ELL 100 – Introduction to Electrical Engineering

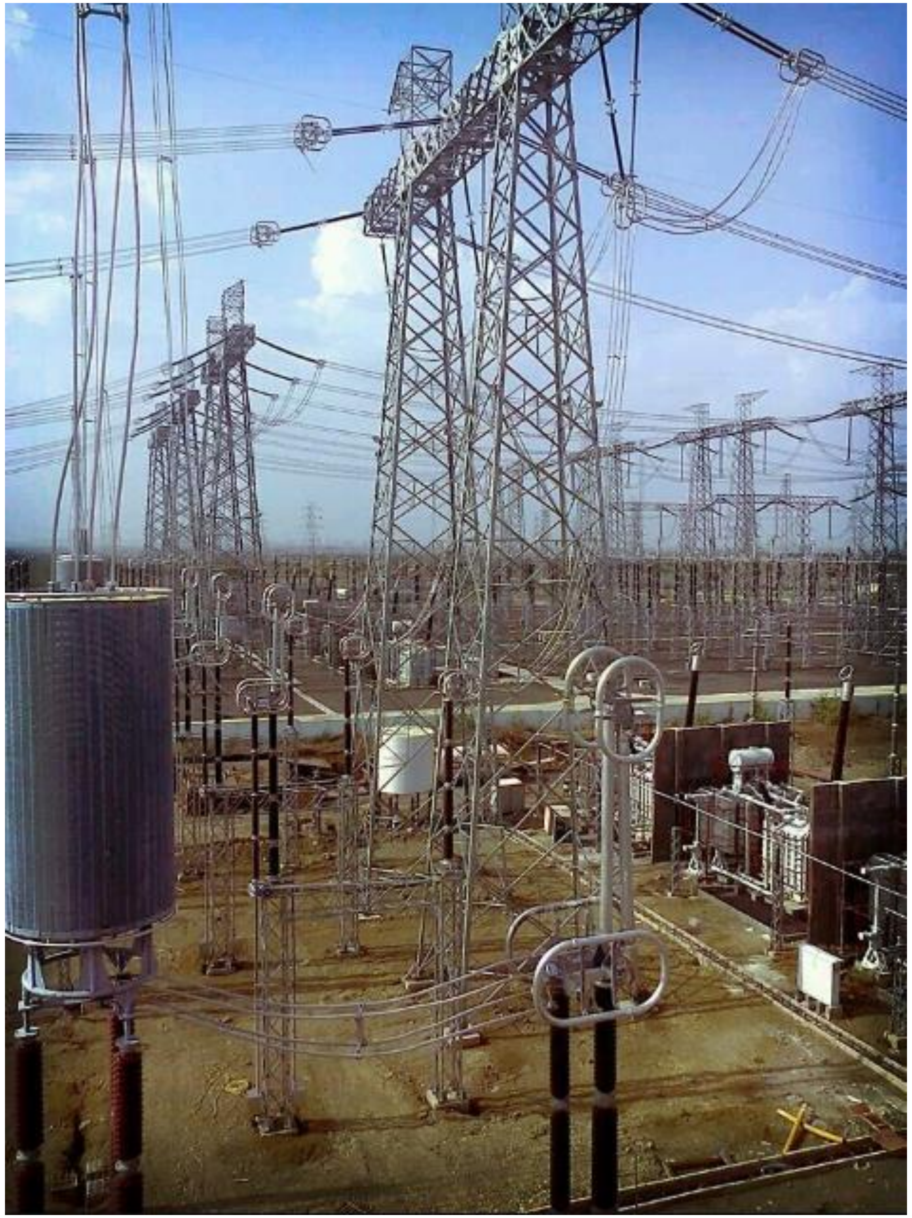
LECTURE 32: TRANSFORMERS - I

Outline

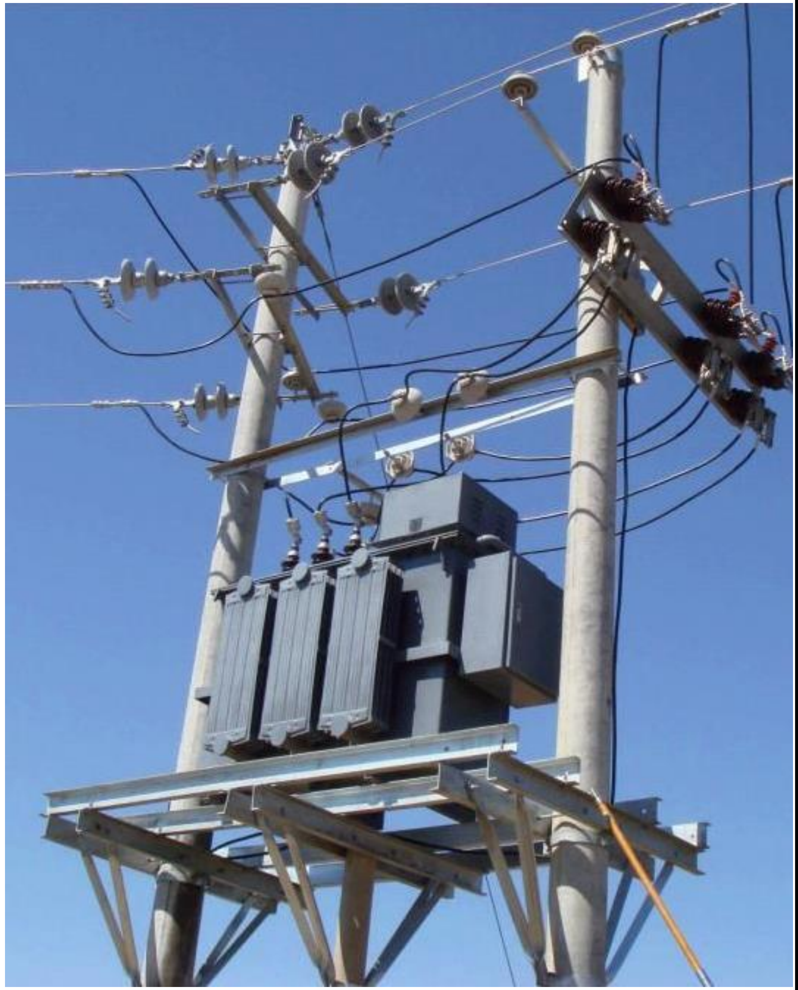
- ❑ Need for and types of transformers
- ❑ Magnetically coupled circuits & ideal transformer
- ❑ Working principle of transformer
- ❑ Practical transformer: various types of losses
- ❑ Numerical examples/exercises

TRANSFORMERS IN POWER GENERATION

TRANSFORMERS IN POWER TRANSMISSION

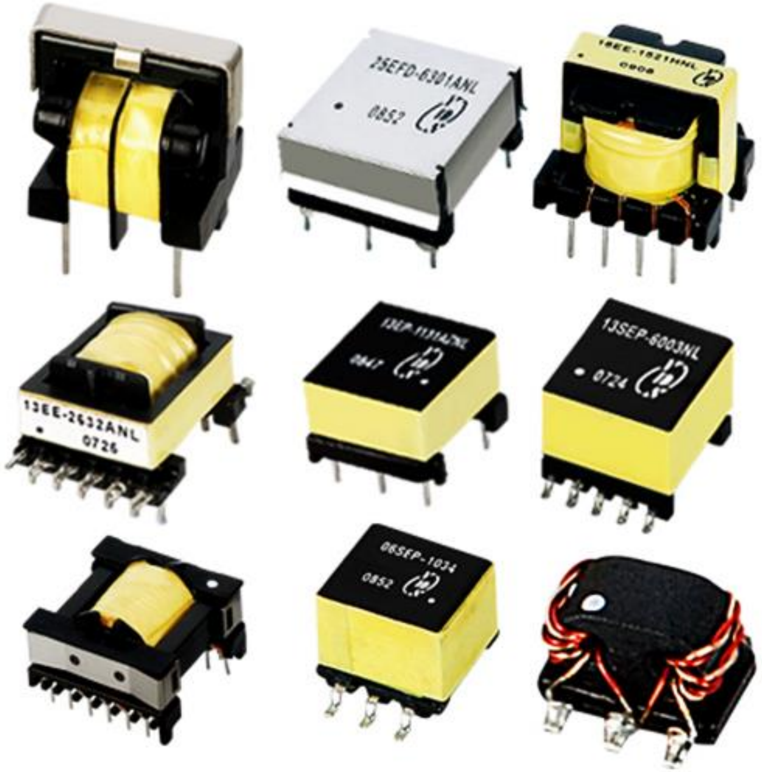


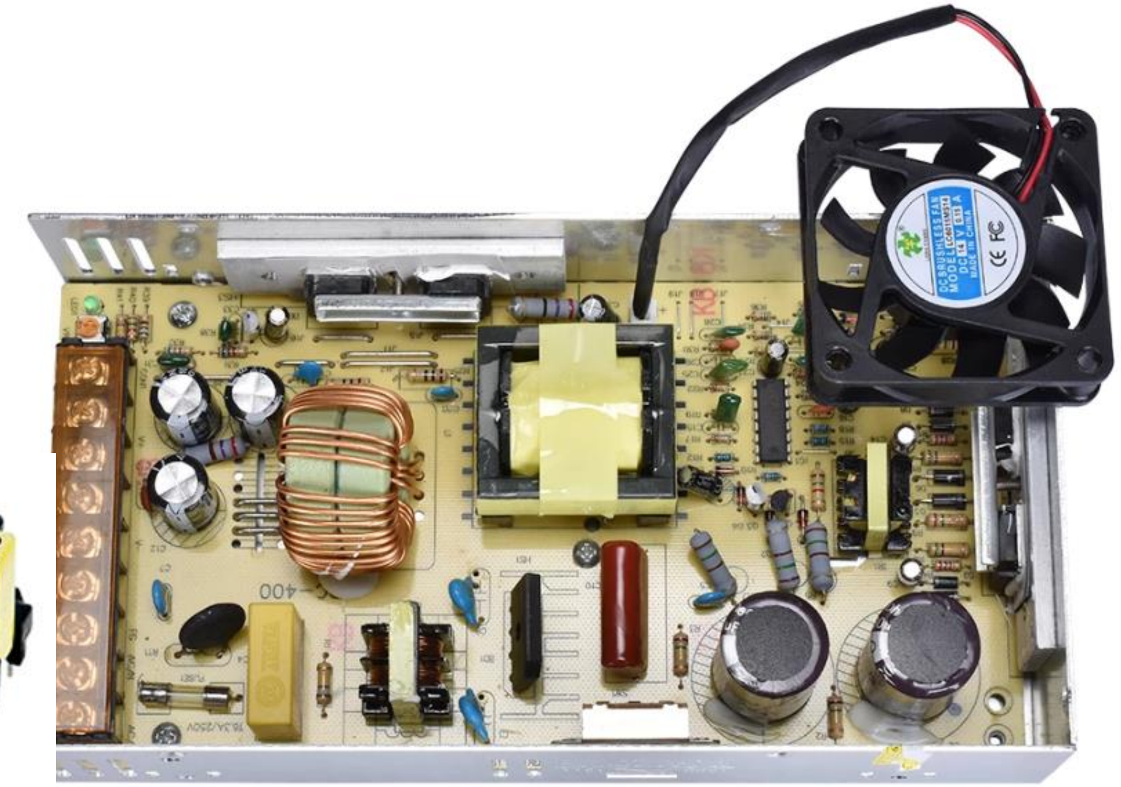
TRANSFORMERS IN POWER DISTRIBUTION



TRANSFORMERS IN POWER UTILIZATION

Welding transformer

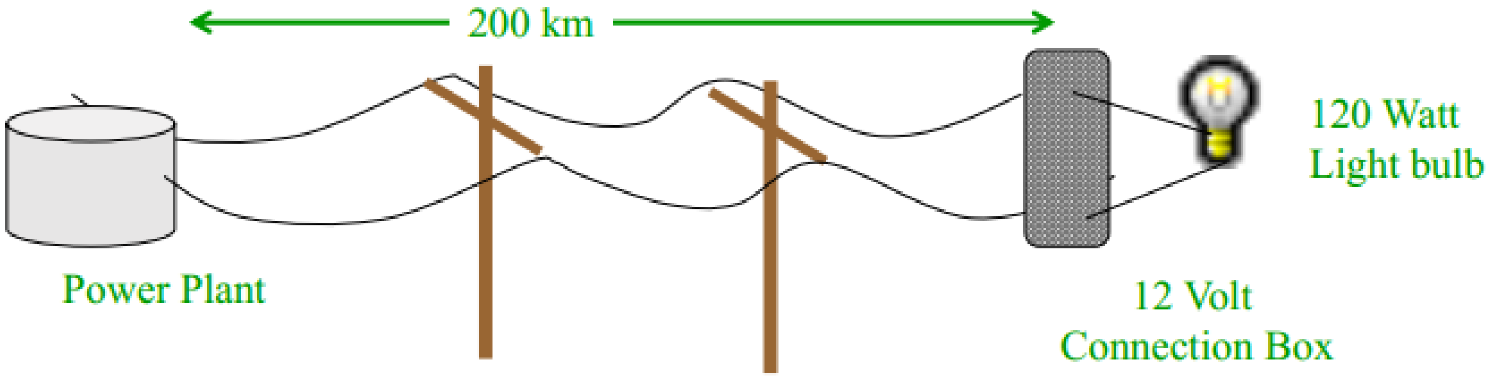




High frequency transformers used in electronic power supplies

Need for Transformers

Power transmission at high voltage while utilization at low voltage



- Estimating resistance of line, typically 0.001 Ω/m => 20 Ω for 200-km
- Current required to glow bulb: $I = P/V = 10A$
- Total power loss in line: $P_{loss} = I^2 R = 10^2 \times 20 = 2000W$
- Efficiency: $\eta = \dfrac{P}{P_{loss} + P} = \dfrac{120}{2000 + 120} \times 100 = 5.66\%$ (very small!)

To **improve** $\boldsymbol{\eta}$, decrease I in **transmission** (i.e. **increase** $\boldsymbol{V}$ for same power)

NEED FOR TRANSFORMERS

- **Transformer**: **Receives AC** electrical **power** at **one voltage** level and delivers/**transfers** it at **another voltage** level (**higher** for “**step-up**” and **lower** for “**step-down**”).

- Need in modern power systems:

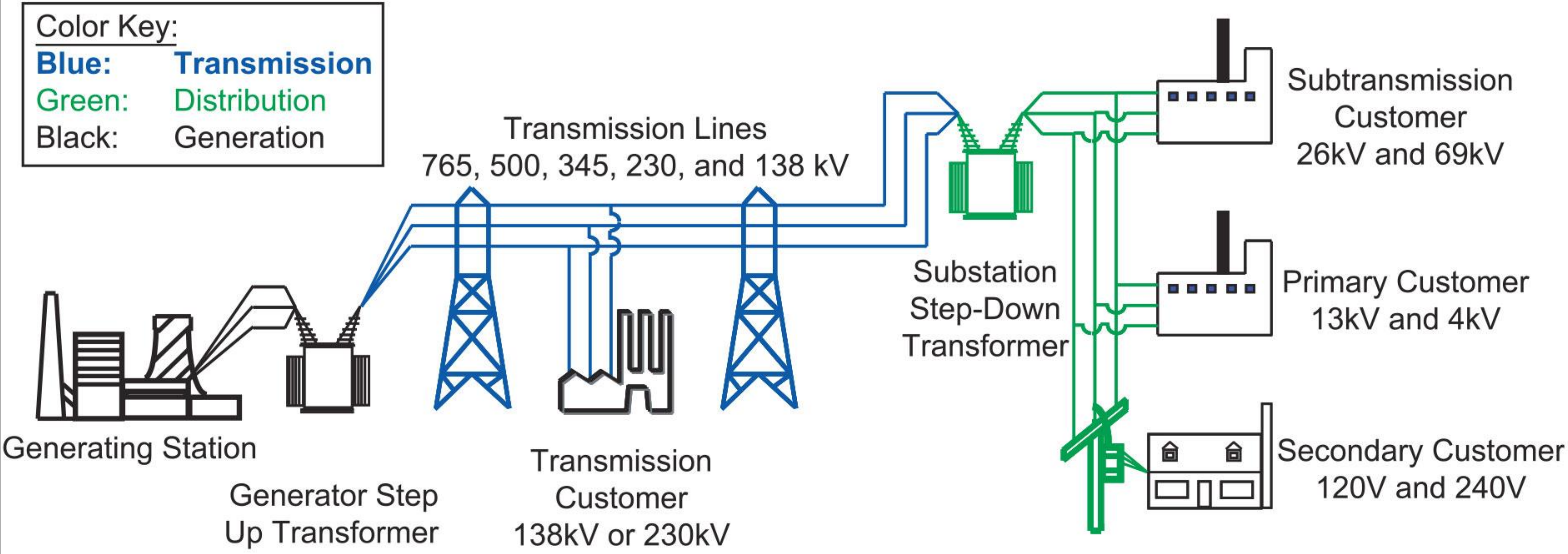
  - **Long distance power lines** can have **significant I^2R losses**.

  - The same power can be delivered by **high-voltage transmission** lines at a **fraction of the current** and with **much higher efficiency**.

  - This feature is one of the main **advantages of AC transmission** and distribution **over DC** system.

