Mechanics of Materials-II

THEORIES OF FAILURES

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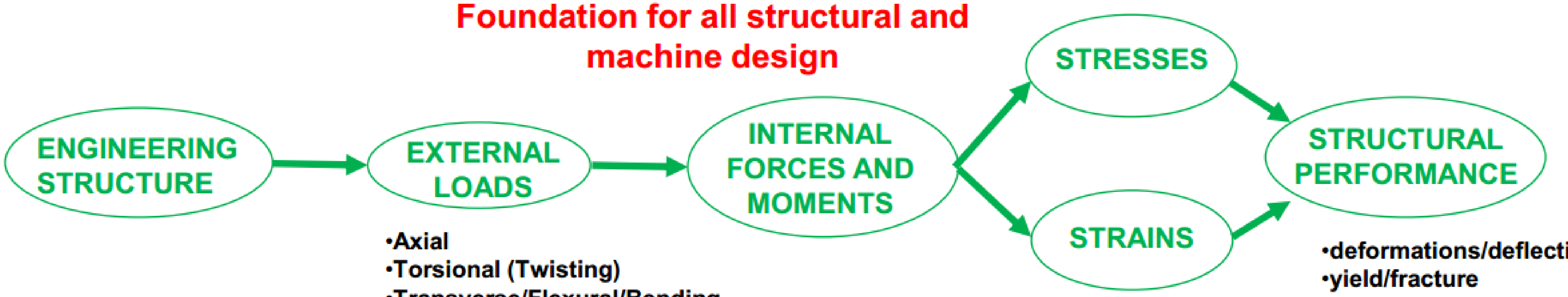
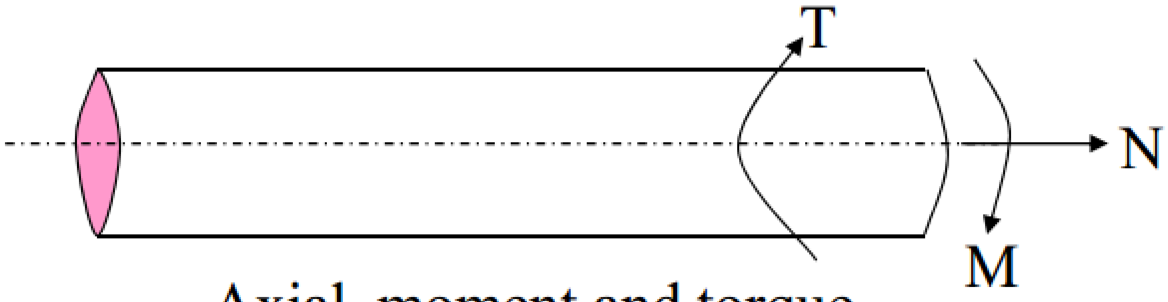
MOM (Course Outcomes)

Foundation for all structural and machine design

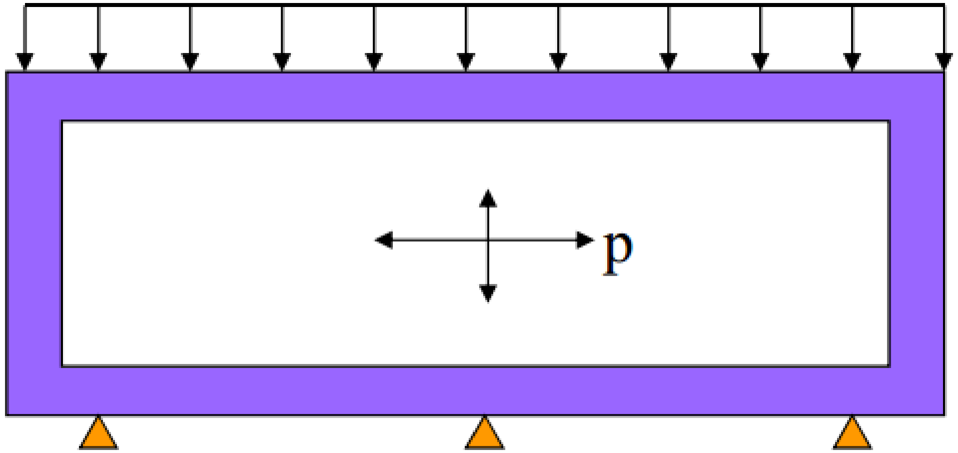


- Axial
- Torsional (Twisting)
- Transverse/Flexural/Bending
- Pressure
- Combined Loading

- deformations/deflections
- yield/fracture
- success/failure?

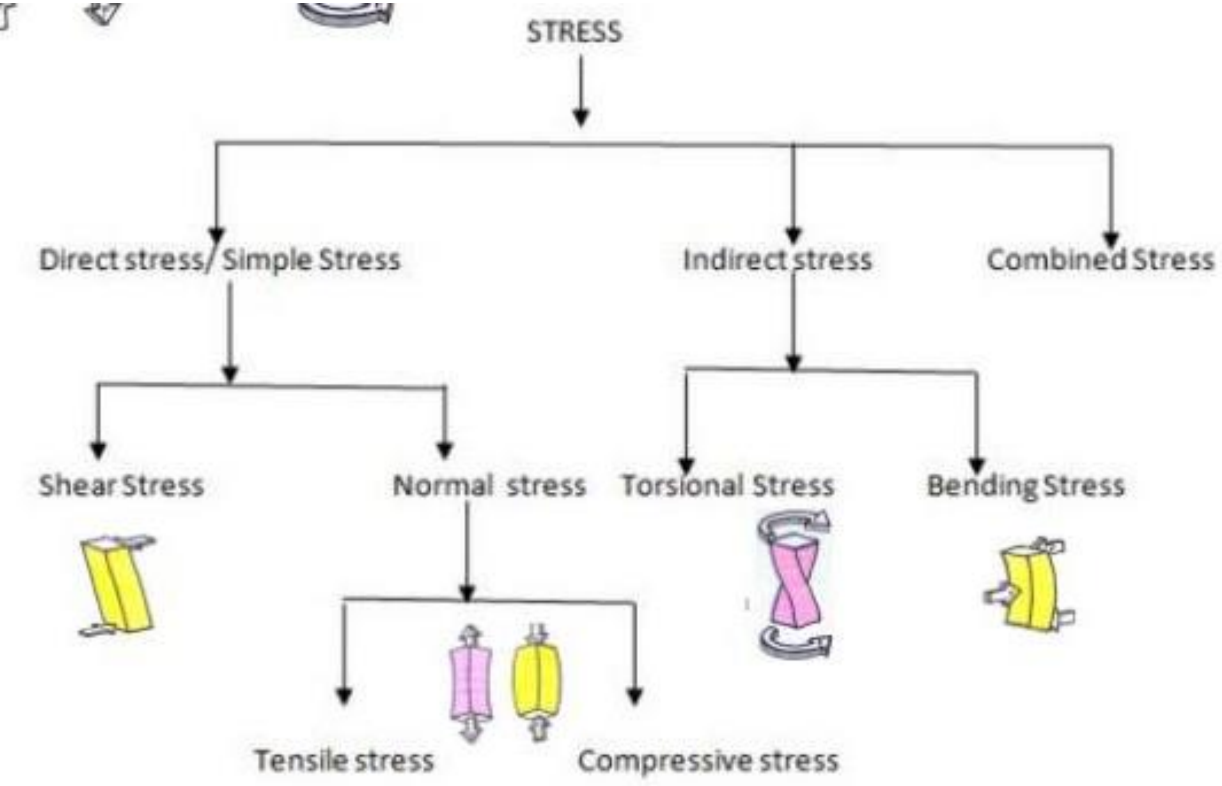


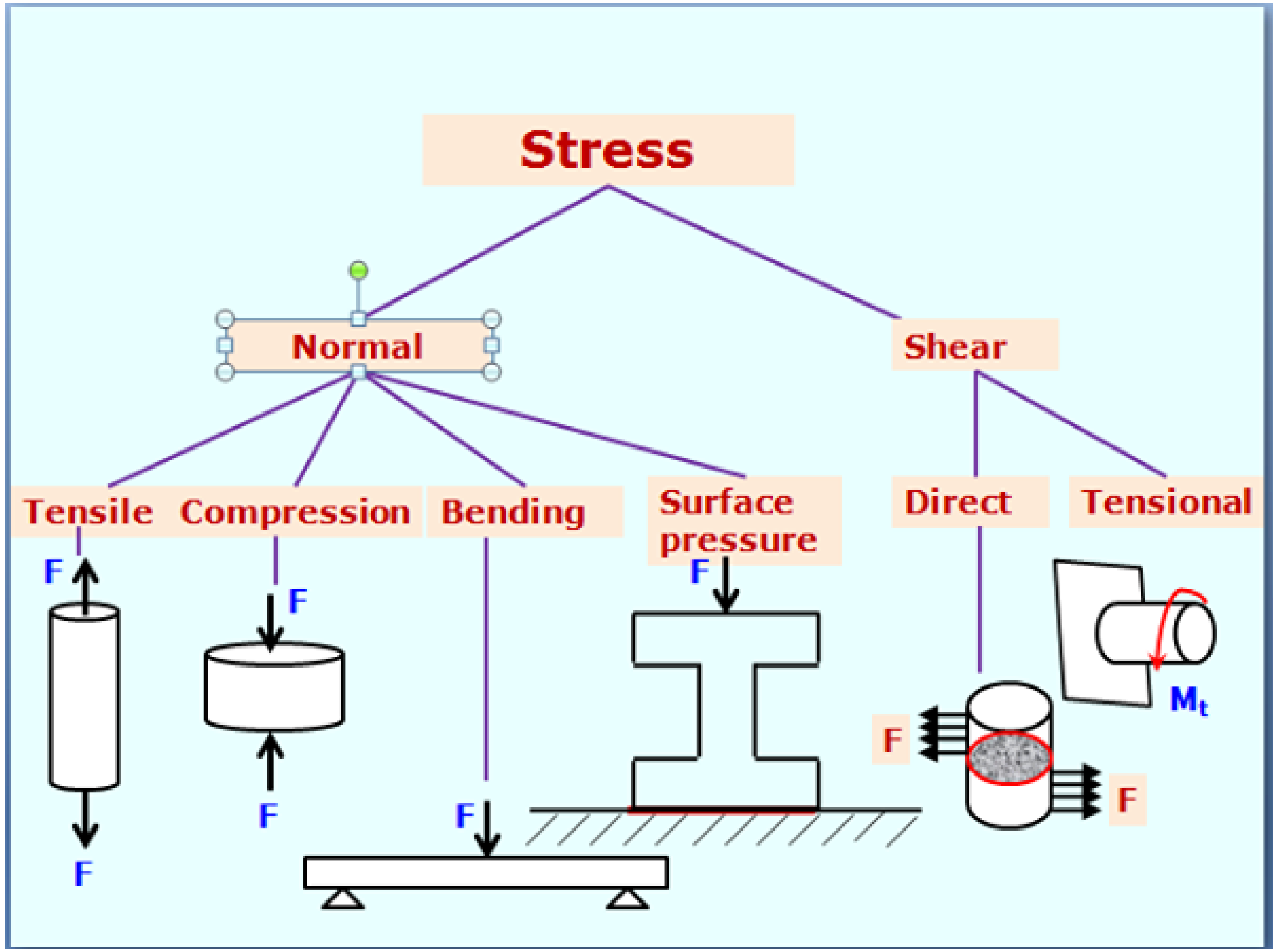
Axial, moment and torque



Internal pressure and external UDL

Theories of Failures



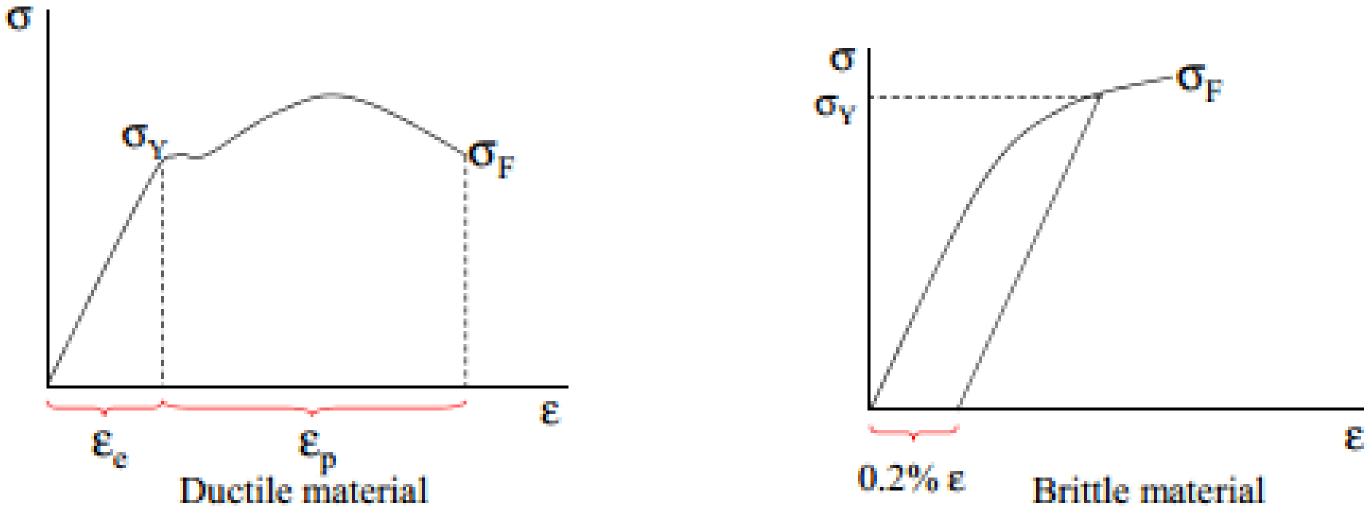


Introduction

- Failure occurs when material starts exhibiting inelastic behavior
- Brittle and ductile materials – different modes of failures – mode of failure – depends on loading
- Ductile materials – **exhibit yielding** – plastic deformation before failure
- Yield stress – material property
- Brittle materials – **no yielding – sudden failure**
- Factor of safety (FS)

