

22nd Oct,
WEDNESDAY

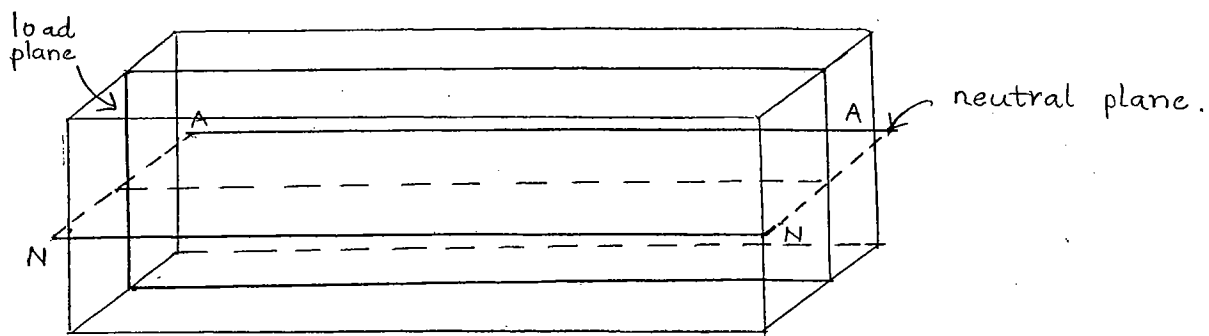
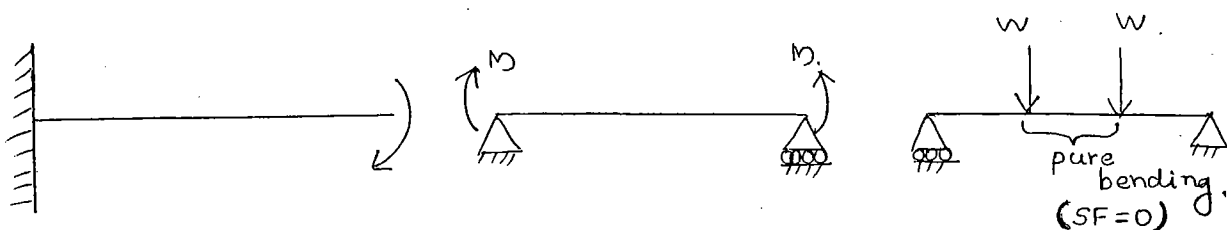
05. THEORY OF SIMPLE BENDING

For pure bending,

$$SF = 0$$

BM = non zero constant & MAX

Elastic curve = arch of a circle.



A line joining centroids of all cross sections along the length of a beam is centroidal axis (or) longitudinal axis (or) axis

- If load is applied, the centroidal axis deflects in the form of elastic curve or deflected shape.
- The axis in the c/s perpendicular to axis of the beam is the neutral axis
- The plane containing neutral axis and the axis of beam is neutral plane. Any point on neutral plane, has no bending stress and no bending strain. (Shear stress and shear strain may be there).

In circular members subj. to torsion, Bernoulli assumption is valid.

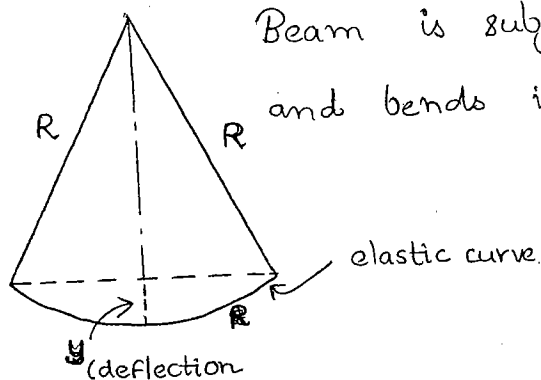
2. It is assumed that beam comprising of layers and they are free to slide one over the other without friction.
 \therefore SF can be eliminated.

3. The material properties are remaining the same in tension and compression. ($E_{\text{tension}} = E_{\text{compression}}$).

4. Radius of curvature is more compared to dimensions of c/s of beam. ($R \gg b \text{ \& } D$).

slopes \downarrow
 deflections \downarrow } superposition is applicable.

5.



→ Flexural Equation (or) Bending Equation.

$$\boxed{\frac{E}{R} = \frac{M}{I} = \frac{f}{y}}$$

$R \rightarrow$ radius of curvature,

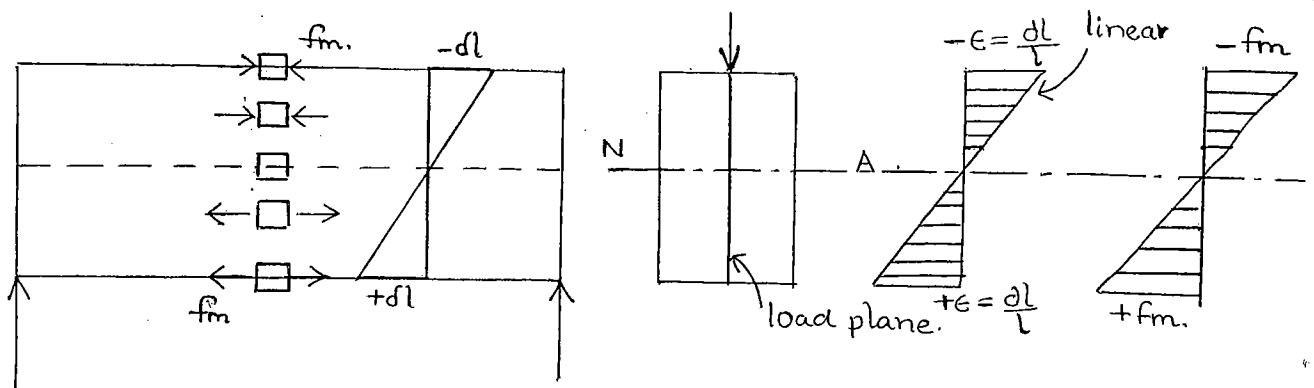
$\frac{1}{R} = \rho \rightarrow$ curvature,

$I \rightarrow$ MI of entire c/s area about NA

$f \rightarrow$ bending stress (indirect normal stress). {tensile or comp}

$y \rightarrow$ linear distance from NA, where f is required.

Due to loading, c/s of beam rotates w.r.t neutral axis. (53)
But NA always remains straight.



- Vertical plane through which load is applied to avoid torsion in the c/s is called 'Load plane'.

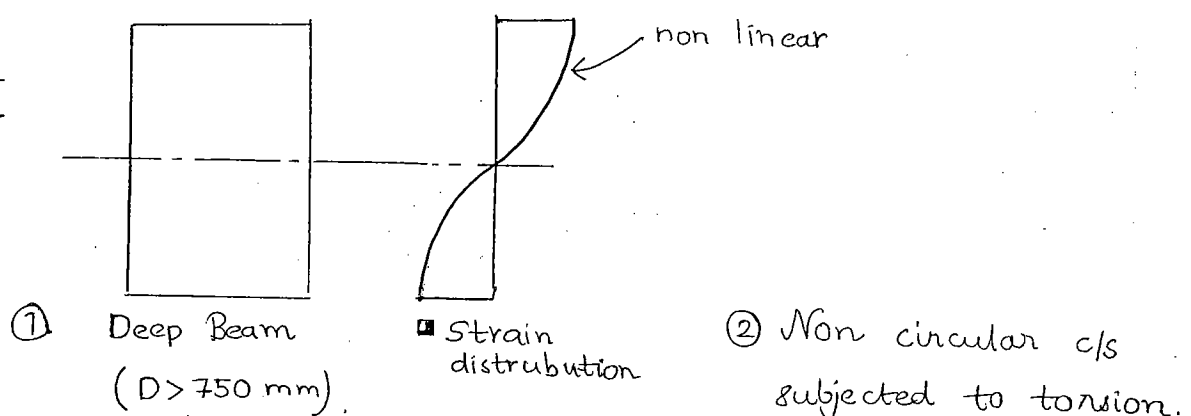
* Assumptions:

1. Euler-Bernoulli:

As per Bernoulli, there is no distortion in the shape of c/s due to bending. As per the assumption, strain distribution is linear along the depth with zero strain at the axis and max. at extreme fibres. As per Bernoulli, the linear distribution of strain is valid in all bending theories upto failure. (WSM of RCC, LSM of RCC, Ultimate Load Method of RCC, Plastic theory in steel)

2. Bernoulli's assumption is valid for composite beams like RCC also. But proper bond is required b/w different materials.

Not valid for:-



$$f = \text{const.} \times y. \Rightarrow f \propto y.$$

(54)

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NOTE:

In a beam, stresses developed are only in longitudinal direction. Even though an element is taken just below the load, no normal stress in the load direction on the element.

* Limitations:

1. Valid only upto PL.
2. Not valid for composites (like RCC).
3. Only gradual load. (no impact loads).
4. Only prismatic beams.

→ Section Modulus (Z)

First moment of area about neutral axis.

$$Z = \frac{I}{y_{\max}} \quad (\text{Unit : } m^3)$$

As $Z \uparrow$, strength in bending \uparrow .

