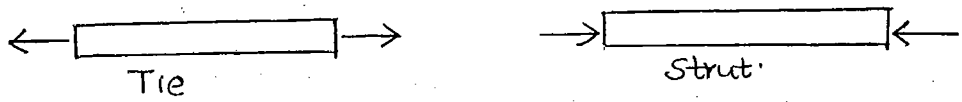
9th Oct,
THURSDAY

03. SHEAR FORCE & BENDING MOMENT

→ Equilibrium Equations.

(i) 1 D



$\Sigma F_{along\ axis} = 0.$

(ii) 2D (Plane Stress).

Eg: Beams, shafts.

$\Sigma F_y = 0$; $\Sigma F_x = 0$; $M_z = 0.$

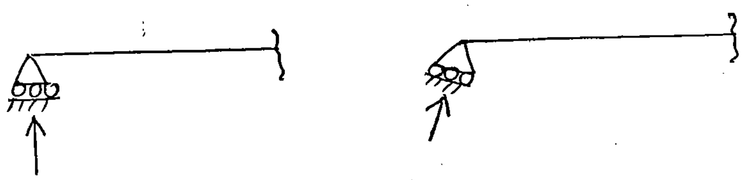
(iii) 3D (spatial)

$\Sigma F_x = 0$; $\Sigma F_y = 0$; $\Sigma F_z = 0.$

$\Sigma M_x = 0$; $\Sigma M_y = 0$; $\Sigma M_z = 0$

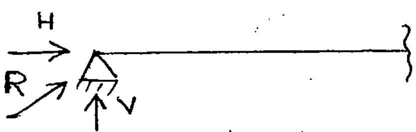
→ Types of Support.

(i) Roller Support.



Eg: Old bridges.

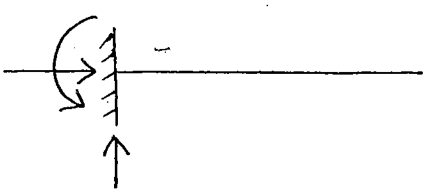
(ii) Hinged Support. (Pinned)



(2 reaction components)

Eg: Old bridge.

(iii) Fixed Support.



(3 reaction components).

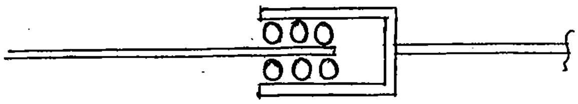
(iv) Internal Hinge.

Eg: Elbow joint

(M = 0)

(v) Shear Hinges

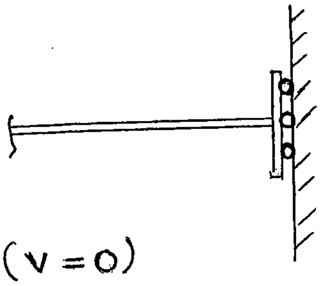
a) Axial Shear Hinge



Eg: Shock absorber.

(H = 0)

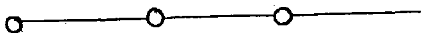
b). Transverse. ($\perp^r$ to axis).



Eg: Rolling Shutters.

(V = 0)

vi) Links.



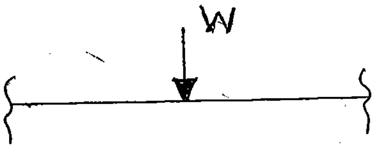
Eg: Truss.

Transfer axial forces

→ Types of Loads.

1. Concentrated or Point Load.

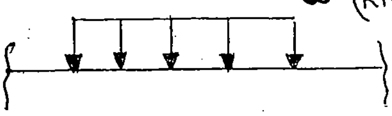
W

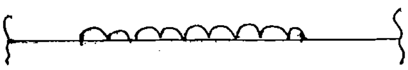


(more stress concentration)

2. Uniformly Distributed load (udl).

ω (kN/m)



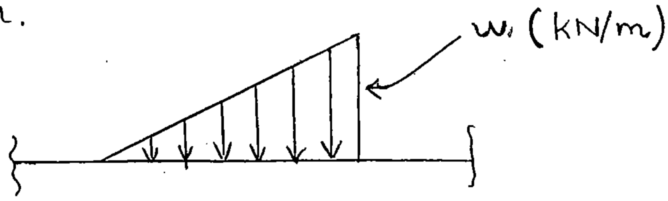


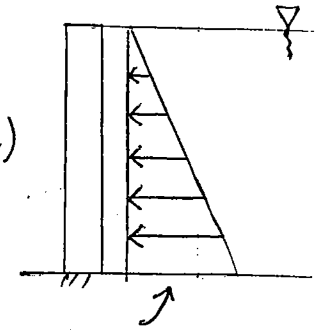
As per IS 875,
(Part 1) DL, (Part 2) LL,
(Part 3) WL, (Part 4) ~~WL~~ SL, acts
as udl.

But as per IS 1893, earthquake load is a random load.

3. Uniformly Varying Load (uvl).

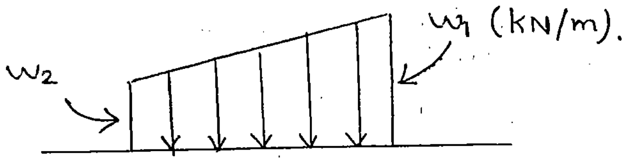
a) Triangular.

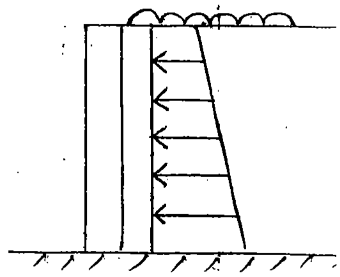




$P = \gamma_w H$

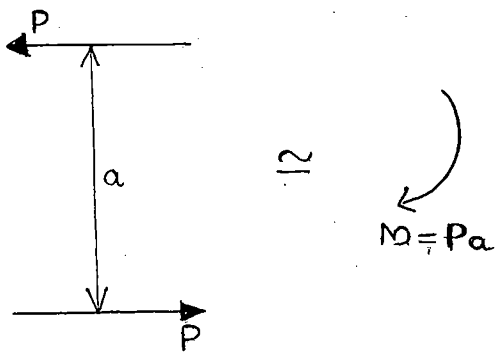
b) Trapezoidal.

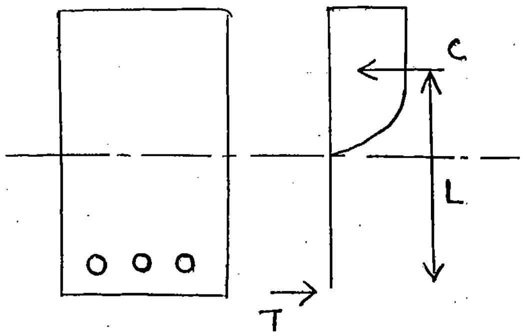




Retaining Wall.

4. Couple

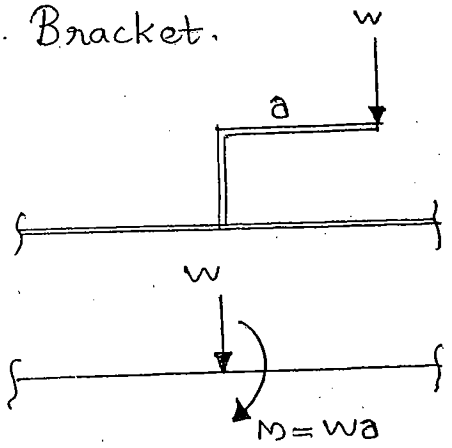




$M = CL$ or TL

$L \rightarrow$ lever arm.

5. Bracket.



→ Types of Beams.

(i) Simply Supported Beam

(ii) Propped (Supported) Cantilever.

→ Shear Force Diagram. & Bending Moment Diagram.

Diagram showing variation of SF along a structure.

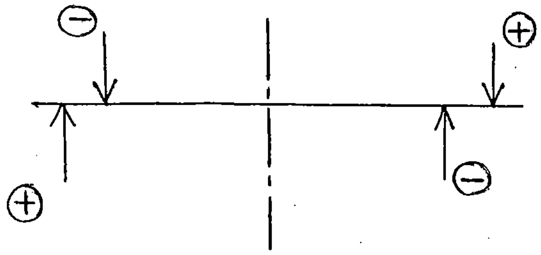
* SF at a point (or) SF @ a section.

Algebraic sum of vertical (or) transverse forces either to the left or to the right of a section

Sign convention:

Clockwise shear — +ve

Anti-clockwise shear — -ve.

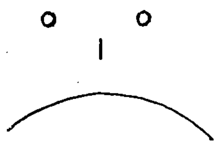


* BM at a Section (or) BM at a Point.

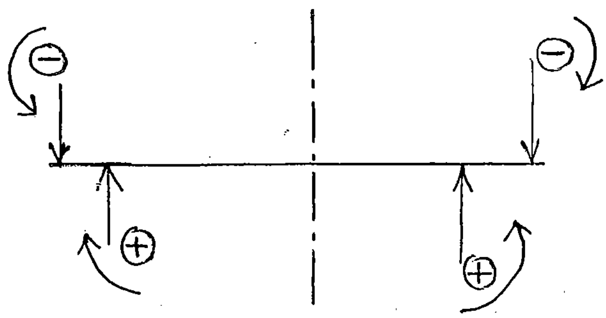
Algebraic sum of moments either to the left or to the right of a section

Sign convention:

sagging
(+ve)



hogging
(-ve)



→ Relation b/w rate of loading, SF & BM

w → rate of loading (kN/m)

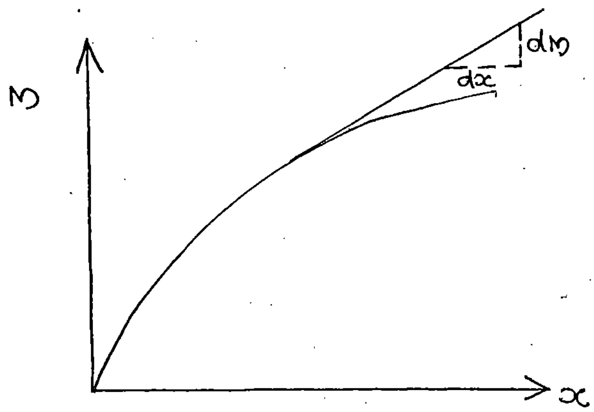
F → SF (kN).

