Unit-3 Graph Theory

3.1

Graphs and their basic properties

We have already studied some preliminary concepts of graph theory in unit 2 where we discussed the digraph of a relation. We have used a digraph as a pictorial representation of a relation and it served only a limited purpose. In this unit we shall extend, and in some cases, generalize these ideas.

Graph theory is applied in such diverse areas as social sciences, linguistics, physical sciences, communication engineering and others. Graph theory also plays an important role in several areas of computer science, such as switching theory and logical design, artificial intelligence, formal languages, computer graphics, operating systems, compiler writing, and information organization and retrieval.

Like many important discoveries, graph theory grew out of an interesting physical problem, the celebrated **Konigsberg Bridge Puzzle**. The outstanding Swiss mathematician *Leonhard Euler* solved the puzzle in 1736, thus laying the foundation for graph theory and earning his title as the father of graph theory.

Graph: A **graph** $G = (V, E)$ consists of a nonempty set V, called the set of **nodes** (or **points** or **vertices**) of the graph and E, called the set of **edges** of the graph, which is a subset of the set of ordered or unordered pairs of element of V.

We shall assume throughout, both the sets V and E of a graph are finite.
If $e \in E$ then e is an **edge**, e is **an ordered pair** $(\boldsymbol{u}, \boldsymbol{v})$ or **an unordered pair** $\{\boldsymbol{u}, \boldsymbol{v}\}$, where $u, v \in V$ and we say that the edge e *connects* or *joins* the nodes u and v. A pair of nodes are said to be **adjacent** if they are connected by an edge.

Directed, undirected and mixed graphs:

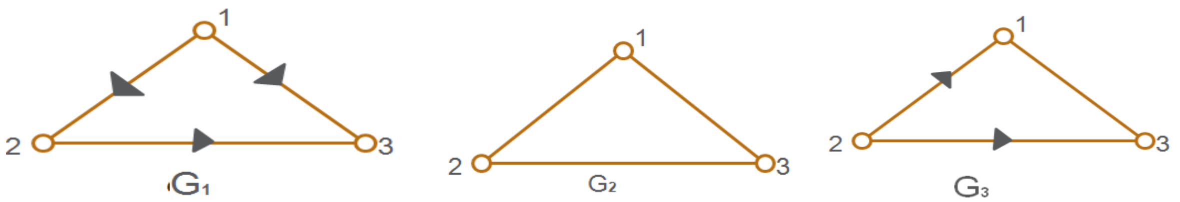
In a graph $G = (V, E)$, an edge which is an ordered pair of $V \times V$ is called a **directed edge**. An edge which is an unordered pair $\{u, v\}, u, v \in V$ is called an

undirected edge. A graph in which every edge is directed is called a **directed graph** or **digraph**. A graph in which every edge is undirected is called a **undirected graph**. If some edges are directed and some are undirected in a graph, then the graph is called a **mixed graph**.

In the diagrams the directed edges are shown by means of arrows which also show directions.

Example 1: Let $V = \{1,2,3\}$, $E_1 = \{(1,2),(1,3),(2,3)\}$, $E_2 = \big\{\{1,2\},\{1,3\},\{2,3\}\big\}$ and $E_3 = \{(2,1),\{1,3\},(2,3)\}$.

Then $G_1 = (V, E_1)$ is a digraph, $G_2 = (V, E_2)$ is an undirected graph and $G_3 = (V, E_3)$ is a mixed graph as shown below.



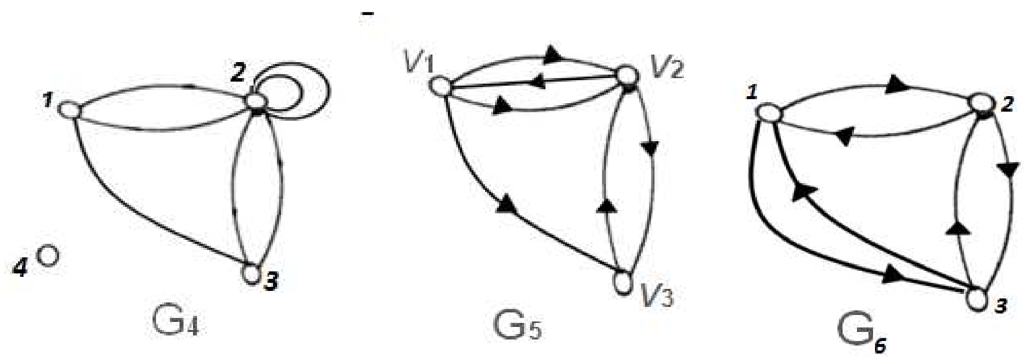
Let $G = (V, E)$ be a graph and $e \in E$ be a directed edge (u, v). Then we say that e is *initiating* or *originating* in the node u and *terminating* or *ending* in the node v. The nodes u and v are called the **initial** and **terminal** nodes of the edge e, respectively. An edge $e \in E$ which joins the nodes u and v, whether it be directed or undirected is said to be **incident** the nodes u and v.

An edge of a graph which joins a node to itself is called a **loop**. The direction of a loop is of no significance; therefore it can be considered either a directed or a an undirected edge.

In the case of directed edges, the two possible edges (u, v) and (v, u) between a pair of nodes u, v which are opposite in direction are considered **distinct**. Two or more distinct edges between a pair of vertices are called **parallel edges**.

Any graph which contains parallel edges is called a **multigraph**.

