2[th] JULY,
TUESDAY

STRENGTH OF MATERIALS (6-8)

Strength : resistance to failure is called strength. It is a material property.

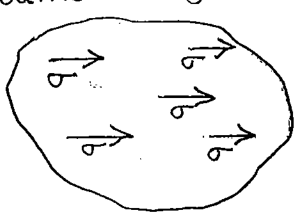
$$\left.\begin{array}{l} M20 \Rightarrow f_{ck} = 20 \text{ MPa} \\ M15 \Rightarrow f_{ck} = 15 \text{ MPa} \end{array}\right\} \text{@ failure, stress developed = strength}$$

Stiffness : resistance against deformation is stiffness. This is a secondary design property. $K \uparrow \ \delta \downarrow$

Assumptions :

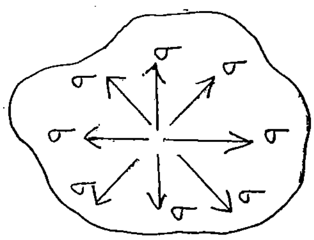
1. Material is continuous. (no voids or no cracks)
2. Material is homogenous and isotropic.

$\underbrace{\text{Homog}}_{\text{same}}\underbrace{\text{enous}}_{\text{origin}}$ - Eg:- wood, iron, gold.
steel, brass, bronze (not homogenous).



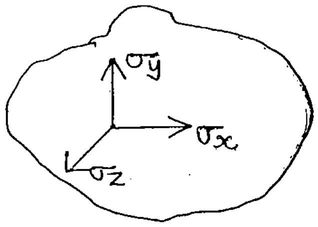
at any point in one direction, same prope

$\underbrace{\text{Iso}}_{\text{same}}\ \underbrace{\text{tropic}}_{\text{directional property}}$ - Eg:- fine grained material (iron, gold, stee
wood (non isotropic).

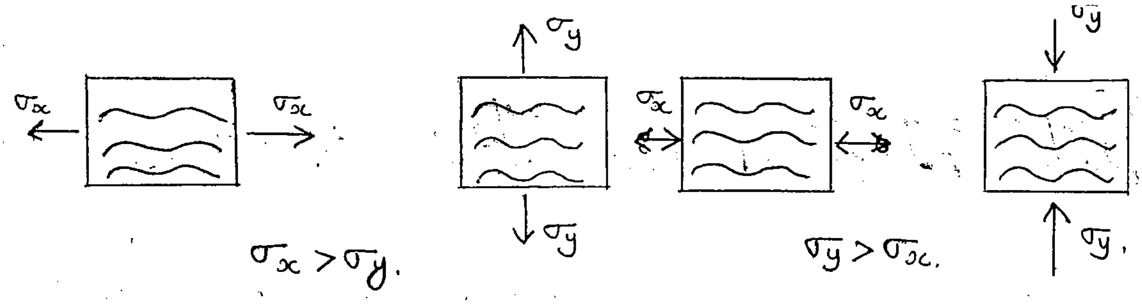


at ~~any~~ one point in any direction, same property.

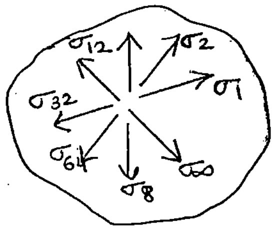
$\underbrace{\text{Ortho}}_{\perp^r}\underbrace{\text{tropic}}_{\text{directional property}}$ - Eg:- Layered material (wood, sedimentary roc
marble, graphite, mica



at one point in $\perp^r$ direction property are different.

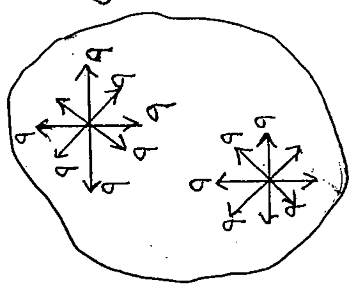


Anisotropic (Non-Isotropic) / Aleotropic



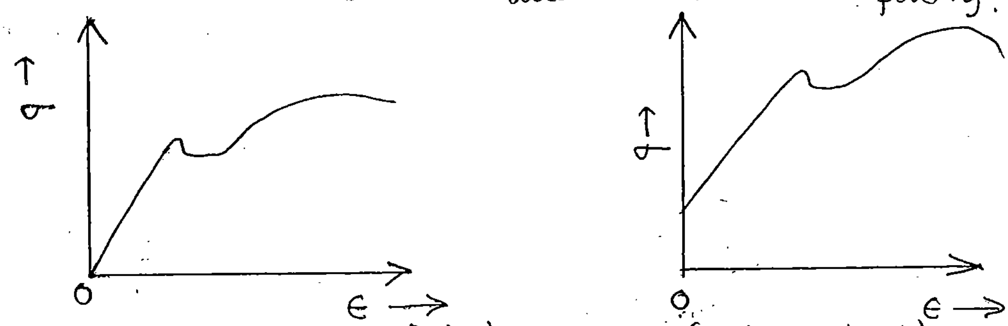
@ one point in different direction property different.
Eg:- Material with cracks and voids

Homogenous + Isotropic – Eg :- Iron, copper, gold.



@ any point in any direction, same property

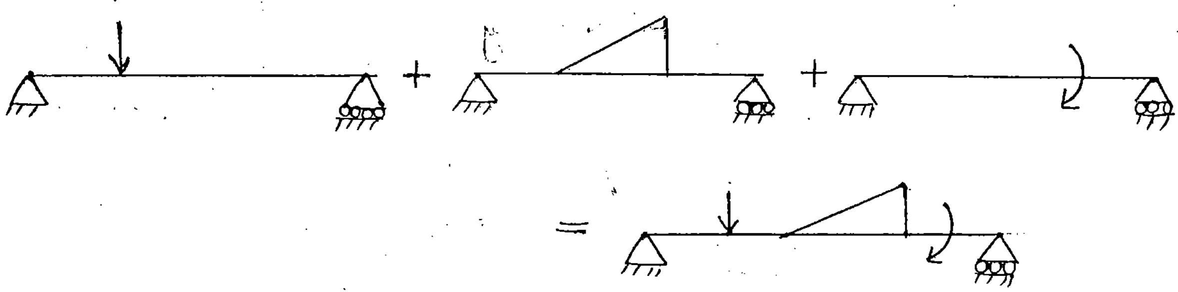
3. Self weight neglected (stress vs strain starts from origin due to this assumption).



(without self weight) (with self wt.)

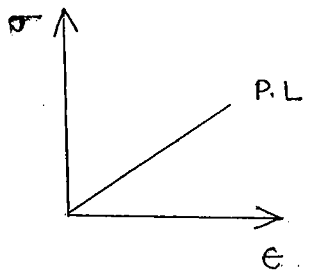
4. Superposition Principle is valid.

Algebraic sum of various effects is equal to the total effect.



Limitations of Superposition Principle :

(i) Linear elastic members.



Robert Hooke's law is valid.

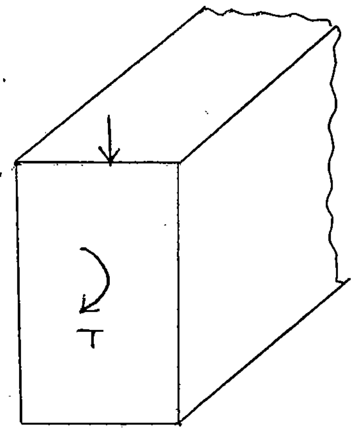
Loads must be upto P.L.

(ii) Deformations are very small.

Not valid for :

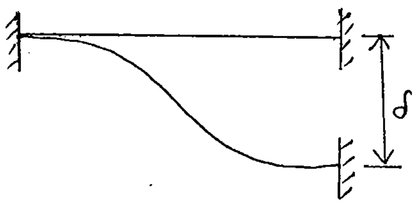
(i) Deep beam.

D > 750 mm.



In deep beams, torsion develops due to loading which causes distortion in shape

(ii) Sinking of supports.

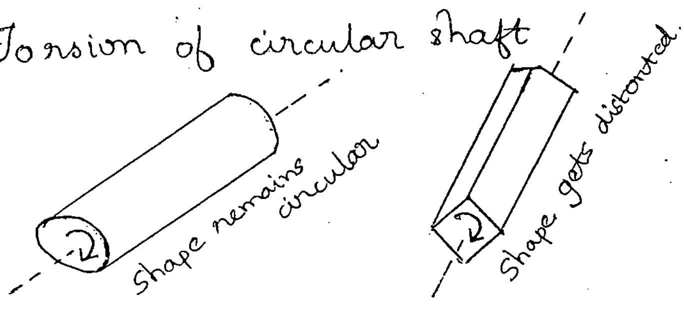


axis gets (curved) distorted.

(iii) Long Columns.

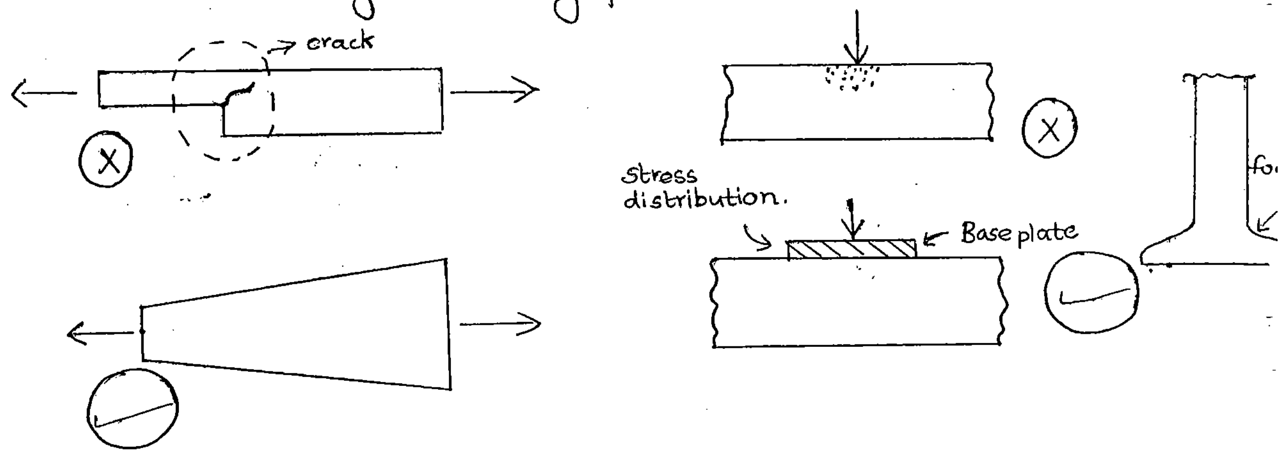
Buckling occurs.

