24-11-2022

1. D.C CIRCUITS

* Basic concepts :-

* Electric circuit :-

→ An 'Electric circuit' is an interconnection of 'electrical elements'.

* Charge :-

→ 'charge' is an electrical property of the atomic particles of which matter consists, measured in 'coulombs' (C).

* Electric current :-

→ It is the time rate of change of charge.

→ Measured in Amperes (A).

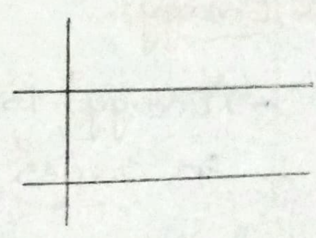
$$\boxed{1 \text{ ampere} = 1 \text{ coulomb/second}}$$

$$\boxed{i = \frac{dq}{dt}}$$

Direct current :-

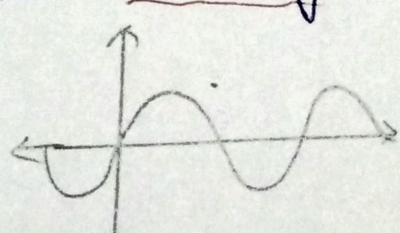
→ D.C is a current that remains 'constant' with time.

→ Here frequency is 'zero'.



Alternating current :-

→ A.C is a current that varies sinusoidally with 'time'.



* Voltage :-

→ 'Voltage' or 'potential difference' is the energy required to move a unit charge through an element.

→ Measured in 'volts' (V).

$$\boxed{V = \frac{dw}{dq}}$$

$$\boxed{1 \text{ volt} = 1 \text{ Joule/coulomb.}}$$

* Power :-

→ It is the time rate of expending or absorbing energy.

→ Measured in 'watts' (W).

$$\boxed{P = \frac{dW}{dt} = \frac{dW}{dq} \cdot \frac{dq}{dt} = VI.}$$

$$\Rightarrow P = \frac{dW}{dt}$$

$$\Rightarrow W = \int P\,dt = \int VI\,t.$$

* Energy :-

→ 'Energy' is the capacity to do work, measured in 'Joules' (J).

$$\boxed{1 \text{ Wh} = 3,600 \text{ J.}}$$

* circuit Elements :-

→ There are two types of Elements.

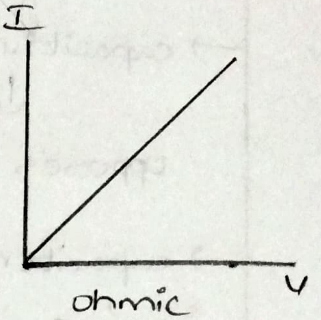
① Passive Elements → Requires/Takes Energy → R, C, L

② Active Elements → Gives Energy

Ex:- Generator, Battery [polarities].

① Passive Elements :-

ⓘ Resistor (R) :-



→ Units 'Ω' ohms

→ opposes the flow of current

→ Made by 'carbon'.

→ $V \propto I \Rightarrow \boxed{V = IR}$.

→ Resistance, '$R \propto l$' & '$R \propto \frac{1}{A}$'

$$\Rightarrow R = \frac{\rho l}{A}.$$

$$\Rightarrow \boxed{\rho = \frac{RA}{l}} \quad \Omega\text{-m}$$

$$\Rightarrow \rho = \frac{1}{\sigma} \quad \Rightarrow \boxed{\sigma = \frac{1}{\rho}}$$

→ Resistivity depends on 'type of material'.

→ Resistance depends on 'material' & 'dimensions'.

→ $\boxed{R = \frac{1}{G}}$ 'G' is 'conductance'.

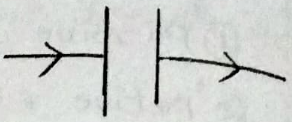
'R' is 'Resistance'.

→ 'current' & 'voltage' are in same phase in Resistor.

(ii) <u>capacitor</u> :- (C)

→ unit is '<u>Faraday</u>'.

→ symbol is '—||—' (or) 'F'.



current passing through capacitor

→ 'capacitor' is 'device'

'capacitance' is property.

→ $\boxed{Q = CV}$, $\boxed{C = \frac{A\varepsilon_0}{d}}$ → parallel plate

→ capacitive Reactance (X_C) → units 'omega' (Ω)

↓

opposes the flow of current

→ capacitor blocks 'D.C current' & allows 'A.C' [because A.C has frequency].

→ D.C acts as '<u>open circuit</u>' [∵ $f = 0$, $X_C = \frac{1}{0} = \infty$]

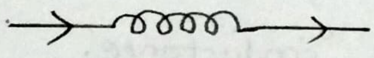
→ I leads V by '90°'.

$$\boxed{X_C = \frac{1}{2\pi f C}}$$

(iii) <u>Inductor</u> :-

→ symbol '—ooo—'

→ current passing through Inductor



→ 'Inductive Reactance' / 'Reluctance' (X_L)

$$\boxed{X_L = 2\pi f L}$$

→ Inductor allows '<u>D.C</u>' [∵ $X_L = 2\pi(0)L = 0$] &

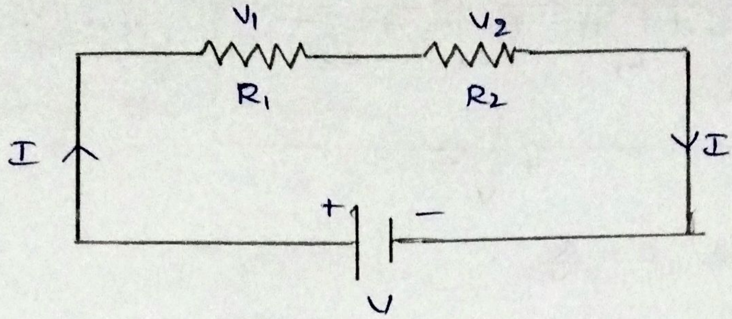
it allows some '<u>A.C</u>' & resists some '<u>A.C</u>'.

→ I '<u>lags</u>' V by '90°'.

→ current is lagging.

* Resistor :-

Series :- [End to End]



i - Same

v - dividing.

$IR_1 + IR_2 = V \longrightarrow (1)$

We know, '$V = IR$' $\longrightarrow (2)$

$\Rightarrow IR_1 + IR_2 = IR$. [From (1) & (2)].

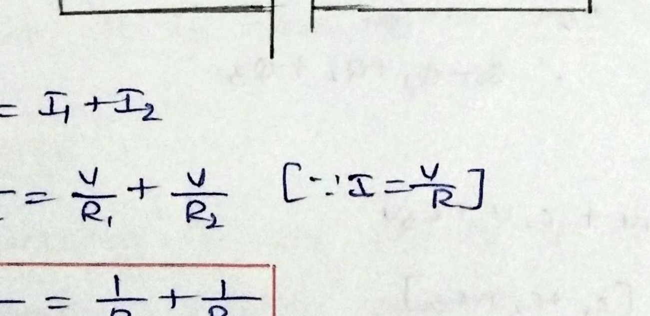
$\Rightarrow I(R_1 + R_2) = IR$

$\therefore \boxed{R_{eq} = R_1 + R_2}$

→ In series, current is 'same' & voltage 'varies'.

→ 'R_{eq}' is higher than others [R_1, R_2].

Parallel :-



V - Same

i - dividing

$I = I_1 + I_2$

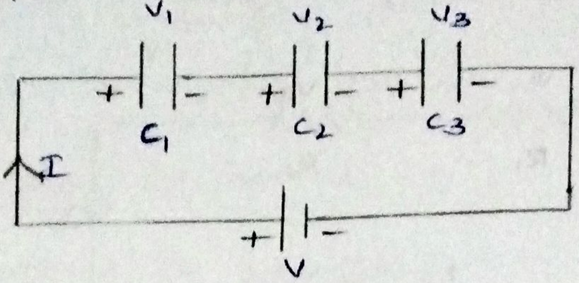
$\Rightarrow \frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2}$ $[\because I = \frac{V}{R}]$

$\Rightarrow \boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}}$

→ R_{eq} is lessar than any other resistances.

* capacitor :-

series :-



We know, $C = \frac{Q}{V}$.

$\Rightarrow V_1 = \frac{Q}{C_1}$, $V_2 = \frac{Q}{C_2}$, $V_3 = \frac{Q}{C_3}$

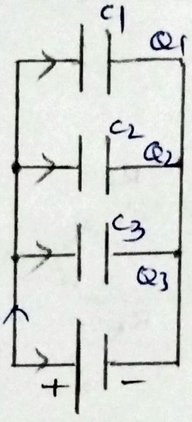
$\Rightarrow V = V_1 + V_2 + V_3$

$\Rightarrow V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]$

$\Rightarrow \frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

$\Rightarrow \boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$

Parallel :-



$Q_1 = C_1 V$

$Q_2 = C_2 V$ $\qquad \therefore Q = Q_1 + Q_2 + Q_3$.

$Q_3 = C_3 V$

$\Rightarrow Q = C_1 V + C_2 V + C_3 V$

$\Rightarrow Q = V[C_1 + C_2 + C_3]$

$\Rightarrow \frac{Q}{V} = C_1 + C_2 + C_3$

$\therefore \boxed{C_{eq} = C_1 + C_2 + C_3}$.

BASIC LAWS

* Ohm's Law :-

→ It states that the 'voltage' across a resistor is directly proportional to the 'current' flowing through the resistor.

$$V \propto I$$

$$\Rightarrow \boxed{V = IR}$$

Here, 'R' is constant

$$\boxed{R = \frac{V}{I}}$$

(or)

$$I \propto V$$

$$\Rightarrow \boxed{I = \frac{V}{R}}$$

* Note :-

Short circuit :-

→ A short circuit is a circuit element with 'resistance' approaching 'zero.'

Open circuit :-

→ An open circuit is a circuit element with 'resistance' approaching 'infinity'.

Resistance :-

→ The Resistance 'R' of an element denotes its ability 'to resist the flow of Electric current,' it is measured in 'ohms' (Ω).

Conductance :-

→ 'conductance' is the ability of an element 'to conduct electric current'.

→ It is measured in 'mhos' ($\mho$) or 'siemens' (S).

* Nodes, Branches and Loops :-

Branch :-

→ A Branch represents a 'single element' such as 'voltage source' or a 'resistor'.

Node :-

→ A Node is the point of connection between 'two' or 'more branches'.

Loop :-

→ A loop is any 'closed path in a circuit'.

* Kirchhoff's Laws :-

① Kirchhoff's current Law [KCL] :-

→ It states that "the algebraic sum of currents entering a node (or a closed boundary) is zero."

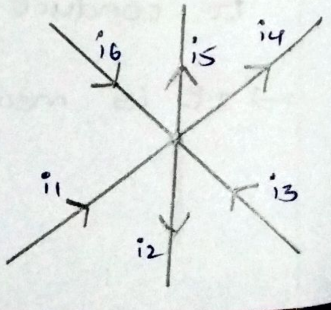
$$\boxed{\sum_{n=1}^{N} i_n = 0}$$

where, 'N' is no. of 'branches'.

→ It is based on "Law of conservation of Energy"

→ It is also called as 'current Law', 'Junction Law' and 'Point Law.'

→ The sum of 'the currents entering a node' is equal to the sum of 'the currents leaving the node'.



$$\Rightarrow \boxed{i_1 - i_2 + i_3 - i_4 - i_5 + i_6 = 0}$$

$$\Rightarrow \boxed{i_1 + i_3 + i_6 = i_2 + i_4 + i_5}$$

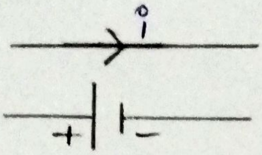
Kirchhoff's Voltage Law [KCL]:-

→ It states that "the algebraic sum of all voltages around a closed path (or loop) is zero."

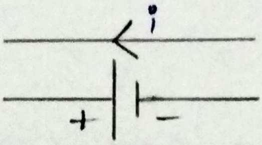
$$\boxed{\sum_{m=1}^{M} V_m = 0}$$

where 'M' is the 'no. of voltages' in the loop.

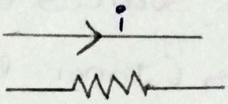
Rules:-

① 

= '−ve'

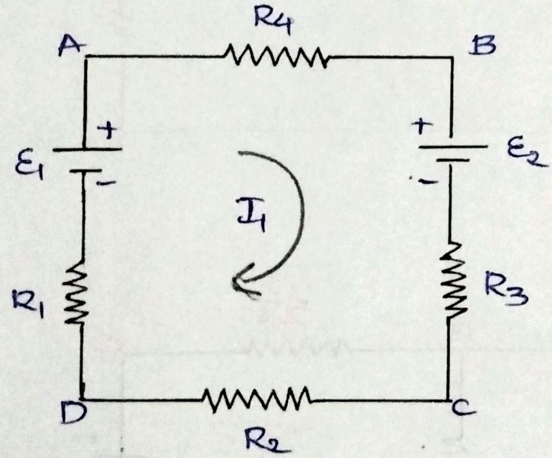
② 

= '+ve'

③ For all resistors, 

= '−ve' = '−iR'

Example:-



$$\boxed{\sum \mathcal{E} + \sum IR = 0}$$

ABCDA,

$\Rightarrow -I_1R_4 - \mathcal{E}_2 - I_1R_3 - I_1R_2 - I_1R_1 + \mathcal{E}_1 = 0.$

* Types [in KVL] :-

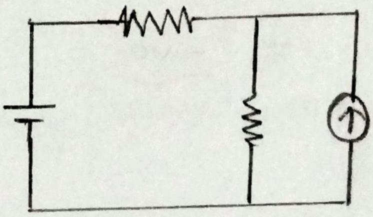
1) Type-1 :- [General model]

→ Having only voltage sources and Resistors [ε, R].

2) Type-2 :-

→ Having voltage sources, Resistors & current sources [outer loops].

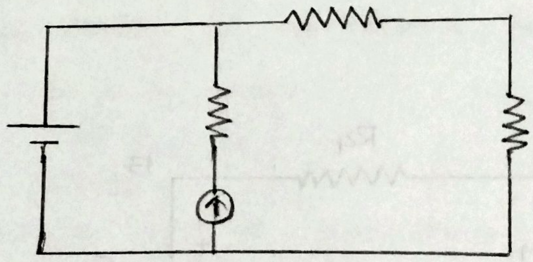
Ex :-



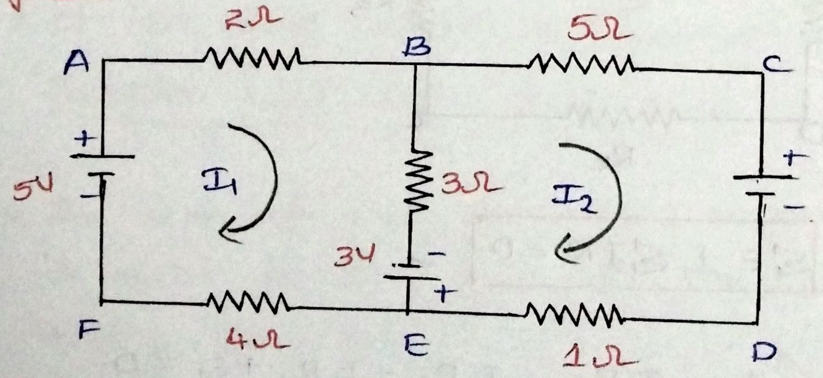
3) Type-3 :- [Super Mesh Model]

→ Having voltage sources (ε), Resistors (R) and current sources (Inner loops).

