UNIT- VI

QUANTUM MECHANICS

Interaction of radiation with matter :-

Black Body radiation

* the electrons in the matter behaves like a oscillators with a characteristics frequency.
* the oscillators, absorb or emit energy in "discrete manner"
* To explain Black body Radiation, max plank proposed Above statement
* According to Hygun's principle, phenomena of Interference diffraction and polarisation, it was established that light behaves like wave.
* According to Einstein photo electric effect, light exhibit particle nature.
* these particles are called "PHOTONS" (quantum Packet).
* De Broglie assumed that particles like electrons, protons, neutrons etc exhibit wave nature
* The concept of wave nature of material particles was experimentally verified by Devisson and his group and G.P. thomson and his group.
* They discovered the wave nature of electrons through diffraction experiments.

Matter Waves :-

* According De Broglie, a wave is associated with a particle in motion called matter wave, which may be regarded as localised with the particle.
* the wavelength of the matter wave is given by

$$\boxed{\lambda = \frac{h}{mv} = \frac{h}{p}}$$

Uncentainity principle :-

Heisenburg Uncentainity principle:-

According to Heisenberg, it is impossible determine the position and momentum of elementary particles simullaneously at arbitrany precision.

$$\Delta x \, \Delta P_x \geq \frac{\hbar}{2} \qquad \left[\frac{\hbar}{2} = \frac{h}{2\pi}\right] \quad \hbar = \frac{h}{2\pi}$$

similarly $\Delta E \, \Delta t \geq \frac{\hbar}{2}$ where E - energy, Δ - uncentainity in time

$\Delta L \, \Delta \theta \geq \frac{\hbar}{2}$ where ΔL - uncertainity in Angular momentum, θ - Angle.

Applications of Uncentainity Relation:-

* The Uncentainity Relation explains the ground stble energy of hydrogen Atom.
* It explains the width of the spectral line.
* It explains the non existence of electrons in nucleus.
* It explains the mass of meson (nuclear force). It is found to be 200 times the mass of electron.

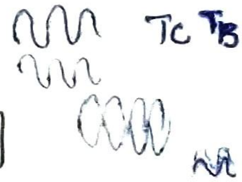
WAVE FUNCTION :-

The trajectory of elementary particles (quantum particles) can be described with the help of wave function.

wave function can be written as

$$\boxed{\psi(x,t) = A e^{i(kx - \omega t)}}$$

where $\vec{k}$ - wave vector $\left[k = \frac{2\pi}{\lambda}\right]$



when two or more waves with slight difference in frequency overlap, it leads to formation of wave packets.

WAVE PACKET:-

* The wave function of matter wave which is confined to small region of space is termed as wave packet and mathematically it can be represented as

$$\psi(x,t) = \int_{-\infty}^{+\infty} A(k) e^{i(kx - \omega t)} dk$$

Time dependent Schrodinger Equation :-

The wave function of localised free particles is given by

$$\psi(x,t) = \int_{-\infty}^{+\infty} A(k)\, e^{i(kx-\omega t)}\, dk$$

* Free particle has only kinetic energy

$$E = \frac{1}{2}mv^2 = \frac{m^2v^2}{2m} = \frac{p^2}{2m} \quad \text{——— (1)}$$

* According to de broglie, $\lambda = \frac{h}{p} \Rightarrow p = \frac{h}{\lambda}$.

$$\boxed{p = \frac{h}{\lambda}}$$

we know that $k = \frac{2\pi}{\lambda}$

$$p = \frac{h}{\frac{2\pi}{k}} \Rightarrow p = \frac{hk}{2\pi}$$

$$\boxed{p = \hbar k} \quad \text{——— (2)}$$

* According plank, $E = h\nu$

$$\cancel{E = h\frac{2\pi}{\omega}} \qquad E = \frac{h\omega}{2\pi}$$

$$\boxed{E = \hbar\omega}$$

*

$$\frac{p^2}{2m} = \hbar\omega$$

$$\frac{\hbar^2 k^2}{2m} = \hbar\omega$$

$$\frac{\hbar k^2}{2m} = \omega \Rightarrow \boxed{\omega = \frac{\hbar k^2}{2m}}$$

* According to wave function $\psi(x,t) = \int_{-\infty}^{\infty} A(k)\, e^{i(kx-\omega t)}\, dk$

NOW $$\frac{\partial\psi}{\partial x} = ik\int_{-\infty}^{\infty} A(k)\, e^{i(kx-\omega t)}\, dk$$

$$\boxed{\frac{\partial\psi}{\partial x} = ik\,\psi(x,t)}$$

$$\frac{\partial^2 \psi}{\partial x^2} = (ik)^2 \psi(x,t)$$

$$\frac{\partial^2 \psi}{\partial x^2} = -k^2 \psi(x,t) \quad \text{——— (4)}$$

$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2}$

NOW $\frac{\partial \psi}{\partial t} = -i\omega \psi(x,t)$

$$\boxed{\frac{\partial \psi}{\partial t} = -i\frac{\hbar k^2}{2m}\psi(x,t)} \quad \text{——— (5)}$$

NOW multiply eq (4) with $-\frac{\hbar^2}{2m}$

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} = \frac{\hbar^2 k^2}{2m}\psi(x,t) \quad \text{——— (6)}$$

NOW multiply eq 5 with $i\hbar$

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = \frac{\hbar^2 k^2}{2m}\psi(x,t) \quad \text{——— (7)}$$

