INTRODUCTION TO CONTROL SYSTEMS

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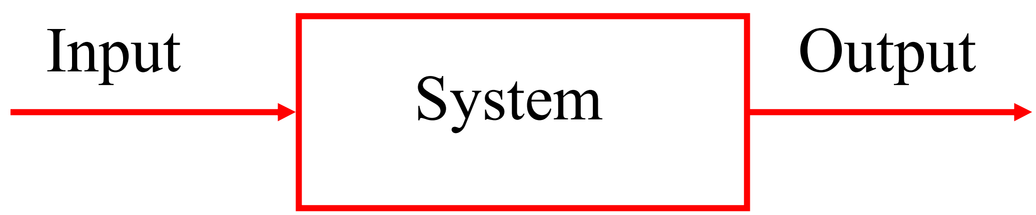
2. What is a control system.
3. Why control systems are important.
4. What are the basic components of a control system.
5. Some examples of control-system applications.
6. Types of control systems.
7. Transfer function
8. Closed loop Vs open loop
9. Block diagram reduction

What is a System

A system is a device or process that takes a given input and produces some output:

- A DC motor takes as input a voltage and produces as output rotary motion
- A chemical plant(like water bottle production) takes in raw chemicals and produces a required chemical product



What is a control system

A control system is considered to be any system which exists for the purpose of regulating or controlling the flow of energy, information, money, or other quantities in some desired fashion.

What is a control system

- An interconnection of components forming a system configuration that will provide a desired system response
- The study of control provides us with a process for analyzing and understanding the behavior of a system for some input
- It also introduces methods for achieving the desired system response

Why control systems are important

- The development and advancement of modern civilization and technology.
- Practically every aspect of our day-to-day activities is affected by some type of control system.
- Control systems are found in abundance in all sectors of industry, such as quality control of manufactured products, automatic assembly lines, machine-tool control, space technology and weapon systems, computer control, transportation systems, power systems, robotics, Micro-Electro-Mechanical Systems, nanotechnology, and many others.

Basic Components of a Control System

1. Objectives of control.
2. Control-system components.
3. Results or outputs.

Examples of Control-System Applications

Steering Control of an Automobile:

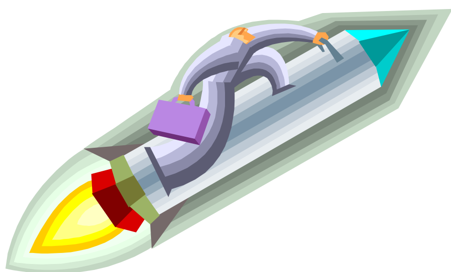
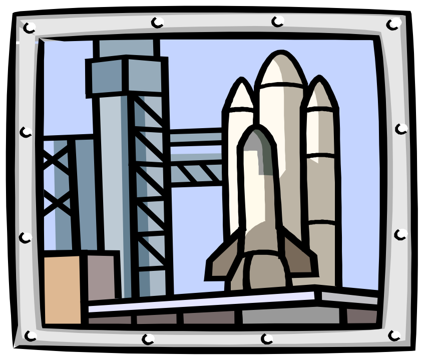
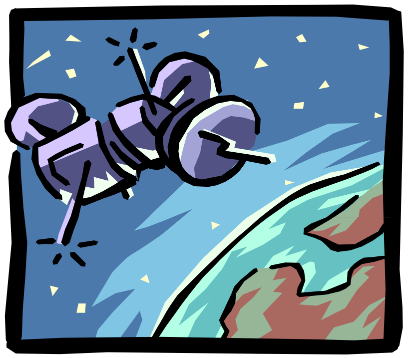
- As a simple example of the control system, consider the steering control of an automobile. The direction of the two front wheels can be regarded as the controlled variable, or the output, y; the direction of the steering wheel is the actuating signal, or the input, u.

- The control system, or process in this case, is composed of the steering mechanism and the dynamics of the entire automobile. However, if the objective is to control the speed of the automobile, then the amount of pressure exerted on the accelerator is the actuating signal, and the vehicle speed is the controlled variable.

Examples of Control-System Applications

Aerospace Applications:

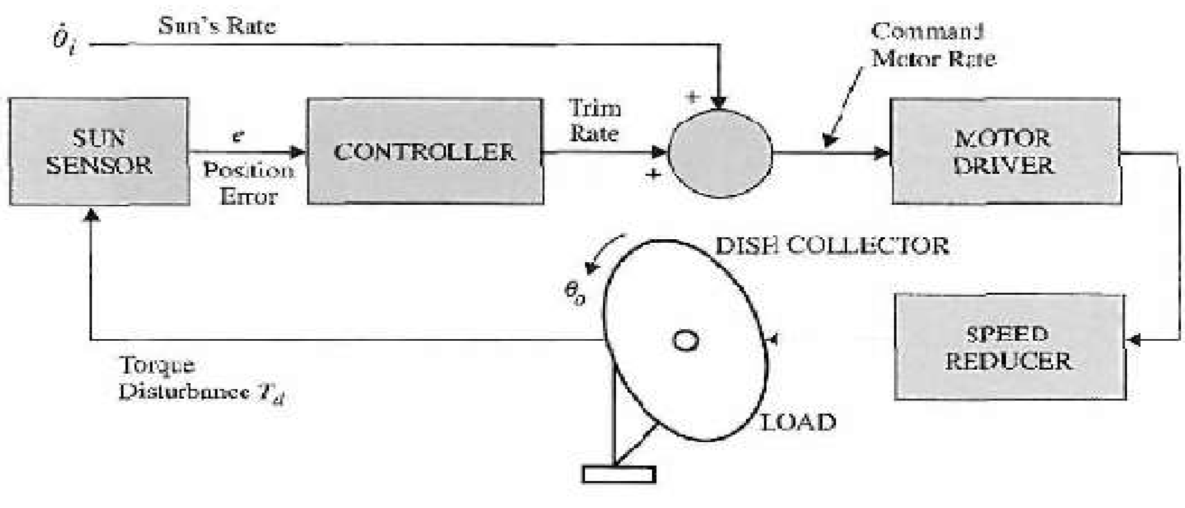
- Aircraft or missile guidance and control
- Space vehicles and structures

Examples of Control-System Applications

Sun-Tracking Control of Solar Array:

To achieve the goal of developing economically feasible non-fossil-fuel electrical power, development of solar power conversion methods, including the solar-cell conversion techniques



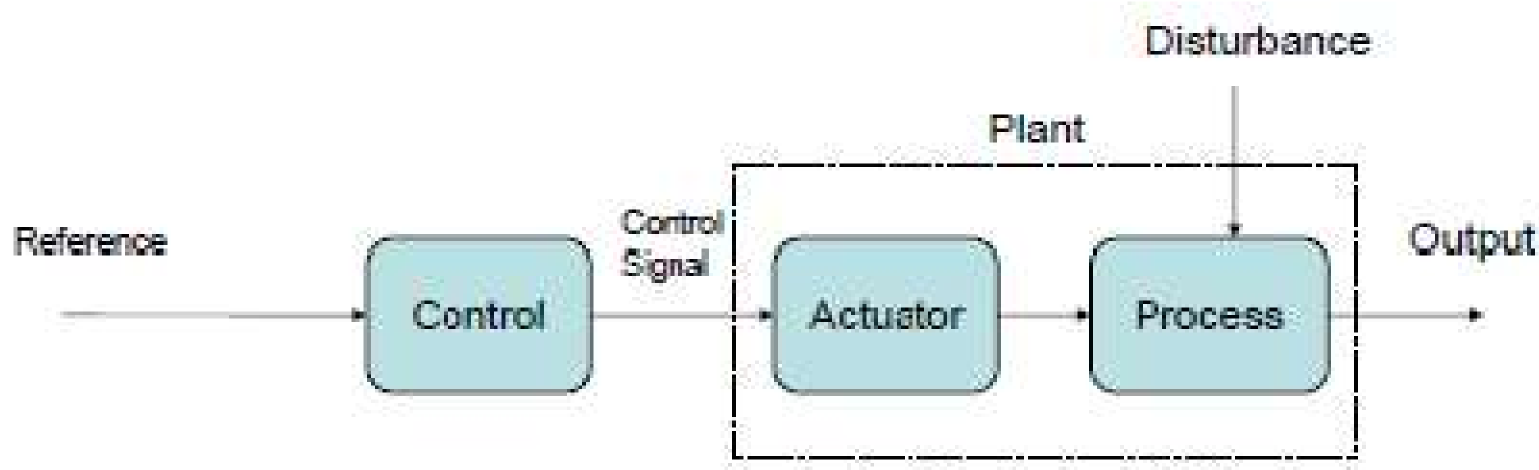
Types of control systems

Control Systems can be classified as :

1) open loop control system
2) closed loop control system

Open-Loop Control Systems (Non feedback Systems)

The elements of an open-loop control system can usually be divided into two parts: the controller and the controlled process, as shown by the block diagram



Open Loop Control System

- ✓A system in which the output has no effect on the control action is known as an open loop control system.
- ✓For a given input the system produces a certain output.
- ✓If there are any disturbances, the output changes and there is no adjustment of the input to bring back the output to the original value.
- ✓Example: ceiling fan

Closed-Loop Control Systems (Feedback Control Systems)

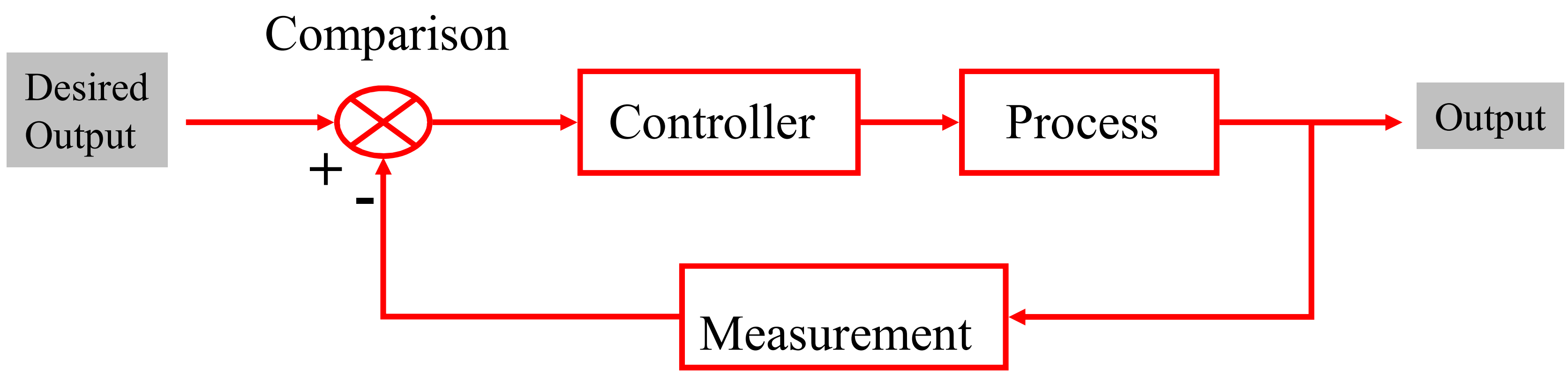
- What is missing in the open-loop control system is a link or feedback from the output to the input of the system.
- To obtain more accurate control, the controlled signal should be fed back and compared with the reference input.

➢ A process to be controlled

➢ A measurement of process output

➢ A comparison between desired and actual output

➢ A controller that generates inputs from comparison



Examples : Washing Machine

- System Requirements
  - Understanding of load sizes
  - Receptacle to hold clothes
  - 'Plumbing'
  - Ease of use, Reliability
  - Low Cost
- Actuators
  - AC or DC Motors
  - Water inlet/drain
- Sensors
  - Water level
  - Load speed/balance
- Control
  - Choice depends on design

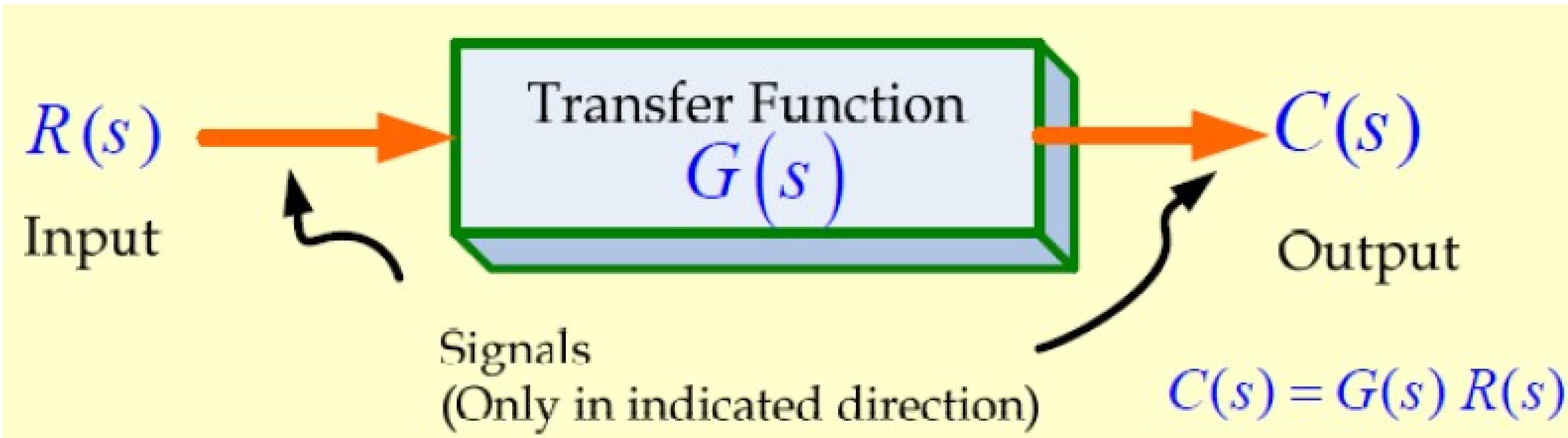
Transfer Function:

- Transfer Function *G(s)* describes system component
- An operator that transfers input to output
- Described as a Laplace transform

$$Y(s) = G(s)U(s)$$

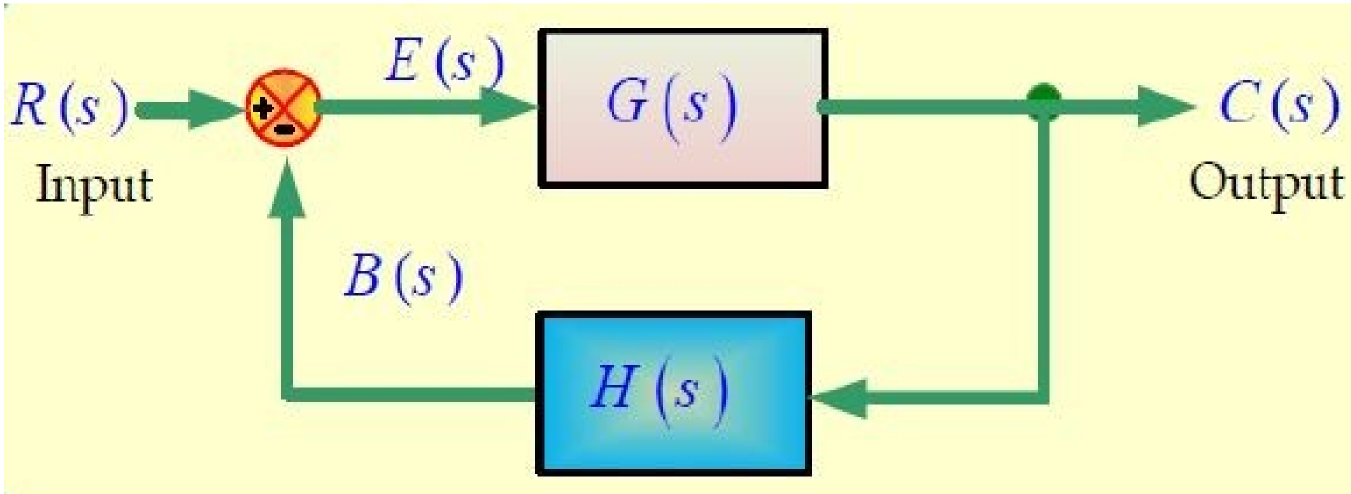
$$X(s) \longrightarrow \boxed{G(s)} \longrightarrow Y(s)$$

Transfer Function: open-loop



Transfer Function, G(s) = C(s)/R(s)

Transfer Function: closed-loop(-ve feed back)



Definitions

R(s) = Input

C(s) = Output

G(s) = Forward transfer function

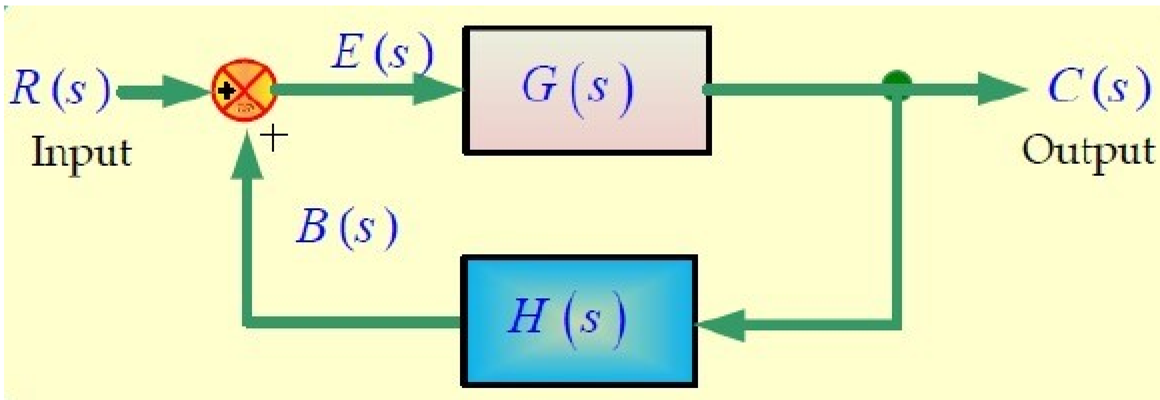
H(s) = Feedback transfer function

B(s) = Feedback

E(s) = Error

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

Transfer Function: closed-loop(+ve feed back)



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s)H(s)}$$

Open loop Vs Closed loop

Open loop

- Accuracy depends on input and process
- More sensitive
- Bandwidth decreased
- More reliability

Open loop Vs Closed loop

<u>Closed loop</u>

- Stability
- Accurate
- Less sensitive
- Less reliable
- Bandwidth increased
- Gain reduced (-ve feedback)

Open loop Vs Closed loop (advantages)

Open loop

1. They are simple and easy to build.
2. They are cheaper, as they use less number of components to build.
3. They are usually stable.
4. Maintenance is easy.

Closed loop

1. More accurate.
2. The effect of external disturbance signals can be made very small.
3. The variations in parameters of the system do not affect the output of the system i.e the output may be made less sensitive to variation in parameter. Hence forward path components can be of less precision. This reduces the cost of the system.
4. Speed of the response can be greatly increased.

Open loop Vs Closed loop (disadvantages)

Open loop

1. They are less accurate.
2. If external disturbances are present, output differs significantly from the desired value.
3. If there are variations in the parameters of the system, the output changes.

Closed loop

1. They are more complex and expensive.
2. They require high forward path gains.
3. The systems are prone to instability (oscillations in the output may occur).
4. Cost of maintenance is high.

Open loop Vs Closed loop

- ✓ For +ve feedback, the phase shift between input and feedback signal is 0^o or $\pm 360^o$
- ✓ For –ve feedback, $\pm 180^o$ i.e out of phase
- ✓ Product of G(s)H(s) is called open loop gain
- ✓ Product of G(s)H(s) represents the actual system which is called loop gain (open loop)
- ✓ The stability of the closed loop system depends on the loop gain. If loop gain = -1, the stability affected. If loop gain > 0 then the closed loop system is more stable than open loop.

Open loop Vs Closed loop

- ✓ With feedback, the bandwidth is increased by the factor of 1+G(s)H(s)

- ✓ Bandwidth α 1/rise time

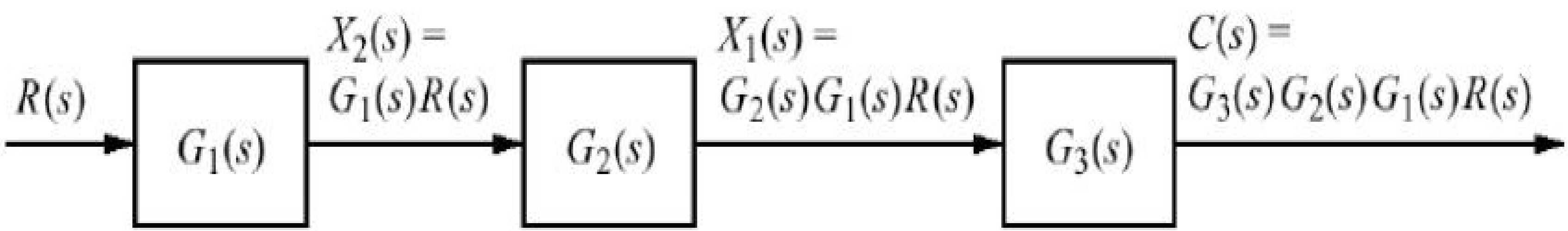
- ✓ Sensitivity α 1/stability

Effects of feedback

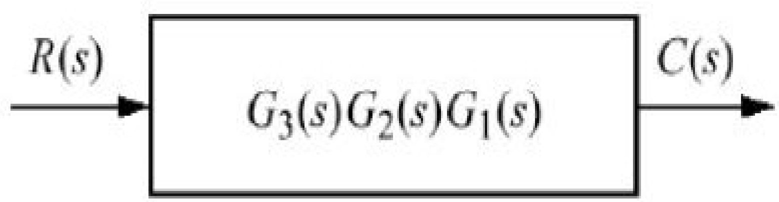
1) Effect on system dynamics: Dynamics can be changed without altering the system parameters.
2) Effect due to parameter variations: Sensitivity decreases.
3) Effect on bandwidth: Bandwidth increases (speed of response is larger)
4) Effect on noise signals: Noise signals can be effectively reduced.

