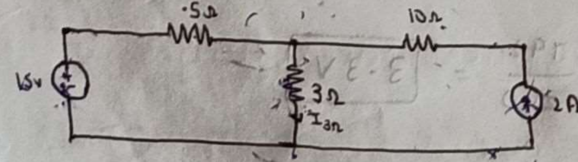
4/11/22

Unit-II {Theorems}

Superposition theorem: In any linear bilateral network consist with the no. of energy sources (i.e current or voltage) & no. of elements. It is possible to determine the response of any element by using superposition theorem directly

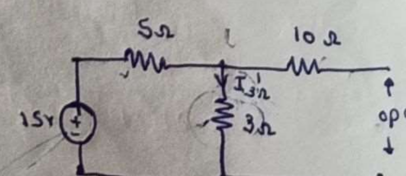
* Response of any element is the algebraic sum of responses caused by individual sources acting alone. {only V & I responses.... not power response directly}
* when individual sources acting alone, the other energy sources must be replaced with their internal resistances.

Q Determine current through 3Ω resistor in given circuit.



* Apply superposition theorem to above circuit.

consider 15V source



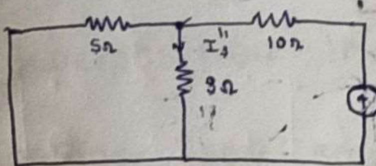
(bcoz resistance in current source in ∞)

current through 10Ω is '0' as it is open.

Let current through 3Ω is $I'_{3\Omega}$

$$I'_{3\Omega} = \frac{V}{R_T} = \frac{15}{8} = \boxed{1.8A}$$

Consider '2A' source



{resistance of voltage circuit is 0}

Let current through 3Ω is $I''_{3\Omega}$.

use current division rule to determine $I''_{3\Omega}$

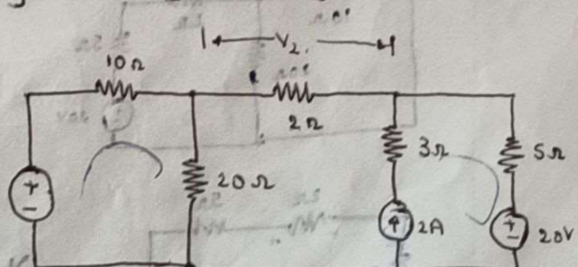
$$I''_{3\Omega} = 2\left(\frac{5}{5+3}\right) = \boxed{1.2 \text{ Amp}}$$

According to superposition principle.

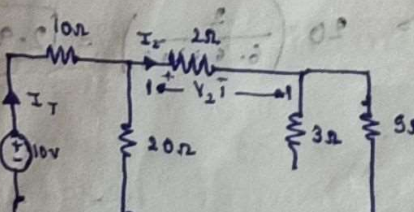
$$I_{3\Omega} = I'_{3\Omega} + I''_{3\Omega}$$

$$I_{3\Omega} = 1.8A + 1.2A = \boxed{3A}$$

2Q Find the voltage across 2Ω resistor in the circuit shown in figure by using superposition theorem.



A Consider '10V' source



{3Ω is neglected}

{5 & 2 ohms are in series & in parallel with 20Ω & in series with 10Ω}

$$I_T = \frac{10}{10 + \left(\frac{20 \times 7}{20+7}\right)} =$$

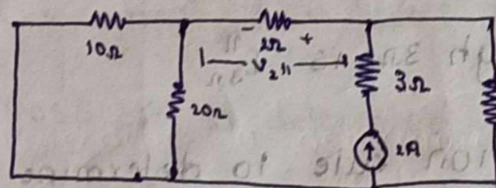
$$\boxed{I_T = 0.65A}$$

So, $I_2 = 0.65\left(\frac{20}{20+7}\right)$ {current division rule

$I_2 = 0.48$ Amp

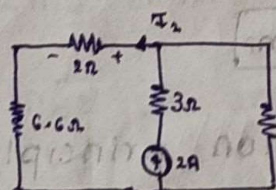
$V_2' = I_2 \times 2\Omega \rightarrow \boxed{0.96\,V} \rightarrow$ ①

* Consider '2A' source



→ 10Ω & 20Ω are in parallel

Effective resistance = 6.6Ω



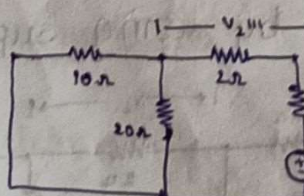
{ 6.6Ω & 2Ω are in Series

Effective resistance = 8.6Ω

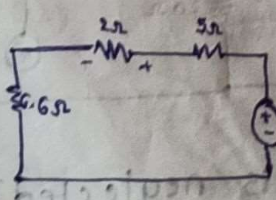
$I_2 = 2\left(\frac{5}{5+8.6}\right) = 0.73\,A$ {5 & 8.6 are in parallel}

$V_2'' = 0.73 \times 2 = \boxed{1.46\,V} \rightarrow$ ②

* Consider 20V source



{ 10Ω & 20Ω are in parallel. Effective resistance = 6.6Ω



$V_2''' = 20\left(\frac{2}{6.6+2+5}\right) = \boxed{2.94\,V} \rightarrow$ ③

According to superposition principle

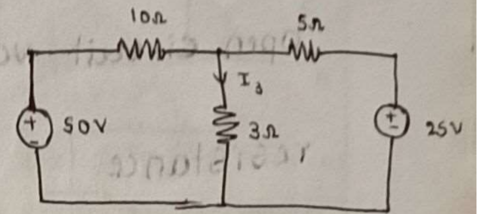
$V_2 = V_2' - V_2'' - V_2''' = 0.96 - 1.46 - 2.94$

$= -3.4\,V$

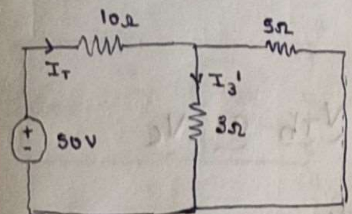
$= 13.41\,V$

$= \boxed{3.4\ V}$

3 Q. For the circuit shown in figure, Determine the current through ~~8~~ 3Ω resistance by using Superposition theorem.



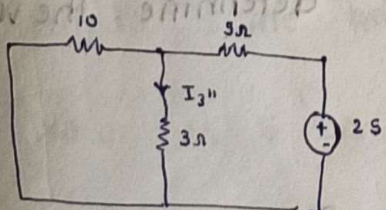
A. Consider '50V' source



$$I_T = \frac{50}{10 + \left(\frac{5\times3}{5+3}\right)} = 4.21\ A$$

$$I_3' = 4.21\left(\frac{5}{5+3}\right) = 2.63\ A \quad \rightarrow ①$$

Consider '25V' source



$$I_T = \frac{25}{\cancel{10+\left(\frac{5\times3}{\ }\right)}\ 5 + \left(\frac{10\times3}{10+3}\right)} = 3.42A$$

$$I_3'' = 3.42\left(\frac{10}{10+3}\right) = 2.63A \quad \rightarrow ②$$

Now According to supperposition principle.

$$I_3 = I_3' + I_3''$$

$$= 2.63 + 2.63$$

$$= \boxed{5.26\ A}$$

* **Thevenin's Theorem**: In any linear bilateral network consist with no. of energy sources (i.e current or voltage} & no. of elements. It is possible to represent the given circuit with

the two terminals equivalent of $\boxed{V\,K.E}$ V_{Th} in series with another two terminals equivalent of R_{Th}, where V_{Th} = Thevinin's voltager or open circuit voltage (V_{oc}). R_{Th} is Thevinin's resistance.

