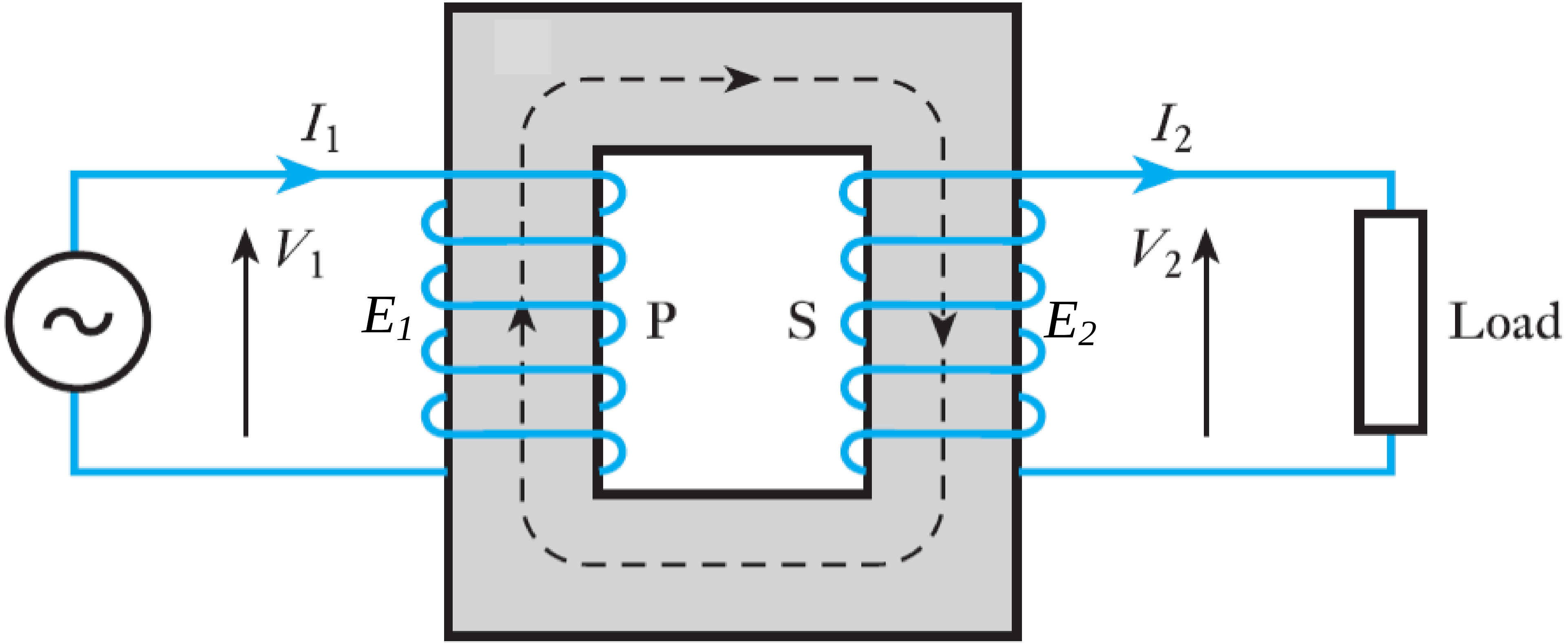
ELL 100 – Introduction to Electrical Engineering

LECTURE 34: TRANSFORMERS - III

OUTLINE

- Phasor diagram for a transformer on load
- Approximate equivalent circuit of a transformer
- Voltage regulation of a transformer
- Efficiency of a transformer
- Open-circuit and short-circuit tests
- Numerical problems

TRANSFORMER

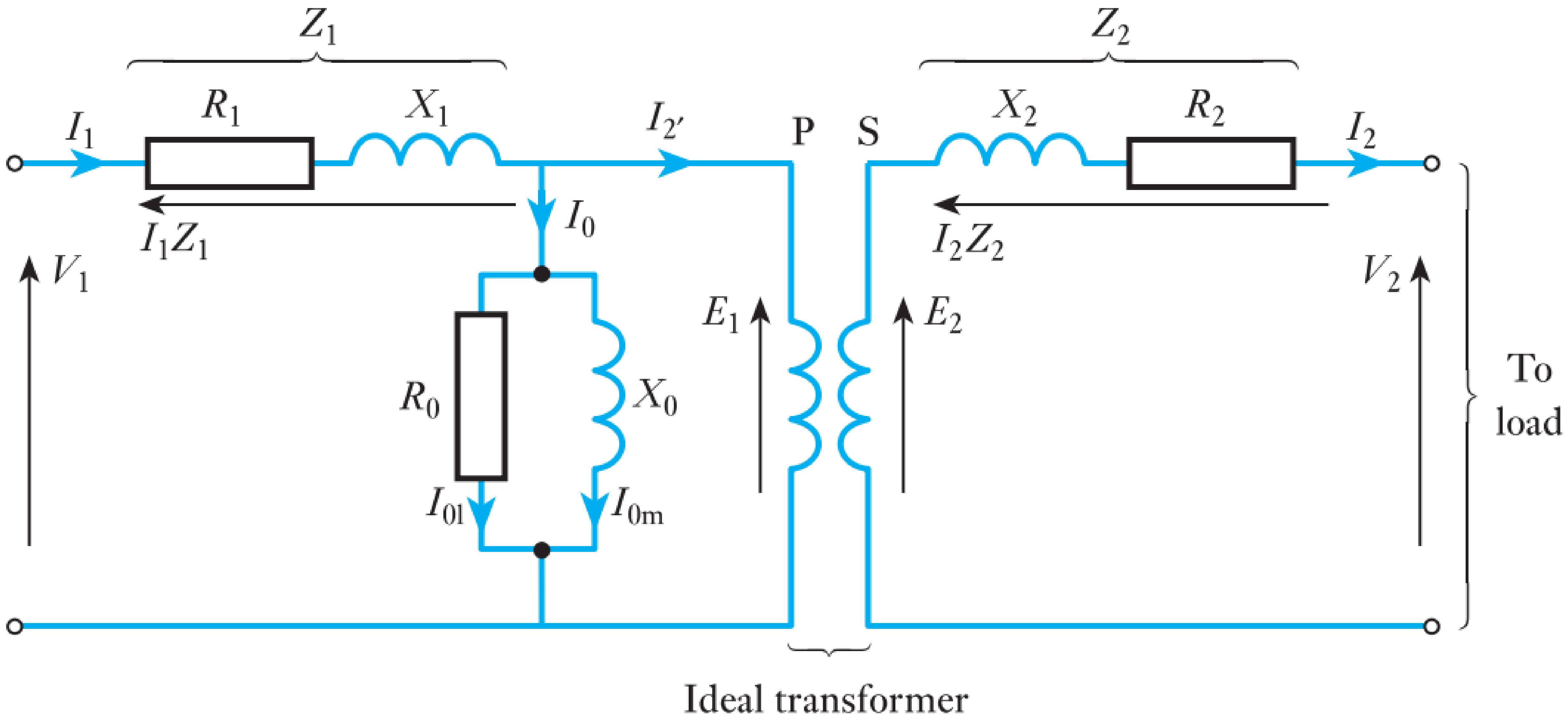


P: Primary winding, **S**: Secondary winding

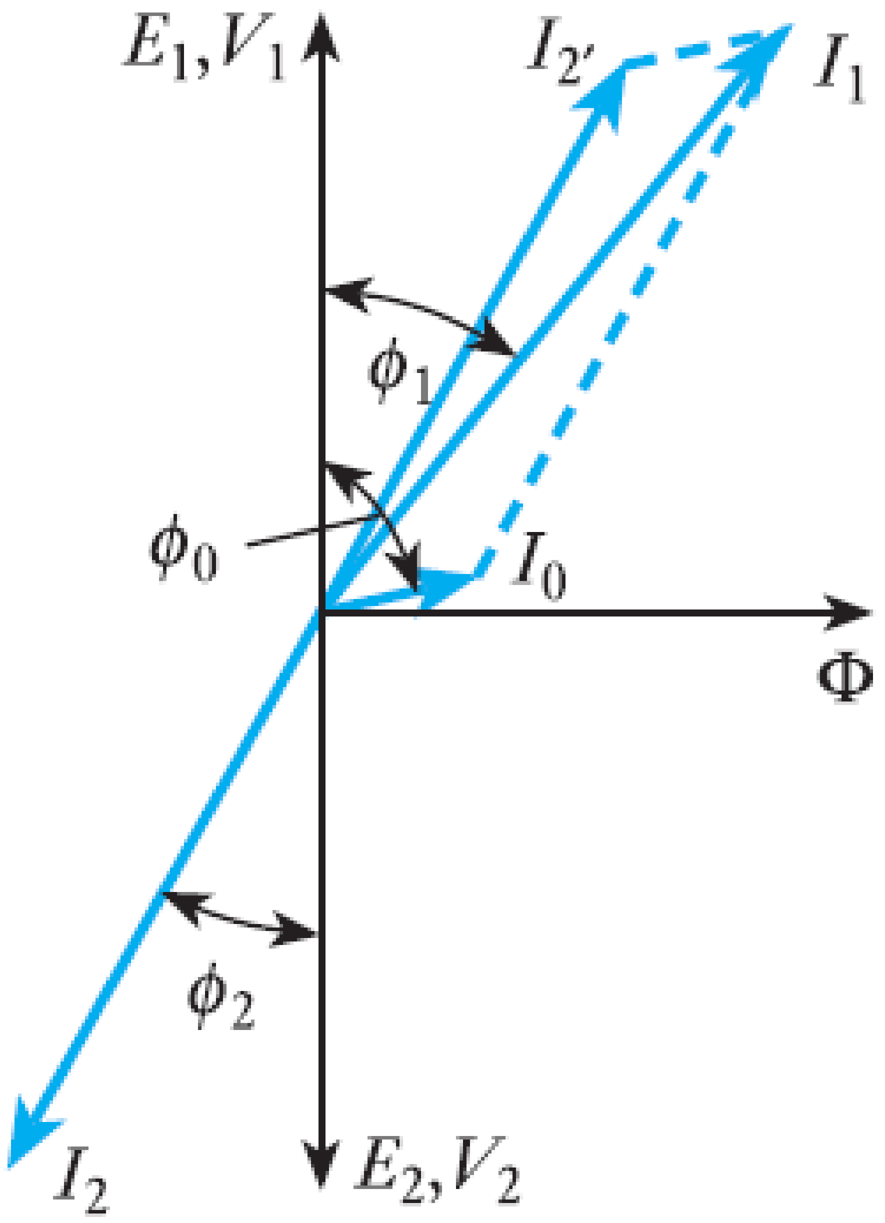
E_1: Primary EMF, E_2: Secondary EMF

I_1: Primary current, I_2: Secondary current

EQUIVALENT CIRCUIT OF TRANSFORMER

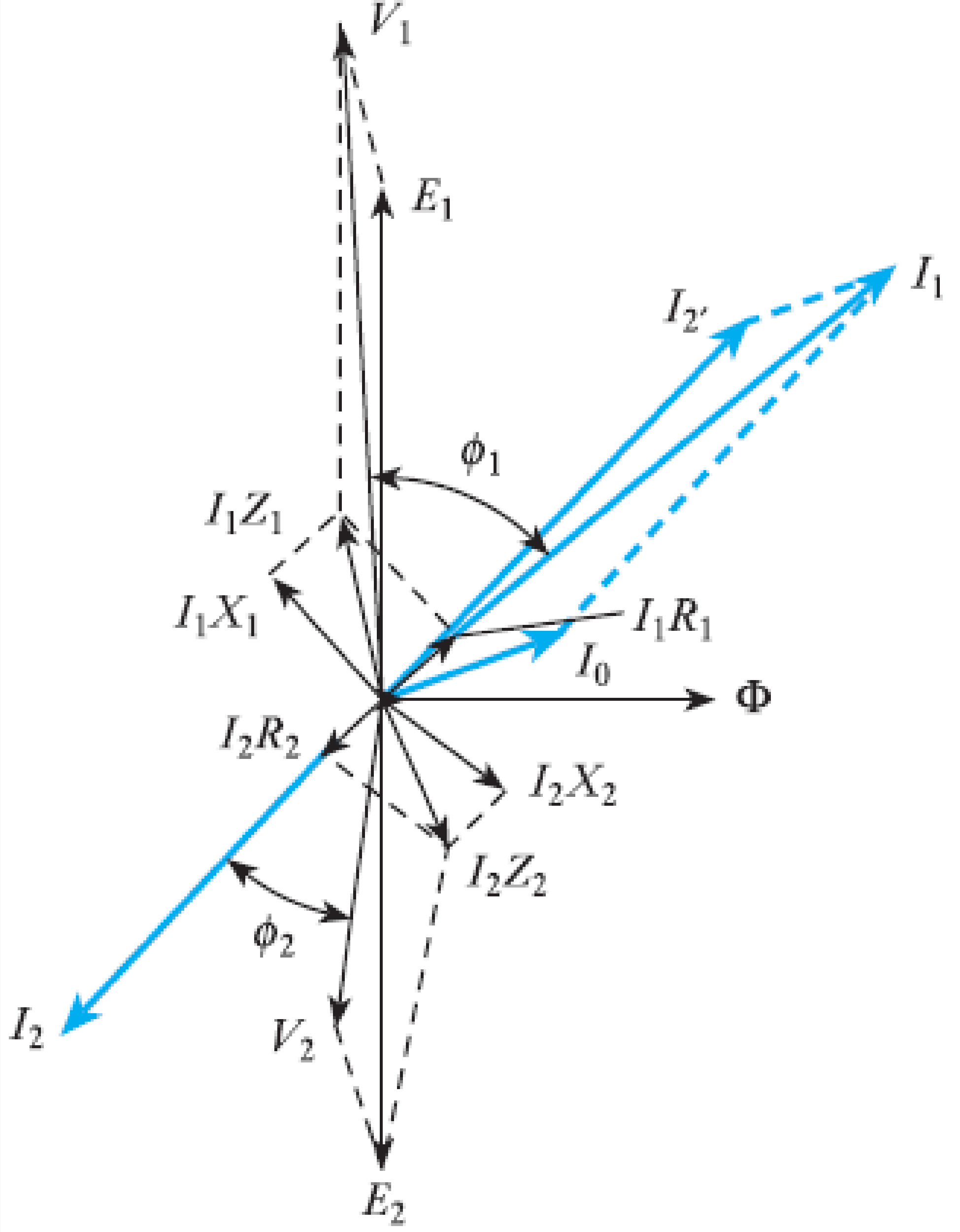


PHASOR DIAGRAM OF LOADED TRANSFORMER



- Recall, mutual **flux Φ** taken as **reference phasor**
- Recall, **EMF E_1** induced in **primary leads** flux by **90^0**, **secondary EMF E_2** drawn in **opposite** direction for **convenience**
- Recall, I_0 is the **no-load primary current**
- Neglecting coil resistances (**$R_{1,2} \sim 0$**) and leakage reactances (**$X_{1,2} \sim 0$**), **$V_1 = E_1$ & $V_2 = E_2$**
- **Load** impedance on the secondary has a **lagging** power factor cos(ϕ_2) i.e. **$Z_L = V_2 / I_2 = |Z_L|\angle\phi_2$**
- **$I_{2'}$** is the **secondary current** I_2 **reflected** onto the **primary side** (via the **transformation ratio *n***)
- **$I_1 = I_0 + I_{2'}$** is the **total primary current** lagging V_1 by an angle ϕ_1 (primary side **p.f. = cos(ϕ_1)**)

