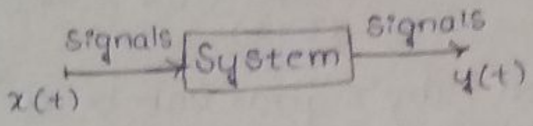
18/01/2021

UNIT-01: Basics of Signals and Systems



Signal:

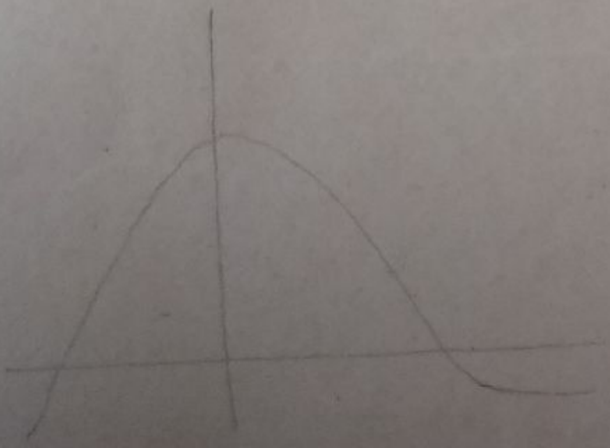
→ Set of data or information
→ Dependent Variable
→ Function of one or more independent variables
→ Single variable function, $f(x)$
→ Multi variable function, $f(x_1, x_2, ----, x_n)$

Systems:

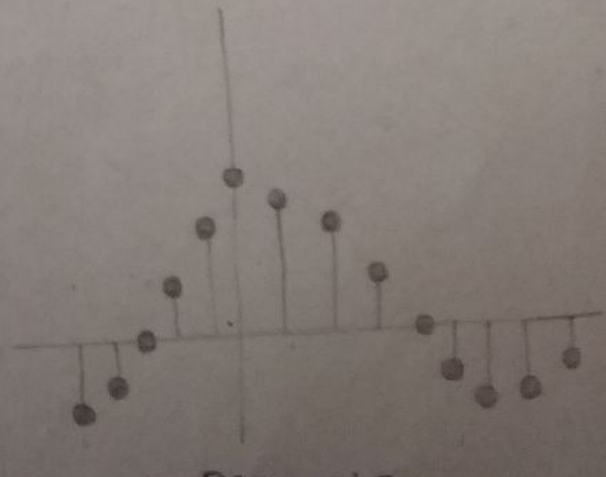
→ Interconnection of physical devices and components.
→ Modify or extract information from signals.
→ Processes set of signals and yields another set of signals.
→ System can be a Electrical system, mechanical or hydraulic.
→ Algorithm Based.

Types of Signals:

- Continuous-time signals - independent variable is continuous.
- Discrete-time signals.

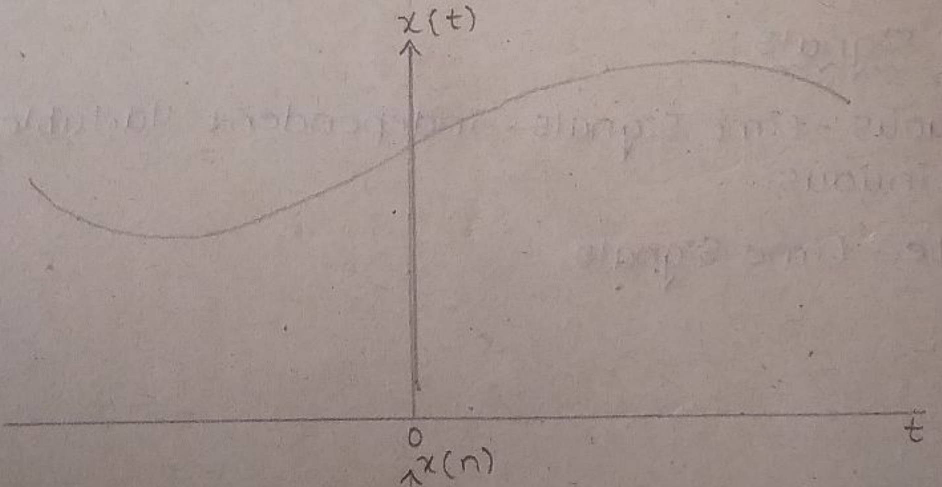


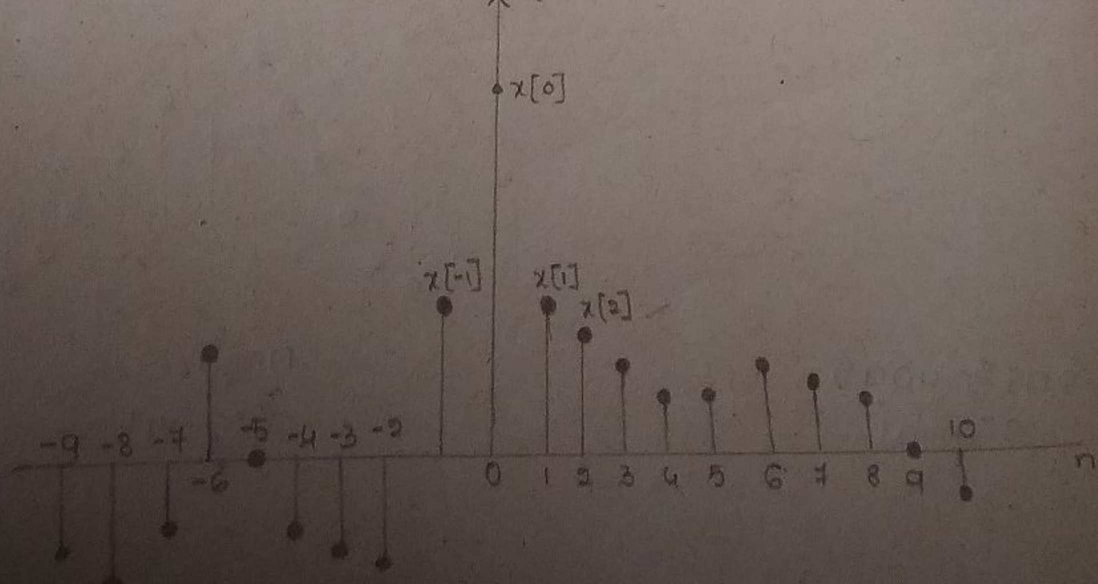
Continuous signal



Discrete signal

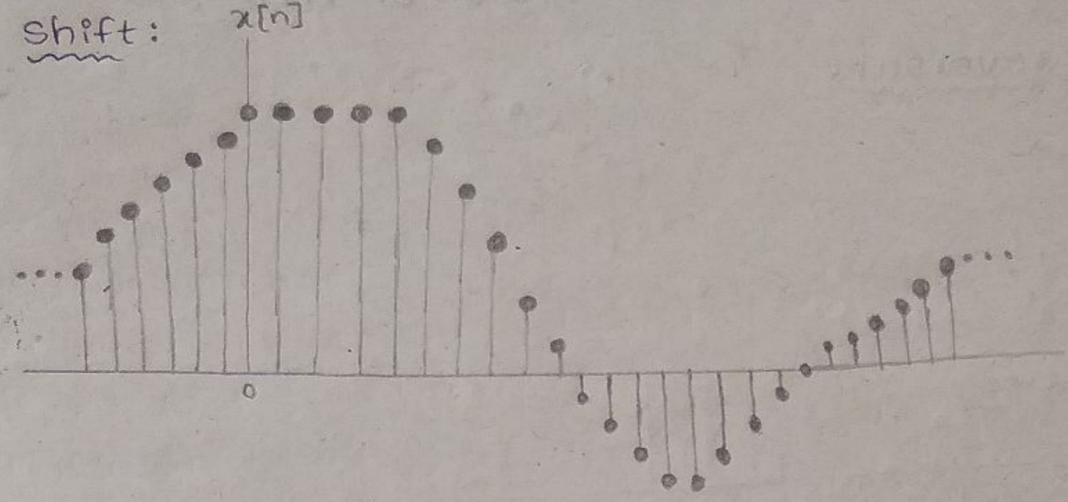
Continuous Time (CTS)	Discrete time (DTS)
The signal can be defined at any time instance and they can take all values in the continuous interval (a,b) where a can be $-\infty$ and b can be ∞.	The signal can be defined only at certain specific Values of time. These time instance need not be equidist-ant but in practice they are usually takes at equally speed intervals.
These are described by differential equations.	These are described by difference equation.
This signal is denoted by $x(t)$.	These signals are denoted by $x(n)$ or notation $x(nT)$ can also be used.
The speed control of a dc motor using a tacho generator feedback or Sine or exponential waveforms..	Microprocessors and computer based systems uses discrete time signals.

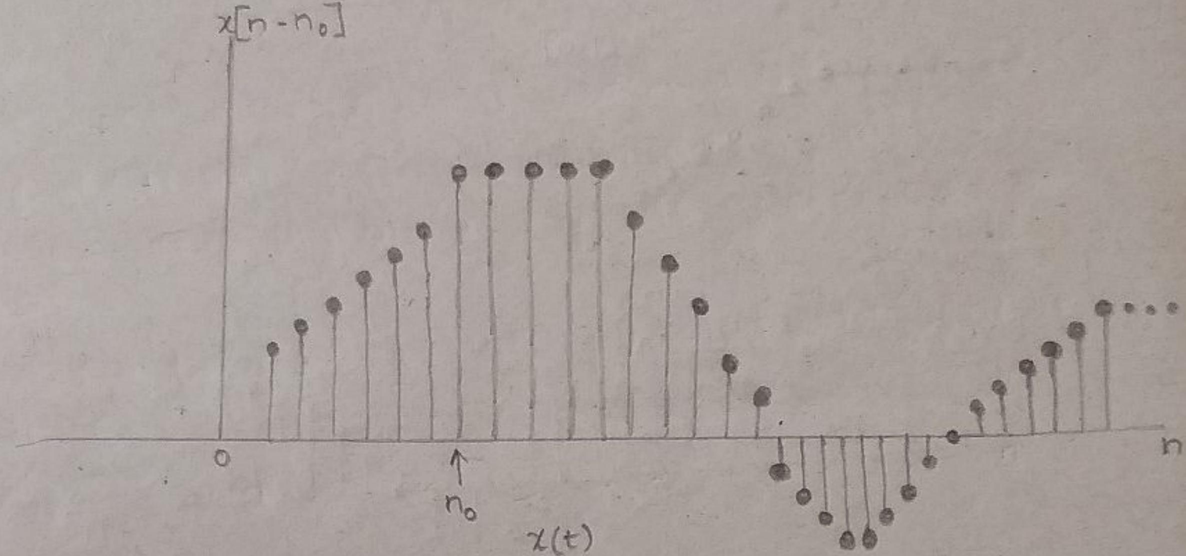


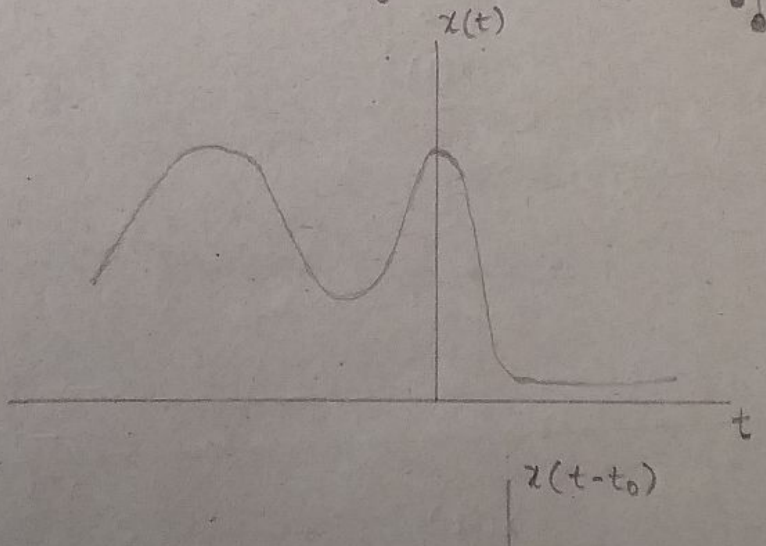


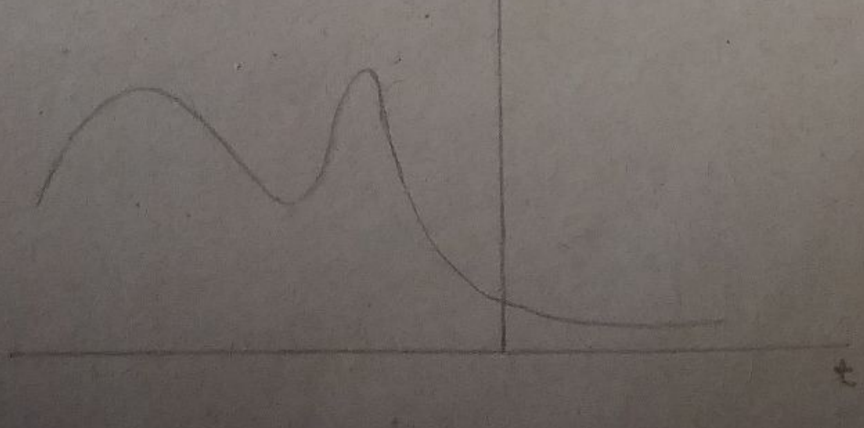
Transformations of the independent variable:

Time shift:



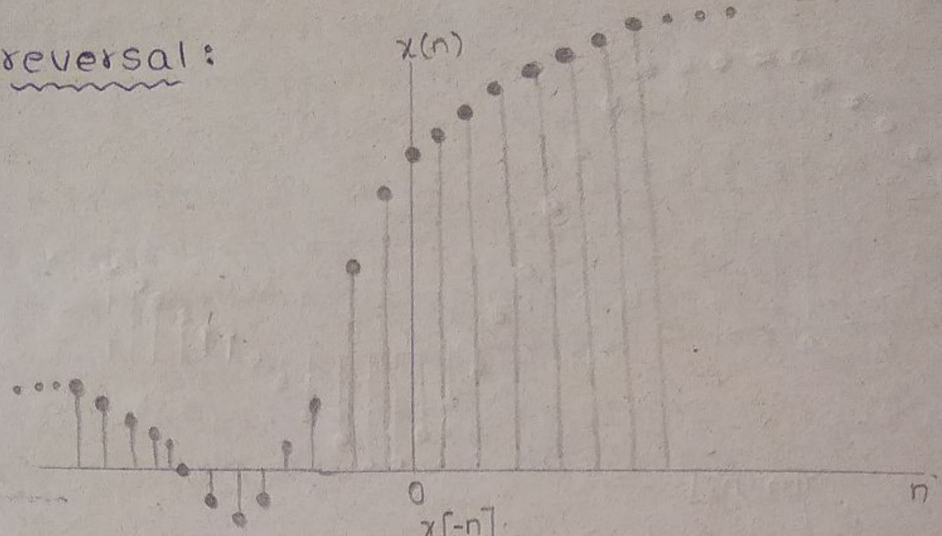


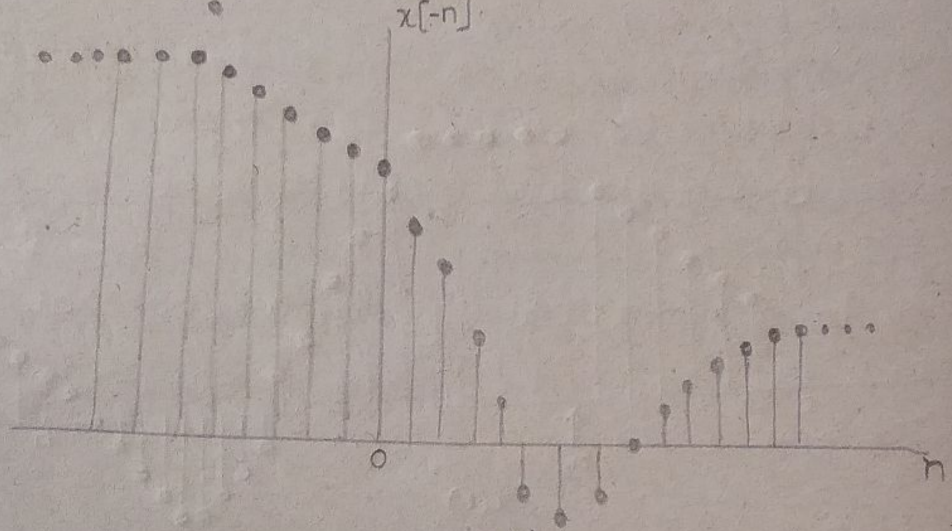


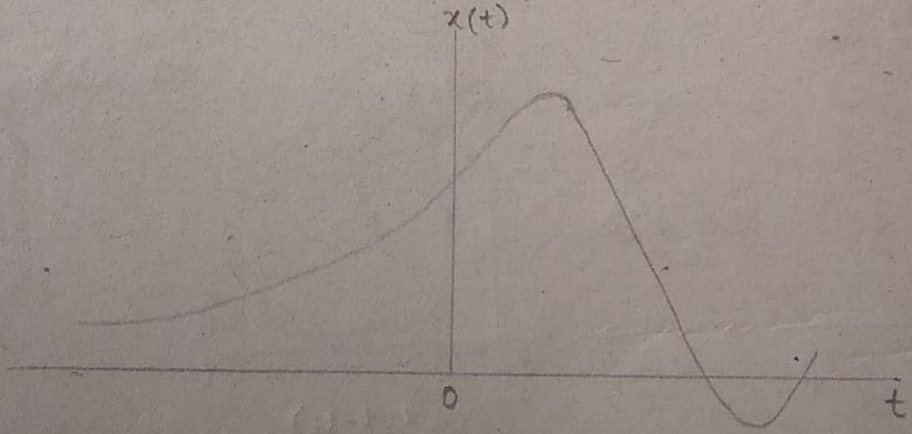


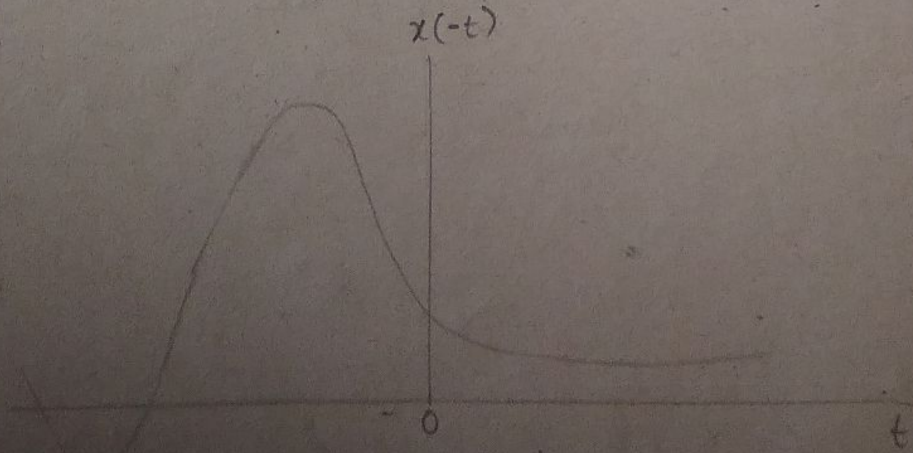
- $x(t-t_0)$ represents a delayed (if t_0 is +ve).
- Advanced (if t_0 is -ve) version of $x(t)$.

Time reversal:

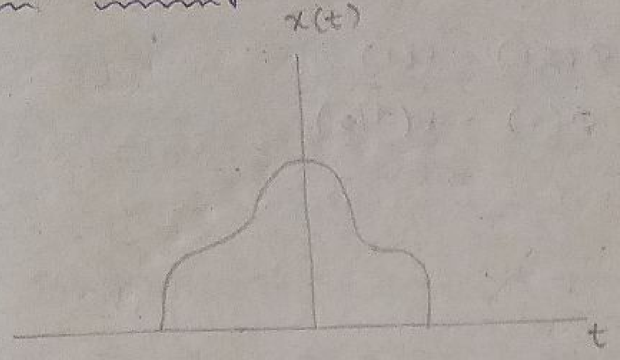


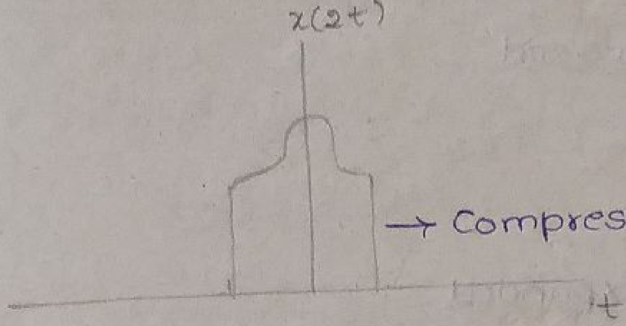


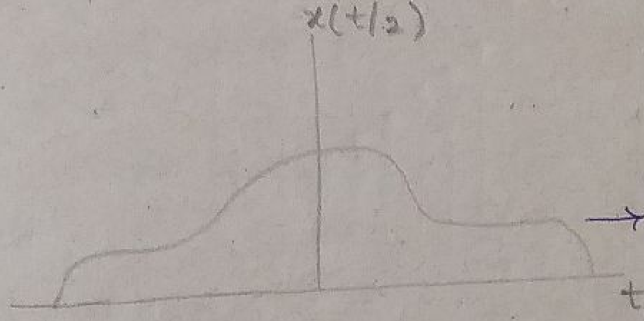




Time Scaling :







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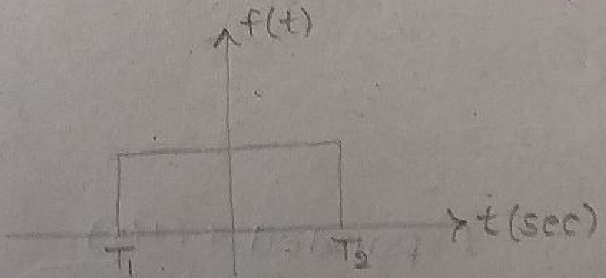
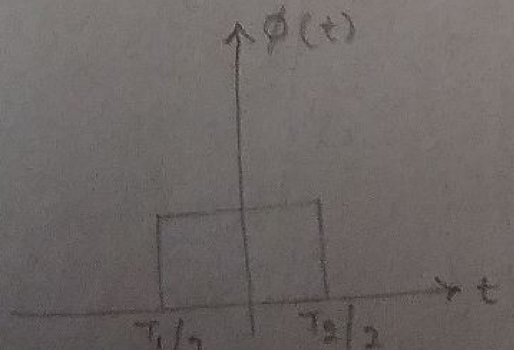
Transformations on signals:

Time Scaling → Compression, → Expansion



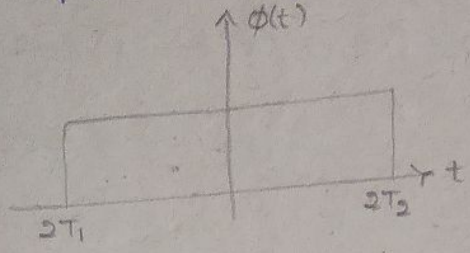
Compression by 2 sec



$$\phi(t) = f(2t)$$

$$\phi\left(\frac{t}{2}\right) = f(t)$$

Expansion by 2 sec



$\phi(2t) = f(t)$

$\phi(t) = f(t/2)$

→ $\phi(t) = f(2t)$

$\phi(t) = f(at)$

here $a = 2 > 1 \Rightarrow$ Compressed

→ $\phi(t) = f(t/2)$

$\phi(t) = f(at)$

here $a = \frac{1}{2} = 0.5 < 1 \Rightarrow$ Expanded

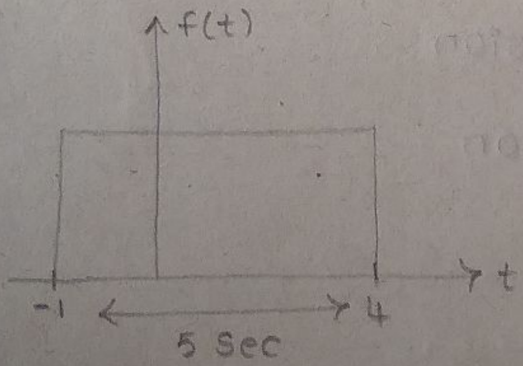
Conclusion:

$$f(t) \xrightarrow[\text{scaling}]{\text{time}} f(at)$$

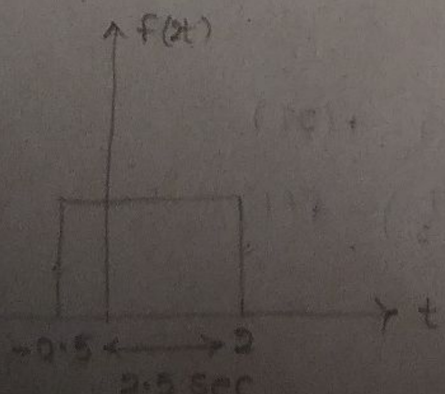
Compressed $a > 1$

Expanded $a < 1$

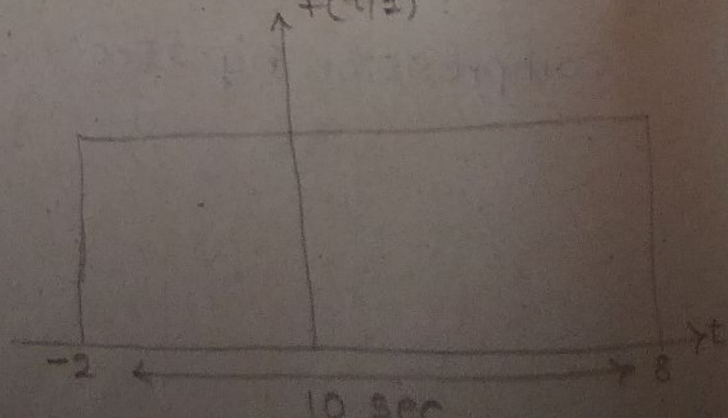
For example:



Compressed $f(2t)$



Expanded $f(t/2)$



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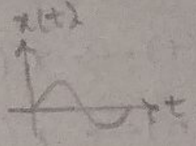
Types of signals → Continuous time signal 

Types of signals → Discrete time signal 

Transformations of independent variable (t or n)

1. Time reversal: (folding / reflection) → reversing the signal → identical in shape

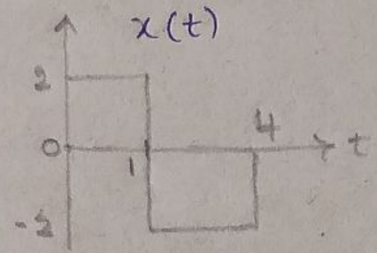
t is replaced -t (continuous time signal)

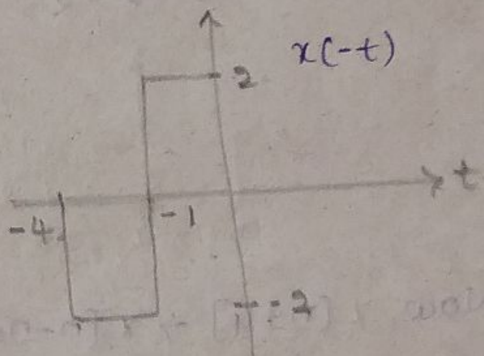
n is replaced -n (Discrete time signal)

$x(t) \longrightarrow x(-t)$

$x(n) \longrightarrow x(-n)$

Ex:





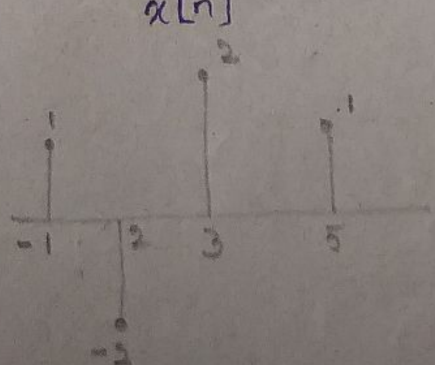
$0 < t < 1 \quad x(t) = 2$

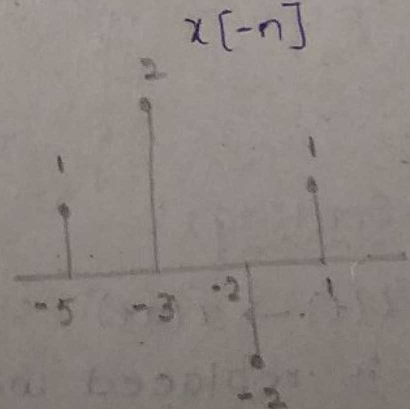
$1 < t < 4 \quad x(t) = -2$

otherwise $x(t) = 0$

$0 > t > -1 \quad x(t) = 2$

$-1 > t > -4 \quad x(t) = -2$





$n = -1 \quad x[n] = 1$

$2 \quad x[n] = -2$

$3 \quad x[n] = 2$

$5 \quad x[n] = 1$

$n = 1 \quad x[n] = 1$

$= -2 \quad x[n] = -2$

$= -3 \quad x[n] = 2$

$= -5 \quad x[-n] = 1$

2. Time shifting:
→ shifted / displaced
→ identical in shape

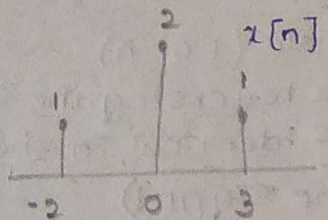
$x(t) \rightarrow x(t-t_0)$ $\quad t_0, n_0 \rightarrow$ amount of shift

$x(n) \rightarrow x(n-n_0)$

$t_0 = +ve \rightarrow$ Right shift (signal is delayed)

$t_0 = -ve \rightarrow$ Left shift (signal is advanced)

Ex:

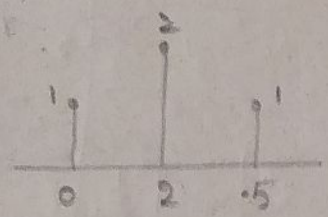


$x[n-2] \rightarrow x[n-n_0]$, here $n_0 = 2$, right shift

$\Rightarrow -2+2 = 0$

$0+2 = 2$

$3+2 = 5$

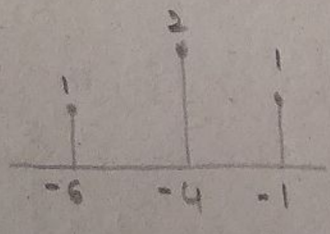


Now, $x[n+4] \rightarrow x[n-n_0]$ $\quad n_0 = -4$, left shift

$\Rightarrow -2-4 = -6$

$0-4 = -4$

$3-4 = -1$



3. Time Scaling:

$x(t) \rightarrow x(at)$

t is replaced with at

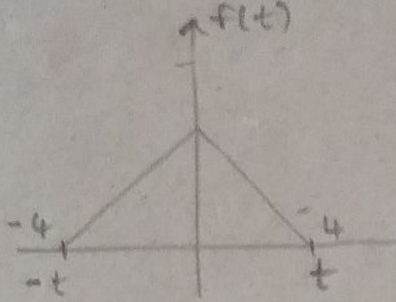
$a > 1 \rightarrow$ compression

$a < 1 \rightarrow$ expansion

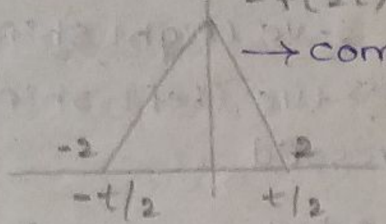
→ If $x(t)$ is an input function, then time scaling transformation will result an output signal $x(at)$.

Ex :

(i)

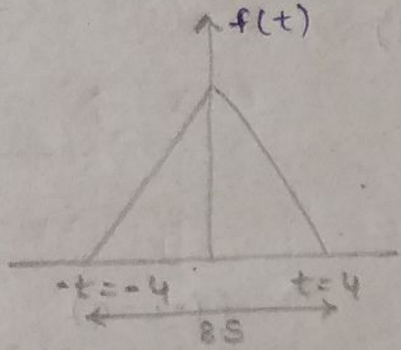


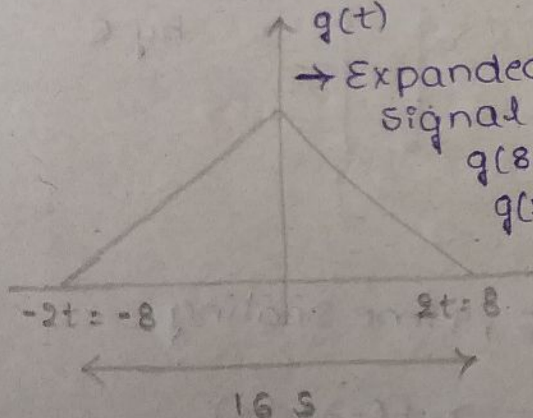
$\phi(t)$ scaled by 2
$= f(2t)$
$\rightarrow$ compressed signal



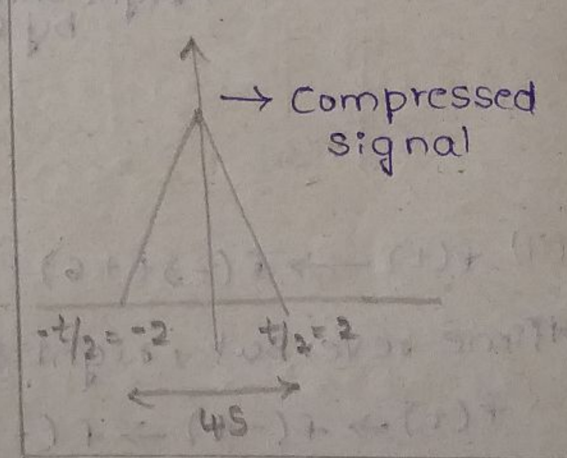
$\phi(t/2) = f(t)$
$\Rightarrow \phi(t) = f(2t)$
$\phi(2s) = f(-4s)$

(ii)

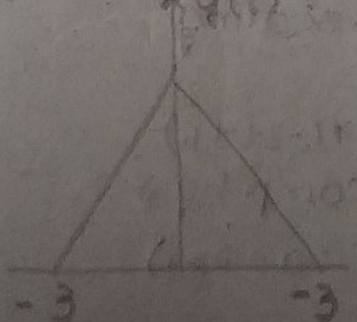


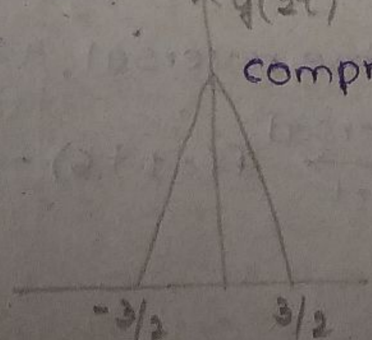


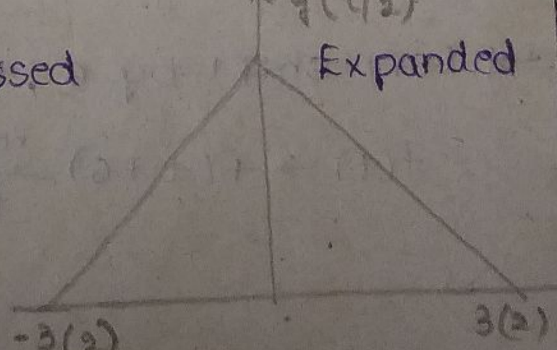
$g(8s) = f(4s)$
$g(2t) = f(t)$
$g(t) = f(t/2)$



In simple:







$f(-at)$ $\rightarrow t \rightarrow t$ (time reversal)

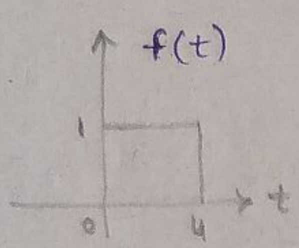
$f(-at)$ $\rightarrow t \rightarrow at$ (time scaling)

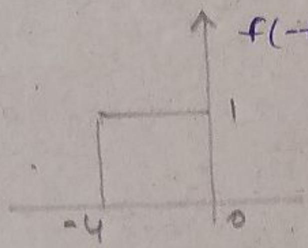
$f(-at+b)$ → time reversal $(-t)$

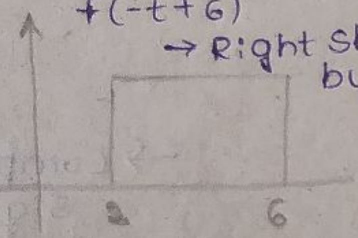
→ time shifting by b → $-ve$ (Right shift) / $+ve$ (Left shift)

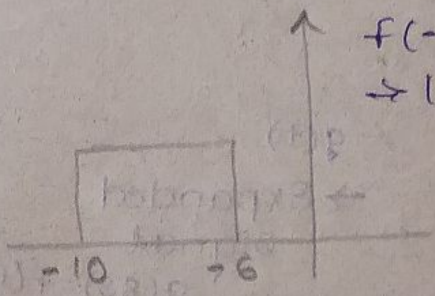
→ time scaling (a) → >1 compressed / <1 Expanded

Ex:
(i)









(ii) $f(t) \longrightarrow f(-3t+6)$

→ Time reversal, right shift by 6, time scaling

$f(t) \rightarrow f(-t) \rightarrow f(-(t-6)) \longrightarrow f(-3t+6)$

$= f(-t+6)$ compressed by 3 $(\because a = 3 > 1)$

→ Left shift by 6, time reversal, time scaling

$f(t) \rightarrow f(t+6) \xrightarrow[t \rightarrow -t]{\text{reversed}} f(-t+6) \longrightarrow f(-3t+6)$

comp by 3 $(\because a = 3 > 1)$

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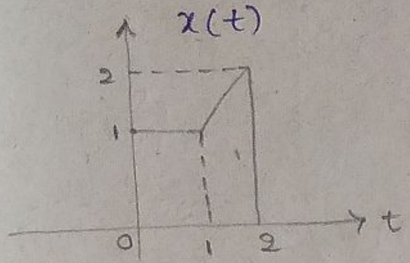
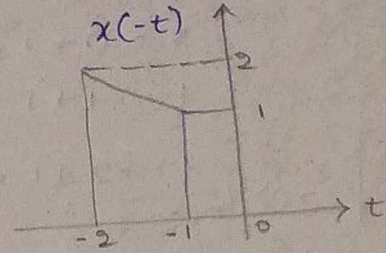
Example :

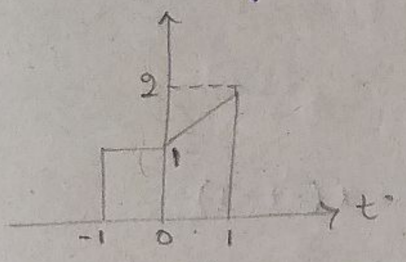


$\longrightarrow$



$x(t)$

Left shift by (1)



LS by (-1)

for $x(t)$

$t = 0 \quad 1$

$t = 1 \quad 1$

$t = 2 \quad 2$

$x(t) \longrightarrow x(-t)$

$(t) \longrightarrow (-t)$

for $x(-t)$

$t = 0 \quad 1$

$t = -1 \quad 1$

$t = -2 \quad 2$

for LS by (1)

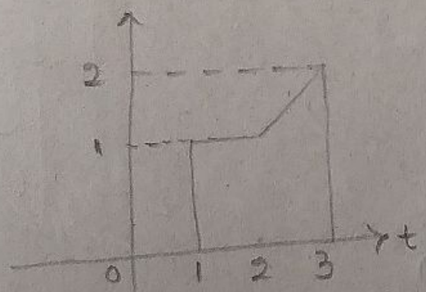
$x(t) \longrightarrow x(t+1)$

$t = 0 - 1 = -1$

$t = 1 - 1 = 0$

$t = 2 - 1 = 1$

Right shift by 1



for rs by (1)

$x(t) \longrightarrow x(t-1)$

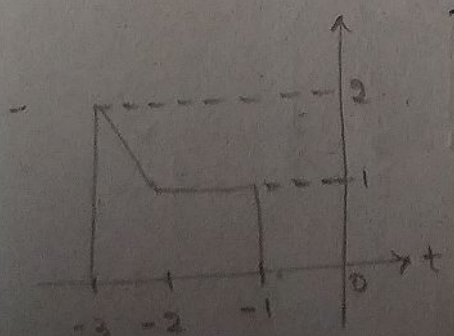
$t = 0 + 1 = 1$

$t = 1 + 1 = 2$

$t = 1 + 2 = 3$

$x(-t)$

Left shift by (1)



