Introduction for SS

Signal : A function which is having one or more independent variables is known as signal.

Basic types of signals:

→ Step signal

→ Delta | impulse

→ Ramp

→ Parabolic

→ Triangular

→ Sinusodial

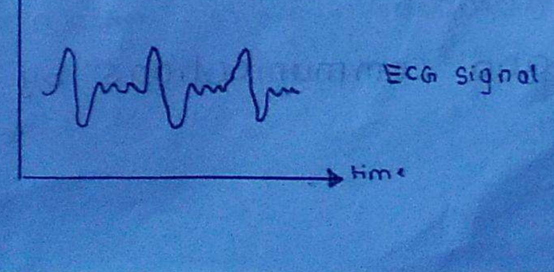
→ Exponential

→ Rectangular

* If signal has one independent variable. It is known as one dimensional signal. Ex: ECG, speech

* If signal has two independent variables It is known as two dimensional signal. Ex: Image → $I(x,y)$

* If signal has three independent variables. It is known as three dimensional signal. Ex: Video - $v(x,y)$.



* Signal types

1) continuous signals

2) discontinuous signals

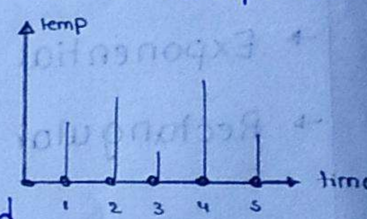
3) Analog signals

4) Digital signals.

* Continuous signals are continuous in time and also continuous in amplitude

Ex: daily life signals.

* Discrete signals are continuous in amplitude but discrete in time. Ex



discrete signals.

* continuous signals are denoted by "$x(t)$" {t = continuous}

* Discrete signals are denoted by "$x[n]$" {n = discrete}

* Digital signals have only values 0's & 1's

Books:

* signals and systems by Alan. V. oppenheim

* Signal, system and communications by B.P. Lathi

* Difference between Analog and Digital

* Analog signals are continuous in nature, whereas digital signals are discrete in nature.

* Analog signal wave type is sinusodial, whereas digital signal is a square

* Analog signal medium of transmission is wire or wireless, whereas a digital signal is a wire.

* Examples of analogs:: Any natural sound, human voice, data read by analog devices

* Examples of digitals:: Electronic signals, computer signals data read by digital devices.

→ * NPTEL for online learning

18/10/22

* Anything that carries information is called signal.

* A signal is a dependent variable or a function with one or more independent variable

$$f(x_1, x_2, \ldots\ldots)$$

f → dependent variable

$x_1, x_2, \ldots$ → independent variable.

* If a signal depends on one independent variable. it is called one dimensional signal → 1D S/g

Ex: ECG, speech signals.

* If a signal depends on two independent variables it is called two dimensional signal → 2D S/g

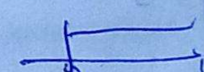
Ex: Image - $I(x, y)$

* If a signal depends on three independent variable it is called three dimensional signal → 3D S/g

Ex: video - $v(x, y, t)$

* Types of signals (Basic) standard

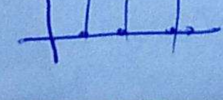
i) → Continuous signals:



When the signal values are continuous and are noted at each continuous time instant. Then they are continuous signals → CTCA (continuous time continuous Amplitude)

2) → Discrete time signals:

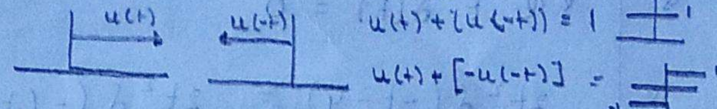
When the signal values are continuous and are noted at discrete time ~~in~~. Then they are Discrete signals → DTCA (Discrete time continuous Amplitude)



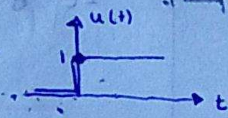
3) → Digital Signals :-

Discrete in time, but quantized in amplitude.

* Basic signals



1) Unit step signal :- $u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$

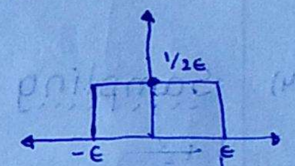


Ex: 1) To check the stability of the system

$\boxed{u(t) + u(-t) = 1}$ {amplitude = 1}

2) Dirac-Delta function :- "{area = 1}"

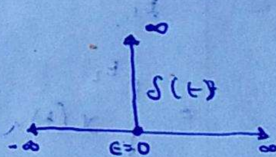
$h(t) = \begin{cases} 1/2\epsilon & -\epsilon \text{ to } \epsilon \ (-\epsilon < t < \epsilon) \\ 0 & \text{ew (else where)} \end{cases}$



$2\epsilon \times \frac{1}{2\epsilon} = 1$

→ $h(t)$ becomes $\delta(t)$ when ϵ becomes zero

$\delta(t) = \lim_{\epsilon \to 0} h(t) = \begin{cases} \infty & t = 0 \\ 0 & ew \end{cases}$



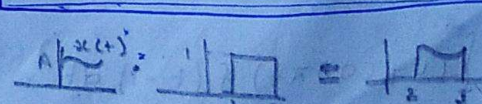
$\delta(t)$ = impulse function. "$\delta(t - 0) = \infty$ at 0"

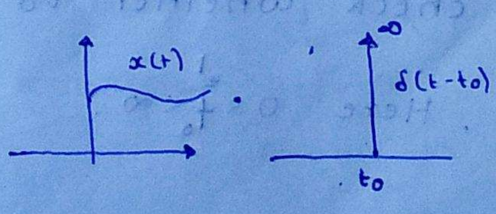
* Properties of unit impulse function:

1) $\int_{-\infty}^{\infty} \delta(t)\, dt = 1$ {area of the graph} = 1

2) $\delta(t)$ is a even function "{$\delta(-t) = \delta(t)$}"

3) Product property :- $\boxed{x(t) \cdot \delta(t - t_0) = x(t_0) \cdot \delta(t - t_0)}$





$= x(t_0)\, \delta(t - t_0)$.

$x(t_0)$ is a point in the graph of $x(t)$ determining the value at $x(t_0)$.

Q $\cos t \cdot \delta(t-\pi/4) :- \cos\frac{\pi}{4} \cdot \delta\left(t-\frac{\pi}{4}\right)$

$= \delta\left(t-\frac{\pi}{4}\right) \cdot \frac{1}{\sqrt{2}}$

Q $t \cdot \delta(t) = 0$

Q $2t^2 \delta(-t-4) = 2t^2 \delta(-(t+4))$

$= 2t^2 \delta(t+4)$ {even function}

$= 2(4)^2 \cdot \delta(t+4)$

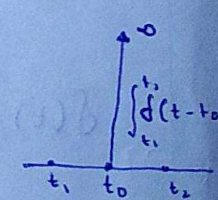
$= 32 \cdot \delta(t+4)$

4) <u>Sampling or shifting property</u> :-

$$\int_{t_1}^{t_2} x(t) \cdot \delta(t-t_0)\, dt = \begin{cases} x(t) & t_1 \le t_0 \le t_2 \\ 0 & \text{ew} \end{cases}$$

$$= \int_{t_1}^{t_2} x(t) \cdot \delta(t-t_0)\, dt = \int_{t_1}^{t_2} x(t_0)\, \delta(t-t_0)\, dt$$

$$= x(t_0) \int_{t_1}^{t_2} \delta(t-t_0)\, dt$$



$= x(t_0) \cdot 1$ t_0 lies b/w t_1 & t_2

** $\int_{t_1}^{t_2} \delta(t-t_0)$ means area of graph of $\delta(t-t_0)$ between intervals t_1 & t_2. As t_0 is in b/w t_1 & t_2 the area of $\delta(t-t_0) = 1$

Q $\int_{0}^{\infty} (t+\cos\pi t)\, \delta(t-1)\, dt$. check whether t_0 is in b/w t_1 & t_2 or not. Here $0 < \overset{1}{t_0} < \infty$.

so, $t + \cos\pi t \,|_{t=1}$

$= 1 + \cos\pi$

$= 0$ //

Q $\int_0^{\infty} y(2-t)\ \delta(3-t)\ dt.$

$\rightarrow \int_0^{\infty} y(2-t)\ \delta(-(t-3))\ dt$

$= \int_0^{\infty} y(2-t)\ \delta(t-3)\ dt$ {even function}

$0 < 3 < \infty$

$= y(2-t)\big|_{t=3}$

$= y(2-3)$

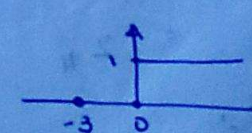
$= y(-1)$ //

Q $\int_0^{\infty} \cos t\ u(t-3)\ \delta(t)\ dt$

Here $t_0 = 0 \rightarrow 0 \leq 0 < \infty$, so

$= \cos t\ u(t-3)\big|_{t=0}$ $\qquad u(t) \Rightarrow$ unit step signal

$= \cos 0\ u(0-3)$ $\qquad u(-3) =$



$= 1(0)$ $\{u(-3) = 0\}$ $\qquad u(-3) = 0$

$= 0$ //

5) Time scaling :