* Procedure to determine V_{Th} or V_C

1) First identify the ~~node~~ resistor in given ckt

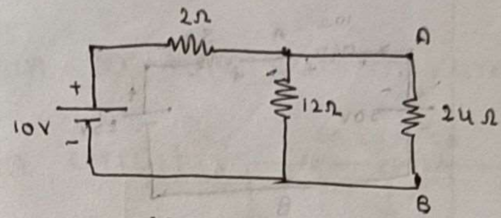
2) Open the ~~node~~ and determine the voltage at open terminal.

* Procedure to determine R_{Th} :

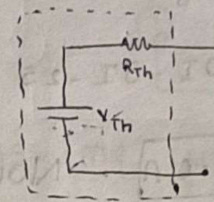
1) Identify the node resistance in given circuit and openit. & Replace all the energy sources with their internal resistances then look back the entire circuit through open circuit terminal.

2) Then calculate the equivalent resistance which is equal to Thevinin's resistance.

Q For the circuit shown below, 24Ω is node resistance. Determine node current by using thevinin's theorem.

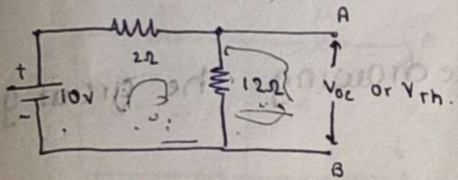


A Here AB is the node.



Thevinins equivalent

For V_{Th} or V_{OC} :



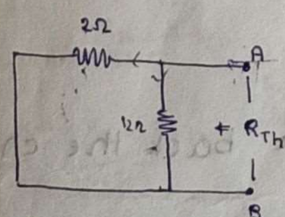
The voltage across AB & Voltage across 12Ω is same bcoz they are in parallel.

* Now 2Ω & 12Ω are in series {AB is open}
=14Ω

So, I (current) = $\frac{10}{14}$ = $\underline{0.71\,A}$

$V_{OC} = V_{Th} = V_{AB} = 0.71 \times 12 = \boxed{8.5V}$

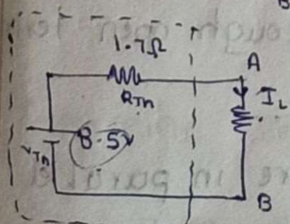
For R_{Th} :



Here 2Ω & 12Ω are in parallel

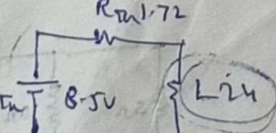
Effective resistance = $\frac{2 \times 12}{2+12} = \boxed{1.72\Omega}$

$R_{Th} = 1.72\,\Omega$



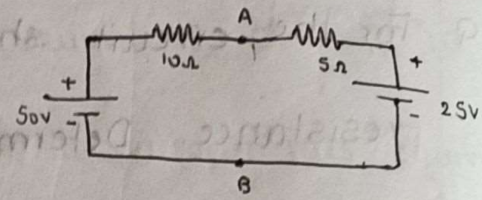
$I_L = \frac{8.5}{1.7+24}$

$\boxed{I_L = 0.33\,A}$

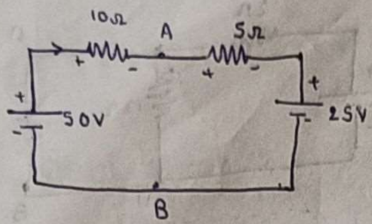


Q Determine the thevinin's equivalent circuit across AB, for the figure shown below.

A. AB is already open.



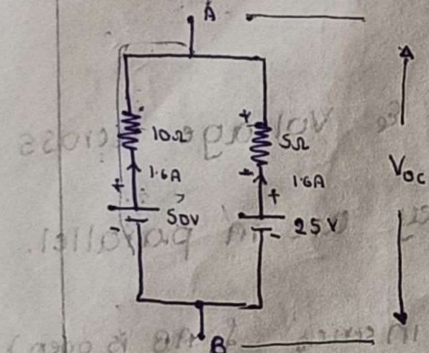
For V_{oc} or V_{Th}:



Assume current in circuit is I. Apply KVL

$50 - 10I - 5I - 25 = 0$

$\boxed{I = 1.6\ A}$. Now Redrawing the circuit gives

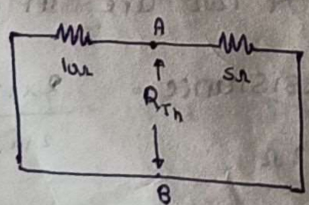


Voltage across A & B is equal to voltage accross 50V & 10Ω & also equal to voltage across 25V & 5Ω

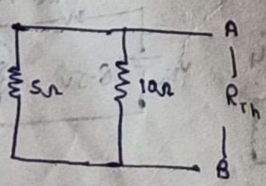
Now $V_{oc} = 50 - 1.66 \times 10 = \boxed{33.33\ V}$

or $V_{oc} = -25 - 5 \times 1.666 = -33.33\ V = |33.33\ V|$

For R_{Th}:



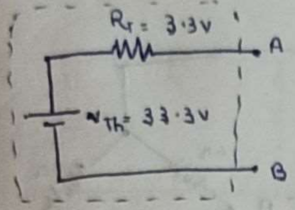
→ Now look back the entire circuit through open terminal



10Ω & 5Ω are in parallel

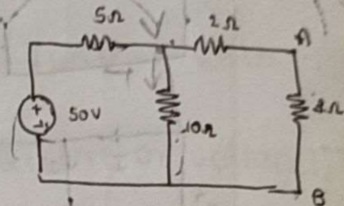
$R_{Th} = \frac{5 \times 10}{5 + 10} = \boxed{3.33\Omega}$

Thevinin's equivalent.

