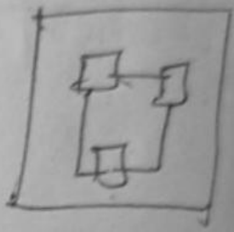
## Systems :-

$x(t) \rightarrow \boxed{\text{process}} \rightarrow y(t)$



Type of signals:-

① $x(t) \rightarrow \boxed{S} \rightarrow y(t)$ CT systems.

② $x[n] \rightarrow \boxed{S} \rightarrow y[n]$ DT systems.

③ $x(t) \rightarrow \boxed{\ } \rightarrow y[n]$

$x[n] \rightarrow \boxed{\ } \rightarrow y(t)$ Hybrid systems

Eg:- A/D conv

$\rightarrow \boxed{\text{system}} \rightarrow D$

$x(t) \rightarrow \boxed{\text{sym}} \rightarrow y(t)$

$x(t) \xrightarrow[S]{\overset{S}{\text{leads}}} y(t)$

$g(t) \xrightarrow{A} h(t)$

$g(t) \rightarrow \boxed{A} \rightarrow h(t)$

CT systems.

(2) Based of no. of i/p's & o/p's.

$x(t) \longrightarrow \boxed{S} \longrightarrow y(t)$

SISO systems

Single input single output.

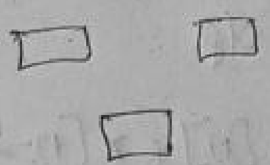
$x(t) \longrightarrow \boxed{\text{system}} \longrightarrow y(t)$

$\begin{bmatrix} x_1(t) \\ \vdots \\ x_n(t) \end{bmatrix} \longrightarrow \text{MIMO} \quad \begin{bmatrix} y_1(t) \\ \vdots \\ y_m(t) \end{bmatrix}$

Multiple input

n input Multiple output in output.

System Interconnection of subsystems



(1) $x(t) \longrightarrow \boxed{S_1} \xrightarrow{y_1(t)} \boxed{S_2} \longrightarrow y(t)$

$y_1(t) = $ o/p of $S_1$

$=$ i/p for $S_2$.

$x(t) \xrightarrow{S_1} y_1(t)$.

$y_1(t) \xrightarrow{S_2} y(t)$

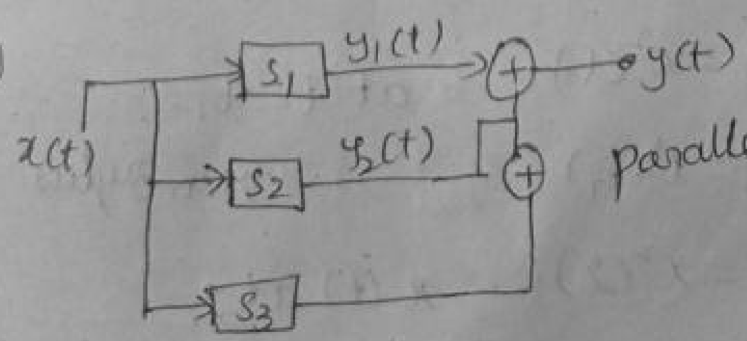
Series

cascade

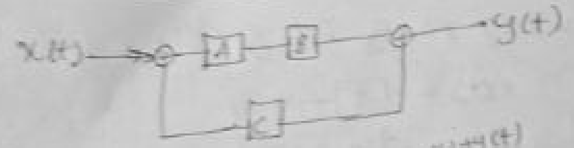
tandem

$x(t) \xrightarrow{S_1} y_1(t) \xrightarrow{S_2} y(t)$

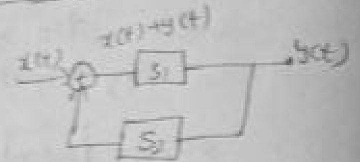
(2)




parallel.




③ Feedback System

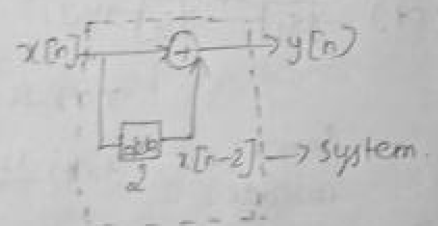



Classification of systems. (Based on behaviour of S).

① Memory / Memoryless System.

A system S is called Memory S if the present o/p depends on present i/p, past i/p, past o/p

Ex:- i) $y[n] = x[n] + x[n-2]$.


at $n=2$;

$y[2] = x[2] + x[0]$ Memory.

ii) $y(t) = x^2(t)$ at $t=4$,

$y(t_1) = x^2(t_1)$ Memoryless

$y(2) = x^2(2)$ $x(t)$ at 2.

iii) $y(t) = x(t^2)$ $t=0.1$; $y(0.1) = x(0.01)$

Memory

$t=2$; $y(2) = x(4)$

non causal

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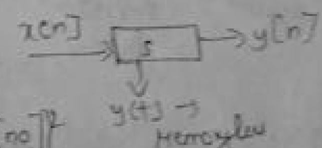
① Memory / memoryless S :- present o/p depends only a present i/p (or) Instantaneous System & static. & dynamic System.

$y(t) \xrightarrow{t=t_1} x(t_1)$

Ex:- ① $y[n] = [2x[n] - x^2[n]]^2$

at $n = n_0$




$y[n_0] = [2x[n_0] - x^2[n_0]]^2$

x at $n_0$

$n=1$; $y[1] = (2x[1] - x^2[1])^2$ $x[1] = 2$

$= (2(2) - 2^2)^2$

memoryless System

② $y[n] = y[n-1] + x[n]$


Put $n=3$;

$y[3] = y[2] + x[3]$

past o/p   present i/p

Memory systems.

② Invertible / Non-Invertible Systems:

A system $S$ is called an Invertible System, if

$x(t) \xrightarrow{A} y(t) \xrightarrow{B} x(t)$

Invertible system — Inverse System

comb of $AB$ = identical System.

Eg ①:

$y(t) = kx(t)$

$x(t) \rightarrow \boxed{\times k} \rightarrow y(t) \rightarrow \boxed{\times \frac{1}{k}} \rightarrow x(t)$.

$\boxed{\times k} \rightarrow k^2 x(t)$

$kx(t)$

Series cascade tandem

Invertible System.

Inverse system.

$k^2 x(t)$

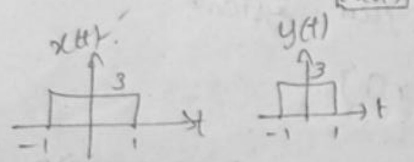
$x(t) = 2$; $y(t) = 4$

$x(t) = -2$; $y(t) = -4$.

②: $y(t) = x(t)$. Identical System

$x(t) \rightarrow \boxed{S_1} \rightarrow y(t) = x(t) \rightarrow \boxed{S_1} \rightarrow x(t)$

Inverse System



Invertible System

| | |
|---|---|
| 4 | 4 |
| 2 | 2 |
| 100 | 100 |

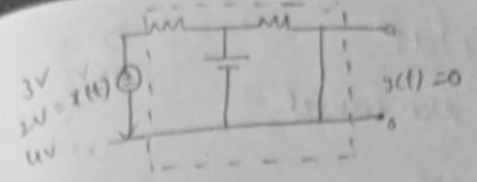
③ $y(t) = 0$. (Non-Invertible system).

$x(t) \rightarrow \boxed{S} \rightarrow y(t) = 0 \rightarrow \boxed{S'} \rightarrow x(t)$.

$V = 0$ SC

A B

A $i=0$ B



$x(t) \rightarrow S \rightarrow y(t)$.

④ $x(t) = x^2(t)$

$x(t) \rightarrow \boxed{\text{Squaring}} \rightarrow y(t) \rightarrow \boxed{\sqrt{\ }} \rightarrow x(t)$

Invertible — Inverse.

If (i) $x(t) = -2$

(ii) $x(t) = +2$

Non-Invertible system.

Invertible System: If distinct i/p's lead to distinct o/p's.

③ causal / Non causal :-

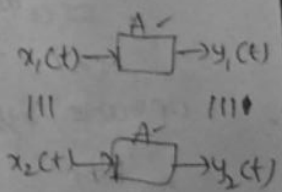
causal :- If its present o/p doesn't depend on ~~future~~ future values. (non-anticipatory system).

Non-causal :- It is anticipatory system.

(∴ Anticipatory - we know what will happen in future)

$x_1(t) \equiv x_2(t) \quad t \le t_0$

$y_1(t) \equiv y_2(t) \quad t \le t_0$.



Eg (i) $y[n] = x[n] - x[n+1]$

Non causal.

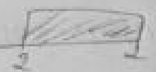
at $n=1$; $y[1] = x[1] - x[2]$ - finite input.

(ii) $y[n] = \sum_{k=-\infty}^{n} x[k]$ causal

③ $y(t) = \int_{t}^{\infty} x(\tau)\,d\tau$

Non causal.

$y(2) = \int_{2}^{\infty} x(\tau)\,d\tau$



④ $y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$

causal.

$y(t) = \int_{-\infty}^{3t} x(\tau)\,d\tau$

(Non-causal).

④ Stable / unstable:

B I B O.

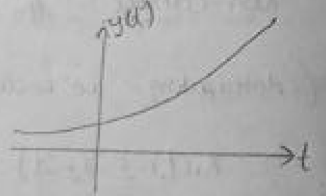
Bounded input Bounded output.

A system S is stable if Bounded i/p leads to Bounded o/p.

Ex: ① $y(t) = e^t$

$\simeq c e^{at}$

$c=1$



unstable system $a = 1 > 0$. $t \to \infty$; $y(t) \to \infty$

BIBO.

② $y[n] = \sum_{k=-\infty}^{n} x[k]$ $-1 < x[k] < 1$

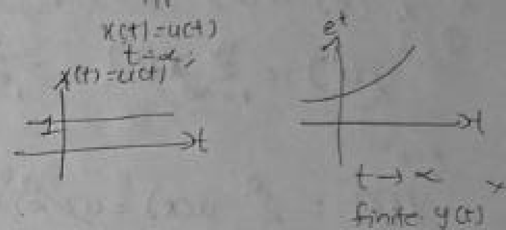
$|x(k)| \leq 1$ $-2 < x(k) < 2$

$= [-\infty - - - 1]$ $x[n] \xrightarrow{\text{accumulate}} \boxed{\ } \to y[n]$ $-\infty$ —— 1

$n=1$ $y[1] = \sum_{k=-\infty}^{1} 2$ $2+ \ldots -\infty$ --- ① $|x(k)| \leq 2$.

$n = 0$ $= \sum_{k=-\infty}^{0}$ unstable system.

③ $y(t) = e^t$



BIBO

unstable.