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

* we know $\int_{-\infty}^{\infty} \delta(at)\ dt$

consider $at = \lambda$

$a\ dt = d\lambda$

$dt = \frac{d\lambda}{|a|}$

$$= \frac{1}{|a|} \int_{-\infty}^{\infty} \delta(\lambda)\, d\lambda$$

Q $$\int_{-\infty}^{\infty} t^2 \, \delta(-t/_2 + 1/_2)\, dt$$

A $$\int_{-\infty}^{\infty} t^2 \, \delta\left(-\left(\frac{t - 1}{2}\right)\right) dt$$ {even function}

$$= \int_{-\infty}^{\infty} t^2 \, \delta\left(\frac{t - 1}{2}\right) dt \quad \int_{0}^{\infty} t^2 \, \delta\left(\frac{1}{2}(t - 1)\right) dt$$

$$= \int_{-\infty}^{\infty} t^2 \, \frac{1}{1/_2} \, \delta(t - 1)\, dt$$ {Time scaling property}

$$= 2 \int_{-\infty}^{\infty} t^2 \, \delta(t - 1)\, dt$$

{ +1 lies b/w $-\infty$ & ∞ }

$$= 2 \; t^2 |_{t=+1}$$

$$= 2 \; (+1)^2$$

$$= 2$$ //

Q $$\int_{-2}^{1} (t + t^2)\, \delta(t-3)\, dt = 0$$ { $t_0 = 3$ doesn't lie b/w -2 & 1 }

Q $$\int_{-1}^{4} (t + t^2)\, \delta(t-3)\, dt$$ { $t_0 = 3$ lies b/w -1 & 4)

So from sampling propert

$$= t + t^2 \,|_{t=3})$$

$$= 12$$ //

Q $$\int_{0}^{3} e^{t-2} \, \delta(2t - 4)\, dt$$

A $\int_0^3 e^{t-2} \delta(2(t-2))\, dt$

$= \int_0^3 e^{t-2} \frac{1}{2} \delta(t-2)\, dt$ (Time scaling property)

$= \frac{1}{2} \int_0^3 e^{t-2} \delta(t-2)\, dt$ (sampling property)

$= \frac{1}{2} e^{t-2} \big|_{t=2}$

$= \frac{1}{2} e^{2-2}$ $\{e^0 = 1\}$

$= 1/2$

Q $\int_0^3 e^{t-2} \delta(2t-2)\, dt$

A $= \int_0^3 e^{t-2} \delta(2(t-1))\, dt$

$= \int_0^3 e^{t-2} \frac{1}{2} \delta(t-1)\, dt$ {Time scaling property}

$= \frac{1}{2} \int_0^3 e^{t-2} \delta(t-1)\, dt$

$= \frac{1}{2} e^{t-2} \big|_{t=1}$

$= \frac{1}{2} e^{1-2}$

$= \frac{1}{2} e^{-1}$

$= \frac{1}{2e}$

Relation b/w $u(t)$ & $\delta(t)$:-

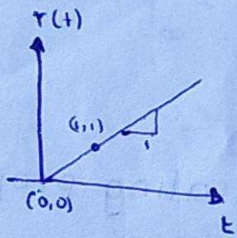
$$\int_{-\infty}^{t} \delta(t)\,dt = 0 \quad \forall \quad t<0$$

$$\int_{-\infty}^{t} \delta(t)\,dt = 1 \quad \forall \quad t>0 \qquad u(t) = \begin{cases} 1 & t>0 \\ 0 & t<0 \end{cases}$$

$$\boxed{u(t) = \int_{-\infty}^{t} \delta(t)\,dt}$$

$$\boxed{\delta(t) = \frac{d}{dt} u(t)}$$

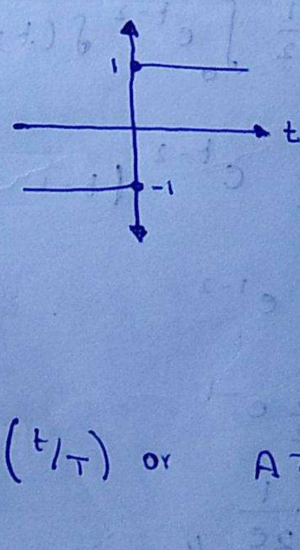
* Unit ramp signal :- {slope = 1}



$$r(t) = \begin{cases} t & t \geq 0 \\ 0 & t<0 \end{cases}$$

* Signum function :-

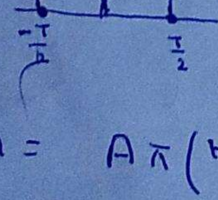
$$sgn(t) = \begin{cases} 1 & t>0 \\ -1 & t<0 \end{cases}$$



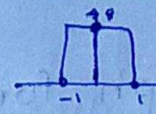
* Rectangular function:

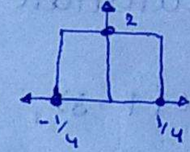
General form = $A\,rect(t/T)$ or $A\pi(t/T)$

Graph



Expression = $A\pi(t/T) = \begin{cases} A & -\frac{T}{2} < t < \frac{T}{2} \\ 0 & ew \end{cases}$

Ex $8 \,\text{rect}(t/2)$ 

Ex $2 \,\text{rect}(t/\tfrac{1}{2})$ or $2 \,\text{rect}(2t)$ = 

Relation b/w $r(t)$ & $u(t)$ = $\boxed{u(t) = \frac{dr(t)}{dt}}$

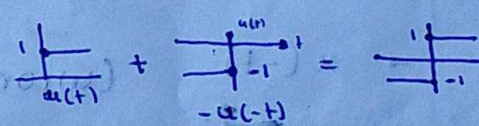
$$\frac{dr(t)}{dt} = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases} = u(t)$$

* $\delta(t) \xrightarrow{\text{integral}} u(t) \xrightarrow{\text{integral}} r(t)$

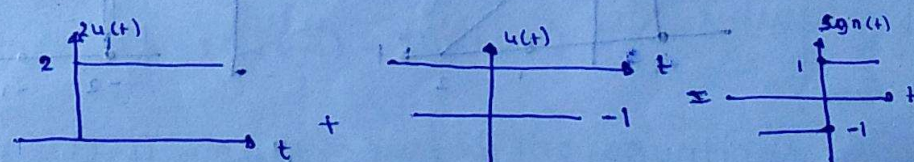
derivative derivative.

* 1) $\delta(t) = \frac{d}{dt} u(t)$

* 2) $r(t) = \int u(t)\, dt = t \cdot u(t)$

* 3) $sgn(t) = u(t) - u(-t)$ 

$= 2u(t) - 1$

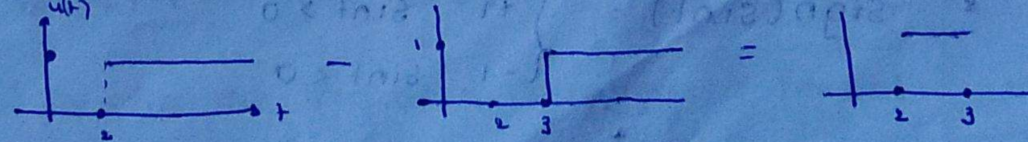


* Speech signal is sum of sinusoids.

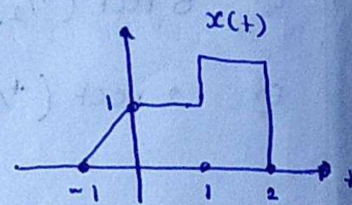
* $u(t - t_0) \to$ Right shift

* $u(t + t_0) \to$ left shift

Ex Graph $u(t-2) - u(t-3)$

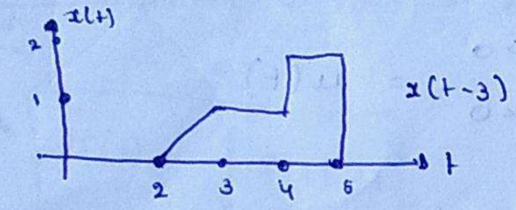


Calculate the graph for $x(t-3)$, $x(t+3)$, $x(t)$

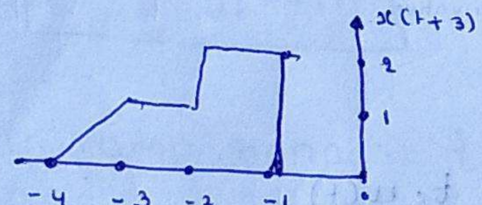


→ $x(t-3)$

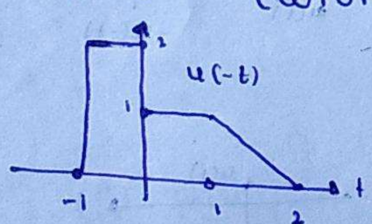
=

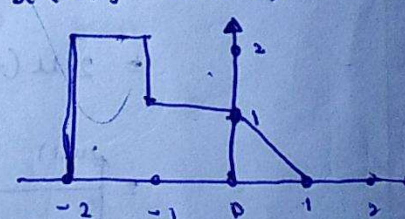


→ $x(t+3)$

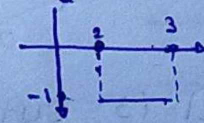


→ $x(-t)$ (wrong) | $x(-t)$ (correct)

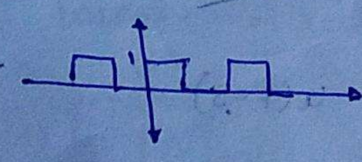


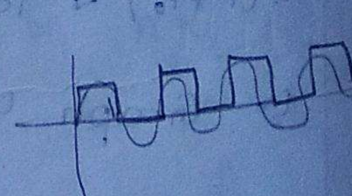


* $u(t-3) - u(t-2)$

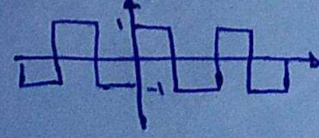


* $u(\sin t) = \begin{cases} 1 & \sin t > 0 \\ 0 & \sin t < 0 \end{cases}$ $\quad u(t) = \begin{cases} & t > 0 \\ 0, & t < 0 \end{cases}$

