Mechanics of Materials-I

SIMPLE STRESS & STRAIN

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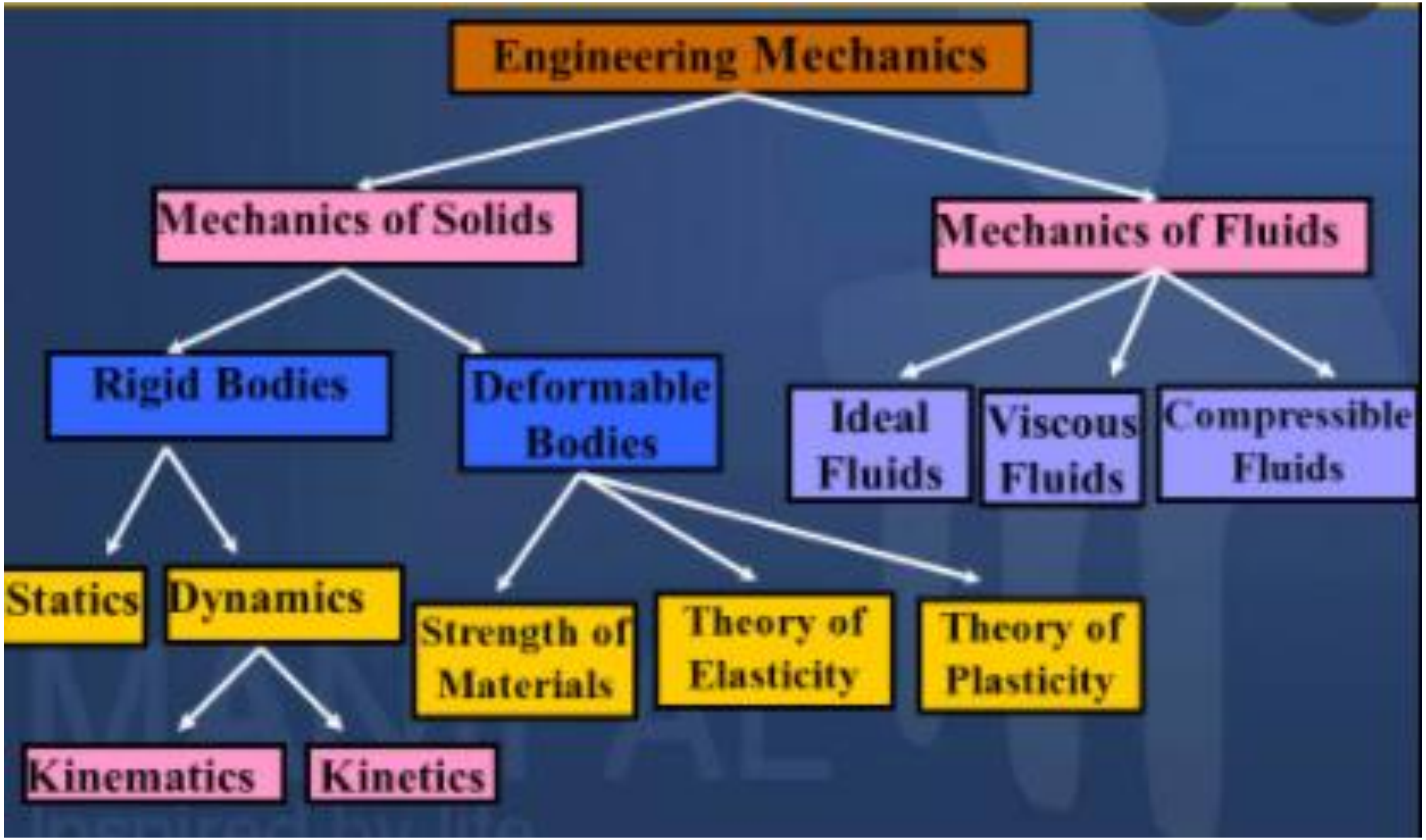
Objective of the course



Objective of the course

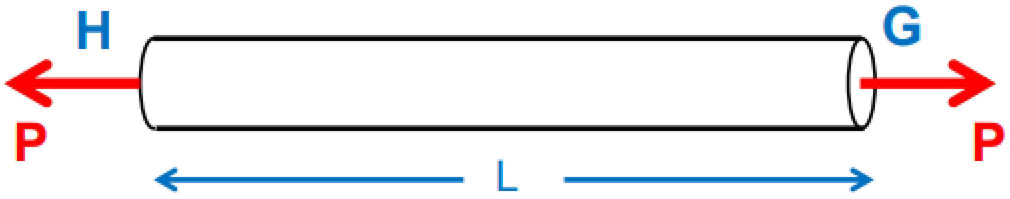
Foundation for all structural and machine design

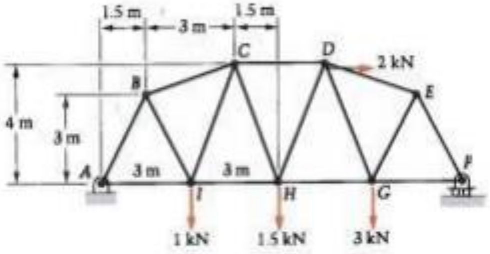
ENGINEERING STRUCTURE → EXTERNAL LOADS → INTERNAL FORCES AND MOMENTS → STRESSES / STRAINS → STRUCTURAL PERFORMANCE

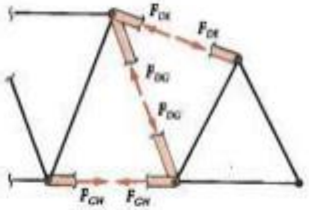
Dead Load,
Live Load,
Wind Load,
Earth quake load

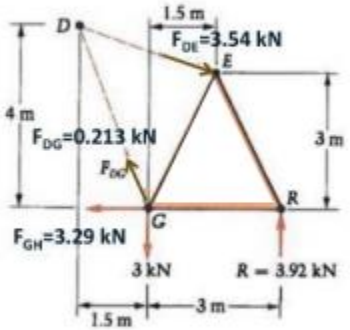
- Axial
- Torsional (Twisting)
- Transverse/Flexural/Bending
- Pressure
- Combined Loading

- deformations/deflections
- yield/fracture
- success/failure?

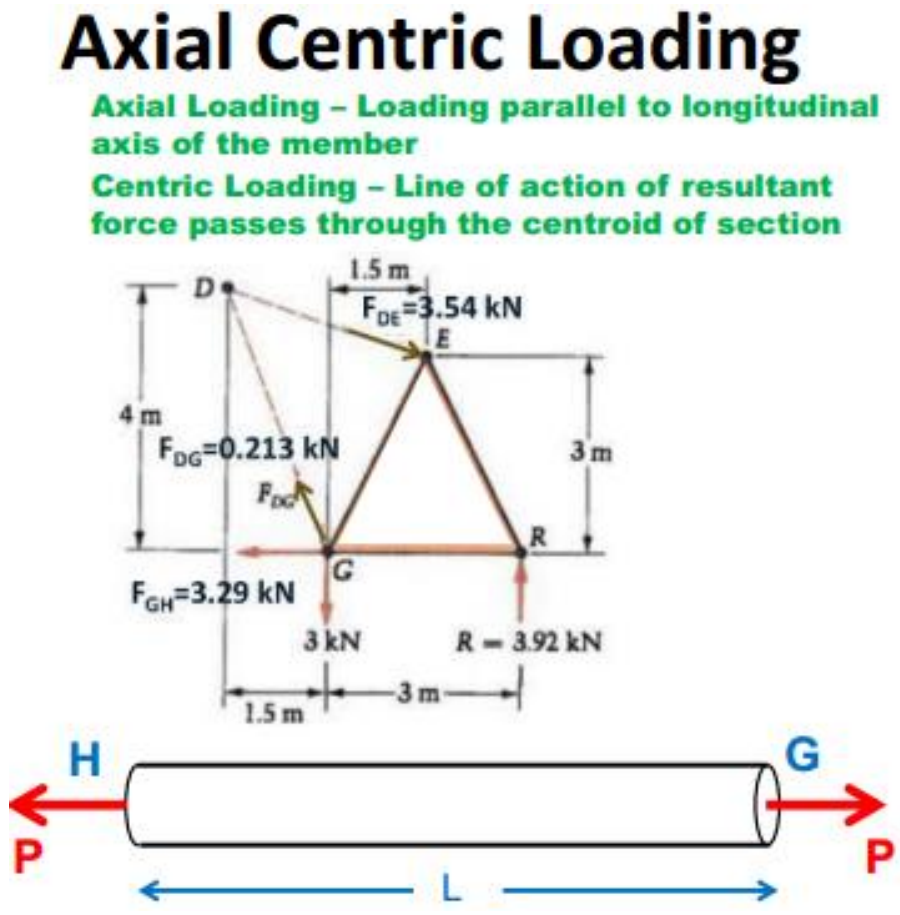






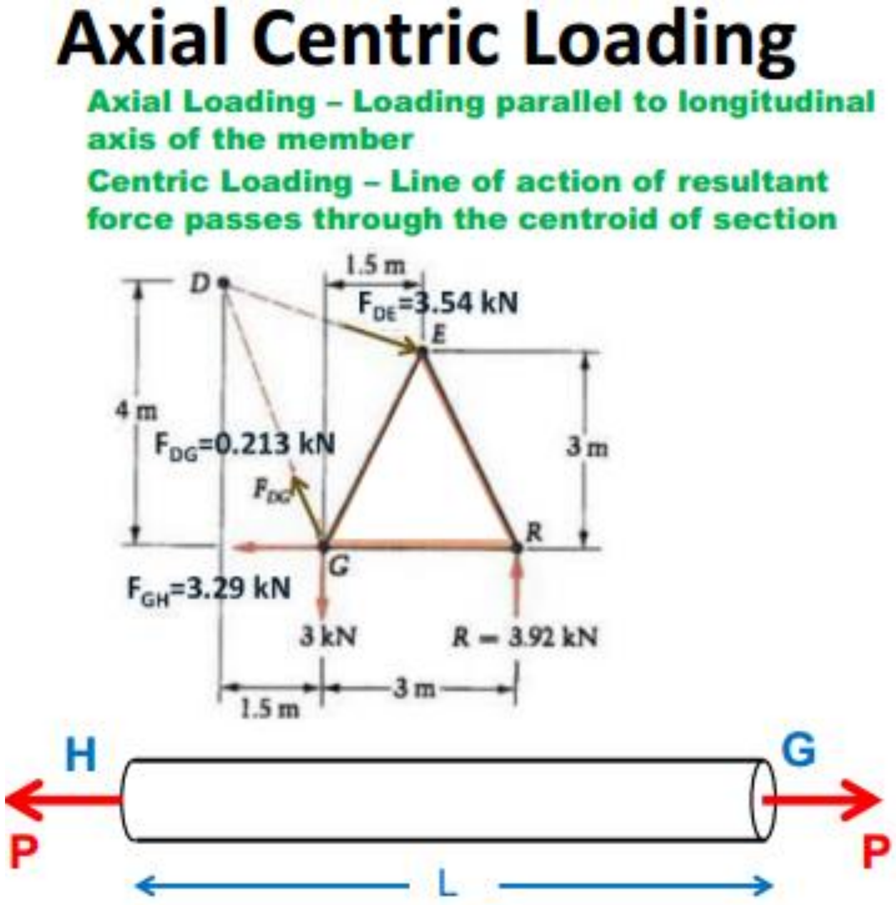


Axial Load


Axial Centric Loading
Axial Loading – Loading parallel to longitudinal axis of the member
Centric Loading – Line of action of resultant force passes through the centroid of section
D
E
G
R
1.5 m
F_DE=3.54 kN
4 m
F_DG=0.213 kN
3 m
F_GH=3.29 kN
3 kN
R = 3.92 kN
3 m
1.5 m
H
G
P
P
L

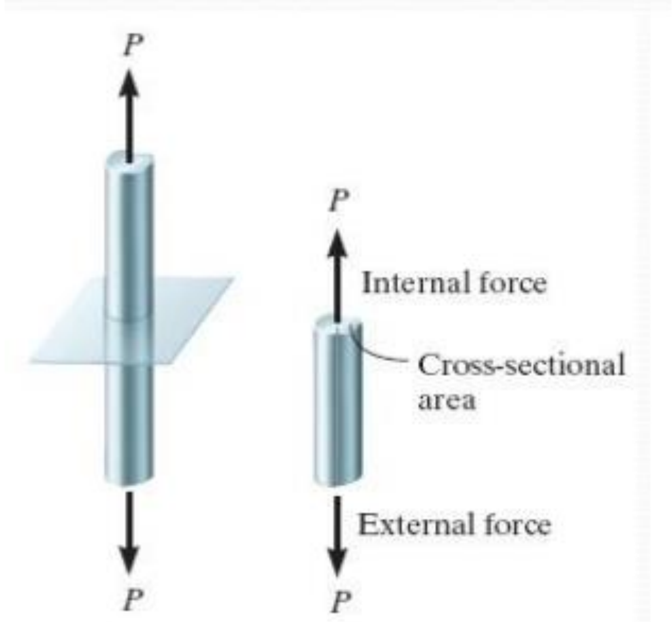
TOPICS

- FBD
- Stress Intensity
- Normal stress
- Shear stress
- State of stress at a point
- Ultimate strength
- Allowable stress
- Factor of safety
- Normal strain
- Shear strain
- Poisson's ratio
- Hooke's law
- Stress-strain characteristics for mild steel



FBD

- The diagramme of a body (or) part of it acted upon by external and internal forces/resisting forces t keep the body in equilibrium condition



Equilibrium of a Deformable Body

Equations of Equilibrium

- Equilibrium of a body requires a ***balance of forces*** and a ***balance of moments***

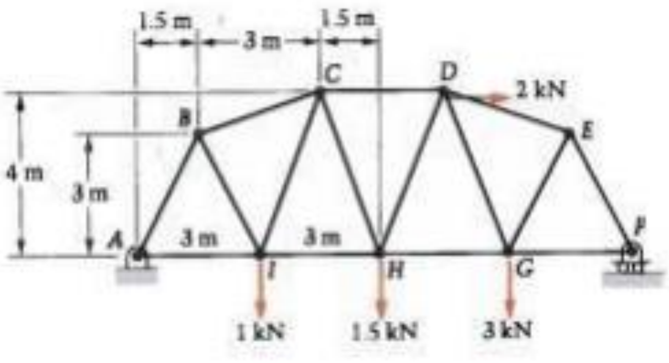
$$\sum \mathrm{F} = 0 \qquad \sum \mathrm{M}_O = 0$$

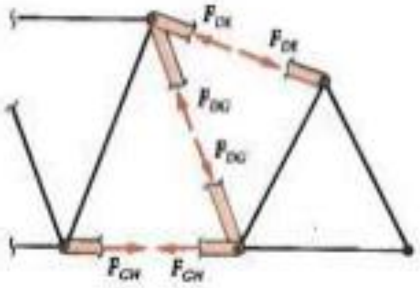
- For a body with *x*, *y*, *z* coordinate system with origin *O*,

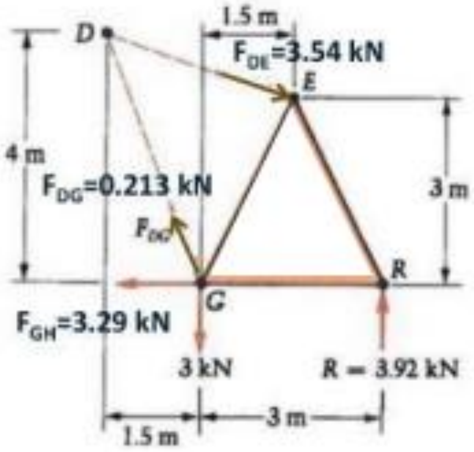
$$\sum F_x = 0,\ \sum F_y = 0,\ \sum F_z = 0$$
$$\sum M_x = 0, \sum M_y = 0, \sum M_z = 0$$

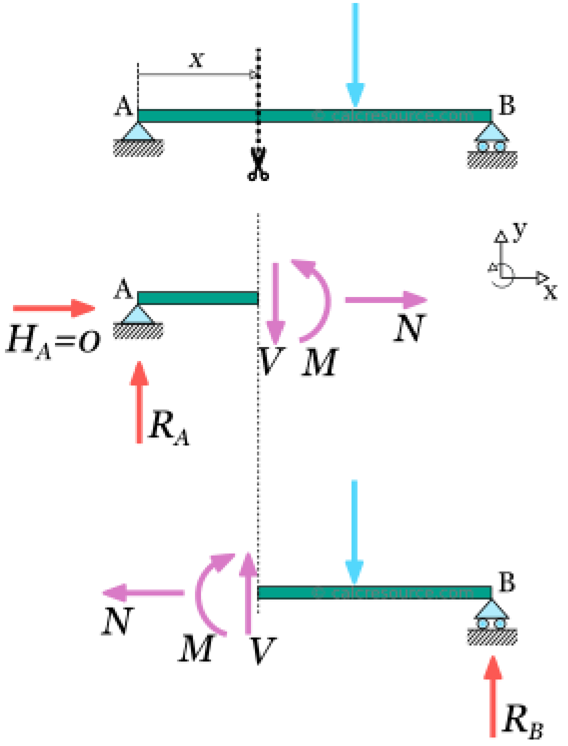
- ***Best way to account for these forces is to draw the body's free-body diagram (FBD).***

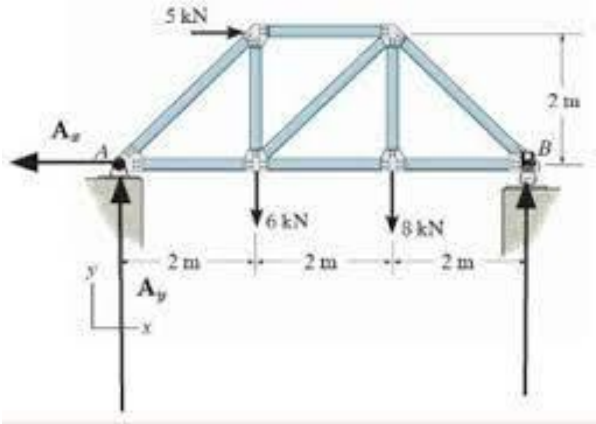
FBD (Examples)







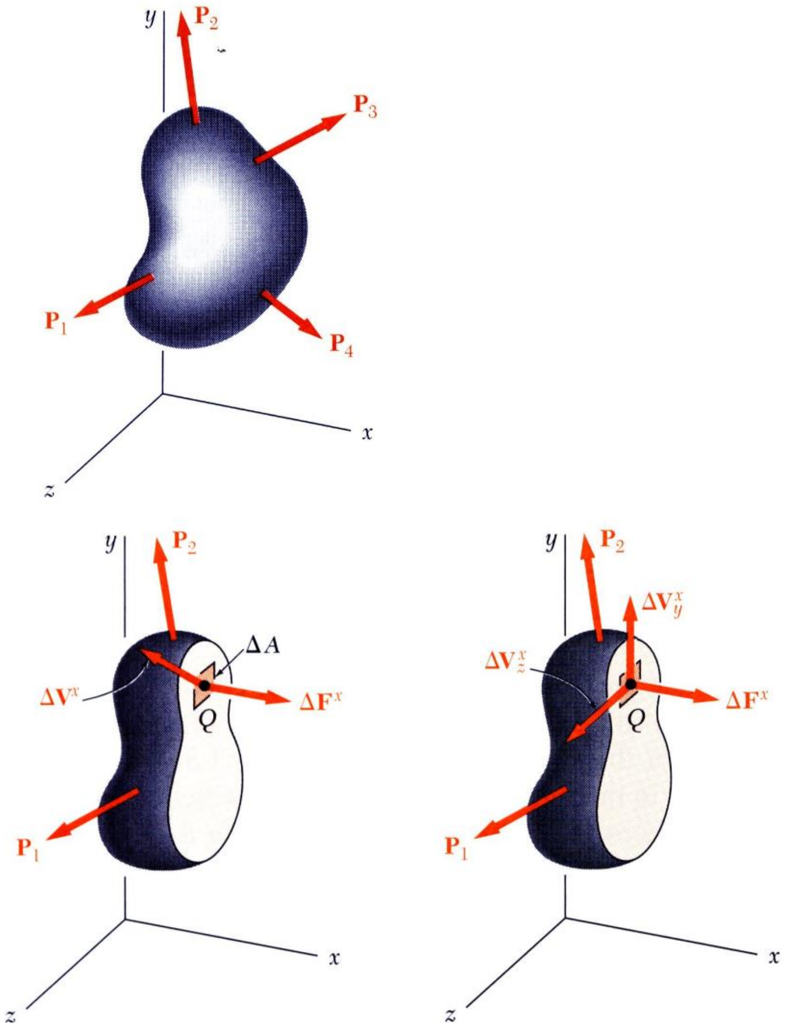




Assumptions

- Material is continuous (No voids and cracks)
- Material Homogeneous and isotropic
  - Homogeneous: At any point one direction, same property
  - Isotropic: At one point in any direction, same property
  - Orthotropic: At one point in perpendicular direction properties are different
  - Anisotropic: @one point in different directions properties are different
- Superposition valid
- Self weight neglected

Stress Intensity

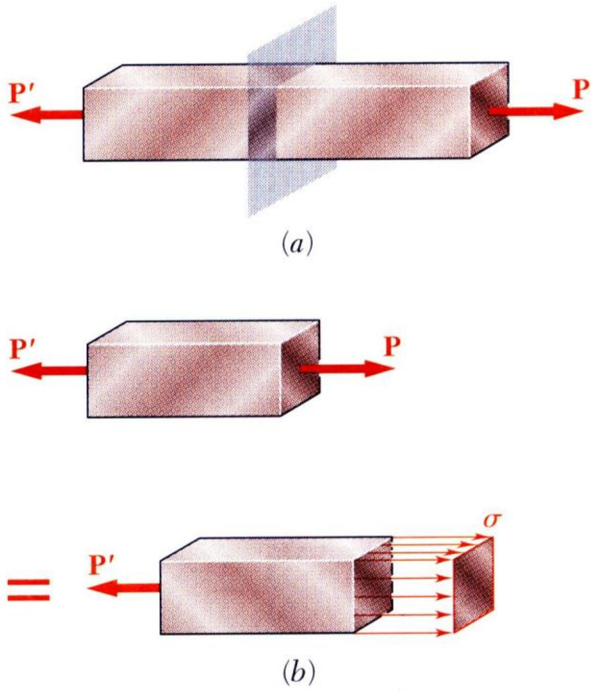


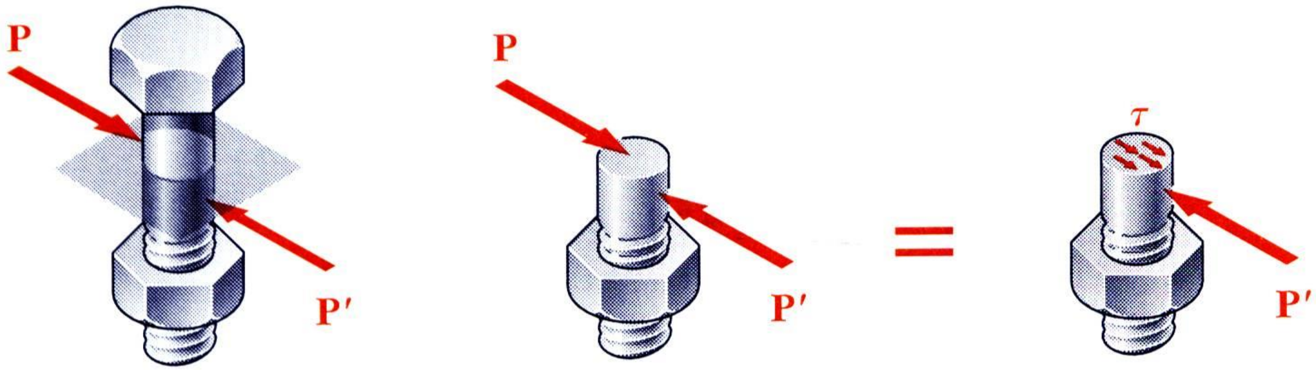
$$\sigma_x = \lim_{\Delta A \to 0} \frac{\Delta F^x}{\Delta A}$$

$$\tau_{xy} = \lim_{\Delta A \to 0} \frac{\Delta V_y^x}{\Delta A}$$

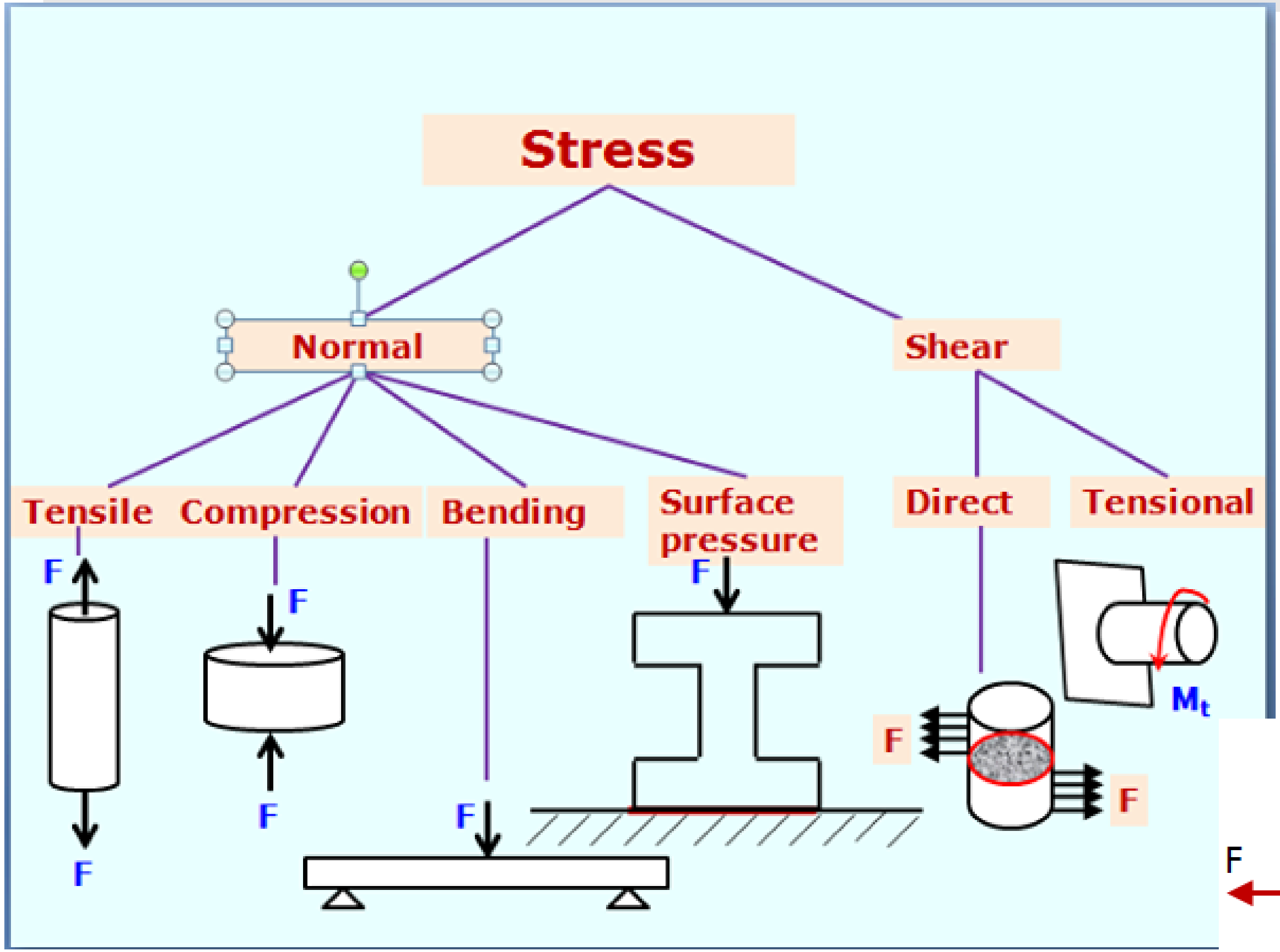
$$\tau_{xz} = \lim_{\Delta A \to 0} \frac{\Delta V_z^x}{\Delta A}$$