Ex :-



1) Type-1 :-



Find :-

ⓐ The current passing through 5Ω resistor.

ⓑ The voltage across 3Ω resistor.

A) Applying KVL for ABEFA loop,

$\Rightarrow -2I_1 - 3(I_1 - I_2) + 3 - 4I_1 + 5 = 0.$

$\Rightarrow -2I_1 - 3I_1 + 3I_2 + 3 - 4I_1 + 5 = 0$

$\Rightarrow -9I_1 + 3I_2 + 8 = 0$

$\Rightarrow -9I_1 + 3I_2 = -8 \longrightarrow (1)$

Applying KVL for BCDEB loop,

$\Rightarrow -5I_2 - 10 - I_2 - 3 - 3(I_2 - I_1) = 0$

$\Rightarrow -5I_2 - I_2 - 13 - 3I_2 + 3I_1 = 0$

$\Rightarrow -9I_2 + 3I_1 - 13 = 0$

$\Rightarrow 3I_1 - 9I_2 = 13 \longrightarrow (2)$

From (1) & (2)

$\Rightarrow -9I_1 + 3I_2 = -8$

$3I_1 - 9I_2 = 13.$

$\therefore I_1 = \frac{11}{24} = 0.46$ amperes.

$\therefore I_2 = \frac{-31}{24} = -1.29$ amperes.

(a) $I_{5\Omega} = I_2 = -1.29$ A.

$V_{5\Omega} = 1.29 \times 5 = 6.45V$ [$\because V = IR$].

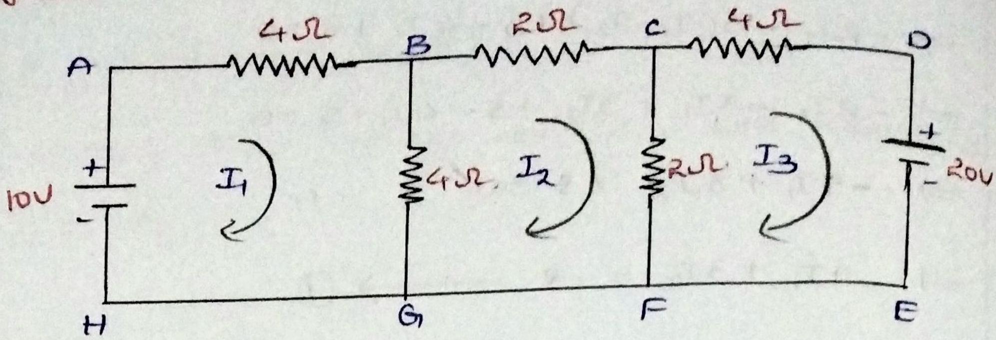
(b) $I_{3\Omega} = I_1 - I_2$

$= 0.46 - (-1.29)$

$= 1.7$ A.

$\therefore V_{3\Omega} = 1.7 \times 3 = 5.1$ V [$\because V = IR$].

* Type-1 :-



Find the current passing through 2Ω (CF) resistor?

A) Applying KVL for 'ABGHA' loop.

$\Rightarrow -4I_1 - 4(I_1 - I_2) + 10 = 0$

$\Rightarrow -4I_1 - 4I_1 + 4I_2 = -10$

$\Rightarrow -8I_1 + 4I_2 = -10 \longrightarrow ①$

Applying KVL for 'BCFGB' loop.

$\Rightarrow -2I_2 - 2(I_2 - I_3) - 4(I_2 - I_1) = 0$

$\Rightarrow -8I_2 + 2I_3 - 4I_1 = 0$

$\Rightarrow -4I_1 + -8I_2 + 2I_3 = 0. \longrightarrow ②$

Applying KVL for 'CDEFC' loop.

$\Rightarrow -4I_3 - 20 - 2(I_3 - I_2) = 0$

$\Rightarrow -6I_3 + 2I_2 = 20$

$\Rightarrow 2I_2 - 6I_3 = 20. \longrightarrow ③$

$\therefore I_1 = \frac{35}{32} = $ '1.093 A'

$I_2 = \frac{-5}{16} = $ '-0.312 A'

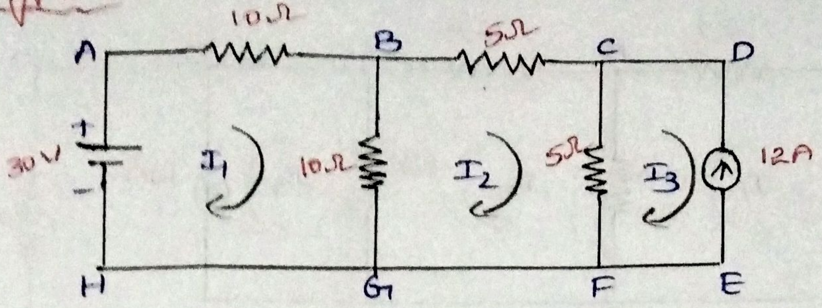
$I_3 = \frac{-55}{16} = $ '-3.437 A'

$\therefore I_{2\Omega}(CF) = I_3 - I_2$

$= -3.437 + 0.312$

$=$ '-3.125 A'

2) Type-2 :-



Find current passing through BC [5Ω resistor] and CF ?

A) Applying 'KVL' for 'ABGHA' loop.

$\Rightarrow -10I_1 - 10(I_1 - I_2) + 30 = 0$

$\Rightarrow -20I_1 + 10I_2 = -30 \longrightarrow (1)$

Applying 'KVL' for 'BCFGB' loop.

$\Rightarrow -5I_2 - 5(I_2 - I_3) - 10(I_2 - I_1) = 0$

$\Rightarrow -10I_2 + 5I_3 - 10I_2 + 10I_1 = 0$

$\Rightarrow 10I_1 - 20I_2 + 5I_3 = 0. \longrightarrow (2)$

Applying 'KVL' for 'CDEFC' loop.

$\Rightarrow I_3 = -12$ [∵ opp direction].

$\longrightarrow (3)$

$\therefore I_1 = 0$ A

$I_2 = -3$ A

$I_3 = -12$ A

$\therefore I_{5\Omega}$ (BC) $= I_2 \cancel{\times I_3}$

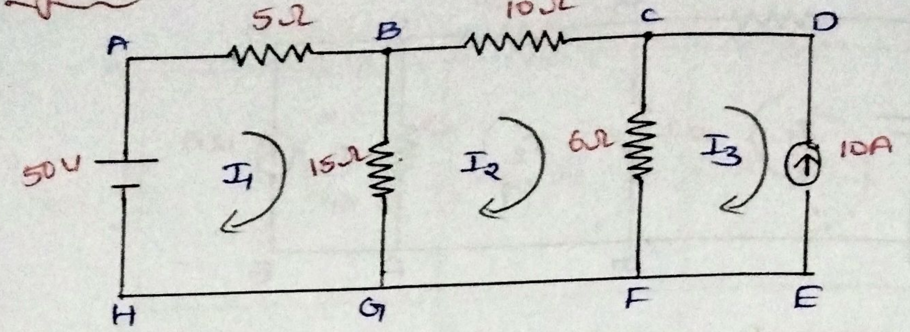
$= -3$ A

$\therefore I_{5\Omega}$ (CF) $= I_2 - I_3$

$= -3 - (-12)$

$= 9$ A.

* Type-2 :-



A) Applying 'KVL' to ABGHA loop.

$\Rightarrow -5I_1 - 15(I_1 - I_2) + 50 = 0$

$\Rightarrow -20I_1 + 15I_2 = -50 \longrightarrow ①$

Applying 'KVL' to BCFGB loop.

$\Rightarrow -10I_2 - 6(I_2 - I_3) - 15(I_2 - I_1) = 0$

$\Rightarrow -10I_2 - 6I_2 + 6I_3 - 15I_2 + 15I_1 = 0$

$\Rightarrow 15I_1 - 31I_2 + 6I_3 = 0 \longrightarrow ②$

Applying 'KVL' to CDEFC loop

$\Rightarrow I_3 = -10 \longrightarrow ③$

$\therefore I_1 = 1.64\ A$

$I_2 = -1.13\ A$

$\therefore I_{5\Omega} = I_1 = 1.64\ A$

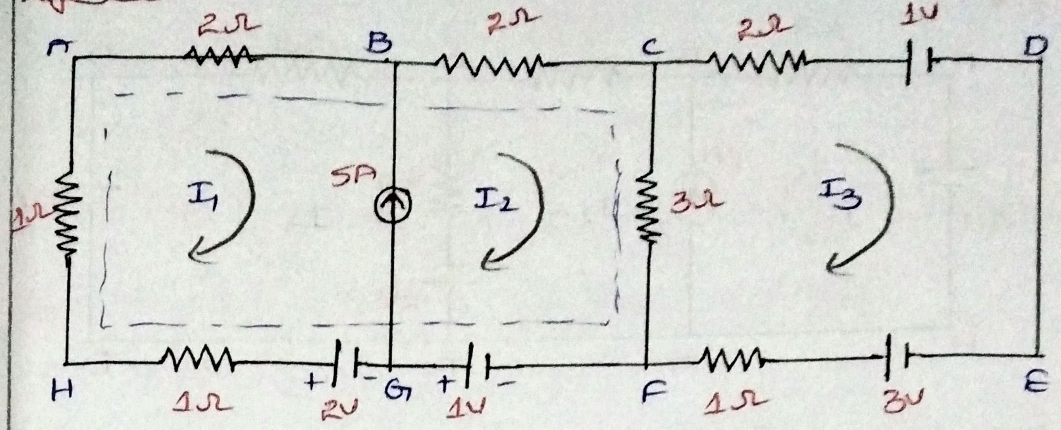
$I_{15\Omega} = I_1 - I_2 = 1.64 + 1.13 = 2.77\ A$

$I_{10\Omega} = I_2 = -1.13\ A$

$\therefore V_{5\Omega} = 1.64 \times 5 = 8.2\ V$

$V_{10\Omega} = -1.13 \times 10 = -13\ V$

* Type-3 :-



A) $I_2 - I_1 = 5$

$\Rightarrow -I_1 + I_2 + (0)I_3 = 5 \longrightarrow ①$

Applying 'KVL' to supermesh ABCFGHA

$\Rightarrow -2I_1 - 2I_2 - 3(I_2 - I_3) + 1 + 2 - I_1 - I_1 = 0$

$\Rightarrow -4I_1 - 5I_2 + 3I_3 = -3 \longrightarrow ②$

Applying 'KVL' to CDEF loop.

$\Rightarrow -2I_3 - 1 + 3 - I_3 - 3(I_3 - I_2) = 0$

$\Rightarrow 3I_2 - 6I_3 = -2 \longrightarrow ③$

From ①, ② & ③

$\Rightarrow -I_1 + I_2 + (0)I_3 = 5$

$-4I_1 - 5I_2 + 3I_3 = -3$

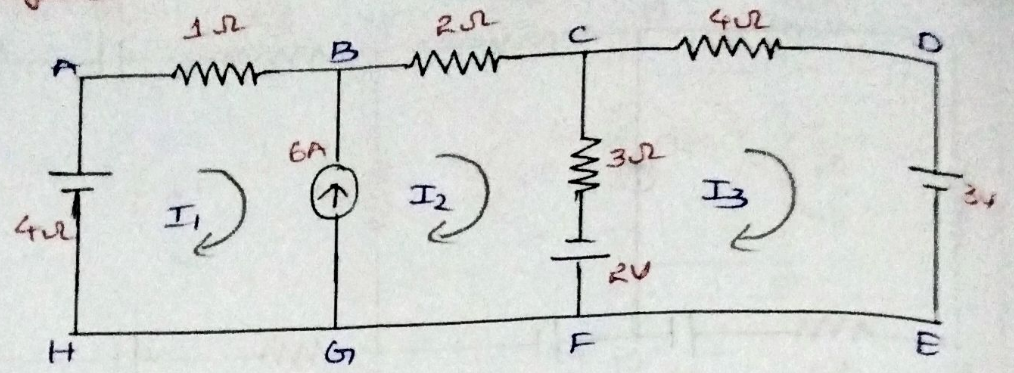
$(0)I_1 + 3I_2 - 6I_3 = -2.$

$\therefore I_1 = -1.8\ A$

$I_2 = 3.2\ A$

$I_3 = 1.9\ A$

* Type-3:-



Find $I_{2\Omega} = ?$

A) $I_2 - I_1 = 6$

$\Rightarrow -I_1 + I_2 + (0) I_3 = 6 \longrightarrow (1)$

Applying 'KVL' to supermesh ABC FGHA.

$\Rightarrow -I_1 - 2I_2 - 3(I_2 - I_3) + 2 + 4 = 0.$

$\Rightarrow -I_1 - 5I_2 + 3I_3 = -6 \longrightarrow (2)$

Applying 'KVL' to CDEFC.

$\Rightarrow -4I_3 - 3 - 2 - 3(I_3 - I_2) = 0$

$\Rightarrow -4I_3 - 3I_3 + 3I_2 = 5$

$\Rightarrow +3I_2 - 7I_3 = 5 \longrightarrow (3)$

$\therefore I_1 = -3.90$ A

$I_2 = 2.09$ A

$I_3 = 0.18$ A

$\therefore I_{2\Omega} = I_2 =$ '2.09A'.

KCL loops

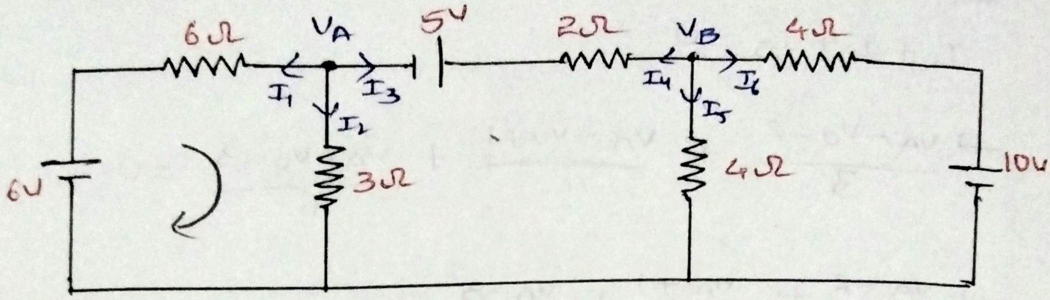
* **steps:-**

① Identify the 'super junction'.

② All the currents are 'leaving' at the junction.

③ Assume junctions are 'high potential'.

* **Example:-**



Reference junction $V_0 = 0$ V.

