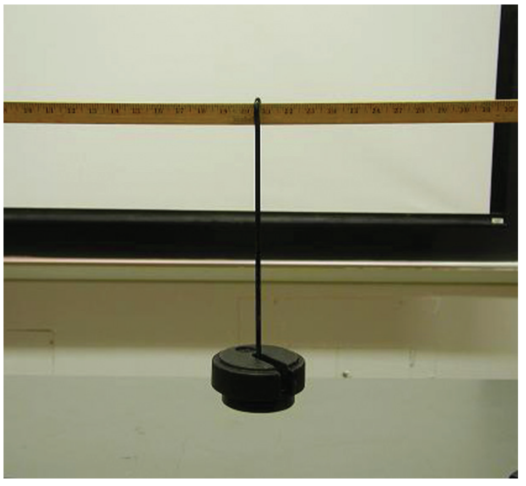
Chapter 14

Bending Stresses in Simple Beams



Contents

14.1. Introduction

We have already discussed in Chapter 13 that the bending moments and shearing forces are set up at all sections of a beam, when it is loaded with some external loads. We have also discussed the methods of estimating the bending moments and shear forces at various sections of the beams and cantilevers.

As a matter of fact, the bending moment at a section tends to bend or deflect the beam and the internal stresses resist its bending. The process of bending stops, when every cross-section sets up full resistance to the bending moment. The *resistance, offered by the internal stresses, to the

* The resistance offered by the internal stresses to the shear force is called shearing stresses. It will be discussed in the next chapter.

bending, is called bending stress, and the relevant theory is called the theory of simple bending.

14.2. Assumptions in the Theory of Simple Bending

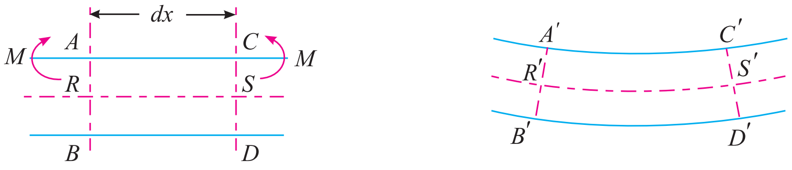
The following assumptions are made in the theory of simple bending:

1. The material of the beam is perfectly homogeneous (*i.e.*, of the same kind throughout) and isotropic (*i.e.*, of equal elastic properties in all directions).
2. The beam material is stressed within its elastic limit and thus, obeys Hooke's law.
3. The transverse sections, which were plane before bending, remains plane after bending also.
4. Each layer of the beam is free to expand or contract, independently, of the layer above or below it.
5. The value of E (Young's modulus of elasticity) is the same in tension and compression.
6. The beam is in equilibrium *i.e.*, there is no resultant pull or push in the beam section.

14.3. Theory of Simple Bending

Consider a small length of a simply supported beam subjected to a bending moment as shown in Fig. 14.1 (*a*). Now consider two sections *AB* and *CD*, which are normal to the axis of the beam *RS*. Due to action of the bending moment, the beam as a whole will bend as shown in Fig. 14.1 (*b*).

Since we are considering a small length of dx of the beam, therefore the curvature of the beam in this length, is taken to be circular. A little consideration will show that all the layers of the beam, which were originally of the same length do not remain of the same length any more. The top layer of the beam has suffered compression and reduced to $A'C'$. As we proceed towards the lower layers of the beam, we find that the layers have no doubt suffered compression, but to lesser degree; until we come across the layer *RS*, which has suffered no change in its length, though bent into $R'S'$. If we further proceed towards the lower layers, we find the layers have suffered tension, as a result of which the layers are stretched. The amount of extension increases as we proceed lower, until we come across the lowermost layer *BD* which has been stretched to $B'D'$.



(*a*) Before bending (*b*) After bending

Fig. 14.1. Simple bending

Now we see that the layers above have been compressed and those below *RS* have been stretched. The amount, by which layer is compressed or stretched, depends upon the position of the layer with reference to *RS*. This layer *RS*, which is neither compressed nor stretched, is known as neutral plane or neutral layer. This theory of bending is called theory of simple bending.

14.4. Bending Stress

Consider a small length dx of a beam subjected to a bending moment as shown in Fig. 14.2 (*a*). As a result of this moment, let this small length of beam bend into an arc of a circle with O as centre as shown in Fig. 14.2 (*b*).

Let M = Moment acting at the beam,

θ = Angle subtended at the centre by the arc and

R = Radius of curvature of the beam.

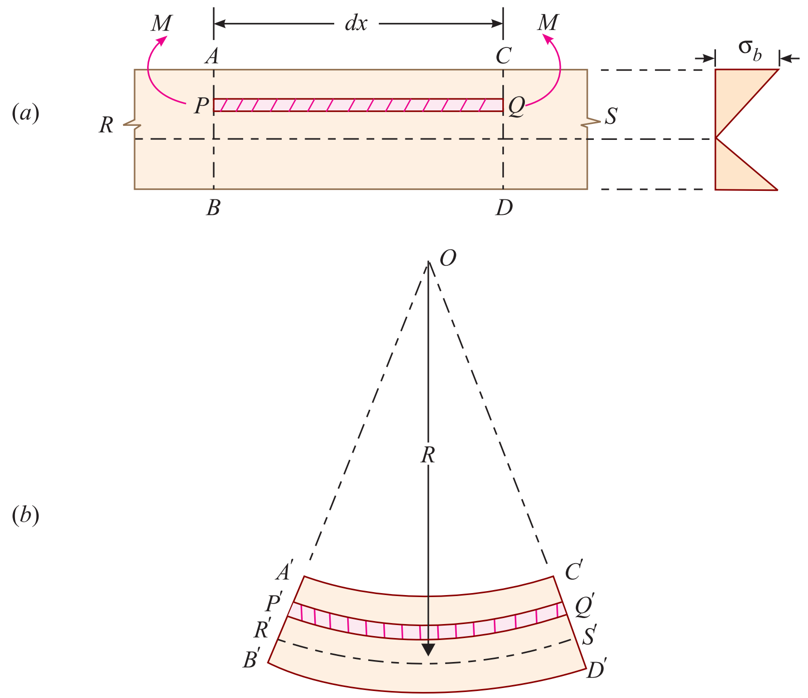


Fig. 14.2. Bending stress

Now consider a layer PQ at a distance y from RS the neutral axis of the beam. Let this layer be compressed to $P'Q'$ after bending as shown in Fig. 14.2 (*b*).

We know that decrease in length of this layer,

$$\delta l = PQ - P'Q'$$

$$\therefore \qquad \text{Strain } \varepsilon = \frac{\delta l}{\text{Original length}} = \frac{PQ - P'Q'}{PQ}$$

Now from the geometry of the curved beam, we find that the two sections $OP'Q'$ and $OR'S'$ are similar.

$$\therefore \qquad \frac{P'Q'}{R'S'} = \frac{R-y}{R}$$

$$\text{or} \qquad 1 - \frac{P'Q'}{R'S'} = 1 - \frac{R-y}{R}$$

$$\text{or} \qquad \frac{R'S' - P'Q'}{PQ} = \frac{y}{R}$$

$$\frac{PQ - P'Q'}{PQ} = \frac{y}{R} \qquad \text{...}(PQ = R'S' = \text{Neutral axis})$$

$$\varepsilon = \frac{y}{R} \qquad \text{...}\left(\because \varepsilon = \frac{PQ - P'Q'}{PQ}\right)$$

It is thus obvious, that the strain (ε) of a layer is proportional to its distance from the neutral axis. We also know that the bending stress,

$$\sigma_b = \text{Strain} \times \text{Elasticity} = \varepsilon \times E$$

$$= \frac{y}{R} \times E = y \times \frac{E}{R} \qquad \ldots\left(\because \varepsilon = \frac{y}{R}\right)$$

Since E and R are constants in this expression, therefore the stress at any point is directly proportional to y, *i.e.*, the distance of the point from the neutral axis. The above expression may also be written as,

$$\frac{\sigma_b}{y} = \frac{E}{R} \quad \text{or} \quad \sigma_b = \frac{E}{R} \times y$$

Note. Since the bending stress is inversely proportional to the radius (R), therefore for maximum stress the radius should be minimum and vice versa.

Example 14.1. *A steel wire of 5 mm diameter is bent into a circular shape of 5 m radius. Determine the maximum stress induced in the wire. Take E = 200 GPa.*

Solution. Given : Diameter of steel wire (d) = 5 mm ; Radius of circular shape (R) = 5 m = 5×10^3 mm and modulus of elasticity (E) = 200 GPa = 200×10^3 N/mm^2.

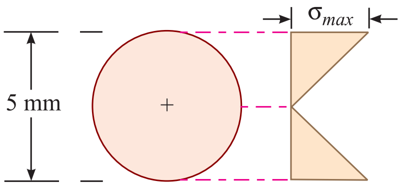


Fig. 14.3

We know that distance between the neutral axis of the wire and its extreme fibre,

$$y = \frac{d}{2} = \frac{5}{2} = 2.5 \text{ mm}$$

and maximum bending stress induced in the wire,

$$\sigma_{b\,(max)} = \frac{E}{R} \times y = \frac{200 \times 10^3}{5 \times 10^3} \times 2.5 = 100 \text{ N/mm}^2 = 100 \text{ MPa} \qquad \textbf{Ans.}$$

Example 14.2. *A copper wire of 2 mm diameter is required to be wound around a drum. Find the minimum radius of the drum, if the stress in the wire is not to exceed 80 MPa. Take modulus of elasticity for the copper as 100 GPa.*

Solution. Given : Diameter of wire (d) = 2 mm ; Maximum bending stress $\sigma_{b\,(max)}$ = 80 MPa = 80 N/mm^2 and modulus of elasticity (E) = 100 GPa = 100×10^3 N/mm^2.

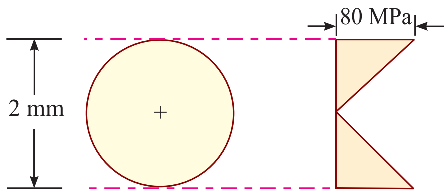


Fig. 14.4

We know that distance between the neutral axis of the wire and its extreme fibre

$$y = \frac{2}{2} = 1 \text{ mm}$$

∴ Minimum radius of the drum

$$R = \frac{y}{\sigma_{b(\max)}} \times E = \frac{1}{80} \times 100 \times 10^3 \qquad \ldots\left(\because \frac{\sigma_b}{y} = \frac{E}{R}\right)$$

$$= 1.25 \times 10^3 \text{ mm} = 1.25 \text{ m} \qquad \textbf{Ans.}$$

Example 14.3. *A metallic rod of 10 mm diameter is bent into a circular form of radius 6 m. If the maximum bending stress developed in the rod is 125 MPa, find the value of Young's modulus for the rod material.*

Solution. Given : Diameter of rod (d) = 10 mm ; Radius (R) = 6 m = 6×10^3 mm and maximum bending stress $\sigma_{b\,(max)}$ = 125 MPa = 125 N/mm^2.

We know that distance between the neutral axis of the rod and its extreme fibre,

$$y = \frac{10}{2} = 5$$

$\therefore$ Value of Young's modulus for the rod material,

$$E = \frac{\sigma_{b(max)}}{y} \times R = \frac{125}{5} \times (6 \times 10^3) \text{ N/mm}^2 \qquad \ldots \left(\because \frac{\sigma_b}{y} = \frac{E}{R} \right)$$

$$= 150 \times 10^3 \text{ N/mm}^2 = 150 \text{ GPa} \qquad \textbf{Ans.}$$

EXERCISE 14.1

1. A copper rod 20 mm diameter is bent into a circular arc of 8 m radius. Determine the intensity of maximum bending stress induced in the metal. Take E = 100 GPa. [**Ans.** 125 MPa]
2. A steel wire of 3 mm diameter is to be wound around a circular component. If the bending stress in the wire is limited to 80 MPa, find the radius of the component. Take Young's modulus for the steel as 200 GPa. [**Ans.** 3.75 m]
3. An alloy wire of 5 mm diameter is wound around a circular drum of 3 m diameter. If the maximum bending stress in the wire is not to exceed 200 MPa, find the value of Young's modulus for the alloy. [**Ans.** 120 GPa]

14.5. Position of Neutral Axis

The line of intersection of the neutral layer, with any normal cross-section of a beam, is known as neutral axis of that section. We have seen in Art. 14.2 that on one side of the neutral axis there are compressive stresses, whereas on the other there are tensile stresses. At the neutral axis, there is no stress of any kind.

Consider a section of the beam as shown in Fig. 14.5. Let be the neutral axis of the section. Consider a small layer PQ of the beam section at a distance from the neutral axis as shown in Fig. 14.5.

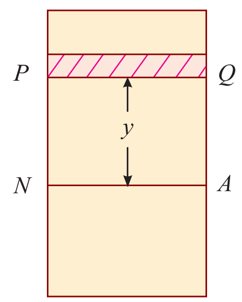


Fig. 14.5. Neutral axis

Let $\qquad \delta a$ = Area of the layer PQ.

We have seen in Art. 14.4 that intensity of stress in the layer PQ,

$$\sigma = y \times \frac{E}{R}$$

$\therefore$ Total stress on the layer PQ

$$= \text{Intensity of stress} \times \text{Area}$$

$$= y \times \frac{E}{R} \times \delta a$$

and total stress of the section.

$$= \Sigma\, y \times \frac{E}{R} \times \delta a = \frac{E}{R} \Sigma\, y . \delta a$$

Since the section is in equilibrium, therefore total stress, from top to bottom, must be equal to zero.

$$\therefore \qquad \frac{E}{R} \Sigma\, y . \delta a = 0$$

$$\text{or} \qquad \Sigma\, y . \delta a = 0 \qquad \ldots \left(\because \frac{E}{R} \text{ cannot be equal to zero} \right)$$

A little consideration will show that $y \times \delta a$ is the moment of the area about the neutral axis and $\Sigma\, y \times \delta a$ is the moment of the entire area of the cross-section about the neutral axis. It is thus obvious that the neutral axis of the section will be so located that moment of the entire area about the axis is zero. We know that the moment of any area about an axis passing through its central axis of a section always passes through its centroid. Thus to locate the neutral axis of a section, first find out the centroid of the section and then draw a line passing through this centroid and normal to the plane of bending. This line will be the neutral axis of the section.

14.6. Moment of Resistance

We have already seen in Art. 14.2 that on one side of the neutral axis there are compressive stresses and on the other there are tensile stresses. These stresses form a couple, whose moment must be equal to the external moment (M). The moment of this couple, which resists the external bending moment, is known as moment of resistance.

Consider a section of the beam as shown in Fig. 14.6. Let NA be the neutral axis of the section. Now consider a small layer PQ of the beam section at a distance y from the neutral axis as shown in Fig. 14.6.

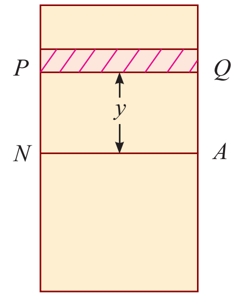


Fig. 14.6. Moment of resistance

Let δa = Area of the layer PQ.

We have seen in Art. 14.4 that the intensity of stress in the layer PQ,

$$\sigma = y \times \frac{E}{R}$$

∴ Total stress in the layer PQ

$$= y \times \frac{E}{R} \times \delta a$$

and moment of this total stress about the neutral axis

$$= y \times \frac{E}{R} \times \delta a \times y = \frac{E}{R}\, y^2 \,.\, \delta a \qquad ...(i)$$

The algebraic sum of all such moments about the neutral axis must be equal to M. Therefore

$$M = \Sigma \frac{E}{R} y^2 \,.\, \delta a = \frac{E}{R} \Sigma\, y^2 \,.\, \delta a$$

The expression $\Sigma\, y^2 \,.\, \delta a$ represents the moment of inertia of the area of the whole section about the neutral axis. Therefore

$$M = \frac{E}{R} \times I \qquad \text{...(where } I = \text{moment of inertia)}$$

or

$$\frac{M}{I} = \frac{E}{R}$$

We have already seen in Art 14.4 that,

$$\frac{\sigma}{y} = \frac{E}{R}$$

∴

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

It is the most important equation in the theory of simple bending, which gives us relation between various characteristics of a beam.

14.7. Distribution of Bending Stress across the Section

We have seen in the previous articles that there is no stress at the neutral axis. In a *simply supported beam, there is a compressive stress above the neutral axis and a tensile stress below it.

* In a cantilever, there is a tensile stress above the neutral axis and compressive stress below it.

We have also discussed that the stress at a point is directly proportional to its distance from the neutral axis. If we plot the stresses in a simply supported beam section, we shall get a figure as shown in Fig. 14.7.

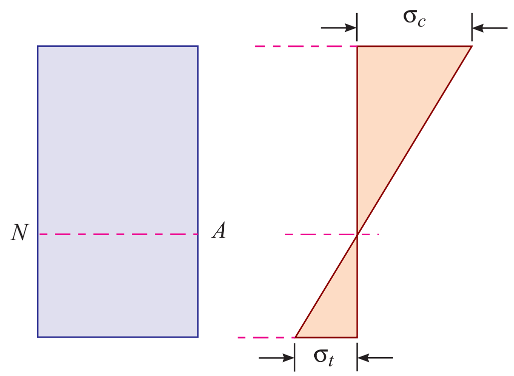


Fig. 14.7. Distribution of Bending Stress

The maximum stress (either compressive or tensile) takes place at the outermost layer. Or in other words, while obtaining maximum bending stress at a section, the value of y is taken as maximum.

14.8. Modulus of Section

We have already discussed in the previous article, the relation for finding out the bending stress on the extreme fibre of a section, *i.e.*,

$$\frac{M}{I} = \frac{\sigma}{y} \quad \text{or} \quad M = \sigma \times \frac{I}{y}$$

From this relation, we find that the stress in a fibre is proportional to its distance from the *c.g.* If y_{max} is the distance between the *c.g.* of the section and the extreme fibre of the stress, then

$$M = \sigma_{max} \times \frac{I}{y_{max}} = \sigma_{max} \times Z$$

where $Z = \dfrac{I}{y_{max}}$. The term 'Z' is known as modulus of section or section modulus. The general practice of writing the above equation is $M = \sigma \times Z$, where σ denotes the maximum stress, tensile or compressive in nature.

We know that if the section of a beam to, is symmetrical, its centre of gravity and hence the neutral axis will lie at the middle of its depth. We shall now consider the modulus of section of the following sections:

1. Rectangular section. **2.** Circular section.

1. ***Rectangular section***

We know that moment of inertia of a rectangular section about an axis through its centre of gravity.

$$I = \frac{bd^3}{12}$$

$\therefore$ Modulus of section
$$Z = \frac{I}{y} = \frac{bd^3}{12} \times \frac{2}{d} = \frac{bd^2}{6} \qquad \ldots\left(\because y = \frac{d}{2}\right)$$

2. *Circular section*

We know that moment of inertia of a circular section about an axis through its *c.g.*,

$$I = \frac{\pi}{64}(d)^4$$

$\therefore$ Modulus of section
$$Z = \frac{I}{y} = \frac{\pi}{64}(d)^4 \times \frac{2}{d} = \frac{\pi}{32}(d)^2 \qquad \ldots\left(\because\ y = \frac{d}{2}\right)$$

Note : If the given section is hollow, then the corresponding values for external and internal dimensions should be taken.

14.9. Strength of a Section

It is also termed as flexural strength of a section, which means the moment of resistance offered by it. We have already discussed the relations :

$$\frac{M}{I} = \frac{\sigma}{y} \quad \text{or} \quad M = \frac{\sigma}{y} \times I \quad \text{and} \quad M = \sigma Z$$

It is thus obvious that the moment of resistance depends upon moment of inertia (or section modulus) of the section. A little consideration will show that the moment of inertia of beam section does not depend upon its cross-section area, but its disposition in relation to the neutral axis.

We know that in the case of a beam, subjected to transverse loading, the bending stress at a point is directly proportional to its distance from the neutral axis. It is thus obvious that a larger area near the neutral axis of a beam is uneconomical. This idea is put into practice, by providing beams of section, where the flanges alone withstand almost all the bending stress.

Example 14.4. *For a given stress, compare the moments of resistance of a beam of a square section, when placed (i) with its two sides horizontal and (ii) with its diagonal horizontal.*

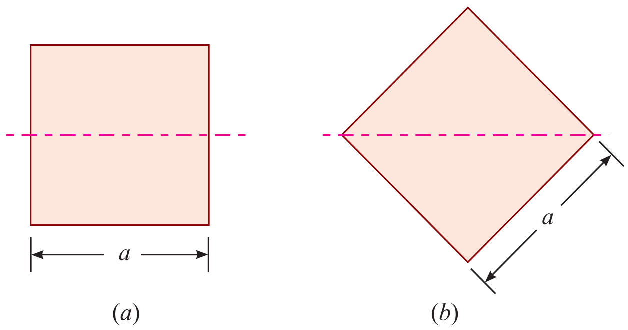


Fig. 14.8

Solution. Given: The square section with its two horizontal sides and with its diagonal horizontal are shown in Fig. 14.8 (*a*) and (*b*).

Let a = Side of the square beam,

M_1 = Moment of resistance of section 1 and

M_2 = Moment of resistance of section 2.

We know that the section modulus of the beam section with its two sides horizontal,

$$Z_1 = \frac{bd^2}{6} = \frac{a \times a^2}{6} = \frac{a^3}{6} \qquad \ldots(i)$$

and moment of inertia of the beam section with its diagonal horizontal may be found out by splitting up the section into two triangles and then adding the moments of inertia of the two triangles about their base.

$$\therefore \qquad I_2 = 2 \times \frac{bh^3}{12} = 2 \times \frac{a\sqrt{2}\left(\frac{a}{\sqrt{2}}\right)^3}{12} = \frac{a^4}{12}$$

and

$$y_{max} = \frac{a}{\sqrt{2}}$$

$$\therefore \qquad Z_2 = \frac{I}{y_{max}} = \frac{\frac{a^4}{12}}{\frac{a}{\sqrt{2}}} = \frac{a^3}{6\sqrt{2}} \qquad ...(ii)$$

Sine the moment of resistance of a section is directly proportional to their moduli of section, therefore

$$\frac{M_1}{M_2} = \frac{Z_1}{Z_2} = \frac{\frac{a^3}{6}}{\frac{a^3}{6\sqrt{2}}} = \sqrt{2} = 1.414 \qquad \textbf{Ans.}$$

EXAMPLE 14.5. *A rectangular beam is to be cut from a circular log of wood of diameter D. Find the ratio of dimensions for the strongest section in bending.*

SOLUTION. Given : Diameter of the circular log of wood = D.

Let b = Breadth of the rectangular beam section and

d = Depth of the rectangular beam section.

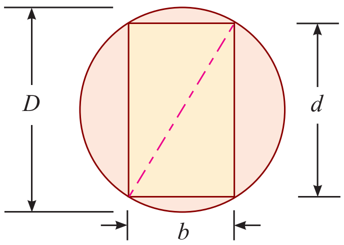


Fig. 14.9

We know that section modulus of the rectangular section.

$$Z = \frac{bd^2}{6}$$

From the geometry of the figure, we find that

$$b^2 + d^2 = D^2$$

or

$$d^2 = D^2 - b^2$$

Substituting the value of d^2 in equation (i),

$$Z = \frac{b \times (D^2 - b^2)}{6} = \frac{bD^2 - b^3}{6}$$

We also know that for the strongest section, let us differentiate the above equation and equate it to zero. *i.e.*,

$$\frac{dZ}{db} = \frac{d}{db}\left[\frac{bD^2 - b^3}{6}\right] = \frac{D^2 - 3b^2}{6}$$

or

$$\frac{D^2 - 3b^2}{6} = 0 \qquad \text{or} \qquad D^2 - 3b^2 = 0 \qquad \text{or} \qquad b = \frac{D}{\sqrt{3}}$$

Substituting this value of b in equation (*ii*),

$$d^2 = D^2 - \frac{D^2}{3} = \frac{2D^2}{3} \quad \text{or} \quad d = D\sqrt{\frac{2}{3}} \quad \textbf{Ans.}$$

EXAMPLE 14.6. *Two beams are simply supported over the same span and have the same flexural strength. Compare the weights of these two beams, if one of them is solid and the other is hollow circular with internal diameter half of the external diameter.*

SOLUTION. Given : Span of the solid beam = Span of the hollow beam and flexural strength of solid beam = Flexural strength of the hollow section.

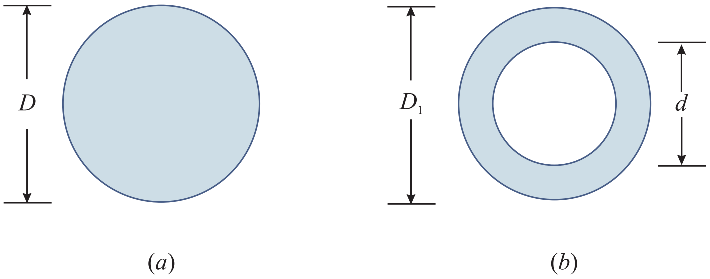


Fig. 14.10

Let D = Diameter of the solid beam and

D_1 = Diameter of the hollow beam.

First of all consider the solid beam as shown in Fig. 14.10 (*a*). We know that section modulus of the solid section,

$$Z_1 = \frac{\pi}{32} \times (D)^3 = \frac{\pi}{32} \times D^3 \quad \text{...(i)}$$

and now consider the hollow beam as shown in Fig. 14.10 (*b*). We also know that section modulus of the hollow section,

$$Z_2 = \frac{\pi}{32\,D_1} \times \left[D_1^4 - d^4\right] = \frac{\pi}{32\,D_1} \times \left[D_1^4 - (0.5\,D_1)^4\right]$$

$$= \frac{\pi}{32} \times 0.9375\,D_1^3 \quad \text{...(ii)}$$

Since both the beams are supported over the same span (l) and have the same flexural strength, therefore section modulus of both the beams must be equal. Now equating equations (*i*) and (*ii*),

$$\frac{\pi}{32} \times D^3 = \frac{\pi}{32} \times 0.9375\,D_1^3 \quad \text{or} \quad D^3 = 0.9375\,(D_1)^3$$

$$\therefore \quad D = (0.9375)^{1/3}\,D_1 = 0.98\,D_1$$

We also know that wights of two beams are proportional to their respective cross-sectional areas. Therefore

$$\frac{\text{Weight of solid beam}}{\text{Weight of hollow beam}} = \frac{\text{Area of solid beam}}{\text{Area of hollow beam}}$$

or

$$= \frac{\frac{\pi}{4} \times D^2}{\frac{\pi}{4} \times \left[(D_1)^2 - d^2\right]} = \frac{D^2}{(D_1)^2 - (0.5\,D_1)^2}$$

$$= \frac{D^2}{0.75(D_1)^2} = \frac{D^2}{(D_1)^2} \times \frac{1}{0.75} = (0.98)^2 \times \frac{1}{0.75} = 1.28 \quad \textbf{Ans.}$$

Example 14.7. *Three beams have the same length, the same allowable stress and the same bending moment. The cross-section of the beams are a square, a rectangle with depth twice the width and a circle as shown in Fig. 14.11.*

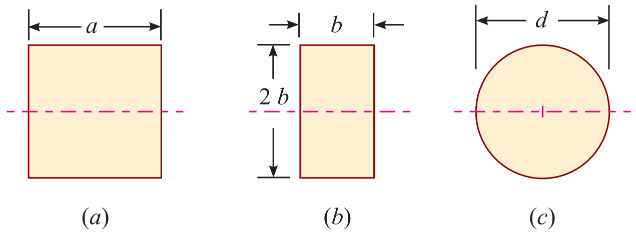


Fig. 14.11

Find the ratios of weights of the circular and the rectanguar beams with respect to the square beam.

