

Rajiv Gandhi University of Knowledge Technology-Nuzvid

DEPARTMENT OF CIVIL ENGINEERING

SOIL MECHANICS

BY

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CONSOLIDATION (Syllabus)

- ☒ Significance of consolidation
- ☒ stress-strain- time relationship for sand and clay
- ☒ e-p and e-log p curves
- ☒ Stress story- Over consolidated and normally consolidated clays
- ☒ Mechanism of consolidation - Spring Analogy
- ☒ Terzaghi's theory of one-dimensional Consolidation
- ☒ Time rate of consolidation and degree of consolidation
- ☒ Determination of coefficient of consolidation (C_v)
- ☒ secondary consolidation



INTRODUCTION

- ⊠ When a structure is placed on a foundation consisting of soil, the loads from the structure cause the soil to be stressed.
 - ⊠ The two most important requirements for the stability and safety of the structure are:
 - ⊠ (1) The **deformation**, especially the vertical deformation, called ‘settlement’ of the soil, should not be excessive and must be within tolerable or permissible limits; and,
 - ⊠ (2) The shear strength of the foundation soil should be adequate to withstand the stresses induced.
 - ⊠ The first of these requirements needs consideration and study of the aspect of the “**Compressibility and Consolidation of soils**”
 - ⊠ The property of a soil by virtue of which volume decrease occurs under applied pressure is termed its ‘**Compressibility**’.
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COMPRESSIBILITY OF SOILS

- ⊠ A soil is a particulate material, consisting of solid grains and void spaces enclosed by the grains. The voids may be filled with air or other gas, with water or other liquid, or with a combination of these.
- ⊠ The volume decrease of a soil under stress might be conceivably attributed to:
 - ⊠ 1. Compression of the solid grains;
 - ⊠ 2. Compression of pore water or pore air;
 - ⊠ 3. Expulsion of pore water or pore air from the voids, thus decreasing the void ratio or porosity.
- ⊠ Under the loads usually encountered in geotechnical engineering practice, the solid grains as well as pore water may be considered to be incompressible.



COMPRESSIBILITY OF SOILS

- ⊠ Thus, compression of pore air and expulsion of pore water are the primary sources of volume decrease of a soil mass subjected to stresses.
- ⊠ A partially saturated soil may experience appreciable volume decrease through the compression of pore air before any expulsion of pore water takes place; the situation is thus more complex for such a soil.
- ⊠ However, it is reasonable to assume that volume decrease of a saturated soil mass is, for all practical purposes, only due to expulsion of pore water by the application of load.
- ⊠ The process of gradual compression due to the expulsion of pore water under steady pressure is referred to as 'Consolidation',



COMPRESSIBILITY OF SOILS

- ⊠ This is a time dependent phenomenon, especially in clays. Thus, the volume change behaviour has two distinct aspects: first, the magnitude of volume change leading to a certain total compression or settlement, and secondly, the time required for the volume change to occur under a particular stress.
- ⊠ The process of mechanical compression resulting in reduction or compression of pore air and consequent densification of soil is referred to as 'Compaction'
- ⊠ In sands, consolidation may be generally considered to keep pace with construction; while, in clays, the process of consolidation proceeds long after the construction has been completed and thus needs greater attention.



One-dimensional Compression and Consolidation

- ✘ The general case is complex, but an analysis of the case in which the compression takes place in one direction only is relatively simple.
- ✘ The compression at shallow elevations underneath a loaded structure is definitely three-dimensional, but the compression in deep strata is essentially one-dimensional.
- ✘ Besides, there are other practical situations in which the compressions approach a truly one-dimensional case. In view of this, one dimensional analysis of compression and consolidation has significant practical applications.
- ✘ Escape of pore water must occur during the compression or one-dimensional consolidation of a saturated soil; this escape takes place according to Darcy's law.
- ✘ The time required for the compression or consolidation is dependent upon the coefficient of permeability of the soil and may be quite long if the permeability is low.

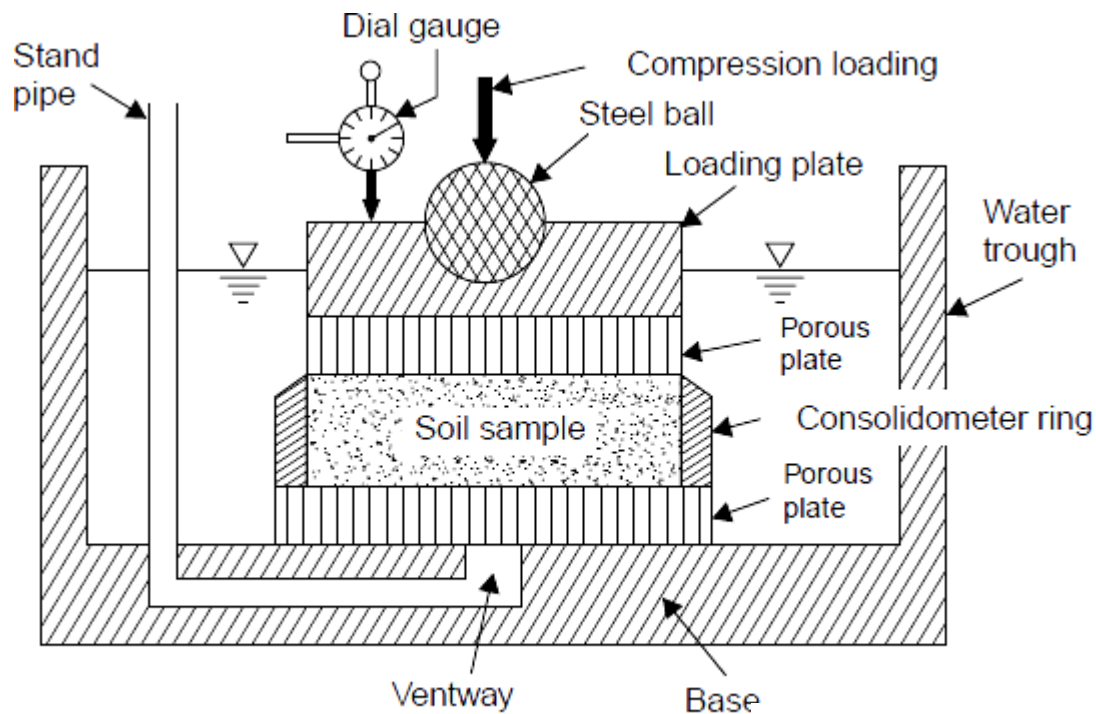
One-dimensional Compression and Consolidation

- ⊠ The applied pressure which is initially borne by the pore water goes on getting transferred to the soil grains during the transient stage and gets fully transferred to the grains as effective stress, reducing the excess pore water pressure to zero at the end of the compression under the applied stress.
- ⊠ Thus, 'Consolidation', may be defined as the gradual and time-dependent process involving expulsion of pore water from a saturated soil mass, compression and stress transfer. This definition is valid for the one-dimensional as well as the general three-dimensional case.
- ⊠ Compressibility is a function of the effective stress.

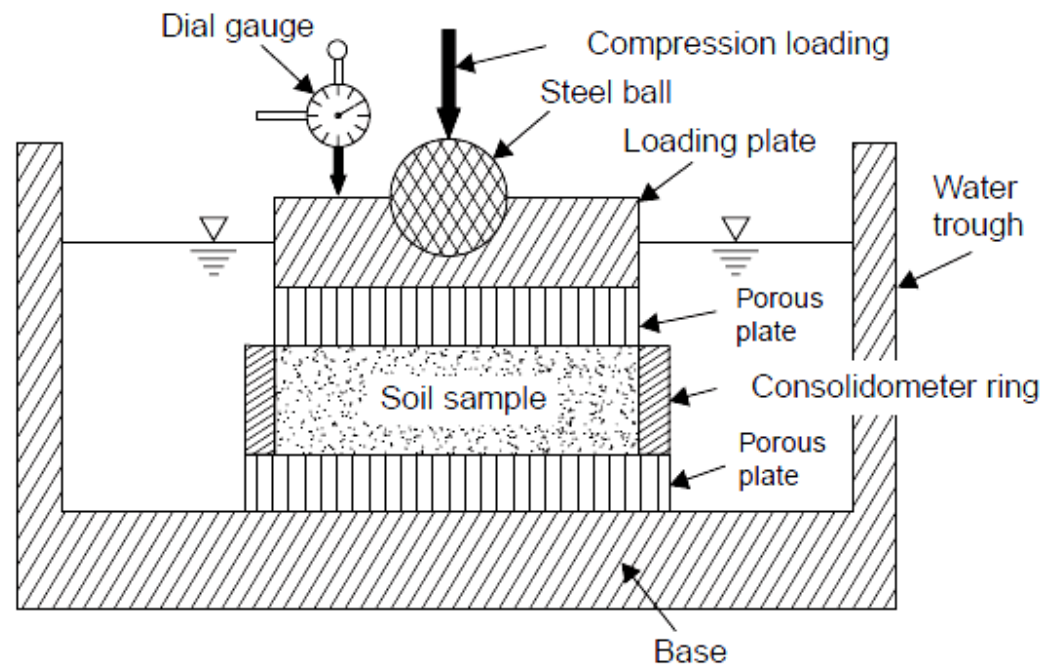


Compressibility and Consolidation Test— Oedometer

- ❑ The apparatus developed by Terzaghi for the determination of compressibility characteristics including the time-rate of compression is called the Oedometer. It was later improved by A. Casagrande and G. Gilboy and referred to as the Consolidometer.
- ❑ There are two types: The **fixed ring type** and the **floating ring type**. In the fixed ring type, the top porous plate alone is permitted to move downwards for compressing the specimen.
- ❑ But, in the floating ring type, both the top and bottom porous plates are free to move to compress the soil sample.
- ❑ Direct measurement of the permeability of the sample at any stage of the test is possible only with the fixed ring type.
- ❑ However, the effect of side friction on the soil sample is smaller in the floating type, while lateral confinement of the sample is available in both to simulate a soil mass in-situ.



(a) Fixed ring type



(b) Floating ring type

Compressibility and Consolidation Test— Oedometer

- ⊠ The consolidation test consists in placing a representative undisturbed sample of the soil in a consolidometer ring, subjecting the sample to normal stress in predetermined stress increments through a loading machine and during each stress increment, observing the reduction in the height of the sample at different elapsed times after the application of the load.
- ⊠ The test is standardised with regard to the pattern of increasing the stress and the duration of time for each stress increment.
- ⊠ Thus the total compression and the time-rate of compression for each stress increment may be determined.
- ⊠ The data permits the study of the compressibility and consolidation characteristics of the soil.



Compressibility and Consolidation Test— Oedometer

- ⊠ The time-rate of volume change differs significantly for cohesionless soils and cohesive soils.
 - ⊠ Cohesionless soils experience compression relatively quickly, often instantaneously, after the load is imposed.
 - ⊠ But clay soils require a significant period before full compression occurs under an applied loading. Relating the time-rate of compression with compression is consolidation.
 - ⊠ Laboratory compression tests are seldom performed on cohesionless soils for two reasons: first, undisturbed soil samples cannot be obtained and secondly, the settlement is rapid, eliminating post-construction problems of settlement. If volume change or settlement characteristics are needed, these are obtained indirectly from in-situ density and density index and other correlations.
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The following procedure is recommended by the ISI for the consolidation test [IS:2720 (Part XV) — 1986]

- ⊠ The specimen shall be 60 mm in diameter and 20 mm thick. The specimen shall be prepared either from undisturbed samples or from compacted representative samples.
 - ⊠ The specimen shall be trimmed carefully so that the disturbance is minimum. The orientation of the sample in the consolidometer ring must correspond to the orientation likely to exist in the field.
 - ⊠ Filter papers are placed above and below the sample and porous stones are placed above and below these. The loading block shall be positioned centrally on the top porous stone.
 - ⊠ This assembly shall be mounted on the loading frame such that the load is applied axially.
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- ⊠ Weights of known magnitude may be hung on the lever system. The holder with the dial gauge to record the progressive vertical compression of the specimen under load, shall then be screwed in place.
- ⊠ The dial gauge shall be adjusted allowing a sufficient margin for the swelling of the soil, if any.
- ⊠ The system shall be connected to a water reservoir with the water level being at about the same level as the soil specimen and the water allowed to flow through and saturate the sample.
- ⊠ An initial setting load of 5 kN/m², which may be as low as 2.5 kN/m² for very soft soils, shall be applied until there is no change in the dial gauge reading for two consecutive hours or for a maximum of 24 hours.
- ⊠ A normal load to give the desired pressure intensity shall be applied to the soil, a stopwatch being started simultaneously with loading.
- ⊠ The dial gauge reading shall be recorded after various intervals of time—0.25, 1, 2.25, 4, 6.25, 9, 12.25, 16, 20.25, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, 500, 600, and 1440 minutes.



