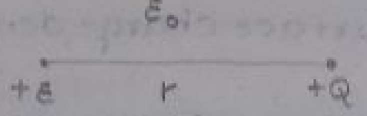
29/01/21

Electrostatics

UNIT-02

Coulomb's Law:



$F \propto qQ$

$F \propto \frac{1}{r^2}$

$$\boxed{F = \frac{K_e |qQ|}{r^2}}$$

here $K = \frac{1}{4\pi\varepsilon_0} = 9\times10^9 \frac{N\text{-}m^2}{c^2}$ → free space

$\varepsilon_0 = 8.85\times10^{-12} \frac{c^2}{N\cdot m^2}$ → permitivity of free space

$K = \frac{\varepsilon}{\varepsilon_0} = \varepsilon_r = n^2$

↓ Dielectric constant

↓ relative permitivity

$\Rightarrow n = \sqrt{\varepsilon_r}$

$\boxed{n = \sqrt{K}}$

$$\boxed{F_e = K_e \frac{qQ}{r^2}\,\hat{r}\ N/c}$$

$\vec{F} = K_e \frac{qQ}{r^2}$

$$\Rightarrow \boxed{\frac{\vec{F}}{Q} = \frac{K_e q}{r^2}\,\hat{r} = \vec{E}}$$ → Electric field strength

↓ force per unit charge

$\vec{E} = \frac{\vec{F}}{Q}$ 'N/c'

↓ Source charge

$r_1, r_2, r_3, r_4, ----r_n$

$q_1, q_2, q_3, q_4, ----q_n$

$$\boxed{\vec{E} = K_e \sum_{i=1}^{n} \frac{q_i}{r_i^2}\,\hat{r}_i}$$

$$= \frac{K_e q_1}{r_1^2}\hat{r}_1 + \frac{K_e q_2}{r_2^2}\hat{r}_2 + \frac{K_e q_3}{r_3^2}\hat{r}_3 + ---- \frac{K_e q_n}{r_n^2}\hat{r}_n$$

$\lambda = \frac{dq}{dl}$ → Line charge density (C/m)

$\sigma = \frac{dq}{dA}$ → surface charge density (C/m²)

$\rho = \frac{dq}{dv}$ → volume charge density (C/m³)

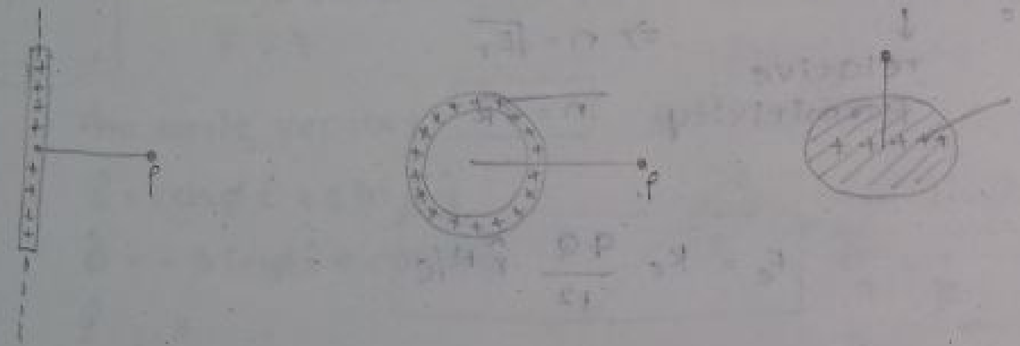
Continuous charge distribution objects

$dq = \lambda dl \Rightarrow Q = \int \lambda dl$ → total charge 'λ'

$dq = \sigma dA \Rightarrow Q = \int \sigma dA$ → " " 'σ'

$dq = \rho dv \Rightarrow Q = \int \rho dv$ → " " 'ρ'

1. Calculate Electric fields due to infinite charge.



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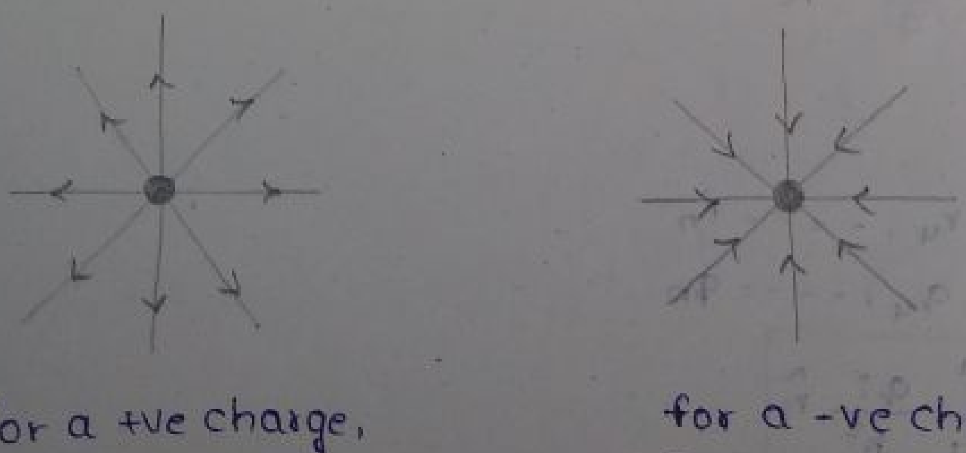
Electric field lines:

(Rules)

→ The lines must begin on a +ve charge and terminate on a -ve charge. In the cases of an excess of one type of charge, some lines will begin or end infinitely far away.

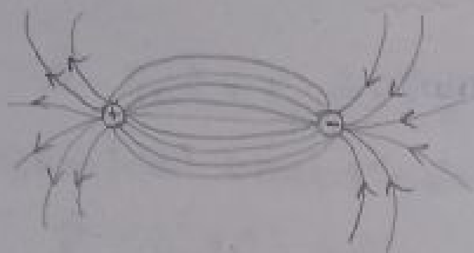


for a +ve charge, field lines are radially outward.

for a -ve charge, it is radially inward.

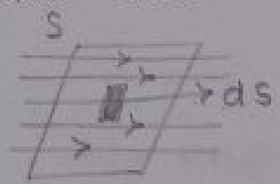
→ The no. of lines drawn leaving a +ve charge or approach-ing a -ve charge is directly proportional to the magnitude of the charge.

→ No two field lines can cross.



The no. of field lines leaving the +ve charge is equals to the number terminating at the -ve charge

Electric flux:

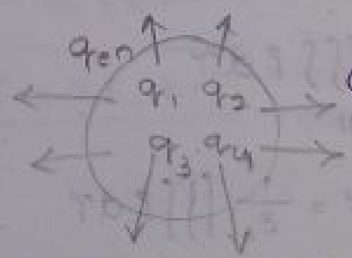


$$d\phi_E = \vec{E}\cdot d\vec{s}$$

$$\int d\phi_E = \iint_S \vec{E}\cdot d\vec{s} = \phi_E \rightarrow \text{scalar quantity}$$

Gauss's law:

$$\iint_S \vec{E}\cdot d\vec{s} = \frac{1}{\varepsilon_0} Q_{en}$$

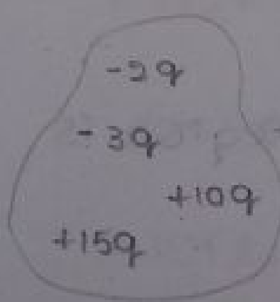


$Q_{en} = q_1 + q_2 + q_3 + q_4$

$Q_{en} \rightarrow$ total charge or net charge enclosed within the surface.

$\varepsilon_0 \rightarrow \varepsilon \rightarrow$ permitivity of the medium

→ Flux through any closed surface is a measure of the total charge inside.



$q = 1.16\ \mu C$

Total electric flux

$= -2q - 3q + 10q + 15q$

$= 20q$

$$\Rightarrow \phi_E = \frac{20q}{\varepsilon_0}$$

$$= \frac{20 \times 1.16 \times 10^{-6}}{8.85 \times 10^{-12}} = 2.62 \times 10^{6}$$

$\boxed{\phi_E \propto Q_{en}}$

$\boxed{\iint \vec{E}\cdot d\vec{s} = \frac{Q_{en}}{\varepsilon_0}}$ → Integral form of Gauss's law

Differential form of Gauss's law:

$$\iint \vec{E}\cdot d\vec{s} = \frac{Q_{en}}{\varepsilon_0} \quad \text{(Gauss's law)}$$

Gauss divergence theorem

$$\iint_S \vec{E}\cdot d\vec{s} = \iiint_V (\vec{\nabla}\cdot\vec{E})\, d\tau$$

$$\iiint_V (\vec{\nabla}\cdot\vec{E})\, d\tau = \frac{Q_{en}}{\varepsilon_0}$$

Volume charge density

$$\rho = \frac{dQ}{d\tau}$$

$$dQ = \rho\, d\tau$$

$$Q_{en} = \iiint_V \rho\, d\tau$$

$$\iiint_V \vec{\nabla}\cdot\vec{E}\, d\tau = \frac{1}{\varepsilon_0}\iiint \rho\, d\tau$$

$$\iiint_V \vec{\nabla}\cdot\vec{E}\, d\tau = \iiint_V \frac{\rho}{\varepsilon_0}\, d\tau$$

$\boxed{\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}}$ → differential form of gauss's law

→ Also known as Maxwell's eq for form

Q. Suppose the electric field in some region is found to be $\vec{E} = kr^3\hat{r}$

(a) Find charge density (ρ).

(b) Find the total charge contained in a sphere of radius 'R'.

Sol. (a) We know, Gauss's law

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\text{div } \vec{E} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 E_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, E_\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(E_\phi)$$

$$\vec{E} = E_r\hat{r} + E_\theta\hat{\theta} + E_\phi\hat{\phi}$$

Spherical coordinates

$$\vec{E} = kr^3\hat{r} + 0\hat{\theta} + 0\phi$$

$$\vec{\nabla}\cdot\vec{E} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 k r^3) = \frac{1}{r^2}\frac{\partial}{\partial r}(kr^5)$$

$$= k\cdot\frac{5r^{\cancel{4}}}{\cancel{r^2}}$$

$$\vec{\nabla}\cdot\vec{E} = 5kr^2$$

We know

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0} \Rightarrow \rho = \varepsilon_0(5kr^2)$$

$$\therefore \boxed{\rho = 5\varepsilon_0 kr^2} \rightarrow \text{charge density}$$

(b) $Q_{en} = \int \rho\, d\tau$

$$= \int_0^R\int_0^\pi\int_0^{2\pi}(5\varepsilon_0 kr^2)\, r^2\sin\theta\, dr\, d\theta\, d\phi$$

$$= 5\varepsilon_0 k\int_0^R r^4 dr\int_0^\pi \sin\theta\, d\theta\int_0^{2\pi} d\phi$$

$$= 5\varepsilon_0 k\left[\frac{r^5}{5}\right]_0^R \cdot -[\cos\theta]_0^\pi \cdot [\phi]_0^{2\pi}$$

$$-\cos\theta]_0^\pi$$
$$= -[\cos\pi - \cos 0]$$
$$= -[-1-1]$$
$$= -[-2] = 2$$

$$= (2\pi)\,\cancel{5}\,\varepsilon_0 k\frac{R^5}{\cancel{5}} \times 2$$

$$= 4\pi\varepsilon_0 kR^5$$

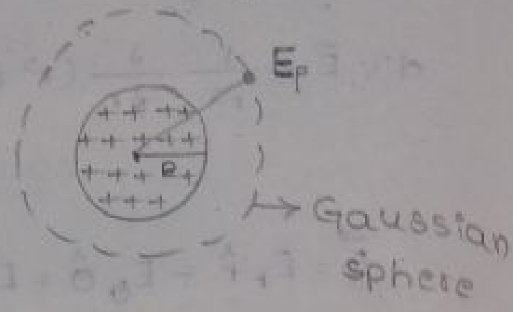
Q. Find the field outside a uniformly charged sphere of radius 'R' and total charge q.

