UNIT - II

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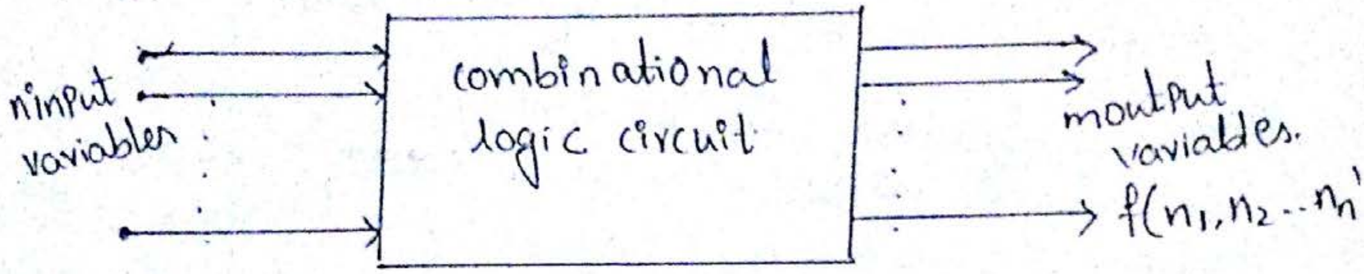
15D k1A05D2

DLD

Gate - level minimization and combination circuits.

combinational circuits:

A combinational circuit consists of logic gates whose outputs at any time are determined directly from the present combination of input without regard to previous inputs and previous outputs, i.e no need of memory.



A combinational circuits consists of inputs variables, logic gates & output variables. logic gate accept signals from the inputs generate signals to the outputs. The input and output data are represented by binary signals (logic 1 or logic 0

Minimization of functions or at Gate level:

Minimization of switching functions $f(x_1, x_2 \ldots x_n)$ is to find an expression $g(x_1, x_2, x_3 \ldots x_n)$ which is equivalent to f which minimises cost criteria. There are various techniques to determine to determine minimum cost. The most common are.

1) Minimum number of appearance of literals (literal is a variable it appears in complemented and uncomplemented form) in the function.

2) Minimum no of literals in sum of products or products of sum.

$$3.\ f(x,y,z) = x'y'z' + xy'z' + xy'z' + x'yz + xyz + xy'z$$
$$= x'y'z' + xy'z' + x'yz + xyz + xy'z \quad -①$$
$$y'z'(x'+x) + yz(x+x') + xy'z$$
$$= y'z' + yz + xy'z$$
$$= y'z' + yz + xy'z \quad -②$$

Eq.① is known as ~~redad~~ redundant expression in Eq② known as irredundant function at is equivalent to F.

Boolean-functions may be simplified by algebraic method as in above process. However this procedure of minimization is awkward because it lacks specific rules to predict each succeding step in the manipulative process. If more number of variables are there it is a tedious process.

Map Method:
The map method provides a simple straight forward procedure for minimizing Boolean-functions.

This method may be regarded either as a pictorial form of a truth table or as an extension of the venndiagram. The map method first proposed by veitch in slightly modified by karnaugh is also known as the veitch diagram or the karnaugh map.

In fact map represents a visual diagram of all possible ways of function.(may be expressed as minterms or maxterms).

Mapping of three variable kmap in SOP, Expression

The minterm has '1' o/p value of the function and 0 is there take complement of the variable, 1 is there take uncomplement of the variable, and in between variables take and operation. Between the minterms take OR operation in order to represent SOP format.

AB \ C	0	1
00	0 m_0	1 m_1
01	2 m_2	3 m_3
11	6 m_6	7 m_7
10	4 m_4	5 m_5

AB \ C	0	1
00	$\bar{A}\bar{B}\bar{C}$	$\bar{A}\bar{B}C$
01	$\bar{A}B\bar{C}$	$\bar{A}BC$
11	$AB\bar{C}$	ABC
10	$A\bar{B}\bar{C}$	$A\bar{B}C$

Mapping of three variables kmap in POS expression.

The maxterms are represented has 0 o/p value of the function. 0 is there unprimed variable & 1 is there primed variable in between take OR operation for variables & in between maxterms take AND operation in order to represent POS form.

AB \ C	0	1
00	$A+B+C$	$A+B+\bar{C}$
01	$A+\bar{B}+C$	$A+\bar{B}+\bar{C}$
11	$\bar{A}+\bar{B}+C$	$A+B+C$
10	$\bar{A}+B+C$	$\bar{A}+B+\bar{C}$

AB \ C	0	1
00	M_0	M_1
01	M_2	M_3
11	M_6	M_7
10	M_4	M_5

Four variable kmap.

Four variable kmap have four variables, so the number of possible combinations are $2^4 = 16$. so the kmap consists of (16 sq) 16 squares.

On column & Row
on column take 2 variables, so four combinations occurs, 00, 01, 11, 10. since kmap uses the Gray code.

CD \ AB	00	01	11	10
00				
01				
11				
10				

On Row take two variables, so four combinations can occur. 00, 01, 11, 10. combine Rows and columns form 4 variable kmap, having 16 number of squares.
write the binary value of each square as shown in figure.

CD \ AB	00	01	11	10
00	0000	0100	1100	1000
01	0001	0101	1101	1001
11	0011	0111	1111	1011
10	0010	0110	1110	1010

fig (a)

CD \ AB	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

Replace the decimal equivalent values of binary valued of each square.

Mapping of four variable kmap in SOP Expression.

The minterm has '1' o/p value of the function and '0' is there take complement of the variable, 1 is there take uncomplement of the variable and in between variable take AND operation, between the minterms take OR operation.

CD \ AB	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

CD \ AB	00	01	11	10
00	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}B\bar{C}\bar{D}$	$AB\bar{C}\bar{D}$	$A\bar{B}\bar{C}\bar{D}$
01	$\bar{A}\bar{B}\bar{C}D$	$\bar{A}B\bar{C}D$	$AB\bar{C}D$	$A\bar{B}\bar{C}D$
11	$\bar{A}\bar{B}CD$	$\bar{A}BCD$	$ABCD$	$A\bar{B}CD$
10	$\bar{A}\bar{B}C\bar{D}$	$\bar{A}BC\bar{D}$	$ABC\bar{D}$	$A\bar{B}C\bar{D}$

Mapping of four variables kmap in POS expression.

The maxterms are represented has '0' o/p value of the function. 0 is the unprimed variable & 1 is the there primed variable, in between take OR operation for variables, in between maxterms take 'AND' operation.

CD \ AB	00	01	11	10
00	M_0	M_4	M_{12}	M_8
01	M_1	M_5	M_{13}	M_9
11	M_3	M_7	M_{15}	M_{11}
10	M_2	M_6	M_{14}	M_{10}

CD \ AB	00	01	11	10
00	$A+B+C+D$	$A+\bar{B}+C+D$	$\bar{A}+\bar{B}+C+D$	$\bar{A}+B+C+D$
01	$A+B+C+\bar{D}$	$A+\bar{B}+C+\bar{D}$	$\bar{A}+\bar{B}+C+\bar{D}$	$\bar{A}+B+\bar{C}+\bar{D}$
11	$A+B+\bar{C}+\bar{D}$	$A+\bar{B}+C+D$	$\bar{A}+\bar{B}+\bar{C}+\bar{D}$	$\bar{A}+B+\bar{C}+\bar{D}$
10	$A+B+\bar{C}+D$	$A+\bar{B}+C+D$	$\bar{A}+\bar{B}+C+D$	$A+\bar{B}+\bar{C}+D$

five variable k-map

ABC \ DE	00	01	11	10
000	0 00000	1 00001	3 00011	2 00010
001	4 00100	5 00101	7 00111	6 00110
011	12 01100	13 01101	15 01111	14 01110
010	8 01000	9 01001	11 01011	10 01010
110	11000 24	11001 25	11011 27	11010 26
111	11100 28	11101 29	11111 31	11110 30
101	10100 20	10101 21	10111 23	10110 22
100	10000 16	10001 17	10011 19	10010 18

On Row take three variables & on column take two variables.
On Row first write two variables 00
01
11
10
10
11 } image
01
00

on upside take write '0' & bottom write 1 as MSB. where at taken the image or reflection the graph is half-of it

DONOT OR DON'T care combination.

so far, the Expressions considered have been completely specified for every combination of the input variables, that is, Each minterm (Maxterm) has been specified as '1'-min or (0)-maxterm. It often occurs that for certain input combinations, the value of output is unspecified Either because the input combinations are invalid or because the precise value of the output is of no conseqnence. The combinations for which the values of the Expression are not specified are called don't care combinations or optional combinations & such Expressions therefore stand incompletly specified

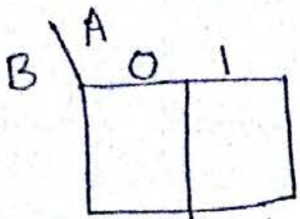
Kmap procedure for minimization.

(1) $P = AB + A\overline{B}$

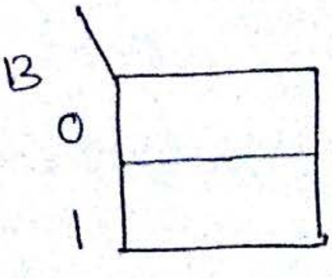
$= A(B+\overline{B}) = A.$

Two variable kmaps: Two variable kmap.

(2) In two variable kmaP, has two variable so the Expression can have $2^2 = 4$ Possible combinations. So the Map consists of four squares.



→ ON the column take the variable as A(one). It is only one variable can occur as either 0 or '1'. ON column represent 0 & 1. Divide the cell has two Parts on column.



→ ON the Row take the B variable. It is only one variable can occur as either 0 or 1. So on Row represent 0 & 1. Divide the cell has two Parts on the Row.

combine the Rows & column of the kmaP it forms the Two variable kmaP, having four squares as in fig(a). write the binary values of Each square as shown in fig(a)

B \ A	0	1
0	00	10
1	01	11

fig(a)

B \ A		
	0	2
	1	3.

fig(b)

Replace the Decimal equivalance values of binary values of each square.

