ELL 100 - Introduction to Electrical Engineering

LECTURE 33: TRANSFORMERS - II

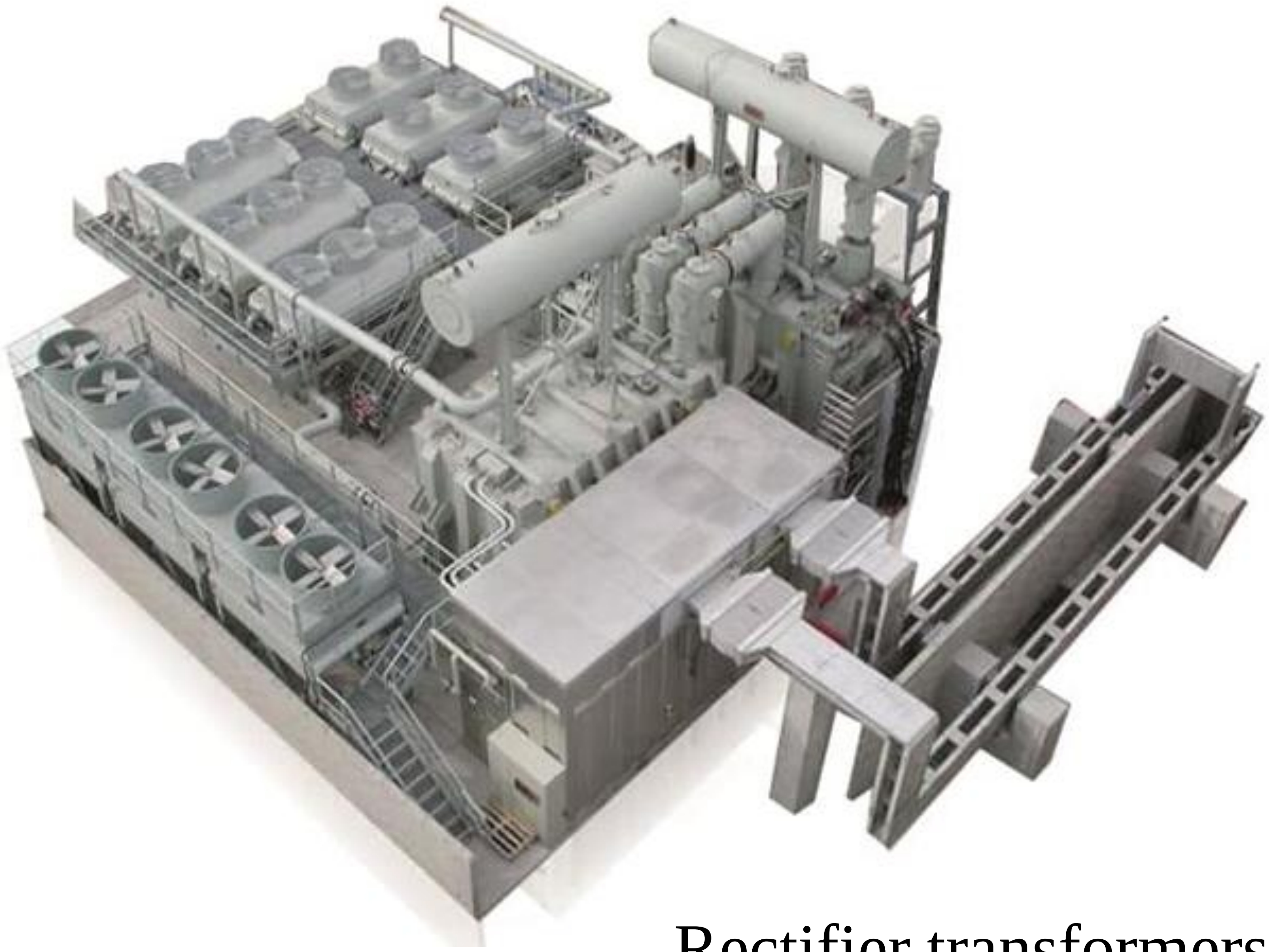
Outline

- ❑ Phasor diagram at ‘no-load’
- ❑ Leakage flux in transformer
- ❑ Transformer equivalent circuit
- ❑ Three-phase transformers
- ❑ Transformer construction
- ❑ Special application-based transformers
- ❑ Numerical examples & practice problems

Industrial Applications of Transformers

Arc furnace transformers for steel furnaces

Industrial Applications of Transformers



Rectifier transformers accompanied by solid state devices for aluminium electrolysis

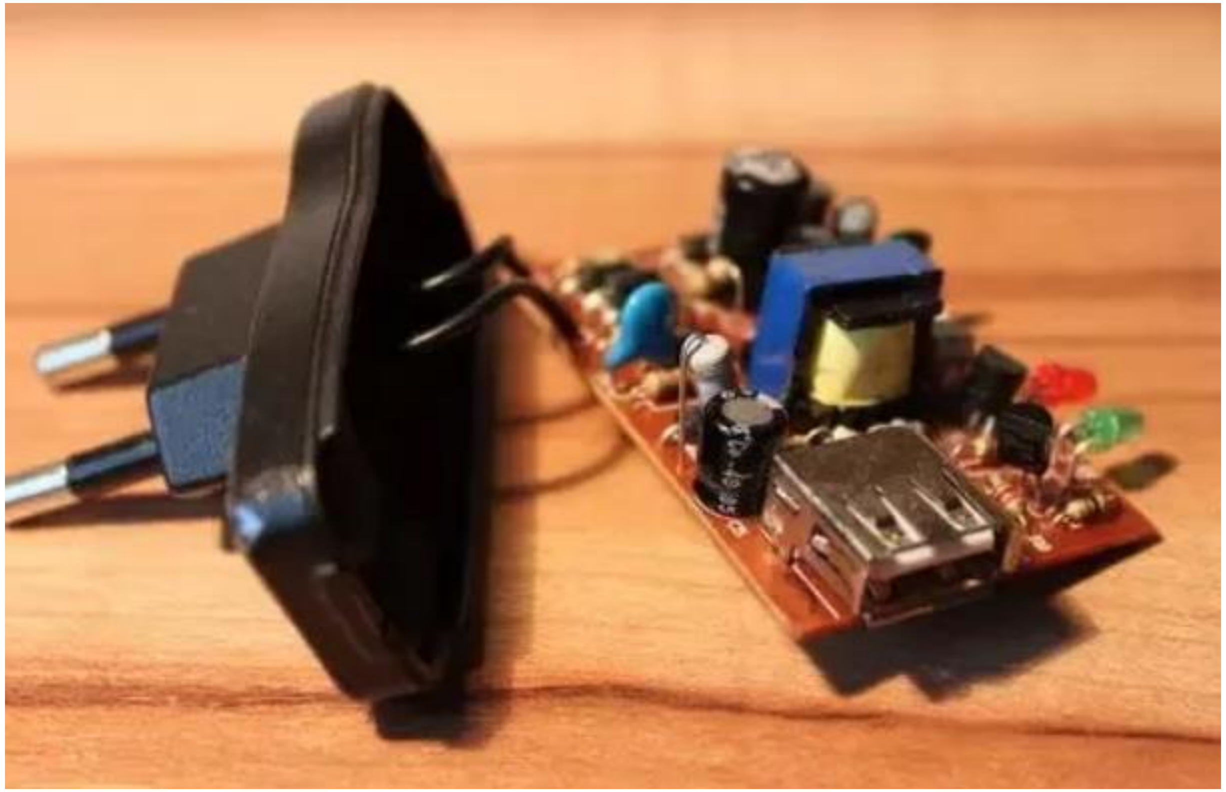
Industrial Applications of Transformers

Converter transformers for variable frequency drives in applications like pumping, mining, etc.

Industrial Applications of Transformers

Booster transformers in traction (locomotion, trains) applications

Transformers in household/electronic appliances



Transformers in mobile chargers and electronic power supplies

Transformers in household/electronic appliances

Transformers for impedance matching in audio and RF devices e.g. loudspeakers

TRANSFORMER AT NO-LOAD

“No-load” operation: Secondary open

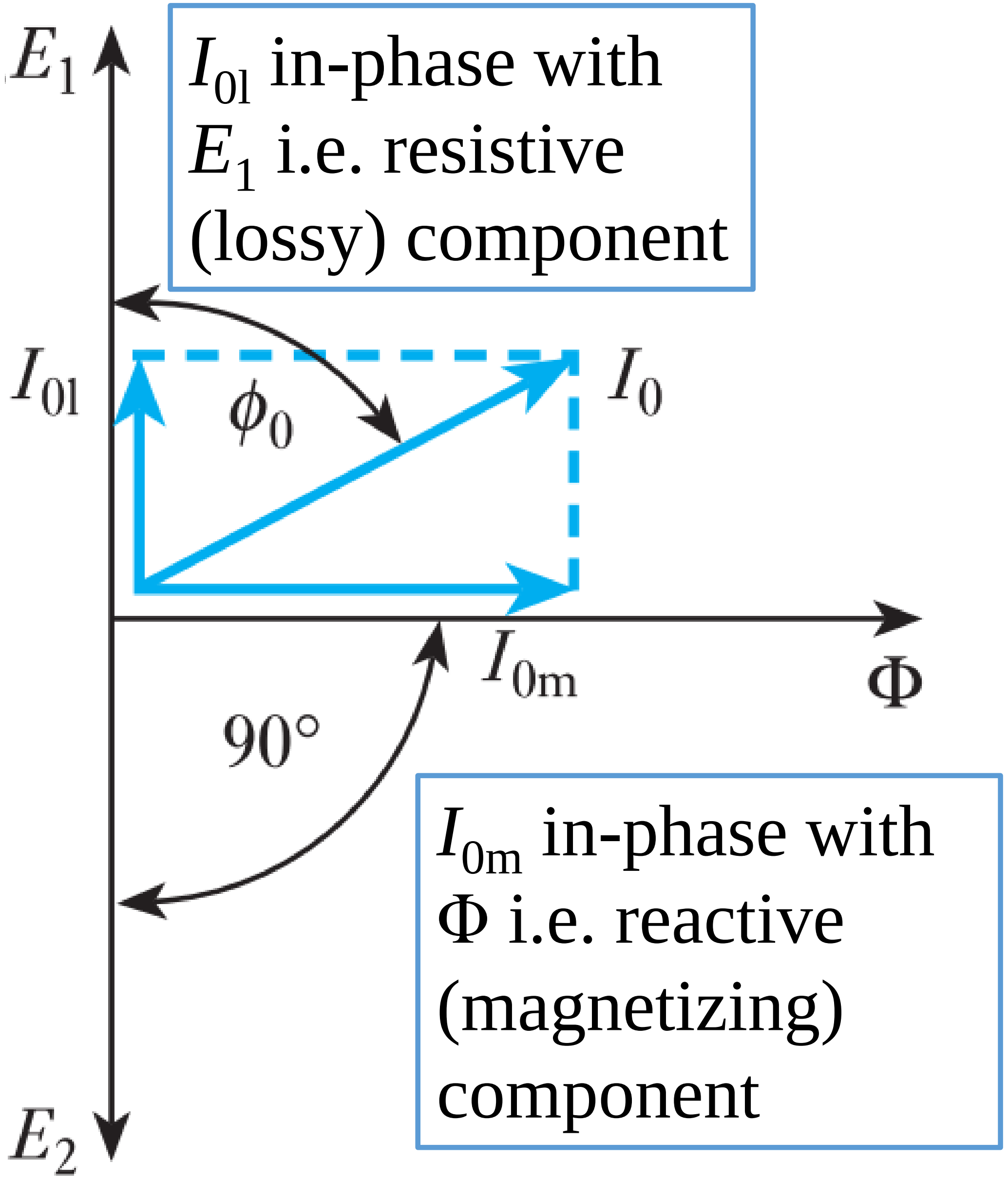
- In a practical (i.e. real) transformer:
  - “No-load losses” (**core losses**) i.e. hysteresis and eddy currents, draw a **finite input current** from **primary** side
  - This current from primary side **produces flux** in the core and hence **EMFs** in the primary and secondary windings
- The above two “**loss**” currents flow even at “**no-load**” on secondary winding, cumulatively called “**no-load current**”.

PHASOR DIAGRAM OF TRANSFORMER AT NO-LOAD

- Mutual **flux** **ϕ** in core is common to primary and secondary => taken **as reference phasor**.
- **EMF E_1** induced in **primary leads flux by 90^0** since $E_1 \propto d\phi/dt$ (**Secondary EMF E_2** is drawn pointing in **opposite direction** to **primary** for **convenience** and clarity, even though it may have same phase depending on the winding orientations i.e. dot convention).
- **Transformation ratio** assumed **one** for **convenience** i.e. magnitude of induced EMF same in both windings.
- $\boldsymbol{I_{0l}}$ = **loss** (hysteresis+eddy current) component of no-load current $\boldsymbol{I_0}$

$\boldsymbol{I_{0m}}$ = **magnetizing** (flux producing) component of no-load current $\boldsymbol{I_0}$

PHASOR DIAGRAM OF TRANSFORMER AT NO-LOAD



- Total "**no-load current**"