TRANSFORMERS IN POWER SYSTEM

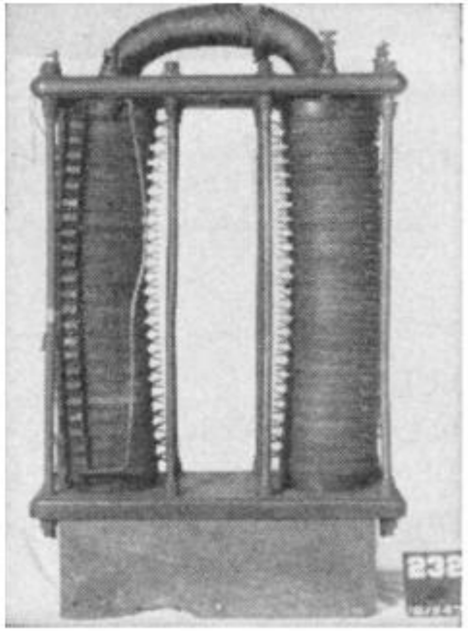
Step-Up Transformer at Generator and Step-Down Transformer at Consumer

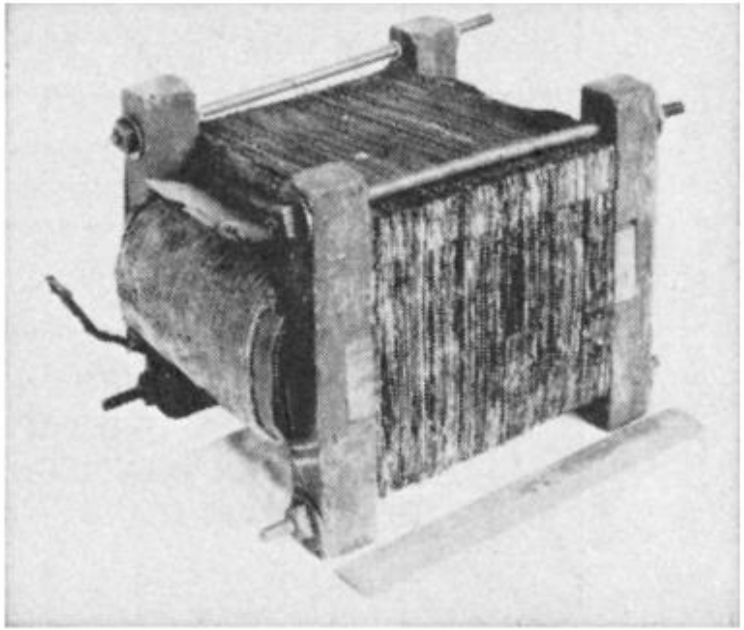


History and Invention

- 1886: Earliest known transformers used for first long-distance AC electric lighting system in Great Barrington, Massachusetts, USA.
- Used to transmit power from 25-hp steam-engine-driven alternator generating 500 V & 12 A to power light bulbs 4000-ft away.

Gaulard and Gibbs transformer





William Stanley's early transformer

Salient Features of Transformer

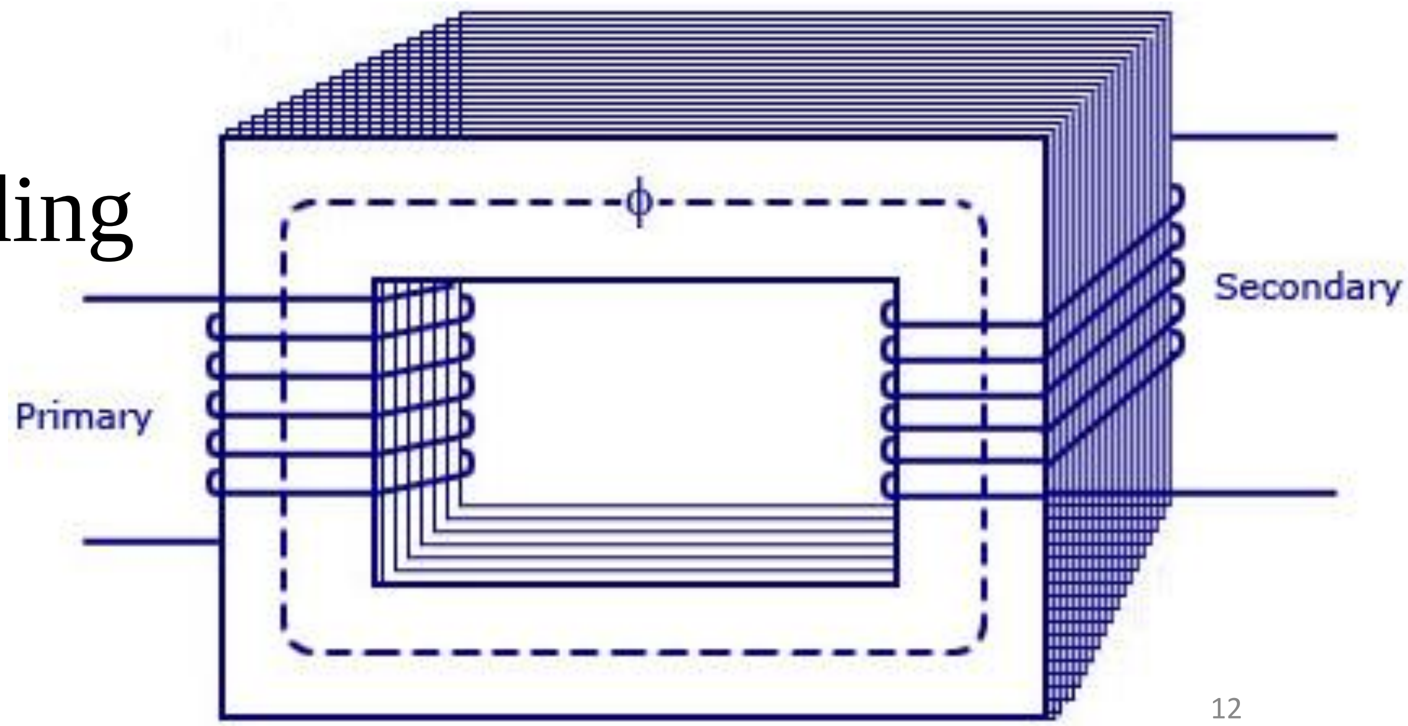
- **Transformation** from a certain (**rms**) **input voltage** (& current) value to a **different output voltage** (& current), keeping the AC signal **frequency constant**.
- Transformation occurs by means of common **magnetic field shared between two coils**.
- **No direct electrical connection** between input & output (exception: auto-transformers).
- Static device with no moving parts => **negligible maintenance**.
- **Highly efficient** power transfer (~98%) from input to output.

TRANSFORMER CONSTRUCTION

- Based on the phenomenon of **Mutual Inductance**.
- Consists of **two** electrical **coils** linked by a **common ferromagnetic core**.

➢ **Primary** (input) winding

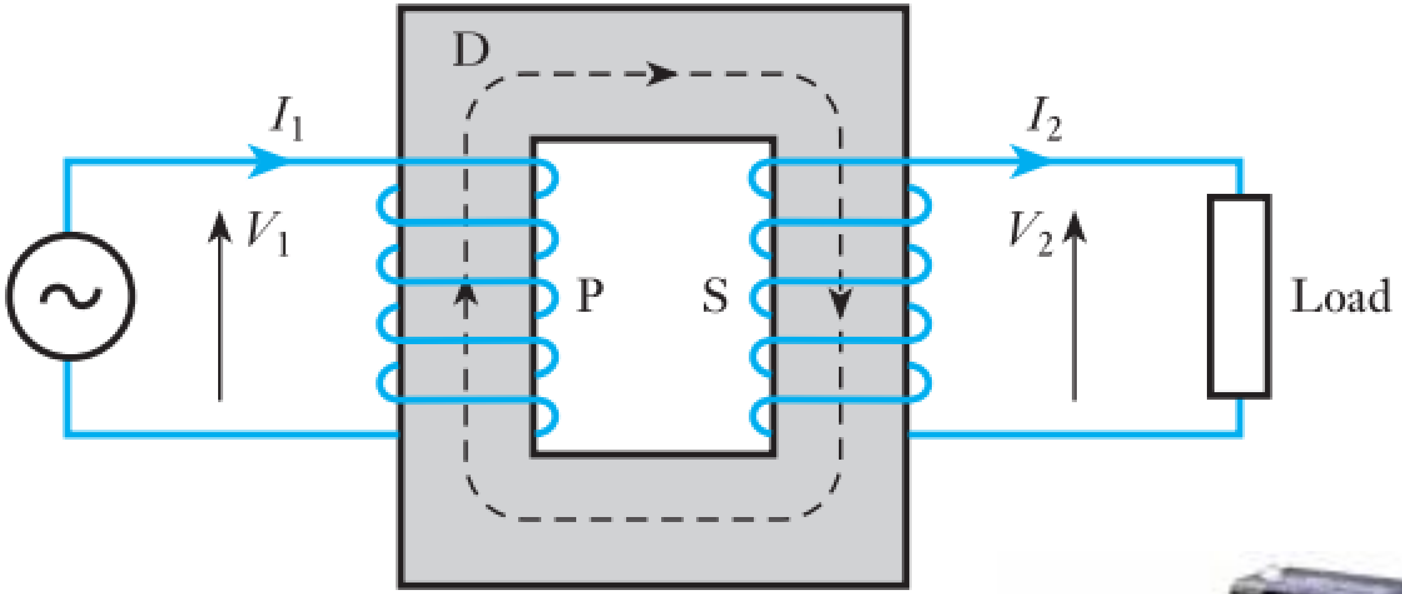
➢ **Secondary** (output) winding



TRANSFORMER CLASSIFICATION

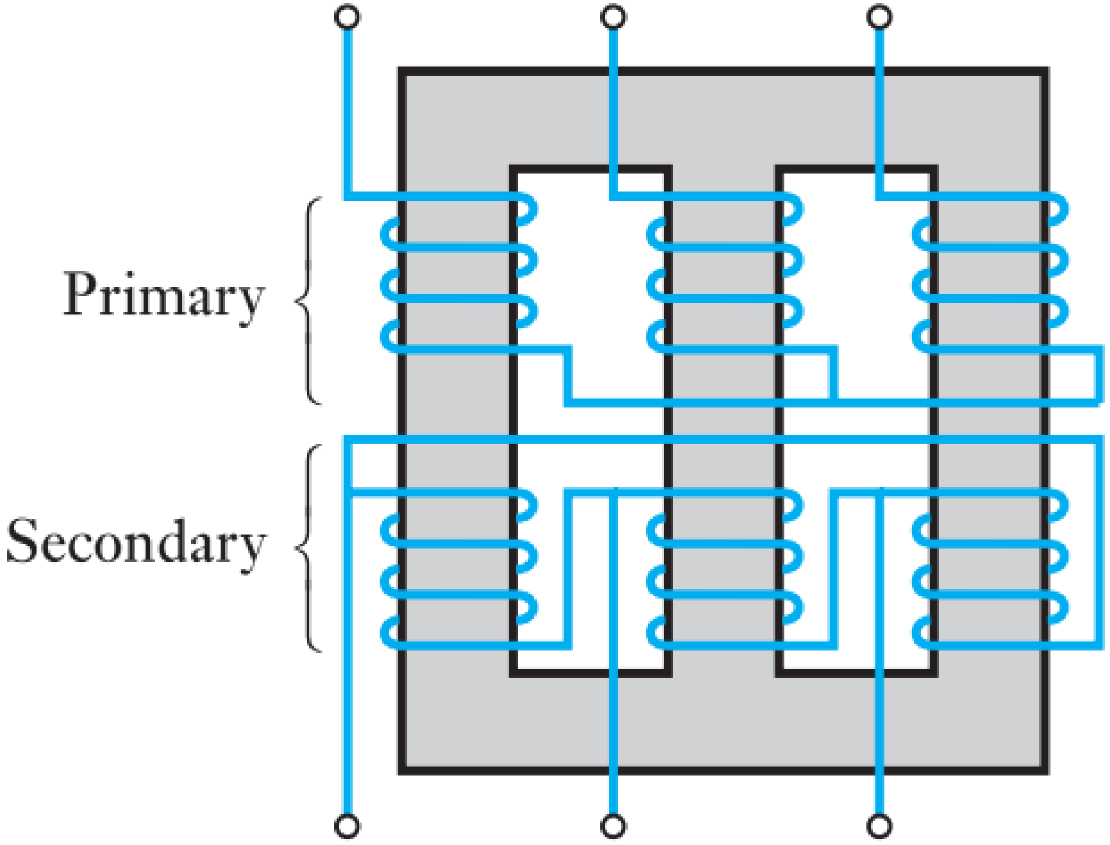
- Based on **number of phases**:
  - ➢ **Single-phase transformer** (low power rating)
    Secondary (output) AC voltage often converted to DC for use in electronic applications.
  - ➢ **Three-phase transformer** (high power rating)
    Can be built as three single-phase transformer units or a single three-phase unit. Used in power systems/electrical grids

TRANSFORMER CLASSIFICATION



Single-phase transformer

TRANSFORMER CLASSIFICATION

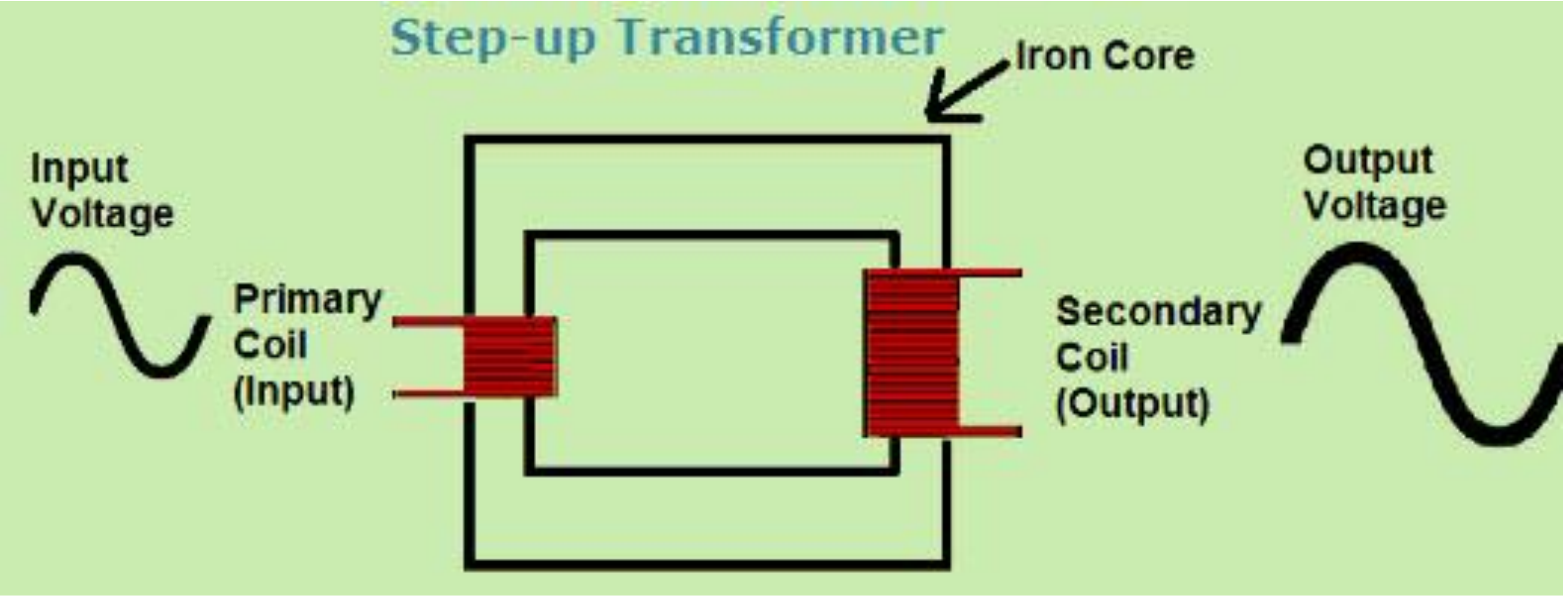


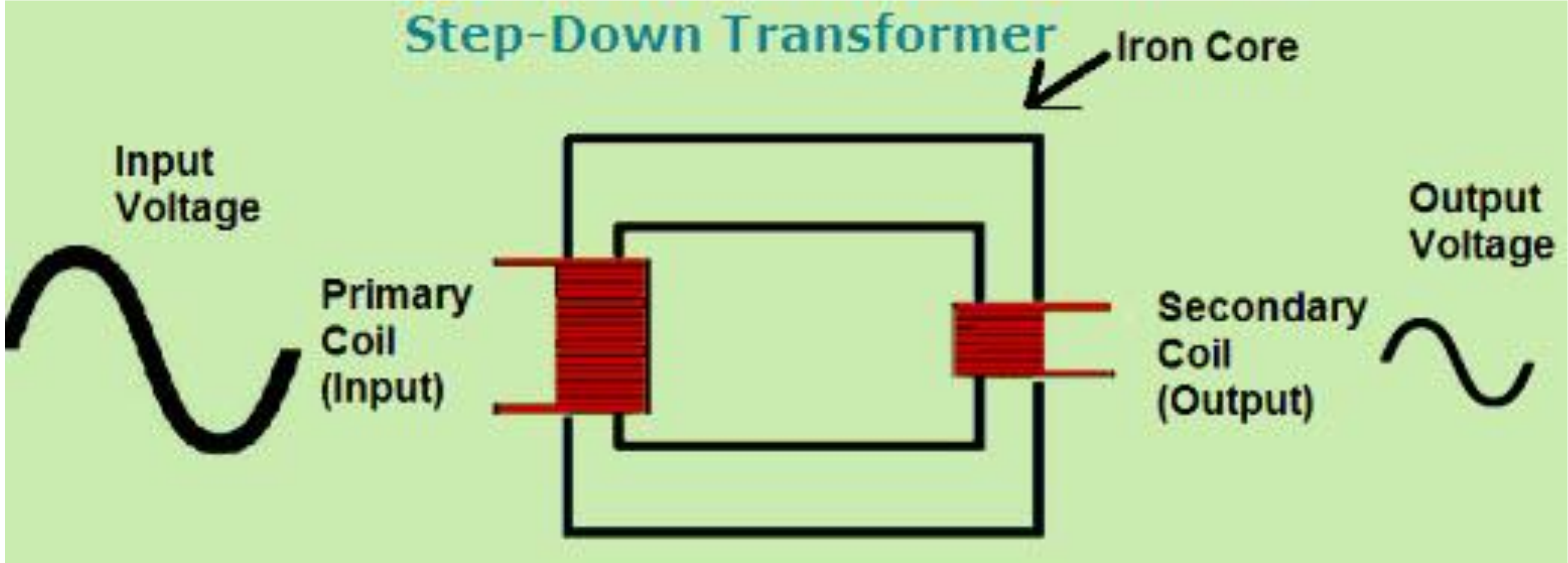
Three-phase transformer

TRANSFORMER CLASSIFICATION

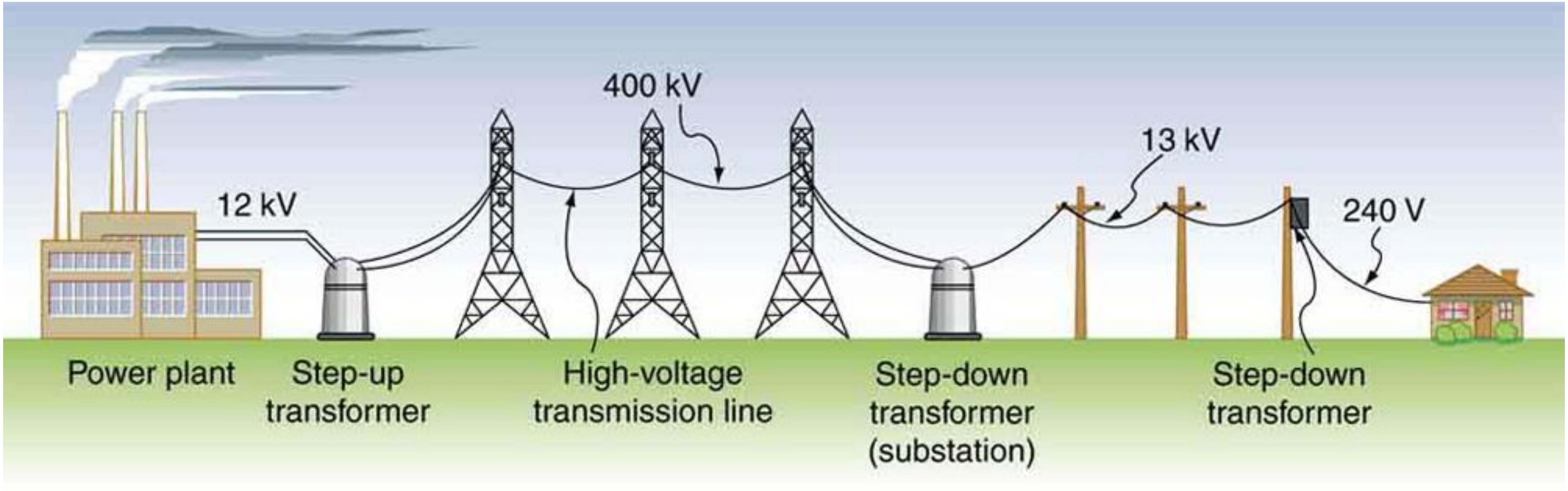
- Based on **voltage transformation level**:
  - **Step-up transformer:**
    Secondary (output) voltage is "stepped up" (**magnified**) by a specific ratio **compared to primary** (input) voltage by using a **greater number of turns in secondary** winding.
  - **Step-down transformer:**
    Secondary (output) voltage is "stepped down" (**attenuated**) by a specific ratio **compared to primary** (input) voltage by using a **greater number of turns in primary** winding.