Introduction

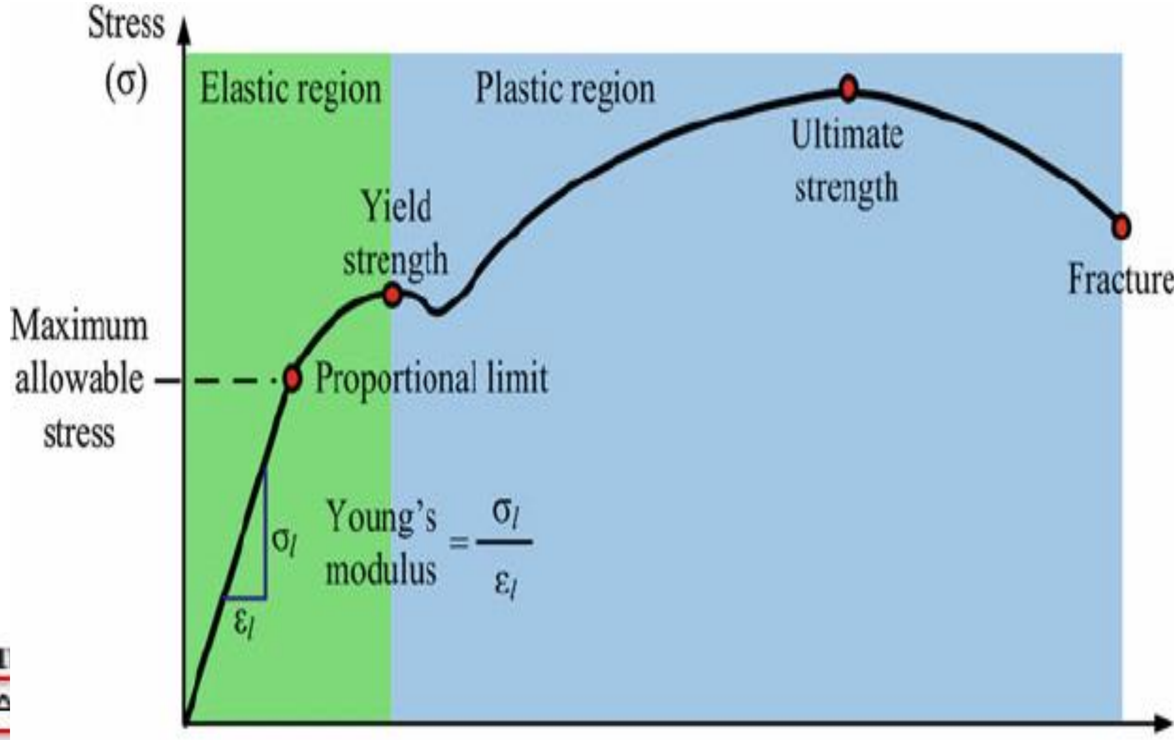
- Ductile and brittle materials



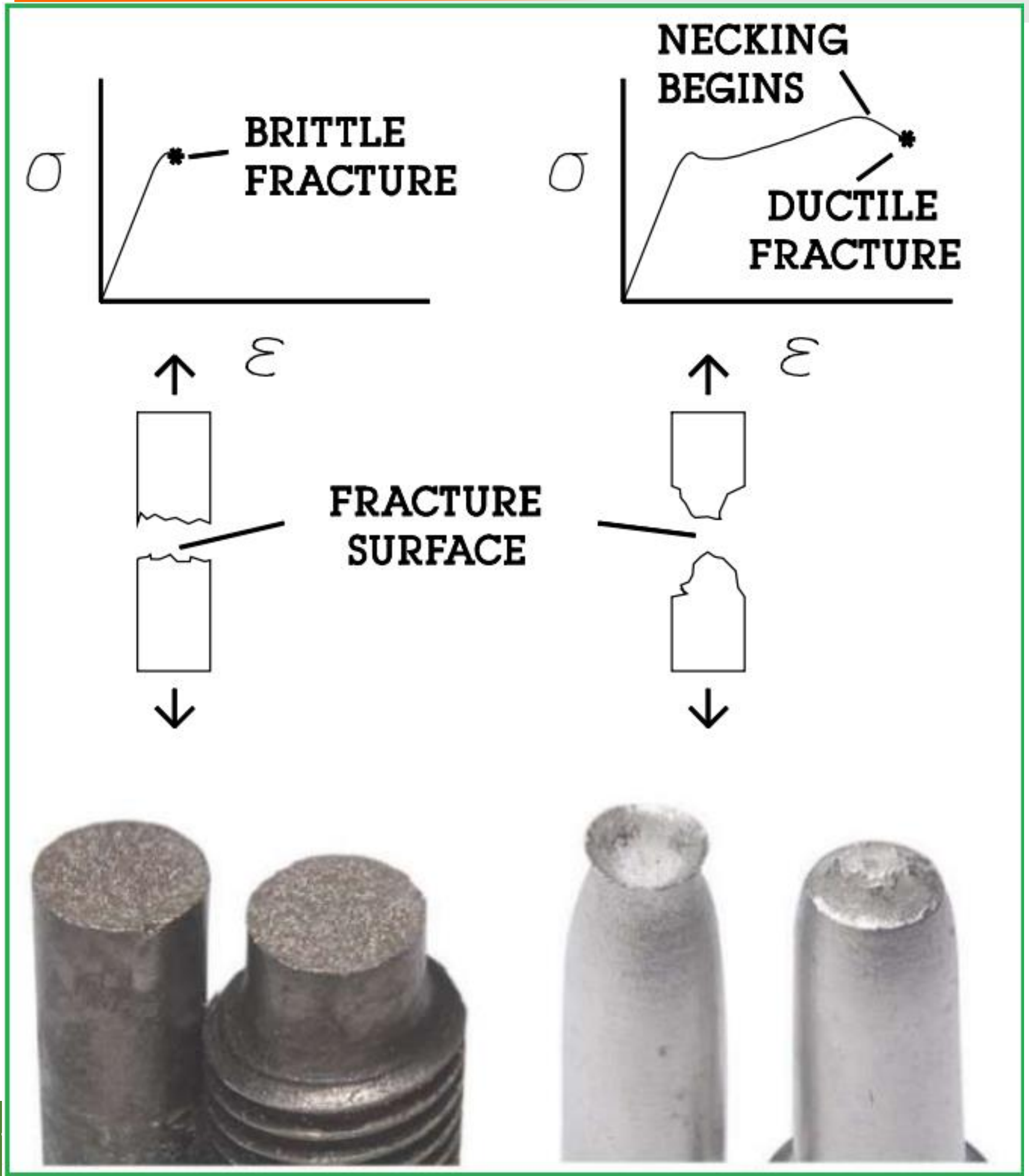
Well – defined yield point in ductile materials – FS on yielding

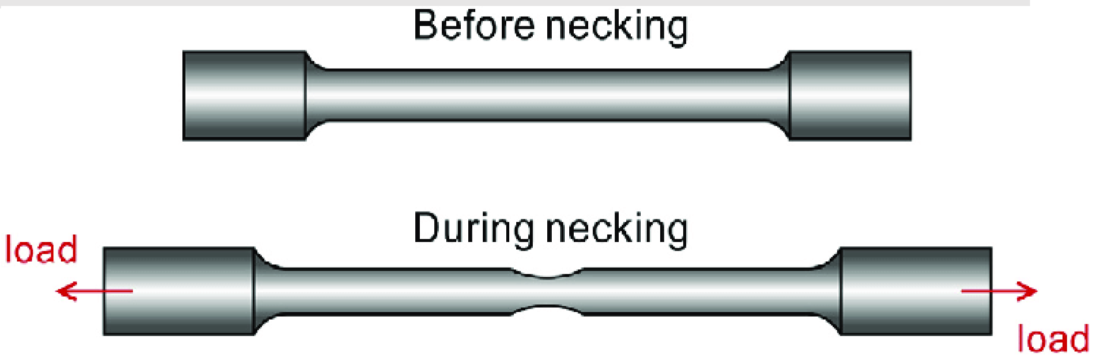
No yield point in brittle materials sudden failure – FS on failure load

Material Characteristics

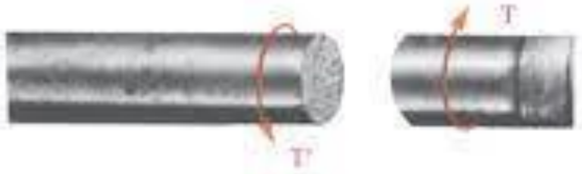


Material Characteristics





Material Characteristics



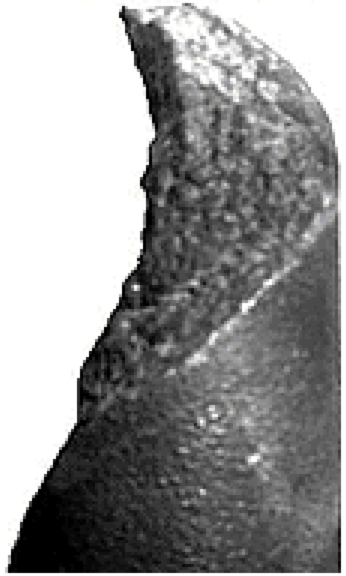
(a) Ductile Failure

(b) Brittle Failure

Ductile Torsion Failure

Brittle Torsion Failure

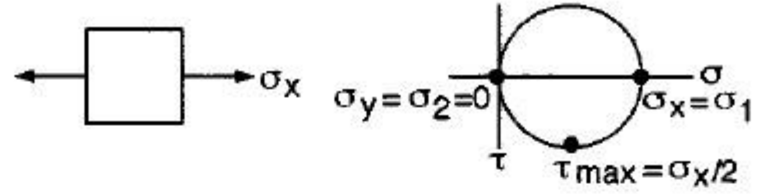
Tensile Failure
in Torsion

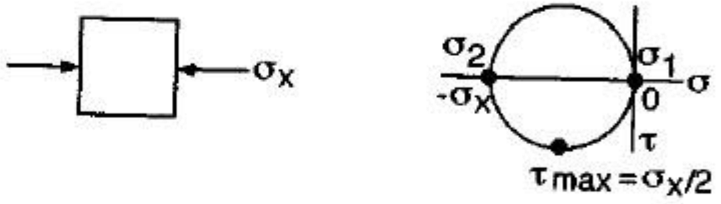


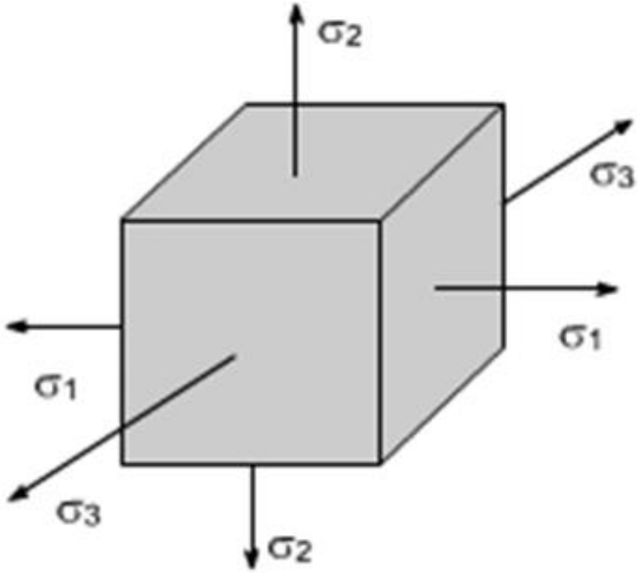
Shear Failure
in Torsion

Objective of Failure Theories

- In case of material subjected to simple state of stress (tension or compression), failure occurs when the stress in the material reaches the elastic limit stress.
- In case of material subjected to complex stresses, the stage of failure is determined either to practically or theoretically .
- Non-applicability of any one theory to all states of stresses and to all materials has resulted in propagation of different theories relating the complex stresses to elastic limit in simple tension or compression.
- Since the complex stress system can be simplified into **three principal stresses**, the problem reduced to linking the **three principal stresses to the stresses at elastic limit in case of simple stresses.**



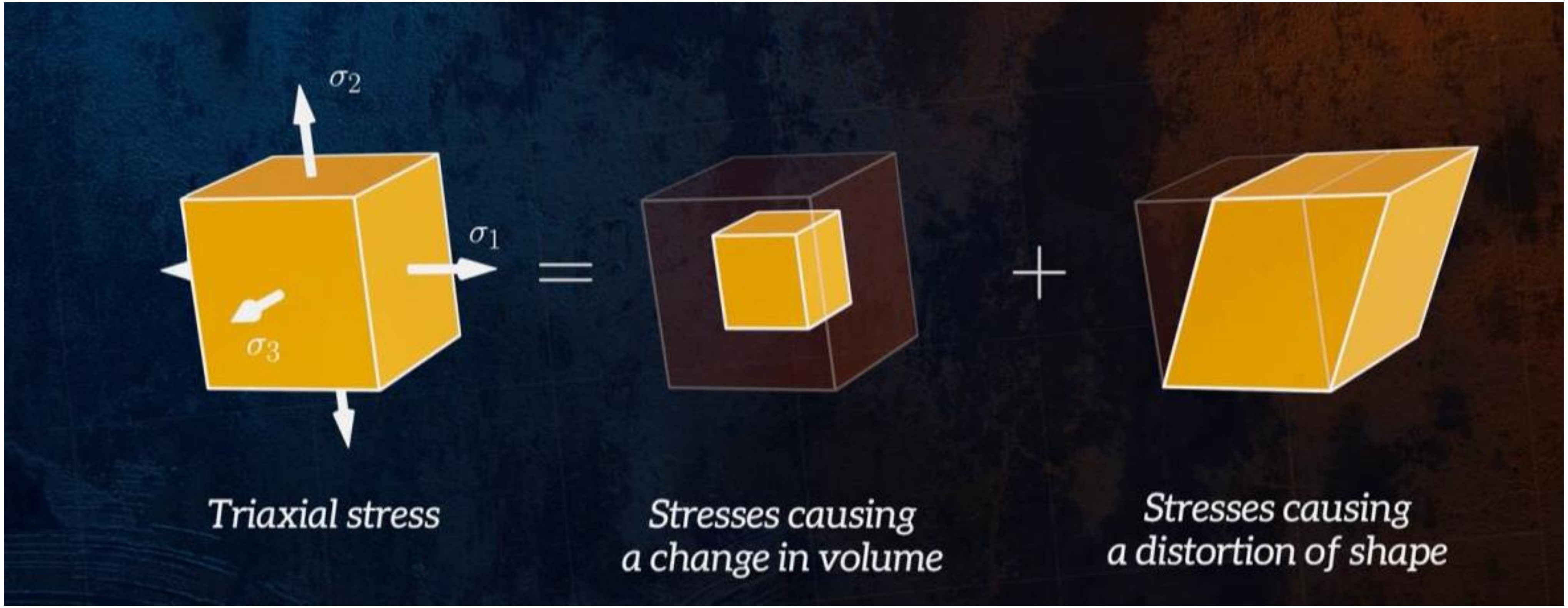




(Triaxial state of stress)

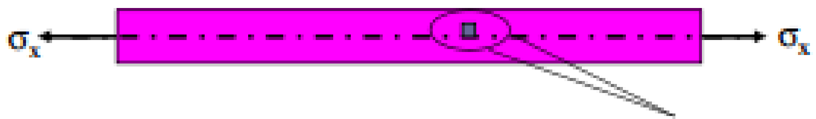
Material Characteristics

TRI AXIAL STRESS



Introduction

- Stress developed in the material < yield stress
- Simple axial load



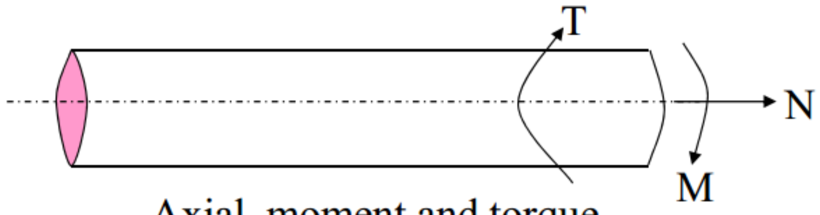
If $\sigma_x = \sigma_Y$ => yielding starts – failure

Yielding is governed by single stress component, σ_x

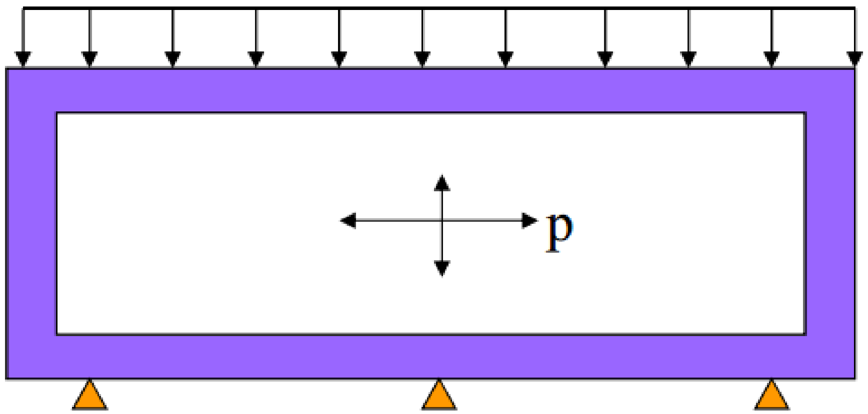
Similarly in pure shear – only shear stress.