From eq (6) and eq (7)

$$i\hbar \frac{\partial \psi}{\partial t}(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2}$$

$$\boxed{-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} = i\hbar \frac{\partial \psi}{\partial t}}$$

$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} = i\hbar \frac{\partial \psi}{\partial t}.$

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi = i\hbar \frac{\partial \psi}{\partial t}.$$

schrodiger introduced the potential term (due to forces on particle). In the equation $\underline{-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + \underline{V\psi} = i\hbar \frac{\partial \psi}{\partial t}}$

we know time dependent schrodinger equation is given by

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi(x,t)}{\partial x^2} + V\psi(x,t) = i\hbar \frac{\partial(\omega/t)}{\partial t}.$$

In 3-D :-

$$-\frac{\hbar^2}{2m}\nabla^2 \frac{\psi(x,t)}{\partial x^2} + V\psi(x,t) = i\hbar \frac{\partial(\omega t)}{\partial t}.$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

wave function is $\psi(x,t) = A e^{i(k_0 r - \omega t)}$

for a free particle :-

$V = 0$

$\cancel{-\frac{\hbar^2}{2m}\frac{\partial^2 \psi(x,t)}{\partial x^2} = i\hbar\frac{\partial}{\partial t}(\psi,t)}$

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) = i\hbar\frac{\partial}{\partial t}\psi(x,t)$$

$$\frac{1}{2m}\left(-i\hbar\frac{\partial}{\partial x}\right)\left(-i\hbar\frac{\partial}{\partial x}\right)\psi = i\hbar\frac{\partial}{\partial t}\psi \qquad \text{——— (1)}$$

we know that $\frac{P^2}{2m}\psi(x,t) = E\,\psi(x,t)$ ——— (2)

Comparing Eq^{n}s (1) & (2)

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* In Quantum mechanics the dynamic variables are, can be calculated from operators.
* the momentum operator : $\hat{P}_x = -i\hbar\frac{\partial}{\partial x}$
* the energy operator : $\hat{E} = i\hbar\frac{\partial}{\partial t}$

* The momentum operator should act on a given wave function, to extract the information regarding the momentum $\hat{p}\,\psi = [\quad]\,\psi$ → momentum eigen value.
* $\hat{E}\,\psi =$ ______ ψ Energy eigen value

for free particle,

$V = 0$.

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) = i\hbar\frac{\partial \psi(x,t)}{\partial t} \quad \text{— ①}$$

$$\left(-i\hbar\frac{\partial}{\partial x}\right)\left(i\hbar\frac{\partial}{\partial x}\right)\psi = i\hbar\frac{\partial\psi}{\partial t}$$

In 3-D,

$$-\frac{\hbar^2}{2m}\nabla^2\psi(x,t) = i\hbar\frac{\partial\psi(x,t)}{\partial t}.$$

$$\frac{1}{2m}\left(-i\hbar\frac{\partial}{\partial x}\right)\left(i\hbar\frac{\partial}{\partial x}\right)\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$\psi = \sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a}$$

$$\psi = \frac{\sqrt{2}}{a}e^{2x}$$

$$E = \frac{p^2}{2m}\frac{\psi(x,t)}{\psi(x,t)} \qquad \frac{p^2}{2m}\psi(x,t) = E\psi(x,t)$$

$$\frac{p^2}{2m}\psi(x,t) = E\psi(x,t)$$

Here $\hat{p} = -i\hbar\frac{\partial}{\partial x}$ — momentum operator

$\hat{E} = i\hbar\frac{\partial}{\partial t}$ — Energy operation.

$$\hat{p}\psi = i\hbar\frac{\partial}{\partial x}\left(\frac{\sqrt{2}}{a}e^{2x}\right) = -i\hbar\frac{\sqrt{2}}{a}2e^{2x} = i\hbar\frac{2\sqrt{2}}{\sqrt{a}}e^{2x}$$

for $\hat{p}\psi = -i\hbar\frac{\partial}{\partial x}\left(\sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a}\right)$

$$= -i\hbar\sqrt{\frac{2}{a}}\frac{\cos n\pi x}{a}(n\pi)$$

$$\boxed{\hat{p}\psi = i\hbar\frac{\sqrt{2}}{a\sqrt{a}}n\pi\cos(n\pi x)}$$

$\hat{p}\psi$ — ψ

$E\psi$ — ψ

Time Independent Schrodinger Equation

By using seperation of variable concept, we can derive time dependent schrodinger equation :-

* In 1-D, schrodinger wave Eqⁿ is given by

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) + V(x)\psi(x,t) = \frac{i\hbar}{2m}\frac{\partial \psi(x,t)}{\partial t} \quad \text{——} ①$$

* In 3-D schrondigen wave Eqⁿ,

$$-\frac{\hbar^2}{2m}\nabla^2\psi(x,t) + V\psi(x,t) = i\hbar\frac{\partial \psi(x,t)}{dt}$$

* $\psi(x,t) = Ae^{i(k_0x - \omega t)}$
* using seperation method :- $\psi(x,t) = \psi(x)\Phi(t)$ —— ②
* substitute Eqⁿ 2 with Eqⁿ ①

