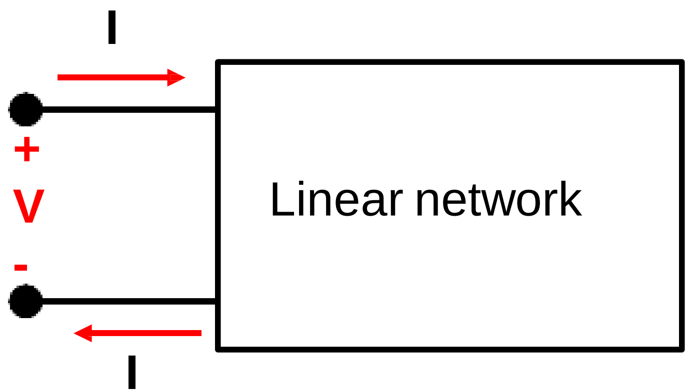
TWO PORT NETWORK PARAMETERS

CONTENTS:

- Definition of a PORT
- Single port and two port networks
- Z Parameters or Impedance parameters
- Y Parameters or Admittance parameters
- Hybrid parameters
- ABCD or Transmission parameters
- Relationship between parameter sets
- Reciprocity and Symmetry
- Interconnection of two port networks
- Reciprocity theorem

What is a **port**?

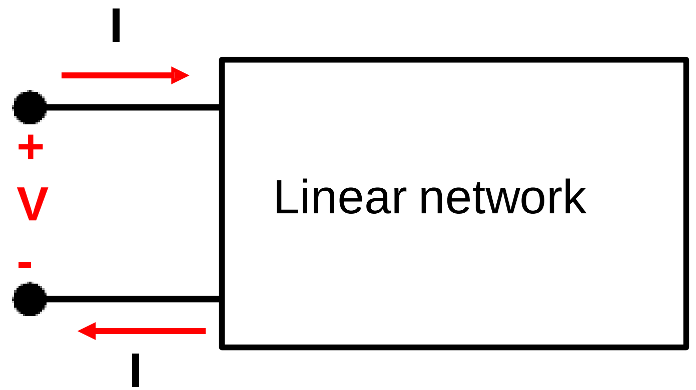
A pair of terminals through which a current may enter or leave a network is known as a ***port***.



One – port network

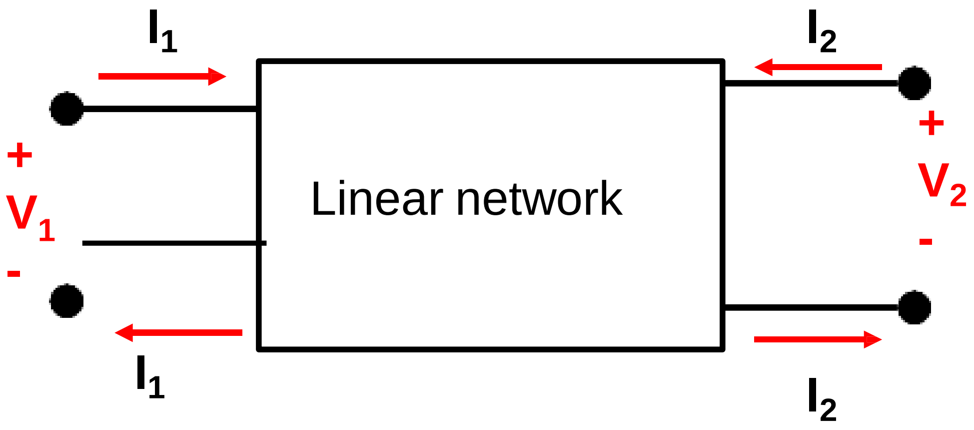
Single port and two port networks:

- Networks with two terminals in which current enters through one of the terminals and leaves from the other are called as Single port networks or One port networks.
- Two terminal devices or elements (such as resistors, capacitors, and inductors) results in one – port network.



One – port network

- Most of the circuits we have dealt with so far are two – terminal or one – port circuits.
- A two – port network is an electrical network with two separate ports for input and output.
- It has two terminal pairs acting as access points. The current entering one terminal of a pair leaves the other terminal in the pair.



Two – port network

Two (2) reason why to study two port – network:

- Such networks are useful in communication, control system, power systems and electronics.
- Knowing the parameters of a two – port network enables us to treat it as a “black box” when embedded within a larger network.

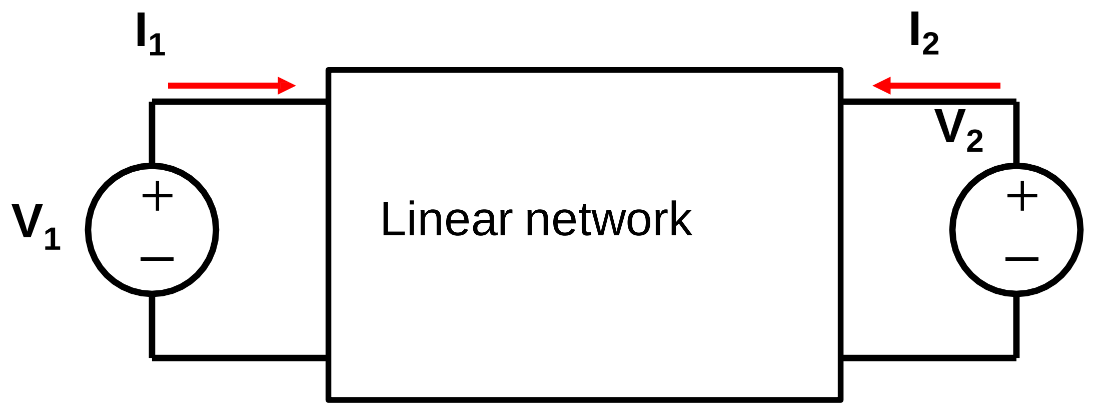
From the network, we can observe that there are 4 variables that is I_1, I_2, V_1 and V_2, which two are independent.

The various term that relate these voltages and currents are called *parameters*.

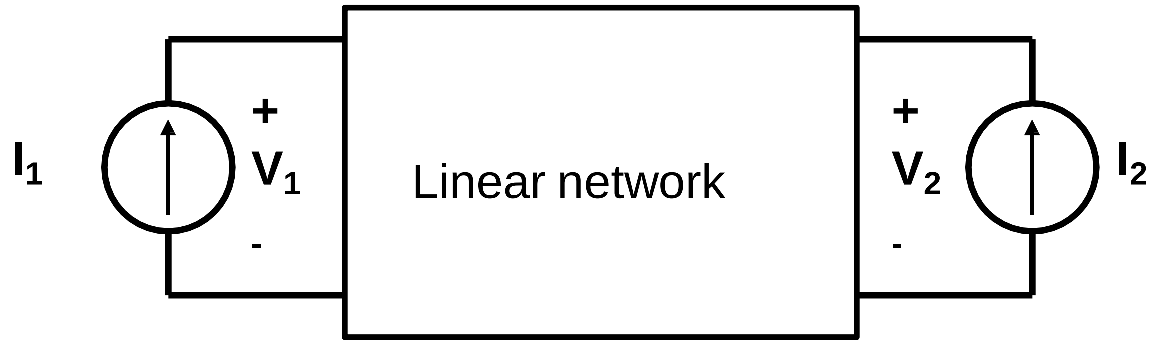
Z – PARAMETER:

- Z – parameter also called as impedance parameter or open circuit impedance parameters and the units is ohm (Ω).
- Impedance parameters is commonly used in the synthesis of filters and also useful in the design and analysis of impedance matching networks and power distribution networks.
- The two – port network may be voltage – driven or current – driven.

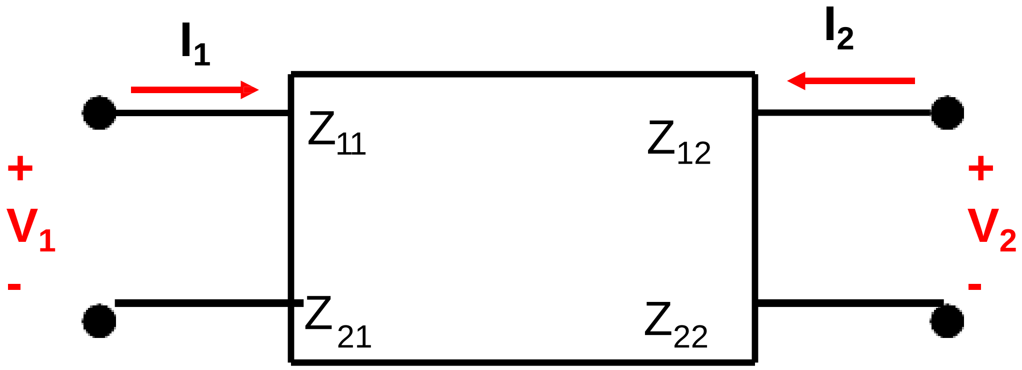
Two – port network driven by voltage source:



Two – port network driven by current sources:



The "black box" is replaced with Z-parameter is as shown below.



The terminal voltage can be related to the terminal current as:

$$V_1 = z_{11} I_1 + z_{12} I_2 \qquad (1)$$

$$V_2 = z_{21} I_1 + z_{22} I_2 \qquad (2)$$

In matrix form as:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

The Z-parameter that we want to determine are z_{11}, z_{12}, z_{21}, z_{22}.

The value of the parameters can be evaluated by setting:

1. $I_1 = 0$ (input port open – circuited)
2. $I_2 = 0$ (output port open – circuited)

Thus,

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \qquad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

Where,

z_{11} = open – circuit input impedance.

z_{12} = open – circuit transfer impedance from port 1 to port 2

z_{21} = open – circuit transfer Impedance from port 2 to port 1

z_{22} = open – circuit output impedance.

- Sometimes, z_{11} and z_{22} are called *driving-point impedances*, while z_{21} and z_{12} are called *transfer impedances.*

- A driving-point impedance is the input impedance of a two-terminal (one-port) device. Thus, z_{11} is the input driving-point impedance with the output port open-circuited.
- While z_{22} is the output driving-point impedance with the input port open circuited.

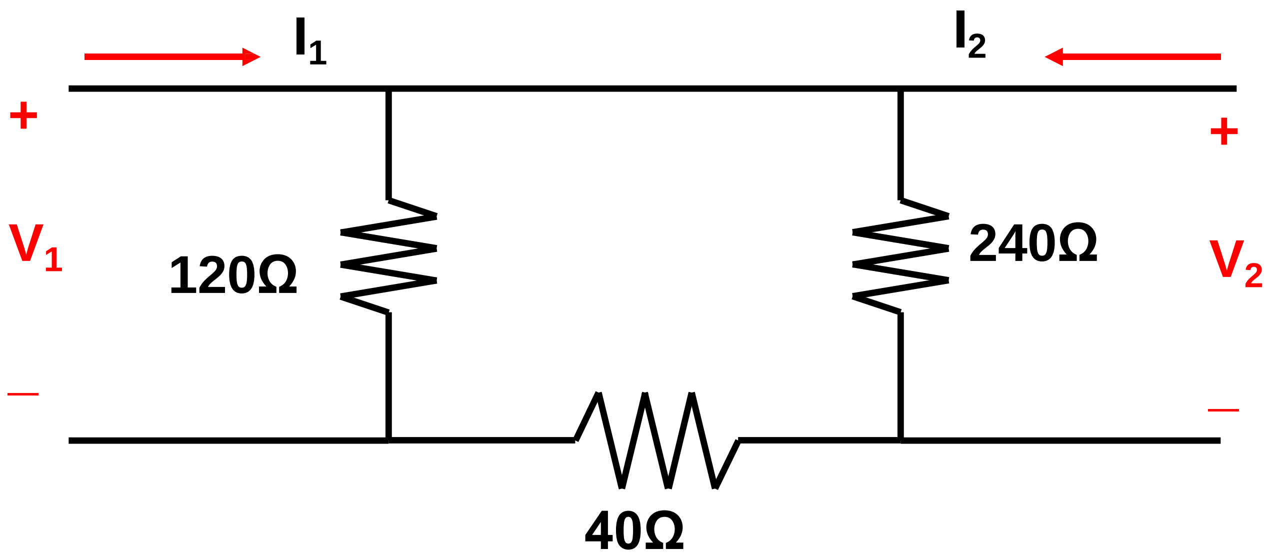
Condition for Symmetry and Reciprocity:

- When $\boxed{z_{21} = z_{12}}$, the two-port network is said to be ***symmetrical***.
- This implies that the network has mirror like symmetry about some center line; that is, a line can be found that divides the network into two similar halves.

- When the two-port network is linear and has no dependent sources, the transfer impedances are equal $\boxed{(z_{21} = z_{12})}$, and the two-port is said to be ***reciprocal***.
- This means that if the points of excitation and response are interchanged, the transfer impedances remain the same.

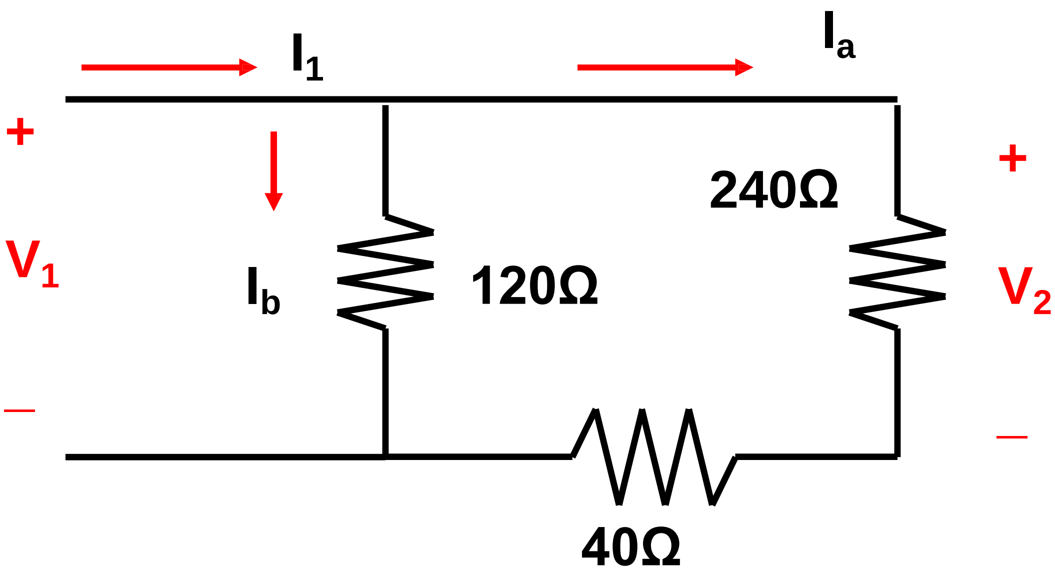
EXAMPLE:

Find the Z – parameter of the circuit below.