If $\tau_{max} = \tau_Y$ => Yielding in shear

Multi-axial stress state ??



Axial, moment and torque



Internal pressure and external UDL

Introduction

- ➤ Multiaxial stress state – six stress components – one
- ➤ representative value
- ➤ Define effective / equivalent stress – combination of components of multiaxial stress state
- ➤ Equivalents stress reaching a limiting value – property of material – yielding occurs – Yield criteria of material – yielding occurs – Yield criteria
- ➤ Yield criteria define conditions under which yielding occurs
- ➤ Single yield criteria – doesn't cater for all materials

- Maximum principal stress

Applied stress => Y

$\sigma_1 = Y, \sigma_2 = 0, \sigma_3 = 0$

- Maximum shear stress

$$\tau_{max} = \left|\frac{\sigma_1 - \sigma_3}{2}\right| = \frac{Y}{2}$$

- Maximum principal strain

$\sigma_1 = Y, \sigma_2 = 0, \sigma_3 = 0$

$$\varepsilon_Y = \frac{\sigma_1}{E} - \frac{\upsilon}{E}(\sigma_2 + \sigma_3) = \frac{Y}{E}$$

- Total strain energy density

Linear elastic material $\quad U = \frac{1}{2} Y \varepsilon_Y = \frac{1}{2}\frac{Y^2}{E}$

Failure Theories

The common most theories are

- Maximum principal stress theory (Rankine)
- Maximum principal strain theory (Saint- Venant)
- Maximum Shear Stress theory (Guest- Tresca)
- Maximum strain energy theory (Beltrami-Haigh)
- Maximum Shear Strain energy or Distortion energy theory(Von mises-Henky)

1. Maximum principal Stress Theory

- Maximum principal stress reaches tensile yield stress (Y)
- For a given stress state, calculate principle stresses, σ_1, σ_2 and σ_3
- Yield function

$$f = \max\left(|\sigma_1|, |\sigma_2|, |\sigma_3|\right) - Y$$

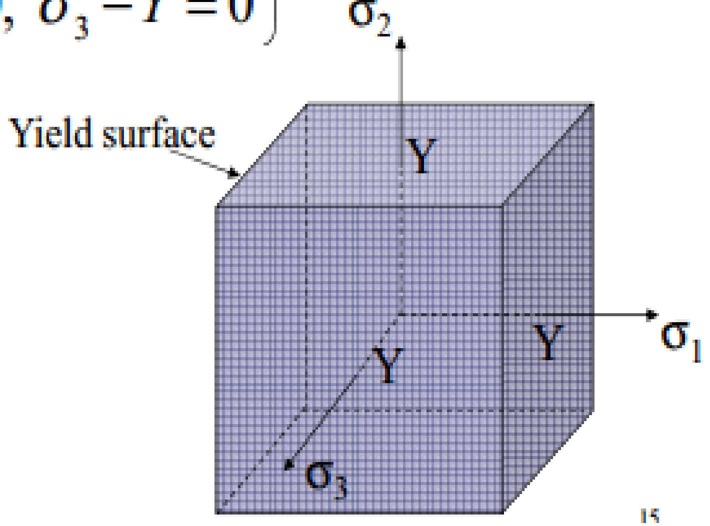
If, $f < 0$ no yielding

$f = 0$ onset of yielding

$f > 0$ not defined

- Yield surface –

$$\left.\begin{aligned} \sigma_1 &= \pm Y \Rightarrow \sigma_1 + Y = 0,\ \sigma_1 - Y = 0 \\ \sigma_2 &= \pm Y \Rightarrow \sigma_2 + Y = 0,\ \sigma_2 - Y = 0 \\ \sigma_3 &= \pm Y \Rightarrow \sigma_3 + Y = 0,\ \sigma_3 - Y = 0 \end{aligned}\right\} \text{Represent six surfaces}$$



Yield strength – same in tension and compression

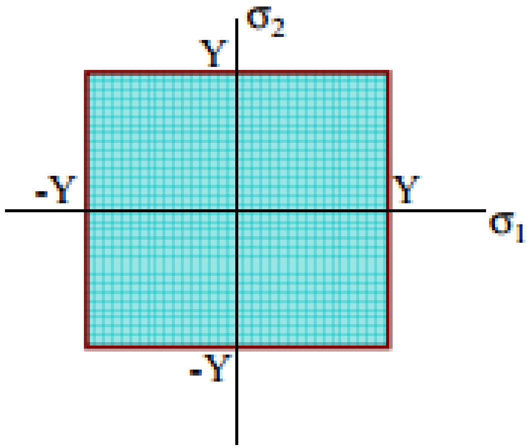
- Working stress, $\sigma = \dfrac{\sigma_y}{F}$

F : Factor of safety

1. Maximum principal Stress Theory

- In 2D case, $\sigma_3 = 0$ – equations become

$$\sigma_1 = \pm Y \Rightarrow \sigma_1 + Y = 0,\ \sigma_1 - Y = 0$$
$$\sigma_2 = \pm Y \Rightarrow \sigma_2 + Y = 0,\ \sigma_2 - Y = 0$$



Closed curve

Stress state inside – elastic, outside => Yielding

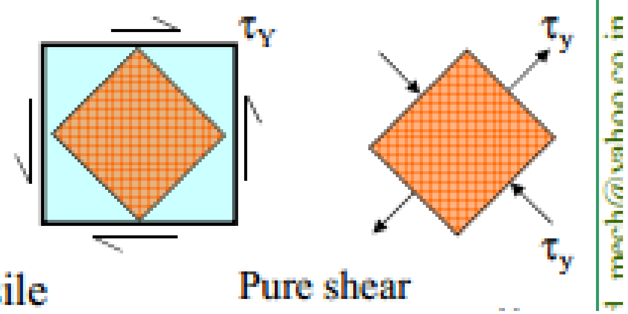
Pure shear test => $\sigma_1 = +\tau_Y$, $\sigma_2 = -\tau_Y$

For tension => $\sigma_1 = +\sigma_Y$

From the above => $\sigma_Y = \tau_Y$

Experimental results – Yield stress in shear is less than yield stress in tension

Predicts well, if all principal stresses are tensile



1. Maximum principal Stress Theory

- "Failure occurs when any one of the three **principal stresses reach the yield or ultimate strengths of the material determined from a uniaxial tension or compression test.**"
- Principal stresses

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

- **Condition for failure:**

Maximum principal stress ≥ Failure stress in Tension test

|Minimum principal stress| ≥ failure stress in compression test

For complex state of stress

Max [|σ1| , |σ2| , |σ3|] > σ_{yt} or σ_{ut}

For planer (biaxial) state of stress, σ3 = 0

Max [|σ1| , |σ2|] > σ_{yt}

1. Maximum principal Stress Theory- Limitations

- On a mild steel specimen when spiel tension test is carried out sliding occurs approximately 45^o to the axis of the specimen; this shows that the failure in this case is due to maximum shear stress rather than the direct tensile stress.
- It has been found that a material which is even though weak in simple compression yet can sustain hydrostatic pressure for in excess of the elastic limit in simple compression

1. Maximum principal Stress Theory- Limitations

- Maximum principal stress ≤ Permissible stress
- Permissible stress = Failure stress in Tension test / Factor of Safety

Tension	σ_{yt} (ductile material)	σ_{ut} (brittle material)
$\sigma 1 > \sigma 2$	$\sigma 1 = \sigma_{yt}$	$\sigma 1 = \sigma_{ut}$
$\sigma 2 > \sigma 1$	$\sigma 2 = \sigma_{yt}$	$\sigma 2 = \sigma_{ut}$
Compression	σ_{yc} **(ductile material)**	σ_{uc} **(brittle material)**
$\sigma 1 > \sigma 2$	$\sigma 1 = -\sigma_{yt}$	$\sigma 1 = -\sigma_{uc}$
$\sigma 2 > \sigma 1$	$\sigma 2 = -\sigma_{yt}$	$\sigma 2 = -\sigma_{uc}$

1. Problem

1. A machine element is subjected to the following stresses: σx = 60 MPa, σy = 45MPa and τxy = 30MPa. Find the factor of safety if it is made of steel having yield stress as 353 MPa using maximum principal stress theory. (Take poisson's ratio as 0.3)

(i) According to maximum principal stress or Rankine's theory of equivalent stress

$$\sigma_e = \frac{1}{2} = \left[(\sigma_x + \sigma_y) + \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}\right]$$

$$\sigma_e = \frac{1}{2} = \left[(60 + 45) + \sqrt{(60 - 45)^2 + 4(30)^2}\right] = 83.42 \text{ MPa}$$

$$\therefore \quad \text{FOS} = \frac{\sigma_{ys}}{\sigma_e} = \frac{353}{83.42} = 4.23$$

Problem-2

- A bolt is subjected to a tensile load of 18kN and a shear load of 12kN. The material has an yield stress of 328.6 MPa. Taking factor of safety as 2.5, determine the core diameter of bolt according to the rankine's theory. Take poisson's ratio as 0.298

Tensile load, $F_T = 18 \text{ kN} = 18 \times 10^3 \text{ N}$

Shear load, $F_s = 12 \text{ kN} = 12 \times 10^3 \text{ N}$

Yield stress, $\sigma_{ys} = 328.6 \text{ MPa}$ FOS = 2.5

$$\text{Allowable stress, } \sigma_e = \frac{\sigma_{ys}}{\text{FOS}} = \frac{328.6}{2.5} = 131.44 \text{ MPa.}$$

$$\text{Tensile stress, } \sigma = \frac{F_T}{A} = \frac{18 \times 10^3}{A} = \sigma_x$$

$$\text{Shear stress, } \tau = \frac{F_s}{A} = \frac{12 \times 10^3}{A} = \tau_{xy}$$

$(\sigma_y = 0$, not given)

Problem-2

- A bolt is subjected to a tensile load of 18kN and a shear load of 12kN. The material has an yield stress of 328.6 MPa. Taking factor of safety as 2.5, determine the core diameter of bolt according to the rankine's theory. Take poisson's ratio as 0.298

$$\sigma_e = \frac{1}{2}\left[\sigma_x + \sqrt{\sigma_x^2 + 4\tau_{xy}^2}\right]$$

$$131.44 = \frac{1}{2}\left[\frac{18\times10^3}{A} + \sqrt{\left(\frac{18\times10^3}{A}\right)^2 + 4\left(\frac{12\times10^3}{A}\right)^2}\right]$$

$$A = 182.59 = \frac{\pi d_c^2}{4}$$

Core dia, $dc = 15.25$ mm

Problem-3 (Assignment)

- Determine the diameter of a solid shaft as per the maximum normal stress theory subjected to a BM of 20 kN.m and Twisting Moment of 12 kN.m. The material of the shaft is yielded under axial tensile stress of 250 MPa. Take E=200 GPa and μ=0.25. Apply FOS as 2.5.

$$\mathrm{MI} = I = \frac{\pi}{64}d^4$$

$$\text{Polar MI} = J = \frac{\pi}{32}d^4$$

$$\tau_{torsional} = \tau = \frac{T}{J}r_{max} = \frac{T}{\frac{\pi}{32}d^4} \times \frac{d}{2} = \frac{16T}{\pi d^3} = \frac{16 \times 12 \times 10^6}{\pi d^3} = \frac{1.92 \times 10^8}{\pi d^3}$$

Maximum bending normal stress at the extreme fibers

$$\sigma_{bending} = \sigma = \frac{M}{I}y_{max} = \frac{M}{\frac{\pi}{64}d^4} \times \frac{d}{2} = \frac{32M}{\pi d^3} = \frac{32 \times 20 \times 10^6}{\pi d^3} = \frac{6.4 \times 10^8}{\pi d^3}$$

Problem-3 (Assignment)

$$\sigma_{1,2} = \left\{\frac{\sigma_x + \sigma_y}{2}\right\} \pm \sqrt{\left\{\frac{\sigma_x - \sigma_y}{2}\right\}^2 + (\tau)^2}$$

$$\sigma_x = \frac{6.4 \times 10^8}{\pi d^3}\ \text{MPa}; \sigma_y = 0, \text{ and } \tau = \frac{1.92 \times 10^8}{\pi d^3}\ \text{MPa}$$

$$\sigma_1 = \left\{\frac{\frac{6.4 \times 10^8}{\pi d^3}}{2}\right\} + \sqrt{\left\{\frac{\frac{6.4 \times 10^8}{\pi d^3}}{2}\right\}^2 + \left\{\frac{1.92 \times 10^8}{\pi d^3}\right\}^2} = \frac{6.93 \times 10^8}{\pi d^3}$$

$$\sigma_2 = \left\{\frac{\frac{6.4 \times 10^8}{\pi d^3}}{2}\right\} - \sqrt{\left\{\frac{\frac{6.4 \times 10^8}{\pi d^3}}{2}\right\}^2 + \left\{\frac{1.92 \times 10^8}{\pi d^3}\right\}^2} = -\frac{0.53 \times 10^8}{\pi d^3}$$

for this combined bending and torsion is

$$[\sigma] = \frac{10^8}{\pi d^3}\begin{bmatrix} 6.93 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -0.53 \end{bmatrix}$$

Problem-3 (Assignment)

Allowable stress will be $\dfrac{\text{failure stress}}{\text{FOS}} = \dfrac{250}{2.5} = 100 \text{ MPa}$

Now, for design of the shaft $\sigma_1 = \dfrac{\sigma_f}{\text{FOS}} = \dfrac{250}{2.5} = 100 \text{ MPa}$

$$\frac{6.93 \times 10^8}{\pi d^3} = 100$$

$$d = 130.2 \text{ mm}$$

Provide a shaft of diameter more than 130.2 mm.

Maximum Principal Strain Theory (Saint Venant's)

- Yielding occurs when maximum **principal strain just exceeds the strain at the tensile yield point in either simple tension or compression**
- If ε_1 and ε_3 are maximum & minimum principal strains corresponding to principal stresses
- $\varepsilon_1 = 1/E(\sigma_1 - \mu\sigma_2 - \mu\sigma_3)$
- $\varepsilon_3 = 1/E(\sigma_3 - \mu\sigma_1 - \mu\sigma_2)$

- Strain due to yield stress in simple tension = 1/E *x* yield stress in tension= 1/E *x* $\boldsymbol{\sigma_{yt}}$
- Strain due to yield stress in simple compression = 1/E *x* yield stress in compression

  = 1/E *x* $\boldsymbol{\sigma_{yc}}$
- Failure happens when $\boldsymbol{\varepsilon 1 \geq \sigma_{yt} /E}$ **or** $\boldsymbol{|\varepsilon 3| \geq \sigma_{yc} /E}$

Maximum Principal Strain Theory (Saint Venant's)

- "Failure occurs at a point in a body when the maximum strain at that point exceeds the value of the maximum strain in a uniaxial test of the material at yield point"
- 'Y' – yield stress in uniaxial tension, yield strain, $\varepsilon_y = Y/E$
- The maximum strain developed in the body due to external loading should be less than this
- Principal stresses => σ_1, σ_2 and σ_3 strains corresponding to these stress => ε_1, ε_2 and ε_3

Maximum Principal Strain Theory (Saint Venant’s)

Strains corresponding to principal stresses -

$$\varepsilon_1 = \frac{\sigma_1}{E} - \frac{\upsilon}{E}(\sigma_2 + \sigma_3)$$

$$\varepsilon_2 = \frac{\sigma_2}{E} - \frac{\upsilon}{E}(\sigma_1 + \sigma_3)$$

$$\varepsilon_3 = \frac{\sigma_3}{E} - \frac{\upsilon}{E}(\sigma_2 + \sigma_1)$$

Maximum of this should be less than ε_y

For onset of yielding

$$|\varepsilon_1| = \frac{Y}{E} \Rightarrow \sigma_1 - \upsilon(\sigma_2 + \sigma_2) = \pm Y$$

$$|\varepsilon_2| = \frac{Y}{E} \Rightarrow \sigma_2 - \upsilon(\sigma_3 + \sigma_1) = \pm Y$$

$$|\varepsilon_3| = \frac{Y}{E} \Rightarrow \sigma_3 - \upsilon(\sigma_1 + \sigma_2) = \pm Y$$

There are six equations – each equation represents a plane

To prevent failure

$$\sigma_1 - \mu(\sigma_2 + \sigma_3) < \sigma_{yt}$$

$$\sigma_3 - \mu(\sigma_1 + \sigma_2) < \sigma_{yc}$$

At the point of failure

$$\sigma_1 - \mu(\sigma_2 + \sigma_3) = \sigma_{yt}$$

$$|\sigma_3 - \mu(\sigma_1 + \sigma_2)| = \sigma_{yc}$$

For the design purposes

$$\sigma_3 - \mu(\sigma_1 + \sigma_2) = \sigma_t$$

$$\sigma_3 - \mu(\sigma_1 + \sigma_2) = \sigma_c$$

Where σ_t and σ_c are the safe stresses.