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x)\Phi(t) + V\psi(x)\Phi(t) = i\hbar\frac{\partial}{\partial t}\psi(x)\Phi(t)$$

$$-\frac{\hbar^2}{2m}\Phi(t)\frac{\partial^2\psi(x)}{\partial x^2} + V\psi(x)\Phi(t) = \psi(x)\, i\hbar\frac{\partial \Phi(t)}{\partial t}$$

devide whole Equation with $\psi(x)\Phi(t)$

$$-\frac{\hbar^2}{2m}\Phi(t)\frac{1}{\psi(x)}\frac{\partial^2\psi(x)}{\partial x^2} + V\Phi(t) = \frac{i\hbar}{\Phi(t)}\frac{\partial\Phi(t)}{\partial t}$$

$$\cancel{-\frac{\hbar^2}{2m} + 1} \qquad -\frac{\hbar^2}{2m}\frac{1}{\psi(x)}\frac{\partial^2\psi(x)}{\partial x^2} + V(x) = \frac{i\hbar}{\phi(t)}\frac{\partial\phi(t)}{\partial t}$$

$$-\frac{\hbar^2}{2m}\frac{1}{\psi(x)}\frac{\partial^2\psi(x)}{\partial x^2} + V(x) = E$$

divide whole Eqⁿ with $\psi(x)$

$$\boxed{-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + V(x)\psi(x) = E\psi(x)}$$

this is independ schodinger Eqⁿ.

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right]\psi(x) = E\,\psi(x)$$

$$\hat{H}\psi = E\psi$$

where $\hat{H}$ can be written as $\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)$

* $\hat{H}$ is Hamiltonion operator.

hence $\cancel{E = \frac{i\hbar}{2m}}$ $\quad E = \frac{i\hbar}{\phi(t)}\frac{\partial \phi(t)}{\partial t}$ $\quad \frac{i\hbar}{\phi(t)}\frac{\partial \phi(t)}{\partial t}$

$$\frac{i\hbar}{\phi(t)}\frac{\partial \phi(t)}{\partial t} = E$$

$$i\hbar\frac{\partial \phi(t)}{\partial t} = E\phi(t) \qquad \phi(t) = Ce^{-\frac{iEt}{\hbar}}$$

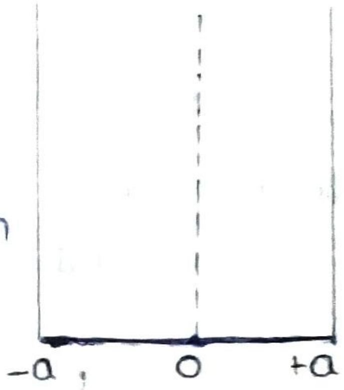
$$\frac{\partial \phi(t)}{\partial t} = \frac{-i}{\hbar}E\phi(t) \qquad Ce^{\frac{iE}{\hbar}t}$$

$$\phi(t) = Ce^{-\frac{iEt}{\hbar}}$$ → eqn used for stationary states

Calculate the eigen values and eigen function in given region of a particle contrastrained in region where potential is

$$V(x) = 0 \qquad -a < x < a$$

$$V(x) = \infty \qquad a \geq |x|$$



Time independent schrodinger wave equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi(x)}{\partial x} + V(x)\psi(x) = E\,\psi(x)$$

since $V(x) = 0$ in $-a < x < a$

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi(x)}{\partial x} = E\,\psi(x)$$

$$\frac{\partial^2 \psi}{\partial x} + \frac{2mE}{\hbar^2}\psi = 0$$

Here, $m, E, \hbar$ are constants

$$\boxed{k^2 = \frac{2mE}{\hbar^2}}$$

$$\frac{\partial^2 \psi}{\partial x} + k^2\psi = 0$$

$\frac{\partial^2\psi}{\partial x^2} + k^2\psi$ is a polynomial. Then the general solution of $\frac{\partial^2\psi}{\partial x^2} + k^2\psi$ is

$$\boxed{\psi(x) = A\cos kx + B\sin kx} \quad \text{—— ①}$$

* $\psi(x=a) = A\cos ka + B\sin ka = 0$
* $\psi(x=-a) = A\cos ka - B\sin ka = 0$

} since particle doesn't exist at edges.

* $A\cos ka = 0$ and $B\sin ka = 0$
* we don't take both A & $B = 0$ since wave function will be ~~Nt~~ zero.

Case (i): $B = 0$ $\quad \cos ka = 0$

$$ka = \frac{n\pi}{2} \quad (n = \text{odd numbers})$$

$$\boxed{k = \frac{n\pi}{2a}} \Rightarrow k^2 = \frac{n^2\pi^2}{4a^2}$$

Since $k^2 = \frac{2mE}{\hbar^2}$

$$\Rightarrow \frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{4a^2} \Rightarrow E_n = \frac{n^2\pi^2\hbar^2}{8ma^2} \quad (n = \text{odd})$$

Case (ii): $A = 0$ $\quad \sin kx = 0$

$$ka = \frac{n\pi}{2} \quad (n = \text{even})$$

$$k = \frac{n\pi}{2a} \rightarrow k^2 = \frac{n^2\pi^2}{4a^2}$$

since $k^2 = \frac{2mE}{\hbar^2} \Rightarrow \frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{4a^2}$

$\psi(x) = A\cos\frac{n\pi}{2a}x$

$\psi(x) = B\sin$

$$\boxed{E_n = \frac{n^2\pi^2\hbar^2}{8ma^2}} \quad n = \text{even}$$

when $B = 0$ $\quad \cos kx = 0$ $\quad \psi_{odd}(x) = A\cos\frac{n\pi}{2a}x$ when n is odd

when $A = 0$ $\quad \sin kx = 0$ $\quad \psi_{even}(x) = A\sin\frac{n\pi}{2a}x$ when n is even

$$\therefore E_n = \frac{n^2\pi^2\hbar^2}{8ma^2} \quad n = 1, 2, 3, 4 \ldots$$

when n is odd, $\psi_{odd} = \sqrt{\frac{1}{a}} \cos \frac{n\pi}{2a} x$,

when n is even $\psi_{even} = \sqrt{\frac{1}{a}} \sin \frac{n\pi x}{2a}$. $A = \sqrt{\frac{1}{a}}$