SOLUTION

i) $I_2 = 0$(open circuit port 2). Redraw the circuit.



$$V_1 = 120 I_b \ldots\ldots\ldots(1)$$

$$I_b = \frac{280}{400} I_1 \ldots\ldots(2)$$

$$\text{sub}(1) \rightarrow (2)$$

$$\therefore Z_{11} = \frac{V_1}{I_1} = 84\Omega$$

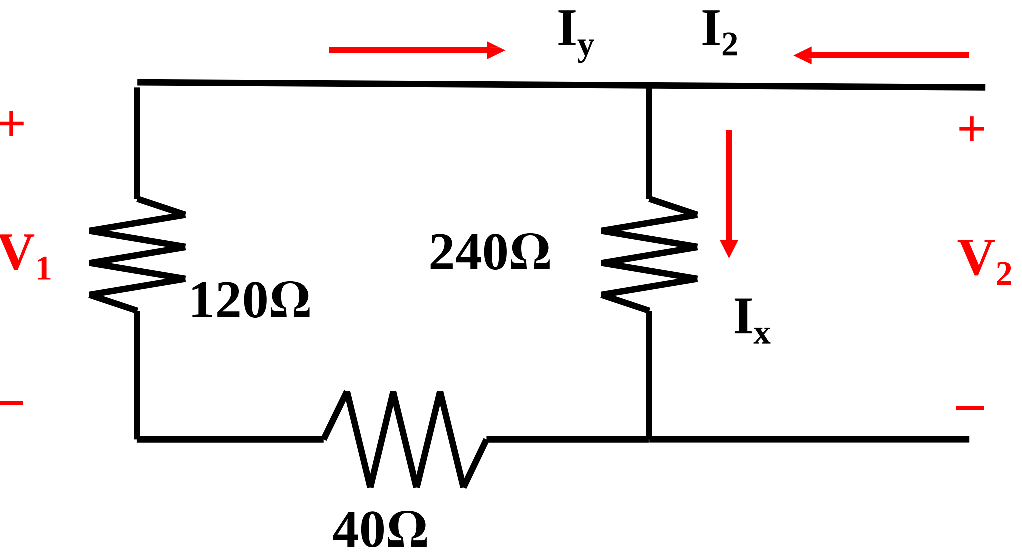
$$V_2 = 240 I_a \ldots\ldots\ldots(3)$$

$$I_a = \frac{120}{400} I_1 \ldots\ldots\ldots(4)$$

$$\text{sub}(4) \rightarrow (3)$$

$$\therefore Z_{21} = \frac{V_2}{I_1} = 72\Omega$$

ii) $I_1 = 0$ (open circuit port 1). Redraw the circuit.



$$V_2 = 240 I_x \ldots\ldots(1)$$

$$I_x = \frac{160}{400} I_2 \ldots\ldots(2)$$

$$\text{sub}\,(1) \rightarrow (2)$$

$$\therefore Z_{22} = \frac{V_2}{I_2} = 96\Omega$$

$$V_1 = 120 I_y \ldots\ldots(3)$$

$$I_y = \frac{240}{400} I_2 \ldots\ldots(4)$$

$$\text{sub}\,(4) \rightarrow (3)$$

$$\therefore Z_{12} = \frac{V_1}{I_2} = 72\Omega$$

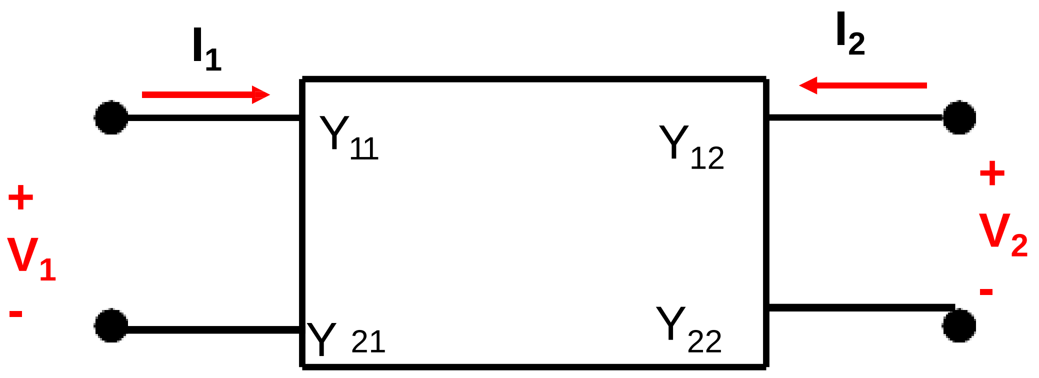
In matrix form:

$$[Z] = \begin{bmatrix} 84 & 72 \\ 72 & 96 \end{bmatrix}$$

Y - PARAMETER:

Y – parameter also called admittance parameter or short circuit admittance parameter and the units is siemens (S) or Mho(℧).

The "black box" that we want to replace with the Y-parameter is shown below.



The terminal current can be expressed in term of terminal voltage as:

$$I_1 = y_{11}V_1 + y_{12}V_2 \quad \text{———} \quad \textbf{(1)}$$

$$I_2 = y_{21}V_1 + y_{22}V_2 \quad \text{———} \quad \textbf{(2)}$$

In matrix form:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

The Y-parameters to be determined are Y_{11}, Y_{12}, Y_{21}, Y_{22}.

The values of the parameters can be evaluated by setting:

i) $V_1 = 0$ (input port short – circuited).

ii) $V_2 = 0$ (output port short – circuited).

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2 = 0} \qquad Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1 = 0}$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2 = 0} \qquad Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1 = 0}$$

Where,

y_{11} = short– circuit input admittance.

y_{12} = short– circuit transfer admittance from port 2 to port 1.

y_{21} = short– circuit transfer admittance from port 1 to port 2.

y_{22} = short– circuit output admittance.

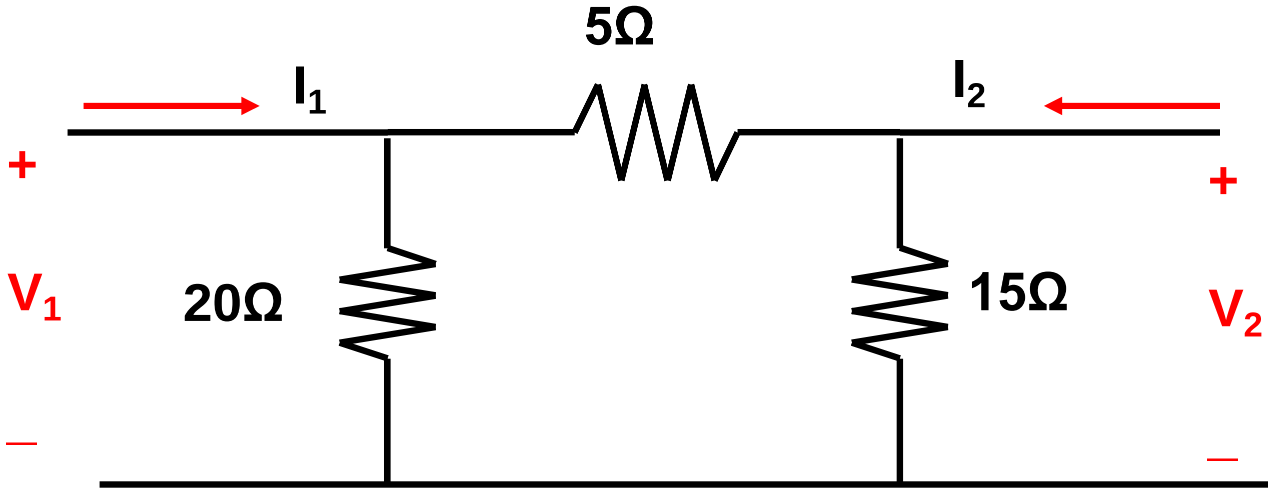
Condition for Symmetry and Reciprocity:

- When $y_{11} = y_{22}$, the two-port network is said to be ***symmetrical***.

- For a two-port network that is linear and has no dependent sources, the transfer admittances are equal ($y_{21} = y_{12}$).

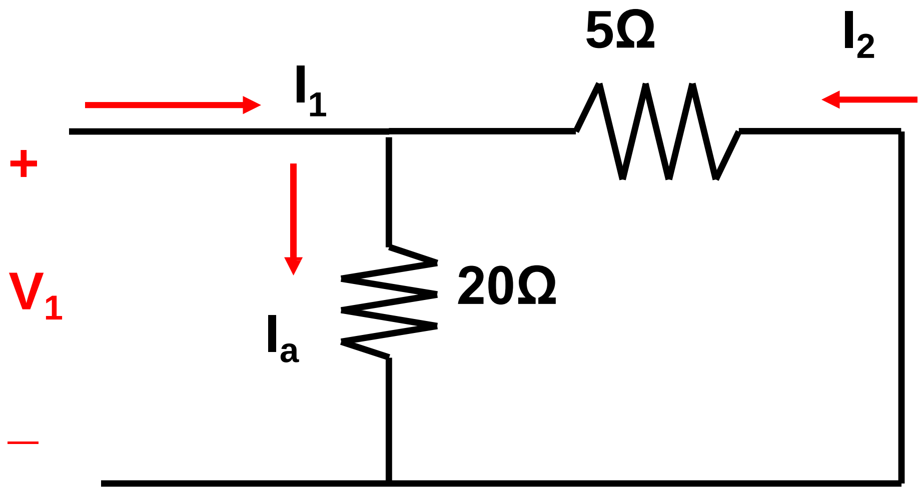
EXAMPLE:

Find the Y – parameter of the circuit shown below.



SOLUTION:

i) $V_2 = 0$



$$V_1 = 20 I_a \ldots\ldots\ldots(1)$$

$$I_a = \frac{5}{25} I_1 \ldots\ldots\ldots(2)$$

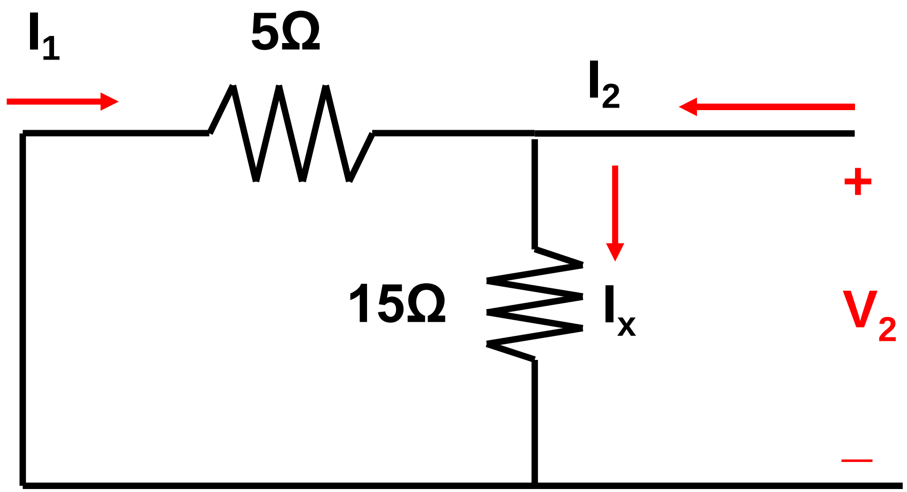
$$\text{sub } (1) \rightarrow (2)$$

$$\therefore Y_{11} = \frac{I_1}{V_1} = \frac{1}{4} S$$

$$V_1 = -5 I_2$$

$$\therefore Y_{21} = \frac{I_2}{V_1} = -\frac{1}{5} S$$

ii) $V_1 = 0$



$$V_2 = 15I_x \ldots\ldots\ldots(3)$$

$$I_x = \frac{5}{25} I_2 \ldots\ldots\ldots(4)$$

sub $(3) \rightarrow (4)$

$$\therefore Y_{22} = \frac{I_2}{V_2} = \frac{4}{15} S$$

$$V_2 = -5I_1$$

$$\therefore Y_{12} = \frac{I_1}{V_2} = -\frac{1}{5} S$$

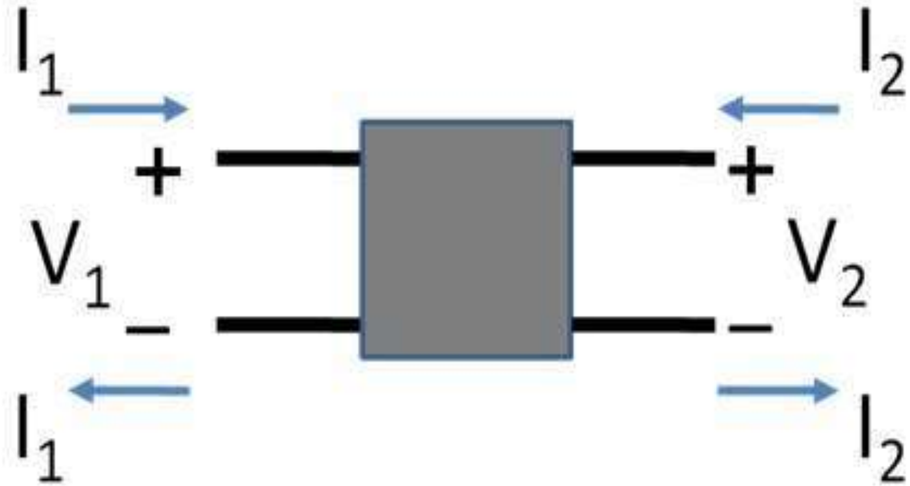
In matrix form;

$$\boxed{[Y] = \begin{bmatrix} \frac{1}{4} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{4}{15} \end{bmatrix} S}$$

H – PARAMETER:

The z and y parameters of a two port network do not always exist. So, there is a need for developing another set of parameters, in which V_1 and I_2 are made dependent variables.

Those four parameters are called the hybrid parameters or H-parameters, one is measured in terms of ohm, one in mho and other two are dimensionless. Since these parameters has mixed dimensions, so they are called as hybrid parameters.



$$\boxed{\begin{aligned} \mathbf{V}_1 &= \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2 \\ \mathbf{I}_2 &= \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2 \end{aligned}}$$

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{h}_{11} & \mathbf{h}_{12} \\ \mathbf{h}_{21} & \mathbf{h}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix} = [\mathbf{h}] \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

$$\mathbf{h}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0}, \quad \mathbf{h}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0}$$

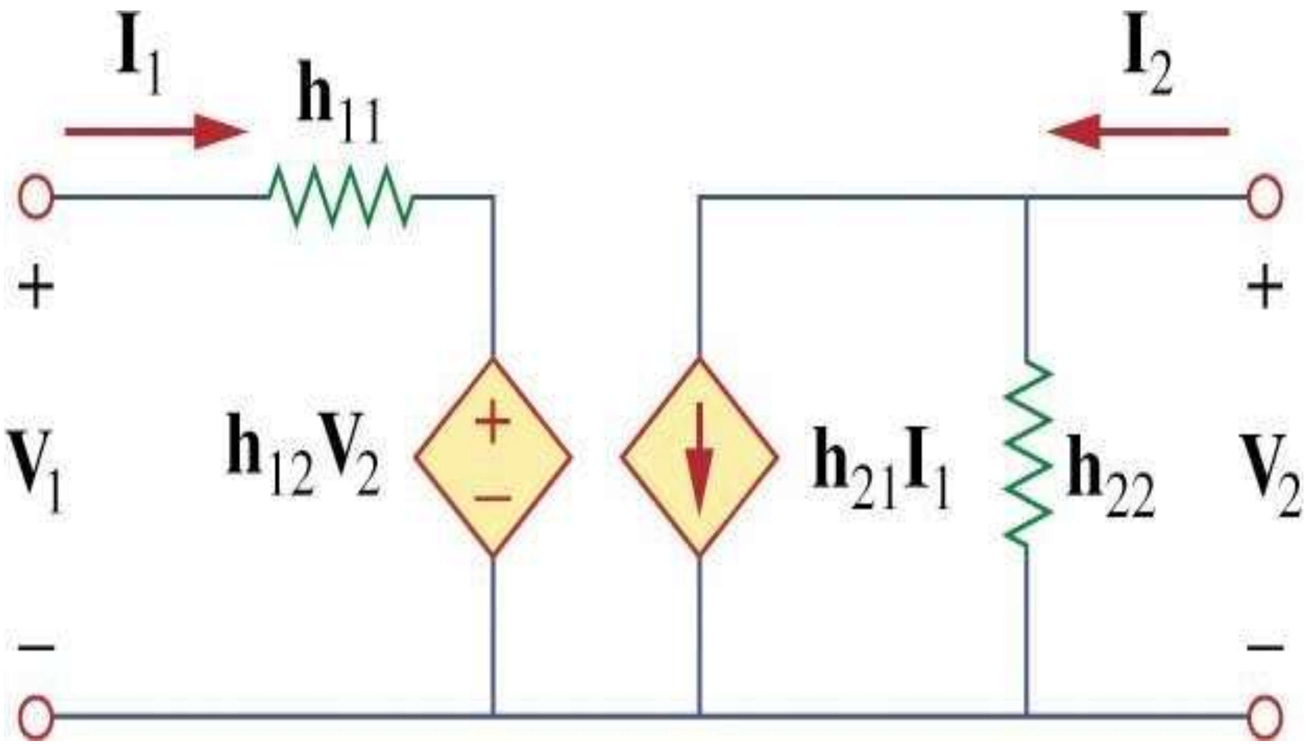
$$\mathbf{h}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0}, \quad \mathbf{h}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0}$$

h_{11} = Short-circuit input impedance

h_{12} = Open-circuit reverse voltage gain

h_{21} = Short-circuit forward current gain

h_{22} = Open-circuit output admittance



Hybrid parameters are very much useful for describing electronic devices such as transistors

Condition for Symmetry and Reciprocity:

- For symmetrical networks, $h_{11} h_{22} - h_{12} h_{21} = 1$

- For reciprocal networks, $h_{12} = -h_{21}$

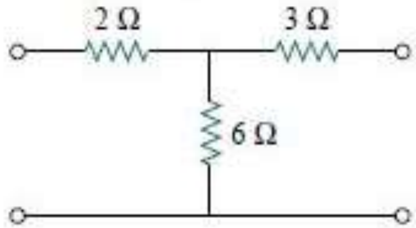


Figure 18.22 For Example 18.5.

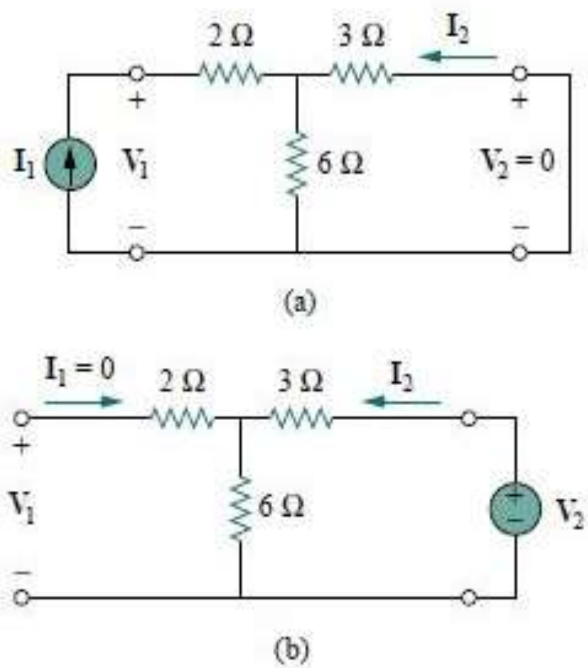


Figure 18.23 For Example 18.5: (a) computing $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$, (b) computing $\mathbf{h}_{12}$ and $\mathbf{h}_{22}$.

