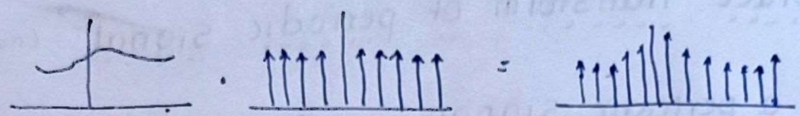
Unit-6 : Sampling

* **Sampling definition**: converting continuous time signal to discrete time signal.

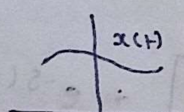


$$x(t) = x(t)\Big|_{t=nTs}$$

* **Sampling interval**: The time interval between 2 samples is known as sampling interval (T_s)

$T_s = 1/f_s$ { f_s = sampling frequency or rate }

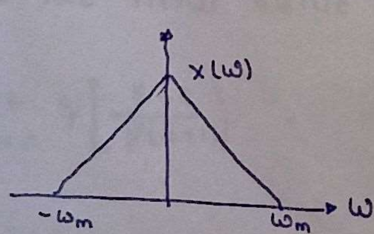
* **Time domain representation of sampling signal.**



$\longrightarrow$ [sampler] $\longrightarrow$ $x_s(t) = x(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nTs)$

* **Sampling theorem of Band limited signals**:

Statement: Any signal $x(t)$ which is band limited to ω_m Hz (ie $X(\omega) = 0$ :: $|\omega| > \omega_m$) can be completely reconstructed back from its sample signal taken at a rate $\boxed{\omega_s \geq 2\omega_m}$ or $\boxed{f_s \geq 2f_m}$ $\omega = 2\pi f$



Band width = $2\omega_m$

Here $2\omega_m$ is the nyquist rate.

Nyquist rate :. It is the minimum sampling rate at which the signal can be sampled and can be completely reconstructed back from its samples without any distortions.

$N \cdot R = 2\omega_m$ or $2f_m$ { ω_m = max frequency of signa

Nyquist interval :. (Time) :. The time interval b/w 2 adjacent samples when the sampling rate is nyquist rate. $N \cdot I = \frac{1}{N \cdot R} = \frac{1}{2f_m} = \frac{1}{2\omega_m}$

* Based on nyquist rate sampling is divided into 3 cases.

1) Over sampling

$\omega_s > 2\omega_m$ or $f_s > 2f_m$

2) Critical sampling :.

$\omega_s = 2\omega_m$ or $f_s = 2f_m$

3) Under sampling :.

$\omega_s < 2\omega_m$ or $f_s < 2f_m$

* frequency domain representation of sampling signal.

we know $x_s(t) = x(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$

Apply Forier transform on both sides.

* $FT\{x_s(t)\} = FT\{x(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nT_s)\}$

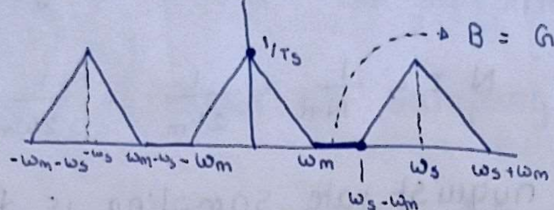
$$X_s(\omega) = \frac{1}{2\pi}\left[FT\{x(t)\} * FT\left\{\sum_{n=-\infty}^{\infty} \delta(t-nTs)\right\}\right.$$

$$= \frac{1}{2\pi}\left[X(\omega) * \frac{2\pi}{T}\sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_s)\right]$$

$$= \frac{1}{T_s}\left[X(\omega) * \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_s)\right]$$

$$= \frac{1}{T_s}\sum_{n=-\infty}^{\infty} X(\omega - n\omega_s]$$

$$X_s(\omega) = \frac{1}{T}\left[X(\omega+n\omega_s) + \dots + X(\omega+\omega_s) + X(\omega) + X(\omega-\omega_s) + \dots X(\omega - n\omega_s) + \dots\right]$$



