Formulation of Transportation problems, Sensitivity analysis in Transportation problems, Assignment problems.

Introduction:-

[Prem Kumar]

The transportation model deals with the transportation of a product available at several sources to a no. of different destinations.

This model can be used for a wide variety of situations such as scheduling, production, investment, plant location, inventory control, and many others

Each shipping source has a certain capacity and each destination has a certain requirement associated with a certain cost of shipping from the sources to the destination.

The objective is to minimize the cost of transportation while meeting the requirements at the destinations.

Transportation problems may also involve movement of a product from plants to warehouses, warehouses to wholesalers, wholesalers to retailers and retailers to customers.

Assumptions in Transportation model:-

1. Total quantity of item available at diff. sources is equal to the total requirement at diff. destinations.
2. Item can be transported conveniently from all sources to destinations.
3. Unit transportation cost of item from all sources to destinations is certainly and precisely known.
4. Transportation cost on a given route is directly proportional to no. of units shipped on that route.
5. The objective is to minimize the total transportation cost for the organization as a whole and not for individual

Matrix form:-

Destinations

Sources (or) Origins	1	2	3	.... j	n	Supply
1	C_{11} x_{11}	C_{12} x_{12}	C_{13} x_{13}	C_{1j} x_{1j}	C_{1n} x_{1n}	a_1
2	C_{21} x_{21}	C_{22} x_{22}	C_{23} x_{23}	C_{2j} x_{2j}	C_{2n} x_{2n}	a_2
3	C_{31} x_{31}	C_{32} x_{32}	C_{33} x_{33}	C_{3j} x_{3j}	C_{3n} x_{3n}	a_3
i	C_{i1} x_{i1}	C_{i2} x_{i2}	C_{i3} x_{i3}	C_{ij} x_{ij}	C_{in} x_{in}	a_i
m	C_{m1} x_{m1}	C_{m2} x_{m2}	C_{m3} x_{m3}	C_{mj} x_{mj}	C_{mn} x_{mn}	a_m
Demand	b_1	b_2	b_3	b_j	b_n	

C_{ij} = Unit shipping cost from i^{th} origin to j^{th} destination

x_{ij} = Quantity shipped from " " "

a_i = Supply available at origin "i".

b_j = Demand at destination "j".

$i = 1, 2, \ldots, m$ & $j = 1, 2, \ldots, n$

DEFINITIONS:-

1. Feasible Solution:- It is a set of non-negative allocations, x_{ij} that satisfies the rim (row & column) restrictions.

2. Basic Feasible Solution:- A feasible solution is said to be a Basic Feasible Solution if it contains not more than $(m+n-1)$ non-negative allocations, where "m" is the number of rows and "n" is the no. of columns of the transportation problem.

3. Optimal Solution:- A feasible solution that minimizes the transportation cost is called an Optimum Solution.

4. Non-degenerate Basic Feasible Solution:- A BFS for a $(m \times n)$ transportation problem is said to be non-degenerate if,

(a) The total no. of non-negative allocations is exactly $(m+n-1)$ (i.e., no. of independent constraint eq's), and

b) These $(m+n-1)$ allocations are in independent positions.

5. Degenerate BFS:- A BFS in which the total no. of non-negative allocations is less than (m+n-1) is called degenerate BFS.

Types of Transportation problem:-

[Panneerselvam]

Classified into: (a) Balanced Transportation Problem &

(b) UnBalanced " "

(a) Balanced Transportation Problem:-

If sum of ~~all~~ supplies of all sources is equal to the sum of the demands of all the destinations, then the problem is termed as "Balanced Transportation Problem".

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

(b) UnBalanced Transportation Problem:-

If sum of supplies of all sources is not equal to sum of demands of all destinations, then the problem is termed as Unbalanced Transportation Problem.

$$\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$$

FORMULATION OF MODEL:-

Ex:- A dairy firm has 3 plants located in a state. Daily milk production at each plant is as follows:

Plant 1 = 6 million Lts

Plant 2 = 1 Million Lts &

Plant 3 = 10 Million Lts

Each day the firm must fulfil the needs of its four distribution centres. Milk requirement at each centre is as follows:

Distribution centre 1 = 7 Million Lts

" " 2 = 5 Million Lts

" " 3 = 3 Million Lts

" " 4 = 2 Million Lts

Cost of shipping One Million Lts of Milk from each plant to each distribution centre is given in the following table in hundreds of rupees:

Distribution Centres

Plants	1	2	3	4
1	2	3	11	7
2	1	0	6	1
3	5	8	15	9

Formulate the mathematical model for the problem.

