Magnetostatics

Magnetostatics discuss magnetic field due to steady current. The magnetic field at any point due to steady current is called magnetostatic field.

In magnetostatics "$\nabla \cdot J = 0$" where J is current density

The magnetic force on a charge 'q' moving with velocity 'v' in a magnetic field 'B' is given by $\vec{F}_{mag} = q(\vec{v} \times \vec{B})$ and in the presence of both electric and magnetic field, the netforce on particle 'q' will be

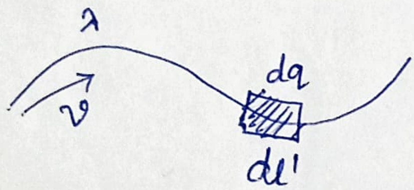
$$F_{net} = q[\vec{E} + (\vec{v} \times \vec{B})] \text{ i.e Lorentz force.}$$

I, J. $\quad I\,dl = K\,da = J\,d\tau$

For isotropic medium I ...
For anisotropic, I is ...

Current in wire: (linear current) I

Lets consider a wire where the linecharge density is defined as 'λ' and charge travelling down in that wire with speed 'v'. The current in the wire



$\lambda = \frac{dq}{dx}$

$\lambda = \frac{q}{x}$

$$I = \frac{dq}{dt} = \frac{d}{dt}(\lambda x) = \lambda \frac{dx}{dt} = \lambda v$$

$$\boxed{I = \lambda v}$$

The magnetic force on a segment of current carrying wire is

$$F_{mag} = \int_l (v \times B)\,dq$$

$$= \int_l (v \times B)\,\lambda\,dl'$$

$$= \int_l (v\lambda \times B)\,\vec{dl'}$$

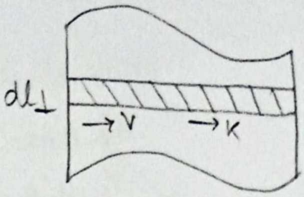
$$= \int_l (I \times \vec{B})\,\vec{dl'}$$

Since I, dl' are in same direction.

$$\boxed{F_{mag} = I \int_l (\vec{dl'} \times \vec{B})}$$

Surface current density ($\vec{K}$)

When a charge flows over a surface, we described it by a surface current 'K'. 'K' is the current per unit width perpendicular to flow.



$$\vec{K} = \frac{dI}{l_\perp} = \sigma v$$

$$\boxed{\vec{K} = \sigma v}$$

Magnetic force on surface current

$q = \frac{\sigma}{a}$

$$F_{mag} = \int_S (v \times B)\, dq$$

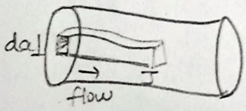
$$= \int_S (v \times B)\, \sigma\, da$$

$$= \int_S (v\sigma \times B)\, da$$

$$\boxed{F_{mag} = \int_S (\vec{K} \times \vec{B})\, da}$$

$K \rightarrow$ vector

Volume current density



When the flow of charge is distributed throughout a 3D region, we describe it by the volume current density J;

i.e $J = \frac{dI}{da_\perp} = \rho v$

$$\boxed{J = \rho v}$$

The magnetic force on volume current is

$$F_{mag} = \int_V (v \times B)\, dq = \int_V (v \times B)\, \rho\, d\tau$$

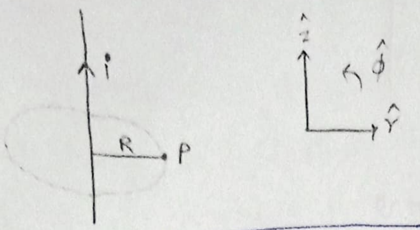
$$= \int_V (v\rho \times B)\, d\tau$$

$$\boxed{F_{mag} = \int_V (\vec{J} \times \vec{B})\, d\tau}$$

* Current crossing a surface 's' is given by

$$\boxed{I = \int_S \bar{J} \cdot d\bar{a}}$$

Biot Savart's Law



prime → bcoz wrt source

dl, B are ⊥er (assuming

F, dl, B are ⊥er mutual

$$\boxed{B = \frac{\mu_0}{4\pi} \int \frac{I \times \hat{R}}{R^2} dl'}$$

$$\boxed{B = \frac{\mu_0 I}{4\pi} \int_l \frac{dl' \times \hat{R}}{R^2}}$$

For surface

$$\boxed{B = \frac{\mu_0}{4\pi} \int_S \frac{\vec{K} \times \hat{R}}{R^2} da}$$

$\mu = 4\pi \times 10^{-7}$ N/m²

For volume

Medium $\mu_0 \rightarrow \mu$

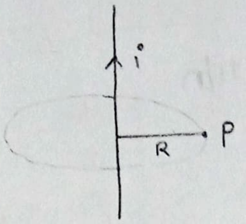
$$\boxed{B = \frac{\mu_0}{4\pi} \int_V \frac{J \times \hat{R}}{R^2} d\tau}$$