→ Flexural Rigidity (EI) (Unit: N-mm²)

As $EI \uparrow$, rigidity in bending \uparrow

Stiffness \uparrow

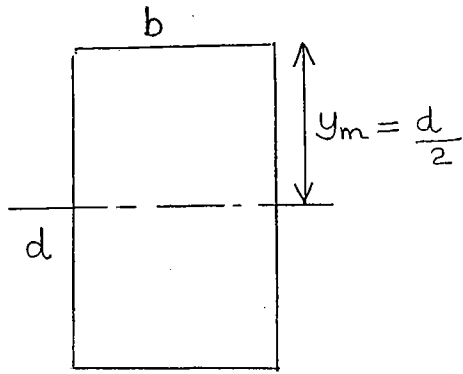
slopes & deflections \downarrow

• In a beam, strength parameter is Z.
stiffness parameter is EI

→ Axial Rigidity (AE)

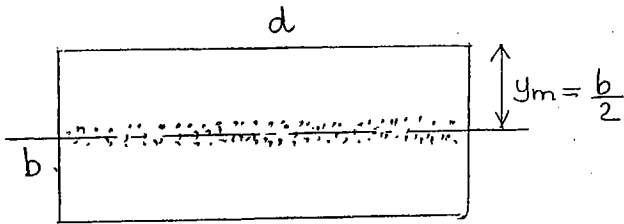
Unit : N

As $AE \uparrow$, axial deformation \downarrow

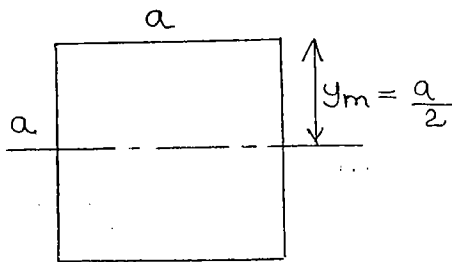


$$Z = \frac{I_{NA}}{y_{max}}$$

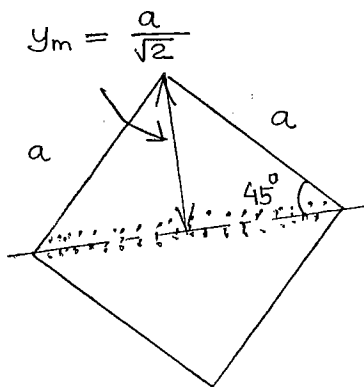
$$= \frac{\left(\frac{bd^3}{12}\right)}{\left(\frac{d}{2}\right)} = \underline{\underline{\frac{bd^2}{6}}}$$



$$Z = \frac{\left(\frac{db^3}{12}\right)}{\left(\frac{b}{2}\right)} = \underline{\underline{\frac{db^2}{6}}}$$



$$Z = \underline{\underline{\frac{a^3}{6}}}$$

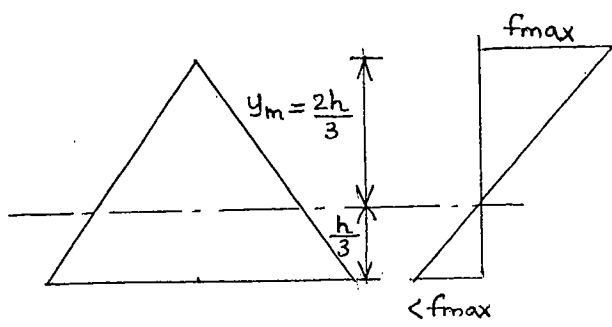


$$Z = \frac{I}{y_{max}} = \frac{\frac{a \cdot a^3}{12}}{\frac{a}{\sqrt{2}}}$$

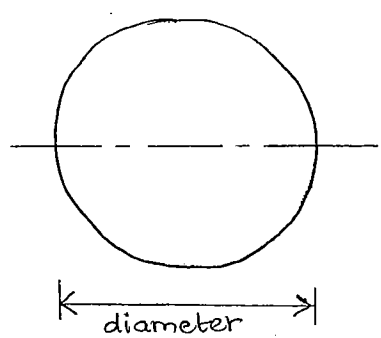
$$\underline{\underline{Z = \frac{a^3}{6\sqrt{2}}}}$$

$$\odot \frac{(\text{Strength})_{sq}}{(\text{Strength})_{di}} = \frac{(Z)_{sq}}{(Z)_{di}} = \sqrt{2} = 1.414$$

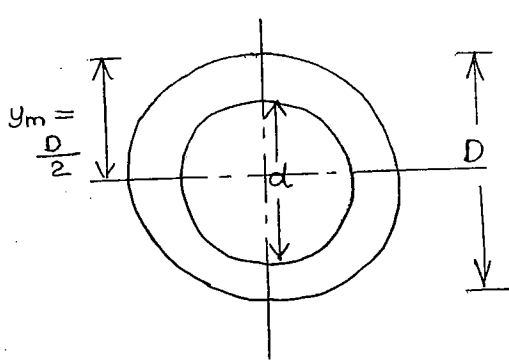
$$(\text{Strength})_{sq} = 41.4\% \uparrow (\text{strength})_{dia}$$



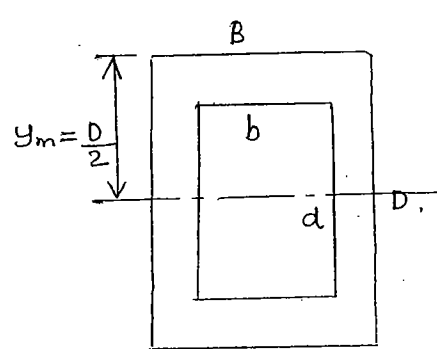
$$Z = \frac{\frac{bh^3}{36}}{\frac{2h}{3}} = \underline{\underline{\frac{bh^2}{24}}}$$



$$Z = \frac{\frac{\pi}{64} d^4}{\frac{d}{2}} = \underline{\underline{\frac{\pi d^3}{32}}}$$

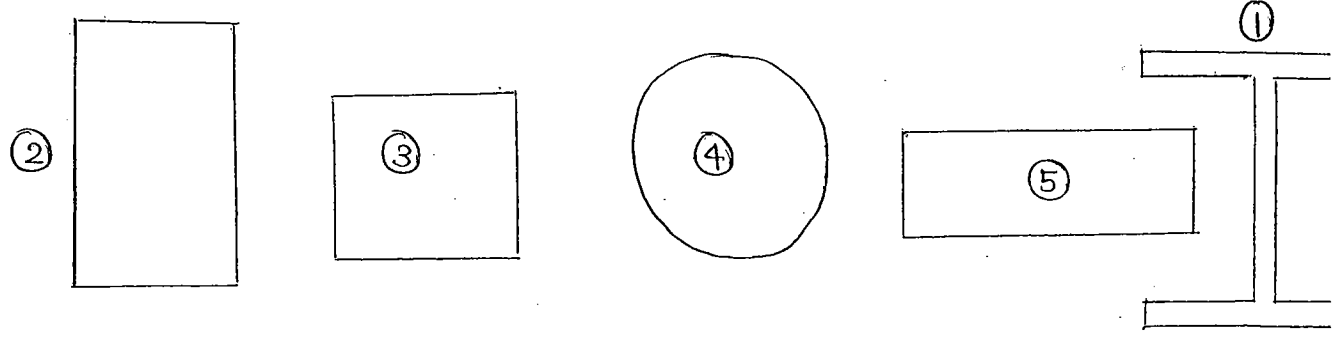


$$Z = \frac{\frac{\pi}{64} (D^4 - d^4)}{\frac{D}{2}} = \frac{\pi (D^4 - d^4)}{32 D}$$

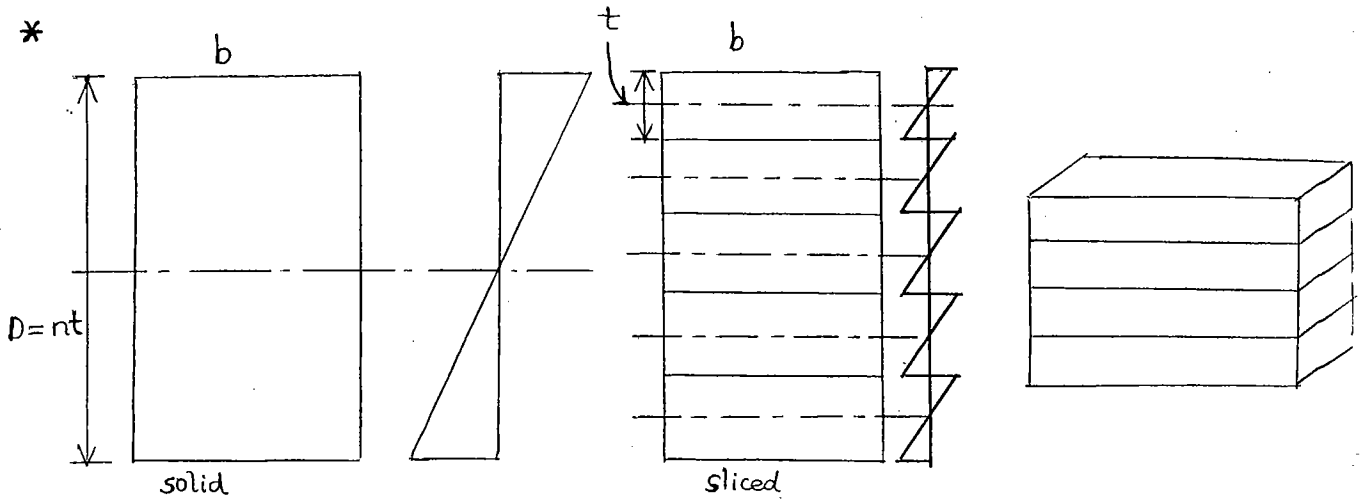


$$Z = \frac{\frac{BD^3}{12} - \frac{bd^3}{12}}{\frac{D}{2}} = \frac{BD^3 - bd^3}{6D}$$

* Same c/s area (Rankings in bending strength).