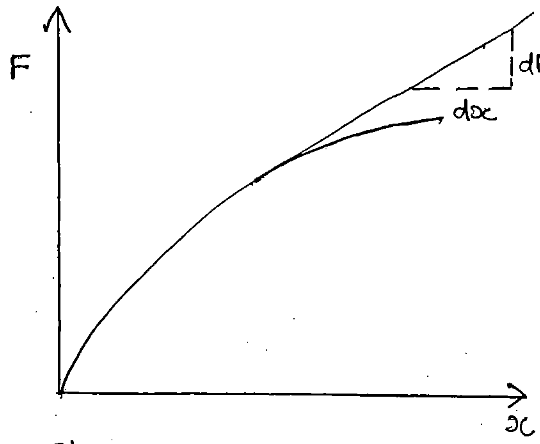
M → BM (kN·m)

$$F = \frac{dM}{dx} \quad \text{---}\rightarrow ①$$

$$w = \frac{dF}{dx} \quad \text{----}\rightarrow ②$$

Rate of change of BM gives SF; and rate of change of SF is rate of loading.

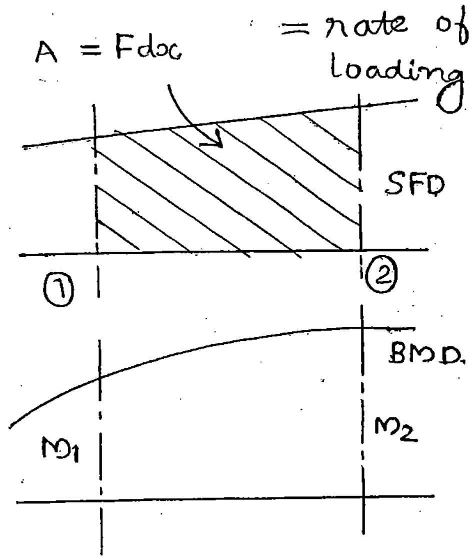




Slope to BMD $= \frac{dM}{dx} = SF$

Slope of SFD $= \frac{dF}{dx}$

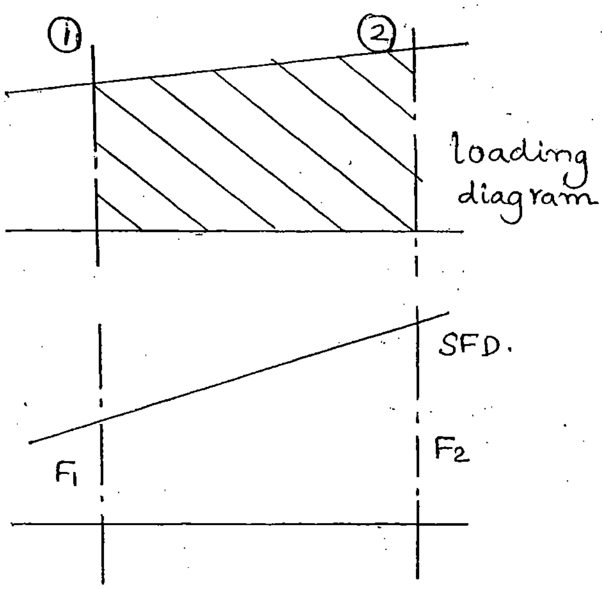
$=$ rate of loading



From ①, $dM = F\,dx$.

$|M_2 - M_1|$ = area of SFD b/w 1 & 2.

From ②, $dF = w.dx$



$|F_2 - F_1|$ = area of loading diagram b/w 1 & 2.

* For M to be maximum

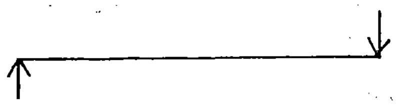
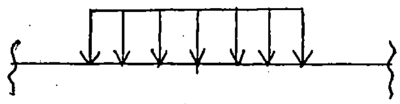
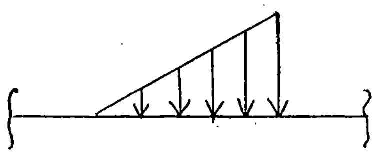
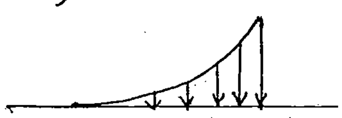
$$\frac{dM}{dx} = 0 \Rightarrow \boxed{F = 0}$$

At the point of maximum magnitude of BM, shear force must be zero. At the point of maximum magnitude of SF, BM need not be zero.

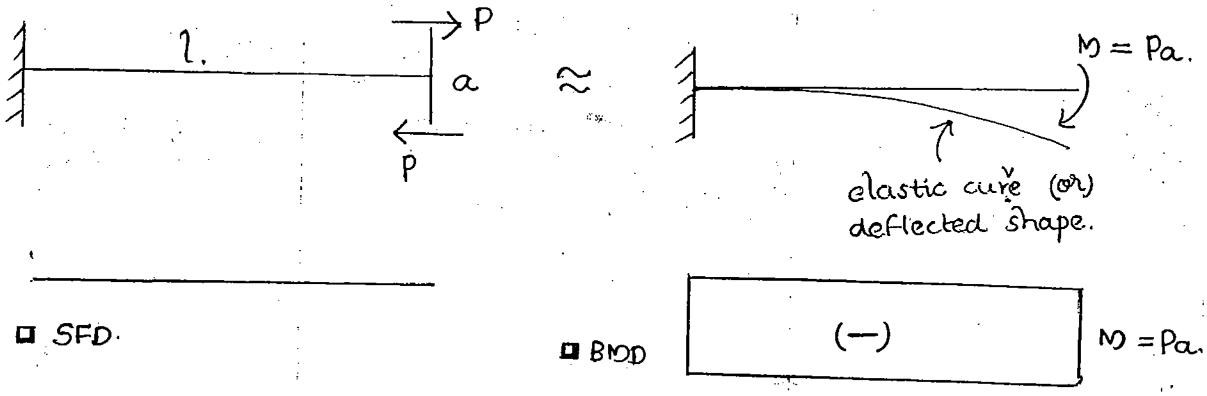
⊙ In a beam, if more than one zero SF point is acting, at all the points BM need not be maximum. (at the point of max BM, SF is zero)

⊙ The above condition is valid only for transverse or vertical or gravity loads. only; not applicable for concentrate moments.

th Oct, WEDNESDAY

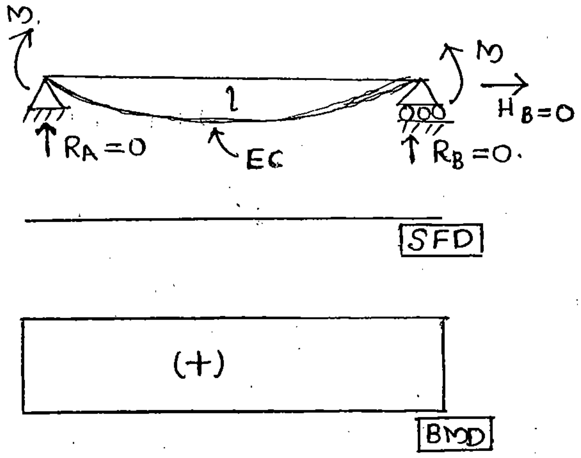
Loading	SFD (kN)	BMD (kN-m)
 No variation of load.	Uniform / Constant / Horizontal st. line (x^0)	Linear / Inclined straight line. (x^1)
 uniformly distri. load (udl) (x^0)	(x^1)	2^0 parabola / Square parabola. (x^2)
 (x^1)	(x^2)	3^0 parabola / Cubic parabola (x^3)
 Parabolic load (x^2)	(x^3)	(x^4)

Q. Draw SFD & BMD :



This is a case of pure bending.
For pure bending, SF = 0
BMD = non zero constant

Q



$\Sigma M_A = 0$

$\Rightarrow R_B \times l - M + M = 0$

$\therefore R_B = 0.$

$\Sigma F_y = 0$

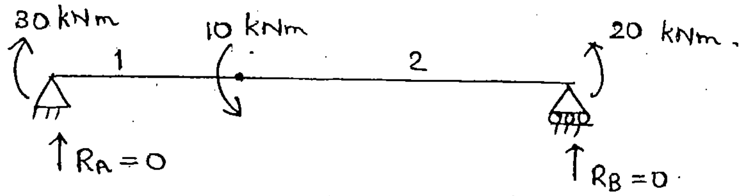
$\Rightarrow R_A + R_B = 0$

$\therefore R_A = 0.$

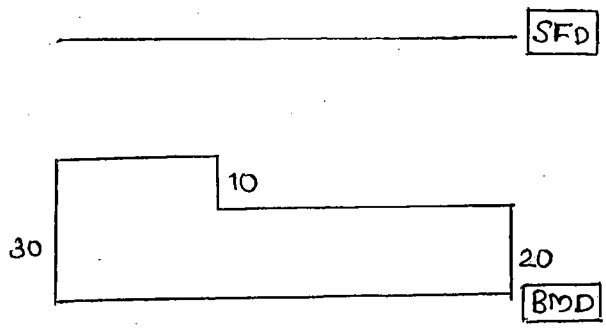
This is a pure bending criterion.
- In real beams, self wt. causes shear force. Therefore pure bending is not possible in practise.

Elastic Curve : It is the deflected shape. For pure bending, it is arc of a circle (R = const), otherwise it is parabola.

Q.

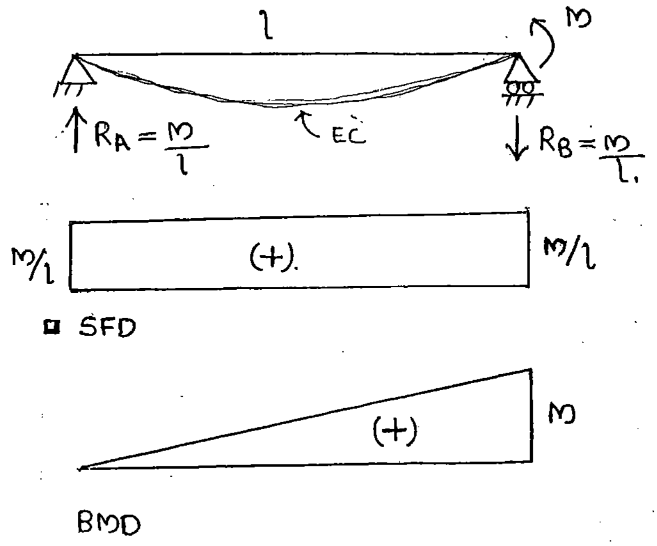


Net moment acting on beam = 30 − 10 − 20 = $\underline{\underline{0}}$



Whenever a concentrated moment acts on the beam, a jump happens in BMD.

Q.