Block diagram reduction/simplification:

1. Combining blocks in cascade (series)



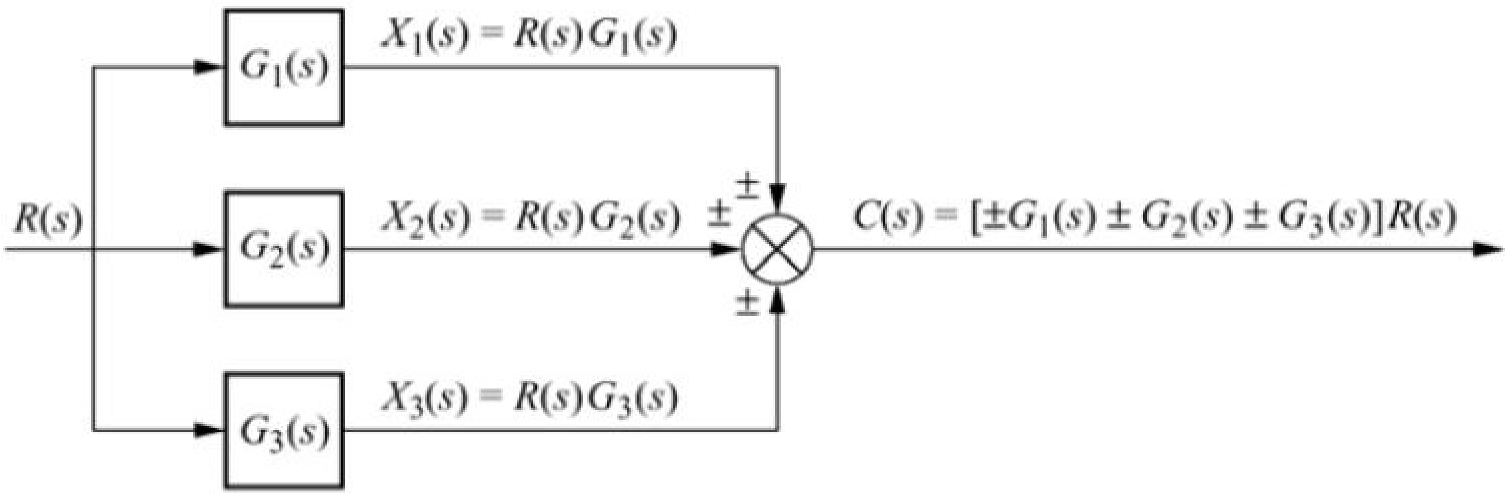
(*a*)



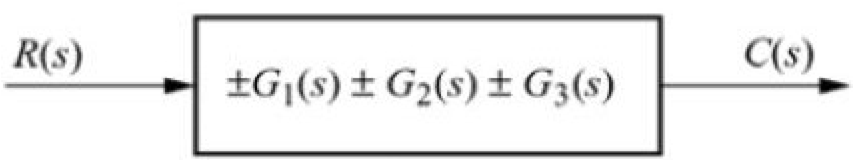
(*b*)

Block diagram reduction/simplification:

1. Combining blocks in cascade (parallel)



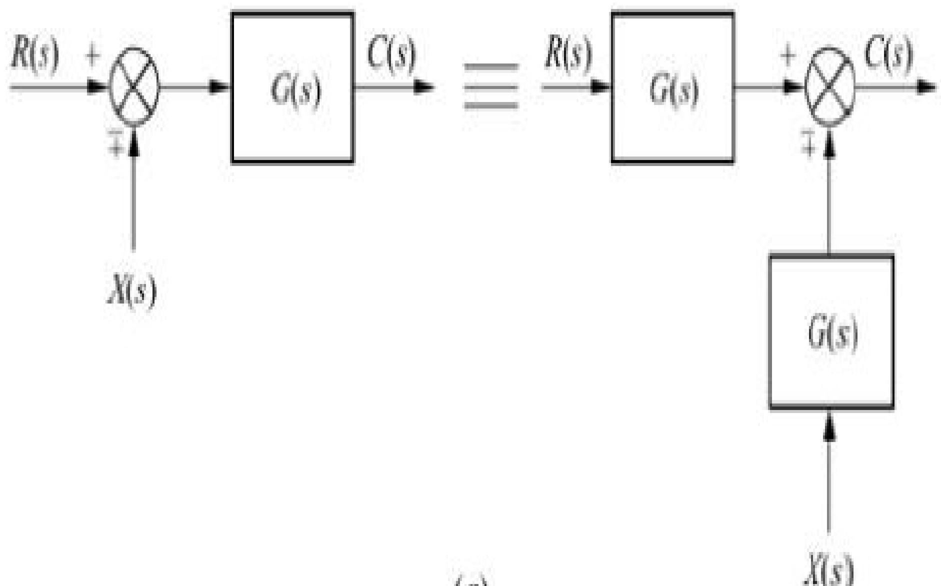
(*a*)



(*b*)

Block diagram reduction/simplification:

2. Moving a summing point (after a block)

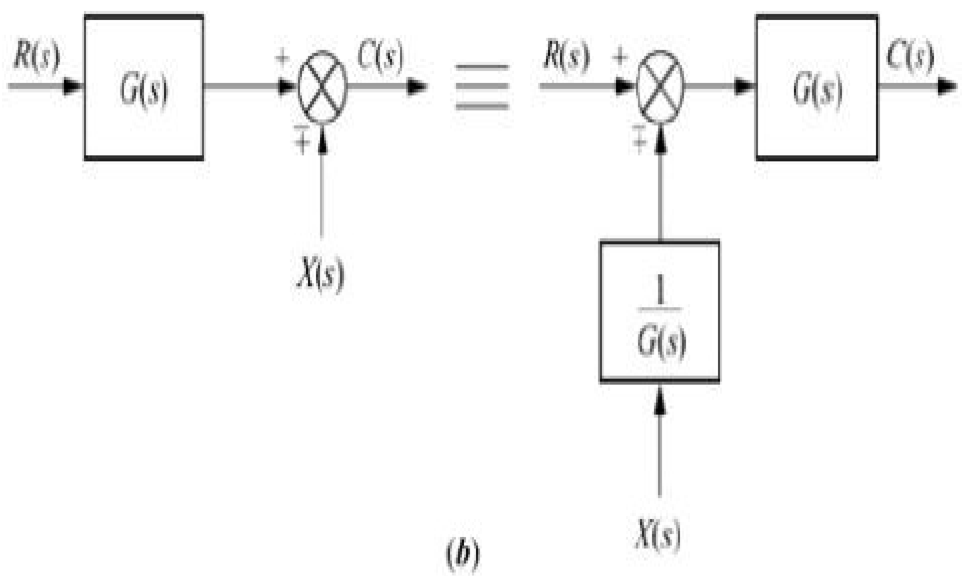


(a)

$$C = G(+R \pm X)$$
$$= +GR \pm GX$$

Block diagram reduction/simplification:

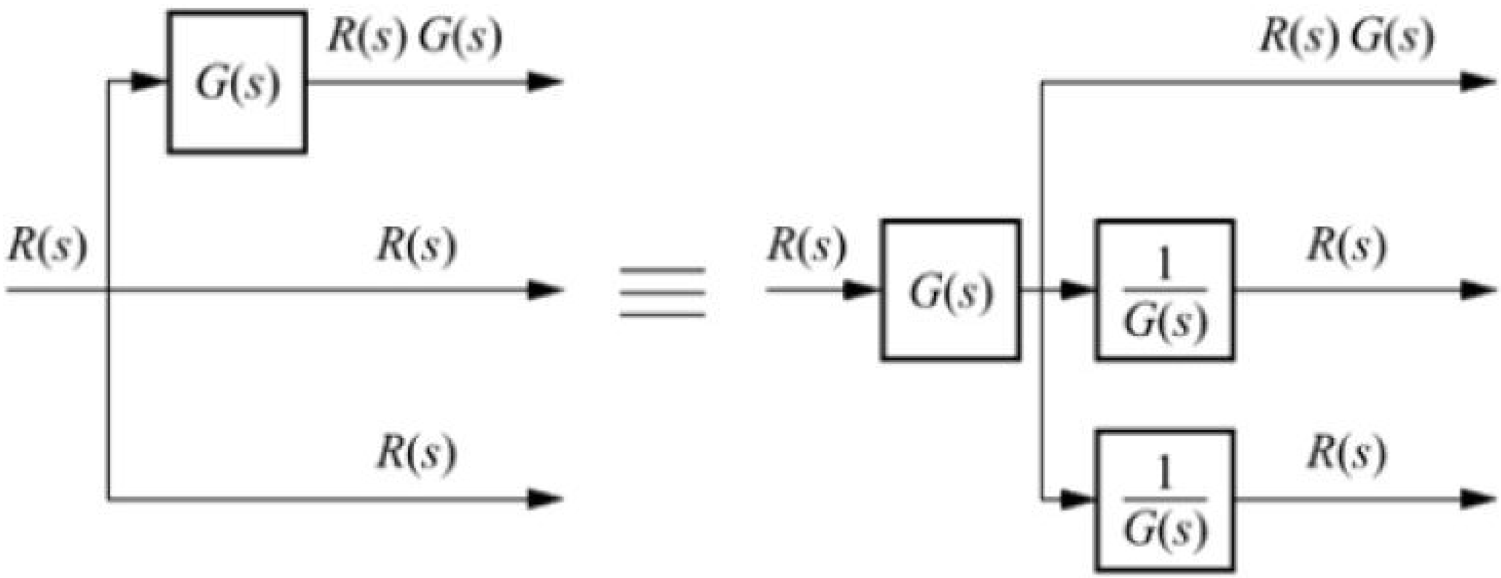
2. Moving a summing point (ahead of a block)



$$C = GR \pm X$$
$$= G(+R \pm X/G)$$

Block diagram reduction/simplification:

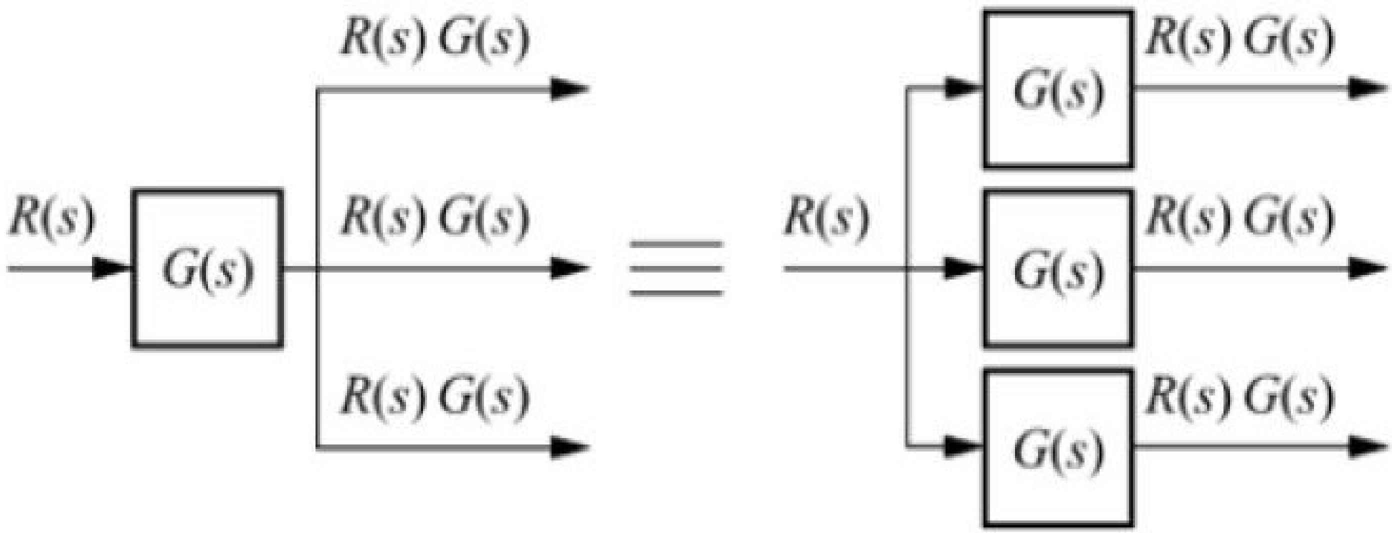
3. Moving a take off (branch) point (after a block)



(a)

Block diagram reduction/simplification:

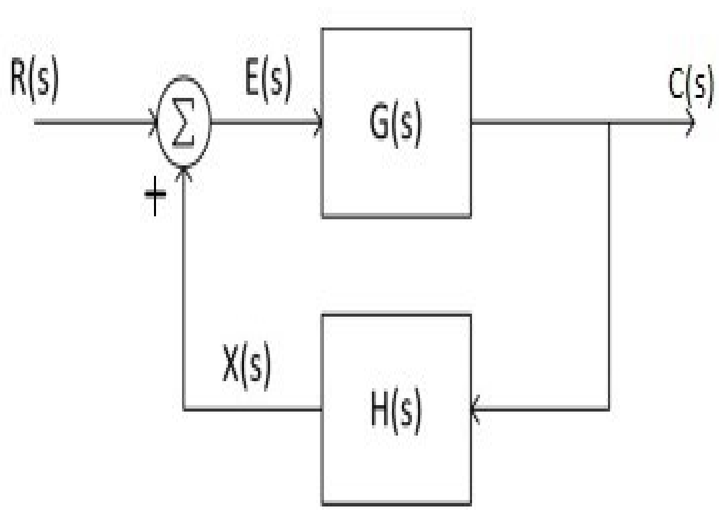
3. Moving a take off (branch) point (ahead of a block)



(b)

Block diagram reduction/simplification:

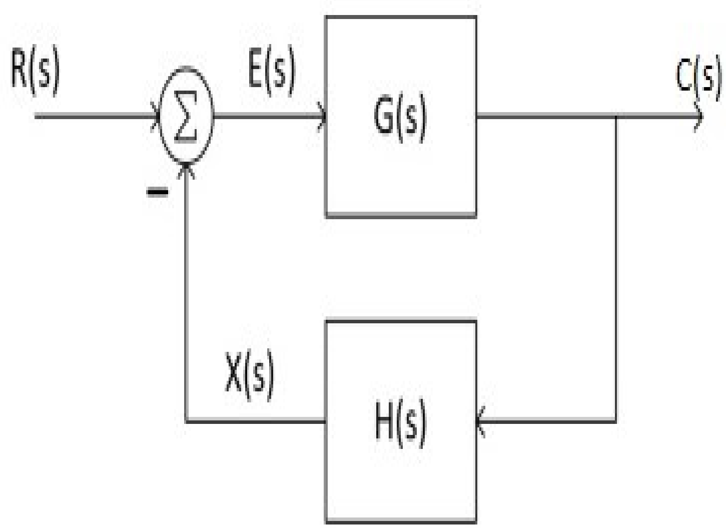
4. Eliminating a feedback loop (+ve feedback)



$$T(s) = \frac{G(s)}{1 - G(s)H(s)}$$

Block diagram reduction/simplification:

4. Eliminating a feedback loop (-ve feedback)



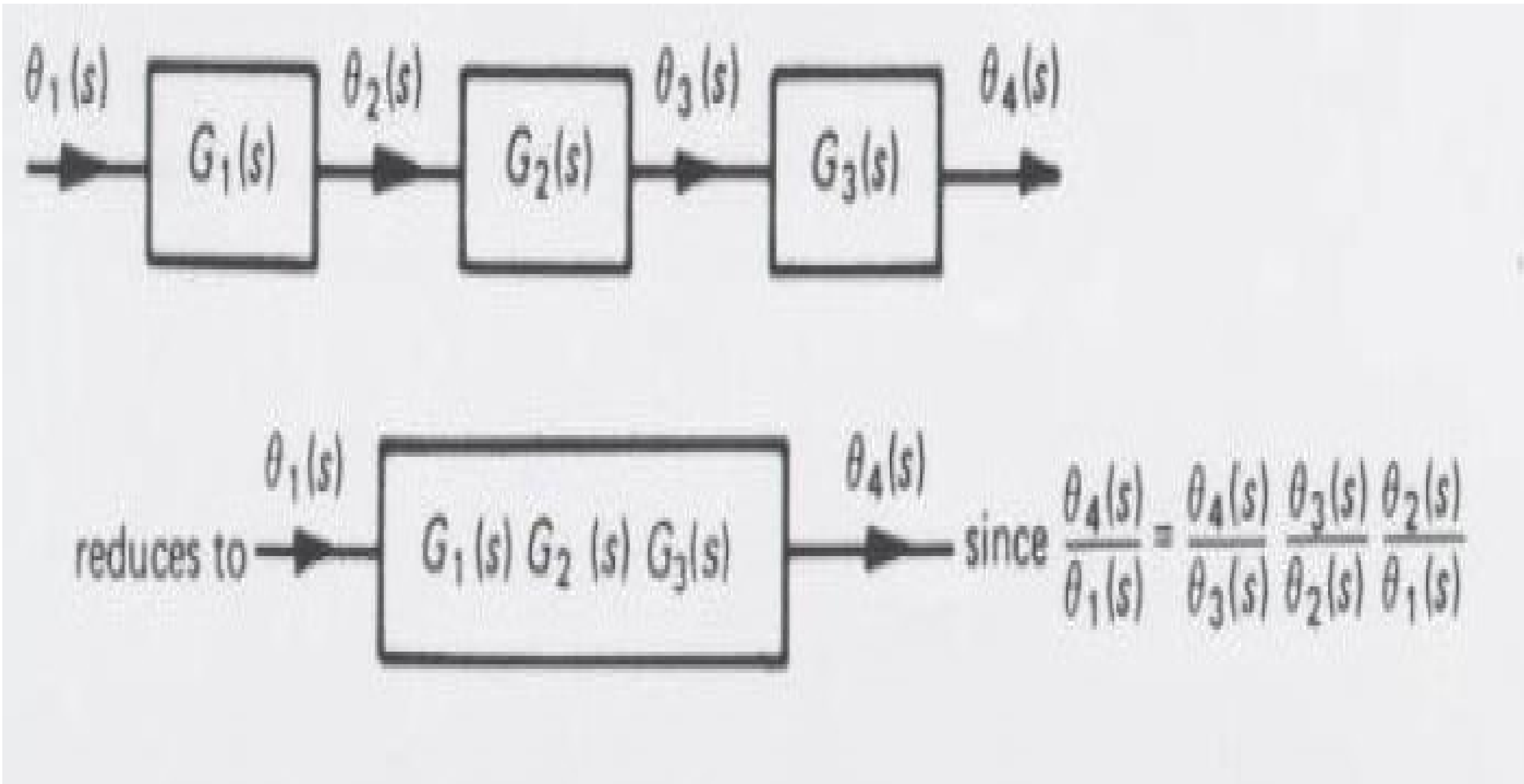
$$T(s) = \frac{G(s)}{1 + G(s)H(s)}$$

Block diagram reduction rules:

1) Check for the blocks connected in series and simplify.
2) Check for the blocks connected in parallel and simplify.
3) Check for the blocks connected in feedback and simplify.
4) If there is difficulty with take-off point while simplifying, shift it towards right.
5) If there is difficulty with summing point while simplifying, shift it towards left.
6) Repeat the above steps till you get the simplified form, i.e. single block.

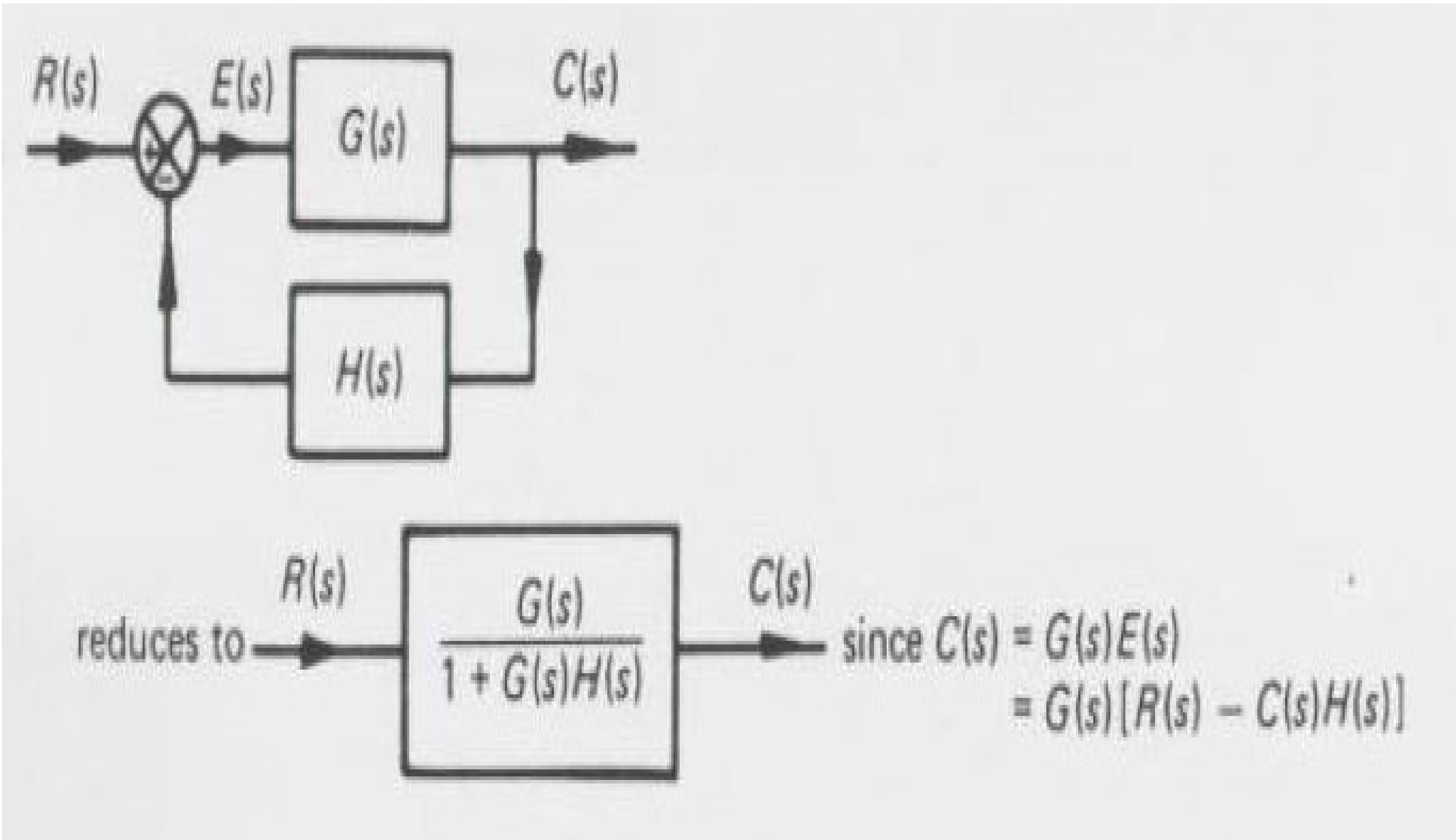
Note: The transfer function present in this single block is the transfer function of the overall block diagram.

Example-1:

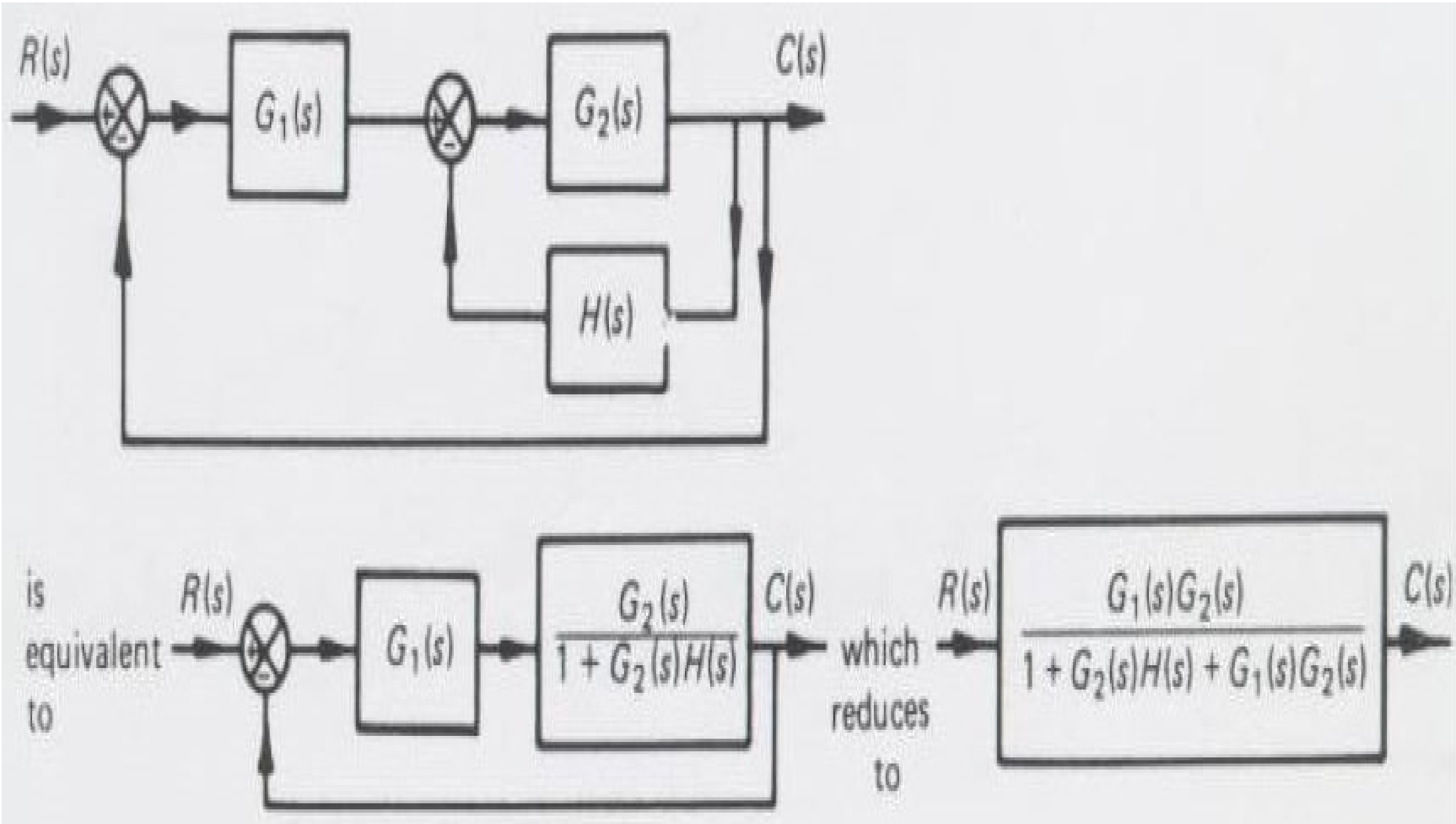


$\theta_1(s)$
$G_1(s)$
$\theta_2(s)$
$G_2(s)$
$\theta_3(s)$
$G_3(s)$
$\theta_4(s)$
reduces to
$\theta_1(s)$
$G_1(s)\,G_2(s)\,G_3(s)$
$\theta_4(s)$
since $\frac{\theta_4(s)}{\theta_1(s)} = \frac{\theta_4(s)}{\theta_3(s)}\frac{\theta_3(s)}{\theta_2(s)}\frac{\theta_2(s)}{\theta_1(s)}$

Example-2:

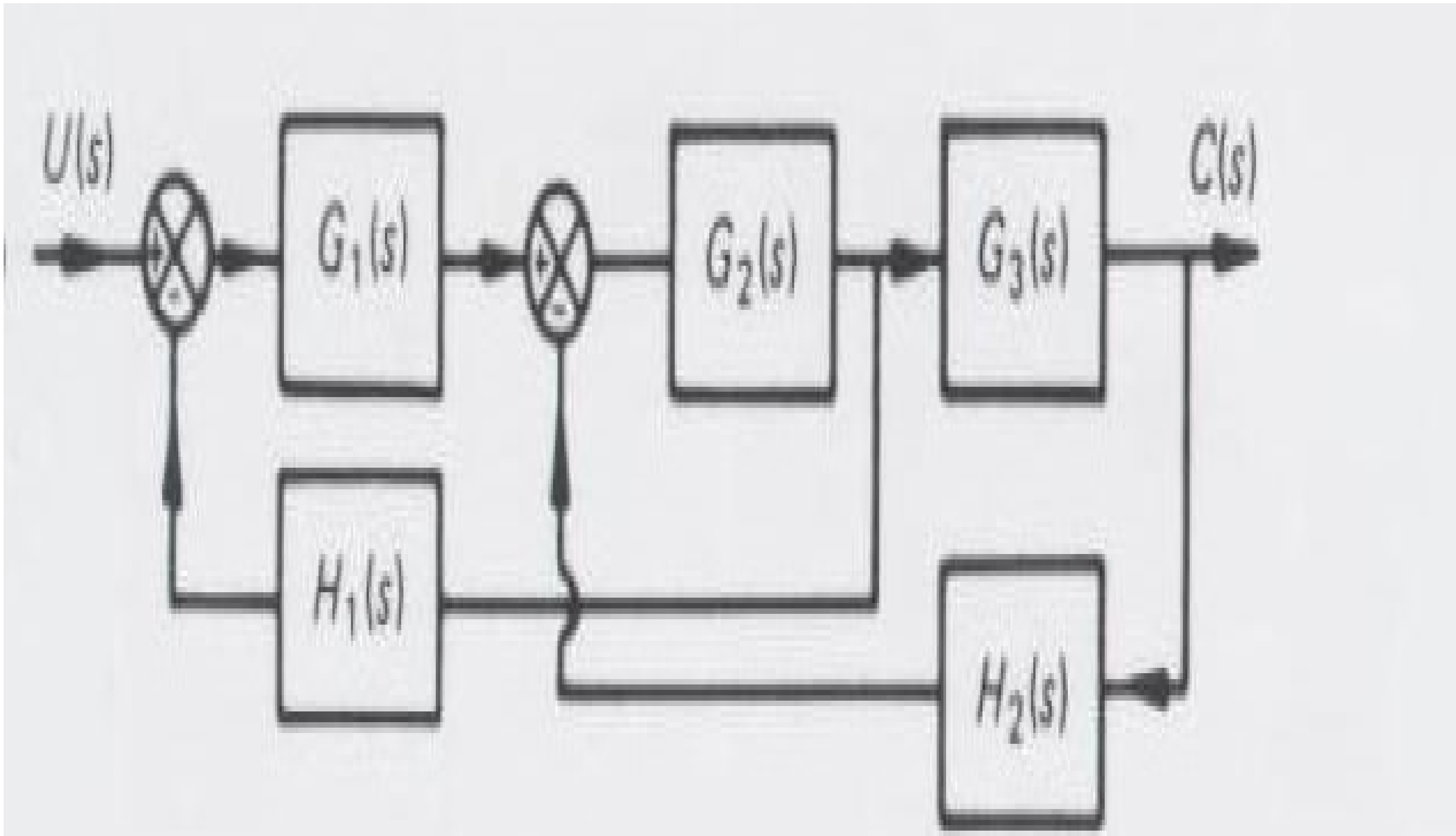


Example-3:

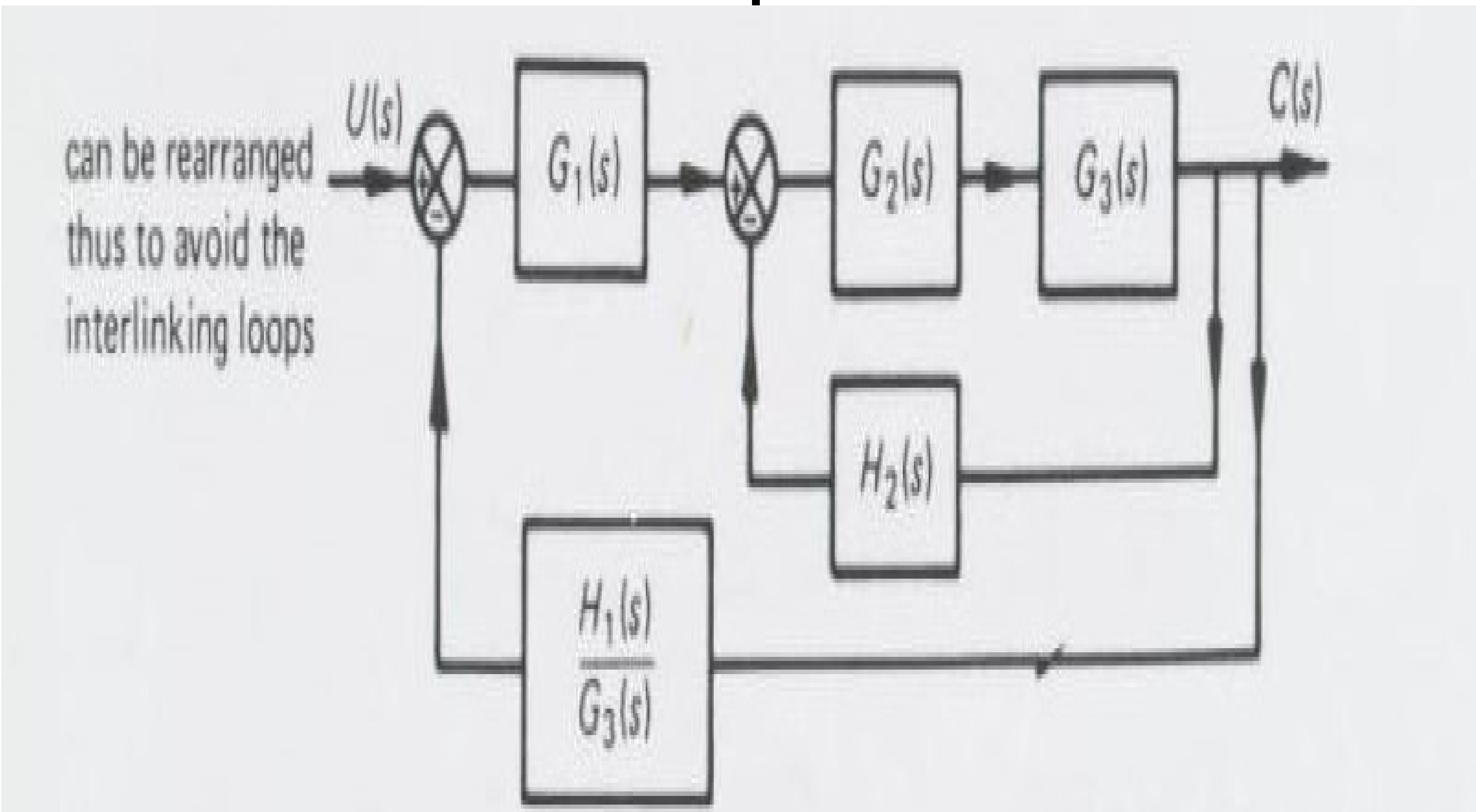


R(s)
G1(s)
G2(s)
C(s)
H(s)
is equivalent to
R(s)
G1(s)
G2(s) / 1 + G2(s)H(s)
C(s)
which reduces to
R(s)
G1(s)G2(s) / 1 + G2(s)H(s) + G1(s)G2(s)
C(s)

Example-4:

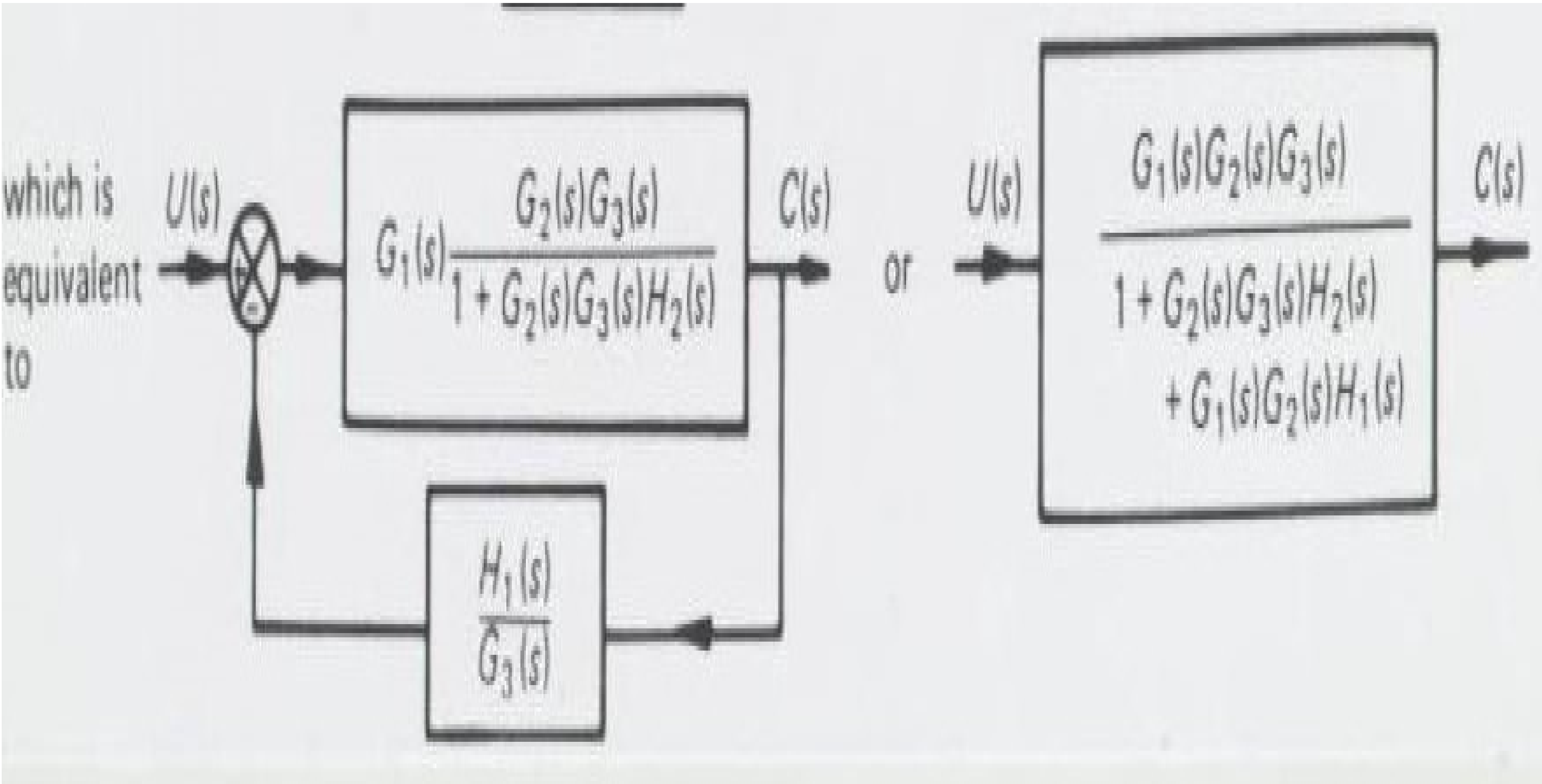


Example-4:



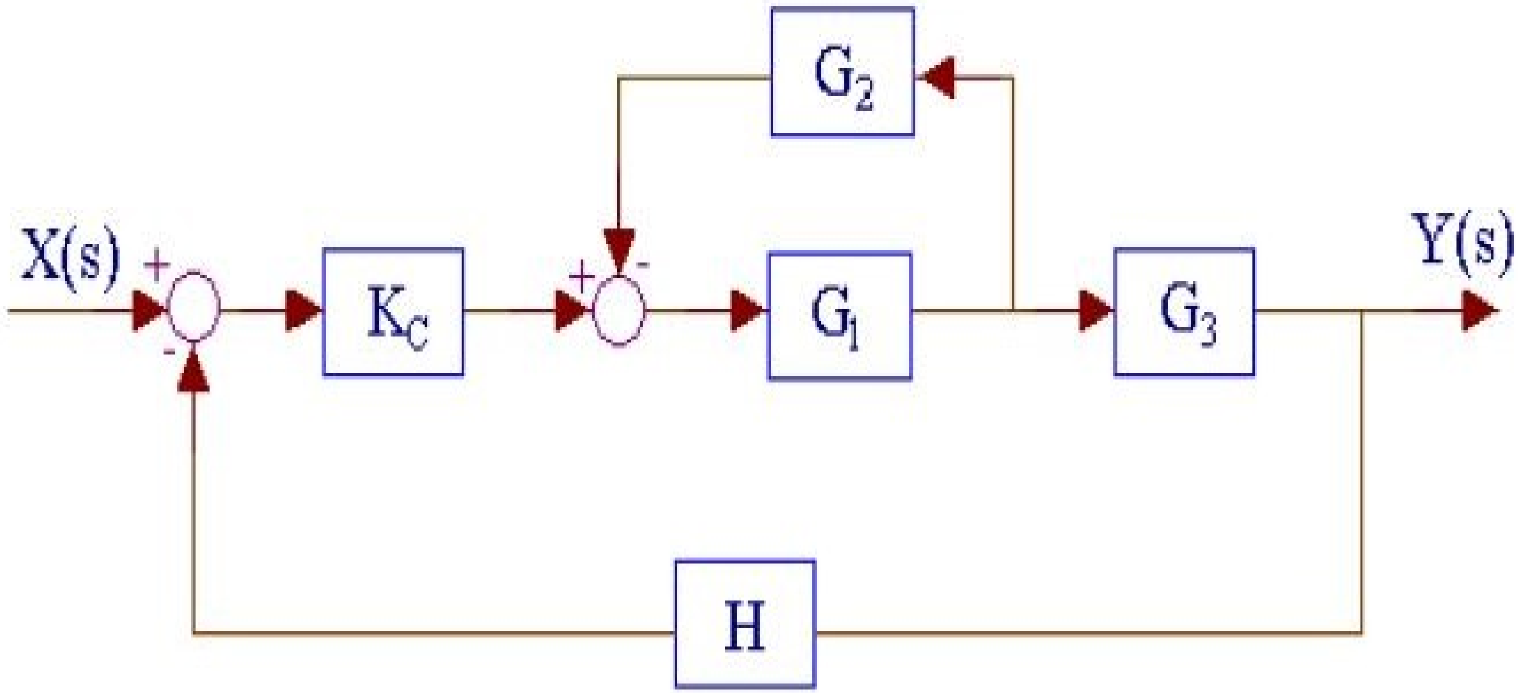
can be rearranged thus to avoid the interlinking loops
U(s)
G1(s)
G2(s)
G3(s)
C(s)
H2(s)
H1(s)
G3(s)

Example-4:

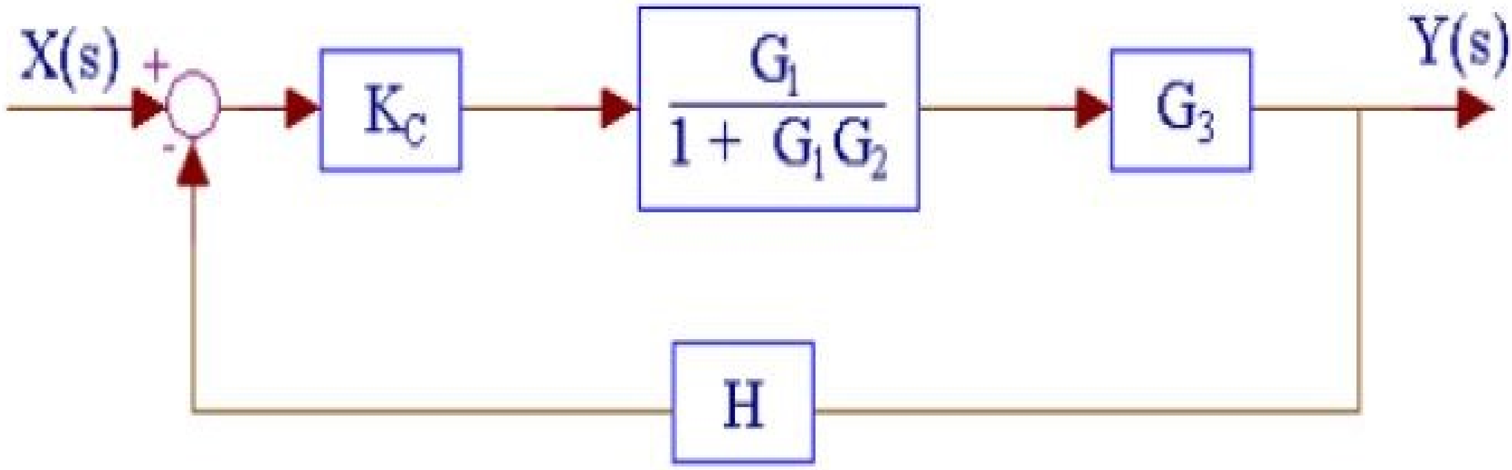


which is equivalent to
U(s)
G1(s) G2(s)G3(s) / 1 + G2(s)G3(s)H2(s)
C(s)
H1(s) / G3(s)
or
U(s)
G1(s)G2(s)G3(s) / 1 + G2(s)G3(s)H2(s) + G1(s)G2(s)H1(s)
C(s)

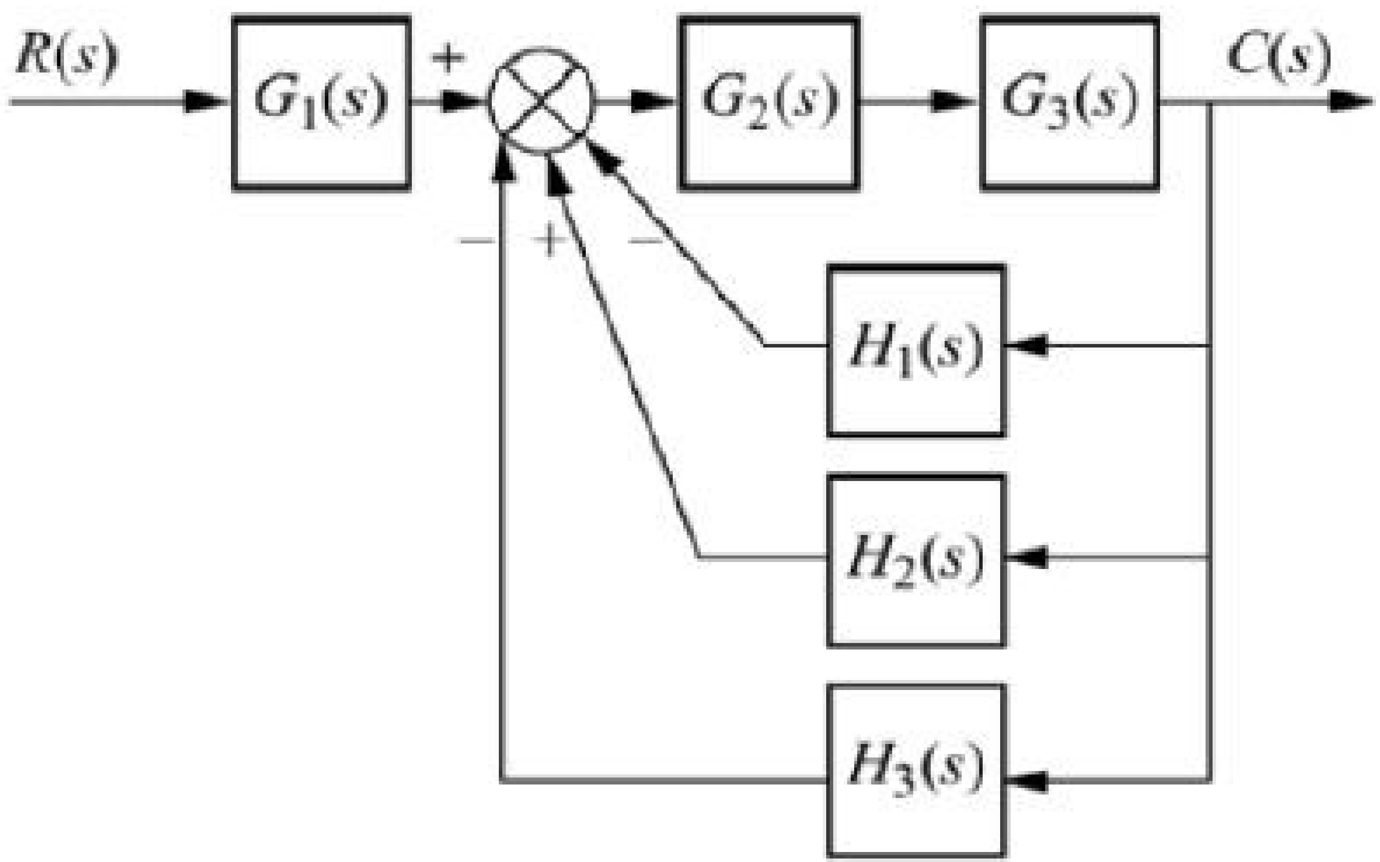
Example-5:



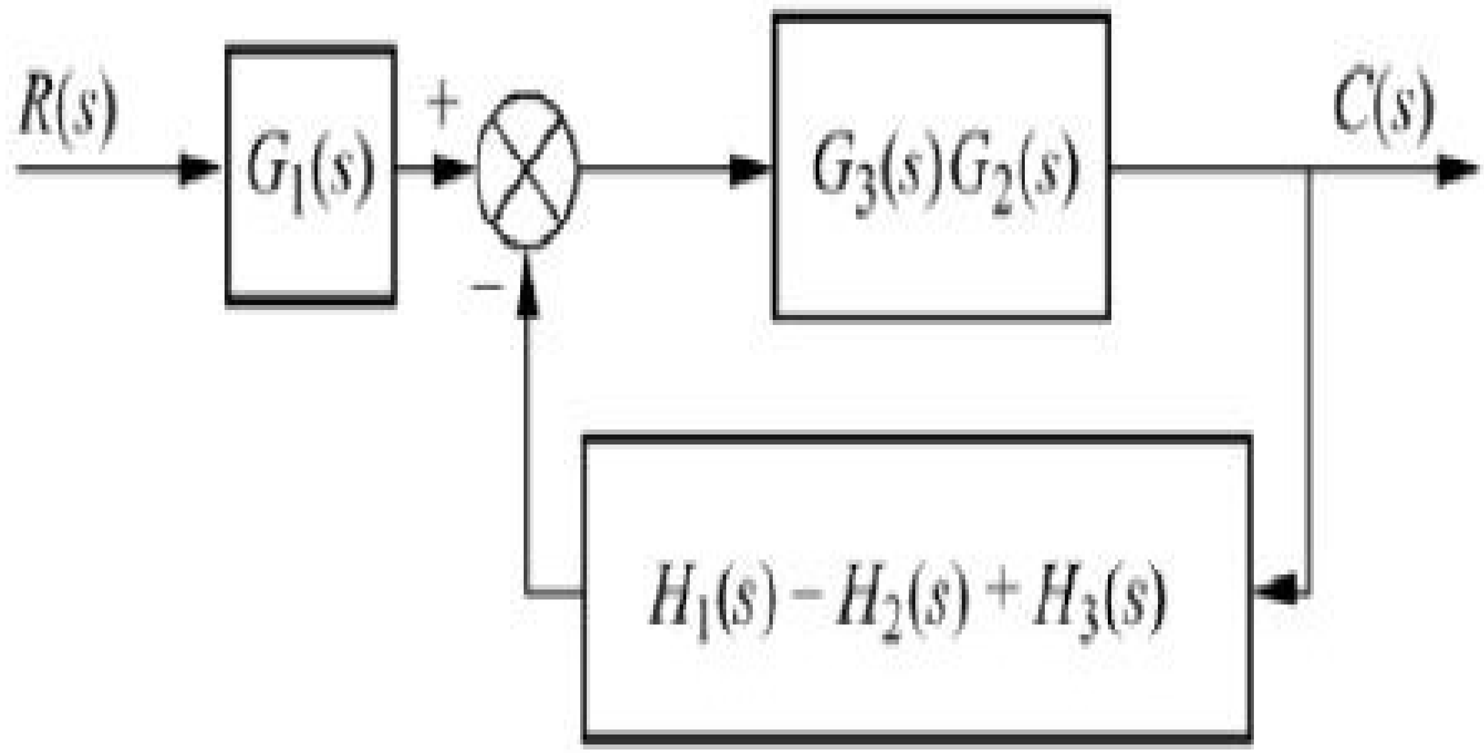
Example-5:



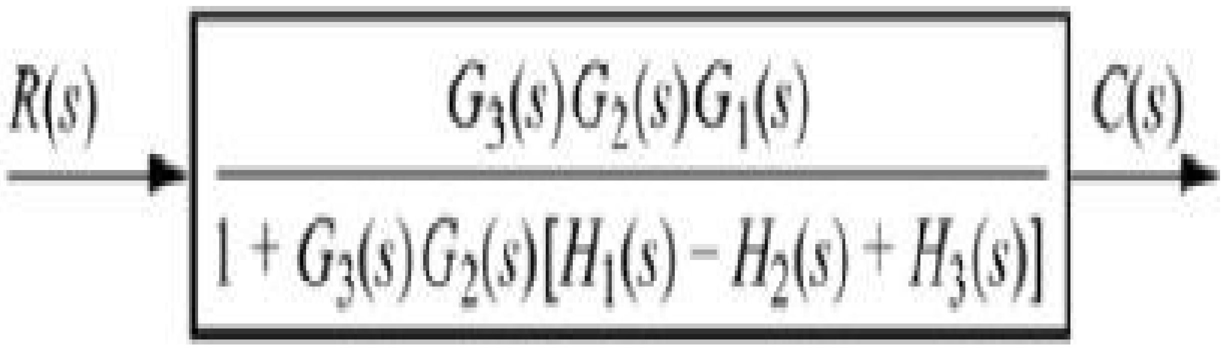
Example-6:



Example-6:

