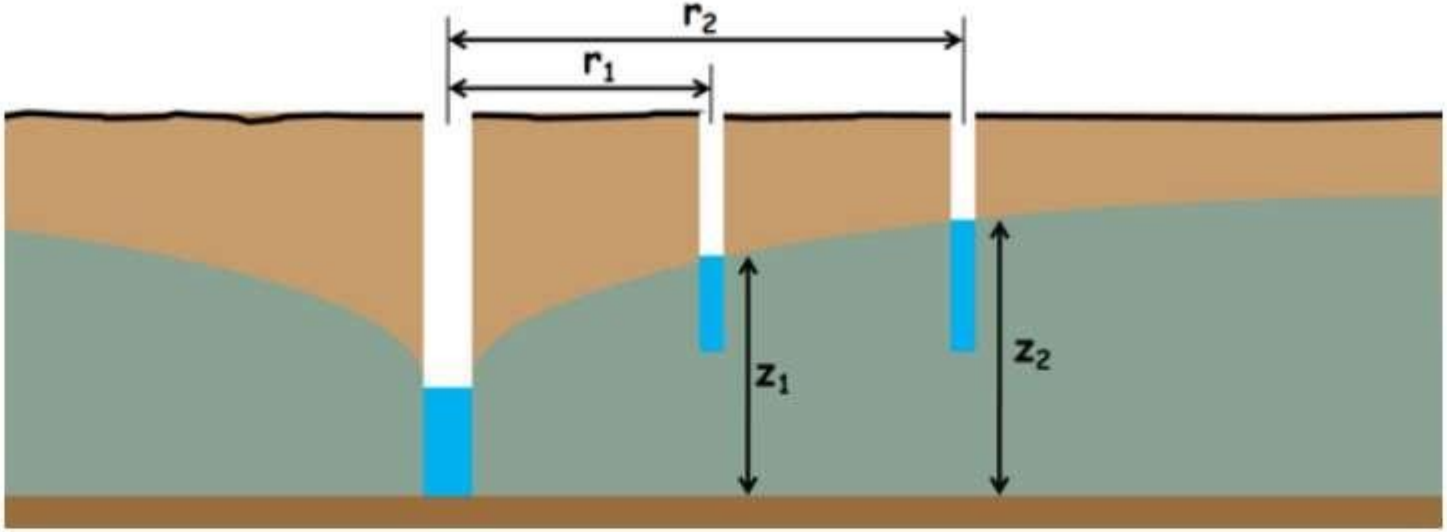


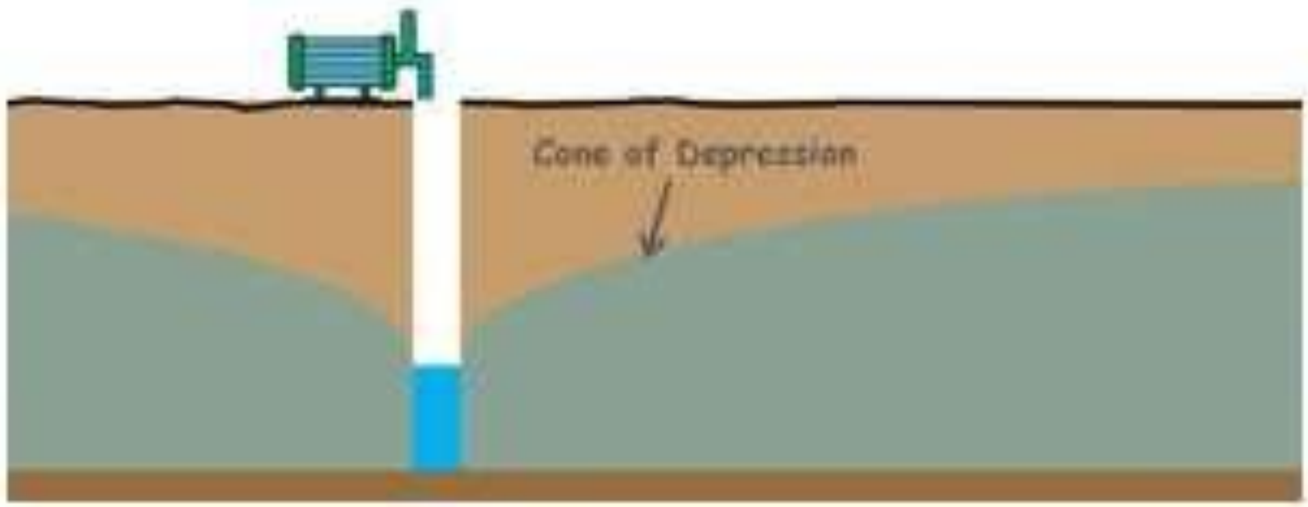
Unconfined Aquifer



Confined Aquifer

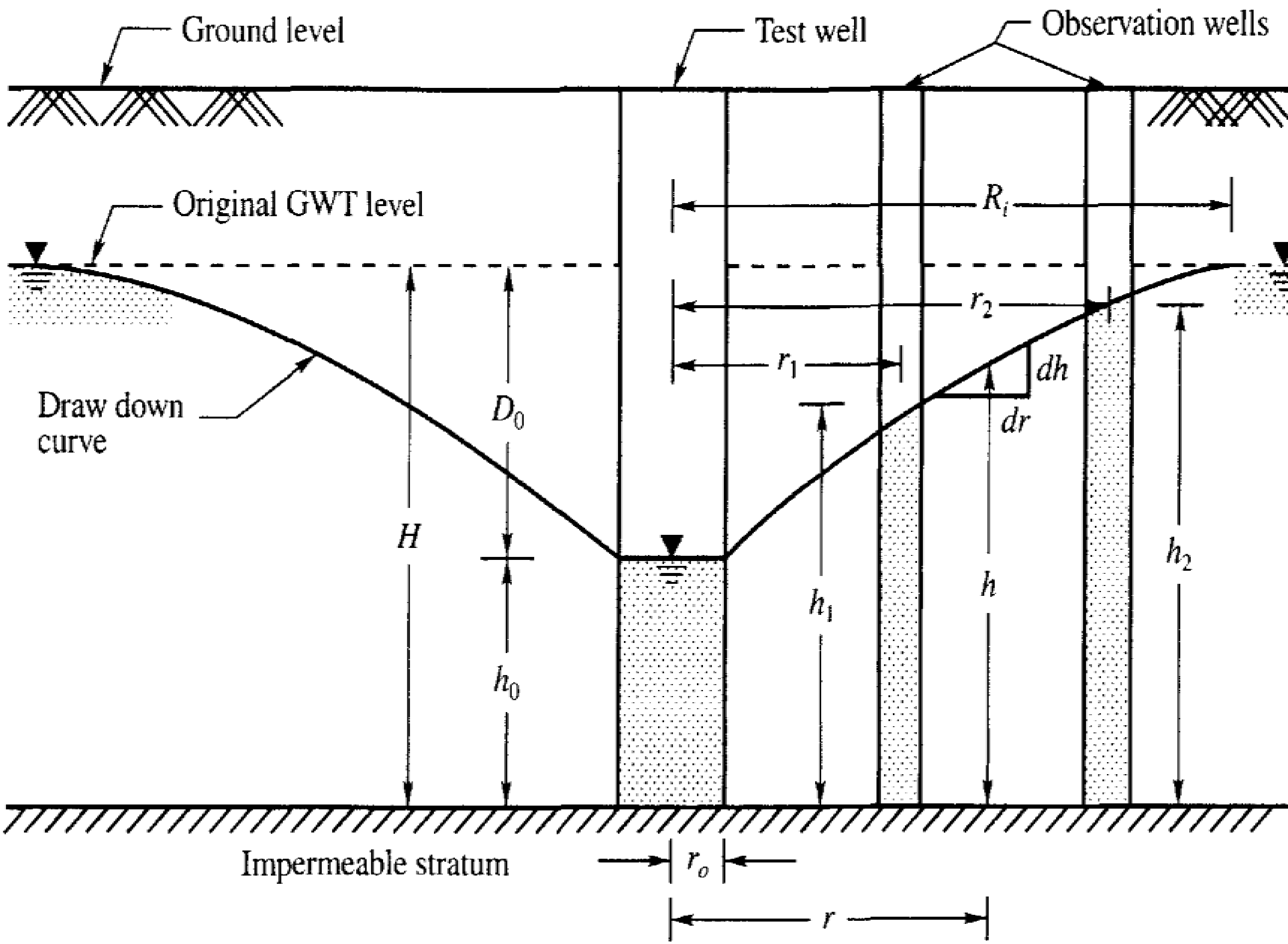
- The test consists of pumping out water continuously at a uniform rate from the test well until the water levels in the test and observation wells remain stationary.
- When this condition is achieved the water pumped out of the well is equal to the inflow into the well from the surrounding strata.
- The water levels in the observation wells and the rate of water pumped out of the well would provide the necessary additional data for the determination of k.
- The drawdown resulting due to pumping is called the **cone of depression.**
- The maximum drawdown $\mathbf{D_0}$ is in the test well. It decreases with the increase in the distance from the test well.
- The depression dies out gradually and forms theoretically, a circle around the test well called the **circle of influence**.
- The radius of this circle, is called the **radius of influence** of the depression cone.


r2
r1
z1
z2


Cone of Depression

Equation for k for an Unconfined Aquifer