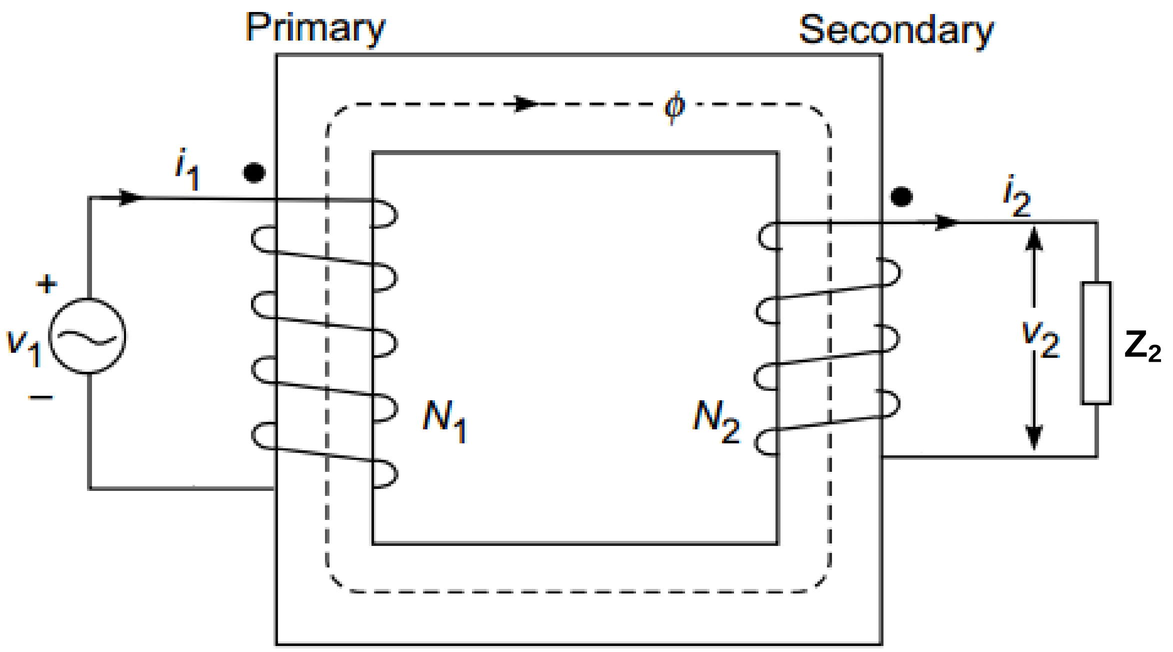
$$I_o = \sqrt{I_{ol}^2 + I_{om}^2}$$

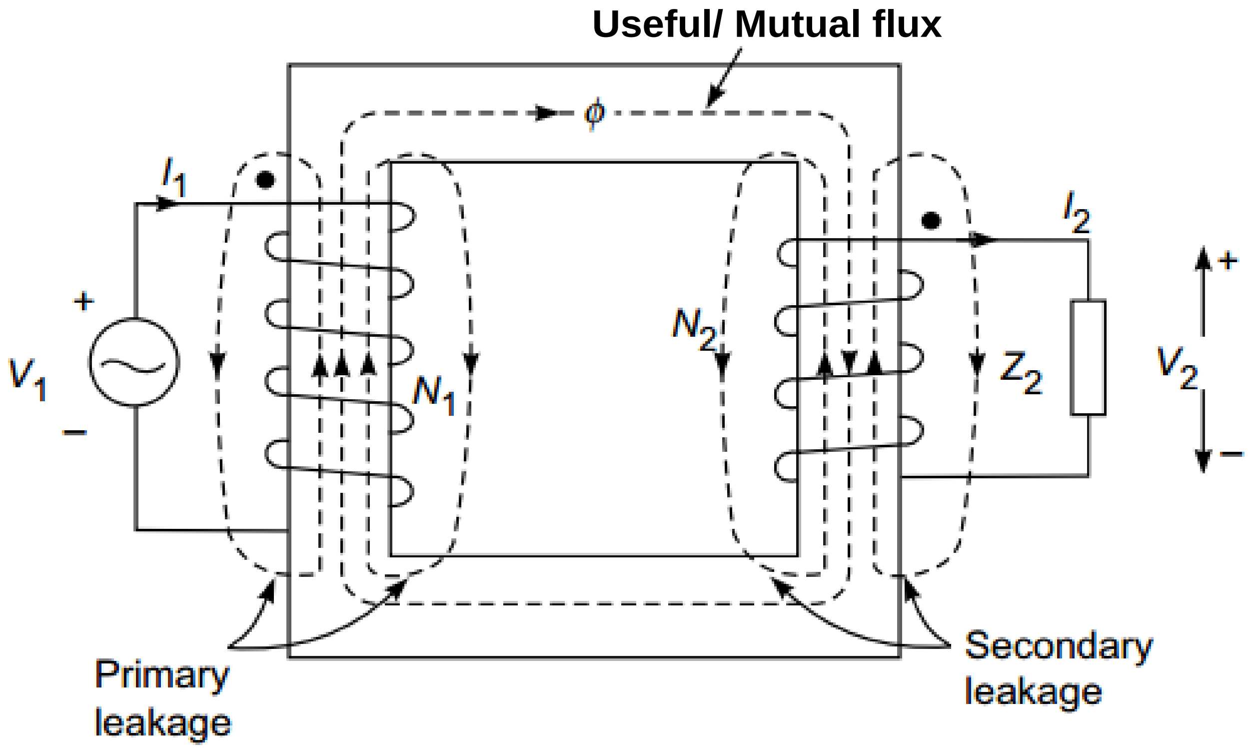
$$I_{ol} = I_o \cos\phi_o; \quad I_{om} = I_o \sin\phi_o$$

- **No-load power factor**
  $= \cos\phi_o$

- **Core power loss**
  $= E_1 I_0 \cos\phi_o$

LEAKAGE FLUX IN TRANSFORMER





Ideal transformer
(no leakage flux)

Practical transformer
(finite leakage flux)

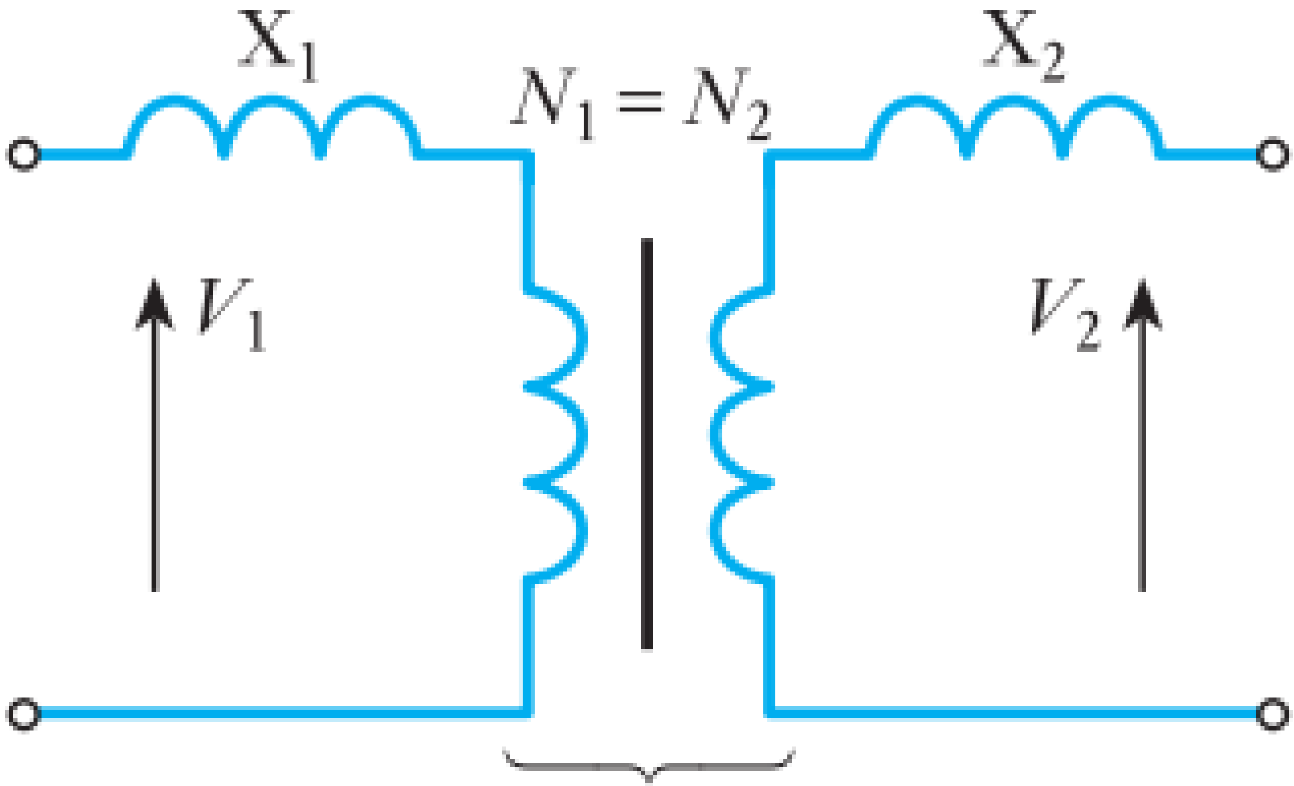
Leakage Flux in Transformer

- **Leakage flux**: The flux **linking** with **only 1 winding**; flows through **air path** between windings.
- **Reluctance** of **air** is **~1000×** that of **core material** => **leakage flux** typically **much less** than **mutual flux**.
- Categorized as:

1. **Primary leakage flux**: **caused** by *mmf* N_1I_1 and **induces voltage** in **primary** winding
2. **Secondary leakage flux**: **caused** by *mmf* N_2I_2 and **induces voltage** in **secondary** winding

Leakage Flux in Transformer

Effect of primary and secondary leakage flux in practical transformer can be modeled as an ideal transformer with **additional inductive reactances (X_1 and X_2) in series** with the two windings.



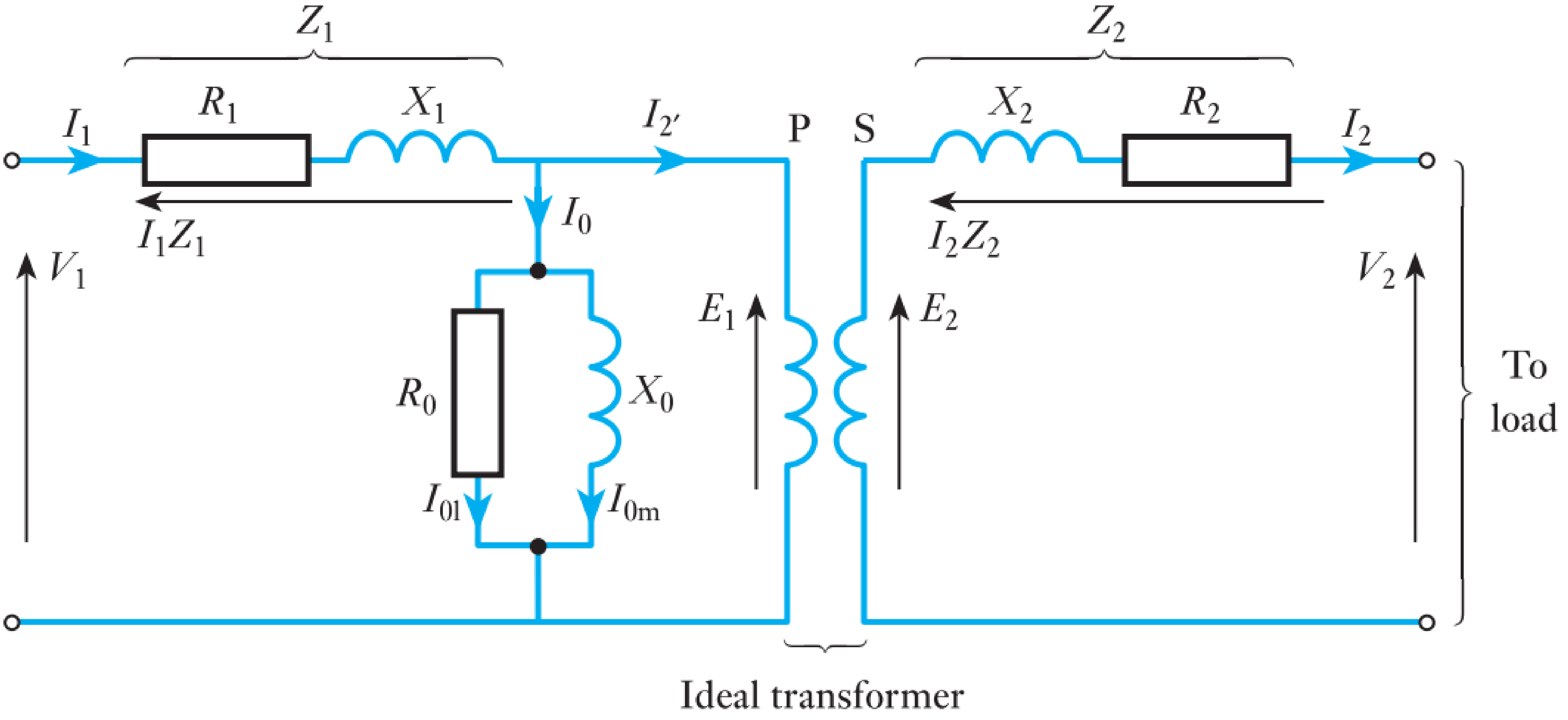
Transformer having no leakage flux

Equivalent Circuit of Transformer

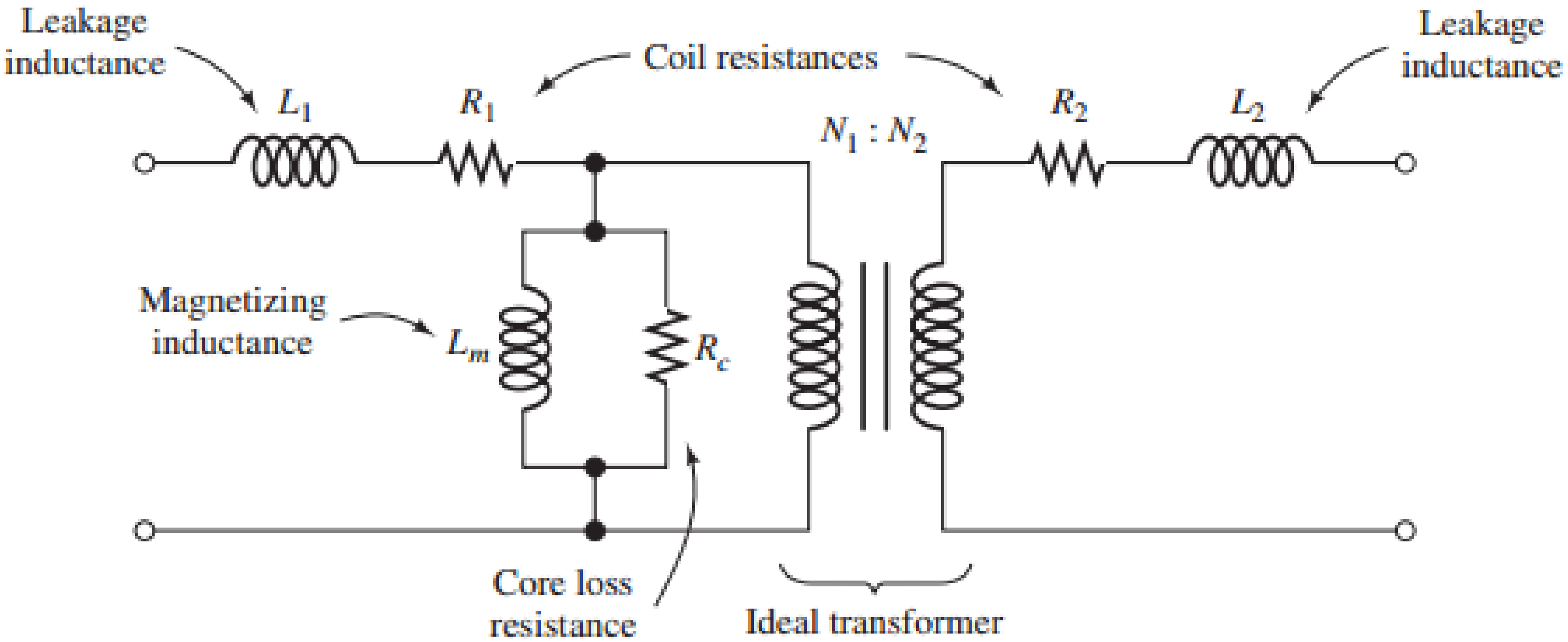
A **practical (real) transformer** can be treated as an **ideal transformer plus additional circuit elements** modeling the various imperfections e.g.