Maximum Principal Strain Theory (Saint Venant's)

Equations – $\sigma_1 - \upsilon\sigma_2 = Y,\ \sigma_1 - \upsilon\sigma_2 = -Y$

$$\sigma_2 - \upsilon\sigma_1 = Y,\ \sigma_2 - \upsilon\sigma_1 = -Y$$

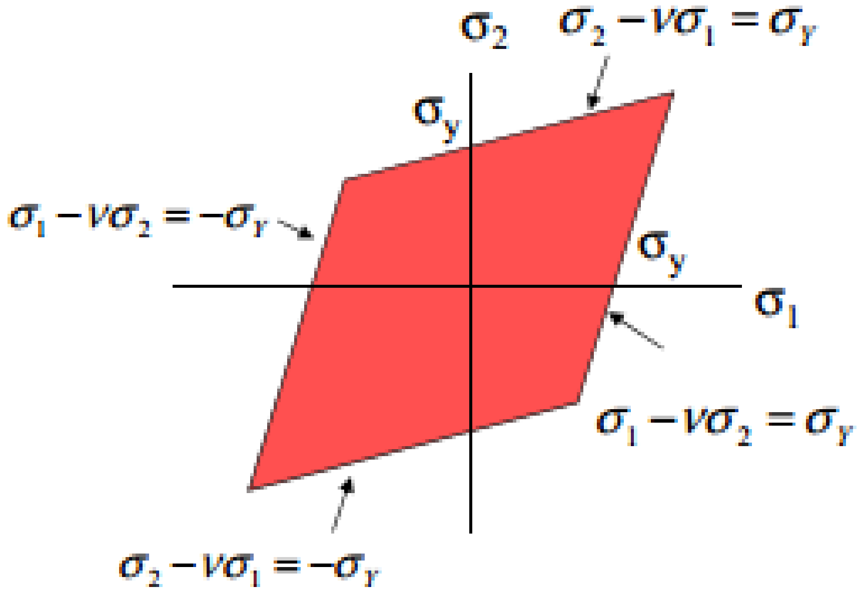
- For 2D case

$$|\sigma_1 - \upsilon\sigma_2| = Y \Rightarrow \sigma_1 - \upsilon\sigma_2 = \pm Y$$

$$|\sigma_2 - \upsilon\sigma_1| = Y \Rightarrow \sigma_2 - \upsilon\sigma_1 = \pm Y$$

There are four equations, each equation represents a straight line in 2D stress space

Plotting in stress space



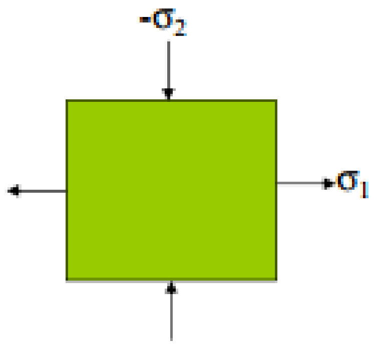
Failure – equivalent stress falls outside yield surface

This theory is more appropriate for ductile materials, brittle materials and materials under hydrostatic pressure.

It does not fit well with the experimental results.

Maximum Principal Strain Theory-Limitations

- **Biaxial loading**



For onset of yielding –

$$Y = \sigma_1 - \upsilon\sigma_2 = \sigma(1+\upsilon)$$
$$Y = \sigma(1+\upsilon)$$

Maximum principal stress theory –

$$\sigma_1 = |\sigma_2| = \sigma$$

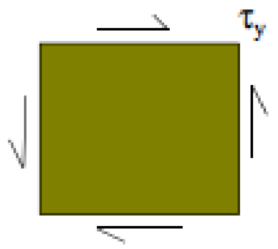
$$Y = \sigma$$

Max. principal strain theory predicts smaller value of stress

than max. principal stress theory

Conservative design

- **Pure shear**



Principal stresses corresponding to shear yield stress

$$\sigma_1 = +\tau_y, \ \sigma_2 = -\tau_y$$

For onset of yielding – max. principal strain theory

$$Y = \tau_y + \upsilon\,\tau_y = \tau_y(1+\upsilon)$$

Relation between yield stress in tension and shear

$$\tau_y = Y/(1+\upsilon) \text{ for } \upsilon = 0.25$$

$\boldsymbol{\tau_y = 0.8Y}$ Not supported by experiments

- The theory overestimates the behaviour of ductile materials
- The theory does no fit well with the experimental results except for brittle materials for biaxial tension
- This theory cannot explain the failure mechanism of a body under hydrostatic condition

Problem-1

A machine element is subjected to the following stresses: $\sigma_x = 60$ MPa, $\sigma_y = 45$MPa and $\tau_{xy} = 30$MPa. Find the factor of safety if it is made of steel having yield stress as 353 MPa using maximum principal strain theory. (Take poisson's ratio as 0.3)

Problem-2

A bolt is subjected to a tensile load of 18kN and a shear load of 12kN. The material has an yield stress of 328.6 MPa. Taking factor of safety as 2.5, determine the core diameter of bolt according to the saint-venant’s theory. Take poisson’s ratio as 0.298

Problem-3 (Assignment)

Determine the diameter of a solid shaft as per the maximum normal strain theory subjected to a BM of 20 kN.m and Twisting Moment of 12 kN.m. The material of the shaft is yielded under axial tensile stress of 250 MPa. Take E=200 GPa and μ=0.25. Apply FOS as 2.5.

Problem-3 (Assignment)

The stress tensor for this combined bending and torsion is

$$[\sigma] = \frac{10^8}{\pi d^3}\begin{bmatrix} 6.93 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -0.53 \end{bmatrix}$$

In this case, maximum normal strain = $\epsilon_{max} = \frac{1}{E}\{\sigma_1 - \mu(\sigma_2 + \sigma_3)\}$

$$\epsilon_{max} = \frac{\frac{10^8}{\pi d^3}}{200 \times 1000}\{6.93 - 0.25(0 - 0.53)\} = \frac{3.53 \times 10^3}{\pi d^3}$$

In case of axial tension, $\sigma_1 = \sigma_f$; and $\sigma_2 = \sigma_3 = 0$.

Applying an FOS 2.5

Allowable stress will be $\frac{\text{failure stress}}{\text{FOS}} = \frac{250}{2.5} = 100$ MPa

Now, for design of the shaft $\sigma_1 = \frac{\sigma_f}{\text{FOS}} = \frac{250}{2.5} = 100$ MPa

Maximum normal strain is $\epsilon_{max} = \frac{1}{E}\{\sigma_1 - \mu(\sigma_2 + \sigma_3)\} = \frac{\sigma_f}{E} = \frac{100}{200 \times 1000} = 5 \times 10^{-4}$

For design, maximum normal strain in the above cases shall be equated.

Maximum Shear Stress Theory(Guest-Treasca)

- This theory also called Coulomb Guest's or Treasca's Theory
- This theory implies that failure will occur when the maximum shear stress τ_{max} in the complex system reaches the value of the maximum shear stress in simple tension at elastic limit
- Shear stress in complex state $\tau_{max} =$
- Maximum shear stress in simple tension at elastic limit $\tau_{max} =$

Maximum Shear Stress Theory(Guest-Treasca)

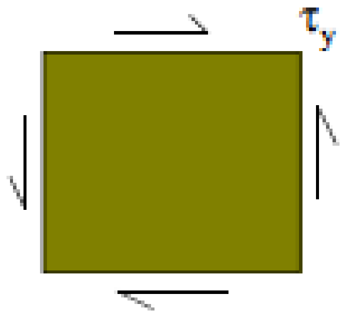
- "Yielding begins when the maximum shear stress at a point equals the maximum shear stress at yield in a uniaxial tension"

$$\tau_{\max} = K_T = \left|\frac{\sigma_1 - \sigma_2}{2}\right| \Rightarrow \tau_{\max} = \frac{Y}{2} = K_T$$

If maximum shear stress < Y/2 => No failure occurs

For pure shear, $\sigma_1 = +\tau_y$, $\sigma_2 = -\tau_y$

$$\tau_{\max} = K_T = \left|\frac{\sigma_1 - \sigma_2}{2}\right| \Rightarrow \tau_{\max} = \tau_y = K_T$$



Shear yield = 0.5 Tensile yield

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Maximum Shear Stress Theory(Guest-Treasca)

- Following equations are obtained

$$\left|\frac{\sigma_1-\sigma_2}{2}\right| = K_T \Rightarrow \frac{\sigma_1-\sigma_2}{2} = \pm K_T$$

$$f_1(\sigma_1,\sigma_2) = \sigma_1-\sigma_2-2K_T; \quad f_2(\sigma_1,\sigma_2) = \sigma_1-\sigma_2+2K_T$$

$$\left|\frac{\sigma_2-\sigma_3}{2}\right| = K_T \Rightarrow \frac{\sigma_2-\sigma_3}{2} = \pm K_T$$

$$f_3(\sigma_2,\sigma_3) = \sigma_2-\sigma_3-2K_T; \quad f_4(\sigma_2,\sigma_3) = \sigma_2-\sigma_3+2K_T$$

$$\left|\frac{\sigma_3-\sigma_1}{2}\right| = K_T \Rightarrow \frac{\sigma_3-\sigma_1}{2} = \pm K_T$$

$$f_5(\sigma_3,\sigma_1) = \sigma_3-\sigma_1-2K_T; \quad f_6(\sigma_3,\sigma_1) = \sigma_3-\sigma_1+2K_T$$

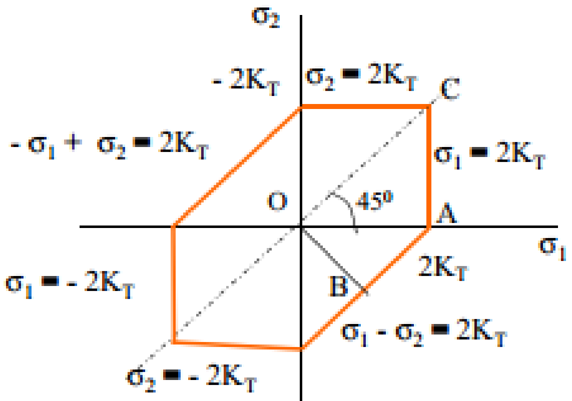
- 2D stress state - $\sigma_3 = 0$

Each equation represents two lines in 2D stress space

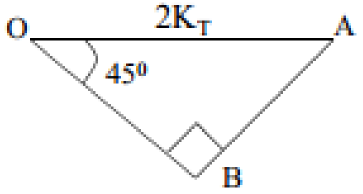
$$\sigma_1-\sigma_2 = \pm 2K_T$$

$$\sigma_2 = \pm 2K_T$$

$$\sigma_1 = \pm 2K_T$$



Yield curve – elongated hexagon



$$OB = OA\cos 45 = 2K_T\frac{1}{\sqrt{2}} = \sqrt{2}K_T$$

$$OC = \frac{OA}{\cos 45} = 2\sqrt{2}K_T$$

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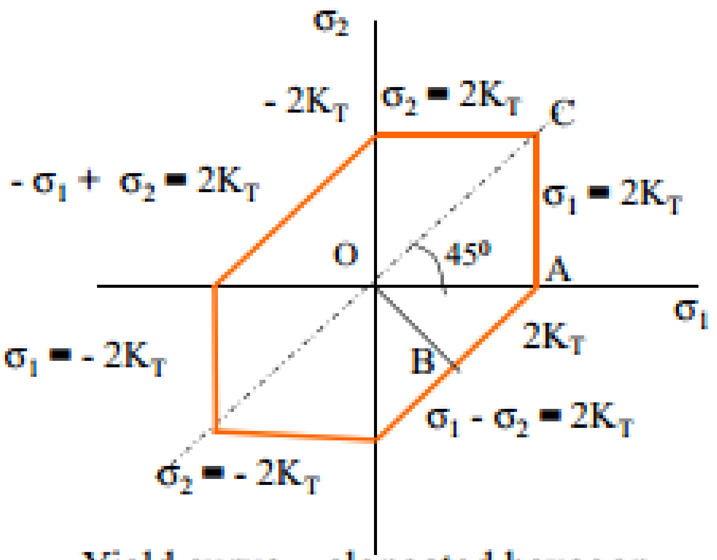
Maximum Shear Stress Theory(Guest-Treasca)

- 2D stress state - $\sigma_3 = 0$

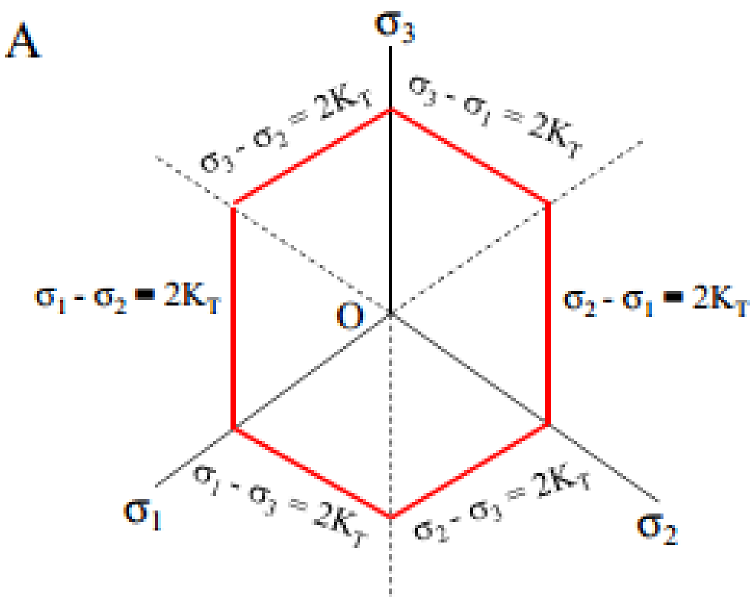
Each equation represents two lines in 2D stress space



Yield curve – elongated hexagon

$$\sigma_1 - \sigma_2 = \pm 2K_T$$
$$\sigma_2 = \pm 2K_T$$
$$\sigma_1 = \pm 2K_T$$

A



Maximum Shear Stress -Limitations

- The theory does not give accurate results for the state of stress of pure shear in which the maximum amount of shear is developed (i.e) Torsion test.
- The theory does not give us close results as found by experiments on ductile materials. However, it gives safe results (It gives satisfactory results for ductile materials particular in case shafts)
- The theory is not applicable in the case where the state of stress consists of triaxial tensile stresses of nearly equal magnitude

Problem-1

A mild steel shaft having yield stress as 232 MPa is subjected to following stresses. σx = 120 MPa,σy = -60 MPa and τxy = 36 MPa. Find factor of safety using Guest's theory.