Sol.

$$\iint \vec{E}\cdot d\vec{s} = \frac{Q_{en}}{\varepsilon_0}$$

$$E\, 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$



$$\left[\iint \vec{E}\cdot d\vec{s} = \iint E\, ds\cos\theta \right.$$

$$= \iint E\, ds$$

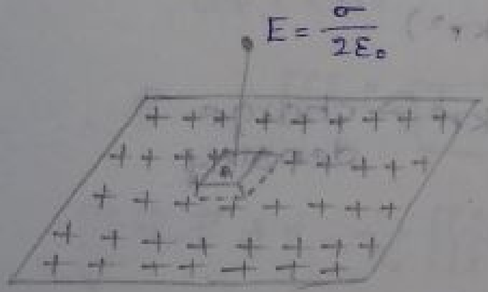
$$= E\iint ds$$

$$= E\cdot 4\pi r^2$$

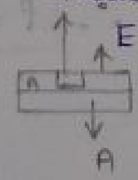
$$\left.\therefore \iint ds = 4\pi r^2\right]$$

Electric field due to infinite plane sheet:

Carries a surface charge density 'σ'



Gaussian pillbox

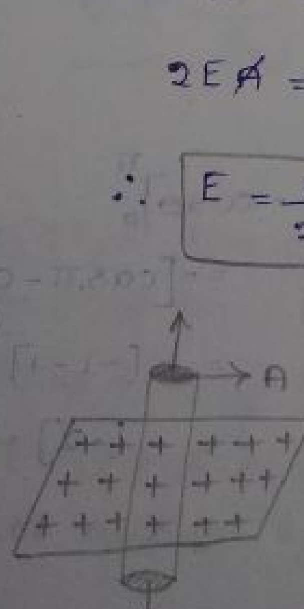


$$\phi_{net} = \int_S \vec{E}\cdot d\vec{s} = \frac{Q_{en}}{\varepsilon_0} \rightarrow \text{Gauss's law} \left(\because \iint E\,dA = 2E\int dA \quad (\phi_E)_{total} = 2EA\right)$$

$$\Rightarrow 2EA = \frac{Q_{en}}{\varepsilon_0}$$

$$2E\cancel{A} = \frac{\sigma\cancel{A}}{\varepsilon_0} \quad (\because Q_{en} = \sigma A) \Rightarrow \frac{Q}{A} = \sigma$$

$$\therefore \boxed{E = \frac{\sigma}{2\varepsilon_0}}$$



$\theta = 0°$

$\vec{E} \parallel \vec{A}$

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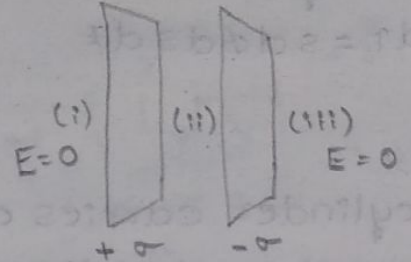
1. Two infinite parallel planes carry equal but opposite uniform charge densities $\pm\sigma$. Find the field in each of the three regions

(i) to the left of both

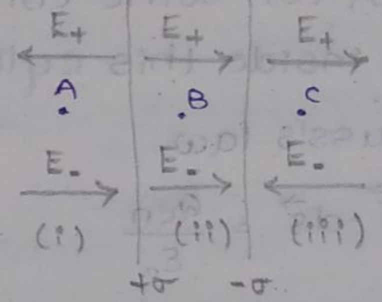
(ii) between them

(iii) to the right of both



Sol. $E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}$

$= \frac{\sigma}{\varepsilon_0}$ (+σ plate to -σ plate)

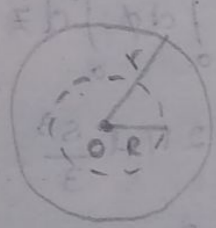


(ii) $\boxed{E = \frac{Q}{\varepsilon_0 A}}$ $\left(\because \sigma = \frac{Q}{A}\right)$

2. Find the electric field inside a sphere that carries a charge density proportional to the distance from the origin, $\rho = kr$, for some constant k.

Sol. Given

$\rho = kr$



Gauss's Law

$$\iint \vec{E}\cdot d\vec{s} = \frac{Q_{en}}{\varepsilon_0} \quad (\vec{E} \parallel d\vec{s})$$

$$E\iint ds = \frac{Q_{en}}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{Q_{en}}{\varepsilon_0}$$

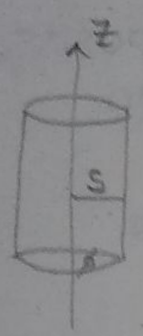
$$Q_{en} = \iiint \rho \, d\tau = \int_0^r\int_0^\pi\int_0^{2\pi} kr \, r^2 \sin\theta \, dr\, d\theta\, d\phi$$

$$= 2\pi \times 2 \int_0^r kr^3 dr$$

$$= \cancel{4}\pi k \frac{r^4}{\cancel{4}} = \pi k r^4$$

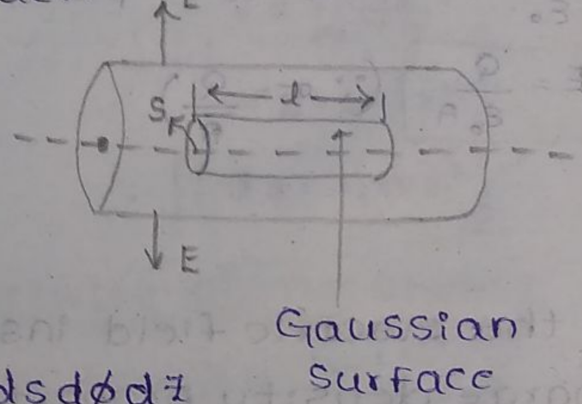
$$E \cdot 4\cancel{\pi} \cancel{r^2} = \frac{\cancel{\pi} k r^4}{\varepsilon_0} \Rightarrow \boxed{E = \frac{kr^2}{4\varepsilon_0}\hat{r}}$$

Note:



for cylinder,
$d\tau = s\, d\phi\, ds\, dz$

3. A long cylinder carries a charge density that is proportional to the distance from the axis: $\rho = ks$, for some constant k. Find the electric field inside this cylinder.



Sol. Gauss's law

$$\iint \vec{E} \cdot d\vec{s} = \frac{Q_{en}}{\varepsilon_0}$$

Now,

$$Q_{en} = \int \rho\, d\tau = \int_0^s \int_0^{2\pi} \int_0^l ks \cdot s\, ds\, d\phi\, dz$$

$$= k \int_0^{2\pi} d\phi \int_0^l dz \int_0^s s^2\, ds$$

$$= k\, 2\pi l \frac{s^3}{3}$$

$$\Rightarrow Q_{en} = \frac{2}{3} k \pi l s^3$$

Now,

$$\iint \vec{E} \cdot d\vec{s} = \frac{Q_{en}}{\varepsilon_0}$$

$$E \cdot \cancel{2}\cancel{\pi}\cancel{s}\cancel{l} = \frac{\frac{\cancel{2}}{3} k \cancel{\pi} \cancel{l} s^{\cancel{3}}}{\varepsilon_0} \quad \left(\because \iint ds = 2\pi s l\right)$$

$$\Rightarrow \boxed{E = \frac{ks^2}{3\varepsilon_0} \hat{s}}$$

4. Use Gauss's law to find the electric field inside and outside a spherical shell of radius R that carries a uniform surface charge density σ.

Sol. $\iint \vec{E} \cdot d\vec{s} = \frac{Q_{en}}{\varepsilon_0}$ $\boxed{\text{Inside}}$

for uniformly charged spherical shell

$E = 0, Q_{en} = 0$

$\boxed{\text{for outside}}$

$\iint \vec{E} \cdot d\vec{s} = \frac{Q_{en}}{\varepsilon_0}$

here $Q_{en} = Q$

$\iint ds = 4\pi r^2$

then

$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2}\hat{r}}$$

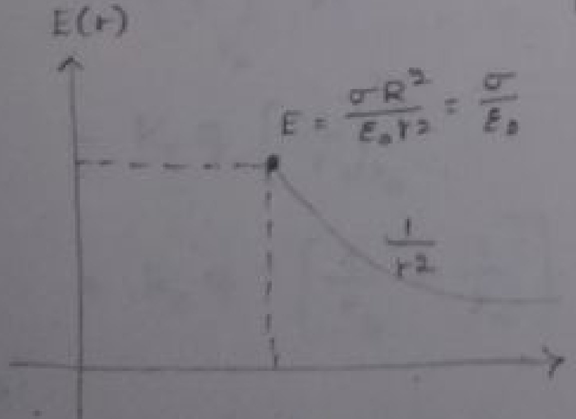
(or) We know $\sigma = \frac{Q}{4\pi R^2}$

$Q = \sigma 4\pi R^2$

$E = \frac{1}{\cancel{4\pi}\varepsilon_0} \cdot \frac{\sigma \cancel{4\pi} R^2}{r^2}$

$$\boxed{\vec{E} = \frac{\sigma R^2}{\varepsilon_0 r^2}\hat{r}}$$

R - constant
r - Variable



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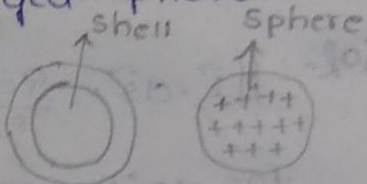
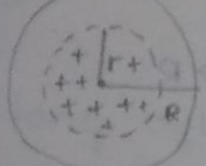
Electric potential - solid sphere

Electric field due to uniformly charged sphere

inside point.





$$\rho = \frac{Q}{\frac{4}{3}\pi R^3} = \text{constant}$$

$$Q_{total} = \rho \frac{4}{3}\pi R^3$$

unit volume $\rightarrow \rho$

$\frac{4}{3}\pi r^3 \rightarrow \rho \frac{4}{3}\pi r^3$

$$\iint \vec{E}\cdot d\vec{s} = \frac{Q_{en}}{\varepsilon_0} \text{ (Gauss's Law)}$$

$$E \cdot 4\pi r^2 = \frac{Q_{en}}{\varepsilon_0} \rightarrow (1)$$

$$Q_{en} = \rho \frac{4}{3}\pi r^3 = \frac{Q}{\cancel{\frac{4}{3}\pi} R^3} \times \cancel{\frac{4}{3}\pi} r^3 = \frac{Qr^3}{R^3} = Q_{en} \rightarrow (2)$$

sub eq (2) in eq(1)

$$\left[Q_{en} = \iiint_0^r \frac{Q}{\frac{4}{3}\pi R^3} r^2 \sin\theta \, dr\, d\theta\, d\phi \right.$$

$$\left. Q_{en} = \frac{Q}{\frac{4}{3}\pi R^3} \cdot \frac{4}{3}\pi r^3 \right]$$

$$E \cdot 4\pi \cancel{r^2} = \frac{1}{\varepsilon_0} \cdot \frac{Q r^{\cancel{3}}}{R^3}$$

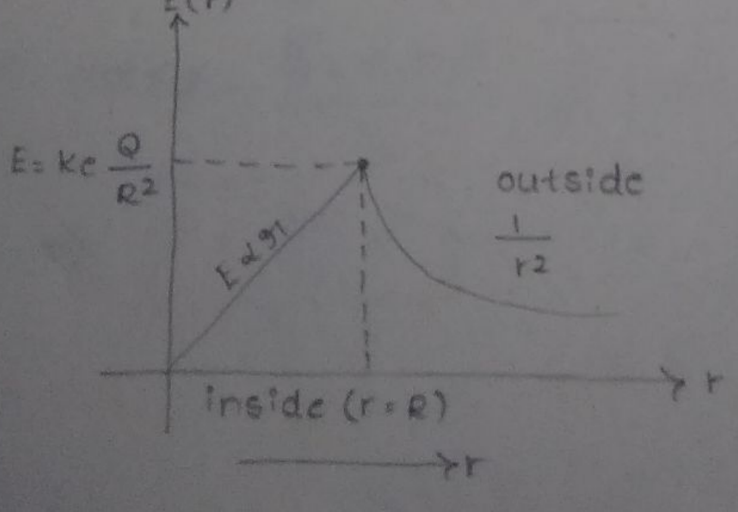
$$\boxed{E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Qr}{R^3}\hat{r}}$$

$< E_{inside} < r$

$$E_{surface} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{QR}{R^{\cancel{3}}}$$
$r = R$

$$E_S = \frac{k_e Q}{R^2}$$

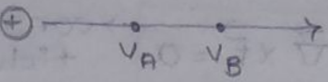
$$E_{outside} = \frac{k_e Q}{r^2}\hat{r}$$



Electric potential (v)

→ It is Scalar quantity.