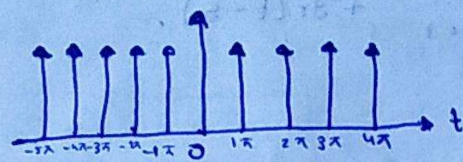
= 

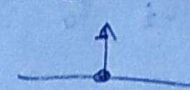


* $\text{sign}(\sin t) = \begin{cases} +1 & \sin t > 0 \\ -1 & \sin t < 0 \end{cases}$

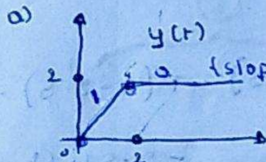


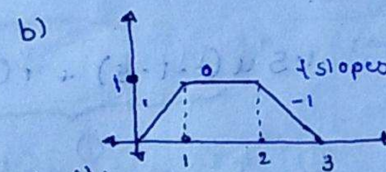
$$\delta(\sin t) = \begin{cases} \infty & \sin t = 0 \\ 0 & \sin t \neq 0 \end{cases} \quad (\pi, 2\pi$$

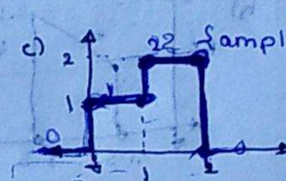




* Graphs are given, write the equation for them respectively.

a)


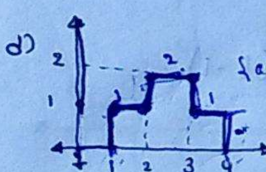
b)


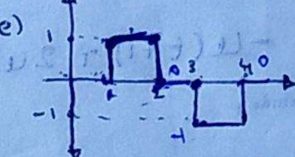
c)


$u(t) + u(t-1) - 2u(t$

Signum of sin t :-

$1u(t-0) + 1u(t-1) + t-2$
$-2u(t-2)$

d)


e)


a) $r(t) - r(t-2)$ $\cancel{+2u(t-2)}$ {is the equation}

c) $u(t) - u(t-1) + 2u(t-1) - 2u(t-2)$

$= u(t) + u(t-1) - 2u(t-2)$ {is the equation}

d) $u(t-1) - u(t-2) + 2u(t-2) - 2u(t-3) + u(t-3) - u(t-4)$

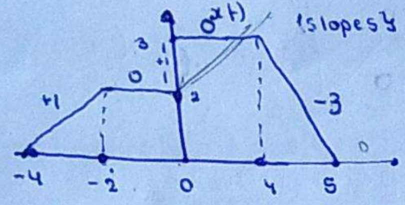
$= u(t-1) + u(t-2) - u(t-3) - u(t-4)$ {is the equation}

e) $u(t-1) - u(t-2) + -[u(t-3) - u(t-4)]$

$= u(t-1) - u(t-2) - u(t-3) + u(t-4)$

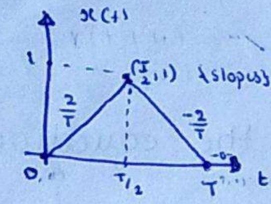
b) $r(t) - r(t-1) - r(t-2) + r(t-3)$.

{This is the required equation. for (b)

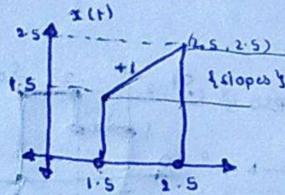
f) 

$r(t+4) - r(t+2) + u(t) - 3r(t-4)$

$+ 3r(t-5)$.

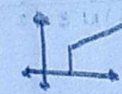
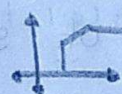
g) 

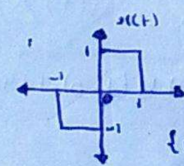
$\frac{2}{T} r(t) - \frac{4}{T} r(t - \frac{T}{2}) + \frac{2}{T} r(t-T)$

h) 

{slopes} $\underbrace{1.5u(t-1.5) + r(t-1.5)}_{①}$ $\overbrace{\quad - r(t-2.5)}^{②}$.

$- 2.5u(t-2.5)$.

①  ② 

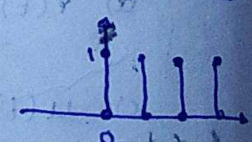
(i) 

$-u(t+1) + 2u(t) - u(t-1)$

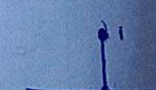
{amplitude}

* Discontinuous Signals {Discrete}

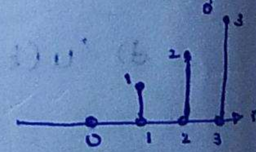
1) unit step signal → $u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$



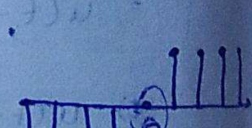
2) Delta dirac (impulse) function: $\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$



3) unit ramp signal → $r[n] = \begin{cases} n & n \geq 0 \\ 0 & n < 0 \end{cases}$



4) signum function → $sg[n] = \begin{cases} 1 & n > 0 \\ -1 & n < 0 \\ 0 & n = 0 \end{cases}$



→ Express $\delta[n]$ in terms of $u[n]$

$$\boxed{\delta[n] = u[n] - u[n-1]}$$

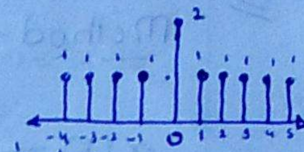
→ Express $u[n]$ in terms of $\delta[n]$

$$\boxed{u[n] = \sum_{k=0}^{\infty} \delta[n-k]}$$

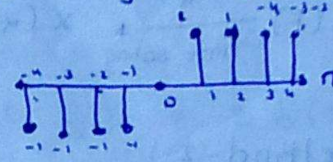
→ Express r[n] in terms of u[n]

$$\boxed{r[n] = n \cdot u[n]}$$

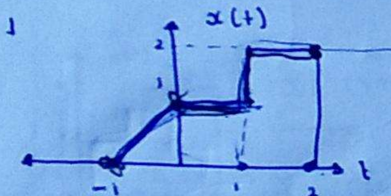
* u[n] + u[-n] = not costant signal



* $$\boxed{u[n] - u[-n] = sg[n]}$$

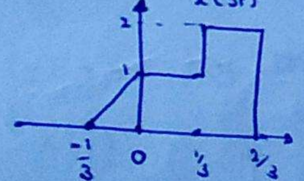


Transformation of signals :

* 

Given x(t), we have to find "x(3t - 5)"

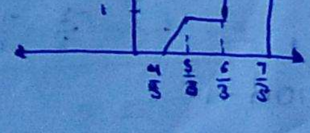
Method 1 : first find x(3t) {Time scaling & shifting}



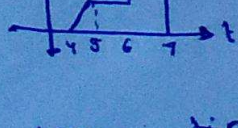
Now $x(3t-5) = x\left[3\left(t - \frac{5}{3}\right)\right]$

find intervals for x(3t - 5)

$t - \frac{5}{3} = \frac{-1}{3}$, $\{t = \frac{4}{3}\}$ to $t - \frac{5}{3} = \frac{2}{3}$ $\{t = \frac{7}{3}\}$

x(3t - 5) = 

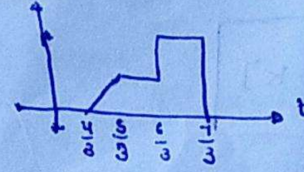
Method-2 : first find x(t - 5) {shifting & Time scaling}



{shifting 5 units}

→ (1)

Now time scaling graph -1

$x(3t-5) =$ 

This the required graph.

Note

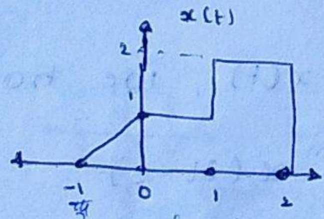
Method - 1 → $x(\alpha t + \beta) = x\left[\alpha\left(t + \beta/\alpha\right)\right]$

$$x(t) \xrightarrow[\text{Time scaling}]{\alpha} x(\alpha t) \xrightarrow[\beta/\alpha]{\text{Time Shifting}} x(\alpha(t + \beta/\alpha))$$

Method - 2

$$x(t) \xrightarrow[\text{Time shifting}]{\beta} x(t+\beta) \xrightarrow[\text{Time scaling}]{\alpha} x(\alpha t + \beta)$$

Q



Find
1) $x(2t+1)$

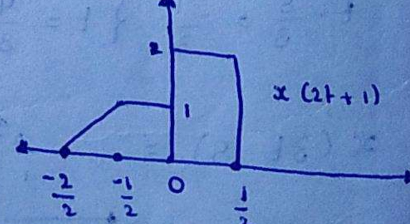
2) $x(5-t)$

3) $x(-t-2)$

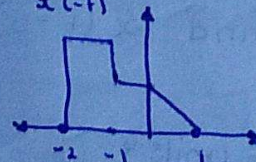
4) $[x(t) + x(-t)]\,u(t)$

5) $x(t) \cdot \delta(t - \frac{3}{2})$

1) $x(2t+1)$ by method - 2

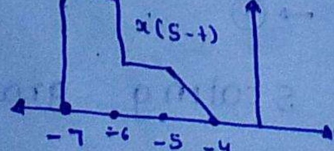


2) $x(-t+5)$ by method - 1

$x(-t) =$ 

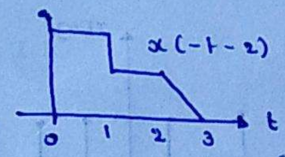
shift 5 units

$\{-7 \text{ to } -4\}$

$x(5-t) =$ 

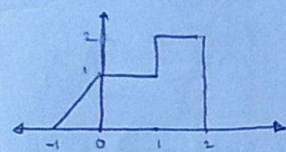
3) $x(-t-2)$ by method-1

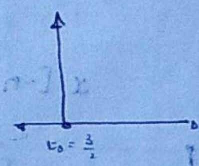
Required graph $\longrightarrow$



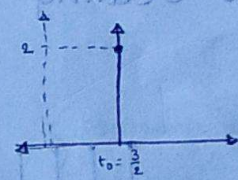
4) $[x(t) + x(-t)]\, u(t)$

5) $x(t) \cdot \delta(t - \frac{3}{2}) = x(\frac{3}{2}) \cdot \delta(t - \frac{3}{2}) = 2\,\delta(t - \frac{3}{2})$