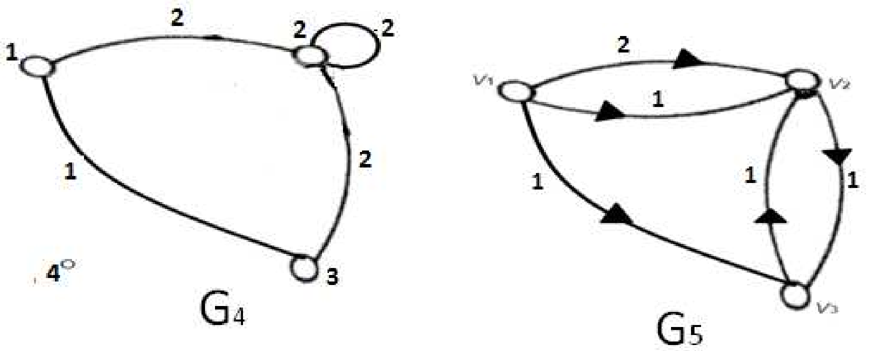
A graph is said to be **simple** if there is no more than one edge between a pair of nodes (no more than one directed edge in the case of a digraph).The graphs G_1, G_2 and G_3 are all simple graphs.



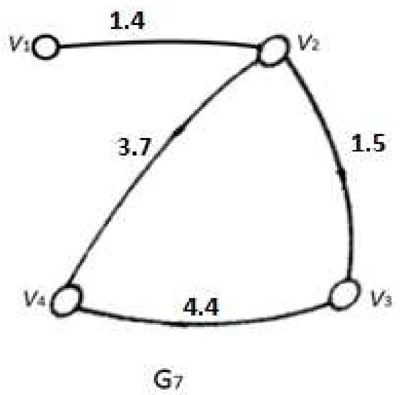
Notice that the graph G_4 , G_5 have parallel edges and so they are multigraphs. The graph G_6 is a simple graph.

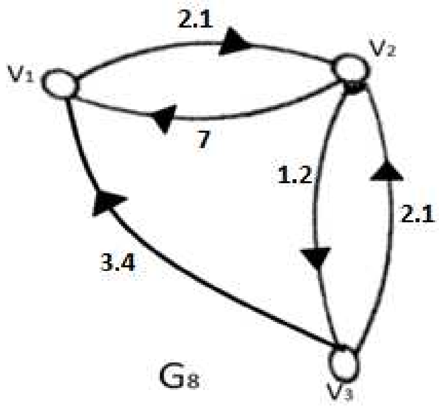
If e denotes the edge joining two nodes u, v and there are n parallel edges between u, v then we say that the **multiplicity** of e is $\boldsymbol{n}$.

The multigraphs may be drawn by displaying their multiplicity on their respective multiple edges (See G_4 , G_5 below with the multiplicities on the edges).



We may also consider the multiplicity as a **weight** assigned to an edge. This interpretation allows us to generalize the concept of weight to numbers which are not necessarily integers (See graphs G_7 and G_8)





A graph in which weights are assigned to every edge is called a **weighted graph**. The graphs G_4 , G_5 , G_7 and G_8 are all weighted graphs.

A graph representing a system of pipelines in which the weights assigned on each pipeline($i.e.$, edge) indicate the *amount of the commodity transferred* through the pipeline is an example of a weighted graph. Similarly, a graph of city streets may be assigned weights according to the *traffic density* on each street.

A node in a graph is said to be an **isolated node** if it is not adjacent to any node. In G_4 , the node 4 is an isolated node.

A graph $G = (V, E)$ is said to be a **null graph** if E is an empty set. That is, a graph containing only isolated nodes is a null graph.

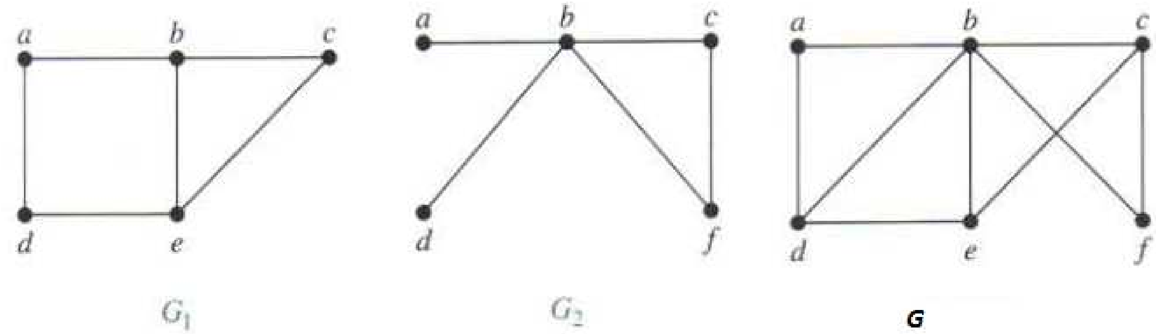
G_0: Empty graph

Subgraph: When edges and vertices are removed from a graph without removing the endpoints of any remaining edges, a smaller graph is obtained such a graph is called a **Subgraph** of the original graph.

Let G and H be graphs. Let $V(G)$ and $V(H)$ be the sets of nodes of G and H respectively. The graph H is said to be a subgraph of the graph G

(written as $H \subseteq G$) if $V(H) \subseteq V(G)$ and every edge of H is also an edge of G. Clearly, the graph G itself, and the null graph obtained from G by deleting all the edges of G are subgraphs of G. Other subgraphs of G are obtained by deleting certain nodes and the edges incidenting with these nodes of G.

Example 2: G_1, G_2 are subgraphs of the graph G.



Union of graphs: The **union** of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is the graph, denoted by $G_1 \cup G_2$, where

$$G = G_1 \cup G_2 = (V, E), V = V_1 \cup V_2,\ E = E_1 \cup E_2$$

In the above diagram $G = G_1 \cup G_2$

Binary relations and simple digraphs:

Let R be a binary relation on V ($i.e.$, $R \subseteq V \times V$). Then the graph of R is a digraph $G = (V, R)$. Notice that G has no parallel edges because R is a set, where the ordered pairs are enlisted only once. Therefore the digraph G is simple. The graph of a binary relation is a simple digraph.