$t \to \infty$

finite $y(t)$ ✗

④ $y(t) = \int_{-\infty}^{3t} x(\tau)\,d\tau$

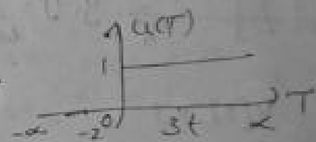
Stable / unstable BIBO

Let $x(t) = u(t) \to B$

u

$x(t) = u(t)$

$y(t) = \int_{-\infty}^{3t} u(\tau)\,d\tau$



unstable

$= \int_{0}^{3t} 1\,d\tau$

$= \tau \Big]_0^{3t}$ to limit the o/p range to the values

$y(t) = 3t$

at $t = -2$; $y(t) = -6$

$\boxed{y(t) = 3t\,u(t)}$

$x(t) = u(t)$

at $t \to \infty$;

at $t = 2$; $y(t) = 6\,u(t)$

$x(n) \rightarrow \boxed{h(n) = u(n)} \xrightarrow[y(n)]{i/p} \boxed{h_1(n)} \rightarrow x(n)$ o/p

Fwd. -y(n-1)

$y(n) = \sum_{k=-\infty}^{n} x(k) \rightarrow x(n)$

$= \sum_{k=-\infty}^{n-1} x(k) + x(n)$

$y(n) = y(n-1) + x(n)$

2nd System.

$\Rightarrow \boxed{x(n) = y(n) - y(n-1)}$

o/p i/p

$\boxed{h_2(n) = \delta(n) - \delta(n-1)}$

$\sum_{k=1}^{4} x(k) = x(1) + x(2) + x(3) + x(4)$

$\sum_{k=1}^{3} x(k) + x(4)$

$x(1) + x(2) + x(3) + x(4)$

$\Rightarrow h_1(n) * h_2(n) = \delta(n)$

$x(n) * \{\delta(n) - \delta(n-1)\} =$

$= [u(n) * \delta(n)] - [u(n) * \delta(n-1)]$

$x(n) * \delta(n) = x(n)$

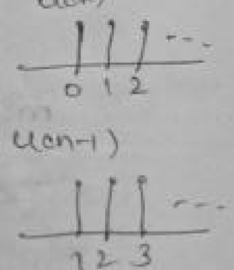
$x(n) * \delta(n-n_0) = x(n-n_0)$.

$= u(n) - u(n-1)$

$= \delta(n)$

u(n)

u(n-1)



LTI $h(n) = u(n)$

is invertible.

$x(n) \rightarrow \boxed{h(n)} \rightarrow \boxed{h_1(n)} \rightarrow y(n)$

$\boxed{h_1(n) * h_2(n)}$

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1. memoryless ; $h(n) = 0$ ; $n \neq 0$.

$h(n) = 0$ ; $t \neq 0$.

2. Invertible ; $h_1(t) * h_2(t) = \delta(t)$

Inverse System

Causal / Non causal :-

$x(n) \rightarrow \boxed{h(n)} \rightarrow$

$y(n) = x(n) * h(n)$

$y(n) =$ past & present i/p.

$y(n)$ depends on $x(k)$ $k \leq n$

for eg : $y(4) \leftarrow x(3), x(4)$

$x(6)$

$\therefore y(n)$ should not depend.

on $x(k)$ $\underline{k > n}$

$y(n) = 0$ ; $k > n$.

$x(n) \rightarrow \boxed{h(n)} \rightarrow y(n) = x(n) * h(n)$

$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$ $k > n$.

$t=3;\ y(3) = 3 \cdot u(3) = 3$

$t=4;\ y(4) = 4 \cdot u(4) = 4.$

Bounded i/p leads to
unbounded o/p.
Unstable system.

***

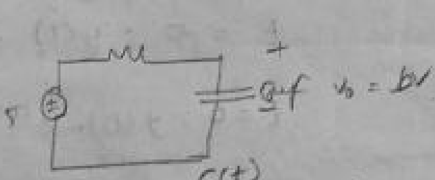
⑤ Time variant / Time invariant System:
↳ o/p doesn't depend change wrt on time.

output

$x(t) \longrightarrow \boxed{S} \longrightarrow y(t)$ Time invariant Systems.

$\downarrow \qquad\qquad\qquad \downarrow$

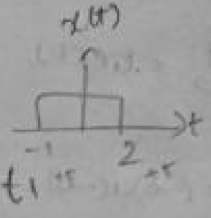
$x(t-t_0) \longrightarrow \boxed{S} \longrightarrow y(t-t_0).$

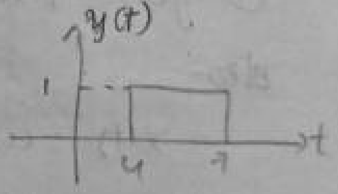



$v_0 = 6V.$

Eg: $x(t) \longrightarrow \boxed{S} \longrightarrow y(t)$

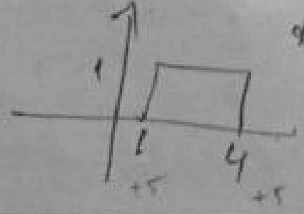
$x(t)$ $\xrightarrow{\text{Rs by 5}}$

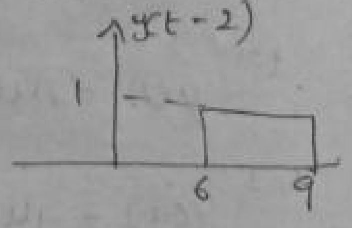



$t_2$ $\quad x(t) \xrightarrow{\text{Rs by 2}} x(t-2)$ $\quad y(t) \xrightarrow{\text{Rs by 2}} y(t-2).$

$x(t-2) \longrightarrow \boxed{S}$



$\therefore$ S is time invariant system.

Examples :-

① $y[n] = nx[n]$

$$x[n] \longrightarrow \boxed{A} \longrightarrow y[n] = nx[n].$$

$$\downarrow n_0 \qquad\qquad \downarrow n_0$$

$$x[n-n_0] \longrightarrow \boxed{A} \longrightarrow ① \Rightarrow y(n-n_0)$$

$$x[n] \xrightarrow{S} y[n] = nx[n] \;;\quad y[n-n_0] = (n-n_0)\,x[n-n_0]$$

Rs by $n_0$

$n \to n - n_0$

Rs by $n_0$

$$x[n-n_0] \xrightarrow[\times n]{S} nx[n-n_0] \neq y[n-n_0].$$

$\therefore$ System is time variant system.

② $y(t) = \sin x(t)$.

$$x(t) \longrightarrow \overset{\text{Sin}}{\boxed{B}} \longrightarrow y(t).$$

$$x(t) \xrightarrow{B} \sin x(t) = y(t) \xrightarrow[t \to t-t_0]{\text{Rs by } t_0} y(t-t_0)$$

Rs by $t_0$ $\quad t \to t-t_0$.

$$= \sin x\,(t-t_0).$$

$$x(t-t_0) \xrightarrow{B} \sin x(t-t_0) = y(t-t_0)$$

$\therefore$ System is Time invariant.

③ $y(t) = x(2t)$

Time scaling compression.

$$x(t) \longrightarrow \boxed{A} \longrightarrow x(2t) = y(t).$$

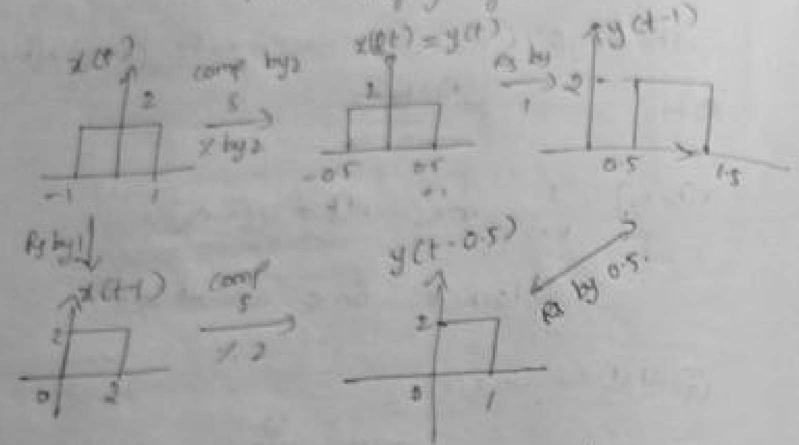
$$x(t) \xrightarrow{A} x(2t) = y(t) \xrightarrow[t \to (t-t_0)]{\text{Rs by } t_0} y(t-t_0)\ \cancel{x(2t-t_0)}$$

$= x[2(t-t_0)]$

$= x(2t-2t_0)$

$x(t) \xrightarrow[t \to t-t_0]{\text{RS by } t_0} x(t-t_0) \xrightarrow[t \to 2t]{A} x(2t-t_0) \neq y(t-t_0)$

Time varying system



$\therefore$ Time variant system.

*** ⑥ Linear / non Linear Systems:

A system is said to be linear if it satisfies two properties.

① Super position (additive) $\longrightarrow$ $x_1(t) \xrightarrow{S} y_1(t)$, $x_2(t) \xrightarrow{S} y_2(t)$

② Homogenity (scaling) $\rightarrow x_1(t) + x_2(t) \xrightarrow{S} y_1(t) + y_2(t)$

$x_1(t) \xrightarrow{S} y_1(t)$

$a x_1(t) \xrightarrow{S} a y_1(t)$

$x_1(t) \xrightarrow{S} y_1(t)$

$a x_1(t) \xrightarrow{S} a y_1(t)$

$b x_1(t) \xrightarrow{S} b y_2(t)$

$\rightarrow a x_1(t) + b x_2(t) \xrightarrow{S} a y_1(t) + b y_2(t)$

Ex:-

① $y(t) = t x(t)$

$x(t) \longrightarrow \boxed{S}^{\,xt} \rightarrow y(t) = t x(t)$

$x_1(t) \xrightarrow{xt} t x_1(t) = y_1(t)$

$x_2(t) \xrightarrow{xt} t x_2(t) = y_2(t)$

Let: $x_3(t) = a x_1(t) + b x_2(t) \xrightarrow{S} t[a x_1(t) + b x_2(t)]$

$= t a x_1(t) + t b x_2(t)$

$= a t x_1(t) + b t x_2(t)$

$a x_1(t) + b x_2(t) \xrightarrow{S} a y_1(t) + b y_2(t)$.

$\therefore$ System is linear.

② $y(t) = 2x(t) + 3$.

$x(t) \longrightarrow \boxed{S}^{\,x2+3} \longrightarrow y(t) = 2x(t) + 3$

$x_1(t) \xrightarrow{S} 2x_1(t) + 3 = y_1(t) \Rightarrow a y_1(t) = 2a x_1(t) + 3a$

$x_2(t) \xrightarrow{S} 2x_2(t) + 3 = y_2(t) \Rightarrow a y_2(t) = 2b x_2(t) + 3b$.