$\Sigma M_A = 0$

$\Rightarrow -R_B \times l - M = 0$

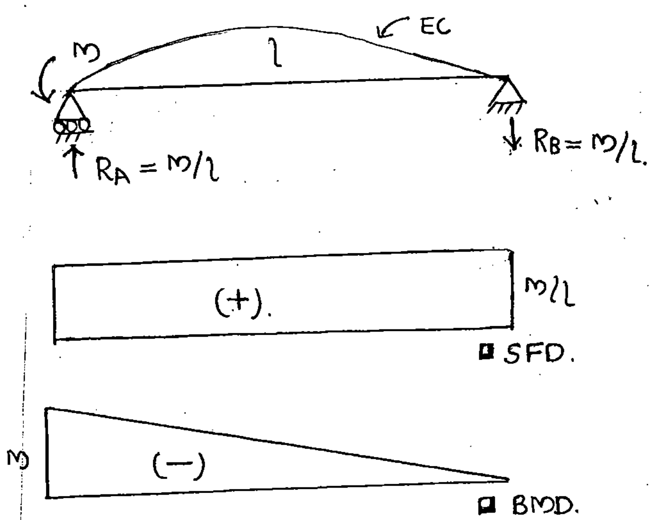
$\therefore R_B = -\frac{M}{l}$

$\Sigma F_y = 0$

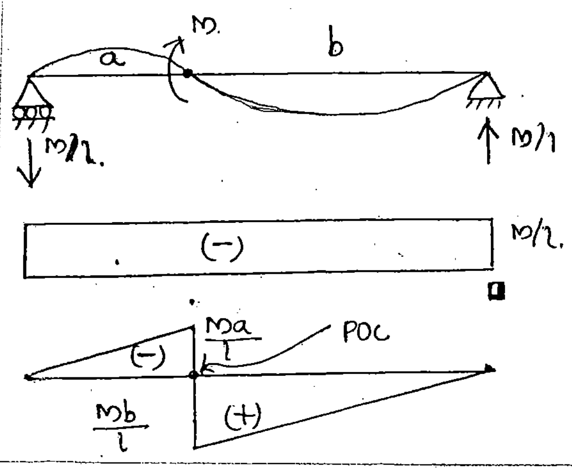
$\Rightarrow R_A + R_B = 0$

$\therefore R_A = \frac{M}{l}$

Q.



Q.



Here $b > a$

$\therefore$ Design BM $= \frac{Mb}{l}$

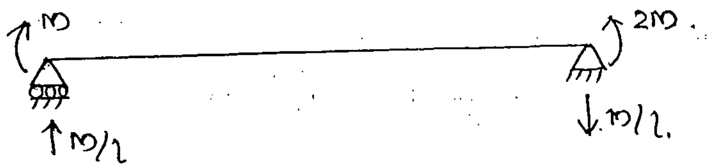
Point of Contraflexure: Point where bending moment changes sign, or curvature of the beam reverses its direction.

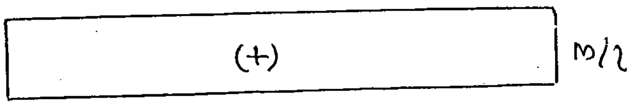
- BMD is always drawn on the tension side. So point of contraflexure determines the portion at which reinforcement is provided. (top or bottom of beam)

* Design BM (or) Absolute BM:

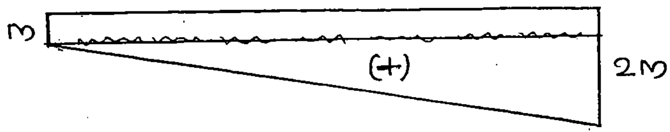
Maximum magnitude of BM over a beam.

Q.





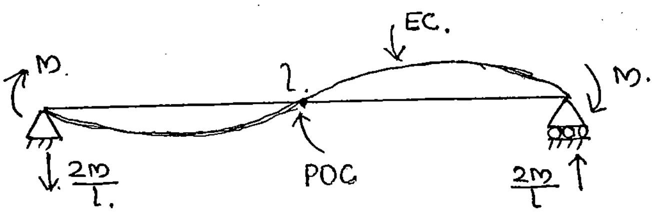
SFD

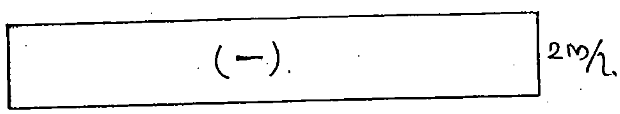


BMD

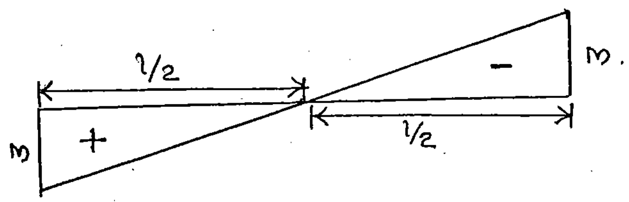
Design BM = 2M.

Q.

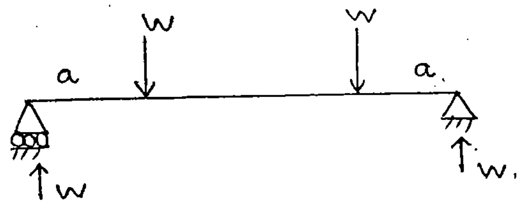


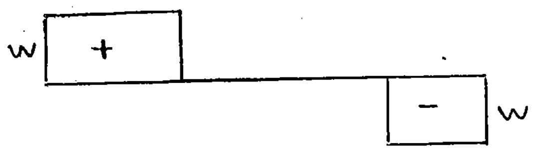


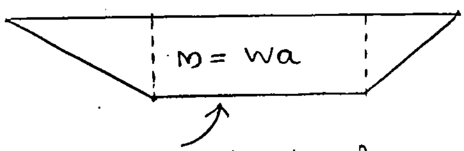
Design BM = M.



Q.



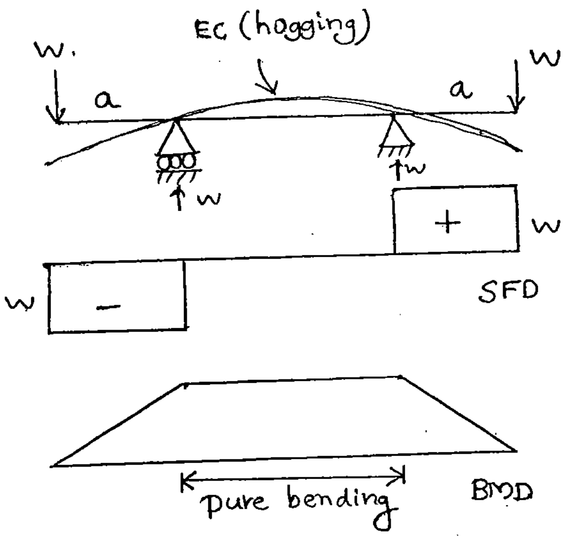




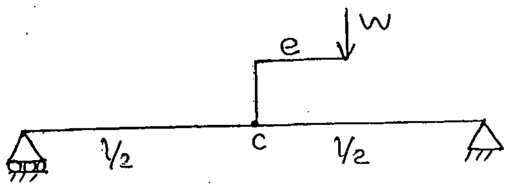
BM is constant where SF is zero (Pure bending).

In laboratories, we apply two-point load systems. It is done to eliminate shear and obtain pure bending criterion. Cracks formed will be due to bending – <u>flexural crack</u>

Q.



Q.



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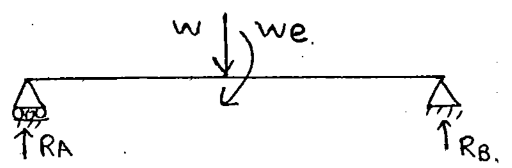
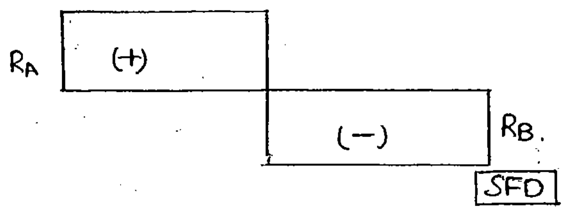
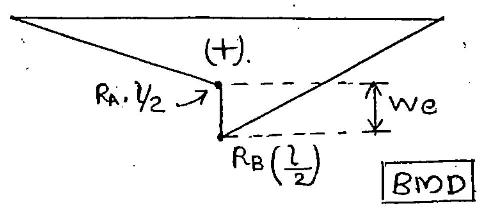






$$R_B \times l = we + \frac{wl}{2}.$$

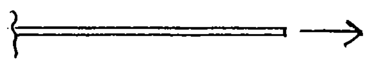
$$R_B = \underline{\underline{\frac{we}{l} + \frac{w}{2}.}}$$

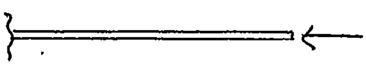
$$R_A + R_B = w.$$

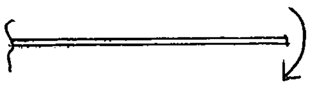
$$R_A = w - \left(\frac{we}{l} + \frac{w}{2}\right).$$

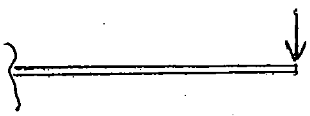
$$= \underline{\underline{\frac{w}{2} - \frac{we}{l}.}}$$

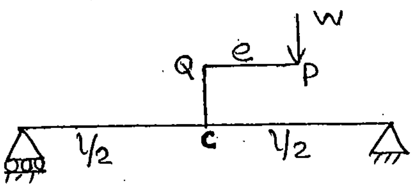
* Design Forces :

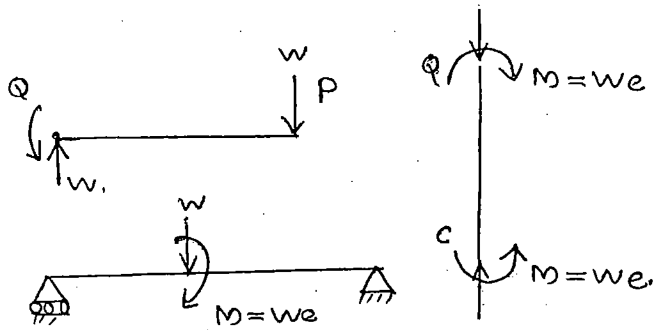
 Axial force – tension

 Axial force – compression.

 Pure bending.

 SF & BM.