Conversely, let $G = (V, E)$ be a simple digraph. Then every edge of E can be expressed by means of an ordered pair of elements of V. Since G is simple, the ordered pairs corresponding to the edges of E are all distinct. Then $E \subseteq V \times V$. Therefore, E is a binary relation on V whose graph is the simple digraph G.Thus we have the following:

If G $= (V, E)$ is a simple digraph then E is a binary relation on V and conversely.

We can define the converse of a simple digraph.

The **converse** of a simple digraph $G = (V, E)$ is the graph $\overline{\boldsymbol{G}} = (\boldsymbol{V}, \overline{\boldsymbol{E}})$, where $\bar{E}$ is the converse of the relation E on V.
note that $\bar{G}$ is also simple. The diagram of $\bar{G}$ is obtained from that of G by simply reversing the directions of the edges of G. The converse $\bar{G}$ is also called the **reversal** or **directional dual** of G.

A simple digraph $G = (V, E)$ is called reflexive, transitive, symmetric, antisymmetric if the relation E on V is respectively reflexive, transitive, symmetric, antisymmetric.

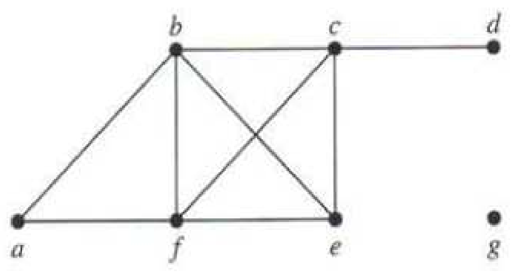
If a simple digraph $G = (V, E)$ is reflexive, symmetric and transitive, then the relation E is an equivalence relation on V and hence V is partitioned into equivalence classes. Any equivalence class of nodes along with the edges connecting them is a subgraph of G. These subgraphs are such that they are pair wise disjoint and the union of these subgraphs is the graph G. In this sense the graph G is partitional into mutually disjoint subgraphs.

Degree of a node in an undirected graph: The **degree** of a node in an undirected graph is the number of edges incident with it, except that a loop at a node contributes a count of 2 to the degree of that node. The degree of a node v is denoted by $\boldsymbol{deg}(\boldsymbol{v})$.

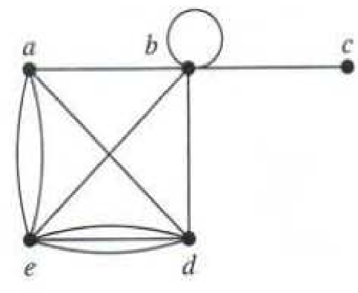
Pendant node: A node is called **pendant** iff it has degree 1.

Note: A pendant node is adjacent to exactly one other node. The degree of an isolated node is 0.

Example 3: What are the degrees of vertices in the graphs $\boldsymbol{G}$ and $\boldsymbol{H}$ given below:



G



H

Solution: In the undirected graph G,

$\deg(a) = 2$, $\deg(b) = \deg(c) = \deg(f) = 4$, $\deg(d) = 1$, $\deg(e) = 3$, $\deg(g) = 0$.

Node d is pendent and node g is an isolated node

In the undirected graph H,

$\deg(a) = 4$, $\deg(b) = 4 + 2(loop) = 6$, $\deg(c) = 1$, $\deg(d) = 5$, $\deg(e) = 6$

Node e in the graph H is pendant

Theorem 1: The Handshaking Theorem

Let $G = (V, E)$ be an undirected graph. Then

$$\sum_{v \in V} \deg(v) = 2|E|$$

Proof: Note that each edge $\{a, b\}$ contributes two to the sum of the degrees of the nodes (because it contributes 1 for the degree of a and 1 for the degree of b). It is true even if $a = b$. This means that the sum of the degrees of the nodes is twice the number of edges. Hence the theorem.

Note:

i. This theorem applies even if multiple edges and loops are present.
ii. This theorem is called the Handshaking Theorem, because of the anology between an edge having two end points and handshake involving two hands.

The following is a consequence of the above theorem.

Theorem 2: An undirected graph has an even number of nodes of odd degree.

Proof: We have $G = (V, E)$, an undirected graph. Let V_1 and V_2 be the sets of nodes of odd and even degree respectively. Clearly $V_1 \cap V_2 = \phi$ and $V_1 \cup V_2 = V$. Therefore,

$$\sum_{v \in V} \deg(v) = \sum_{v \in V_1} \deg v + \sum_{v \in V_2} \deg(v)$$

By Handshaking theorem

$$2|E| = \sum_{v \in V_1} \deg(v) + \sum_{v \in V_2} \deg(v)$$

Note that for each $v \in V_2, \deg(v)$ is even and so $\sum_{v \in V_2} \deg(v)$ is even. From the above, it follows that

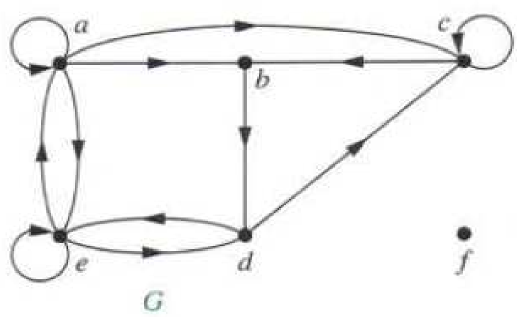
$$\sum_{v \in V_1} \deg(v) \text{ is even.}$$

If $|V_1|$ is odd then $\sum_{v \in V_1} \deg(v)$ is odd – a contradiction. Therefore $|V_1|$ must be even.Thus, the number of vertices of odd degree is even. Therefore, G has an even number of vertices of odd degree. Hence the theorem.