$\vec{E} = \vec{\nabla} V$

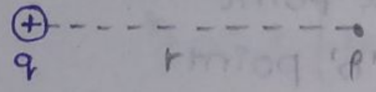


$\vec{V} \rightarrow -\vec{\nabla} V = \vec{E}$

$V_A > V_B$

$\Delta V = IR$

$\vec{E} = E_x \hat{x} + E_y \hat{y}$



$$\vec{E}_p = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \hat{r}$$

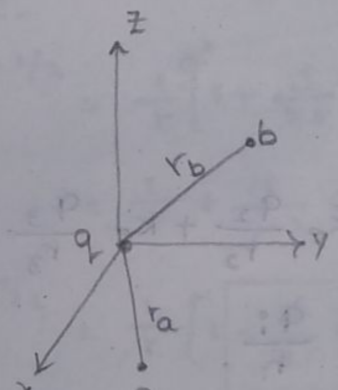
$d\vec{l} = dr\,\hat{r} + r d\theta\,\hat{\theta} + r\sin\theta\, d\phi\,\hat{\phi}$

$$\vec{E} \cdot d\vec{l} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} dr (\hat{r} \cdot \hat{r})$$

$$\left\{ \begin{matrix} \hat{r} \perp \hat{\theta} \\ \hat{\phi} \perp \hat{r} \end{matrix} \right\}$$

$$\vec{E} \cdot d\vec{l} = \frac{k_e q}{r^2} dr$$

$$\int_a^b \vec{E} \cdot d\vec{l} = \int_a^b \frac{k_e q}{r^2} dr$$



$$= \int_{r_a}^{r_b} \frac{k_e q}{r^2} dr$$

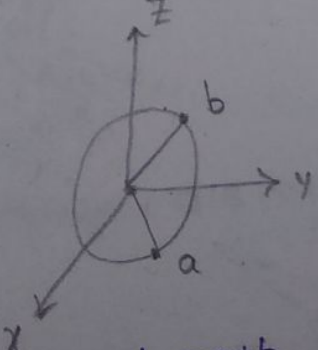
$$= k_e q \int_{r_a}^{r_b} \frac{1}{r^2} dr$$

$$\left(\int \frac{-1}{r^2} dr = \frac{-1}{r} \right)$$

$$= k_e q \left[\frac{-1}{r} \right]_{r_a}^{r_b}$$

$$= k_e q - \left[\frac{1}{r_b} - \frac{1}{r_a} \right]$$

$$\int_a^b \vec{E} \cdot d\vec{l} = \frac{k_e q}{r_a} - \frac{k_e q}{r_b}$$



$$\oint \vec{E} \cdot d\vec{l} = 0$$

If $r_a = r_b$ $\quad \int \vec{E} \cdot d\vec{l} = 0$

→ Integral around a closed path

Stokes theorem

$$\oint \vec{E}\cdot d\vec{l} = \oiint \text{curl}\,\vec{E}\cdot d\vec{s} = \iint (\vec{\nabla}\times\vec{E})\cdot d\vec{s}$$ Electrostatic field

$$\therefore \boxed{\oint \vec{E}\cdot d\vec{l} = 0 \Rightarrow \vec{\nabla}\times\vec{E} = 0} \rightarrow \text{conservative field}$$

if $\vec{\nabla}\times\vec{E} = 0$

$(\because \vec{E} = -\text{grad } v)$

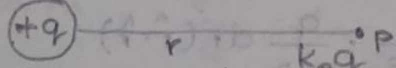
$$\int \vec{E}\cdot d\vec{l} = \frac{k_e q}{r_a} - \frac{k_e q}{r_b} = V_a - V_b$$

here V_a = Electric potential at 'a' point

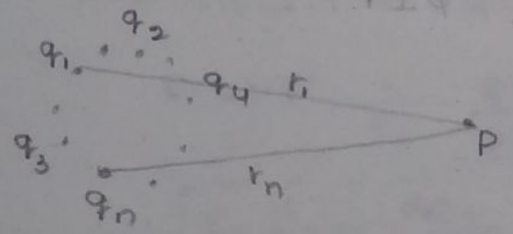
V_b = Electric potential at 'b' point

$$\boxed{V(r) = \frac{k_e q}{r}} \rightarrow \text{Electric potential}$$

units: J/c - 1 volt



$$V = \frac{k_e q}{r}$$

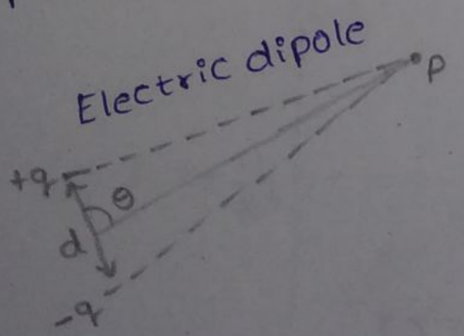


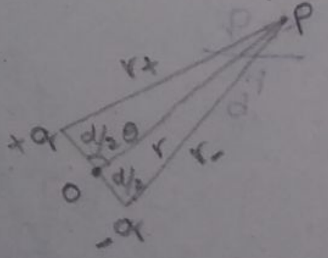
$$V = k_e \frac{q}{r_1} + k_e \frac{q_2}{r_2} + k_e \frac{q_3}{r_3} + \cdots + k_e \frac{q_n}{r_n}$$

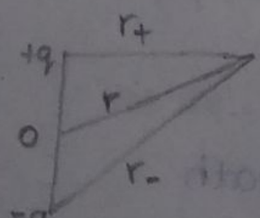
$$\boxed{V = k_e \sum_{i=1}^{n} \frac{q_i}{r_i}}$$

Super position principle holds good for 'V' also along with $\vec{E}$, $\vec{F}$

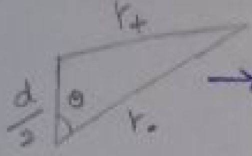
Electric potential at a point due to an electric dipole :







$$V(r) = \frac{k_e q}{r_+} - \frac{k_e q}{r_-} \rightarrow (1)$$



$\rightarrow$ Cosines Law

$$r^2 = r^2 + \left(\frac{d}{2}\right)^2 - \cancel{2}r\left(\frac{d}{\cancel{2}}\right)\cos\theta$$

$$= r^2\left(1 + \frac{d^2}{4r^2} - \frac{\cancel{r}d}{r\cancel{2}}\cos\theta\right)$$

$$r_+ = r\left[1 + \frac{d^2}{4r^2} - \frac{d}{r}\cos\theta\right]^{1/2}$$

$d <<<< r \quad \frac{d^2}{r^2} \rightarrow$ neglected

$$r_+ = r\left[1 - \frac{d}{r}\cos\theta\right]^{1/2}$$

$$V(r) = \frac{k_e q}{r_+} - \frac{k_e q}{r_-} = k_e q\left[\frac{1}{r_+} - \frac{1}{r_-}\right]$$

$$\rightarrow \frac{1}{r_+} = \frac{1}{r}\left[1 - \frac{d}{r}\cos\theta\right]^{-1/2} = \frac{1}{r}\left[1 + \frac{d}{2r}\cos\theta\right]$$

$$= \frac{1}{r} + \frac{d}{2r^2}\cos\theta$$

$$\rightarrow \frac{1}{r_-} = \frac{1}{r}\left[1 + \frac{d}{r}\cos\theta\right]^{-1/2} = \frac{1}{r}\left[1 - \frac{d}{2r}\cos\theta\right]$$

$$= \frac{1}{r} - \frac{d}{2r^2}\cos\theta$$

$$\frac{1}{r_+} - \frac{1}{r_-} = \cancel{\frac{1}{r}} - \cancel{\frac{1}{r}} + \frac{\cancel{2}d\cos\theta}{\cancel{2}r^2} = \frac{d\cos\theta}{r^2}$$

$$V = \frac{k_e q d\cos\theta}{r^2} \Rightarrow V = \frac{k_e P\cos\theta}{r^2} \quad (\because P = qd)$$

$$\boxed{V_{dipole} \propto \frac{1}{r^2}}$$

$\rightarrow$ dipolemoment $(P) = qd$

= charge $\times$ distance

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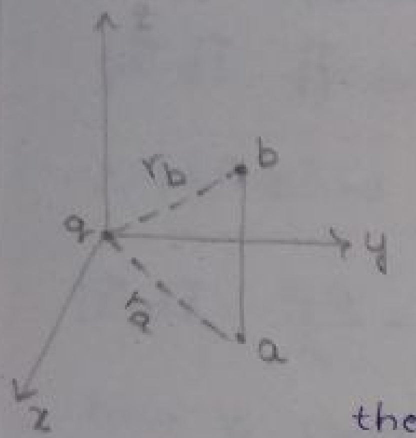


If $r_a = r_b$

$\oint \vec{E} \cdot d\vec{l} = 0$

$\oint \vec{E} \cdot d\vec{l} = \oiint (\vec{\nabla} \times \vec{E}) \cdot d\vec{s}$

$\Downarrow$

equals to zero

then $\vec{E}$ = conservative field / Electrostatic field

$$\int \vec{E} \cdot d\vec{l} = \frac{k_e q}{r_a} - \frac{k_e q}{r_b}$$

$$= -\left[\frac{k_e q}{r_b} - \frac{k_e q}{r_a}\right]$$

$$= -(V_b - V_a)$$

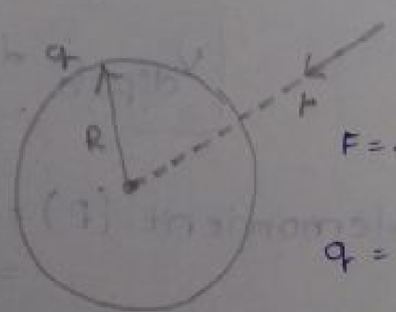
$$\Rightarrow -\int_a^b \vec{E} \cdot d\vec{l} = (V_b - V_a)$$

* $\boxed{V(r) = -\int_O^r \vec{E} \cdot d\vec{l}}$ here O = reference point

$V_b - V_a = \int_a^b \vec{\nabla} V \cdot d\vec{l}$ → Fundamental theorem for the gradients

$$\boxed{\vec{E} = -\vec{\nabla} V}$$

Ex. Find the potential inside and outside a spherical shell of radius R, that carries a uniform surface charge. Set the reference point at infinity.



$F = \frac{q}{4\pi R^2}$

$q = \frac{\sigma 4\pi R^2}{r}$

$d\vec{r} = dr\,\hat{r}$

Sol. Let

uniform surface charge = σ

$V(r) = -\int_{ref}^{r} \vec{E} \cdot d\vec{l}$ → formula

$E(r) = \frac{k_e q}{r^2}\hat{r}$

$$v(r) = -\int_{ref}^{r} \frac{k_e q}{r^2}\,\hat{r}\cdot dr\,\hat{r}$$

$$= -\int_{\infty}^{r} \frac{k_e q}{r^2}\,dr$$

$$= k_e q\left[\frac{1}{r}\right]_{\infty}^{r}$$

$$= \frac{k_e q}{r} - \frac{k_e q}{\infty}$$

$$= \frac{k_e \sigma 4\pi R^2}{r} \quad \left[\because V(r) = \frac{k_e q}{r}\right]$$

$$\Rightarrow \frac{1}{\cancel{4}\cancel{\pi}\varepsilon_0}\cdot\frac{\sigma\cancel{4}\cancel{\pi}R^2}{r}$$

$$\therefore \boxed{V_{out}^{(r)} = \frac{\sigma R^2}{\varepsilon_0 r}}$$

For inside :

$$V(r) = -\int_{o}^{r}\vec{E}\cdot d\vec{r}$$

↳ ref point (∞)

$$= -\int_{\infty}^{r}\vec{E}(r)\cdot d\vec{r}$$

$$= -\left[\int_{\infty}^{R}\vec{E}(r)\cdot d\vec{r} + \int_{R}^{r}\vec{E}(r)\cdot d\vec{r}\right]$$

$$= -\int_{\infty}^{R}\vec{E}(r)\cdot d\vec{r} - \int_{R}^{r}\vec{E}(r)\cdot d\vec{r}$$

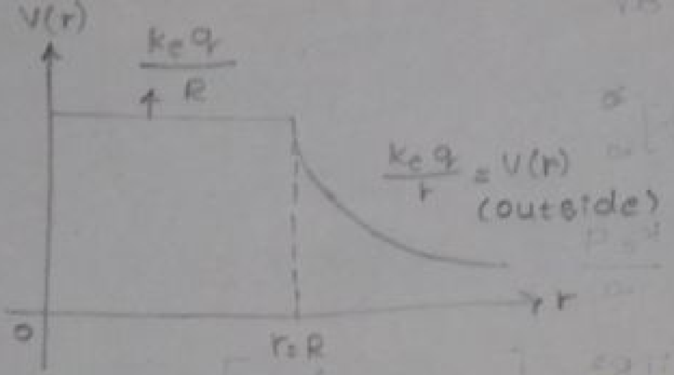
$$= -\int_{\infty}^{R}\frac{k_e q}{r^2}\,dr - 0$$ ($\because$ Elecric field inside the spherical shell is zero)

$$= k_e q\int_{\infty}^{R} -\frac{1}{r^2}\,dr$$

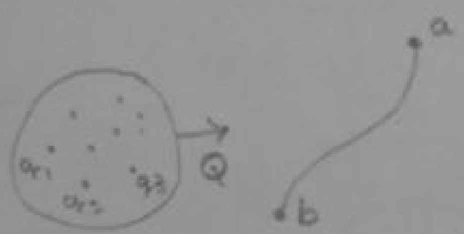
$$= k_e q\left[\frac{1}{r}\right]_{\infty}^{R}$$

$$= k_e q\left[\frac{1}{R} - \underbrace{\frac{1}{\infty}}_{0}\right] = \frac{k_e q}{R}$$

$$\boxed{V(r)_{inside} = \frac{k_e q}{R} = V_R}$$



→ Inside the potential is constant & why because R is constant and outside potential may varies why because r varies from surface to infinity.