- Only two observation wells at radial distances of r_1 and r_2 from the test well are shown.
- When the inflow of water into the test well is steady, the depths of water in these observation wells are h_1 and h_2 respectively.
- Let h be the depth of water at radial distance r. The area of the vertical cylindrical surface of radius r and depth h through which water flows is $\mathbf{A = 2\pi rh}$
- The hydraulic gradient is $\mathbf{i = dh/dr}$
- As per Darcy's law the rate of inflow into the well when the water levels in the wells remain stationary is $\mathbf{q = kiA}$



Ground level
Test well
Observation wells
Original GWT level
R_i
r_2
r_1
dh
dr
Draw down curve
D_0
H
h_0
h_1
h
h_2
Impermeable stratum
r_o
r

- Substituting for A and i the rate of inflow across the cylindrical surface is

$$q = k \frac{dh}{dr} 2\pi r h$$

- Rearranging the terms, we have

$$\frac{dr}{r} = \frac{2\pi k h dh}{q}$$

- The integral of the equation within the boundary limits is

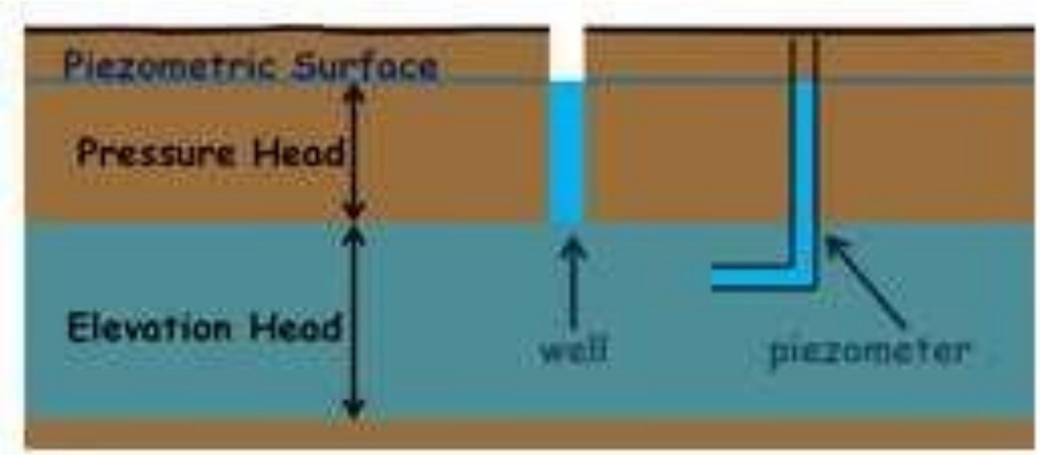
$$\int_{r_1}^{r_2} \frac{dr}{r} = \frac{2\pi k}{q} \int_{h_1}^{h_2} h\, dh$$