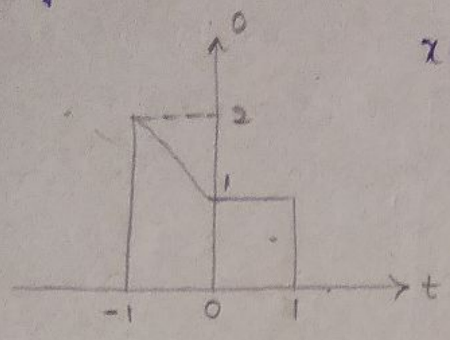
$x(-t) \longrightarrow x(-t+1)$

$t = 0 - 1 = -1$

$t = -1 - 1 = -2$

$t = -2 - 1 = -3$

Right shift by (1)



$x(-t) \rightarrow x(-t-1)$

$t = 0+1 = 01$

$t = -1+1 = 0$

$t = -2+1 = -1$

Hence

$x(t) \xrightarrow{RS} x(t-1)$

$x(t) \xrightarrow{LS} x(t+1)$

$x(t) \xrightarrow{rev} x(-t) \xrightarrow{L.S} x(-(t+1)) = x(-t-1)$

$x(t) \xrightarrow{rev} x(-t) \xrightarrow{R.S} x(-(t-1)) = x(-t+1)$

1. $x(t) \xrightarrow{rev} x(-t)$

$x(t+2) \rightarrow x(-t+2)$

$x(t) \xrightarrow{Rs\ by\ 1} x(t-1)$

$x(-t) \xrightarrow{Rs\ by\ 1} x(-(t-1)) \rightarrow x(-t+1)$

$x(t) \xrightarrow{Ls\ by\ 3} x(t+3)$

$x(-t) \xrightarrow{Ls\ by\ 3} x(-(t+3)) \rightarrow x(-t-3)$

2. $x(-3t+6) \rightarrow$ 6 differentways

(i) reversal, shifting, Scaling

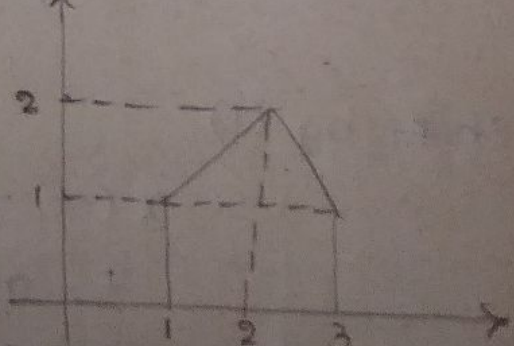
(ii) reversal, scaling, shifting

(iii) Scaling, rev, shifting

(iv) scaling, shift, rev

(v) Shift, scaling, rev

(vi) Shift, rev, Scaling



Now,

(i) reversal, shifting, scaling

$$x(t) \xrightarrow{rev} x(-t) \xrightarrow[by\,6]{R\text{-}Shift} x(-t+6) \xrightarrow[by\,3]{scaling} x(-3t+6)$$

(ii) reversal, scaling, shifting

$$x(t) \xrightarrow{rev} x(-t) \xrightarrow[by\,3]{scaling} x(-3t) \xrightarrow[Shift\ by\ 2]{Right} x(-3t+6)$$

(iii) Scaling, rev, shifting

$$x(t) \xrightarrow[by\,3]{scaling} x(3t) \xrightarrow{rev} x(-3t) \xrightarrow{RS\ by\ 2} x(-3(t-2)) = x(-3t+6)$$

(iv) Scaling, shifting, reversal

$$x(t) \xrightarrow[by\,3]{Scaling} x(3t) \xrightarrow{LS\ by\ 2} x(3(t+2)) \xrightarrow{rev} x(-3t+6)$$

$$x(3t+6)$$

(v) shifting, scaling, reversal

$$x(t) \xrightarrow{LS\ by\ 6} x(t+6) \xrightarrow[by\,3]{scaling} x(3t+6) \xrightarrow{rev} x(-3t+6)$$

(vi) shifting, reversal, scaling

$$x(t) \xrightarrow{LS\ by\ 6} x(t+6) \xrightarrow{rev} x(-t+6) \xrightarrow[by\,3]{Scaling} x(-3t+6)$$

3. $x(3-n) = x[-n+3]$

(i) reversal, shifting

$$x(n) \xrightarrow{rev} x(-n) \xrightarrow{R\text{-}S\ by\ 3} x[-(n-3)] = x[-n+3]$$

(ii) shifting, reversal

$$x(n) \longrightarrow x[n+3] \longrightarrow x[-n+3]$$

(iii) RS, reversal

$$x(n) \longrightarrow x(n-3) \longrightarrow x(-n-3) \times$$

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$x(n) \to$ integer $(-\infty$ to $\infty)$

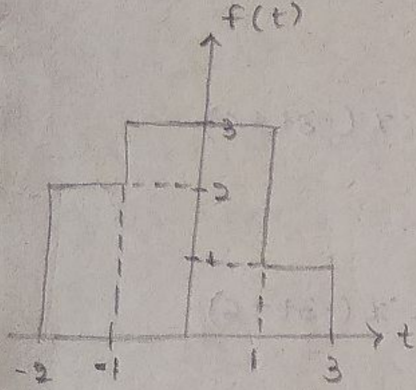
$x(t) \to (-\infty$ to $\infty)$

4. $f(-3t-7)$

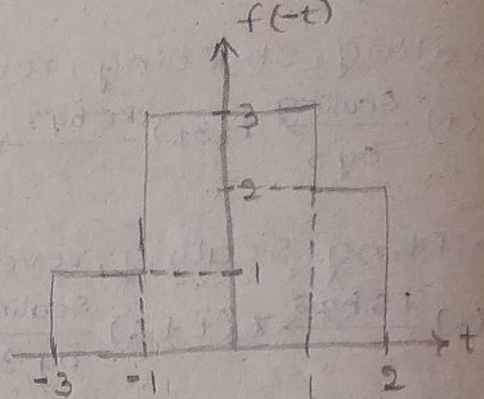
→ time shifting
→ time scaling
→ time reversal

(i) reversal, shifting, scaling

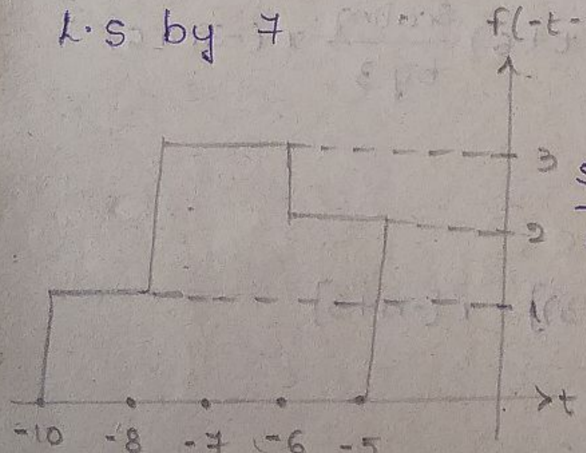
$$f(t) \longrightarrow f(-t) \xrightarrow[(t+7)]{\text{L.S by 7}} f(-(t+7)) = f(-t-7) \xrightarrow[\text{by 3 (comp)}]{\text{scaling}} f(-3t-7)$$



rev →



L.S by 7

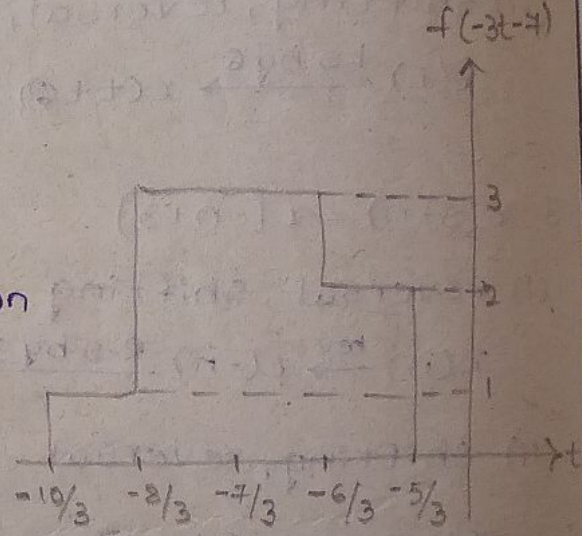


Scale by 3 → Compression

↓

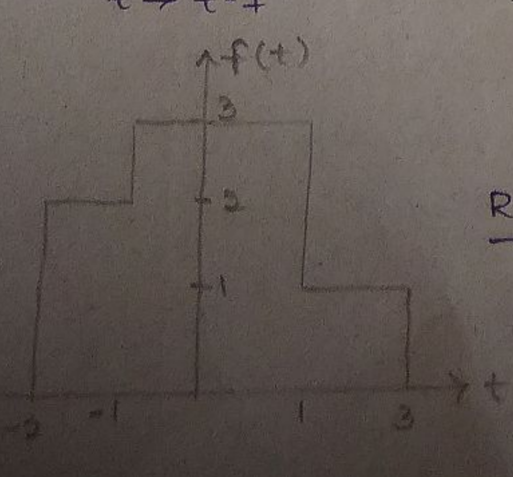
$a > 1$

$3 > 1$

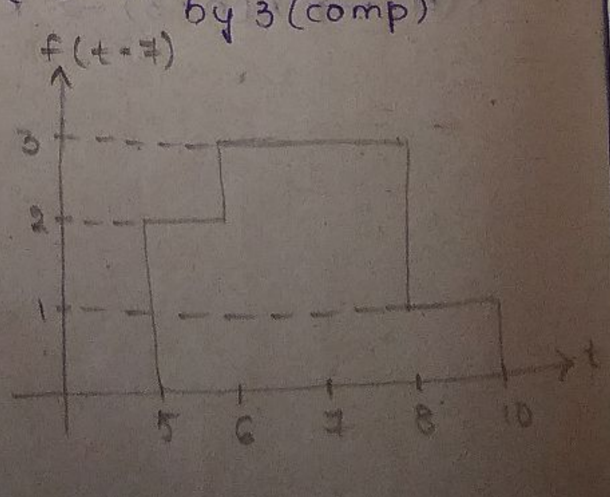


(ii) shifting, reversal, scaling

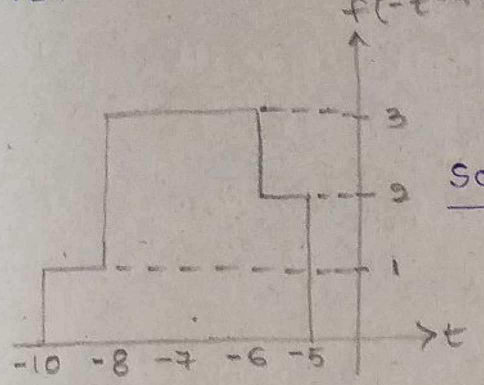
$$f(t) \xrightarrow[t \to t-7]{\text{R.S by 7}} f(t-7) \xrightarrow[t \to -t]{\text{rev}} f(-t-7) \xrightarrow[\text{by 3 (comp)}]{\text{Scaling}} f(-3t-7)$$



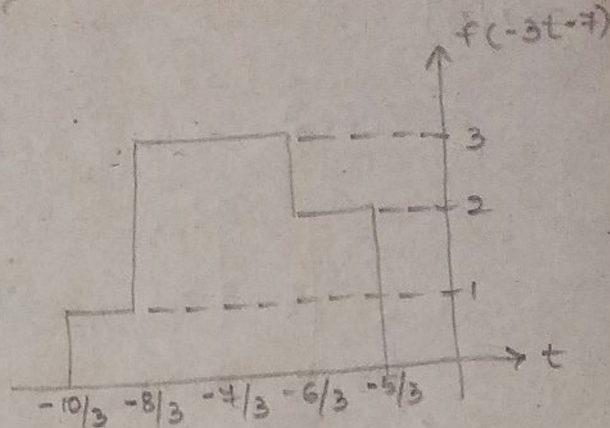
R.S by 7 →



rev



Scaling by 3 →

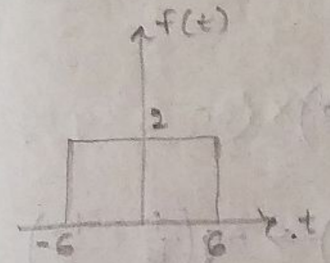


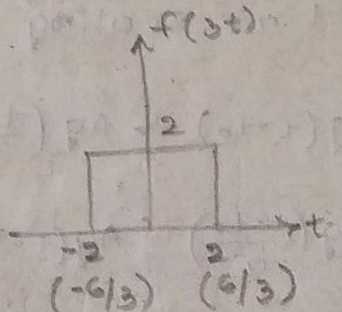
Amplitude Scaling:

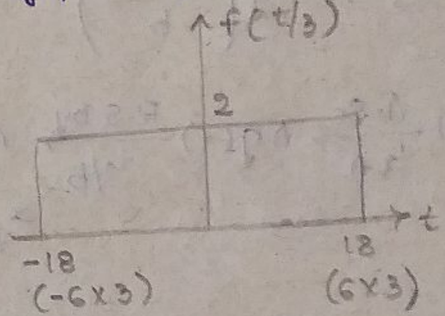
$f(t) \rightarrow f(t/3) \rightarrow$ time scaling / expansion

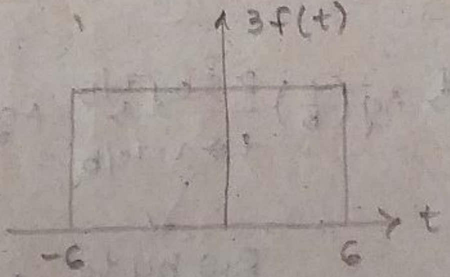
$f(t) \rightarrow 3f(t) \rightarrow$ Amplitude scaling

$f(t) \rightarrow f(3t) \rightarrow$ time scaling / compression



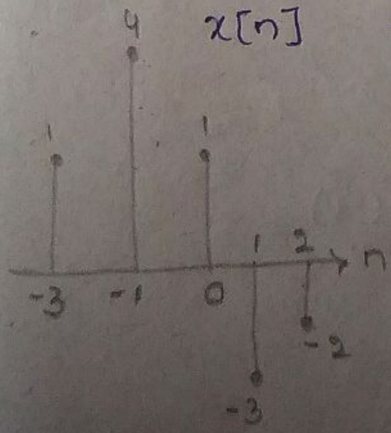


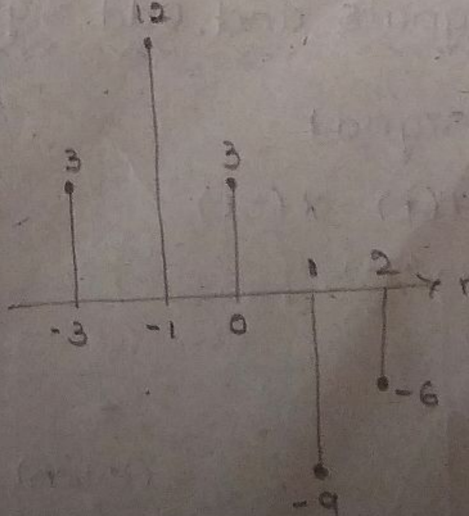




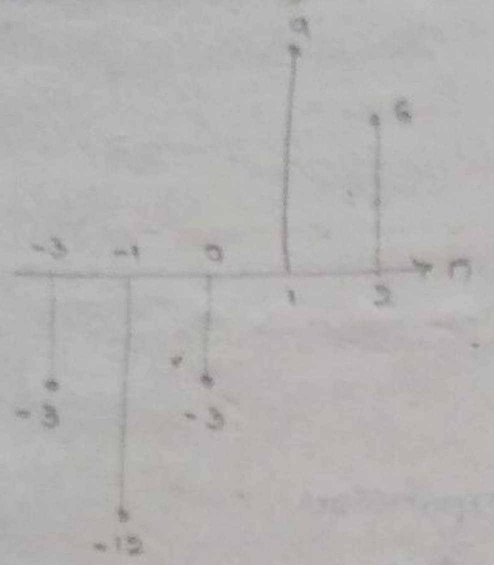
For example:

$x[n] \longrightarrow 3x[n]$





$-3x[n]$



$$g(t) \longrightarrow Ag\left(\frac{t-t_0}{b}\right)$$

Amplitude scaling
time shifting
time scaling

(i) $g(t) \xrightarrow[\times A]{A.Sc} Ag(t) \xrightarrow[t_0/b]{R.S\ by} Ag(t-t_0) \rightarrow Ag\left(\frac{t-t_0}{a}\right) \times$ (shifting by t_0)

$\rightarrow Ag\left(\frac{t-t_0}{b}\right)$ $\left\{ Ag\left(t-\frac{t_0}{b}\right) \rightarrow Ag\left(\frac{t}{b}-\frac{t_0}{b}\right) = Ag\left(\frac{t-t_0}{b}\right) \right.$

(ii) $g(t) \xrightarrow{A.SC} Ag(t) \xrightarrow[by\ b]{Scaling} Ag\left(\frac{t}{b}\right) \xrightarrow[t \to t-t_0/b]{R.S\ by\ t_0/b} Ag\left(\frac{t-t_0/b}{b}\right) \times$ (shifting by t_0/b)

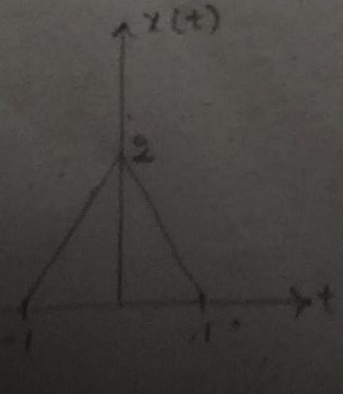
$\downarrow$

$\xrightarrow[t \to t-t_0]{R.S\ by\ t_0} Ag\left(\frac{t-t_0}{b}\right)$

Even signals and odd signals

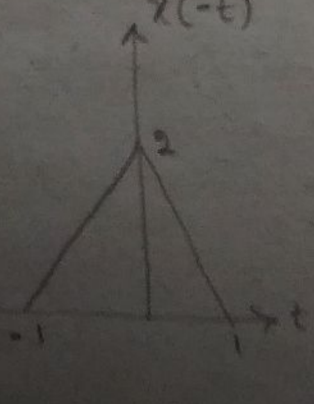
(i) Even signal

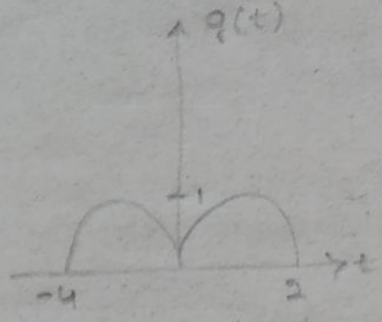
$$x(t) = x(-t)$$



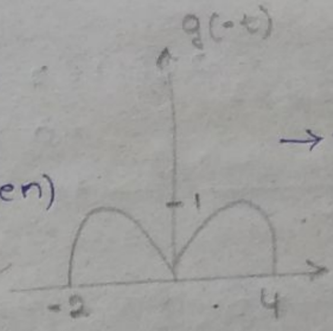
=

(even)

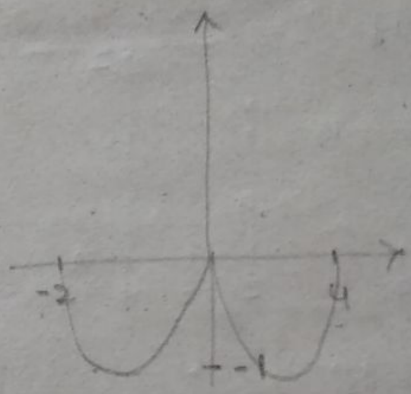




≠
(not even)

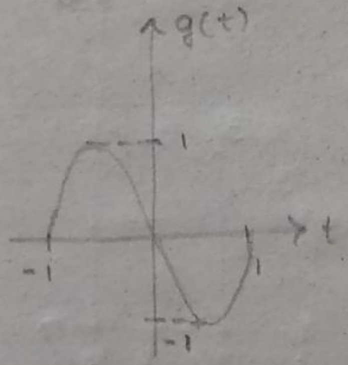


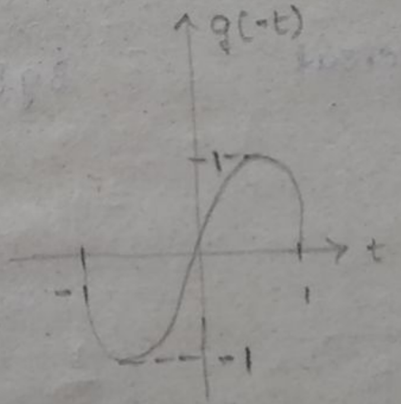
→ Amplitude scaling
$-g(-t)$

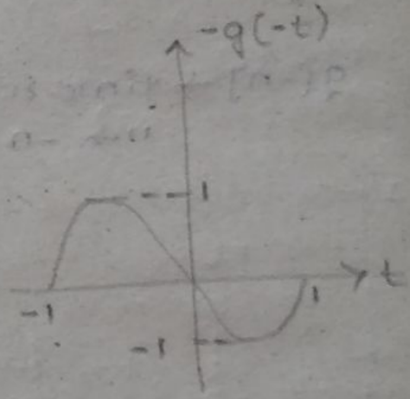


(ii) Odd signal

$x(t) = -x(-t)$







So, $g(t)$ is an odd signal

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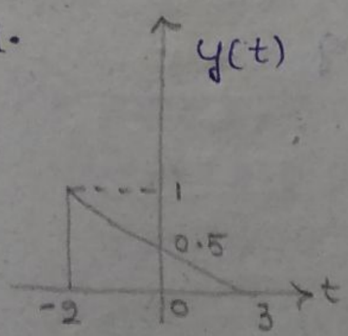
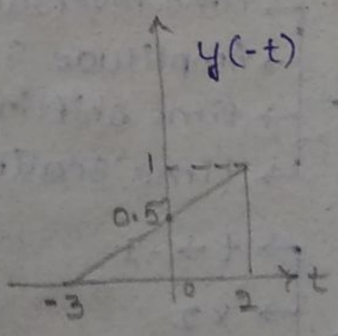
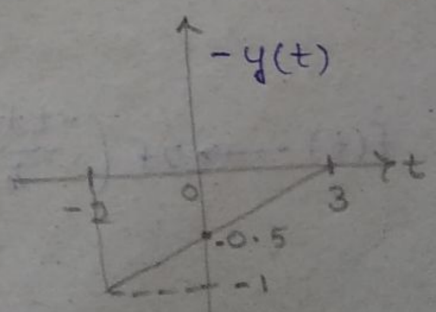
input signal } $y(t)$ → $y(-t)$ time reversal $t \to -t$
→ $-y(t)$ Amplitude scaling → +ve / → -ve

1.



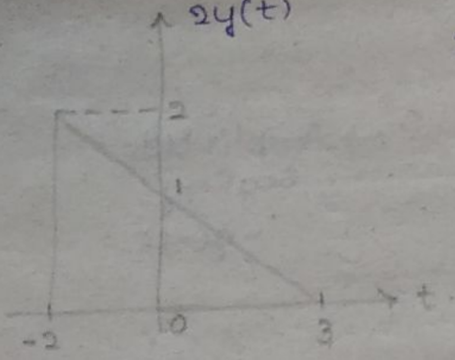




$y(t) = 1$ at $t = -2$
$= 0$ at $t = 3$
$= 0.5$ at $t = 0$

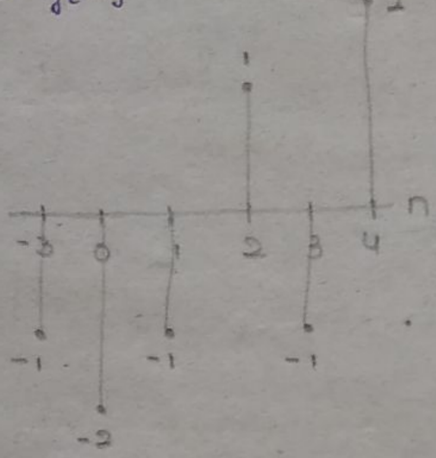
$y(-t) = 1$ at $t = 2$
$= 0$ at $t = -3$
$= 0.5$ at $t = 0$

$-y(t) = -1$ at $t = -2$
$= 0$ at $t = 3$
$= -0.5$ at $t = 0$

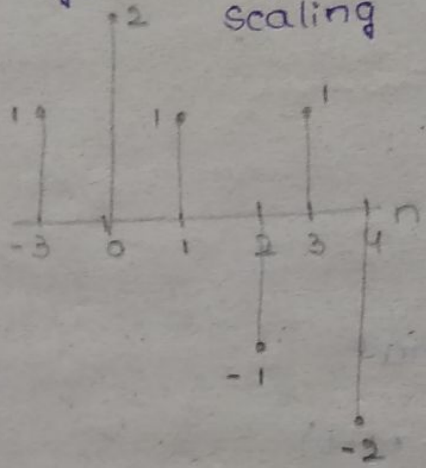


$$2y(t) = 2 \text{ at } t = -2$$
$$= 0 \text{ at } t = 3$$
$$= 1 \text{ at } t = 0$$

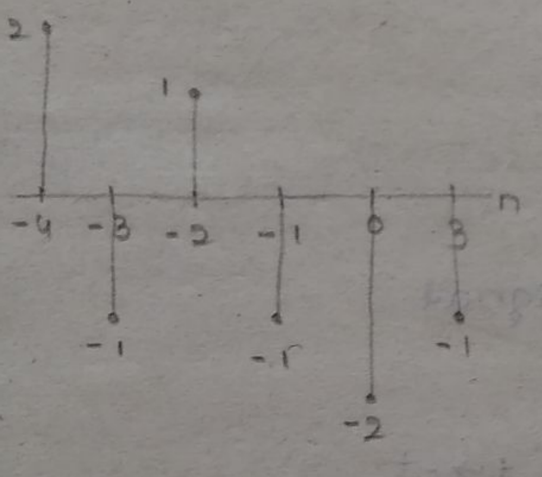
2. $g[n]$



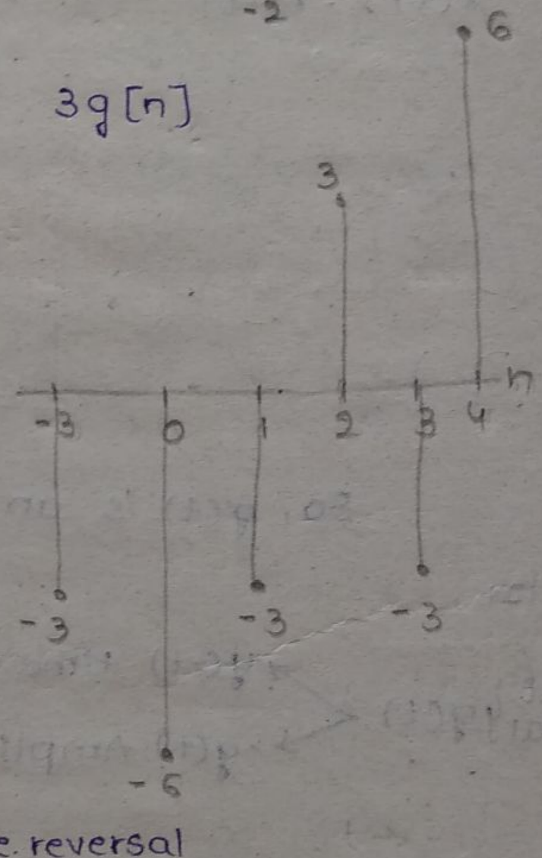
$-g(n) \rightarrow$ Amplitude scaling



$g[-n] \rightarrow$ time reversal
$n \rightarrow -n$



$3g[n]$



$$f(t) \longrightarrow 2f\left(\frac{-t+4}{5}\right)$$

- time reversal
- Amplitude scaling
- time shifting
- time scaling

$\Rightarrow$
- $t \rightarrow -t$
- $\times 2$
- $t \rightarrow t \pm t_0$
- $t \rightarrow at$

1. $f(t) \xrightarrow[t \to t+\frac{4}{5}]{\text{LS by } 4/5} f\left(t+\frac{4}{5}\right) \xrightarrow[t \to -t]{\text{time rev}} f\left(-t+\frac{4}{5}\right) \xrightarrow[t \to t/5]{\text{time scaling}} f\left(\frac{-t}{5}+\frac{4}{5}\right)$

$\xrightarrow[\text{Amp SC}]{\times 2} 2f\left(\frac{-t}{5}+\frac{4}{5}\right) = 2f\left(\frac{-t+4}{5}\right)$

2. $f(t) \xrightarrow[t \to -t]{\text{time rev}} f(-t) \xrightarrow[t \to t-\frac{4}{5}]{\text{RS by } 4/5} f\left(-t+\frac{4}{5}\right) \xrightarrow[\text{by } 1/5]{\text{Scaling}} f\left(\frac{-t}{5}+\frac{4}{5}\right)$

$\xrightarrow[\text{Amp SC}]{\times 2} 2f\left(\frac{-t}{5}+\frac{4}{5}\right) = 2f\left(\frac{-t+4}{5}\right)$

$f(t) \to f(2t) \xrightarrow[\text{time SC}]{a=2>1}$ compressed

$f(t) \to f(t/2) \xrightarrow[a=\frac{1}{2}<1]{}$ Expansed

$f(at) \to a>1$ comp $\div a$

$f(at) \to a<1$ exp $\times a$

Ex:

(i) $f\left(\frac{3}{2}t\right) \sim f(at)$

$a = \frac{3}{2} = 1.5 > 1$ compression

(ii) $f\left(\frac{2}{3}t\right)$

$a = \frac{2}{3} < 1$ expansion

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Even and odd signals:

Even - $x(t) = x(-t)$; $x_e(-t) = x_e(t) \to (3)$

Odd - $x(t) = -x(-t)$; $x_o(-t) = -x_o(t) \to (4)$

Any signal

$x(t) = x_e(t) + x_o(t) \to (1)$

Apply time reversal $(t \to -t)$

$x(-t) = x_e(-t) + x_o(-t)$

$= x_e(t) + (-x_o(t))$

$= x_e(t) - x_o(t) \to (2)$

eq(1) + (2)

$$2x_e(t) = x(t) + (x(-t))$$

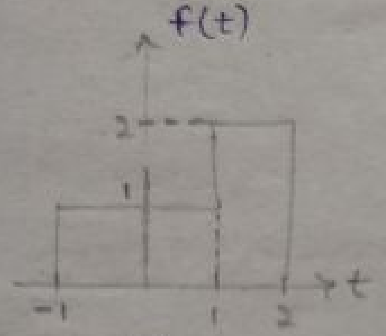
$$x_e(t) = \frac{x(t) + x(-t)}{2} \rightarrow (5)$$

eq(1) - (2)

$$2x_o(t) = x(t) - x(-t)$$

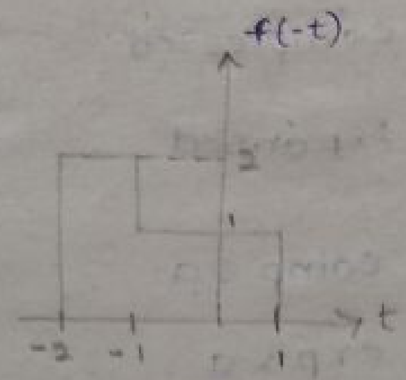
$$x_o(t) = \frac{x(t) - x(-t)}{2} \rightarrow (6)$$

Example:



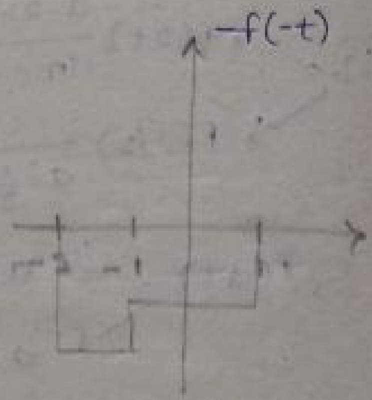
(i)

$t = \{-1, 1, 2)$



(ii)

$t = \{-2, -1, 1\}$



(iii)

$t = \{-2, -1, 1, 2\}$

for continuous function

we have to take time interval instead of time instance.

=> $-2 < t < -1 = 0$	$-2 < t < -1 = 2$	$-2 < t < -1 = -2$
$-1 < t < 0 = 1$	$-1 < t < 0 = 1$	$-1 < t < 0 = -1$
$0 < t < 1 = 1$	$0 < t < 1 = 1$	$0 < t < 1 = -1$
$1 < t < 2 = 2$	$1 < t < 2 = 0$	$1 < t < 2 = 0$

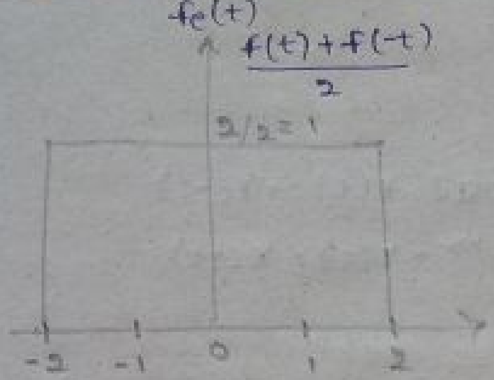
f(t) is neither odd nor even

$$f(t) = f_e(t) + f_o(t)$$

find even and odd parts of f(t)

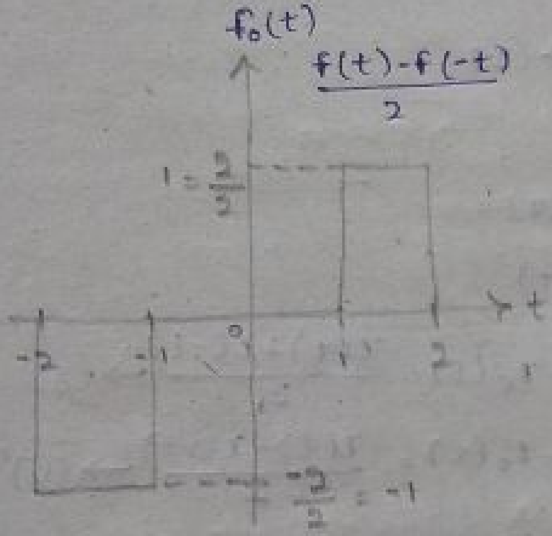
$$f_e(t) = \frac{f(t) + f(-t)}{2}$$

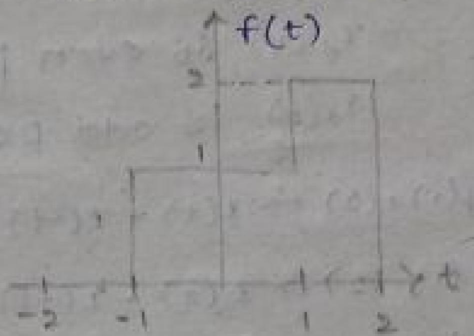
$$f_o(t) = \frac{f(t) - f(-t)}{2}$$



$$\begin{bmatrix} t=-2, \ f(t)=0 \ ; \ f(-t)=2 \Rightarrow \frac{0+2}{}=2 \\ t=-1, \ f(t)=1 \ ; \ f(-t)=1 \Rightarrow 2 \\ t=0, \ f(t)=1 \end{bmatrix}$$

			+	−
$-2<t<-1$	$f(t)=0$	$f(-t)=2$	2	−2
$-1<t<0$	$f(t)=1$	$f(-t)=1$	2	0
$0<t<1$	$f(t)=1$	$f(-t)=1$	2	0
$1<t<2$	$f(t)=2$	$f(-t)=0$	2	2





$f(t) = f_e(t) + f_o(t)$

$\boxed{\begin{matrix} f(t) \to Af(t) \\ \times A \end{matrix}}$ $\quad f(t) \to 2f(t)$, $A = 2$

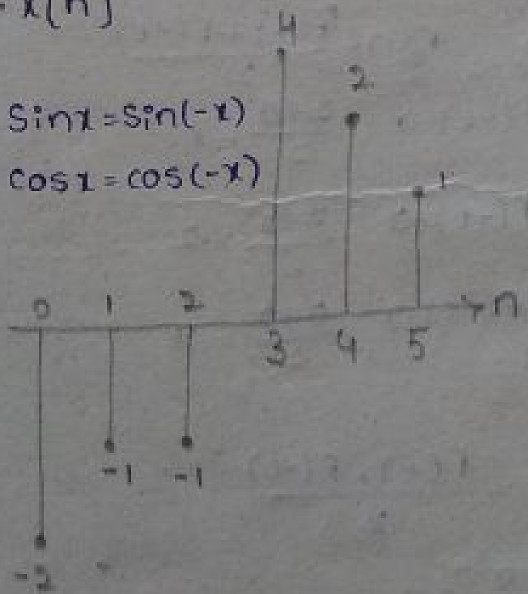
$f(t) \to \frac{f(t)}{2} \left[\times \frac{1}{2}\right]$

$A = 1/2 \Rightarrow f(t) \times 1/2$

2. $x[n]$

$\sin x = \sin(-x)$

$\cos x = \cos(-x)$



$$x_e[n] = \frac{x[n] + x[-n]}{2} \text{ (even)}$$

$$x_o[n] = \frac{x[n] - x[-n]}{2} \text{ (odd)}$$

1/02/21

Even and odd signals:

Even signal $x(t) = x(-t)$ odd signal $x(t) = -x(-t)$
$\Rightarrow -x(t) = x(-t)$

$$x(t) = x_e(t) + x_o(t) \rightarrow (1)$$

$$x(-t) = x_e(-t) + x_o(-t)$$

$$x(-t) = x_e(t) - x_o(t) \rightarrow (2)$$

from eq (1)

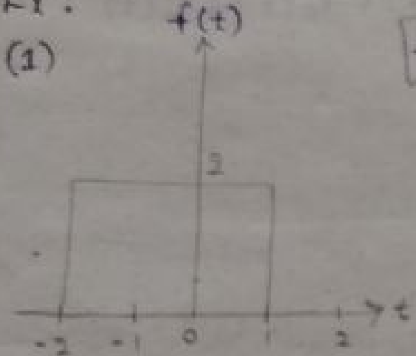
$x_e(t)$ is even part of $x(t)$

$x_o(t)$ is odd part of $x(t)$

$$eq(1)+(2) \Rightarrow x(t) + x(-t) = 2x_e(t) \Rightarrow x_e(t) = \frac{x(t)+x(-t)}{2} \rightarrow (3)$$

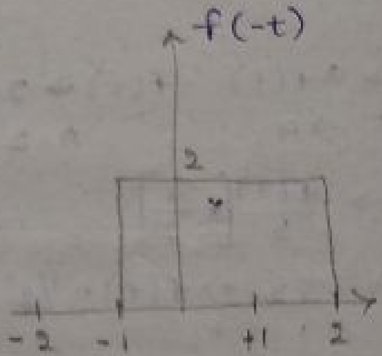
$$eq(1)-(2) \Rightarrow x(t) - x(-t) = 2x_o(t) \Rightarrow x_o(t) = \frac{x(t)-x(-t)}{2} \rightarrow (4)$$

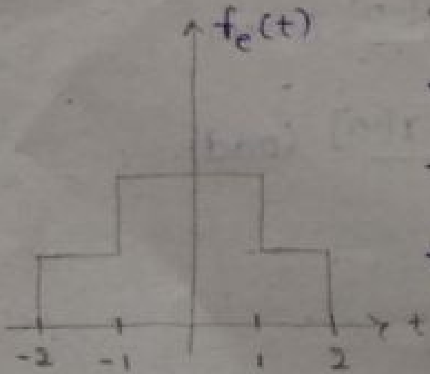
Ex:
(1)