Q = unit +ve charge

$\vec{F} = -Q\vec{E} = -\vec{F_e}$

$\Rightarrow \vec{F_e} = -Q\vec{E}$

$$W = \int_a^b \vec{F} \cdot d\vec{l}$$

$$= \int_a^b -Q\vec{E} \cdot d\vec{l}$$

$$= -Q \int_a^b \vec{E} \cdot d\vec{l}$$

$$W = Q\left[\frac{k_e q}{r_b} - \frac{k_e q}{r_a}\right]$$

$$W = Q\left[V(b) - V(a)\right]$$

$$\frac{W}{Q} = V_b - V_a \Rightarrow J/C = 1V$$

→ potential difference between two points a and b is equals to the work done per unit +ve charge.

$$W = Q(V_b - V_a)$$

$$\boxed{W = Q\Delta V}$$

$$W = Q(V_b - V_a)$$

If a is at infinity

$$V_a = 0$$

$$W = Q V_b \quad \left(\because V_b = \frac{k_e q}{r}\right)$$

$$W = \frac{k_e Q q}{r}$$

$$\Rightarrow \boxed{W = U = \frac{k_e Q q}{r}} \rightarrow \text{Electric potential energy}$$

$$\Rightarrow U = \frac{k_e q_1 q_2}{r}$$

Poisson's equation:

Relation between E and V

$$E = -\text{grad}\, V$$

$$\Rightarrow \vec{E} = -\vec{\nabla} V$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \rightarrow \text{Gauss's law in differential form}$$

$$\vec{\nabla} - \vec{\nabla} V = \frac{\rho}{\varepsilon_0}$$

$$(-\vec{\nabla} \cdot \vec{\nabla} V) = \frac{\rho}{\varepsilon_0}$$

$$-\nabla^2 V = \frac{\rho}{\varepsilon_0}$$

$$\Rightarrow \boxed{\nabla^2 V = \frac{-\rho}{\varepsilon_0}} \rightarrow \text{Poisson's equation}$$

If $\rho = 0 \Rightarrow$ free size, no charges $\rho = 0$

$$\boxed{\nabla^2 V = 0} \rightarrow \text{Laplace's equation}$$

$$V(r) = -\int_0^r \vec{E}\cdot d\vec{l}$$

$V(r) \rightarrow$ path independent

$$V(b) - V(a) = -\int_a^b \vec{E}\cdot d\vec{l}$$

$$\oint \vec{E}\cdot d\vec{l} = 0 \Rightarrow \vec{\nabla}\times\vec{E} = 0$$

$$\Rightarrow \vec{E} = -\vec{\nabla}V$$

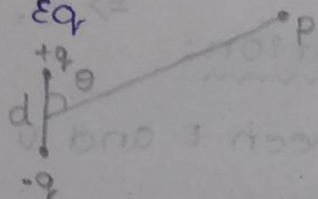
$$W = Q(V(r))$$

$$\Rightarrow V(r) = \frac{W}{Q}$$

$\nabla^2 V = \frac{-\rho}{\varepsilon_0} \rightarrow$ poisson's Eq

$\nabla^2 V = 0 \rightarrow$ Laplace's Eq

$$V_{dipole} = \frac{k_e q d\cos\theta}{r^2}$$



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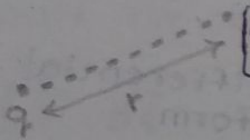


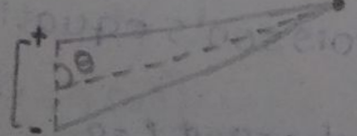
$$\boxed{V_P = \frac{k_e q}{r}}$$

$$V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{l}$$

$$\Rightarrow V_b - V_a = \int_a^b \vec{\nabla}V\cdot d\vec{l}$$

$$\boxed{\vec{E} = -\vec{\nabla}V}$$

(V_{dipole}) d



$$\boxed{V_P = \frac{k_e q d\cos\theta}{r^2}}$$

Hence

$$V_{Monopole} \propto \frac{1}{r}$$

$$V_{dipole} \propto \frac{1}{r^2}$$

Electric field due dipole from electric potential:

$$V_p = \frac{k_e P \cos\theta}{r^2}$$

$$\vec{E} = -\vec{\nabla} V$$

$$\vec{\nabla} V = \frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{\phi} \rightarrow \text{In spherical polar coordinates}$$

$$\vec{E} = \frac{-\partial}{\partial r}\left(\frac{k_e P\cos\theta}{r^2}\right)\hat{r} - \frac{1}{r}\frac{\partial}{\partial\theta}\left(\frac{k_e P\cos\theta}{r^2}\right)$$

$$= -k_e P\cos\theta \frac{\partial}{\partial r}\left(\frac{1}{r^2}\right)\hat{r} - \frac{1}{r}\frac{k_e P}{r^2}\frac{\partial}{\partial\theta}(\cos\theta)\hat{\theta}$$

$$= \frac{2k_e P\cos\theta}{r^3}\hat{r} + \frac{k_e P}{r^3}\sin\theta\,\hat{\theta}$$

$$\frac{\partial}{\partial r}\left(\frac{1}{r^2}\right) = \frac{-2}{r^3}$$

$$= \frac{k_e P}{r^3}(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta})$$

$$\frac{\partial}{\partial\theta}(\cos\theta) = -\sin\theta$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{qd}{r^3}(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta})$$

$\hookrightarrow$ vector form

$$|\vec{E}| = \frac{1}{4\pi\varepsilon_0}\frac{qd}{r^3}\sqrt{4\cos^2\theta + \sin^2\theta} \rightarrow \text{Magnitude}$$

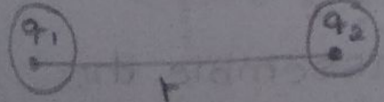
$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0} \rightarrow \text{Gauss's law in differential form}$$

$$\vec{\nabla}\cdot(-\vec{\nabla}V) = \frac{\rho}{\varepsilon_0} \rightarrow \nabla^2 V = \frac{-\rho}{\varepsilon_0} \rightarrow \text{Poisson's eq}$$

If $\rho = 0$

$$\nabla^2 V = 0 \rightarrow \text{Laplace's eq}$$

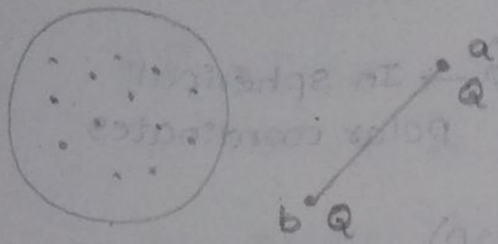
$$\left[\frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2} + \frac{\partial}{\partial z^2}\right]V = 0 \rightarrow \text{Laplace's eq for Cartesian coordinates}$$



$$U = \frac{k_e q_1 q_2}{r}$$

$$\Rightarrow \frac{U}{q_2} = \frac{k_e q_1}{r}$$

$\Rightarrow \boxed{U = q_2 V_P}$ (or) $\boxed{W = qV_P}$



$$W = \int_a^b \vec{F}_{ext} \cdot \vec{dl}$$

$$W = \int_a^b -Q\vec{E} \cdot \vec{dl} \quad (\because \vec{F}_e = Q\vec{E})$$

$$\vec{F}_{ext} = -\vec{F}_e = -Q\vec{E}$$

$$= -Q \int_a^b \vec{E} \cdot \vec{dl}$$

$$= Q\left[-\int_a^b \vec{E} \cdot \vec{dl}\right]$$

$$= Q(V_b - V_a)$$

$$W = Q\Delta V$$

$$\Rightarrow \Delta V = \frac{W}{Q} = \frac{U}{Q}$$

$$\Rightarrow \boxed{U = Q\Delta V}$$

Problem :

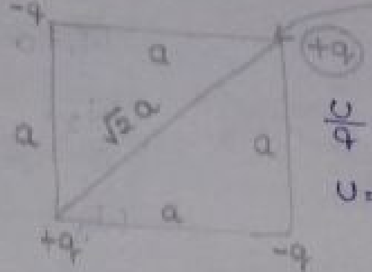
(a) Three charges are situated at the corners of a square (side a). How much work does it take to bring in another charge, +q, from far away and place it in the fourth corner?

(b) How much work does it take to assemble the whole configuration of four charges?

Sol: $U = \frac{k_e q_1 q_2}{r}$

$\Sigma V_i = V$

$\Rightarrow V_1 + V_2 + V_3$



$\frac{U}{q} = V$

$U = Vq$

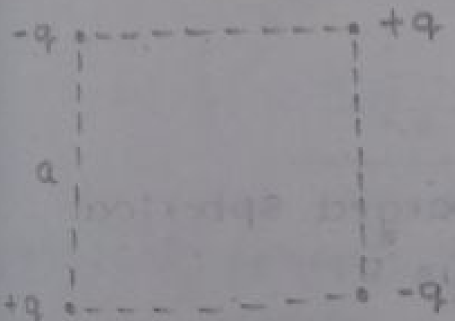
$$V = \frac{k_e(-q)}{a} + \frac{k_e(+q)}{\sqrt{2}a} + \frac{k_e(-q)}{a}$$

$$= -\frac{2k_e q}{a} + \frac{k_e q}{\sqrt{2}a}$$

$$= \frac{k_e q}{a}\left[\frac{1}{\sqrt{2}} - 2\right]$$

$$U = Vq = \frac{k_e q^2}{a}\left(\frac{1}{\sqrt{2}} - 2\right)$$

$$\therefore \boxed{U = \frac{k_e q^2}{a}\left(\frac{1}{\sqrt{2}} - 2\right)}$$



$W_1 = 0$

$W_2 = -\frac{k_e q}{a}$

$W_3 = \frac{-k_e q^2}{a} + \frac{k_e q^2}{\sqrt{2}a}$

$W_4 = \frac{-k_e q^2}{a} - \frac{k_e q^2}{a} + \frac{k_e q^2}{\sqrt{2}a}$

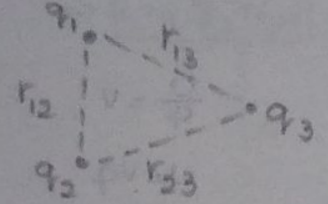
Now,

$$W_1 + W_2 + W_3 + W_4 = \frac{-k_e q^2}{a} - \frac{k_e q^2}{a} + \frac{k_e q^2}{\sqrt{2}a} - \frac{k_e q^2}{a} - \frac{k_e q^2}{a} + \frac{k_e q^2}{\sqrt{2}a}$$

$$= \frac{2k_e q^2}{\sqrt{2}a} - \frac{4k_e q^2}{a}$$

$$W = \frac{2k_e q^2}{a}\left[\frac{1}{\sqrt{2}} - 2\right]$$

$$\Rightarrow \boxed{W = \frac{1}{2}\sum_{i=1}^{n} q_i V(r_i)}$$



$$W_1 = 0$$

$$W_2 = \frac{k_e q_1 q_2}{r_{12}}$$

$$W_3 = \frac{k_e q_1 q_3}{r_{13}} + \frac{k_e q_2 q_3}{r_{23}}$$

$$W_{net} = W_1 + W_2 + W_3 + \text{-----} + W_n$$

$$\boxed{W = \frac{1}{2} \sum_{i=1}^{n} q_i \, v(r_i)}$$ → Total work done to assemble of point charges.

(discrete charge distribution)

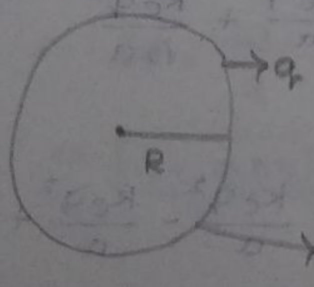
→ here $\rho = \frac{dq}{d\tau} \Rightarrow dq = \rho \, d\tau \Rightarrow q = \int \rho \, d\tau$

$(\because q = \int \rho d\tau)$ $v \rightarrow$ $$\boxed{W = \frac{1}{2} \int \rho v \, d\tau}$$ → Energy / work done of a continuous charge distribution

$E \rightarrow$ $$\boxed{W = \frac{\varepsilon_0}{2} \int_{\text{all space}} E^2 \, d\tau}$$

Find the energy of a uniformly charged spherical shell of total charge q and radius 'R'.

