# Formulas & imp. points.

## Signals and systems:

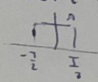
* $u(t)$ unit step signal

**Basic signals**

1) unit step signal ($u(t)$), 2) Dirac-delta func → $h(t)$

3) Impulse funct → $\delta(t)$, 4) Ramp signal $r(t)$

5) Signum func. → $\mathrm{sgn}(t)$, 6) Rectangular funct. → $A\,\mathrm{rect}(t/T)$



* **Properties of impulse function**

→ area under graph $\int_{-\infty}^{\infty} \delta(t)\,dt = 1$

→ $\delta(t) = \delta(-t)$

→ product property $x(t)\cdot\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$

→ sampling property $\int_{t_1}^{t_2} x(t)\cdot\delta(t-t_0) = x(t_0)$ $\{t_1 \le t \le t_2\}$

→ Time scaling $\delta(at) = \frac{1}{|a|}\delta(t)$

* **Relation b/w $u(t)$ & $\delta(t)$**

1) $\int_{-\infty}^{t} \delta(t)\,dt = u(t)$ → $\frac{d}{dt}u(t) = \delta(t)$

2) $\int u(t)\,dt = r(t)$ → $\frac{d}{dt}r(t) = u(t)$

$\delta(t) \xrightarrow{\text{integral}} u(t) \xrightarrow{\text{integral}} r(t)$

$r(t) \xrightarrow{\text{derivative}} u(t) \xrightarrow{\text{derivative}} \delta(t)$

$\{ r(t) = t\cdot u(t) \}$

$u(t-t_0) \rightarrow$ Right shift ($t_0$ units)

$u(t+t_0) \rightarrow$ left shift ($t_0$ units)

* To ~~draw~~ write equation of (combination of unit signals) calculate amplitudes & multiply with $u(t)$ with amplitude change at their respective time (point) & add them

* To write equation of (combination of ramp-signals) calculate slopes & multiply $r(t)$ with slope change at their respective time (point) & add them.

$$\left.\begin{aligned}
&\rightarrow r[n] = u[n] \cdot n \\
&\rightarrow u[n] = \sum_{k=0}^{\infty} \delta[n-k] \\
&\rightarrow \delta[n] = u[n] - u[n-1] \\
&\rightarrow u[0] = 1
\end{aligned}\right\} \text{Discrete signal relations}$$

* Transformation of signals { convert * transform $x(\alpha t+\beta)$ }

Method-1 Time scaling → Time shifting

Method-2 Time shifting → Time Scaling

m-1 * $x(t) \xrightarrow[\text{Time scaling}]{\alpha} x(\alpha t) \xrightarrow[\beta/\alpha]{\text{Time shift}} x(\alpha(t+\beta/\alpha))$

m-2 $x(t) \xrightarrow[\text{Time shift}]{\beta} x(t+\beta) \xrightarrow[\text{Time scale}]{\alpha} x(\alpha t+\beta)$

Total 5 classification

* If Energy is finite, power = 0
* If Energy is infinite, power = finite

} vice versa.

**Energy** $\lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 \, dt$ (or) $\lim_{N \to \infty} \sum_{n=-N}^{N} |x[n]|^2$

continuous  discrete

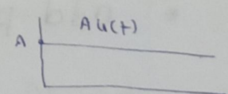
**Power** $\lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$ (or) $\lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$

**cases**

**case-1** :: If $t \to \infty$ or $t \to -\infty$ and amplitude tends to zero, then it is energy signal

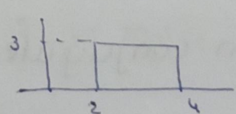
{ It should also have finite amplitude }

**case-2** :: Signal having constant amplitude over infinite duration is power signal. Ex




**case-3** :: All periodic signals are power signals

**case-4** : Signal having finite amplitude for finite length is a energy signal Ex




**case-5** :: If $T \to \infty$ then Amplitude $\to \infty$ or

If $T \to -\infty$, then amplitude $\to \infty$ (or)

If $T \to 0$, then amplitude $\to \infty$, then

The signals are neither energy nor power signals

## Energy formulas,

If the energy of $x(t) \to E$, then

- $x(-t) \to E$
- $-x(t) \to E$
- $x(t \pm T) \to E$
- $x(at) \Rightarrow \frac{E}{|a|}$
- $ax(t) = a^2 E$
- $x(at-b) = \frac{E}{a}$