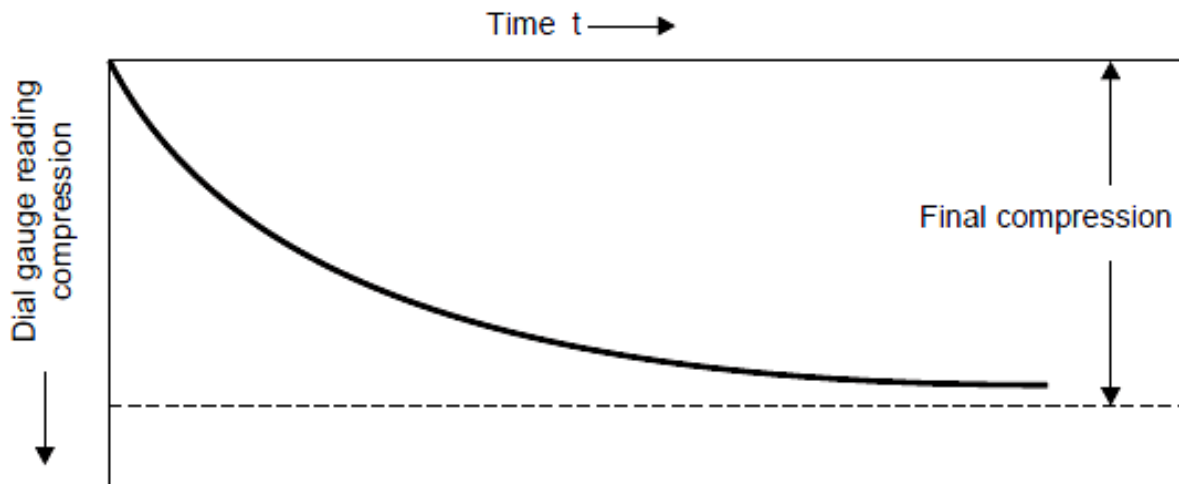
- ⊠ The dial gauge readings are noted until 90% consolidation is reached. Thereafter, occasional observations shall be continued. For soils which have slow primary consolidation, loads should act for at least 24 hours and in extreme cases or where secondary consolidation must be evaluated, much longer.
 - ⊠ At the end of the period specified, the load intensity on the soil specimen is doubled. Dial and time readings shall be taken as earlier.
 - ⊠ Then successive load increments shall be applied and the observations repeated for each load till the specimen has been loaded to the desired intensity.
 - ⊠ The usual sequence of loading is of 10, 20, 40, 80, 160, 320 and 640 kN/m². Smaller increments may be desirable for very soft soil samples. Alternatively, 6, 12, 25, 50, 100 and 200 per cent of the maximum field loading may be used.
 - ⊠ After the last load has been on for the required period, the load should be decreased to 1/4 the value of the last load and allowed to stand for 24 hours.
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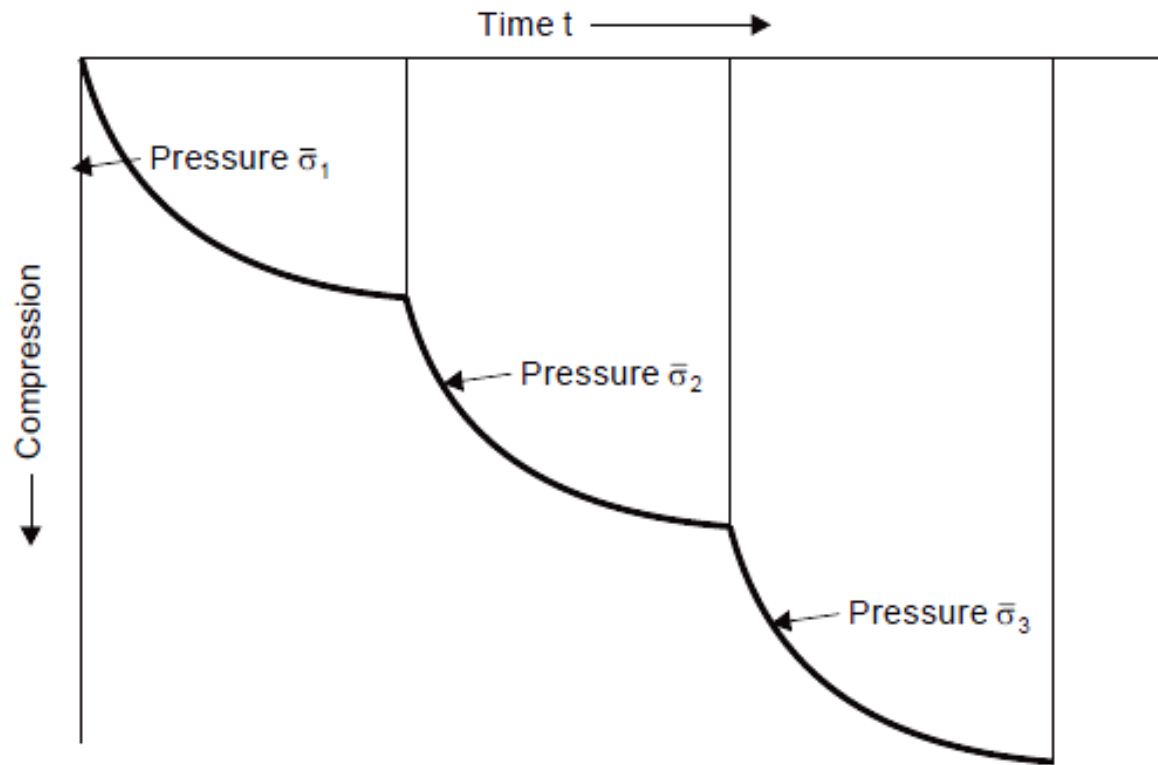
- ⊠ No time-dial readings are normally necessary during the rebound, unless information on swelling is required. The load shall be further reduced in steps of one-fourth the previous intensity till an intensity of 10 kN/ m² is reached.
- ⊠ After the final reading has been taken for 10 kN/m² the load shall be reduced to the initial setting load, kept for 24 hours and the final reading of the dial gauge noted.
- ⊠ When the observations are completed, the assembly shall be quickly dismantled and the ring with the consolidated soil specimen weighed.
- ⊠ The soil shall then be dried to constant weight in an oven maintained at 105° to 110° C and the dry weight recorded.
- ⊠ The following data should also be obtained:
 - ⊠ 1. Moisture content and weight of the soil sample before the commencement of the test.
 - ⊠ 2. Moisture content and weight of the sample after completion of the test.
 - ⊠ 3. The specific gravity of the solids.
 - ⊠ 4. The temperature of the room where the test is conducted.

Presentation and Analysis of Compression Test Data

- ⊠ The consolidation is rapid at first, but the rate gradually decreases. After a time, the dial reading becomes practically steady, and the soil sample may be assumed to have reached a condition of equilibrium.
- ⊠ For the common size of the soil sample, this condition is generally attained in about twenty-four hours, although, theoretically speaking, the time required for complete consolidation is infinite.
- ⊠ This variation of compression or the dial gauge reading with time may be plotted for each one of the stress increments.



- ⊠ The time-compression curves for consecutive increments of stress appear somewhat as shown in Fig



PRESSURE-VOID RATIO CURVES

- ⊠ Since compression is due to decrease in void spaces of the soil, it is commonly indicated as a change in the void ratio.
- ⊠ Therefore, the final stress-strain relationships, are presented in the form of a graph between the pressure and void ratio, with a point on the curve for the final condition of each pressure increment.
- ⊠ Accurate determinations of void ratio are essential and may be computed from the following data:
 1. The cross-sectional area of the sample A , which is the same as that of the brass ring.
 2. The specific gravity, G_s , of the solids.
 3. The dry weight, W_s , of the soil sample.
 4. The sample thickness, h , at any stage of the test.
- ⊠ **A) Height of solids method:**
- ⊠ Let V_s = volume of the solids in the sample where
- ⊠ Where γ_w - unit weight of water

$$V_s = \frac{W}{G_s \gamma_w}$$



⊠ We can also write $V_s = h_s A$ or $h_s = \frac{V_s}{A}$

⊠ where, h_s = thickness of solid matter.

⊠ If e is the void ratio of the sample, then $e = \frac{Ah - Ah_s}{Ah_s} = \frac{h - h_s}{h_s}$

⊠ **B) Change of Void-Ratio Method**

⊠ In one-dimensional compression the change in height Δh per unit of original height h equals the change in volume ΔV per unit of original volume V . $\frac{\Delta h}{h} = \frac{\Delta V}{V}$

⊠ V may now be expressed in terms of void ratio e .

$$V_v = eV_s, \quad V = V_s(1 + e), \quad V'_v = e'V_s, \quad V' = V_s(1 + e')$$

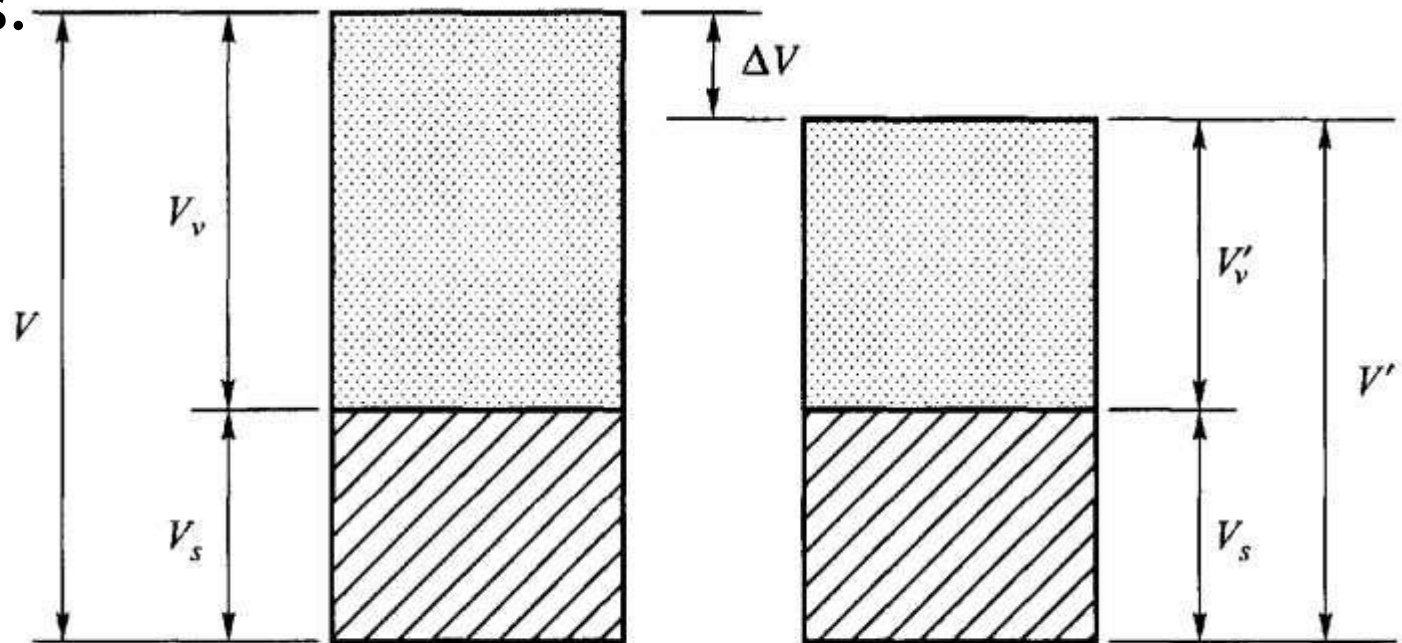
$$\frac{\Delta V}{V} = \frac{V - V'}{V} = \frac{V_s(1 + e) - V_s(1 + e')}{V} = \frac{e - e'}{1 + e} = \frac{\Delta e}{1 + e}$$

⊠ Therefore, $\frac{\Delta h}{h} = \frac{\Delta e}{1 + e}$

$$\Delta e = \frac{1 + e}{h} \Delta h$$

$$\Delta e = \frac{1+e}{h} \Delta h$$

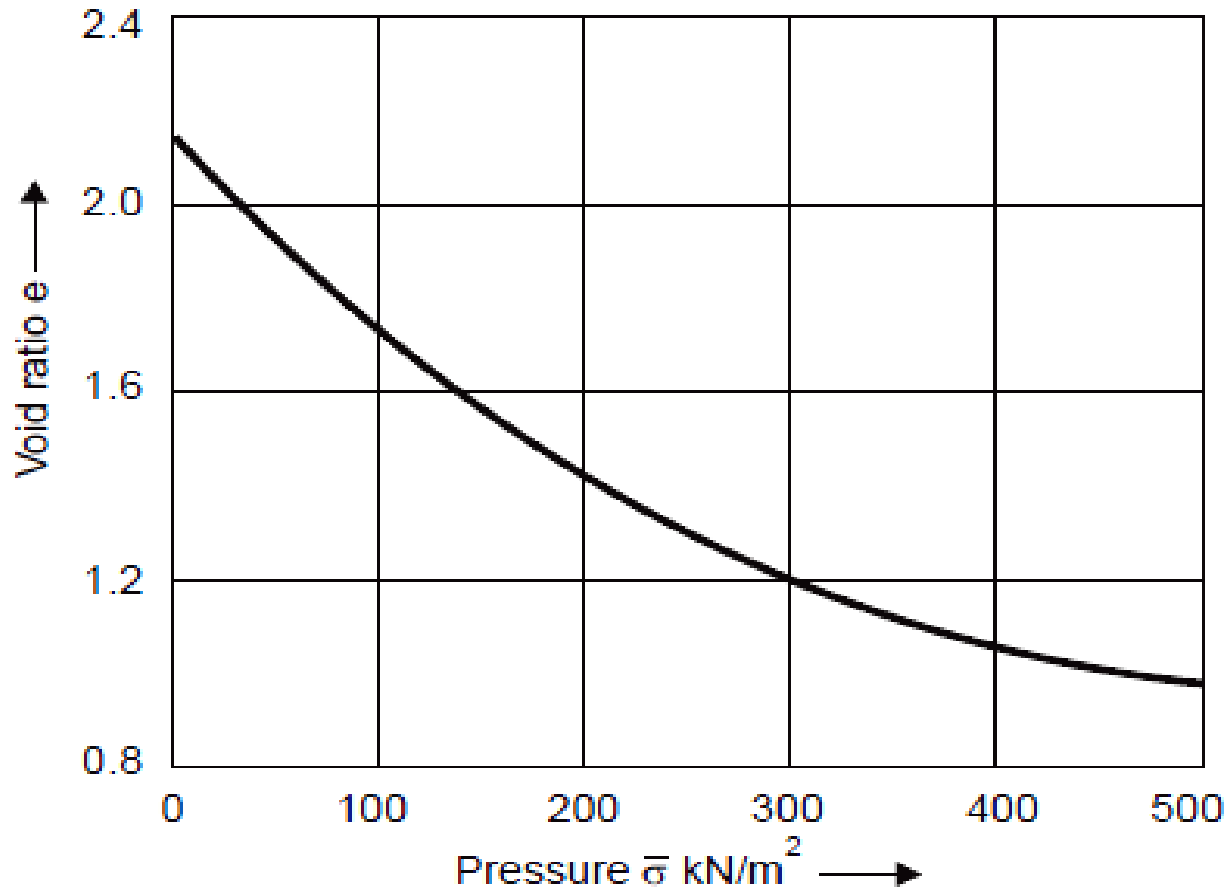
- ⌘ wherein, Δe = change in void ratio under a load, h = initial height of sample, e = initial void ratio of sample, e' = void ratio after compression under a load, Δh = compression of sample under the load which may be obtained from dial gauge readings.