$a x(t) + b x_2(t) \xrightarrow{S} 2(a x_1(t) + b x_2(t)) + 3$

$= 2a x_1(t) + 2b x_2(t) + 3$

$= a \cdot 2x_1(t) + b \cdot 2x_2(t) + 3.$

$\boxed{ay_1(t) + by_2(t)}$

$= 2ax_1(t) + 3a +$
$2bx_2(t) + 3b.$

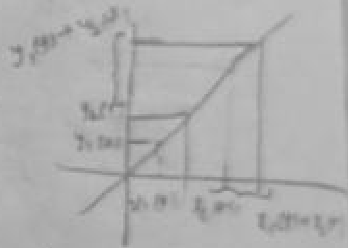
$$ax_1(t) + bx_2(t) \xrightarrow{S} ay_1(t) + by_2(t).$$

S is non linear

$$x_1(t) \longrightarrow 2x_1(t) + 3 = y_1(t)$$

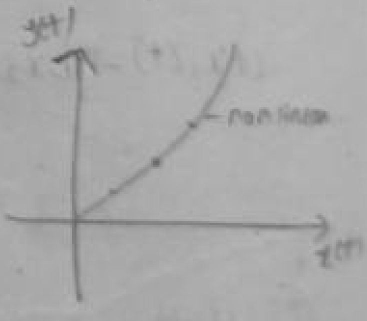
$$x_2(t) \longrightarrow 2x_2(t) + 3 = y_2(t)$$

$$x_1(t) + x_2(t) \xrightarrow[S]{x2+3} 2[x_1(t) + x_2(t)] + 3$$



(4) $y(t) = x^2(t).$ $y = mx.$

$2 \rightarrow 4$
$3 \rightarrow 9$
$4 \rightarrow 16.$




System is non-linear.

Linear (or) Time invariant

linear
2) Time Invariant systems:-

LTI systems.
useful practical systems.

linear:- i) superposition (additive)
ii) homogenous (scaling).

$$x(t) = \sum a\, x_i(t)$$
$$y(t) = \sum a\, y_i(t)$$

$\delta[t] \rightarrow \boxed{CTS} \rightarrow h(t)$ Impulse response.

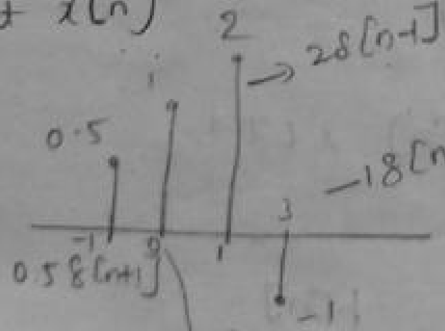
$\delta[n] \rightarrow \boxed{DTS} \rightarrow h[n]$ Impulse response.

$$x[n] \rightarrow \sum a\, \delta[n]$$
$$\Big\downarrow \text{ linear.}$$
$$y[n] \leftarrow \sum a\, h[n].$$

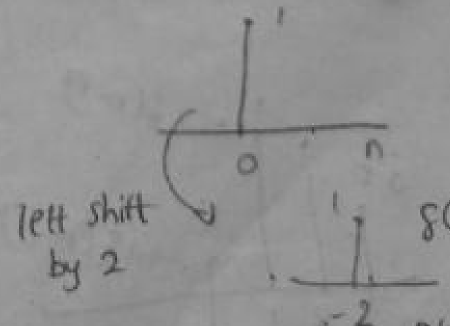
Discrete LTI systems:-

$x(n) \rightarrow \delta(n)$

let $x[n]$




$x[n] = 1\delta[n]$.




$n+2=0$
$\Rightarrow n=-2.$

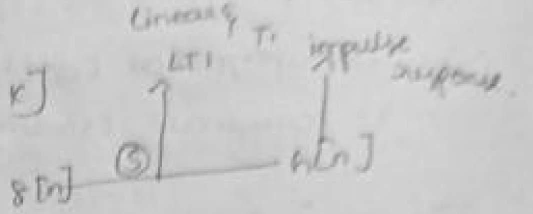
$\delta[n]$ ↳ by 2 $\longrightarrow$

$$x[n] = 0.5\,\delta[n+1] + \delta[n] + 2\delta[n-1] + (-1)\delta[n-3]$$

$x[n] = x[-1]\delta[n+1] + x(0)\delta[n] + x(1)\delta(n-1) + x(3)\delta[n-3]$

$x[n]$ at $n=-1$ ; $n+1=0$, $n=-1$ ; $n=0$ ; $n-1=0$, $n=1$ ; $n=3$, $n-3=0$

$$x[n] = \sum_{k=-\infty}^{\infty} x(k)\,\delta[n-k]$$




Time variant.

$\delta[n-k] \longrightarrow h[n-k]$

$\delta[n-1] \longrightarrow h[n-1]$

Linear: homogeneity $2\delta[n] \longrightarrow 2h[n]$

additive $2\delta[n] + \delta[n-1] \xrightarrow{S} 2h[n] + h[n-1]$

$\delta(n) \longrightarrow h(n)$

$$\underbrace{\sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)}_{x(n)} \xrightarrow{LTI} \underbrace{\sum_{k=-\infty}^{\infty} x(k)\,h(n-k)}_{y(n)}$$

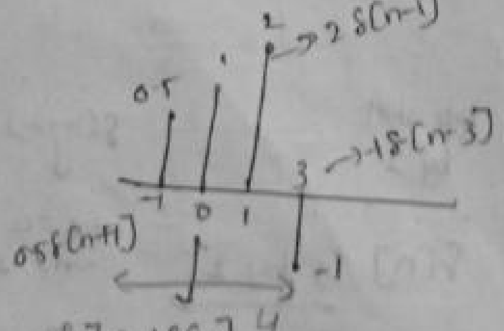
shifted impulse response by k.

$$y[n] = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$$

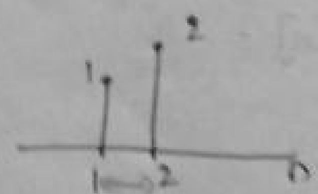
L convolution sum.

$= x(n) * h(n)$

Example: calculate $y[n]$, LTI.




$x[n] = 1\delta[n]$ 4



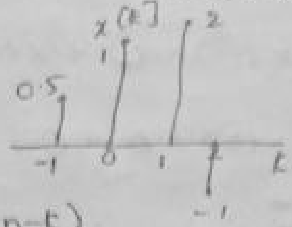

$h[n] = 1$

$h[2] = 2$

$x[n] = 0.5\delta[n+1] + \delta[n] + 2\delta[n-1] + (-1)\delta[n-3]$

① $x[n] = x[-1]\delta[n+1] + x(0)\delta[n] + x(1)\delta(n-1) + x(3)\delta[n-3]$.

② LTI




③ h[n]

$$y[n] = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

$k=-3$

$x(-3)=0.$

$n=0$

$$y(0) = \sum_{k=-\infty}^{\infty} x(k)\,h(-k).$$

$-1, 0, 1, 2, 3$

$$y(k) = \sum_{k=-1}^{3} x(k)\,h(n-k)$$

$n=0$

$$y(0) = \underset{k=-1}{x(-1)h(1)} + \underset{k=0}{x(0)h(0)} + \underset{k=1}{x(1)h(-1)} + \underset{k=2}{x(2)h(-2)} + \underset{k=3}{x(3)h(-3)}$$

$(0.5).\quad (0.5)(1) + (1)(0) + 2\cdot 0 + 0 + 0.$

$n=1$

$$y(1) = \sum_{k=-1}^{3} x(k)\,h(1-k)$$

$=2$

$$= \underset{k=-1}{x(-1)h(2)} + \underset{k=0}{x(0)h(1)} + \underset{k=1}{x(1)h(0)} + \underset{k=2}{x(2)h(-1)} + \underset{k=3}{x(3)h(-2)}$$

$(0.5)\ (2)\qquad (1)\ (1)$

$$y(2) = \sum_{k=-1}^{3} x(k)\,h(2-k)$$

$=4.$

$$y(2) = x(-1)h(3) + x(0)h(2) + x(1)h(1) + x(2)h(0) + x(3)h(-1)$$

$(1)\ (2)\qquad (2)\ (1)$

$n=-1$

$$y(-1) = \sum_{k=-1}^{3} x(k)\, h(-1-k)$$

$(-1-(-1))$

$k=-1 \quad k=0 \quad k=1$

$$= x(-1)h(0) + x(0)\,h(-1) + x(1)\,h(-2) + \ldots$$

$$= (0.5)\,0 + 0$$

$n=3 \quad \sum_{k=-\infty}^{\infty} x(k)\, h(3-k) \quad k=1$

$y[3]=4.$

$h(3)=0$

$$= x(-1)\,h(4) + x(0)\,h(3) + h(2)\,x(1)$$

$$+ h(1)\,x(2) + h(0) + h(-)$$

$h(2-k) = h(1)$

$h(1)=1$

$h(2)=2$

$x[n] \qquad h[n]$

$N_1 \qquad N_2$

no. of non zero samples in $y[n]$

$N = N_1 + N_2 - 1$

$= 4+1-1 = 4.$

$y(0) = 0.5$

$y(1) = 2$

$y(2) = 4$

$y(3) = 4$

$y(-1) = 0$

$y(4) = 0.$

Let

$n=4$

$y(4) =$

11/3/21

$\delta[n] \rightarrow h[n] \rightarrow$ Impulse response

$\delta[n-k] \rightarrow h[n-k]$ Time invariant.