Problem-2

- Determine the diameter of a solid shaft as per the maximum normal strain theory subjected to a BM of 20 kN.m and Twisting Moment of 12 kN.m. The material of the shaft is yielded under axial tensile stress of 250 MPa. Take E=200 GPa and μ=0.25. Apply FOS as 2.5.

this case, $\tau_{max} = \frac{1}{2}\{\sigma_{max} - \sigma_{min}\} = \frac{1\times10^8}{2\pi d^3}(6.93-(-0.53)) = \frac{7.46\times10^8}{2\pi d^3}$

case of axial tension, $\sigma_1 = \sigma_f;\ \sigma_2 = \sigma_3 = 0$.

plying an FOS 2.5

llowable stress will be $\sigma_f = \frac{\text{failure stress}}{\text{FOS}} = \frac{250}{2.5} = 100\text{ MPa}$

$$\tau_{max\,f} = \frac{1}{2}\{\sigma_{max} - \sigma_{min}\} = \frac{\sigma_f}{2} = \frac{100}{2}$$

r design equating maximum shear stress from two cases

$$\frac{100}{2} = \frac{7.46\times10^8}{2\pi d^3}$$

$$d = 133.41\text{ mm}$$

Provide a shaft of diameter more than 133.41 mm.

Problem-3 (Assignment)

- **A shaft is subjected to a maximum torque of 10KN-m and a maximum bending moment of 8KN-m at perpendicular section. if the allowable equivalent stress in simple is 160MN/m2 , find the diameter of the shaft according to the maximum shear stress theory**

Problem-4

A machine element is subjected to the following stresses: $\sigma x = 60$ MPa, $\sigma y = 45$MPa and $\tau xy = 30$MPa. Find the factor of safety if it is made of steel having yield stress as 353 MPa using maximum shear stress theory. (Take poisson's ratio as 0.3)

Problem-5

A bolt is subjected to a tensile load of 18kN and a shear load of 12kN. The material has an yield stress of 328.6 MPa. Taking factor of safety as 2.5, determine the core diameter of bolt according to the Tresca's theory. Take poisson's ratio as 0.298

Maximum Strain Energy Theory(Beltrani-Haigh)

- Failure at any point in a body subjected to a state of stress begins only when the energy density absorbed at that point is equal to the energy density absorbed by the material when subjected to elastic limit in a uni-axial stress
- Haigh's theory
- $U = \frac{1}{2} \boldsymbol{\sigma} \boldsymbol{\varepsilon}$
- $U = \frac{1}{2} (\sigma)^2/E$

$$U = \int_0^{\varepsilon y} \sigma . d\varepsilon$$

- $U = \frac{1}{2} (\boldsymbol{\sigma_1} \boldsymbol{\varepsilon_1} + \boldsymbol{\sigma_2} \boldsymbol{\varepsilon_2} + \boldsymbol{\sigma_3} \boldsymbol{\varepsilon_3})$
- $\varepsilon_1 = 1/E(\sigma_1 - \mu\sigma_2 - \mu\sigma_3)$
- $\varepsilon_2 = 1/E(\sigma_2 - \mu\sigma_1 - \mu\sigma_3)$
- $\varepsilon_3 = 1/E(\sigma_3 - \mu\sigma_1 - \mu\sigma_2)$

Maximum Strain Energy Theory(Beltrani-Haigh)

- Assumption: strains are recoverable upto elastic limit and energy absorbed at failure is independent on stress system

- Total strain energy per unit volume in 3D-system

  $= 1/2E\,(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 \; -2\,\mu(\sigma_1\sigma_2 + \sigma_3\sigma_1 + \sigma_2\sigma_3\,)$

- Strain energy per unit volume corresponding at elastic limit = ½ $\boldsymbol{\sigma_{yt}\,\varepsilon_{yt}}$

  $\mathbf{= ½\,\sigma_{yt} \; x\,(\sigma_{yt}\,/E)}$

  $\mathbf{= 1/2E\,(\sigma_{yt}^2)}$

Maximum Strain Energy Theory(Beltrani-Haigh)

In a 3D stress system, the strain energy per unit volume is given by

$$U = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]$$

At the point of failure

$$\frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right] = \frac{\sigma_y^2}{2E}$$

$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) = \sigma_y^2$$

In $2D$ stress system ($\sigma_2 = 0$), the above equation reduce to

$$\sigma_1^2 + \sigma_3^2 - 2\mu\sigma_1\sigma_3 = \sigma_y^2$$

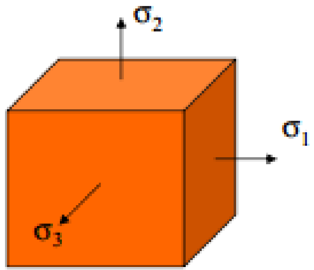
For the design

$$\sigma_1^2 + \sigma_3^2 - 2\mu\sigma_1\sigma_3 \le \sigma^2$$

Maximum Strain Energy Theory(Beltrani-Haigh)

- Body subjected to external loads => principal stresses

Strain energy associated with principal stresses



$$U = \frac{1}{2}(\sigma_1\varepsilon_1 + \sigma_2\varepsilon_2 + \sigma_3\varepsilon_3)$$

$$\varepsilon_1 = \frac{\sigma_1}{E} - \frac{\upsilon}{E}(\sigma_2 + \sigma_3)$$

$$\varepsilon_2 = \frac{\sigma_2}{E} - \frac{\upsilon}{E}(\sigma_3 + \sigma_1)$$

$$\varepsilon_3 = \frac{\sigma_2}{E} - \frac{\upsilon}{E}(\sigma_1 + \sigma_2)$$

$$U = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\upsilon(\sigma_1\sigma_2 + \sigma_3\sigma_1 + \sigma_2\sigma_3)\right]$$

For onset of yielding,

$$\frac{Y^2}{2E} = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\upsilon(\sigma_1\sigma_2 + \sigma_3\sigma_1 + \sigma_2\sigma_3\right.$$

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- Yield function –

$$f = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \upsilon(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) - Y^2$$

$$f = \sigma_e^2 - Y^2$$

Equivalent stress => $\sigma_e^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \upsilon(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)$

Yielding => $f = 0$, safe $f < 0$

- For 2D stress state => $\sigma_3 = 0$ – Yield function becomes

$$f = \sigma_1^2 + \sigma_2^2 - \upsilon\sigma_1\sigma_2 - Y^2$$

For onset of yielding => f = 0 $\quad \sigma_1^2 + \sigma_2^2 - \upsilon\sigma_1\sigma_2 - Y^2 = 0$

Plotting this in principal stress space

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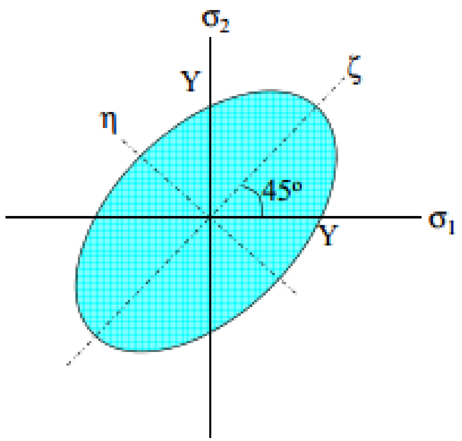
Maximum Strain Energy Theory(Beltrani-Haigh)

Rearrange the terms – $\left(\frac{\sigma_1}{Y}\right)^2+\left(\frac{\sigma_2}{Y}\right)^3-2\upsilon\left(\frac{\sigma_1}{Y}\frac{\sigma_2}{Y}\right)=1$



Equivalent stress inside – no failure

This represents an ellipse – Transform to ζ-η csys

$$\sigma_1=\zeta\cos 45-\eta\sin 45=\frac{1}{\sqrt{2}}(\zeta-\eta)$$

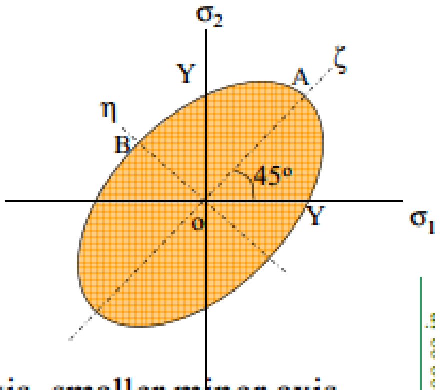
$$\sigma_2=\zeta\sin 45+\eta\cos 45=\frac{1}{\sqrt{2}}(\zeta+\eta)$$

Substitute these in the above expression

Simplifying, $\frac{\zeta^2}{\frac{Y^2}{(1-\upsilon)}}+\frac{\eta^2}{\frac{Y^2}{(1+\upsilon)}}=1 \Rightarrow \frac{\zeta^2}{a^2}+\frac{\eta^2}{b^2}=1$

Semi major axis – OA => $a=\frac{Y}{\sqrt{(1-\upsilon)}}$

Semi minor axis – OB => $b=\frac{Y}{\sqrt{(1+\upsilon)}}$



Higher Poisson ratio – bigger major axis, smaller minor axis

If $\upsilon = 0$ => circle of radius 'Y'

Maximum Strain Energy Theory-Limitations

- It is applicable for ductile materials particularly in case of pressure vessel
- The theory does not applicable to materials for which **σ_{yt} different from σ_{yc}**
- **The theory does not give results exactly equal to the experimental results even for ductile materials**

According to this theory material fails when

$$1/2E\,(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\,\mu(\sigma_1\sigma_2 + \sigma_3\sigma_1 + \sigma_2\sigma_3) \geq 1/2E\,(\sigma_{yt}^2)$$

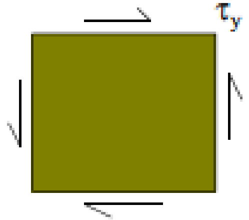
$$(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\,\mu(\sigma_1\sigma_2 + \sigma_3\sigma_1 + \sigma_2\sigma_3) \geq (\sigma_{yt}^2)$$

For design purpose, $(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\,\mu(\sigma_1\sigma_2 + \sigma_3\sigma_1 + \sigma_2\sigma_3) \geq (\sigma_{yt} / FOS)^2$

Maximum Strain Energy Theory(Beltrani-Haigh)

- Pure shear

Principal stresses corresponding to shear yield stress



$\sigma_1 = +\tau_y,\ \sigma_2 = -\tau_y$

$$\varepsilon_1 = \frac{\tau_y}{E}(1+\upsilon),\ \varepsilon_2 = -\frac{\tau_y}{E}(1+\upsilon)$$

Strain energy, $U_\tau = \frac{(1+\upsilon)}{2E} 2\tau_y^2 = \frac{1}{2E} Y^2 \Rightarrow Y = \sqrt{2(1+\upsilon)}\tau_y$

$$\tau_y = 0.632\ Y$$

Problem-1

A mild steel shaft having yield stress as 232 MPa is subjected to following stresses. σx = 120 MPa, σy = -60 MPa and τxy = 36 MPa. Find factor of safety using Haigh's theory.

Problem-2

- The principal stresses at a point in an elastic material are 200MPa (tensile), 100MPa(Tensile) and 50MPa (compressive). If the stress at the elastic limit in simple tension is 200 MPa. Determine whether the failure will occur or not according to maximum strain energy theory.

$\sigma_1 = 200$ N/mm^2 (tensile)

$\sigma_2 = 100$ N/mm^2 (tensile)

$\sigma_3 = 50$ N/mm^2 (compressive) $= -50$ N/mm^2

$\mu = 0.3$

Elastic limit in simple tension, $\sigma_t{}^* = 200$ N/mm^2

The total strain energy absorbed per unit volume in the material is given by equation (24.5).

$\therefore$ Total strain energy per unit volume in the material

$$= \frac{1}{2E}[\sigma_1{}^2 + \sigma_2{}^2 + \sigma_3{}^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)]$$

$$= \frac{1}{2E}[200^2 + 100^2 + (-50)^2 - 2 \times 0.3 \{200 \times 100 + 100 \times (-50) + (-50) \times 200\}]$$

$$= \frac{1}{2E}[40000 + 10000 + 2500 - 0.6(20000 - 5000 - 10000)]$$

$$= \frac{1}{2E}[52500 - 0.6 \times 5000] = \frac{1}{2E}[49500] \quad \ldots(i)$$

Problem-2

- The principal stresses at a point in an elastic material are 200MPa (tensile), 100MPa(Tensile) and 50MPa (compressive). If the stress at the elastic limit in simple tension is 200 MPa. Determine whether the failure will occur or not according to maximum strain energy theory.

Strain energy per unit volume corresponding to stress at elastic limit in simple tension is given by equation (24.6).

∴ Strain energy per unit volume at elastic limit in simple tension

$$= \frac{1}{2E} \times \sigma_t^{*2}$$

$$= \frac{1}{2E} \times 200^2 \qquad (\because \ \sigma_t^* = 200)$$

$$= \frac{40000}{2E} \qquad \ldots(ii)$$

Now apply the theory of maximum strain energy.

Problem-3 (Assignment)

- Determine the diameter of a solid shaft as per the maximum normal strain theory subjected to a BM of 20 kN.m and Twisting Moment of 12 kN.m. The material of the shaft is yielded under axial tensile stress of 250 MPa. Take E=200 GPa and μ=0.25. Apply FOS as 2.5.

In this case, total strain energy density

$$U = \frac{1}{2E}\{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu\sigma_1\sigma_2 - 2\mu\sigma_2\sigma_3 - 2\mu\sigma_3\sigma_1\}$$

$$U = \frac{10^{16}}{2E\pi^2 d^6}\{6.93^2 + 0^2 + (-0.53)^2 + 2 \times 0.25 \times 6.93 \times 0.53\} = \frac{50.16 \times 10^{16}}{2E\pi^2 d^6}$$

In case of axial tension, $\sigma_1 = \sigma_f$; $\sigma_2 = \sigma_3 = 0$.

Applying an FOS 2.5

Allowable stress will be $\sigma_f = \frac{\text{failure stress}}{\text{FOS}} = \frac{250}{2.5} = 100$ MPa

$$U_f = \frac{1}{2E}\{\sigma_f^2\}.$$

Now, for design of the shaft

$$U_f = \frac{1}{2E}\{\sigma_f^2\} = \frac{100 \times 100}{2E} = \frac{10^4}{2E}$$

For design equating total strain energy density from two cases

$$\frac{10^4}{2E} = \frac{50.16 \times 10^{16}}{2E\pi^2 d^6}$$

$$d = 131.12 \text{ mm}$$

Provide a shaft of diameter more than 131.12 mm.

Problem-3 (Assignment)

- Determine the diameter of a solid shaft as per the maximum normal strain theory subjected to a BM of 20 kN.m and Twisting Moment of 12 kN.m. The material of the shaft is yielded under axial tensile stress of 250 MPa. Take E=200 GPa and μ=0.25. Apply FOS as 2.5.