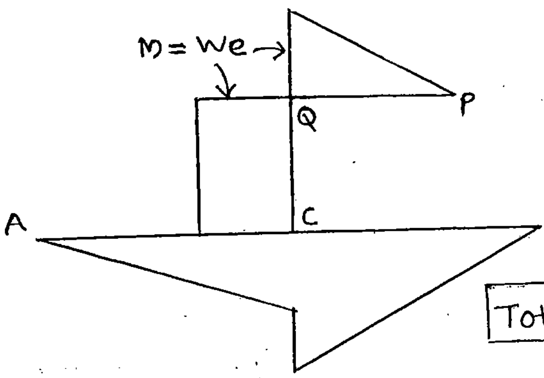


PQ → SF, BM.

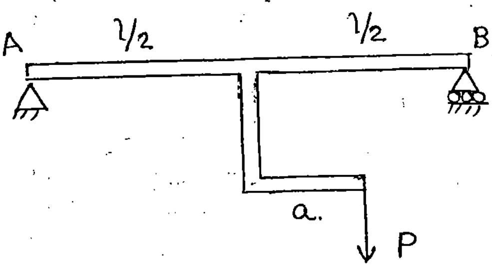
QC → AF(comp), BM

AB → SF, BM.

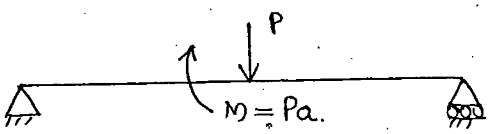
$\boxed{\text{Vertical jump in SFD indicates conc. load or reaction.}}$



Total BMD.

Q. – 

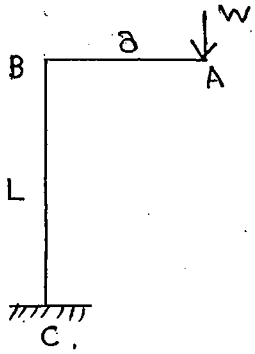
Find design BM on beam AB.

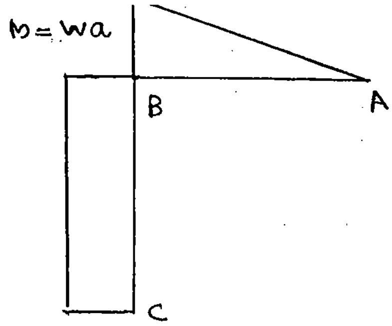


⇒ Now its same as previous.

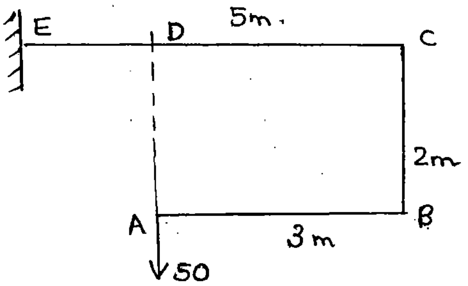
$$\text{Design BM} = R_B\left(\frac{l}{2}\right) = \left(\frac{Pa}{l} + \frac{P}{2}\right)\frac{l}{2} = \underline{\underline{\frac{Pa}{2}}} + \frac{Pl}{4}$$

Q Draw BMD.





Q.

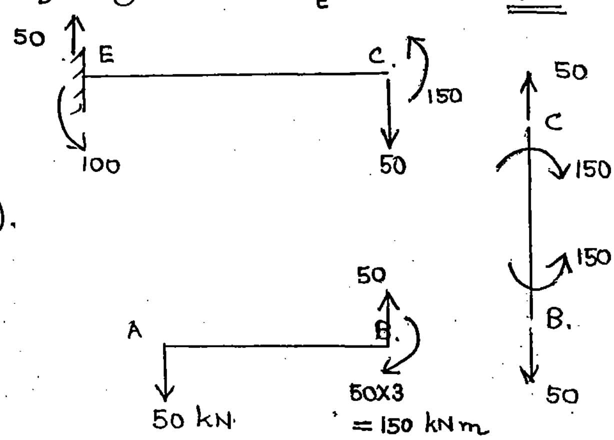


$M_A = 0$

$M_B = 50 \times 3 = 150$

$M_C = 50 \times 3 = 150$

$M_D = \text{zero}$ & $M_E = 50 \times 2 = \underline{\underline{100}}$



Design Forces:

AB $\longrightarrow$ SF, BM.

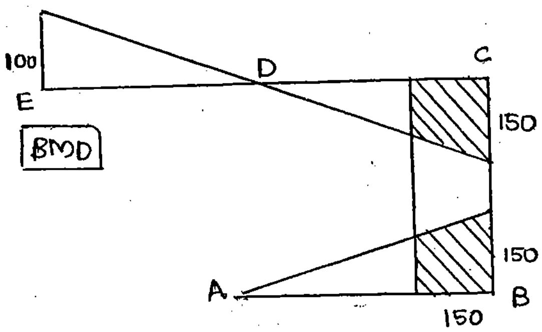
BC $\longrightarrow$ Pure BM, AF (tension).

CE $\longrightarrow$ SF, BM

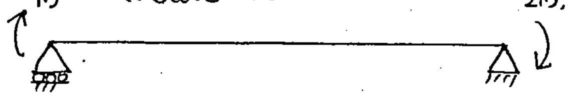
@ E:

$M_E = +150 - 50 \times 5$

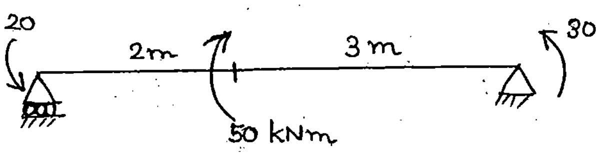
$= -100$ kNm (hogging).

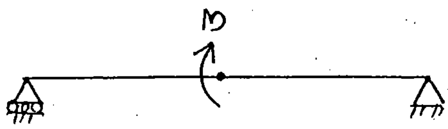


locate POC

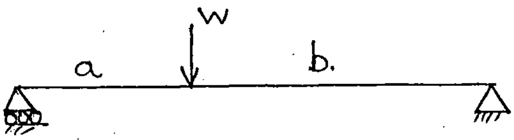


Q.

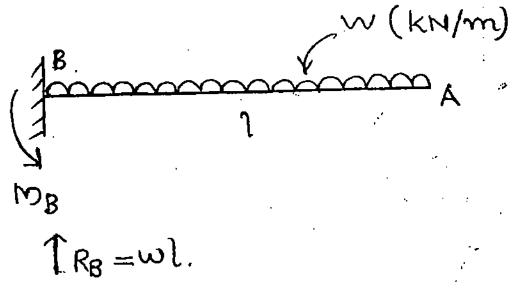




Q.



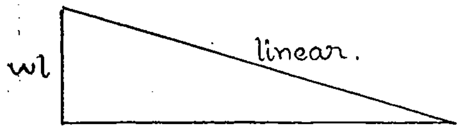
Q.

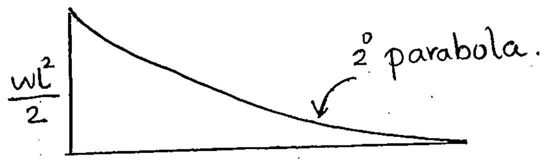


Shear Force, $F = \frac{dM}{dx}$.

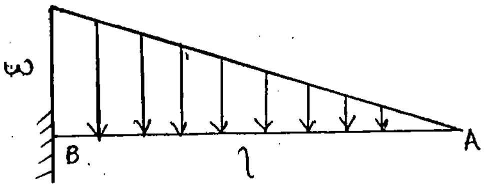
where $\frac{dM}{dx}$ is slope of BMD

So, shape of BMD is (positive slope) concave. and not convex.





Q.



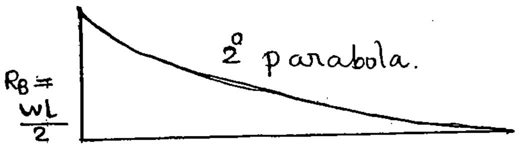
$(SF)_A = 0.$

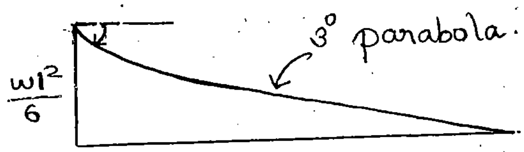
$(SF)_B = \frac{1}{2} \times w \times l = \frac{wl}{2}$

$w = \frac{dF}{dx}$

rate of loading

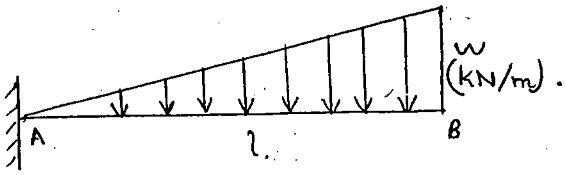
slope of SFD.





$M_B = -\frac{1}{2} wl \times \left(\frac{1}{3} l\right) = -\frac{wl^2}{6}$ (hog)

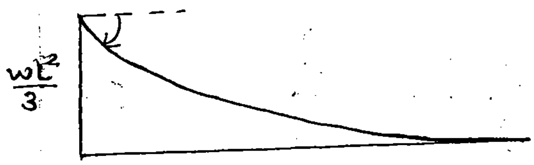
Q.



Rate of loading max at B.

$\therefore \frac{dF}{dx} =$ slope max at B.

Similarly min. rate of loading at A. $\therefore$ slope = zero at A.

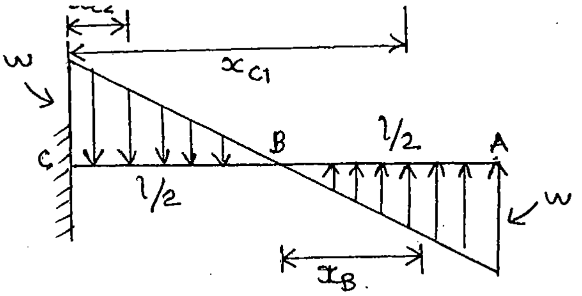


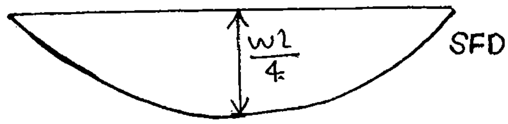
$M_B = -\frac{1}{2} \times wl \times \frac{2}{3} \times l = \frac{wl^2}{3}.$

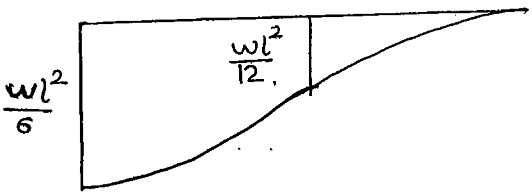
$(SF)_A = \text{max.} \Rightarrow \frac{dM}{dx} = \text{max.}$

$(SF)_B = 0 \Rightarrow \frac{dM}{dx} = 0.$

Q.







