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DEPARTMENT OF CIVIL ENGINEERING

SOIL MECHANICS

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Contents

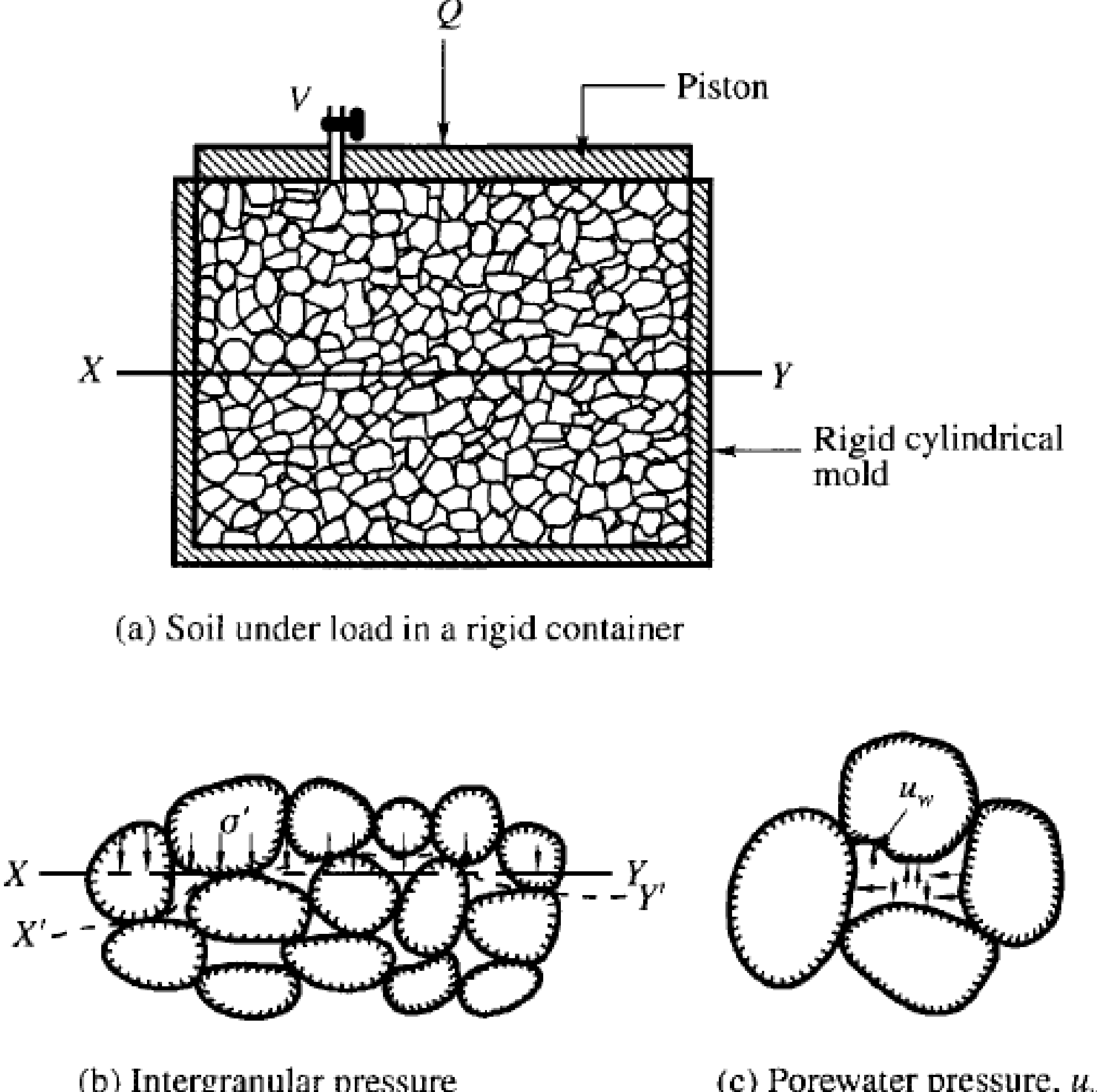
Introduction

- The pressure transmitted through grain to grain at the contact points through a soil mass is termed as intergranular or **effective pressure**.
- It is known as effective pressure since this pressure is responsible for the decrease in the void ratio or increase in the frictional resistance of a soil mass.
- If the pores of a soil mass are filled with water and if a pressure induced into the pore water, tries to separate the grains, this pressure is termed as **pore water pressure or neutral stress.** The effect of this pressure is to increase the volume or decrease the frictional resistance of the soil mass.

Effective stress

- Consider a rigid cylindrical mold, in which dry sand is placed. Assume that there is no side friction.
- Load Q is applied at the surface of the soil through a piston. The load applied at the surface is transferred to the soil grains in the mold through their points of contact.
- If the load is quite considerable, it would result in the compression of the soil mass in the mold.
- The compression might be partly due to the elastic compression of the grains at their points of contact and partly due to relative sliding between particles.
- If the sectional area of the cylinder is A, the average stress at any level XY may be written as

$$\sigma_a = \frac{Q}{A}$$



(a) Soil under load in a rigid container

(b) Intergranular pressure

(c) Porewater pressure, u_w

Figure 5.1 Effective and pore water pressures

Effective Stress

- The stress σ_a is the average stress and not the actual stress prevailing at the grain to grain contacts which is generally very high.
- Any plane such as XY will not pass through all the points of contact and many of the grains are cut by the plane.
- The actual points of contact exhibit a wavy form. However, for all practical purposes the average stress is considered.
- Since this stress is responsible for the deformation of the soil mass, it is termed the intergranular or effective stress.
- We may therefore write,

$$\sigma_a = \sigma'$$

where σ' is the effective stress.

Pore water pressure

- Consider now another experiment. Let the soil in the mold be fully saturated and made completely watertight. If the same load Q is placed on the piston, this load will not be transmitted to the soil grains as in the earlier case.
- If we assume that water is incompressible, the external load Q will be transmitted to the water in the pores.
- This pressure that is developed in the water is called the pore water or neutral stress u_w.
- This pore water pressure u_w prevents the compression of the soil mass. The value of this pressure is

$$u_w = \frac{Q}{A}$$

Total Pressure

- If the valve V provided in the piston is opened, immediately there will be expulsion of water through the hole in the piston. The flow of water continues for some time and then stops.
- The expulsion of water from the pores decreases the pore water pressure and correspondingly increases the intergranular pressure.
- At any stage the total pressure Q/A is divided between water and the points of contact of grains.
- Total pressure $\sigma_t = \dfrac{Q}{A}$ = Intergranular pressure + pore water pressure

$$\sigma_t = \sigma' + u_w$$

- Final equilibrium will be reached when there is no expulsion of water.
- At this stage the porewater pressure $u_w = 0$. All the pressure will be carried by the soil grains. Therefore, we can write, $\sigma_t = \sigma'$

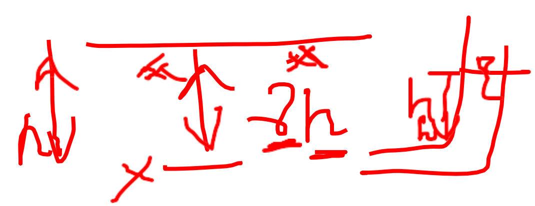
Total Pressure

- The pore water pressure u_w can be induced in the pores of a soil mass by a head of water over it.
- When there is no flow of water through the pores of the mass, the intergranular pressure remains constant at any level.
- But if there is flow, the intergranular pressure increases or decreases according to the direction of flow.

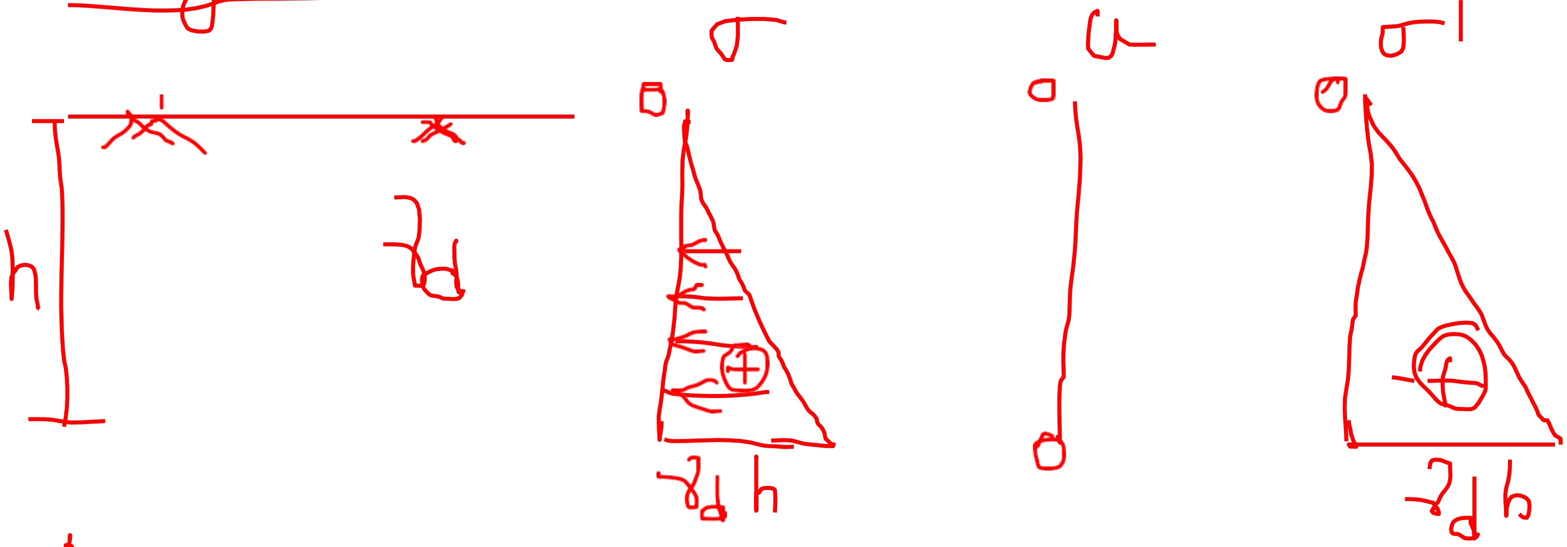
σ = wt of soil = γh

u = PWP = $\gamma_w h_w$

$\sigma' = \sigma - u$



① Dry Soil



At GL, $= 0$, $\sigma = \gamma \times 0 = 0$

$- 0$

$= 0$

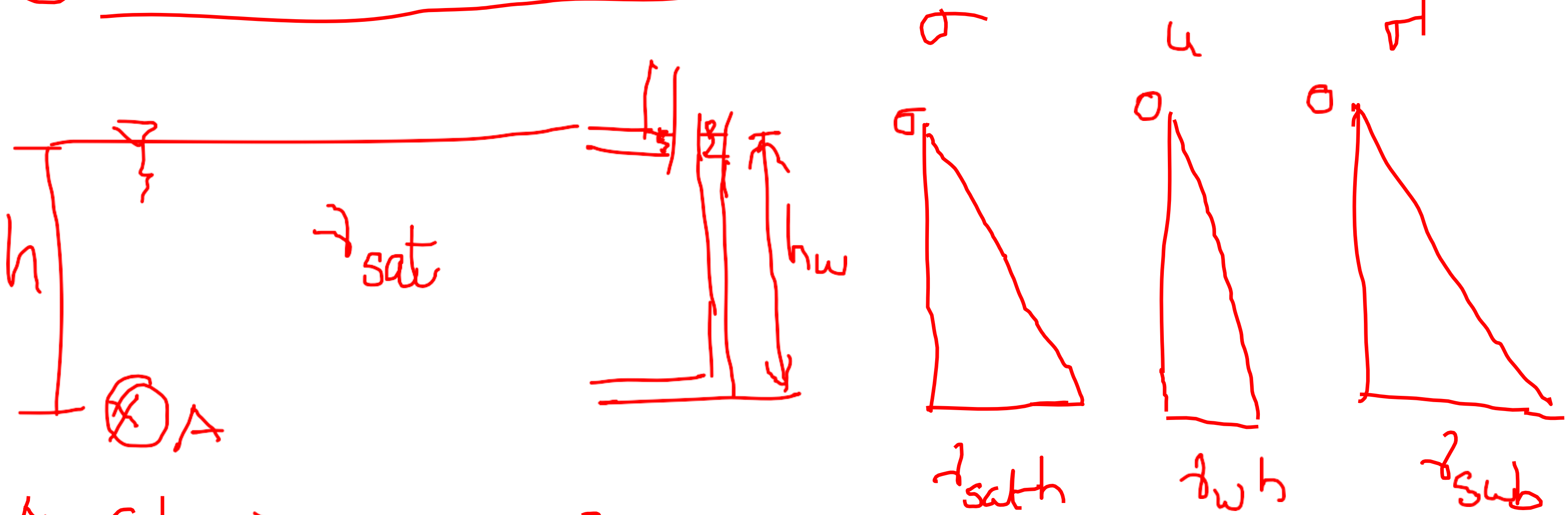
At m depth, $\sigma = \gamma_d x = \gamma_d$

$- 0$

$\sigma' = \sigma - u = \gamma_d$

h

② Saturated Soil



At GL, $Z=0$, $\sigma = \gamma \times 0 = 0$

$u = \gamma_w \times 0 = 0$

$\sigma' = \sigma - u = 0$

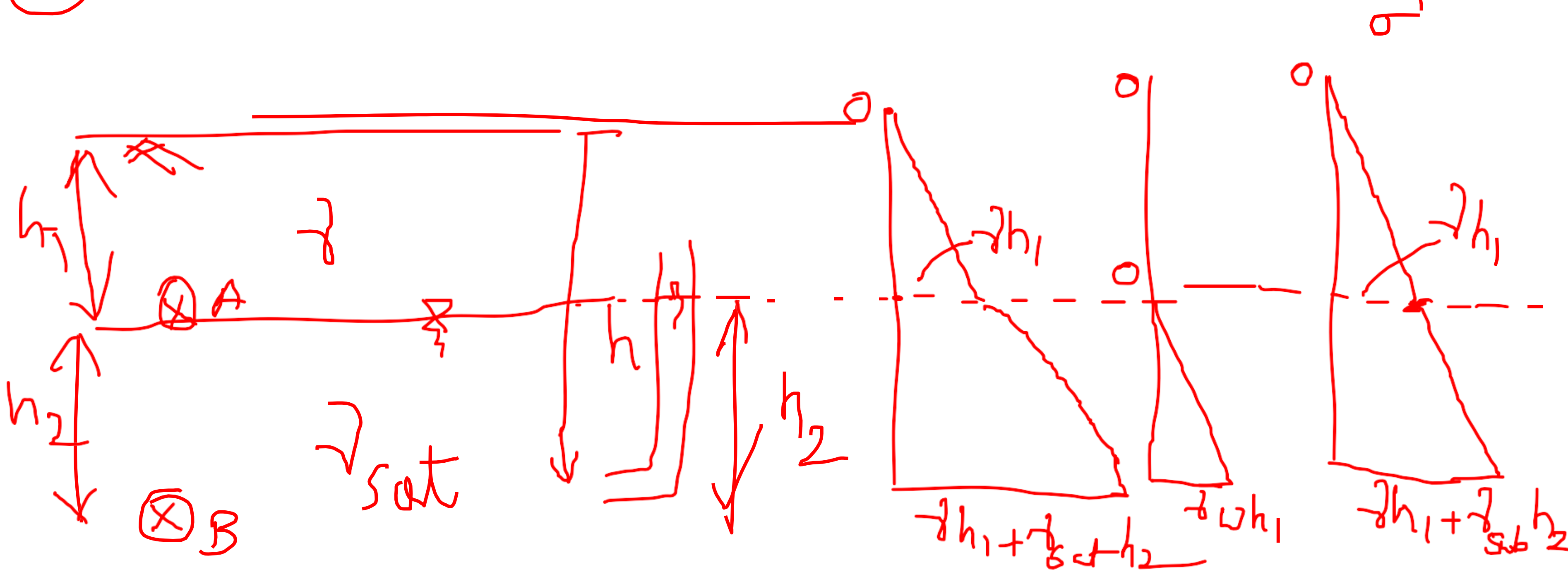
~~At~~ A, =

$\sigma = \gamma_{sat}x = \gamma_{sat}h$

$u = \gamma_w$

$\sigma' = \gamma_{sat}h - \gamma_w h = (\gamma_{sat} - \gamma_w)h = \underline{\underline{\gamma_{sub}h}}$

③ Unsaturated Soil



At GL, $z = 0$, $\sigma = 0$, $u = 0$, $\sigma' = 0$

At $z =$, $\sigma = \gamma h$, $u = 0$, $\sigma' = \sigma - u = \gamma h_1$

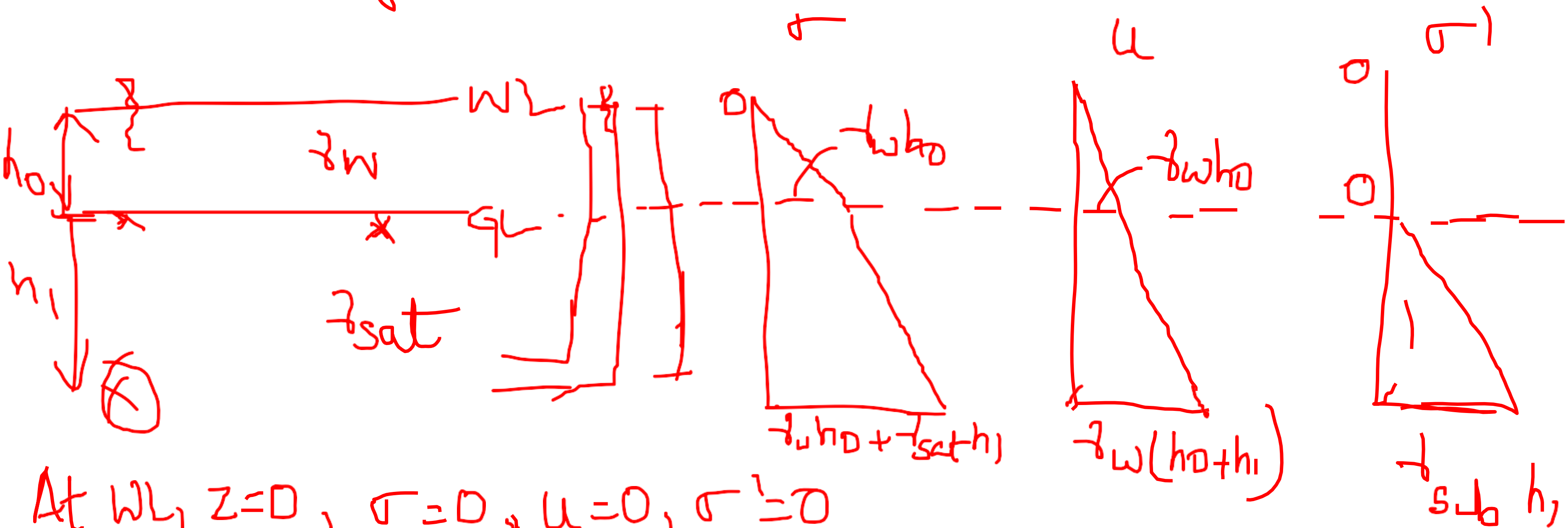
At $z = h_1 + h_2 =$, $\sigma = \gamma \times h_1 + \gamma_{sat} \times h_2$