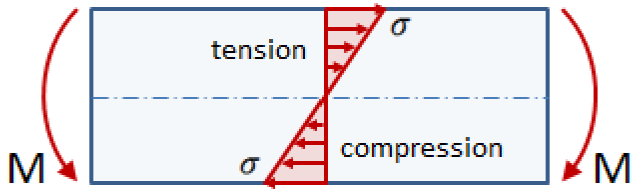
Stress Intensity

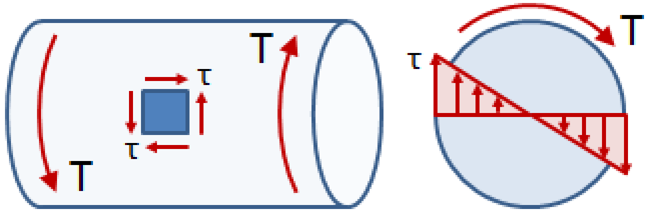


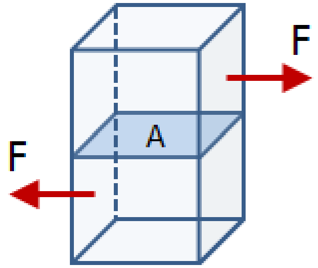


Classification of Stress

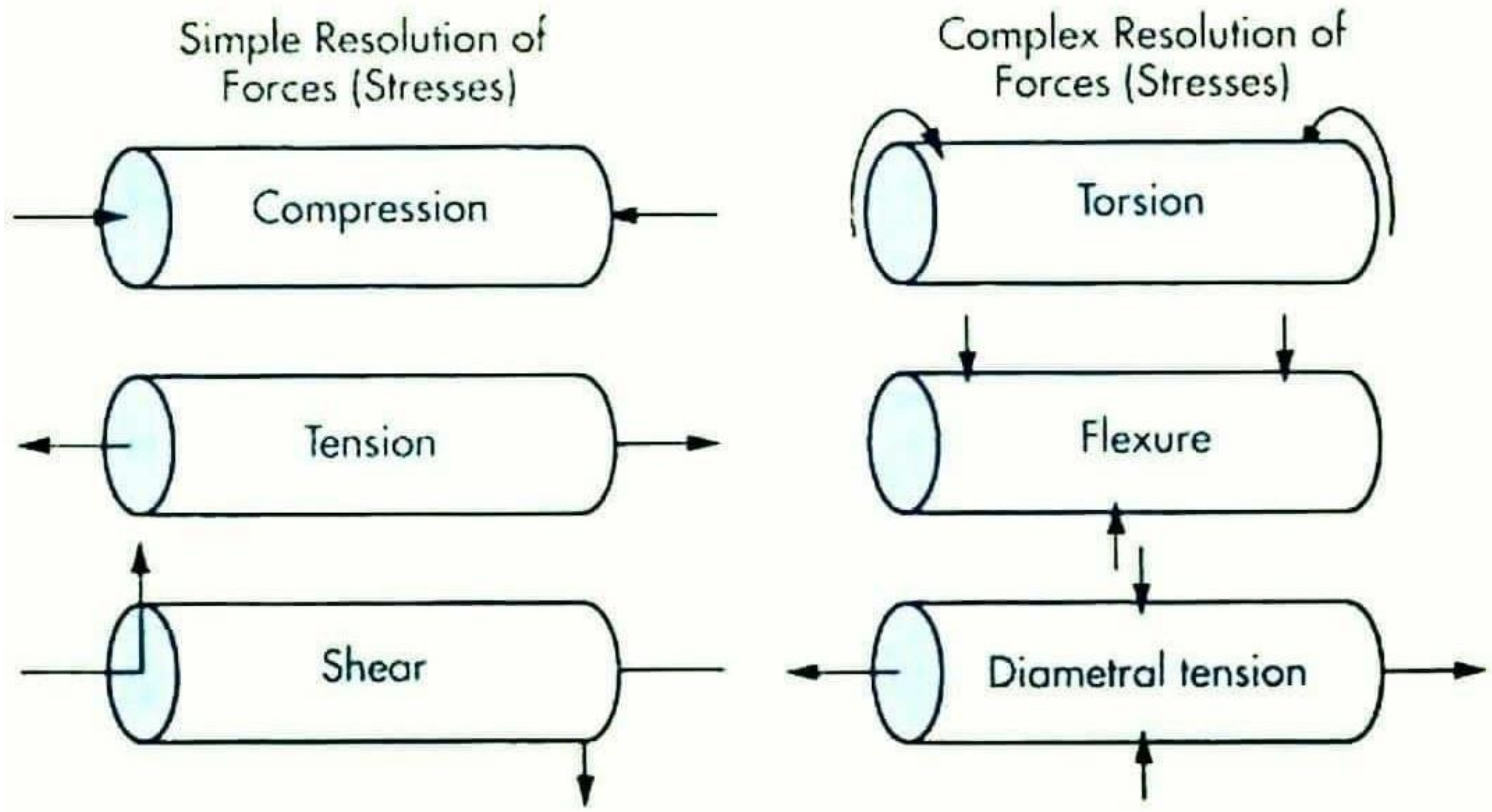




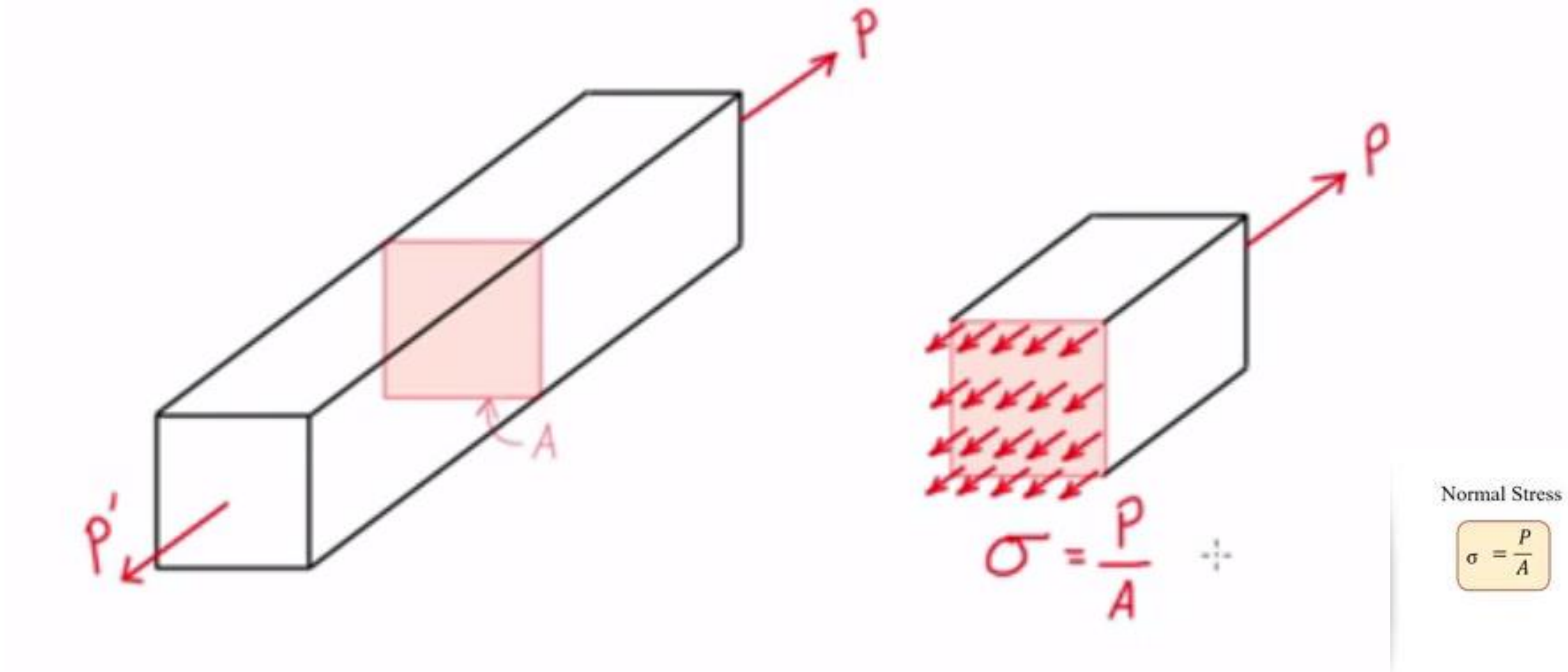


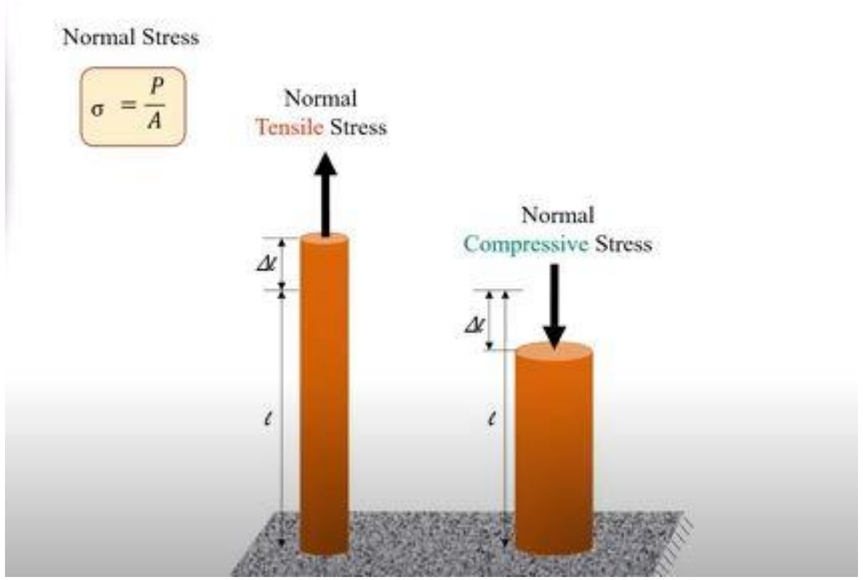


Classification of Stress



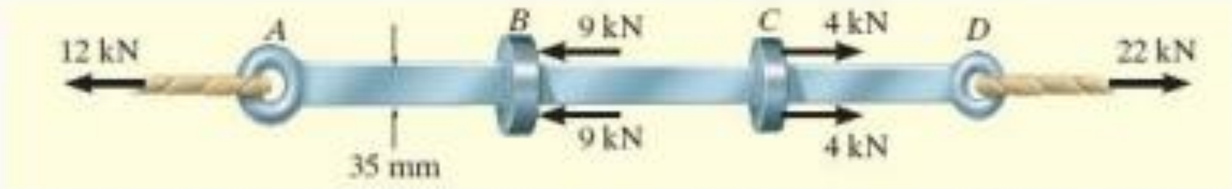
Normal Stress

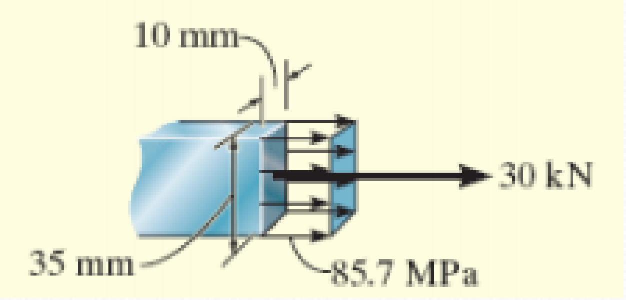


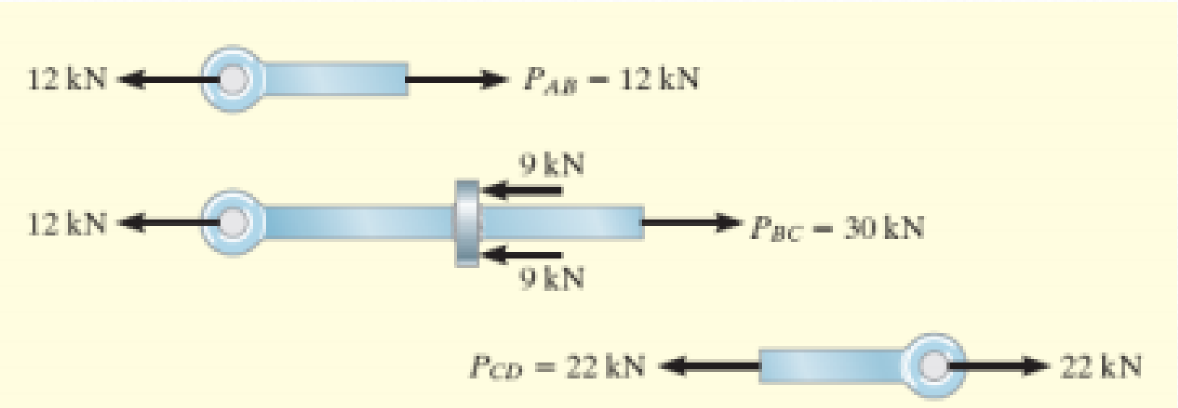


Normal Stress

The bar has a constant width of 35 mm and a thickness of 10 mm. Determine the maximum average normal stress in the bar when it is subjected to the loading shown.



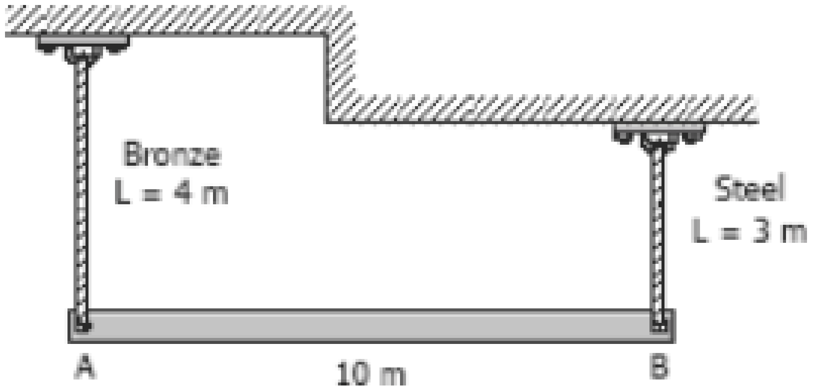




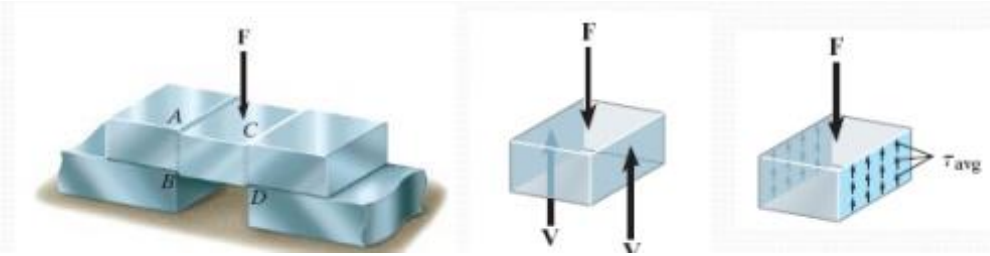
Normal Stress

A homogeneous 800 kg bar AB is supported at either end by a cable as shown in below fig. Calculate the smallest area of each cable if the stress is not exceed 90 MPa in bronze and 120 MPa in steel

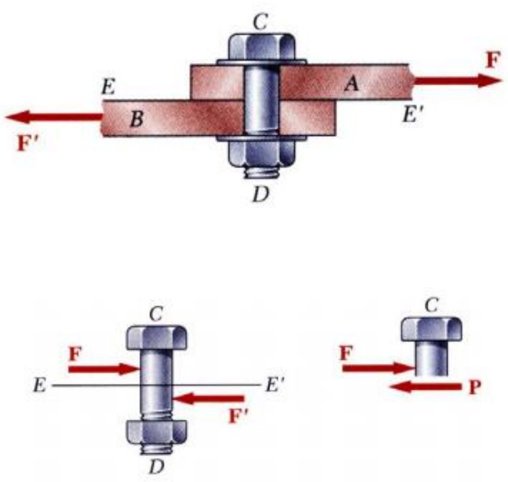
Figure P-105



Shear Stress

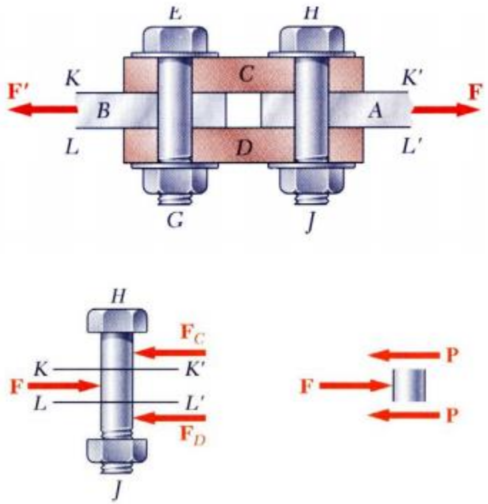


Single Shear



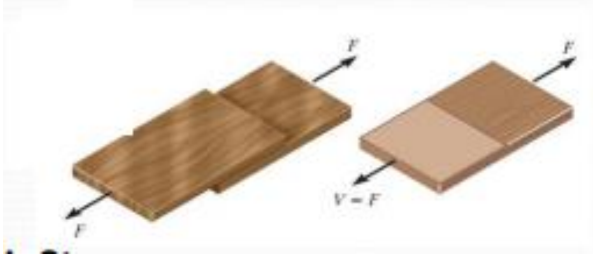
$$\tau_{ave} = \frac{P}{A} = \frac{F}{A}$$

Double Shear

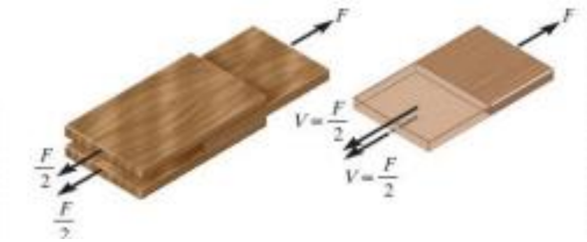


$$\tau_{ave} = \frac{P}{A} = \frac{F}{2A}$$

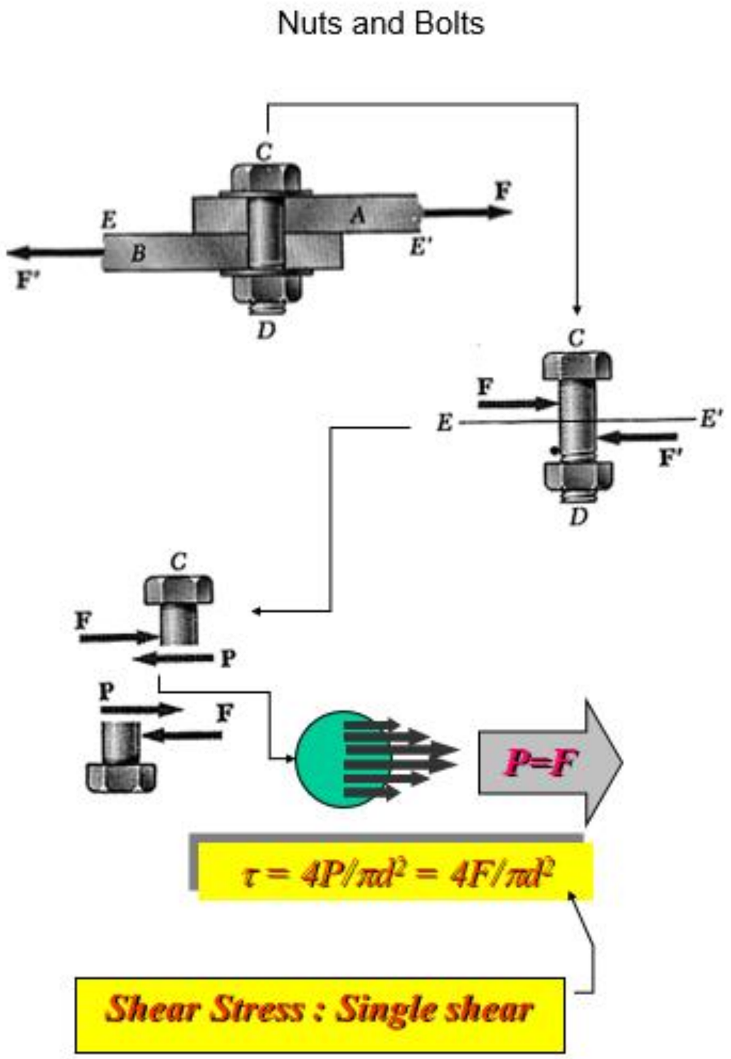
a) Single Shear

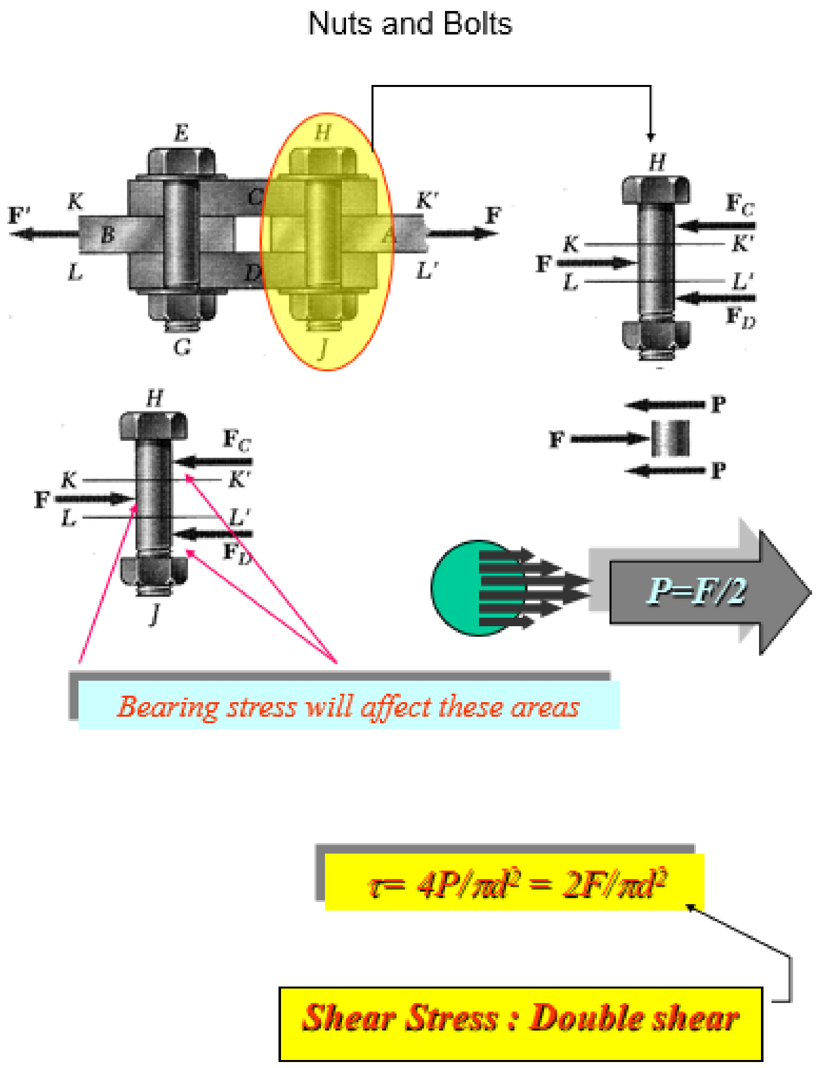


b) Double Shear



Shear Stress





Shear Stress

1. If load is P=400 kN, 20 mm diameter of bolt is used in the clevis shown in fig. Find the shear stress?
2. What is maximum diameter of the bolt can be used if the load P=400 kN & Shear strength of bolt is 300 MPa?

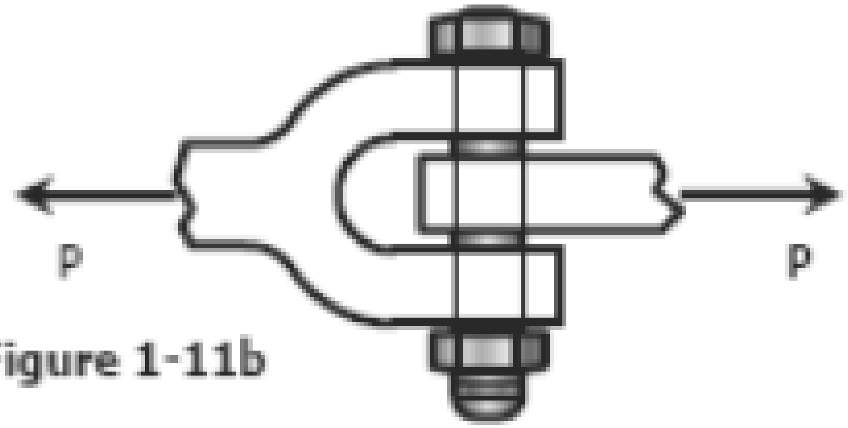
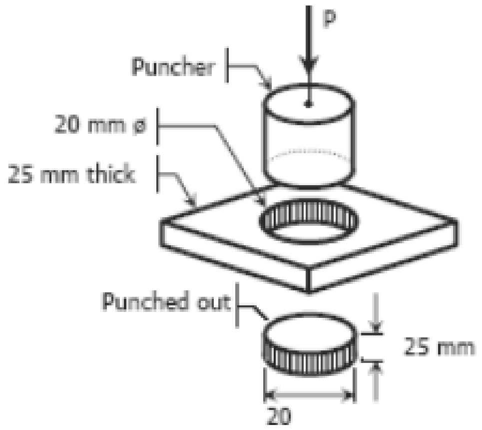
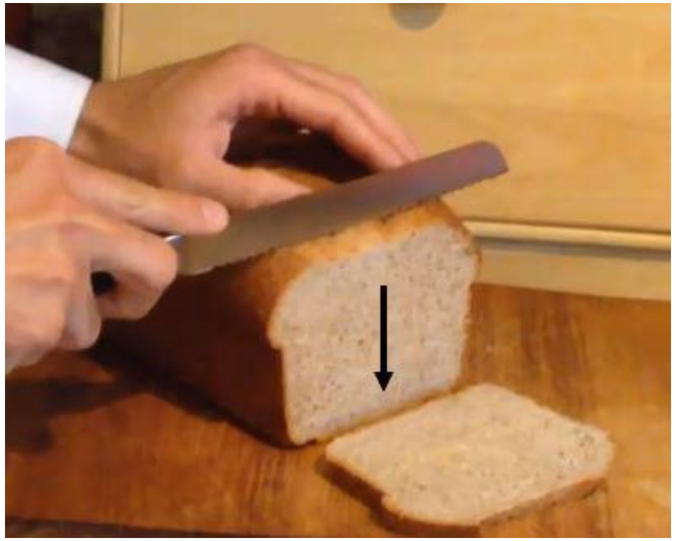


Figure 1-11b

Shear Stress

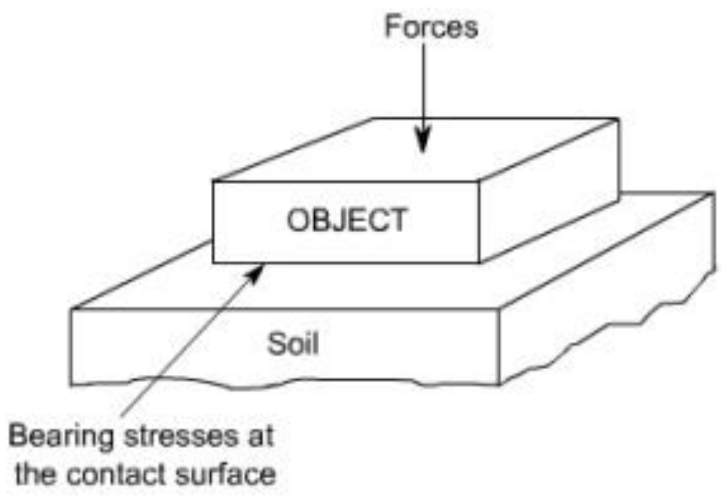
What is the force required to punch 20 mm diameter hole in a plate of thickness 25 mm? Shear strength of plate is 350 MPa

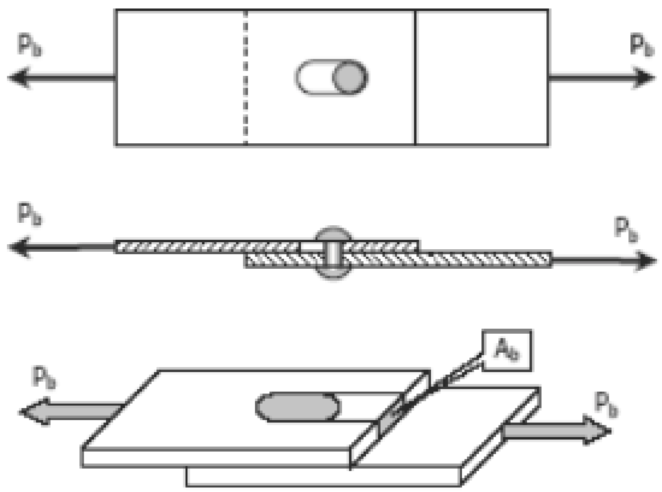




Bearing Stress

Bearing Stress: When one object presses against another, it is referred to a bearing stress (They are in fact the compressive stresses).