V_A & V_B are super junctions

At Junction - A:-

$I_1 + I_2 + I_3 = 0$

$$\Rightarrow \frac{V_A - V_0 - 6}{6} + \frac{V_A - V_0}{3} + \frac{V_A - V_B + 5}{2} = 0$$

$$\Rightarrow \frac{V_A - 6}{6} + \frac{V_A}{3} + \frac{V_A - V_B + 5}{2} = 0 \quad [\because V_0 = 0]$$

$$\Rightarrow \frac{V_A - 6 + 2V_A + 3V_A - 3V_B + 15}{6} = 0$$

$$\Rightarrow 6V_A + 3V_B = -9 \longrightarrow ①$$

At Junction - B:-

$I_4 + I_5 + I_6 = 0$

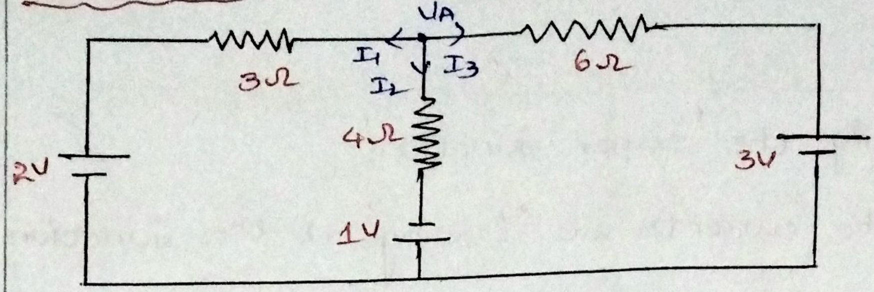
$$\Rightarrow \frac{V_B - V_A - 5}{2} + \frac{V_B - V_0}{4} + \frac{V_B - V_0}{4} = 0.$$

$$\Rightarrow -2V_A + 4V_B = 20 \longrightarrow ②$$

$\therefore$ '$V_A = 1.333$ V'

'$V_B = 5.666$ V'.

* Home work :-

1) 

Find $I_{4\Omega} = ?$ using KCL?

A) At Junction V_A :-

$I_1 + I_2 + I_3 = 0$

$$\Rightarrow \frac{V_A - V_0 - 2}{3} + \frac{V_A - V_0 + 1}{4} + \frac{V_A - V_0 - 3}{6} = 0$$

$$\Rightarrow \frac{V_A - 2}{3} + \frac{V_A + 1}{4} + \frac{V_A - 3}{6} = 0$$

2	4, 6
2	2, 3
3	1, 3
	1, 1

$$\Rightarrow \frac{4(V_A - 2) + 3(V_A + 1) + 2(V_A - 3)}{12} = 0$$

$$\Rightarrow 4V_A - 8 + 3V_A + 3 + 2V_A - 6 = 0$$

$$\Rightarrow 9V_A - 14 + 3 = 0$$

$$\Rightarrow 7V_A = \cancel{7} + 11$$

$$\Rightarrow V_A = \frac{+71}{9} = \frac{+11}{9} = +1.22 \text{ volts.}$$

$$\therefore I_{4\Omega} = \frac{V_A}{R}$$

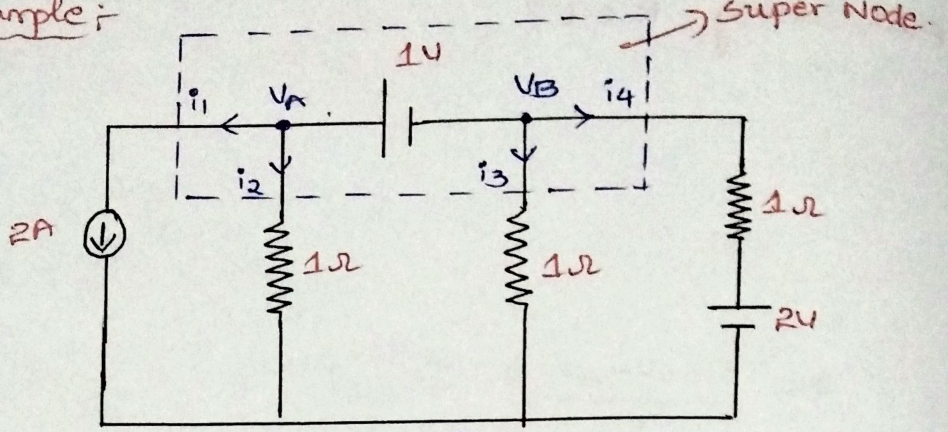
$$= \frac{1.22}{4}$$

$$I_{4\Omega} \simeq 0.305 \text{ Amperes.}$$

* Super Node :-

→ If the voltage source is in between two Non-reference junctions, then it is called "Super Node".

Example :-



Applying 'KCL' at 'V_A' :-

$$\Rightarrow \frac{2}{1} + \frac{V_A - V_0}{1} + \frac{V_B - V_0}{1} + \frac{V_B - V_0 - 2}{1} = 0$$

$$\Rightarrow \cancel{2} + V_A + V_B + V_B - \cancel{2} = 0$$

$$\Rightarrow V_A + 2V_B = 0 \longrightarrow (1)$$

Applying 'KVL' at 'Super Node' :-

$$\Rightarrow V_A - V_B = 1 \longrightarrow (2)$$

$$\Rightarrow V_A = 1 + V_B.$$

Substitute (2) in eq^n (1)

$$\Rightarrow 1 + V_B + 2V_B = 0.$$

$$\Rightarrow 1 + 3V_B = 0.$$

$$\Rightarrow 3V_B = -1$$

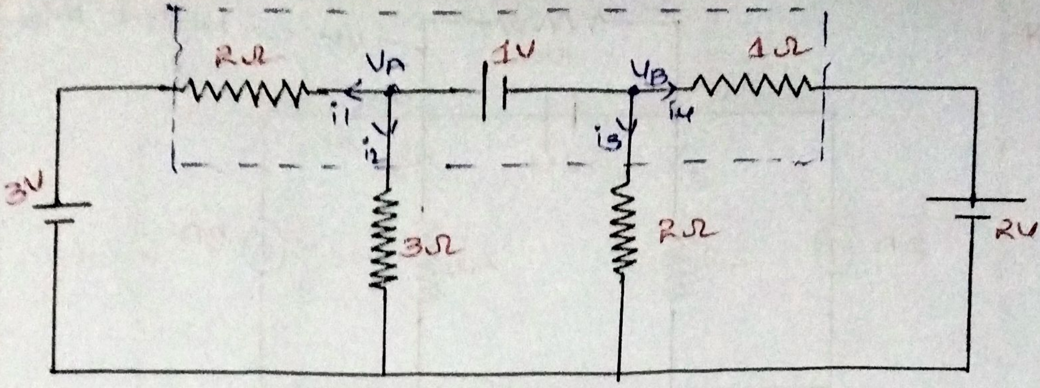
$$\Rightarrow 'V_B = \frac{-1}{3}'$$

put $V_B = -\frac{1}{3}$ in eq^n (2)

$$\Rightarrow V_A - \left(-\frac{1}{3}\right) = 1 \qquad \Rightarrow V_A = 1 \mp \frac{1}{3} = '\frac{2}{3}'$$

$$\therefore I_{1\Omega} = \frac{V_1}{R} = \frac{\frac{2}{3}}{1}, \quad 'I_{1\Omega} = \frac{2}{3}'.$$

*



Find $I_{3\Omega} = ?$

A) Applying 'KCL' at 'V_A'.

$$\Rightarrow \frac{V_A - 3}{2} + \frac{V_A}{3} + \frac{V_B}{2} + \frac{V_B - 2}{1} = 0$$

$$\Rightarrow \frac{3V_A - 9 + 2V_A + 3V_B + 6V_B - 12}{6} = 0$$

$$\Rightarrow 5V_A + 9V_B - 21 = 0.$$

$$\Rightarrow 5V_A + 9V_B = 21. \longrightarrow (1)$$

Applying 'KVL' at 'super Node'.

$$\Rightarrow V_A - V_B = 1. \longrightarrow (2)$$

substitute (2) in (1)

$$\Rightarrow 5(1 + V_B) + 9V_B = 21$$

$$\Rightarrow 5 + 5V_B + 9V_B = 21$$

$$\Rightarrow 14V_B = 16 \quad \Rightarrow V_B = \frac{16}{14} = '1.14V.'$$

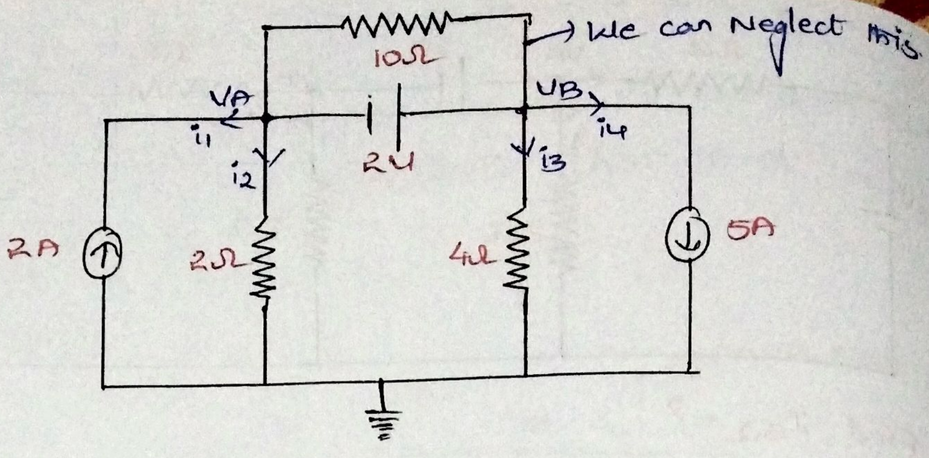
Now, $V_A = 1 + V_B = 1 + 1.14 = '2.14V.'$

$$\therefore I_{3\Omega} = \frac{V_A}{R}$$

$$= \frac{2.14}{3}$$

$$= '0.71A'.$$

*



A) Applying 'KCL' at 'VA'

$\Rightarrow \frac{-2}{1} + \frac{V_A}{2} + \frac{V_B}{4} + \frac{5}{1} = 0$

$\Rightarrow \frac{-8 + 2V_A + V_B + 20}{4} = 0$

$\Rightarrow 2V_A \cancel{+} V_B = -12$

$\Rightarrow 2V_A + V_B = -12 \longrightarrow ①$

Applying 'KVL' at 'SuperNode'.

$\Rightarrow V_B - V_A = 2. \longrightarrow ②$

$\Rightarrow V_B = 2 + V_A$

Substitute ② in ①

$\Rightarrow 2V_A + 2 + V_A = -12$ $\qquad V_B = 2 + V_A$

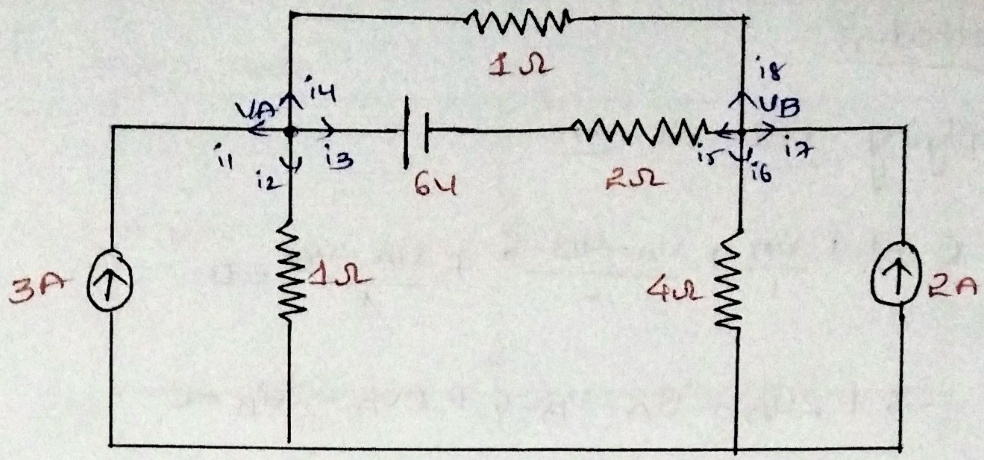
$\Rightarrow 3V_A = -14$ $\qquad \Rightarrow V_B = 2 + (-4.667)$

$\Rightarrow V_A = \frac{-14}{3} = -4.667\ V.$ $\qquad \Rightarrow V_B = -2.667V.$

$I_{2\Omega} = \frac{V_A}{R} = \frac{-4.667}{2} = -2.3335\ A$

$I_{4\Omega} = \frac{V_B}{R} = -\frac{2.667}{4} = -0.66675\ A.$

*



Method-1:-

A) Applying KCL at V_A.

$$\Rightarrow -3 + \frac{V_A}{1} + \frac{V_A - V_B - 6}{2} + \frac{V_A - V_B}{1} = 0$$

$$\Rightarrow -6 + 2V_A + V_A - V_B - 6 + 2V_A - 2V_B = 0$$

$$\Rightarrow 5V_A - 3V_B = 12 \longrightarrow (1)$$

Applying KCL at V_B.

$$\Rightarrow \frac{V_B - V_A + 6}{2} + \frac{V_B}{4} + \frac{V_B - V_A}{1} - 2 = 0$$

$$\Rightarrow 2V_B - 2V_A + 12 + V_B + 4V_B - 4V_A - 8 = 0) \, 4$$

$$\Rightarrow -6V_A + 7V_B + 4 = 0$$

$$\Rightarrow -6V_A + 7V_B = -4 \longrightarrow (2)$$

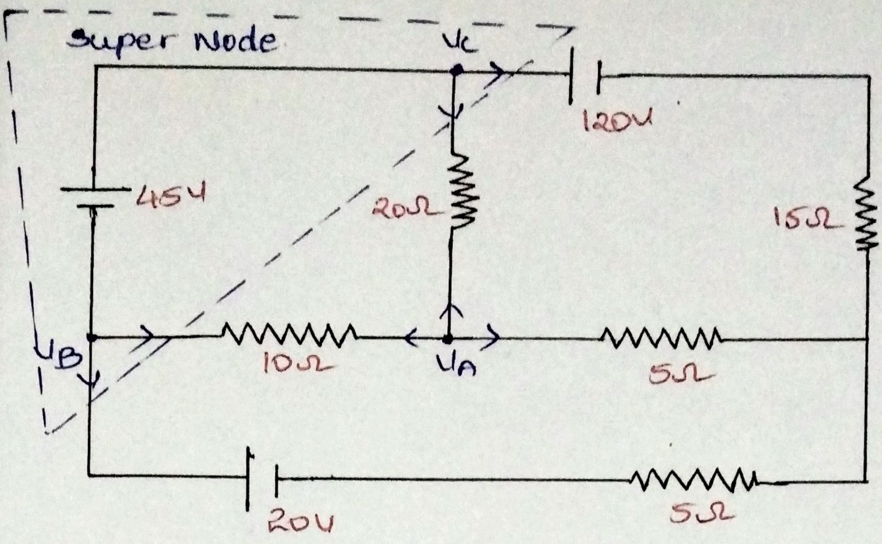
By solving (1) & (2)

