

# Mechanics of Materials-II

## SPRINGS

**Botsa Srinivasa Rao**

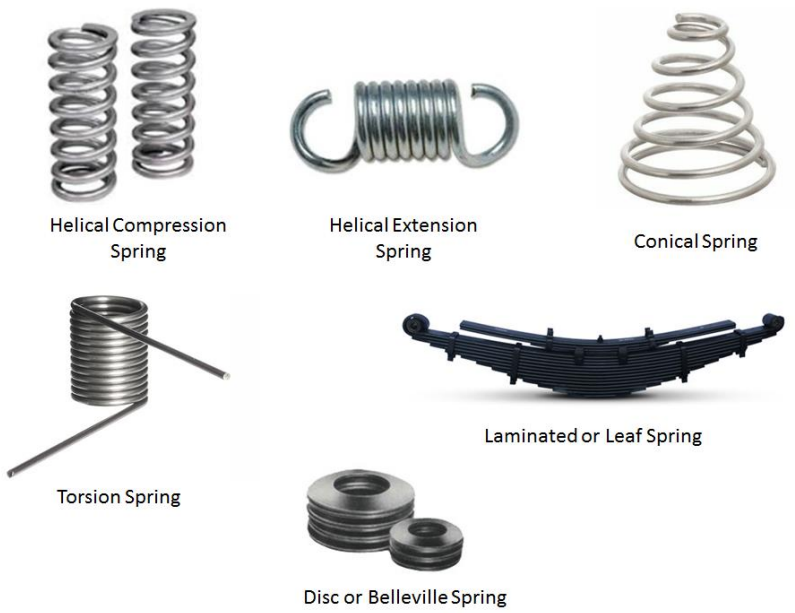
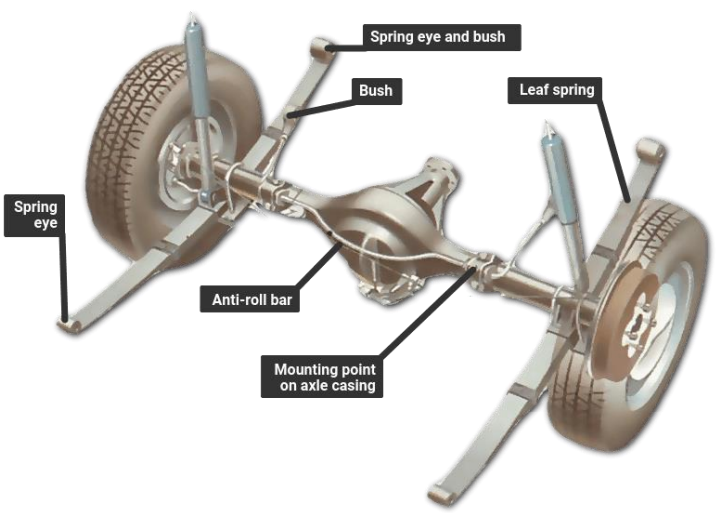
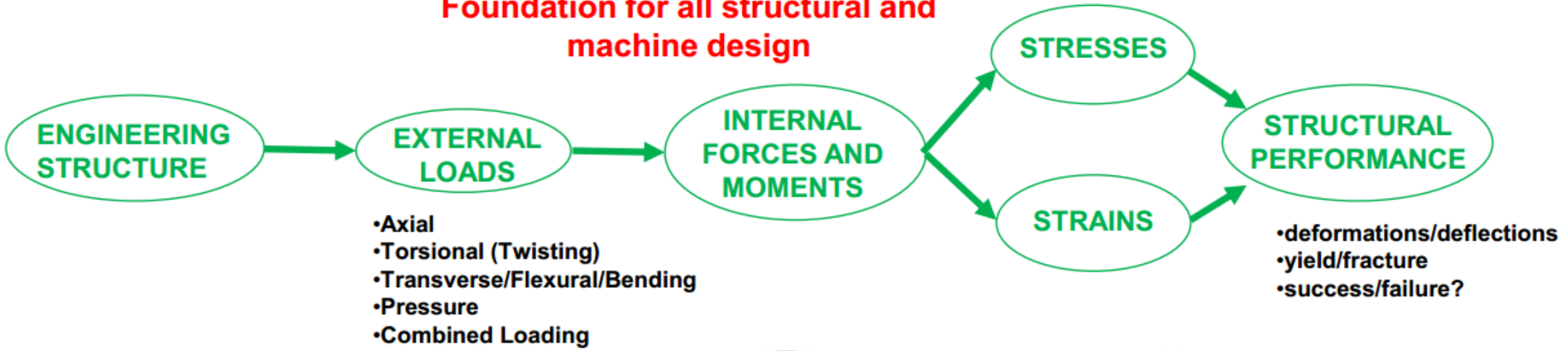
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# MOM (Course Outcomes)

Foundation for all structural and machine design



# *Springs*

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- Springs are elastic bodies (Generally metal) that can be twisted, pulled or stretched by some force. They can return to their original shape when the force is released.

## Function of springs

1. To store energy and release when required
2. To reduce shock, impact and vibration among moving parts
3. Reduce the effect of impact loading
4. A carriage spring used to absorb shocks

Energy absorption of these elements is due to ability to deform. Members deform more under the loads and absorb more energy.

# Types of Springs

## 1. Helical Springs

- a. Close Coil (Tension) Helical Springs
- b. Open Coil (Compression) Helical Springs
- c. Torsion spring
- d. Spiral spring

## 2. Leaf Spring (or) Carriage Spring

## 3. Disc Springs



Tension Spring



Compression Spring



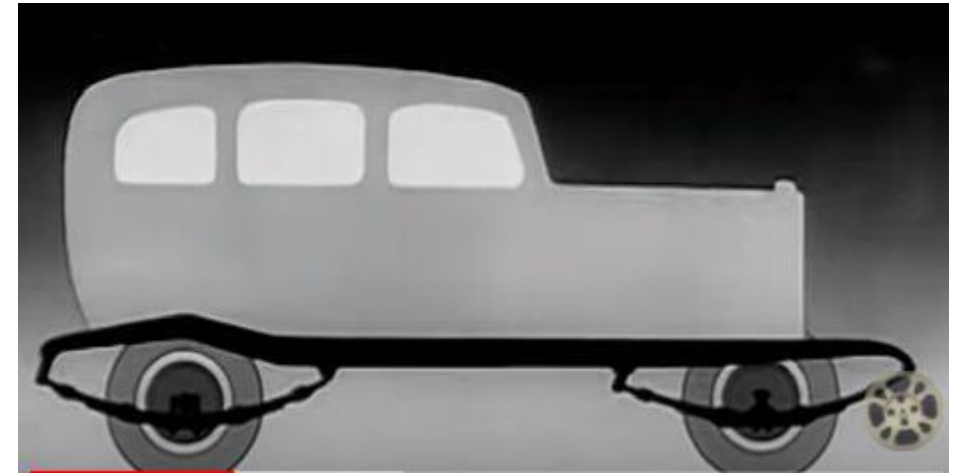
Torsion Spring



Spiral Spring



Disc or Belleville Spring



# Springs



**Helical Compression Spring**



**Garter Spring**



**Helical Extension Spring**



**Coil Spring**



**Torsion Spring**



**Spring Belt**



**Leaf Spring**



**Oil Seal Spring**

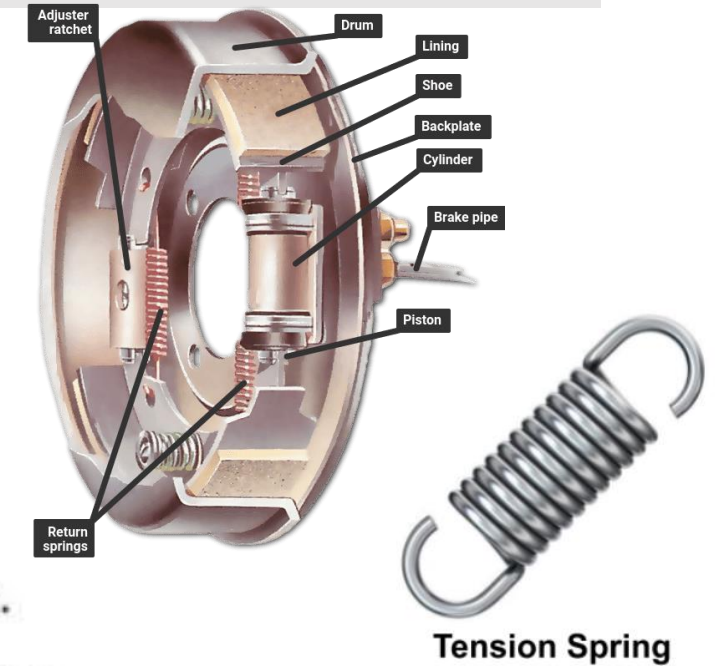
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# *Springs- Applications*

## APPLICATIONS OF SPRINGS

- 1) To apply forces and controlling motion, as in brakes and clutches.
- 2) Measuring forces, as in the case of a spring balance.
- 3) Storing energy, as in the case of springs used in watches and toys.
- 4) Reducing the effect of shocks and vibrations in vehicles and machine foundations.



LEAF SPRING IN SUSPENSION SYSTEM

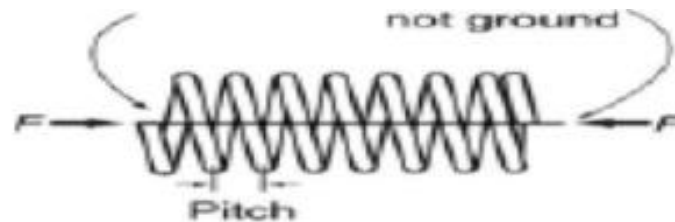


# Springs- Applications

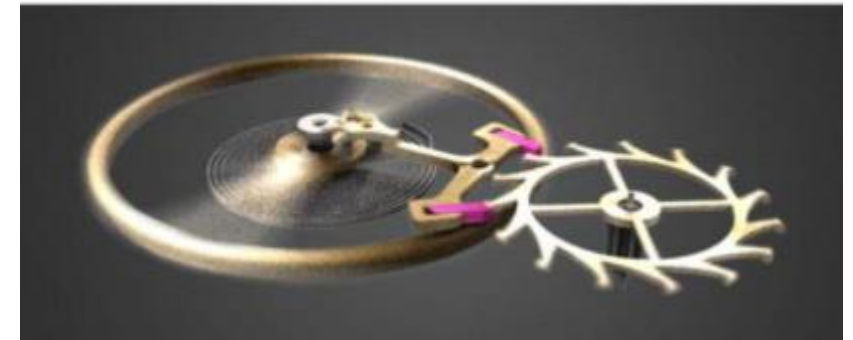
Extension Spring



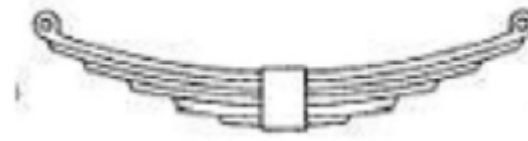
Compression Spring



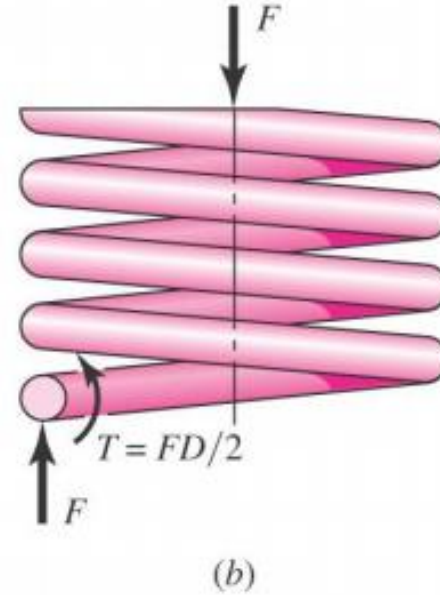
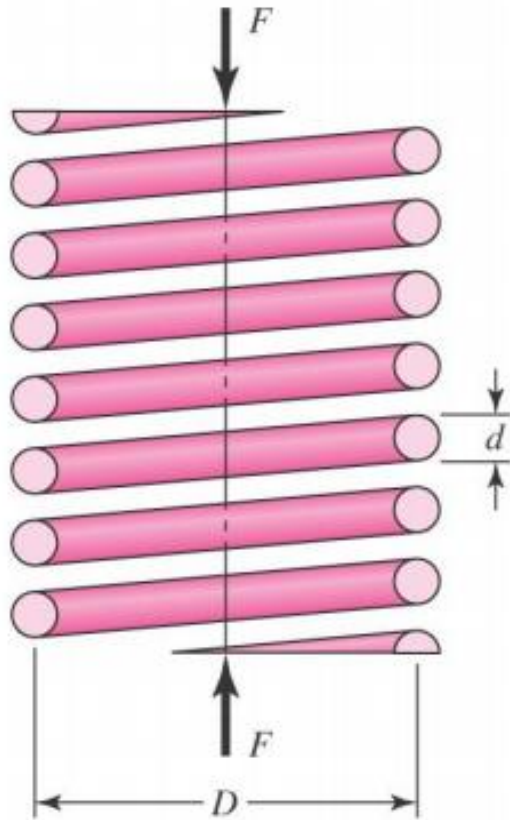
Spiral Spring