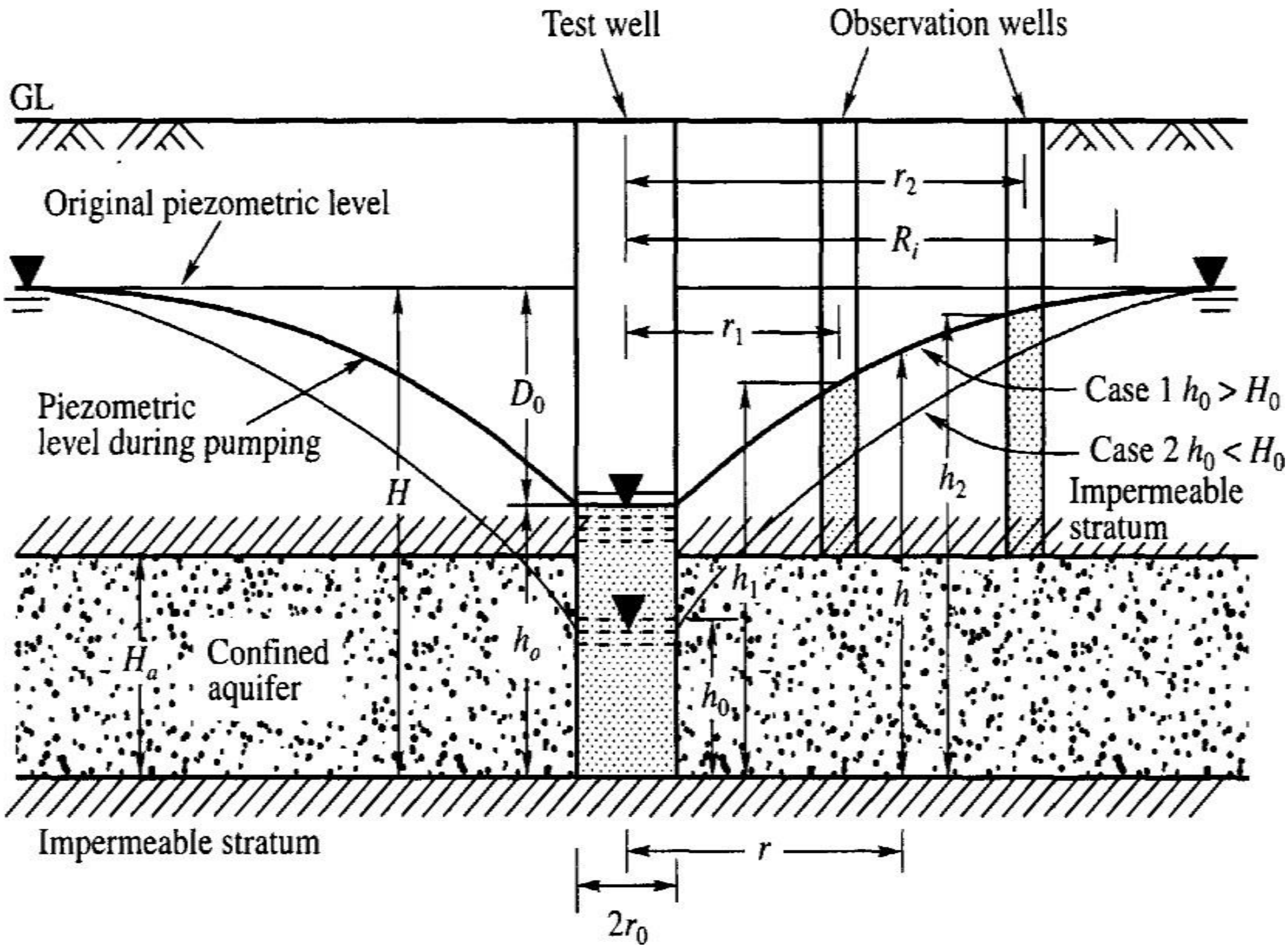
- The equation for k after integration and rearranging is

$$\boxed{k = \frac{2.3q}{\pi (h_2^2 - h_1^2)} \log \frac{r_2}{r_1}}$$

Equation for k in a Confined Aquifer

- The water in the observation wells rises above the top of the aquifer due to artesian pressure. When pumping from such an artesian well two cases might arise. They are:
- **Case 1.** The water level in the test well might remain above the roof level of the aquifer at steady flow condition.
- **Case 2.** The water level in the test well might fall below the roof level of the aquifer at steady flow condition.
- If Ho is the thickness of the confined aquifer and ho is the depth of water in the test well at the steady flow condition Case 1 and Case 2 may be stated as— **Case l.** When $h_o > H_o$. **Case 2.** When $h_o < H_o$.





Test well
Observation wells
GL
r_2
Original piezometric level
R_i
r_1
Piezometric level during pumping
D_0
Case 1 $h_0 > H_0$
Case 2 $h_0 < H_0$
H
h_2
Impermeable stratum
h_1
h
H_a
Confined aquifer
h_o
h_0
Impermeable stratum
r
$2r_0$

Case 1. When $h_0 > H_0$

- In this case, the area of a vertical cylindrical surface of any radius r does not change, since the depth of the water bearing strata is limited to the thickness Ho. Therefore, the discharge surface area is $\mathbf{A = 2\pi r H_o}$

- As per Darcy's law, $q = kiA = k\frac{dh}{dr}2\pi r H_0$
- The integration of the equation after rearranging the terms yields

$$\int_{h_1}^{h_2} dh = \frac{q}{2\pi k H_0}\int_{r_1}^{r_2}\frac{dr}{r} \quad \text{or} \quad (h_2 - h_1) = \frac{q}{2\pi H_0}\log_e\frac{r_2}{r_1}$$

