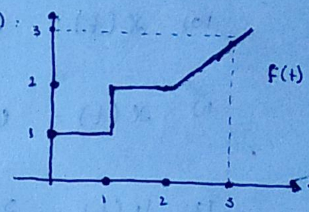
28/10/22

Assignment - Problems :

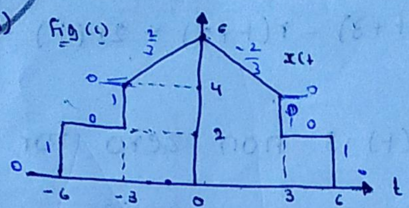
1) Express the following signal interms of $u(t)$

2) Express the following signals interms of step & ramp function fig(a), fig(b)

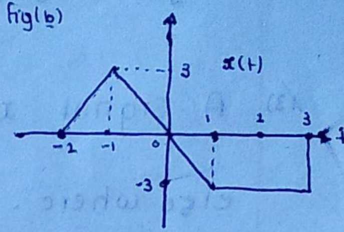
fig(a) :



3) Fig (c)



fig(b)



4) Find the value of $\int_{-\infty}^{} \frac{\sin 3T}{T} \delta(t-T)\, dT$

5) Find the value of $\int_{-\infty}^{\infty} \delta(1-t)(t^3+4)\, dt$

6) The integral value of $\int_{-\infty}^{t} [\delta(T+2) - \delta(T-2)]\, dT$ is $\underline{0}$.

7) The value of integral $\int_{0}^{\infty} \cos(t-T)\, \delta(T+3)\, dT$

8) Let $x(t) = \begin{cases} -t+1 & ; -1 \le t \le 0 \\ t & ; 0 \le t < 2 \\ 2 & ; 2 \le t < 3 \\ 0 & ; e.w \end{cases}$

Then sketch the graph of

(i) $x(t)$

(ii) $x(t-2)$

(iii) $x(t+3)$

(iv) $x(-3t-2)$

v) $x(2t/3 + 1/2)$

Sketch the wave form of the following signals.

9) $x(t) = u(t) - u(t-6)$

10) $x(t) = r(t+1) - r(t) + r(t-2)$

11) $x(t) = u(-t+2) + r(t+1) - r(t-1)$

12) $y(t) = 3u(t+3) - r(t+2) + 2r(t) - 2u(t-2) - r(t-3)$
$- 2u(t-4)$

13) A signal $x(t)$ is non zero for $-1 \leq t \leq 2$ & zero else where. Then the signal $y(t) = 2x\left(\left[\frac{2-t}{2}\right]\right)$ is non zero for duration ....

14) A discrete time signal $x[n]$ is $x[n] = (6-n)\left[u[n] - u[n-6]\right]$

Sketch

(i) $x[n+3]$

(ii) $x[6-n]$

(iii) $x[3n+1]$

(iv) $x[n/2]$

(v) $x[n]\, u[2-n]$

vi) $x[n-1]\, \delta[n-3]$

15) Sketch the following signals

(i) $u(t-3)$

(ii) $u(t+3)$

(iii) $u(-t-3)$

(iv) $u(-t+3)$

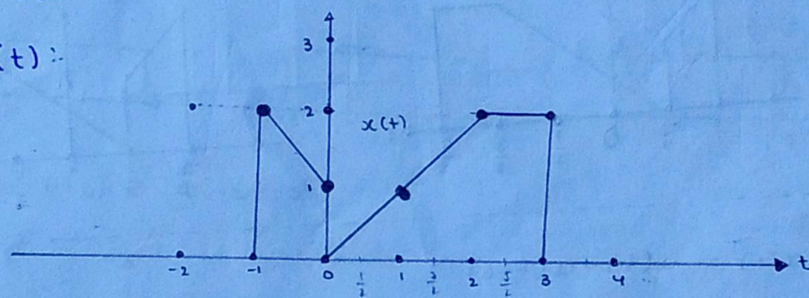
10) Express the following functions in general form of unit step function $u(\pm t - t_0)$

(i) $u(t/4 - 4)$

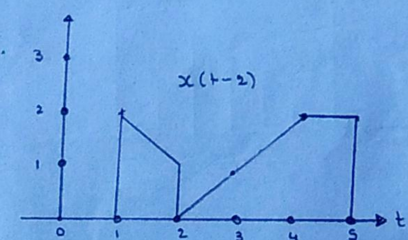
(ii) $u(-3t + 6)$

Answers:

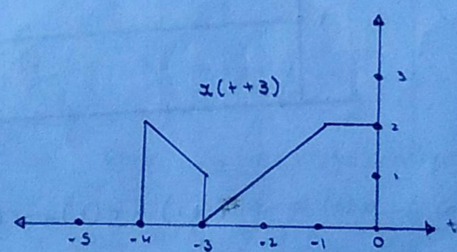
8 (i) $x(t)$:-



(ii) $x(t-2)$



(iii) $x(t+3)$

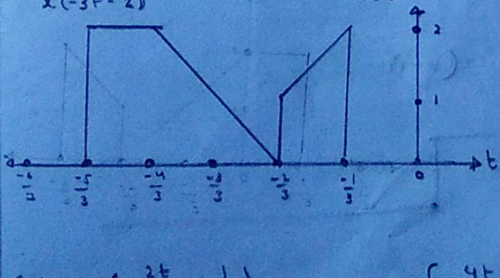


(iv) $x(-3t-2)$:-

$x\left(-3\left(t + \frac{2}{3}\right)\right)$ {Time scaling & Time shifting}

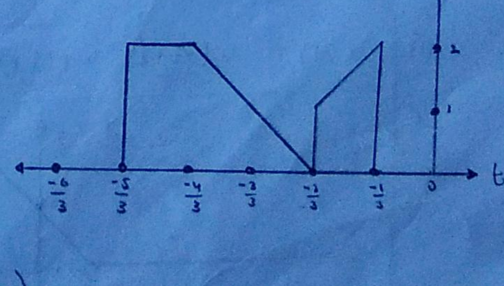
method - 1

$x(-3t-2)$



$x(-3t-2)$ by method - 2

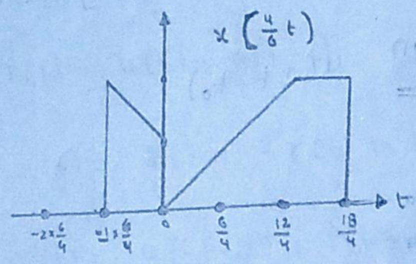
Time shifting & Time scaling

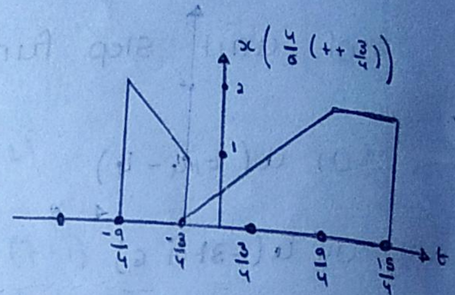


(v) $x\left(\frac{2t}{3} + \frac{1}{2}\right) \rightarrow x\left(\frac{4t+3}{6}\right)$

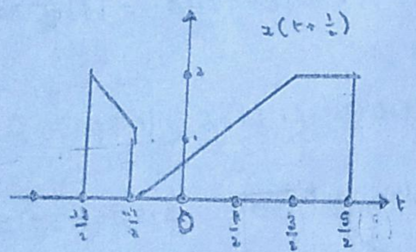
$\rightarrow x\left[\frac{1}{6}(4t+3)\right] \rightarrow x\left[\frac{4}{6}\left(t + \frac{3}{4}\right)\right]$

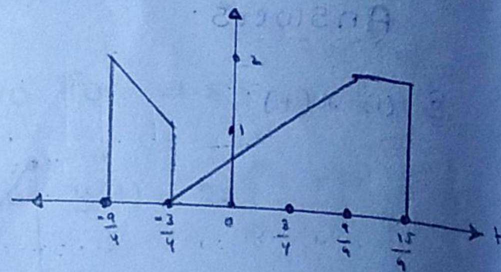
By method-1 Time scaling → Time shifting.





method-2 : Time shifting → Time scaling.





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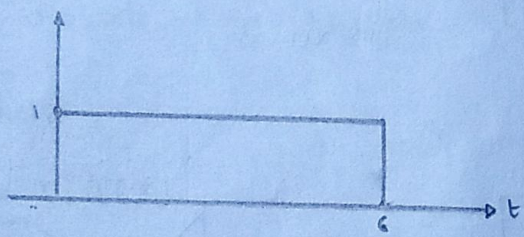
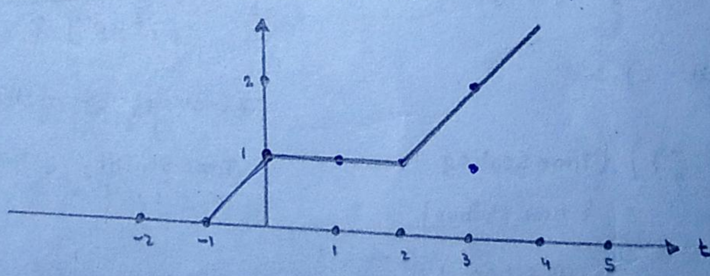
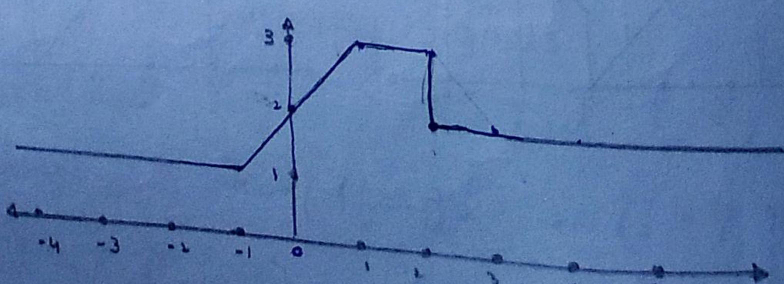
9) $x(t) = u(t) - u(t-6)$



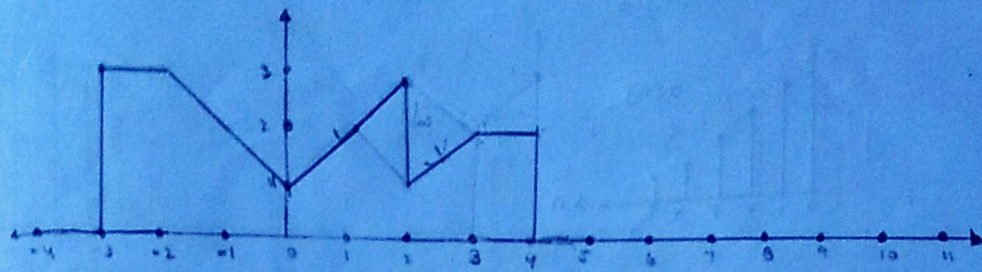
10) $x(t) = r(t+1) - r(t) + r(t-2)$



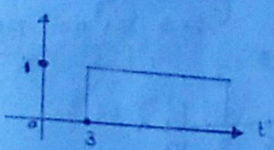
11) $x(t) = u(-t+2) + r(t+1) - r(t-1)$



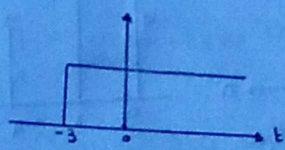
12) $y(t) = 3u(t+3) - r(t+2) + 2r(t) - 2u(t-2) - \underline{r(t-3)} - 2u(t-4)$



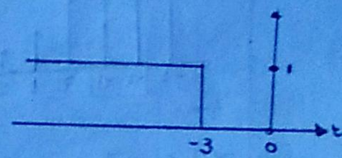
15) (i) $u(t-3)$



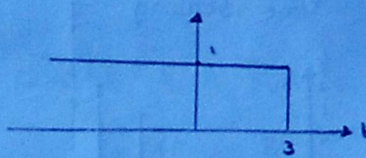
(ii) $u(t+3)$



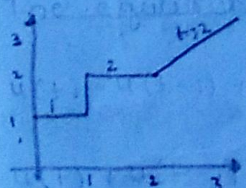
(iii) $u(-t-3) = u(-(t+3))$



(iv) $u(-t+3) = u(-(t-3))$



1)



first write the intervals where amplitude changes w.r.t to t (t) & $(t-1)$ $(t-2)$ & write amplitude change. Now write the equation with $u(t)$

$= \underline{u(t) + u(t-1) + (t-2)u(t-2)}$

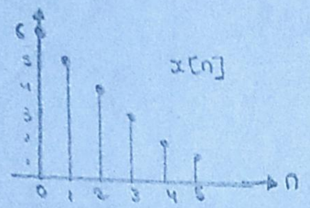
= Is the required equation

2) $3r(t+2) - 6r(t+1) + 3r(t-1) + 3u(t-3)$ {fig(b)}

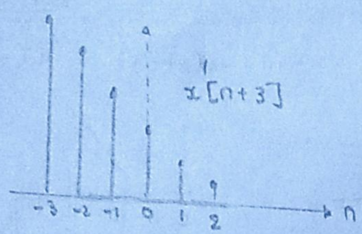
3) $2u(t+6) + 2u(t+3) + \frac{2}{3}r(t+3) - \frac{4}{3}r(t) + \frac{2}{3}r(t-3)$

$- 2u(t-3) - 2u(t-6)$

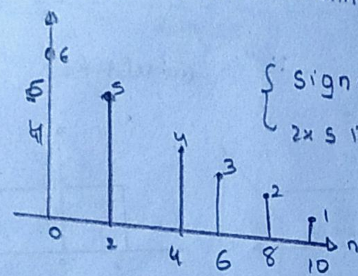
(4) $x[n] = (6-n)\,[u[n] - u[n-6]]$



(i) $x[n+3]$



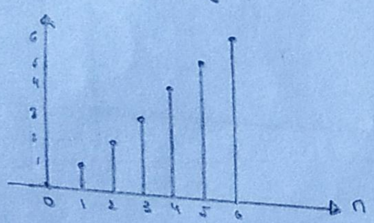
(iv) $x[\frac{n}{2}]$ {Time Scaling}



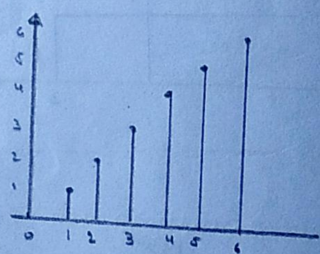
{signal at 2×3, 2×4, 2×5 is not present}

(ii) $x[6-n] = x[-(n-6)]$

Method-1: Time saling to Time shifting.