Leaf (or) carriage (or) Semi elliptical spring

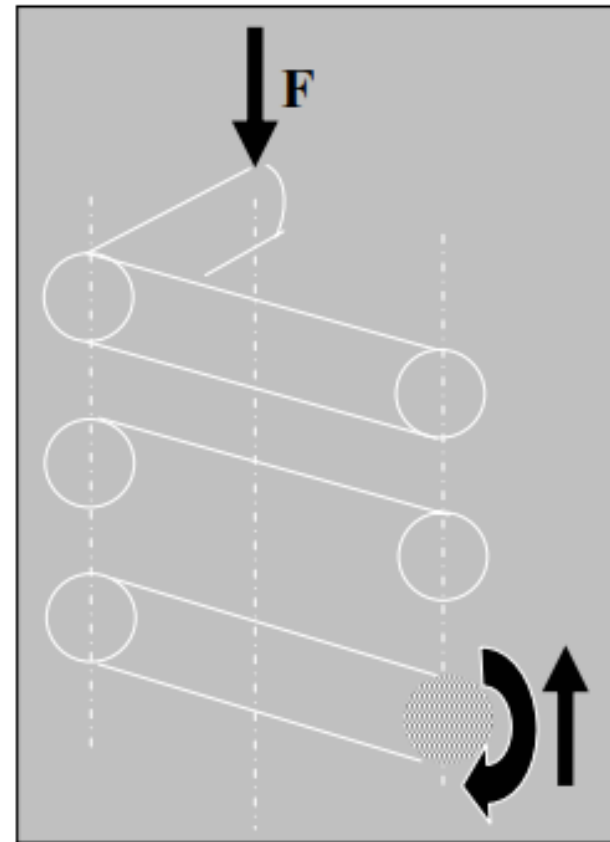
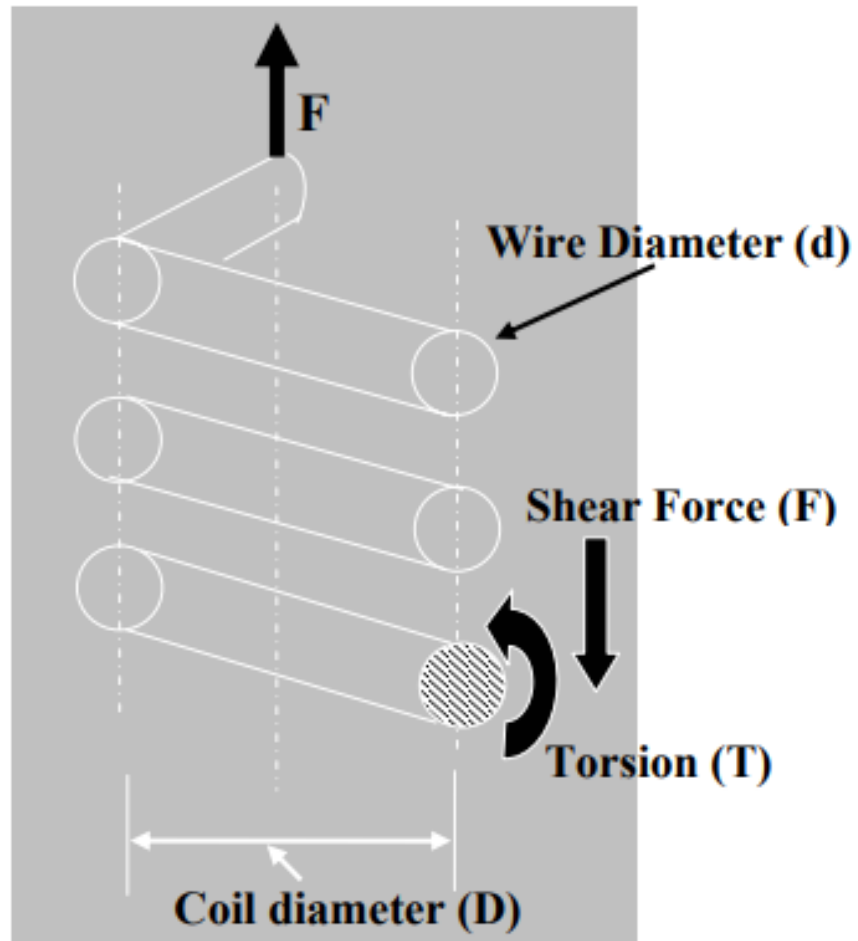


# Helical Springs- Axial Load

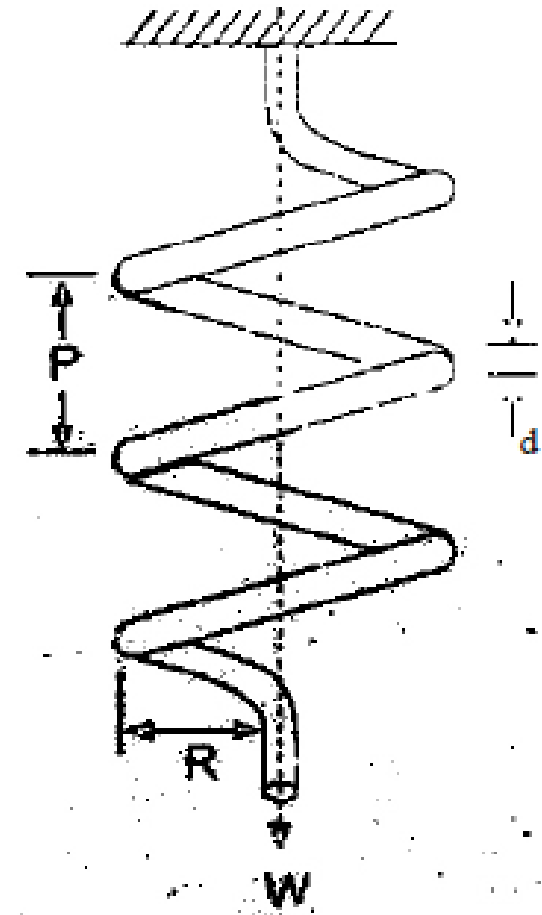
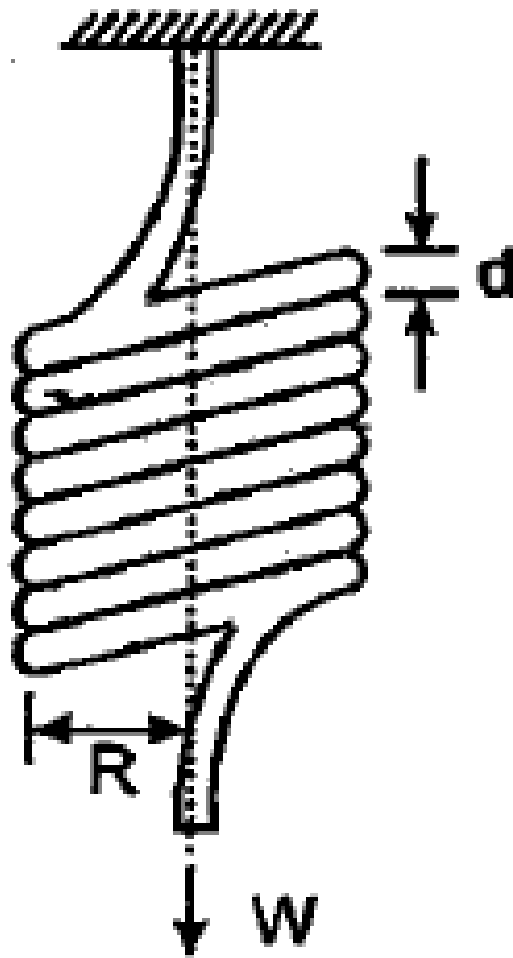




# Helical Springs- Axial Load



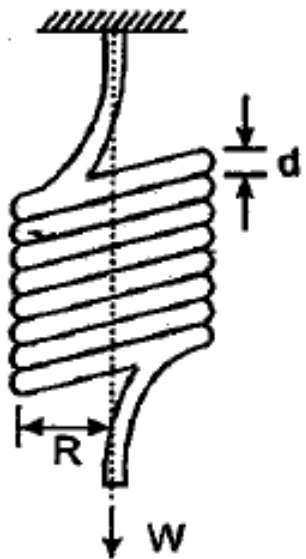
# Helical Springs



# Close Coiled Spring Under Axial Load

- Deflection in the spring due to load  $W$ ,

$$\delta = \frac{64WR^3n}{Cd^4}$$



Where,

$W$  = axial load

$n$  = no. of turns of spring

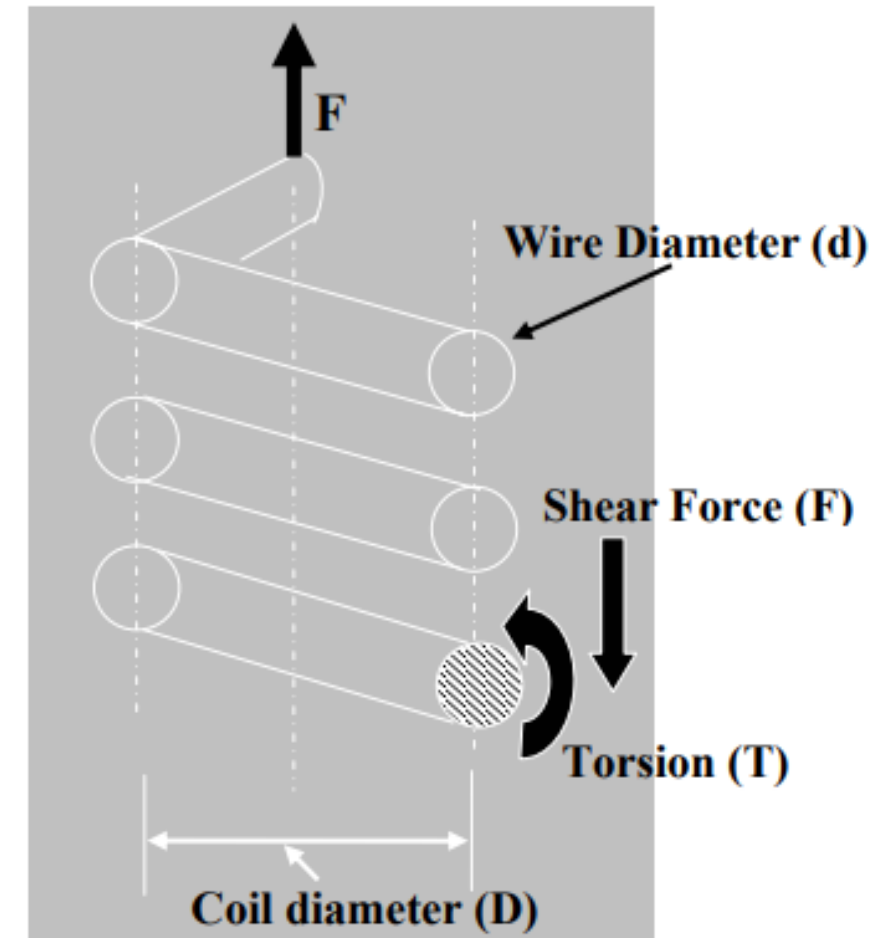
$d$  = diameter of the rod of the spring