- **A shaft is subjected to a maximum torque of 10KN-m and a maximum bending moment of 8KN-m at perpendicular section. if the allowable equivalent stress in simple is 160MN/m2 , find the diameter of the shaft according to the maximum shear stress theory**

Problem-4

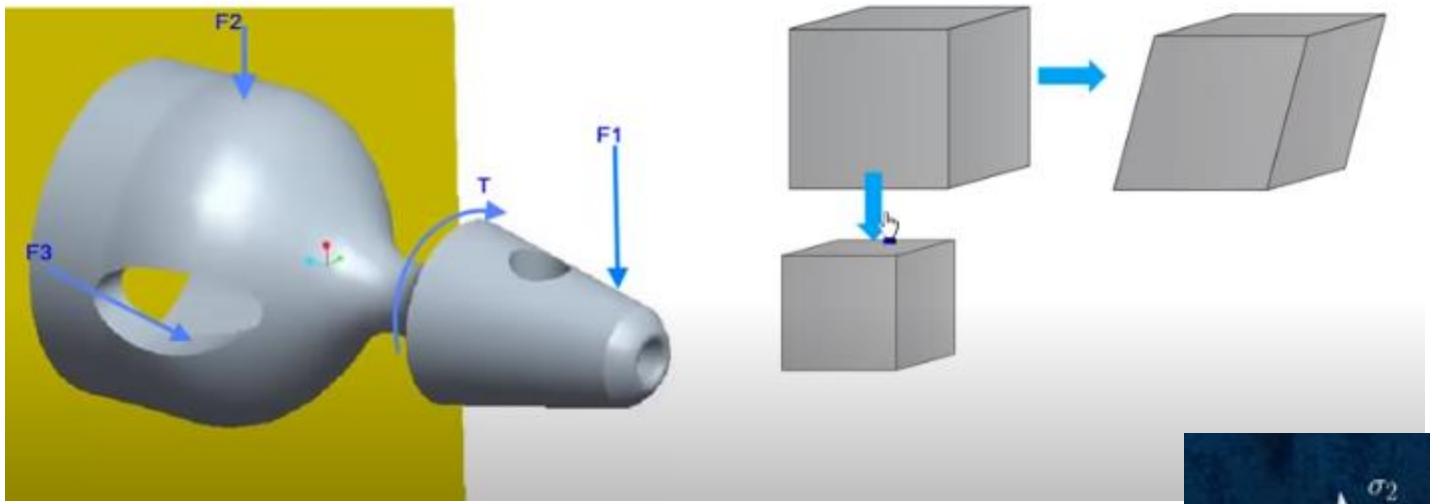
A machine element is subjected to the following stresses: $\sigma x = 60$ MPa, $\sigma y = 45$MPa and $\tau xy = 30$MPa. Find the factor of safety if it is made of steel having yield stress as 353 MPa using maximum shear stress theory. (Take poisson's ratio as 0.3)

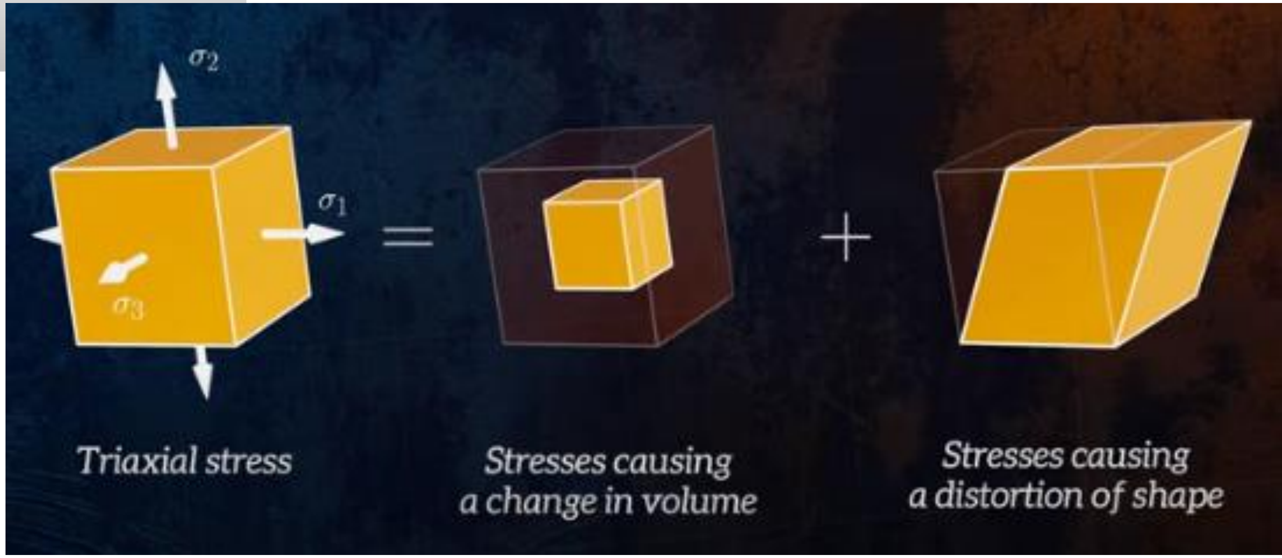
Maximum Shear Strain Energy Theory(Von Misses-Henky)

- It states that the elastic failure occurs when the shear strain energy per unit volume in the stresses material reaches a value equal to the shear strain energy per unit volume at the elastic limit point in simple tension.
- Failure of a material occurs when the distortion energy in a multi axial stress state is greater than that of the distortion energy in a tensile test at yielding
- Total strain energy (U)= volumetric strain energy(U_V) + distortional energy(U_D)
- From above theory

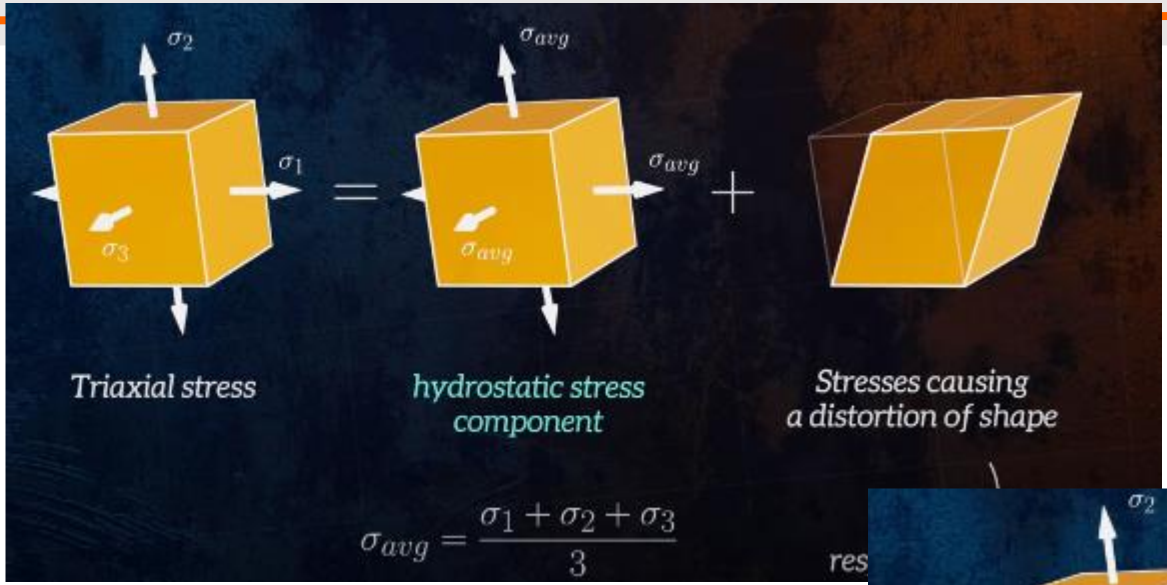
$$U = 1/2E\,(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 \;-2\,\mu(\sigma_1\sigma_2 + \sigma_3\sigma_1 + \sigma_2\sigma_3\,)$$

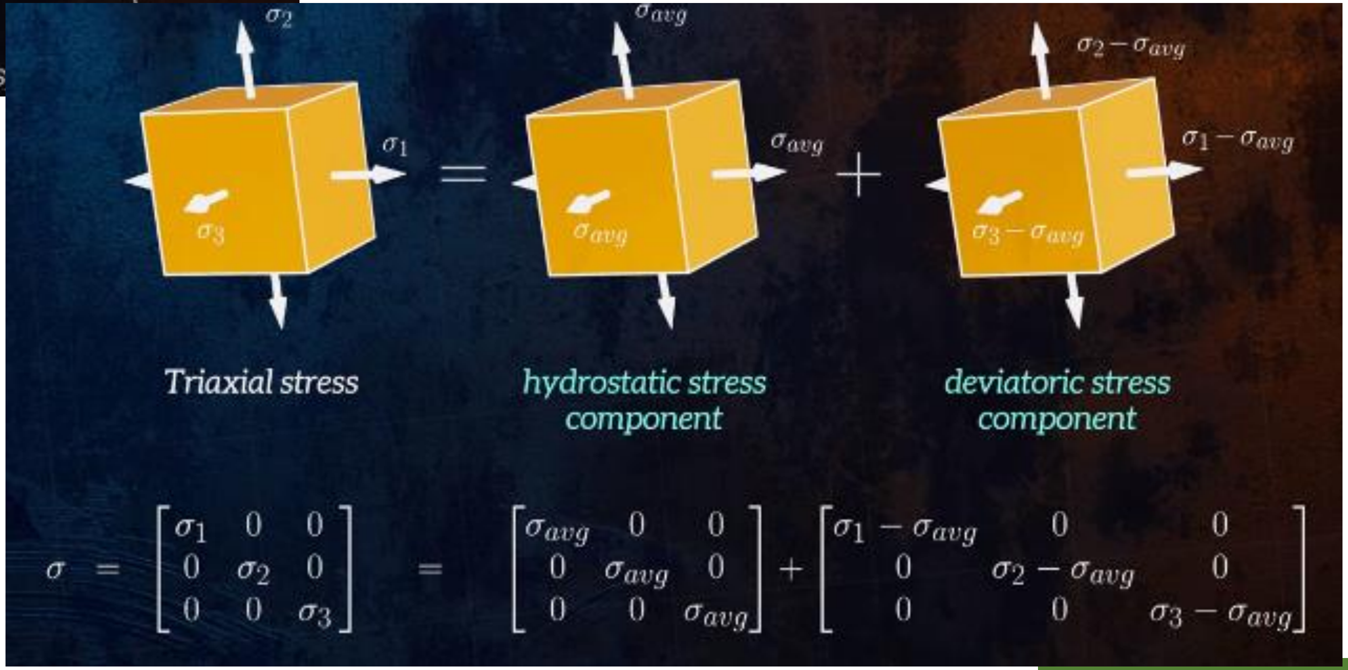
Maximum Shear Strain Energy Theory-Limitations





Maximum Shear Strain Energy Theory





Maximum Shear Strain Energy Theory-

$$\varepsilon_V = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \frac{1}{E}\{(\sigma_1 + \sigma_2 + \sigma_3) - 2\upsilon(\sigma_1 + \sigma_2 + \sigma_3)\}$$

$$\varepsilon_V = \frac{(1-2\upsilon)}{E}(\sigma_1 + \sigma_2 + \sigma_3) = \frac{3(1-2\upsilon)}{E}p$$

$$\text{Volumetric strain energy,} \quad U_V = \frac{1}{2}p\varepsilon_V$$

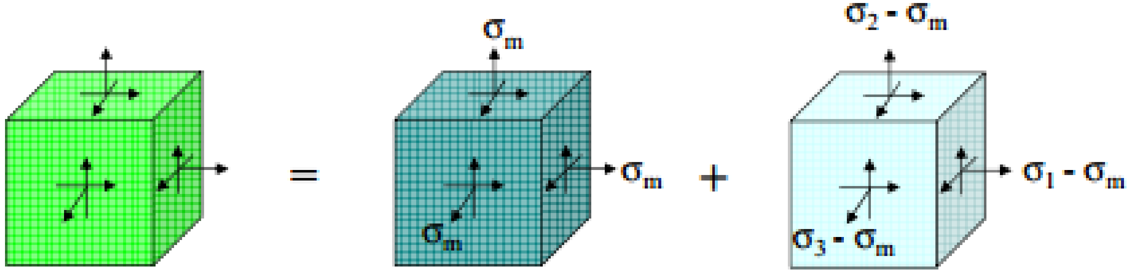
$$U_V = \frac{1}{2}p\frac{3(1-2\upsilon)}{E}p = \frac{3(1-2\upsilon)}{2E}p^2 = \frac{(1-2\upsilon)}{6E}(\sigma_1 + \sigma_2 + \sigma_3)^2$$

U = strain energy due to principal stresses & strains

$$U = \frac{1}{2E}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - \frac{\upsilon}{E}(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)$$

Maximum Shear Strain Energy Theory-

- Distortional energy –



$$U_D = U - U_V$$

$$U_D = \frac{1}{2E}\left[\left(\sigma_1^2+\sigma_2^2+\sigma_3^2\right)-2\upsilon\left(\sigma_2\sigma_1+\sigma_2\sigma_3+\sigma_3\sigma_1\right)\right]-\left(\frac{1-2\upsilon}{6E}\right)\left(\sigma_1+\sigma_2+\sigma_3\right)^2$$

Simplifying this

$$U_D = \frac{1}{12G}\left[\left(\sigma_1-\sigma_2\right)^2+\left(\sigma_2-\sigma_3\right)^2+\left(\sigma_3-\sigma_1\right)^2\right]$$

Maximum Shear Strain Energy Theory

- Compare this with distortion in uniaxial tensile stress

$$U_D = \frac{Y^2}{6G} = \frac{1}{12G}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]$$

$$\Rightarrow 2Y^2 = (\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2$$

Yield function,

$$f = \sigma_e^2 - Y^2$$

Equivalent stress, $\sigma_e^2 = \frac{1}{2}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]$

- 2D stress state => $\sigma_3 = 0$

Yield function, $f = \sigma_1^2+\sigma_2^2-\sigma_1\sigma_2-Y^2$

Onset of yielding, $\sigma_1^2+\sigma_2^2-\sigma_1\sigma_2 = Y^2$

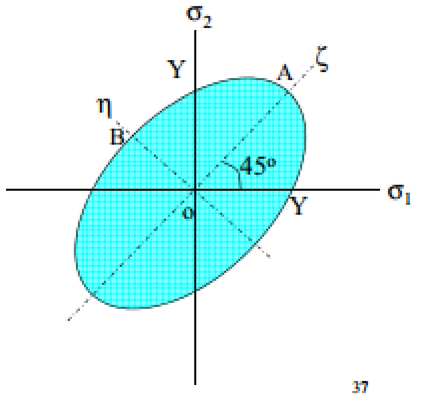
Re-arrange the terms –

$$\left(\frac{\sigma_1}{Y}\right)^2+\left(\frac{\sigma_2}{Y}\right)^2-\left(\frac{\sigma_1\sigma_2}{Y^2}\right)=1$$

This represents an ellipse

Semi - major axis, OA $=\sqrt{2}Y$

Semi - minor axis, OB $=\sqrt{\frac{2}{3}}Y$



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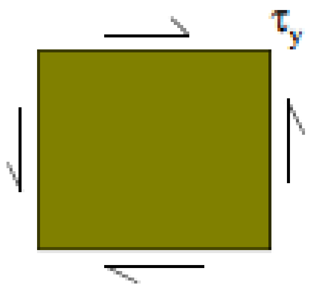
Maximum Shear Strain Energy Theory-Limitations

- The theory gives best results for ductile material particularly in case of pure shear (or) σ_{yt} equal to σ_{yc}
- This theory is regarded as one to which conform most of the ductile material under the action of various types of loading.

- Pure shear –

Principal stresses corresponding to shear yield stress



$\sigma_1 = +\tau_y\,,\ \sigma_2 = -\tau_y$

$$Y^2 = \sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = 3\tau_y^2 \Rightarrow \tau_y = 0.577Y$$

Shear yield = 0.577 * Tensile yield

Suitable for ductile materials

Problem-1

1. A mild steel shaft having yield stress as 232 MPa is subjected to following stresses. σx = 120 MPa,σy = -60 MPa and τxy = 36 MPa. Find factor of safety using Von Mises theory.
2. A material has yield stress of 600 Mpa. Compute the factor of safety for the von-mises theory. σ1 = 420 MPa, σ2 = 410 MPa and σ3 = 0

$$\text{Von-mises theory, } \sigma_e = \sqrt{\frac{(\sigma_1-\sigma_2)^{-2}+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}}$$

$$\sigma_e = \sqrt{\frac{(420-180)^2+(180-0)^2+(420-0)^2}{2}} = 364.97 \text{ MPa}$$

$$FOS = \frac{\tau_{ys}}{\tau_e} = \frac{600}{364.97} = 1.644$$

Problem-2

- The principal stresses at a point in an elastic material are 200MPa (tensile), 100MPa(Tensile) and 50MPa (compressive). If the stress at the elastic limit in simple tension is 200 MPa. Determine whether the failure will occur or not according to maximum shear strain energy theory.

