

Q6: Mathematical Representation of Recursively enumerable Language is Turing Machine

→ Turing Machine is a 7-tuple variable

$$M = (Q, \Sigma, \Gamma, \beta, \delta, q_0, F)$$

where

Q = Set of all states

Σ = Set of input symbols

Γ = Set of all tape symbols

β = Blank symbol (B)

q_0 = Initial state

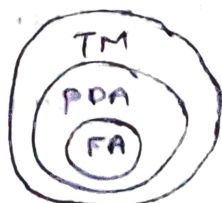
F = Set of all final states

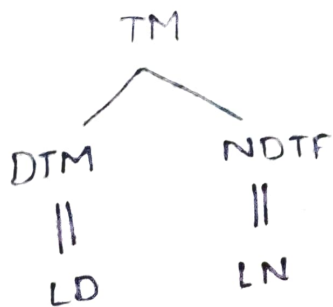
$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{L/R\} \rightarrow$ Transition Function

Note:-

- ① FA with memory and with Read and write capability is known as Turing Machine.
- ② The abstract model of real computer system is Turing machine
- ③ The expressive power of Turing Machine and real computer system is same

- ④ The TM has infinite tape which is 2-way infinite and is divided into cells such that each cell contain only one input symbol.
- ⑤ The empty cells of infinite tape is filled with blank symbol B.
- ⑥ Tape header of the TM is bidirectional
- ⑦ It can read data from the tape and write the data over the tape
- ⑧ TM works as
 - ① Language Acceptor
 - ② Language Generator
 - ③ Transducer
- ⑧ The language which is accepted by TM is known as Recursively Enumerable Language.
- ⑨ The language accepted by FA and PDA is also accepted by TM.
- ⑩ TM also accepts some of languages which are not accepted by PDA





$$LD = LN$$

Instantaneous Description (ID) TM:

ID describe the next move of the Turing Machine

→ Next move of TM depends on 2 entities

$$QXT \longrightarrow QXT \times L/R$$

$$\delta(q_i, a) = (q_j, x, L/R)$$



Acceptance by TM

After taking the string TM has 3 possibilities

- ① May go to Final Halt
- ② May go to Non-final Halt
- ③ May go to infinite loop

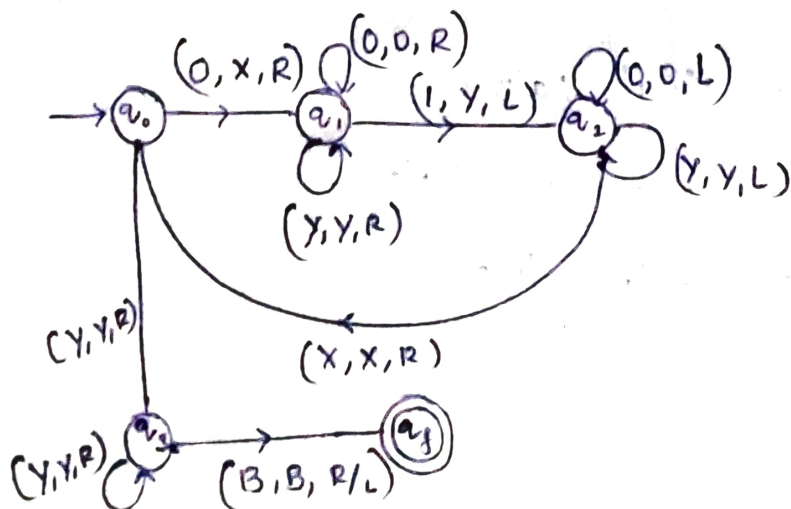
→ Final Halt (FH)
 After processing input string if the TM reaches the final halt then the input string is accepted by TM

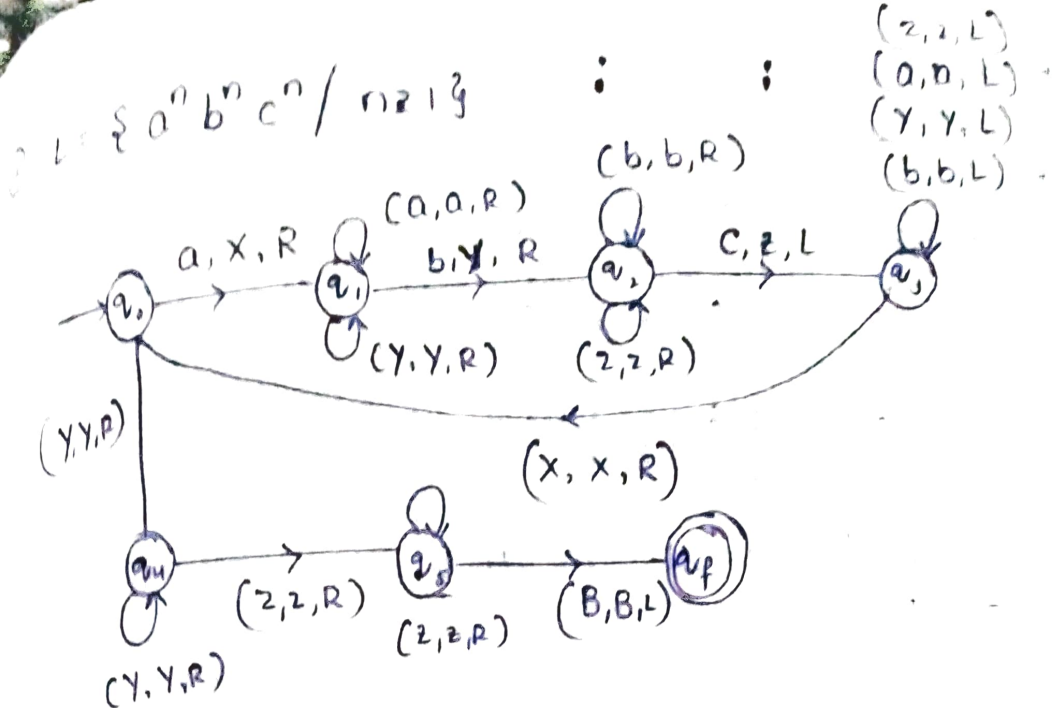
→ Non-Final Halt (NFH)
 The TM reaches Non-final Halt then input string is rejected by TM.