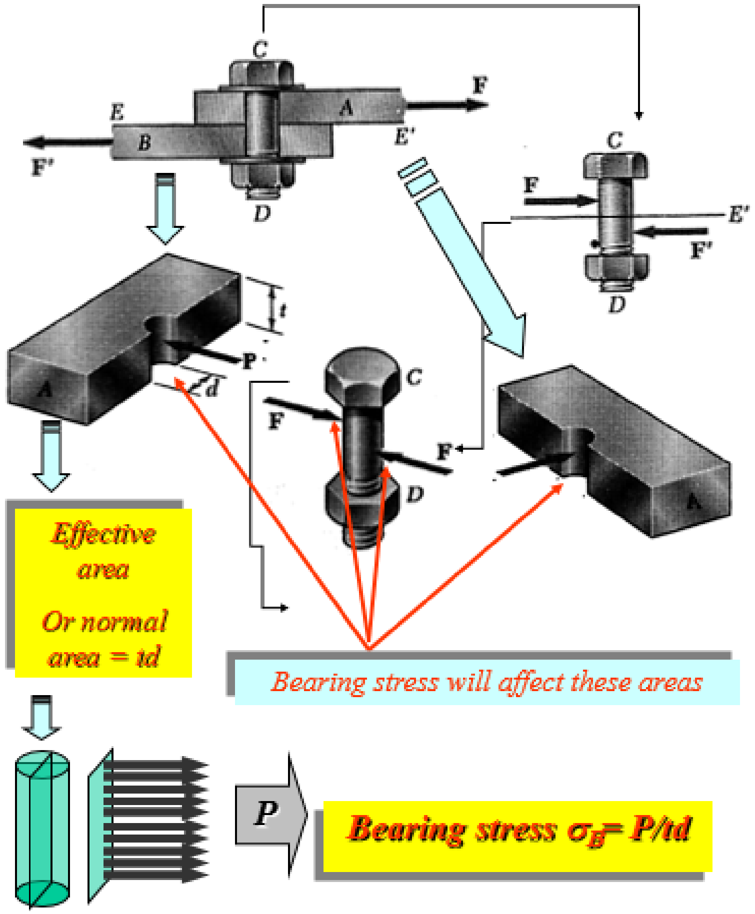


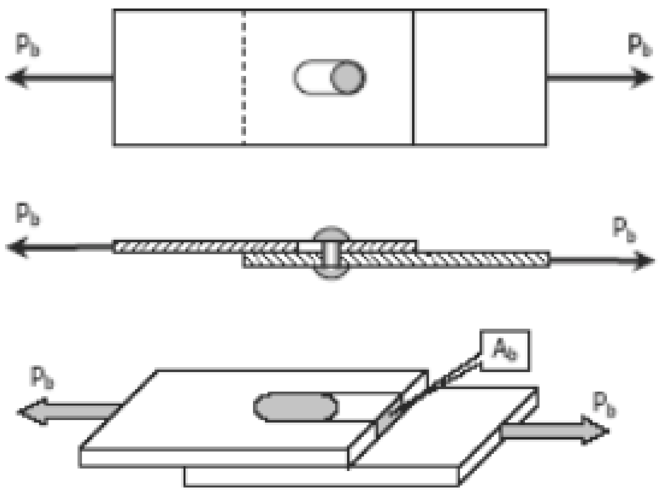


$$\sigma_b = \frac{P_b}{A_b}$$

Bearing Stress

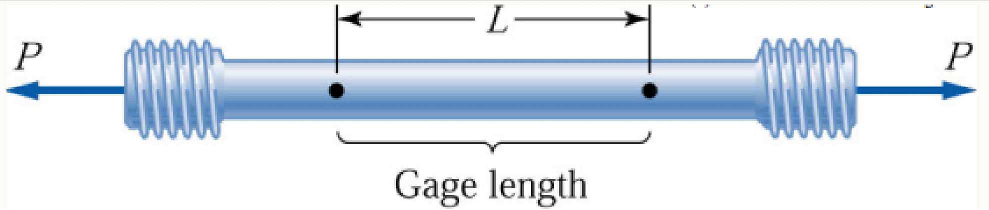
Nuts and Bolts

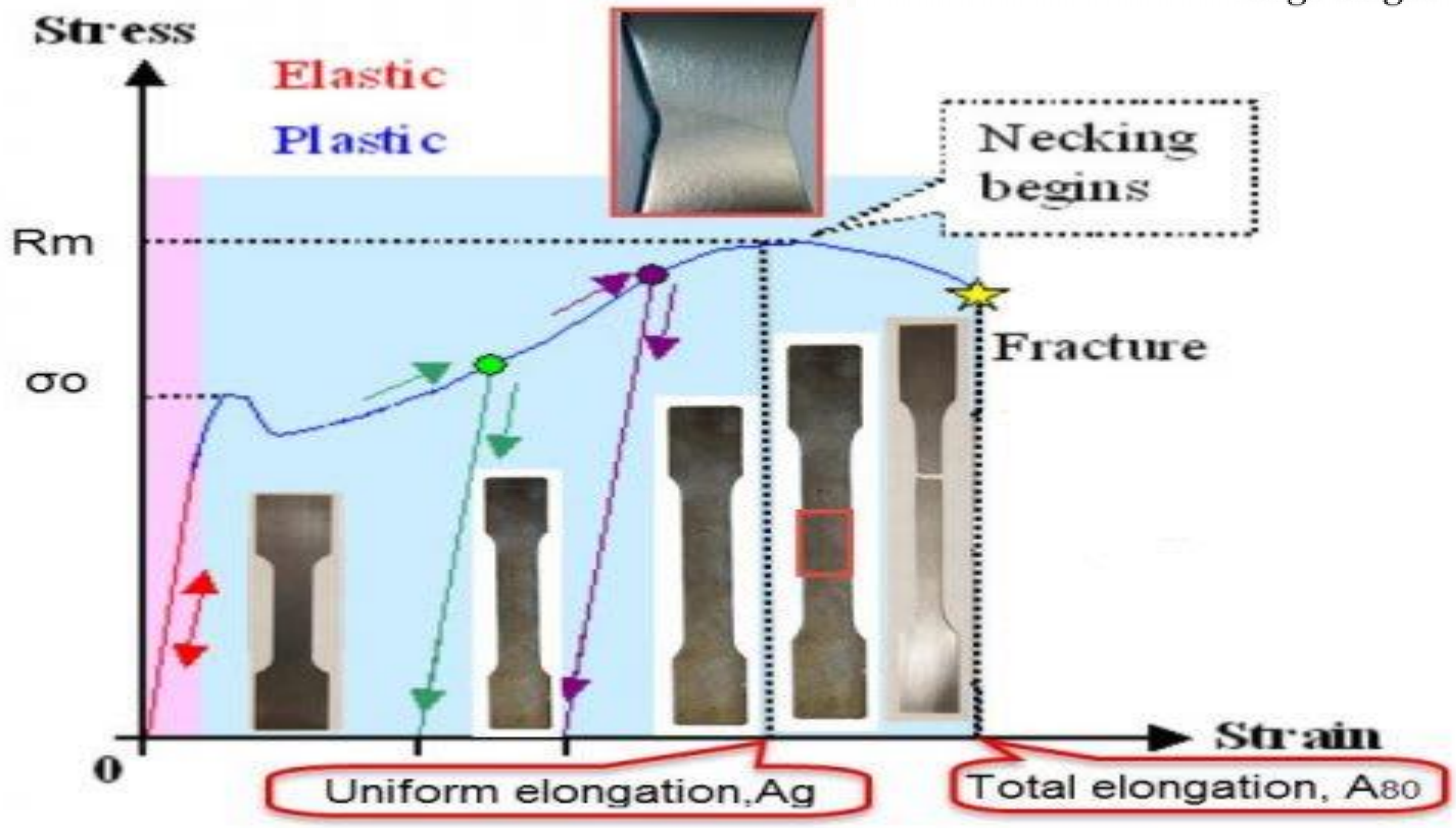


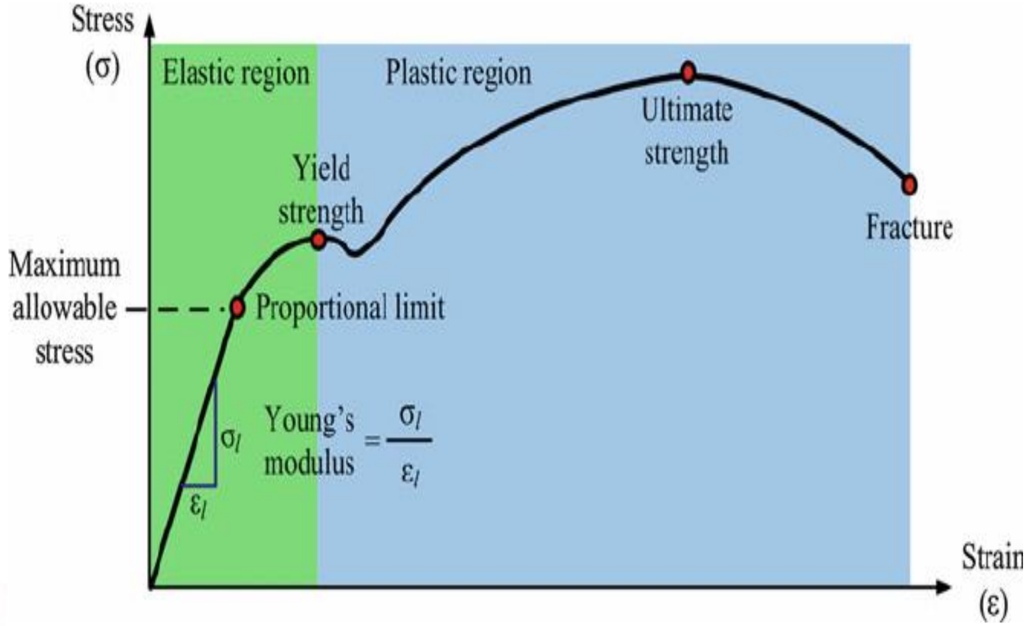


$$\sigma_b = \frac{P_b}{A_b}$$

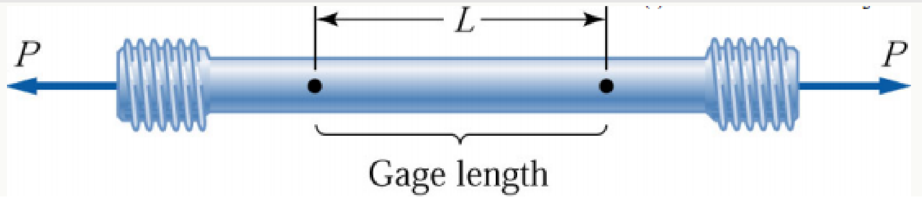
Material Characteristics

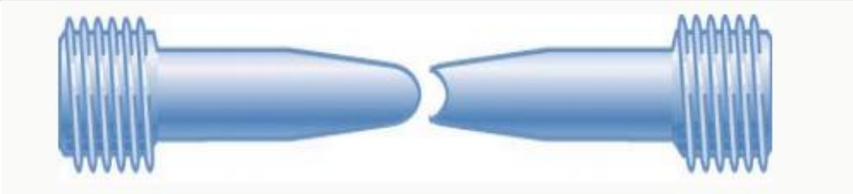


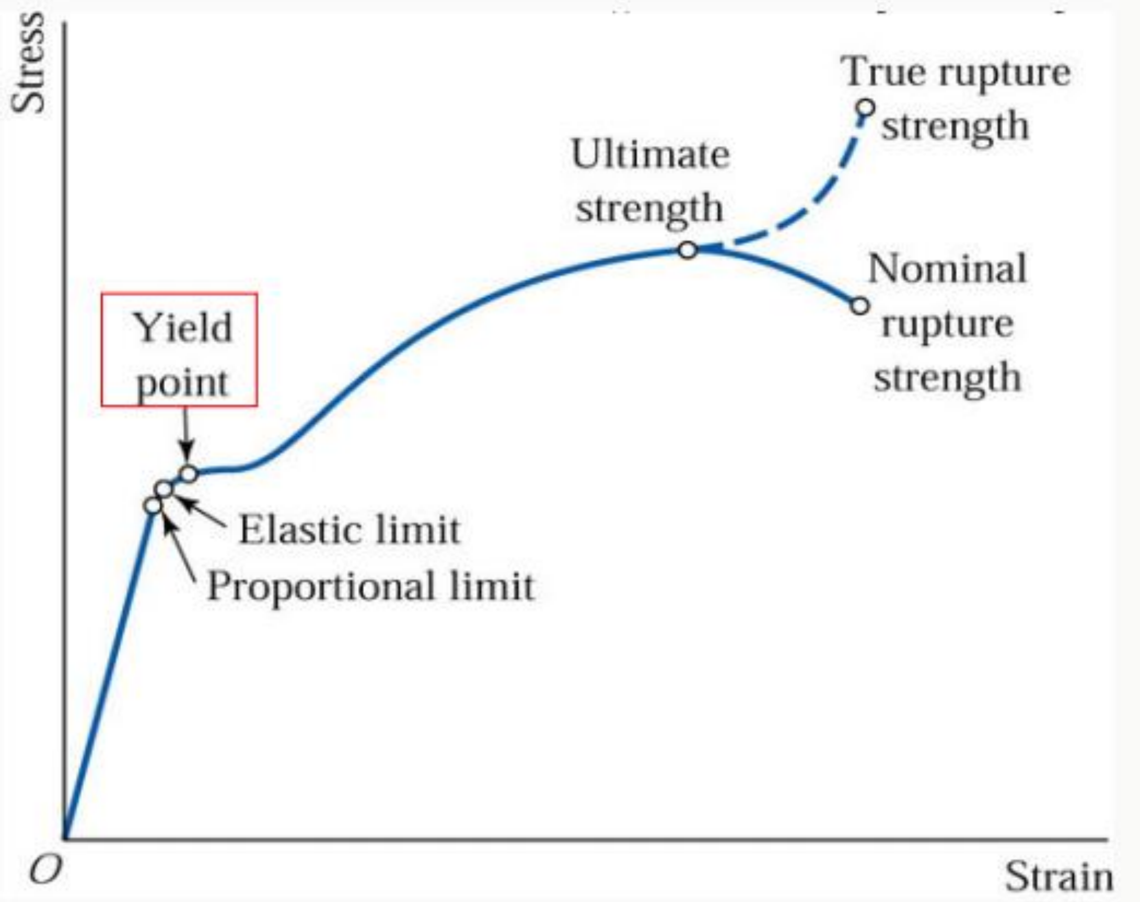


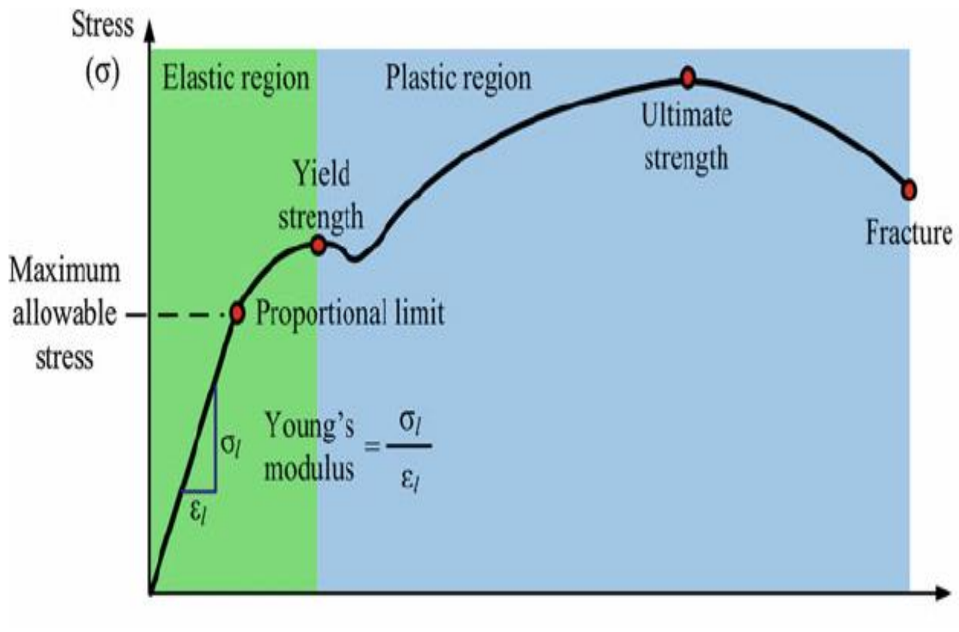


Material Characteristics









Material Characteristics (Hook’s Law)

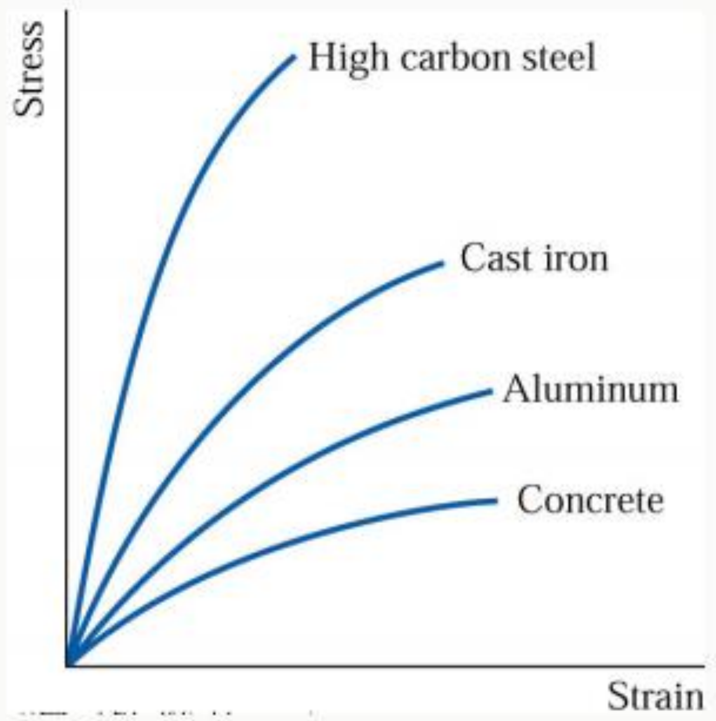


Figure 2.4 Stress-strain diagrams for various materials that fail without significant yielding.

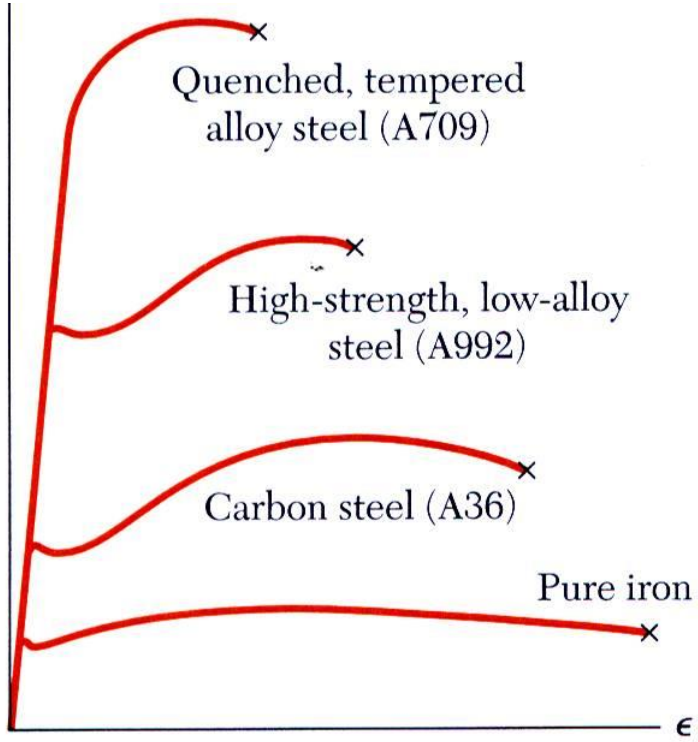
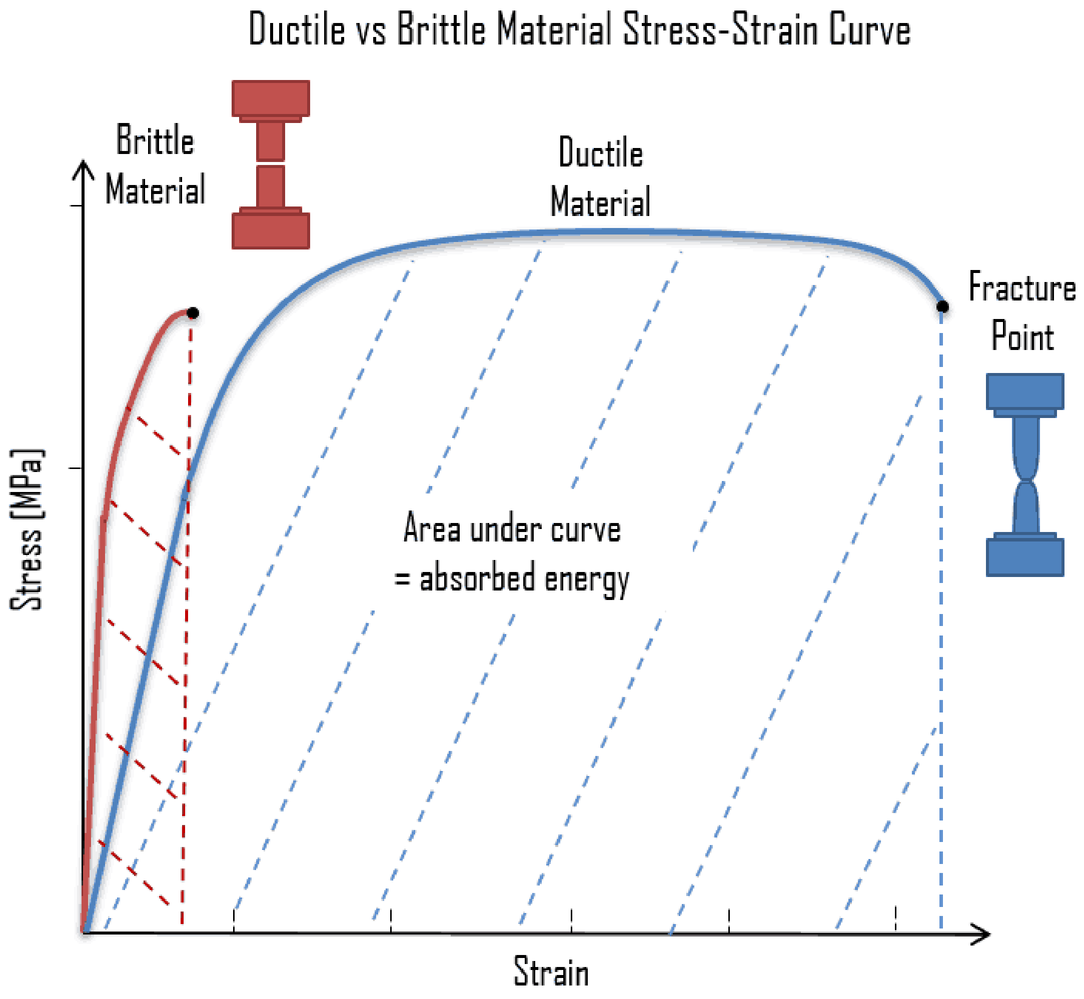
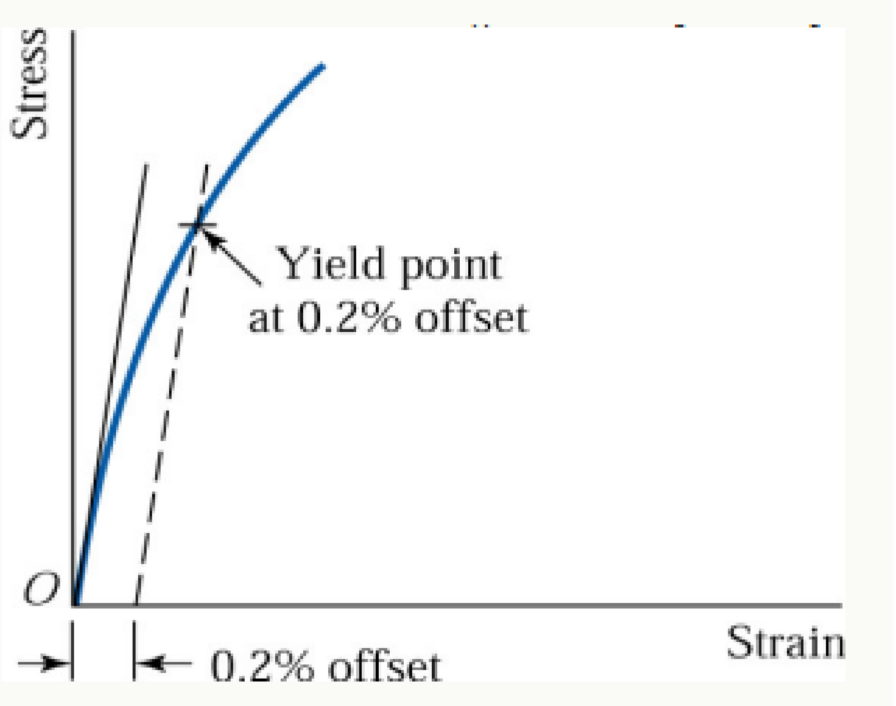


Fig. 2.16 Stress-strain diagrams for iron and different grades of steel.

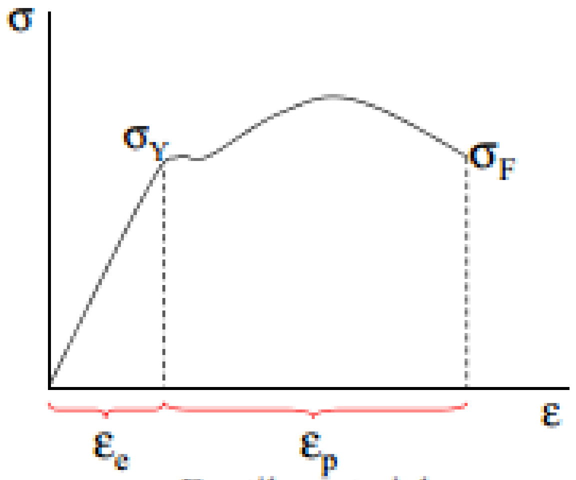
Materials- Stress vs Strain Curve



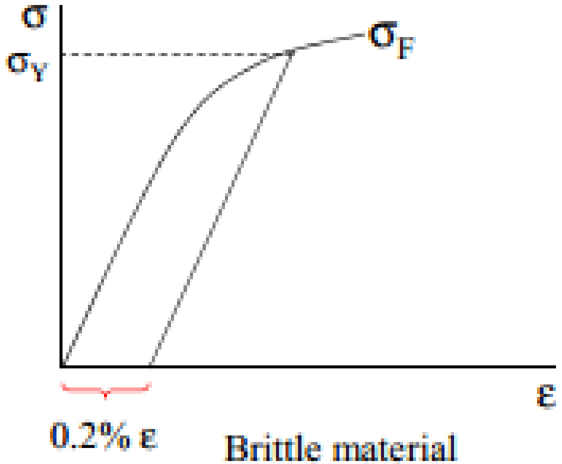


Materials- Factor of Safety (FOS)

- Ductile and brittle materials



Ductile material



Brittle material

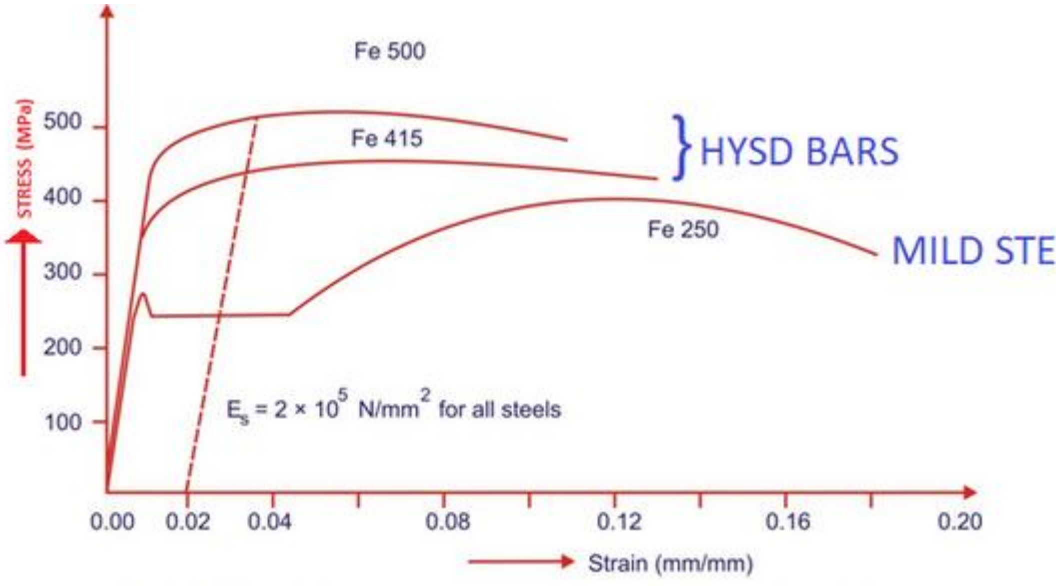


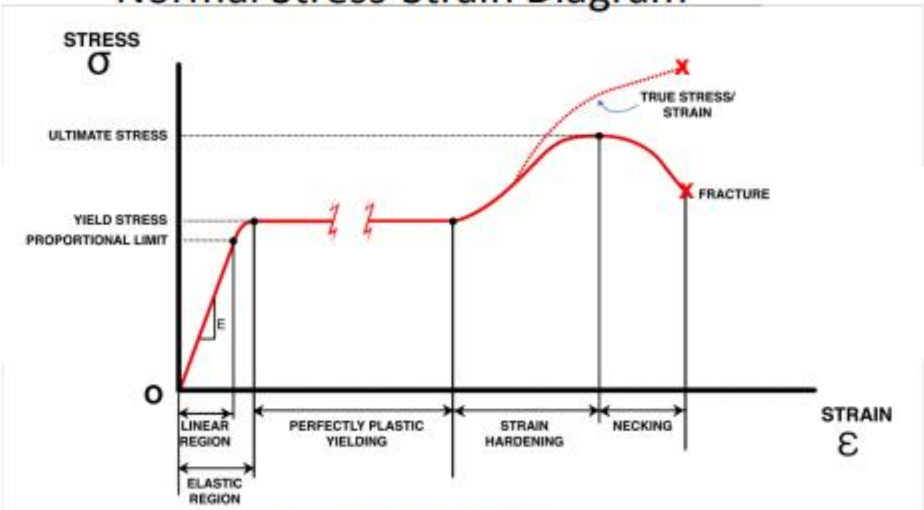
Fig. 1.1 Typical stress strain curves for various types of steel.

Well – defined yield point in ductile materials – FS on yielding

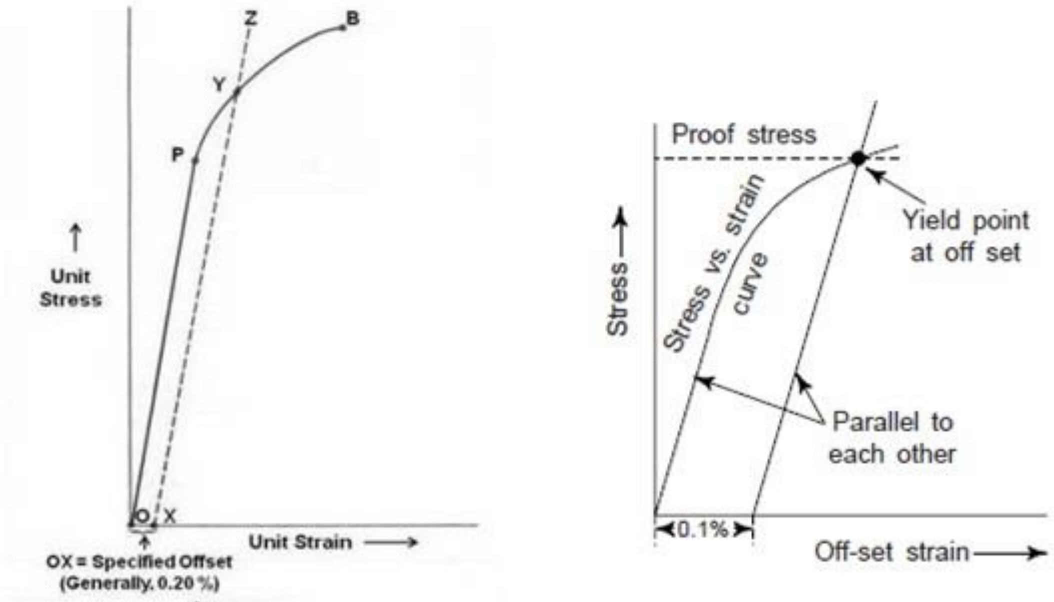
No yield point in brittle materials sudden failure – FS on failure load

Materials-

Normal Stress-Strain Diagram

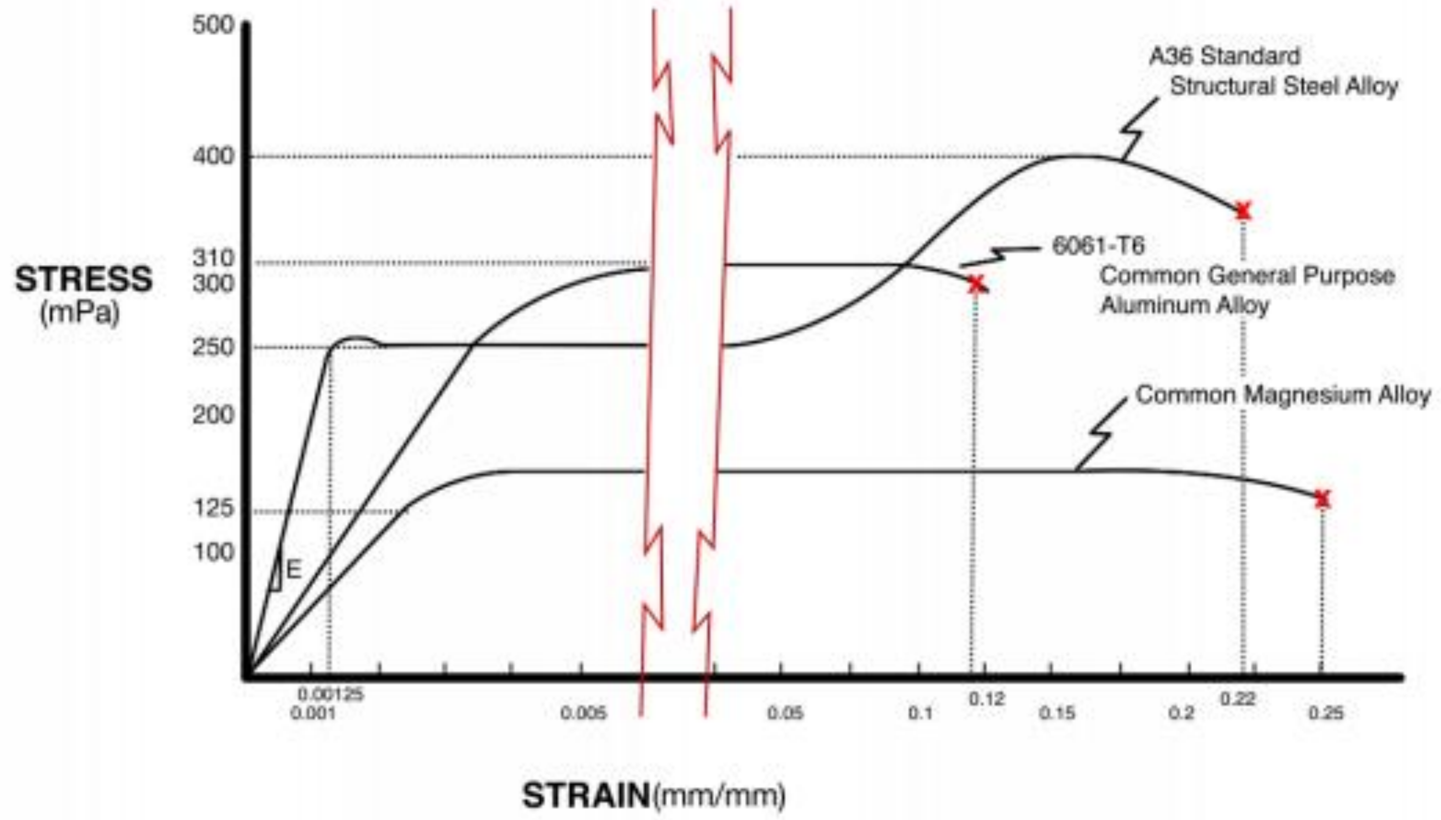


Strength:	Capacity for high stress/ultimate stress
Toughness:	Capacity for energy absorption (area under stress-strain curve)
Resilience:	Capacity for deforming elastically (area under elastic region)
Ductility:	Capacity for high deformation/strain
Brittleness:	Low capacity for deformation/strain