(iv) Torsion of circular shaft



5. St. Venent's Principle is valid.

Sudden change in any parameter causes stress concentra

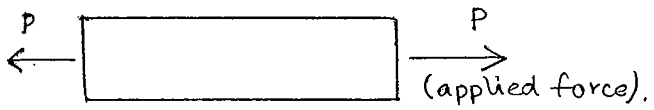


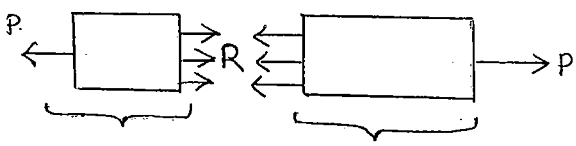
Stress

The Internal resistance developed against deformation per unit area. is called stress.

$$\sigma = \frac{\text{resisting force}}{\text{Unit area}}$$;

Unit of stress $= N/m^2$
$kPa = kN/m^2$
* $MPa = N/mm^2$
$GPa = 10^3\ N/mm^2$
$= 10^3\ MPa$





$\Sigma F_x = 0$ $\qquad$ $\Sigma F_x = 0$

$P = R$ $\qquad$ $P = R$

$$\therefore \quad \sigma = \frac{P}{A} = \frac{R}{A}.$$

NOTE: ⊙ A member free to deform without showing reaction or resistance will have zero stress.

⊙ A member free to move away without any frictional resistance, stress developed is zero.

⊙ A member free to expand or contract due to temperat. change, there will be no stress.

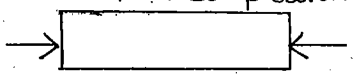
Classification of Stress :

- Classification.
  - Normal
    - Direct
    - Indirect — Bending / flexural stress.
  - Shear.
    - Direct — Tangential force/unit area.
    - Indirect — Torsional Shear stress.

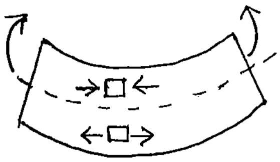
TIE : Tension

STRUT : Compression

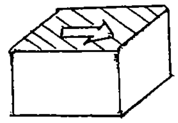
$$\sigma = \frac{R}{A} = \frac{P}{A}$$



$$f = \frac{M}{Z}$$



$$\tau = \frac{F}{A}$$



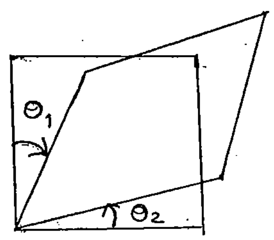
$$\tau = \frac{T}{Z_p}$$

Strains :

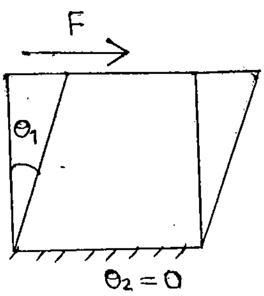
(i) Normal strain (due to normal force),

$$e, \epsilon = \frac{\text{Change in dimension}}{\text{Original dimension}} \text{; unitless.}$$

(ii) Shear strain (due to shear force) → angular change or distortion b/w any two mutually perpendicular planes in radian is Shear Strain.



$\phi = \theta_1 + \theta_2$



$\phi = \theta_1 + 0$ (angle coming alone, ∴ it should be in radians)

NOTE : As radian is a secondary unit, its dimensionless.

(iii) Volumetric Stress (due to normal force),

$$e_v = \epsilon_v = \frac{\delta V}{V} \text{ ; No unit}$$

NOTE: ⊙ Normal forces can cause change in dimensions as well as volume.

⊙ Shear forces can change the shape without change in volume.

⊙ External force ⟶ Deformation ⟶ Resistance ⟶ Stress

$\downarrow$

Strain.

Strain is independent & stress depends on strain.

Material Properties:

1. Elasticity → ability to regain shape on removal of external force.

2. Plasticity → member undergoes permanent or plastic deforma at constant load.

3. Ductility ⟶ material can be made into thin wires.

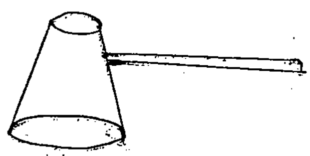
   Eg:- All soft metals (Au, Ag, Al, Cu, steel)

   Ductility is related to tension. Ductile materials are strong in tension and weak in shear. They are moderate in compression.

4. Malleability ⟶ pressed into thin sheets.

   Eg: all ductile materials.

   Properties of malleable and ductile are the same.



— mallet. (malleability).

It's related to compression.

5. Brittle ⟶ fails suddenly

   Eg: Cast Iron, concrete, glass.

   All brittle materials are strong in compression and weak in tension. and moderate in shear.

6. Creep – The plastic or permanent deformation due to constant load with time

Aug, SATURDAY

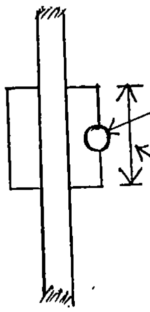
Stress-Strain Curves

* Low Carbon Steels

a) Mild Steel (Fe 250)

Carbon ($\leq 0.15\%$) : Carbon is the strength parameter.

Manganese : increases toughness. (resistance to impact loading)



strain gauge (Extenso meter)

Gauge length, $GL = 5.65\sqrt{A}$ (Emperical Formula).
where $A \rightarrow$ nominal/initial c/s area

U.T.M (Universal Testing Machine)

[UTM can be used for measuring shear, tension, compression, flexure, torsion etc and ∴ called as Universal.]

Gauge length is independent of length of bar, shape of c/s, rate of loading.

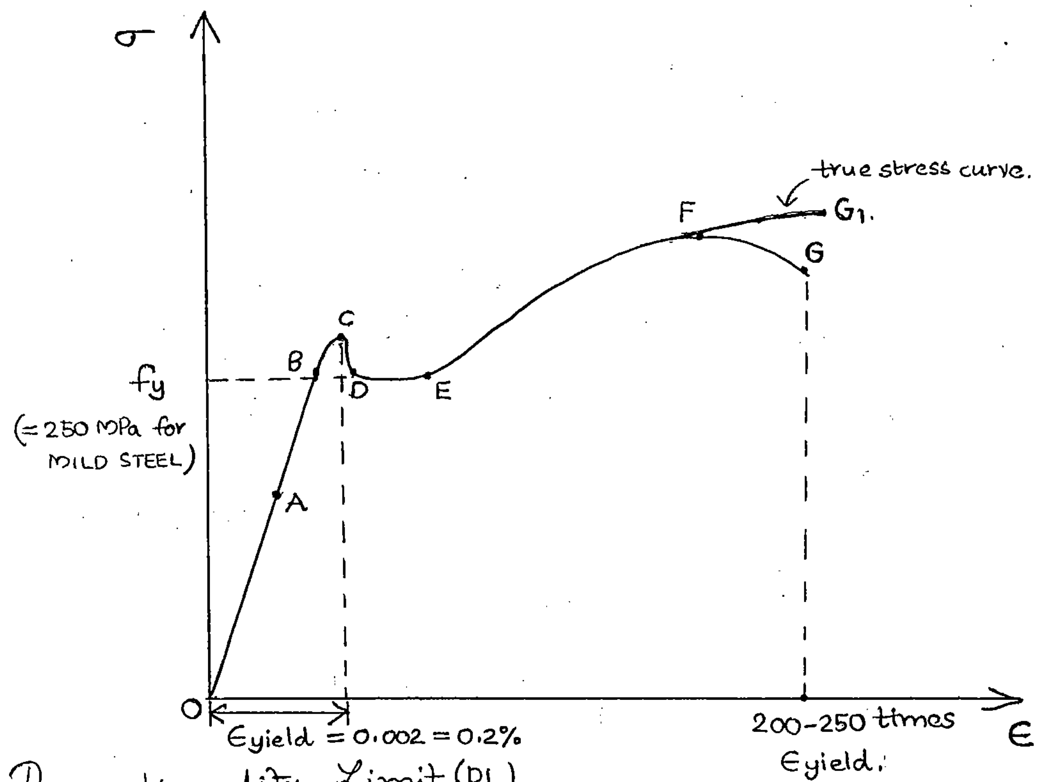
UTM is strain oriented. Resistance offered by the bar is given by Load Dial.

$$\text{Strain} = \frac{\delta(GL)}{GL}$$

$\sigma = \frac{P}{A}$ ← load dial reading ; σ = nominal stress / Initial stress / Engg. stress / Stre

True stress or Instantaneous or Actual stress, $\sigma_0 = \frac{P}{A_0}$

$A_0 \longrightarrow$ true / instantaneous / actual area.



A : Proportionality Limit (PL)

ie upto A, $\sigma \propto \epsilon$

OA is a straight line.

OA is linear elastic.

Hooke's Law is valid upto PL only.

B : Elastic Limit (EL)

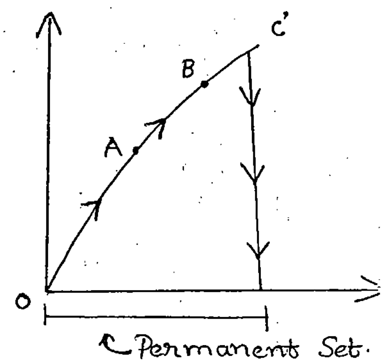
ie upto B, material is elastic.

A to B : graph is slightly curved.

Hooke's Law not valid.

AB : Non linear elastic zone.

NOTE : Loading Beyond Elastic limit causes 'permanent set' or 'Plastic Deformation' or 'Residual strain' in the material.



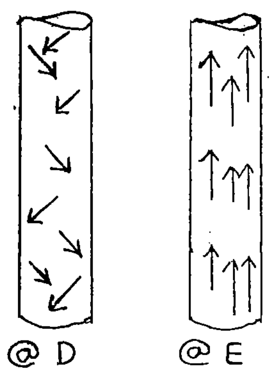
C : Upper Yield Point.

At yield point, resistance of the material suddenly drops down, which occurs at a strain of 0.002 in most of the metals.