$\boxed{f_e(t), f_o(t) = ?}$

$$t \rightarrow -t$$
time reversal



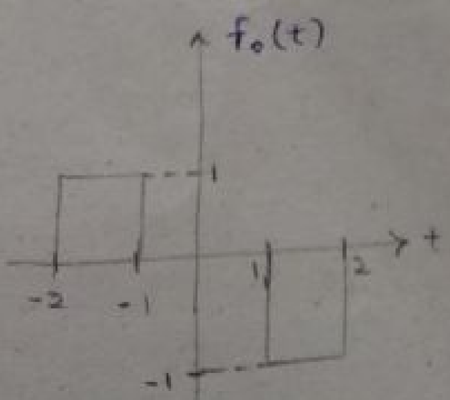


$$f_e(t) = \frac{f(t)+f(-t)}{2}$$

$-2 < t < -1$ $f(t) = 2$ $f(-t) = 0$ $\frac{2+0}{2} = 1$

$-1 < t < 1$ $f(t) = 2$ $f(-t) = 2$ $\frac{2+2}{2} = 2$

$+1 < t < 2$ $f(t) = 0$ $f(-t) = 2$ $\frac{0+2}{2} = 1$

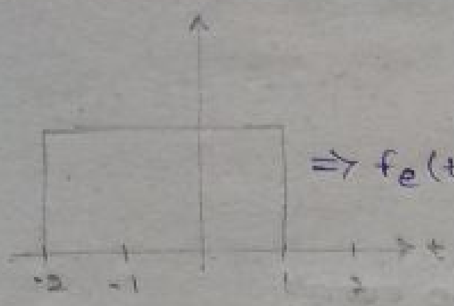


$$f_o(t) = \frac{f(t) - f(-t)}{2}$$

$-2 < t < -1$ $\frac{2-0}{2} = 1$

$-1 < t < 1$ $\frac{2-2}{2} = 0$

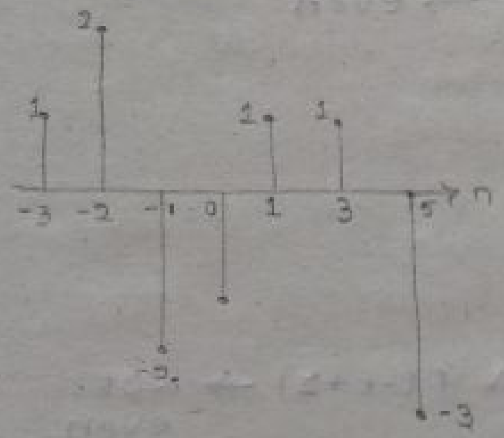
$1 < t < 2$ $\frac{0-2}{2} = -1$



$\Rightarrow f_e(t) + f_o(t) = f(t)$

(2)

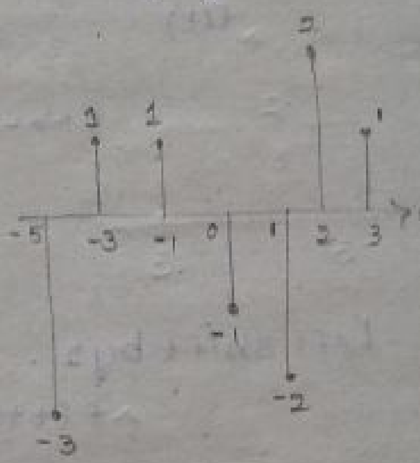
$x[n]$



$\boxed{x_e(n), x_o(n) = ?}$

Time reversal
$n \to -n$

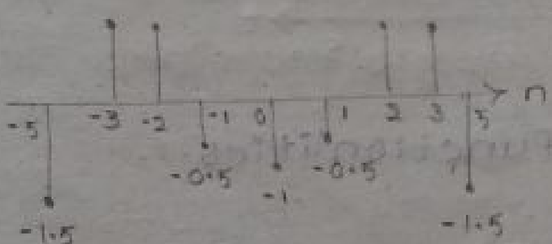
$x[-n]$



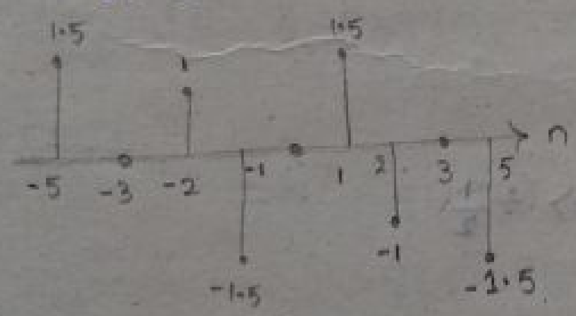
→ We have to take only at particular instant of time.

n	$x(n)$	$x(-n)$	$x_e(n)$	$x_o(n)$
-5	0	-3	-1.5	1.5
-3	1	1	1	0
-2	2	0	1	1
-1	-2	1	-0.5	-1.5
0	-1	-1	-1	0
1	1	-2	-0.5	1.5
2	0	2	1	-1
3	1	1	1	0
5	-3	0	-1.5	-1.5

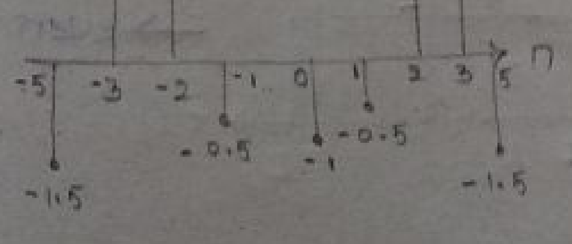
$x_e[n]$



$x_o[n]$



$x_e[-n]$



$\therefore \boxed{x_e[n] \text{ is even}}$

→ $g(t)$ is an odd signal

then $g_e(t) = 0$

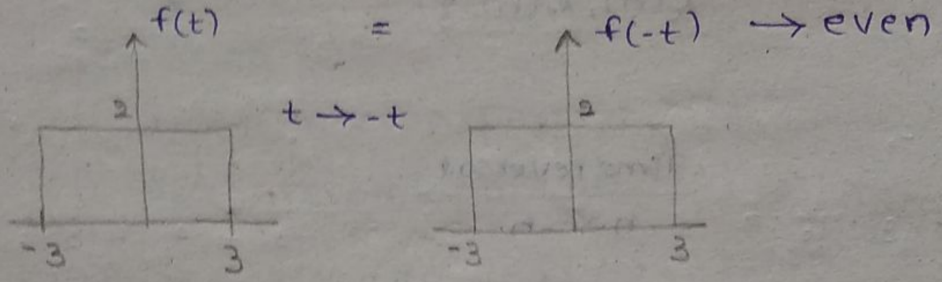
$g_o(t) = g(t)$

2/02/21

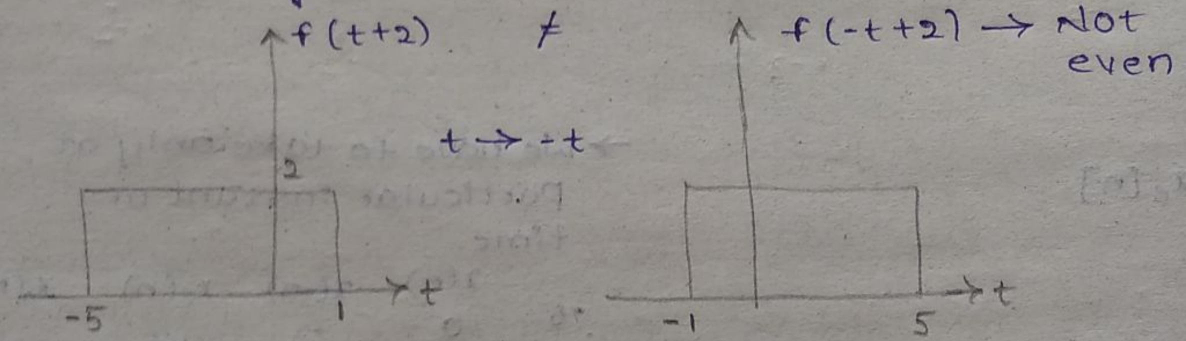
1. $f(t)$ is even function, Is $f(t+2)$ also even?

Answer. No

proof:



Left shift by 2



→ shifting changes even/odd functionlities.

2. $f(t)$ is even function, Is $f(\frac{t}{2})$ also even?

A. Yes

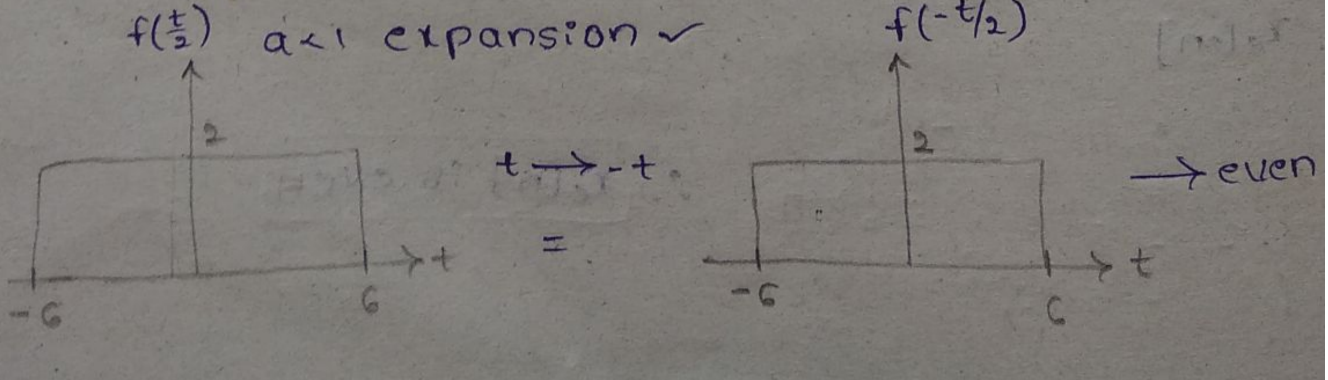
proof:

Scaling $f(t) \rightarrow f(\frac{t}{2})$

$a = \frac{1}{2} \sim f(at)$; $\div a \Rightarrow \div \frac{1}{2}$

$a > 1$ compression

$a < 1$ expansion ✓

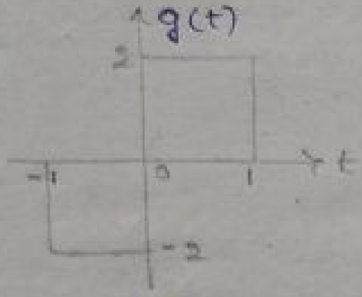


→ Scaling doesn't effect even/odd functionalities.

3. odd signal

$x(t) = -x(-t) \rightarrow$ TR

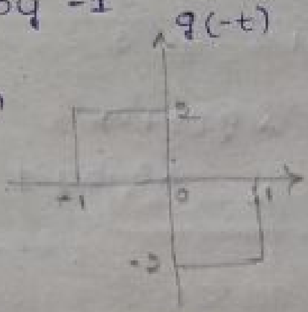
$\rightarrow$ AS by -1



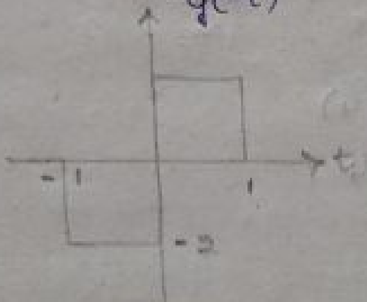
(i)

not even

$\neq$



(ii)



(iii)

$\therefore g(t)$ is odd

<u>Periodic Signals:</u>

CT $\quad x(t) = x(t+T)$ → It can be any value; → period

DT $\quad x[n] = x[n+N]$ → integer

→ Unchanged by a time shift by T.

→ Start at $-\infty$ and continuous forever(∞)

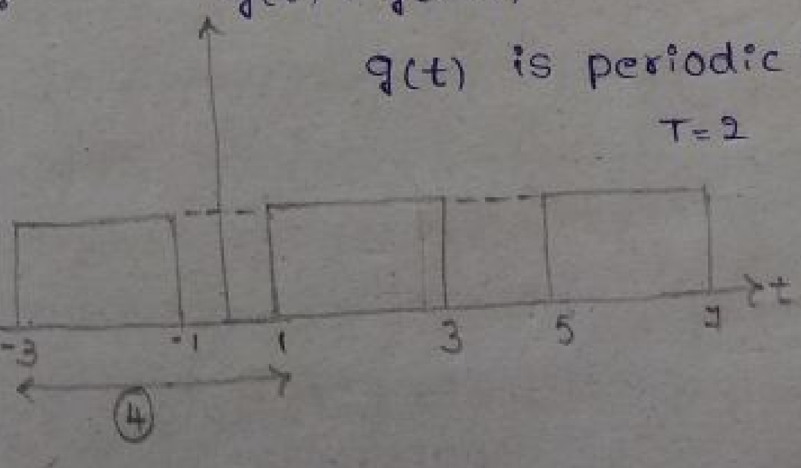
→ Periodic segment extend $-\infty$ ↳ ∞

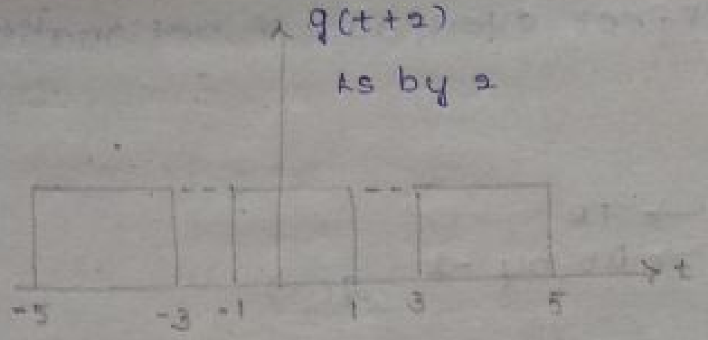
Ex:

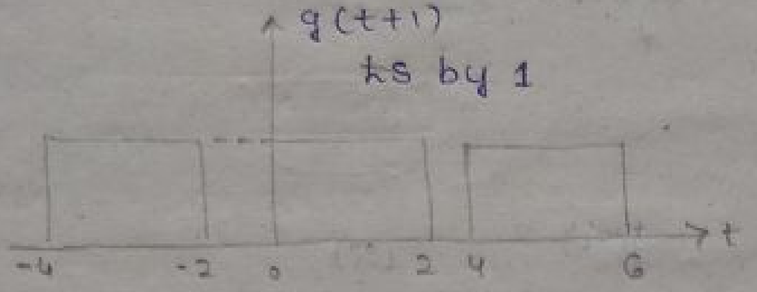
1. $g(t) = g(t+T)$

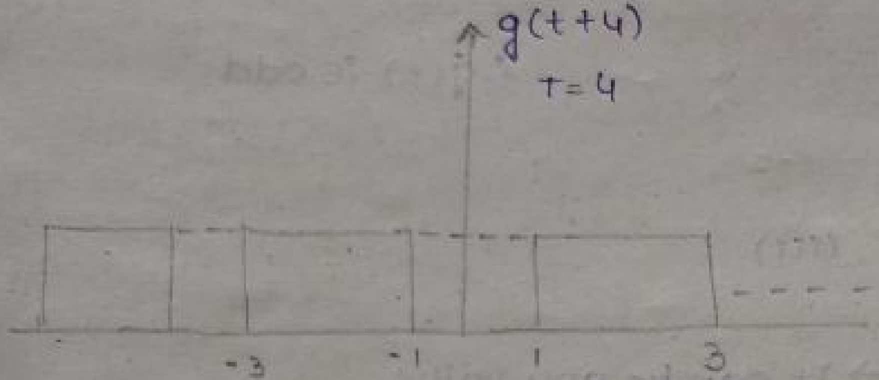
$g(t)$ is periodic for T

$T = 2$









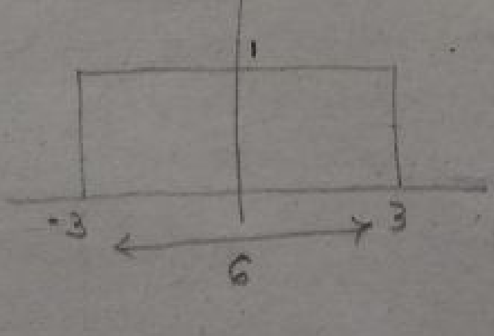
g(t) is periodic with $\boxed{T=4}$ → fundamental period

$T = 8, 12, 16$ ---- integral multiples

$$g(t) = g(t+T)$$

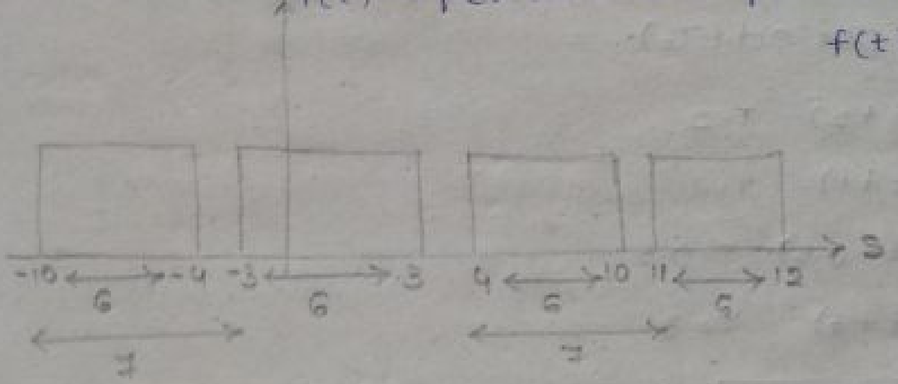
$$g(t+T) = g(t+T+T)$$
$$= g(t+2T)$$
$$= g(t+3T)$$
$$= g(t+\underbrace{m}_{\text{integer}}T)$$

2.



$f(t) \rightarrow$ periodic with period $T = 7, 14, 21, ----$

$f(t) = f(t+T)$



$f(t+7),\ f(t+14),\ f(t+21), ----$

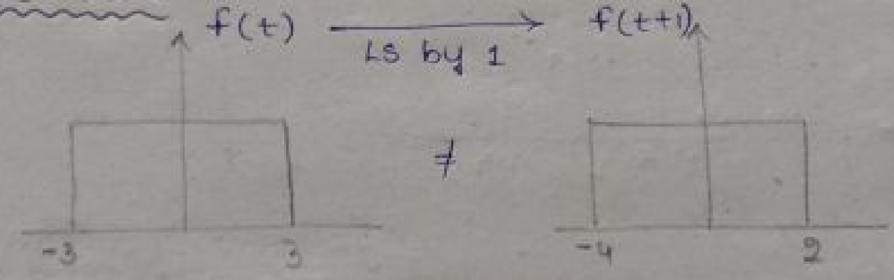
fundamental period 7

↓

complete one cycle

Aperiodic:

Ex:



05-02-21

If $x(t) = x(t+T) \rightarrow (1)$ CT

where T is a +ve value

then it is said to be periodic signal

And for DT

$x[n] = x(n+N)$

where N is a +ve integer

let

$x(t+T) = x(t+T+T) = x(t+2T) = x(t+3T) = x(t+4T) \rightarrow (2)$

$\Rightarrow x(t+mT)$

$\longrightarrow$ m is an integer

that means

$x(t) = x(t+T)$

shifting

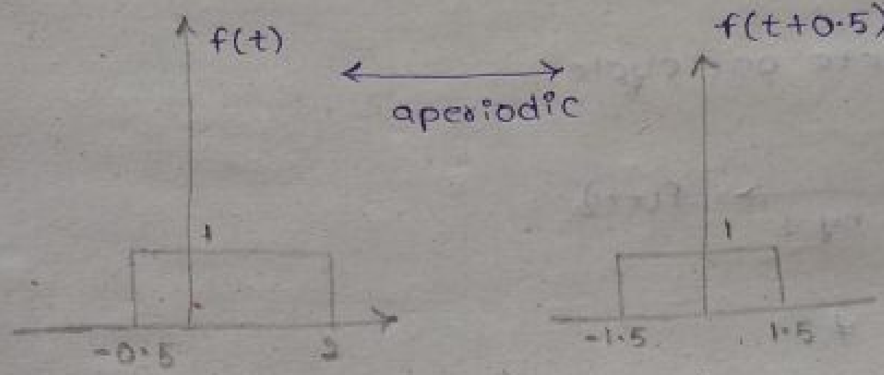
$t \rightarrow t+T$

→ Smallest value which satisfies eq (1) is called fundamental period (T_0).

$$\Rightarrow f(t) = f(t+2) \quad T=2$$
$$= f(t+1) \quad T=1$$
$$= f(t+0.5) \quad T=0.5$$
$$= f(t+3) \quad T=3$$

$$\boxed{f(t) = f(t+0.5)} = f(t+2(0.5)) = f(t+1)$$
$$= f(t+3(0.5)) = f(t+1.5)$$
$$= f(t+4(0.5)) = f(t+2) \ldots$$

• Is f(t) is periodic?



Left shift

Periodic:

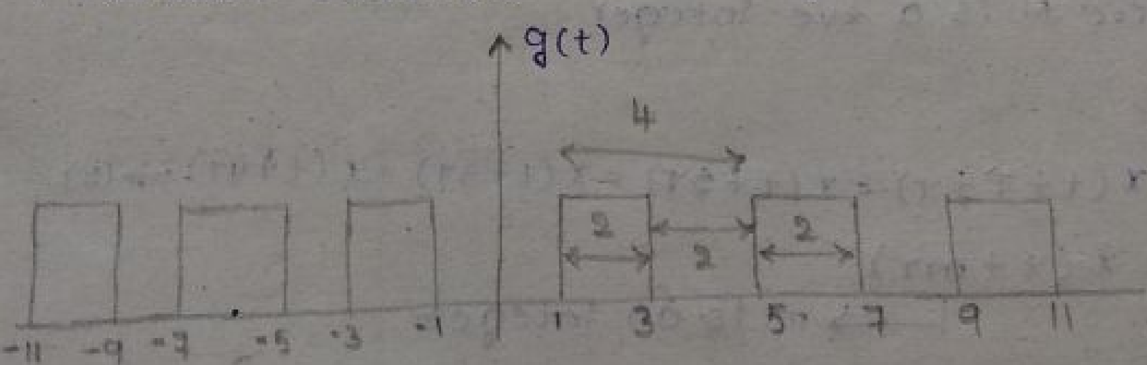
$x(t) = x(t+T)$ $\qquad$ $t \rightarrow t+T$ (shifting operation)

→ unchanged after shifting by T → +ve value

g(t) is periodic with period of 'I'

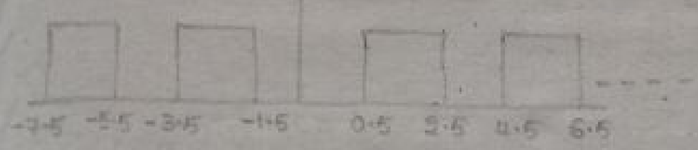
→ $-\infty$ to continuous forever (∞)

→ Periodic extension of a segment



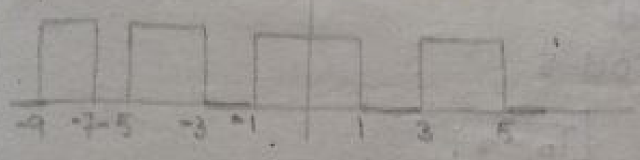
$g(t+0.5)$

$\boxed{T=0.5}$ ×



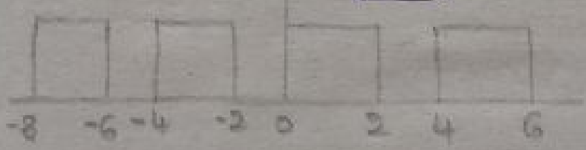
$g(t+2)$

$\boxed{T=2}$ ×



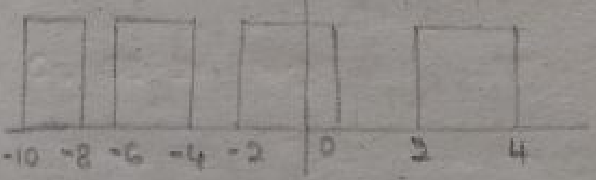
$g(t+1)$

$\boxed{T=1}$ ×



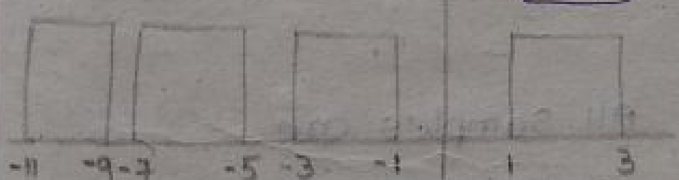
$g(t+3)$

$\boxed{T=3}$ ×



$g(t+4)$

$\boxed{T=4}$ → $g(t)$ is periodic with period 4



$\boxed{T_0 = 4}$ → fundamental period

here $T=4 \Rightarrow f = \frac{1}{T} = \frac{1}{4} = 0.25$ Hz

$g(t) = g(t+4)$

$= g(t+8), g(t+12), g(t+16)$

Periodic segmendt

extend

$-\infty \leftarrow \qquad \longrightarrow +\infty$

T= period

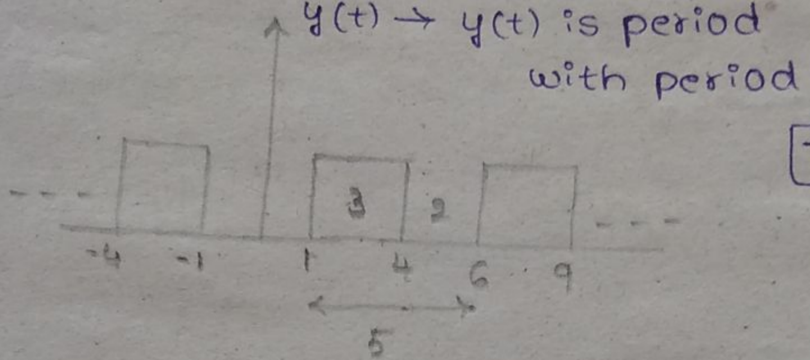
T_0 = Smallest value of T

$\frac{1}{T} = f$ = no. of cycles/s

↓

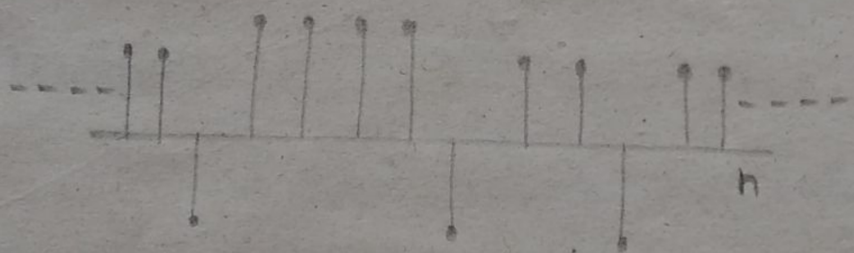
Frequency

Units: Hertz (Hz)

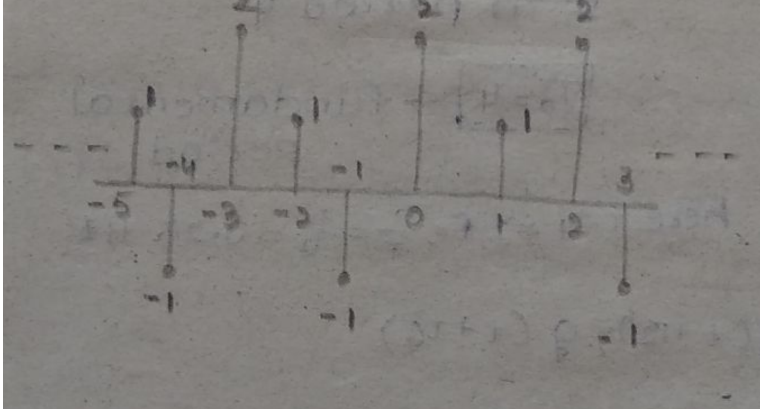


$\boxed{T_0 = 5}$

(i) DT Is x[n] periodic?



(ii) Is y(n) periodic?



All samples are equidistant =1

6|2|21

Exponential Signals:

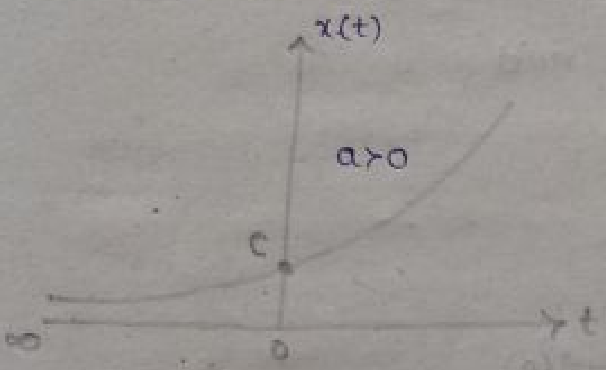
$$\boxed{x(t) = ce^{at}}$$

Case(i) - aperiodic

C-real and a-real

→ $a>0$

→ $a<0$



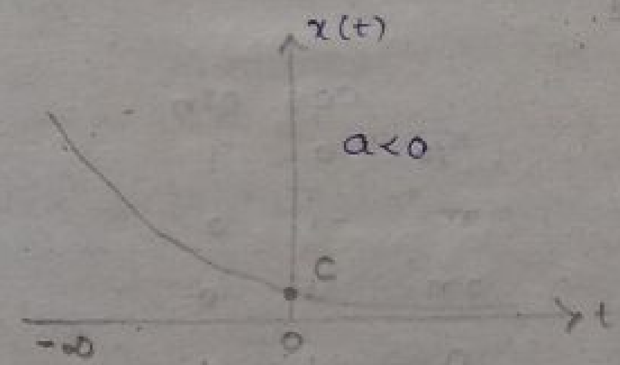
Growing exponential

as t ↑es x(t) ↑es

at $t=0$

$x(t) = ce^{a(0)}$

$= ce^{0} = c(1) = c$



Decaying exponential

t ↑es x(t) ↓es

Case(ii)

C-real a-imaginary

$\Rightarrow a = j\omega_0$

ω_0 = angular freque-ncy

$x(t) = ce^{j\omega_0 t} \rightarrow (1)$ Is x(t) periodic?

$x(t) = x(t+T)$ ↓ periodic

↓ replace $t \rightarrow t+T$

$x(t+T) = ce^{j\omega_0 (t+T)}$

$e^{a+b} = e^{a}.e^{b}$

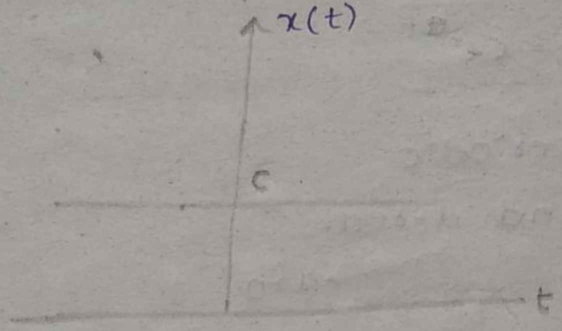
$= ce^{j\omega_0 t} . e^{j\omega_0 T} \rightarrow (2)$

$e^{x} = 1$

$x = 0$

x(t) is periodic when $e^{j\omega_0 T} = 1$

$$e^{j\omega_0 T} = 1 \begin{cases} \omega_0 = 0 \; ; \; x(t) = c \text{ (constant signal) i.e periodic } \forall T \\ \omega_0 \neq 0 \end{cases}$$



Note: Constant signal is periodic $\forall T$

$t \to t + T$

$\Rightarrow T_0 = \text{undefined}$

Eulers relation

$e^{jx} = \cos x + j \sin x$

$$e^{j\omega_0 T} = \cos\omega_0 T + j \underbrace{\sin\omega_0 T}_{0} = 1 + j(0) = 1$$

So $\boxed{e^{j\omega_0 T} = 1}$

When $\omega_0 T = 2\pi$

$\omega_0 T = 2\pi(m)$

$$\boxed{T = \frac{2\pi m}{\omega_0}}$$

where m is integer

	cos	sin
$\pi/2$	0	1
π	-1	0
2π	1	0
0	1	0
3π	-1	0
4π	1	0
5π	-1	0
6π	1	0

and so on.

$x(t)$ is periodic with $T = \frac{2\pi m}{\omega_0}$

T_0 = (smallest value of T)

$\Rightarrow m = 1$

$$\boxed{T_0 = \frac{2\pi}{\omega_0}} \to \text{fundamental period}$$

$$\text{frequency} = \frac{1}{T} = \frac{\omega_0}{2\pi m}$$

Let $x_1(t) = e^{j\omega_0 t} \rightarrow f_1 = \frac{1}{T} = \frac{\omega_0}{2\pi} \rightarrow$ fundamental frequency

$x_2(t) = e^{j2\omega_0 t} \rightarrow f_2 = \frac{2\omega_0}{2\pi} = 2\left(\frac{\omega_0}{2\pi}\right) = 2f_1$

then $x_1(t)$ and $x_2(t)$ are called "harmonically related signals" and $x_2(t)$ is known as 2^{nd} harmonic of $x_1(t)$ i.e f_1

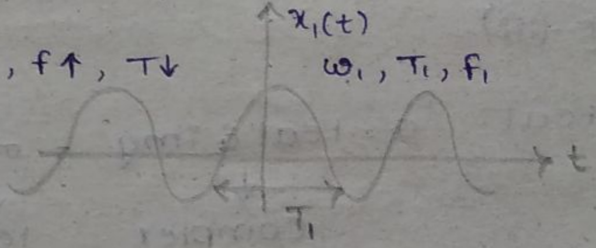
$x_3(t) = e^{j3\omega_0 t} \rightarrow f_3 = 3f_1$

$x_4(t) = e^{j4\omega_0 t} \rightarrow f_4 = 4f_1 \rightarrow$ 4^{th} harmonic of f_1

$x_m(t) = e^{jm\omega_0 t} \rightarrow f_m = mf_1 \rightarrow$ m^{th} harmonic of f_1

here $x_2(t), x_3(t), x_4(t), \text{-----} x_m(t)$ are called "harmonics".

$\omega_0\uparrow, f\uparrow, T\downarrow$



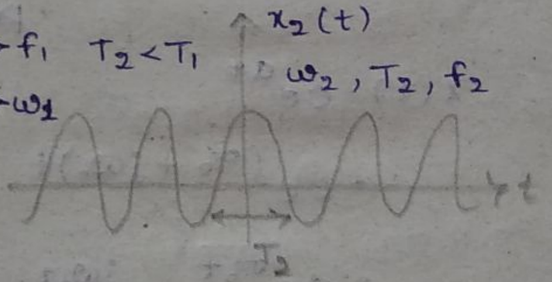
Let 3 signals be

$\underbrace{1K, 2K, 3K}_{\text{harmonics}}$ Hz

$$\boxed{f_0 = 1\text{ KHz}}$$

$f_2 > f_1$ $T_2 < T_1$

$\omega_2 > \omega_1$



$x(t) = ce^{j\omega_0 t}$ special case $\rightarrow$ a sinusoidal signal $\downarrow$ combination of sin/cos terms

$= c[\cos\omega_0 t + j\sin\omega_0 t]$

$\cos\omega_0 t = Re\{e^{j\omega_0 t}\}$

$\sin\omega_0 t = Img\{e^{j\omega_0 t}\}$

General sinusoidal signal

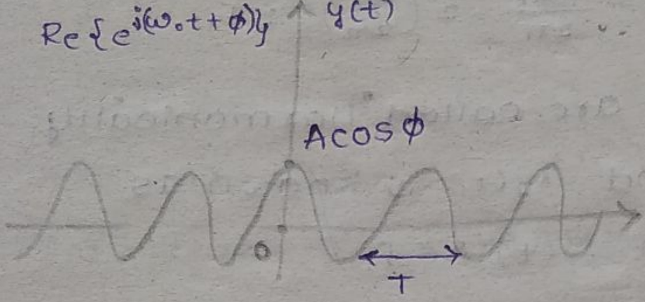
$$y(t) = A\cos(\omega_0 t + \phi)$$ $\omega_0 \rightarrow$ angular frequency

$A \rightarrow$ Amplitude

$\phi \rightarrow$ Phase shift

$T = \frac{2\pi}{\omega_0} \rightarrow$ Period (time taken to complete 1 cycle)

$Re\{e^{j(\omega_0 t + \phi)}\}$

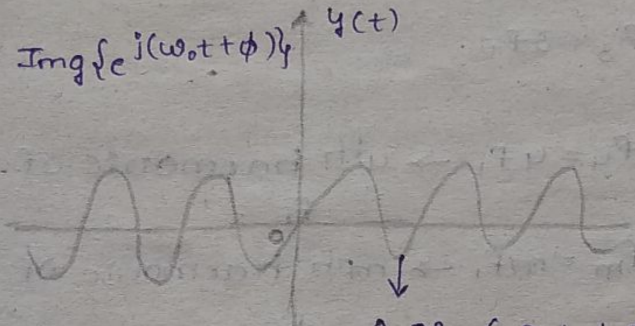


at $t=0$

$y(t) = A\cos(\omega_0(0)+\phi)$

$= A\cos\phi$

$Img\{e^{j(\omega_0 t+\phi)}\}$



$A\sin(\omega_0 t + \phi)$

9/2/21

Case -(iii)

c real $\quad a = \text{real} \,\&\, \text{img} = \sigma_0 + j\omega_0$

$a \rightarrow$ complex, $\sigma_0 \rightarrow$ real part of a, $j\omega_0 \rightarrow$ img part of a

$$x(t) = ce^{at}$$

$$= ce^{(\sigma_0 + j\omega_0)t}$$

$$e^{a+b} = e^a \cdot e^b$$

$$= ce^{\sigma_0 t} \cdot e^{j\omega_0 t}$$

(i) $ce^{at} \rightarrow$ real

(ii) $ce^{j\omega_0 t} \rightarrow$ imag

$\Rightarrow ce^{\sigma_0 t} \cdot e^{j\omega_0 t}$

$ce^{\sigma_0 t} \rightarrow$ Amplitude

Hence case (i) $a = \sigma_0 = \text{real}$

case (ii) $c = ce^{\sigma_0 t}$

Is this periodic? Yes, it is period with $T = \frac{2\pi}{\omega_0}m$

$x(t) = Ae^{j\omega_0 t}$ where $A = ce^{\sigma_0 t} \Rightarrow$ Amplitude

According to Euler's relation

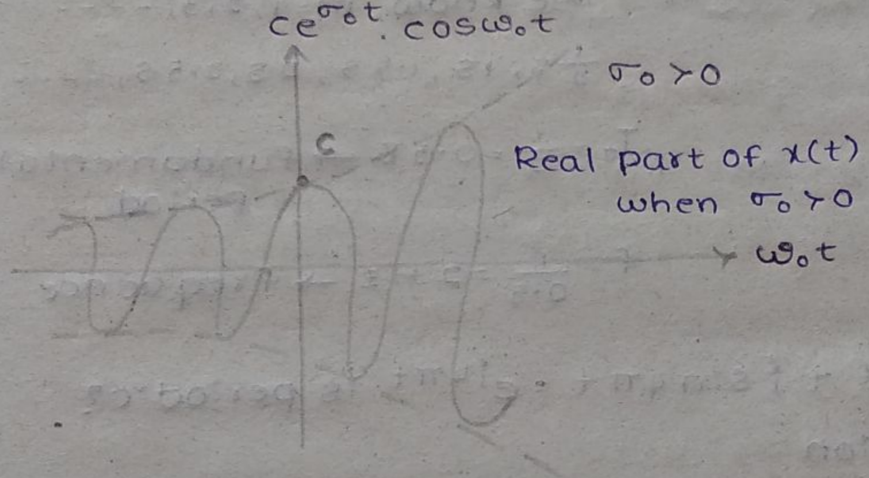
$= A[\cos\omega_0 t + j\sin\omega_0 t]$

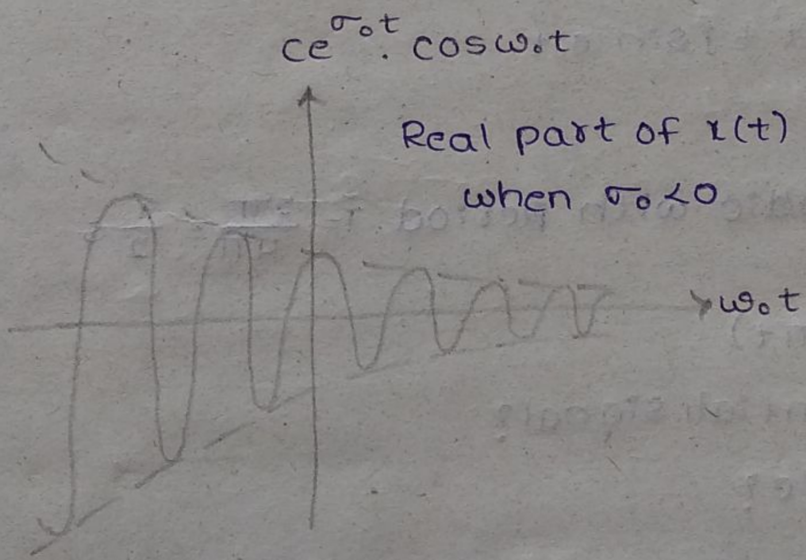
$= ce^{\sigma_0 t}\cos\omega_0 t + j\,ce^{\sigma_0 t}\sin\omega_0 t$

$=$ Real $+$ Imaginary

$ce^{\sigma_0 t}.\cos\omega_0 t =$ real part of $x(t)$ — $\sigma_0 > 0$ (growing exp), $\sigma_0 < 0$ (decaying exp)

$ce^{\sigma_0 t}.\sin\omega_0 t =$ Imag part of $x(t)$





[• Sin graphs should be drawn]

12/2/21

$x(t) = ce^{at}$

$x(t) = 1e^{j4\pi t}$

here $c = 1$; $a = j4\pi \Rightarrow j\omega_0$

$\downarrow$ real $\quad \downarrow$ imag $\quad \Rightarrow \omega_0 = 4\pi$

case(ii) is periodic

$T = \frac{2\pi}{\omega_0} m$

$T_0 = \frac{2\pi}{\omega_0}$

here,

$T = \frac{2\pi}{4\pi} m = \frac{1}{2} m$

We know, $m = 1, 2, 3, ----$

$\frac{1}{2}s, 1s, 1.5s, 2s, 2.5s, -----$

$T_0 = \frac{1}{2} = 0.5s \rightarrow$ fundamental period

$f = \frac{1}{0.5} = 2\,Hz \rightarrow$ frequence

Ex.

