RESOLVING POWER

When two objects (or their images) are very near to each other, they may appear as one it may be impossible for the eyes to see them separate. The separation of such closed objects is terms as *resolutions*. *The ability of an optical instrument to form distinctly separate images of two objects very close is called its resolving power.*

Rayleigh Criterion of Resolution:

It is defined as the inverse of the distance or angular separation between two objects which can be just resolved when viewed through the optical instrument.

dip in the middle of two central maxima. Thus the two wavelengths can be distinguished from one another and according to Rayleigh they are said to be *"Just Resolved"*.

If the difference in wavelengths is such that their principal maxima are separately visible, then there is a distinct point of zero intensity in between the two wavelengths. Hence according to Rayleigh they are said to be *"Resolved"*.

When the difference in wavelengths is so small that the central maxima corresponding to two wavelengths come still closer as shown in figure, then the resultant intensity curve is quite smooth without any dip. This curve is as if there is only one wavelength Rayleigh Criterion of Resolution

Statement: Two sources are resolvable by an optical instrument when the central maximum of one diffraction pattern falls over the first minimum of the other diffraction pattern and vice versa.

For example:

Let us consider the resolution of two wavelengths and by a grating. When the difference in wavelengths is smaller and such that the central maximum of the wavelength coincides with the first minimum of the other as shown in figure, then the resultant intensity curve is as shown by the thick curve. The curve shows a distinct somewhat bigger and stronger.

Hence according to Rayleigh the two wavelengths are *"Not Resolved"*.

Thus the two spectral lines can be resolved only up to a certain limit expressed by Rayleigh Criterion.

Resolved

Just Resolved

Not Resolved
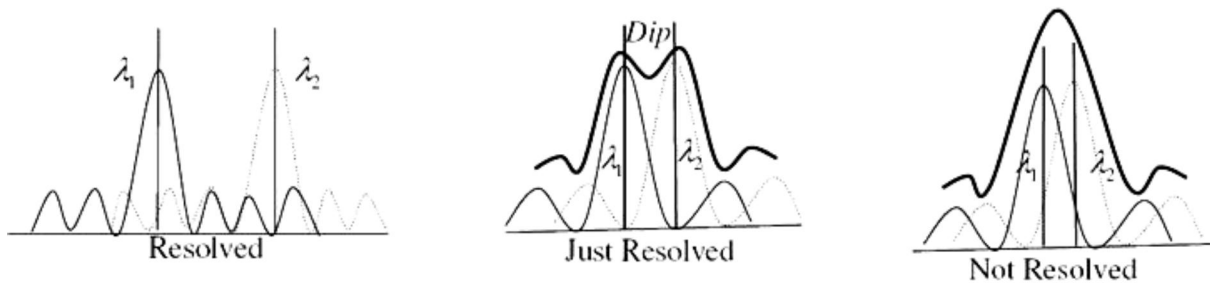
Resolving Power of Telescope

In telescopes, very close objects such as binary stars or individual stars of galaxies subtend very small angles on the telescope. To resolve them we need very large apertures.
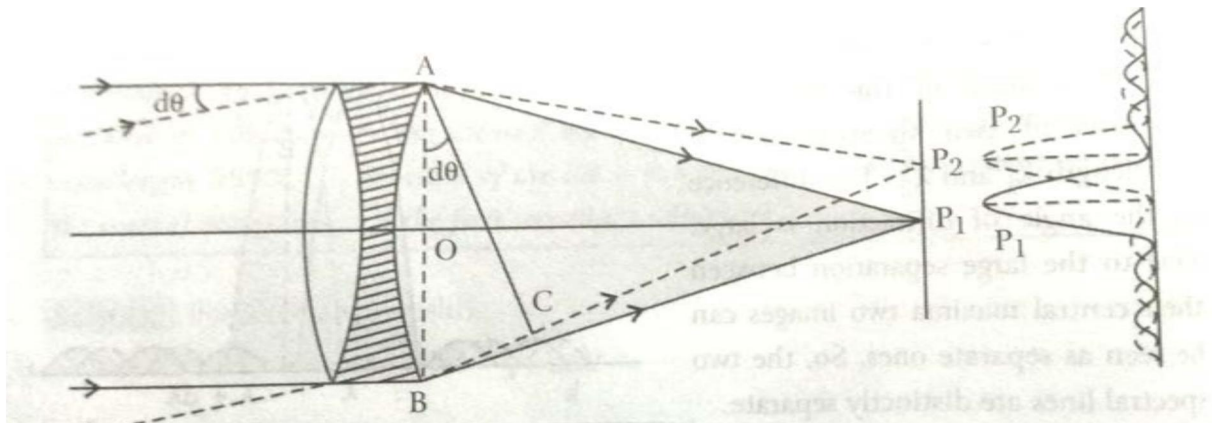
In above fig. the image of each point object is a Fraunhofer diffraction Patten and lies in the focal plane of the telescope objective AB. Let P1 and P2 be the position of centre of maxima of the two images. We can use Rayleigh's to determine the resolving power. The angular separation between two objects must be