Phasor Diagram of Loaded Transformer



- If we now include the coil resistances $\boldsymbol{R_{1,2}}$ and leakage reactances $\boldsymbol{X_{1,2}}$, then $\boldsymbol{V_1 \neq E_1}$ & $\boldsymbol{V_2 \neq E_2}$

$$V_2 = E_2 - I_2Z_2 = E_2 - I_2R_2 - jI_2X_2$$

$$\Rightarrow E_2 = V_2 + I_2R_2 + jI_2X_2$$

$$V_1 = E_1 + I_1Z_1 = E_1 + I_1R_1 + jI_1X_1$$

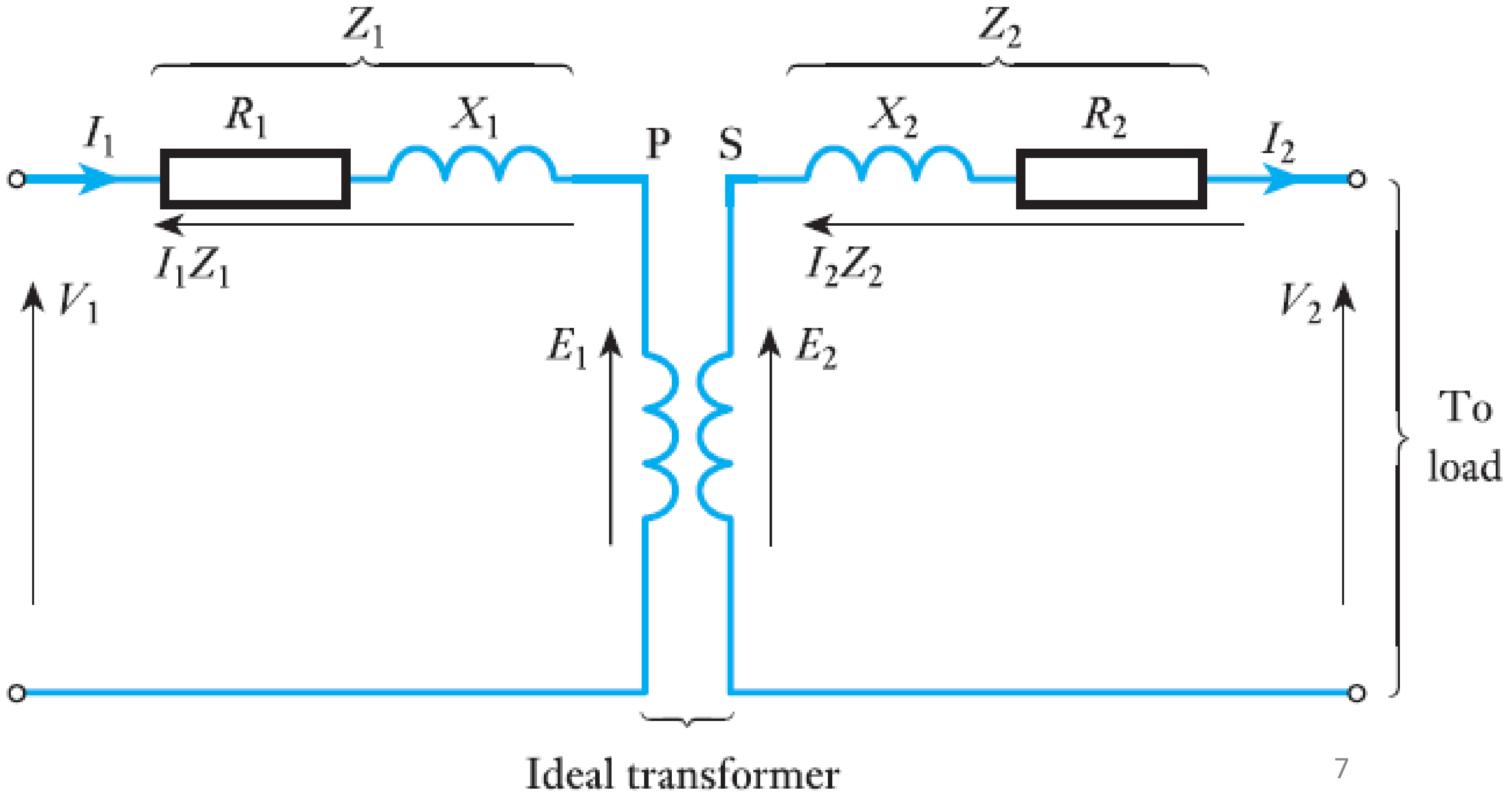
- Again, **load** impedance on the secondary is taken to have **lagging p.f.** $\cos(\phi_2)$ i.e. $\boldsymbol{Z_L = V_2 / I_2 = |Z_L|\angle\phi_2}$

- **Primary** side **p.f. = $\boldsymbol{\cos(\phi_1)}$** i.e. total primary current $\boldsymbol{I_1}$ **lags** supply voltage $\boldsymbol{V_1}$ by angle $\boldsymbol{\phi_1}$

APPROXIMATE EQUIVALENT CIRCUIT OF TRANSFORMER

- **No-load current I_0** of a transformer is typically only about **3–5%** of the **full-load primary current I_1**
- Thus, the **parallel core impedances R_0 and X_0 can be omitted** without introducing an appreciable error when considering the behaviour of the transformer **near full-load**

Transformer equivalent circuit near full-load includes only coil resistances (copper loss) and leakage reactances



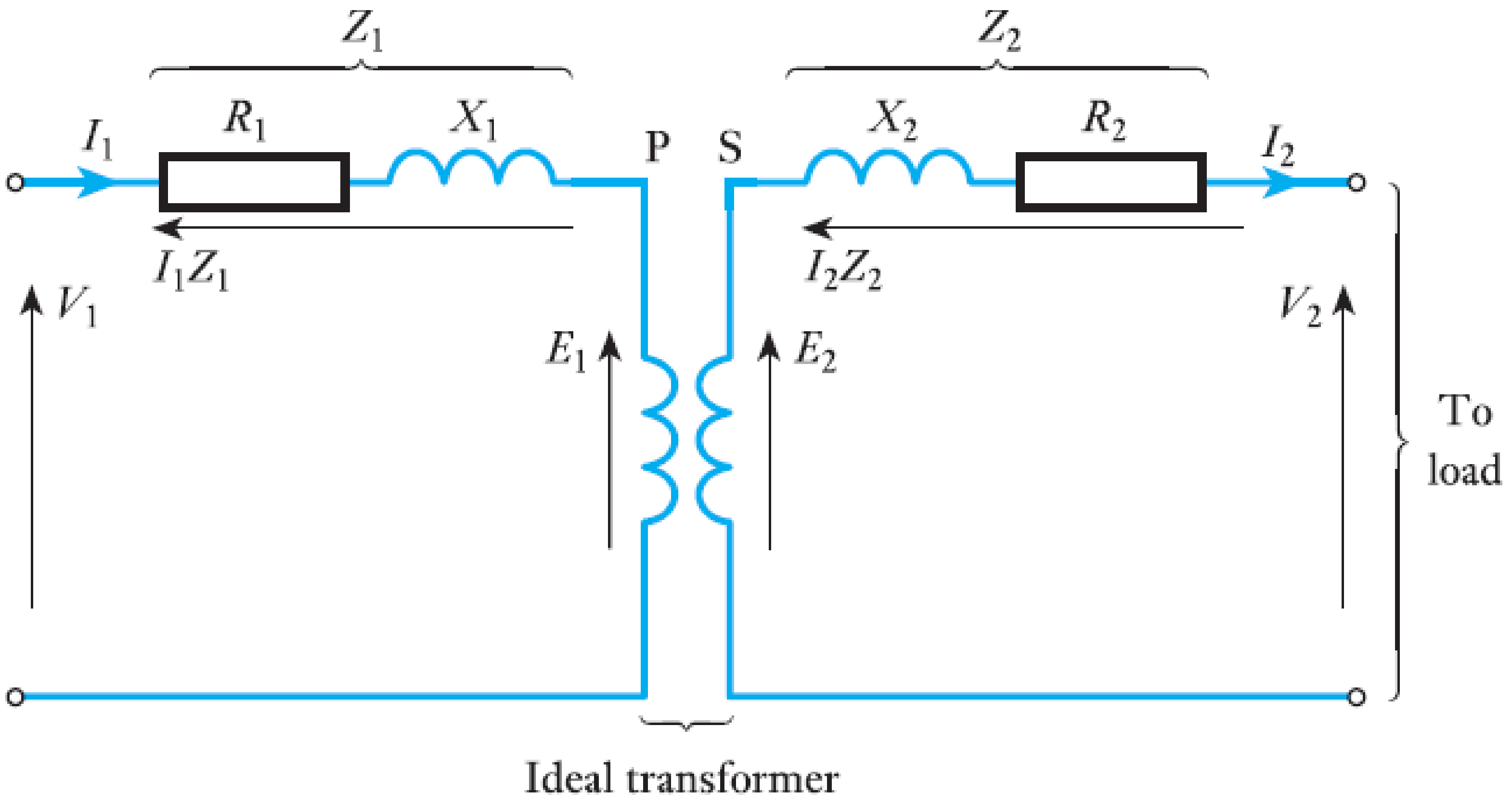
Approximate Equivalent Circuit of Transformer

The **coil resistance** $\boldsymbol{R_2}$ and **leakage reactance** $\boldsymbol{X_2}$ of the **secondary** winding are **transferred to** the **primary** side as a **resistance** $\boldsymbol{R_2'}$ and a **reactance** $\boldsymbol{X_2'}$ via the transformation ratio ("reflected impedance")

$$R_2' = R_2/n^2$$

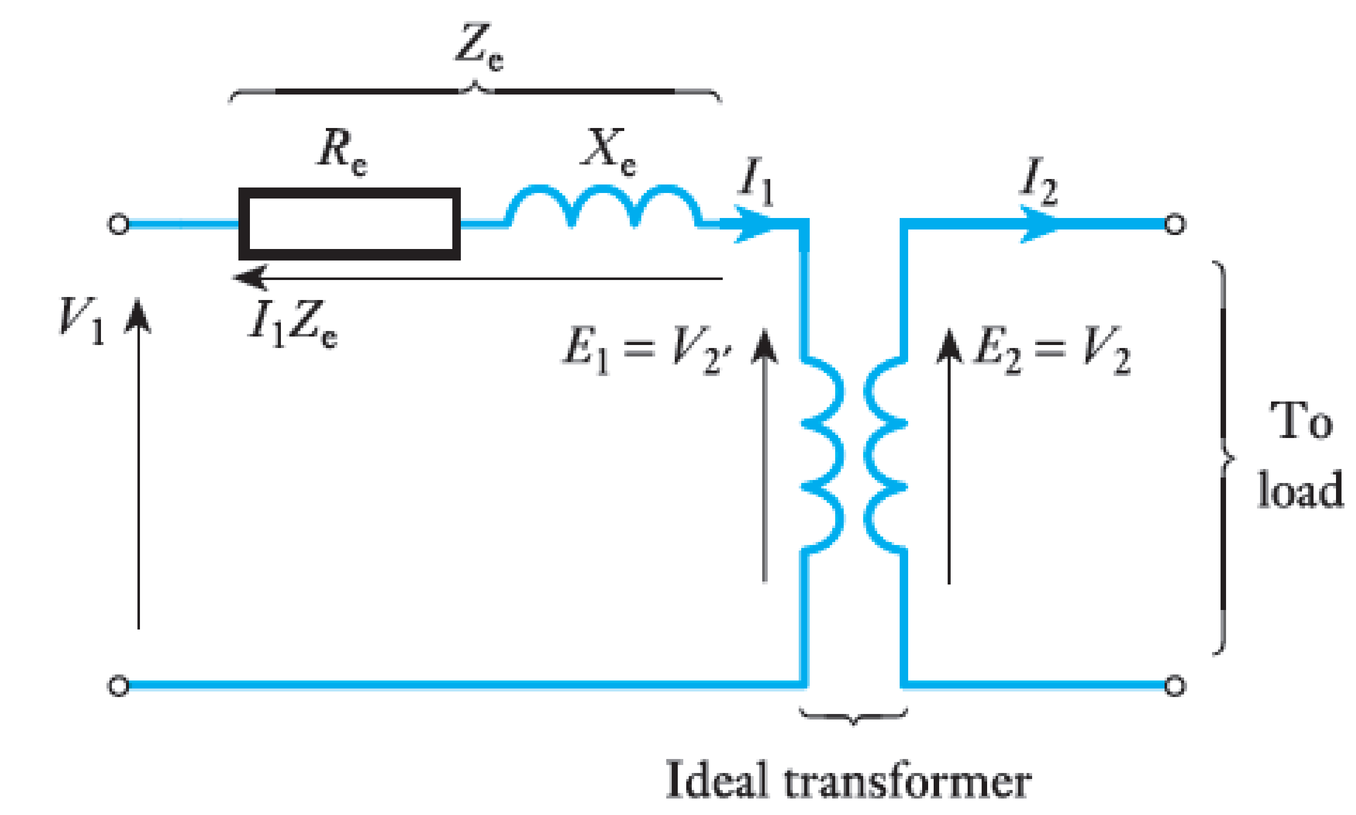
$$X_2' = X_2/n^2$$

where $n = N_2/N_1$
(transformation ratio)



Approximate Equivalent Circuit of Transformer

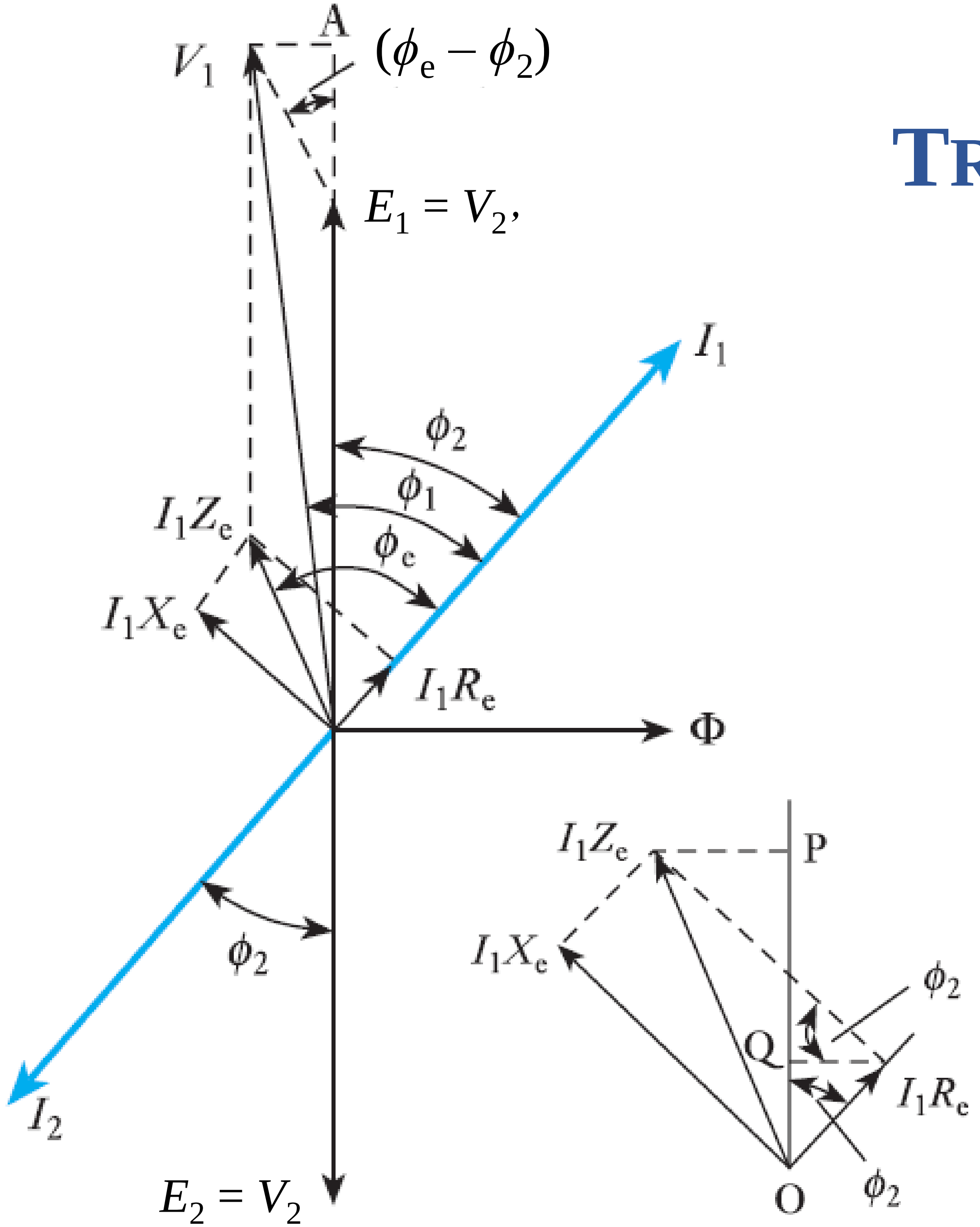
- $\boldsymbol{R_e = R_1 + R_2/n^2}$ is a single **resistance** in the **primary** circuit **equivalent** to R_1 and R_2 of the actual transformer
- $\boldsymbol{X_e = X_1 + X_2/n^2}$ is a single **reactance** in the **primary** circuit **equivalent** to X_1 and X_2 of the actual transformer
- $\boldsymbol{Z_e = R_e + jX_e}$ the **equivalent impedance** of the primary and secondary windings referred to the **primary circuit**