$= \gamma_w \times h_2 = \gamma_w h_2$

$\sigma' = \sigma - u = \gamma h_1 + \gamma_{sat} h_2 - \gamma_w h_2$

$= \gamma h_1 + (\gamma_{sat} - \gamma_w) h_2 = \gamma h_1 + \gamma_{sub} h_2$

④ Submerged Soil



At WL, $z = 0$, $\sigma = 0$, $u = 0$, $\sigma' = 0$

At GL, $z = h_0$ $\sigma = \gamma_w h_0$ $u = \gamma_w h_0$, $\sigma' = \sigma - u = 0$

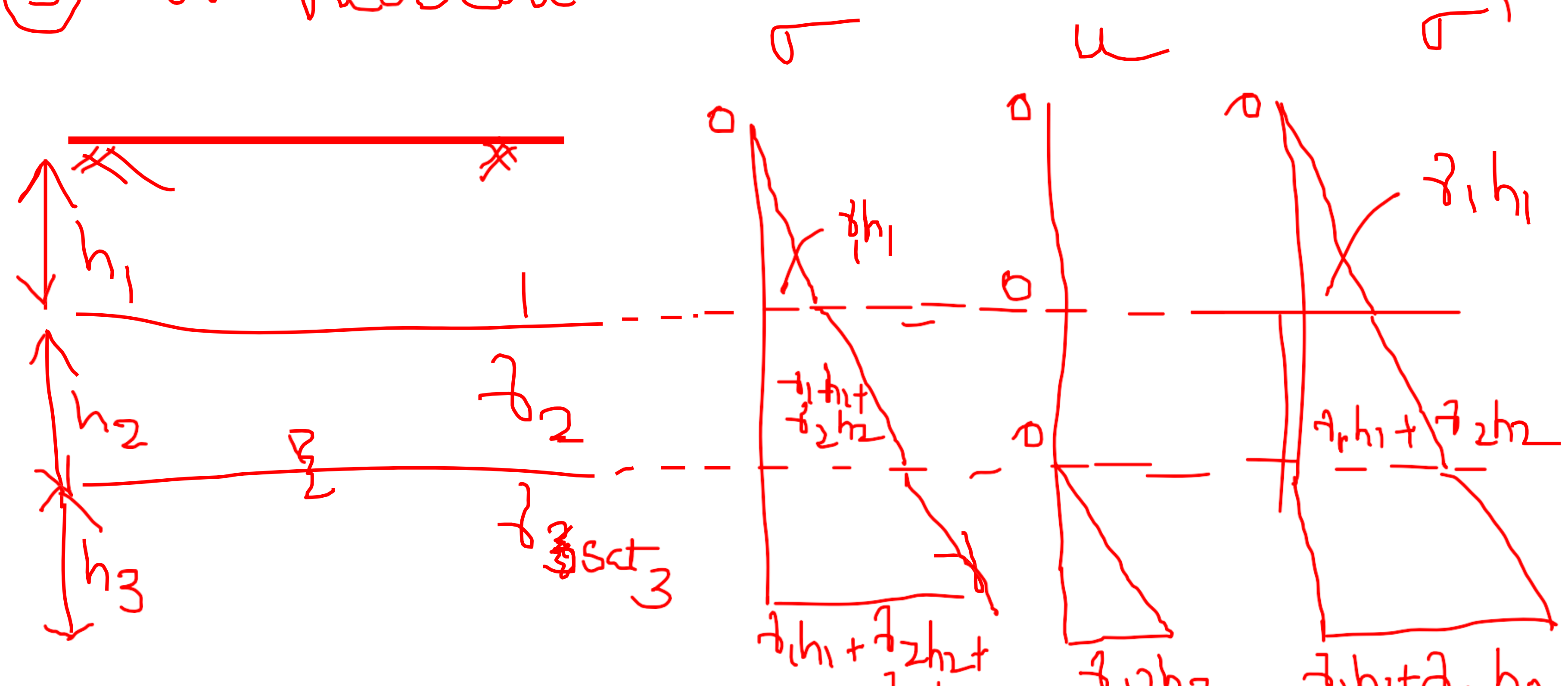
At $= h_0 + h_1$, $\sigma = \gamma_w h_0 + \gamma_{sat} \times h_1$

$u = \gamma_w (h_0 + h_1)$

$\sigma' = \cancel{\gamma_w h_0} + \gamma_{sat} h - \cancel{\gamma_w} \; {}_0 - \gamma_w$

$= \gamma_{sub} h_1$

⑤ Stratified soil



At $z = 0$, $\sigma = 0$, $u = 0$, $\sigma' = 0$

$z = h_1$, $\sigma = \gamma_1 h_1$, $u = 0$, $\sigma' = \gamma_1 h_1$

$z = h_1 + h_2$, $\sigma = \gamma_1 h_1 + \gamma_2 h_2$, $u = 0$, $\sigma' = \gamma_1 h_1 + \gamma_2 h_2$

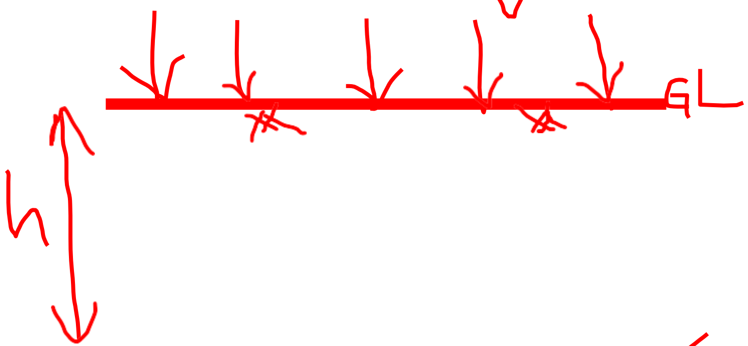
$z = h_1 + h_2 + h_3$, $\sigma = \gamma_1 h_1 + \gamma_2 h_2 + \gamma_{sat_3} h_3$

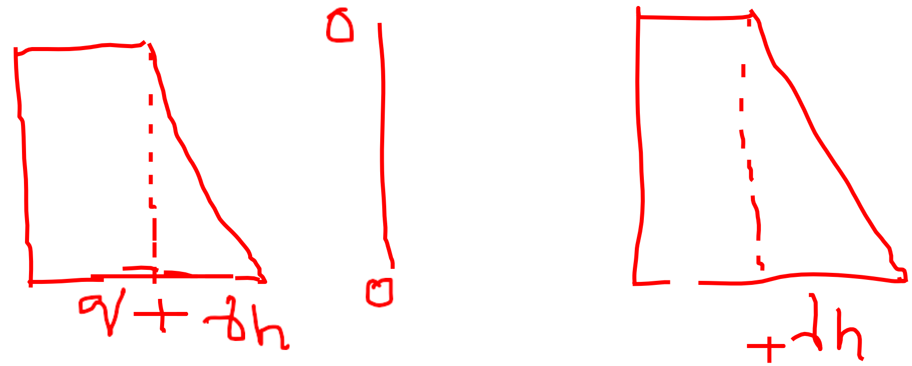
$u = \gamma_w h_3$

$\sigma' = \gamma_1 h_1 + \gamma_2 h_2 + \gamma_{sub} h_3$

6 Surcharge effect

q kN/m^2





At $z = 0$, $= q$, $u = 0$, $\bar{=}$

$z = h$, $\sigma = \; + \gamma$, $u = 0$, $\bar{\sigma} = \; +$
$\quad z \qquad\qquad\qquad\qquad\qquad q \qquad\qquad h$

Capillary rise in soils

- When rainfall occurs on this soil surface the rain water percolates through these voids and flows down through the soil under the influence of gravity.
- This water under the ground surface exists in two forms,
- 1. Free water
- 2. Held water
- Free water is ground water that moves inside the soil under the influence of gravity. So it is also called Gravitational water. It completely fills and saturates the voids present in the soil.
- To measure the level of free water underground we make observation wells into the ground. The level up to which the underground water rises in the well is the level of free water. This level is called ground water table.
- The water present at the water table experience **atmospheric pressure** and below this level **pressure on the water increases with depth** and all the water present in the voids below water table is in **compression.**

- If the water contained in the soil were subjected to no force other than gravity, the soil above the water table would be perfectly dry.
- If the lower part of the mass of dry soil comes into contact with water, the water rises in the voids to a certain height above the freewater surface.
- The upward flow into the voids of the soil is attributed to the **surface tension** of the water.
- The height to which water rises above the water table against the force of gravity is called **capillary rise.**
- **Capillary effect** is the ability of a liquid to flow in narrow spaces without the assistance of external forces.

Rise of Water in Capillary Tubes

- The phenomenon of capillary rise can be demonstrated by immersing the lower end of a very small diameter glass tube into water. Such a tube is known as capillary tube.
- As soon as the lower end of the tube comes into contact with water, the attraction between the glass and the water molecules combined with the surface tension of the water pulls the water up into the tube to a height h_c above the water level. The height h_c is known as the height of capillary rise.
- The upper surface of water assumes the shape of a cup, called the 'meniscus' that joins the walls of the tube at an angle α known as the contact angle.

Surface Tension

- Surface tension is a force that exists at the surface of the meniscus. Along the line of contact between the meniscus in a tube and the walls of the tube itself, the surface tension**,** $\mathbf{T_s}$**,** is expressed as the force per unit length acting in the direction of the tangent
- The components of this force along the wall and perpendicular to the wall are
- Along the wall = $\mathbf{T \cos \alpha}$ per unit length of wall
- Normal to the wall = $\mathbf{T_s \sin \alpha}$ per unit length of wall
- The force normal to the wall tries to pull the walls of the tube together and the one along the wall produces a compressive force in the tube below the line of contact.

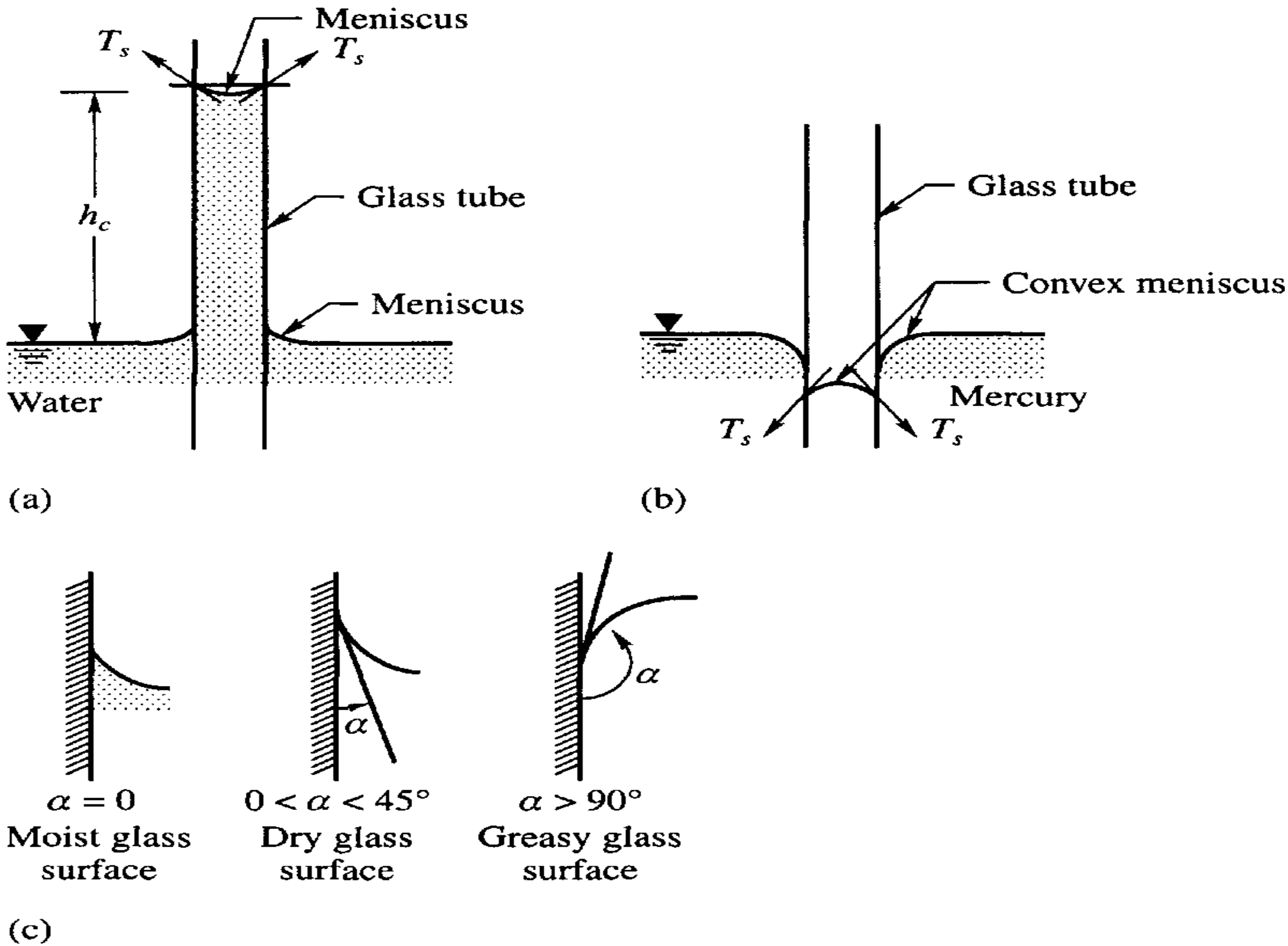


Figure Capillary rise and meniscus

Rise of Water in Capillary Tubes

- If the meniscus has stopped moving upward in the tube, then there must be equilibrium between the weight of the column of water suspended from the meniscus and the force with which the meniscus is clinging to the wall of the tube. We can write the following equation of equilibrium

$$\pi d T_s \cos\alpha = \frac{\pi d^2 h_c \gamma_w}{4} \quad \text{or} \quad h_c = \frac{4T_s \cos\alpha}{d\gamma_w}$$

- The surface tension Ts for water at 20 °C can be taken as equal to 75×10^{-8} kN per cm. The above equation can be simplified by assuming $\alpha = 0$ for moist glass and by substituting for T_s.
- Therefore, for the case of water, the capillary height h_c can be written as

$$h_c = \frac{4T_s}{d\gamma_w} = \frac{4 \times 75 \times 10^{-8} \times 10^6}{d \times 9.81} = \frac{0.3}{d}$$

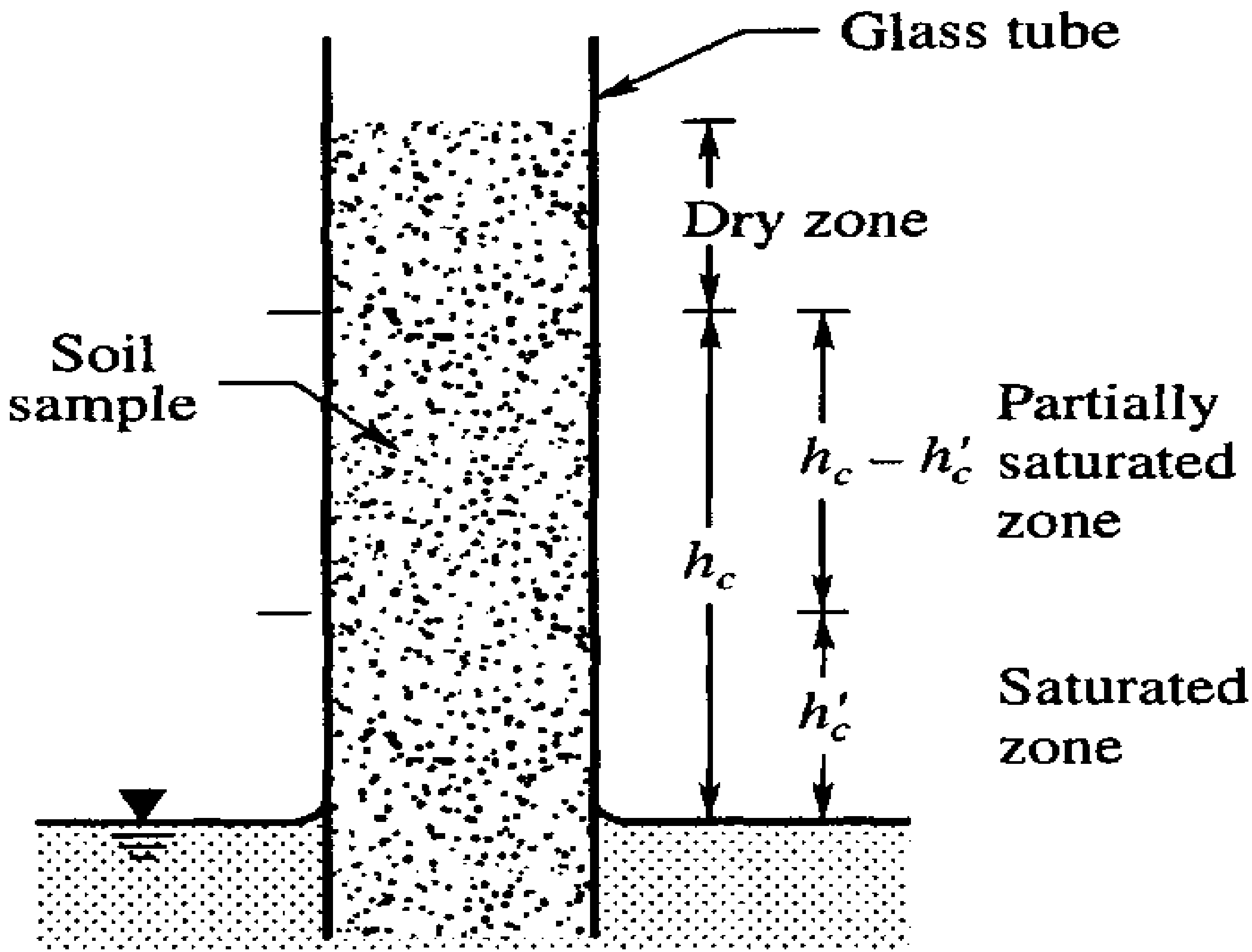
- h and d are expressed in cm, and, $\gamma_w = 9.81$ kN/m^3

- In contrast to capillary tubes the continuous voids in soils have a variable width.
- They communicate with each other in all directions and constitute an intricate network of voids. When water rises into the network from below, the lower part of the network becomes completely saturated.
- In the upper part, however, the water occupies only the narrowest voids and the wider areas remain filled with air.
- Sand would remain fully saturated only up to a height h' which is considerably smaller than hc.
- A few large voids may effectively stop capillary rise in certain parts.
- The water would rise, therefore, to a height of hc only in the smaller voids. The zone between the depths (hc - h‘) will remain partially saturated.
- The height of the capillary rise is greatest for very fine grained soils materials, but the rate of rise in such materials is slow because of their low permeability.