$\boxed{\epsilon_{yield} = 0.002 = 0.2\%}$

D : lower yield point.

DE : Plastic Zone / Permanent Deformation

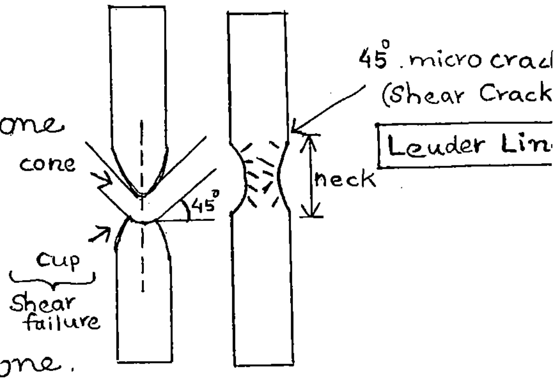


In plastic zone, reorientation of molecules occur. Due to this material becomes nearly homogenous and start resisting the loading

F : Ultimate point (Ultimate stress)

G : Brittle Point (Brittle stress).

Zones :



OA = linear elastic zone

AB = non-linear elastic zone

CD = yield zone.

DE = plastic zone

EF = strain hardening zone.

FG = necking zone / Strain softening zone

In strain hardening zone (EF), material undergoes higher strain to resist little external forces.

Lower yield point (D) is the design stress. in all the designs like Working Stress method, Plastic Theory, Ultimate Load method, Limit State method etc. It is the yield stress corresponding to D. The position of upper yielding point is not stable which may change based on shape and size of specimen used. ∴ lower yield point is preferred in design.

Ductility Factor, $DF = \dfrac{\epsilon_{fail}}{\epsilon_{yield}}$

For mild steel,

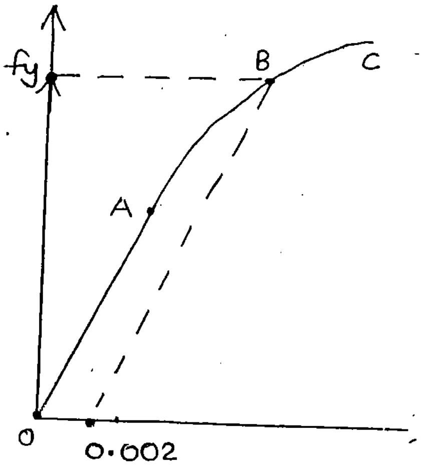
$DF = 200 \text{ to } 250$

14th Sept, SUNDAY

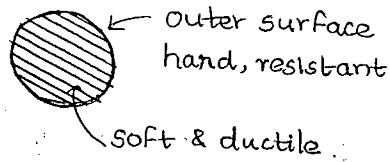
* High Carbon Steel

- Carbon increases strength and hardness but decreases ductility and toughness.

Eg : HYSD Fe 415, Fe 500 (not used nowadays)
TMT Fe 415, Fe 500 (used widely)



TMT – Thermo Mechanically Treated steel.

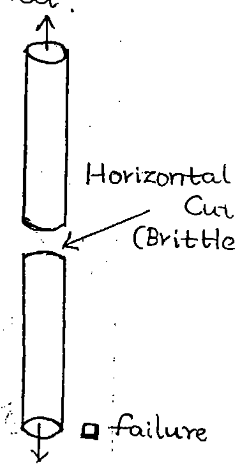


- Manganese increases toughness.

- Proof Stress or Yield Stress.

It is the stress corresponding to fixed strain (0.2%) is called Proof stress. It is used when exact yield stress is not known. It is obtained by 'Offset method'.

f_y → yield or proof stress.



Zones :

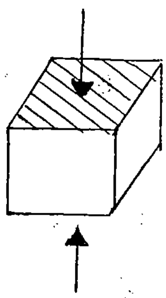
OA = linear elastic (Hooke's Law is valid)

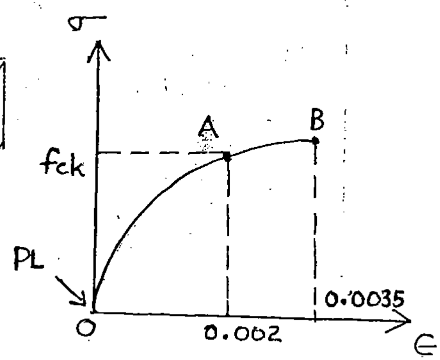
AB = non linear elastic (Hooke's Law is not valid)

BC = strain hardening zone.

→ Brittle Material.

- Stronger in compression
- moderate in shear
- weak in tension.





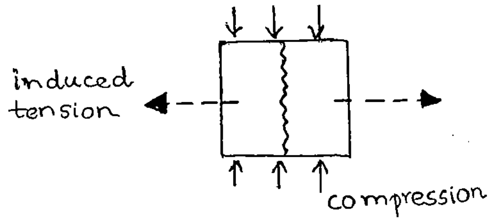
Eg: Concrete, Cast iron, glass.

- Brittle materials are tested in compression whereas ductile materials are tested in tension.

In case of brittle materials, PL will be very close to origin.

A = First cracking point.

B = Failure point.



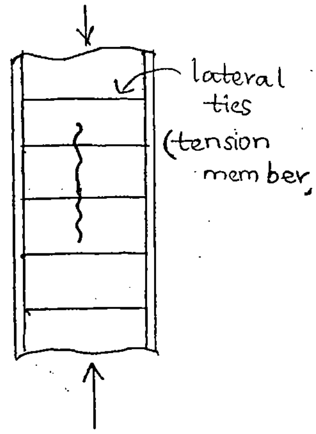
Stress corresponding to A = fck.

fck = first cracking stress (or) ultimate stress.

Stress corresponding to A = Stress corresponding to B.

Crack formation is due to induced tension.

Lateral ties are used for the confinement of concrete.



- Zones:

  OA = non linear elastic

  AB = strain hardening zone.
  (crack widening zone)

- Ductility Factor $= \frac{\epsilon_{fail}}{\epsilon_{first\ crack}} = \frac{0.0035}{0.002} = 1.75$

- Factor of Safety:

  Ductile, $FS = \frac{\text{yield stress}}{\text{Working stress}}$

  Brittle, $FS = \frac{\text{ultimate stress}}{\text{working stress}}$.

- Margin of safety:

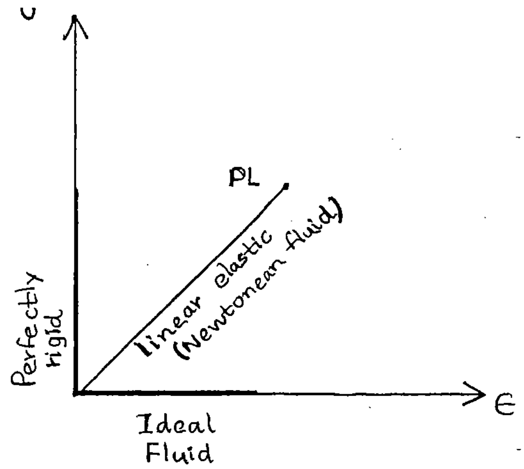
  Margin of safety = FS − 1.

used by aerospace engineers where high ductile materials are used in the aeroplane construction. ∵ high ductile materials are used, less FS is required.

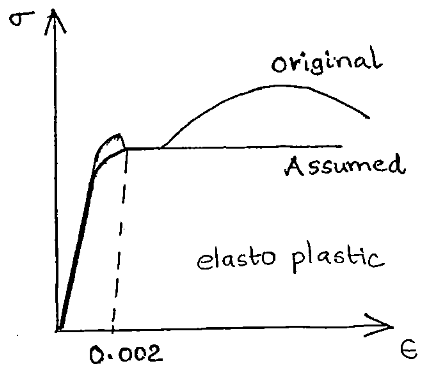
→ Idealised $\sigma - \epsilon$ curves

- assumed
- can be used in designs directly.
- For a perfectly rigid body, there won't be any dimension changes or volumetric changes. ($\delta V = 0$)

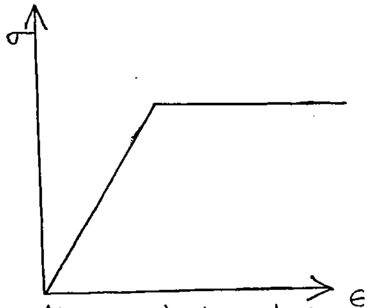
Eg: Diamond, glass.



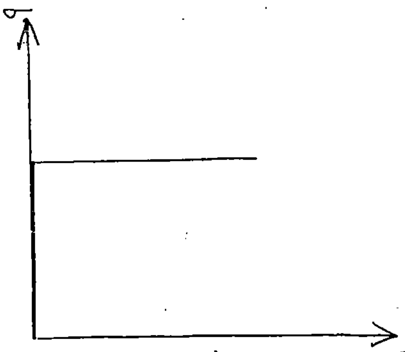
- Ideal Fluid will have dimension changes but no volume changes. as an Ideal fluid has no viscosity, no surface tension, incompressible ($\delta V = 0$), irrotational.



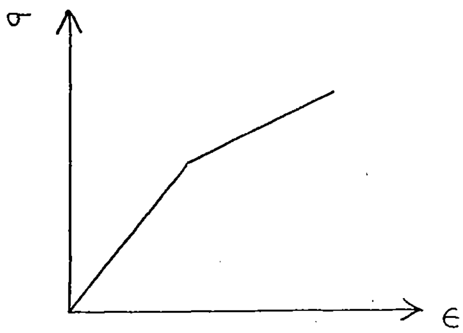
LSM → Idealised $\sigma - \epsilon$ curve for MS.



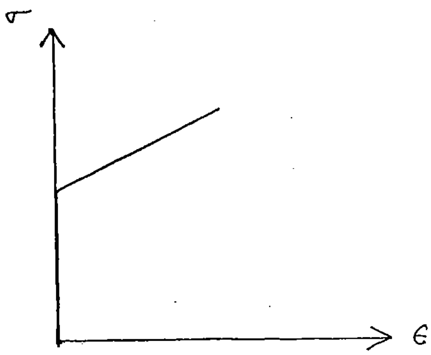
linear elastic-plastic
elasto-plastic.