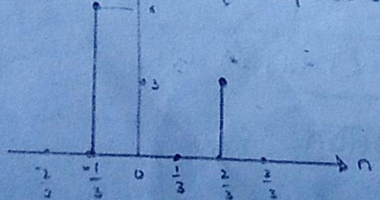
method-2: Time shifting to Time scaling.



(iii) $x[3n+1] = x[3(n+\frac{1}{3})]$

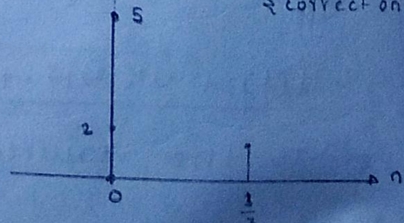
method-1: Time scaling to Time shifting.

{not possible}

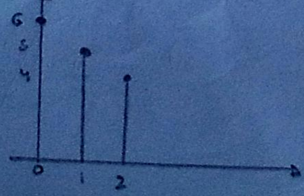


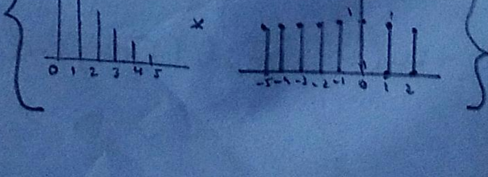
method-2: Time shifting to Time scaling.

{correct one}

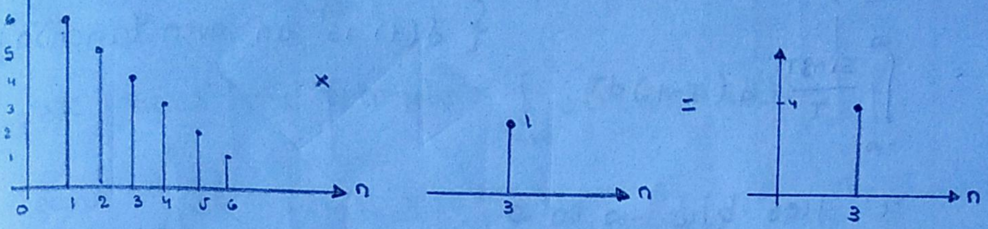


v) $x[n]\,u[2-n]$





(xi) $x[n-1]\,\delta[n-3]$



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13) we know $x(t)$ is non-zero at $-1 \le t \le 2$

$x\left(\frac{2-t}{2}\right)$ will be non-zero at

$-1 \le t \le 2$ { multiply with '-1' }

$-2 \le -t \le 1$ { Add 2 on each side }

$0 \le 2-t \le 3$ { Divide with 2 on each side }

$0 \le \frac{2-t}{2} \le \frac{3}{2}$

$x\left(\frac{2-t}{2}\right)$ is non-zero b/w 0 to $\frac{3}{2}$

so, The signal $y(t) = 2x\left(\frac{2-t}{2}\right)$ is non zero

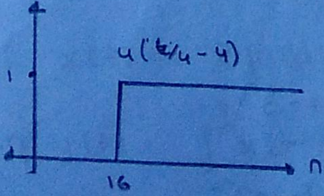
at $\boxed{0 \le t \le \frac{3}{2}}$

---

16 $u\left(\frac{1}{4}(t-16)\right)$

This signal can be written

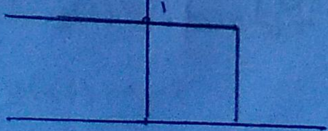
as $\boxed{u(t-16)}$ $\boxed{t_0 = 16}$



$u(-3t+6) = u(-3(t-2))$

This signal can be written

as $\boxed{u(-t+2)}$ $\boxed{t_0 = -2}$



4) $\int_{-\infty}^{\infty} \frac{\sin 3T}{T} \delta(t-T)\, dT$

$\{\delta(t)$ is an even function$\}$

$= \int_{-\infty}^{\infty} \frac{\sin 3T}{T} \delta(T-t)\, dT$

't' lies b/w $-\infty$ to ∞

so, from $\int_{t_1}^{t_2} x(t)\cdot\delta(t-t_0)\, dt = x(t_0)$ $\{$if $t_1 \le t_0 \le t_2\}$

so $\int_{-\infty}^{\infty} \frac{\sin 3T}{T} \delta(t-T)\, dT = \frac{\sin 3T}{T}$ $\{$at $T=t\}$

$= \frac{\sin 3t}{t}$

$= \boxed{\frac{1}{t}\cdot \sin 3t}$

5) $\int_{-\infty}^{\infty} \delta(1-t)\,(t^3+4)\, dt = \int_{-\infty}^{\infty} (t^3+4)\,\delta(t-1)\, dt$ $\{\delta(t)$ - even function$\}$

1 lies b/w $-\infty$ to ∞, so

$\int_{-\infty}^{\infty} (t^3+4)\,\delta(t-1)\, dt = (t^3+4)_{\text{at } t=1}$

$= (1)^3+4$

$= \boxed{5}$

7) $\int_{0}^{\infty} \cos(t-T)\,\delta(T+3)\, dT$ $\{t_0 = -3\}$

Here -3 doesn't lies b/w 0 & ∞

so, $\int_{0}^{\infty} \cos(t-T)\,\delta(T+3)\, dT = \boxed{0}$

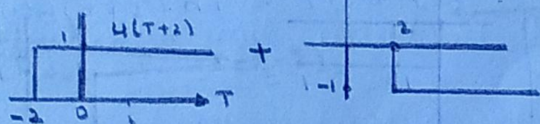
6) $\int_{-\infty}^{t} \delta(T+2)\, dT - \int_{-\infty}^{t} \delta(T-2)\, dT$

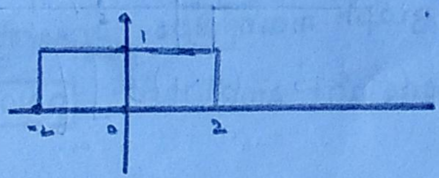
A) We have the formula $\int_{-\infty}^{t} \delta(t)\, dt = u(t)$

Now $\int_{-\infty}^{t} \delta(T+2)\, dT = u(T+2)$

Now $\int_{-\infty}^{t} \delta(T-2)\, dT = u(T-2)$

$u(T+2) - u(T-2)$ {consider $T = t$}





$\int_{-\infty}^{t} (\delta(T+2) - \delta(T-2))\, dT$ If

$t > -2,\ = 1$

$t < -2,\ = 0$

1/11/22 Assignment - 2

1) $\alpha^n u[n]$

a) $\alpha = -1$

b) $\alpha = \frac{1}{2}$

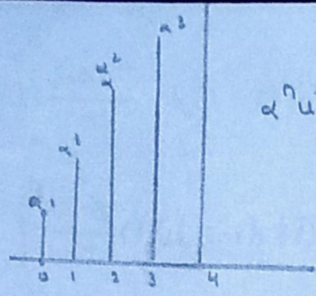
c) $\alpha = 1$

d) $\alpha = 2$

2) If $x(t)$ is equal to $\delta(t+2) - \delta(t-2)$ find energy in $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

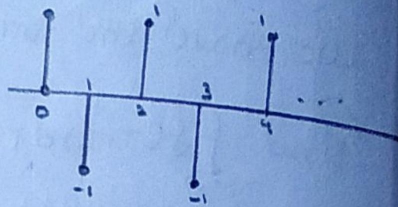
3) $u(-t) + Ae^{-t}$ is power or energy?

4) If $y(t)$'s energy is (E) what is the energy of $y(\alpha t) \rightarrow$?



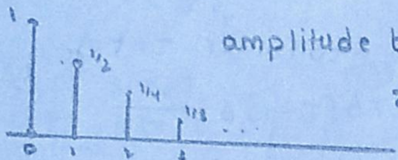
a) $\alpha = -1$

(It is a periodic function)



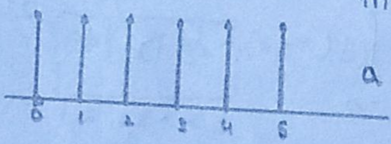
So, a is a $\boxed{\text{power}}$ $\boxed{\text{signal}}$

b) $\alpha = \frac{1}{2}$



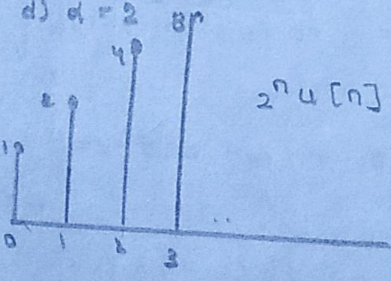
As $t \to \infty$, the amplitude becomes zero, so, it is a $\boxed{\text{energy}}$ $\boxed{\text{signal}}$

c) $\alpha = 1$



the graph maintains a constant amplitude, so it is a $\boxed{\text{power signal}}$

d) $\alpha = 2$



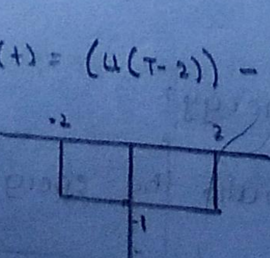
when n tends to infinity amplitude becomes infinite so, it is neither power nor energy.

2) Given $x(t) = \delta(t+2) - \delta(t-2)$, so

$$x(T) = \delta(T+2) - \delta(T-2)$$

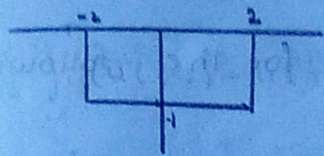
$$y(t) = \int_{-\infty}^{t} \delta(T+2)\,dT - \int_{-\infty}^{t} \delta(T-2)\,dT$$

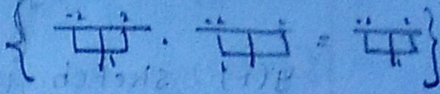
$$y(t) = (u(T-2)) - u(T+2)$$



It is an energy signal because of constant amplitude

$y(t) = u(T-2) - u(T+2)$





$$\text{Energy} = \lim_{T\to\infty} \int_{-T}^{T} y(t)^2\, dt$$

Here limits are -2 to 2

$$\text{Energy} = \int_{-2}^{2} |y(t)|^2\, dt = \int_{-2}^{2} (-1)^2\, dt = \int_{-2}^{2} dt$$

$$[t]_{-2}^{2} = 2-(-2)$$

$$\text{Energy} = \boxed{4}$$

3a $u(-t) + Ae^{-t}$

let t tends to ∞, then amplitude becomes zero, and t tends to $-\infty$, the amplitude becomes non-zero. So, it is not an energy signal.

when we substitute $-\infty$ in t, the function's amplitude becomes infinite. So

It is neither energy nor power signal

4A Given Energy of $y(t) = E$

$y(\alpha t) = \dfrac{E}{\alpha}$ and α can be negative, but

Energy can't be negative, so,

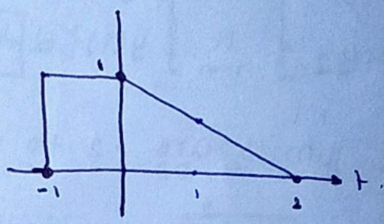
Energy of $y(\alpha t) = \dfrac{E}{|\alpha|}$.