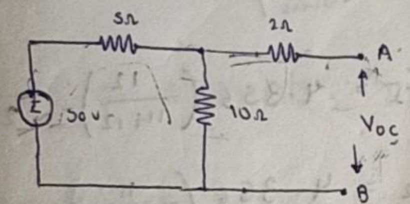


$$I = \frac{33.3\,V}{3.33V} = \boxed{10\text{ Amp}}$$

Q. Use thevinin's theorem to find out the current through 3Ω in given circuit.



* For V_{oc} or V_{Th}.



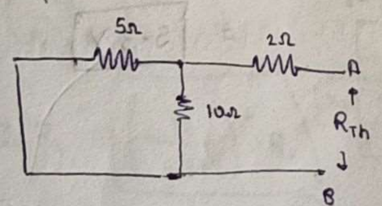
Assume current I

$$I = \frac{50}{10+5} = 3.33\,A$$

V_{oc} or V_{Th} or V_{AB} = $3.33 \times 10 = \boxed{33.33\text{ V}}$

bcoz voltage across V_{AB} & Voltage across 10Ω is equal as they are in parallel.

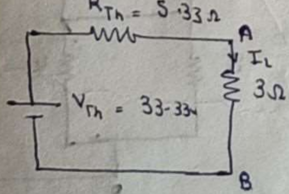
For R_{Th}:



$$R_{Th} = 2 + \left(\frac{5 \times 10}{5+10}\right)$$

$$R_{Th} = \boxed{5.33\,\Omega}$$

Thevinin's equivalent :



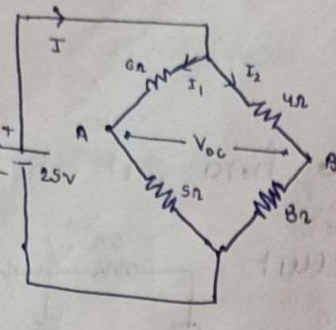
$$I_L = \frac{33.33}{5.33+3}$$

$$\boxed{I_L = 4\text{ amp}}$$

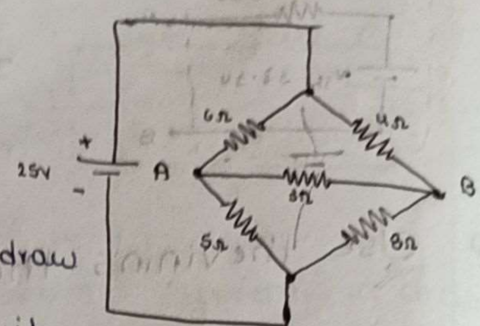
Q. Determine Thevinin's equivalent of given circuit and also determine current through load resistor

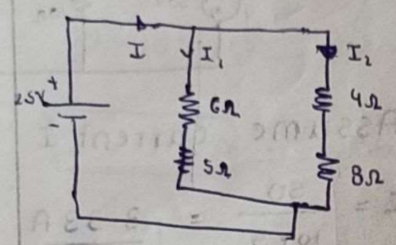
A. In the given circuit AB is the load & (rxo&o/)

A For V_{oc} or V_{Th}



Now redraw the circuit.



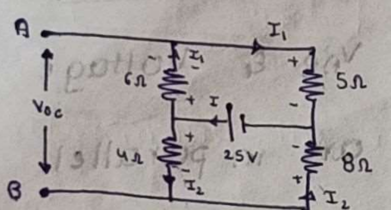


$$I = \frac{25}{\left(\frac{11 \times 12}{11+12}\right)} = 4.356 \text{ Amp}$$

$$I_1 = 4.356\left(\frac{12}{11+12}\right) = 2.272 \text{ Amp}$$

$$I_2 = 4.356\left(\frac{11}{11+12}\right) = 2.083 \text{ Amp}$$

Redraw the circuit

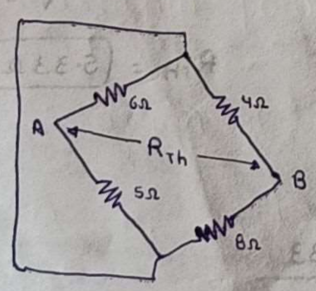


$$V_{oc} = V_{Th} = V_{AB} =$$

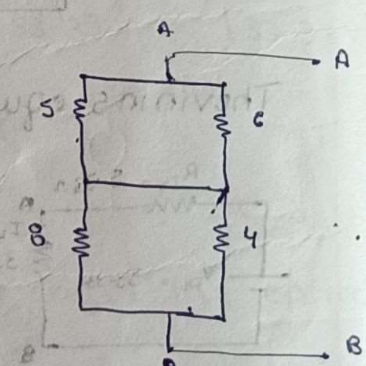
$$= 6 \times 2.27 - 4 \times 2.083$$

$$= \boxed{5.3\text{ V}}$$

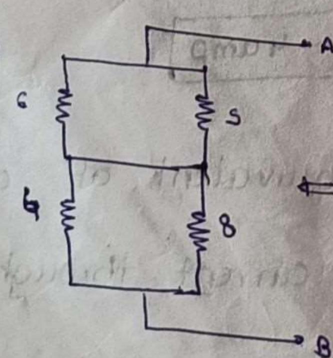
For R_{Th}:



Redraw the circuit



Now

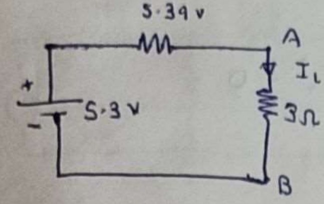


$\Leftarrow R_{Th}$

$$R_{Th} = \left(\frac{6 \times 5}{6+5}\right) + \left(\frac{5 \times 4}{5+4}\right)$$

$$R_{Th} = 5.39\,\Omega$$

Thevinin's equivalent, ..



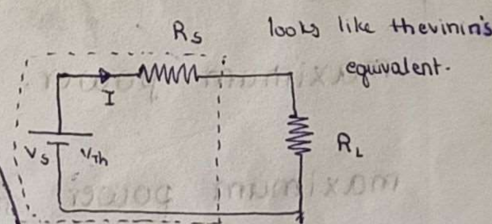
$$I_L = \frac{5.3}{5.39 + 3} = \boxed{0.63\ A}$$

* maximum power transfer theorem:-

In any linear bilateral network consist with the no. of energy sources (i.e current or voltage), & no. of elements. It is possible to transfer the maximum power from source to the load, when the load resistance = source resistance.

* Variable resistance = Rheostat.