Solution. Square, rectangular and circular sections are shown in Fig. 14.11 (*a*), (*b*) and (*c*).

Let a = Side of the square beam,

b = Width of a rectangular beam,

∴ $2b$ = Depth of the rectangular beam and

d = Diameter of a circular section.

Since all the three beams have the same allowable stress (σ) and bending moment (M), therefore the modulus of section of the three beams must be equal.

We know that the section modulus for a square beam,

$$Z_1 = \frac{bd^2}{6} = \frac{a \times a^2}{6} = \frac{a^3}{6} \quad ...(i)$$

Similarly, modulus of section for rectangular beam,

$$Z_2 = \frac{bd^2}{6} = \frac{b(2b)^2}{6} = \frac{2b^3}{3} \quad ...(ii)$$

and modulus of section for a circular beam,

$$Z_3 = \frac{\pi}{32} \times d^3 \quad ...(iii)$$

Equating equations (*i*) and (*ii*),

$$\frac{a^3}{6} = \frac{2b^3}{3} \quad \text{or} \quad a^3 = 6 \times \frac{2b^3}{3} = 4b^3$$

$$\therefore \quad b = 0.63\, a \quad ...(iv)$$

Now equating equations (*i*) and (*iii*),

$$\frac{a^3}{6} = \frac{\pi}{32} \times d^3$$

$$\therefore \quad a^3 = 6 \times \frac{\pi}{32} \times d^3 = \frac{3\pi}{16} \times d^3$$

$$\text{or} \quad d = 1.19\, a \quad ...(v)$$

We know that weights of all the beams are proportional to the cross sectional areas of their sections. Therefore

$$\frac{\text{Weight of square beam}}{\text{Weight of rectangular beam}} = \frac{\text{Area of square beam}}{\text{Area of rectangular beam}}$$

$$= \frac{a^2}{2b^2} = \frac{a^2}{2 \times (0.63a)^2} = \frac{1}{0.79} \quad \textbf{Ans.}$$

and
$$\frac{\text{Weight of square beam}}{\text{Weight of circular beam}} = \frac{\text{Area of square beam}}{\text{Area of circular beam}}$$

$$= \frac{a^2}{\frac{\pi}{4} \times d^2} = \frac{a^2}{\frac{\pi}{4} \times (1.19a)^2} = \frac{1}{1.12} \quad \textbf{Ans.}$$

EXAMPLE 14.8. *Prove that moment of resistance of a beam of square section, with its diagonal in the plane of bending is increased by flatting top and bottom corners as shown in Fig. 14.12. Also prove that the moment of resistance is a maximum when y = 8Y/9.*

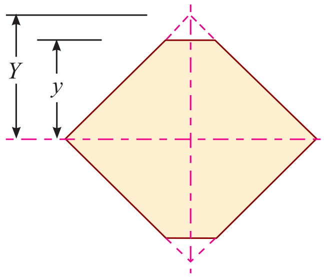


Fig. 14.12

SOLUTION. First of all, let us divide the section into a square with diagonal $2y$ and a rectangle with sides as $2y$ and $2(Y - y)$ as shown in Fig. 14.13 (*a*) and (*b*).

The moment of inertia of the square section with its diagonal in the plane of bending may be found out by splitting up the section into two triangles, and then adding the moments of inertia of the two triangles about its base.

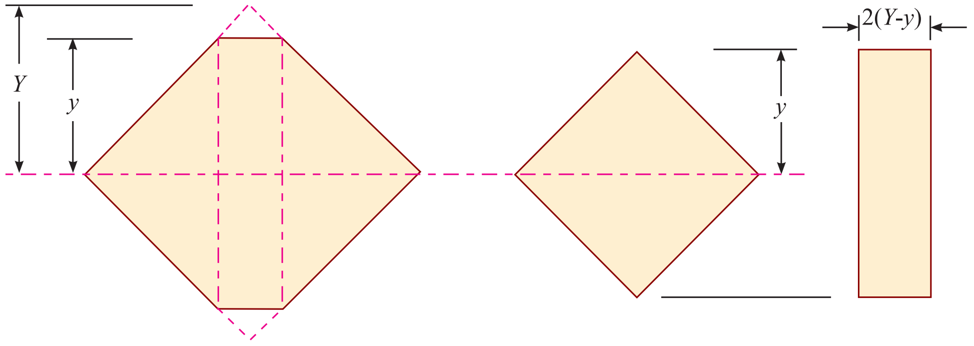


Fig. 14.13. (*a*) and (*b*)

We know that moment of inertia for the square section,

$$I_1 = 2 \times \frac{bh^3}{12} = 2 \times \frac{2y \times y^3}{12} = \frac{y^4}{3}$$

and moment of inertia for the rectangular section,

$$I_2 = \frac{2(Y - y) \times (2y)^3}{12} = \frac{4}{3}(Yy^3 - y^4)$$

∴ Total moment of inertia of the section,

$$I = I_1 + I_2 = \frac{y^4}{3} + \frac{4}{3}(Yy^3 - y^4) = \frac{4}{3}Yy^3 - y^4$$

We also know that the bending stress at a distance x from the neutral axis,

$$\sigma_b = \frac{M}{I} \times y = \frac{M}{\frac{4}{3}Yy^3 - y^4} \times y = \frac{M}{\frac{4}{3}Yy^2 - y^3}$$

Now for maximum bending stress, differentiating the above equation and equating the same to zero,

$$\frac{d}{dy}\left(\frac{M}{\frac{4}{3}Y \cdot y^2 - y^3}\right) = 0 \qquad \text{or} \qquad \frac{4}{3}Y \times 2y - 3y^2 = 0$$

$$\frac{8Y}{3} - 3y = 0 \qquad \text{or} \qquad y = \frac{8Y}{9} \qquad \textbf{Ans.}$$

EXAMPLE 14.9. *A wooden floor is required to carry a load of 12 kN/m² and is to be supported by wooden joists of 120 mm × 250 mm in section over a span of 4 metres. If the bending stress in these wooden joists is not to exceed 8 MPa, find the spacing of the joists.*

SOLUTION. Given : Load on the floor = 12 kN/m² = 12×10^{-3} N/mm² ; Width of joist (b) = 120 mm ; Depth of joist (d) = 250 mm ; Span (l) = 4 m = 4×10^3 mm and maximum bending stress $\sigma_{b\,(max)}$ = 8 MPa = 8 N/mm².

Let x = Spacing of the joists in mm.

We know that rate of loading on the joist,

$$w = 12 \times 10^{-3} \times x \times 1 = 12 \times 10^{-3}\,x \text{ N/mm}$$

and maximum bending moment at the centre of a simply supported beam subjected to a uniformly distributed load,

$$M = \frac{wl^2}{8} = \frac{(12x \times 10^{-3}) \times (4 \times 10^3)^2}{8} = 24 \times 10^3\,x \text{ N-m} \qquad ...(i)$$

We also know that section modulus of each rectangular joist,

$$Z = \frac{bd^2}{6} = \frac{120 \times (250)^2}{6} = 1.25 \times 106 \text{ mm}^3$$

and moment of resistance,

$$24 \times 10^3\,x = \sigma_{b\,(max)} \cdot Z = 8 \times 1.25 \times 10^6 = 10 \times 10^6$$

$$\therefore \quad x = \frac{10 \times 10^6}{24 \times 10^3} = 417 \text{ mm} \qquad \textbf{Ans.}$$

14.10. Bending Stresses in Symmetrical Sections

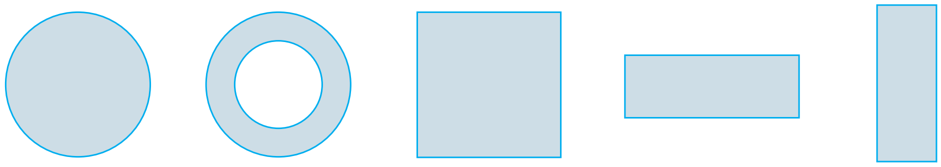


Fig. 14.14. Symmetrical sections.

We know that in a symmetrical section (*i.e.*, circular, square or rectangular), the centre of gravity of the section lies at the geometrical centre of the section as shown in Fig. 14.14. Since the neutral axis of a section passes through its centre of gravity, therefore neutral axis of a symmetrical section passes through its geometrical centre. In such cases, the outermost layer or extreme fibre is at a distance of $d/2$ from its geometrical centre, where d is the diameter (in a circular section) or depth (in square or rectangular sections).

Note : In most or the cases, we are required to find the maximum bending stress in the section. We know that the bending stress at a point, in a section is directly proportional to its distance from the neutral axis. Therefore, maximum bending stress in a section will occur in the extreme fibre of the section.

Example 14.10. *A rectangular beam 60 mm wide and 150 mm deep is simply supported over a span of 6 m. If the beam is subjected to central point load of 12 kN, find the maximum bending stress induced in the beam section.*

Solution. Given : Width (b) = 60 mm ; Depth (d) = 150 mm ; Span (l) = 6 × 10^3 mm and load (W) = 12 kN = 12 × 10^3 N.

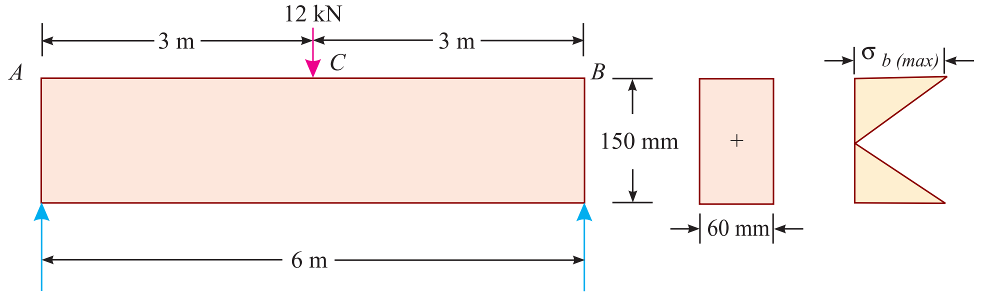


Fig. 14.15

We know that maximum bendint moment at the centre of a simply supported beam subjected to a central point load,

$$M = \frac{Wl}{4} = \frac{(12 \times 10^3) \times (6 \times 10^3)}{4} = 18 \times 10^6 \text{ N-mm}$$

and section modulus of the rectangular section,

$$Z = \frac{bd^2}{6} = \frac{60 \times (150)^2}{6} = 225 \times 10^3 \text{ mm}^3$$

∴ Maximum bending stress,

$$\sigma_{max} = \frac{M}{Z} = \frac{18 \times 10^6}{225 \times 10^3} = 80 \text{ N/mm}^2 = 80 \text{ MPa} \qquad \textbf{Ans.}$$

Example 14.11. *A rectangular beam 300 mm deep is simply supported over a span of 4 metres. What uniformly distributed load the beam may carry, if the bending stress is not to exceed 120 MPa. Take I = 225 × 10^6 mm^4.*

Solution. Given : Depth (d) = 300 mm ; Span (l) = 4 m = 4 × 10^3 mm ; Maximum bending stress (σ_{max}) = 120 MPa = 120 N/mm^2 and moment of inertia of the beam section (I) = 225 × 10^6 mm^4.

Let w = Uniformly distributed load the beam can carry.

We know that distance between the neutral axis of the section and extreme fibre,

$$y = \frac{d}{2} = \frac{300}{2} = 150 \text{ mm}$$

and section modulus of the rectangular section,

$$Z = \frac{I}{y} = \frac{225 \times 10^6}{150} = 1.5 \times 10^6 \text{ mm}^3$$

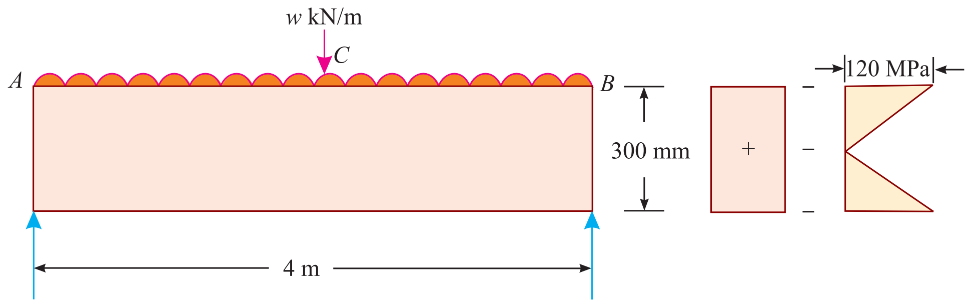


Fig. 14.16

∴ Moment of resistance,

$$M = \sigma_{max} \times Z = 120 \times (1.5 \times 10^6) = 180 \times 10^6 \text{ N-mm.}$$

We also know that maximum bending moment at the centre of a simply supported beam subjected to a uniformly distributed load (M),

$$180 \times 10^6 = \frac{wl^2}{8} = \frac{w \times (4 \times 10^3)^2}{8} = 2 \times 10^6\, w$$

$$\therefore \quad w = \frac{180}{2} = 90 \text{ N/mm} = 90 \text{ kN/m} \qquad \textbf{Ans.}$$

EXAMPLE 14.12. *A cantilever beam is rectrangular in section having 80 mm width and 120 mm depth. If the cantilever is subjected to a point load of 6 kN at the free end and the bending stress is not to exceed 40 MPa, find the span of the cantilever beam.*

SOLUTION. Given : Width (b) = 80 mm ; Depth (d) = 120 mm ; Point load (W) = 6 kN = 6×10^3 N and maximum bending stress (σ_{max}) = 40 MPa = 40 N/mm^2.

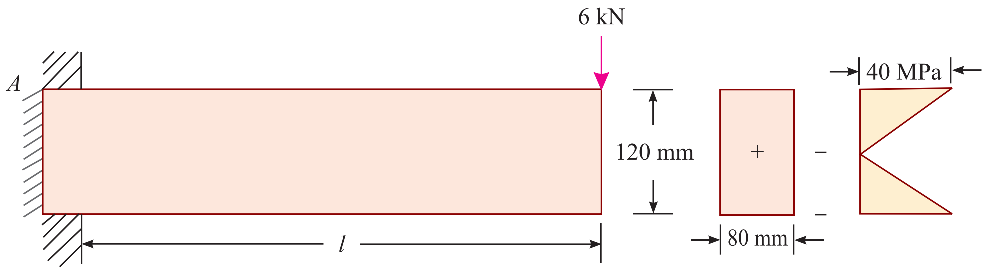


Fig. 14.17

Let $\quad l$ = Span of the cantilever beam.

We know that section modulus of the rectangular section,

$$Z = \frac{bd^2}{6} = \frac{80 \times (120)^2}{6} = 192 \times 10^3 \text{ mm}^3$$

and maximum bending moment at the fixed end of the cantilever subjected to a point load at the free end,

$$M = Wl = (6 \times 10^3) \times l$$

∴ Maximum bending stress [$\sigma_{b\,(max)}$]

$$40 = \frac{M}{Z} = \frac{6 \times 10^3 \times l}{192 \times 10^3} = \frac{l}{32}$$

or $\quad l = 40 \times 32 = 1280 \text{ mm} = 1.28 \text{ m} \qquad$ **Ans.**

EXAMPLE 14.13. *A rectangular beam 60 mm wide and 150 mm deep is simply supported over a span of 4 metres. If the bneam is subjected to a uniformly distributed load of 4.5 kN/m, find the maximum bending stress induced in the beam.*

SOLUTION. Given : Width (b) = 60 mm ; Depth (d) = 150 mm ; Span (l) = 4 m = 4×10^3 mm and uniformly distributed load (w) = 4.5 kN/m = 4.5 N/mm.

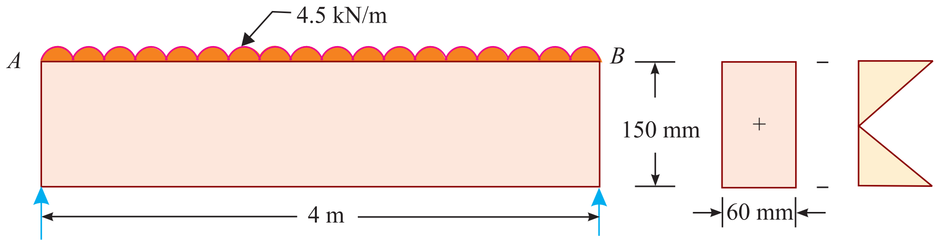


Fig. 14.18

We know that section modulus of the rectangular section,

$$Z = \frac{bd^2}{6} = \frac{60 \times (150)^2}{6} = 225 \times 10^3 \text{ mm}^3$$

and maxmum bending moment at the centre of a simply supported beam subjected to a uniformly distributed load,

$$M = \frac{wl^2}{8} = \frac{4.5 \times (4 \times 10^3)^2}{8} = 9 \times 10^6 \text{ N-mm}$$

∴ Maximum bending stress,

$$\sigma_{max} = \frac{M}{Z} = \frac{9 \times 10^6}{225 \times 10^3} = 40 \text{ N/mm}^2 = 40 \text{ MPa} \qquad \textbf{Ans.}$$

EXAMPLE 14.14. *A timber beam of rectangular section supports a load of 20 kN uniformly distributed over a span of 3.6 m. If depth of the beam section is twice the width and maximum stress is not to exceed 7 MPa, find the dimensions of the beam section.*

SOLUTION. Given : Total load (W) = 20 kN = 20×10^3 N ; Span (l) = 3.6×10^3 mm ; Depth of beam section (d) = $2b$ and (σ_{max}) = 7 MPa = 7 N/mm^2.

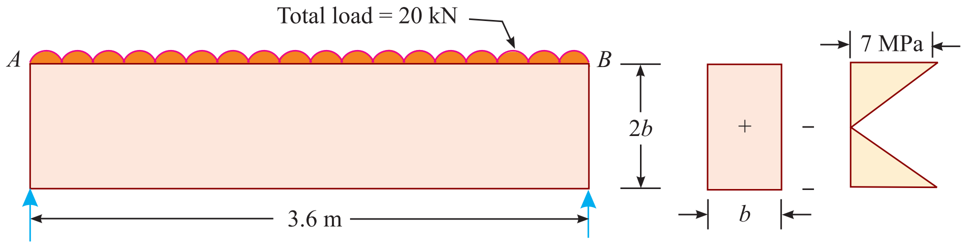


Fig. 14.19

We know that section modulus of the rectangular section,

$$Z = \frac{bd^2}{6} = \frac{b \times (2b)^2}{6} = \frac{2b^3}{3}$$

and maximum bending moment at the centre of a smiply supported beam subject to a uniformly distributed load,

$$M = \frac{wl^2}{8} = \frac{Wl}{8} = \frac{(20 \times 10^3) \times (3.6 \times 10^3)}{8} = 9 \times 10^6 \text{ N-mm}$$

∴ Maximum bending stress (σ_{max}),

$$7 = \frac{M}{Z} = \frac{9 \times 10^6}{\frac{2b^2}{3}} = \frac{13.5 \times 10^6}{b^3}$$

or
$$b^3 = \frac{(13.5 \times 10^6)}{7} = 1.93 \times 10^6$$

∴ $b = 1.25 \times 10^2 = 125$ mm **Ans.**

and $d = 2b = 2 \times 125 = 250$ mm **Ans.**

EXAMPLE 14.15. *A hollow square section with outer and inner dimensions of 50 mm and 40 mm respectively is used as a cantilever of span 1 m. How much concentrated load can be applied at the free end of the cantilever, if the maximum bending stress is not to exceed 35 MPa?*

SOLUTION. Given : Outer width (or depth) (B) = 50 mm ; Inner width (or depth) = (b) = 40 mm; Span (l) = 1×10^3 mm and maximum bending stress $\sigma_{b\,(max)}$ = 35 MPa = 35 N/mm^2.

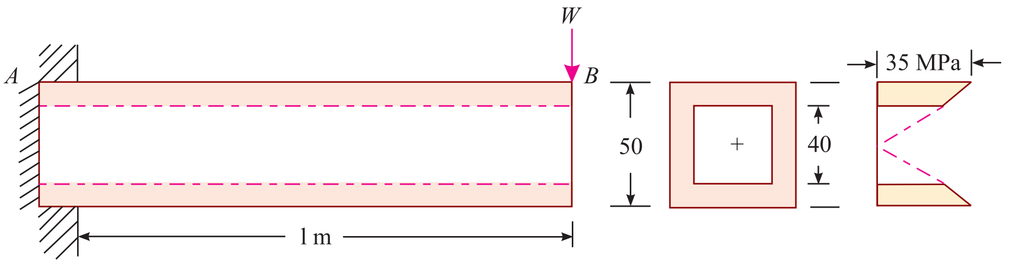


Fig. 14.20

Let W = Concentreated load that be applied at the free end of the cantilever.

We know that moment of inertia of the hollow square section,

$$I = \frac{BD^3}{12} - \frac{bd^3}{12} = \frac{BB^3}{12} - \frac{bb^3}{12} = \frac{B^4}{12} - \frac{b^4}{12} = \frac{(50)^4}{12} - \frac{(40)^4}{12} \text{ mm}^4$$
$$= 307.5 \times 10^3 \text{ mm}^4$$

∴ Modulus of section,
$$Z = \frac{I}{y} = \frac{307.5 \times 10^3}{25} = 12300 \text{ mm}^3$$

and maximum bending moment at the fixed end of the cantilever subjected to a point load at the free end,

$$M = Wl = W \times (1 \times 10^3) = 1 \times 10^3\, W$$

∴ Maximum bending stress (σ_{max}),

$$35 = \frac{M}{Z} = \frac{1 \times 10^3\, W}{12300}$$

or
$$W = \frac{35 \times 12300}{1 \times 10^3} = 430.5 \text{ N}$$ **Ans.**

EXAMPLE 14.16. *A hollow steel tube having external and internal diameter of 100 mm and 75 mm respectively is simply supported over a span of 5 m. The tube carries a concentrated load of W at a distance of 2 m from one of the supports. What is the value of W, if the maximum bending stress is not to exceed 100 MPa?*

SOLUTION. Given : External diameter (D) = 100 mm ; Internal diameter (d) = 75 mm ; Span (l) = 5 m = 5 × 10^3 mm ; Distance AC (a) = 2m = 2 × 10^3 mm or Distance BC (b) = 5 – 2 = 3 m = 3 × 10^3 mm and maximum bending stress (σ_{max}) = 100 MPa = 100 N/mm^2.

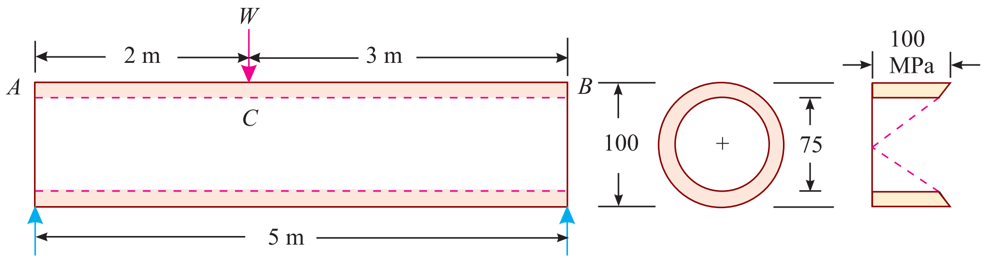


Fig. 14.21

We know that maximum bending moment over a simply supported beam subjected to an eccentric load,

$$M = \frac{Wab}{l} = \frac{W \times (2 \times 10^3) \times (3 \times 10^3)}{5 \times 10^3} = 1.2 \times 10^3\ W$$

and section modulus of a hollow circular section,

$$Z = \frac{\pi}{32 \times D} \times \left[D^4 - d^4\right] = \frac{\pi}{32 \times 100} \times \left[(100)^4 - (75)^4\right] \text{mm}^3$$

$$= 67.1 \times 10^3 \text{ mm}^3$$

We also know that maximum bending stress [$\sigma_{b\,(max)}$],

$$100 = \frac{M}{Z} = \frac{1.2 \times 10^3\ W}{67.1 \times 10^3} = 0.018\ W$$

$$\therefore \quad W = \frac{100}{0.018} = 5.6 \times 10^3 \text{ N} = 5.6 \text{ kN} \qquad \textbf{Ans.}$$

EXAMPLE 14.17. *A cast iron water pipe of 500 mm inside diameter and 20 mm thick is supported over a span of 10 meters. Find the maximum stress in the pipe metal, when the pipe is running full. Take density of cast iron as 70.6 kN/m^3 and that of water as 9.8 kN/m^3.*

SOLUTION. Given : Inside diameter (d) = 500 mm ; Thickness (t) = 20 mm or outside diameter (D) = $d + 2t$ = 500 + (2 × 20) = 540 mm ; Span (l) = 10 m = 10 × 10^3 mm ; density of cast iron = 70.6 kN/m^3 = 70.6 × 10^{-6} N/mm^2 and density of water = 9.8 kN/m^3 = 9.8 × 10^{-6} N/mm^2.

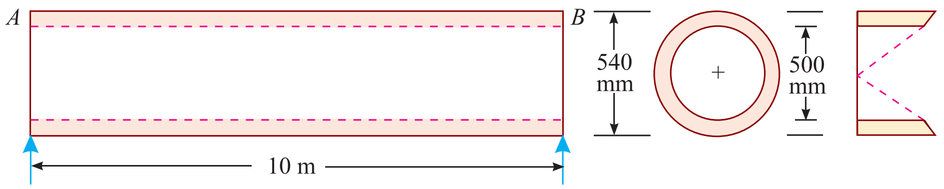


Fig. 14.22

We know that cross-sectional area of the cast iron pipe,

$$= \frac{\pi}{4} \times \left[D^2 - d^2\right] = \frac{\pi}{4} \times \left[(540)^2 - (500)^2\right] = 32.67 \times 10^3 \text{ mm}^2$$

and its weight (w_1) = (70.6 × 10^{-6}) × (32.67 × 10^3) = 2.31 N/mm

We also know that cross-sectional area of the water section

$$= \frac{\pi}{4} \times (d)^2 = \frac{\pi}{4} \times (500)^2 = 196.35 \times 10^3 \text{ mm}^2$$

and its weight $(w_2) = (9.8 \times 10^{-6}) \times (196.35 \times 10^3) = 1.92$ N/mm

$\therefore$ Total weight of the cast iron pipe and water section

$$w = w_1 + w_2 = 2.31 + 1.92 = 4.23 \text{ N/mm}$$

We also know that maximum bending moment at the centre of the beam subjected to a uniformly distributed load,

$$M = \frac{wl^2}{8} = \frac{4.23 \times (10 \times 10^3)^2}{8} = 52.9 \times 10^6 \text{ N-mm}$$

and section modulus of a hollow circular section,

$$Z = \frac{\pi}{32\,D} \times \left[D^2 - d^4\right] = \frac{\pi}{32 \times 540} \times \left[(540)^4 - (500)^4\right] \text{ mm}^3$$

$$= 4.096 \times 10^6 \text{ mm}^3$$

$\therefore$ Maximum bending stress,

$$\sigma_{b\,(max)} = \frac{M}{Z} = \frac{52.9 \times 10^6}{4.096 \times 10^6} = 12.9 \text{ N/mm}^2 = 12.9 \text{ MPa} \qquad \textbf{Ans.}$$

EXERCISE 14.2

1. A beam 3 m long has rectangular section of 80 mm width and 120 mm depth. If the beam is carrying a uniformly distributed load of 10 kN/m, find the maximum bending stress developed in the beam. [**Ans.** 58.6 MPa]
2. A rectangular beam 200 mm deep is simply supported over a beam of span 2 m. Find the uniformly distributed load, the beam can carry if the bending stress is not to exceed 30 MPa. Take I for the beam as 8×10^6 mm^4. [**Ans.** 4.8 N/mm]
3. A rectangular beam, simply supported over a span of 4 m, is carrying a uniformly distributed load of 50 kN/m. Find the dimensions of the beam, if depth of the beam section is 2.5 times its width. Take maximum bending stress in the beam section as 60 MPa. [**Ans.** 125 mm; 300 mm]
4. Calculate the cross-sectional dimensions of the strongest rectangular beam, that can be cut out of a cylindrical log of wood whose diameter is 500 mm. [**Ans.** 288.5 mm × 408.5 mm]

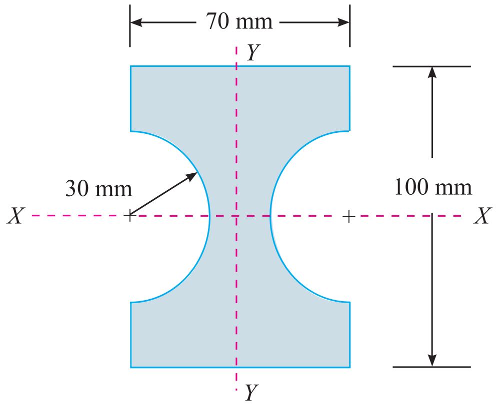


Fig. 14.23

5. Fig. 14.23 shows the section of a beam. What is the ratio of its moment of resistance to bending in the plane *Y-Y* to that for bending in the plane *X-X*, if the maximum stress due to bending is same in both the cases.