Q. For an LTI system, an input $x(t)$ produces output $y(t)$. sketch the outputs for the following inputs

a) $5x(t)$

b) $x(t+1) - x(t-1)$

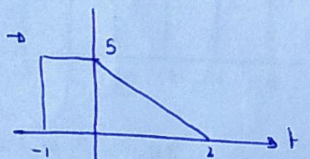
c) $\frac{d}{dt} x(t)$



a)

A. As the system is linear time invariant.

$5x(t)$ gives the output $5y(t)$



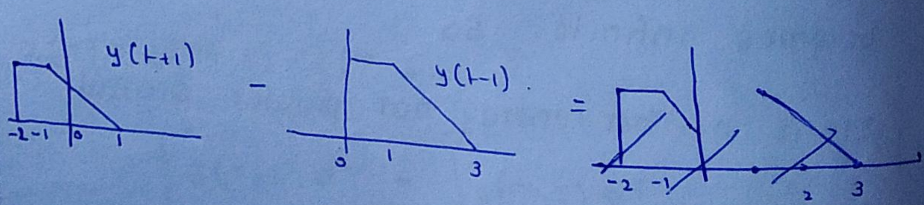
$5x(t)$ is input

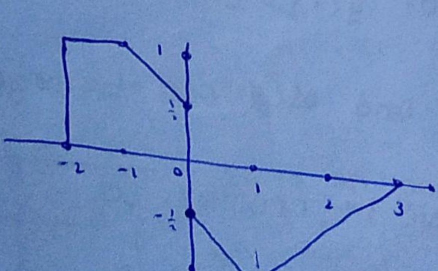
$5y(t)$ is output.

b) $x(t+1)$ gives the output $y(t+1)$

& $+x(t-1)$ gives the output $+y(t-1)$

Now $x(t+1) - x(t-1)$ gives the output $y(t+1) - y(t-1)$



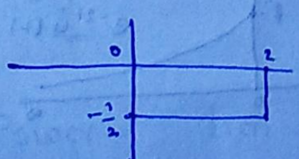


c) $\frac{d}{dt} x(t)$ gives us the output $\frac{d}{dt} y(t)$

{Because of Linear Time invariant system}.

$$y(t) = \begin{cases} 1 , & -1 \leq t \leq 0 \\ 1 - \frac{t}{2} , & 0 \leq t \leq 2 \end{cases}$$

$$\frac{dy(t)}{dt} = \begin{cases} 0 , & -1 \leq t \leq 0 \\ -\frac{1}{2} , & 0 \leq t \leq 2 \end{cases}$$



$\frac{d}{dt} y(t)$.

* Basic problems with ans

1.3 Determine the values of P_∞ and E_∞ for each of the following signals:

a) $x_1(t) = e^{-2t}u(t)$

b) $x_2(t) = e^{j(2t+\pi/4)}$

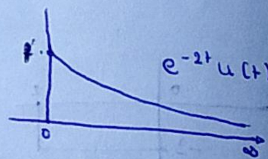
c) $x_3(t) = \cos(t)$

d) $x_1[n] = \left(\frac{1}{2}\right)^n u[n]$

e) $x_2[n] = e^{j(\pi/2n + \pi/8)}$

f) $x_3[n] = \cos\left(\frac{\pi}{4}n\right)$

* a) $x_1(t) = e^{-2t}u(t)$ Graph =



→ The limits are 0 to ∞. So,

$$\text{Energy} = \int_0^\infty [e^{-2t}u(t)]^2 dt = \int_0^\infty e^{-4t} dt$$

$$\to \frac{e^{-4t}}{-4}\Big|_0^\infty = \frac{-1}{4} e^{-4t}\Big|_0^\infty$$

& As energy is finite power = 0

$$= \frac{-1}{4}[e^{-4\infty} - e^{-4(0)}] = \frac{-1}{4}\left[\frac{1}{e^{4\infty}} - e^0\right] = \frac{1}{4}$$

$\boxed{P_\infty = 0}$ & $\boxed{E_\infty = \frac{1}{4}}$ $\left\{ P_\infty = \frac{1}{2T}\int_{-T}^{T} (x_1(t))^2 dt \quad \{T = \infty\} \right\}$

b) $x_2(t) = e^{j(2t+\pi/4)}$

$$= e^{2jt} + e^{j\pi/4}$$

c) $x_3(t) = \cos(t)$

Energy $= \lim\limits_{t\to\infty} \int_{-t}^{t} |x_3(t)|^2\, dt = \lim\limits_{t\to\infty} \int_{-t}^{t} \cos^2 t\, dt$

$= \frac{1}{2} \int_{-\infty}^{\infty} \cos 2t + 1 = \frac{1}{2}(\infty) = \infty$, $\boxed{E_\infty = \infty}$

As energy is infinite power is finite.

Power $= \lim\limits_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x_3(t)|^2\, dt$

$= \lim\limits_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \frac{1}{2}(1+\cos 2t)\, dt$

$= \lim\limits_{T\to\infty} \frac{1}{4T} \left[t + \frac{\sin 2t}{2} \right]_{-T}^{T}$

$= \lim\limits_{T\to\infty} \frac{1}{4T} \left[2T + \cancel{\frac{\sin 2T}{2}} - \cancel{\frac{\sin 2T}{2}} \right]$

$= \lim\limits_{T\to\infty} \frac{1}{2}$

$= \frac{1}{2}$ $\boxed{P_\infty = \frac{1}{2}}$

d) $x_1[n] = \left[\frac{1}{2}\right]^n u[n]$ {u[n] is defined from 0}

Energy $= \lim\limits_{N\to\infty} \sum_{n=-N}^{N} |x[n]|^2 = \lim\limits_{N\to\infty} \sum_{n=-N}^{N} \left[\frac{1}{2}\right]^{2n}$

$= \lim\limits_{N\to\infty} \sum_{n=-N}^{N} \left[\frac{1}{4}\right]^n = \lim\limits_{N\to\infty} \sum_{n=0}^{N} \left[\frac{1}{4}\right]^n = \left(\frac{1}{4}\right)^0 + \left(\frac{1}{4}\right)^1 + \left(\frac{1}{4}\right)^2 + \dots \left(\frac{1}{4}\right)^\infty$

It is in G.P $a = 1$, $r = \frac{1}{4}$ & numerator is in AP $d = 0$

$$S_\infty = \frac{a}{1-r} \times \frac{dr}{(1-r)^2}$$

$$S_\infty = \frac{1}{1-\frac{1}{4}} \times \frac{0\left(\frac{1}{4}\right)}{\left(1-\frac{1}{4}\right)^2}$$

$$S_\infty = \frac{1}{\frac{3}{4}}$$

$S_\infty = \frac{4}{3}$ $\therefore$ Energy $\boxed{E_\infty = \frac{4}{3}}$

As energy is finite, power = 0 $\boxed{P_\infty = 0}$

e) $x[n] = e^{j\left[\frac{\pi}{2}n + \frac{\pi}{8}\right]}$

f) $x_3[n] = \cos(\frac{\pi}{4} n)$

* Energy $E = \lim_{t \to \infty} \int_{-t}^{t} |\cos^2(\frac{\pi}{4} n)| \, dt$

$$E_\infty = \lim_{N \to \infty} \sum_{n=-N}^{N} |\cos^2(\tfrac{\pi}{4} n)| = \sum_{n=-\infty}^{\infty} \cos^2(\tfrac{\pi}{4} n)$$

becomes infinity $\rightarrow$ $\boxed{E_\infty = \infty}$

* Energy is infinite so power is finite.

$$\text{power} = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

$$\lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left(\frac{1 + \cos(\frac{\pi}{2} n)}{2} \right) \neq \frac{1}{2}$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \left(\frac{1}{2} \right) \sum_{n=-N}^{N} 1 + \cos(\tfrac{\pi}{2} n)$$

1.4 Let $x[n]$ be a signal with $x[n]=0$ for $n<-2$ and $n>4$. For each signal given below, determine the values of n for which it is guaranteed to be zero a) $x[n-3]$ b) $x[n+4]$ c) $x[-n]$ d) $x[-n+2]$ e) $x[-n-2]$

* a) $x[n-3]$ ie $x[n]$ is right shifted 3 units

So limits should also be shifted

$n < -2+3 \rightarrow \boxed{n < +1}$

$n > 4+3 \rightarrow \boxed{n > 7}$

* b) $x[n+4]$ i.e $x[n]$ is left shifted 4 units.

So limits should also be shifted

$n < -2-4 \rightarrow \boxed{n < -6}$

$n > 4-4 \rightarrow \boxed{n > 0}$

* c) $x[-n]$ ie $x[n]$ is reversed by n.

So limits should be revesed

$n < -2 \rightarrow \boxed{n > 2}$

$n > 4 \rightarrow \boxed{n < -4}$

* e) $x[-n-2]$ ie $x[-n]$ is shifted left ~~left~~ right 2 units

$[-(n+2)]$

So limits should be ~~reversed~~ shifted

$n \geq 2-2$ $\quad$ $n \geq 0$ $\quad$ $n > 0$

$n \leq -4-2$ $\quad$ $n \geq -6$ $\quad$ $n < -6$

d) $x[-n+2]$ i.e $x[n]$ is ~~left~~ shifted 2 units. right

$x[-(n-2)]$

so limits should be shifted

$n > 2+2$ $\quad$ $\boxed{n > 4}$

$n < -4+2$ $\quad$ $\boxed{n < -2}$

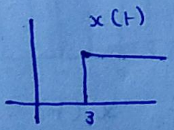
1.5: Let $x(t)$ be a signal with $x(t) = 0$ for $t < 3$.

For each signal given below, determine the values of t for which it is gauranteed to be zero.

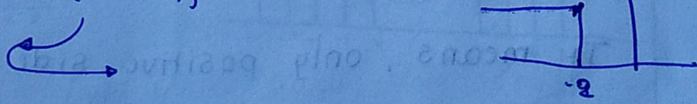
a) $x(1-t)$ $\quad$ b) $x(1-t) + x(2-t)$ $\quad$ c) $x(1-t)\,x(2-t)$

d) $x(3t)$ $\quad$ e) $x(t/3)$

A Let $x(t)$ be $u(t-3)$, which satisfies the signal.

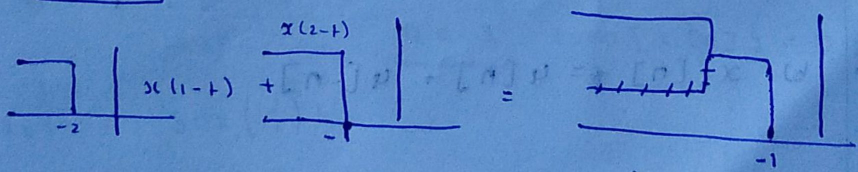


a) $x(-(t-1)) = x(-t+1)$



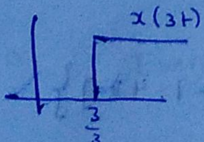
i.e $\boxed{t > -2}$, $x(1-t) = 0$

b)

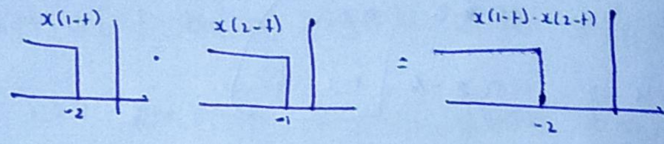


i.e $\boxed{t > -1}$, $x(1-t) + x(2-t) = 0$.

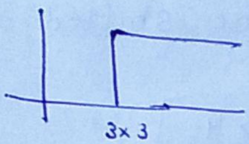
d)



$= \boxed{t < 1}$, $x(3t) = 0$

c) 

$\boxed{t > -2}$ then $x(1-t) \cdot x(2-t)$ becomes zero

e) $x(t/3)$ 

$\boxed{t < 9}$, $x(t/3)$ becomes 0

1.6 Determine whether or not each of the following signals is periodic.

a) $x_1(t) = 2e^{j(t+\pi/4)}u(t)$

b) $x_2[n] = u[n] + u[-n]$

c) $x_3[n] = \sum_{k=-\infty}^{\infty} \{\delta[n-4k] - \delta[n-1-4k]\}$

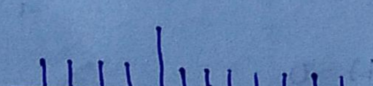
→ a) $x_1(t) = 2e^{j(t+\pi/4)}u(t)$.

$x_1(t) = 2 \cdot e^{jt} \cdot e^{j\frac{\pi}{4}} \cdot u(t)$

$x_1(t) = 2(\cos t + j\sin t) \cdot e^{j\frac{\pi}{4}} u(t)$

It means, only positive side of $x_1(t)$, when multiplied with $u(t)$. so it is aperiodic signal.

→ b) $x_2[n] = u[n] + u[-n]$



It is aperiodic signal.

→ c) $x_3[n] = \sum_{k=-\infty}^{\infty} \{\delta[n-4k] - \delta[n-1-4k]\}$.

already done in class.

1.7 For each signal given below, determine all the values of the independent variable at which the even part of the signal is guaranteed to be zero.

a) $x_1[n] = u[n] - u[n-4]$

b) $x_2(t) = \sin(\frac{1}{2}t)$

c) $x_3[n] = (\frac{1}{2})^n u[n-3]$

d) $x_4(t) = e^{-5t} u(t+2)$.