$\therefore$ * $E_n = \frac{n^2 \pi^2 \hbar^2}{2m(2a)^2}$

$E_1 = \frac{\pi^2\hbar^2}{8ma^2}$ $E_2 = 4\frac{\pi^2\hbar^2}{8ma^2}$ $E_3 = 9\frac{\pi^2\hbar^2}{8ma^2}$ $E_4 = 16\frac{\pi^2\hbar^2}{8ma^2}$

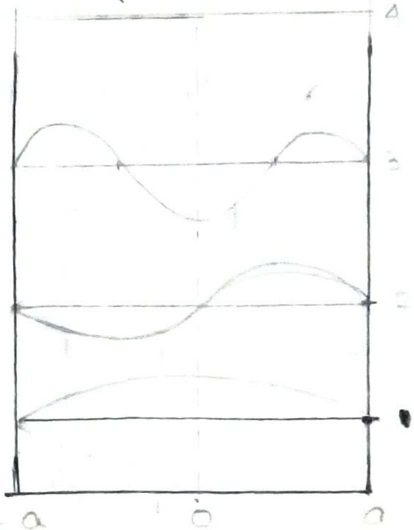
$E_{100} = (100)^2 \frac{\pi^2\hbar^2}{8ma^2}$ $E_{101} = (101)^2 \frac{\pi^2\hbar^2}{8ma^2}$ .....

$\Delta E = E_{101} - E_{100} = 201 \frac{S}{(2a)^2}$ $\left[\because S = \frac{\pi^2\hbar^2}{2m} \right]$

* $x = -a$ $\psi(x) = A\cos\frac{n\pi x}{2a}$

$A\cos \pi/2 = 0$

$x = 0$ $\psi(x) = A\cos 0$



$$\boxed{E_n = n^2 \frac{\pi^2\hbar^2}{2m(2a)^2}}$$

* As n increases discreteness decreases, it become continous, so Quantum problem behaves as classical problem

* If a particle is constrained to a region of 0 to L.

$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$

$n = \frac{\pi^2\hbar^2}{8ma^2}$

$$\boxed{\psi_{even} = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{L}}$$

Normalisation Functions :-

$x(t) = \sin t$ $\quad\quad$ $x(t) = \cos t$

$x^*(t) = \sin t$ $\quad\quad$ $x^*(t) = \cos t$

we have wave function, $\psi(x,t) = Ae^{i(kx-\omega t)}$

we have wave packet, $\psi(x,t) = \int Ae^{i(kx-\omega t)}$

* The probality of finding particle in mode is given by

~~P = N sin nπx/L~~ = 1

$$P = \int_{-\infty}^{\infty} \psi^* \psi \, d\tau = 1$$

* $\psi = N \sin\left(\frac{n\pi x}{L}\right)$ is a normalised function. — ①

* $P = N^2 \int_{-\infty}^{\infty} \sin^2 \frac{n\pi x}{L} dx = N^2 \int_0^L \sin^2 \frac{n\pi x}{L} dx = 1$

$$P = \frac{N^2}{2} \int_0^L 1 - \cos\left(\frac{2n\pi x}{L}\right) dx = 1$$

$$P = \frac{N^2}{2} \left[x - \frac{\sin \frac{n\pi x}{L}}{n\pi/L} \right]_0^L = 1$$

$$P = \frac{N^2}{2} L = 1 \quad \Rightarrow N = \sqrt{\frac{2}{L}}$$

where N is normalised value

from ①

$$\boxed{\psi = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}}$$ — Normalised function.

Densisty of Energy states:-

* In 1-D $\quad E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$

* In 3-D $\quad E_n = (n_x^2 + n_y^2 + n_z^2)\frac{\pi^2\hbar^2}{2mL^2}$

volume = $\frac{4}{3}\pi n^3$ - $\frac{4}{3}\pi n^3$

where n is energy level

$$n^2 = \frac{2mL^2 E}{\pi^2 \hbar^2} \qquad n^3 = \left(\frac{2mL^2 E}{\pi^2 \hbar^2}\right)^{3/2}$$

* Energy values won't be Negative, so levels are all in first ~~volume~~ = octanct only.

$$\text{volume} = \frac{1}{8}\,\frac{4}{3}\pi\left(\frac{2mL^2 E}{\pi^2 \hbar^2}\right)^{3/2}$$

$$\text{volume} = \frac{1}{6}\pi\left(\frac{2mL^2 E}{\pi^2 \hbar^2}\right)^{3/2}$$

* For unit volume of each level, & total volume be n then

No. of energy level = $N(E) = \frac{\text{volume of total surface}}{\text{volume of each layer}}$

* for total volume = 1 & volume of each level = 1

then

$$N(E) = \frac{\frac{1}{6}\pi\left(\frac{2mL^2 E}{\pi^2 \hbar^2}\right)^{3/2}}{1}$$