Sol.



$$W = \frac{1}{2} \int \rho v \, d\tau$$

$$E_{out} = \frac{k_e q}{r^2} \hat{r}$$

$$W = \frac{\varepsilon_0}{2} \int E^2 \, d\tau$$

$d\tau = r^2 \sin\theta \, dr \, d\theta \, d\phi$

$$= \frac{\varepsilon_0}{2} \int \left(\frac{k_e q}{r^2}\right)^2 r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= \frac{\varepsilon_0}{2} k_e^2 q^2 \int_R^{\infty} \int_0^{\pi} \int_0^{2\pi} \frac{1}{r^4} r^2 dr \, \underbrace{\sin\theta \, d\theta}_{2} \, \underbrace{d\phi}_{2\pi}$$

$$= \frac{\varepsilon_0}{2} k_e^2 q^2 (4\pi) \int_R^{\infty} \frac{1}{r^2} dr$$

$$= \frac{\varepsilon_0}{2} k_e^2 q^2 (4\pi) \left[\frac{-1}{r}\right]_R^{\infty}$$

$$= \frac{\cancel{\varepsilon_0}}{2} \cdot \frac{1}{\underset{4}{\cancel{16}}\pi^{\cancel{2}} \varepsilon_0^{\cancel{2}}} \cdot q^2 (\cancel{4\pi}) - \left[\frac{1}{\infty} - \frac{1}{R}\right]$$

$$\boxed{W_{total} = \frac{1}{8\pi\varepsilon_0} \cdot \frac{q^2}{R}}$$

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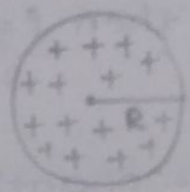
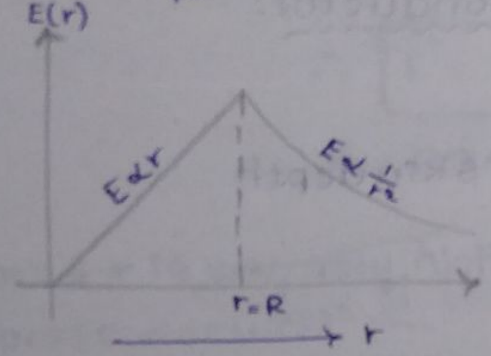
Find the energy stored in an uniformly charged solid sphere of radius 'R' and charge q.

Sol. $E_{inside} = \frac{k_e q r}{R^3}\hat{r}$ here $r<R$



$E_{outside} = \frac{k_e q}{r^2}\hat{r}$ here $r>R$



$$W = U = \frac{\varepsilon_0}{2}\int_{\text{All space}} E^2 d\tau = \frac{\varepsilon_0}{2}\int_0^R E_{in}^2 r^2 (4\pi) dr + \frac{\varepsilon_0}{2}\int_R^{\infty} E_{out}^2 4\pi r^2 dr$$

$$= \frac{\varepsilon_0}{2}\int_0^R \left(\frac{k_e q r}{R^3}\right)^2 4\pi r^2 dr + \frac{\varepsilon_0}{2}\int_R^{\infty}\left(\frac{k_e q}{r^2}\right)^2 4\pi r^2 dr$$

$$= \frac{4\pi\varepsilon_0}{2} \cdot \frac{k_e^2 q^2}{R^6}\int_0^R r^4 dr + \frac{4\pi\varepsilon_0}{2} \cdot k_e^2 q^2 \int_R^{\infty} \frac{1}{r^4} \times \cancel{r^2} dr$$

$$= \left[\frac{k_e^2 q^2}{R^{\cancel{6}}} \times \frac{4\pi\varepsilon_0}{2} \times \frac{R^5}{5}\right] + \left[\frac{4\pi\varepsilon_0}{2} \cdot k_e^2 q^2 \left[\frac{-1}{r}\right]_R^{\infty}\right]$$

$$= \frac{k_e^2 q^2 4\pi\varepsilon_0}{10R} - k_e^2 q^2 \frac{4\pi\varepsilon_0}{2}\left[\underset{\Downarrow\, 0}{\frac{1}{\infty}} - \frac{1}{R}\right]$$

$$= \frac{k_e^2 q^2 4\pi\varepsilon_0}{R}\left[\frac{1}{10} + \frac{1}{2}\right]$$

$$= \frac{3}{5} \times \frac{1}{(4\pi\varepsilon_0)^2} \cdot \frac{q^2 (4\pi\varepsilon_0)}{R}$$

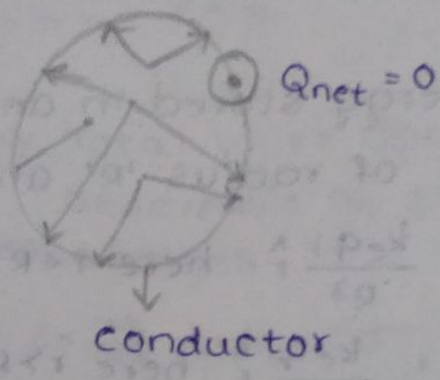
$$\boxed{W = \frac{1}{4\pi\varepsilon_0} \times \frac{3}{5} \times \frac{q^2}{R}}$$

Fields inside a perfect conductor:

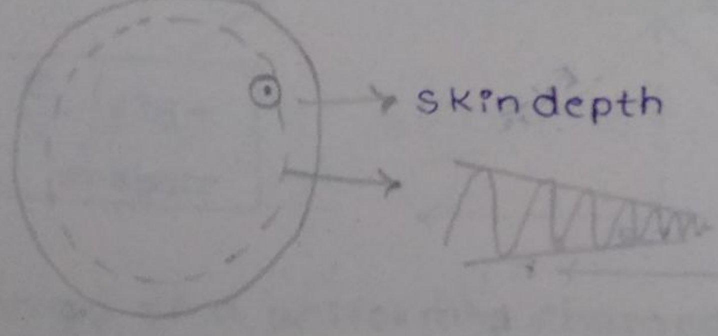
(i) $E_{inside} = 0$

$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$

if $\vec{E} = 0$ then $\rho = 0$



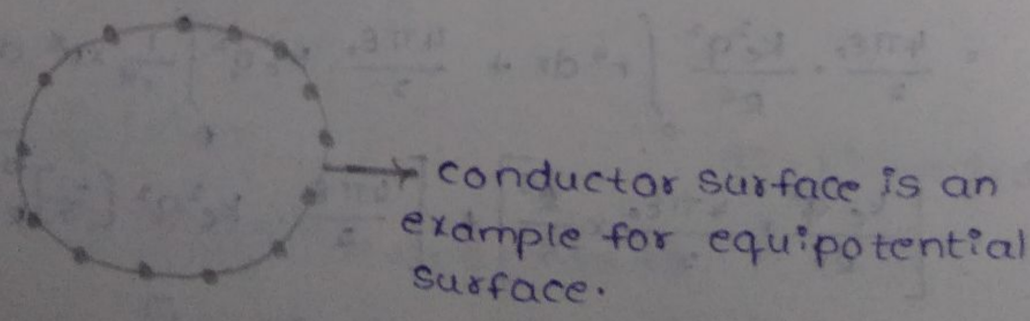
EM wave propagation of conductor:



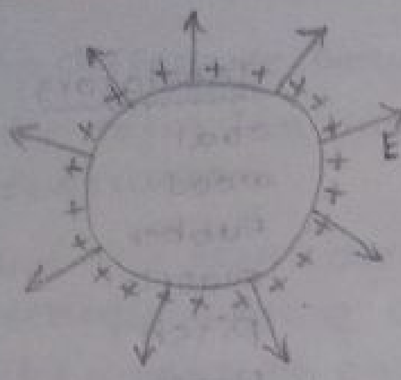
$E_{inside} = 0$

$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \Rightarrow \rho_{inside} = 0 \rightarrow$ for a conductor

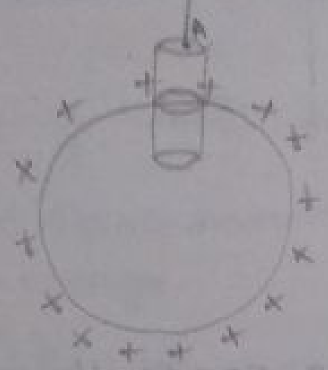
→ Any net charge resides on the surface



Here, the points on the surface are called equipotential points.



Just outside the conductor
$\vec{E}$ is $\perp$er to the Surface



$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{en}}{\varepsilon_0}$$

$$E \cancel{A} \cos 0^\circ = \frac{\sigma \cancel{A}}{\varepsilon_0} \quad \left(\because \sigma = \frac{Q}{A}\right)$$

$$E = \frac{\sigma}{\varepsilon_0}$$

$$\therefore \boxed{\vec{E} = \frac{\sigma}{\varepsilon_0} \hat{n}}$$

ε_0 = permitivity of free space

Conductors = σ is very very high
Insulators (Dielectrics)
semiconductors

Material	Dielectric constant 'K'
Air (dry)	1.00059
Bakelite	4.9
Fused quartz	3.78
Mylar	3.2
Neoprene rubber	6.7
Nylon	3.4
Paper	3.7
Paraffin-impregnated paper	3.5
Polystyrene	2.56
Polyvinyl chloride	3.4

Porcelain	6
Pyrex glass	5.6
Silicone oil	2.5
Strontium titanate	233
Teflon	2.1
Vaccum	1,00,0 00
Water	80

Insulators:

coal
wood
Rubber
Plastic
Pitch
Resin

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* for the Application of electric field it responds then it is said to be dielectris and if doesn't respond then it is said to be Insulators.

Dielectrics:

→ It is an insulating material or a very poor conductor.

→ When dielectrics are placed in an electric field, practically no current flows in them because unlike metals, they have no loosely bound, or free, electrons that may drift through the material.

→ Instead, electric polarization occurs.

Insulators:

→ An electric insulator is a material in which the electron doesn't flow freely or the atom of the insulator have tightly bound electrons whose internal electric charges don't flow freely; very little electric current will flow through it under the influence of electric field.

The difference between insulator and dielectric:

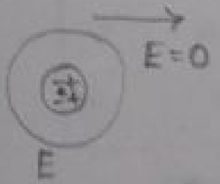
→ The term insulator is used to indicate electrical obstruction.

→ But the term dielectric is used to indicate the energy storing capacity of the material (by means of polarization).

$-q \rightarrow +q$

Electric dipole moment, it is directed from -ve charge to +ve charge.

$$|\vec{P}| = qd$$



Centre of +ve charge and centre of -ve charge do coincide at the centre.

→ Electrical dipole moment ($\vec{P}$) is proportional to applied field $\vec{E}$.

$$\vec{P} \propto \vec{E}$$

$$\vec{P} = \alpha \vec{E}$$

↓

Atomic polarizability

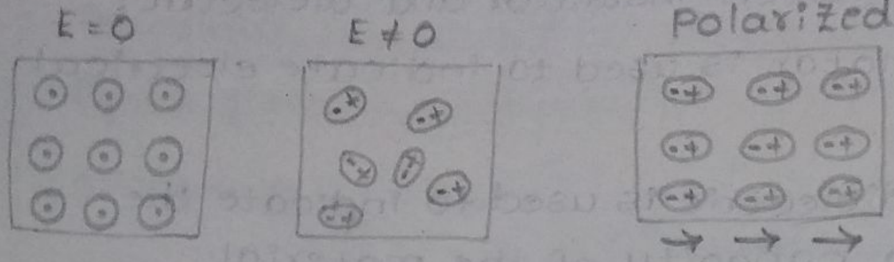
If

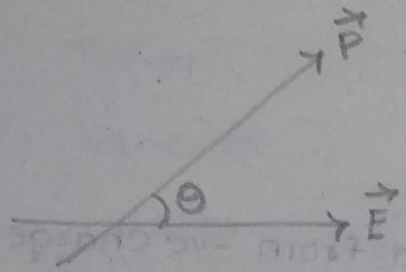
$P = 0 \rightarrow$ non-polar molecule

$P \neq 0 \rightarrow$ polar molecule

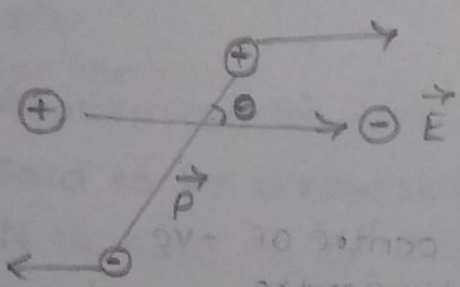
CO_2, HCl, NaCl, NH_3, H_2S, H_2O } Polar

H_2, Ne → Non-polar





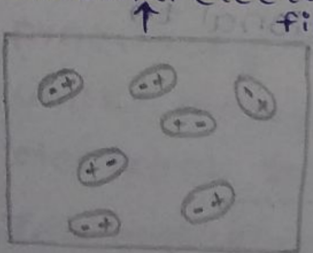
$$\vec{\tau} = \vec{p} \times \vec{E}$$



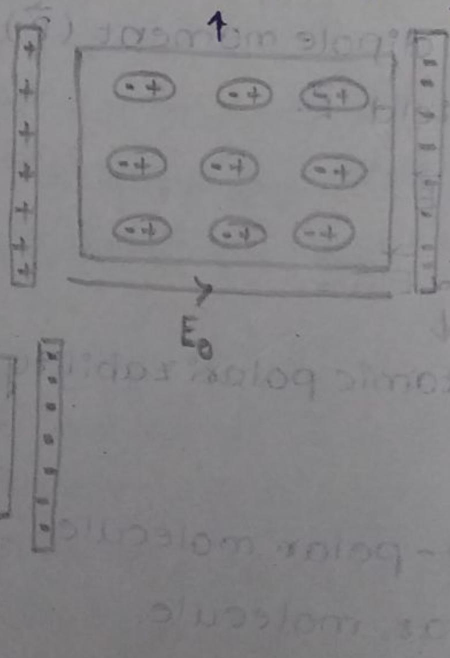
Potential energy of an electric dipole

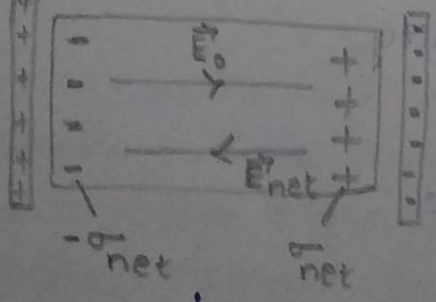
$$U = -\vec{p} \cdot \vec{E}$$

Polar molecules are randomly oriented in the absence of an external electric field.