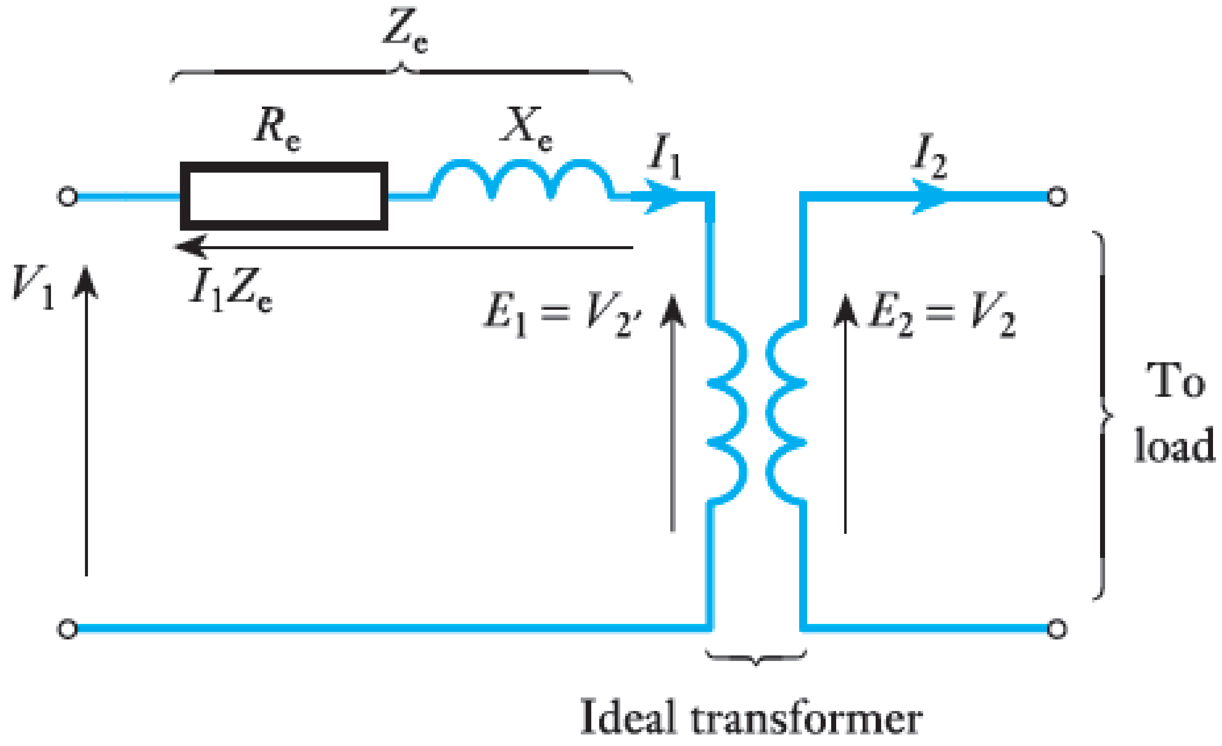


$$Z_e = \sqrt{R_e^{\,2} + X_e^{\,2}}$$

$$R_e = Z_e \cos\phi_e, X_e = Z_e \sin\phi_e$$

Phasor Diagram Of Loaded Transformer using Approximate Equivalent Circuit





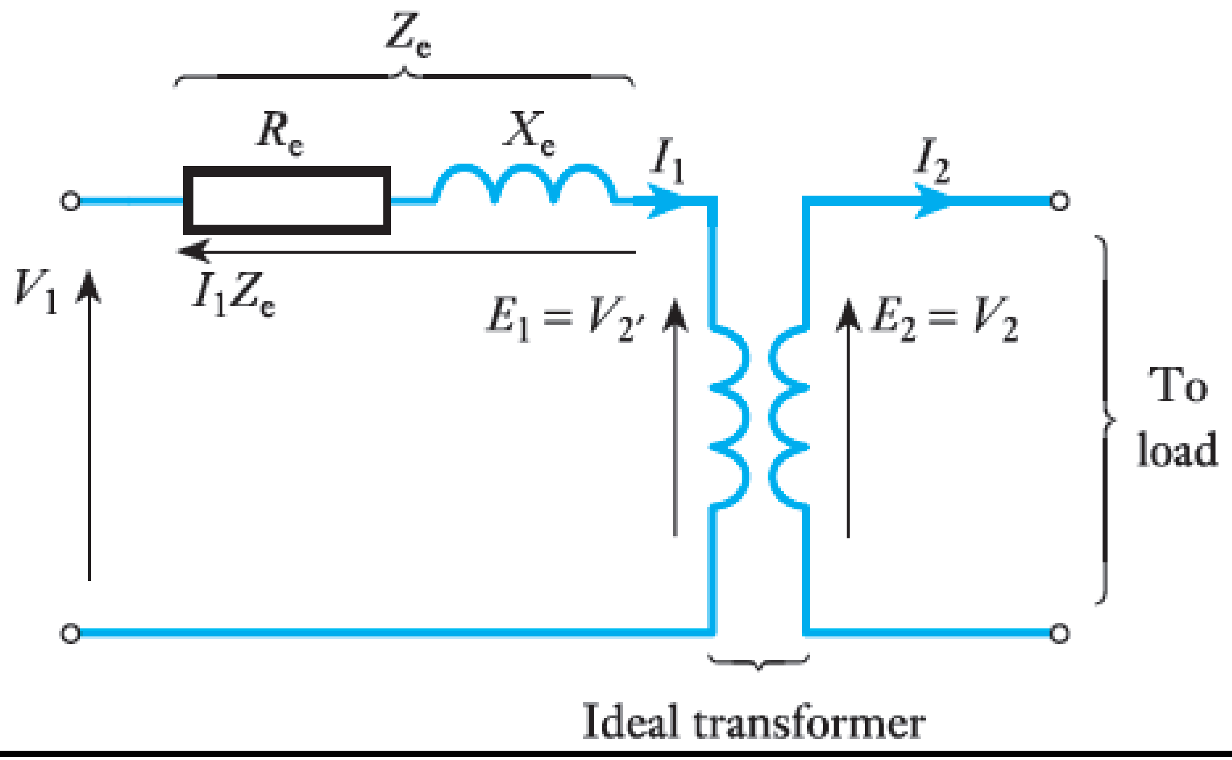
$$Z_e = \sqrt{R_e^{\,2} + X_e^{\,2}}$$

$$R_e = Z_e \cos\phi_e, X_e = Z_e \sin\phi_e$$

VOLTAGE REGULATION OF A TRANSFORMER

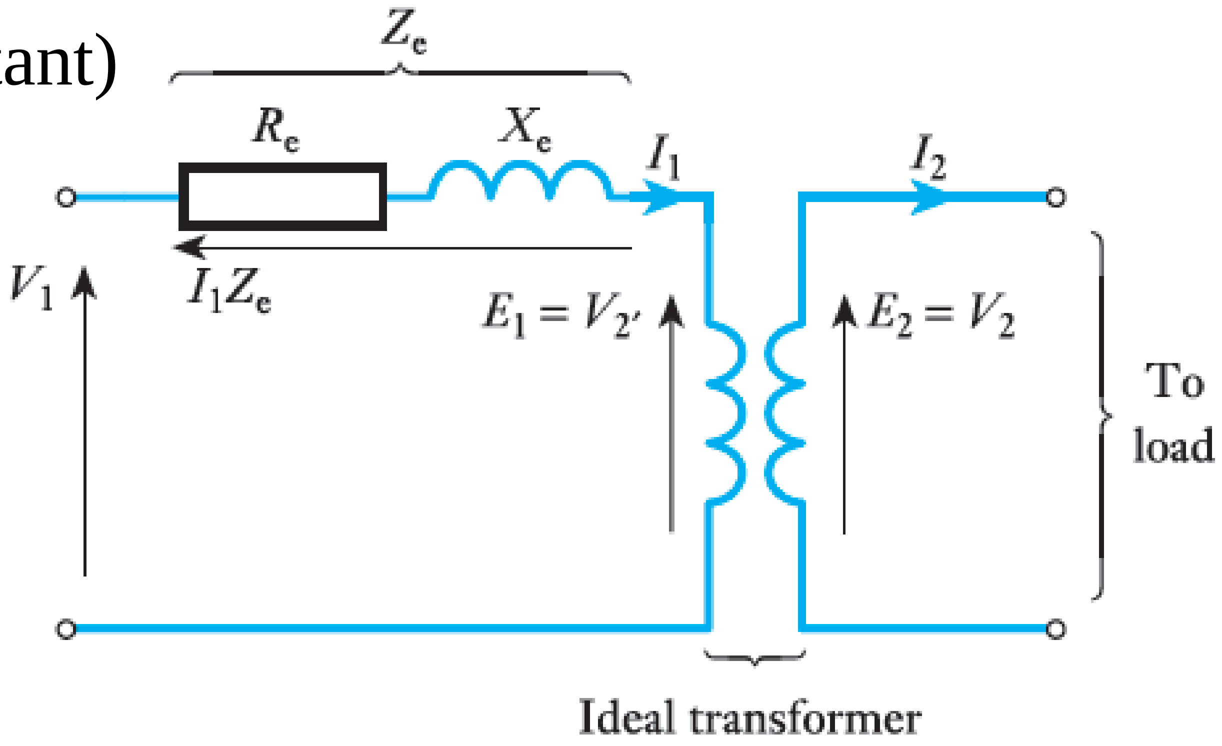
Defined as the *variation of the secondary voltage between no-load* ($V_{2,n\text{-}l}$) *and full-load* ($V_{2,f\text{-}l}$), expressed as either a per-unit or a percentage of the no-load secondary voltage, the *primary voltage being held constant*

$$\textbf{Voltage Regulation} = [V_{2,n\text{-}l} - V_{2,f\text{-}l}] / V_{2,n\text{-}l} \quad \textit{at constant } V_1$$



VOLTAGE REGULATION OF A TRANSFORMER

$\boxed{V_1}$ is $\boxed{\text{applied } \textbf{primary voltage}}$ (held constant)



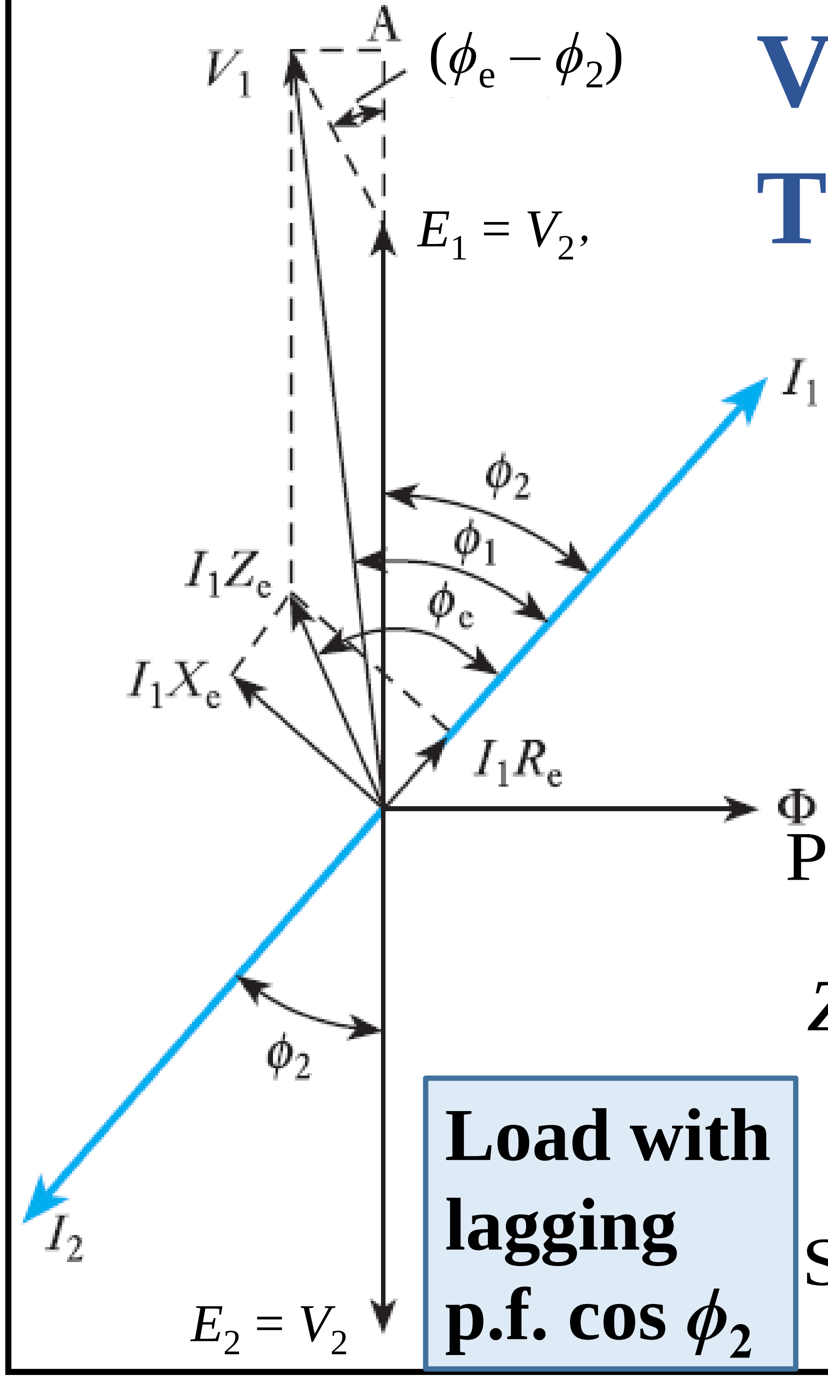
At **no-load**, $I_2 = 0 \Rightarrow I_1 = 0 \Rightarrow E_1 = V_1$
But $E_1 = V_{2'} = V_{2,nl} / n$, where $\boxed{\boldsymbol{n = N_2/N_1}}$ is the **transformation ratio**
$\Rightarrow V_{2,nl} = \boldsymbol{V_1 . n}$
(no-load secondary voltage)

Let the $\boxed{\textbf{full-load secondary} \text{ terminal } \textbf{voltage}}$ be $V_{2,fl} = \boxed{\boldsymbol{V_2}}$

$\Rightarrow$ $\boxed{\textbf{Voltage Regulation}}$ $= (V_{2,nl} - V_{2,fl}) / V_{2,nl} = (V_1.n - V_2) / (V_1.n)$