*

$$N(E) = \int D(E)\, dE.$$

* $D(E) = \frac{d}{dE} N(E).$ where $D(E) \Rightarrow$ density of energy level

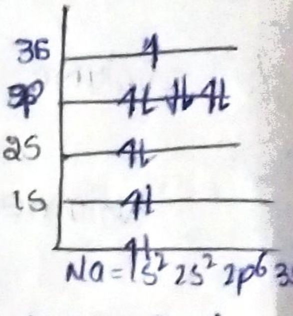
$$\boxed{D(E) = \frac{1}{6}\pi\left(\frac{2mL^2}{\pi^2 \hbar^2}\right)^{3/2} \frac{3}{2} E^{1/2}}$$

$$D(E) = \frac{1}{4}\pi\left(\frac{2mL^2}{\pi^2 \hbar^2}\right)^{3/2} \sqrt{E}.$$

Fermi level :-

* The highest occupied energy level at absolute zero temperature (0K) is called "FERMI LEVEL".
* At absolute zero, the fermi energy represents the energy which divides the filled states from the empty states.
* The function is given by

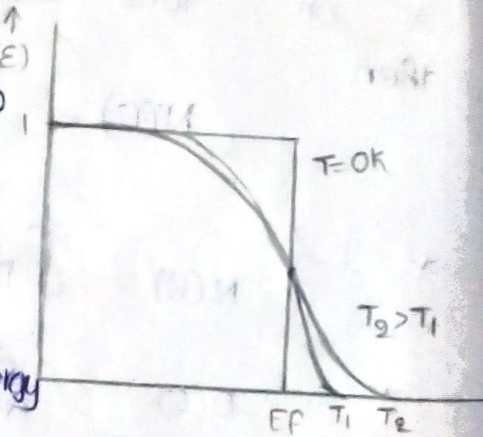
$$f(E) = \frac{1}{e^{(E-E_F)/K_BT}+1}$$



where E - energy of particle
E_F - fermi energy

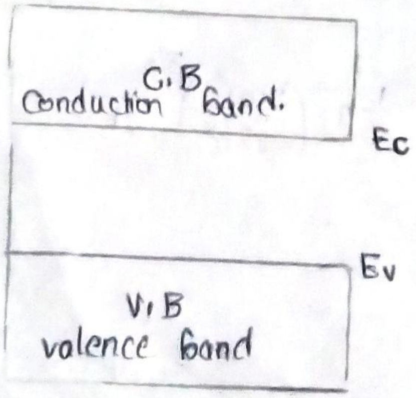
* The Above equation gives the probability that particular Quantum state at ~~part~~ energy E is filled or not
* If $f(E) = 1$, that means Energy state is occupied by electron
* If $f(E) = 0$, means Energy state is empty.
* when $E_F = E$ $f(E) = 1/2$
  some of electrons ($1/2$ no) are exited to empty levels.

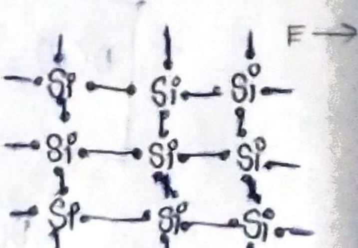
Note:- The e^-s near to fermi level are participated in excitation.

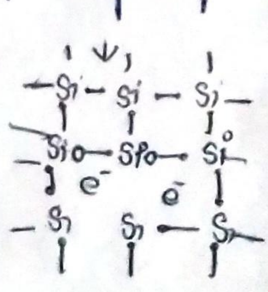


Band Theory of solids

* In solids, instead of single energy levels, energy bands will form.
* The band structure can be shown in the figure







* Quantum mechanics explained the existence of forbidden energy band between two bands.

* If the width of forbidden band between valence band and conduction band is in order of 6eV or more, the excited electrons(e's for conduction) not exist. such materials are called "INSULATORS". [6eV >

* If the width of forbidden band is is order of 1-2eV a few electrons which are near to the fermi level can excited to the conduction Band. Such materials are called SEMICONDUCTORS.

* In the conducters, the conduction band overlaps on the valence band.

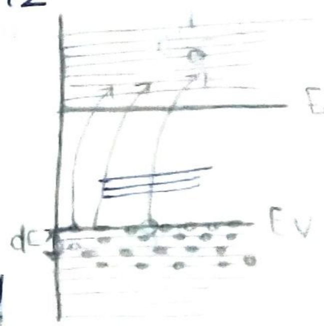
Concerntration of electrons and Holes in a SEMI CONDUCTER.

$\frac{2^{3/2}}{4}$

we know density of energy states is given by

$6 \; 2^{3-2} \; 2^{-1/2}$

$$D(E) = \frac{L^3}{4} \cdot \frac{1}{\hbar^3 \pi^2} (2m)^{3/2} \cdot E^{1/2}$$



The No. of states is given by

$$N(E) = D(E)\, dE$$

No of occupied states is given by

$$\boxed{N(E) = D(E)\, dE \times f(E)}$$

Number of states per unit volume is given by in range of energy E and E+dE is

$$\boxed{dn_0 = \frac{N(E)}{V} = \frac{1}{4\pi^2 \hbar^3} (2m)^{3/2} (E-E_c)^{1/2} f(E)}$$

$$n_0 = \int_{E_c}^{\infty} dn_0 = \int_{E_c}^{\infty} \frac{8\sqrt{2}\pi m^{3/2}}{h^3} \frac{(E-E_c)^{1/2}}{e^{(E-E_f)/k_B T}+1} dE$$