$x(k)\,\delta[n-k] \rightarrow x(k)\,h(n-k]$ homogeneous

$$\left.\sum_{k=-\infty}^{\infty} x(k)\,\delta[n-k] \longrightarrow \sum_{k=-\infty}^{\infty} x(k)\,h(n-k]\right\}$$ - linear superposition

$$\int_{-\infty}^{\infty} x(t)\,\delta(t) = x(t) \text{ at } t=0 = x(0).$$

shifting property

$\delta[n-k] = n-k=0 \rightarrow n=k.$

$x(k)$ at $k=n = x(n)$

$$x[n] \xrightarrow{LTI} y[n] = \sum_{-\infty}^{\infty} x(k)\,h(n-k)$$

$||$

$$\sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k).$$

convolution sum

$y(n) = x[n] + h(n)$

↓ Input ↓ impulse response

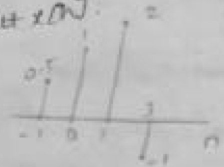
$$x[n] \xrightarrow{\text{LTI system}} y[n]$$

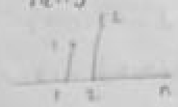
$$y[n] = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$$

$$= x(n) * h(n)$$

Let $x[n]$ =   calculate $y[n]$ LTI

$h[n]$ – impulse response





$0.5\,\delta[n+1] + \delta[n] + 2\delta[n-1] - 1\delta[n-3]$

$y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$   $h_k \xrightarrow{TR} h(-k) \xrightarrow{shift} h_{n-k}$.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| ... -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 | ... | | k. |
| 0 | 0 | 0.5 | 1 | 2 | 0 | -1 | 0 | ... | | $x(k) = x_k$. |
| $h_{-3}$ | $h_{-2}$ | $h_{-1}$ | $h_0$ | $h_1$ | $h_2$ | $h_3$ | | | | |
| 0 | 0 | 0 | 0 | 0 | 1 | 2 | 0 | 0 | $h(k)$ | $y(0)$ |
| $h_3$ | $h_2$ | $h_1$ | | $h_{-1}$ | $h_{-2}$ | $h_{-3}$ | | | | |
| 0 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | $h_{-k}$ | $y(1)$ rs by 1 $y(2)$ |
| 0 | 0 | 2 | 1 | 0 | 0 | 0 | ... | | $h_{1-k}$ rs by 1 | $y(1)$ rs by 2 |
| 0 | 0 | 0 | 2 | 1 | 0 | 0 | ... | | $h_2 - k$ | $y(2) = 0$ |
| 0 | 0 | 0 | 0 | 2 | 1 | 0 | ... | | $h_3 - k$ | $y(-3) = 0$ |
| 0 | 2 | 1 | 0 | 0 | 0 | ... | | | $h_{-1-k}$ ls by 1 | |
| 2 | 1 | 0 | 0 | 0 | 0 | ... | | | $h_{-2-k}$ ls by 2. | |

0 2 1 - -
0 -1 - -
0 2 1 - -
0 -1 2 - -
0 0 2 1 - -
0 -1 0 0

$y(0) = 0.5$

$y(1) = 2$

$y(2) = 4$

$y(3) = 4$ → $y(4) = -1$

$y(5) = -2$

$y(-1) = 0$

$y(-3) = 0$

$\sum_{k=\infty}^{\infty} x(k)\,h(n-k)$

no. of non zero values is $y(n) = N_1 + N_2 - 1$ (length of $h(n)$; length of $x(n)$)

$= 5 + 2 - 1 = 6$

② calculate $y[n]$ for a LTI system.

$x[n] = \alpha^n u[n] = h[n]$.

$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$.

$x(k) = \alpha^k u(k)$;   $h(k) \longrightarrow h(n-k)$

$h(k) = \alpha^k u(k)$

↓

$h(-k) = \alpha^{-k} u(-k)$

$h(n-k) = \alpha^{n-k} u(n-k)$

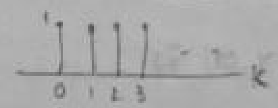
$y[n] = \sum_{k=-\infty}^{\infty} \alpha^k u(k)\,\alpha^{n-k} u(n-k)$   Indep. of k.

$= \sum_{k=-\infty}^{\infty} \alpha^n u(k)\,u(n-k)$

$= \alpha^n \sum_{k=-\infty}^{\alpha} u(k)\,u(n-k)$

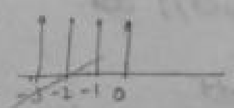
$\boxed{a^m \times a^{-m} = 1 \quad a^{m-n} = a^m / a^n}$

$u(k)$ (0 to $\alpha$)

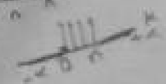


↓ TR.

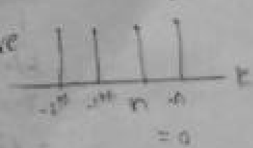
$u(-k)$ ($-\infty$ to 0)



rs by n. → $u(n-k)$   n is +ve.   $-\alpha$ to n.



ls by n → n is -ve



$$y[n] = \alpha^n \sum_{k=-\infty}^{\infty} u(k)\, u(n-k)$$

$$= \alpha^n \sum_{k=0}^{n} 1$$

$$y[n] = \alpha^n (n+1)\, u[n].$$

6

$$\sum_{i=0}^{2} 2 = \underset{i=0}{2} + \underset{i=1}{2} + \underset{i=2}{2}$$

3

$$\sum_{k=0}^{2} 1 = \underset{k=0}{1} + \underset{k=1}{1} + \underset{k=2}{1}$$

$$\sum_{k=0}^{3} 1 = 1+1+1+1$$

d) $x$

Assignment: 2

16/3/21 **Discrete LTI systems:**

Calculate $y(n)$

① $x(n) = \alpha^n u(n)$ applied to LTI systems

$0 < \alpha < 1$ $\qquad h(n) = u(n)$

$$y(n) = \left(\frac{1-\alpha^{n+1}}{1-\alpha}\right) u(n)$$

$x(n) \xrightarrow{\text{LTI}} \boxed{h(n)} \rightarrow y(n) = x(n) * h(n)$

$$= \sum_{k=-\infty}^{\infty} x(k)\, h(n-k) \longrightarrow ①$$

Convolution form.

$\delta(n) \longrightarrow \boxed{\text{LTI}} \longrightarrow h(n)$

$x(n) = \alpha^n u(n) \qquad h(n) = u(n)$

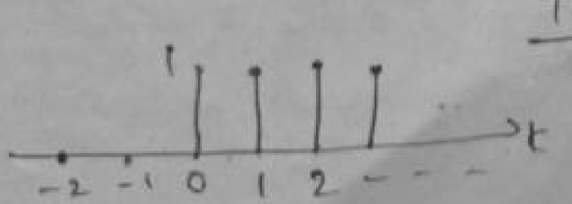
$\downarrow$

$x(k) = \alpha^k u(k) \qquad h(k) = u(k) \Rightarrow h(n-k) = u(n-k)$

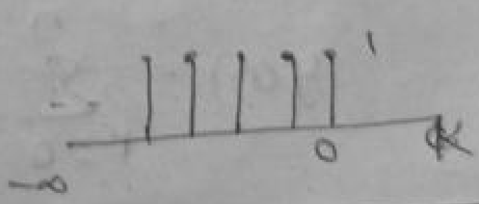
$$\boxed{y(n) = \sum_{k=-\infty}^{\infty} \cancel{\alpha}^{k} \, (u(k))\, u(n-k)}$$

$u(k) = \begin{cases} 1 & k \geq 0 \\ 0 & k < 0 \end{cases}$ $\qquad u(k) \xrightarrow{TR} u(-k) \xrightarrow{TS\,by\,n} u(n-k)$

0 to $\alpha$ $\qquad u(-k)$

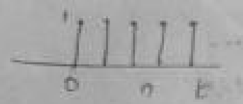


$\xrightarrow{TR}$



$u(n-k)$ $n>0$ (Right shift)

$u(k)$



ⓐ

$u(n-k)$

ⓑ

$n<0$ (left shift)

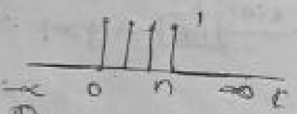
ⓒ

$n<0$

$u(k)\,u(n-k) = ⓐ \cdot ⓒ = 0$

$0$ to $\alpha$ $\quad -\alpha$ to $-n$

$n>0$



$u(k)\,u(n-k) = ⓐ \cdot ⓑ$ $\quad 0\cdot 1 \quad 1\cdot 1 \quad 1\cdot 0$

$k>0$ $\quad -\alpha$ to $n$

$u(k)\,u(n-k) = 1$ ; $0<k<n$

$$y(n) = \sum_{k=-\infty}^{\infty} \alpha^k\, u(k)\, u(n-k)$$

$$= \sum_{k=-\infty}^{0} \cancel{\alpha^k}\, 0 + \sum_{k=0}^{n} \alpha^k\, 1 + \cancel{\sum_{k=n}^{\alpha} \alpha^k\, 0}$$

$n>0$

$$y(n) = \sum_{k=0}^{n} \alpha^k$$

$= \alpha^0 + \alpha^1 + \alpha^2 + \alpha^3 + \dots \alpha^n$

$a=1$ ; $r=\alpha$

geometric series

$a + ar + ar^2 + ar^3 + \dots + ar^{n-1}$ $\quad$ $(n+1)$ terms / $n$ terms

$$S_{n+1} = \frac{1(1-\alpha^{n+1})}{1-\alpha}$$

$$S_\infty = \frac{a}{1-r}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$n>0$

$$y(n) = \frac{1-\alpha^{n+1}}{1-\alpha}$$

$n<0$

$y(n) = 0$

$n>0$ $\quad g=1$

$n<0,\ g=0$

$\forall n$

$$y(n) = \left(\frac{1-\alpha^{n+1}}{1-\alpha}\right) u(n)$$

$u(n)$ $n \geq 0 = 1$

$n<0 = 0$

$u(k) \xrightarrow{TR} u(-k) \xrightarrow[\text{LS by } 2]{\text{Shift by } 2} u(2-k)$

$n$ +ve $= 2$

$k \Rightarrow -k$

$k \to k+2$

$\to u(-(k+2)) = u(-k-2)$ ×

$\xrightarrow{\text{RS by } 2} u(-(k-2)) = u(-k+2)$

$= u(2-k)$ ✓

$u(k)$ $\quad k \to k-2$

$\downarrow$ TR RS by $n$

$u(n-k)$

$n$ is +ve.

$n$ is -ve ⓐ

$u(k) \xrightarrow{TR} u(-k) \xrightarrow{\text{RS by } 4} u(-4-k)$ ✓

$k \to k-4$

$u(-(k-4)) = u(-k+4)$ ×

$\underline{k \to k+4}$ $u(-(k+4)) = u(-k-4)$ ✓
LS by 4

$u(k) \xrightarrow{TR} u(n-k)$
- $n$ is $-ve$ ; LS by $n$
- $n$ is $+ve$ ; RS by $n$

$u(t) \to u(t-0)$
- $n$ is $+ve$ : RS by $n$
- $n$ is $-ve$ : RS by $n$

$u(k) \xrightarrow[k \to k-4]{RS} u(k-4)$  $n = 4 = +ve$.

**Continuous LTI Systems :-**

$\delta(t) \xrightarrow{S} h(t)$

$\delta(t-T) \to h(t-T)$  Time invariant.