Sol:- Step-1:-

Key Decision: To find how much ~~milk~~ quantity of milk from which plant to which distribution centre be shipped so as to satisfy the constraints and minimize the cost.

~~Variables;~~

Origins: $i = 1, 2, 3, \ldots m$ &

Destinations: $j = 1, 2, 3, 4 \ldots n$

Variables: x_{ij} & $i = 3, j = 4$

$x_{11}, x_{12}, x_{13}, x_{14},$

$x_{21}, x_{22}, x_{23}, x_{24}$

$x_{31}, x_{32}, x_{33}, x_{34}$

Matrix form:

Distribution Centres

Plants	1	2	3	4
1	x_{11}	x_{12}	x_{13}	x_{14}
2	x_{21}	x_{22}	x_{23}	x_{24}
3	x_{31}	x_{32}	x_{33}	x_{34}

Step-2:- Feasible alternatives are sets of values of x_{ij} where, $x_{ij} \geq 0$.

Step-3:- **Objective:** To minimize the cost of transportation.

i.e., Minimize $Z = 2x_{11} + 3x_{12} + 11x_{13} + 7x_{14}$
$+ x_{21} + 0x_{22} + 6x_{23} + x_{24}$
$+ 5x_{31} + 8x_{32} + 15x_{33} + 9x_{34}$

If C_{ij} is unit cost of shipping from i^{th} source to j^{th} destination, the objective is

$$\text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} x_{ij}$$

Step-4:- **Constraints** are

(i) Because of availability or supply:

$x_{11} + x_{12} + x_{13} + x_{14} = 6$ (For Milk plant 1)

$x_{21} + x_{22} + x_{23} + x_{24} = 1$ (" " " 2)

$x_{31} + x_{32} + x_{33} + x_{34} = 10$ (" " " 3)

There are 3 constraints (= No. of plants)

In general, $\sum_{j=1}^{m} x_{ij} = a_i$, $i = 1, 2, 3, \ldots, m$

(ii) Because of requirement or demand:

$x_{11} + x_{21} + x_{31} = 7$ (For distribution centre 1)

$x_{12} + x_{22} + x_{32} = 5$ (" " " 2)

$x_{13} + x_{23} + x_{33} = 3$ (" " " 3)

$x_{14} + x_{24} + x_{34} = 2$ (" " " 4)

In general, $\sum_{i=1}^{m} x_{ij} = b_j$, $j = 1, 2, 3, \ldots, n$

The given situation involves $(3 \times 4 = 12)$ variables and $(3 + 4 = 7)$ constraints.

In general, no. of variables $= (m \times n)$
no. of constraints $= (m + n)$

Since, the transportation model is always balanced, one of these constraints must be redundant.

Thus, No. of independent constraint eq.'s $= (m + n - 1)$
No. of basic variables in BFS $= (m + n - 1)$

NOTE:-

- All supply and demand constraints are of equality type.
- They are expressed in terms of only one kind of unit.
- Each variable occurs only once in supply constraints and only once in demand constraints.
- Each variable in the constraints has unit coefficient only.

Solution of Transportation Model:-

Steps involved in solving a transportation model:

(i) To check whether the given problem is of standard type or not.

Minimisation type is considered as standard type.

(ii) To check whether the given problem is of balanced type or not.

- Balanced: Total supply = Total demand
- If not, a dummy origin or destination is added to balance the supply and demand.

(iii) To find Initial Basic Feasible Solution (IBFS)

Methods: (a) North West Corner Method (NWCM)

(b) Row minima

(c) Column minima

(d) Least Cost Method (LCM)

(e) Vogel's Approximation Method (VAM)

(iv) To perform Optimality Test.

- To do this test, the following conditions must be satisfied.

(a) The IBFS should be a non-degenerate BFS.

A BFS is said to be non-degenerate when

No. of allocations $\geq (m+n-1)$

where, m - no. of rows

n - no. of columns.

(b) All the allocated cells must be in independant positions.

- Optimality test can be done by the following methods:
  - (a) Stepping stone method.
  - (b) Modified Distribution Method (MODI) (or) u-v method.