$\therefore V_A = 4.411$ volts

$V_B = 3.352$ volts

$$I_{1\Omega} = \frac{V_A}{1} = 4.411 \text{ A}$$

$$I_{4\Omega} = \frac{V_B}{4} = \frac{3.352}{4} = 0.838 \text{ A}$$

* 

A) Applying 'KCL' at 'VA':

$$\Rightarrow \frac{V_A - V_B}{10} + \frac{V_A - V_C}{20} + \frac{V_A}{5} = 0$$

$$\Rightarrow \frac{2V_A - 2V_B + V_A - V_C + 4V_A}{20} = 0$$

$$\Rightarrow 7V_A - 2V_B - V_C = 0 \longrightarrow (1)$$

Applying 'KCL' at 'SuperNode' [VB & VC]:

$$\Rightarrow \frac{V_B - 20}{5} + \frac{V_B - V_A}{10} + \frac{V_C - V_A}{20} + \frac{V_C - 120}{15} = 0$$

$$\Rightarrow \frac{12V_B - 240 + 6V_B - 6V_A + 3V_C - 3V_A + 4V_C - 480}{60} = 0$$

$$\Rightarrow 18V_B - 9V_A + 7V_C = 720$$

$$\Rightarrow -9V_A + 18V_B + 7V_C = 720 \longrightarrow (2)$$

Applying 'KVL' at 'SuperNode':

$$\Rightarrow V_C - V_B = 45 \longrightarrow (3)$$

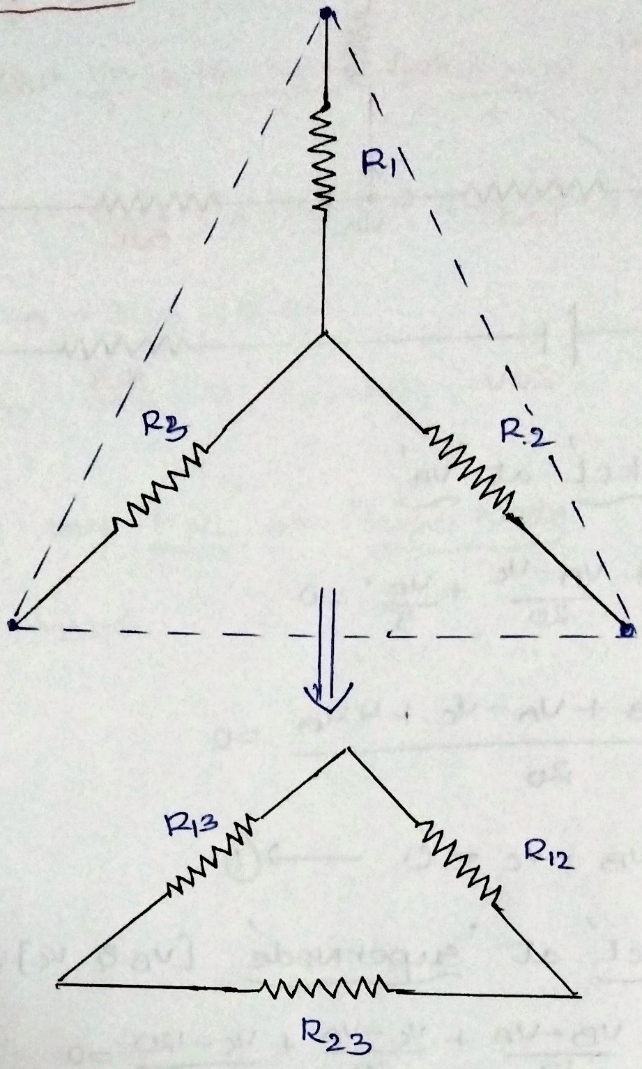
$\therefore$ $V_A = 15.81\ V$

$V_B = 21.89\ V$

$V_C = 66.89\ V$.

* Star - Delta Networks :-

Star ⇒ Delta :-

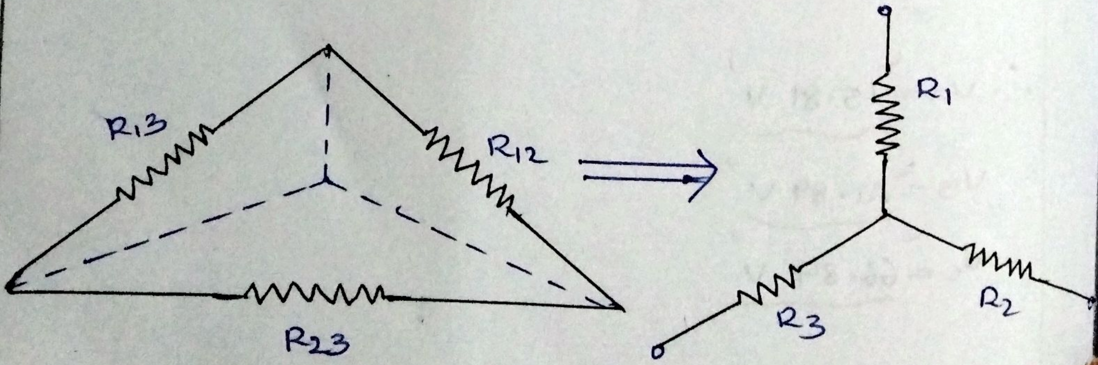


$\rightarrow R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3}$ (or) $\frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$

$R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1}$ (or) $\frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$

$R_{13} = R_1 + R_3 + \frac{R_1 R_3}{R_2}$ (or) $\frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$

Delta ⇒ Star :-



$$R_1 = \frac{R_{13} \cdot R_{12}}{R_{13} + R_{12} + R_{23}}$$

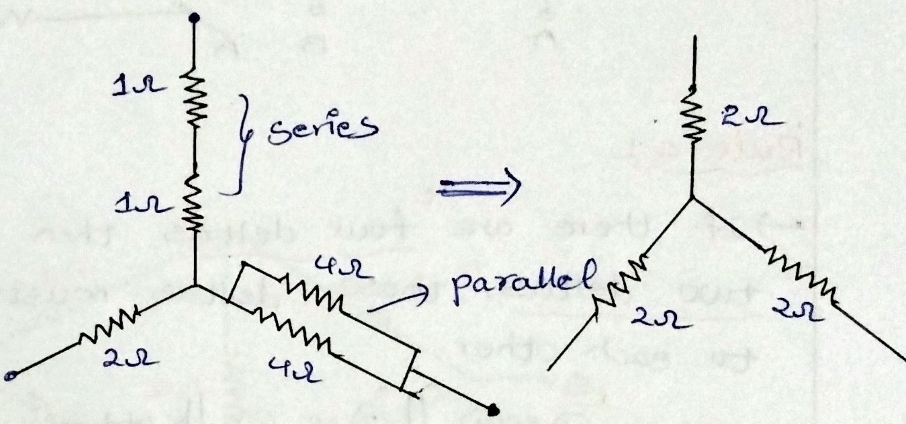
$$R_2 = \frac{R_{12} \cdot R_{23}}{R_{13} + R_{12} + R_{23}}$$

$$R_3 = \frac{R_{13} \cdot R_{23}}{R_{13} + R_{12} + R_{23}}.$$

Rules :-

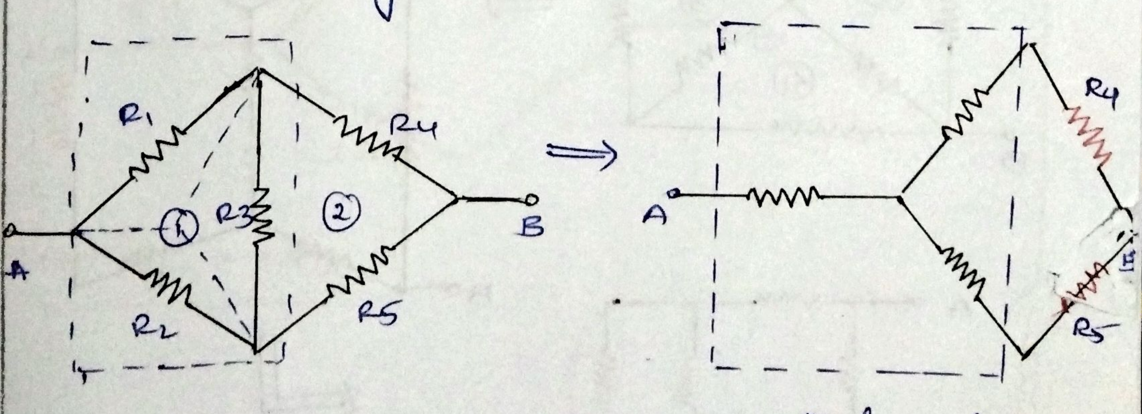
① If there is any "series" (or) "parallel" combination in a given Network, then reduce it.

Ex :-



→ convert it into a single Network then apply delta to it.

② If there are two deltas then convert any one delta only.

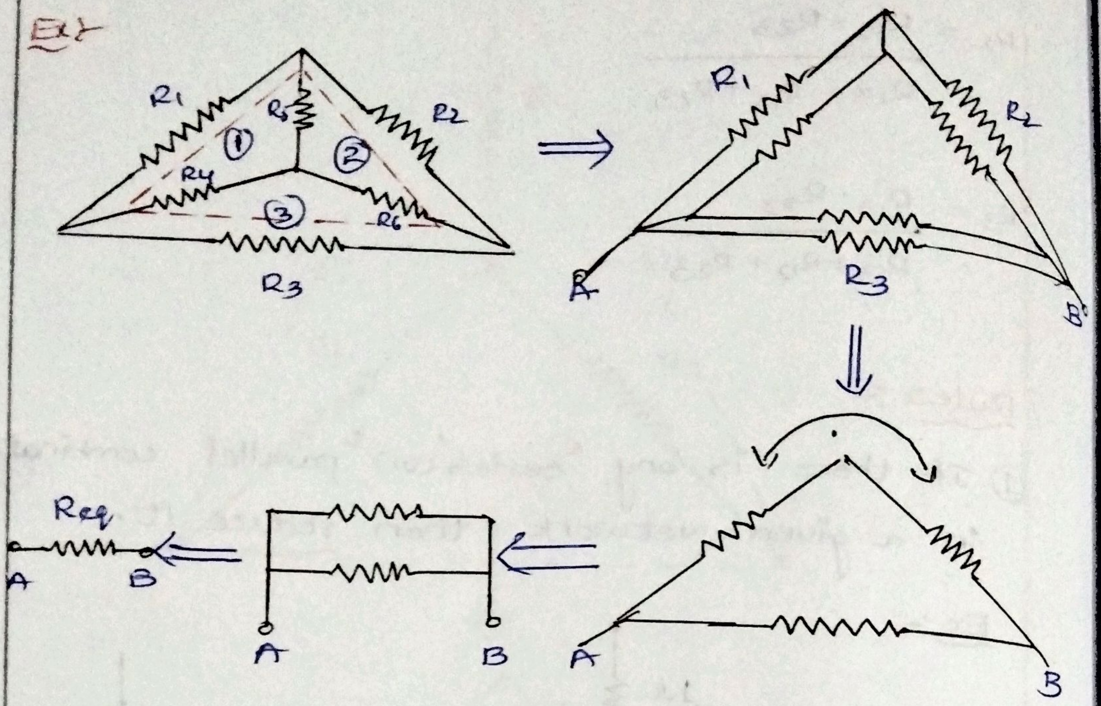


Apply Rule-1 & solve it.

Rule-3 :-

→ If there are 3 deltas, then convert the star into delta.

Ex:-

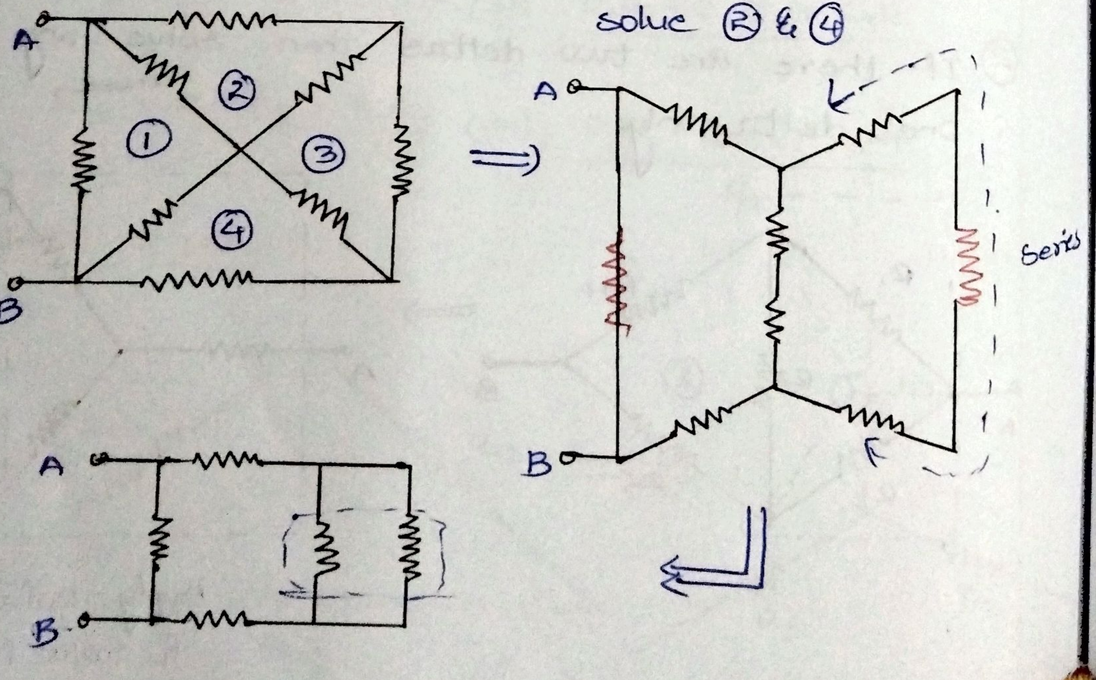


Rule-4 :-

→ If there are four deltas then convert any two deltas, those deltas must be opposite to each other.

i.e, ① && ③ || ② && ④ // other.

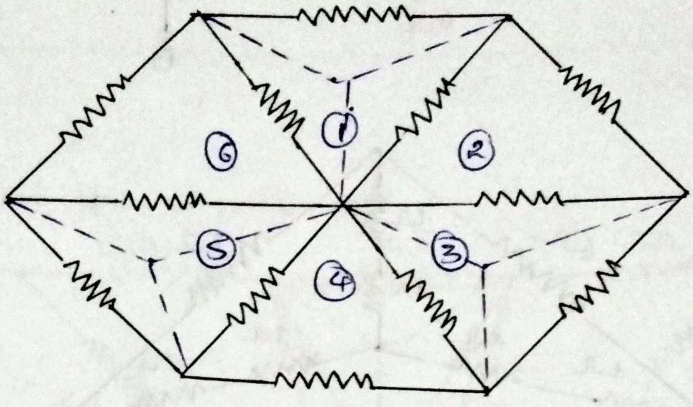
Ex:-



Rule-5 :-

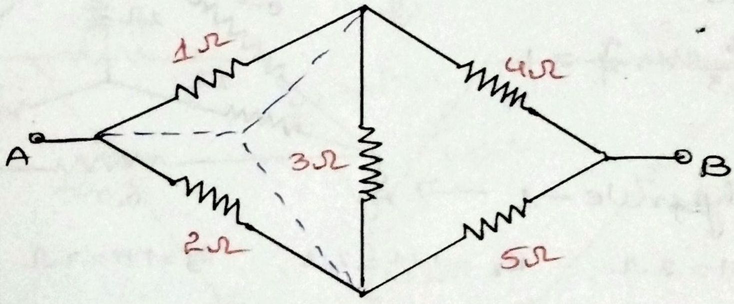
→ If there are 5 deltas, then convert/solve any three alternate deltas i.e, ①, ③, ⑤ [or] ②, ④, ⑥

Ex :-



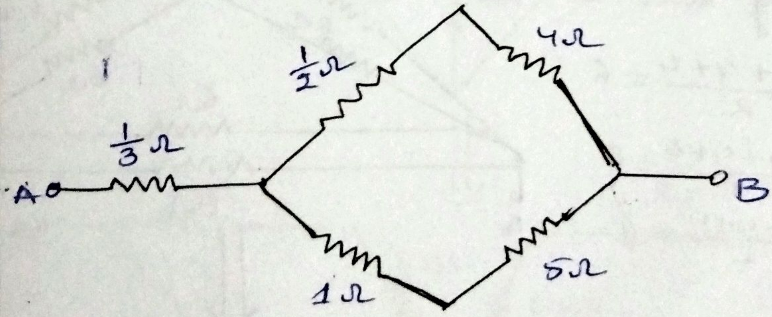
* Problems :-

1)



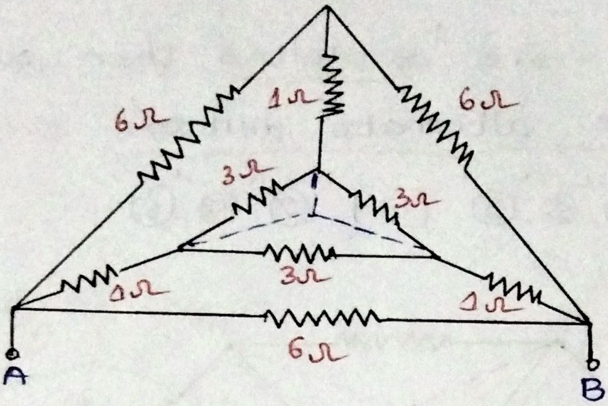
Find $R_{AB} = ?$

A)

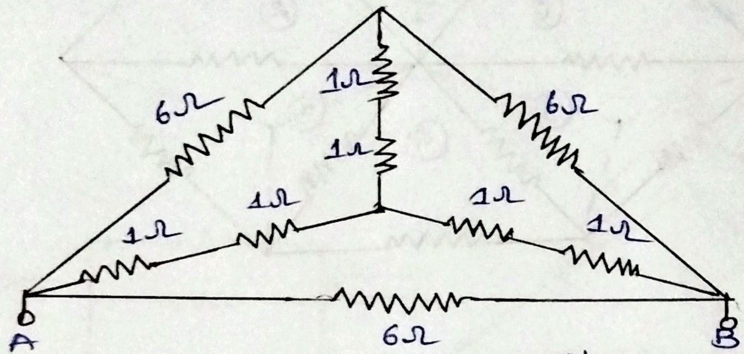


$\Rightarrow R_{AB} = 2.9\,\Omega$.