→ Loop
 The TM goes to infinite loop then the accessibility is undecidable.

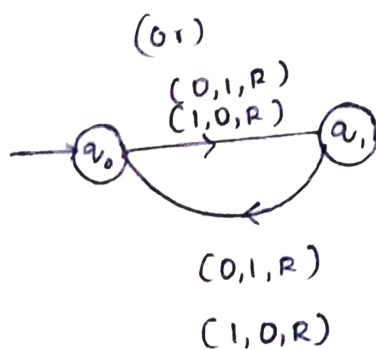
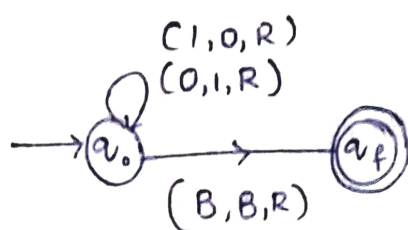
P: Construct Turing Machine for the language
 $L = \{0^n 1^n / n \geq 1\}$

Sol: $L = \{01, 0011, 000111, 00001111, \dots\}$

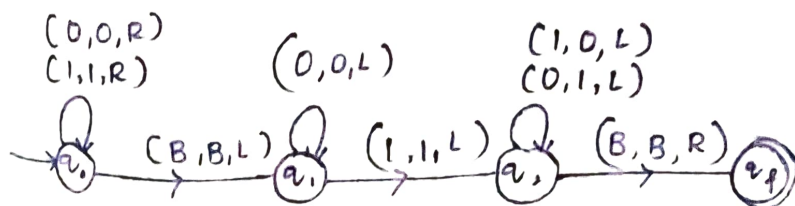




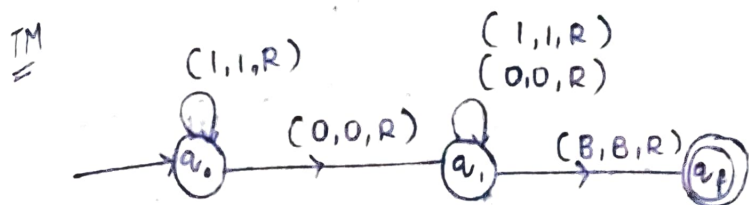
③ Turing Machine generates 1's complement



④ Turing Machine generates 2's complement



S	0	1	B
q_0	$a, 1, D, R$	$a, 0, 1, R$	Halt
a_1	$a, 0, R$	$a, 1, 1, R$	a_f, B, R
f	Halt	Halt	Halt



Ex: $L = \{ a^n b^n \mid 1 < n < 2021 \}$

- | | |
|--------|--------------------|
| a) RL | ① only a |
| b) CFL | ② a, b, but not c |
| c) CSL | ③ b, c, but not d |
| d) REL | ④ All a, b, c, d ✓ |

Recursive set / Recursively enumerable set

The language which is accepted by TM is known as TM recognisable language

TM recognisable language is of 2 types

- ① Recursive set or Recursive language
- ② Recursively Enumerable set

Recursive Set (RS)

The language L which is accepted by TM for which the membership property is decidable

(or)

The language L is accepted by TM is said to be RS if for every $x \in L$, Turing Machine goes to Final Halt, for every $x \notin L$ TM goes to Non Final Halt.

Recursive Enumerated Set (RES)

The language which is accepted by TM for which the membership property is undecidable is RES.

(or)

The language L accepted by TM is said to be RES if for every $x \in L$, TM goes to Final Halt or for every $x \notin L$, TM goes to Non-final Halt or Infinite loop

closure properties of RS

closed	Not closed
① Union	① Kleen closure
② Intersection	② Homomorphism
③ Concatenation	③ Substitution
④ Complement	④ Quotient with RL
⑤ Inverse Homomorphism	
⑥ Reversal	
⑦ Intersection with RL	

closure properties of RES

closed	Not closed
① Union	① Compliment
② Concatenation	
③ Kleen closure	
④ Intersection	
⑤ Substitution	
⑥ Homomorphism	
⑦ Inverse Homomorphism	
⑧ Intersection with RL	
⑨ Quotient with RL	