The path difference will be equal to,

$$BP_2-AP_2= \lambda$$

But from fig.

$$BP_2-AP_2=BC=AB \, d\theta$$

[the angle is small, sinθ= dθ]

=D dθ

[AB=D=diameter of objective lens of telescope]

dθ= λ /D

for circular apertures, Airy showed that 1.22 λ may be used instead of λ .

dθ = 1.22 λ /D

Thus higher the diameter D, better the resolution. The best astronomical optical telescopes have mirror diameters as large as 10m to achieve the best resolution. Also, larger wavelengths reduce the resolving power and consequently radio and microwave telescopes need larger mirrors.

Relation between magnifying power and the resolving power of telescope:

From the theory of telescope, we know that magnifying power of a telescope is given by

M= Diameter of objective (entrance pupil)/diameter of exit pupil (eye ring)

 = D/d   ……………..(1)

The magnifying power is said to be normal if diameter d of the eye ring is equal to the diameter d_e, of the pupil of eye. Hence,

$$\text{Normal magnifying power} = \frac{\text{Diameter of the objective}}{\text{Diameter of the eye}} = \frac{D}{d_e}$$

Further, the limit of resolution of the telescope objective of diameter D is given by

$$d\theta = \frac{1.22\lambda}{D} \qquad ….(2)$$

and the limit of resolution of the unaided eye is given by

$$d\theta' = \frac{1.22\lambda}{d_e} \qquad ….(3)$$

$$\because \frac{\text{Limit of resolution of the eye}}{\text{Limit of resolution of the telescope}} = \frac{d\theta'}{d\theta} = \frac{1.22\lambda}{d_e} \Big/ \frac{1.22\lambda}{D}$$

$$= \frac{D}{d_e} = \text{Normal magnifying power of the telescope} \quad ….(4)$$

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Thus, <u>the limit of resolution of telescope, multiplied by its normal magnifying power is equal to the limit of resolution of the unaided eye.</u>

If the wavelength of light is taken as 5000 Å and the pupil diameters of the eye as 2 mm, the minimum resolving angle between two distant point objects by the eye

$$d\theta' = \frac{1.22\lambda}{d_e} = \frac{1.22 \times 5000 \times 10^{-8}}{0.2} \text{ radians}$$

$$= \frac{1.22 \times 5 \times 10^{-5}}{0.2} \times \frac{180}{\pi} \times 60 \text{ minutes}$$

=1 minute of arc = 60 seconds of arc.

Similarly the resolvable angle of the 40 inch refracting telescope

$$d\theta' = \frac{1.22\lambda}{D} = \frac{1.22 \times 5 \times 10^{-5}}{40 \times 2.54} \times \frac{180}{\pi} \times 60 \times 60 \text{ sec.}$$

$$= \frac{5}{40} = \frac{1}{8} \text{ second}$$

<u>Thus, we can say that the least angular separation (in seconds) resolvable by a telescope is</u>

$$\frac{5}{\text{aperture in inches}}$$

which is Dawe's rule.

Hence, the normal magnifying power of the telescope with 40 inch objective

$$= \frac{d\theta'}{d\theta} = \frac{60}{1/8} = 480.$$

Thus to bring out the full details of the object, the magnifying power of about 480 is required. If the magnifying power is greater than this, it will serve no useful purpose because it would not be accompanied by any more resolution and if it is less than this, the full advantage of high resolving power of the telescope will not be taken.