$\boxed{= \boldsymbol{(V_1 - V_2/n) / V_1}}$

VOLTAGE REGULATION IN TERMS OF TRANSFORMER CIRCUIT PARAMETERS



$$V_1^2 = (V_{2'} + V_{2'}A)^2 + (V_1A)^2$$

$$= \{V_{2'} + I_1 Z_e \cos(\phi_e - \phi_2)\}^2 + \{I_1 Z_e \sin(\phi_e - \phi_2)\}^2$$

$$I_1 Z_e \sin(\phi_e - \phi_2) << V_{2'} \text{ (typically)}$$

$$\text{So, } \boxed{V_1 \approx V_{2'} + I_1 Z_e \cos(\phi_e - \phi_2)}$$

$$\text{Per Unit } \boxed{\text{voltage Regulation} = \frac{V_1 - V_{2'}}{V_1} = \frac{I_1 Z_e \cos(\phi_e - \phi_2)}{V_1}}$$

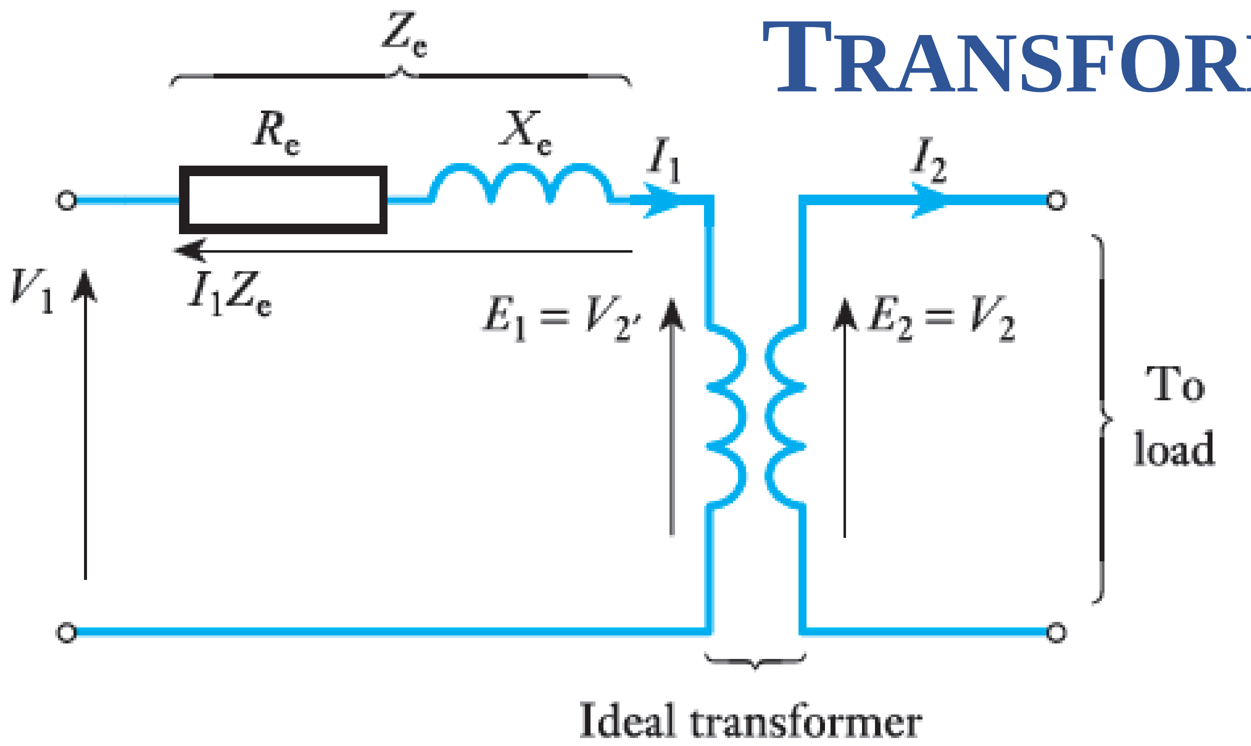
$$Z_e \cos(\phi_e - \phi_2) = Z_e(\cos\phi_e . \cos\phi_2 + \sin\phi_e . \sin\phi_2)$$

$$= R_e \cos\phi_2 + X_e \sin\phi_2$$

Load with lagging p.f. cos ϕ_2

$$\text{So, } \boxed{\text{per unit voltage regulation} = \frac{I_1(R_e \cos\phi_2 + X_e \sin\phi_2)}{V_1}}$$

Voltage Regulation in terms of Transformer Circuit Parameters



If the secondary is connected to a **load** with **leading p.f. cos ϕ_2**, then

$$Z_e \cos(\phi_e + \phi_2) = Z_e(\cos\phi_e.\cos\phi_2 - \sin\phi_e.\sin\phi_2)$$
$$= R_e \cos\phi_2 - X_e \sin\phi_2$$

So, $\boxed{\text{per unit voltage regulation} = \dfrac{I_1(R_e \cos\phi_2 - X_e \sin\phi_2)}{V_1}}$

Efficiency of a Transformer

Efficiency of a transformer

$$= \boxed{\frac{\text{Output Power}}{\text{Input Power}}} = \boxed{\frac{\text{Output Power}}{\text{Output Power} + \text{losses}}} = \boxed{\frac{\text{Input Power} - \text{losses}}{\text{Input Power}}}$$

$$= \boxed{1 - \frac{\text{losses}}{\text{Input Power}}}$$

Types of losses incurred in a transformer:

- Copper losses (I^2R losses)
  - ➢ Copper losses in primary ($I_1{}^2R_1$)
  - ➢ Copper losses in secondary ($I_2{}^2R_2$)
- Core losses (P_c)
  - ➢ Hysteresis loss
  - ➢ Eddy current loss

EFFICIENCY OF A TRANSFORMER

CONDITION FOR MAXIMUM EFFICIENCY OF A TRANSFORMER

- Let $\boldsymbol{R_{2e}}$ be the **total equivalent resistance** of the primary and secondary windings **referred to the secondary** circuit, i.e.

$$\boxed{R_{2e} = R_1\left(\frac{N_2}{N_1}\right)^2 + R_2}$$

($R_{1,2}$ is the coil resistance of primary,secondary)

- => For a secondary load current I_2, **total copper loss** = $\boldsymbol{I_2^2R_{2e}}$
- Let the **core loss** (independent of load current I_2) be = $\boldsymbol{P_c}$

$$\boxed{\text{Efficiency} = \frac{I_2V_2 \times \text{p.f.}}{I_2V_2 \times \text{p.f.} + P_c + I_2^2R_{2e}}}$$

Output power

Total loss

- Load power factor is **p.f. = cos $\boldsymbol{\phi}$**

CONDITION FOR MAXIMUM EFFICIENCY OF A TRANSFORMER

Divide numerator & denominator of Efficiency expression by I_2 to get

$$\boxed{\text{Efficiency} = \frac{V_2 \cos\phi}{V_2 \cos\phi + (P_c / I_2) + I_2 R_{2e}}}$$

Only denominator above depends on I_2 => differentiate w.r.t I_2 & equate to zero

$$\frac{\mathrm{d}}{\mathrm{d}I_2}\left(V_2 \times \text{p.f.} + \frac{P_\mathrm{c}}{I_2} + I_2 R_{2\mathrm{e}}\right) = 0$$

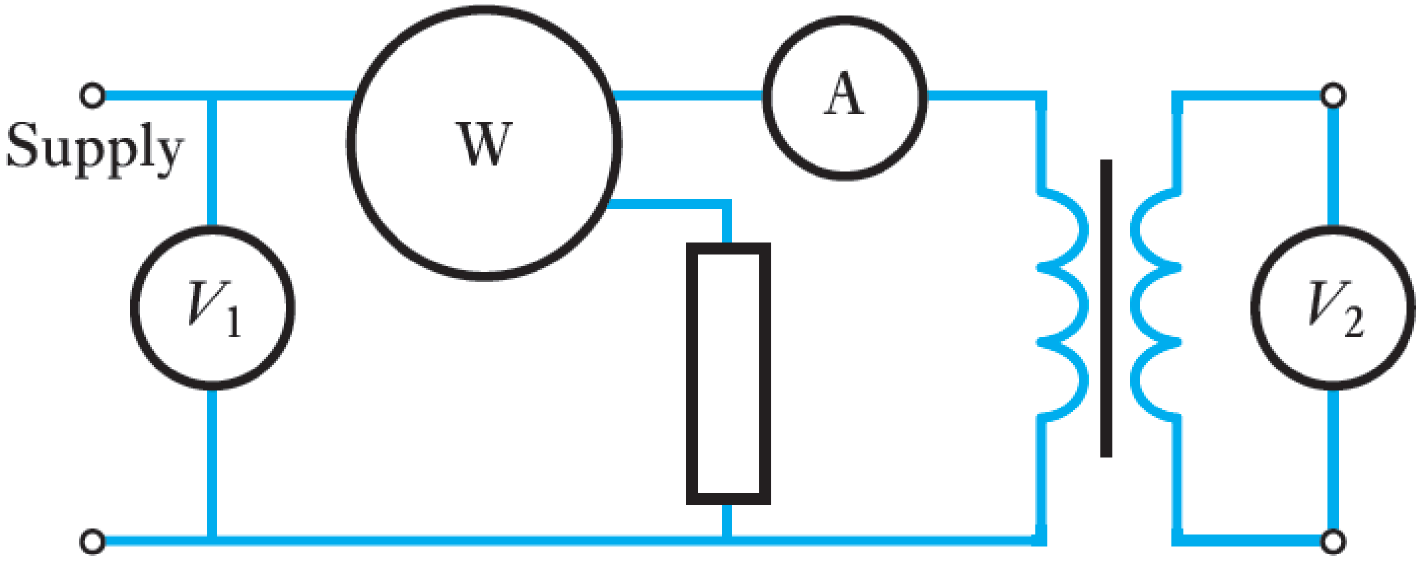
$$\therefore \qquad -\frac{P_\mathrm{c}}{I_2^2} + R_{2\mathrm{e}} = 0 \qquad \boxed{\Rightarrow I_2^{\,2} R_{2e} = P_c}$$

=> The **efficiency** of a transformer is **maximum** when the **variable copper (I^2R) loss** is **equal to** the **constant core loss**

OPEN-CIRCUIT AND SHORT-CIRCUIT TESTS ON A TRANSFORMER

These two tests **enable** the **efficiency** and the **voltage regulation** to be calculated **without** actually **loading** the **transformer**

OPEN-CIRCUIT TEST: Carried out at rated voltage of transformer

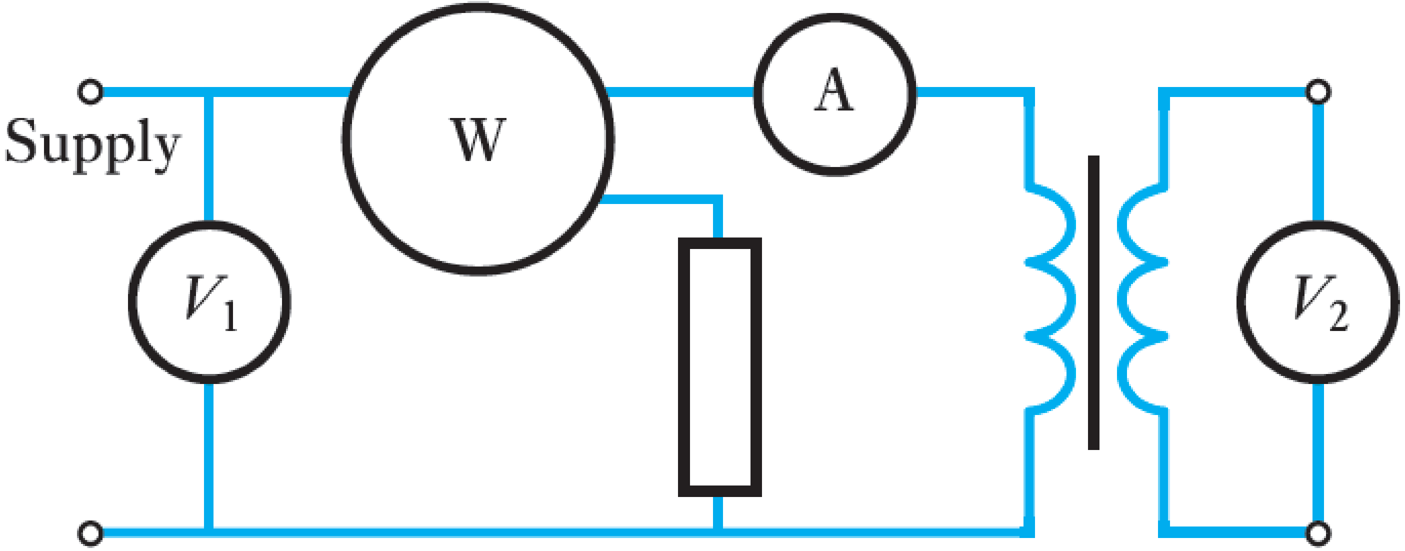


$V_{1,2}$: Voltmeter readings ($V_2/V_1 \approx N_2/N_1 = n$)

A: Ammeter (measures **no-load current I_0 ~ 0**)

W: Wattmeter (measures **core loss P_c**)

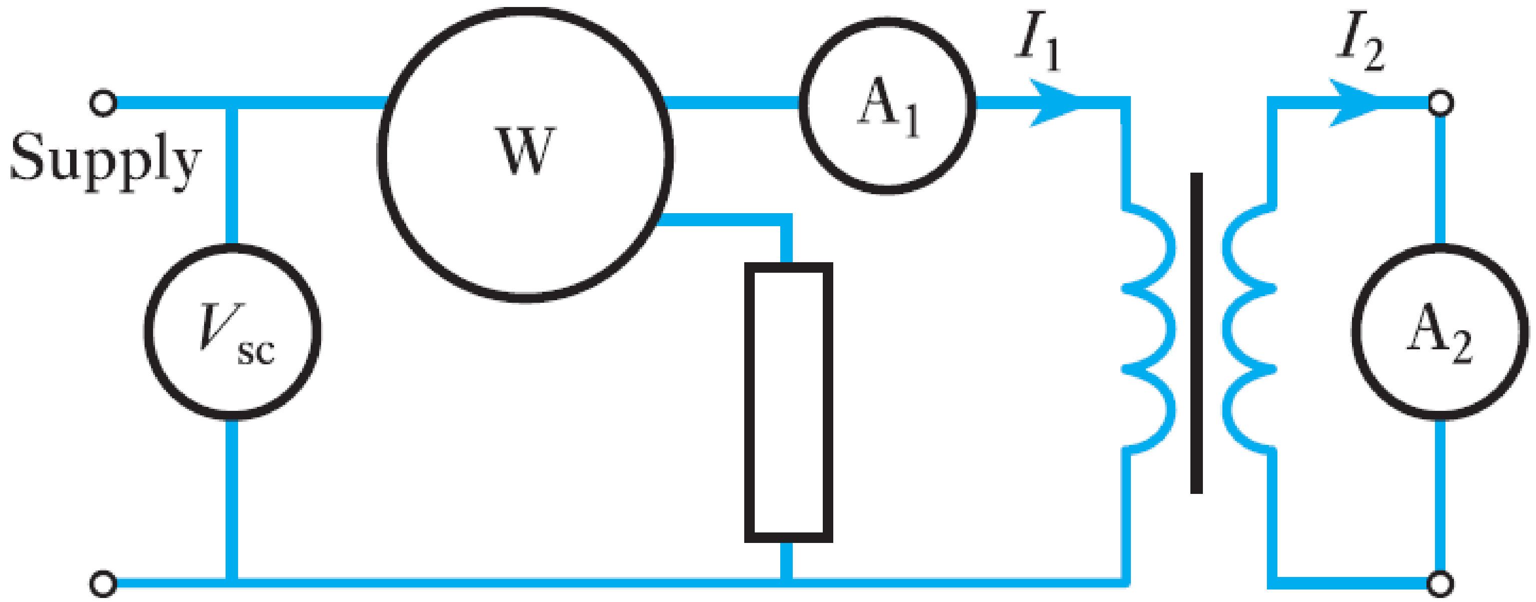
OPEN-CIRCUIT TEST OF TRANSFORMER



The primary **current** on **no-load** (I_0) is **< 5%** of the **full-load current**, so that the I^2R **copper loss** on **no-load < 0.25%** of the I^2R **copper loss** on **full load,** and is therefore **negligible** compared with the core loss. Hence the **wattmeter reading** $\boldsymbol{P_{oc}}$ can be considered as the **core loss** $\boldsymbol{P_c}$ of the transformer.

