Mechanics of Materials-II

ANALYSIS OF CYLINDERS

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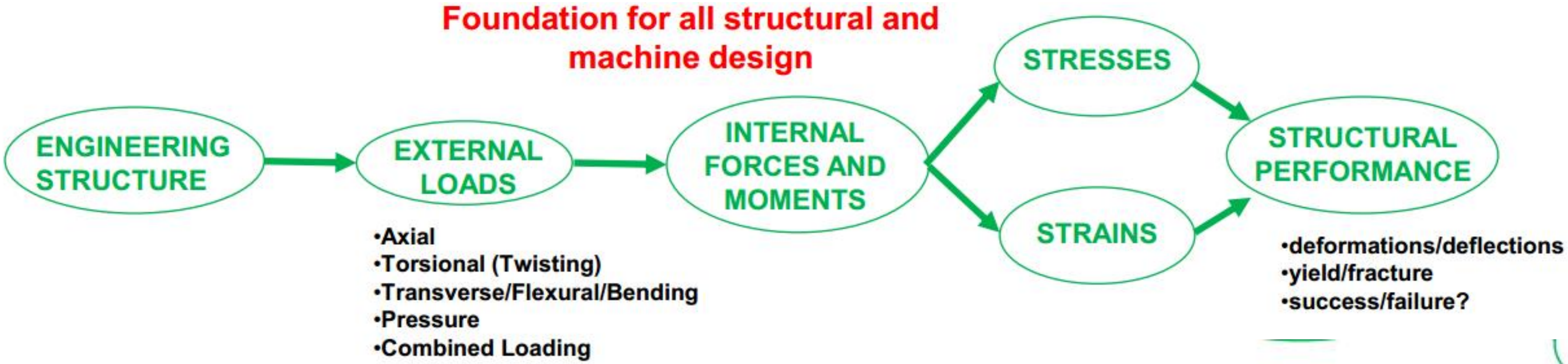
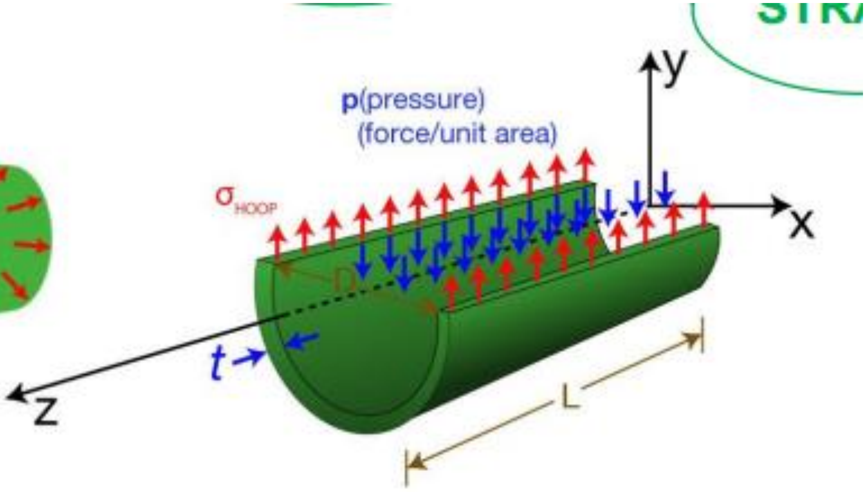
MOM (Course Outcomes)





Cylinders: Outcome

- Types of pressure vessels
- Stresses in Thin and Thick wall Pressure vessels
- Strain and change in deformation pressure vessels due to internal pressure
- Lame’s Equation in Thick Pressure vessels

Commonly used cylinders (transporting or Storage of liquids, gases of fluids)

1. Water tank
2. Gas Cylinder
3. Oxygen Cylinders

Cylinders: Applications

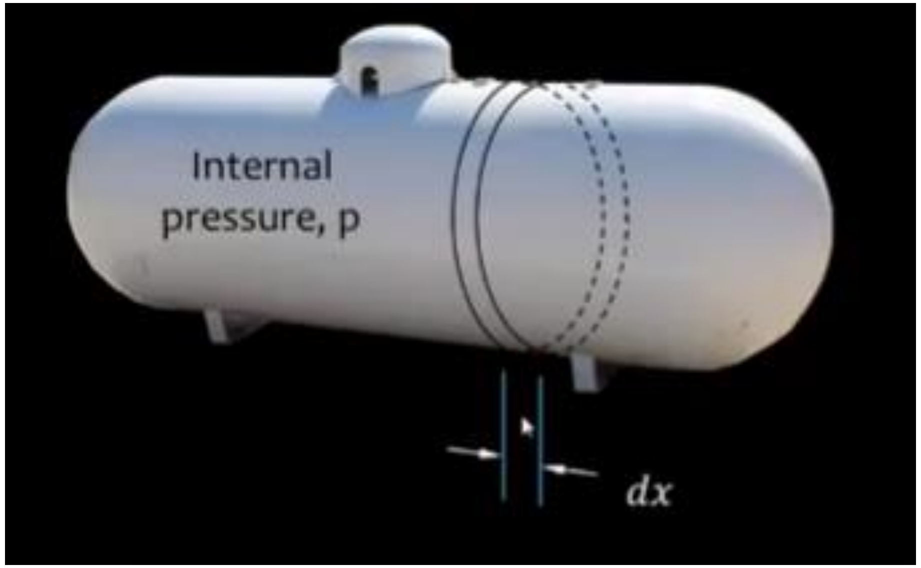
Cylinders: Types

Pressure Vessels

Thin Pressure Vessels
$[^{D}/_{t} \geq 20]$

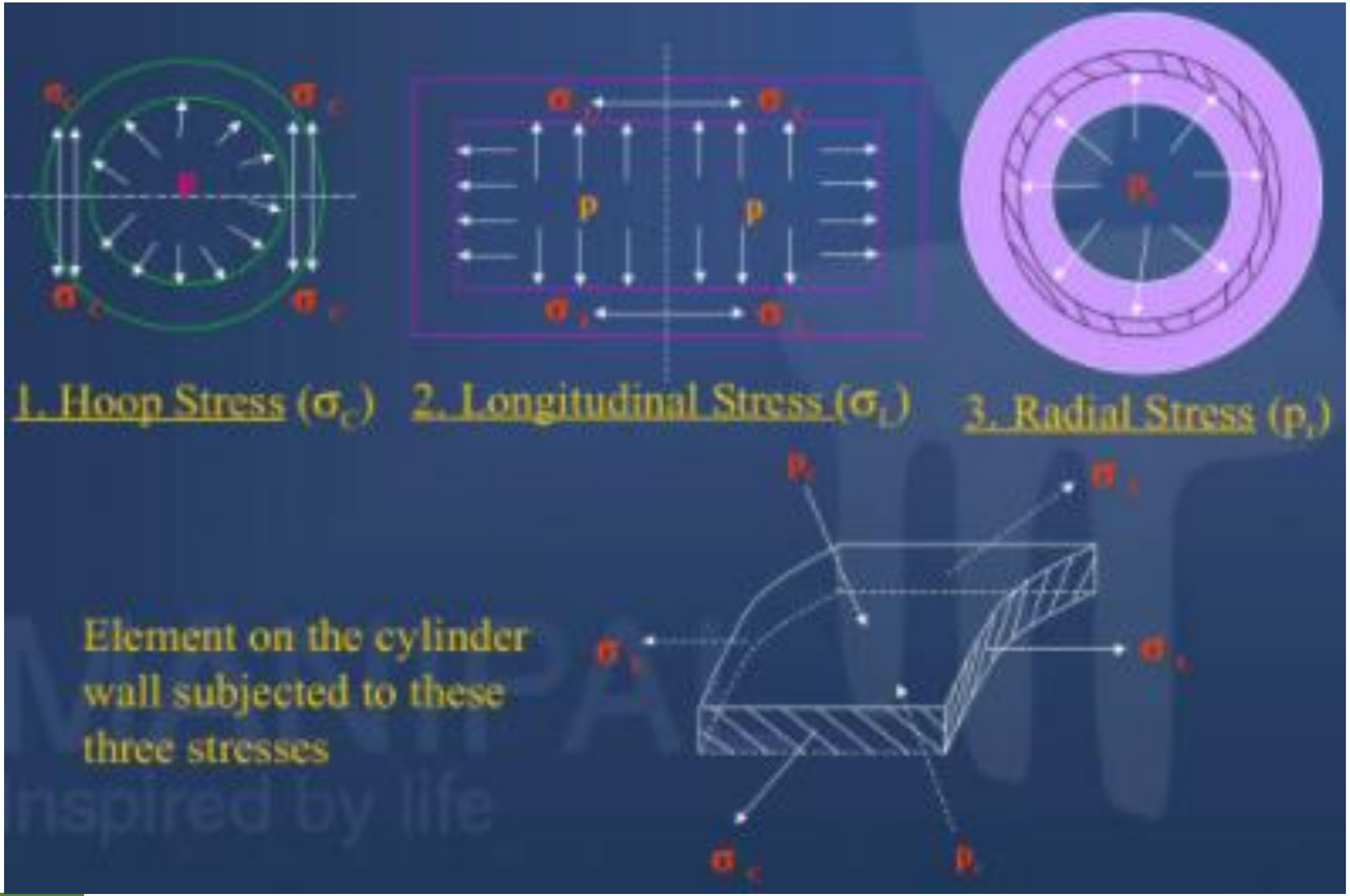
Thick Pressure Vessels
$[^{D}/_{t} < 20]$

1. **Boilers**
2. **Gas Storage tanks**
3. **Blimps**
4. **Pipelines**



1. **Gun Barrels**
2. **Submarine**
3. **Liquid/gas carrying pipe**
4. **High pressure pipes**

Cylinders: Stresses

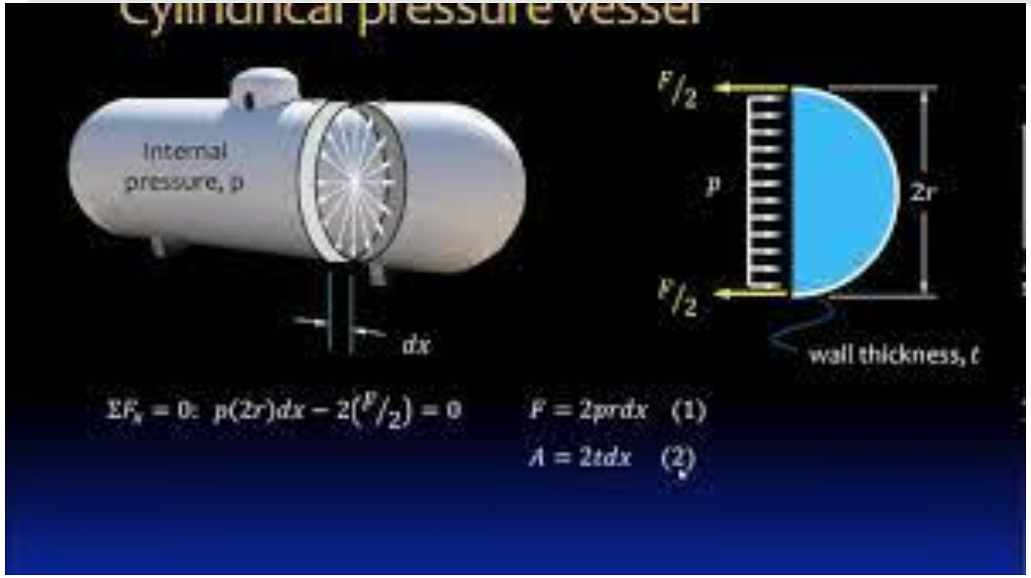


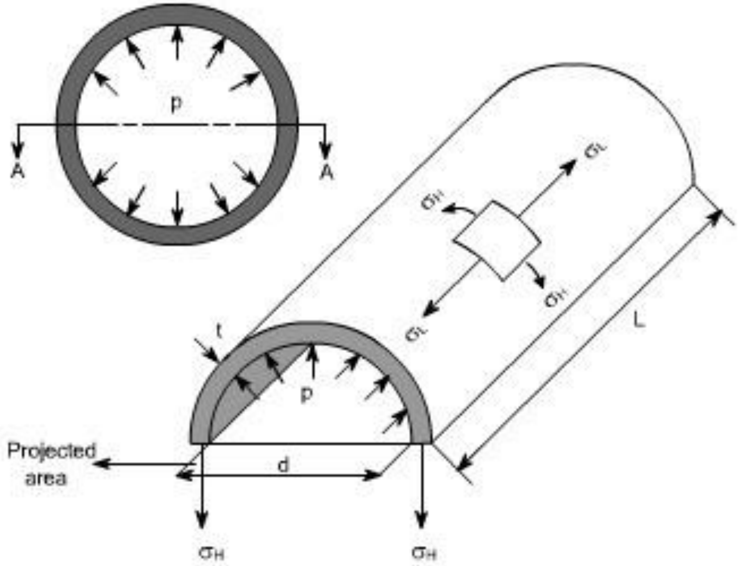
Cylinders: Stresses

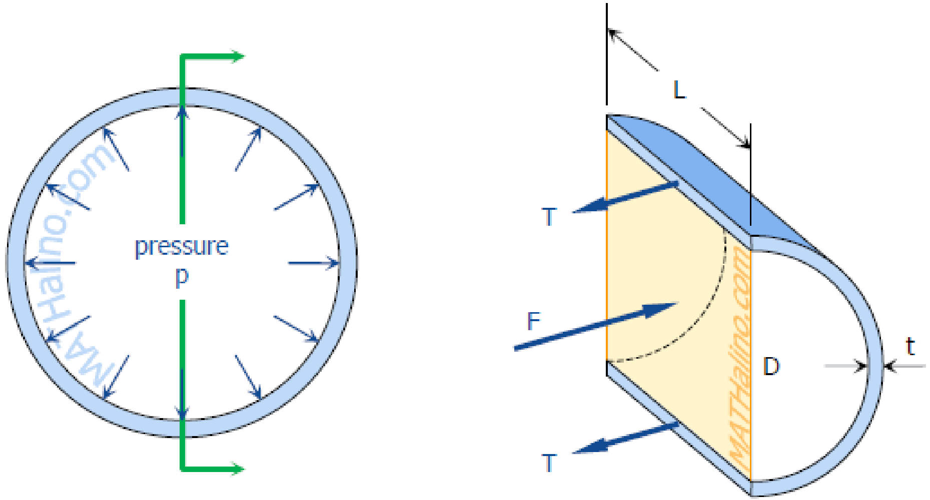
- These cylinders are subjected to fluid pressures. When a cylinder is subjected to a internal pressure, at any point on the cylinder wall, three types of stresses are induced on three **mutually perpendicular planes**

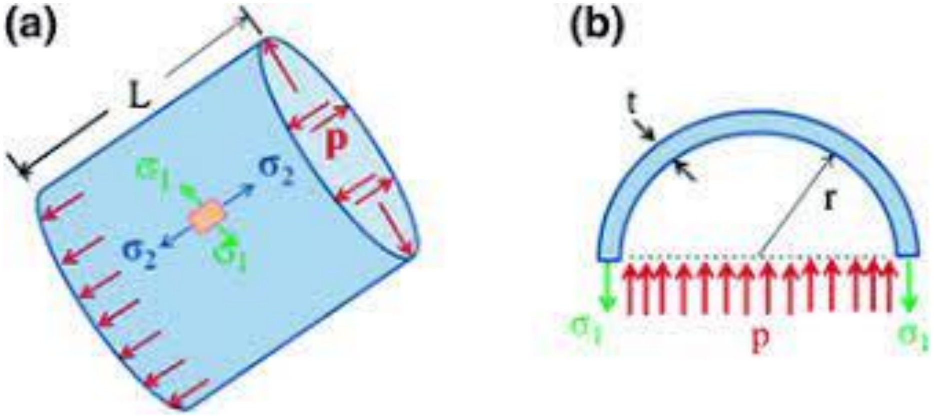
- **Hoop or Circumferential Stress** – This is directed along the tangent to the circumference and **tensile** in nature. Thus, there will be **increase in diameter**
- **Longitudinal Stress:** This stress is directed along the length of the cylinder. This is also **tensile** in nature and tends to **increase the length.**
- **Radial Stress:** It is **compressive** in nature.
  - Its equal to fluid pressure on the inside wall and zero on the outer wall if its open to atmosphere.