Find the hybrid parameters for the two-port network of Fig. 18.22.

Solution:

To find $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$, we short-circuit the output port and connect a current source $\mathbf{I}_1$ to the input port as shown in Fig. 18.23(a). From Fig. 18.23(a),

$$\mathbf{V}_1 = \mathbf{I}_1(2 + 3 \parallel 6) = 4\mathbf{I}_1$$

Hence,

$$\mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 4\ \Omega$$

Also, from Fig. 18.23(a) we obtain, by current division,

$$-\mathbf{I}_2 = \frac{6}{6+3}\mathbf{I}_1 = \frac{2}{3}\mathbf{I}_1$$

Hence,

$$\mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = -\frac{2}{3}$$

To obtain $\mathbf{h}_{12}$ and $\mathbf{h}_{22}$, we open-circuit the input port and connect a voltage source $\mathbf{V}_2$ to the output port as in Fig. 18.23(b). By voltage division,

$$\mathbf{V}_1 = \frac{6}{6+3}\mathbf{V}_2 = \frac{2}{3}\mathbf{V}_2$$

Hence,

$$\mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{2}{3}$$

Also,

$$\mathbf{V}_2 = (3 + 6)\mathbf{I}_2 = 9\mathbf{I}_2$$

Thus,

$$\mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{9}\ \mathrm{S}$$

Determine the h parameters for the circuit in Fig. 18.24.

Answer: $\mathbf{h}_{11} = 1.2\ \Omega$, $\mathbf{h}_{12} = 0.4$, $\mathbf{h}_{21} = -0.4$, $\mathbf{h}_{22} = 0.4$ S.

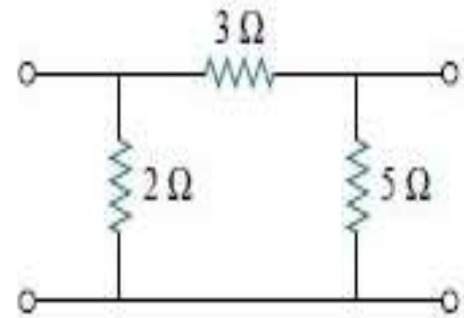
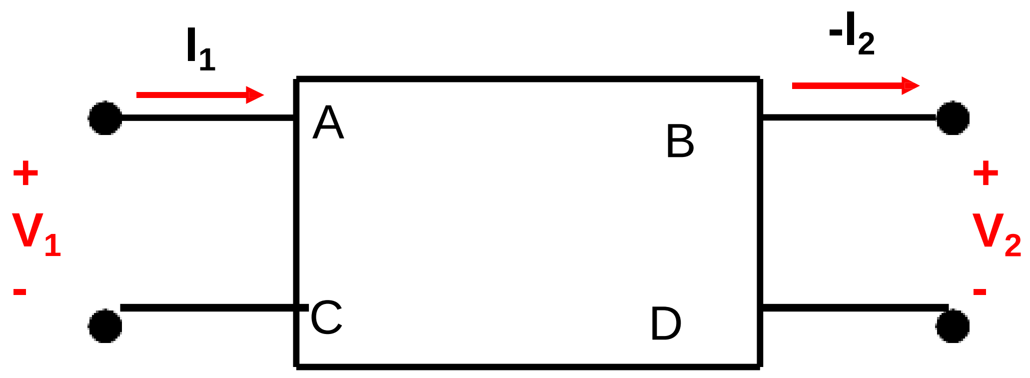


Figure 18.24 For Practice Prob. 18.5.

T (ABCD) PARAMETER

- T – parameter or ABCD – parameter is a another set of parameters relates the variables at the input port to those at the output port.
- T – parameter also called *transmission parameters* because this parameter are useful in the analysis of transmission lines because they express sending – end variables (V_1 and I_1) in terms of the receiving – end variables (V_2 and $-I_2$).

The "black box" that we want to replace with T – parameter is as shown below.



The equation is:

$$V_1 = AV_2 - BI_2 \ldots\ldots(1)$$

$$I_1 = CV_2 - DI_2 \ldots\ldots(2)$$

In matrix form is:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

The T – parameter that we want determine are A, B, C and D where A and D are dimensionless, B is in ohm (Ω) and C is in siemens (S).

The values can be evaluated by setting

i) $I_2 = 0$ (input port open circuit)

ii) $V_2 = 0$ (output port short circuit)

Thus;

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad B = \left.\frac{V_1}{I_2}\right|_{V_2=0}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} \qquad D = \left.\frac{I_1}{I_2}\right|_{V_2=0}$$

In terms of the transmission parameter a network is reciprocal if,

$$\boxed{AD - BC = 1}$$

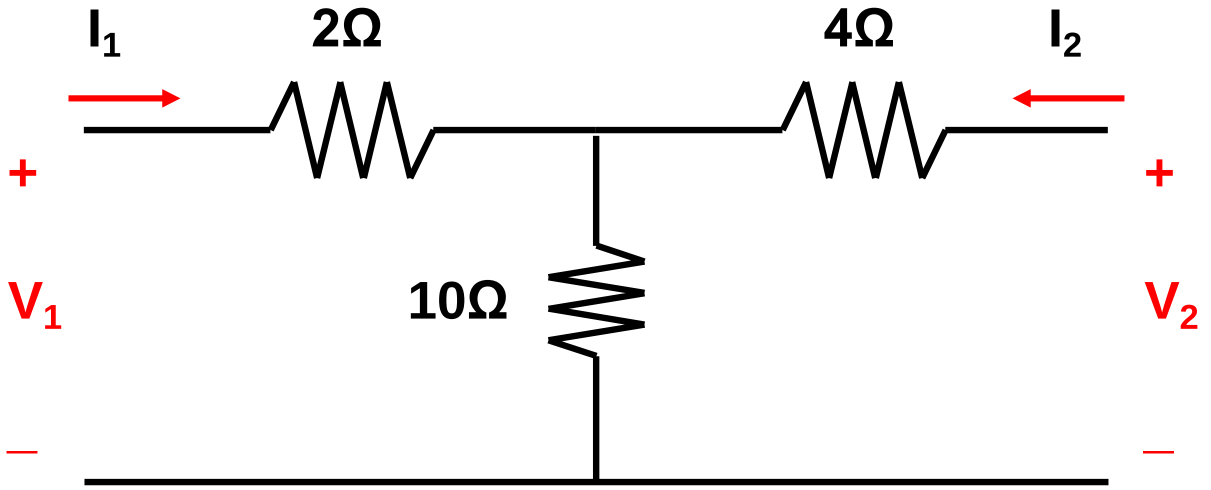
Condition for Symmetry and Reciprocity:

- For symmetrical networks, $A=D$

- For reciprocal networks, $AD-BC=1$

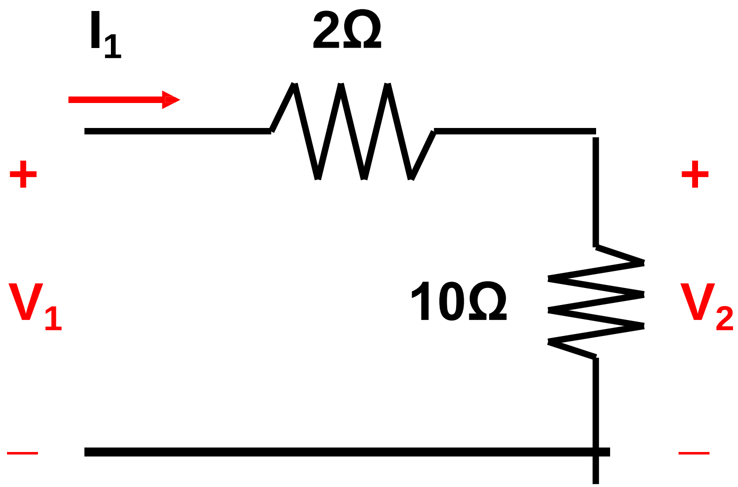
EXAMPLE:

Find the ABCD – parameter of the circuit shown below.



SOLUTION:

i) $I_2 = 0$,



$$V_2 = 10I_1$$

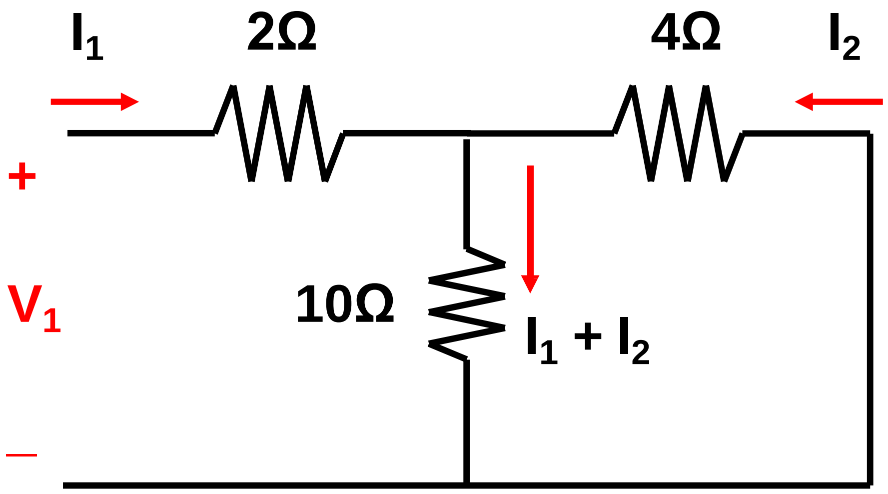
$$\therefore C = \frac{I_1}{V_2} = 0.1S$$

$$V_1 = 2I_1 + V_2$$

$$V_1 = 2\left(\frac{V_2}{10}\right) + V_2 = \frac{6}{5}V_2$$

$$\therefore A = \frac{V_1}{V_2} = 1.2$$

ii) $V_2 = 0$,



$$\left[T\right] = \begin{bmatrix} 1.2 & 6.8 \\ 0.1 & 1.4 \end{bmatrix}$$

$$I_2 = -\frac{10}{14} I_1$$

$$\therefore D = -\frac{I_1}{I_2} = 1.4$$

$$V_1 = 2I_1 + 10\left(I_1 + I_2\right)$$

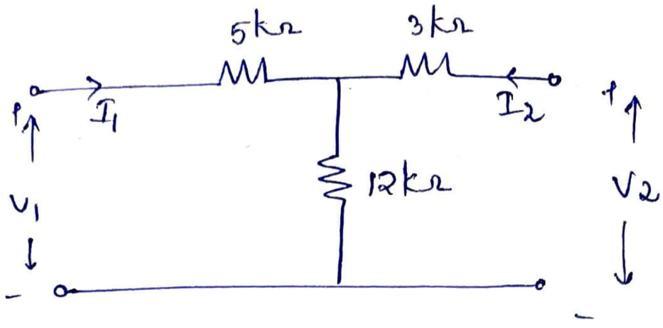
$$V_1 = 12I_1 + 10I_2$$

$$V_1 = 12\left(-\frac{14}{10} I_2\right) + 10I_2$$

$$\therefore B = -\frac{V_1}{I_2} = 6.8\Omega$$

Numericals

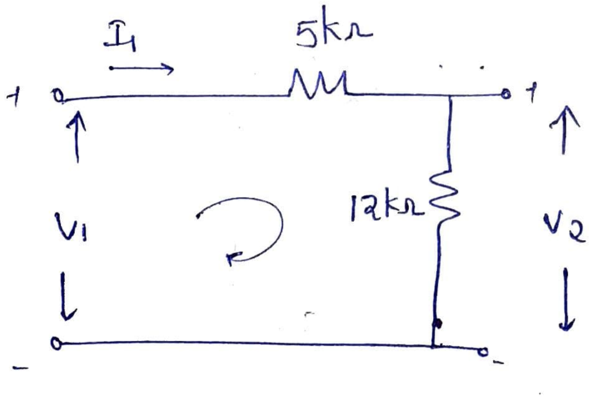
1Q) find the z-parameters for the network shown.



Sol) Governing equations of Z-parameter network,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

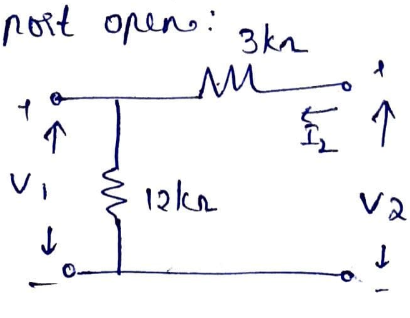
i) $I_2 = 0$ ie output port open :



$$Z_{11} = \frac{V_1}{I_1} = \frac{(5k + 12k)\, I_1}{I_1} = 17\,k\Omega$$

$$Z_{21} = \frac{V_2}{I_1} = \frac{(12k)\, I_1}{I_1} = 12\,k\Omega$$

ii) $I_1 = 0$ ie input port open :



$$Z_{12} = \frac{V_1}{I_2} = \frac{(12k)I_2}{I_2}$$

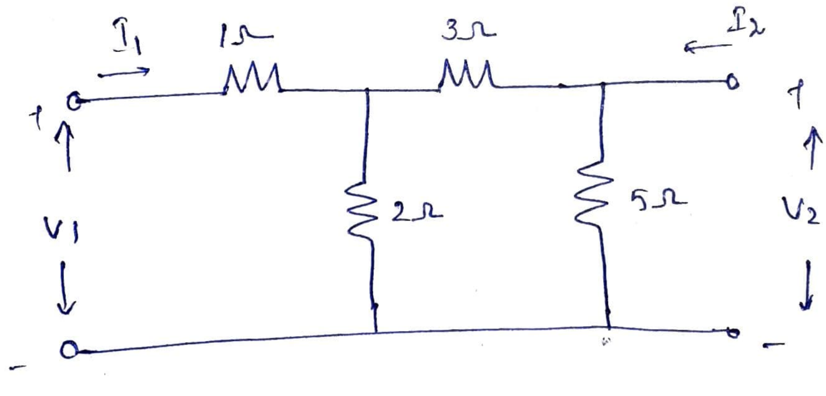
$$= 12k\Omega$$

$$Z_{22} = \frac{V_2}{I_2} = \frac{(3k+12k)\,I_2}{I_2} = 15k\,\Omega$$

$\therefore$ $Z_{11} = 17\,k\Omega$ $\quad$ $Z_{12} = 12\times10^3\,\Omega$

$Z_{21} = 12k\Omega$ $\quad$ $Z_{22} = 15k\Omega.$

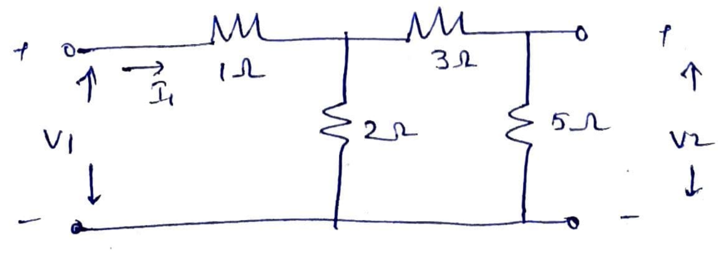
2Q) find the z-parameters for the circuit shown:



Sol) z-parameter matrix equation:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [Z]\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

i) Put $I_2 = 0$ ie make output port open:



$$Z_{11} = \frac{V_1}{I_1} = \frac{\left(1 + 2 \parallel (3+5)\right) I_1}{I_1}$$

$$= 1 + (2 \parallel 8)$$

$$= 1 + \frac{2 \times 8}{10}$$

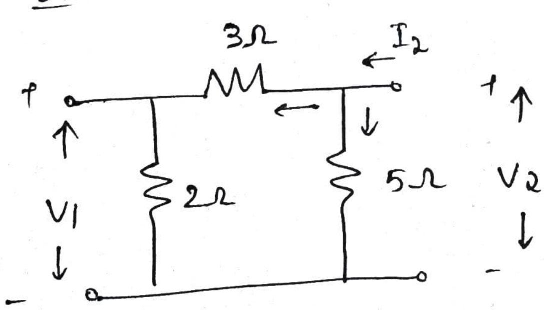
$$Z_{11} = \boxed{2.6\,\Omega}$$

$$Z_{21} = \frac{V_2}{I_1} = \frac{\left(I_1 \times \frac{2}{2+8}\right) \times 5}{I_1}$$

$$= \frac{2}{10} \times 5$$

$$= \boxed{1\,\Omega}$$

ii, $I_1 = 0$ ie input open circuited :



$$Z_{12} = \frac{V_1}{I_2} = \cancel{\frac{5 \times I_1}{5+5}} \frac{\left(\frac{5 \times I_2}{5+3+2}\right) \times 2}{I_2}$$

$$= \frac{5}{10} \times 2 = \boxed{1\,\Omega}$$

$$Z_{22} = \frac{V_2}{I_2} = \frac{\left(\frac{(3+2)}{3+2+5} I_2\right) \times 5}{I_2}$$

$$= \frac{5}{10} \times 5 = \boxed{2.5\,\Omega}$$

$$\therefore [Z] = \begin{bmatrix} 2.6 & 1 \\ 1 & 2.5 \end{bmatrix} \Omega$$

Q3) The following readings are obtained experimentally for an unknown two port network:

	V_1	V_2	I_1	I_2
o/p open:	100V	60V	10A	0
i/p open:	30V	40V	0A	3A

Compute Z-parameters.