For a semi-circle of radius *r*, the centroid is at a distance of $4r/3\pi$ from the centre. [**Ans.** 2.85]

QUESTIONS

1. Define the term 'bending stress' and explain clearly the theory of simply bending.

2. State the assumptions made in the theory of simple bending.

3. Prove the relations,

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

where M = Bending moment,

I = Moment of inertia,

σ = Bending stress in a fibre, at a distance y from the neutral axis,

E = Young's modulus, and

R = Radius of curvature.

4. Discuss the procedure in finding out the bending stress in a symmetrical section.

5. How will you find the bending stress in a hollow circular section?

OBJECTIVE TYPE QUESTIONS

1. The neutral axis of a section is an axis, at which the bending stress is

(*a*) minimum (*b*) zero (*c*) maximum (*d*) infinity

2. In the theory of simply bending, the bending stress in the beam section varies

(*a*) linearly (*b*) parabolically (*c*) elliptically (*d*) none of them

3. When a cantilever is loaded at its free end, maximum compressive stress shall develop at

(*a*) bottom fibre (*b*) top fibre (*c*) neutral axis (*d*) centre of gravity

4. The section modulus of a rectangular section having width (*b*) and depth (*d*) is

(*a*) $\frac{bd}{6}$ (*b*) $\frac{bd^2}{6}$ (*c*) $\frac{bd^3}{6}$ (*d*) $\frac{b^2d}{6}$

5. The section modulus of a circular section of diameter (*d*) is

(*a*) $\frac{\pi}{32}(d)^2$ (*b*) $\frac{\pi}{32}(d)^3$ (*c*) $\frac{\pi}{64}(d)^3$ (*d*) $\frac{\pi}{64}(d)^4$

ANSWERS

1. (*b*) **2.** (*a*) **3.** (*a*) **4.** (*b*) **5.** (*b*)

Chapter **15**

Bending Stresses in Composite Beams

Contents

15.1. Introduction

In the last chapter, we have discussed the bending stresses in simple beams, and the pattern in which these stresses vary along the symmetrical sections. But sometimes we come across beams of composite sections. And we are required to study the pattern in which these stresses vary along such sections.

15.2. Types of Composite Beams

Though there are many types of composite beams that we come across, yet the following are important from the subject point of view:

1. Beams of unsymmetrical sections
2. Beams of uniform strength
3. Flitched beams.

15.3. Beams of Unsymmetrical Sections

We have already discussed in the last chapter that in a symmetrical section, the distance of extreme fibre from the c.g. of the section $y = d/2$. But this is not the case, in an unsymmetrical section (L, I, T, etc.), since the neutral axis of such a section does not pass through the geometrical centre of the section. In such cases, first the centre of gravity of the section is obtained as discussed in Chapter 6 and then the values of y, in the tension and compression sides, is studied. For obtaining the bending stress in a beam, the bigger value of y (in tension or compression) is used in the equation. This will be illustrated by the following examples.

EXAMPLE 15.1. *Two wooden planks 150 mm × 50 mm each are connected to form a T-section of a beam. If a moment of 6.4 kN-m is applied around the horizontal neutral axis, inducing tension below the neutral axis, find the bending stresses at both the extreme fibres of the cross- section.*

SOLUTION. Given: Size of wooden planks = 150 mm × 50 mm and moment (M) = 6.4 kN-m = 6.4 × 10^6 N-mm.

Two planks forming the T-section are shown in Fig. 15.1. First of all, let us find out the centre of gravity of the beam section. We know that distance between the centre of gravity of the section and its bottom face,

$$\bar{y} = \frac{(150 \times 50)\,175 + (150 \times 50)\,75}{(150 \times 50) + (150 \times 50)} = \frac{1875000}{15000} = 125 \text{ mm}$$

∴ Distance between the centre of gravity of the section and the upper extreme fibre,

$$y_t = 20 - 125 = 75 \text{ mm}$$

and distance between the centre of gravity of the section and the lower extreme fibre,

$$y_c = 125 \text{ mm}$$

We also know that Moment of inertia of the T section about an axis passing through its c.g. and parallel to the botom face,

$$I = \left[\frac{150 \times (50)^3}{12} + (150 \times 50)\,(175 - 125)^2\right] + \left[\frac{50 \times (150)^3}{12} + (150 \times 50\,(125 - 75)^2\right] \text{mm}^4$$

$$= (20.3125 \times 10^6) + (32.8125 \times 10^6) \text{ mm}^4$$

$$= 53.125 \times 10^6 \text{ mm}^4$$

∴ Bending stress in the upper extreme fibre,

$$\sigma_1 = \frac{M}{I} \times y_t = \frac{6.4 \times 10^6}{53.125 \times 10^6} \times 125 \text{ N/mm}^2$$

$$= 15.06 \text{ N/mm}^2 = 15.06 \text{ MPa (compression)}$$ **Ans.**

and bending stress in the lower extreme fibre,

$$\sigma_2 = \frac{M}{I} \times y_c = \frac{6.4 \times 10^6}{53.125 \times 10^6} \times 75 \text{ N/mm}^2$$

$$= 9.04 \text{ N/mm}^2 = 9.04 \text{ MPa (tension)}$$ **Ans.**

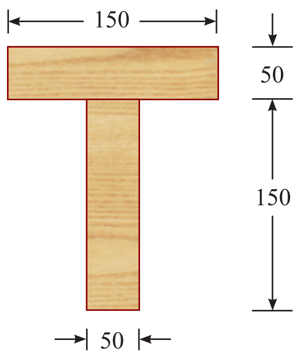


Fig. 15.1

EXAMPLE 15.2. *Figure 15.2 shows a rolled steel beam of an unsymmetrical I-section.*

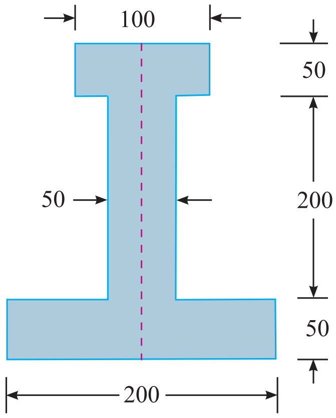


Fig. 15.2

If the maximum bending stress in the beam section is not to exceed 40 MPa, find the moment, which the beam can resist.

SOLUTION. Given: Maximum bending stress (σ_{max}) = 40 MPa = 40 N/mm^2.

We know that distance between the centre of gravity of the section and bottom face,

$$\bar{y} = \frac{(100 \times 50)\,275 + (200 \times 50)\,150 + (200 \times 50)\,25}{(100 \times 50) + (200 \times 50) + (200 \times 50)} = 125 \text{ mm}$$

$\therefore \quad y_1 = 300 - 125 = 175$ mm and $y_2 = 125$ mm

Thus we shall take the value of y = 175 mm (*i.e.*, greater of the two values between y_1 and y_2). We also know that moment of inertia of the *I*-section about an axis passing through its centre of gravity and parallel to the bottom face,

$$I = \left[\frac{100 \times (50)^3}{12} + (100 \times 50)\,(275 - 125)^2\right] + \left[\frac{50 \times (200)^3}{12} + (50 \times 200)\,(150 - 125)^2\right]$$

$$+ \left[\frac{200 \times (50)^3}{12} + (200 \times 50)\,(125 - 25)^2\right] \text{mm}^4$$

$$= 255.2 \times 10^6 \text{ mm}^4$$

and section modulus of the *I*-section,

$$Z = \frac{I}{y} = \frac{255.2 \times 10^6}{175} = 1.46 \times 10^6 \text{ mm}^3$$

$\therefore$ Moment, which the beam can resist,

$$M = \sigma_{max} \times Z = 40 \times (1.46 \times 10^6) \text{ N-mm}$$

$$= 58.4 \times 10^6 \text{ N-mm} = 58.4 \text{ kN-m} \qquad \textbf{Ans.}$$

EXAMPLE 15.3. *A simply supported beam and its cross-section are shown in Fig. 15.3. The beam carries a load of 10 kN as shown in the figure. Its self weight is 3.5 kN/m. Calculate the maximum bending stress at X-X.*

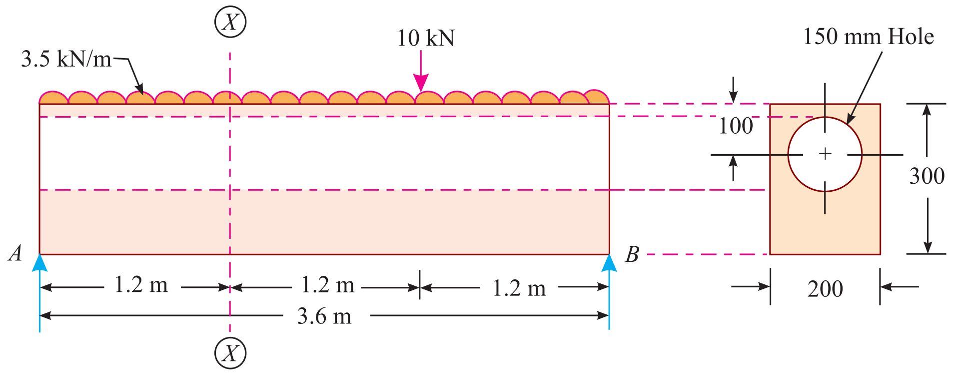


Fig. 15.3

Solution. Given: Point load (W) = 10 kN = 10×10^3 N and self weight of the beam (w) = 3.5 kN/m = 3.5 N/mm.

First of all, let us find out the centre of gravity of the beam section. We know that distance between the centre of gravity of the section and its bottom face,

$$\bar{y} = \frac{[(200 \times 300)\ 150] - \left[\frac{\pi}{4}(150)^2 \times 200\right]}{[200 \times 300] - \left[\frac{\pi}{4}(150)^2\right]} = 129 \text{ mm}$$

∴ Distance between centre of gravity of the section and the upper extreme fibre,

$$y_t = 30 - 129 = 171 \text{ mm}$$

and distance between the centre of gravity of the section and the lower extreme fibre,

$$y_c = 129 \text{ mm}$$

Therefore for maximum bending stress, we shal use the value of y equal to 171 mm (*i.e.*, greater of the two values of y_t and y_c). We know that moment of inertia of the section passing through its centre of gravity and parallel to *x-x* axis,

$$I = \left[\frac{200\,(300)^3}{12} + (200 \times 300) \times (150 - 129)^2\right] - \left[\frac{\pi}{64}(150)^4 + \frac{\pi}{4} \times (150)^2 \times (200 - 129)^2\right] \text{mm}^4$$

$$= (476.5 \times 10^6) - (113.9 \times 10^6) = 362.6 \times 10^6 \text{ mm}^4$$

Now let us find out the bending moment at *x-x*. Taking moments about A and equating the same,

$$R_B \times 3.6 = (3.5 \times 3.6 \times 1.8) + (10 \times 2.4) = 46.68$$

$$\therefore \quad R_B = \frac{46.68}{3.6} = 13.0 \text{ kN}$$

or $\quad R_A = [(3.5 \times 3.6) + 10] - 13.0 = 9.6$ kN

and bending moment at X,

$$M = (9.6 \times 1.2) - (3.5 \times 1.2 \times 0.6) = 9 \text{ kN-m} = 9 \times 10^6 \text{ N-mm}$$

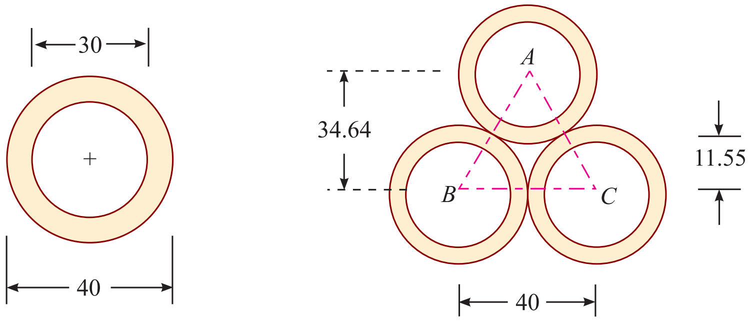
∴ Maximum bending stress at X,

$$\sigma_b = \frac{M}{I} \times y = \frac{9 \times 10^6}{362.6 \times 10^6} \times 171 = 4.24 \text{ N/mm}^2$$

$$= 4.24 \text{ MPa} \qquad \textbf{Ans.}$$

Example 15.4. *A steel tube 40 mm outside diameter and 30 mm inside diameter is simply supported over a 6 m span and carries a central load of 200 N. Three such tubes and firmly joined together, to act as a single beam, in such a way that their centres make an equilateral triangle of side 40 mm. Find the central load, the new beam can carry, if the maximum bending stress is the same in both the cases.*

Solution. Given: Outside diameter (D) = 40 mm ; Inside diameter (d) = 30 mm ; Span (l) = 6 m = 6×10^3 mm and central point load in case of single tube (W_1) = 200 N.



(a) Single tube (a) Three tubes

Fig. 15.4

First of all, consider a single tube as shown in Fig. 15.4 (a). We know that maximum bending moment over simply supported load subjected to a central point load

$$M = \frac{Wl}{4} = \frac{200 \times (6 \times 10^3)}{4} = 300 \times 10^3 \text{ N-mm}$$

and section modulus of a hollow circular section

$$Z = \frac{\pi}{32\,D} \times \left[D^4 - d^4\right] = \frac{\pi}{32 \times 40} \times \left[(40)^4 - (30)^4\right] \text{mm}^3$$

$$= 4.295 \times 10^3 \text{ mm}^3$$

$\therefore$ Maximum bending stress,

$$\sigma_{max} = \frac{M}{Z} = \frac{300 \times 10^3}{4.295 \times 10^3} = 69.85 \text{ N/mm}^2$$

Now consider these tubes firmly joined together as shown in Fig. 15. 4 (b). We know that vertical height of the equilateral triangle,

$$= AB \sin 60° = 40 \times 0.866 = 34.64 \text{ mm}$$

$\therefore$ Centre of gravity of the section will lie at a height of 34.64/3 = 11.5 mm from the base BC. Thus distance between the centre of gravity of the section and upper extreme fibre,

$$y_c = (34.64 - 11.55) + 20 = 43.09 \text{ mm}$$

and distance between the centre of gravity of the section and the lower extreme fibre,

$$y_t = 11.55 + 20 = 31.55 \text{ mm}$$

Therefore for maximum bending stress, we shall use the value of y equal to 43.09 mm (*i.e.*, greater of two values of y_c and y_t). We know that cross-sectional area of one tube,

$$A = \frac{\pi}{4} \times \left[D^2 - d^2\right] = \frac{\pi}{4} \times \left[(40)^2 - (30)^2\right] = 549.8 \text{ mm}^2$$

and moment of inertia of one hollow tube

$$= \frac{\pi}{64} \times \left[D^4 - d^4\right] = \frac{\pi}{64} \times \left[(40)^4 - (30)^4\right] = 85.9 \times 10^3 \text{ mm}^4$$

∴ Moment of inertia of whole section passing through its centre of gravity and parallel to X-X axis,

$$I = 2\left[85.9\times10^3 + 549.8\,(11.55)^2\right] + \left[85.9\times10^3 + 549.8\,(34.64-11.55)^2\right]$$

$$= (318.5 \times 10^3) + (379.0 \times 10^3) = 697.5 \times 10^3 \text{ mm}^4$$

and maximum bending moment at the centre of beam due to the central load W_2,

$$M = \frac{W_2 l}{4} = \frac{W_2 \times (6\times10^3)}{4} = 1.5 \times 10^3\ W_2 \text{ N-mm}$$

We know that maximum bending stress (σ_{max})

$$69.85 = \frac{M}{I} \times y = \frac{1.5\times10^3\,W_2}{697.5\times10^3} \times 43.09 = 0.093\ W_2$$

$$\therefore \quad W_2 = \frac{69.85}{0.093} = 751 \text{ N} \qquad \textbf{Ans.}$$

EXAMPLE 15.5. *Figure 15.5 shows a rolled steel beam of an unsymmetrical I-section.*

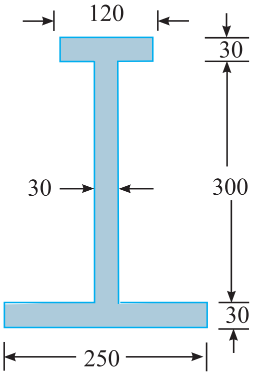


Fig. 15.5

If a similar I-section is welded on the top of it to form a symmetrical section, determine the ratio of the moment of resistance of the new section to that of the single section. Assume the permissible bending stress in tension and compression to be the same.

SOLUTION. Given: Permissible bending stess in tension = Permissible bending stress in compression.

First of all, let us find the centre of gravity of the section. We know that distance between the centre of gravity of the section and bottom face,

$$\bar{y} = \frac{(120\times30)\,345 + (300\times30)\,180 + (250\times30)\,15}{(120\times30) + (300\times30) + (250\times30)} \text{ mm}$$

$$= \frac{2974500}{20100} = 148 \text{ mm}$$

$$\therefore \quad y_1 = 360 - 148 = 212 \text{ mm} \qquad \text{and} \qquad y_2 = 148 \text{ mm}$$

Thus for the prupose of calculating moment of resistance of the section, we shall take the value of y equal to 212 mm (*i.e.*, greater of the two values between y_1 and y_2). We also know that moment of inertia of the I-section about an axis through its centre of gravity and parallel to its x-x axis,

$$I_1 = \left[\frac{120\times(30)^3}{12} + (120\times30)\,(345-148)^2\right] + \left[\frac{30\times(300)^3}{12} + (30\times300)\,(180-148)^2\right]$$

$$+ \left[\frac{250\times(30)^3}{12} + (250\times30)\,(148-15)^2\right] \text{mm}^4$$

$$= 350 \times 10^6 \text{ mm}^4$$

∴ Section modulus of the *I*-section,

$$Z_1 = \frac{1}{y} = \frac{350 \times 10^6}{212} = 1.65 \times 10^6 \text{ mm}^3$$

and moment of resistance of the *I*-section

$$M_1 = \sigma \times Z_1 = \sigma \times 1.65 \times 10^6 = 1.65 \times 10^6\, \sigma \qquad ...(i)$$

Now, let us consider the double section as shown in Fig. 15.6. We know that in this case, centre of gravity of the section will lie at the junction of the two sections.

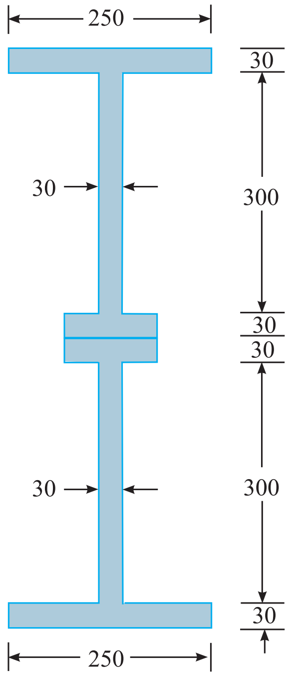


Fig. 15.6

Therefore moment of inertia of the double section about its axis through its c.g. and parallel to *x-x* axis,

$$I_2 = 2\,[(350 \times 10^6) + 20100 \times (212)^2] \text{ mm}^4$$

$$= 2\,[(350 \times 10^6) + (903.4 \times 10^6)] = 2506.8 \times 10^6 \text{ mm}^4$$

∴ Section moulus of the double section,

$$Z_2 = \frac{I}{y} = \frac{2506.8 \times 10^6}{360} = 6.96 \times 10^6 \text{ mm}^3$$

and moment of resistance of the double I-section

$$M_2 = \sigma \times Z_2 = \sigma \times 6.96 \times 10^6\, \sigma \qquad ...(ii)$$

∴ Ratio of moments of resistances

$$\frac{M_2}{M_1} = \frac{6.96 \times 10^6\, \sigma}{1.65 \times 10^6\, \sigma} = 4.22 \qquad \textbf{Ans.}$$

EXAMPLE 15.6. *The cross-section of a beam is shown in Fig. 15.7. The beam is made of material with permissible stress in compression and tension equal to 100 MPa and 140 MPa respectively.*

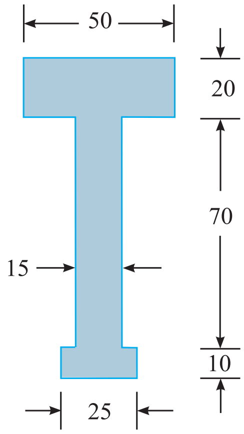


Fig. 15.7

Calculate the moment of resistance of the cross-section, when subjected to a moment causing compression at the top and tension at the bottom.

Solution. Given: Permissible stress in compression $(\sigma_c) = 100$ MPa $= 100$ N/mm^2 and permissible stress in tension $(\sigma_t) = 140$ MPa $= 140$ N/mm^2.

Moment of resistance of the cross-section

First of all, let us find the centre of gravity of the section. We know that the distance between the centre of gravity of the section and its bottom face,

$$\bar{y} = \frac{(50\times 20)\,90 + (70\times 15)\,45 + (25\times 10)5}{(50\times 20) + (70\times 15) + (25\times 10)} = 60.2 \text{ mm}$$

$$\therefore \qquad y_1 = 100 - 60.2 = 39.8 \text{ mm} \qquad \text{and} \qquad y_2 = 60.2 \text{ mm}$$

Thus for the purpose of calculating moment of resistance of the section, we shall take the value of y equal to 60.2 mm (*i.e.*, greater of the two values between y_1 and y_2). We also know that moment of inertia of the section about an axis through its c.g. and parallel to x-x axis,

$$I = \left[\frac{50\times(20)^3}{12} + (50\times 20)(90-60.2)^2\right] + \left[\frac{15\times(70)^3}{12} + (70\times 15)(60.2-45)^2\right]$$

$$+ \left[\frac{25\times(10)^3}{12} + (25\times 10)\,(60.2-5)^2\right] \text{mm}^4$$

$$= 2356.6 \times 10^3 \text{ mm}^4$$

$\therefore$ Section modulus of the section (in compression zone),

$$Z_1 = \frac{I}{y_1} = \frac{2356.6\times 10^3}{39.8} = 59.2 \times 10^3 \text{ mm}^3$$

and moment of resistance of the compression zone,

$$M_1 = \sigma_c \times Z_1 = 100 \times 59.2 \times 10^3 = 5920 \times 10^3 \text{ N-mm}$$

Similarly, section modulus of the section (in tension zone),

$$Z_2 = \frac{M}{y_2} = \frac{2356.6\times 10^3}{60.2} = 39.1 \times 10^3 \text{ mm}^3$$

and moment of resistance of the tension zone,

$$M_2 = \sigma_t \times Z_2 = 140 \times 39.1 \times 10^3 = 5474 \times 10^3 \text{ N-mm}$$

$\therefore$ Moment of resistance of the cross-section is the least of the two values *i.e.*,

5474×10^3 N-mm **Ans.**

EXERCISE 15.1

1. Cantilever beam of span 2.5 m has a T-section as shown in Fig. 15.8. Find the point load, which the cantilever beam can carry at its free end, if the bending stress is not to exceed 50 MPa. (**Ans.** 1.6 kN)
2. An I-section shown in Fig. 15.9 is simply supported over a span of 5 metres. If the tensile stress is not to exceed 20 MPa, find the safe uniformly distributed load, the beam can carry. (**Ans.** 6.82 kN/m)

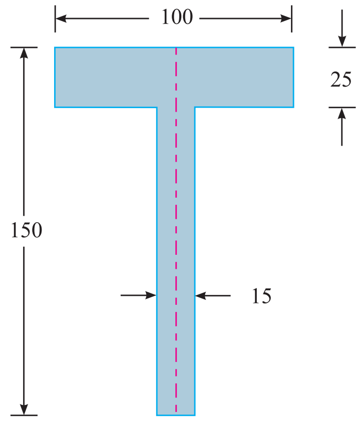


Fig. 15.8

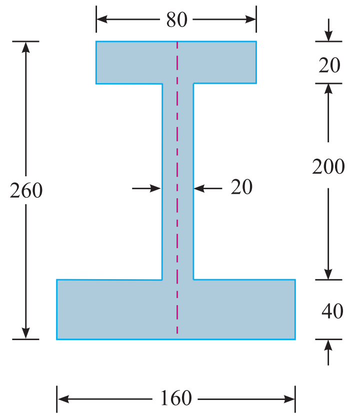


Fig. 15.9

3. Two beams are simply supported over the same span and have the same flexural strength. Compare the weights of these two beams, if one of them is solid circular and the other hollow circular with internal diameter half of the external diameter. (**Ans.** 1.28)

15.4. Beams of Uniform Strength

We have already discussed that in a simply supported beam, carrying a *uniformly distributed load, the maximum bending moment will occur at its centre. It is thus obvious that the bending stress is also maximum at the centre of the beam. As we proceed, from the centre of the beam towards the supports, the bending moment decreases and hence the maximum stress developed is below the permissible limit. It results in the wastage of material. This wastage is negligible in case of small spans, but considerable in case of large spans.

The beams of large spans are designed in such a way that their cross-sectional area is decreased towards the supports so that the maximum bending stress developed is equal to the allowable stress (as is done at the centre of the beam). Such a beam, in which bending stress developed is constant and is equal to the allowable stress at every section is called a beam of uniform strength. The section of a beam of uniform strength may be varied in the following ways:

1. By keeping the width uniform and varying the depth.
2. By keeping the depth uniform and varying the width.
3. By varying both width and depth.

The most common way of keeping the beam of uniform strength is by keeping the width uniform and varying the depth.

EXAMPLE 15.7. *A simply supported beam of 2.4 meters span has a constant width of 100 mm throughout its length with varying depth of 150 mm at the centre to minimum at the ends as shown in Fig. 15.10. The beam is carrying a point load W at its mid-point.*

* This is the most practical case. However, if a beam is carrying some other type loading, the maximum bending moment will occur, at a point, near its centre.

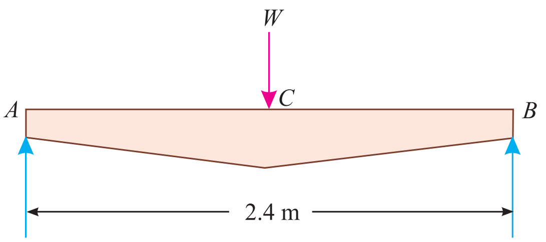


Fig. 15.10

Find the minimum depth of the beam at a section 0.6 m from the left hand support, such that the maximum bending stress at this section is equal to that at the mid-span of the beam.