- The equation for *k is*

$$\boxed{k = \frac{2.3q}{2\pi H_0 (h_2 - h_1)}\log\frac{r_2}{r_1}}$$

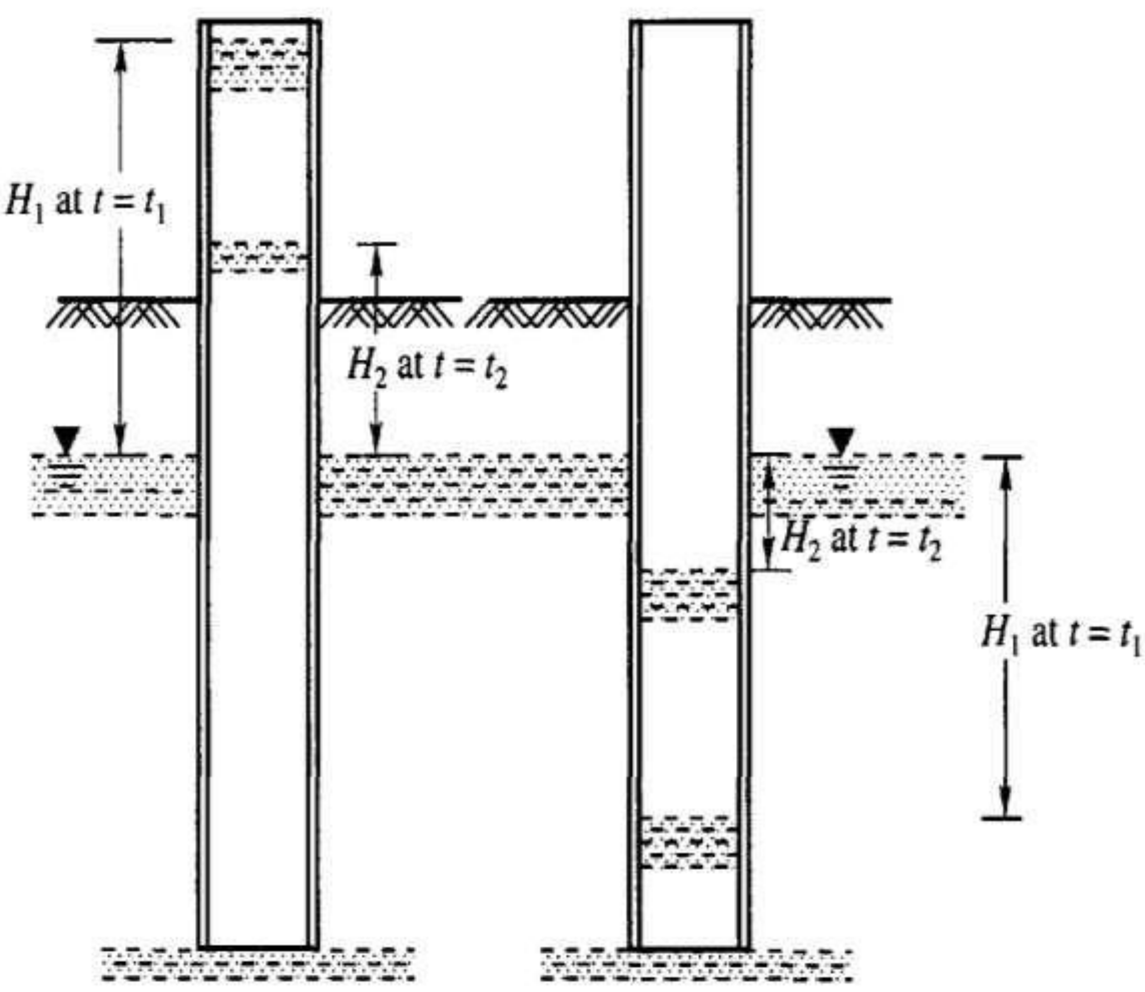
Case 1.When $h_0 < H_0$

- Under the condition when h_o is less than H_o, the flow pattern close to the well is similar to that of an unconfined aquifer whereas at distances farther from the well the flow is artesian.
- Muskat (1946) developed an equation to determine the hydraulic conductivity.
- The e

$$k = \frac{2.3q}{\pi(2HH_0 - H_0^2 - h_0^2)} \log\frac{R_i}{r_0}$$

BOREHOLE PERMEABILITY TESTS

- Two types of tests may be carried out in auger holes for determining k. They are
- (a) Falling water level method
- (b) Rising water level method



(a) Falling water head method (b) Rising water head method

Falling Water Level Method (cased hole and soil flush with bottom)

- In this test auger holes are made in the field that extend below the water table level. Casing is provided down to the bottom of the hole. The casing is filled with water which is then allowed to seep into the soil.
- The rate of drop of the water level in the casing is observed by measuring the depth of the water surface below the top of the casing at 1, 2 and 5 minutes after the start of the test and at 5 minutes intervals thereafter.
- These observations are made until the rate of drop becomes negligible or until sufficient readings have been obtained. The coefficient of permeability is computed as

$$k = \frac{2.3\,\pi r_0}{5.5(t_2 - t_1)} \log \frac{H_1}{H_2}$$

where, H_1 = piezometric head at $t = t_l$, H_2 = piezometric head at $t = t_2$

Rising Water Level Method (cased hole and soil flush with bottom)

- This method, most commonly referred to as the time-lag method, consists of bailing the water out of the casing and observing the rate of rise of the water level in the casing at intervals until the rise in water level becomes negligible.
- The rate is observed by measuring the elapsed time and the depth of the water surface below the top of the casing.
- The intervals at which the readings are required will vary somewhat with the permeability of the soil.
- The same equation is applicable in this case also.

Example: A pumping test was made in pervious gravels and sands extending to a depth of 50 ft, where a bed of clay was encountered. The normal ground water level was at the ground surface. Observation wells were located at distances of 10 and 25 ft from the pumping well. At a discharge of 761 ft^3 per minute from the pumping well, a steady state was attained in about 24 hr. The draw-down at a distance of 10 ft was 5.5 ft and at 25 ft was 1.21 ft. Compute the hydraulic conductivity in ft/sec.

Solution:

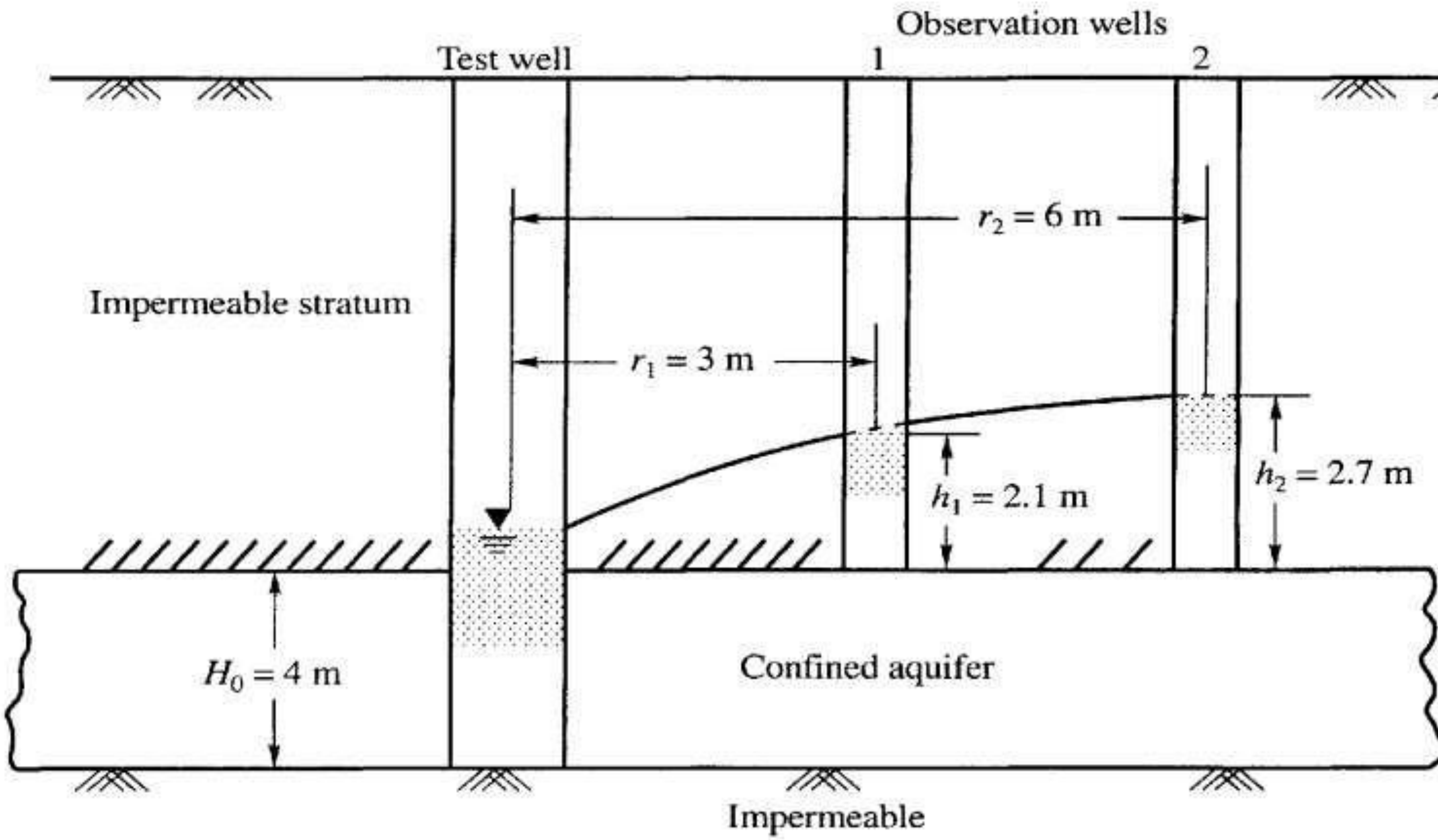
$$k = \frac{2.3q}{\pi\,(h_2^2 - h_1^2)} \log \frac{r_2}{r_1}$$