Resolving Power Of Microscope:

For microscopes, the resolving power is the inverse of the distance between two objects that can be just resolved.
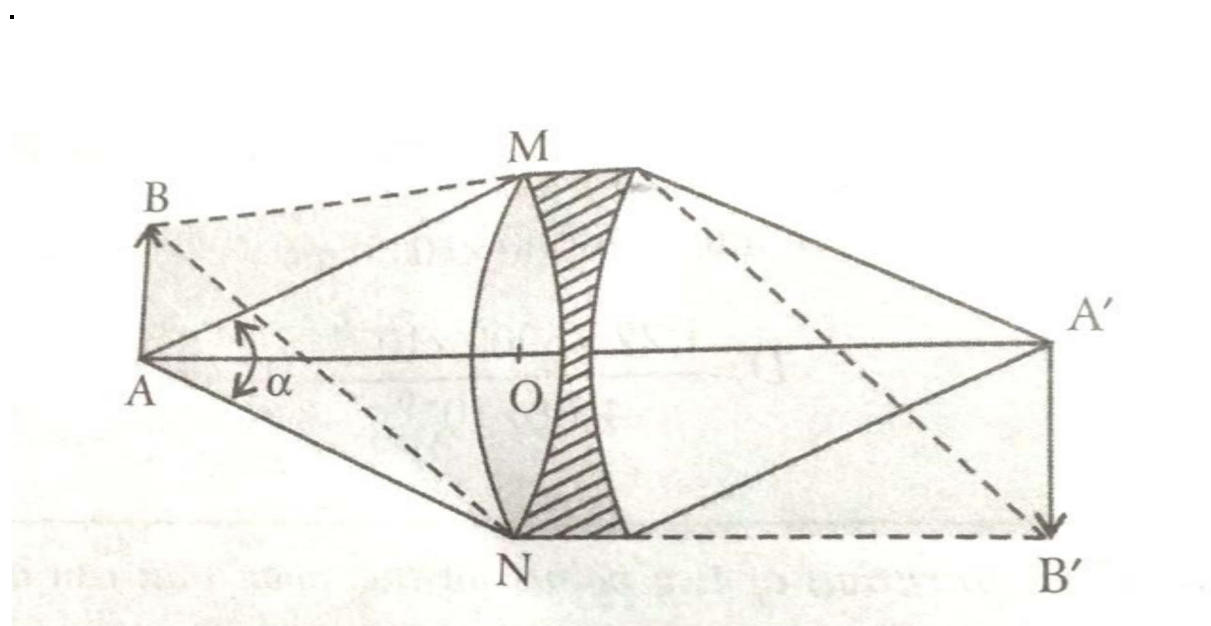
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Fig.(1)

In fig.(1), MN is the aperture of the objective of a microscope and A and B are two object points at a distance d apart. A' and B' are the corresponding Fraunhofer diffraction pattern of the two images. A' is the position of the central maximum of A and B' is the position of the central maximum of B'. the two image are said to be just resolved if the position of the central maximum B' also correspond to the first minimum of the image of A'.