SOLUTION. Given: Span $(l) = 2.4 \text{ m} = 2.4 \times 10^3$ mm ; Width $(b) = 100$ mm and depth at the centre $(d_C) = 150$ mm.

Let d_X = Depth at the section X *i.e.*, 0.6 m (*i.e.*, 600 mm) from the left end.

f_X = Bending stress at X and

f_C = Bending stress at C.

Since the beam is carrying a central point load, therefore the reaction at A,

$$R_A = R_B = \frac{W}{2}$$

Bending moment at C,
$$M_C = \frac{W}{2} \times 1200 = 600 \text{ W}$$

Similarly,
$$M_X = \frac{W}{2} \times 600 = 300 \text{ W}$$

We know that section modulus at the centre of beam,

$$Z_X = \frac{b.d_X^2}{6} = \frac{100 d_X^2}{6} = 50 \frac{d_X^2}{3} \text{ mm}^3$$

and
$$Z_C = \frac{b.d_C^2}{6} = \frac{100 \times (150)^2}{6} = 375\ 000 \text{ mm}^3$$

We also know that bending moment at C (M_C),

$$600\ W = \sigma_C \times Z_C = \sigma_C \times 375\ 000$$

$$\therefore \quad \sigma_C = \frac{600\ W}{375\ 000} \qquad ...(i)$$

Similarly bending moment at X (M_X)

$$300\ W = \sigma_X \times Z_X = \sigma_X \times \frac{50\ d_X^2}{3}$$

$$\therefore \quad \sigma_X = 300\ W \times \frac{3}{50\ d_X^2} = \frac{18\ W}{d_X^2} \qquad ...(ii)$$

Since σ_C is equal to σ_X, therefore equating (*i*) and (*ii*),

$$\frac{600 W}{375000} = \frac{18 W}{d_X^2}$$

$$\therefore \quad d_X^2 = \frac{18 \times 375000}{600} = 11250 \text{ mm}^2$$

or
$$d_X = 106.01 \text{ mm} \qquad \textbf{Ans.}$$

EXAMPLE 15.8. *A horizontal cantilever 3 m long is of rectangular cross-section 60 mm wide throughout its length, and depth varying uniformly from 60 mm at the free end to 180 mm at the fixed end. A load of 4 kN acts at the free end as shown in Fig. 15.11.*

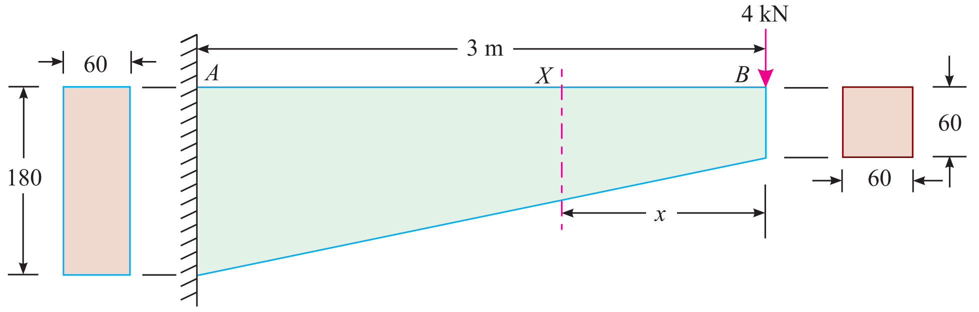


Fig. 15.11

Find the position of the highest stressed section and the value of the maximum bending stress induced. Neglect the weight of the cantilever itself.

SOLUTION. Given: Span $(l) = 3 \text{ m} = 3 \times 10^3$ mm and point load at the free end $(W) = 4 \text{ kN} = 4 \times 10^3$ N.

Position of the highest stressed section

Let x = Distance in metres of the section from B, which is highest stressed.

We know that the moment at X,

$$M_X = (4 \times 10^3)(x \times 10^3) = 4 \times 10^6 \, x \text{ N-mm} \qquad ...(i)$$

and depth of the cantilever at X,

$$d = 60 + \frac{180 - 60}{3} x = 60 + 40\,x \text{ mm} \qquad ...(ii)$$

∴ Section modulus at X,

$$Z_X = \frac{bd^2}{6} = \frac{60}{6}(60 + 40\,x)^2 \text{ mm}^3$$

$$= 10\,[20\,(3 + 2\,x)]^2 = 4000\,(3 + 2\,x)^2 \text{ mm}^3 \qquad ...(iii)$$

We also know that bending stress at X,

$$\sigma = \frac{M_X}{Z_X} = \frac{4 \times 10^6\, x}{4000\,(3 + 2\,x)^2} = \frac{10^3\, x}{(3 + 2\,x)^2} \text{ N/mm}^2 \qquad ...(iv)$$

Now for σ to be maximum, differentiate the above equation and equate it to zero, *i.e.*,

$$\frac{d\sigma}{dx} = \frac{d}{dx}\left(\frac{10^3\, x}{(3 + 2\,x)^2}\right) = 0 \qquad \text{or} \qquad 2\,(3 + 2\,x) = 0$$

∴ $x = 1.5$ m **Ans.**

Value of the maximum bending stress

Now substituting the value of x in equation (iv),

$$\sigma_{max} = \frac{10^3 \times 1.5}{(3 + 2 \times 1.5)^2} = 41.7 \text{ N/mm}^2 = 41.7 \text{ MPa} \qquad \textbf{Ans.}$$

15.5 Beams of Composite Section (Flitched Beams)

A composite section may be defined as a section made up of two or more different materials, joined together in such a manner that they behave like a single piece and, each material bends to the same radius of curvature. Such beams are used when a beam of one material, if used alone, requires quite a large cross-sectional area; which does not suit the space available. A material is then reinforced with some other material, of higher strength, in order to reduce the cross-sectional area of the beam and to suit the space available (as is done in the case of reinforced cement concrete beams).

In such cases, the total moment of resistance will be equal to the sum of the moments of individual sections.

Consider a beam of a composite section made up of two different materials as shown in Fig. 15.12.

Let E_1 = Modulus of elasticity of part 1,

I_1 = Moment of inertia of the part 1,

M_1 = Moment of resistance for part 1,

σ_1 = Stress in part 1,

Z_1 = Modulus of section for part 1,

$E_2, I_2, M_2, \sigma_2, Z_2$ = Corresponding values for part 2 and

R = Radius of the bend up beam.

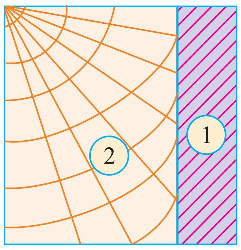


Fig. 15.12

We know that the moment of resistance for beam 1,

$$M_1 = \sigma_1 \times Z_1 \qquad (\because M = \sigma \times Z)$$

Similarly,

$$M_2 = \sigma_2 \times Z_2$$

∴ Total moment of resistance of the composite section,

$$M = M_1 + M_2 = (\sigma_1 \times Z_1) + (\sigma_2 \times Z_2) \qquad ...(i)$$

We also know that at any distance from the neutral axis, the strain in both the materials will be the same.

$$\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_1} \qquad \text{or} \qquad \sigma_1 = \frac{E_1}{E_2} \times \sigma_2 = m \times \sigma_2$$

where $m = \frac{F_1}{E_2}$ *i.e.*, Modulus ratio.

From the above two relations, we can find out the total moment of resistance of a composite beam or stresses in the two materials. But, if the sections of both the materials are not symmetrical, then one area of the components is converted into an equivalent area of the other.

EXAMPLE 15.9. *A flitched timber beam made up of steel and timber has a section as shown in Fig. 15.13.*

Determine the moment of resistance of the beam. Take σ_S = 100 MPa and σ_T = 5 MPa.

SOLUTION. Width of each timber section (b_T) = 60 mm ; Depth of each timber section (d_T) = 200 mm ; Stress in timber (σ_T) = 5 MPa = 5 N/mm^2 ; Width of steel section (b_S) = 15 mm ; Depth of steel section (d_S) = 20 mm and stress in steel (σ_S) = 100 MPa = 100 N/mm^2.

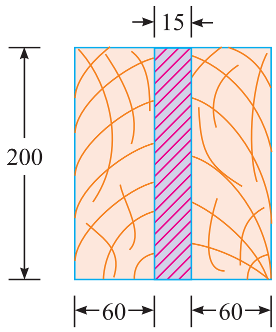


Fig. 15.13

We know that the section modulus of a rectangular body,

$$Z = \frac{bd^2}{6}$$

∴ Modulus of section for both the timber sections,

$$Z_T = 2\left[\frac{60 \times (200)^2}{2}\right] = 800 \times 10^3 \text{ mm}^3 \qquad ...(\because \text{ of two sections})$$

Similarly, modulus of section for the steel section

$$Z_S = \frac{15 \times (200)^2}{6} = 100 \times 10^3 \text{ mm}^3$$

We also know that moment of resistance for timber,

$$M_T = \sigma_T \times Z_T = 5 \times (800 \times 10^3) = 4 \times 10^6 \text{ N-mm}$$

Similarly,
$$M_S = \sigma_S \times Z_S = 100 \times (100 \times 10^3) = 10 \times 10^6 \text{ N-mm}$$

∴ Total moment of resistance of the beam,

$$M = M_T + M_S = (4 \times 10^6) + (10 \times 10^6) = 14 \times 10^6 \text{ N-mm}$$

$$= 14 \text{ kN-m} \qquad \textbf{Ans.}$$

EXAMPLE 15.10. *A timber beam 100 mm wide and 200 mm deep is strengthened by a steel plate 100 mm wide and 100 mm thick, screwed at the bottom surface of the timber beam as shown in Fig. 15.14.*

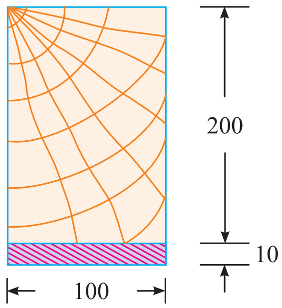


Fig. 15.14

Calculate the moment of resistance of the beam, if the safe stresses in timber and steel are 10 MPa and 150 MPa respectively. Take $E_S = 20\ E_T$.

SOLUTION. Given : Width of timber section $(b_T) = 100$ mm ; Depth of timber section $(d_T) = 200$ mm ; Safe stress in timber $(\sigma_T) = 10$ MPa = 10 N/mm^2 ; Width of steel section $(b_S) = 100$ mm ; Depth of steel section $(d_S) = 10$ mm ; Safe stress in steel $(\sigma_S) = 150$ MPa = 150 N/mm^2 and modulus of elasticity for steel $(E_S) = 20\ E_T$.

We know that stress in steel is m times (20 times in this case) the stress in timber at the same level. Hence the resistance offered by the steel is also equal to m times the resistance offered by the timber of an equal area. It is thus obvious that if we replace steel by timber (or *vice versa*) of an area equal to m times the area of the steel, the total resistance to bending offered will remain unchanged; provided the distribution of the area about the neutral axis also remains unchanged. This can be done, by keeping the depth of the area unchanged and by increasing the breadth of the timber m times the breadth of the steel. The section thus obtained is called equivalent section and its moment of resistance is equal to that of the given section.

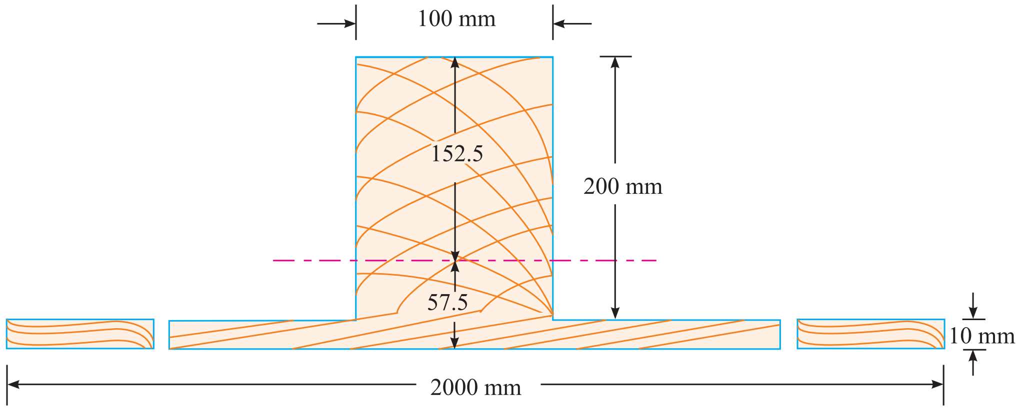


Fig. 15.15

In this case the equivalent section (of wood) is shown in Fig. 15.15. The bottom steel plate has been replaced by an equivalent timber of 100 × 20 = 2000 mm.

We know that distance between the centre of gravity of the equivalent timber section and its bottom face,

$$\bar{y} = \frac{(2000 \times 10) \times 5 + (100 \times 200) \times 110}{(2000 \times 10) + (100 \times 200)} = 57.5 \text{ mm}$$

Therefore distance between the centre of gravity of the equivalent timber section and the upper extreme fibre,

$$y_C = 210 - 57.5 = 152.5 \text{ mm}$$

and $y_T = 57.5$ mm

Therefore we shall take the value of $y = 152.5$ mm (*i.e.*, greater of the two values among, y_T and y_C). Now when the stress in uppermost fibre is 10 N/mm^2 (given safe stress), then the stress in the lowermost fibre,

$$= \frac{10 \times 57.5}{152.5} = 3.77 \text{ N/mm}^2$$

∴ Actual stress in steel at this fibre = 3.77 × 20 = 75.4 N/mm^2

It is below the given safe stress (*i.e.*, 150 N/mm^2). We also know that moment of inertia of the equivalent timber section about an axis passing through its centre of gravity and parallel to *x-x* axis,

$$I = \left[\frac{2000 \times (10)^3}{12} + (2000 \times 10)(57.5 - 5)^2\right] + \left[\frac{100 \times (200)^3}{12} + (100 \times 200)(110 - 57.5)^2\right] \text{mm}^4$$

$$= (55.3 \times 10^6) + (121.8 \times 10^6) = 177.1 \times 10^6 \text{ mm}^4$$

and section modulus of the equivalent section,

$$Z = \frac{I}{y} = \frac{177.1 \times 10^6}{152.5} = 1.16 \times 10^6 \text{ mm}^3$$

∴ Moment of resistance of the equivalent section,

$$M = \sigma_1 \times Z = 10\,(1.16 \times 10^6) = 11.6 \times 10^6 \text{ N-mm}$$

$$= 11.6 \text{ kN-m}$$ **Ans.**

Alternate method

Let us convert the section into an equivalent steel section as shown in Fig. 15.16. The upper timber beam has been replaced by an equivalent steel beam of thickness

$$\bar{y} = \frac{(100 \times 10) \times 5 + (200 \times 5) \times 110}{(100 \times 10) + (200 \times 5)} \text{ mm}$$

$$= 57.5 \text{ mm (same as in first method)}$$

Therefore distance between the centre of gravity of the equivalent steel section and the upper extreme fibre,

$$y_c = 210 - 57.5 = 152.5 \text{ mm}$$

and $$y_t = 57.5 \text{ mm}$$

Therefore we shall take the value of $y = 152.5$ mm (*i.e.*, greater of the two values *i.e.*, y_t and y_c). Now when the stress in the uppermost fibre is $10 \times 20 = 200$ N/mm^2 (given safe stress), the stress in the lowermost fibre

$$= \frac{200 \times 57.5}{152.5} = 75.4 \text{ N/mm}^2$$

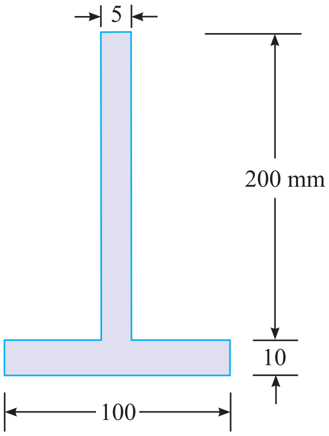


Fig. 15.16

It is below the given safe stress (*i.e.*, 150 N/mm^2). We also know that moment of inertia of the equivalent steel section, about an axis passing through its c.g. and parallel to *x-x* axis,

$$I = \left[\frac{100 \times (10)^3}{12} + (100 \times 10)(57.5 - 5)^2\right] + \left[\frac{5 \times (200)^3}{12} + (5 \times 200)(110 - 57.5)^2\right] \text{mm}^4$$

$$= (2.76 \times 10^6) + (6.09 \times 10^6) = 8.85 \times 10^6 \text{ mm}^4$$

and section modulus of the equivalent section,

$$Z = \frac{I}{y} = \frac{8.85 \times 10^6}{152.5} = 0.058 \times 10^6 \text{ mm}^3$$

∴ Moment of resistance of the equivalent section,

$$M = \sigma_2 \times Z = (20 \times 10) \times (0.058 \times 10^6) = 11.6 \times 10^6 \text{ N-mm}$$

$$= 11.6 \text{ kN-m} \quad \textbf{Ans.}$$

EXAMPLE 15.11. *A compound beam is formed by joining two bars, one of brass and the other of steel, each 40 mm wide and 10 mm deep. This beam is supported over a span of 1 mm with the brass bar placed over the steel bar as shown in Fig. 15.17.*

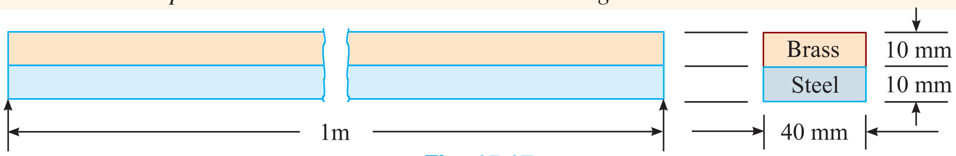


Fig. 15.17

Determine the maximum load, which can be applied at the centre of the beam, when the bars are:

(*a*) *separate and can beand independently,*

(*b*) *firmly secured to each other, throughout their length.*

Take $E_S = 200$ *GPa ;* $E_B = 80$ *GPa and* $\sigma_S = 112.5$ *MPa ;* $\sigma_B = 75$ *MPa*

SOLUTION. Given: Width $(b) = 40$ mm ; Depth of brass bar $(d_B) = d_S = 10$ mm ; Span $(l) = 1$ m $= 1 \times 10^3$ mm ; Modulus of elasticity for steel $(E_S) = 200$ GPa $= 200 \times 10^3$ N/mm^2 ; Modulus of elasticity for brass $(E_B) = 80$ GPa $= 80 \times 10$ N/mm^2 ; Allowable stress in steel $(\sigma_S) = 112.5$ MPa $= 112.5$ N/mm^2 and allowable stress in brass $\sigma_B = 75$ MPa $= 75$ N/mm^2.

When the bars are separate and can bend independently

Let W = Maximum load, which can be applied at the centre of the beam.

We know that section modulus for steel,

$$Z_S = Z_B = \frac{bd^2}{6} = \frac{40 \times (10)^2}{6} = \frac{2000}{3} \text{ mm}^3$$

A little consideration will show that each bar will bend about its own axis independently. But for the sake of simplicity, let us assume that each bar has the same radius of curvature. We know that

$$\frac{M}{I} = \frac{E}{R} \quad \text{or} \quad R = \frac{EI}{M}$$

$$\therefore \quad \frac{E_S \cdot I_S}{M_S} = \frac{E_B \cdot I_B}{M_B}$$

$$\frac{M_S}{M_B} = \frac{E_S}{E_B} = \frac{200 \times 10^3}{80 \times 10^3} = 2.5 \quad \text{...}(\because I_S = I_B)$$

or $\quad M_S = 2.5\, M_B$

$\therefore \quad \sigma_S \cdot Z_S = 2.5\, \sigma_B \cdot Z_B$

$$\sigma_S = 2.5\, \sigma_B \quad \text{...}\left(\because Z_S = Z_B = \frac{2000}{3}\right)$$

Thus stress in brass when the *stress in steel is 112.5 N/mm^2,

$$\sigma_B = \frac{\sigma_S}{2.5} = \frac{112.5}{2.5} = 45 \text{ N/mm}^2$$

It is below the permissible stress (*i.e.*, 75 N/mm^2). Therefore moment of resistance of the steel beam,

$$M_S = \sigma_S \times Z_S = 112.5 \times \frac{2000}{3} = 75\,000 \text{ N-mm}$$

and

$$M_B = \sigma_B \times Z_B = 45 \times \frac{2000}{3} = 30\,000 \text{ N-mm}$$

Therefore total moment of resistance,

$$M = M_S + M_B = 75\,000 + 30\,000 = 105\,000 \text{ N-mm} \quad \text{...}(i)$$

We know that maximum bending moment at the centre, when it is to support a load W at the centre,

$$M = \frac{Wl}{4} = \frac{W \times (1 \times 10^3)}{4} = 250\, W \quad \text{...}(ii)$$

* If the maximum stress in brass is considered to be 75 N/mm^2, then the stress in steel

$$\sigma_S = 2.5\, \sigma_B = 2.5 \times 75 = 187.5 \text{ N/mm}^2$$

But it is more than the permissible limit. Therefore we shall consider stress in steel as 112.5 N/mm^2.

Equating equations (*i*) and (*ii*),

$$105\,000 = 250\,W$$

$$\therefore \quad W = \frac{105\,000}{250} = 420 \text{ N} \quad \textbf{Ans.}$$

When the bars are firmly secured to each other throughout their length

Now let us convert the whole section into an equivalent *brass section as shown in Fig. 15.18.

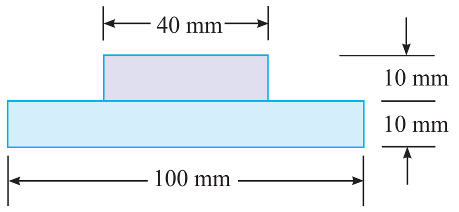


Fig. 15.18

The bottom steel plate has been replaced by an equivalent brass plate of thickness

$$= 40 \times \frac{200 \times 10^3}{80 \times 10^3} = 100 \text{ mm}$$

We know that distance between the centre of gravity of the section and bottom face of the equivalent brass section,

$$\bar{y} = \frac{(100 \times 10)\,5 + (40 \times 10)\,15}{(100 \times 10) + (40 \times 10)} = 7.86 \text{ mm}$$

∴ Distance of centre of gravity from the upper extreme fibre,

$$y_1 = 20 - 7.86 = 12.14 \text{ mm} \qquad \text{and} \qquad y_2 = 7.86 \text{ mm}$$

Therefore we shall take the value of y = 12.14 mm (*i.e.*, greater of the two values among y_T and y_C).

Now when the stress in the uppermost fibre is 75 N/mm^2 (given stress) then the stress in the lowermost fibre is

$$= \frac{75 \times 7.86}{12.14} = 48.6 \text{ N/mm}^2$$

Therefore actual stress in steel in the lowermost fibre

$$= 48.6 \times 2.5 = 121.5 \text{ N/mm}^2$$

It is more than the given safe stress in steel (*i.e.*, 112.5 N/mm^2). It is thus obvious that the brass cannot be fully stressed. Now taking maximum stress in steel at the bottom to be 112.5 N/mm^2, we find that the stress in brass at the bottom fibre,

$$\sigma_B = \frac{\sigma_S}{2.5} = \frac{112.5}{2.5} = 45 \text{ N/mm}^2$$

We also know that moment of inertia of the equivalent section about an axis passsing through its centre of gravity and parallel to x-x axis,

$$I = \left[\frac{100 \times (10)^3}{12} + (100 \times 10)(7.86 - 5.0)^2\right] + \left[\frac{40 \times (10)^3}{12} + (40 \times 10)(15 - 7.86)^2\right] \text{mm}^4$$

* We may also convert the whole section into an equivalent steel section.

$$= 40.24 \times 10^3 \text{ mm}^4$$

and section modulus of the equivalent section,

$$Z = \frac{I}{y} = \frac{40.24 \times 10^3}{12.14} = 3.31 \times 10^3 \text{ mm}^3$$

∴ Moment of resistance of the equivalent section,

$$M = \sigma \times Z = 45 \times (3.31 \times 10^3) = 149 \times 10^3 \text{ N-mm} \quad ...(iii)$$

We know that the maximum bending moment at the centre, when it is to support a load W at the centre,

$$M = \frac{Wl}{4} = \frac{W \times (1 \times 10^3)}{4} = 250\,W \quad ...(iv)$$

Equating equations (*iii*) and (*iv*)

$$149 \times 10^3 = 250\,W$$

$$\therefore \quad W = \frac{149 \times 10^3}{250} = 596 \text{ N} \quad \textbf{Ans.}$$

EXERCISE 15.2

1. A cantilever beam 2.5 m long has 50 mm width throughout its length and depth varying uniformly from 50 mm at the free end to 150 mm at the fixed end. If a load of 3 kN acts at the free end, find the position of highest stressed section and value of maximum bending stress induced. Neglect the weight of the beam itself. (**Ans.** 1.25 m ; 45 MPa)
2. A timber beam 150 mm deep and 150 mm wide is reinforced by a steel plate 100 mm wide and 10 mm deep attached at the lower face of the timber beam. Calculate the moment of resistance of the beam, if allowable stresses in timber and steel are 6 MPa and 60 MPa respectively. Take $E_s = 166\ E_t$. (**Ans.** 9.45 kN-m)
3. A timber joist 100 mm wide and 150 mm deep is reinforced by fixing two steel plates each 100 mm wide and 10 mm thick attached symmetrically at the top and the bottom. Find the moment of resistance of the beam, if allowable stresses in timber and steel are 7 MPa and 100 MPa respectively. Take $E_s = 16\ E_t$. (**Ans.** 17.15 kN-m)

QUESTIONS

1. Discuss the difference of procedure in finding out the bending stress in (*a*) symmetrical section, and (*b*) an unsymmetrical section.
2. Explain the term ‘strength of a section’.
3. Illustrate the term ‘beam of uniform strength’. Explain its necessity.
4. What do you understand by the term flitched beam? How would you find out the bending stresses in such a beam when it is of (*a*) a symmetrical section and (*b*) an unsymmetrical section?
5. Define the term ‘equivalent section’ used in a flitched beam.

OBJECTIVE TYPE QUESTIONS

1. Which of the following is a composite section?
   (*a*) hollow circular section (*b*) *T*-section
   (*c*) *Z*-section (*d*) both '*b*' and '*c*'
2. A beam of uniform strength has constant
   (*a*) shear force (*b*) bending moment
   (*c*) cross-sectional area (*d*) deflection
3. In a flitched beam, one section is reinforced with another section. The purpose of such a beam is to improve
   (*a*) shear force over the section (*b*) moment of resistance over the section
   (*c*) appearance of the section (*d*) all of these

ANSWERS

1. (*d*) **2.** (*b*) **3.** (*b*)

Chapter

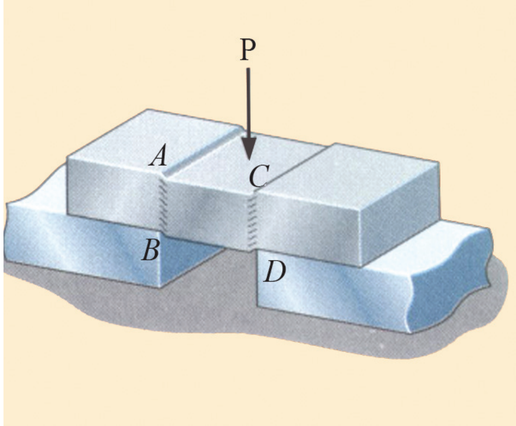
16

Shearing Stresses in Beams

Contents

Note :

Important Results related to this chapter are given at the end of this book See Appendix Table 3



16.1. Introduction

In the previous chapter, we discussed the theory of simple bending. In this theory, we assumed that no shear force is acting on the section. But in actual practice when a beam is loaded, the shear force at a section always comes into play, alongwith the bending moment. It has been observed that the effect of shearing stress, as compared to the bending stress, is quite negligible, and is not of much importance. But, sometimes, the shearing stress at a section assumes much importance in the design criterion. In this chapter, we shall discuss the shearing stress for its own importance.