Problems

① Find IBFS using: (i) North West Corner Method
(ii) Row Minima
(iii) Column Minima
(iv) Least Cost Method
(v) Vogel's Approximation Method

for the following matrix in which the transportation costs are given in hundreds of rupees. Also, find corresponding transportation cost.

Distribution Centres

		1	2	3	4	Supply
	1	2	3	11	7	6
Plants	2	1	0	6	1	1
	3	5	8	15	9	10
Demand		7	5	3	2	

Sol:- Step-1:- To check whether it is a std. type of problem or not.

Here, cost is given. Cost is to be minimised.

So, the given problem is a std. type of problem.

Step-2:- To check whether it is a balanced problem or not.

Total supply = 6+1+10 = 17

Total demand = 7+5+3+2 = 17

∵ Total supply = Total demand, the given problem is a balanced one.

Step-3:- To find IBFS

For solving the given transportation problem, the following cell representation is followed:

a_i

$\boxed{x_{ij} \quad \boxed{c_{ij}}}$

b_j

where, C_{ij} - Cost from $i \to j$

- Unit transportation cost from source "i" to destination "j".

x_{ij} - No. of units (Min. of supply & demand. $i \to j$

a_i - No. of supply units available at source

b_j - No. of demand units req. at destination.

(i) North-West Corner Method (NWCM):-

This rule cors method may be stated as follows:

(a) Start in North-West (Upper Left) corner of table. and compare the supply (S_1) of plant 1 with the requirement (demand) (D_1) of distribution centre 1.

- If $D_1 < S_1$, take $x_{11} = D_1$, find the balance supply and demand and proceed to cell which is North-West in remaining cells.
- If $D_1 = S_1$, take $x_{11} = D_1 (= S_1)$, find the balance supply and demand & repeat the procedure.
- If $D_1 > S_1$, take $x_{11} = S_1$, compute the balance supply and demand & repeat the procedure for remaining cells.

Distribution Centres

Plants	1	2	3	4	Supply
1	6 $\boxed{2}$	$\boxed{3}$	$\boxed{11}$	$\boxed{7}$	6/0
2	1 $\boxed{1}$	$\boxed{0}$	$\boxed{6}$	$\boxed{1}$	1/0
3	$\boxed{5}$	5 $\boxed{3}$	3 $\boxed{15}$	2 $\boxed{9}$	10/5/2/0
Demand	7/1/0	5/0	3/0	2/0	

Transportation cost $= [(6\times2)+(1\times1)+(5\times8)+(3\times15)+(2\times9)]\times100$
$= [12+1+40+45+18]\times100$
$=$ Rs. 11,600/-

(ii) Row Minima Method :-

This method consists in allocating as much as possible in the lowest cost cell of first row so that either capacity of first plant is satisfied or requirement at j^{th} distribution centre is satisfied or both.

3 Cases: (a) If capacity of first plant is completely exhausted cross 1^{st} row & proceed to 2^{nd} row.

(b) If requirement at j^{th} distribution centre is satisfied, cross j^{th} column and reconsider 1^{st} row with remaining capacity.

(c) If capacity of first plant as well as requirement at j^{th} distribution centre are completely satisfied, cross off the row as well as j^{th} column and move down to second row.

Continue the process until all the rim conditions are satisfied.

Distribution Centres

Plants	1	2	3	4	Supply
1	[2] 6	[3]	[11]	[7]	6/0
2	[1]	[0]	[6]	[1]	1/0
3	[5] 1	[8] 4	[15] 3	[9] 2	10/9/5/3/0
Demand	7/1/0	5/4/0	3/0	2/0	

Transportation cost $= [(6\times2)+(1\times0)+(1\times5)+(4\times8)+(3\times15)$
$+(2\times9)]\times100$

$= [12+0+5+32+45+18]\times100$

$=$ Rs. 11,200/-

(iii) Column Minima Method:-

This method consists in allocating as much as possible in the lowest cost cell of first column so that either demand of first distribution centre is satisfied or capacity of i^{th} plant is exhausted or both. In case of tie among the lowest cost cells in the column, select arbitrarily.

3 Cases: (a) If requirement of 1^{st} distribution centre is satisfied, cross off 1^{st} column and move right to 2^{nd} column.

(b) If capacity of i^{th} plant is satisfied, cross off i^{th} row and reconsider first column with remaining requirement.

