5. ~~Thermo~~ Electro dynamics

ohm's law :-

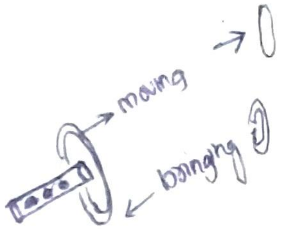
we know that $V = IR$ & $V = El$ & $R = \rho\frac{l}{A}$

$$El = IR$$

$$El = I\,\rho\frac{l}{A}$$

$$El = JA\,\rho\frac{l}{A}$$

$$E = J\rho$$



$$E\frac{1}{\rho} = J$$

$$\boxed{J = \sigma E}$$

EMF :-

* the electromotive force (EMF) can be defined as the driving force which drives charges from one point to another.
* the driving force can be denoted by f_s & written as $f_s + E = f$ where f_s is driving force & E is electric force.
* within in the ideal source of EMF $(R=0)$ [i.e (Resistance less battery) in the sense the electric force i.e the net force (f) is 0 i.e $f=0$;

$$f_s + E = 0$$

$$\boxed{f_s = -E} \quad -(1)$$

* The potential difference between the two terminals (V) is given by

$$V = -\int E\cdot dl = +\int f_s\,dl \quad [\because f_s = -E]$$

$$\boxed{V = \int f_s\,dl}$$

$$\boxed{V = \int f_s\,dl = \mathcal{E}}$$

* In general we can EMF & potential are same.

Motional EMF (INDUCED EMF):-

* When we move conducting wire through uniform magnetic field, then EMF will be induced which is equal to the rate change in magnetic flux lines

$\int B . d\ell$ / dt $\quad \varepsilon = \frac{d\phi}{dt}$ $\quad \phi = \int B\, da$

$$\boxed{\varepsilon = \frac{-d\phi}{dt}} = \oint B . dl$$

where ϕ is flux

* According to faraday law, change in magnetic field ~~induced~~ induces an electric field.

$\varepsilon = \frac{-d\phi}{dt}$

* To understand faraday's law,

Experiment 1: pull the conducting loop of wire from the uniform magnetic field.

observation Current will be induced in loop and this is because of motional emf ($\varepsilon = \frac{-d\phi}{dt}$)

Experiment 2: move the magnet by holding the loop in same position.

observation: current will flow in the loop and this is because of ~~mo~~ the electric field exerts a force on charge particle

* If the loop moves, it is the magnetic field that setup EMF but as stationary charges experience No magnetic field, forces. The forces won't be magnetic in the stationary loop condition.

Faraday's Laws:-

Faraday quoted that change in magnetic field induces an electric field and emphirically it can be written as

$$\varepsilon = \oint E . dl = \frac{-d\phi}{dt} \quad \text{where } \int \frac{dB}{dt} da = \frac{d\phi}{dt}$$

$$\varepsilon = \oint E . dl = -\int \frac{\partial B}{\partial t} da \; ; \quad \int (\nabla \times E)\, da = -\int \frac{\partial B}{\partial t} da$$

$$\boxed{\nabla \times E = \frac{-\partial B}{\partial t}} \text{ differential form.}$$

$\varepsilon = \oint E\, dl$ / $\frac{-d\phi}{dt}$

Maxwell's Equation :-

① $\oint E \cdot da = \frac{Q}{\varepsilon_0}$ $\qquad \nabla \cdot E = \frac{\rho}{\varepsilon_0}$

② $\oint B \cdot da = 0$ $\qquad \nabla \cdot B = 0$

③ $\oint E \cdot dl = -\int \frac{dB}{dt} da$ $\qquad \nabla \times E = -\frac{\partial B}{\partial t}$

④ $\oint B \cdot dl = \mu_0 i$ $\qquad \nabla \times B = \mu_0 J$

* From continouity equation :-

$$\nabla \cdot J = -\frac{\partial \rho}{\partial t} \qquad \nabla \cdot E = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot J = -\frac{\partial}{\partial t}(\varepsilon_0 \nabla \cdot E)$$

$$\nabla \left(J + \varepsilon_0 \frac{\partial E}{\partial t} \right) = 0$$

here $\varepsilon_0 \frac{\partial E}{\partial t}$ is called displacement current

* we know that,

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\underbrace{\nabla \cdot (\nabla \times E)}_{0} = -\frac{\partial}{\partial t}\underbrace{(\nabla \cdot B)}_{0}$$

* also $\nabla \times B = \mu_0 J$

$$\boxed{\nabla \cdot (\nabla \times B) = \mu_0 \nabla \cdot J}$$

'0' when volume charge density is constant.