(a) Initial condition

(b) Compressed condition

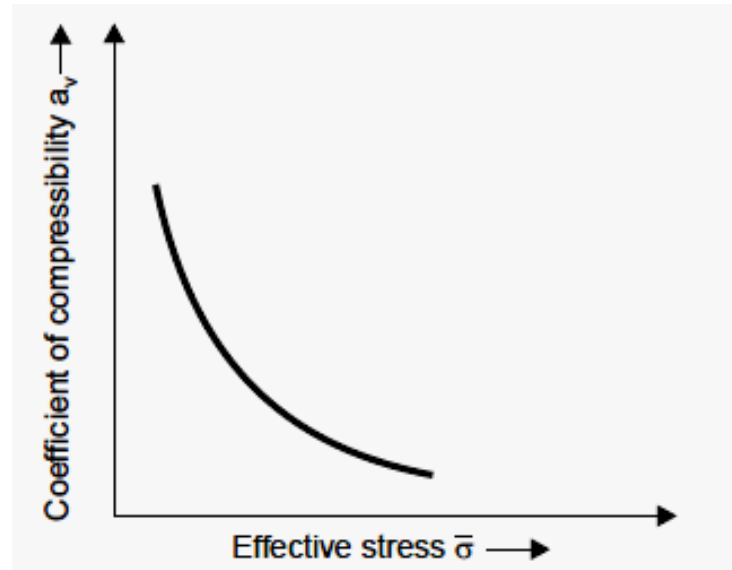
$$\Delta e = [(1 + e)/H] \cdot \Delta H$$



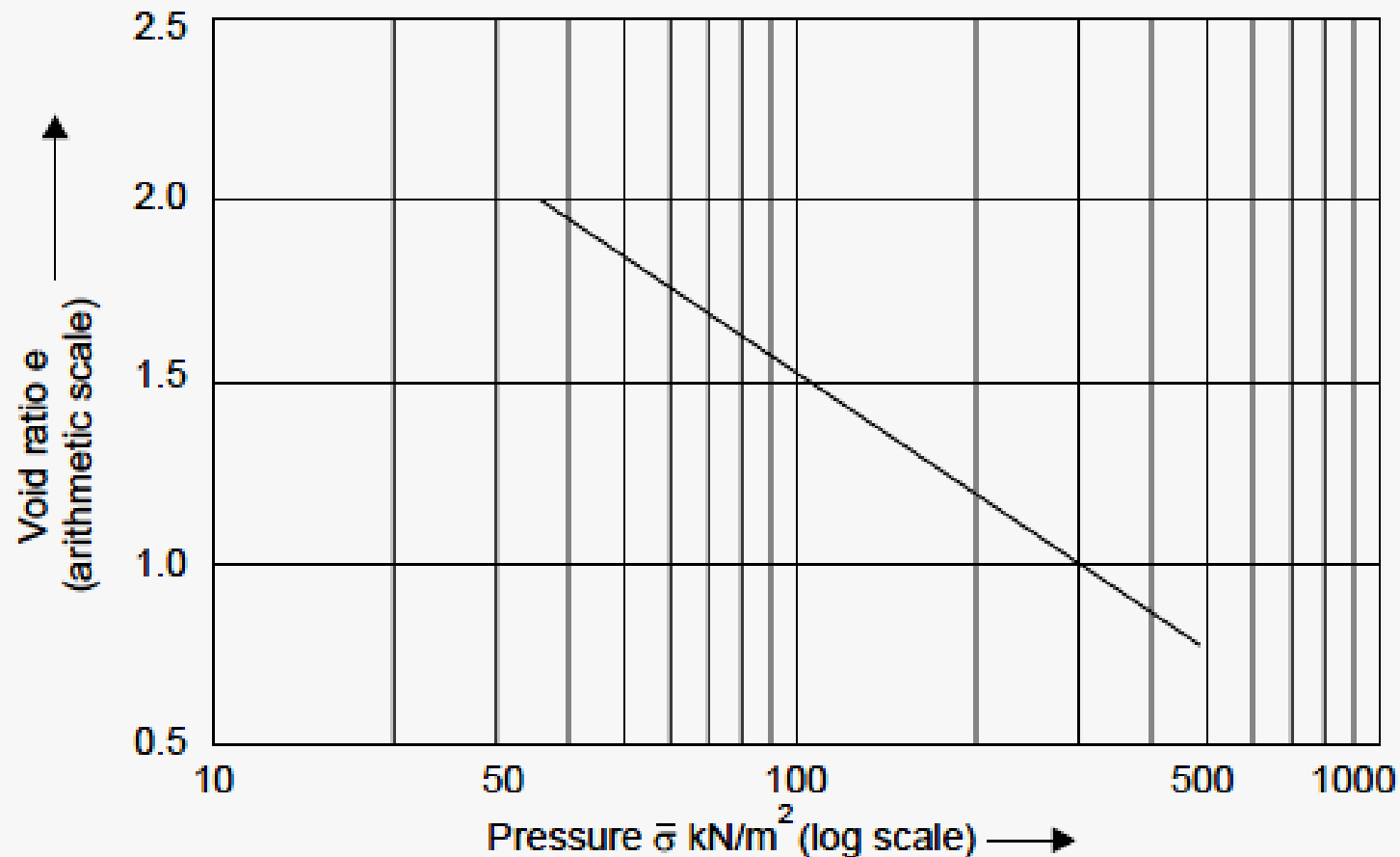
Working backwards from the known value of the final void ratio, the void ratio corresponding to each pressure may be computed. A typical pressure-void ratio curve is shown in Fig



- ⊠ The slope of this curve at any point is defined as the coefficient of compressibility, a_v .
- ⊠ Mathematically speaking,
$$a_v = - \frac{\Delta e}{\Delta \bar{\sigma}}$$
- ⊠ (Alternatively, the curve may be approximated to a straight line between this point and another later point of pressure and its slope may be taken as a_v).
- ⊠ It is difficult to use a_v in a mathematical analysis, because of the constantly changing slope of the curve. This leads us to the fact that compressibility is a function of the effective stress as shown in Fig

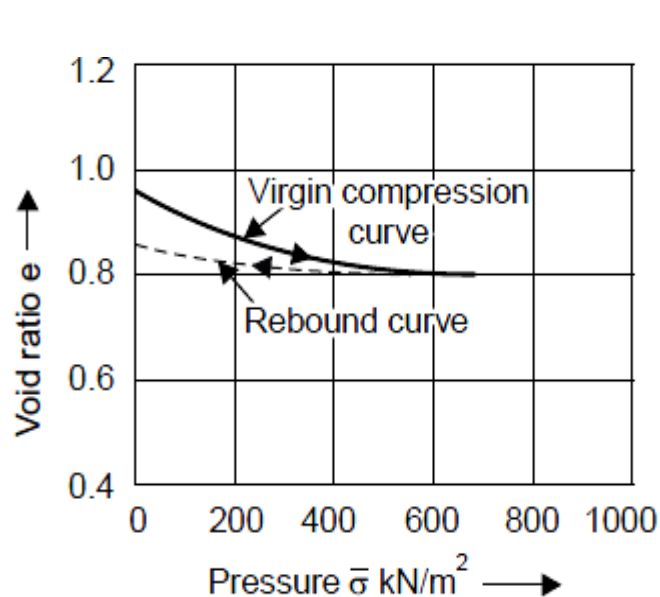


- ⌘ If the void-ratio is plotted versus the logarithm of the pressure, the data will plot approximately as a straight line (or as a series of straight lines, as described later), as shown in Fig. In this form the test data are more adaptable to analytical use.

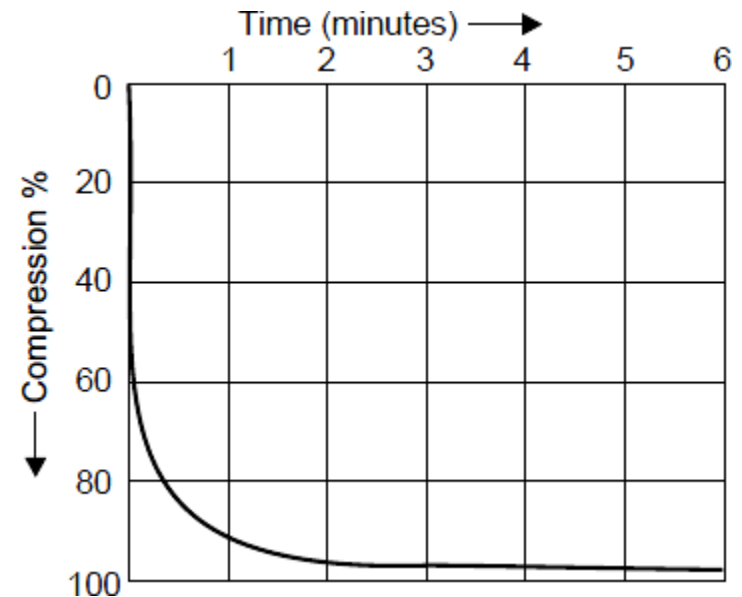


Compressibility of Sands

- ⊠ The pressure-void ratio relationship for a typical sand under one-dimensional compression and typical time-compression curve for an increment of stress is shown in Fig.



Pressure void-ratio relationship for a typical sand



Typical time-compression curve for a sand

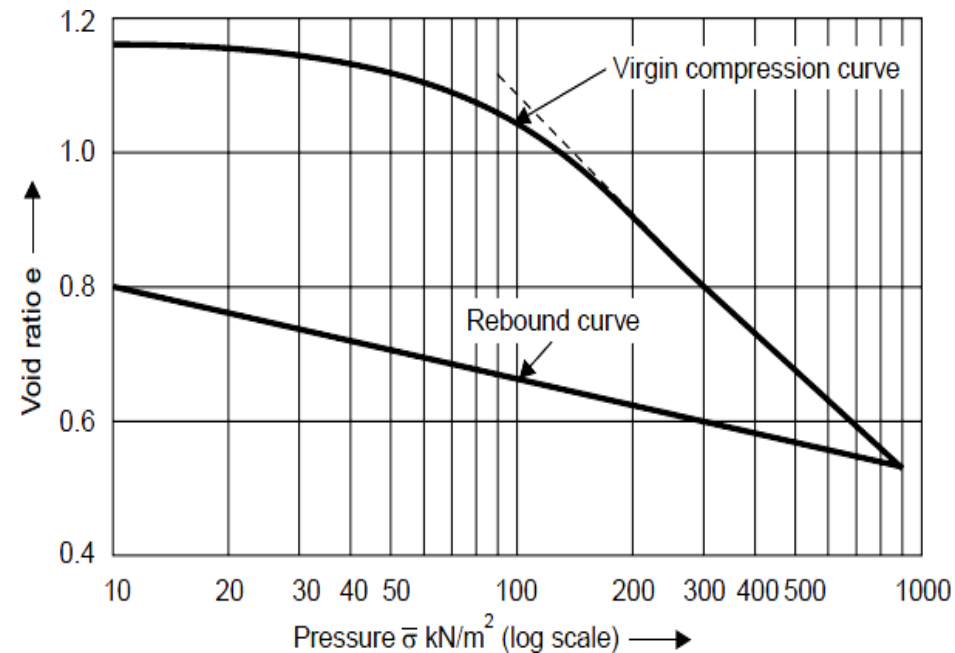
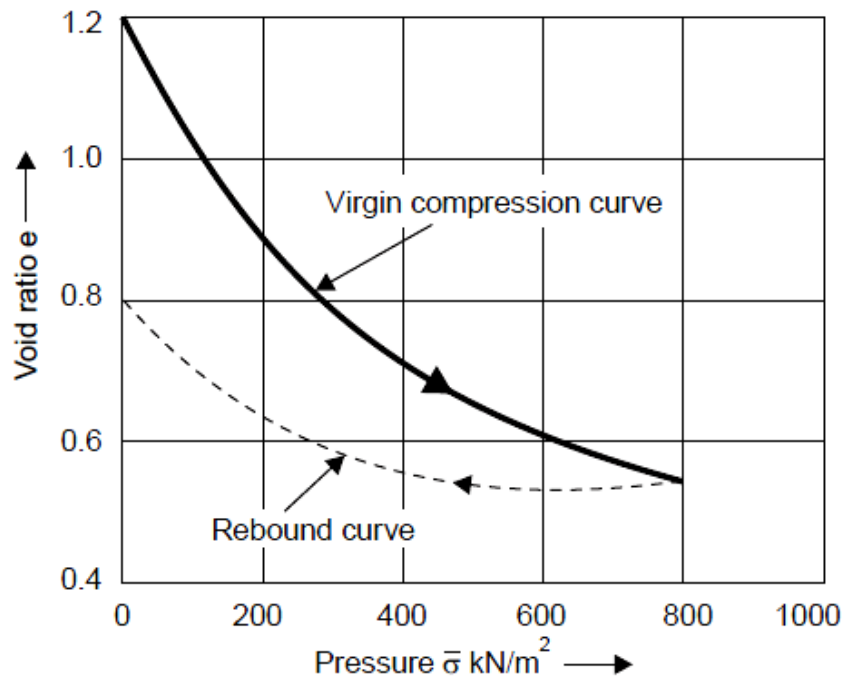
Compressibility of Sands

- ⊠ It is observed that although there is some rebound on release of pressure, it is never cent per cent; as such, the pressure-void ratio curves for initial loading and unloading, which are respectively referred to as the 'Virgin Compression Curve' and 'Rebound Curve', will be somewhat different from each other.
- ⊠ It is also observed that not much of reduction in void ratio occurs in the sands, indicating that their compressibility is relatively very low.
- ⊠ It is also observed from the time-compression curve that the major part of the compression takes place almost instantaneously.
- ⊠ In about one minute about 95% of the compression has occurred in this particular case.
- ⊠ The time-lag during compression is largely of a frictional nature in the case of sands.