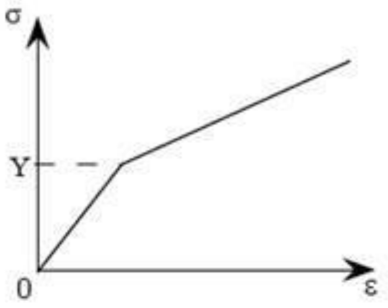
Stress vs. strain curve for a brittle material

Materials-

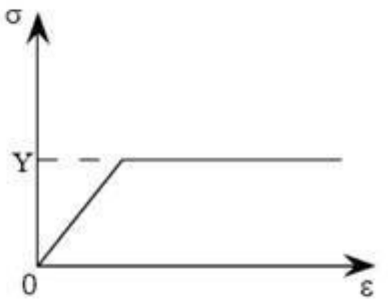


1) What is the approximate Modulus of Elasticity for A36 Steel?
2) What is the approximate Ultimate Strength of A36 Steel?
3) What is the approximate Ultimate Strength of 6061-T6 Aluminum?
4) What is the approximate Proportional Limit of the common Magnesium Allo
5) What is the approximate Yield Stress of the A36 Steel?
6) Which of these material is the strongest? Why?
   Aluminum or Magnesium
7) Which is the most ductile material? Why?
   Steel or Aluminum or Magnesium
8) Which is the most brittle material? Why?
   Steel or Aluminum or Magnesium
9) Which material is the stiffest? Why?
   Steel or Aluminum or Magnesium

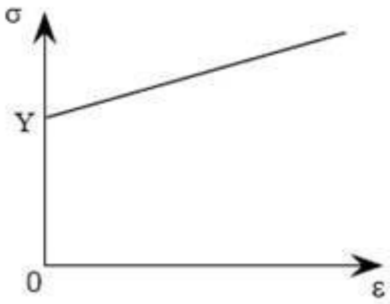
Materials-



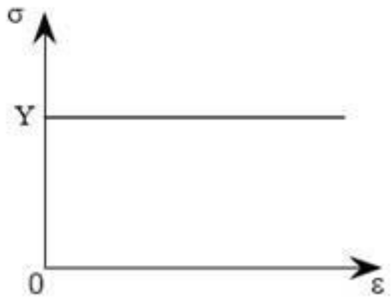
(a) Linear Elastic-Plastic



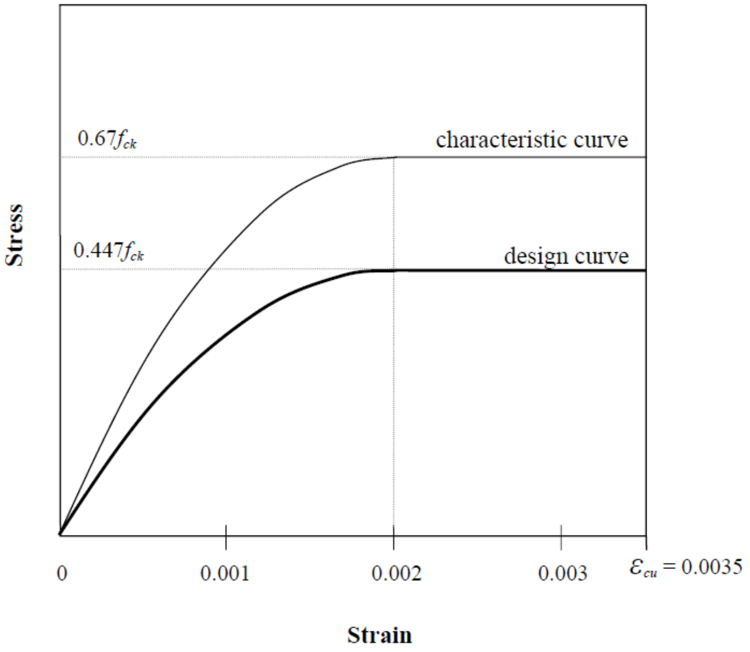
(b) Elastic/Perfectly-Plastic



(c) Rigid/Linear Hardening



(d) Rigid-Perfectly-Plastic



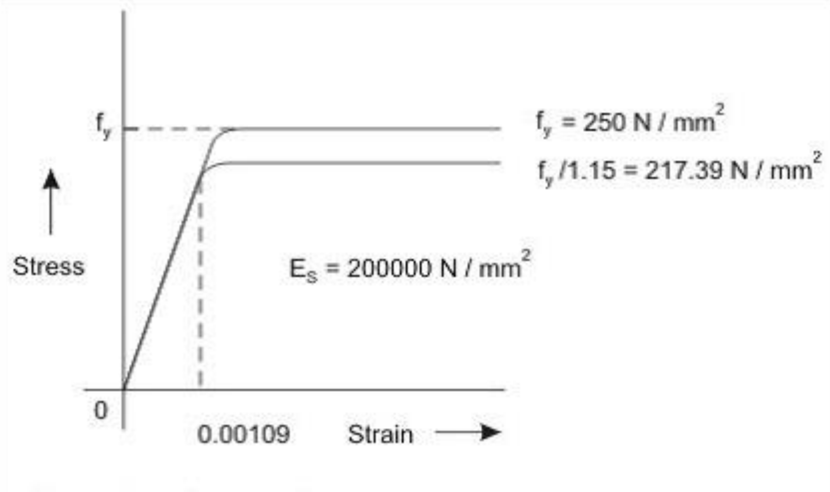


Fig. 1.2.3 : Stress - Strain curve for mild steel (idealised) (Fe 250) with definite yield point.

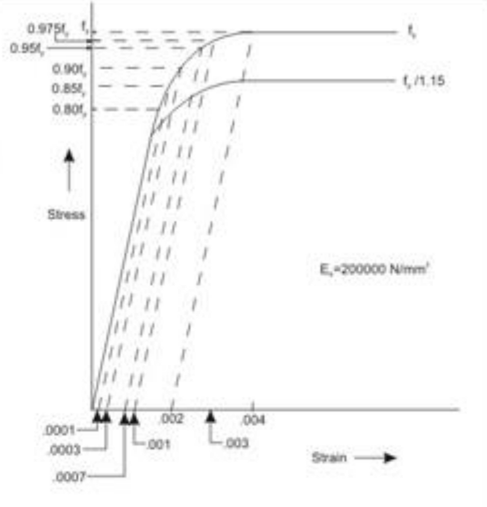
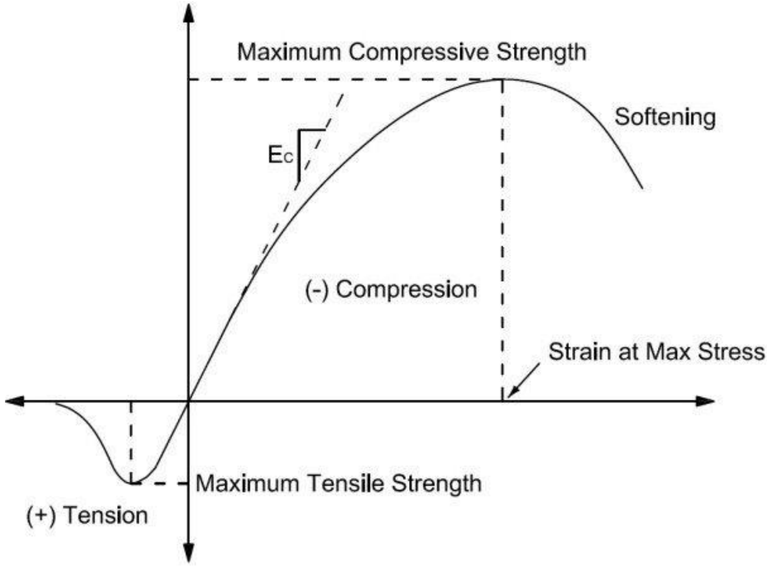
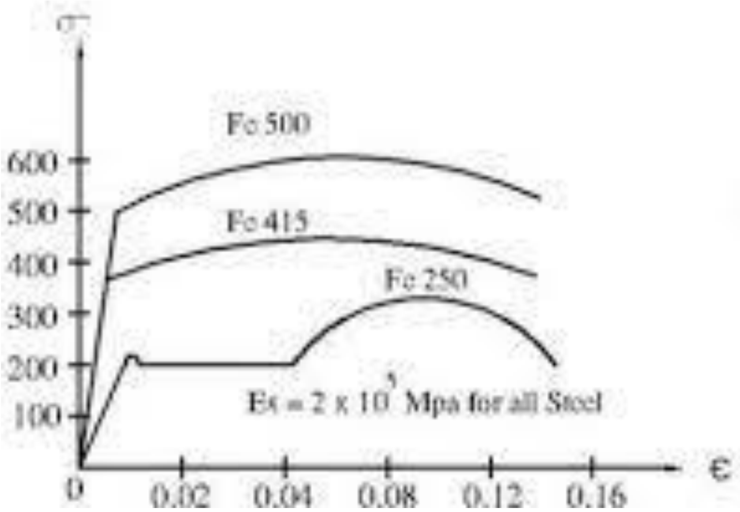
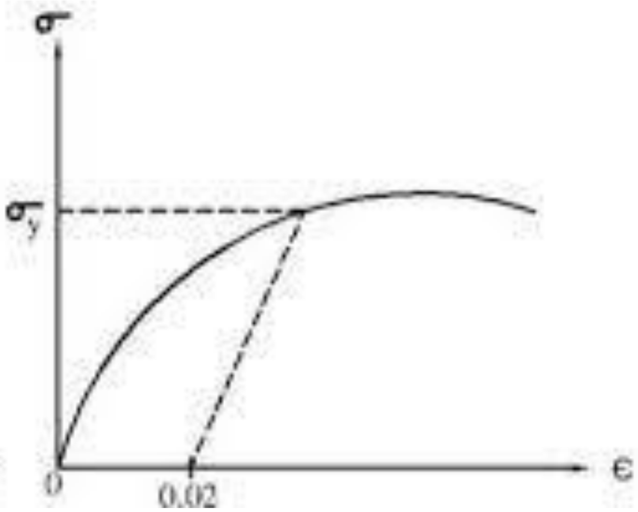


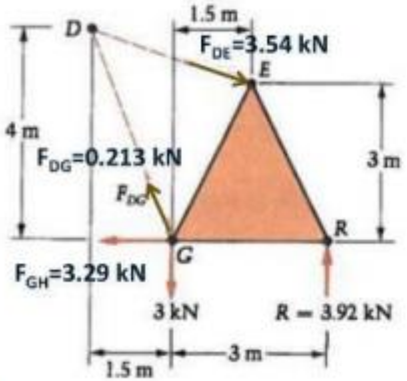
Fig. 1.2.4 : Stress-Strain Curve for cold worked deformed bar

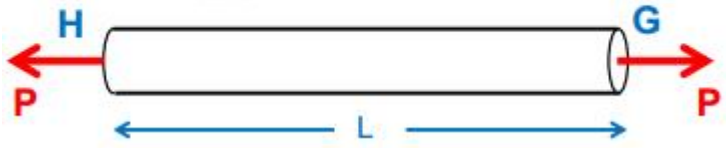




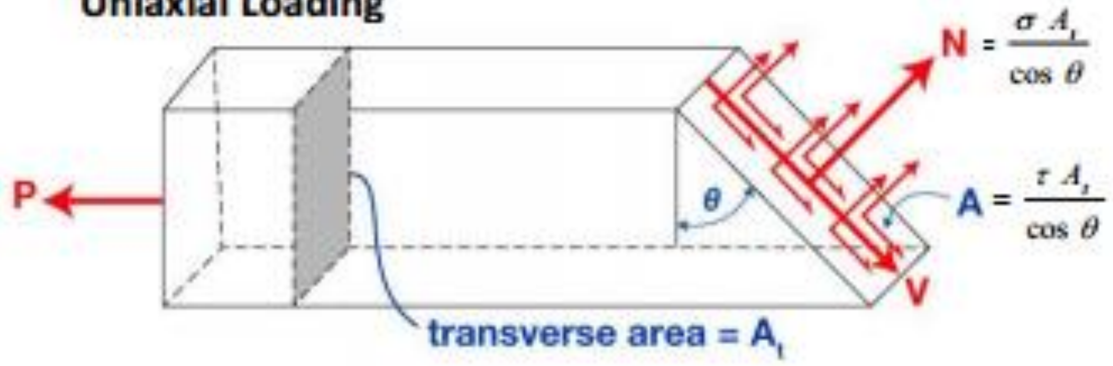


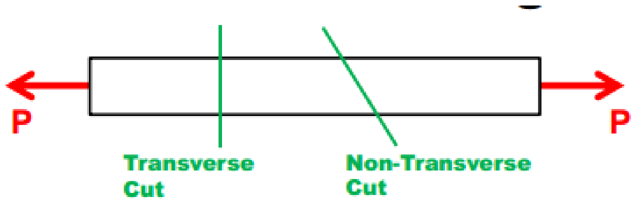
State of Stress



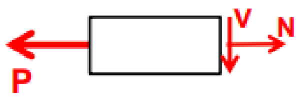


Maximum Normal and Shear Stresses on Inclined Planes for Uniaxial Loading

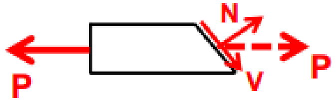




Transverse Cut

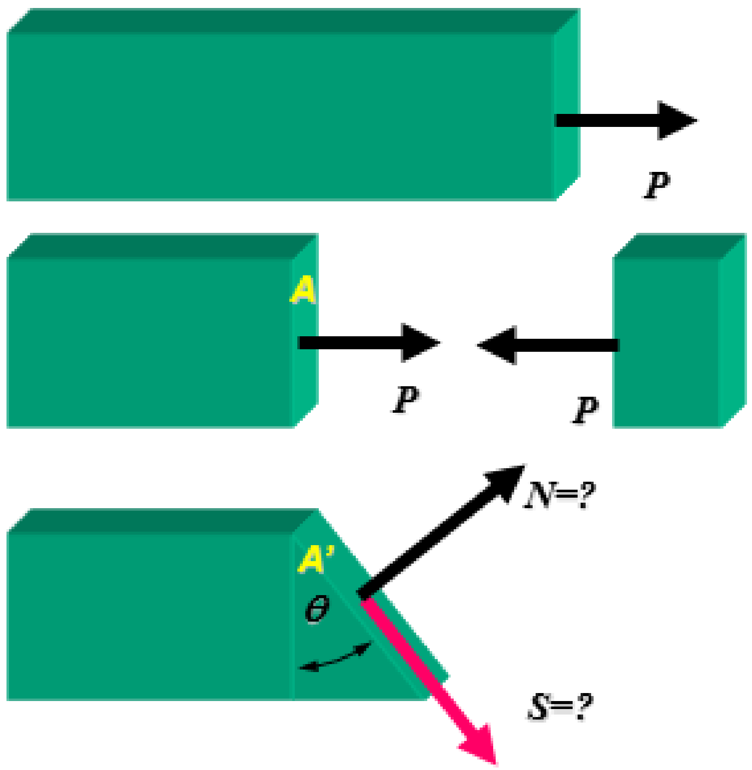


Non-Transverse Cut



State of Stress

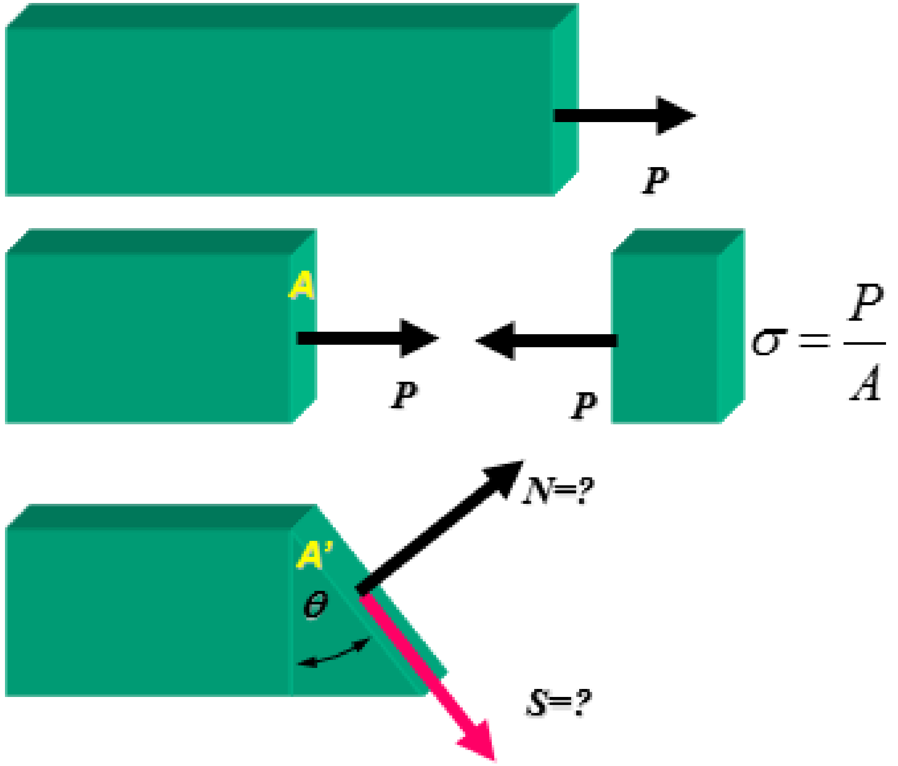
Normal stress and Shear Stress



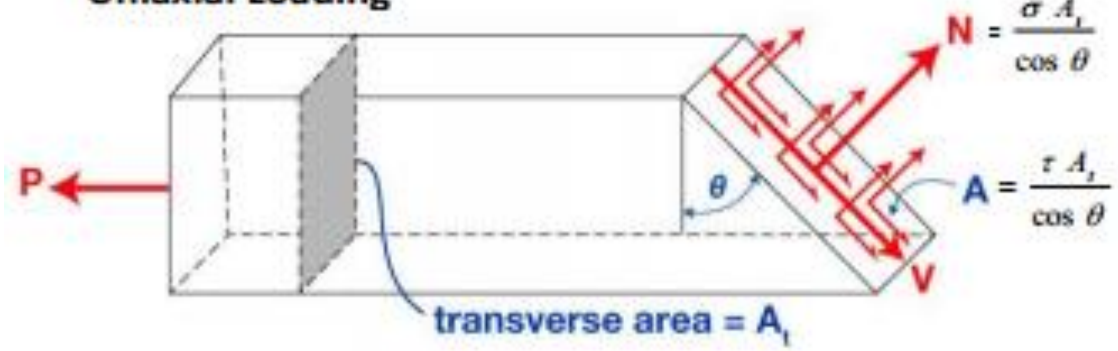
$$\sigma = \frac{P}{A}$$

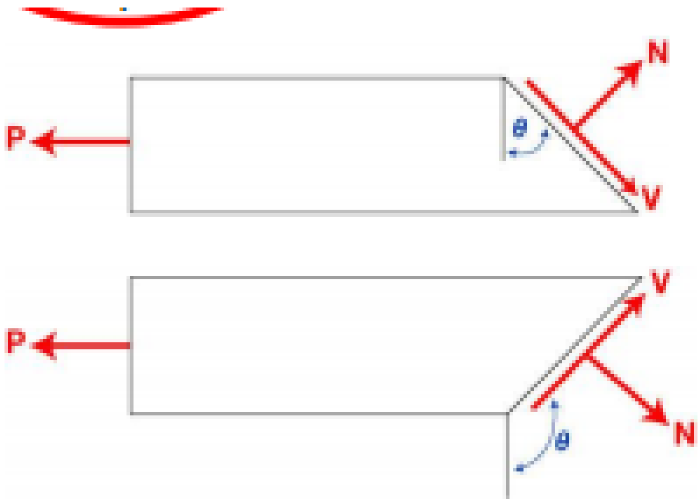
State of Stress

Normal stress and Shear Stress

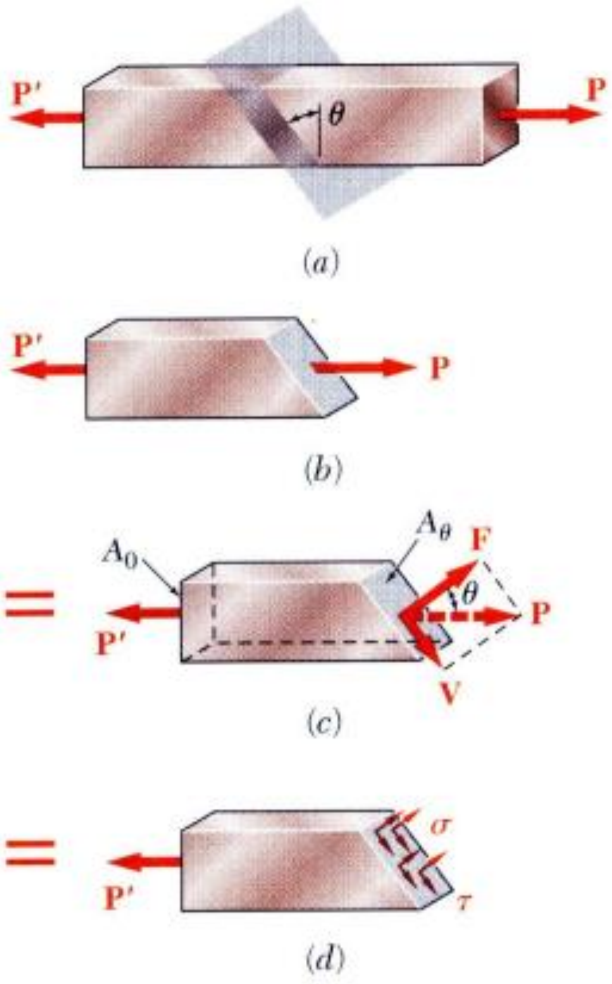


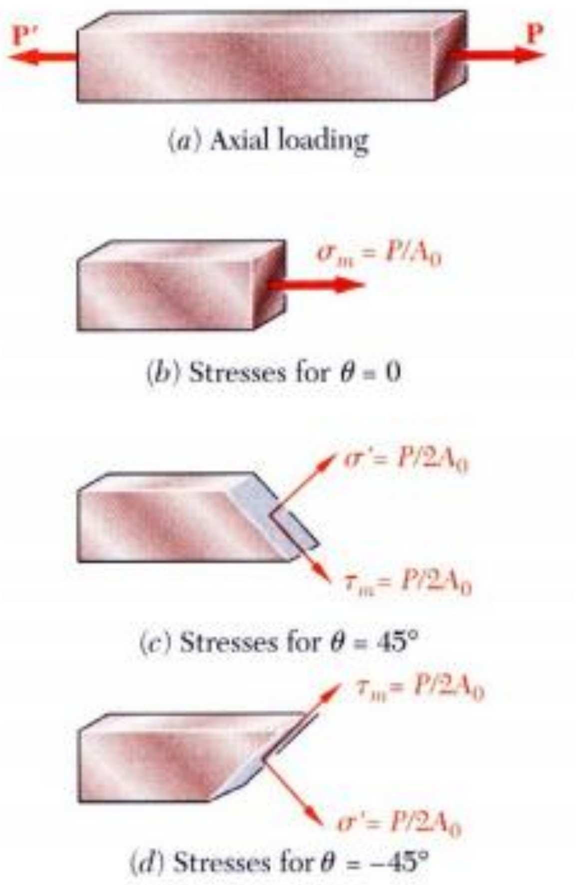
Maximum Normal and Shear Stresses on Inclined Planes for Uniaxial Loading



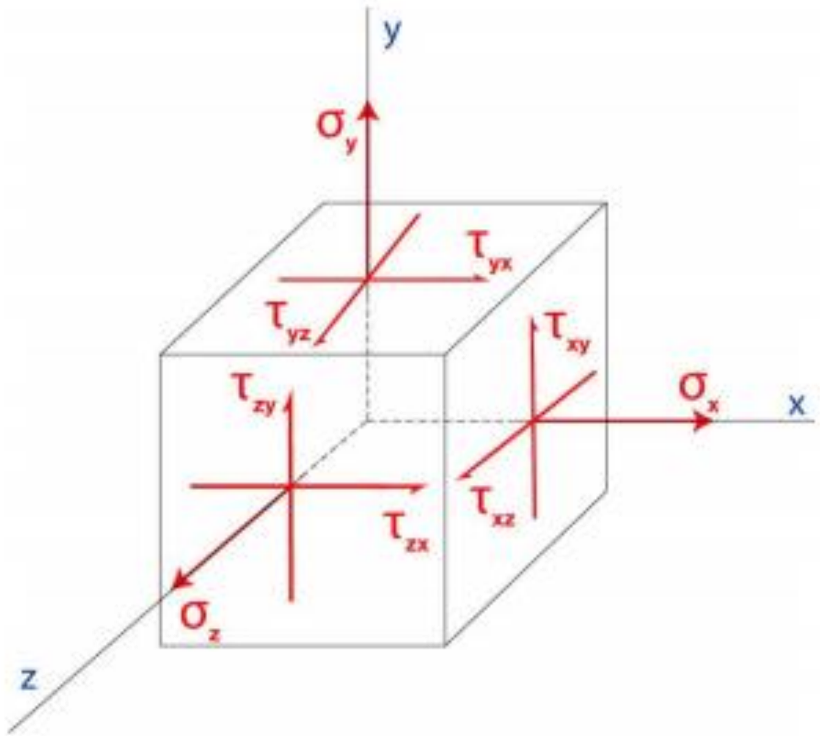


Stress on inclined plane

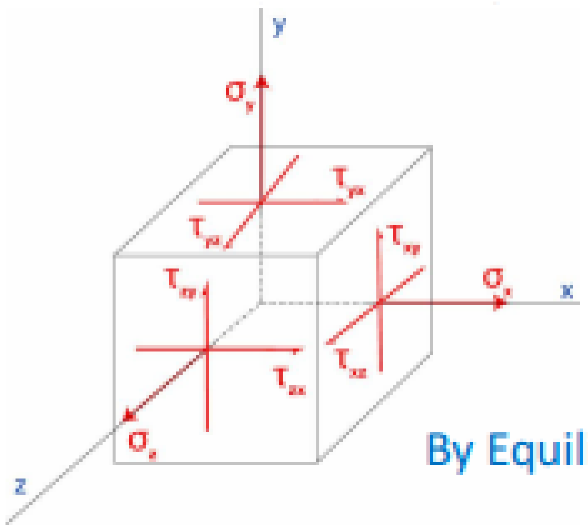




State of Stress



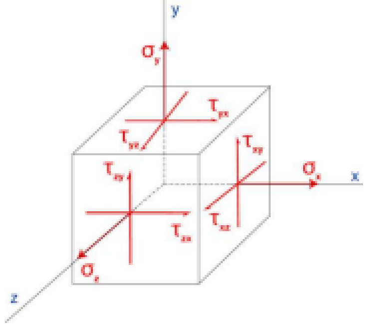
State of Stress



By Equilibrium:

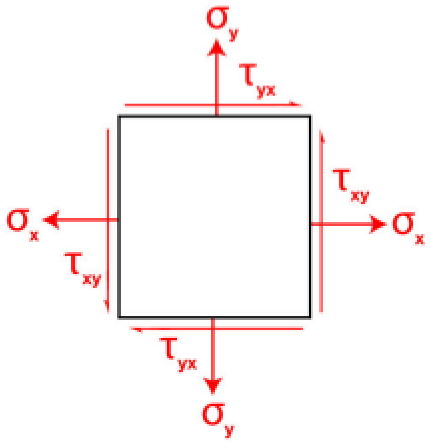
State of Stress

3D State of Stress at a Point
(shown in positive sign convention)

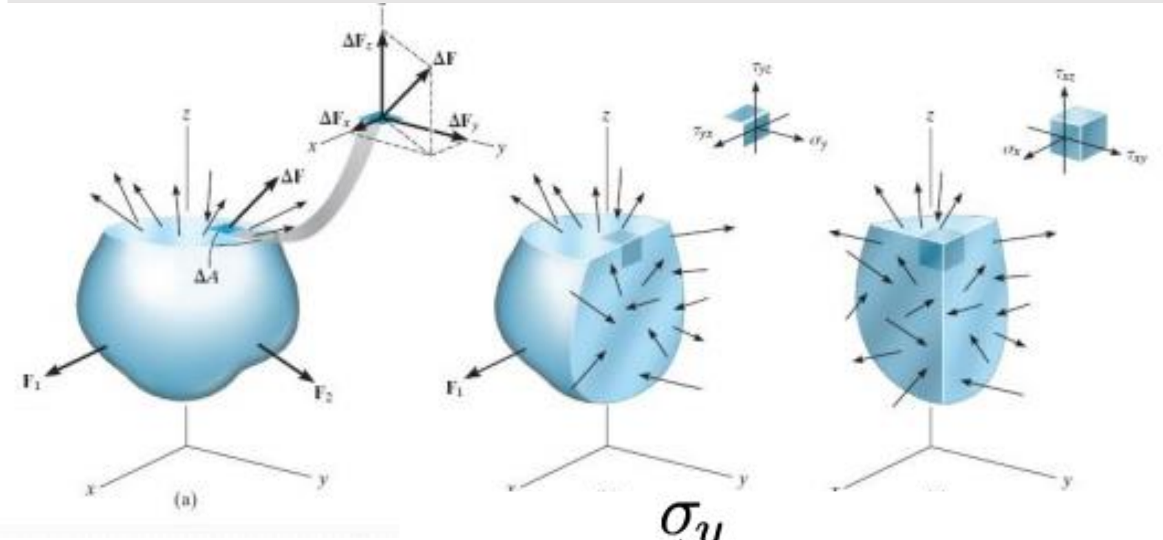


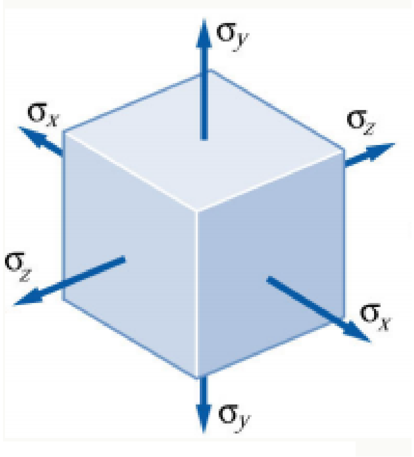
Two-Dimensional (2D) or Plane Stress
(shown in positive sign convention)