(1) $y(t) = \cos 4\pi t + j \sin 4\pi t = e^{j4\pi t}$ is periodic?

A. Euler's relation

$e^{j\omega_0 t} = \cos \omega_0 t + j \sin \omega_0 t$

$\Rightarrow \omega_0 = 4\pi$

So, it is periodic with period $T = \frac{2\pi}{4\pi} = \frac{1}{2}s$

(2) $q(t) = 8\cos(400\pi t)$

(a) Is it exponential signal?

(b) Is it periodic?

A. (a) No, it is not exponential signal.

It is a cosine signal

$\Rightarrow A\cos(\omega_0 t + \phi)$

(b) Here $A = 8$

$\omega_0 = 400\pi$

$\phi = 0$

then $T = \frac{2\pi}{\omega_0} m$

$\Rightarrow T = \frac{2\cancel{\pi}}{400\cancel{\pi}} m$

$= \frac{1}{200} s, \frac{2}{200} s, \frac{3}{200} s, \frac{4}{200} s, \ldots$

$T_0 = \frac{1}{200} s$

$f = 200 \text{ Hz}$

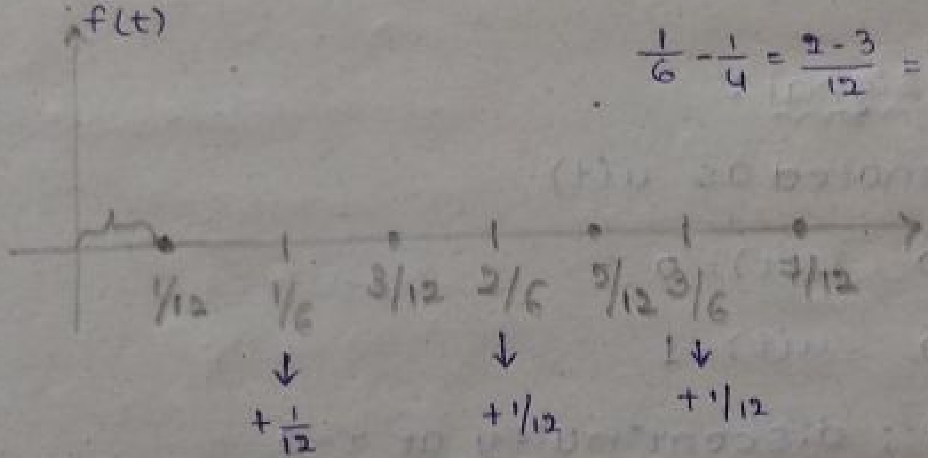
∴ It is a periodic signal.

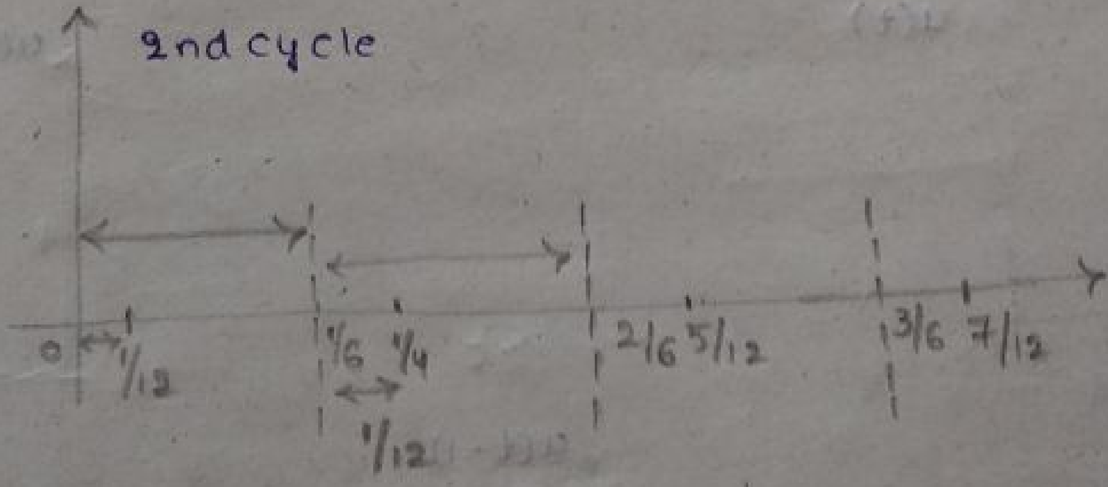
(3) $f(t) = 10 \sin 12\pi t + 10 \cos 12\pi t \rightarrow$ Periodic signal

A. $T = \frac{2\pi}{12\pi} m$

$= \frac{1}{6} s, \frac{2}{6} s, \frac{3}{6} s, \frac{4}{6} s, \ldots$ one cycle $= 1/6$ s

$\frac{1}{6} - \frac{1}{4} = \frac{2-3}{12} = -1/12$





(4) $g(t) = 5 \sin 12\pi t + 10 \sin 18\pi t$ is periodic?

A. $T_1 = \frac{2\pi}{12\pi} = \frac{1}{6}\,s$

$T_2 = \frac{2\pi}{18\pi} = \frac{1}{9}\,s$

Yes, it is periodic.

$T = $ L.C.M of T_1 and T_2

$= \frac{1}{6} + \frac{1}{9} = \frac{3+2}{18} = \frac{5}{18}\,s$

Elementary Signals: → DT, CT

1. Exponential signal (ce^{at})
2. Unit step signal ($u(t)$)
3. Impulsive signal ($\delta(t)$)

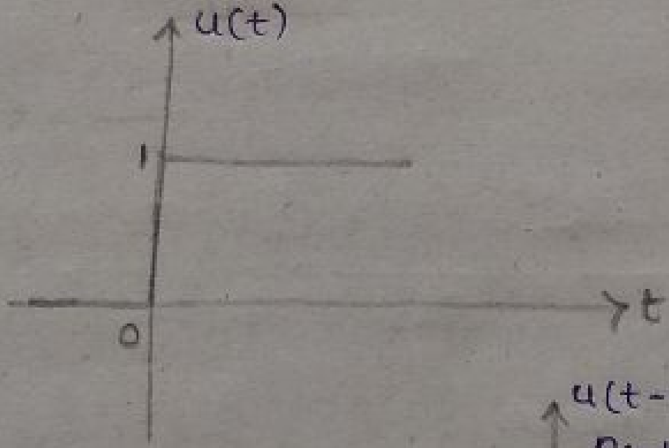
Unit step signal:

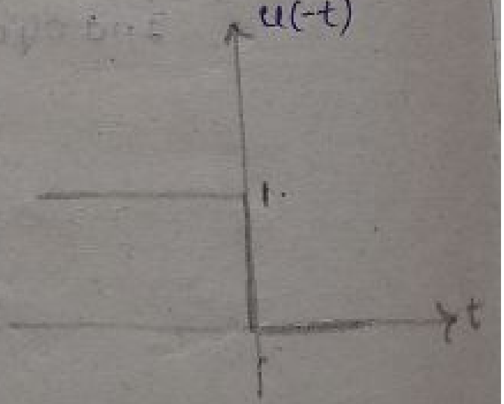
→ It is denoted as $u(t)$

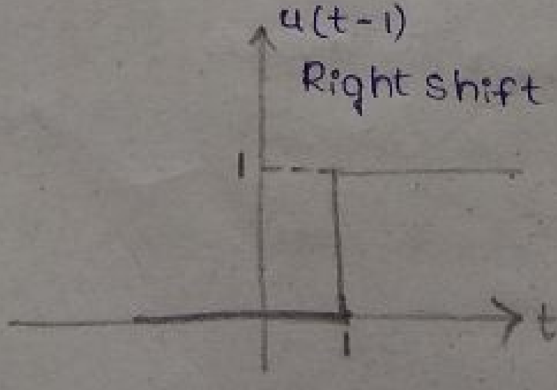
→ If $t<0$ $u(t)=0$

$t>0$ $u(t)=1$

$t=0$; discontinuity at $t=0$







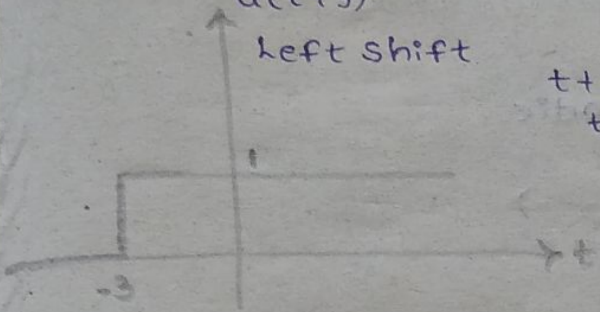
$t-1=0$

$\Rightarrow t=1$

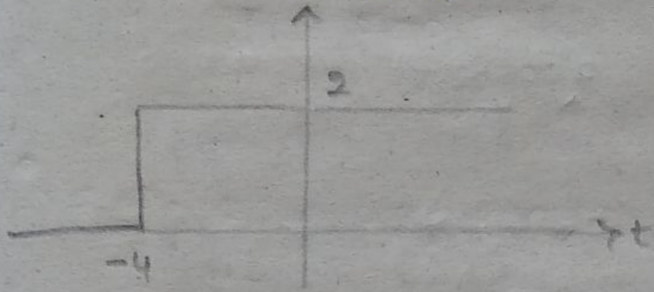
u(t+3)

Left shift

$t+3=0$
$t=-3$

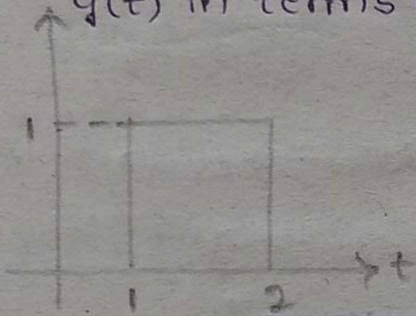


2u(t+4)



Ex

(1) g(t) in terms of u(t).



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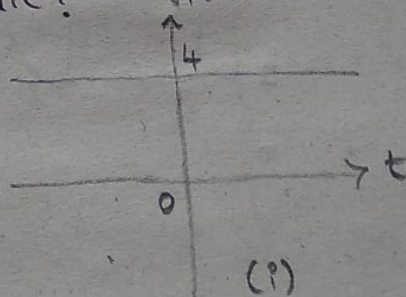
1. $g(t) = 4 + 2\cos 5\pi t$; Is g(t) periodic?

A. $\underbrace{4}_{g_1(t)} + \underbrace{2\cos 5\pi t}_{g_2(t)}$

$g_1(t)$



(i)

$g_1(t) = 4 \ \forall t$

at $t=9$, 4
$=11$, 4
$=100$, 4

constant signal

$x(t) = x(t+T)$ — period

T_0 = undefined

All sinusoidal signals $\lt$ sin, cos

$A \cos/\sin(\omega_0 t + \phi) \rightarrow$ periodic

$$T = \frac{2\pi}{\omega_0} m \quad (\because m = 1, 2, ----)$$

$$T_0 = \frac{2\pi}{\omega_0} \quad (m = 1)$$

$$f = \frac{\omega_0}{2\pi}$$

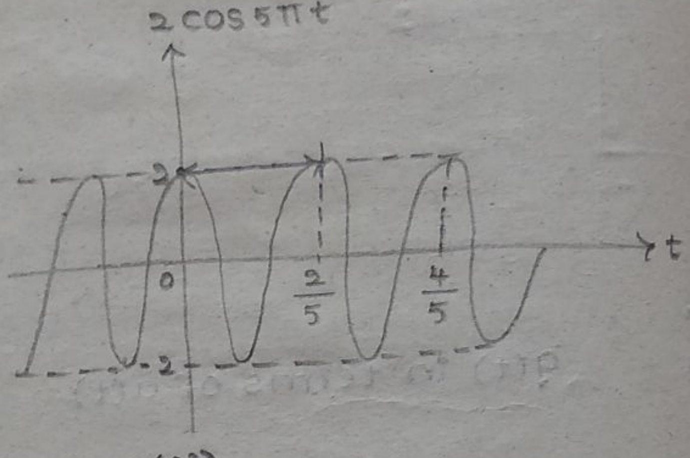
$g_2(t) = 2\cos 5\pi t$

Amplitude $(A) = 2$

$\omega_0 = 5\pi$

$\phi = 0°$

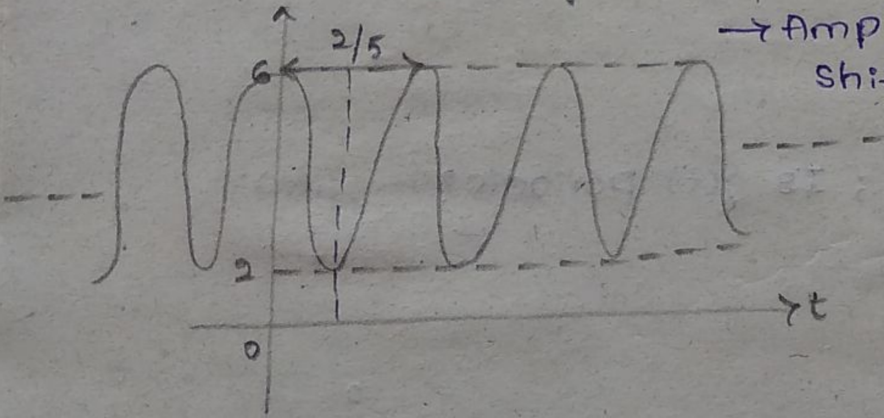
$$T_2 = \frac{2\pi m}{5\pi} = \frac{2m}{5} s$$



(ii)

Now (i) + (ii)

$4 + 2\cos 5\pi t = g(t)$



$\rightarrow$ Amplitude shifting

<u>Note:</u>

ap + ap $\rightarrow$ ap

ap + p $\rightarrow$ ap

p + p $\rightarrow$ periodic (depends on) $\rightarrow$ f_1/f_2, ω_1/ω_2, T_1/T_2 $\rightarrow$ rational (periodic)

$\rightarrow$ Irrational (aperiodic)

2. $g(t) = 4\cos 2t + 2\sin 4\pi t$

A.

$$\underbrace{4\cos 2t}_{g_1(t)} + \underbrace{2\sin 4\pi t}_{g_2(t)}$$

$\omega_{01} = 2$ $\qquad$ $\omega_{02} = 4\pi$ $\qquad$ $\left[f = \frac{1}{T} \quad T = \frac{1}{f}\right]$

$T_1 = \frac{2\pi}{2}$ $\qquad$ $T_2 = \frac{2\pi}{4\pi}$

$T_1 = \pi$ $\qquad$ $T_2 = \frac{1}{2}$

$f_1 = \frac{1}{\pi}$ $\qquad$ $f_2 = 2$

$T\{g(t)\} = LCM\{T_1, T_2\}$

$f\{g(t)\} = HCF\{f_1, f_2\}$

$\Rightarrow \frac{T_1}{T_2} = 2\pi = \text{irrational}$

So, given signal is aperiodic.

3. $y(t) = 2\cos 4\pi t + 4\sin 7\pi t$ is $y(t)$ periodic?

If yes, find T_0?

A. Given

$$y(t) = \underbrace{2\cos 4\pi t}_{y_1(t)} + \underbrace{4\sin 7\pi t}_{y_2(t)}$$

$T_1 = \frac{2\pi}{4\pi}$ $\qquad$ $T_2 = \frac{2\pi}{7\pi}$

$T_1 = \frac{1}{2}$ $\qquad$ $T_2 = \frac{2}{7}$

$\frac{T_1}{T_2} = \frac{1/2}{2/7} = \frac{7}{4} = 1.75 \rightarrow$ rational number

$\therefore$ $y(t)$ is periodic

$T = LCM\{T_1, T_2\} = LCM(1/2, 2/7)$

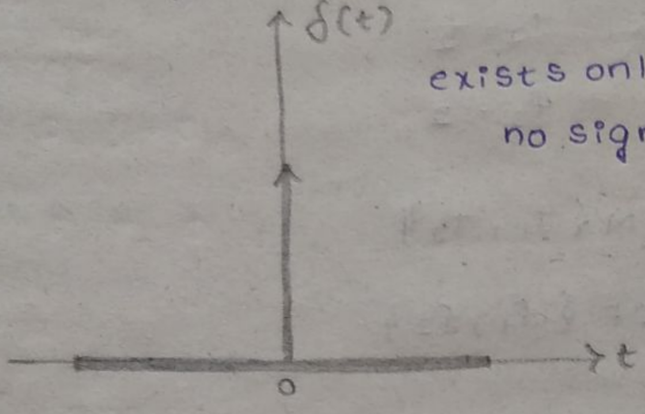
$= \frac{LCM\{1,2\}}{HCF\{2,7\}} = \frac{2}{1} = 2$

Unit

Impulse $\delta(t)$:

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1 \rightarrow \text{area}$$
$\rightarrow \neq$ Amplitude

$\Rightarrow$ area of impulse $= 1$



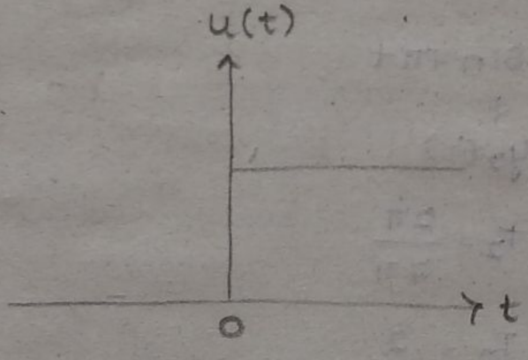
exists only at $t=0$

no signal when $t \neq 0$

$$\int_{-\infty}^{t} \delta(\tau)\, d\tau = 0 \qquad t < 0^- \text{ (before 0)}$$

$$\int_{\infty}^{t} \delta(\tau)\, d\tau = 1 \qquad t \geq 0^+ \text{ (at 0 and above)}$$

(positive value of t)



$$u(t) = \begin{cases} 1 & \text{at } t > 0 \\ 0 & \text{at } t < 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = u(t)$$

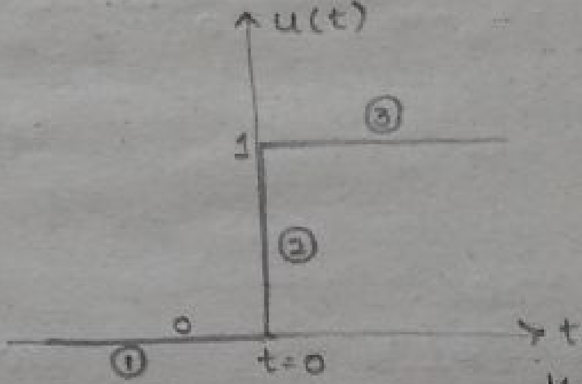
$\Rightarrow$ $0 = 0 \quad t < 0$

$1 = 1 \quad t \geq 0^+$

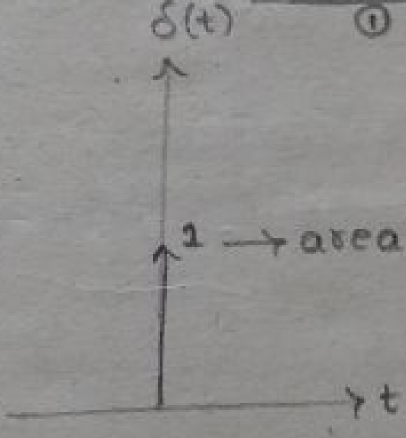
$$\Rightarrow \frac{du(t)}{dt} = \delta(t)$$

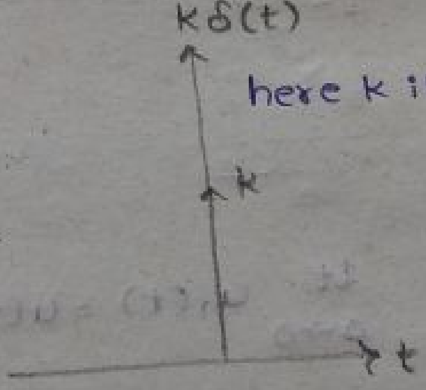
$$\frac{du(t)}{dt} = \frac{\text{change in } u(t)}{\text{chane in time} = 0}$$

$$\left.\begin{aligned}\frac{du(t)}{dt} &= 0 \quad t > 0 \\ &= \infty \quad t = 0 \\ &= 0 \quad t < 0\end{aligned}\right\} \delta(t)$$



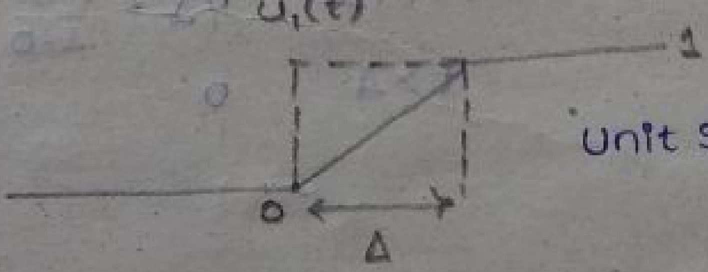
0 to 1 → within no time = Zero time





here k is amplitude

* $x(t) \xrightarrow{Amp} 2\delta(t)$ here 2 is amplitude



Δ = 0 to 1

Unit step

$$u(t) = \lim_{\Delta \to 0} u_1(t)$$

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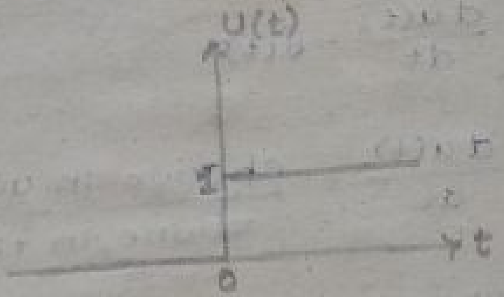
Unit-Step:

$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$



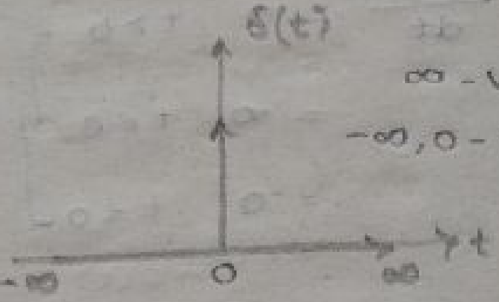
Unit-Impulse:

$$\delta(t) = 0 \quad t \neq 0$$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1 \quad t = 0$$

↓

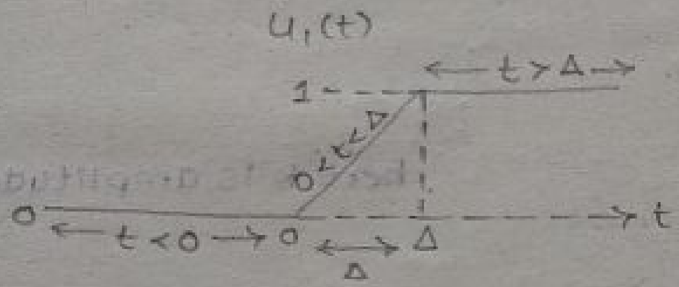
area

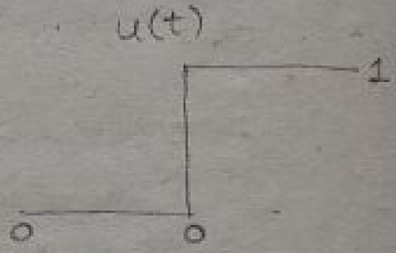


∞ - very high

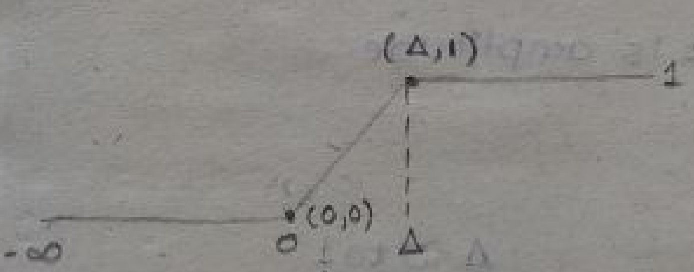
-∞, 0 - very low

Limiting Signals:





$$\lim_{\Delta \to 0} u_1(t) = u(t)$$



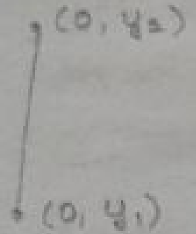
	Slopes of $u_1(t)$
$t < 0$	0
$0 < t < \Delta$	$1/\Delta = \frac{1-0}{\Delta - 0}$
$t > \Delta$	0

We know

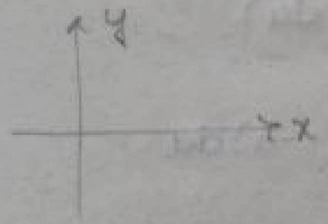
(i) Slope between a horizontal line

(x_1, y_1) —— (x_2, y_2) $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad (x_1, 0)$ —— $(x_2, 0)$ $\quad m = \frac{0-0}{x_2 - x_1} = 0$

(ii) slope b/w a vertical line

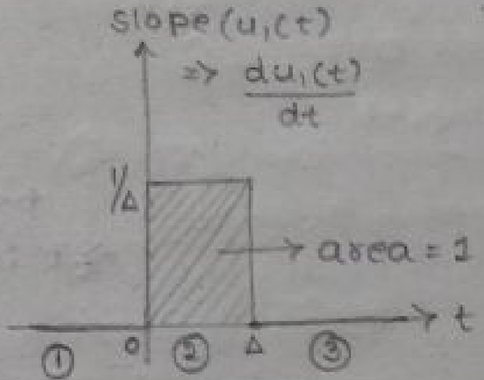


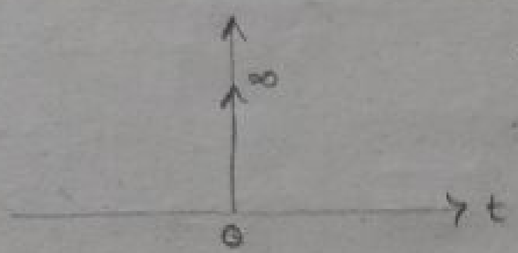
$$m = \frac{y_2 - y_1}{0 - 0} = \infty$$



$$\text{slope}(m) = \frac{dy}{dx} = \frac{\text{change in } y}{\text{change in } x}$$

→ Now, Graph of $u_1(t)$ slopes:





$$\lim_{\Delta \to 0} \delta_1(t) = \delta(t) \longrightarrow \text{Impulse signals}$$

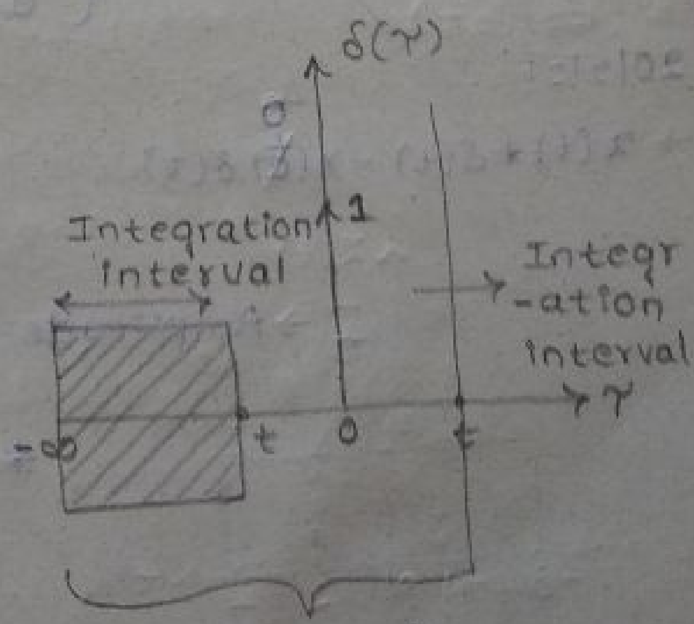
$u(t) \quad \delta(t)$

$$\int \delta(t)\,dt = u(t)$$

$$\frac{du(t)}{dt} = \delta(t)$$

Properties of $\delta(t)$:

→ $$\int_{-\infty}^{t} \delta(\tau)\,d\tau = 1 \quad t > 0$$



Since

if $t < 0$ then $\int_{-\infty}^{t} \delta(\tau)\,d\tau = 0$

here, τ = variable and

t = constant

$\rightarrow \boxed{\tau = t-a}$

$d\tau = dt - da$

$= 0 - da = -da$

Lower limit

$t = -\infty$

$\Rightarrow -\infty = t - a$

$a = t + \infty = \infty$

$$\int_{\infty}^{0} \delta(t-a)(-da)$$

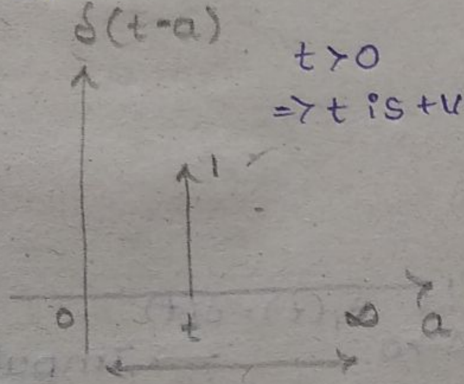
$$u(t) = \int_{0}^{\infty} \delta(t-a)\,da$$

Upper limit

$t = t$

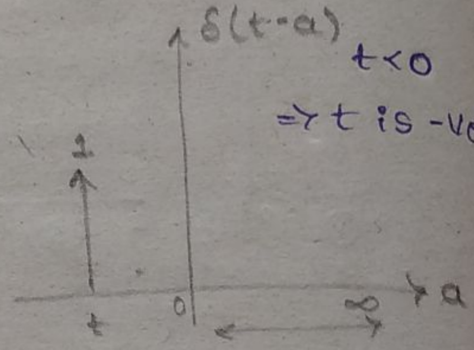
$\Rightarrow t = t - a$

$a = 0$



$t > 0$

$\Rightarrow t$ is +ve



$t < 0$

$\Rightarrow t$ is -ve

Hence

$$\int_{0}^{\infty} \delta(t-a)\,da = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

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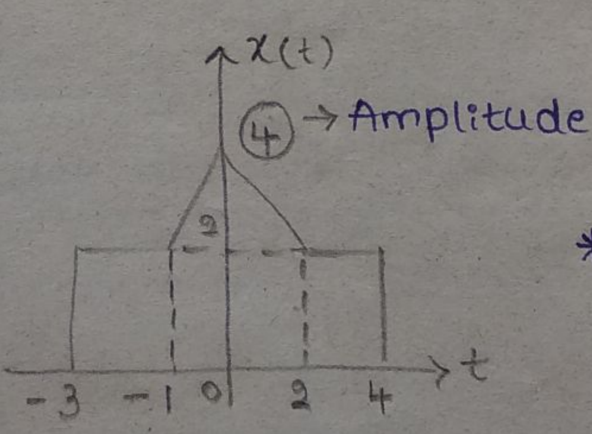
$\rightarrow x(t) * \delta(t) = x(\overset{0}{\cancel{t}})\,\delta(t)$



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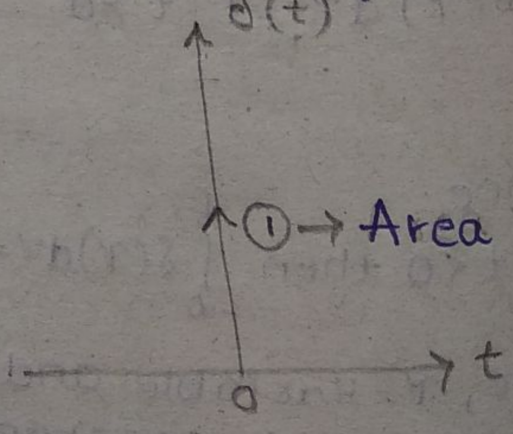
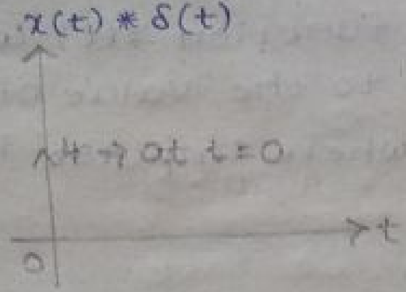


$= $ 

$x(t)$		$\delta(t)$	$x(t) * \delta(t)$
$-3<t<-1$	2	0	0
at $t=0$	4	1 → Area	4 → area
$-1<t<0^-$	$2 \to 4$	0	0
$0^+<t<2$	$4 \to 2$	0	0
$2<t<4$	2	0	0

$$x(t)\,\delta(t-2) = x(2)\,\delta(t-2)$$

$\Downarrow$ at $t=2$; $\Downarrow$ only at $t-2=0$, $\boxed{t=2}$

→ $x(t)\,\delta(t-\tau) = x(\tau)\,\delta(t-\tau)$

$\Downarrow$ at τ ; $\Downarrow$ at $t=\tau$

Apply $\int$ on both sides w.r.t 't'

$$\int_{-\infty}^{\infty} x(t)\,\delta(t-\tau)\,dt = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,dt$$

$$= x(\tau)\underbrace{\int_{-\infty}^{\infty} \delta(t-\tau)\,dt}_{1}$$

$$\boxed{\int_{-\infty}^{\infty} x(t)\,\delta(t-\tau)\,dt = x(\tau) \quad \text{at } t=\tau}$$

⇒ Sifting (or) Sampling property

Area of the product of function $x(t)$ with unit impulse $\delta(t)$ is equal to the value of the function $x(t)$ at the instant where impulse is defined or exists.

$$\int_{-\infty}^{\infty} x(t)\,\delta(t-\tau)\,dt = \text{Value of } x(t) \text{ at } t=\tau \Rightarrow x(\tau)$$

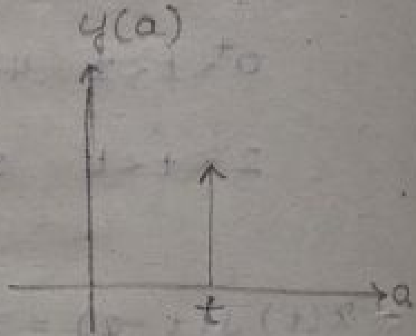
$$\rightarrow \int_{-\infty}^{\infty} x(t)\,\delta(t) = x(t) \text{ at } t=0 \Rightarrow x(0)$$

Ex: $\int_{-\infty}^{\infty} y(a)\,\delta(t-a)\,da \Rightarrow y(a)$ at $a=t$

$y(t)$

$t-a=0$

$\Rightarrow a=t$



$y(a) \Rightarrow a=t$ $\Downarrow$

then $y(t)$

Problems:

1. $\int_{-\infty}^{\infty} \delta(t)\,e^{-j\omega t}\,dt = x(t)$ at $t=0$

$\delta(t) \downarrow$ impulse at $t=0$; $e^{-j\omega t} \rightarrow x(t)$

$$\Rightarrow x(t) = e^{-j\omega t} \text{ at } t=0$$

$$= e^{-j\omega(0)}$$

$$= e^{-0} \Rightarrow e^{0} = 1$$

$$\therefore \int_{-\infty}^{\infty} \delta(t)\,e^{-j\omega t}\,dt = 1$$

2. $\int_{-\infty}^{\infty} \delta(t+4)\cos\left(\frac{\pi t}{4}\right) dt = x(t)$ at $t=-4$

$t+4=0$ $\qquad x(t)$

$t=-4$

$$x(t) = \cos\left(\frac{\pi t}{4}\right) dt$$

$$x(-4) = \cos\left(\frac{-4\pi}{4}\right)$$

$$= \cos(-\pi)$$

$$= \cos\pi \Rightarrow -1$$

$$\therefore \int_{-\infty}^{\infty} \delta(t+4)\cos\left(\frac{\pi t}{4}\right) dt = -1$$

3. $\delta(t+5)e^{-j\omega t} = ?$

$$\delta(t+5)e^{-j\omega t} = e^{-j\omega(-5)}\delta(t+5)$$

$t+5=0$

$t=-5$

$$= e^{j\omega 5}\delta(t+5)$$

Elementary signals $\rightarrow$ DT, CT

continuous signals

$u(t)$ - No, it's not periodic

$\delta(t)$ - No, " "

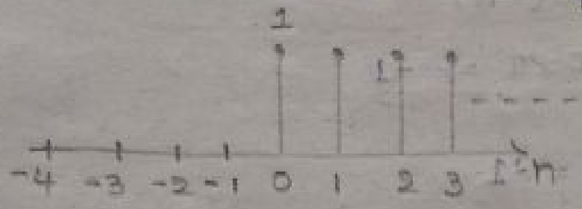
$^{\times}$($ce^{at} \Rightarrow$ yes, a is an imaginary)

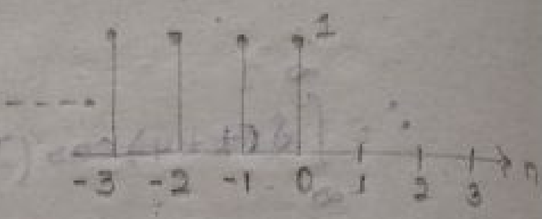
Discrete:

Unit Step:

Unit step $u[n]$ (n → integer) $= \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$

$u[n] \xrightarrow[n \to -n]{\text{time rev}} u[-n]$

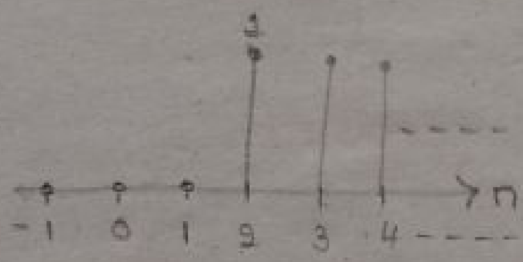




$u[n-2]$ (Right shift)

$\Rightarrow n-2=0$

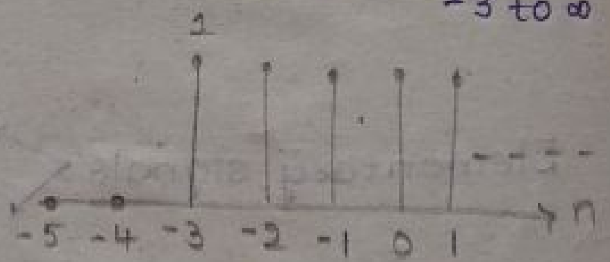
$\Rightarrow n=2 \rightarrow$ Starts at $n=2$ to ∞