When an external $\vec{E}$ is applied, the molecules partially align with the field.





The charged edges of the dielectric can be modeled as an additional pair of parallel plates (polarising) establishing an electric field $\vec{E}_{net}$ in the direction opposite that of $\vec{E}_0$.

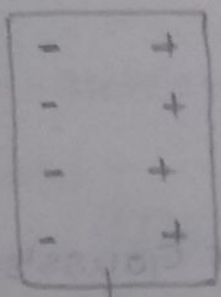
$\vec{P}$ = Electric dipolemoment per volumes is called "Electric polarization".

$$\frac{\vec{p}}{\Delta V} = \frac{\vec{p}}{\Delta \tau} = \vec{P} \rightarrow \text{Electric polarization}$$

$$\vec{P} \propto \vec{E}$$

$$\Rightarrow \boxed{\vec{P} = \varepsilon_0 \chi_e \vec{E}}$$

$\chi_e \rightarrow$ Electric susceptibility

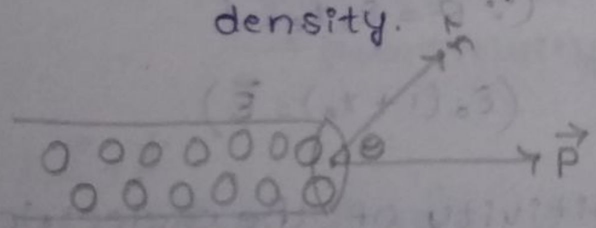


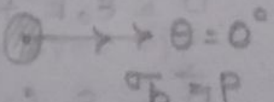
$\rightarrow$ Bound surface charge

Uniform polarization:

$\sigma_b = \vec{P}\cdot\hat{n} = P\cos\theta$ $\qquad\qquad \vec{P} = \frac{P}{\Delta V}$

Where σ_b = Bound surface charge density.

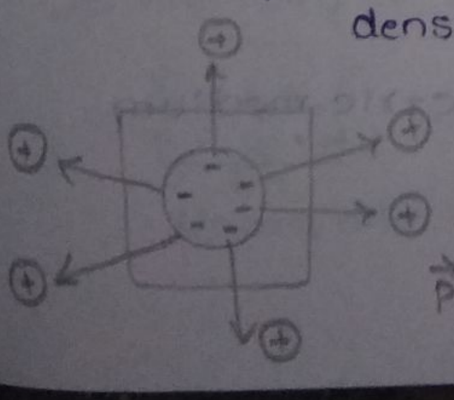




Non-uniform polarization:

$$\rho_b = -\vec{\nabla}\cdot\vec{P}$$

Where ρ_b = Bound volume charge density.



$$\vec{P} = \frac{\vec{P}}{\Delta V}$$

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Gauss's Law in dielectrics:

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\Rightarrow \varepsilon_0\, \vec{\nabla}\cdot\vec{E} = \rho$$

In case of dielectrics

$$\rho = \rho_b + \rho_f \qquad \text{(free electrons / ions)}$$

↳ free volume charge density

$$\varepsilon_0\, \vec{\nabla}\cdot\vec{E} = \rho_b + \rho_f$$

$$\vec{\nabla}\cdot\varepsilon_0\vec{E} = -\vec{\nabla}\cdot\vec{P} + \rho_f$$

$$\vec{\nabla}\cdot(\varepsilon_0\vec{E} + \vec{P}) = \rho_f$$

* $\boxed{\vec{\nabla}\cdot\vec{D} = \rho_f}$ → Differential form of Gauss's Law in dielectrics.

where

$\vec{D} = \varepsilon_0\vec{E} + \vec{P}$ → Electric displacement vector

$$\vec{D} = \varepsilon_0\vec{E} + \varepsilon_0\chi_e\vec{E} \quad (\because \vec{P} = \varepsilon_0\chi_e\vec{E})$$

$$\vec{D} = \varepsilon_0(1+\chi_e)\vec{E} \quad (\varepsilon_0(1+\chi_e) = \bar{\varepsilon})$$

$\boxed{\vec{D} = \varepsilon\vec{E}}$ → Permitivity of the dielectric medium

$\boxed{1 + \chi_e = \dfrac{\varepsilon}{\varepsilon_0} = k}$ → Dielectric constant

↳ $\vec{D} \propto \vec{E}$

$\Rightarrow \vec{D} = \varepsilon\vec{E}$ → Linear dielectric medium.

$\vec{\nabla}\cdot\vec{D} = \rho_f$

$\iiint_V \vec{\nabla}\cdot\vec{D}\, d\tau = \iiint \rho_f\, d\tau = (Q_{enc})_f$ $\qquad \rho_f = \frac{dQ_f}{d\tau} \Rightarrow dQ_f = \rho_f\, d\tau$

$\Downarrow$

Gauss's divergence theorem

$\iiint \vec{\nabla}\cdot\vec{D}\, d\tau = \oiint \vec{D}\cdot d\vec{A}$

$\boxed{\oiint_S \vec{D}\cdot d\vec{A} = (Q_f)_{enc}}$ → Integral form of Gauss's law in dielectrics

k = dielectric constant = $\frac{\text{Permitivity of the medium}}{\text{permitivity of the free space}}$

$\boxed{k = 1 + \chi_e = \frac{\varepsilon}{\varepsilon_0}}$

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$k = 1 + \chi_e = \frac{\varepsilon}{\varepsilon_0}$

$\vec{P} \propto \vec{E} \Rightarrow \vec{P} = \varepsilon_0 \chi_e \vec{E}$

→ Electric susceptibility (χ_e)

→ permittivity of the free space (ε_0)

$\vec{D} = \varepsilon_0\vec{E} + \vec{P}$

↳ $\vec{P} = 0$, if there is no matter to polarize

$\Rightarrow \boxed{\vec{D} = \varepsilon_0\vec{E}}$ → free space

$\boxed{\varepsilon = \varepsilon_0(1+\chi_e)}$

↓

Permittivity of the medium

$\vec{P} = \varepsilon_0\chi_e\vec{E}$ $\qquad \vec{P} = \varepsilon_0\chi_e\left[\vec{E} + \frac{E_0^2}{2} + \frac{E_0^3}{3} + \cdots\right]$

Linear dilectrics

Gauss's law in dielectrics:

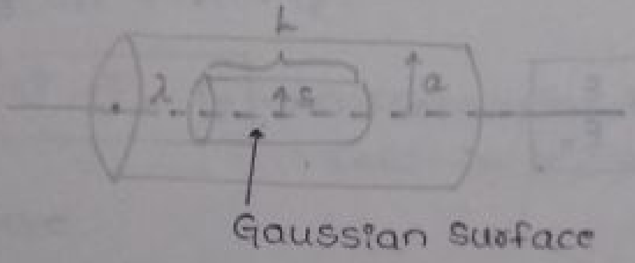
$\vec{\nabla}\cdot\vec{D} = \rho_f$ → differential form

ρ_f ↓ free volume charge density

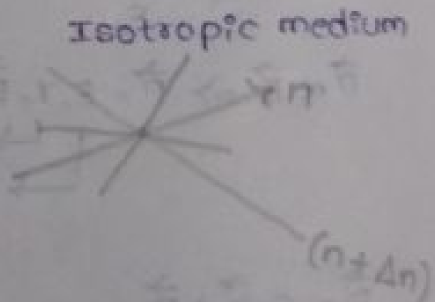
$$\iiint \vec{\nabla}\cdot\vec{D}\, d\tau = \iiint_V \rho_f\, d\tau = (Q_f)_{enc}$$

$$\oint\!\!\oint \vec{D}\cdot d\vec{s} = (Q_f)_{enc}$$ → Integral form of gauss's law

P1. A long straight wire, carrying uniform line charge λ, is surrounded by rubber insulation out to a radius a. Find the electric displacement.



Sol. Long straight (line) wire



$$\lambda = \frac{Q}{L}$$

$$\Rightarrow Q = \lambda L$$

$$\oint\!\!\oint \vec{D}\cdot d\vec{A} = (Q_f)_{enc}$$

$$D(2\pi s L)\cos 0^\circ = \lambda L$$

$$\boxed{\vec{D} = \frac{\lambda}{2\pi s}\hat{s}}$$

Note:

→ $\frac{Q_b}{A} = \sigma_b = \vec{P}\cdot\hat{n}$ → Bound surface charge density

→ $\frac{Q_b}{V} = \rho_b = -\vec{\nabla}\cdot\vec{P}$ → Bound volume charge density

B. A sphere of radius R carries a polarization

$P(r) = kr$

where k is a constant and r is the vector from the center.

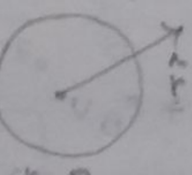
(a) Calculate the bond charges σ_b and ρ_b.

(b) Find the field inside and outside the sphere.

Sol.

(a) $\sigma_b = \vec{P}\cdot\hat{n}$

$= kr\cdot\hat{r}\cdot\hat{r}$



since, $\vec{P}(r) = kr\,\hat{r}$

$\Rightarrow \boxed{\sigma_b = kR}\ (r = R)$

$\rho_b = -(\vec{\nabla}\cdot\vec{P})$

$$\nabla\cdot V = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 V_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta V_\theta) + \frac{1}{r\sin\theta}\frac{\partial V_\phi}{\partial\phi}$$

here P_θ and $P_\phi = 0$

$\Rightarrow \vec{P} = kr.\hat{r}$

$$\rho_b = -\left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 kr)\right]$$

$$= -\left[\frac{1}{r^2}\frac{\partial}{\partial r}(kr^3\right] = -\frac{k}{r^2}\frac{\partial}{\partial r}(r^3)$$

$$= -\frac{k}{\cancel{r^2}}.3\cancel{r^2}$$

$$= -3k$$

$$\Rightarrow \boxed{\rho_b = -3k}$$

(b) $E_{inside} = \frac{1}{3\varepsilon_0}\rho r\hat{r}$ → Electric field inside sphere

$$= \frac{1}{\cancel{3}\varepsilon_0}\cdot\frac{Qr}{(\frac{4}{\cancel{3}}\pi R^3)}\cdot\hat{r} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qr}{R^3}\hat{r}$$

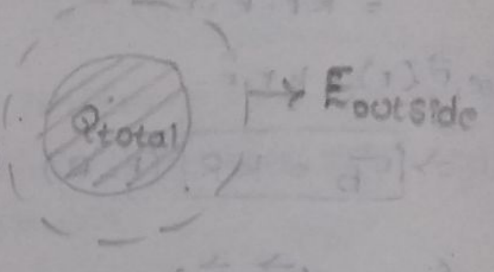
$$\left[= \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qr}{R^3}\hat{r}\right]$$

$$\boxed{E_{inside} = \frac{k_e Q r}{R^3} \cdot \hat{r}}$$

Since here $\rho = -3k$

$$\Rightarrow E_{inside} = \frac{-\cancel{3} k r}{\cancel{3} \varepsilon_0} \cdot \hat{r} = -\left(\frac{k}{\varepsilon_0}\right) r \cdot \hat{r}$$

$$\boxed{E_{inside} = \left(\frac{-k}{\varepsilon_0}\right) \cdot r \cdot \hat{r}}$$



$$Q_{total} = \sigma_b A + \rho_b V$$

$$= kR(4\pi R^2) + (-\cancel{3}k)(\tfrac{4}{\cancel{3}}\pi R^3)$$

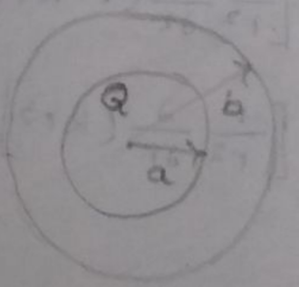
$$= 4\pi k R^3 - 4\pi k R^3$$

$$= 0$$

$\boxed{E_{outside} = 0}$ as $Q_{total} = 0$

P3. A metal sphere of radius 'a' carries a charge 'Q'. It is surrounded, out to radius b, by linear dielectric material of permittivity ε. Find the potential at the center (relative to infinity).