* Even signal if $x(t) = x(-t)$

* odd signal if $x(t) = -x(-t)$

* Even path ~~or~~ ~~symmetric path~~ $= \frac{x(t) + x(-t)}{2}$ $x_e(t)$ (or) $\frac{x[n] + x[-n]}{2}$

* odd path or anti ~~symmetric path~~ $= \frac{x(t) - x(-t)}{2}$ $x_o(t)$ (or) $\frac{x[n] - x[-n]}{2}$

* Even conjugate, if $x[n] = x^*[-n]$

* odd conjugate, if $x[n] = -x^*[-n]$

* Even conjugate path $x_{ec}[n] = \frac{x[n] + x^*[-n]}{2}$

* odd conjugate path $x_{oc}[n] = \frac{x[n] - x^*[-n]}{2}$

* Symmetric path $\to$ even path

* Anti symmetric path $\to$ odd odd path

- Period signal, if $x(t) = x(t \pm T)$ {T = Time period}

* Steps to find period of sum of signals:

consider signal $x_1(t) + x_2(t) + x_3(t) + \dots$

1) calculate individual time periods, $T_1, T_2, T_3, \dots$

2) calculate $\frac{T_1}{T_2}, \frac{T_1}{T_3}, \frac{T_1}{T_4}, \dots$

3) If ratioes of step 2 are rational, then $x(t)$ is periodic

4) If the signal is periodic, find lcm of denominators of step 2 ratioes

{continuous}

5) Time period $T = LCM \times T_1$

- Period signal, if $x[n] = x[n \pm N]$

* calculate the ratio $\frac{\omega_0}{2\pi} = \frac{M}{N}$

* If the ratio is rational, then it is periodic and N is the time period

* For periodicity of sum of 2 or more discrete time signals,

calculate N for each signal, ie $N_1, N_2, N_3 \dots$

then LCM of $N_1, N_2, N_3 \dots$ is the periodicity

## General forms

1) $\cos\omega_0 t$, where $\omega_0 = \frac{2\pi}{T}$

2) $\sin\omega_0 t$, where $\omega_0 = \frac{2\pi}{T}$

3) $e^{j\omega_0 t} = \cos\omega_0 t + j\sin\omega_0 t$

4) $|e^{j\omega_0 t}| = \sqrt{\cos^2\omega_0 t + \sin^2\omega_0 t} = 1$

5) $e^{-j\omega_0 t} = \cos\omega_0 t - j\sin\omega_0 t$

6) $|e^{-j\omega_0 t}| = 1$

Ex: $e^{j10t}$; then $\omega_0 = 10$ { from $e^{j\omega_0 t}$ }

7) $\sin A \cdot \cos B = \sin(A+B) + \sin(A-B)$

* $x = a+jb$, then $x^* = a-jb$ {conjugate}

* If we asked to calculate time period of signal $x[n]$ which is in the form of summation, then put the values $-1, 0, 1$ in summation variable and determine time period.

* $\sum_{k=0}^{n} 1 = n+1$ $\left\{ \sum_{k=0}^{n} k^0 = \underbrace{0^0 + 1^0 + 2^0 + \cdots n^0}_{n} \right\}$ similarly

* $\sum_{n=-N}^{N} 1 = 2N+1$

* $\sum_{n=-N}^{N} 3 = 3 \sum_{n=-N}^{N} 1 = 3[2N+1]$

Unit - 2

* 2 types of systems

1) continuous time systems (continuos time inputs gives CT output

2) Discrete time systems (Discrete time input gives DT output)

* 6 classifications of systems:

1) Linear & non linear systems {additivity & scaling property}

* If continuous consider $x(t) = \alpha x_1(t) + \beta x_2(t)$

* If discrete, consider $x[n] = x_1[n] + x_2[n]$ and also

If $x[n]$ is in power

2) Time invariant & Time variant systems

* Shift $x(t)$ with $t_0$ units $\rightarrow x(t - t_0)$

* Replace $t$ with $t_0$ { Delay input gives delay output }

3) Causal & non-causal system.