2]



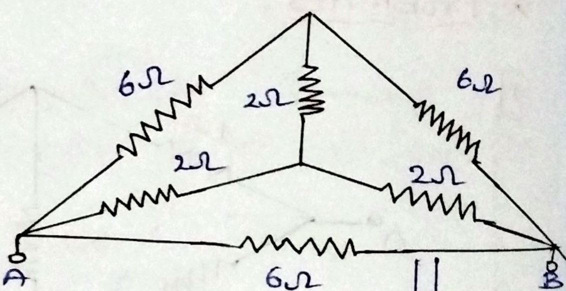
A)



$R_1 = \frac{3\times3}{3+3+3} = \frac{9}{9} = 1$

$R_2 = \frac{3\times3}{3+3+3} = \frac{9}{9} = 1$

$R_3 = \frac{3\times3}{3+3+3} = \frac{9}{9} = 1$



→ By applying rule − 1 →

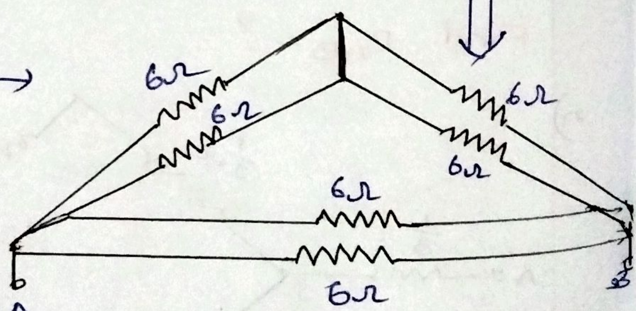
$R_1 = 1+1 = 2\Omega$, $R_2 = 1+1 = 2\Omega$, $R_3 = 1+1 = 2\Omega$

→ By applying Rule − 3 →

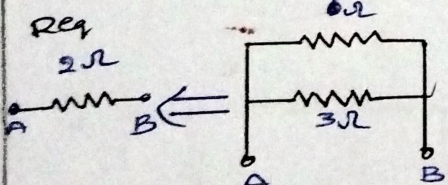
$\Rightarrow R_{12} = \frac{4+4+4}{2} = 6$

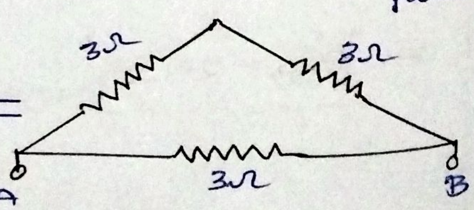
$\Rightarrow R_{23} = \frac{4+4+4}{2} = 6$

$\Rightarrow R_{13} = \frac{4+4+4}{2} = 6$



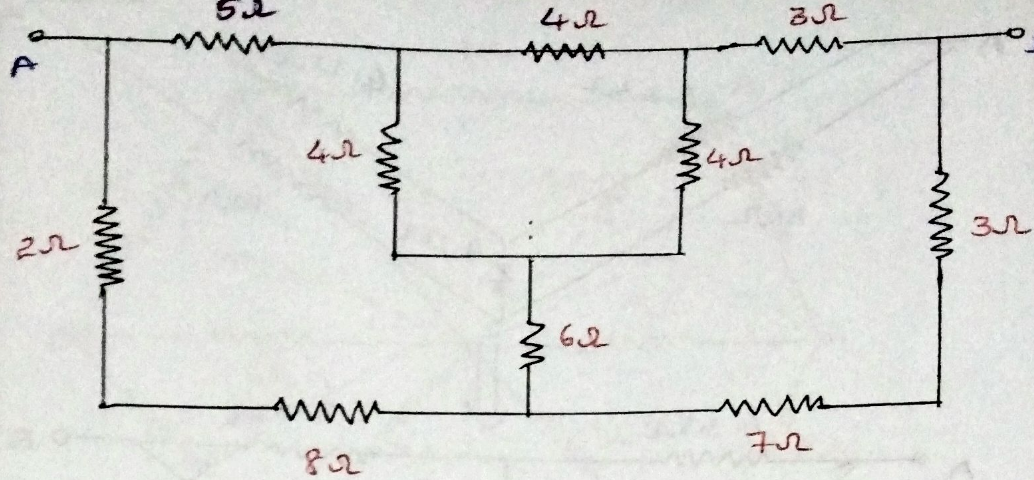
parallel



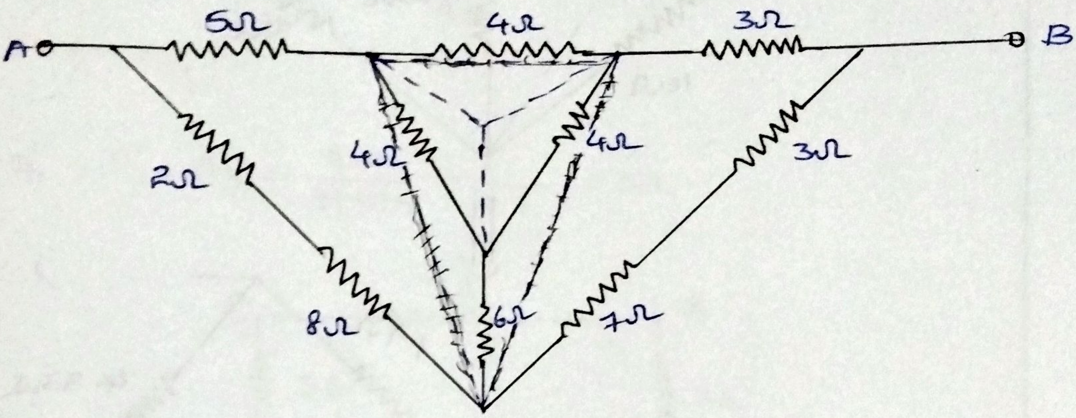


$\therefore R_{eq} = 2\Omega$

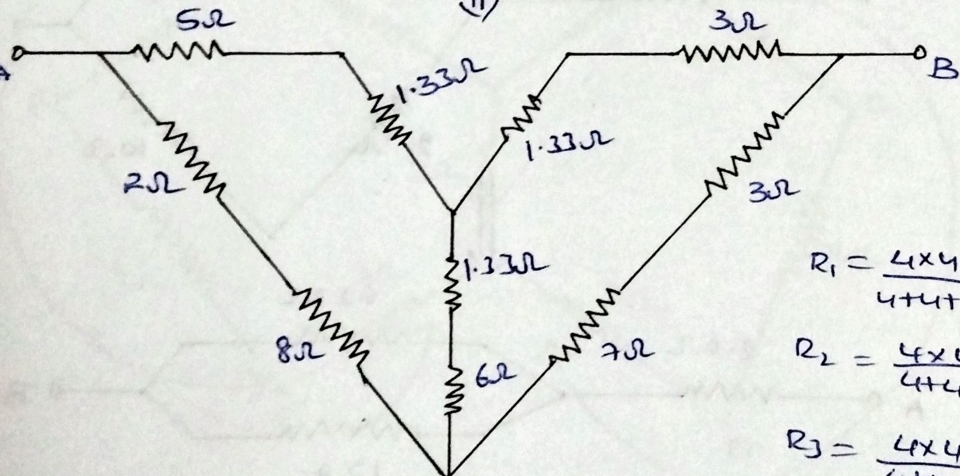
3



A) Redraw the circuit as.