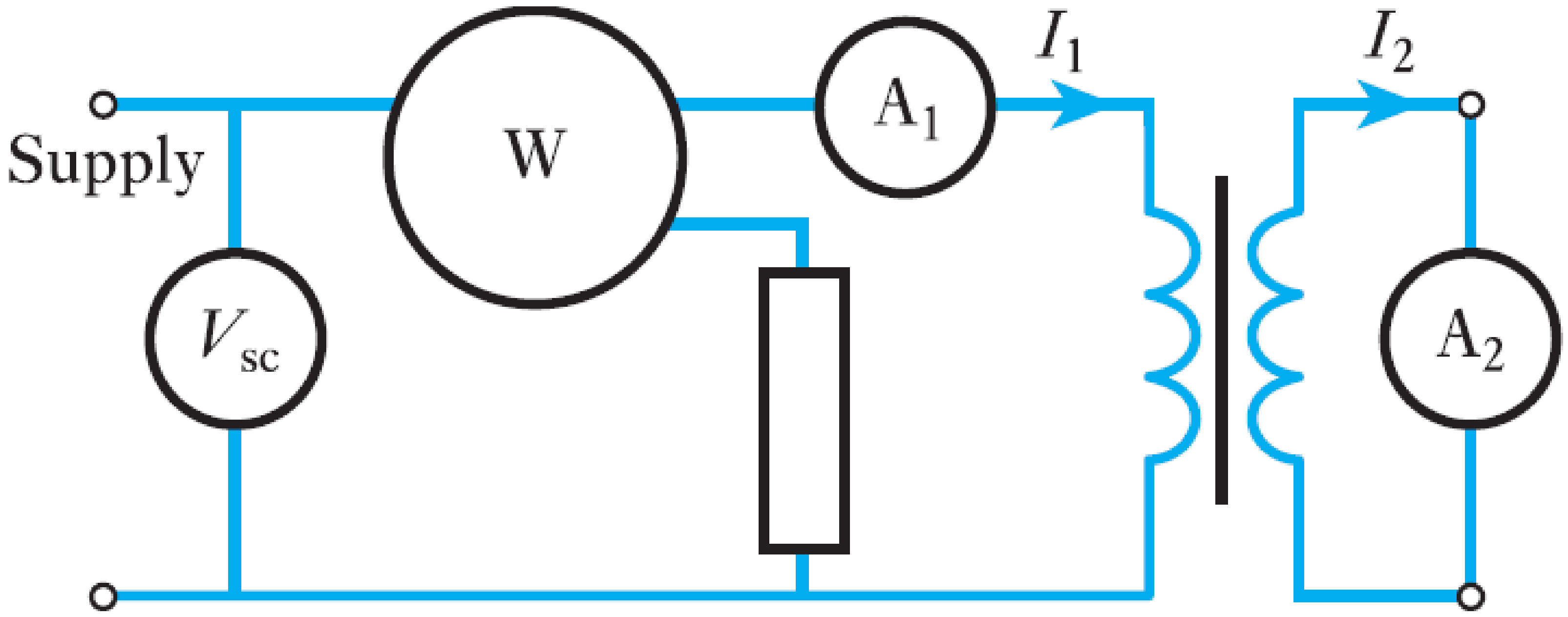
SHORT-CIRCUIT TEST OF TRANSFORMER

- The **secondary** is **short-circuited** through **ammeter A_2**, as shown below, and a **low voltage V_{sc}** is applied to the **primary** side
- The applied voltage V_{sc} is **adjusted** until the **full-load current I_2** (I_1) is measured in the **secondary** (primary) circuit



SHORT-CIRCUIT TEST OF TRANSFORMER

The **core loss** is **negligibly small**, since the **applied voltage** and therefore the **flux** are only **~ 3 - 5%** of the **rated voltage** and **flux**, and the core loss is approximately proportional to the square of the flux. Hence the **power** registered on **wattmeter** $\boldsymbol{P_{sc}}$ can be taken as the $\boldsymbol{I^2R}$ **copper loss** in the windings at full-load.



Efficiency of Transformer from Open-Circuit And Short-Circuit Tests

$\boldsymbol{P_{oc}}$ = input power on the open-circuit test at rated voltage = **core loss**

$\boldsymbol{P_{sc}}$ = input power on the short-circuit test with full-load currents
= total I^2R **copper loss** on full load

Total loss on full-load = $\boldsymbol{P_{oc}} + \boldsymbol{P_{sc}}$

If $\boldsymbol{S}$ is the **apparent rated power** (in VA) and **p.f.** is the **load power factor**,

=> $$\boxed{\text{Efficiency on full load} = \frac{\text{full-load } S \times \text{p.f.}}{(\text{full-load } S \times \text{p.f.}) + P_{oc} + P_{sc}}}$$

Efficiency of Transformer from Open-Circuit and Short-Circuit Tests

- For any **load equal to *m* × full-load** $(0 < m < 1)$, corresponding **total loss =** $\boldsymbol{P_{oc} + m^2 P_{sc}}$ (core loss is independent of load while copper loss $\propto I^2$)

- The corresponding efficiency = $\boxed{\dfrac{m \times \text{full-load } S \times \text{p.f.}}{(m \times \text{full-load } S \times \text{p.f.}) + P_{oc} + m^2 P_{sc}}}$

SOLVED NUMERICAL PROBLEMS

Q1. A single-phase transformer has 1000 turns on the primary and 200 turns on the secondary. The no-load current drawn is 3 A at a power factor 0.2 lagging. Calculate the total primary current and power factor at primary when the secondary is loaded to a current of 280 A at a power factor of 0.8 lagging. Assume the voltage drop in the windings to be negligible.

Solution: $I_{2'} = I_2.n = I_2 \times (N_2/N_1)$

$$I_{2'} \times 1000 = 280 \times 200$$

$$\Rightarrow \boxed{I_{2'} = 56\,A}$$

$$\cos\,\phi_2 = 0.8, \cos\,\phi_0 = 0.2$$

$$I_1 \times \cos\,\phi_1 = I_{2'} \times \cos\,\phi_2 + I_0 \times \cos\,\phi_0$$

$$= 56 \times 0.8 + 3 \times 0.2 = 45.4\,A$$

$$I_1 \times \sin\,\phi_1 = I_{2'} \times \sin\,\phi_2 + I_0 \times \sin\,\phi_0$$

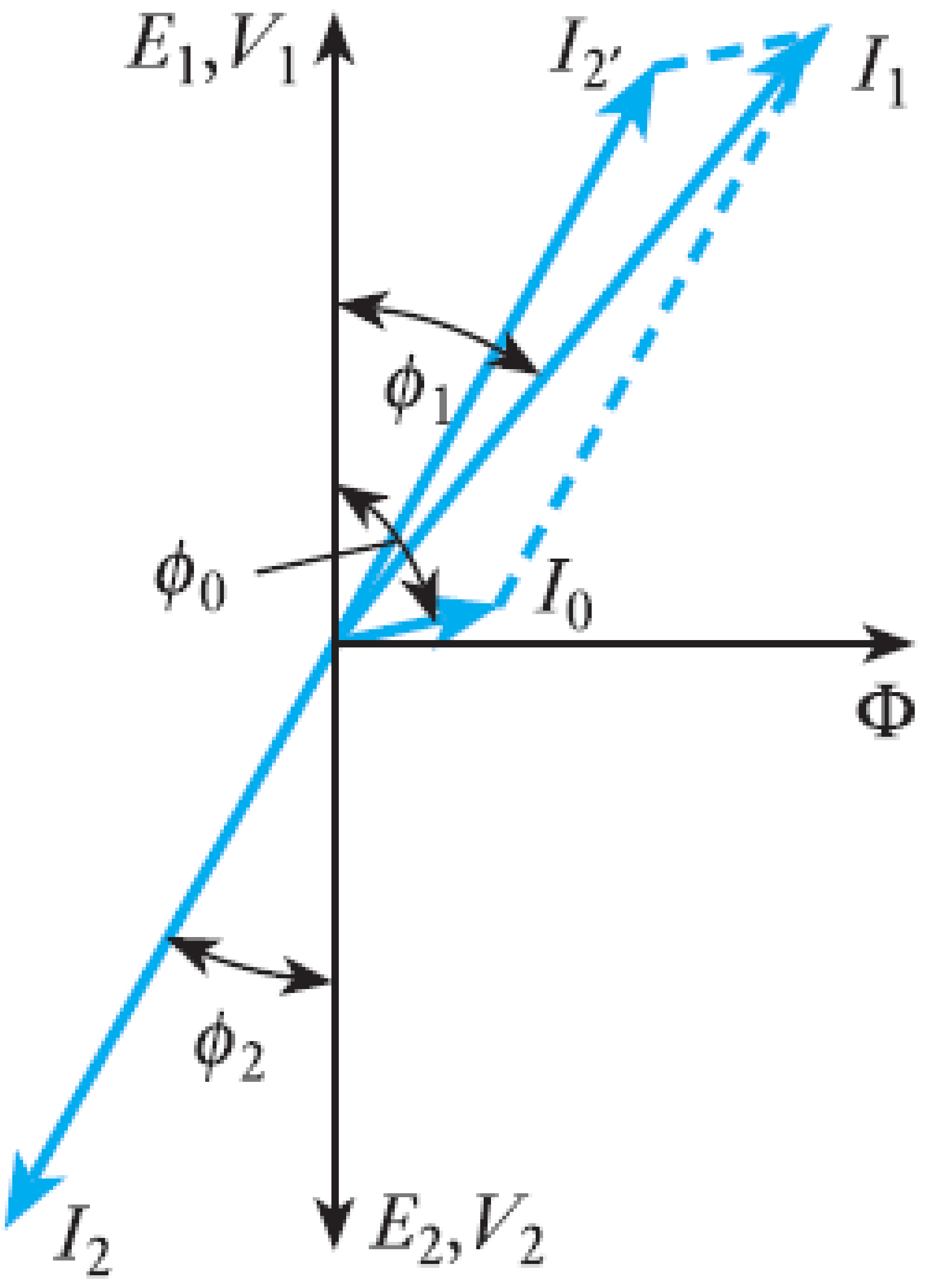
$$= 56 \times 0.6 + 3 \times 0.98 = 36.54\,A$$

So, $\boxed{I_1 = 58.3\,A}$

$$\tan\phi_1 = \frac{36.54}{45.4} = 0.805$$

$$\boxed{\phi_1 = 38^0 50'}$$

The primary power factor $= \boxed{\cos\phi_1 = 0.78 \text{ lagging}}$



Q2. A 100 kVA transformer has 400 turns on the primary and 80 turns on the secondary. The primary and secondary resistances are 0.3 Ω and 0.01 Ω respectively, and the corresponding leakage reactances are 1.1 Ω and 0.035 Ω respectively. The supply voltage is 2200 V. Calculate:

(a) the equivalent impedance referred to the primary circuit

(b) the voltage regulation and the secondary terminal voltage at full-load having a power factor of (i) 0.8 lagging and (ii) 0.8 leading.

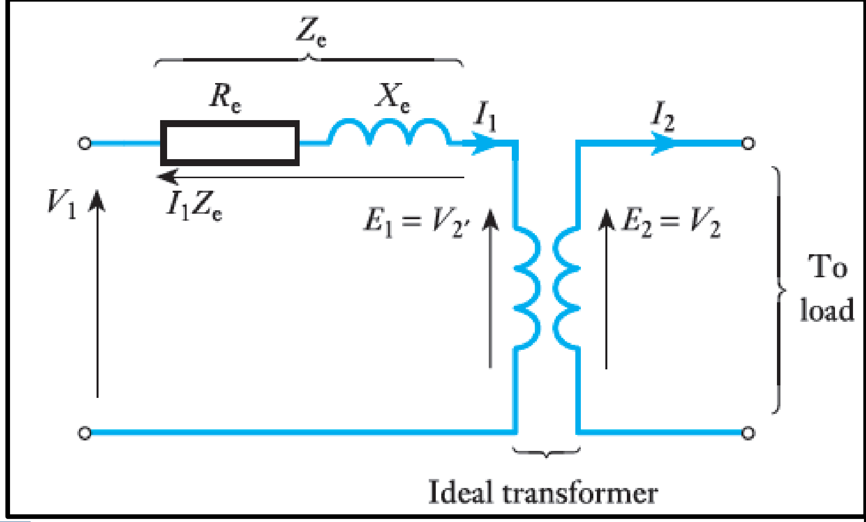
Soln: a) Equivalent resistance referred to primary is

$$\boxed{R_e = 0.3 + 0.01\left(\frac{400}{80}\right)^2 = 0.55\Omega}$$

$$\boxed{\begin{aligned} R_e &= R_1 + R_2/n^2 \\ X_e &= X_1 + X_2/n^2 \end{aligned}}$$

Equivalent leakage reactance referred to primary is

$$\boxed{X_e = 1.1 + 0.035\left(\frac{400}{80}\right)^2 = 1.975\ \Omega}$$



Equivalent impedance referred to primary is

$$\boxed{Z_e = \sqrt{0.55^2 + 1.975^2} = 2.05\Omega}$$

b) i) Since $\cos\phi_2 = 0.8, \sin\phi_2 = 0.6$

$$\boxed{\text{Full load primary current} = \frac{100 \times 1000}{2200} = 45.45\,\text{A}}$$

Voltage regulation for power factor 0.8 lagging is

$$= \frac{45.45(0.55 \times 0.8 + 1.975 \times 0.6)}{2200} = 3.36\%$$

Voltage Regulation = $I_1(R_e\cos\phi_2 + X_e\sin\phi_2)/V_1$

Secondary terminal voltage on no load

$$= 2200 \times \frac{80}{400} = 440\,\mathrm{V}$$

Therefore decrease of secondary terminal voltage between no load and full load = $440 \times 0.0336 = 14.8\,\mathrm{V}$

Therefore secondary terminal voltage on full load = 440-14.8 = 425 V

ii) Voltage regulation for power factor 0.8 leading is

$$= \frac{45.45(0.55 \times 0.8 - 1.975 \times 0.6)}{2200} = -1.54\,\%$$

Voltage Regulation = $I_1(\mathrm{R_e}\cos\phi_2 - \mathrm{X_e}\sin\phi_2)/V_1$

Increase of secondary terminal voltage between no load and full load=440 × 0.0154 = 6.78 V

Therefore secondary terminal voltage on full load =440 +6.78=447 V

Q3. Calculate the per-unit and percentage resistance and leakage reactance drops of the transformer in the previous numerical (Q2).

Solution:

$$\boxed{\text{Per-unit resistance drop of a transformer}}$$

$$\boxed{= \frac{\begin{array}{c}\text{full-load primary} \\ \text{current}\end{array} \times \begin{array}{c}\text{equivalent resistance} \\ \text{referred to primary}\end{array}}{\text{primary voltage}}}$$

$$\boxed{= \frac{\begin{array}{c}\text{full-load secondary} \\ \text{current}\end{array} \times \begin{array}{c}\text{equivalent resistance} \\ \text{referred to secondary}\end{array}}{\text{secondary voltage on no load}}}$$

Full-load primary current =45.45 A

Equivalent resistance referred to primary circuit = 0.55 Ω

$$\text{Resistance drop} = \frac{45.45 \times 0.55}{2200} = 1.14\%$$

Full-load secondary current$=45.45 \times 400 / 80 = 227.2\,\text{A}$

Equivalent resistance referred to secondary circuit

$$=0.01+0.3\left(\frac{80}{400}\right)^2 = 0.022\,\Omega$$

$$\text{Secondary Voltage on no load} = 440\,\text{V}$$

$$\text{Resistance Drop} = \frac{227.2 \times 0.022}{440} = 1.14\%$$

Leakage reactance drop of a transformer

$$= \frac{\text{full-load primary current} \times \text{equivalent leakage resistance referred to primary}}{\text{primary voltage}}$$

$$= \frac{45.45 \times 1.975}{2200} = 4.08\%$$

Q4. The primary and secondary windings of a 500 kVA transformer have resistances of 0.42 Ω and 0.0019 Ω respectively. The primary and secondary voltages are 11 kV and 400 V respectively, and the core loss is 2.9 kW. Assuming the load to have a p.f. of 0.8, calculate the efficiency at

(a) full load (b) half load.