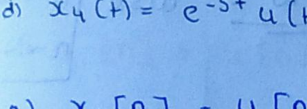
a) $x_1[n] = u[n] - u[n-4]$

Even part is $\frac{1}{2}\{x_1[n] + x_1[-n]\}$

$= \frac{1}{2}\{u[n] - u[n-4] + u[-n] - u[-n-4]\}$

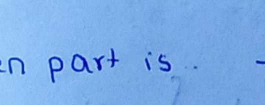
$= \frac{1}{2}\{u[n] + u[-n] - (u[n-4] + u[-n-4])\}$

$u[n] + u[-n]$



$-$

$\rightarrow$



$n > 3$, $\text{evn}\{x_1[n]\} = 0$

$n < -3$, $\text{even}\{x_1[n]\} = 0$

b) $x_2(t) = \sin(\frac{1}{2})t$

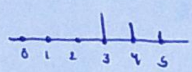
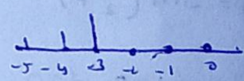
Even part, $\frac{1}{2}[x_2(t) + x_2(t)] = \frac{1}{2}[\sin(\frac{1}{2}t) - \sin(\frac{1}{2}t)]$

$= 0$, Even of $x_2(t)$ is 0 at $-\infty$ to ∞

c) $x_3[n] = \left(\frac{1}{2}\right)^n u[n-3]$

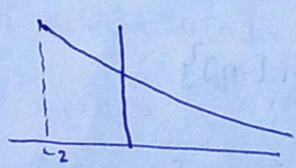
$\rightarrow$ Even $= \frac{1}{2}\left[x_3[n] + x_3[-n]\right]$

$= \frac{1}{2}\left[\left(\frac{1}{2}\right)^n u[n-3] + \left(\frac{1}{2}\right)^{-n} u[-n-3]\right]$

$= \frac{1}{2}\Bigg[$  $+$  $\Bigg]$

= Even part is 0 at $\underline{-2 \text{ to } 2}$ also $\boxed{\begin{matrix} n \to \infty \\ n \to -\infty \end{matrix}}$

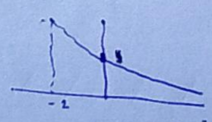
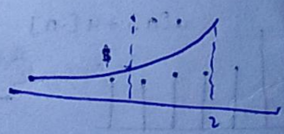
d) $x_4(t) = e^{-st} u(t+2)$



$t/c + 2$,

even $= \frac{1}{2}\left\{e^{-st} u(t+2) + e^{st} u(-t+2)\right\}$

$= \frac{1}{2}\left\{e^{-st} u(t+2) + e^{st} u(-t+2)\right\}$

$= \frac{1}{2}\Bigg[$  $+$  $\Bigg]$

It is even when only $\underline{t \to \infty}$ & $\underline{t \to -\infty}$

Basic problems on unit - 2

1) Let $x[n] = \delta[n] + 2\delta[n-1] - \delta[n-3]$ and

$h[n] = 2\delta[n+1] + 2\delta[n-1]$

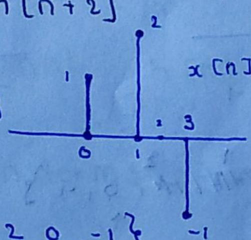
compute the plot each of the following convolutions

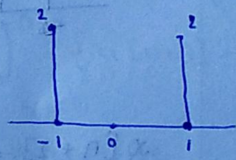
a) $y_1[n] = x[n] * h[n]$

b) $y_2[n] = x[n+2] * h[n]$

c) $y_3[n] = x[n] * h[n+2]$

A Graph $x[n]$ & $h[n]$





Writing $x[n] = \{\underset{\uparrow}{1}, 2, 0, -1\}$

and $h[n] = \{2 \ \underset{\uparrow}{0} \ 2\}$

a) $y_1[n] = x[n] * h[n]$

$x[n] = \{0, \underset{\uparrow}{1}, 2, 0, -1\}$

$h[n] = \{2, \underset{\uparrow}{0}, 2, 0, 0\}$

$y_1[n] =$

	0	1	2	0	-1
2	0	2	4	0	-2
0	0	0	0	0	0
2	0	2	4	0	-2
0	0	0	0	0	0
0	0	0	0	0	0

$\{0, 2, 4, 2, 2, 0, -2, 0, 0\}$

$y_1[n] =$

$\{2, \underset{\uparrow}{4}, 2, 2, 0, -2\}$

$y_1[n] =$ 

$\rightarrow 2\delta[n+1] + 4\delta[n] + 2\delta[n-1] + 2\delta[n-2] - 2\delta[n-4]$

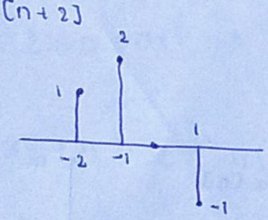
b) $y_2[n] = x[n+2] * h[n]$

$x[n+2] = \delta[n+2] + 2\delta[n+2-1] - \delta[n+2-3]$

$x[n+2] = \delta[n+2] + 2\delta[n+1] - \delta[n-1]$

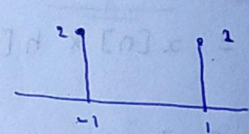
$h[n] = 2\delta[n+1] + 2\delta[n-1]$

$x[n+2]$



&

$h[n]$



$x[n+2] = \{ 1, 2, 0, -1 \}$ (↑ at 0)

$h[n] = \{ 2, 0, 2 \} = \{ 0, 2, 0, 2 \}$

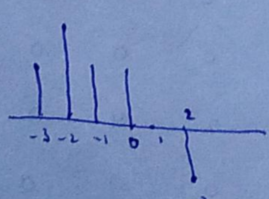
$y_2[n]$

	1	2	0	-1
0	0	0	0	0
2	2	4	0	-2
0	0	0	0	0
2	2	4	0	-2

$= \{ \cancel{0}, -4, \cancel{4}, \cancel{4}, 0, 0, 2 \}$

$= \{ 0, 2, 4, 2, 2, 0, -2 \}$ (↑ at 4)

$= \{ 2, 4, 2, 2, 0, -2 \}$ (↑ at 2)



$y_2[n]$

→ Here we observe $y_2[n] = y_1[n+2]$

$\therefore\ y_2[n] = y_1[n+2]$

c) $y_3[n] = x[n] * h[n+2]$

we get $y_3[n] = y_1[n+2]$, because, we have

formulas.. as (Time Shifting property)

when $x[n] * h[n] = y[n]$, then

$x[n+n_0] * h[n] = y[n+n_0]$

$x[n] * h[n+n_0] = y[n+n_0]$

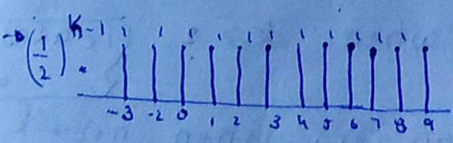
So, $y_3[n] = y_1[n+2]$

2 Consider the signal $h[n] = \left(\frac{1}{2}\right)^{n-1} \{u[n+3] - u[n-10]\}$

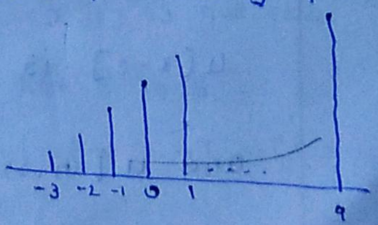
Express A and B in terms of n so that the following equation holds:

$$h[n-k] = \begin{cases} \left(\frac{1}{2}\right)^{n-k-1}, & A \le k \le B \\ 0, & ew \end{cases}$$

$h[k] = \left[\frac{1}{2}\right]^{k-1} \{u[k+3] - u[k-10]\}$, draw graph



= $h[k]$



$h[n-k]$

$h[k]$ is

Here $h[k]$ is valid

for the interval $-3 \le k \le 9$, so $h[-k]$ is valid for the interval $-9 \le -k \le 3$, and

$h[n-k]$ is valid for interval $n-9 \le n-k \le n+3$

So, $\underline{A = n-9}$, $\underline{B = n+3}$

2.3 Consider an input $x[n]$ and a unit impulse response $h[n]$ given by $x[n] = \left(\frac{1}{2}\right)^{n-2} u[n-2]$

$h[n] = u[n+2]$

Determine ~~thre~~ and plot the output $y[n] = x[n] * h[n]$

$x[k] = \left(\frac{1}{2}\right)^{k-2} u[k-2]$

$h[n-k] = u[n-k+2]$

$y[n] = x[n] * h[n]$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$$y[n] = \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^{k-2} u[k-2]\; u[n-k+2]$$

$$y[n] = \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^{k-2} u[k-2]\; u[n-k+2]$$

$u[k-2]$ is non zero when $k \geq 2$ $\left\{ \text{i.e. } u[n] \text{ is nonzero of } 2 \text{ to } n+2 \right.$

and $u[(n+2)-k]$ is non zero when $n+2 > k$

$$y[n] = \sum_{k=2}^{n+2} \left(\frac{1}{2}\right)^{k-2} \cdot u[n]$$

$$y[n] = \left\{ \left(\frac{1}{2}\right)^0 + \left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^2 + \dots \left(\frac{1}{2}\right)^n \right\} u[n]$$

$$y[n] = \left\{ 1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \dots \left(\frac{1}{2}\right)^n \right\} u[n]$$ for n

Sum of n terms in GP = $S_n = \frac{a(1-r^n)}{1-r}$ $r<1$

Here $r = \frac{1}{2}$ & $a = 1$

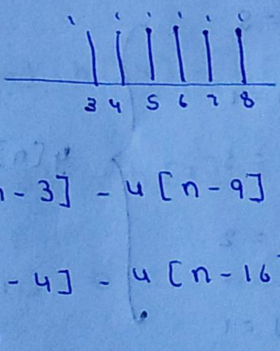
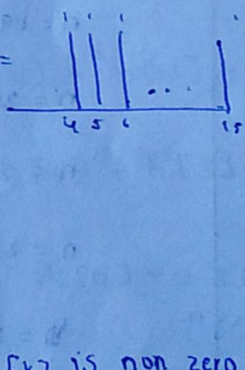
$$y[n] = \frac{1\left(1-\left(\frac{1}{2}\right)^n\right)}{1-\frac{1}{2}}$$

$$y[n] = 2\left[1-\left(\frac{1}{2}\right)^n\right]$$

2.4 Compute and plot $y[n] = x[n] * h[n]$, when

$$x[n] = \begin{cases} 1, & 3 \le n \le 8 \\ 0 & \text{ew} \end{cases}$$

$$h[n] = \begin{cases} 1, & 4 \le n \le 15 \\ 0 & \text{ew} \end{cases}$$

A $x[n] =$  & $h[n] =$ 

$$x[n] = u[n-3] - u[n-9]$$

$$h[n] = u[n-4] - u[n-16]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$ $x[k]$ is non zero from 3 to 8

$$y[n] = \sum_{k=-\infty}^{\infty} (u[k-3] - u[k-9])(u[n-k-4] - u[n-k-16])$$

$$y[n] = \sum_{k=-\infty}^{\infty} \Big[u[k-3]\,u[n-k-4] - u[k-3]\,u[n-k-16]$$
$$- u[k-9]\,u[n-k-4] + u[k-9]\,u[n-k-16]\Big]$$

$$y[n] = \sum_{k=3}^{8} x[k]\, h[n-k]$$

$= x[3]\,h[n-3] + x[4]\,h[n-4] + x[5]\,h[n-5] +$

$x[6]\,h[n-6] + x[7]\,h[n-7] + x[8]\,h[n-8]$

$= h[n-3] + h[n-4] + h[n-5] + h[n-6] + h[n-7] + h[n-8]$

$\{x[n] = 1, \; 3 \le n \le 8\}$

If $n = 7, \; y[n] = 1$

$n = 8, \; y[n] = 2$

$n = 9, \; y[n] = 3$ $\qquad y[n] = n-6, \quad 7 \le n \le 11$

$n = 10, \; y[n] = 4$

$n = 11, \; y[n] = 5$

$n = 12, \; y[n] = 6$

$n = 13, \; y[n] = 6$ $\qquad y[n] = 6, \quad 12 \le n \le 18$

.