$x(T)dT\,\delta(t-T) \to x(T)dT\,h(t-T)$  homogeneous

$$\int_{-\infty}^{\infty} x(T)\,\delta(t-T)\,dT \to \int_{-\infty}^{\infty} x(T)\,h(t-T)\,dT$$

shifting
$T = t$.
$x(T)$ at $T = t$.
$x(t) \xrightarrow{LTI} y(t) = \int_{-\infty}^{\infty} x(T)\,h(t-T)\,dT$

$y(t) = x(t) * h(t)$
convolution integral.

continuous LTI

$f(a) \to \boxed{g(a)} \to y(a) = f(a) * g(a)$

$f(a)$: input; $g(a)$: impulse response.

$y(a) = f(a) * g(a)$

(a) $\int_{-\infty}^{\infty} f(a)\,g(a-t)\,dt = y(a)$
(b) $\int_{-\infty}^{\infty} f(a)\,g(a-t)\,dt = y(a)$ ✓
(c) $\int_{-\infty}^{\infty} f(a)\,g(t-a)\,da = y(a)$
(d) $\int_{-\infty}^{\infty} f(a)\,g(a+t)\,da = y(a)$

$y(t) = x(t) * h(t)$
$= \int_{-\infty}^{\infty} x(T)\,h(t-T)\,dT$

**Properties of LTI systems:-**
(Discrete & continuous)

① commutative

input, impulse
$x(t) * h(t) = h(t) * x(t)$

$h(n) \to \boxed{x(n)} \to y(n)$

$\int h(t-T)\,x(T)\,dT$
$\int x(t-T)\,h(T)$

$\int_{-\infty}^{\infty} x(T)\,h(t-T)\,dT = \int_{-\infty}^{\infty} h(T)\,x(t-T)\,dT$

$y(t) = x(t) * h(t) = h(t) * x(t)$

2) **Associative**

$x(n) * [h_1(n) * h_2(n)]$
$= [x(n) * h_1(n)] * h_2(n)$

$\boxed{\begin{array}{l} a \times (b \times c) = (a \times b) \times c \\ 2 \times (4 \times 3) = (2 \times 4) \times 3 \\ 2 \times 12 = 8 \times 3 \end{array}}$

$x(n) \to \boxed{h_1 n} \to \boxed{h_2 n} \to y(n)$

cascade series

(i) $y_1(n) = x(n) * h_1(n) = y_1(n) * h_2(n)$

$x(n) \to \boxed{h_2(n) * h_1(n)} \to y(n)$

$y(n) = [x(n) * h_1(n)] * h_2(n)$

$\Rightarrow y(n) = x(n) * [h_1(n)] * h_2(n)]$

$= x(n) * [h_2(n) * h_1(n)]$

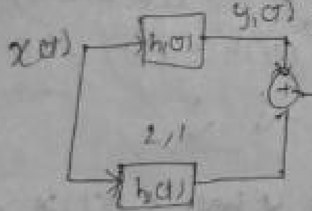
(ii)

$x(n) \to \boxed{h_2(n)} \xrightarrow{y_1(n)} \boxed{h_1(n)} \to y(n)$

③ Distributive:

$x(t) * [h_1(t) + h_2(t)] = [x(t) * h_1(t)] +$
$[x(t) * h_2(t)]$

$y_1(t) = x(t) * h_1(t)$ parallel connection




$y(t) = y_1(t) + y_2(t)$

$y_2(t) = x(t) * h_2(t)$

$= [x(t) * h_1(t)] +$
$[x(t) * h_2(t)]$

⇕

①, ①

$x(t) \to \boxed{h_1(t) + h_2(t)} \to y(t)$

$\Rightarrow x(t) * [h_1(t) + h_2(t)]$

$y(t) = x(t) * [h_1(t) + h_2(t)]$

commutative:

$x(n) \to \boxed{h(n)} \to y(n)$

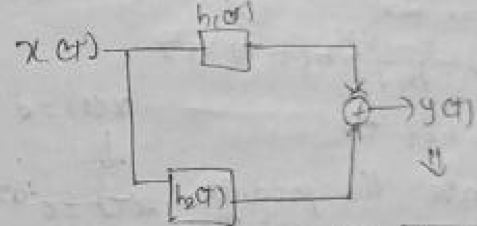
$x(t) \to \boxed{h(t)} \to y(t)$

$h(n) \to \boxed{x(n)} \to y(n)$

$h(t) \to \boxed{x(t)} \to y(t)$

Distributive:

$x(n) * [h_1(n) + h_2(n)] = [x(n) * h_1(n)] +$
$[x(n) * h_2(n)]$




⇓

$x(t) \to \boxed{h_1(t) + h_2(t)} \to y(t)$

Associative:-

$[x(n) * h_1(n)] * h_2(n) =$

$x(n) * [h_2(n) * h_1(n)]$

**Example:** consider an LTI system whose $h(t) = u(t)$. Given $x(t) = e^{-at}u(t)$; $a > 0$. find $y(t)$?

$\delta(t) \longrightarrow \boxed{LTI} \longrightarrow h(t) = u(t)$

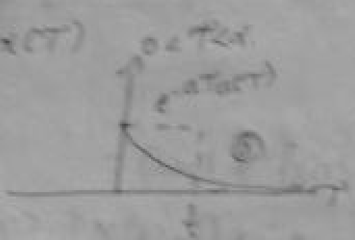
$x(t) \xrightarrow{h(t)=u(t)} \boxed{LTI} \longrightarrow y(t) = x(t) * h(t) = h(t) * x(t)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

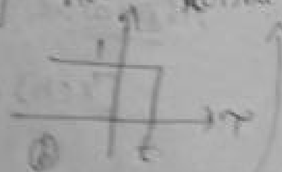
1) $x(\tau)$ vs $\tau$
2) $h(\tau)$ vs $\tau$
↓
Time Rev
$h(-\tau) \xrightarrow[\text{by } t]{\text{Time shift}} h(t-\tau)$ vs $\tau$
3) $x(\tau)\, h(t-\tau)$
4) Integrate the product wrt $\tau$.

$x(t) = e^{-at}u(t)$ vs $t$
↓
$x(\tau) = e^{-a\tau}u(\tau)$ vs $\tau$.

$x(\tau)$ $0 < \tau < \infty$



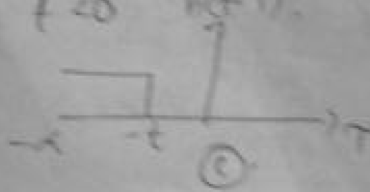

$\boxed{t > 0}$ $h(t-\tau)$




$e^{-a\tau}u(\tau)$ $0 < \tau < t$ $x(\tau)h(t-\tau)$

$-\infty < \tau < t$
$a < b$

$t < 0$ $h(t-\tau)$





$t < 0$
$x(\tau)h(t-\tau) = 0$
$t > 0$
$x(\tau)\, h(t-\tau) = e^{-a\tau}u(\tau)$
$0 < \tau < t$.

$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

$= \int_{-\infty}^{0} 0 + \int_{0}^{t} e^{-a\tau}u(\tau)\, d\tau + \int_{t}^{\infty} 0$

$= \int_0^t e^{-a\tau} u(\tau)\, d\tau$

$= \int_0^t e^{-a\tau}\, d\tau = \left[\frac{e^{-a\tau}}{-a}\right]_0^t$

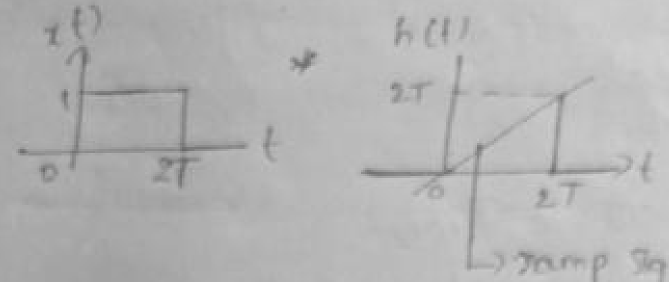
$= \frac{-1}{a}\left[e^{-at} - e^{-a0}\right] = \frac{-1}{a}(e^{-at} - 1)$

$\forall t$ $\boxed{y(t) = \frac{1}{a}(1 - e^{-at})\, u(t)}$

② $x(t) = \begin{cases} 1 & 0 < t < 2T \\ 0 & \text{otherwise}. \end{cases}$

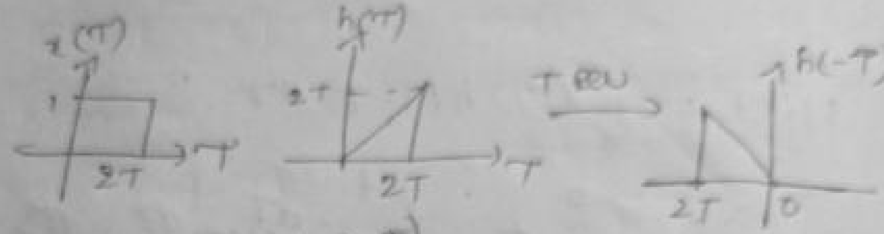
$h(t) = \begin{cases} t & 0 < t < 2T \\ 0 & \text{otherwise}. \end{cases}$

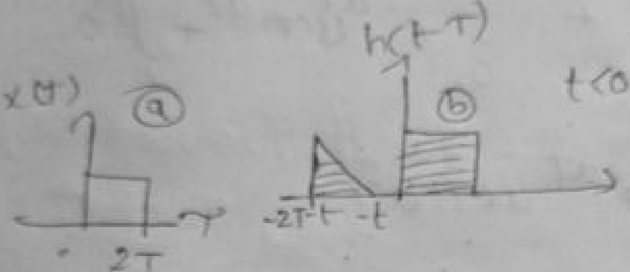
$y(t) = x(t) * h(t)$
$= \int_{-\infty}^{\infty} x(\tau) h(t-\tau)\, d\tau$



$t=0,\ h(t)=0$
$t=1,\ h(t)=1$
$t=2T,\ h(t)=2T$

ramp signal.

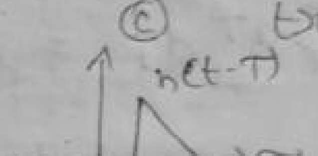




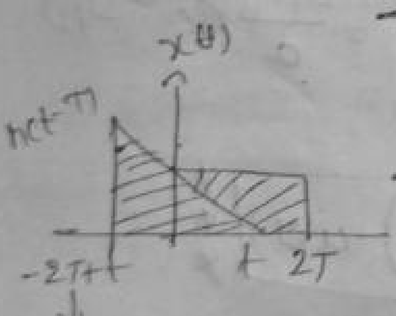
$t<0$; is by $-1$

Time shift by $t$

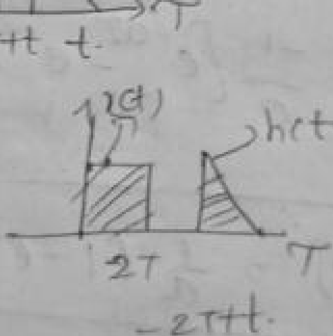
① $x(\tau)\,h(t-\tau)=0$



$t>0$.



exceeds.