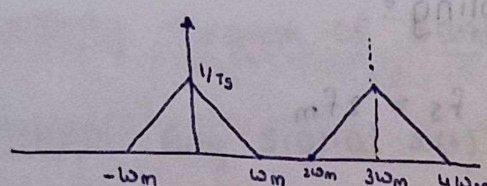
$\omega_s - \omega_m - \omega_m$

"$[B = \omega_s - 2\omega_m]$"

case-i :- Over sampling :- $\omega_s > 2\omega_m$

Let $\omega_s = 3\omega_m$

$$X_s(\omega) = \frac{1}{T_s}\left[\dots + X(\omega) + X(\omega - 3\omega_m) + X(\omega - 6\omega_m) + \dots\right]$$

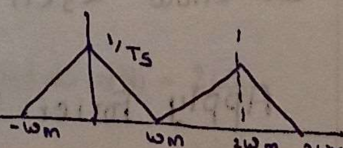


$B = \omega_s - 2\omega_m = 3\omega_m - 2\omega_m$

$\boxed{B = \omega_m}$

case-ii :- Critical sampling :-

Let $\omega_s = 2\omega_m$



$$X_s(\omega) = \frac{1}{T_s}\left[\dots + X(\omega) + X(\omega - 2\omega_m) + X(\omega - 4\omega_m) + \dots\right]$$

$B = \omega_s - 2\omega_m$

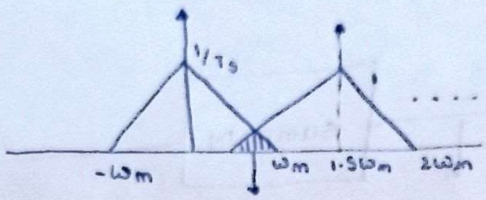
$B = 2\omega_m - 2\omega_m$ $\rightarrow$ $\boxed{B = 0}$

case-iii : Under sampling

$\omega_s < 2\omega_m$

Let $\omega_s = 1.5\,\omega_m$

$$X_s(\omega) = \frac{1}{T_s}\left[\cdots + X(\omega) + X(\omega - 1) + X(\omega - 2) + \cdots\right]$$



Aliazing effect

Aliazing effect : Overlapping of replicas with original signal. In under sampling, lower frequencies of $X_s(\omega)$ overlap with higher frequencies of shifted $x(\omega)$. This is known as Aliazing effect. This overlapping leads to distortion & this is known as Aliazing effect.

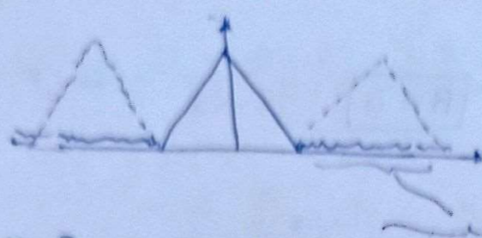
* Aliazing occurs due to...

1) when sampling rate is less than nyquist rate

$\omega_s <$ N.R or $2\omega_m$

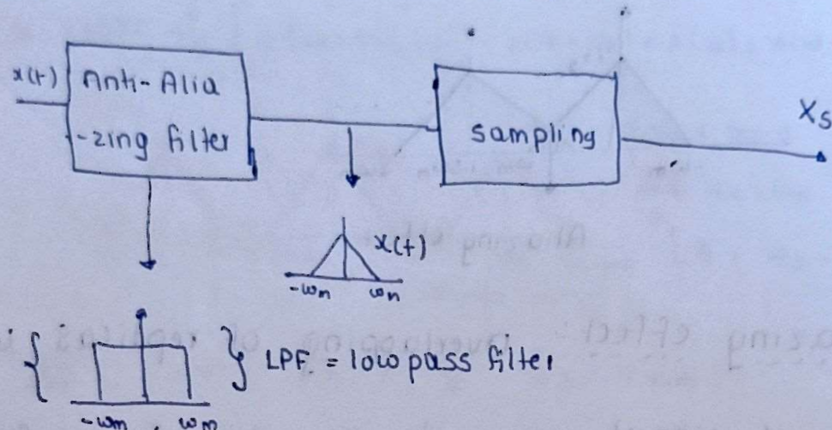
2) If the signal is not band limited to finite range

* for example, signal is like this... →



while replicating... the original signal is not formed.