Mapping of two variable in SOP expression.

A \ B	0	1
0	0 $\bar{A}\bar{B}$	1 $\bar{A}B$
1	2 $A\bar{B}$	3 AB.

A \ B		
	m_0	m_1
	m_2	m_3

The minterms are (known as) Represented by '1' as o/p function

0 —— Primed or complemented

1 —— uncomplemented

in between variables take AND operation.

These squares are called cells. Each square on the kmap represents a unique minterm. SOP form = $m_0 + m_1 + m_2 + m_3$

Mapping of two variables in POS expression.

A \ B	0	1
0	0 $(A+B)$	1 $(A+\bar{B})$
1	2 $(\bar{A}+B)$	3 $(\bar{A}+\bar{B})$

The maxterms are represented by '0' as output function.

0 — unprimed or uncomplemented variable

1 — complemented or Primed variable.

In between variables take OR operation.

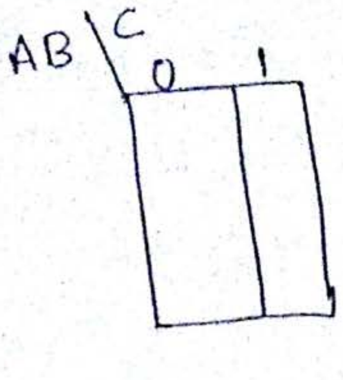
A \ B	0	1
0	M_0	M_1
1	M_2	M_3

POS form is $M_0 \cdot M_1 \cdot M_2 \cdot M_3$.

three variable kmap.

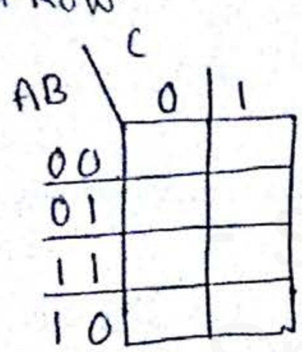
Three variable kmap have three variables so the number of possible combinations are $2^3 = 8$. So the kmap consists of 8 squares.

ON column



ON the column take one variable on the Row take two variables.
On the column on variable can occur as Either '0 or 1'. Divide the cell has two parts on column as shown in fig.

ON Row.



ON the Row take two variables so four combinations are there, 00, 01, 11, 10. The kmap is Represented or uses the cyclic code or ~~[illegible]~~ code. cyclic code means two consecutive numbers differ in only one bit position. cyclic codes ~~[illegible]~~ are unit distance code.

combine Rows and columns form 3 variable kmap having 8 number of squares. write the binary value of each square as shown in fig(a)

AB \ C	0	1
00	000	001
01	010	011
11	110	111
10	100	101

fig(a)

AB \ C	0	1
00	0	1
01	2	3
11	6	7
10	4	5

fig(b)

Replace the Decimal Equivalent values of binary valued of each square.

1. The o/p of a function is notable to predict are
as don't cares.

2. For certain input combinations can't occur or invalid i,
then the o/p of the function are indicated as don't care

3. Don't care terms are denoted by X, d or Ø Ø

4. Don't care value can be either '0' or '1' since it is
unspecified value.

Minimization of switching functions:

For reducing the Boolean expressions in SOP or POS form
on the kmap, look at the 1's (0s) present on the map. These
represent the minterms (maxterms). Look for the minterms (maxterms)
adjacent to each other, in order to combine them in to larger
squares.

combining of adjacent squares in a k-map containing
1s or (0's) for the purpose of simplification of a SOP or
(POS) expression is called looping.

Some of the minterms (maxterms) may have many adjacencies
always start with the minterm (maxterm) with the least number
of adjacencies and try to form as large as a square as possible
And number of minterms are grouped or looping in 2^n.
The cells are said to be adjacent if they differ by only
one cell.

(6)

are
valid
if
t.care

acent cells:
The cells are said to be adjacent if they differ by one bit.

Ex 001, 011 these two cells are adjacent since only one bit change.

The kmap uses the Gray code so every (those) cells are side by side they are adjacent as corner cells are also adjacent to each other.

Groupping of cells:

→ Groupping of cells are done if they are adjacent to each other.

→ In groupping the number of cells should be 2^n. ie $2^0, 2^1, 2^2, 2^3, 2^4, 2^5 \ldots\ldots 2^n$.

→ Always try to form large group ie 2^n we have to check first, $2^{n-1} \ldots$ as 32 no of cells check not possible try to form group for 16 not possible check for 8, ...
at last form as 1

- If 1 cell is there then the group is called as 'isolated'
- If 2 cells are gropped then the group is called as 'Pairs'
- If 4 cells are groupped then the group is called as 'quads'
- If 8 cells are groupped then the group is called as 'octets'

Finding procedure for minimal form:
minimization by map procedure.

$$f = AB + A\overline{B}$$
$$= A(B+\overline{B}) = A.$$

Here B variable is changed the value as A variable has a constant value. So eliminate the changed variable

and unchanged variable will be there in the o/p. ie check the number
minimal form of POS or SOP. the name of minimal or check
minimal form of SOP.
The value of variable is changed then eliminate variable in minimal-form of SOP.
The value of variable is unchanged then write the variable in minimal form as if '1' is there unprimed variable if 0 is there then primed variable. In between variable we 'And' operation are known as minterms & between minterms we OR operation.

minimal form of POS.
The value of variable is changed then eliminate variable in minimal form of SOP.
The value of variable is unchanged then write the variable in minimal form as if '1' is there primed variable & if 0 is there the unprimed variable. In between variables we OR operation are known as maxterms & between maxterms we AND operation.

Steps to find minimal form by using kmap.

1. check for maximum number of minterm or maxterm value depending on that choose number of variable kmap required.

2. Plot the kmap.

3. If minterms are given plot place value 1 in k-map corresponding value ~~given~~ of cell.
Maxterms are given then place value '0' in k-map corresponding value of cell.
Don't cares are given the place 'd' 'x' & in kmap corresponding value of cell.

check for adjacent cells, try to form larger group and the number of cells should be 2^n. ($2^n, 2^{n-1} \ldots 2^5, 2^4, 2^3, 2^2, 2^1, 2^0$

5. Name groups as $S_1, S_2, S_3 \ldots S_n$ & group minimal minterms or minimal maxterms.
6. check positions of cells groupped in Row wise & column wise, eliminate changed variable whose value is change. write the variable whose value is constant or notchanged.
7. if minterms are used then write minimal form SOP.
8. By using procedure as '1' is there unprimed variable '0' is there primed variable in between take 'AND' operation so we get minimal minterms & between minterms take OR operations forms minimal SOP form.

if maxterms are used in kmap write minimal form SOP By using procedure as '1' is ther primed variable, '0' is there unprimed variable inbetween take 'OR' operation so we get minimal maxterms & between maxterms take AND operation forms minimal POS-form.

Note

1. In Kmaps '1' & '0' are must to be covered in any one of the group.
2. Don't care are need not to be covered, if they are required to form larger group utilize them, ~~other wise~~ cover and form a group. otherwise donot cover ther them.
3. Always try to them cover ~~larger~~ more number of '1' or '0' & ~~for~~ try to form larger group.

Problems

without don't care condition.

① $f = \bar{A}B + A\bar{B} + AB$

The above expression is in the form of SOP.

$f = \bar{A}B + A\bar{B} + AB$ (① ② ③)

① minterm → $\bar{A}B$ ⇒ 01 ⇒ 1

② minterm → $A\bar{B}$ ⇒ 10 ⇒ 2

③ minterm → AB ⇒ 11 ⇒ 3

$f = \Sigma(1,2)$

step-1 It requires two variable k-map.

step-2

A \ B	0	1
0	0	1
1	2	3

step-3

A \ B	0	1
0		1
1	1	1

place 1 in 1,2,3 place

step-4

A \ B	0	1
0		1 → S_2
1	1	1 → S_1

cells (2,3) are adjacent to each other.
(10,11) only one bit change so they are adjacent to each other.

And S_1 group formed.
cells (1,3) are adjacent to eachother. Its groupname is S_2

step-5

Pairs are formed S_1 — (2,3)
S_2 — (1,3).

-6
check positions of cells groupped in Row wise & column wise. eliminate variable whose value is changed. write the variable whose value is constant or not changed.

S_1 covered the positions (2, 3)

step-7

Row wise

A = 1 constant (not changed) These are minterms so A

columnwise

B = (0, 1) → changed value so eliminate B variable

$\left.\begin{matrix}0\\1\end{matrix}\right\}$

$S_1 = A$.

S_2 covered the positions (1, 3)

Row wise

A = $\left.\begin{matrix}0\\1\end{matrix}\right\}$ changed value eliminate

columnwise $\underset{1}{B}$ so unchanged value, comes at O/P.

$S_2 = B$

SoP form $S_1 + S_2 = A + B$.

minimal form $S_1 + S_2 = A + B$

② $F(x, y, z) = \Sigma(3, 4, 6, 7)$

step-1

Σ is given so this Expresion is SoP form. The maximum number of minterm is 7. so 3 variable kmap is required.

$\therefore 2^3 = 8$.

step-2

Plot the k map.

$x \backslash yz$	00	01	11	10
0	0	1	3	2
1	4	5	7	6

step-3 & 4 & 5

$x \backslash yz$	00	01	11	10
0			1 (S_2)	
1	1 (S_1)		1	1

step-6 & 7

S_1 Pair $\rightarrow$ (4,6)

Row

$x=1$ constant $\Rightarrow$ x

on column

y	z
0	0
1	0

y is changed eliminate & z is const.

$z=0 \Rightarrow \bar{z} \Rightarrow \boxed{S_1 = x \cdot \bar{z}}$

S_2 Pair $\rightarrow$ (3,7)

Row

x: 0, 1 } changed eliminate

on column

yz 1 1 } unchanged $\Rightarrow$ yz. $\boxed{S_2 = y \cdot z}$

Minimal SOP form is $\underline{S_1 + S_2 = x \cdot \bar{z} + yz}$

③ $F = A'C + A'B + AB'C + BC$. find the minimal sop form by using k-map.