(c) If requirement of first distribution centre as well as capacity of i^{th} plant are completely satisfied, cross off 1^{st} column, i^{th} row and move right to the 2^{nd} column.

Continue the process until all the rim conditions are satisfied.

Plants	1	2	3	4	Supply
1	6 [2]	[3]	[11]	[7]	6/0
2	1 [1]	[10]	[6]	[1]	1/0
3	[5]	5 [8]	3 [15]	2 [9]	10/5/2/0
Demand	7/6/0	5/0	3/0	2/0	

Transportation cost $= [(6\times2)+(1\times1)+(5\times8)+(3\times15)+(2\times9)]\times100$

$= (12+1+40+45+18)\times100$

$=$ ₹. 11,600/-

(iv) Least-Cost Method (or Matrix Minima Method or Lowest Cost Entry Method):-

This method consists in allocating as much as possible in the lowest cost cell ~~leas~~ and then further allocation is done in the cell with the second lowest and so on. In case of tie among the cost, select the cell where allocation of more number of units can be made.

	1	2	3	4	Supply
1	[2] 6	[3]	[11]	[7]	6/0
2	[1]	[0]	[6]	[1]	1/0
3	[5] 1	[8] 4	[15] 3	[9] 2	10/9/5/3/0
Demand	7/1/0	5/4/0	3/0	2/0	

Transportation cost $= [(2\times6)+(1\times0)+(1\times5)+(4\times8)+(3\times15)$
$+(2\times9)]\times100$

$=(12+0+5+32+45+18)\times100$

$=₹.\ 11,200/-$

(v) Vogel's Approximation Method (VAM) (or) Penalty Method (or) Regret Method:-

The difference b/n the two lowest costs for each row and column is the opportunity cost. It would be more economi-cal to make allocation against the row or column with the highest opportunity cost.

This method consists of following steps:

(a) Write down the cost matrix. Enter the difference b/n the smallest and second smallest element in each column below the corresponding column and to the right of each row. Put these numbers in brackets.

(b) Select the row (or) column with greatest difference and allocate as much as possible within the restrictions of the rim conditions to the lowest cost cell in the row or column selected.

In case of tie among the highest penalties, select the row or column having minimum cost.

In case of tie among min. cost also, select the cell which can have max. allocation.

If there is tie among max. allocation cells also, select the cell arbitrarily for allocation.

(c) Cross off the row/column completely satisfied by the allocation just made.

(d) Repeat all steps a to c until all assignments have been made.

		DC 1	DC 2	DC 3	DC 4	Supply	
Plant	1	[2] 1	[3] 5	[11]	[7]	6/1/0	[1] [1] [5] ←
	2	[1]	[0]	[6]	[1] 1	1/0	[1]
	3	[5] 6	[8]	[15] 3	[9]	10/7/6/0	[3] [3] [4] [10] [4] [5]
Demand		7/6/0	5/0	3/0	2/1/0		
		[1]	[3]	[5]	[6] ↑		
		[3]	[5] ↑	[4]	[2]		
		[3]		[4]	[2]		
		[5]		[15] ↑	[9]		
		[5]			[9] ↑		
		[5] ↑					

Transportation cost $= [(1\times2)+(5\times3)+(1\times1)+(6\times5)+(3\times15)+(1\times9)]\times100$

$= [2+15+1+30+45+9]\times100$

$= 9300+900$

$=$ ₹ 10,200

It can be concluded that VAM is best suited for the given problem as it is yielding less transportation cost as compared to other methods.

(2) Find the IBFS of the following transportation problem by VAM.

Warehouse

Factory	W_1	W_2	W_3	W_4	Capacity
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
Requirement	5	8	7	14	34 (Total)

Sol:-

Warehouse

Factory	W_1	W_2	W_3	W_4	Capacity	
F_1	$\boxed{19}$ 5	$\boxed{30}$	$\boxed{50}$	$\boxed{10}$ 2	7/2/0	[9] [9] [40] [40]
F_2	$\boxed{70}$	$\boxed{30}$	$\boxed{40}$ 7	$\boxed{60}$ 2	9/7/0	[10] [20] [20] [20] [20]
F_3	$\boxed{40}$	$\boxed{8}$ 8	$\boxed{70}$	$\boxed{20}$ 10	18/10/0	[12] [20] [50]←
Requirement	5/0	8/0	7/0	14/4/2/0	34 (Total)	
	[21]	[22] ↑	[10]	[10]		
	[21] ↑		[10]	[10]		
			[10]	[10]		
			[10]	[50] ↑		
			[40]	[60] ↑		