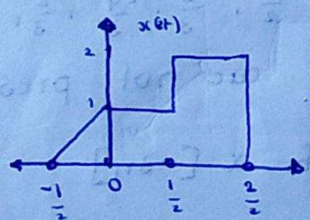




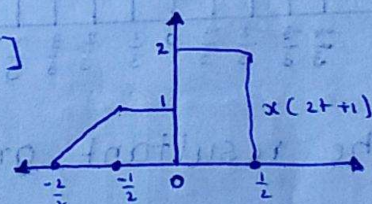
=



1) $x(2t+1)$ by method-1



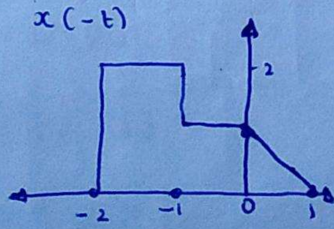
$x[2(t+\frac{1}{2})]$ $\longrightarrow$



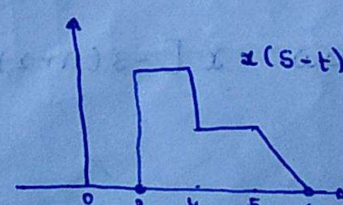
Time scaling　　　　Shifting

2) $x(-t+5)$ by method-1　　$x(-(t-5))$

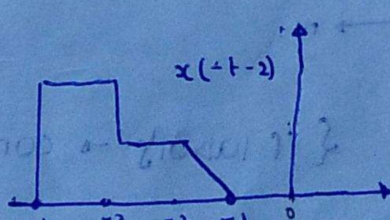
$x(-t)$



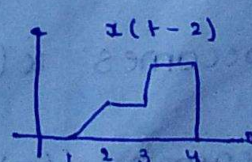
$\longrightarrow$



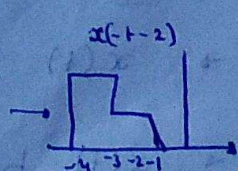
3) $x(-t-2)$ by method-1　　$x(-(t+2))$



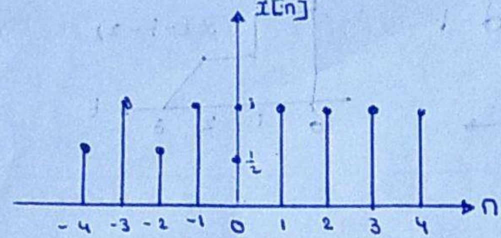
By method-2



$\rightarrow$



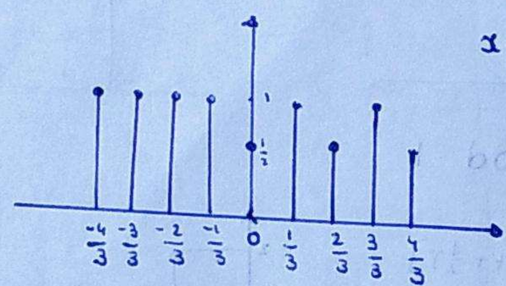
Q



Now draw $x[-6-3n] = x[-(3n+6)]$

Method-1 = $x[-(3(n+2))]$

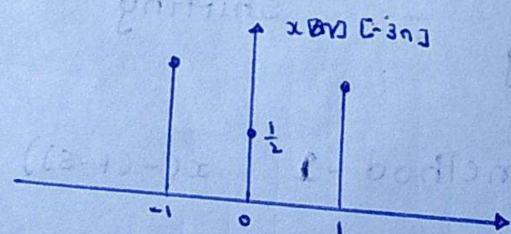
Time scaling.



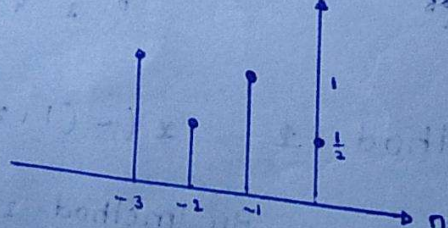
{ But signals at $\frac{-1}{2}$, $\frac{-2}{3}$, $\frac{-4}{3}$, $\frac{2}{3}$, $\frac{1}{3}$, $\frac{4}{3}$ are not present }

so the resultant graph of $x[-3n]$

=



Now $x[-3(n+2)]$ =

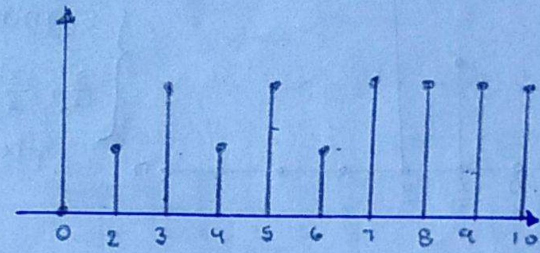


Note :-

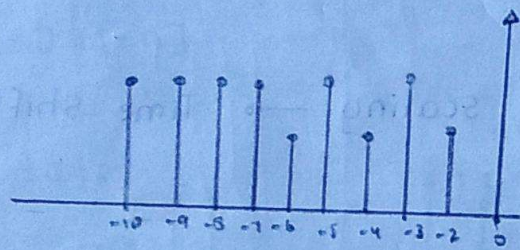
→ $x(t)$ becomes $x(at)$ {if $|a|>1$} → compression

→ $x(t)$ becomes $x(at)$ {if $|a|<1$} → expansion

Method - 2 $x[-6]$

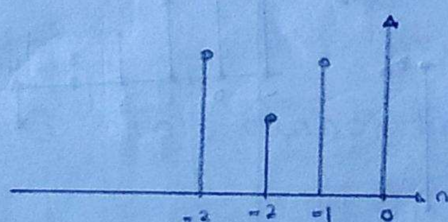


$x[-n-6]$



Singnals at $\frac{-2}{3}$, $\frac{-4}{3}$, $\frac{-5}{6}$, $\frac{-7}{8}$, $-\frac{8}{3}$, $-\frac{10}{3}$ are absent

$x[-3n-6]$



This is the solution.

Q $x[n] = n \cdot [u(n+5) - u(n-5)]$

1) $x[2n]$

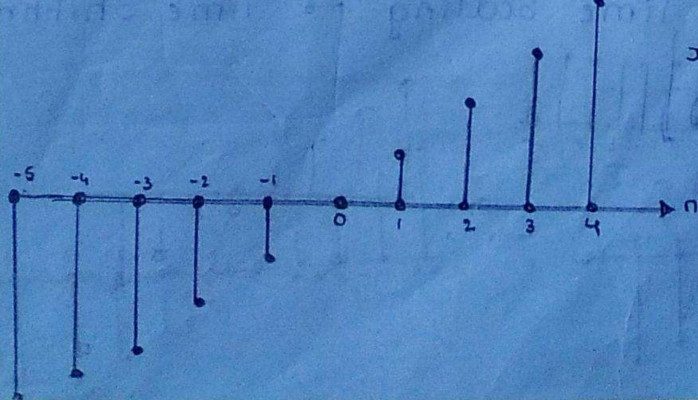
2) $x[3-n]$

4) $x\left[\frac{3-n}{3}\right]$

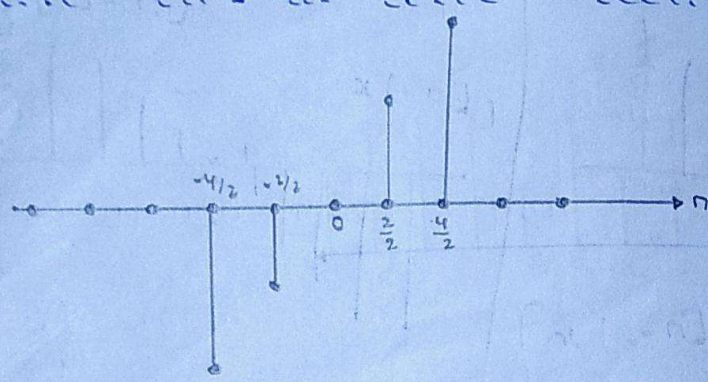
5) $x[n]\,u[-n]$

6) $x[4-3m]$

$x[n] = n[u(n+5) - u(n-5)]$



$x[2n]$ {only one method possible}

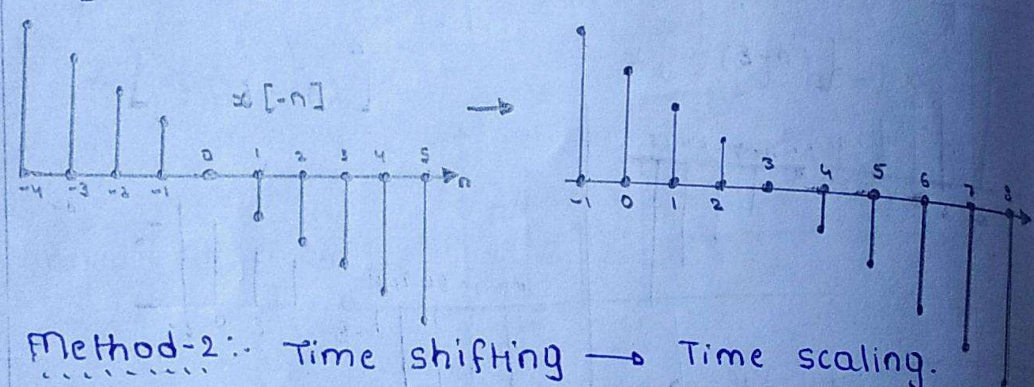


{ signals at $\frac{1}{2}, \frac{-1}{2}$, $\frac{3}{2}, \frac{-3}{2}, \frac{-5}{2}$ are not present