The above expression is not a canonical form. The above expression has to change in to canonical form.

step-4 check for adjacent cells try to form larger group

→ (1, 3, 5, 7) ⇒ s quad.

⇒ (3, 2) → Pair

step-5 Name the group

A \ BC	00	01	11	10
0		1	1	1
1		1	1	

$S_1 = (1, 3, 5, 7) \Rightarrow$ quad

$S_2 = (3, 2). \Rightarrow$ Pair.

step-6 & 7

S_1 ⇒ ON Row A
0
1 } changed eliminate

ON column BC
0 1
1 1 } B in changed & C in constant ⇒ C = 1.
⇒

$\boxed{S_1 = C}$

S_2 group (2, 3)

ON Row A
0 } unchanged A = 0 ⇒ $\bar{A}$

ON column BC
1 1
1 0 } C in changed & B in constant
B = 1 ⇒ B.

$\boxed{S_2 = \bar{A} \cdot B}$

minimal SOP in $S_1 + S_2 = C + \bar{A} \cdot B$.

(4) $F = \Sigma(0, 1, 2, 4, 5, 6, 8, 9, 12, 13)$

step-1 The maximum number in 13 so it required four variable k-map.

The expression is said to be canonical-form if it has all variables of a function. The function has three so that the every minterm should have three variab...

$$A'C\cdot 1 + A'B\cdot 1 + AB'C + B\cdot C\cdot 1$$

→ C missing (in $A'B\cdot 1$)
L B missing (in $A'C\cdot 1$)
L A missing (in $B\cdot C\cdot 1$)

By Idempotency law $A+A'=1$.

$$A'C(B+B) + A'B(C+C') + AB'C + BC(A+A')$$

$$\underset{1}{\underline{A'BC}} + \underset{2}{\underline{A'B'C}} + \underset{3}{\underline{A'BC}} + \underset{4}{\underline{A'BC'}} + \underset{5}{\underline{AB'C}} + \underset{6}{\underline{ABC}} + \underset{7}{\underline{A'BC}}$$

1 = 7 = 3 — write only once

$$\Rightarrow \underset{1}{\underline{A'BC}} + \underset{2}{\underline{A'B'C}} + \underset{3}{\underline{A'BC'}} + \underset{4}{\underline{AB'C}} + \underset{5}{\underline{ABC}}$$

1 minterm $A'BC \Rightarrow 011 \Rightarrow 3$
2 ,, $A'B'C \Rightarrow 001 \Rightarrow 1$
3 ,, $A'BC' \Rightarrow 010 \Rightarrow 2$
4 ,, $AB'C \Rightarrow 101 \Rightarrow 5$
5 ,, $ABC \Rightarrow 111 \Rightarrow 7$.

Primed — 0
unprimed — 1

$$f = \Sigma(1, 2, 3, 5, 7).$$

Step-1 check maximum number → 7 ⇒ 3 variables req.

Step-2 Plot 3:kmap.

A \ BC	00	01	11	10
0	0	1	3	2
1	4	5	7	6

Step-3

	00	01	11	10
0		1	1	1
1		1	1	

Ex-2 plot 4-variable k-map.

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

step-3 & 4

AB \ CD	00	01	11	10
00	1	1		1
01	1	1		1
11	1	1		
10	1	1		

step-5

$S_1 \Rightarrow (0, 1, 4, 5, 8, 9, 12, 13) \Rightarrow$ octet

$S_2 \Rightarrow (0, 2, 4, 6) \Rightarrow$ quad.

step-6

$S_1 = (0, 1, 4, 5, 8, 9, 12, 13)$

on Row

A	B
0	0
0	1
1	1
1	0

} A value & B value are changed Eliminate.

on column

C	D
0	0
0	1

} D is changed & C is const $C=0 \Rightarrow \bar{C}$

$\boxed{S_1 = \bar{C}}$

$S_2 = (0, 2, 4, 6)$

ON ROW

A	B
0	0
0	1

} B is changed & A is const $A = 0 \Rightarrow \bar{A}$

ON column

C	D
0	0
1	0

} C is changed & D is constant $D=0 \Rightarrow \bar{D}$

$\Rightarrow \bar{D}$

$\Rightarrow \bar{A} \cdot \bar{D}$

$\boxed{S_2 = \bar{A} \cdot \bar{D}}$

minimal sop is $S_1+S_2 = \bar{C} + \bar{A}\bar{D}$

$F = A'B'C' + B'CD' + A'BCD' + AB'C'$

The above expression is not a canonical form convert into standard or canonical form.

-6, 7

$S_1 =$ (0,

$S_2 =$

S_3

$F = A'B'C' \cdot 1 + 1 \cdot B'CD' + A'BCD' + AB'C' \cdot 1$

A missing

D is missing

D is missing

$= A'B'C'(D+D') + (A+A')B'CD' + A'BCD' + AB'C'(D+D')$

$\Rightarrow\; = \underbrace{A'B'C'D}_{1} + \underbrace{A'B'C'D'}_{2} + \underbrace{AB'CD'}_{3} + \underbrace{A'B'CD'}_{4} + \underbrace{A'BCD'}_{5} + \underbrace{AB'C'D}_{6} + \underbrace{AB'C'D'}_{7}$

① minterm $A'B'C'D \Rightarrow 0001 \Rightarrow 1$

② ... $A'B'C'D' \Rightarrow 0000 \Rightarrow 0$

③ ... $AB'CD' \Rightarrow 1010 \Rightarrow 10$

④ ... $A'B'CD' \Rightarrow 0010 \Rightarrow 2$

⑤ ... $A'BCD' \Rightarrow 0110 \Rightarrow 6$

⑥ ... $AB'C'D \Rightarrow 1001 \Rightarrow 9$

⑦ ... $AB'C'D' \Rightarrow 1000 \Rightarrow 8$

$f = \Sigma(0,1,2,6,8,9,10)$.

Step-1 2, 3, 4

AB \ CD	00	01	11	10
00	0: 1	1: 1	3	2: 1
01	4	5	7	6: 1
11	12	13	15	14
10	8: 1	9: 1	11	10: 1

S2 S1 S3

convert P-6.7

$S_1 = (0,1,8,9) \Rightarrow$ Quad
$S_2 = (0,2,8,10) \Rightarrow$ Quad
$S_3 = (2,6) \Rightarrow$ Pair.

S_1 group (0,1,8,9)

on Row

$\overset{\times}{A}B$, 00, 10 } A is changed & B is const. $B=0 \Rightarrow \overline{B}$

on column

$C\overset{\times}{D}$, 00, 01 } D is changed & C is const. $C=0 \Rightarrow \overline{C}$

$$\boxed{S_1 = \overline{B}\cdot\overline{C}}$$

S_2 group (0,2,8,10)

on Row

$\overset{\times}{A}B$, 00, 10 } A is changed, B is const. $B=0 \Rightarrow \overline{B}$

on column

$\overset{\times}{C}D$, 00, 10 } C is changed & D is constant $D=0 \Rightarrow \overline{D}$.

$$\boxed{S_2 = \overline{B}\cdot\overline{D}}$$

$S_3 \Rightarrow$
minimal SOP form is $S_1 + S_2 + S_3 = \cancel{\overline{B}\overline{C} + \overline{B}\overline{D}}$

S_3 group (2,6)

on Row

$A\overset{\times}{B}$, 00, 01 } B is changed & A is const $A=0 \Rightarrow \overline{A}$

on column

CD, 10 } const C & D are const $\Rightarrow C=1\ D=0 \Rightarrow C\overline{D}$

$$\boxed{S_3 = \overline{A}\cdot C\cdot\overline{D}}$$

minimal SOP $\boxed{S_1 + S_2 + S_3 = \overline{B}\overline{C} + \overline{B}\overline{D} + \overline{A}C\cdot\overline{D} = F}$

$f = (0, 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 13)$

step-1, 2, 3, 4

AB \ CD	00	01	11	10
00	0: 1	1: 1	3: 1	2: 1
01	4: 1	5: 1	7	6
11	12: 1	13: 1	15	14
10	8: 1	9: 1	11: 1	10: 1

$\rightarrow S_2$ $\quad S_1 \leftarrow 01$

step-5, 6, 7

$S_1 = (0, 1, 4, 5, 8, 9, 12, 13) = \overline{C}$ $\quad \boxed{S_1 = \overline{C}}$

$S_2 = (0, 1, 2, 3, 8, 9, 10, 11) \Rightarrow \overline{B}$ $\quad \boxed{S_2 = \overline{B}}$

minimal SOP is $S_1 + S_2 = \overline{B} + \overline{C} = f$

$\rightarrow$ $f(A, B, C, D, E) = (0, 2, 4, 6, 9, 13, 21, 23, 25, 29, 31)$

step-1, 2, 3, 4

ABC \ DE	00	01	11	10
000	0: 1	1	3	2: 1
001	4: 1	5	7	6: 1
011	12	13: 1	15	14
010	8	9: 1	11	10
110	24	25: 1	27	26
111	28	29: 1	31: 1	30
101	20	21: 1	23: 1	22
100	16	17	19	18

S_1 $\quad S_2$ $\quad S_3$

$S_1 (0, 2, 4, 6)$

$S_1 = \overline{A}\,\overline{B}\,\overline{E}$

$S_2 = (9, 13, 25, 29)$

$S_2 = B \cdot \overline{D} E$

$S_3 (21, 23, 29, 31)$

$S_3 = ACE$

minimal SOP is $S_1 + S_2 + S_3$

$\boxed{A'B'E' + BD'E + ACE = f}$

Donot cover:

$f(w,x,y,z) = \Sigma(1,3,10) + \Sigma_d(0,2,8,12)$

step-1, 2, 3, 4

AB \ CD	00	01	11	10
00	0: d	1: 1	3: 1	2: d
01	4	5	7	6
11	12: d	13	15	14
10	8: d	9	11	10: 1

$\rightarrow S_1$

S_2

step, 5, 6, 7

$S_1(0,1,2,3)$

$S_1 = \bar{A}\bar{B}$

$S_2(0,2,8,10)$

$S_2 = \bar{B}\bar{D}$

minimal SOP is $S_1 + S_2$

$\bar{A}\bar{B} + \bar{B}\bar{D}$

Note: Donot cares are need not to be covered. if they are required to form larger group utilize them to form a group. otherwise donot cover them.

i.e why 12 minterm don't care is left ~~as~~ i.e ungroupped it.