→ Sliced Beams.



$$\frac{(\text{Strength})_{\text{solid}}}{(\text{Strength})_{\text{sliced}}} = \frac{(Z)_{\text{solid}}}{(Z)_{\text{sliced}}} = \frac{\frac{b(nt)^2}{6}}{n \left(\frac{bt^2}{6} \right)} = n.$$

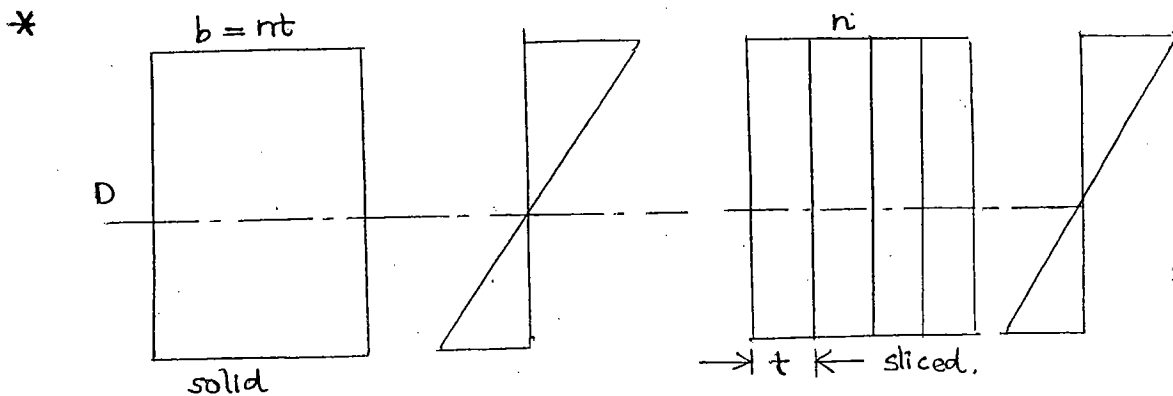
$$P = \frac{l}{R} = \frac{M}{EI}$$

$$\Rightarrow P \propto \frac{1}{I}$$

$$\frac{P_{\text{solid}}}{P_{\text{sliced}}} = \frac{I_{\text{sliced}}}{I_{\text{solid}}} = \frac{n \left(\frac{bt^3}{12} \right)}{\frac{b(nt)^3}{12}} = \frac{1}{n^2}$$

$$P_{\text{sliced}} = P_{\text{solid}} \times n^2 \quad (\text{Take the example of a book})$$

$$(\text{Stiffness})_{\text{solid}} = (\text{Stiffness})_{\text{sliced}} \times n^2$$



$$\frac{(\text{Strength})_{\text{solid}}}{(\text{Strength})_{\text{sliced}}} = \frac{(Z)_{\text{solid}}}{(Z)_{\text{sliced}}} = \frac{(nt)D^2}{6} \div \frac{n\left(\frac{tD^2}{6}\right)}{6} = 1$$

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$$\frac{P_{\text{solid}}}{P_{\text{sliced}}} = \frac{I_{\text{sliced}}}{I_{\text{solid}}} = \frac{n\left(\frac{tD^3}{12}\right)}{(nt)\frac{D^3}{12}} = 1$$

$$\therefore P_{\text{solid}} = P_{\text{sliced}}$$

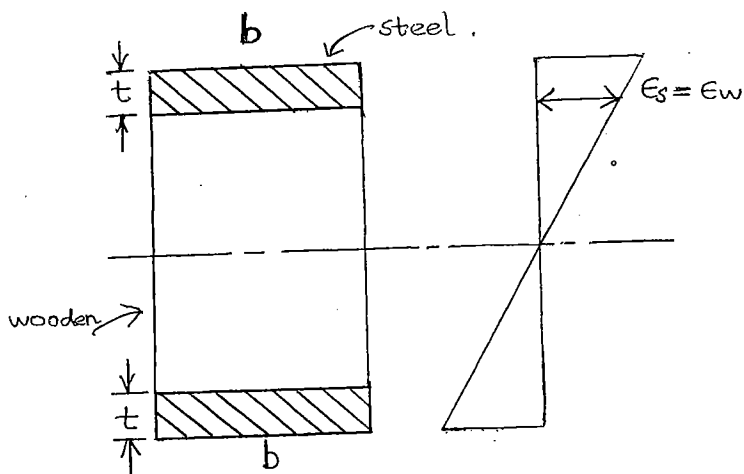
$$I_{\text{sliced}} = I_{\text{solid}}$$

$$(\text{Stiffness})_{\text{solid}} = (\text{Stiffness})_{\text{sliced}}$$

→ Fitched Beams (composite beams)

Example : RCC

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$$E_s = E_w$$

$$\frac{f_s}{E_s} = \frac{f_w}{E_w}$$

$$f_s = \left(\frac{E_s}{E_w}\right) f_w$$

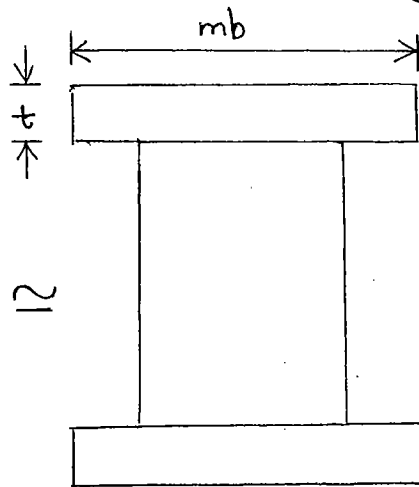
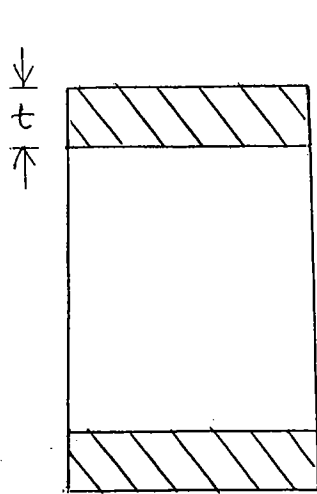
In a composite beam, different material should be bonded together so that the load can be shared.

• Bernoulli's assumption is valid for composite beams.

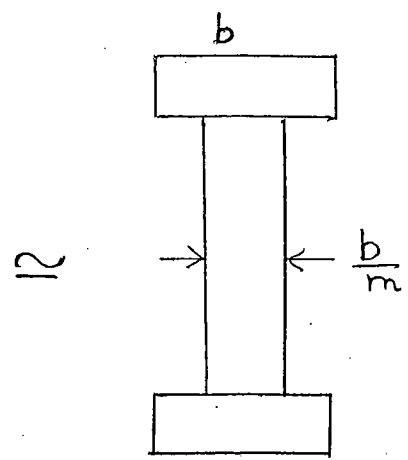
$$\text{Modular ratio, } m = \frac{E_{\text{strong}}}{E_{\text{weak}}}$$

$$f_s = m f_w$$

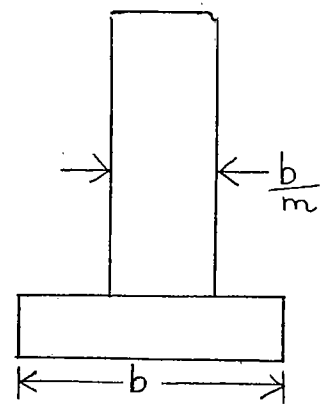
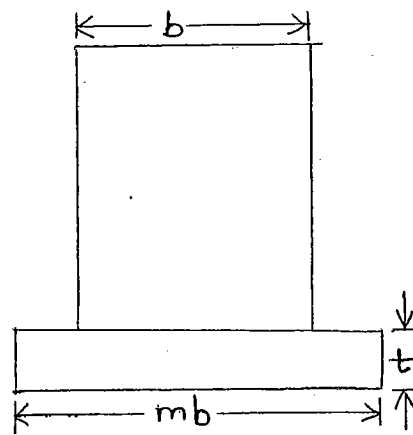
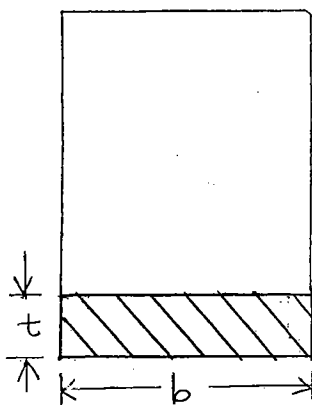
For the analysis of composite beams, equivalent area method is used. Total c/s is divided into equivalent material area of single material and analysed using bending equation.



■ Equivalent in Wood

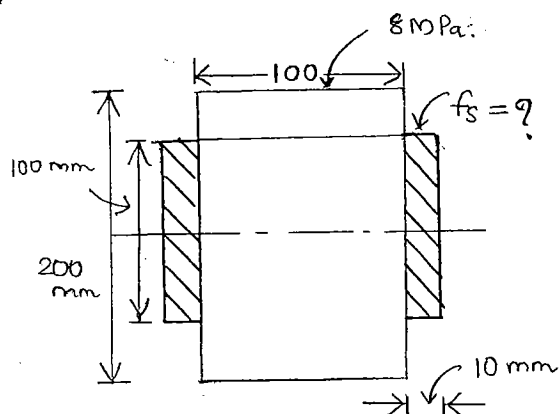


■ Equivalent in steel.