Magnetic Energy ~~Destiny~~ Density :-

* the change in current in a loop induces an EMF in that loop (same loop). the flux is proportional to the current.

$\phi \propto i$

$$\boxed{\Phi = Li} \quad -①$$ where L is self inductance.

* we know that Induced Emf $\varepsilon = -\frac{d\phi}{dt}$ and $\phi = \int B.ds$

$$\varepsilon = -\frac{d}{dt}(LI)$$

$$\boxed{\varepsilon = -L\frac{dI}{dt}.}$$

$V = -L\frac{dI}{dt}$

* the work done on a unit charge againest back emf in one trip around the circuit is equal to $-\varepsilon$.
* the amount of charge per unit time passing down the wire
* so the total work done is,

$-\varepsilon$

$\frac{q}{t}$

$$\varepsilon = -\frac{d\phi}{dt} = -L\frac{dI}{dt}$$

$\phi \propto i$

$\phi = LI$

Self inductance.

$\varepsilon = -\frac{d\phi}{dt}$

$\varepsilon = -\frac{d}{dt}(LI)$

$\varepsilon = -L\frac{dI}{dt}$

$$w \propto \varepsilon$$

$$w = -\varepsilon q$$

$$\frac{dw}{dt} = -\varepsilon\frac{dq}{dt}$$

$$\frac{dw}{dt} = -\varepsilon . I$$

$$\frac{dw}{dt} = LI\frac{dI}{dt}$$

$$\int\frac{dw}{dt} = \int LI\frac{dI}{dt}$$

$$\boxed{w = \frac{1}{2}LI^2}$$

$W = -Vq$

$V = \frac{W}{q}$

$W = Vq$

$$W = \frac{1}{2} L I^2$$

$$W = \frac{1}{2}(LI)I = \frac{1}{2}\Phi I$$

we know that $\Phi = \int_{Surface} B \cdot ds$

we know $\underline{B} = \int \nabla \times A \Rightarrow B = \nabla \times \vec{A}$

$\nabla \times A$

$$\therefore \Phi = \int (\nabla \times A) \cdot ds$$

$$\Phi = \int_{line} A \cdot d\vec{l} \quad \text{(using stoke's theorem)}$$

$$W = \frac{1}{2} \Phi I$$

$\phi = \int B \cdot ds$

$(\nabla \times A) ds$

$$W = \frac{1}{2} I \int_{line} A \cdot dl$$

$$W = \frac{1}{2} \int_{line} (A \cdot I) dl \quad \text{(for linear current density)}$$

* For Volume current density :- $W = \frac{1}{2} \int (A \cdot J) d\tau$

$$W = \frac{1}{2} \int_V (A \cdot J) d\tau$$

$\nabla \times B = \mu_0 J$

* To represent work done in terms of magnetic field we know that

$$\nabla \times B = \mu_0 J \quad \text{(due to Amphere law)}$$

$$W = \frac{1}{2\mu_0} \int (A \cdot \mu_0 J) d\tau$$

$$\frac{\frac{1}{2\mu_0} \int B^2 d\tau}{\int d\tau}$$

$$W = \frac{1}{2\mu_0} \int (A \cdot \nabla \times B) d\tau$$

$$\frac{B^2}{2\mu_0}$$

$$\nabla \cdot (A \times B) = B \cdot (\nabla \times A) - A \cdot (\nabla \times B)$$

$$\boxed{A \cdot \nabla \times B = B \cdot (\nabla \times A) - \nabla \cdot (A \times B)}$$

$$W = \frac{1}{2\mu_0} \int B \cdot (\nabla \times A) d\tau - \frac{1}{2\mu_0} \int \nabla \cdot (A \times B) d\tau$$

$$W = \frac{1}{2\mu_0} \int_V B^2 d\tau - \frac{1}{2\mu_0} \int_S (A \times B) da$$

work done/ Energy in magnetic field) is

$$W = \frac{1}{2\mu_0}\int B^2 d\tau$$

$$\boxed{W = \frac{1}{2}\int \frac{B^2}{\mu_0} d\tau}$$

we know energy in electric field $E = \frac{1}{2}\int \varepsilon_0 E^2 d\tau$

* for Electromagnetic waves, total energy will be,

workdone $= W + E$

$$\boxed{W_{EB} = \frac{1}{2}\int \frac{B^2}{\mu_0} d\tau + \frac{1}{2}\int \varepsilon_0 E^2 d\tau}$$

Maxwell's Equations

write maxwell's equations in free space, dielectrics, vaccum

① $\int E.da = \frac{Q_{en}}{\varepsilon_0}$ $\qquad \nabla.E = \frac{\rho}{\varepsilon_0}$

② $\oint B.da = 0$ $\qquad \nabla.B = 0$

③ $\int E.dl = -\frac{d}{dt}\int_S B.da$ $\qquad \nabla \times E = -\frac{\partial B}{\partial t}$.