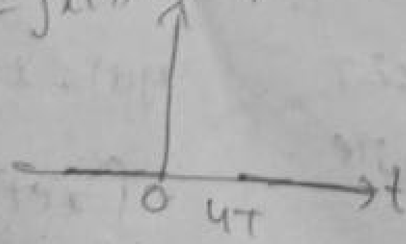
$t>0$

RS $= \boxed{0}$

$$-2T+t>2T$$

$$\Rightarrow t>4T=0.$$

$$y(t)=\int x(\tau)\,h(t-\tau)$$



$x(t) = e^{-at} \; u(t)$

$\downarrow$ exp $\quad \downarrow$ unit step

$a>0$

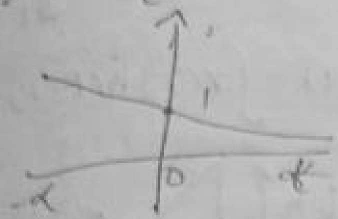
$e^{at} \approx ce^{at}$.

$C=1$; $a=-a=-ve$

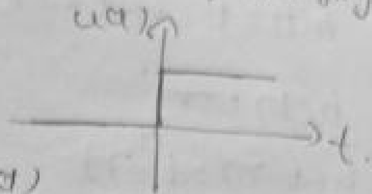
case (i): $a>0$, +ve : ↑ sing exp

$a<0$ : -ve ; decaying exp

$e^{-at}$

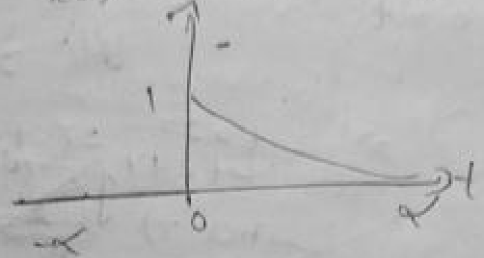


$u(t)$



$x(t) = e^{-at} u(t)$

$x(T)$ vs $T$



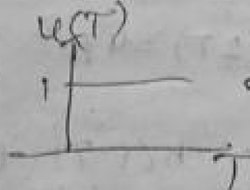
$2\times1=2$

$3\times1=3$

$h(t) = u(t)$

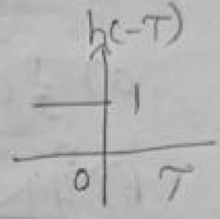
$\Rightarrow h(T) = h(T)$

$u(T)$



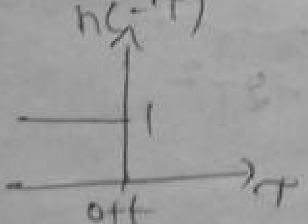
$\xrightarrow{TR}$

$h(-T)$



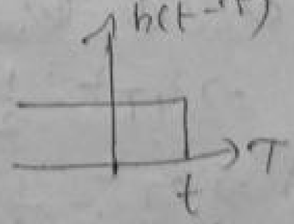
$h(T) \xrightarrow{T\,Rev} h(-T) \xrightarrow[\text{by } t]{T\,shift}$ $T>0 \rightarrow$ RS $h(t-T)$ ; $T<0 \rightarrow$ LS $h(t-T)$

$h(-T)$



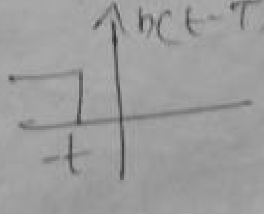
$\xrightarrow[\text{by } t]{RS}$

$h(t-T)$



$\xrightarrow[\text{by } t]{LS}$

$h(t-T)$



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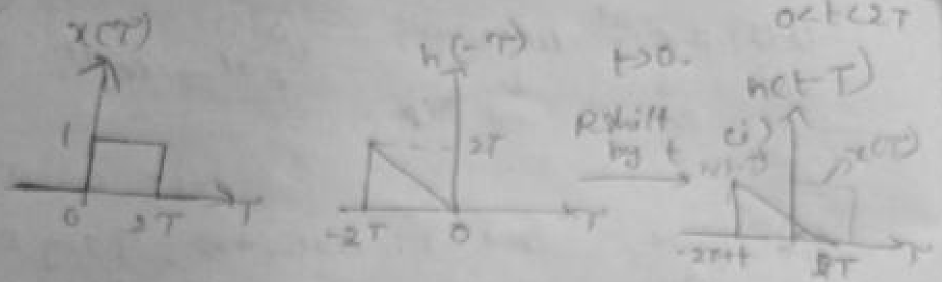



$h(t) = t \quad 0 < t < 2T$ $\qquad y(t) = \int x(\tau)\, h(t-\tau)$

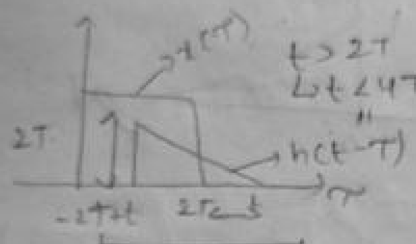
$h(\tau) = \tau$

$h(t-\tau) = t - \tau$

at $\tau = 0 = t$


$= \frac{1}{2} bh$

$y(t) = \frac{1}{2}\, t \cdot t = \frac{1}{2} t^2$

② 


$t > 2T$

$2T < t < 4T$

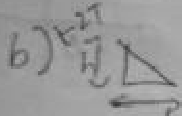
$2T < t < 4T.$


$c = ⓐ - ⓑ$


$t - (-2T + t)$

$\cancel{t} + 2T - \cancel{t}$

a) $\frac{1}{2}(2T)(2T) = 4T^2$

b) 


$h(t-\tau) = t - \tau = t - 2T$

at $\tau = 2^+$

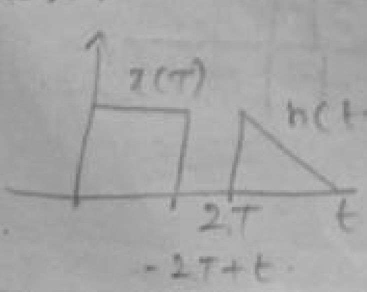
$= \frac{1}{2}(t-2T)^2 = \frac{1}{2}\left[t^2 - 4Tt + 4T^2\right]$

c) ⇒ ⓐ - ⓑ

$\Rightarrow 4T^2 - \frac{1}{2} t^2 + 2Tt - 2T^2$

$y(t) \Rightarrow 2T^2 - \frac{1}{2} t^2 + 2Tt$

③ case 3 $\qquad \int x(\tau)\, h(t-\tau) = y(t)$

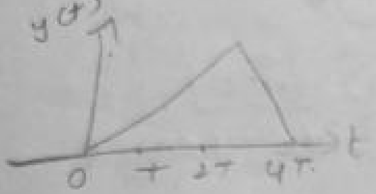



$-2T + t > 2T$

$t > 4T.$

$$y(t) = \begin{cases} 0 & t<0 \\ \frac{1}{2}t^2 ; & 0<t<2T \\ -\frac{1}{2}t^2+2Tt/2 & T<t<4T \\ 0 & t>4T. \end{cases}$$

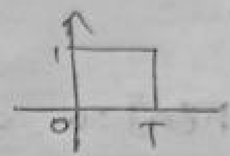
$\boxed{y = x^2}$

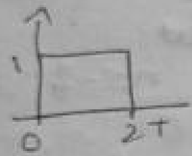


$$\int_{-\infty}^{\infty} \overset{x(t)}{\overset{\downarrow}{x(\tau)}} \; \overset{h(t)}{h(t-\tau)} \, d\tau$$

$t<0$ LS, $t>0$ RS.

$$x(t) = \begin{cases} 1 & 0<t<T \\ 0 & \text{otherwise.} \end{cases}$$





$$y(t) = x(t) * h(t) := \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

Memoryless, Inv, causal/NC, TI, TV, stable, linear/non linear.

LTI :- linear / Time invariant.

1) Memory / memoryless :-

$$y(n) \xrightarrow{\text{depends}} x(n)$$

$x(n-1)$, $(n+2)$.

if $x(n) = \delta(n) \xrightarrow{\text{only at}} \boxed{n=0}$ $\quad \delta(n) \rightarrow \boxed{LTI} \rightarrow h(n)$

$y(n) = n(0) \Rightarrow n = 0$ ; $\boxed{n=0}$ memoryless $\boxed{h(n)}$ $n \neq 0$

$\delta(n) \Rightarrow h(n) = 0 ;\ n \neq 0$ *

$\boxed{n=0}$

$\Rightarrow h(n) = k\,\delta(n)$ $\boxed{n=0}$

at $n \neq 0$ ; any $n \neq 0$ ; $h(n) = 0$

$n = 0$

$h(0) = k\,\delta(0) = k(1)$

$\Rightarrow k = h(0)$ *

$h[n] = 0 ;\ n \neq 0$ ; $h[n] \neq 0 ;\ n \neq 0$.

$\delta(n) \rightarrow h[n] = 2$ ; at $n = -2$ Memory

$\boxed{n=0}$ $\quad h[n] = 0 \quad n \neq 0$

C LTI - $h(t) = 0 \ \forall\ t \neq 0$ $\quad \delta(n)$

$h(t) = k\,\delta(t)$ ; $t = 0$

$k = \int_{-\infty}^{\infty} h(t)\,dt$ $\qquad h(0) = k\,\delta(0)$

$$\int_{-\infty}^{\infty} h(t)\,dt = \int_{-\infty}^{\infty} k\,\delta(t)\,dt = k \int_{-\infty}^{\infty} \delta(t)\,dt$$

$$\int_{-\infty}^{\infty} h(t)\,dt = k$$

LTI Systems are memoryless,

if $h[n] = 0$ where $n \neq 0$.

$h(t) = 0 \qquad t \neq 0$.

$h(n) = k\,\delta(n)$

$y(t) = k\,x(t)$ ; $\boxed{h(t) = k\,\delta(t)}$

$\boxed{k=1}$ ; $y(t) = x(t)$

$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$ $\qquad$ Identity System $\quad x(t) \rightarrow \boxed{\ } \rightarrow y(t) = x(t)$.

$y(t) = x(t) * h(t)$

$= x(t) * \delta(t)$

$\boxed{x(t) * \delta(t) = x(t)}$ *

$x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau$

shifting $\quad$ prop

$t = \tau$

$x(\tau)$ at $\tau = t = x(t)$

$y$ at $t$ $\qquad x$ at $t$

$\boxed{y(t) = k\,x(t)}$ $\quad | \quad y(n) = k\,x(n)$

$k = 1$ : Identity systems

Are LTI identity systems memoryless? Yes.