16.2. Shearing Stress at a Section in a Loaded Beam

Consider a small portion *ABDC* of length *dx* of a beam loaded with uniformly distributed load as shown in Fig. 16.1 (*a*).

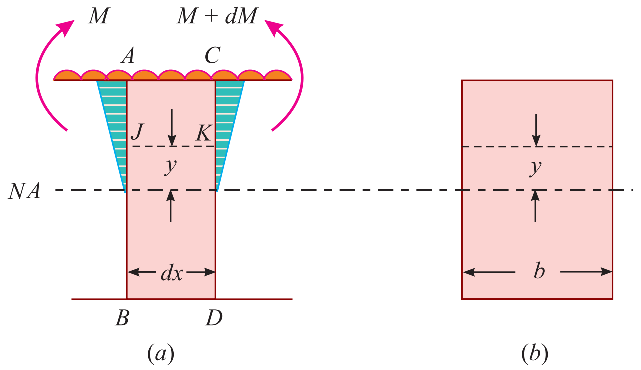


Fig. 16.1. Shearing stress

We know that when a beam is loaded with a uniformly distributed load, the shear force and bending moment vary at every point along the length of the beam.

Let M = Bending moment at *AB*,

$M + dM$ = Bending moment at *CD*,

F = Shear force at *AB*,

$F + dF$ = Shear force at *CD*, and

I = Moment of inertia of the section about its neutral axis.

Now consider an elementary strip at a distance y from the neutral axis as shown in Fig. 16.1 (*b*).

Now let σ = Intensity of bending stress across *AB* at distance y from the neutral axis and

a = Cross-sectional area of the strip.

We have already discussed that

$$\frac{M}{I} = \frac{\sigma}{y} \quad \text{or} \quad \sigma = \frac{M}{I} \times y \qquad \text{... (See Art. 14.6)}$$

Similarly, $$\sigma + d\sigma = \frac{M + dM}{I} \times y$$

where $\sigma + d\sigma$ = Intensity of bending stress across *CD*.

We know that the force acting across *AB*

$$= \text{Stress} \times \text{Area} = \sigma \times a = \frac{M}{I} \times y \times a \qquad ...(i)$$

Similarly, force acting across *CD*

$$= (\sigma + d\sigma) \times a = \frac{M + dM}{I} \times y \times a \qquad ...(ii)$$

$\therefore$ Net unbalanced force on the strip

$$= \frac{M + dM}{I} \times y \times a - \frac{M}{I} \times y \times a = \frac{dM}{I} \times y \times a$$

The total *unbalanced force (F) above the neutral axis may be found out by integrating the above equation between 0 and $d/2$.

or $$= \int_0^{\frac{d}{2}} \frac{dM}{I} a \cdot y \cdot dy = \frac{dM}{I} \int_0^{\frac{d}{2}} a \cdot y \cdot dy = \frac{dM}{I} A\bar{y} \qquad ...(iii)$$

where A = Area of the beam above neutral axis, and $\bar{y}$ = Distance between the centre of gravity of the area and the neutral axis.

We know that the intensity of the shear stress,

$$\tau = \frac{\text{Total force}}{\text{Area}} = \frac{\frac{dM}{I} \cdot A\bar{y}}{dx \cdot b} \qquad \text{...(Where } b \text{ is the width of beam)}$$

$$= \frac{dM}{dx} \times \frac{A \cdot \bar{y}}{Ib}$$

$$= F \times \frac{A\bar{y}}{Ib} \qquad \left(\text{Substituting } \frac{dM}{dx} = F = \text{Shear force}\right)$$

16.3. Distribution of Shearing Stress

In the previous article, we have obtained a relation, which helps us in determining the value of shear stress at any section on a beam. Now in the succeeding articles, we shall study the distribution of the shear stress along the depth of a beam. For doing so, we shall calculate the intensity of shear stress at important sections of a beam and then sketch a shear stress diagram. Such a diagram helps us in obtaining the value of shear stress at any section along the depth of the beam. In the following pages, we shall discuss the distribution of shear stress over the following sections:

1. Rectangular sections,
2. Triangular sections,
3. Circular sections,
4. *I*-sections,
5. *T*-sections and
6. Miscellaneous sections.

16.4. Distribution of Shearing Stress over a Rectangular Section

Consider a beam of rectangular section *ABCD* of width and depth as shown in Fig. 16.2 (*a*). We know that the shear stress on a layer *JK* of beam, at a distance *y* from the neutral axis,

$$\tau = F \times \frac{A\,\bar{y}}{Ib} \qquad ...(i)$$

* This may also be found out by splitting up the beam into number of strips at distance of from the neutral axis.

We know that unbalanced force on strip 1 $= \frac{dM}{I} \times a_1 \cdot y_1$

Similarly, unbalanced force on strip 2 $= \frac{dM}{I} \times a_2 \cdot y_2$

and unbalanced force on strip 3 $= \frac{dM}{I} \times a_3 \cdot y_3$ and so on

∴ Total force, $$F = \frac{dM}{I} \times a_1 \cdot y_1 + \frac{dM}{I} \times a_2 \cdot y_2 + \frac{dM}{I} \times a_3 \cdot y_3 + ...$$

$$= \frac{dM}{I}(a_1 . y_1 + a_2 . y_2 + a_3 . y_3 + ...) = \frac{dM}{I} A\,\bar{y}$$

where
F = Shear force at the section,
A = Area of section above y (*i.e.*, shaded area *AJKD*),
$\bar{y}$ = Distance of the shaded area from the neutral axis,
∴ $A\bar{y}$ = Moment of the shaded area about the neutral axis,
I = Moment of inertia of the whole section about its neutral axis, and
b = Width of the section.

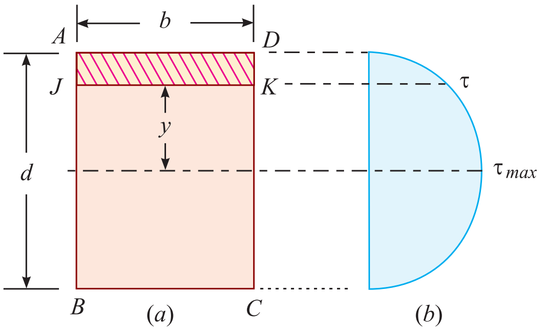


Fig. 16.2. Rectangular section

We know that area of the shaded portion *AJKD*,

$$A = b\left(\frac{d}{2} - y\right) \quad ...(ii)$$

$$\therefore \quad \bar{y} = y + \frac{1}{2}\left(\frac{d}{2} - y\right) = y + \frac{d}{4} - \frac{y}{2}$$

$$= \frac{y}{2} + \frac{d}{4} = \frac{1}{2}\left(y + \frac{d}{2}\right) \quad ...(iii)$$

Substituting the above values of A and $\bar{y}$ in equation (*i*),

$$\tau = F \times \frac{A\bar{y}}{Ib} = F \times \frac{b\left(\frac{d}{2} - y\right) \times \frac{1}{2}\left(y + \frac{d}{2}\right)}{Ib}$$

$$= \frac{F}{2I}\left(\frac{d^2}{4} - y^2\right) \quad ...(iv)$$

We see, from the above equation, that τ increase as y decreases. At a point, where $y = d/2$, $\tau = 0$; and where y is zero, τ is maximum. We also see that the variation of τ with respect to y is a parabola.

At neutral axis, the value of τ is maximum. Thus substituting $y = 0$ and $I = \dfrac{bd^3}{12}$ in the above equation,

$$\tau_{max} = \frac{F}{2 \times \frac{ba^3}{12}}\left(\frac{d^2}{4}\right) = \frac{3F}{2bd} = 1.5\,\tau_{av} \quad ...\left(\because \ \tau_{av} = \frac{F}{\text{Area}} = \frac{F}{bd}\right)$$

Now draw the shear stress distribution diagram as shown in Fig. 16.2 (*b*).

EXAMPLE 16.1. *A wooden beam 100 mm wide, 250 mm deep and 3 m long is carrying a uniformly distributed load of 40 kN/m. Determine the maximum shear stress and sketch the variation of shear stress along the depth of the beam.*

SOLUTION. Given: Width (b) = 100 mm ; Depth (d) = 250 mm ; Span (l) = 3 m = 10^3 mm and uniformly distributed load (w) = 40 kN/m = 40 N/mm.

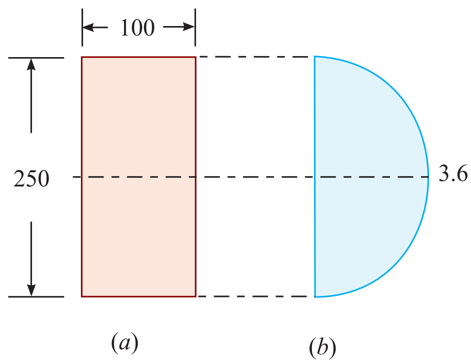


Fig. 16.3

We know that shear force at one end of the beam,

$$F = \frac{wl}{2} = \frac{40 \times (3 \times 10^3)}{2} \text{ N}$$

$$= 60 \times 10^3 \text{ N}$$

and area of beam section,

$$A = b \cdot d = 100 \times 250 = 25\,000 \text{ mm}^2$$

∴ Average shear stress across the section,

$$\tau_{av} = \frac{F}{A} = \frac{60 \times 10^3}{25\,000} = 2.4 \text{ N/mm}^2 = 2.4 \text{ MPa}$$

and maximum shear stress,

$$\tau_{max} = 1.5 \times \tau = 1.5 \times 2.4 = 3.6 \text{ MPa} \quad \textbf{Ans.}$$

The diagram showing the variation of shear along the depth of the beam is shown in Fig. 16.3 (b).

16.5. Distribution of Shearing Stress over a Triangular Section

Consider a beam of triangular cross-section ABC of base b and height h as shown in Fig. 16.4 (a).

We know that the shear stress on a layer JK at a distance y from the neutral axis,

$$\tau = F \times \frac{A\bar{y}}{Ib} \quad ...(i)$$

where F = Shear force at the section,

$A\bar{y}$ = Moment of the shaded area about the neutral axis and

I = Moment of inertia of the triangular section about its neutral axis.

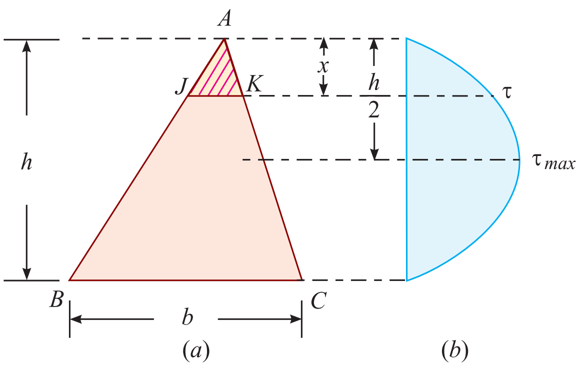


Fig. 16.4. Triangular section.

We know that width of the strip JK,

$$b = \frac{bx}{h}$$

$\therefore$ Area of the shaded portion AJK,

$$A = \frac{1}{2} JK \times x = \frac{1}{2}\left(\frac{bx}{h} \times x\right) = \frac{bx^2}{2h}$$

and

$$\bar{y} = \frac{2h}{3} - \frac{2x}{3} = \frac{2}{3}(h - x)$$

Substituting the values of b, A and $\bar{y}$ in equation (i),

$$\tau = F \times \frac{\left(\frac{bx^2}{2h}\right) \times \frac{2}{3}(h - x)}{I \times \frac{bx}{h}} = \frac{F}{3I} \times [x(h - x)]$$

$$= \frac{F}{3I} \times \left[hx - x^2\right] \qquad ...(ii)$$

Thus we see that the variation of τ with respect to x is parabola. We also see that as a point where $x = 0$ or $x = h$, $\tau = 0$. At neutral axis, where $x = \frac{2h}{3}$,

$$\tau = \frac{F}{3I}\left[h \times \frac{2h}{3} - \left(\frac{2h}{3}\right)^2\right] = \frac{F}{3I} \times \frac{2h^2}{9} = \frac{2Fh^2}{27I}$$

$$= \frac{2Fh^2}{27 \times \frac{bh^3}{36}} = \frac{8F}{3bh} \qquad ...\left(\because\ I = \frac{bh^3}{36}\right)$$

$$= \frac{4}{3} \times \frac{F}{\text{Area}} = 1.33\,\tau_{av} \qquad ...\left(\because\ \text{Area} = \frac{bh}{2}\right)$$

Now for maximum intensity, differentiating the equation (ii) and equating to zero,

$$\frac{d\tau}{dx}\left[\frac{F}{3I}(hx - x^2)\right] = 0$$

$$\therefore \qquad h - 2x = 0 \quad \text{or} \quad x = \frac{h}{2}$$

Now substituting this value of x in equation (ii),

$$\tau_{max} = \frac{F}{3I}\left[h \times \frac{h}{2} - \left(\frac{h}{2}\right)^2\right] = \frac{Fh^2}{12I} = \frac{Fh^2}{12 \times \frac{bh^3}{36}} \qquad ...\left(\because\ I = \frac{bh^3}{36}\right)$$

$$= \frac{3F}{bh} = \frac{3}{2} \times \frac{F}{\text{Area}} = 1.5\,\tau_{av}$$

Now draw the shear stress distribution diagram as shown in Fig. 16.4 (b).

EXAMPLE 16.2. *A beam of triangular cross section having base width of 100 mm and height of 150 mm is subjected to a shear force of 13.5 kN. Find the value of maximum shear stress and sketch the shear stress distribution along the depth of beam.*

SOLUTION. Given: Base width (b) = 100 mm ; Height (h) = 150 mm and shear force (F) = 13.5 kN = 13.5×10^3 N

We know that area of beam section,

$$A = \frac{b \cdot h}{2} = \frac{100 \times 150}{2} \text{ mm}^2$$

$$= 7500 \text{ mm}^2$$

∴ Average shear stress across the section,

$$\tau_{av} = \frac{F}{A} = \frac{13.5 \times 10^3}{7500} \text{ N/mm}^2$$

$$= 1.8 \text{ N/mm}^2 = 1.8 \text{ MPa}$$

and maximum shear stress,

$$\tau_{av} = 1.5 \times \tau_{av} = 1.5 \times 1.8 = \mathbf{2.7\ MPa} \quad \textbf{Ans.}$$

The diagram showing the variation of shear stress along the depth of the beam is shown in Fig. 16.5(*b*).

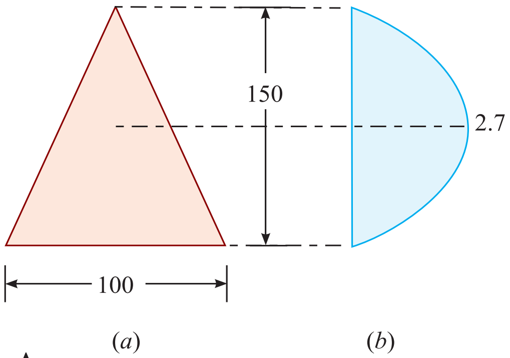


Fig. 16.5

16.6. Distribution of Shearing Stress over a Circular Section

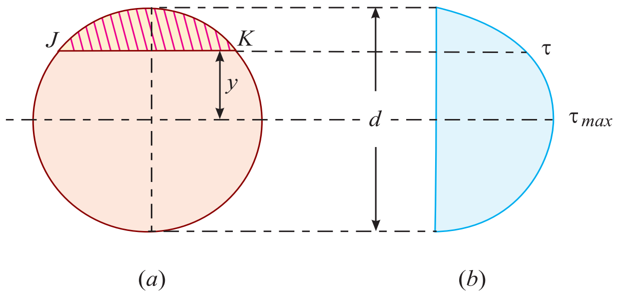


Fig. 16.6. Circular section.

Consider a circular section of diameter *d* as shown in Fig. 16.6 (*a*). We know that the shear stress on a layer *JK* at a distance *y* from the neutral axis,

$$\tau = F \times \frac{A\bar{y}}{Ib}$$

where
F = Shear force at the section,
$A\bar{y}$ = Moment of the shaded area about the neutral axis,
r = Radius of the circular section,
I = Moment of inertia of the circular section and
b = Width of the strip *JK*.

We know that in a circular section,

width of the strip *JK*, $b = 2\sqrt{r^2 - y^2}$

and area of the shaded strip,

$$A = 2\sqrt{r^2 - y^2} \cdot dy$$

∴ Moment of this area about the neutral axis

$$= 2y\sqrt{r^2 - y^2} \cdot dy \qquad \text{...}(i)$$

Now moment of the whole shaded area about the neutral axis may be found out by integrating the above equation between the limits y and r, *i.e.*,

$$A\bar{y} = \int_y^r 2y\sqrt{r^2 - y^2} \cdot dy$$

$$= \int_y^r b \cdot y \cdot dy \qquad \text{... } (\because\ b = 2\sqrt{r^2 - y^2}) \quad \text{...}(ii)$$

We know that width of the strip JK,

$$b = 2\sqrt{r^2 - y^2}$$

or

$$b^2 = 4\sqrt{r^2 - y^2} \qquad \text{... (Squaring both sides)}$$

Differentiating both sides of the above equation,

$$2b \cdot db = 4\,(-2y)\,dy = -8y \cdot dy$$

or

$$y \cdot dy = -\frac{1}{4}\,b \cdot db$$

Substituting the value of $y \cdot dy$ in equation (ii),

$$A\bar{y} = \int_y^r b\left(-\frac{1}{4}b \cdot db\right) = -\frac{1}{4}\int_y^r b^2 \cdot db \qquad \text{... }(iii)$$

We know that when $y = y$, width $b = b$ and when $y = r$, width $b = 0$. Therefore, the limits of integration may be changed from y to r, from b to zero in equation (iii),

or

$$A\bar{y} = -\frac{1}{4}\int_b^0 b^2 \cdot db$$

$$= \frac{1}{4}\int_0^b b^2 \cdot db \qquad \text{... (Eliminating –ve sign)}$$

$$= -\frac{1}{4}\left[\frac{b^3}{3}\right]_0^b = \frac{b^3}{12}$$

Now substituting this value of $A\bar{y}$ in our original formula for the shear stress, *i.e.,*

$$\tau = F \times \frac{A\bar{y}}{Ib} = F \times \frac{\frac{b^3}{12}}{Ib} = F \times \frac{b^2}{12I}$$

$$= F \times \left[\frac{(2\sqrt{r^2 - y^2})^2}{12I}\right] \qquad \text{... } (\because\ b = 2\sqrt{r^2 - y^2})$$

$$= F \times \frac{r^2 - y^2}{3I}$$

Thus we again see that τ increases as y decreases. At a point, where $y = r$, $\tau = 0$, $= 0$ and where y is zero, τ is maximum. We also see that the variation of τ with respect to y is a parabolic curve. We see that at neutral axis τ is maximum.

Substituting $y = 0$ and $I = \frac{\pi}{64} \times d^4$ in the above equation,

$$\tau_{max} = F \times \frac{r^2}{3 \times I} = F \times \frac{\left(\frac{d}{2}\right)^2}{3 \times \frac{\pi}{64} \times d^4} = \frac{4F}{3 \times \frac{\pi}{4} \times d^2} = 1.33\,\tau_{av}$$

Now draw the shear stress distribution diagram as shown in Fig. 16.6 (*b*).

EXAMPLE 16.3. *A circular beam of 100 mm diameter is subjected to a shear force of 30 kN. Calculate the value of maximum shear stress and sketch the variation of shear stress along the depth of the beam.*

SOLUTION. Given: Diameter (d) = 100 mm and shear force (F) = 30 kN = 30×10^3 N

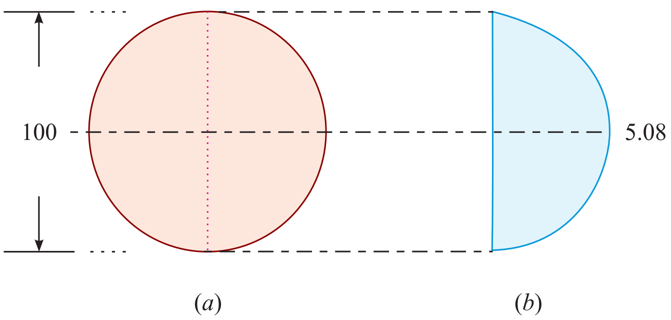


Fig. 16.7

We know that area of the beam section,

$$A = \frac{\pi}{4}(d)^2 = \frac{\pi}{4}(100)^2 \text{ mm}^2 = 7854 \text{ mm}^2$$

$\therefore$ Average shear stress across the section

$$\tau_{av} = \frac{F}{A} = \frac{30 \times 10^3}{7854} = 3.82 \text{ N/mm}^2 = 3.82 \text{ MPa}$$

and maximum shear stress,

$$\tau_{max} = 1.33 \times \tau_{av} = 1.33 \times 3.82 = \mathbf{5.08\ MPa} \qquad \textbf{Ans.}$$

The diagram showing the variation of shear stress along the depth of the beam is shown in Fig. 16.7.

EXERCISE 16.1

1. A rectangular beam 80 mm wide and 150 mm deep is subjected to a shearing force of 30 kN. Calculate the maximum shear stress and draw the distribution diagram for the shear stress.]
   [**Ans.** 3.75 MPa]
2. A rectangular beam 100 mm wide is subjected to a maximum shear force of 50 kN. Find the depth of the beam, if the maximum shear stress is 3 MPa. [**Ans.** 250 mm]

3. A triangular beam of base width 80 mm and height 100 mm is subjected to a shear force of 12 kN. What is the value of maximum shear stress? Also draw the shear stress distribution diagram over the beam section. [**Ans.** 4.5 MPa]

4. A circular beam of diameter 150 mm is subjected to a shear force of 70 kN. Find the value of maximum shear stress and sketch the shear stress distribution diagram over the beam section. [**Ans.** 5.27 MPa]

16.7. Distribution of Shearing Stress over an *I*-Section

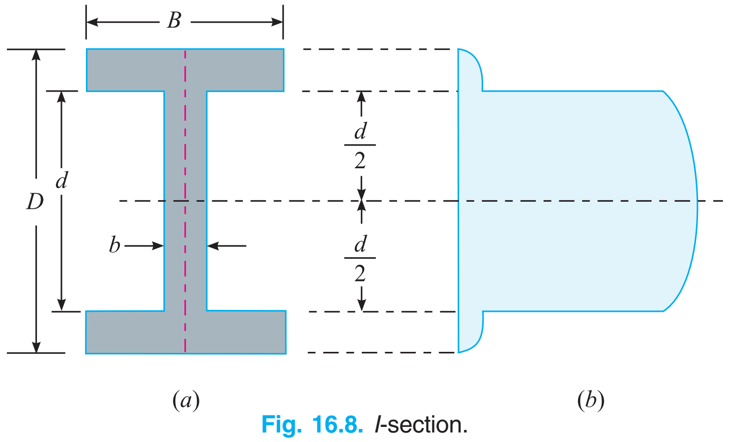


Fig. 16.8. *I*-section.

Consider a beam of an *I*-section as shown in Fig. 16.8 (*a*)

Let B = Overall width of the section,

D = Overall depth of the section,

d = Depth of the web, and

b = Thickness of the web.

We know that the shear stress on a layer *JK* at a distance *y* from the neutral axis,

$$\tau = F \times \frac{A\bar{y}}{Ib} \quad \text{... (i)}$$

Now we shall discuss two important cases

(*i*) when *y* is greater than $\frac{d}{2}$

(*ii*) when *y* is less than $\frac{d}{2}$.

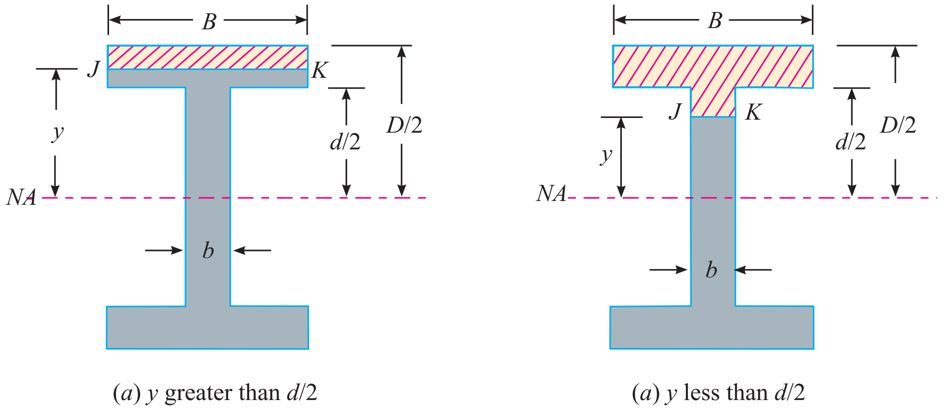


Fig. 16.9

(i) When y is greater than $\frac{d}{2}$

It means that y lies in the flange as shown in Fig. 16.9 (*a*). In this case, shaded area of the flange,

$$A = B\left(\frac{D}{2} - y\right)$$

and

$$\bar{y} = y + \frac{1}{2}\left(\frac{D}{2} - y\right)$$

Now substituting these values of A and $\bar{y}$ from the above equations, in our original equation (*i*) of shear force, *i.e.*,

$$\tau = F \times \frac{A\bar{y}}{Ib} = F \times \frac{B\left(\frac{D}{2} - y\right) \times \left[y + \frac{1}{2}\left(\frac{D}{2} - y\right)\right]}{IB}$$

$$= \frac{F}{2I}\left(\frac{D^2}{4} - y^2\right)$$

Thus we see that τ increases as y decreases. We also see that the variation of τ with respect to y is a parabolic curve. At the upper edge of the flange, where $y = \frac{D}{2}$, shear stress is zero and at the lower edge where $y = \frac{d}{2}$, the shear stress,

$$\tau = \frac{F}{2I}\left[\frac{D^2}{4} - \left(\frac{d}{2}\right)^2\right] = \frac{F}{8I}(D^2 - d^2)$$

(ii) When y is less than $\frac{d}{2}$

It means that y lies in the web as shown in Fig. 10.9 (*b*). In this case, the value of $A\bar{y}$ for the flange

$$= B\left(\frac{D}{2} - \frac{d}{2}\right) \times \left[\frac{d}{2} + \frac{1}{2}\left(\frac{D}{2} - \frac{d}{2}\right)\right]$$

$$= B\left(\frac{D-d}{2}\right)\left[\frac{1}{2}\left(\frac{D+d}{2}\right)\right] = B\,\frac{(D^2 - d^2)}{8} \qquad ...(i)$$

and the value of $A\bar{y}$ for the web above AB

$$= b\left(\frac{d}{2} - y\right) \times \left[y + \frac{1}{2}\left(\frac{d}{2} - y\right)\right]$$

$$= b\left(\frac{d}{2} - y\right) \times \left[\frac{1}{2}\left(\frac{d}{2} + y\right)\right] = \frac{b}{2}\left(\frac{d^2}{4} - y^2\right) \qquad ...(ii)$$

$$\therefore \qquad \text{Total } A\bar{y} = \frac{B(D^2 - d^2)}{8} + \frac{b}{2}\left(\frac{d^2}{4} - y^2\right)$$

Now substituting the value of $A\bar{y}$ from the above equation, in our original equation of shear stress on a layer at a distance y from the neutral axis, *i.e.*,

$$\tau = F \times \frac{A\bar{y}}{Ib} = F \times \frac{\frac{B(D^2 - d^2)}{8} + \frac{b}{2}\left(\frac{d^2}{4} - y^2\right)}{Ib}$$

$$= \frac{F}{Ib}\left[\frac{B\,(D^2-d^2)}{8}+\frac{b}{2}\left(\frac{d^2}{4}-y^2\right)\right]$$

Thus we see that in the web also τ increases as y decreases. We also see that the variation of τ with respect to y in the web also is a parabolic curve. At neutral axis where $y = 0$, the shear stress is maximum.