TRANSFORMER CLASSIFICATION





TRANSFORMER CLASSIFICATION



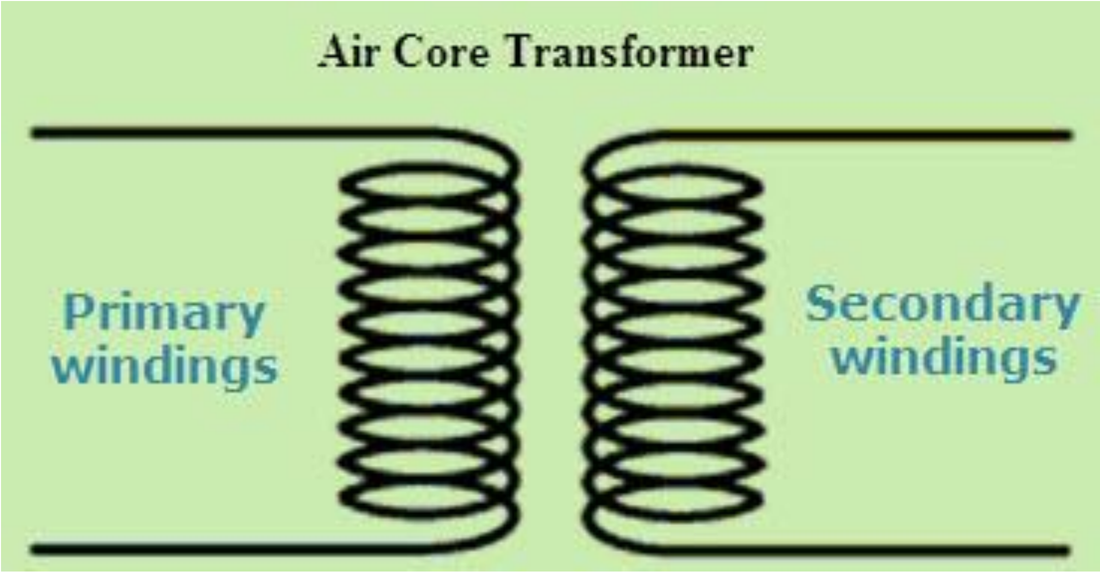
Usage of Step-up and Step-down transformers in Power Systems

TRANSFORMER CLASSIFICATION

- Based on **core medium** used:
  - **Air core transformer:** Coils wound around hollow plastic or ceramic material (**μ_r ~ 1**) => less flux linkage between coils. Used for **high frequency** applications (radio and television receivers).
  - **Iron / Steel core transformer:** High permeability (**μ_r ~ 1500**) => mostly used in transformers in **power systems/electrical grids**. Susceptible to **Hysteresis loss** and **Eddy Current loss**.
  - **Ferrite core and other alloys** (e.g. AlNiCo): Used in **high frequency** transformers as they offer **reduced losses** in core.

TRANSFORMER CLASSIFICATION

Steel/Iron core transformer



Air core transformer

Ferrite core transformer

TRANSFORMER CLASSIFICATION

- Based on **usage in power system**:
  - **Power transformer:** Large rating transformers used in electrical **power transmission** networks for step up/down applications (e.g. **400 kV, 200 kV, 110 kV, 66 kV, 33kV**). Generally designed at operated at full load.
  - **Distribution transformer:** Comparatively small rating transformers used in **power distribution** networks (e.g. **11 kV, 6.6 kV, 3.3 kV, 440 V, 230 V**). Rarely operated at full load.

TRANSFORMER CLASSIFICATION

Power transformers

TRANSFORMER CLASSIFICATION

Distribution transformers

TRANSFORMER CLASSIFICATION

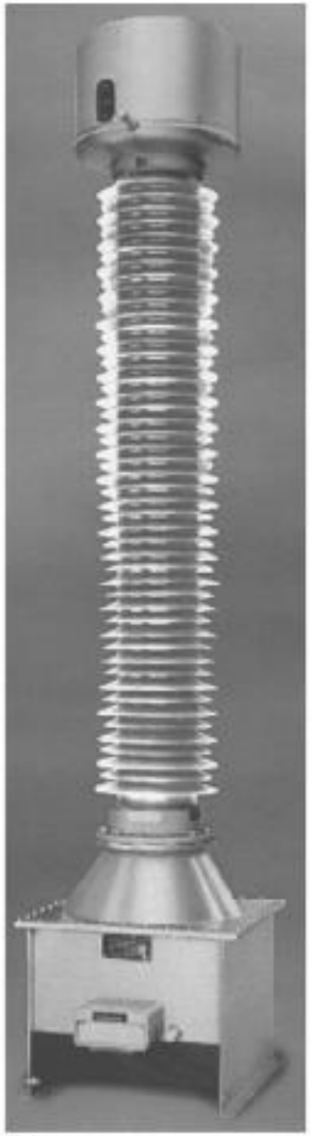
- Based on **application**:

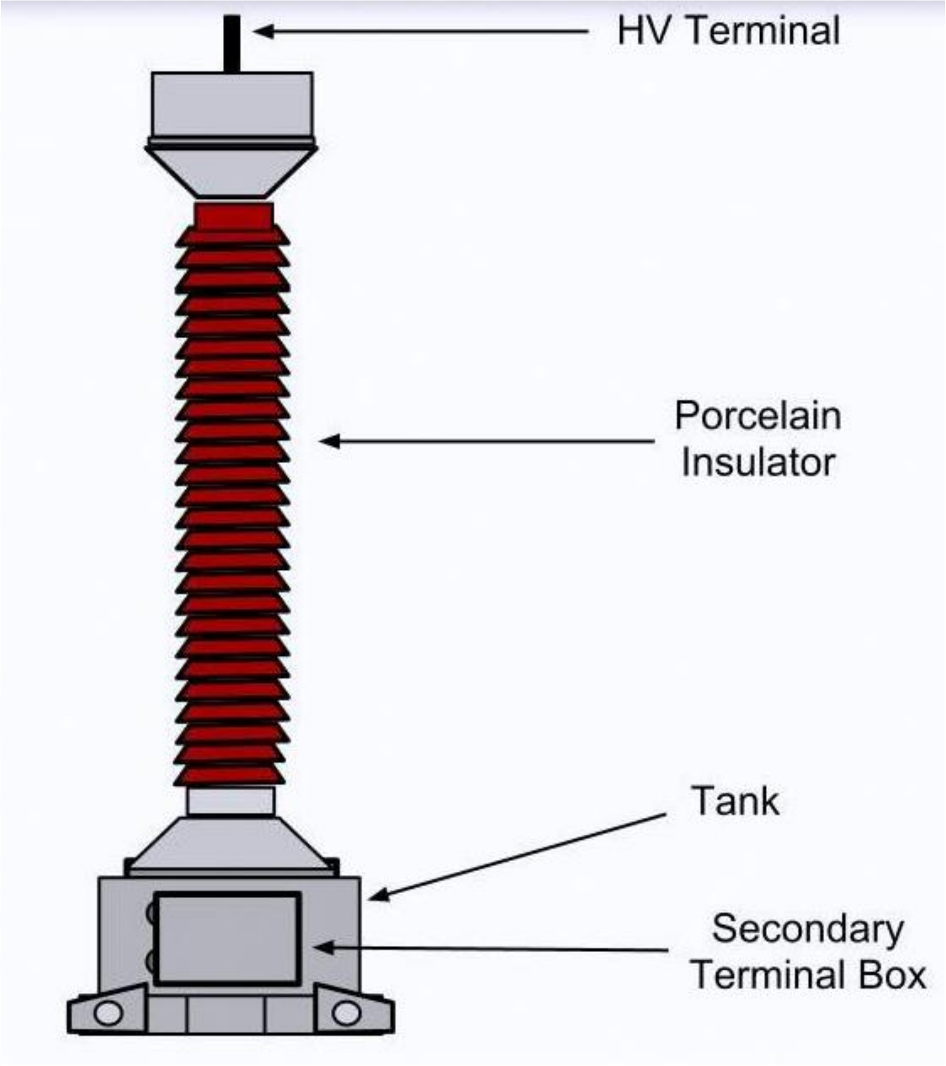
  ➢ **Measurement Transformers**

  1. **Current transformer** (CT) is used for **measuring current** in a circuit. **Primary** winding is connected in **series** with the **monitored circuit**.

  2. **Voltage transformer** (VT) is used for **measuring voltage** in a circuit. **Primary** winding is connected in **parallel** with the **monitored circuit**.