[Depends on present and past input & not on future]

[also time (t) in $x(t)$ & $y(t)$ should be same]

4) Stable & unstable system

Bounded input gives bounded output

5) Static and dynamic systems

[Time (t) in $x(t)$ & $y(t)$ should be same]

6) Invertable & non invertable systems

different inputs givens different output.

## Convolution:

### Continuous signals convolution:

$$y(t) = x(t) * h(t)$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau)\, d\tau$$

### Graphical method

1) Calculate limits of $y(t)$

i.e Sum of lower limits of $x(t)$ & $h(t)$ $< t <$ Sum of upper limits of $x(t)$ & $h(t)$

2) Convert 't' domain to '$\tau$' domain $(t \to \tau)$

3) Time reversal of $h(\tau)$ i.e obtain $h(-\tau)$.

This is known as folding or flipping operation.

4) Shifting 't' units of $h(-\tau)$ i.e obtain $h(t-\tau)$

5) multiply $x(\tau)$ & $h(t-\tau)$ and use formula

$$\int_{-\infty}^{\infty} x(\tau) h(t-\tau)\, d\tau$$

* Properties of convolution:

1) $x(t) * h(t) = h(t) * x(t)$ {commutative property}

2) $x(t) * [h_1(t) + h_2(t)] = x(t) * h_1(t) + x(t) * h_2(t)$

{Distributive property}

3) $(x(t) * h_1(t)) * h_2(t) = x(t) * (h_1(t) * h_2(t))$

{Associative property}

* $x(t) * h(t-t_0) = y(t-t_0)$

$x(t-t_0) * h(t) = y(t-t_0)$ } Time shifting property

$x(t-t_1) * h(t-t_2) = y(t-t_1-t_2)$

* $\frac{d}{dt} x(t) * h(t) = \frac{d}{dt} y(t)$

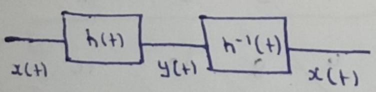
$x(t) * \frac{d}{dt} h(t) = \frac{d}{dt} y(t)$ } Differentiation property

→ In an LTI system

$h[n] * h^{-1}[n] = \delta[n]$

* Invertibility of LTI system.

A system is invertable when connected in series with the original system produces an output eqiual to the input.

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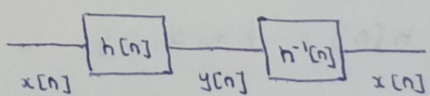


$h[n] * h^{-1}[n] = \delta[n]$

* We know $y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$

* System is invertable, if "$h(t) = 0$" or "$h[n]=0$" {n & t<0}

* System is stable, if $\sum_{k=-\infty}^{\infty} |h[k]| < \infty$ or $\int_{-\infty}^{\infty} h(t)\,dt < \infty$

* Convolution in discrete.

$$x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

| | continuous LTI | Discrete LTI |
|---|---|---|
| Property | $h(t) = 0, t < 0$ | $h[n] = 0, n < 0$ |
| Stability | $\sum_{k=-\infty}^{\infty} \lvert h(k) \rvert < \infty$ | $\int_{-\infty}^{\infty} h(t)dt < \infty$ |
| Inequality | $h(t) * h_1(t) = \delta(t)$ | $h[n] * h_1[n] = \delta[n]$ |
| memoryless /static | $h(t) = 0, t \neq 0$ $= k\delta(t)$ | $h[n] = 0, n \neq 0$ $= k\delta[n]$ |

* A system is causal if it lies only on +ve side of t or n
* A system is stable if amplitude is finite on every time interval.
* $\sum_{k=0}^{n} a^k = \left(\frac{1-a^{n+1}}{1-a}\right)$
* $x[n] = \{1, 2, 3, 4\}$

$h[n] = \{4, 5, 6\}$

$x[n] * y[n] =$

$\{4, 13, 28, 43, 38, 24\}$

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 4 | 4 | 8 | 12 | 16 |
| 5 | 5 | 10 | 15 | 20 |
| 6 | 6 | 12 | 18 | 24 |

* $S(t) = \int_{-\infty}^{t} h(t)\,dt$
* $h(t) = \frac{d}{dt} S(t)$

where $S(t)$ = Step response

And $h(t)$ = impulse response

Similarly $S[n] = \sum_{k=-\infty}^{n} h[k]$