1. **Copper losses** in primary and secondary
2. **Core losses**
3. **Leakage reactances** in primary and secondary
4. **No-load magnetizing current**

EQUIVALENT CIRCUIT OF TRANSFORMER



EQUIVALENT CIRCUIT OF TRANSFORMER



Typical values for a 60-Hz 20-kVA 2400/240-V Transformer

Element Name	*Symbol*	*Ideal*	*Real*
Primary resistance	R_1	0	3.0 Ω
Secondary resistance	R_2	0	0.03 Ω
Primary leakage reactance	$X_1 = \omega L_1$	0	6.5 Ω
Secondary leakage reactance	$X_2 = \omega L_2$	0	0.07 Ω
Magnetizing reactance	$X_m = \omega L_m$	∞	15 kΩ
Core-loss resistance	R_c	∞	100 kΩ

EQUIVALENT CIRCUIT OF TRANSFORMER

Equivalent circuit parameters at different voltage / power (kVA) levels

Element nameplate	Ideal model	#1—2 kVA 230:115 V 50 Hz	#2—10 kVA 2300:230 V 60 Hz	#3—100 kVA 11,000:2200 V 60 Hz
Magnetizing reactance $X_m = \omega L_m$, Ω	∞	1437.5	69,400	57,300
Core loss resistance, R_c, Ω	∞	294.2	75,600	124,000
Primary leakage reactance $X_{l1} = \omega L_{l1}$, Ω	0	0.430	12	31.2
Secondary leakage reactance, $X_{l2} = \omega L_{l2}$, Ω	0	0.006	0.12	1.25
Primary ohmic resistance R_1, Ω	0	0.428	5.80	6.1
Secondary ohmic resistance R_2, Ω	0	0.123	0.0605	0.29

THREE-PHASE TRANSFORMERS

- To meet the demand for **three-phase power transmission**.

- Can be realized in two ways:

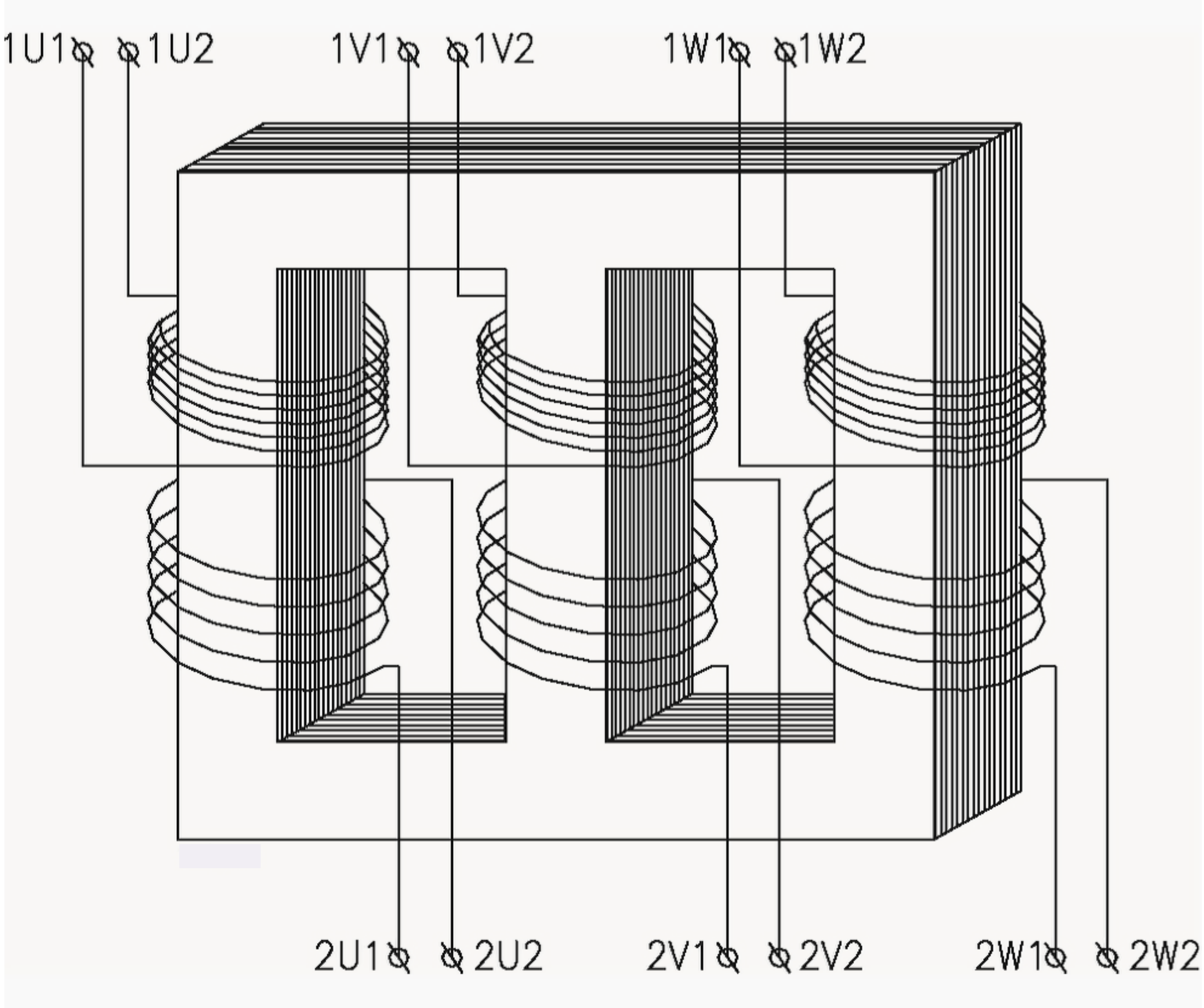
➢ Connecting **three single-phase transformers** appropriately

➢ Using a specially constructed **three-phase transformer**

- For the **same kVA rating**, a **three-phase transformer** turns out to be **smaller and cheaper** than using **three separate single-phase transformers**.

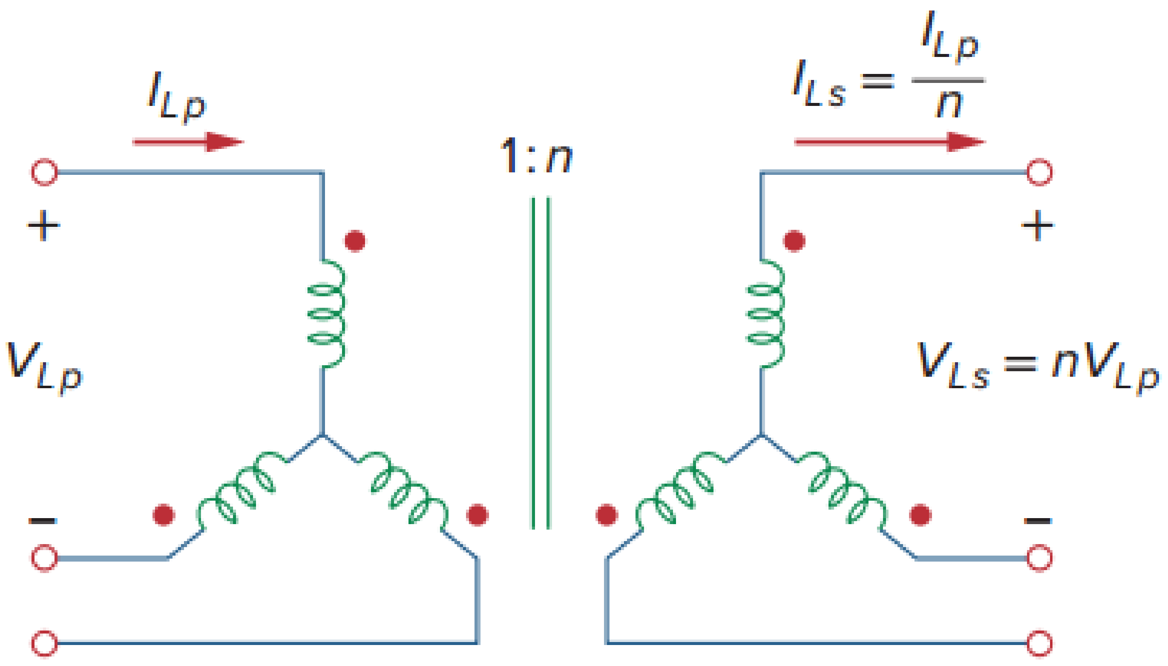
THREE-PHASE TRANSFORMERS

The primary and secondary **windings** of three-phase transformers can be connected in either **star or delta**.

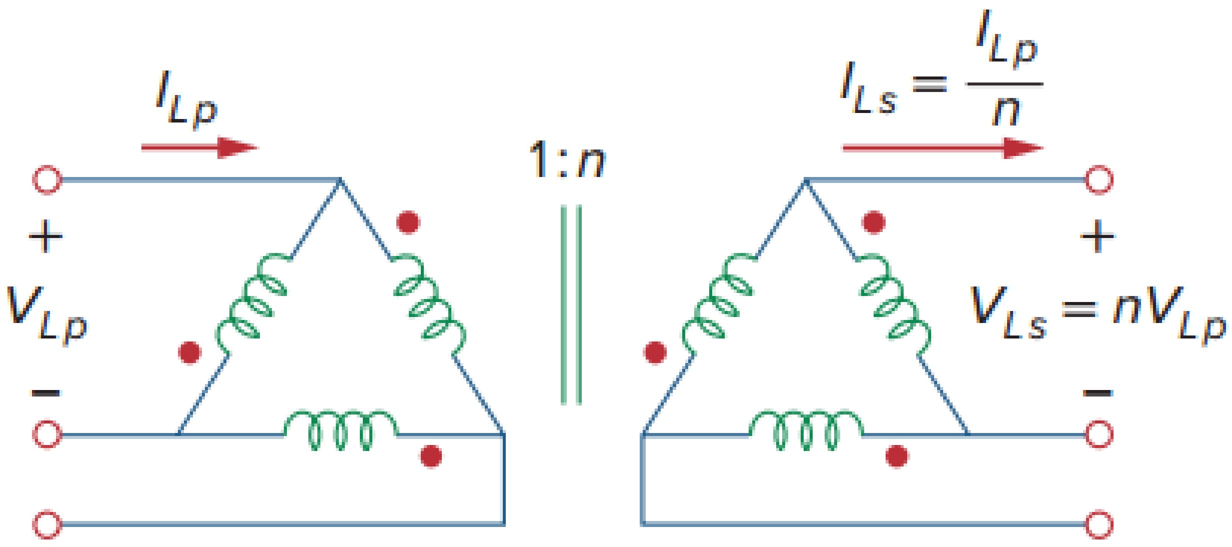


THREE-PHASE TRANSFORMERS

Types of winding arrangements in 3-phase transformers



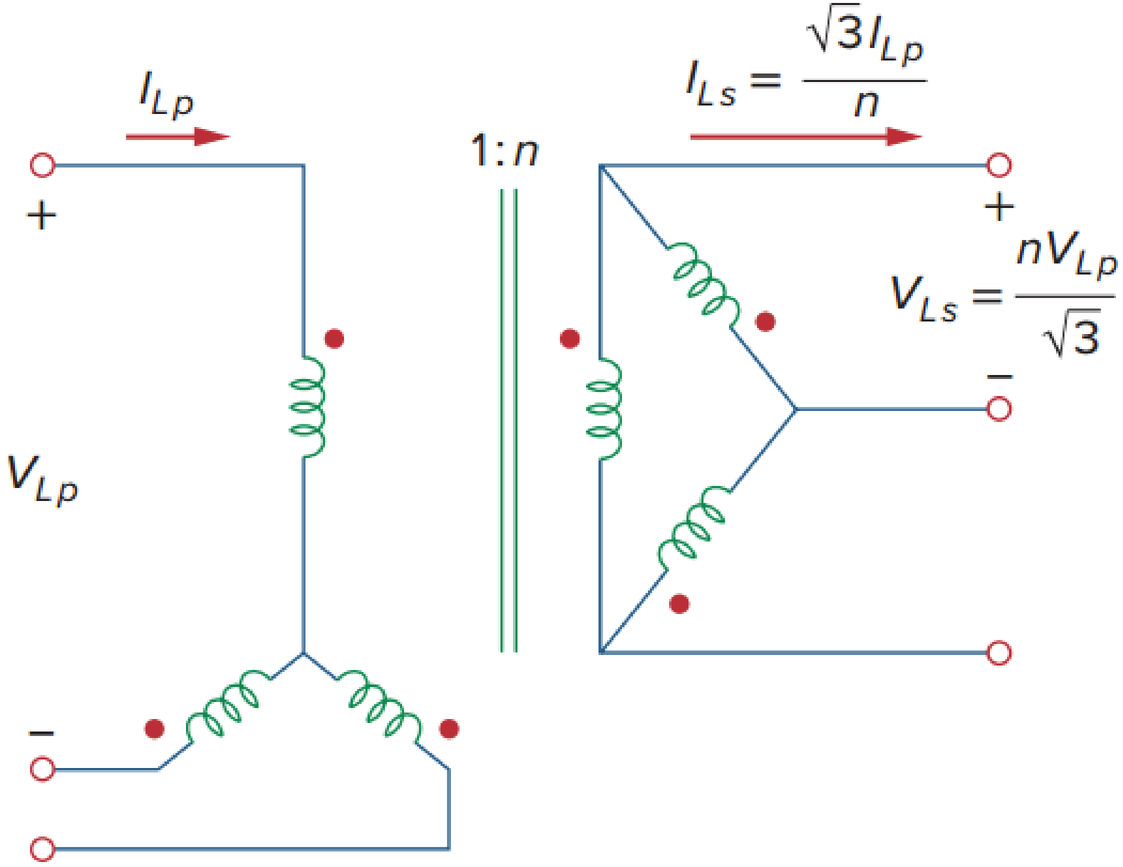
Star-Star connected
three phase transformer