Transportation Cost $= (5\times19)+(8\times8)+(7\times40)+(2\times10)$
$+(2\times60)+(10\times20)$
$= 95+64+280+20+120+200$
$= 779$

(2) Solve the following transportation problem:

		W₁	W₂	W₃	W₄	Capacity
	F_1	19	30	50	10	7
Factory	F_2	70	30	40	60	9
	F_3	40	8	70	20	18
Requirement		5	8	7	14	

(Warehouse: W_1, W_2, W_3, W_4)

Sol:- Step-1:- To check whether the given transportation problem is standard type or not.

Minimisation type is considered as standard type.

Since, the transportation cost is to be minimised. The given problem is of standard type.

Step-2:- To check whether the given transportation problem is balanced or not.

Total factory capacity $= 7+9+18 = 34$

Total warehouse req. $= 5+8+7+14 = 34$

Total factor capacity = Total warehouse req. $= 34$.

The given problem is balanced.

Step-3:- To find Initial Basic Feasible Solution (IBFS) by Vogel's Approximation Method (VAM).

	W_1	W_2	W_3	W_4	Capacity					
F_1	19 (5)	30	50	10 (2)	7/2/0	[9]	[9]	[40]	[40]	
F_2	70	30	40 (7)	60 (2)	9/7/0	[10]	[20]	[20]	[20]	[20]
F_3	40	8 (8)	70	20 (10)	18/10/0	[12]	[20]	[50] ←		
Requirement	5/0	8/0	7/0	14/4/2/0						
	[21]	[22] ↑	[10]	[10]						
	[21]		[10]	[10]						
			[10]	[10]						
			[10]	[50] ↑						
			[40]	[60] ↑						

Transportation cost = $(5\times19)+(2\times10)+(7\times40)+(2\times60)+(8\times8)+(10\times20)$

$= 95+20+280+120+64+200$

$= ₹ \cdot 779$

Step-4:- To perform optimality test by MODI method.

CONDITIONS for optimality test

1. No. of allocations = 6

$m+n-1 = 3+4-1 = 6$

$\because$ No. of allocations $= m+n-1 = 6$

So, it is non-degenerate

2. All the allocated cells are in independent positions.

So, optimality test can be performed by MODI method.

MODI method:

Sub Step-1:- Cost matrix for allocated cells

u_i \ v_j	19	-2	-10	10
0	19			10
50			40	60
10		8		20

Sub step-2:- (u_i+v_j) matrix for unallocated cells

	-2	-10	
69	48		
29		0	

Sub step-3:- Cell Evaluation Matrix $[c_{ij} - (u_i+v_j)]$ for unallocated cells c_{ij} - Cost in Original Cost matrix

.	30-(-2)	50-(-10)	.
70-69	30-48	.	.
40-29	.	70-0	.

=

	32	60	
1	-18		
11		70	

From CEM, identify the cell with most negative cell evaluation. & mark it ✓.

(a) Trace a closed path in the matrix. This closed path has the following characteristics:

(i) It begins & ends in the identified cell.

(ii) It consists of a series of alternate hzl. & vertical lines only (no diagonals).

(iii) It can be traced CW or ACW.

(iv) All other corners of the path lie in allocated cells only.

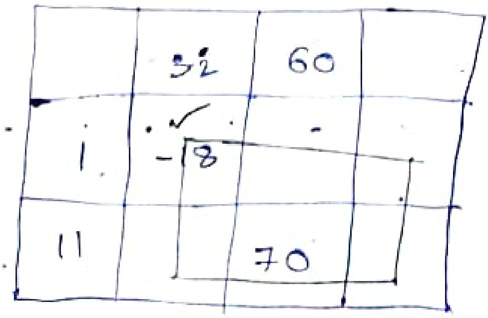
(v) The path may skip over any number of allocated or vacant cells.

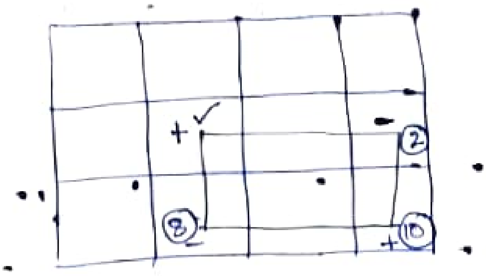
(vi) There will always be one and only one closed path which may be traced.

