THE METHOD OF FALSE POSITION

This is the oldest method for finding the real root of a nonlinear equation $f(x) = 0$ and closely resembles the bisection method. In this method, also known as regular falsi or the method of chords, we choose two points a and b such that $f(a)$ and $f(b)$ are of opposite sings. Hence, a root must lie in between these points. Now, the equation of the chord joining the two points $[a, f(a)]$ and $[b, f(b)]$ is given by

$$\frac{y-f(a)}{x-a} = \frac{f(b)-f(a)}{b-a}. \tag{1}$$

The method consists in replacing the part of the curve between the points $[a, f(a)]$ and $[b, f(b)]$ by means of the chord joining these points, and taking the point of intersection of the chord with the $x-axis$ as an approximation to the root. The point of intersection in the present case is obtained by putting $y = 0$ in (1). Thus, we obtain

$$x_1 = a - \frac{f(a)}{f(b)-f(a)}(b-a) = \frac{af(b)-bf(a)}{f(b)-f(a)}, \tag{2}$$

which is the first approximation to the root of $f(x) = 0$. If now $f(x_1)$ and $f(a)$ are of opposite sings, then the root lies between a and x_1, and we replace b by x_1 in (2) , and obtain the next approximation. Otherwise, we replace a by x_1 and generate the next approximation. The procedure is repeated till the root is obtained to the desired accuracy. Figure gives a graphical representation of the method. The error criterion can be used in this case also.

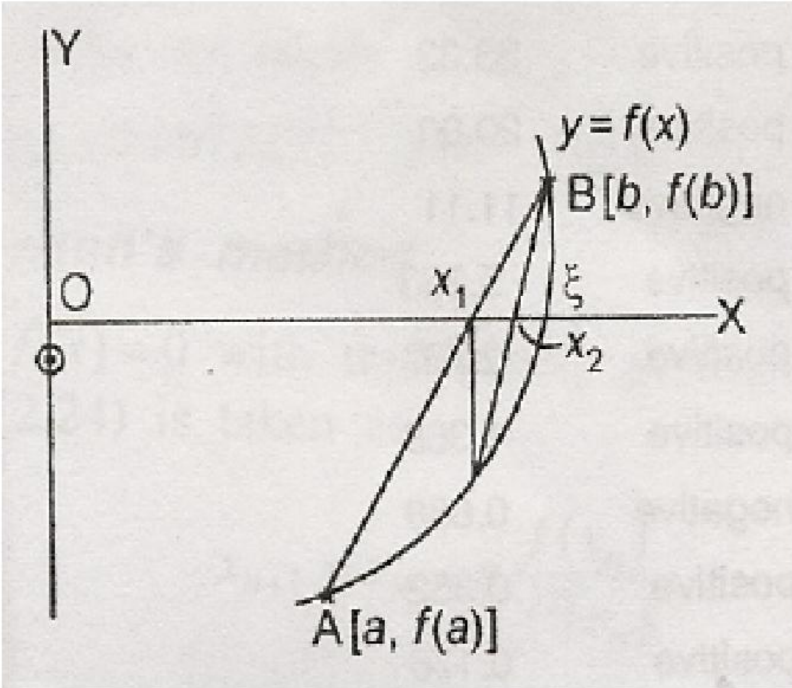

Figure Method of false position.

Example 1

Find a real root of the equation $f(x) = x^3 - 2x - 5 = 0$.

Solution: We find $f(2) = -1$ and $f(3) = 16$. Hence $a = 2, b = 3$, and a root lies between 2 and 3. Equation (2) gives

$$x_1 = \frac{2(16) - 3(-1)}{16 - (-1)} = \frac{35}{17} = 2.058823529.$$

Now, $f(x_1) = -0.390799917$ and hence the root lies between 2.058823529 and 3.0. Using formula (2), we obtain

$$x_2 = \frac{2.058823529(16) - 3(-0.147204057)}{16.147204057} = 2.089639211.$$

Since $f(x_2) = -0.147204057$, it follows that the root lies between 2.08126366 and 3.0. Hence, we have

$$x_3 = \frac{2.08126366(16) - 3(-0.147204057)}{16.147204057} = 2.089639211.$$

Proceeding in this way, we obtain successively:

$$x_4 = 2.092739575, x_5 = 2.09388371, x_6 = 2.094305452,$$

$$x_7 = 2.094460846, \ldots$$

The correct value is 2.0945..., so that x_7 is correct to five significant figures.

Example 2

Given that the equation $x^{2.2} = 69$ has a root between 5 and 8. Use the method of regula-falsi to determine it.

Solution: Let $f(x) = x^{2.2} - 69$. We find

$$f(5) = -34.50675846 \text{ and } f(8) = 29.00586026.$$

Hence

$$x_1 = \frac{5(28.00586026) - 8(-34.50675846)}{28.00586026 + 34.50675846} = 6.655990062.$$

Now, $f(x_1) = -4.275625415$ and therefore, the root lies between 6.655990062 and 8.0. We obtain

$$x_2 = 6.83400179, x_3 = 6.850669653.$$

The correct root is 6.85..., so that x_3 is correct to three significant figures.

Example: A root of the equation $xe^x - 1 = 0$ lies in the interval $(0.5, 1)$. Determine this root correct to three decimal places using regula-falsi method.

Solution: The root lies in $(0.5, 1)$. We have $x_0 = 0.5, x_1 = 1, f(x_0) = -0.17564, f(x_1) = 1.71828$.

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$= \frac{0.5(1.71828) - 1(-0.17564)}{1.71828 + 0.17564} = 0.54637.$$

$f(x_2) = f(0.54637) = -0.05643.$

Since $f(x_1)f(x_2) < 0$, the root lies in the interval $(0.54637, 1)$.

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)} = 0.56079\,.$$

$f(x_3) = f(0.56079) = -0.01746.$

Since $f(x_1)f(x_3) < 0$, the root lies in the interval $(0.56079, 1)$.

$$x_4 = \frac{x_1 f(x_3) - x_3 f(x_1)}{f(x_3) - f(x_1)} = 0.56521.$$

$f(x_4) = f(0.56521) = -0.00533.$

Since $f(x_1)f(x_4) < 0$, the root lies in the interval $(0.56521, 1)$.

$$x_5 = \frac{x_1 f(x_4) - x_4 f(x_1)}{f(x_4) - f(x_1)} = 0.56654.$$

$f(x_5) = f(0.56654) = -0.00167.$

Since $f(x_1)f(x_5) < 0$, the root lies in the interval $(0.56654, 1)$.

$$x_6 = \frac{x_1 f(x_5) - x_5 f(x_1)}{f(x_5) - f(x_1)} = 0.56696.$$

Hence, the root correct to three decimal places is 0.567. Note that the right end point x_1, of the initial interval is fixed in all iterations.

Exercise

1. Use the method of false position to obtain a root, correct to three decimal places, of each of the following equations
   a. $x^3 + x^2 + x + 7 = 0$
   b. $x^3 - x^2 - 1 = 0$
   c. $x^3 - x - 4 = 0$
2. Find a root of the equation $x \log_{10} x = 1.2$ using false position method.
3. Find a real root of $xe^x = 2$ using regula falsi method.
4. By using regula falsi method, find an approximate root of the equation $x^4 - x - 10 = 0$ that lies between 1.8 and 2. Carry out three approximations.

Answers

1.
   a. -2.104

b. 1.465

c. 1.796

2. 2.74
3. 0.85260
4. 1.8555