Here $I = \left(\frac{V_s}{R_L + R_s}\right)$



Power at load $= I^2 R_L = \boxed{\left(\frac{V_s}{R_L + R_s}\right)^2 \cdot R_L} \rightarrow (1)$

Now differentiate w.r.t R_L & equate to zero.

$$\frac{dP}{dR_L} = 0 \rightarrow \frac{d}{dR_L}\left(\frac{V_s^2}{(R_L+R_s)^2}\right) R_L = 0$$

$$\frac{V_{Th}^2}{4R_L^2}$$

R_T

$$V_s^2 \frac{d}{dR_L}\left(\frac{R_L}{(R_L + R_s)^2}\right) = 0$$

$$V_s^2 \left[\frac{(R_L + R_s)^2 - R_L 2(R_L + R_s)}{(R_L + R_s)^4}\right] = 0$$

$$(R_L + R_s)^2 - 2R_L(R_L + R_s) = 0$$

$$R_L^2 + \cancel{2R_sR_L} + R_s^2 - \cancel{2R_LR_s} - 2R_L^2 = 0$$

$$R_s^2 - R_L^2 = 0$$

$$R_s^2 = R_L^2$$

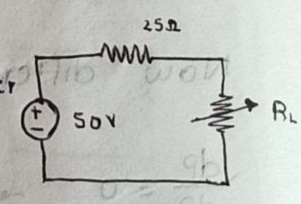
$\boxed{R_s = R_L}$ This means, when $R_s = R_L$, the power is maximum {condition is $R_s = R_L$}

Now

$$\boxed{P_{max} = \frac{V_s^2}{4R_L}} \quad \{R_L = R_s \text{ \& from eq } 1\}$$

or) $$\boxed{P_{max} = \frac{V_{Th}^2}{4R_L}}$$

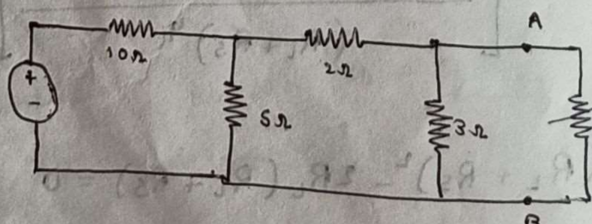
Q* Determine the value of R_L when the load draw maximum power. Also find the value of maximum power for the circuit shown below.



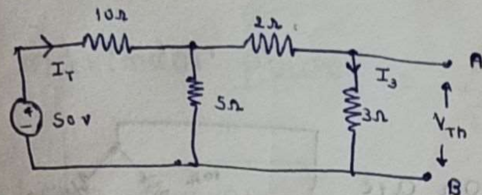
A Condition of max power transfer is $R_L = R_s = 25\Omega$

Max Power $= \frac{V_s^2}{4R_L} = \frac{(50)^2}{4 \times 25} = \frac{2500}{100} = \boxed{25 \text{ watts}}$

Q Determine max power delivered to load in given circuit.



* For V_{Th}.



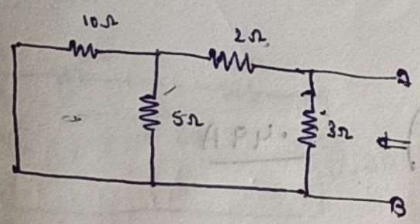
$$I_T = \frac{50\,v}{10 + \left(\frac{5 \times 5}{5+5}\right)} = \underline{\underline{4A}}$$

{2 & 3Ω are in series & parallel with 5Ω & in series with 10Ω}

Now $I_3 = 4\left(\frac{5}{5+5}\right) = \boxed{2A}$

$V_{Th} = 2 \times 3 = \boxed{6}\ v$

For R_{Th}:



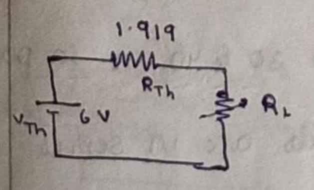
Look back the entire circuit through open terminal

$$R_{Th} = \left[\left(\frac{10 \times 5}{10+5}\right) + 2\right] \parallel 3\Omega$$

{10 & 5 are in parallel & in series with 2Ω & total in parallel with 3Ω}

$R_{Th} = \underline{1.919\Omega}$

Arrange the circuit in thevinin's model.



By max power transfer theorem,

for max power, $R_L = R_S = R_{Th}$

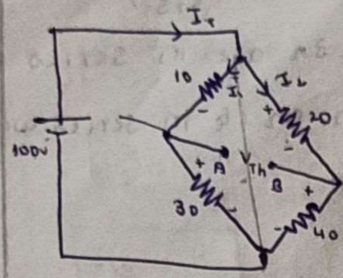
$\boxed{R_L = 1.919\Omega}$

& $P_{max} = \frac{6^2}{4(1.919)}$

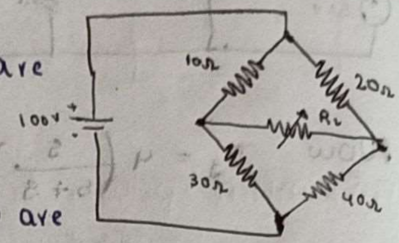
$\boxed{= 4.68}$ watts

Q Determine the maximum power transfered to the load in given circuit.

A For V_{Th}



{10 & 30Ω are in series & 20 & 40 are in series. They both are in parallel.



$$I_T = \frac{100}{\left(\frac{40\times 60}{40+60}\right)} = \boxed{4.16\,A} \quad \&$$

$$I_{31} = (4.16)\left(\frac{60}{60+40}\right) = \underline{2.49\,A}$$

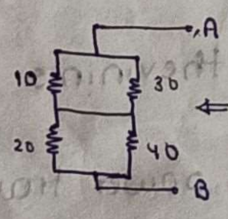
$$I_2 = (4.16)\left(\frac{40}{60+40}\right) = \underline{1.66\,A}$$

$$V_{Th} = 10\times 2.49 - 20\times 1.66$$

$$V_{Th} = |18.24\,V|$$

For R_{Th}:

Diagram circuit



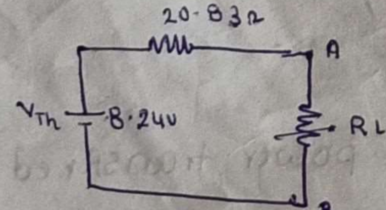
← R_{Th}

Now 10 & 20 are in parallel also 30 & 40 are in parallel. Totals are in series.

$$\text{So } R_{Th} = \left(\frac{10\times 30}{10+30}\right) + \left(\frac{20\times 40}{20+40}\right)$$

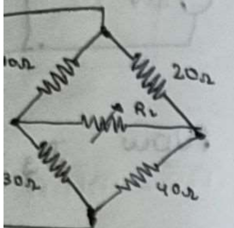
$$R_{Th} = \underline{20.83\,\Omega}$$

Draw the circuit as thevinin's equivalent



Here $R_L = R_S = R_{Th}$

$$\boxed{R_L = 20.83}$$



are in parallel.