(b) Mark the identified cell as +ve & each occupied cell at the corners of the path alternately -ve, +ve, -ve & so on.

(c) Make a new allocation in the identified cell by entering the smallest allocation on path that has been assigned a -ve sign. Add & subtract this new allocation from the cells at the corners of the path, maintaining row and column requirements.

This causes one basic cell to become zero and other cells remain non-negative. The basic cell whose allocation has been made zero, leaves the solution





5 [19]	[30]	[50]	2 [10]	7
[70]	2 [30]	7 [40]	[60]	9
[40]	6 [8]	[70]	12 [20]	18
5	8	7	14	

Second Feasible Solution

Transportation Cost $= (5\times19)+(2\times30)+(7\times40)+(6\times8)$
$+(12\times20)+(10\times2)$
$= 95+60+280+48+240+20$
$= \cancel{667}\ 743$

Conditions for optimality test:

1. No. of allocations $=6$

$m+n-1 = 3+4-1 = 6$

$\therefore$ No. of allocations $= m+n-1 = 6$

It is non-degenerate.

2. All the allocated cells are in independant positions.

MODI method:

Sub step-1:- Cost matrix for allocated cells.

$u_i \downarrow$ \ $v_j \rightarrow$	19	-2	8	10
0	[19]			[10]
32		[30]	[40]	
10		[8]		[20]

Sub step-2:- (u_i+v_j) matrix for unallocated cells.

	-2	8	
51			42
29		18	

Substep-3:- Cell evaluation matrix for unallocated cell

$C_{ij} - (u_i + v_j)$

	50-(-2)	50-8	
70-51			60-42
40-29		70-18	

=

	32	42	
19			18
11		52	

The CEM contains all +ve values. So, second BFS is the optimum solution.

∴ Transportation cost = ₹. 743.

STEPPING STONE Method: (RamaMurthy)

- Consider the matrix giving IBFS.
- Start with any arbitrary empty cell and allocate "+ve" to this cell
- Starting with empty cell, draw a loop moving horizontally and vertically from allocated cell to allocated cell. (There should not be any diagonal movement).
- After completing the loop, mark "−" & "+" alternatively
- Now, calculate cell evaluation of the empty cell by adding all the cost values with respective sign allotted at each corner.
- Similarly, calculate cell evaluation of all empty cells.

If any cell evaluation is "−ve", cost can be reduced i.e., solution is not optimal. If all cell evaluations are "+ve" (or) "zero", solution is optimal.

(a) Cell evaluation:

For (1,2) = 30 − 10 + 20 − 8 = 50 − 18
= 32

For (1,3) = 50 − 10 + 60 − 40 = 60

	1	2	3	4
1	5 $\boxed{19}$	+ $\boxed{30}$	$\boxed{50}$	2 $\boxed{10}$
2	$\boxed{70}$	$\boxed{30}$	7 $\boxed{40}$	2 $\boxed{60}$
3	$\boxed{40}$	8 $\boxed{8}$	$\boxed{70}$	10 $\boxed{20}$
	5	8	7	14

For (2,1) = 70 − 19 + 10 − 60 = 1

For (2,2) = 30 − 60 + 20 − 8 = 50 − 68 = −18

	1	2	3	4
1	5 $\boxed{19}$	$\boxed{30}$	$\boxed{50}$	2 $\boxed{10}$
2	$\boxed{70}$	$\boxed{30}$	7 $\boxed{40}$	2 $\boxed{60}$
3	$\boxed{40}$	8 $\boxed{8}$	$\boxed{70}$	10 $\boxed{20}$

For (3,1) = 40 − 20 + 10 − 19 = 50 − 39 = 11

For (3,3) = 70 − 20 + 60 − 40 = 70

	1	2	3	4
1	5 $\boxed{19}$	$\boxed{30}$	$\boxed{50}$	2 $\boxed{10}$
2	$\boxed{70}$	$\boxed{30}$	7 $\boxed{40}$	2 $\boxed{60}$
3	$\boxed{40}$	8 $\boxed{8}$	$\boxed{70}$	10 $\boxed{20}$

∵ Cell (2,2) is having −ve value. The IBFS is not optimal.