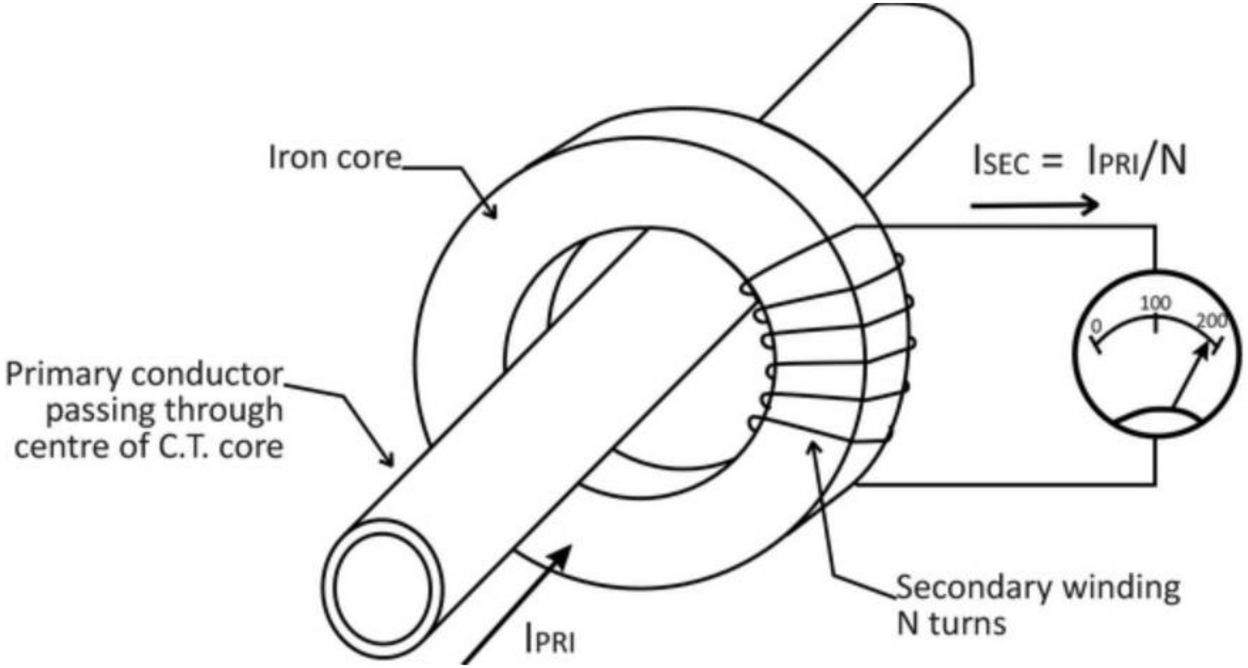
TRANSFORMER CLASSIFICATION

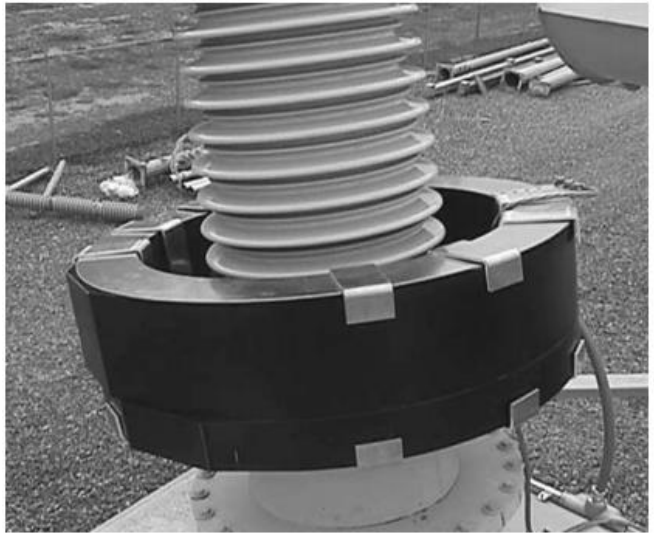




Voltage transformers

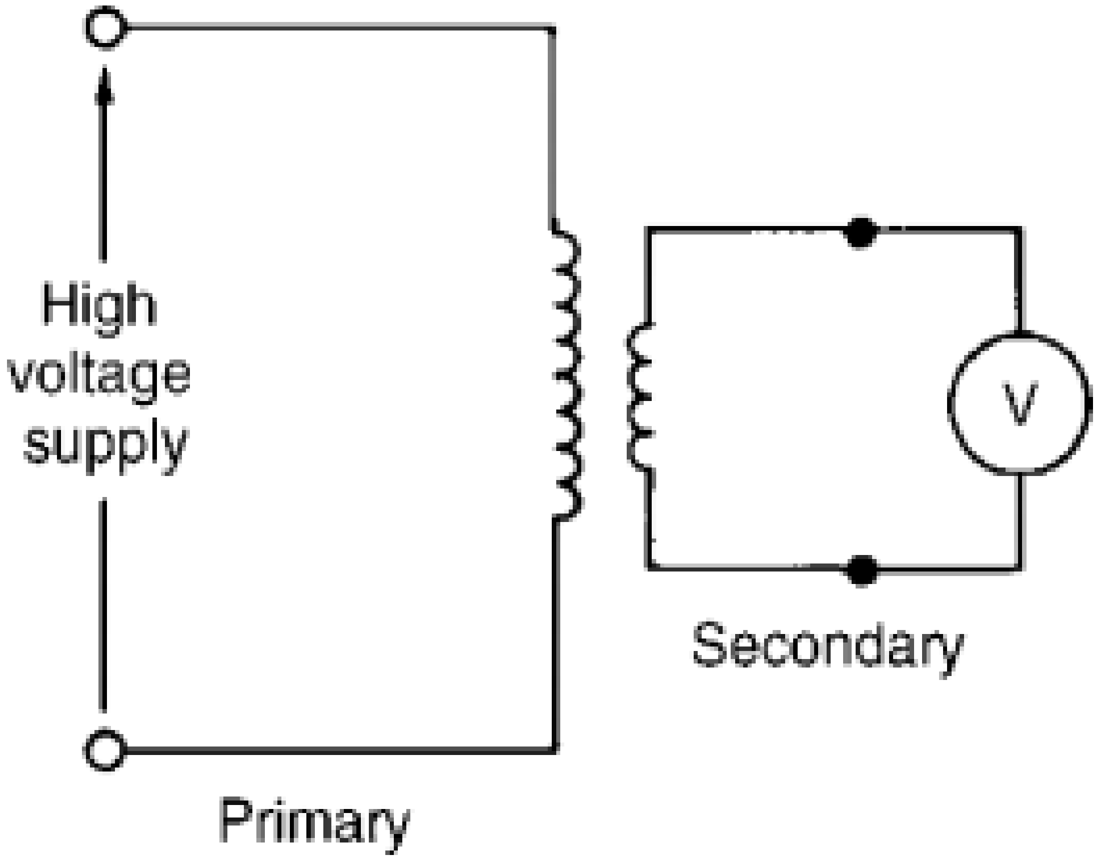
TRANSFORMER CLASSIFICATION

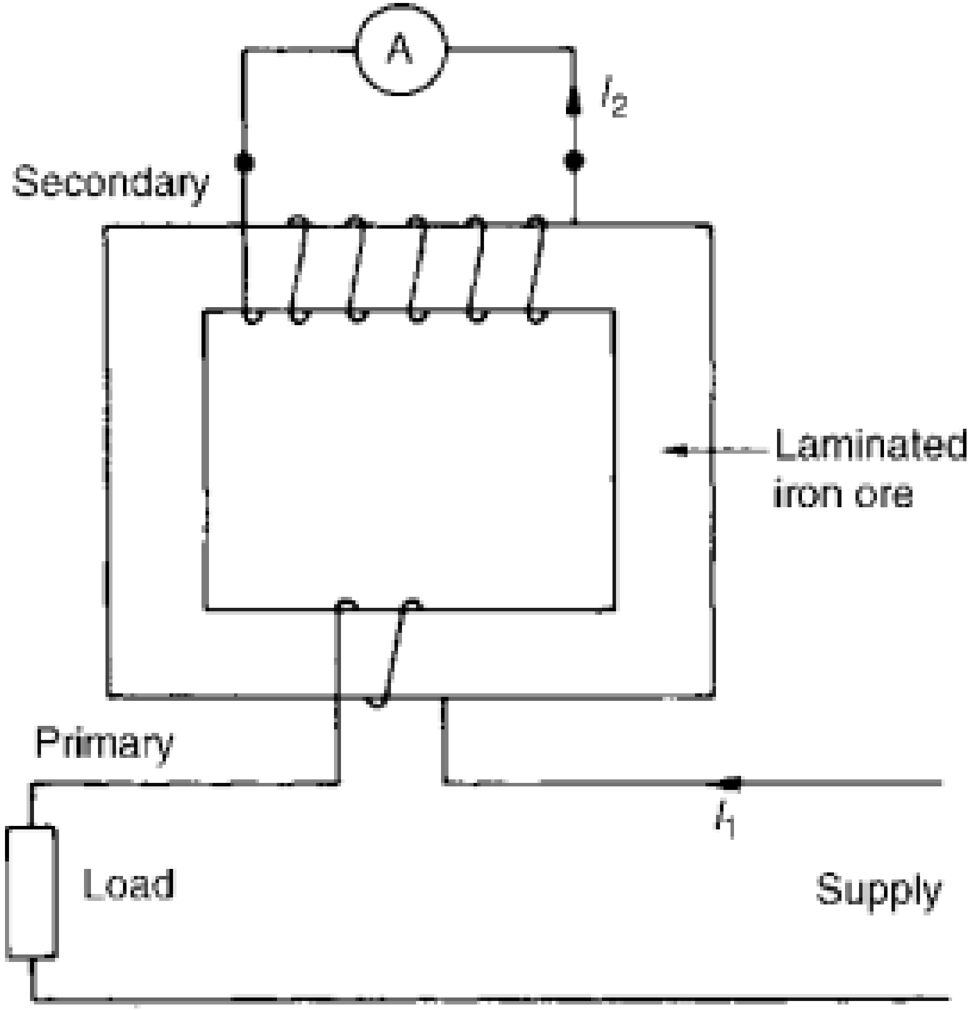




Current transformers

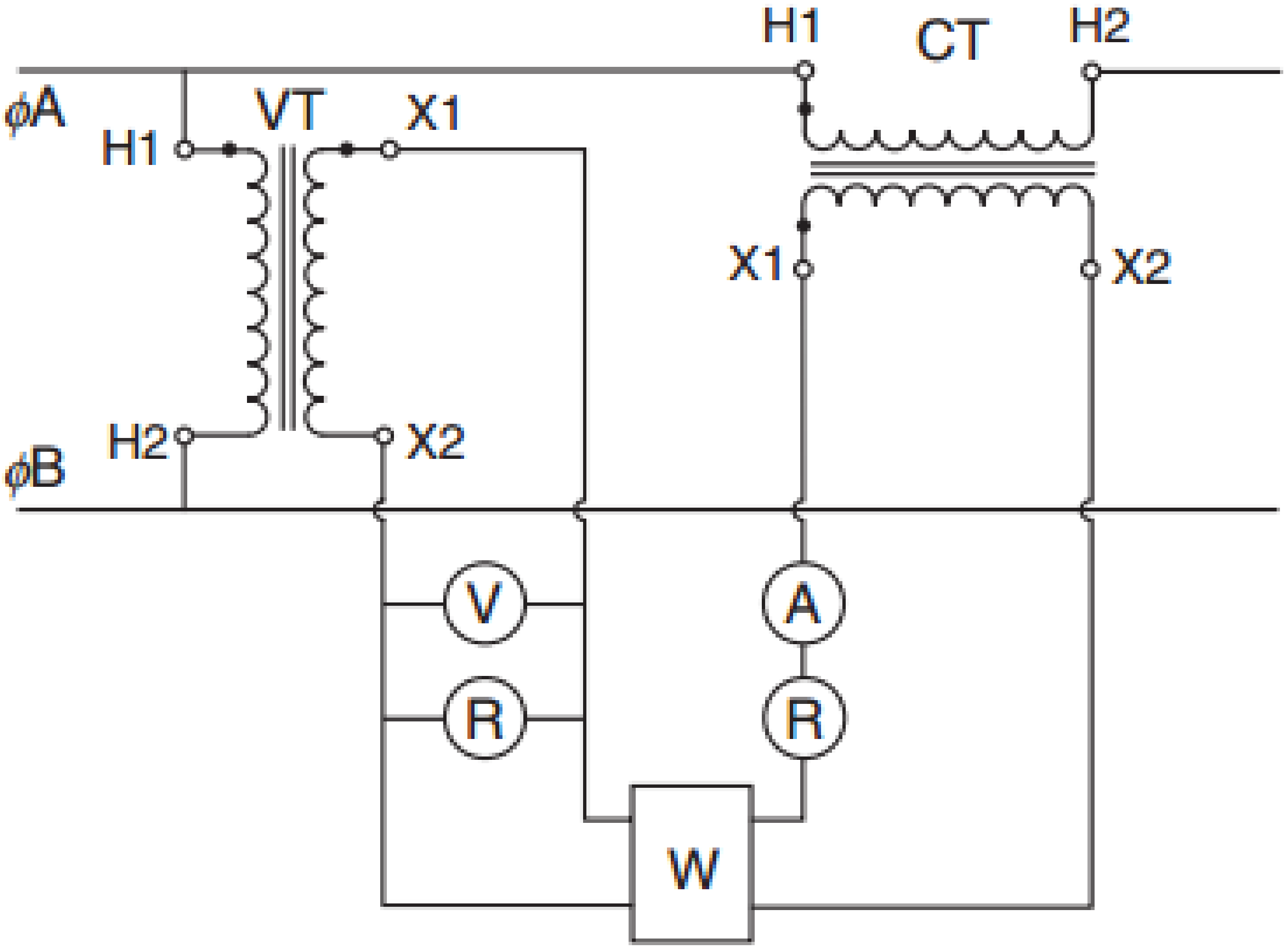
TRANSFORMER CLASSIFICATION





Simplified wiring diagram of VT (left) and CT (right)

TRANSFORMER CLASSIFICATION

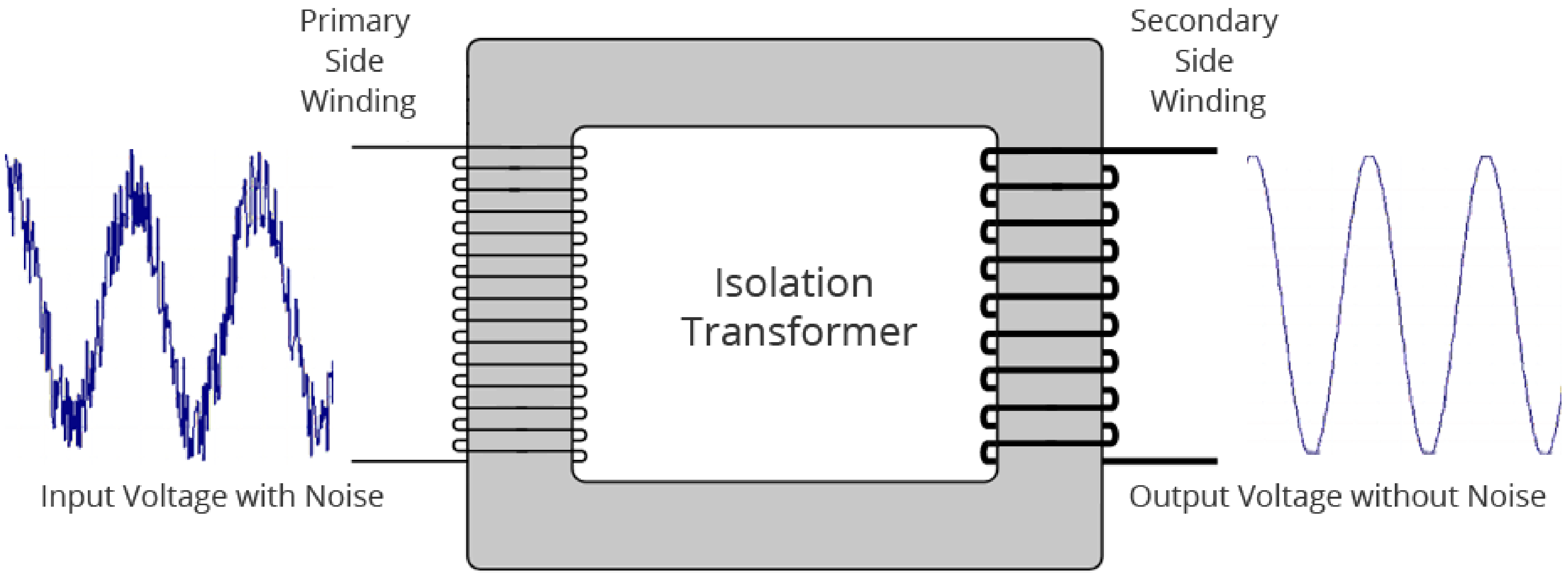


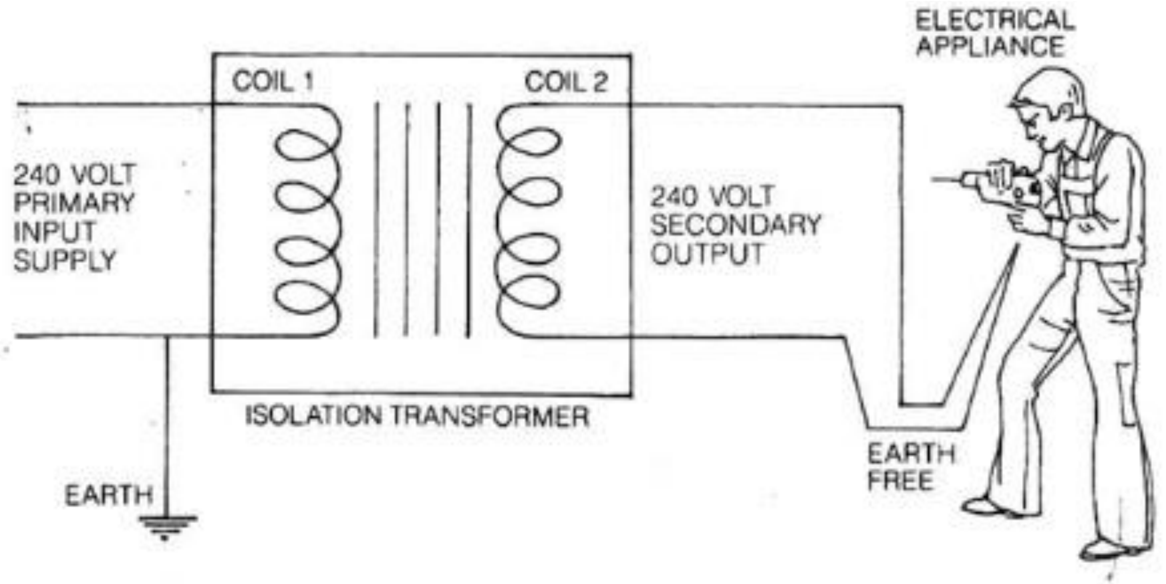
VT connected in parallel and CT in series with circuit

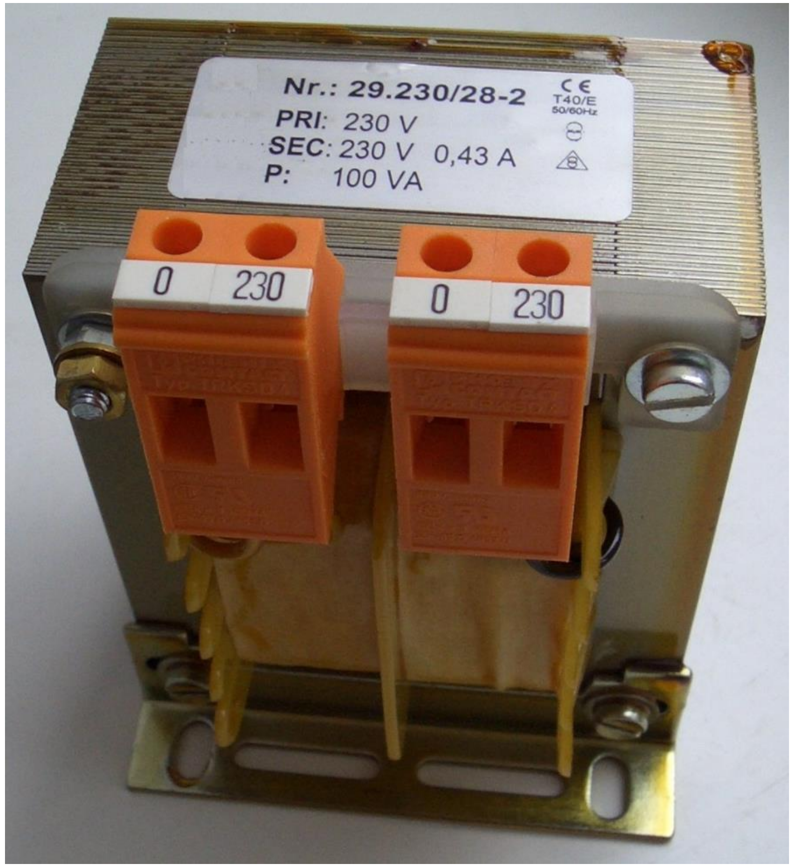
TRANSFORMER CLASSIFICATION

- Based on **application**:
- **Isolation transformers:**
  - ➢ **Secondary voltage** is **equal to primary voltage** i.e. number of turns in both windings equal.
  - ➢ Used to **provide electrical isolation** and **protection against electric shock** e.g. bathroom shaver-sockets, portable electric tools, etc.
  - ➢ Also used to **suppress electrical noise** in sensitive devices e.g. mobile/wireless communication

TRANSFORMER CLASSIFICATION







Isolation transformers

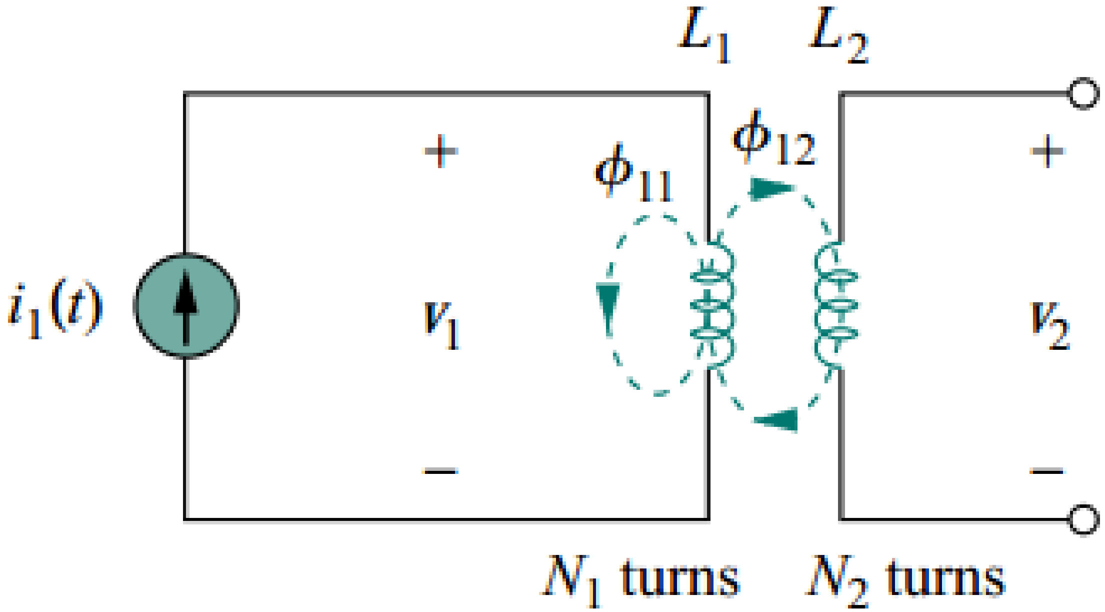
MAGNETICALLY COUPLED CIRCUITS

- **Transformer action** is based on **magnetic coupling**: **two** current carrying **coils** (with or without electrical contacts between them) affect each other through their **mutual magnetic field**.