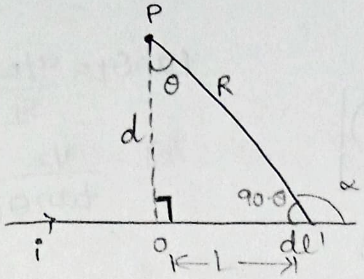
$$B(r)_{air} = \frac{\mu_0}{4\pi} \int \frac{I \times \hat{R}}{R^2} dl'$$

$$B(r)_{medium} = \frac{\mu}{4\pi} \int \frac{I \times \hat{R}}{R^2} dl'$$

Permeability (μ): Ability of material which allows magnetic flux through it.

Magnetic field due to a current carrying wire





$$B = \frac{\mu_0 I}{4\pi} \int_l \frac{dl' \times \hat{R}}{R^2}$$

$\cos\theta = \frac{d}{R}$

$\frac{1}{R} = \frac{\cos\theta}{d}$

$\frac{1}{R^2} = \frac{\cos^2\theta}{d^2}$

$$dl \times \hat{R} = |dl|\,|\hat{R}| \sin\alpha = dl' \cos\theta$$

$$\tan\theta = \frac{L}{d} \Rightarrow L = d\tan\theta$$

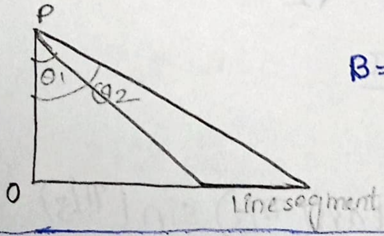
$$dl' = d\sec^2\theta\, d\theta$$

$$\Rightarrow dl \times \hat{R} = dl' \cos\theta$$

$$= d\sec^2\theta\, d\theta \cdot \cos\theta$$

$$= d\sec\theta\, d\theta = \frac{d}{\cos\theta} d\theta$$

$$B(x) = \frac{\mu_0 I}{4\pi} \int_l \frac{d\, d\theta}{\cos\theta} \cdot \frac{\cos^2\theta}{d^2} = \frac{\mu_0 I}{4\pi d} \int_l \cos\theta\, d\theta$$



$$B = \frac{\mu_0 I}{4\pi d} \int_{\theta_1}^{\theta_2} \cos\theta\, d\theta$$

$$\boxed{B = \frac{\mu_0 I}{4\pi d} [\sin\theta_2 - \sin\theta_1]\, \hat{\phi}}$$

1) For infinite long wire, $\theta_1 = 90°$, $\theta_2 = -90°$

$$B = \frac{\mu_0 I}{2\pi d}$$

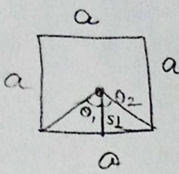
2) Semi Infinite wire $\theta_1 = 0$, $\theta_2 = 90°$

$$B = \frac{\mu_0 I}{4\pi d}.$$

* Magnetic fields at center of different shapes.

$\boxed{n \neq 1, 11}$

Eg:



$\theta_2 = -\theta_1 = \pi/4 = \pi/n$

$$B = \frac{\mu_0 I}{4\pi S_\perp} 2\sin\left(\frac{\pi}{n}\right)$$

For n sides,

$\tan\theta_1 = \frac{a/2}{S_\perp}$

$$\boxed{B = \frac{n\mu_0 I}{2\pi S_\perp} \sin\left(\frac{\pi}{n}\right)}$$

$S_\perp = \frac{a/2}{\tan\theta_1}$

$$B = \frac{\mu_0 I}{2S_\perp} \frac{\sin(\pi/n)}{\pi/n}$$

$$\boxed{B = \frac{\mu_0 I}{a} \tan(\pi/n) \frac{\sin(\pi/n)}{(\pi/n)}}$$

* Calculate the B at center of

a) square
b) Triangle
c) Hexagon
d) Pentagon

a) Square

$$B = \frac{\mu_0 I}{a} \tan(\pi/4) \frac{\sin(\pi/4)}{\pi/4}$$

$$= \frac{4\mu_0 I}{a\pi} \cdot 1 \cdot \frac{1}{\sqrt{2}}$$

$$B = \frac{2\sqrt{2}\mu_0 I}{a\pi}$$

b) Δ'le

$$B = \frac{\mu_0 I}{a} \tan(\pi/3) \frac{\sin(\pi/3)}{\pi/3}$$

$$= \frac{3\mu_0 I}{a} \sqrt{3} \cdot \frac{\sqrt{3}}{2}$$

$$B = \frac{9\mu_0 I}{2a\pi}$$

c) Hexagon:

$$B = \frac{\mu_0 I}{a}\tan(\pi/6)\,\frac{\sin(\pi/6)}{\pi/6}$$

$$= \frac{\overset{3}{\cancel{6}}\,\mu_0 I}{a\pi}\left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{2}\right)$$

$$B = \frac{\sqrt{3}\,\mu_0 I}{a\pi}$$

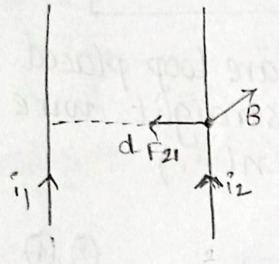
d) Pentagon:

$$B = \frac{\mu_0 I}{a}\tan(\pi/5)\,\frac{\sin(\pi/5)}{\pi/5}$$

$$B = \frac{5\mu_0 I}{a\pi}\tan(\pi/5)\sin(\pi/5)$$

Two parallel wires

Find the force of attraction b/w two long parallel wires at distance 'd' apart carrying current i_1, i_2 in the same direction.



Here these two wires attract.

$$B_1 = \frac{\mu_0 I_1}{2\pi d}$$

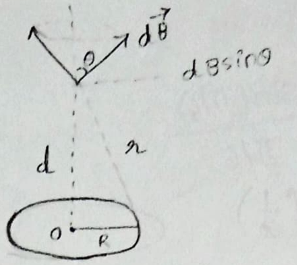
$$F = \int (I \times B)\, dl'$$

$$F_{21} = I_2 \int (dl' \times B)$$

$$= I_2 B_1 \cdot L$$

$$F_{21} = \frac{\mu_0 I_1 I_2}{2\pi d} L$$

* Circular ring



dl, r are $\perp$ er

$R, r \rightarrow \theta$

$$\bar{B} = \frac{\mu_0 I}{4\pi}\int \frac{dl' \times \hat{r}}{r^2}$$

$B = B\cos\theta + B\sin\theta$

$B\cos\theta$

$$B = \frac{\mu_0 I}{4\pi}\int \frac{|dl'|}{r^2} = \frac{\mu_0 I}{4\pi r^2} 2\pi R$$

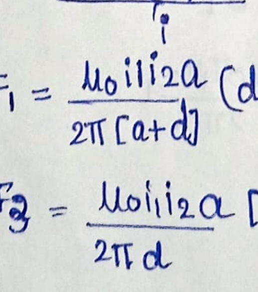
$$\therefore \quad B = \frac{\mu_0 I}{4\pi r^2} 2\pi R\cos\theta .$$

$$\boxed{B = \frac{\mu_0 I}{2}\frac{R^2}{(R^2+d^2)^{3/2}}}$$

At center $d = 0$

$$\boxed{B = \frac{\mu_0 I}{2R}\hat{z}}$$

* Find the force on a square loop placed as shown in the figure near an infinite straight wire, both the loop, wire carry a steady current 'i')



②, ④ are at same ~~direction~~ distance and in opp directio So they will cancel out.

$$F_1 = \frac{\mu_0 i_1 i_2 a}{2\pi[a+d]} \text{ (down)}$$

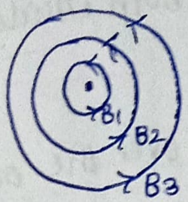
$$F_3 = \frac{\mu_0 i_1 i_2 a}{2\pi d} \text{ [up]}$$

$$F_{net} = F_3 - F_1 = \frac{\mu_0 i_1 i_2 a}{2\pi}\left[\frac{1}{d} - \frac{1}{a+d}\right]$$

$$= \frac{\mu_0 i_1 i_2}{2\pi}\left[\frac{a^2}{d(a+d)}\right]$$

$$\boxed{F = \frac{\mu_0 i_1 i_2}{2\pi}\left[\frac{a^2}{d(a+d)}\right]}$$

Ampere's law: (only for steady currents)



$\odot \rightarrow$ infinite wire carrying current outside the page.

$$B = \frac{\mu_0 i}{2\pi r}\hat{\phi} \qquad \vec{dl} = dr\hat{r} + r d\phi\hat{\phi} + dz\hat{z}$$

$$\oint \vec{B}\cdot\vec{dl} = \oint_l \frac{\mu_0 i}{2\pi r} r\,d\phi = \frac{\mu_0 i}{2\pi}\oint d\phi = \mu_0 i$$

$$\boxed{\oint_l \vec{B}\cdot\vec{dl} = \mu_0 i_{enclosed}}$$

$$\boxed{\begin{array}{l}\nabla\cdot(\nabla\times B) = 0 \\ \nabla\cdot J = -\frac{\partial\rho}{\partial t}\end{array}}$$

W.K.T.