$R$  = mean radius of the coil

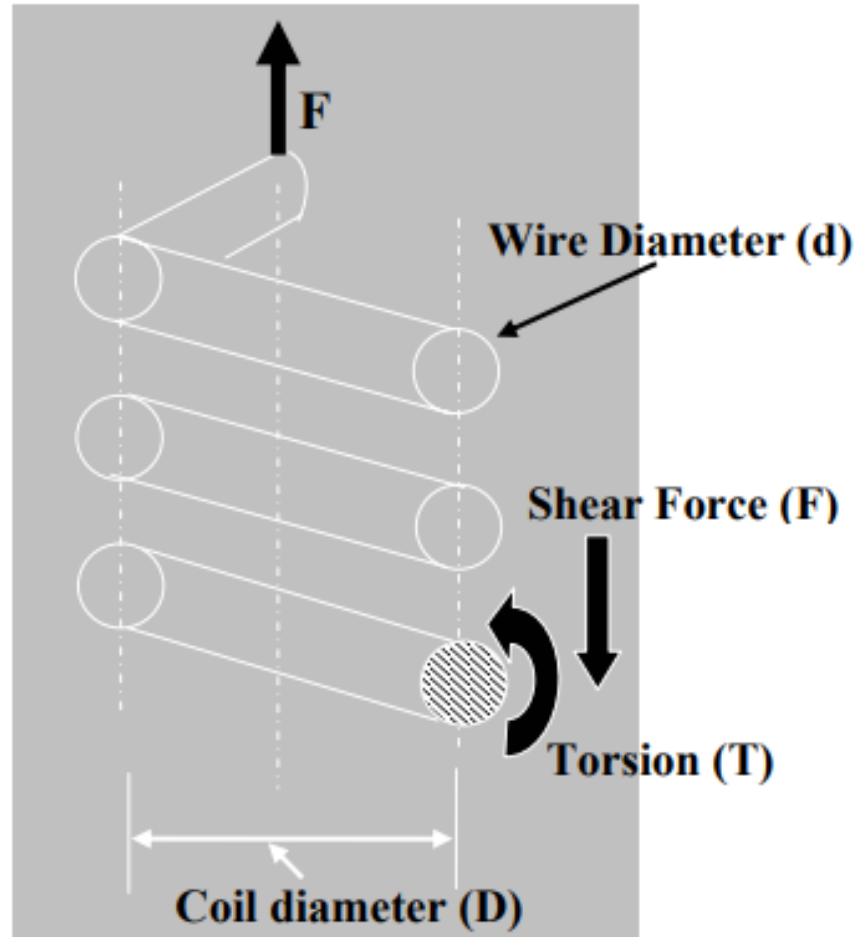
$G$  = modulus of rigidity for the spring material.

- Stiffness of the spring,

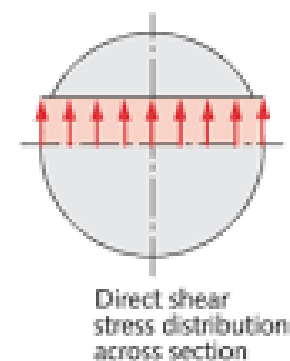
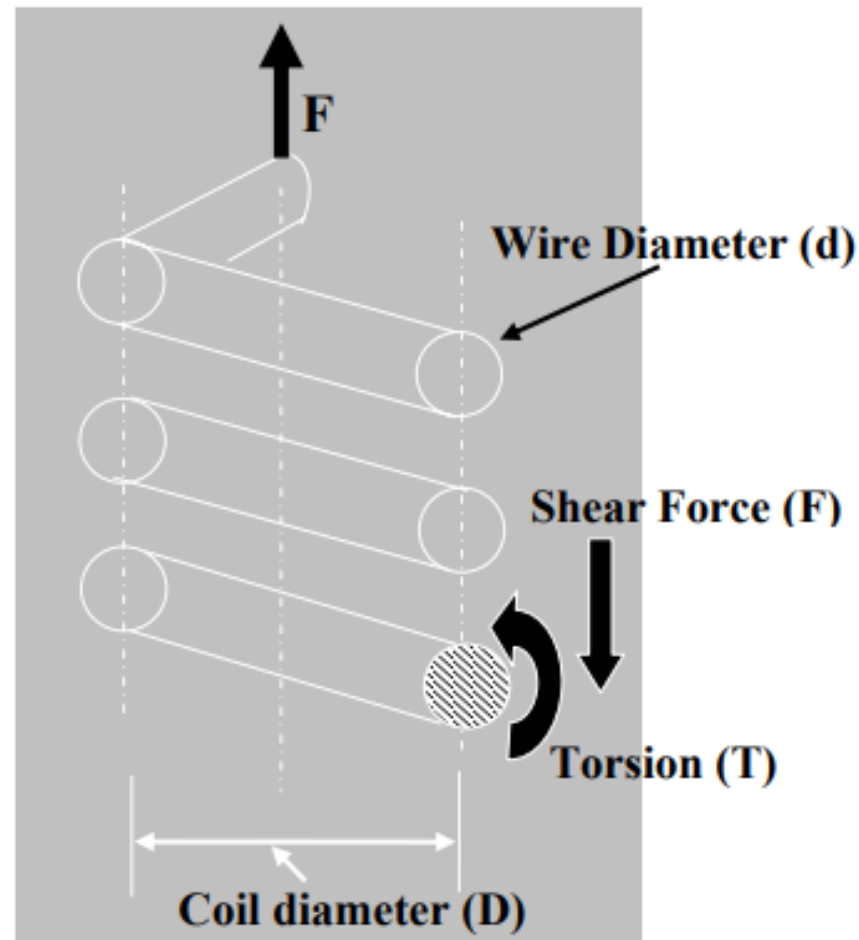
$$k = \frac{W}{\delta} = \frac{Gd^4}{64R^3n}$$



# Close Coiled Spring Under Axial Load



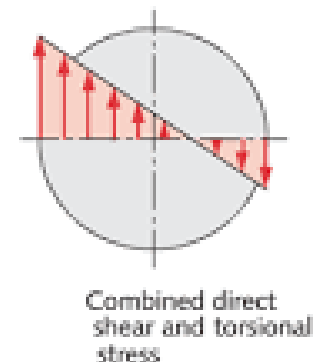
# Close Coiled Spring Under Axial Load



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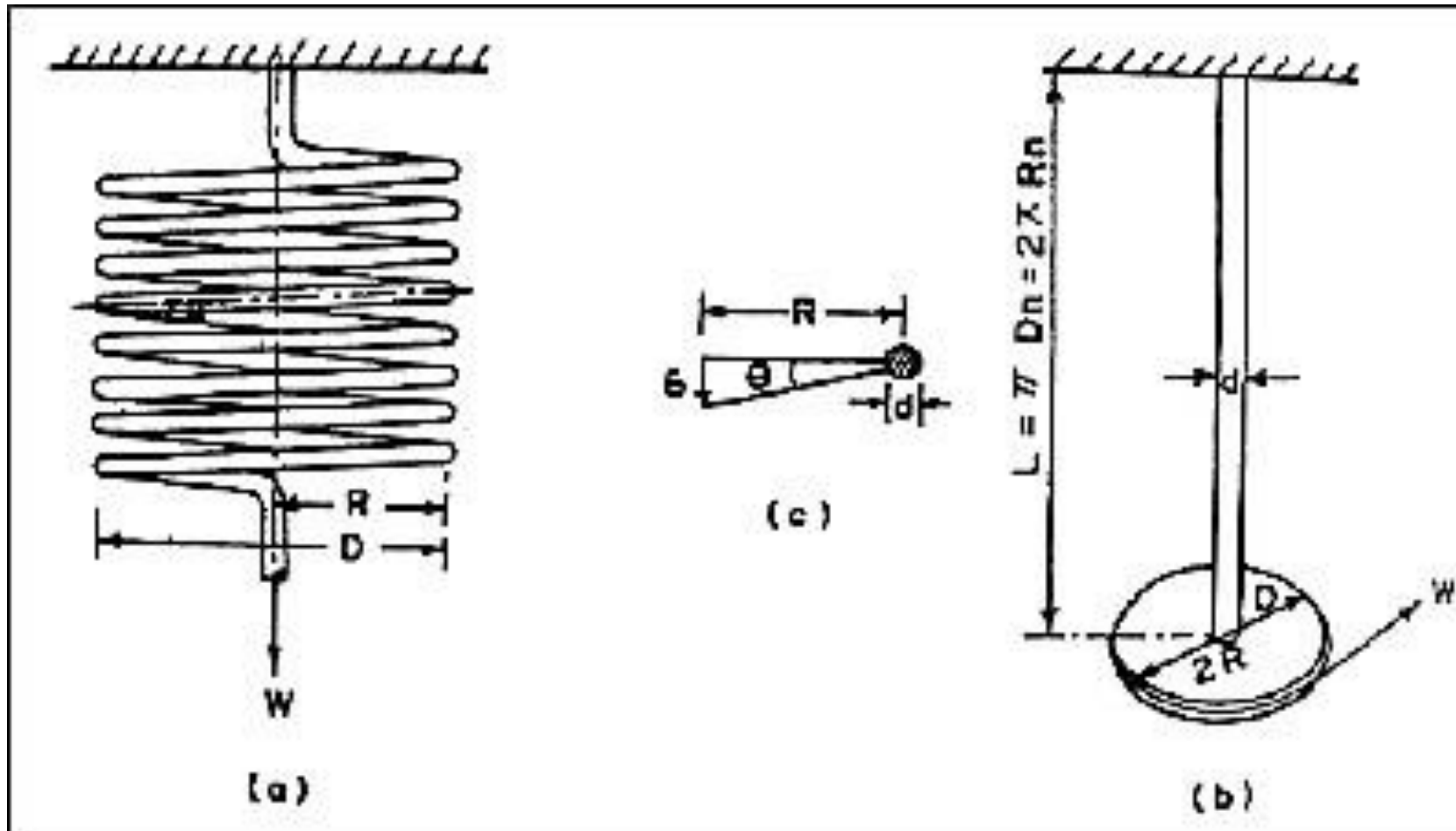


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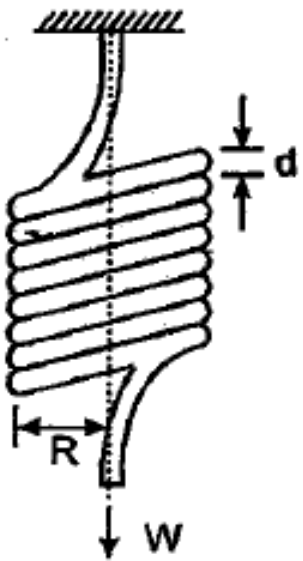
# Helical Springs



# Close Coiled Spring Under Axial Load

- Deflection in the spring due to load  $W$ ,

$$\delta = \frac{64WR^3n}{Cd^4}$$



- Stiffness of the spring,

$$k = \frac{W}{\delta} = \frac{Cd^4}{64R^3n}$$

- Stiffness of the spring,

$$k = \frac{W}{\delta} = \frac{Cd^4}{64R^3n}$$

Where,

$W$  = axial load

$n$  = no. of turns of spring

$d$  = diameter of the rod of the spring

$R$  = mean radius of the coil

$G$  = modulus of rigidity for the spring material.