& 20 are in parallel

e 40 are in parallel.

e in series

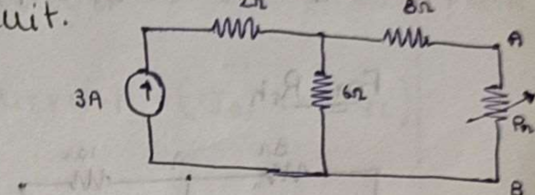
equivalent

$R_S = R_{Th}$

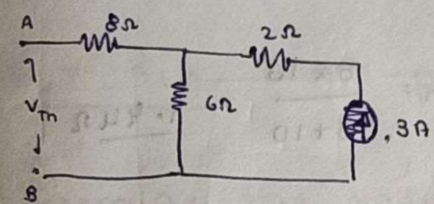
maximum power $= \dfrac{8.24^2}{4(20.83)}$

$$\boxed{P_{max} = 0.81 W}$$

Q Determine the maximum power transfered to the load in given circuit.



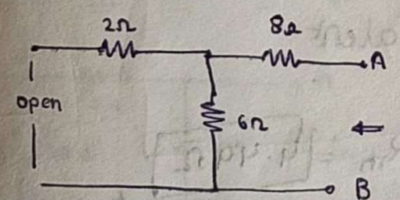
A For V_{Th} :-



{8Ω is neglected}

$V_{Th} = 3(6) = 18V$ { V_{Th} = Voltage across 6Ω}

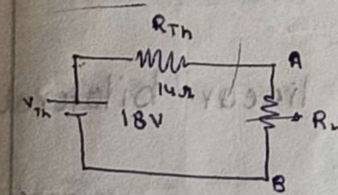
For R_{Th}



{2Ω is ignored}

Now $R_{Th} = 8 + 6 = 14Ω$

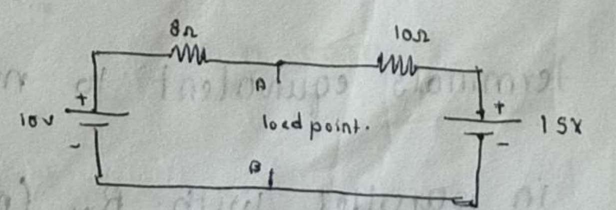
Bring to thevinin's equivalent



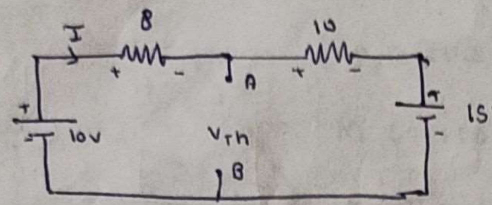
$R_L = R_{Th} = 14Ω$

$P_{max} = \dfrac{18^2}{4 \times 14} = \boxed{5.78 \text{ watts}}$

Q Determine the value of load resistance in given circuit for the condition to transfer the maximum power.



A For V_{Th}



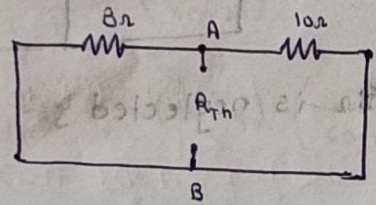
$10 - 8I_1 - 10I - 15 = 0$

$I = -0.271$ Amp.

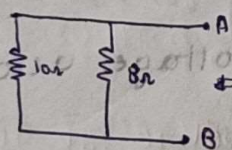
$I = -0.27$ A

$V_{Th} = 10 - 8(-0.27) = \underline{12.16V}$ {V_{Th} = voltage across 10V & 8Ω}

For R_{Th}

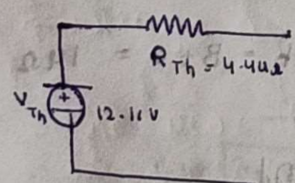


$R_{Th} = \frac{8 \times 10}{8+10} = \underline{4.44Ω}$



$R_{Th} = \frac{8 \times 10}{8+10} = \underline{4.4Ω}$

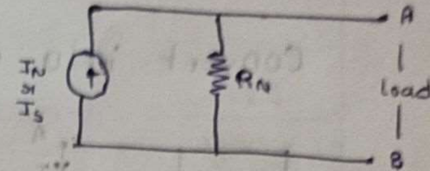
Draw thevinin's equivalent



$R_L = R_{Th} = \boxed{4.44Ω}$

* Norton's Theorem: In any linear bilateral network consist the no. of energy sources (: current or voltage) & no. of elements. It is possible to replace the given circuit with two terminals equivalent to nortons current I_N & in parallel with R_N (Nortons resistance)

Norton's Equivalent →



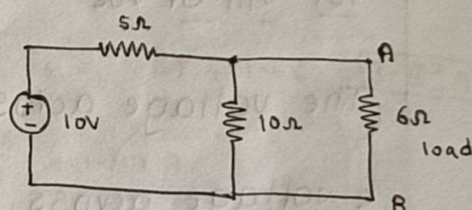
* Procedure to find I_N

1) Identify the load given in the circuit and make it as short circuit.

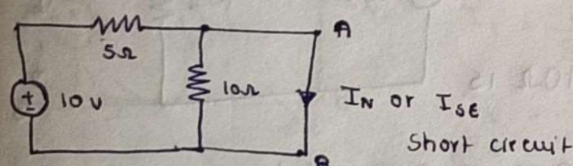
2) Now the current through short circuit path is equal to Norton current (I_N or I_{sc})

* $R_N = R_{Th}$. same procedure to find R_{Th} & R_N.

Q Determine the value of load current in given circuit by using norton's theorem, w.r.t.



* ~~Aor~~ For I_N or I_{sc}:

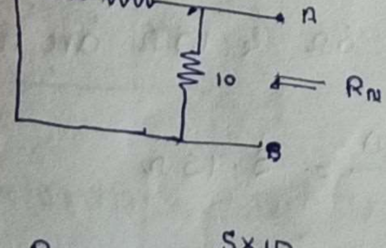


10Ω & and short circuit (0Ω) are in parallel.