$$i = \int_S J\cdot da$$

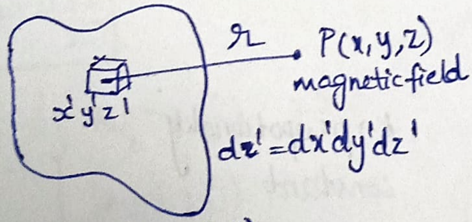
$$\oint B\cdot dl = \mu_0 \int_S J\,\vec{da}$$

$$\int_S (\nabla\times B)\,da = \int_S \mu_0 J\,da$$

$$\therefore \boxed{\nabla\times B = \mu_0 J}$$

Divergence of magnetic field

$$\nabla\cdot(A\times B) = B\cdot(\nabla\times A) - A\cdot(\nabla\times B)$$



$J(x', y', z')$

$B(x, y, z)$

$$r = (x-x')\hat{x} + (y-y')\hat{y} + (z-z')\hat{z}$$

$$B = \frac{\mu_0}{4\pi}\int_V \frac{J\times\hat{r}}{r^2}\,d\tau'$$

$$\nabla\cdot B = \frac{\mu_0}{4\pi}\int_V \nabla\cdot\left(J(r')\times\frac{\hat{r}}{r^2}\right)d\tau'$$

$$\nabla\cdot B = \frac{\mu_0}{4\pi}\int_V \left[\frac{\hat{r}}{r^2}\cdot(\nabla\times J(r')) - J(r')\cdot\left[\nabla\times\frac{\hat{r}}{r^2}\right]\right]d\tau'$$

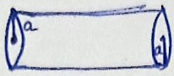
$\boxed{\nabla\cdot B = 0}$ [$\because$ magnetic monopoles doesn't exist.

* A steady current 'i' flow down a long cylindrical wire of radius 'a'. Find the magnetic field both inside and outside the wire, if

i) The current is uniformly distributed over the outside suface of the wire.

ii) The current is distributed in such a way that 'j' is propotional to r, the distance from the axis.

i)



1) $r > a$

$$\oint B\cdot dl = \mu_0 i_{en}$$

$$B(2\pi r) = \mu_0 i$$

$$\boxed{B = \frac{\mu_0 i}{2\pi r}\hat{\phi}}$$

2) $r < a$

$$i_{en} = 0$$

$$\therefore \boxed{B = 0}$$

2) $r < a$

$$\oint B\cdot dl = \mu_0 i_{en}$$

$$B(2\pi r) = \mu_0 \int_0^r J\cdot da$$

$$B(2\pi r) = \mu_0 K(2\pi)\frac{r^3}{3}$$

$$B = \frac{\mu_0 r^2}{3}K$$

$$B = \frac{\mu_0 r^2}{3}\cdot\frac{3 I_{en}}{2\pi a^3}$$

$$\boxed{B = \frac{\mu_0 r^2 i_{en}}{2\pi a^3}}$$

ii) 1) $r > a$

$$J \propto r$$

$J = Kr$ K→ proportionality constant

$$I_{enc} = \int_0^a J\cdot da$$

$$= \int_0^a Kr\cdot 2\pi r\, dr$$

$$= 2\pi K\int_0^a r^2\, dr$$

$$I_{en} = \frac{2\pi K a^3}{3} \Rightarrow \boxed{K = \frac{3 I_{en}}{2\pi a^3}}$$

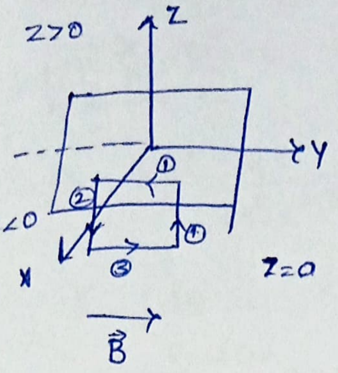
$$\therefore \oint B\cdot dl = \mu_0 i_{en}$$

$$B(\cancel{2\pi} r) = \mu_0\cdot\frac{\cancel{2\pi} K a^3}{3}$$

$$B = \frac{\mu_0 K a^3}{3r}$$

$$\boxed{B = \frac{\mu_0 i_{en}}{2\pi r}\hat{\phi}}$$

Magnetic field in sheet



Here current is flowing through x axis.

Here for 2^{nd}, 4^{th} line $B \cdot dl = 0$

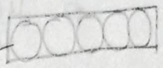
Since they are ⊥el.