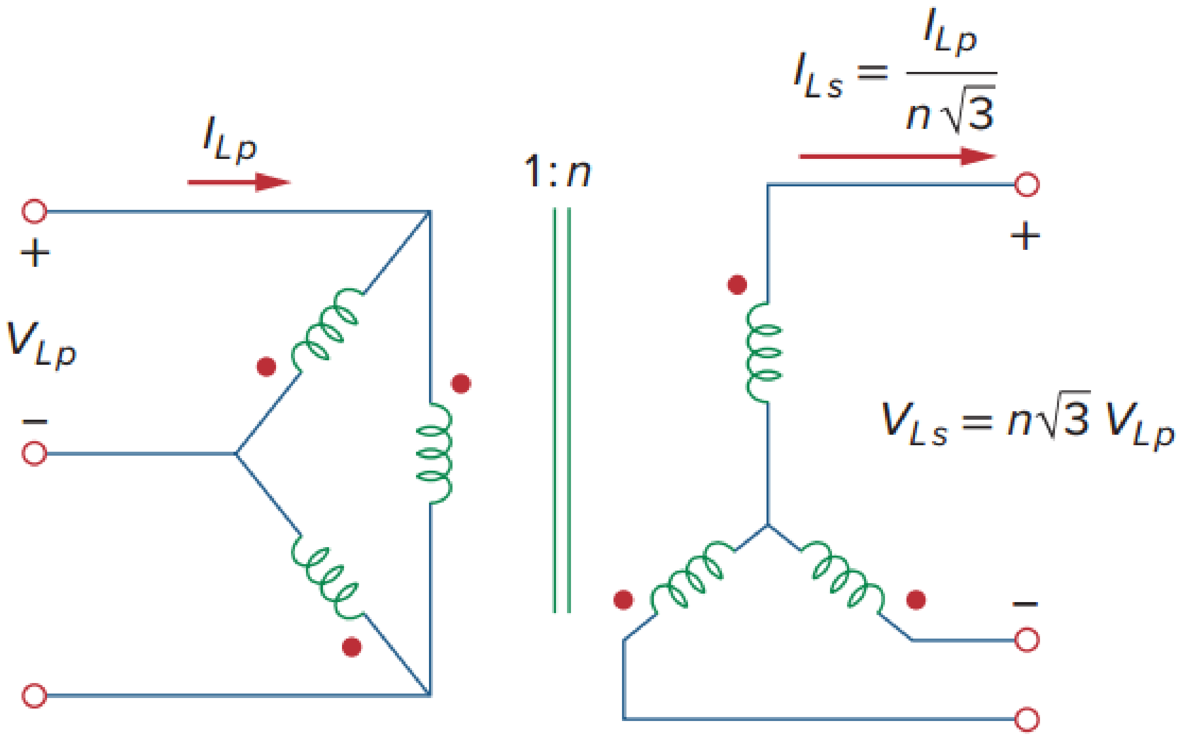
Delta-Delta connected
three phase transformer

Three Phase Transformers

Types of winding arrangements in 3-phase transformers



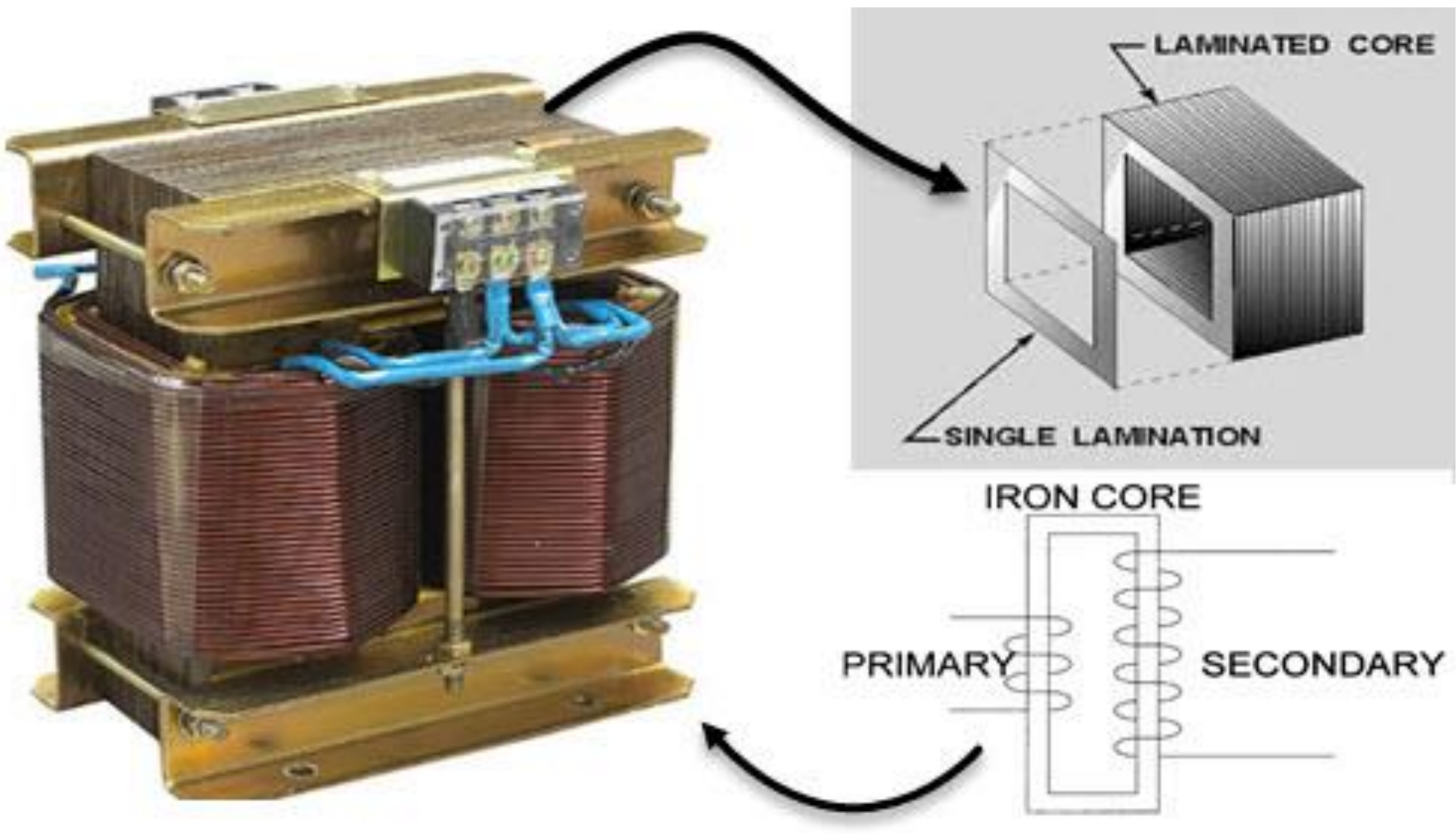
Star-Delta connected three phase transformer

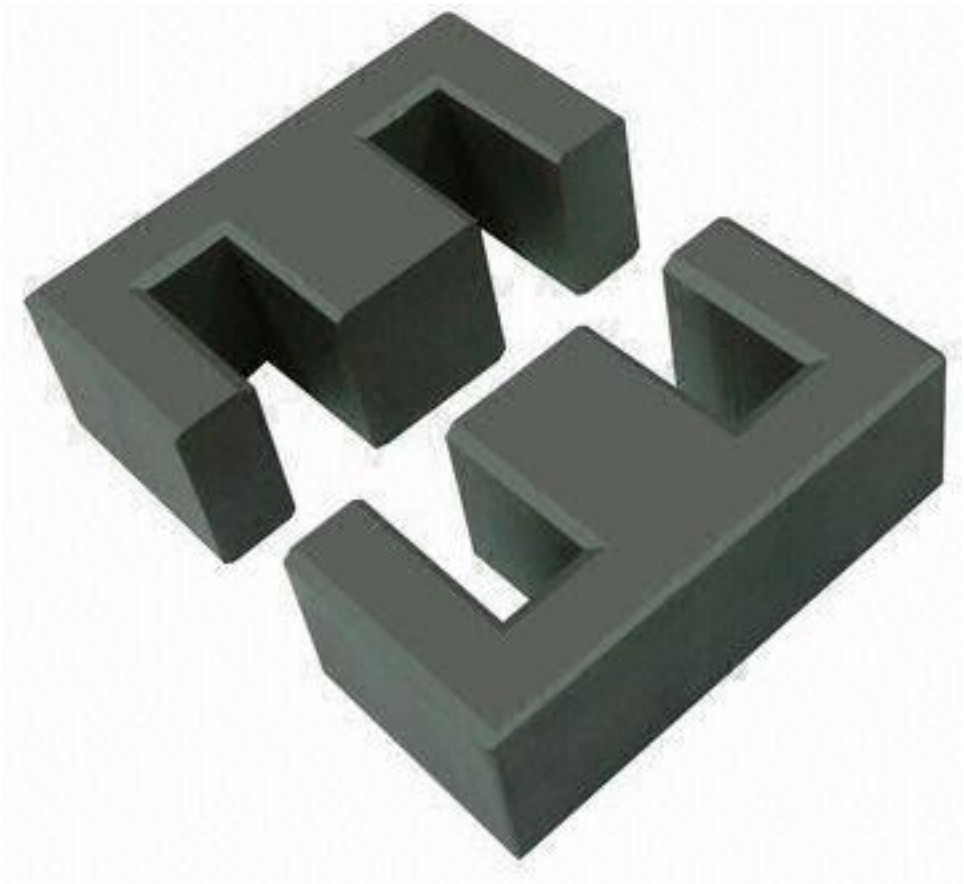


Delta-Star connected three phase transformer

TRANSFORMER CONSTRUCTION

To reduce transformer core losses:



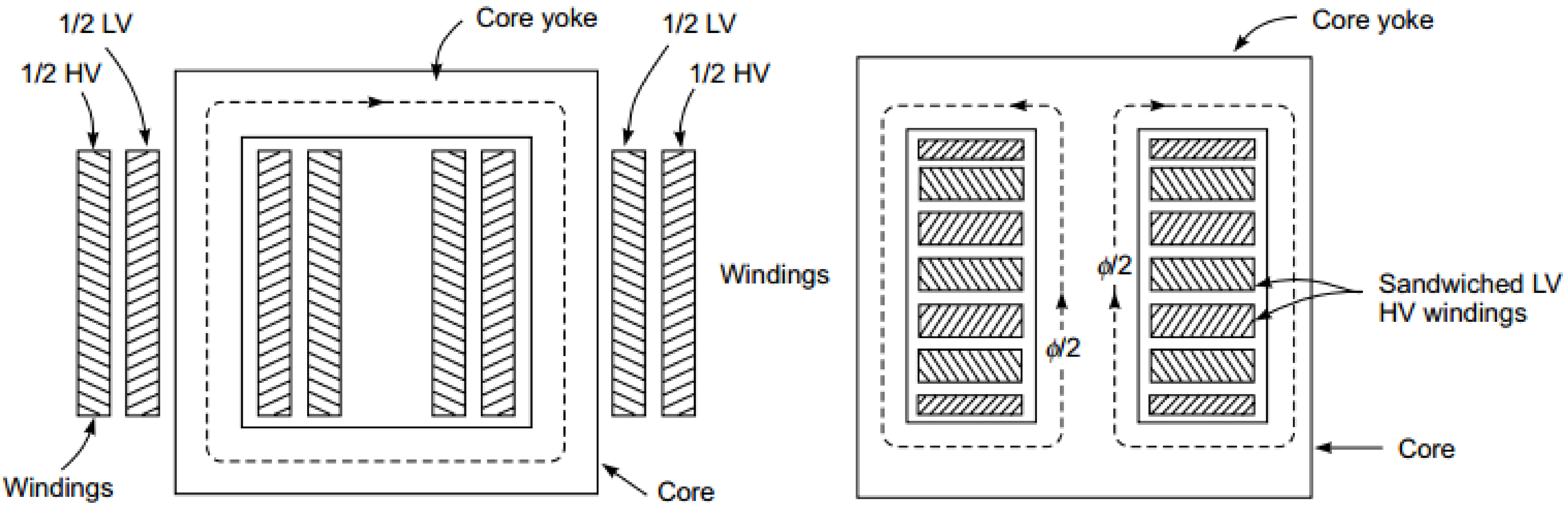


Transformer core lamination
(reduces eddy current loss)

High permeability core
(reduces hysteresis loss)

TRANSFORMER CONSTRUCTION

To reduce leakage flux, designs shown below are used:

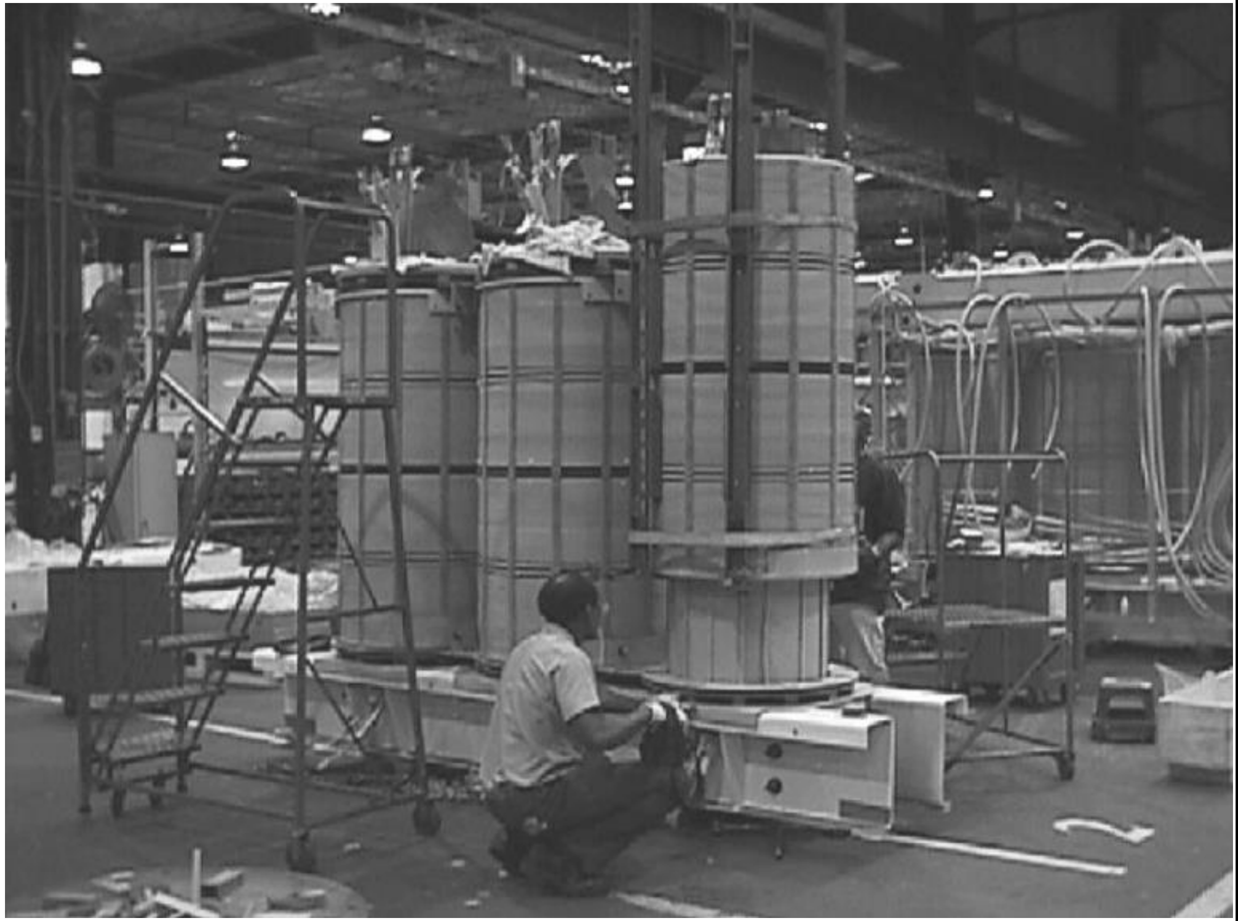


(a) Core-type transformer

(b) Shell-type transformer

Transformer Construction

“E”-assembly, prior to addition of coils and insertion of top yoke

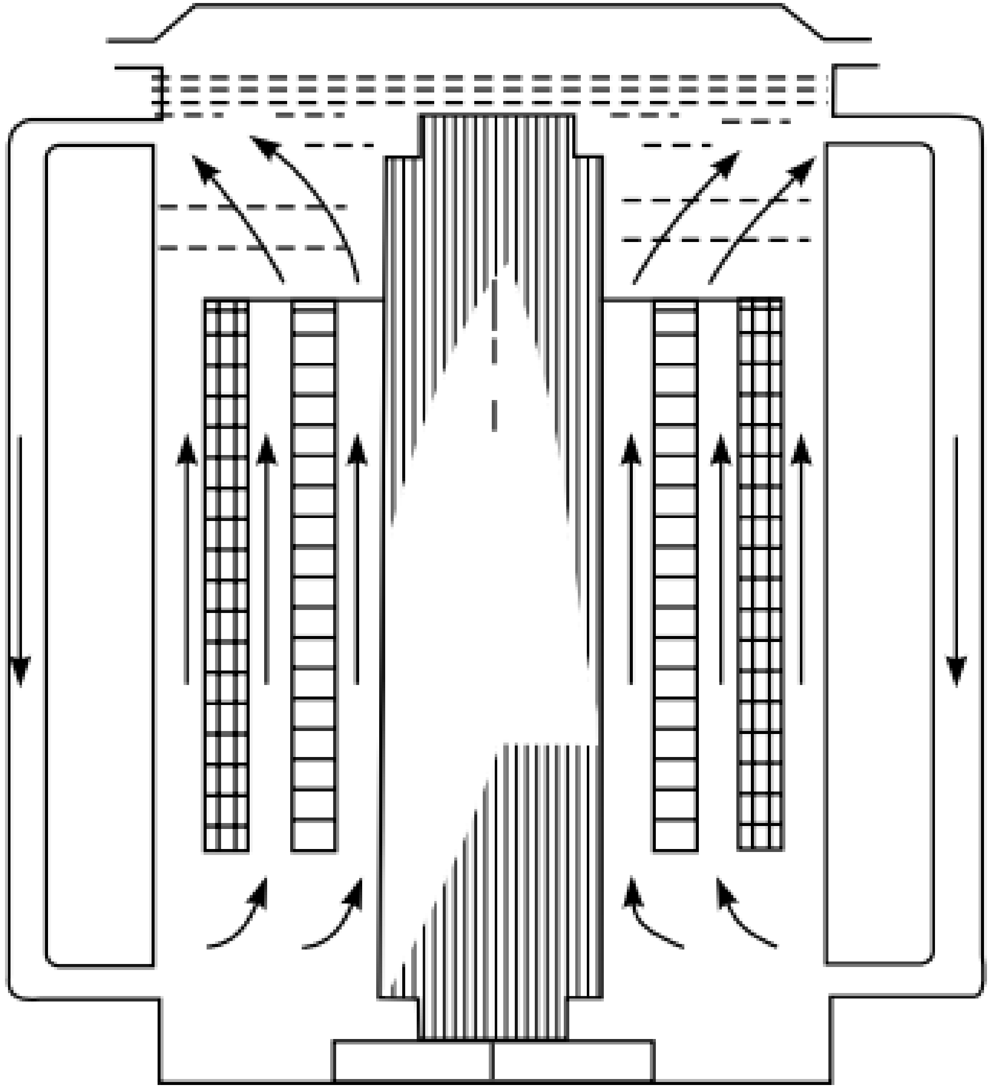


Concentric arrangement, outer coil being lowered onto core leg over top of inner coil

TRANSFORMER CONSTRUCTION

Cooling of winding and core through oil in transformer tank:

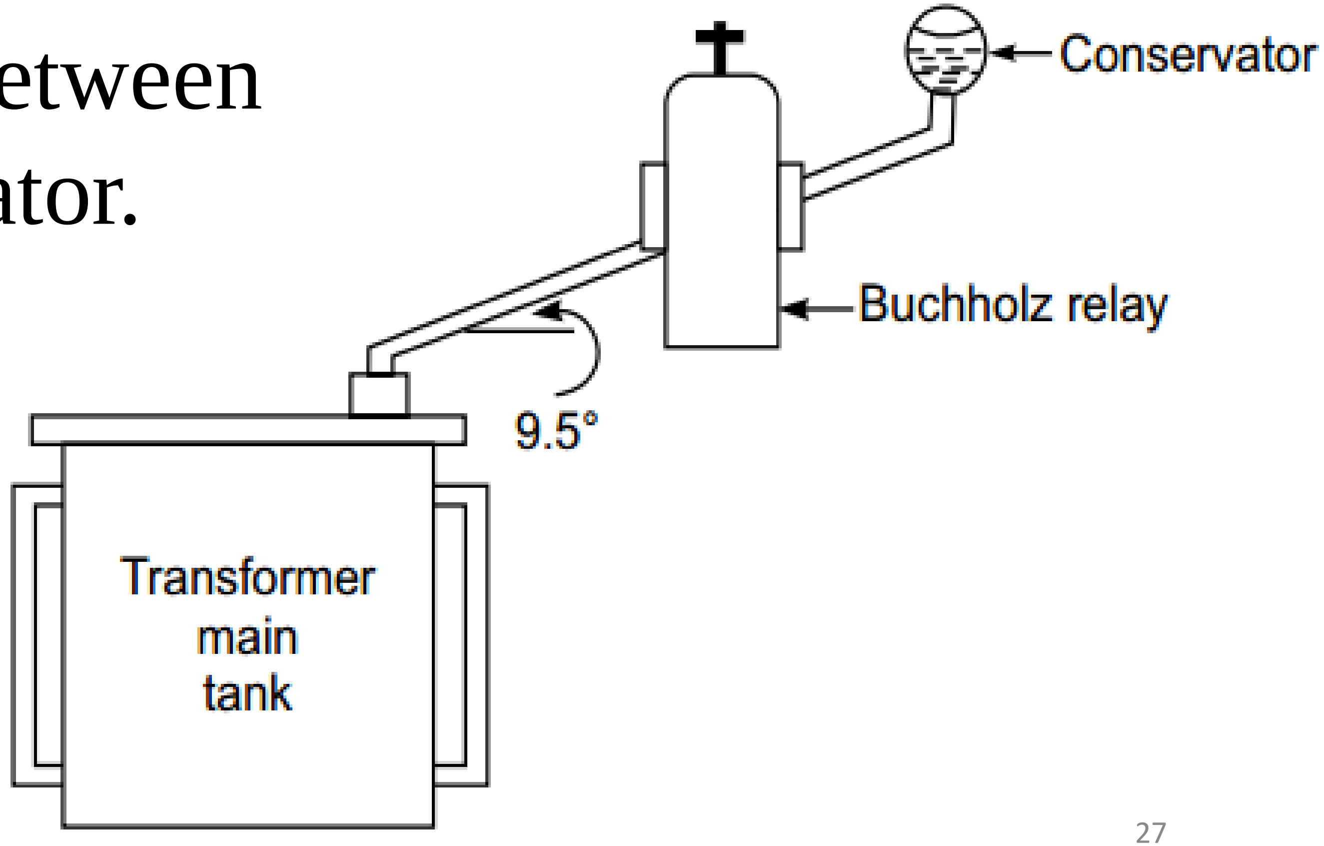
Oil around core and windings gets heated due to transformer losses => oil expands and moves upwards => then flows downwards by side walls and cools => cycle repeats.



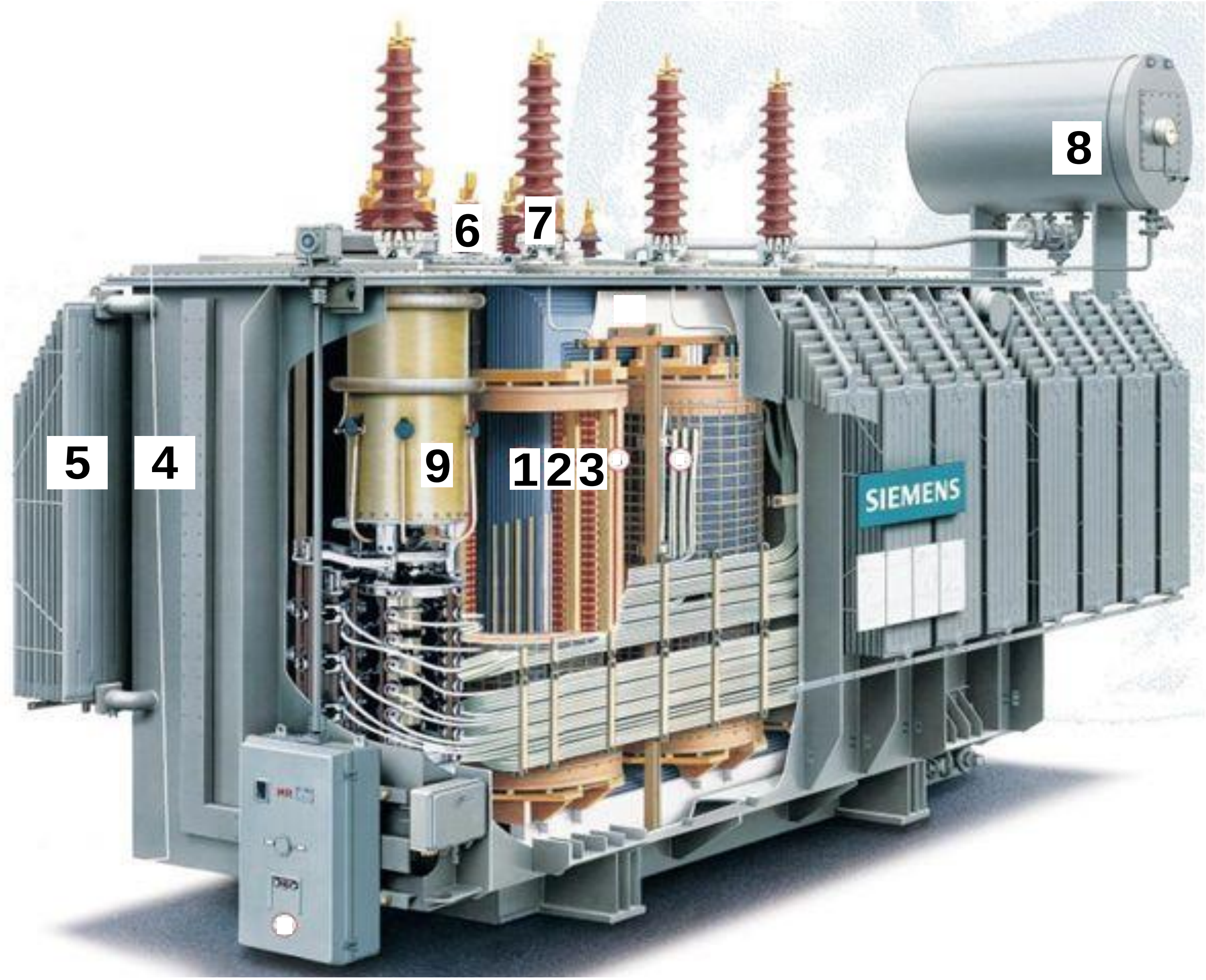
TRANSFORMER CONSTRUCTION

Transformer protection:

Buchholz relay: A gas actuated relay switch installed in oil-immersed transformers, connected between transformer tank and conservator.