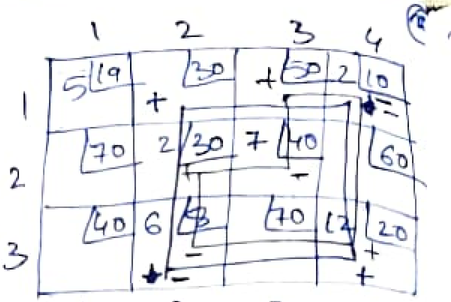
- Select the unoccupied cell having the highest "−ve" cell evaluation & draw a closed path.
- Select min. allocated value among all "−ve" position on closed path.
- Assign this value to selected unoccupied cells.
- Add this value to other occupied cells marked "+ve".
- Subtract this value to other occupied cells marked "−ve".

5 $\boxed{19}$	$\boxed{30}$	$\boxed{50}$	2 $\boxed{10}$
$\boxed{70}$	+ 2 $\boxed{30}$	7 $\boxed{40}$	2 0 $\boxed{60}$
$\boxed{40}$	8 − 6 $\boxed{8}$	$\boxed{70}$	10 + 12 $\boxed{20}$

Second BFS

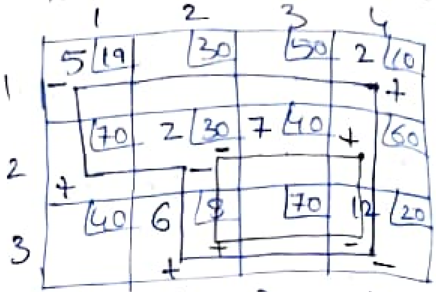
5 $\boxed{19}$	$\boxed{30}$	$\boxed{50}$	2 $\boxed{10}$
$\boxed{70}$	2 $\boxed{30}$	7 $\boxed{40}$	$\boxed{60}$
$\boxed{40}$	6 $\boxed{8}$	$\boxed{70}$	12 $\boxed{20}$

Cell evaluation:



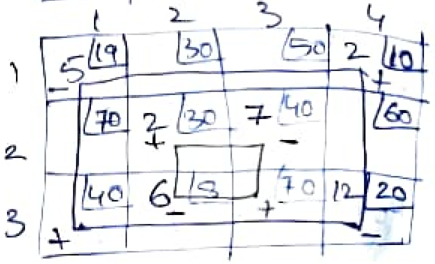
For (1,2) = 30 − 10 + 20 − 8
= 50 − 18 = 32

For (1,3) = 50 − 10 + 20 − 8 + 30 − 40
= 100 − 58 = 42



For (2,1) = 70 − 19 + 10 − 20 + 8 − 30
= 88 − 69 = 19

For (2,4) = 60 − 20 + 8 − 30
= 68 − 50 = 18



For (3,1) = 40 − 19 + 10 − 20
= 50 − 39 = 11

For (3,3) = 70 − 40 + 30 − 8
= 100 − 48 = 52

∵ All cell evaluations are positive, the second BFS yields optimal solution.

Transportation cost = (5×19) + (2×10) + (2×30) + (7×40) + (6×8) + (12×20)

= 95 + 20 + 60 + 280 + 48 + 240

= 7.743

③ A company has 4 warehouses & 6 stores. The warehouses altogether has a surplus of 22 units of a given commodity divided among them.

Warehouses	1	2	3	4
Surplus	5	6	2	9

The 6 stores altogether need 22 units of commodity. Individual requirements of stores 1, 2, 3, 4, 5 & 6 are 4, 4, 6, 2, 4 & 2 respectively. Cost of shifting of one commodity from warehouse to store is given below.

Store

	1	2	3	4	5	6	Surplus
Warehouse 1	9	12	9	6	9	10	5
2	7	3	7	7	5	5	6
3	6	5	9	11	3	11	2
4	6	8	11	2	2	10	9

Require How should the product be shipped so that the transportation cost is minimum?

Sol:- Step-1:- To check whether the given problem is std. type or not.

Since, transportation cost is to be minimised, the given problem is standard type.

Step-2:- To check whether the given problem is balanced or not.

Total surplus = 5+6+2+9 = 22

Total requirement = 4+4+6+2+4+2 = 22

∵ Total surplus = Total requirement, the given problem is balanced.

Step-3:- To find IBFS using VAM.