$\vec{K} = K\hat{x} \quad \left[\hat{K} = \frac{dI}{dl_{\perp}}\right]$

For direction of B

$B = \frac{\mu_0}{4\pi} \int_S \frac{\bar{K} \times \hat{\mathscr{r}}}{\mathscr{r}^2} da$

The sheet edge [no of wires]

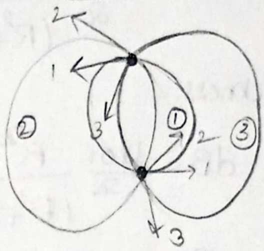


wire current is out of page

$\oint B \cdot dl = \mu_0 i_{en}$

$= \mu_0 \vec{K} L$

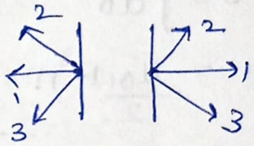
Consider 3 wires:



For loop:

$\int_1 B\,dl + \underbrace{\int_2 B \cdot dl}_{0} + \int_3 B\,dl + \underbrace{\int_4 B\,dl}_{0} = \mu_0 K L$

$\int B(-\hat{y})(-\hat{y})dy + \int B(\hat{y})(\hat{y})dy = \mu_0 K L$



$BL + BL = \mu_0 K L$

$\boxed{B = \frac{\mu_0 K}{2}}$

$\therefore \quad B = \begin{cases} \frac{\mu_0 K}{2}(-\hat{y}) : z>0 \\ \frac{\mu_0 K}{2}(\hat{y}) : z<0 \end{cases}$

Solenoid: Device used to produce magnetic flux

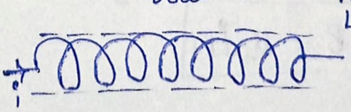
i) Infinite solenoid | Very long solenoid:

$B_{out} = 0$

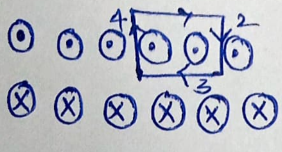
N turns
L length

$n = \frac{N}{L}$

By amphae's law



For N coils : $i_{en} = i(N)$



2, 4 ⊥el
1 → out

$\oint B \cdot dl = \mu_0 i_{en}$

$\int_1 B\,dl + \int_2 B\,dl + \int_3 B\,dl + \int_4 B\,dl = \mu_0 i_{en}$

$\int_3 B \cdot dl = \mu_0 i_{en}$

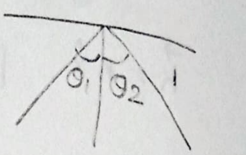
$$\int B(-\hat{y})\cdot(-\hat{y})\,dy = \mu_0 i_{en}$$

$$BL = \mu_0 i_{en}$$

$$BL = \mu_0 i(N)$$

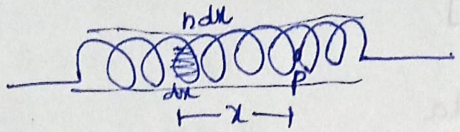
$$B\cancel{L} = \mu_0 i(n\cancel{L})$$

$$\boxed{B = \mu_0 n i}$$



So θ_1, θ_2 can be $90°$ (max)

By biot savart's law (Infinite)



$$B = \frac{\mu_0 i}{2}\frac{R^2}{(R^2+d^2)^{3/2}}$$

For small part

1 coil: $$dB = \frac{\mu_0 i}{2}\frac{R^2}{(R^2+x^2)^{3/2}}$$

ndx thickness:

$$dB = \frac{\mu_0 i}{2}\frac{R^2}{(R^2+x^2)^{3/2}}\,n\,dx$$

$$B = \int dB$$

$$= \frac{\mu_0 i R^2 n}{2}\int_{-\infty}^{\infty}\frac{dx}{(R^2+x^2)^{3/2}}$$

Assume: $x = R\tan\theta$

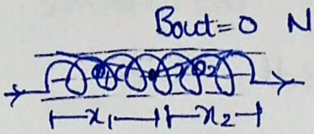
$dx = R\sec^2\theta\, d\theta$

$$B = \frac{\mu_0 i n \cancel{R^2}}{2}\int\frac{\cancel{R}\,\cancel{\sec^2}\theta\, d\theta}{\cancel{R^3}\sec^3\theta}$$

$$= \frac{\mu_0 i n}{2}\int_{-\pi/2}^{\pi/2}\cos\theta\, d\theta$$

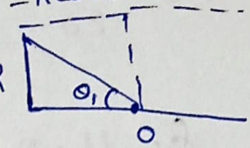
$$\boxed{B = \mu_0 i n}$$

Finite solenoid



For solenoid, rather than ends the field at center is uniform

$$dB = \frac{\mu_0 i}{2} \frac{R^2}{(R^2+x^2)^{3/2}} n\,dx.$$



$$B = \int dB$$

$$B = \int_{-R\cot\theta_1}^{R\cot\theta_2} \frac{\mu_0 i}{2} \frac{R^2}{(R^2+x^2)^{3/2}} n\,dx$$

$x = R\tan\theta$

$dx = R\sec^2\theta\, d\theta.$

$$= \frac{\mu_0 i R^2 n}{2} \int_{-R\cot\theta_1}^{R\cot\theta_2} \frac{dx}{(R^2+x^2)^{3/2}}$$

$$= \frac{\mu_0 i \cancel{R^2} n}{2} \int_{-R\cot\theta_1}^{R\cot\theta_2} \frac{R\sec^{\cancel{2}}\theta\, d\theta}{R^3\sec^3\theta}$$