$n = 18, \; y[n] = 6$

$n = 19, \; y[n] = 5$

$n = 20, \; y[n] = 4$

$n = 21, \; y[n] = 3$ $\qquad y[n] = 24-n \quad 19 \le n \le 23$

$n = 22, \; y[n] = 2$

$n = 23, \; y[n] = 1$

$n = 24, \; y[n] = 0$

$$y[n] = \begin{cases} n-6, & 7 \le n \le 11 \\ 6, & 12 \le n \le 18 \\ 24-n, & 19 \le n \le 23 \\ 0, & \text{ew} \end{cases}$$

2.5 Let $x[n] = \begin{cases} 1, & 0 \le n \le 9 \\ 0, & \text{else where} \end{cases}$ & $h[n] = \begin{cases} 1, & 0 \le n \le N \\ 0, & \text{ew} \end{cases}$

where $N \le 9$ is an integer. Determine the value of

N given that, $y[n] = x[n] * h[n]$ and $y[4]=5$

& $y[14] = 0$

$y[n] = x[n] * h[n]$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k], \quad x[k] \text{ is non zero for } 0 \le k \le 9$$

$$y[n] = \sum_{k=0}^{9} x[k]\, h[n-k]$$

$$y[n] = h[n] + h[n-1] + h[n-2] + h[n-3] + \cdots h[n-9]$$

$$y[4] = h[4] + h[3] + h[2] + h[1] + h[0]$$

$= 5$,,

$$y[14] = h[14] + h[13] + h[12] + h[11] + h[10]$$

$$+ h[9] + h[8] + h[9] + h[7] + h[6] + h[5] = 0$$

i.e $h[n] = 0$ If $n \geq 9$, Here $h[n] = 0$ If $n \geq 5$

& $h[n]$ is non zero for $0 \le n \le 4$

so $\boxed{N=4}$

Let compute and plot the convolution $y[n] = x[n] * h[n]$, where $x[n] = \left(\frac{1}{3}\right)^{-n} u[-n-1]$ and $h[n] = u[n-1]$

$y[n] = x[n] * h[n]$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$$y[n] = \sum_{k=-\infty}^{\infty}\left(\left(\frac{1}{3}\right)^{-k} u[-k-1]\right) u[n-k-1]$$

$y[n]$ = Here $u[-k-1]$ is non zero for $k \le -1$

& $u[n-1-k]$ is non zero for $k > (n-1)$

$$y[n] = \sum_{n-1}^{-1}\left(\frac{1}{3}\right)^{-k} u[n-k+1]$$

$$y[n] = \left(\frac{1}{3}\right)^{1-n} + \left(\frac{1}{3}\right)^{2-n} + \cdots \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{3}\right)^{-1}$$

$$y[n] = 3 + 3^2 + \cdots 3^{n-2} + 3^{n-1}$$

$$y(n) = \frac{a(r^n - 1)}{r-1} = \frac{3(3^n - 1)}{3-1} - 3^n$$

upton $(n+1)$ terms only

$$y[n] = \frac{3 \cdot 3^n - 3}{2} - 3^n$$

$$y[n] = \frac{3 \cdot 3^n - 3 - 2 \cdot 3^n}{2}$$

$$y[n] = \frac{3^n - 3}{2}$$

2.7 A linear system S has the relationship

$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, g[n-2k]$, between its input

$x[n]$ and its output $y[n]$, where $g[n] =$

$u[n] - u[n-4]$

a) Determine $y[n]$ when $x[n] = \delta[n-1]$

b) Determine $y[n]$ when $x[n] = \delta[n-2]$

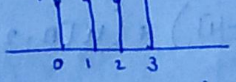
c) IS S LTI?

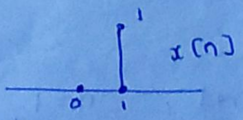
d) Determine $y[n]$ when $x[n] = u[n]$

Given $y[n] = \sum_{k=-\infty}^{\infty} x[k]\, g[n-2k] = x[n] * g[n-k]$

and $g[n] = u[n] - u[n-4]$

$\rightarrow \sum_{k=-\infty}^{\infty} x[k]\left(u[n-2k] - u[n-2k-4]\right)$

Here, $g[n] =$ 

a) If $x[n] = \delta[n-1]$ 

y $x[n] = \{\underset{\uparrow}{0}, 1\}$

$\rightarrow \sum_{k=-\infty}^{\infty} g[n-2k]\, \delta[k-1]$

we have the formula $\int_{t_1}^{t_2} x(t)\, \delta(t-t_0) = x(t_0)$

similarly $\sum_{k=-\infty}^{\infty} g[n]\, \delta[n-n_0] = g[n_0]$

$\cdot \sum_{k=-\infty}^{\infty} g[n-2k]\, \delta[k-1] = g[n-2]$

$= u[n-2] - u[n-4-2]$

$= \underline{u[n-2] - u[n-6]}$ //

b) If $x[n] = \delta[n-2]$

$\rightarrow \sum_{k=-\infty}^{\infty} g[n-2k]\, \delta[k-2] = g[n-2(2)] = g[n-4]$

$= \underline{u[n-4] - u[n-8]}$ //

2-8

d) If $x[n] = u[n]$

$$\rightarrow \sum_{k=-\infty}^{\infty} u[k]\, g[n-2k]$$

$$= \sum_{k=0}^{\infty} g[n-2k] = \sum_{k=0}^{\infty} \cancel{u[n] + u[n-4]}\ u[n-2k] - u[n-2k-4]$$

$$= g[n-0] + g[n-2] + g[n-4] + \cdots$$

$$= (u[n] + u[n-4]) + (u[n-2] + u[n-6]) + (u[n-4] + u[n-8])$$

$+ \cdots$

$\rightarrow n=0 \rightarrow 1,\quad n=1 \rightarrow 1,\quad n=2 \rightarrow 2,\quad n=3 \rightarrow 2$

$n=4 \rightarrow 2,\quad n=5 \rightarrow 2 \quad + n=6 \rightarrow 2 \ \cdots$

$$y[n] = \begin{cases} 1, & n=0,1 \\ 2 & n>1 \end{cases}$$

c) Here $x[n]$ is input & $y[n]$ is output.

Here $y[n] = \sum_{k=-\infty}^{\infty} x[k]\, g[n-2k] = x[n] * g[n-k]$

$y[n] = x[n] * g[n-k]$

$\cancel{y_2[n]}$ & $\cancel{x}$ $y_1[n] = x[n-n_0] * g[n-k] \rightarrow ①$

$y_2[n] = x[n-n_0] * g[n-n_0-k] \rightarrow ②$

Here ① & ② are not equal

So the system is Time variant.

Determine and sketch the convolution of the following two signals: $x(t) = \begin{cases} t+1 & 0 \le t \le 1 \\ 2-t & 1 < t \le 2 \\ 0 & \text{elsewere} \end{cases}$

& $h(t) = \delta(t+2) + 2\delta(t+1)$

We have to calculate $x(t) * h(t)$ -

$\rightarrow \int_{-\infty}^{\infty} x(t) \left(\delta(t+2) + 2\delta(t+1) \right) d$

= we know $x(t) * h(t) = h(t) * x(t)$

$= h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$

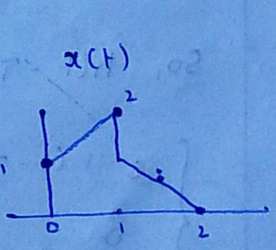
$= \int_{-\infty}^{\infty} [\delta(\tau+2) + 2\delta(\tau+1)]\, x(t-\tau)\, d\tau$

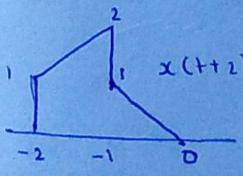
$= \int_{-\infty}^{\infty} x(t-\tau)\, \delta(\tau+2)\, d\tau + 2\int_{-\infty}^{\infty} x(t-\tau)\, \delta(\tau+1)\, d\tau$

$\left\{ \text{we know } \int_{t_1}^{t_2} x(t)\, \delta(t-t_0)\, dt = x(t_0) \right\}$

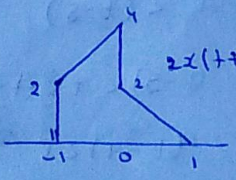
$= x(t-(-2)) + 2x(t-(-1))$

$= x(t+2) + 2x(t+1)$





+



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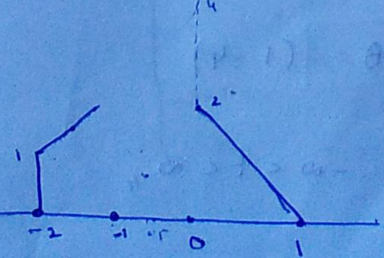
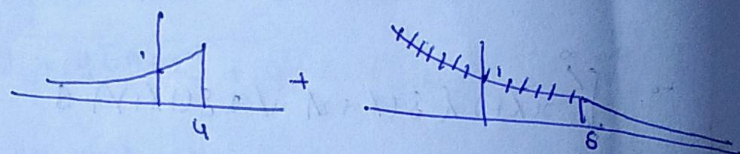


2.9 2.9 : Let $h(t) = e^{2t}u(-t+4) + e^{-2t}u(t-5)$ 2.10

Determine A & B such that

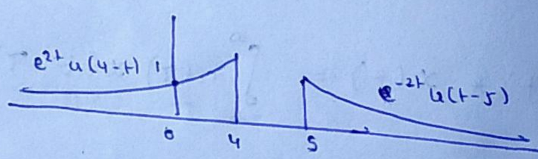
$$h(t-\tau) = \begin{cases} e^{-2(t-\tau)} & , \tau < A \\ 0 & , A < \tau < B \\ e^{2(t-\tau)} & , B < \tau \end{cases}$$

A $h(t) =$ A

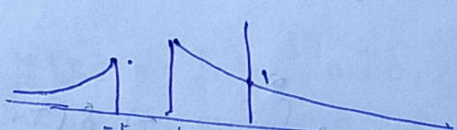


i.e $h(t)$ is not zero for $-\infty$ to ∞

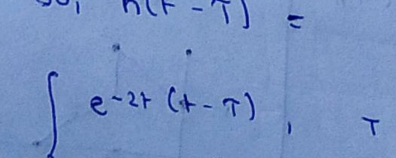
$h(t) =$



$h(\tau) =$ & $h(-\tau)$



Now draw $h(t-\tau)$



Now we can write

If $\tau < t-5$, we get $e^{-2t(t-\tau)}$

If $\tau > t-4$, we get $e^{2(t-\tau)}$

So, $h(t-\tau) =$

$$\begin{cases} e^{-2t(t-\tau)} & , \tau < (t-5) \\ 0 & , (t-5) < \tau < (t-4) \\ e^{2(t-\tau)} & , (t-4) < \tau \end{cases}$$

$A = (t-5)$ and $B = (t-4)$

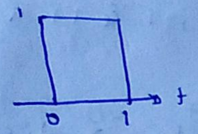
Hence, where $-\infty < t < \infty$

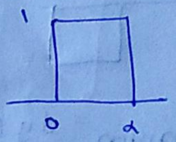
Suppose that

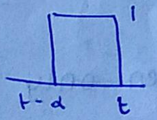
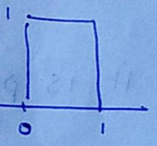
$$x(t) = \begin{cases} 1, & 0 \le t \le 1 \\ 0, & \text{elsewhere} \end{cases} \quad \text{and } h(t) = x\left(\frac{t}{\alpha}\right), \text{ where } 0 < \alpha \le 1$$

a) Determine and sketch $y(t) = x(t) * h(t)$.

b) $\frac{dy(t)}{dt}$ contains only three discontinuities, what is the value of α?

$x(t) =$ 

$h(t) =$ 

$h(t-\tau) =$  & $x(\tau)$ 

multiplication..

$y(t) = 0$ at $t < 0$

$y(t$ If $t > 0$, so that limits are 0 to t

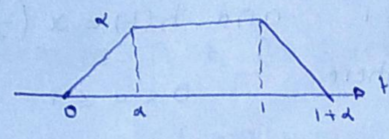
$$\int_0^t x(\tau)\, h(t-\tau)\, d\tau = (\tau)_0^t = t \rightarrow (1)$$

Now limits are.. $\int_{t-\alpha}^{t} 1\, d\tau = [\tau]_{t-\alpha}^{t} = t - t + \alpha = \alpha \rightarrow (2)$

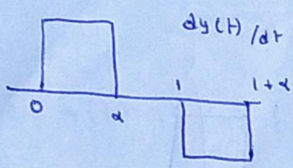
Now limits are $\int_{t-\alpha}^{1} 1\, d\tau = [\tau]_{t-\alpha}^{1} = 1 - t + \alpha \rightarrow (3)$

So, $$y(t) = \begin{cases} 0, & t < 0 \\ t, & 0 \le t \le \alpha \\ \alpha, & \alpha \le t \le 1 \\ 1-t+\alpha, & 1 \le t \le \alpha+1 \\ 0, & t > \alpha+1 \end{cases}$$

$y(t)$



$$\frac{dy(t)}{dt} = \begin{cases} 1 & 0 \le t \le \alpha \\ 0 & \alpha \le t \le 1 \\ -1 & 1 \le t \le (1+\alpha) \\ 0 & ew \end{cases}$$



the signal discontinued at

$0, \alpha, 1$ & $1+\alpha$

But, given $\frac{dy(t)}{dt}$ has only 3 discontinuties

it is possible when only $\boxed{\alpha = 1}$

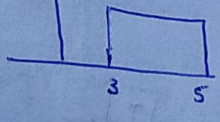
2.11 Let $x(t) = u(t-3) - u(t-5)$ and $h(t) = e^{-3t}u(t)$

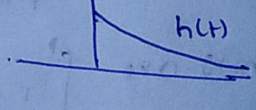
a) compute $y(t) = x(t) * h(t)$

b) compute $g(t) = \left[\frac{d}{dt} x(t)\right] * h(t)$

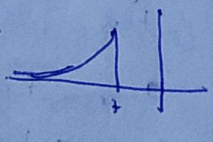
c) How is $g(t)$ related to $y(t)$

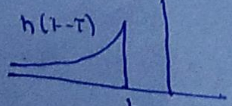
A $x(t) =$

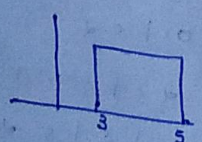




→ $x(\tau) * h(t-\tau) =$







→ multiplication.