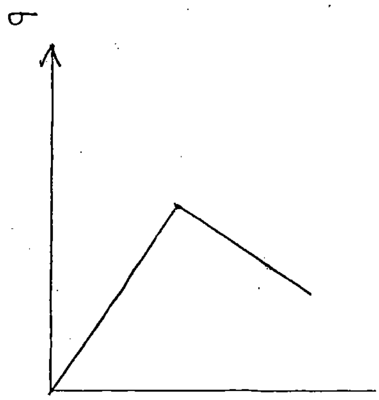
rigid - plastic



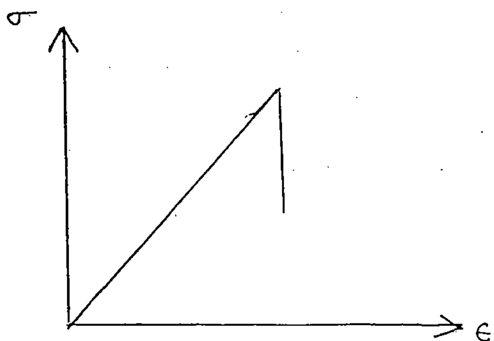
Linear elastic- strain hardening



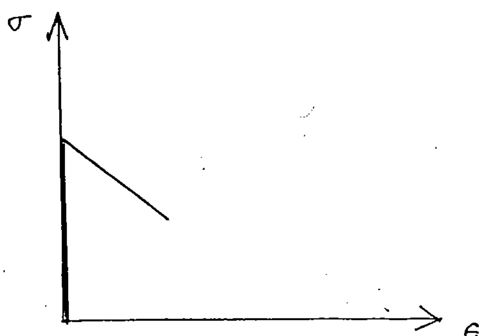
Rigid-strain hardening



linear elastic-
strain softening
(necking)



Linear elastic - yielding



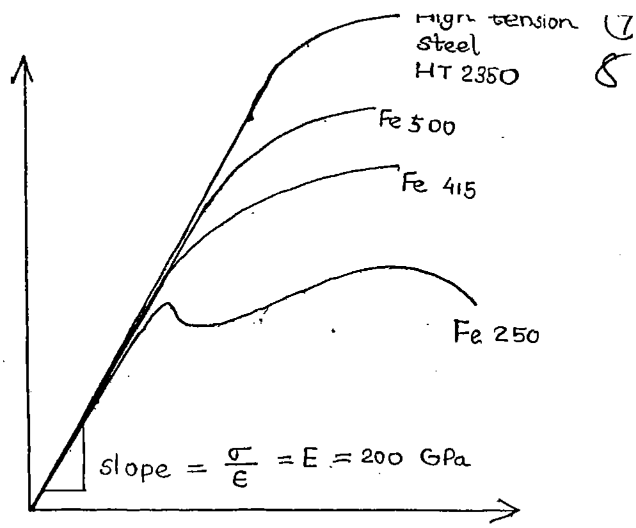
rigid - necking

→ Elastic Constants

- Within elastic limit

$\sigma \propto \epsilon$

- valid exactly upto PL.



$$\sigma = E\epsilon$$

$$\therefore E = \frac{\sigma}{\epsilon}$$

E → Young's modulus (or) Modulus of Elasticity.

It is a non zero positive value and constant for a given material under any conditions.

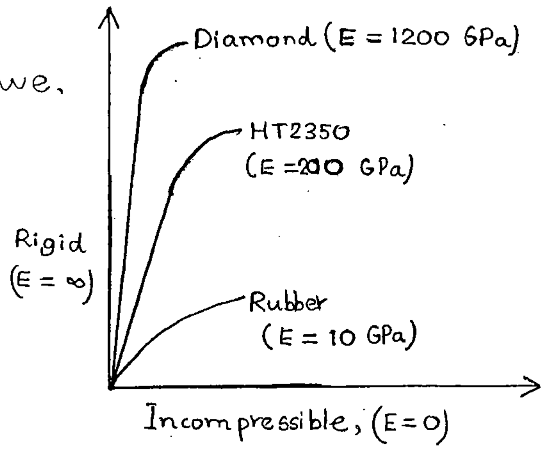
E (steel) = 200 GPa
= 200×10^3 MPa

} For all grades irrespective of carbon.

- E is the slope of $\sigma - \epsilon$ curve.

As slope increases, E also increases.

- Higher the E value, higher will be the elasticity.



- within elastic limit,

Hooke's law in shear stress gives,

$\tau \propto \gamma$ (valid upto PL)

$$\tau = G\gamma$$

$$C, N, G = \frac{\tau}{\gamma}$$

G, N, C → shear modulus, (or) rigidity modulus (or) modulus of rigidity

$\uparrow G \Rightarrow \downarrow \gamma$ (shear strain)

$\downarrow$ distortion in shape.

- volumetric stress $\propto$ volumetric strain.

$$\sigma \propto \epsilon_v$$

$\sigma \rightarrow$ Uniform Normal Stress acting all around volume (or) Volumetric stress (or) hydrostatic pressure.

⊙ On a submerged body with hydrostatic pressure, there will be only volumetric changes without change in shape.
$\therefore$ shear stress is zero.

$$\sigma = K \cdot \epsilon_v$$

Bulk modulus (or) Dilation constant } $K = \dfrac{\sigma}{\epsilon_v}$

Dilation means change in volume.

-K is used only for hydrostatic pressure conditions.

$$\uparrow K \Rightarrow \epsilon_v \downarrow \text{ ie, } \partial v \downarrow \qquad \left\{ \epsilon_v = \frac{\partial v}{V} \right\}$$

$$\downarrow K \Rightarrow \delta v \uparrow$$

$\rightarrow \dfrac{1}{K}$ = compressibility.

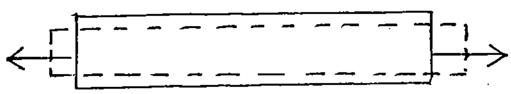
Rigid body $(\delta v = 0)$, $K = \infty$

Incompressible material, $(\delta v = 0)$, $K = \infty$.

$$\boxed{E > K > G}$$; for isotropic material.

→ Poisson's Ratio $(\mu, \gamma, 1/m)$

$$\mu = -\left(\frac{\epsilon_{lat}}{\epsilon_{lin}}\right)$$

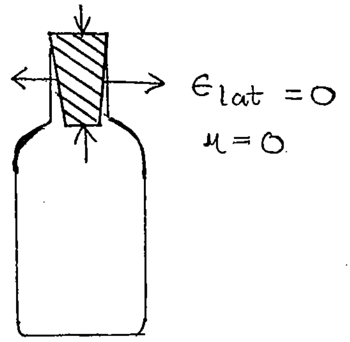


μ has no units.

Range of μ : -ve to 0.5

For genetic material, μ is -ve.

For engg. material, $0 \leq \mu \leq 0.5$



- μ (cork) $= 0$
- $\mu = 0.5$; for incompressible, non dilatant ($dv = 0$)

Eg: Ideal fluids, water.

For rubber, clay, paraffin wax, mercury, μ is nearly 0.5

For $\partial V = 0$, $\mu = 0.5$

- μ (isotropic) $= 0.25$
- μ (soft metals) $\gtrsim 0.25$

More the softness, more the ductility and hence more poisson's rat

$\mu_{(steel)} = 0.3$; $\mu_{(gold)} = 0.44$.

$\uparrow \mu \Rightarrow \uparrow$ ductility

$\uparrow E \Rightarrow \uparrow$ elasticity

- μ (brittle) < 0.25

  μ (concrete) $= 0.15$.

- μ (rigid) $= \dfrac{\epsilon_{lat}}{\epsilon_{lin}} = \dfrac{0}{0}$; not defined.

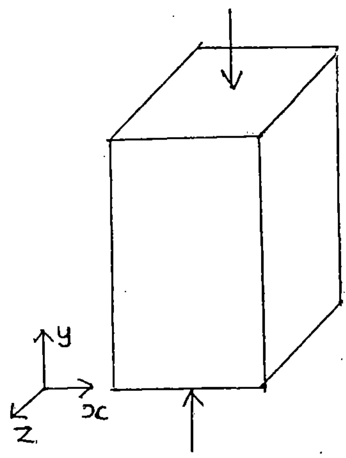
For incompressible material (ideal),

$\epsilon_{lin} = \epsilon_y = 1$ unit

as no friction b/w molecules,

$\epsilon_{lat} = \epsilon_x = \epsilon_z = \frac{1}{2}$ unit.

$$\mu = \frac{\epsilon_{lat}}{\epsilon_{lin}} = \frac{(1/2)}{1} = 0.5$$

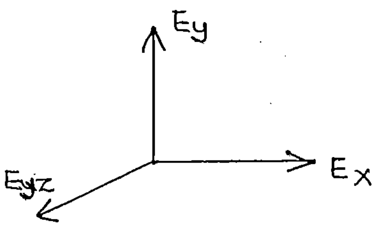


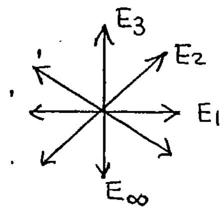
→ Relations b/w E, G, K & μ

$E = 2G(1+\mu)$
$E = 3K(1-2\mu)$
$\mu = \dfrac{3K-2G}{6K+2G}$
$E = \dfrac{9KG}{3K+G}$

Of the four elastic constants, E & μ are independent constar for homogeneous + isotropic materials.

Material	Total EC.	Independent EC
Homogeneous + Isotropic	4	2 (E, μ)
Homogeneous + Orthotropic	12	9
Homogeneous + Anisotropic	∞	21





$E_x \neq E_y \neq E_z$

$G_x \neq G_y \neq G_z$

$K_x \neq K_y \neq K_z$

$\mu_x \neq \mu_y \neq \mu_z$.