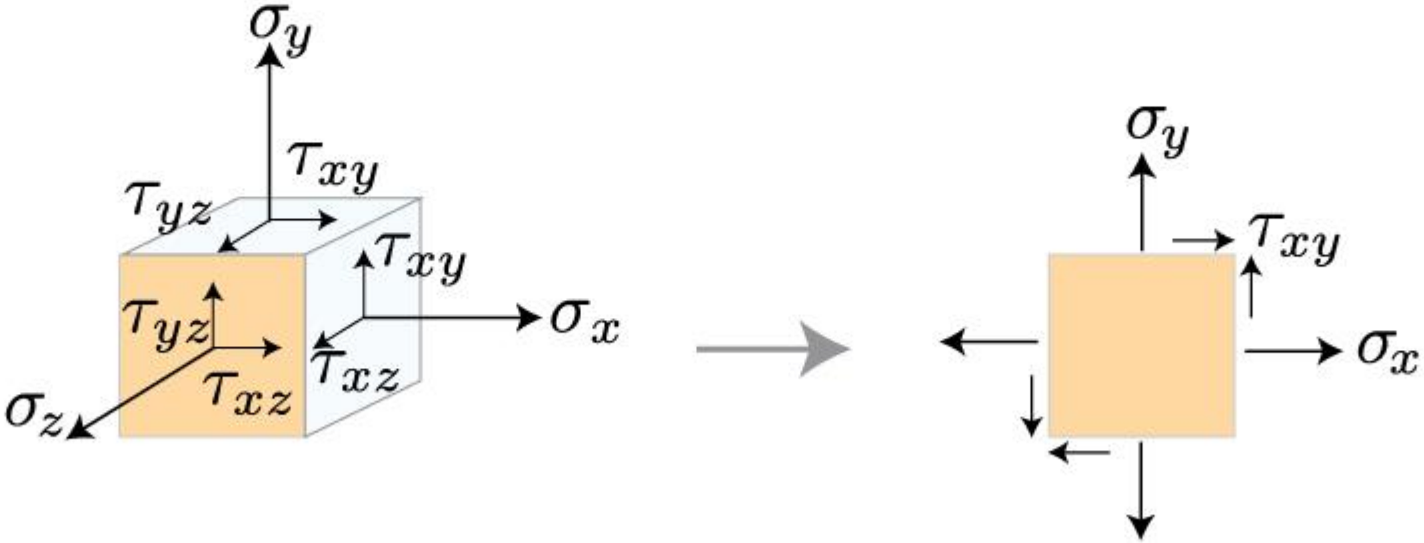
$$\sigma_z = \tau_{xz} = \tau_{zx} = \tau_{yz} = \tau_{zy} = 0$$



State of Stress



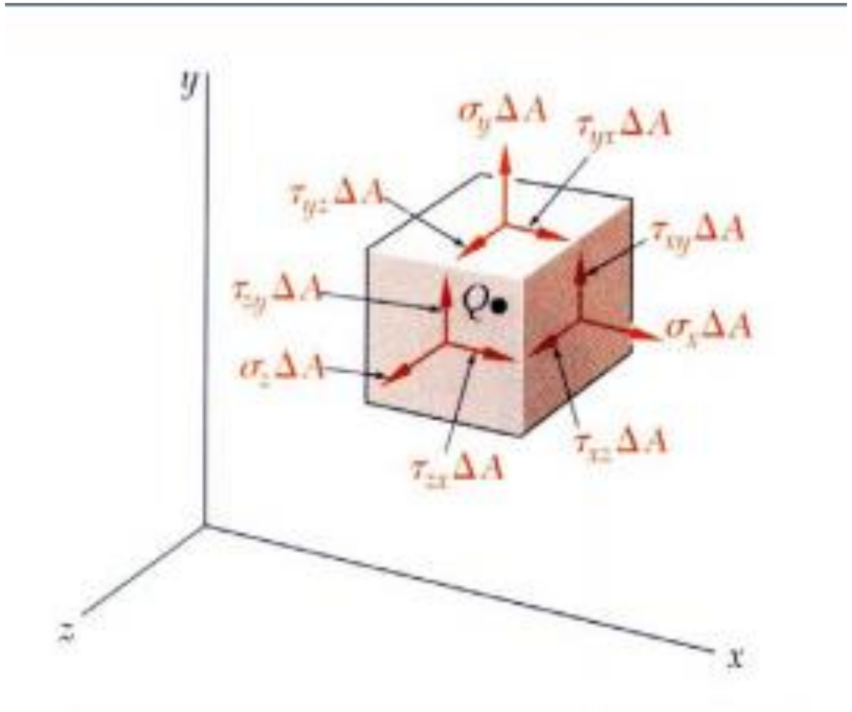


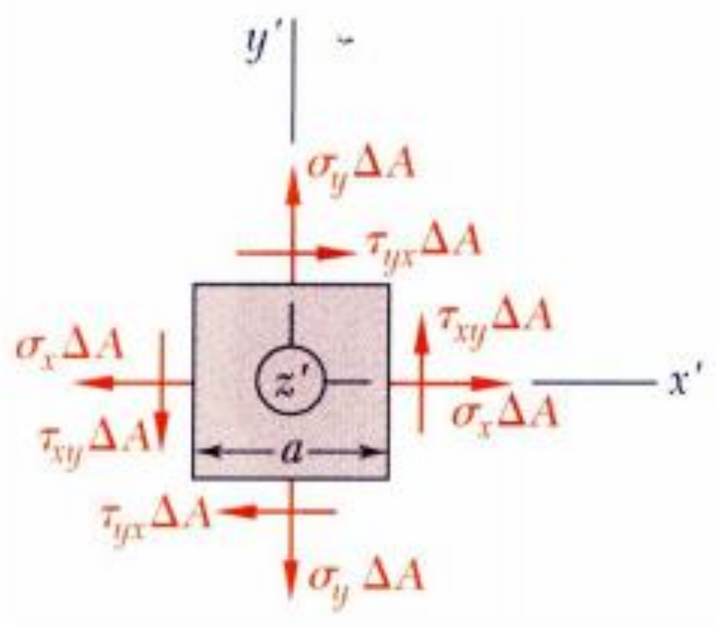


3D Stress State

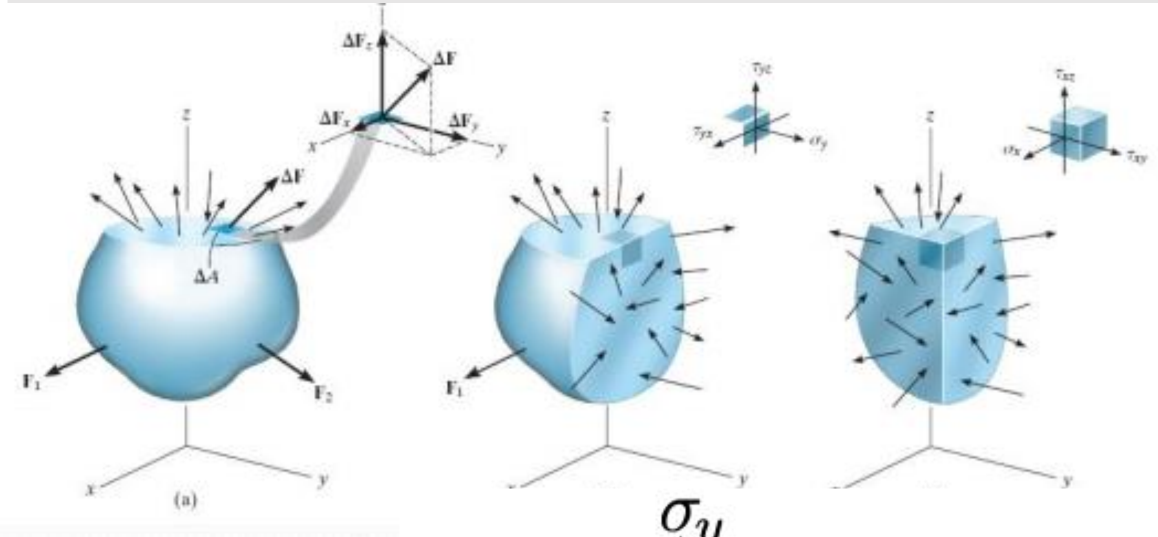
Plane Stress

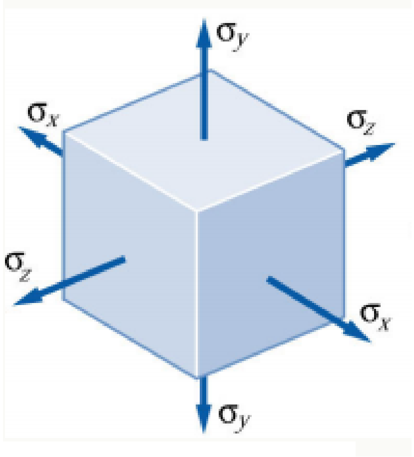
State of Stress

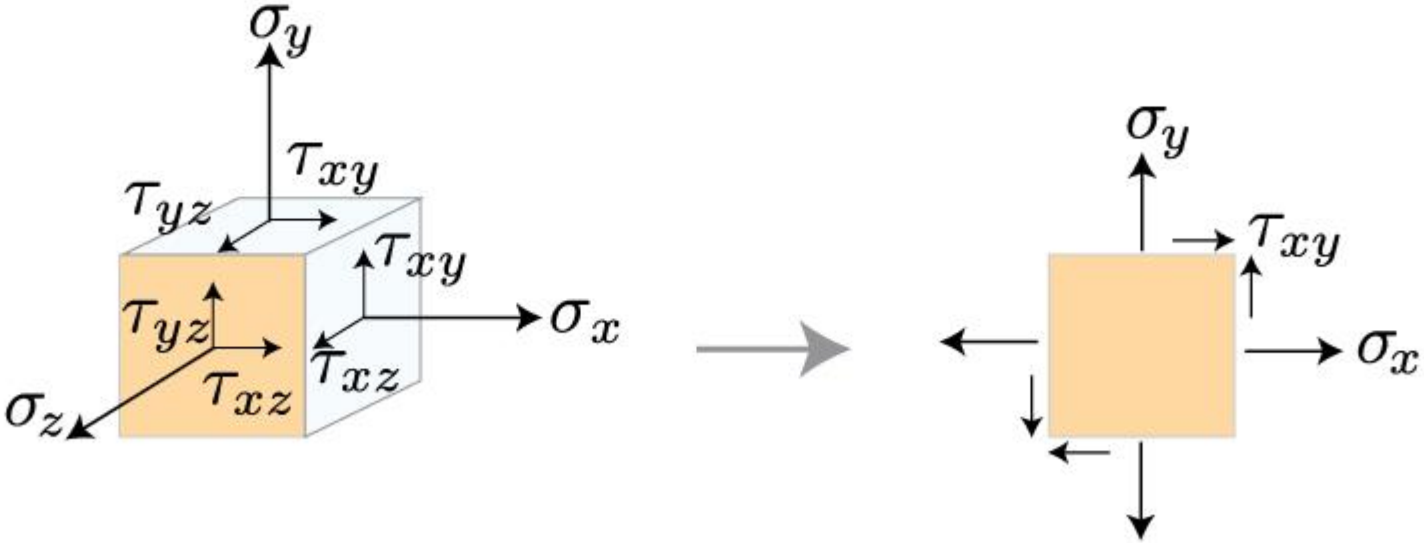




State of Stress



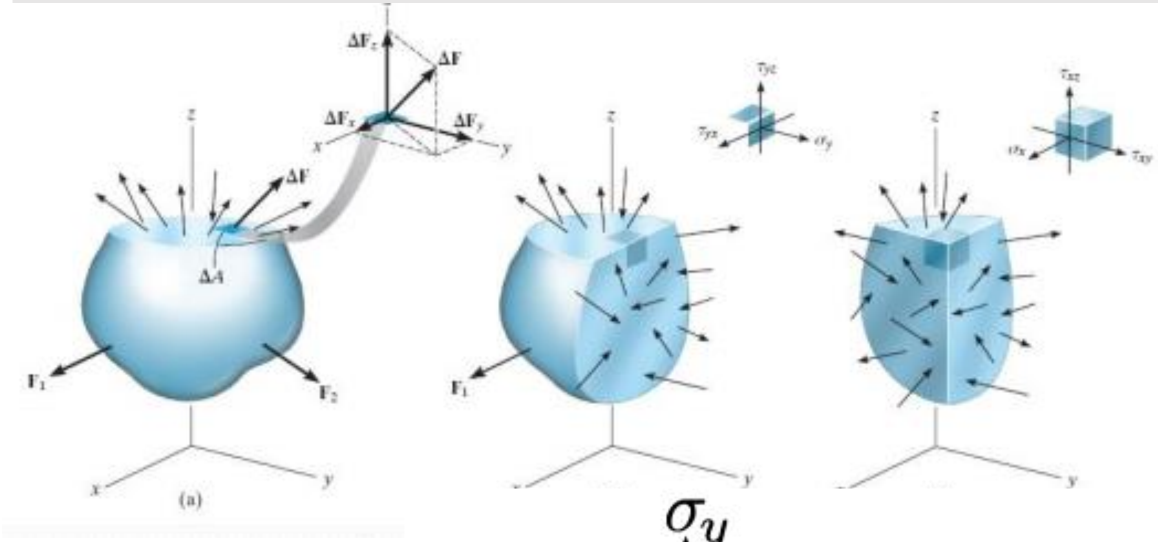


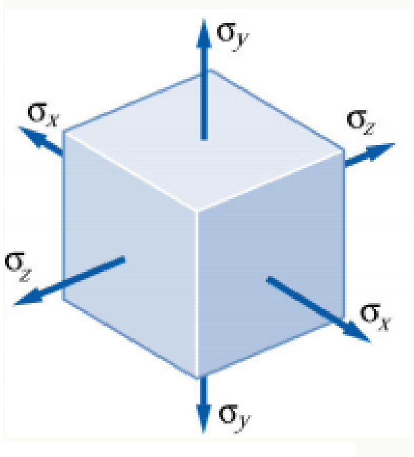


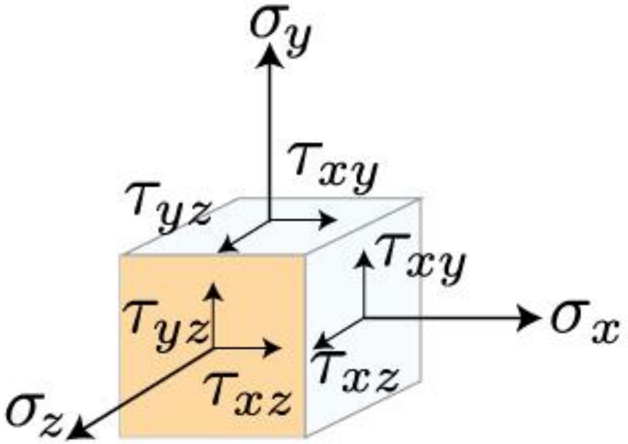
3D Stress State

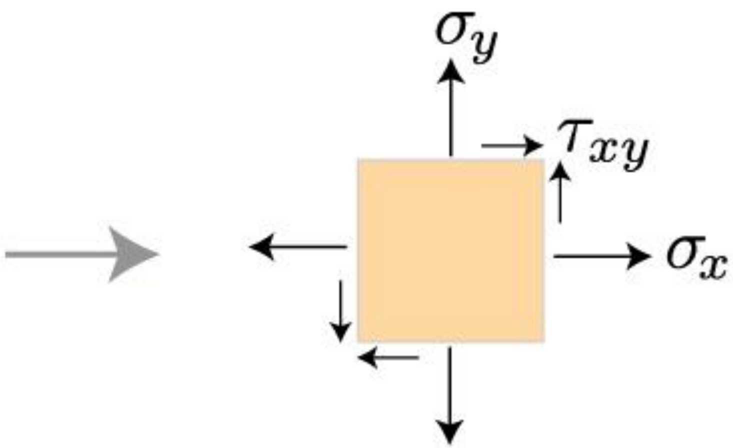
Plane Stress

State of Stress





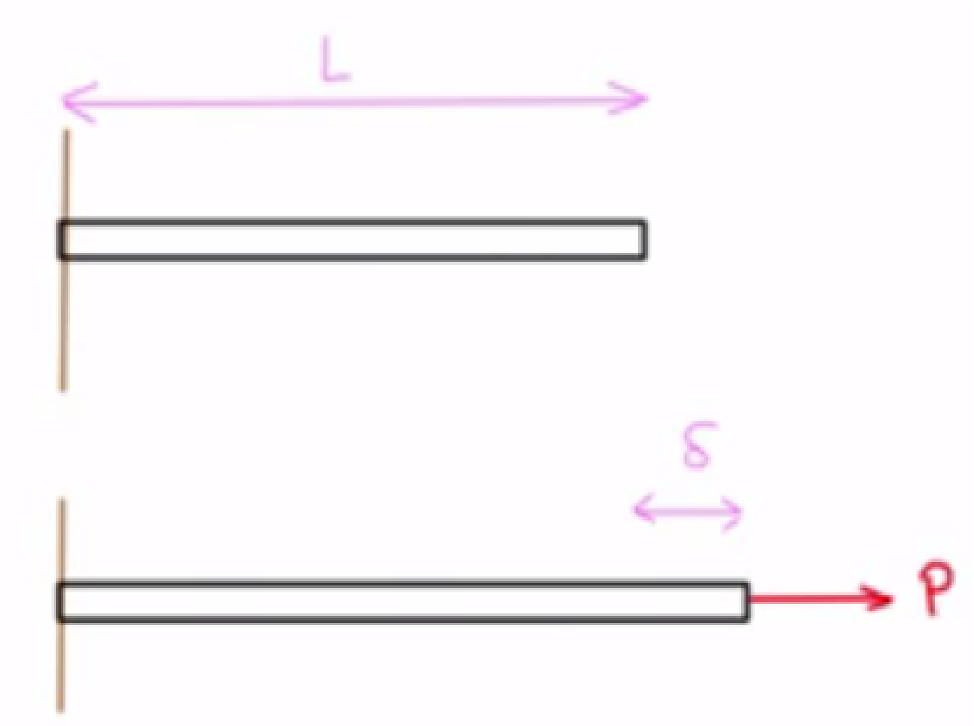


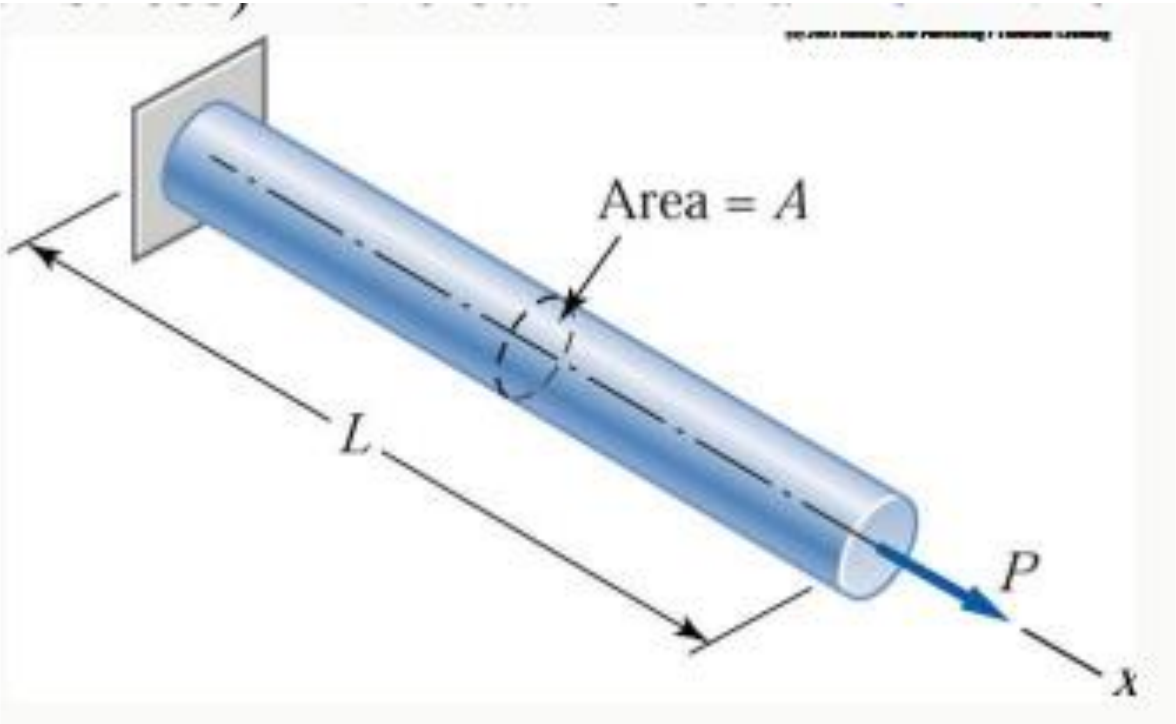


3D Stress State

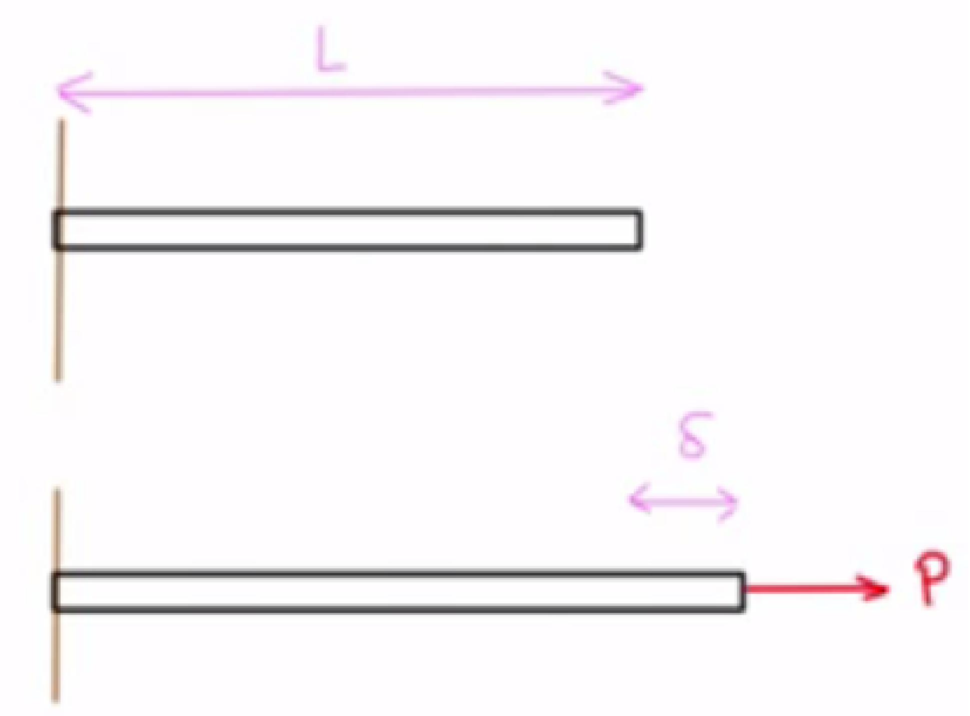
Plane Stress

Normal Strain

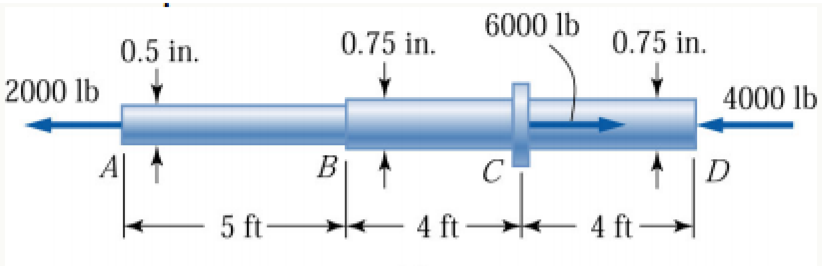




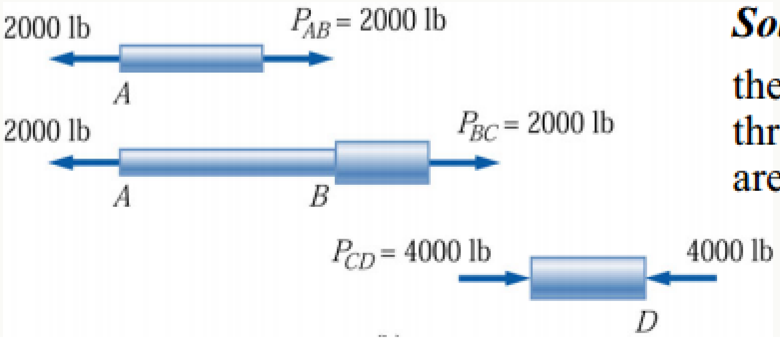
Normal Strain



Problems



(a)



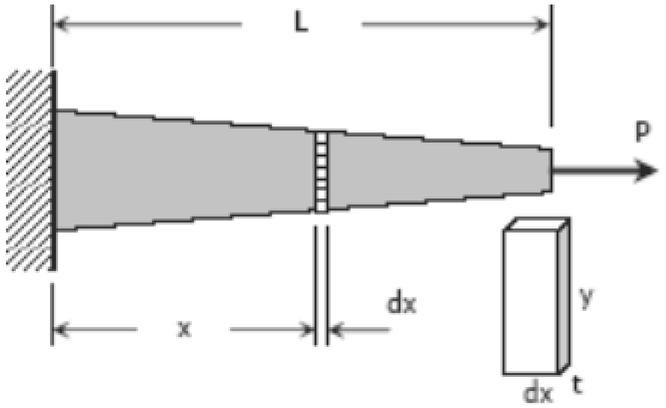
Solution

the internal forces in the three segments of the shaft are

$P_{AB} = P_{BC} = 2000$ lb (T)

$P_{CD} = -4000$ lb (C)

Problems

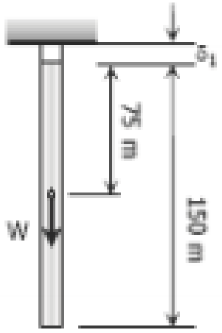


$$\delta = \frac{P}{E}\int_0^L \frac{dx}{A}$$

Problems

A steel rod having a cross-sectional area of 300 mm^2 and a length of 150 m is suspended vertically from one end. It supports a tensile load of 20 kN at the lower end. If the unit mass of steel is 7850 kg/m^3 and E = 200 × 10^3 MN/m^2, find the total elongation of the rod.

Solution 206



Let δ = total elongation

δ_1 = elongation due to its own weight

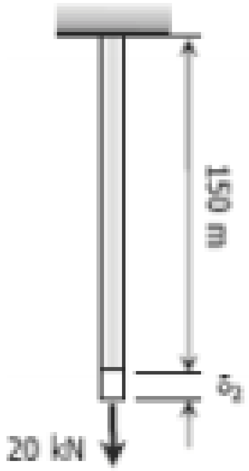
δ_2 = elongation due to applied load

$$\delta = \delta_1 + \delta_2$$

$$\delta_1 = \frac{PL}{AE}$$

Where: P = W = 7850(1/1000)3(9.81)[300(150)(1000)]
P = 3465.3825 N
L = 75(1000) = 75 000 mm
A = 300 mm^2
E = 200 000 MPa

$$\delta_1 = \frac{3465.3825\ (75000)}{300\ (200\ 000)} = 4.33\ \text{mm}$$



$$\delta_2 = \frac{PL}{AE}$$

Where: P = 20 kN = 20 000 N
L = 150 m = 150 000 mm
A = 300 mm^2
E = 200 000 MPa

$$\delta_2 = \frac{20000(150000)}{300(200000)} = 50\ \text{mm}$$

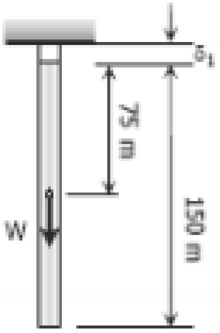
Total elongation:

$$\delta = 4.33 + 50 = 54.33\ \text{mm}$$

Problems

A steel rod having a cross-sectional area of 300 mm^2 and a length of 150 m is suspended vertically from one end. It supports a tensile load of 20 kN at the lower end. If the unit mass of steel is 7850 kg/m^3 and E = 200 × 10^3 MN/m^2, find the total elongation of the rod.