- Energy stored,

$$U = \frac{1}{2}W\delta = \frac{32W^2R^3n}{Cd^4}$$

# *Close Coiled Spring-Problem-1*

A close coiled helical spring is made of 5 mm diameter wire. It is made up of 30 coils, each of mean diameter 75 mm. If the maximum stress in the spring is not to exceed 200 MPa, then determine

(a) the proof load

(b) the extension of the spring when carrying this load.

Take  $G = 80 \text{ GPa}$ .

## Close Coiled Spring-Problem-3

A closely coiled helical spring of round steel wire 5 mm in diameter having 12 complete coils of 50 mm mean diameter is subjected to an axial load of 100 N. Find the deflection of the spring and the maximum shear stress in the material.  $G=80$  GPa

$$\delta = \frac{64 WR^3 n}{Cd^4} = \frac{64 \times 100 \times (25)^3 \times 12}{(80 \times 10^3) \times (5)^4} = 24 \text{ mm}$$

$$\tau = \frac{2500}{24.54} = 101.9 \text{ N/mm}^2 = 101.9 \text{ MPa}$$

## Close Coiled Spring-Problem-3

A closely coiled helical spring of round steel rod required to carry a load of 1000N for a stress of 400 MPa, The spring stiffness being 20 N/mm the dia of the helix is 100mm and G for the material is 80 GPa. Calculate

- (1) Diameter of wire
- (2) Number of turns required for the spring

$$\delta = \frac{64 WR^3 n}{Cd^4} = \frac{64 \times 100 \times (25)^3 \times 12}{(80 \times 10^3) \times (5)^4} = 24 \text{ mm}$$

$$\tau = \frac{2500}{24.54} = 101.9 \text{ N/mm}^2 = 101.9 \text{ MPa}$$



## *Close Coiled Spring-Problem-2*

A helical spring in which the slope of the helix may be assumed small, is required to transmit a maximum pull of 1 kN and to extend 10 mm for 200 N load. If the mean diameter of the coil is to be the 80 mm, find the suitable diameter for the wire and number of coils required. Take  $G = 80$  GPa and allowable shear stress as 100 MPa.

## Close Coiled Spring-Problem-2

18) The stiffness of spring is 10N/mm. What is the axial deformation in the spring when a load of 50N is acting?

19) A helical spring is made of 4mm steel wire with a mean radius of 25mm and number of turns of coil 15. What will be deflection of the spring under a load of 6N. Take  $C = 80 \times 10^3 \text{ N/mm}^2$

16) A close-coiled helical spring is to have a stiffness of 900 N/m in compression, with a max. Load of 45N and a max. Shearing stress of  $120\text{N/mm}^2$ . The solid length of the spring (i.e coils touching) is 45 mm. Find

(i) the wire dia

(ii) the mean coil radius

(iii) the number of coils. Take  $c = 0.4 \times 10^5 \text{ N/mm}^2$

(Apr/May 2015) 16 Marks

$k = 900 \text{ N/m}$     $w = 45 \text{ N}$     $\tau = 120\text{N/mm}^2$

# Closely Coiled Helical Spring – Axial Twist

## 28.12. Closely-coiled Helical Springs Subjected to an Axial Twist

Consider a closely-coiled helical spring subjected to an axial twist as shown in Fig. 28.4.

Let

- $d$  = Diameter of the spring wire,
- $R$  = Mean radius of the spring coil,
- $n$  = No. of turns of coils,
- $C$  = Modulus of rigidity for the spring material and
- $M$  = Moment or axial twist applied on the spring.

A little consideration will show that the number of spring coils will tend to increase or decrease depending upon the sense of the moment. Moreover, if the number of turns tend to increase then the mean radius of the spring coil will decrease. Now let us consider that the number of turns increase from  $n$  to  $n'$  and the mean radius decreases from  $R$  to  $R'$ .

Now length of the spring,

$$l = 2\pi Rn = 2\pi R'n' \quad \dots(i)$$

$$\therefore \frac{1}{R} = \frac{2\pi n}{l} \quad \text{and} \quad \frac{1}{R'} = \frac{2\pi n'}{l}$$

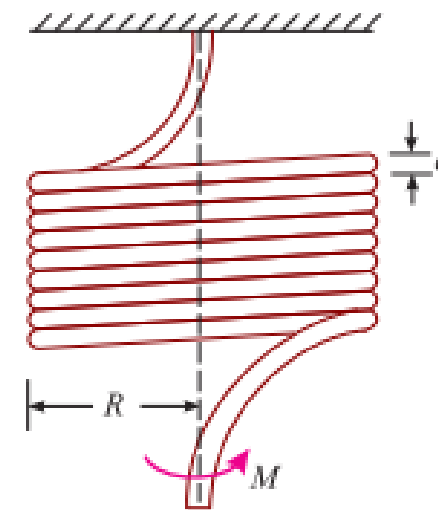


Fig. 28.4. Closely-coiled helical spring

# Closely Coiled Helical Spring – Axial Twist

We know that

$$\frac{M}{I} = E \times \text{Change of curvature}$$

$$= E \left( \frac{1}{R'} - \frac{1}{R} \right) = E \left( \frac{2\pi n'}{l} - \frac{2\pi n}{l} \right) = \frac{2\pi E}{l} (n' - n)$$

$$2\pi (n' - n) = \frac{Ml}{EI} \quad \dots(ii)$$

We also know that the total angle of bend,

$$\phi = 2\pi (n' - n)$$

Substituting the value of  $2\pi (n' - n)$  from equation (ii),

$$\phi = \frac{Ml}{EI}$$

Differentiating the above equation with respect to  $l$ ,

$$\frac{d\phi}{dl} = \frac{M}{EI}$$

It is thus obvious that the change in curvature or angle of bend per unit length, is constant throughout the spring.

We know that the energy stored in the spring,

$$U = \frac{1}{2} M \cdot \phi$$

# *Closely Coiled Helical Spring – Axial Twist*

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# Closely Coiled Helical Spring – Axial Twist

Principal stress  $\sigma_1 = \sigma_b$

Maximum shear stress

$$\tau_{\max} = \sigma_b/2$$

Angle of twist  $\Theta = 0$

Axial deflection  $\delta = 0$

Resilience

$$U = M^2 L / 2EI$$

But  $M/I = \sigma/y$

$$M = \sigma I / y = \sigma I / (d/2) = 2\sigma I / d$$

$$M^2 = (2\sigma I / d)^2 = 2\sigma^2 I^2 / d^2$$

$$\text{Therefore } U = (2\sigma^2 I^2 / d^2) L / 2EI$$

$$= 2\sigma^2 (\pi/64) d^4 / E d^2$$

$$= U = (\sigma^2 / 8E) (\pi/4) d^2 L$$

$$= (\sigma^2 / 8E) (\text{Volume of spring wire})$$

$$\text{Resilience } = u = U/V = \sigma^2 / 8E$$

Stiffness

$$\text{Stiffness of spring } = k = M / \Phi = Ed^4 / 64Dn$$

FINAL RESULTS

$$\text{Stress } \sigma = 32M / \pi d^3$$

$$\text{Rotation of free end } = \Phi = 64WDn / Ed^4$$

$$\text{Resilience } = u = \sigma^2 / 8E$$

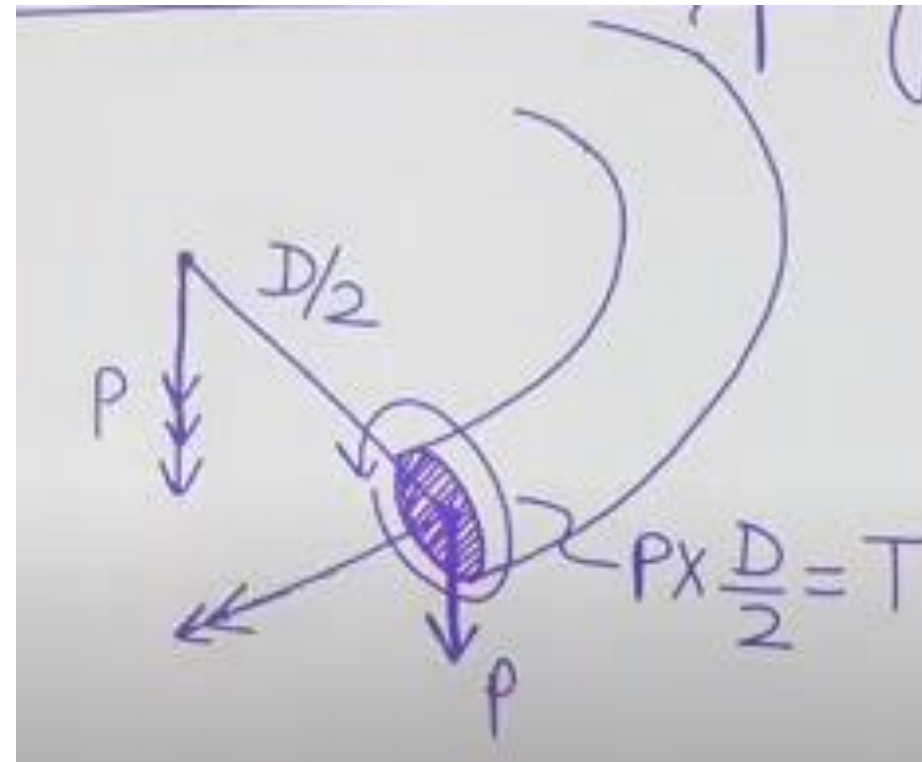
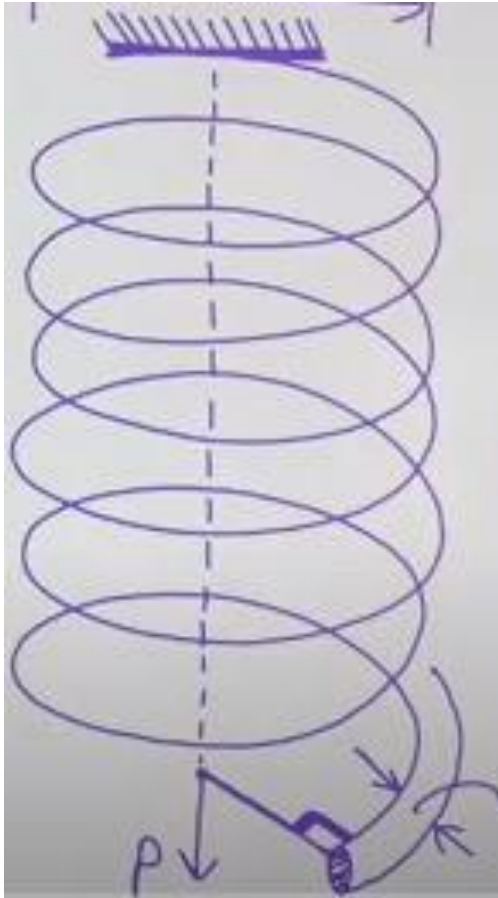
$$\text{Axial deflection } = \delta = 0$$

# *Closely Coiled Helical Spring-Problem*

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A closely coiled helical spring is made up of 10 mm diameter steel wire having 10 coils with 80 mm mean diameter. If the spring is subjected to an axial twist of 10 kN.mm. Determine the bending stress and increase in the number of turns. Take  $E=200$  GPa

# Closely Coiled Helical Spring-Problem



# Closely Coiled Helical Spring-Problem

$$\frac{\tau_{\text{Twisting}}}{\tau_{\text{direct}}} = 2C = 2 \frac{D}{d}$$

usually,  $D \gg d \Rightarrow C \gg 1$ .

$$\frac{\tau_{\text{Twisting}}}{\tau_{\text{direct}}} \gg 1 \left\{ \Rightarrow \tau_{\text{Twisting}} \gg \tau_{\text{direct}} \right.$$

# Closely Coiled Helical Spring-Problem

Wahl  $\rightarrow k_w = \text{Wahl factor}$

$$k_w = \left[ \frac{4C-1}{4C-4} + \frac{0.615}{C} \right]$$
$$\tau_{\max} = k_w \frac{16T}{\pi d^3}$$

$\Rightarrow$  It takes into account the effect of direct shear and curvature



# Open Coiled Helical Spring

Now consider an open coiled helical spring subjected to an axial load as shown in Fig. 28.5.