Cylinders: Stresses in Thin Cylinders

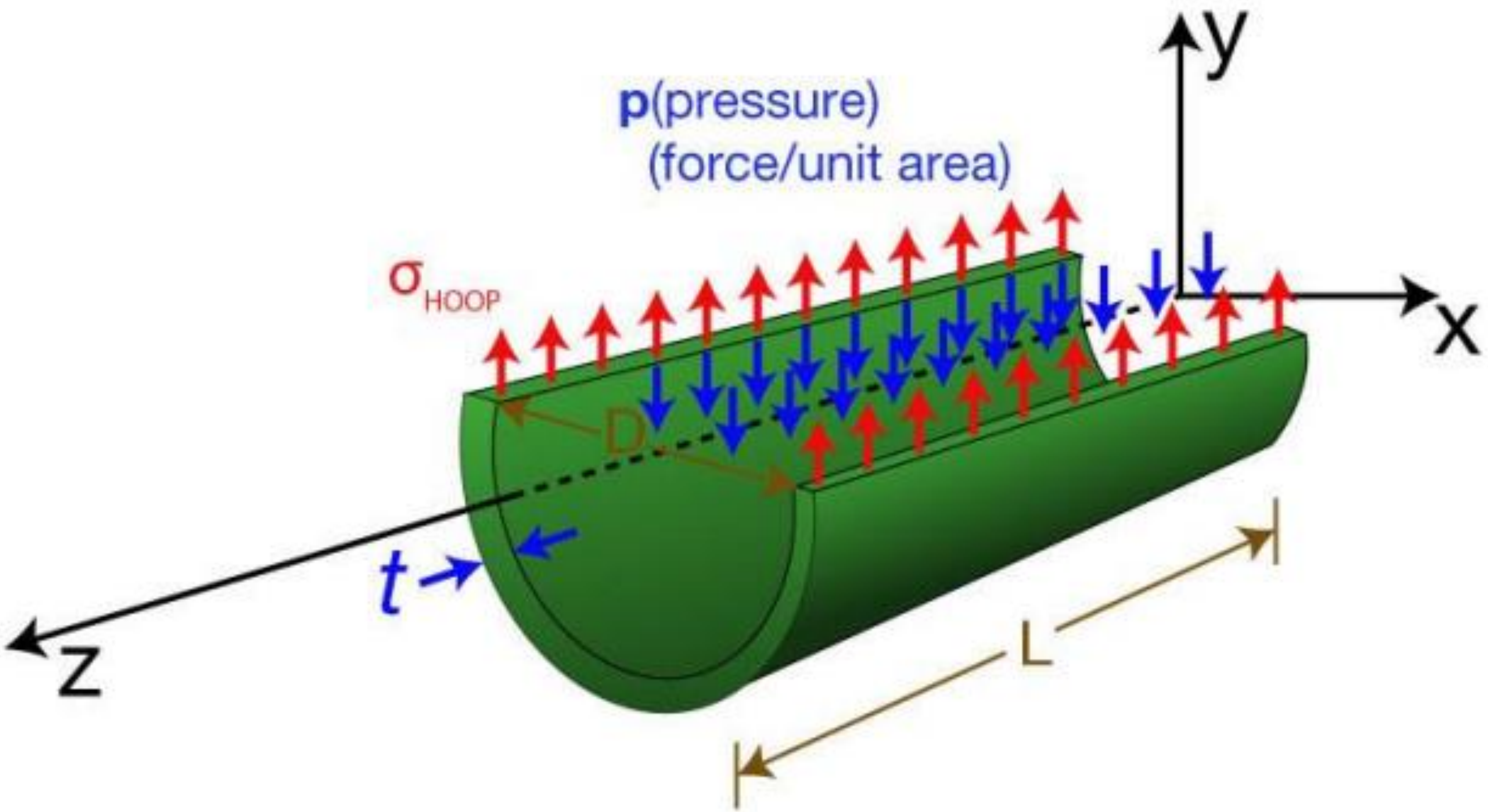




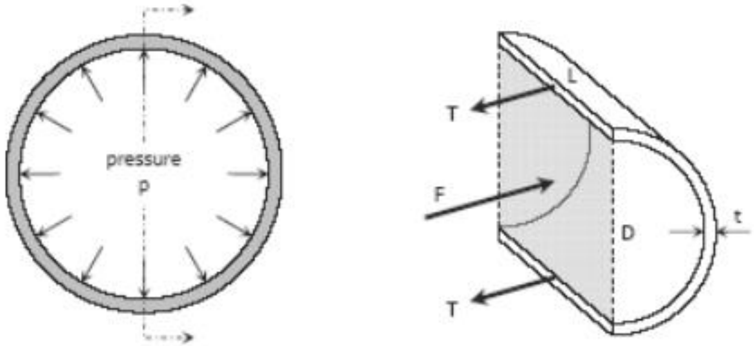




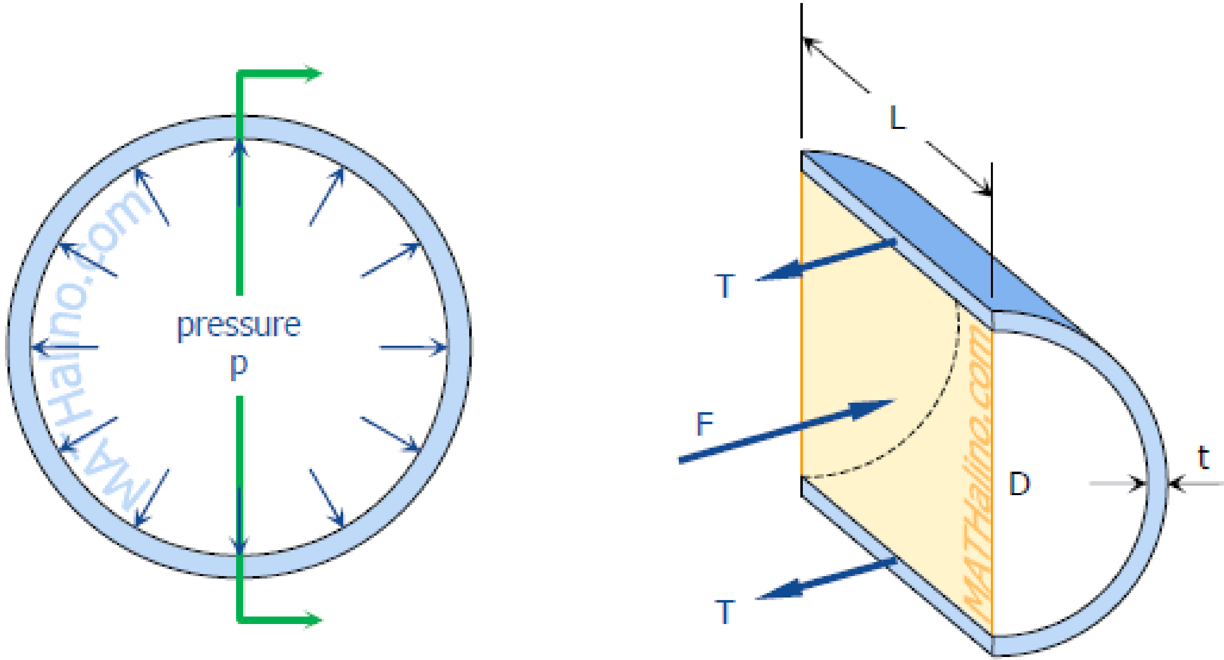
Thin Cylinders: Circumferential/Hoop Stress



Thin Cylinders: Circumferential/Hoop Stress



$$F = pA = pDL$$
$$T = \sigma_t A_{wall} = \sigma_t tL$$
$$[\,\Sigma F_H = 0\,]$$
$$F = 2T$$
$$pDL = 2(\sigma_t tL)$$
$$\sigma_t = \frac{pD}{2t}$$

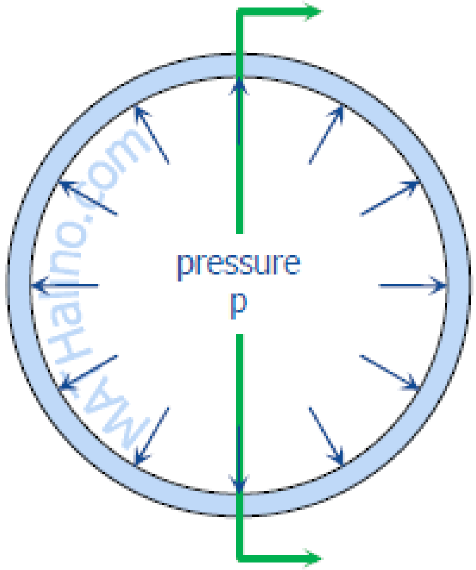


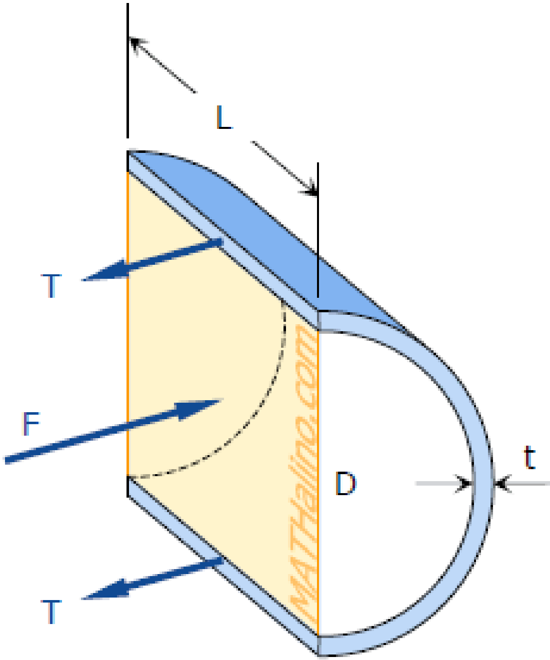
If there exist an external pressure p_o and an internal pressure p_i, the formula may be expressed as:

$$\sigma_t = \frac{(p_i - p_o)D}{2t}$$

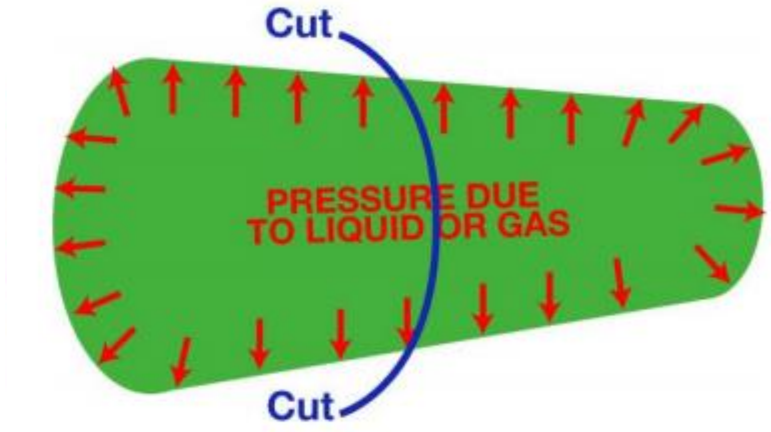
Thin Cylinders: Circumferential/Hoop Stress

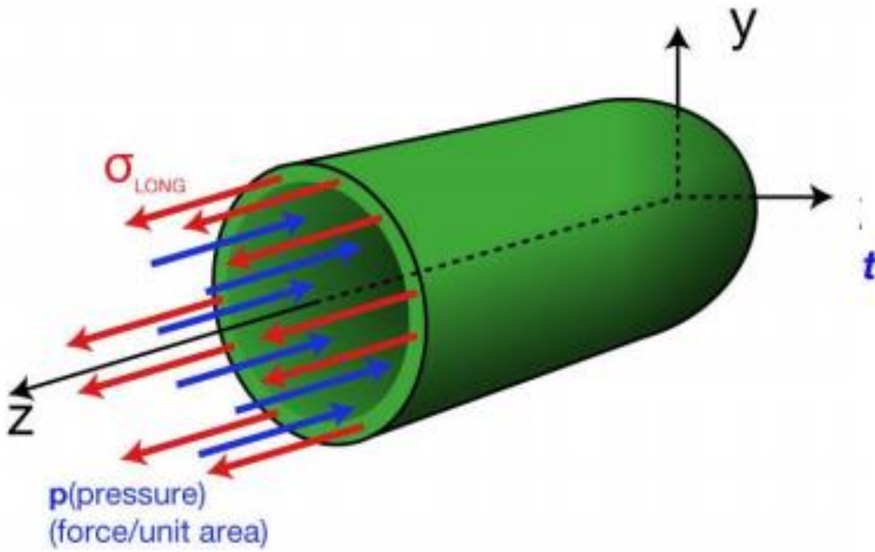
$$\sigma_1 = \frac{pr}{t}$$

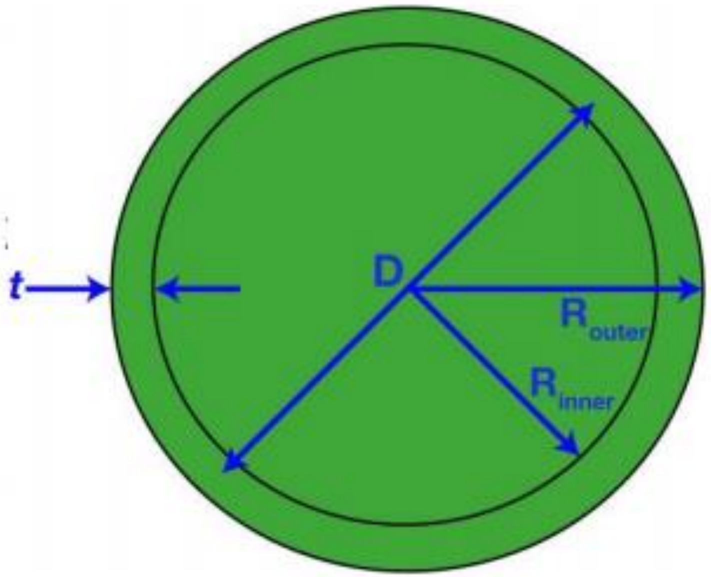




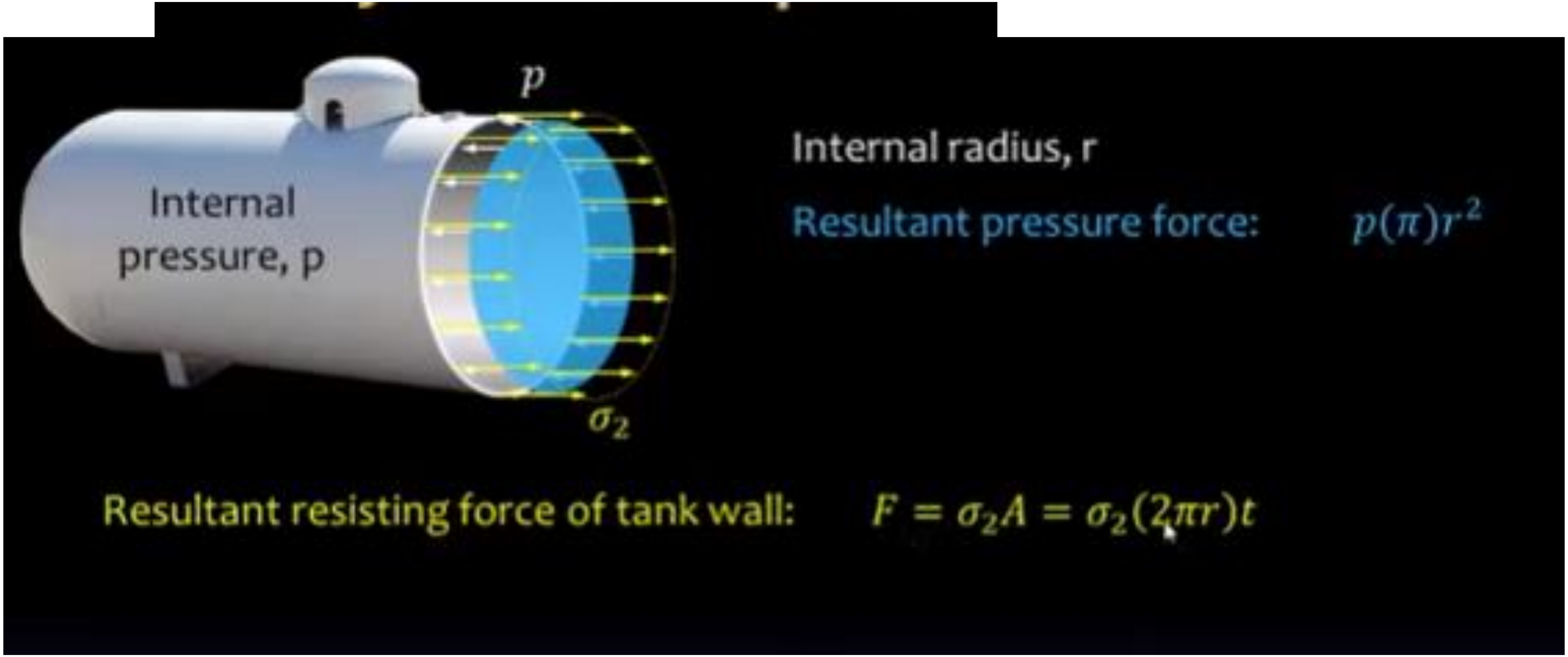
Thin Cylinders: Longitudinal /Axial Stress