since $e^{(E-E_f)/K_BT} >>> 1$ we neglect ~~this term~~. 1

$$n_0 = \int_{E_c}^{\infty} \frac{8\sqrt{2}\pi m^{*3/2}}{h^3} \frac{(E-E_c)^{1/2}}{e^{E/K_BT} \cdot e^{-E_f/K_BT}} dE$$

$$n_0 = \int_{E_c}^{\infty} 8\sqrt{2}\pi m^{*3/2} \frac{e^{E_f/K_BT}}{e^{E/K_BT}} \times \frac{e^{E_c/K_BT}}{e^{E_c/K_BT}} (E-E_c)^{1/2} dE$$

$$n_0 = \int_{E_c}^{\infty} \frac{8\sqrt{2}\pi m^{*3/2}}{h^3} e^{E_f/K_BT} e^{-E_c/K_BT} e^{\left(\frac{E_c-E}{K_BT}\right)} (E-E_c)^{1/2} dE$$

let $e^{\frac{E_c-E}{K_BT}} = x_0$

$\cancel{E_c - E = x_0 K_BT}$

$$\frac{E-E_c}{K_BT} = x_c$$

$$E - E_c = x_c K_BT$$

$$\boxed{dE = dx_c K_BT}$$

$$n_0 = \frac{8\sqrt{2}\pi m_n^{*3/2}}{h^3} e^{-(E_c-E_f)/K_BT} \int_0^{\infty} x_c^{1/2} (K_BT)^{1/2} e^{-x_c}(K_BT)\, dx_c$$

$$n_0 = \frac{8\sqrt{2}\, m_n^{*3/2}}{h^3} (K_BT)^{3/2} \frac{\sqrt{\pi}}{2} e^{-\frac{(E_c-E_f)}{K_BT}}$$

Electron Concentration: $n_0 = 2\left(\frac{2\pi m_n^* K_BT}{h^2}\right)^{3/2} e^{-(E_c-E_f)/K_BT}$

$$\boxed{n_0 = N_c\, e^{-(E_c-E_f)/K_BT}}$$

Hole Concentration:-

$$P_0 = N_v\, e^{-(E_f-E_v)/K_BT}$$

$$N_v = 2\left(\frac{2\pi m_p^* K_BT}{h^2}\right)^{3/2}$$

m_n^* effective mass of electron

m_p^* effective mass of hole.

LAW OF MASS ACTION :-

the no of electrons per unit volume in the conducting Semiconducting band is given by $n_0 = 2\left(\frac{2\pi m_n^* k_B T}{h^2}\right)^{3/2} e^{-\frac{(E_c - E_F)}{k_B T}}$

The no of holes per unit volume in the valence band is given by $p_0 = 2\left(\frac{2\pi m_p^* k_B T}{h^2}\right)^{3/2} e^{-(E_F - E_v)/k_B T}$

$2^{3/2} \cdot 2^{3/2}$

2^3

$$n_0 p_0 = 2^5 \left(\frac{\pi^2 k_B^2 m_p^* m_n^*}{h^4}\right)^{3/2} T^3 e^{-(E_c - E_v)/k_B T}$$

$$\boxed{n_0 p_0 = A T^3 e^{-\left(\frac{E_c - E_v}{k_B T}\right)}}$$

* The product $n_0 p_0$ is seem to be independent of position of fermi level and has same values for intrinsic semiconductors (for pure semiconductors) and for extrinsic (impure)

$$n_0 P_0 = n_i P_i$$

since pure conductor $(n_i = P_i)$

$$\boxed{n_0 P_0 \quad n_i^2 = P_i^2}$$

$$n_i^2 = P_i^2 = A T^3 e^{-(E_c - E_v)/k_B T}$$

where E_g ~ energy gap

$$n_i^2 = P_i^2 = A T^3 e^{-E_g / k_B T}$$

* $n_i = P_i = A^{1/2} T^{3/2} e^{\frac{-E_g}{2 k_B T}}$

at 300K,

For silicon (Si) $E_g = 1.1 eV$

$$\boxed{n_i = P_i = 1.6 \times 10^{10} \text{ atom/cm}^3}$$

n_i - no. of electrons in conduction band

P_i - no of holes in valence band

For Germanium (Ge) $E_g = 0.7 eV$

$$\boxed{n_i = P_i = 2.5 \times 10^{13} \text{ atom/cm}^3}$$

calculate Fermi level position in Intrisic semiconducters

$$n_i = P_i \quad (\text{for intrinsic})$$

$$n_0 = P_0$$

$$N_c\, e^{-(E_c - E_f)/K_BT} = N_v\, e^{-(E_f - E_v)/K_BT}$$

$$N_c \log e^{-(E_c - E_f)/K_BT} = N_v \log e^{-(E_f - E_v)/K_BT}$$

$$\cancel{-N_c \frac{(E_c - E_f)}{K_BT} = -N_v \frac{E_f - E_v}{K_BT}}$$

$$\cancel{N_c\,(E_c - E_f) = N_v\,(E_f - E_v)}$$

$$\cancel{N_c E_c - N_c E_f = N_v E_f - N_v E_v}$$

since $(N_c = N_v)$

$$e^{-(E_c - E_f)/K_BT} = e^{-(E_f - E_v)/K_BT}$$

$$(E_c - E_f)/K_BT = \frac{E_f - E_v}{K_BT} \qquad \frac{dw}{dt}$$

$$E_c - E_f = E_f - E_v$$

$$\boxed{E_f = \frac{E_c + E_v}{2}}$$

Position of fermi level in Intrinsic SemiConducters

Velocity And Effective mass:-

* When a electron is accelarated inside a crystal, its mass is generally appears different from free electron mass and is usually referred as "<u>effective mass of electron</u>".