P-10

01 $\sigma = \dfrac{P}{A} = \dfrac{16000}{4\times4} = 1000 \text{ kg/cm}^2$

$\epsilon = \dfrac{dl}{l} = \dfrac{0.1}{200} = 5\times10^{-4}$

$\Rightarrow E = \dfrac{\sigma}{\epsilon} = \dfrac{1000}{5\times10^{-4}} = 2\times10^{6}$

$$E = 2G(1+\mu)$$

$$2\times10^6 = 2G\left(1+\frac{1}{4}\right)$$

$$\therefore\ G = \underline{\underline{0.8\times10^6}}\ kg/cm^2$$

5. $$\sigma = \frac{50000}{\frac{\pi}{4}d^2} = 994.718\ kg/cm^2$$

$$\epsilon_{lin} = \frac{\sigma}{E} = \frac{994.718}{10^6} = 9.947\times10^{-4}.$$

$$\mu = \frac{\epsilon_{lat}}{\epsilon_{lin}}$$

$$\epsilon_{lat} = 0.25\times\epsilon_{lin} = 2.487\times10^{-4}.$$

$$\frac{\partial D}{D} = 2.487\times10^{-4}$$

$$\therefore\ \partial D = 2.487\times10^{-4}\times8 = \underline{\underline{0.002}}\ cm$$

2. $$\epsilon_{lin} = \frac{0.03}{20}$$

$$\epsilon_{lat} = \frac{0.0018}{4} = 4.5\times10^{-4}$$

$$\mu = \frac{4.5\times10^{-4}}{0.03/20} = \underline{\underline{0.3}}$$

3. $$K = \frac{\sigma}{\epsilon_v} = \frac{\sigma}{(\partial V/V)}$$

$$2.5\times10^5 = \frac{200}{\partial V/20}$$

$$\partial V = \underline{\underline{0.016}}\ m^3$$

4. $E = 2\times10^5\ N/mm^2$

$\mu = \frac{1}{4}$

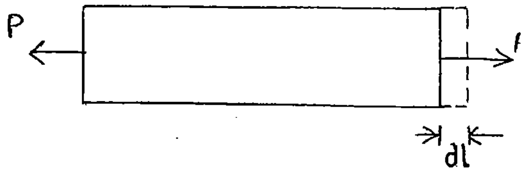
$$E = 2G(1+\mu)$$

$$2\times10^5 = 2G\left(1+\frac{1}{4}\right) \Rightarrow G = \underline{\underline{0.8\times10^5}}\ N/mm^2$$

→ Linear & Volumetric Changes

* Prismatic Bar Subjected to Axial Force

$$\sigma = \frac{P}{A} \; ; \quad \epsilon = \frac{\partial l}{l}$$



$$E = \frac{\sigma}{\epsilon} = \frac{(P/A)}{(\partial l/l)}$$

$$\boxed{\partial l = \frac{Pl}{AE}}$$

– Limitations :-

(i) Prismatic sections only.

(ii) Load upto P.L only

(iii) Gradual loads only (Hooke's Law not valid for impac load

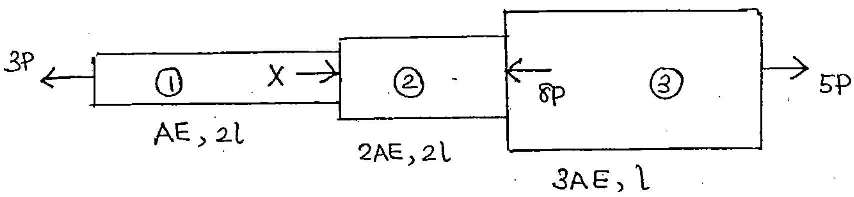
The term 'AE' is called Axial Rigidity.

Unit : $m^2 \cdot \frac{N}{m^2} = \underline{\underline{N}}$

$\uparrow AE \Rightarrow \uparrow$ rigid & stiff bar : $\downarrow \partial l$.

For perfectly rigid bodies, $AE = \infty$

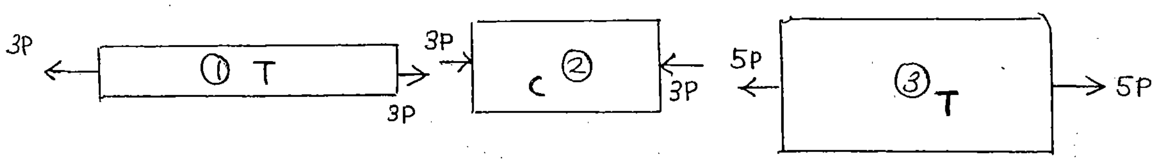
* Composite Bars



$\Sigma F_x = 0$

$\Rightarrow 5P - 8P + X - 3P = 0$

$X = +6P$ (assumed direction is correct).

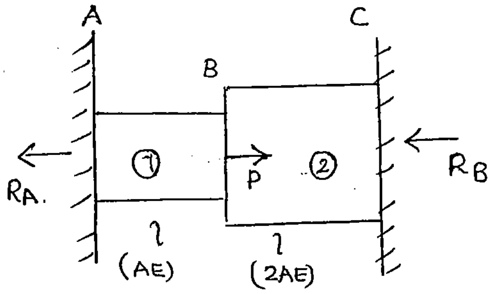


$$\delta l = \delta l_{①} + \delta l_{②} + \delta l_{③} \quad \{\text{use tension as +ve}\}$$

$$= \frac{3P \times 2l}{AE} - \frac{3P \times 2l}{2AE} + \frac{5P \times l}{3AE}$$

$$= + \underline{\underline{\frac{14Pl}{3AE}}} \quad \text{(increase in length)}$$

Q

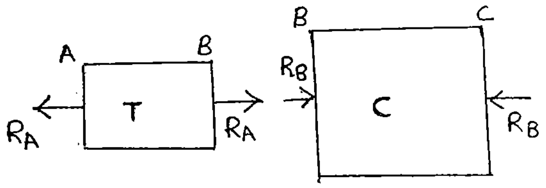


Equilibrium equation, $\Sigma F_x = 0$

$$R_A + R_B = P.$$

Compatibility condition, $\delta_{AC} = 0.$

$$\delta l_{AB} + \delta l_{BC} = 0.$$



$$\Rightarrow \frac{R_A l}{AE} + \frac{(-R_B)l}{2AE} = 0.$$

$$R_A + -\frac{R_B}{2} = 0.$$

$$\therefore R_A = \frac{P}{3}$$

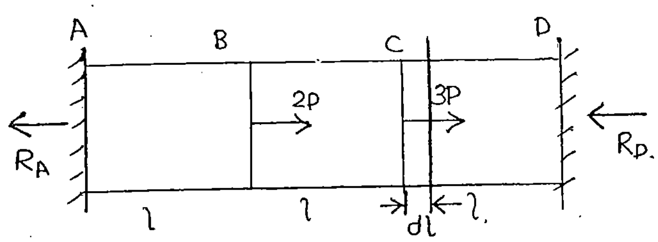
$$R_B = \frac{2P}{3}$$

Stress in $AB = \dfrac{R_A}{A} = \underline{\underline{\dfrac{P}{3A}}}$

Displacement of $B = \delta l_{AB}$ or δl_{BC}

$$= \frac{R_A l}{AE} = \underline{\underline{\frac{Pl}{3AE}}} \quad \text{(towards right)}$$

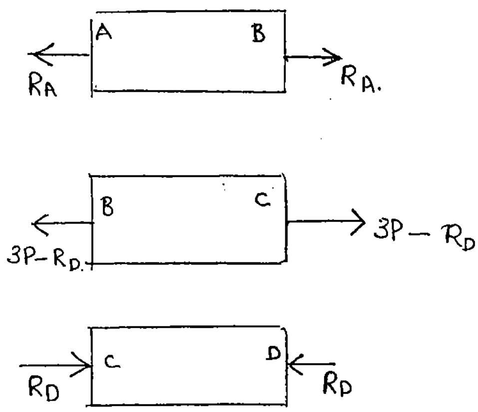
Q.



AE = const.

Find reactions ?

Equilibrium equations : $(\Sigma F_x = 0)$



$$R_A + R_D = 3P + 2P = 5P$$

Compatibility Conditions : $(\delta l_{AD} = 0)$

$$\frac{R_A l}{AE} + \frac{(3P - R_D) l}{AE} + \frac{-R_D l}{AE} = 0.$$

$$R_A - 2R_D = -3P.$$

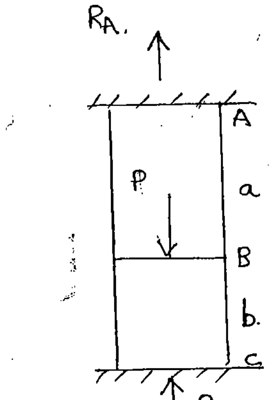
$$R_D = \frac{8P}{3}.$$

$$\underline{\underline{R_A = \frac{7P}{3}.}}$$

Displacement of B $= \delta l_{AB} = \dfrac{R_A l}{AE} = \dfrac{7PL}{3AE}$ (towards right).

Displacement of C $= \delta l_{CD} = \dfrac{R_D l}{AE} = \dfrac{8PL}{3AE}$ (towards right)

Q.



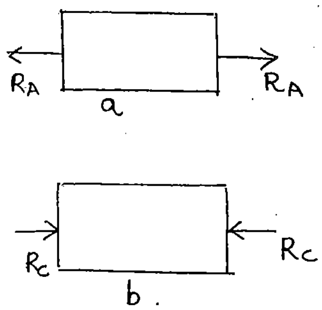
$l = a + b$

AE = constant.

Reactions = ?

$$R_A + R_C = P.$$

$$\frac{R_A \cdot a}{AE} + \frac{-R_C \cdot b}{AE} = 0.$$



$$a R_A - b R_C = 0.$$

$$a R_A = (l - a) R_C$$

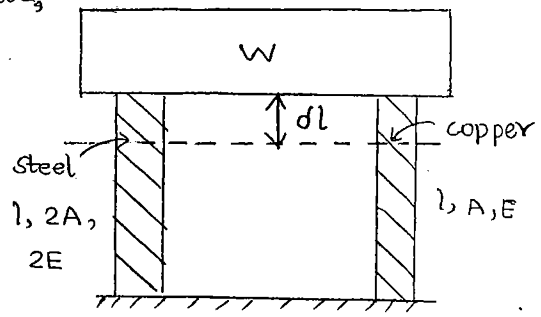
$$R_A = \left(\frac{l-a}{a}\right) R_C.$$

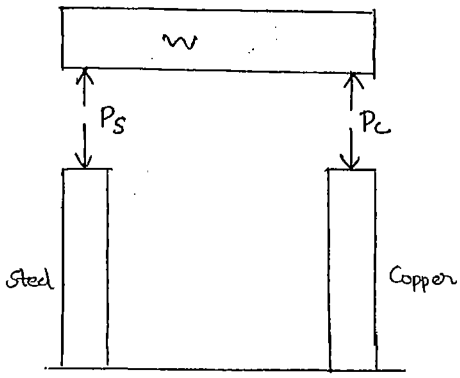
$$\left(\frac{l-a}{a} + 1\right) R_C = P.$$

$$\frac{l}{a} R_C = P \Rightarrow \underline{\underline{R_C = \frac{Pa}{l}.}}$$

$$\underline{\underline{R_A = \frac{Pb}{l}}}$$

Q. To keep the rigid body horizontal, determine the stress in steel and copper column.





$P_s + P_c = w$ (Eqbm eqn).