TRANSFORMER CONSTRUCTION



Main parts of a three-phase transformer:

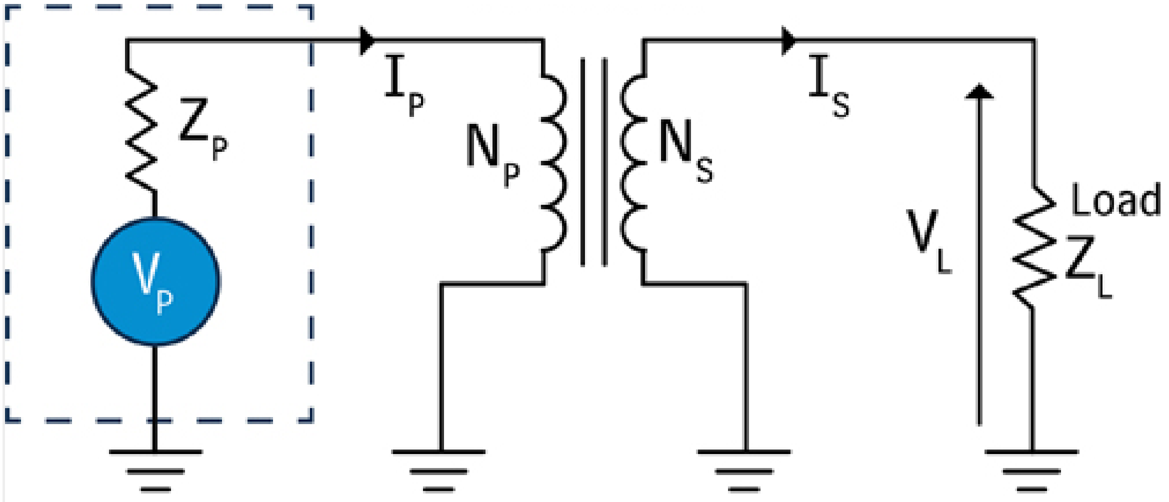
1. Laminated core
2. Low voltage winding
3. High voltage winding
4. Transformer tank
5. Radiator and fan
6. Low voltage bushing
7. High voltage bushing
8. Conservator

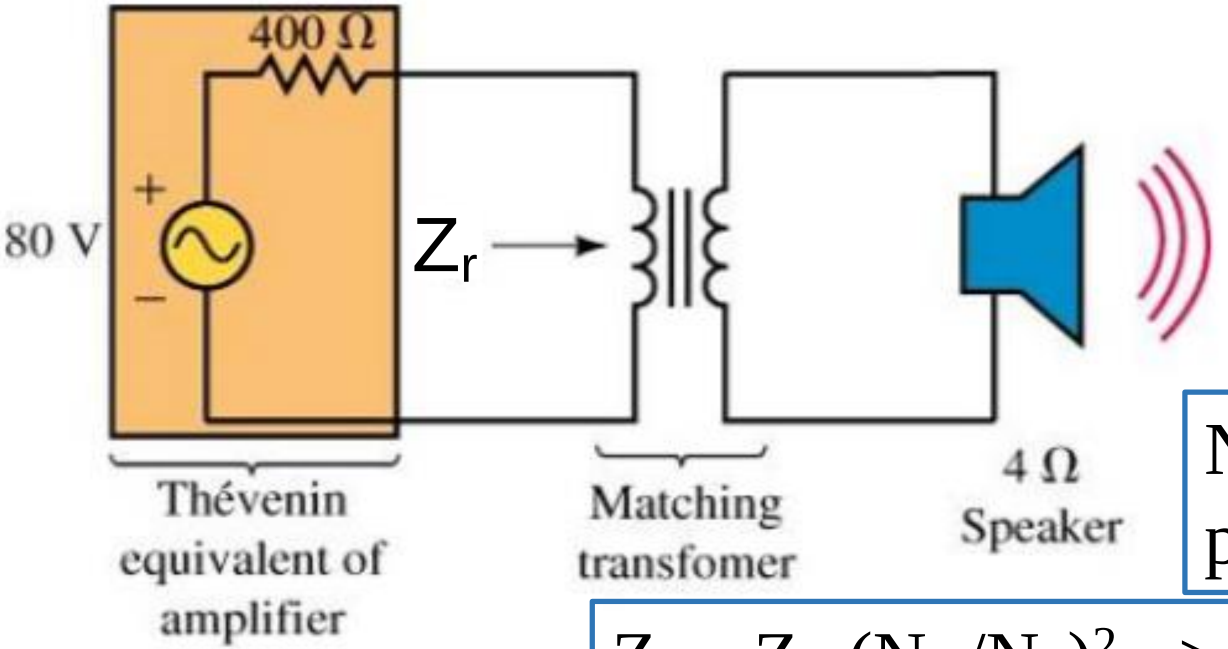
Special Application Based Transformers

Impedance-matching transformers:

- Low-voltage (secondary) winding has less turns (N_S) and drives a low impedance (load). High-voltage winding (primary) has more turns ($N_P > N_S$) and sees a “reflected” load impedance larger by a factor of $(N_P/N_S)^2$.
- Aim is to maximize power transferred from source (connected to primary side) to load (recall maximum power transfer theorem).
- Applications – audio equipment, microphones, amplifiers, data networks and systems, telephone grids, etc.

IMPEDANCE-MATCHING TRANSFORMER





Need $Z_r = Z_p = 400\ \Omega$ for max power transfer to speaker

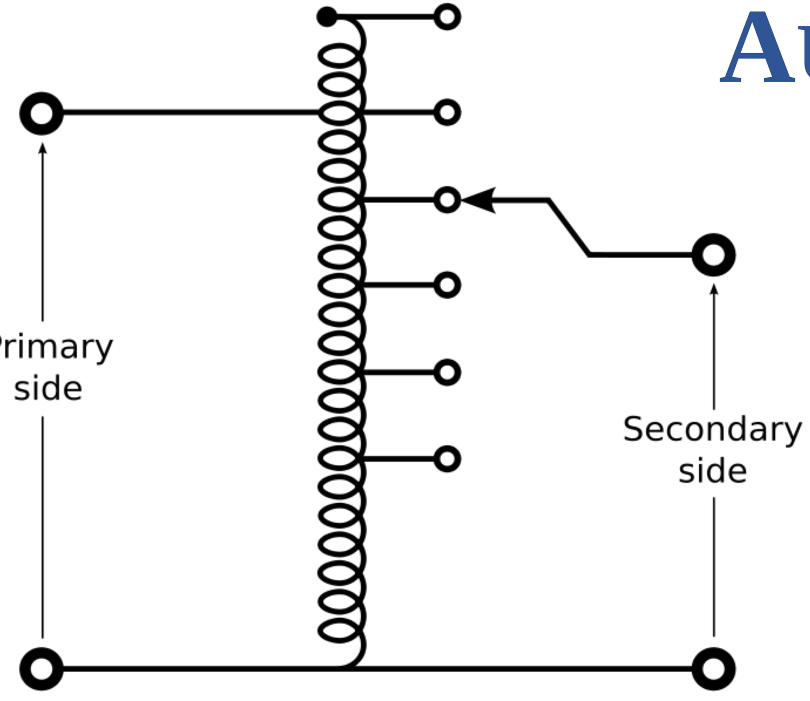
$Z_r = Z_L.(N_P/N_S)^2 \Rightarrow$ need $N_P/N_S = (Z_r/Z_L)^{1/2} = (400/4)^{1/2} = 10$

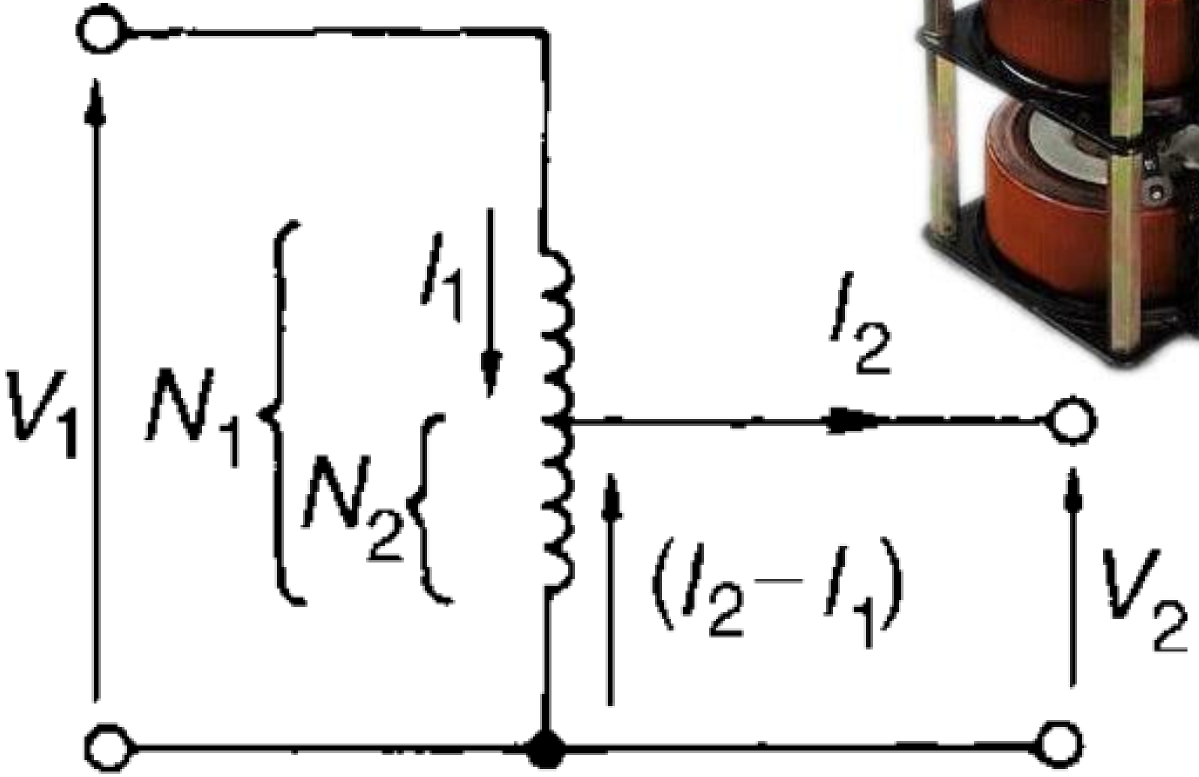
SPECIAL APPLICATION BASED TRANSFORMERS

Auto-transformers:

- Single winding transformer i.e. portions of same winding act as primary and secondary => no electrical isolation.
- Continuously variable output voltage using sliding contact.
- Can be made single-phase or three-phase.
- Applications: Voltage stabilizers, Variable DC source (with a diode bridge rectifier), Motor starters
- Advantages: Lower copper loss (I^2R) as compared to 2-winding transformer. Also has less weight and volume.

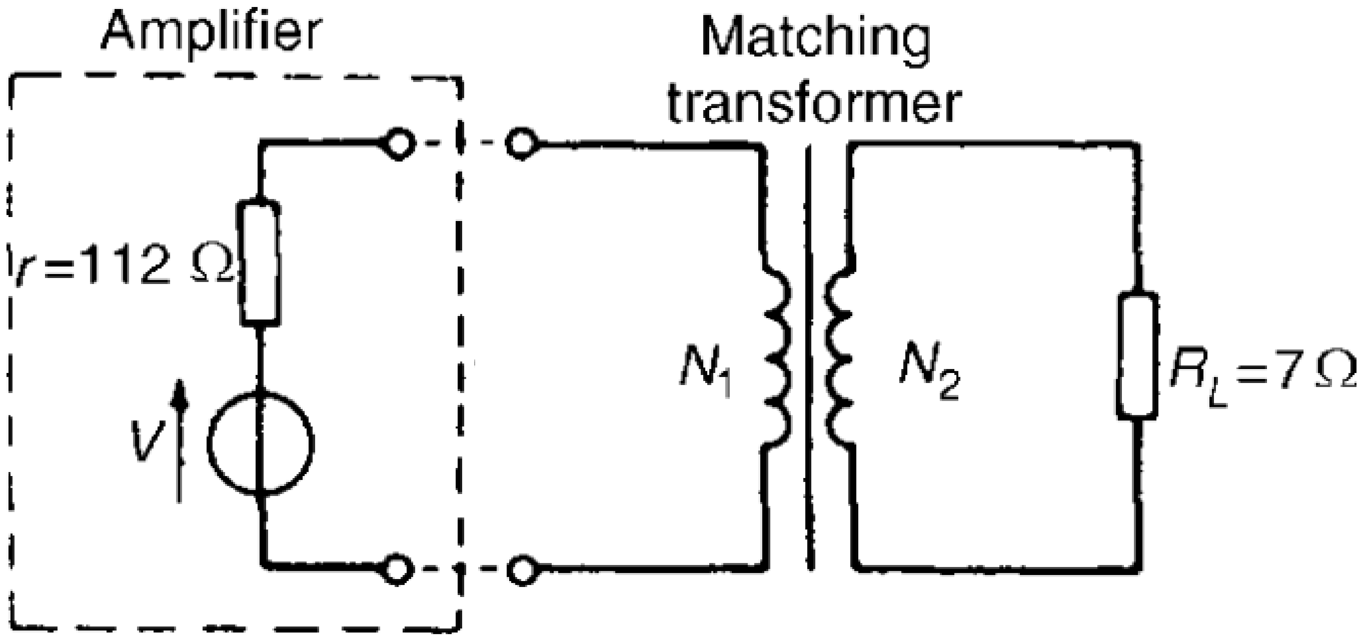
Auto-Transformer





NUMERICAL PROBLEMS

1. The output stage of an amplifier has an output resistance of 112 Ω. Calculate the optimum turns ratio of a transformer that would transfer maximum power from the amplifier to a load resistance of 7 Ω.

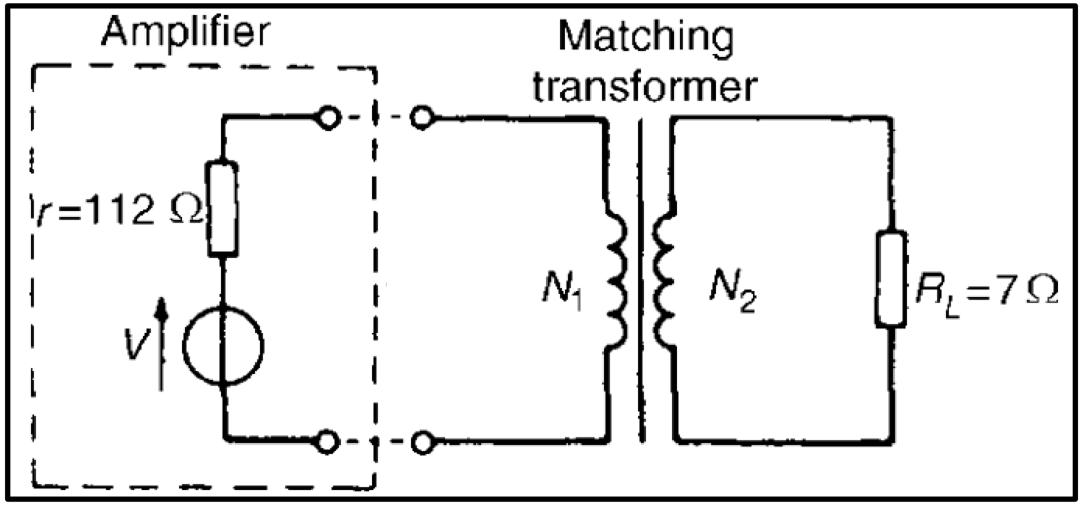


NUMERICAL PROBLEMS

Soln: The reflected resistance value of load (7 Ω) to primary side is,

$$R_{r\_7\Omega} = \frac{v_1}{I_1} = \frac{v_2/n}{nI_2} = \frac{v_2}{n^2 I_2} = \frac{R_L}{n^2} = \frac{7}{n^2}$$