So, effective resistance $= \frac{10 \times 0}{10+0} = 0\Omega$

I_N or $I_S = \frac{10}{5} = \underline{2A}.$

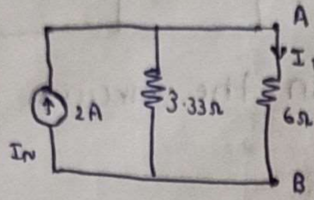
For R_N or R_{Th} :-



lookback, Now 10Ω & 5Ω are in parallel

$R_{Th} = \frac{5 \times 10}{10+5} = 3.33\,\Omega$

Convert into norton's equivalent

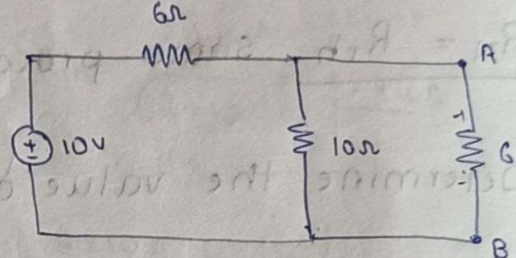


Now $I_L = 2\left(\frac{3.33}{3.33+6}\right)$

$I_L = \boxed{0.41A}$

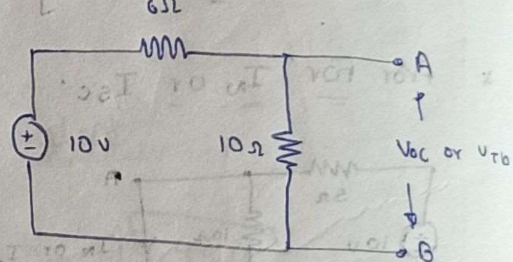
Q Determine the load current for same circuit using thevinin's theorem

Given circuit



considering AB is the required load

<u>For V_{Th} or V_{oc}</u>



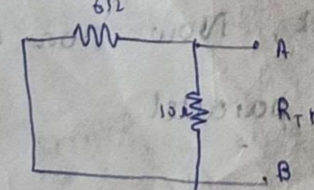
The voltage across AB & voltage across 10Ω is same bcoz they are in parallel

Now 6Ω & 10Ω are in series, $R_{eq} = 6 + 10 = 16Ω$

$V_{oc} \neq$ Current $= \frac{10}{16} = 0.625$ Amp

$V_{oc} = 10 \times 0.625 = 6.25$ Volts

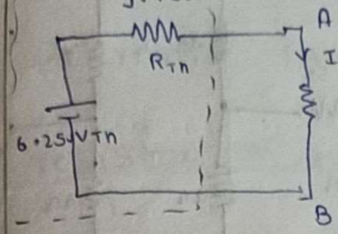
<u>For R_{Th} :</u>



Here 6Ω & 10Ω are in parallel

$\frac{6 \times 10}{16} = 3.75Ω$

The load current is

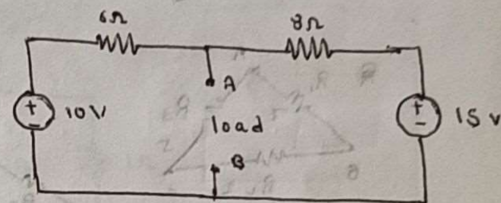


The load current = $\frac{6.25}{3.75 + 6\Omega}$

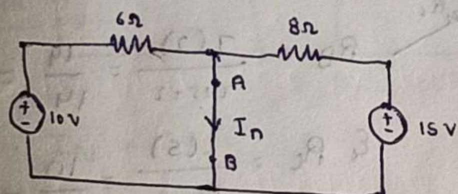
{load is 6Ω}

$I_L = \frac{6.25}{9.75} = \underline{0.64\ A}$

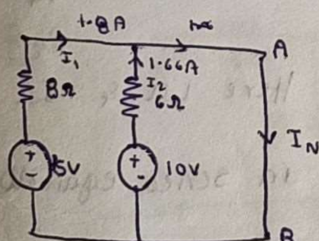
Q Determine the Nortons equivalent for given circuit



A For I_N or I_{sc} :



Now Redraw the circuit as follows

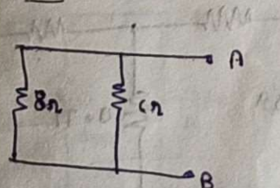


I_N or I_S = 1.87 + 1.66 = $\boxed{3.53\ A}$

$I_1 = \frac{15}{8} = 1.87\ A$

$I_2 = \frac{10}{6} = 1.66\ A$

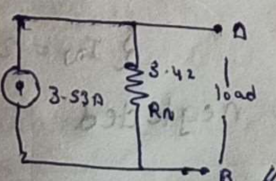
For R_N



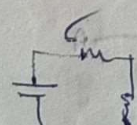
$R_N = \frac{8 \times 6}{8 + 6} = \boxed{3.42\ A}$

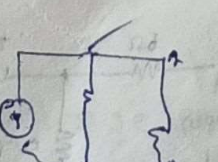
{8Ω & 6Ω are in parallel}

Now draw Norton's equivalent.



//.

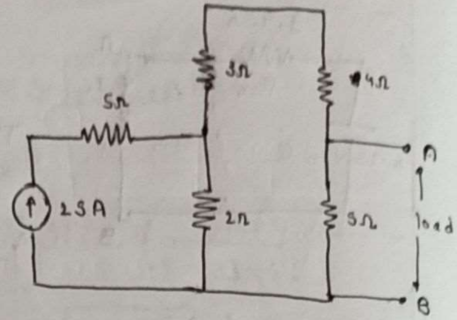




I_N (R_N)

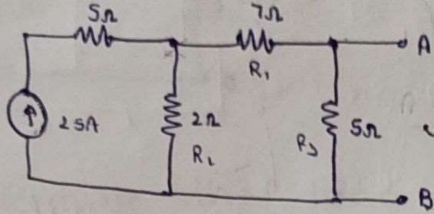
Q Determine the Nortons equivalent circuit for the circuit shown in figure.

A Here 3Ω & 4Ω are in



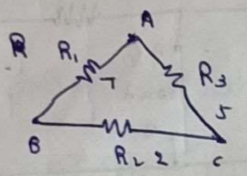
Series, $E_q = 7\Omega$, Circuit

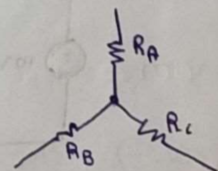
is



Let current flowing through AB is I_N.

converting 7Ω, 2Ω, 5Ω is starconnection.



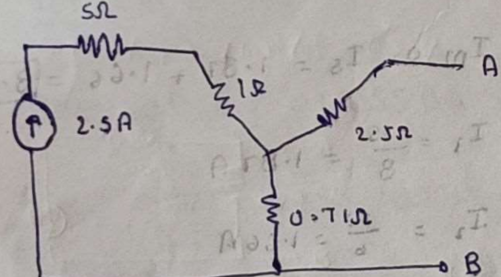


$$R_A = \frac{(7)(5)}{7+5+2} = \frac{35}{14} = 2.5\Omega$$

$$R_B = \frac{7(2)}{7+5+2} = \frac{14}{14} = 1\Omega$$

$$\& \; R_C = \frac{2(5)}{7+5+2} = \frac{10}{14} = 0.71\Omega$$

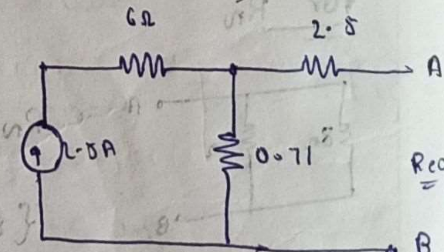
Redraw the circuit



Here 1Ω & 5Ω are in series equivalent =

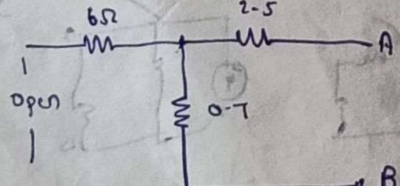
6Ω

so Redraw circuit



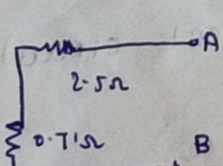
2.5Ω & 0.71Ω are in parallel for R_N

Replace source with its internal resistance.