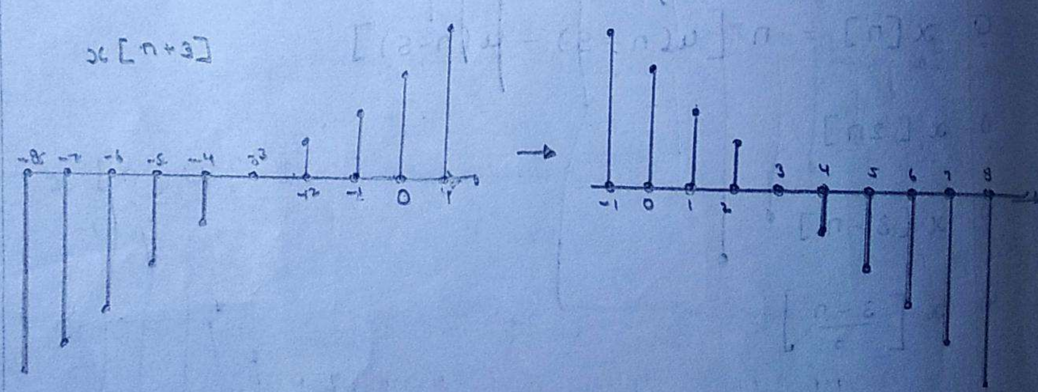
* $x[3-n]$

Method-1 : Time Scaling $\rightarrow$ Time Shifting

$x[-(n-3)]$



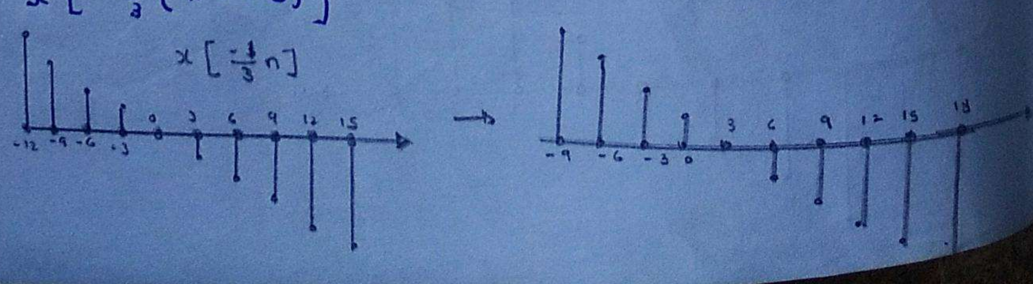
Method-2 : Time shifting $\rightarrow$ Time scaling.



* $x\left[\frac{3-n}{3}\right] = x\left[1 - \frac{n}{3}\right]$

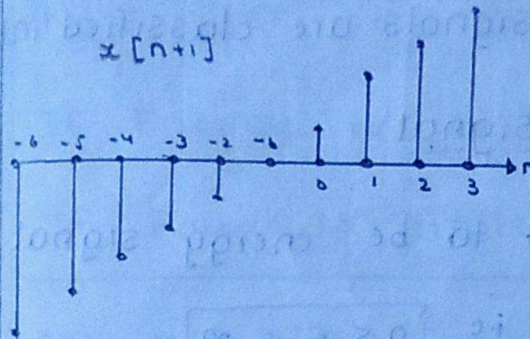
Method-1 : Time Scaling $\rightarrow$ Time Shifting.

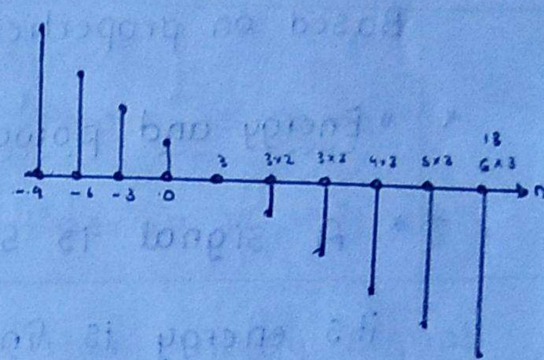
$x\left[-\frac{1}{3}(n-3)\right]$



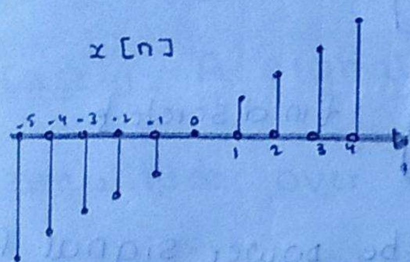
Method-2 : Time shifting $\longrightarrow$ Time scaling.

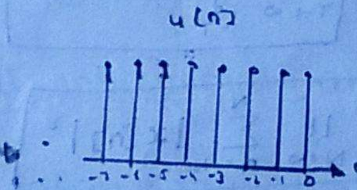
$x[1-\frac{n}{3}]$

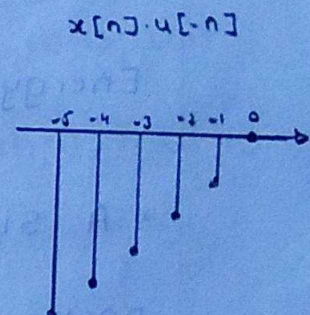




* $x[n]u[-n]$

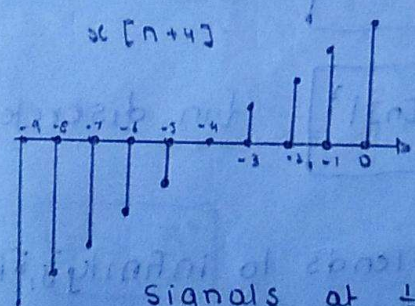


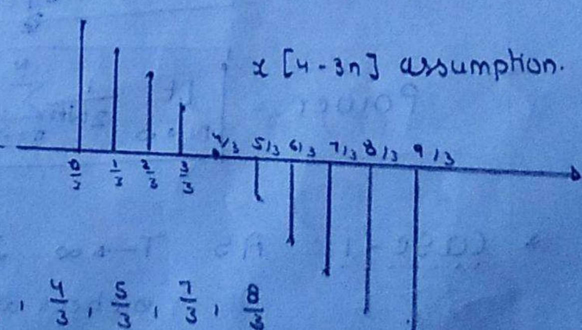




* $x[4-3n]$ This cannot be done in method-1

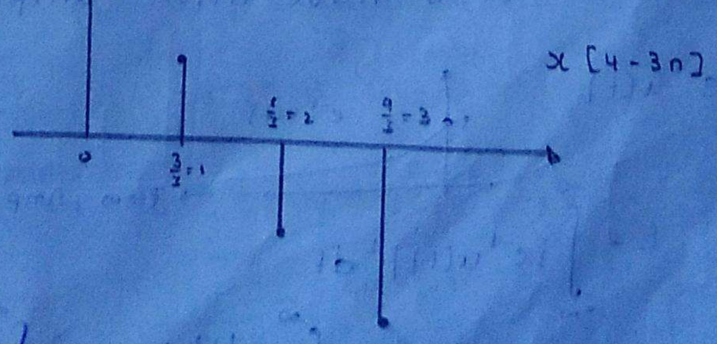
Method-2 → Time scaling ↔ Time shifting.





signals at $\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{5}{3}, \frac{7}{3}, \frac{8}{3}$

are not present, the resultant signal is



* ## Classification of signals :-

Based on properties, signals are classified into 5 types

* 1) **Energy and power signal**:

* A signal is said to be energy signal, if its energy is finite. i.e $\boxed{0 < E < \infty}$

Energy = $\boxed{\lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 dt}$ {in continuous}

Energy = $\boxed{\lim_{N \to \infty} \sum_{n=-N}^{N} |x[n]|^2}$ {in discrete}

* A signal is said to be power signal if its power is finitie i.e $\boxed{0 < P < \infty}$

Power = $\boxed{\lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt}$ {in continuous}

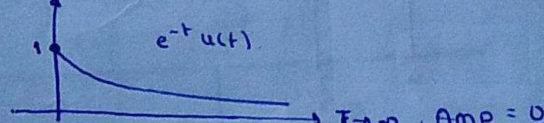
Power = $\boxed{\lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2}$ {in discrete}

* case - i : As $T \to \infty$ {T tends to infinity}, if Amplitude tends to 0, then it is the energy signal

check at $-\infty$ also.

{It should also have finite amplitude}

Ex: $e^{-t}u(t)$



Now $E = \int_{-\infty}^{\infty} |e^{-t}u(t)|^2 dt$

$= \int_{0}^{\infty} (e^{-t})^2 dt = \int_{0}^{\infty} e^{-2t} dt$

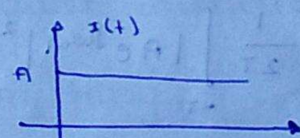
$$= \frac{-1}{2} e^{-2t} \Big|_0^{\infty} \quad 7b \ \boxed{\frac{1}{2}} \quad \{e^{-\infty} = 0\}$$

NOTE:

* If Energy is finite, Power is zero &
* If Power is finite, Energy is infinite.