⇓

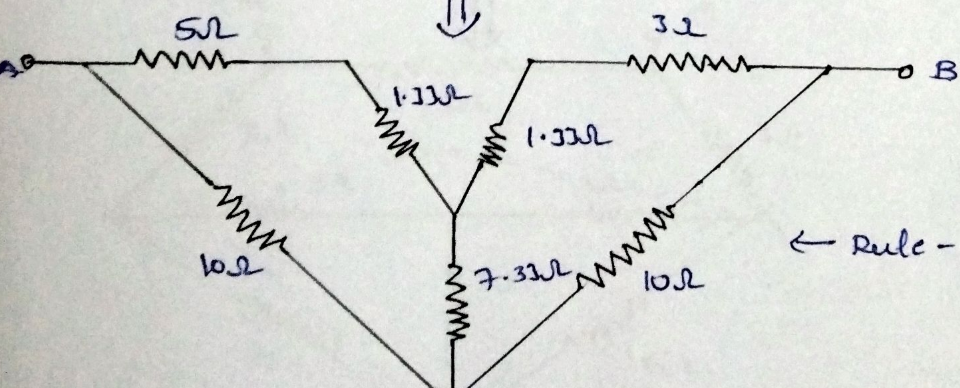


$R_1 = \frac{4 \times 4}{4+4+4} = 1.33$

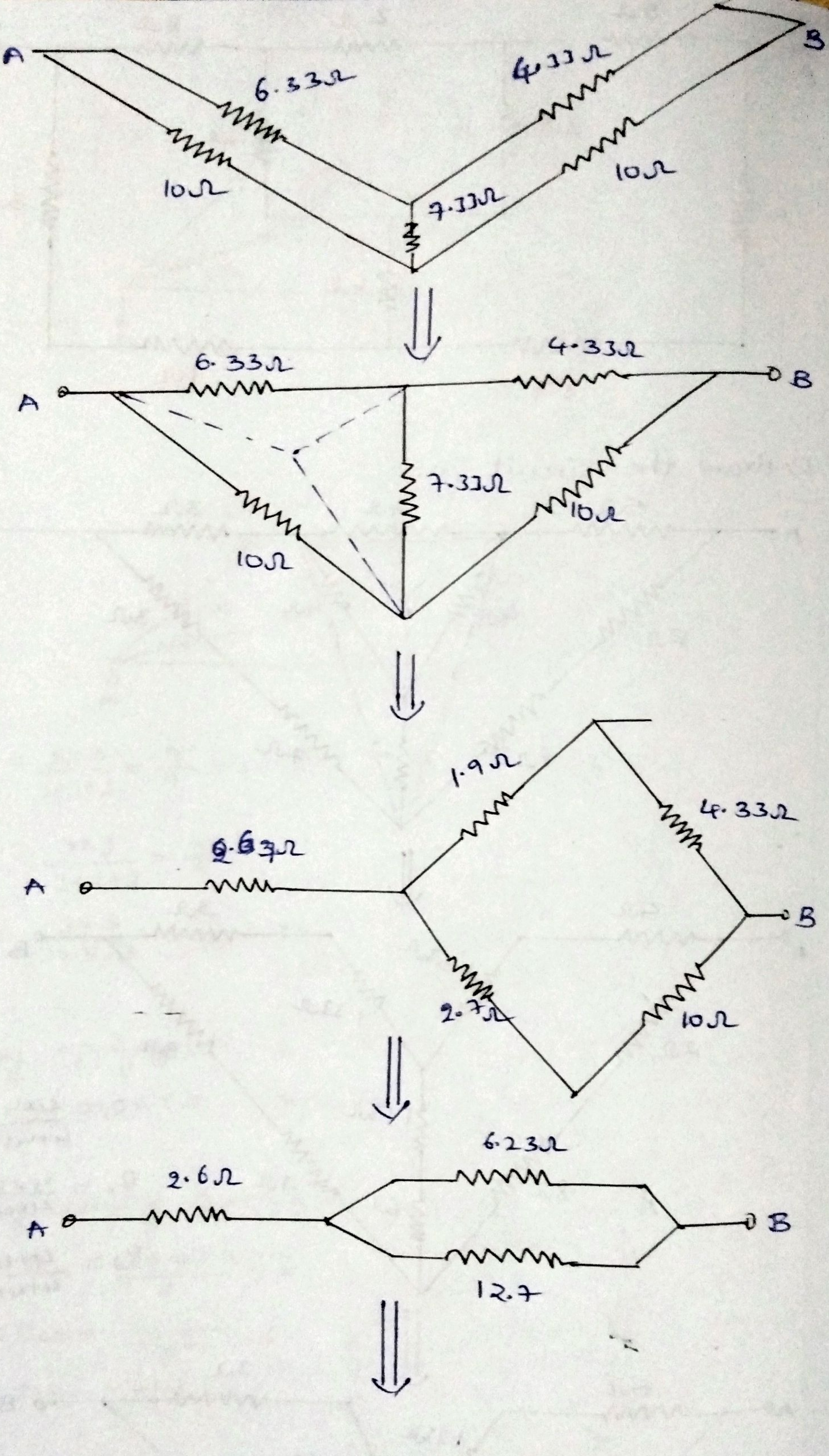
$R_2 = \frac{4 \times 4}{4+4+4} = 1.33$

$R_3 = \frac{4 \times 4}{4+4+4} = 1.33.$

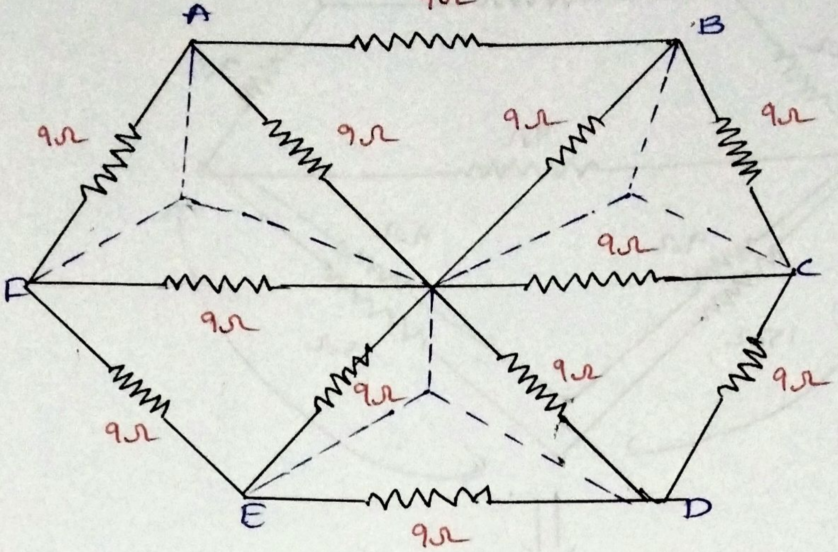
⇓



← Rule - 1

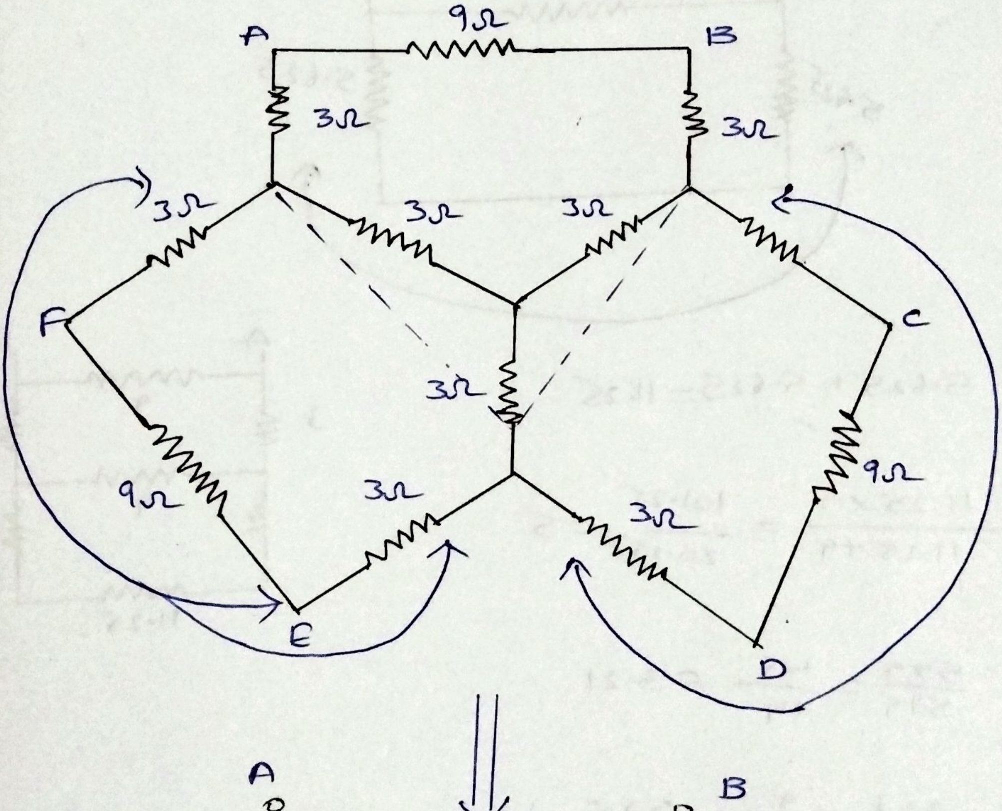

A
B
6.33Ω
10Ω
4.33Ω
10Ω
7.33Ω
A
6.33Ω
4.33Ω
B
7.33Ω
10Ω
10Ω
1.9Ω
4.33Ω
A
B
2.7Ω
10Ω
6.23Ω
2.6Ω
A
B
12.7

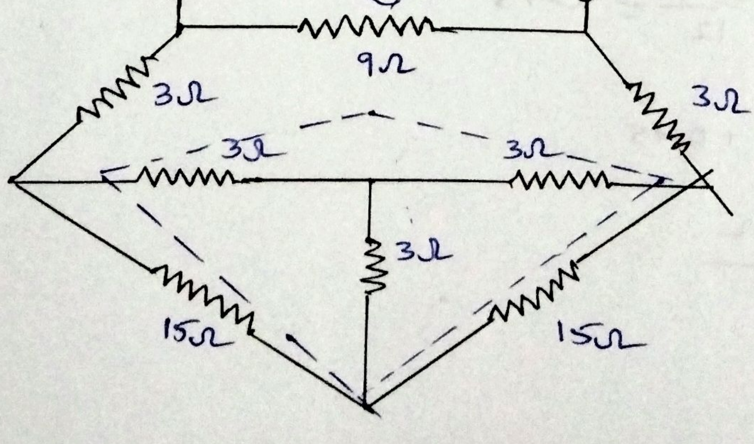
*

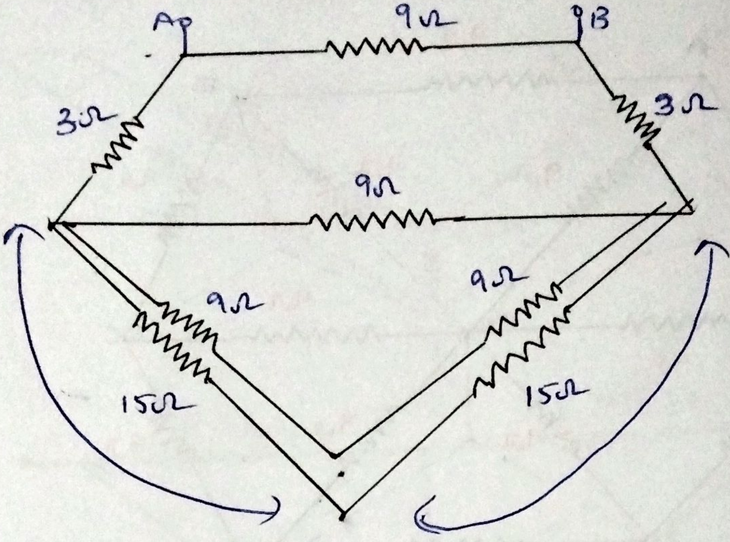


Find Req?

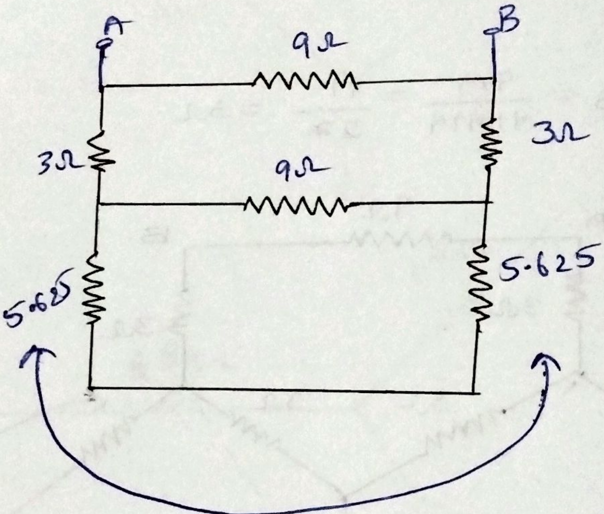
A) $R_1 = R_2 = R_3 = \frac{9 \times 9}{9+9+9} = \frac{81}{27} = 3\Omega$.

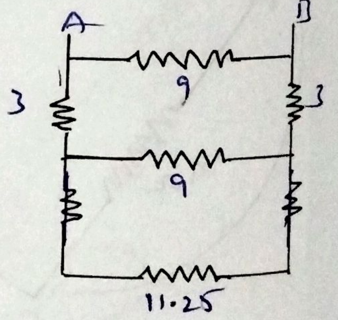






⇓





$\Rightarrow 5.625 + 5.625 = 11.25$

$\Rightarrow \frac{11.25 \times 9}{11.25 + 9} = \frac{101.25}{20.25} = 5$

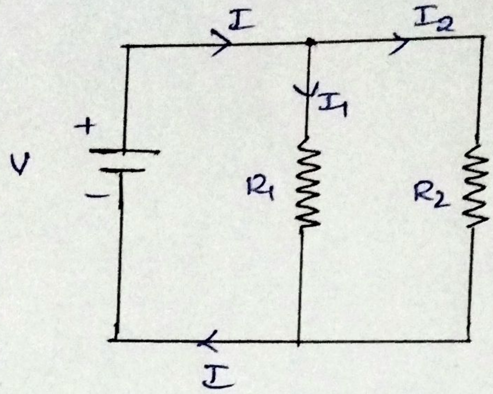
$\Rightarrow \frac{5 \times 9}{5 + 9} = \frac{45}{14} = 3.21$

$\Rightarrow \frac{3 \times 3}{3 + 9} = \frac{9}{12} = 0.75$

$\Rightarrow 3.21 + 0.75$

$\Rightarrow \underline{4.2\,\Omega}$

* Current Division Rule (or) Branch current :-



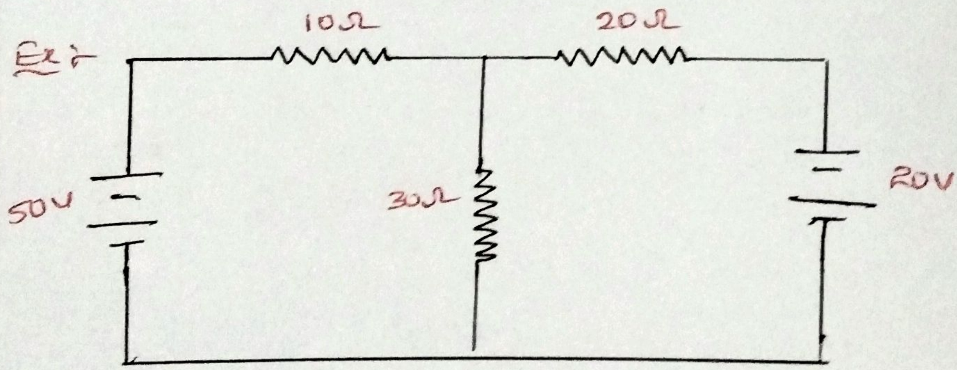
" $I = I_1 + I_2$ "

$$\Rightarrow \boxed{I_1 = \left(\frac{R_2}{R_2 + R_1}\right) \times I}$$

$$\Rightarrow \boxed{I_2 = \left(\frac{R_1}{R_1 + R_2}\right) \times I}$$

* Superposition Theorem :-

Ex :-



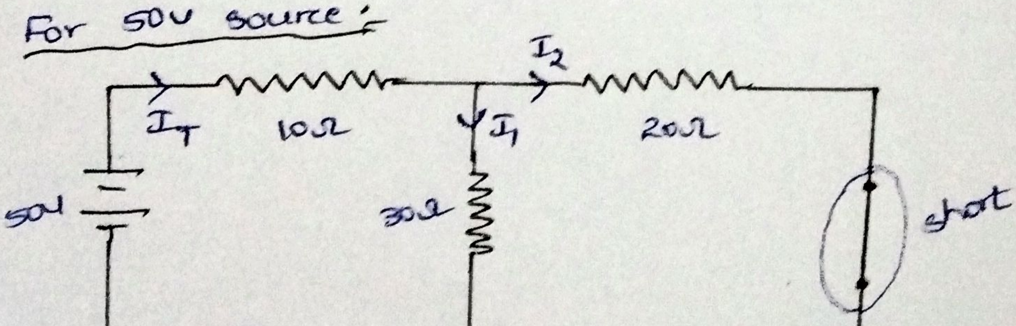
calculate $I_{30\Omega} = ?$

'2 sources'

'2+1 = 3 steps'

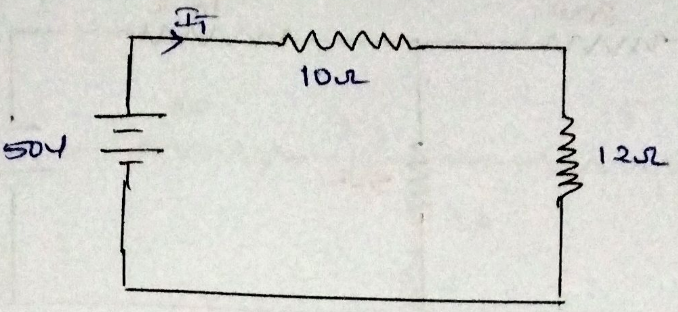
A) Step-1 :-

For 50V source :-



V → short

I → open.



Because $\frac{20 \times 30}{20+30} = \frac{\overset{12}{\cancel{600}}}{\cancel{50}} = 12\Omega$ [parallel].

Now, In series $10 + 12 = 22\Omega$.

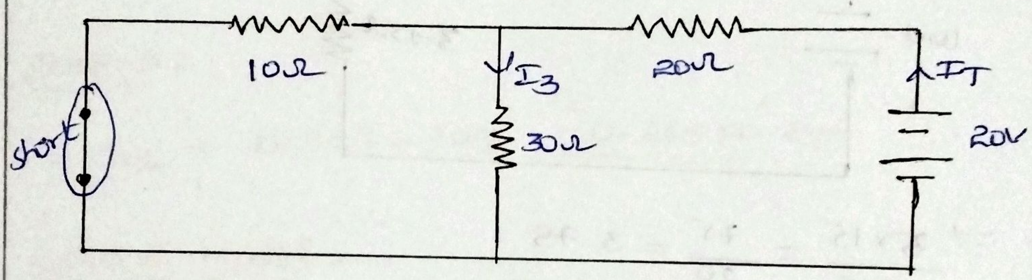
$\therefore I_T = \frac{50}{22} = 2.27\,A$. $[\because I = \frac{V}{R}]$.

$\therefore I_{30} = 2.27 \times \left(\frac{20}{20+30}\right) = 0.908\,A$.