where $q = \dfrac{761}{60} = 12.683 \text{ ft}^3/\text{sec}$

$r_1 = 10$ ft, $r_2 = 25$ ft, $h_2 = 50 - 1.21 = 48.79$ ft, $h_1 = 50 - 5.5 = 44.5$ ft

$$k = \frac{2.3 \times 12.683}{3.14(48.79^2 - 44.5^2)} \log \frac{25}{10} = 9.2 \times 10^{-3} \text{ ft/sec.}$$

Example: A field pumping test was conducted from an aquifer of sandy soil of 4 m thickness confined between two impervious strata. When equilibrium was established, 90 liters of water was pumped out per hour. The water elevation in an observation well 3.0 m away from the test well was 2.1 m and another 6.0 m away was 2.7 m from the roof level of the impervious stratum of the aquifer. Find the value of k of the soil in m/sec.



Solution:

$$k = \frac{2.3q}{2\pi H_0\,(h_2 - h_1)} \log \frac{r_2}{r_1}$$

$q = 90 \times 10^3\ \text{cm}^3/\text{hr} = 25 \times 10^{-6}\ \text{m}^3/\text{sec}$

$$k = \frac{2.3 \times 25 \times 10^{-6}}{2 \times 3.14 \times 4\,(2.7 - 2.1)} \log \frac{6}{3} = 1.148 \times 10^{-6}\ \text{m/sec}$$

Example: A pumping test was carried out for determining the hydraulic conductivity of soil in place. A well of diameter 40 cm was drilled down to an impermeable stratum. The depth of water above the bearing stratum was 8 m. The yield from the well was 4 m^3/min at a steady drawdown of 4.5 m. Determine the hydraulic conductivity of the soil in m/day if the observed radius of influence was 150m.

Solution:

$$k = \frac{2.3q}{\pi D_0 (2H - D_0)} \log \frac{R_i}{r_0}$$

$$q = 4 \text{ m}^3/\text{min} = 4 \times 60 \times 24 \text{ m}^3/\text{day}$$

$$D_0 = 4.5 \text{ m}, \quad H = 8 \text{ m}, \quad R_i = 150 \text{ m}, \quad r_0 = 0.2 \text{ m}$$

$$k = \frac{2.3 \times 4 \times 60 \times 24}{3.14 \times 4.5 (2 \times 8 - 4.5)} \log \frac{150}{0.2} = 234.4 \text{ m/day}$$

HYDRAULIC CONDUCTIVITY IN STRATIFIED LAYERS OF SOILS

- Soils may be stratified by the deposition of different materials in layers which possess different permeability characteristics.
- In such stratified soils engineers desire to have the average permeability either in the horizontal or vertical directions.
- The average permeability can be computed if the permeabilities of each layer are determined in the laboratory.
- For calculations and analysis purpose it is assumed that in stratified soils each individual layer is homogeneous and isotropic. That means each single layer has similar structure and physical properties in all the directions.
- The average coefficient of permeability of the whole soil deposit varies with the direction of the flow. So we calculate the permeability of such deposit for flow of water in two directions.
- 1. Horizontal Flow that is Flow Parallel to Bedding Planes
- 2.Vertical Flow that is Flow Perpendicular to Bedding Planes

- The procedure is as follows:
- $k_1, k_2, k_3, \ldots k_n$ = hydraulic conductivities of individual strata of soil either in the vertical or horizontal direction.
- $z_1, z_2, z_3, \ldots z_n$ = thickness of the corresponding strata.
- $Z = z_1 + z_2 + z_3 + \ldots + z_n$
- k_h = average hydraulic conductivity parallel to the bedding planes (usually horizontal).
- k_v = average hydraulic conductivity perpendicular to the bedding planes (usually vertical).

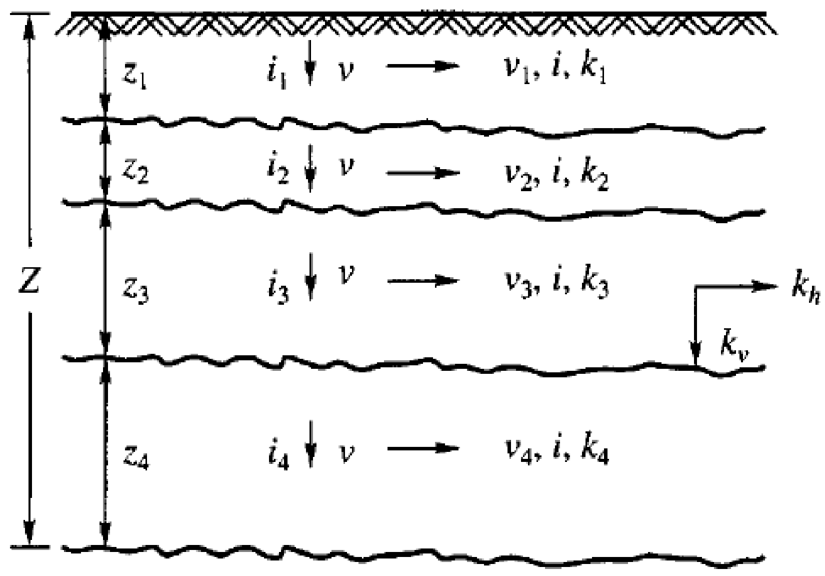
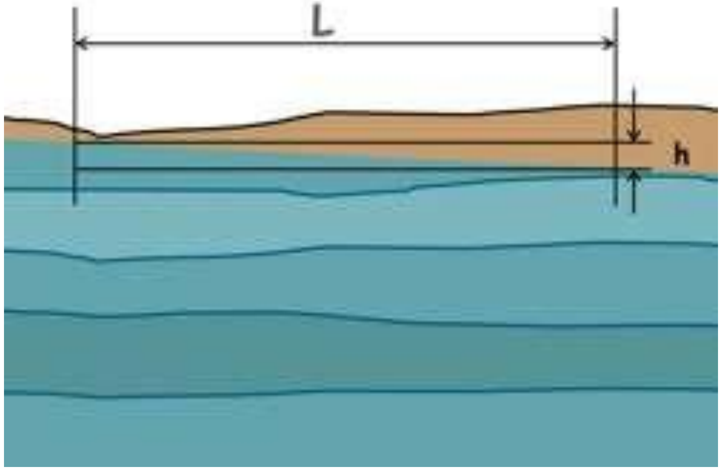