$t < 3, \quad y(t) = 0$

$t > 3$, so limits are 3 to t

$$\int_3^t x(\tau)\, h(t-\tau)\, d\tau = \int_3^t 1\, e^{-3(t-\tau)}\, u(t-\tau)\, d\tau$$

$$= \int_3^t e^{-3(t-\tau)}\, d\tau \qquad \{ \text{put } t-\tau = u, \ d\tau = -du \}$$

$$\to t-3 \to 0$$

$$= -\int_{t-3}^{0} e^{-3u}\, du$$

$$= \int_0^{t-3} e^{-3u}\, du = \left. \frac{e^{-3u}}{-3} \right|_0^{t-3} = \frac{e^{-3(t-3)}}{-3} + \frac{e^0}{3}$$

$$= \frac{1 - e^{-3(t-3)}}{3}$$

= Now limits 3 to 5

$$= \int_3^5 e^{-3(t-\tau)}\, d\tau \qquad \left\{ \begin{array}{l} \text{put } t-\tau = m \\ t-3 \ \& \ t-5 \end{array} \right\}$$

$$= -\int_{t-3}^{t-5} e^{-3m}\, dm = \int_{t-5}^{t-3} e^{-3m}\, dm = \left. \frac{e^{-3m}}{-3} \right|_{t-5}^{t-3}$$

$$= \frac{e^{-3(t-3)}}{-3} + \frac{e^{-3(t-5)}}{3}$$

$$= \frac{e^{-3(t-5)} - e^{-3(t-3)}}{3} = \frac{e^{-3(t-5)}\,(1 - e^{-6})}{3}$$

$$y(t) = \begin{cases} \dfrac{1 - e^{-3(t-3)}}{3}, & 0 \le t \le t-3 \\[2ex] \dfrac{e^{-3(t-5)}\,(1 - e^{-6})}{3}, & t-3 < t < t-5 \end{cases}$$

We have a convolution property that

If $y(t) = x(t) * h(t)$

then $\frac{d\,x(t)}{dt} * h(t) = \frac{d}{dt} y(t)$

So $\frac{d}{dt} x(t) * h(t) = g(t) = \frac{d}{dt} y(t)$

c) $g(t) = \frac{d}{dt} y(t)$

b) $g(t) = \begin{cases} \frac{d}{dt}\left[\frac{1-e^{-3(t-3)}}{3}\right] & , 0 \le t \le t-3 \\ \frac{d}{dt}\left[\frac{e^{-3(t-5)}(1-e^{-6})}{3}\right] & , t-3 \le t \le t-5 \end{cases}$

$g(t) = \begin{cases} \frac{-1}{3}\left[-3e^{-3(t-3)}\right] & , 0 \le t \le t-3 \\ (e^{-6}-1)\,e^{-3(t-5)} & , t-3 \le t \le t-5 \end{cases}$

$g(t) = \begin{cases} e^{-3(t-3)} & 0, \le t \le t-3 \\ (e^{-6}-1)\,e^{-3(t-5)} & t-3 \le t \le t-5 \end{cases}$

2.12 Let $y(t) = e^{-t}u(t) * \sum_{k=-\infty}^{\infty} \delta(t-3k)$

Show that $y(t) = Ae^{-t}$ for $0 \le t < 3$, and determine the value of A.

A $y(t) = \dots + e^{-(t+6)}u(t+6) + e^{-(t+3)}u(t+3) + e^{-t}u(t)$

$+ e^{-(t-3)}u(t-3) + e^{-(t-6)}u(t-6) + \dots$ Here $t<3$ so $u(t-3)$ & higher terms become zero

$= e^{-t} + e^{-(t+3)} + e^{-(t+6)} + \dots$

$= e^{-t}(1 + e^{-3} + e^{-6} + \dots)$

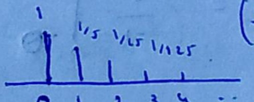
$= e^{-t}\,\frac{1}{\;}$ {sum of GP upto ∞ terms} so $\boxed{A = \dfrac{1}{1-e^{-3}}}$ $\left\{ S_\infty = \dfrac{a}{1-r} \times \dfrac{dr}{(1-r)^2} \right.$

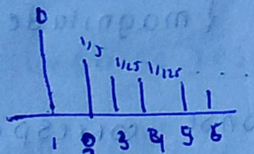
$\left. a = 1,\ r = e^{-3}\ \&\ d = 0 \right\}$

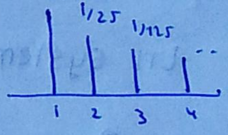
Consider a discrete-time system S_1 with impulse response.. $h[n] = \left(\frac{1}{5}\right)^n u[n]$

a) Find the integer A such that $h[n] - A h[n-1] = \delta[n]$

b) Using the result from part (a), determine the impulse response $g[n]$ of an LTI system S_2 which is the inverse system of S_1

a) $h[n] =$  $\left(\frac{1}{5}\right)^n u[n]$

Now  $= \left(\frac{1}{5}\right)^{n} K[n-1] = h[n-1]$

multiply with $\frac{1}{5}$ $= \frac{1}{5} h[n-1] =$ 

now $h[n] - \frac{1}{5} h[n-1]$ gives $\delta[n]$

so $\boxed{A = \frac{1}{5}}$

b) → we know $h[n] - \frac{1}{5} h[n-1] = \delta[n]$

from invertibility $h[n] * h^{-1}[n] = \delta[n]$

given $h^{-1}[n] = g[n]$

2.14 which of the following impulse responses corresponds to stable LTI systems?

a) $h_1(t) = e^{-(1-2j)t}u(t)$

b) $h_2(t) = e^{-t}\cos(2t)u(t)$

* If $\int_{-\infty}^{\infty} |h(t)|dt$ is finite, then the impulse response function corresponds to stable LTI system.

a) $\int_{-\infty}^{\infty} |h_1(t)| = \int_{0}^{\infty} e^{-t} dt = \frac{e^{-t}}{-1}\Big|_{0}^{\infty} = -[e^{-\infty} - e^{0}]$

$-[0-1] = 1$ = finite {magnitude of $e^{-(1-2j)t} = e^{-t}$}

So, this impulse response corresponds to stable LTI system.

b) $\int_{-\infty}^{\infty} |h_2(t)| dt = \int_{0}^{\infty} e^{-t}\cos(2t)dt$

This cannot be infinite is because cosine function lies between 0 to 1 only and e^{-t} is decreasing so value of $\int_{0}^{\infty} e^{-t}\cos(2t)dt$ is finite.

So, this impulse response corresponds to stable LTI system.

2.15 which of the following impulses responses corresponds to stable LTI systems.

a) $h_1[n] = n\cos\left(\frac{\pi}{4}n\right)u[n]$

b) $h_2[n] = 3^n u[-n+10]$

A If $\sum_{k=-\infty}^{\infty} |h[k]|$ is finite, then the impulse response corresponds to stable LTI system.

a) $\sum_{k=-\infty}^{\infty} k\cos\left(\frac{\pi}{4}k\right)u[k]$ $\{k>0\}$

$= \sum_{k=0}^{\infty} k\cos\left(\frac{\pi}{4}k\right)$ = infinite, bcoz k is increasing to infinity

so, the impulse response do not corresponds to stable LTI system.

b) $\sum_{k=-\infty}^{\infty} 3^k u[-k+10]$

$\sum_{k=-\infty}^{10} 3^k = 3^{-\infty} + \cdots 3^{-n} + \cdots 3^{-2} + 3^{-1} + 1 + 3 + \cdots 3^{10}$

we get a finite value when we do sum of infinite terms.

$= 3^{10} + 3^9 + 3^8 + \cdots$ $S_\infty = \frac{3^{10}}{1-\frac{1}{3}} \times 0$ $a = 3^{10}$ $r = \frac{1}{3}$ $(d=0)$

$= \frac{3^{10}}{1-\frac{1}{3}} = \frac{3^{10}}{\frac{2}{3}} = \boxed{\frac{3^{11}}{2}}$ $S_\infty = \left\{\frac{a}{1-r} \times \frac{dr}{(1-r)^2}\right\}$

So, the impulse response corresponds to stable LTI system.

2.16 For each of the following impulse responses corres-ponds to stable LTI systems

2.16 For each of the following statements, determine whether it is true or false:

a) If $x[n]=0$ for $n<N_1$ and $h[n]=0$ for $n<N_2$ then $x[n]*h[n]=0$ for $n<N_1+N_2$

b) If $y[n]=x[n]*h[n]$, then $y[n-1]=x[n-1]*h[n-1]$

c) If $y(t)=x(t)*h(t)$, then $y(-t)=x(-t)*h(-t)$

d) If $x(t)=0$ for $t>T_1$ and $h(t)=0$ for $t>T_2$, then $x(t)*h(t)=0$ for $t>T_1+T_2$.

b) Given $y[n]=x[n]*h[n]$

Now $y[n]=\sum_{k=-\infty}^{\infty} x[k]h[n-k]$.

$y[n-1]=\sum_{k=-\infty}^{\infty} x[k]h[n-1-k]$

$y[n-1]=x[n]*h[n-1]$

So the statement is false.

c) Given $y(t) = x(t) * h(t)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$y(-t) = \int_{-\infty}^{\infty} x(\tau)\, h(-t-\tau)\, d\tau$$

so $y(-t) = x(t) * h(-t)$

so the statement is false

a) we know $x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$

Here k is zero for $k < N_1$ (in $x[k]$)

& and $n-k$ is zero for $n-k < N_2$ [in $h[n-k]$]

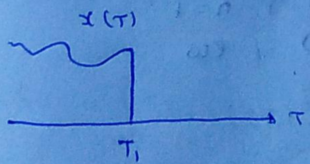
→ i.e $k > n - N_2$

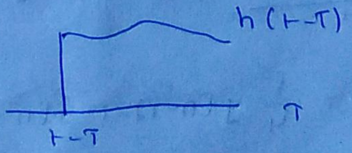
→ $n - N_2 < k < N_1$

→ By this condition, we can write $n - N_2 < N_1$

→ $n < N_1 + N_2$, True.

d) $x(t) = 0$ for $t > T_1$ & $h(t) = 0$ for $t > T_2$





multiply, then $x(\tau)\, h(t-\tau) = 0$ for $t - T_2 > T_1$

$t > T_1 + T_2$

So, It is true.

2.17 Consider an LTI system whose input $x(t)$ and output $y(t)$ are related by differential equation

$$\frac{d}{dt} y(t) + 4y(t) = x(t)$$

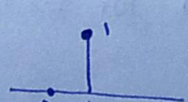
The system also satisfies the condition of initial rest a) If $x(t) = e^{(-1+3j)t} u(t)$, what is $y(t)$?
b) Note that $Re\{x(t)\}$ will satify the above equation with $Re\{y(t)\}$. Determine the output $y(t)$ of the LTI system if

$$x(t) = e^{-t} \cos(3t) u(t).$$

2.18 Consider a causal LTI system whose input $x[n]$ and out-put $y[n]$ are related by the differentiala equation

$$y[n] = \frac{1}{4} y[n-1] + x[n],$$ Determine $y[n]$ if $x[n] = \delta[n-1]$

A Given $x[n] = \delta[n-1]$



i.e $x[n] = \begin{cases} 1, & n=1 \\ 0, & ew \end{cases}$

$$y[n] = \frac{1}{4} y[n-1] + \delta[n-1]$$

As system is causal linear time invariant system.