→ But in kmaps '1' & '0' are must to be covered in any one of the group.

⇒ $F(x,y,z) = \Sigma(2,3,4,6,7)\ \Sigma d(x,y,z)$

solution

Σd

$x \cdot 1 \cdot 1 + y \cdot 1 \cdot 1 + z \cdot 1 \cdot 1$

(z missing, y missing; z mis, x min; y mis, x mis)

$x(y+\bar{y})(z+\bar{z}) + (x+\bar{x})y\cdot(z+z') + z(x+\bar{x})(y+\bar{y})$

$(xy+x\bar{y})(z+\bar{z}) + (xy+\bar{x}y)(z+\bar{z}) + zx + z\bar{x}(y+\bar{y})$

$= xyz + xy\bar{z} + x\bar{y}z + x\bar{y}\bar{z} + xyz + \bar{x}y\bar{z} + \bar{x}yz + \bar{x}y\bar{z}$
$+ xyz + x\bar{y}z + \bar{x}yz + \bar{x}\bar{y}z.$

$= 111 + 110 + 101 + 100 + 111 + 110 + 011 + 010$
$+ 111 + 101 + 011 + 001$

$= 7 + 6 + 5 + 4, + \cancel{7} + 6 + 3 + 2 + 7 + 6 + 3, + 1$

$\Sigma d = (0,1,2,3,4,5,6,7)$ but the minterms are as $\Sigma(2,3,4,6,7)$ so eliminate the minterms for don't cares.

0-7 for S_1 group Row

$\Sigma(2,3,4,6,7) + \Sigma d(0,1,5) = f$

$x \backslash yz$	00	01	11	10
0	(0) ϕ	(1) ϕ	(3) 1	(2) 1
1	(4) 1	(5) ϕ	(7) 1	(6) 1

$\rightarrow S_1$

$S_1 = 1.$

Minimize the following Boolean function.

$Y(A,B,C,D) = \Sigma m(1,3,5,8,9,11,15) + d(2,13)$.

Step-1, 2, 3

The function requires four variable kmap.

Plot 4-kmap

$AB \backslash CD$	00	01	11	10
00	(0)	(1) 1	(3) 1	(2) ϕ
01	(4)	(5) 1	(7)	(6)
11	(12)	(13) ϕ	(15) 1	(14)
10	(8) 1	(9) 1	(11) 1	(10)

$\rightarrow S_1$

S_2

S_3

S_4

Step-4, 5

$S_1(1,5,9,13)$ forms quad

$S_2(9,11,13,15)$ form quad

$S_3(1,3,9,11)$ forms quard

$S_4(8,9)$ forms Pair.

26-7

S_1 group (1, 5, 9, 13)

ROW AB 00, 01, 11, 10 } changed aliminate A & B

column CD 01 } const $\Rightarrow \bar{C}D$.

$\boxed{S_1 = \bar{C}D}$

S_2 On Row: AB 11, 10 } A is const $A = 1 \Rightarrow A$ on column CD 00, 01 } C = const $C = 0 \Rightarrow \bar{C}$

$\boxed{S_2 = A \cdot \bar{C}}$

S_3 (1, 3, 9, 11)

on Row AB 00, 10 } B = const $B = 0 \Rightarrow \bar{B}$

on column CD 01, 11 } D is const $D = 1 \Rightarrow D$

$\boxed{S_3 = \bar{B} \cdot D}$

S_4 (8, 9)

on ROW AB 10 } $A \cdot \bar{B}$

on column CD 00, 01 } $\Rightarrow$ C = const $\Rightarrow \bar{C}$

$\boxed{S_4 = A\bar{B} \cdot \bar{C}}$

minimal form of SOP is $S_1 + S_2 + S_3 + S_4 = \bar{C}D + A\bar{C} + \bar{B}D + A\bar{B}\bar{C}$

Minimal form for POS

$$f(A, B, C, D) = \pi(1, 2, 3, 8, 9, 10, 11, 14) \cdot d(7, 15).$$

Solution:

step1: Max no. is 15 to represent we require four variable kmap.

step-2 Plot 4 k-map.

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

Step-3

Given symbol is π it represents these are are represented by value '0'

Step- 4 & 5

AB \ CD	00	01	11	10
00	0	1 0	3 0	2 0
01	4	5	7 d	6
11	12	13	15 d	14 0
10	8 0	9 0	11 0	10 0

S1 S2 S3 S4

Step-5

S1 forms (8, 9, 10, 11) ⟶ quad

S2 (2, 3, 10, 11) ⟶ quad

S3 (9, 11, 1, 3) ⟶ quad

S4 (10, 11, 14, 15) ⟶ quad.

Step-6

The given terms are maxterms so if variables have value of '1' represents primed variable if variable value '0' represents unprimed variable, between variable take 'OR' operation & between maxterms or groups are 'And' operation.

S1

Row

AB 10 } unchanged $A=1$ $B=0$ } $\Rightarrow (\bar{A}+B)$
$\bar{A}$ B

column CD 00, 01, 11, 10 ⟶ changed eliminate.

$$\boxed{S_1 = \bar{A}+B}$$

3 group $(2, 3, 10, 11)$

on Row

$\overset{\times}{A}B$
0 0
1 0

} A is changed w Bis const $B=0 \Rightarrow B$

on column

$\overset{\times}{CD}$
1 1
1 0

} C is const w D changed $C=1 \Rightarrow \bar{C}$

$$\boxed{S_2 = B+\bar{C}}$$

S_3 on Row $(9, 11, 1, 3)$

$\overset{\times}{A}B$
0 0
1 0

} A is changed w B is const $B=0 \Rightarrow B \Rightarrow B$

on column

$\overset{\times}{C}D$
0 1
1 1

} D is const w C is changed $D=1 \Rightarrow \bar{D}$

$$S_3 = (B+\bar{D})$$

S_4 $(10, 11, 14, 15)$

on Row

$A\overset{\times}{B}$
1 1
1 0

} B is changed w A is const $\Rightarrow A=1 \Rightarrow \bar{A}$

on column

$C\overset{\times}{D}$
1 1
1 0

} C is const w D is changed. $C=1 \Rightarrow \bar{C}$

$$S_4 = \bar{A}+\bar{C}$$

minimal POS form is $S_1 \cdot S_2 \cdot S_3 \cdot S_4 = (\bar{A}+B)(B+\bar{C})(B+\bar{D})(\bar{A}+\bar{C})$

Note: Donot cares need not to be covered so 7 don't care (lefted) not considered.

$\rightarrow F(A,B,C) = (A'+B'+C')(A'+B')(A'+C')$
$= (A'+B'+C')(A'+B')(A'+C')$

The above function is not a standerad POS function. convert above function to standarad POS. standarad POS function is a function that every maxterm should have all (three) variables of the function.

$F = (A'+B'+C')(A'+B')(A'+C')$

$= (A'+B'+C') \cdot (A'+B'+0)\ (A'+0+C')$ → B missing, ↳ C is missing

$A \cdot A' = 0$ — By Idempotency law.

$= (A'+B'+C)\ (A'+B'+C \cdot C')\ (A'+B \cdot B'+C')$

$= (A'+B'+C')\ (A'+B'+C)\ (A'+B'+C')\ (A'+B+C')(A'+B'+C')$

$= \underset{①}{(A'+B'+C')}\ \underset{②}{(A'+B'+C)}\ \underset{③}{(A'+B+C')}\ \underset{\cancel{④}}{\cancel{(A'+B'+C')}}$

maxterm · ① $A'+B'+C' = 111 \Rightarrow 7$

" ② $A'+B'+C = 110 \Rightarrow 6$

③ $A'+B+C' = 101 \Rightarrow 5$

~~④ $A'+B'+C'$ = 1~~

$A' = 1$
complemented
uncom = 0

$P = \pi(5, 6, 7)$

3-Kmap

A \ BC	00	01	11	10
0	0	1	3	2
1	4	5: 0	7: 0	6: 0

→ S_1
↳ S_2

$S_1(6,7) \Rightarrow (\bar{A}+\bar{B})$

$S_2(5,7) \Rightarrow (\bar{A}+\bar{C})$

minimal POS form is $(\bar{A}+\bar{B})(\bar{A}+\bar{C})$.

Find the minimal form of SOP for following function.

$\pi(5, 6, 7)$.

SOP function and POS functions are complement to each other. In the question asked find the minimal SOP form so first find minterms numbers (decimal numbers which are not in SOP form those (will) are known as maxterms.

other than minterm numbers are known as maxterm numbers

$\pi(5,6,7)$

$\Sigma(0,1,2,3,4)$. ⇒ SOP → $f = (0,1,2,3,4)$.

A \ BC	00	01	11	10
0	1	1	1	1
1	1			

S_1 = minterm group.

$S_1 = \bar{A}\ (0,1,2,3)$ $\boxed{S_1 = \bar{A}}$

$S_2 = (0,4) \Rightarrow \bar{B}\bar{C}$ $\boxed{S_2 = \bar{B}\bar{C}}$

minimal SOP $S_1 + S_2 = \bar{A} + \bar{B}\bar{C} = f$

obtain minimal forms by using kmap.
a) sum of Product and
b) Product of sum expressions for the given function below

$F(A,B,C,D) = \Sigma(0,1,2,5,8,9,10)$.

a) $F = \Sigma(0,1,2,5,8,9,10)$ sum of minterm form.

b) $\pi(3,4,6,7,11,12,13,14,15)$.