- **Voltage induced in coil-1** (with coil-2 open circuit) can be written as,

$$\boxed{v_1} = N_1 \frac{d\phi_1}{dt} = N_1 \frac{d\phi_1}{dt}\frac{di_1}{di_1}$$

$$= \left(N_1 \frac{d\phi_1}{di_1}\right)\frac{di_1}{dt} = \boxed{L_1 \frac{di_1}{dt}}$$

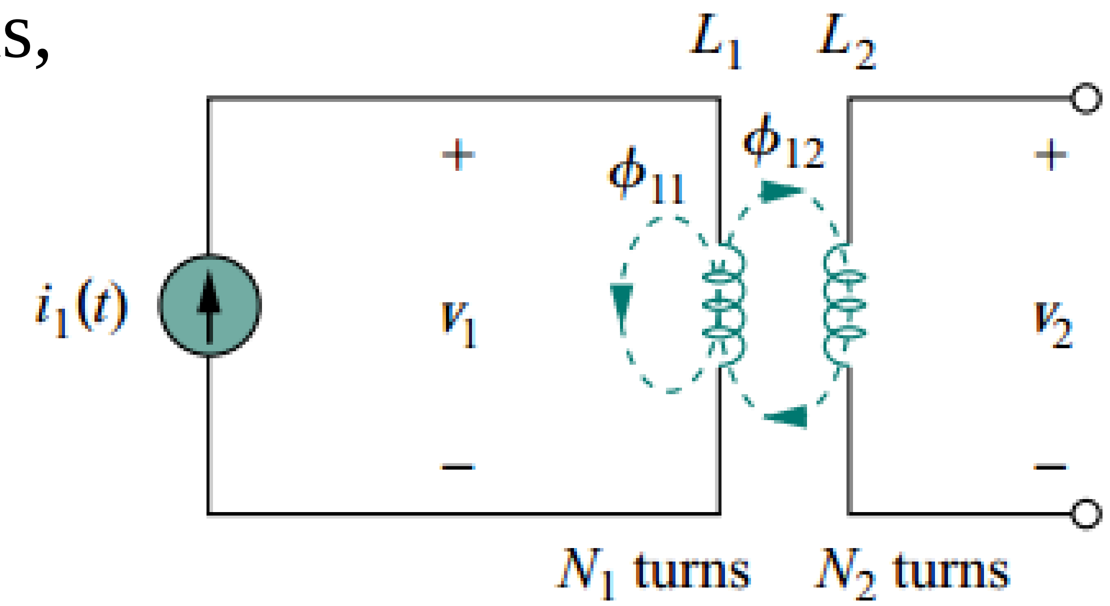


MAGNETICALLY COUPLED CIRCUITS

- **Voltage induced in coil-2** due to i_1 is,

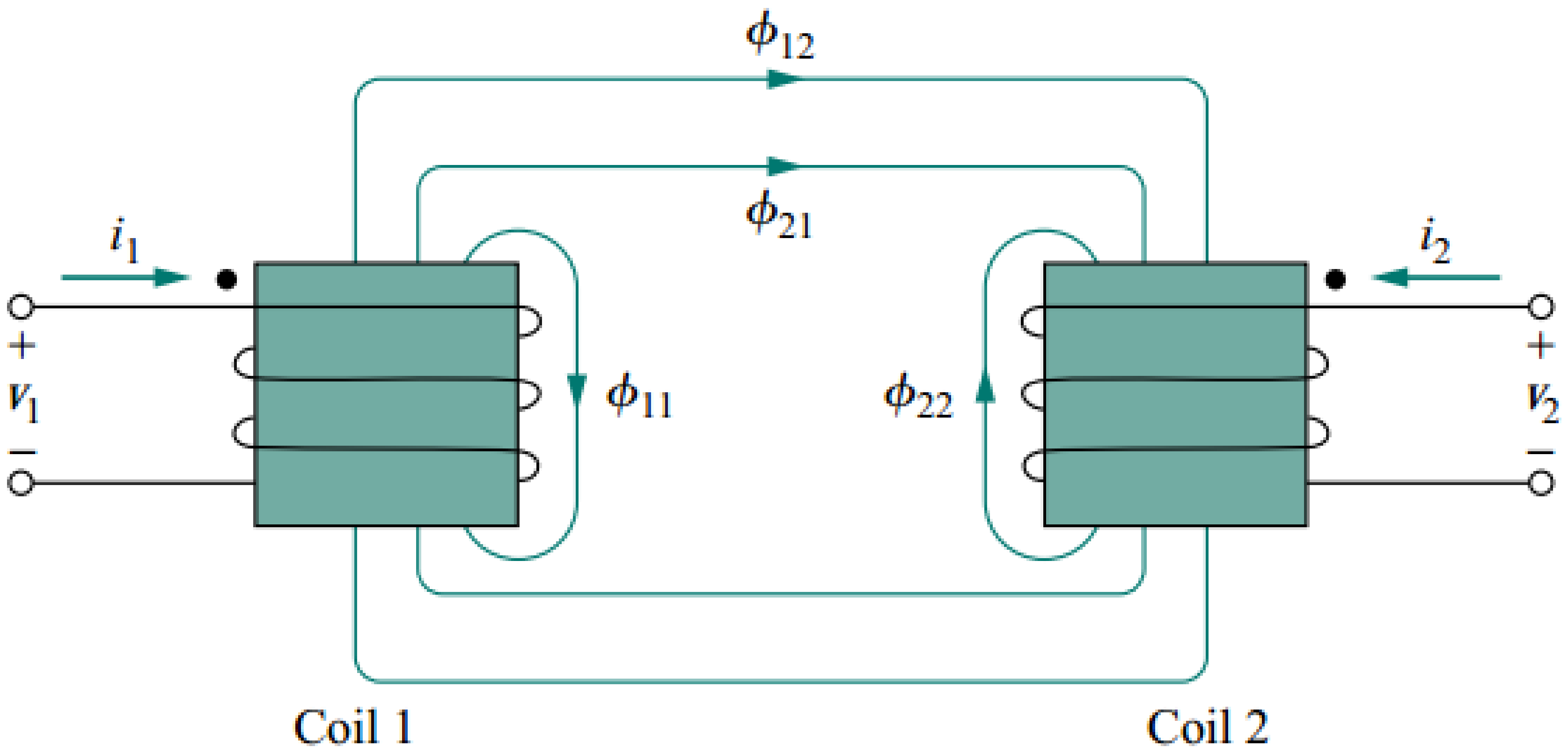
$$\boxed{v_2} = N_2 \frac{d\phi_{12}}{dt} = N_2 \frac{d\phi_{12}}{dt}\frac{di_1}{di_1}$$

$$= \left(N_2 \frac{d\phi_{12}}{di_1} \right) \frac{di_1}{dt} = \boxed{M \frac{di_1}{dt}}$$



- This phenomenon of a **current carrying coil** (inductor) **inducing** a **voltage** across a **neighboring coil** (inductor) is '**mutual inductance**'.
- The **polarity of voltage** induced depends on the **orientation/winding** of the **two coils**, represented using "**dot convention**".

MAGNETICALLY COUPLED CIRCUITS



Dot Convention:

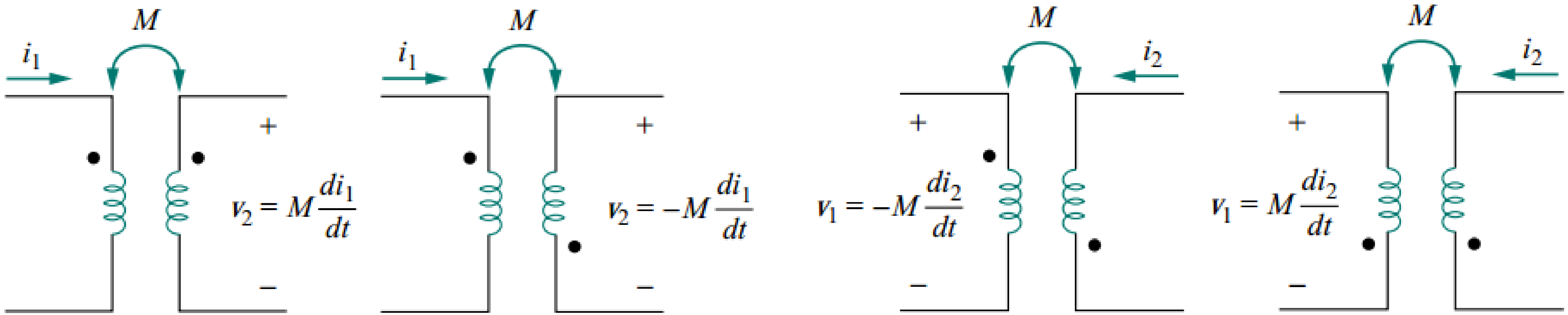
If **current enters** (**leaves**) the **dotted terminal** of **one coil**, the **polarity** of the induced **mutual voltage** in the **second coil** is **positive** (**negative**) at the **dotted terminal**.

MAGNETICALLY COUPLED CIRCUITS

Dot Convention:

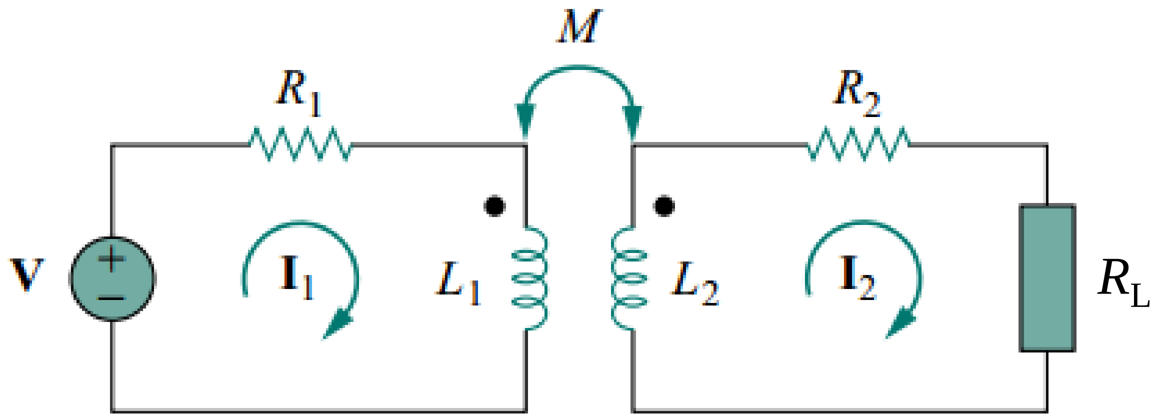
If **current enters** (**leaves**) the **dotted terminal** of **one coil**, the **polarity** of the induced **mutual voltage** in the **second coil** is **positive** (**negative**) at the **dotted terminal**.

Possible configurations with current directions and polarity of induced mutual voltages are shown below



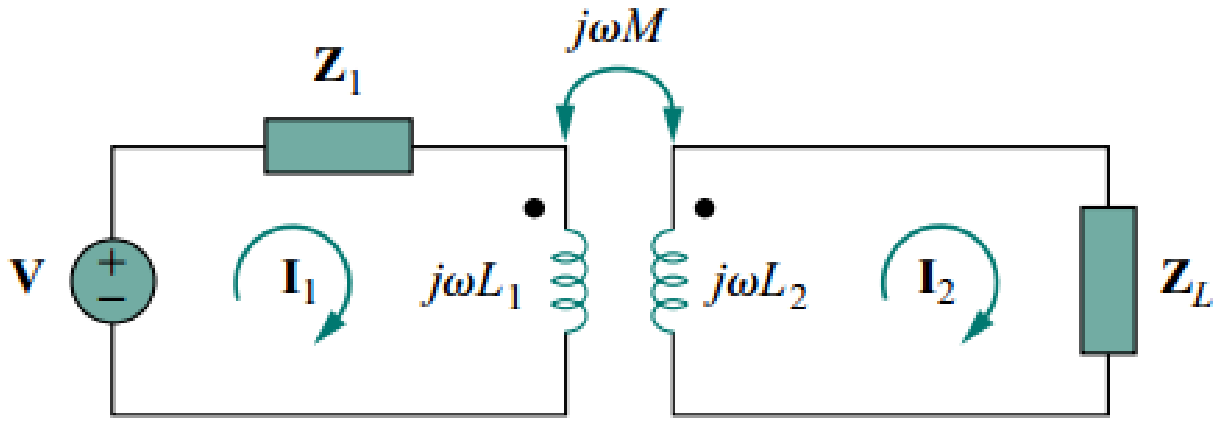
MAGNETICALLY COUPLED CIRCUITS

Time domain and equivalent frequency domain descriptions



$V(t) = I_1(t)R_1 + V_1(t)$
$= I_1(t)R_1 + L_1(dI_1/dt) + M(-dI_2/dt)$
$= I_1(t)R_1 + L_1(dI_1/dt) - M(dI_2/dt)$

$V_2(t) = I_2(t)(R_2 + R_L)$
$=> L_2(-dI_2/dt) + M(dI_1/dt) = I_2(t)(R_2 + R_L)$
$=> -L_2(dI_2/dt) + M(dI_1/dt) = I_2(t)(R_2 + R_L)$

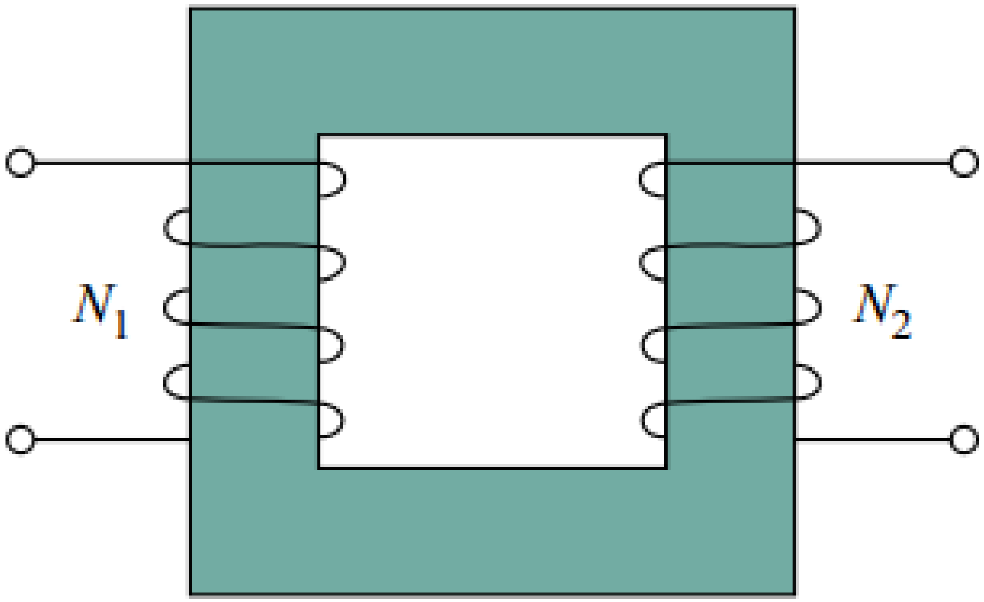


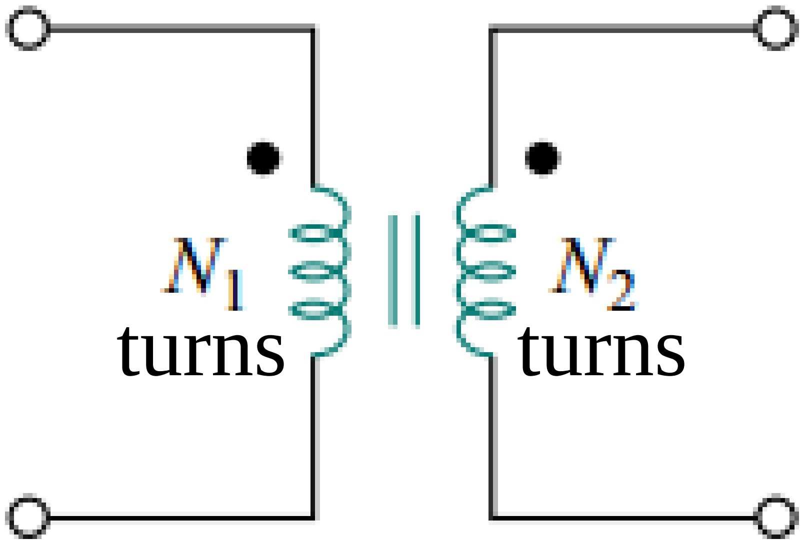
$\mathbf{V} = \mathbf{I_1 Z_1} + V_1$ (bold denotes phasor)
$= \mathbf{I_1 Z_1} + j\omega L_1.\mathbf{I_1} + j\omega M.(-\mathbf{I_2})$
$= \mathbf{I_1 Z_1} + j\omega L_1 \mathbf{I_1} - j\omega M \mathbf{I_2}$

$V_2 = \mathbf{I_2 Z_L}$
$=> j\omega L_2.(-\mathbf{I_2}) + j\omega M \mathbf{I_1} = \mathbf{I_2 Z_L}$
$=> -j\omega L_2 \mathbf{I_2} + j\omega M \mathbf{I_1} = \mathbf{I_2 Z_L}$

IDEAL TRANSFORMER

- **Perfect coupling** i.e. coefficient of coupling $\boxed{k = M\big/\sqrt{L_1 L_2} = 1}$
- Primary and secondary **windings are lossless** i.e. $\boldsymbol{R_1 = R_2 = 0}$
- Primary and secondary winding have **large inductive reactances** $\boldsymbol{(L_1 \propto (N_1)^2 >> 1,\ L_2 \propto (N_2)^2 >> 1)}$.



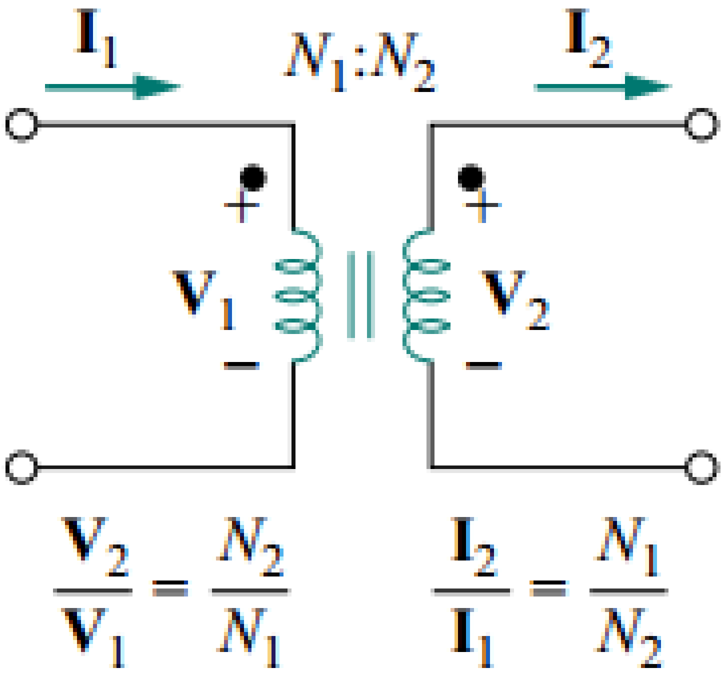


IDEAL TRANSFORMER

- Frequency domain analysis of ideal transformer:

$$\mathbf{V_1} = j\omega L_1\mathbf{I_1} - j\omega M\mathbf{I_2} \;\; ; \quad \mathbf{V_2} = -j\omega L_2\mathbf{I_2} + j\omega M\mathbf{I_1}$$

- For ideal transformer, $\boxed{M = \sqrt{L_1 L_2}}$



$$=> \mathbf{V_1} = j\omega\sqrt{L_1}\left(\sqrt{L_1}\mathbf{I_1} - \sqrt{L_2}\mathbf{I_2}\right)$$

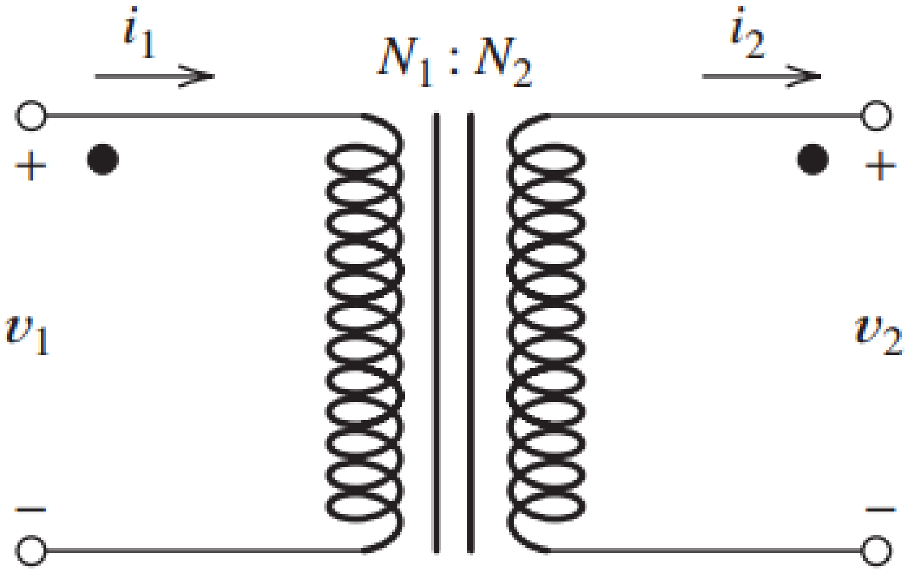
$$\mathbf{V_2} = j\omega\sqrt{L_2}\left(\sqrt{L_1}\mathbf{I_1} - \sqrt{L_2}\mathbf{I_2}\right)$$