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Problem-3 (Assignment)

- Determine the diameter of a solid shaft as per the maximum normal strain theory subjected to a BM of 20 kN.m and Twisting Moment of 12 kN.m. The material of the shaft is yielded under axial tensile stress of 250 MPa. Take E=200 GPa and μ=0.25. Apply FOS as 2.5.

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- **A shaft is subjected to a maximum torque of 10KN-m and a maximum bending moment of 8KN-m at perpendicular section. if the allowable equivalent stress in simple is 160MN/m2 , find the diameter of the shaft according to the maximum shear stress theory**

Problem-4

A machine element is subjected to the following stresses: σx = 60 MPa, σy = 45MPa and τxy = 30MPa. Find the factor of safety if it is made of steel having yield stress as 353 MPa using maximum shear stress theory. (Take poisson's ratio as 0.3)

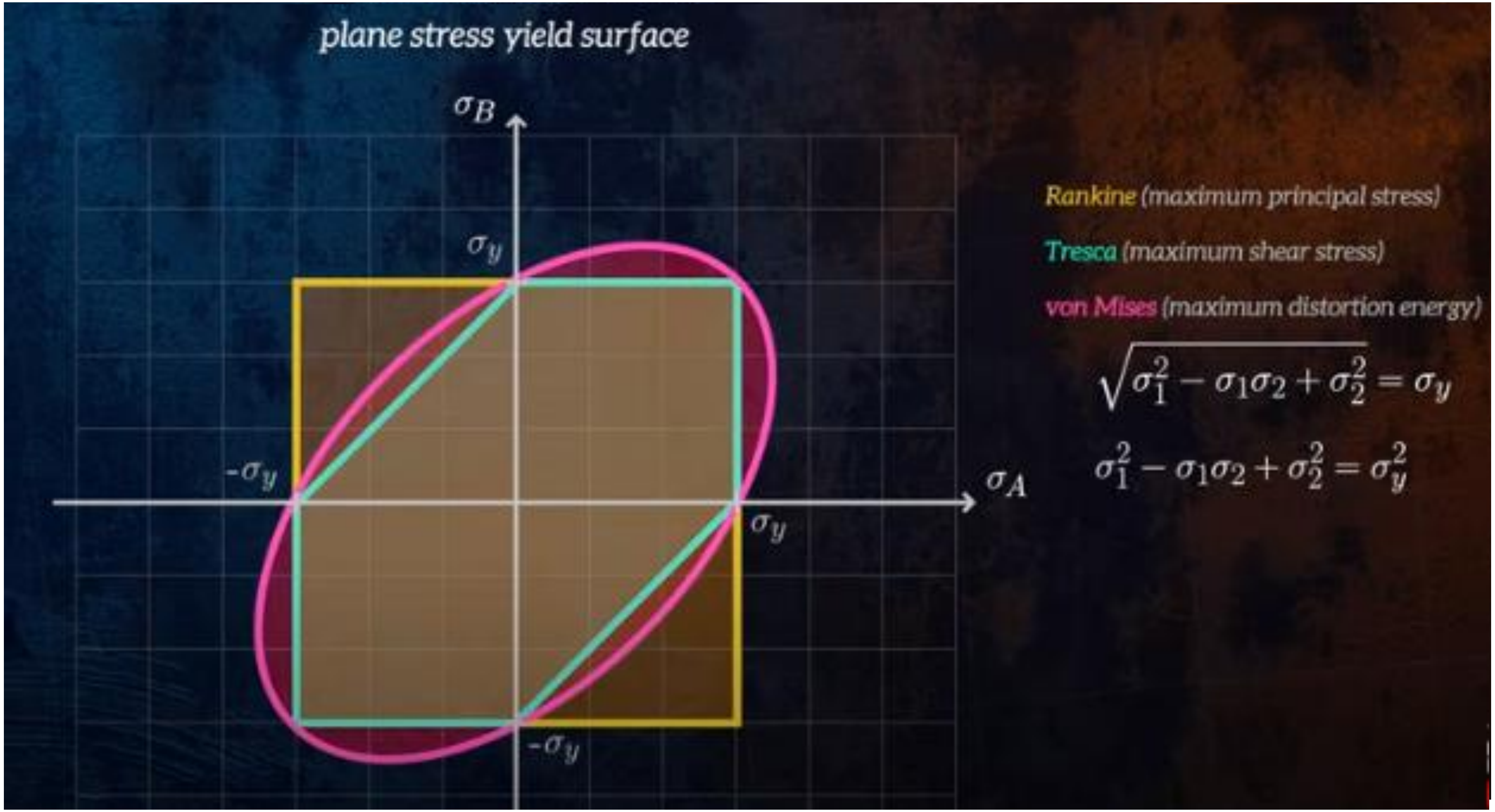
Theories of Failure

Design conditions for various failure theory

Failure theory	Proposed by	Condition for design
Maximum principal stress theory	Rankine, Lame	$\sigma_1 \le \sigma$
Maximum principal strain theory	Saint Venants	$\frac{1}{E}(\sigma_1 - \mu\sigma_2) \le \frac{\sigma_y}{E}$
Maximum shear stress theory	Coulomb Guest, Tresca	$\frac{\sigma_1 - \sigma_2}{2} \le \frac{\sigma_y}{2}$
Maximum strain energy theory	Beltrami-Haigh	$(\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2) \le \sigma_y^2$
Distortion energy theory	Huber-Henky-Von Mises	$(\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2) \le \sigma_y^2$

- When one of the principal stresses at a point is large in comparison to the other, all the failure theories gives nearly the same result.
- When a member is subjected to uni-axial tension, all the failure theories gives the same result.

Graphs of all theories



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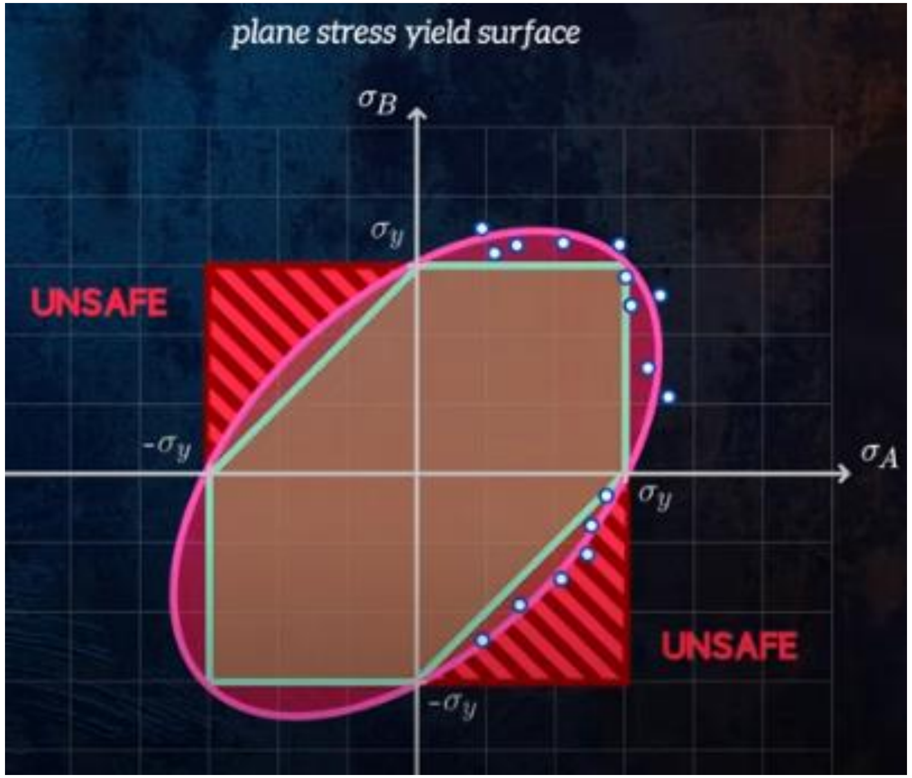

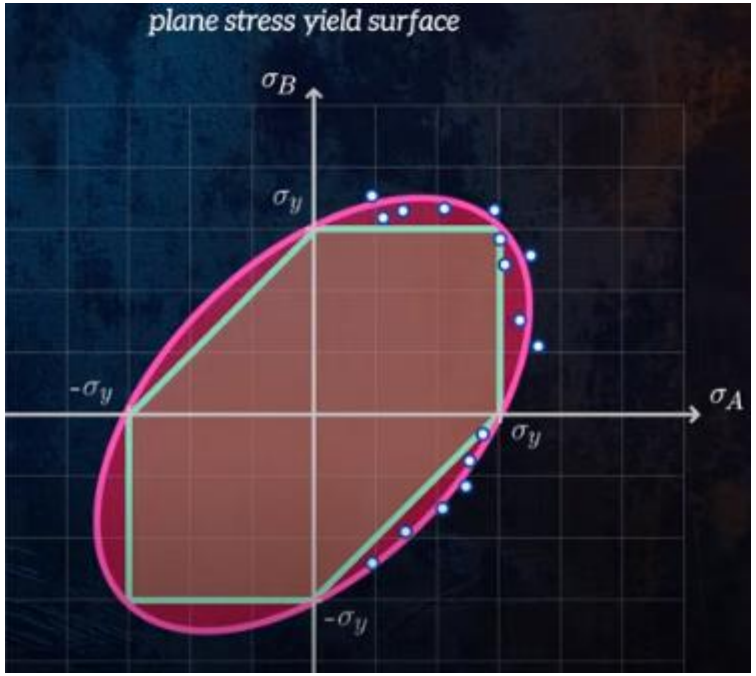

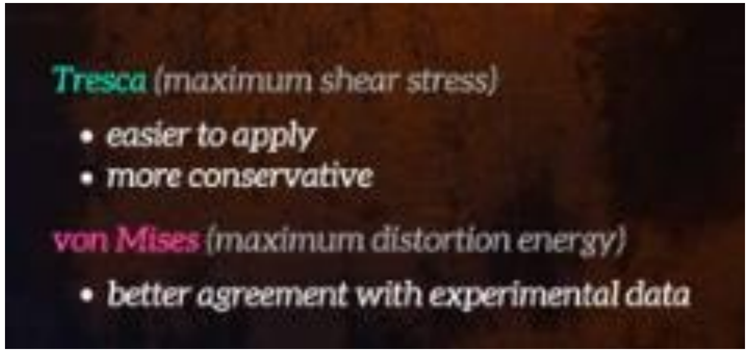

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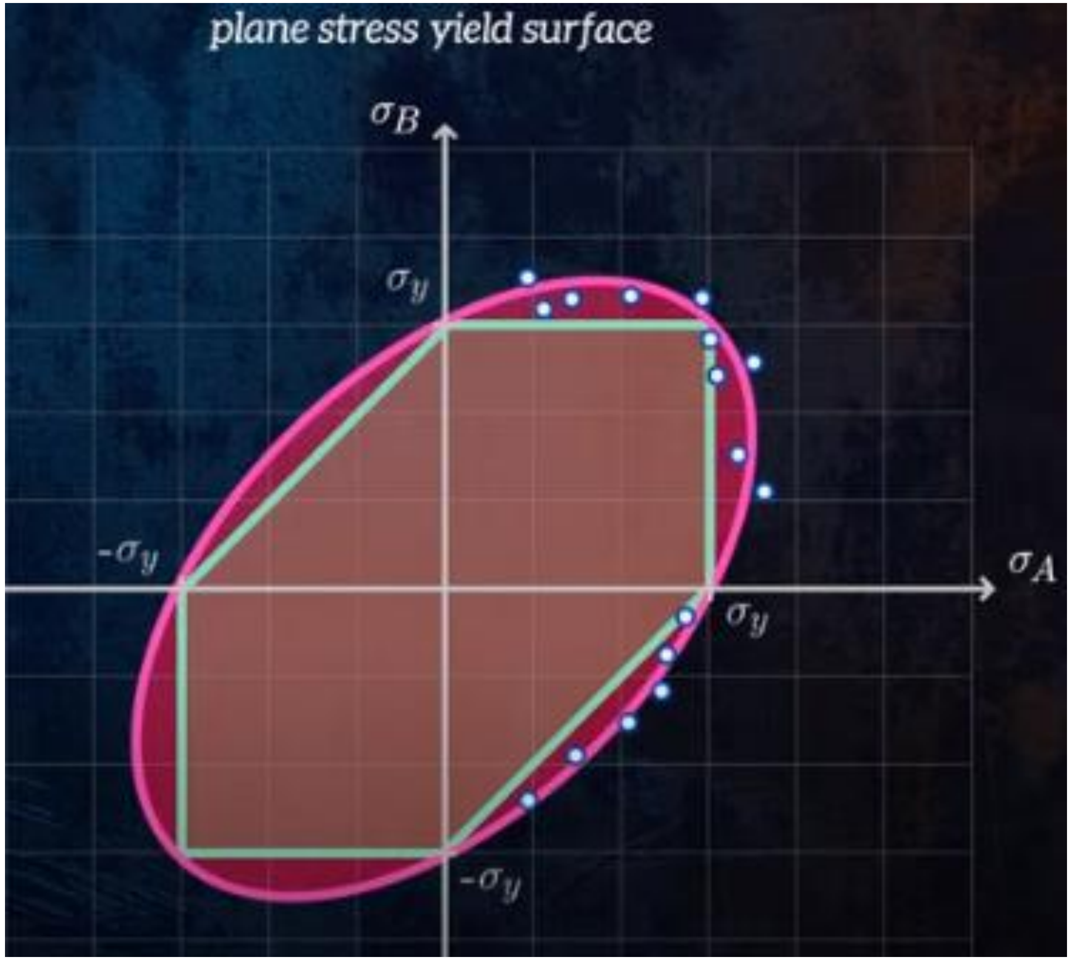