Let

$d$  = Diameter of the spring wire,

$R$  = Mean radius of the spring coil,

$P$  = Pitch of the spring coils,

$n$  = No. of turns of coils,

$C$  = Modulus of rigidity for the spring materials,

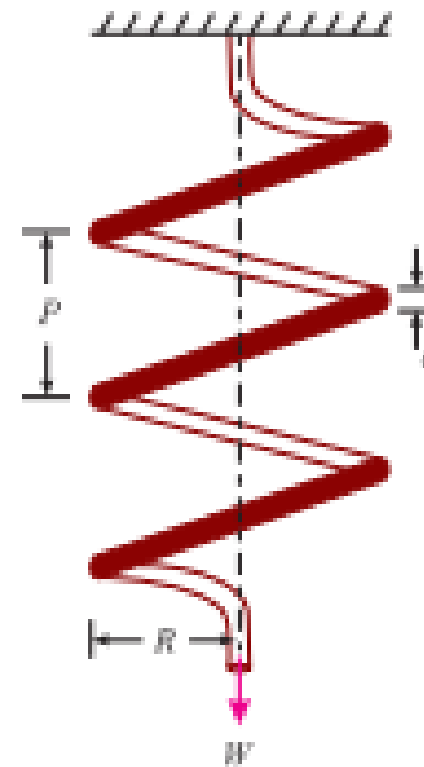
$W$  = Axial load on the spring,

$\tau$  = Maximum shear stress induced in the spring wire due to loading,

$\sigma_b$  = Bending stress induced in the spring wire due to bending,

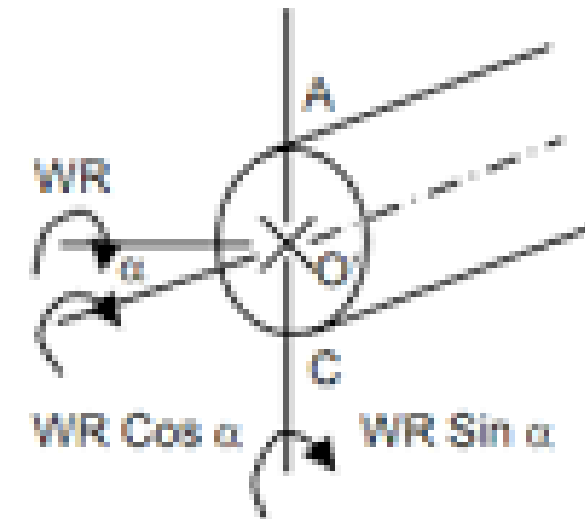
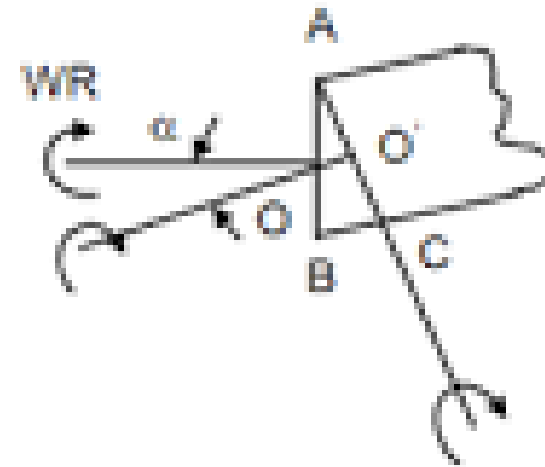
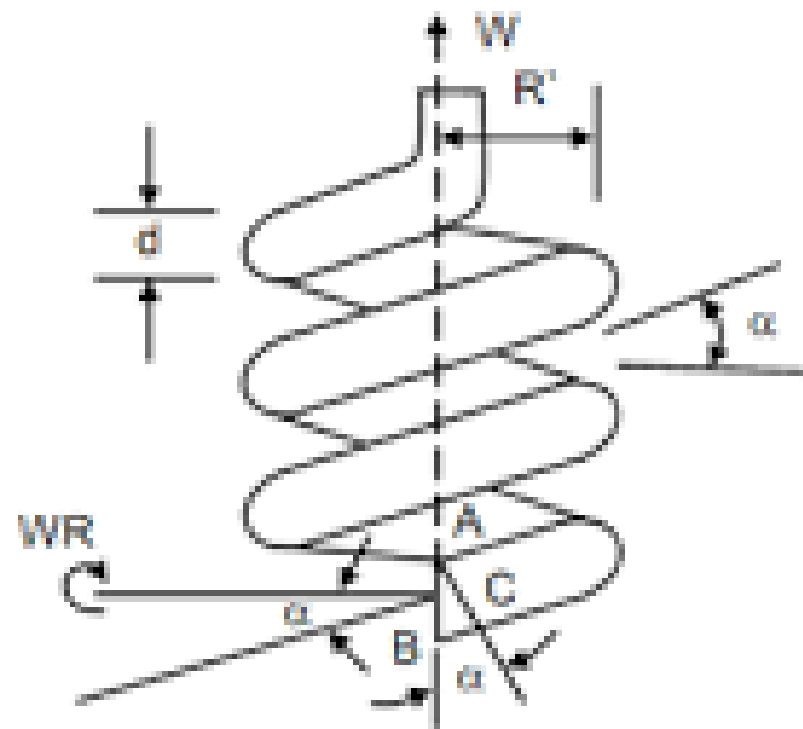
$\delta$  = Deflection of the spring as a result of axial load and

$\alpha$  = Angle of helix.

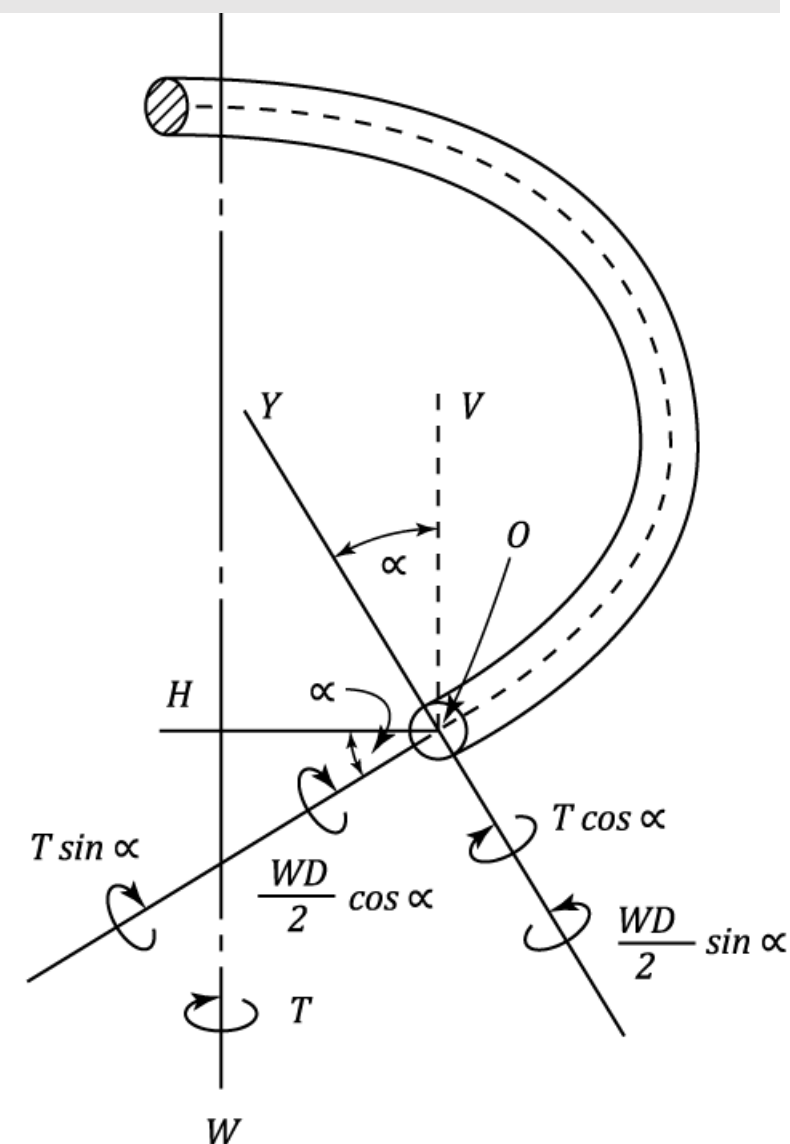
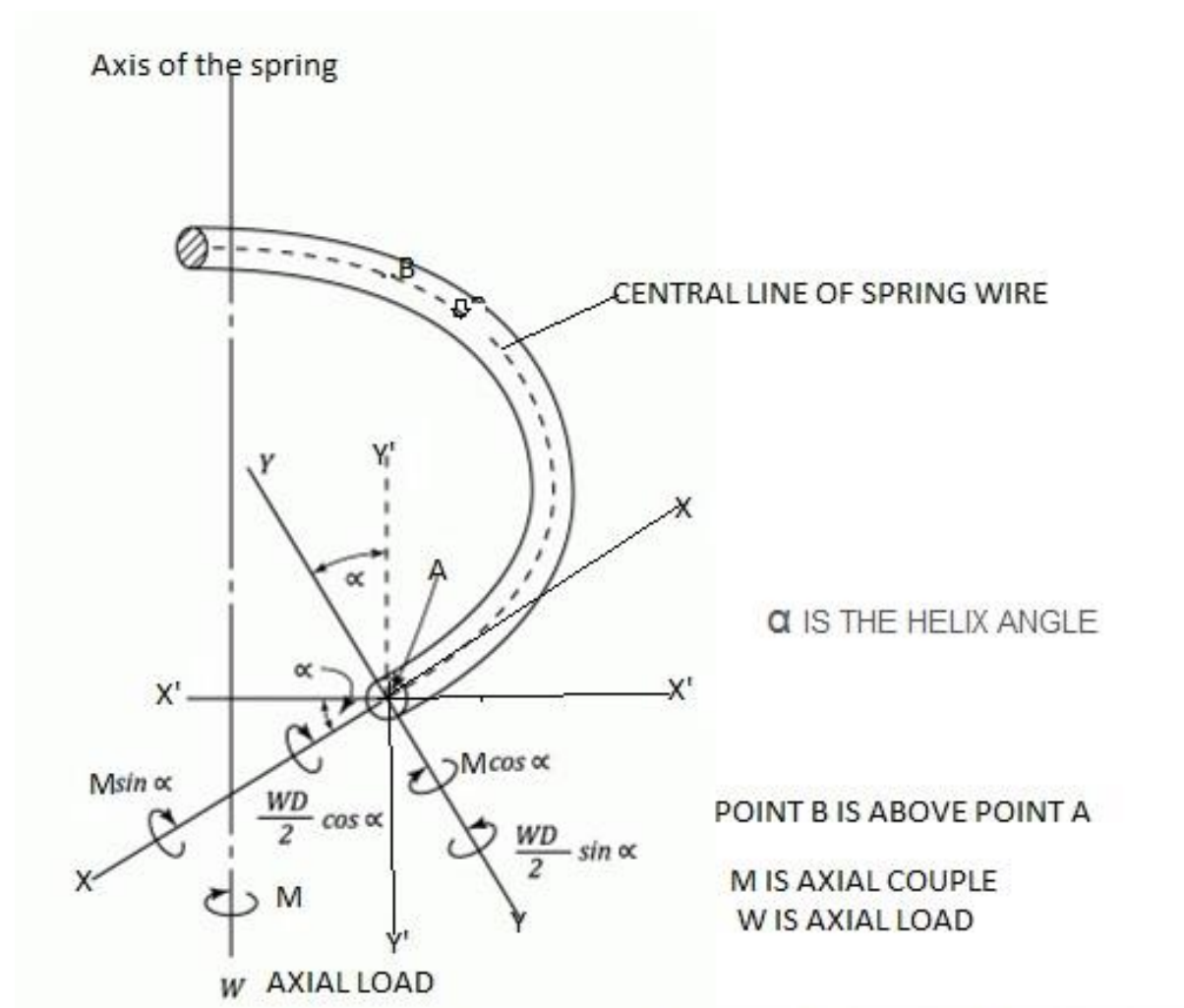