$u[n+3]$

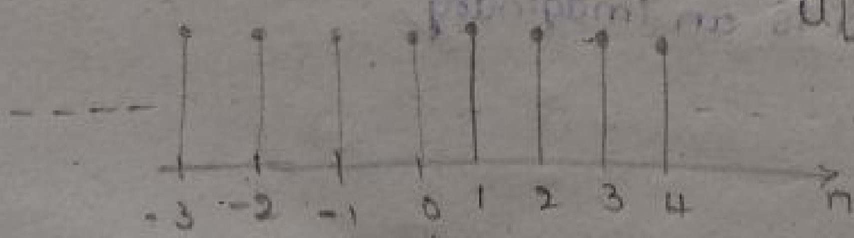
$\Rightarrow n+3=0$

$\Rightarrow n=-3 \rightarrow$ Starts at -3 to ∞



$u[n] \xrightarrow{\text{Time Rev}} u[-n] \xrightarrow[n \to n-4]{\text{R.S by 4}} u[-(n-4)]$

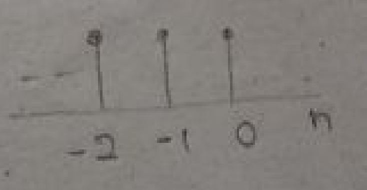
$\Rightarrow u[-n+4]$



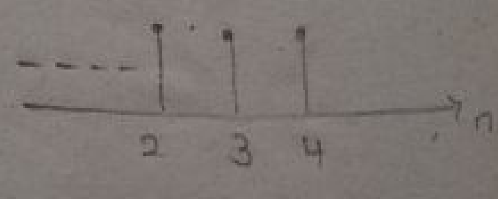
$u[4-n] = u[-n+4]$

$\hookrightarrow 4-n=0$

$\Rightarrow n=4$ to ∞



$\xrightarrow{+4}$



Impulse signal :

$$\delta[n] = \begin{cases} 0 & n \neq 0 \\ \underset{\downarrow}{1} & n = 0 \end{cases}$$

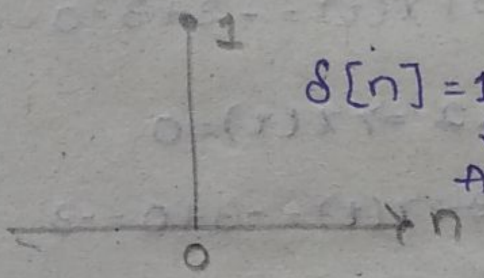
(Amplitude)

$\delta(t)$ at $t=0$

$$\int_{-\infty}^{\infty} \delta(t)\,dt = \underset{\downarrow}{1}$$

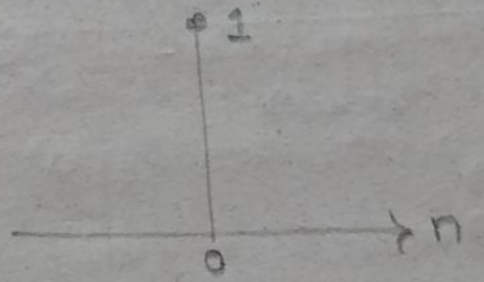
Area

$\delta[n]$



$\delta[n] = 1$ at 0
↓
Amp

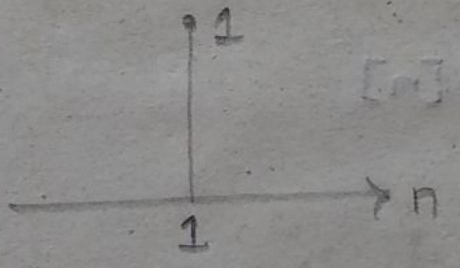
$\delta[-n]$



$\delta[n-1]$

$\hookrightarrow n-1=0$

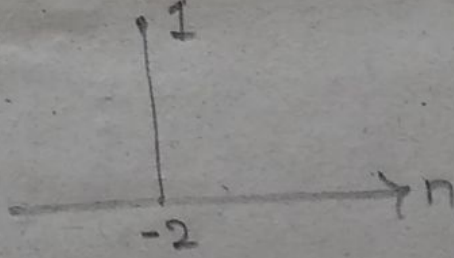
$n=1$



$\delta[n+2]$

$\hookrightarrow n+2=0$

$n=-2$



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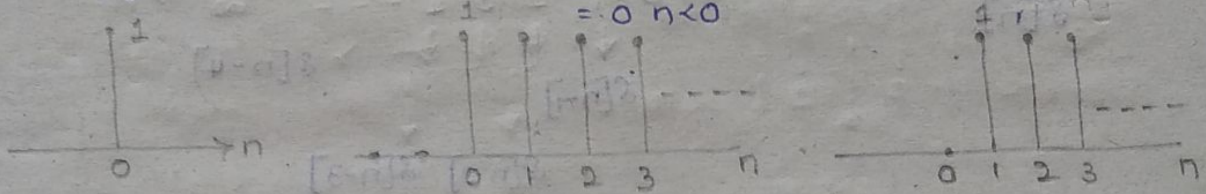
Relation between δ[n] and u[n]:

$$\delta[n] = \{u[n]\}$$

δ[n]

$u[n] = 1 \quad n \geq 0$

$= 0 \quad n < 0$



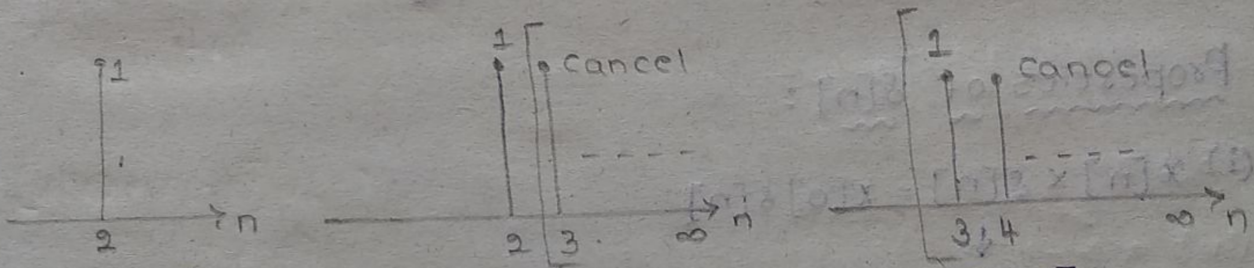
u[n-1]

Starts at n=1 to ∞

1. $\boxed{\delta[n] = u[n] - u[n-1]}$

Ex:

$$\delta[n-2] = u[n-2] - u[n-3]$$



δ[n-2] u[n-2] u[n-3]

2. $u[n] \longleftarrow \delta[n]$

$$= \delta[n] + \delta[n-1] + \delta[n-2] + \cdots\cdots \delta[n-\infty]$$

$$\boxed{u[n] = \sum_{n_0=0}^{\infty} \delta[n-n_0]}$$

$\delta[n] = 1 \quad n = 0$

$\delta[n] = 0 \quad n \neq 0$

Ex: at n=4

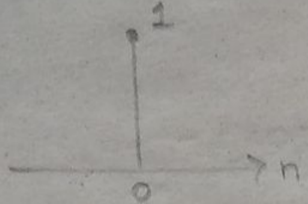
$u[n] = u[4]$

$\Rightarrow u[n]$ at $n = 4 = 1$

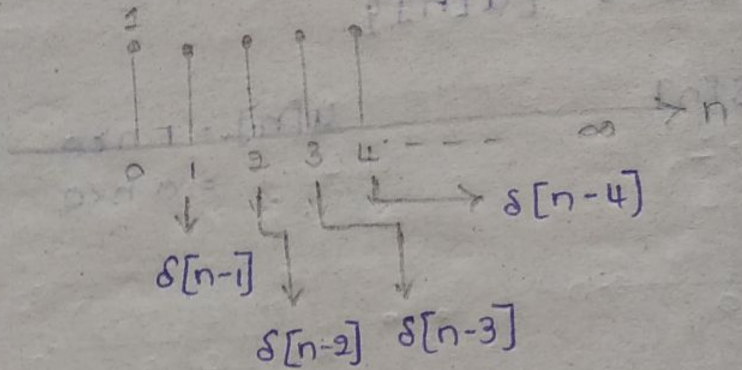
$$= \delta[\overset{0}{4-0}] + \delta[\overset{0}{4-1}] + \delta[\overset{0}{4-2}] + \delta[\overset{0}{4-3}] + \delta[\overset{1}{4-4}]$$

$$+ \delta[\overset{0}{4-5}] + \cdots\cdots \delta[4-\infty]$$

= 1



$\delta[n]$



D.T	C.T
$\delta[n] = u[n] - u[n-1]$	$\delta[t] = \dfrac{du(t)}{dt}$
$u[n] = \sum_{n_0=0}^{\infty} \delta[n-n_0]$	$u[t] = \int_{-\infty}^{\infty} \delta(t)\,dt$

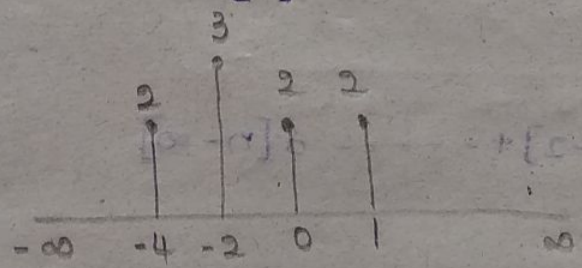
Properties of $\delta[n]$:

(i) $x[n] \times \delta[n] = x[0]\,\delta[n]$

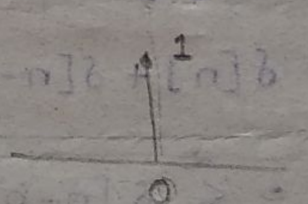
$\downarrow$

$n=0$

$x[n]$



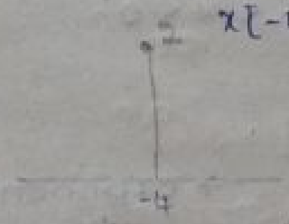
$\delta[n]$



	$x[n]$	$\delta[n]$	
$n=-4$	2	0	0
$n=-2$	3	0	0
$\boxed{n=0}$	2	1	2
$n=1$	2	0	0

$x[n]\,\delta[n+4] = x[-4]\,\delta[n+4]$

$= 2\,\delta[n+4]$



Discrete time

Exponential Signals:

Case(i):

$x[n] = C\alpha^{n} = Ce^{\beta n} \quad (\alpha = e^{\beta})$

2

$e^{(\ln 2)n}$ $\quad (e^{\beta})^{n}$

$(\Rightarrow e)$ $\quad \Rightarrow e^{\beta} = 2$

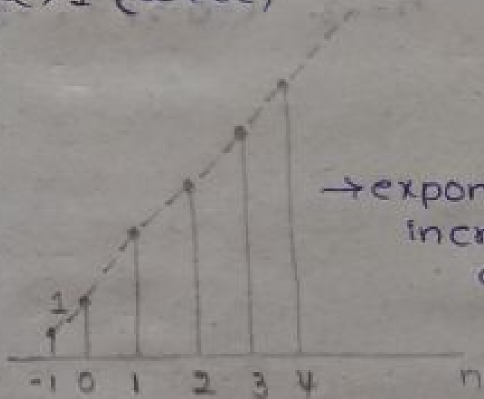
$\beta = \ln 2$

C-real ; α-real → +ve $\alpha > 0$; → -ve $\alpha < 0$

↓

C = 1

(i) $\alpha > 1$ (α +ve)



→ exponential increasing graph

$C\alpha^{n}$ here $C = 1$

$\Rightarrow 1 \cdot 2^{n} = 2^{n}$

$n = -1 \Rightarrow 2^{-1} = \frac{1}{2} = 0.5$

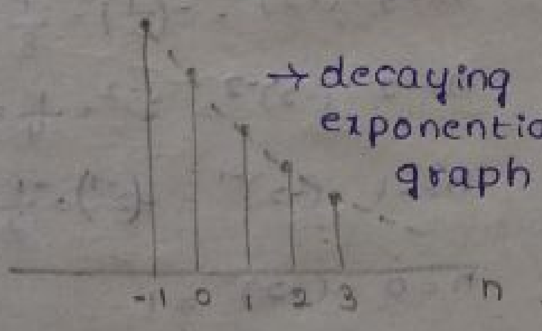
$n = 0 \Rightarrow 2^{0} = 1$

$n = 1 \Rightarrow 2^{1} = 2$

$n = 2 \Rightarrow 2^{2} = 4$

$n = 3 \Rightarrow 2^{3} = 8$

(ii) $0 < r < 1$ (α +ve)



→ decaying exponential graph

$\alpha = 0.5$

$n = -1 \Rightarrow (0.5)^{-1} = \left(\frac{5}{10}\right)^{-1}$

$= \left(\frac{10}{5}\right) = 2$

$n = 0 \Rightarrow (0.5)^{0} = 1$

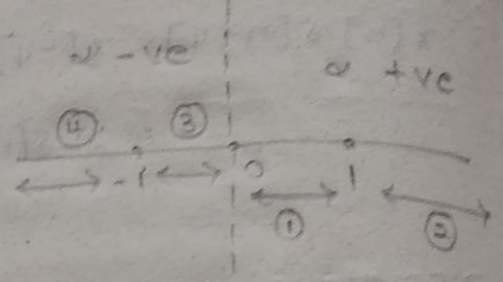
$n = 1 \Rightarrow (0.5)^{1} = 0.5$

$n = 2 \Rightarrow (0.5)^{2} = 0.25$

$n = 3 \Rightarrow (0.5)^{3} = 0.125$

(iii) 0

$1 - < \alpha < 0$

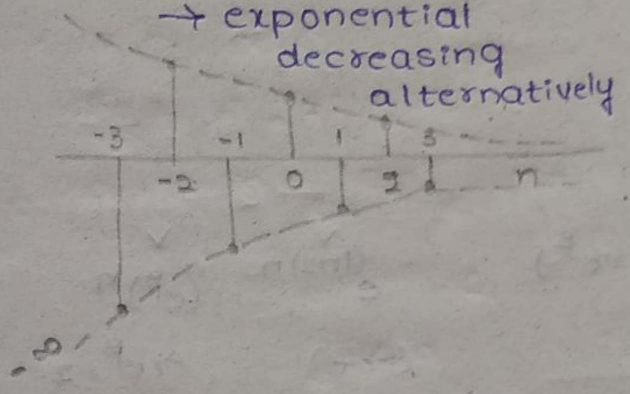


→ exponential decreasing alternatively

$c\alpha^n$

$c = 1$

$\alpha = -0.5$



	$\alpha = -0.5$
$n=0$	$(-0.5)^0 = 1$
$n=1$	$(-0.5)^1 = -0.5$
$n=2$	$(-0.5)^2 = 0.25$
$n=3$	$(-0.5)^3 = -0.125$
$n=4$	$(-0.5)^4 = 0.0625$
$n=-1$	$(-0.5)^{-1} = -2$
$n=-2$	$(-0.5)^{-2} = 4$
$n=-3$	$(-0.5)^{-3} = -8$

if $\alpha = -2$

$n = 0, \ (-2)^0 = 1$

$n = 1, \ (-2)^{-1} = -0.5$

$n = 2, \ (-2)^{-2} = 0.25$

$n = 3, \ (-2)^{-3} = -0.125$

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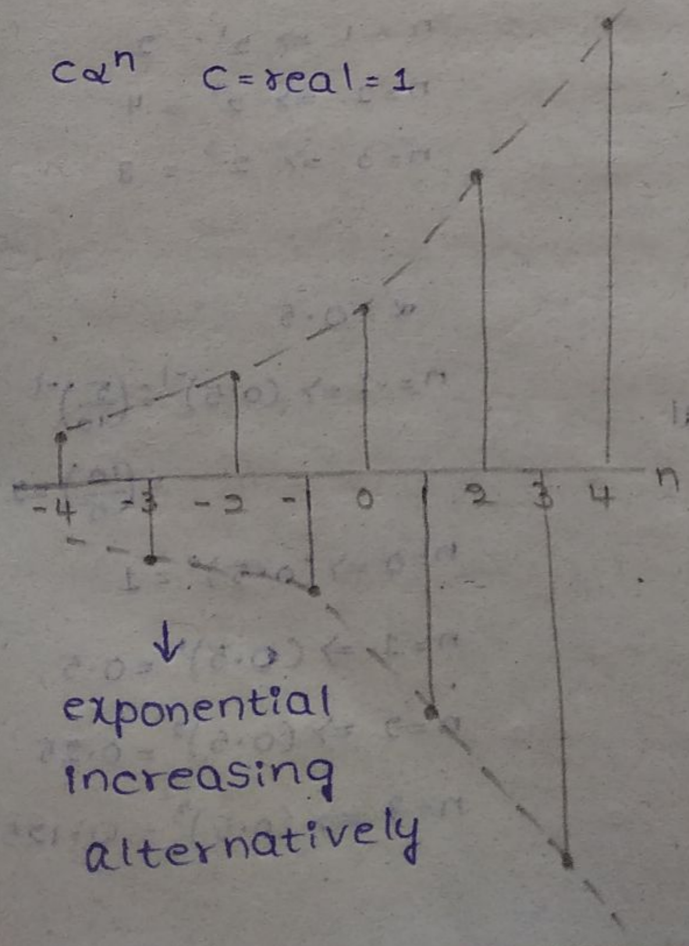
(iv) $\alpha < -1$

$c\alpha^n$ $c = \text{real} = 1$



↓
exponential increasing alternatively

	$\alpha = -2$
$n=-4$	$(-2)^{-4} = \left(\frac{1}{2}\right)^4 = \frac{1}{16} = 0.0625$
$n=-3$	$(-2)^{-3} = -\left(\frac{1}{2}\right)^3 = \frac{-1}{8} = -0.125$
$n=-2$	$(-2)^{-2} = 2^{-2} = \frac{1}{4} = 0.25$
$n=-1$	$(-2)^{-1} = -(2^{-1}) = \frac{-1}{2} = -0.5$
$n=0$	$(-2)^0 = 1$
$n=1$	$(-2)^1 = -2$
$n=2$	$(-2)^2 = 4$
$n=3$	$(-2)^3 = -8$
$n=4$	$(-2)^4 = 16$

$c\alpha^n$ $c=1$ α-real $\begin{cases} +ve \\ -ve \end{cases}$

$\alpha < -1$	$-1 < \alpha < 0$	$0 < \alpha < 1$	$\alpha > 1$
increasing alternatively (+ve/-ve)	decreasing alternatively (+ve/-ve)	decreas -ing	increasing

Ex:

1. $(0.8)^n$

A. $C=1$, $\alpha=0.8$

$\Rightarrow 0<r<1$

Exponential decreasing

2. e^{-2n}

A. $c\alpha^n$, $c=1$

$\alpha = e^{-2} = (0.135)^n$

here $\alpha = 0.135$; $0<\alpha<1$

Exponential decreasing

3. 2^k

Is 2^k an exponential signal?

A. $2^k = (e^{\beta})^k$

$e^{\beta} = 2$

$\beta = \ln 2 = 0.693$

$\Rightarrow 2^k = e^{0.693k}$ is an exponential signal.

Case (ii):

$C\alpha^n = Ce^{\beta n}$ $\quad \alpha = e^{\beta} \Rightarrow \beta = j\omega_0$

$x[n] = e^{j\omega_0 n}$

It is periodic.

when, $\omega \pm 2\pi = \omega_0$

$e^{j(\omega_0 + 2\pi)n} = e^{j\omega_0 n} \cdot e^{j2\pi n}$

$= e^{j\omega_0 n} \cdot 1$

$= e^{j\omega_0 n}$

$\boxed{CT}$

$e^{j\omega_0 t}$

It's periodic

And $T = \frac{2\pi}{\omega_0}$

Euler's relation

$e^{\pm ix} = \cos x \pm i \sin x$

$e^{\pm j\omega_0 n} = \cos \omega_0 n \pm j \sin \omega_0 n$

$\Rightarrow e^{\pm j2\pi n} = \cos 2\pi n \pm j \sin 2\pi n$

$= 1 \pm j(0)$

$e^{\pm j2\pi n} = 1$

Difference between CT exp / DT exp:

CT:

1. $e^{j\omega_0 t}$; periodic and $T = \frac{2\pi}{\omega_0}$

$\Downarrow$

$\cos \omega_0 t + i \sin \omega_0 t$

$\left.\begin{matrix}\cos \omega_0 t \\ \sin \omega_0 t\end{matrix}\right\}$ Sinusoidals are always periodic

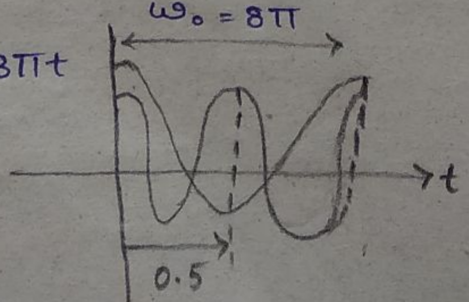
$T = \frac{2\pi}{\omega_0} \quad \forall \omega_0$

2. CT exponential signal is unique for each value of ω_0.

(i) $\cos \omega_0 t \Rightarrow \omega_0 = 2\pi$; $\cos 2\pi t$ yes ; $T = \frac{2\pi}{2\pi} = 1\,s$

(ii) $\cos 4\pi t \Rightarrow \omega_0 = 4\pi$; $\cos 4\pi t$ yes ; $T = \frac{2\pi}{4\pi} = 0.5\,s$

cos 8πt



$\omega_0 \uparrow$, rate of oscillations $\uparrow$

$\uparrow T = \frac{2\pi}{\omega_0 \downarrow}$

$\uparrow f = \frac{1}{T} = \frac{\omega_0 \uparrow}{2\pi}$

DT:

1. $e^{j\omega_0 n} = e^{j(\omega_0+2\pi)n}$

$= e^{j\omega_0 n} \cdot \underbrace{e^{j2\pi n}}_{1} = e^{j\omega_0 n}$

$\omega_0 + 4\pi$; $e^{j(\omega_0+4\pi)n} = e^{j\omega_0 n} \cdot \underbrace{e^{j4\pi n}}_{1} = e^{j\omega_0 n}$

$\omega_0 - 2\pi$; $e^{j(\omega_0-2\pi)n} = e^{j\omega_0 n} \cdot \underbrace{e^{-j2\pi n}}_{1} = e^{j\omega_0 n}$

$\omega_0 - 4\pi$; $e^{j(\omega_0-4\pi)n} = e^{j\omega_0 n} \cdot \underbrace{e^{-j4\pi n}}_{1} = e^{j\omega_0 n}$

DT is periodic, $\boxed{\omega_0 \pm (2\pi)m} \Rightarrow N = \frac{2\pi}{\omega_0} \cdot m$

2. $\omega_0 \pm (2\pi)m$; signal is same for ω_0 = integral multiple of 2π.

DT	CT
$e^{j\omega_0 n}$	$e^{j\omega_0 t}$
$\omega_0 \pm 2\pi$	$\forall \omega_0$
n is only integer	t is any value
$N = \left(\frac{2\pi}{\omega_0}\right) \cdot m$	$T = \left(\frac{2\pi}{\omega_0}\right) \cdot m$

Periodicity:

$x[n] = x[n+N]$

CT

$x(t) = x(t+T)$

here N = integer

N_0 = fundamental time period

Let

$x[n] = e^{j\omega_0 n}$

$$e^{j\omega_0 n} = e^{j\omega_0 [n+N]}$$

$$= e^{j\omega_0 n} \cdot \underbrace{e^{j\omega_0 N}}_{1}$$

$\because e^{j\omega_0 N} = 1$

$N = -4, -3, -2, -1, 0, 1, 2, \ldots$

$\Rightarrow e^{j\omega_0 (-4)}$, (-3)

and so on

$\therefore x[n]$ is periodic.

Eulers relation

$$e^{j\omega_0 N} = \cos\omega_0 N + j\sin\omega_0 N$$

$$= 1 + j(0) = 1$$

$$\omega_0 N = 0, 2\pi, 4\pi, -2\pi, -4\pi$$

$\Rightarrow \omega_0 N = (2\pi)m$ m = integer

$$\frac{\omega_0}{2\pi} = \frac{m \rightarrow \text{int}}{N \rightarrow \text{int}} \longleftarrow \omega_0 = \frac{2\pi m}{N}$$

$\Rightarrow N = \dfrac{2\pi m}{\omega_0} \rightarrow$ period

$$\boxed{\frac{\omega_0}{2\pi} = \text{rational number}}$$

$N_0 = \dfrac{2\pi m}{\omega_0} \rightarrow$ Smallest value of m

'Fundamental period'.

Ex:

1. $\cos\frac{n}{8} \approx \cos\omega_0 n$

A. here $\omega_0 = \frac{1}{8}$

$$\frac{\omega_0}{2\pi} = \frac{1/8}{2\pi} = \frac{1}{16\pi} = \text{irrational}$$

$\therefore$ Given signal is aperiodic

$\cos\frac{t}{8} \approx \cos\omega_0 t$

here $\omega_0 = \frac{1}{8}$

$\therefore$ Given signal is periodic

$$T = \frac{2\pi}{1/8} = 16\pi$$

2. $\cos \frac{7\pi n}{4}$

A. Here $\omega_0 = \frac{7\pi}{4}$

$$\frac{\omega_0}{2\pi} = \frac{7\pi/4}{2\pi}$$

$$= \frac{7\cancel{\pi}}{4} \times \frac{1}{2\cancel{\pi}}$$

$$= \frac{7}{8} \rightarrow \text{rational}$$

$\therefore \cos \frac{7\pi n}{4}$ is periodic

$$N = \frac{2\pi m}{\omega_0}$$

$$= \frac{2\cancel{\pi} m}{7\cancel{\pi}/4} = \frac{8m}{7} \Rightarrow \boxed{N=8} \left(\because \frac{8\times\cancel{7}}{\cancel{7}} = 8\right)$$

We know

$$\omega_0 = \frac{2\pi m}{N} = 2\pi(7/8) \Rightarrow \boxed{m=7}$$

N = integer

$$\frac{7\pi}{4} = 2\pi\left(\frac{7}{8}\right)$$

$\cos \frac{7\pi t}{4}$

here $\omega_0 = \frac{7\pi}{4}$

$\therefore$ It is periodic

3. $\cos \frac{5\pi n}{3}$ is periodic? $N_0 = ?$

A. Here $\omega_0 = \frac{5\pi}{3}$

$$\frac{\omega_0}{2\pi} = \frac{5\pi/3}{2\pi}$$

$$= \frac{5\cancel{\pi}}{3} \times \frac{1}{2\cancel{\pi}} = \frac{5}{6} \rightarrow \text{rational}$$

$\therefore \cos \frac{5\pi n}{3}$ is periodic.

$$N_0 = \frac{2\pi m}{\omega_0} = \frac{2\pi m}{5\pi/3}$$

$$= 2\cancel{\pi} m \times \frac{3}{5\cancel{\pi}}$$

$$= \frac{6m}{5} = \frac{6\times\cancel{5}}{\cancel{5}}$$

$$\Rightarrow \boxed{N_0 = 6}$$

$$\omega_0 = \frac{2\pi m}{N}$$

$$\frac{5\pi}{3} = 2\pi\left(\frac{5}{6}\right)$$

$$\Rightarrow \boxed{m=5}$$

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Exponential signals

$Ce^{at} = e^{j\omega_0 t}$ − CT $\Rightarrow a = j\omega_0$	DT − $e^{j\omega_0 n}$
1. It is always periodic $\forall \omega_0$ By euler's relation $e^{j\omega_0 t} = \cos\omega_0 t + j\sin\omega_0 t$ Sinusoidal signals are always period $\forall\omega_0$ $T = \frac{2\pi}{\omega_0} m$; 1,2,3,---- FTP: Smallest value of m	1. $e^{j\omega_0 n} = \cos\omega_0 n + i\sin\omega_0 t$ It is periodic when $N\omega_0 = \pm 2\pi m$ $\frac{\omega_0}{2\pi} = \frac{m}{N}$ = rational number
2. Exponential signal is distinct for different values of ω_0 $x_1(t) = e^{j\omega_{01} t}$ $x_2(t) = e^{j(\omega_{01}+2\pi)t}$ $= e^{j\omega_{01} t} \cdot \underbrace{e^{j2\pi t}}_{\neq 1}$	2. Here the signal will be same. $x_1[n] = e^{j\omega_{01} n}$ $x_2[n] = e^{j(\omega_{01}+2\pi)n}$ n = 1,2,3,... $= e^{j\omega_{01} n} \cdot \underbrace{e^{j2\pi n}}_{1}$ $= e^{j\omega_{01} n}$ $N = \frac{2\pi m}{\omega_0}$ FTP: N_0 = smallest value of m, which makes N_0 integer

1. $e^{j\frac{7\pi}{10}n} = x[n]$. Is $x[n]$ periodic?

A. $\frac{\omega_0}{2\pi} = \frac{7\pi/10}{2\pi} = \frac{7\cancel{\pi}}{10} \times \frac{1}{2\cancel{\pi}} = \frac{7}{20}$ and $\omega_0 = \frac{14\pi}{20} = \frac{7\pi}{10}$

$\therefore x[n]$ is periodic.

$N = \frac{2\pi m}{\omega_0} = \frac{2\pi}{7\pi/10} m = \frac{20}{7} m$ ($\because m = 1,2,3,----$)

$N_0 = (m=7) = 20$ s $\rightarrow$ FTP

$\omega_0 = \frac{2\pi m}{N}$

$\frac{14\pi}{20} = 2\pi\left(\frac{7}{20}\right)$

$\Rightarrow m = 7$

→ We can also $x[n]$ is periodic with period 40.

Since,

if $m = 14 \Rightarrow \frac{20}{\cancel{7}_1} \times \cancel{14}^2 = 40S$

Note:

Rational number

$Q = P/q$ where $q \neq 0$

2. $e^{j\frac{2\pi}{3}n} + e^{j\frac{3\pi}{8}n} = x[n]$. Is $x[n]$ periodic? If yes, find FTP?

A. $e^{j\frac{2\pi}{3}n}$

$x_1[n]$

$\downarrow$

$\omega_{01} = \frac{2\pi}{3}$

$\frac{\omega_{01}}{2\pi} = \frac{2\cancel{\pi}/3}{2\cancel{\pi}}$

$= \frac{1}{3}$

Rational

$\Rightarrow x_1[n]$ is periodic

$e^{j\frac{3\pi}{8}n}$

$x_2[n]$

$\downarrow$

$\omega_{02} = \frac{3\pi}{8}$

$\frac{\omega_{02}}{2\pi} = \frac{3\cancel{\pi}/8}{2\cancel{\pi}}$

$= \frac{8}{16}$ → Rational

$\Rightarrow x_2[n]$ is periodic

$$\underset{(P)}{x_1[n]} + \underset{(P)}{x_2[n]} = \underset{(P)}{x[n]}$$

$x[n]$ is periodic, if $\frac{T_1}{T_2}, \frac{f_1}{f_2}, \frac{\omega_{01}}{\omega_{02}} =$ rational

$$\Rightarrow \frac{2\pi/3}{3\pi/8} = \frac{16}{9} \rightarrow \text{rational}$$

$\therefore x[n]$ is also periodic.

Now, finding the period

$N_1 = \frac{2\pi m}{\omega_{01}}$

$= \frac{2\pi \cdot m}{2\pi/3} = 3m$

$N_{01} = 3$ [m=1]

$N_2 = \frac{2\pi m}{\omega_{02}}$

$= \frac{2\pi \cdot m}{3\pi/8}$

$= \frac{16}{3} m$

$N_{02} = 16$ [m=3]

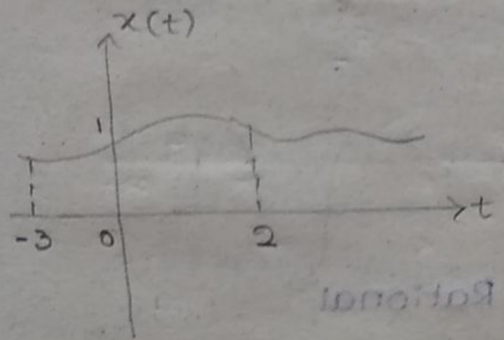
DT are defined $x[n] = x[n+N]$ i.e only at integers

Period of $x[n] = \{L.C.M \text{ of } N_{01}, N_{02}\}$

$= LCM\{3, 16\}$

$= 48$

$\therefore x[n]$ is periodic with period 48.



$x(t)$ at $t=0$ 1

at $t=2$ 0.5

at $t=-3$ 0.2

All physical Q Voltage = 2V

Current = 3A

Energy and power:

CT

$$\text{Energy of } x(t) = \int_{-\infty}^{\infty} [x(t)]^2 dt$$

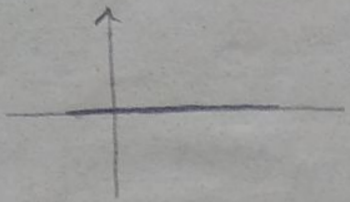
DT

$$\text{Energy of } x[n] = \sum_{n=-\infty}^{\infty} x[n]^2$$

Power:

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} [x(t)]^2 dt$$

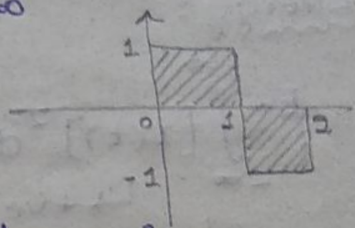
$$\text{Area} = \int_{-\infty}^{\infty} x(t)\,dt$$



Power   Energy

↓   ↓

W ⟷ J



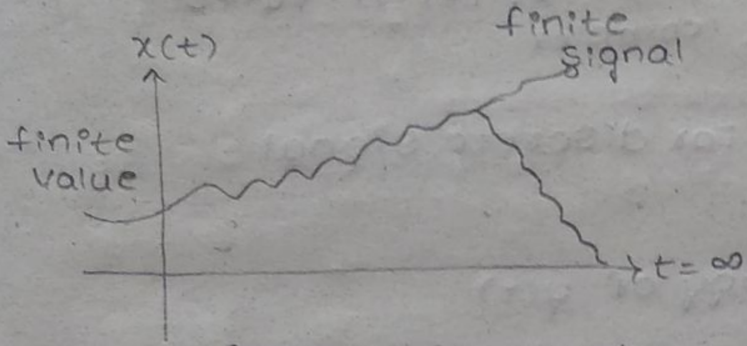
$$\text{area} = \int_0^1 1\,dt + \int_1^2 (-1)\,dt$$

$$= [t]_0^1 + -1\,[t]_1^2$$

$$= (1-0) + (-1)(2-1)$$

$$= 1 + (-1)(1)$$

$$= 1 - 1 = 0$$



energy ≠ finite value

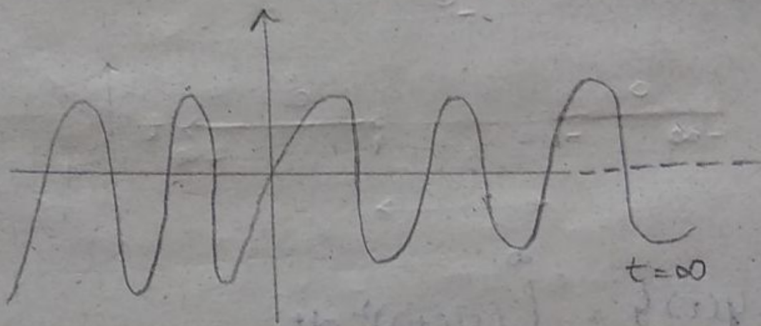
$t \to \infty$

$x(t) \to 0$

$x(t)$

energy = 5

energy = ∞



E = periodic signals = ∞

Power:

$$P = \lim_{t\to\infty} \frac{1}{2T} \int_{-T}^{T} [x(t)]^2\,dt$$

$$= \lim_{t\to\infty} \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} [x(t)]^2\,dt$$

$$= \lim_{t\to\infty} \frac{1}{T-(-T)} \int_{-T}^{T} [x(t)]^2\,dt$$

$$= \lim_{t\to\infty} \frac{1}{2T} \int_{-T}^{T} [x(t)]^2\,dt$$

$$\lim_{t\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} [x(t)]^2\,dt$$

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Energy:

$$E = \int_{-\infty}^{\infty} [x(t)]^2 \, dt \rightarrow x(t) \text{ is real}$$

↳ +ve

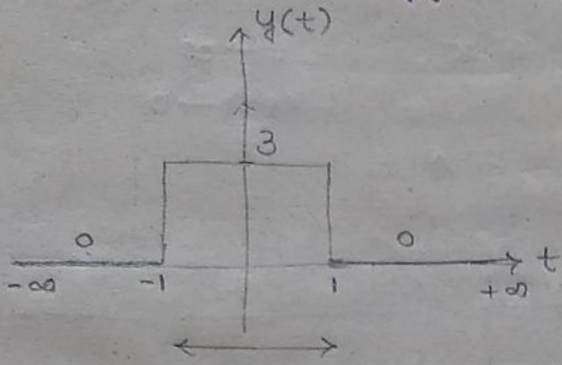
$$= \int_{-\infty}^{\infty} |x(t)|^2 \, dt \rightarrow x(t) = a + ib, \ |x(t)| = \sqrt{a^2+b^2}$$

complex

$$= \sum_{n=-\infty}^{\infty} [x[n]]^2 \rightarrow \text{For discrete signal}$$

Ex. Calculate the energy of $y(t)$.

1



Converges to 0 at $|t| = \infty$

A. $E\{y(t)\} = \int_{-\infty}^{\infty} (y(t))^2 \, dt$

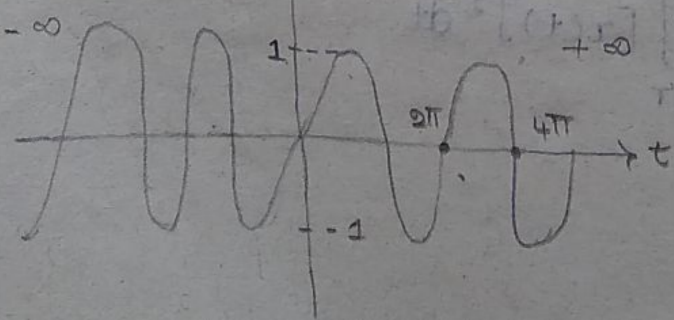
$$= \int_{-\infty}^{-1} \cancel{0^2}\, dt + \int_{-1}^{1} 3^2 \, dt + \int_{1}^{\infty} \cancel{0^2}\, dt$$

$$= \int_{-1}^{1} 9 \, dt = [9t]_{-1}^{1} = 9[1-(-1)]$$

$$= 9(2) = 18$$

$\therefore$ $\boxed{\text{Energy } (E) = 18\,J}$

2. Calculate the energy of $x(t) = \sin t$.



Periodic signal

A. $E = \int_{-\infty}^{\infty} \sin^2 t \, dt$

$= \int_{-\infty}^{\infty} \left(\frac{1-\cos 2t}{2}\right) dt$

$= \int_{-\infty}^{\infty} \frac{1}{2} dt + \int_{-\infty}^{\infty} \frac{-\cos 2t}{2} dt$

$= \left[\frac{t}{2}\right]_{-\infty}^{\infty} + \frac{-1}{2}\int_{-\infty}^{\infty} \cos 2t \, dt$

$\downarrow$ Area $= 0$

$= \frac{1}{2}\left[\infty - (-\infty)\right]$

$\boxed{E = \infty}$

$\sin t$ $\downarrow$ All sinusoidal CT are periodic

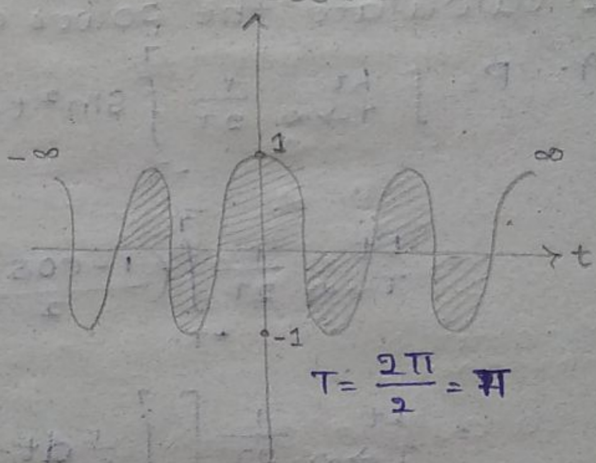
$T = \frac{2\pi}{4\pi} = 2\pi$

Here $a = t$

$\cos 2a = 1 - \sin^2 a$

$\sin^2 a = \frac{1-\cos 2a}{2}$

cos 2t



$T = \frac{2\pi}{2} = \pi$

Power:

$$\underset{T\to\infty}{Lt} \frac{1}{2T}\int_{-T}^{T} [x(t)]^2 dt$$

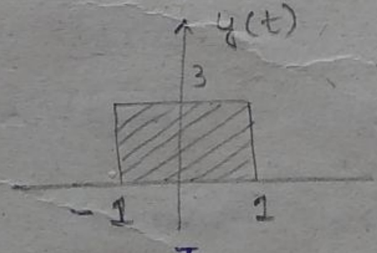
$$\underset{T\to\infty}{Lt} \frac{1}{T}\int_{-T/2}^{T/2} [x(t)]^2 dt$$

$$\underset{N\to\infty}{Lt} \frac{1}{2N+1}\sum_{n=-N}^{N} |x(t)|^2$$

$\downarrow$ For discrete signal

Ex.