$\{y[0] = 0\}$ because $x[0]=0$

$$y[1] = \frac{1}{4} y[0] + x[0] = 0 + 1 = 1$$

$y[2] = \frac{1}{4} y[1] + x[2] = \frac{1}{4}(1) + 0 = \frac{1}{4}$ $\{y[1]=1\}$

$y[3] = \frac{1}{4} y[2] + x[3] = \frac{1}{4}\left[\frac{1}{4}\right] + 0 = \frac{1}{16}$ $\{y[2] = \frac{1}{4}\}$

$y[4] = \frac{1}{4} y[3] + x[4] = \frac{1}{4}\left[\frac{1}{16}\right] + 0 = \frac{1}{64}$ $[y(3) = \frac{1}{16}]$

we can write general term as .. $y[n] = \left(\frac{1}{4}\right)^{n-1}$

for $n \geq 1$

$= y[n] = \left(\frac{1}{4}\right)^{n-1} \quad \forall \; n \geq 1$

Evaluate the following integrals:

a) $\int_{-\infty}^{\infty} u_0(t)\cos(t)\,dt$

b) $\int_{0}^{5} \sin(2\pi t)\,\delta(t+3)\,dt$

c) $\int_{-5}^{5} u_1(1-\tau)\cos(2\pi\tau)\,d\tau$

b) $\int_{0}^{5} \sin(2\pi t)\,\delta(t+3)\,dt$

it is in the form of $\int_{t_1}^{t_2} x(t)\,\delta(t-t_0)\,dt$

Here $t_0 = -3$, Here -3 is not between 0 to 5

so $\int_{0}^{5} \sin(2\pi t)\,\delta(t+3)\,dt = 0$ //

c) $\int_{-5}^{5} \cos(2\pi\tau)\,u_1(1-\tau)\,d\tau$

consider $\cos(2\pi\tau)$ as $x(\tau)$

then $\int_{-5}^{5} x(\tau)\, u_1(1-\tau)\, d\tau$ i.e $x(\tau) \cdot u_1(1-\tau)$ is defined only for -5 to 5

- It is in the form of $\int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

to change the limits from $(-5$ to $5)$ to $(-\infty$ to $\infty)$

consider $x(\tau)$ as $\cos(2\pi\tau)\big[\underbrace{u(\tau+5) - u(\tau-5)}_{\text{defined for only for } -5 \text{ to } 5}\big]$

then $\int_{-\infty}^{\infty} x(\tau)\, u_1(1-\tau)\, d\tau$

$= x(t) * u_1(t)$

a) $\int_{-\infty}^{\infty} u_0(t)\cos(t)\, dt$

2.21 Compute the convolution $y[n] = x[n] * h[n]$ of the following pairs of signals.

a) $x[n] = \alpha^n u[n]$ & $h[n] = \beta^n u[n]$

$\alpha = \beta$

A $y[n] = x[n] * h[n]$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k].h[n-k]$$

$$y[n] = \sum_{k=-\infty}^{\infty} \alpha^k u[k]\, \beta^{n-k}\, u[n-k]$$

~~$y[n]$~~ Here $\alpha^k u[k]$ is non zero for $k > 0$

& $\beta^{n-k} u[n-k]$ is non zero for $n-k>0$

$k < n$

So, $y[n] = \sum_{k=0}^{n} \alpha^k . \frac{\beta^n}{\beta^k}$

$= y[n] = \beta^n \sum_{k=0}^{n} \left(\frac{\alpha}{\beta}\right)^k$ for $n >$

$= y[n] = \beta^n \left[1 + \frac{\alpha}{\beta} + \left(\frac{\alpha}{\beta}\right)^2 + \cdots \left(\frac{\alpha}{\beta}\right)^n \right]$

Sum of n terms in G.P is..

$= y[n] = \beta^n \left[\frac{1 - \left(\frac{\alpha}{\beta}\right)^n}{1 - \frac{\alpha}{\beta}} \right] = \cancel{\beta^n} \frac{\frac{\beta^n - \alpha^n}{\cancel{\beta^n}}}{\frac{\beta - \alpha}{\beta}}$

$= \frac{\beta^{n+1} - \beta\alpha^n}{\beta - \alpha}$ for $n \geq 0$

b) $x[n] = h[n] = \alpha^n u[n]$

$$y[n] = \sum_{k=-\infty}^{\infty} x[n]\, h[n-k]$$

$$= \sum_{k=-\infty}^{\infty} \alpha^k u[k]\, \alpha^{n-k} u[n-k]$$

$$= \sum_{k=-\infty}^{\infty} \frac{\alpha^k}{\alpha^k} \alpha^n\, u[k]\, u[n-k]$$

$u[k]$ is non zero for $k > 0$

$u[n-k]$ is no zero for $n-k > 0 \quad k < n$

$$= \sum_{k=0}^{n} \alpha^n \qquad n \geq 0$$

$$= \alpha^n \sum_{k=0}^{n} 1 \qquad n \geq 0$$

$$= \alpha^n \sum_{k=0}^{n} k^0$$

$$= \alpha^n [\underset{\text{consider }1}{0^0} + \underbrace{1^0 + 2^0 + 3^0 + \dots n^0}_{1+1+\text{-sum upto } n \text{ terms}}]$$

$$= \alpha^n [1+n]$$

$$= \alpha^n [1+n] \qquad n \geq 0$$

c) $x[n] = \left(\frac{-1}{2}\right)^n u[n-4]$

$h[n] = 4^n u[2-n]$

A) $$y[n] = \sum_{k=-\infty}^{\infty} x[n]\, h[n-k]$$

$$= \sum^{\infty} \left(\frac{-1}{2}\right)^k u[k-4] \cdot 4^{n-k} u[2-n+k]$$

u[k-4] is non zero for k > 4

u[2-n+k] is non zero for ~~k < 2-n~~.

2 - n + k > 0

k > n - 2

$$\rightarrow \sum_{4}^{k<n-2} \left(\frac{-1}{2}\right)^k 4^{n-k}$$

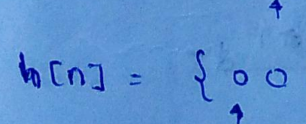
$$= \sum_{k=4}^{\infty} \frac{(-1)^k}{2^k} \frac{4^n}{2^{2k}}$$

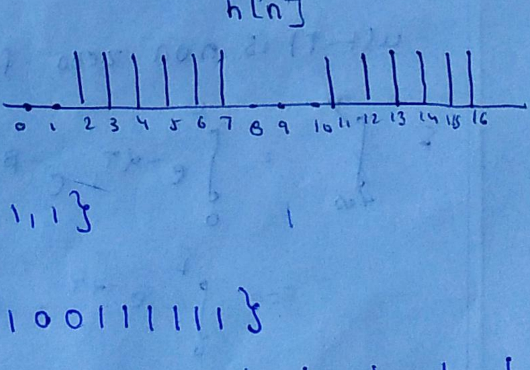
$$= 4^n \sum_{k=4}^{\infty} \left(\frac{-1}{8}\right)^k$$

$$= 4^n \left[\left(\frac{-1}{8}\right)^4 + \left(\frac{-1}{8}\right)^5 + \left(\frac{-1}{8}\right)^6 + \dots \right]$$

$= 4^n$ [sum of terms in upto ∞] $a = \left(\frac{-1}{8}\right)^4$, $r = \frac{-1}{8}$

x[n] and h[n] are ..



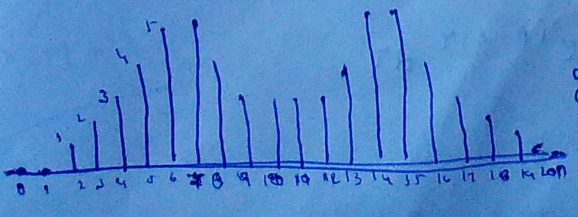


$x[n] = \{1, 1, 1, 1, 1\}$

$h[n] = \{0\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 1\}$

$x[n] * h[n] =$

$\{0, 0, 1, 2, 3, 4, 5, 5, 4, 3, 3, 3, 3, 4, 5, 5, 4, 3, 2, 1\}$



2.22 For each of the following pairs of wave forms, use the convolution integral to find the response $y(t)$ of the LTI system with impulse response $h(t)$ to the input $x(t)$. Sketch your results

20

a) $x(t) = e^{-\alpha t} u(t)$
$h(t) = e^{-\beta t} u(t)$ } Do both when $\alpha \neq \beta$ & $\alpha = \beta$ }

A Convolution of continuous signal is .. $\int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$

$$= \int_{-\infty}^{\infty} \cancel{e^{-\alpha t} u(t) \cdot e^{-\beta t} u(t)} \qquad \alpha \neq \beta$$

$$= \cancel{\int_{0}^{\infty} e^{-\alpha t} \cdot e^{\beta t} dt} = \int_{-\infty}^{\infty} e^{-\alpha \tau} u(\tau) \cdot e^{-\beta(t-\tau)} u(t-\tau) d\tau$$

$u(\tau)$ is non zero for $\tau > 0$

$u(t-\tau)$ is non zero for $t-\tau > 0$ $\quad \tau < t$

$$\cancel{\int_{-\infty}^{\infty}} = \int_{0}^{t} e^{-\alpha \tau} \cdot e^{-\beta(t-\tau)} d\tau$$

$$= \int_{0}^{t} e^{-\alpha \tau} \cdot e^{(-\beta t + \beta \tau)} d\tau$$

$$= \int_{0}^{t} e^{\beta \tau} \cdot e^{-\alpha \tau} \cdot e^{-\beta t} d\tau$$

$$= e^{-\beta t} \int_{0}^{t} e^{(\beta \tau - \alpha \tau)} d\tau$$

$$= e^{-\beta t} \int_{0}^{t} e^{(\beta - \alpha)\tau} d\tau.$$

put $(\beta-\alpha)\tau = u$

$\Rightarrow (\beta-\alpha)\, d\tau = du$

$\Rightarrow d\tau = \dfrac{du}{\beta-\alpha}$

$= e^{-\beta t}\displaystyle\int_0^t e^{u}\,\frac{du}{\beta-\alpha}$

$= \dfrac{e^{-\beta t}}{\beta-\alpha}\left[e^{u}\right]_0^t = \dfrac{e^{-\beta t}}{\beta-\alpha}\left[e^{(\beta-\alpha)t}\right]_0^t$

$= \dfrac{e^{-\beta t}}{\beta-\alpha}\left[e^{(\beta-\alpha)t} - 1\right]$

If $\alpha = \beta$

$= e^{-\beta t}\displaystyle\int_0^t e^{\alpha\tau}\cdot e^{-\alpha\tau}\, d\tau$

$= e^{-\beta t}\displaystyle\int_0^t 1\, d\tau$

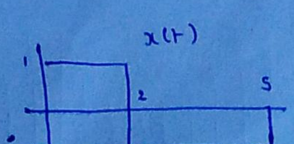
$= e^{-\beta t}\left[\tau\right]_0^t$

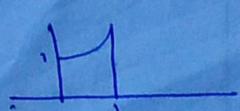
$= e^{-\beta t}\cdot t$

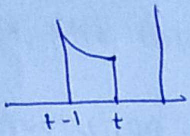
$$y(t) = \begin{cases} \dfrac{e^{-\beta t}\left[e^{(\beta-\alpha)t} - 1\right]}{\beta-\alpha}, & \alpha \neq \beta \\ t\cdot e^{-\beta t}, & \alpha = \beta \end{cases}$$

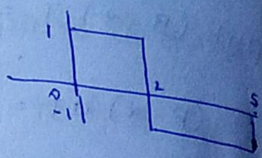
b) $x(t) = u(t) - 2u(t-2) + u(t-5)$

$h(t) = e^{2t}u(1-t)$

A $x(t) =$ 

& $h(t) =$ 

$h(t-\tau) =$ 

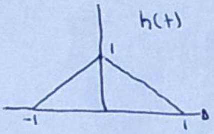
$x(\tau) =$ 

2.23 Let $h(t)$ be the triangular pulse shown in fig (a) & let $x(t)$ be the impulse train depicted in fig (b). That is

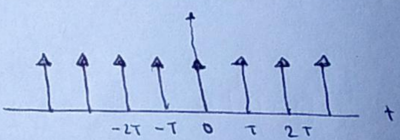
$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$$

Determine and sketch $y(t) = x(t) * h(t)$ for the following values of T.

a) $T = 4$, b) $T = 2$, c) $T = \frac{3}{2}$. d) $T = 1$



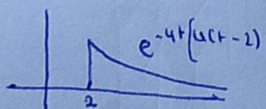
(a)



Don't know..

2.9 The following are the impulse responses of continuous-time LTI systems. Determine whether each system is causal and / or stable. Justify your answer.

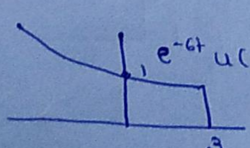
a) $h(t) = e^{-4t} u(t-2)$



causal

Stable

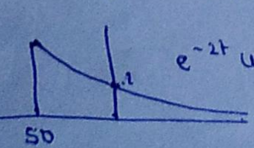
b) $h(t) = e^{-6t} u(3-t)$



non-causal

non-stable

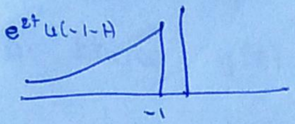
c) $h(t) = e^{-2t} u(t+50)$



non-causal

Stable

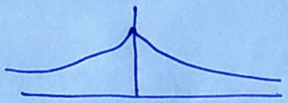
d) $h(t) = e^{2t}u(-1-t)$



non-causal

Stable

e) $h(t) = e^{-6|t|}$



non-causal

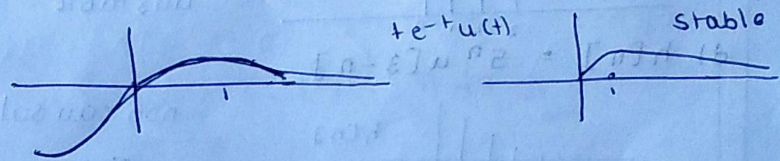
Stable

f) $h(t) = te^{-t}u(t)$

causal

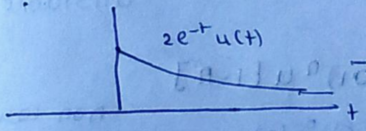
stable

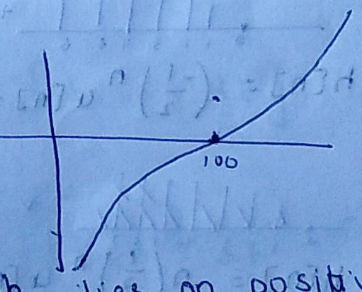
$te^{-t} =$



g) $h(t) = \left(2e^{-t} - e^{(t-100)/100}\right)u(t)$

$h(t) = 2e^{-t}u(t) - e^{(t-100)/100}u(t)$



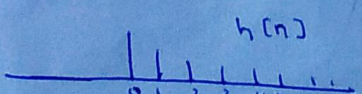


= The resultant graph lies on positive axis and have $-\infty$ amplitude at a point, so the system is causal and non-stable.