Sol) $I_2 = 0$ i.e. o/p opened:

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{100}{10} = 10\Omega.$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{60}{10} = 6\Omega$$

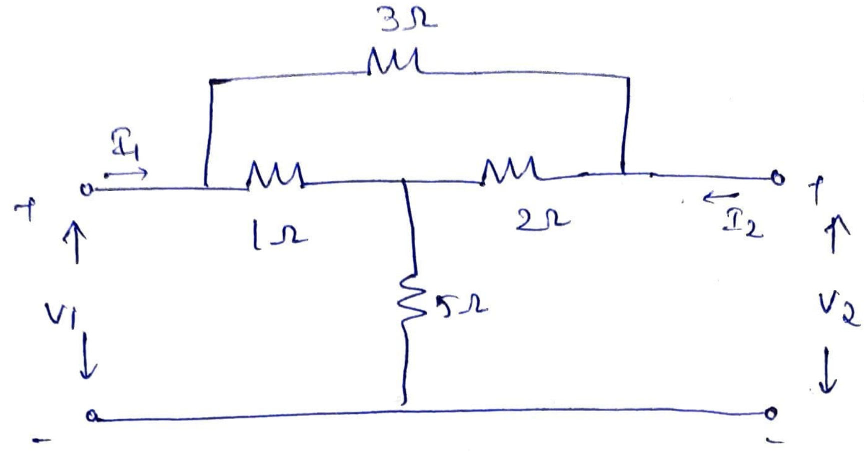
$I_1 = 0$ i.e. i/p opened:

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{30}{3} = 10\Omega$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{40}{3} = 13.33\Omega$$

$$\therefore [Z] = \begin{bmatrix} 10 & 10 \\ 6 & 13.33 \end{bmatrix} \Omega$$

Q4) Obtain the open circuit parameters for the below n/w.



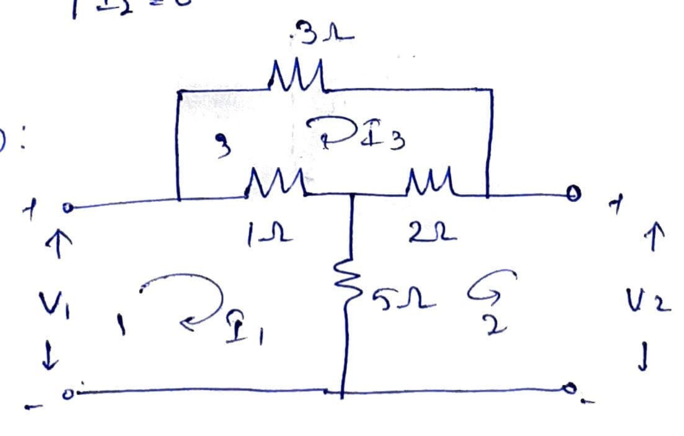
Sol) Governing equations for Z-parameter network are,

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \quad \text{and} \quad Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$I_2 = 0$:



KVL:

loop 1 $V_1 = (I_1 - I_3)1 + I_1(5) \Rightarrow V_1 = 6I_1 - I_3 \rightarrow (1)$

loop 3: and $3I_3 + 2(I_3) + 1(I_3 - I_1) = 0$

$$\Rightarrow 6I_3 - I_1 = 0$$

$$\Rightarrow I_3 = \frac{1}{6} I_1 \rightarrow (2)$$

Substitute eq ② in eq ①,

$$V_1 = 6I_1 - I_3$$

$$= 6I_1 - \frac{1}{6}I_1$$

$$\Rightarrow V_1 = \frac{35}{6}I_1$$

$$\Rightarrow \boxed{Z_{11} = \frac{V_1}{I_1} = \frac{35}{6}\,\Omega}$$

Apply KVL in loop 2,

$$V_2 = 2I_3 + 5I_1$$

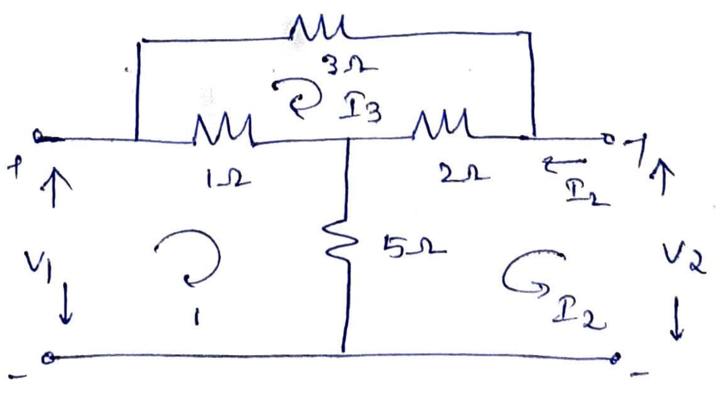
$$= 2\left(\frac{1}{6}I_1\right) + 5I_1$$

$$\Rightarrow V_2 = I_1\left(\frac{16}{3}\right)$$

$$\Rightarrow \boxed{Z_{21} = \frac{V_2}{I_1} = \frac{16}{3}\,\Omega}$$

Now, $Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$ $\qquad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$

$I_1 = 0$:



Apply KVL in loop 3 and 2:

loop 3:

$$3I_3 + 2(I_3 + I_2) + 1I_3 = 0$$

$$\Rightarrow 6I_3 + 2I_2 = 0$$

$$\Rightarrow I_3 = -\frac{1}{3} I_2 \longrightarrow (3)$$

loop 2:

$$V_2 = 2(I_2 + I_3) + 5I_2$$

$$\Rightarrow V_2 = 7I_2 + 2I_3$$

$$= 7I_2 + 2\left(\frac{-1}{3}\right) I_2$$

$$= \left(7 - \frac{2}{3}\right) I_2$$

$$= \frac{19}{3} I_2$$

$$\Rightarrow \boxed{Z_{22} = \frac{V_2}{I_2} = \frac{19}{3}\,\Omega}$$

loop 1:

$$V_1 = -I_3(1) + 5I_2$$

$$\Rightarrow V_1 = -\left(-\frac{1}{3} I_2\right) + 5I_2$$

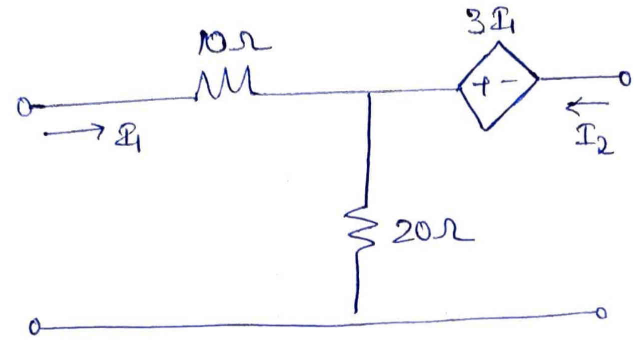
$$\Rightarrow V_1 = \left(5 + \frac{1}{3}\right) I_2 = \cancel{\frac{16}{3}} \quad \frac{16}{3} I_2$$

$$\Rightarrow \boxed{Z_{12} = \frac{V_1}{I_2} = \frac{16}{3}\,\Omega}$$

$$\therefore [Z] = \begin{bmatrix} 35/6 & 16/3 \\ 16/3 & 19/3 \end{bmatrix} \Omega$$

Numericals on T-parameters (or) ABCD parameters:

Q1) Find the transmission parameters for the two port network shown.

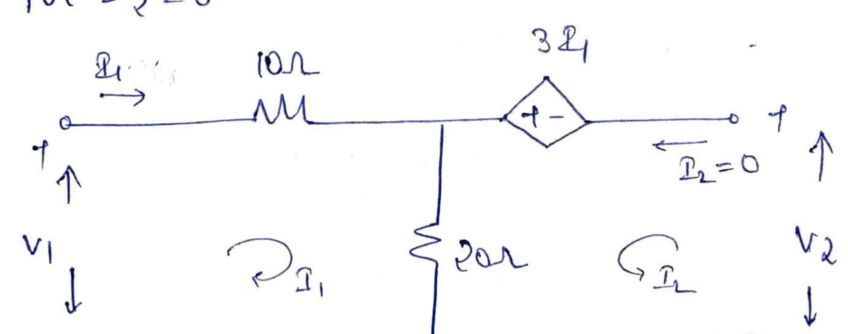


Sol) Transmission parameter n/w equations are,

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

i) Put $I_2 = 0$



Apply KVL in loop 2,

$$V_2 = -3I_1 + 20I_1 \Rightarrow V_2 = 17I_1 \rightarrow (1)$$

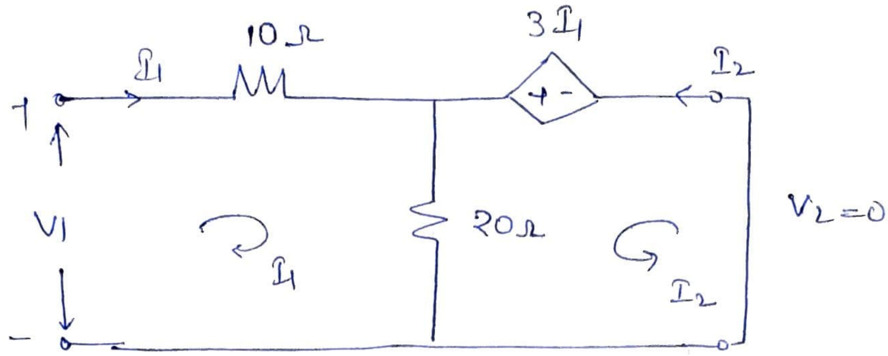
$$\Rightarrow \frac{I_1}{V_2} = \frac{1}{17}$$

$$\Rightarrow \boxed{C = \frac{1}{17}\ \mho}$$

$$A = \frac{V_1}{V_2} = \frac{(10+20)\, I_1}{\cancel{20\, I_1}\ 17 I_1} = \frac{30\, I_1}{17\, I_1}$$

$$\Rightarrow \boxed{A = \frac{30}{17}}$$

ii, $\underline{V_2 = 0}$: ie short circuit the output port:



Apply KVL in loop 2,

$$-3I_1 + 20(I_1 + I_2) = 0.$$

$$\Rightarrow -3I_1 + 20 I_1 = -20 I_2$$

$$\Rightarrow 17 I_1 = -20 I_2$$

$$\Rightarrow \frac{I_1}{I_2} = -\frac{20}{17} \longrightarrow (2)$$

$$\therefore D = -\frac{I_1}{I_2} = -\left(-\frac{20}{17}\right) = \frac{20}{17} \qquad \boxed{\therefore D = \frac{20}{17}}$$

Apply KVL in loop 1,

$$V_1 = 10 I_1 + 20(I_1 + I_2) \Rightarrow V_1 = 30 I_1 + 20 I_2$$

$$\Rightarrow \quad V_1 = 30\left(\frac{-20}{17} I_2\right) + 20 I_2 \qquad (\because \text{from eq } ②)$$

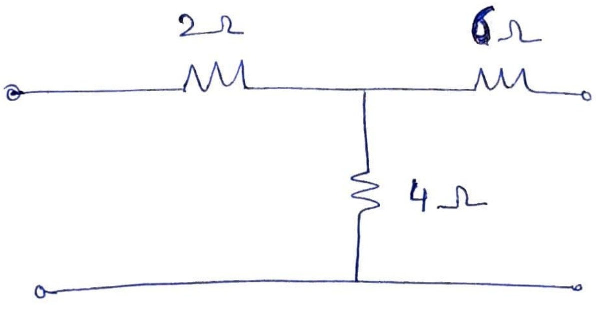
$$= \left(-\frac{600}{17} + 20\right) I_2$$

$$\Rightarrow V_1 = \frac{-260}{17} I_2$$

$$\Rightarrow \quad B = -\frac{V_1}{I_2} = \frac{260}{17} \Rightarrow \boxed{B = \frac{260}{17}\ \Omega}$$

$$\therefore \quad [T] = \begin{bmatrix} \frac{30}{17} & \frac{260}{17}\ \Omega \\ \frac{1}{17}\ \mho & \frac{20}{17} \end{bmatrix}$$

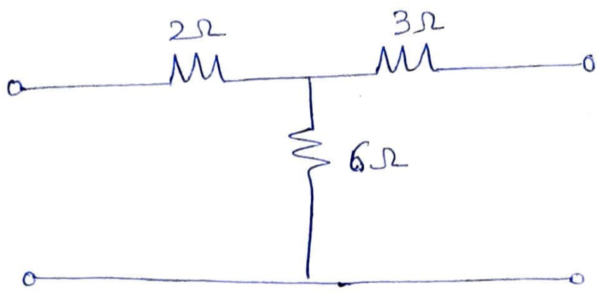
Q2) find the transmission parameters for the below circuit.



<u>Answer</u>: $A = 1.5$, $B = 11\,\Omega$, $C = 0.25\,S$, $D = 2.5$

Numericals on h-parameters:

Q1) Find the hybrid parameters for the twoport network shown.

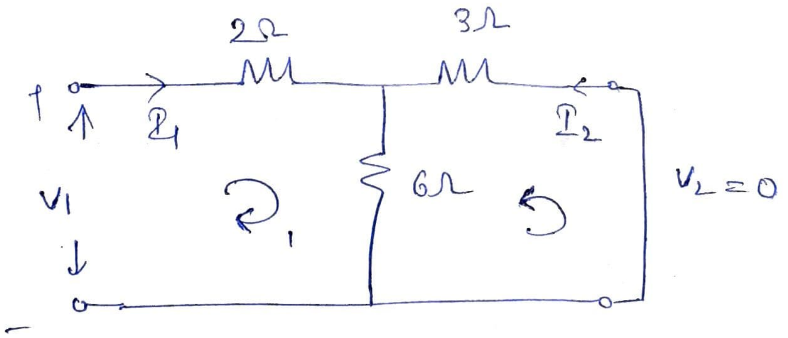


Sol) The governing equation for h-parameter n/w is,

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2 .$$

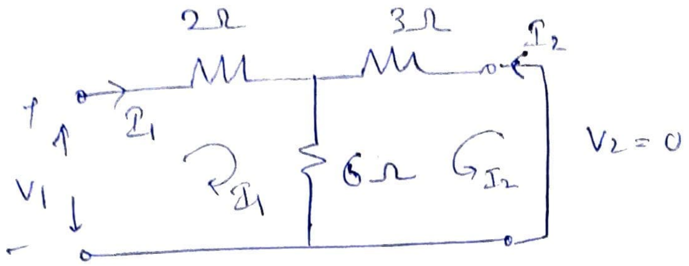
i) Make $V_2 = 0$ i.e. short circuit the output port:



Apply KVL in loop 1,

$$V_1 = 2I_1 + 6(I_1 + I_2)$$

$$\Rightarrow V_1 = 8I_1 + 6I_2$$



$$h_{11} = \frac{V_1}{I_1} = \frac{(2 + 3 \| 6)\, I_1}{I_1}$$

$$= \frac{\left(2 + \frac{3 \times 6}{3 + 6}\right) I_1}{I_1}$$

$$= 2 + 2$$

$$= 4$$

$$\Rightarrow \boxed{h_{11} = 4}$$

Apply KVL in loop 1.

$$V_1 = 2 I_1 + 6 (I_1 + I_2)$$

$$V_1 = 8 I_1 + 6 I_2$$

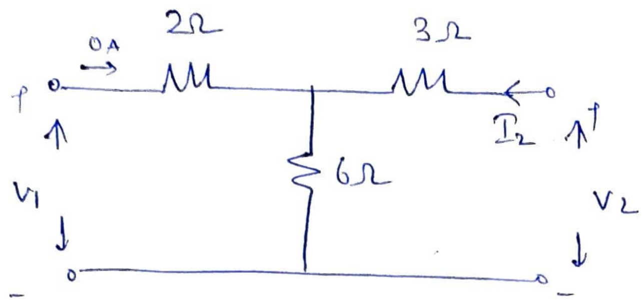
$$\Rightarrow 4 I_1 = 8 I_1 + 6 I_2$$

$$\Rightarrow -4 I_1 = 6 I_2$$

$$\Rightarrow \frac{I_2}{I_1} = -\frac{2}{3}$$

$$\Rightarrow \boxed{h_{21} = -\frac{2}{3}}$$

Now, (ii) $I_1 = 0$ ie input is opened,



$$h_{22} = \cancel{\frac{V_2}{I_2}} \quad \frac{I_2}{V_2} = \frac{I_2}{(3+6)I_2} = \boxed{\frac{1}{9}\ \mho = h_{22}}$$