$\therefore I_{30} = I_1 = 0.908\,A$ $(\downarrow)$

Step 2 :-

For 20V source:



Parallel : $\frac{10 \times 30}{10+30} = \frac{\overset{7.5}{\cancel{300}}}{\cancel{40}}$

Series :- $7.5 + 20 = 27.5$

$\therefore I_T = \frac{V}{R} = \frac{20}{27.5} = 0.72\,A$

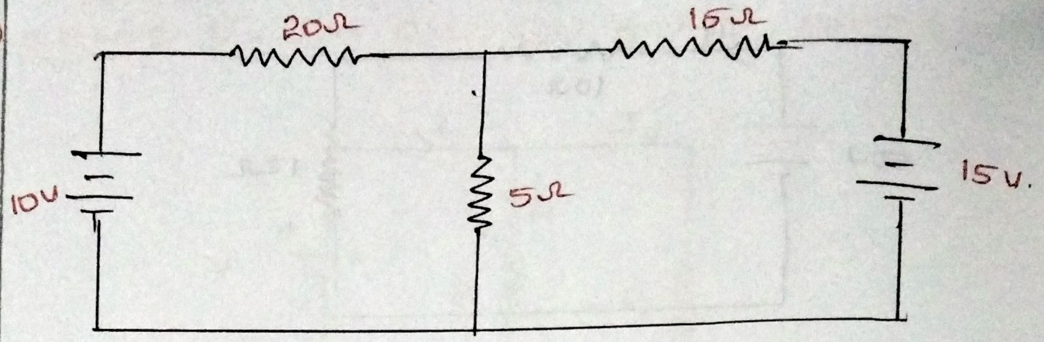
$\therefore I_3 = (0.72) \times \left(\frac{10}{10+30}\right) \doteq 0.18\,A \downarrow$

$\therefore I_3 = 0.18\,A$ $(\downarrow)$

Step 3 :-

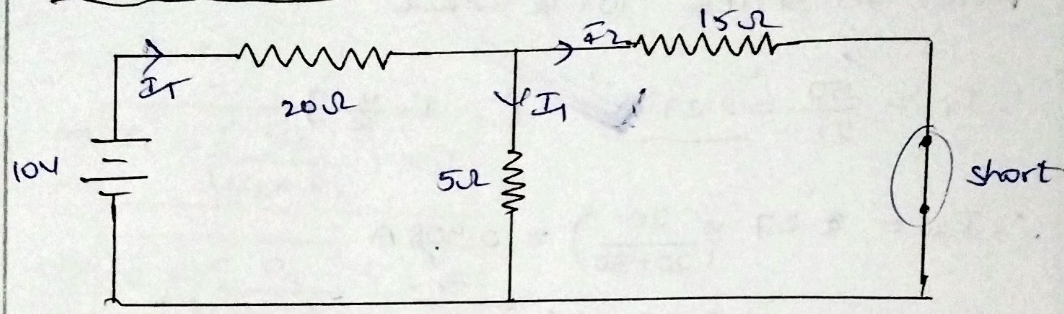
$I_{30\Omega} = I_1 + I_3 = 0.908\,A + 0.18\,A = 1.088\,A \downarrow$

②

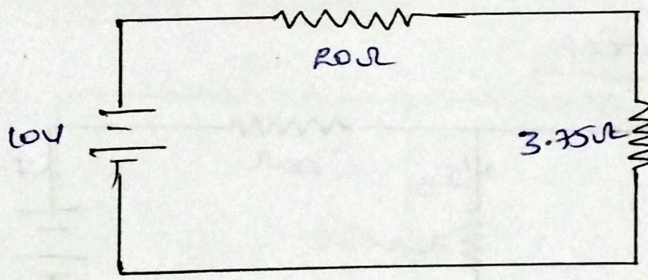


A) <u>step-1:-</u>

<u>For 10V source:-</u>



⇓



$$\Rightarrow \frac{5\times15}{5+15} = \frac{75}{20} = 3.75$$

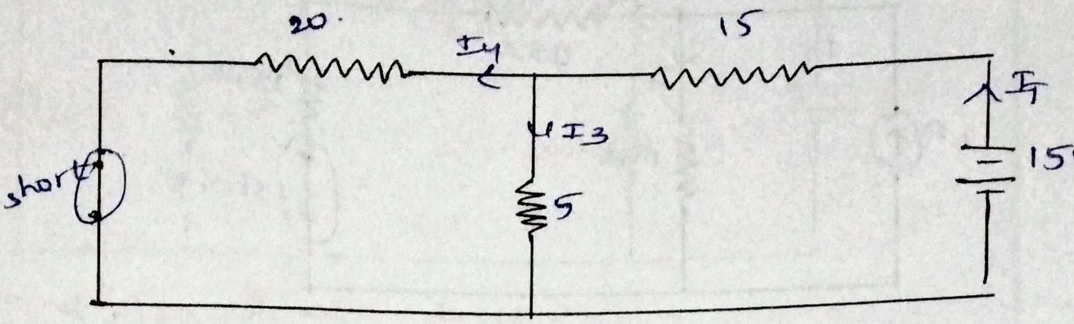
series :- $20 + 3.75 = 23.75$

$$\therefore I = \frac{V}{R} = \frac{10}{23.75} = 0.421A$$

$$\therefore I_2 = 0.421 \times \left(\frac{5}{5+15}\right)$$

$I_2 = 0.105A$ (⟶)

step-2 :



Parallel :

$$\Rightarrow \frac{20\times5}{20+5} = \frac{100}{25} = 4.$$

series :

$$\Rightarrow 15+4=19.$$

$$\therefore I_T = \frac{V}{R} = \frac{15}{19} = 0.789\ A$$

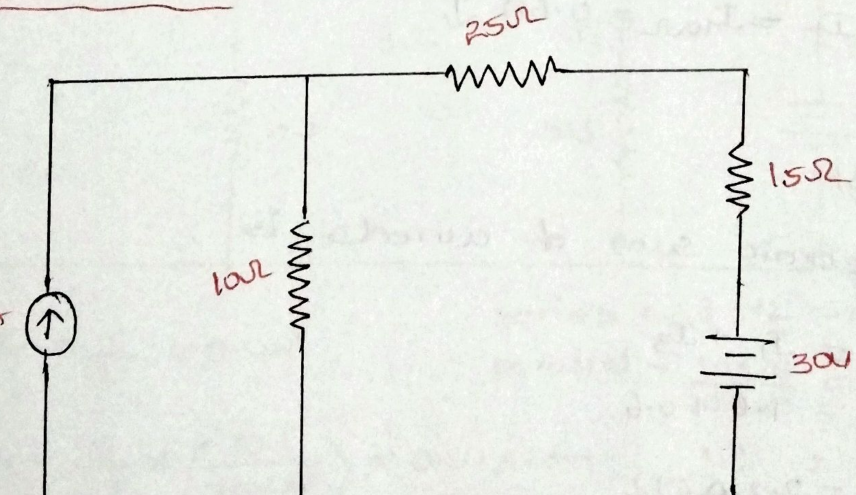
$$\therefore I_{15} = I_T = 0.789(A)\ (\longleftarrow)$$

step-3 :

$$\therefore I_{15} = 0.789 - 105 = 0.685\ A \leftarrow$$

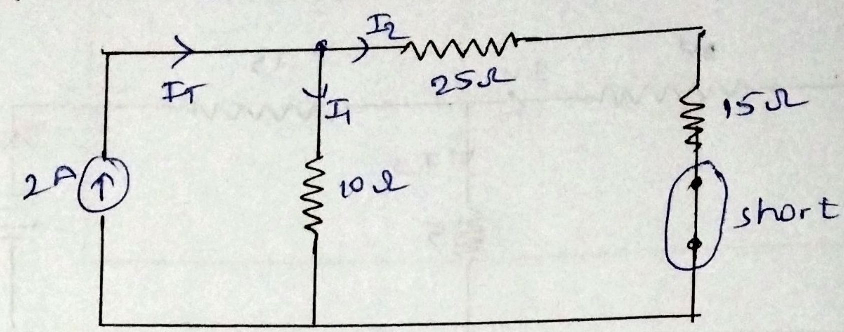
second Model :

3



Find $I_{10\Omega} = ?$

A) step-1 :- (For 2A)

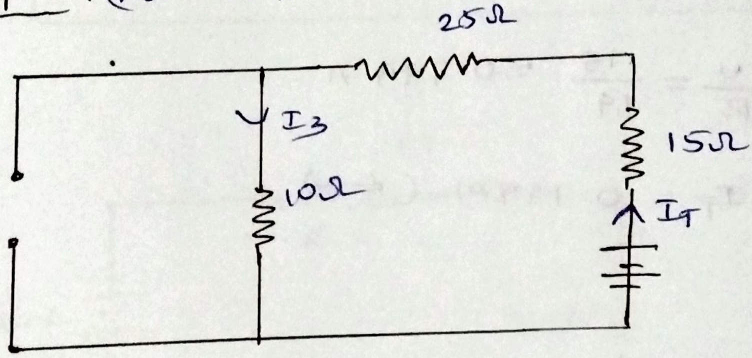


series :- $25 + 15 = 40$.

$I_T = 2A$.

$\therefore I_1 = 2 \times \left(\frac{40}{40+10}\right) = 1.6\,A. \downarrow$

step-2 :- (For 30 V)



$I_T = \frac{30}{15+25+10} = \frac{30}{50} = 0.6\,A$

$I_3 = I_T = I_{10\Omega} = 0.6A \downarrow.$

step-3 :-

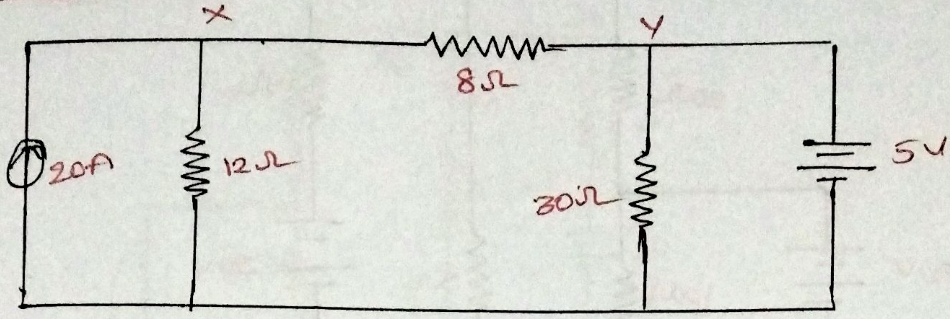
→ Algebraic sum of currents is

$I_{10} = I_1 + I_3$

$= 1.6 + 0.6$

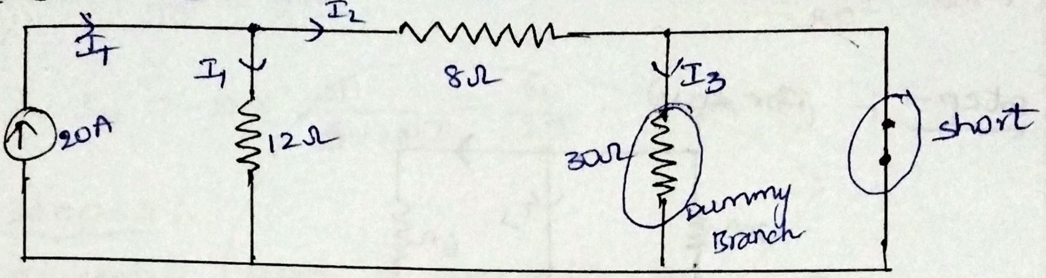
$\therefore I_{10\Omega} = 2.2\,A\ (\downarrow).$

Third Model :-



Find $I_{8\Omega} = ?$, with direction?

A) step-1 :- (For 20A)



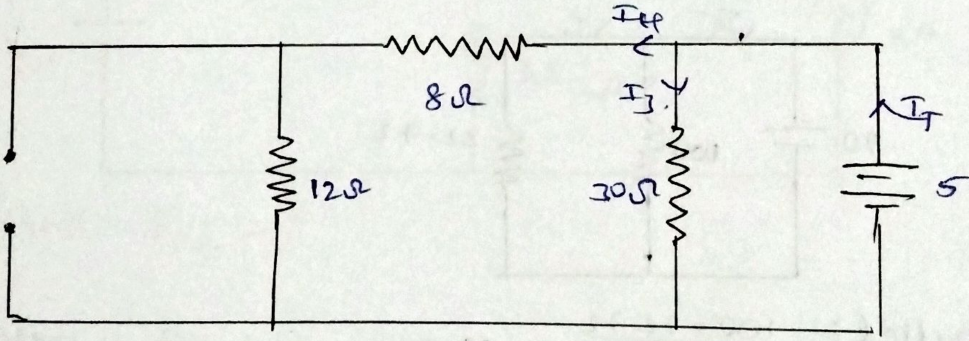
$$\frac{0\times 30}{0+30} = 0.$$

→ parallel :- $\frac{12\times 8}{12+8} = \frac{96}{20} = 4.8\Omega$ (No need)

$I_T = 20A$

$\therefore I_2 = 20 \times \left(\frac{12}{12+8}\right) = 20 \times \frac{12}{20} = 12A$ →

step-2 :-



series = 8 + 12 = 20

parallel = $\frac{30\times 20}{30+20} = 12$

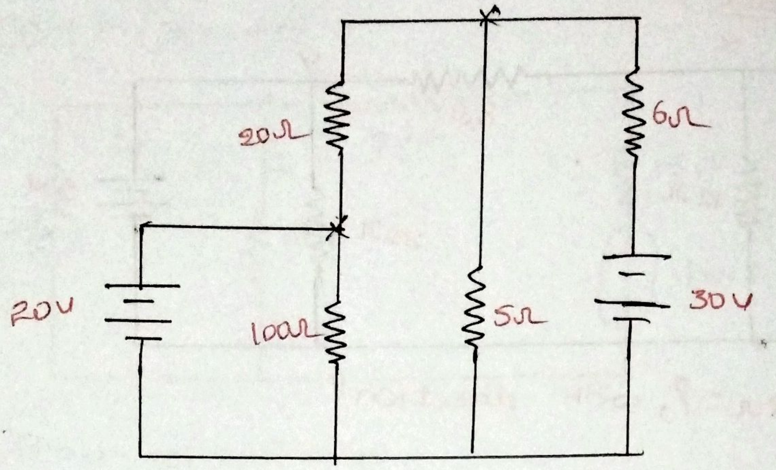
$I_T = \frac{5}{12} = 0.41A$

$I_4 = \frac{5}{12} \times \left(\frac{30}{30+20}\right) = 0.24A$ ←

step-3 :-

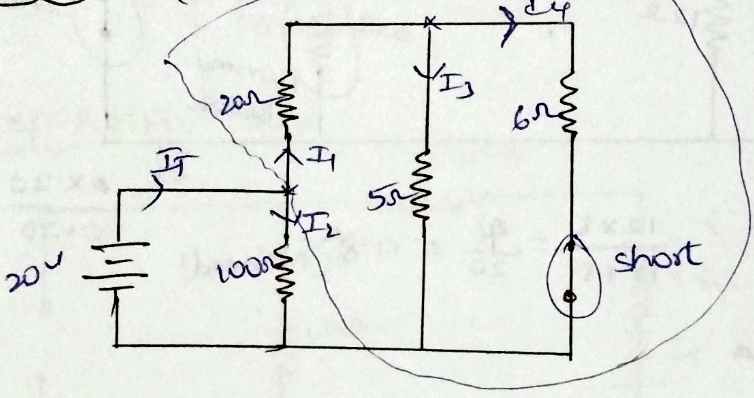
$I_{8\Omega} = I_2 + I_4 = 12 - 0.24 = 11.76A$ (→) X → Y

*



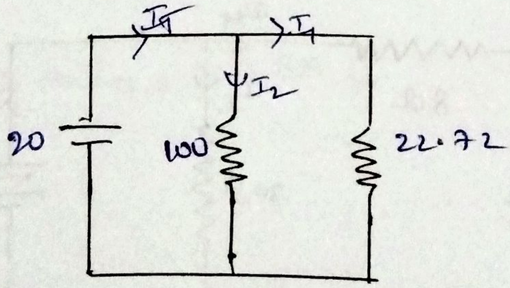
Find $I_{5\Omega} = ?$

A) step-1 : (For 20V)