∴ Maximum shear stress,

$$\therefore \qquad \tau_{max} = \frac{F}{Ib}\left[\frac{B}{8}(D^2-d^2)+\frac{bd^2}{8}\right] \qquad \text{...(Substituting } y = 0)$$

Now, shear stress at the junction of the top of the web and bottom of the flange

$$= \frac{F}{Ib}\left[\frac{B}{8}(D^2-d^2)\right] \qquad \text{...}\left(\text{Substituting } y = \frac{d}{2}\right)$$

$$= \frac{F}{8I}\times\frac{B}{b}(D^2-d^2)$$

Notes: 1. We see that the shear stress at the junction of the top of web and bottom of the flange is different from both the above expressions $\left(i.e., \text{ when } y > \frac{d}{2} \text{ and } y < \frac{d}{2}\right)$.

We also see that the shear stress changes, abruptly from $\frac{F}{8I}(D^2-d^2)$ to $\frac{F}{8I}\times\frac{B}{b}(D^2-d^2)$.

Thus the shear stress at this junction, suddenly increases by B/b times as shown in Fig. 16.8(*b*).

2. If the I-section is symmetrical, the shear stress distribution diagram will also be symmetrical.

3. From the shear stress distribution diagram, we see that most of the shear stress is taken up by the web. It is an important factor in the design of various important structures.

Example 16.4. *An I-sections, with rectangular ends, has the following dimensions:*

Flanges =150 mm × 20 mm, Web = 300 mm 10 mm.

Find the maximum shearing stress developed in the beam for a shear force of 50 kN.

Solution. Given: Flange width (B) = 150 mm ; Flange thickness = 20 mm ; Depth of web (d) = 300 mm; Width of web = 10 mm; Overall depth of the section (D) = 340 mm and shearing force (F) = 50 kN = 50×10^3 N.

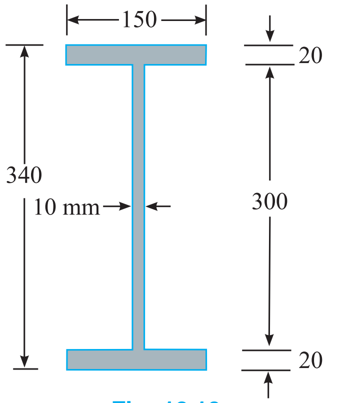


Fig. 16.10

We know that moment of inertia of the *I*-section about its centre of gravity and parallel to *x-x* axis,

$$I_{XX} = \frac{150\times(340)^3}{12}-\frac{140\times(300)^3}{12}\text{ mm}^4$$

$$= 176.3\times10^6 \text{ mm}^4$$

and maximum shearing stress,

$$\tau_{max} = \frac{F}{Ib}\left[\frac{B}{8}(D^2-d^2)+\frac{bd^2}{8}\right]$$

$$= \frac{50\times10^3}{(176.3\times10^6)\times10}\left[\frac{150}{8}[(340)^2-(300)^2]+\frac{10\times(300)^2}{8}\right]\text{N/mm}^2$$

$= 16.8$ N/mm^2 = **16.8 MPa** **Ans.**

EXAMPLE 16.5. *An I-section beam 350 mm × 200 mm has a web thickness of 12.5 mm and a flange thickness of 25 mm. It carries a shearing force of 200 kN at a section. Sketch the shear stress distribution across the section.*

SOLUTION. Given: Overall depth (D) = 350 mm ; Flange width (B) = 200 mm ; Width of Web = 12.5 mm ; Flange thickness = 25 mm and the shearing force (F) = 200 kN = 200×10^3 N.

We know that moment of inertia of the I-section about it centre of gravity and parallel to x-x axis,

$$I_{XX} = \frac{200 \times (350)^3}{12} - \frac{187.5 \times (300)^3}{12} = 292.7 \times 10^6 \text{ mm}^4$$

We also know that shear stress at the upper edge of the upper flange is zero. And shear stress at the joint of the upper flange and web

$$= \frac{F}{8I}[D^2 - d^2] = \frac{200 \times 10^3}{8 \times (292.7 \times 10^6)}[(350)^2 - (300)^2] \text{ N/mm}^2$$

$$= 2.78 \text{ N/mm}^2 = 2.78 \text{ MPa}$$

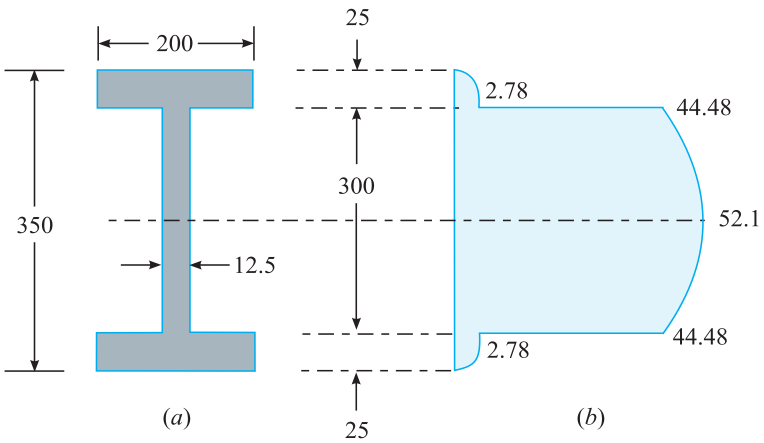


Fig. 16.11

The shear stress at the junction suddenly increases from 2.78 MPa to $2.78 \times \frac{200}{12.5} = 44.48$ MPa.

We also know that the maximum shear stress,

$$\tau_{max} = \frac{F}{I \cdot b}\left[\frac{B}{8}(D^2 - d^2) + \frac{bd^2}{8}\right]$$

$$= \frac{200 \times 10^3}{(292.7 \times 10^6) \times 12.5}\left[\frac{200}{8}(350)^2 - (300)^2 + \frac{12.5 \times (300)^2}{8}\right]$$

$$= 52.1 \text{ N/mm}^2 = 52.1 \text{ MPa}$$

Now complete the shear stress distribution diagram across the section as shown in Fig 16.11 (b).

16.8. Distribution of Shearing Stress over a T-section

The procedure for determining the distribution of stress over a *T*-section is the same as discussed in Art. 16.7. In this case, since the section is not symmetrical about *x-x* axis, therefore, the shear stress distribution diagram will also not be symmetrical.

EXAMPLE 16.6. *A T-shaped cross-section of a beam shown in Fig. 16.12 is subjected to a vertical shear force of 100 kN. Calculate the shear stress at important points and draw shear stress distribution diagram. Moment of inertia about the horizontal neutral axis is mm^4.*

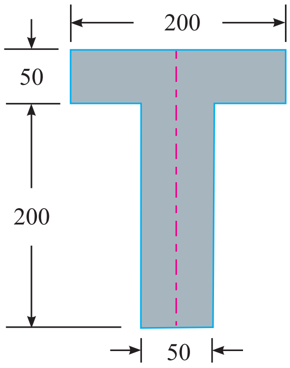


Fig. 16.12

SOLUTION. Given: Shear stress (F) = 100 kN = 100×10^3 N and moment of inertia (I) = 113.4×10^6 mm^4.

First of all let us find out the position of the neutral axis. We know that distance between the centre of gravity of the section and bottom of the web,

$$\bar{y} = \frac{[(200 \times 50) \times 225] + [(200 \times 50) \times 100]}{(200 \times 50) + (20 \times 50)}$$

$$= 162.5 \text{ mm}$$

∴ Distance between the centre of gravity of the section and top of the flange,

$$y_C = (200 + 50) - 162.5 = 87.5 \text{ mm}$$

We know that shear stress at the top of the flanges is zero. Now let us find out the shear stress at the junction of the flange and web by considering the area of the *flange of the section. We know that area of the upper flange,

$$A = 200 \times 50 = 10000 \text{ mm}^2$$

$$\bar{y} = 87.5 - \frac{50}{2} = 62.5 \text{ mm}$$

$$B = 200 \text{ mm}$$

∴ Shear stress at the junction of the flange and web,

$$\tau = F \times \frac{A \cdot \bar{y}}{I \cdot B} = 100 \times 10^3 \times \frac{10000 \times 62.5}{(113.4 \times 10^6) \times 200} \text{ N/mm}^2$$

$$= 2.76 \text{ N/mm}^2 = 2.76 \text{ MPa}$$

* It may also be found out by considering the area of web of the section as discussed below. We know that area of the web,

$$A = 200 \times 50 = 10000 \text{ mm}^2$$

$$\bar{y} = 162.5 - 200/2 = 62.5 \text{ mm}, \; b = 50 \text{ mm}$$

∴ Shear stress at the junction of the flange and web,

$$\tau = F \times \frac{A \cdot \bar{y}}{I \cdot b} = 100 \times 10^3 \times \frac{10000 \times 62.5}{(113.4 \times 10^6) \times 50}$$

$$= 11.04 \text{ N/mm}^2 = 11.04 \text{ MPa}$$

In this case, the shear stress at the junction suddenly decreases from 11.04 MPa to $11.04 \times \frac{50}{200} = 2.76$ MPa.

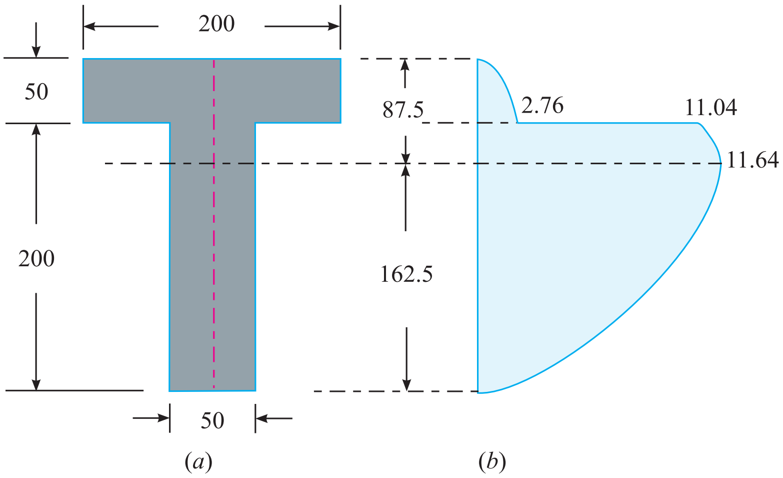


Fig. 16.13

The shear stress at the junction suddenly increases from 2.76 MPa to $2.76 \times \frac{200}{50} = 11.04$ MPa.

Now let us find out the shear stress at the neutral axis, where the shear stress is maximum.

Considering the area of the T-section above the neutral axis of the section, we know that

$$* A\bar{y} = [(200 \times 50) \times 62.5] + \left[(37.5 \times 50) \times \frac{37.5}{2}\right] \text{mm}^3$$

$$= 660.2 \times 10^3 \text{ mm}^3$$

and $\qquad b = 50$ mm

∴ Maximum shear stress,

$$\tau_{max} = F \times \frac{A \cdot \bar{y}}{I \cdot b} = 100 \times 10^3 \times \frac{660.2 \times 10^3}{(113.4 \times 10^6) \times 50} \text{ N/mm}^2$$

$$= 11.64 \text{ N/mm}^2 = 11.64 \text{ MPa}$$

Now draw the shear stress distribution diagram across the section as shown in Fig. 16.13(b).

16.9. Distribution of Shearing Stress over a Miscellaneous Section

The procedure for determining the distribution of shear stress over a miscellaneous section, is the same as discussed in the previous articles. The shear stress at all the important points should be calculated and then shear stress distribution diagram should be drawn as usual.

* It may also be found out by considering the area below neutral axis as discussed below. We know that

$$A\bar{y} = (162.5 \times 50) \times \frac{162.5}{2} = 660.2 \times 10^3 \text{ mm}^3$$

EXAMPLE 16.7. *A cast-iron bracket subjected to bending, has a cross-section of I-shape with unequal flanges as shown in Fig. 16.14.*

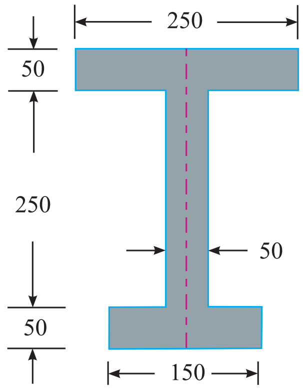


Fig. 16.14

If the compressive stress in top flange is not to exceed 17.5 MPa, what is the bending moment, the section can take? If the section is subjected to a shear force of 100 kN, draw the shear stress distribution over the depth of the section.

SOLUTION. Given: Compressive stress (σ_c) = 17.5 MPa = 17.5 N/mm^2 and shear force (F) = 100 kN = 100×10^3 N

Bending moment the section can take

First of all, let us find out the position of the neutral axis. We know that distance between centre of gravity of the section and bottom face,

$$\bar{y} = \frac{(250 \times 50)\,325 + (250 \times 50)\,175 + (150 \times 50)\,25}{(250 \times 50) + (250 \times 50) + (150 \times 50)}$$

$$= \frac{6\,437\,500}{32\,500} = 198 \text{ mm}$$

∴ Distance of centre of gravity from the upper extreme fibre,

$$y_c = 350 - 198 = 152 \text{ mm}$$

and moment of inertia of the section about an axis passing through its centre of gravity and parallel to x-x axis,

$$I = \left[\frac{250 \times (50)^3}{12} + (250 \times 50)\,(325 - 198)^2\right]$$

$$+ \left[\frac{50 \times (250)^3}{12} + (50 \times 250)\,(198 - 175)^2\right]$$

$$+ \left[\frac{150 \times (50)^3}{12} + (150 \times 50)\,(198 - 25)^2\right] \text{mm}^4$$

$$= 502 \times 10^6 \text{ mm}^4$$

∴ Bending moment the section can take

$$= \frac{\sigma_c}{y_c \times I} = \frac{17.5}{152} \times 502 \times 10^6 = 57.8 \times 10^6 \text{ N-mm}$$

$= 57.8$ kN-m **Ans.**

Shear stress distribution diagram

We know that the shear stress at the extreme edges of both the flanges is zero. Now let us find out the shear stress at the junction of the upper flange and web by considering the area of the upper flange. We know that area of the upper flange,

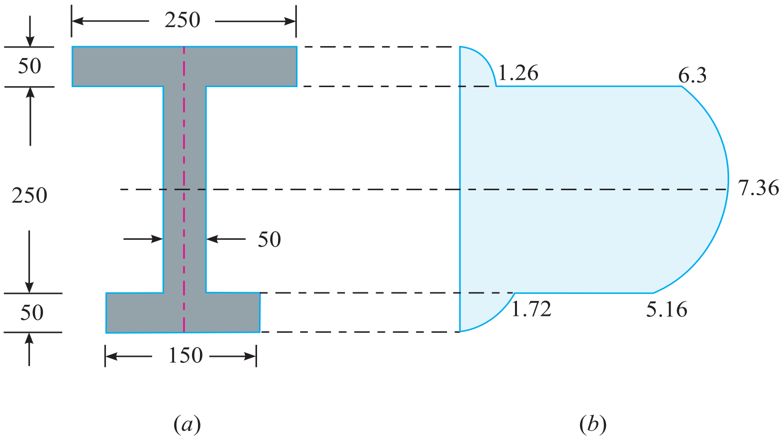


Fig. 16.15

$$A = 250 \times 50 = 12500 \text{ mm}$$

$$\bar{y} = 152 - \frac{50}{2} = 127 \text{ mm}$$

and
$$B = 250 \text{ mm}$$

∴ Shear stress at the junction of the upper flange and web,

$$\tau = F \times \frac{A \cdot \bar{y}}{I \cdot B} = 100 \times 10^3 \frac{12500 \times 127}{(502 \times 10^6) \times 250} \text{ N/mm}^2$$

$$= 1.26 \text{ N/mm}^2 = 1.26 \text{ MPa}$$

The shear stress at the junction suddenly increases from 1.26 MPa to $1.26 \times \frac{250}{50} = 6.3$ MPa. Now let us find out the shear stress at the junction of the lower flange and web by considering the area of the lower flange. We know that area of the lower flange,

$$A = 150 \times 50 = 7500 \text{ mm}^2$$

$$\bar{y} = 198 - \frac{50}{2} = 173 \text{ mm}$$

and
$$B = 150 \text{ mm}$$

∴ Shear stress at the junction of the lower flange and web,

$$\tau = F \times \frac{A \cdot \bar{y}}{I \cdot B} = 100 \times 10^3 \times \frac{7500 \times 173}{(502 \times 10^6) \times 150}$$

$$= 1.72 \text{ N/mm}^2 = 1.72 \text{ MPa}$$

The shear stress at the function suddenly increases from 1.72 MPa to $1.72 \times \frac{150}{50} = 5.16$ MPa.

Now let us find out the shear stress at the neutral axis, where the shear stress is maximum. Considering the area of the *I*-section above neutral axis, we know that

$$A\bar{y} = [(250 \times 50) \times 127] + \left[(102 \times 50) \times \frac{102}{2}\right] \text{mm}^3$$

$$= 1.848 \times 106 \text{ mm}^3$$

and $\qquad b = 50 \text{ mm}$

∴ Maximum shear stress,

$$\tau_{max} = F \times \frac{A \cdot \bar{y}}{I \cdot b} = 100 \times 10^3 \times \frac{1.848 \times 10^6}{(502 \times 10^6) \times 50} \text{ N/mm}^2$$

$$= 7.36 \text{ N/mm}^2 = 7.36 \text{ MPa}$$

Now draw the shear stress distribution diagram over the depth of the section as shown in Fig. 16.15.

EXAMPLE 16.8. *A steel section shown in Fig. 16.16 is subjected to a shear force of 20 kN.*

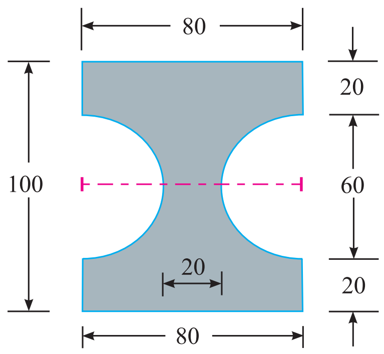


Fig. 16.16

Determine the shear stress at the important points and sketch the shear distribution diagram.

SOLUTION. Given: Shear force $(F) = 20 \text{ kN} = 20 \times 10^3 \text{ N}$

Since the section is symmetrical about x-x and y-y axes therefore, centre of the section will lie on the geometrical centroid of the section. For the purpose of moment of inertia and shear stress, the two semi-circular grooves may be assumed to be together and considered as one circular hole of 60 mm diameter. Therefore moment of inertia of the section about an axis passing through its centre of gravity and parallel to x-x axis,

$$I = \left[\frac{80 \times (100)^3}{12}\right] - \left[\frac{\pi}{64}(60)^4\right] = 6.03 \times 10^6 \text{ mm}^4$$

We know that shear stress at the extreme edges of A and E of the section is zero. Now let us find out the shear stress at B by considering the area between A and B.

We know that area of the upper portion between A and B

$$A = 80 \times 20 = 1600 \text{ mm}^2$$

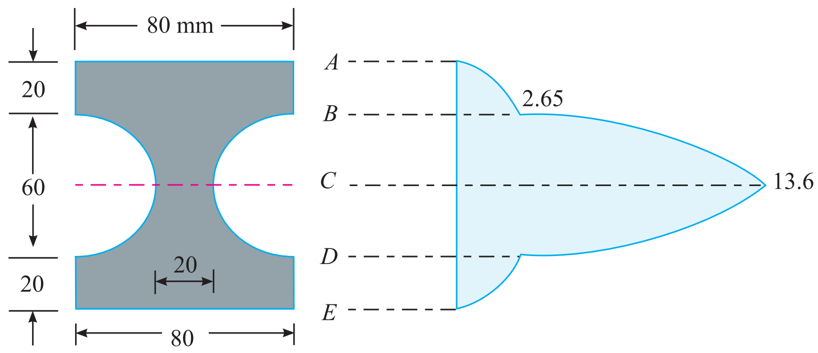
$$\bar{y} = 30 + \frac{20}{2} = 40 \text{ mm}$$

and $\qquad B = 80 \text{ mm}$

∴ Shear stress at B,

$$\tau = F \times \frac{A \cdot \bar{y}}{I \cdot B} = 20 \times 10^3 \times \frac{1600 \times 40}{(6.03 \times 10^6) \times 80} \text{ N/mm}^2$$

$$= 2.65 \text{ N/mm}^2 = 2.65 \text{ MPa}$$



(*a*) (*b*)

Fig. 16.17

Now let us find out the shear stress at the neutral axis, where the shear stress is maximum. Considering the area above the neutral axis, we know that

$$A\bar{y} = [(80\times 50)\times 25] - \left[\frac{\pi}{2}(30)^2 \times \frac{4\times 30}{3\pi}\right] \text{ mm}^3$$

$$= 100\ 000 - 18\ 000 = 82\ 000 \text{ mm}^3$$

and $b = 20$ mm

∴ Maximum shear stress,

$$\tau_{max} = F\times\frac{A\cdot\bar{y}}{I\cdot b} = 20\times 10^3 \times \frac{82\,000}{(6.03\times 10^6)\times 20} \text{ N/mm}^2$$

$$= 13.6 \text{ N/mm}^2 = 13.6 \text{ MPa}$$

Now draw the shear stress distribution diagram over the section as shown in Fig. 16.17 (*b*).

EXAMPLE 16.9. *A beam of square section is used as a beam with one diagonal horizontal. Find the maximum shear stress in the cross section of the beam. Also sketch the shear stress distribution across the depth of the section.*

SOLUTION. Given: A square section with its diagonal horizontal.

The beam with horizontal diagonal is shown in Fig. 16.18 (*a*).

Let $2b$ = Diagonal of the square, and

F = Shear force at the section.

Now consider the shaded strip *AJK* at a distance x from the corner *A*. From the geometry of the figure, we find that length $JK = 2x$

∴ Area of *AJK*, $A = \frac{1}{2}\times 2x\cdot x = x^2$

and $\bar{y} = b - \frac{2x}{3}$

We know that moment of inertia of the section *ABCD* about the neutral axis,

$$I = 2\times\frac{2b\times b^3}{12} = \frac{b^4}{3}$$

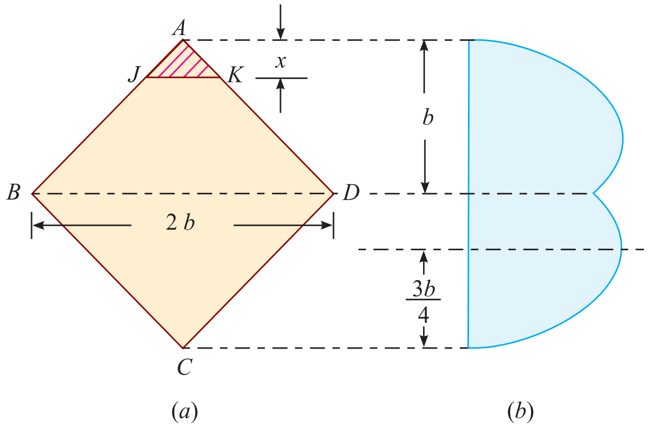


Fig. 16.18

and shearing stress at any point,

$$\tau = F \times \frac{A\bar{y}}{Ib} = F \times \frac{x^2\left(b - \frac{2x}{3}\right)}{\frac{b^4}{3} \times 2x} \qquad \text{(Here } b = JK = 2x\text{)}$$

$$= \frac{F}{2b^4}(3bx - 2x^2) \qquad \dots (i)$$

We also know that when $x = 0$, $\tau = 0$ and when $x = b$, then

$$\tau = \frac{F}{2b^2} = \frac{F}{\text{Area}} = \tau_{mean}$$

Now for maximum shear stress, differentiating the equation (*i*) and equating it to zero.

$$\frac{d\tau}{dx} = \frac{d}{dx}\left[\frac{F}{2b^4}(3bx - 2x^2)\right] = 0$$

$$\therefore \qquad 3b - 4x = 0 \qquad \text{or} \qquad x = \frac{3b}{4}$$

Substituting this value of x in equation (*i*),

$$\tau_{max} = \frac{F}{2b^4}\left[3b \times \frac{3b}{4} - 2\left(\frac{3b}{4}\right)^2\right] = \frac{F}{2b^4} \times \frac{9b^2}{8}$$

$$= \frac{9}{8} \times \frac{F}{2b^2} = \frac{9}{8} \times \frac{F}{\text{Area}} = \frac{9}{8} \times \tau_{mean}$$

Now complete the shear stress distribution diagram as shown in Fig. 16.18 (*b*).

EXAMPLE 16.10. *A rolled steel joist 200 mm × 160 mm wide has flange 22 mm thick and web 12 mm thick. Find the proportion, in which the flanges and web resist shear force.*

SOLUTION. Given : Overall depth (D) = 200 mm ; Flange width (B) = 160 mm ; Flange thickness (t_f) = 22 mm ; Web thickness (b) = 12 mm and web depth (d) = 156 mm.

Let $\qquad F$ = Shear force resisted by the section.

From the geometry of the figure, we find that the moment of inertia of the section through its c.g. and parallel to x-x axis,

$$I = \frac{1}{12}[(160) \times (200)^3 - (148)(156)^3] \text{ mm}^4$$

$$= 59.84 \times 10^6 \text{ mm}^4$$

Now consider an elementary strip of thickness dy of the flange at a distance y from the neutral axis. Therefore area of the elementary strip,

$$dA = 160\,dy$$

We know that the intensity of shear stress at the strip,

$$\tau = \frac{F}{2I}\left(\frac{D^2}{4} - y^2\right) = \frac{F}{2I}\left(\frac{(200)^2}{4} - y^2\right)$$

$$= \frac{F}{2I}(10000 - y^2)$$

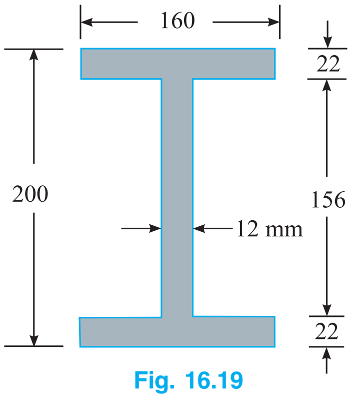


Fig. 16.19

∴ Resistance offered to shear by this strip

$$= \tau \cdot dA = \frac{F}{2I}(10\,000 - y^2) \times 160\,dy$$

$$= 160\,dy \times \frac{F}{2I}(10\,000 - y^2) = \frac{80F}{I}(10\,000 - y^2)\,dy$$

Now total resistance offered to shear by the flange

$$= \int_{78}^{100} \frac{80F}{I}(10\,000 - y^2)\,dy$$

$$= \frac{80F}{I}\left[10\,000\,y - \frac{y^3}{3}\right]_{78}^{100}$$

$$= \frac{80F}{I}\left[\frac{2 \times 10^6}{3} - \frac{1.865 \times 10^6}{3}\right]$$

$$= \frac{80F}{I} \times \frac{0.135 \times 10^6}{3}$$

$$= \frac{80F}{59.84 \times 10^6} \times 0.045 \times 10^6 = 0.06\,F$$

∴ Total resistance offered to shear by both the flanges

$$= 0.06\,F \times 2 = 0.12\,F$$

and total resistance offered to shear by the web

$$= F - 0.12\,F = 0.88\,F$$

It is obvious that the resistance offered by flanges is 12% and by web is **88%** **Ans.**

EXERCISE 16.2

1. An *I*-section beam consists of two flanges 150 mm × 20 mm and a web of 310 mm × 10 mm. Find the magnitude of maximum shear stress when it is subjected to a shear force of 40 kN and draw the shear stress distribution diagram over the depth of the section. [**Ans.** 13.1 MPa]
2. A *T*-section beam with 100 mm × 15 mm flange and 150 × 15 mm web is subjected to a shear force of 10 kN at a section. Draw the variation of shear stress across the depth of the beam and obtain the value of maximum shear stress at the section. [**Ans.** 6.3 MPa]

3. An *I*-section consists of the following sections:

Upper flange = 130 mm × 50 mm

Web = 200 mm × 50 mm

Lower flange = 200 mm × 50 mm

If the beam is subjected to a shearing force of 50 kN, find the maximum shear stress across the section. Also draw the shear stress distribution diagram. Take I as 284.9×10^6 mm^4.