Ex2: $x(t) = |1/t|$ It is neither energy signal nor power signal {bcoz its amplitude $= \infty$ at $t=0$}

* Case-ii: A signal which maintains constant amplitude over infinite duration, then that is a power signal.



Ex: $x(t) = A \cdot u(t)$.

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |A\,u(t)|^2 \, dt = \int_0^T |A\,u(t)|^2 \, dt$$

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_0^T A^2 \, dt = \frac{1}{2T} \cdot A^2 (T)$$

$$\boxed{P = \frac{A^2}{2}}$$

* Case iii: All periodic signals are power signals

Ex: $A\cos(\omega_0 t)$ is a periodic signal. so, it is a power signal.

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-\infty}^{\infty} (A\cos(\omega_0 t))^2 \, dt = \frac{1}{2T} \cdot A^2 \sin \omega_0 t$$

$$P = \lim_{T \to \infty} \frac{1}{2T} A^2 \int_{-\infty}^{\infty} \cos^2 \omega_0 t \, dt \quad \left\{ \int_{-\infty}^{\infty} \frac{\cos 2\omega_0 t + 1}{2} \right\}$$

$$P = \lim_{T\to\infty} \frac{A^2}{4T} \int_{-T}^{T} (\cos 2\omega_0 t + 1)\, dt$$

$$P = \lim_{T\to\infty} \frac{A^2}{4T} \left[\frac{\sin 2\omega_0 t}{2\omega_0} + t \right]_{-T}^{T} = \lim_{T\to\infty} \frac{A^2}{4T} [t]_{-T}^{T}$$

$$P = \lim_{T\to\infty} \frac{A^2}{4T} (2T)$$

$$\boxed{P = \frac{A^2}{2}}$$

Ex 2 : $A e^{j\omega_0 t} = A(\cos\omega_0 t + j \sin\omega_0 t)$ {formula}

This is a periodic signal so, it is power signal.

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |A e^{j\omega_0 t}|^2\, dt$$

$\left\{ |a+ib| = \sqrt{a^2+b^2} \right.$

$|\cos\omega_0 t + j \sin\omega_0 t| = \sqrt{\cos^2\omega_0 t + \sin^2\omega_0 t}$

$\left. = 1 \right\}$

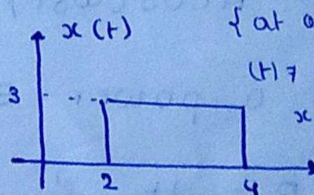
$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} A^2\, dt$$

$$P = \lim_{T\to\infty} \frac{1}{2T} \cdot A^2 (2T)$$

$$\boxed{P = A^2}$$

* Case-iv : Finite amplitude with finite length, such signals are energy signals.

$$\text{Energy} = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2\, dt$$



{at any interval (t) the value of $x = 3$ so $x(t) = 3$

$$= \int_{2}^{4} |3|^2\, dt$$

$$= 9(2) = 18$$

* Case V : If $T \to \infty$, Amplitude $\to \infty$ (or)

If $T \to -\infty$, Amplitude $\to \infty$ (or)

If $T \to 0$, Amplitude $\to \infty$. Then

These signals are neither energy nor power signal.

Ex: e^{-3t} {is neither energy nor power}

A: when we substitute $t = -\infty$, the amplitude becomes constant (not 0). So, it is not energy.

Ex: $x(t) = r(t)$ {is neither energy nor power}

→ * Product of two power signals is undefined and it can be anything.

Q 1) e^{-2t^2} is an energy signal.

Q→ $\alpha^n u[n]$

a) $\alpha = -1$

b) $\alpha = \frac{1}{2}$

c) $\alpha = 1$

d) $\alpha = 2$

Energy

1) $x(t) \to E$
2) $x(-t) \to E$
3) $-x(t) \to E$
4) $x(t \pm T) \to E$
5) $x(at) \to \frac{E}{|a|}$
6) $a\,x(t) = a^2 E$
7) $x(at-b) \to \frac{E}{a}$

Q→ If $x(t)$ is equal to $\delta(t+2) - \delta(t-2)$. Find energy in $y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$

Q→ $u(-t) + Ae^{-t}$ is power or energy?

Q→ If $y(t)$'s energy is (E) what is the energy $y(\alpha t) \longrightarrow ?$

Answers are backside:

* Addition of power signal & Energy signal is power signal.

* For Power signal, $E = \infty$, $P =$ finite

* For Energy signal, $P = 0$, $E =$ finite.

2) Even and odd signals:

$x(t) = x(-t) \rightarrow$ even

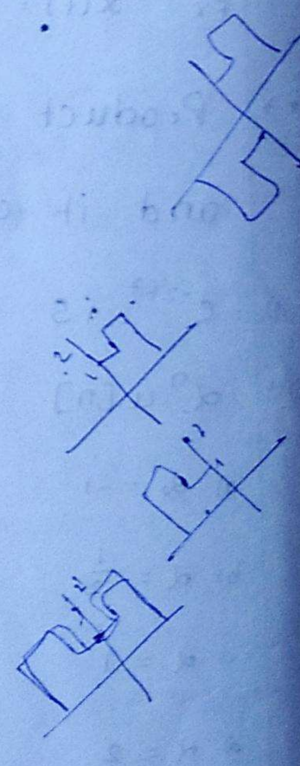
$x(t) = -x(-t) \rightarrow$ odd

$x(t) = x_e(t) + x_o(t) \rightarrow (1)$

$x(-t) = x_e(t) - x_o(t) \rightarrow (2)$

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$



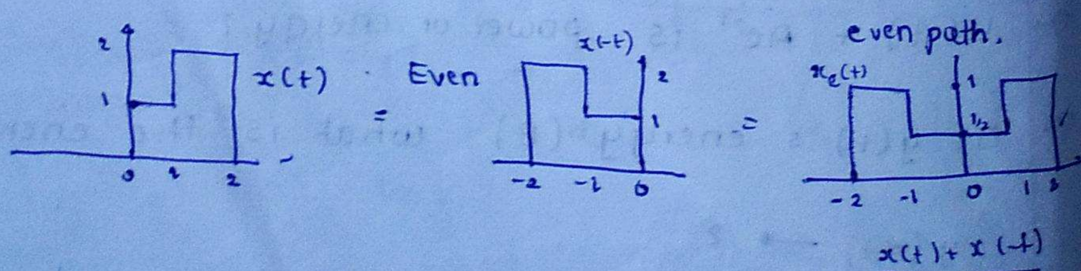
* $x[n] = x[-n] \rightarrow$ even

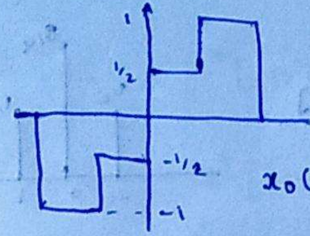
* $x[n] = -x[-n] \rightarrow$ odd

* $x_e[n] = \dfrac{x[n] + x[-n]}{2}$

* $x_o[n] = \dfrac{x[n] - x[-n]}{2}$

Q



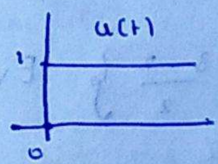


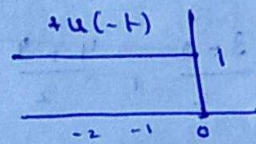
odd path → $\frac{x(t) - x(-t)}{2}$

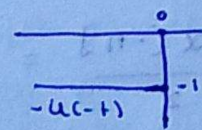
$x_o(t)$.

Q Find even and odd parts of signal $u(t)$

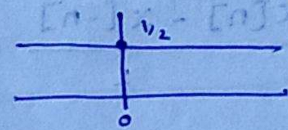
A





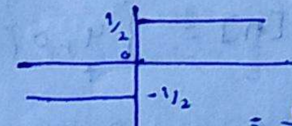


even path = $\frac{x(t) + x(-t)}{2}$ =



$x_e(t)$

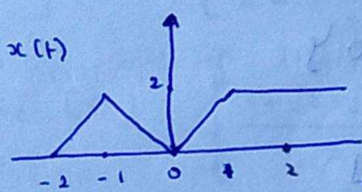
Odd path = $\frac{x(t) - x(-t)}{2}$ =

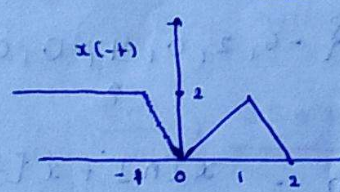


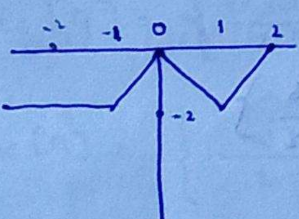
$x_o(t)$

$= \frac{1}{2}\,\text{sgn}(t)$.