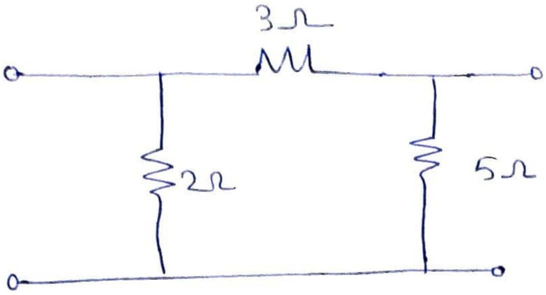
$$h_{12} = \frac{V_1}{V_2} = \frac{6(I_2)}{(3+6)I_2}$$

$$= \frac{6}{9} = \frac{2}{3}$$

$$\therefore \boxed{h_{12} = \frac{2}{3}}$$

$$\therefore [h] = \begin{bmatrix} 4\ \Omega & 2/3 \\ -2/3 & \frac{1}{9}\mho \end{bmatrix}$$

Q2) Determine the h-parameters for the circuit,

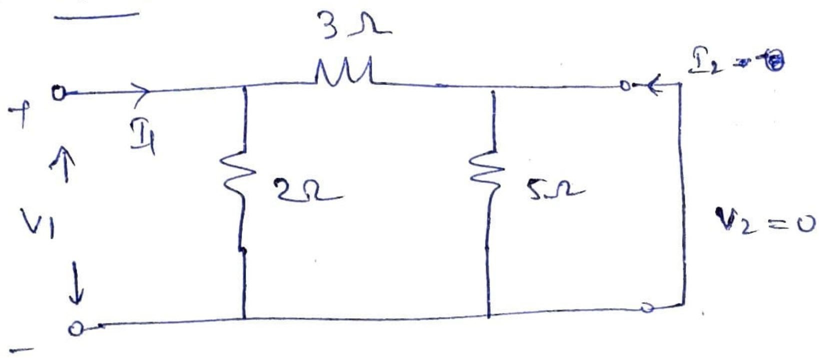


Sol) H-parameter equations are,

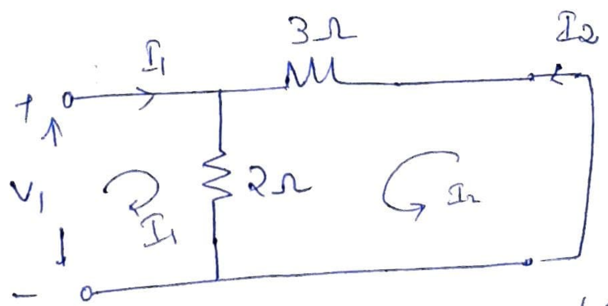
$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

i, $V_2 = 0$:



⇓



$$h_{11} = \frac{V_1}{I_1} = \frac{(2 \| 3)\, I_1}{I_1} = \frac{2 \times 3}{5} = \frac{6}{5}$$

$$\therefore \boxed{h_{11} = \frac{6}{5}}\,\Omega \Rightarrow \boxed{h_{11} = 1.2\,\Omega}$$

Apply KVL in loop 1,

$$V_1 = 2(I_1 + I_2)$$

$$\Rightarrow \frac{6}{5} I_1 = 2I_1 + 2I_2 \qquad \left(\because \text{ from } h_{11} = \frac{V_1}{I_1} = \frac{6}{5} \right.$$

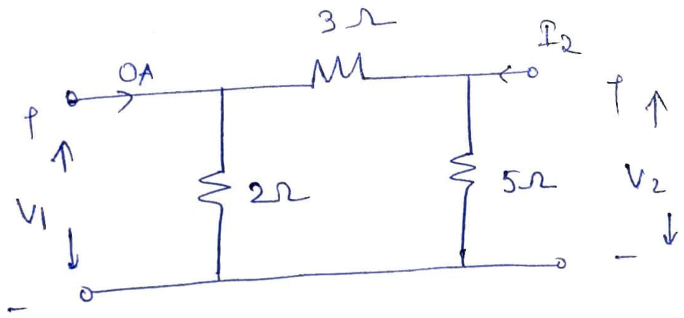
$$\left. \Rightarrow V_1 = \frac{6}{5} I_1 \right)$$

$$\Rightarrow \left(\frac{6}{5} - 2\right) I_1 = 2I_2$$

$$\Rightarrow -\frac{4}{5} I_1 = 2I_2$$

$$\Rightarrow h_{21} = \frac{I_2}{I_1} = -\frac{2}{5} = \boxed{-0.4 = h_{21}}$$

ii) $\underline{I_1 = 0}$ i.e., $\underline{\text{input opened}}$:-



$$h_{22} = \frac{I_2}{V_2} = \cancel{\frac{I_2}{I_2((5 \| (3+2))/5}}$$

(where V_2 = current flowing through 5 Ω × resistance 5Ω

$$= I_2 \times \frac{(3+2)}{(3+2)+5} \times 5$$

$$= I_2 \times \frac{25}{10} \quad)$$

$$\therefore h_{22} = \frac{I_2}{V_2} = \frac{I_2}{I_2 \times \frac{25}{10}} = \frac{10}{25} \Rightarrow \boxed{h_{22} = 0.4\,S}$$

$$h_{12} = \frac{V_1}{V_2} = \frac{2 \times (\text{current flowing through } 2\Omega)}{5 \times (\text{current flowing through } 5\Omega)}$$

$$= \frac{2 \times \frac{I_2}{2}}{5 \times \frac{I_2}{2}}$$

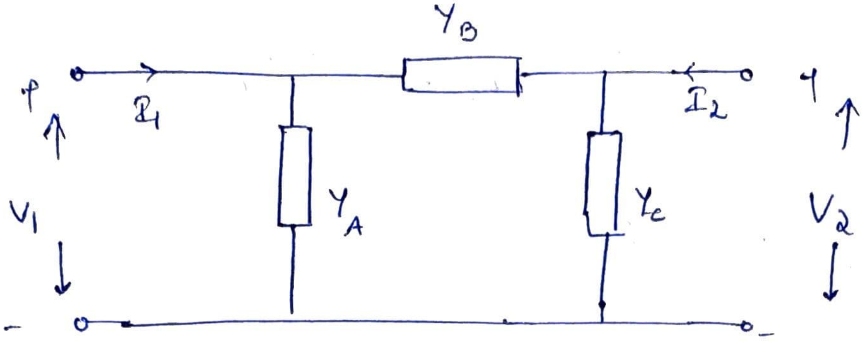
$$= \frac{2}{5} = 0.45$$

$$\therefore \boxed{h_{12} = 0.45}$$

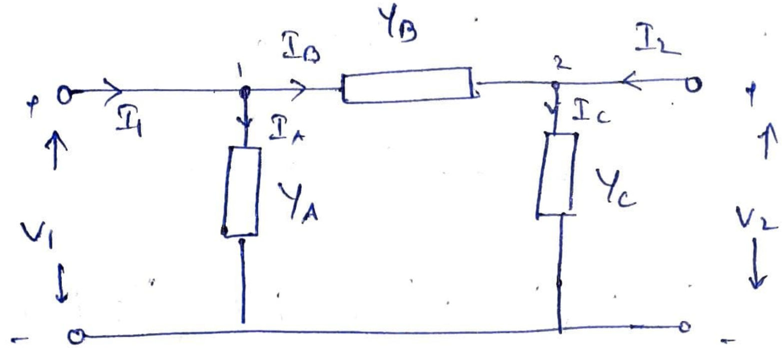
$$\therefore [h] = \begin{bmatrix} 1.2\,\Omega & 0.4 \\ -0.4 & 0.45 \end{bmatrix}$$

Numericals on Y-parameters:

Q1) Find the Y-parameters of the following π circuit and draw the Y-parameter model.



Sol) Note currents in each branch.



Apply KCL at node 1,

$$I_1 = I_A + I_B$$

$$\Rightarrow I_1 = V_1 Y_A + (V_1 - V_2) Y_B$$

$$\Rightarrow I_1 = V_1 (Y_A + Y_B) + V_2 (-Y_B) \rightarrow (1)$$

Apply KCL at node 2,

$$I_B + I_2 = I_C$$

$$\Rightarrow (V_1 - V_2) Y_B + I_2 = V_2 Y_C$$

$$\Rightarrow - I_2 = (V_1 - V_2) Y_B - Y_C V_2$$

$$\Rightarrow I_2 = V_2 Y_C - (V_1 - V_2) Y_B$$

$$\Rightarrow I_2 = V_1 (-Y_B) + V_2 (Y_C + Y_B) \rightarrow (2)$$

Equation (1) and (2) are in the form,

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \rightarrow (3) \quad \text{and} \quad I_2 = Y_{21} V_1 + Y_{22} V_2 \rightarrow (4)$$

Comparing eq (3) and (4), with eq (1) and (2),

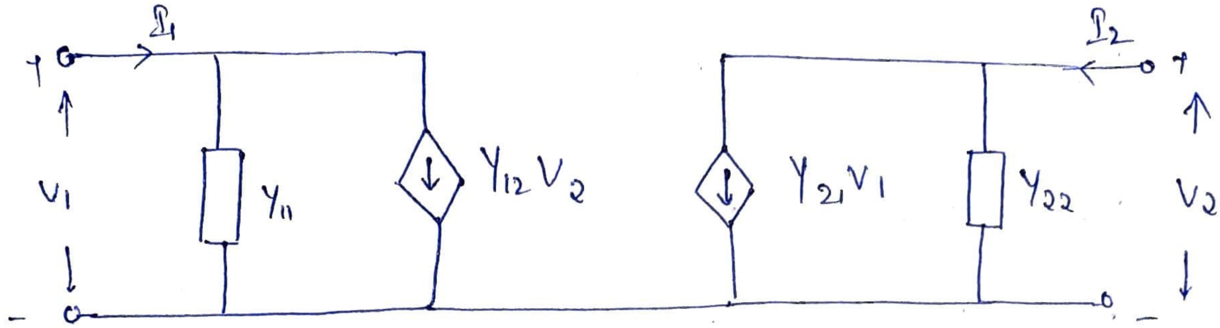
$$Y_{11} = Y_A + Y_B \qquad Y_{12} = -Y_B$$

$$Y_{21} = -Y_B \qquad Y_{22} = Y_B + Y_C$$

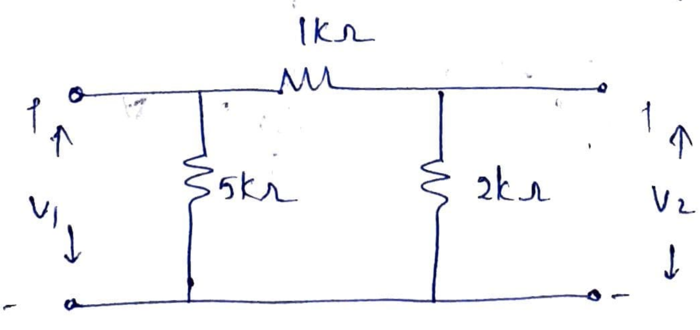
- $(Y_A + Y_B)V_1$ ie $Y_{11}V_1$ is the current through admittance Y_{11}

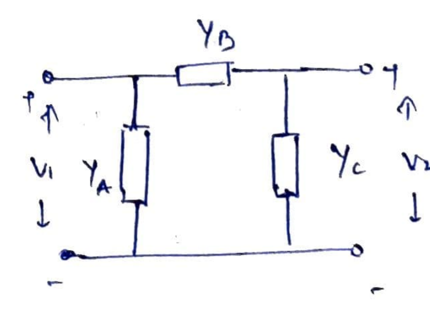
- $(Y_B + Y_C)V_2$ ie $Y_{22}V_2$ is the current through admittance Y_{22}.

- $Y_{12}V_2$ ie $-Y_BV_2$ and $Y_{21}V_1$ ie $-Y_BV_2$ are the controlled current sources. The equivalent circuit can be drawn as shown.



Q2) An π-attenuator has been shown. Find Y-parameters and draw the equivalent Y parameter circuit.





Sol) Comparing the given circuit with π network.

$Y_A = \frac{1}{5k}$ mho $= 0.2\,m℧$

$Y_B = \frac{1}{1k}$ mho $= 1\,m℧$

$Y_C = \frac{1}{2k}$ mho $= 0.5\,m℧$

The Y-parameters are,

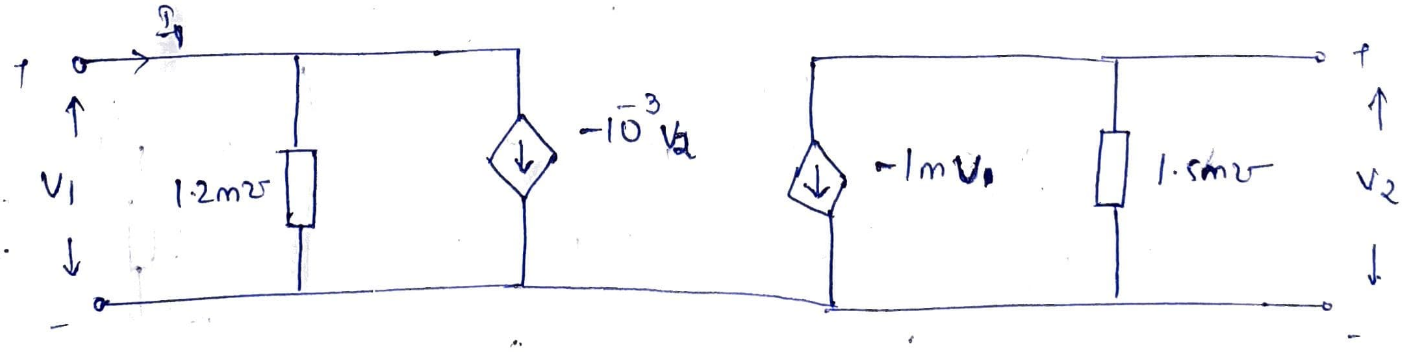
$Y_{11} = Y_A + Y_B = 1.2\,m℧$

$Y_{12} = -Y_B = -1\,m℧$

$Y_{21} = -Y_B = -1\,m℧$

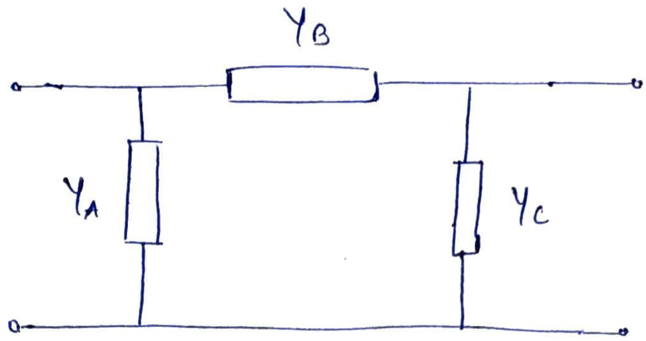
$Y_{22} = Y_B + Y_C = 1.5\,m℧$

The Y-parameter equivalent circuit can be drawn as shown.



Q3) In a π network, the series arm impedance is $0.05 \times 10^{-3} \angle -90^\circ$ mho and shunt arm impedances are $0.1 \times 10^{-3} \angle 0^\circ$ and $0.2 \times 10^{-3} \angle 90^\circ$ mho find the Y-parameters.

Sol) Standard π model network,



Given, $Y_B = 0.05 \times 10^{-3} \angle -90 \, ℧$

$Y_A = 0.1 \times 10^{-3} \angle 0^\circ \, ℧$

$Y_C = 0.2 \times 10^{-3} \angle 90^\circ \, ℧.$

$Y_{11} = Y_A + Y_B = 0.1 \times 10^{-3} \angle 0^\circ + 0.05 \times 10^{-3} \angle -90^\circ = (1 - j0.05) \times 10^{-3} \, ℧$

$Y_{12} = -Y_B = j0.05 \times 10^{-3} \, ℧$

$Y_{21} = -Y_B = j0.05 \times 10^{-3} \, ℧$

$Y_{22} = Y_B + Y_C = 0.05 \times 10^{-3} \angle -90^\circ \, ℧ + 0.2 \times 10^{-3} \angle 90^\circ$

$= j0.15 \times 10^{-3} \, ℧$

Q4) Following short circuit currents and voltages are obtained experimentally for a two port network.

(a) with output port short circuited,

$I_1 = 5mA, \; I_2 = -0.3mA, \; V_1 = 25V.$

b) With input port short circuited,

$I_1 = -5mA, \; I_2 = 10mA, \; V_2 = 30V.$

Determine Y-parameters.