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$$m = 20$$

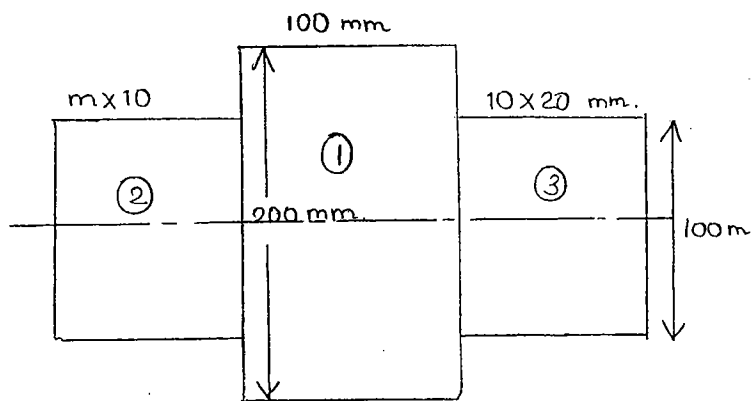
From linear variation of stress,

$$100 \text{ mm} \longrightarrow 8 \text{ MPa}$$

$$50 \text{ mm} \longrightarrow ? \quad (\text{From NA})$$

$$= 8 \times \frac{50}{100} = \underline{\underline{4 \text{ MPa}}}$$

$$\begin{aligned} f_s &= m \cdot f_w \\ &= 20 \times 4 = \underline{\underline{80 \text{ MPa}}} \end{aligned}$$



MI of equivalent wooden beam about NA

$$I = I_1 + 2 I_2$$

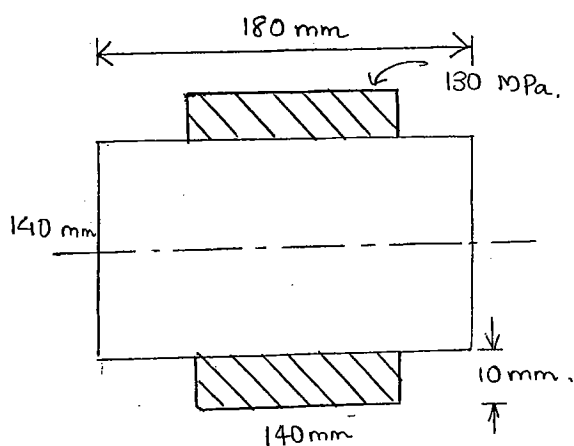
$$= 100 \times \frac{200^3}{12} + 2 \times 200 \times \frac{100^3}{12}$$

$$= \underline{\underline{10^8 \text{ mm}^4}}$$

$$y_{\max} = \frac{200}{2} = 100 \text{ mm}$$

$$\Rightarrow \frac{M}{I} = \frac{f}{y}$$

$$M = \frac{f I}{y} = 8 \times \frac{1 \times 10^8}{100} = \underline{\underline{8 \text{ kN m}}}$$



$$\left. \begin{aligned} f_w &= 8 \text{ MPa} \\ f_s &= 130 \text{ MPa} \end{aligned} \right\} \text{max. values}$$

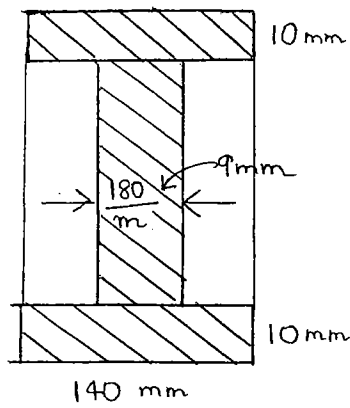
$$80 \text{ mm} \rightarrow 130 \text{ MPa}$$

$$70 \text{ mm} \rightarrow ?$$

$$f_s = \frac{70 \times 130}{80} = 113.75 \text{ MPa}$$

$$\text{Stress in wood, } f_w = \frac{f_s}{m} = \frac{113.75}{20} = \underline{\underline{5.6875 \text{ MPa} < 8 \text{ MPa}}}$$

If $f_w = 8 \text{ MPa}$, stress in steel (f_s) goes beyond 130 MPa, which is practically not possible as steel fails if its stress = 130 MPa. \therefore in the design stress in the steel is the deciding criteria.



MI of equivalent steel beam about NA,

$$I = \frac{140 \times 160^3}{12} - \frac{(140-9) 140^3}{12}$$

$$= \underline{\underline{17.82 \times 10^6 \text{ mm}^4}}$$

From bending equation, (using eq. steel section).

$$\frac{M}{I} = \frac{f}{y} \Rightarrow M = \frac{130 \times 17.82 \times 10^6}{80} = 28.95 \times 10^6 \text{ Nmm.}$$

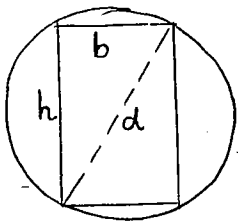
$$= \underline{\underline{28.95 \text{ kNm}}}$$

→ Beam of Uniform Strength.

Along the length of a beam, if the bending stress developed is const, it is the beam of uniform strength.

3rd Oct,
THURSDAY

-43. ① In order to obtain a rectangle of maximum strength in pure bending from a circular log of wood,



$$d^2 = b^2 + h^2$$

$$h^2 = d^2 - b^2 \rightarrow \textcircled{1}$$

$$Z = \frac{bh^2}{6} = \frac{b(d^2 - b^2)}{6}$$

For strongest rectangular section, Z should be maximum.

$$\frac{dz}{db} = 0$$

$$= \frac{d^2 - 3b^2}{6} = 0.$$

$$\Rightarrow b = \frac{d}{\sqrt{3}} \rightarrow \textcircled{2}$$

$$h^2 = d^2 - b^2$$

$$= d^2 - \left(\frac{d}{\sqrt{3}}\right)^2$$

$$h = \sqrt{\frac{2}{3}} d \rightarrow \textcircled{3}$$

$$\Rightarrow \boxed{\frac{h}{b} = \sqrt{2}}$$

Area of strongest rectangle = bh

$$= \left(\frac{1}{\sqrt{3}} d\right) \times \left(\sqrt{\frac{2}{3}} d\right)$$

$$= \underline{\underline{\frac{\sqrt{2}}{3} d^2}}$$

p-44

$$9. \quad \frac{M}{I} = \frac{f}{y}$$

$$M = f \cdot \frac{I}{y} = fz = f \cdot \frac{bd^2}{6}$$

Given $f = \text{const.}$ & $d = \text{const.}$

$$\therefore \underline{\underline{M \propto b}}$$

45.

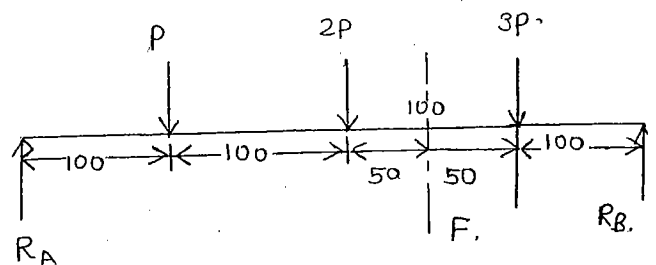
$$03. \quad R_B \times 400 = P \times 100 + 2P \times 200 + 3P \times 300$$

$$R_B = \frac{14}{4} P$$

$$R_A = \frac{5}{2} P$$

$$M_F = R_B \times 150 - 3P \times 50$$

$$= \frac{14}{4} P \times 150 - 3P \times 50 = \underline{\underline{375 P}}$$

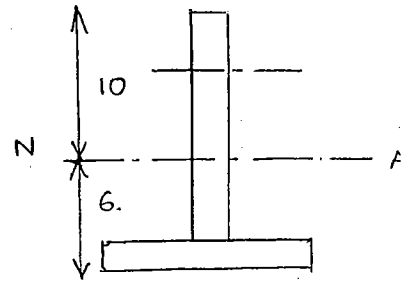
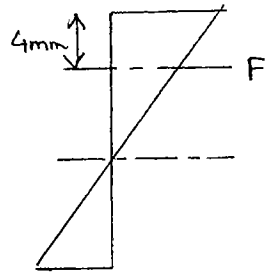