Figure Flow through stratified layers of soil

Flow in the Horizontal Direction

- When water flows through such soils in parallel or horizontal direction, let's say head loss which is the energy loss of water is observed h over a distance of L. So it generate the hydraulic gradient i which is h by L.
- We can see the head loss is same for all the layers and also the length of flow is same so the hydraulic gradient of total soil mass i will be similar to the hydraulic gradient of each layer. Hence $i = i_1 = i_2 = i_3$
- So, when the flow is in the horizontal direction the hydraulic gradient / remains the same for all the layers.



- From the continuity equation we can write the total discharge through soil mass is equal to the sum of discharge through each layer.
- $q = q_1 + q_2 + q_3 + \ldots\ldots\ldots +q_n$
- Let $v_1, v_2, v_3, \ldots v_n$ be the discharge velocities in the corresponding strata. Then

$$Q = kiZ = (v_1 z_1 + v_2 z_2 + \cdots + v_n z_n) \quad = (k_1 i z_1 + k_2 i z_2 + \cdots + k_n i z_n)$$

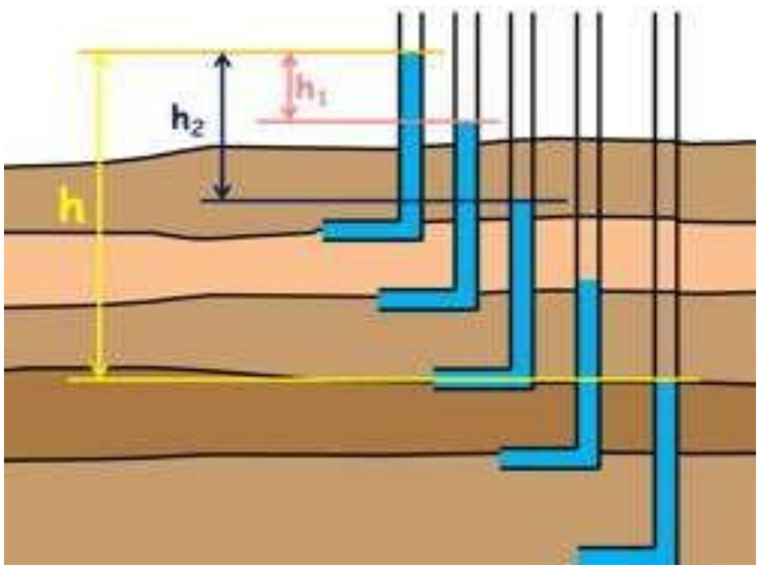
- Therefore,

$$\boxed{k_h = \frac{1}{Z}(k_1 z_1 + k_2 z_2 + \cdots + k_n z_n)}$$

Flow in the Vertical Direction

- When flow is in the vertical direction, the hydraulic gradients for each of the layers are different.
- Let these be denoted by i_1, i_2, i_3,…i_n
- Let *h* be the total loss of head as the water flows from the top layer to the bottom through a distance *of Z*.
- The average hydraulic gradient is *h*/*Z*. The principle of continuity of flow requires that the downward velocity be the same in each layer. Therefore,

$$v = k_v \frac{h}{Z} = k_1 i_1 = k_2 i_2 = \cdots = k_n i_n$$



- If $h_1, h_2, h_3, \ldots h_n$ are the head losses in each of the layers, we have

$$h = h_1 + h_2 + \cdots + h_n$$

$$\text{or} \quad h = z_1 i_1 + z_2 i_2 + \cdots + z_n i_n$$

- Solving the above equations we have

$$k_v = \frac{Z}{\dfrac{z_1}{k_1} + \dfrac{z_2}{k_2} + \cdots + \dfrac{z_n}{k_n}}$$

- It should be noted that in all stratified layers of soils the horizontal permeability is generally greater than the vertical permeability.

APPROXIMATE VALUES OF THE HYDRAULIC CONDUCTIVITY OF SOILS

- The coefficients of permeability of soils vary according to their type, textural composition, structure, void ratio and other factors.Therefore, no single value can be assigned to a soil purely on the basis of soil type.
- The possible coefficients of permeability of some soils are

Table 4.1 Hydraulic conductivity of some soils (after Casagrande and Fadum, 1939)

k (cm/sec)	Soils type	Drainage conditions
10^{1} to 10^{2}	Clean gravels	Good
10^{1}	Clean sand	Good
10^{-1} to 10^{-4}	Clean sand and gravel mixtures	Good
10^{-5}	Very fine sand	Poor
10^{-6}	Silt	Poor
10^{-7} to 10^{-9}	Clay soils	Practically impervious

Factors Affecting Permeability Of Soil

- Coefficient of permeability of a soil depends on both the properties of the soil and properties of the water which is flowing through it.
- let us consider this equation which is the general expression for the coefficient of permeability.

$$K = C.D^2 \frac{e^3}{1+e} \frac{\gamma_w}{\mu}$$

- It has been obtained by comparing Poiseuille equation with Darcy's equation.
- C is a shape factor dependant on grain shape.

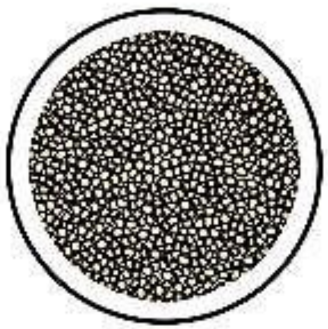
Particle size:

- When a soil mass contains coarse grained particles it contains large volume of voids and those voids are interconnected. So, high amount of water may flow through these interconnected voids easily. Hence such soils have higher value of permeability.
- While soil mass with fine grains have poorly connected void structure consequently lower value of permeability is observed.
- In the general equation that permeability of the soil is **directly proportional to the square of particle diameter.**
- If the soil particle size is large its permeability will be high and if particle size is small the permeability will be low.