$$\Rightarrow n = \sqrt{\frac{7}{R_{r\_7\Omega}}}$$



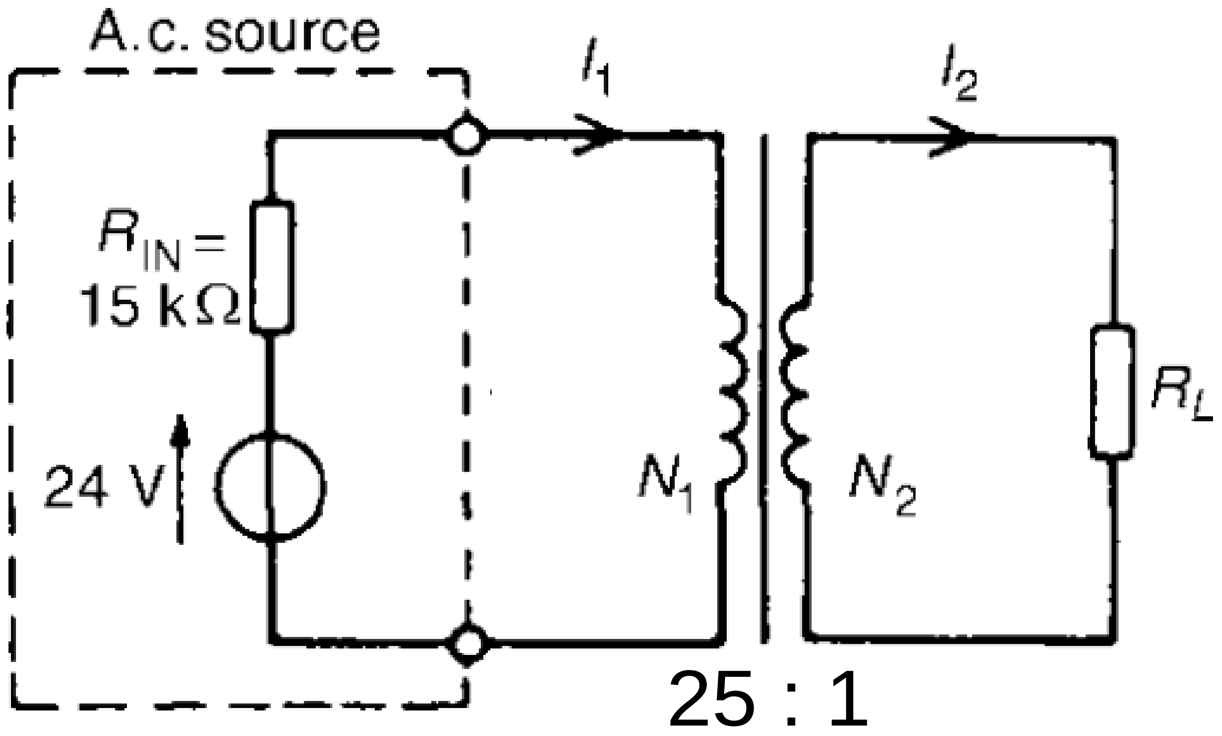
Applying the maximum power transfer theorem, $\boxed{R_{r\_7\Omega} = R_{in} = 112\Omega}$

The optimum transformation ratio of transformer is calculated as,

$$\boxed{n = \sqrt{7/R_{r\_7\Omega}} = \sqrt{7/112} = 0.25}$$ i.e. $\boxed{N_2/N_1 = 1/4}$

NUMERICAL PROBLEMS

2. An AC source of 24-V and internal resistance 15-kΩ is matched to a load by a 25:1 ideal transformer. Determine (a) the load resistance value and (b) the power dissipated in the load.



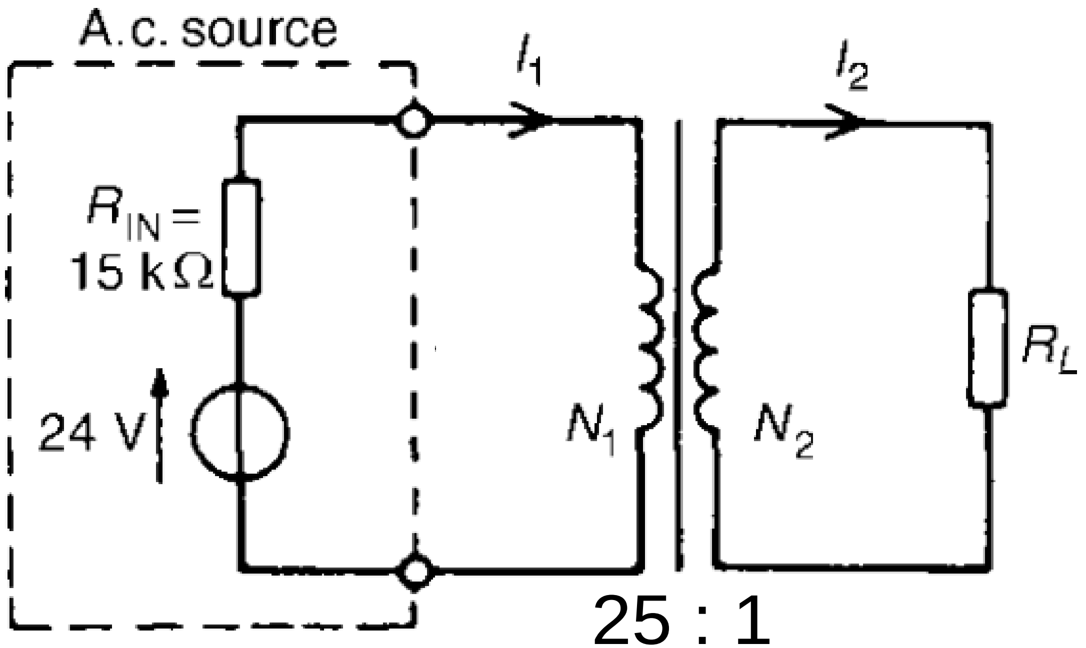
NUMERICAL PROBLEMS

Soln: (a) The reflected resistance value of the load to primary side is,

$$R_{r\_R_L} = \frac{v_1}{I_1} = \frac{v_2/n}{nI_2} = \frac{v_2}{n^2 I_2} = \frac{R_L}{0.04^2}$$

$$\Rightarrow R_L = R_{r\_R_L} \times 0.04^2$$

$n = N_2/N_1 = 1/25 = 0.04$



For maximum power transfer,

$$R_{r\_R_L} = R_{in} = 15k\Omega$$

The value of load resistance is thus,

$$R_L = R_{r\_R_L} \times 0.04^2 = 24\Omega$$

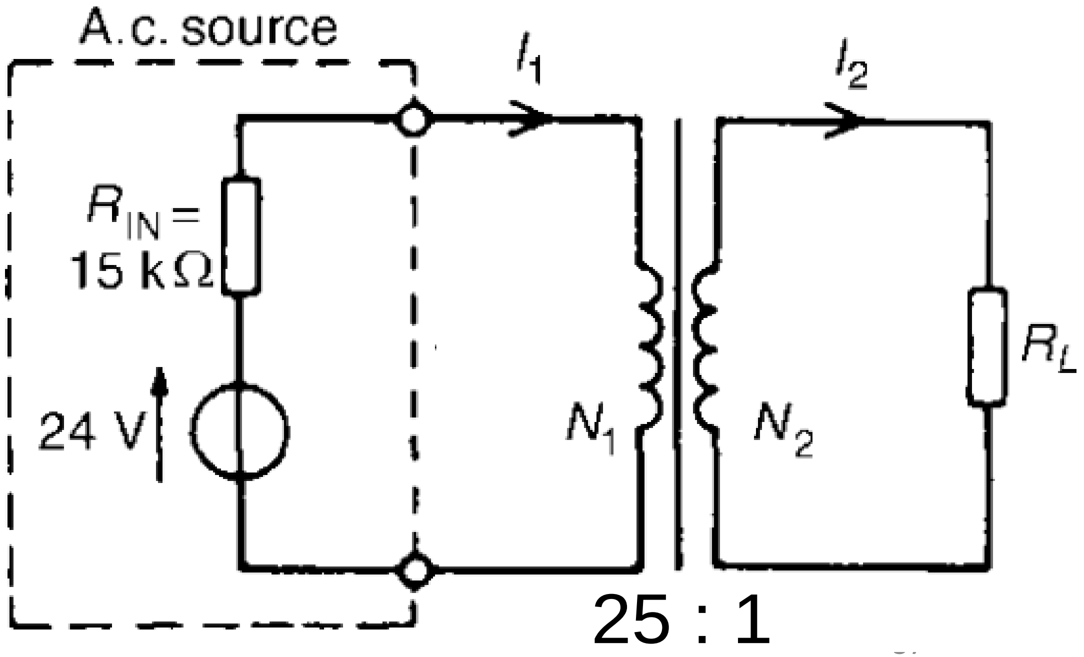
NUMERICAL PROBLEMS

(b) The effective input resistance is $R_{eff} = R_{in} + R_{r\_R_L} = 15k\Omega + 15k\Omega = 30k\Omega$

Thus the primary side current is,

$$I_1 = \frac{V}{R_{eff}} = \frac{24}{30000} = 0.8mA$$

$$n = N_2/N_1 = 1/25 = 0.04$$



=> The secondary current is,

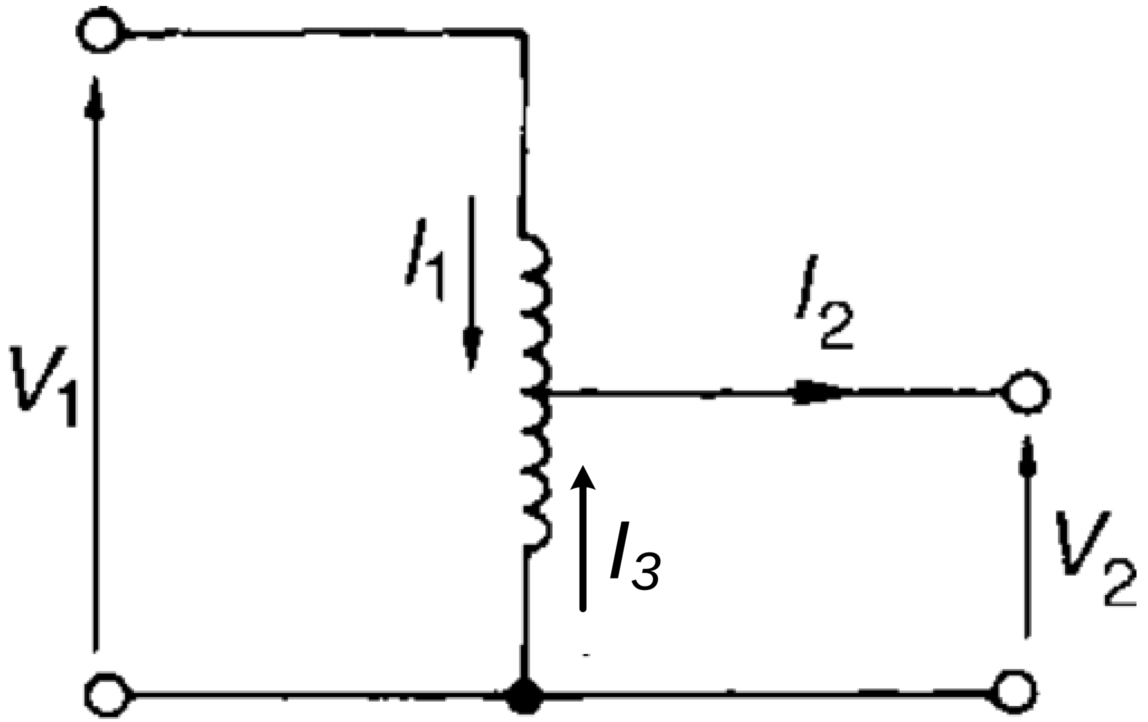
$$I_2 = I_1/n = 0.8/0.04 = 20 \text{ mA}$$

=> The power dissipated in load is,

$$P_L = I_2^2 R_L = 9.6mW$$

NUMERICAL PROBLEMS

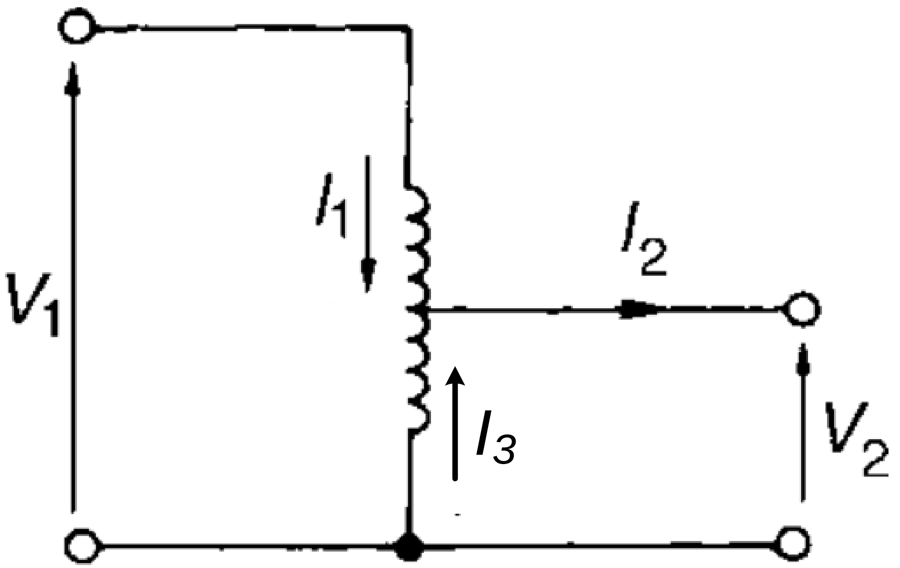
3. A single-phase auto-transformer has a voltage ratio 320V:250V and supplies a load of 20 kVA at 250V. Assuming an ideal transformer, determine the current in each section of the winding.