Sol) For Y-parameter representation,

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \qquad Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \qquad Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}.$$

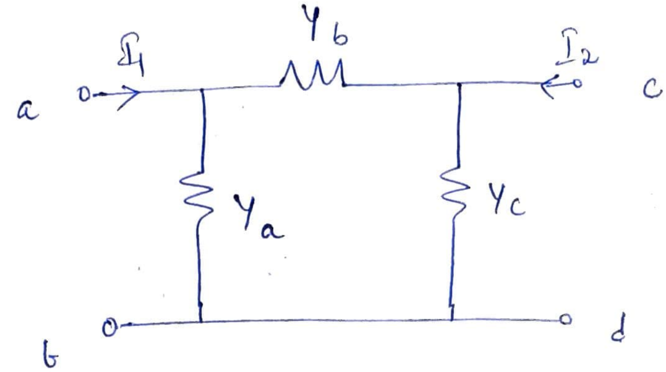
$\therefore$ from case (i) $\quad Y_{11} = \dfrac{5m}{25} = 0.2\ m℧$

$$Y_{21} = \frac{-0.3m}{25} = -12\ \mu ℧ = -0.012m℧$$

from case (ii), $\quad Y_{12} = \dfrac{-5m}{30} = -0.17\ m℧$

$$Y_{22} = \frac{10m}{30} = 0.33\ m℧$$

Q 5) Find the Y parameters Y_a, Y_b and Y_c of the equivalent π-network shown below, to represent a two-port network for which the following measurements were taken:



(a) With terminal pairs c-d short circuited, a voltage of $10\angle 0^\circ$ V applied at terminal pairs a-b resulting in $I_1 = 2.50\angle 0^\circ$ A and $I_2 = -0.5\angle 0^\circ$ A

(b) With the terminal pair ab short circuited, the same applied voltage at terminal pair c-d, the current at output port resulted is $I_2 = 1.50\angle 0^\circ$ A.

Sol) With terminals c-d short circuited, ie $V_2 = 0$ V.

$V_1 = 10\angle 0^\circ$, $I_1 = 2.5\angle 0^\circ$ A, $I_2 = -0.5\angle 0^\circ$ A

$$Y_{11} = \frac{I_1}{V_1} \Rightarrow Y_a + Y_b = \frac{I_1}{V_1}$$

$$\Rightarrow Y_a + Y_b = \frac{2.5\angle 0^\circ}{10}$$

$$\Rightarrow Y_a + Y_b = 0.25\ \mho \rightarrow (1)$$

and $Y_{21} = \cancel{-Y_B} =$

and $\cancel{Y_{21} = \frac{V_2}{I_1}}$

$\Rightarrow \cancel{-Y_b = \frac{V_2}{I_1}}$

$\Rightarrow -Y_b =$

and $Y_{21} = \frac{I_2}{V_1}$

$$\Rightarrow -Y_b = \frac{I_2}{V_1}$$

$$\Rightarrow -Y_b = \frac{-0.5\angle 0^\circ}{10}$$

$$\Rightarrow Y_b = 0.05\angle 0^\circ\ \mho$$

$$\boxed{Y_b = 0.05\ \mho}$$

Substitute the value of Y_b in eq (1)

$$Y_a = 0.25 - Y_b = 0.25 - 0.05$$

$$\boxed{Y_a = 0.2\ \mho}$$

With terminal pair a-b shorted,

$$Y_{22} = Y_b + Y_c \Rightarrow \cancel{V_2}\ \frac{I_2}{V_2} = Y_b + Y_c$$

$$\Rightarrow \frac{1.5\angle 0^\circ}{10\angle 0^\circ} = Y_b + Y_c$$

$$\Rightarrow 0.15 = Y_b + Y_c$$

$$\Rightarrow Y_c = 0.15 - Y_b = 0.15 - 0.05 = \boxed{0.1\ \mho}$$

$\therefore \quad Y_a = 0.2℧$

$Y_b = 0.05℧$

$Y_c = 0.10℧$

Relationship between parameter sets:

→ Z-parameters in terms of
- Y-parameters
- h-parameters
- ABCD parameters

→ Y-parameters in terms of
- Z-parameters
- h-parameters
- ABCD parameters.

→ h-parameters in terms of
- Z-parameters
- Y-parameters
- ABCD parameters.

→ ABCD parameters in terms of
- Z-parameters
- Y-parameters
- h-parameters.

1) i, Z-parameters in terms of Y-parameters.

From $[V] = [Z][I]$

$\Rightarrow [I] = [Z^{-1}][V] \longrightarrow ①$

from $[I] = [Y][V] \longrightarrow ②$

Comparing eq ① and ②,

$[Z^{-1}] = [Y]$

$\Rightarrow [Z] = [Y^{-1}]$

$$\Rightarrow \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \frac{1}{\Delta Y}\begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}$$

$$\Rightarrow \quad Z_{11} = \frac{Y_{22}}{\Delta Y}$$

$$Z_{12} = -\frac{Y_{12}}{\Delta Y}$$

(where $\Delta Y = Y_{11}Y_{22} - Y_{12}Y_{21}$)

$$Z_{21} = \frac{-Y_{21}}{\Delta Y}$$

$$Z_{22} = \frac{Y_{11}}{\Delta Y}$$

ii, Z- parameters in terms of ABCD parameters.

Consider equation, $V_1 = AV_2 - BI_2 \longrightarrow (1)$

and $I_1 = CV_2 - DI_2 \longrightarrow (2)$

from (2), $I_1 = CV_2 - DI_2$

$\Rightarrow CV_2 = I_1 + DI_2$

$\Rightarrow V_2 = \frac{1}{C} I_1 + \frac{D}{C} I_2 \longrightarrow (3)$

Substitute eq (3) in eq (1),

$$V_1 = A\left(\frac{1}{C} I_1 + \frac{D}{C} I_2\right) - BI_2$$

$$V_1 = \frac{A}{C} I_1 + \left(\frac{AD}{C} - B\right) I_2 \longrightarrow (4)$$

Comparing eq (4) and eq (3) with z-parameter equations,

i.e., $V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

$\therefore \quad Z_{11} = \frac{A}{C}$

$Z_{12} = \frac{AD - BC}{C}$

$Z_{21} = \frac{1}{C}$

$Z_{22} = \frac{D}{C}$

iii, Z-parameters in terms of hybrid parameters:

Equation for h-parameter network are,

$$V_1 = h_{11} I_1 + h_{12} V_2 \longrightarrow ①$$
$$I_2 = h_{21} I_1 + h_{22} V_2 \longrightarrow ②$$

from eq ②, $h_{22} V_2 = -h_{21} I_1 + I_2$

$$\Rightarrow V_2 = -\frac{h_{21}}{h_{22}} I_1 + \frac{I_2}{h_{22}} \longrightarrow ③$$

Sub eq ③ in ①,

$$V_1 = h_{11} I_1 + h_{12}\left(-\frac{h_{21}}{h_{22}} I_1 + \frac{I_2}{h_{22}}\right)$$

$$V_1 = \left(h_{11} - \frac{h_{12} h_{21}}{h_{22}}\right) I_1 + \frac{h_{12}}{h_{22}} I_2. \longrightarrow ④$$

Comparing eq ④ and eq ③ with Z-network equations, i.e.,

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \; ; \; V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$\therefore Z_{11} = \frac{h_{11} h_{22} - h_{12} h_{21}}{h_{22}} = \frac{\Delta h}{h_{22}} \qquad (\because \Delta h = h_{11} h_{22} - h_{12} h_{21})$$

$$Z_{12} = \frac{h_{12}}{h_{22}}$$

$$Z_{21} = \frac{-h_{21}}{h_{22}}$$

$$Z_{22} = \frac{1}{h_{22}}$$

2) i, Y-parameters in terms of Z-parameters:

from $[Z] = [Y]^{-1} \Rightarrow [Y] = [Z^{-1}]$

$$\Rightarrow \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \frac{1}{\Delta z} \begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}$$

$$\Rightarrow \quad Y_{11} = \frac{Z_{22}}{\Delta z} \qquad (\because \Delta z = Z_{11}Z_{22} - Z_{12}Z_{21}).$$

$$Y_{12} = \frac{-Z_{12}}{\Delta z}$$

$$Y_{21} = \frac{-Z_{21}}{\Delta z}$$

$$Y_{22} = \frac{Z_{11}}{\Delta z}$$

ii, Y-parameters in terms of hybrid parameters:

The governing equations of h-parameter network are,

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$\Rightarrow h_{11} I_1 = V_1 - h_{12} V_2$$

$$\Rightarrow I_1 = \frac{V_1}{h_{11}} - \frac{h_{12}}{h_{11}} V_2 \longrightarrow (1)$$

and $I_2 = h_{21} I_1 + h_{22} V_2$

$$\Rightarrow I_2 = h_{21}\left(\frac{V_1}{h_{11}} - \frac{h_{12}}{h_{11}} V_2\right) + h_{22} V_2 \quad (\because \text{substituting eq (1)})$$

$$\Rightarrow I_2 = \frac{h_{21}}{h_{11}} V_1 + \left(-\frac{h_{12} h_{21}}{h_{11}} + h_{22}\right) V_2$$

$$I_2 = \frac{h_{21}}{h_{11}} V_1 + \left(\frac{h_{11} h_{22} - h_{12} h_{21}}{h_{11}}\right) V_2 \longrightarrow (2)$$

Comparing eq (1) and eq (2) with Y-parameter network eq ree,

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{and} \quad I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$\therefore Y_{11} = \frac{1}{h_{11}} \qquad \text{and} \quad Y_{22} = \frac{\Delta h}{h_{11}}$$

$$Y_{12} = \frac{-h_{12}}{h_{11}} \qquad \because \Delta h = h_{11} h_{22} - h_{12} h_{21}.$$

$$Y_{21} = \frac{h_{21}}{h_{11}}$$

iii, Y-parameters in terms of ABCD parameters:

from ABCD n/w eq,
$$V_1 = AV_2 - BI_2 \longrightarrow (1)$$
$$I_1 = CV_2 - DI_2 \longrightarrow (2)$$

from eq (1), $BI_2 = AV_2 - V_1$

$$\Rightarrow I_2 = \frac{A}{B}V_2 - \frac{V_1}{B} \longrightarrow (3)$$

Sub eq (3) in (2),

$$I_1 = CV_2 - D\left(\frac{A}{B}V_2 - \frac{1}{B}V_1\right)$$

$$\Rightarrow I_1 = \left(C - \frac{AD}{B}\right)V_2 + \frac{D}{B}V_1 \longrightarrow (4)$$

Comparing eq (3) and eq (4) with Y-parameter n/w equations,

i.e., $I_1 = Y_{11}V_1 + Y_{12}V_2$ and $I_2 = Y_{21}V_1 + Y_{22}V_2$,

$$\therefore \quad Y_{11} = \frac{D}{B}$$

$$Y_{12} = \frac{BC - AD}{B}$$

$$Y_{21} = \cancel{\frac{A}{B}} \; -\frac{1}{B}$$

$$Y_{22} = \cancel{\frac{d}{B}} \; \frac{A}{B}$$

3) i) h-parameters in terms of z-parameters:

from z-parameter network equations,

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \longrightarrow (1)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2. \longrightarrow (2)$$

from eq (2) $Z_{22} I_2 = V_2 - Z_{21} I_1$

$$\Rightarrow I_2 = \frac{1}{Z_{22}} V_2 - \frac{Z_{21}}{Z_{22}} I_1$$

$$\Rightarrow I_2 = -\frac{Z_{21}}{Z_{22}} I_1 + \frac{1}{Z_{22}} V_2 \longrightarrow (3)$$

Sub eq (3) in eq (1),

$$V_1 = Z_{11} I_1 + Z_{12}\left(-\frac{Z_{21}}{Z_{22}} I_1 + \frac{1}{Z_{22}} V_2\right)$$

$$\Rightarrow V_1 = \left(Z_{11} - \frac{Z_{12} Z_{21}}{Z_{22}}\right) I_1 + \frac{Z_{12}}{Z_{22}} V_2 \longrightarrow (4)$$

Comparing eq (4) and (3) with governing equations of h-parameter network, ie $V_1 = h_{11} I_1 + h_{12} V_2$

and $I_2 = h_{21} I_1 + h_{22} V_2$

$$\therefore \quad h_{11} = Z_{11} - \frac{Z_{12} Z_{21}}{Z_{22}} = \frac{\Delta Z}{Z_{22}}$$

$$h_{12} = \frac{Z_{12}}{Z_{22}}$$

$$h_{21} = \frac{-Z_{21}}{Z_{22}}$$

$$h_{22} = \frac{1}{Z_{22}}$$

ii) h-parameters in terms of Y-parameters:

The governing equations of the Y-parameter equations are,

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \longrightarrow (1)$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \longrightarrow (2)$$

From eq (1), $Y_{11} V_1 = I_1 - Y_{12} V_2$

$$\Rightarrow V_1 = \frac{1}{Y_{11}} I_1 - \frac{Y_{12}}{Y_{11}} V_2 \longrightarrow (3)$$

Sub eq (3) in (2),

$$I_2 = Y_{21}\left(\frac{1}{Y_{11}} I_1 - \frac{Y_{12}}{Y_{11}} V_2\right) + Y_{22} V_2$$

$$\Rightarrow I_2 = \left(\frac{Y_{21}}{Y_{11}}\right) I_1 + \left(-\frac{Y_{21} Y_{12}}{Y_{11}} + Y_{22}\right) V_2$$

$$\Rightarrow I_2 = \frac{Y_{21}}{Y_{11}} I_1 + \frac{Y_{11} Y_{22} - Y_{21} Y_{12}}{Y_{11}} V_2 \longrightarrow (4)$$

Comparing eq (3) and (4) with h-network eq (1) and (2) equation

ie $V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

$$\therefore \quad h_{11} = \frac{1}{Y_{11}}$$

$$h_{12} = -\frac{Y_{12}}{Y_{11}}$$

$$h_{21} = \frac{Y_{21}}{Y_{11}}$$

$$h_{22} = \frac{\Delta Y}{Y_{11}}$$

iii) h-parameters in terms of ABCD parameters :

The ABCD parameter equations are given by,

$$V_1 = AV_2 - BI_2 \longrightarrow (1)$$

$$I_1 = CV_2 - DI_2 \longrightarrow (2)$$

from eq (2), $DI_2 = CV_2 - I_1$

$$\Rightarrow I_2 = \frac{C}{D} V_2 - \frac{1}{D} I_1 \longrightarrow (3)$$

Sub eq (3) in eq (1), $V_1 = AV_2 - B\left(\frac{C}{D} V_2 - \frac{1}{D} I_1\right)$

$$\Rightarrow V_1 = \left(A - \frac{BC}{D}\right) V_2 + \frac{B}{D} I_1$$

$$\Rightarrow V_1 = V_2\left(\frac{AD - BC}{D}\right) + \frac{B}{D} I_1 \longrightarrow (4)$$

comparing eq (4) and (3) with standard h-parameter equation

i.e., $V_1 = h_{11} I_1 + h_{12} V_2$ and $I_2 = h_{21} I_1 + h_{22} V_2$

$$\therefore h_{11} = \frac{B}{D}$$

$$h_{12} = \frac{AD - BC}{D}$$

$$h_{21} = \cancel{\frac{C}{D}} \; -\frac{1}{D}$$

$$h_{22} = \cancel{\frac{C}{D}} \; \frac{C}{D}.$$

4) i, ABCD parameters in terms of Z-parameters:

from the governing equations of Z-parameter network,

we have $$V_1 = Z_{11} I_1 + Z_{12} I_2 \longrightarrow (1)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \longrightarrow (2)$$

from eq (2), $$Z_{21} I_1 = V_2 - Z_{22} I_2$$

$$\Rightarrow I_1 = \frac{1}{Z_{21}} V_2 - \frac{Z_{22}}{Z_{21}} I_2 \longrightarrow (3)$$

Sub eq (3) in eq (1),

$$V_1 = Z_{11}\left(\frac{1}{Z_{21}} V_2 - \frac{Z_{22}}{Z_{21}} I_2\right) + Z_{12} I_2$$

$$\Rightarrow V_1 = \frac{Z_{11}}{Z_{21}} V_2 + \left(\frac{-Z_{11} Z_{22}}{Z_{21}} + Z_{12}\right) I_2 \longrightarrow (4)$$

Comparing eq (4) and (3) with ~~Z~~ ABCD network equation,

i.e., $V_1 = A V_2 - B I_2$ ~~and~~ ~~$V_2 = \ldots$~~

and $I_1 = C V_2 - D I_2$

$$\therefore A = \frac{Z_{11}}{Z_{21}}$$

$$B = -\left(\frac{-Z_{11} Z_{22} + Z_{12} Z_{21}}{Z_{21}}\right) = \frac{\Delta Z}{Z_{21}}$$

$$C = \frac{1}{Z_{21}}$$

$$D = +\frac{Z_{22}}{Z_{21}}$$

ii. ABCD parameters in terms of Y-parameters:

Y-parameter equations are given by, $I_1 = Y_{11} V_1 + Y_{12} V_2 \rightarrow (1)$

$I_2 = Y_{21} V_1 + Y_{22} V_2 \rightarrow (2)$

from eq (2), $Y_{21} V_1 = I_2 - Y_{22} V_2$

$$\Rightarrow V_1 = -\frac{Y_{22}}{Y_{21}} V_2 + \frac{1}{Y_{21}} I_2 \rightarrow (3)$$