The definition of the degree of a node in a digraph can be refined in the following way:

In-degree, out-degree and degree of a node in a digraph: Let $G = (V, E)$ be a digraph. The **in-degree** of a node $v \in V$, denoted by $\deg^{-}(v)$, is the number of edges with v as their terminal node. The outdegree of v, denoted by $\deg^{+}(v)$, is the number of edges with v as their initial node. A loop at a vertex v contributes 1 to both the in-degree and the out-degree of V.The sum of the in-degree and out-degree of a node v is called the **degree** of the node v

Example 4: Find the in-degree and out-degree of each node in the digraph G given below:



Solution:

The in-degrees of nodes of G are:

$\deg^{-}(a) = 1 + 1(loop) = 2$, $\deg^{-}(b) = 2$, $\deg^{-}(c) = 1 + 1(loop) = 2$

$\deg^{-}(c) = 2 + 1(loop) = 3$, $\deg^{-}(d) = 2$, $\deg^{-}(e) = 2 + 1(loop) = 3$

and $\deg^{-}(f) = 0$.

The out-degrees of nodes of G are:

$\deg^{+}(a) = 4$, $\deg^{+}(b) = 1$, $\deg^{+}(c) = 2$, $\deg^{+}(d) = 2$, $\deg^{+}(e) = 3$ and $\deg^{+}(f) = 0$.
Note that the sum of the in-degrees of all nodes equals to the sum of out-degrees of all nodes and each is equal to the number of edges. This is infact true in a diagraph and is proved in the following theorem.

Theorem 3: If $\boldsymbol{G = (V, E)}$ is a digraph, then

$$\sum_{v \in V} \mathbf{deg}^{-}(\boldsymbol{v}) + \sum_{v \in V} \mathbf{deg}^{+}(\boldsymbol{v}) = |\boldsymbol{E}|$$

Proof: Since each directed edge has an initial node and a terminal node, the sum of the in-degrees and the sum of the out-degrees of all nodes in a digraph are the same. Both of these sums are the number of edges in the digraph. Hence the result .

Degree sequence: The degree sequence of a graph is the sequence of the degrees of the vertices of the graph is nonincreasing order.

A sequence d_1 , d_2 , … , d_n is called **graphic** if it is the degree sequence of a simple graph.

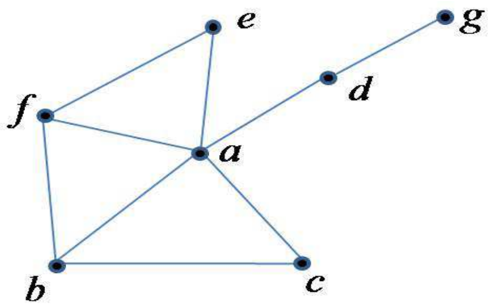
Example 5: Determine whether the degree sequence $5, 3, 3, 2, 2, 2, 1$ is a graphic? If it is, how many edges does this graph have? Draw a graph having this degree sequence.

Solution: By *Handshaking theorem,* the sum of the degrees of all vertices is equal to twice the number of edges. Therefore

$$2|E| = 5 + 3 + 3 + 2 + 2 + 2 + 1 = 18$$

$$\Longrightarrow |E| = 9$$

Therefore, there are 9 edges in the graph. The diagram of such a graph is given below:



In this graph the vertex g is a pendant.

Remark: The definition of graph contains no reference to the length or the shape and positioning of the edges joining any pair of nodes, nor does it prescribe any ordering of positions of nodes. Therefore, for a given graph, there is no unique diagram which represents the graph. We can obtain a variety of diagrams by locating the nodes in an arbitrary number of different positions and also by showing the edges by arcs or lines of different shapes. Due to this arbitrariness, it can happen that two diagrams which look entirely different from one another may represent the same graph. We now has the concept of *sameness* is graph theory.

Isomorphic graphs: Two graphs are said to be **isomorphic** if there exists a one – to – one correspondence between the nodes of the two graph which preserve adjacency of the nodes as well as the directions of the edges, if any.

Note: By the above definition of isomorphism a one – to – one correspondence exists between the edges as well.

Two graphs $G = (V, E)$ and $G' = (V', E')$ are said to be isomorphic, written as $G \cong G'$, if there exists an adjacency preserving bijective map between their vertices. That is, if there exists a bijective map $f: V \longrightarrow V'$ such that

(i) $(a, b) \in E \implies \big(f(a), f(b)\big) \in E'$, for all $(a, b) \in E$ in the case of diagraphs

(ii) $\{a, b\} \in E \implies \{f(a), f(b)\} \in E'$, for all $\{a, b\} \in E$ in the case of undirected graph

In such a case, f is called an **isomorphism** between G and G'

Note: It is essential that two graphs which are isomorphic have the same number of nodes and edges, however, this is not sufficient condition for an isomorphism to exist.

Note: It is often difficult to determine whether two graphs are isomorphic. There are $n!$ possible one-to-one correspondences between the node sets of two graphs with n nodes. Testing each such correspondence to see whether it preserves adjacency and is impractical if n is large.

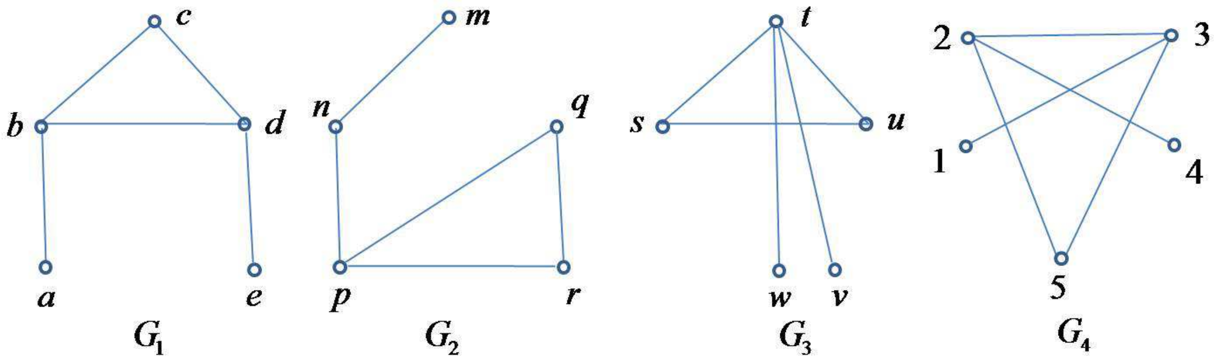
Sometimes it is not hard to show that two graphs are not isomorphic. In particular we can show that two graphs are not isomorphic if we can find a property, only one of the two graphs has, but that is preserved by isomorphism.