NUMERICAL PROBLEMS

Soln: $\boxed{Rating = 20kVA = V_1 I_1 = V_2 I_2}$

V_1 = 320 V, V_2 = 250 V



=> Primary current $\boxed{I_1 = \frac{20000}{320} = 62.5A}$

=> Secondary current $\boxed{I_2 = \frac{20000}{250} = 80A}$

Current in common part of winding $\boxed{I_3 = I_2 - I_1 = 80 - 62.5 = 17.5A}$

NUMERICAL PROBLEMS

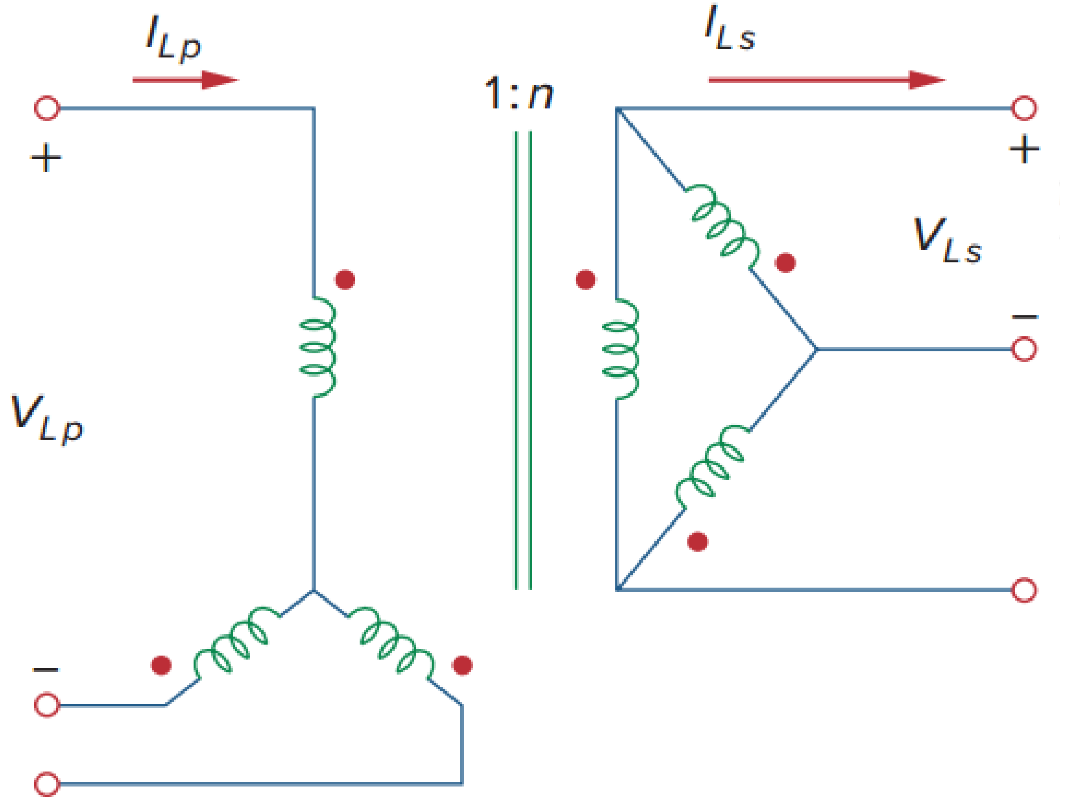
4. A three-phase transformer has 500 primary turns and 50 secondary turns. If the supply voltage is 2.4 kV, find the secondary line voltage at no-load when the windings are connected in (a) star-delta and (b) delta-star.

Soln:

(a) Star-Delta

$n = 50/500 = 0.1$

$V_{lp} = 2400\ \text{V}$



$$\boxed{V_{ls} = \frac{nV_{lp}}{\sqrt{3}} = 138.56V}$$

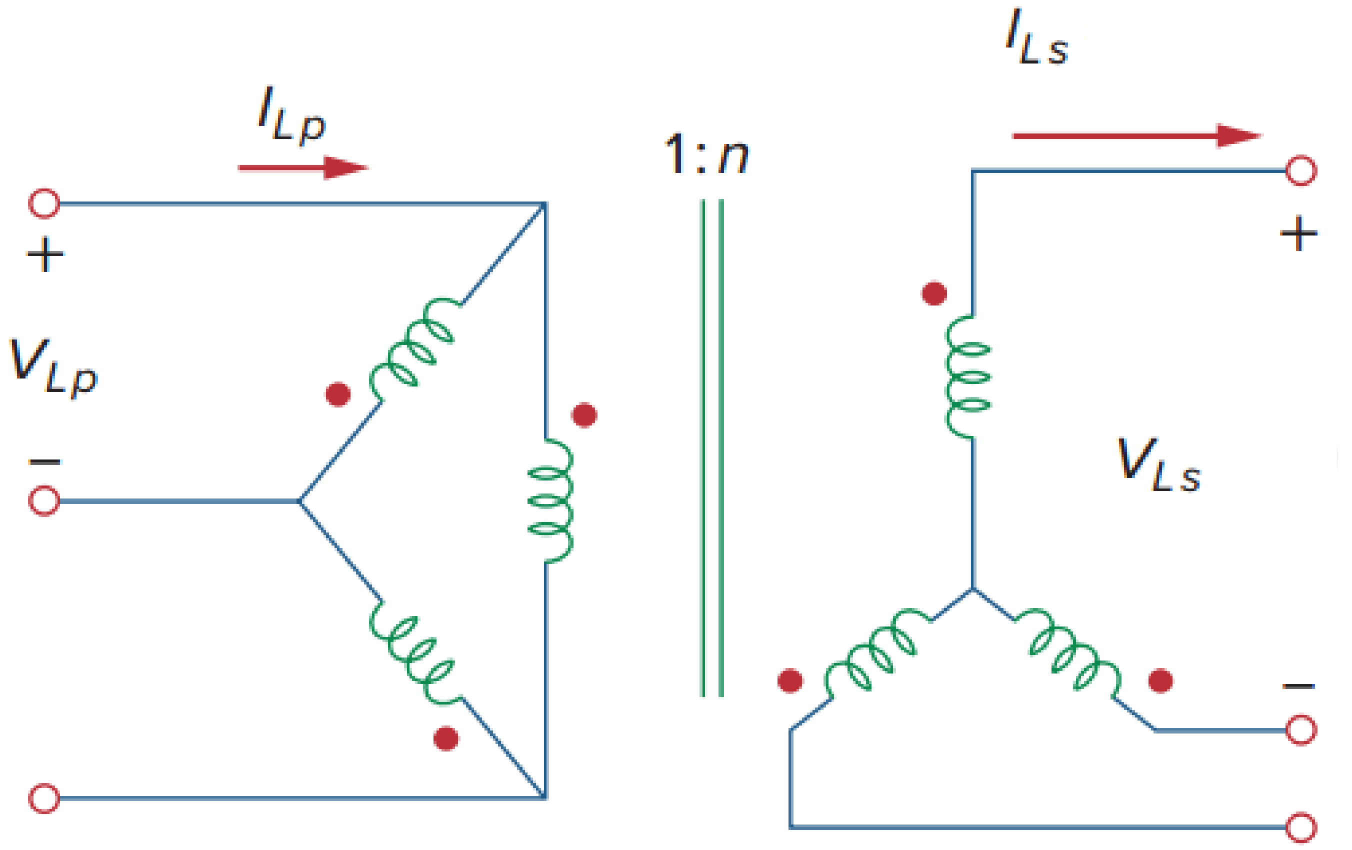
NUMERICAL PROBLEMS

(b) Delta-Star

V_{lp} = 2400 V

n = 50/500 = 0.1

$$\boxed{V_{ls} = \sqrt{3} n V_{lp} = 415.69 V}$$



NUMERICAL PROBLEMS

5. A current transformer has a single turn on the primary winding and a secondary winding of 60 turns. The secondary winding is connected to an ammeter with a resistance of 0.15 Ω. The resistance of the secondary winding is 0.25 Ω. If the current in the primary winding is 300 A, determine (a) the reading on the ammeter, (b) the potential difference across the ammeter and (c) the total load (in VA) on the secondary.

Soln: (a) $n = 60 \Rightarrow \boxed{I_2 = I_1/n = 300/60 = 5\text{ A}}$ (current through ammeter)

(b) $\boxed{V_{amm} = I_2 R_{amm} = 5 \times 0.15 = 0.75V}$ (potential difference across ammeter)

(c) $R_T = R_{amm} + R_s = 0.15 + 0.25 = 0.4\Omega$ (total resistance across secondary)

=> Load on secondary: $\boxed{P_s = I_2^2 R_T = 5^2 \times 0.4 = 10VA}$

NUMERICAL PROBLEMS

6. A voltage transformer has 4000 primary turns and 20 secondary turns. If the voltmeter reading in the secondary shows 110 V, how much is the voltage at the primary winding?

Soln: The primary winding voltage is,

$$\boxed{V_1 = \frac{V_2}{n} = \frac{110}{20/4000} = 22kV}$$

NUMERICAL PROBLEMS

7. A 2400V/400V single-phase transformer takes a no-load current of 0.5A and has a core loss of 400W. Determine the values of the magnetizing and core loss components of the no-load current.

Soln: $P_{core\_loss} = V_1 I_o \cos\phi_o$

$$\Rightarrow \boxed{\cos\phi_o = \frac{P_{core\_loss}}{V_1 I_o} = \frac{400}{2400 \times 0.5} = 0.33}$$ (No-load power factor)

Core loss component of no-load current: $\boxed{I_{ol} = I_o \cos\phi_o = 0.5 \times 0.33 = 0.16A}$

Magnetizing component of no-load current: $\boxed{I_{om} = I_o \sin\phi_o = 0.47A}$

Numerical Problems

8. A transformer takes a current of 0.8A when its primary is connected to a 240V, 50Hz supply with the secondary being open circuit. If the power absorbed is 72 W, determine (a) the iron loss current, (b) the power factor on no-load, and (c) the magnetizing current.

Soln: No-load current I_o = 0.8 A, Core loss = 72 W

$$P_{core\_loss} = V_1 I_o \cos\phi_o$$

$$\Rightarrow \boxed{\cos\phi_o = \frac{P_{core\_loss}}{V_1 I_o} = \frac{72}{240\times 0.8} = 0.375}$$ ((b) no-load power factor)

(a) Iron loss component of no-load current: $\boxed{I_{ol} = I_o \cos\phi_o = 0.8\times 0.375 = 0.3A}$

(c) Magnetizing current: $\boxed{I_{om} = I_o \sin\phi_o = 0.74A}$

Numerical Problems

9. A single-phase transformer has 480 turns on the primary and 90 turns on the secondary. The mean length of the flux path in the core is 1.8 m and the joints are equivalent to an airgap of 0.1 mm. The value of the magnetic field strength in the core is 400 A/m for a flux density of 1.1 T, and the corresponding core loss is 1.7 W/kg at 50 Hz. The density of the core material is 7800 kg/m^3. If the maximum value of flux density achieved is 1.1 T when a potential difference of 2200 V (rms) at 50 Hz is applied to the primary, calculate: (a) the cross-sectional area of the core, (b) the secondary voltage at no-load, and (c) the primary current and power factor at no-load.

NUMERICAL PROBLEMS

Soln:

(a) $N_1 = 480$, $E_1 = 2200$ V, $f = 50$ Hz, $B_m = 1.1$ T

Using EMF equation of transformer,

$$E_1 = 4.44 f \phi_m N_1 \Rightarrow \boxed{\phi_m = \frac{2200}{4.44 \times 50 \times 480} = 0.0206 Wb}$$ (peak flux in core)

=> The cross sectional area of core is,

$$\boxed{A = \frac{\phi_m}{B_m} = \frac{0.0206}{1.1} = 0.0187 m^2}$$

(b) $N_2 = 90$, $N_1 = 480$, $V_1 = 2200$ V

=> No-load secondary voltage is $\boxed{V_2 = nV_1 = (90/480)2200 = 412.5V}$

(c) $B_m = 1.1$ T, $H_{m,C} = 400$ A/m (in core), $l_\mathrm{C} = 1.8$ m (length of core), $l_\mathrm{A} = 10^{-4}$ m (length of air gap)

mmf drop for the core = $H_{m,C} \times l_\mathrm{C}$ => $\boxed{mmf_C = 400 \times 1.8 = 720A}$

mmf drop for the airgap = $H_{m,A} \times l_\mathrm{A} = (B_m/\mu_o) \times l_\mathrm{A}$

$$=> \boxed{mmf_A = \frac{1.1}{4\pi \times 10^{-7}} \times 0.0001 = 87.53A}$$

Therefore total *mmf* applied to produce the given flux is,

$$\boxed{mmf_T = mmf_C + mmf_A = 720 + 87.53 = 807.53A}$$

The applied $mmf_T = N_1 \times I_{om}$, where I_{om} is the magnetizing current

=> $I_{om} = mmf_T / N_1$

Therefore, peak value of magnetizing current is $\boxed{I_{om\_pk} = \frac{807.53}{480} = 1.68A}$

=> RMS value of magnetizing current is $\boxed{I_{om} = \frac{I_{om\_pk}}{\sqrt{2}} = 1.19A}$

Volume of core: $Volume = 0.0187 \times 1.8 = 0.0336m^3$

Mass of core: $Mass = Volume \times Density = 0.0336 \times 7800 = 262.55Kg$

Core loss is 1.7 W/kg => Total core loss: $\boxed{P_{core} = 262.55 \times 1.7 = 446.3W}$

$P_{core} = V_1 I_{ol}$ (where V_1 and I_{ol} are rms quantities)

$$\Rightarrow \boxed{I_{ol} = \frac{P_{core}}{V_1} = \frac{446.3}{2200} = 0.203A}$$ (Core loss component of no-load current)

$\Rightarrow$ Total no-load primary current is $\boxed{I_o = \sqrt{I_{ol}^2 + I_{om}^2} = \sqrt{0.203^2 + 1.19^2} = 1.207A}$

=> No-load power factor is $\boxed{\cos\phi_o = \frac{I_{ol}}{I_o} = \frac{0.203}{1.207} = 0.168}$

UNSOLVED PRACTICE PROBLEMS

1. A single-phase, 240V/2880V ideal transformer is supplied from a 240V source through a cable of resistance 3Ω. If the load across the secondary winding is 720Ω determine (a) the primary current flowing and (b) the power dissipated in the load resistance.

2. An AC source of 20V and internal resistance 20kΩ is matched to a load by a 16:1 single-phase transformer. Determine (a) the value of the load resistance and (b) the power dissipated in the load.

UNSOLVED PRACTICE PROBLEMS

3. A single-phase auto transformer has a voltage ratio of 480V:300V and supplies a load of 30kVA at 300V. Assuming an ideal transformer, calculate the current in each section of the winding.

4. A three-phase transformer has 600 primary turns and 150 secondary turns. If the supply voltage is 1.5 kV determine the secondary line voltage on no-load when the windings are connected (a) star-star, (b) star-delta (c) delta-star (d) delta-delta.

UNSOLVED PRACTICE PROBLEMS

5. A current transformer has two turns on the primary winding and a secondary winding of 260 turns. The secondary winding is connected to an ammeter with a resistance of 0.2Ω. The resistance of the secondary winding is 0.3Ω. If the current in the primary winding is 650A, determine (a) the reading on the ammeter, (b) the potential difference across the ammeter, and (c) the total load in VA on the secondary.

6. A 3300V/440V, single-phase transformer takes a no-load current of 0.8A and the iron loss is 500 W. Draw the no-load phasor diagram and determine the values of the magnetizing and core loss components of the no-load current.

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