* $h(n) = 0$ for $n \neq 0$

* $h(t) = 0 \ \forall\ t \neq 0$ $\quad$ LTI systems are called memoryless

$y(t) = k\,x(t)$

$h(t) = k\,\delta(t)$

$k = 1$.

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Memoryless :-

$y(n) \propto x(n)$
$\downarrow$
at n.

$x(n) \to \boxed{h(n)} \to y(n) = x(n) * h(n)$.

$\delta(n) \to \boxed{LTI} \to h(n)$.

$x(n) = \delta(n) \to$ unit impulse

$y(n) = h(n) \to$ unit impulse response.

$h(n) \propto \delta(n)$

exists only at $n=0$ ; exists only at $n=0$

$\Rightarrow \boxed{h(n) = 0;\ n \neq 0}$ such systems are memoryless.

from g $h(n) = K\delta(n)$ only at $n=0$

$n=0;\ n\neq 0\ ;\ \delta(n) = 0$

$\Rightarrow h(n) = 0$.

$n=0;\ h(0) = K\delta(0) = K(1)$

$\Rightarrow \boxed{K = h(0)}$ *

o/p

If $h(n) = K\delta(n)$ * o/p

$\Rightarrow y(n) = Kx(n)$ *

let $K=1;\ h(n) = \delta(n)$

$\delta(n) \to \boxed{\ } \to h(n) = \delta(n)$

$y(n) = x(n)$

$x(n) \to \boxed{\ } \to y(n) = x(n)$ Identity System.

$g(n) \to \boxed{h(n)}_{LTI} \to y(n)$ identity system

$y(n) = g(n) * h(n) = g(n)$

$= g(n) * \delta(n)$

$= \sum_{k=-\infty}^{\infty} g(k)\,\delta(n-k)$ $\to k=n$

$\downarrow$ shifting p

$g(k)$ at $k=n = g(n)$.

$\delta(n) * h(n) = \delta(n)$

* $g(n) * \delta(n) = g(n)$
* $x(n) * \delta(n) = x(n)$

Continuous LTI Systems:

$\boxed{h(t) = 0 \quad \forall\ t \neq 0}$

can take form $h(t) = K\delta(t)$ only $t=0$

$K = \int_{-\infty}^{\infty} h(t)\, dt$.

$K = 1$

$h(t) = \delta(t) \Rightarrow y(t) = x(t) * h(t)$

$y(t) = x(t)$ $\quad = x(t) * \delta(t)$

$y(t) = x(t)$

② Invertible systems / Non Invertible Systems:-

$x(n) \to \boxed{S\ h(t)}_{LTI} \to y(n) \to \boxed{S'}_{h_2(t)} \to x(n)$ series cascade

Invertible S $\quad$ Inverse System

Identity System    Identity system

$x(n) \rightarrow \boxed{h_1(n) * h_2(n)} \rightarrow x(n) = y(n)$

$h(n) = \delta(n)$

$x(n) = x(n) * h(n)$

$x(n) = x(n) * \delta(n)$

$\boxed{h_1(n) * h_2(n) = \delta(n)}$ $\therefore$ System $S$ is invertible.

Impulse R of $S$ invertible.    imp R of $S'$ inverse.

$x(n) \rightarrow \boxed{S}_{h_1(n)} \rightarrow \boxed{\ }_{h_2(n)} \rightarrow x(n)$

$h_1(n) * h_2(n) = \delta(n)$. Invertible

$x(t) \rightarrow \boxed{h_1(t)}_{S} \rightarrow \boxed{h_2(t)}_{S'} \rightarrow y(t)$

$x(t) \rightarrow \boxed{\delta(t)}_{h_1(t) * h_2(t)} \rightarrow y(t) = x(t)$

$\Rightarrow \boxed{h_1(t) * h_2(t) = \delta(t)}$ $S$ is invertible. $S^{1}$ is inverse.

__Example:__ consider an LTI system,

$y(t) = x(t - t_0)$    Invertible?

$x(t) \rightarrow \boxed{\ }_{\text{Time shift by } t_0} \rightarrow y(t) = x(t - t_0)$

$h_1(t) * h_2(t) = \delta(t)$

$t > 0$.

$\delta(t) \rightarrow \boxed{\ } \rightarrow h(t) = \delta(t + t_0)$

$\delta(t) \rightarrow \boxed{h(t)}_{\delta(t + t_0)} \rightarrow \boxed{h_2(t)}_{h_2(t) = \delta(t + t_0)} \rightarrow \delta(t)$

RS by 2.

$\delta(t) \rightarrow \delta(t-2) \xrightarrow{\text{LS by } 2} \delta(t + \cancel{2} - \cancel{2}) = \delta(t)$.

$t \rightarrow t+2$

$\Rightarrow h_1(t) * h_2(t) = \delta(t)$

$\Rightarrow \underset{(i)}{\delta(t - t_0)} * \underset{(ii)}{\delta(t + t_0)}$

$\rightarrow \int_{-\infty}^{\infty} \delta(\tau - t_0)\, \delta(t - \tau + t_0)\, d\tau$

$S$ is invertible

$\underset{(i)}{x(t)} * \underset{(ii)}{h(t)}$

$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$

$h(t) \rightarrow h(\tau) \xrightarrow{TR} h(-\tau)$

$-\tau \rightarrow t - \tau$ TS

$h(t - \tau)$

$\delta(t + t_0) \xrightarrow{t \rightarrow \tau} \delta(\tau - t_0) \xrightarrow{\tau \rightarrow -\tau} \delta(-\tau + t_0)$

TS $\downarrow$ $-\tau \rightarrow t - \tau$

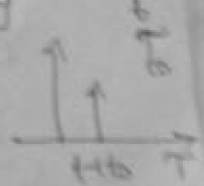
$\boxed{\delta(t - \tau + t_0)}$

$\Rightarrow \int_{-\infty}^{\infty} \overset{x(t)}{\delta(\tau - t_0)}\, \delta(t - \tau + t_0)\, d\tau$    shifting

$\Downarrow$

$t - \tau + t_0 = 0$

$\tau = t + t_0$




$\delta(\tau - t_0)$ at $\tau = t + t_0 \Rightarrow \delta(t + \cancel{t_0} - \cancel{t_0})$

$= \delta(t)$

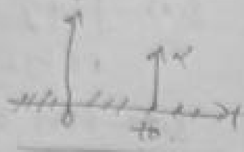
$\therefore$ LTI System is invertible.

$h_2(t) = \delta(t-t_0)$

Such that $\boxed{h_1(t) * h_2(t) = \delta(t)}$

(3) $h(t) = 0 \; ; \; t \neq 0$

$h(t) = \delta(t-t_0) = t = t_0$

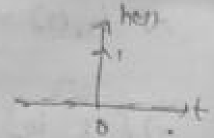


$t_0$.

$\delta(t) \to 0$ ∴ not memoryless

(1) $t_0 = 0 \; ; \; h(t) = \delta(t)$

$h(t) = 0 ; t \neq 0$



∴ memoryless.

at $t=4$ ; $h(t) = \delta(t-t_0) = \delta(4-5)$

$t_0 = 5$ ↓ $h(4) = \delta(-1)$

$h(4) \neq \delta(4)$.

$\delta(4) \Rightarrow \delta(t) \to t=0$

o/p at $t=4$ depending on

i/p at $= -1$

∴ memory.

$\boxed{t_0 = 0}$ ; $h(t) = \delta(t-0)$

$t=4$ ; $h(4) = \delta(4) \Rightarrow$ memoryless

(2) $h(n) = u(n)$ ; memoryless ?

Invertable ?

$h(n) = 0 ; n \neq 0$ ∴ memory

$h(4) = n(4) = 1 \neq 0$ $n=4$

$h_1(n) = \; ; \; h(n) * h_1(n) = \delta(n)$

$* \; \delta(n) \to \boxed{\phantom{x}} \to h(n) = u(n)$ Given

$x(n) \to \boxed{\phantom{x}} \to y(n) = x(n) * h(n)$

$= \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$.

$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases}$

$= \sum_{k=-\infty}^{\infty} x(k)\, u(n-k)$

$u(n-k) = \begin{cases} 1 & n-k \geq 0 \\ 0 & \text{otherwise} \end{cases}$

$= \sum_{k=-\infty}^{n} x(k) + \sum_{k=n}^{\infty} x(k) \cdot 0$

$n-k \geq 0$

$\Rightarrow k \leq n$

(1) $k \to -\infty$ to $n$

0 $k \to n$ to $\infty$

$y(n) = \sum_{k=-\infty}^{n} x(k)$

Accumulate

$y(1) = \sum_{k=-\infty}^{1} x(k)$

memory.

$= x(-\infty) + \dots + x(1)$

$y(2) = x(-\infty) + \dots + x(1) + x(2)$.

$x(n) \rightarrow \boxed{\begin{matrix} h(n) \\ = u(n) \end{matrix}}$ Inv $\xrightarrow[y(n)]{i/p} \boxed{h_1(n)} \rightarrow x(n)$ $-y(n-1)$ o/p

$$y(n) = \sum_{k=-\infty}^{n} x(k) \longrightarrow x(n)$$

$$= \sum_{k=-\infty}^{n-1} x(k) + x(n)$$

$$y(n) = y(n-1) + x(n)$$

$$\sum_{k=1}^{4} x(k) = x(1) + x(2) + x(3) + x(4)$$

$$\sum_{k=1}^{3} x(k) + x(4)$$

$$x(1) + x(2) + x(3) + x(4)$$

2nd system.

$\Rightarrow \boxed{x(n) = y(n) - y(n-1)}$

o/p i/p

$$\boxed{h_2(n) = \delta(n) - \delta(n-1)}$$

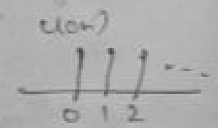
$\Rightarrow h_1(n) * h_2(n) = \delta(n)$

$-x(n) * \{\delta(n) - \delta(n-1)\} =$

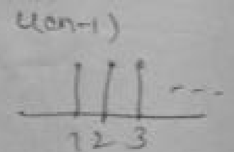
$$\boxed{\begin{matrix} x(n) * \delta(n) = x(n) \\ x(n) * \delta(n-n_0) = \\ x(n-n_0). \end{matrix}}$$

$= [u(n) * \delta(n)] - [u(n) * \delta(n-1)]$

$= u(n) - u(n-1)$



$= \delta(n)$



LTI $h(n) = u(n)$

is invertible.

$x(n) \rightarrow \boxed{h_1(n)} \rightarrow \boxed{h_2(n)} \rightarrow y(n)$

$\boxed{h_1(n) * h_2(n)}$

1. memoryless ; $h(n) = 0$ ; $n \neq 0$.