Solution: a)

$$\text{Full load secondary current} = \frac{500 \times 1000}{400} = 1250\,\text{A}$$

$$\text{Full load primary current} = \frac{500 \times 1000}{11000} = 45.45\,\text{A}$$

Therefore secondary I^2R loss on full load

$=1250^2 \times 0.0019 = 2969\,\text{W}$

Primary I^2R loss on full load $=45.5^2 \times 0.42 = 870\,\text{W}$

Total I^2R loss on full load = 3.84 kW

Total loss on full load = 3.84+2.9= 6.74 kW (total loss = copper loss + core loss)

Output power on full load = 500 × 0.8 = 400 kW

Input power on full load = 400 + 6.74 = 406.74 kW

Efficiency on full load $=1-\dfrac{6.74}{406.74} = 0.983\,\text{pu} = 98.3\%$

(b) Since the I^2R loss varies as the square of the current,

Total I^2R loss on half load = 3.84 × $(0.5)^2$ = 0.96 kW

Total loss on half load = 0.96 + 2.9 = 3.86 kW

Efficiency on half load $=1-\dfrac{3.86}{203.86} = 98.1\%$

Q5. For the transformer in the previous numerical (Q4), assuming the power factor of the load to be 0.8 again, find the power output at which the efficiency of the transformer is maximum and calculate its value.

Solution: With the full-load output of 500 kVA, the total I^2R loss is 3.84 kW. Let ***m* = fraction of full-load** apparent power (in kVA) at which the **efficiency** is a **maximum** => corresponding **total I^2R loss = m^2 × 3.84 kW** Also, given that **core loss = 2.9 kW**

For **maximum efficiency**, **core loss = I^2R (copper) loss**

$\Rightarrow m^2 \times 3.84 = 2.9$ => $\mathbf{m = 0.87}$

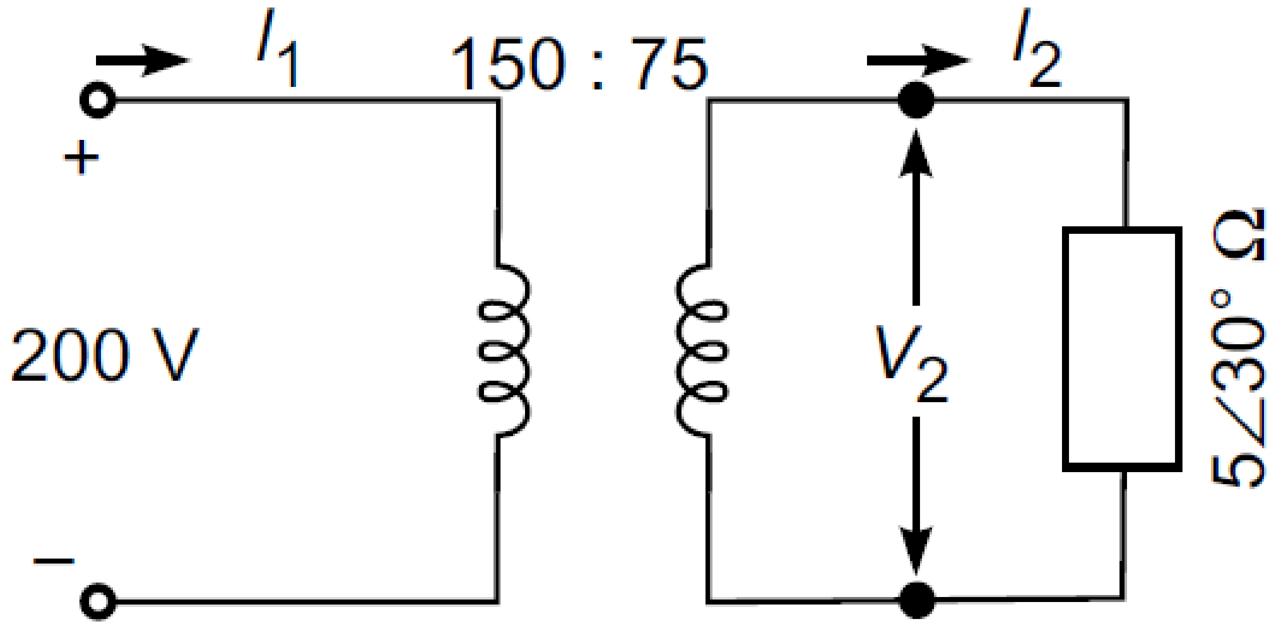
$\Rightarrow$ kVA output at maximum efficiency = 0.87 × 500 = 433 kVA

$\Rightarrow$ **Output power** = 433 × 0.8 = **346.4 kW** (load p.f. = 0.8)

Total loss = 2 × 2.9 = **5.8 kW**

=> **Maximum efficiency** = $\boxed{1 - \dfrac{5.8}{346.4 + 5.8} = 0.984\,\text{pu} = 98.4\%}$

Q6. Consider the transformer shown below. The secondary is connected to a load impedance $5\angle30^0$. Calculate the primary side input impedance, secondary terminal voltage, primary & secondary currents, and their respective power factors and real powers.



Solution:

$\bar{Z}_2 = 5\angle 30°\ \Omega$

$a = N_1/N_2 = 150/75 = 2$

$\bar{Z}_1 = \bar{Z}_2' = (2)^2\, 5\angle 30° = 20\angle 30°\ \Omega$

$V_2 = 200/2 = 100$ V;

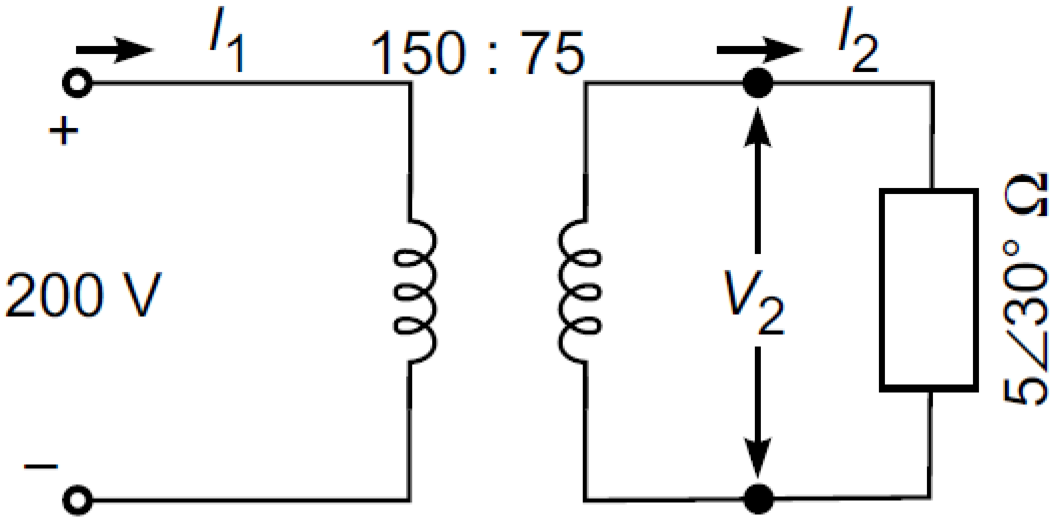
(secondary terminal voltage)

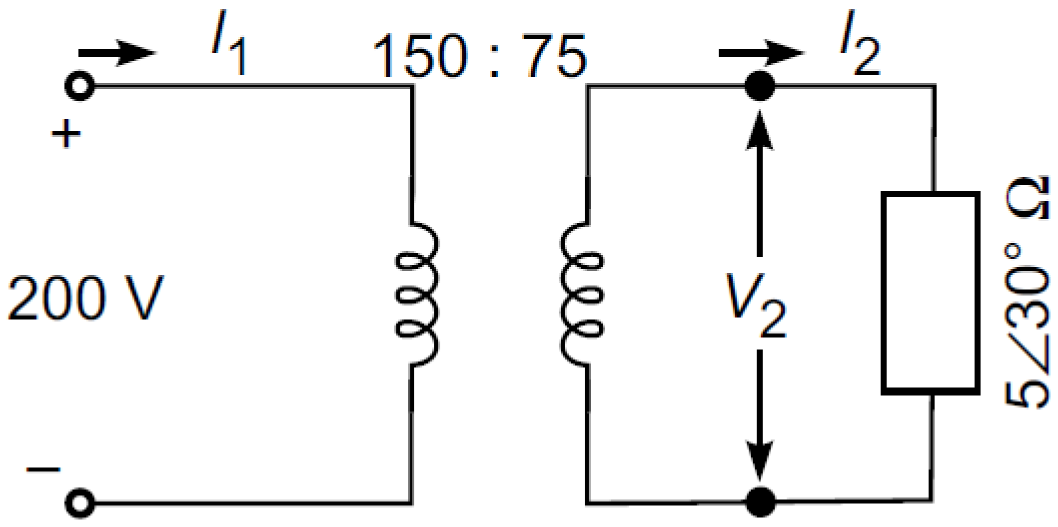
$\bar{I}_2 = 100\angle 0°/5\angle 30° = 20\angle{-30°}$ A

$I_2 = 20$ A; pf = cos 30° = 0.866 lagging

$\bar{I}_1 = \bar{I}_2' = 20\angle 30°/2 = 10\angle{-30°}$ A

$I_1 = 10$ A; pf = cos 30° = 0.866 lagging





$$\boxed{\begin{aligned} P_2 \text{ (secondary power output)} &= (20)^2 \times \text{Re } 5\ \angle 30° \\ &= 400 \times 4.33 = 1.732 \text{ kW} \end{aligned}}$$

$$\boxed{\begin{aligned} P_1 \text{ (primary power input)} &= P_2 \text{ (as the transformer is lossless)} \\ &= 1.732 \text{ kW} \\ P_1 &= V_1 I_1 \cos \theta_1 = 200 \times 10 \times 0.866 \\ &= 1.732 \text{ kW} \end{aligned}}$$

Q7. The following test results were obtained on a 50 kVA transformer:

Open-circuit test: primary voltage, 3300 V; secondary voltage, 400 V; primary power, 430 W.

Short-circuit test: primary voltage, 124 V; primary current, 15.3 A; primary power, 525 W; secondary current, full-load value.

Calculate:

(a) the efficiencies at full load and half load for 0.7 power factor;

(b) the voltage regulations for a load with power factor 0.7,

(i) lagging, (ii) leading;

(c) the secondary terminal voltages corresponding to (i) and (ii).

Solution:

(a) Core loss = 430 W (primary power reading in open-circuit test)

I^2R loss on full-load = 525 W (primary power in short-circuit test)

∴ Total loss on full load = 955 W = 0.955 kW

$$\text{Efficiency at full-load} = \frac{50\times 0.7}{(\underbrace{50}_{\text{Rated kVA}}\times \underbrace{0.7}_{\text{p.f.}})+0.955} = 1-\frac{0.955}{35.95} = 0.973\,\text{pu} = 97.3\%$$

I^2R loss on half load = 525 × $(0.5)^2$ = 131 W

∴ Total loss on half load = 430 + 131 = 561 W = 0.561 kW

$$\text{Efficiency at half-load} = \frac{25\times 0.7}{(25\times 0.7)+0.561} = 1-\frac{0.561}{18.06} = 0.969\,\text{pu} = 96.9\%$$

(b) $\cos\phi_e = \dfrac{525}{124\times 15.3} = 0.2765$

So, $\phi_e = 73^0 57'$

Short-circuit test ($V_2 = 0$): p.f. on primary side only due to equivalent transformer impedance Z_e)

$\cos\phi_2 = 0.7$ (load p.f.)

So, $\phi_2 = 45^0 34'$

For lagging p.f. load,

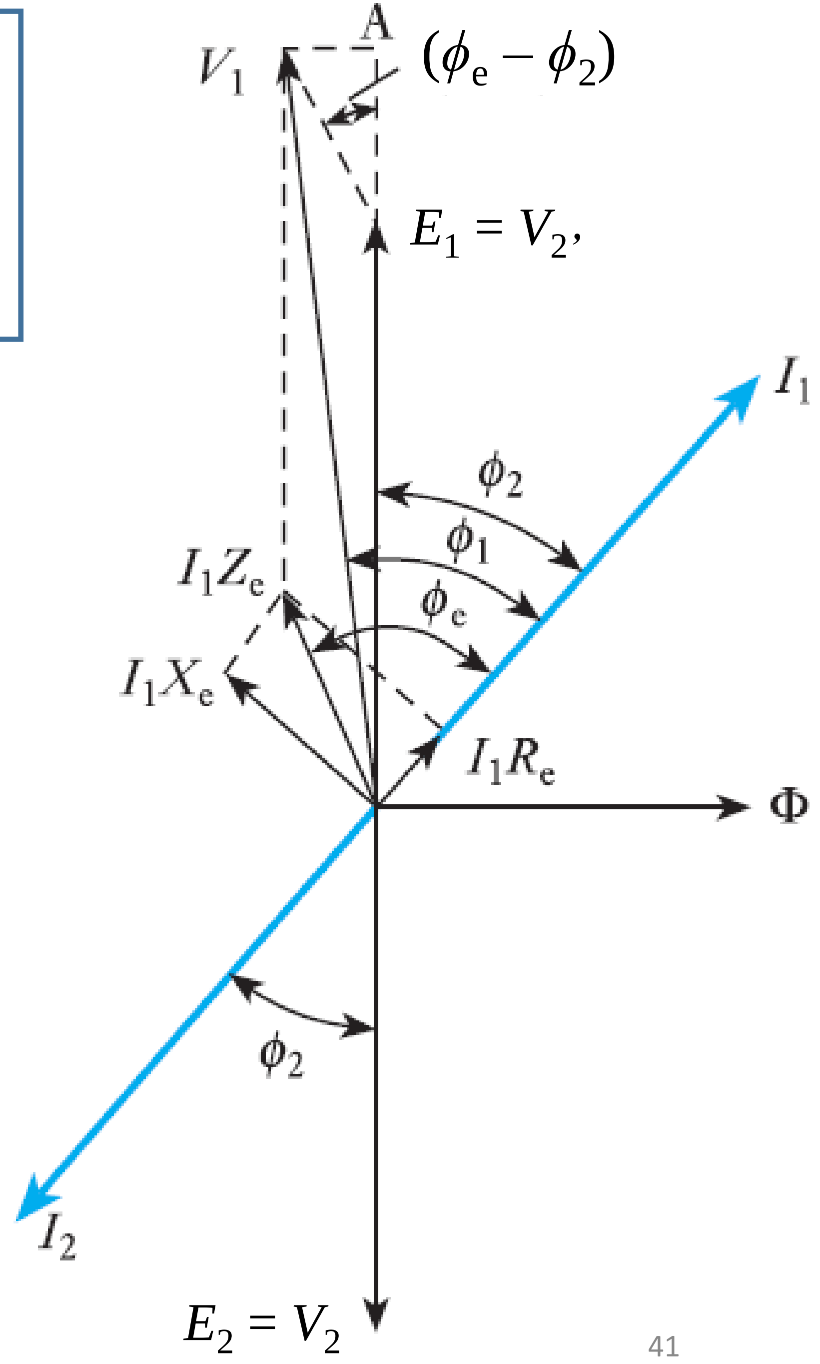
voltage regulation = $I_1 Z_e \cos(\phi_e - \phi_2)/V_1$

Where I_1 = full-load primary current

V_1 = rated primary voltage

$$\text{Voltage regulation} = \frac{124\cos(73^0 57' - 45^0 34')}{3300}$$

$$= 0.033\,\text{pu} = 3.3\%$$



For leading p.f. load,

voltage regulation = $I_1 Z_e \cos(\phi_e + \phi_2)/V_1$

Where I_1 = full-load primary current

V_1 = rated primary voltage

$$\text{Voltage regulation} = \frac{124\cos(73^0 57' + 45^0 34')}{3300}$$

$$= -0.0185\, pu = -1.85\%$$

(c) Secondary voltage on open-circuit = 400 V.