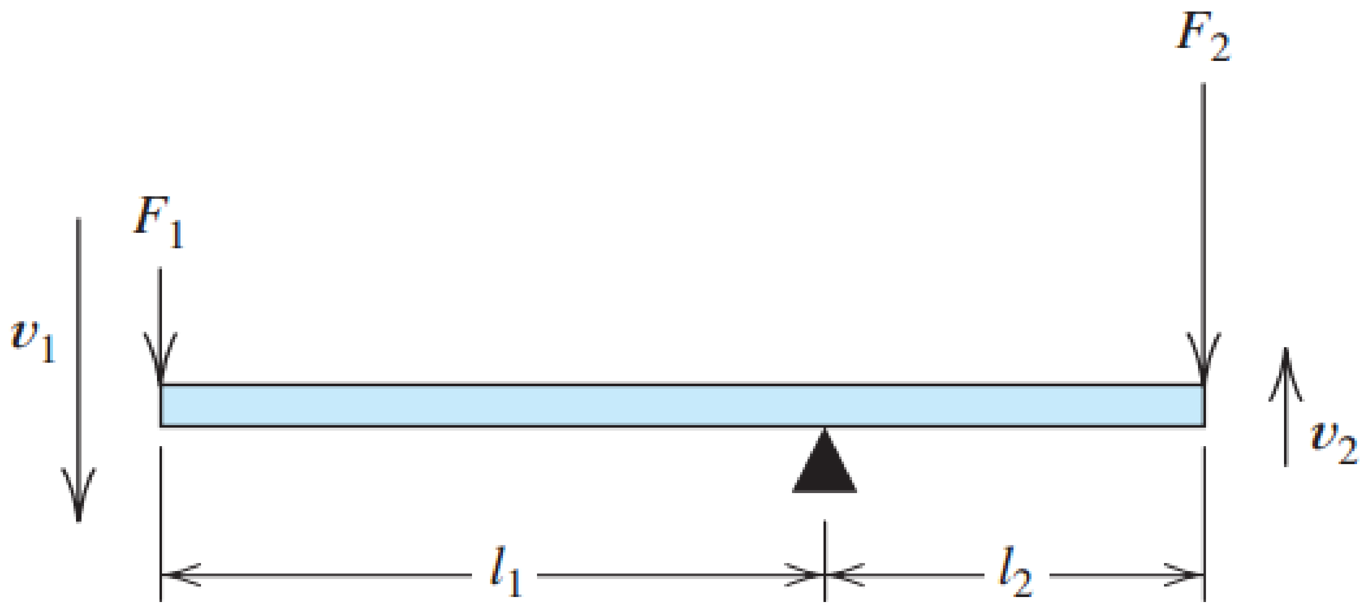
- Dividing above two equations, $\mathbf{V_2} = \sqrt{L_2/L_1}\,\mathbf{V_1}$ => $\boxed{\mathbf{V_2} = (N_2/N_1)\mathbf{V_1}}$
- For ideal (lossless) transformer, input power = output power

$$=> \mathbf{V_2}\mathbf{I_2} = \mathbf{V_1}\mathbf{I_1} \quad => \quad \boxed{\mathbf{I_2} = (N_1/N_2)\mathbf{I_1}}$$

IDEAL TRANSFORMER

Mechanical analogue of the transformer: The Lever





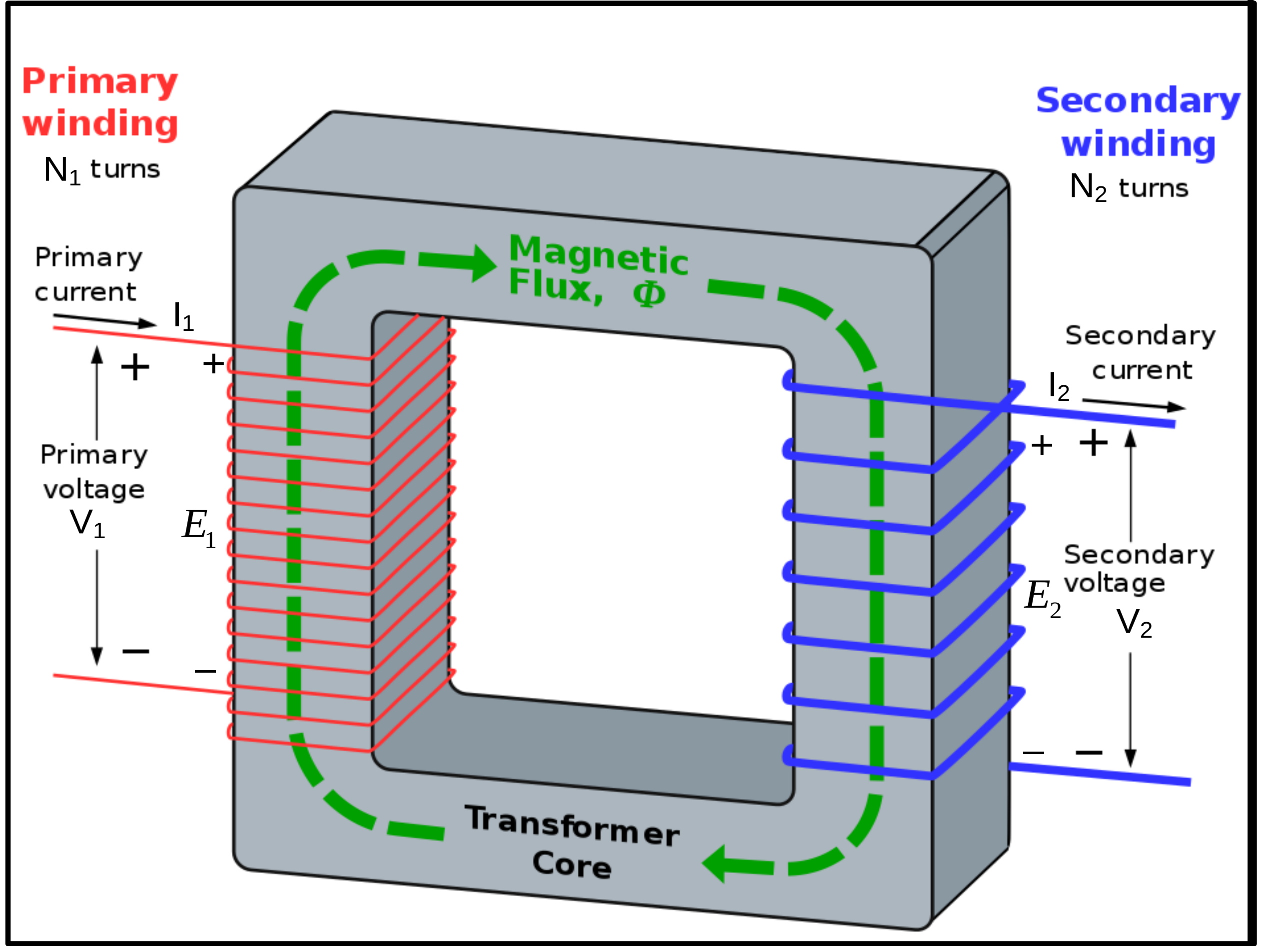
$$v_2 = \frac{N_2}{N_1} v_1 \qquad i_2 = \frac{N_1}{N_2} i_1$$

$$v_2 = \frac{l_2}{l_1} v_1 \qquad F_2 = \frac{l_1}{l_2} F_1$$

WORKING PRINCIPLE OF TRANSFORMER

- When **AC voltage** is applied to **primary** winding with **secondary open**, a "**no-load**" **current** flows in **primary** which sets up a **magnetic flux** in the core.

- This **flux** links with both primary and secondary coils => **induces EMF** by mutual (in secondary) and self (in primary) induction.



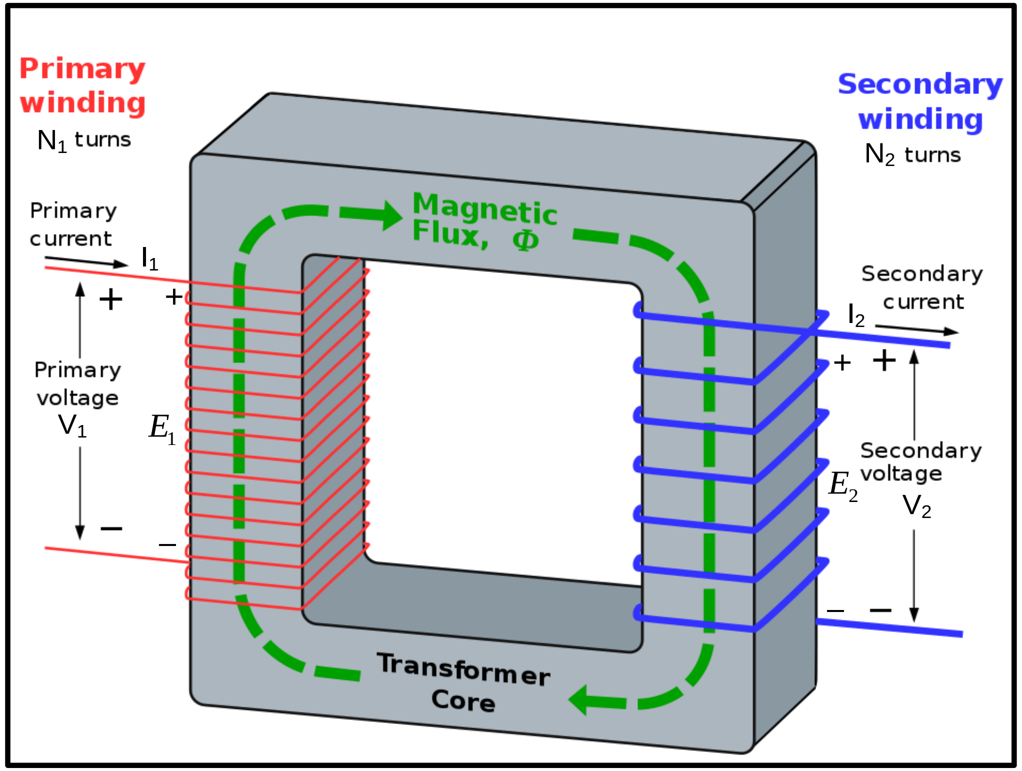
- If **secondary** is **loaded**, then **current** flows through secondary and this is **reflected** back in **primary** winding as **EMF** through mutual induction.

Working Principle of Transformer

- **Ideal transformer**: same flux ϕ linked with both windings
  => induced EMF in windings: $\boxed{E_1 = N_1\, d\phi/dt \;\; ; \quad E_2 = N_2\, d\phi/dt}$

- **Transformation ratio**:

$$\boxed{\frac{E_2}{E_1} = \frac{N_2}{N_1} = n}$$



- **Lossless operation**:

$$E_1 = V_1 \;\; ; \quad E_2 = V_2$$

$$V_1 I_1 = V_2 I_2$$

$$\Rightarrow \quad \boxed{\frac{V_2}{V_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1} = n}$$

WORKING PRINCIPLE OF TRANSFORMER

EMF Equation:

AC voltage across primary => AC flux generated: $\boxed{\phi = \phi_m \sin(\omega t)}$

=> EMF induced in primary:

$$\boxed{E_1 = N_1 \frac{d\phi}{dt} = N_1 \phi_m \omega \cos(\omega t) = 2\pi f N_1 \phi_m \sin\left(\omega t + \frac{\pi}{2}\right)}$$ (and $E_2 = (N_2/N_1)E_1$)

$\Rightarrow$ Induced EMFs are 90^0 phase-shifted from flux

RMS values can be computed as: $\boxed{E_{1_{RMS}} = 4.44\, f N_1 \phi_m}$

$$\boxed{E_{2_{RMS}} = 4.44\, f N_2 \phi_m}$$

ISSUES IN PRACTICAL TRANSFORMER

- Primary and secondary **windings** are **not lossless**. There are **resistive losses** in **core** as well as **windings**.

- **Perfect coupling** (i.e. $k = M/\sqrt{L_1 L_2} = 1$) **not possible** due to **flux leakage** outside core and windings (typically $\boldsymbol{k < 1}$).

- **Temperature rise** due to **resistive losses** results in formation of **hot spots**, that can **damage winding** insulation and **core** properties.

- **Short circuit forces** due to **leakage flux**/magnetic field in **between** the **windings** carrying AC current. The **conductors** in the winding **exert force** on each other, leading to **mechanical stress** in winding.

Transformer Losses

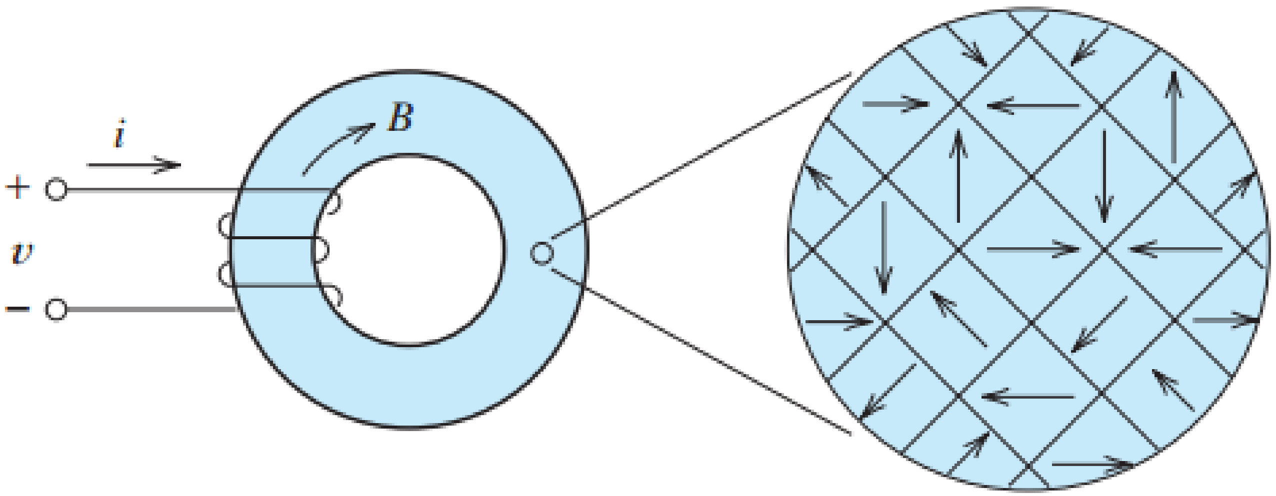
- **<u>Core Loss</u>:** Depends on operating frequency and flux density

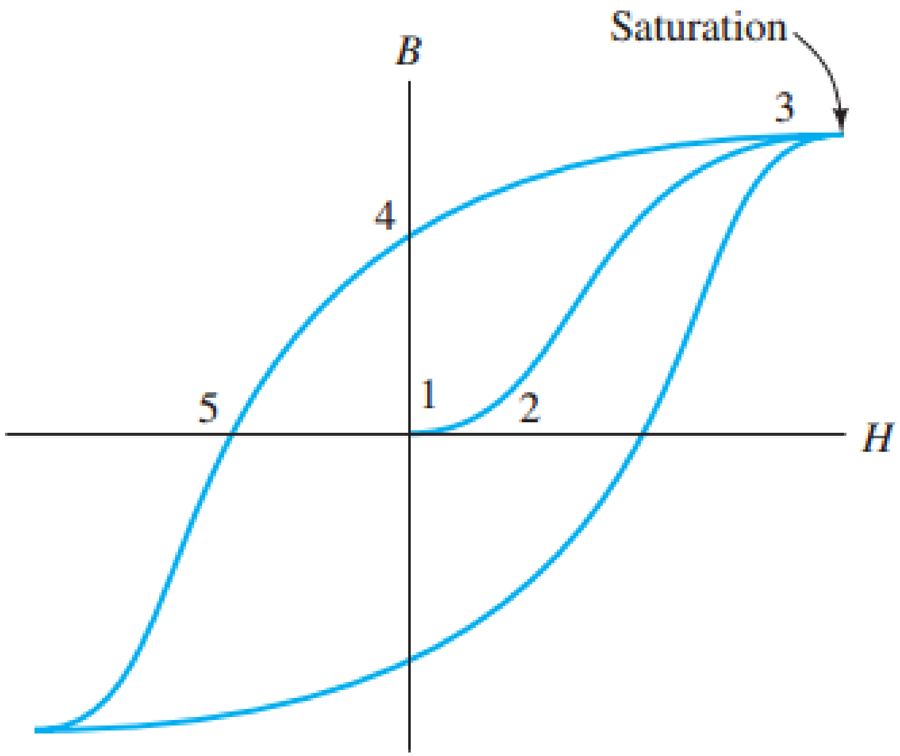
1. Hysteresis loss: $\boxed{P_h = k_h f B_m^{\chi}}$

- Internal molecular structure (magnetic domains) of core material reverses as the magnetic flux alternates
- This results in heating of the core
- Power loss proportional to area of hysteresis loop (B-H curve)
- High permeability material like Ni-Fe alloys used in core construction to reduce this loss