Thin Cylinders: Longitudinal /Axial Stress



Thin Cylinders: Longitudinal /Axial Stress

$$F = pA = p\frac{\pi}{4}D^2$$

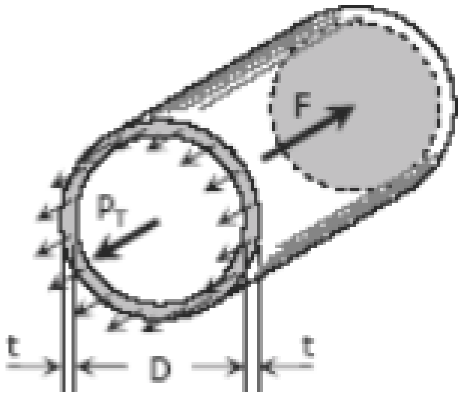
$$P_T = \sigma_L \pi D t$$

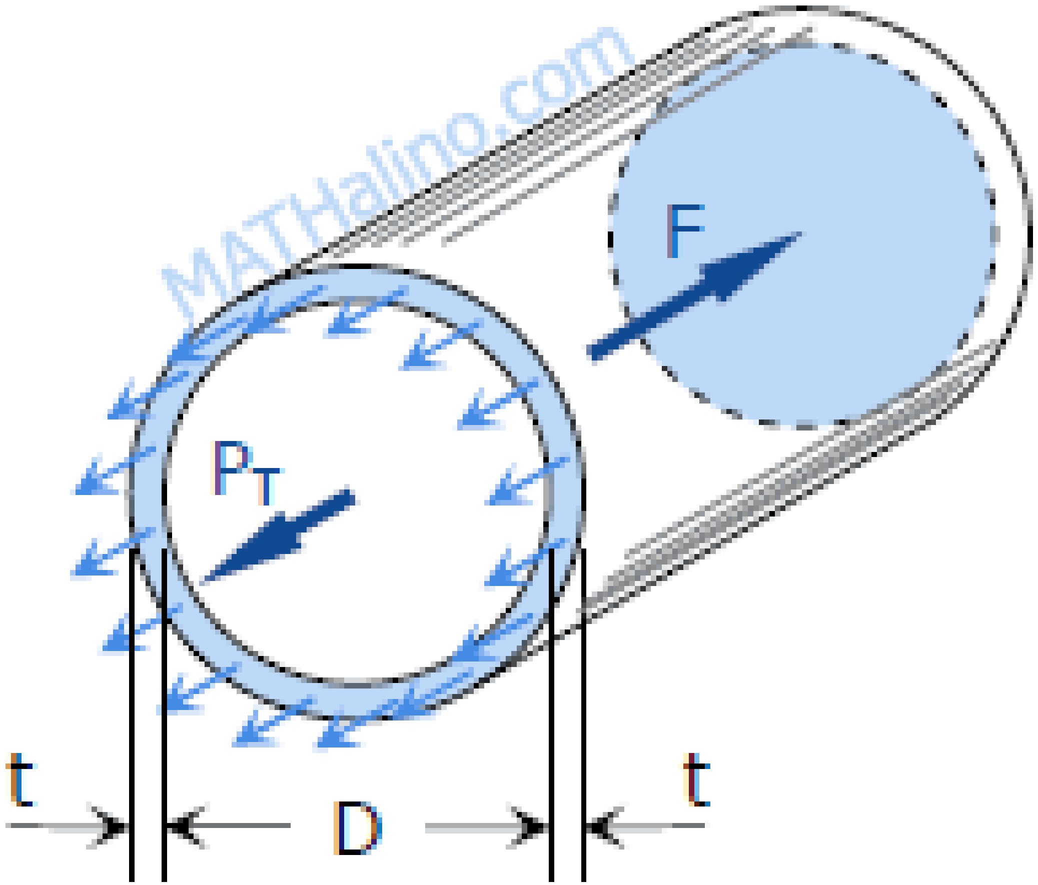
$$[\ \Sigma F_H = 0\ ]$$

$$P_T = F$$

$$\sigma_L \pi D t = p\frac{\pi}{4}D^2$$

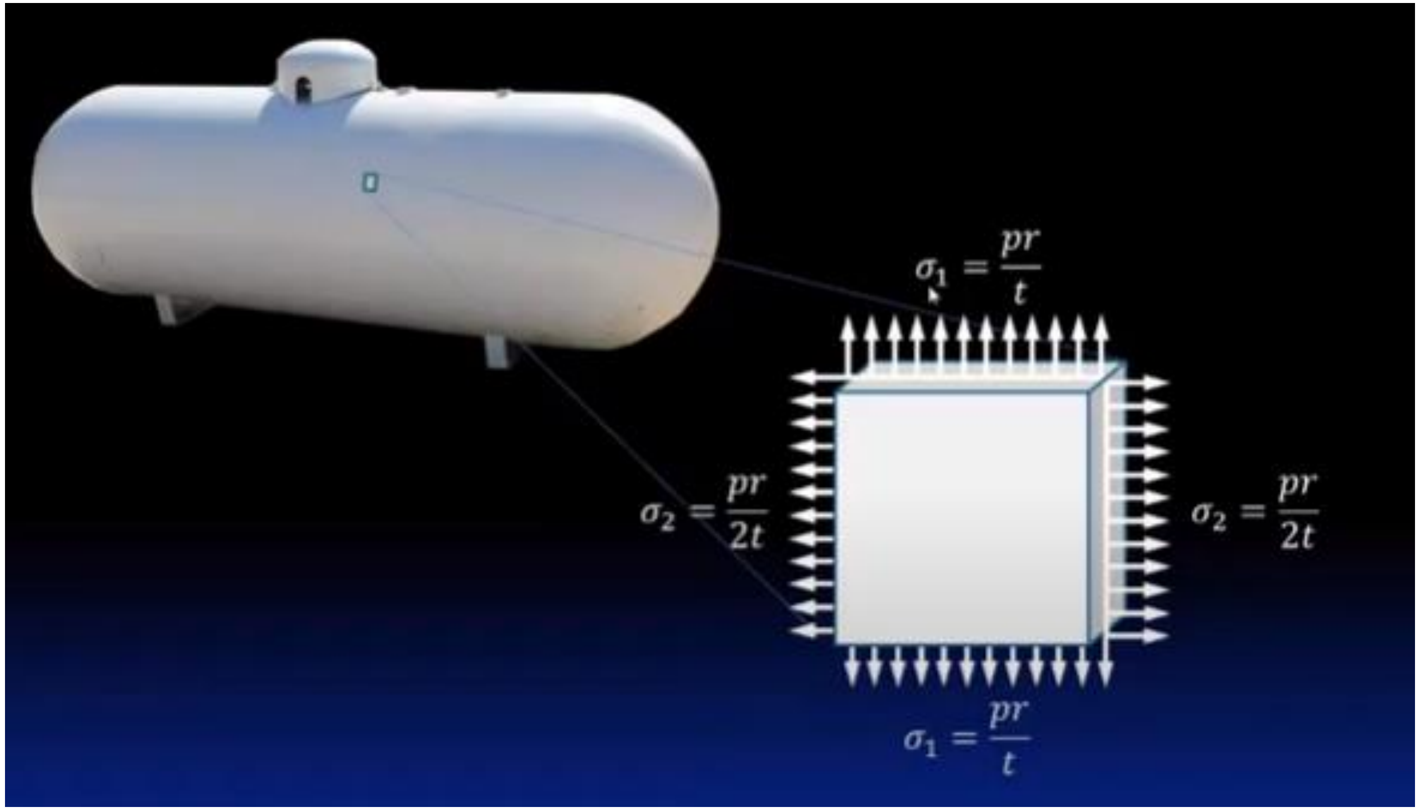
$$\sigma_L = \frac{pD}{4t}$$



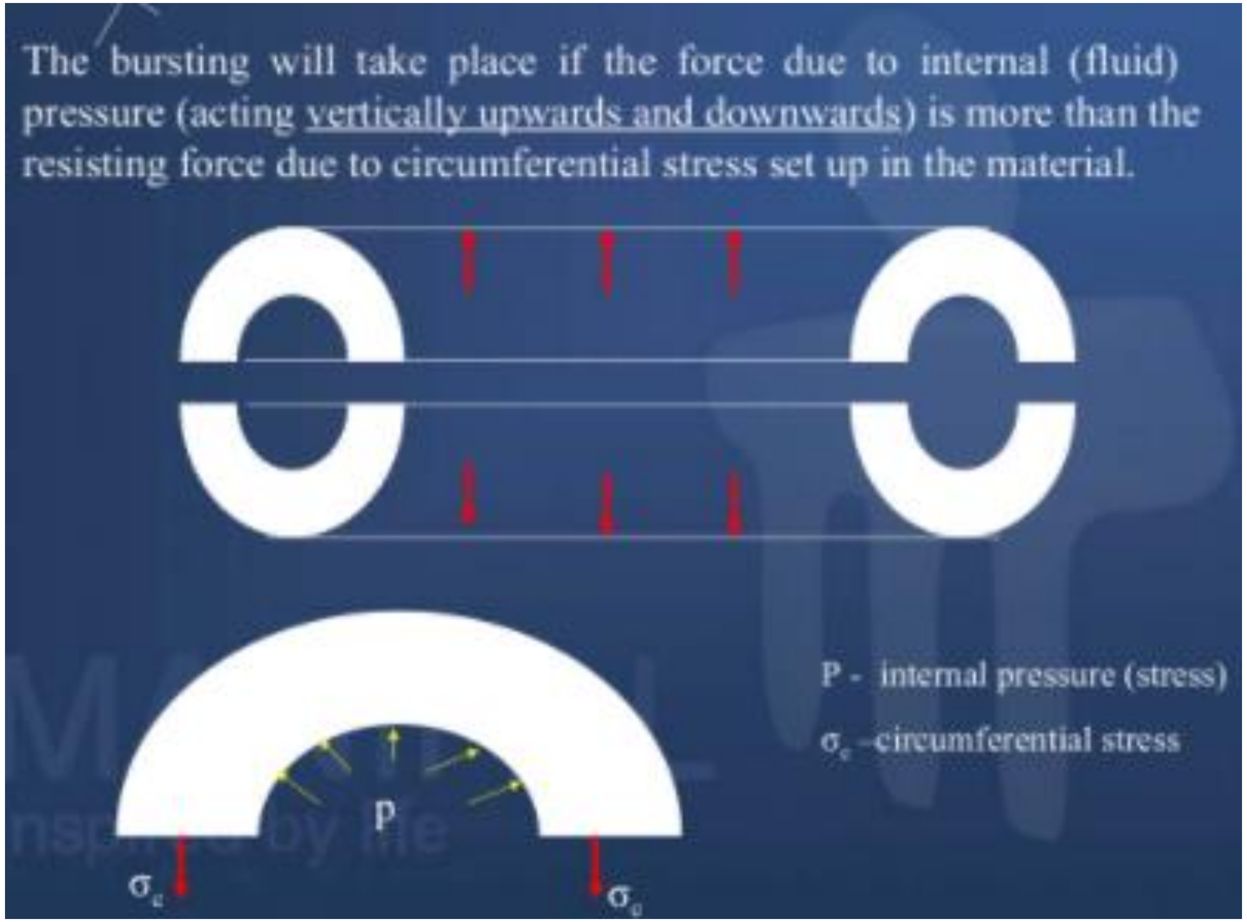


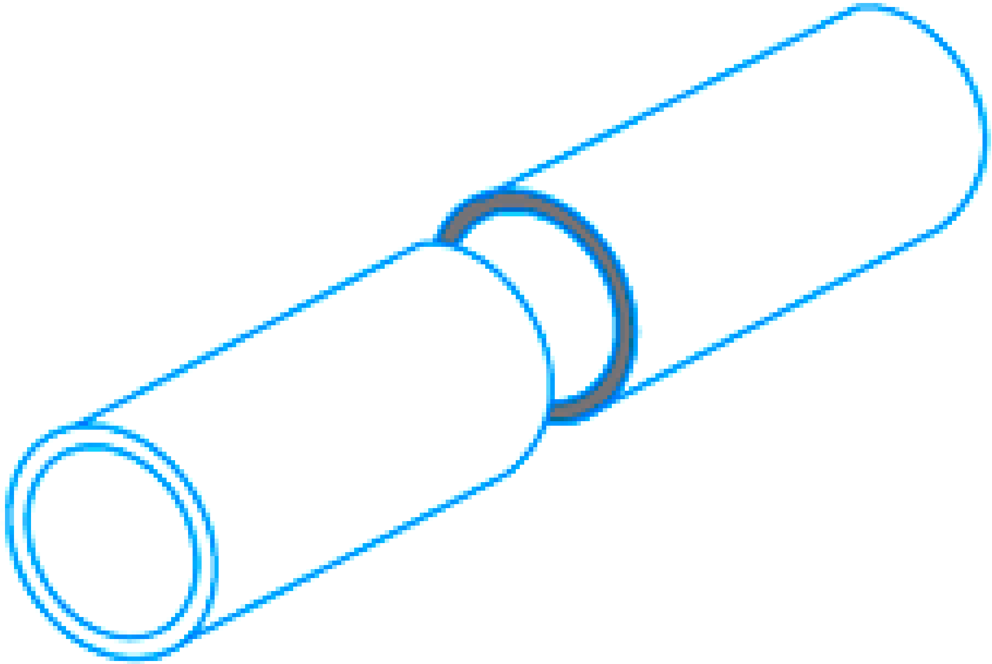
$$\sigma_L = \frac{(p_i - p_o)D}{4t}$$

Thin Cylinders: Longitudinal /Axial Stress



Thin Cylinders: Failures

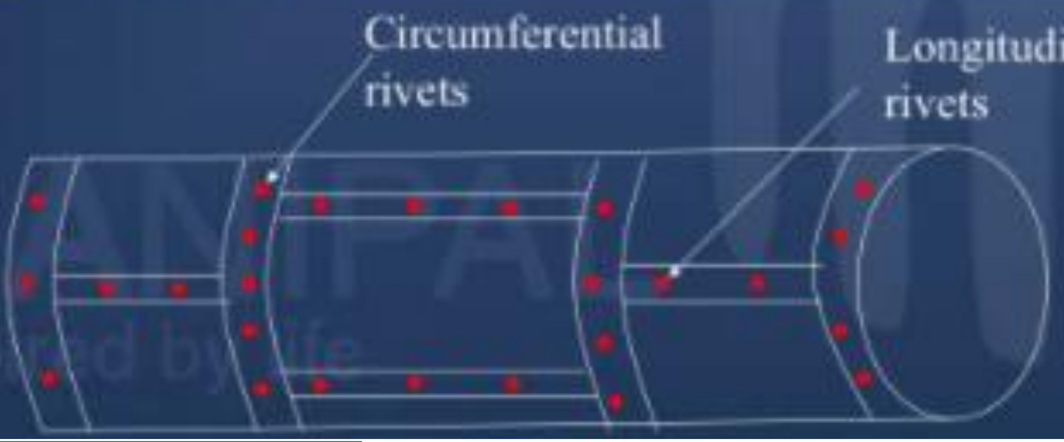




Thin Cylinders: Joint Efficiency

Steel plates of only particular lengths and width are available. Hence whenever larger size cylinders (like boilers) are required, a number of plates are to be connected. This is achieved by using riveting in circumferential and longitudinal directions as shown in figure. Due to the holes for rivets, the net area of cross section decreases and hence the stresses increase.