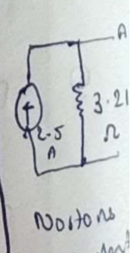
$\{ I_N = 2.5A \}$

$\{6\Omega$ is neglected



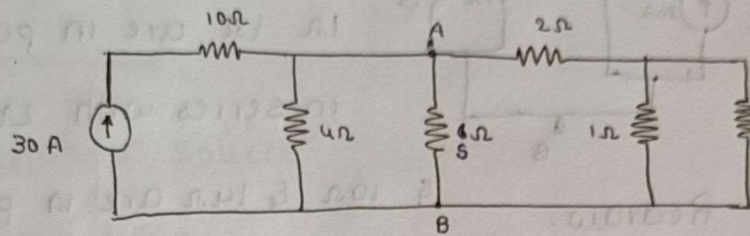
$\{2.5\Omega$ & 0.75 are in series$\}$

equivalent $= 2.50 + 0.71 = 3.21\Omega$

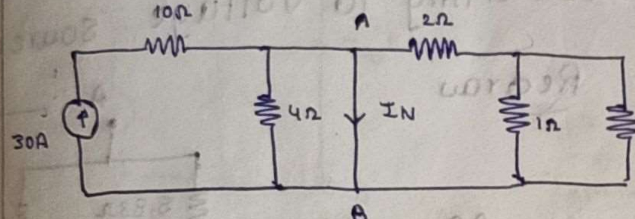


Nortons equivalent

Q. Determine the current flowing through 5Ω resistor in a given circuit by using norton's theorem.

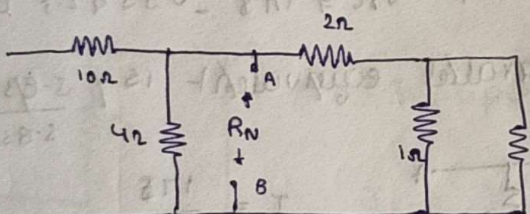


A. I_N or I_{sc}



$I_N = 30A$

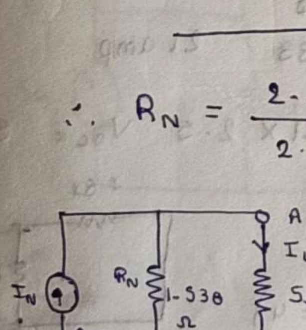
R_N :



1Ω & 1Ω are in parallel & in Series with 2Ω

Total resistance = 2.5Ω

$$\therefore R_N = \frac{2.5 \times 4}{2.5 + 4} = 1.538\,\Omega$$

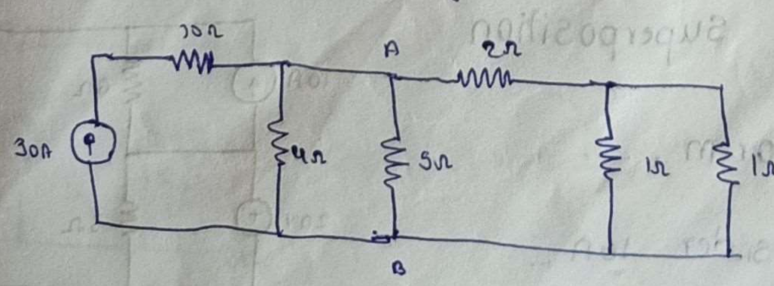


$$I_L = 30\left(\frac{1.538}{1.538 + 5}\right)$$

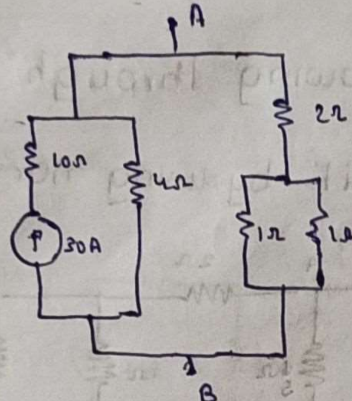
$$\underline{I_L = 7.05A}$$

* Solve it by another method.

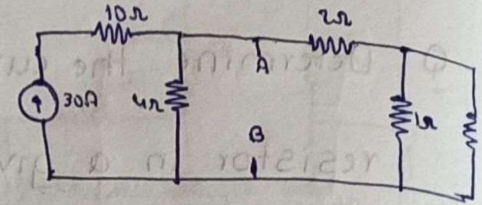
Given circuit converting into voltage source



open AB



Redraw.



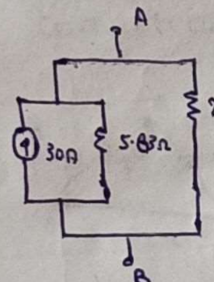
1Ω 1Ω are in parallel, eq = 0.5Ω

in series with 2Ω = eq = 2.5Ω

& 10Ω & 14Ω are in parallel = $\frac{10 \times 14}{24}$

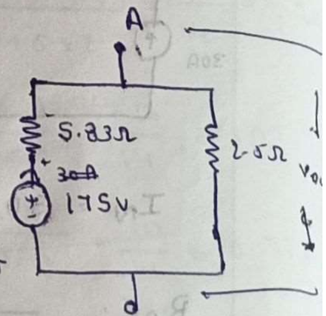
= 5.83Ω

Redraw.