$(SF)_A = 0$

$(SF)_B = -\frac{1}{2} \times \frac{l}{2} \times w = -\frac{wl}{4}$

$(SF)_C = 0.$

$w = \frac{dF}{dx}$

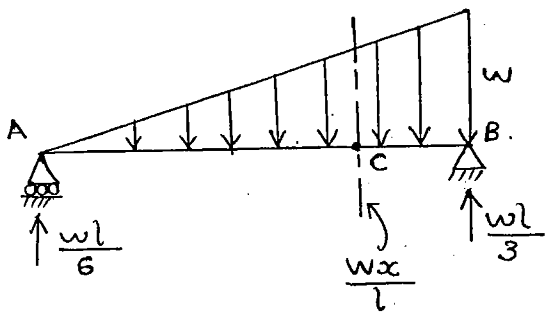
$M_A = 0.$

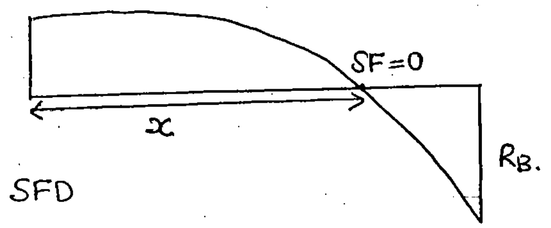
$M_B = \frac{wl}{4} \times \frac{2}{3} \times \frac{l}{2}$

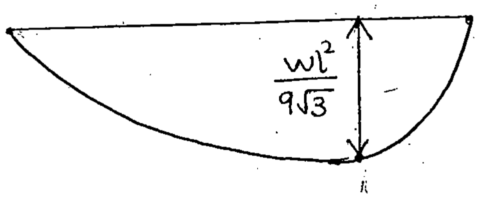
$= \frac{wl^2}{12}$ (sagging)

$M_C = \frac{1}{2} \times \frac{l}{2} \times w\left(\frac{2}{3} \cdot \frac{l}{2} + \frac{l}{2}\right) - \frac{1}{2} \times \frac{l}{2} w\left(\frac{1}{3} \times \frac{l}{2}\right) = \underline{\underline{\frac{wl^2}{6}}}$

Q.







$R_A + R_B = \frac{wl}{2}.$

$l \to w$
$x \to \frac{w}{l}x$

$\Sigma M_A = 0$

$R_B \times l = \frac{wl}{2}\left(\frac{2}{3} \times l\right).$

$\frac{wx^2}{2l} =$

$R_B = \frac{wl}{3}.$

$(SF)_C = R_A -$ hatched area of Δ^{le}

$0 = \frac{wl}{6} - \frac{1}{2}x\left(\frac{wx}{l}\right)$

$\Rightarrow x = l/\sqrt{3}$ (from A).

$M_A = M_B = 0.$

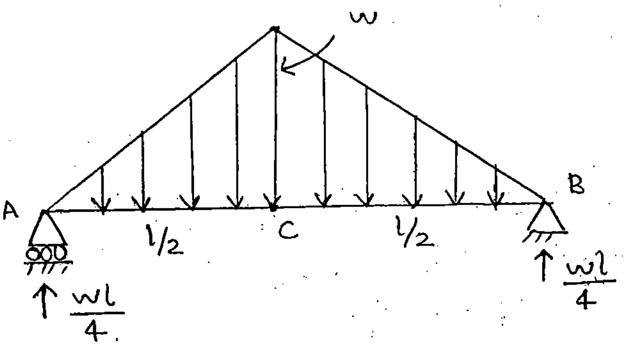
Max BM @ zero SF point $\Big\}$ $M_C = R_A \cdot x -$ hatched area $\times \frac{x}{3}.$

$M_C = \frac{wl}{6}x - \frac{1}{2}x\left(\frac{wx}{l}\right)\frac{x}{3}.$

$= \frac{wl^2}{6\sqrt{3}} - \frac{wl^2}{18\sqrt{3}} = \underline{\underline{\frac{wl^2}{9\sqrt{3}}}}$

$\frac{l^3}{3\sqrt{3}}$

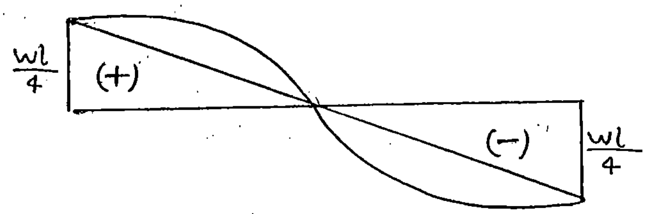
Q.



$$R_A + R_B = \frac{wl}{2}$$

$$R_B \times l = \frac{wl}{2} \times \frac{l}{2}$$

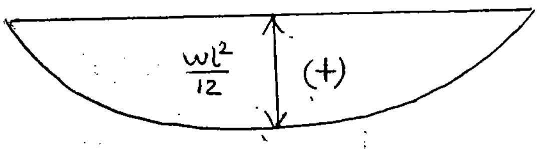
$$R_B = \frac{wl}{4} = R_A.$$



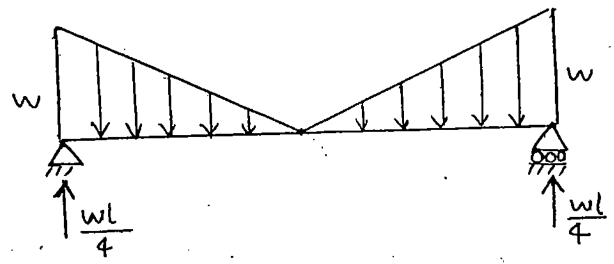
$$M_C = R_A \times \frac{l}{2} - w \times \frac{l}{2} \times \frac{1}{2} \times \frac{l/2}{3.}$$

$$= \frac{wl}{4} \times \frac{l}{2} - \frac{wl^2}{24.}$$

$$= \underline{\underline{\frac{wl^2}{12}}}$$

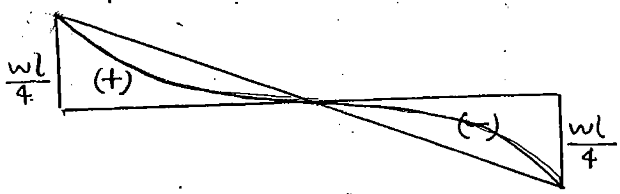


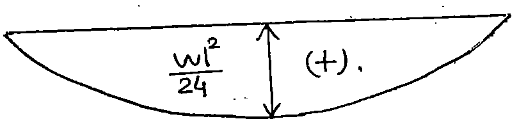
Q.



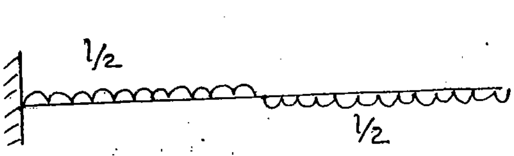
$$M_C = \frac{wl}{4} \times \frac{l}{2} - w \times \frac{l}{2} \times \frac{1}{2} \times \frac{2}{3} \times \frac{l}{2}$$

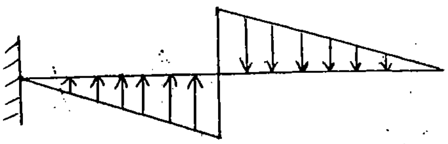
$$= \frac{wl^2}{8} - \frac{wl^2}{12} = \frac{wl^2}{24.}$$

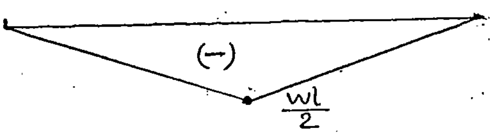


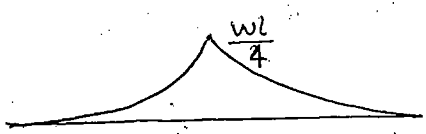


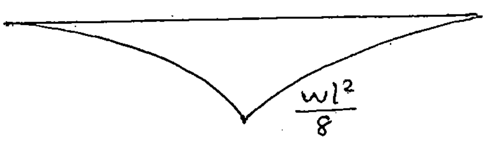
Q.

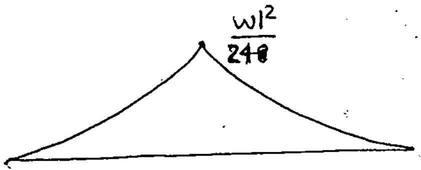










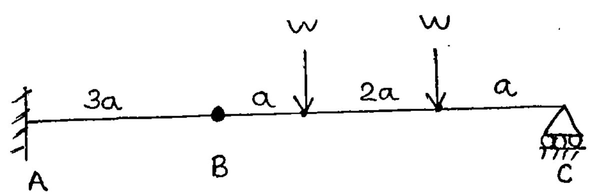


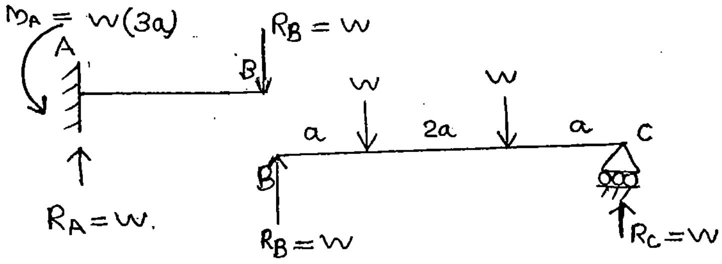
Oct, THURSDAY

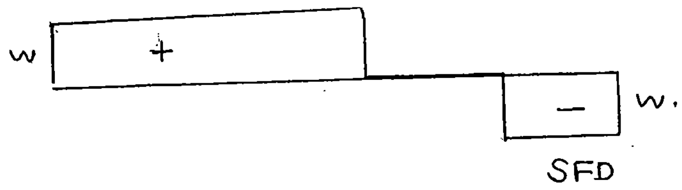
→ Beams with Internal Hinges :

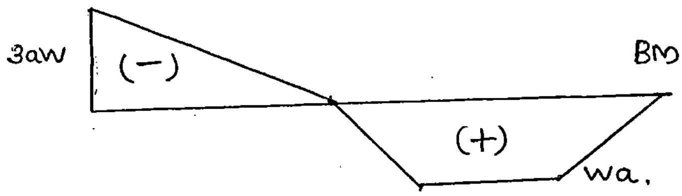
Internal hinge is also called as 'Moment Hinge' ($M=0$).

Q.