$$\epsilon_F = 1.5 \times 10^{-6}$$

$$f_F = \epsilon_F \times E$$

$$= (1.5 \times 10^{-6}) (200 \times 10^3)$$

$$= 0.3 \text{ N/mm}^2$$



Using bending equation (@ F),

$$\frac{M}{I} = \frac{f_F}{y_F}$$

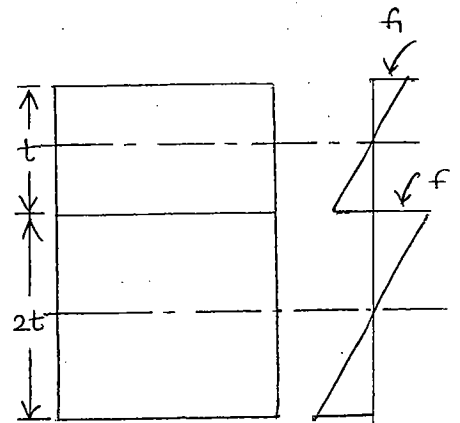
$$\frac{375P}{2176} = \frac{0.3}{6}$$

$$P = \underline{\underline{0.290 \text{ N}}}$$

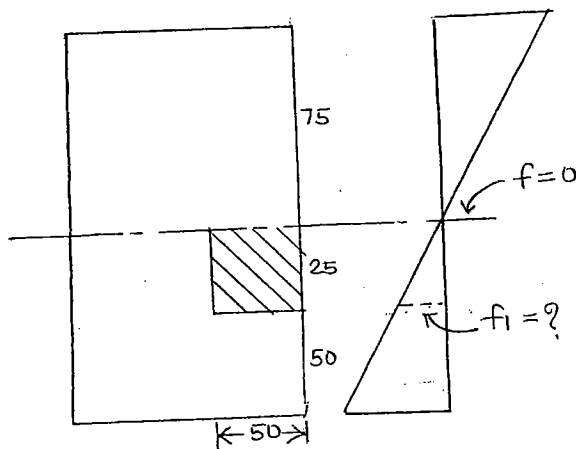
9. $\frac{E}{R} = \frac{M}{I} = \frac{f}{y} = \text{const.}$

$$f = ky$$

$$\frac{f_1}{f_2} = \frac{(y_{\max})_1}{(y_{\max})_2} = \frac{t/2}{2t/2} = \underline{\underline{\frac{1}{2}}}$$



14.

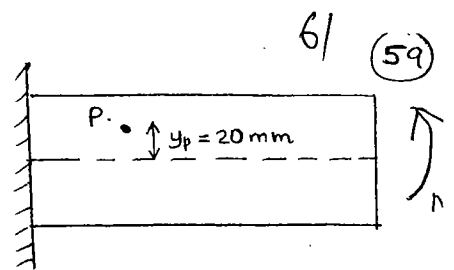
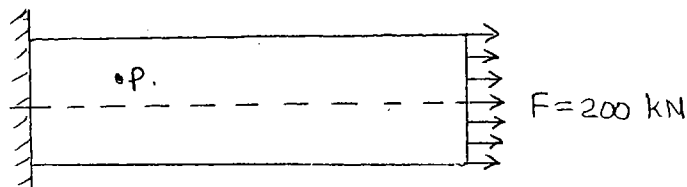


$$\frac{f_1}{y_1} = \frac{M}{I}$$

$$\frac{f_1}{25} = \frac{16 \times 10^6}{\left(\frac{100 \times 150^3}{12} \right)}$$

$$f_1 = \underline{\underline{14.2 \text{ MPa}}}$$

$$\begin{aligned} \text{Force on hatched area} &= \text{avg stress} \times \text{hatched area} \\ &= \frac{1}{2} (0 + f_1) \times 25 \times 50 = \underline{\underline{8.9 \text{ kN}}} \end{aligned}$$



$$\begin{array}{c} 2000 \text{ N/m}^2 \leftarrow \boxed{P} \rightarrow \end{array} \quad \sigma = \frac{F}{A} \text{ (tensile).}$$

$$= \frac{200}{0.1} = 2000 \text{ N/m}^2$$

$$\begin{array}{c} 3007 \text{ N/m}^2 \rightarrow \boxed{P} \leftarrow f_p. \end{array}$$

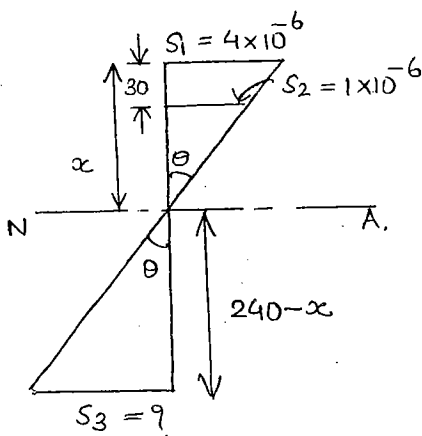
$$f_p = \frac{M}{I} y_p.$$

$$= \frac{200}{1.33 \times 10^{-3}} \left(\frac{20}{1000} \right).$$

$$= 3007 \text{ N/m}^2.$$

Resultant stress @ P:

$$\begin{array}{c} 1007 \text{ N/m}^2 \rightarrow \boxed{P} \leftarrow 1007 \text{ N/m}^2 \end{array}$$



$$\tan \theta = \frac{4 \times 10^{-6}}{x} = \frac{1 \times 10^{-6}}{x - 30} = \frac{S_3}{240 - x}.$$

$$x = 40 \text{ mm.}$$

$$\underline{\underline{S_3 = 20 \times 10^{-6}}}$$

$$\frac{E}{R} = \frac{f}{y}.$$

$$\frac{2 \times 10^5}{500/2} = \frac{f}{0.5/2} \Rightarrow \underline{\underline{f = 200 \text{ N/mm}^2}}$$

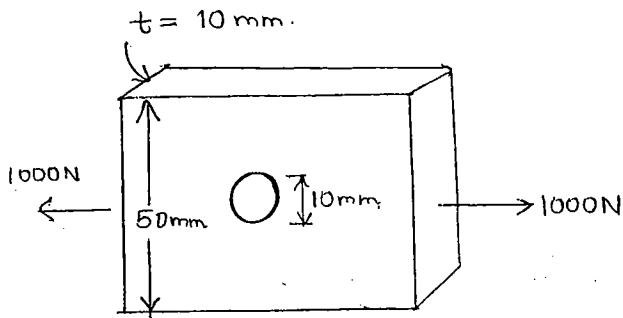
P-14.

$$4. (dl)_{sw} = \frac{wl}{2AE} \text{ (elongation)}$$

$$(dl)_{ext} = \frac{wl}{AE} \text{ (contraction).}$$

$$(dl)_{net} = dl_{sw} - dl_{ext} = \frac{wl}{2AE} - \frac{wl}{AE} = \ominus \frac{wl}{2AE} \text{ (contraction).}$$

8.

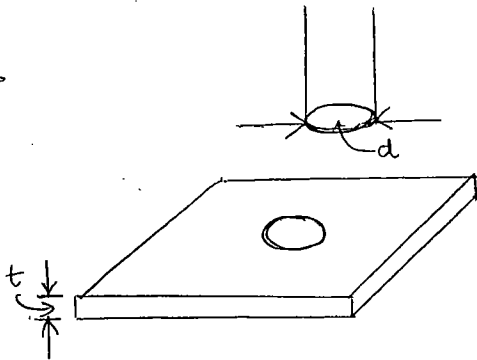


$$\sigma_{\max} = \frac{P}{A_{\min.}}$$

$$= \frac{1000}{(50-10) 10} = \underline{\underline{2.5 \text{ MPa}}}$$

Level 2

5.



Punching head force = shear resistance.

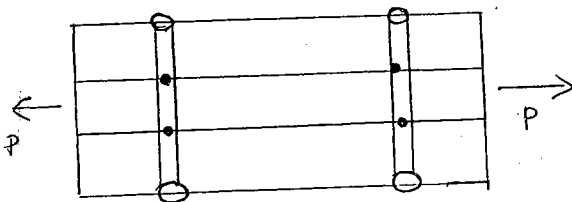
 σ (cls area of head) = τ (shearing area)

$$\sigma \left(\frac{\pi}{4} d^2 \right) = \tau (\pi d t)$$

$$47 \left(\frac{\pi}{4} d^2 \right) = \tau (\pi d t)$$

$$\Rightarrow \underline{\underline{\tau = d = 10 \text{ mm}}}$$

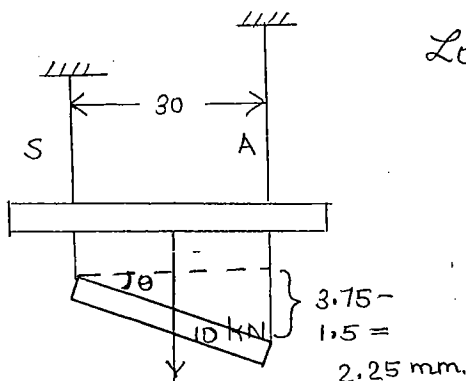
7



Rivet in double shear.

$$\text{Force for each cut} = \underline{\underline{\frac{P}{2}}}$$

16.