Solution 206



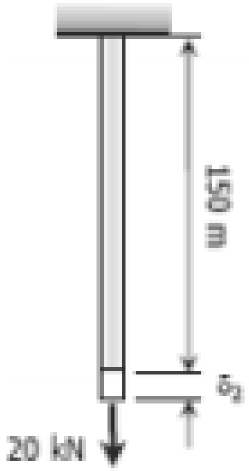
Let δ = total elongation
δ_1 = elongation due to its own weight
δ_2 = elongation due to applied load

$$\delta = \delta_1 + \delta_2$$

$$\delta_1 = \frac{PL}{AE}$$

Where:
P = W = 7850(1/1000)3(9.81)[300(150)(1000)]
P = 3465.3825 N
L = 75(1000) = 75 000 mm
A = 300 mm^2
E = 200 000 MPa

$$\delta_1 = \frac{3465.3825\,(75000)}{300\,(200\,000)} = 4.33 \text{ mm}$$



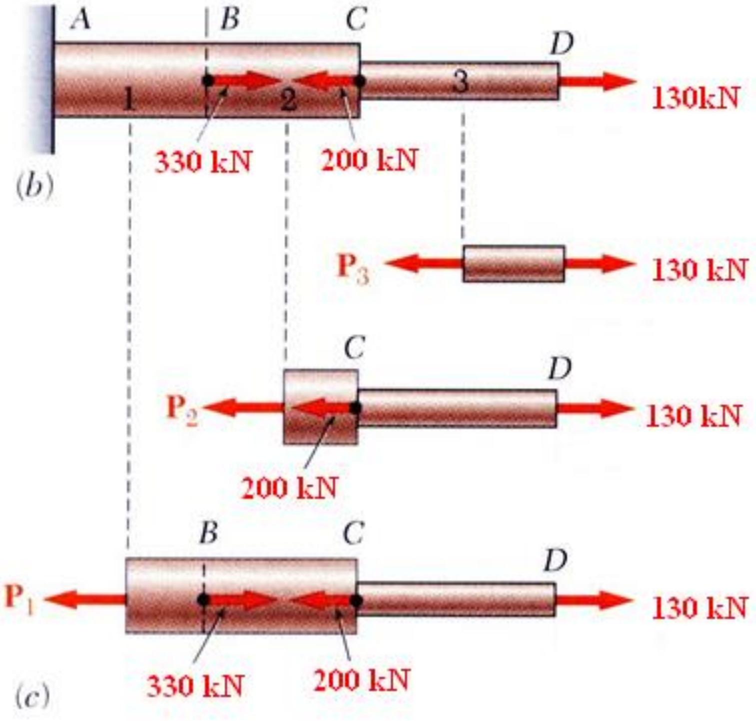
$$\delta_2 = \frac{PL}{AE}$$

Where:
P = 20 kN = 20 000 N
L = 150 m = 150 000 mm
A = 300 mm^2
E = 200 000 MPa

$$\delta_2 = \frac{20000(150000)}{300(200000)} = 50 \text{ mm}$$

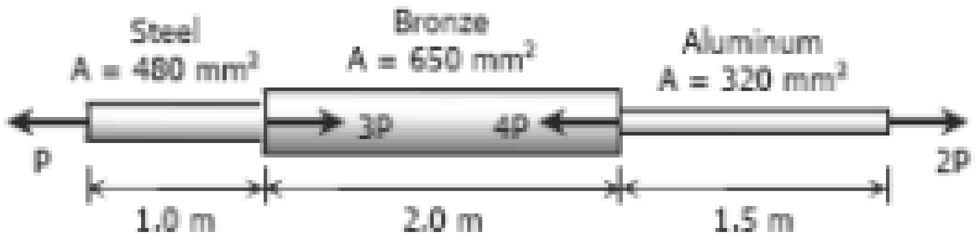
Total elongation:
$\delta = 4.33 + 50 = 54.33$ mm

Problems

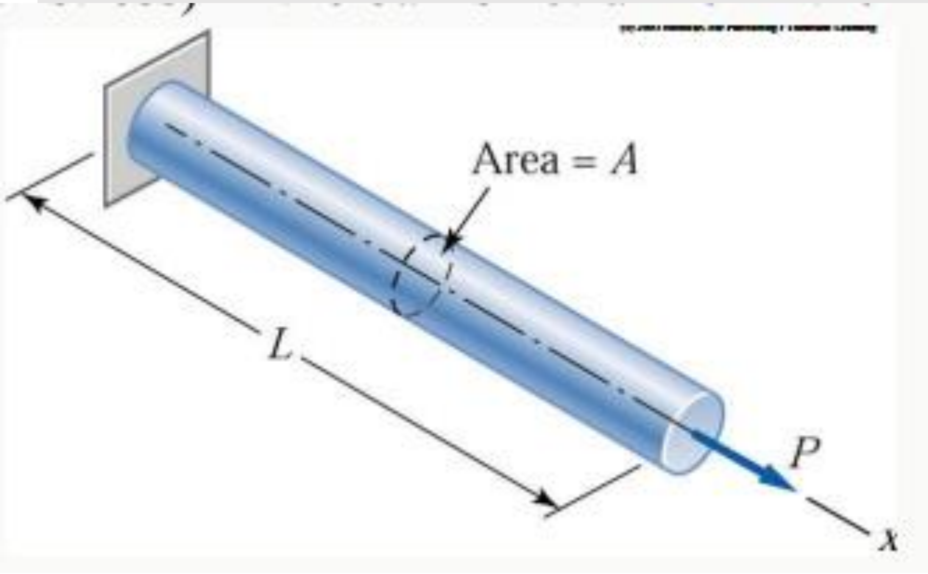

A
B
C
D
1
2
3
130kN
330 kN
200 kN
(b)
P3
130 kN
C
D
P2
130 kN
200 kN
B
C
D
P1
130 kN
330 kN
200 kN
(c)

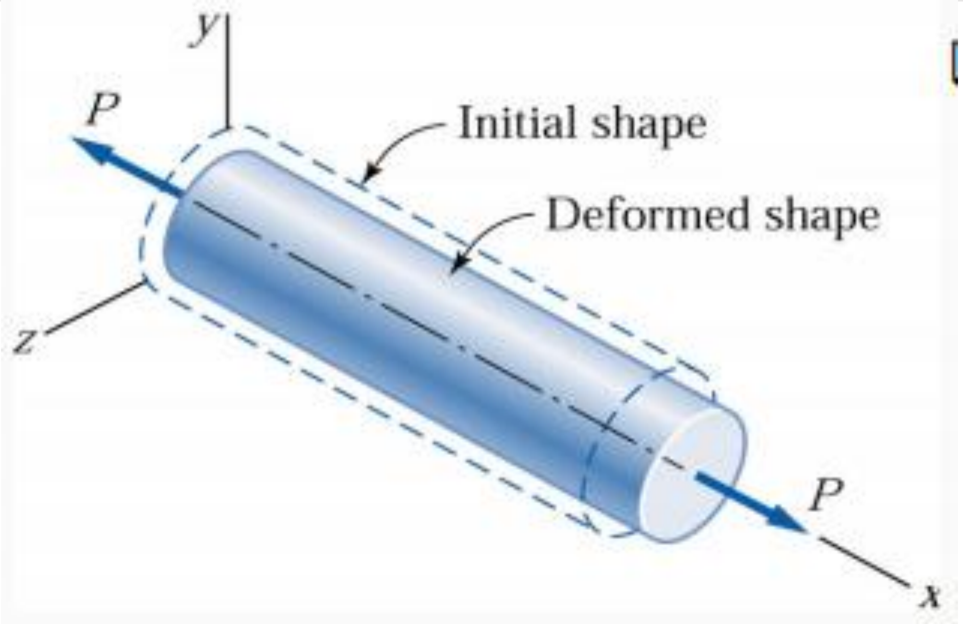
Problems

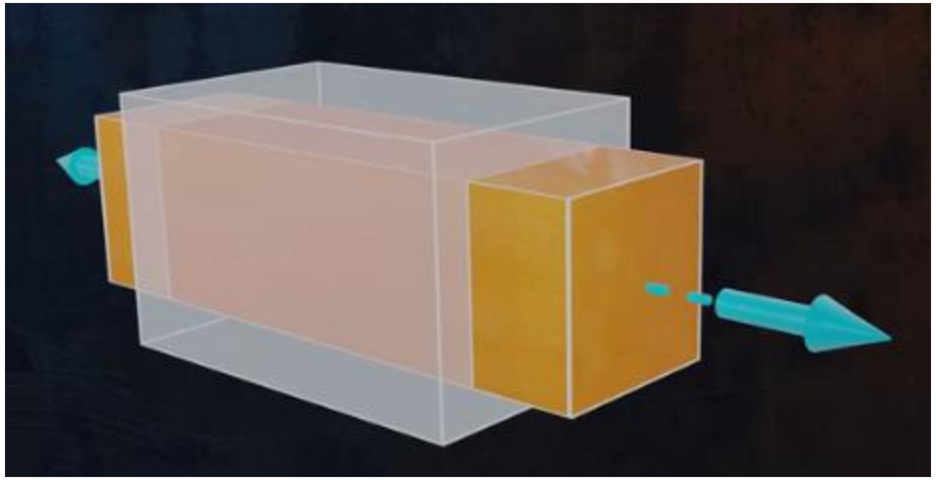
A bronze bar is fastened between a steel bar and an aluminum bar as shown in Fig. P-211. Axial loads are applied at the positions indicated. Find the largest value of P that will not exceed an overall deformation of 3.0 mm, or the following stresses: 140 MPa in the steel, 120 MPa in the bronze, and 80 MPa in the aluminum. Assume that the assembly is suitably braced to prevent buckling. Use E_{st} = 200 GPa, E_{al} = 70 GPa, and E_{br} = 83 GPa.



Poisson’s Ratio

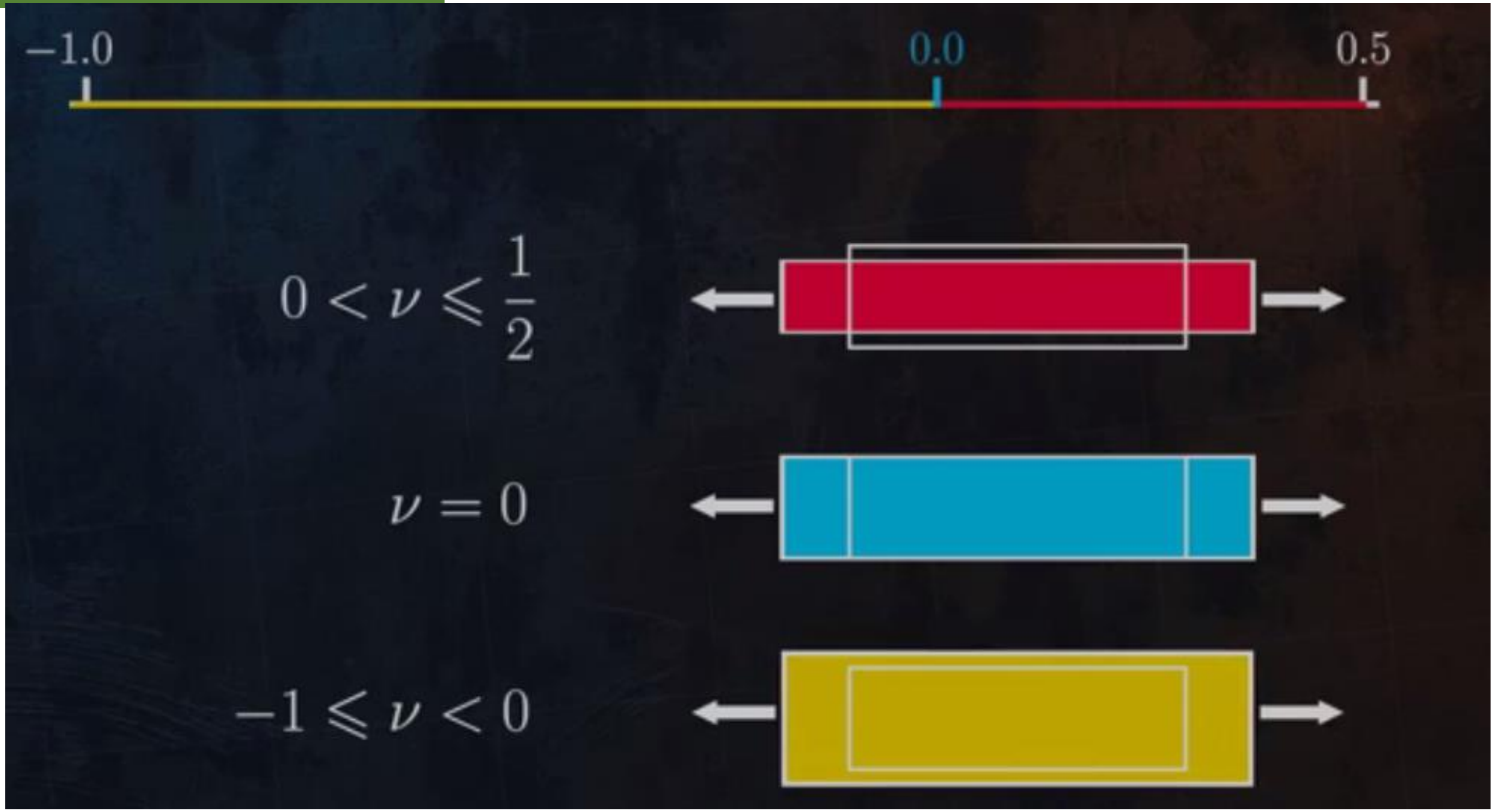


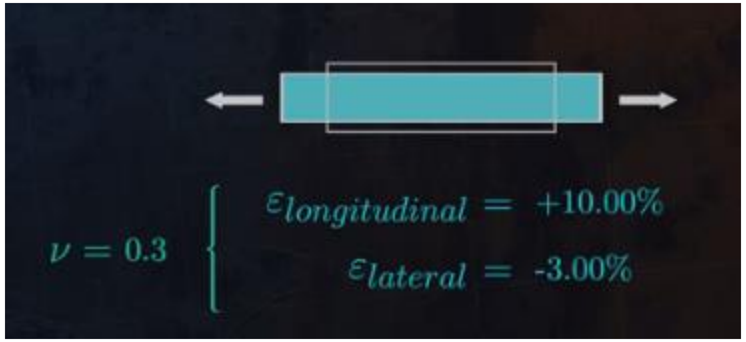


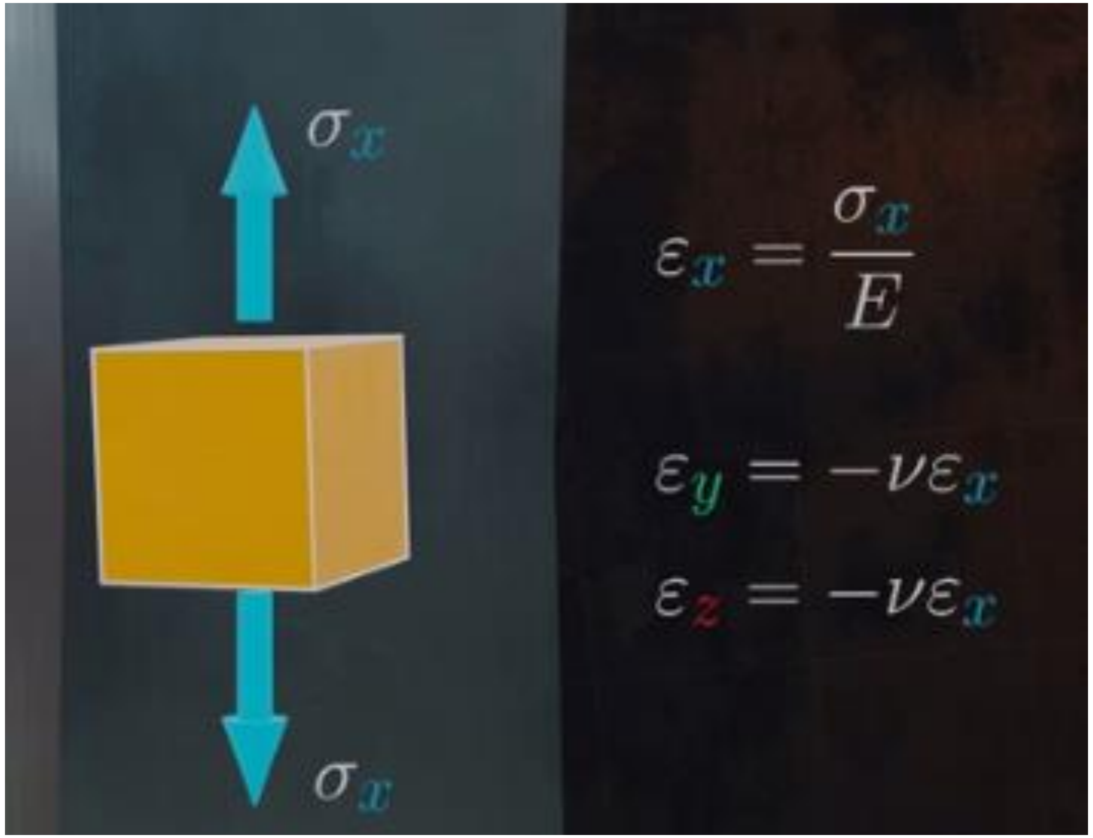


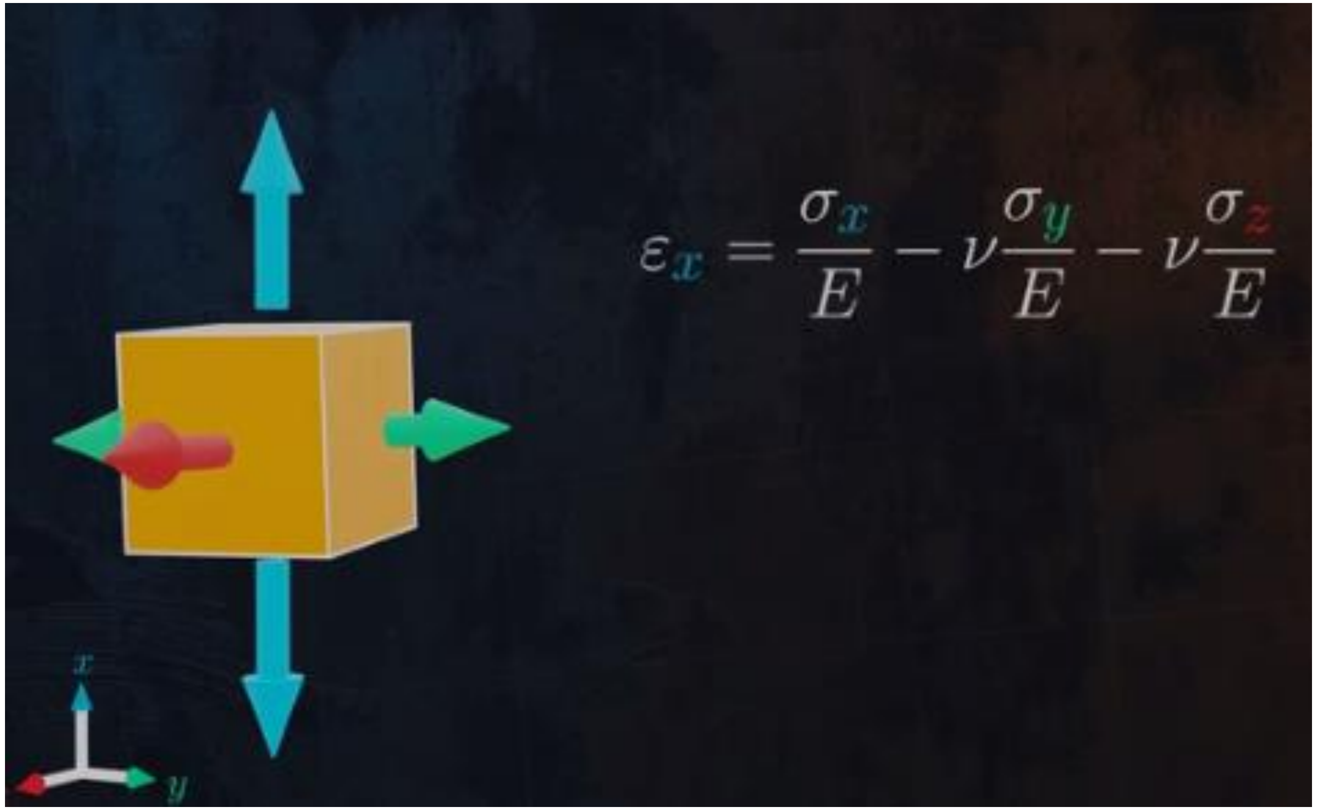
POISSON'S RATIO

$$\nu = \frac{-\varepsilon_{lateral}}{\varepsilon_{longitudinal}}$$


−1.0
0.0
0.5
$0 < \nu \leqslant \frac{1}{2}$
$\nu = 0$
$-1 \leqslant \nu < 0$


$\nu = 0.3$
$\varepsilon_{longitudinal} = +10.00\%$
$\varepsilon_{lateral} = -3.00\%$





volumetric strain

$$e_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$e_v = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z)$$

↳ if $\nu = 0.5 \rightarrow e_v = 0$

↳ material is incompressible

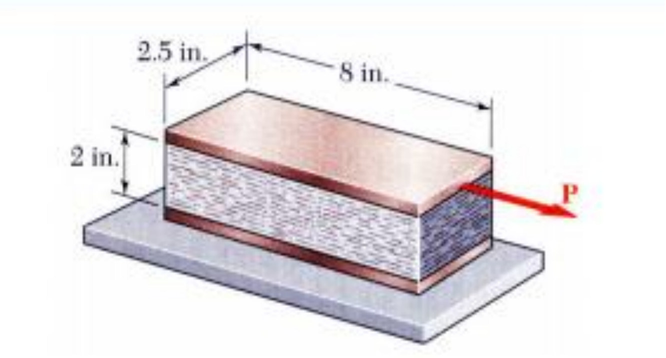
GENERALIZED HOOKE'S LAW

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

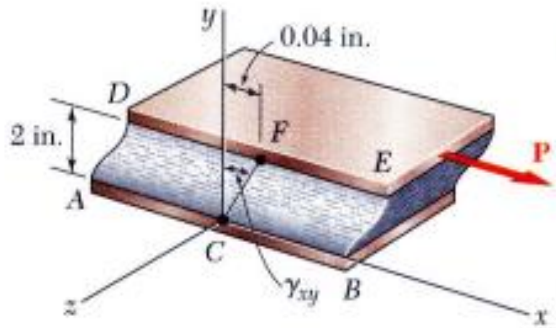
$$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$$

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$$

Relation Between Constants



A rectangular block of material with modulus of rigidity $G = 90$ ksi is bonded to two rigid horizontal plates. The lower plate is fixed, while the upper plate is subjected to a horizontal force P. Knowing that the upper plate moves through 0.04 in. under the action of the force, determine a) the average shearing strain in the material, and b) the force P exerted on the plate.



- Determine the average angular deformation or shearing strain of the block.

$$\gamma_{xy} \approx \tan \gamma_{xy} = \frac{0.04\,\text{in.}}{2\,\text{in.}} \qquad \gamma_{xy} = 0.020\,\text{rad}$$

- Apply Hooke's law for shearing stress and strain to find the corresponding shearing stress.

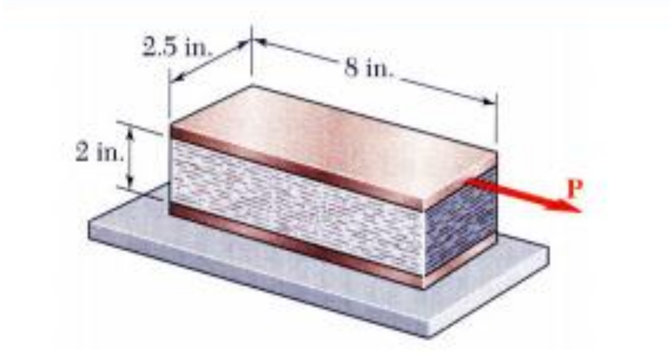
$$\tau_{xy} = G\gamma_{xy} = \left(90\times10^{3}\,\text{psi}\right)(0.020\,\text{rad}) = 1800\,\text{psi}$$

- Use the definition of shearing stress to find the force P.

$$P = \tau_{xy}A = (1800\,\text{psi})(8\,\text{in.})(2.5\,\text{in.}) = 36\times10^{3}\,\text{lb}$$

$$\boxed{P = 36.0\,\text{kips}}$$

Relation Between Constants



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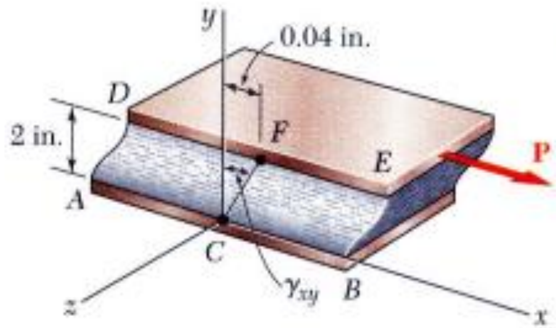


- Determine the average angular deformation or shearing strain of the block.

$$\gamma_{xy} \approx \tan \gamma_{xy} = \frac{0.04\,\text{in.}}{2\,\text{in.}} \qquad \gamma_{xy} = 0.020\,\text{rad}$$

- Apply Hooke's law for shearing stress and strain to find the corresponding shearing stress.

$$\tau_{xy} = G\gamma_{xy} = \left(90 \times 10^3\,\text{psi}\right)(0.020\,\text{rad}) = 1800\,\text{psi}$$

- Use the definition of shearing stress to find the force P.

$$P = \tau_{xy} A = (1800\,\text{psi})(8\,\text{in.})(2.5\,\text{in.}) = 36 \times 10^3\,\text{lb}$$

$$\boxed{P = 36.0\,\text{kips}}$$

Relation Between Constants

- It can be shown that the relationship between shear stress τ and shear strain γ is linear within the elastic range ; that is,

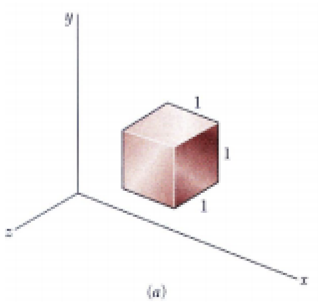
$$\tau = G\gamma \tag{2.13}$$

Which is Hooke´s law for shear. The material constant G is called ***the shear modulus of elasticity*** (or simply *shear modulus*), or the *modulus of rigidity*. The shear modulus has the same units as the modulus of elasticity (Pa or psi).

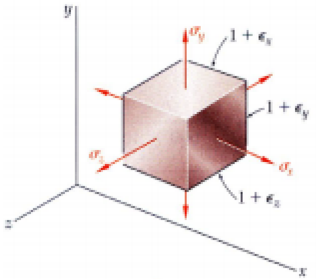
- The ***shear modulus of elasticity G*** is related to the modulus of elasticity ***E*** and poisson´s ratio ν by

$$\boxed{G = \frac{E}{2(1+\nu)}} \tag{2.14}$$

Relation Between Constants



(a)



(b)

- Relative to the unstressed state, the change in volume is

$$e = 1 - \left[(1+\varepsilon_x)(1+\varepsilon_y)(1+\varepsilon_z)\right] = 1 - \left[1+\varepsilon_x+\varepsilon_y+\varepsilon_z\right]$$

$$= \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$= \frac{1-2\nu}{E}\left(\sigma_x+\sigma_y+\sigma_z\right)$$

$$= \text{dilatation (change in volume per unit volume)}$$

- For element subjected to uniform hydrostatic pressure,

$$e = -p\frac{3(1-2\nu)}{E} = -\frac{p}{k}$$

$$k = \frac{E}{3(1-2\nu)} = \text{bulk modulus}$$

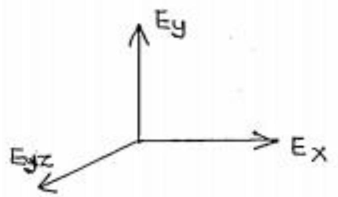
- Subjected to uniform pressure, dilatation must be negative, therefore

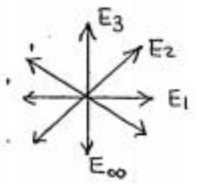
$$0 < \nu < \tfrac{1}{2}$$

Elastic Contsants

Of the four elastic constants, E & μ are independent constar for homogeneous + isotropic materials.

Material	Total EC.	Independent EC
Homogeneous + Isotropic	4	2 (E, μ)
Homogeneous + Orthotropic	12	9
Homogeneous + Anisotropic	∞	21





$E_x \neq E_y \neq E_z$

$G_x \neq G_y \neq G_z$

$K_x \neq K_y \neq K_z$

$\mu_x \neq \mu_y \neq \mu_z$.

Relation Between Constants

→ Relations b/w E, G, K & μ

$$E = 2G(1+\mu)$$

$$E = 3K(1-2\mu)$$

$$\mu = \frac{3K-2G}{6K+2G}$$

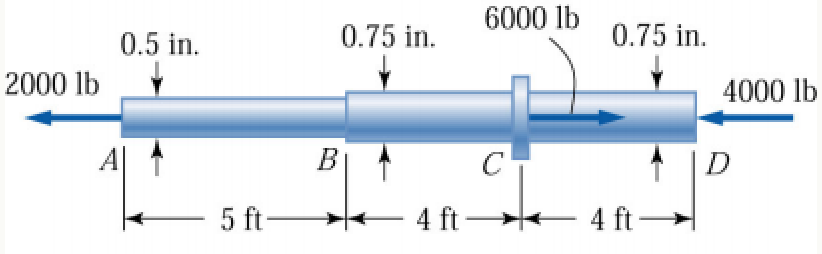
$$E = \frac{9KG}{3K+G}$$

Relation Between Constants

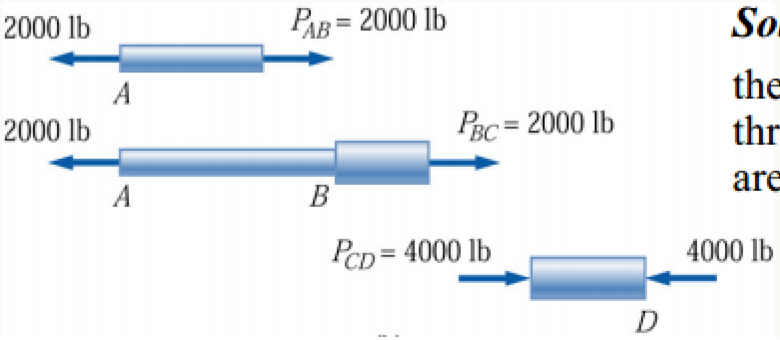
→ Relations b/w E, G, K & μ

$E = 2G(1+\mu)$
$E = 3K(1-2\mu)$
$\mu = \dfrac{3K-2G}{6K+2G}$
$E = \dfrac{9KG}{3K+G}$

Problems



(a)

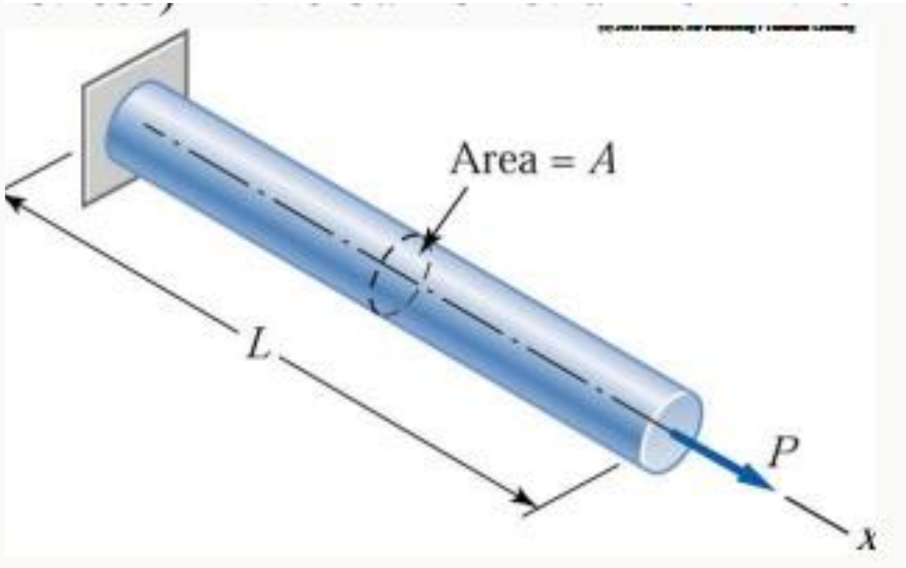


Solution

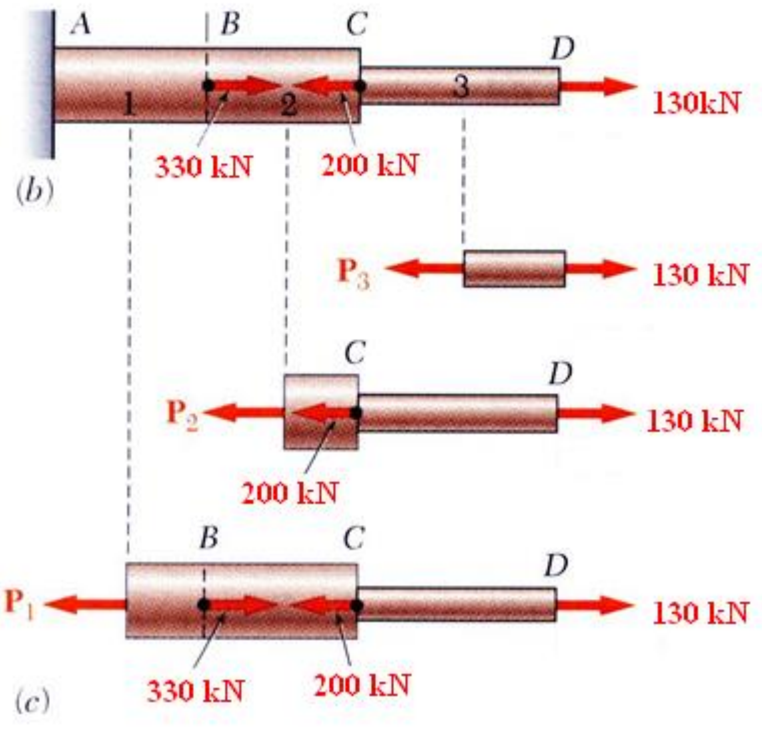
the internal forces in the three segments of the shaft are

$P_{AB} = P_{BC} = 2000$ lb (T)

$P_{CD} = -\ 4000$ lb (C)

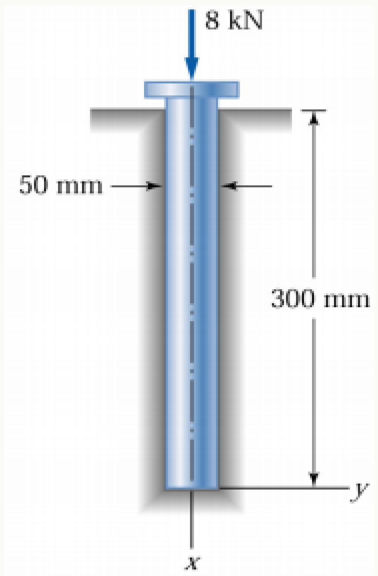


Problems



Sample problem 2.4

The 50-mm-diameter rubber rod is place in a hole with rigid, lubricated walls. There is no clearance between the rod and the sides of the hole. Determine the change in the length of the rod when the 8-kN load is applied. Use $E = 40$ MPa and $\nu = 0.45$ for rubber.