[**Ans.** 4.42 MPa]

QUESTIONS

1. Derive an expression for the shear stress at any point in the cross-section of a beam.
2. Show that for a rectangular section, the distribution of shearing stress is parabolic.
3. The cross-section of a beam is a circle with the diameter D. If F is the total shear force at the cross-section, show that the shear stress at a distance y from the neutral axis.

$$= \frac{16F}{3\pi D^2}\left[1-\left(\frac{2v}{D}\right)^2\right]$$

4. Explain by mathematical expression, that the shear stress abruptly changes at the junction of the flange and web of an *I*-section and a *T*-section.
5. Describe the procedure for drawing the shear stress distribution diagram for composite sections.

OBJECTIVE TYPE QUESTIONS

1. When a rectangular section of a beam is subjected to a shearing force, the ratio of maximum shear stress to the average shear stress is
(*a*) 2.0 (*b*) 1.75 (*c*) 1.5 (*d*) 1.25
2. In a triangular section, the maximum shear stress occurs at
(*a*) apex of the triangle (*b*) mid of the height
(*c*) 1/3 of the height (*d*) base of the triangle
3. A square with side x of a beam is subjected to a shearing force of F. The value of shear stress at the top edge of the section is
(*a*) zero (*b*) $0.5\ F/a^2$ (*c*) F/a^2 (*d*) $1.5\ F/a^2$
4. An inverted *T*-section is subjected to a shear force F. The maximum shear stress will occur at
(*a*) top of the section (*b*) neutral axis of the section
(*c*) junction of web and flange (*d*) none of these

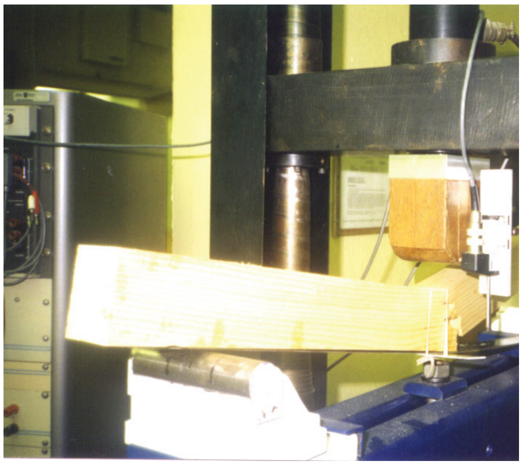
ANSWERS

1. (*c*) **2.** (*b*) **3.** (*a*) **4.** (*b*)

Chapter

17

Direct and Bending Stresses



Contents

17.1. Introduction

We have already discussed in Chapter 2, that whenever a body is subjected to an axial tension or compression, a direct stress comes into play at every section of the body. We also know that whenever a body is subjected to a bending moment a bending stress comes into play. It is thus obvious that if a member is subjected to an axial loading, along with a transverse bending, a direct stress as well as a bending stress comes into play. The magnitude and nature of these stresses may be easily found out from the magnitude and nature of the load and the moment. A little consideration will show that since both these stresses act normal to a cross-section, therefore the two stresses may be algebraically added into a single resultant stress.

17.2. Eccentric Loading

A load, whose line of action does not coincide with the axis of a column or a strut, is known as an eccentric load. A bucket full of water, carried by a person in his hand, is an excellent example of an eccentric load. A little consideration will show that the man will feel this load as more severe than the same load, if he had carried the same bucket over his head. The simple reason for the same is that if he carries the bucket in his hand, then in addition to his carrying bucket, he has also to lean or bend on the other side of the bucket, so as to counteract any possibility of his falling towards the bucket. Thus we say that he is subjected to :

1. Direct load, due to the weight of bucket (including water) and
2. Moment due to eccentricity of the load.

17.3. Columns with Eccentric Loading

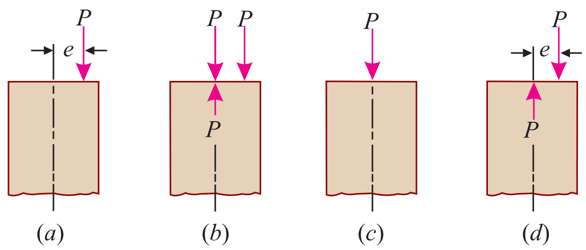


Fig. 17.1

Consider a column subjected to an eccentric loading. The eccentric load may be easily analysed as shown in Fig. 17.1 and as discussed below :

1. The given load P, acting at an eccentricity of e, is shown in Fig. 17.1 (*a*).
2. Let us introduce, along the axis of the strut, two equal and opposite forces P as shown in Fig. 17.1 (*b*).
3. The forces thus acting, may be split up into three forces.
4. One of these forces will be acting along the axis of the strut. This force will cause a direct stress as shown in Fig. 17.1 (*c*).
5. The other two forces will form a couple as shown in Fig. 17.1 (*d*). The moment of this couple will be equal to $P \times e$ (This couple will cause a bending stress).

Note : A column may be of symmetrical or unsymmetrical section and subjected to an eccentric load, with eccentricity about one of the axis or both the axes. In the succeeding pages, we shall discuss these cases one by one.

17.4. Symmetrical Columns with Eccentric Loading about One Axis

Consider a column *ABCD* subjected to an eccentric load about one axis (*i.e.*, about *y-y* axis) as shown in Fig. 17.2

Let P = Load acting on the column,

e = Eccentricity of the load,

b = Width of the column section and

d = Thickness of the column.

∴ Area of column section,

$$A = b \cdot d$$

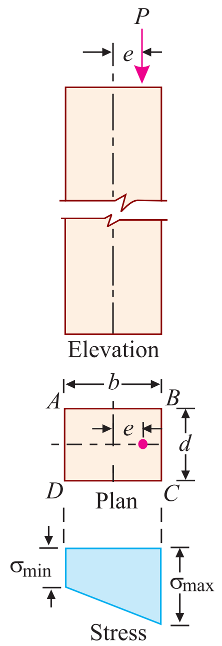


Fig. 17.2

and moment of inertia of the column section about an axis through its centre of gravity and parallel to the axis about which the load is eccentric (*i.e.*, *y-y* axis in this case),

$$I = \frac{d \cdot b^2}{12}$$

and modulus of section, $$Z = \frac{I}{y} = \frac{db^2/12}{b/2} = \frac{db^2}{6}$$

We know that direct stress on the column due to the load,

$$\sigma_0 = \frac{P}{A}$$

and moment due to load, $$M = P \cdot e$$

∴ Bending stress at any point of the column section at a distance *y* from *y-y* axis,

$$\sigma_b = \frac{M \cdot y}{I} = \frac{M}{Z} \qquad \ldots\left(\because Z = \frac{I}{y}\right)$$

Now for the bending stress at the extreme, let us substitute $y = \frac{b}{2}$ in the above equation,

$$\sigma_b = \frac{M \cdot \frac{b}{2}}{I} = \frac{M \cdot \frac{b}{2}}{\frac{db^3}{12}} \qquad \ldots\left(\because I = \frac{db^3}{12}\right)$$

$$= \frac{6M}{db^3} = \frac{6P \cdot e}{db^2} \qquad \ldots(\because M = P \cdot e)$$

$$= \frac{6P \cdot e}{A \cdot b} \qquad \ldots(\text{Substituting } db = A)$$

We have already discussed in the previous article, that an eccentric load causes a direct stress as well as bending stress. It is thus obvious that the total stress at the extreme fibre,

$$= \sigma_0 \pm \sigma_b = \frac{P}{A} \pm \frac{6P \cdot e}{A \cdot b} \qquad \ldots(\text{In terms of eccentricity})$$

$$= \frac{P}{A} \pm \frac{M}{Z} \qquad \ldots(\text{In terms of modulus of section})$$

The +ve or –ve sign will depend upon the position of the fibre with respect to the eccentric load. A little consideration will show that the stress will be maximum at the corners *B* and *C* (because these corners are near the load), whereas the stress will be minimum at the corners *A* and *D* (because these corners are away from the load). The total stress along the width of the column will vary by a straight line law. The maximum stress,

$$\sigma_{max} = \frac{P}{A} + \frac{6P \cdot e}{Ab} = \frac{P}{A}\left(1 + \frac{6e}{b}\right) \qquad \ldots(\text{In terms of eccentricity})$$

$$= \frac{P}{A} + \frac{M}{Z} \qquad \ldots(\text{In terms of section modulus})$$

and $$\sigma_{min} = \frac{P}{A} - \frac{6P \cdot e}{Ab} = \frac{P}{A}\left(1 - \frac{6e}{b}\right) \qquad \ldots(\text{In terms of eccentricity})$$

$$= \frac{P}{A} - \frac{M}{Z} \qquad \ldots(\text{In terms of section modulus})$$

NOTES : From the above equations, we find that

1. If σ_0 is greater than σ_b, the stress throughout the section, will be of the same nature (*i.e.*, compressive).
2. If σ_0 is equal to σ_b, even then the stress throughout the section will be of the same nature. The minimum stress will be equal to zero, whereas the maximum stress will be equal to $2 \times \sigma_0$.
3. If σ_0 is less than σ_b, then the stress will change its sign (partly compressive and partly tensile).

EXAMPLE 17.1. *A rectangular strut is 150 mm and 120 mm thick. It carries a load of 180 kN at an eccentricity of 10 mm in a plane bisecting the thickness. Find the maximum and minimum intensities of stress in the section.*

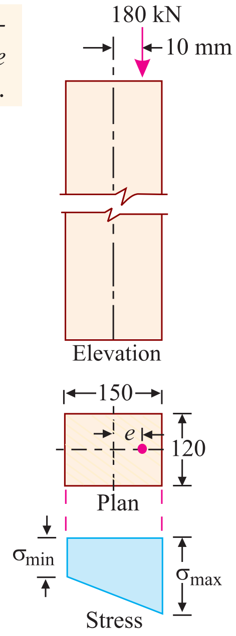


Fig. 17.3

SOLUTION. Given: Width (b) = 150 mm ; Thickness (d) = 120 mm ; Load (P) = 180 kN = 180×10^3 N and eccentricity (e) = 10 mm.

Maximum intensity of stress in the section

We know that area of the strut,

$$A = b \times d = 150 \times 120 = 18\,000 \text{ mm}^2$$

and maximum intensity of stress in the section,

$$\sigma_{max} = \frac{P}{A}\left(1+\frac{6e}{b}\right) = \frac{1800 \times 10^3}{18\,000}\left(1+\frac{6 \times 10}{150}\right) \text{ N/mm}^2$$

$$= 10\,(1 + 0.4) = 14 \text{ N/mm}^2 = \mathbf{14\ MPa} \quad \textbf{Ans.}$$

Minimum intensity of stress in the section

We also know that minimum intensity of stress in the section,

$$\sigma_{min} = \frac{P}{A}\left(1-\frac{6e}{b}\right) = \frac{1800 \times 10^3}{18\,000}\left(1-\frac{6 \times 10}{150}\right) \text{ N/mm}^2$$

$$= 10\,(1 - 0.4) = 6 \text{ N/mm}^2 = \mathbf{6\ MPa} \quad \textbf{Ans.}$$

EXAMPLE 17.2. *A rectangular column 200 mm wide and 150 mm thick is carrying a vertical load of 120 kN at an eccentricity of 50 mm in a plane bisecting the thickness. Determine the maximum and minimum intensities of stress in the section.*

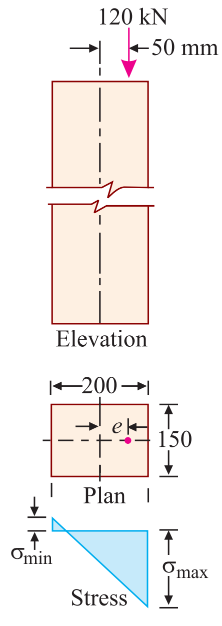


Fig. 17.4

SOLUTION. Given: Width (b) = 200 mm; Thickness (d) = 150 mm ; Load (P) = 120 kN = 120×10^3 N and eccentricity (e) = 50 mm.

Maximum intensity of stress in the section

We know that area of the column,

$$A = b \times d = 200 \times 150 = 30\,000 \text{ mm}^2$$

and maximum intensity of stress in the section,

$$\sigma_{max} = \frac{P}{A}\left(1+\frac{6e}{b}\right) = \frac{120 \times 10^3}{30\,000}\left(1+\frac{6 \times 50}{200}\right) \text{ N/mm}^2$$

$$= 4\,(1 + 1.5) = 10 \text{ N/mm}^2 = \mathbf{10\ MPa} \quad \textbf{Ans.}$$

Minimum intensity of stress in the section

We also know that minimum intensity of stress in the section,

$$\sigma_{min} = \frac{P}{A}\left(1-\frac{6e}{b}\right) = \frac{120 \times 10^3}{30\,000}\left(1-\frac{6 \times 50}{200}\right) \text{ N/mm}^2$$

$$= 4\,(1 - 1.5) = 4\,(-0.5) = -2 \text{ N/mm}^2$$

$$= 2 \text{ N/mm}^2 \text{ (tension)} = \mathbf{2\ MPa\ (tension)} \quad \textbf{Ans.}$$

EXAMPLE 17.3. *In a tension specimen 13 mm in diameter the line of pull is parallel to the axis of the specimen but is displaced from it. Determine the distance of the line of pull from the axis, when the maximum stress is 15 per cent greater than the mean stress on a section normal to the axis.*

SOLUTION. Given: Diameter (d) = 13 mm and maximum stress (σ_{max}) = 1.15 σ_{mean}

We know that area of the specimen,

$$A = \frac{\pi}{4}(d)^2 = \frac{\pi}{4}(13)^2 = 132.7 \text{ mm}^2$$

and its section modulus,

$$Z = \frac{\pi}{32}(d)^3 = \frac{\pi}{32}(13)^3 = 215.7 \text{ mm}^3$$

Let P = Pull on the specimen in N, and

e = Distance of the line of pull from the axis in mm.

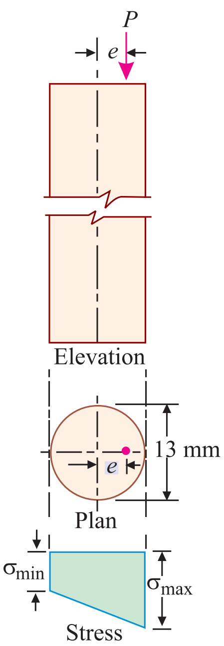


Fig. 17.5

$\therefore$ Moment due to load,

$$M = P \cdot e$$

We also know that the mean stress,

$$\sigma_{mean} = \frac{P}{A} = \frac{P}{132.7} \text{ N/mm}^2 \qquad \text{...}(i)$$

and maximum stress,

$$\sigma_{max} = \sigma_{mean} + \frac{M}{Z} = \frac{P}{132.7} + \frac{P \cdot e}{215.7}$$

Since σ_{max} is 15% greater than σ_{mean}, therefore

$$\frac{P}{132.7} + \frac{P \cdot e}{215.7} = \frac{P}{132.7} \times \frac{115}{100}$$

or

$$\frac{1}{132.7} + \frac{e}{215.7} = \frac{115}{13270}$$

$$\therefore \quad e = \left(\frac{115}{13270} - \frac{1}{132.7}\right) \times 215.7 = \mathbf{0.25 \text{ mm}} \qquad \textbf{Ans.}$$

EXAMPLE 17.4. *A hollow rectangular masonry pier is 1.2 m × 0.8 m wide and 150 mm thick. A vertical load of 2 MN is transmitted in the vertical plane bisecting 1.2 m side and at an eccentricity of 100 mm from the geometric axis of the section.*

Calculate the maximum and minimum stress intensities in the section.

SOLUTION. Given: Outer width (B) = 1.2 m = 1.2×10^3 mm ; Load (P) = 2 MN = 2×10^6 N ; Outer thickness (D) = 0.8 m = 0.8×10^3 mm ; Thickness (t) = 150 mm and eccentricity (e) = 100 mm.

Maximum stress intensity in the section

We know that area of the pier,

$$A = (BD - bd)$$
$$= [(1.2 \times 10^3) \times (0.8 \times 10^3)] - [(0.9 \times 10^3) \times (0.5 \times 10^3)]$$
$$= (0.96 \times 10^6) - (0.45 \times 10^6) = 0.51 \times 10^6 \text{ mm}^2$$

and its section modulus,

$$Z = \frac{1}{6}[BD^2 - bd^2] = \frac{1}{6}[(1.2 \times 10^3) \times (0.8 \times 10^3)^2]$$
$$- [(0.9 \times 10^3) \times (0.5 \times 10^3)^2] \text{ mm}^3$$
$$= \frac{1}{6}[(768 \times 10^6) - (225 \times 10^6) = 90.5 \times 10^6 \text{ mm}^3$$

We know that moment due to eccentricity of load,

$$M = P \cdot e = (2 \times 10^6) \times 100 = 200 \times 10^6 \text{ N-mm}$$

∴ Maximum stress intensity in the section,

$$\sigma_{max} = \frac{P}{A} + \frac{M}{Z} = \frac{2 \times 10^6}{0.51 \times 10^6} + \frac{200 \times 10^6}{90.5 \times 10^6} \text{ N/mm}^2$$
$$= 3.92 + 2.21 = 6.13 \text{ N/mm}^2 = \mathbf{6.13 \text{ MPa}} \quad \textbf{Ans.}$$

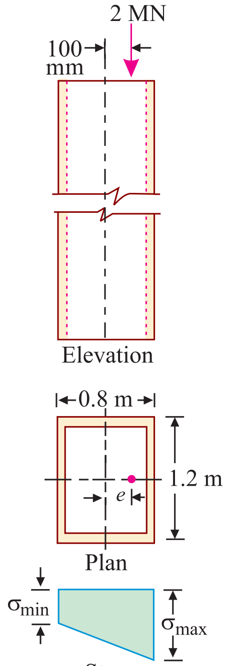


Fig. 17.6

Minimum stress intensity in the section

We also know that minimum stress intensity in the section,

$$\sigma_{min} = \frac{P}{A} - \frac{M}{Z} = \frac{2 \times 10^6}{0.51 \times 10^6} - \frac{200 \times 10^6}{90.5 \times 10^6} \text{ N/mm}^2$$
$$= 3.92 - 2.21 = 1.71 \text{ N/mm}^2 = \mathbf{1.71 \text{ MPa}} \quad \textbf{Ans.}$$

EXAMPLE 17.5. *A hollow circular column having external and internal diameters of 300 mm and 250 mm respectively carries a vertical load of 100 kN at the outer edge of the column. Calculate the maximum and minimum intensities of stress in the section.*

SOLUTION. Given: External diameter (D) = 300 mm ; Internal diameter (d) = 250 mm and load (P) = 100 kN = 100×10^3 N

Maximum intensity of stress in the section

We know that area of the column,

$$A = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}[(300)^2 - (250)^2] \text{ mm}^2$$
$$= 21.6 \times 10^3 \text{ mm}^2$$

and its section modulus,

$$Z = \frac{\pi}{32} \times \left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{32} \times \left[\frac{(300)^4 - (250)^4}{300}\right] \text{ mm}^3$$
$$= 1372 \times 10^3 \text{ mm}^3$$

Since the column carries the vertical load at its outer edge, therefore eccentricity,

$$e = 150 \text{ mm}$$

and moment due to eccentricity of load,

$$M = P \cdot e = (100 \times 10^3) \times 150 = 15 \times 10^6 \text{ N-mm}$$

∴ Maximum intensity of stress in the section,

$$\sigma_{max} = \frac{P}{A} + \frac{M}{Z} = \frac{100 \times 10^3}{21.6 \times 10^3} + \frac{15 \times 10^6}{1372 \times 10^3} \text{ N/mm}^2$$
$$= 4.63 \times 10.93 = 15.56 \text{ N/mm}^2 = \mathbf{15.56 \text{ MPa}} \quad \textbf{Ans.}$$

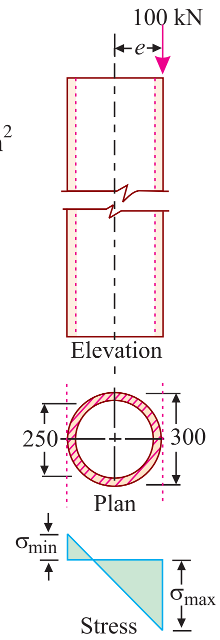


Fig. 17.7

Minimum intensity of stress in the section

We also know that minimum intensity of stress in the section,

$$\sigma_{min} = \frac{P}{A} - \frac{M}{Z} = \frac{100 \times 10^3}{21.6 \times 10^3} - \frac{15 \times 10^6}{1372 \times 10^3} \text{ N/mm}^2$$

$$= 4.63 - 10.93 = -6.3 \text{ N/mm}^2$$

$$= 6.3 \text{ N/mm}^2 \text{ (tension)} = \mathbf{6.3 \text{ MPa (tension)}} \qquad \textbf{Ans.}$$

EXERCISE 17.1

1. A rectangular strut 200 mm wide and 150 mm thick carries a load of 60 kN at an eccentricity of 20 mm in a plane bisecting the thickness. Find the maximum and minimum intensities of stresses in the section. (**Ans.** 3200 kPa ; 800 kPa)
2. A circular column of 200 mm diameter is subjected to a load of 300 kN, which is acting 5 mm away from the geometric centre of the column. Find the maximum and minimum stress intensities in the section. (**Ans.** 11.94 MPa ; 7.16 MPa)
3. A rectangular hollow masonry pier of 1200 mm × 800 mm with wall thickness of 150 mm carries a vertical load of 100 kN at an eccentricity of 100 mm in the plane bisecting to 1200 mm side. Calculate the maximum and minimum stress intensities in the section (**Ans.** 291.6 kPa ; 100.4 kPa)
4. A hollow circular column of 200 mm external diameter and 180 mm internal diameter is subjected to a vertical load of 75 kN at an eccentricity of 35 mm. What are the maximum and minimum stress intensities ? (**Ans.** 22.28 MPa ; 2.84 MPa)

17.5. Symmetrical Columns with Eccentric Loading about Two Axes

In the previous articles, we have discussed the cases of eccentric loading about one axis only. But, sometimes the load is acting eccentrically about two axes as shown in Fig. 17.8. Now consider a column *ABCD* subjected to a load with eccentricity about two axes as shown in Fig. 17.8.

Let P = Load acting on the column ,

A = Cross-sectional area of the column,

e_X = Eccentricity of the load about *X-X* axis,

Moment of the load about *X-X* axis,

$$M_X = P \cdot e_X$$

Let I_{XX} = Moment of inertia of the column section about *X-X* axis and

e_Y, M_Y, I_{YY} = Corresponding values of *Y-Y* axis.

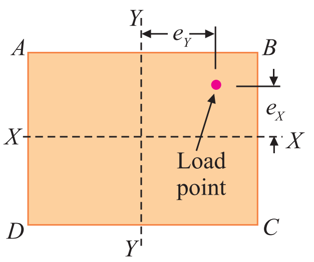


Fig. 17.8

The effect of such a load may be split up into the following three parts :

1. Direct stress on the column due to the load,

$$\sigma_0 = \frac{P}{A} \qquad ...(i)$$

2. Bending stress due to eccentricity e_X,

$$\sigma_{bX} = \frac{M_X \cdot y}{I_{XX}} = \frac{P \cdot e_X \cdot y}{I_{XX}} \qquad ...(ii)$$

3. Bending stress due to eccentricity e_Y,

$$\sigma_{bY} = \frac{M_Y \cdot x}{I_{YY}} = \frac{P \cdot e_Y \cdot x}{I_{YY}} \qquad ...(iii)$$

∴ Total stress at the extreme fibre

$$= \sigma_0 \pm \sigma_{bX} \pm \sigma_{bY} = \frac{P}{A} \pm \frac{M_X \cdot y}{I_{XX}} \pm \frac{M_Y \cdot x}{I_{YY}}$$

The +ve or –ve sign depends upon the position of the fibre with respect to the load. A little consideration will show that the stress will be maximum at *B*, where both the +ve signs are to be adopted. The stress will be minimum at *D*, where both the –ve signs are to be adopted. While calculating the stress at *A*, the value of M_X is to be taken as +ve, whereas the value of M_Y as –ve. Similarly for the stress at *C*, the value of M_Y is to be taken as +ve, whereas the value of M_X as –ve.

EXAMPLE 17.6. *A column 800 mm × 600 mm is subjected to an eccentric load of 60 kN as shown in Fig. 17.9.*

What are the maximum and minimum intensities of stresses in the column ?

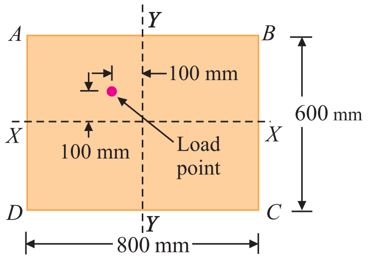


Fig. 17.9

SOLUTION. Given: Width (*b*) = 800 mm ; Thickness (*d*) = 600 mm ; Load (*P*) = 60 kN = 60 × 10^3 N ; Eccentricity along *X-X* axis (e_X) = 100 mm and eccentricity along *Y-Y* axis (e_Y) = 100 mm.