Load is acting at centre.

$$P_S = P_A = \frac{P}{2} = \frac{10}{2} = 5 \text{ kN.}$$

$$\sigma_S = \frac{P_S}{A_S} = \frac{5 \times 10^3}{0.1 \times 10^2} = 500 \text{ kN/mm}^2$$

$$\sigma_A = \frac{P_A}{A_A} = \frac{5 \times 10^3}{0.2 \times 10^2} = 250 \text{ kN/mm}^2$$

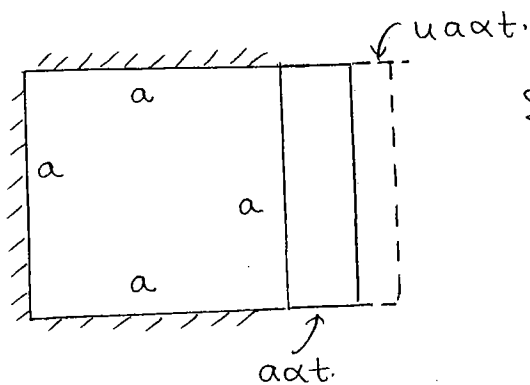
$$\delta l_A = \left(\frac{PL}{AE} \right)_A = \frac{5 \times 10^3 \times 1000}{(0.2 \times 10^2) (66667)} = 3.75 \text{ mm}$$

$$\delta l_S = \left(\frac{PL}{AE} \right)_S = \frac{5 \times 10^3 \times 600}{0.1 \times 10^2 \times 2 \times 10^5} = 1.5 \text{ mm.}$$

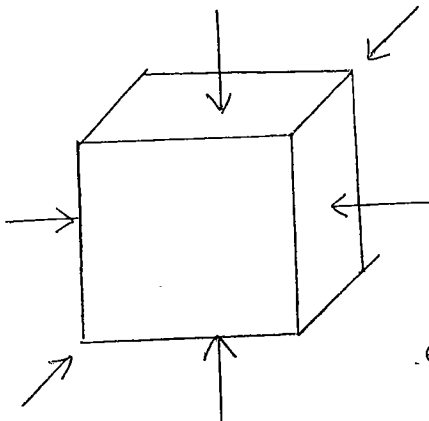
$$\sin \theta = \frac{2.25}{300} \Rightarrow \theta = \underline{\underline{0.43}} \text{ (cw)}$$

60

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$$\begin{aligned} \text{Total expansion} &= a\alpha t + u a\alpha t \\ &= \underline{\underline{a\alpha t(1+u)}} \end{aligned}$$



Due to temperature change,

$$\epsilon_x = \epsilon_y = \epsilon_z = \alpha \Delta T \rightarrow \textcircled{1}$$

Due to expansion prevented,

$$\epsilon_x = \epsilon_y = \epsilon_z = \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E} - \mu \frac{\sigma_z}{E}$$

$$\epsilon_x = \frac{-\sigma}{E} - \mu \left(\frac{-\sigma}{E} \right) - \mu \left(\frac{-\sigma}{E} \right) \rightarrow \textcircled{2}$$

Equating $\textcircled{1}$ & $\textcircled{2}$,

$$-\frac{\sigma}{E} + \mu \frac{\sigma}{E} + \mu \frac{\sigma}{E} = \alpha \Delta T$$

$$\sigma = \frac{\alpha E \Delta T}{(1-2\mu)}$$

If cube is free to expand in all directions, what is the temperature stress developed?

Zero

23 Oct,
THURSDAY

06 SHEAR STRESS IN BEAMS

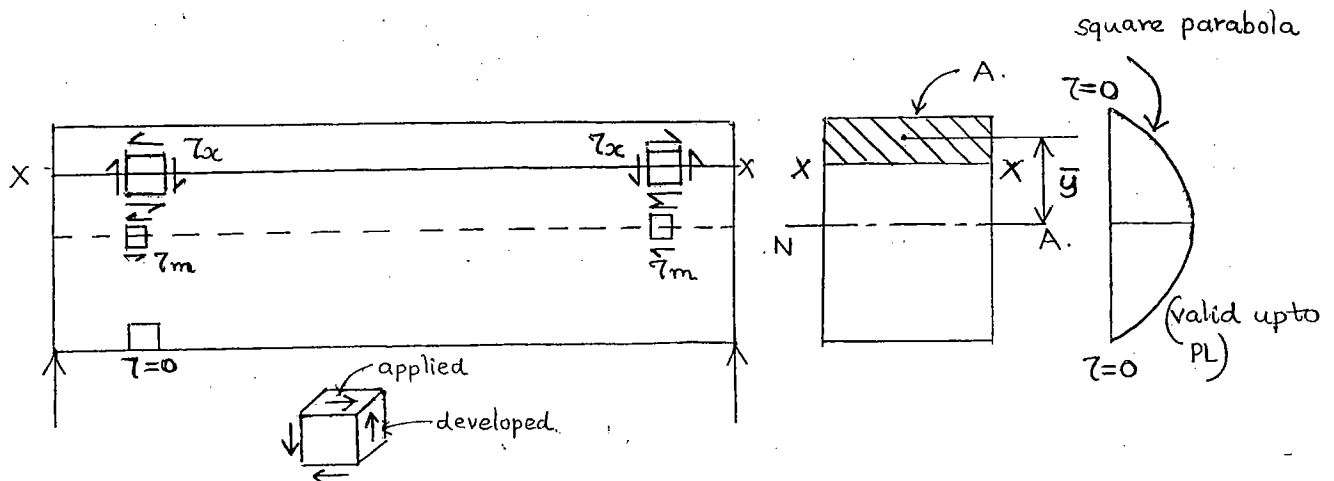
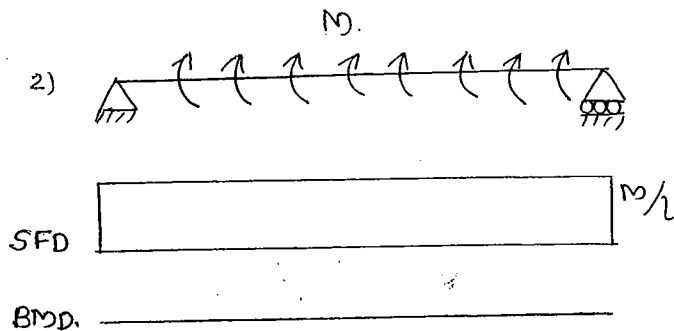
• Flexural shear stress (or) Indirect shear stress due to bending action in a beam.

• Pure shear occurs when;

SF = non zero const. and maximum.

BM = 0

Eg: 1) Deep beam ($D > 750$ mm) { Bending moment is almost ignored }



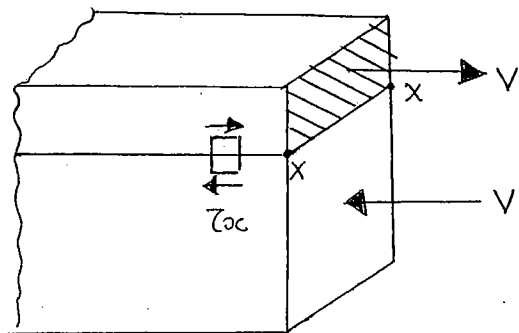
In a beam, the loading will be in transverse direction which causes layers of the beam move one over the other in the axial or longitudinal direction.

\therefore the critical shear stress in a beam is in axial direction of beam only.

To balance this shear, a complementary shear stress of

equal magnitude and opposite in direction develops on vertical planes as shown in fig

(6)
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$$\tau_{xc} = \frac{V A \bar{y}}{I b}$$

where $V \rightarrow$ SF at a c/s due to vertical or transverse loading.

$A \rightarrow$ the area either above or below the section X-X in the c/s. $\left\{ \begin{array}{l} A \text{ above NA} - (+ve) \\ A \text{ below NA} - (-ve) \end{array} \right\}$ net area is considered

$\bar{y} \rightarrow$ Distance to centroid of area from NA.

$I \rightarrow$ MI of entire c/s area (not the hatched area) about NA

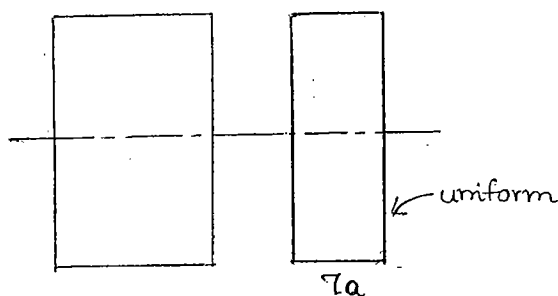
$b \rightarrow$ width of c/s parallel to NA where shear stress is required

$$\tau_{xc} = \frac{V}{I} \frac{A \bar{y}}{b}$$

const. \rightarrow variables @ a c/s $\left\{ \text{unit: } \frac{m^2 \cdot m}{m} = m^2 \right\}$

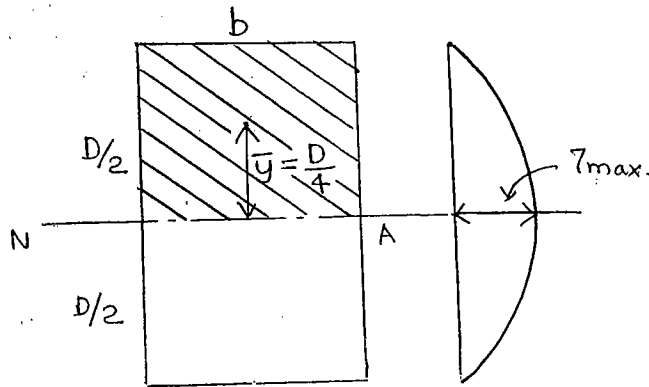
* Average Shear Stress:

$$\tau_a = \frac{V}{\text{c/s area}} ; \text{uniform in c/s}$$



→ Relation b/w τ_m & τ_{avg} .