Compressibility and Consolidation of Clays

- ⊠ A typical pressure versus void ratio curve for a clay to natural pressure scale and to the logarithmic pressure scale are shown in



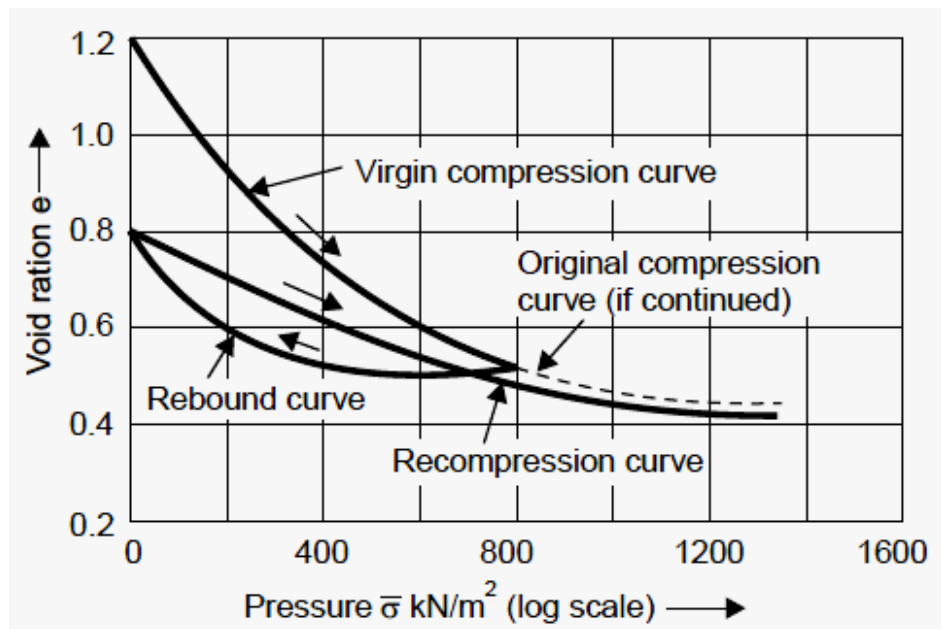
- ⊠ It is clear that a clay shows greater compressibility than a sand for the same pressure range. It is also clear that the rebound on release of pressure during unloading is much less.
- ▮ The second mode of semi-log plotting yields straight lines in certain zones of loading and unloading.

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- ⊠ In the semi-logarithmic plot, it can be seen that the virgin compression curve in this case approximates a straight line from about 200 kN/m² pressure.
 - ⊠ The numerical value of the slope of this straight line, C_c , which is obviously negative in view of the decreasing void ratio for increasing pressure, is called the 'Compression index':

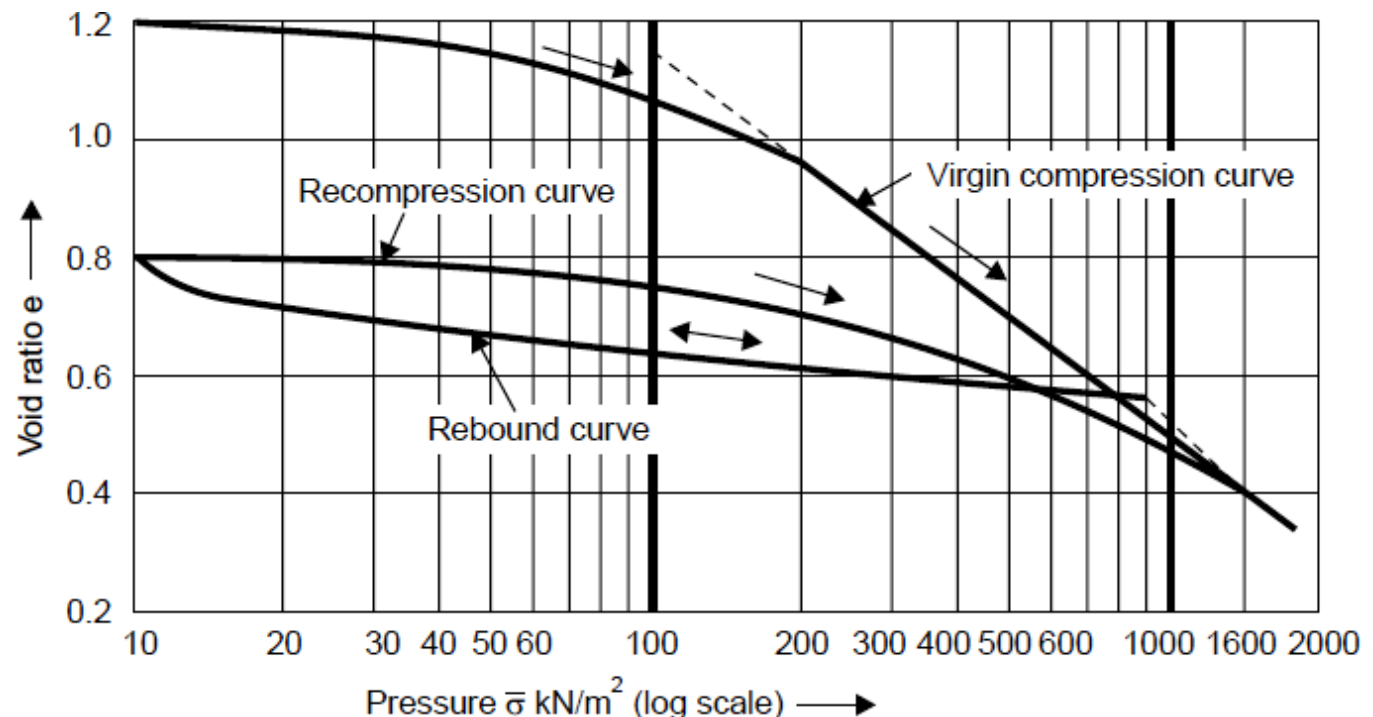
$$C_c = \frac{(e - e_0)}{\log_{10} \frac{\bar{\sigma}}{\bar{\sigma}_0}}$$

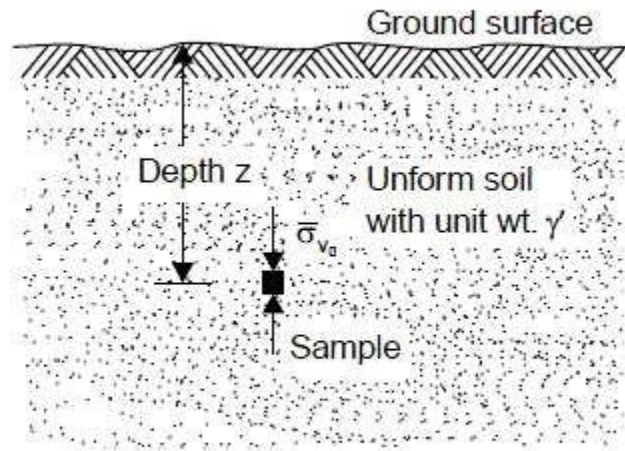


- ⌘ If, after complete removal of all loads, the sample is reloaded with the same series of loads as in the initial cycle, a different curve, called the 'recompression curve' is obtained.
- ⌘ Some of the volume change due to external loading is permanent. The difference in void ratios attained at any pressure between the virgin curve and the recompression curve is predominant at lower pressures and gets decreased gradually with increasing pressure.
- ⌘ The two curves are almost the same at the pressure from which the original rebound was made to occur during unloading. The recompression curve is less steep than the virgin curve.

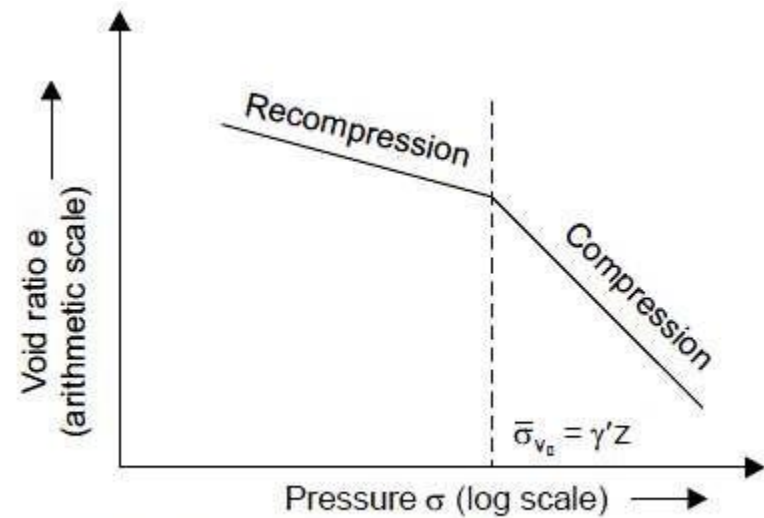


- It may be noted from Fig. that the curvature of the virgin compression curve at pressures smaller than about 200 kN/m² resembles the curvature of the recompression curve at pressures smaller than about 800 kN/m² from which the rebound occurred.
- This resemblance indicates that the specimen was probably subjected to a pressure of about 150 to 200 kN/m² at some time before its removal from the ground. Therefore, the initial curved portion of the so-called virgin curve can be visualised as a recompression curve; it may also be concluded that a convex curvature on this type of semi-logarithmic plot always indicates recompression.

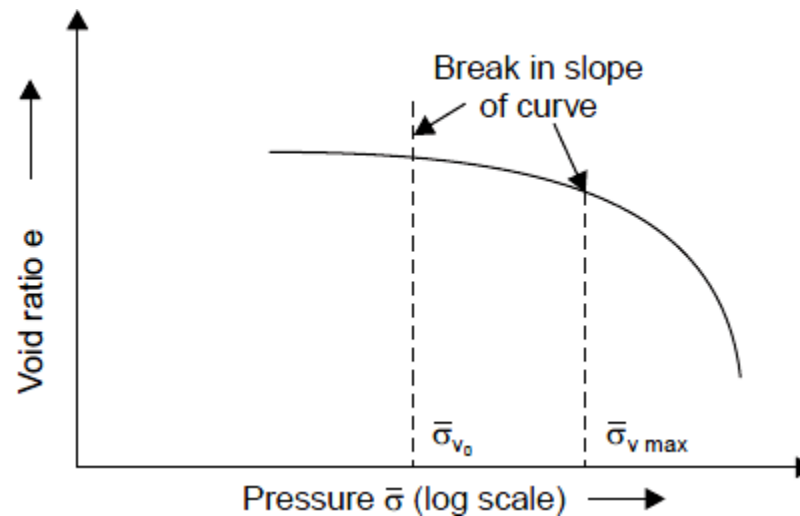




(a) Location of sample taken for compression test



(b) Compression test results for the sample



↓ Compression test results where past pressure exceeds present overburden pressure

Normally Consolidated Soil and Overconsolidated Soil

- ⊠ A soil for which the existing effective stress is the maximum to which it has ever been subjected in its stress history, is said to be ‘normally consolidated’.
 - ⊠ The straight portion of the virgin compression curve shown in Fig. corresponds to such a situation.
 - ⊠ A soil is said to be ‘overconsolidated’ if the present effective stress in it has been exceeded sometime during its stress history. The curved portion of the virgin compression curve in Fig. prior to the straight line portion corresponds to such a situation.
 - ⊠ An overconsolidated soil is also said to be a ‘pre-compressed’ soil. Thus the compressibility of a soil in an overconsolidated condition is much less than that for the same soil in a normally consolidated condition.
 - ⊠ A soil which is not fully consolidated under the existing overburden pressure is said to be ‘underconsolidated’.
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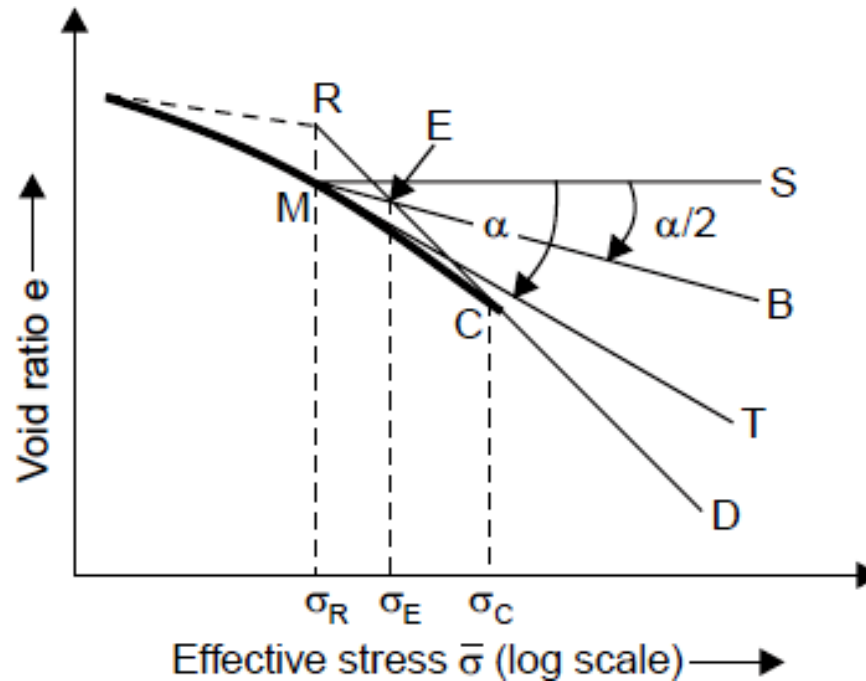


- ⊠ A number of agencies in nature transform normally consolidated clays to overconsolidated or precompressed ones.
- ⊠ For example, geological agencies such as glaciers apply pressures on advancing and unload on receding.
- ⊠ Human agencies such as engineers load through construction and unload through demolition of structures.
- ⊠ Environmental agencies such as climatic factors cause loading and unloading through ground-water movements and the phenomenon of capillarity.
- ⊠ A quantitative measure of the degree of overconsolidation is what is known as the 'Overconsolidation Ratio', OCR. It is defined as follows:

$$\text{OCR} = \frac{\text{Maximum effective stress to which the soil has been subjected in its stress history}}{\text{Existing effective stress in the soil}}$$

- ⊠ Thus, the maximum OCR of normally consolidated soil equals 1.

- ⊠ A. Casagrande (1936) proposed a geometrical technique to evaluate past maximum effective stress or preconsolidation pressure from the e versus $\log \sigma$ plot obtained by loading a sample in the laboratory.
- ⊠ The steps in the geometrical construction are:
- ⊠ 1. The point of maximum curvature M on the curved portion of the e vs. $\log \sigma'$ plot is located.
- ⊠ 2. A horizontal line MS is drawn through M .
- ⊠ 3. A tangent MT to the curved portion is drawn through M .
- ⊠ 4. The angle SMT is bisected, MB being the bisector.
- ⊠ 5. The straight portion DC of the plot is extended backward to meet MB in E .
- ⊠ 6. The pressure corresponding to the point E , σ_E , is the most probable past maximum effective stress or the preconsolidation pressure.