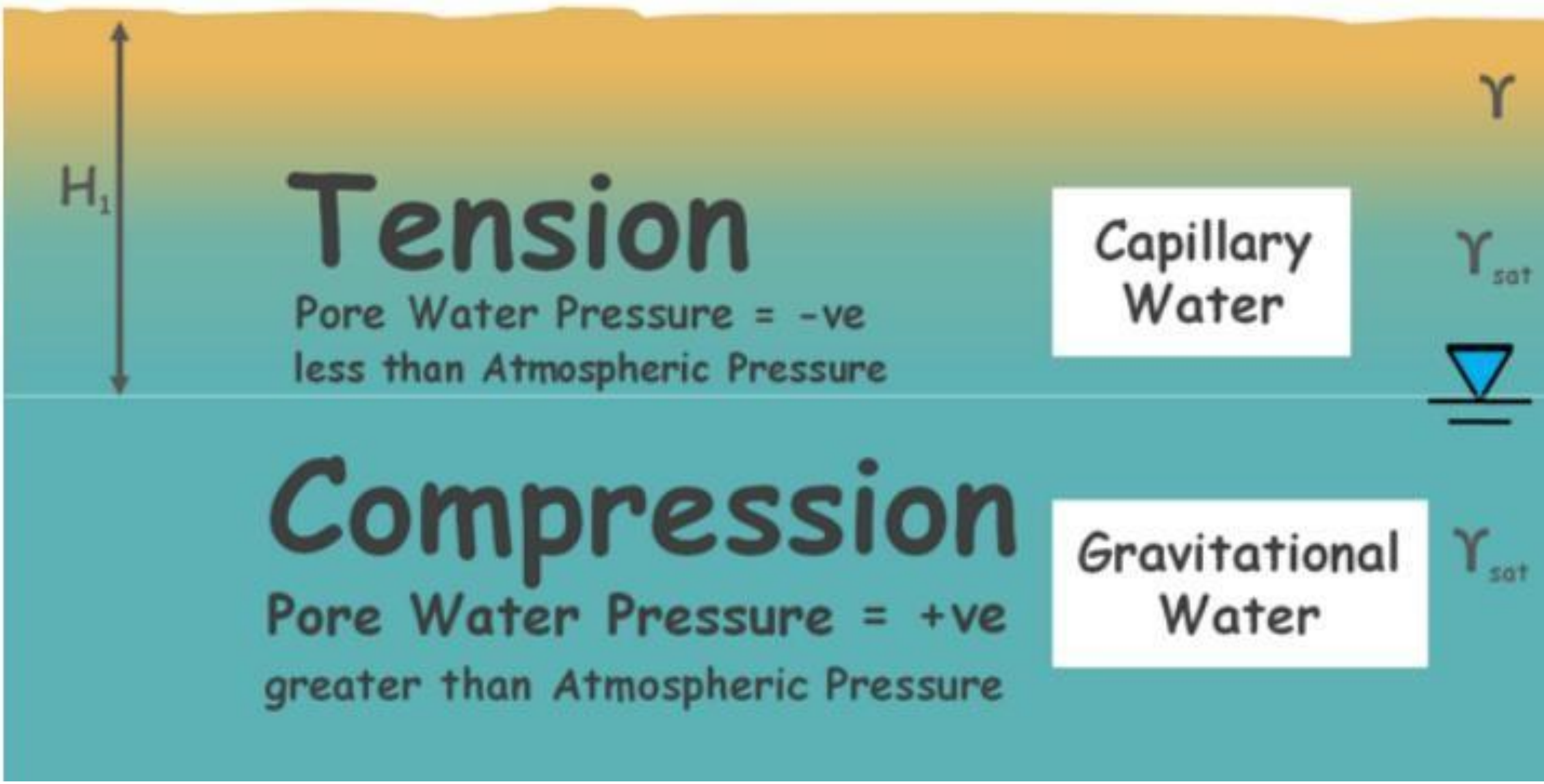
- As the effective grain size decreases, the size of the voids also decreases, and the height of capillary rise increases.
- A rough estimation of the height of capillary rise can be determined from the equation,

$$h_c = \frac{C}{eD_{10}}$$

- in which e is the void ratio, D10 is Hazen's effective diameter in centimeters, and C is an empirical constant which can have a value between 0.1 and 0.5 sq. cm.

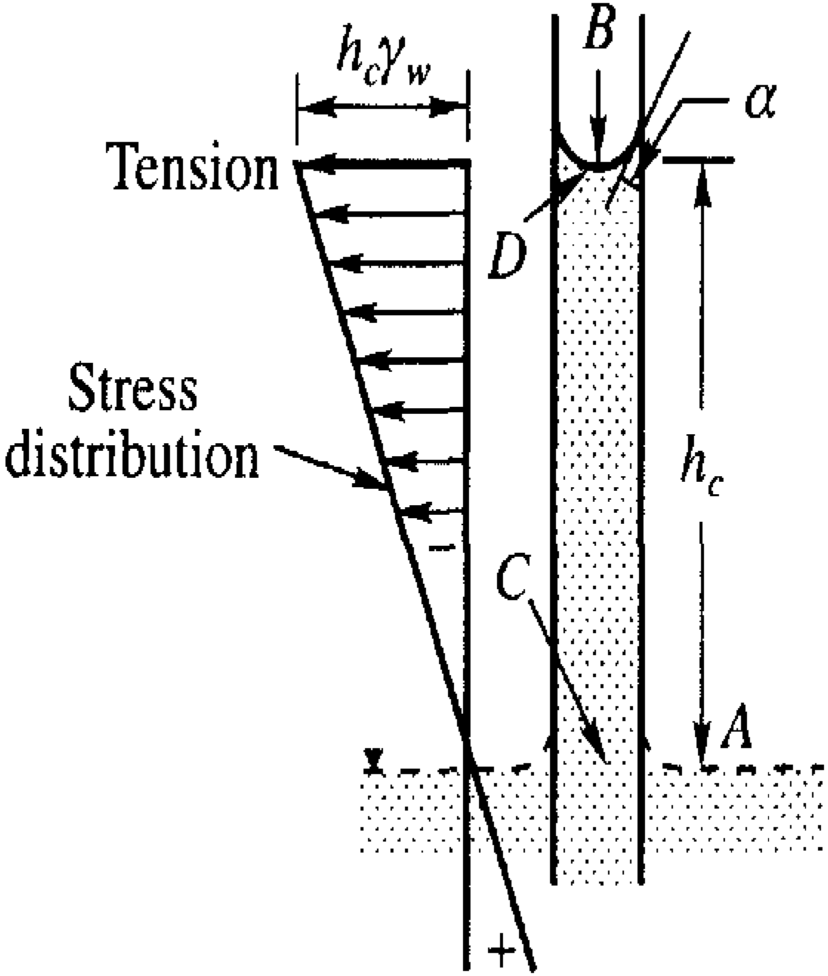


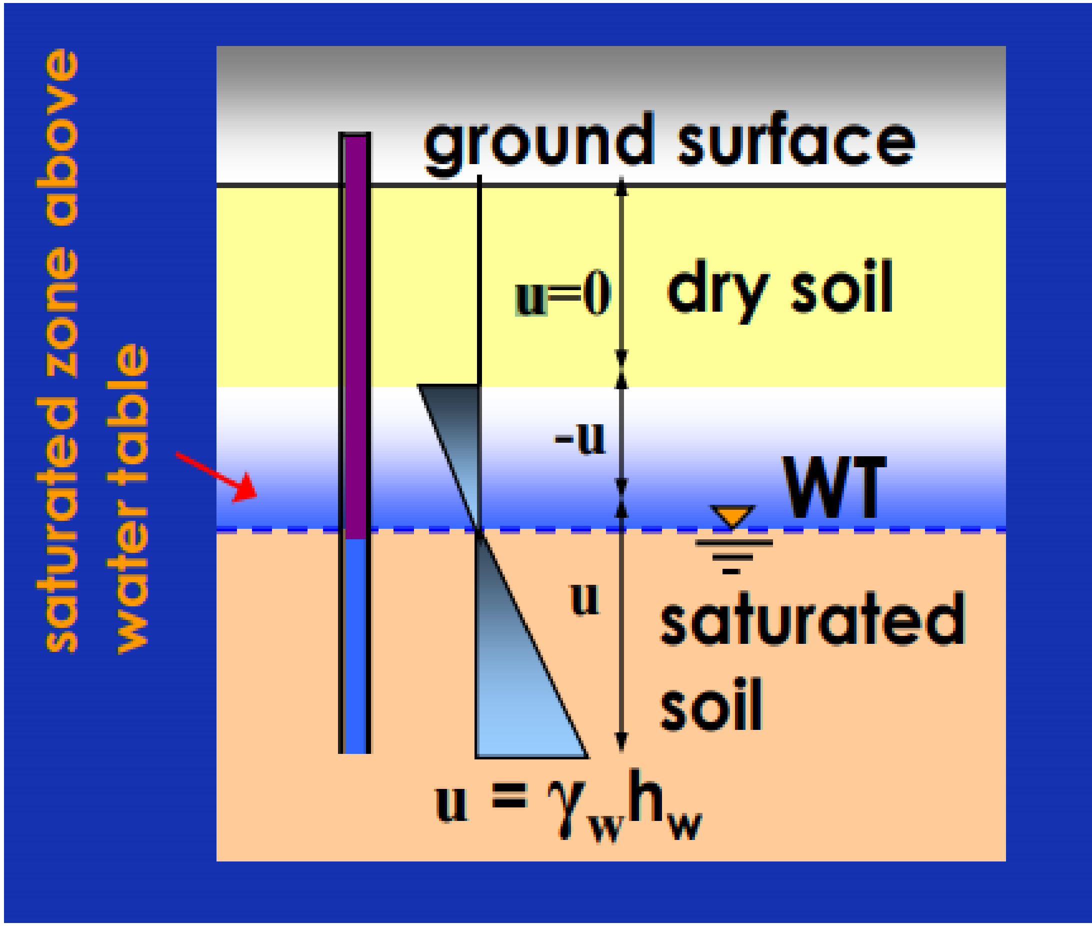
(a) Height of capillary rise


H1
Tension
Pore Water Pressure = -ve
less than Atmospheric Pressure
Capillary Water
γ
γsat
Compression
Pore Water Pressure = +ve
greater than Atmospheric Pressure
Gravitational Water
γsat

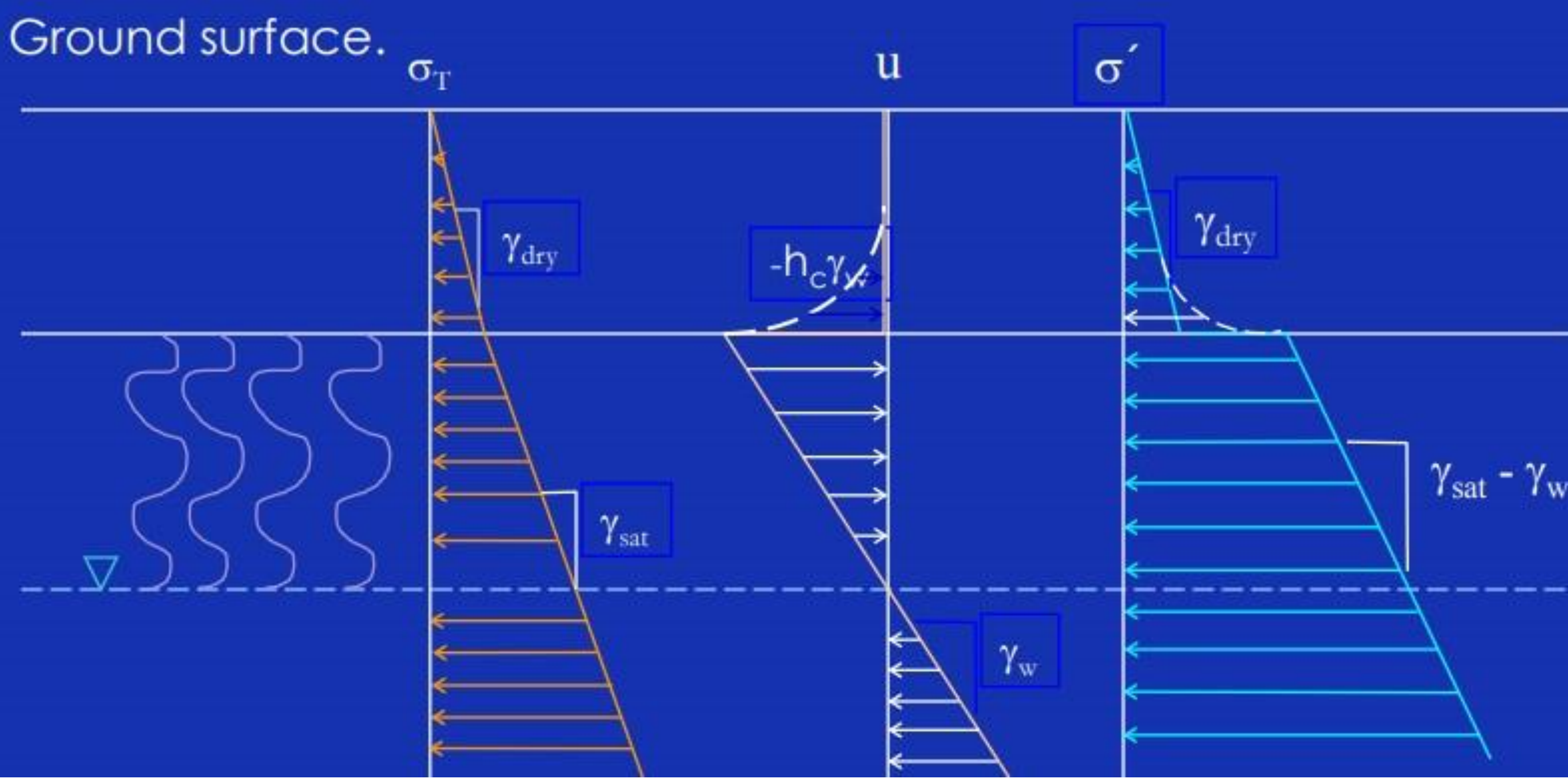
Stress distribution

Capilliary Pressure, u_w = (-) h_c x γ_w

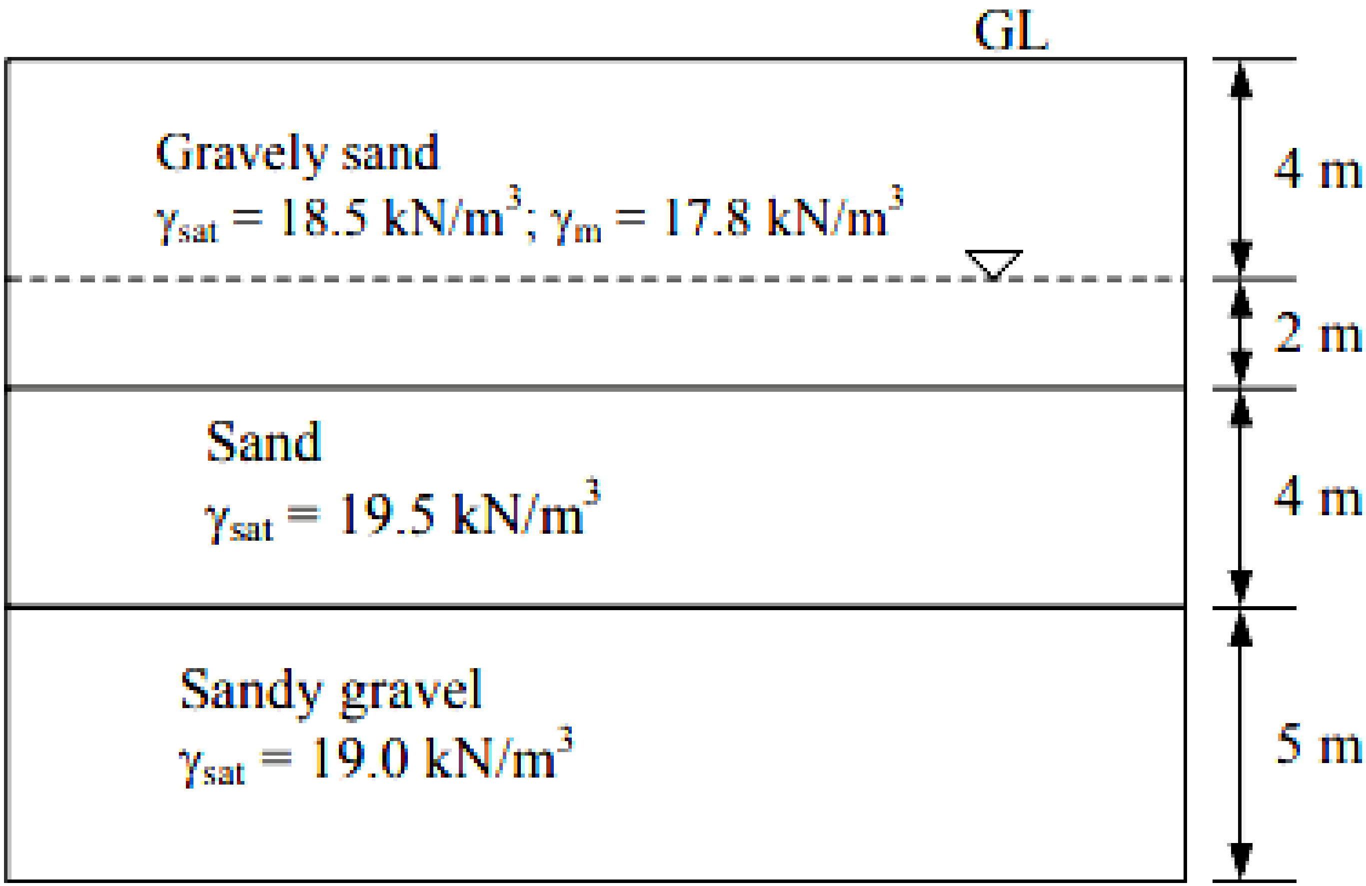




In the capillary zone: $\sigma' = \sigma - (-u_c) = \sigma + u_c$



Example 1: Plot the variation of total and effective vertical stresses, and pore water pressure with depth for the soil profile shown below



Solution:

Within a soil layer, the unit weight is constant, and therefore the stresses vary linearly. Therefore, it is adequate if we compute the values at the layer interfaces and water table location, and join them by straight lines.