$\tan\theta = \frac{x}{R}$

$\sin\theta = \frac{x}{\sqrt{x^2+R^2}}$

$$= \frac{\mu_0 i n}{2} \int_{-R\cot\theta_1}^{R\cot\theta_2} \cos\theta\, d\theta$$

$$= \frac{\mu_0 i n}{2} [\sin\theta]_{-R\cot\theta_1}^{R\cot\theta_2}$$

$$= \frac{\mu_0 i n}{2} \left[\frac{x}{\sqrt{x^2+R^2}}\right]_{R\cot\theta_1}^{R\cot\theta_2}$$

$$= \frac{\mu_0 i n}{2} \left[\frac{R\cot\theta_2}{R\csc\theta_2} + \frac{R\cot\theta_1}{R\csc\theta_1}\right]$$

$$\boxed{B = \frac{\mu_0 n i}{2}[\cos\theta_2 + \cos\theta_1]}$$

Electrostatics Vs Magnetostatics

$$\oint E\, da = \frac{q}{\varepsilon_0}$$

$$\nabla . E = \frac{\rho}{\varepsilon_0}$$

$$\oint B\, dl = \mu_0 i_{en}$$

$$\nabla \times B = \mu_0 J$$

$$\oint_l E . dl = 0$$

$$\nabla \times E = 0$$

$$\oint_S B . da = 0$$

$$\nabla . B = 0$$

$$\nabla \times E = 0$$

$$E = -\nabla V$$

$V \rightarrow$ scalar potential

$$\vec{B} = \nabla \times \vec{A}$$

$A \rightarrow$ magnetic potential

$(\nabla . \vec{A} = 0)$

$$V(r) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho}{r} d\tau$$

$$\nabla^2 V = \frac{-\rho}{\varepsilon_0}$$

$$\nabla^2 V = 0$$

$$\therefore \nabla \times B = \mu_0 J$$

$$\nabla (\nabla \times \vec{A}) = \mu_0 J)$$

$$\nabla (\underbrace{\nabla A}_{0}) - \nabla^2 A = \mu_0 J$$

$$\boxed{\nabla^2 A = -\mu_0 J}$$

on comparing with Poisson eqn

$\rho \rightarrow J$

$\mu_0 \rightarrow \frac{1}{\varepsilon_0}$

* Just as curl of E is zero permitted us to introduce a scalar potential V in electrostatics. Divergence B is equal to zero invites the introduction of vector potential A in magnetostatics. Since the $\nabla . B = 0$ indicates the flux lines (B) will curl from north to south. This curl can represent mathematically as curl of some vector $\vec{A}$. i.e $\vec{B} = \nabla \times \vec{A}$ where $\vec{A}$ is defined magnetic vector potential.

From ampheres law, we define $\nabla \times B = \mu_0 J$, if we introduce $B = \nabla \times A$ then

$$\nabla (\nabla \times A) = \mu_0 J$$

$$\nabla (\underbrace{\nabla A}_{0}) - \nabla^2 A = \mu_0 J$$

$$\boxed{\nabla^2 A = -\mu_0 J}$$

Volume: $A(r) = \frac{\mu_0}{4\pi} \int \frac{J(r')}{r} d\tau'$

loop: $A(r') = \frac{\mu_0 I}{4\pi} \int \frac{dl'}{r}$

Surface: $A(r') = \frac{\mu_0}{4\pi} \int \frac{K(r')}{r} da'$

Dimensions of vector potential:

$$\vec{B} = \nabla \times \vec{A}$$

On integrating with 'l'

$$[B]L = [A]$$

$$[A] = \frac{[F][L]}{[V][q]}$$

$F = qVB$
$\Rightarrow B = \frac{F}{qV}$

$$[A][q] = [F][t]$$

$$[A][q] = \text{Momentum}$$

$$\boxed{[A] = \text{Momentum}/\text{Charge}}$$

Magnetic dipole:

$$A(r) = \frac{\mu_0 I}{4\pi} \int \frac{dl'}{r}$$

General expansion:

$$A(r) = \frac{\mu_0 I}{4\pi} \Big\{ \underbrace{\frac{1}{r}\int dl'}_{\text{Monopole}} + \underbrace{\frac{1}{r^2}\int r'\cos\theta' dl}_{\text{dipole}} + \underbrace{\frac{1}{r^2}\int (r')^2\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right) dl}_{\text{Tripole}} + \cdots$$