Coarse grained soils

Large particle size

Fine grained soils



small particle size

- Allen Hazen, 1911 gave that for clean sands with less than 5% fines and with D_{10} size in between 0.1 and 3.0mm,

$$\mathbf{k = CD_{10}^{2}}$$

Where k is in cm/s, $D_{10}$is in mm, C varies from 0.4 to 1.2.

- If k in cm and D_{10} is in cm, an approximate relation of

$k = 100D_{10}^{2}$ can be used for soil having $k<10^{-3}$ cm/s.

Specific surface area of soil particles:

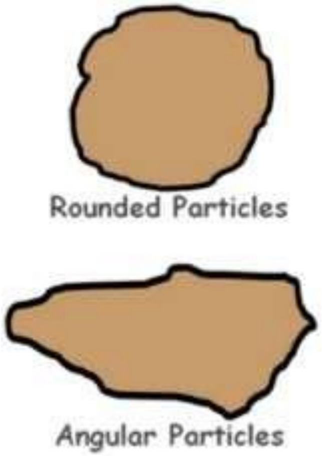
- The surface of the soil particles offers a resistance to the flow of water. The more surface is in contact with the flowing water greater is the resistance posed to the water. Hence lower will be the permeability.

$$\text{Specific Surface Area} = \frac{\text{Surface Area}}{\text{Volume}}$$

- If volume of a particle is kept constant but its surface area is increased then specific surface area of that particle will also increase.
- Coarse grained particles have relatively less specific surface area so they pose less resistance to flow of water consequently offer relatively high coefficient of permeability, while fine soil particles have larger specific surface area so resistance will be more hence permeability will be less.
- So the permeability is inversely proportional to the specific surface.

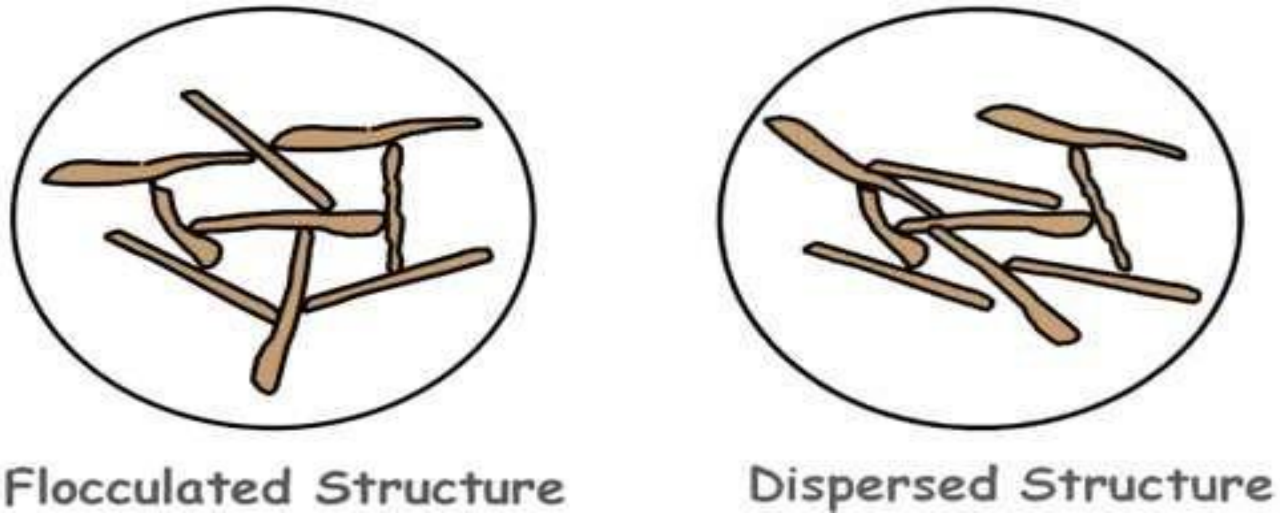
Shape of soil particles:

- Specific surface area of a particle also depends on its shape. Hence permeability also depends on the shape of the soil particles.
- Rounded soil particles have relatively less specific surface area when compared to angular particles.
- So when water flows through soil mass consists of rounded particles it will face less resistance and permeability will be relatively higher than if water flows through soil mass of angular particles.



Soil Structure:

- Permeability also depends on how these particles arrange themselves in a soil mass that is in the structure of the soil mass.
- Water flows through voids and the connectivity of these voids depends upon the structural arrangement of the soil particles.
- In case of fine grained soils if the soil particles are arranged in flocculated structure then its permeability will be more when compared to that if the particles are arrange in the dispersed structure as we know flocculated structure has more voids than in the dispersed structure.



- If a soil mass has stratified structure then its permeability varies according to the direction of flow.
- It has been observed that when the flow of water is parallel to the bedding planes the permeability of the soil mass is higher than when water flows perpendicular to the bedding planes.
- Including these factors permeability of soil deposit also depends upon the structural defects like cracks or fissures in the soil mass.

Void Ratio:

$$\text{Void Ratio} = \frac{\text{Volume of Voids}}{\text{Volume of Solids}}$$

- Void ratio of a soil mass is the volume of voids present in it divided by the volume of solids.
- If volume of voids in the soil increases the flow path becomes wider and voids interconnectivity increases. Hence permeability of soil increases.
- So in general we can say permeability of a soil mass increases with the increase in its void ratio and decreases when voids ratio decrease.
- But it is not true for all types of soils. Clay soils have higher void ratio because of their flocculated structure but still their permeability is very low because the flow path through voids in case of clays is extremely small and poorly connected.

Adsorbed Water:

- Fine grained soil particles carry charges on their surface and because of that dipolar water molecules are strongly attached to their surface. This attached water is called adsorbed water. As this water is bounded by electrical forces it is not free to move under gravity.
- This adsorbed water layer causes an obstruction to the flow of water in the pores by blocking the voids or reducing their effective size and hence reduces the permeability of soils.

Degree of Saturation:

- The saturation of the soil mass also affects its permeability. If the soil is partially saturated then it may have some voids which contain entrapped air.
- These entrapped air pockets may block the flow path which may reduce the permeability of the soil.
- While if the soil is fully saturated there will not be any such blockage.
- Hence we can say the permeability of a partially saturated soil is smaller than that of fully saturated soil.