=> secondary voltage on full-load, p.f. 0.7 lagging

= 400(1 − 0.033) = 387 V (since voltage regulation = 3.3%)

=> secondary voltage on full-load, p.f. 0.7 leading

= 400(1 + 0.0185) = 407 V (since voltage regulation = -1.85%)

Q8. The following data were obtained on a 20 kVA, 50 Hz, 2000/200 V distribution transformer. Draw the equivalent circuits of the transformer referred to the HV (high-voltage) and LV (low-voltage) sides respectively.

	Voltage (*V*)	*Current* (*A*)	*Power* (*W*)
OC test with HV open-circuited	200	4	120
SC test with LV short-circuited	60	10	300

Solution:

OC test (LV side) : Gives core admittance Y_0 (parallel combination of core loss conductance G_i and magnetizing susceptance B_m)

$$Y_0 = \frac{4}{200} = 2 \times 10^{-2}\ \mho\ ; G_i = \frac{120}{(200)^2} = 0.3 \times 10^{-2}\ \mho$$

$$B_m = \sqrt{Y_0^2 - G_i^2} = 1.98 \times 10^{-2}\ \mho$$

SC test (HV side) : Gives coil/winding impedance Z (series combination of coil resistance R and leakage reactance X)

$$Z = \frac{60}{10} = 6\ \Omega\ ; R = \frac{300}{(10)^2} = 3\ \Omega$$

$$X = \sqrt{Z^2 - R^2} = 5.2\ \Omega$$

Transformation ratio, $\boxed{\dfrac{N_H}{N_L} = \dfrac{2000}{200} = 10}$

=> Equivalent circuit referred to the HV side:

$$G_i\,(\text{HV}) = 0.3 \times 10^{-2} \times \frac{1}{(10)^2} = 0.3 \times 10^{-4}\ \mho$$

$$B_m\,(\text{HV}) = 1.98 \times 10^{-2} \times \frac{1}{(10)^2} = 1.98 \times 10^{-4}\ \mho$$

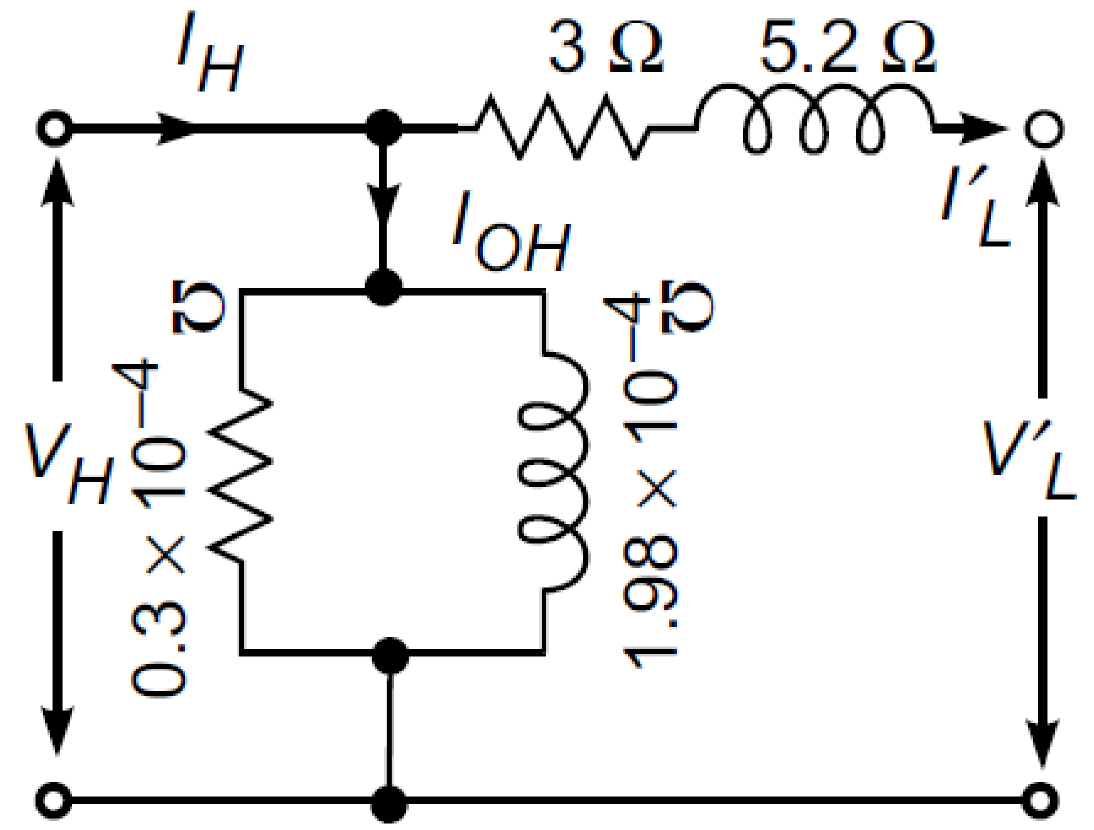
Note: G_i and B_m have units of admittance (inverse of impedance), so transform as inverse of transformation ratio

=> Equivalent circuit referred to the LV side:

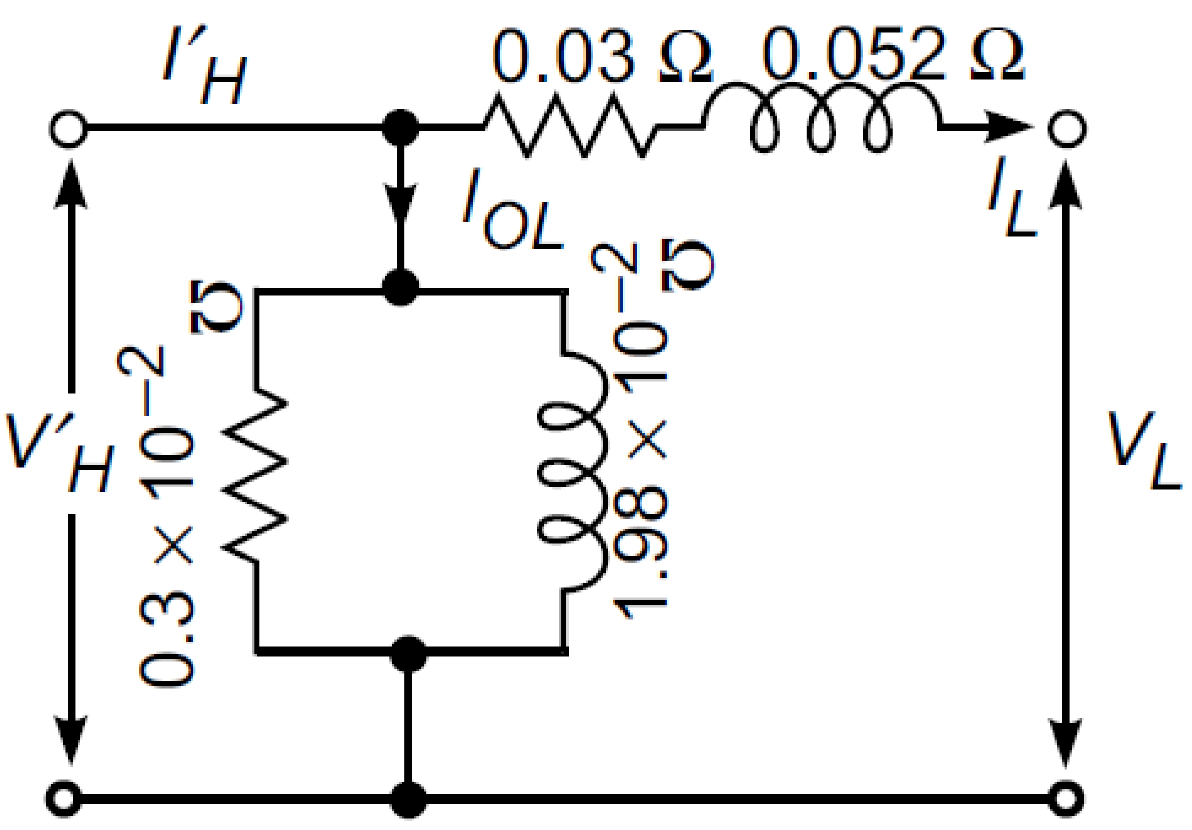
$$R(\text{LV}) = 3 \times \frac{1}{(10)^2} = 0.03\ \Omega$$

$$X(\text{LV}) = 5.2 \times \frac{1}{(10)^2} = 0.052\ \Omega$$

Transformer equivalent circuits



(a) Referred to HV



(b) Referred to LV

Q9. The equivalent circuit parameters of a 150 kVA, 2400/240 V transformer are:

$$R_1 = 0.2\ \Omega,\ R_2 = 2\times10^{-3}\ \Omega$$

$$X_1 = 0.45\ \Omega,\ X_2 = 4.5\times10^{-3}\ \Omega$$

$$R_i = 10\ \text{k}\Omega\ , X_m = 1.6\ \text{k}\Omega \text{ (as seen from 2400-V side)}$$

Calculate:

(a) Open-circuit current, power and p.f. when the LV (low-voltage) side is excited at rated voltage

(b) The voltage at which the HV (high-voltage) side should be excited to conduct a short-circuit test (LV shorted) with full-load current flowing. What is the input power and its p.f. under this condition?

Solution: $R_i = \dfrac{1}{G_i}, \quad X_m = \dfrac{1}{B_m}$

$$\text{Ratio of transformation, } a = \frac{2400}{240} = 10$$

(a) Referring the shunt parameters to LV side

$$R_i\,(\text{LV}) = \frac{10 \times 1000}{(10)^2} = 100\ \Omega$$

$$X_m\,(\text{LV}) = \frac{1.6 \times 1000}{(10)^2} = 16\ \Omega$$

=> No-load current

$$\bar{I}_0\,(\text{LV}) = \frac{240\angle 0^\circ}{100} - j\,\frac{240\angle 0^\circ}{16}$$

$$= 2.4 - j\,15 = 15.2\angle{-80.9^\circ}\ \text{A}$$

$$I_0 = 15.2\ \text{A, pf} = \cos 80.9^\circ = 0.158\ \text{lagging}$$

(b) LV shorted, HV excited, full-load current flowing:

Shunt parameters can be ignored under this condition

Equivalent series parameters referred to HV side:

$$R = 0.2 + 2 \times 10^{-3} \times (10)^2 = 0.4\ \Omega$$

$$X = 0.45 + 4.5 \times 10^{-3} \times (10)^2 = 0.9\ \Omega$$

$$\bar{Z} = 0.4 + j\,0.9 = 0.958\ \angle 66^\circ\ \Omega$$

Full-load current on HV side: $I_{fl}\,(\mathrm{HV}) = \dfrac{150 \times 1000}{2400} = 62.5\ \mathrm{A}$

=> Voltage drop on HV side: $V_{\mathrm{SC}}\,(\mathrm{HV}) = 62.5 \times 0.958 = 59.9\ \mathrm{V}$ or $60\ \mathrm{V}$ (say)

=> Power drawn on HV side: $P_{\mathrm{SC}} = (62.5)^2 \times 0.4 = 1.56\ \mathrm{kW}$

Power factor on HV side: $pf_{\mathrm{SC}} = \cos 66^\circ = 0.406$ lagging

Q10. A 500 kVA transformer has an efficiency of 95% at full-load, and the same efficiency also at 60% of full-load; both loads at upf (unity power factor) .

(a) Compute the iron and copper losses of the transformer.

(b) Determine the efficiency of the transformer at 3/4th of full-load.

Solution: (a) Let P_i be the iron (core) loss and P_c be the copper (I^2R) loss

At full-load: $$\frac{500 \times 1}{500 \times 1 + P_i + P_c} = 0.95 \quad \text{(i)}$$

At 0.6× full-load: $$\frac{500 \times 0.6}{500 \times 0.6 + P_i + (0.6)^2 P_c} = 0.95 \quad \text{(ii)}$$

Solving Eqs (i) and (ii) we get $P_i = 9.87$ kW

$P_c = 16.45$ kW

(b) At 3/4th full load upf

$$\eta = \frac{500 \times 0.75}{500 \times 0.75 + 9.87 + (0.75)^2 \times 16.45}$$

$$= 95.14\%$$

Q11. For a 150 kVA, 2400/240 V transformer, the equivalent circuit parameters are given as

$$R_1 = 0.2\ \Omega,\ R_2 = 2\times10^{-3}\ \Omega$$

$$X_1 = 0.45\ \Omega,\ X_2 = 4.5\times10^{-3}\ \Omega$$

$$R_i = 10\ \mathrm{k}\Omega\ ,X_m = 1.6\ \mathrm{k}\Omega \text{ (as seen from 2400-V side)}$$

(a) Draw the circuit model as seen from the HV side.

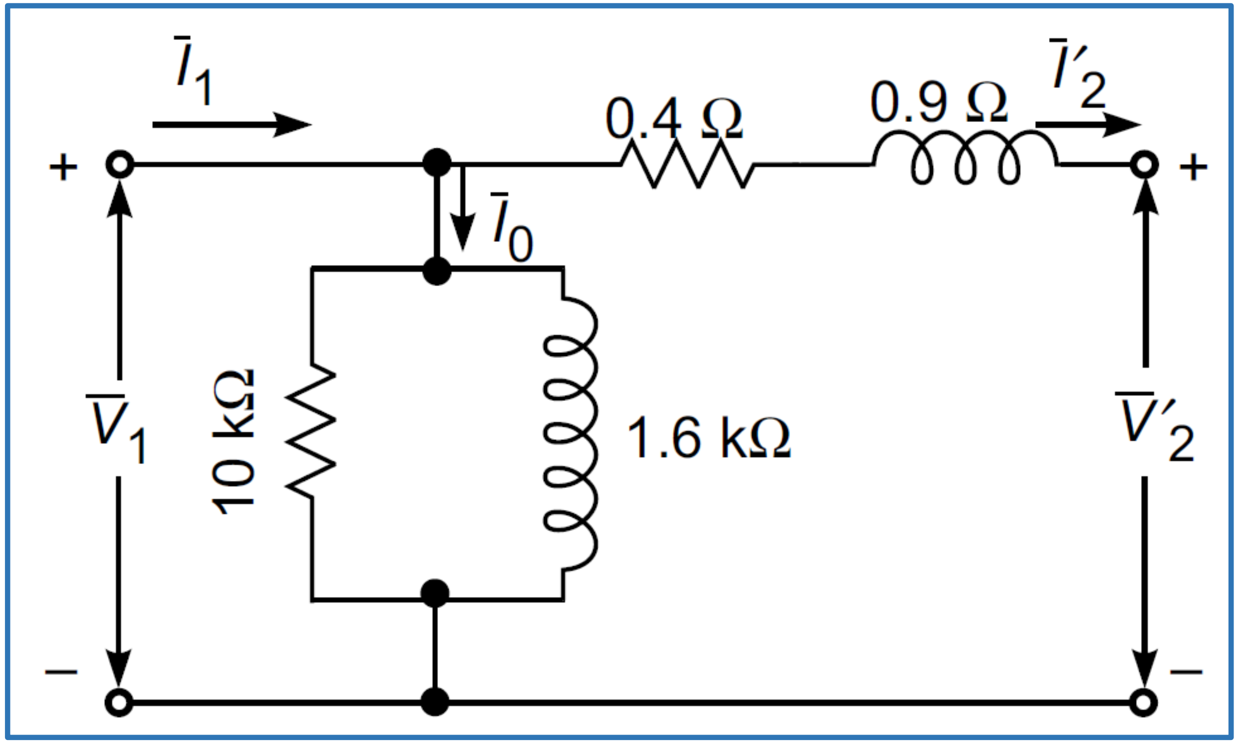
(b) Determine therefrom the voltage regulation and efficiency when the transformer is supplying full-load at 0.8 lagging p.f. on the secondary side at rated voltage.