At the ground level,

$\sigma_v = 0$; $\sigma_v' = 0$; and $u=0$

At 4 m depth,

$\sigma_v = (4)(17.8) = 71.2$ kPa; $u = 0$

$\therefore \sigma_v' = 71.2$ kPa

At 6 m depth,

$\sigma_v = (4)(17.8) + (2)(18.5) = 108.2$ kPa

$u = (2)(9.81) = 19.6$ kPa

$\therefore \sigma_v' = 108.2 - 19.6 = 88.6$ kPa

At 10 m depth,

$\sigma_v = (4)(17.8) + (2)(18.5) + (4)(19.5) = 186.2$ kPa

$u = (6)(9.81) = 58.9$ kPa

$\therefore \sigma_v' = 186.2 - 58.9 = 127.3$ kPa

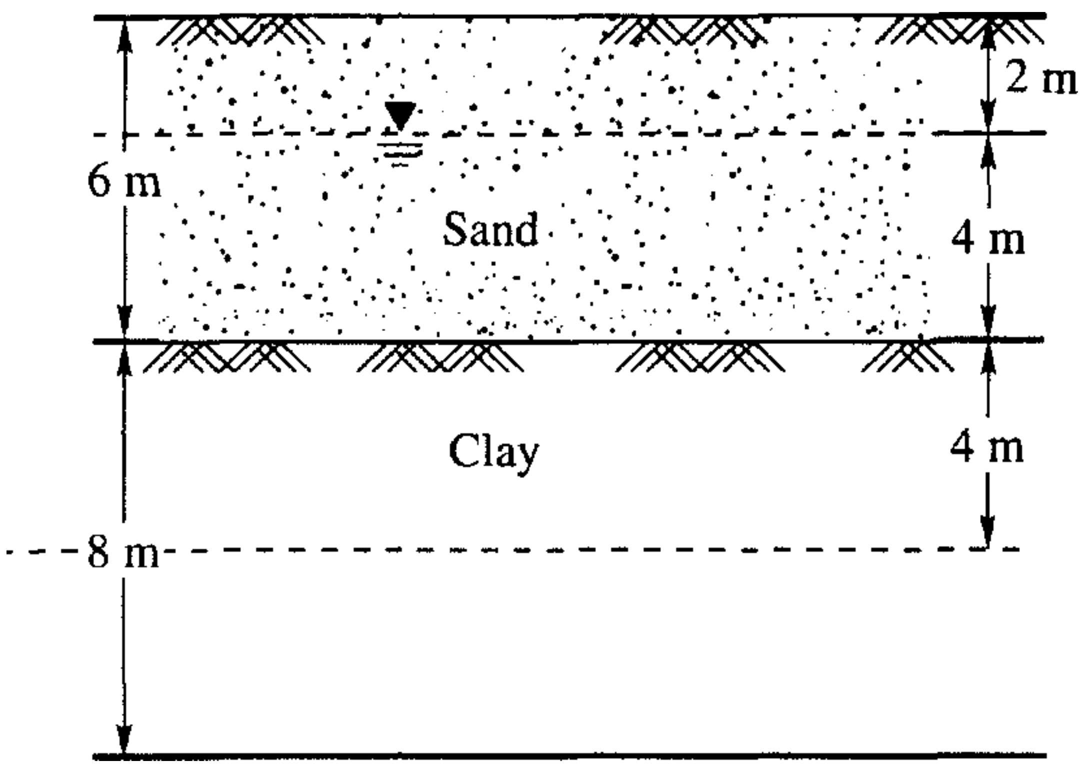
At 15 m depth,

$\sigma_v = (4)(17.8) + (2)(18.5) + (4)(19.5) + (5)(19.0) = 281.2$ kPa

$u = (11)(9.81) = 107.9$ kPa

$\therefore \sigma_v' = 281.2 - 107.9 = 173.3$ kPa

Example 2: A clay stratum 8.0 m thick is located at a depth of 6 m from the ground surface. The natural moisture content of the clay is 56% and G = 2.75. The soil stratum between the ground surface and the clay consists of fine sand. The water table is located at a depth of 2 m below the ground surface. The submerged unit weight of fine sand is 10.5 kN/m^3, and its moist unit weight above the water table is 18.68 kN/m^3. Calculate the effective stress at the center of the clay layer.



Solution

Fine sand:

Above water table: $\gamma_t = 18.68$ kN/m^3

Below WT: $\gamma_b = 10.5$ kN/m^3

$$\gamma_{sat} = 10.5 + 9.81 = 20.31 \text{ kN/m}^3$$

Clay stratum:

For $S = 1.0$,

$$e = wG_s = 0.56 \times 2.75 = 1.54$$

$$\gamma_{sat} = \frac{\gamma_w (G_s + e)}{1 + e} = \frac{9.81(2.75 + 1.54)}{1 + 1.54} = 16.57 \text{ kN/m}^3$$

$$\gamma_b = 16.57 - 9.81 = 6.76 \text{ kN/m}^3$$

At a depth 10.0 m from GL, that is, at the center of the clay layer,

$$\sigma_t = 2 \times 18.68 + 4 \times 20.31 + 4 \times 16.57$$

$$= 37.36 + 81.24 + 66.28 = 184.88 \text{ kN/m}^2$$

$$u_w = 4 \times 9.81 + 4 \times 9.81 = 39.24 + 39.24 = 78.48 \text{ kN/m}^2$$

Effective stress, $\sigma' = \sigma_t - u_w = 184.88 - 78.48 = 106.40$ kN/m^2

Example 3:

The diameter of a clean capillary tube is 0.08 mm. Determine the expected rise of water in the tube.

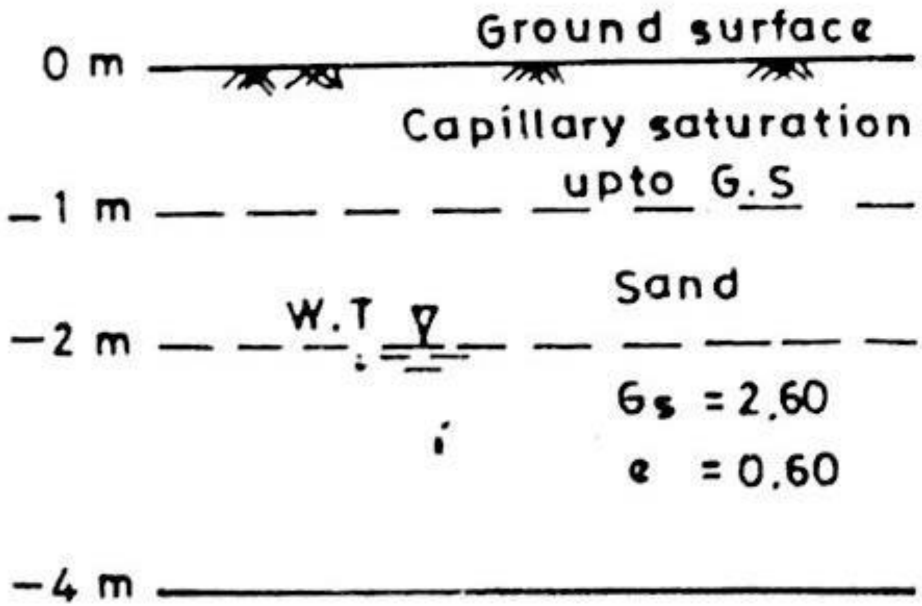
Solution

Per Eq. (5.22), the expected rise, h_c, in the capillary tube is

$$h_c = \frac{0.3}{d} = \frac{0.3}{0.008} = 37.5 \text{ cm}$$

where, d is in centimeters

Example 6.3 For the subsoil conditions shown in Fig. 6.21 (a) what are the effective strees values at 1 m, 2 m and 4 m depths ? Assume $\gamma_w = 10\ kN/m^3$.



(a) Subsoil conditions

Solution:

$$\gamma_{sat\ (sand)} = \frac{G+e}{1+e}\gamma_w = \frac{2.6+0.6}{1+0.6} \times 10 = 20 \text{ kN/m}^3$$

The sand is saturated by gravity flow below W.T. and by capillary flow upto a height of 2 m above W.T.

El. — 1 m: $\sigma = 1 \times 20 = 20 \text{ kN/m}^2$ (total stress is the same whether the soil is saturated by gravity flow or capillary flow

$u = -1 \times 10 = -10 \text{ kN/m}^2$ (capillary flow, hence –ve porewater pressure)

$\bar{\sigma} = \sigma - u = 20 - (-10) = 30 \text{ kN/m}^2$

El. — 2 m: $\sigma = 2 \times 20 = 40 \text{ kN/m}^2$

$u = 0$ ($\because$ porewater pressure = 0 at W.T.)

$\bar{\sigma} = 40 \text{ kN/m}^2$

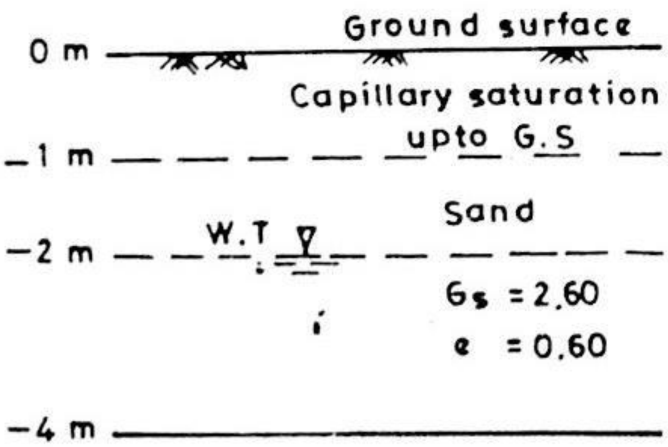
El. —4 m : $\sigma = 4 \times 20 = 80 \text{ kN/m}^2$

$u = 2 \times 10 = 20 \text{ kN/m}^2$

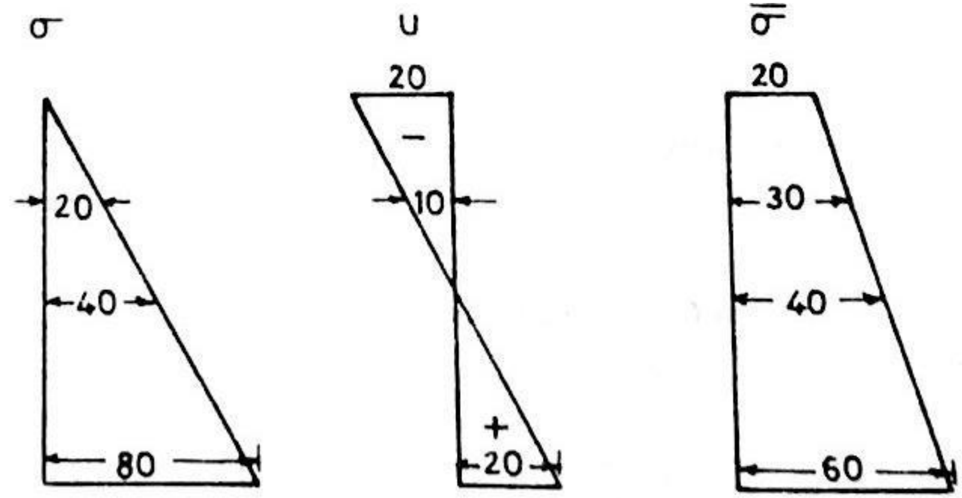
$\bar{\sigma} = 80 - 20 = 60 \text{ kN/m}^2$

At El. 0 m : $u = -2 \times 10 = -20 \text{ kN/m}^2$

and $\bar{\sigma} = 0 - (-20) = 20 \text{ kN/m}^2$



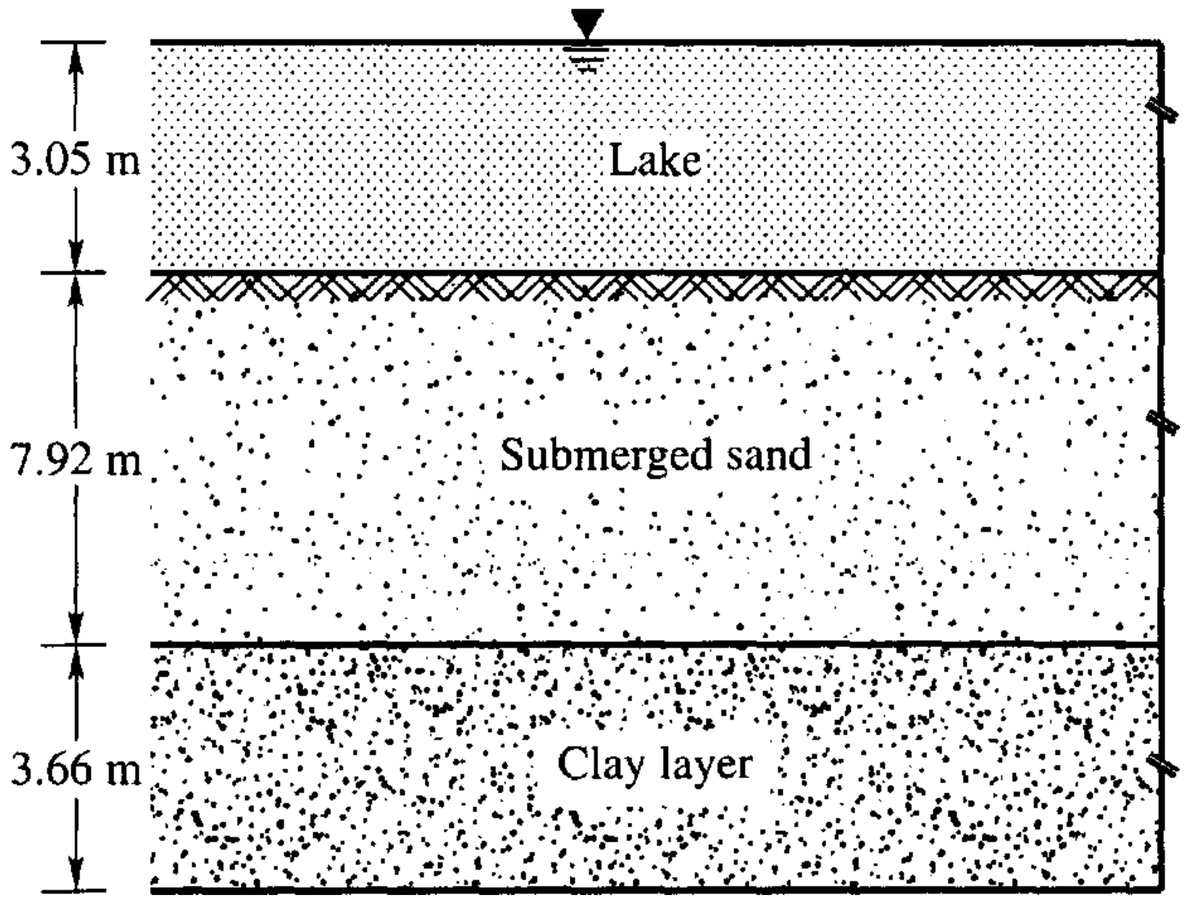
(a) Subsoil conditions



(b) Stress-distribution diagrams

Example 5: A clay layer 3.66 m thick rests beneath a deposit of submerged sand 7.92 m thick. The top of the sand is located 3.05 m below the surface of a lake. The saturated unit weight of the sand is 19.62 kN/m^3 and of the clay is 18.36kN/m^3.

Compute (a) the total vertical pressure, (b) the pore water pressure, and (c) the effective vertical pressure at mid height of the clay layer



(a) Total pressure

The total pressure σ, over the midpoint of the clay is due to the saturated weights of clay and sand layers plus the weight of water over the bed of sand, that is

$$\sigma_t = \frac{3.66}{2} \times 18.36 + 7.92 \times 19.62 + 3.05 \times 9.81 = 33.6 + 155.4 + 29.9 = 218.9 \text{ kN/m}^2$$

(b) Pore water pressure is due to the total water column above the midpoint. That is

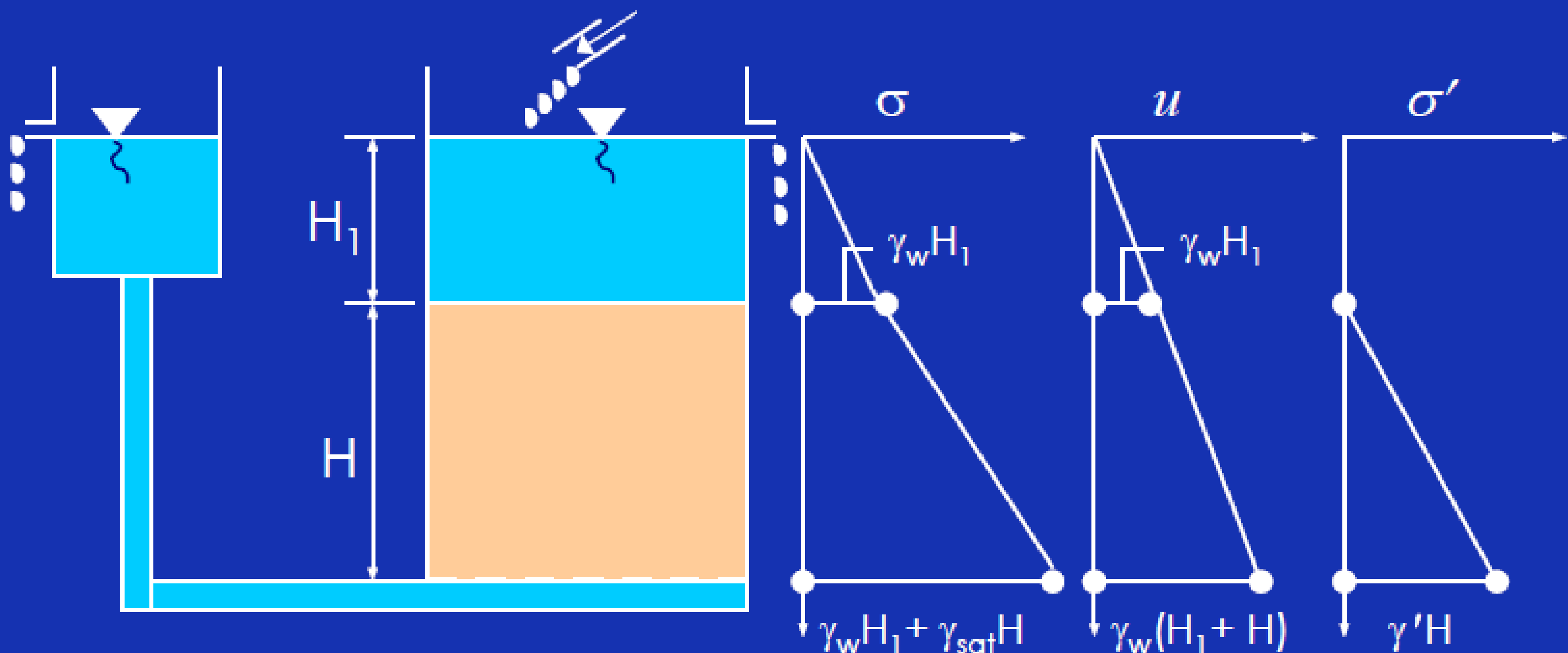
$$u_w = \frac{3.66}{2} \times 9.81 + 7.92 \times 9.81 + 3.05 \times 9.81 = 125.6 \text{ kN/m}^2$$