AB \ CD	00	01	11	10
00	1	1		1
01		1		
11				
10	1	1		1

$S_1(0,1,8,9)$

$S_1 = \bar{B}\cdot\bar{C}$

$S_2(0,2,8,10)$

$S_2 = \bar{B}\bar{D}$

$S_3(1,5)$

$S_3 = \bar{A}\bar{C}D$

$F = S_1 + S_2 + S_3 = \bar{B}\bar{C} + \bar{B}\bar{D} + \bar{A}\bar{C}D$.

(b) $\pi(3,4,6,7,11,12,13,14,15)$

AB \ CD	00	01	11	10
00	0:	1:	3: 0	2:
01	4: 0	5:	7: 0	6: 0
11	12: 0	13: 0	15: 0	14: 0
10	8:	9:	11: 0	10:

$S_1(12,13,14,15)$
$(\bar{A}+\bar{B})$

$S_2(3,7,11,15)$
$(\bar{C}+\bar{D})$

$S_3(4,6,12,14)$
$\bar{B}+D$

minimal POS is $(\bar{A}+\bar{B})(\bar{C}+\bar{D})(\bar{B}+D)$

find minimal form of POS.

$Y(A,B,C,D) = \Sigma(1,3,5,8,9,11,15) + \Sigma d(2,13)$

solution:

Don't cares will be same in both POS & SOP form. other than minterms are maxterms.

$\pi(0,2,4,6,7,10,12,14)\cdot d(2,13)$

$\pi(0,4,6,7,10,12,14)\cdot d(2,13)$

AB \ CD	00	01	11	10
00	0: 0	1:	3:	2: d
01	4: 0	5:	7: 0	6: 0
11	12: 0	13: d	15:	14: 0
10	8:	9:	11:	10: 0

$S_1(2,6,10,14)$

$S_1 = \overline{C}+D = \bar{C}+D$

$S_2(6,14,4,12)$

$S_2 = \bar{B}+D = \bar{B}+D$

$S_3(0,4,2,6)$

$S_3 = A+D$

minimal POS $S_1 \cdot S_2 \cdot S_3 = (\bar{C}+D)\cdot(\bar{B}+D)\cdot(A+D)$.

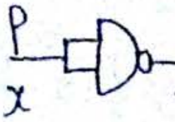
(4,5) Conversion of Basic gates to NAND gate: NAND

NOT gate

$f = x' \rightarrow$ it is a single variable

Nand $f = \overline{xy}$

$f = \overline{xy} \rightarrow$ nand function $f = \overline{x \cdot x} = x'$



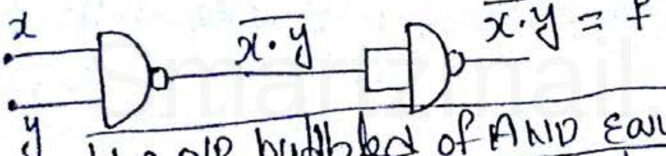
$x' = f \Rightarrow$ function of Not

AND gate AND function has to be implemented by Nand gate

$f = x \cdot y \rightarrow$ And gate function. Nand $f = \overline{x \cdot y}$

Take double time complement $f = x \cdot y$ in order to get same function value ($\because x = 0$ assume $\overline{x} = 1 \Rightarrow \overline{\overline{x}} = 0$)

$f = \overline{\overline{x \cdot y}} \Rightarrow \overline{\text{Nand}} = f$



$\Rightarrow f = x \cdot y$. =

$\boxed{\text{so o/p bubbled of AND Equivalent to Nand gate}}$

OR gate OR function has to be implemented by Nand gate.

$f = x + y \rightarrow$ OR gate function Nand gate $f = \overline{x \cdot y}$

Take double time complement $f = x+y$ in order to get same function value.

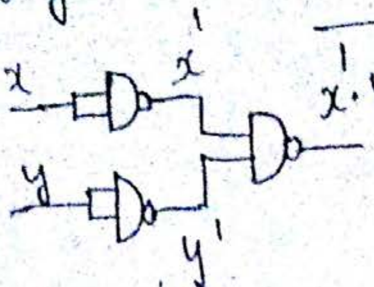
$f = \overline{\overline{x+y}} = \overline{x' \cdot y'}$ assume $x' = A$ $y' = B$ $f = \overline{A \cdot B} \rightarrow$ nand functi

$f = \overline{A \cdot B} \Rightarrow$ Nand function functionality.

$f = \overline{x' \cdot y'}$

$\boxed{\text{so i/p bubbled of OR gate is Equivalent to Nand gate function.}}$

$f = \overline{x' \cdot y'}$



$\overline{x' \cdot y'} = f \Rightarrow x+y$

conversion of Basic gates NOR gates.

NOT

$f = x'$

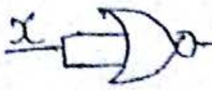
$f = \overline{x+x} = \overline{x}$ $\quad x+x = x.$

NOR

$f = \overline{x+y}$

NOR symbol

NOT



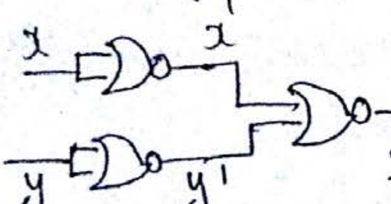
$x' = f$

AND gate

$f = x \cdot y$

Take double time complement for the AND function.

$f = \overline{\overline{x \cdot y}} = \overline{x' + y'} \Rightarrow A = x' \quad B = y' \Rightarrow \overline{A+B} \Rightarrow$ NOR functionality.



$f = \overline{x' + y'}$ or $x \cdot y$.

OR gate

$f = x+y$

Take double complement for the OR function.

$f = \overline{\overline{x+y}} = \overline{x+y} = A \Rightarrow \overline{A} = f$

$\overline{\overline{x+y}} = x+y.$

Equivalent Representation of Basic gates in universal gates.

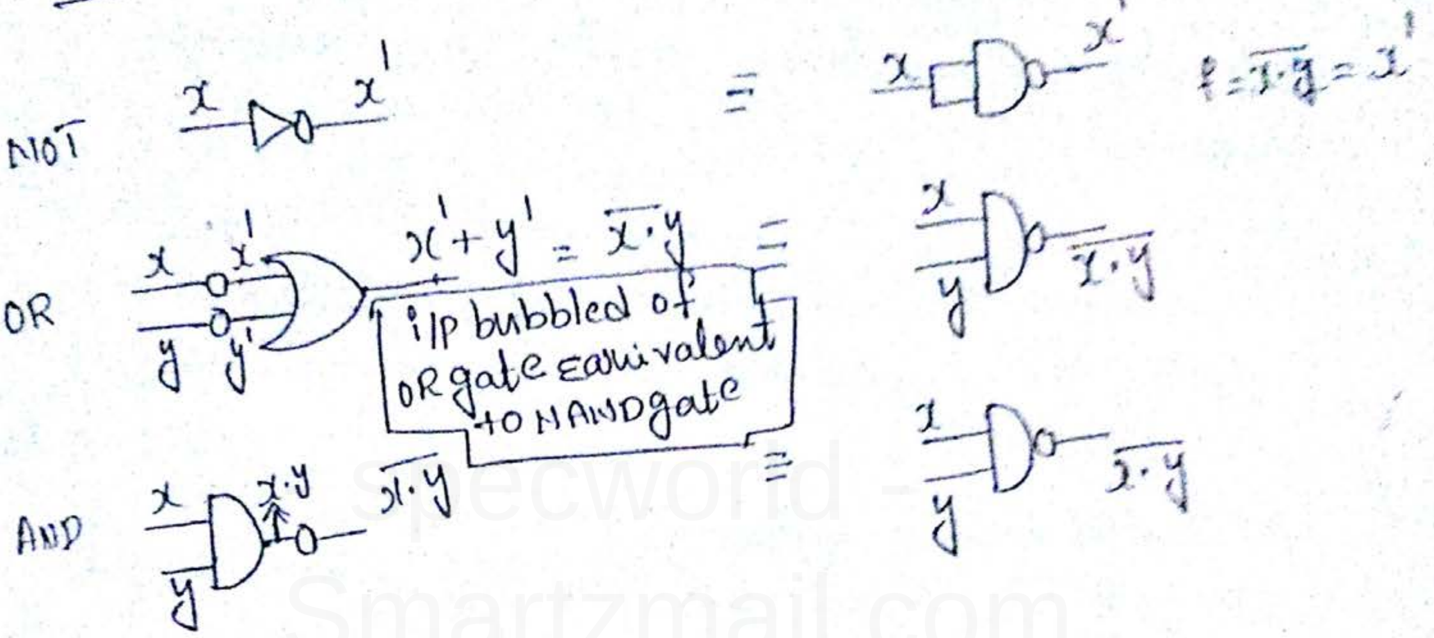
AND-OR-NOT are basic gates

NAND-NOR - are universal gates - Any function can be implemented by using these gates