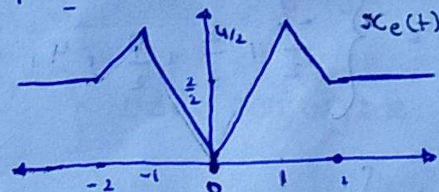
2Q



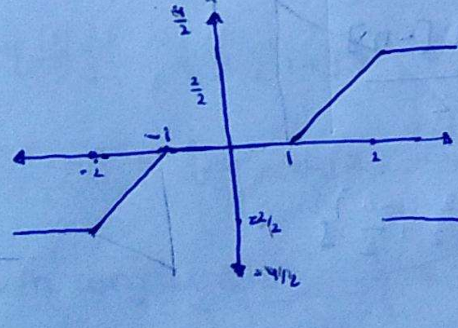




Even path



odd path

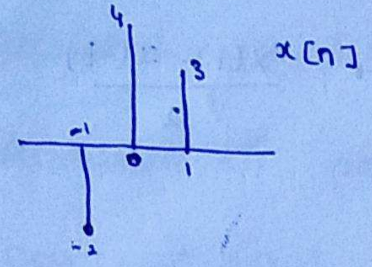


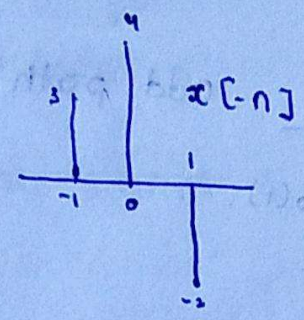
$x_o(t)$.

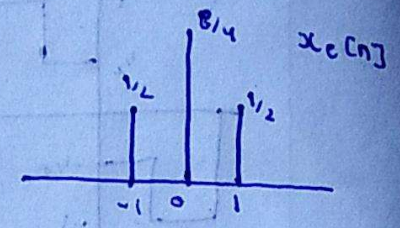
Q→ $x[n] = \{-2, \underset{\uparrow}{4}, 3\}$

$x_e[n]$

$x_o[n]$







$x[n] = \{3, 4, -2\}$

$$\frac{x[n] + x[-n]}{2} = \left\{ \frac{3-2}{2}, \frac{4+4}{2}, \frac{3-2}{2} \right\} \text{ Even path.}$$

$$\frac{x[n] - x[-n]}{2} = \left\{ \frac{-5}{2}, 0, \frac{5}{2} \right\} \text{ odd path.}$$

Q $x[n] = \{4, 0, 2, -6\}$ $\underset{\uparrow}{}$ $= x[n] = \{0, 0, 0, 4, 0, 2, -6\}$ $\underset{\uparrow}{}$

Q $x[-n] = \{-6, 2, 0, 4, 0, 0, 0\}$ $\underset{\uparrow}{}$

$$\text{Even path} = \frac{x[n] + x[-n]}{2}$$

$$= \left\{ \frac{-6}{2}, \frac{2}{2}, \frac{0}{2}, 4, \frac{0}{2}, \frac{2}{2}, \frac{-6}{2} \right\}$$

$$= \{-3, 1, 0, 4, 0, 1, -3\}$$

$$\text{odd path} = \frac{x[n] - x[-n]}{2}$$

$$= \left\{ \frac{6}{2}, \frac{-2}{2}, 0, 0, 0, \frac{2}{2}, \frac{-6}{2} \right\}$$

$$= \{3, -1, 0, 0, 0, 1, -3\}$$

* $x[n] = x^*[-n]$ → even conjugate

* $x[n] = -x^*[-n]$ → odd conjugate

* $x_{ec}[n] = \dfrac{x[n] + x^*[-n]}{2}$

* $x_{oc}[n] = \dfrac{x[n] - x^*[n]}{2}$

Q $x[n] = \{1+j2, \underset{\uparrow}{2}, j5\}$

$x[-n] = \{j5, \underset{\uparrow}{2}, 1+j2\}$

$\left\{\begin{array}{l} x = a + iz \\ \bar{x} = a - iz \end{array}\right\}$ conjugate.

$x^*[-n] = \{-j5, \underset{\uparrow}{2}, 1-j2\}$

$x_{ec}[n] = \left[\dfrac{1+j2-j5}{2}, \dfrac{2+2}{2}, \dfrac{j5+1-j2}{2}\right]$

$x_{ec}[n] = \left\{\dfrac{1-j3}{2}, 2, \dfrac{1+j3}{2}\right\}$ Even conjugate

$x_{oc}[n] = \left\{\dfrac{1+j7}{2}, 0, \dfrac{j7-1}{2}\right\}$ odd conjugate.

Q $x[n] = \{-4, -j5, \underset{\uparrow}{1+j2}, 4\}$

$x[-n] = \{4, 1+j2, -4-j5\}$

$x^*[-n] = \{4, 1-j2, -4+j5\}$

Even conjugate = $\left\{\dfrac{-j5}{2}, 1, \dfrac{j5}{2}\right\}$

odd conjugate. = $\left\{\dfrac{-8-j5}{2}, \dfrac{j2}{}, \dfrac{8-j5}{2}\right\}$ Ans.

Q consider the sequence $x[n]$, find the conjugate anti-symmetric part of the sequence. {odd conjugate}

Symmetric path → even

Antisymmetric path → odd.

Ans. $\{-4-2.5j, \ 2j, \ 4-2.5j\}$

3) Periodic & aperiodic signal:

- A signal is said to be periodic, when.

$\boxed{x(t) = x(t+T)}$ T = Time period.

* Steps to find period of sum of signals.

1) * Calculate the individual time periods. $T_1, T_2, T_3 \ldots$

2) * Calculate $\frac{T_1}{T_2}, \frac{T_1}{T_3}, \frac{T_1}{T_4} \ldots$

3) * The ratioes in step 2 are rational, then the signal is periodic signal.

→ Sum of signals is $x_1(t) + x_2(t) + x_3(t) \ldots$

4) * If the signal is periodic, find LCM of denominators in step 2.

5) * Time period $T = LCM \times T_1$

1Q $\cos(50\pi t) + \sin(60\pi t) + \cos(80\pi t)$ $\{T_1, T_2, T_3\}$

General form is $\sin(\omega_0 t)$ or $\cos \omega_0 t$

* $\omega_0 = \frac{2\pi}{T} = 50\pi = \boxed{T_1 = \frac{1}{25}}$

② $\omega_0 = \frac{2\pi}{T_2} = 60\pi$. $\boxed{T_2 = \frac{1}{30}}$

$\omega_0 = \frac{2\pi}{T_3} = 80\pi$ $\boxed{T_3 = \frac{1}{40}}$

Now: $\frac{T_1}{T_2} = \frac{6}{5}$ & $\frac{T_1}{T_3} = \frac{8}{5}$ & is rational

③ The signal is periodic

④ $\text{Lcm}(5, 5) = 5$

⑤ $T = 5 \times 1/25$ $\boxed{T = 1/5}$

(2g) $y(t) = e^{j(\pi/3)t} + e^{j(\frac{5\pi}{6})t}$

A 1) $T_1 = 6$ $T_2 = 12/5$ $\{ \omega_0 = \frac{5\pi}{6} = \frac{2\pi}{T_2} \}$

General form $e^{j\omega_0 t}$ $\{ \omega_0 = \frac{\pi}{3} = \frac{2\pi}{T_1} \}$

2) $\frac{T_1}{T_2} = \frac{6}{12/5} = \frac{30}{12} = \frac{5}{2}$ rational

3) The signal is periodic

4) Lcm of 2 = 2

5) $T = 2 \times 6 = \boxed{12 = T}$

(3a) $\sin\left(\frac{2\pi}{3}\right)t \cdot \cos\left(\frac{4\pi}{5}\right)t$

= $\sin A \cdot \sin B = \frac{1}{2}$ 8

$\frac{1}{2}\left(2\sin\left(\frac{2\pi}{3}\right)t\right) \cdot \cos\left(\frac{4\pi}{5}\right)t$

$\frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right)$

$\frac{1}{2}\left(\sin\left(\frac{22\pi}{15}\right) + \sin\left(\frac{-2\pi}{15}\right)\right)$

$\frac{1}{2}\sin\left(\frac{22\pi}{15}\right)t + \frac{1}{2}\sin\left(\frac{-2\pi}{15}\right)$

1) $T_1 = \frac{15}{11}$ $T_2 = \cancel{\frac{-1}{15}}\ -15$

2) $\frac{T_1}{T_2} = \frac{15/11}{-15/1} = \frac{-1}{11}$ Rational

3) This is periodic signal.

4) Lcm of 11 = 11

5) $T = 11 \times \frac{15}{11} = \boxed{15 = T}$

4Q $\cos(13t) + \sin(17\pi t)$

1) $T_1 = \frac{2\pi}{13}$ 2) $T_2 = \frac{2}{17}$

It is aperiodic signal bcoz $\frac{T_1}{T_2}$ is not rational

5Q je^{j10t}

* It is a periodic signal.

$e^{jx} = \cos x + j\sin x$

$j \cdot e^{j10t} = e^{j\pi/2} \cdot e^{j10t}$

$= e^{j(10t + \pi/2)}$ {in the form of $e^{j\omega_0}$}

$= \omega_0 = 10$, $\frac{2\pi}{T} = 10$

$\boxed{T = \frac{\pi}{5}}$

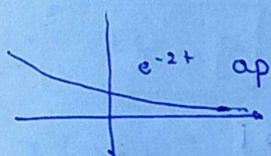
* Sum of signals, T_1/T_2 ratio should be rational for periodic signal.

* For single signal, T can be in π's also for periodic signal.

5Q $e^{-(2+j3)t}$

A It is aperiodic signal

→ $e^{-2t} \cdot e^{-3jt}$ $\{e^{-3jt}$ = periodic signal$\}$



6Q $x(t) = \text{even}\{\cos 3\pi t\, u(t)\}$

Even of $\cos 3\pi t \cdot u(t)$ is $= x(t)$ let

$$\frac{x(t_0) + (x(-t_0))}{2} = \frac{\cos 3\pi t \cdot u(t) + \cos 3\pi(-t) \cdot u(-t)}{2}$$

$$= \frac{\cos 3\pi t \cdot u(t) + \cos 3\pi(t) \cdot u(-t)}{2} = \frac{\cos 3\pi t\,(u(t)+u(-t))}{2}$$

$$= \boxed{\frac{\cos 3\pi t}{2} = x(t)} \quad \{u(t) + u(-t) = 1\}$$

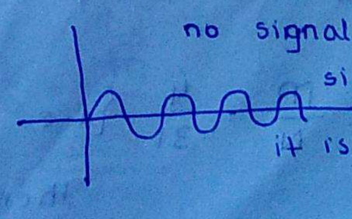
= In the form of $\cos(\omega_0 t)$.