(c) Effective vertical pressure

$$\sigma_t - u_w = \sigma' = 218.9 - 125.6 = 93.3 \text{ kN/m}^2$$

Stresses when no flow takes place

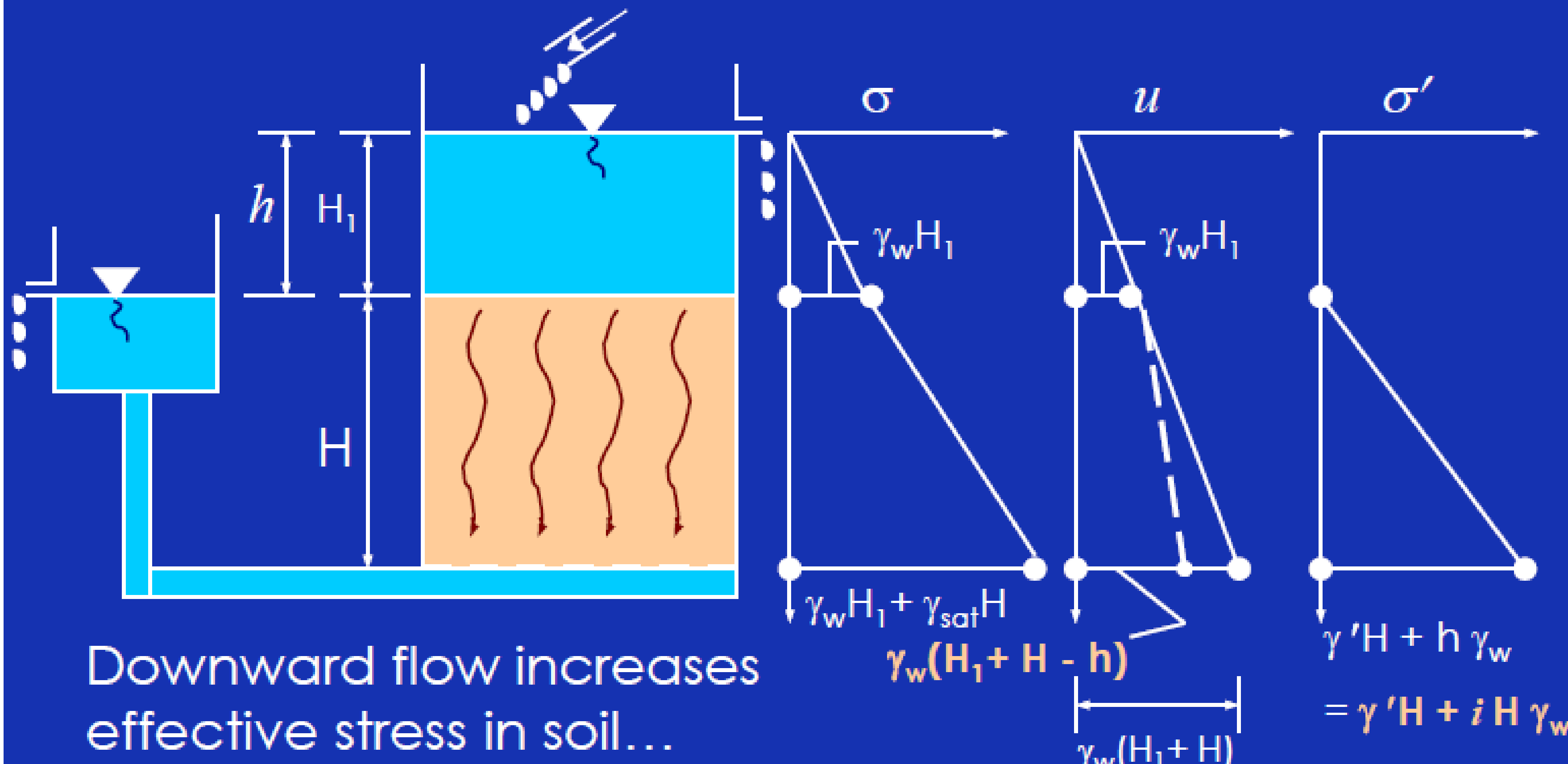
Case –I : When no flow takes place through soil (Hydrostatic condition)



No flow; Head loss ΔH = 0; No change in effective stress

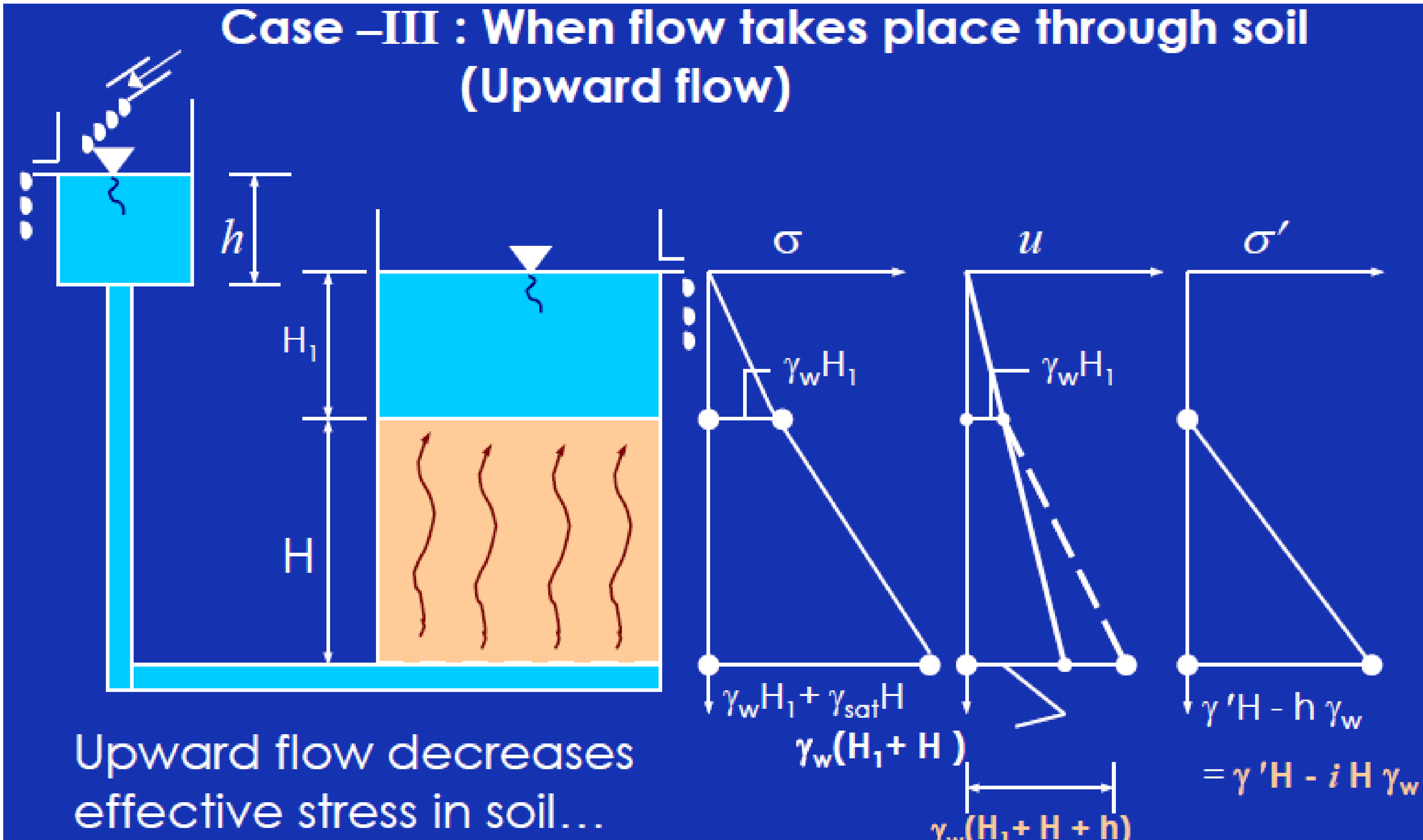
Stresses when flow takes place through the soil from **top to bottom**

Case –II : When flow takes place through soil (Downward flow)



Downward flow increases effective stress in soil…

Stresses when flow takes place through the soil from **bottom to top**



Effective stress

Downward seepage increases the effective stress.

- $\boldsymbol{\sigma' = \gamma' H + p_s H}$

Upward seepage decreases the effective stress.

- $\boldsymbol{\sigma' = \gamma' H - p_s H}$

  where seepage pressure [kN/m^3]

- $\boldsymbol{p_s = i\,\gamma_w}$

Critical Hydraulic Gradient, i_c

- The hydraulic gradient at which the effective stress becomes zero is known as **Critical Hydraulic gradient.**
- In the case of upward flow:

  When $i \rightarrow i_c$ $\quad \boldsymbol{\sigma' = \gamma' H + H\, i_c \gamma_w = 0 \Rightarrow i = i_c = \gamma' / \gamma_w}$
- Under these circumstances, cohesion-less soils can not support any weight.
- Moreover, as $i \rightarrow i_c$ soil becomes much looser and $k \uparrow$

Quick condition or Boiling condition in cohesion-less soils

- In such cases, cohesionless soils lose all of their shear strength and bearing capacity and a visible agitation of soil grains is observed.
- This phenomenon is known as ***boiling or a quick sand condition***.
- Substituting $\gamma_b = \frac{\gamma_w (G_s - 1)}{1+e}$, we get $i_C = \frac{\gamma'}{\gamma_w} = \frac{G_s - 1}{1+e}$
- The critical gradient of natural granular soil deposits can be calculated if the void ratios of the deposits are known.
- For all practical purposes the specific gravity of granular materials can be assumed as equal to 2.65.
- It should be remembered that a quick condition does not occur in clay deposits since the cohesive forces between the grains prevent the soil from boiling.

Conditions favourable for the formation of quick sand

- Quick sand is not a type of sand but a flow condition occurring within a cohesion-less soil when its effective stress is reduced to zero due to upward flow of water.
- Quick sand occurs in nature when water is being forced upward under pressurized conditions.
- In this case, the pressure of the escaping water exceeds the weight of the soil and the sand grains are forced apart.
- The result is that the soil has no capability to support a load.

Points to remember:

- Quick sand is not a type of sand but a hydraulic condition.
- Cohesionless soil becomes quick when effective stress becomes zero.
- For quick sand conditions to take place prevailing hydraulic gradient should be about unity.
- It occurs mainly in fine sands. A cohesive soil does not become quick even effective normal stress becomes zero because clay possesses some shear strength due to cohesion. In very pervious Sand and gravels, the large discharge required to maintain a quick condition make it improbable of occurrence.
- Human being and animals are not sucked into a quick sand.
- High artesian pressure in a coarse sand is one of the most important reasons for the development of quick sand condition.

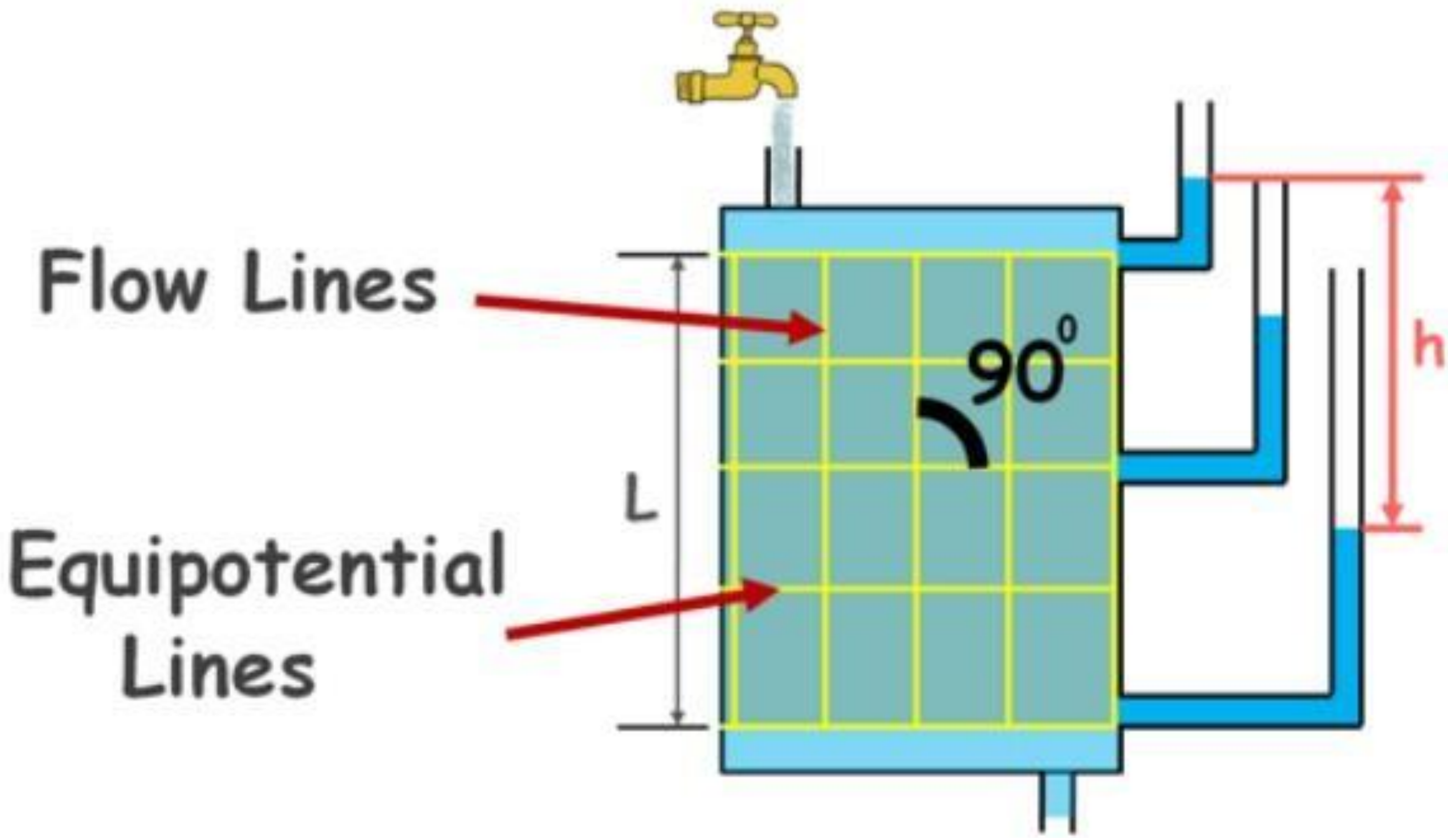
Seepage

- The interaction between soils and percolating water has an important influence on:
- 1 . The design of foundations and earth slopes,
- 2. The quantity of water that will be lost by percolation through a dam or its subsoil.
- Foundation failures due to 'piping' are quite common.
- **Piping** is a phenomenon by which the soil on the downstream sides of some hydraulic structures get lifted up due to excess pressure of water.
- The pressure that is exerted on the soil due to the seepage of water is called the **seepage force or pressure**.
- In the stability of slopes, the seepage force is a very important factor. Shear strengths of soils are reduced due to the development of neutral stress or pore pressures.

Flow Net

- The computation of seepage loss under or through a dam, the uplift pressures caused by the water on the base of a concrete dam and the effect of seepage on the stability of earth slopes can be studied by constructing flow nets.
- A flow net is a graphical representation of how the hydraulic energy is dissipated as water flows through a pervious medium.
- A flow net for an isometric medium is a network of flow lines and equipotential lines intersecting at right angles to each other.
- The path which a particle of water follows in its course of seepage through a saturated soil mass is called a flow line.
- Equipotential lines are lines that intersect the flow lines at right angles. At all points along an equipotential line, the water would rise in piezometric tubes to the same elevation known as the piezometric head

Flow net for One-dimensional flow



Flow net construction

- Generally the flow of water in soil is three dimensional and analysis of such flow is too complex and difficult. So we simplify the flow situations to two dimensional and analyze the flow.
- There are many methods that are in use for the construction of flow nets. Some of the important methods are:
- 1. Analytical method,
- 2. Electrical analog method,
- 3. Scaled model method,
- 4. Graphical method.

Analytical method

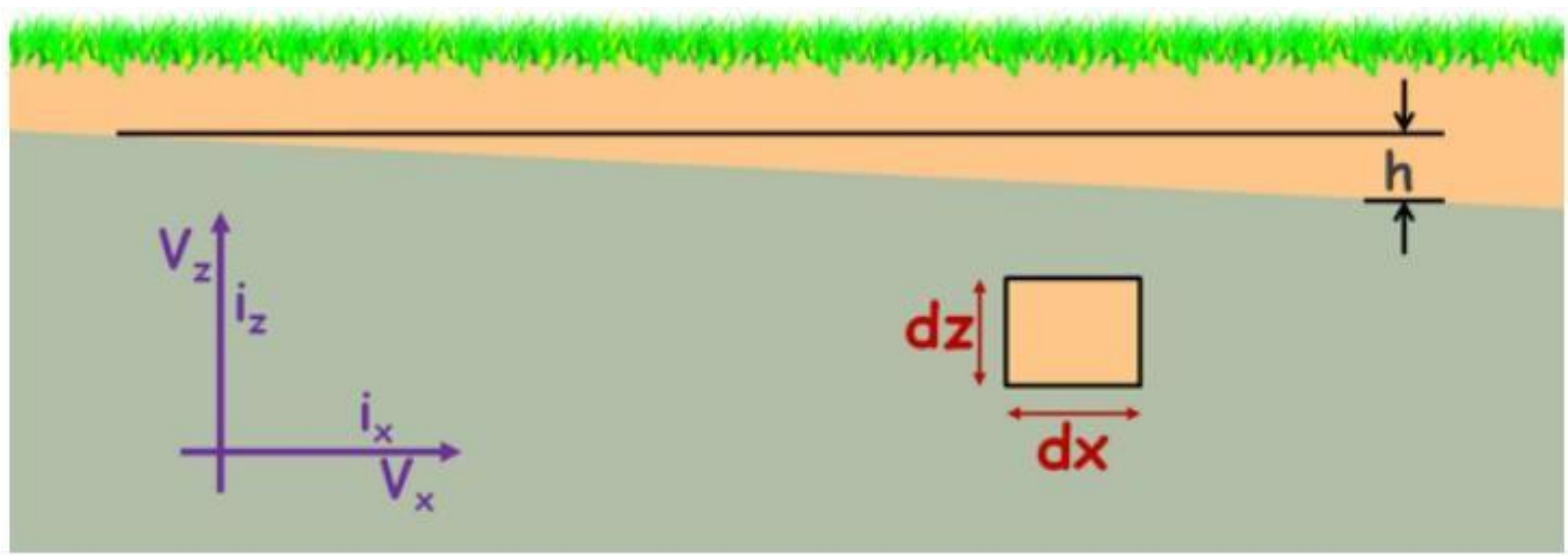
- **LAPLACE EQUATION:**
- Analytical method of obtaining a flow net for a flow of water in a soil mass is a mathematical solution to an equation that is obtained by the flow conditions. It can be used in relatively simple cases of flow, where the boundary conditions are known and can be expressed by equations.
- Let's consider a soil mass which is completely saturated by water flowing through it. We assume velocity of flowing water in x and z directions are v_x and v_z respectively.
- Let us consider a small soil element of dimension dx, dy and dz. y direction is normal to the plane.
- Let's say water is flowing in the soil because of a hydraulic head h and the hydraulic gradient in the x and z directions are i_x and i_z respectively.
- Water is not flowing in y direction as we are analysing flow only in two dimensions.

➢Now using continuity equation we can write the amount of water going in the soil element is equal to the amount of water coming out of it.

$$v_x dz + v_z dx = (v_x + \frac{\partial v_x}{\partial x} dx)dz + (v_z + \frac{\partial v_z}{\partial z} dz)dx$$

➢After solving it we arrive at simpler equation of continuity.

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} = 0$$



- After applying some assumptions like the Darcy's law is valid we can write velocity of water as permeability times hydraulic gradient.
- **v = ki**
- Where permeability in x-direction is k_x and z-direction is k_z and corresponding hydraulic gradients across these elements are $i_x = \partial h/\partial x$ and $i_z = \partial h/\partial z$
- so we can write the equation as : $K_x \frac{\partial^2 h}{\partial x^2} + K_z \frac{\partial^2 h}{\partial z^2} = 0$
- if soil is isotropic then permeability in x direction is equal to the permeability in z direction. $\mathbf{k_x = k_z}$
- So finally we arrive at this neat and clean partial differential equation.

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

- This equation is called Laplace's equation for homogeneous soil. It says that the change of gradient in the x-direction plus the change of gradient in the z-direction is zero. The solution of this equation gives a family of curves meeting at right angles to each other. One family of these curves represents flow lines and the other equipotential lines.