- Sometimes the lower and upper bound for the preconsolidation pressure are also mentioned.
- If the tangent to the initial recompression portion and the straight line portion of the virgin curve DC meet at R, the pressure σ_R corresponding to R is said to be the minimum preconsolidation pressure, while that corresponding to C, σ_C , is said to be the maximum preconsolidation pressure.

Relationship between Compressibility and Liquid Limit

⊠ A.W. Skempton and his associates have established a relationship between the compressibility of a clay, as indicated by its compression index, and the liquid limit, by conducting experiments with clays from various parts of the world.

⊠ The equation of this straight line may be approximately written as:

$$C_c = 0.007 (w_L - 10)$$

⊠ we may write for consolidation of field deposits of clay:

$$C_c = 0.009 (w_L - 10)$$



Modulus of Volume Change and Consolidation Settlement

- ⌘ The 'modulus of volume change, is defined as the change in volume of a soil per unit initial volume due to a unit increase in effective stress. It is also called the 'coefficient of volume change' or 'coefficient of volume compressibility' and is denoted by the symbol, m_v .

$$m_v = - \frac{\Delta e}{(1 + e_0)} \cdot \frac{1}{\Delta \bar{\sigma}}$$

Δe represents the change in void ratio and represents the change in volume of the saturated soil occurring through expulsion of pore water, and $(1 + e_0)$ represents initial volume, both for unit volume of solids.

- ⌘ But we know that

$$- \frac{\Delta e}{\Delta \bar{\sigma}} = a_v, \text{ the coefficient of compressibility.}$$

\therefore

$$m_v = \frac{a_v}{(1 + e_0)}$$

When the soil is confined laterally, the change in volume is proportional to the change in height, ΔH of the sample, and the initial volume is proportional to the initial height H_0 of the sample.

$$\therefore m_v = -\frac{\Delta H}{H_0} \cdot \frac{1}{\Delta \bar{\sigma}}$$
$$\Delta H = m_v \cdot H_0 \cdot \Delta \bar{\sigma}$$

Thus, the consolidation settlement, S_c , of a clay for full compression under a pressure increment $\Delta \bar{\sigma}$, is given by above equation

This is under the assumption that $\Delta \bar{\sigma}$ is transmitted uniformly over the thickness. However, it is found that $\Delta \bar{\sigma}$ decreases with depth non-linearly. In such cases, the consolidation settlement may be obtained as:

$$S_c = \int_0^H m_v \cdot \Delta \bar{\sigma} \cdot dz$$

This integration may be performed numerically by dividing the stratum of height H into thin layers and considering $\Delta \bar{\sigma}$ for the mid-height of the layer as being applicable for the thin layer. The total settlement of the layer of height H will be given by the sum of settlements of individual layers.



The consolidation settlement S_c , may also be put in a different, but more common form, as follows:

$$m_v = \frac{e}{(1+e_0)} \cdot \frac{1}{\Delta \bar{\sigma}}, \text{ ignoring sign.}$$

$$\frac{\Delta H}{H_0} = \frac{\Delta e}{(1+e_0)}$$

$$S_c = \Delta H = \frac{\Delta e}{(1+e_0)} \cdot H_0$$

Substituting for Δe in terms of the compression index, C_c

$$S_c = \Delta H = H_0 \cdot \frac{C_c}{(1+e_0)} \cdot \log_{10} \frac{\bar{\sigma}}{\bar{\sigma}_0}$$

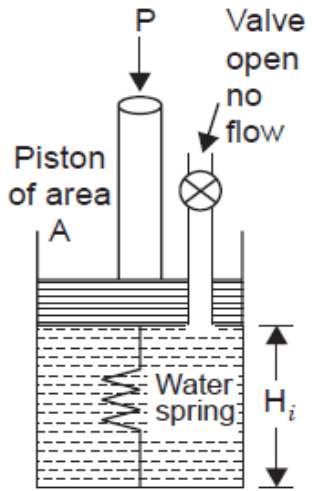
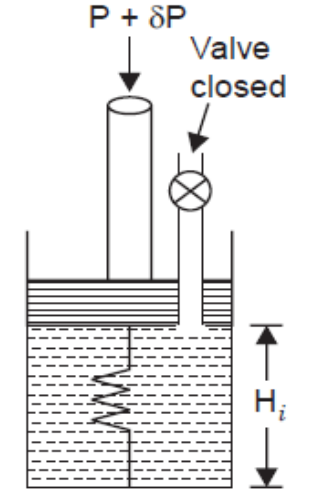
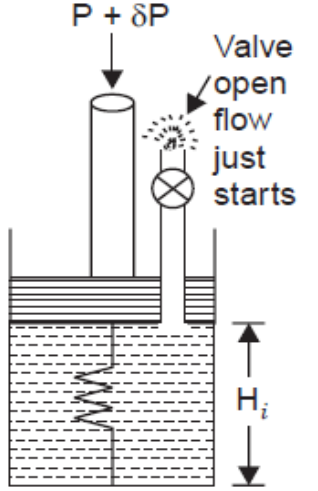
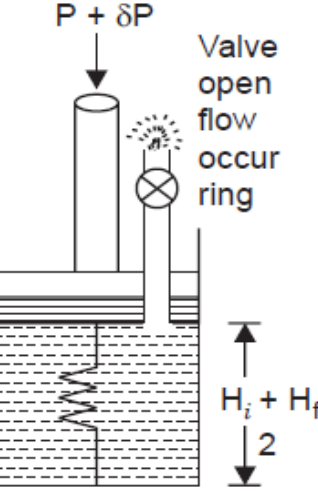
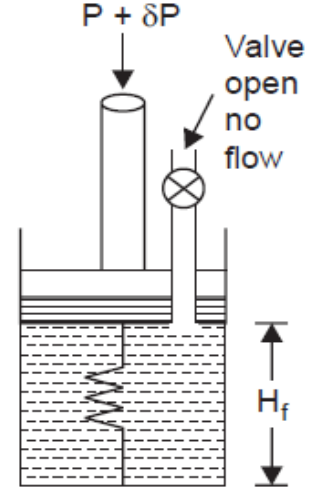
$$S_c = H_0 \cdot \frac{C_c}{(1+e_0)} \cdot \log_{10} \left(\frac{\bar{\sigma}_0 + \bar{\sigma}}{\bar{\sigma}_0} \right)$$



A MECHANISTIC MODEL FOR CONSOLIDATION

- ⊠ A mechanistic model for the phenomenon of consolidation was given by Taylor (1948), by which the process can be better understood.
 - ⊠ A spring of initial height H_i is surrounded by water in a cylinder. The spring is analogous to the soil skeleton and the water to the pore water.
 - ⊠ The cylinder is fitted with a piston of area A through which a certain load may be transmitted to the system representing a saturated soil.
 - ⊠ The piston, in turn, is fitted with a vent, and a valve by which the vent may be opened or closed.
 - ⊠ In this mechanistic model, the compressible soil skeleton is characterised by the spring and the pore water by the water in the cylinder.
 - ⊠ The more compressible the soil, the longer the time required for consolidation; the more permeable the soil, the shorter the time required.
-



				
$\sigma = \frac{P}{A}$ $u = 0$ $\bar{\sigma} = \frac{P}{A}$	$\sigma = \frac{P}{A} + \frac{\delta P}{A}$ $u = 0 + \frac{\delta P}{A}$ $\bar{\sigma} = \frac{P}{A} + 0$	$\bar{\sigma} = \frac{P}{A} + \frac{\delta P}{A}$ $u = 0 + \frac{\delta P}{A}$ $\bar{\sigma} = \frac{P}{A} + 0$	$\sigma = \frac{P}{A} + \frac{\delta P}{A}$ $u = 0 + \frac{1}{2} \frac{\delta P}{A}$ $\bar{\sigma} = \frac{P}{A} + \frac{1}{2} \frac{\delta P}{A}$	$\sigma = \frac{P}{A} + \frac{\delta P}{A}$ $u = 0$ $\bar{\sigma} = \frac{P}{A} + \frac{\delta P}{A}$
<p>Equilibrium under load P</p>	<p>Equilibrium under load $P + \delta P$</p>	<p>Beginning of transient flow; excess u just starts to reduce and $\bar{\sigma}$ just starts to increase. $t = 0$, 0% consolidation</p>	<p>Half-way of transient flow; 50% of excess u dissipated; σ increased by $\frac{1}{2} \cdot \frac{\delta p}{A}$ $0 < t < t_f$ 50% consolidation</p>	<p>End of transient flow; excess u fully dissipated; $\bar{\sigma}$ increased to $\frac{P}{A} + \frac{\delta P}{A}$ $t = t_f$ equilibrium under load $(P + \delta P)$ 100% consolidation</p>
(a)	(b)	(c)	(d)	(e)

- ☒ Referring to Fig. (a), let a load P be applied on the piston. Let us assume that the valve of the vent is open and no flow is occurring. This indicates that the system is in equilibrium under the total stress P/A which is fully borne by the spring, the pressure in the water being zero.
- ☒ Referring to Fig. (b), let us apply an increment of load δP to the piston, the valve being kept closed. Since no water is allowed to flow out, the piston cannot move downwards and compress the spring; therefore, the spring carries the earlier stress of P/A , while the water is forced to carry the additional stress of $\delta P/A$ imposed on the system, the sum counteracting the total stress imposed. This additional stress $\delta P/A$ in the water is known as the hydrostatic excess pressure.
- ☒ Referring to Fig. (c), let us open the valve and start reckoning time from that instant. Water just starts to flow under the pressure gradient between it and the atmosphere seeking to return to its equilibrium or atmospheric pressure. The excess pore pressure begins to diminish, the spring starts getting compressed as the piston descends consequent to expulsion of pore water. It is just the beginning of transient flow, simulating the phenomenon of consolidation. The openness of the valve is analogous to the permeability of soil.

- ⊠ Referring to Fig. (d), flow has occurred to the extent of dissipating 50% of the excess pore pressure. The pore water pressure at this instant is half the initial value, i.e., $0.5(\delta P/A)$. This causes a corresponding increase in the stress in the spring of $0.5(\delta P/A)$, the total stress remaining constant at $[(P/A) + (\delta P/A)]$. This stage refers to that of “50% consolidation”.
- ⊠ Referring to Fig. (e), the final equilibrium condition is reached when the transient flow situation ceases to exist, consequent to the complete dissipation of the pore water pressure. The spring compresses to a final height $H_f < H_i$, carrying the total stress of $(P + \delta P)/A$, all by itself, since the excess pore water pressure has been reduced to zero, the pressure in it having equaled the atmospheric. The system has reached the equilibrium condition under the load $(P + \delta P)$. This represents “100% consolidation” under the applied load or stress increment. We may say that the “soil” has been consolidated to an effective stress of $(P + \delta P)/A$.



TERZAGHI'S THEORY OF ONE-DIMENSIONAL CONSOLIDATION

- ☒ Terzaghi (1925) advanced his theory of one-dimensional consolidation based upon the following assumptions, the mathematical implications being given in parentheses:
- ☒ 1. The soil is homogeneous (k_z is independent of z).
- ☒ 2. The soil is completely saturated ($S = 100\%$).
- ☒ 3. The soil grains and water are virtually incompressible (γ_w is constant and volume change of soil is only due to change in void ratio).
- ☒ 4. The behaviour of infinitesimal masses in regard to expulsion of pore water and consequent consolidation is no different from that of larger representative masses.
- ☒ 5. The compression is one-dimensional (u varies with z only).



- ⊠ 6. The flow of water in the soil voids is one-dimensional, Darcy's law being valid.

$$\left(\frac{\partial v_x}{\partial x} = \frac{\partial v_y}{\partial y} = 0 \text{ and } v_z = k_z \cdot \frac{\partial h}{\partial z} \right).$$

Also, flow occurs on account of hydrostatic excess pressure ($h = u/\gamma_w$).

- ⊠ 7. Certain soil properties such as permeability and modulus of volume change are constant; these actually vary somewhat with pressure. (k and m_v are independent of pressure).
- ⊠ 8. The pressure versus void ratio relationship is taken to be the idealised one, as shown in Fig. (a_v is constant).
- ⊠ 9. Hydrodynamic lag alone is considered and plastic lag is ignored, although it is known to exist. (The effect of k alone is considered on the rate of expulsion of pore water).



Basic differential equation of consolidation according to Terzaghi's theory of one-dimensional consolidation

$$\frac{\partial u}{\partial t} = \frac{k}{\gamma_w \cdot m_v} \cdot \frac{\partial^2 u}{\partial z^2}$$

$$\frac{\partial u}{\partial t} = c_v \cdot \frac{\partial^2 u}{\partial z^2}$$

where $c_v = \frac{k}{\gamma_w \cdot m_v}$

c_v is known as the “Coefficient of consolidation”. u represents the hydrostatic excess pressure at a depth z from the drainage face at time t from the start of the process of consolidation.

The coefficient of consolidation may also be written in terms of the coefficient of compressibility:

$$c_v = \frac{k}{\gamma_w m_v} = \frac{k(1 + e_0)}{\alpha_v \gamma_w}$$

The coefficient of consolidation combines the effect of permeability and compressibility characteristics on volume change during consolidation. Its units can be shown to be mm^2/s or L^2T^{-1} .



SOLUTION OF TERZAGHI'S EQUATION FOR ONE-DIMENSIONAL CONSOLIDATION

- ☒ Terzaghi solved the differential equation for a set of boundary conditions which have utility in solving numerous engineering problems and presented the results in graphical form using dimensional parameters.
- ☒ The following are the boundary conditions:
 - ☒ 1. There is drainage at the top of the sample: At $z = 0$, $u = 0$, for all t .
 - ☒ 2. There is drainage at the bottom of the sample: At $z = 2H$, $u = 0$, for all t .
 - ☒ 3. The initial hydrostatic excess pressure u_i is equal to the pressure increment, $\Delta\sigma$ $u = u_i = \Delta\sigma$, at $t = 0$.
- ☒ Terzaghi chose to consider this situation where $u = u_i$ initially throughout the depth, although solutions are possible when u_i varies with depth in any specified manner.
- ☒ The thickness of the sample is designated by $2H$, the distance H thus being the length of the longest drainage path, i.e., maximum distance water has to travel to reach a drainage face because of the existence of two drainage faces. (In the case of only one drainage face, this will be equal to the total thickness of the clay layer).