④ $\int B.dl = \mu_0 I_{en} + \mu_0\varepsilon_0 \int_S \frac{dE}{dt} da$ $\qquad \nabla \times B = \mu_0 J + \mu_0\varepsilon_0\frac{\partial E}{\partial t}$.

In free space ($\rho = 0$ and $J = 0$)

① $\int E.da = 0$ and $\nabla.E\ 0$; ② $\oint B.da = 0$ and $\nabla.B = 0$.

③ $\int E.dl = -\frac{\partial}{\partial t}\int_S B.da$; ③ $\nabla \times E = -\frac{\partial B}{\partial t}$.

④ $\int B.dl = \mu_0\varepsilon_0 \int_S \frac{dE}{dt} da$ and $\int$ $\nabla \times B = \mu_0\varepsilon_0 \frac{\partial E}{\partial t}$.

Consider Equation ④ & ③

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times (\nabla \times E) = \frac{-\partial}{\partial t}(\nabla \times B)$$

$$-\nabla^2 E = \frac{-\partial}{\partial t}\left(\mu_0\varepsilon_0 \frac{\partial E}{\partial t}\right)$$

$$\nabla^2 E = +\mu_0\varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$\boxed{\nabla^2 E - \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} = 0}$$

this is similan to the generalised form of wave equation

$$\frac{\partial^2 F}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 F}{\partial t^2} = 0$$

* The equation is called as Electromagnetic wave equation.

$$\boxed{\nabla^2 E - \mu_0\varepsilon_0 \frac{\partial^2 E}{\partial t^2} = 0}$$

$$\nabla^2 B - \mu_0\varepsilon_0 \frac{\partial^2 B}{\partial t^2} = 0$$

$$\frac{1}{v^2} = \mu_0\varepsilon_0$$

$$v = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

$$v = c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \approx 2.98\times10^8 \text{ m/s}.$$

maxwell's Equations in Dielectric Medium

$$\nabla\cdot E = \frac{\rho_{free}}{\varepsilon_0}$$

In matter,

$$\cancel{J = J_f + J_b}$$

$$\cancel{J_b = \nabla\times M}$$

$$\rho = \rho_{free} + \rho_b$$

$$\rho_b = -\nabla\cdot P$$

$$\cancel{P = \frac{\rho}{\varepsilon_0} - \nabla\cdot P}$$

$$\cancel{\rho = \frac{1}{\varepsilon_0}(\rho_f - \nabla\cdot P)}$$

$$\rho = \rho_f + \rho_b$$

$$\rho = \rho_f - \nabla\cdot P$$

$$\nabla\cdot E = \frac{\rho}{\varepsilon_0}$$

$$\nabla\cdot E = \frac{1}{\varepsilon_0}(\rho_f - \nabla\cdot P)$$

we know tha $\nabla \cdot E = \frac{\rho}{\varepsilon_0}$

in matter $\rho = \rho_f + \rho_b$

$\rho = \rho_f - \nabla \cdot \vec{P}$

$$\nabla \cdot E = \frac{1}{\varepsilon_0}(\rho_f - \nabla \cdot \mathbb{P})$$

$$\underbrace{\nabla \cdot (\varepsilon_0 E + \vec{P})}_{\vec{D}} = \rho_f$$

$$\nabla \cdot D = \rho_{free}$$

$$\varepsilon = (1+\chi_e)\varepsilon_0$$

$$\boxed{\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}} \qquad \boxed{\varepsilon_r = 1+\chi_e}$$

$$\frac{\nabla \times B}{\mu_0} = J_f + \nabla \times M + \frac{\partial P}{\partial t} + \varepsilon_0 \frac{\partial E}{\partial t}$$

$$\nabla \times \left(\frac{B}{\mu_0} - M\right) = J_f + \frac{\partial}{\partial t}(P + \varepsilon_0 E)$$

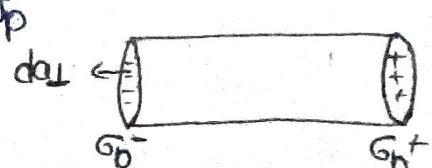
$$\boxed{\nabla \times H = J_f + \frac{\partial}{\partial t}(\varepsilon_0 E + P)}$$

$$\rho_f = \nabla \cdot (\varepsilon_0 E + \mathbb{P})$$

In matter, $J = J_f + J_b + J_p$

$J_b = \nabla \times M$

$J_p = \frac{\partial P}{\partial t}$.