Helical Compression Spring

# Open Coiled Helical Spring



# Open Coiled Helical Spring



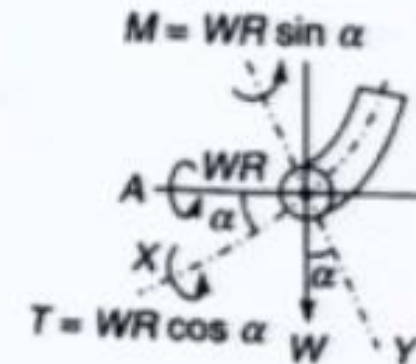
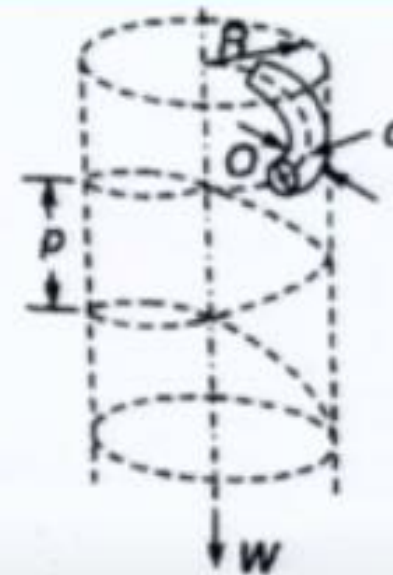
# Open Coiled Helical Spring

We know  $\theta = \frac{TL}{Gl_p} = \frac{WR \cos \alpha \cdot L}{Gl_p}$

We know  $\frac{M}{I} = \frac{E}{R}$  and  $\frac{L}{R} = \phi \quad \therefore \frac{1}{R} = \frac{\phi}{L}$

$\therefore \frac{M}{I} = \frac{E\phi}{L} \quad \therefore \phi = \frac{ML}{EI}$

Where  $\phi$  is the angle made by the bent wire at the centre of curvature due to bending moment  $M$



# Open Coiled Helical Spring

$$\frac{1}{2} W \cdot \delta = \frac{1}{2} T \cdot \theta + \frac{1}{2} M \cdot \phi \quad W \cdot \delta = T \cdot \theta + M \cdot \phi$$

$$W \cdot \delta = WR \cos \alpha \times \frac{WRL \cos \alpha}{GI_p} + WR \sin \alpha \times \frac{WRL \sin \alpha}{EI}$$

$$\delta = WR^2 L \left[ \frac{\cos^2 \alpha}{G I_p} + \frac{\sin^2 \alpha}{EI} \right] \quad \delta = WR^2 \left[ \frac{\cos^2 \alpha}{G \times \frac{\pi}{32} d^4} + \frac{\sin^2 \alpha}{E \times \frac{\pi}{64} d^4} \right] \times 2\pi n R \sec \alpha$$

$$\delta = \frac{64 WR^3 n \sec \alpha}{d^4} \left[ \frac{\cos^2 \alpha}{G} + \frac{2 \sin^2 \alpha}{E} \right]$$

# Open Coiled Helical Spring- Problem-1

A open coiled helical spring is made up of 10 mm diameter steel wire having 12 coils with 100 mm mean diameter. Angle of helix being  $15^\circ$ . Determine the axial deflection and the intensities of bending and shear stresses under axial load of 500 N. Take  $G=80$  GPa and  $E=200$  GPa

$$\begin{aligned}\delta &= WR^2 \times 2\pi nR \sec \alpha \left[ \frac{\cos^2 \alpha}{\frac{\pi}{32} d^4 C} + \frac{\sin^2 \alpha}{E \times \frac{\pi}{64} d^4} \right] \\ &= \frac{64 WR^3 n \sec \alpha}{d^4} \left[ \frac{\cos^2 \alpha}{C} + \frac{2 \sin^2 \alpha}{E} \right]\end{aligned}$$

**Note.** If we substitute  $\alpha = 0$  in the above equation, it gives deflection of a closed coiled spring i.e.,

$$\delta = \frac{64 WR^2 n}{Cd^4}$$

# Closely Coiled Helical Spring-Problem

$$\begin{aligned}\delta &= WR^2 \times 2\pi nR \sec \alpha \left[ \frac{\cos^2 \alpha}{\frac{\pi}{32} d^4 C} + \frac{\sin^2 \alpha}{E \times \frac{\pi}{64} d^4} \right] \\ &= \frac{64 WR^3 n \sec \alpha}{d^4} \left[ \frac{\cos^2 \alpha}{C} + \frac{2 \sin^2 \alpha}{E} \right]\end{aligned}$$

**Note.** If we substitute  $\alpha = 0$  in the above equation, it gives deflection of a closed coiled spring i.e.,

$$\delta = \frac{64 WR^3 n}{Cd^4}$$

# Open Coiled Helical Spring

A closely coiled helical spring is made up of 10 mm diameter steel wire having 12 coils with 100 mm mean diameter. Angle of helix being  $15^\circ$ . Determine the axial deflection and the intensities of bending and shear stresses under axial load of 500 N. Take  $G=80$  GPa and  $E=200$  GPa

$$\begin{aligned}\delta &= \frac{64 WR^3 n \sec \alpha}{d^4} \left[ \frac{\cos^2 \alpha}{C} + \frac{2 \sin^2 \alpha}{E} \right] \\ &= \frac{64 \times 500 \times (50)^3 \times 12 \sec 15^\circ}{(10)^4} \left[ \frac{\cos^2 15^\circ}{80 \times 10^3} + \frac{2 \sin^2 15^\circ}{200 \times 10^3} \right] \text{ mm} \\ &= 4\,800\,000 \times 1.0353 \left[ \frac{(0.9659)^2}{80 \times 10^3} + \frac{2 \times (0.2588)^2}{200 \times 10^3} \right] \text{ mm} \\ &= 4969\,440 \times \frac{2.467}{200 \times 10^3} = 61.3 \text{ mm} \quad \text{Ans.}\end{aligned}$$

$$I = \frac{\pi d^4}{64} = \frac{\pi (10)^4}{64} = 490.9 \text{ mm}^4$$

$\therefore$  Bending stress in the section

$$\sigma_b = \frac{M}{I} \times y = \frac{6470}{490.9} \times 5 = 65.9 \text{ N/mm}^2 = 65.9 \text{ MPa}$$

**Shear stress induced in the wire**

Let  $\tau$  = Shear stress induced in the wire in  $\text{N/mm}^2$ .

We know that twisting moment (or torque) in the coil,

$$\begin{aligned}T &= WR \cos \alpha = 500 \times 50 \cos 15^\circ \text{ N-mm} \\ &= 25\,000 \times 0.9659 = 24\,150 \text{ N-mm}\end{aligned}$$

We also know that twisting moment ( $T$ )

$$24\,150 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times \tau \times (10)^3 = 196.4 \tau$$

$$\therefore \tau = \frac{24150}{196.4} = 123 \text{ N/mm}^2 = 123 \text{ MPa} \quad \text{Ans.}$$



## *Open Coiled Helical Spring*

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A open coil helical spring made of 10 mm diameter wire has 15 coils of 50 mm radius with a  $20^\circ$  angle of helix. Determine the deflection of the spring, when subjected to an axial load of 300 N. Take  $E = 200$  GPa and  $G = 80$  GPa.

# *Springs are in Parallel*

A closely coiled helical spring is made up of 10 mm diameter steel wire having 10 coils with 80 mm mean diameter. If the spring is subjected to an axial twist of 10 kN.mm. Determine the bending stress and increase in the number of turns. Take  $E=200$  GPa

## *Bending stress in the wire*

We know that moment of inertia of the spring wire section,

$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64} \times (10)^4 = 490.9 \text{ mm}^4$$

$\therefore$  Bending stress in the wire

$$\sigma = \frac{M}{I} \times y = \frac{10 \times 10^3}{490.9} \times 5 = 101.9 \text{ N/mm}^2 = 101.9 \text{ MPa}$$

## *Increase in the number of turns*

We know that length of the coil,

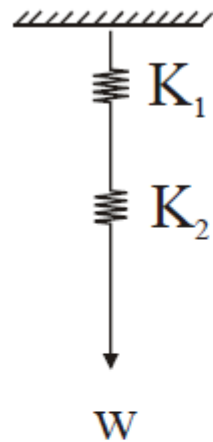
$$l = 2\pi Rn = 2\pi \times 40 \times 10 = 800\pi \text{ mm}$$

and increase in the no. of turns,

$$n' - n = \frac{Ml}{EI} \times \frac{1}{2\pi} = \frac{(10 \times 10^3) \times 800\pi}{(200 \times 10^3) \times 490.9} \times \frac{1}{2\pi} = 0.04 \quad \text{Ans.}$$

# *Springs are in Series*

## (i) Springs in series



- Total extension =  $\Delta$  (individual extensions)

$$\Delta = \Delta_1 + \Delta_2$$

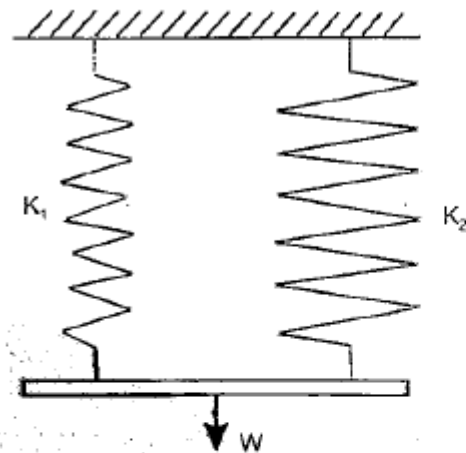
- load applied ( $W$ ) will be same on both the springs.
- The equivalent stiffness is given by

$$\frac{1}{k_{eq}} = \frac{1}{K_1} + \frac{1}{K_2}$$

where  $K_1$  &  $K_2$  are individual stiffness of the springs.

# *Springs are in Parallel*

(ii) Springs in parallel



- Both the springs have same extension.

$$d = d_1 = d_2$$

- Load applied ( $W$ ) is shared by the springs.