SOLUTION OF TERZAGHI'S EQUATION FOR ONE-DIMENSIONAL CONSOLIDATION

The general solution for the above set of boundary conditions has been obtained on the basis of separation of variables and Fourier Series expansion and is as follows:

$$U = 1 - \sum_{m=0}^{\infty} \frac{2 \int_0^{2H} u_i \cdot \sin \frac{Mz}{H} \cdot dz}{M \int_0^{2H} u_i \cdot dz} \cdot e^{-M^2 T}$$

the symbol M represents $(\pi/2) (2m + 1)$

- ⌘ Three-dimensionless parameters are introduced for convenience in presenting the results in a form usable in practice.
- ⌘ The first is **Drainage path ratio**, z/H , relating to the location of the point at which consolidation is considered, H being the maximum length of the drainage path.



- ⊠ The second is the **consolidation ratio or degree of consolidation**, U_z , to indicate the extent of dissipation of the hydrostatic excess pressure in relation to the initial value:

$$U_z = (u_i - u)/u_i = \left(1 - \frac{u}{u_i}\right)$$

- ⊠ The third dimensionless parameter, relating to time, and called '**Time-factor**', T , is defined as follows:

$$T = \frac{c_v t}{H^2}$$

where c_v is the coefficient of consolidation,

H is the length drainage path,

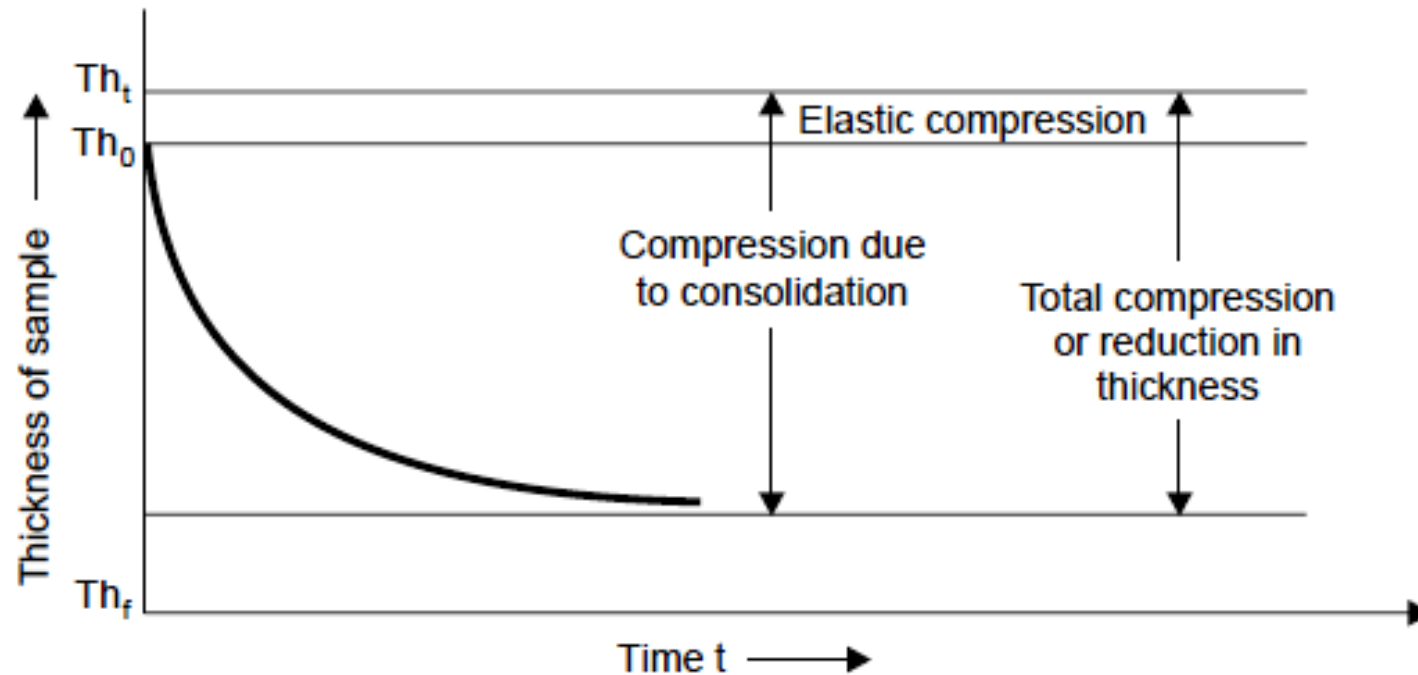
and t is the elapsed time from the start of consolidation process.

When $U < 60\%$, $T = (\pi/4)U^2$

▶ When $U > 60\%$, $T = -0.9332 \log_{10} (1 - U) - 0.0851$.

EVALUATION OF COEFFICIENT OF CONSOLIDATION FROM OEDOMETER TEST DATA

- ⊠ The more generally used fitting methods are the following:
- ⊠ (a) The square root of time fitting method
- ⊠ (b) The logarithm of time fitting method



(a) The square root of time fitting method

- ⊠ This method was devised by Taylor (1948). In this method, the dial readings are plotted against the square root of time as given in Fig.
 - ⊠ On the theoretical curve a straight line exists up to 60 percent consolidation while at 90 percent consolidation the abscissa of the curve is 1.15 times the abscissa of the straight line produced.
 - ⊠ The fitting method consists of first drawing the straight line which best fits the early portion of the laboratory curve. Next a straight line is drawn which at all points has abscissa 1.15 times as great as those of the first line.
 - ⊠ The intersection of this line and the laboratory curve is taken as the 90 percent consolidation point. Its value may be read and is designated as t_{90} .
-



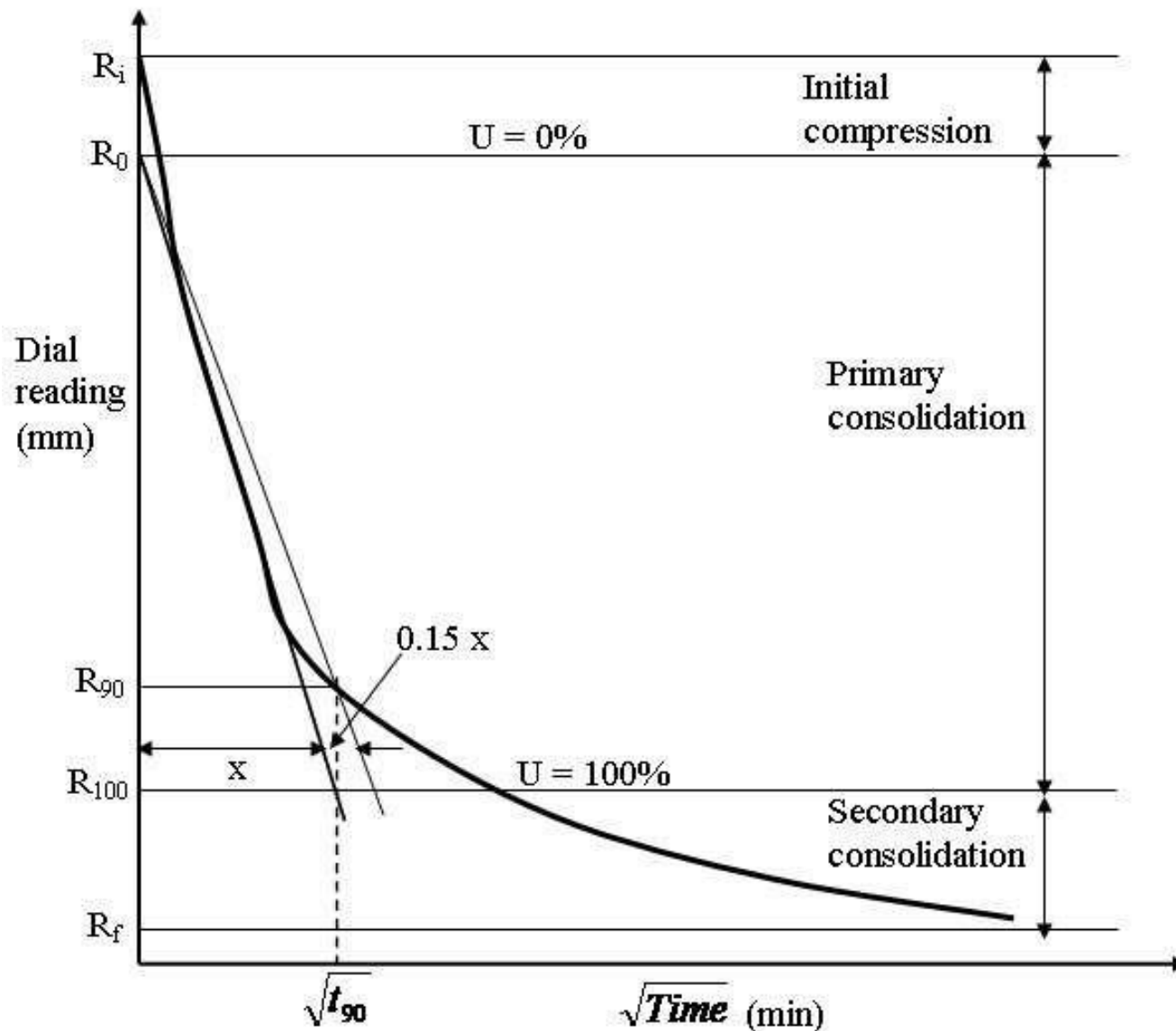
(a) The square root of time fitting method

- ⊠ Usually the straight line through the early portion of the laboratory curve intersects the zero time line at a point (R_o) differing somewhat from the initial point (R_i). This intersection point is called the corrected zero point.
- ⊠ If one-ninth of the vertical distance between the corrected zero point and the 90 per cent point is set off below the 90 percent point, the point obtained is called the "100 percent primary compression point" (R_{100}).
- ⊠ The compression between zero and 100 per cent point is called "primary compression".
- ⊠ At the point of 90 percent consolidation, the value of $T = 0.848$. The equation of c_v may now be written as

$$c_v = \frac{T_{90} H^2}{t_{90}}$$



The Square Root of Time Fitting Method



$$c_v = \frac{T_{90} H^2}{t_{90}}$$

(b) The logarithm of time fitting method

- ⊠ This method was proposed by Casagrande and Fadum (1940).
- ⊠ On the laboratory curve, the intersection formed by the final straight line produced backward and the tangent to the curve at the point of inflection is accepted as the 100 per cent primary consolidation point and the dial reading is designated as R_{100} .
- ⊠ The time-compression relationship in the early stages is also parabolic just as the theoretical curve.
- ⊠ The dial reading at zero primary consolidation R_0 can be obtained by selecting any two points on the parabolic portion of the curve where times are in the ratio of 1 : 4.
- ⊠ The difference in dial readings between these two points is then equal to the difference between the first point and the dial reading corresponding to zero primary consolidation.



SECONDARY CONSOLIDATION

- ⊠ When the hydrostatic excess pressure is fully dissipated, no more consolidation should be expected.
- ⊠ However, in practice, the decrease in void ratio continues, though very slowly, for a long time after this stage, called 'Primary Consolidation'.
- ⊠ The effect or the phenomenon of continued consolidation after the complete dissipation of excess pore water pressure is termed 'Secondary Consolidation' and the resulting compression is called 'Secondary Compression'.
- ⊠ During this stage, plastic readjustment of clay platelets takes place and other effects as well as colloidal-chemical processes and surface phenomena such as induced electro-kinetic potentials occur. These are, by their very nature, very slow.
- ⊠ Secondary consolidation of mineral soils is usually negligible but it may be considerable in the case of organic soils due to their colloidal nature.



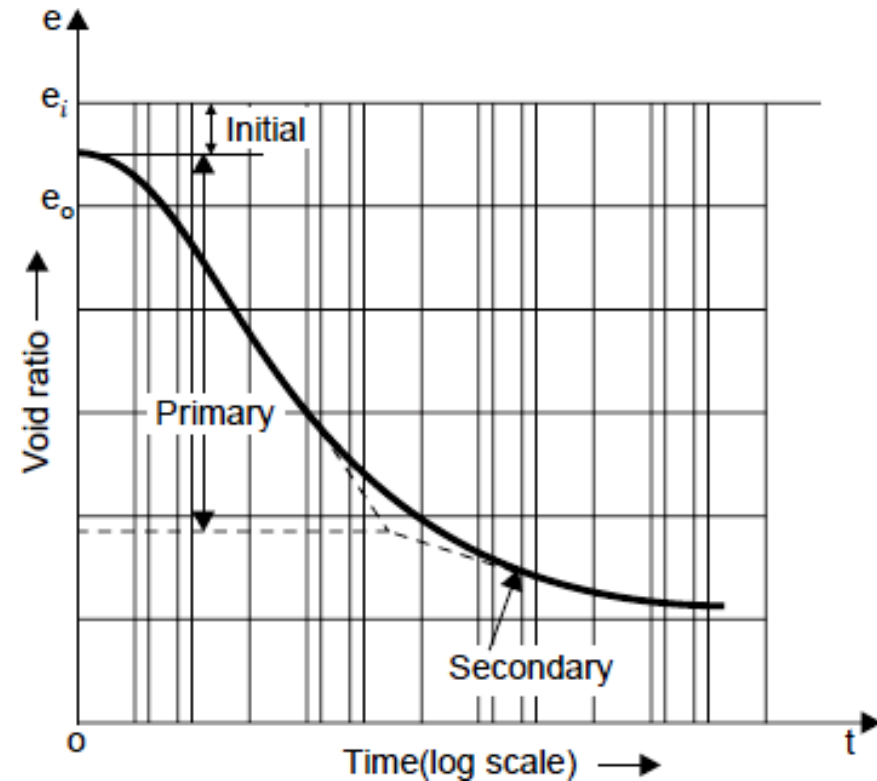
☒ Hence, the secondary compression can be identified on a plot of void ratio versus logarithm of time

➤ Secondary compression appears as a straight line sloping downward or, in some cases, as a straight line followed by a second straight line with a flatter slope.