Store

	1	2	3	4	5	6	Surplus						
Warehouse 1	[9]	[12]	[9] 5	[6]	[9]	[10]	5/0	[3]	[3]	[0]	[0]	[0]	[0]
2	[7]	[3] 4	[7]	[7]	[5] E	[5] 2	6/4/0	[2]	[2]	[2]	[4] ←		
3	[6] 1	[5]	[9] 1	[11]	[3]	[11]	2/1/0	[2]	[2]	[2]	[1]	[3]	[3]
4	[6] 3	[8]	[11]	[2] 2	[2] 4	[10]	9/7/3/0	[0]	[0]	[4] ←	[2]	[5] ←	
Requirement	4/1/0	4/0	6/1/0	2/0	4/0	2/0							
	[0]	[2]	[2]	[4]	[1]	[5] ↑							
	[0]	[2]	[2]	[4] ↑	[1]								
	[0]	[2]	[2]		[1]								
	[0]	[2]	[2]										
	[0]		[0]										
	[3] ↑		[0]										
			[0]										

Transportation Cost $= (5\times9)+(4\times3)+(2\times5)+(1\times6)+(1\times9)+(3\times6)+(2\times2)+(4\times2)$

$= 45+12+10+6+9+18+4+8$

$= ₹112$

Step-4:- To perform optimality test by MODI method

1. No. of allocations $= 8$

$m+n-1 = 4+6-1 = 9$

$\because$ No. of allocations $\neq m+n-1$. The IBFS is degenerate solution.

Convert this into non-degenerate by allocating "ε" ($\varepsilon \to 0$) in the cell which has least cost & also doesn't form a closed loop.

Let us allocate "ε" in cell (2, 5).

No. of allocations $= 8 = m+n-1$.

So, it is non-degenerate.

2. All allocations are in independant positions.

MODI Method:

Substep-1:- Cost matrix for allocated cells.

$u_i \downarrow$ / $v_j \rightarrow$	6	0	9	2	2	2
0			$\boxed{9}$			
3		$\boxed{3}$			$\boxed{5}$	$\boxed{5}$
0	$\boxed{6}$		$\boxed{9}$			
0	$\boxed{6}$			$\boxed{2}$	$\boxed{2}$	

Substep-2:- (u_i+v_j) matrix for unallocated cells.

6	0		2	2	2
9		12	5		
	0		2	2	2
	0	9			2

Substep-3:- CEM for unallocated cells. $[c_{ij} - (u_i + v_j)]$

3	12		4	7	8
-2		-5	0		
	5		9	1	9
	8	2			8

∵ Some of the CEM values are -ve, the IBFS is not optimal.

3	12		4	7	8
-2		+ -5	0	-	
+	5	-	9		9
-	8	2	-	+	8

				ε -	
		+			
+ 1		1 -			
3 -				4 +	

		5			
	4	+ε		ε-ε	2
1+ε		1-ε			
3-ε			2	4+ε	

		5 [9]			
	4 [3]	ε [7]			2 [5]
1 [6]		1 [9]			
3 [6]			2 [2]	4 [2]	

→ Second BFS.

Transportation cost = $(5\times9)+(4\times3)+(2\times5)+(1\times6)+(1\times9)$
$+(3\times6)+(2\times2)+(4\times2)$ = ₹.112

To perform optimality test.

1. No. of allocations = m+n-1 = 8 → Non-degenerate
2. All allocations are in independant position.

MoDI Method:-

Substep-1:- Cost matrix for allocated cells.

u_i \ v_j	6	5	9	2	2	7
0			$\boxed{9}$			
-2		$\boxed{3}$	$\boxed{7}$			$\boxed{5}$
0	$\boxed{6}$		$\boxed{9}$			
0	$\boxed{6}$			$\boxed{2}$	$\boxed{2}$	

Substep-2:- (u_i+v_j) matrix for unallocated cells.

6	5		2	2	7
4			0	0	
	5		2	2	7
	5	9			7

~~∵ All CEM values are +ve, second BFS is optimal~~

~~Transportation cost = ₹ 112/-~~

Substep-3 :- Cell Evaluation Matrix $[C_{ij} - (u_i+v_j)]$

3	7		4	7	3
3			7	5	
	0		9	1	4
	3	2			3

∵ All CEM values are +ve, second BFS is optimal

Transportation cost = ₹. 112/-