1. Calculate the power of $y(t)$



A. $P = \underset{T\to\infty}{Lt} \frac{1}{2T}\int_{-T}^{T} y^2(t)\, dt$

$= \underset{T\to\infty}{Lt} \frac{1}{2T}\int_{-1}^{1} 3^2 \, dt$

$= \underset{T\to\infty}{Lt} \frac{1}{2T}\left[9t\right]_{-1}^{1}$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T} 9[1-(-1)]$$

$$= \underset{T\to\infty}{Lt} \frac{\cancel{18}\,9}{\cancel{2}T}$$

$$= \underset{T\to\infty}{Lt} \frac{9}{T} \to E$$

$$\boxed{Power = 0}$$

2. Calculate the power of sint

A. $P = \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} \sin^2 t\, dt$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} \left(\frac{1-\cos 2t}{2}\right) dt$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T} \left[\int_{-T}^{T} \frac{1}{2}\, dt - \int_{-T}^{T} \cancel{\frac{\cos 2t}{2}}\, dt \right]$$

$$0$$

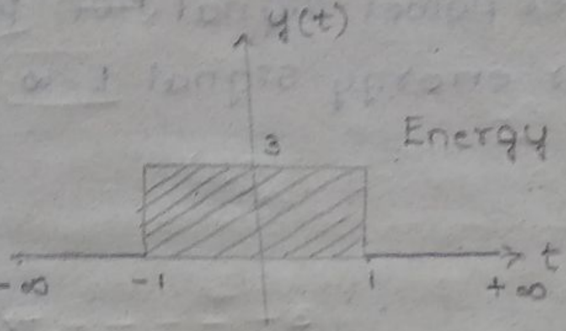
$$= \underset{T\to\infty}{Lt} \frac{1}{2T} \cdot \frac{1}{2} [t]_{-T}^{T}$$

$$= \underset{T\to\infty}{Lt} \frac{1}{4T} [T-(-T)]$$

$$= \underset{T\to\infty}{Lt} \frac{1}{4\cancel{T}} (2\cancel{T})$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2}$$

$$\boxed{P = \frac{1}{2}}$$

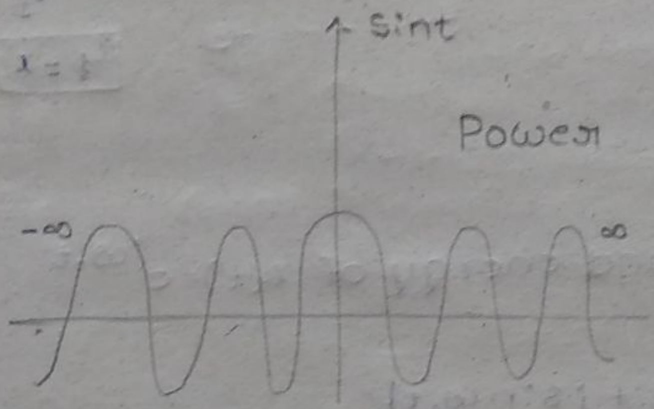


Here, Area is finite

converges to 0 at $|t|=\infty$

Energy $(E) = 18\,J$

Power $(P) = 0$



Energy $= \infty$

Power $= 1/2$

Classification based on Energy / Power:

1. A signal is called as

   Energy signal if energy is finite $\boxed{0 < E < \infty}$

   Power signal if power is finite $\boxed{P > 0}$

2. A signal cannot be both energy signal and power signal.

   if it is energy signal $\boxed{0 < E < \infty \Rightarrow P = 0}$

   Ex: $P = \underset{T \to \infty}{Lt} \frac{E}{2T}$ → finite

   $= \frac{5}{2T} = 0$

   if it is power signal $\boxed{P > 0 ,\ E = \infty}$

3. A signal can be neither power signal ~~nor~~ $\boxed{P=\infty, 0}$ nor energy signal $\boxed{E=\infty}$

Ex: Ramp signal

$g(t) = t$

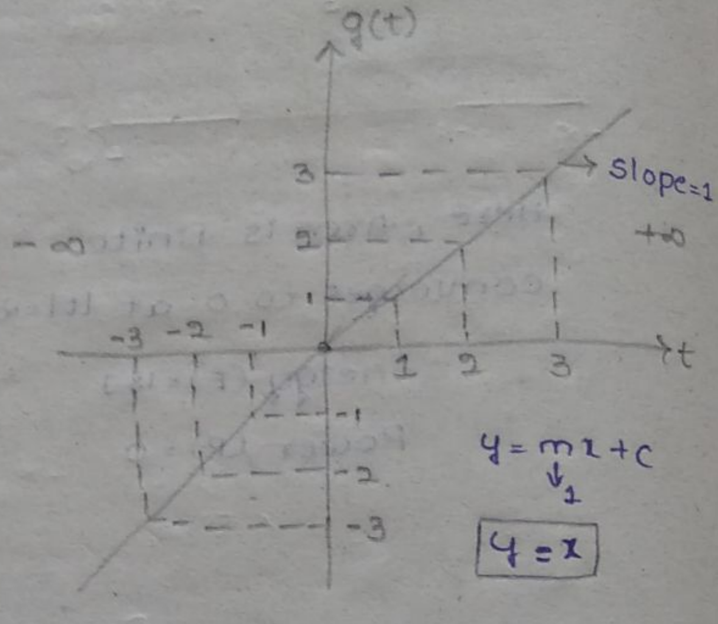
at $t=1$ $g(t)=1$

$t=2$ $g(t)=2$

$t=-3$ $g(t)=-3$

here $E=\infty$

$P=\infty$



Problems:

1. Calculate power and energy of $x(t) = e^{j\omega_0 t}$

A. It is periodic

$$|e^{j\omega_0 t}| = |\cos\omega_0 t + j\sin\omega_0 t|$$

$$= a + jb$$

$$= \sqrt{\cos^2\omega_0 t + \sin^2\omega_0 t} = 1$$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$= \int_{-\infty}^{\infty} 1 \, dt = 1\,[t]_{-\infty}^{\infty}$$

$$= 1\,[\infty - (-\infty)]$$

$$= \infty$$

$$P = \int \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} 1 \, dt$$

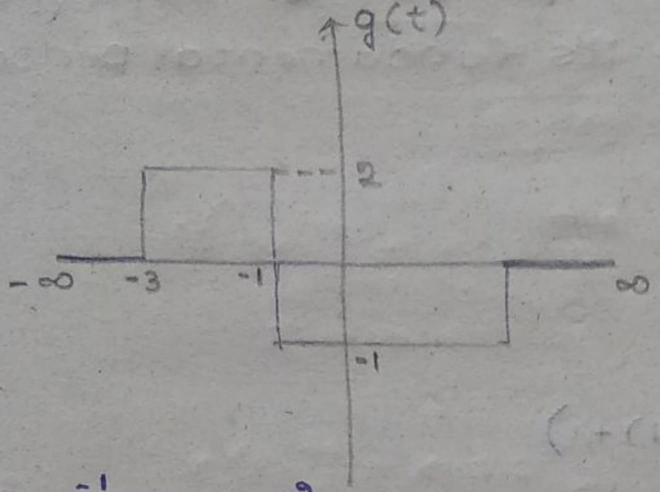
$$= \underset{T\to\infty}{Lt} \frac{1}{2T} [t]_{-T}^{T}$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T}\left[T-(-T)\right]$$

$$= \underset{T\to\infty}{Lt} \frac{1}{\cancel{2T}} \cdot \cancel{2T}$$

$$= 1\ W$$

2.



Energy

(a) Finite ✓

(b) ∞

(c) 0

Power

(a) 1

(b) ∞

(c) 0

(d) 5 ✓

A. $$E = \int_{-3}^{-1} 2^2\, dt + \int_{-1}^{2} (-1)^2\, dt$$

$$= 4\left[t\right]_{-3}^{-1} + 1\left[t\right]_{-1}^{2}$$

$$= 4\left[-1+3\right] + 1\left[2+1\right]$$

$$= 4\times 2 + 1\times 3 = 8+3$$

$$= 11$$

$$\boxed{E = 11}$$

$$P = \underset{T\to\infty}{Lt} \frac{1}{2T}\int_{-T}^{T}\left[2^2 dt + (-1)^2 dt\right]$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T}\left[\int_{-T}^{T} 2^2\, dt + \int_{-T}^{T}(-1)^2\, dt\right]$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T}\left[4\left[T-(-T)\right] + 1\left[T-(-T)\right]\right]$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T}\left[4\times 2T + 1\cdot 2T\right]$$

$$= \underset{T\to\infty}{Lt} \frac{1}{\cancel{2T}_{1}} \times \cancel{10T}^{5}$$

$$= \underset{T\to\infty}{Lt} 5$$

$$\boxed{P = 5}$$

1|3|21

Systems:

input → System → output

(i) $x(t) \rightarrow$ CTS $\rightarrow y(t)$

CTS ↓ Continuous time system

i/p and o/p both are CT signals

(ii) $x[n] \rightarrow$ DTS $\rightarrow y[n]$

DTS ↓ Discrete time system

(iii) Hybrid system - combination of both CTS and DTS

$x[n] \rightarrow$ ☐ $\rightarrow y(t)$

Ex: A/D converter
Analog
Digital

Analog: The range of amplitude can take continuous.

↑ y (amplitude)
Values

Digital: Amplitude can take only finite no. of values.

CT: Time is continuous.

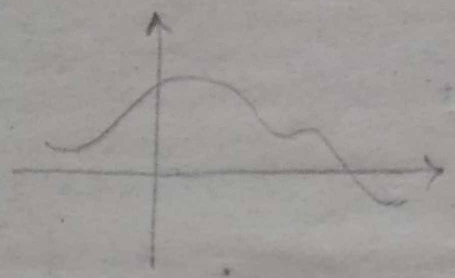
→ x

DT: Time is discrete.
And specific value (integer) should be taken.

→ x

Examples:

(i)



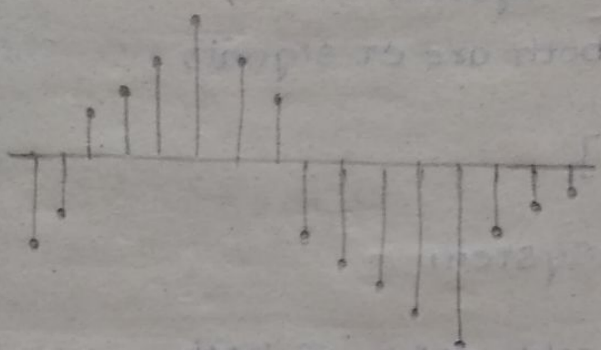
Analog ✓

Digital

CTS ✓

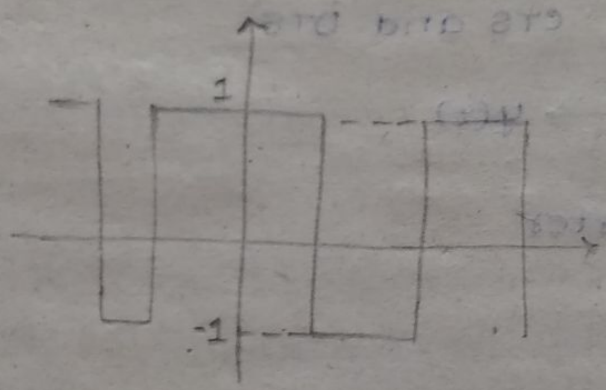
DTS

(ii)



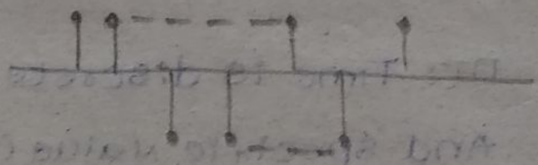
DTS, Analog

(iii)



CTS, Digital

(iv)



DTS, Digital

Analog - Digital system:

N = 2 => Binary signal   Ex: Computer (0,1)

2|3|21

Systems:

1.

(i) $x(t) \longrightarrow \boxed{CTS} \longrightarrow y(t)$

$x(t) \rightarrow y(t)$

(ii) $x[n] \rightarrow \boxed{DTS} \rightarrow y[n]$

$x[n] \rightarrow y[n]$

(iii) $x(t) \rightarrow \boxed{} \rightarrow y[n]$

$x[n] \rightarrow \boxed{} \rightarrow y(t)$

Hybrid systems

Ex: A/D converter

D/A converter

2.

(i) $x(t) \longrightarrow \boxed{S} \rightarrow y(t)$

SISO systems

(single input single output)

(ii) $\underline{x}(t) \rightarrow \boxed{S} \rightarrow \underline{y}(t)$

MIMO systems

(Multiple input multiple output)

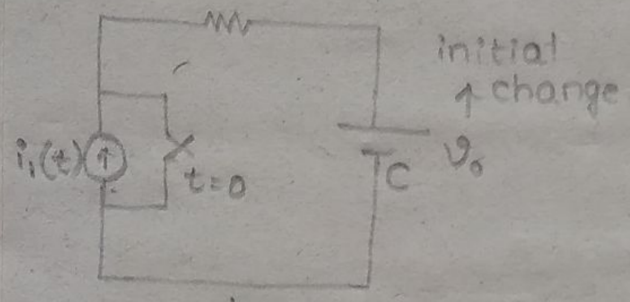
$$\left.\begin{array}{l} x_1(t) \\ x_2(t) \\ x_3(t) \\ \vdots \\ x_n(t) \end{array}\right\} \text{Input} \qquad \left.\begin{array}{l} y_1(t) \\ y_2(t) \\ y_3(t) \\ \vdots \\ y_n(t) \end{array}\right\} \text{Output}$$

3. Types of systems:

1. Memory | Memoryless:

Dynamic systems — Instantaneous (or) Static systems

A system S is said to be memory S, if the present o/p depends on present input and past inputs, (out depends on) past inputs.



↓
Memory system

$C = \frac{Q}{V}$

$\rightarrow v = \frac{Q}{C}$

$i = \frac{dQ}{dt}$

$Q = \int i\,dt$

$\Rightarrow v = \frac{\int i\,dt}{C}$

$x(t) = \frac{1}{C}\int i_1(t)\,dt + v_0$

Examples:

1. $y[n] = x[n] - x^2[n]$

A. It is DT system

$x[n] \rightarrow y[n]$

At $n=0$

$y[0] = x[0] - x^2[0]$

→ Memoryless system

$\boxed{n=0}$

$\left.\begin{array}{l} y[n] \text{ at } n=0 \\ x[n] \text{ at } n=0 \end{array}\right\} \begin{array}{l} y[0] \\ x[0] \end{array}$

2. $y[n] = x[n] - x[n-3]$

A. $y[1] = x[1] - x[-2]$

→ Memory system

$\boxed{n=1}$

$y[n]$ at $n=1$

$x[n]$ at $n=1$

and $x[n-3]$ at $n=-2$

3. $y[n] = \sum_{k=-\infty}^{n} x(k)$

A. $\boxed{n=2}$

$y[2] = \sum_{k=-\infty}^{2} x(k)$

$y[2] = x(-\infty) + n(----) + ------ + x(2)$

$x(-1) + x(0) + x(1)$

→ Memory system.

4. $y(n) = y(n-1) + x(n)$

A. $y(n)$ depends on present i/p, past o/p.

$n=2$

$y(2) = y(1) + x(2)$

→ Memory system

5. $y(t) = x(t^2)$

A. $y(0.1) = x(0.01)$; $t = 0.1$, $y(2) = x(4)$; $t = 2$

→ Memory system

N casual

(Future values should not be consider)

2. Invertible / Non-Invertible:

A system 's' is said to be invertible s

if $x(t) \xrightarrow{\text{Produces}} y(t)$

and $y(t) \xrightarrow{\text{Produces}} x(t)$

$$x(t) \longrightarrow \boxed{S} \xrightarrow{y(t)} \boxed{S'} \longrightarrow x(t)$$

Invertible system (S) Inverse system (S')

Eg: $y(t) = kx(t)$

$$x(t) \longrightarrow \underset{\times k}{\overset{A}{\boxed{\text{System}}}} \xrightarrow{y(t)} \overset{B}{\boxed{\div k}} \longrightarrow x(t)$$

A is invertible system B is inverse system

1. $y(t) = x(t)$

$x(t) \rightarrow \boxed{A} \xrightarrow{y(t)} \boxed{B} \rightarrow x(t)$

A ↓ Invertible system

B ↓ Inverse system

$x(t), y(t)$

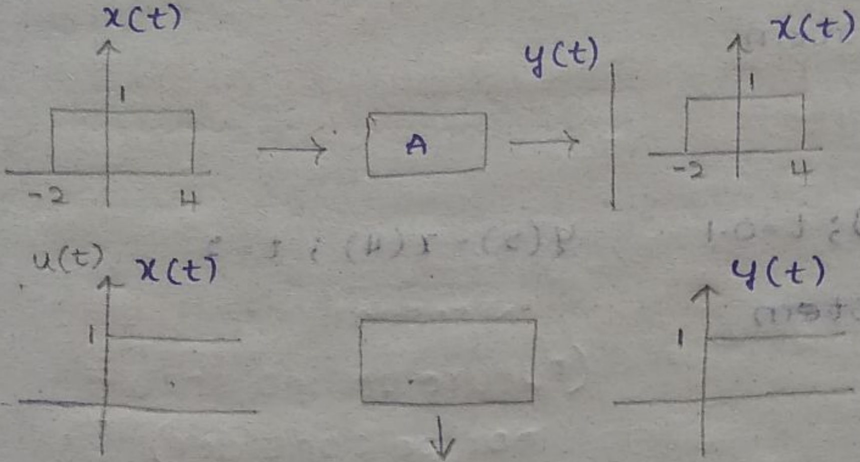
$x(t) \rightarrow \boxed{S} \rightarrow y(t)$

$x(t) \xrightarrow{S} y(t)$

$\boxed{y(t) = 2x(t)}$

$x(t) \rightarrow \boxed{\times 2} \rightarrow y(t)$

Identical system $x(t) \rightarrow y(t)$



Are identical system and invertible.

2. $y(t) = 0$

$x(t) \rightarrow \boxed{A} \rightarrow y(t) = 0 \rightarrow \boxed{?} \rightarrow x(t)$

A ↓ Invertible system

? ↓ Inverse system

→ Non-invertible system

3. $y(t) = x^2(t)$

square

$x(t) \rightarrow \boxed{S} \rightarrow y(t) = x^2(t)$

$x(t) = -2 \quad ; \quad y(t) = 4$

$x(t) = +2 \quad ; \quad y(t) = 4$

$\sqrt{\text{Square}}$

$\underset{y(t)}{4} \rightarrow \boxed{} \rightarrow \underset{x(t)}{\pm 2}$

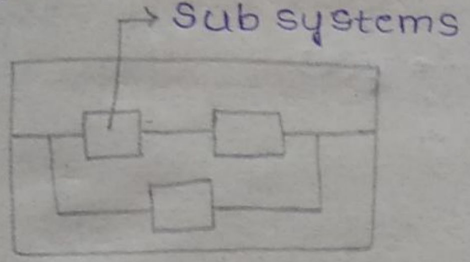
→ Non-invertible system.

$-2 \rightarrow \boxed{A} \rightarrow 4$

$4 \rightarrow \boxed{B} \rightarrow -2$

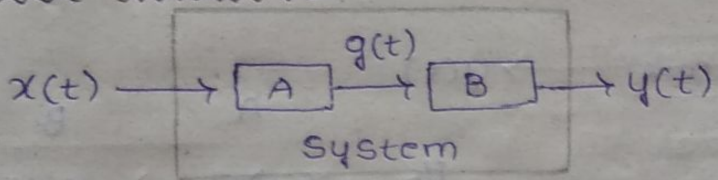
5|3|21

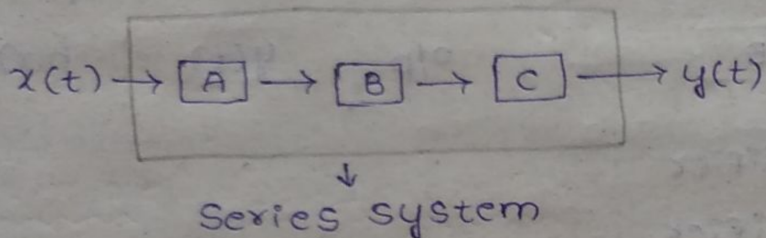
Systems:



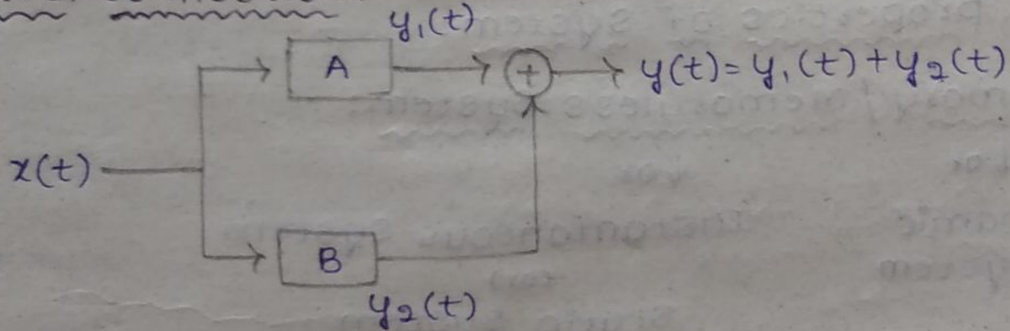
Interconnections of Subsystems:

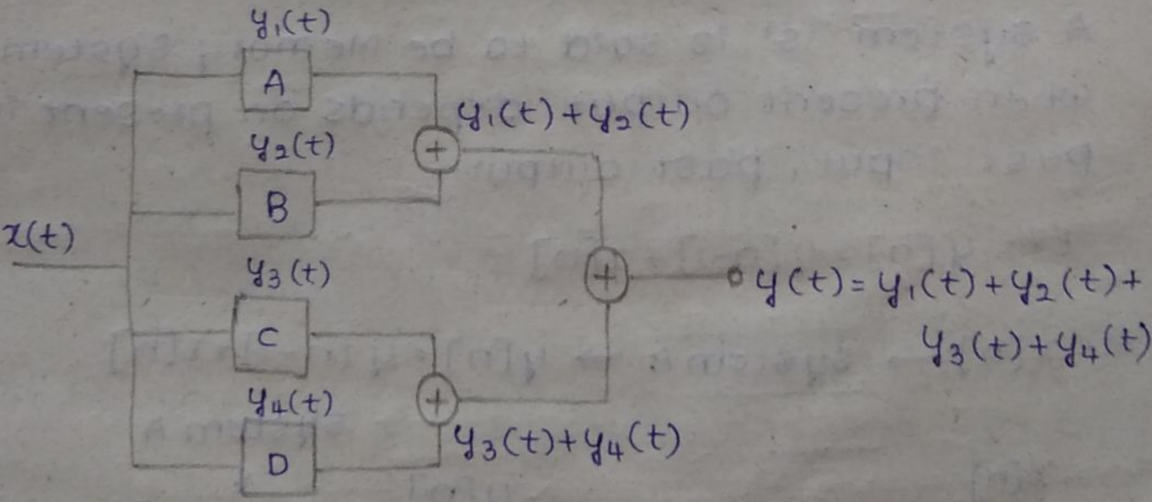
1. Series Connection:



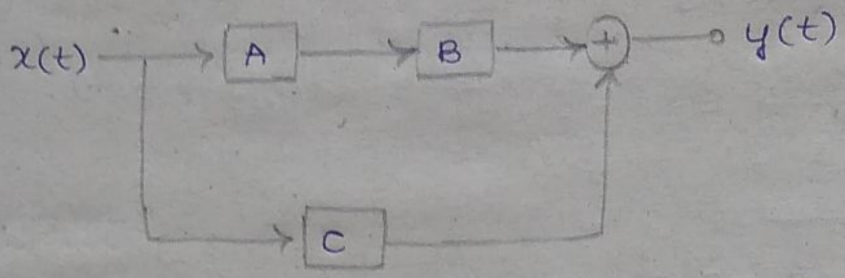


2. Parallel Connection:

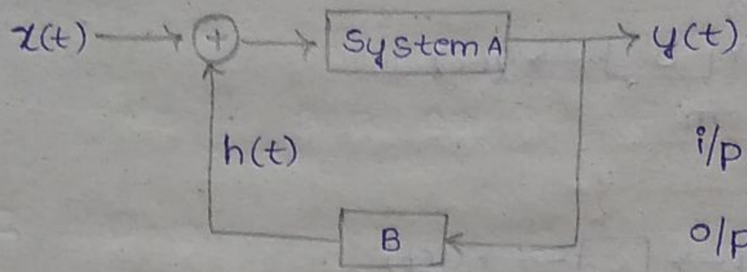




3. Series-parallel connection:



4. Feedback interconnection:



	A	B
i/p	$x(t)+h(t)$	$y(t)$
o/p	$y(t)$	$h(t)$

Feedback Amplifier

Cascade Amplifier

Basic properties of system:

1. Memory / Memoryless System:

Memory ↓ or Dynamic system

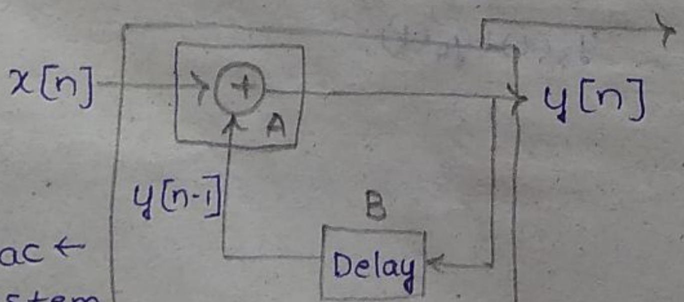
Memoryless ↓ or Instantaneous system (or) Static system

A system 's' is said to be Memory system, when present output depends on present input, past input, past output.

Ex: $y[n] = y[n-1] + x[n]$

A. $x[n] \longrightarrow \boxed{\text{system A}} \rightarrow y[n] = y[n-1] + x[n]$.

Feedbac-k system ←



→ system A

↓

Two subsystems

↓

1. Add

2. Delay

Given $y[n] = y[n-1] + x[n]$

Let

$n = 3$

$y[3] = y[2] + x[3]$

$y[3]$ → Present o/p

$y[2]$ → o/p at $n=2$ → Past o/p

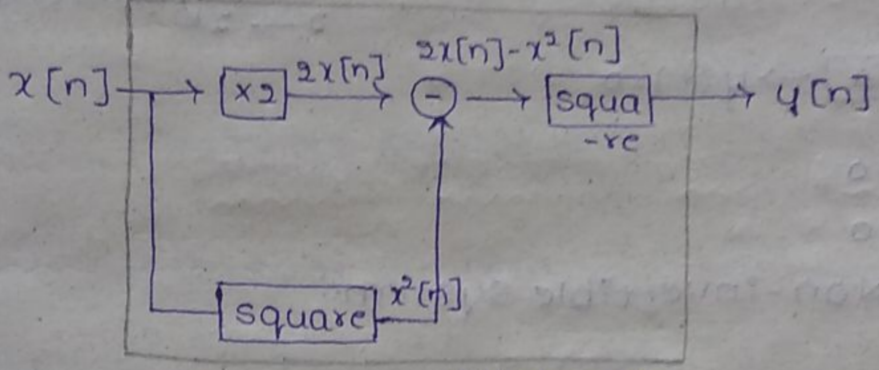
$x[3]$ → input $x[n]$ at $n=3$ → present i/p

∴ Given system is Memory System

Ex-(ii): $y[n] = \{2x[n] - x^2[n]\}^2$

A.

$x[n] \longrightarrow \boxed{A} \longrightarrow y[n] = \{2x[n] - x^2[n]\}^2$



System A

Given $y[n] = \{2x[n] - x^2[n]\}^2$

At $n = 2$

$y[2] = \{2x[2] - x^2[2]\}^2$

$y[2]$ → present o/p

$x[2]$ → present i/p

$x[0] = 1$

$x[1] = 2$

$\boxed{x[2] = 4}$

$x[3] = 6$

$x[4] = 7$

$= (2(4) - 4^2)^2$

$= (8 - 16)^2 = 64$

∴ It is memoryless system $(\because x^2[n] \neq x[n^2])$

2. Invertible / Non-invertible system:

$$x(t) \rightarrow \boxed{A} \xrightarrow{y(t)} \boxed{B} \rightarrow x(t)$$

$\boxed{A} \downarrow$ Invertible

$\boxed{B} \downarrow$ Inverse system

Examples:

1. $y(t) = x(t)$

A. $x(t) \rightarrow \boxed{A} \rightarrow y(t) = x(t)$

"Identity system"

→ o/p is identical to i/p

2. $y(t) = 0$

A. $x(t) \rightarrow \boxed{A} \rightarrow y(t) = 0$

$2 \rightarrow 0$

$3 \rightarrow 0$

So, it is Non-invertible system.

$\left.\begin{matrix} 1 \rightarrow 1 \\ 2 \rightarrow 2 \end{matrix}\right\}$ Invertible

3. $y(t) = x^2(t)$

A. $x(t) \rightarrow \boxed{A} \rightarrow y(t) = x^2(t)$

$-2 \rightarrow 4$

$2 \rightarrow 4$

So, it is Non-invertible system.

Note:

Invertible systems: If distinct input leads to distinct outputs.

3. Casual / Non-casual system:

Casual ↓ Non-Anticipatory system

Non-casual ↓ Anticipatory system

A system A is casual, if present output doesn't depends on future values and only depends on present and past.

Examples:

1. $y[n] = x[n] - x[n+1]$

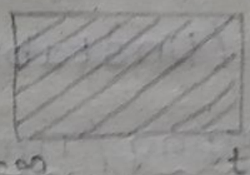
A. $\xrightarrow{x[n]} \boxed{A} \longrightarrow y[n] = x[n] - x[n+1]$

At $n=1$

$$y[1] = x[1] - \underset{\text{Future i/p}}{\underset{\downarrow}{x[2]}}$$

∴ It is Non-casual system.

2. $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$



A. $y(2) = \int_{-\infty}^{2} x(\tau)\, d\tau \rightarrow$ at $t=2$

∴ It is casual system

3. $y[n] = \sum_{k=-\infty}^{3n} x[k]$

A. At $n=1$

$$y[1] = \sum_{k=-\infty}^{3} x[k] = x[-\infty] + \text{-----} + x[1] + x[\cancel{2}] + x[\cancel{3}]$$

∴ It is Non-casual system.

4. $y(k) = x(-k)$

A. At $n=4$; $y(4) = x(-4)$

At $n=-4$; $y(-4) = x(4)$

∴ It is Non-casual System.

5. $y(t) = x(t)\cos(t+1)$

A. $x(t) \longrightarrow \boxed{\text{System}} \longrightarrow y(t) = x(t)\cos(t+1)$

$\times \cos(t+1)$

$\downarrow$

time Varying

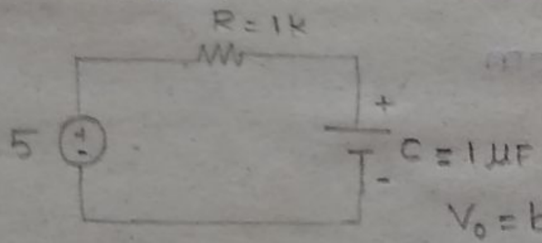
at $t=1$

$y(1) = x(1)\cos(2)$

$\therefore$ It is casual system.

6|3|21

1. Memory / Memoryless $\rightarrow$ Dynamic / static systems
2. Invertible / Non-Invertible systems
3. Casual / Non casual systems
4. stable / Unstable systems
5. Time Varying / Time invariant systems:



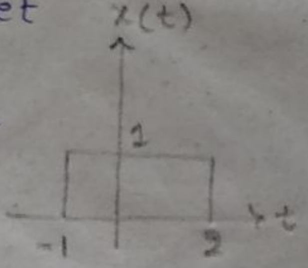
$$
\begin{array}{ccc}
x(t) & \xrightarrow{S} & y(t) \\
t_0 \downarrow & & \downarrow t_0 \\
x(t-t_0) & \xrightarrow{S} & y(t-t_0)
\end{array}
\Rightarrow S \text{ is time invariant system}
$$

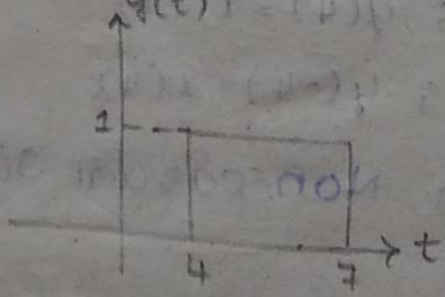
Ex:

(i) $x(t) \longrightarrow \boxed{S} \longrightarrow y(t)$ Rs by 5

A. Let

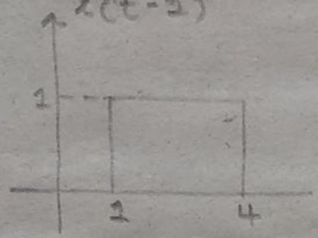
t_1

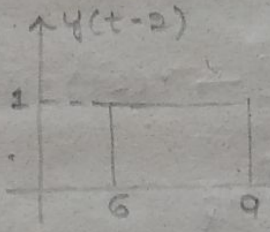




$x(t)$ Rs by 2 $\qquad\qquad$ $y(t)$ Rs by 2

t_2





$$x(t-2) \longrightarrow \boxed{S} \xrightarrow{\text{Rs by 5}} y(t-2)$$

$\therefore$ S is time invariant system.

(ii) $y[n] = n\,x[n]$

A. $x[n] \longrightarrow \boxed{A} \longrightarrow y[n]$

$\downarrow n_0 \qquad\qquad \downarrow n_0$

$x[n-n_0] \rightarrow \boxed{A} \rightarrow ? = y(n-n_0)$

$$x[n] \xrightarrow[\times n]{S} y[n] = n\,x[n] \;;\; y[n-n_0] = (n-n_0)\,x[n-n_0] \rightarrow (1)$$

Rs by n_0

$n \rightarrow n - n_0$

Rs by n_0

$$x[n-n_0] \xrightarrow[\times n]{S} n\,x[n-n_0] \neq y[n-n_0]$$

(2)

$\therefore$ System is time varying system.

(iii) $y(t) = \sin x(t)$

A. $x(t) \longrightarrow \boxed{B} \longrightarrow y(t)$

sin

$$x(t) \xrightarrow{B} \sin x(t) = y(t) \xrightarrow[t \rightarrow t - t_0]{\text{Rs by } t_0} y(t-t_0) = \sin x(t-t_0)$$

Rs by t_0

$t \rightarrow t - t_0$

$$x(t-t_0) \xrightarrow{B} \sin x(t-t_0) = y(t-t_0)$$

$\therefore$ System is time invariant system.

(iv) $y(t) = x(2t)$

A. $x(t) \longrightarrow \boxed{A} \longrightarrow x(2t) = y(t)$

$\downarrow$

Time
scaling
(compressible)

$x(t) \longrightarrow x(at)$

here $a = 2 > 1$

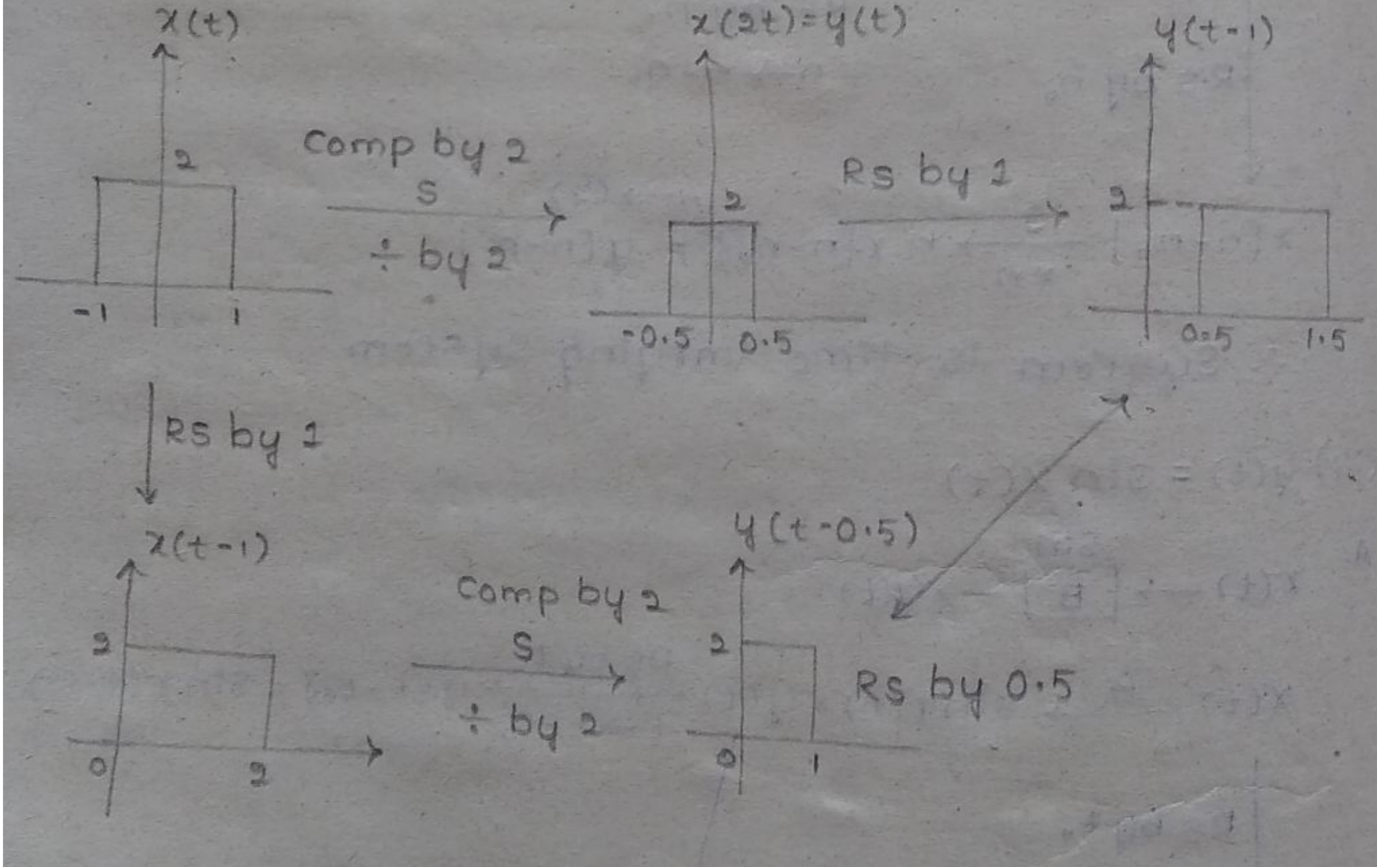
$$x(t) \xrightarrow[t \to 2t]{\text{comp by 2} \; A} x(2t) = y(t) \xrightarrow[t \to t - t_0]{\text{Rs by } t_0} y(t - t_0) = x[2(t - t_0)]$$

$$= x(2t - 2t_0)$$

$\downarrow$ Rs by t_0, $t \to t - t_0$

$$x(t - t_0) \xrightarrow[t \to 2t]{A} x(2t - t_0) \neq y(t - t_0)$$

$\therefore$ System is time varying system.



$\therefore$ It is time varying system.

Linear / Non-Linear Systems: A system is said to be linear if it satisfies two properties.