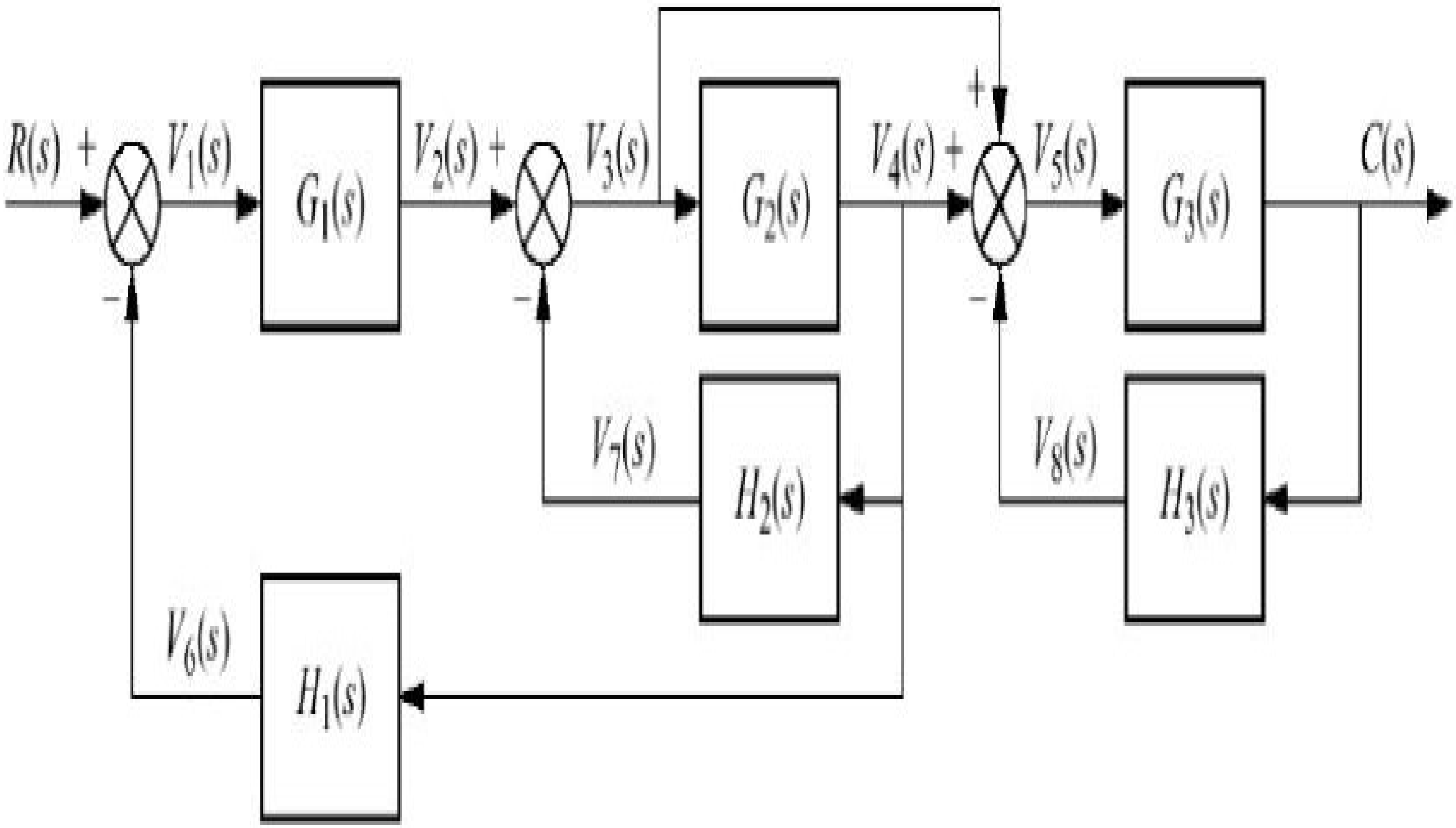


Example-6:

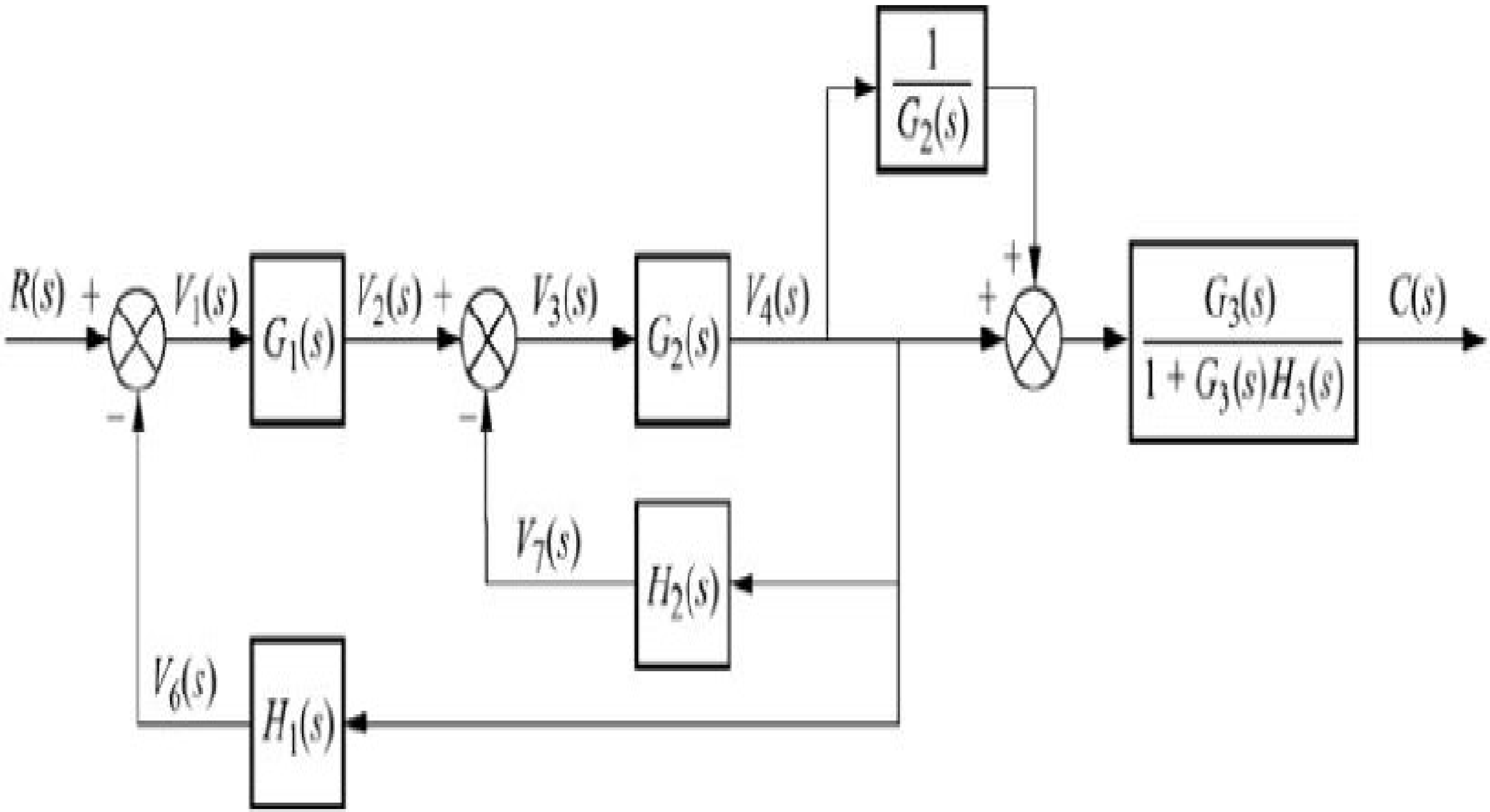


$$\frac{C(s)}{R(s)} = G_1\left[\frac{G_3G_2}{1+G_3G_2\left(H_1 - H_2 + H_3\right)}\right]$$

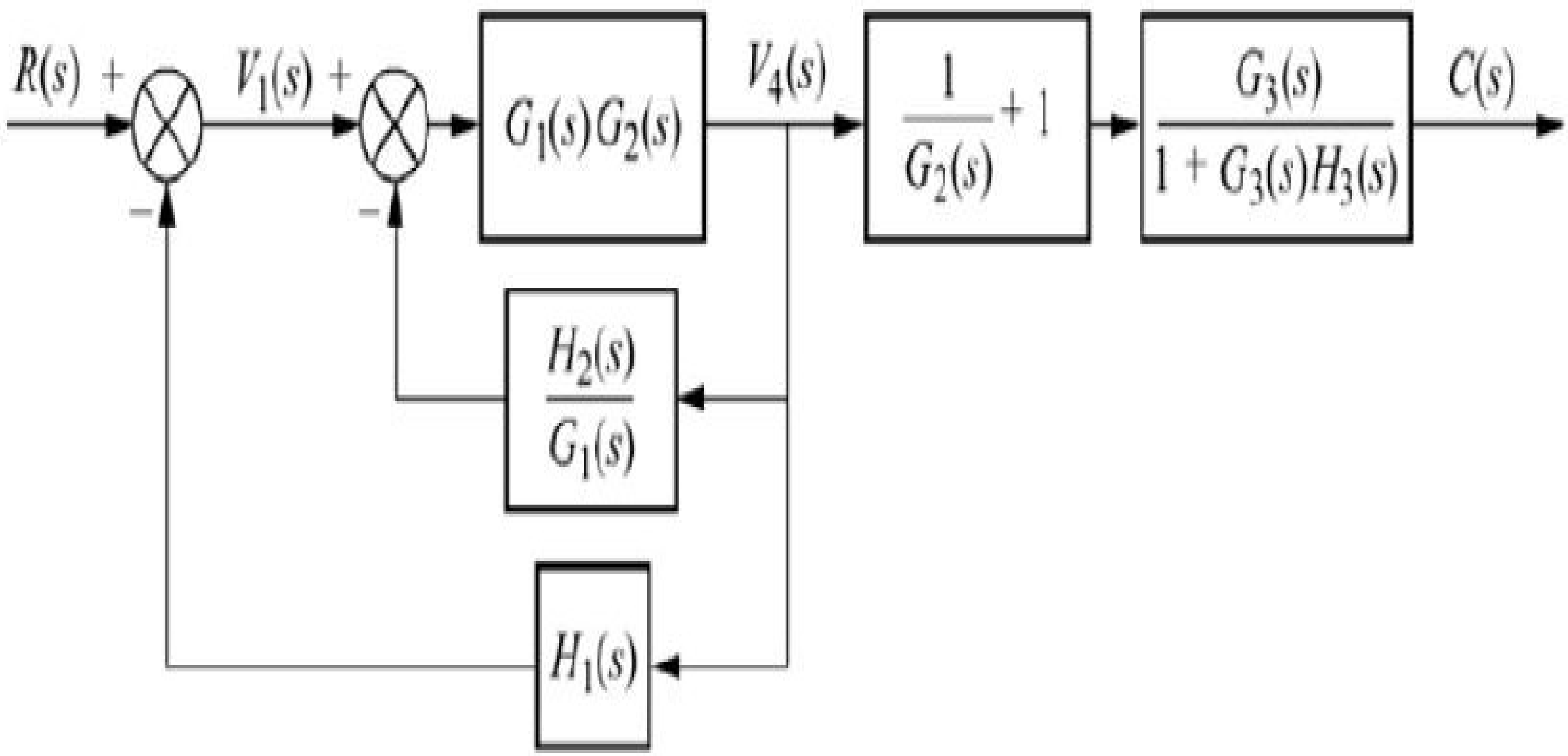
Example-7:



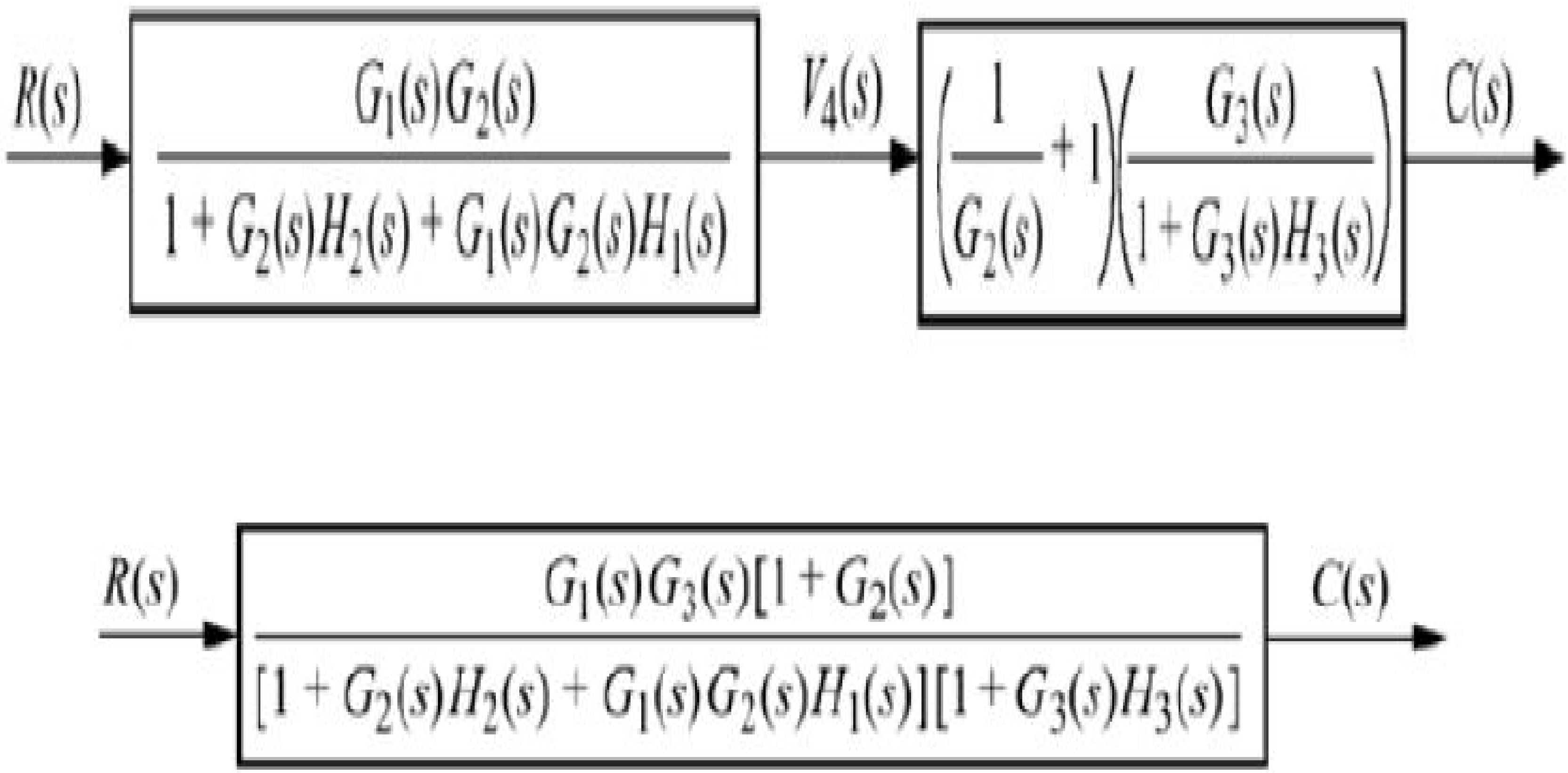
Example-7:



Example-7:

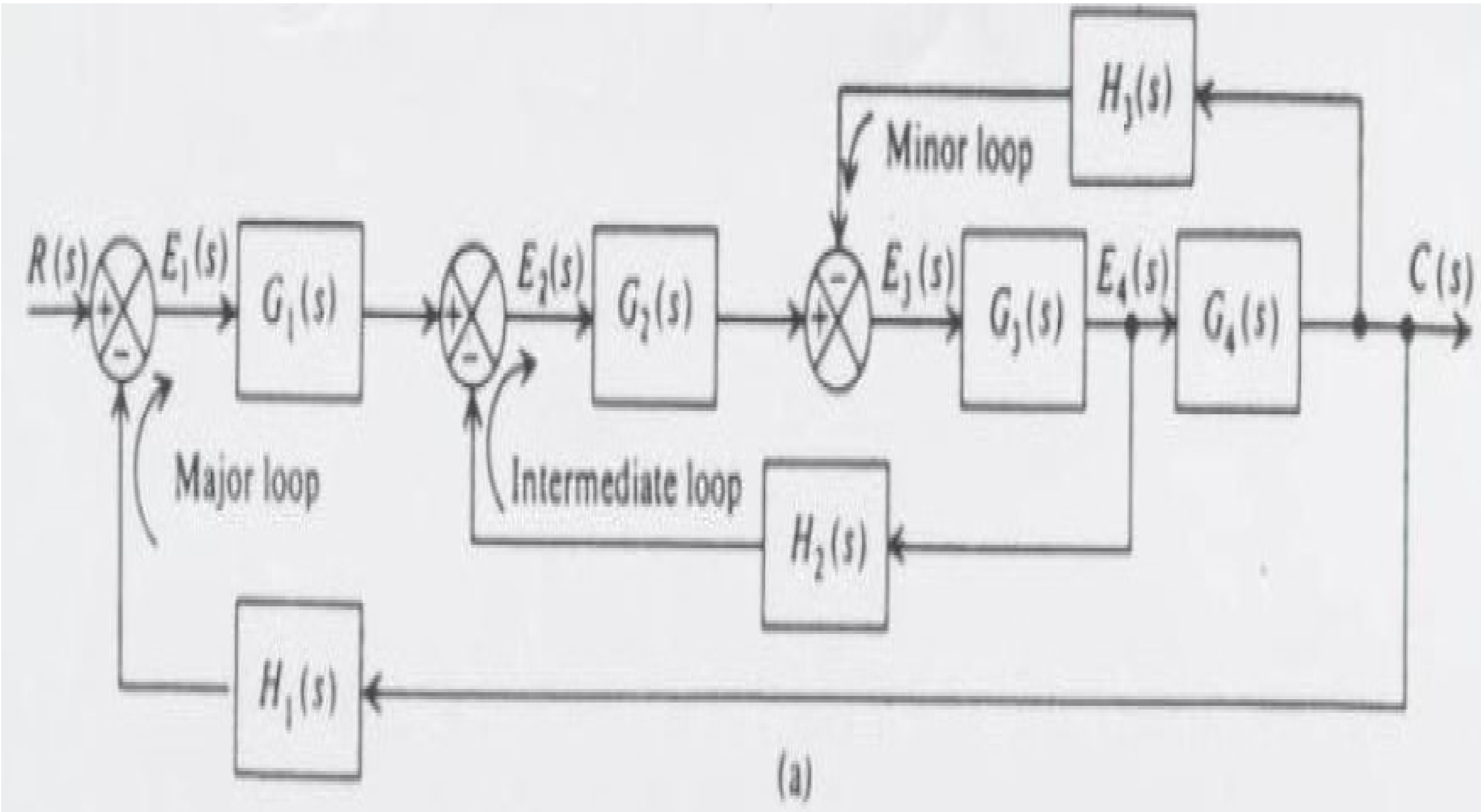


Example-7:



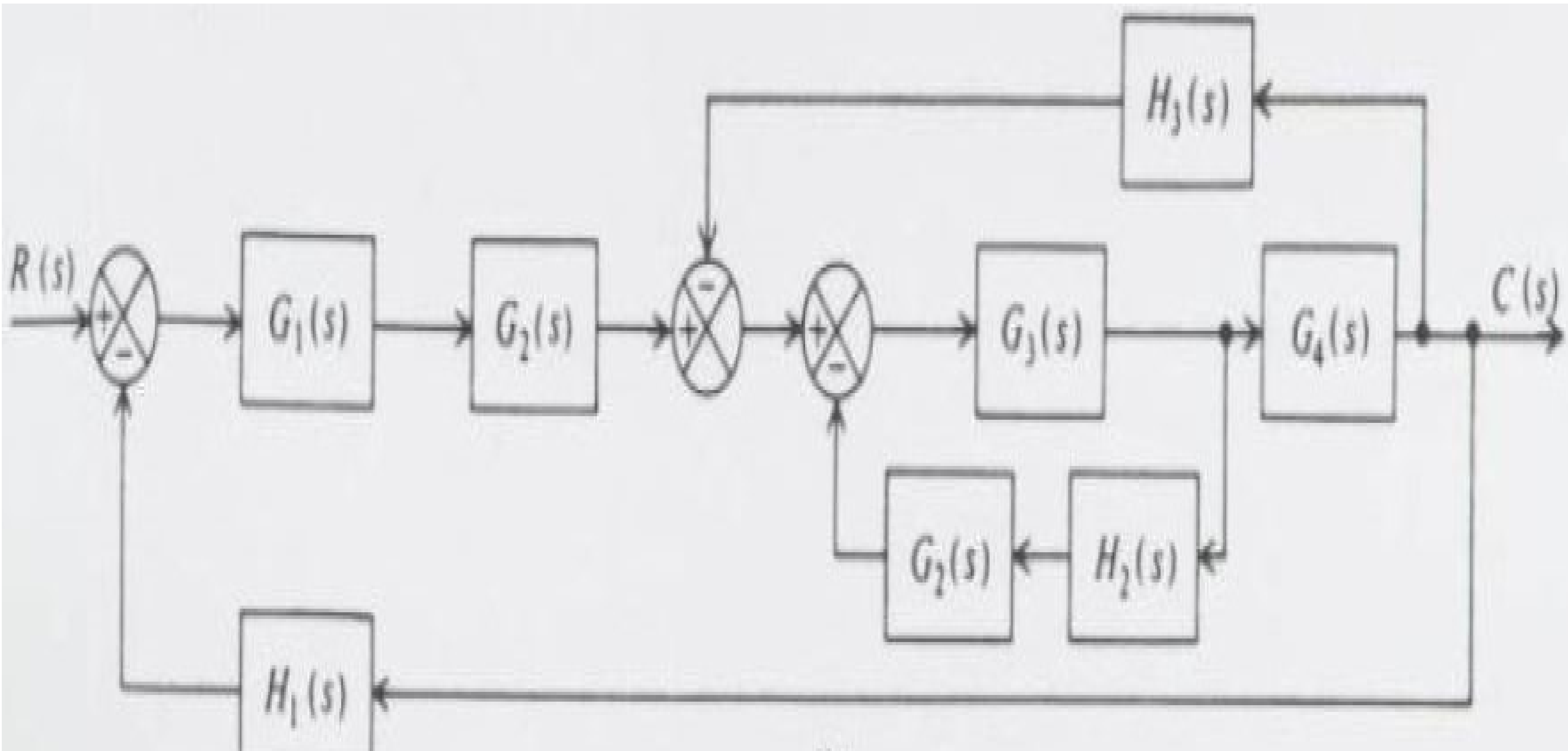
$R(s)$
$\frac{G_1(s)G_2(s)}{1+G_2(s)H_2(s)+G_1(s)G_2(s)H_1(s)}$
$V_4(s)$
$\left(\frac{1}{G_2(s)}+1\right)\left(\frac{G_3(s)}{1+G_3(s)H_3(s)}\right)$
$C(s)$
$R(s)$
$\frac{G_1(s)G_3(s)[1+G_2(s)]}{[1+G_2(s)H_2(s)+G_1(s)G_2(s)H_1(s)][1+G_3(s)H_3(s)]}$
$C(s)$

Example-8:



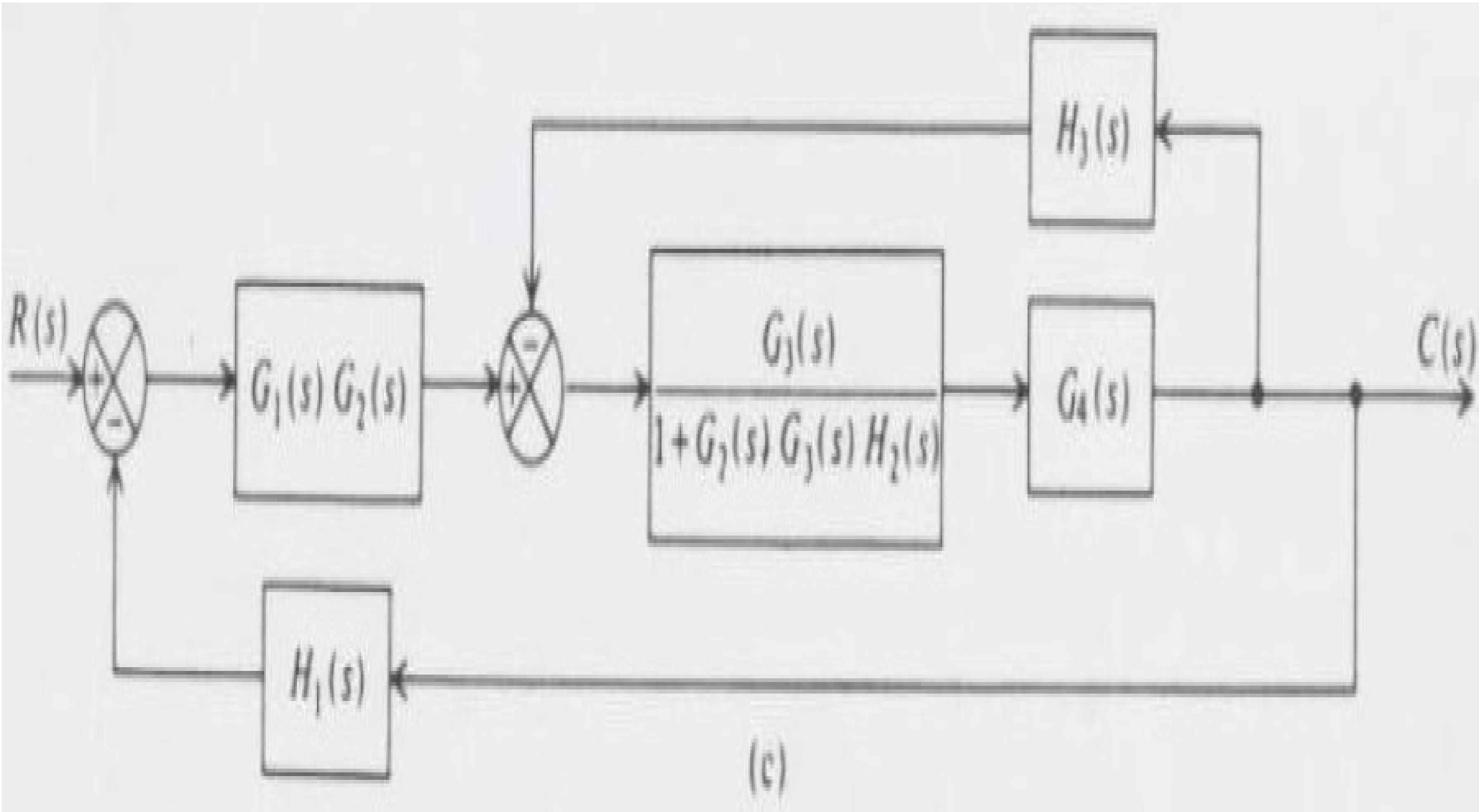
$H_3(s)$
Minor loop
$R(s)$
$E_1(s)$
$G_1(s)$
$E_2(s)$
$G_2(s)$
$E_3(s)$
$G_3(s)$
$E_4(s)$
$G_4(s)$
$C(s)$
Major loop
Intermediate loop
$H_2(s)$
$H_1(s)$
(a)