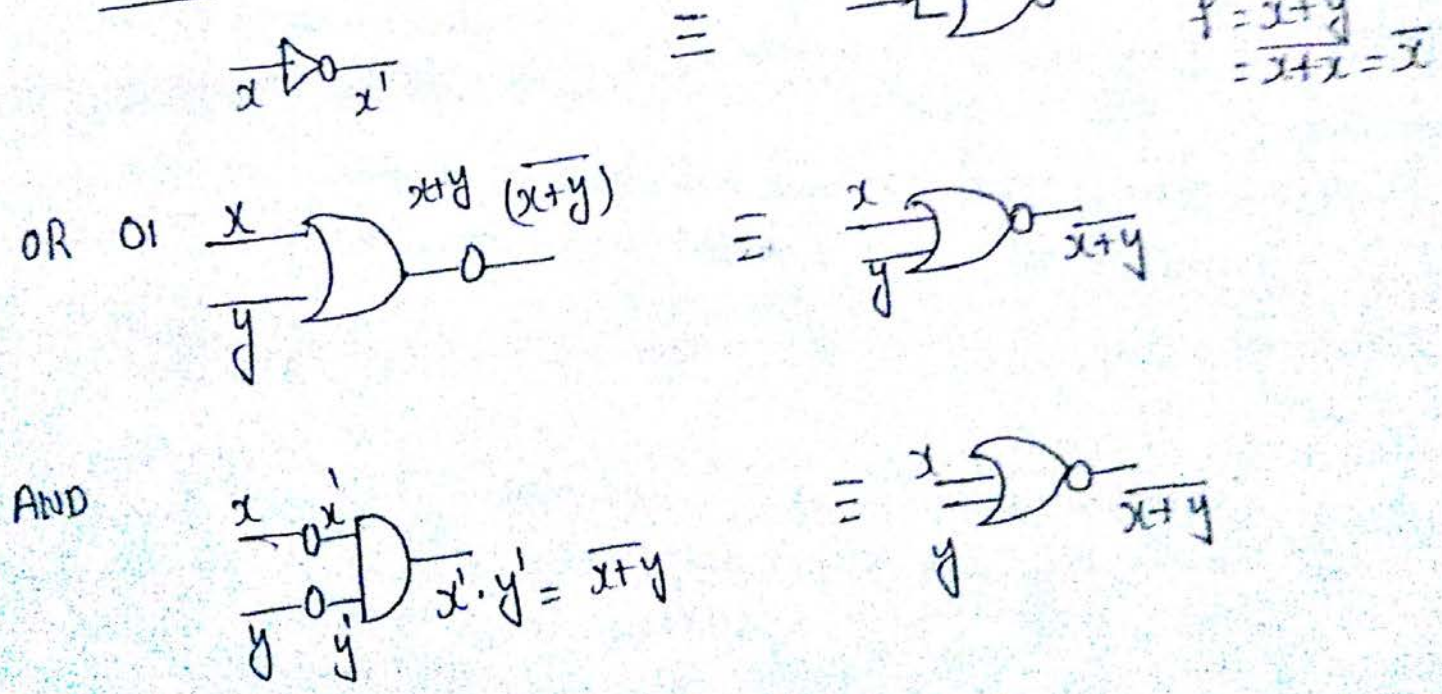
EXOR-EXNOR are derived gates

↳ These function can be implemented by using basic gates.

NAND



NOR



Note

1. i/p bubbled of OR gate is Equivalent to NAND gate
2. o/p bubbled of AND gate is Equivalent to NAND gate
3. o/p bubbled of OR gate is Equivalent to NOR gate
4. i/p bubbled of AND gate is Equivalent to NOR gate.

NAND – NOR Implementation or
converting AND-OR-Invert logic to NAND/NOR logic.

Steps:

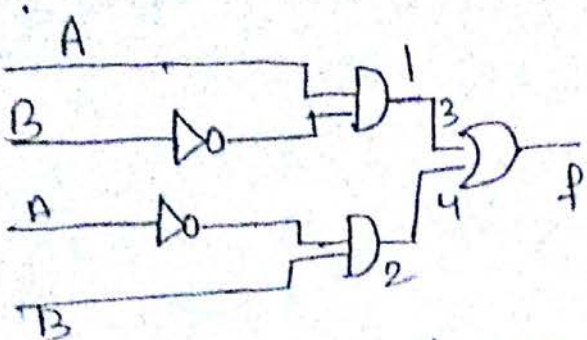
1. Draw the circuit in AND-OR-Invert logic
2. If NAND hardware is chosen, add a bubble at the output of each AND gate and at the inputs to all the OR gates.
3. If NOR hardware is choosen, add a bubble at the o/p of each OR gate and at the inputs to all the AND gates.
4. Add or subtract an inverter on each line that received a bubble in step 2 & 3 so that the polarity of signals on those lines remains unchanged from that of the original diagram.
5. Replace the i/p bubbled OR by NAND & Replace the i/p bubbled AND by NOR.
6. Eliminate double inversions or NOT or bubbles.

Note:

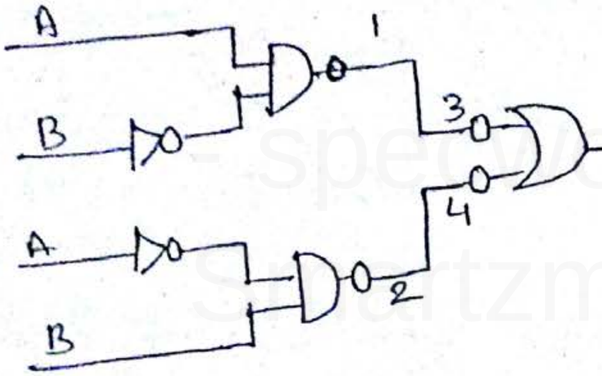
1. If the function is in SOP then it is Easy to implement by NAND logic
2. If the function is in POS form then the function is can be Easily implemented by NOR logic.

$f = A\bar{B} + \bar{A}B$. Implement the function by using NAND logic.

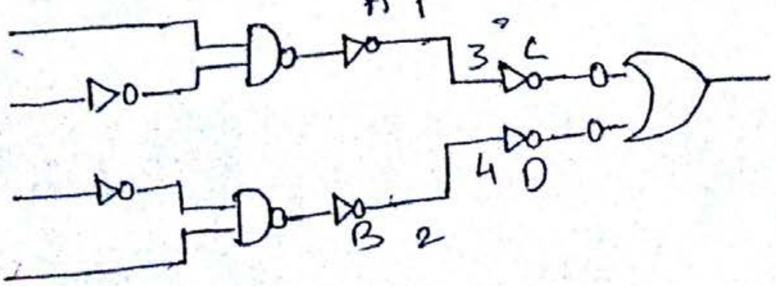
step-1 AOI logic



step-2 NAND logic choosen add bubbles at o/p of AND ∴ bubbles at i/p of OR gate.



step-3 Add or subtract an inversion on each line that received bubble. so that polarities remains unchanged

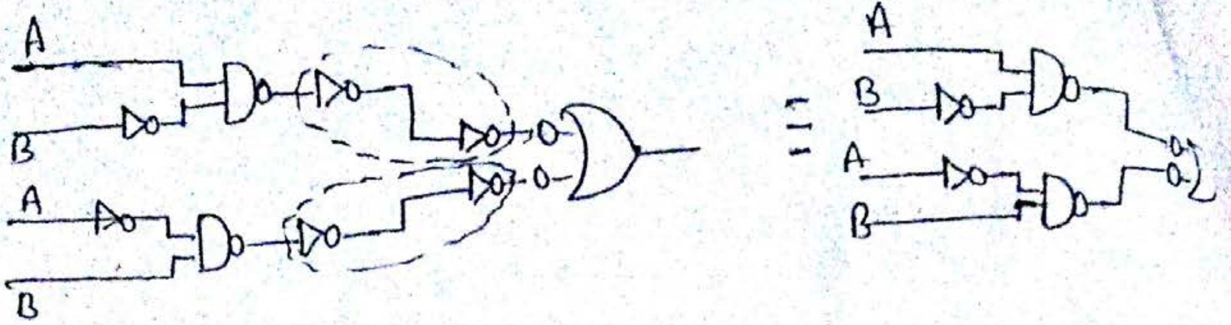


at 1, 2, added bubbles to eliminate A & B NOT gates are added.

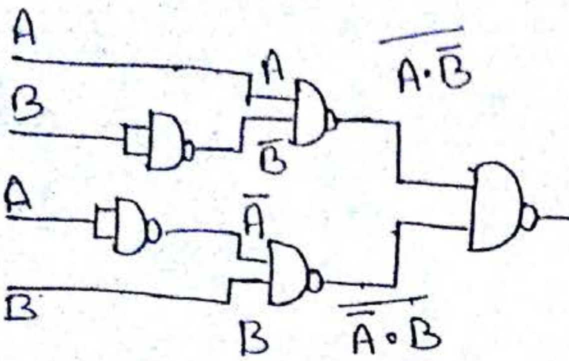
at 3,4 added bubbles to eliminate or get equal functionality C & D NOT gates are added.

step-4

eliminate double inversions 1–3 ⇒ 1-NOT & 3-NOT
eliminate double inversions 2 2-NOT & 4-NOT



step-5



$f = \overline{\left(\overline{A\cdot\overline{B}}\right)\cdot\left(\overline{\overline{A}\cdot B}\right)}$

Verification.

$\overline{(\overline{A}+B)\cdot(A+\overline{B})}$

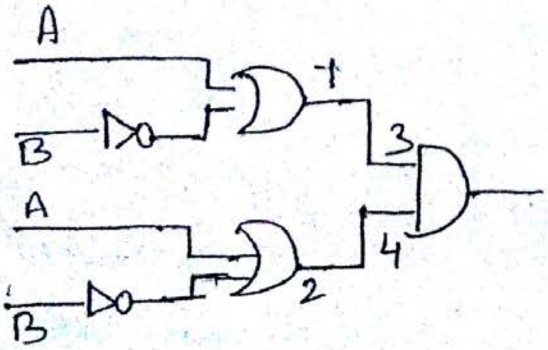
$\left(\overline{\overline{A}}\cdot\overline{B}\right)+\left(\overline{A}\cdot\overline{\overline{B}}\right)$

$A\cdot\overline{B}+\overline{A}\cdot B$.