A property preserved by isomorphism of graphs is called a **graph invariant**.

The degrees of the nodes in isomorphic graphs G, H must be same. That is, a node v of degree d in G must correspond to a node $f(v)$ of degree d in H.

The number of nodes, number of edges and the number of vertices of each degree are all invarient under isomophism. If any of these differ in two graphs, then these graphs cannot be isomorphic. However, when these invariants are the same, it does not necessarily mean that the graphs are isomorphic.

Example 6: Which of the following graphs are isomorphic?



Solution:

All the graphs G_k , $k = 1,2,3,4$ are undirected graphs with 5 vertices and 5 edges.

The degree sequences are as follows:

$$G_1 : 3,3,2,1,1$$

$$G_2 : 3,2,2,2,1$$

$$G_3 : 4,2,2,1,1$$

$$G_4 : 3,3,2,1,1$$

Since the degree sequence is a graph invariant property, possibly G_1 , G_4 are isomorphic. It also confirms G_1, G_2, G_3 are pair wise nonisomorphic and G_2, G_3, G_4 are pairwise nonisomorphic.

Construction of an isomorphism f from G_1 to G_4. The vertex sets of G_1 and G_4 are $V_1 = \{a, b, c, d, e\}$, $V_4 = \{1,2,3,4,5\}$ respectively. Now f must map c onto 5(why?). Therefore $f(c) = 5$. Since the vertex c is adjacent to vertices b, d of degree 3 in G_1 and the vertex 5 is adjacent to vertices 2,3 of degree 3 in G_4 ;

we take $f(b) = 2$, $f(d) = 3$.Since b is adjacent to a of degree 1 in G_1 and 2 is adjacent to 4 of degree 1 in G_4; we take $f(a) = 4$.

We are left with the option $f(a) = 1$. Thus, we have set a bijection $f : V_1 \longrightarrow V_4$ in the following way

$$\begin{aligned} a &\xrightarrow{f} 4 \\ b &\longrightarrow 2 \\ c &\longrightarrow 5 \\ d &\longrightarrow 3 \\ e &\longrightarrow 1 \end{aligned}$$

We see that f preserves the adjacency as shown below:

$$\{a, b\} \xrightarrow{f} \{f(a), f(b)\} = \{4,2\}$$

$$\{b, c\} \xrightarrow{f} \{2,5\}$$

$$\{b, d\} \xrightarrow{f} \{2,3\}$$

$$\{c, d\} \xrightarrow{f} \{5,3\}$$

$$\{d, e\} \xrightarrow{f} \{3,1\}$$

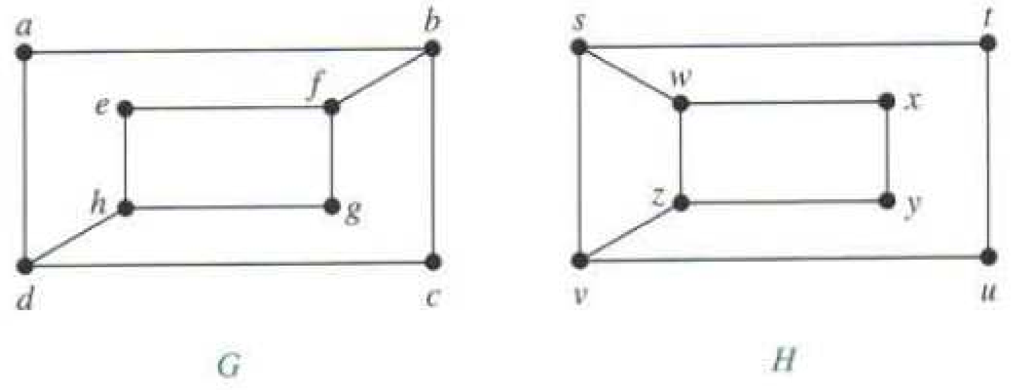
Thus, f is an isomorphism from G_1 to G_4 and $G_1 \cong G_4$.

Note: The map $g : V_1 \longrightarrow V_4$ defined by

$g(a) = 1$, $g(b) = 3$, $g(c) = 5$, $g(d) = 2$ and $g(e) = 4$

is also an isomorphism (verify!).Are these only two isomorphisms between the graphs G_1 and G_4 ? (Investigate!).

Example 7: Determine whether the following graphs are isomorphic?



Solution: Both the graphs G and H are undirected graphs; both have 8 vertices and 10 edges.

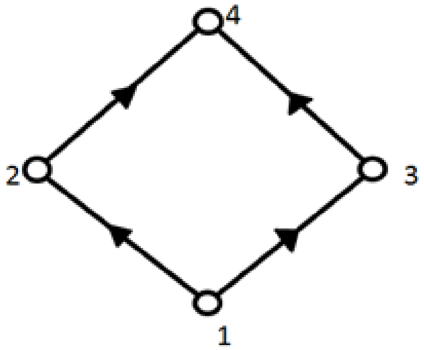
G		H	
Vertex	Degree	Vertex	Degree
a	2	s	3
b	3	t	2
c	2	u	2
d	3	v	3
e	2	w	3
f	3	x	2
g	2	y	2
h	3	z	3

The degree sequence of G : 2,2,2,2,3,3,3,3.