Example-8:

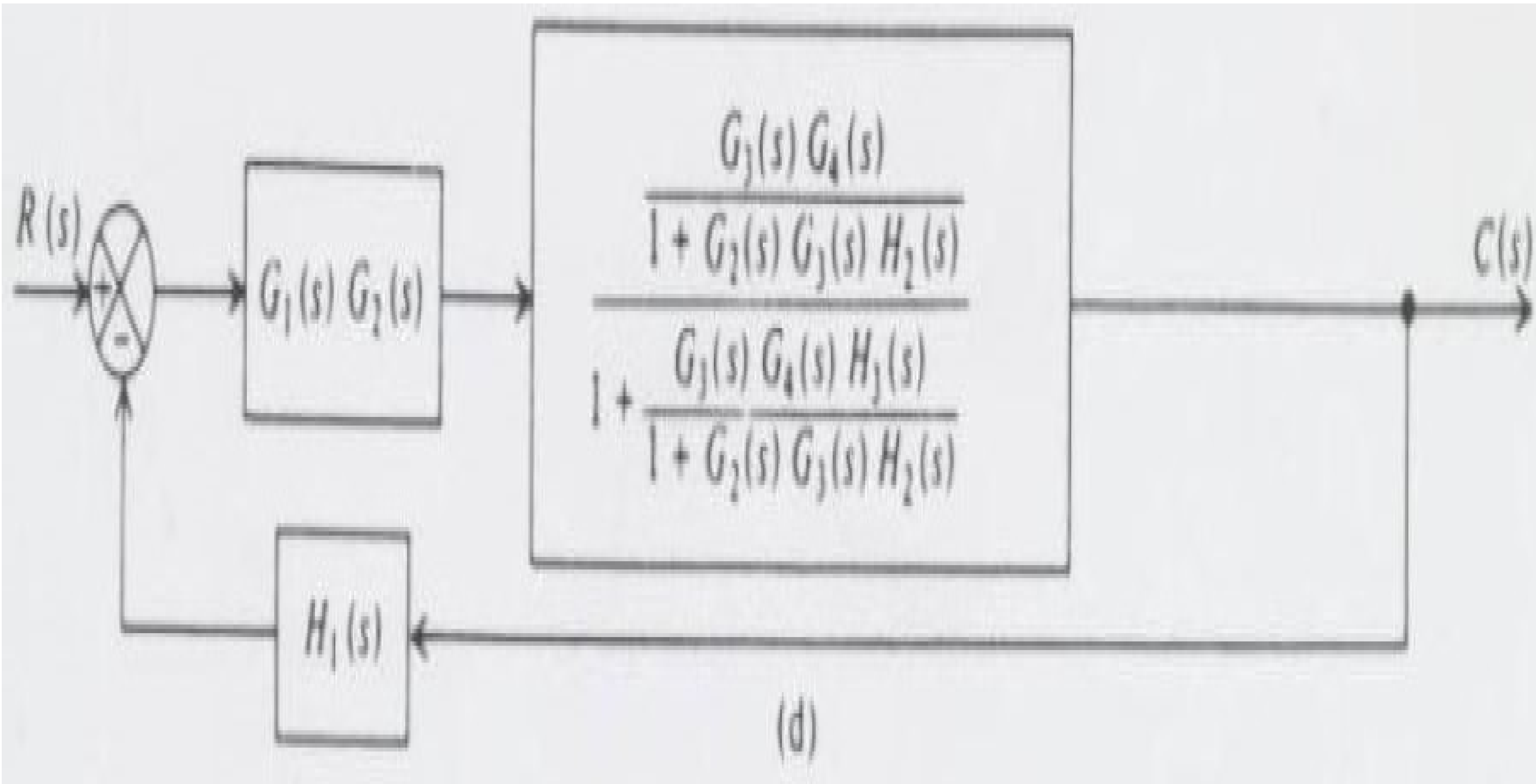


(b)

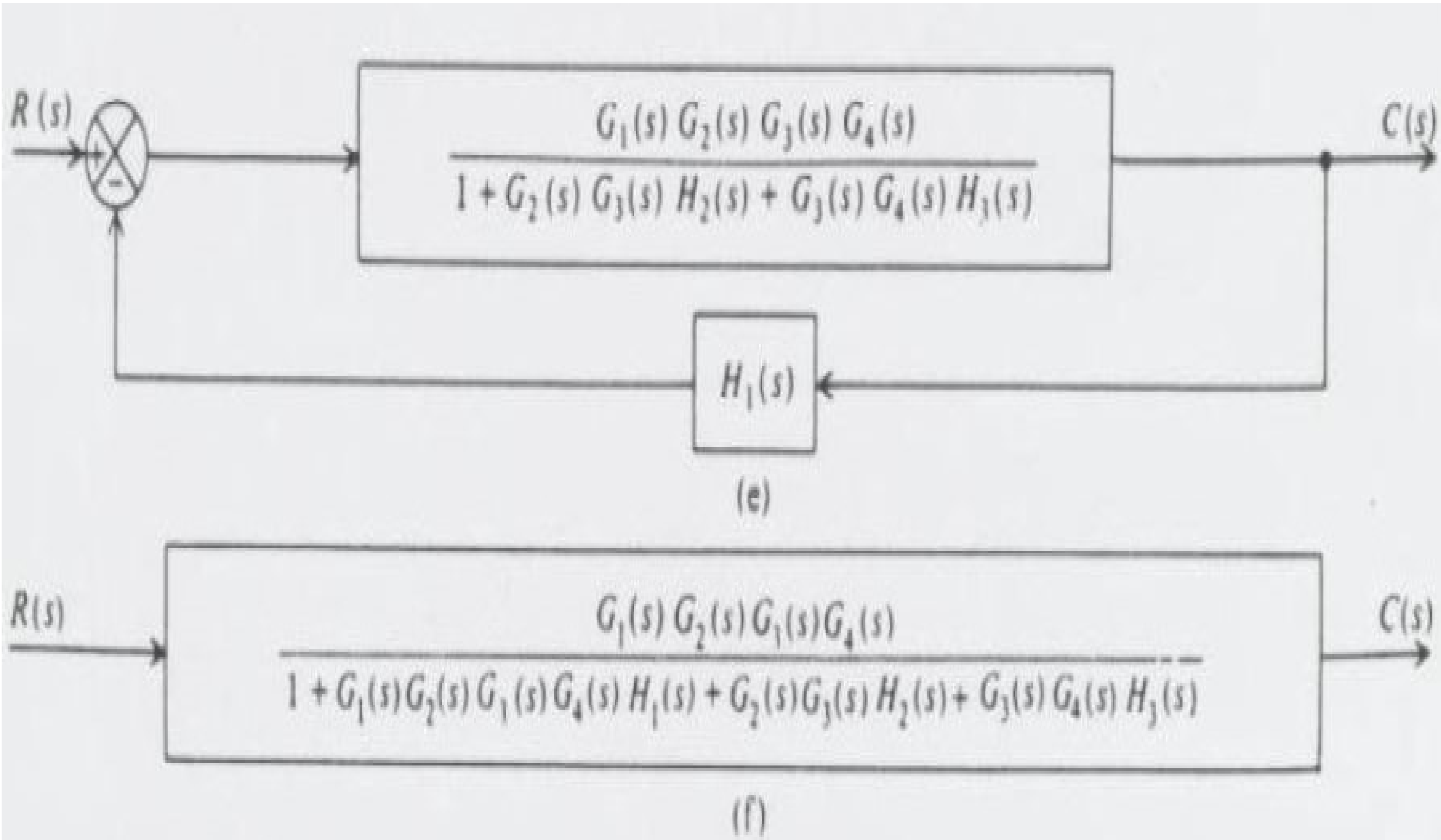
Example-8:



Example-8:


R(s)
+
-
$G_1(s)\,G_2(s)$
$\dfrac{\dfrac{G_3(s)\,G_4(s)}{1+G_2(s)\,G_3(s)\,H_2(s)}}{1+\dfrac{G_3(s)\,G_4(s)\,H_3(s)}{1+G_2(s)\,G_3(s)\,H_2(s)}}$
C(s)
$H_1(s)$
(d)

Example-8:


R(s)
C(s)
$\frac{G_1(s)G_2(s)G_3(s)G_4(s)}{1+G_2(s)G_3(s)H_2(s)+G_3(s)G_4(s)H_1(s)}$
$H_1(s)$
(e)
R(s)
C(s)
$\frac{G_1(s)G_2(s)G_1(s)G_4(s)}{1+G_1(s)G_2(s)G_1(s)G_4(s)H_1(s)+G_2(s)G_3(s)H_2(s)+G_3(s)G_4(s)H_3(s)}$
(f)

Signal flow graphs:

- ✓ Used to represent the control system graphically and it was developed by **S.J.Mason**.

Definitions:

Node: A node is a point representing a variable or signal

Branch: A branch is directed line segment joining two nodes. The arrow on the branch indicates the direction of signal flow and the gain of a branch is the transmittance.

Transmittance: The gain acquired by the signal when it travels from one node to another is called transmittance. The transmittance can be real or complex.

Input node (Source): It is a node that has only outgoing branches.

Output node (Sink): It is a node that has only incoming branches.

Mixed node: It is a node that has both incoming and outgoing branches.

Path: A path is traversal of connected branches in the direction of the branch arrows. The path should not cross a node more than once.

Signal flow graphs:

Open path: A open path starts at a node and ends at another node

Closed path: A closed path starts and ends at same node.

Forward path: It is a path from an input node to an output node that does not cross any node more than once.

Forward path gain: It is the product of the branch transmittances (gains) of a forward path.

Individual loop: It is closed path starting from a node and after passing through a certain part of a graph arrives at same node without crossing any node more than once.

Loop gain: It is the product of the branch transmittances (gains) of a loop.

Non-touching loops: If the loops does not have a common node then they are said to be non-touching loops.

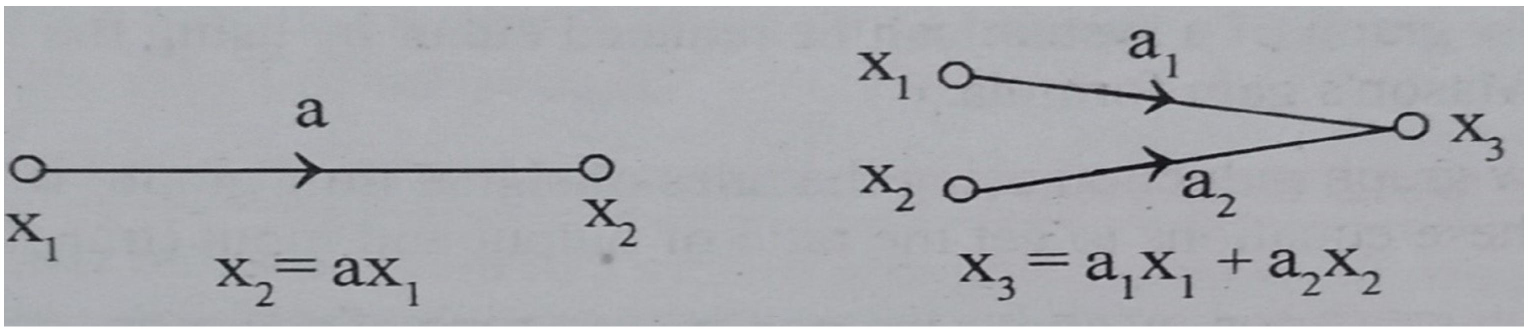
Properties of Signal flow graph:

1) The algebraic equations which are used to construct signal flow graph must be in the form of cause and effect relationship.
2) Signal flow graph is applicable to linear systems only.
3) A node in the signal flow graph represents the variable or signal.
4) A node adds the signals of all incoming branches and transmits the sum to all outgoing branches.
5) A mixed node which has both incoming and outgoing signals can be treated as an output node by adding an outgoing branch of unity transmittance.
6) A branch indicates functional dependence of one signal on the other.
7) The signals travel along branches only in the marked direction and when it travels it gets multiplied by the gain or transmittance of the branch.
8) The signal flow graph of system is not unique. By arranging the system equations different types of signal flow graphs can be drawn for a given system.

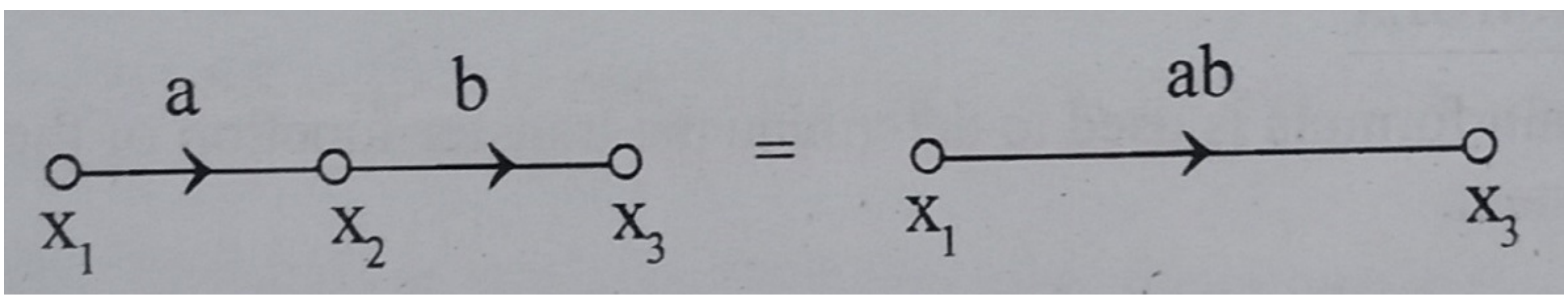
Signal flow graph algebra:

- Signal flow graph for a system can be reduced to obtain the transfer function of the system using the following rules.

1) Incoming signal to a node through a branch is given by the product of a signal at previous node and the gain of the branch.

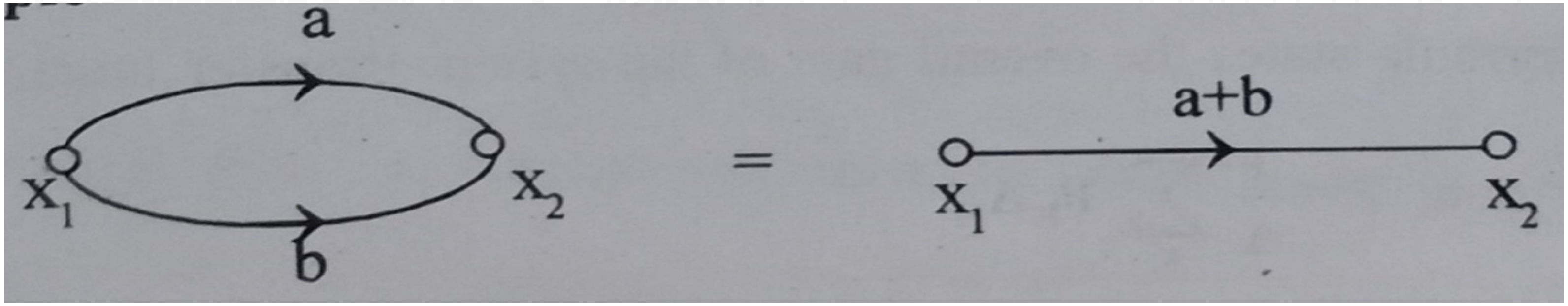


2) Cascaded branches can be combined to give a single branch whose transmittance is equal to the product of individual branch transmittance.

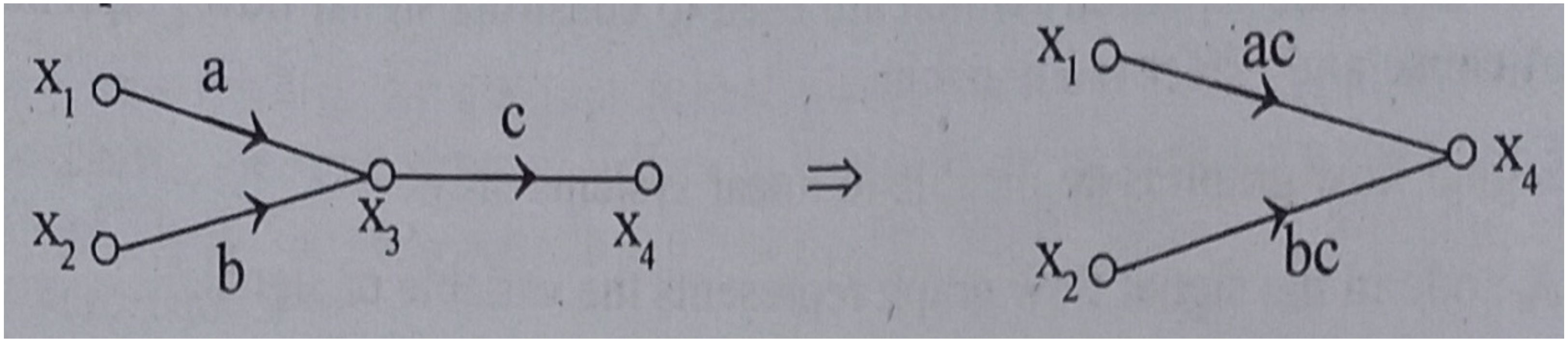


Signal flow graph algebra:

3) Parallel branches may be represented by single branch whose transmittance is the sum of individual branch transmittances.

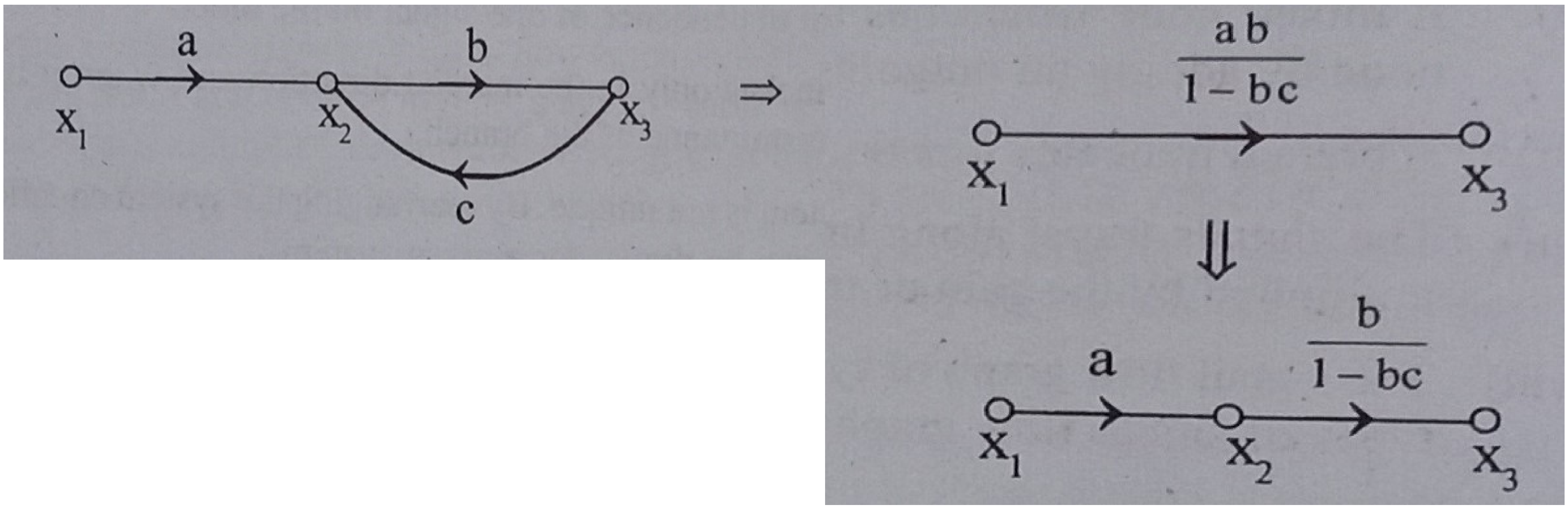


4) A mixed node can be eliminated by multiplying the transmittance of outgoing branch (from the mixed node) to the transmittance of all incoming branches to the mixed node.



Signal flow graph algebra:

5) A loop may be eliminated by writing equations at the input and output node and rearranging the equations to find the ratio of output to input. This ratio gives the gain of resultant branch.



MASON'S GAIN FORMULA:

✓ Used to determine the transfer function of the system from the signal flow graph of the system.

$$T = \frac{C(s)}{R(s)} = \frac{\Sigma_{i=1}^{N} P_i \Delta_i}{\Delta}$$

Where,

- **C(s)** is the output node
- **R(s)** is the input node
- **T** is the transfer function or gain between $R(s)$ and $C(s)$
- **P_i** is the i^{th} forward path gain

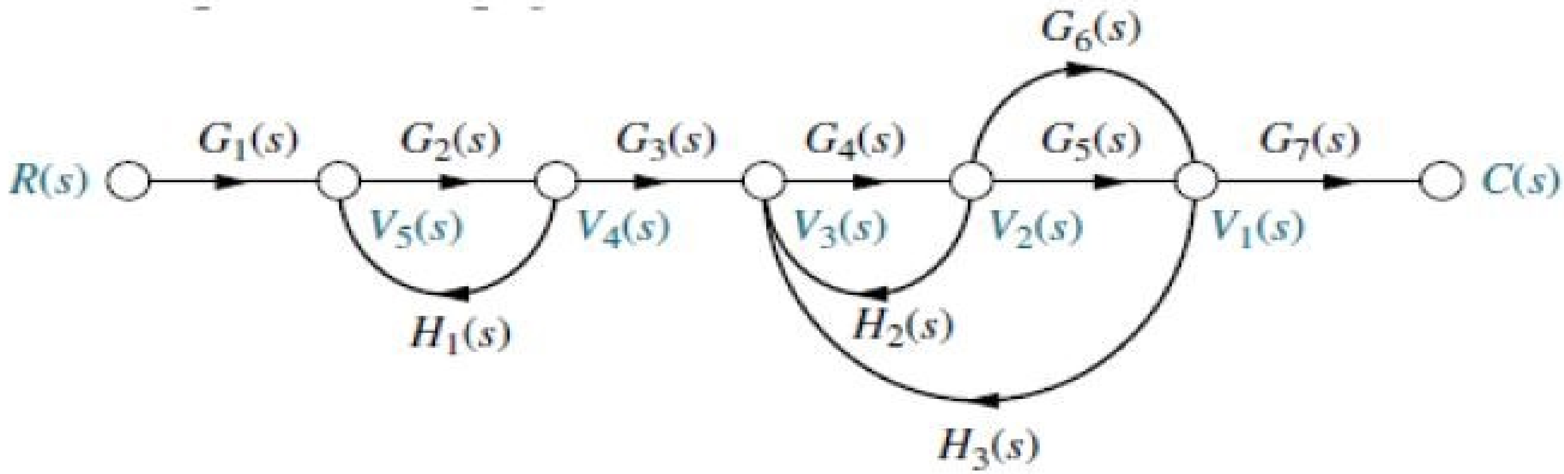
$$\Delta = 1 - (sum\ of\ all\ individual\ loop\ gains)$$

$$+(sum\ of\ gain\ products\ of\ all\ possible\ two\ nontouching\ loops)$$

$$-(sum\ of\ gain\ products\ of\ all\ possible\ three\ nontouching\ loops)+\ldots$$

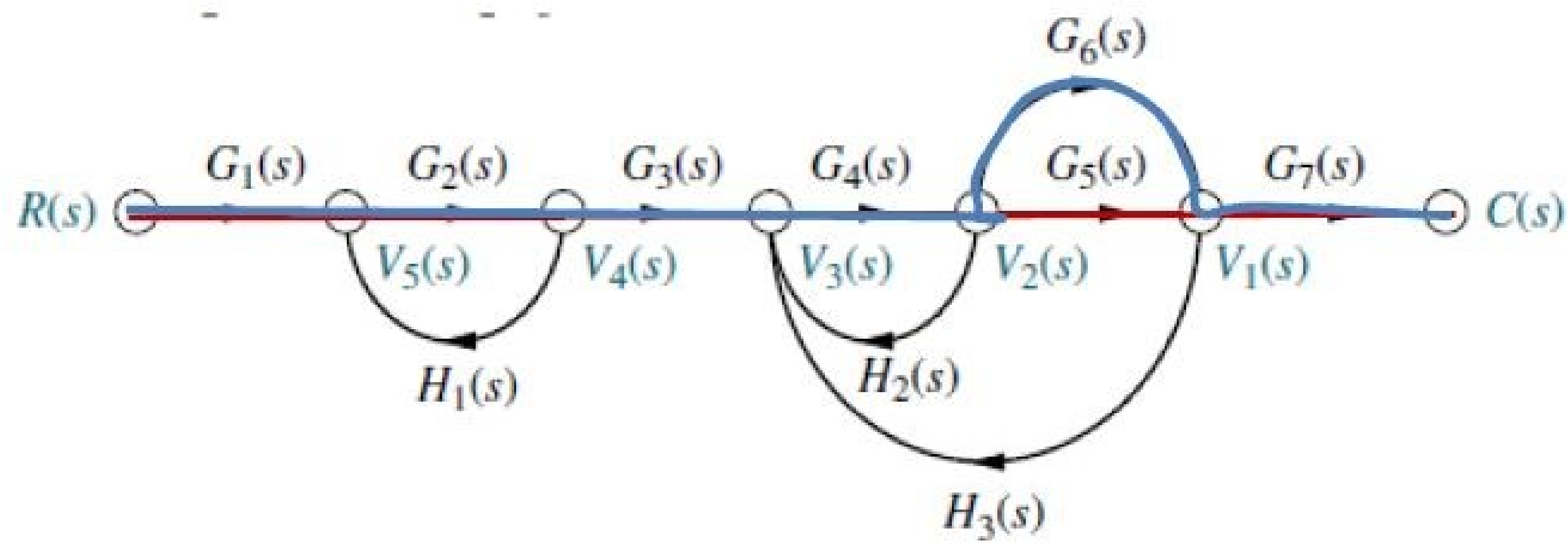
Δ_i is obtained from Δ by removing the loops which are touching the i^{th} forward path.