$f=(A+\overline{B})\cdot(A+\overline{B})$ Implement the above function by NOR gates

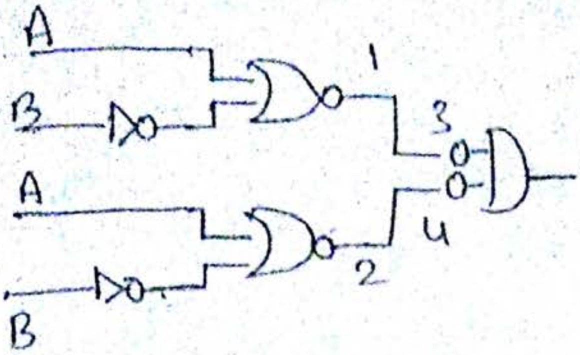
The above function is in POS form so it can be easily implemented by NOR logic.

step-1 AOI logic



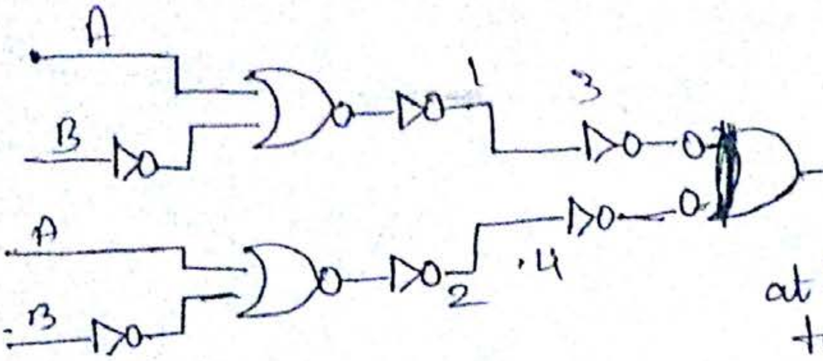
step-2 NOR logic is choosen add bubbles at o/p of OR gate and add bubbles at i/p of AND gate.

at 1, 2, 3, 4 positions add bubbles.



step-3

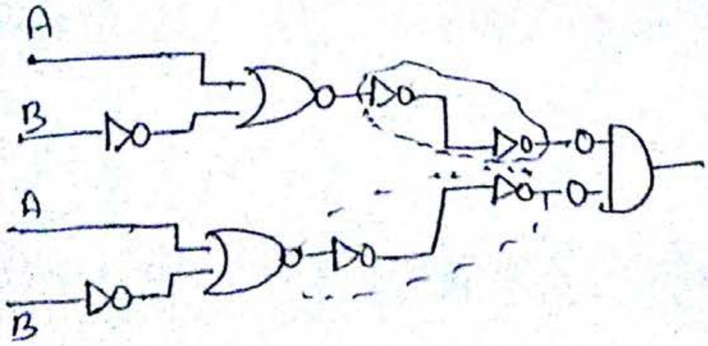
Add or subtract an inversion on each line that received bubble. so that polarities remains unchanged.



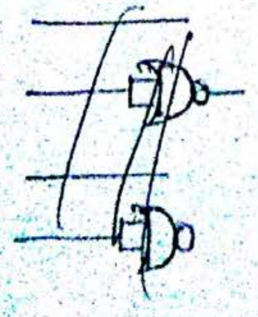
at 1, 2, added bubbles to get Equal functionality add NOT gates.
at 3,4 added bubbles to eliminate or get Equal functionality add NOT gate

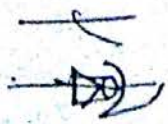
step- 4

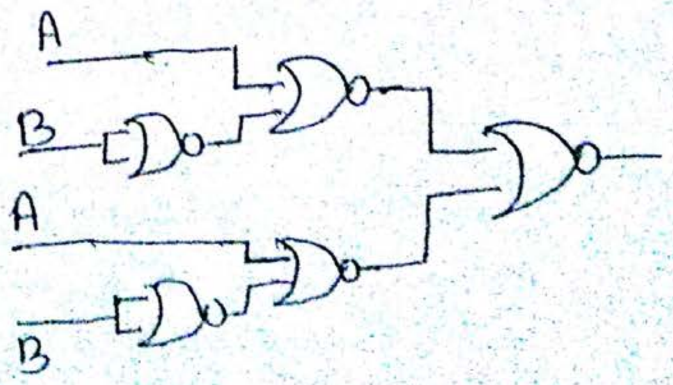
eliminate double inversions (1,3) 1-NOT & 3-Not
eliminate double inversions (2,4) 2-Not & 4-Not



step-5







Minimize the Boolean Equation
$F(A,B,C,D) = \Sigma m(7,9,10,11,12,13,14,15)$ using K
Also realize the simplified Expression using
a) AND-OR   b) OR-AND   c) NAND-NAND
d) NOR-NOR.

Solution

For AND-OR (or) NAND-NAND: Realization.

And-OR is represented for SOP: For SOP nand realization is Easy. So for And-OR (or) Nand-Nand logic can be implemented by 1 for SOP form.

$F = \Sigma m(7,9,10,11,12,13,14,15)$

Step-1 & 2, 3

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7: 1 (S4)	6
11	12: 1	13: 1	15: 1	14: 1 (S1)
10	8	9: 1	11: 1	10: 1 (S2)

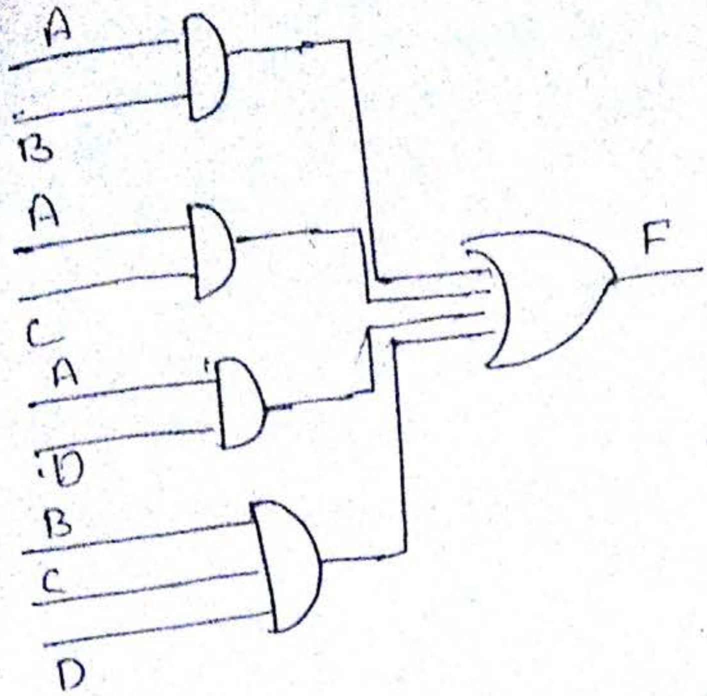
S3

Step-4

$S_1(12,13,14,15) \Rightarrow AB$
$S_2(10,11,14,15) \Rightarrow AC$
$S_3(9,11,13,15) \Rightarrow AD$
$S_4(7,15) \Rightarrow BCD$

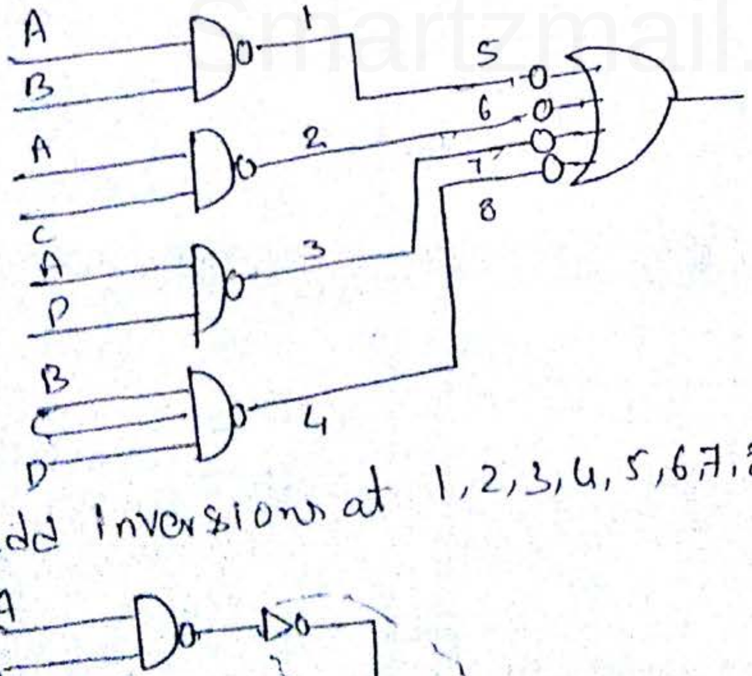
minimal SOP is $AB + AC + BCD$.

AND-OR logic

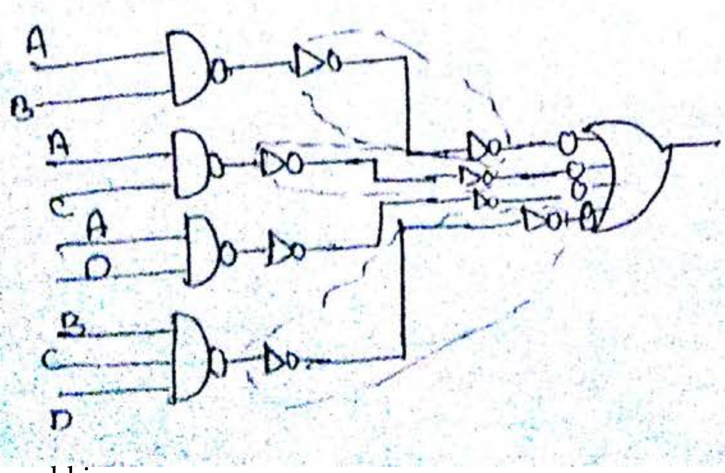


NAND-NAND

add bubbles of o/p of And gate & add bubbles i/p of OR gate.



add inversions at 1,2,3,4,5,6,7,8.



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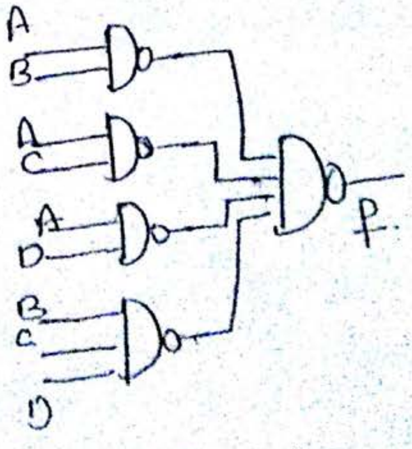


if it is not adjacent to any other 1's.
Two adjacent 1's form a Prime Implicant provided that are not within a group of four adjacent squares.
Four adjacent 1's form Prime Implicant if they are not with in a group of Eight adjacent squares. 4 so....
The essential prime implicant are found by looking at each square marked with a 1 & checking the number of Prime implicants that cover it. The prime implicant is essential if it is the only Prime implicants that covers minterms.

find Prime implicants & essential Prime implicants by kmap for following function.