* According to DeBroglie, the particle velocity 'v' is equal to the Group velocity of wave packet represented by an electron is given by

$$V_p = \frac{\omega}{K}$$

$$V_g = \frac{dw}{dK}$$

$E = h\nu$

$E = \hbar\omega$

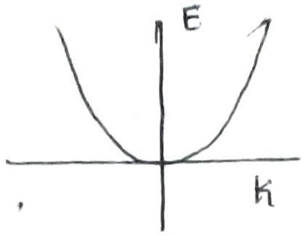
$F = h$

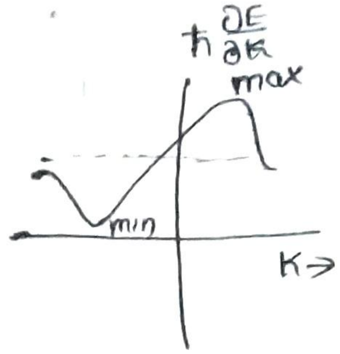
$c = \lambda\nu$

$\nu = \frac{c}{\lambda}$

$$\frac{dE}{dk} = \hbar\frac{d\omega}{dk}$$

$$\boxed{\frac{d\omega}{dk} = \frac{1}{\hbar}\left(\frac{dE}{dk}\right)}$$





$$\frac{d\omega}{dk} = \frac{1}{\hbar}\left(\frac{dE}{dk}\right)$$

* Let an $\bar{e}$ is subjected to applied electric field with strength E. for a time interval t,

* If velocity of $\bar{e}$ is 'v', & distance travelled by $\bar{e}$ in dt ~~then~~ dt time interval 'dt' is given by

distance = vdt.

* ~~Electric field is given by~~

* Energy gained ~~dE~~ by an electron is given by dE

$$\boxed{dE = (e\bar{E})(vdt)} \Rightarrow dE = (e\bar{E})\frac{1}{\hbar}\frac{\partial E}{\partial k}dt$$

where dE is energy gained by $\bar{e}$ in time dt

* Accelaration of free electron is given by

$$F = ma = eE$$

$$\boxed{\therefore a = \frac{eE}{m}} \quad \text{— ①}$$

$\frac{dv}{dt} = \frac{E}{m}$

$dv = \frac{E}{m}dt$

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{1}{\hbar}\frac{\partial E}{\partial k}\right) = \frac{1}{\hbar}\frac{\partial}{\partial t}\left(\frac{\partial E}{\partial k}\right)$$

$$\boxed{a = \frac{1}{\hbar}\frac{\partial^2 E}{\partial k^2}\frac{dk}{dt}}$$

we know $dE = \frac{(e\bar{E})}{\hbar}\frac{\partial E}{\partial k}dt$

$$\frac{dk}{dt} = \frac{e\bar{E}}{\hbar}$$

$$\boxed{a = \frac{1}{\hbar}\frac{e\bar{E}}{\hbar}\frac{\partial^2 E}{\partial k^2}}$$

$$\boxed{a = \frac{eE}{\hbar^2}\frac{\partial^2 E}{\partial k^2}}$$ where $\frac{\partial E}{\partial k}$ value at maximum

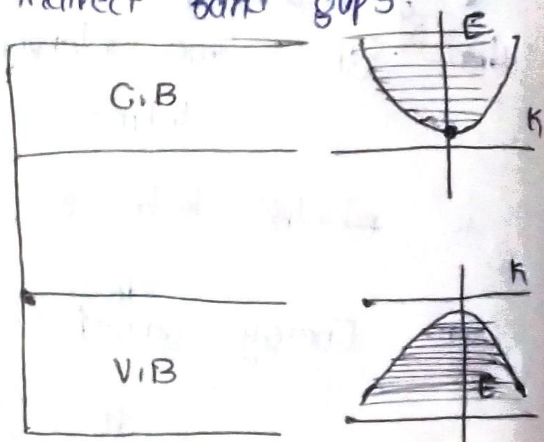
WKT $a = \frac{eE}{m}$

$$\therefore \frac{eE}{m} = \frac{eE}{\hbar^2}\frac{\partial^2 E}{\partial k^2}$$

$$\boxed{m_e^* = \frac{\hbar^2}{\left(\frac{\partial^2 E}{\partial k^2}\right)}}$$ effective mass

* For a free e^- $m_e^* = m$
* For an intrinsic semiconducters, no. of electrons = no of holes
* Germinium, Silicon have indirect Band gaps.
* more 3eV, behaves as Insulator.