TRANSFORMER LOSSES

Hysteresis loss

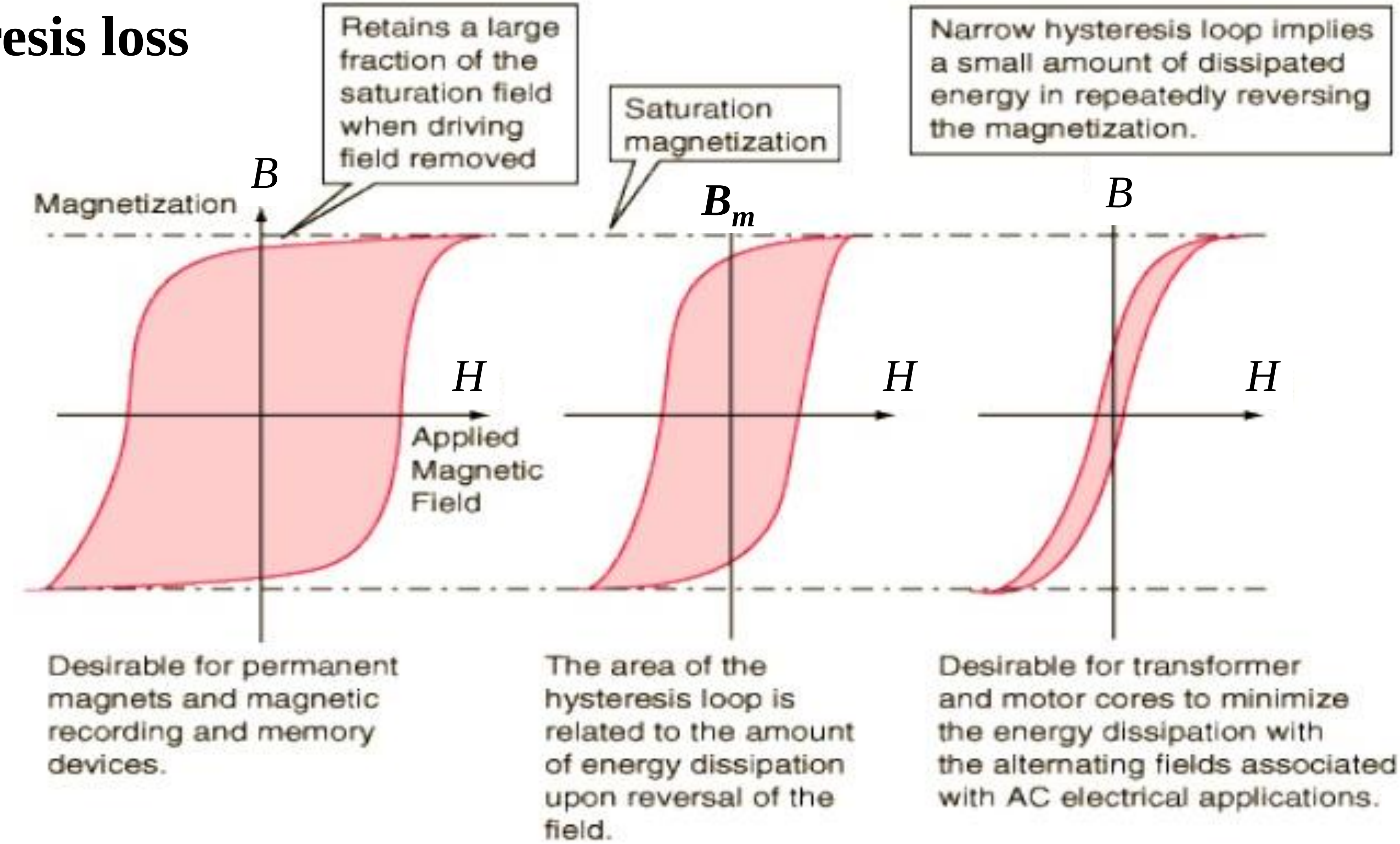




Core material with initial un-aligned magnetic domains and B-H curve

TRANSFORMER LOSSES

Hysteresis loss



TRANSFORMER LOSSES

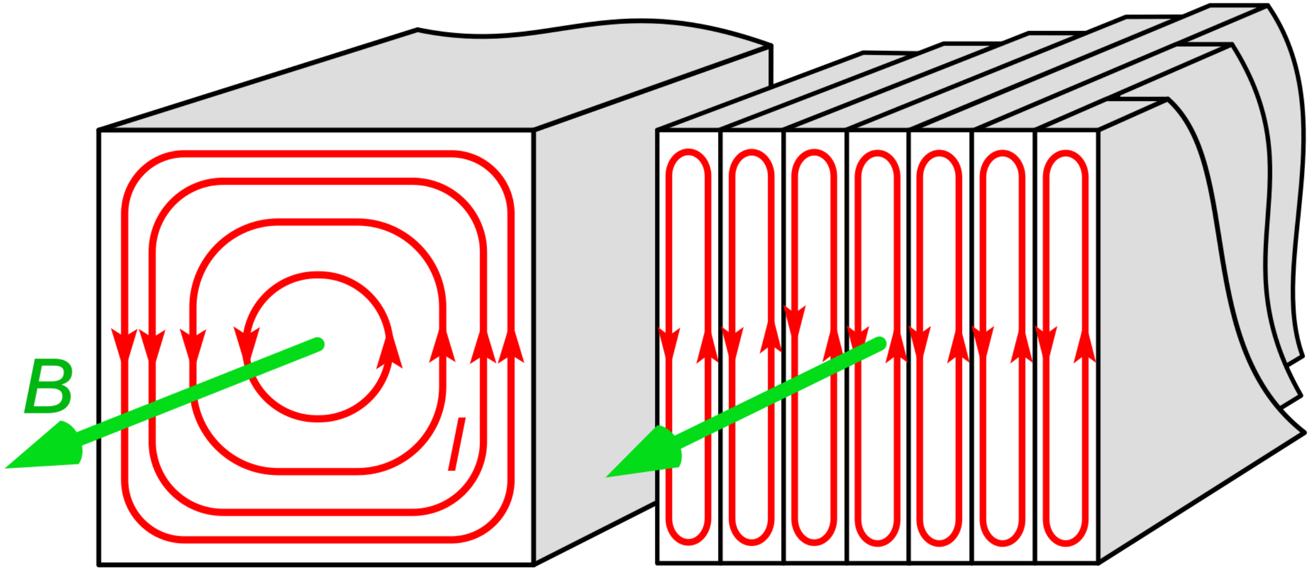
- <u>**Core Loss**</u>**:** Depends on operating frequency and flux density

2. Eddy current loss: $\boxed{P_e = k_e f^2 B_m^2}$

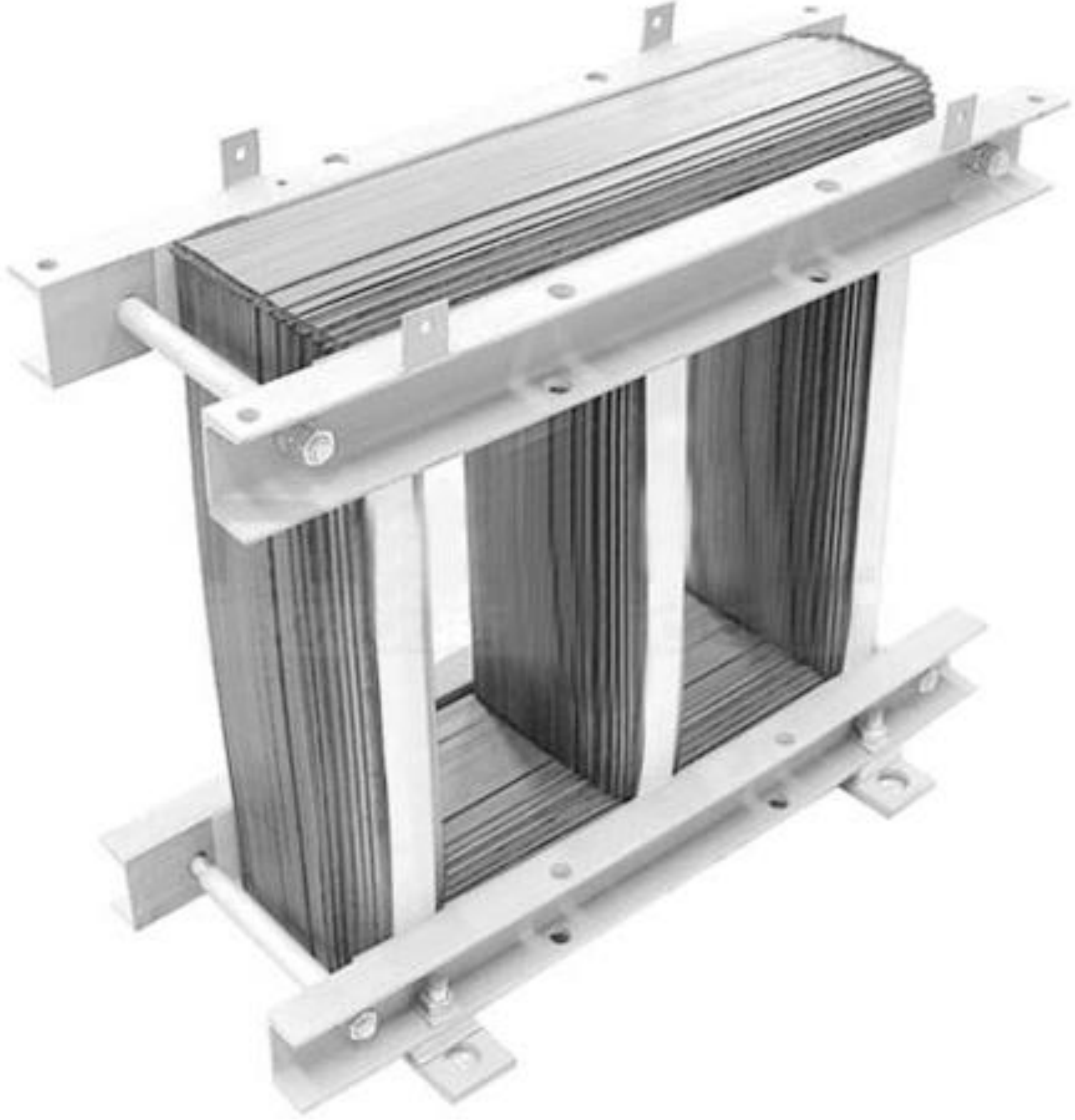
  - Circulating currents due to induced EMF in core
  - Results in excessive heating and power loss in core
  - Can be reduced by increasing the resistivity of core
  - Achieved by laminating the core and inserting thin layers of insulating material between them

TRANSFORMER LOSSES

Eddy current loss



Eddy currents in normal and laminated core



Laminations in core

TRANSFORMER LOSSES

- **Copper Loss:** Depends on the current flowing through the windings
  - If R_1 and R_2 are the primary and secondary winding resistances,

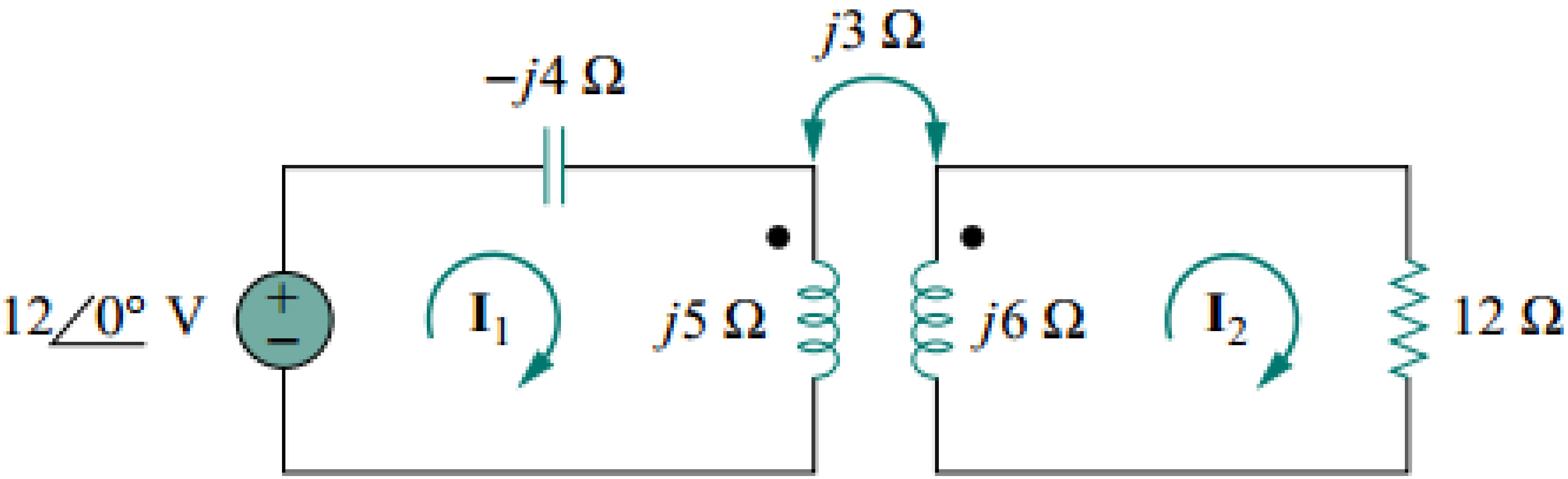
$$\text{Copper loss } P_C = \mathbf{I_1}^2 R_1 + \mathbf{I_2}^2 R_2$$

  - Non-uniform distribution of current in the conductors further increases the heating

- **Stray Loss**: Due to leakage flux/fields inducing eddy-currents in frame members, tank walls, bushing flanges, etc.

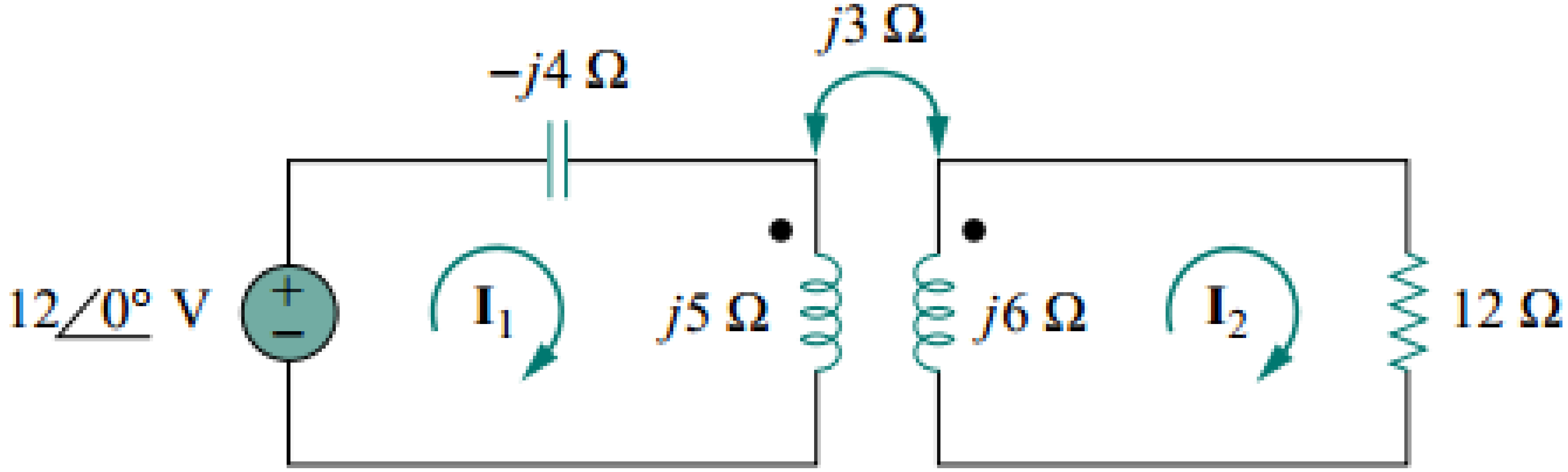
NUMERICAL PROBLEMS

1. Calculate the phasor currents I_1 and I_2 in the given circuit.



Soln: KVL in coil 1 => $12+(-4j+5j)I_1-3jI_2=0$ => $\boxed{jI_1-3jI_2=12}$ …(1)

KVL in coil 2 => $-3jI_1+(12+6j)I_2=0$ => $\boxed{I_1=(2-4j)I_2}$ …(2)



Substituting (2) into (1), we get $(2j+4-3j)I_2 = 12$

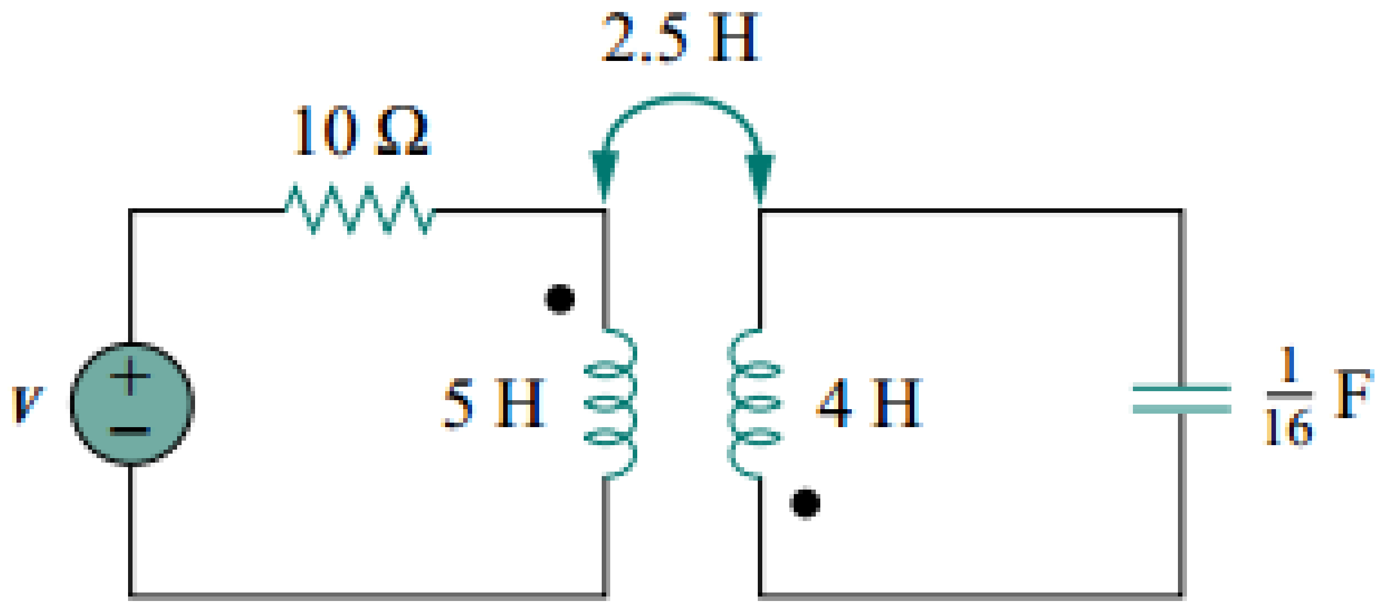
$$\Rightarrow \boxed{I_2 = \frac{12}{4-j} = 2.91\angle 14.04^0\,A}$$

Substituting above into (2),we get $I_1 = (2-4j)I_2$

$$\Rightarrow \boxed{I_1 = 13.01\angle -49.39^0\,A}$$

Numerical Problems

2. Determine the coupling coefficient for the transformer below.



Soln: The coupling coefficient is $k = \dfrac{M}{\sqrt{L_1 L_2}}$ => $\boxed{k = \dfrac{2.5}{\sqrt{5 \times 4}} = 0.56}$