Sub eq (3) in eq (1),

$$I_1 = Y_{11}\left(-\frac{Y_{22}}{Y_{21}} V_2 + \frac{1}{Y_{21}} I_2\right) + Y_{12} V_2$$

$$\Rightarrow I_1 = \left(-\frac{Y_{11} Y_{22}}{Y_{21}} + Y_{12}\right) V_2 + \frac{Y_{11}}{Y_{21}} I_2 . \rightarrow (4)$$

Comparing eq (3) and (4) with T-network equations,

i.e, $V_1 = AV_2 - BI_2$ and $V_2 = CV_2 - DI_2$.

$$\therefore A = -\frac{Y_{22}}{Y_{21}}$$

$$B = -\frac{1}{Y_{21}}$$

$$C = \frac{-Y_{11}Y_{22} + Y_{12}Y_{21}}{Y_{21}} = \frac{-\Delta y}{Y_{21}}$$

$$D = -\frac{Y_{11}}{Y_{21}}$$

iii) ABCD parameters in terms of h-parameters:

The governing equations of h-parameters are,

$$V_1 = h_{11} I_1 + h_{12} V_2 \longrightarrow (1)$$

and $$I_2 = h_{21} I_1 + h_{22} V_2 \longrightarrow (2)$$

from eq (2), $h_{21} I_1 = I_2 - h_{22} V_2$

$$\Rightarrow I_1 = -\frac{h_{22}}{h_{21}} V_2 + \frac{1}{h_{21}} I_2 \longrightarrow (3)$$

Sub eq (3) in eq (1),

$$V_1 = h_{11}\left(-\frac{h_{22}}{h_{21}} V_2 + \frac{1}{h_{21}} I_2\right) + h_{12} V_2$$

$$V_1 = \left(-\frac{h_{11} h_{22}}{h_{21}} + h_{12}\right) V_2 + \frac{h_{11}}{h_{21}} I_2 \longrightarrow (4)$$

Comparing eq (4) and (3) with governing equations of ABCD network equations, i.e., $V_1 = A V_2 - B I_2$

$I_1 = C V_2 - D I_2$

$$\therefore \quad A = \frac{-h_{11} h_{22} + h_{12} h_{21}}{h_{21}} = \frac{-\Delta h}{h_{21}}$$

$$B = -\frac{h_{11}}{h_{21}}$$

$$C = -\frac{h_{22}}{h_{21}}$$

$$D = -1/h_{21}$$

Numericals on relationship between parameter sets:

1Q) Find [Z], [Y] and [h] of a two port network, if,

$$[T] = \begin{bmatrix} 10 & 1.5\,\Omega \\ 2\,S & 4 \end{bmatrix}$$

Ans:

2Q) Determine [Y] and [T] of a two port network whose Z-parameters are:

$$[Z] = \begin{bmatrix} 6 & 4 \\ 4 & 6 \end{bmatrix} \Omega$$

Answer: $$[Y] = \begin{bmatrix} 0.3 & -0.2 \\ -0.2 & 0.3 \end{bmatrix} S$$

and $$[T] = \begin{bmatrix} 1.5 & 5\,\Omega \\ 0.25\,S & 1.5 \end{bmatrix}$$

Interconnection of two port networks:

Different ways of interconnection of two port networks are:

i) Series connection

ii) Cascade connection

iii) Parallel connection.

1) Series connection:

- The open circuit impedance parameter is highly useful in characterizing the series connected two port networks.

- Let A and B be the two port networks connected in series

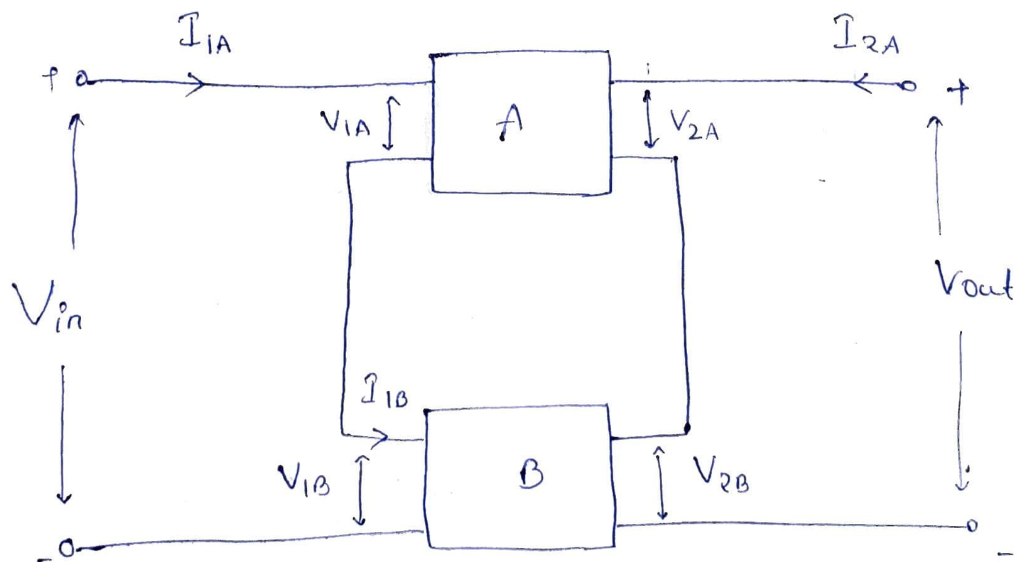


fig: Series connection of two 2-port networks.

for network A, $V_{1A} = Z_{11A} I_{1A} + Z_{12A} I_{2A}$

$$V_{2A} = Z_{21A} I_{1A} + Z_{22A} I_{2A}$$

for network B,

$$V_{1B} = Z_{11B} I_{1B} + Z_{12B} I_{2B}$$

$$V_{2B} = Z_{21B} I_{1B} + Z_{22B} I_{2B}.$$

from the figure, referring to the way the networks are interconnected,

$$I_1 = I_{1A} = I_{1B}$$

$$I_2 = I_{2A} = I_{2B}$$

$$V_2 = V_{2A} + V_{2B}$$

However, $V_1 = V_{1A} + V_{1B}$.

$$= Z_{11A} I_{1A} + Z_{12A} I_{2A} + Z_{11B} I_{1B} + Z_{12B} I_{2B}$$

$$= Z_{11A} I_1 + Z_{12A} I_2 + Z_{11B} I_1 + Z_{12B} I_2$$

$$V_1 = I_1 (Z_{11A} + Z_{11B}) + I_2 (Z_{12A} + Z_{12B}) \longrightarrow (1)$$

and $V_2 = V_{2A} + V_{2B}$

$$= Z_{21A} I_{1A} + Z_{22A} I_{2A} + Z_{21B} I_{1B} + Z_{22B} I_{2B}$$

$$= Z_{21A} I_1 + Z_{22A} I_2 + Z_{21B} I_1 + Z_{22B} I_2$$

$$V_2 = I_1(Z_{21A} + Z_{21B}) + I_2(Z_{22A} + Z_{22B}) \longrightarrow (2)$$

Thus, the equations for a 2 two-port networks, which are connected in series, are,

$$V_1 = (Z_{11A} + Z_{11B}) I_1 + (Z_{12A} + Z_{12B}) I_2$$

$$V_2 = (Z_{21A} + Z_{21B}) I_1 + (Z_{22A} + Z_{22B}) I_2$$

In matrix form,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11A} + Z_{11B} & Z_{12A} + Z_{12B} \\ Z_{21A} + Z_{21B} & Z_{22A} + Z_{22B} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$= \left\{ \begin{bmatrix} Z_{11A} & Z_{12A} \\ Z_{21A} & Z_{22A} \end{bmatrix} + \begin{bmatrix} Z_{11B} & Z_{12B} \\ Z_{21B} & Z_{22B} \end{bmatrix} \right\} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \left\{ [Z]_A + [Z]_B \right\} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [Z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

where, $[Z]$ is the Z parameter matrix for series connected 2 two-port networks.

$$[Z] = [Z]_A + [Z]_B .$$

Illy for n number of two port networks, connected in series,

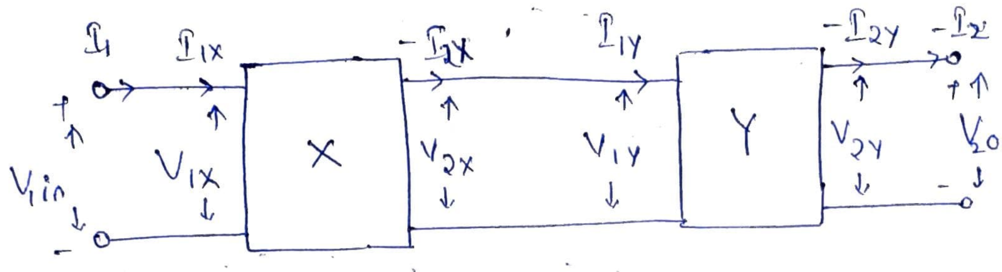
$$[Z] = [Z]_A + [Z]_B + \dots + [Z]_n .$$

Thus, it can be observed that the overall Z-parameter matrix for series connected two port networks is ; the sum of Z matrices of each individual network.

2) Cascade connection:

- ABCD parameters are highly useful in characterising cascaded two port networks.

- Let X and Y be two networks connected in cascade.



for network X, $V_{1X} = A_X V_{2X} - B_X I_{2X}$

$I_{1X} = C_X V_{2X} - D_X I_{2X}$

For network Y, $V_{1Y} = A_Y V_{2Y} - B_Y I_{2Y}$

$I_{1Y} = C_Y V_{2Y} - D_Y I_{2Y}$

for the cascade connection,

$I_1 = I_{1X}$; $-I_{2X} = I_{1Y}$; $I_2 = I_{2Y}$

$V_1 = V_{1X}$; $V_{2X} = V_{1Y}$; $V_2 = V_{2Y}$

The overall transmission parameters for the combined network as shown in matrix form, becomes

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} V_{1X} \\ I_{1X} \end{bmatrix}$$

$$= \begin{bmatrix} A_X & B_X \\ C_X & D_X \end{bmatrix} \begin{bmatrix} V_{2X} \\ -I_{2X} \end{bmatrix}$$

$$= \begin{bmatrix} A_X & B_X \\ C_X & D_X \end{bmatrix} \begin{bmatrix} V_{1Y} \\ I_{1Y} \end{bmatrix}$$

$$= \begin{bmatrix} A_X & B_X \\ C_X & D_X \end{bmatrix} \begin{bmatrix} A_Y & B_Y \\ C_Y & D_Y \end{bmatrix} \begin{bmatrix} V_{2Y} \\ -I_{2Y} \end{bmatrix}$$

$$= \begin{bmatrix} A_X & B_X \\ C_X & D_X \end{bmatrix} \begin{bmatrix} A_Y & B_Y \\ C_Y & D_Y \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$= \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

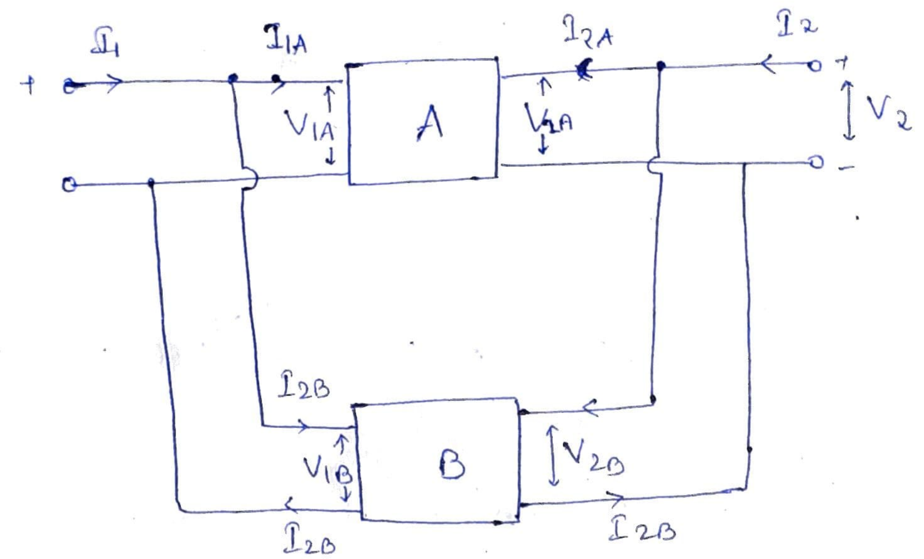
where $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_X & B_X \\ C_X & D_X \end{bmatrix} \begin{bmatrix} A_Y & B_Y \\ C_Y & D_Y \end{bmatrix}$

$\therefore$ The overall ABCD parameter network matrix for cascaded network is the matrix product of ABCD matrices of individual networks.

3) Parallel connection:

- Y-parameter representation is useful for parallel connection.

- Let A and B be two 2 port networks connected in parallel as shown.



For network A, $I_{1A} = Y_{11A} V_{1A} + Y_{12A} V_{2A}$

$$I_{2A} = Y_{21A} V_{1A} + Y_{22A} V_{2A}$$

for network B, $I_{1B} = Y_{11B} V_{1B} + Y_{12B} V_{2B}$

$$I_{2B} = Y_{21B} V_{1B} + Y_{22B} V_{2B}.$$

for the parallel connection as shown,

$$V_1 = V_{1A} = V_{1B} \; ; \; V_2 = V_{2B} = V_{2A}$$

$$I_1 = I_{1A} + I_{1B} \; ; \; I_2 = I_{2A} + I_{2B}.$$

Thus, $I_1 = I_{1A} + I_{1B}$

$$= \left(Y_{11A} V_{1A} + Y_{12A} V_{2A}\right) + \left(Y_{11B} V_{1B} + Y_{12B} V_{2B}\right)$$

$$= \left(Y_{11A} V_1 + Y_{12A} V_2\right) + \left(Y_{11B} V_1 + Y_{12B} V_2\right)$$

$$I_1 = \left(Y_{11A} + Y_{11B}\right) V_1 + \left(Y_{12A} + Y_{12B}\right) V_2 \longrightarrow (1)$$

and $I_2 = I_{2A} + I_{2B}$

$$= \left(Y_{21A} V_{1A} + Y_{22A} V_{2A}\right) + \left(Y_{21B} V_{1B} + Y_{22B} V_{2B}\right)$$

$$= \left(Y_{21A} V_1 + Y_{22A} V_2\right) + \left(Y_{21B} V_1 + Y_{22B} V_2\right)$$

$$I_2 = \left(Y_{21A} + Y_{21B}\right) V_1 + \left(Y_{22A} + Y_{22B}\right) V_2 \longrightarrow (2)$$

from eq (1), (2), the combined network parameter equations in matrix form, will be as shown,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11A} + Y_{11B} & Y_{12A} + Y_{12B} \\ Y_{21A} + Y_{21B} & Y_{22A} + Y_{22B} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$= \left\{ \begin{bmatrix} Y_{11A} & Y_{12A} \\ Y_{21A} & Y_{22A} \end{bmatrix} + \begin{bmatrix} Y_{11B} & Y_{12B} \\ Y_{21B} & Y_{22B} \end{bmatrix} \right\} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$= \left\{ [Y]_A + [Y]_B \right\} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

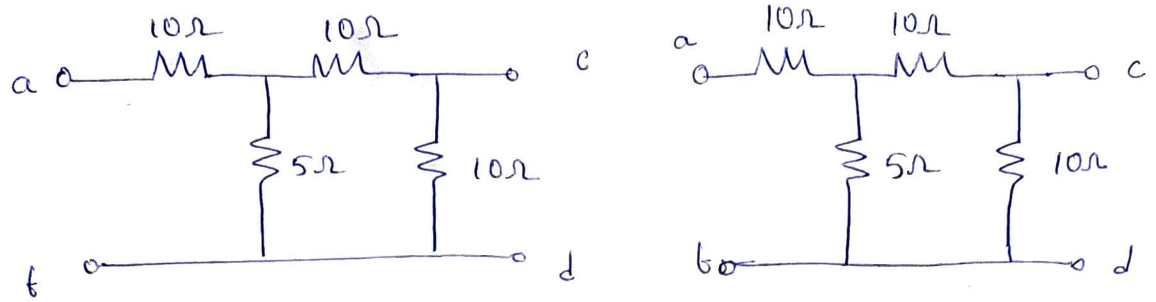
$$\Rightarrow \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [Y] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

where $[Y] = [Y]_A + [Y]_B$

This result may be generalized for any number of Y parameter network paralleling.

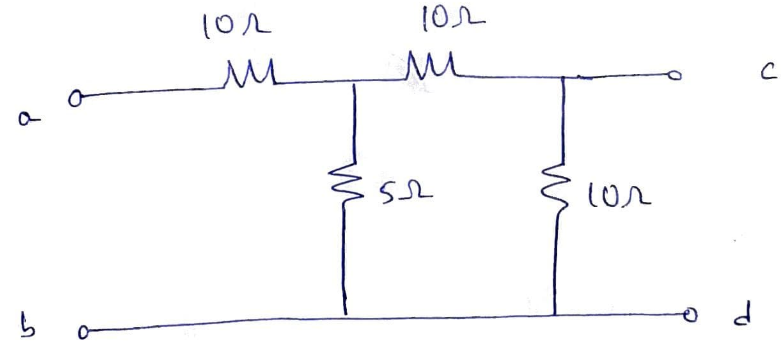
The overall Y parameter matrix is then simply the summation of Y-matrices of each individual two port networks.