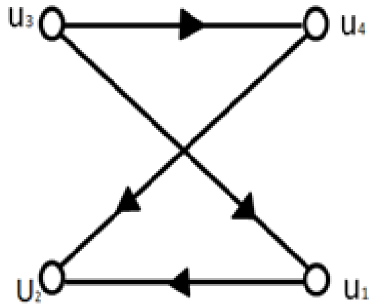
The degree sequence of H : 2,2,2,2,3,3,3,3.

Thus, all the invariants are agreeing. Note that $\deg(a) = 2$ in G and it must correspond to either t, u, x or y (because these are vertices of degree 2 in H). Observe that each of these four vertices t, u, x, y in H is adjacent to another vertex of degree 2 in H , where as a is adjacent to the vertices b, d of degree 3 and not adjacent to any vertex of degree 2 in G.This shows that G and H are not isomorphic.

Example 8: Show that the following graphs are isomorphic.



G



G'

Solution: We have diagraphs

$G = (V, E)$, where $V = \{1, 2, 3, 4,\}$, $E = \{(1,2), (1,3), (2,4), (3,4)\}$

$G' = (V', E')$, where $V' = \{u_1, u_2, u_3, u_4\}$

$$E' = \{(u_1, u_2), (u_3, u_1), (u_3, u_4), (u_4, u_2)\}$$

The following are in-degree and out-degree tables for the vertices of G and G'.

	G	
$vertex$	deg^-	deg^+
1	0	2
2	1	1
3	1	1
4	2	0

	G'	
$vertex$	deg^-	deg^+
u_1	1	1
u_2	2	0
u_3	0	2
u_4	1	1

Since the in-degree and out-degree is in variatiant under an isomorphism, we must map 1 onto u_3 and 4 onto u_2. Now we have two possibilities
(i) $2 \longrightarrow u_1$, $3 \longrightarrow u_4$ (ii) $2 \longrightarrow u_4$, $3 \longrightarrow u_1$.

Let $f : V \longrightarrow V'$ be defined by $(1) = u_3$, $f(2) = u_1$, $f(3) = u_4$, $f(4) = u_2$.

This bijection f preserves the adjacency of nodes as shown below:

$$(1,3) \xrightarrow{f} \big(f(1), f(3)\big) = (u_3, u_4)$$

$$(1,2) \xrightarrow{f} (u_3, u_1)$$

$$(2,4) \xrightarrow{f} (u_1, u_2)$$

$$(3,4) \xrightarrow{f} (u_4, u_2)$$

This shows f is an isomorphism between G and $G'$and $G \cong G'$.

Let $g : V \longrightarrow V'$ be defined by

$$g(1) = u_3 , g(2) = u_4 , g(3) = u_1 , g(4) = u_2$$

It may be verified that g is also an isomorphism between G and G' and $G \cong G'$ (verify!).

Are these only two isomorphisms between the graphs G and G'? (Explore!)

P1:

How many edges are there in an undirected graph with 10 nodes each of degree six?

Solution:

We have $\sum_{v \in V} \deg(v) =$ The sum of the degrees of all nodes $= 10 \times 6 = 60.$

By Handshaking Theorem, $2|E| = \sum_{v \in V} \deg(v) = 60$. Therefore, $|E| = 30$. Thus the number of edges in the graph is 30.

P2:

Determine whether the degree sequences 1,1,1,1,1 is graphic. If it is, draw a graph having the given degree sequence.

Solution:

The given degree sequence is 1,1,1,1,1. By *Handshaking theorem*,
$2|E| =$ Sum the degrees of all the vertices

$$= 1 + 1 + 1 + 1 + 1 = 5$$

This is not possible. Therefore the given degree sequence is not a graphic.

P3:

Determine whether the degree sequences 3,2,2,1,0 is graphic. If it is, draw a graph having the given degree sequence.

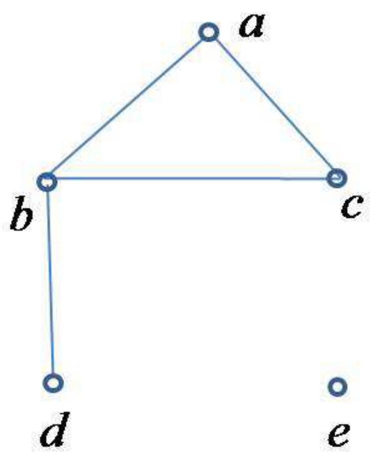
Solution:

The given degree sequence is 3,2,2,1,0.By Handshaking theorem

$2|E| =$ Sum of the degrees of all the vertices $= 3 + 2 + 2 + 1 + 0 = 8$

$\Longrightarrow |E| = 4$

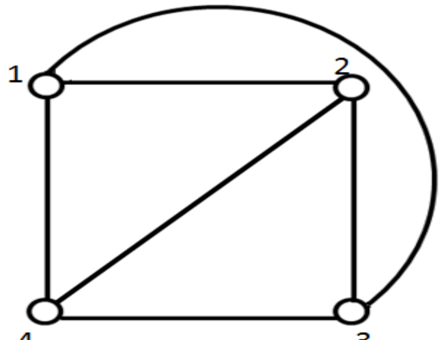
We have to draw a simple graph with 5 nodes and 4 edges with the given degree sequence.



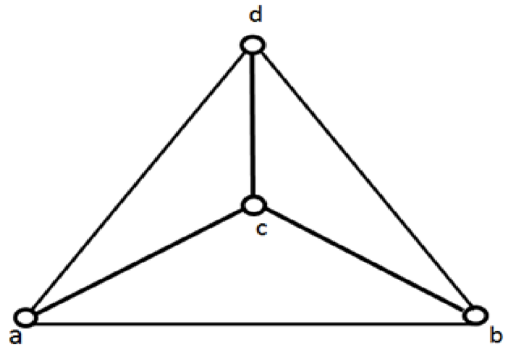
The given sequence is graphic. In the above graph d is pendant and e is an isolated node.