Compatibility condition : $dl_s = dl_c$.

$$\frac{P_s\, l}{2A \cdot 2E} = \frac{P_c\, l}{AE}$$

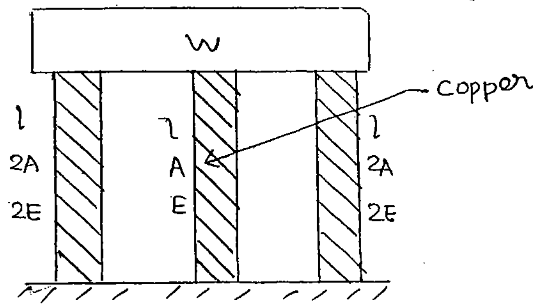
$$P_s = 4P_c.$$

$$\therefore \quad P_c = \frac{w}{5} \quad \& \quad P_s = \frac{4w}{5}$$

Stress in steel column $= \dfrac{P_s}{A} = \dfrac{4w/5}{2A} = \dfrac{2w}{5A}$ (compression).

Stress in copper column $= \dfrac{P_c}{A} = \dfrac{w/5}{A} = \dfrac{w}{5A}$ (compression).

Q. Two steel bars and a copper bar are supporting a rigid bar of weight w. Calculate stresses.



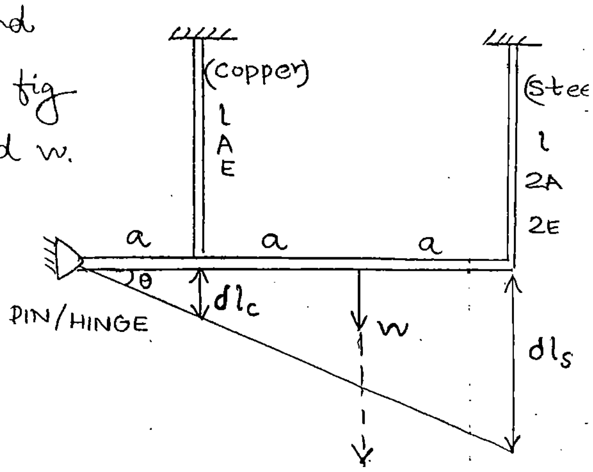
$2P_s + P_c = w.$ $(\Sigma F_x = 0)$

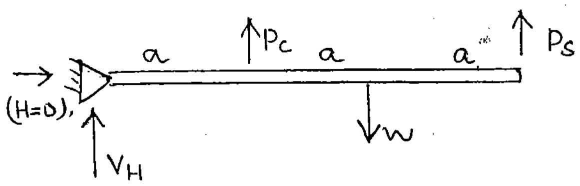
$$\frac{P_s l}{2A \cdot 2E} = \frac{P_c l}{AE}$$

$$P_s = 4P_c$$

$$\therefore P_c = \frac{w}{9} \quad \& \quad P_s = \frac{4w}{9}$$

Q A rigid bar is hinged at one end and supported by two wires as shown in fig. Determine stresses developed due to load w.





Taking moments about hinge,

$$P_c \cdot a + P_s \times 3a = w \times 2a$$

$$P_c + 3P_s = 2w$$

Using similar triangles,

$$\frac{dl_c}{a} = \frac{dl_s}{3a}$$

$$dl_c = \frac{dl_s}{3}$$

$$\frac{P_c \cdot l}{AE} = \frac{P_s \cdot l}{4AE \cdot 3}$$

$$P_s = 12 P_c$$

$$\therefore P_c = \frac{2w}{37} \quad \& \quad P_s = \frac{24w}{37} \quad \text{(tension)}$$

Stress in steel wire, $\sigma_s = \dfrac{24w}{37 \times 2A} = \dfrac{12w}{37A}$

Stress in copper wire, $\sigma_c = \dfrac{2w}{37A}$

$$P_s + P_c = \frac{24w}{37} + \frac{2w}{37} = \frac{26w}{37}$$

$$P_s + P_c + V_H = w$$

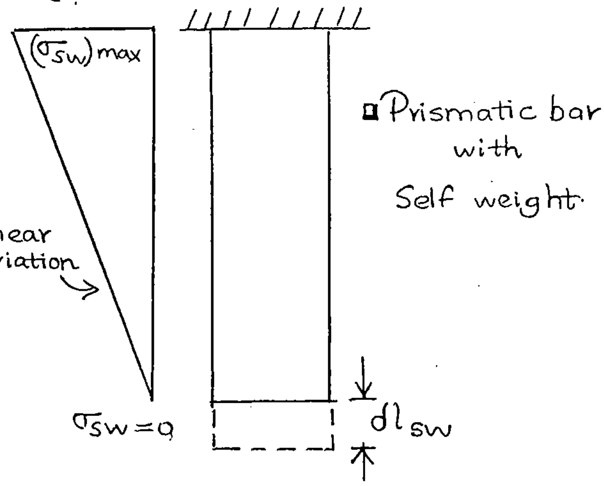
$$\therefore V_H = w - \frac{26w}{37} = \frac{11w}{37}$$

* Self weight Deformation

$$dl_{sw} = \frac{wl}{2AE}$$

$$= \frac{(\gamma A l)\, l}{2AE}$$

$$\boxed{(dl)_{sw} = \frac{\gamma l^2}{2E}}$$

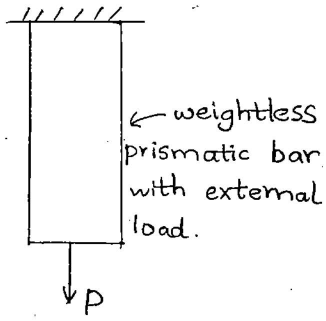


NOTE :

- Self weight deformation is independent of shape and area of c/s, directly proportional to square of length
- Self weight deformation is half that of same self weight attached at the end of a similar weightless bar.

$$P = w$$

$$(dl)_{ext} = \frac{PL}{AE} = \frac{wL}{AE}$$



- Stress due to self weight, $\sigma_{sw} = \frac{w}{A}$

$w \rightarrow$ wt below a c/s, where stress is required.

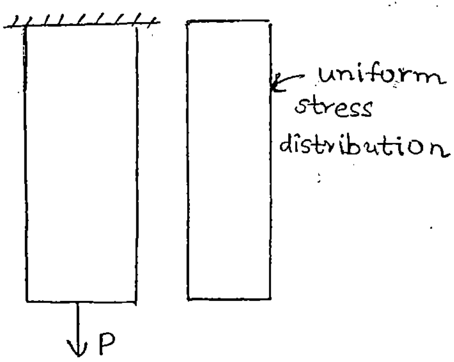
$$(\sigma_{sw})_{\text{free end}} = 0$$

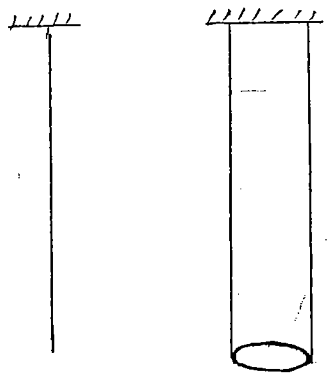
$$(\sigma_{sw})_{\text{fixed end}} = \frac{w}{A} = \frac{\gamma A l}{A} = \gamma l$$

- Stress due to self weight is also independent of shape and area of c/s, directly proportional to length.

Weightless prismatic bar with external load, $\sigma_{ext} = \frac{P}{A}$

Uniform stress distribution which is independent of length.





l = same

E, γ = same

$(\delta l)_{sw} = \frac{\gamma l^2}{2E} \rightarrow$ same

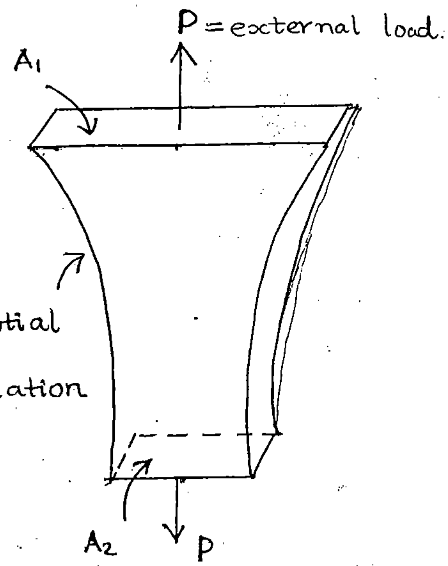
$(\sigma)_{sw} = \gamma l \rightarrow$ same.

→ Bar of Uniform Strength.

Along the length of a bar, if stress developed is constant then it is bar of uniform strength.

Eg:- weightless prismatic bar subjected to external loading. In practise weightless members are not possible. Self weight will be acting along with external load. In such a case, prismatic members cannot be bar of uniform strength

* Bar of Uniform Strength with Self wt + External load.



$$\frac{A_1}{A_2} = e^{(\gamma l/\sigma)}$$

$$\ln\left(\frac{A_1}{A_2}\right) = \frac{\gamma l}{\sigma}$$

$\gamma \rightarrow$ wt. density.

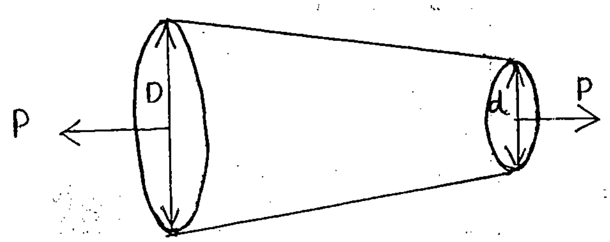
$l \rightarrow$ length of bar.

$\sigma \rightarrow$ const. / uniform stress along the length of bar.