Identifying terminologies from SFG:



a) Input node.
b) Output node.
c) Forward paths.
d) Feedback paths (loops).
e) Determine the loop gains of the feedback loops.
f) Determine the path gains of the forward paths.
g) Non-touching loops

Identifying terminologies from SFG:

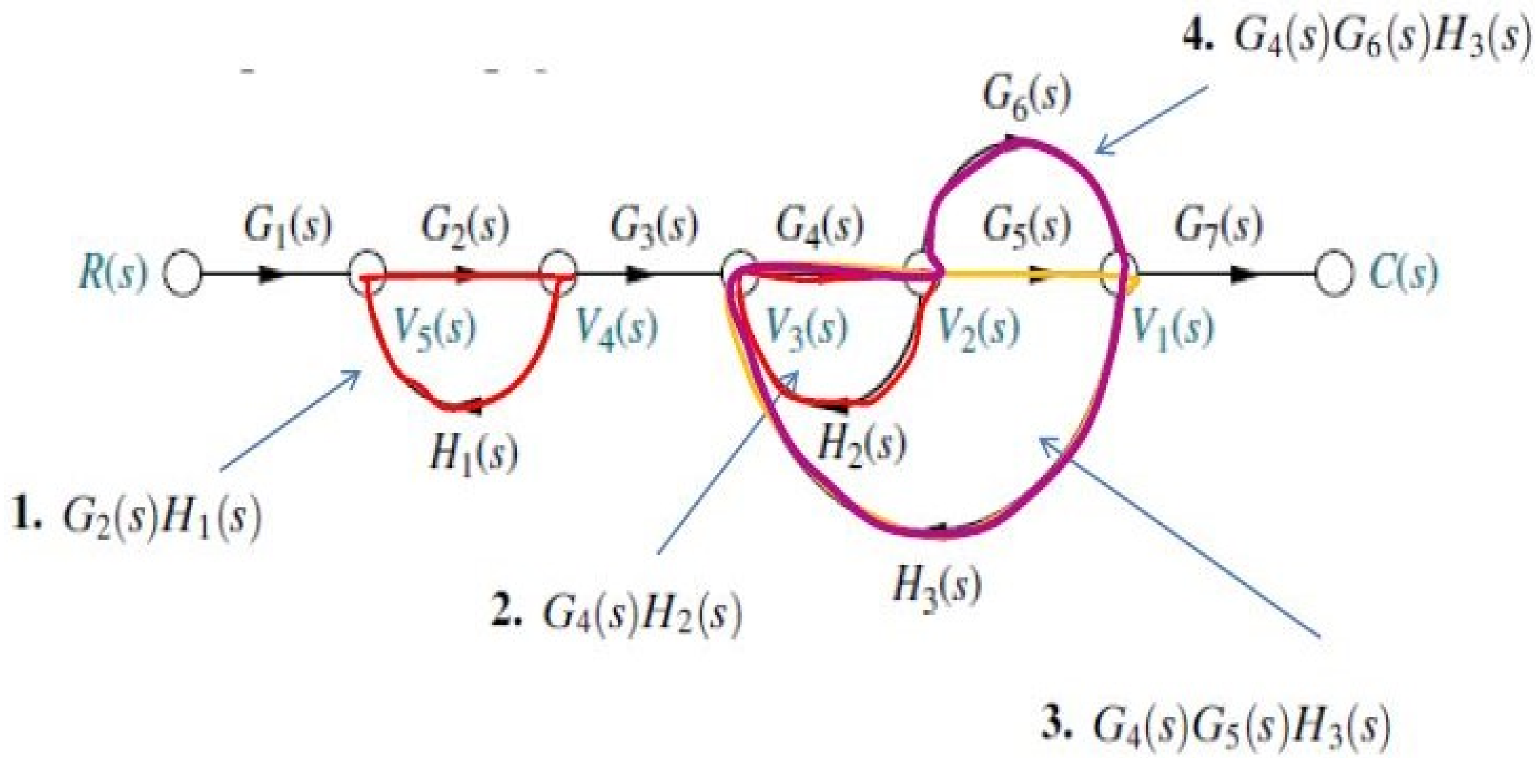


There are two forward path gains;

1. $G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)G_7(s)$ **2.** $G_1(s)G_2(s)G_3(s)G_4(s)G_6(s)G_7(s)$

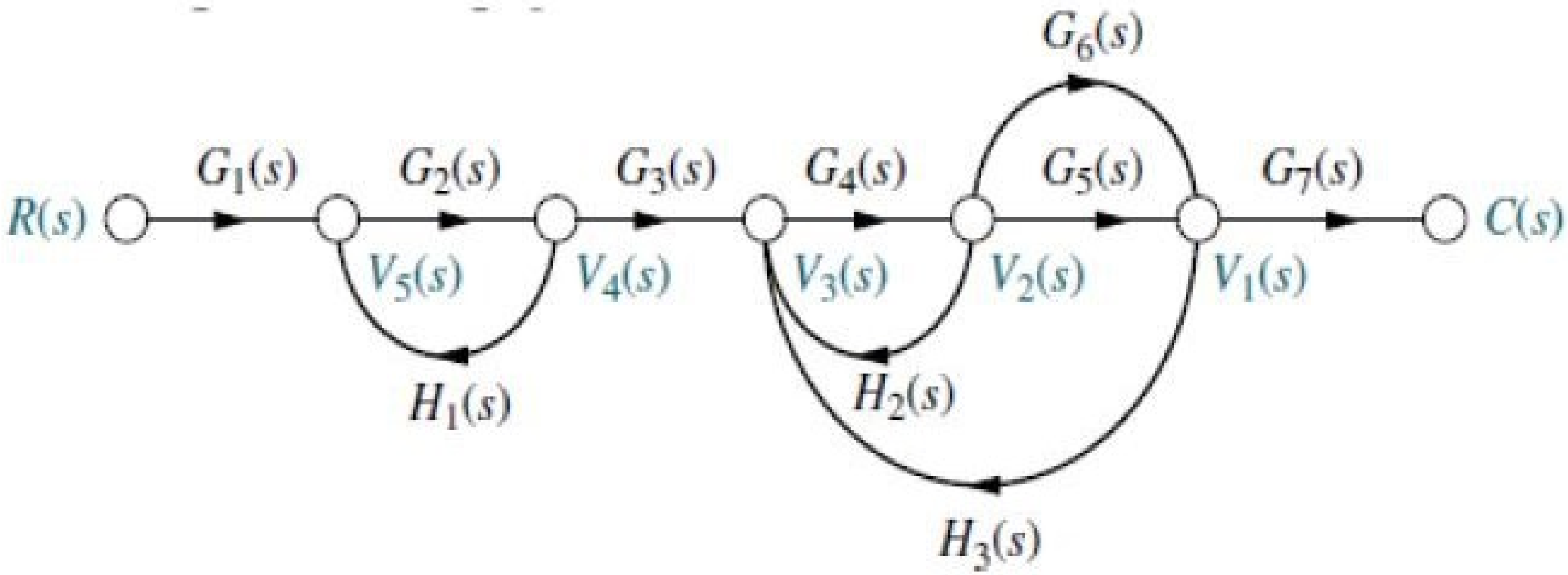
Identifying terminologies from SFG:

- There are four loops



Identifying terminologies from SFG:

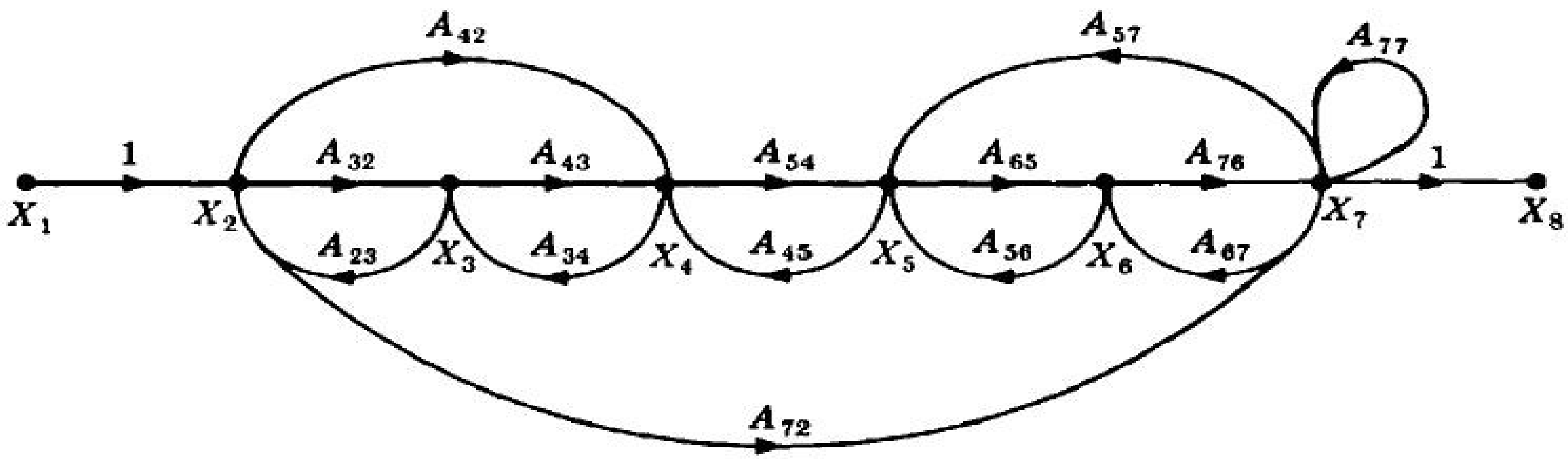
- Non touching loop gains



1. $[G_2(s)H_1(s)][G_4(s)H_2(s)]$
2. $[G_2(s)H_1(s)][G_4(s)G_5(s)H_3(s)]$
3. $[G_2(s)H_1(s)][G_4(s)G_6(s)H_3(s)]$

Identifying terminologies from SFG:

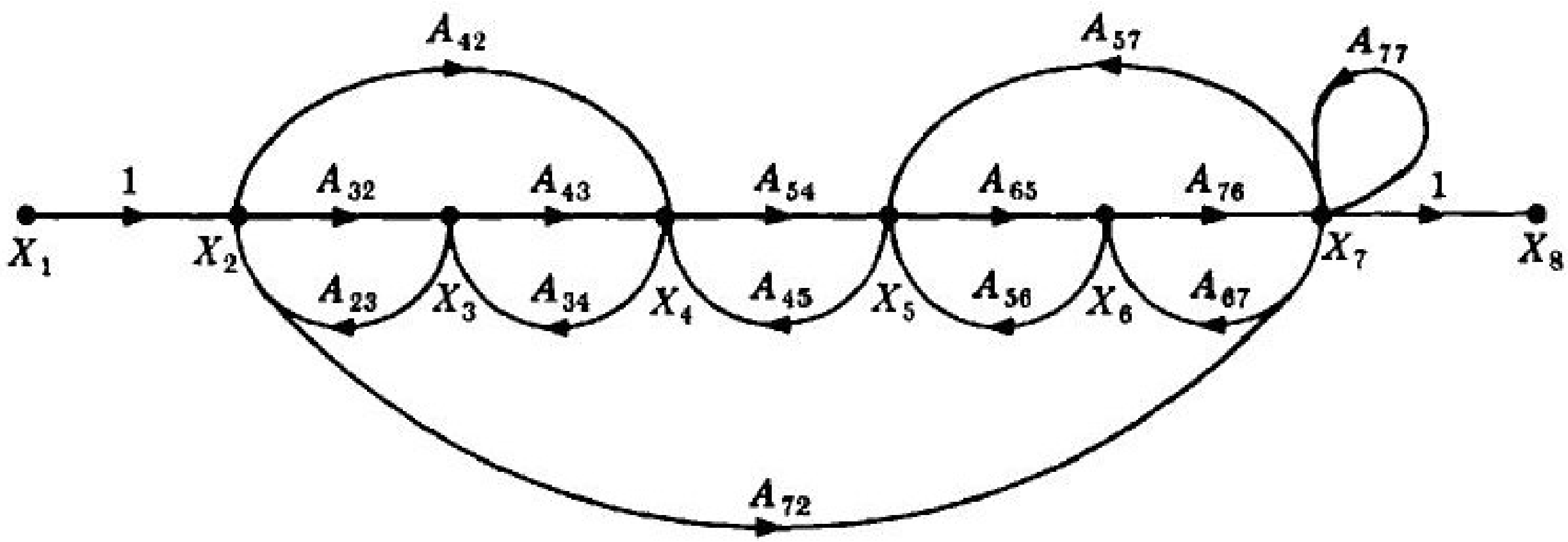
Consider the signal flow graph below and identify the following



a) Input node.
b) Output node.
c) Forward paths.
d) Feedback paths.
e) Self loop.
f) Determine the loop gains of the feedback loops.
g) Determine the path gains of the forward paths.

Identifying terminologies from SFG:

Input and output Nodes

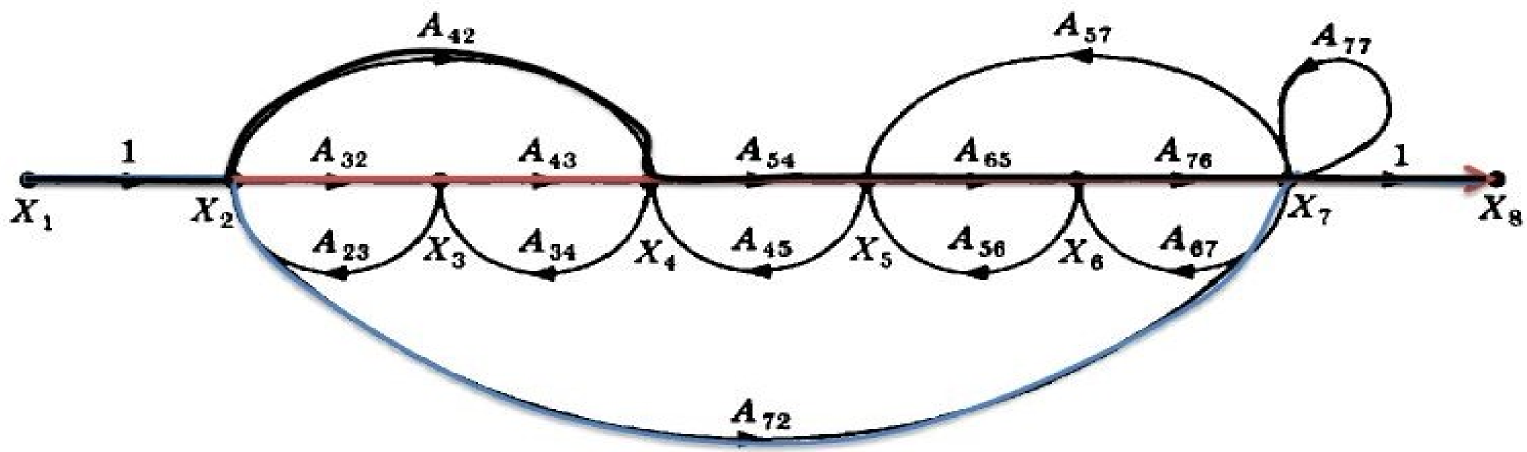


a) Input node X_1

b) Output node X_8

Identifying terminologies from SFG:

(c) Forward Paths



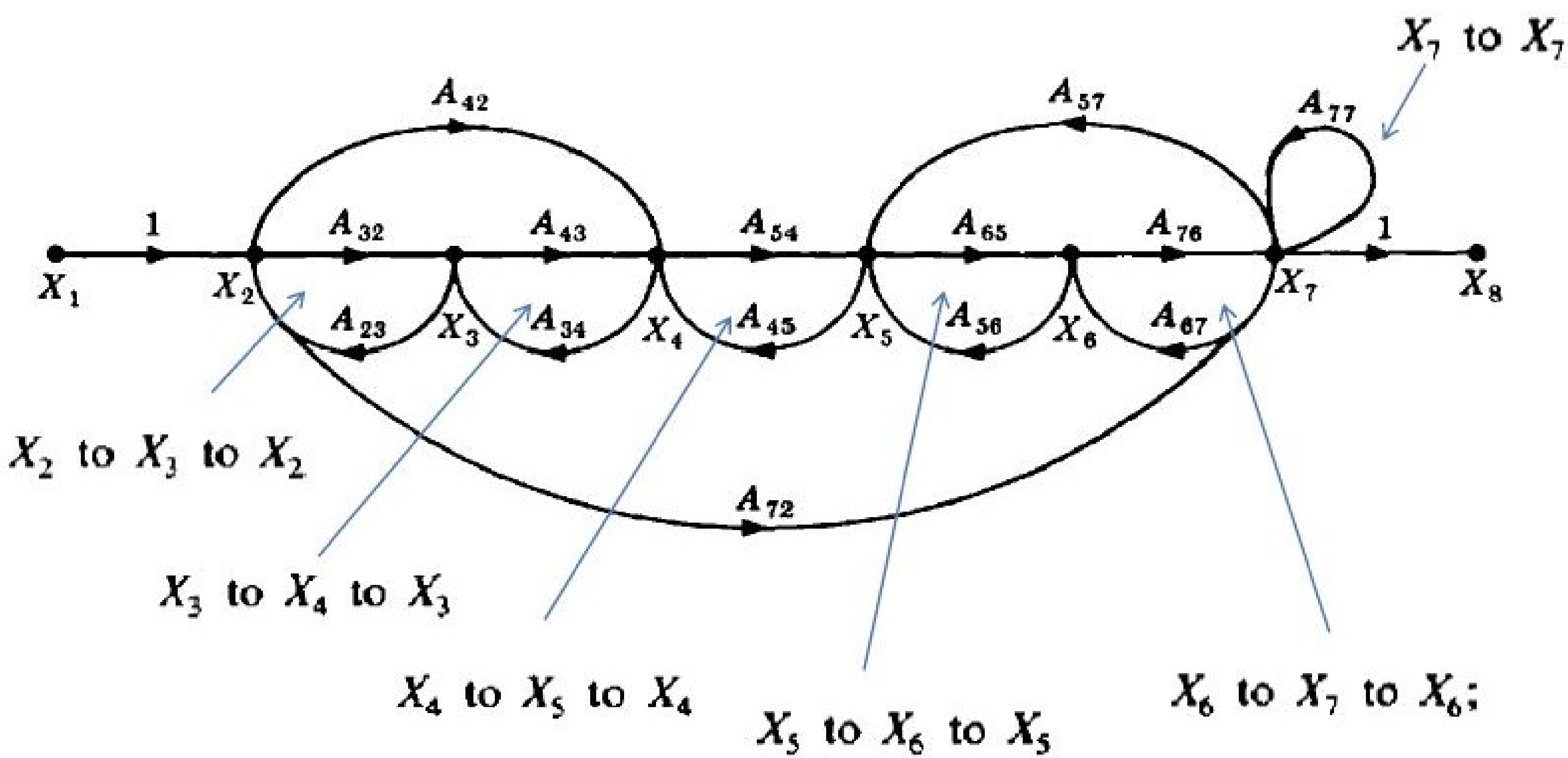
X_1 to X_2 to X_3 to X_4 to X_5 to X_6 to X_7 to X_8

X_1 to X_2 to X_7 to X_8

X_1 to X_2 to X_4 to X_5 to X_6 to X_7 to X_8

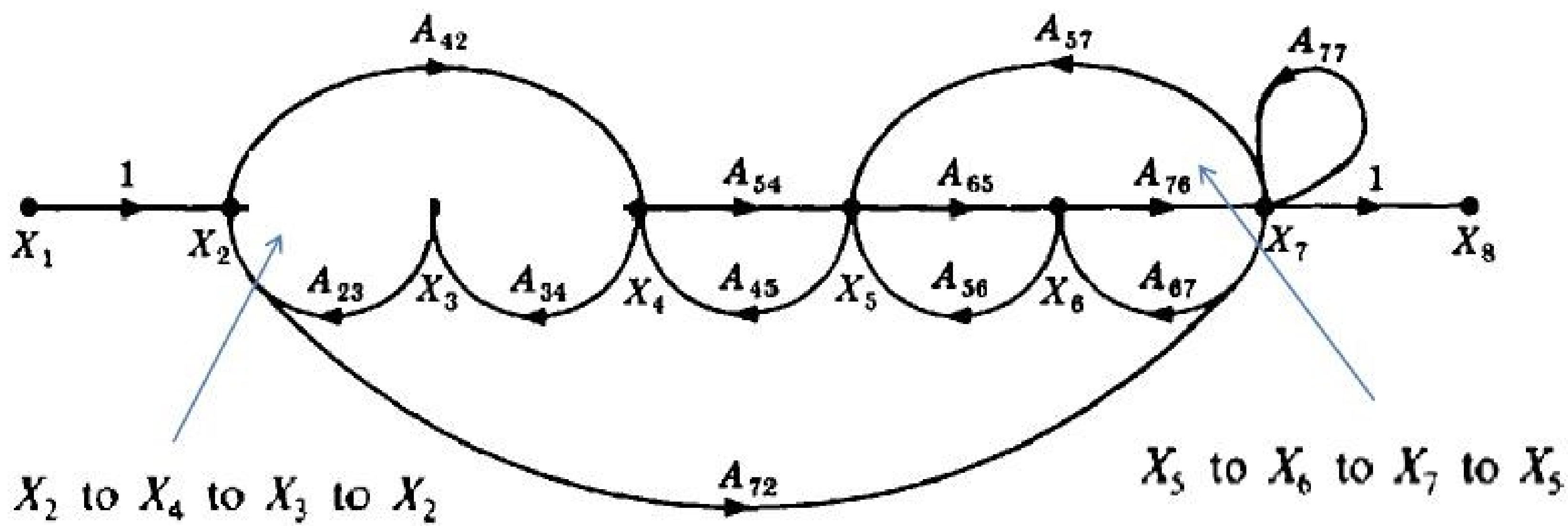
Identifying terminologies from SFG:

(d) Feedback Paths or Loops



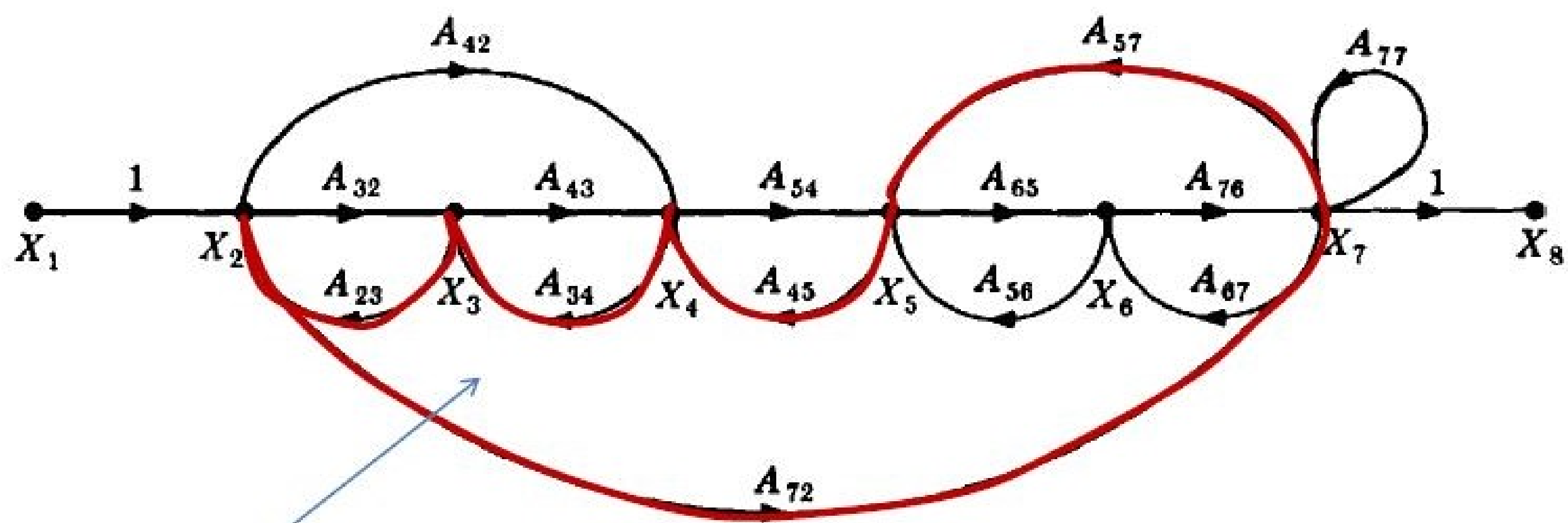
Identifying terminologies from SFG:

(d) Feedback Paths or Loops



Identifying terminologies from SFG:

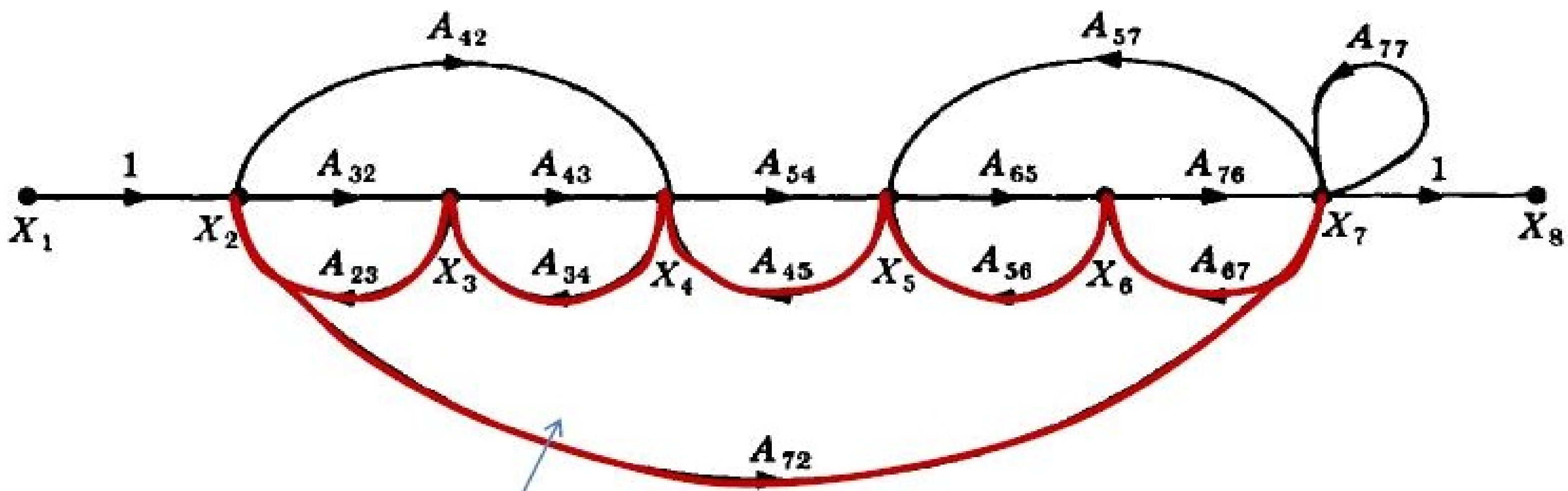
(d) Feedback Paths or Loops



X_2 to X_7 to X_5 to X_4 to X_3 to X_2

Identifying terminologies from SFG:

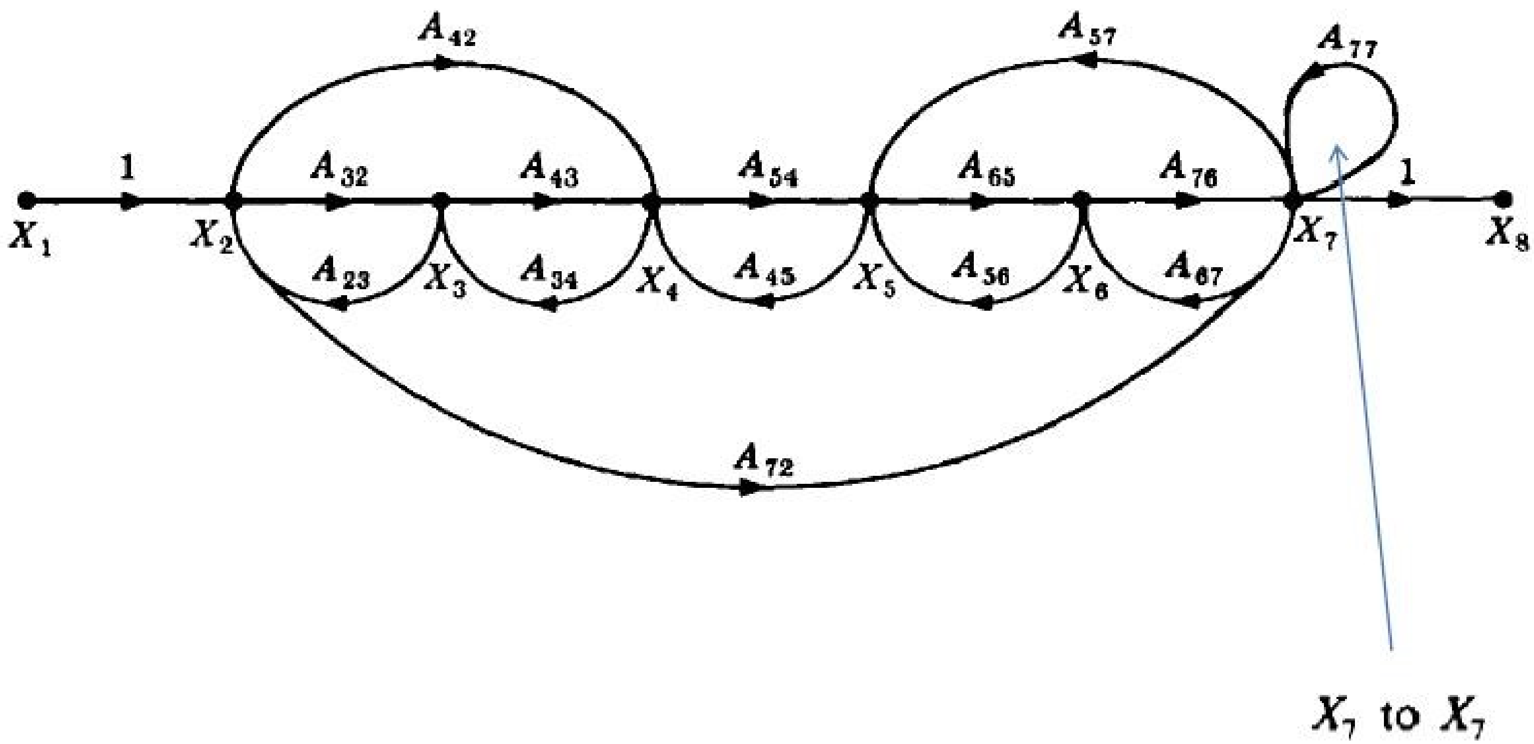
(d) Feedback Paths or Loops



X_2 to X_7 to X_6 to X_5 to X_4 to X_3 to X_2

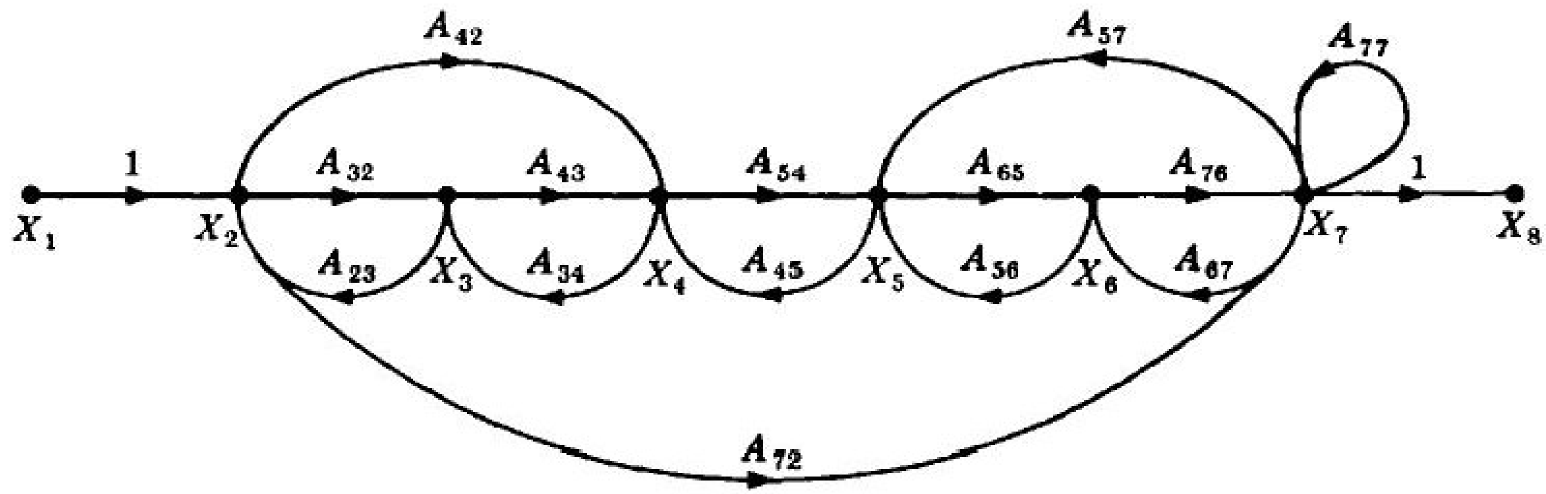
Identifying terminologies from SFG:

(e) Self Loop(s)



Identifying terminologies from SFG:

(f) Loop Gains of the Feedback Loops



$A_{32}A_{23}$ $\quad$ $A_{76}A_{67}$; $\quad$ $A_{72}A_{57}A_{45}A_{34}A_{23}$;

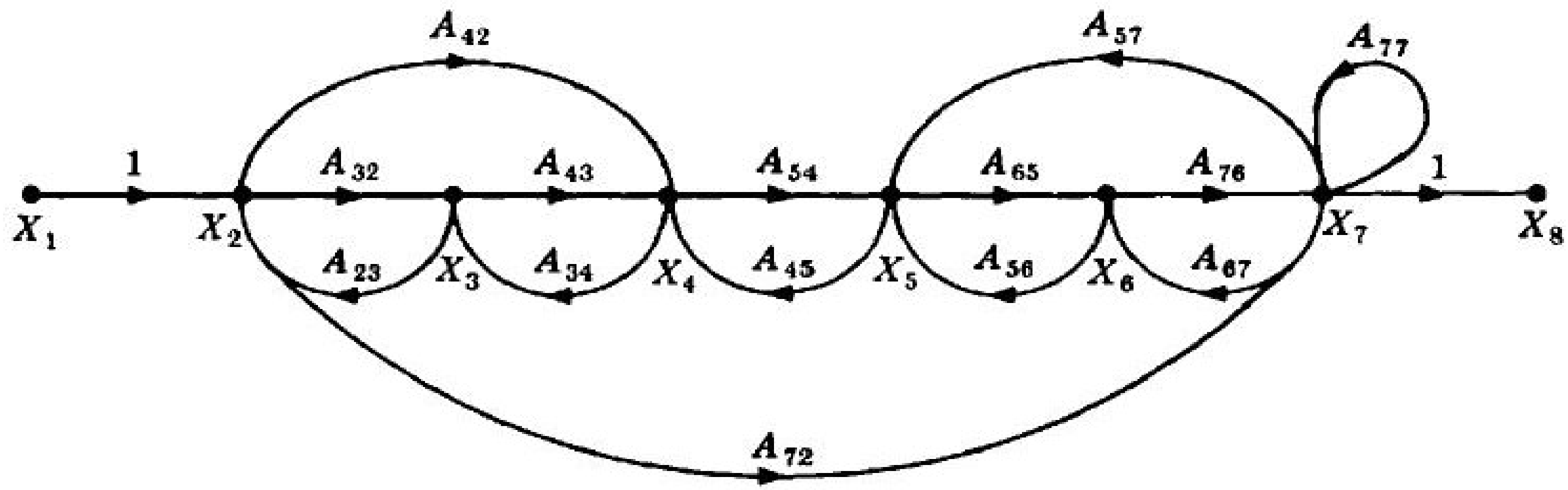
$A_{43}A_{34}$; $\quad$ $A_{65}A_{76}A_{57}$; $\quad$ $A_{72}A_{67}A_{56}A_{45}A_{34}A_{23}$

$A_{54}A_{45}$; $\quad$ A_{77};

$A_{65}A_{56}$ $\quad$ $A_{42}A_{34}A_{23}$

Identifying terminologies from SFG:

(g) Path Gains of the Forward Paths



$A_{32}A_{43}A_{54}A_{65}A_{76}$

A_{72}

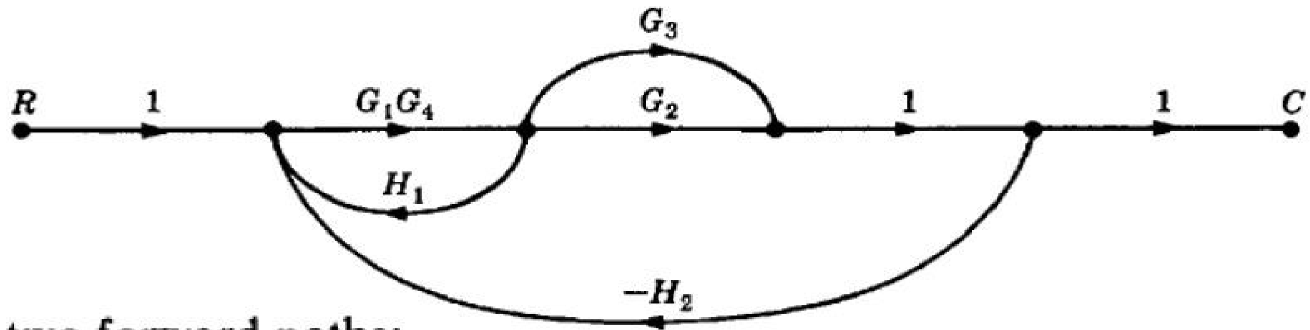
$A_{42}A_{54}A_{65}A_{76}$

Systematic approach for SFG:

1. Calculate forward path gain P_i for each forward path *i*.
2. Calculate all loop transfer functions
3. Consider non-touching loops 2 at a time
4. Consider non-touching loops 3 at a time
5. etc
6. Calculate Δ from steps 2,3,4 and 5
7. Calculate Δ_i as portion of Δ not touching forward path *i*

Example-1

calculate the transfer function of the following signal flow graph



There are two forward paths:

$P_1 = G_1G_2G_4 \qquad P_2 = G_1G_3G_4$

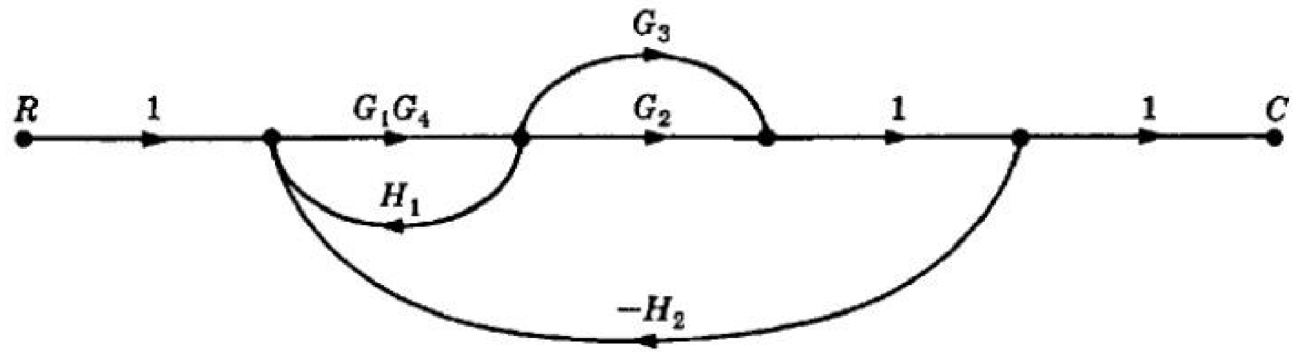
Therefore,

$$\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

There are three feedback loops

$$L_1 = G_1G_4H_1, \quad L_2 = -G_1G_2G_4H_2, \quad L_3 = -G_1G_3G_4H_2$$

Example-1



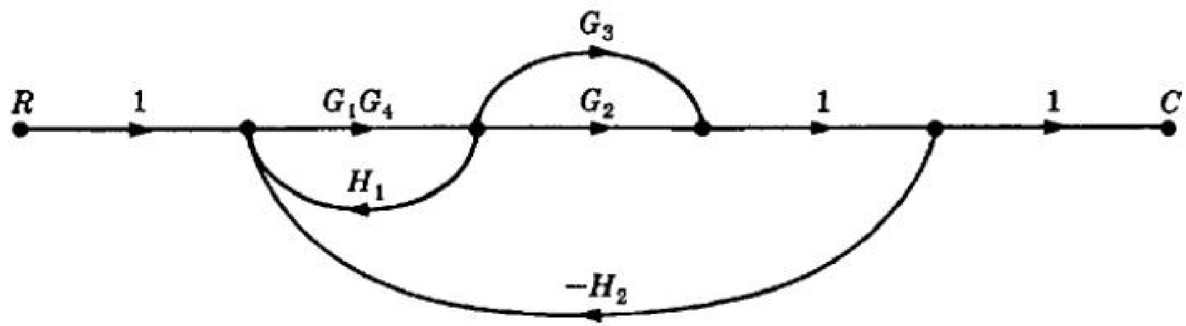
There are no non-touching loops, therefore

Δ = 1- (sum of all individual loop gains)

$$\Delta = 1 - \left(L_1 + L_2 + L_3\right)$$

$$\Delta = 1 - \left(G_1G_4H_1 - G_1G_2G_4H_2 - G_1G_3G_4H_2\right)$$

Example-1



Eliminate forward path-1

Δ_1 = 1- (sum of all individual loop gains)+...

$\Delta_1 = 1$

Eliminate forward path-2

Δ_2 = 1- (sum of all individual loop gains)+...

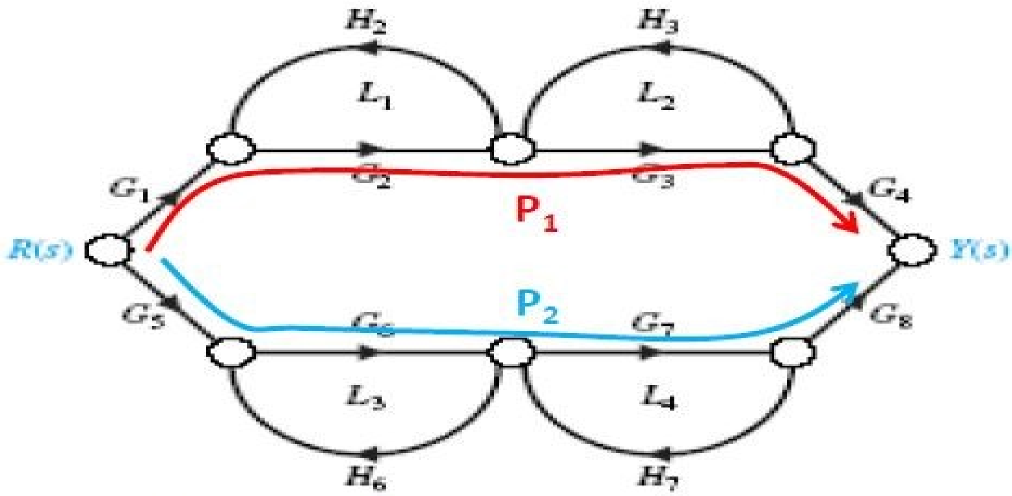
$\Delta_2 = 1$

Example-1

$$\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_4 + G_1G_3G_4}{1 - G_1G_4H_1 + G_1G_2G_4H_2 + G_1G_3G_4H_2}$$

$$= \frac{G_1G_4(G_2 + G_3)}{1 - G_1G_4H_1 + G_1G_2G_4H_2 + G_1G_3G_4H_2}$$

Example-2



1. Calculate forward path gains for each forward path.

$$P_1 = G_1G_2G_3G_4 \text{ (path 1)} \quad \text{and} \quad P_2 = G_5G_6G_7G_8 \text{ (path 2)}$$

2. Calculate all loop gains.

$$L_1 = G_2H_2, \quad L_2 = H_3G_3, \quad L_3 = G_6H_6, \quad L_4 = G_7H_7$$

3. Consider two non-touching loops.

L_1L_3 L_1L_4

L_2L_4 L_2L_3

Example-2

- Three non-touching loops – No loops

$$\Delta = 1 - \left(L_1 + L_2 + L_3 + L_4\right) + \left(L_1L_3 + L_1L_4 + L_2L_3 + L_2L_4\right)$$

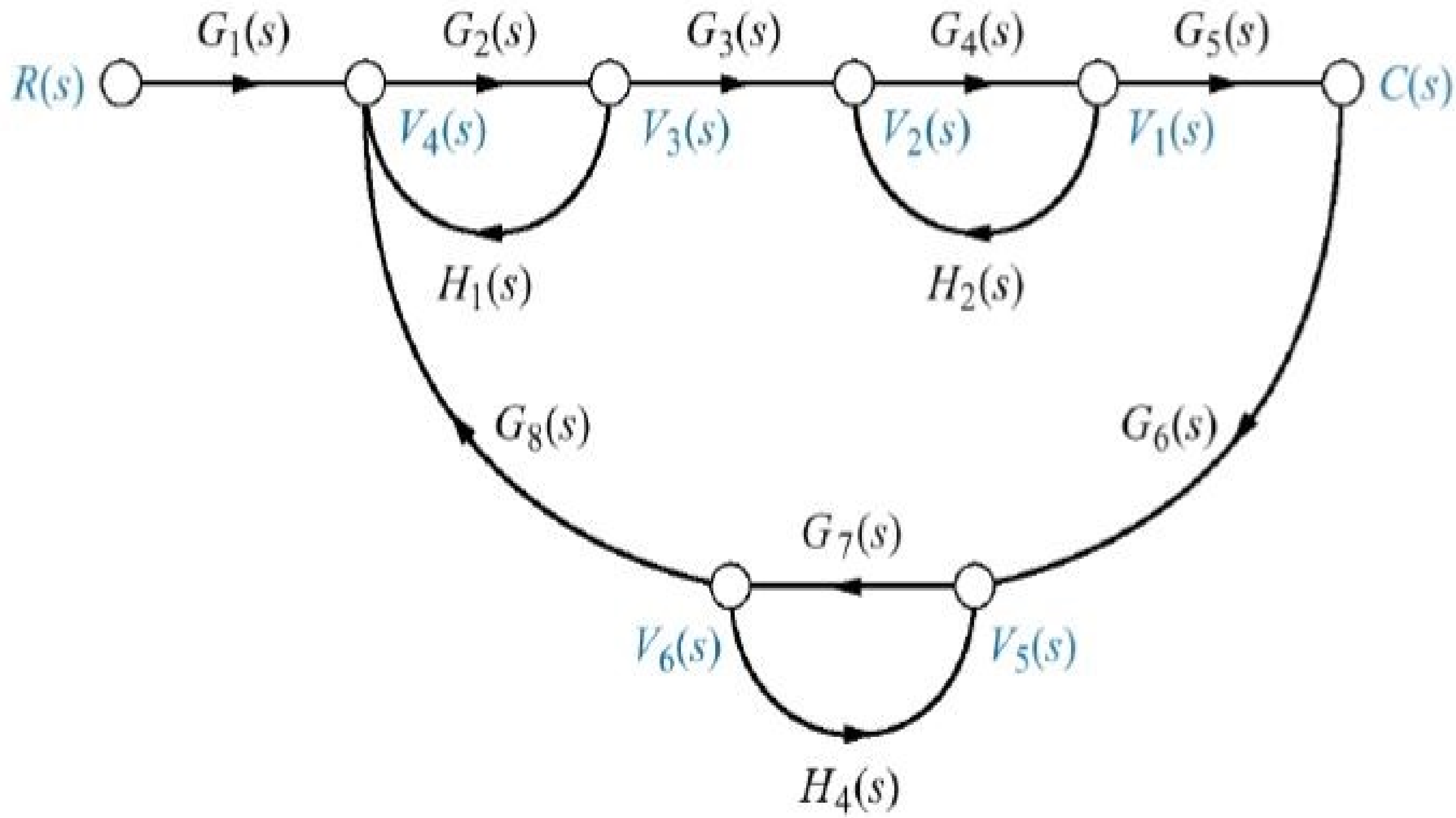
$$\Delta = 1 - \left(G_2H_2 + H_3G_3 + G_6H_6 + G_7H_7\right) + \left(G_2H_2G_6H_6 + G_2H_2G_7H_7 + H_3G_3G_6H_6 + H_3G_3G_7H_7\right)$$

Example-2

$$\frac{Y(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$\frac{Y(s)}{R(s)} = \frac{G_1G_2G_3G_4\left[1-\left(G_6H_6+G_7H_7\right)\right]+G_5G_6G_7G_8\left[1-\left(G_2H_2+G_3H_3\right)\right]}{1-\left(G_2H_2+H_3G_3+G_6H_6+G_7H_7\right)+\left(G_2H_2G_6H_6+G_2H_2G_7H_7+H_3G_3G_6H_6+H_3G_3G_7H_7\right)}$$