The following are the impulse responses of discrete-time LTI systems. Determine whether each system is causal and/or stable

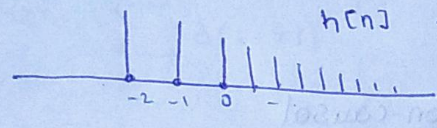
a) $h[n] = \left(\frac{1}{5}\right)^n u[n]$

causal



stable.

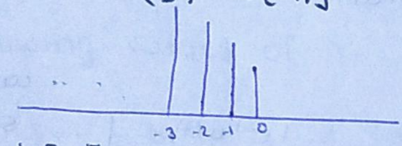
b) $h[n] = (0.8)^n u[n+2]$



$h[n]$ = non causal

stable.

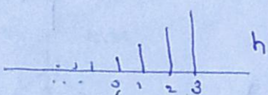
c) $h[n] = \left(\frac{1}{2}\right)^n u[-n]$



non causal

unstable

d) $h[n] = 5^n u[3-n]$

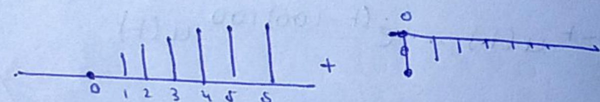


$h[n]$

non causal

stable

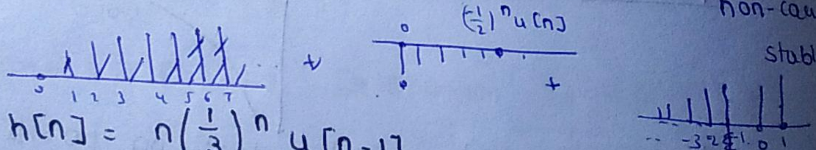
e) $h[n] = \left(\frac{-1}{2}\right)^n u[n] + (1.01)^n u[n-1]$



causal

unstable

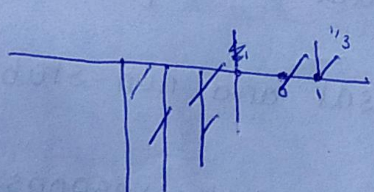
f) $h[n] = \left(\frac{-1}{2}\right)^n u[n] + (1.01)^n u[1-n]$



non-causal

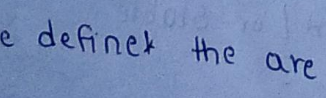
stable

g) $h[n] = n\left(\frac{1}{3}\right)^n u[n-1]$



non-causal

unstable



causal.

stable

2.27 We define the are under a continuous-time signal

$v(t)$ as $A_v = \int_{-\infty}^{+\infty} v(t)\, dt$

Show that if $y(t) = x(t) * h(t)$, then $A_y = A_x A_h$

we know $y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

Also given $A_v = \int_{-\infty}^{\infty} v(t)\, dt$

then $A_y = \int_{-\infty}^{\infty} y(t)\, dt$

$$A_y = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau \right] dt$$

$$A_y = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} x(\tau) \left(h(t-\tau)\, dt \right) \right] d\tau$$

$$A_y = \int_{-\infty}^{\infty} x(\tau)\, d\tau \left[h(t-\tau)\, dt \right]$$

$$A_y = \int_{-\infty}^{\infty} x(\tau)\, d\tau \cdot \int_{-\infty}^{\infty} h(t-\tau)\, dt$$

$$\boxed{A_y = A_x \cdot \int_{-\infty}^{\infty} h(t-\tau)\, dt}$$

Here $h(t-\tau)$ ~~be~~ means the graph $h(t)$ shifted τ units right so its area doesn't change

so $\int_{-\infty}^{\infty} h(t-\tau)\, dt = A_h$

$$A_y = A_x \cdot A_h$$

2.26 Consider the evaluation of $y[n] = x_1[n] * x_2[n] * x_3[n]$

where $x_1[n] = (0.5)^n u[n]$

and $x_2[n] = u[n+3]$

& $x_3[n] = \delta[n] - \delta[n-1]$

a) Evaluate the convolution $x_1[n] * x_2[n]$

A: $x_1[n] * x_2[n] = \sum_{k=-\infty}^{\infty} (0.5)^k u[k] \cdot u[n-k+3]$

$$= \sum_{k=-\infty}^{\infty} (0.5)^k u[k] \cdot u[(n+3)-k]$$

$u[k]$ is non zero for $k > 0$, &

$u[n+3-k]$ is non zero for $n+3 > k$

$$= \sum_{0}^{n+3} (0.5)^k = \frac{1 - a^{(n+3)+1}}{1-a}$$

$$= \frac{1 - (0.5)^{n+4}}{1 - \frac{1}{2}} = \boxed{2 \left\{ 1 - \left(\tfrac{1}{2}\right)^{n+4} \right\}} \quad \forall\ n \geq 0$$

b) Convolute the result part of (a) with $x_3[n]$ in order to evaluate $y[n]$

A: Consider $2\left[1 - \left(\frac{1}{2}\right)^{n+4}\right]$ as $y_1[n]$

Now $y_1[n] * x_3[n] = \sum_{k=-\infty}^{\infty} (\delta[n] - \delta[n-1])\, y_1[n-k]$

(or) $x_3[n] * y_1[n]$

$$= \sum_{k=-\infty}^{\infty} \delta[n]\, y_1[n-k] - \sum_{k=-\infty}^{\infty} \delta[n-1]\, y_1[n-k]$$

$$\left\{ \text{we have } \sum_{k=-\infty}^{\infty} x[n]\,\delta[n-n_0] = x[n_0] \right\}$$

$$\text{so} = \left[y_1[n-0] - y_1[n-1] \right]$$

$$\text{So} = y_1[n] - y_1[n-1]$$

$$\Rightarrow 2\left\{1-\left(\tfrac{1}{2}\right)^{n+4}\right\} - 2\left\{1-\left(\tfrac{1}{2}\right)^{n+3}\right\}$$

$$= \cancel{2} - \left(\tfrac{1}{2}\right)^{n+3} - \cancel{2} + \left(\tfrac{1}{2}\right)^{n+2}$$

$$= \left(\tfrac{1}{2}\right)^{n+2} - \left(\tfrac{1}{2}\right)^{n+3}$$

$$= \left(\tfrac{1}{\cancel{2}}\right)^{n+\cancel{3}+1(\cancel{n}\,\cancel{+3})} \qquad \frac{\left(\cancel{\tfrac{1}{2}}\right)^{\cancel{n+2}}}{\left(\tfrac{1}{2}\right)^{\cancel{n+3}}} = \boxed{\left[\tfrac{1}{2}\right]^{n+3}}$$

$$y[n] = \left(\tfrac{1}{2}\right)^{n+2} - \left(\tfrac{1}{2}\right)^{n+3} = \left(\tfrac{1}{2}\right)^{n+2} - \tfrac{1}{2}\left(\tfrac{1}{2}\right)^{n+2}$$

$$= \left(\tfrac{1}{2}\right)^{n+2} \cdot \left(1-\tfrac{1}{2}\right) = \tfrac{1}{2}\left(\tfrac{1}{2}\right)^{n+2}$$

c) Evaluate the convolution $x_2[n] * x_3[n]$

A. $x_3[n] * x_2[n] = x_2[n] * x_3[n]$

$$= \sum_{k=-\infty}^{\infty} \delta[n] - \delta[n-1] \left[x_2[n \cancel{-k}] \right]$$

we get $x_2[n] - x_2[n-1]$

$$= u[n+3] - u[n+3-1] = u[n+3] - u[n+2]$$

d) Convolute the result of part (c) with $x_1[n]$ in order to evaluate $y[n]$.

A $x_1[n] = (0.5)^n u[n]$ & $y_2[n] = u[n+3] - u[n+2]$

$(x_1[n]) * [u[n+3] - u[n+2]]$

from distributive property

$\rightarrow x_1[n] * u[n+3] - x_1[n] * u[n+2]$

$$= \sum_{k=-\infty}^{\infty} (0.5)^k u[k]\, u[n-k+3] - \sum_{k=-\infty}^{\infty} (0.5)^k u[k]\, u[n-k+2]$$

$$= \sum_{k=0}^{n+3} (0.5)^k - \sum_{k=0}^{n+2} (0.5)^k$$

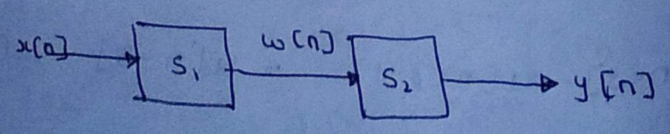
$$= \frac{1-(0.5)^{n+4}}{1-0.5} - \frac{1-(0.5)^{n+3}}{1-0.5} \quad \text{for } n+3 \geq 0$$

$$= 2\left\{1-\left(\frac{1}{2}\right)^{n+4}\right\} - 2\left\{1-\left(\frac{1}{2}\right)^{n+3}\right\} \quad n \geq -3$$

Already simplified

$$= \left(\frac{1}{2}\right)^{n+3} \quad n \geq -3$$

2.19 Consider the cascade of the following two systems S_1 & S_2 as depicted figure



S_1: causal LTI, $w[n] = \frac{1}{2}[w[n-1] + x[n]]$;

S_2: causal LTI, $y[n] = \alpha y[n-1] + \beta w[n]$

The difference equation relating $x[n]$ & $y[n]$ is.

$y[n] = -\frac{1}{8} y[n-2] + \frac{3}{4} y[n-1] + x[n]$

a) Determine α and β

b) show the impulse response of the cascade connection of S_1 & S_2.

a) Given $w[n] - \frac{1}{2} w[n-1] = x[n]$

Now from $y[n] = \alpha y[n-1] + \beta w[n]$

$\to \beta w[n] = y[n] - \alpha y[n-1]$

$\to w[n] = \frac{1}{\beta} y[n] - \frac{\alpha}{\beta} y[n-1] \to \cancel{1} \to (a)$

$\to w[n-1] = \frac{1}{\beta} y(n-1) - \frac{\alpha}{\beta} y(n-2) \to (1)$

multiply with $\frac{1}{2}$ = $\frac{1}{2} w[n-1] = \frac{1}{2\beta} y[n] - \frac{\alpha}{2\beta} y(n-2)$ $\to (2)$

a - 2

(a) - (2) gives $= x[n]$

$= -\left(\frac{1}{2\beta} y[n-1] \pm \frac{\alpha}{2\beta} y(n-2]\right) + \frac{1}{\beta} y[n] - \frac{\alpha}{\beta} y(n-1)$

$= \frac{1}{\beta} y[n] - \frac{\alpha}{\beta} y[n-1] - \frac{1}{2\beta} y[n-1] + \frac{\alpha}{2\beta} y[n-2]$ $\to (b)$

$x[n]$

$y[n] = \frac{-1}{8} y(n-2] + \frac{3}{4} y[n-1] + x[n] \to (3)$

from (b)

taking $\frac{-1}{\beta}$ common

$$\rightarrow \frac{-1}{\beta}\left[-y[n] + \alpha[n-1] + \frac{1}{2}[n-1] - \frac{\alpha}{2}y[n-2]\right] = x[n]$$

$$= -y[n] + \left(\alpha + \frac{1}{2}\right)y[n-1] - \frac{\alpha}{2}y[n-2] = -\beta x[n]$$

$$= \left(\alpha + \frac{1}{2}\right)y[n-1] - \frac{\alpha}{2}y[n-2] + \beta x[n] = y[n]$$

comparing with ③

$$\left(\alpha + \frac{1}{2}\right) = \frac{3}{4} \quad \& \quad \frac{-\alpha}{2} = \frac{-1}{8} \quad , \boxed{\beta = 1}$$

$$\boxed{\alpha = \frac{1}{4}}$$

<u>2.25</u> let the signal $y[n] = x[n] * h[n]$

where $x[n] = 3^n u[-n-1] + \left(\frac{1}{3}\right)^n u[n]$

and $h[n] = \left(\frac{1}{4}\right)^n u[n+3]$

a) Determine $y[n]$ without using the distributive property of convolution

b) Determine $y[n]$ with using distributive property of convolution.

<u>A</u> $y[n] = x[n] * h[n]$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$$= y[n] = \sum_{k=-\infty}^{\infty} \left[3^k u[-k-1] + \left(\frac{1}{3}\right)^k u[k]\right]\left(\frac{1}{4}\right)^{n-k} u[n-k+3]$$

$$= \sum_{k=-\infty}^{\infty} 3^k u[-k-1] \left(\frac{1}{4}\right)^{n-k} u[n-k+3] + \sum_{k=-\infty}^{\infty} \left(\frac{1}{3}\right)^k u[k] \left(\frac{1}{4}\right)^{n-k} u[n-k+3]$$

$$= \sum_{k=-\infty}^{-1} 3^k \left(\frac{1}{4}\right)^{n-k}$$