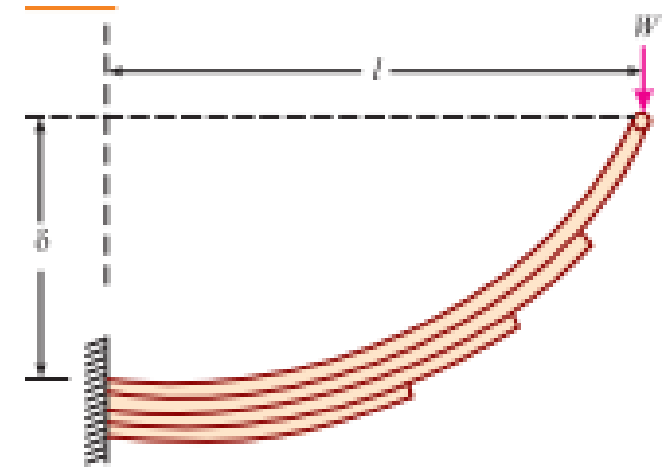
$$W = W_1 + W_2$$

- The equivalent stiffness is given by,

$$\text{eq. } K = K_1 + K_2$$

# Leaf Springs/ Carriage under Load

- These are also called as bending springs and laminated springs
- These are two Types
  - Semi elliptical springs (Simply supported at its ends subjected to central load)
  - Quarter elliptical springs (Cantilever type)



## *Leaf Springs under Load*

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- Carriage springs – Railway wagons, coaches and vehicles – used absorb the shocks – to avoid unpleasant feeling to the passengers.
- The energy absorbed by a laminated spring , during a shock- released without doing any useful work.
- Number of parallel strips of metal having different lengths but same width and placed one over the other in laminations

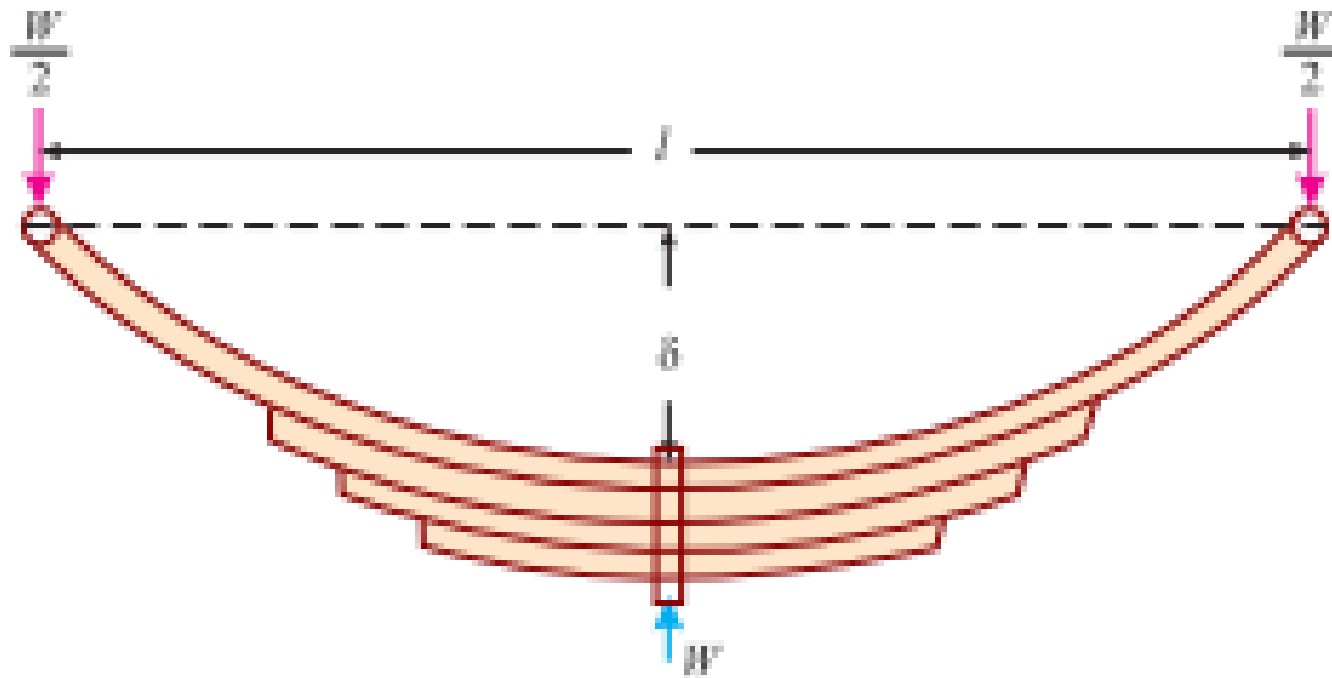
## *Leaf Springs under Load*

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- All the plates are initially bent to the same radius and are free to slide one over the other.
- When the spring is loaded to the designed load, all the plates become flat and the central deflection disappears.
- The purpose of this type of arrangement of plates is to make the spring of uniform strength throughout.
- This is achieved by tapering the ends of the laminations. The semi-elliptical type spring rests on the axis of the vehicle and its top plate is pinned at the ends to the chassis of the vehicle.

# Leaf Springs under Load

Now consider a carriage spring pinned at its both ends, and carrying an upward load at its centre

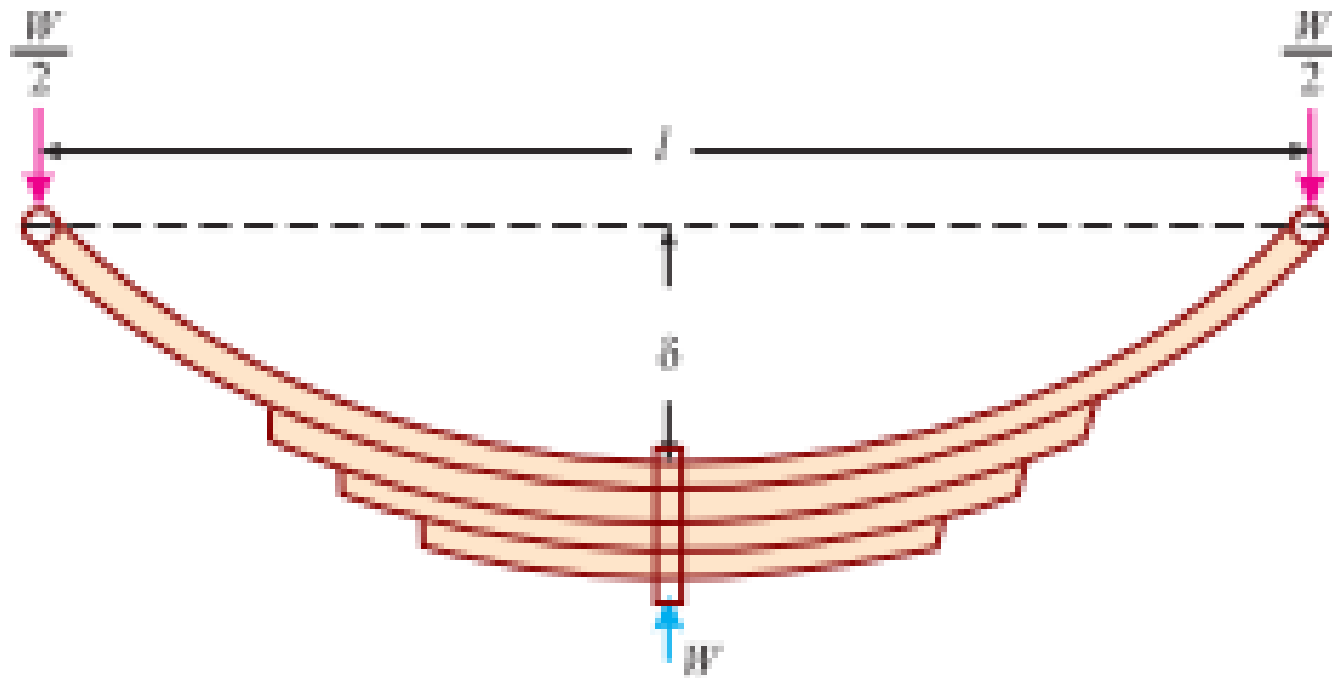


- $l$  = Span of the spring,
- $t$  = Thickness of plates,
- $b$  = Width of the plates,
- $n$  = Number of plates,
- $W$  = Load acting on the spring,
- $\sigma$  = Maximum bending stress developed in the plates,
- $\delta$  = Original deflection of the top spring and
- $R$  = Radius of the spring.



# Leaf Springs under Load

Now consider a carriage spring pinned at its both ends, and carrying an upward load at its centre



and moment resisted by one plate

$$\begin{aligned}
 M &= \frac{Wl}{4} \\
 &= \frac{\sigma \cdot I}{y} \\
 &= \frac{\sigma \times \frac{bt^3}{12}}{\frac{t}{2}} = \frac{\sigma \cdot bt^2}{6}
 \end{aligned}$$

∴ Total moment resisted by  $n$  plates,

$$M = \frac{n \sigma bt^2}{6}$$

# Leaf Springs under Load

$$\delta = \frac{l^2}{8R}$$

We also know that in the case of a bending beam,

$$\frac{\sigma}{y} = \frac{E}{R} \quad \text{or} \quad R = \frac{E \cdot y}{\sigma} = \frac{Et}{2\sigma}$$

Substituting this value of  $R$  in equation (iii)

$$\delta = \frac{l^2}{8 \times \frac{Et}{2\sigma}} = \frac{\sigma l^2}{4Et}$$

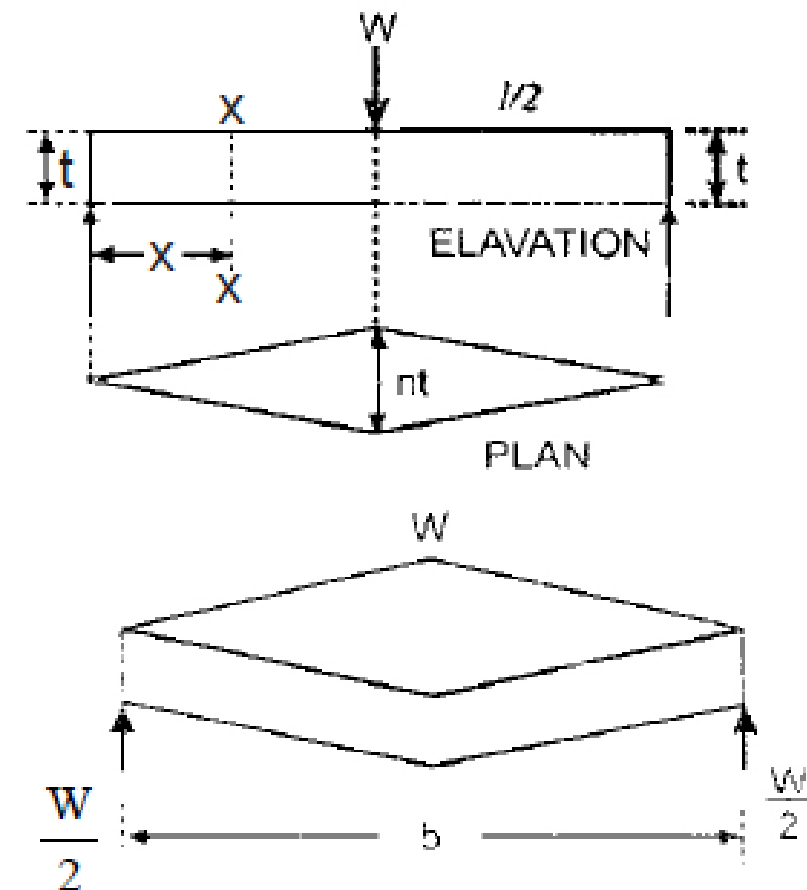
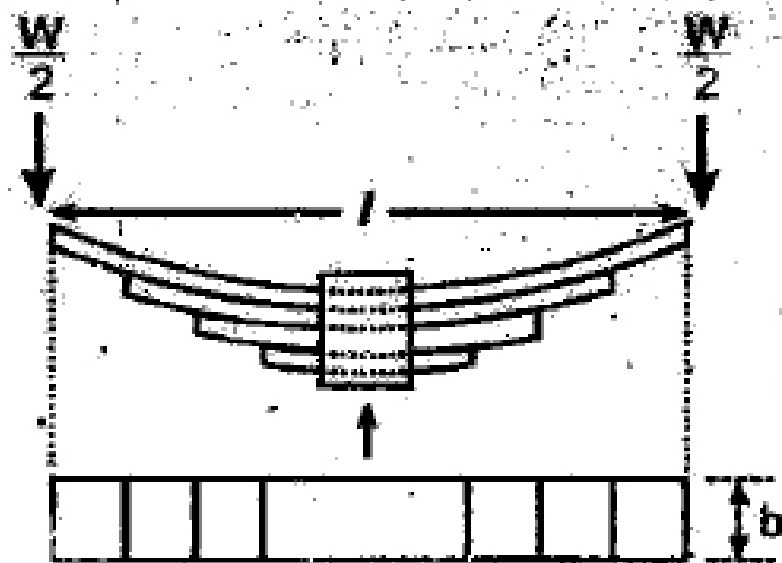
Now substituting the value of  $\sigma$  in the above equation,

$$\delta = \frac{3Wl}{2nbt^2} \times \frac{l^2}{4Et} = \frac{3Wl^3}{8Enbt^3}$$

# Leaf Springs under Load

Central deflection is given by,

$$d = \frac{3 W l^3}{8 E n b t^3}$$



## *Leaf Springs Problem-1*

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A leaf spring is to be made of seven steel plates **65 mm wide** and **6.5 mm thick**. Calculate the length of the spring, so that it may carry a central load of **2.75 kN**, the bending stress being limited to **160 MPa**. Also calculate the deflection at the centre of the spring. **Take E** for the spring material **as 200 GPa**.