$q = \sigma_b \, da_\perp$

$I = \frac{dq}{dt} = \frac{d}{dt}(\sigma_b da_\perp)$

$I = \frac{dP}{dt} da_\perp$

$\frac{I}{da_\perp} = \frac{dP}{dt}$

$J_P = \frac{\partial P}{\partial t}$.

According to maxwell equation,

$$\nabla \times B = \mu_0 J$$

$$\nabla \times B = \mu_0 (J_f + J_b + J_p) + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

$$\nabla \times \frac{B}{\mu_0} = J_f + J_b + J_p + \varepsilon_0 \frac{\partial E}{\partial t}$$

maxwell Equations in Dielectric medium

1) $\int D \cdot da = Q_{en}$ $\qquad \nabla \cdot D = \rho_f$

2) $\int B \cdot da = 0$ $\qquad \nabla \cdot B = 0$

3) $\oint E \cdot dl = -\frac{d}{dt}\int B \cdot da$ $\qquad \nabla \times E = -\frac{\partial B}{\partial t}$

4) $\int H \cdot dl = \mu_0 i_{enc} + \mu_0 \int_S \frac{dD}{dt} da$ $\qquad \nabla \times H = J_f + \frac{\partial D}{\partial t}$

In dielectric medium $J_f = 0$ and $\rho_f = 0$

$$\nabla^2 E - \mu\epsilon \frac{\partial^2 E}{\partial t^2} = 0 \qquad \boxed{v = \frac{1}{\sqrt{\epsilon\mu}}} \qquad \boxed{c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}}$$

$$n = \frac{c}{v} = \sqrt{\frac{\epsilon\mu}{\epsilon_0\mu_0}}$$

$$\boxed{n = \sqrt{\epsilon_r \mu_r}}$$

($\mu \approx \mu_0$ for dielectric medium)

$\frac{\mu}{\mu_0} \approx 1 \rightarrow$ (for dielectrics)

Maxwell's Equations for Conducters :-

In conducters

$$J = \sigma E \quad \& \quad \rho(t) = \rho(0) e^{-\frac{\sigma}{\epsilon_0} t}.$$

$$\sigma \rightarrow \infty \quad \text{as} \quad \rho \rightarrow 0$$

1) $\nabla \cdot E = 0$ 1) $\oint E \cdot da = 0$

2) $\nabla \cdot B = 0$ 2) $\oint B \cdot da = 0$

3) $\nabla \times E = \frac{-\partial B}{\partial t}$ 3) $\oint E \cdot dl = \frac{-\partial}{\partial t} \int_S \frac{\partial B}{\partial t} da$

4) $\nabla \times B = \mu_0 \epsilon_0 \frac{\partial E}{\partial t} + \mu_0 \sigma \bar{E}$ 4) $\int B \cdot dl = \mu_0 \sigma E t \; \mu_0 \epsilon_0 \frac{d}{dt} \int_S \frac{dE}{dt} da$

Poynting's Theorem

* the poynting's theorem is the work energy theorem of electrodynamics.
* the poytings theorem says, then, that the work done on the charges by the electromagnetic force is equal to the decrease in energy stored in the field, less the energy that flow out through the surface.
* the energy density in electromagnetic field (the total energy energy stored in the electromagnetic field) is given by

$$U_{em} = \frac{1}{2} \int \left(\epsilon_0 E^2 + \frac{B^2}{\mu_0} \right) d\tau \qquad U_{em} = \frac{1}{2} \int \epsilon_0 E^2$$

* The workdone by the electromagnetic forces on the charge charges in the intervel of dt can be written as

$$dw = F \cdot dl$$

$$dw = [qE + (v \times B)q] \cdot dl$$

$$dw = q[E + (v \times B)] \cdot dl$$

we know $dl = v\,dt$

$$dw = q[E \cdot v\,dt + \cancel{(v \times B) \cdot v\,dt}]$$

$$(A \times B) \cdot A = 0$$

$$dw = qE \cdot v\,dt$$

$$dw = E \cdot (vq)\,dt$$

$$\frac{dw}{dt} = E \cdot (vq) \qquad q = \int_V \rho\, d\tau$$

$$\frac{dw}{dt} = E \cdot v \int_V \rho\, d\tau$$

$$\frac{dw}{dt} = E \cdot \int_V (\rho v)\, d\tau$$

$$\frac{dw}{dt} = E \cdot \int_V J \cdot d\tau$$

$$\boxed{\frac{dw}{dt} = \int_V (E \cdot J)\, d\tau} \quad -① $$

* "E.J" is the workdone per unit time per unit volume which is known as "power delivered per unit volume".
* By using Amphere's law (4th) we can eliminate 'J'