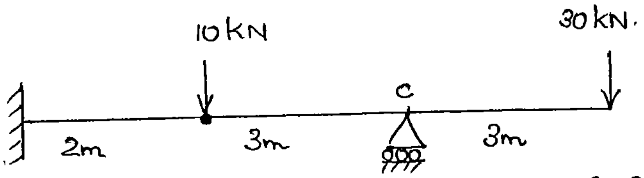


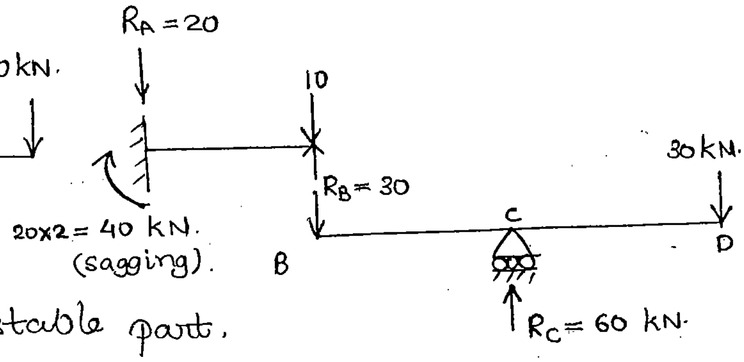






Q.

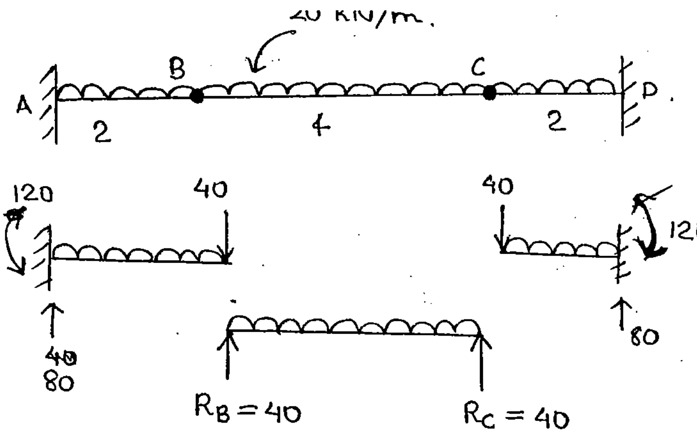




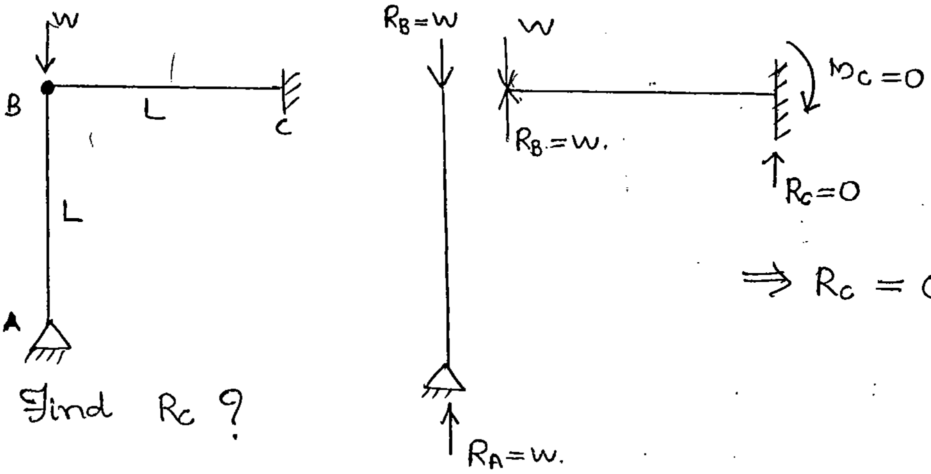
Cantilever portion is the stable part.

Apply 10 kN in the stable part.

Q.



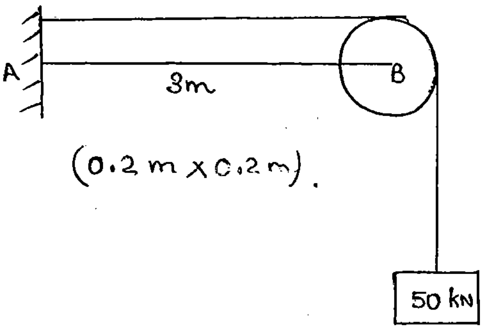
Q.



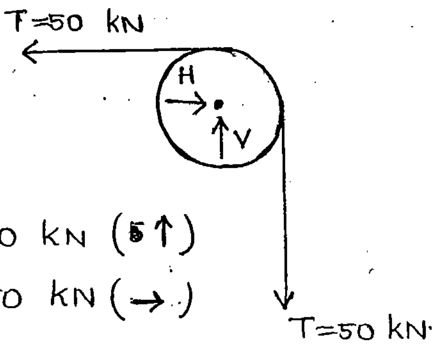
Find R_c ?

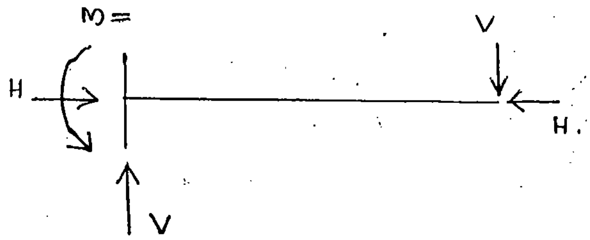
$\Rightarrow R_c = 0.$

Q.



Find σ, M, SF/ V at A.





$V = 50$ kN (↑)

$H = 50$ kN (→)

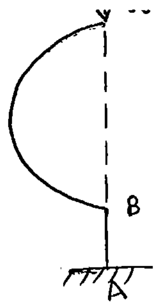
Axial force $= H = 50$ kN.

$M = 50 \times 3 = 150$ kNm. (hogging).

$SF = V = 50$ kN.

$$\sigma = \frac{AF}{\text{c/s area of beam}} = \frac{50}{0.2 \times 0.2} = \underline{\underline{1250 \text{ kN/m}^2}}$$

Q



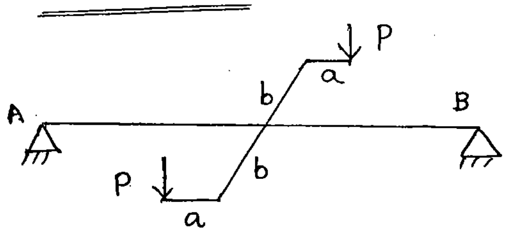
Line of action of w passes through A.

$\therefore M_A = 0$

Design force in AB = Axial force (compression only)

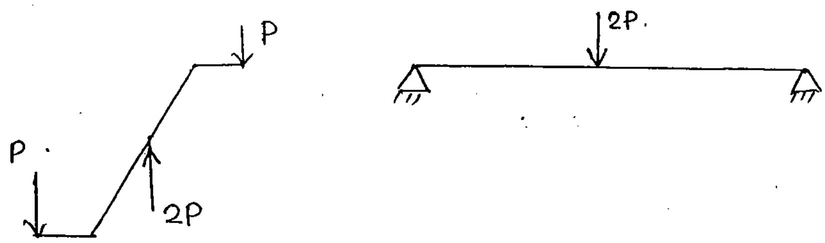
No BM, No SF

Q.

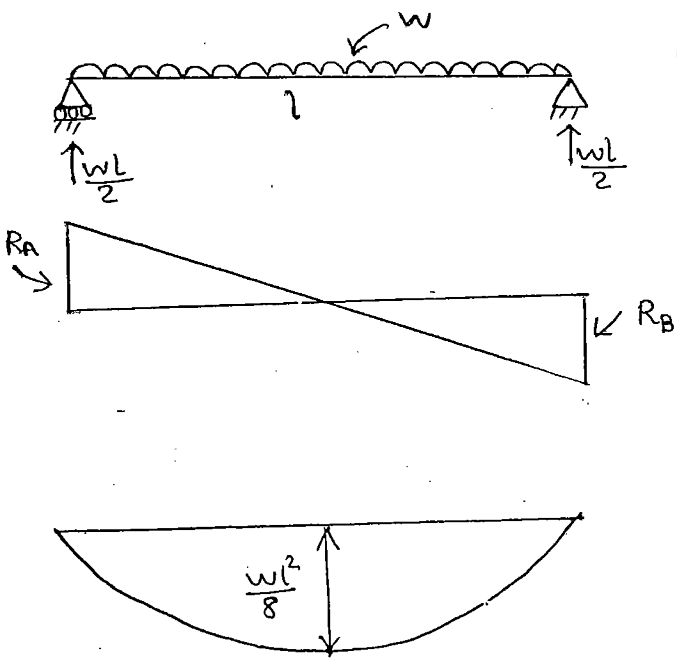


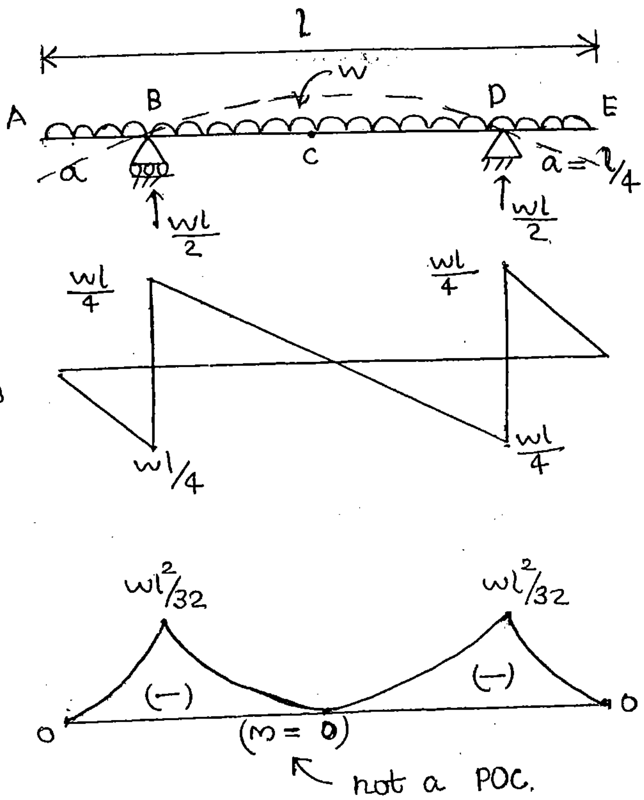
. Calculate design force in AB

Design force in AB = SF & BM



Q.





So providing a overhang, (a = l/4),

the design BM can be reduced for SSB with udl.