Note: In longitudinal joint, the circumferential stress is developed and in circumferential joint, longitudinal stress is developed.



Circumferential stress is given by,

$$\sigma_c = \frac{p \times d}{2 \times t \times \eta_L}$$

Longitudinal stress is given by,

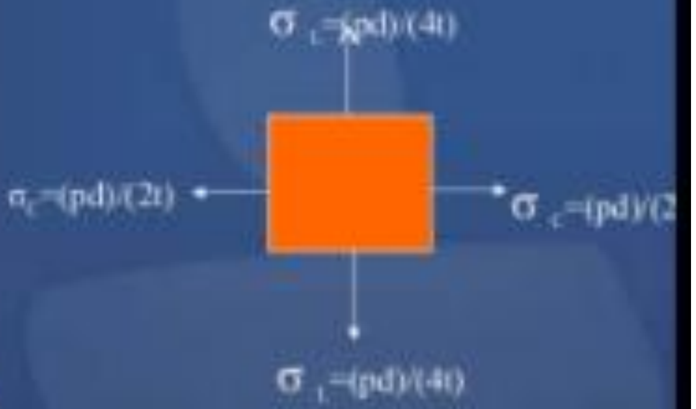
$$\sigma_L = \frac{p \times d}{4 \times t \times \eta_c}$$

Thin Cylinders: Circumferential Strain(Hoop Strain)

$$\varepsilon_C = \frac{\sigma_C}{E} - \mu \times \frac{\sigma_L}{E}$$

$$= 2 \times \frac{\sigma_L}{E} - \mu \times \frac{\sigma_L}{E}$$

$$= \frac{\sigma_L}{E} \times (2-\mu)$$



i.e., $\boxed{\varepsilon_C = \frac{\delta d}{d} = \frac{p \times d}{4 \times t \times E} \times (2-\mu)}$ ....(3)

Note: Let δd be the change in diameter. Then

$$\varepsilon_c = \frac{final\ circumference - original\ circumference}{original\ circumference}$$

$$= \left[\frac{\pi(d+\delta d) - \pi d}{\pi d}\right] = \frac{\delta d}{d}$$

Thin Cylinders: Longitudinal Strain(Axail Strain)

$$\varepsilon_L = \frac{\sigma_L}{E} - \mu \times \frac{\sigma_C}{E}$$

$$= \frac{\sigma_L}{E} - \mu \times \frac{(2\times\sigma_L)}{E} = \frac{\sigma_L}{E} \times (1 - 2\times\mu)$$

i.e., $\varepsilon_L = \frac{\delta l}{L} = \frac{p \times d}{4 \times t \times E} \times (1 - 2\times\mu)$ ....(4)

VOLUMETRIC STRAIN, $\frac{\delta v}{V}$

Change in volume = δV = final volume – original volume

original volume = V = area of cylindrical shell × length

$$= \frac{\pi d^2}{4} L$$

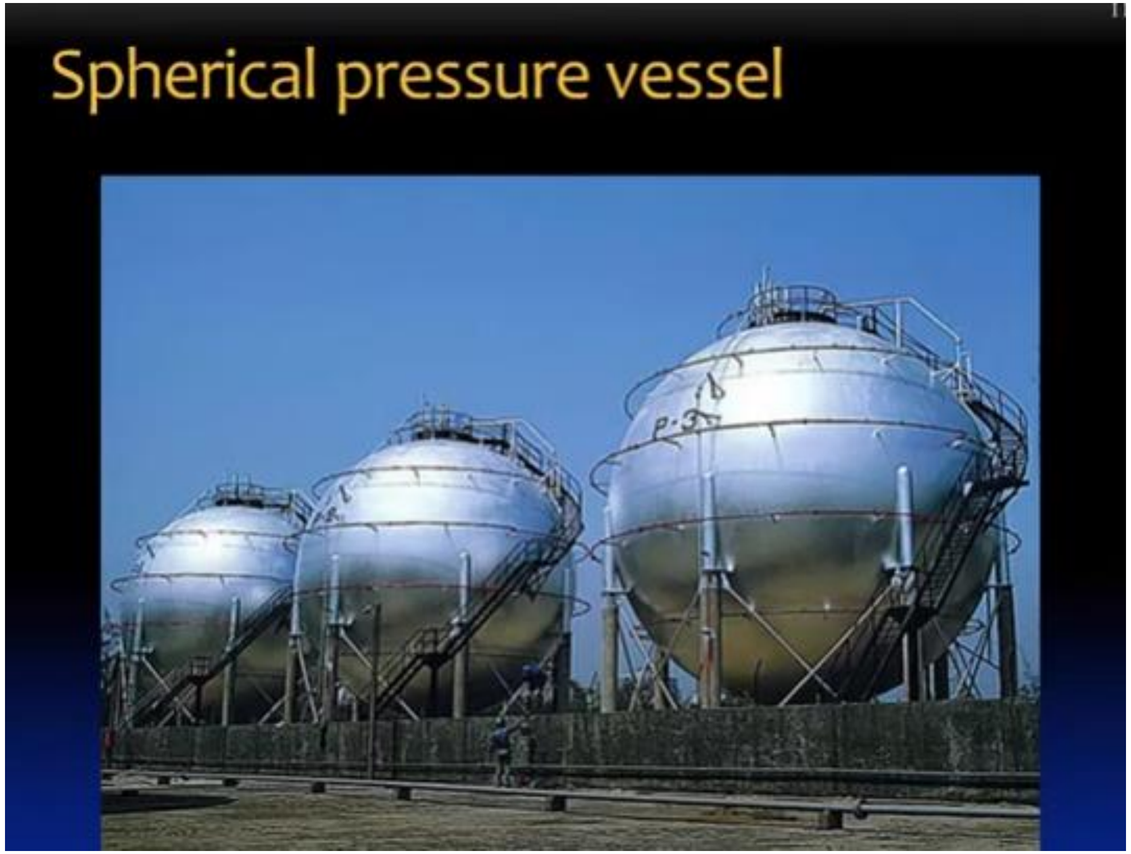
$$= \frac{\delta L}{L} + 2 \times \frac{\delta d}{d}$$

$$\frac{dV}{V} = \varepsilon_L + 2 \times \varepsilon_C$$

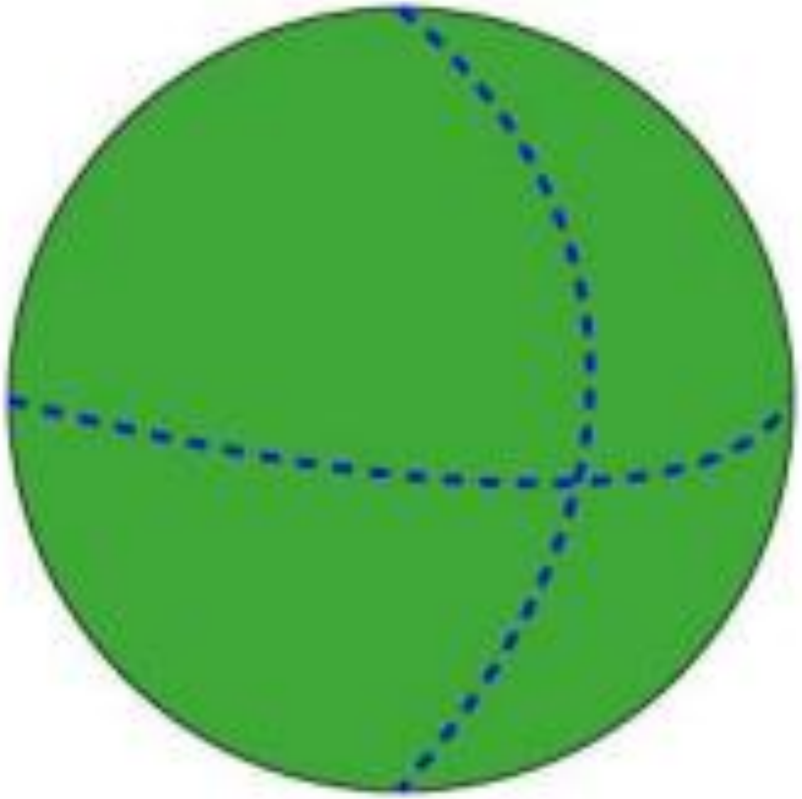
$$= \frac{p \times d}{4 \times t \times E}(1 - 2\times\mu) + 2 \times \frac{p \times d}{4 \times t \times E}(2 - \mu)$$

$\frac{dv}{V} = \frac{p \times d}{4 \times t \times E}(5 - 4\times\mu)$ ....(5)

Thin Spherical Vessel:



Thin Spherical Vessel:



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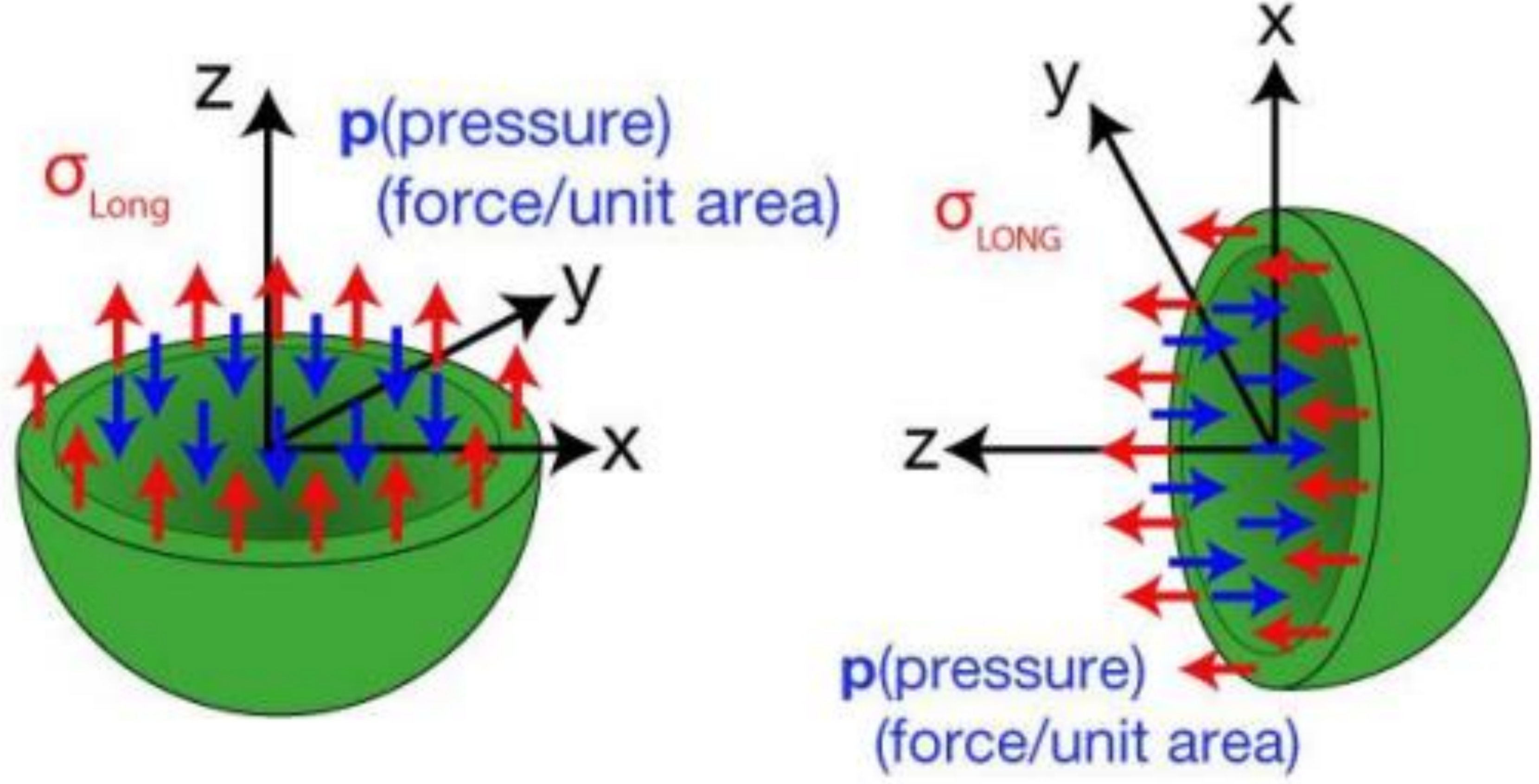
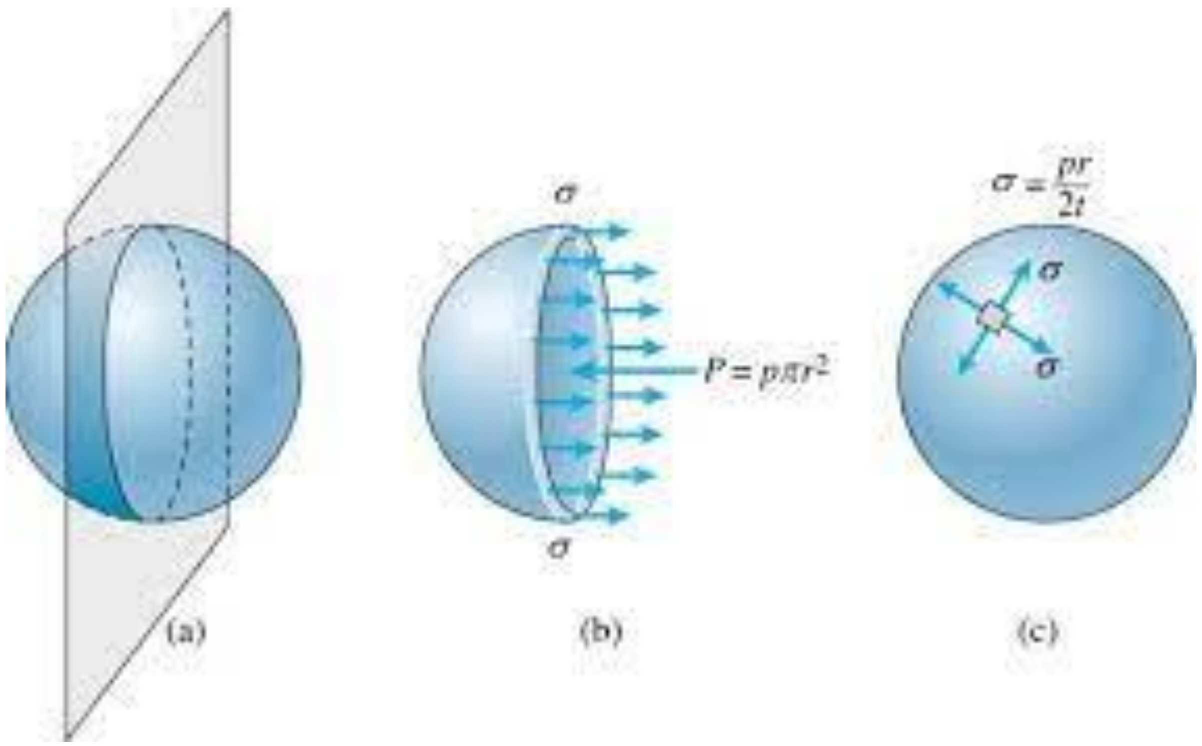
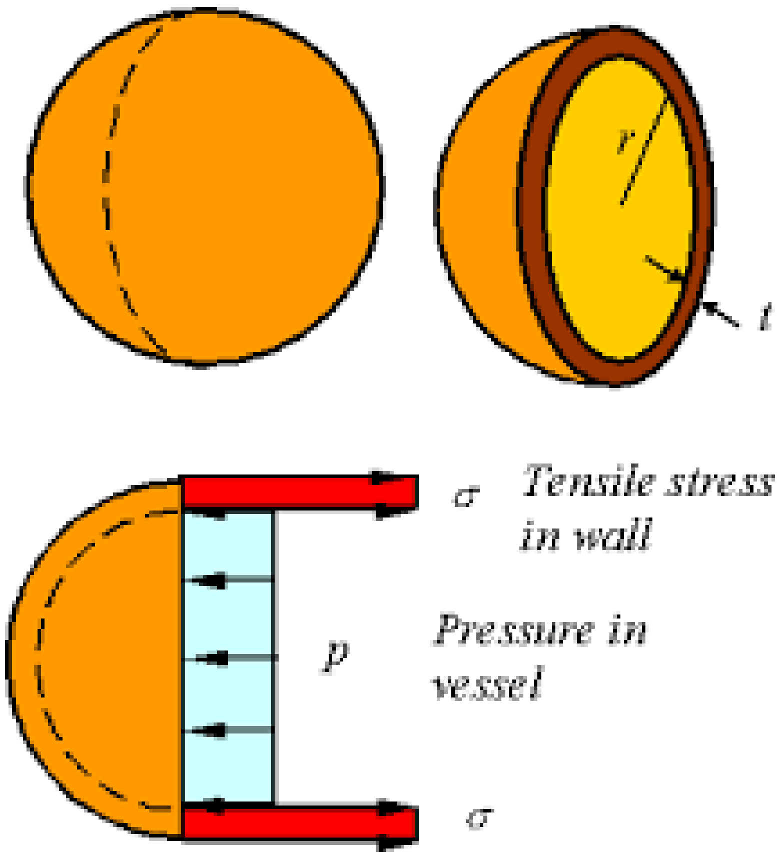