1. Super position (Additive)

$x_1(t) \xrightarrow{S} y_1(t)$

$x_2(t) \xrightarrow{S} y_2(t)$

$\Rightarrow x_1(t)+x_2(t) \xrightarrow{S} y_1(t)+y_2(t)$

5V i/p → 9V o/p
2V i/p → 7V o/p
5+2 i/p → 9+7 o/p
7V 16V

2. Homogenity (Scaling)

$x_1(t) \xrightarrow{S} y_1(t)$

$ax_1(t) \xrightarrow{S} ay_1(t)$

5V → 8V
↓×3
15V → 24V = o/p

$\Rightarrow$ $x_1(t) \xrightarrow{S} y_1(t)$

$ax_1(t) \xrightarrow{S} ay_1(t)$

$bx_2(t) \xrightarrow{S} by_2(t)$

$ax_1(t)+bx_2(t) \xrightarrow{S} ay_1(t)+by_2(t)$

Ex: $y(t)=t\,x(t)$ ×t

(i) $x(t) \longrightarrow \boxed{S} \longrightarrow y(t)=t\,x(t)$

A. $x_1(t) \longrightarrow t x_1(t) = y_1(t)$

$x_2(t) \longrightarrow t x_2(t) = y_2(t)$

Let

$x_3(t) = ax_1(t)+bx_2(t) \xrightarrow{S} t\left[ax_1(t)+bx_2(t)\right]$

$= tax_1(t) + tbx_2(t)$

$= a t x_1(t) + b t x_2(t)$

$ax_1(t)+bx_2(t) \xrightarrow{S} a\cdot y_1(t) + b\cdot y_2(t)$

∴ System is Linear

(ii) $y(t) = 2x(t) + 3$

A. $x(t) \longrightarrow \underset{+3}{\overset{\times 2}{\boxed{S}}} \longrightarrow y(t) = 2x(t) + 3$

$x_1(t) \xrightarrow{S} 2x_1(t) + 3 = y_1(t) \Rightarrow a y_1(t) = 2a x_1(t) + 3a$

$x_2(t) \xrightarrow{S} 2x_2(t) + 3 = y_2(t) \Rightarrow b y_2(t) = 2b x_2(t) + 3b$

$a x_1(t) + b x_2(t) \xrightarrow{S} 2(a x_1(t) + b x_2(t)) + 3$

$= 2a x_1(t) + 2b x_2(t) + 3$

$= a \cdot 2x_1(t) + b \cdot 2x_2(t) + 3$

$a y_1(t) + b y_2(t)$

$= 2a x_1(t) + 3a +$
$2b x_2(t) + 3b$

$a x_1(t) + b x_2(t) \overset{S}{\not\rightarrow} a y_1(t) + b y_2(t)$

$\therefore$ System is Non-linear

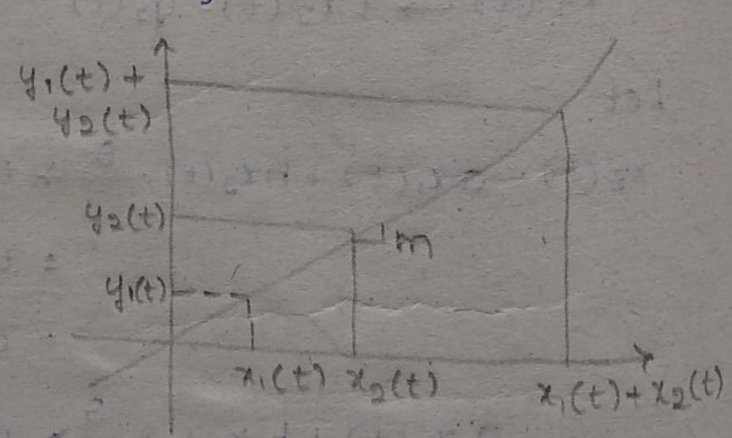
$x_1(t) \xrightarrow{S} 2x_1(t) + 3 = y_1(t)$

$x_2(t) \xrightarrow{S} 2x_2(t) + 3 = y_2(t)$

$x_1(t) + x_2(t) \xrightarrow[\times 2 \;\; +3]{S} 2[x_1(t) + x_2(t)] + 3$

$y(t) = 2x(t)$

$\Rightarrow y = mx$

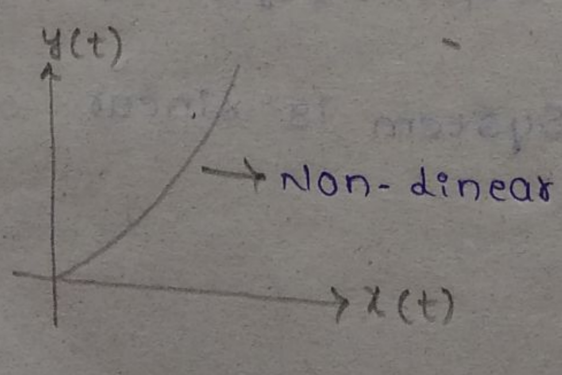


(iii) $y(t) = x^2(t)$

$2 \longrightarrow 4$

$3 \longrightarrow 9$

$4 \longrightarrow 16$



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[Assignment-3 written in Notes-02] 7|3|21

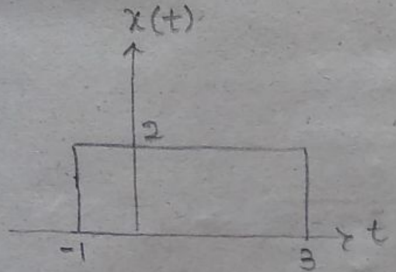
Stable / Unstable Systems: A system is said to be stable, if output is under control.

BIBO - Bounded input Bounded output.

If bounded input leads to bounded output.

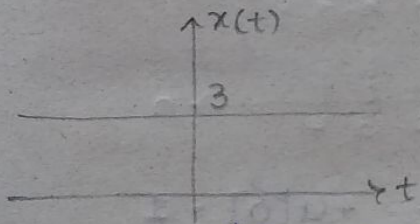
$x(t) \longrightarrow \boxed{S} \longrightarrow y(t)$

$\downarrow$ bounded $\qquad \downarrow$ bounded



$-1 < t < 3$

$x(t) = 2$ at $t = 0$



bounded signal

$-\infty < t < \infty$

$x(t) = 3 \ \forall t$

Is $u(t)$ a bounded signal?

A. Yes

$t = -\infty \ ; \ u(t) = 0$

$t = \infty \ ; \ u(t) = 1$

Examples:

1. $y(t) = t\,x(t)$

A. Let $x(t)$ be a bounded i/p

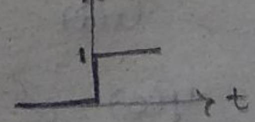
$x(t) \longrightarrow \boxed{\text{System}} \longrightarrow y(t)$

$\times t$

$\downarrow$

as a unit step signal $u(t)$

$\Rightarrow x(t) = u(t)$



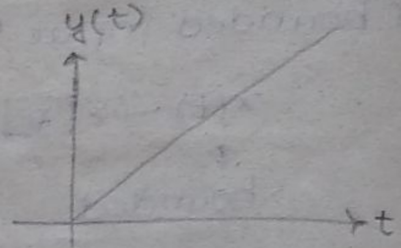
$u(t)$ at $t=-\infty$

at $t=-\infty$, $y(t) = t\,x(t) = (-\infty)\,u(-\infty) = 0$

$t=\infty$, $y(t) = t\,x(t) = (\infty)\,u(\infty) = \infty$

$t=0$, $y(t) = t\,x(t) = 0 \cdot u(0) = 0$

$\therefore$ The system is Unstable system.



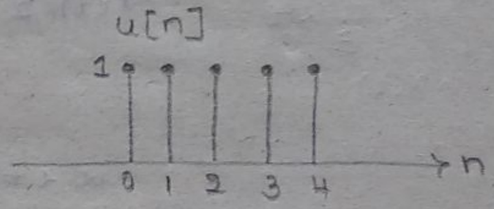
$t=1$, $y(t) = 1 \cdot u(1) = 1$

$t=2$, $y(t) = 2 \cdot u(2) = 2$

$t=3$, $y(t) = 3 \cdot u(3) = 3$ ----

2. $y[n] = \sum_{k=-\infty}^{n} x(k)$

A. Let $x(k)$ should be bounded



$x(k) = u(k)$

$y[n] = \sum_{k=-\infty}^{n} u(k)$

$y[0] = \sum_{k=-\infty}^{0} u[k] = \overset{0}{u[-\infty]} + \text{------} + u[0] = 1$

$y[1] = \sum_{k=-\infty}^{1} u[k] = u[-\infty] + \text{-----} + \cancel{u[-1]} + \overset{1}{u[0]} + \overset{1}{u[1]} = 2$

$y[2] = \qquad\qquad = 3$

$\vdots$

$y[\infty] = \infty$

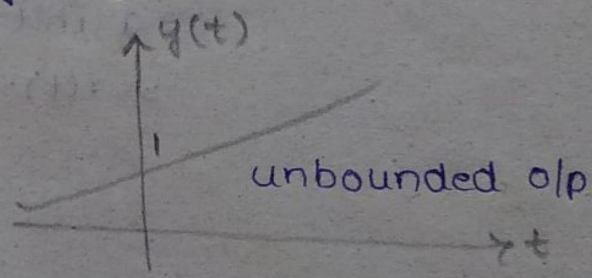
$\therefore$ Unstable system

3. $y(t) = e^t$

A. $x(t) \longrightarrow \boxed{S} \rightarrow y(t) = e^t$

$\downarrow$

$u(t)$



$y(0) = e^0 = 1$

$y(-\infty) = e^{-\infty} = 0$

$y(\infty) = e^{\infty} = \infty$

$\therefore$ Unstable system

4. $y(t) = e^{x(t)}$

A. $x(t) \longrightarrow \boxed{S} \longrightarrow y(t)$

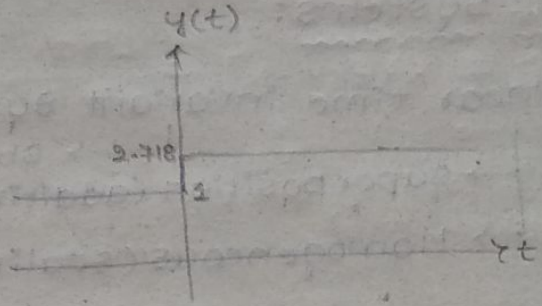
$x(t)$ is bounded

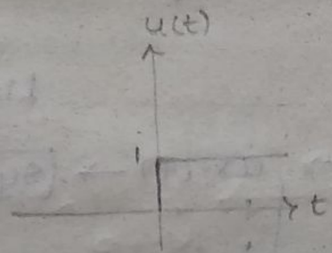
$\hookrightarrow u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$

$y(-\infty) = e^{u(-\infty)} = e^{0} = 1$

$y(\infty) = e^{u(\infty)} = e^{1} = 2.7182$

$y(0^{+}) = e^{u(0^{+})} = e^{1}$





Bounded i/p bounded o/p

$\therefore$ Stable system

5. $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

A. Let $x(\tau)$ be bounded

$\Rightarrow x(\tau) = u(\tau)$

$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$

$y(t) = \int_{-\infty}^{t} u(\tau)\, d\tau$

$= \int_{-\infty}^{0} u(\tau)\, d\tau + \int_{0}^{t} u(\tau)\, d\tau$

$= 0 + \int_{0}^{t} 1\, d\tau$

$= 0 + [\tau]_{0}^{t} = 0 + [t - 0] = t$

$y(t) = t \quad ; \quad x(t) = u(t)$

$t = -\infty \quad y(t) = -\infty \qquad t = -\infty \quad x(t) = 0$

$t = \infty \quad y(t) = \infty \qquad t = \infty \quad x(t) = 1$

$\downarrow \qquad\qquad \downarrow$

Unbounded o/p bounded i/p $\therefore$ Unstable system

→ All memoryless systems can be casual systems.

↓

present o/p

depends on present i/p

9/3/21 **UNIT-02: Linear Time Invariant systems**

LTI systems:

Linear time invariant systems

→ shifting in i/p - shifting in o/p

→ superposition (additivity)

→ Homogeneous (scaling)

$$x(t) = \sum_i a x_i(t)$$

$$\downarrow \qquad \downarrow$$

$$y(t) = \sum_i a y_i(t)$$

Linear

$$\Rightarrow a x_i(t) \longrightarrow \boxed{\text{system}} \longrightarrow a y_i(t)$$

h(t)

$$x(t) = \{ + \ldots \} \qquad \{ + \ldots \} \; y(t)$$

Unit Impulse Signal:

$$\delta(t) \longrightarrow \boxed{CTS} \rightarrow h(t)$$

$$\delta[n] \rightarrow \boxed{DTS} \longrightarrow h[n]$$

Impulse response

$$x(t) \longrightarrow \delta(t) \rightarrow \boxed{S} \rightarrow y(t)$$

$$\downarrow$$

$$h(t)$$

$$\delta(t) \longrightarrow \boxed{S} \longrightarrow h(t)$$

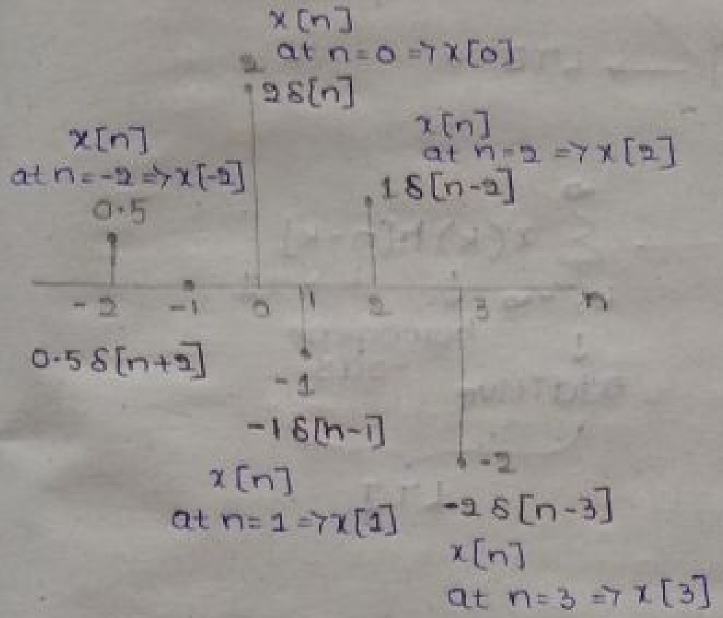
$$x(t) = \{ + \ldots \} \qquad \{ + \ldots \} = y(t)$$

$$\delta[n] \longrightarrow \boxed{DTS} \rightarrow h[n]$$

$$x[n] = \{ \qquad \} = y[n]$$

Discrete LTI:

$$x[n] \xrightarrow{\text{in terms of}} \delta[n]$$



LTI

$$x[n] \to \boxed{S} \to y[n]$$

$$x[n] = 0.5\,\delta[n+2] + 2\,\delta[n] - \delta[n-1] + \delta[n-2] - 2\,\delta[n-3]$$

$$= \underset{n=-2}{x[-2]\,\delta[n+2]} + \underset{n=0}{x[0]\,\delta[n]} + \overset{n=1}{x[1]\,\delta[n-1]} + \overset{n=2}{x[2]\,\delta[n-2]} + \underset{n=3}{x[3]\,\delta[n-3]}$$

LTI

$$x[n] \longrightarrow \boxed{S} \to ?\ y[n]$$

$$\delta[n] \to \boxed{S} \to h[n]$$

1. $x[n]$ = sum of impulse signals

$$(n=0)\ \delta[n] \xrightarrow{S} h[n]$$

Linear - $2\,\delta[n] \xrightarrow{S} 2\,h[n]$

TI - $(n=2)\ \delta[n-2] \xrightarrow{S} h[n-2]$

$$\delta[n+3] \xrightarrow{S} h[n+3]$$

$$2\,\delta[n] + \delta[n+3] \longrightarrow 2h[n] + h[n+3]$$

Linear

$$\begin{array}{ccc} 2 \to \boxed{S} & \xrightarrow{TI} & 4 \\ \downarrow \times 2 & & \downarrow \times 2 \\ 4 \to \boxed{S} & \to & 8 \end{array}$$

1. $\delta[n] \xrightarrow{S} h[n]$
2. LTI

$$x[n] \xrightarrow{S} y[n]$$

$$\delta[n] \xrightarrow{S} h[n] \quad \text{Impulse response}$$

$$n-k \qquad\qquad n-k$$

$$\delta[n-k] \xrightarrow{S} h[n-k] \longrightarrow \text{LTI}$$

$$\sum_{k=-\infty}^{\infty} x(k)\,\delta[n-k] \xrightarrow[S]{\text{LTI}} \sum_{k=-\infty}^{\infty} x(k)\,h[n-k] \rightarrow \text{TI}$$

$k=-\infty$ ↓ Additive; ↓ Scaling (amp); ↓ TI $(n=k)$

$k=-\infty$ ↓ Additive; ↓ homogeneous

$$\underbrace{\text{Additive} \quad \text{homogeneous}}_{\text{LTI}}$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k) \longrightarrow \sum_{k=-\infty}^{\infty} x(k)\,h(n-k) = y(n)$$

↓

Convolution Sum

LTI

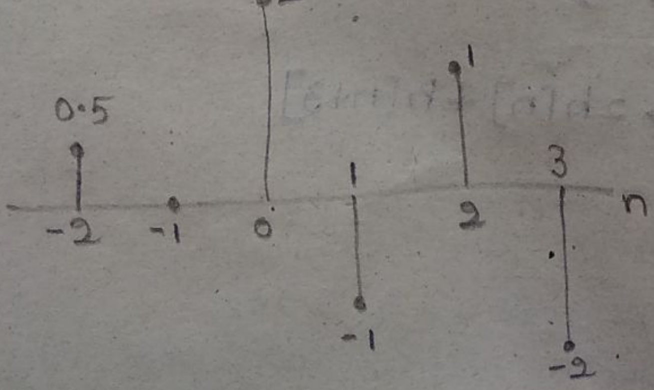
$$\boxed{y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)} = x(n) * h(n) \quad \text{Convoluted}$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$$

$x(k)$: Amplitude of signal at any k

$h(n-k)$: Shifted impulse response (by k)

Given $x[n]$

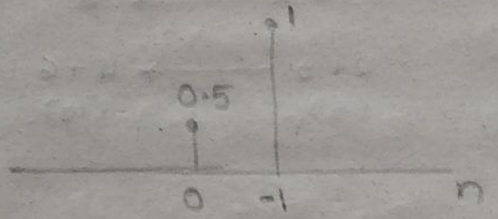


Given LTI

$$x[n] \rightarrow \boxed{S} \rightarrow y[n]$$

$\delta[n] \xrightarrow{S} h[n]$

Given: $h[n]$



$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

$---- \; -3 \;\; -2 \;\; -1 \;\; 0 \;\; 1 \;\; 2 \;\; 3 \; --- \; k$

$---- \; x_{-3} \;\; x_{-2} \;\; x_{-1} \;\; x_0 \;\; x_1 \;\; x_2 \;\; x_3 \; ---- \; x_k = x(k)$

$\downarrow$ $x(n)$ at $n=k$

$---- \; h_{-3} \;\; h_{-2} \;\; h_{-1} \;\; h_0 \;\; h_1 \;\; h_2 \;\; h_3 \; ---- \; h_k = h(k)$

$\downarrow$ Time rev

$---- \; h_3 \;\; h_2 \;\; h_1 \;\; h_0 \;\; h_{-1} \;\; h_{-2} \;\; h_{-3} \; --- \; h_{-k} = h(-k)$

$----- \; h_4 \;\; h_3 \;\; h_2 \;\; h_1 \;\; h_0 \;\; h_{-1} \;\; h_{-2} \; --- \; h_{n-k} = h(1-k)$

Rs by 1

$-------- \; h_3 \;\; h_2 \;\; h_1 \;\; h_0 \;\; h_{-1} \; --- = h(2-k)$

$$\boxed{y[n] = \sum_{k=-\infty}^{\infty} x(k)h(n-k)}$$

$$n=0 \rightarrow y(0) = \sum_{k=-\infty}^{\infty} x(k)h(-k) = \underbrace{x(-1)h(1)}_{k=-1} + \underbrace{x(0)h(0)}_{k=0} + \underbrace{x(1)h(-1)}_{k=1} + -----$$

$$n=1 \rightarrow y(1) = \sum_{k=-\infty}^{\infty} x(k)h(1-k) = \underbrace{x(-1)h(2)}_{k=-1} + \underbrace{x(0)h(1)}_{k=0} + \underbrace{x(1)h(0)}_{k=1} + ----$$

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Discrete LTI:

TI $\rightarrow$ time shift is same in i/p and o/p

L $\downarrow$ Linear

Linear $\rightarrow$ Homogeneous, Super position

$x(t) \xrightarrow{S} y(t)$

$ax(t) \xrightarrow{S} ay(t)$

Homogeneous

$5 \rightarrow \boxed{} \rightarrow 10$

$\downarrow \times 3 \qquad\qquad \downarrow \times 3$

$15 \rightarrow \boxed{} \rightarrow 30$

Super position

$2 \xrightarrow{S} 4$

$3 \xrightarrow{S} 6$

$\underset{(5)}{2+3} \longrightarrow \underset{(10)}{4+6}$

LTI system

$\delta[n] \rightarrow \boxed{} \rightarrow h[n]$

$x[n] \rightarrow \boxed{\text{LTI}} \rightarrow$ Impulse response

$\left.\begin{array}{l} x[n] \rightarrow \text{interms of impulse signals} \\ h[n] \rightarrow \text{impulse response} \end{array}\right\}$ LTI o/p for an i/p

$\delta[n] \xrightarrow{LTI} h[n]$

$\delta[n-k] \longrightarrow h[n-k]$ TI

$x(k)\,\delta[n-k] \longrightarrow x(k)\,h[n-k]$ homogenity

$\sum_{k=-\infty}^{\infty} x(k)\,\delta[n-k] \longrightarrow \sum_{k=-\infty}^{\infty} x(k)\,h[n-k]$ superposition

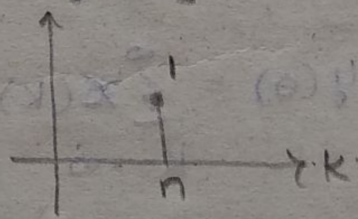
$\downarrow$
$k=n$

$\Downarrow$

Sifting property

$x(k)$ at $k=n$

$\delta[n-k]$

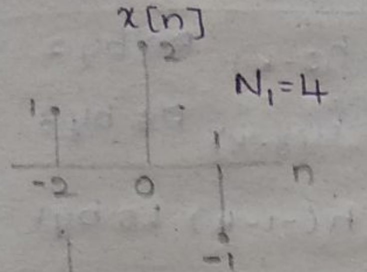


$x(n) \longrightarrow y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$

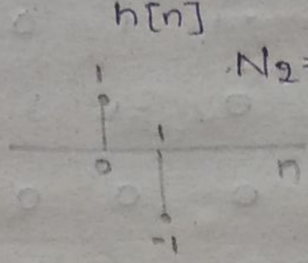
Convolution sum $\boxed{y(n) = x(n) * h(n)}$

Ex. Calculate the $y[n]$

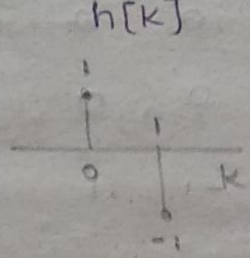
1.



$N_1 = 4$



$N_2 = 2$



A.

$x[3] = x[n]$ at $n=3$

$\delta[n] \longrightarrow \boxed{LTI} \longrightarrow h[n]$ ② ③

$\rightarrow \delta[n+2] + 2\delta[n] - 1\delta[n-1] = x[n]$ ①

$y[n] = x[n] * h[n]$

$$y[n] = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

$n=0$

$$y(0) = \sum_{k=-\infty}^{\infty} x(k)h(-k) = \underset{k=0}{x(0)h(0)} + \underset{k=1}{x(1)h(-1)} + \underset{k=2}{x(-2)h(2)}$$

$h(k) \xrightarrow{TR} h(-k)$

$\downarrow$ Rs by n

$h(n-k)$

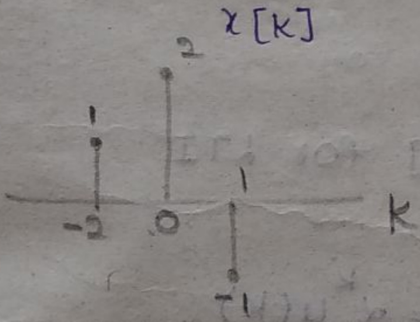
the no. of non-zero values in $y(n)$

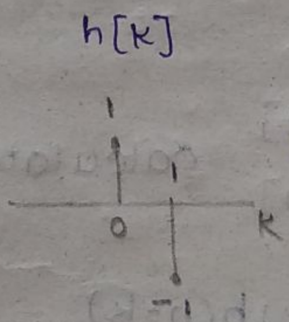
$= N_1 + N_2 - 1 = 4 + 2 - 1 = 5$

$\downarrow$

Length of $x[n]$

(Length of $h[n]$)





$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

-3	-2	-1	0	1	2	3	k	$\Sigma x(k)h(n-k)$
0	1	0	2	-1	0	0	$x_k = x(k)$	$y(0) = 0$
0	0	0	1	-1	0	0	$h_k = h(k)$	$y(1) = +1$ $y(-1) = 2$
0	0	-1	1	0	0	0	$h_{-k} = h(-k)$	Rs by 1
0	0	0	-1	1	0	0	$h(1-k)$	

-3	-2	-1	0	1	2	3	k	
0	0	0	0	-1	1	0	$h(2-k)$ Rs by 2	
0	0	0	0	0	-1	1	$h(3-k)$ Rs by 3	
0	-1	1	0	0	0	0	$h(-1-k)$ Ls by 1	$y(-1)=-$
-1	1	0	0	0	0	0	$h(-2-k)$ Ls by 2	$y(-2)=$
-1 1	0	0	0	0	0	0	$h(-3-k)$ Ls by 3	$y(-3)=$

$n=0 \quad y(0) = \sum_{k=-\infty}^{\infty} x(k)h(-k)$

$\cdot\ n=1$

$$y(1) = \sum_{k=-\infty}^{\infty} x(k)h(1-k)$$

$$h(k) \xrightarrow{TR} h(-k) \xrightarrow[\text{Ls by } 1]{\text{shifting by } -n} h(-(k+1)) = h(-k-1)$$

$$= h(-1-k)$$

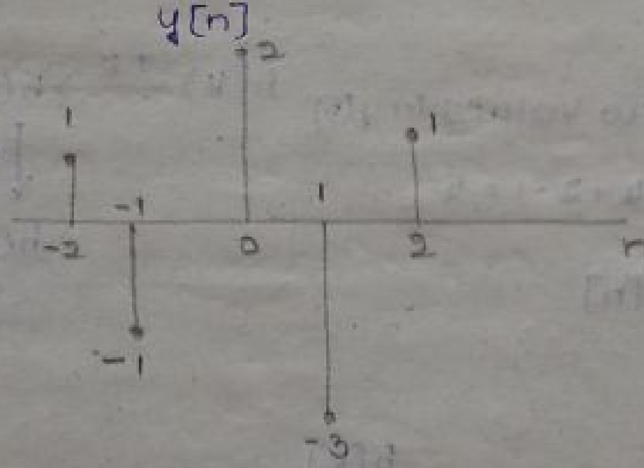
$\downarrow$

$k \to k+1$

$\downarrow$

$h(n-k)$

$\Rightarrow n=-1$



2. $x[n] = \alpha^n u[n]$

$h[n] = \alpha^n u[n]$ calculate $y[n]$ for LTI

A. $$y[n] = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

$x(k) = \alpha^k u(k)$

$h(k) = \alpha^k u(k)$

$\downarrow$

$h(-k) = \alpha^{-k} u(-k)$

$h(n-k = \alpha^{n-k} u(n-k)$

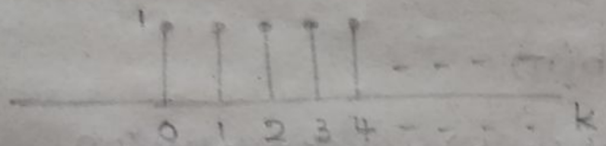
$$= \sum_{k=-\infty}^{\infty} \alpha^k u(k) \cdot \alpha^{n-k} u(n-k) \quad \left[\because \alpha^{a-b} = \frac{\alpha^a}{\alpha^b}\right] (\because -k+n = n-k)$$

$$= \sum_{k=-\infty}^{\infty} \alpha^n u(k)\, u(n-k)$$

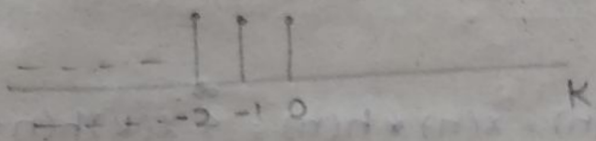
$$= \alpha^n \sum_{k=-\infty}^{\infty} u(k)\, u(n-k)$$

$$\left[\; = \alpha^a \cdot \alpha^{-b} \qquad \alpha^a \cdot \alpha^{-a} = 1 \qquad \frac{\alpha^a}{\frac{1}{\alpha^a}} = 1 \;\right]$$

$u(k)$

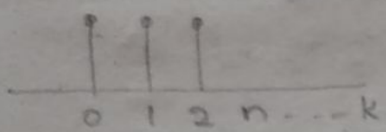


$u(-k)$



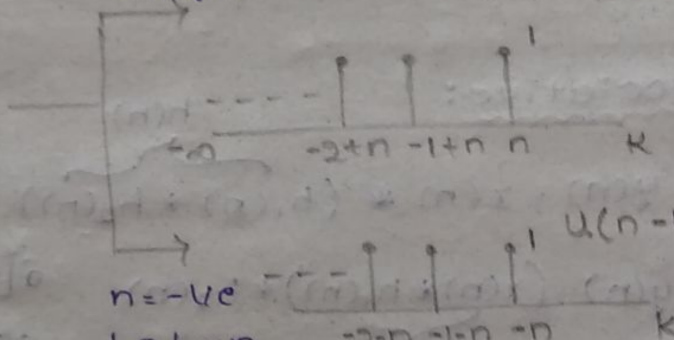
$u(k)$

$n = 0 \to \infty$



(i)

$n = +ve$
Rs by n $\longrightarrow u(n-k) \Rightarrow \boxed{-\infty \text{ to } n}$

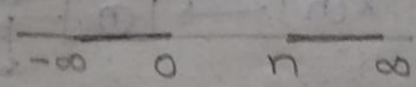


$u(n-k)$

$n = -ve$
Ls by n

(ii)

$u(k)u(n-k) = 0 \quad n$ is $-ve$



$1 \times 0 = 0$

$1 \times 1 = 1 \quad n \geq 0 ; \sum_{k=0}^{n} 1$

$$y(n) = \alpha^n \boxed{\sum_{k=0}^{n} 1} = \underset{k=0}{1} + \underset{k=1}{1} + \cdots \underset{k=n}{1}$$

$$\therefore y(n) = \alpha^n (n+1)$$

$$\sum_{k=0}^{2} 2 = \underset{k=0}{2} + \underset{k=1}{2} + \underset{k=2}{2}$$

$$\sum_{k=0}^{3} 1 = \underset{k=0}{1} + \underset{k=1}{1} + \underset{k=2}{1} + \underset{k=3}{1} = 4$$

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Discrete LTI Systems:

$$x[n] \longrightarrow \boxed{\text{LTI}} \longrightarrow y[n] = x(n) * h(n)$$

(the box is labelled "Discrete")

$$\delta[n] \longrightarrow \boxed{\text{D LTI}} \longrightarrow h[n]$$

$$y(n) = x(n) * h(n)$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k) \longrightarrow \text{Convolution sum}$$

Properties:

1. **Commutative:** $y(n) = x(n) * h(n) = \sum\limits_{k=-\infty}^{\infty} x(k)h(n-k)$

$$y(n) = h(n) * x(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k)$$

2. **Associative:**

$$y(n) = x(n) * (\overbrace{h_1(n) * h_2(n)}^{h(n)})$$

$$y(n) = (\underbrace{x(n) * h_1(n)}_{x_2[n]}) * h_2(n)$$

$$\delta[n] \longrightarrow \boxed{1} \longrightarrow h_1[n]$$

$$\delta[n] \longrightarrow \boxed{2} \longrightarrow h_2[n]$$

$$x[n] \longrightarrow \underset{h_1[n]}{\boxed{1}} \xrightarrow{y_1[n]} \underset{h_2[n]}{\boxed{2}} \longrightarrow y_2[n] = x_2[n] * h_2[n]$$

$$= x[n] * h_1[n] * h_2[n]$$

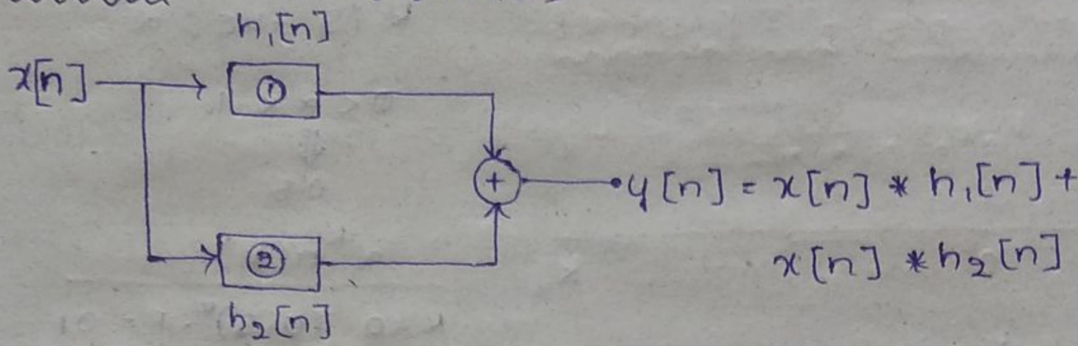
$$x_2[n] = x[n] * h_1[n]$$

$$x[n] \longrightarrow \underset{h_2[n]}{\boxed{2}} \longrightarrow \underset{h_1[n]}{\boxed{1}} \longrightarrow y_2[n] = y_1[n] * h_1[n]$$

$$= x[n] * h_2[n] * h_1[n]$$

$$y_1[n] = x[n] * h_2[n]$$

3. Distributive: $x[n] * h_1[n]$



$x[n] * h_2[n]$

$\delta[n] \longrightarrow \boxed{A} \longrightarrow y_1[n]$

$\delta[n] \longrightarrow \boxed{A} \longrightarrow y_2[n]$

$$x[n] * h_1[n] + x[n] * h_2[n] = x[n] * [h_1[n] + h_2[n]]$$

P3. $x[n] = \alpha^n u[n]$ &

$h[n] = u[n]$

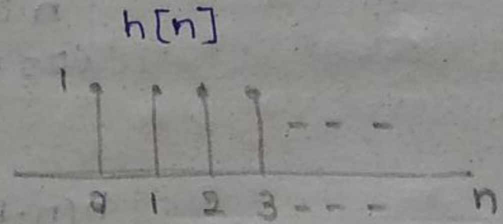
calculate $y[n]$ for the discrete LTI system.