Solution

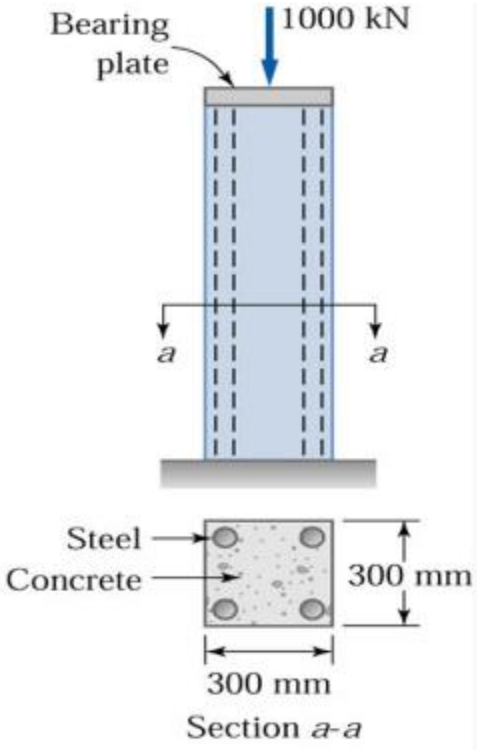
Lubrication allows the rod to contract freely in the axial direction, so that the axial stress throughout the bar is

$$\sigma_x = \frac{P}{A} = -\frac{8000}{\frac{\pi}{4}(0.05)^2} = -4.074\times10^6\ pa$$

Problems on Compatibility Equations

Sample Problem 2.6

The concrete post in Fig. (a) is reinforced axially with four symmetrically placed steel bars, each of cross-sectional area 900 mm². Compute **the stress in each material** when the 1000-kN axial load is applied. The moduli of elasticity are 200 Gpa for steel and 14 Gpa for concrete.

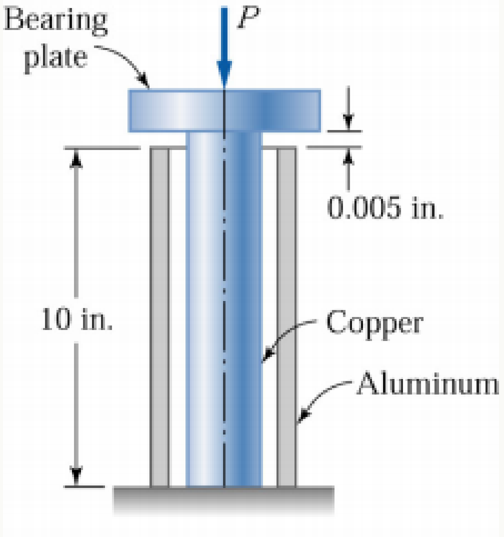


Let the allowable stresses in the post described in Sample Problem 2.6 be σ_{st} =120 Mpa and σ_{co} = 6 Mpa. Compute the maximum safe axial load *P* and may be applied.

Problems on Compatibility Equations

Sample Problem 2.8

Figure (a) shows a copper rod that is placed in an aluminum sleeve. The rod is 0.005 in. longer than the sleeve. Find the maximum safe load P that can be applied to the bearing plate, using the following date：



(a)

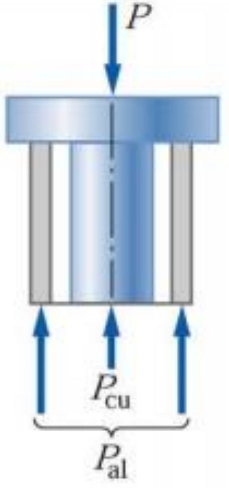
	Copper	Aluminum
Area (in.^{2})	2	3
E(psi)	17×10^6	10×10^6
Allowable stress (ksi)	20	10

Solution

Equilibrium in Fig. (b). From this FBD we get

$$\Sigma F = 0 + \uparrow \quad P_{cu} + P_{al} - P = 0 \qquad \text{(a)}$$

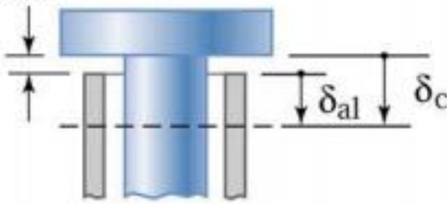
Because no other equations of equilibrium are available, the forces P_{cu} and P_{al} are statically indeterminate.



(b) FBD

Compatibility Figure (c) shows the changes in the lengths of the two material, the compatibility equation is

$$\delta_{cu} = \delta_{al} + 0.005 \text{ in.} \qquad \text{(b)}$$



Problems on Varying crosssection

5. A member *ABCD* is subjected to point load as shown in Fig. 3.14.

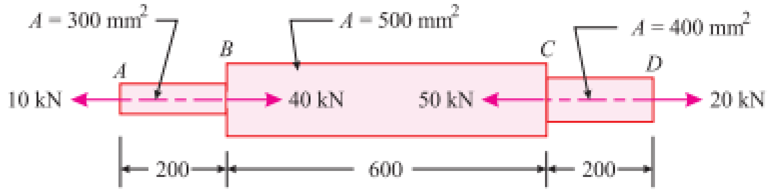


Fig. 3.14

Determine the total change in length of the member. Take $E = 200$ GPa.

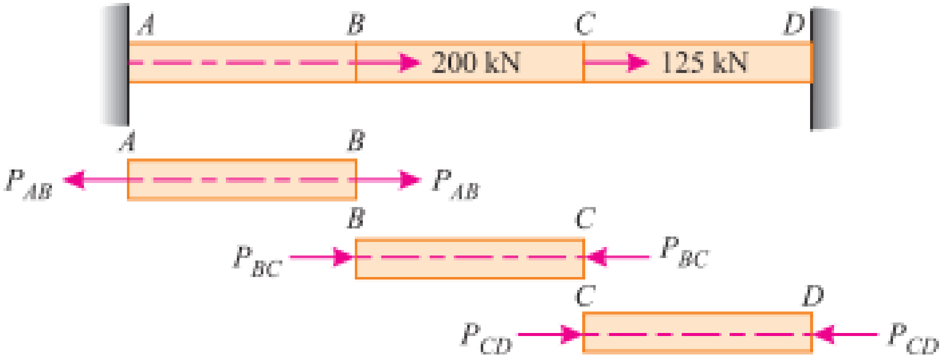
[Ans. 0.096 mm (decrease)]

Problems on Compatibility Equations

EXAMPLE 4.2. *An aluminium bar 3 m long and 2500 mm² in cross-section is rigidly fixed at A and D as shown in Fig. 4.3.*

Fig. 4.3

Determine the loads shared and stresses in each portion and the distances through which the points B and C will move. Take E for aluminium as 80 GPa.



Problems on Composite Materials

3. A uniform rigid block weighing 160 kN is to be supported on three bars as shown in Fig. 4.24.

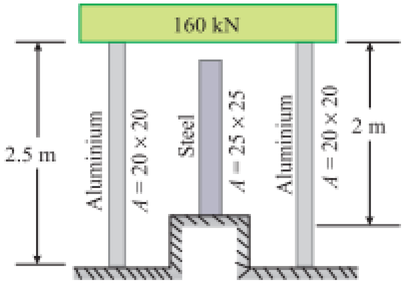


Fig. 4.24

There is 4 mm gap between the block and the top of the steel bar. Find the stresses developed in the bars. Take E_S = 200 GPa and E_A = 80 GPa. [**Ans.** σ_A = 148.9 MPa ; σ_S = 65.3 MPa]

Thermal Stress and Strain

Thermal Effects

Most engineering materials:
- Expand when heated
- Contract when cooled

$\alpha \equiv$ coefficient of thermal expansion

= strain per 1° temperature change

Thermal Strain

$$\varepsilon_T = \alpha(\Delta T)$$

We will assume α is constant
(actually it is generally higher at higher temperatures)
For homogeneous, isotropic materials,
α is the same coefficient in all directions

Thermal Stress

exists when the member is restrained

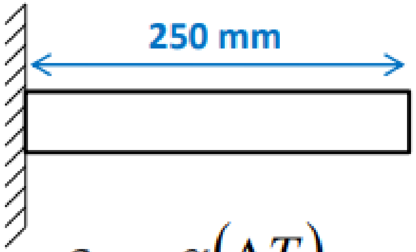
Bronze

$\Delta T = 40^\circ$ *increase*

$\alpha = 16.9 \ x \ 10^{-6} / {}^\circ C$

E=100 GPa

Unrestrained



$$\varepsilon_T = \alpha(\Delta T)$$

$$\delta_T = \varepsilon_T L = \alpha(\Delta T)L = 16.0 \ x \ 10^{-6}$$

$$\delta_T = 0.169 \, mm$$

$\sigma = 0$ **unrestrained**

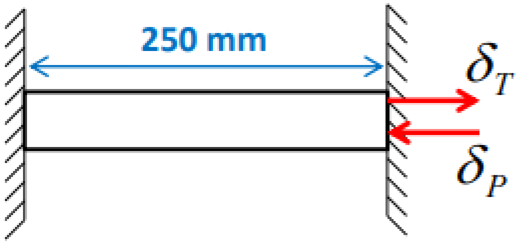
Bronze

$\Delta T = 40^\circ$ *increase*

$\alpha = 16.9 \ x \ 10^{-6} / {}^\circ C$

E=100 GPa

Fully restrained



$$\delta_{total} = \delta_T - \delta_P = 0$$

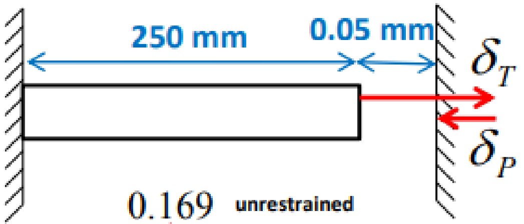
Bronze

$\Delta T = 40^\circ$ *increase*

$\alpha = 16.9 \ x \ 10^{-6} / {}^\circ C$

E=100 GPa

Partially restrained



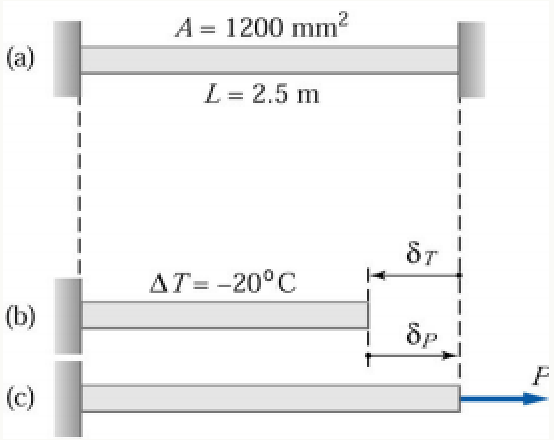
0.169 unrestrained

$$\delta_{total} = \delta_T - \delta_P = 0.05$$

Problems on Thermal stress

Sample problem 2.10

The horizontal steel rod, 2.5 m long and 1200 mm^2 in cross-sectional area, is secured between two walls as shown in Fig. (a). If the rod is stress-free at 20 ℃, compute the stress when the temperature has dropped to -20℃. Assume that (1) the walls do not move and (2) the walls move together a distance $\triangle = 0.5$ mm. Use $\alpha = 11.7\times10^{-6}$/℃ and E =200 GPa.



Solution

Part 1

Compatibility $\delta_T = \delta_P$

Hooke's law $\delta_T = \alpha(\triangle T)L$ and

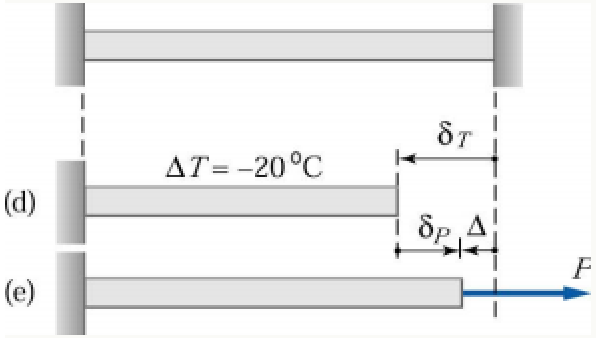
$\delta_P = PL/(EA) = \sigma L/E$,

$$\frac{\sigma L}{E} = \alpha(\Delta T)L$$

$\sigma = \alpha(\triangle T)E = (11.7\times10^{-6})(40)(200\times10^9) = 93.6\times10^6$ Pa = 93.6 MPa *Answer*

Part 2

Compatibility when the walls move together a distance Δ,



Compatibility $\delta_T = \delta_P + \Delta$

Hooke's law Substituting for δ_T and δ_P as in Part 1, we obtain

$$\alpha(\Delta T)L = \frac{\sigma L}{E} + \Delta$$

the stress $\sigma = E\left[\alpha(\Delta T) - \frac{\Delta}{L}\right] = (200\times10^9)\left[(11.7\times10^{-6})(40) - \frac{0.5\times10^{-3}}{2.5}\right]$

$= 53.6\times10^6$ Pa = 53.6 MPa *Answer*

the movement of the walls reduces the stress considerably.

Thermal stresses in Composite Bars

5.5. Thermal Stresses in Composite Bars

Whenever there is some increase or decrease in the temperature of a bar, consisting of two or more different materials, it causes the bar to expand or contract. On account of different coefficients of linear expansions the two materials do not expand or contract by the same amount, but expand or contract by different amounts.

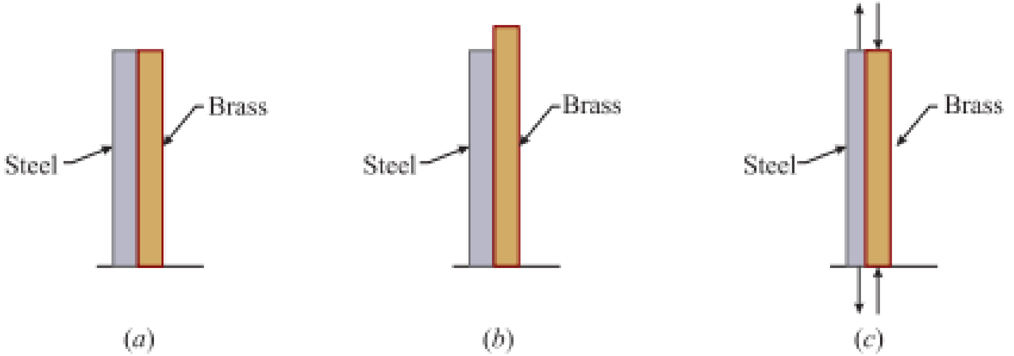
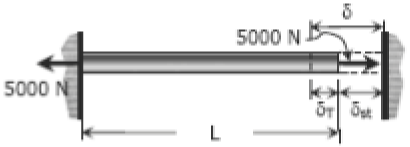


Fig. 5.6. Composite bars

A steel rod is stretched between two rigid walls and carries a tensile load of 5000 N at 20°C. If the allowable stress is not to exceed 130 MPa at -20°C, what is the minimum diameter of the rod? Assume α = 11.7 μm/(m·°C) and E = 200 GPa.

Solution 262



$$\delta = \delta_T + \delta_{st}$$

$$\frac{\sigma \cancel{L}}{E} = \alpha \cancel{L}(\Delta T) + \frac{P\cancel{L}}{AE}$$

$$\sigma = \alpha E(\Delta T) + \frac{P}{A}$$

$$130 = (11.7 \times 10^{-6})(200\,000)(40) + \frac{5000}{A}$$

$$A = \frac{5000}{36.4} = 137.36 \text{ mm}^2$$

$$\tfrac{1}{4}\pi d^2 = 137.36; \quad d = 13.22 \text{ mm}$$

Thermal stresses problems

EXAMPLE 5.7. *A composite bar made up of aluminium and steel, is held between two supports as shown in Fig. 5.4.*

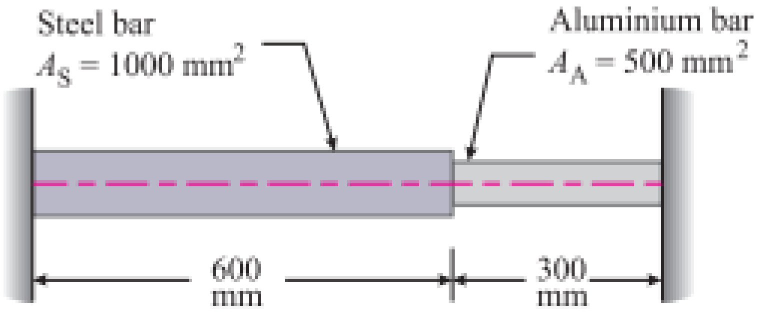
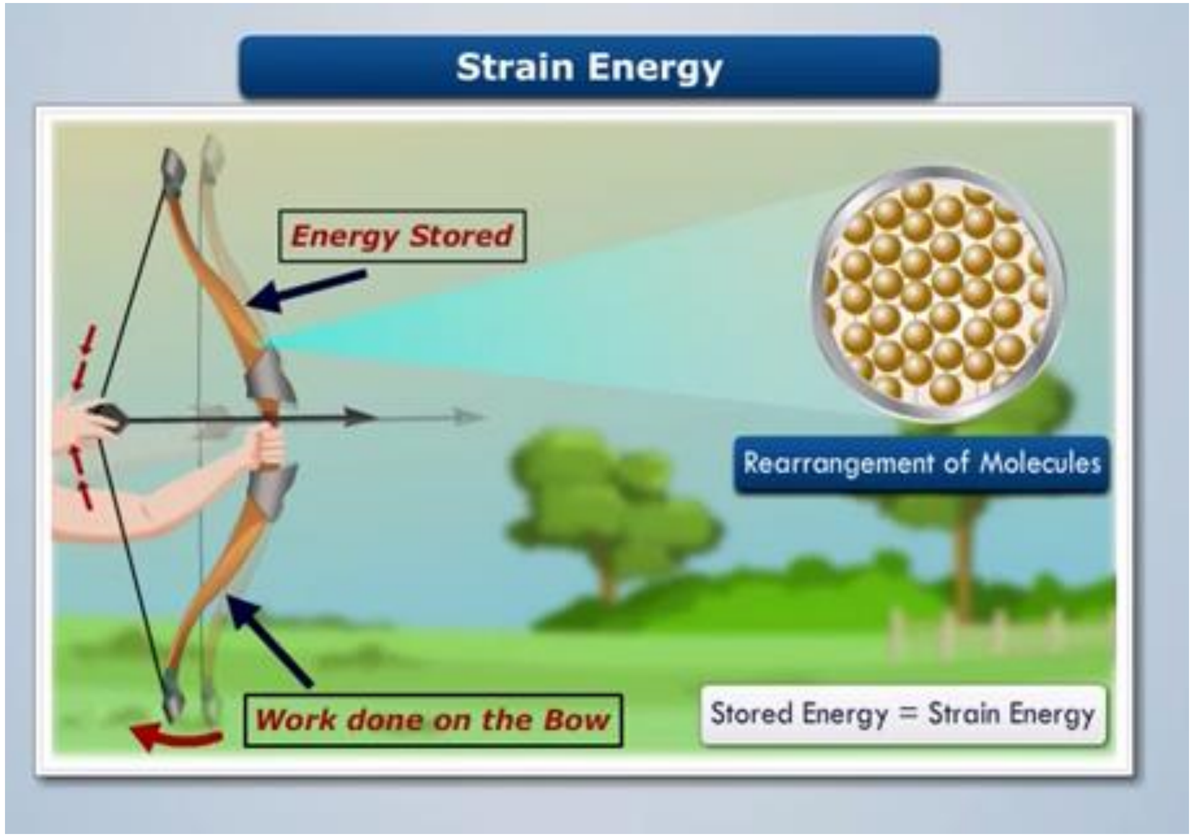


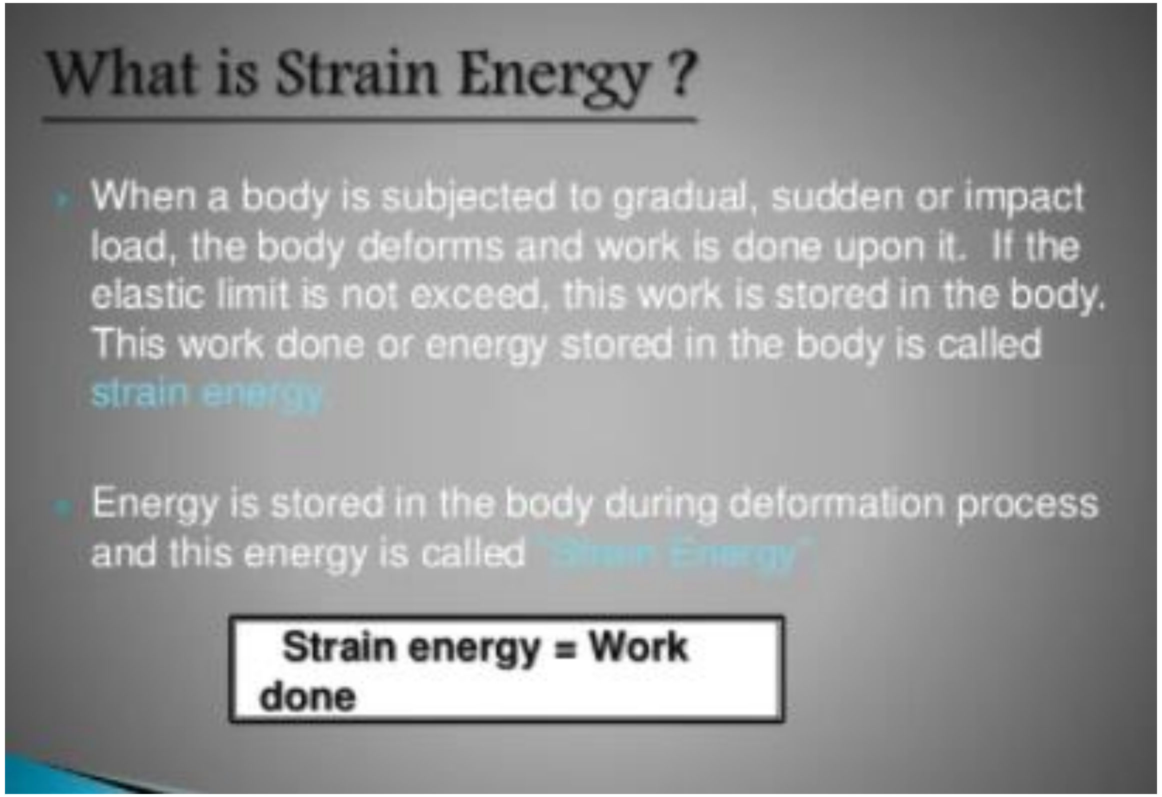
Fig. 5.4

The bars are stress-free at a temperature of 38°C. What will be the stresses in the two bars, when the temperature is 21°C, if (a) the supports are unyielding, (b) the supports come nearer to each other by 0.1 mm? It can be assumed that the change of temperature is uniform all along the length of the bar.

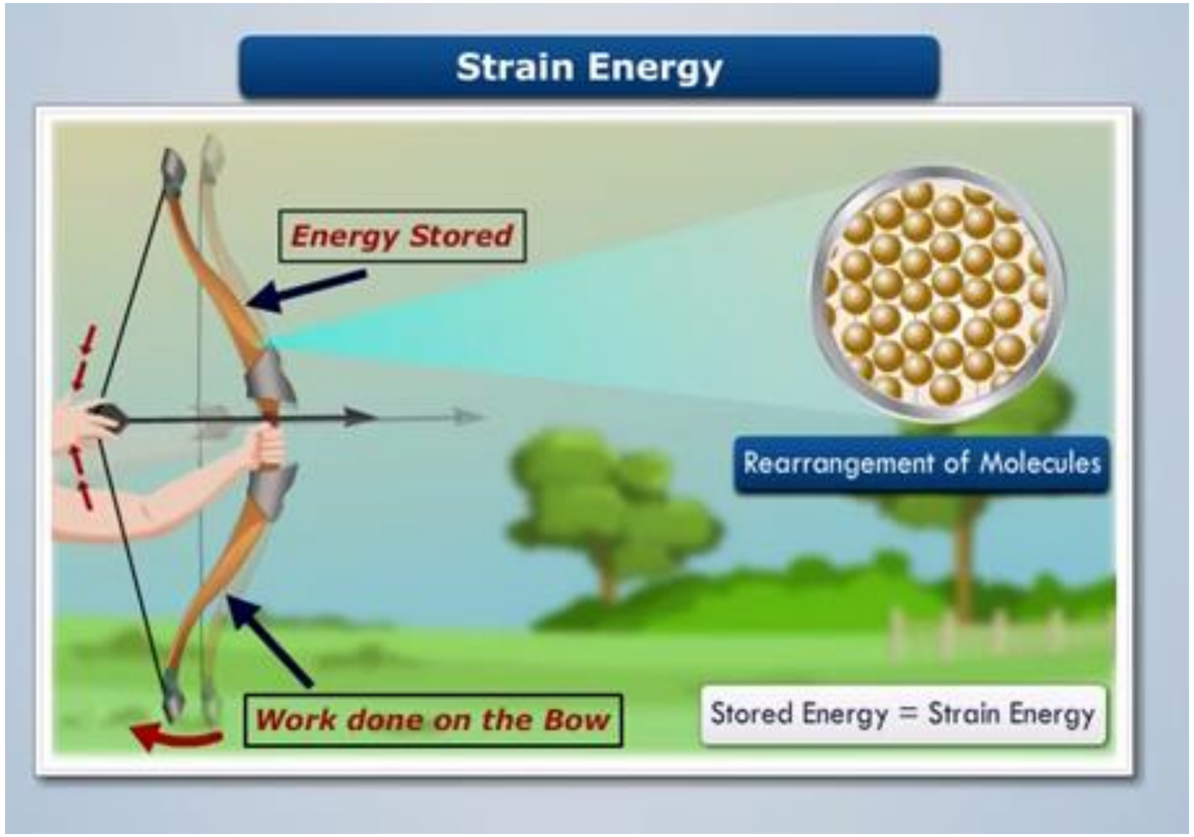
Take E for steel as 200 GPa; E for aluminium as 75 GPa and coefficient of expansion for steel as 11.7×10^{-6} per °C and coefficient of expansion for aluminium as 23.4×10^{-6} per °C.

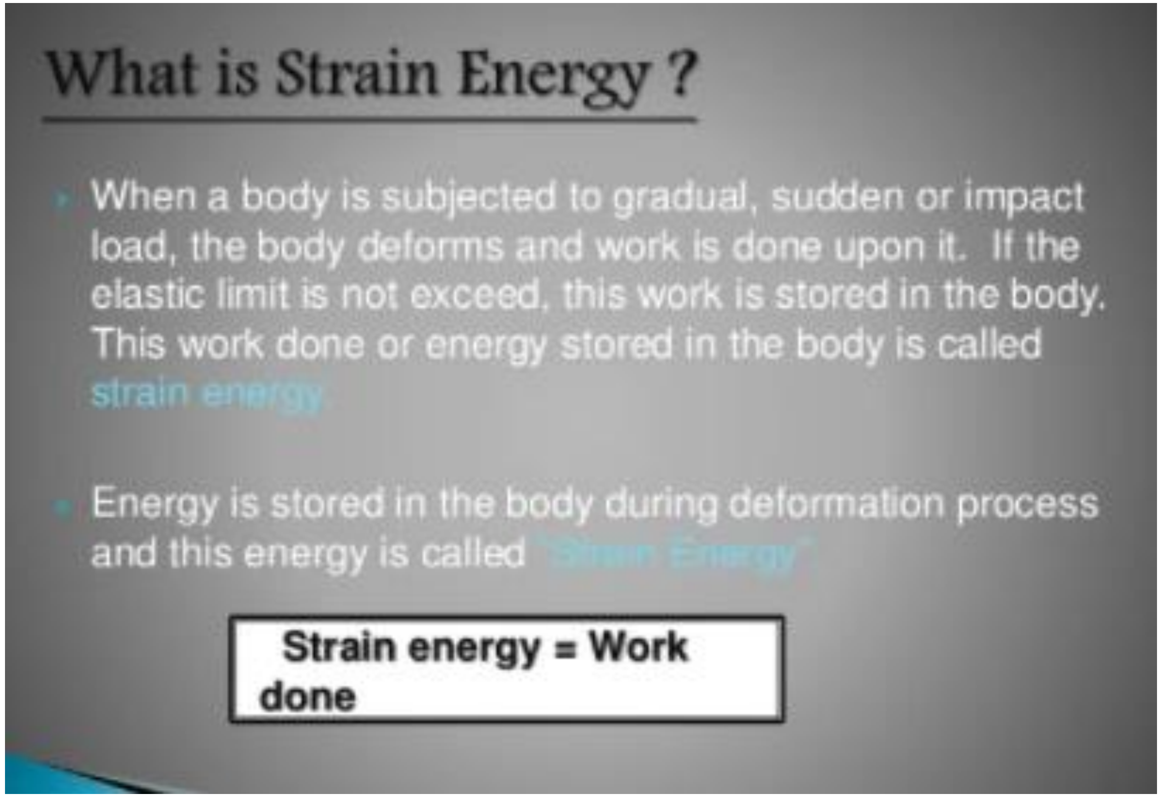
Strain Energy





Strain Energy





Strain Energy

Strain energy:-

The energy stored in a member due to external workdone is the strain energy.

$$\boxed{U = \frac{1}{2} W \cdot \delta}$$

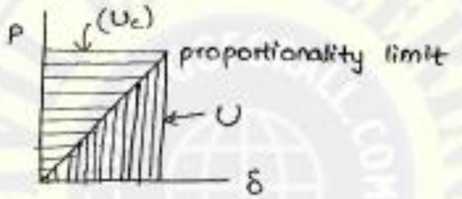
Unit :- N-m (or) Joule

Energy :- Scalar

Resilience:-(U)

The energy stored in a member within proportionality limit is Resilience.