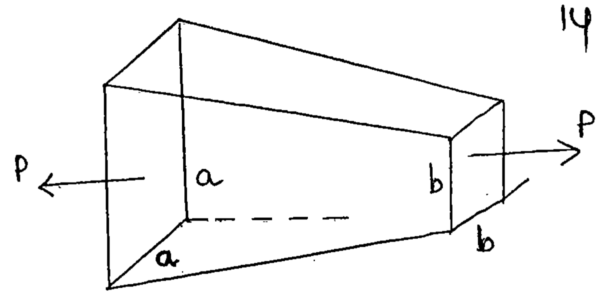
19th Sept, FRIDAY

→ Tapering Bars

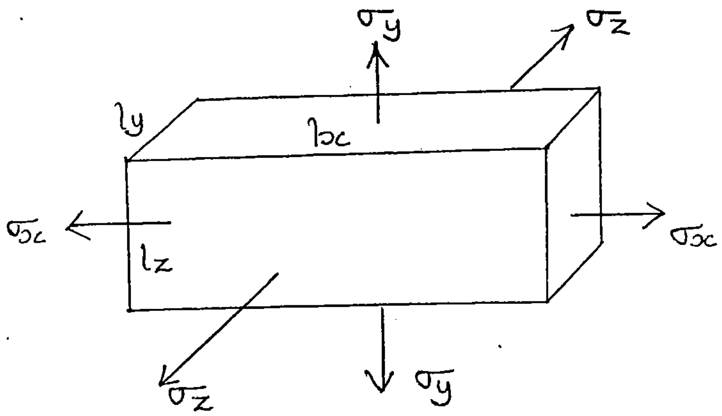
* $$\delta l = \frac{Pl}{\frac{\pi}{4}(Dd)E}$$



$$* \ \delta l = \frac{Pl}{(a.b)E}$$



→ Volumetric Strain.

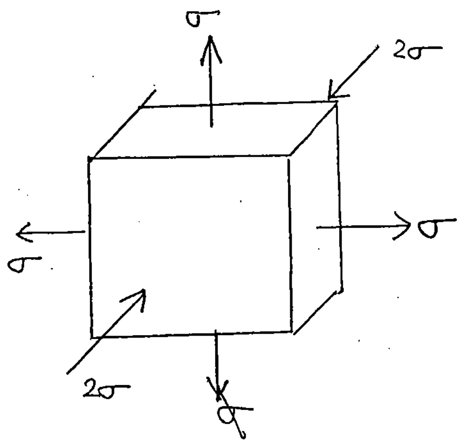


$$\frac{\delta l_x}{l_x} = \epsilon_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}$$

$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E} - \mu\frac{\sigma_z}{E}$$

$$\epsilon_z = \frac{\sigma_z}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_y}{E}$$

Q. Find δv for the cube shown...?



$$\boxed{\frac{\partial v}{V} = \epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z.}$$

Put $\sigma_x = +\sigma$, $\sigma_y = \sigma$, $\sigma_z = -2\sigma$.

$$\epsilon_x = \frac{\sigma}{E} - \frac{\mu\sigma}{E} + \mu\frac{2\sigma}{E} = \frac{\sigma}{E} + \mu\frac{\sigma}{E}$$

$$\epsilon_y = \frac{\sigma}{E} - \mu\frac{\sigma}{E} + \frac{\mu 2\sigma}{E} = \frac{\sigma}{E} + \mu\frac{\sigma}{E}.$$

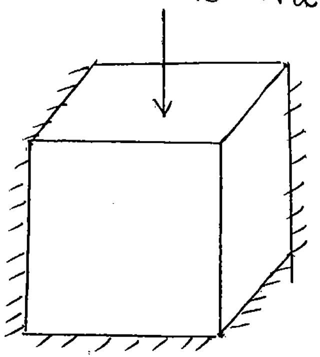
$$\epsilon_z = -2\cancel{\mu}\frac{\sigma}{E} - \mu\frac{\sigma}{E} - \mu\frac{\sigma}{E} = -\frac{2\sigma}{E} - 2\mu\frac{\sigma}{E}$$

$$\frac{\partial v}{V} = \epsilon_x + \epsilon_y + \epsilon_y = \underline{\underline{0}}$$

Q A cube of size 'a' is restrained in all directions and free at the top. A compressive stress of 10 MPa is applied in y direction as shown in fig. Determine ① Uniform stress developed in x & z directions. ② strain in y direction

$\sigma_x = ?,\ \sigma_y = -10\ \text{MPa},\ \sigma_z = ?$.

~~200~~ 10 MPa



E = 200 GPa.

$\mu = 0.3$.

$$\frac{\partial V}{V} = \epsilon_x + \epsilon_y + \epsilon_z .$$

$$\epsilon_x = +\frac{\mu\sigma_y}{E} + \frac{\sigma_x}{E} - \frac{\mu\sigma_z}{E}$$

$$\epsilon_y = -\frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_z}{E}$$

$$\epsilon_z = +\frac{\mu\sigma_y}{E} - \frac{\mu\sigma_x}{E} + \frac{\sigma_z}{E}$$

But $\epsilon_x = \epsilon_z = 0$.

$$0 = \frac{0.3\times 10}{2\times 10^5} + \frac{\sigma_x}{E} - \frac{0.3\,\sigma_z}{E}$$

$$\Rightarrow \sigma_x - 0.3\,\sigma_z + 3 = 0 .$$

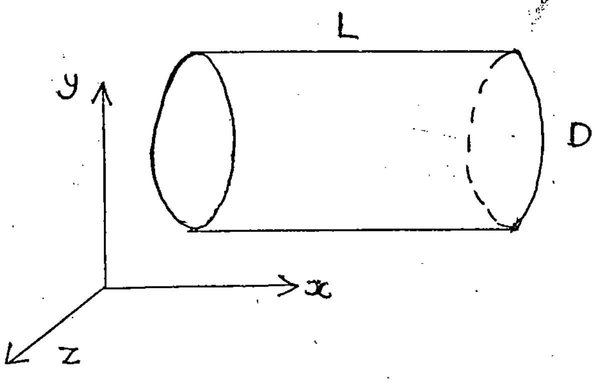
$$\sigma_x = \sigma_z = \sigma$$

$$\therefore\ \sigma_x = \sigma_y = -4.29\ \text{MPa (compressive)}.$$

$$\epsilon_y = -\frac{10}{E} - \frac{0.3\times -4.29}{E} - \frac{0.3\times -4.29}{E}$$

$= -3.713\times 10^{-5}$ ~~mm~~ = ~~0.0037~~ mm (–ve mean ↓ in dimension)

→ Cylinder



$$\epsilon_V = \epsilon_x + \epsilon_y + \epsilon_z.$$

$$= \frac{\partial l}{l} + \frac{\partial D}{D} + \frac{\partial D}{D.}$$

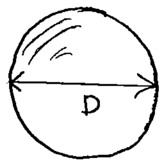
$$= \epsilon_l + \epsilon_h + \epsilon_h$$

$$\boxed{\epsilon_v = \epsilon_l + 2\epsilon_h}$$

$\epsilon_l \rightarrow$ linear / axial / longitudinal strain.

$\epsilon_h \rightarrow$ hoop / circumferential strain

$\rightarrow$ Sphere



$$\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z$$

$$= \frac{\partial D}{D} + \frac{\partial D}{D} + \frac{\partial D}{D}.$$

$$\boxed{\epsilon_v = 3\epsilon_h}$$

P-4

Q1. Scalar : Magnitude + No direction. Eg: distance, speed.

Vector : Magnitude + One direction. Eg: displacement, velocity.

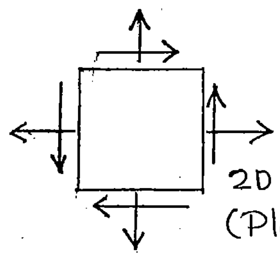
Tensor : Magnitude + more than one direction.

Eg:- stress, strain, MI

$$\begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}_{3\times3}$$

component of stresses (spatial)

Tensors can be expressed in Matrix form for computer applicatio



2D tensor (Plane stress)

Q-6

Q6. Visco-elastic $\rightarrow$ Elasto plastic.

Q7. Tenacity — maximum tensile strength.

P-9

3. $E = 2G(1+\mu)$.

when $\mu = 0$, $\frac{G}{E} = 0.5$

when $\mu = 0.5$, $\frac{G}{E} = 0.33$

$\Rightarrow G = (0.33 \text{ to } 0.5)E$

→ Temperature Stresses :

- Indirect stress.
- external loads are direct stresses.

α → coeffecient of linear (thermal) expansion.

It is the strain developed per unit change in temperatu
'α' is a material property and is constant for given material.

$$\alpha_{steel} = \alpha_{concrete} = 12 \times 10^{-6} \ /^\circ C.$$

$\uparrow \alpha$: $\uparrow$ active for temperature.

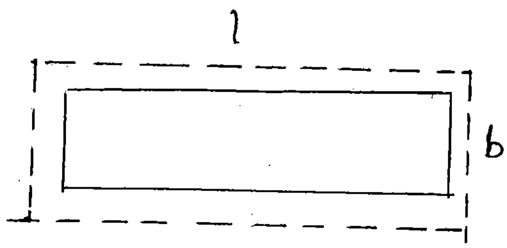
$$\epsilon_t = \alpha t.$$

$$\frac{\partial l}{l} = \epsilon_t = \alpha_t.$$

$$\Rightarrow \partial l = l \alpha t$$

$$\frac{\partial b}{b} = \epsilon_t = \alpha t.$$

$$\Rightarrow \partial b = b(\alpha t)$$



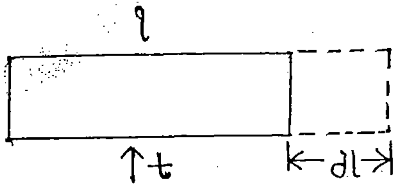
Prismatic steel bar ($\uparrow t$).

⊙ As temperature increases due to uniform heating, all the dimensions increase. Due to uniform cooling, all the dimensi decrease.

(i) Prismatic bar free to expand or contract.

$$\epsilon_t = \alpha t.$$

$$\frac{\partial l}{l} = \alpha t$$

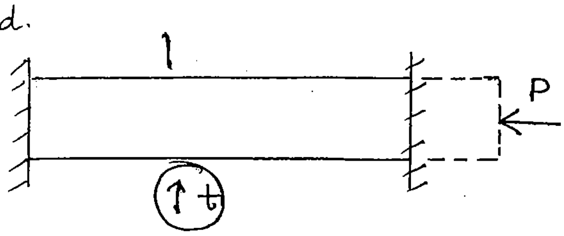


⇒ Free expansion along length, $\partial l = l \alpha t$

Member is free to expand or contract, therefore no stress will be induced.