Parallel : $\frac{5 \times 6}{5+6} = \frac{30}{11} = 2.72\,\Omega$

series : $2.72 + 20 = 22.72\,\Omega$



parallel : $\frac{100 \times 22.72}{100 + 22.72} = 18.51$

$\therefore I_T = \frac{V}{R} = \frac{20}{18.51} = 1.08\text{ A}$

$I_1 = \overset{(I_T)}{1.08} \times \frac{100}{100 + 22.72}$

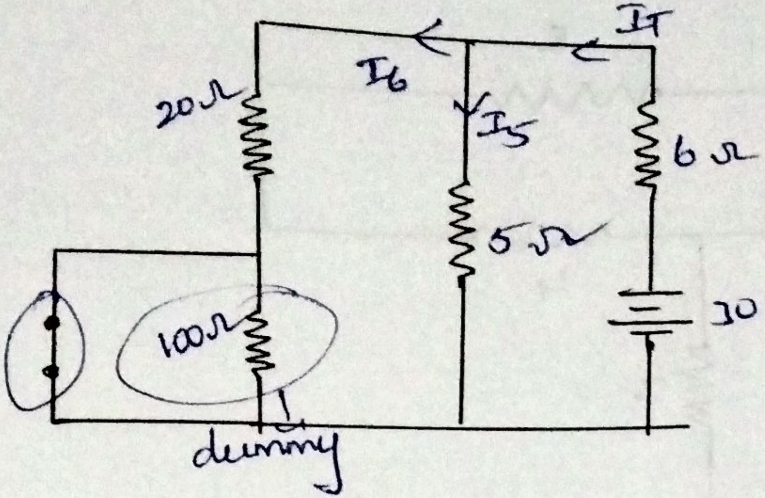
$= 0.88\text{ A}$

$\therefore I_{3\Omega} = I_1 \times \left(\frac{6}{5+6}\right)$

$= 0.88 \times 0.54$

$= 0.48\text{ A}$

step-2 (For 30V)



parallel $= \frac{20 \times 5}{20+5} = 4$

series $= 4+6 = 10$

$I_T = \frac{30}{10} = 3A.$

$\therefore I_5 = 3 \times \frac{20}{5+20} = \frac{60}{25} = 2.4A \downarrow$

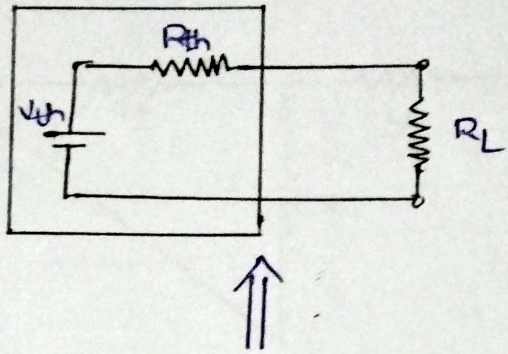
step-3 :

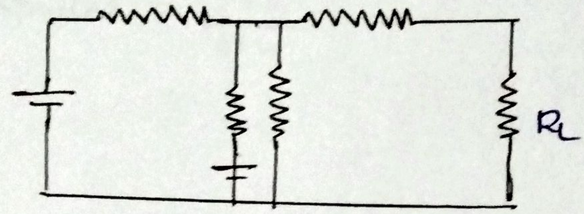
Algebraic sum of currents is

$\Sigma I. = I_3 + I_5 = 0.48 + 2.4 = 2.88 A. \downarrow$

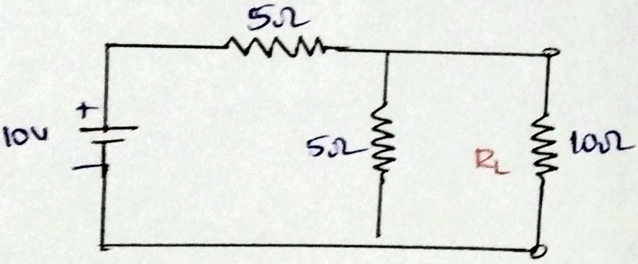
* Thevenins Theorem :-

one voltage } series
one Resistor }



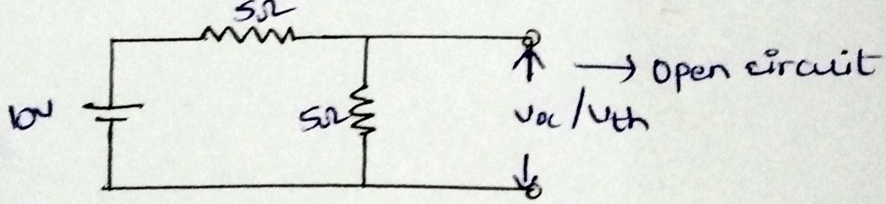


Example :-



$I_{10\Omega} = ?$

step-1 :- Identify & Remove Load source (R_L).



step-2 :- calculate voltage drop (V_{oc}) at open circuit

5Ω & 5Ω are in series

$\Rightarrow R = 10\Omega$

$\Rightarrow V = 10V$

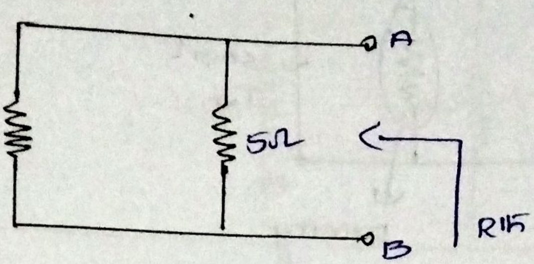
$\therefore I = \frac{V}{R} = \frac{10}{10} = 1$

$\therefore V_{oc} / V_{th} = IR$

$= (1)(5)$ [$\because R = 5\Omega$]

$= 5$

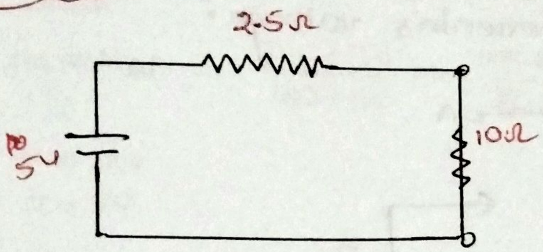
step-3 : calculate thevenins Resistance.



$\therefore R_{AB} = \frac{5 \times 5}{5+5} = \frac{25}{10} = 2.5\,\Omega$

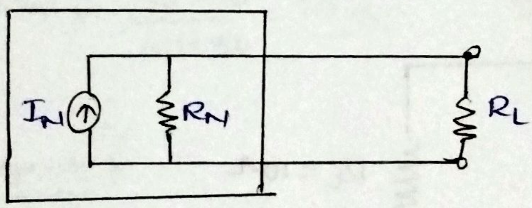
$\therefore R_{TH} = 2.5\,\Omega$.

step-4 : Equivalent circuit of Thevenin.



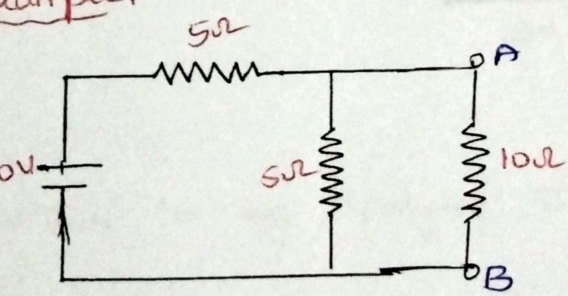
$I_{10\Omega} = \frac{5}{2.5+10} = \frac{5}{12.5} = 0.4A.$

* Norton's Theorem :



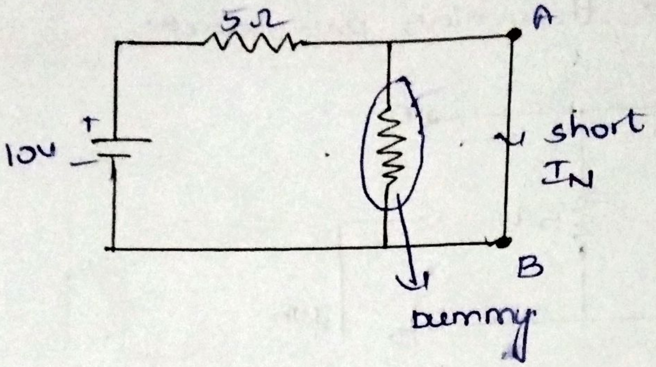
One current source } parallel
One Resistor

Example :



step-1 :

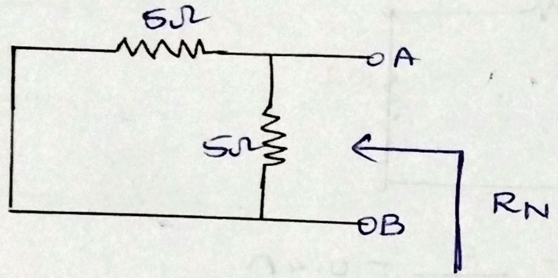
Identify & short the Load Resistor (R_L).



step-2 :- Calculate the current I_N.

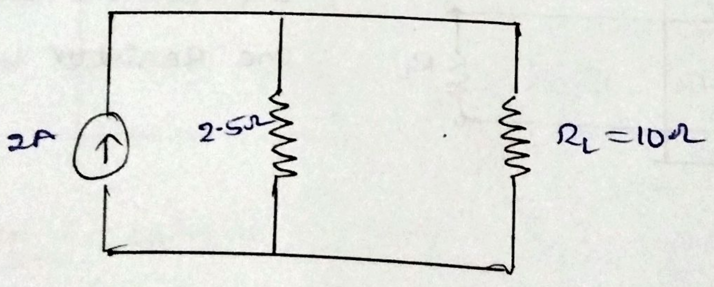
$\Rightarrow I_N = \frac{V}{R} = \frac{10}{5} = 2A$

step-3 :- Calculate ~~Thevenin's~~ Norton's voltage.



$R_N = 2.5\Omega$ $\quad \left[\because R_N = \frac{5\times5}{5+5}\right]$

step-4 :- Equivalent circuit



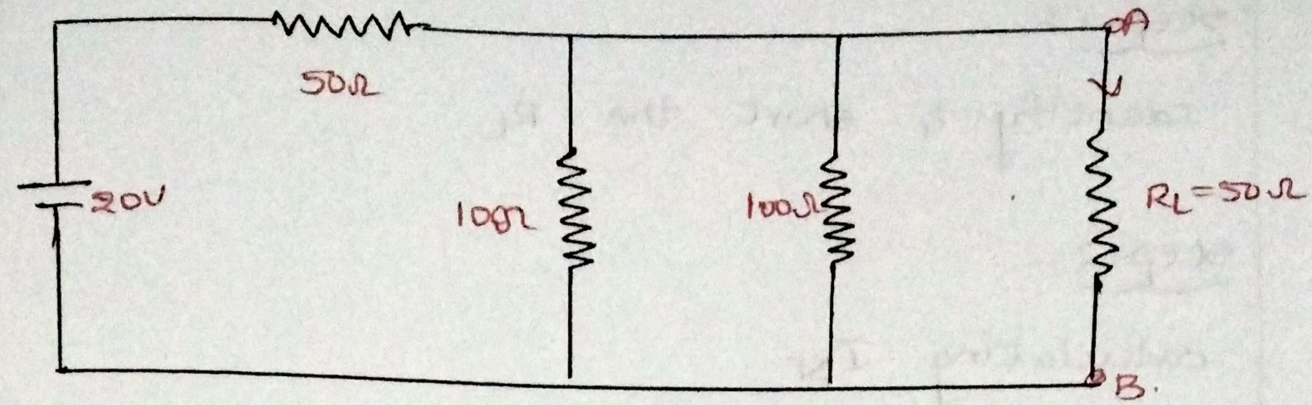
$I_{10\Omega} = 2\times\left(\frac{2.5}{2.5+10}\right)$

$I_{10\Omega} = \frac{5}{12.5}$

$\therefore I_{10\Omega} = 0.4 A$

$\therefore \boxed{I_{Thevenin's} = I_{Norton's}}$

*



Find $I_L = ?$

A) **Step-1 :-**

Identify & Remove R_L.

Step-2 :-

$$\text{parallel} = \frac{100 \times 100}{100 + 100} = \frac{10000}{200} = 50\,\Omega$$

$V = 20$

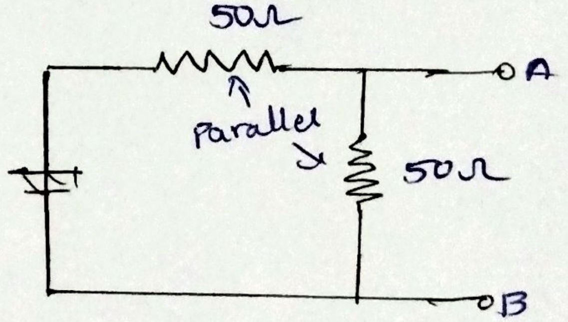
$R = 50$

$\therefore I = \frac{20}{100} = 0.2\text{ A}$ $\qquad I_{100\Omega} = (0.2) \times \left(\frac{100}{100+100}\right) = 0.1$

$\therefore V_R = IR = (0.1) \times 100 = 10\text{ V}.$

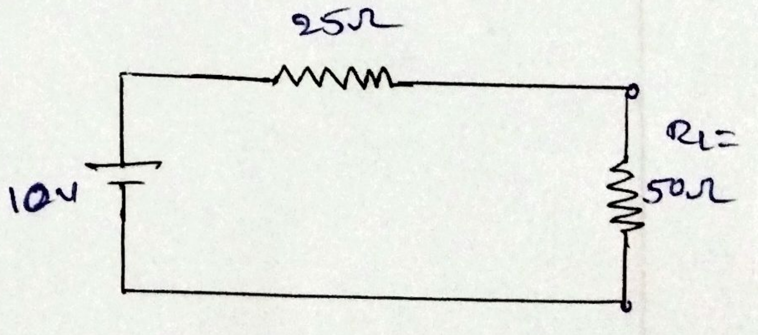
Step-3 :-

$$R_{TH} = \frac{50 \times 50}{50 + 50} = 25\,\Omega$$



Step-4 :-

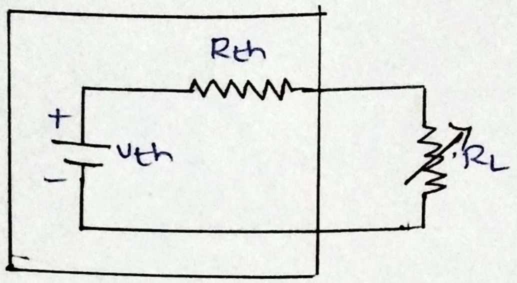
$$I_L = \frac{10}{50+25} = \frac{10}{75} = 0.13\text{ A}$$

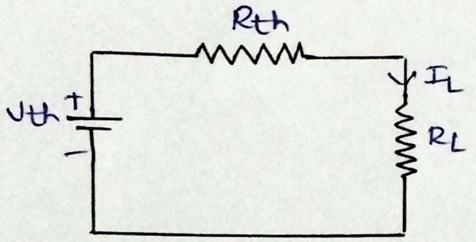


* Maximum Power Transfer Theorem:-

→ In a given circuit, we can draw maximum power, when load is resistance is equal to internal resistance of all the sources.

Ex:-





$$I_L = \frac{V_{th}}{R_{th}+R_L} \longrightarrow (1)$$

power, $P = I_L^2 R_L \longrightarrow (2)$

From (1) & (2)

$$\Rightarrow P = \frac{(V_{th})^2}{(R_{th}+R_L)^2} R_L \longrightarrow (3)$$

For maximum power, $\frac{dP}{dR_L} = 0$.

$$\Rightarrow (V_{th})^2 \left[\frac{(R_L+R_{th})^2 (1) - R_L(2(R_L+R_{th})(1+0))}{((R_{th}+R_L)^2)^2} \right] = 0$$

$$\Rightarrow (R_L+R_{th})^2 - 2R_L(R_L+R_{th}) = 0$$

$$\Rightarrow (R_L+R_{th})^2 = 2R_L(R_L+R_{th}).$$

$$\Rightarrow R_L + R_{th} = 2R_L.$$

$\therefore \boxed{R_{th} = R_L} \rightarrow$ when $R_L = R_{th}$, we can draw Maximum power.