we know that $\nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$

$$\nabla \times \frac{B}{\mu_0} = J + \varepsilon_0 \frac{\partial E}{\partial t}$$

$$J = \nabla \times \frac{B}{\mu_0} - \varepsilon_0 \frac{\partial E}{\partial t}$$

$$E \cdot J = E \cdot \left(\nabla \times \frac{B}{\mu_0}\right) - \varepsilon_0 E \cdot \frac{\partial E}{\partial t} \quad — ②$$

$$\nabla \cdot \left(E \times \frac{B}{\mu_0}\right) = \frac{B}{\mu_0} \cdot (\nabla \times E) - E \cdot \left(\nabla \times \frac{B}{\mu_0}\right)$$

$$\boxed{E \cdot \nabla \times \frac{B}{\mu_0} = \frac{B}{\mu_0} \cdot (\nabla \times E) - \nabla \cdot \left(E \times \frac{B}{\mu_0}\right)}$$

$$E\cdot\left(\nabla\times\frac{B}{\mu_0}\right) = -\frac{B}{\mu_0}\cdot\frac{\partial B}{\partial t} - \nabla\cdot\left(E\times\frac{B}{\mu_0}\right) \quad \text{--- (3)}$$

substiting eq^n (3) in eq^n (2)

$$E\cdot J = E\cdot\left(\nabla\times\frac{B}{\mu_0}\right) - \varepsilon_0 E\cdot\frac{\partial E}{\partial t}$$

$$E\cdot J = \frac{-1}{\mu_0} B\frac{\partial B}{\partial t} - \nabla\left(E\times\frac{B}{\mu_0}\right) - \varepsilon_0 E\cdot\frac{\partial E}{\partial t}$$

since $\quad B\frac{\partial B}{\partial t} = \frac{1}{2}\frac{\partial B^2}{\partial t}$ and $E\frac{\partial E}{\partial t} = \frac{1}{2}\frac{\partial E^2}{\partial t}$

$$E\cdot J = \frac{-1}{2\mu_0}\frac{\partial B^2}{\partial t} - \frac{1}{2}\varepsilon_0\frac{\partial E^2}{\partial t} - \nabla\cdot\left(E\times\frac{B}{\mu_0}\right)$$

$$\boxed{E\cdot J = -\frac{1}{2}\left(\frac{\partial}{\partial t}\left(\frac{B^2}{\mu_0} + \varepsilon_0 E^2\right)\right) - \nabla\cdot\left(E\times\frac{B}{\mu_0}\right)} \quad \text{--- (4)}$$

Equ (4) in Eq (1)

$$\frac{dw}{dt} = \int_V (E\cdot J)\, d\tau$$

$$\frac{\partial w}{\partial t} = \frac{-1}{2}\frac{\partial}{\partial t}\int_V\left(\frac{B^2}{\mu_0} + \varepsilon_0 E^2\right)d\tau - \int_V \nabla\cdot\left(E\times\frac{B}{\mu_0}\right)d\tau$$

$$\boxed{\frac{\partial w}{\partial t} = \frac{-1}{2}\frac{\partial}{\partial t}(u_{en}) - \frac{1}{\mu_0}\int_S (E\times B)\,da} \quad \text{--- (5)}$$

$$S = \frac{1}{\mu_0}(E\times B)$$

$$\boxed{\frac{\partial w}{\partial t} = \frac{-\partial}{\partial t}u_{en} - \int_S S\cdot da} \quad \text{--- (6)}$$

from eq^n (5)

$$\frac{\partial w}{\partial t} = \frac{-1}{2}\frac{\partial}{\partial t}(u_{en}) - \frac{1}{\mu_0}\int_S (E\times B)\,da$$

The first intergral in the above equation $-\frac{\partial}{\partial t}u_{en}$ is charge of total energy stored in the fields & second term which $\frac{-1}{\mu_0}\int_S (E\times B)da$ represents the rate at which energy is carried out of volume 'v' across its boundary surface.

The ~~Energy~~ per unit time per unit area ~~perpendicular~~ transported by the feld is called the poyitng vector

$$S = \frac{1}{\mu_0}(E \times B)$$

'S' represents the direction of propagating and perpendicular to Electric & magnetic fields,

'S' the poynting vector