$\omega_0 = 3\pi$, $\frac{2\pi}{T} = 3\pi$

$\boxed{T = \frac{2}{3}}$. The time period of periodic signal.

7Q $\sin\left(\frac{\pi}{6} t\right) \cdot u(t) = x(t)$

It is a-periodic signal



no signal on -ve side, so it is aperiodic signal.

8Q $v(t) = 30\sin 100t + 10\cos 300t + 6\sin\left(500t + \frac{\pi}{4}\right)$

A $T_1 = \{\omega_0 = 100\}$ $T_2 = \{\omega_0 = 300\}$ $T_3 = \{\omega_0 = 500\}$

1) $T_1 = \frac{\pi}{50}$ $T_2 = \frac{\pi}{150}$ $T_3 = \frac{\pi}{250}$

2) $\frac{T_1}{T_2} = \frac{3}{1}$, $\frac{T_1}{T_3} = \frac{5}{1}$.

3) It is a periodic signal

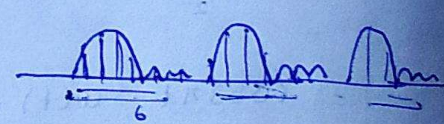
4) LCM of denomitators (1, 1) is 1

5) Time period $= T_1 \times LCM = \boxed{\frac{\pi}{50}}$

→ In Discrete Signals :- {periodic or aperiodic}

Solving method

$f = \frac{1}{T}$ $\frac{\omega_0}{2\pi} = \frac{M}{N}$



M = no. of cycles involved in periodic repetetion.

N = no. of samples in M.

* Continuous sinusoids and complex sinusoids are periodic for any value of ω_0, whereas equivalent discrete terms are periodic if $\frac{\omega_0}{2\pi}$ ratio is a rational number i.e $\left(\frac{M}{N}\right)$.

Ex :- $\frac{M}{N} = \frac{4}{31}$ {i.e. in 4 cycles, 31 samples are there & this set repeats}

* To make N as period of discrete signal, 'M' number of full cycles in continous signal is repeated.

* Periodic in discrete :

if $\boxed{x[n] = x[n+N]}$, then it is periodic signal.

Ex : Let $x[n] = \sin \omega_0 n \rightarrow (1)$

$x[n+N] = \sin \omega_0 [n+N] = \sin(\omega_0 n + \omega_0 N)$

From $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$x[n+N] = \sin \omega_0 n \cos \omega_0 N + \cos \omega_0 n \sin \omega_0 N \rightarrow (2)$

From the definition $x[n] = x[n+N]$ i.e (1) = (2)

$$\sin \omega_0 n = \sin \omega_0 n \underbrace{\cos \omega_0 N}_{\text{should become } 1} + \underbrace{\cos \omega_0 n \sin \omega_0 N}_{\text{should become } 0}$$

for making equation satisfy.

$\cos \omega_0 N = \cos 2\pi M$ { $\cos \omega_0 N$ is 1 at every point of $2n\pi$ { Here n = M) }

$\omega_0 N = 2\pi M$

$$\boxed{\frac{\omega_0}{2\pi} = \frac{m}{N}}$$

{N is the time period}.

1Q. $\sin[(3\pi/5)n]$

Here $\omega_0 = \frac{3\pi}{5}$

From $\frac{\omega_0}{2\pi} = \frac{3\pi/5}{2\pi/1} = \frac{3}{10} = \frac{m}{N}$

N is the time period $\boxed{N = 10}$

2Q $\sin[\pi^2 n]$ it is aperiodic signal.

$\frac{\omega_0}{2\pi}$ becomes $\frac{\pi^2}{2\pi}$ $\{\omega_0 = \pi^2\}$

$\frac{m}{N} = \frac{\pi}{2}$ is irrational, so it is apperiodic signal.

3Q $x[n] = (j)^{n/2}$

A We know $\boxed{j = \cancel{1}\cos\cancel{x} + j\sin x}$ $\boxed{j = e^{j\cdot\frac{\pi}{2}}}$

$x[n] = (e^{j\frac{\pi}{2}})^{n/2}$

$= e^{j\frac{\pi}{4}n}$ in the form of $e^{j\omega_0 n}$

$\omega_0 = \frac{\pi}{4}n$, $\frac{\omega_0}{2\pi} = \frac{\frac{\pi}{4}}{2\pi} = \frac{m}{N} = \frac{1}{8} \rightarrow \boxed{N = 8}$

→ Procedure to check periodicity of sum of 2 or more discrete time signals.

* 1) Identify the individual time periods N_1, N_2, N_3 ... for eq $x[n] = x_1[n] + x_2[n] + \dots$

2) Total time period = LCM of Time periods N_1, N_2 ....

4Q $y[n] = \sin\left(\frac{\pi}{3}n\right) + \cos\left(\frac{\pi}{4}n\right)$

A $N_1 = \left\{\frac{\omega_0}{2\pi} = \frac{m}{N} = \frac{\pi/3}{2\pi} = \frac{1}{6}\right\} = 6$

$N_2 = \left\{\frac{\omega_0}{2\pi} = \frac{m}{N} = \frac{\pi/4}{2\pi} = \frac{1}{8}\right\} = 8$

Lcm of N_1 & N_2 {8, 6} is 24

So, periodicity = $\underline{\underline{24}}$

5) $\cos(3\pi n) + \sin(7\pi n) + \cos(2.5\pi n)$

A: $N_1 = \left\{ \frac{\omega_0}{2\pi} = \frac{m}{N} = \frac{3\pi}{2\pi} = \frac{3}{2} \right\} = 2$

$N_2 = \left\{ \frac{\omega_0}{2\pi} = \frac{m}{N} = \frac{7\pi}{2\pi} = \frac{7}{2} \right\} = 2$

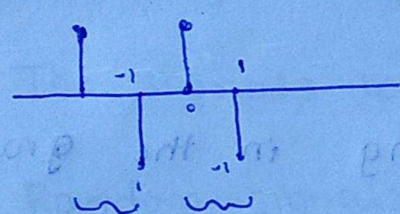
$N_3 = \left\{ \frac{\omega_0}{2\pi} = \frac{m}{N} = \frac{2.5\pi}{2\pi} = \frac{5}{4} \right\} = 4$

Lcm of N_1 & N_2 & N_3 {2, 2, 4} is $\boxed{4}$

So, periodicity of eq is $\boxed{4}$

6) $x[n] = (-1)^{n^2}$

A: It is a periodic signal.

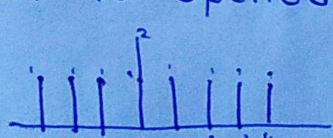


2 samples are repeating

so, its is periodicity is $\boxed{2}$.

7) $x[n] = u[n] + u[-n]$

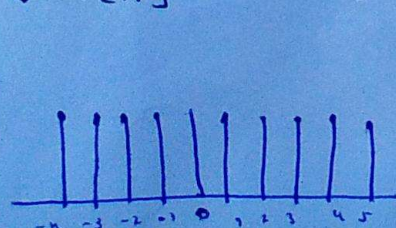
It is aperiodic signal, at 0, amplitude is $\underline{\underline{2}}$



8) $x[n] = u[n] + u[-n-1]$

It is periodic signal.

periodicity 1.



Q Find the time period of $x[n] = \sum_{k=-\infty}^{\infty} [\delta[n-4k] - \delta[n-1-4k]]$

A $x[n] = \sum_{k=-\infty}^{\infty} [\delta[n-4k] - \delta[n-1-4k]]$

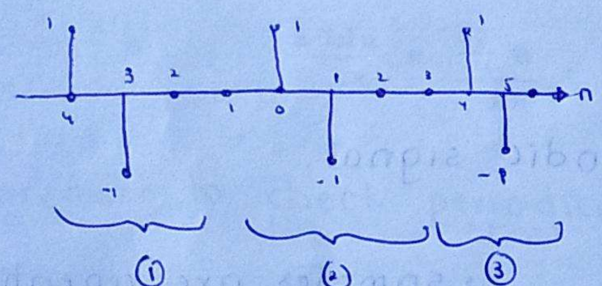
= substitute $-1, 0, 1$ in k, then

$k=-1$, $x[n] = \delta[n+4] - \delta[n+3] \rightarrow ①$

$k=0$, $x[n] = \delta[n] - \delta[n-1] \rightarrow ②$

$k=1$, $x[n] = \delta[n-4] - \delta[n-5] \rightarrow ③$

Draw graph of ①, ② & ③



4 samples are repeating in the graph

i.e (+1, -1, 0, 0)

So, the Time period of the given equation is $\boxed{4}$.

(4) Causal & non-causal signals.

* Causal signals are defined only in positive intervals.

Ex: $u(t)$, $u(t-1)$

* $u(t+1)$ is non-causal signal.

* In non-causal, both negative & positive intervals are present.

* The signals which are defined only in negative intervals are known as Anti-causal signal.

5) Deterministic and Random signals

* The signals which are predictable is Deterministic signals. Ex: unit step signal, voice

* The signals which are unpredictable are Random signals Ex: Noise.

→ Deterministic signals are characterised by mathemetical representation.