$$A_{dipole} = \frac{\mu_0 I}{4\pi r^2}\left\{ \int r'\cos\theta' dl \right\}$$

$$= \frac{\mu_0 I}{4\pi r^2}\left\{ \int \hat{r}\cdot r' dl \right\}$$

$$\boxed{\int \hat{r}\cdot r' dl' = -\hat{r}\times\int da}$$

$$A_{dipole} = \frac{\mu_0 I}{4\pi r^2}\left[-\hat{r}\times\int da\right] \Rightarrow \frac{\mu_0 I}{4\pi r^2}\left[\int da\times\hat{r}\right]$$

$$= \frac{\mu_0}{4\pi r^2}\left[I\int da\times\hat{r}\right] = \frac{\mu_0}{4\pi r^2}(m\times\hat{r})$$

where $\boxed{m = I\int da = IA}$

$$\boxed{\vec{A}_{dipole} = \frac{\mu_0}{4\pi r^2}(m\times\hat{r})}$$ $m \rightarrow$ magnetic moment

Magnetostatics boundary conditions

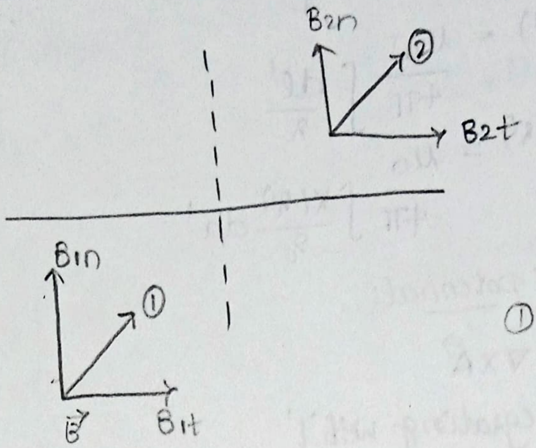
$\nabla \cdot E = \frac{\rho}{\epsilon_0}$

$\oint E \cdot da = \frac{q}{\epsilon_0}$

Now: If

$\nabla \cdot B = 0$

$\oint B\, da = 0$



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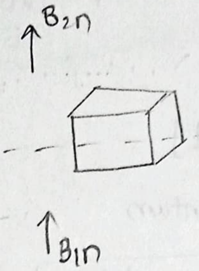
Normal components



Here only top, bottom faces contribute bcoz remaining faces are ⊥ to field, so the $\oint B\, da$ is 0

$$\int B_{2n}\, da - \int B_{1n}\, da = 0$$

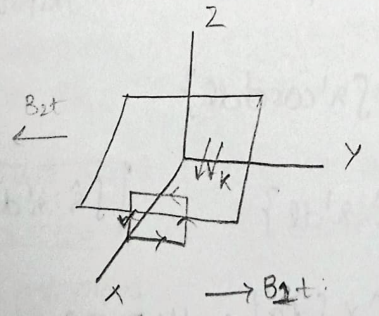
down face (area)

$$\boxed{B_{2n} - B_{1n} = 0}$$

∴ Normal components of magnetic field are continuous.

Tangential components

$\vec{n}$ unit vector normal to interface.



μ_0? μKL

$$\int B_{2t}(-\hat{y}) \cdot (-\hat{y}\, dy) + 0 + \int B_{1t}(\hat{y})(\hat{y} \cdot dy) + 0 = \mu_0 KL$$

$$B_{2t} L + B_{1t} L = \mu_0 KL$$

$$\boxed{B_{2t} - B_{1t} = \mu_0 (\bar{K} \times \hat{n})}$$

Here if we consider either up/down $\hat{n}$ then once we get $-\hat{y}, \hat{y}$. So '-' is taken

∴ Tangential components are discontinuous by $\mu_0(\bar{K} \times \hat{n})$

Bound currents

Magnetisation $M = \frac{m}{V}$

$\nabla \times B = \mu_0 J$

$\nabla \times B = \mu_0 [J_f + J_b]$

$$A_{dip} = \frac{\mu_0}{4\pi} \frac{m \times \hat{r}}{r^2}$$

$$M(r) = \frac{m}{V} \qquad \therefore m = \int_V M(r')\, d\tau'$$

$$A_{dip} = \frac{\mu_0}{4\pi} \int M(r') \times \left(\frac{\hat{r}}{r^2}\right) d\tau' \qquad \nabla\left(\frac{1}{r}\right) = -\frac{\hat{r}}{r^2}$$

$$A_{dip} = -\frac{\mu_0}{4\pi} \int_V M(r') \times \nabla\left(\frac{1}{r}\right) d\tau$$

$$\boxed{\nabla \times \left(\frac{M(r')}{r}\right) = \frac{1}{r} \nabla \times M(r') + M(r') \times \nabla\left(\frac{1}{r}\right)}$$

$$A_{dip} = \frac{\mu_0}{4\pi} \int_V \frac{\nabla \times M(r')}{r} d\tau - \frac{\mu_0}{4\pi} \int_V \nabla \times \left(\frac{M(r')}{r}\right) d\tau$$