$$(BN-NA') - (BM-MA')$$

$$NA'=MA'$$

Path difference = BN-BM

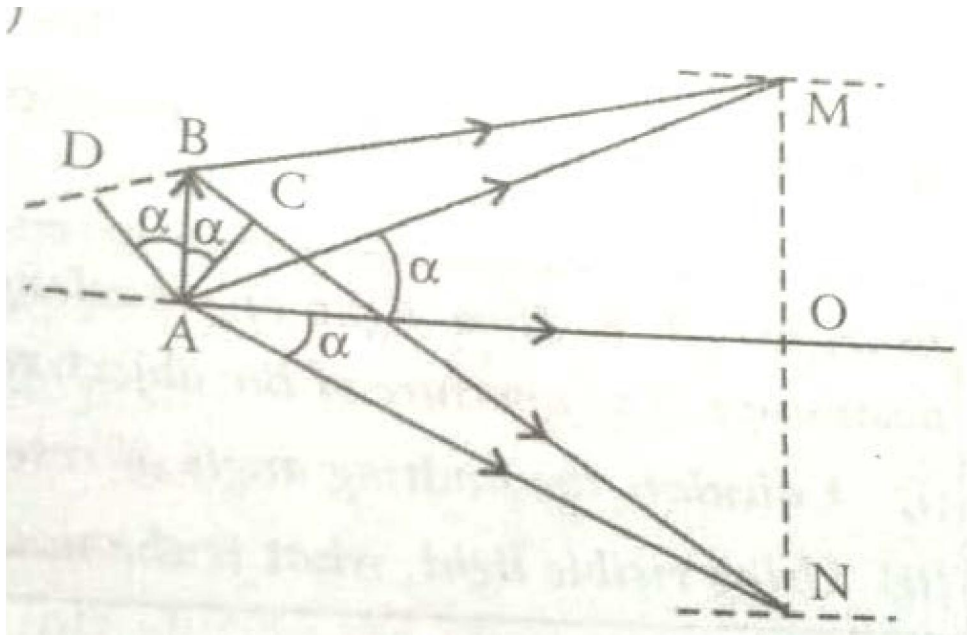In fig.(2), AD is perpendicular to DM and AC is perpendicular to BN.

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Fig.(2)

$$BN-BM = (BC+CN)-(DM-DB)$$

$$CN=AN=AM=DM$$

Path difference=BC+DB

From theΔ ACB andΔ ADB

$$BC=AB \sin\alpha =d \sin\alpha$$

$$DB=AB \sin\alpha=d \sin\alpha$$

Path difference=2d $\sin\alpha$

If this Path difference 2d $\sin\alpha$=1.22 λ, then, A' corresponds to the first minimum of the image B' and the two image appear just resolved.

$$2d \sin\alpha=1.22 \lambda$$

d=1.22 λ /2 $\sin\alpha$

Where *n* is the refractive index of the medium separating object and aperture. Note that to achieve high-resolution *n sin θ* must be large. This is known as the Numerical aperture.

Thus, for good resolution:

1. sinθ must be large. To achieve this, the objective lens is kept as close to the specimen as possible.
2. A higher refractive index (n) medium must be used. Oil immersion microscopes use oil to increase the refractive index. Typically for use in biology studies, this is

limited to 1.6 to match the refractive index of glass slides used. (This limits reflection from slides). Thus the numerical aperture is limited to just 1.4-1.6. Thus, optical microscopes (if you do the math) can only image to about 0.1 microns. This means that usually organelles, viruses and proteins cannot be imaged.

3. Decreasing the wavelength by using X-rays and gamma rays. While these techniques are used to study inorganic crystals, biological samples are usually damaged by x-rays and hence are not used.

Resolving power of prism

Ability of a prism to separated two close spectral lines is known as resolving power of the prism. It is measured by $\frac{\lambda}{d\lambda}$, where d λ is the smallest wavelength difference that can be just resolved by the wavelength λ.

Let a plane wave front BP of light wavelength λ and λ +d λ be incident on the prism ABC which is placed on position of minimum deviation. Let CQ and CQ' be the emergent wavefront for wavelength λ and λ +d λ as shown in below fig. let δ and $\delta + d\delta$ be the angle of deviation, μ and $\mu + d\mu$ these refractive induces corresponding to wavelength λ and λ +d λ respectively.