  Ge - 0.7eV Si - 1.1eV



* when energy given to atom to emit e^-, if total energy is consumed to emit electron from V.B is called "direct Band"
* when energy provided to atom is used to heat lattice called "Indirect Band"
* In intrinsin semiconducters,

no. of e^-s = no. of holes in valency band in all the temperatures

$$n_p = n_i$$

$$N_c\, e^{-(E_c - E_{fi})/k_BT} = N_v\, e^{-(E_{fi} - E_v)/k_BT}$$

$$\frac{N_c}{N_v} = e^{(-E_{fi} + E_v + E_c - E_{fi})/k_BT}$$

$$\left(\frac{m_n^*}{m_p^*}\right)^{3/2} = e^{(E_c + E_v - 2E_{fi})/k_BT}$$

Applying ln on B.T

$$(E_c + E_v - 2E_{fi}) = k_BT \ln\left(\frac{m_n^*}{m_p^*}\right)^{3/2}$$

$$E_{Fi} = \left(\frac{E_C + E_V}{2}\right) - \frac{K_B T}{2} \ln\left(\frac{m_n^*}{m_p^*}\right)^{3/2}$$

(or)

$$E_{Fi} = \left(\frac{E_C + E_V}{2}\right) + \frac{K_B T}{2} \ln\left(\frac{m_p^*}{m_n^*}\right)^{3/2}$$

* If ~~eleffctive~~ effective mass of hole = effective mass of electron, fermi level will exactly at the middle of the forbidden

Position of fermi level in Extrinsic ~~for~~ Semi Conductors.

$$n_0 = N_C\, e^{-(E_C - E_F)/K_B T}$$ (e^- charge density in N type)

$$P_0 = N_V\, e^{-(E_F - E_V)/K_B T}$$

* $$\frac{n_0}{P_0} = \frac{N_C}{N_V} e^{-(E_C + E_V - 2f_n)/K_B T}$$

$$\frac{n_0}{P_0} = \frac{n_0 \times n_0}{P_0 \times n_0} = \frac{n_0^2}{P_0 \times n_0} = \frac{n_0^2}{n_i^2}$$ [∵ $n_i = P_i$ in intristic semiconducters]

* $$\frac{n_0^2}{n_i^2} = \frac{N_C}{N_V} \cdot e^{-(E_C + E_V - 2f_n)/K_B T}$$

$$\frac{N_C}{N_V} = e^{+(E_C + E_V - 2f_i)/K_B T}$$

$$\frac{n_0^2}{n_i^2} = e^{\left(\frac{E_C + E_V - 2f_i}{K_B T}\right)} e^{-\left(\frac{E_C + E_V - 2f_n}{K_B T}\right)}$$

$$\frac{n_0^2}{n_i^2} = e^{2\left[\frac{f_n - f_i}{K_B T}\right]}$$

$$n_0^2 = n_i^2\, e^{2(f_n - f_i)/K_B T}$$

f_n - fermi level of N type
f_i - fermi level of intrinsic

$$\boxed{n_0 = n_i\, e^{(f_n - f_i)/K_B T}}$$

$n_0 >> n_i$

n_0 - electron charge concentration in N type.

Since $n_i > n_0$

$E_{fn} > E_{fi}$

* Since no. of electrons (n_0) in conduction band of N type semiconducter is greater than n_i.
* The fermi level in N type semiconductor is close to conductio Band.

* Similarly for p type semiconducters

$$\boxed{P_0 = P_i e^{(E_{fn} - E_F)/K_BT}}$$

the fermi level close to valence band.

* Exact position of fermilevel in N type semiconductor can be made if

$$N_D = N_C e^{-(E_C - E_{fn})/K_BT}$$

N_D - donar concentration
N_a - Acceptor Concentration

$$\ln\left(\frac{N_D}{N_C}\right) = \frac{-E_C + E_{fn}}{K_BT}$$

$$-E_C + E_{fn} = K_BT \ln\left(\frac{N_D}{N_C}\right)$$

$$\boxed{E_{fn} = E_C + K_BT \ln\left(\frac{N_D}{N_C}\right)} \quad \text{—} (1)$$

$$\boxed{E_{fn} = E_C - K_BT \ln\left(\frac{N_C}{N_D}\right)}$$

$$N_a = N_V e^{-(E_{fp} - E_V)/K_BT}$$

$$\ln\left(\frac{N_a}{N_V}\right) = \frac{-E_{fp} + E_V}{K_BT}$$

$$-E_{fp} + E_V = K_BT \ln\left(\frac{N_a}{N_V}\right)$$

$$\boxed{E_{fp} = E_V - \ln\left(\frac{N_a}{N_V}\right) K_BT}$$

$$\boxed{E_{fp} = E_V + \ln\left(\frac{N_V}{N_a}\right) K_BT} \quad \text{—} (2)$$

Adding ① & ②

$$E_{fn} + E_{fp} = E_C + K_BT \ln\left(\frac{N_D}{N_C}\right) + E_V + \ln\left(\frac{N_C}{N_a}\right)$$

$$\boxed{E_{fn} + E_{fp} = E_C + E_V + K_BT \ln\left(\frac{N_D}{N_a}\right)}$$

Si° Ge In

Transiston is used to contool flow of $\bar{e}$s.

Ga As direct band gap

Hall Effect

$$F_B = v(q \times B)$$

$$F_E = qE$$

$$qE = v(q \times B) \qquad [\because B \& E \perp n]$$

$$qE = qvB.$$

$$v_e B = \frac{V_H}{w} \qquad w - \text{thickness of semiconducten}$$

$$V_H = v_e B w \qquad I = nAve \qquad v = \frac{I}{neA}.$$

$$V_H = \frac{I}{neA} B w = \frac{I}{ne\,wt} B w = \frac{IB}{net}$$

$$V_H = \frac{IB}{net} \qquad R_H = \frac{1}{ne} = \frac{V_H t}{BI} \qquad \mu = R_H \sigma.$$

Exceptation values: $\langle P_x \rangle = \int \psi^* \left(-i\hbar \frac{\partial}{\partial t}\right) \psi$

$$\int \psi^* \psi \, d\tau = 1$$

$$\psi = \sqrt{\frac{1}{L}} \sin \frac{n\pi x}{L}$$

→