* Due to those small signals under it, Now we take a filter called low pass filter to cut the fluctuating signals. The filter considered should have frequencies from $-\omega_m$ to ω_m



$\left\{ \right\}$ LPF = low pass filter

* Those small fluctuating signals will be cut by low pass filter.

Q Find the nyquist rate of $x(t) = \frac{\sin 4000\pi t}{\pi t}$

A $\omega_m = 4000\pi$

Nyquist $\omega_s = 2\omega_m$

$\omega_s = 2(4000\pi)$

$\omega_s = 8000\pi$

$\left\{ \begin{array}{l} \omega_s = 2\pi f_s \\ \omega_s = 2\pi f_m \end{array} \right\}$

Q Find the nyquist rate of $x(t) = 1 + \cos 2000\pi t + \sin 4000\pi t$.

A $\omega_{m_1} = 2000\pi$ & $\omega_{m_2} = 4000\pi$

ω_m = maximum frequency

so $\omega_m = 4000\pi$

$\omega_s = 2\omega_m = 8000\pi$

Q Find the nyquist rate of sinc 2000t.

A $\text{sinc } 2000t = \frac{\sin 2000\pi t}{2000\pi t}$

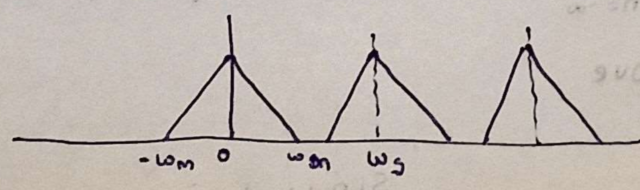
$\omega_m = 2000\pi$

$\omega_s = 2\omega_m = 4000\pi$

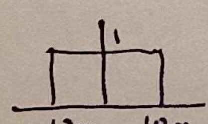
Signal deconstruction or Recovery :

The process of getting back the original signal from the sample signal is called signal recon-struction or signal recovery.

consider sample signal.

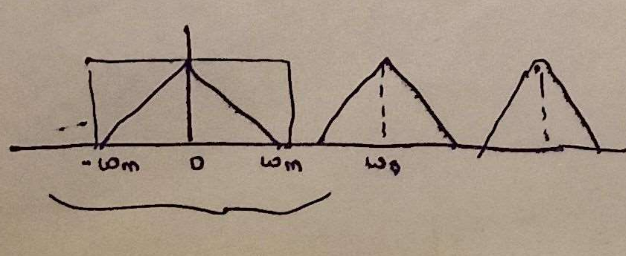


To retrive original signal, consider a low pass filter of frequency $-\omega_m$ to ω_m of amplitude 1

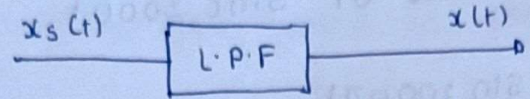


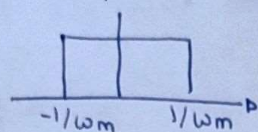
. Now we apply it to sample signal

we get the original signal



Here the band width of low pass filter should be of the range $\omega_m < B < \omega_s - \omega_m$ to retrieve the original.





$$x_3(t) = x_s(t) * h(t)$$

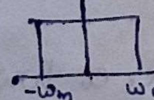
$$= \left(x(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nT_s)\right) * h(t)$$

$$= \left[\sum_{n=-\infty}^{\infty} x(t) \cdot \delta(t - nT_s) * h(t)\right]$$

$$= \left[\sum_{n=-\infty}^{\infty} x(nT_s)\, h(t - nT_s)\right]$$

$$x_r(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\, 2f_m \cdot \sin\left(2f_m(t - nT_s)\right]$$

In above

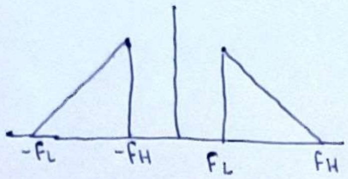


$$\longrightarrow \frac{\sin \omega_m t}{\pi t}$$

$$h(t) \longrightarrow 2f_m \frac{\sin 2f_m t}{2f_m t}$$

* Band Pass signal:-

A signal which has band of frequency from non-zero value to another non-zero value.



$f_s \geq 2B$ $\quad B_w = f_H - f_L = B$