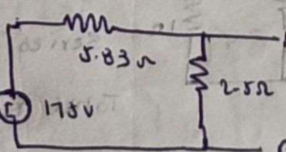
converting to voltage source

Redraw



* voltage across AB = voltage across 2.5Ω., V_{oc} ≠ 175 _ 5.83 & 2.5

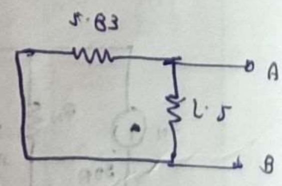
are in parallel equivalent is ($\frac{5.83 \times 2.5}{5.83 + 2.5}$



$I = \frac{175}{8.33} = 21$ amp

$V_{oc} = 21 \times 2.5$ $V_{oc} = 52.5$ V

By calculating $R_{Th} = 1.74\Omega$



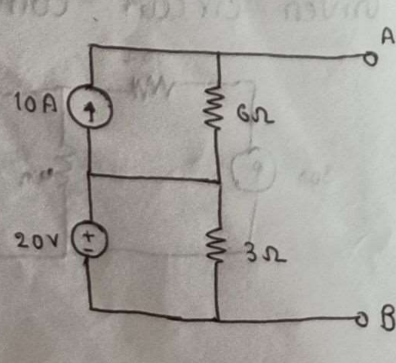
By thevinins = $\frac{52.5V}{1.74 + 5}$ {load = 4}

$R_{Th} = \frac{5.83 \times 2.5}{5.83 + 2.5} = 1.74$

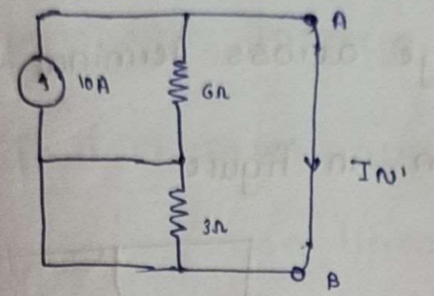
→ $I_L = \frac{52.5V}{6.74} = \boxed{7.7 \text{ amp}}$

Q Replace the given network shown in figure in a current source in parallel with resistance

A By superposition theorem,

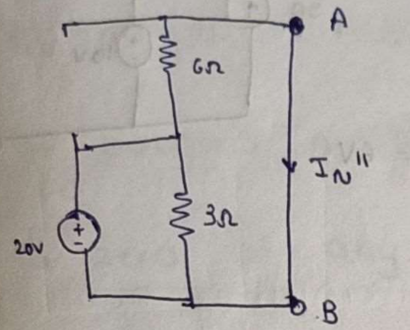


consider 10A ..



$\underline{I_N' = 10\ A}$

Consider 20V source.

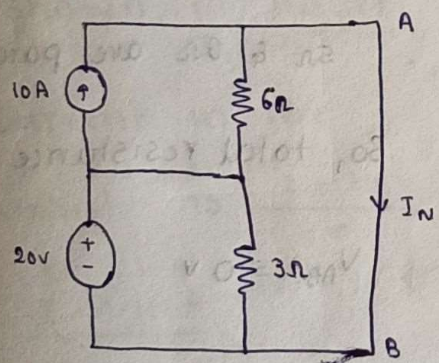


$$I = \frac{20}{\frac{6\times 3}{6+3}} \quad \{6 \,\&\, 3 \text{ are } \| \text{el}\}$$

$$I = 10A$$

$$I_N'' = 10\text{Amp}\left(\frac{3}{6+3}\right) = \frac{30}{9} = \underline{\underline{3.33\text{ am}}}$$

Now

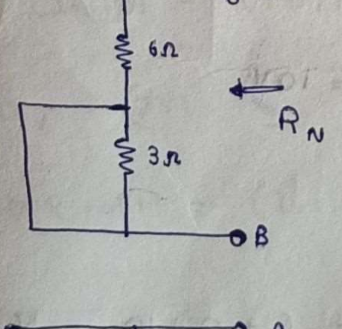


Superposition priciple

$$I_N = I_N' + I_N'' = 10 + 3.333$$

$$= \underline{\underline{13.33\ A}}$$

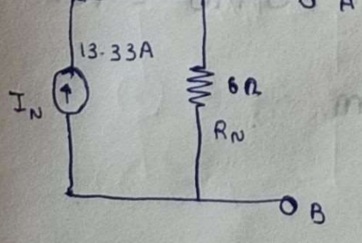
For R_N



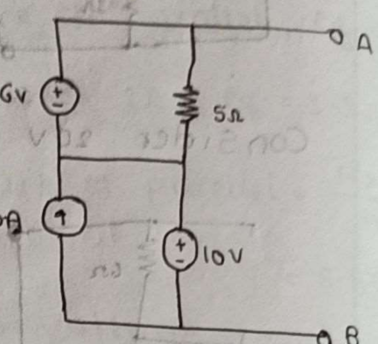
{ 3Ω & 0Ω are in parallel

total = 0Ω

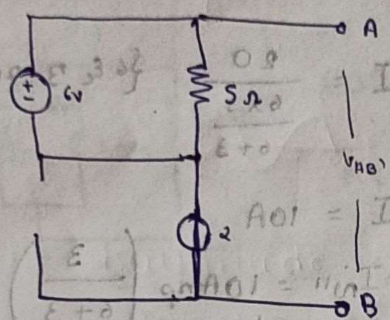
so $\underline{\underline{R_N = 6\Omega}}$



Q Determine the voltage across terminal A & B in the circuit shown in figure.

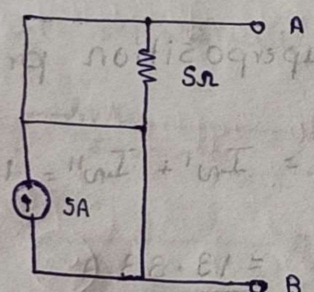


A: Consider 6V source, by superposition theorem.



$V_{AB}' = 6V$

Consider 5Amp source,

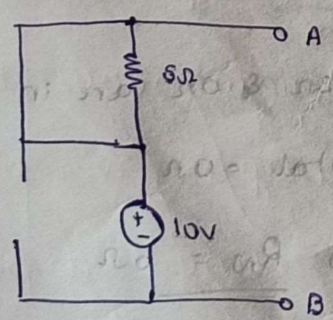


5Ω & 0Ω are parallel

So, total resistance = 0

$V_{AB}'' = 0\ V$

Consider 10 V source,



$V_{AB}''' = 10V.$

So, voltage b/w $V_{AB} = 6 + 0 + 10 = 16V$

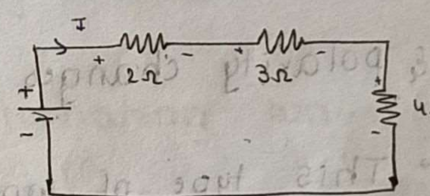
By superposition system.

* Tellegen's Theorem

This theorem applicable to lumped elements, linear & non linear networks & also time variant & time invariant circuits.

* For a given circuit, the algebraicr sum of powers in all branches in a circuit is zero at any instant

Q Verify the tellegen's theorem for the circuit shown in figure.



* Current in the circuit is $\frac{2+3+4}{10} = \frac{1}{I}$

$\Sigma VI = 0$

$\underline{I = 1.111\,A}$

$p = 0$

By tellegens theorem,

$10(1.111) - (1.111)^2(2) - 3(1.111)^2 - 4(1.111)^2$

$= 0$, (or)

$10 \times 1.111 = (1.111)^2(2) + 3(1.111)^2 + 4(1.111)^2$