Anisotropic Soil

- Soils in nature do possess permeabilities which are different in the horizontal and vertical directions.
- The permeability in the horizontal direction is greater than in the vertical direction in sedimentary deposits and in most earth embankments.
- The study of flow nets would be of little value if this variation in the permeability is not taken into account.
- applies for a soil mass where anisotropy exists.
- This equation may be written in the form

$$\frac{\partial^2 h}{\frac{k_z}{k_x}\,\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

- If we consider a new coordinate variable x_c measured in the same direction as x multiplied by a constant, expressed by:

$$x_c = x_x\sqrt{\frac{k_z}{k_x}}$$

- The equation becomes: $$\frac{\partial^2 h}{\partial x_c^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

- This is a Laplace equation in the coordinates x_c and z. This equation indicates that a cross-section through an anisotropic soil can be transformed to an imaginary section which possesses the same permeability in all directions.

- The transformation of the section can be effected by multiplying the x-coordinates by √(kz /kx) and keeping the z-coordinates at the natural scale. The flow net can be sketched on this transformed section.

- The permeability to be used with the transformed section is:

$$k_e = \sqrt{k_x k_z}$$

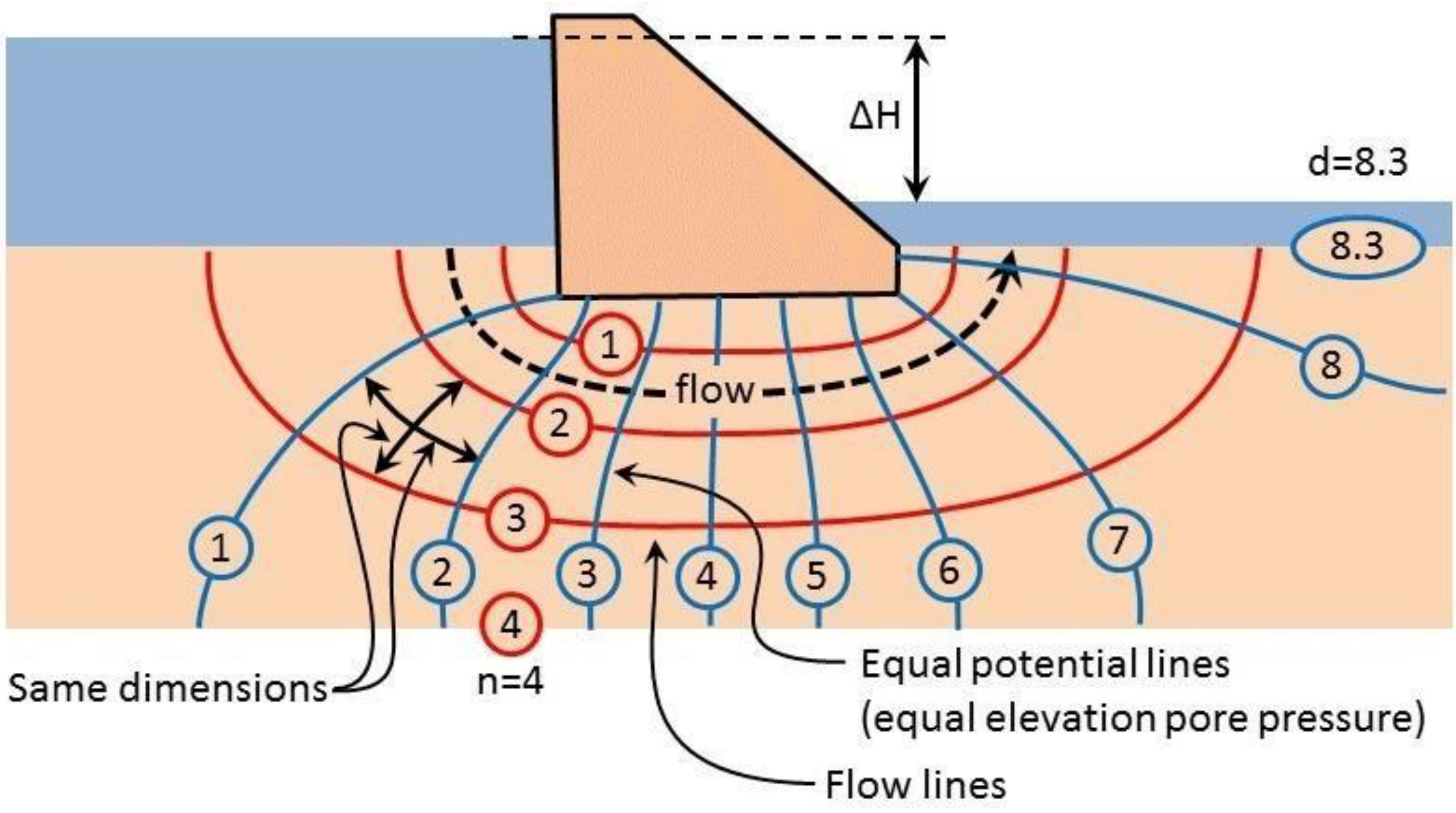
- **Note:** The analytical method, based on the Laplace equation although rigorously precise, is not universally applicable in all cases because of the complexity of the problem involved.

Scaled model method

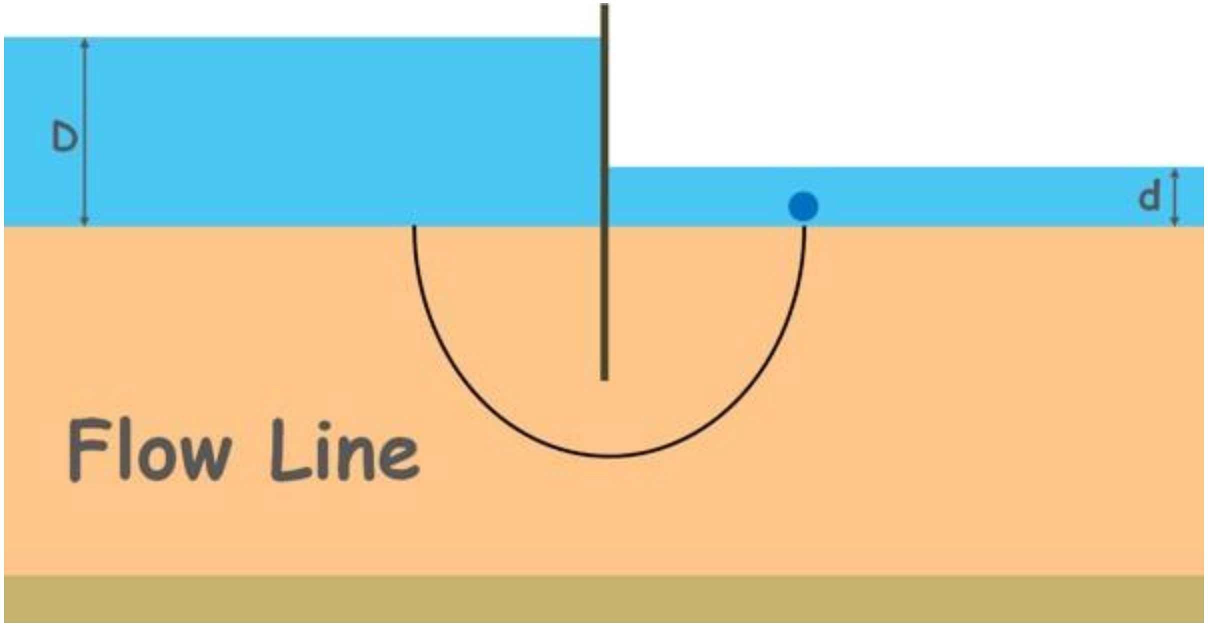
- Scaled models are very useful to solve seepage flow problems.
- Soil models can be constructed to depict flow of water below concrete dams or through earth dams.
- These models are very useful to demonstrate the fundamentals of fluid flow, but their use in other respects is limited because of the large amount of time and effort required to construct such models.

Graphical method

- This is the most commonly used method of flow net construction because it is easy and it provides nearly accurate results. This method is one of the solutions to the Laplace equation.



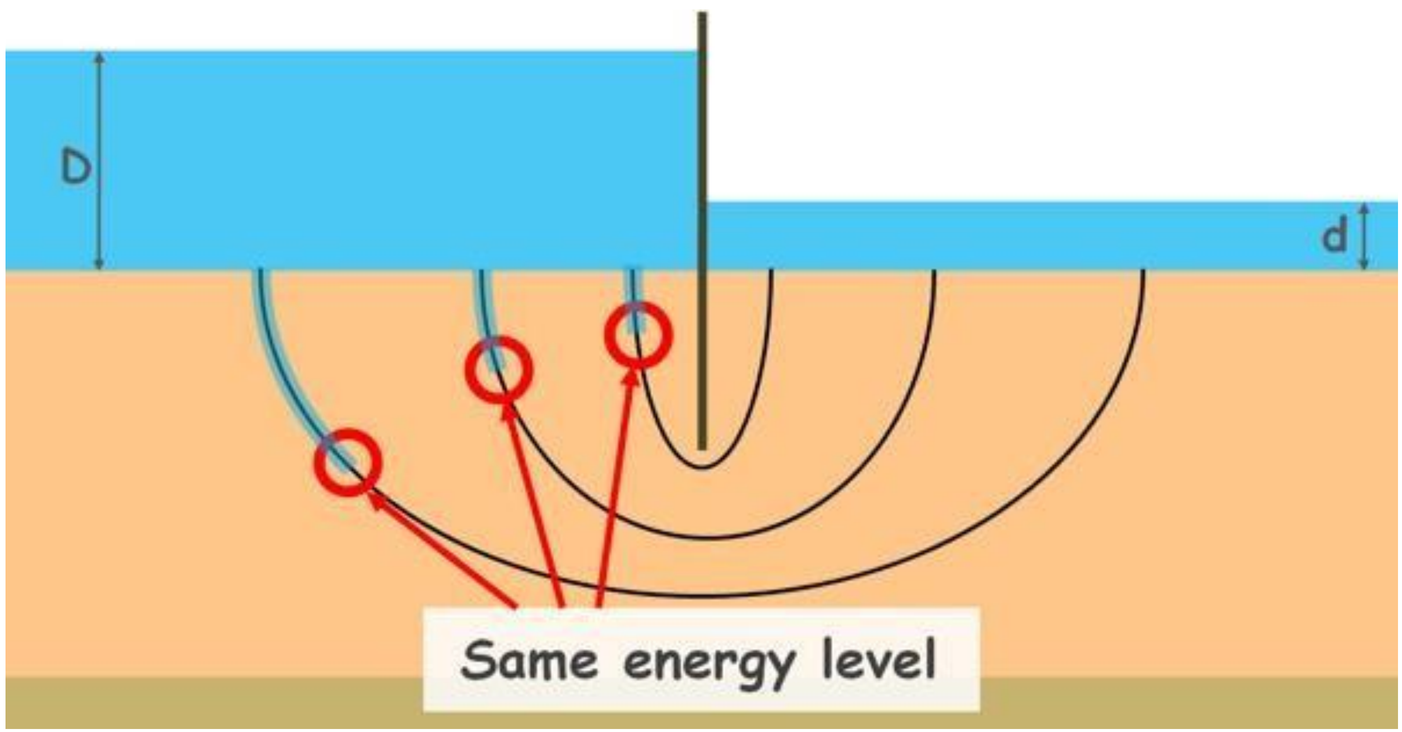
- Let's take a soil mass of some thickness and it lies upon an impermeable strata. A sheet pile is driven into the soil up to some depth. The sheet has water on its one side of depth capital D and on another side small d.



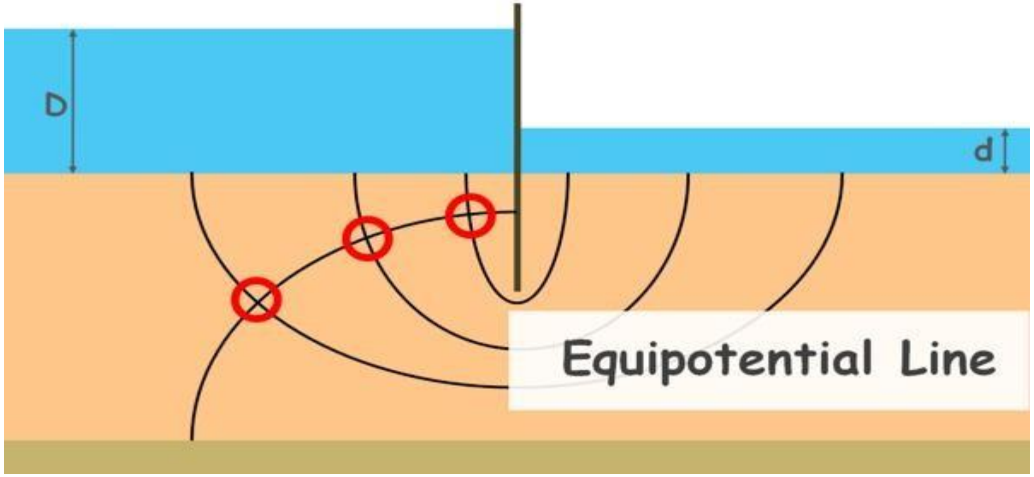
- We can see there is imbalance of head on the different side of the sheet pile, so water will flow from high head to low head. But water cannot pass through this sheet pile so flow will take place through seepage via soil below.

Let us consider a water molecule enters the soil at some point on the upstream surface, goes below the tip of the driven sheet and ends up at some point on the downstream surface. The flow path assumed by a water molecule is the flow line. Similarly many particles will leave the upstream and reach the downstream forming different flow lines.

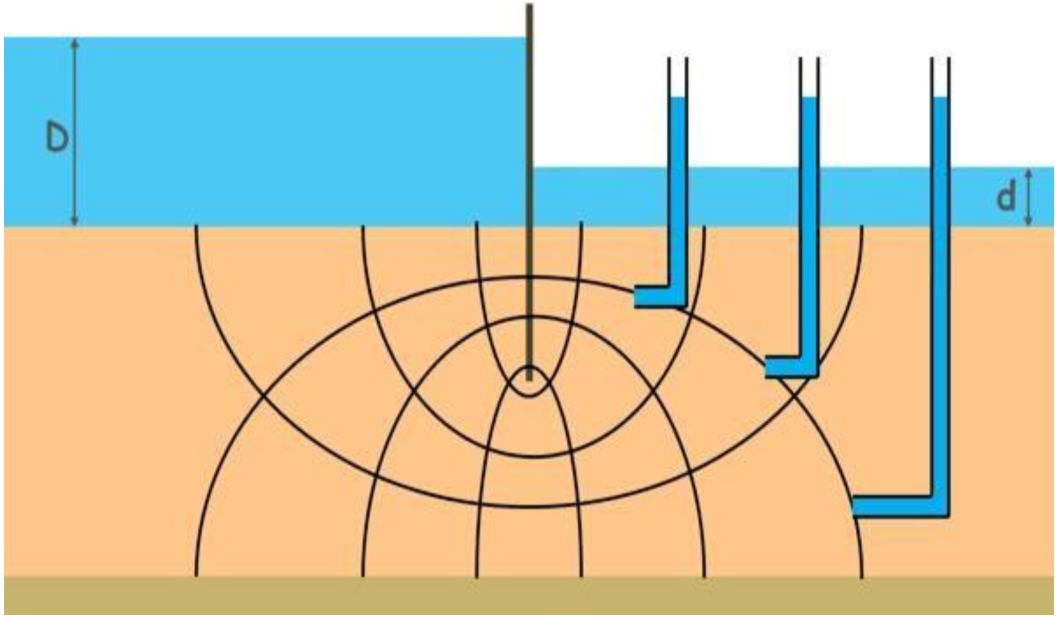
Note that each flow line begins from the upstream surface, which is at pressure $\gamma_w D$, and travels through the soil, constantly losing its energy and terminates at the downstream surface, which is at pressure $\gamma_w d$.



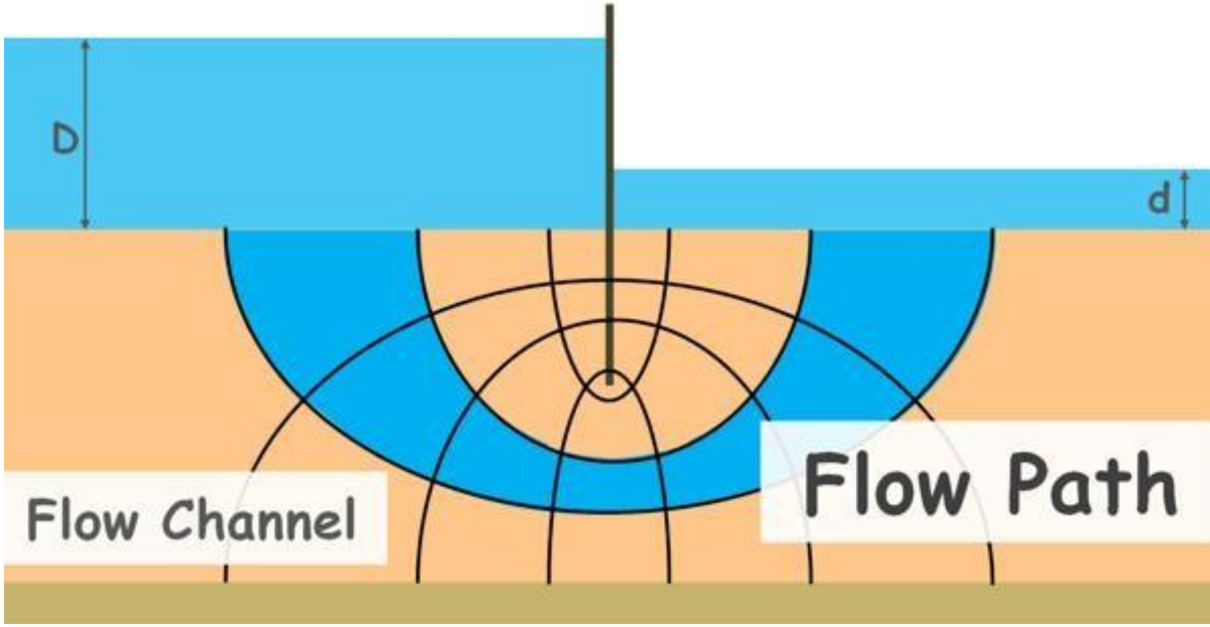
So, we can pick up certain points on the different flow lines where total energy lost is equal, or we can say points of same energy level. When we join such points together, the line so formed is an equipotential line.