# Leaf Springs Problem-1

A leaf spring is to be made of seven steel plates **65 mm wide** and **6.5 mm thick**. Calculate the length of the spring, so that it may carry a central load of **2.75 kN**, the bending stress being limited to **160 MPa**. Also calculate the deflection at the centre of the spring. **Take E** for the spring material **as 200 GPa**.

We know that bending stress ( $\sigma_b$ ),

$$160 = \frac{3WL}{2nbt^2} = \frac{3 \times (2.75 \times 10^3) \times l}{2 \times 7 \times 65 \times (6.5)^2} = 0.215 l$$

$$\therefore l = \frac{160}{0.215} = 744.2 \text{ mm} \quad \text{Ans.}$$

*Deflection at the centre of the spring*

We also know that deflection at the centre of the spring,

$$\delta = \frac{\sigma l^2}{4Et} = \frac{160 \times (744.2)^2}{4 \times (200 \times 10^3) \times 6.5} = 17.0 \text{ mm}$$

## *Leaf Springs Problem-5 (Assignment)*

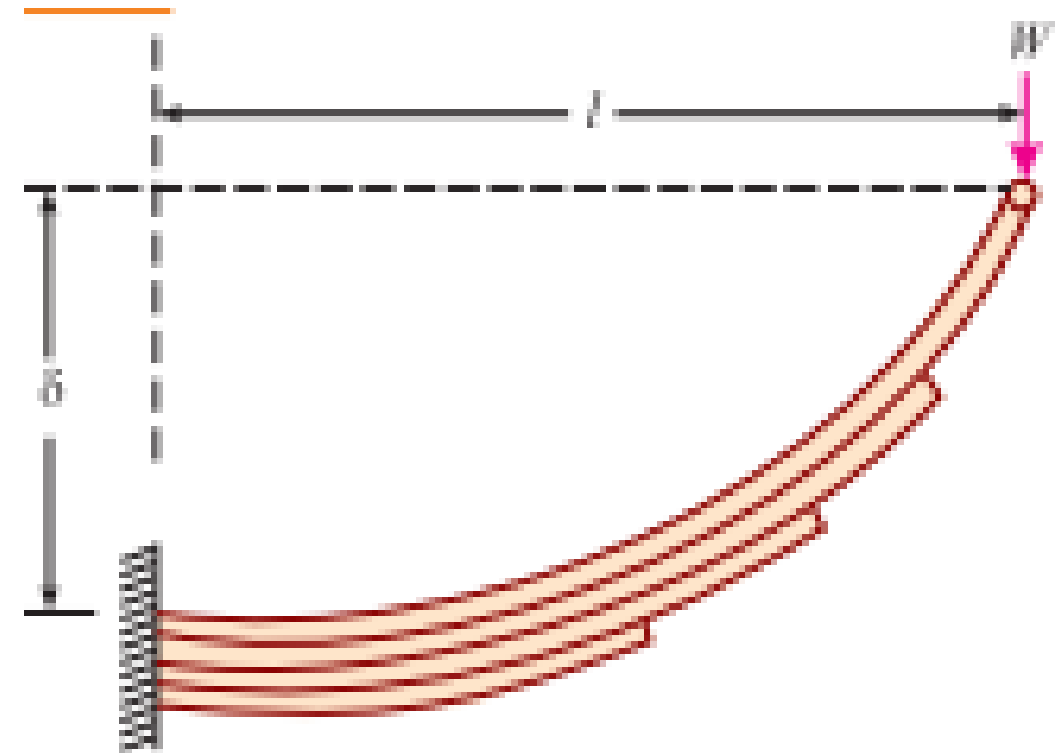
A leaf spring 750 mm long is required to carry a central point load of 8 kN. If the central deflection is not to exceed 20 mm and the bending stress is not greater than 200 MPa, determine the thickness, width and number of plates.

Also compute the radius, to which the plates should be curved. Assume width of the plate equal to **12 times its thickness** and  **$E=200 \text{ GPa}$**

# Leaf Springs/ Carriage

## Quarter elliptical springs (Cantilever type)

- The quarter-elliptical type leaf springs are rarely used, except as certain parts in some machines.
- Like a carriage spring, a quarter elliptical type leaf spring consists of a **number of parallel strips** of a metal **having different lengths but same width and placed one over the other** in laminations
- The plates are initially bent to the same radius and are free to slide one over the other.



# Leaf Springs/ Carriage

$$WT = \frac{n\sigma bt^2}{6} \quad \text{or} \quad \sigma = \frac{6WT}{nbt^2}$$

ry of the spring figure, we know that

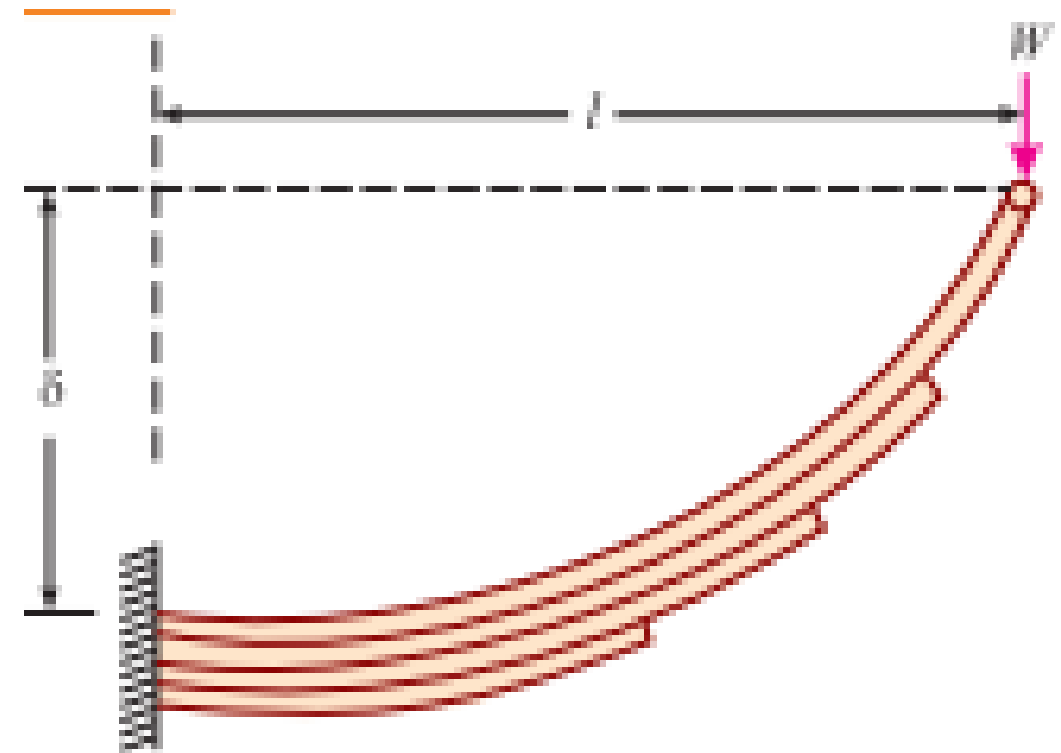
$$\delta (2R - \delta) = l \cdot l = l^2$$

$$\delta = \frac{l^2}{2R}$$

...(Neglecting  $\delta^2$ )

the case of a bending cantilever,

$$\frac{\sigma}{y} = \frac{E}{R}$$





# Leaf Springs/ Carriage

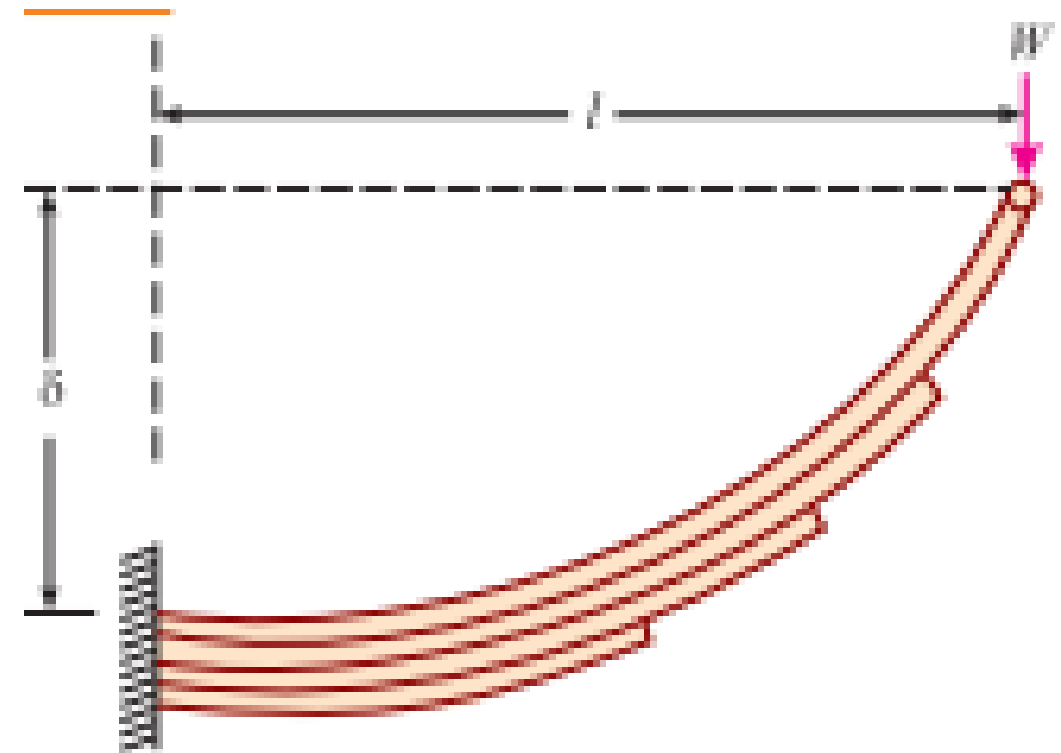
$$R = \frac{E \cdot y}{\sigma} = \frac{Et}{2\sigma}$$

value of  $R$  in equation (iii),

$$\delta = \frac{l^2}{2 \times \frac{Et}{2\sigma}} = \frac{\sigma l^2}{Et}$$

value of  $\sigma$  in the above equation,

$$\delta = \frac{6Wl}{nbt^2} \times \frac{l^2}{Et} = \frac{6Wl^3}{Enbt^3}$$



## *Leaf Springs Problem-6 (Assignment)*

A quarter-elliptic leaf spring **800 mm long** is subjected to a point load of **10 kN**. If the bending stress and deflection is not to exceed **320 MPa and 80 mm** respectively, find the suitable size and number of plates required by taking the width as **8 times the thickness**. Take  **$E=200 \text{ GPa}$**

# *Springs (Assignment)*

16) A close-coiled helical spring is to have a stiffness of 900 N/m in compression, with a max. Load of 45N and a max. Shearing stress of  $120\text{N/mm}^2$ . The solid length of the spring (i.e coils touching) is 45 mm. Find

(i) the wire dia

(ii) the mean coil radius

(iii) the number of coils. Take  $c = 0.4 \times 10^5 \text{ N/mm}^2$  (Apr/May 2015) 16 Marks

$k = 900 \text{ N/m}$     $w = 45 \text{ N}$     $\tau = 120\text{N/mm}^2$

A closely coiled helical spring is made up of 10 mm diameter steel wire having 10 coils with 80 mm mean diameter. If the spring is subjected to an axial twist of 10 kN.mm. Determine the bending stress and increase in the number of turns. Take  $E=200 \text{ GPa}$

A open coil helical spring made of 10 mm diameter wire has 15 coils of 50 mm radius with a  $20^\circ$  angle of helix. Determine the deflection of the spring, when subjected to an axial load of 300 N. Take  $E = 200 \text{ GPa}$  and  $G = 80 \text{ GPa}$ .

# *Springs*

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# *Queries?*