(ii) Fixed Rigidly (along length)

Free expansion = Expansion prevented.



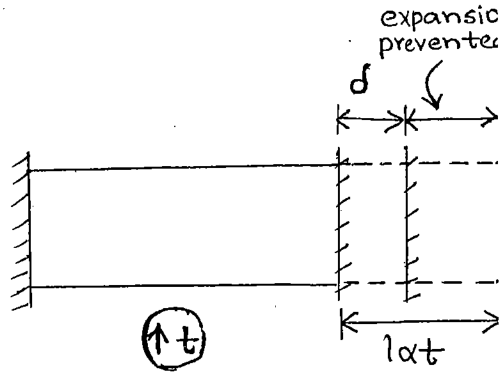
$$l\alpha t = \frac{Pl}{AE}$$

$$\boxed{\sigma_t = (\alpha t)E}$$

↑t : compression

↓t : tension.

(iii) Yielding Supports.



If $l\alpha t \leq \delta$; no stress developed.

If $l\alpha t > \delta$; stress developed.

$$\text{Expansion prevented}, = \frac{Pl}{AE}$$

$$l\alpha t - \delta = \frac{Pl}{AE}$$

$$\sigma_t = \frac{(l\alpha t - \delta)E}{l}$$

- Due to ~~yielding~~ heating supports move outward, come closer due to cooling

Q. A steel bar of 5m length is at a room temp of 30°c. The bar is uniformly heated to 90°c. Determine temperature stress developed if bar is:

(i) free to expand.
(ii) expansion prevented. along length.
(iii) Supports yield by 1.5 mm along length.
(iv) Supports yield by 5 mm along length.
use $E = 200$ GPa, $\mu = 0.3$.

Ans: (i) zero.

(ii) $\sigma_t = \alpha t E = 12 \times 10^{-6} \times (90-30) \times 200 \times 10^3$ MPa.

$= 144$ MPa.

(iii) $\delta l = l\alpha t = 5 \times 12 \times 10^{-6} \times 60 = 3.6$ mm.

$$\sigma_t = \frac{(l\alpha t - \delta)E}{l} = \frac{(3.6 - 1.5)}{5000} \times 2 \times 10^5$$

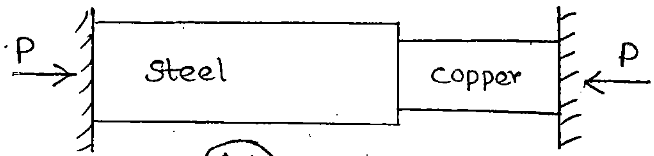
$$= \underline{\underline{84 \text{ MPa}}}$$

(iv) $\delta l < \delta \Rightarrow$ $\underline{\underline{\text{no}}}$ stress.

→ Composite Bars

– made of different materials.

* Series :



Free expansion of both bars = expansion prevented by both bars

$\uparrow t$ $\alpha_c > \alpha_s$

$$(l\alpha t)_s + (l\alpha t)_c = \left(\frac{Pl}{AE}\right)_s + \left(\frac{Pl}{AE}\right)_c$$

$$P_s = P_c = P$$

$$\sigma_s = \frac{P}{A_s} \quad ; \quad \sigma_c = \frac{P}{A_c}$$

For rigid supports, $\uparrow t$: compression.

$\downarrow t$: tension.

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Q.6. $L_s = L_a = 1$ m; $\alpha_s = 11 \times 10^{-6}/°C$; $\alpha_a = 24 \times 10^{-6}/°C$

$E_s = 200$ GPa, $E_a = 70$ GPa ; $A_s = 100$ mm^2, $A_a = 200$ mm^2

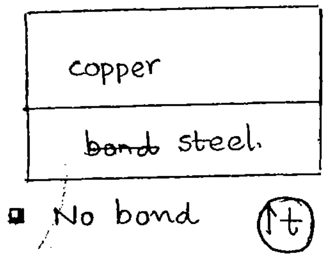
$\Delta t = 58° - 38° = 20°$.

$$1 \times 11 \times 10^{-6} \times 20 + 20 \times 1 \times 24 \times 10^{-6} = \frac{P \times 1}{100 \times 10^{-6} \times 200 \times 10^3} + \frac{P \times 1}{200 \times 70 \times 10^3}$$

$$P = \underline{\underline{5.76 \text{ kN}}}$$

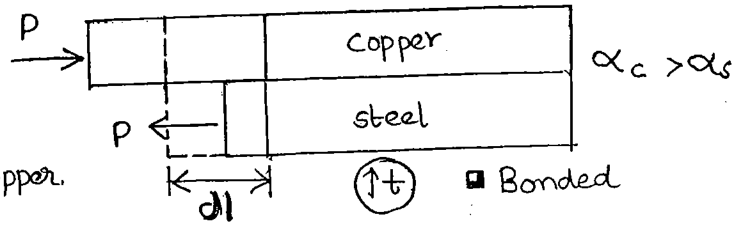
* Parallel.

- Uniform heating: no warping



As there is no bond and no supports, both copper and steel will expand individually upon heating and ∴ no stresses are induced.

- Net change in length of steel = net change in length of copper.

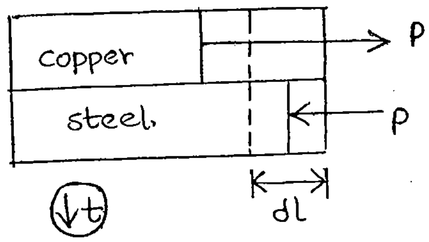


$$\left(l\,\alpha\,t\right)_s + \left(\frac{Pl}{AE}\right)_s = \left(l\,\alpha\,t\right)_c - \left(\frac{Pl}{AE}\right)_c$$; (compatibility condition)

$(\alpha\downarrow)$ $(\alpha\uparrow)$

$$P_s = P_c = P$$

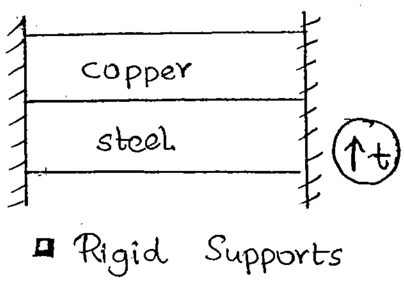
Same compatibility equation can be used for both increase and decreased in temperature, the nature of stresses should be changed accordingly.



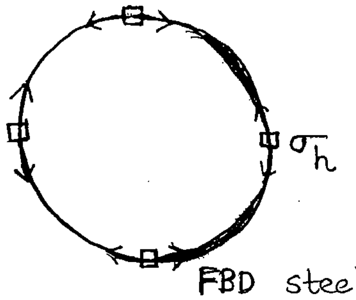
- For ideal composite material, α must be nearly equal.

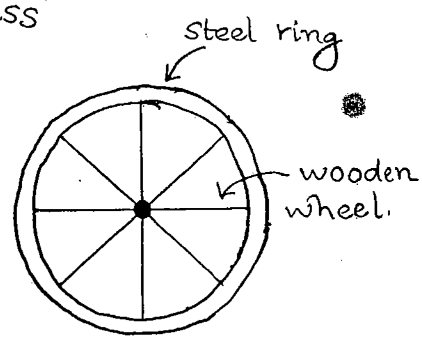
Eg: Concrete & Steel

- Both in compression if between rigid supports.



→ Hoop Stress (or) Circumferential Stress





$d \rightarrow$ initial diameter of steel ring
$D \rightarrow$ diameter of rigid wooden wheel.
$D \rightarrow$ final diameter of steel ring

- Hoop strain $= \epsilon_h = \dfrac{\pi D - \pi d}{\pi d}$.
- Hoop stress, $\sigma_h = \epsilon_h E$

$$= \left(\frac{D-d}{d}\right)E.$$

∴ tension in steel ring & compression in wooden wheel.

- Min increase in temperature for fixing,

$$\epsilon_h = \epsilon_t.$$

$$\frac{D-d}{d} = \alpha t.$$

$$\Rightarrow \boxed{t = \frac{D-d}{\alpha d}}$$

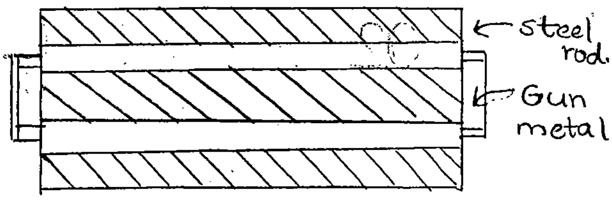
Q. A steel ring of 499 mm ϕ is to be fitted over a wooden wheel 500 mm ϕ. E of steel = 200 GPa, $\alpha_s = 12 \times 10^{-6}$ /°C. Determine (i) hoop stress developed. (ii) min increase in temp. for fixing.

(i) $\sigma_h = \left(\dfrac{D-d}{d}\right)E = \dfrac{(500-499)}{499} \times 2 \times 10^5 = 400.8$ MPa

(ii) Min. $t = \dfrac{D-d}{d\alpha} = \dfrac{500-499}{499 \times 12 \times 10^{-6}} = \underline{\underline{167°C}}$

Q.08 Parallel $(\alpha_g > \alpha_s)$

$(\downarrow t = 200\,^\circ F)$.



$$(l\alpha t)_g - \left(\frac{Pl}{AE}\right)_g = (l\alpha t)_s + \left(\frac{Pl}{AE}\right)_s$$

$$10\times10^{-6}\times200 - \frac{P}{200\times100\times10^3} = 6\times10^{-6}\times200 + \frac{P}{100\times200\times10^3}$$

$$\underline{\underline{P = 8\ kN}}$$

Q.09. $\sigma_s = \frac{P}{A_s} = \frac{8000}{100} = \underline{\underline{80\ MPa}}$

$\sigma_{gm} = \frac{P}{A_g} = \frac{8000}{200} = \underline{\underline{40\ MPa}}$

Q.05. $(\alpha t)_a - \left(\frac{Pl}{AE}\right)_a = (\alpha t)_s + \left(\frac{Pl}{AE}\right)_s.$

$25\times10^{-6}\times80 - \frac{P}{\quad}$