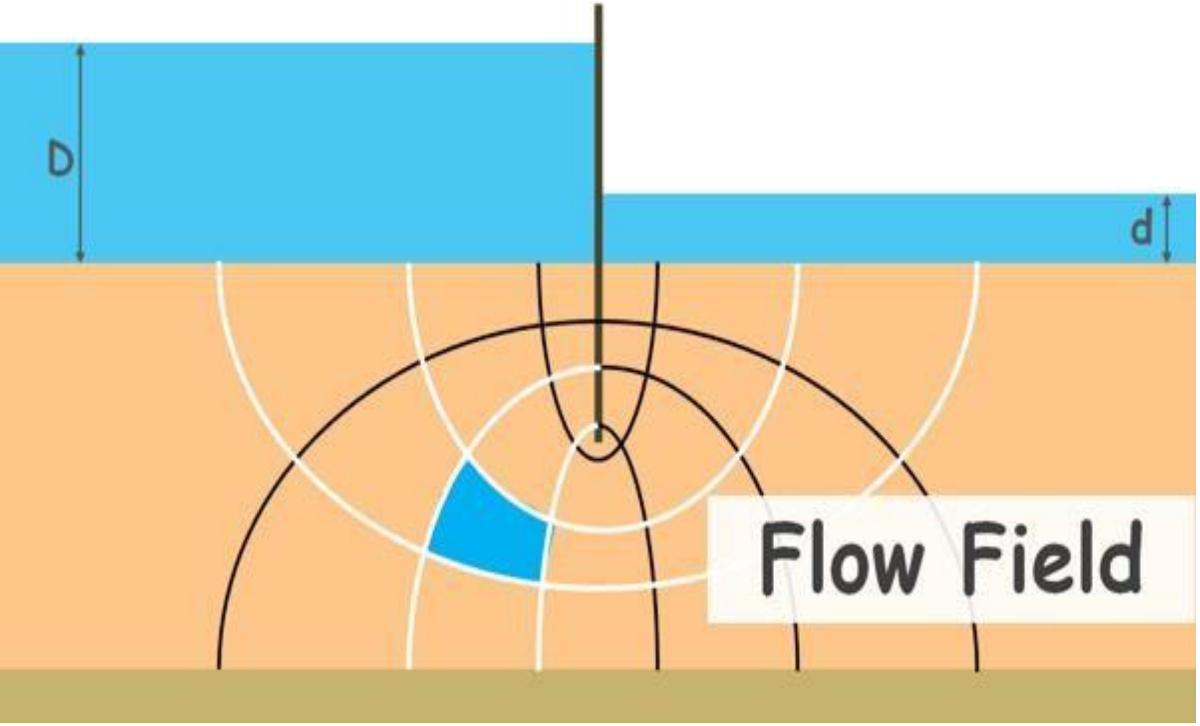


Similarly many different points of the same energy on different flow lines can be observed and many such equipotential lines can be drawn. If we insert piezometers into the soil at different points along an equipotential line we will notice water rises to the same elevation in all these piezometers.



The space between two adjacent flow lines is called the flow path or flow channel.

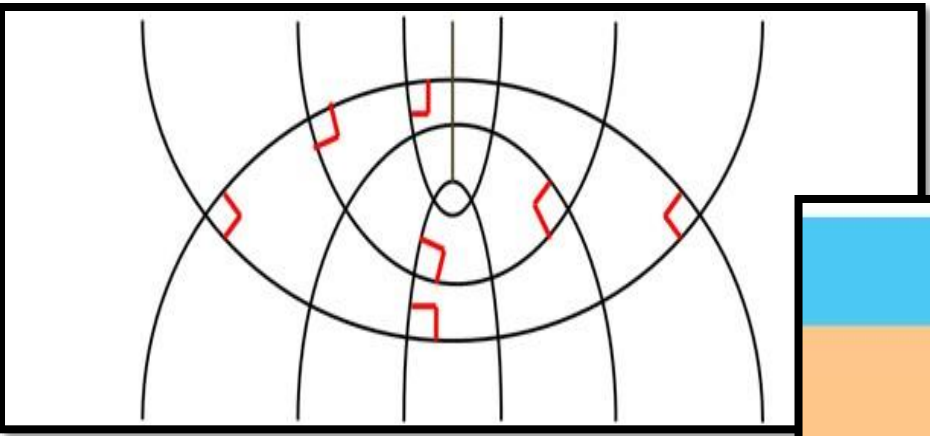


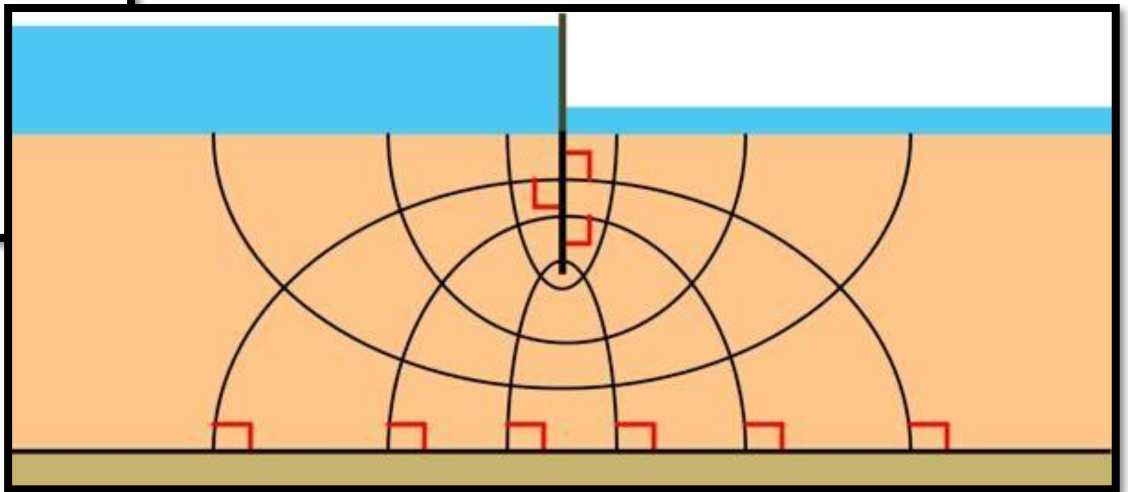


And the area enclosed between any two adjacent flow lines and adjacent equipotential lines is called Flow field.

Properties of Flow Net

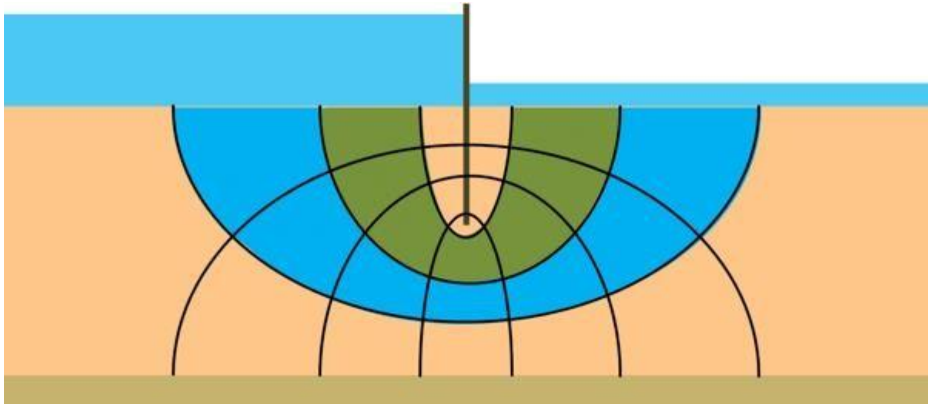
1. The angle of intersection between each flow line and an equipotential line must be 90° which means they should be orthogonal to each other.
2. Two flow lines or two equipotential lines can never cross each other.



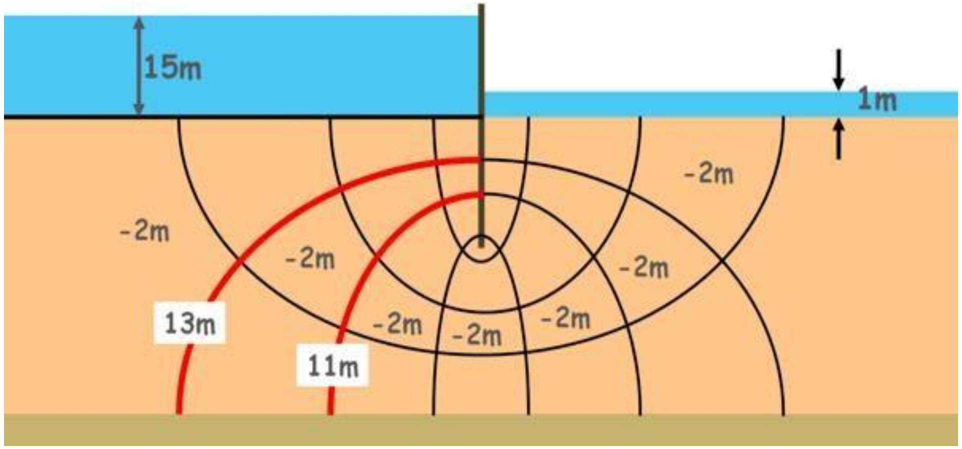


Properties of Flow Net

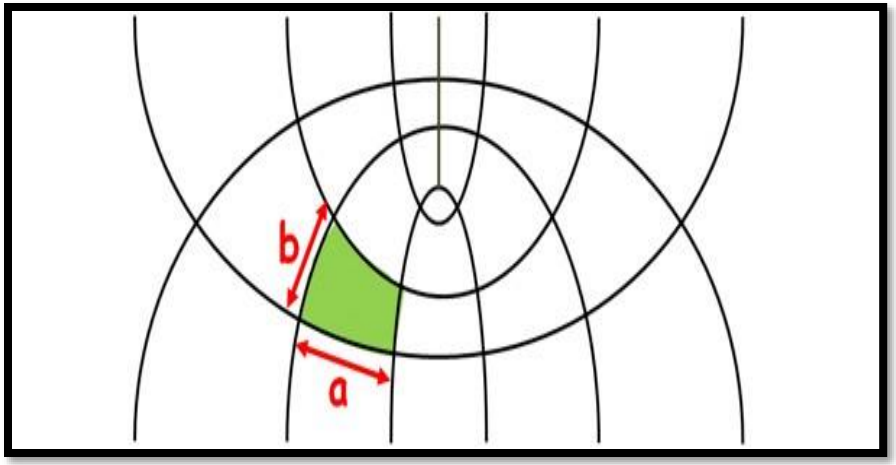
3. Equal quantity of seepage occurs in each flow channel. A flow channel is a space between two flow lines.

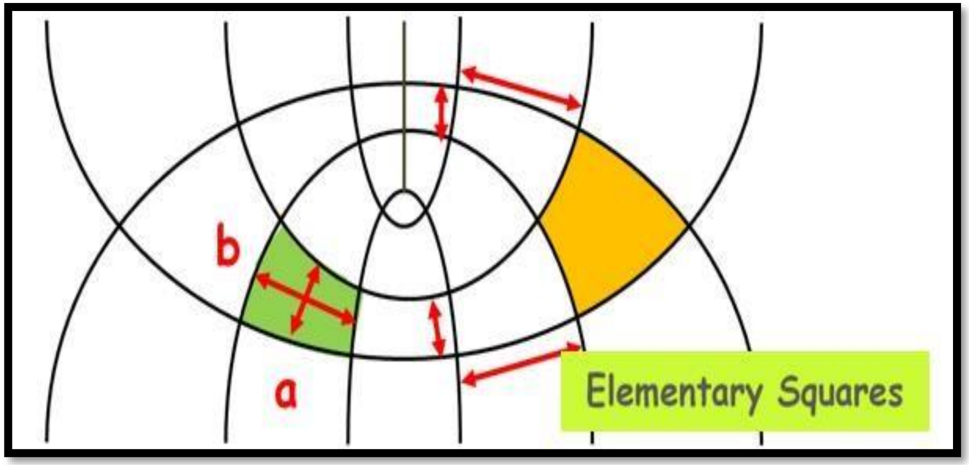


4. Head loss is the same between two adjacent potential lines.



Properties of Flow Net





5. The space formed between two flow lines and two equipotential lines is called a flow field. It should be in a square form.

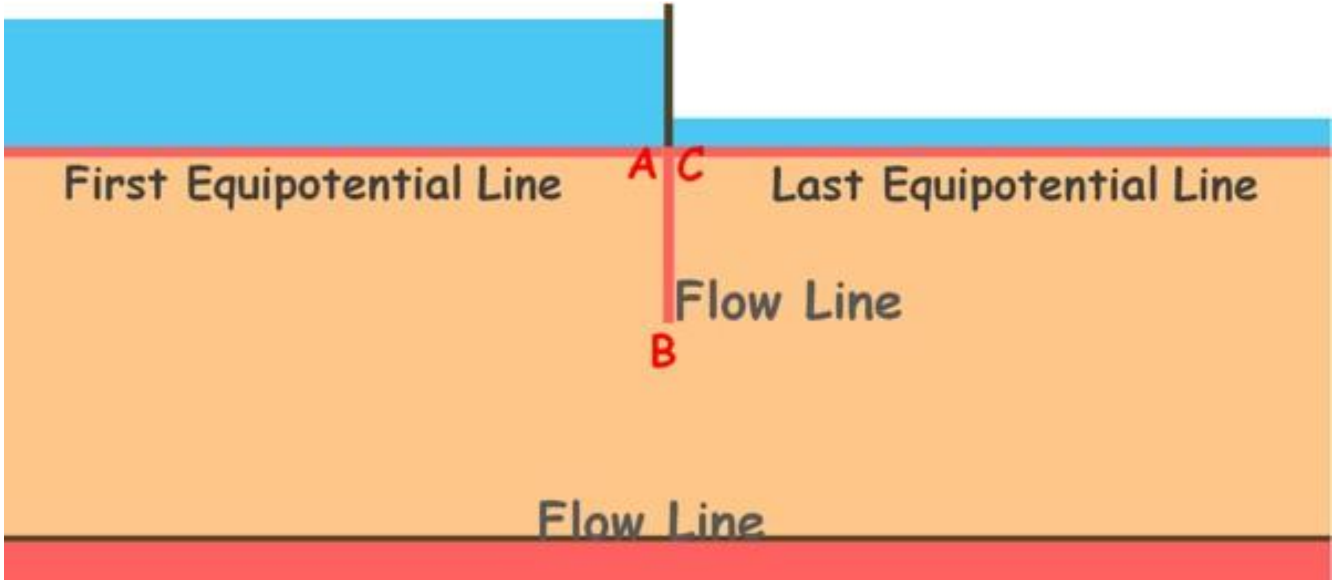
6. Either flow lines or equipotential lines are smoothly drawn curves.

7. Flow nets are drawn based on the boundary conditions only. They are independent of the permeability of soil and the head causing flow.

Boundary Conditions

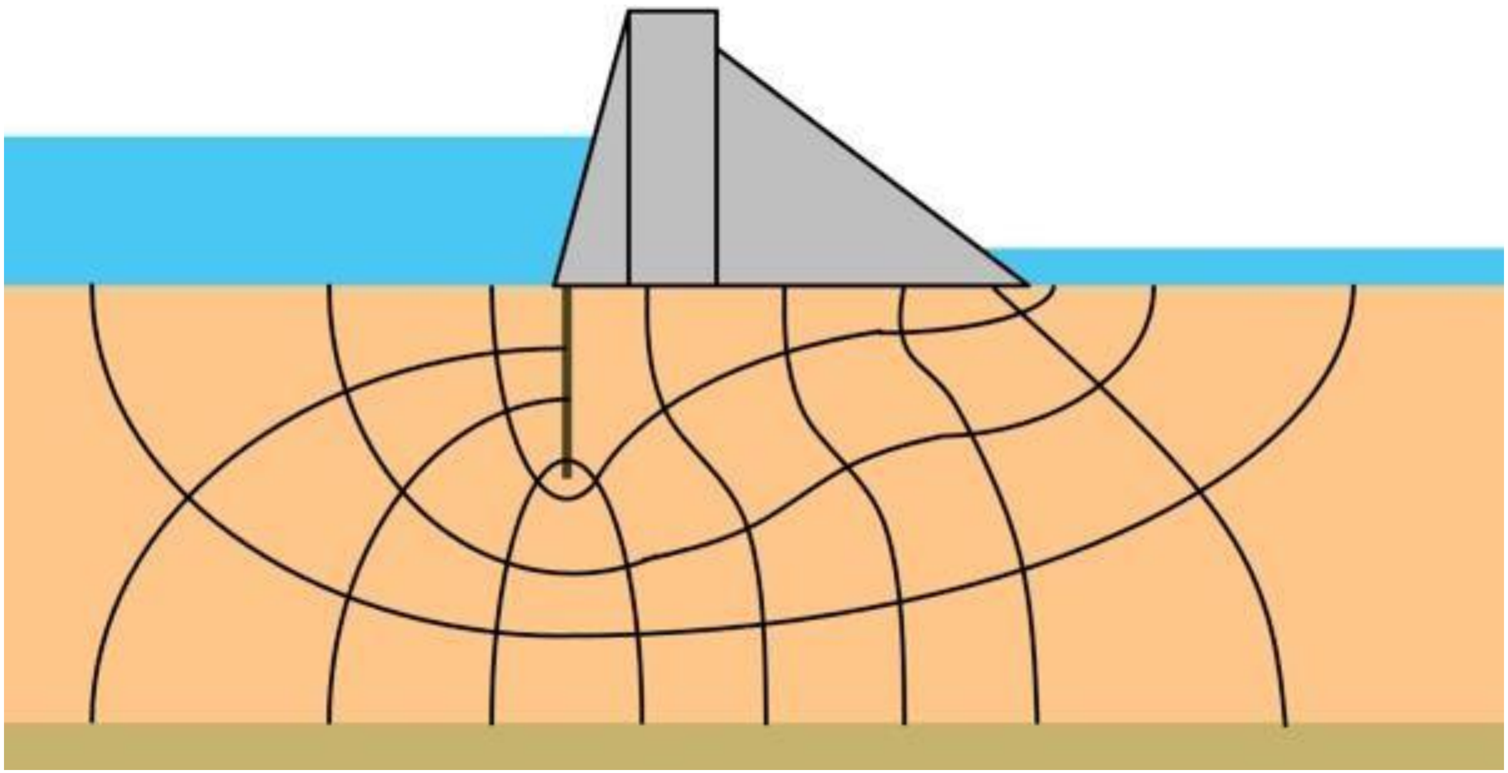
- Flow of water through earth masses is in general three dimensional.
- Since the analysis of three-dimensional flow is too complicated, the flow problems are solved on the assumption that the flow is two-dimensional.
- All flow lines in such a case are parallel to the plane of the figure, and the condition is therefore known as two-dimensional flow.
- All flow studies dealt with herein are for the steady state case.
- The expression for boundary conditions consists of statements of head or flow conditions at all boundary points.
- To construct a flow net we also need to identify the boundary conditions present for the flow. Boundary conditions are the restrictions that limit the flow in a certain space or area.
- A flow net is unique for a given set of boundary conditions. If the geometry of the flow space changes, the boundary conditions will be changed and hence the flow net will be changed.

- First boundary condition is the upstream surface, from where the flow starts and if we notice, it is the first equipotential line of our flow net as at every point on this line the total head is same.
- Second boundary condition is similar and that is the downstream surface, it is the last equipotential line of our flow net.
- The third boundary is the sheet pile. Water molecule cannot cross this sheet, it flows from a point which is near to the sheet on the upstream and moves vertically downward. and after crossing the sheet pile it vertically ascends. The sheet pile is also tracing the flow path of the molecule so this boundary, so ABC, is a flow line.
- Fourth boundary is the bottom most impermeable surface. Water molecules cannot cross it. This line is also a flow line as the water will flow along this surface from one side to other.



But if we change the soil in which the water is flowing, the flow net will not change. Only the permeability of the soil k will change.

Flow net will also remain unchanged even if the upstream and downstream water levels are reversed; only the direction of the flow will be reversed.