➤ The void ratio, e_f , at the end of primary consolidation can be found from the intersection of the backward extension of the secondary line with a tangent drawn to the curve of primary compression, as shown in the figure.

➤ The rate of secondary compression, depends upon the increment of stress and the characteristics

➤ of the soil.



- the time-rate of secondary compression is through the 'coefficient of secondary compression', C_α , in terms of strain or percentage of settlement as follows

$$\epsilon = \frac{\Delta H}{H} = -C_\alpha \log_{10} \left(\frac{t_2}{t_1} \right)$$

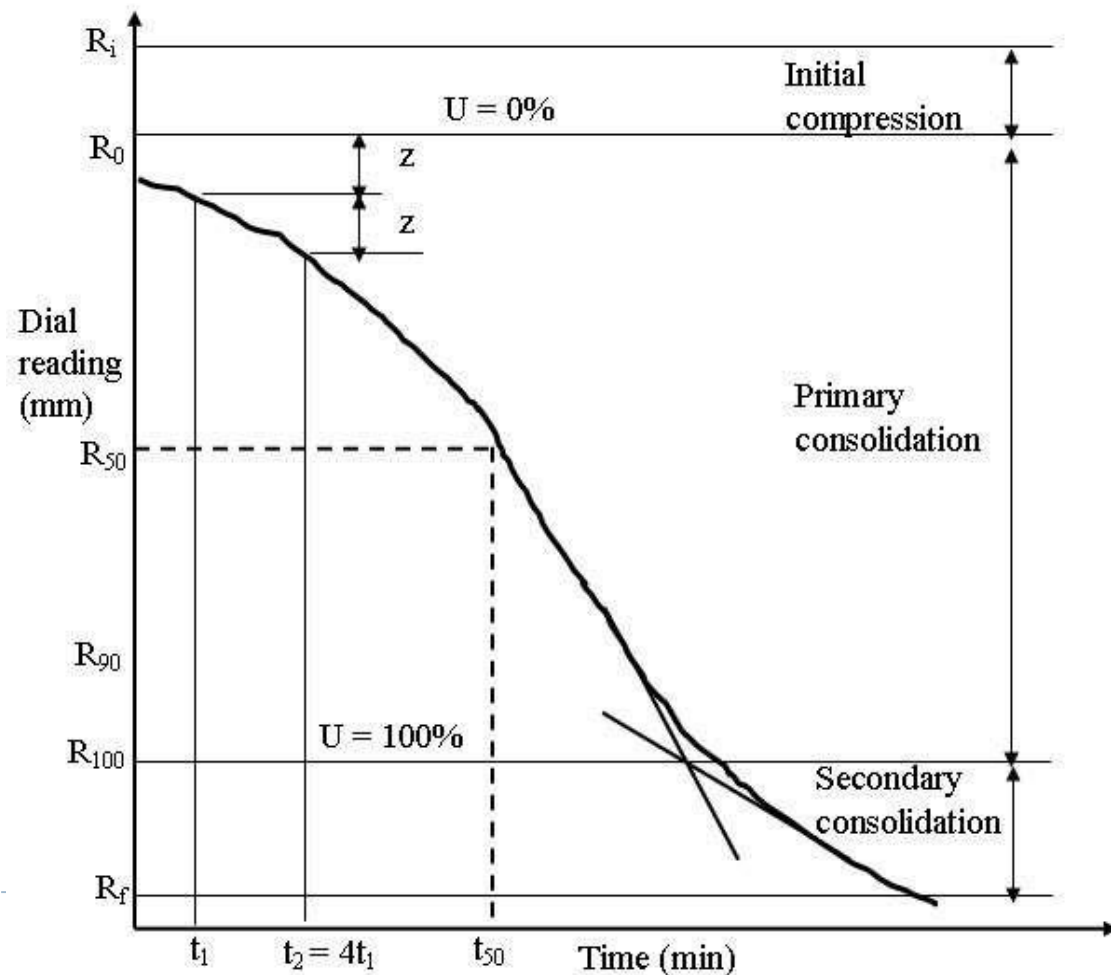
- Here, t_1 is the time required for the primary compression to be virtually complete, t_2 any later time, and Δe is the corresponding change in void ratio. α is a coefficient expressing the rate of secondary compression.
- In other words, C_α may be taken to be the slope of the straight line representing the secondary compression on a plot of strain versus logarithm of time.
- The relation between α and C_α is

$$C_\alpha = \frac{\alpha}{(1 + e)}$$



- ⊠ The interval between 0 and 100% consolidation is divided into equal intervals of percent consolidation. Since it has been found that the laboratory and the theoretical curves have better correspondence at the central portion, the value of c_v is computed by taking the time t and time factor T at 50 percent consolidation.
- ⊠ The equation to be used is

$$C_v = \frac{T_{50} H^2}{t_{50}}$$



In a consolidation test the following results have been obtained. When the load was changed from 50 kN/m² to 100 kN/m², the void ratio changed from 0.70 to 0.65. Determine the coefficient of volume decrease, m_v and the compression index, C_c .

$$e_0 = 0.70$$

$$\bar{\sigma}_0 = 50 \text{ kN/m}^2$$

$$e_1 = 0.65$$

$$\bar{\sigma} = 100 \text{ kN/m}^2$$

Coefficient of compressibility, $a_v = \frac{\Delta e}{\Delta \bar{\sigma}}$, ignoring sign.

$$= \frac{(0.70 - 0.65)}{(100 - 50)} \text{ m}^2/\text{kN} = 0.05/50 \text{ m}^2/\text{kN} = 0.001 \text{ m}^2/\text{kN}.$$

Modulus of volume change, or coefficient of volume decrease,

$$m_v = \frac{a_v}{(1 + e_0)} = \frac{0.001}{(1 + 0.70)} = \frac{0.001}{1.7} \text{ m}^2/\text{kN}.$$

$$= 5.88 \times 10^{-4} \text{ m}^2/\text{kN}$$

Compression index, $C_c = \frac{\Delta e}{\Delta (\log \bar{\sigma})} = \frac{(0.70 - 0.65)}{(\log_{10} 100 - \log_{10} 50)}$

$$= \frac{0.05}{\log_{10} \frac{100}{50}} = \frac{0.05}{\log_{10} 2} = \frac{0.050}{0.301} = \mathbf{0.166}.$$

A sand fill compacted to a bulk density of 18.84 kN/m³ is to be placed on a compressible saturated marsh deposit 3.5 m thick. The height of the sand fill is to be 3 m. If the volume compressibility m_v of the deposit is 7×10^{-4} m²/kN, estimate the final settlement of the fill.

$$\text{Ht. of sand fill} = 3 \text{ m}$$

$$\text{Bulk unit weight of fill} = 18.84 \text{ kN/m}^3$$

$$\begin{aligned}\text{Increment of the pressure on top of marsh deposit } \Delta \bar{\sigma} &= 3 \times 18.84 \\ &= 56.52 \text{ kN/m}^2\end{aligned}$$

$$\text{Thickness of marsh deposit, } H_0 = 3.5 \text{ m}$$

$$\text{Volume compressibility } m_v = 7 \times 10^{-4} \text{ m}^2/\text{kN}$$

$$\begin{aligned}\text{Final settlement of the marsh deposit, } \Delta H &= m_v \cdot H_0 \cdot \Delta \bar{\sigma} \\ &= 7 \times 10^{-4} \times 3500 \times 56.52 \text{ mm} \\ &= 138.5 \text{ mm.}\end{aligned}$$

A layer of soft clay is 6 m thick and lies under a newly constructed building. The weight of sand overlying the clayey layer produces a pressure of 260 kN/m² and the new construction increases the pressure by 100 kN/m². If the compression index is 0.5, compute the settlement. Water content is 40% and specific gravity of grains is 2.65.



Initial pressure, $\bar{\sigma}_0 = 260 \text{ kN/m}^2$

Increment of pressure, $\Delta\bar{\sigma} = 100 \text{ kN/m}^2$

Thickness of clay layer, $H = 6 \text{ m} = 600 \text{ cm.}$

Compression index, $C_c = 0.5$

Water content, $w = 40\%$

Specific gravity of grains, $G = 2.65$

Void ratio, $e = wG$, (since the soil is saturated) $= 0.40 \times 2.65 = 1.06$

This is taken as the initial void ratio, e_0 .

Consolidation settlement,

$$\begin{aligned} S &= \frac{H \cdot C_c}{(1 + e_0)} \log_{10} \left(\frac{\bar{\sigma}_0 + \Delta\bar{\sigma}}{\bar{\sigma}_0} \right) \\ &= \frac{600 \times 0.5}{(1 + 1.06)} \log_{10} \left(\frac{260 + 100}{260} \right) \text{ cm} \\ &= \frac{300}{2.06} \log_{10} \left(\frac{360}{260} \right) \text{ cm} \\ &= \mathbf{21.3 \text{ cm.}} \end{aligned}$$

Example 7.3: The following results were obtained from a consolidation test:

Initial height of sample $H_i = 2.5$ cm

Height of solid particles $H_s = 1.25$ cm

<i>Pressure in kN/m^2</i>	<i>Dial reading in cm</i>
0	0.000
13	0.000
27	0.004
54	0.016
108	0.044
214	0.104
480	0.218
960	0.340
1500	0.420

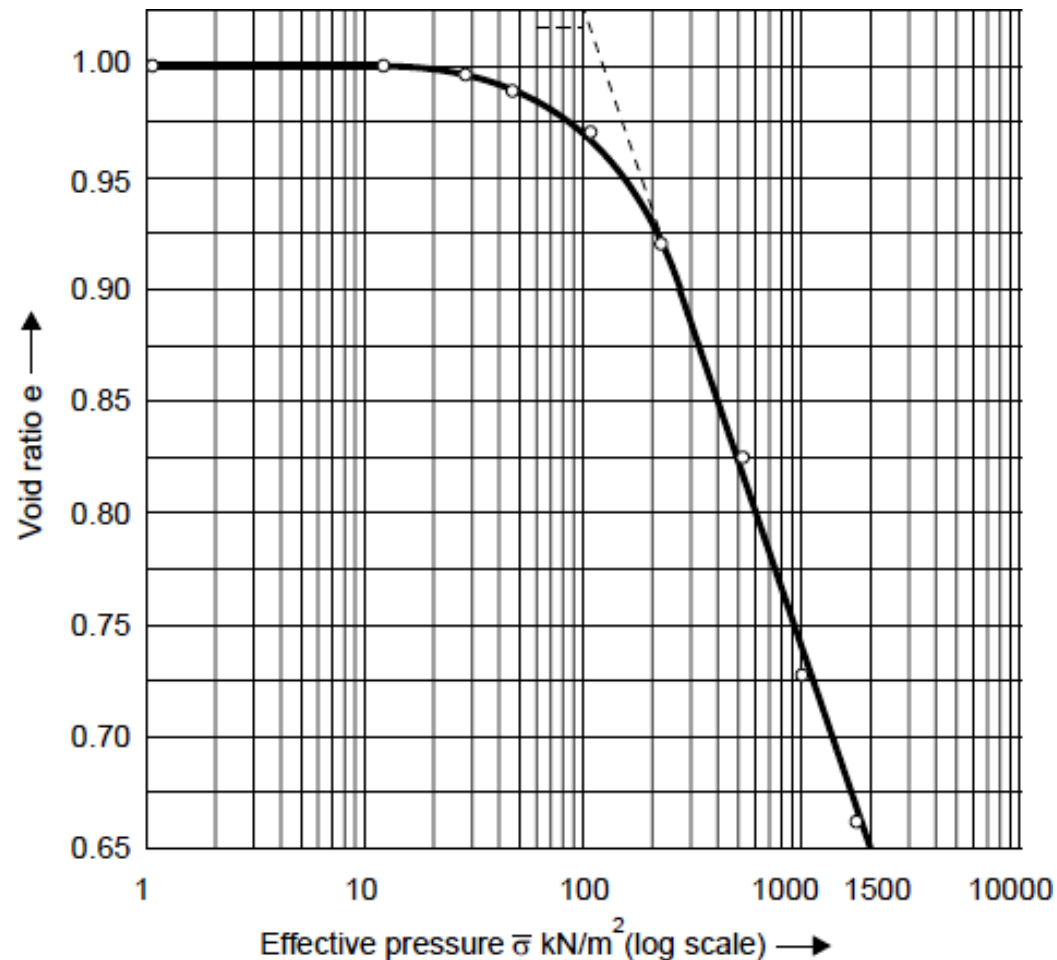
Plot the pressure-void ratio curve and determine (a) *the compression index* and (b) *the preconsolidation pressure*.

$$\text{Initial void ratio, } e_0 = \frac{(H_i - H_s)}{H_s} = \frac{(2.50 - 1.25)}{1.25} = \frac{1.25}{1.25} = 1.000$$

The heights of sample and the void ratios at the end of each pressure increments are tabulated below:

<i>Pressure in kN/m²</i>	<i>Dial reading in cm</i>	<i>Height of sample, H in cm</i>	<i>Void ratio</i>
0	0.000	2.500	1.000
13	0.000	2.500	1.000
27	0.004	2.496	0.997
54	0.016	2.484	0.987
108	0.044	2.456	0.965
214	0.104	2.396	0.917
480	0.218	2.282	0.826
960	0.340	2.160	0.728
1500	0.420	2.080	0.664





$$(a) C_c = \Delta e / \log(\bar{\sigma}_2 / \bar{\sigma}_1)$$

= Δe , if one logarithmic cycle of pressure is chosen as the base.

= **0.303**, in this case.

(b) The pre-consolidation pressure by A. Casagrande's method

= **180 kN/m^2 .**

There is a bed of compressible clay of 4 m thickness with pervious sand on top and impervious rock at the bottom. In a consolidation test on an undisturbed specimen of clay from this deposit 90% settlement was reached in 4 hours. The specimen was 20 mm thick.

Estimate the time in years for the building founded over this deposit to reach 90% of its final settlement.



This is a case of one-way drainage in the field.

∴ Drainage path for the field deposit, $H_f = 4 \text{ m} = 4000 \text{ mm}$. In the laboratory consolidation test, commonly it is a case of two-way drainage.

∴ Drainage path for the laboratory sample, $H_l = 20/2 = 10 \text{ mm}$

Time for 90% settlement of laboratory sample = 4 hrs.