$h(n) = 0$ ; $k \neq 0$.

2. Invertible ; $h_1(t) * h_2(t) = \delta(t)$

Inverse system

Causal / Non causal :-

$x(n) \rightarrow \boxed{h(n)} \rightarrow$ $y(n) = x(n) * h(n)$

$y(n) =$ past & present i/p.

$y(n)$ depends on $x(k)$ $k \leq n$

for eg : $y(4) \leftarrow x(3), x(4)$

$x(6)$

$\therefore y(n)$ should not depend

on $x(k)$ $\underline{k > n}$

$y(n) = 0$ ; $k > n$

$x(n) \rightarrow \boxed{h(n)} \rightarrow y(n) = x(n) * h(n)$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k) \quad k > n$$

## causal

$y(n)$ $\quad h(n-t)=0 \;;\; t>n \;;\; n-t<0.$

$$\Rightarrow \boxed{h(n)=0 \;;\; n<0}$$

$\delta(n) \rightarrow \boxed{LTI} \rightarrow h(n)=0 \quad \boxed{n<0}$ $\quad \boxed{n=0}$

causal

$n=0$ Initial rest

non-linear

$*\; y(n)=2x(n)+3$

$n=2$

$y(2)=2x(2)+3$

causal

$x(n)=0,$

$y(n)=2(0)+3$

$=\underline{3}$

## continuous :-

$$\boxed{h(t)=0 \text{ for } t<0}$$

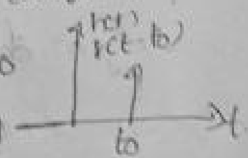
## Example :

① $\delta(t-t_0)=h(t)$ memory, Invertible, causal?

$h(t)=0 \;;\; t<0$

$\delta(t-t_0)=0 \;;\; t<0.$

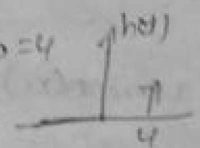
$t_0\geq 0$

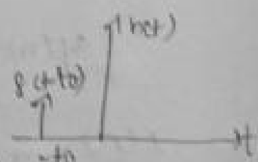


causal

$h(t)=\delta(t-4)$

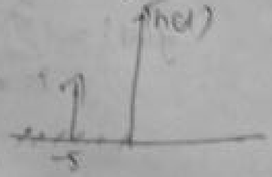
$\cdot\; t_0=4$





non causal

$h(t)=\delta(t+5)$

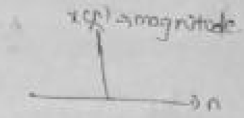


NC $\quad t_0=-5<0.$

## Stable : BIBO

Bounded i/p, Bounded o/p.

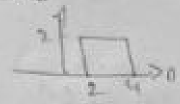
$x(n) \rightarrow \boxed{h(n)} \rightarrow y(n)$



$|x(n)| < L < \infty$

$\Rightarrow -L < x(n) < L \Rightarrow$ bounded

$-L < x(n-2) < L$



$x(n) \rightarrow \boxed{h(n)} \rightarrow y(n) = x(n) * h(n)$

$$=\sum_{t=-\infty}^{\infty} x(t)\,h(n-t)$$

$x(n)$ is bounded $\quad = h(n) * x(n)$

$|x(n)| < L < \infty$

$$y(n) = \sum_{t=-\infty}^{\infty} h(t)\,x(n-t)$$

$$|y(n)| = \left|\sum_{t=-\infty}^{\infty} h(t)\,x(n-t)\right|$$

$x(n)$ is bounded

$$|y(n)| \leq \sum_{t=-\infty}^{\infty} |h(t)|\,|x(n-t)|$$

$$|y(n)| \leq L \sum_{t=-\infty}^{\infty} |h(t)| < \infty.$$

$\downarrow$

bounded o/p

$=|1|=0$

$=5.$

$$\boxed{\sum_{t=-\infty}^{\infty} |h(t)| < \infty}$$ : absolutely summable.

$|y(n)| < \infty \Rightarrow$ bounded o/p
bounded i/p.

also

<u>continuous</u> : $\int_{-\infty}^{\infty} |h(t)|dt < \infty$ ; absolutely integrable.

<u>LTI System</u>

| | DT | CT |
|---|---|---|
| memoryless ; | $h(n)=0$ ; $n \neq 0$ | $h(t)=0$ ; $t \neq 0$. |
| Invertible ; | $h(n) * h_2(n) = \delta(n)$ | $h(t) * h_2(t) = \delta(t)$ |
| causal ; | $h(n)=0$ ; $n<0$ | $h(t)=0$ ; $t<0$. |
| stable ; | $\sum_{t=-\infty}^{\infty} \|h(t)\| < \infty$ | $\int_{-\infty}^{\infty} h(t)\,dt < \infty$. |

<u>Example</u> :

$\delta(t-t_0) = h(t)$

$$\int_{-\infty}^{\infty} h(t)\,dt = \int_{-\infty}^{\infty} \delta(t-t_0)\,dt = 1 < \infty$$

$\therefore$ System is stable

$\delta(n) \rightarrow \boxed{\phantom{x}} \rightarrow h(n)$

$\delta(t) \rightarrow \boxed{\phantom{x}} \rightarrow h(t)$

<u>Unit response</u>

$u(n) \rightarrow \boxed{\phantom{x}} \rightarrow s(n)$ $\qquad h(n)$

$u(t) \rightarrow \boxed{\phantom{x}} \rightarrow s(t)$ $\qquad h(t)$

$\delta(t) \rightarrow h(t)$ ; $u(t) \rightarrow s(t)$

$\delta(n) \rightarrow h(n)$ ; $u(n) \rightarrow s(n)$

<u>Discrete</u> <u>LTI</u> :-

$$\begin{array}{ccc} \delta(n) & \longrightarrow & u(n) \\ \downarrow & & \downarrow \\ h(n) & \longleftrightarrow & s(n) \end{array}$$

$\delta(n) = u(n) - u(n-1)$

$$\boxed{u(n) = \sum_{t=-\infty}^{n} \delta(t)}$$

$u(n) \rightarrow \boxed{h(n)} \rightarrow s(n) = u(n) * h(n)$

$= h(n) * u(n).$

$= \sum_{t=-\infty}^{\infty} h(t)\,u(n-t)$

$$s(n) = \sum_{t=-\infty}^{n} h(t) + \cancel{\sum_{t>n}^{\infty} h(t)}\,0$$

$n-k \geq 0$

$$\boxed{u(n-t) = 1 ; \Rightarrow k \leq n}$$

$$\Rightarrow \boxed{s(n) = \sum_{t=-\infty}^{n} h(t)}$$

$u(n) = \sum_{t=-\infty}^{n} \delta(k)$

$s(n) = u(n) - u(n-1)$

$$\Rightarrow s(n) = \sum_{t=-\infty}^{n-1} h(k) + h(n)$$

$= s(n-1) + h(n).$

$\rightarrow \boxed{h(n) = S(n) - S(n-1)}$

$$\sum_{t=1}^{3} x(t) = x(1) + x(2) + x(3)$$
$$= \sum_{t=1}^{2} x(t) + x(3)$$
$$= x(1) + x(2) + x(3)$$

$\boxed{\delta(n) = u(n) - u(n-1)}$

Continuous LTI Systems:

$\delta(t) \rightarrow h(t)$ $\quad$ $u(t) \rightarrow s(t)$

$u(t) = \int_{-\infty}^{t} \delta(\tau) d\tau$ $\quad$ | $\quad$ $s(t) = \int_{-\infty}^{t} h(\tau) d\tau$

$\delta(t) = \frac{du(t)}{dt}$ $\quad$ | $\quad$ $h(t) = \frac{ds(t)}{dt}$

(7) $s(t)$ = Given

$$h(t) = \frac{ds(t)}{dt}$$

$$\int_{-\infty}^{\infty} |h(t)| < \infty$$

Stable

Example: LTI

(1) $x(t) = u(t)$ ; $h(t) = e^{-at} u(t)$

$y(t)$

$t \rightarrow t - \tau$

$y(t) = x(t) * h(t)$

$$= \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

$$= \int u(\tau) e^{-a(t-\tau)} u(t-\tau) d\tau$$

$$= e^{-at} \int_{-\infty}^{\infty} e^{a\tau} \boxed{u(\tau) u(t-\tau)} d\tau$$

$1 ; \tau > 0$

$1 ; t - \tau > 0 \Rightarrow \boxed{\tau < t}$

$$= e^{-at} \int_{0}^{t} e^{a\tau} d\tau$$

$$= e^{-at} \left[ e^{a\tau} \right]_0^t$$

$$= \frac{e^{-at}}{a} \left[ e^{at} - 1 \right]$$

$$= \frac{1}{a} (e^{0} - e^{-at}) = \frac{1}{a} (1 - e^{-at})$$

(5) $x(n) = 2^n u(-n)$ $\quad h(n) = u(n)$ $\quad y(n) = ?$

$$y(n) = x(n) * h(n)$$

$$= \sum_{k=-\infty}^{\infty} x(k) h(n-k)$$

$$= \sum_{k=-\infty}^{\infty} 2^k \boxed{u(-k) u(n-k) =}$$

$= 1 ; -k \geq 0 \Rightarrow k \leq 0$

$= 1 ; n - k \geq 0$

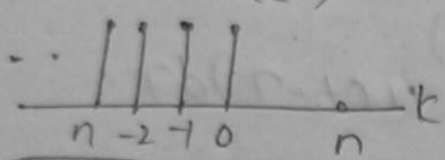
$k \leq n.$

$$y(n) = \sum_{k=-\infty}$$

$u(-k) \quad u(n-k)$

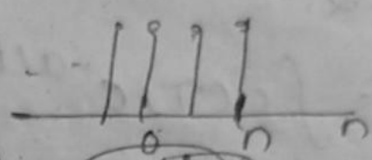
$n>0$ $\quad$ $n$ is $-ve$;

$u(-k)$ $\quad$ $u(n-k)$




$-\alpha$ to $0$

(a)




(b)

$-\alpha$ to $n$

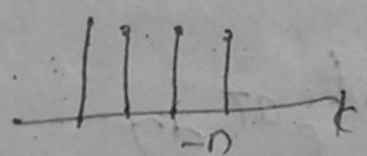
$n<0$:-

$u(-k)u(n-k) =$

$k$ $\boxed{-\alpha \text{ to } -n}$

(a) & (c)

$u(n-k)$




(c)

$n>0$ ; $=1$

(a) * (b).

$$y(n) = \sum_{k=-\alpha}^{0} 2^k$$

$$= \sum_{k=-\alpha}^{-n} 2^k$$