Thin Spherical Vessel:

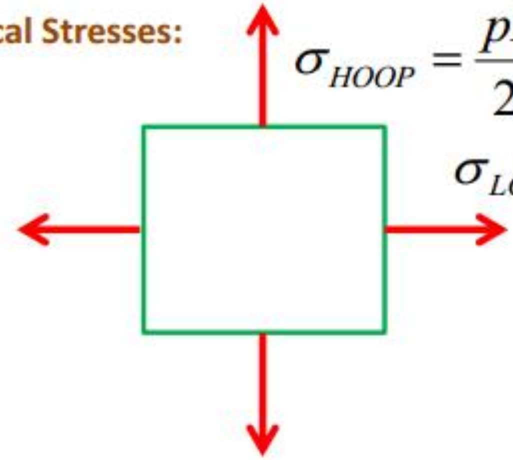




Thin Cylinders/Sphere (Mohr's Circle)

Thin-Walled Pressure Vessels

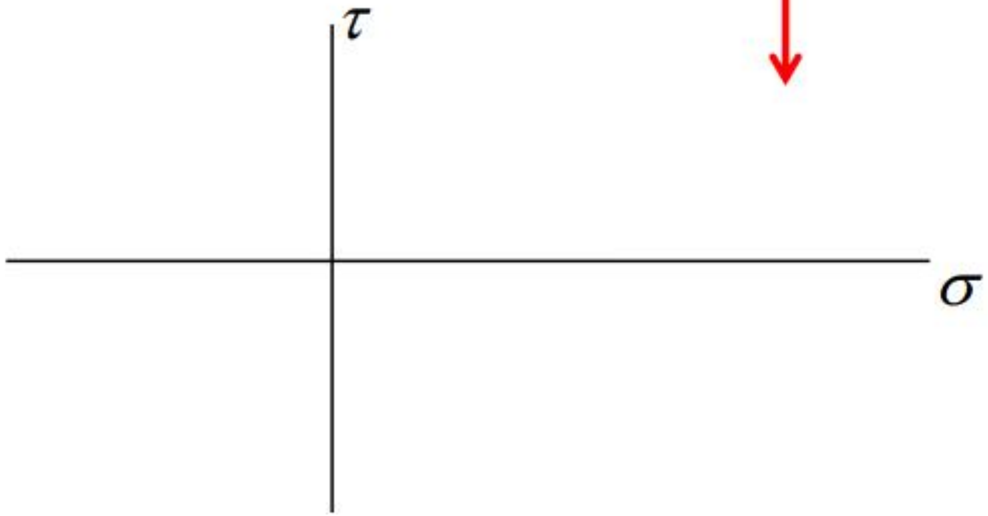
Cylindrical Stresses:



$$\sigma_{HOOP} = \frac{pD}{2t}$$

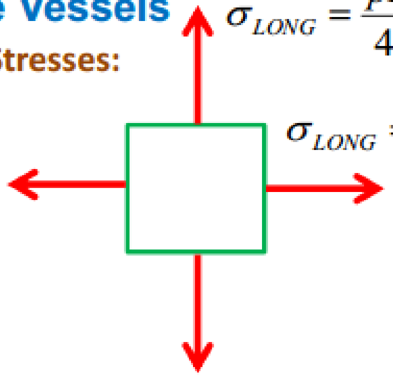
$$\sigma_{LONG} = \frac{pD}{4t}$$

Mohr's circle for Plane Stress



Thin-Walled Pressure Vessels

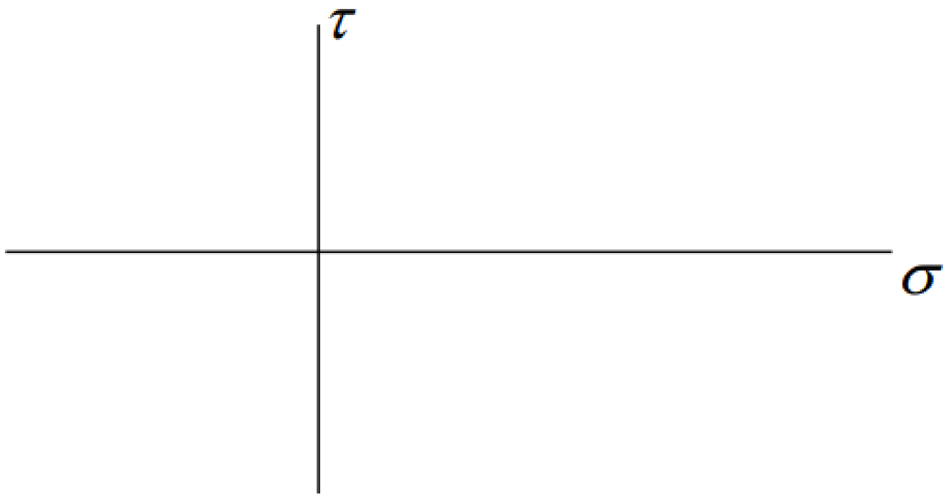
Spherical Stresses:



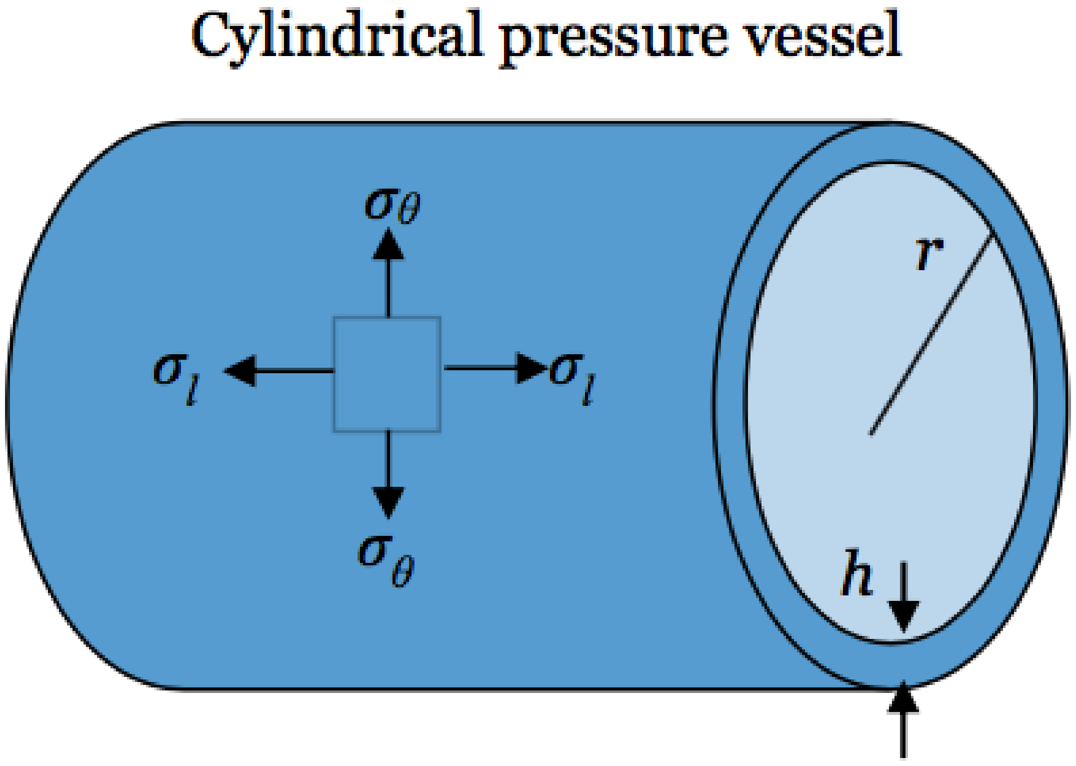
$$\sigma_{LONG} = \frac{pD}{4t}$$

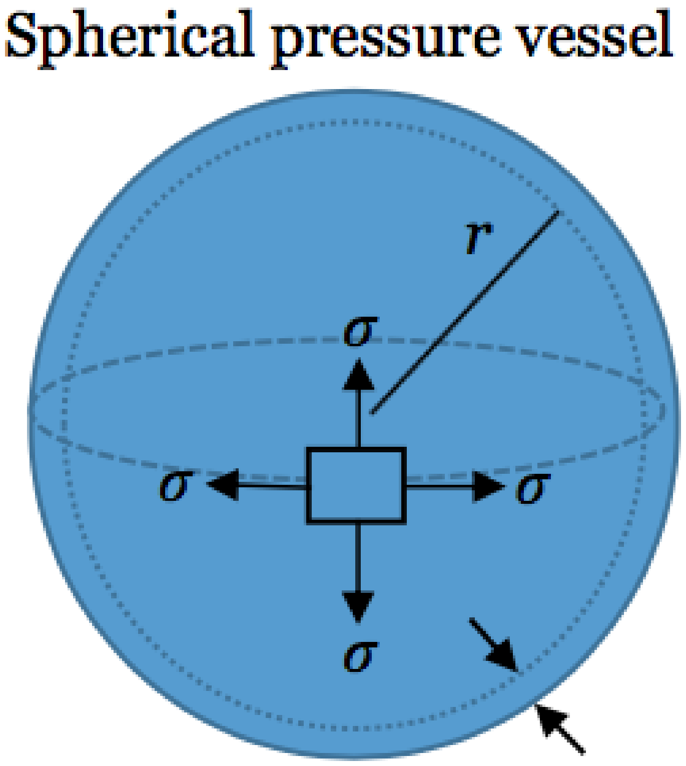
$$\sigma_{LONG} = \frac{pD}{4t}$$

Mohr's circle for Plane Stress



Thin Cylinders/Sphere





Thin Cylinders: Problem-1

A Thin cylindrical shell is 3m long and 1 m in internal diameter. It is subjected to internal pressure of 1.2 MPa. If the thickness of the sheet is 12 mm. Find the stresses in thin shell?

Thin Cylinders: Problem-2

A Thin cylindrical shell is 3m long and 1 m in internal diameter. It is subjected to internal pressure of 1.2 MPa. If the thickness of the sheet is 12 mm. Find the stresses in thin shell, Changes in diameter, length and volume. Take E=200 GPa and poisson’s ratio is 0.3.

Thin Cylinders: Problem-3

Boiler shell is to be made of 15 mm thick plate having a limiting tensile stress of 120 MPa. If the efficiencies of longitudinal and circumferential joints are 70% and 30% respectively. Determine:

(i) The maximum permissible diameter of the shell for an internal pressure of 2 MPa?

(ii) Permissible intensity of internal diameter when the shell diameter is 1.5m.

Thin Cylinders: Problem-4

A cylindrical tank of Di=750 mm, t=12mm and L=1.5 m is completely filled with an oil of specific weight 7.85 kN/m^3 at atmospheric pressure. If the efficiency of the longitudinal joint is 75% and that of circumferential join is 45%. Find the pressure head of oil in the tank. Also calculate change in volume. Permissible tensile stress is 120 MPa, E=200 GPa & poisson's ratio is 0.3.

SOLUTION:

Let p = max permissible pressure in the tank.

Then we have, $\sigma_L = (p\times d)/(4\times t)\ \eta_C$

$120 = (p\times 750)/(4\times 12)\ 0.45$

$p = 3.456$ MPa.

Also, $\sigma_C = (p\times d)/(2\times t)\ \eta_L$

$120 = (p\times 750)/(2\times 12)\ 0.75$

$p = 2.88$ MPa.

Max permissible pressure in the tank, p = 2.88 MPa.

$$\text{Vol. Strain, } \frac{dv}{V} = \frac{(p\times d)}{(4\times t\times E)}\times(5-4\times\mu)$$

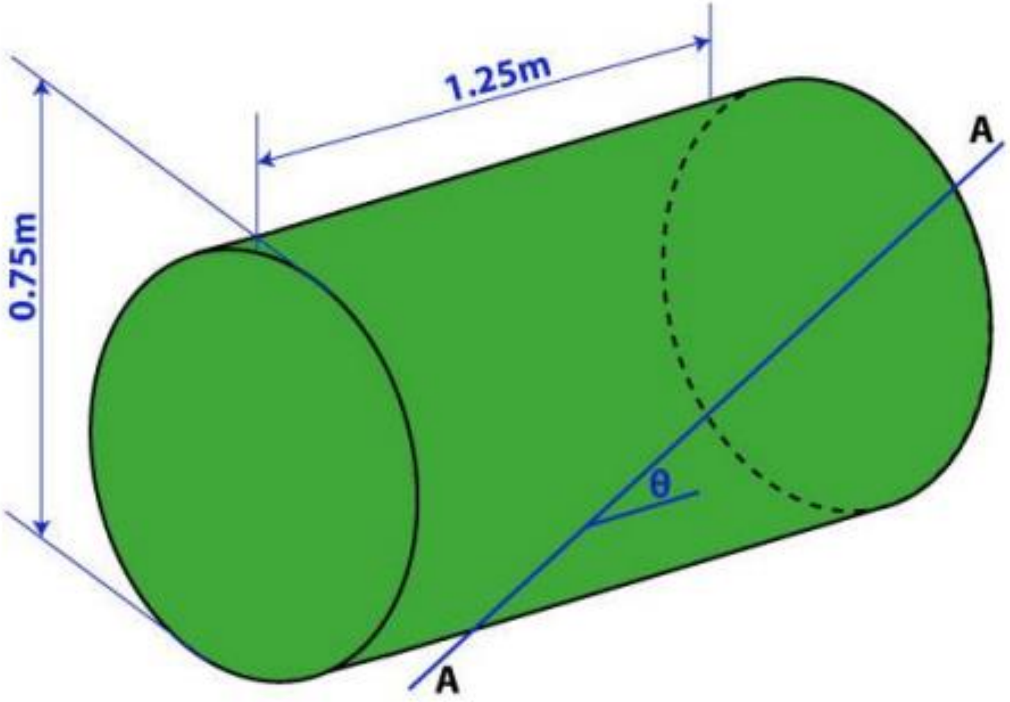
$$= \frac{(2.88\times 750)}{(4\times 12\times 200\times 10^3)}\times(5-4\times 0.3) = 8.55\times 10^{-4}$$

$$dv = 8.55\times 10^{-4}\times V = 8.55\times 10^{-4}\times\frac{\pi}{4}\times 750^2\times 1500 = 0.567\times 10^6\ \text{mm}^3$$