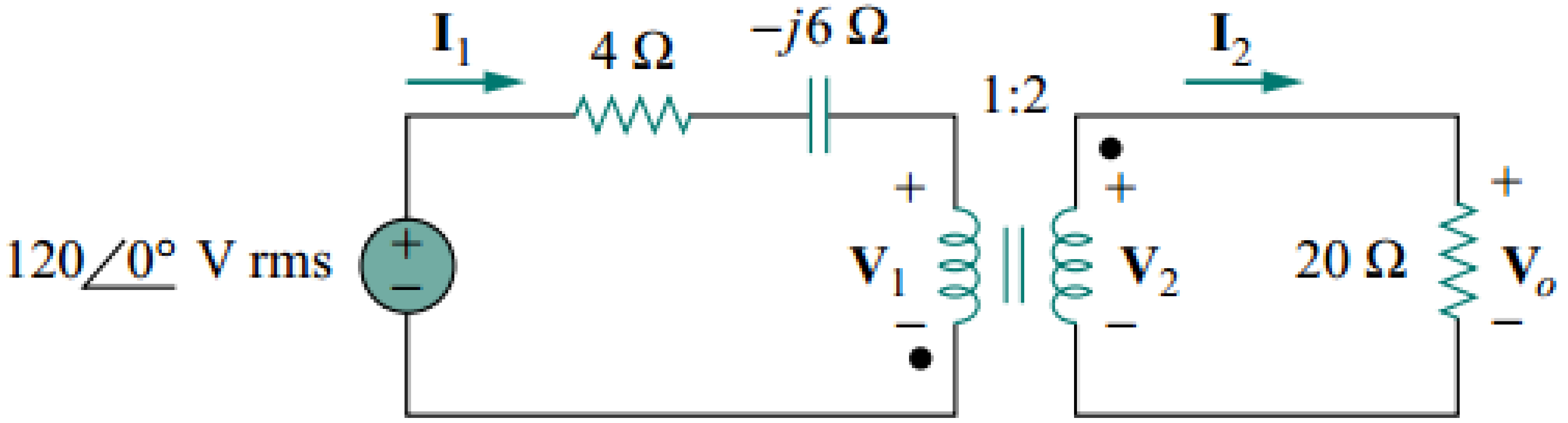
NUMERICAL PROBLEMS

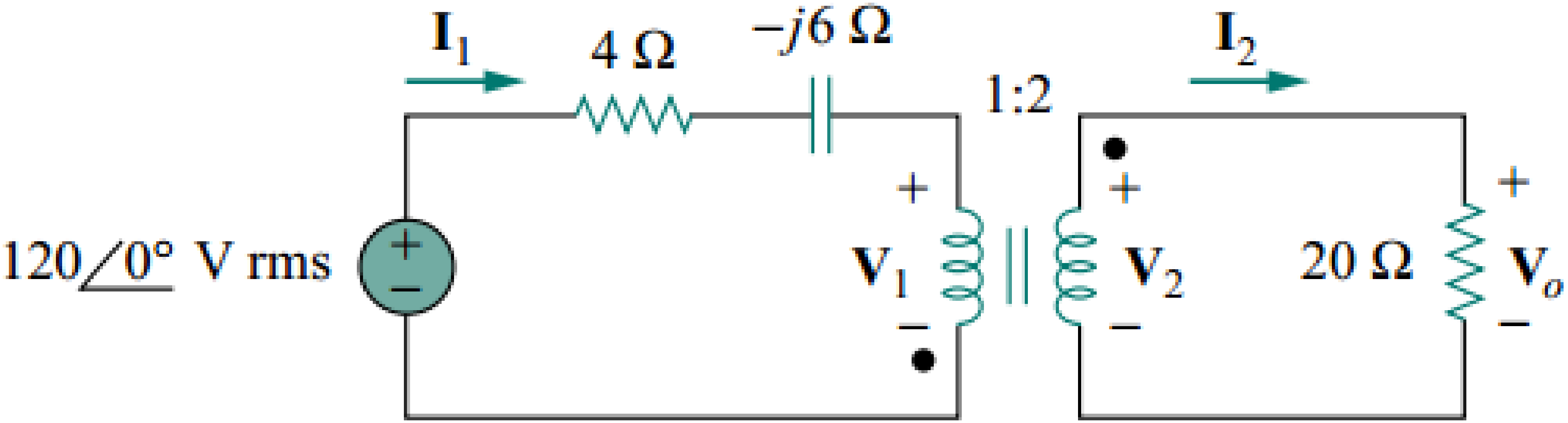
3. For the transformer circuit below, find:

(a) the effective input impedance Z_{in} and source current I_1,

(b) the output voltage V_o,

(c) the complex power S supplied by the source.





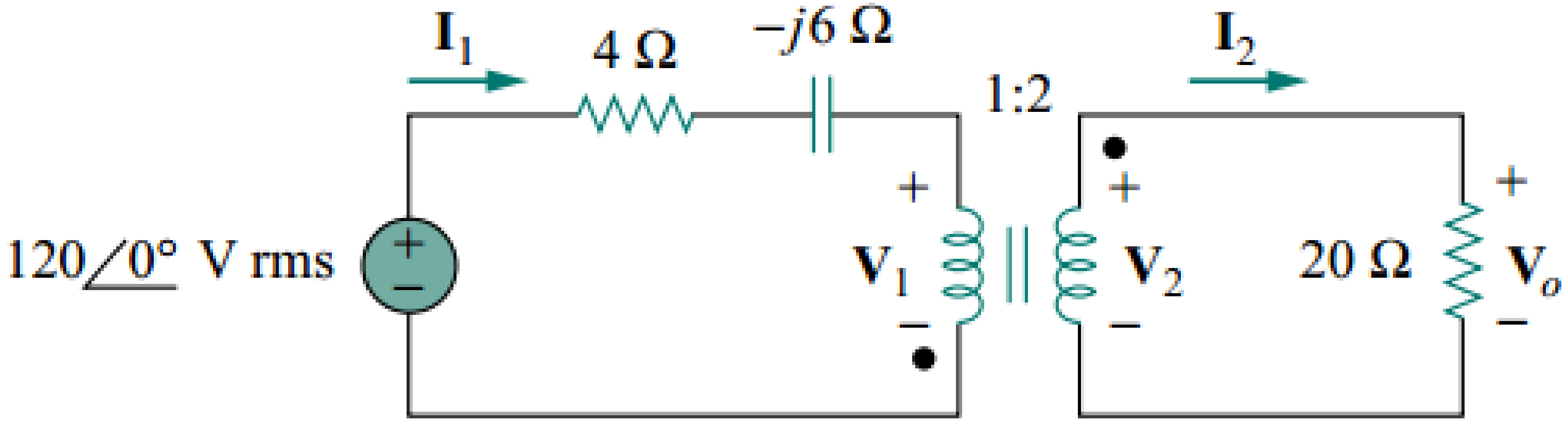
Soln:

(a) We first calculate the 20 Ω output resistance reflected to the primary side,

$$\boxed{Z_{r\_20\Omega} = \frac{v_1}{I_1} = \frac{v_2/n}{nI_2} = \frac{v_2}{n^2 I_2} = \frac{Z_L}{n^2} = \frac{20}{2^2} = 5\Omega}$$

Effective input impedance: $\boxed{Z_{in} = 4 - 6j + Z_{r\_20\Omega} = 9 - 6j = 10.82\angle -33.69^0 \Omega}$

Source current: $\boxed{I_1 = 120\angle 0^0 / Z_{in} = 120\angle 0^0 / 10.82\angle -33.69^0 = 11.09\angle 33.69^0 A}$



(b) Since both I_1 and I_2 leave the dotted terminals,

$$\boxed{I_2 = -I_1/n = -11.09\angle 33.69^0/2 = -5.545\angle 33.69^0\,A}$$

Output voltage: $\boxed{V_0 = 20I_2 = 110.9\angle -146.31^0\,A}$

(c) Complex Power from source:
$$\boxed{\begin{aligned} S &= V_s I_1^* = \left(120\angle 0^0\right)\left(11.09\angle 33.69^0\right)^* \\ &= 1330.8\angle -33.69^0 VA \end{aligned}}$$

NUMERICAL PROBLEMS

4. An ideal transformer has a transformation ratio of 0.125 and the primary current is 3 A when it is supplied with 240 V. Calculate the secondary voltage and current.

Soln: Transformation ratio is $\boxed{n = \frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{I_1}{I_2} = 0.125}$

The secondary voltage is $\boxed{V_2 = V_1 \times 0.125 = 30V}$

The secondary current is $\boxed{I_2 = \frac{I_1}{0.125} = 24A}$

Numerical Problems

5. An ideal transformer, connected to a 240 V mains, supplies power to a 12 V, 150 W lamp. Calculate the transformation ratio and the current drawn from the mains.

Soln: The secondary current is $\boxed{I_2 = \frac{P}{V_2} = \frac{150}{12} = 12.5A}$

The transformation ratio is $\boxed{n = \frac{V_2}{V_1} = \frac{12}{240} = 0.05}$

The current drawn from supply is $\boxed{I_1 = I_2 \times n = 12.5 \times 0.05 = 0.625A}$

NUMERICAL PROBLEMS

6. A 5-kVA single-phase transformer has a transformation ratio of 0.1 and is fed from a 2.5-kV supply. Neglecting losses, determine (a) the full load secondary current, (b) the minimum load resistance which can be connected across the secondary winding, (c) the primary current at full load kVA.

Soln: (a) The secondary voltage is $\boxed{V_2 = V_1 \times n = 2500 \times 0.1 = 250V}$

=> At full load, $\boxed{5kVA = V_2 \times I_2 \Rightarrow I_2 = 5000/250 = 20A}$

(b) The minimum load resistance is $\boxed{R_L = V_2/I_2 = 250/20 = 12.5\Omega}$

(c) The primary current at full load is $\boxed{I_1 = I_2.n = 20 \times 0.1 = 2\text{ A}}$

Numerical Problems

7. A 100 kVA, 4000V/200V, 50 Hz single-phase ideal transformer has 100 secondary turns. Determine (a) the primary and secondary current, (b) the number of primary turns, and (c) maximum value of the flux developed in the core.

Soln: (a)

$$\boxed{\begin{aligned} &100KVA = V_1 \times I_1 = V_2 \times I_2 \\ &\Rightarrow I_1 = 100000/V_1 = 100000/4000 = 25A \\ &\quad\; I_2 = 100000/V_2 = 100000/200 = 500A \end{aligned}}$$

(b)

$$\boxed{\begin{aligned} &N_2/N_1 = V_2/V_1 \\ &\Rightarrow N_1 = \left(V_1/V_2\right) N_2 = 2000 \end{aligned}}$$

(c) EMF equation for an ideal transformer is,

$$\boxed{\begin{aligned} &E_1 = 4.44 f \phi_m N_1 \\ &\Rightarrow \phi_m = \frac{4000}{4.44 \times 50 \times 2000} = 9.01 \times 10^{-3} Wb = 9.01 mWb \end{aligned}}$$

NUMERICAL PROBLEMS

8. A single-phase, 50 Hz ideal transformer has 25 primary turns and 300 secondary turns. The cross sectional area of the core is 300 cm^2. When the primary winding is connected to a 250 V supply, determine

(a) the maximum value of the flux density in the core, and

(b) the voltage induced in the secondary winding.

Soln: (a) EMF equation for an ideal transformer is,

$$\boxed{E_1 = 4.44 f \phi_m N_1 \Rightarrow \phi_m = \frac{250}{4.44 \times 50 \times 25} = 0.045 Wb}$$

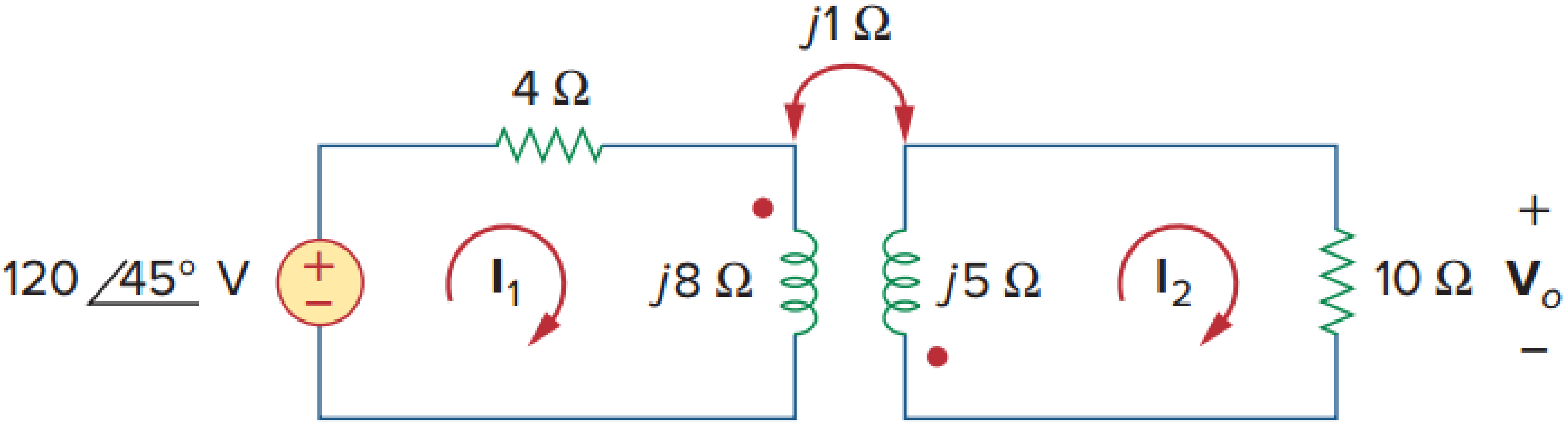
The maximum flux density in core is $\boxed{B_m = \frac{\phi_m}{A} = \frac{0.045}{300 \times 10^{-4}} = 1.5T}$

(b) EMF induced in secondary winding can be computed as,

$$\boxed{\begin{aligned} & N_2/N_1 = V_2/V_1 \\ & \Rightarrow V_2 = \left(N_2/N_1\right)V_1 = 3000V \end{aligned}}$$

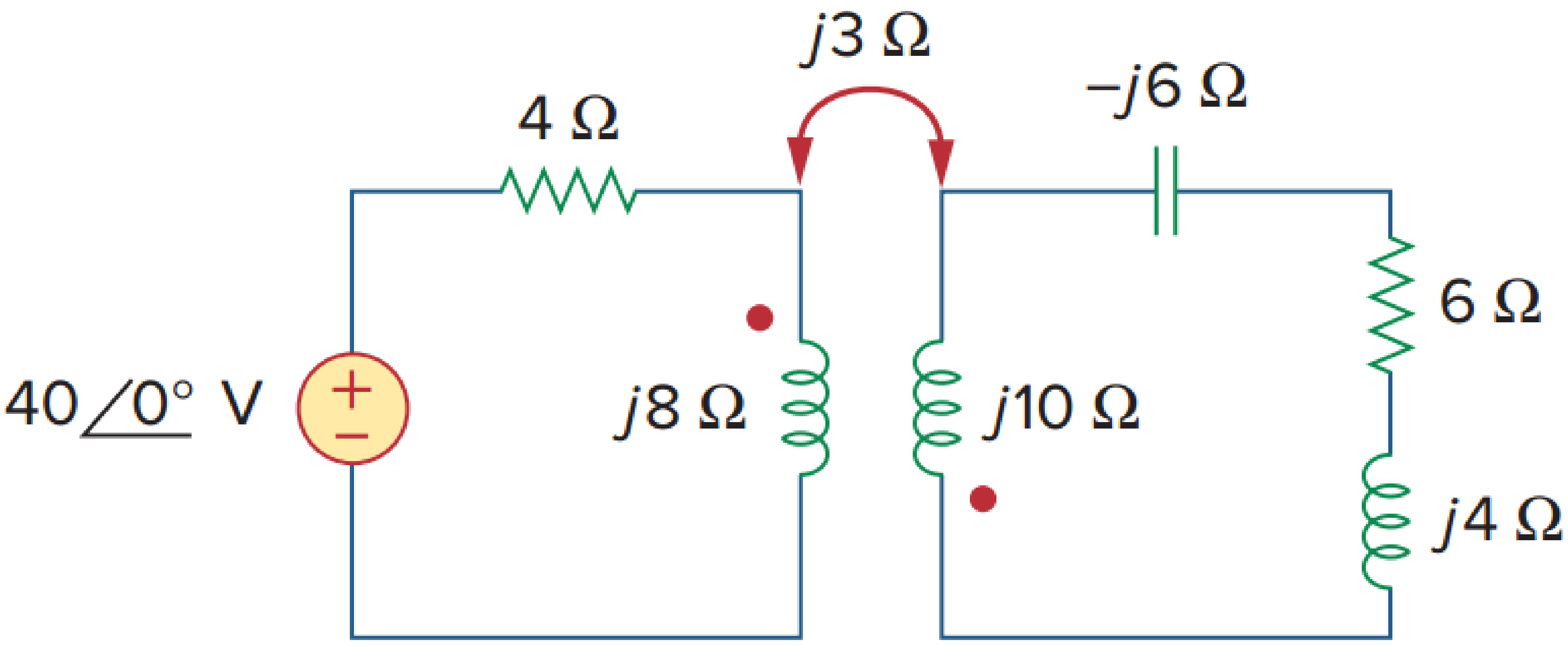
Unsolved Practice Problems

1. Determine the voltage V_o in the given circuit.



UNSOLVED PRACTICE PROBLEMS

2. Find the input impedance and the current from voltage source in the given circuit.



Unsolved Practice Problems

3. A step-down transformer having a transformation ratio of 0.05 has a primary voltage of 4 kV and a load of 10 kW. Neglecting losses, calculate the value of the secondary current.

4. A 10 kVA, single-phase transformer has a transformation ratio of 0.083 and is supplied from a 2.4 kV supply. Neglecting losses, determine (a) the full load secondary current, (b) the minimum value of load resistance which can be connected across the secondary winding without the kVA rating being exceeded, and (c) the primary current.

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1. Edward Hughes, *Electrical and Electronic Technology*, Pearson Education Limited, Essex, 2008.

2. Charles K. Alexander and Matthew N. O. Sadiku, *Fundamentals of Electric Circuits*, McGraw-Hill Education, New York, 2017.

3. John Bird, *Electrical Circuit Theory and Technology*, Elsevier Science, Oxford, UK, 2007.