Sol. $V = -\int_{\infty}^{0} \vec{E} \cdot d\vec{l}$



$$= -\left[\int_{\infty}^{b} \vec{E} \cdot d\vec{l} + \int_{b}^{a} \vec{E} \cdot d\vec{l} + \int_{a}^{0} \vec{E} \cdot d\vec{l}\right]$$

↓ Surface of the dielectric

↓ dielectric to metal surface

↳ Metal surface to center

$$= -\left[\int_{\infty}^{b} \frac{k_e Q}{r^2} dr + \int_{b}^{a} \frac{Q}{4\pi\varepsilon} \frac{1}{r^2} dr + \int_{a}^{0} (0)\, dr\right]$$

↓ outside

↓ inside the dielectric

↓ inside the metal surface (sphere)

$$= k_e Q \int_{\infty}^{b} \left(\frac{-1}{r^2} dr\right) + \frac{Q}{4\pi\varepsilon} \int_{b}^{a} \left(\frac{-1}{r^2} dr\right)$$

$$= k_e Q \left[\frac{1}{r}\right]_{\infty}^{b} + \frac{Q}{4\pi\varepsilon} \left[\frac{+1}{r}\right]_{b}^{a}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot Q \left[\frac{1}{b} - \frac{1}{\infty}\right] + \frac{Q}{4\pi\varepsilon_0} \left[\frac{1}{a} - \frac{1}{b}\right]$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{b} + \frac{Q}{4\pi\varepsilon} \cdot \frac{1}{a} - \frac{Q}{4\pi\varepsilon_0} \cdot \frac{1}{b}$$

$$\therefore \boxed{V = \frac{Q}{4\pi} \left(\frac{1}{\varepsilon_0 b} + \frac{1}{\varepsilon a} - \frac{1}{\varepsilon b} \right)}$$

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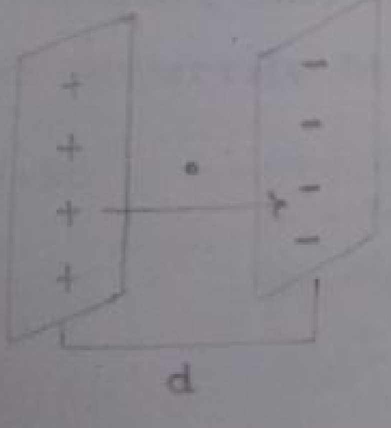
Parallel Capacitor:



$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} \quad \left(\because \sigma = \frac{Q}{A}\right)$$

$$\Delta V = Ed$$

$$\Delta V = \left(\frac{Q}{\varepsilon_0 A}\right) d$$

$E \propto Q$

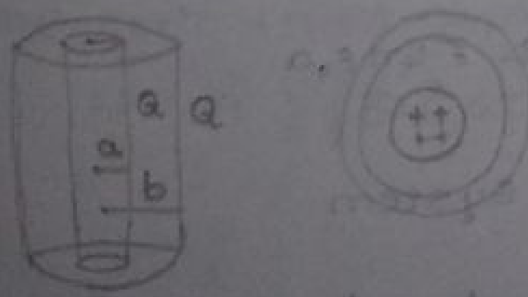
$\Delta V \propto Q$

$$C = \frac{\varepsilon_0 A}{d} = \frac{Q}{\Delta V}$$

$$\Delta V = \frac{1}{C} Q \Rightarrow \boxed{C = \frac{Q}{\Delta V}}$$

$$1 \text{ Faraday} = \frac{1C}{1V}$$

Cylindrical Capacitor:



$$C = \frac{Q}{\Delta V} = \frac{l}{2 k_e \ln\left(\frac{b}{a}\right)} \Rightarrow C \propto l$$

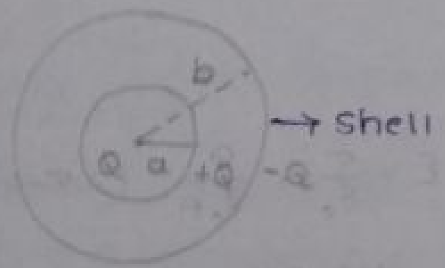
$$\because k_e = \frac{1}{4\pi\varepsilon_0}$$

$$\boxed{C = \frac{2\pi\varepsilon_0 l}{\ln(b/a)}}$$

→ It depends on the radii of the two cylindrical coordinates.

$$\frac{C}{l} = \frac{\lambda}{\Delta V} = \frac{2\pi\varepsilon_0}{\ln(b/a)}$$

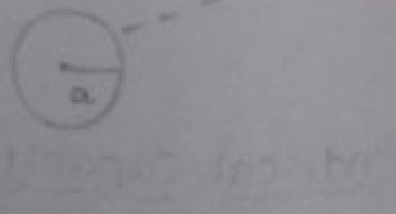
Spherical capacitor:



$$C = \frac{ab}{k_e(b-a)} = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$$

$$\boxed{C_{spherical} = \frac{4\pi\varepsilon_0}{\left(\frac{1}{a}-\frac{1}{b}\right)}}$$

$$b = \infty$$



$$C_{spherical} = \frac{4\pi\varepsilon_0}{\left(\frac{1}{a}-\frac{1}{\infty}\right)}$$

$$= \frac{4\pi\varepsilon_0}{\frac{1}{a}} = 4\pi\varepsilon_0 a$$

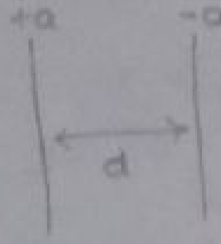
Capacitance of an Earth system

6400 km

$\Rightarrow 6400 \times 10^3$ m $\qquad k_e = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$

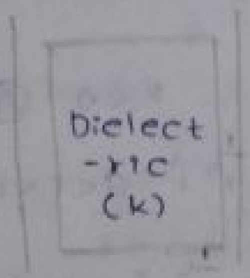
$C_{earth} = 712 \times 10^{-4}$ F

$\Rightarrow \boxed{C_{earth} = 712\,\mu F}$



$$C = \frac{\varepsilon_0 A}{d}$$

d - increases then c decreases and viceversa



$C = kC_0$ → capacitance increases

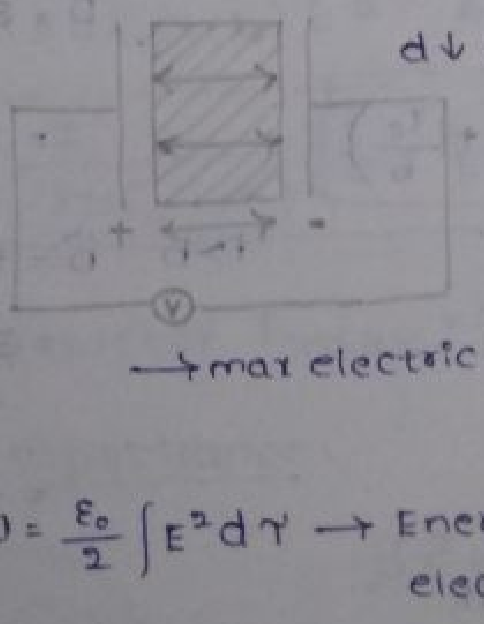
increase in max operating voltage

→ Mechanical support

Dielectric strength: The maximum electric field increase in strength (V/m) can a dielectric withstand is called dielectric strength.

$C = \frac{k\varepsilon_0 A}{d}$ → serway

$\Delta V = Ed$



$d \downarrow\ C \uparrow$

Teflon → $k = 2.1$

60×10^6 V/m

Porcelain → $k = 2.56$

12×10^6 V/m

→ max electric field

$W = U = \frac{\varepsilon_0}{2}\int E^2\, d\tau$ → Energy stored or electrostatic in any system

Dielectric filled capacitor

$W = U = \frac{1}{2}\int \vec{D}\cdot\vec{E}\, d\tau$

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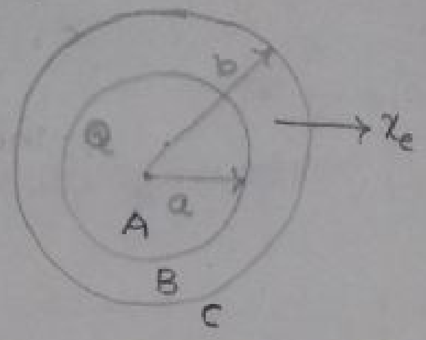
Problem. A spherical conductor, of radius a, carries a charge Q. It is surrounded by linear dielectric material of susceptibility χ_e, out to radius b. Find the energy of this configuration.

Sol. $\oint\!\!\oint \vec{E}\cdot d\vec{S} = \frac{Q_{enc}}{\varepsilon_0}$



Formula for energy

$$\boxed{W = \frac{1}{2}\int \vec{D}\cdot\vec{E}\, d\tau}$$

$\Rightarrow d\tau = 4\pi r^2 dr$

θ, ϕ

$$E = \begin{cases} 0 & r<a \quad (A) \\ \frac{Q}{4\pi\varepsilon}\cdot\frac{1}{r^2}\hat{r} & a<r<b \quad (B) \\ \frac{Q}{4\pi\varepsilon_0}\cdot\frac{1}{r^2}\hat{r} & r>b \quad (C) \end{cases}$$

$\vec{D} = \varepsilon_0\vec{E} + \vec{P}$

$= \varepsilon_0\vec{E} + \varepsilon_0\chi_e\vec{E}$

$= \varepsilon_0\vec{E}(1+\chi_e)$

$= \varepsilon_0(1+\chi_e)\vec{E}$

$\boxed{\vec{D} = \varepsilon\vec{E}} \quad \because \varepsilon = \varepsilon_0(1+\chi_e)$

$r<a \quad \vec{D} = 0$

$r>a \quad \vec{D} = \varepsilon E$

$\vec{D} = \cancel{\varepsilon}\cdot\frac{1}{4\pi\cancel{\varepsilon}}\cdot\frac{Q}{r^2}$

$= \frac{Q}{4\pi r^2}\cdot\hat{r}$

$$\boxed{W = \frac{Q^2}{8\pi\varepsilon_0(1+\chi_e)}\left(\frac{1}{a} + \frac{\chi_e}{b}\right)}$$

$\downarrow$

Final Answer

$r>b \quad \vec{D} = \varepsilon_0 E$

$= \cancel{\varepsilon_0}\cdot\frac{1}{4\pi\cancel{\varepsilon_0}}\cdot\frac{Q}{r^2}$

$= \frac{Q}{4\pi r^2}\cdot\hat{r}$

$$W = \frac{1}{2}\int_a^{\infty}\vec{D}\cdot\vec{E}\cdot 4\pi r^2 dr$$

$$(\vec{D}\cdot\vec{E}) = \frac{1}{2}\int_a^b \vec{D}\cdot\vec{E}\, 4\pi r^2 dr + \int_b^{\infty}\vec{D}\cdot\vec{E}\, 4\pi r^2 dr$$

$$= \frac{4\pi}{2}\int_a^b \underbrace{\frac{Q}{4\pi r^2}\hat{r}}_{\vec{D}}\cdot\underbrace{\frac{Q}{4\pi\varepsilon r^2}\hat{r}}_{\vec{E}}\, r^2 dr + \frac{4\pi}{2}\int_b^{\infty}\underbrace{\frac{Q}{4\pi r^2}\hat{r}}_{\vec{D}}\cdot\underbrace{\frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}}_{\vec{E}}\, r^2 dr$$

$a<r<b \qquad\qquad r>b$

$$= \frac{1}{2}4\pi \cdot \frac{Q^2}{\underset{8}{\cancel{16}}\pi^{\cancel{2}}\varepsilon}\int_a^b \frac{1}{r^2}\,dr + \frac{1}{2}4\pi \cdot \frac{Q^2}{\underset{8}{\cancel{16}}\pi^{\cancel{2}}\varepsilon_0}\int_b^\infty \frac{1}{r^2}\,dr$$

$$= \frac{Q^2}{8\pi\varepsilon}\left[\frac{-1}{r}\right]_a^b + \frac{Q^2}{8\pi\varepsilon_0}\left[\frac{-1}{r}\right]_b^\infty$$

$$= \frac{Q^2}{8\pi\varepsilon}\left[-\left[\frac{1}{b}-\frac{1}{a}\right]\right] + \frac{Q^2}{8\pi\varepsilon_0}\left[-\left[\underset{\Downarrow \atop 0}{\frac{1}{\infty}}-\frac{1}{b}\right]\right]$$

$$= \frac{Q^2}{8\pi\varepsilon}\left(\frac{1}{a}-\frac{1}{b}\right) + \frac{Q^2}{8\pi\varepsilon_0}\left(\frac{1}{b}\right)$$

We know $\varepsilon = \varepsilon_0(1+\chi_e)$

$$\Rightarrow \frac{Q^2}{8\pi\varepsilon_0(1+\chi_e)}\left(\frac{1}{a}-\frac{1}{b}\right) + \frac{Q^2}{8\pi\varepsilon_0}\left(\frac{1}{b}\right)$$