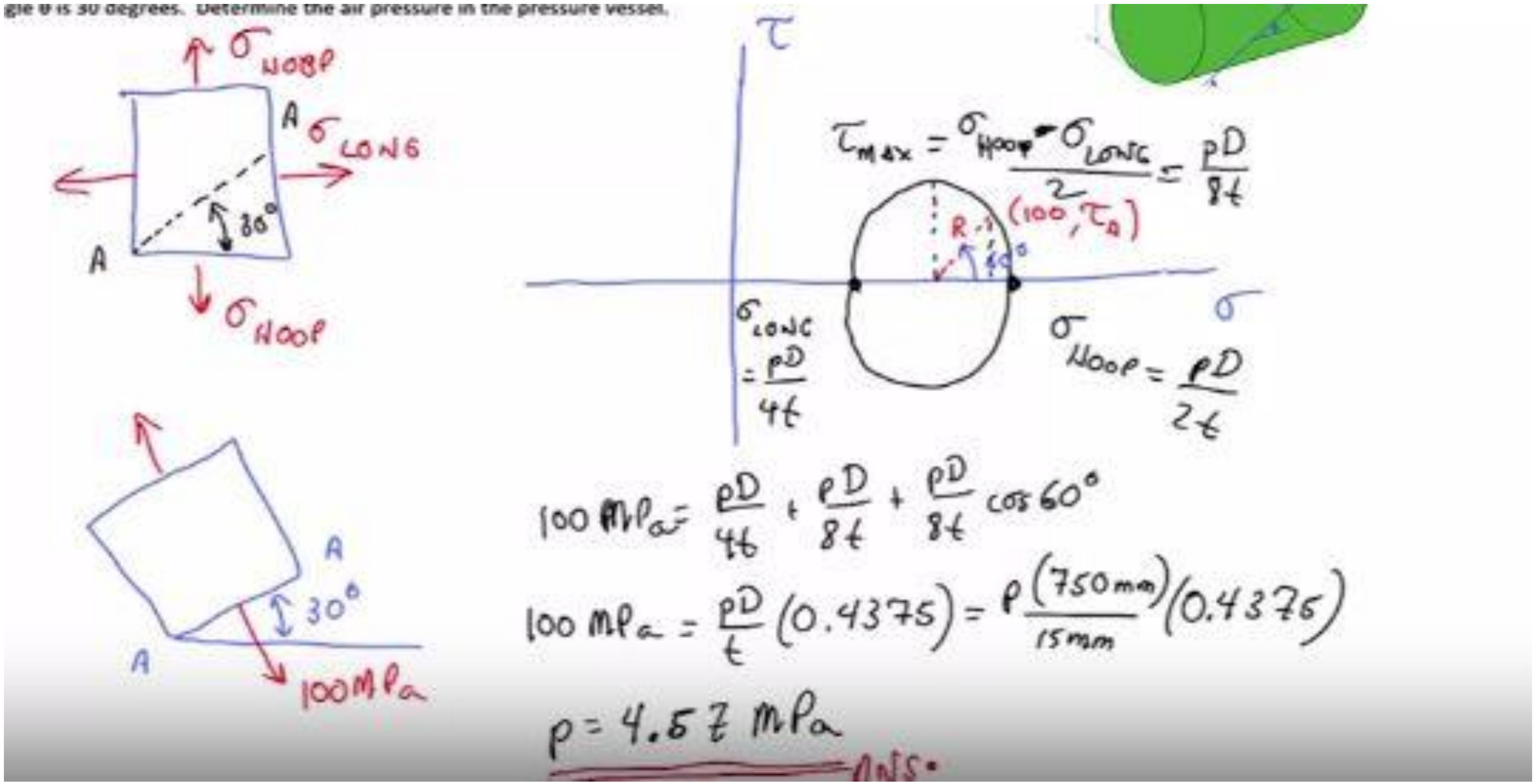
$$= 0.567\times 10^{-3}\ \text{m}^3 = \underline{0.567}\ \text{litres.}$$

Thin Cylinders: Problem-5

A steel cylindrical pressure vessel has the dimensions shown below. The wall thickness of the vessel is 15 mm. The normal stress on a plane cut A-A (perpendicular to the surface of the vessel) is 100 MPa in tension. The angle θ is 30 degrees. Determine the air pressure in the pressure vessel.



Thin Cylinders: Assignment



Thin Cylinders: Assignment

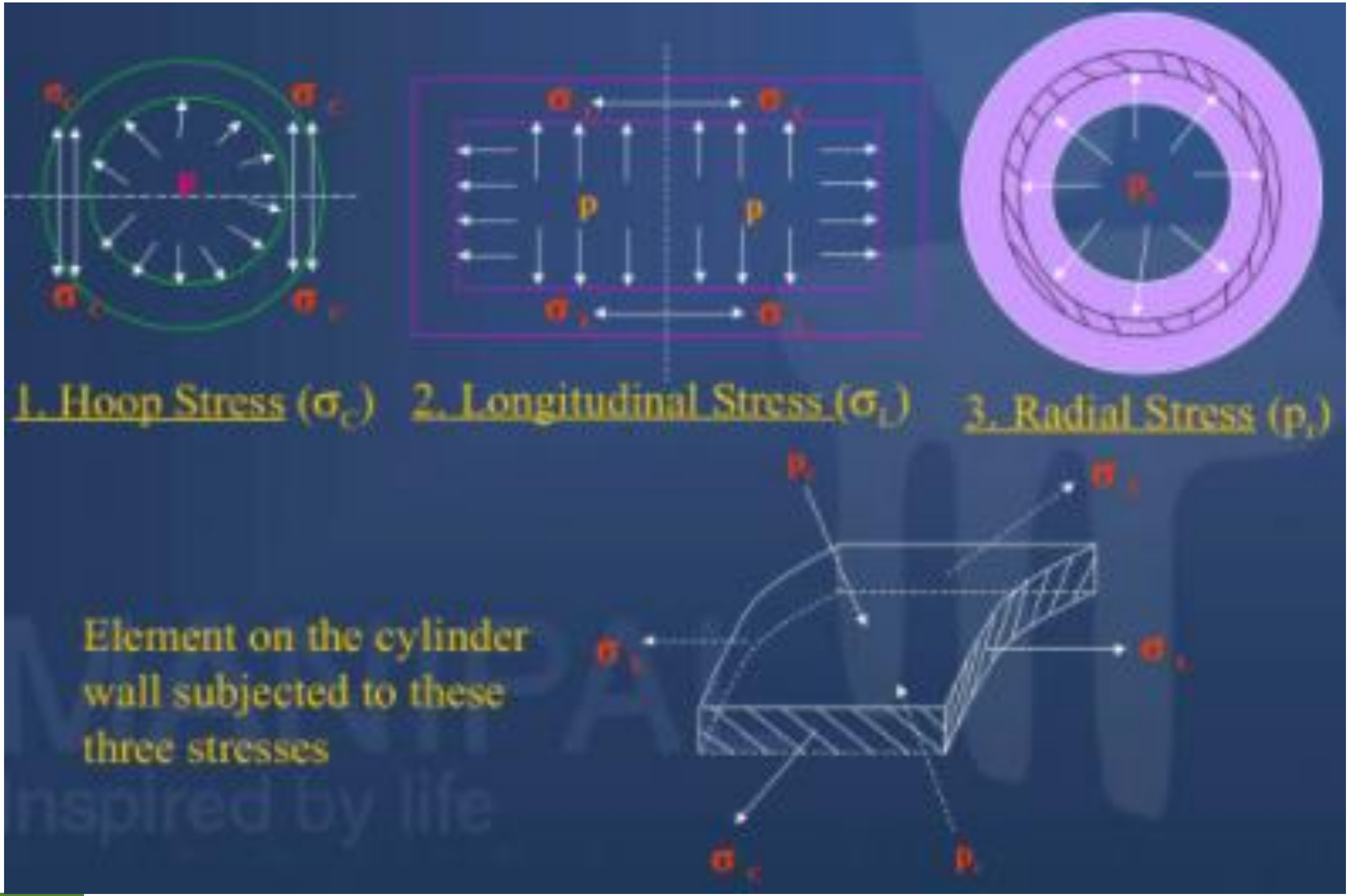
1. A steel cylindrical pressure vessel subjected to internal pressure of 1 MPa. The radius of the cylinder is 1500 mm and thickness of wall is 10 mm. (A) Determine the hoop stress and longitudinal stress in cylindrical walls (B) Calculate the change in diameter due to internal pressure
2. A rubber ball is inflated to pressure of 80 kPa. At that pressure diameter of the ball is 208 mm ad wall thickness is 12 mm. The rubber has E=3.5 MPa and poisson's ratio=0.45. Determine the maximum stress and strain in the ball.
3. A spherical weather balloon is made up of 0.2 mm thick fabric that has tensile strength of 10 MPa. The balloon is designed to reach an altitude where the internal pressure is 2000 Pa above the atmospheric pressure. Find the largest allowable diameter?

Thin Cylinders: Assignment

4. A spherical stainless steel tank having a diameter of 400 mm is used store propane gas at a pressure of 2.4 MPa. Determine the permissible thickness of steel tank when yield stress of steel in tension limited to 112 MPa and normal strain must not exceed 1000×10^{-6}. E=200 GPa, Poisson's ratio is 0.28.

5. The cylindrical pressure vessel with hemi spherical end caps is made of steel. The vessel has uniform thickness of 20 mm and outer diameter of 400 mm. When the vessel is pressurised to 4.5 MPa and length of cylindrical part is 600 mm. determine the change in overall length of thin pressure vessel (E =200 GPa, Poisson's ratio 0.3)

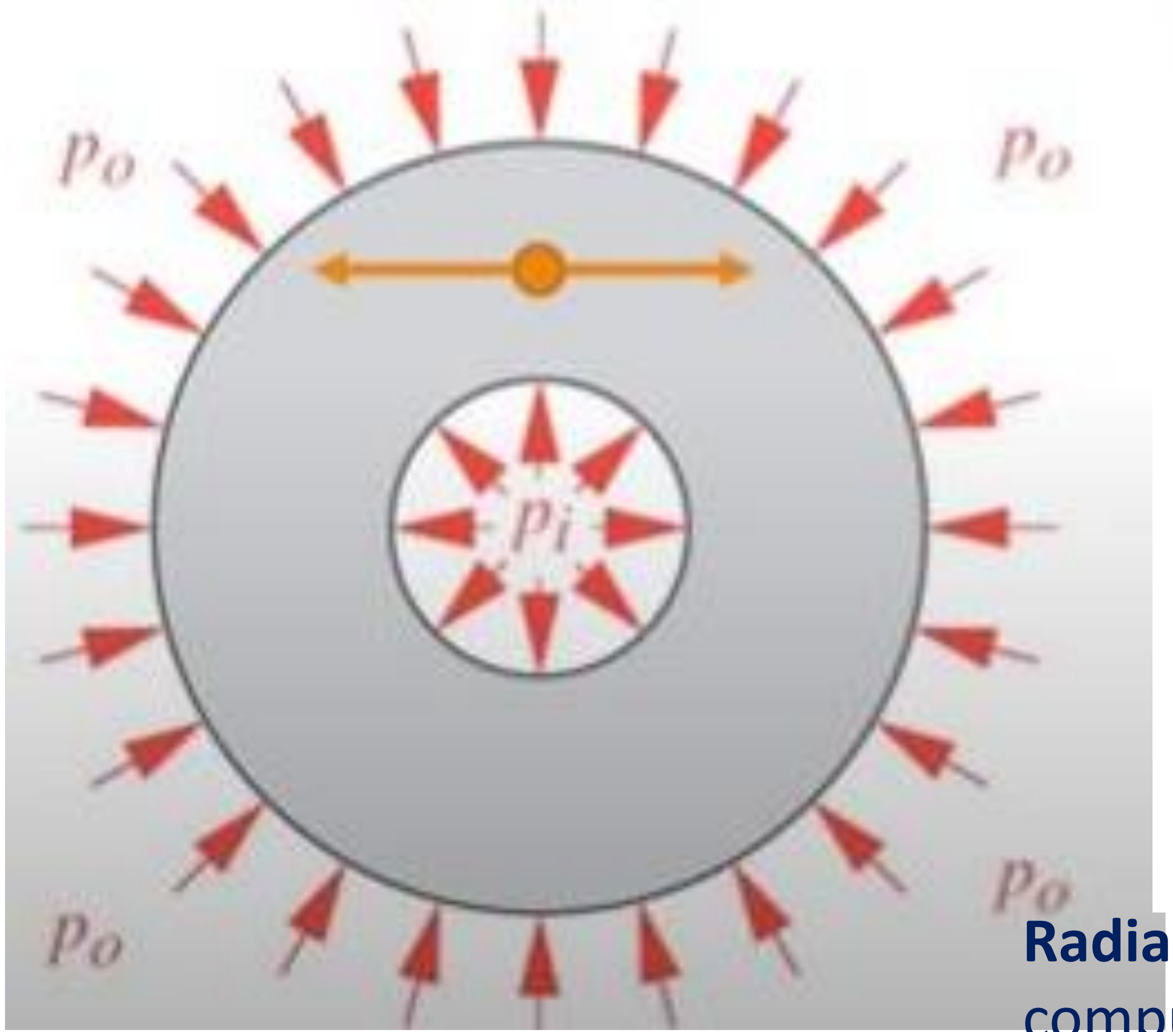
Cylinders: Stresses

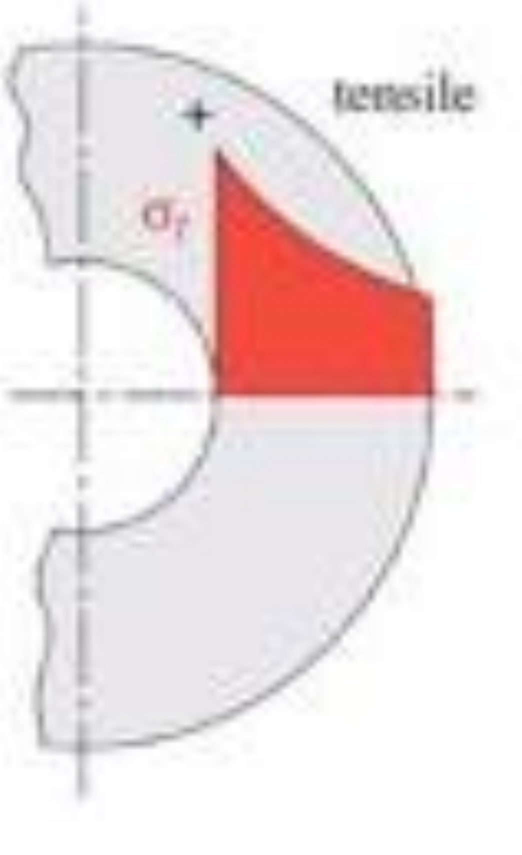


Pressure Vessel: Stresses

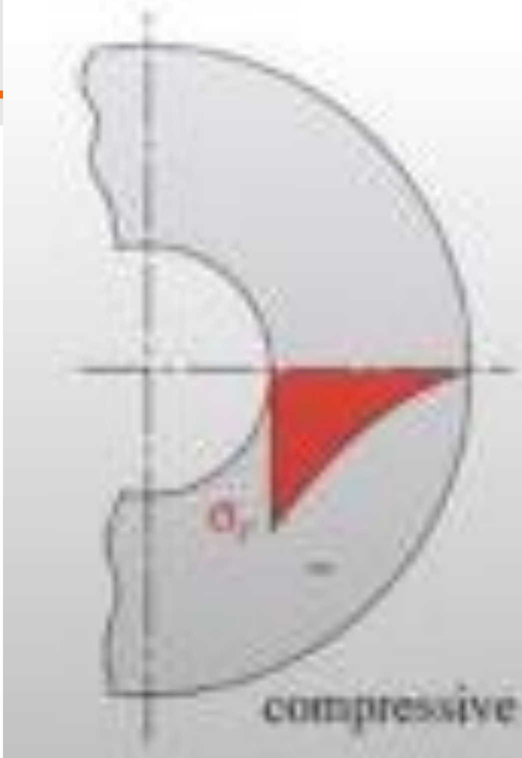
- Thick cylinders are designed to withstand **high internal pressure about 40 to 60 MPa**
- **Hoop or Circumferential Stress** – This is directed along the tangent to the circumference and **tensile** in nature. Thus, there will be **increase in diameter. (not uniform across the cylindrical wall)**
- **Longitudinal Stress:** This stress is directed along the length of the cylinder. This is also **tensile** in nature and tends to **increase the length.**
- **Radial Stress:** It is **compressive** in nature.
  - Its equal to fluid pressure on the inside wall and zero on the outer wall if its open to atmosphere. Magnitude of radial stress is large and hence it cannot neglected
  - Radial stress and Circumferential stresses are computed by using ‘Lame’s Equations’.