According to Fermat's principle, the optical path between the incident and emergent wavelength for any wavelength must be the same. Hence for the wavelength λ.
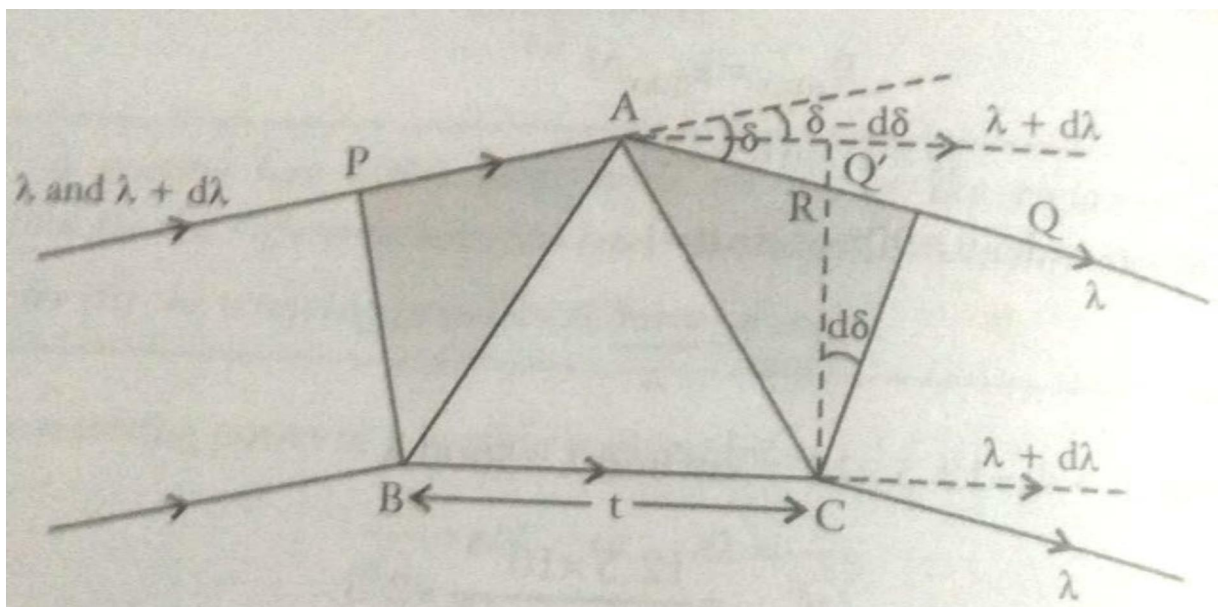
$$PA+AQ= \mu \ BC \qquad\qquad\qquad ….. (1)$$

Similarly, for the wavelength $\lambda + d\lambda$

$$PA+AQ'= (\mu - d\mu)BC \qquad\qquad …. (2)$$

From eq.(1) and (2), we have

$$AQ-AQ'= d\mu \ BC$$

but from above fig.

AQ=AR, so that AQ-AQ'=AQ-AR=RQ

$$RQ= d\mu BC$$

Again, from fig.

$$Q'CQ = d\delta$$

$$RQ=CQ \ d\delta$$

$$CQ \ d\delta = \ d\mu \ BC$$

The emergent beam has rectangular section. The section may also be considered as a rectangular aperture of width e which is the width of the emergent be CQ.

Hence,

$$e d\delta =t \ d\mu$$

$$d\delta = \frac{t d\mu}{e} \qquad\qquad\qquad (3)$$

Where, t is the width of the base of prism.

Hence, the diffraction pattern for each wavelength is equivalent to that due to a rectangular aperture of width e. according to Rayleigh criterion, if two spectral lines are just resolved, then principal maximum of one should fall on the principle maximum of the other. it means that angular separation $d\delta$ betaween the principal maxima of λ

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and $\lambda + d\lambda$ must be equal to the angle $d\theta$ between the principal maximum and first minimum of λ. The angle $d\theta$ is $given$ by the Fraunhofer diffraction at a single slit.

$$e \sin d\theta = \lambda$$

$$\mathrm{e}d\theta = \lambda$$

$$d\theta = \frac{\lambda}{e} \qquad \ldots\ldots\ldots (4)$$

For just resolution $d\delta = d\theta$. So, from eq.(3) and (4), we get

$$\frac{td\mu}{e} = \frac{\lambda}{e}$$

$$td\mu = \lambda$$

Hence, the resolving power of the prism,

$$\frac{\lambda}{d\lambda} = t\frac{d\mu}{d\lambda}$$

It show that the resolving power of a prism is directly proportional to the width of the base of the prism and also proportional to the rate of change of refractive index with wavelength. As we have $\frac{d\mu}{d\lambda}$= **dispersive power.**

So, **Resolving power= t* dispersive power**

References:

1. A textbook of Optics by Brij Lal and Subramaniam
2. Optics by Ajay Ghatak
3. Physical Optics and Lasers by J.P. Agrawal
4. Physical Optics and Lasers by Tripathi and Singh