Q1) Two port networks are shown below. Obtain the transmission parameters of the resulting circuit when both the circuits are in cascade.



Sol) Since both the networks are identical, ABCD parameters of any one network can be calculated.

The network is as shown,

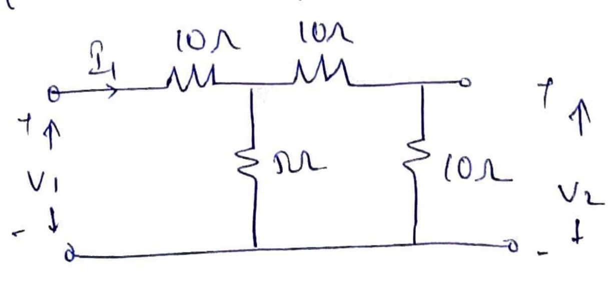


The governing equations for ABCD network parameters are,

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

i Put $I_2 = 0$ ie output opened.



$$A = \left.\frac{V_1}{V_2}\right|_{I_2 = 0}$$

$$= \frac{(10 + 5 \parallel 20)\, I_1}{10 \times i}$$

(Let i be the current flowing through 10Ω, whose voltage is V_2)

$$= \frac{\left(10 + \frac{5 \times 20}{25}\right) I_1}{10 \times \left(I_1 \times \frac{5}{5+10+10}\right)}$$

$$= \frac{(10 + 4)\, I_1}{10 \times I_1 \times \frac{1}{5}}$$

$$= \frac{14}{2} = 7$$

$\therefore$ ~~A = 7~~ $\boxed{A = 7}$

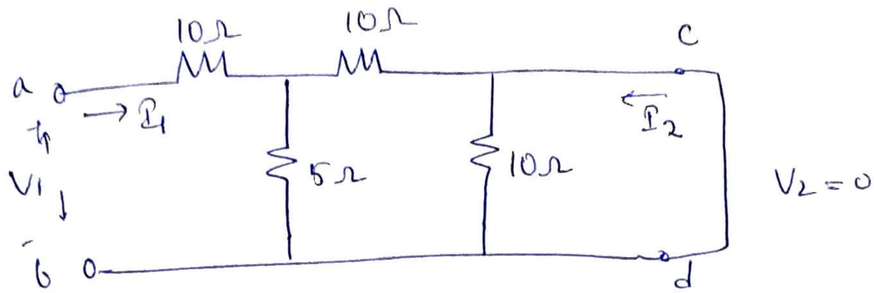
$$C = \left.\frac{I_1}{V_2}\right|_{I_2 = 0}$$

$$= \frac{I_1}{10 \times i}$$

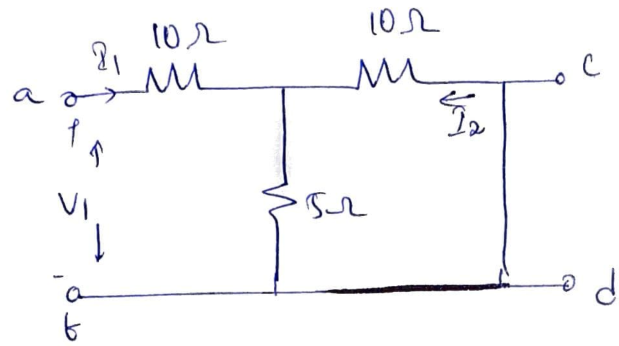
$$= \frac{I_1}{10 \times \left(I_1 \times \frac{5}{5+10+10}\right)}$$

$$= \frac{I_1}{10 \times I_1 \times \frac{1}{5}} = \frac{1}{2} \qquad \therefore \boxed{C = \frac{1}{2}\,\mho}$$

ii, $V_2 = 0$:

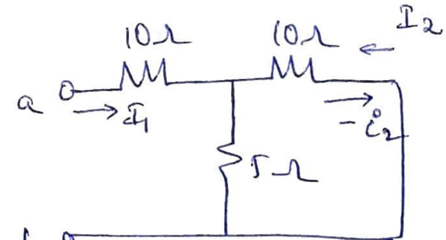


⇓



$$B = -\frac{V_1}{I_2} = -\left(\frac{10I_1 + 5(I_1+I_2)}{I_2}\right) \rightarrow ①$$

$$D = -\frac{I_1}{I_2}$$



Let us assume a current $-i_2$ which is having same magnitude but its direction is opposite to that of I_2

$\therefore I_2 = -i_2$

$$= -\frac{I_1}{(-i_2)}$$

$$= \frac{I_1}{i_2}$$

$$= \frac{I_1}{I_1 \times \frac{5}{5+10}}$$

$$= \frac{15}{5} = 3 \qquad \boxed{\therefore D = 3} \rightarrow ②$$

from eq ②, $D = 3$

$$-\frac{I_1}{I_2} = 3$$

$$\Rightarrow I_1 = -3I_2 \rightarrow ③$$

Sub ③ in ①

$$B = -\left(\frac{10I_1 + 5(I_1+I_2)}{I_2}\right)$$

$$= -\frac{(15I_1 + 5I_2)}{I_2}$$

$$= -\frac{(15(-3I_2) + 5I_2)}{I_2}$$

$$= -\frac{(-15+5)I_2}{I_2}$$

$$= 10$$

$$\therefore \boxed{B = 10}$$

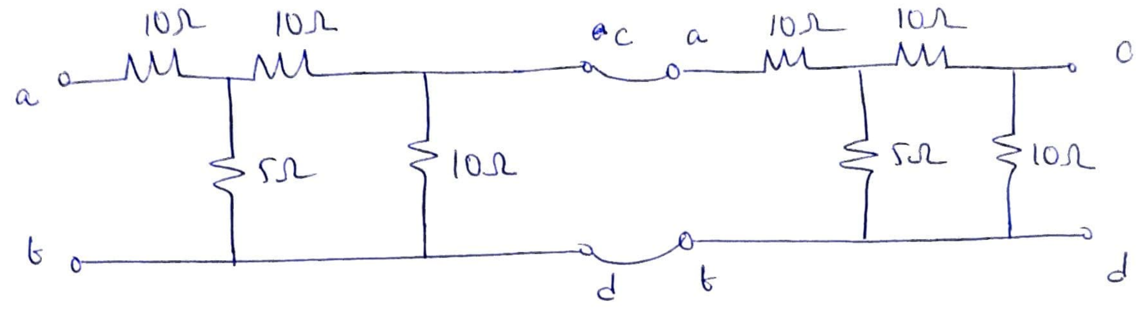
$$\therefore \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} = \begin{bmatrix} 7 & \\ \frac{1}{2} & 3 \end{bmatrix}$$

and

$$\begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} = \begin{bmatrix} 7 & \\ \frac{1}{2} & 3 \end{bmatrix}$$

$\because$ Both circuits are identical.

Now, the two network in cascade will be as shown,



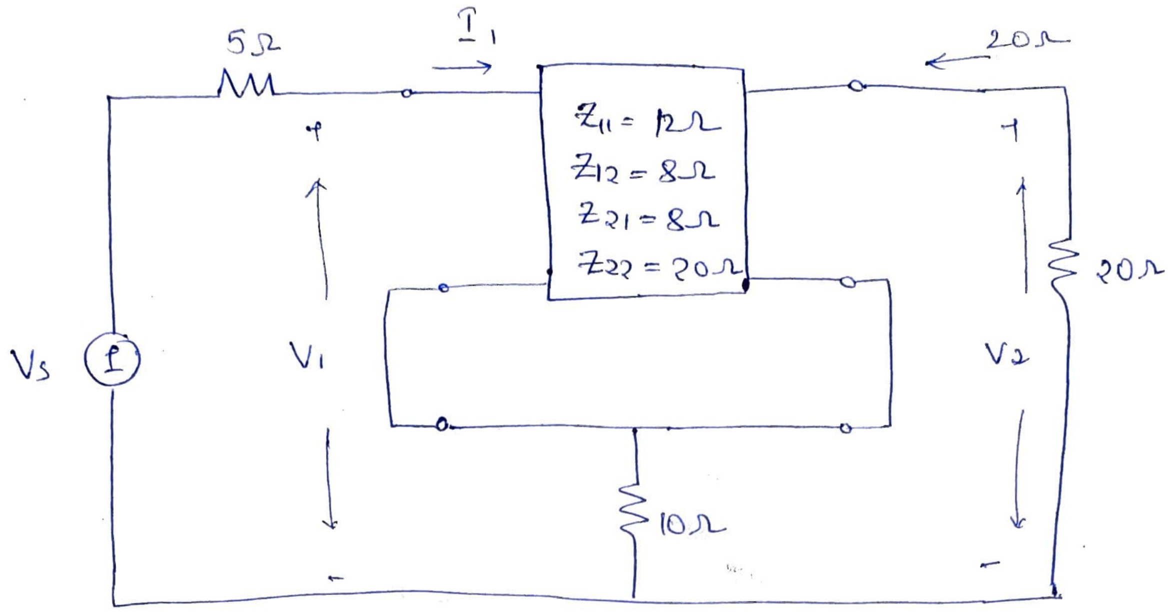
The resultant ABCD parameters for the cascaded network are,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & \\ \frac{1}{2} & 3 \end{bmatrix} \begin{bmatrix} 7 & \\ \frac{1}{2} & 3 \end{bmatrix}$$

$$= \begin{bmatrix} & \\ & \end{bmatrix}$$

2(a) Evaluate $\frac{V_2}{V_s}$ in the circuit below.



Sol) This can be regarded as a two 2-port networks, connected in series.

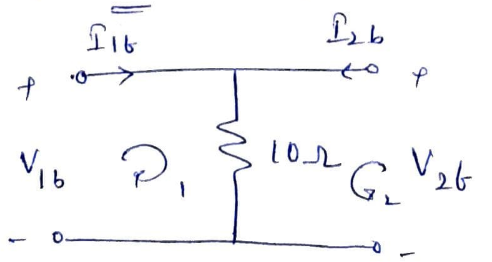
Let the network be Na & Nb.

Z-parameters for Na :

$$[Z]_a = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}_a$$

$$= \begin{bmatrix} 12 & 8 \\ 8 & 20 \end{bmatrix}$$

Z-parameters for Nb:



Apply KVL in Loop 1 & 2,

loop: $V_{1b} = (I_{1b} + I_{2b})10$

$V_{1b} = I_{1b}(10) + I_{2b}(10)$ → (1)

Loop 2: $V_{2b} = 10(I_{1b} + I_{2b})$

$$V_{2b} = 10(I_{1b}) + 10(I_{2b}) \rightarrow (2)$$

Comparing eq (1) and (2) with standard z-parameter equations,

we have,

$$[Z]_b = \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix}$$

Now, because, both networks Na & Nb are connected in series,

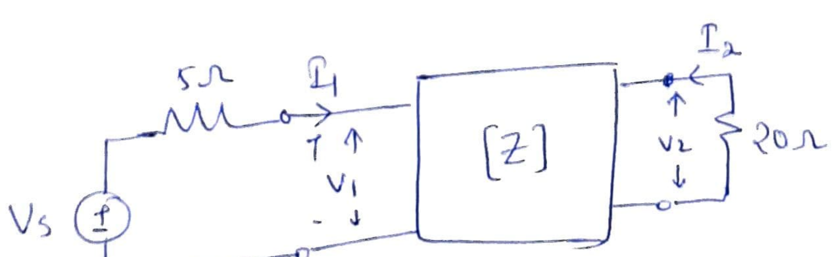
[Z] i.e. Z matrix for the series connected network

$$[Z] = [Z]_a + [Z]_b$$

$$= \begin{bmatrix} 12 & 8 \\ 8 & 20 \end{bmatrix} + \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & 18 \\ 18 & 30 \end{bmatrix}$$

Now, equivalent n/w can be re-drawn as,



$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$\Rightarrow V_1 = 22 I_1 + 18 I_2 \longrightarrow (3)$$

$$\& \; V_2 = 18 I_1 + 30 I_2 \longrightarrow (4)$$

at input port, apply KVL,

$$V_s = 5 I_1 + V_1$$

$$\Rightarrow V_1 = V_s - 5 I_1 \longrightarrow (5)$$

at output port, apply KVL,

$$V_2 = -20 I_2 \quad \Rightarrow I_2 = \frac{-V_2}{20}. \longrightarrow (6)$$

Sub (5), (6) in eq (3)

$$V_s - 5 I_1 = 22 I_1 + 18\left(\frac{-V_2}{20}\right)$$

$$\Rightarrow V_s = 27 I_1 - \frac{18}{20} V_2$$

$$\Rightarrow V_s = 27 I_1 - 0.9 V_2 \longrightarrow (7)$$

Sub eq (6) in eq (2).

$$V_2 = 18 I_1 + 30\left(-\frac{V_2}{20}\right)$$

$$\Rightarrow V_2 = 18 I_1 - 1.5 V_2$$

$$\Rightarrow 2.5 V_2 = 18 I_1$$

$$\Rightarrow I_1 = \frac{2.5}{18} V_2 \longrightarrow (8) \; \cancel{= \frac{25}{10 \times 18} V_2}$$

$$\cancel{\Rightarrow I_1}$$

Substitute eq ⑧ in ⑦

$$V_s = 27\left(\frac{2.5}{18}\right)V_2 - 0.9V_2$$

$$\Rightarrow V_s = \frac{3 \times 2.5}{2} V_2 - 0.9V_2$$

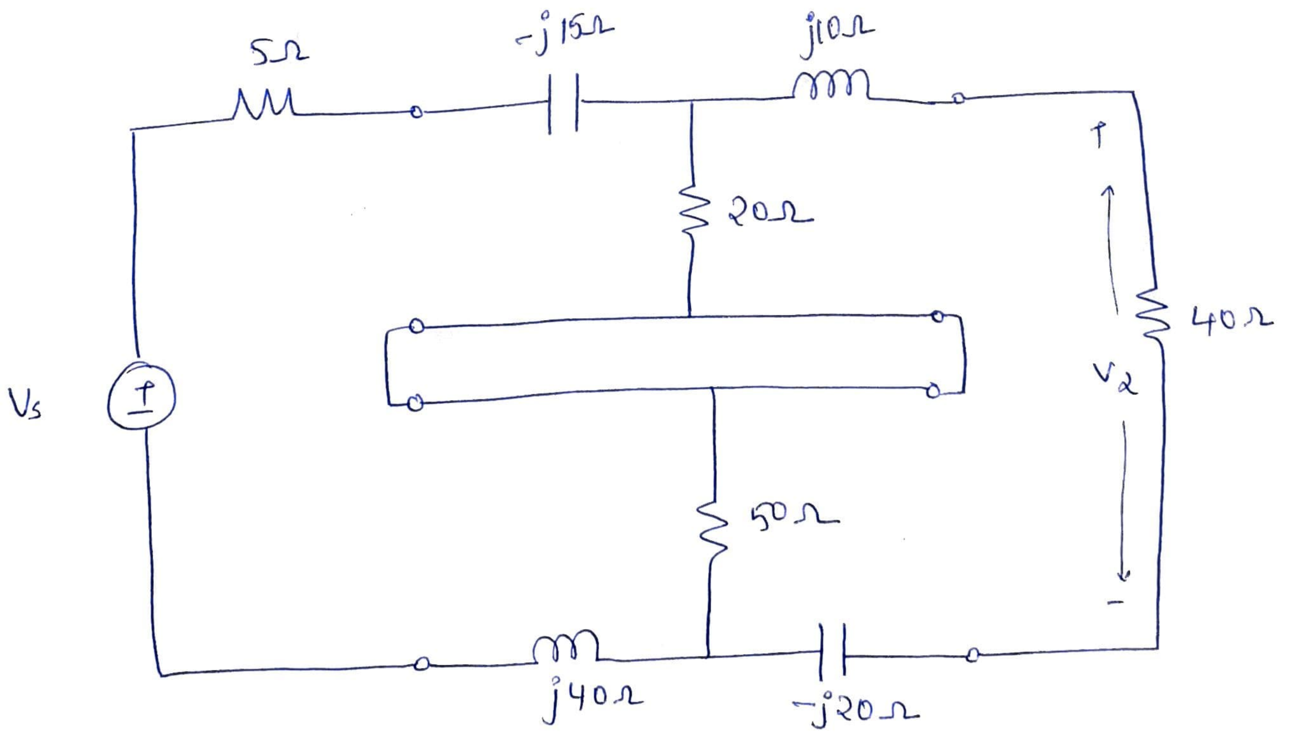
$$= 3.75V_2 - 0.9V_2$$

$$= 2.85V_2$$

$$\Rightarrow \frac{V_2}{V_s} = \frac{1}{2.85} = 0.3509.$$

$$\therefore \boxed{\frac{V_2}{V_s} = 0.3509}$$

3Q) find V_2/V_s in the below circuit.



Ans: $0.58\angle{-40^\circ}$