$$\boxed{\int_V \nabla \times (v)\, d\tau = -\int_S v \times da}$$

$$A_{dip} = \frac{\mu_0}{4\pi} \int_V \frac{\nabla \times M(r')}{r} d\tau + \frac{\mu_0}{4\pi} \int_S \frac{M(r') \times da}{r}$$

$$\boxed{J_b = \nabla \times M(r')}$$

$$\boxed{K_b = M(r) \times \hat{n}}$$

Now,

$$\nabla \times B = \mu_0 J$$

$$\nabla \times B = \mu_0 [J_f + J_b]$$

$$\nabla \times B = \mu_0 [J_f + \nabla \times M].$$

$M \rightarrow$ Magnetisation

$$\nabla \times \frac{B}{\mu_0} = J_f + \nabla \times M$$

$$\nabla \times \left(\frac{B}{\mu_0} - M\right) = J_f$$

Let $\frac{B}{\mu_0} - M = H$

$H \rightarrow$ Auxilary field

$$\boxed{\nabla \times H = J_f}$$

$$\because \boxed{H = \frac{B}{\mu_0} - M}$$

$$\chi_m = \frac{M}{H} \Rightarrow M = \chi_m H$$

$\chi_m \rightarrow$ magnetic susceptibility

$$\therefore \quad H = \frac{B}{\mu_0} - \chi_m H$$

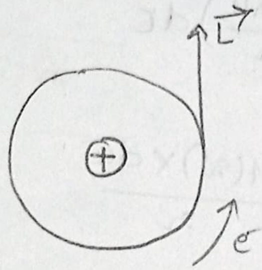
$$\boxed{B = \mu_0(1+\chi_m)H}$$

$$\boxed{B = \mu H}$$

$$\because \mu = \mu_0(1+\chi_m)$$

$$\mu_r = \frac{\mu}{\mu_0} = 1 + \chi_m$$

Magnetic momentum and angular momentum



magnetic moment due to orbital

due to spin of e^-

$\vec{L}$ angular momentum $= m_e v_e r$

e^- so $-ve$

$$m = I\int da = IA = \frac{-e}{t}(\pi r^2)$$

$$= \frac{-e v_e}{2\cancel{\pi} \cancel{r}}(\cancel{\pi} r^2)$$

$$\boxed{m = \frac{-e v_e r}{2}}$$

$$m = \left(\frac{-e}{2m_e}\right)(m_e v_e r)$$

$$\boxed{m = \left(\frac{-e}{2m_e}\right)\vec{L}}$$

$\vec{L} = \frac{h}{2\pi} l$

$l \to$ azimuthal quantum number.

$\vec{L} = \frac{h}{2\pi}, \frac{2h}{2\pi}, \frac{3h}{2\pi} \ldots$ $l = 1, 2, 3$

$\therefore m = \left(\frac{-eh}{4m_e\pi}\right) l$

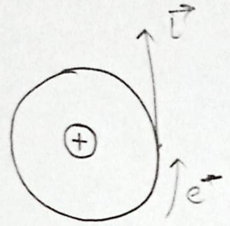
$\boxed{m = \mu_B l}$ μ_B = Bohr magneton

$m \to$ magnetic momentum

$m_L = \mu_B l \Rightarrow$ orbital magnetic moment

$m_S = \mu_B S \Rightarrow$ spin magnetic moment

Total magnetic moment = orbital + spin



without magnetic field:

The e^- is balanced by electrostatic force and centripetal force.

diamagnet
✓
No permanent magnetic moments

$$\frac{Ke^2}{r^2} = \frac{mv_e^2}{r} \quad -①$$

with magnetic field:

The e^- changes its velocity to balance the forces such that

$$F_B + F_e = \frac{mv_e^2}{r}$$

$$\frac{Ke^2}{r^2} + e\bar{v}_e B = \frac{m\bar{v}_e^2}{r} \quad -②$$

① & ②

$$e\bar{v}_e\bar{B} = \frac{m_e[\bar{v}_e^2 - v_e^2]}{r}$$

$$e\bar{v}_e\bar{B} = \frac{m_e}{r}[\bar{v}_e + v_e][\bar{v}_e - v_e]$$

If $v_e \simeq \bar{v}_e$

$$e \cancel{\bar{V}_e} \bar{B} = \frac{m_e}{r} [2 \cancel{\bar{V}_e}] \Delta V_e$$

$$\boxed{eB = \frac{m_e}{r} 2 \Delta V_e} \Rightarrow \boxed{\Delta V_e = \frac{eBr}{2m_e}}$$

Now,

WKT:
$$m = \frac{-eVr}{2} \Rightarrow \Delta m = -\frac{e}{2} \Delta V r$$

$$\boxed{\Delta m = -\frac{e^2 r^2 B}{4m_e}}$$

paramagnetic
↓
extra field to align