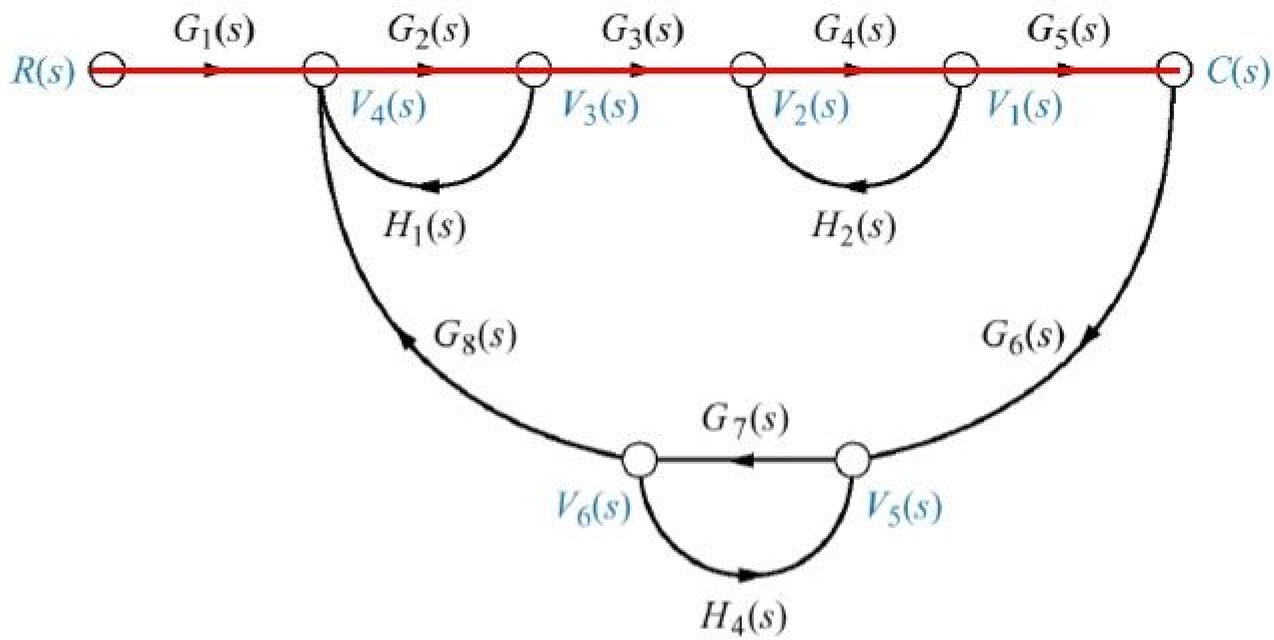
Example-3



$G_1(s)$
$G_2(s)$
$G_3(s)$
$G_4(s)$
$G_5(s)$
$R(s)$
$C(s)$
$V_4(s)$
$V_3(s)$
$V_2(s)$
$V_1(s)$
$H_1(s)$
$H_2(s)$
$G_8(s)$
$G_6(s)$
$G_7(s)$
$V_6(s)$
$V_5(s)$
$H_4(s)$

Example-3

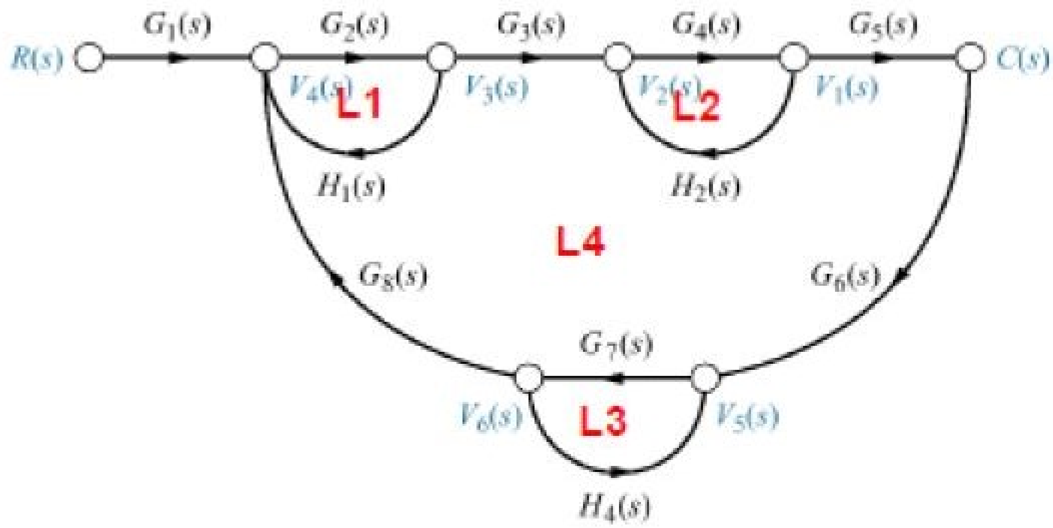
- ✓ There is only one forward path



$$P_1 = G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)$$

Example-3

✓There are four feedback loops



L1. $G_2(s)H_1(s)$

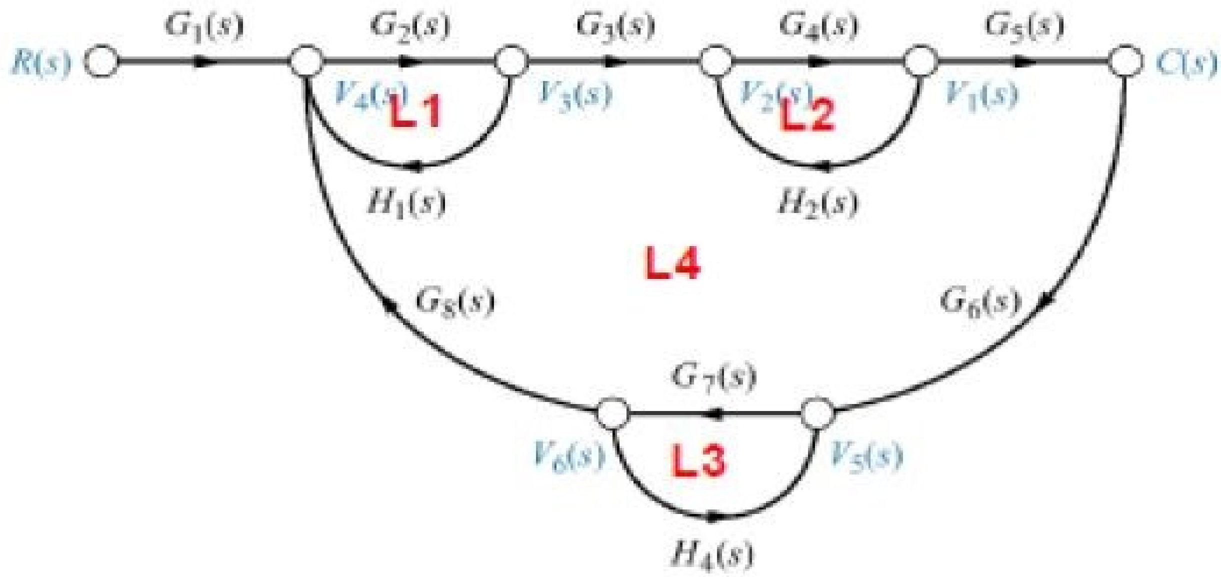
L2. $G_4(s)H_2(s)$

L3. $G_7(s)H_4(s)$

L4. $G_2(s)G_3(s)G_4(s)G_5(s)G_6(s)G_7(s)G_8(s)$

Example-3

- ✓Non-touching loops(two)

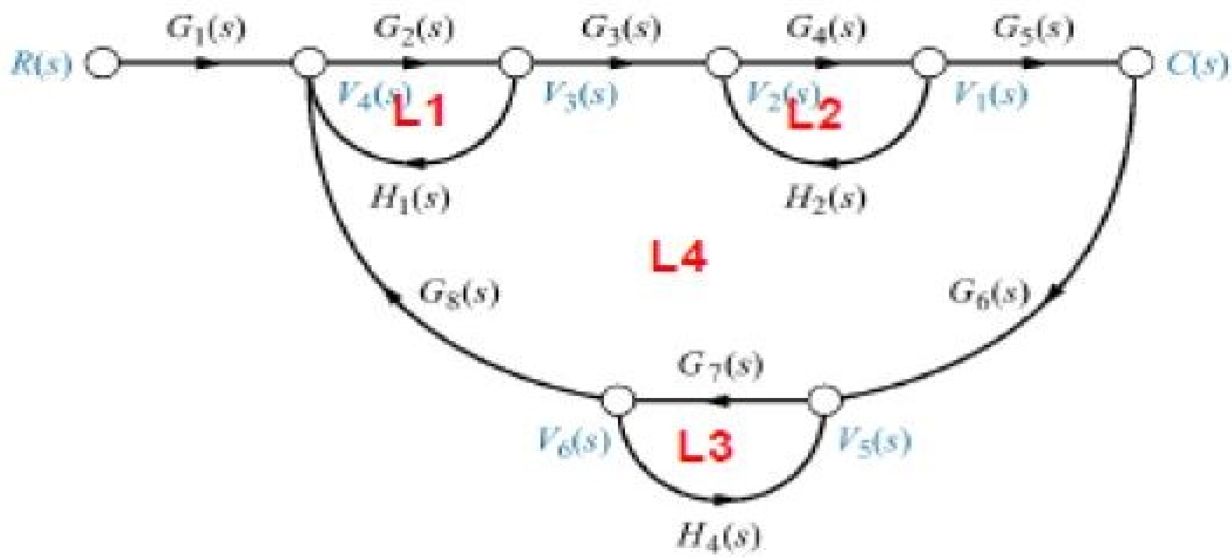


L1 and L2: $G_2(s)H_1(s)G_4(s)H_2(s)$ L2 and L3: $G_4(s)H_2(s)G_7(s)H_4(s)$

L1 and L3: $G_2(s)H_1(s)G_7(s)H_4(s)$

Example-3

✓Non-touching loops(three)



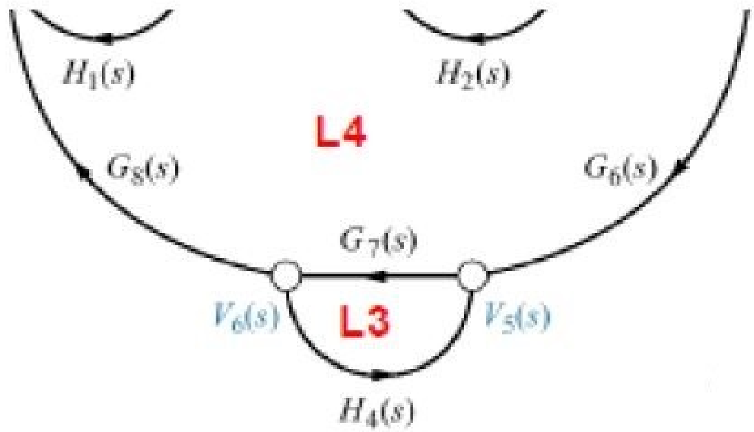
L1, L2, L3: $G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s)$

Example-3

$$\Delta = 1 - [G_2(s)H_1(s) + G_4(s)H_2(s)$$
$$+ G_7(s)H_4(s) + G_2(s)G_3(s)G_4(s)G_5(s)G_6(s)G_7(s)G_8(s)]$$
$$+ [G_2(s)H_1(s)G_4(s)H_2(s) + G_2(s)H_1(s)G_7(s)H_4(s)$$
$$+ G_4(s)H_2(s)G_7(s)H_4(s)]$$
$$- [G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s)]$$

Eliminate forward path-1

$$\Delta_1 = 1 - G_7(s)H_4(s)$$

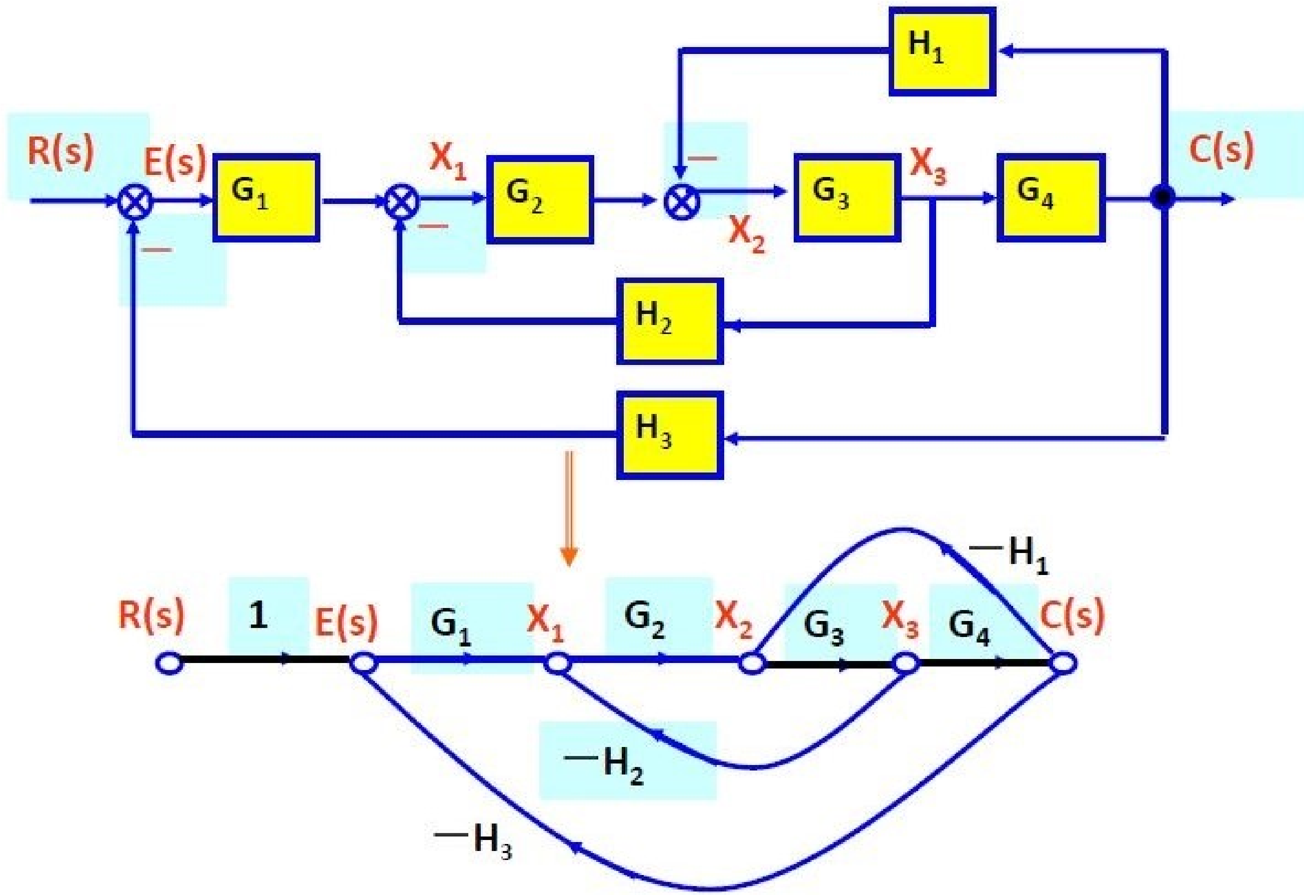


Example-3

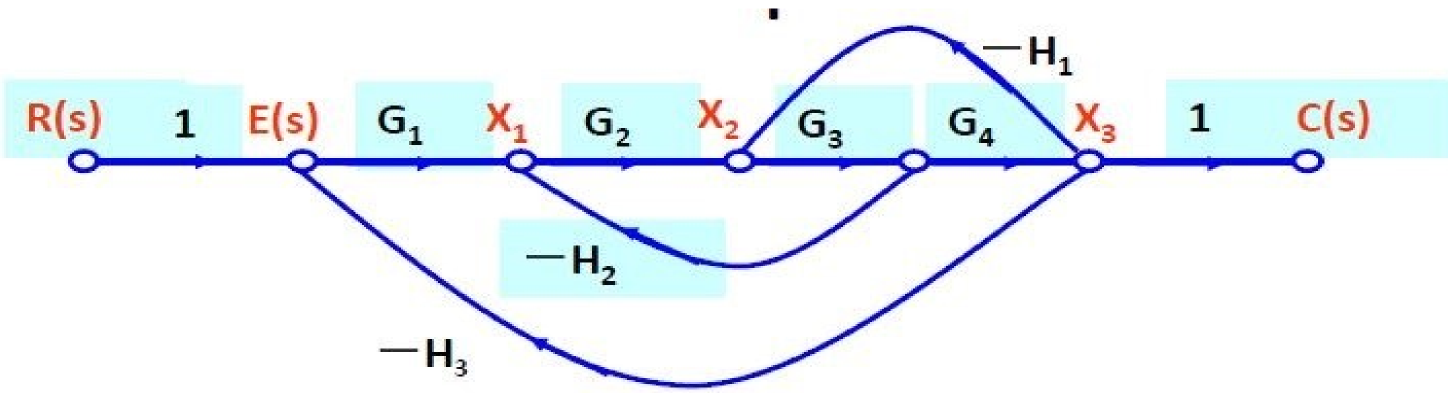
$$T(s) = \frac{P1\Delta1}{\Delta}$$

$$T(s) = \frac{G1(S)G2(S)G3(S)G4(S)G5(S)[1 - G7(S)H4(S)]}{1 - \begin{bmatrix} G2(S)H1(S) + G4(S)H2(S) + G7(S)H4(S) + \\ G2(S)G3(S)G4(S)G5(S)G6(S)G7(S)G8(S) \end{bmatrix} + \\ [G2(S)H1(S)G4(S)H2(S) + G2(S)H1(S)G7(S)H4(S) + G4(S)H2(S)G7(S)H4(S)] \\ -G2(S)H1(S)G4(S)H2(S)G7(S)H4(S)}$$

Block diagram to Signal flow graph: EXAMPLE-1



EXAMPLE-1

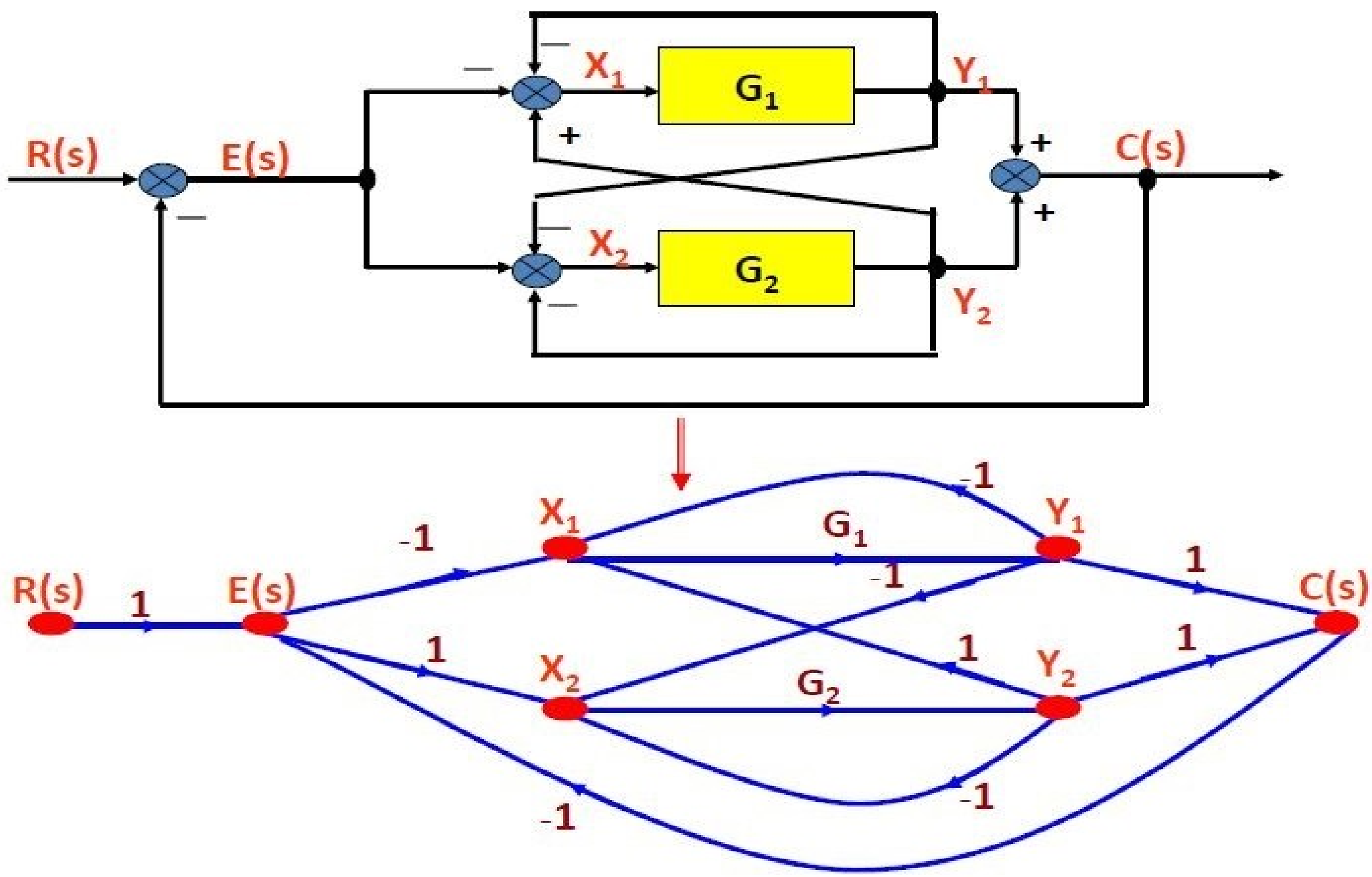


$$\Delta = 1 + (G_1G_2G_3G_4H_3 + G_2G_3H_2 + G_3G_4H_1)$$

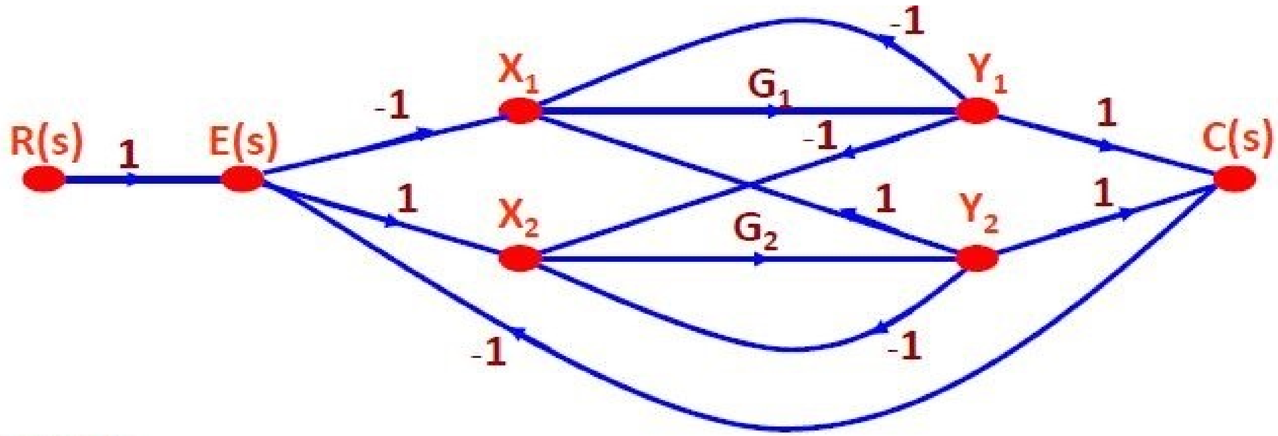
$$P_1 = G_1G_2G_3G_4 ; \qquad \Delta_1 = 1$$

$$G = \frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1 + G_1G_2G_3G_4H_3 + G_2G_3H_2 + G_3G_4H_1}$$

EXAMPLE-2



EXAMPLE-2



7 loops:

$[G_1 \cdot (-1)]$; $[G_2 \cdot (-1)]$; $[G_1 \cdot (-1) \cdot G_2 \cdot 1]$; $[(-1) \cdot G_1 \cdot 1 \cdot (-1)]$;

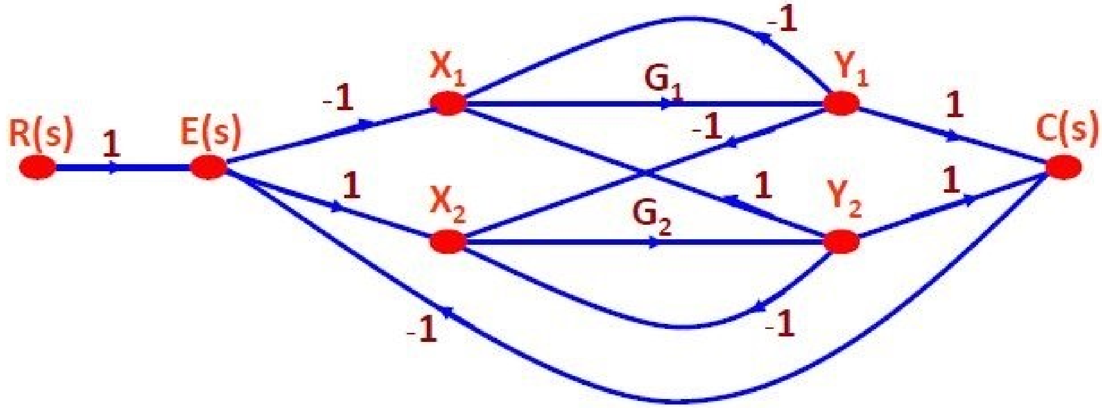
$[(-1) \cdot G_1 \cdot (-1) \cdot G_2 \cdot 1 \cdot (-1)]$; $[1 \cdot G_2 \cdot 1 \cdot (-1)]$; $[1 \cdot G_2 \cdot 1 \cdot G_1 \cdot 1 \cdot (-1)]$.

3 '2 non-touching loops' :

$[G_1 \cdot (-1)] \cdot [G_2 \cdot (-1)]$; $[(-1) \cdot G_1 \cdot 1 \cdot (-1)] \cdot [G_2 \cdot (-1)]$;

$[1 \cdot G_2 \cdot 1 \cdot (-1)] \cdot [G_1 \cdot (-1)]$.

EXAMPLE-2



$$\frac{C(s)}{R(s)} = \frac{\sum p_k \Delta_k}{\Delta}$$

Then: $\Delta = 1 + 2G_2 + 4G_1G_2$

$$= \frac{G_2 - G_1 + 2G_1G_2}{1 + 2G_2 + 4G_1G_2}$$

4 forward paths:

$$p_1 = (-1)\cdot G_1 \cdot 1 \qquad \Delta_1 = 1 + G_2$$

$$p_2 = (-1)\cdot G_1 \cdot (-1)\cdot G_2 \cdot 1 \qquad \Delta_2 = 1$$

$$p_3 = 1 \cdot G_2 \cdot 1 \qquad \Delta_3 = 1 + G_1$$

$$p_4 = 1 \cdot G_2 \cdot 1 \cdot G_1 \cdot 1 \qquad \Delta_4 = 1$$