A. $\delta[n] \longrightarrow \boxed{\text{Discrete LTI}} \longrightarrow h[n] = u[n]$

$x[n] \longrightarrow \boxed{\text{Discrete LTI}} \longrightarrow y[n]$

$\downarrow$

$\alpha^n u[n]$



$x[n] \longrightarrow \underset{h[n]}{\boxed{}} \longrightarrow y[n] = x[n] * h[n]$

$$= \sum_{K=-\infty}^{\infty} x(K) h(n-K) \longrightarrow (1)$$

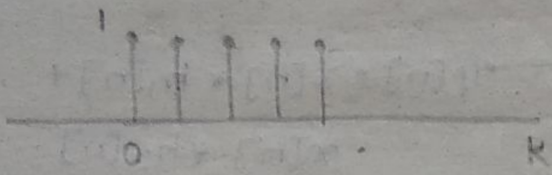
$x[n] = \alpha^n u(n)$

$x(K) = \alpha^K u(K)$ $\qquad h(K) \rightarrow h(-K) \rightarrow h(n-K)$

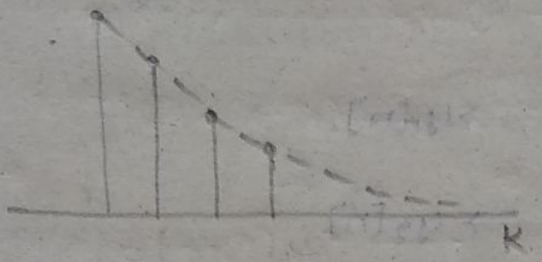
$h[n] = u[n] \Rightarrow h(K) = u(K)$

$h(-K) = u(-K)$; $h(n-K) = u(n-K)$

$u(k) = x(k)$



$\alpha^k u(k) = x(k)$



k is +ve : α^k

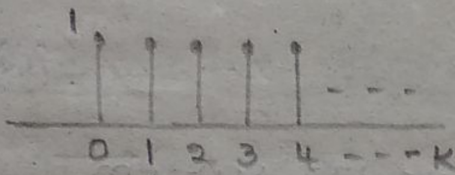
$0 < \alpha < 1$

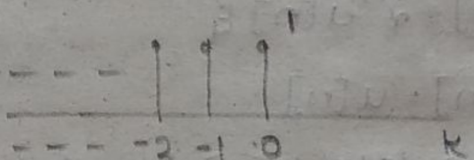
$\Downarrow$

0.5

$\alpha^k u(k)$	
$k=0$	$(0.5)^0 \cdot 1 = 01$
$k=1$	$(0.5)^1 \cdot 1 = 0.5$
$k=2$	$(0.5)^2 \cdot 1 = 0.25$

$h(k) = u(k) \xrightarrow{TR} h(-k)$





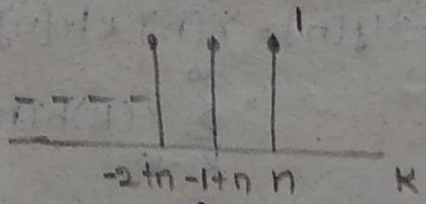
$\downarrow n$

$h(n-k)$

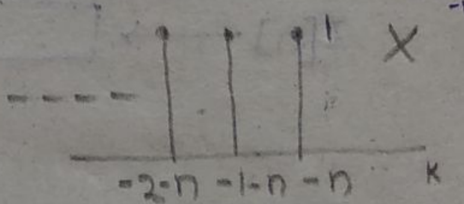
n is +ve (RS by n) → $h(n-k)$

n is -ve (LS by n) → $h(n-k) \rightarrow -\infty$ to $-n$

$x(k) =$ 0 to ∞



(i)

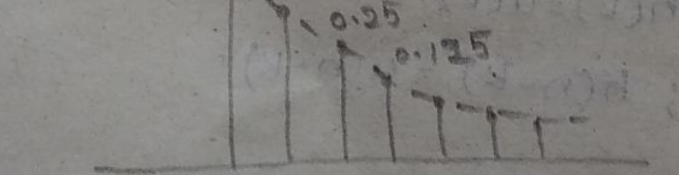


(ii)

$x(k) = \alpha^k u(k)$

$\downarrow$

$-\infty$ to $-n$

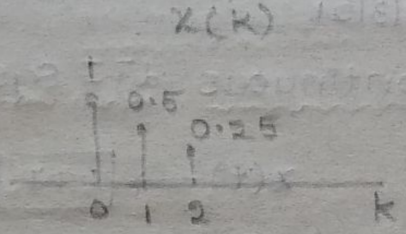


(iii)

$$y(n) = \boxed{\sum_{k=-\infty}^{\infty}} x(k) h(n-k)$$

$\boxed{-\infty \text{ to } 0}$ x $\boxed{0 \text{ to } \infty}$

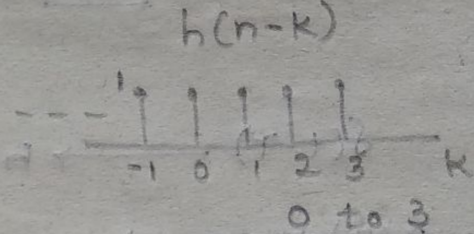
$$x(k)h(n-k) = \begin{cases} \alpha^k & 0 \le k \le n \\ 0 & \text{otherwise} \end{cases}$$

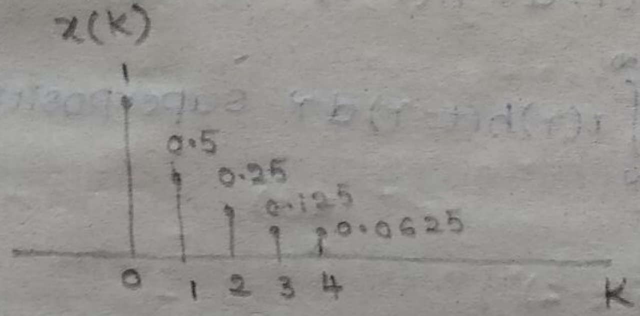


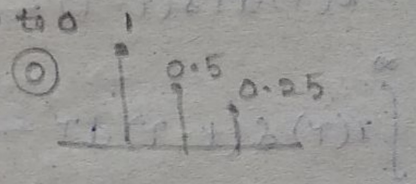
$$= \begin{cases} \alpha^k & k = 2;\ n = 3 \\ \text{otherwise} & \end{cases}$$

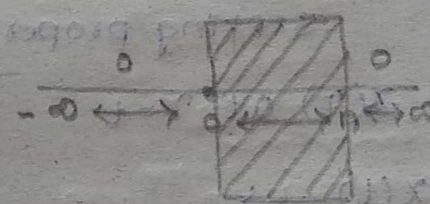
(k = 4) (−∞)

(n = 3)









$$x(k)h(n-k) = \begin{cases} \alpha^k & 0 \le k \le n \\ 0 & \text{otherwise} \end{cases}$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k) = \sum_{k=0}^{n} \alpha^k$$

$$= \alpha^0 + \alpha^1 + \alpha^2 + \alpha^3 + \cdots + \alpha^n$$

↓

Geometric series

$$\boxed{\begin{aligned} S_\infty &= \frac{a}{1-r} \\ S_n &= a\left(\frac{1-r^n}{1-r}\right) \end{aligned}}$$

here

$a = 1,\ r = \infty$

$$y[n] = \left[\frac{1-\alpha^n}{1-\alpha}\right] u[n]$$

↓

$n \ge 0,\ u[n] = 1$

$n < 0,\ u[n] = 0$

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Continuous LTI systems:

$$x(t) \rightarrow \boxed{LTI} \rightarrow y(t)$$

$$\delta(t) \rightarrow \boxed{LTI} \rightarrow h(t)$$

$h(t)$ ↓ impulse response

$\delta(t-\tau) \longrightarrow h(t-\tau)$ Time invariant

$x(\tau)d\tau\,\delta(t-\tau) \longrightarrow x(\tau)\,d\tau\, h(t-\tau)$ homogeneous

$\int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau \longrightarrow \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$ superposition

linear system

↓

sifting property

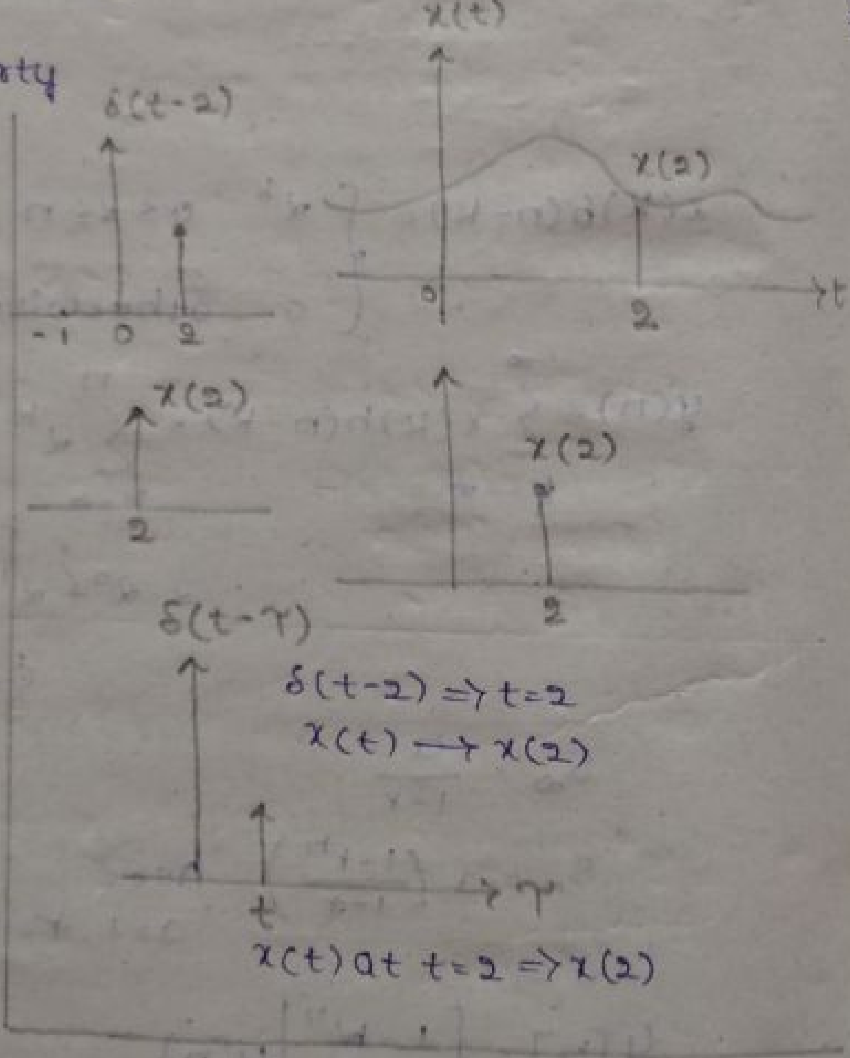
$x(\tau)$ at $\tau = t$

$x(t) \longrightarrow$

$$\boxed{\int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau = y(t)}$$

↓

Convolution (Sum) integral



$$\boxed{\begin{aligned} y(t) &= x(t) * h(t) \\ &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau \end{aligned}}$$

Convolution Integral

$\rightarrow g(a) * f(a) = y(a)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau = x(t) * h(t)$$

(a) $\int_{-\infty}^{\infty} g(a) f(a-\tau) da = y(a)$ ✗

(b) $y(a) = \int_{-\infty}^{\infty} g(a-\tau) f(a) d\tau$ ✗

(c) $y(a) = \int_{-\infty}^{\infty} f(\tau) g(a-\tau) da$ ✗

(d) $y(a) = \int_{-\infty}^{\infty} f(\tau) g(a-\tau) d\tau$ ✓

Properties of Continuous LTI systems:

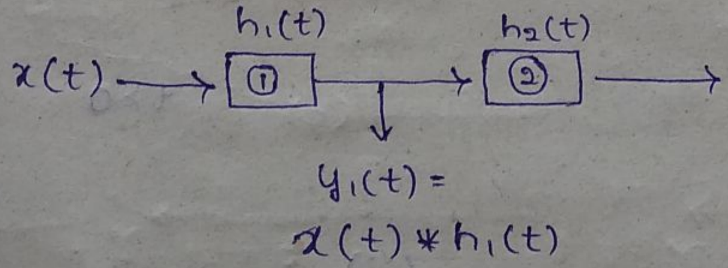
1. Commutative: $(a \times b = b \times a)$

$$y(t) = x(t) * h(t)$$
$$= \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

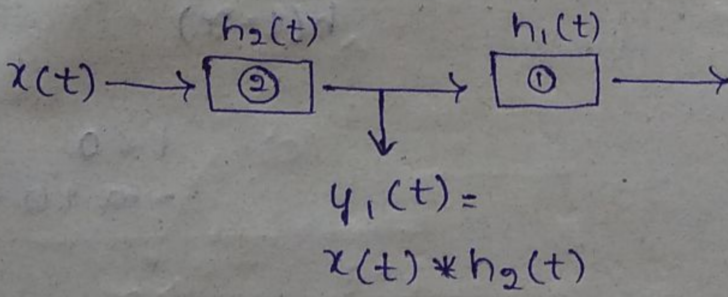
$$y(t) = h(t) * h(t) = \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau$$

2. Associative: $[(a+b)+c = a+(b+c)]$

$$y(t) = x(t) * (h_1(t) * h_2(t))$$
$$= (x(t) * h_1(t)) * h_2(t)$$



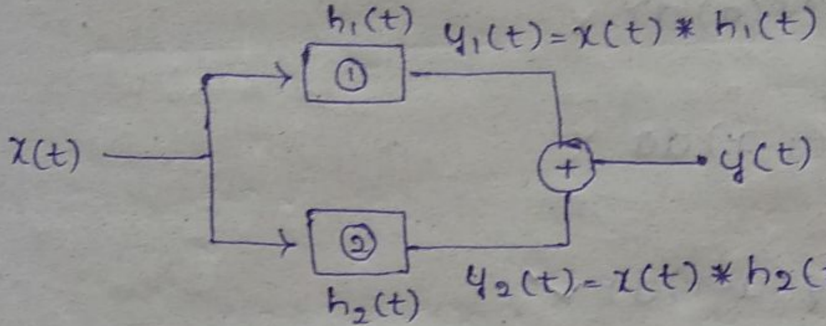
$$y(t) = y_1(t) * h_2(t)$$
$$= (x(t) * h_1(t)) * h_2(t)$$



$$y(t) = y_1(t) * h_1(t)$$
$$= (x(t) * h_2(t)) * h_1(t)$$

3. Distributive:

$$y(t) = x(t) * [h_1(t) + h_2(t)]$$
$$= [x(t) * h_1(t)] + [x(t) * h_2(t)]$$



$$\Rightarrow y(t) = y_1(t) + y_2(t)$$
$$y(t) = [x(t) * h_1(t)] + [x(t) * h_2(t)]$$
$$= x(t) * [h_1(t) + h_2(t)]$$

Examples:

1. $x(t) = e^{-at}u(t)$
   $h(t) = u(t)$ $\quad a > 0 \quad y(t) = ?$

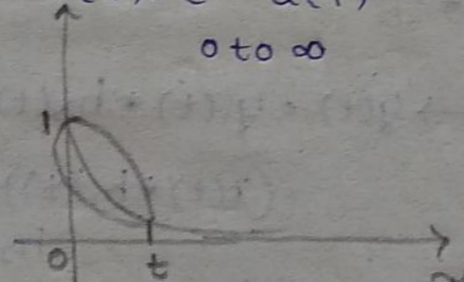
A. $y(t) = x(t) * h(t)$ $\qquad x(\tau) = e^{-a\tau}u(\tau)$

$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$ $\qquad h(t-\tau) = u(t-\tau)$

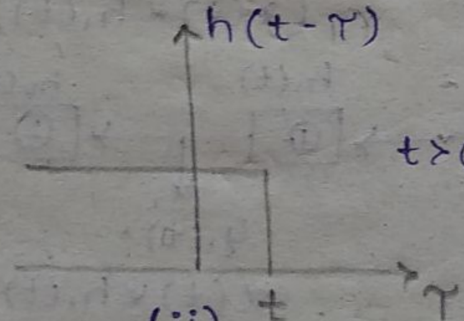
$h(\tau) = u(\tau)$

$x(\tau) = e^{-a\tau}u(\tau)$

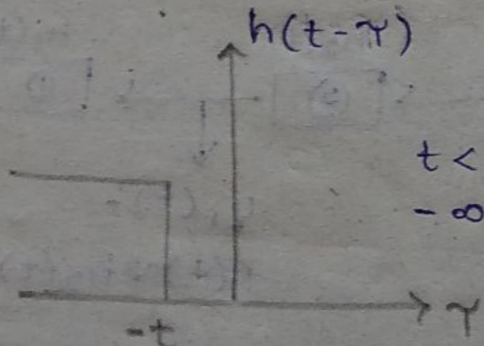
0 to ∞



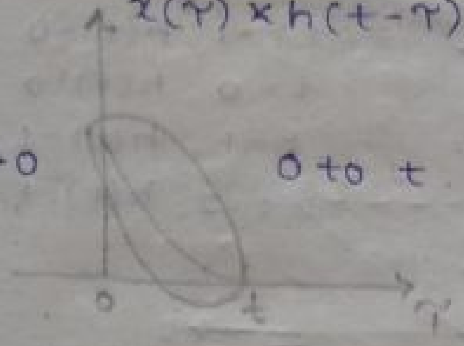
(i)



(ii)



(iii)



$t>0$ 0 to t

$x(\tau)$ 0 to ∞

$h(t-\tau)$ $-\infty$ to t

$t<0$ $0 = x(\tau) \times h(t-\tau)$

$$x(\tau)h(t-\tau) = \begin{cases} 0 & t<0 \text{ ; otherwise} \\ e^{-a\tau} & 0<\tau<t \text{ ; } t>0 \end{cases}$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$$

$$= \int_0^t e^{-a\tau}d\tau = \frac{\left[e^{-a\tau}\right]_0^t}{-a}$$

$$= \frac{-1}{a}\left[e^{-at} - e^{-0}\right]$$

$$= \frac{-1}{a}\left(e^{-at} - 1\right)$$

$$y(t) = \frac{1}{a}\left(1-e^{-at}\right) \text{ ; } t>0$$

$t>0$; 1

$t<0$; 0

$\forall t$

$$\boxed{y(t) = \frac{1}{a}\left(1-e^{-at}\right)u(t)}$$

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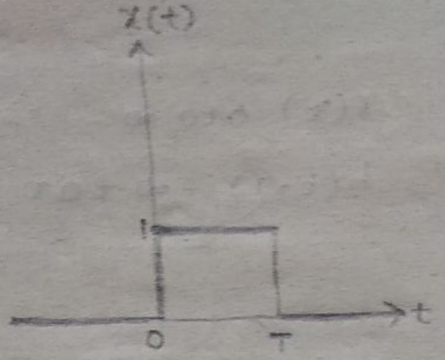
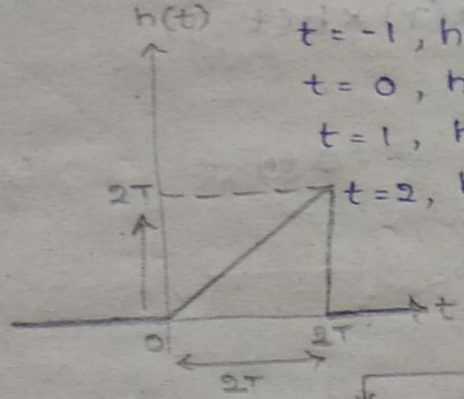
2. Given on LTI system $x(t) = \begin{cases} 1 & 0<t<T \\ 0 & \text{otherwise} \end{cases}$ $y(t) = ?$

$$h(t) = \begin{cases} t & 0<t<2T \\ 0 & \text{otherwise} \end{cases}$$

A.





$t=-1,\ h(t)=0$

$t=0,\ h(t)=0$

$t=1,\ h(t)=t=1$

$t=2,\ h(t)=t=2$

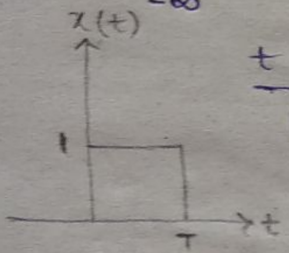
$t=2T,\ h(t)=t=2T$

$y(t)=x(t)*h(t)$

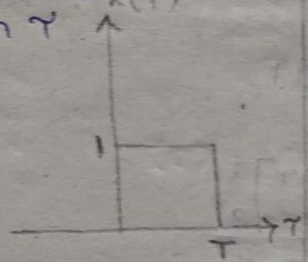
$$y(t)=\int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$$

$x(t)$ vs t, $h(t)$ vs t

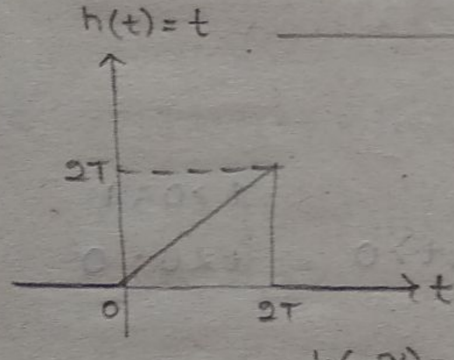
1. $x(\tau)$ vs τ
2. $h(\tau)$ vs $\tau \xrightarrow[\text{①}]{TR} h(-\tau)$ ② T shift by t: $h(t-\tau)=h(-\tau+t)$ $t>0$, RS; $t<0$, LS
3. $x(\tau)h(t-\tau)$
4. Area of product $=y(t)$

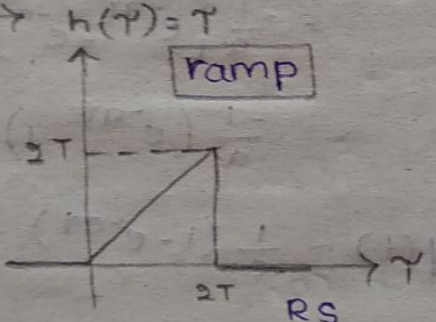


$\xrightarrow[\text{with } \tau]{t \text{ is replaced}}$

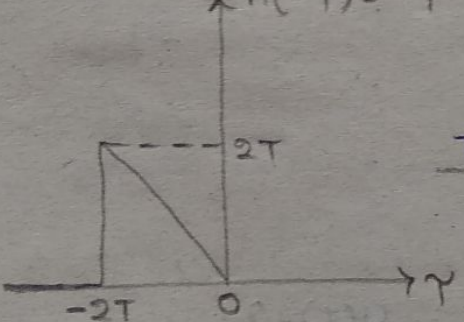


$h(t)=t \longrightarrow h(\tau)=\tau$ $\boxed{\text{ramp}}$

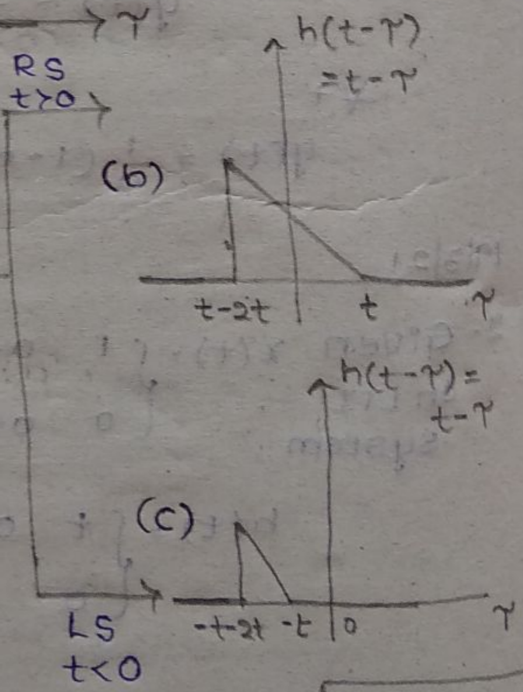


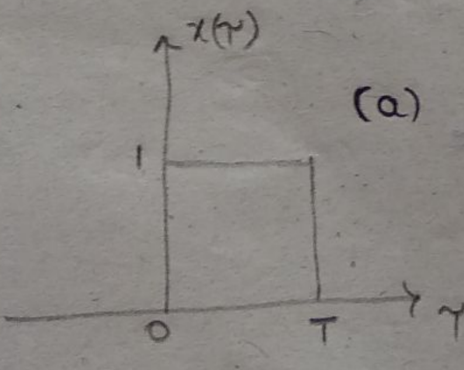


$\xrightarrow[TR]{\tau \text{ is } -\tau}$



Time shift by t





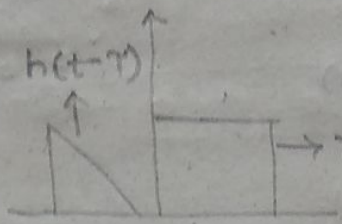
$h(t)=t$

$h(\tau)=\tau$

$h(t-\tau)=t-\tau$

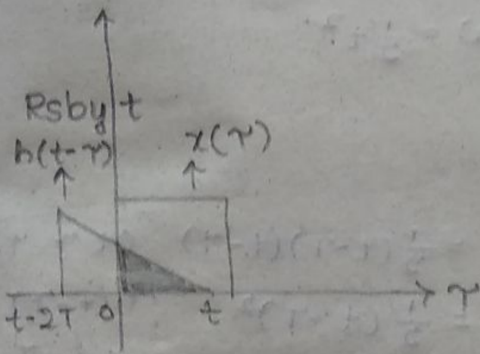
$y(t) = \int x(\tau) h(t-\tau)$

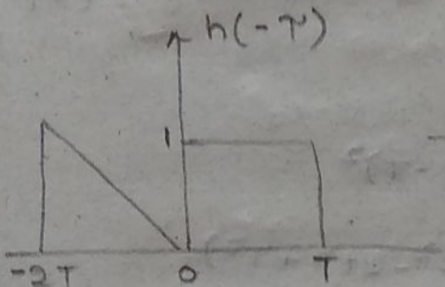
$t<0$ = (a) x (c) →



→ $x(\tau) = 0 = y(t)$

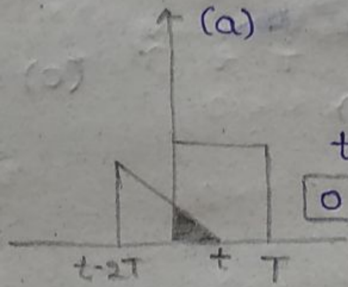
$t>0$ = (a) x (b) → present





RS →

$t>0$



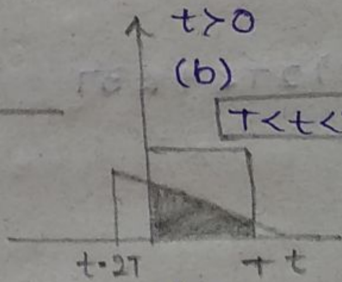
$t<T$

$\boxed{0<t<T}$

$t>0$

(b)

$\boxed{T<t<2T}$



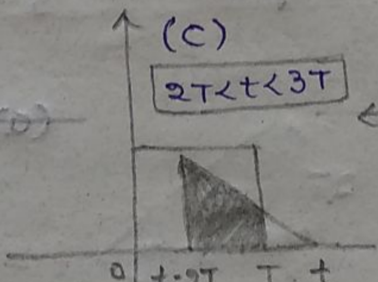
$t>T$

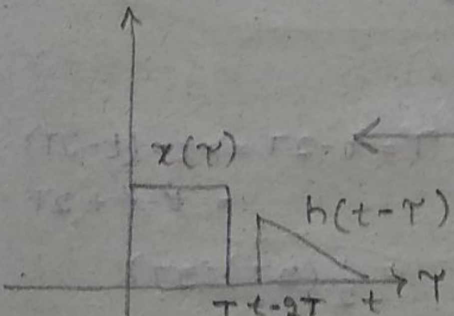
$t-2T<0$

$\Rightarrow t<2T$

(c)

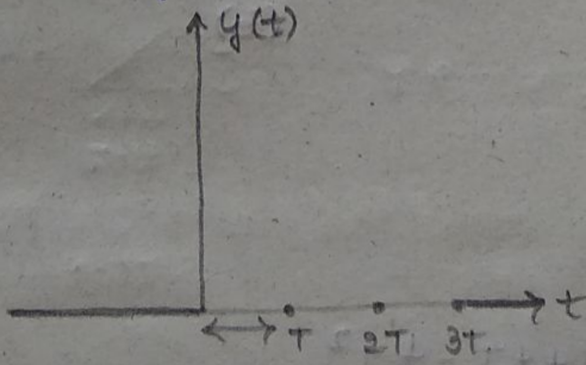
$\boxed{2T<t<3T}$



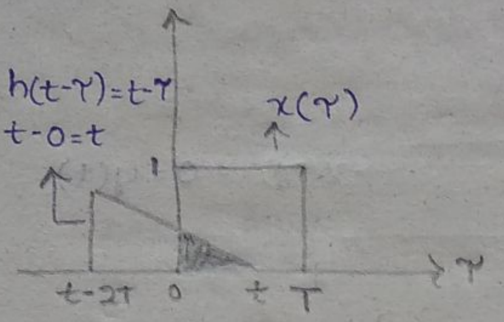


$t-2T>T$

$t>3T \Rightarrow y(t)=0$



(a) $0 < t < T$

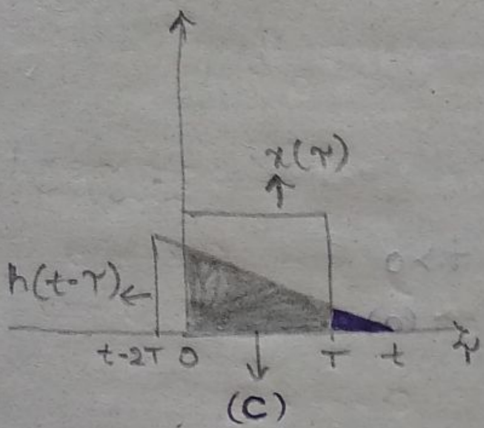


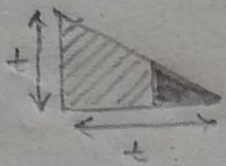
$$y(t) = \int x(\tau)h(t-\tau)$$

$$= \frac{1}{2}bh$$

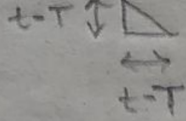
$$\Rightarrow y(t) = \frac{1}{2}(t)(t) = \frac{1}{2}t^2$$

(b) $T < t < 2T$





$$(a) = \frac{1}{2}t^2$$



$$(b) = \frac{1}{2}(t-T)(t-T) \qquad \left[\begin{array}{l} \text{at } \tau = T; \\ h(t-\tau) \\ = t - T \end{array}\right]$$

$$= \frac{1}{2}(t-T)^2$$

$$(c) = (a) - (b)$$

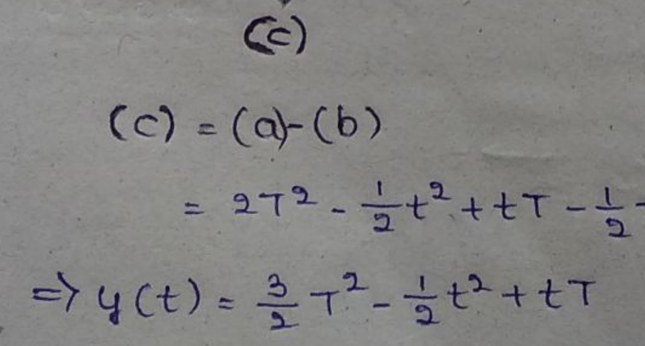
$$= \frac{1}{2}t^2 - \frac{1}{2}(t-T)^2$$

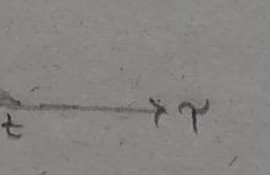
$$= \frac{1}{2}t^2 - \frac{1}{2}(t^2 - 2tT + T^2)$$

$$= \cancel{\frac{1}{2}t^2} - \cancel{\frac{1}{2}t^2} + tT - \frac{1}{2}T^2$$

$$\Rightarrow y(t) = tT - \frac{1}{2}T^2$$

(c) $2T < t < 3T$





$$(a)\ \tau = t - 2T \Rightarrow t - (t - 2T)$$

$$= \cancel{t} - \cancel{t} + 2T$$

$$= \frac{1}{2}(2T)(2T)$$

$$= 2T^2$$

(b)

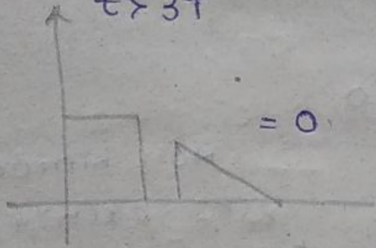
$$= \frac{1}{2}(t-T)^2$$

$$(c) = (a) - (b)$$

$$= 2T^2 - \frac{1}{2}t^2 + tT - \frac{1}{2}T^2$$

$$\Rightarrow y(t) = \frac{3}{2}T^2 - \frac{1}{2}t^2 + tT$$

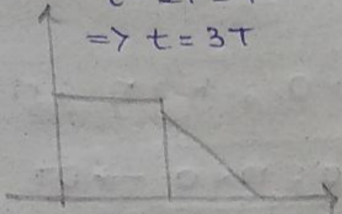
$t > 3T$



$= 0$

$t - 2T = T$

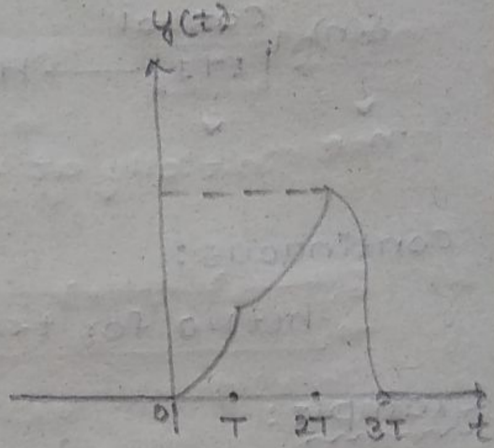
$\Rightarrow t = 3T$



$= \frac{3}{2}T^2 - \frac{9}{2}T^2 + 3T^2$

$= \frac{9}{2}T^2 - \frac{9}{2}T^2 = 0$

$$y(t) = \begin{cases} 0 & t < 0 \\ \frac{1}{2}t^2 & 0 < t < T \\ Tt - \frac{1}{2}T^2 & T < t < 2T \\ \frac{3}{2}T^2 - \frac{1}{2}t^2 + tT & 2T < t < 3T \\ 0 & t > 3T \end{cases}$$



20|3|21

1. Memoryless

$h(n) = 0 \; ; \; n \neq 0 \rightarrow DT$

$h(t) = 0 \; ; \; t \neq 0 \rightarrow CT$

2. Invertible

$h_1(t) * h_2(t) = \delta(t) \rightarrow CT \qquad h_1(n) * h_2(n) = \delta(n) \rightarrow DT$

$\downarrow$

inverse system

3. Casual / Non casual:

$x(n) \longrightarrow \boxed{h(n)} \longrightarrow y(n) = x(n) * h(n)$

$y(n)$ = past and present i/p

$y(n)$ depends on $x(k)$ $k \leq n$

for eg : $y(4) < x(3), x(4)$

$\therefore y(n)$ should not depend on

$x(k)$ $k > n$

$y(n) = 0 \; ; \; k > n$

$\xrightarrow{x(n)} \boxed{h(n)} \longrightarrow y(n) = x(n) * h(n)$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, \underbrace{h(n-k)}_{0} \qquad k > n$$

Casual

$y(n) \quad h(n-k) = 0 \; ; \; k > n \; ; \; n-k < 0.$

$\Rightarrow \boxed{h(n) = 0 \; ; \; n < 0} \longrightarrow$ DT

$\delta(n) \xrightarrow{} \boxed{\text{LTI}} \longrightarrow h(n) = 0 \; ; \; n < 0$

Casual

$n = 0$ initially at rest $n = 0$

Non Linear

$y(n) = 2x(n) + 3$

$n = 2$

$y(2) = 2x(2) + 3$

Casual ✓

$x(n) = 0$

$y(n) = 2(0) + 3$

$= 3$

Continuous:

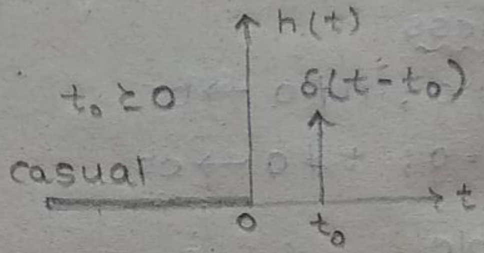
$\boxed{h(t) = 0 \text{ for } t < 0}$

Example:

1. $\delta(t - t_0) = h(t)$ (check whether it is Memory, Invertible, casual?)*

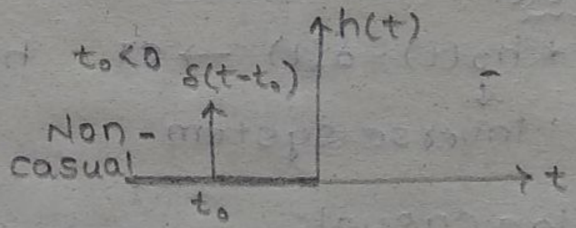
A. $\boxed{h(t) = 0 \; ; \; t < 0}$

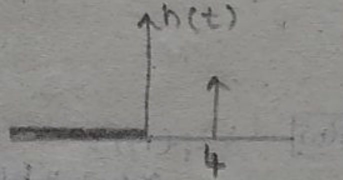
$\delta(t - t_0) = 0 \; ; \; t < 0$



$h(t) = \delta(t - 4)$

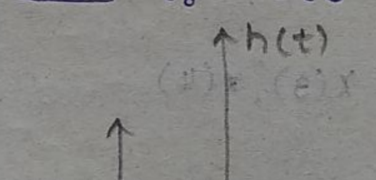
$\Rightarrow t_0 = 4$





$h(t) = \delta(t + 5)$

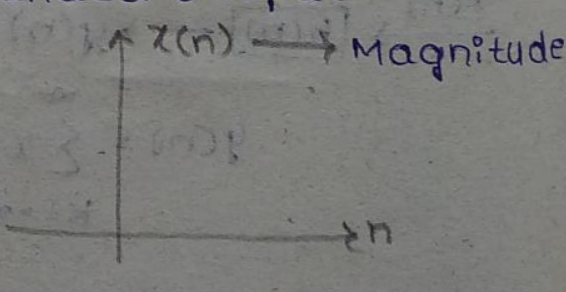
$\boxed{\text{NC}}$ $t_0 = -5 < 0$



4. **Stable:** BIBO

Bounded input Bounded output

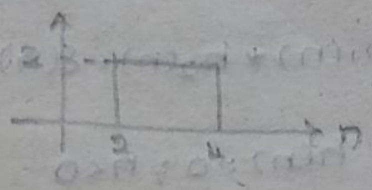
$x(n) \longrightarrow \boxed{h(n)} \longrightarrow y(n)$



$|x(n)| < L < \infty$

$-L < x(n) < L \Rightarrow$ bounded

$-L < x(n-2) < L$



$x(n) \rightarrow \boxed{h(n)} \rightarrow y(n) = x(n) * h(n)$

$$= \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$= h(n) * x(n)$$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$|y(n)| = \left| \sum_{k=-\infty}^{\infty} h(k)\, x(n-k) \right|$$

$x(n)$ is bounded

$|x(n)| < L < \infty$

$$|y(n)| \leq \sum_{k=-\infty}^{\infty} |h(k)|\, |x(n-k)|$$

$$|y(n)| \leq L \sum_{k=-\infty}^{\infty} |h(k)| < \infty$$

$\downarrow$

bounded output

$$\boxed{\sum_{k=-\infty}^{\infty} |h(k)| < \infty}$$ absolutely summable $\rightarrow$ DT

$|y(n)| < \infty \Rightarrow$ bounded o/p

bounded i/p

Continuous:

$$\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$$; absolutely integrable

LTI System	DT	CT
Memoryless	$h(n)=0 ; n \neq 0$	$h(t)=0 ; t \neq 0$
Invertible	$h_1(n) * h_2(n) = \delta(n)$	$h_1(t) * h_2(t) = \delta(t)$
casual	$h(n)=0 ; n<0$	$h(t)=0 ; t<0$
Stable	$\sum_{k=-\infty}^{\infty} \lvert h(k) \rvert < \infty$	$\int_{-\infty}^{\infty} h(t)\,dt < \infty$

Ex: $\delta(t-t_0) = h(t)$

A. $$\int_{-\infty}^{\infty} h(t)\,dt = \int_{-\infty}^{\infty} \delta(t-t_0)\,dt = 1 < \infty$$

$\therefore$ system is stable.

21/03/21

$\delta(n) \longrightarrow \boxed{} \longrightarrow h(n)$

$\delta(t) \longrightarrow \boxed{} \longrightarrow h(t)$

Unit step response:

$u(n) \longrightarrow \boxed{} \longrightarrow \delta(n) \quad h(n)$

$u(t) \longrightarrow \boxed{} \longrightarrow \delta(t) \quad h(t)$

$\delta(t) \longrightarrow h(t) \; ; \; u(t) \longrightarrow \delta(t)$

$\delta(n) \longrightarrow h(n) \; ; \; u(n) \longrightarrow \delta(n)$

Discrete LTI:

$$\begin{array}{ccc} \delta(n) & \longleftrightarrow & u(n) \\ \downarrow & & \downarrow \\ h(n) & \longleftrightarrow & \delta(n) \end{array}$$

$\delta(n) = h(n) - h(n-1)$

$$\boxed{u(n) = \sum_{k=-\infty}^{n} \delta(k)}$$

$u(n) \longrightarrow \boxed{h(n)} \longrightarrow \delta(n) = u(n) * h(n)$

$= h(n) * u(n)$

$= \sum_{k=-\infty}^{\infty} h(k)\, u(n-k)$

$$\boxed{\begin{aligned} u(n-k) &= 1; \\ n-k &\geq 0 \\ \Rightarrow k &\leq n \end{aligned}}$$

$$s(n) = \sum_{k=-\infty}^{n} h(k) + \sum_{k=n}^{\infty} h(k)$$

$$\Rightarrow \boxed{S(n) = \sum_{k=-\infty}^{n} h(k)}$$

$u(n) = \sum_{k=-\infty}^{n} \delta(k)$

$\delta(n) = h(n) - h(n-1)$

$$\Rightarrow \underline{\delta}(n) = \sum_{k=-\infty}^{n-1} h(k) + h(n)$$

$= s(n-1) + h(n)$

$$\Rightarrow \boxed{h(n) = \underline{\delta}(n) - S(n-1)}$$

$$\boxed{\begin{aligned} \sum_{i=1}^{3} x(i) &= x(1) + x(2) + x(3) \\ &= \sum_{i=1}^{2} x(i) + x(3) \\ &= x(1) + x(2) + x(3) \end{aligned}}$$

$\Rightarrow \boxed{\delta(n) = u(n) - u(n-1)}$

Continuous LTI systems:

$\delta(t)$

$\downarrow$

$h(t)$

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$

$$\delta(t) = \frac{d\, u(t)}{dt}$$

$h(t)$

$\downarrow$

$S(t)$

$$S(t) = \int_{-\infty}^{t} h(\tau)\, d\tau$$

$$h(t) = \frac{d\, S(t)}{dt}$$

* $S(t)$ = given

$\boxed{\text{Stable}}$ $\qquad h(t) = \frac{d\, S(t)}{dt}$

$\downarrow$

$$\int_{-\infty}^{\infty} |h(t)| < \infty$$

Examples:

1. $x(t) = u(t)$; $h(t) = e^{-at} u(t)$; $y(t) = ?$

A. $y(t) = x(t) * h(t)$

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau \qquad (\because t \rightarrow t-\tau)$$

$$= \int u(\tau)\, e^{-a(t-\tau)}\, u(t-\tau)\, d\tau$$

$$= e^{-at} \int_{-\infty}^{\infty} e^{a\tau}\, u(\tau)\, u(t-\tau)\, d\tau$$

$u(\tau) \downarrow$ $1 ; \boxed{\tau > 0}$ $\qquad u(t-\tau) \downarrow$ $1 ; t-\tau > 0$ $\Rightarrow \boxed{\tau < t}$

$$= e^{-at} \int_{0}^{t} e^{a\tau}\, d\tau$$

$$= e^{-at} \left[\frac{e^{a\tau}}{a} \right]_{0}^{t}$$

$$= \frac{e^{-at}}{a} \left[e^{at} - 1 \right] = \frac{1}{a}(e^{0} - e^{-at}) = \frac{1}{a}(1 - e^{-at})$$

9. $x(n) = 2^n u(-n)$; $h(n) = u(n)$; $y(n) = ?$

A. $y(n) = x(n) * h(n)$

$$= \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$= \sum_{k=-\infty}^{\infty} 2^k\, \underbrace{u(-k)}_{\substack{1\,;\,-k \geq 0 \\ \Rightarrow k \leq 0}}\; \underbrace{u(n-k)}_{\substack{1\,;\,n-k \geq 0 \\ \Rightarrow k \leq n}}$$

u(-k)

(a)

u(n-k)

n>0

(b) $\boxed{-\infty \text{ to } n}$

$n<0$: $u(-k)u(n-k)$
$=1$
k $\boxed{-\infty \text{ to } -n}$
(a) x (c)

$n>0$:
$=1$
$-\infty$ to 0
(a) x (b)

u(n-k)

n<0

(c) $\boxed{-\infty \text{ to } -n}$

$$y(n) = \sum_{k=-\infty}^{0} 2^k \qquad n \geq 0 =$$

$$y(n) = \sum_{k=-\infty}^{-n} 2^k \qquad n < 0 =$$