P4:

Show that the following graphs are isomorphic.



G



G'

Solution:

The given graphs are undirected graphs.

We have, $G = (V, E)$, where $V = \{1,2,3,4\}$ and

$$E = \{\{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}\},$$

$G' = (V', E')$, where $V' = V$, $E' = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}\}$.

Define a map $\boldsymbol{f} : \boldsymbol{V} \longrightarrow \boldsymbol{V}'$ by , $f(1) = a\ f(2) = b$, $f(3) = c$, $f(d) = 4$.

Now, f preserves the adjacency as shown below:

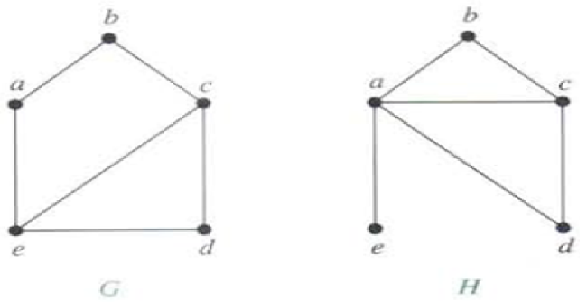
$$\{1,2\} \xrightarrow{f} \{f(1), f(2)\} = \{a, b\}$$
$$\{1,3\} \rightarrow \{a, c\}$$
$$\{1,4\} \rightarrow \{a, d\}$$
$$\{2,3\} \rightarrow \{b, c\}$$
$$\{2,4\} \rightarrow \{b, d\}$$
$$\{3,4\} \rightarrow \{c, d\}$$

Therefore f is an isomorphism from G to G' and $G \cong G'$.

Note: Every bijections from V to V' is an isomorphism(Why?)

P5:

Show that the following graphs are not isomorphic.



Solution:

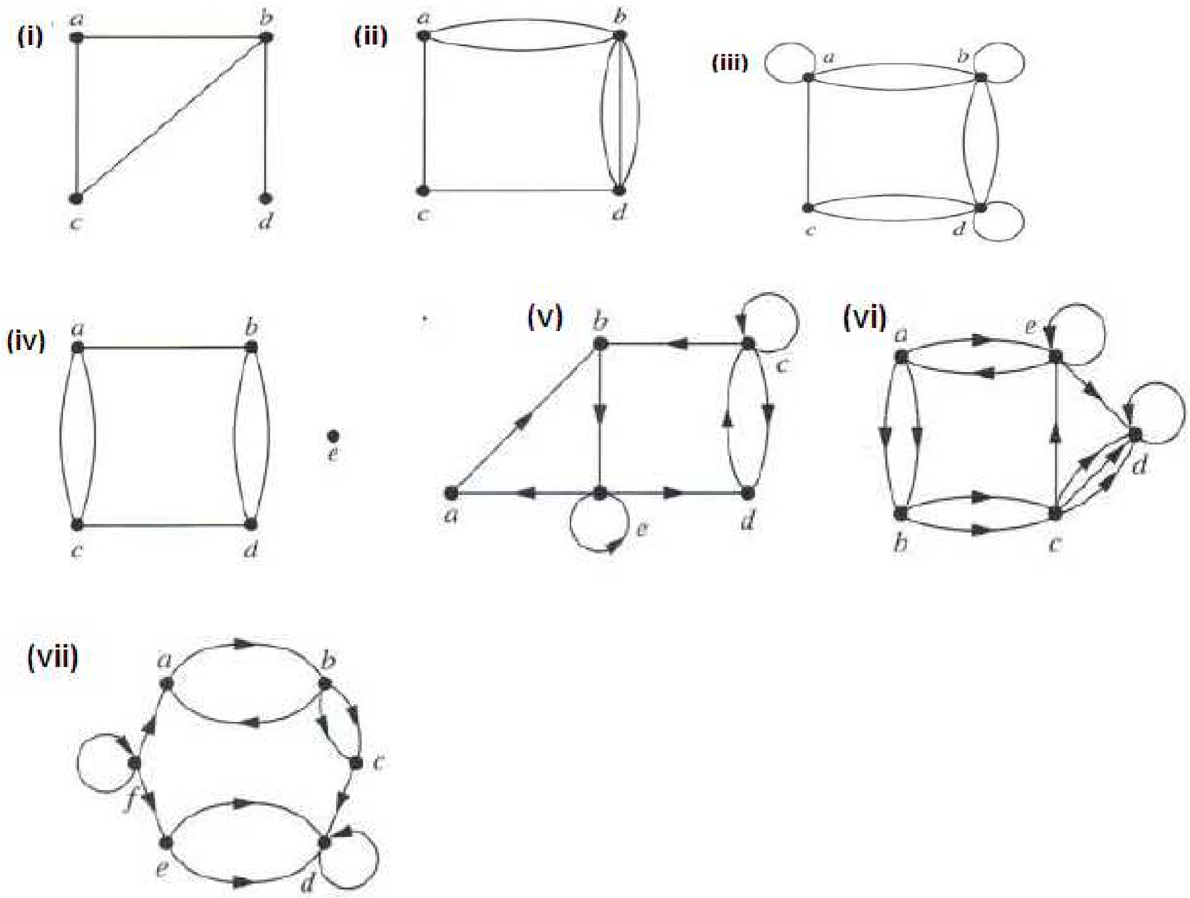
The given graphs G and H are undirected. Both have 5 vertices and 6 edges. Note that the graph H has a vertex e of degree 1 and G has no vertices degree 1. Therefore, G and H are not isomorphic.

Note: The degree sequence of G is $3, 3$,$3, 2, 2, 2$ and the degree sequence of H is $3, 3, 3, 2, 2, 1$.Therefore G and H are not isomorphic .