Thick Cylinders/Sphere





(a) Tangential stress



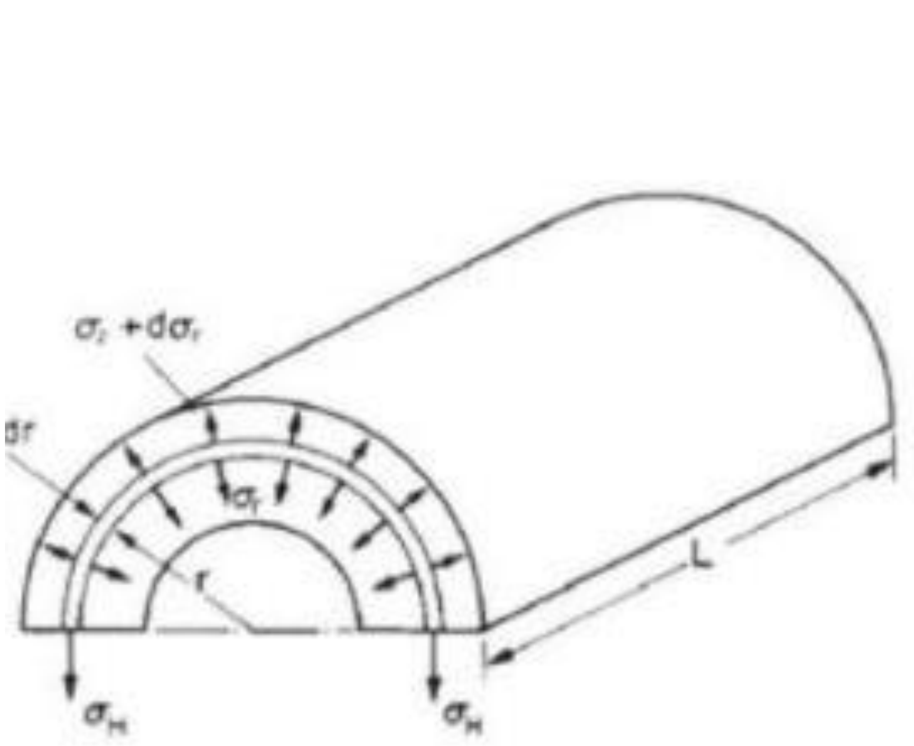
(b) Radial stress

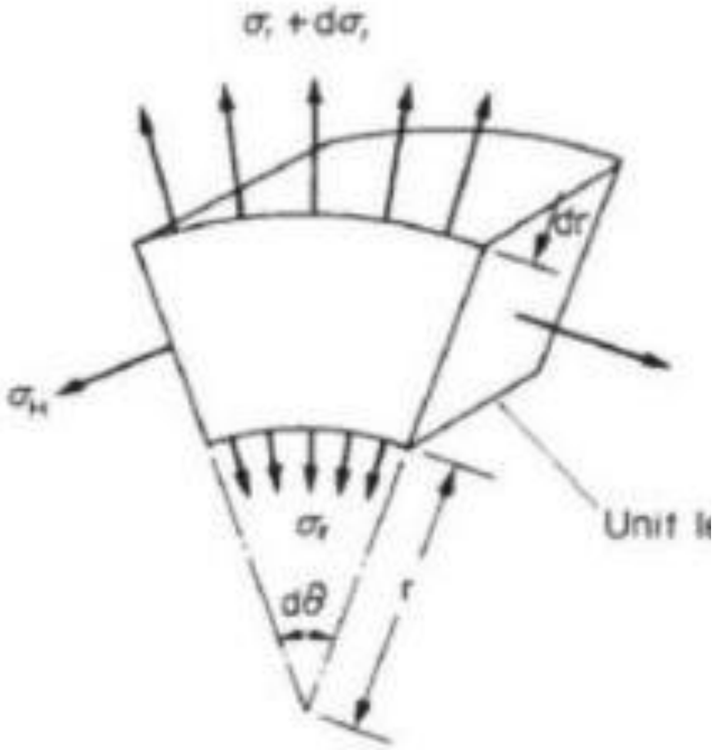
Radial stress and **Circumferential** stresses are computed by using **'Lame's Equations'**.

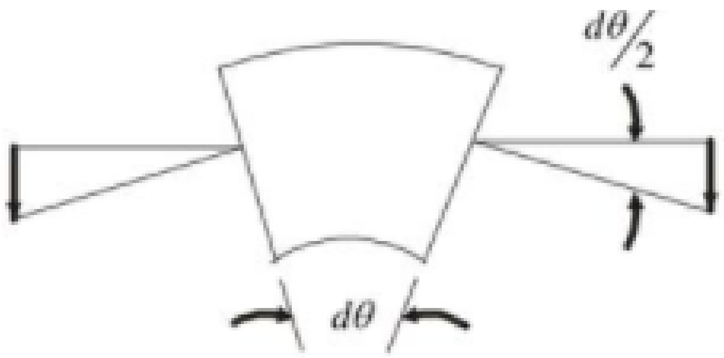
Thick Pressure Vessel: Lame's Equation

- **Radial stress** and **Circumferential** stresses are computed by using **'Lame's Equations**' developed by Gabriel Lame in 1833.
- **Assumptions:**
  - **Plane sections of the cylinder normal to its axis remain plane and normal even under pressure**
  - **Longitudinal stress and strain remain constant throughout the thickness of the wall**
  - **Since longitudinal stress and strain are constant. It follows that the difference in magnitude of hoop stress and radial stress at any point on the cylinder wall is a constant**
  - **The material is homogeneous, isotropic and obeys Hooke's Law**

Thick Pressure Vessel: Lame's Equation







For radial equibrium of the element:

$$(\sigma_R + d\sigma_R)(r + dr)d\theta dz - \sigma_R \times rd\theta \times dz - 2(\sigma_H \times dr \times dz \times \sin\frac{d\theta}{2}) = 0$$

Thick Pressure Vessel: Lame's Equation

The force balance for equilibrium in the radial direction:

$$(\sigma_R + d\sigma_R)(r+dr)d\theta dz - \sigma_R \times rd\theta \times dz - 2(\sigma_H \times dr \times dz \times \sin\frac{d\theta}{2}) = 0$$

$$(r\sigma_R + rd\sigma_R + \sigma_R dr + d\sigma_R dr)d\theta - \sigma_R rd\theta - 2\sigma_H dr\left(\frac{d\theta}{2}\right) = 0 \_\_\_\_\_\_(1)$$

$$(\textit{for small angle, } \sin\frac{d\theta}{2} \approx \frac{d\theta}{2})$$

Dividing equation (1) by $d\theta$ or neglecting second-order small quantities

$$\sigma_R dr + rd\sigma_R - \sigma_H dr = 0 \_\_\_\_\_\_\_\_(2)$$

Dividing equation (2) by dr and rearrange the equation :-

$$\sigma_H = \sigma_R + r\frac{d\sigma_R}{dr} \_\_\_\_\_\_\_\_\_\_\_(3)$$

Assuming now that plane section remains plane, i.e the longitudinal str *is* constant across the wall of the cylinder, then

$$\varepsilon_z = \varepsilon_L = \frac{1}{E}[\sigma_L - \nu(\sigma_R + \sigma_H)] = \text{constant} \_\_\_\_\_\_\_\_(\text{eq.4})$$

$$\sigma_R + \sigma_H = \frac{1}{\upsilon}(\sigma_L - E\varepsilon_L) = \text{constant} \_\_\_\_\_\_\_\_(\text{eq.5})$$

It is also assumed that the longitudinal stress, σ_L is constant across the walls at points remote from the ends :

$$\sigma_R + \sigma_H = 2A\ (say) \_\_\_\_\_\_\_\_\_\_(\text{eq.6})$$

Substitute σ_H from (eq. 3) into (eq. 6) :-

$$\sigma_R + \left(\sigma_R + r\frac{d\sigma_R}{dr}\right) = 2A \_\_\_\_\_\_\_\_(\text{eq.7})$$

Multiplying through by r and rearranging :-

$$2r\sigma_R + r^2\frac{d\sigma_R}{dr} - 2Ar = 0 \_\_\_\_\_\_\_(\text{eq.8})$$

Thick Pressure Vessel: Lame's Equation

Eq. 8 can be written in the differential equation form :-

$$\frac{d}{dr}\left(r^2\sigma_R - Ar^2\right) = 0 \_\_\_\_\_\_\_\_ (eq.9)$$

Intergrate eq. 9 over r :-

$$r^2\sigma_R - Ar^2 = \text{constant} = -B(say) \_\_\_\_\_\_\_\_\_\_ (eq.10)$$

Therefore :–

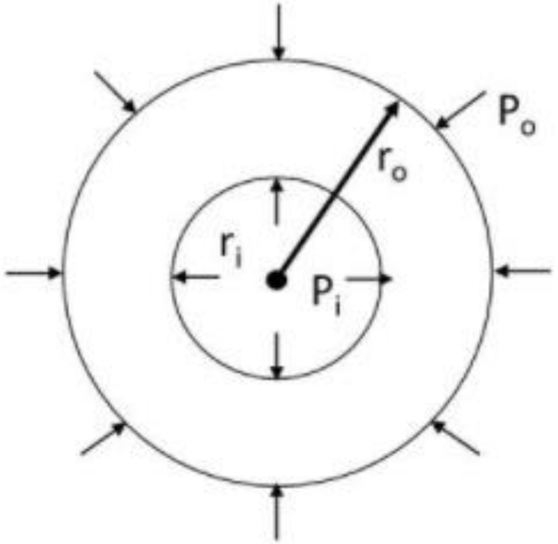
$$\boxed{\sigma_R = A - \frac{B}{r^2}} \_\_\_\_\_\_\_\_\_\_\_\_ (eq.11)$$

By substituting eq. 11 into eq. 6 :-

$$\boxed{\sigma_H = A + \frac{B}{r^2}} \_\_\_\_\_\_\_\_\_\_\_\_ (eq.12)$$

Lame's Equation

Boundary conditions for



$$\sigma_R = -P_i \quad at \quad r = r_i$$
$$\sigma_R = -P_o \quad at \quad r = r_o$$

Note: pressure is compression

Thick Pressure Vessel: Lame’s Equation

Constants A and B can be determined from the boundary conditions $r=r_i$ and $r=r_o$

$$(i)\ at\ r = r_i, \sigma_R = -P_i$$
$$(ii)\ at\ r = r_o, \sigma_R = -P_o$$

Substituting in equation (11), we get :-

$$-P_i = A - \frac{B}{r_i^2}$$

$$-P_o = A - \frac{B}{r_o^2}$$

$$-P_i + P_o = -\frac{B}{r_i^2} + \frac{B}{r_o^2}$$

$$P_i - P_o = \frac{B}{r_i^2} - \frac{B}{r_o^2}$$

$$= B\left(\frac{r_o^2 - r_i^2}{r_i^2 r_o^2}\right)$$

Therefore :-

$$B = \frac{r_i^2 r_o^2 (P_i - P_o)}{(r_o^2 - r_i^2)}$$

- Then

$$A = \frac{B}{r_i^2} - P_i$$

$$A = \frac{r_o^2 (P_i - P_o)}{(r_o^2 - r_i^2)} - P_i$$

$$A = \frac{r_o^2 P_i - r_o^2 P_o - r_o^2 P_i + r_i^2 P_i}{(r_o^2 - r_i^2)}$$

$$A = \frac{r_i^2 P_i - r_o^2 P_o}{(r_o^2 - r_i^2)}$$

- Substituting the constants

$$\sigma_R = \frac{r_i^2 P_i - r_o^2 P_o}{(r_o^2 - r_i^2)} - \frac{r_i^2 r_o^2 (P_i - P_0)}{r^2 (r_o^2 - r_i^2)}$$

$$\sigma_H = \frac{r_i^2 P_i - r_o^2 P_o}{(r_o^2 - r_i^2)} + \frac{r_i^2 r_o^2 (P_i - P_0)}{r^2 (r_o^2 - r_i^2)}$$

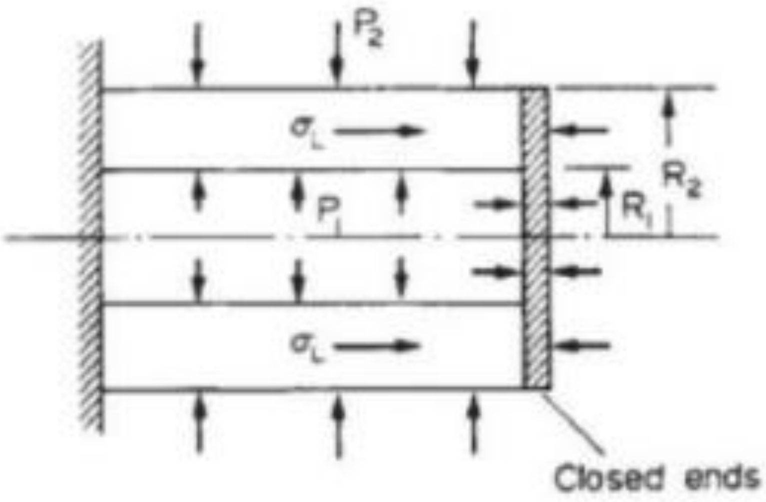
Thick Pressure Vessel: Lame's Equation

Now consider the cross-section of a thick cylinder with closed ends subjected to an internal pressure P_i and external pressure P_o.

For horizontal equilibrium:-

$$P_o \pi r_o^2 - P_i \pi r_i^2 = \sigma_L \pi (r_o^2 - r_i^2)$$

$$\sigma_L = \frac{P_o r_o^2 - P_i r_i^2}{r_o^2 - r_i^2}$$



Thick Cylinders/Sphere

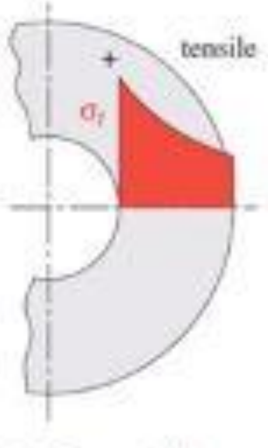
There are three types of stresses:

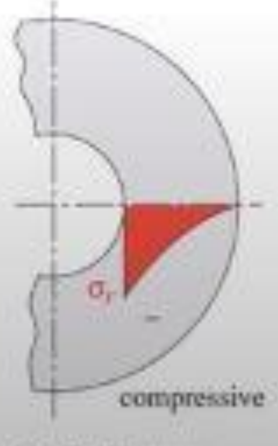
- Tangential
- Radial
- Axial (If Closed)

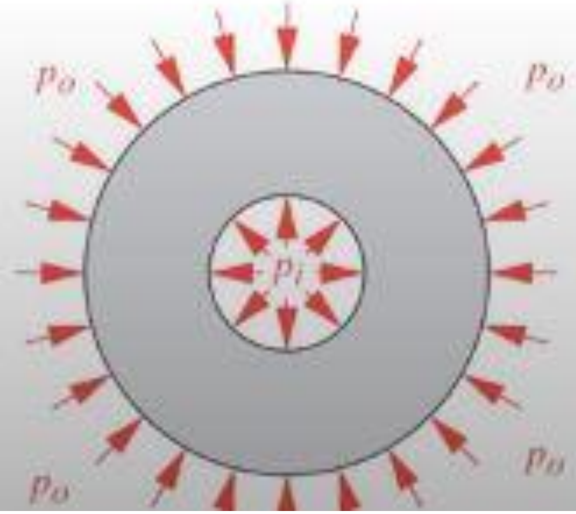
$$\sigma_t = \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2} + \frac{r_i^2 r_o^2 (p_i - p_o)}{r^2 (r_o^2 - r_i^2)}$$

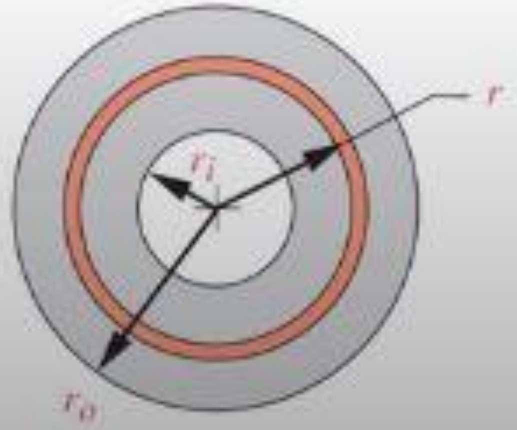
$$\sigma_r = \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2} - \frac{r_i^2 r_o^2 (p_i - p_o)}{r^2 (r_o^2 - r_i^2)}$$

$$\sigma_a = \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2}$$









Thick Cylinders/Sphere

There are three types of stresses:

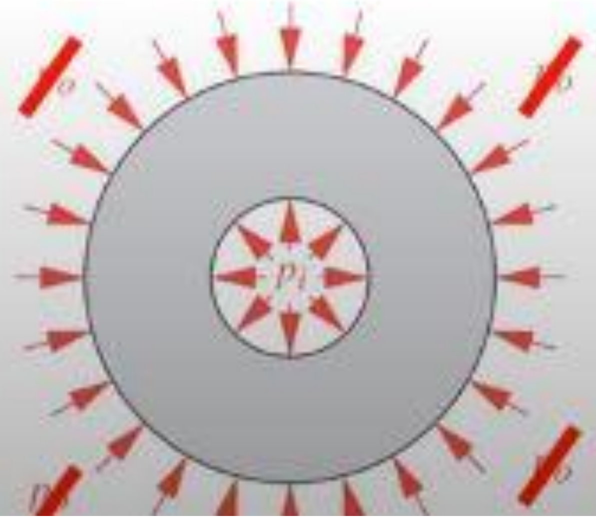
- Tangential
- Radial
- Axial (If Closed)

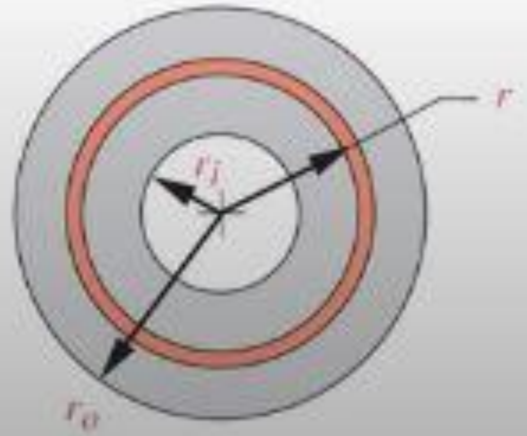
$$\sigma_t = \frac{p_i r_i^2}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right)$$

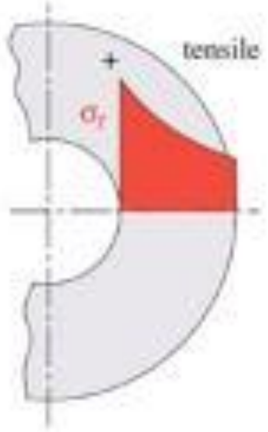
$$\sigma_r = \frac{p_i r_i^2}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r^2}\right)$$

- If $p_0 = 0$

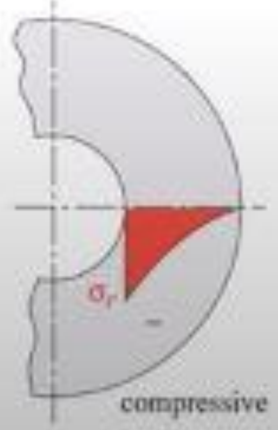
$$\sigma_a = \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2}$$







(a) Tangential stress



(b) Radial stress

Thick Cylinders: Assignment

Maximum Shear Stress

$$\tau_{max} = \frac{\sigma_H - \sigma_R}{2}$$

Since σ_H is normally tensile, whilst σ_R is compressive and both exceed σ_L in magnitude:-

$$\tau_{max} = \frac{1}{2}\left[\left(A + \frac{B}{r^2}\right) - \left(A - \frac{B}{r^2}\right)\right]$$

$$\tau_{max} = \frac{B}{r^2}$$

The greatest valueof τ_{max} thus normally occurs at the inside radius where $r = R_1$ or inner radius

Thick Cylinders: Assignment

NOTE:

1. Variations of Hoop stress and Radial stress are parabolic across the cylinder wall.
2. At the inner edge, the stresses are maximum.
3. The value of 'Permissible or Maximum Hoop Stress' is to be considered on the inner edge.
4. The maximum shear stress (σ_{max}) and Hoop, Longitudinal and radial strains (ε_c, ε_L, ε_r) are calculated as in thin cylinder but separately for inner and outer edges.

Thick Cylinders: Problem-1

- A thick cylindrical shell with inner radius 10 cm and outer radius 16 cm is subjected to an internal pressure of 70 MPa. Find the maximum and minimum hoop stresses
- Given a =10 cm and b = 16 cm

  the hoop stress at $r=r_i=a=10$ cm (Maximum at the inner radius) is

$$\sigma_H = \frac{r_i^2 P}{(r_o^2 - r_i^2)}\left(1+\frac{r_o^2}{r^2}\right) = \frac{0.1^2 \times 70(10^6)}{0.16^2 - 0.1^2}\left(1+\frac{0.16^2}{0.1^2}\right) = 159.73 MPa$$

- Similarly the hoop stress at $r=r_o$ =16 cm is

$$\sigma_H = \frac{r_i^2 P}{(r_o^2 - r_i^2)}\left(1+\frac{r_o^2}{r^2}\right) = \frac{0.1^2 \times 70(10^6)}{0.16^2 - 0.1^2}\left(1+\frac{0.16^2}{0.16^2}\right) = 89.74 MPa$$

Thick Cylinders: Problem-1

Thick Cylinders: Problem-2

Calculate the thickness of metal necessary for a cylindrical shell of internal radius 160mm to withstand an internal pressure of 25 MPa, if maximum permissible tensile stress is 125 MPa

- Given P_i= P=25 MPa and Maximum hoop stress , i.e., σ_H at r = r_i is 125 MPa.
- Considering P_o as zero (no external pressure) , using the formula for hoop stress at r=r_i

$$\sigma_H = \frac{r_i^{\,2} P}{\left(r_o^{\,2} - r_i^{\,2}\right)}\left(1 + \frac{r_o^{\,2}}{r^2}\right) = \frac{0.16^2 \times 25(10^6)}{r_o^{\,2} - 0.16^2}\left(1 + \frac{r_o^{\,2}}{0.16^2}\right) = 125 MPa$$

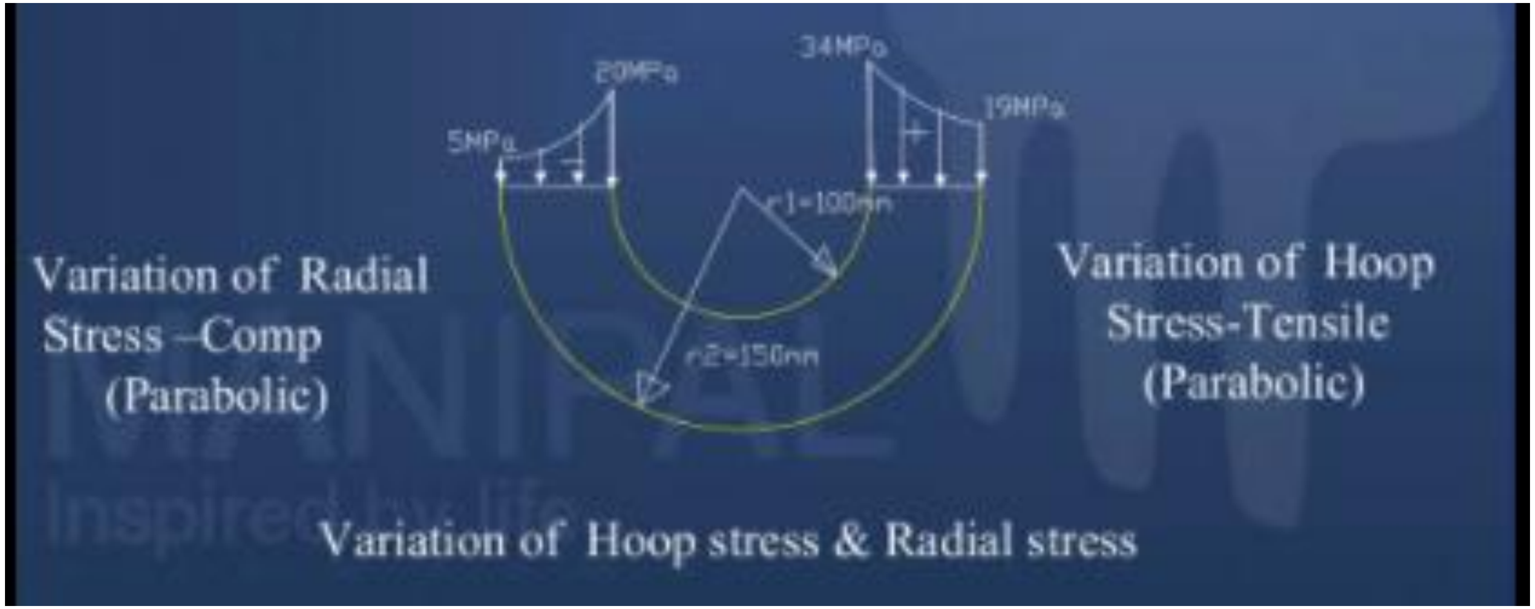
$$r_o = 0.18m$$

So the thickness is b-a = 180-160=20 mm

Thick Cylinders: Problem-2

Thick Cylinders: Problem-3

- A thick cylindrical pipe of external diameter 300mm and internal diameter 200mm is subjected to an internal fluid pressure of 20 MPa and external pressure of 5 MPa. Determine the maximum hoop stress developed and draw the variation of hoop stress and radial stress across the thickness. Show at least four points for the case.



Thick Cylinders: Problem-3

- A thick cylindrical pipe of external diameter 300mm and internal diameter 200mm is subjected to an internal fluid pressure of 20 MPa and external pressure of 5 MPa. Determine the maximum hoop stress developed and draw the variation of hoop stress and radial stress across the thickness. Show at least four points for the case.

Thick Cylinders: Problem-4

- Find the thickness of the metal required for a thick cylindrical shell of internal diameter 160 mm to withstand an internal pressure of 8 MPa. The maximum hoop stress in the section is not to exceed 35 MPa.
- Hint: **Lami's Constants and**
- **radial stress in outer face is zero**

On the outer face, pressure = 0.

i.e., $p_r = 0$ at $r = r_1$.

$$\therefore \quad 0 = \frac{1,37,600}{r_1^2} - 13.5$$

$$\therefore \quad r_1 = \underline{100.96\text{mm}}$$

$$\boxed{\therefore \quad \text{Thickness of the metal} = r_1 - r_2 = \underline{20.96\text{mm}}}$$

Thick Cylinders: Problem-5

- A thick cylindrical pipe of external diameter 300mm and internal diameter 200mm is subjected to an internal fluid pressure of 14 MPa. Determine the maximum hoop stress developed in the cross section. What is the percentage error if the maximum hoop stress is calculated by the equation for thin cylinder?

Max hoop stress on the inner face (where x=100mm):

$$\sigma_{max} = \frac{252000}{100^2} + 11.2 = \underline{36.4\text{ MPa}}.$$

By thin cylinder formula, $\sigma_{max} = \dfrac{p \times d}{2 \times t}$

where D = 200mm, t = 50mm and p = 14MPa.

$$\therefore \sigma_{max} = \frac{14 \times 200}{2 \times 50} = \underline{28\text{MPa}}.$$

$$\text{Percentage error} = \left(\frac{36.4 - 28}{36.4}\right) \times 100 = \underline{23.08\%}.$$

Thick Cylinders: Problem-6

The principal stresses at the inner edge of a cylindrical shell are 81.88 MPa (T) and 40MPa (C). The internal diameter of t
cylinder is 180mm and the length is 1.5m. The longitudir
stress is 21.93 MPa (T). Find,

(i) Max shear stress at the inner edge.
(ii) Change in internal diameter.
(iii) Change in length.
(iv) Change in volume.

Take E=200 GPa and μ=0.3.

SOLUTION:

i) Max shear stress on the inner face :

$$\tau_{max} = \frac{\sigma_C - p_r}{2} = \frac{81.88-(-40)}{2}$$

$$= 60.94 \text{ MPa}$$

ii) Change in inner diameter :

$$\frac{\delta d}{d} = \frac{\sigma_C}{E} - \frac{\mu}{E}\times p_r - \frac{\mu}{E}\times\sigma_L$$

$$= \frac{81.88}{200\times10^3} - \frac{0.3}{200\times10^3}\times21.93 - \frac{0.3}{200\times10^3}\times(-40)$$

$$= 4.365\times10^{-4}$$

$$\therefore \quad \delta d = +0.078\text{mm.}$$

iii) Change in Length :

$$\frac{\delta l}{L} = \frac{\sigma_L}{E} - \frac{\mu}{E}\times p_r - \frac{\mu}{E}\times\sigma_C$$

$$= \frac{21.93}{200\times10^3} - \frac{0.3}{200\times10^3}\times(-40) - \frac{0.3}{200\times10^3}\times81.8$$

$$= 46.83\times10^{-6}$$

$$\therefore \quad \delta l = +0.070\text{mm.}$$

Thick Cylinders: Problem-6

ii) Change in inner diameter :

$$\frac{\delta d}{d} = \frac{\sigma_c}{E} - \frac{\mu}{E} \times p_r - \frac{\mu}{E} \times \sigma_L$$

$$= \frac{81.88}{200\times10^3} - \frac{0.3}{200\times10^3} \times 21.93 - \frac{0.3}{200\times10^3} \times (-40)$$

$$= 4.365\times10^{-4}$$

$$\therefore \quad \delta d = \underline{+0.078\text{mm.}}$$

iii) Change in Length :

$$\frac{\delta l}{L} = \frac{\sigma_L}{E} - \frac{\mu}{E} \times p_r - \frac{\mu}{E} \times \sigma_c$$

$$= \frac{21.93}{200\times10^3} - \frac{0.3}{200\times10^3} \times (-40) - \frac{0.3}{200\times10^3} \times 81.88$$

$$= 46.83\times10^{-6}$$

$$\therefore \quad \delta l = \underline{+0.070\text{mm.}}$$

iv) Change in volume :

$$\frac{\delta V}{V} = \frac{\delta l}{L} + 2\times\frac{\delta d}{D}$$

$$= 9.198 \times 10^{-4}$$

$$\therefore \quad \delta V = 9.198\times10^{-4} \times \left(\frac{\pi\times180^2\times1500}{4}\right)$$

$$= \underline{35.11\times10^3}\ \text{mm}^3.$$

Thick Cylinders: Problem-6

ind the max internal pressure that can be allowed into a thick pipe of uter diameter of 300mm and inner diameter of 200mm so that tensile tress in the metal does not exceed 16 MPa if, (i) there is no external luid pressure, (ii) there is a fluid pressure of 4.2 MPa.

i.e., $16 = \frac{b}{100^2} + a$ ....(1)

$0 = \frac{b}{150^2} - a$ ....(2)

Solving we get, a = 4.92 & b=110.77×10³

so that $\sigma_c = \frac{110.77\times10^3}{r^2} + 4.92$ ....(3)

$p_r = \frac{110.77\times10^3}{r^2} - 4.92$ ....(4)

Fluid pressure on the inner face where r = 100mm,

$p_r = \frac{110.77\times10^3}{100^2} - 4.92 = \underline{6.16\text{ MPa}}$

Boundary conditions:

At r=100mm , σ_c= 16 N/mm²

At r=150mm , p_r= 4.2 MPa.

i.e., $16 = \frac{b}{100^2} + a$ ....(1)

$4.2 = \frac{b}{150^2} - a$ ....(2)

Solving we get, a = 2.01 & b=139.85×10³

so that $\sigma_c = \frac{139.85\times10^3}{r^2} + 2.01$ ....(3)

$p_r = \frac{139.85\times10^3}{r^2} - 2.01$ ....(4)

Thick Cylinders: Assignment

PROBLEM 1:

A pipe of 150mm internal diameter with the metal thickness of 50mm transmits water under a pressure of 6 MPa. Calculate the maximum and minimum intensities of circumferential stresses induced.

(Ans: 12.75 MPa, 6.75 MPa)

PROBLEM 2:

Determine maximum and minimum hoop stresses across the section of a pipe of 400mm internal diameter and 100mm thick when a fluid under a pressure of $8 N/mm^2$ is admitted. Sketch also the radial pressure and hoop stress distributions across the thickness.

(Ans: σ_{max}=20.8 N/mm^2, σ_{min}=12.8 N/mm^2)

PROBLEM 3:

A thick cylinder with external diameter 240mm and internal diameter 'D' is subjected to an external pressure of 50 MPa. Determine the diameter 'D' if the maximum hoop stress in the cylinder is not to exceed 200 MPa.

(Ans: 169.7 mm)

Thick Cylinders: Over View

Thick Cylinders

Queries?