Maximum intensity of stress in the column

We know that area of the column,

$$A = b \times d = 800 \times 600 = 480 \times 10^3 \text{ mm}^2$$

and moment of inertia of the column about *X-X* axis,

$$I_{XX} = \frac{bd^3}{12} = \frac{800 \times (600)^3}{12} = 14.4 \times 10^9 \text{ mm}^4$$

Similarly,
$$I_{YY} = \frac{bd^3}{12} = \frac{600 \times (800)^3}{12} = 25.6 \times 10^9 \text{ mm}^4$$

We also know that moment due to eccentricity of load along *X-X* axis,

$$M_X = P \cdot e_X = (60 \times 10^3) \times 100 = 6 \times 10^6 \text{ N-mm}$$

Similarly,
$$M_Y = P \cdot e_Y = (60 \times 10^3) \times 100 = 6 \times 10^6 \text{ N-mm}$$

From the geometry of the loading, we find that distance between *Y-Y* axis and corners *A* and *B* (or *D* and *C*).

$$x = 400 \text{ mm}$$

Similarly, distance between *X-X* axis and corners *A* and *D* (or *B* and *C*).

$$y = 300 \text{ mm}$$

We know that maximum intensity of stress at *A*,

$$\sigma_A = \frac{P}{A} + \frac{M_X y}{I_{XX}} + \frac{M_Y x}{I_{YY}}$$

$$= \frac{60 \times 10^3}{480 \times 10^3} + \frac{(6 \times 10^6) \times 300}{14.4 \times 10^9} + \frac{(6 \times 10^6) \times 400}{25.6 \times 10^9} \text{ N/mm}^2$$

$$= 0.125 + 0.125 + 0.094 = 0.344 \text{ N/mm}^2 = \mathbf{0.344 \text{ MPa}}$$ **Ans.**

Minimum intensity of stress in the column

We also know that minimum intensity of stress in the column,

$$\sigma_C = \frac{P}{A} - \frac{M_X \cdot y}{I_{XX}} - \frac{M_Y \cdot x}{I_{YY}}$$

$$= \frac{60 \times 10^3}{480 \times 10^3} - \frac{(6 \times 10^6) \times 300}{14.4 \times 10^9} - \frac{(6 \times 10^6) \times 400}{25.6 \times 10^9} \text{ N/mm}^2$$

$$= 0.125 - 0.125 - 0.094 = -0.094 \text{ N/mm}^2$$

$$= 0.094 \text{ N/mm}^2 \text{ (tension)} = \mathbf{0.094 \text{ MPa (tension)}} \qquad \textbf{Ans.}$$

EXAMPLE 17.7. *A masonry pier of 3 m × 4 m supports a vertical load of 80 kN as shown in Fig. 17.10.*

(*a*) *Find the stresses developed at each corner of the pier.*

(*b*) *What additional load should be placed at the centre of the pier, so that there is no tension anywhere in the pier section ?*

(*c*) *What are the stresses at the corners with the additional load in the centre.*

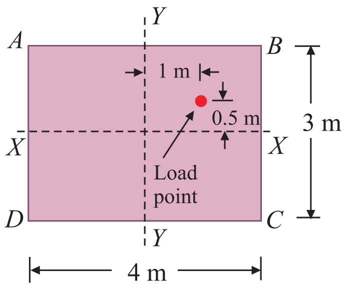


Fig. 17.10

SOLUTION. Given: Width (b) = 4 m ; Thickness (d) = 3 m ; Load (P) = 80 kN ; Eccentricity along X-X axis (e_X) = 0.5 m and eccentricity along Y-Y axis (e_Y) = 1 m.

(a) Stresses developed at each corner

We know that area of the pier,

$$A = b \times d = 4 \times 3 = 12 \text{ m}^2$$

and moment of inertia of the pier about X-X axis,

$$I_{XX} = \frac{bd^3}{12} = \frac{4 \times (3)^3}{12} = 9 \text{ m}^4$$

Similarly,

$$I_{YY} = \frac{bd^3}{12} = \frac{3 \times (4)^3}{12} = 16 \text{ m}^4$$

We also know that moment due to eccentricity of load along X-X axis,

$$M_X = P \cdot e_X = 80 \times 0.5 = 40 \text{ kN-m}$$

Similarly,

$$M_Y = P \cdot e_Y = 80 \times 1.0 = 80 \text{ kN-m}$$

From the geometry of the loading, we find that distance between Y-Y axis and the corners A and B,

$$x = 2 \text{ m}$$

Similarly distance between X-X axis and the corners A and D,

$$y = 1.5 \text{ m}$$

We know that stress at A,

$$\sigma_A = \frac{P}{A} + \frac{M_X \cdot y}{I_{XX}} - \frac{M_Y \cdot x}{I_{YY}} = \frac{80}{12} + \frac{40 \times 1.5}{9} - \frac{80 \times 2}{16} \text{ kN/m}^2$$

$$= 6.67 + 6.67 - 10 = 3.34 \text{ kN/m}^2 = \mathbf{3.34 \text{ kPa}} \qquad \textbf{Ans.}$$

Similarly,

$$\sigma_B = \frac{P}{A} + \frac{M_X \cdot y}{I_{XX}} + \frac{M_Y \cdot x}{I_{YY}} = \frac{80}{12} + \frac{40 \times 1.5}{9} + \frac{80 \times 2}{16} \text{ kN/m}^2$$

$$= 6.67 + 6.67 + 10.0 = 23.34 \text{ kN/m}^2 = \mathbf{23.34 \text{ kPa}} \qquad \textbf{Ans.}$$

$$\sigma_C = \frac{P}{A} - \frac{M_X \cdot y}{I_{XX}} + \frac{M_Y \cdot x}{I_{YY}} = \frac{80}{12} - \frac{40 \times 1.5}{9} + \frac{80 \times 2}{16} \text{ kN/m}^2$$

$= 6.67 - 6.67 + 10.0 = 10.0$ kN/m^2 = **10.0 kPa** **Ans.**

and
$$\sigma_D = \frac{P}{A} - \frac{M_X \cdot y}{I_{XX}} - \frac{M_Y \cdot x}{I_{YY}} = \frac{80}{12} - \frac{40 \times 1.5}{9} - \frac{80 \times 2}{16} \text{ kN/m}^2$$

$= 6.67 - 6.67 - 10.0 = -10.0$ kN/m^2 = **10 kPa (tension)** **Ans.**

(b) Additional load at the centre for no tension in the pier section

Let W = Additional load (in kN) that should be placed at the centre for no tension in the pier section.

We know that the compressive stress due to the load

$$= \frac{W}{A} = \frac{W}{12} \text{ kN/m}^2$$

We also know that for no tension, in the pier section the compressive stress due to the load. W should be equal to the tensile stress at D, *i.e.*, 10.0 kN/m^2.

$$\therefore \quad \frac{W}{12} = 10.0$$

or $W = 10.0 \times 12 =$ **120 kN** **Ans.**

(c) Stresses at the corners with the additional load in the centre

We find that the stress due to the additional load

$$= \frac{W}{A} = \frac{120}{12} = 10.0 \text{ kN/m}^2$$

∴ Stress at A, $\sigma_A = 3.34 + 10.0 =$ **13.34 kPa** **Ans.**

Similarly, $\sigma_B = 23.34 + 10.0 =$ **33.34 kPa** **Ans.**

$\sigma_C = 10.0 + 10.0 =$ **20.0 kPa** **Ans.**

and $\sigma_D = 10.0 + 10.0 =$ **0** **Ans.**

17.6. Unsymmetrical Columns with Eccentric Loading

In the previous articles, we have discussed the symmetrical column sections subjected to eccentric loading. But in an unsymmetrical column, first *c.g.* and then moment of inertia of the section is found out. After that the distances between the *c.g.* of the section and its corners are calculated. The stresses on the corners are then found out as usual, by using the respective values of moment of inertia and distance of the corner from the *c.g.* of the section.

EXAMPLE 17.8. *A hollow cylindrical shaft of 200 mm external diameter has got eccentric bore of 140 mm diameter, such that the thickness varies from 20 mm at one end to 40 mm at the other. Calculate the extreme stress intensities, if the shaft is subjected to a load of 400 kN along the axis of the bore.*

SOLUTION. Given: External diameter (D) = 200 mm ; Internal diameter (d) = 140 mm and load (P) = 400 kN = 400 × 10^3 N.

We know that net area of the shaft,

$$A = \frac{\pi}{4}[(200)^2 - (140)^2] = 5\ 100\ \pi \text{ mm}^2$$

First of all, let find out the centre of gravity of the section. Let the left end A be the point of reference.

(*i*) ***Main circle***

$$a_1 = \frac{\pi}{4} \times D^2 = \frac{\pi}{4} \times (200)^2 = 10\,000\,\pi \text{ mm}^2$$

$$x_1 = \frac{200}{2} = 100 \text{ mm}$$

(*ii*) ***Bore***

$$a_2 = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times (140)^2 = 4\,900\,\pi \text{ mm}^2$$

$$x_2 = 40 + \frac{140}{2} = 110 \text{ mm}$$

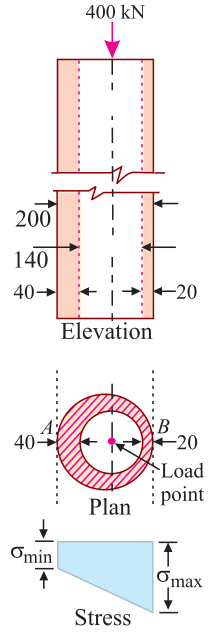


Fig. 17.11

We know that distance between the centre of gravity of the section and the left end *A*,

$$\bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2} = \frac{(10000\,\pi \times 100) - (4900\,\pi \times 100)}{10000\,\pi \times 4900\,\pi}$$

$$= 90.4 \text{ mm}$$

From the geomety of the figure, we find that the eccentricity of the load,

$$e = 110 - 90.4 = 19.6 \text{ mm}$$

∴ Moment due to eccenticity of load,

$$M = P \cdot e = (400 \times 10^3) \times 19.6$$

$$= 7.84 \times 10^6 \text{ N-mm}$$

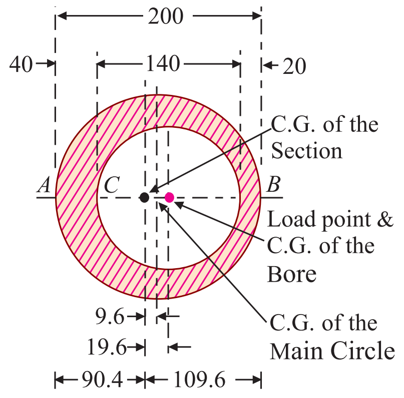


Fig. 17.12

Distance of corner *A* from the centre of gravity of the section,

$$y_A = 90.4 \text{ mm}$$

Similarly, $y_B = 200 - 90.4 = 109.6$ mm

We know that the moment of inertia of the main circle about its centre of gravity,

$$I_{G1} = \frac{\pi}{64} \times (200)^4 = 25 \times 10^6\,\pi \text{ mm}^4$$

and distance between the centre of gravity of the main circle and centre of gravity of the section,

$$h_1 = 100 - 90.4 = 9.6 \text{ mm}$$

∴ Moment of inertia of the main circle about the centre of gravity of the section

$$= I_{G1} + a_1\,h_1^2 = (25 \times 10^6\,\pi) + (10\,000\,\pi)\,(9.6)^2 \text{ mm}^4$$

$$= 25.92 \times 10^6\,\pi \text{ mm}^4$$

Similarly, moment of inertia of the bore about its centre of gravity

$$I_{G2} = \frac{\pi}{64} \times (140)^4 = 6.0 \times 10^6\,\pi \text{ mm}^4$$

and distance between the centre of gravity of the bore and the centre of gravity of the section,

$$h_2 = 110 - 90.4 = 19.6 \text{ mm}$$

∴ Moment of inertia of the bore about the centre of gravity of the section

$$= I_{G2} + a_2\,h_2^2 = (6.0 \times 10^6\,\pi) + (4\,900\,\pi)\,(19.6)^2 \text{ mm}^4$$

$$= 7.88 \times 10^6\,\pi \text{ mm}^4$$

and net moment of inertia of the section about its centre of gravity,

$$I = (25.92 \times 10^6\,\pi) - (7.88 \times 10^6\,\pi) = 18.04 \times 10^6\,\pi \text{ mm}^4$$

We know that maximum stress intensity,

$$\sigma_{max} = \frac{P}{A} + \frac{M \cdot y_B}{I} = \frac{400 \times 10^3}{5100\,\pi} + \frac{(7.84 \times 10^6) \times 109.6}{18.04 \times 10^6\,\pi}\ \text{N/mm}^2$$

$$= 24.97 + 15.16 = 40.13\ \text{N/mm}^2 = \mathbf{0.13\ MPa} \qquad \textbf{Ans.}$$

and minimum stress intensity,

$$\sigma_{min} = \frac{P}{A} - \frac{M \cdot y_A}{I} = \frac{400 \times 10^3}{5100\,\pi} - \frac{(7.84 \times 10^6) \times 90.4}{18.04 \times 10^6\,\pi}\ \text{N/mm}^2$$

$$= 24.97 - 12.51 = 12.46\ \text{N/mm}^2 = \mathbf{12.46\ MPa} \qquad \textbf{Ans.}$$

EXAMPLE 17.9. *A short C.I. column has a rectangular section 160 mm 200 mm with a circular hole of 80 mm diameter as shown in Fig. 17.13.*

It carries an eccentric load of 100 kN, located as shown in the figure. Determine the values of the stresses at the four corners of the section.

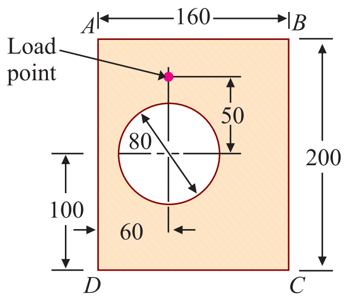


Fig. 17.13

SOLUTION. Given: Width (B) = 160 mm ; Depth (D) = 200 mm; Diameter of circular hole (d) = 80 mm and load (P) = 100 kN = 100×10^3 N.

We know that area of the column section,

$$A = (200 \times 160) - \left(\frac{\pi}{4} \times (80)^2\right) = 26\,970\ \text{mm}^2$$

First of all, let us find out the centre of gravity of the section. Let AD be the line of reference.

(i) ***Outer rectangle***

$$a_1 = 200 \times 160 = 32\,000\ \text{mm}^2$$

$$x_1 = 160/2 = 80\ \text{mm}$$

(ii) ***Circular hole***

$$a_2 = \frac{\pi}{4} \times (80)^2 = 5\,027\ \text{mm}^2$$

$$x_2 = 60\ \text{mm}$$

We know that distance between the centre of gravity of the section and AD,

$$\bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2} = \frac{(32000 \times 80) - (5027 \times 60)}{(32000 - 5027)} = 83.7\ \text{mm}$$

From the geometry of the figure, we find that eccentricity of load about X-X axis

$$e_X = 50\ \text{mm} \quad \text{and} \quad e_Y = 83.7 - 60 = 23.7\ \text{mm}$$

∴ Moment due to eccentricity of load along X-X axis,

$$M_X = P \cdot e_X = (100 \times 10^3) \times 50 = 5 \times 10^6\ \text{N-mm}$$

Similarly $$M_Y = P \cdot e_Y = (100 \times 10^3) \times 23.7 = 2.37 \times 10^6\ \text{N-mm}$$

and distance of the corner A from X-X axis passing through centre of gravity of the section,

$$y_A = y_B = y_C = y_D = 100\ \text{mm}$$

Similarly, distance of corner A from Y-Y axis passing through centre of gravity of the section,

$$x_A = x_D = 83.7\ \text{mm}$$

and $$x_B = x_C = 160 - 83.7 = 76.3\ \text{mm}$$

We know that the moment of inertia of the main rectangle *ABCD*, passing through its centre of gravity and parallel to *X-X* axis,

$$I_{G1} = \frac{160 \times (200)^2}{12} = 106.7 \times 10^6 \text{ mm}^4$$

and moment of inertia of the circular hole, passing through its centre of gravity and parallel to *X-X* axis,

$$I_{G2} = \frac{\pi}{4} \times (80)^2 = 2.01 \times 10^6 \text{ mm}^4$$

Since the centre of gravity of the rectangle and the circular hole coincides with the *X-X* axis, therefore moment of inertia of the section about *X-X* axis,

$$I_{XX} = (106.7 \times 10^6) - (2.01 \times 10^6) = 104.69 \times 10^6 \text{ mm}^4 \quad ...(i)$$

We also know that the moment of inertia of the main rectangle *ABCD*, passing through its centre of gravity and parallel to *Y-Y* axis,

$$I_{G3} = \frac{200 \times (160)^3}{12} = 68.26 \times 10^6 \text{ mm}^4$$

and distance between the centre of gravity of the rectangle from *Y-Y* axis,

$$h_1 = 83.7 - 80 = 3.7 \text{ mm}$$

∴ Moment of inertia of the rectangle through centre of gravity of the section and about *Y-Y* axis

$$= I_{G3} + a_1 h_1^2 = (68.26 \times 10^6) + 32\,0000 \times (3.7)^2 \text{ mm}^4$$
$$= 68.7 \times 10^6 \text{ mm}^4$$

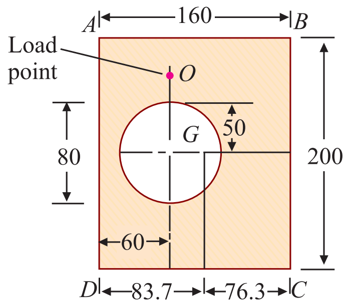


Fig. 17.14

Similarly, moment of inertia of the circular hole through its centre of gravity and parallel to *Y-Y* axis,

$$I_{G4} = \frac{\pi}{64} \times (80)^4 = 2.01 \times 10^6 \text{ mm}^4$$

and distance between the centre of gravity of the circular section from *Y-Y* axis,

$$h_2 = 83.7 - 60 = 23.7 \text{ mm}$$

∴ Moment of inertia of the circular hole through centre of gravity of the section and about *Y-Y* axis

$$= I_{G4} + a_2 h_2^2 = (2.01 \times 10^6) + 5\,027 \times (23.7)^2 \text{ mm}^4$$
$$= 4.84 \times 10^6 \text{ mm}^4$$

and net moment of inertia of the section about *Y-Y* axis,

$$I_{YY} = (68.7 \times 10^6) - (4.84 \times 10^6) = 63.86 \times 10^6 \text{ mm}^4 \quad ...(ii)$$

Now from the geometry of the figure, we find that stress at *A*,

$$\sigma_A = \frac{P}{A} + \frac{M_X \cdot y_A}{I_{XX}} + \frac{M_Y \cdot x_A}{I_{YY}}$$

$$= \frac{100 \times 10^3}{26970} + \frac{(5 \times 10^6) \times 100}{104.69 \times 10^6} + \frac{(2.3 \times 10^6) \times 83.7}{63.86 \times 10^6} \text{ N/mm}^2$$

$$= 11.5 \text{ N/mm}^2 = \mathbf{11.5 \text{ MPa}} \qquad \textbf{Ans.}$$

Similarly, $$\sigma_B = \frac{P}{A} + \frac{M_X \cdot y_B}{I_{XX}} - \frac{M_Y \cdot x_B}{I_{YY}}$$

$$= \frac{100 \times 10^3}{26970} + \frac{(5 \times 10^6) \times 100}{104.69 \times 10^6} - \frac{(2.3 \times 10^6) \times 76.3}{63.86 \times 10^6} \text{ N/mm}^2$$

$$= 5.74 \text{ N/mm}^2 = \mathbf{5.74 \text{ MPa}} \qquad \textbf{Ans.}$$

$$\sigma_C = \frac{P}{A} - \frac{M_X \cdot y_C}{I_{XX}} - \frac{M_Y \cdot x_C}{I_{YY}}$$

$$= \frac{100 \times 10^3}{26970} - \frac{(5 \times 10^6) \times 100}{104.69 \times 10^6} - \frac{(2.3 \times 10^6) \times 76.3}{63.86 \times 10^6} \text{ N/mm}^2$$

$$= -3.82 \text{ N/mm}^2 = \mathbf{3.82 \text{ MPa (tensile)}} \qquad \textbf{Ans.}$$

and
$$\sigma_D = \frac{P}{A} - \frac{M_X \cdot y_D}{I_{XX}} + \frac{M_Y \cdot x_D}{I_{YY}}$$

$$= \frac{100 \times 10^3}{26970} - \frac{(5 \times 10^6) \times 100}{104.69 \times 10^6} + \frac{(2.3 \times 10^6) \times 83.7}{63.86 \times 10^6} \text{ N/mm}^2$$

$$= 1.95 \text{ N/mm}^2 = \mathbf{1.95 \text{ MPa}} \qquad \textbf{Ans.}$$

17.7. Limit of Eccentricity

We have seen in Art. 17.2 and 17.3, that when an eccentric load is acting on a column, it produces direct stress as well as bending stress. On one side of the neutral axis there is a maximum stress (equal to the sum of direct and bending stress) and on the other side of the neutral axis there is a minimum stress (equal to direct stress minus bending stress). A little consideration will show that so long as the bending stress remains less than the direct stress, the resultant stress is compressive. If the bending stress is equal to the direct stress, then there will be a zero stress on one side. But if the bending stress exceeds the direct stress, then there will be a tensile stress on one side. Though cement concrete can take up a small tensile stress, yet it is desirable that no tensile stress should come into play.

We have seen that if the tensile stress is not to be permitted to come into play, then bending stress should be less than the direct stress, or maximum, it may be equal to the direct stress, *i.e.*,

$$\sigma_b \leq p_0$$

$$\frac{P \cdot e}{Z} \leq \frac{P}{A} \qquad ...(\because M = P \cdot e)$$

or
$$e \leq \frac{Z}{A}$$

It means that for no tensile condition, the eccentricity e should be less than $\frac{Z}{A}$ or equal to $\frac{Z}{A}$. Now we shall discuss the limit for eccentricity in the following cases :

1. For a rectangular section,
2. For a hollow rectangular section,
3. For a circular section and
4. For a hollow circular section.

(a) Limit of eccentricity for a rectangular section

Consider a rectangular section of width (b) and thickness (d) as shown in Fig. 17.15. We know that the modulus of section,

$$Z = \frac{1}{6} bd^2 \qquad ...(i)$$

and area of the section,
$$A = bd \qquad ...(ii)$$

We also know that for no tension condition,

$$e \leq \frac{Z}{A}$$

$$\leq \frac{\frac{1}{6} bd^2}{bd}$$

$$\leq \frac{1}{6} d$$

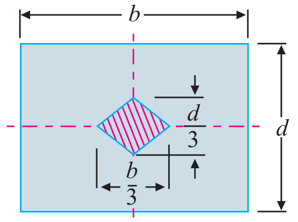


Fig. 17.15

It means that the load can be eccentric, on either side of the geometrical axes, by an amount equal to $d/6$. Thus if the line of action of the load is within the middle third, as shown by the dotted area in Fig. 17.15, then the stress will be compressive throughout.

(b) Limit of eccentricity for a hollow rectangular section

Consider a hollow rectangular section with B and D as outer width and thickness and b and d internal dimensions respectively. We know that the modulus of section,

$$Z = \frac{(BD^3 - bd^3)}{6D} \qquad \text{...}(i)$$

and area of the hollow rectangular section,

$$A = BD - bd \qquad \text{...}(ii)$$

We also know that for no tension condition,

$$e \leq \frac{Z}{A}$$

$$\leq \frac{\frac{(BD^3 - bd^3)}{6D}}{BD - bd}$$

$$\leq \frac{(BD^3 - bd^3)}{6D\,(BD - bd)}$$

It means that the load can be eccentric, on either side of the geometrical axis, by an amount equal to $\dfrac{(BD^3 - bd^3)}{6D\,(BD - bd)}$.

(c) Limit of eccentricity of a circular section

Consider a circular section of diameter d as shown in Fig. 17.16. We know that the modulus of section,

$$Z = \frac{\pi}{32} \times d^3 \qquad \text{...}(i)$$

and area of circular section, $A = \dfrac{\pi}{4} \times d^2$ ...(*ii*)

We also know that for no tension condition,

$$e \leq \frac{Z}{A}$$

$$\leq \frac{\frac{\pi}{32} \times d^3}{\frac{\pi}{4} \times d^2}$$

$$\leq \frac{d}{8}$$

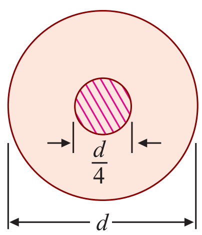


Fig. 17.16

It means that the load can be eccentric, on any side of the geometrical centre, by an amount equal to $d/8$. Thus, if the line of action of the load is within a circle of diameter equal to one-fourth of the main circle as shown by the dotted area in Fig. 17.16, then the stress will be compressive throughout.

(*d*) *Limit of eccentricity for hollow circular section*

Consider a hollow circular section of external and internal diameters as D and d respectively. We know that the modulus of section,

$$Z = \frac{\pi}{32} \times \frac{(D^4 - d^4)}{D} \qquad ...(i)$$

and area of hollow circular section,

$$A = \frac{\pi}{4} \times (D^2 - d^2) \qquad ...(ii)$$

We also know that for no tension condition,

$$e \leq \frac{Z}{A}$$

$$\leq \frac{\frac{\pi}{32} \times \frac{(D^4 - d^4)}{D}}{\frac{\pi}{4} \times (D^2 - d^2)}$$

$$\leq \frac{(D^2 - d^2)}{8D} \qquad ...[\because (D^4 - d^4) = (D^2 + d^2)(D^2 - d^2)]$$

It means that the load can be eccentric, on any side of the geometrical centre, by an amount equal to $\dfrac{(D^2 - d^2)}{8D}$.

EXERCISE 17.2

1. A rectangular pier is 1500 mm × 1000 mm is subjected to a compressive load of 450 kN as shown in Fig. 17.17.

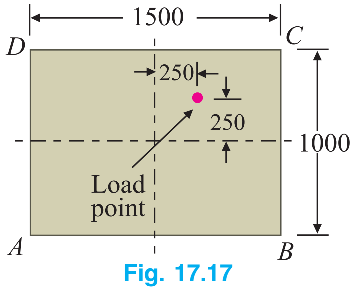


Fig. 17.17

Find the stress intensities on all the four corners of the pier.

[**Ans.** $\sigma_A = -4.5$ kPa ; $\sigma_B = +1.5$ kPa ; $\sigma_C = 10.5$ kPa ; $\sigma_D = 4.5$ kPa]

2. A hollow square column has 1.5 m outside length and 1 m inside length. The column is subjected to a load of 7 kN located on a diagonal and at a distance of 0.8 m from the vertical axis of the pier. Determine the stress intensities on the outside corners of the column.

[**Ans.** 23.15 kPa ; 5.6 kPa ; 11.95 kPa ; 5.6 kPa]

3. A short hollow cylindrical cast iron column of outside diameter 300 mm and inside diameter 200 mm was casted. On inspection, it was found the bore is eccentric in such a way that the thickness on one side is 70 mm and 30 mm on the other. If the column is subjected to a load of 80 kN at the axis of the bore, find the extreme intensities of stresses in the base.

[**Ans.** 3.66 kPa ; 0.73 MPa]

QUESTIONS

1. Distinguish clearly between direct stress and bending stress.
2. What is meant by eccentric loading? Explain its effects on a short column.
3. Derive the relation for the maximum and minimum stress intensities due to eccentric loading.
4. Obtain a relation for the maximum and minimum stresses at the base of a symmetrical column. When it is subjected to
   (*a*) an eccentric load about one axis and (*b*) an eccentric load about two axes.
5. Show that for no tension in the base of a short column, the line of action of the load should be within the middle third.
6. Define the term limit of eccentricity. How will you find out this limit in case of a hollow circular section?

OBJECTIVE TYPE QUESTIONS

1. The maximum stress intensity at the base of a square column of area A and side b subjected to a load W at an eccentricity e is equal to

   (*a*) $\frac{W}{A}\left(1+\frac{2e}{b}\right)$ (*b*) $\frac{W}{A}\left(1-\frac{4e}{b}\right)$ (*c*) $\frac{W}{A}\left(1+\frac{6e}{b}\right)$ (*d*) $\frac{W}{A}\left(1+\frac{8e}{b}\right)$

2. The minimum stress intensity in the above case is

   (*a*) $\frac{W}{A}\left(1-\frac{e}{b}\right)$ (*b*) $\frac{W}{A}\left(1-\frac{2e}{b}\right)$ (*c*) $\frac{W}{A}\left(1-\frac{3e}{b}\right)$ (*d*) $\frac{W}{A}\left(1-\frac{6e}{b}\right)$

3. The maximum eccentricity of a load on a circular section to have same type of stress is
   (*a*) one-eighth of diameter (*b*) one-sixth of diameter
   (*c*) one-fourth of diameter (*d*) one-third of diameter

ANSWERS

1. (*c*) 2. (*d*) 3. (*c*)