3.1. **Graphs and their basic properties**

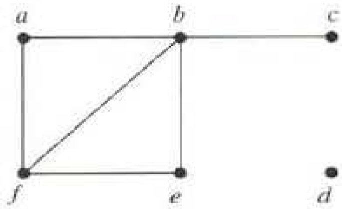
Exercise:

1. Determine whether the graphs shown below has directed or undirected edges, whether it has multiple edges, and whether it has one or more loops. Use your answer to determine the type of graph .

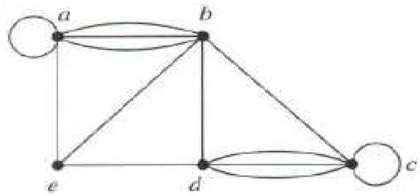


2. Find the number of vertices, the number of edges, and the degree of each vertex in the given undeircted graph. Identify all isolated and pendant vetices.

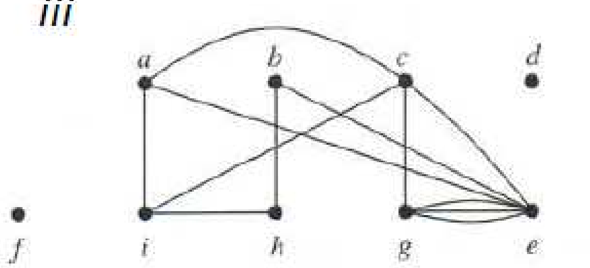
i



ii



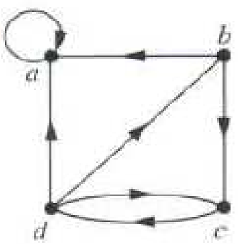
iii



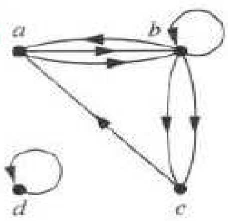
3. Can a simple graph exist with 15 vertices each of degree five?

4. Determine the number of vertices and edges and find the in – degree and out – degree of each vertex for the given directed multigraph.

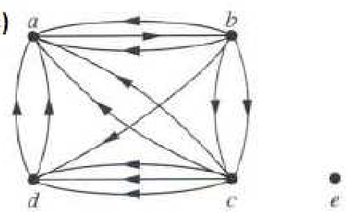
(a)



(b)



(c)

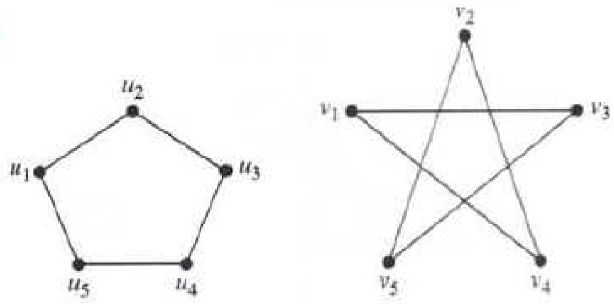


5. Determine whether each of these sequences is graphic. For those that are, draw a graph having the given degree sequence.

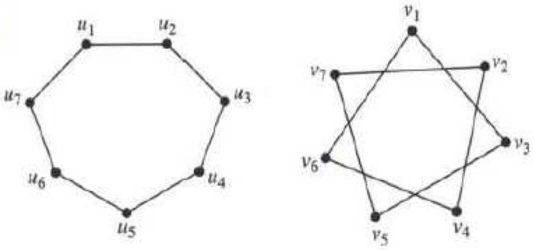
a) 3, 3, 3, 3, 2
b) 5, 4, 3, 2, 1
c) 4, 4, 3, 2, 1
d) 4, 4, 3, 3, 3

6. Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.

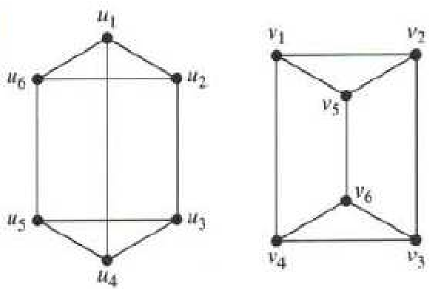
a)



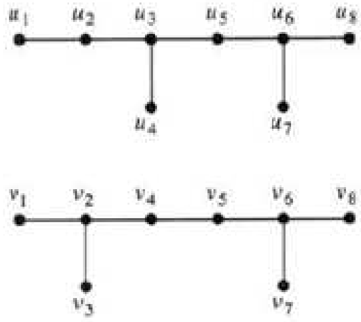
b)



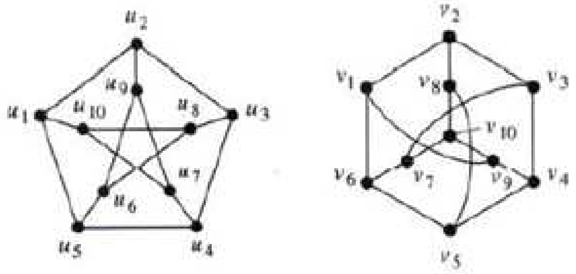
c)



d

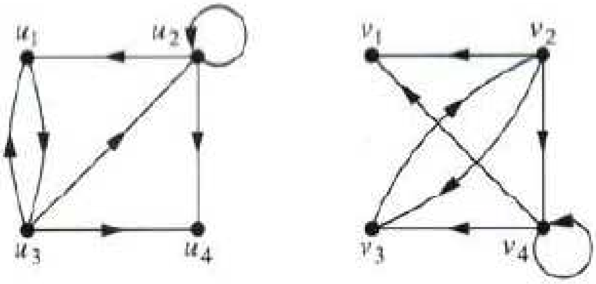


e



7. Determine whether the given pair of directed graphs are isomorphic.

a)



b)