Time factor for 90% settlement, $T_{90} = 0.848$

$$\therefore T_{90} = \frac{C_v t_{90_f}}{H_f^2} = \frac{C_v t_{90_l}}{H_l^2}$$

or

$$\frac{t_{90_f}}{H_f^2} = \frac{t_{90_l}}{H_l^2}$$

$$\begin{aligned}\therefore t_{90_f} &= \frac{t_{90_l}}{H_l^2} \times H_f^2 = \frac{4 \times (4000)^2}{10^2} \text{ hrs} \\ &= \frac{4 \times 400}{24 \times 365} \text{ years} \\ &\approx \mathbf{73 \text{ years.}}\end{aligned}$$

A saturated soil has a compression index of 0.25. Its void ratio at a stress of 10 kN/m² is 2.02 and its permeability is 3.4×10^{-7} mm/s. Compute:

- (i) Change in void ratio if the stress is increased to 19 kN/m²;
- (ii) Settlement in (i) if the soil stratum is 5 m thick; and
- (iii) Time required for 40% consolidation if drainage is one-way.



Compression index, $C_c = 0.25$

$e_0 = 2.02$

$\bar{\sigma}_0 = 10 \text{ kN/m}^2$

$k = 3.4 \times 10^{-7} \text{ mm/s}$

$\bar{\sigma}_1 = 19 \text{ kN/m}^2$

$$(i) \quad C_c = \frac{\Delta e}{\log_{10} (\bar{\sigma}_1 / \bar{\sigma}_0)} \therefore 0.25 = \frac{\Delta e}{\log_{10} (19 / 10)}$$

$$\therefore \Delta e = 0.25 \log_{10} (1.9) \approx \mathbf{0.07}$$

or Void ratio at a stress of $19 \text{ kN/m}^2 = 2.02 - 0.07 = 1.95$

$$a_v = \Delta e / \Delta \bar{\sigma} = 0.07/9 = 0.00778 \text{ m}^2/\text{kN}$$

$$m_v = a_v / (1 + e_0) = 0.00778 / (1 + 2.02) = 2.575 \times 10^{-3} \text{ m}^2/\text{kN}$$

(ii) Thickness of soil stratum, $H = 5 \text{ m}$.

$$\begin{aligned} \text{Settlement, } S &= \frac{H \cdot C_c}{(1 + e_0)} \log_{10} \left(\frac{\bar{\sigma}_0 + \Delta \bar{\sigma}}{\bar{\sigma}_0} \right) = \frac{H \cdot C_c}{(1 + e_0)} \log_{10} \left(\frac{\bar{\sigma}_1}{\bar{\sigma}_0} \right) \\ &= \frac{5 \times 1000 \times 0.25}{(1 + 2.02)} \log_{10} (19/10) \text{ mm} \approx \mathbf{115.4 \text{ mm}} \end{aligned}$$

(iii) If drainage is one way, drainage path, H = thickness of stratum = 5 m

$$T_{40} = \frac{C_v t_{40}}{H^2}; \quad T_{40} = (\pi/4)U^2 = (\pi/4) \times (0.40)^2 = 0.04 \pi = 0.125664$$

$$C_v = k/m_v \cdot \gamma_w$$

$$= \frac{3.4 \times 10^{-7} \times 10^{-3}}{2.575 \times 10^{-3} \times 9.81} \text{ m}^2/\text{s} = 1.346 \times 10^{-8} \text{ m}^2/\text{s}$$

$$\begin{aligned} \therefore t_{40} &= \frac{T_{40} \cdot H^2}{C_v} \\ &= \frac{0.125664 \times 5 \times 5}{1.346 \times 10^{-8} \times 60 \times 60 \times 24} \text{ days} \\ &\approx \mathbf{270.14 \text{ days.}} \end{aligned}$$



The settlement analysis (based on the assumption of the clay layer draining from top and bottom surfaces) for a proposed structure shows 2.5 cm of settlement in four years and an ultimate settlement of 10 cm. However, detailed sub-surface investigation reveals that there will be no drainage at the bottom. For this situation, determine the ultimate settlement and the time required for 2.5 cm settlement.

The ultimate settlement is not affected by the nature of drainage, whether it is one-way or two-way.

Hence, the ultimate settlement = 10 cm.

However, the time-rate of settlement depends upon the nature of drainage.

Settlement in four years = 2.5 cm.

$$T = \frac{C_v t}{H^2}$$
$$U = \frac{2.5}{10.0} = 25\%$$

Since the settlement is the same, $U\%$ is the same;

hence, the time-factor is the same.

$$\therefore T/C_v = t/H^2 = \text{Constant.}$$

or

$$\frac{t_2}{H_2^2} = \frac{t_1}{H_1^2},$$

t_2 and H_2 referring to double drainage, and t_1 and H_1 referring to single drainage. The drainage path for single drainage is the thickness of the layer itself, while that for double drainage is half the thickness.

$$\therefore H_1 = 2H_2$$

$$\therefore \frac{t_2}{H_2^2} = \frac{t_1}{4H_2^2},$$

$$\therefore t_1 = 4t_2 = 4 \times 4 \text{ yrs} = 16 \text{ yrs.}$$

The void ratio of clay A decreased from 0.572 to 0.505 under a change in pressure from 120 to 180 kg/m². The void ratio of clay B decreased from 0.612 to 0.597 under the same increment of pressure. The thickness of sample A was 1.5 times that of B. Nevertheless the time required for 50% consolidation was three times longer for sample B than for sample A. What is the ratio of the coefficient of permeability of A to that of B ?



Clay A

$$e_0 = 0.572$$

$$e_1 = 0.505$$

$$\bar{\sigma}_0 = 120 \text{ kN/m}^2$$

$$\bar{\sigma}_1 = 180 \text{ kN/m}^2$$

$$a_{v_A} = \frac{\Delta e}{\Delta \bar{\sigma}} = \frac{0.067}{60} \text{ m}^2/\text{kN}$$

$$\begin{aligned} m_{v_A} &= \frac{0.067}{60} / (1 + 0.572) \\ &= 7.10 \times 10^{-4} \text{ m}^2/\text{kN} \end{aligned}$$

Clay B

$$e_0 = 0.612$$

$$e_1 = 0.597$$

$$\bar{\sigma}_0 = 120 \text{ kN/m}^2$$

$$\bar{\sigma}_1 = 180 \text{ kN/m}^2$$

$$m_{v_B} = \frac{\Delta e}{\Delta \bar{\sigma}} = \frac{0.015}{60} \text{ m}^2/\text{kN}$$

$$\begin{aligned} m_{v_B} &= \frac{0.015}{60} / (1 + 0.612) \\ &= 1.55 \times 10^{-4} \text{ m}^2/\text{kN} \end{aligned}$$

$$H_A/H_B = 1.5 \text{ and } t_{50_B}/t_{50_A} = 3$$

$$T_{50} = C_v t_{50}/H^2$$

\therefore

$$T_{50} = \frac{C_v \cdot t_{50_A}}{H_A^2} = \frac{C_{v_B} \cdot t_{50_B}}{H_B^2}$$

$$\frac{C_{v_A}}{C_{v_B}} = \frac{t_{50_B}}{t_{50_A}} \cdot \frac{H_A^2}{H_B^2} = 3 \times (1.5)^2 = 6.75$$

But
or

$$C_v = k/m_v \gamma_w$$

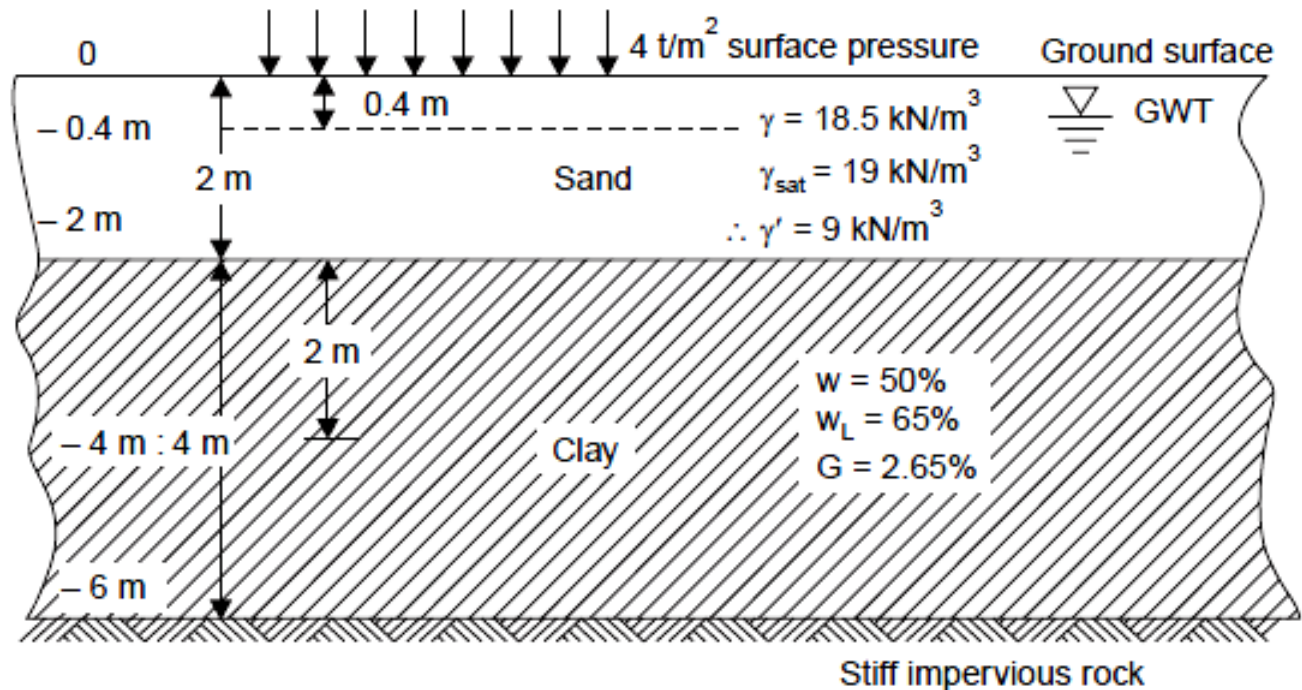
$$k = C_v m_v \gamma_w$$

\therefore

$$k_A/k_B = \frac{c_{v_A} \cdot m_{v_A}}{c_{v_B} \cdot m_{v_B}} = 6.75 \times \frac{7.10 \times 10^{-4}}{1.55 \times 10^{-4}} = 30.92 \approx \mathbf{31}.$$

(a) The soil profile at a building site consists of dense sand up to 2 m depth, normally loaded soft clay from 2 m to 6 m depth, and stiff impervious rock below 6 m depth. The ground-water table is at 0.40 m depth below ground level. The sand has a density of 18.5 kN/m^3 above water table and 19 kN/m^3 below it. For the clay, natural water content is 50%, liquid limit is 65% and grain specific gravity is 2.65. Calculate the probable ultimate settlement resulting from a uniformly distributed surface load of 40 kN/m^2 applied over an extensive area of the site.

(b) In a laboratory consolidation test with porous discs on either side of the soil sample, the 25 mm thick sample took 81 minutes for 90% primary compression. Calculate the value of coefficient of consolidation for the sample.



For the clay stratum:

$$w = 50\%$$

$$G = 2.65$$

Since it is saturated, $e = w.G = 0.50 \times 2.65 = 1.325$

This is the initial void ratio, e_0 .

$$\begin{aligned}\gamma_{\text{sat}} &= \frac{(G + e)}{(1 + e)} \gamma_w \approx \frac{(2.65 + 1.325)}{(1 + 1.325)} \times 10 \text{ kN / m}^3 = \frac{3.975 \times 10}{2.325} \text{ kN / m}^3 \\ &= 17.1 \text{ kN/m}^3 \\ \gamma &= (\gamma_{\text{sat}} - \gamma_w) = 7.1 \text{ kN/m}^3\end{aligned}$$

Initial effective overburden pressure at the middle of the clay layer:

$$\begin{aligned}\bar{\sigma}_0 &= (0.4 \times 18.5 + 1.6 \times 9.0 + 2 \times 7.1) \text{ t/m}^2 \\ &= 36 \text{ kN/m}^2\end{aligned}$$

Let us assume that the applied surface pressure of 4.0 t/m^2 gets transmitted to the middle of the clay layer undiminished.

$$\therefore \Delta \bar{\sigma} = 40 \text{ kN/m}^2$$

The compression index, C_c may be taken as:

$$C_c = 0.009 (w_L - 10)$$

$$\therefore C_c = 0.009 (65 - 10) = 0.495$$

The consolidation settlement, S , is given by:

$$\begin{aligned} S &= \frac{H \cdot C_c}{(1 + e_0)} \log_{10} \left(\frac{\bar{\sigma}_0 + \Delta \bar{\sigma}}{\bar{\sigma}_0} \right) \\ &= \frac{400 \times 0.495}{(1 + 1.325)} \log_{10} \frac{(36 + 40)}{36} \text{ cm} \\ &= \mathbf{27.64 \text{ cm.}} \end{aligned}$$

(b) Thickness of the laboratory sample = 25 mm.

Since it is two-way drainage with porous discs on either side, the drainage path,

$$H = 25/2 = 12.5 \text{ mm.}$$

Time for 90% primary compression, $t_{90} = 81$ minutes.

Time factor, T_{90} , for $U = 90\%$ is known to be 0.848.

$$\begin{aligned} \text{(Alternatively, } T &= -0.9332 \log_{10} (1 - U) - 0.0851 \\ &= -0.9332 \log_{10} 0.10 - 0.0851 = 0.9332 - 0.0851 = 0.8481 \end{aligned}$$

$$\therefore T_{90} = \frac{C_v t_{90}}{H^2}$$

Coefficient of consolidation

$$\begin{aligned} C_v &= \frac{T_{90} \cdot H^2}{t_{90}} = \frac{0.848 \times (125)^2}{81} \text{ cm}^2/\text{min.} \\ &= \frac{0.848 \times (125)^2}{81 \times 60} \text{ cm}^2/\text{s} \\ &= \mathbf{2.726 \times 10^{-4} \text{ cm}^2/\text{s.}} \end{aligned}$$