Applications of Flow Net

- Flow net is useful to determine the following parameters in seepage analysis of soil :

- Quantity of seepage
- Seepage Pressure
- Uplift Pressure
- Exit Gradient

Quantity of seepage

- The quantity of seepage **q** is calculated per unit length of the section. The flow through any square can be written as

$$\Delta q = k\Delta h$$

- Let the number of flow channel and equipotential drops in a section be N, and Nd, respectively. Since all drops are equal, we can write

$$\Delta h = \frac{h}{N_d}$$

- Since the discharge in each flow channel is the same we can write,

$$q = N_f \Delta q$$

- Substituting for Δq and Δh, we have

$$\boxed{q = kh\frac{N_f}{N_d}}$$

SEEPAGE PRESSURE

- Seepage pressure at any point is determined by using the below mentioned formula :

$$P_s = \gamma_w . h$$

- where,
- h = Hydraulic potential after "n" potential drops. It can be expressed as :

$$h = H - n.\Delta H$$

$$\Delta H = \frac{H}{N_d}$$

- Where ΔH= Potential drop or drop in head between 2 adjacent equipotential lines.
- This force exerts a drag on the element known as the seepage pressure.
- It has the dimension of unit weight, and at any point its line of action is tangent to the flow line.
- The seepage pressure is a very important factor in the stability analysis of earth slopes.

Uplift Pressure

- The uplift pressure at any point within the soil mass can be found using the undermentioned formula. It is also called as hydrostatic pressure.

$$P_u = \gamma_w . h_w$$

- Pressure head = Total head – elevation head.

$$h_w = h \pm z$$

Exit Gradient

- The exit gradient is the hydraulic gradient at the downstream end of flow line where seepage water from the soil mass joins with free water at the downstream. Exit gradient can be expressed as :

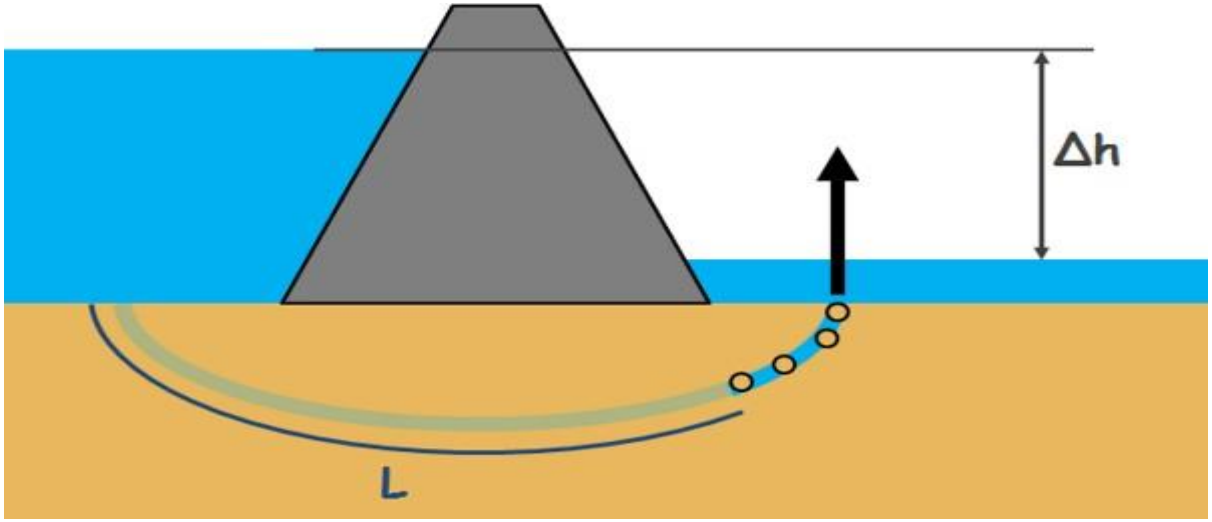
$$i_{exit} = \frac{\Delta H}{\Delta L} \qquad i_c = \frac{G_s - 1}{1 + e}$$

ΔL = Length of the flow field

ΔH = Potential drop or Drop in head between two adjacent equipotential lines.

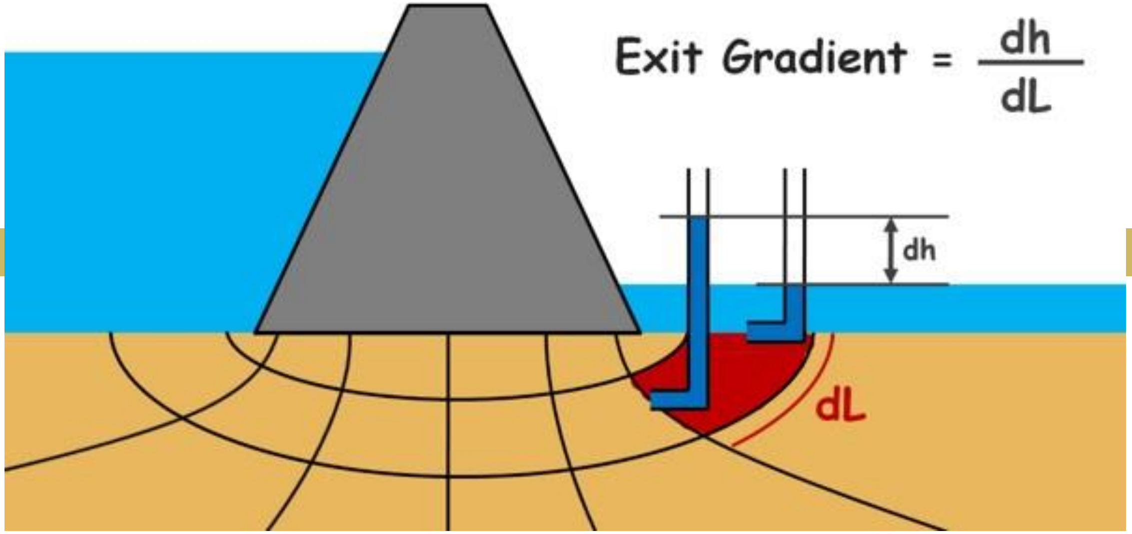
Piping Failure In Hydraulic Structures

- When water flows through the pervious foundation of any dam with a very high hydraulic gradient, it may carry soil particles with it causing to form pipe-shaped channels in its foundation and the structure may fail because of piping failure.



- There are two types of piping failures:
- 1. Backward-erosion piping failure
- 2. Heave piping failure

Backward-erosion piping failure:

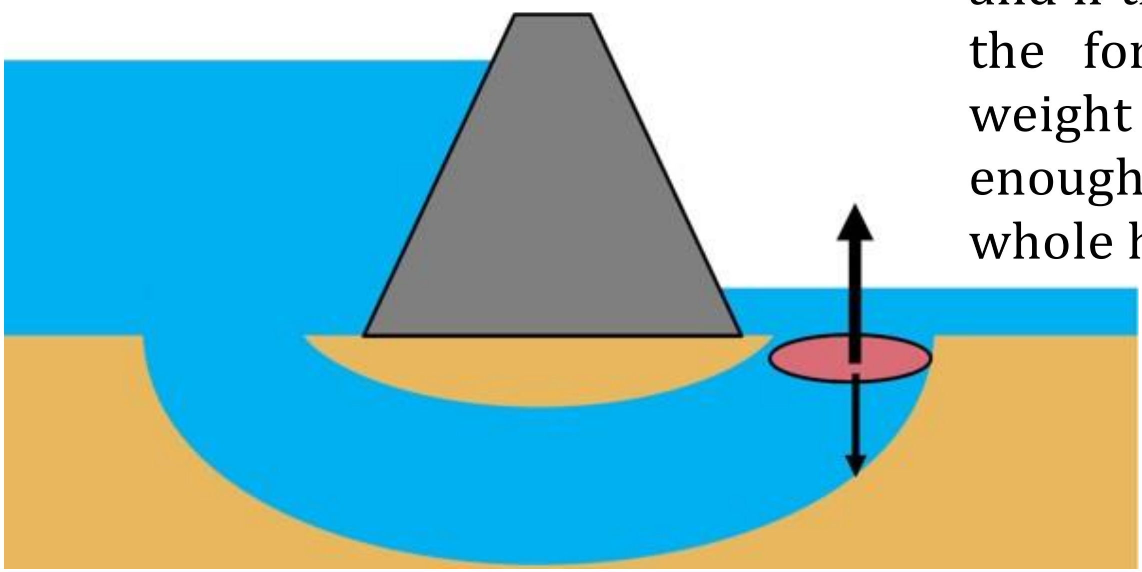


- When water percolates through soil under any hydraulic structure it may happen to exit vertically upwards at the downstream.

- If this water moves with a high hydraulic gradient, the seepage force or drag exerted by it on soil particles may push them up and soil particles at the exit point of the water may be removed by the force of water.
- This removal of soil particles decreases the length of flow and that increases the hydraulic gradient even more causing further removal of the soil. This process of erosion of soil in the backward direction continues towards the upstream and a pipe like opening is formed below the dam.
- Hydraulic structure becomes unstable and may fail. This type of Piping is called backward erosion piping.
- Generally, backward erosion piping failure occurs when the exit gradient becomes greater than the critical hydraulic gradient.
- The failure is so serious that for safety against piping a factor of safety of at least 6 is recommended.

Heave piping failure:

- Heave meaning is to lift or raise something with great force.
- This kind of failure may also occur on the downstream side of a hydraulic structure when water is coming out vertically upwards.
- It occurs when the upward seepage force acting on the particles over an area exceeds the downward force because of submerged weight of the soil above that area.



- This condition destabilizes the soil and if this uplift force acts opposite the force of gravity due to the weight of the dam and if it is high enough, it may destabilize the whole hydraulic structure.

Factor of safety:

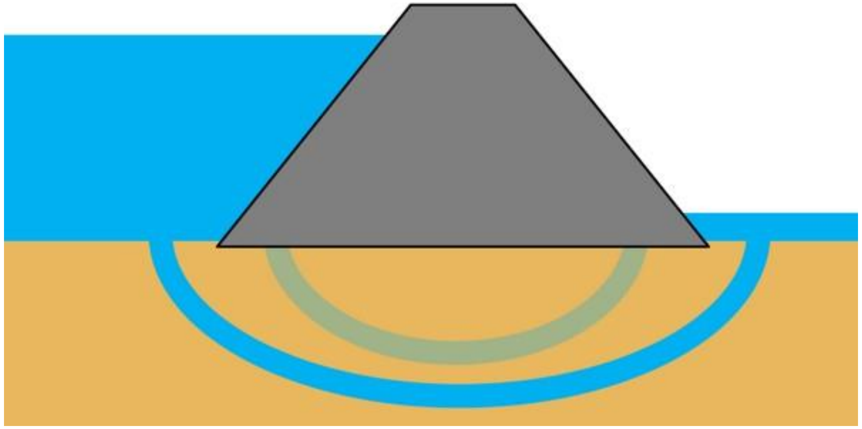
- To prevent backward erosion piping or heave piping failure the upward seepage force exerted by water on soil particles should be less than the downward force due to weight of soil or structure or any other arrangement.
- So factor of safety against piping in the soil can be written as weight of everything above which is downward force divided by upward seepage force applied by water.

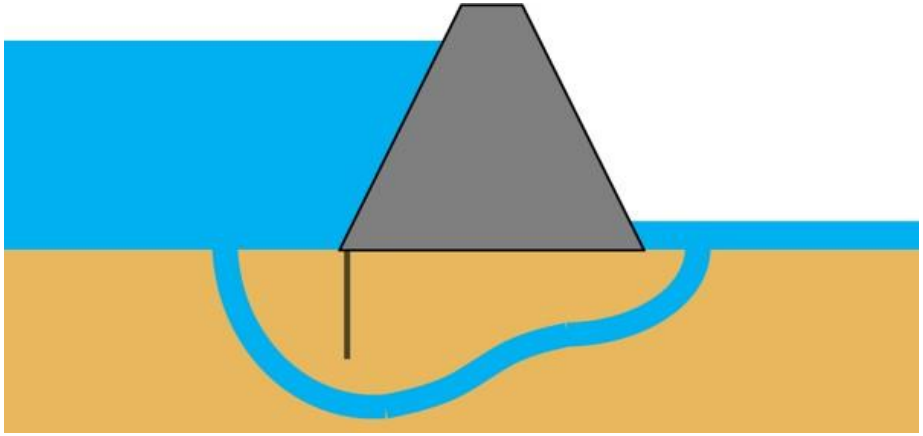
$$\text{Factor of safety} = \frac{W}{U}$$

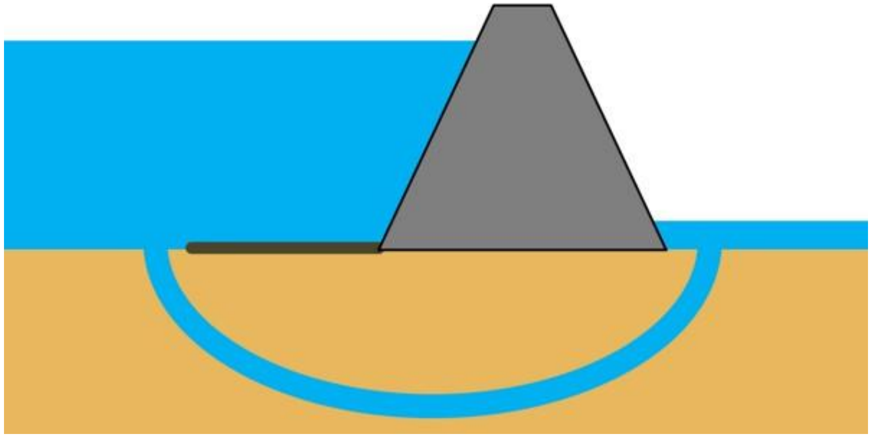
- If value of downward force that is weight of overlying material is twice the value of upward force we receive the factor of safety as 2. This means the structure is twice safe. Downward force is greater than uplift force so structure is safe and is twice strong.

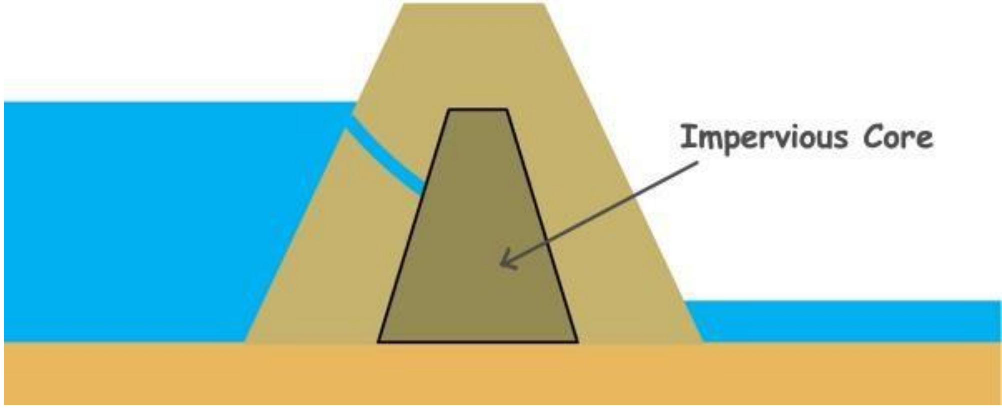
- There are few methods through which we can prevent the piping failure.
- **Increasing the path of flow:**
- We know the hydraulic gradient is head loss over the length of the flow. We can see in a dam it is difficult to decrease the head loss but we have control over the length of the flow over which it is percolating.
- If we increase the length of the flow, the hydraulic gradient will decrease and that also cause to decrease the exit gradient. We can increase the length of flow so that exit gradient becomes well below the critical hydraulic gradient and structure will be safe.
- There are few methods through which we can increase this flow path
- 1. We can increase the base width of the hydraulic structure so water has to travel more to reach the downstream and length of flow is increased.

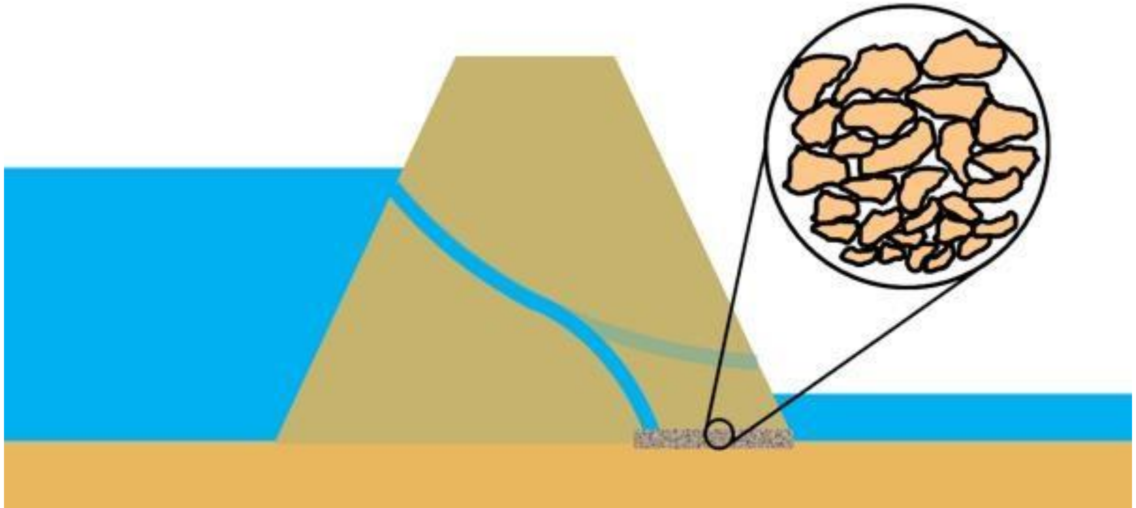
- 2. We can provide vertical cut off walls below the structure at the upstream end there by water has to go below this wall to reach the downstream.
- 3. We can also provide an impervious blanket on the upstream.
- **Reducing seepage:**
- If our hydraulic structure is an earth dam, we can reduce possibility of piping failure through the body of the dam by the reduction of seepage through its body. For that we provide an impervious core through which water cannot flow
- We can also provide the drainage filter near the toe of the earth dam. Drainage filter consists of layers of pervious material which permit flow of water but prevent the movement of soil particles. Drainage filter changes the direction of flow away from the downstream face and avoid piping through the body of the dam.
- **Loaded filter:**
- It is provided at the downstream from where the water emerges out. A loaded filter consists of pervious material such as graded sand and gravels.
- The loaded filter increases the downward force to counter the upward seepage force at the point where water emerges out from the soil. Hence the loaded filter increases the factor of safety against piping and the factor of safety is given by weight of soil plus weight of filter divided by upward seepage force.








Impervious Core



In order to compute the seepage loss through the foundation of a cofferdam, flownets were constructed. The result of the flownet study gave N_f= 6, N_d = 16. The head of water lost during seepage was 19.68 ft. If the hydraulic conductivity of the soil is k = 13.12 x 10^{-5} ft/min, compute the seepage loss per foot length of dam per day.

Solution

The equation for secpage loss is

$$q = kh\frac{N_f}{N_d}$$

Substituting the given values,

$$q = 13.12\times10^{-5}\times19.68\times\frac{6}{16} = 9.683\times10^{-4}\ \text{ft}^3/\text{min} = 1.39\ \text{ft}^3/\text{day} \text{ per ft length of dam.}$$