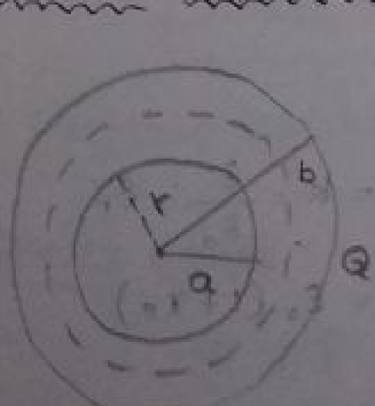
$$= \frac{Q^2}{8\pi\varepsilon_0(1+\chi_e)}\left[\frac{1}{a}-\frac{1}{b}+\frac{(1+\chi_e)}{b}\right]$$

$$= \frac{Q^2}{8\pi\varepsilon_0(1+\chi_e)}\left[\frac{1}{a}-\cancel{\frac{1}{b}}+\cancel{\frac{1}{b}}+\frac{\chi_e}{b}\right]$$

$$\therefore \boxed{W = \frac{Q^2}{8\pi\varepsilon_0(1+\chi_e)}\left[\frac{1}{a}+\frac{\chi_e}{b}\right]}$$

* For a conductor $E_{inside} = 0$

Spherical capacitance:



$$\oiint \vec{E}\cdot\vec{dS} = \frac{Q_{enc}}{\varepsilon_0}$$

$$\vec{E}_r \cdot 4\pi r^2 = \frac{Q_{enc}}{\varepsilon_0}$$

$$\Rightarrow \vec{E}_r = \frac{1}{4\pi\varepsilon_0 r^2}\cdot Q$$

We know

$$C = \frac{Q}{\Delta V}$$

$$\Delta V = V_b - V_a = -\int_a^b E(r)\,dr$$

$$= -\int_a^b \frac{Q}{4\pi\varepsilon_0 r^2}\cdot dr$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_a^b$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right)$$

$$V_b - V_a = \frac{1}{4\pi\varepsilon_0}\left(\frac{a-b}{ab}\right)$$

$$\Delta V = V_a - V_b = \frac{1}{4\pi\varepsilon_0}\left(\frac{b-a}{ab}\right)$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{\frac{1}{4\pi\varepsilon_0}\left(\frac{b-a}{ab}\right)} = \frac{4\pi\varepsilon_0}{\left(\frac{1}{a} - \frac{1}{b}\right)}$$

$$\therefore \boxed{C = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)}$$

→ Energy stored between the plates of a capacitor

$$W = \frac{1}{2}CV^2$$

→ In the absence of the dielectric

$$C_0 = \frac{\varepsilon_0 A}{d}$$

→ In the presence of the dielectric

$$C' = \frac{\varepsilon A}{d} \qquad 1 + \chi_e = K = \frac{\varepsilon}{\varepsilon_0} = \varepsilon_r$$

$$= \frac{\varepsilon_r \varepsilon_0 A}{d} \qquad \varepsilon = \varepsilon_0(1 + \chi_e)$$

$$\boxed{C' = K C_0}$$

Energy

→ $U \text{ (or) } W = \frac{Q^2}{2C} \text{ (or) } \frac{1}{2}CV^2 \text{ (or) } \frac{1}{2}Q\Delta V$

↓ (potential difference)

Electric Current:

$$I_{avg} = \frac{\Delta Q}{\Delta t}$$

as $\Delta t \to 0$ then

$$I_{instantaneous} = \lim_{\Delta t \to 0} I_{avg} = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t}$$

$$\boxed{I_{ins} = \frac{dQ}{dt}}$$

$$I_{avg} = nqv_d A$$

Where A = area

v_d = drift speed

n = concentration of electrons

q = charge

Current density:

$$\vec{J} = \frac{I_{avg}}{A}$$

$$\boxed{\vec{J} = nqv_d}$$

→ current density is a vector quantity.

Ohm's Law:

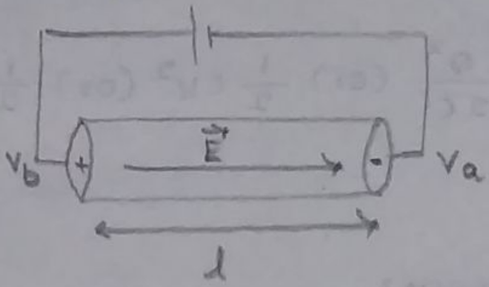
$\vec{J} \propto \vec{E}$ at constant temperature

$$\vec{J} = \sigma \vec{E}$$

$$\frac{\vec{J}}{\vec{E}} = \sigma$$

Where σ = conductivity

$\Delta V = V_b - V_a = El$

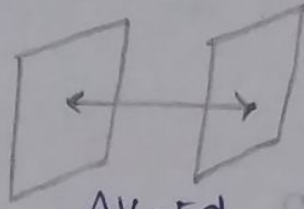


$$\boxed{V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l}}$$

$$\Delta V = -E[l]_b^a$$

$$= -E(a-b)$$

$$\Rightarrow E = \frac{\Delta V}{l}$$



$\Delta V = Ed$

then, $J = \sigma\left(\frac{\Delta V}{l}\right)$

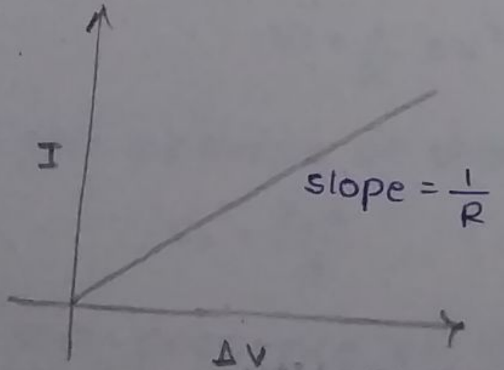
$$\frac{I}{A} = \sigma\left(\frac{\Delta V}{l}\right)$$

$$\Delta V = \left(\frac{l}{A\sigma}\right) I = RI$$

$\boxed{R = \frac{l}{\sigma A}}$ → Resistivity (or) specific resistance

$\Rightarrow R = \frac{\rho l}{A}$ where $\frac{1}{\sigma} = \rho$ (resistivity)

As per ohm's law



$V = IR$ → At constant temperature

$$\frac{\Delta V}{\Delta I} = \frac{1}{\text{slope}} = R$$

$$\sigma = \frac{nq^2\tau}{m_e}$$

$$\rho = \frac{1}{\sigma} = \frac{m_e}{nq^2\tau}$$

Where,

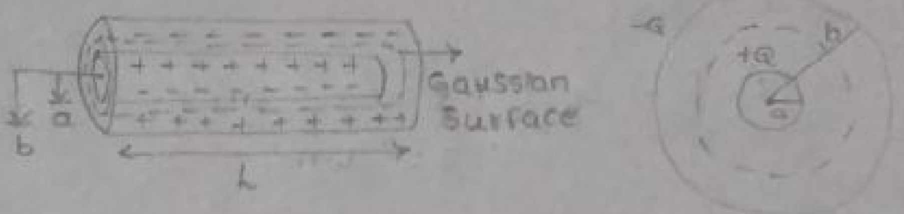
m_e = mass of electrons

τ = Avg interval b/w two successive collisions

19/2/21

P. Find the capacitance per unit length of two coaxial metal cylindrical tubes, of radii a and b.

A.



$$\oiint \vec{E}\cdot d\vec{S} = \frac{Q_{enc}}{\varepsilon_0}$$

$$\phi_E = \oiint E\cdot ds\cos\theta = \frac{Q_{enc}}{\varepsilon_0}$$

$\theta = 0^\circ$, $\vec{ds}$ and $\vec{E}$ are parallel

$$E\int ds = \frac{Q_{en}}{\varepsilon_0} \Rightarrow E(2\pi r \cancel{L}) = \frac{\lambda \cancel{L}}{\varepsilon_0} \qquad \because \left(\lambda = \frac{Q}{L}\right)$$

$$\boxed{\vec{E}(r) = \frac{1}{2\pi\varepsilon_0}\times\frac{\lambda}{r}\,\hat{r}}$$

$$V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{r} = -\int_a^b \frac{\lambda}{2\pi\varepsilon_0}\cdot\frac{1}{r}\,dr$$

$$= \frac{-\lambda}{2\pi\varepsilon_0}\int_a^b \frac{dr}{r}$$

$$= \frac{-\lambda}{2\pi\varepsilon_0}\left[\ln r\right]_a^b$$

$$= \frac{-\lambda}{2\pi\varepsilon_0}\ln\left(\frac{b}{a}\right)$$

$$\Delta V = V_a - V_b = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{b}{a}\right)$$

$$C = \frac{Q}{\Delta V} = \frac{\cancel{Q}}{\frac{\cancel{Q}}{2\pi\varepsilon_0 L}\times\ln\left(\frac{b}{a}\right)} = \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{b}{a}\right)} \qquad \left(\because \lambda = \frac{Q}{L}\right)$$

$$\therefore \boxed{\frac{C}{L} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{b}{a}\right)}}$$

$R \propto l$

$\propto \frac{1}{A} \Rightarrow R = \frac{\rho l}{A}$ $\quad A = 1\,cm^2$

$l = 1\,cm$

$\rho = R\left(\frac{A}{l}\right)$

$\boxed{\rho = R} \rightarrow$ specific resistance

units: '$\Omega \cdot m$'

→ Current density:

$\vec{J} = \frac{I}{A}$

units: Amp/cm^2 , Amp/m^2

→ Ohm's law

$\left.\begin{array}{l}\vec{J} = \sigma \vec{E} \\ \Delta V = IR\end{array}\right\}$ at constant temperature

σ = conductivity

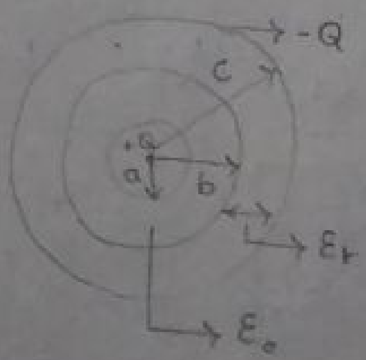
$\sigma = \frac{1}{\rho} = \frac{1}{\text{resistivity}}$

→ $\vec{\nabla} \cdot \vec{D} = \rho_f$ Where, ρ_f = free volume charge density

$\oint\!\!\oint \vec{D} \cdot d\vec{A} = (Q_f)_{enc}$ → Integral form

P. A certain coaxial cable consists of a copper wire, radius a, surrounded by a concentric copper tube of inner radius c. The space between is partially filled (from b out to c) with material of dielectric constant ε_r, as shown. Find the capacitance per unit length of this cable.

A.



$\vec{D}$ - $a < r < b$

$\vec{D}$ - $b < r < c$

$\Delta V = -\int \vec{E} \cdot d\vec{l}$

$$\iint \vec{E} \cdot \vec{ds} = \frac{Q_{enc}}{\varepsilon_0}$$

$$E(2\pi r \cancel{L}) = \frac{\lambda \cancel{L}}{\varepsilon_0} \quad \left(\because \lambda = \frac{Q}{L} \Rightarrow Q = \lambda L\right)$$

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0} \cdot \frac{1}{r} \hat{r} \quad a<r<b$$

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon} \cdot \frac{1}{r} \hat{r} \quad b<r<c$$

ΔV

$\downarrow$

$$V_c - V_a = + \int_a^c \vec{E} \cdot \vec{dl}$$

$$\hookrightarrow \cancel{-}(V_a - V_c) = \cancel{-}\left(-\int_c^a \vec{E} \cdot \vec{dl}\right)$$

$$V = + \int_a^b \vec{E} \cdot \vec{dl} + \int_b^c \vec{E} \cdot \vec{dl}$$

$$= \int_a^b \frac{Q}{2\pi\varepsilon_0 L r} dr + \int_b^c \frac{Q}{2\pi\varepsilon L r} \cdot dr$$

$$= \frac{Q}{2\pi\varepsilon_0 L} \ln\left(\frac{b}{a}\right) + \frac{Q}{2\pi\varepsilon L} \ln\left(\frac{c}{b}\right) \quad \left(\because \int_a^b \frac{dr}{r} = [\ln r]_a^b\right)$$

$$C = \frac{Q}{V}$$

$$= \frac{Q}{\frac{Q}{2\pi\varepsilon_0 L} \ln\left(\frac{b}{a}\right) + \frac{Q}{2\pi\varepsilon L} \ln\left(\frac{c}{b}\right)}$$

$$= \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{b}{a}\right)} + \frac{2\pi\varepsilon L}{\ln\left(\frac{c}{b}\right)}$$

$$\therefore \boxed{\frac{C}{L} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{b}{a}\right)} + \frac{2\pi\varepsilon}{\ln\left(\frac{c}{b}\right)}}$$