1. Rectangular / Square.



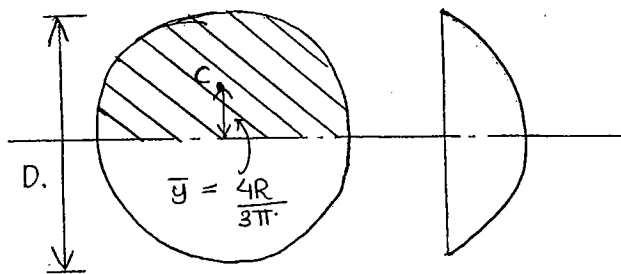
$$\begin{aligned}\tau_m &= \frac{V A \bar{y}}{I b} \\ &= \frac{V \cdot \left(b \cdot \frac{D}{2}\right) \left(\frac{D}{4}\right)}{\frac{b D^3}{12} \cdot b}\end{aligned}$$

$$\tau_a = \frac{V}{b D}$$

$$\Rightarrow \boxed{\frac{\tau_m}{\tau_a} = \frac{3}{2}}$$

$$\tau_m = 1.5 \tau_a \quad (50\% \text{ more than } \tau_a)$$

2. Solid Circular.



$$\begin{aligned}\tau_m &= \frac{V \cdot \frac{\pi d^2}{8} \times \frac{2d}{3\pi}}{\frac{\pi d^4}{64} \cdot d}\end{aligned}$$

$$\tau_a = \frac{V}{\frac{\pi d^2}{4}}$$

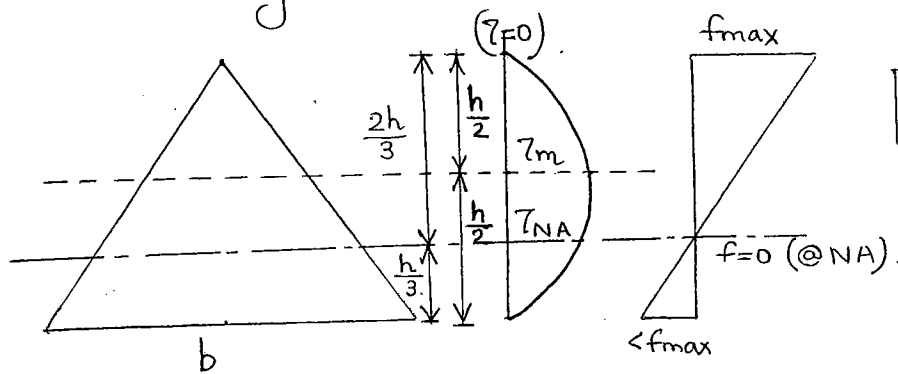
$$\boxed{\frac{\tau_m}{\tau_a} = \frac{4}{3}}$$

$$\tau_m = 1.33 \tau_a \quad (33\% \text{ more than } \tau_a)$$

⊙ In a beam, shear stress is secondary criteria, and main design criteria is bending. So τ_a is considered instead of τ_m .

3. Triangular.

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$$\tau_{NA} < \tau_{max}$$

$$\frac{\tau_m}{\tau_a} = \frac{3}{2} = \text{same as square/rect.}$$

$$\frac{\tau_{NA}}{\tau_a} = \frac{4}{3} = \text{same as solid circular section}$$

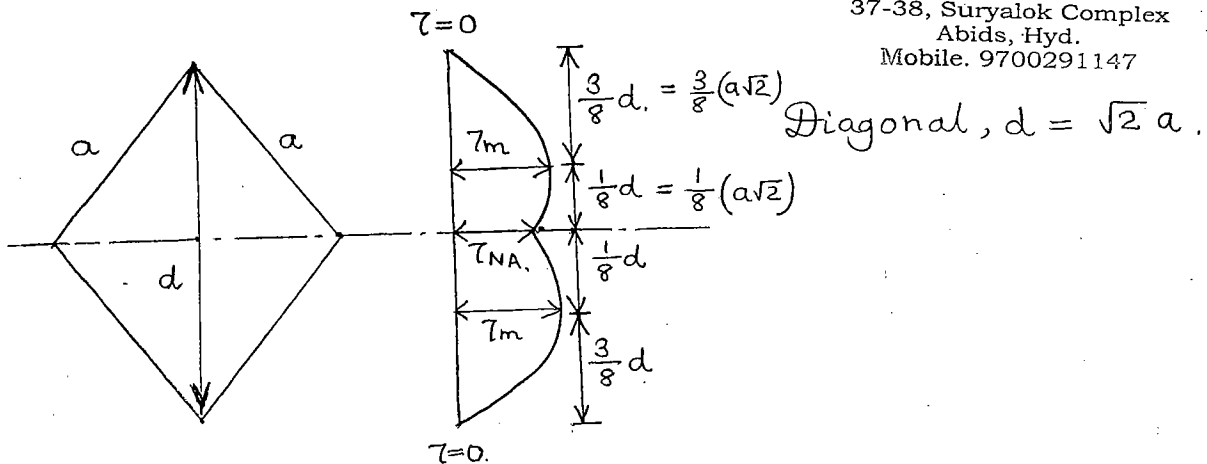
$$\frac{\tau_m}{\tau_{NA}} = \frac{9}{8}$$

At the point of max bending stress, (f_{max}), shear stress must be zero ($\tau=0$).

At the point of max shear stress (τ_m), bending stress need not be zero.

4. Diamonds.

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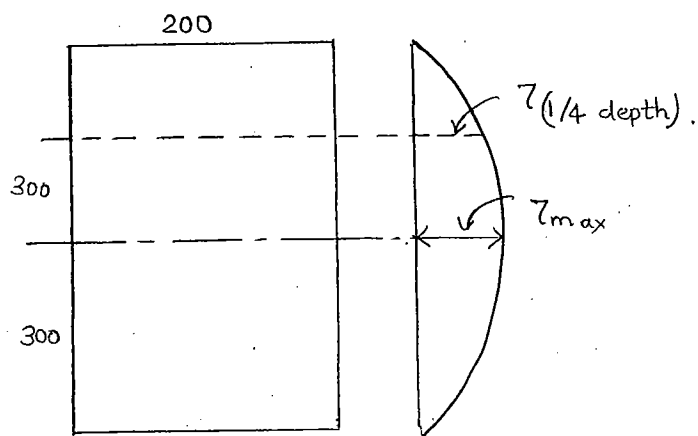
$$\frac{\tau_m}{\tau_a} = \frac{9}{8}$$

$$\frac{\tau_{NA}}{\tau_{avg}} = 1$$

$$\frac{\tau_m}{\tau_{NA}} = \frac{9}{8}$$

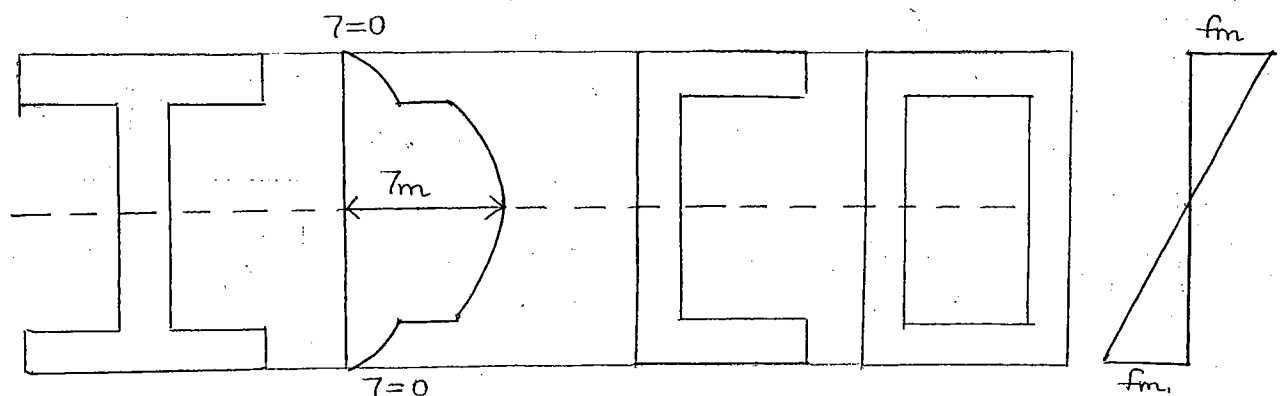
$$\tau_m = \frac{9}{8} \tau_a = 1.125 \tau_a \text{ (12.5\% more than } \tau_a)$$

Section	τ_m/τ_a	τ_n/τ_a
Rectangular/ Square	$3/2$	$3/2$
Circular	$4/3$	$4/3$
Triangle.	$3/2$	$4/3$
Diamond.	$9/8$	1