Viscosity:

- Let us consider the viscosity of the water. If we look at the general permeability equation we notice that permeability is inversely proportional to the viscosity of the fluid.
- The more viscous the liquid is more resistance it will pose to flow and slower it will move into the voids. Consequently lower will be the permeability.
- On the other hand if the liquid flowing through the soil is less viscous, less resistance it will offer to flow hence higher will be the permeability.

$$\boxed{K} = C.D^2 \frac{e^3}{1+e} \frac{\gamma_w}{\mu}$$

μ = viscosity

Temperature: $K \; \alpha \; \frac{1}{\mu}$ $\mu \; \alpha \; \frac{1}{T}$

- With it let’s consider the temperature of the liquid. Permeability is dependent on the viscosity and we know viscosity is inversely dependent on the temperature. Hence, Permeability is directly related to temperature.
- We know as the temperature of the liquid increases its viscosity decreases, consequently it permeability increases. Similarly when temperature of the liquid flowing through soil decreases, its viscosity increases hence its permeability decreases.

- There is a relationship between temperature of the permeant and the permeability of the soil which can be given as this:

$$k_{27} = k_T \frac{\mu_T}{\mu_{27}}$$

- Here k_{27} and K_T are permeability of soil at temperature 27 degree and test temperature T degree centigrade respectively. And μ_{27} and μ_T are viscosity of liquid at temperature and 27 degree and test temperature T degree centigrade respectively.

Impurities and Organic Matter in water:

- The presence of impurities or any organic matter in the water or in the soil tends to block the flow path by blocking the voids that result in decreased permeability of the soil.

A sand deposit contains three distinct horizontal layers of equal thickness. The hydraulic conductivity of the upper and lower layers is 10^{-3} cm/sec and that of the middle is 10^{-2} cm/sec. What are the equivalent values of the horizontal and vertical hydraulic conductivities of the three layers, and what is their ratio?

Solution

Horizontal flow

$$k_h = \frac{1}{Z}(k_1 z_1 + k_2 z_2 + k_3 z_3) = \frac{1}{3}(k_1 + k_2 + k_3) \quad \text{since} \quad z_1 = z_2 = z_3$$

$$k_h = \frac{1}{3}(10^{-3} + 10^{-2} + 10^{-3}) = \frac{1}{3}(2 \times 10^{-3} + 10^{-2}) = 4 \times 10^{-3} \text{ cm/sec}$$

Vertical flow

$$k_v = \frac{Z}{\frac{z_1}{k_1} + \frac{z_2}{k_2} + \frac{z_3}{k_3}} = \frac{3}{\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}} = \frac{3}{\frac{2}{k_1} + \frac{1}{k_2}}$$

$$= \frac{3k_1 k_2}{2k_2 + k_1} = \frac{3 \times 10^{-3} \times 10^{-2}}{2 \times 10^{-2} + 10^{-3}} = 1.43 \times 10^{-3} \text{ cm/sec}$$

$$\frac{k_h}{k_v} = \frac{4 \times 10^{-3}}{1.43 \times 10^{-3}} = 2.8$$

The following details refer to a test to determine the value of A; of a soil sample: sample thickness = 2.5 cm, diameter of soil sample = 7. 5 cm, diameter of stand pipe = 10mm, initial head of water in the stand pipe =100 cm, water level in the stand pipe after 3 h 20 min = 80 cm. Determine the value of k if e = 0.75. What is the value of k of the same soil at a void ratio e = 0.90?

Solution

Use Eq. (4.13) where, $k = \dfrac{2.3aL}{A(t_1 - t_0)} \log \dfrac{h_0}{h_1}$

$$a = \frac{3.14}{4}(1)^2 = 0.785 \text{ cm}^2$$

$$A = \frac{3.14}{4}(7.5)^2 = 44.16 \text{ cm}^2$$

$t = 12000$ sec

By substituting the value of k for $e_1 = 0.75$

$$k = k_1 = \frac{2.3 \times 0.785 \times 2.5}{44.16 \times 12000} \times \log \frac{100}{80} = 0.826 \times 10^{-6} \text{ cm/sec}$$

For determining k at any other void ratio, use Eq. (4.35)

$$\frac{k_1}{k_2} = \frac{e_1^3}{1+e_1} \bigg/ \frac{e_2^3}{1+e_2} = \frac{1+e_2}{1+e_1} \times \left(\frac{e_1}{e_2}\right)^3$$

$$\text{Now,} \quad k_2 = \frac{1+e_1}{1+e_2} \times \quad {\frac{e_2}{e_1}}^{\quad 3} \times k_1$$

For $e_2 = 0.90$

$$k_2 = \frac{1.75}{1.90} \times \left(\frac{0.9}{0.75}\right)^3 \times 0.826 \times 10^{-6} = 1.3146 \times 10^{-6} \ \text{cm/sec}$$

The data given below relate to two falling head permeameter tests performed on two different soil samples:

(a) stand pipe area = 4 cm^2,

(b) sample area = 28 cm^2,

(c) sample height = 5 cm,

(d) initial head in the stand pipe =100 cm,

(e) final head = 20 cm,

(f) time required for the fall of water level in test 1, t = 500 sec,

(g) for test 2, t = 15 sec.

Determine the values of k for each of the samples. If these two types of soils form adjacent layers in a natural state with flow

(a) in the horizontal direction, and

(b) flow in the vertical direction, determine the equivalent permeability for both the cases by assuming that the thickness of each layer is equal to 150 cm.

$$k = \frac{2.3aL}{At}\log\frac{h_1}{h_2}$$

For test 1

$$k_1 = \frac{2.3\times4\times5}{28\times500}\log\frac{100}{20} = 2.3\times10^{-3}\ \text{cm/sec}$$

For test 2

$$k_2 = \frac{2.3\times4\times5}{28\times15}\log\frac{100}{20} = 76.7\times10^{-3}\ \text{cm/sec}$$

Flow in the horizontal direction

Use Eq. (4.27)

$$k_h = \frac{1}{Z}(k_1z_1 + k_2z_2) = \frac{1}{300}(2.3\times150 + 76.7\times150)\times10^{-3} = 39.5\times10^{-3}\ \text{cm/sec}$$

Flow in the vertical direction

Use Eq. (4.28)

$$k_v = \frac{Z}{\dfrac{z_1}{k_1}+\dfrac{z_2}{k_2}} = \frac{300}{\dfrac{150}{2.3\times10^{-3}}+\dfrac{150}{76.7\times10^{-3}}} = 4.46\times10^{-3}\ \text{cm/sec}$$

The figure shows a cross-section through the strata underlying a site. Calculate the equivalent permeability of the layered system in the vertical and horizontal direction. Assume that each layer is isotropic.