$(SF)_A = 0$

$(SF)_{B,\,left} = -wa = -w\dfrac{l}{4}$

$(SF)_{B,\,right} = -wa + \dfrac{wl}{2} = -\dfrac{wl}{4} + \dfrac{wl}{2} = \dfrac{wl}{4}$

$(SF)_C = \dfrac{wl}{2} - \dfrac{wl}{2} = 0.$

$M_A = 0.$

$M_B = -wa \times \dfrac{a}{2} = -w\dfrac{l}{4} \times \dfrac{l}{8} = \underline{\underline{-\dfrac{wl^2}{32}}}$

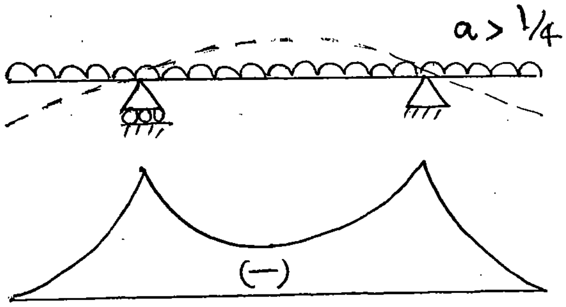
$M_C = \dfrac{wl}{2} \times \dfrac{l}{4} - \dfrac{wl}{2} \times \dfrac{l}{4} = 0$

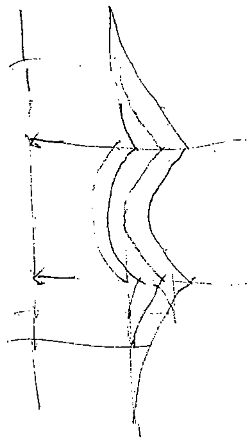
⊙ Point of Inflection: The point where BM just becomes zero.

All POCs are POIs; the converse may not be true.

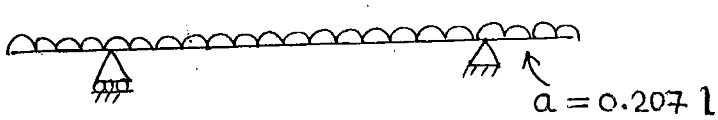
⊙ Compared to simply supported beam, BM <u>decreases by 4 times</u> for a beam with overhang $(= l/4)$.

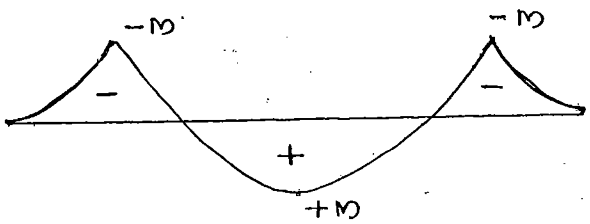
Q.





Q.





$+M = -M$

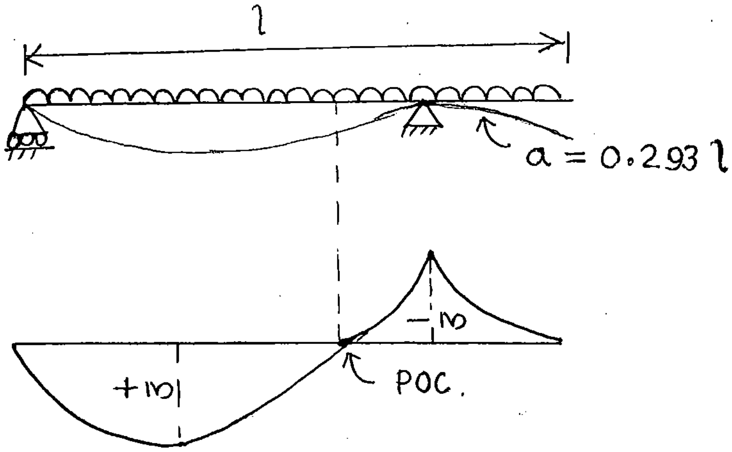
$\Rightarrow$ Sagging BM = Hogging BM.

$M = wa\dfrac{a}{2} = \underline{\underline{\dfrac{wl^2}{46.67}}}$

Compared to SSB, BM decreases by $\frac{46.67}{8} = 5.8$ times.

So this is the least design BM when overhang provided on both sides.

Q.

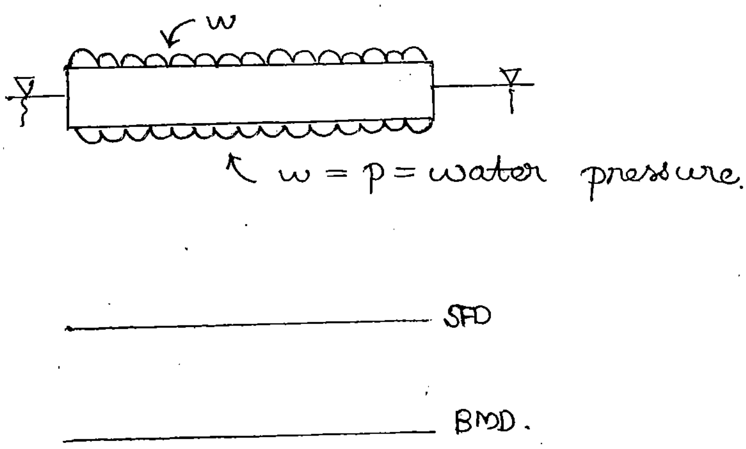


Design criteria:

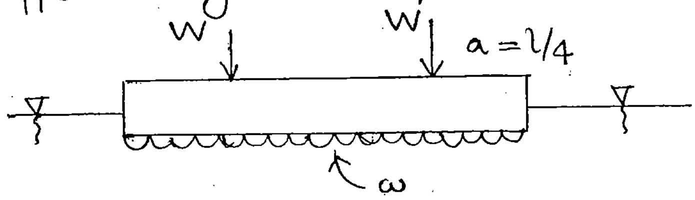
Sagging BM = hogging BM $= wa\left(\frac{a}{2}\right) = \frac{wl^2}{23.3}$

Compared to SSB, BM decreases by $\frac{23.3}{8} = 2.9$ times

Q. A wooden log of uniform c/s is floating on water with self weight. Draw SFD & BMD.

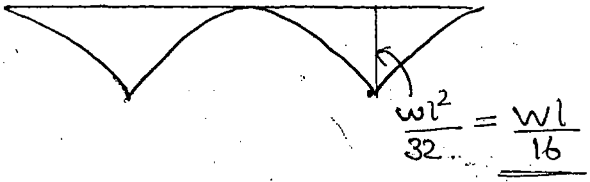


Q. A wooden log floats on water as shown in fig and supported by two equal point loads. Draw BMD

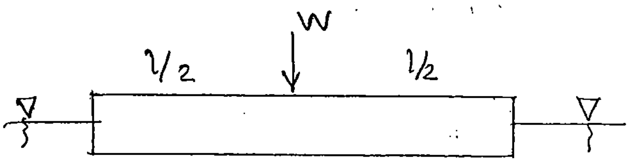


$$wl = 2w$$

$$\frac{wl^2}{32} = \frac{2wl}{32} = \underline{\underline{\frac{wl}{16}}}$$

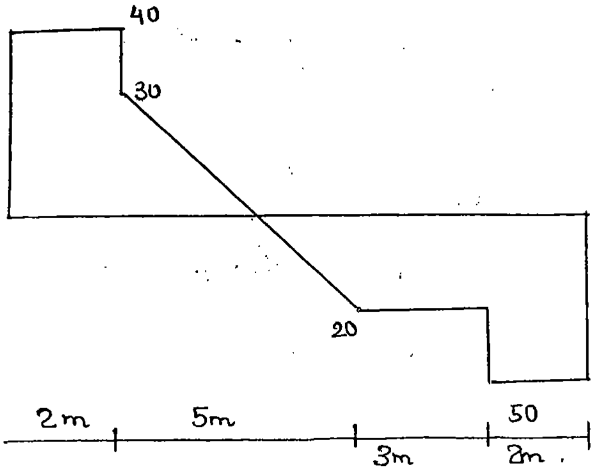


Q. A wooden log is floating on water with central load W. Draw SFD & BMD.



→ Convertion of SFD to Loading

Q.

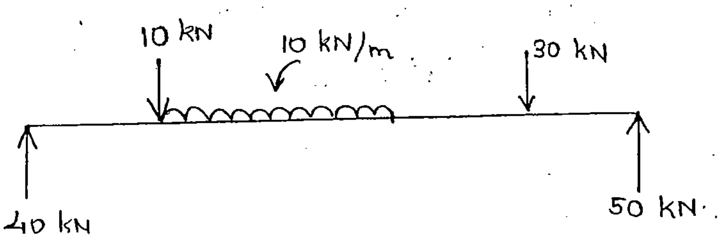


Intensity of loading,

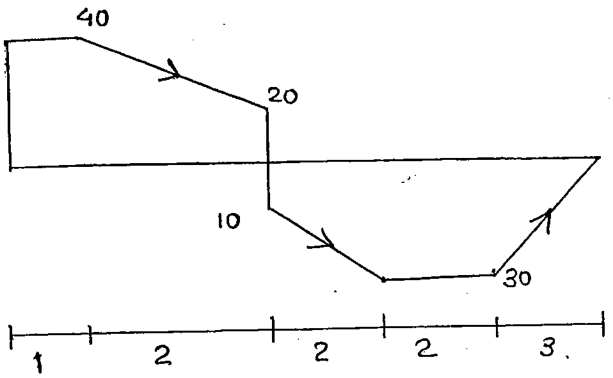
$$w = \frac{dF}{dx}$$

$$= \frac{30 - (-20)}{5}$$

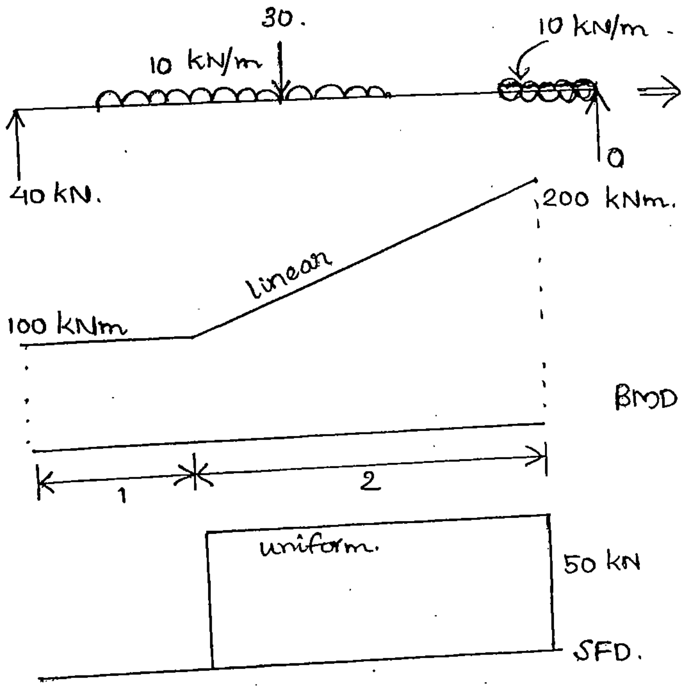
$$= 10 \text{ kN/m}.$$



Q.