The recoverable strain energy is Resilience



The area under load and deformation curve upto proportionality limit is also Resilience.

$$\boxed{U = \frac{1}{2} P \cdot \delta} \rightarrow (1)$$

U_c - complimentary Resilie

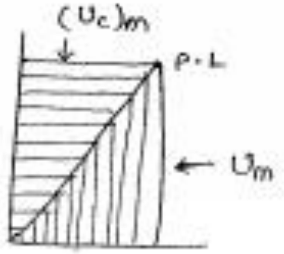
Proof Resilience:-

The maximum resilience stored in a member which can be obtain by loading upto proportionality limit

* Modular Resilience:-

Resilience per unit volume (or) Area under stress strain curve upto proportionality limit is called Modulus of Resilience

$$\boxed{U_m = \frac{U}{V}}$$



From (2) $U_m = \frac{1}{2} \sigma \cdot \varepsilon$

From (3) $U_m = \frac{\sigma^2}{2E}$

Units:- Unit of stress (or) M·pa

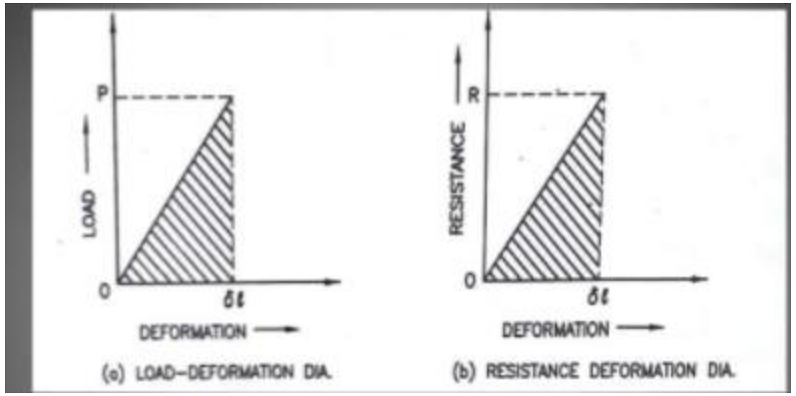
Strain Energy

Type of Loading:-

1. Gradual load:-

All the loads by default are gradual loads only

$$\left.\begin{aligned} \sigma &= \frac{P}{A} \\ \delta &= \frac{Pl}{AE} \end{aligned}\right\} \text{for gradual loads only}$$

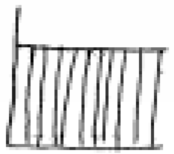


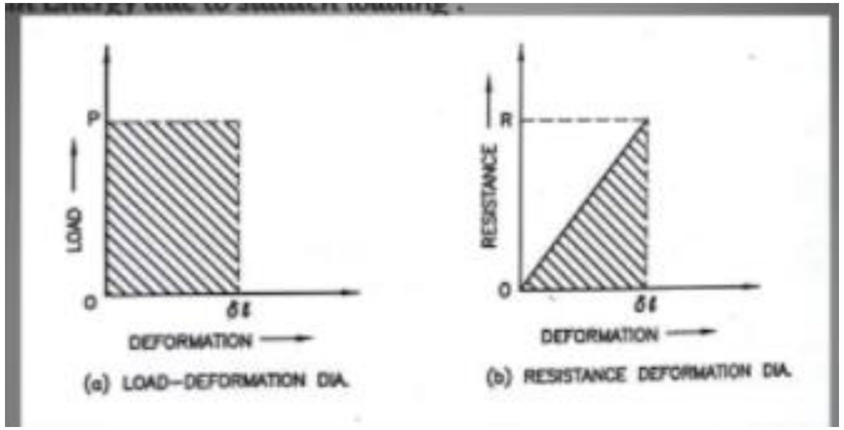
Strain Energy

3. Sudden load :- (Imaginary load) :-

$$\sigma_{sudden} = 2(\sigma_{gradual}) = \frac{2P}{A}$$

$$\delta_{sudden} = 2(\delta_{gradual}) = \frac{2PL}{AE}$$





8.8. Strain Energy Stored in a Body, when the Load is Applied with Impact

Sometimes in factories and workshops, the impact load is applied on a body *e.g.*, when we lower a body with the help of a crane, and the chain breaks while the load is being lowered the load falls through a distance, before it touches the platform. This is the case of a load applied with impact.

Now consider a bar subject to a load applied with impact as shown in Fig 8.1.

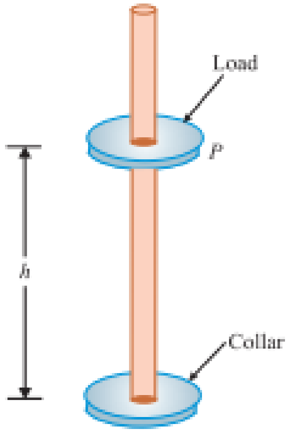


Fig. 8.1

Let P = Load applied with impact,
A = Cross-sectional area of the bar,
E = Modulus of elasticity of the bar material,
l = Length of the bar,
δl = Deformation of the bar, as a result of this load,
σ = Stress induced by the application of this load with impact, and
h = Height through which the load will fall, before impacting on the collar of the bar.

$\therefore$ Work done = Load × Distance moved
$= P\,(h + \delta l)$

and energy stored, $U = \dfrac{\sigma^2}{2E} \times A\,l$

Since energy stored is equal to the work done, therefore

$$\frac{\sigma^2}{2E} \times Al = P\,(h + \delta l) = P\left(h + \frac{\sigma}{E}.l\right) \qquad \ldots\left(\because \delta l = \frac{\sigma}{E}.l\right)$$

$$\frac{\sigma^2}{2E} \times Al = Ph + \frac{P\sigma l}{E}$$

$$\therefore \quad \sigma^2\left(\frac{Al}{2E}\right) - \sigma\left(\frac{Pl}{E}\right) - Ph = 0$$

Multiplying both sides by $\left(\dfrac{E}{Al}\right)$,

$$\frac{\sigma^2}{2} - \sigma\left(\frac{P}{A}\right) - \frac{PEh}{Al} = 0$$

This is a quadratic equation. We know that

$$\sigma = \frac{P}{A} \pm \sqrt{\left(\frac{P}{A}\right)^2 + \left(4 \times \frac{1}{2}\right)\left(\frac{PEh}{Al}\right)}$$

$$= \frac{P}{A}\left[1 \pm \sqrt{1 + \frac{2AEh}{Pl}}\right]$$

Once the stress (σ)is obtained, the corresponding instantaneous deformation (δl) or the strain energy stored may be found out as usual.

Cor. When δ is very small as compared to h, then

$$\text{Work done} = Ph$$

$$\therefore \quad \frac{\sigma^2}{2E}Al = Ph$$

or $$\sigma^2 = \frac{2EPh}{Al}$$

$$\therefore \quad \sigma = \sqrt{\frac{2EPh}{Al}}$$

EXAMPLE 8.4. *A 2 m long alloy bar of 1500 mm² cross-sectional area hangs vertically and has a collar securely fixed at its lower end. Find the stress induced in the bar, when a weight of 2 kN falls from a height of 100 mm on the collar. Take E = 120 GPa. Also find the strain energy stored in the bar.*

SOLUTION. Given : Length of bar $(l) = 2\text{ m} = 2 \times 10^3$ mm ; Cross-sectional area of bar $(A) = 1500$ mm^2 ; Weight falling on collar of bar $(P) = 2\text{ kN} = 2 \times 10^3$ N ; Height from which weight falls $(h) =$ 100 mm and modulus of elasticity $(E) = 120\text{ GPa} = 120 \times 10^3\text{ N/mm}^2$.

Stress induced in the bar

We know that in this case, extension of the bar will be small and negligible as compared to the height (h) from where the weight falls on the collar (due to small value of weight *i.e.*, 2 kN and a large value of h *i.e.*, 100 mm). Therefore stress induced in the bar

$$\sigma = \sqrt{\frac{2EPh}{A.l}} = \sqrt{\frac{2 \times (120 \times 10^3) \times (2 \times 10^3) \times 100}{1500 \times (2 \times 10^3)}}\ \text{N/mm}^2$$

$$= 126.5\text{ N/mm}^2 = 126.5\text{ MPa} \qquad \textbf{Ans.}$$

Strain energy stored in the bar

We also know that volume of the bar,

$$V = l\,.\,A = (2 \times 10^3) \times 1500 = 3 \times 10^6\text{ mm}^3$$

and strain energy stored in the bar,

$$U = \frac{\sigma^2}{2E} \times V = \frac{(126.5)^2}{2 \times (120 \times 10^3)} \times (3 \times 10^6)\text{ N-mm}$$

$$= 200 \times 10^3\text{ N-mm} = 200\text{ N-m} \qquad \textbf{Ans.}$$

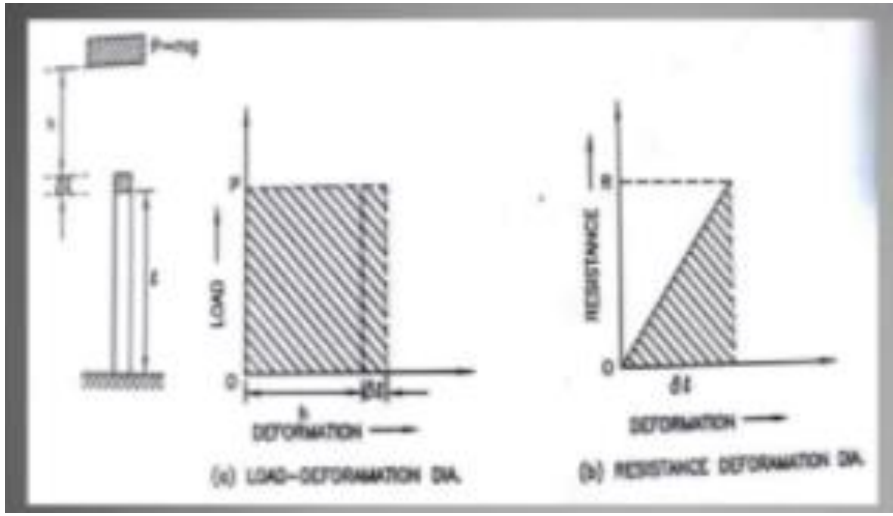
Strain Energy

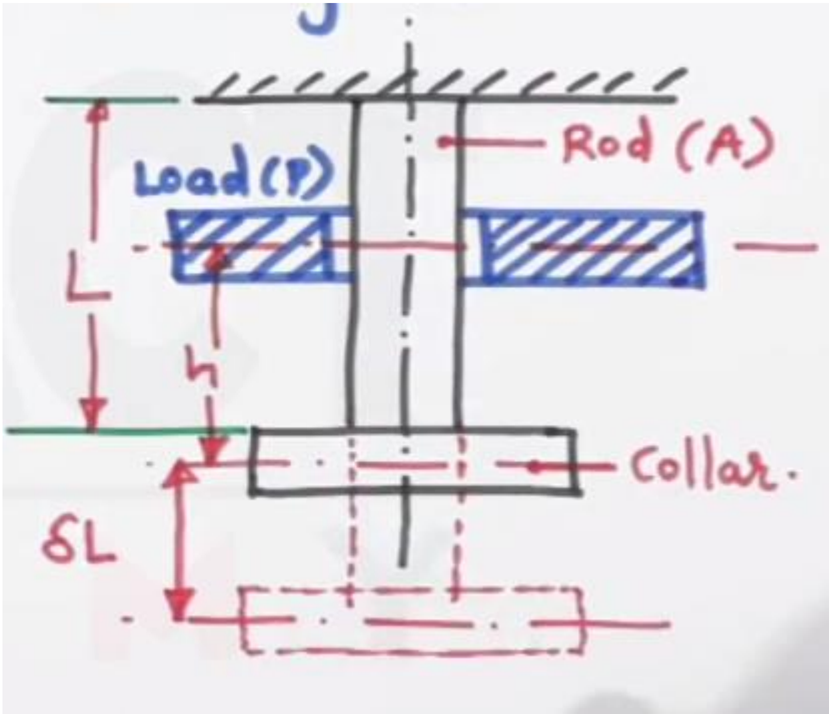
2. Impact load:-

Work done = strain energy stored

$$P \cdot h = \frac{\sigma^2}{2E} \cdot V$$

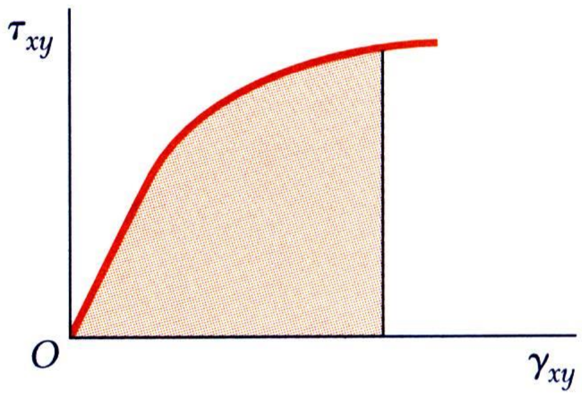
$$\sigma_{impact} = \sqrt{\frac{2Ph \cdot E}{V}}$$

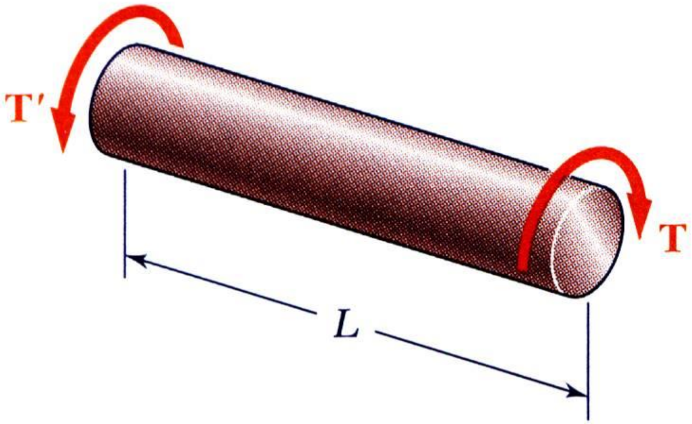




$$\therefore \sigma = \frac{P}{A} + \sqrt{\frac{2EPh}{Al} + \frac{p^2}{A^2}}$$

Strain Energy



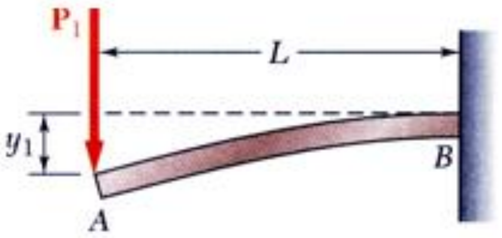


- In the case of a uniform shaft,

$$U = \frac{T^2 L}{2GJ}$$

Stress Formulae

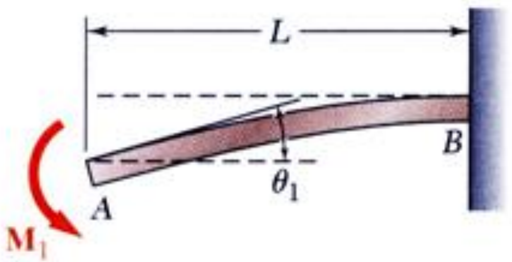
- Transverse load



$$U = \int_0^{y_1} P\,dy = \tfrac{1}{2} P_1 y_1$$

$$= \tfrac{1}{2} P_1 \left(\frac{P_1 L^3}{3EI} \right) = \frac{P_1^2 L^3}{6EI}$$

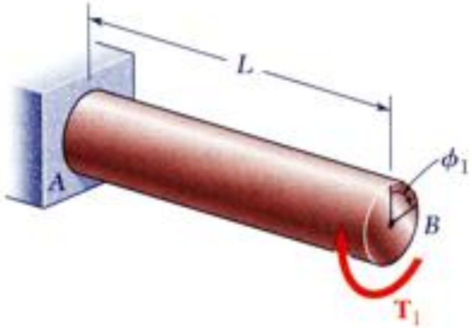
- Bending couple



$$U = \int_0^{\theta_1} M\,d\theta = \tfrac{1}{2} M_1 \theta_1$$

$$= \tfrac{1}{2} M_1 \left(\frac{M_1 L}{EI} \right) = \frac{M_1^2 L}{2EI}$$

- Torsional couple



$$U = \int_0^{\phi_1} T\,d\phi = \tfrac{1}{2} T_1 \phi_1$$

$$= \tfrac{1}{2} T_1 \left(\frac{T_1 L}{JG} \right) = \frac{T_1^2 L}{2JG}$$

Strain Energy

Strain energy:-

The energy stored in a member due to external workdone is the strain energy.

$$\boxed{U = \frac{1}{2} W \cdot \delta}$$

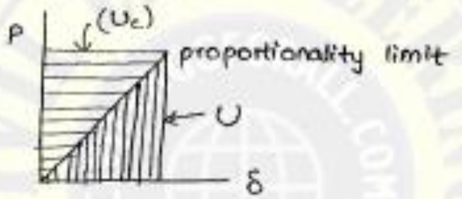
Unit :- N-m (or) Joule

Energy :- Scalar

Resilience:-(U)

The energy stored in a member within proportionality limit is Resilience.

The recoverable strain energy is Resilience



The area under load and deformation curve upto proportionality limit is also Resilience.

$$\boxed{U = \frac{1}{2} P \cdot \delta} \rightarrow ①$$

U_c - complimentary Resilie

Proof Resilience:-

The maximum resilience stored in a member which can be obtain by loading upto proportionality limit

* Modular Resilience:-

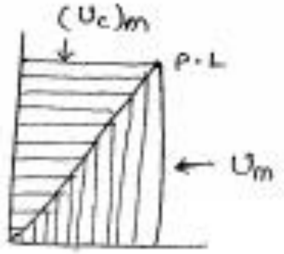
Resilience per unit volume (or) Area under stress strain curve upto proportionality limit is called Modulus of Resilience

$$\boxed{U_m = \frac{U}{V}}$$

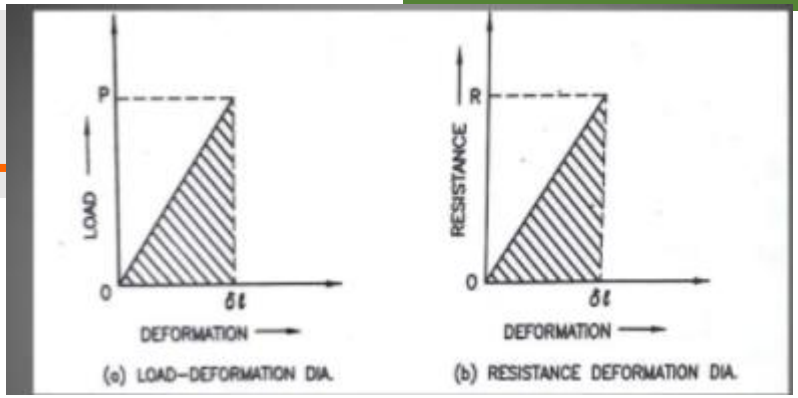
From ② $U_m = \frac{1}{2} \sigma \cdot \varepsilon$

From ③ $U_m = \frac{\sigma^2}{2E}$

Units:- Unit of stress (or) M·pa



Strain Energy



Type of Loading:-

1. Gradual load:-

All the loads by default are gradual loads only

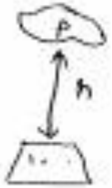
$$\left.\begin{aligned} \sigma &= \frac{P}{A} \\ \delta &= \frac{Pl}{AE} \end{aligned}\right\} \text{ for gradual loads only}$$

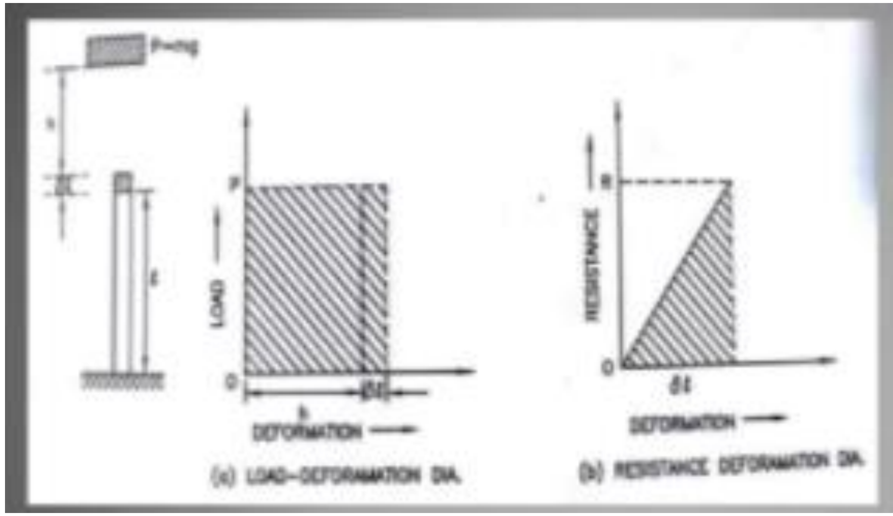
2. Impact load:-

Work done = strain energy stored

$$P \cdot h = \frac{\sigma^2}{2E} \cdot V$$

$$\sigma_{impact} = \sqrt{\frac{2Ph \cdot E}{V}}$$

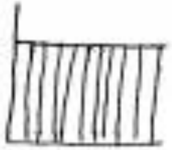




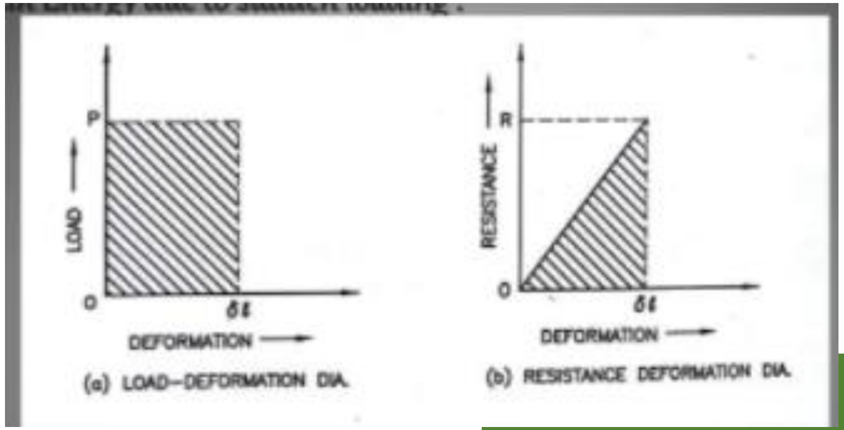
3. Sudden load:- (Imaginary load):-

$$\sigma_{sudden} = 2(\sigma_{gradual}) = \frac{2P}{A}$$

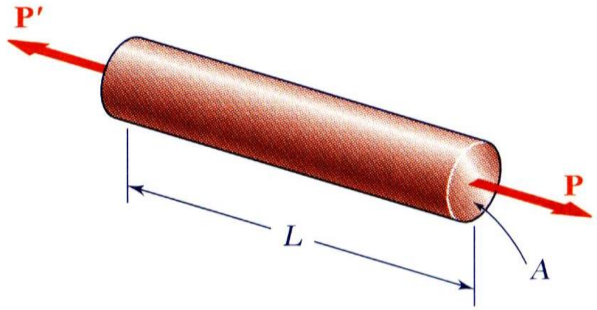
$$\delta_{sudden} = 2(\delta_{gradual}) = \frac{2PL}{AE}$$



$$\therefore \sigma = \frac{P}{A} + \sqrt{\frac{2EPh}{Al} + \frac{P^2}{A^2}}$$

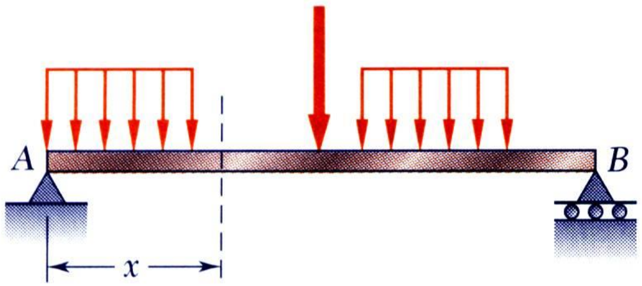


Strain Energy



•For a rod of uniform cross-section,

$$U = \frac{P^2 L}{2AE}$$

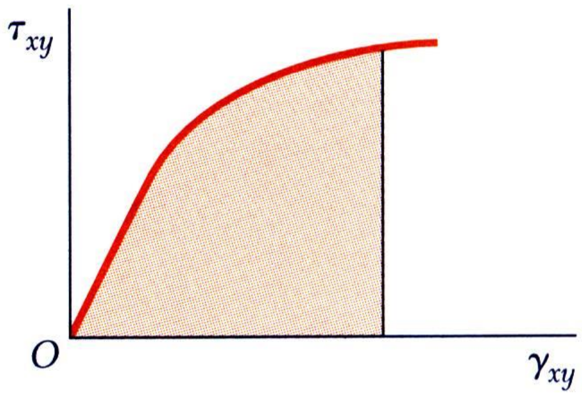


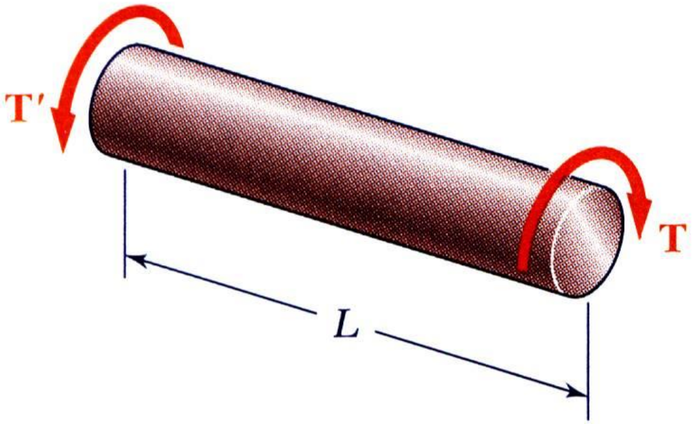
•For a beam subjected to a bending load,

$$U = \int \frac{\sigma_x^2}{2E} dV = \int \frac{M^2 y^2}{2EI^2} dV$$

$$\sigma_x = \frac{M\,y}{I}$$

Strain Energy



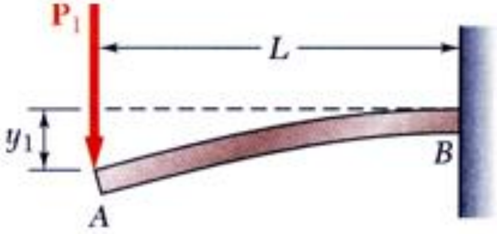


•In the case of a uniform shaft,

$$U = \frac{T^2 L}{2GJ}$$

Stress Formulae

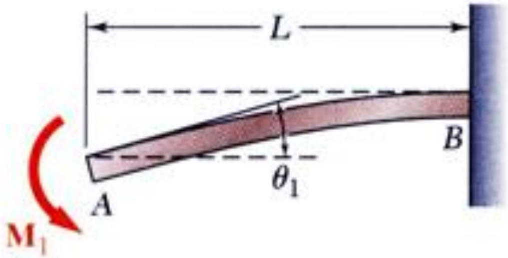
- Transverse load



$$U = \int_0^{y_1} P\,dy = \tfrac{1}{2}P_1 y_1$$

$$= \tfrac{1}{2}P_1\left(\frac{P_1 L^3}{3EI}\right) = \frac{P_1^2 L^3}{6EI}$$

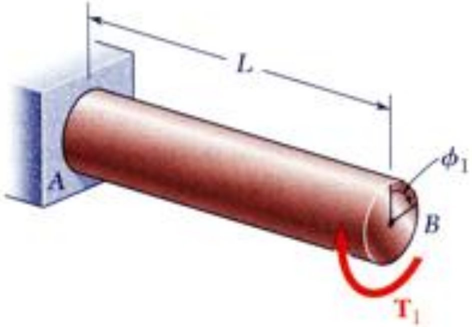
- Bending couple



$$U = \int_0^{\theta_1} M\,d\theta = \tfrac{1}{2}M_1\theta_1$$

$$= \tfrac{1}{2}M_1\left(\frac{M_1 L}{EI}\right) = \frac{M_1^2 L}{2EI}$$

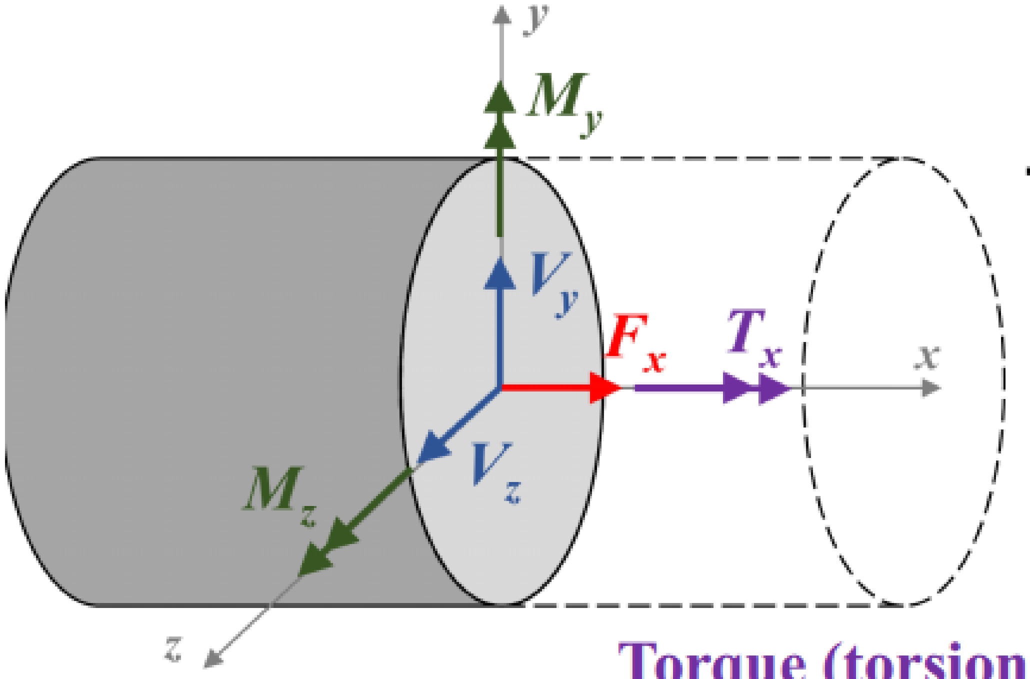
- Torsional couple



$$U = \int_0^{\phi_1} T\,d\phi = \tfrac{1}{2}T_1\phi_1$$

$$= \tfrac{1}{2}T_1\left(\frac{T_1 L}{JG}\right) = \frac{T_1^2 L}{2JG}$$

Stress Formulae



Load	Type of stress	Stress distribution
Axial force F_x	Normal	$\sigma_x = F_x / A$
Shear force V_y	Shear	$\tau_{xy} = \frac{V_y Q}{I_{zz} t}$
Shear force V_z	Shear	$\tau_{xz} = \frac{V_z Q}{I_{yy} t}$
Torque (torsional moment) T_x	Shear	$\tau = T\rho / I_p$
Bending moment M_y	Normal	$\sigma_x = M_y z / I_{yy}$
Bending moment M_z	Normal	$\sigma_x = -M_z y / I_{zz}$

Simple Stress-Strain & Strain Energy

Simple Stress-Strain & Strain Energy

Queries?