$\Sigma(3,4,5,7,9,13,14,15)$

AB \ CD	00	01	11	10
00	0	1	1	2
01	4: 1	1	1	6
11	12	13: 1	15: 1	14: 1
10	8	9: 1	11	10

$S_1(5,7,13,15) = BD$
$S_2(3,7) = \bar{A}CD$
$S_3(14,15) = ABC$
$S_4(9,13) = A\bar{C}D$
$S_5(4,5) = \bar{A}B\bar{C}$

Prime Implicants $S_1 = BD$, $S_2 = \bar{A}CD$ $S_3 = ABC$
$S_4 = A\bar{C}D$. $S_5 = \bar{A}B\bar{C}$

essential Prime implicants
$S_2 = \bar{A}CD$ $S_3 = ABC$ $S_4 = A\bar{C}D$ $S_5 = \bar{A}B\bar{C}$

ner two level implementation:

Represent the following function all two level possible implementations.

$$F(A,B,C,D) = \Sigma m(0,1,2,3,4,8,9,12)$$

solution: By using SOP format the functions we get (in) 4 implemented by Basic AND-OR logic

For the given function find minimal form of SOP.

AB \ CD	00	01	11	10
00	1	1	1	1
01	1			
11	1			
10	1	1		

$S_1(0,1,2,3) = \bar{A}\bar{B}$

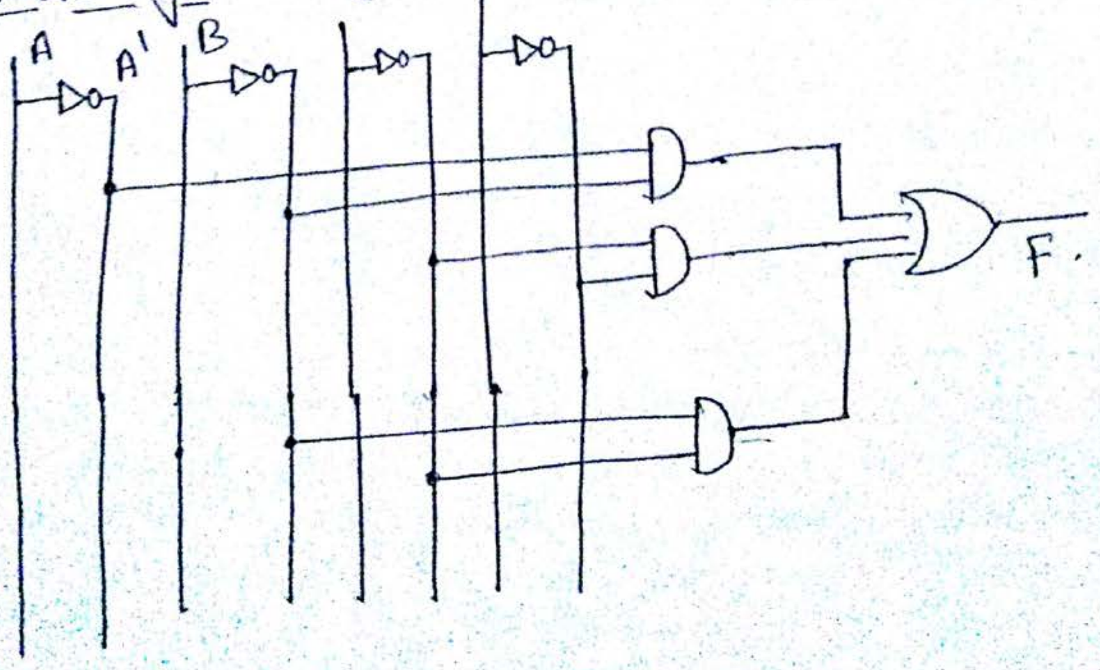
$S_2(0,4,8,12) = \bar{C}\bar{D}$

$S_3(0,1,8,9) = \bar{B}\bar{C}$

minimal SOP $= \bar{A}\bar{B} + \bar{C}\bar{D} + \bar{B}\bar{C} = F$ —(1)

implement above equation by only AND-OR logic

AND-OR logic



Demorgans law
operation $\begin{cases} AND \rightarrow OR \\ OR - AND \end{cases}$
variables $\begin{cases} \text{Primed} - \text{unPrimed} \\ \text{unPrimed} - \text{Primed} \end{cases}$

$F = \bar{A}\bar{B} + \bar{C}\bar{D} + \bar{B}\bar{C}$

$\bar{\bar{F}} = F \Rightarrow$ By Demorgans law.

$F = \bar{A}\bar{B} + \bar{C}\bar{D} + \bar{B}\bar{C}$

Take double complementation to get same function.

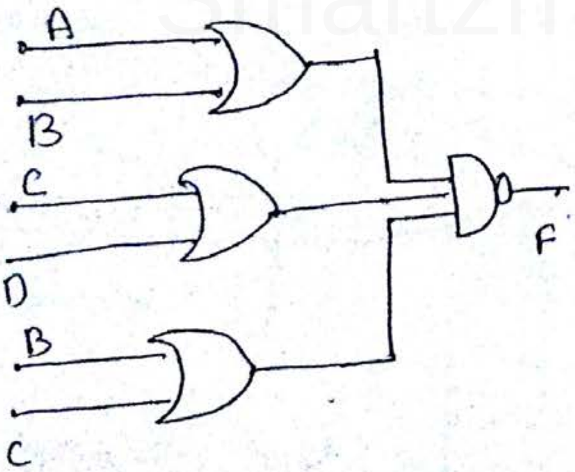
$$\bar{\bar{F}} = \overline{\overline{\bar{A}\bar{B} + \bar{C}\bar{D} + \bar{B}\bar{C}}}$$

$$= \overline{(\overline{\bar{A}\bar{B}})\cdot(\overline{\bar{C}\bar{D}})\cdot(\overline{\bar{B}\bar{C}})} = \overline{(\bar{\bar{A}}+\bar{\bar{B}})\cdot(\bar{\bar{C}}+\bar{\bar{D}})(\bar{\bar{B}}+\bar{\bar{C}})}$$

$$\bar{\bar{F}} = \overline{(A+B)\cdot(C+D)\cdot(B+C)} = F \quad - ②$$

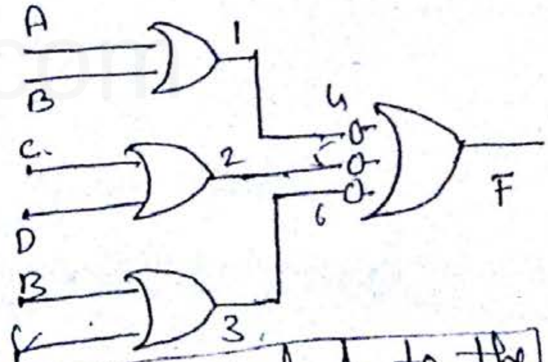
Eq. ② in b/n variables have OR operator & b/n terms have AND followed a complement. So above function can be implemented by OR–NAND.

OR–NAND



≡

OR-NOT-OR



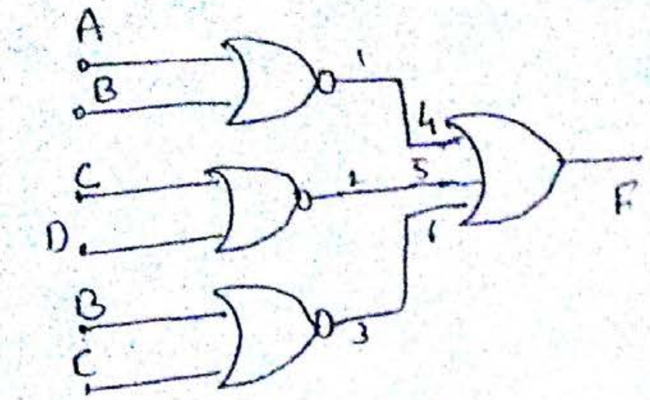
∵ NAND Equivalent to the i/p bubbled of OR gate.

The bubble of 4th position transfer to 1 Position & bubble of 5 Position transfer to 2 Position, bubble of 6 Position transfer to 3 Position.

OR-NOT-OR

NOR-OR

NOR – OR



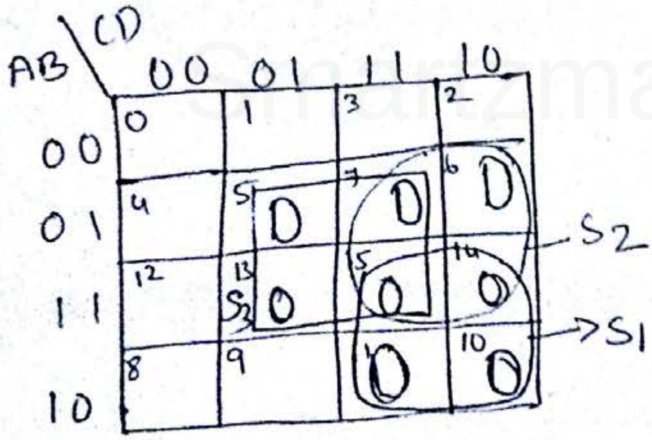
~~Next~~ POS implementation designing done for POS.

By using POS format the function can be designed by Basic OR-AND logic.

Σ (given minterms) $= \Sigma(0,1,2,3,4,8,9,12)$

π (remaining terms) $= \pi(5,6,7,10,11,13,14,15)$

$F = \pi(5,6,7,10,11,13,14,15)$



$S_1(10,11,14,15) = \bar{A}+\bar{C}$

$S_2(6,7,14,15) = \bar{B}+\bar{C}$

$S_3(5,7,13,15) = \bar{B}+\bar{D}$

minimal POS in $S_1 \cdot S_2 \cdot S_3$

$(\bar{A}+\bar{C})(\bar{B}+\bar{C})(\bar{B}+\bar{D})$

OR-AND logic