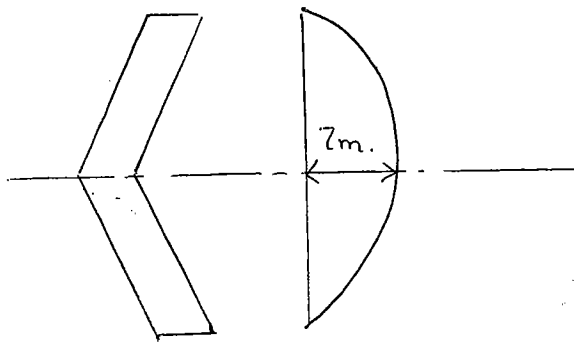
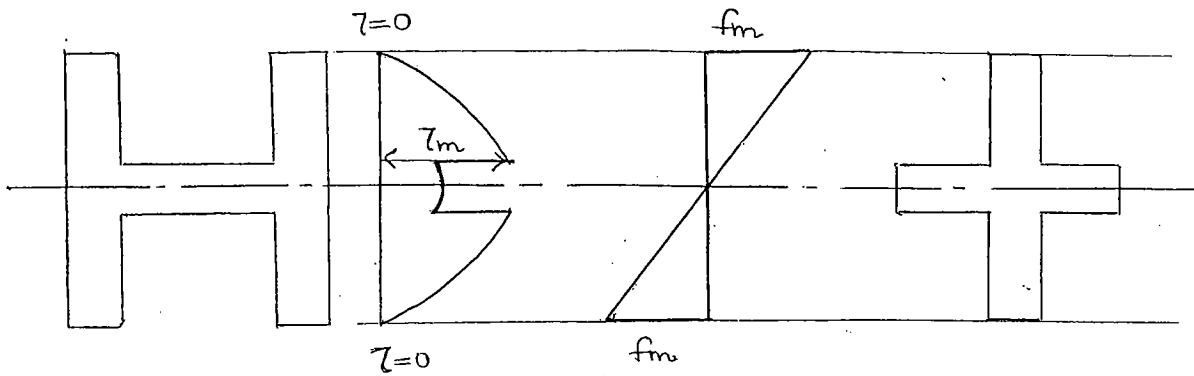
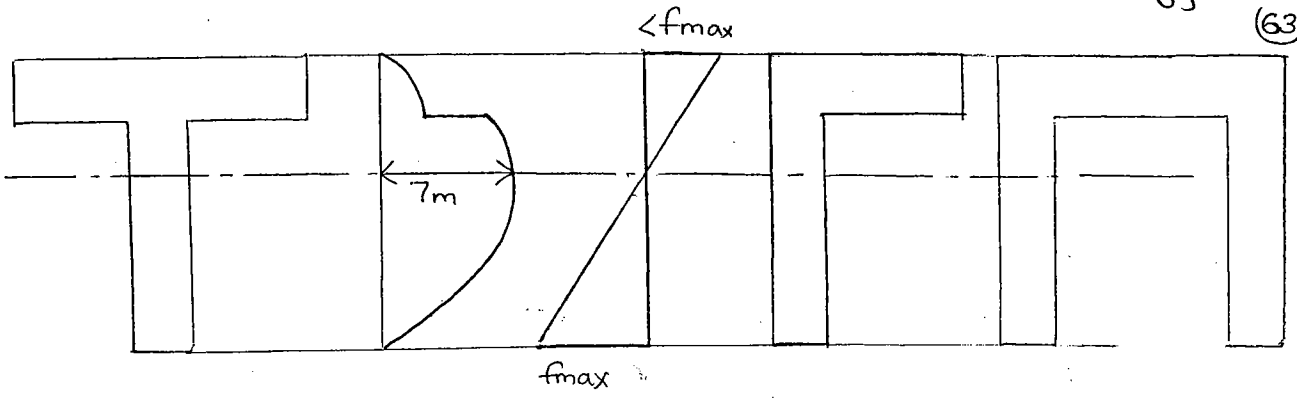


$$\frac{\tau_{(1/4 \text{ depth})}}{\tau_m} = \frac{V \cdot (200 \times 150) (75 + 150)}{I_b} \div \frac{V (200 \times 300) (150)}{I_b} = \underline{\underline{\frac{3}{4}}}$$

→ Flanged Beams



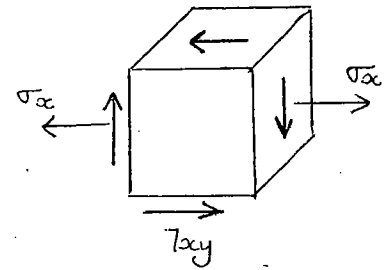
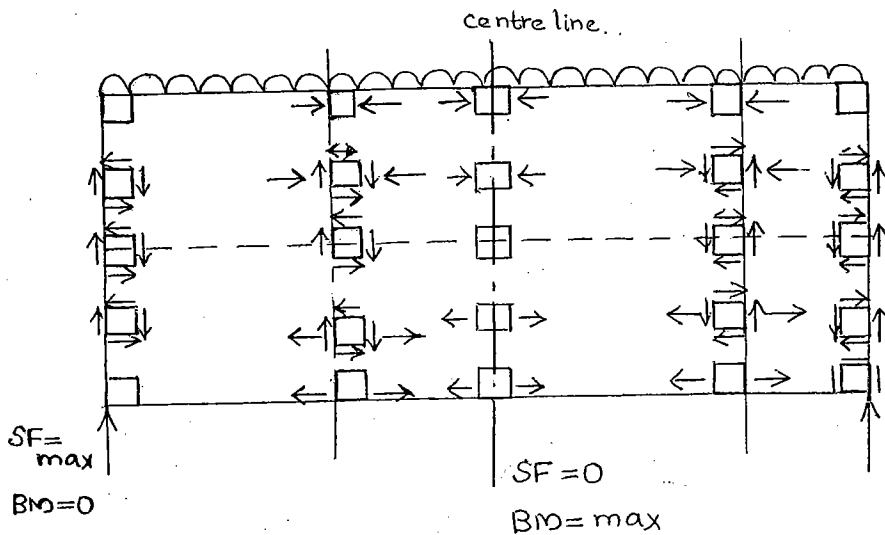
In flanged beams, max. shear stress is taken by web, max bending stress taken by flange.



$$\tau = \frac{VA\bar{y}}{Ib} \Rightarrow \tau \propto \frac{1}{b}$$

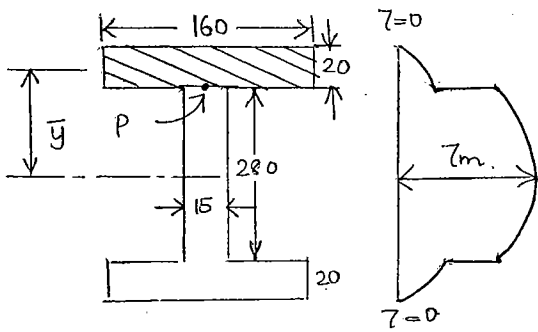
ie

$b \uparrow$	$\tau \downarrow$
$b \downarrow$	$\tau \uparrow$



$\sigma_x = f ; \tau_{xy} = \tau_{yx} = \tau$
 $\sigma_y = 0 ; \tau_{xz} = 0 = \tau_z$
 $\sigma_z = 0 ; \tau_{xyz} = 0 = \tau_{zy}$

But $\epsilon_x \neq 0$
 $\epsilon_y \neq 0$
 $\epsilon_z \neq 0$



NOTE :

⊙ In a beam, stresses in the width direction (z direction) will be zero. ∴ beam can be taken as a plane stress system. However, the strain in the width of (or z direction) direction is not zero.

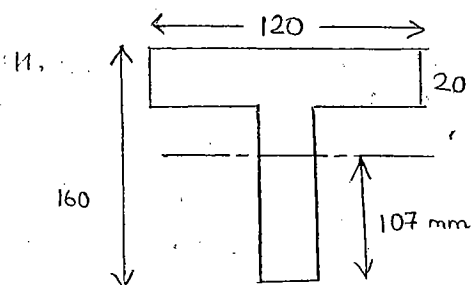
$$8. \quad I_{NA} = \frac{160 \times 320^3}{12} - \frac{145 \times 280^3}{12} = \underline{\underline{171.6 \times 10^6 \text{ mm}^4}}$$

$$\begin{aligned} \tau_p &= \frac{VA\bar{y}}{I b_p} = \frac{200 \times 10^3 \times (160 \times 20) \cdot (140 + 10)}{171.6 \times 10^6 \times (15)} \\ &= \underline{\underline{37.296 \text{ MPa}}} \quad \rightarrow \text{(in web).} \end{aligned}$$

$$10. \quad \tau_p = \frac{200 \times 10^3 \times 160 \times 20 (150)}{171.6 \times 10^6 \times \frac{160}{2}} = \underline{\underline{3.496 \text{ MPa}}} \quad \rightarrow \text{(in flange).}$$

$$9. \quad \bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{160 \times 20 \times 150 + 140 \times 15 \times 70}{160 \times 20 + 140 \times 15} = 118.30 \text{ mm}$$

$$\tau_m = \frac{200 \times 10^3 \times (160 \times 20 + 140 \times 15) \cdot 118.30}{171.6 \times 10^6 \times 15} = \underline{\underline{48.71 \text{ MPa}}}$$



$$\begin{aligned} \tau_{\max} &= \frac{VA\bar{y}}{I b} = \frac{140 \times 10^3 \times 107 \times 20 \times \frac{107}{2}}{13 \times 10^6 \times 20} \\ &= \underline{\underline{61.65 \text{ MPa}}} \end{aligned}$$

$$f = \frac{M}{Z} = \frac{wl/4}{\frac{bd^2}{6}}$$

$$= \frac{3wl}{2bd^2} = 12.$$

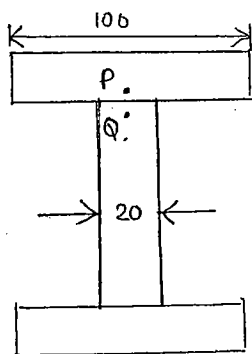
$$q = \frac{VA\bar{y}}{Ib} = \frac{\frac{w}{2} \times bd \times \frac{d}{2}}{\frac{bd^3}{12} \times b} = 1.2.$$

$$= \frac{3w}{bd} = 1.2.$$

$$\frac{f}{q} = \frac{12}{1.2} = \frac{3wl/bd^2}{2 \times 3w/bd}.$$

$$\frac{bd}{bd^2}$$

$$\frac{10}{2} = \frac{l}{d} \Rightarrow \frac{l}{d} = 5$$



$$\tau_Q = \tau_p \times \frac{100}{20} = \underline{60 \text{ MPa}}$$