(c) For the conditions specified in (b), calculate also the HV side current and its p.f.

Solution: (a) $\boxed{N_1/N_2 = E_1/E_2 = 2400/240 = 10}$

=> $\boxed{\begin{aligned} R(\text{HV}) &= 0.2 + 2 \times 10^{-3} \times (10)^2 = 0.4\ \Omega \\ X(\text{HV}) &= 0.45 + 4.5 \times 10^{-3} \times (10)^2 = 0.9\ \Omega \end{aligned}}$

=> The circuit model as seen from HV side is drawn below



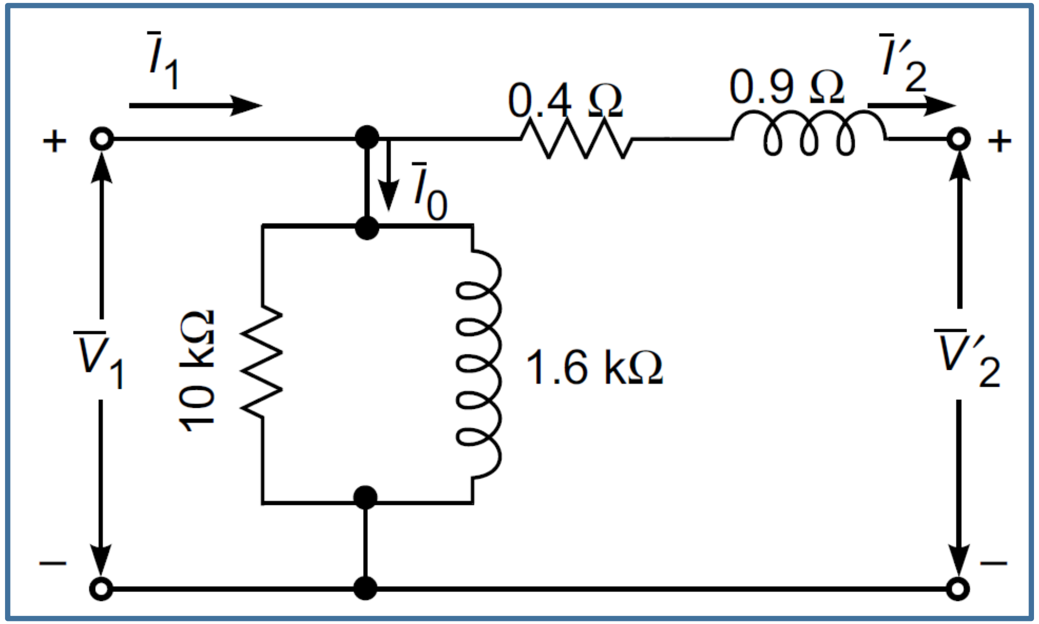
(b)

$$I_{2(fl)} = \frac{150 \times 1000}{240} = 625 \text{ A, 0.8 pf lagging}$$

$$V_2 = 240 \text{ V}$$

$$I_2' = \frac{625}{10} = 62.5 \text{ A, 0.8 pf lagging}$$

$$V_2' = 2400 \text{ V}$$



$$\text{Voltage drop} = 62.5(0.4 \times 0.8 + 0.9 \times 0.6) = 53.75 \text{ V}$$

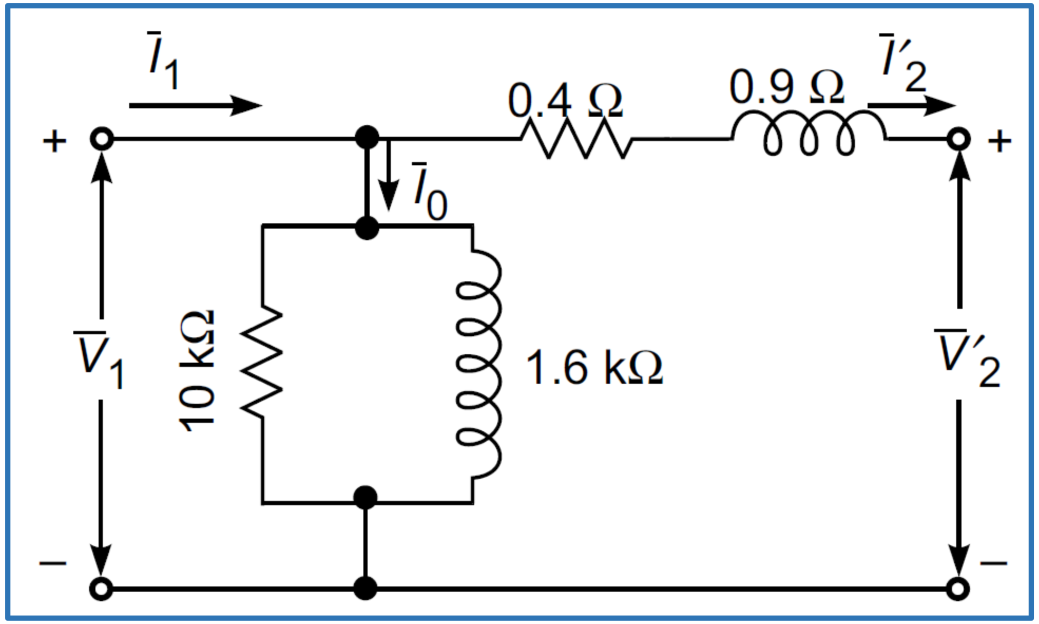
Voltage Regulation = $I_2'(R\cos\phi_2 + X\sin\phi_2)/V_2'$

$\cos\phi_2 = 0.8 \Rightarrow \sin\phi_2 = 0.6$

$$\text{Voltage regulation} = \frac{53.75}{2400} \times 100 = 2.24\%$$

(b)

$$V_1 = 2400 + 53.75 = 2453.75 = 2454 \text{ V}$$



Output power:

$$\boxed{P_{(\text{out})} = 150 \times 0.8 = 120 \text{ kW}}$$

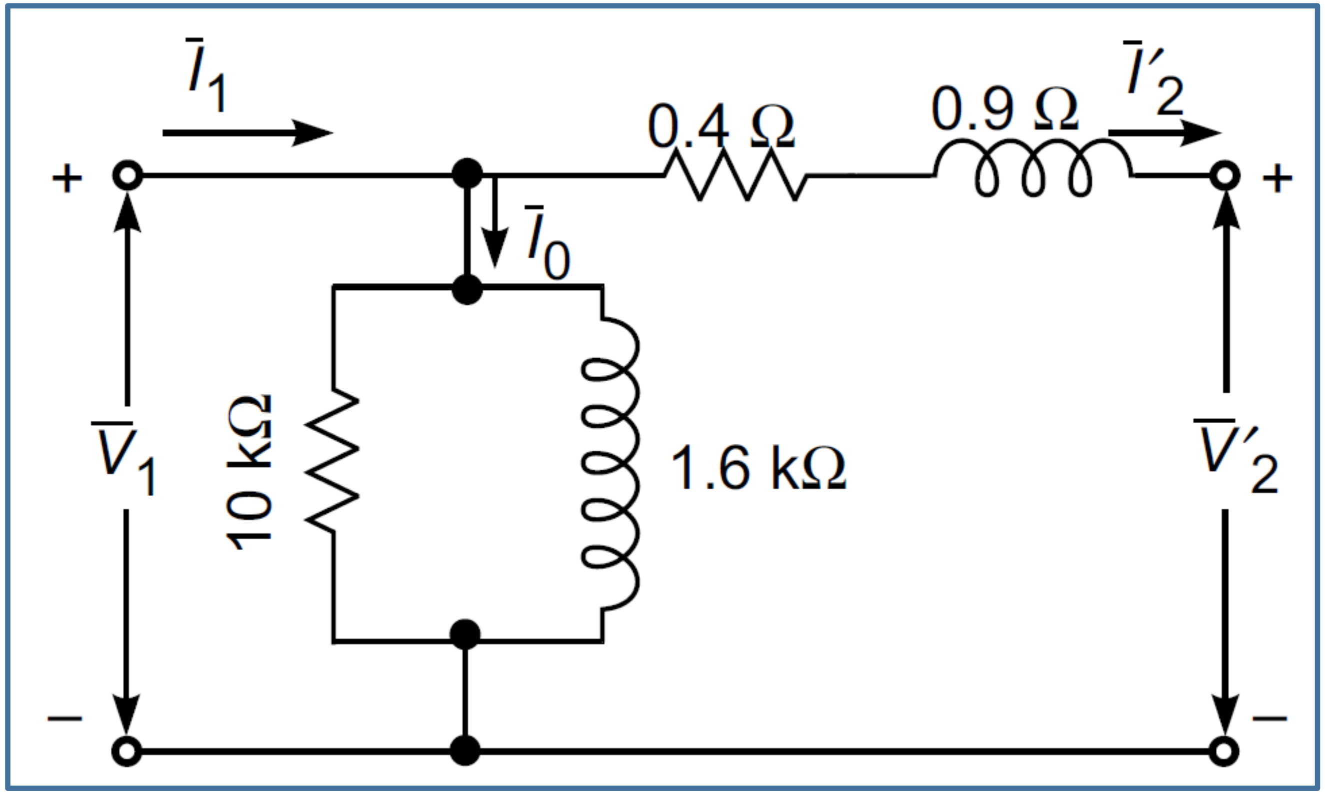
$$P_{c(\text{copper loss})} = (62.5)^2 \times 0.4 = 1.56 \text{ kW}$$

$$P_i(\text{core loss}) = \frac{(2454)^2}{10 \times 1000} = 0.60 \text{ kW}$$

=> Total loss $\boxed{P_L = P_i + P_c = 0.60 + 1.56 = 2.16 \text{ kW}}$

=> Efficiency $\boxed{\eta = \dfrac{120}{120 + 2.16} = 98.2\%}$

(c)



$$\bar{I}_0 = \frac{2454\angle 0^\circ}{10\times 1000} - j\frac{2454\angle 0^\circ}{1.6\times 1000}$$

$$= 0.245 - j\,1.53 \text{ A}$$

$$I_2' = 62.5\,(0.8 - j\,0.6) = 50 - j\,37.5 \text{ A}$$

$$\bar{I}_1 = \bar{I}_0 + \bar{I}_2 = 50.25 - j\,39.03$$

$$= 63.63\ \angle{-37.8^\circ} \text{ A}$$

$$I_1 = 63.63 \text{ A, pf} = 0.79 \text{ lagging}$$

$$I_2{}' = 62.5\angle -cos^{-1}\,(0.8)$$

i.e. 62.5 A at 0.8 p.f. lagging

Unsolved Practice Problems

Q1. The ratio of turns of a single-phase transformer is 8, the resistances of the primary and secondary windings are 0.85 Ω and 0.012 Ω respectively, and the leakage reactances of these windings are 4.8 Ω and 0.07 Ω respectively. Determine the voltage to be applied to the primary to obtain a current of 150 A in the secondary when the secondary terminals are short circuited. Ignore the magnetizing current.

Ans: 176.5 V

Q2. A single-phase transformer operates from a 230 V supply. It has an equivalent resistance of 0.1 Ω and an equivalent leakage reactance of 0.5 Ω referred to the primary. The secondary is connected to a coil having a resistance of 200 Ω and a reactance of 100 Ω. Calculate the secondary terminal voltage. The secondary winding has four times as many turns as the primary. **Ans: 928 V**

Q3. A 230 V/400 V single-phase transformer absorbs 35 W when its primary winding is connected to a 230 V, 50 Hz supply, the secondary being on open circuit. When the primary is short-circuited and a 10 V, 50 Hz supply is connected to the secondary winding, the power absorbed is 48 W when the current has the full-load value of 15 A. Estimate the efficiency of the transformer at half load, 0.8 power factor lagging.

Ans: 0.981 p.u.

Q4. Calculate the voltage regulation at 0.8 lagging power factor for a transformer which has an equivalent resistance of 2 per cent and an equivalent leakage reactance of 4%.

Ans: 4%

Q5. A 10 kVA single-phase transformer, for 2000 V/400 V at no load, has resistances and leakage reactances as follows:

Primary winding: resistance, 5.5 Ω; reactance, 12 Ω.

Secondary winding: resistance, 0.2 Ω; reactance, 0.45 Ω.

Determine the approximate value of the secondary voltage at full load, 0.8 power factor (lagging), when the primary supply voltage is 2000 V.

Ans: 377.6 V

Q6. A 75 kVA transformer, rated at 11 kV/230 V on no load, requires 310 V across the primary to circulate full-load currents on short circuit, the power absorbed being 1.6 kW. Determine: (a) the percentage voltage regulation; (b) the full-load secondary terminal voltage for power factors of (i) unity, (ii) 0.8 lagging and (iii) 0.8 leading. If the input power to the transformer on no load is 0.9 kW, calculate the per-unit efficiency at full load and at half load for power factor 0.8 and find the load (in kV A) at which the efficiency is maximum.

Ans: 2.13 per cent, 225.1 V; 0.41 per cent, 223.5 V;
2.81 per cent, 228.7 V; 0.960 p.u., 0.958 p.u.;
56.25 kVA

Q7. The primary and secondary windings of a 30 kVA, 11,000/230 V transformer have resistances of 10 Ω and 0.016 Ω respectively. The total reactance of the transformer referred to the primary is 23 Ω. Calculate the percentage regulation of the transformer when supplying full-load current at a power factor of 0.8 lagging. **Ans: 3.08%**

Q8. A 50 kVA, 6360 V/230 V transformer is tested on open and short-circuit to obtain its efficiency, the results of the test being as follows. Open circuit: primary voltage, 6360 V; primary current, 1 A; power input, 2 kW. Short-circuit: voltage across primary winding, 180 V; current in secondary winding, 175 A; power input, 2 kW. Find the efficiency of the transformer when supplying full load at a power factor of 0.8 lagging and draw a phasor diagram (neglecting impedance drops) for this condition.
Ans: 0.887 p.u.

Q9. A single-phase transformer is rated at 10 kVA, 230 V/100 V. When the secondary terminals are open-circuited and the primary winding is supplied at normal voltage (230 V), the current input is 2.6 A at a power factor of 0.3. When the secondary terminals are short-circuited, a voltage of 18 V applied to the primary causes the full-load current (100 A) to flow in the secondary, the power input to the primary being 240 W. Calculate:

(a) the efficiency of the transformer at full load, unity power factor;

(b) the load at which maximum efficiency occurs;

(c) the value of the maximum efficiency.

Ans: 0.96 p.u., 8.65 kVA, 0.96 p.u. at unity power factor

Q10. Each of two transformers, A and B, has an output of 40 kVA. The core losses in A and B are 500 and 250 W respectively, and the full-load I^2R losses are 500 and 750 W respectively. Tabulate the losses and efficiencies at quarter, half and full load for a power factor of 0.8. For each transformer, find the load at which the efficiency is a maximum.

Ans: A, 93.77, 96.24, 96.97 per cent; B, 96.42, 97.34, 96.97 per cent;
A, 40 kVA; B, 23.1 kVA

Q11. A 40 kVA transformer has a core loss of 450 W and a full-load I^2R loss of 850 W. If the power factor of the load is 0.8, calculate: (a) the full-load efficiency; (b) the maximum efficiency; (c) the load at which maximum efficiency occurs.

Ans: 0.961 p.u., 0.9628 p.u., 23.3 kW

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