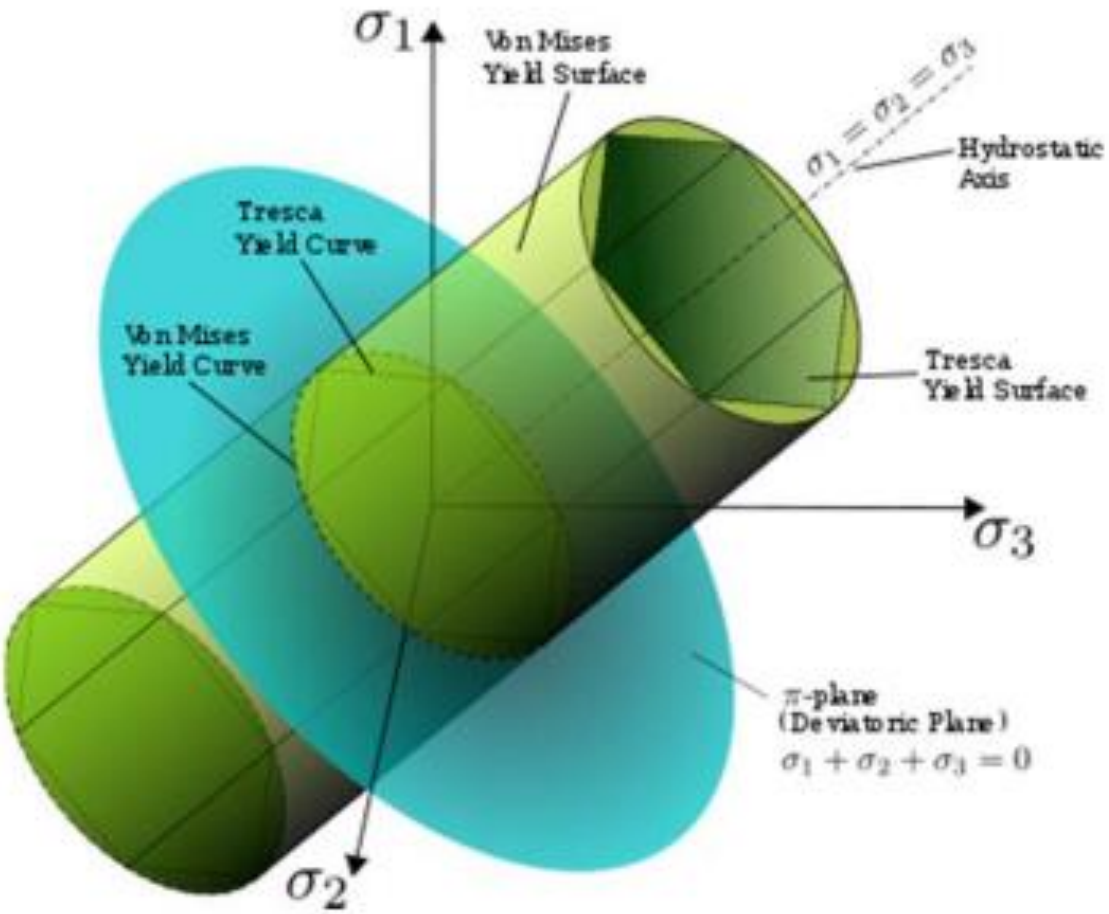

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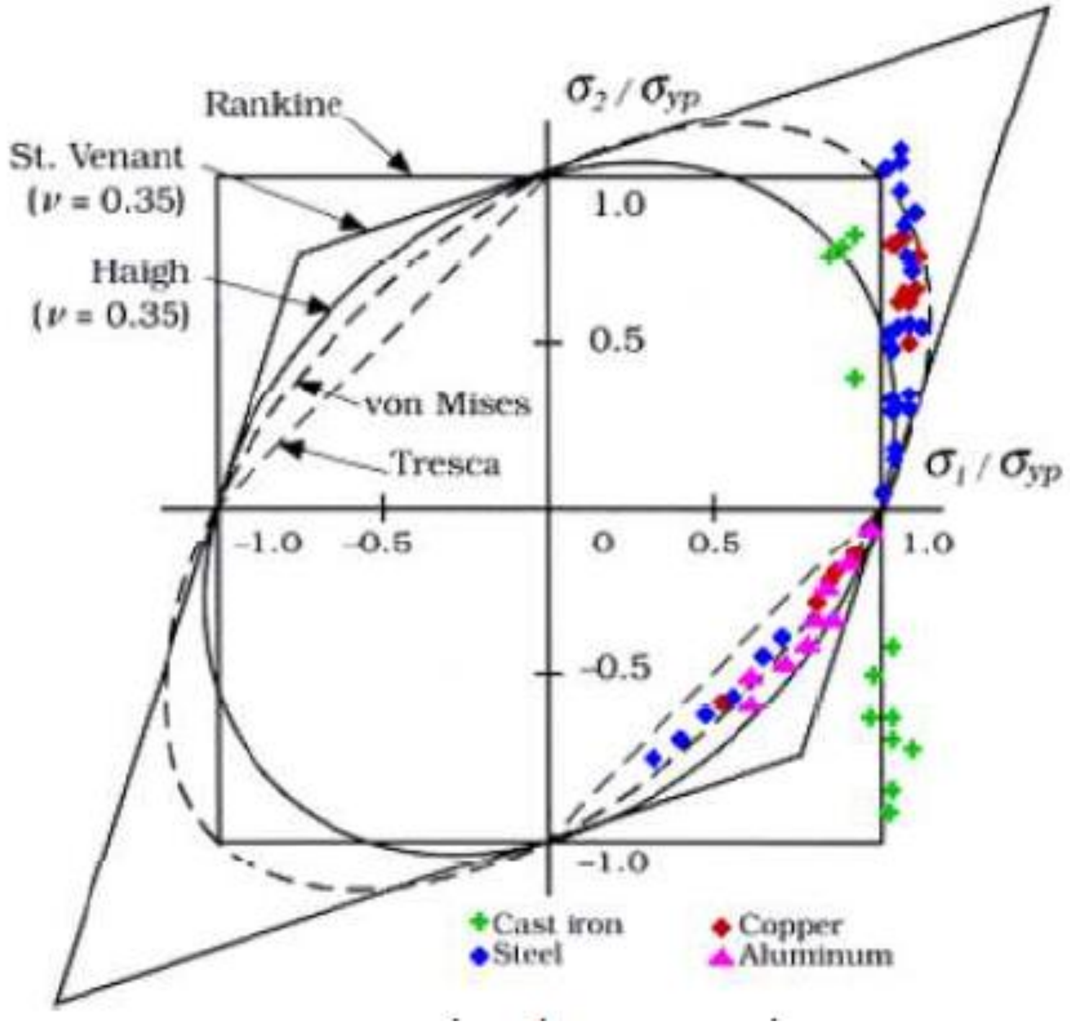

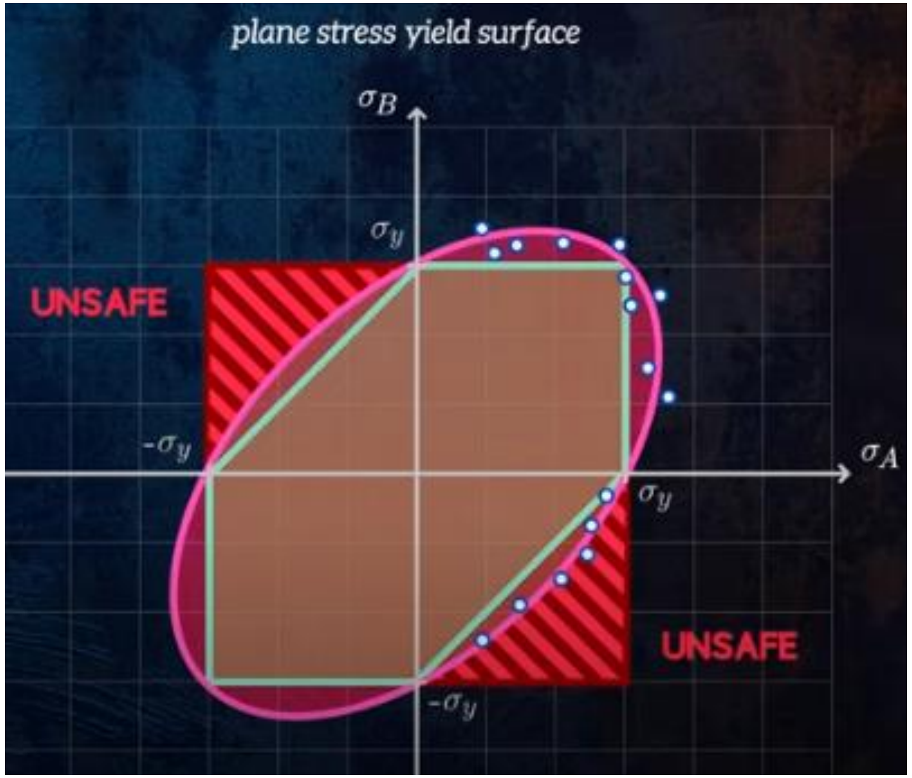

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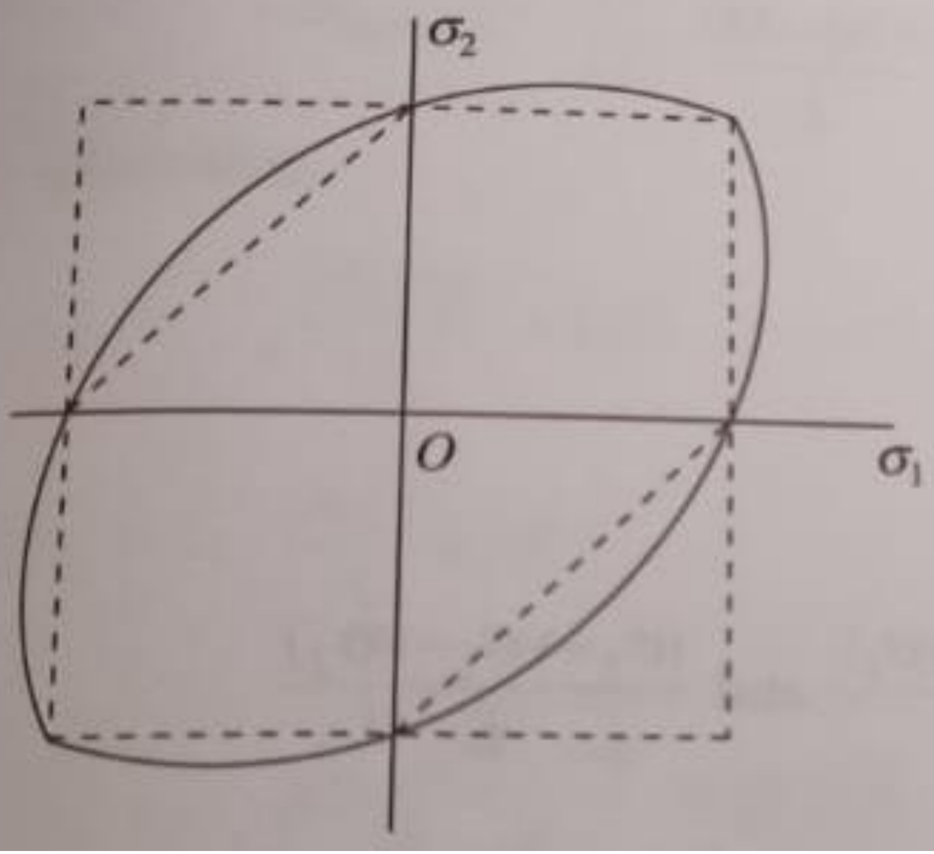

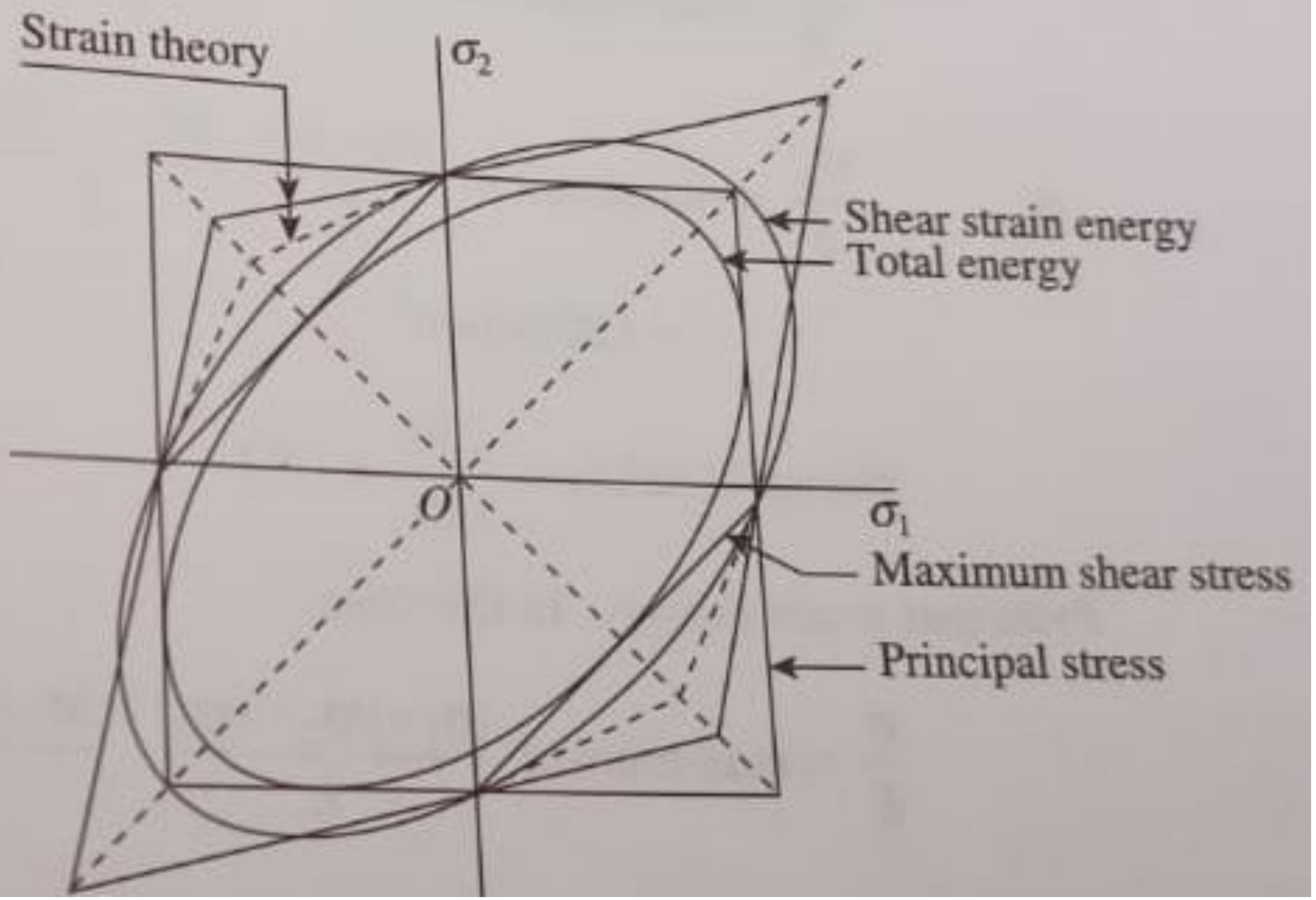

Graphs of all theories

Determine the FOS as different failure theories of a body subjected to the following principal stresses. σ_1 = 100 MPa,σ_2 = 20 MPa and σ_3 = -20 MPa. The material of the body yielded under uniaxial tensile stress of 300 MPa. (μ=0.25, E=200 GPa)

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Graphs of all theories

- Each failure theory gives a relation between yielding in tension and shear ($\upsilon = 0.25$)

Failure Criteria

Theory	Loading: Uniaxial	Loading: Pure Shear	Relationship
Maximum principal stress	$\sigma_{max} = \sigma_{YP}$	$\sigma_{max} = \tau_{YP}$	$\tau_{YP} = \sigma_{YP}$
Maximum principal strain	$\varepsilon_{max} = \sigma_{YP} / E$	$\varepsilon_{max} = 5\tau_{YP} / 4E$	$\tau_{YP} = 0.8\ \sigma_{YP}$
Maximum octahedral shear stress	$\tau_{oct} = \sigma_{YP} \frac{\sqrt{2}}{3}$	$\tau_{oct} = \tau_{YP} \sqrt{\frac{2}{3}}$	$\tau_{YP} = 0.577\ \sigma_{YP}$
Maximum distortional energy density			$\tau_{YP} = 0.577\ \sigma_{YP}$
Maximum shear stress	$\tau_{max} = \sigma_{YP} / 2$	$\tau_{max} = \tau_{YP}$	$\tau_{YP} = 0.5\ \sigma_{YP}$

Theories of Failures Conclusion

- **Maximum principal stress theory:**

  Best theory of failure for brittle material design
- **Maximum shear stress theory:**

  Gives safe design for ductile
- **Maximum distortion energy theory:**

  Best for ductile material design

Theories of Failures

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