Q



$\Rightarrow \Sigma F_y = 0$

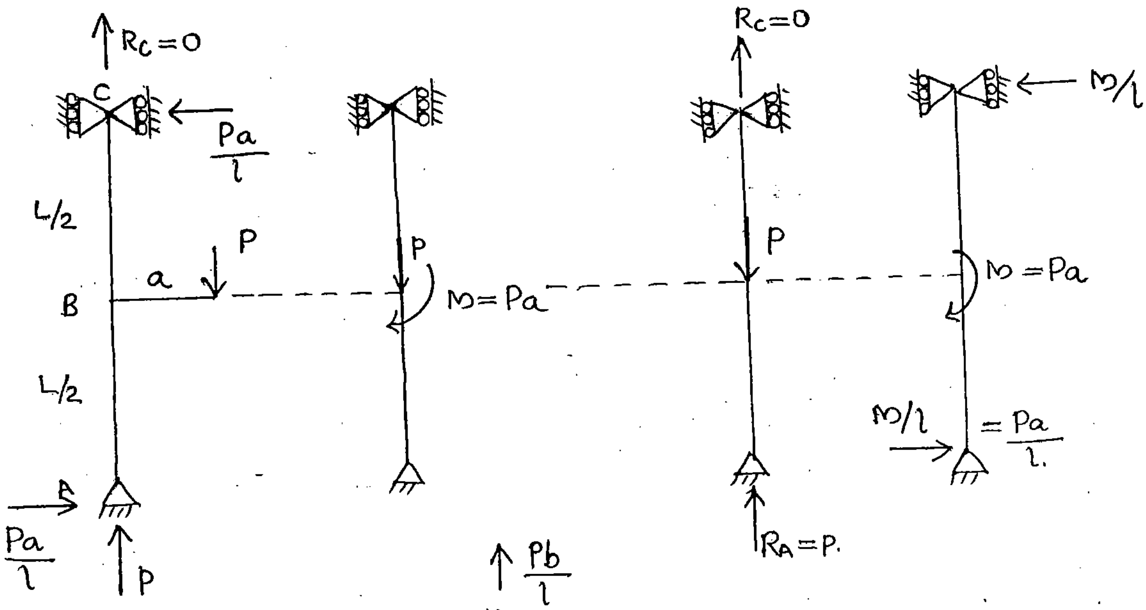
$$F = \frac{dM}{dx}$$

Slope of BMD $= 0$

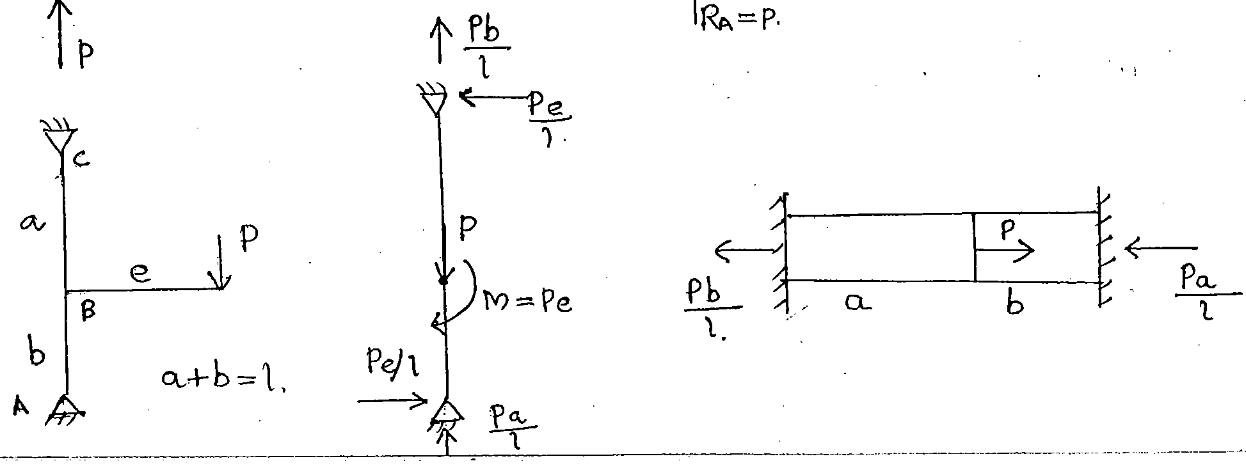
$$F = \frac{\text{Change in BM.}}{\text{length of beam.}}$$

$$= \frac{200-100}{2} = \underline{\underline{50 \text{ kN}}}$$

Q.



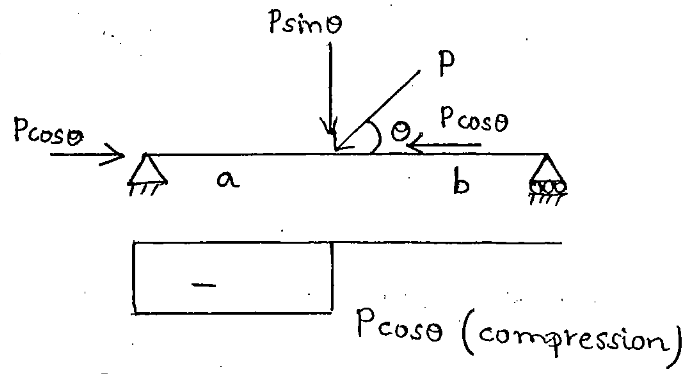
Q



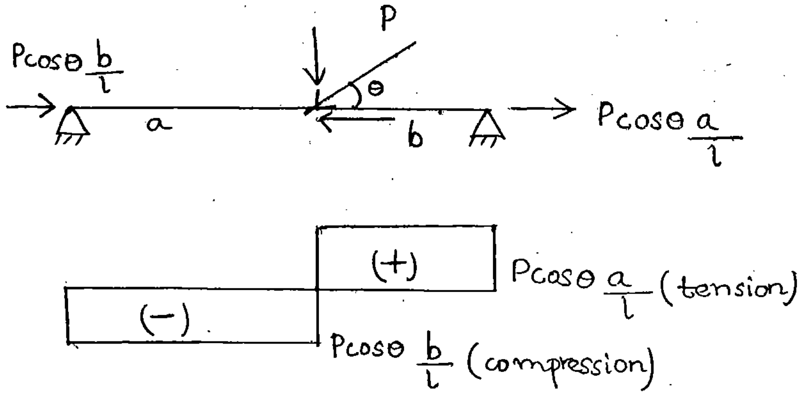
→ Axial Force Diagram.

- due to axial loads.
- inclined loads.

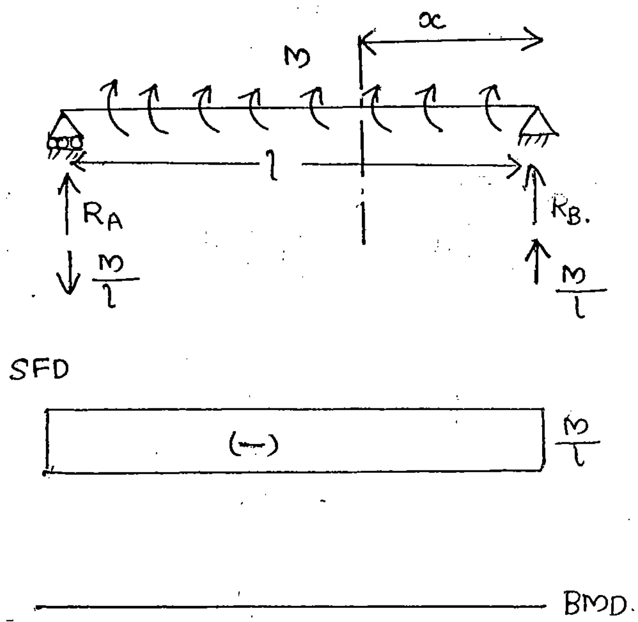
Q



Q.



Q.



(Pure Shear)

Total distributed moment $= M$

$l \longrightarrow M$

$x \longrightarrow \frac{Mx}{l}$

$$M_x = R_B x - \frac{Mx}{l}$$

$$= \frac{M}{l}x - \frac{Mx}{l} = 0$$

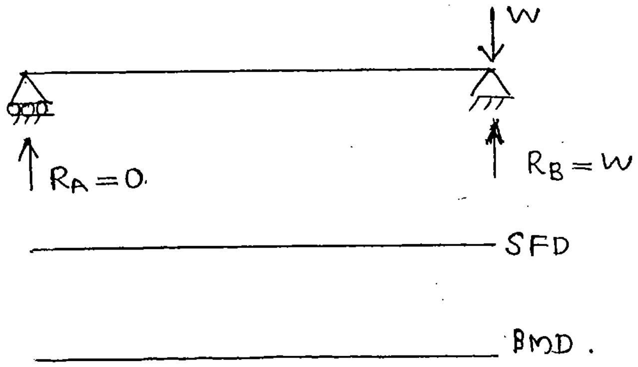
- Pure Shear :-

SF → non zero constant & max.

BM = 0.

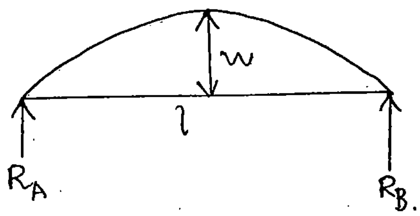
- Only example of Pure Shear Condition.

Q.



P-29

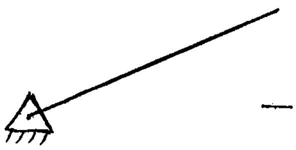
11.



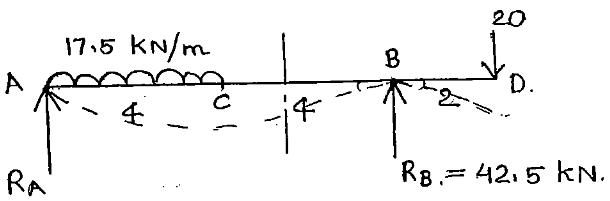
Max SF = Max reaction.

$$R_A = R_B = \frac{\underline{\text{Total load}}}{2} = \frac{\text{area}}{2}$$

$$= \frac{\frac{2}{3} l w}{2} = \frac{wl}{3}$$

12.  — purely axial load.

06.



$$M_x = -20x + R_B(x-2)$$

$$0 = -20x + 42.5(x-2) \Rightarrow x = \underline{3.78\ m}$$

7th Oct,
FRIDAY.