21-03-2023.

2: WAVE OPTICS

* **Interference**:- superposition of Two waves.

· **Essential conditions**:-

(1) coherent source.

(a) constant phase difference.

(b) Same Frequency.

Desirable conditions:- [same Amplitude].

→ In Young's Experiment.

(1) Distance b/w two slits is small.

(2) Distance b/w screen and slits is Large.

Ex for Interference:-

Let $y_1 = A_1 \sin\omega t$

$y_2 = A_2 \sin(\omega t + \phi)$.

$y = y_1 + y_2$

$\Rightarrow y = A_1 \sin\omega t + A_2(\sin\omega t \cdot \cos\phi + \cos\omega t \cdot \sin\phi)$.

$\Rightarrow y = (A_1 + A_2\cos\phi)\sin\omega t + A_2 \sin\phi \cos\omega t \longrightarrow (1)$

Let $A_1 + A_2\cos\phi = R\cos\theta$.
$A_2\sin\phi = R\sin\theta$ } (2)

substitute (2) in (1)

$\Rightarrow y = R\sin(\omega t + \theta)$. $\longrightarrow$ (A) [Here, R & θ are Arbitary constants]

From (2),

$\Rightarrow R^2 = (A_1 + A_2\cos\phi)^2 + A_2^2\sin^2\phi$

$\Rightarrow R = \sqrt{(A_1 + A_2\cos\phi)^2 + A_2^2\sin^2\phi}$.

$\Rightarrow R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi\,(\cos^2\phi + \sin^2\phi)}$

$\Rightarrow \boxed{R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}} \longrightarrow (3)$

substitute (3) & (4) in eqn (A).

$\therefore \boxed{y = R\sin(\omega t + \theta)}$

$\dfrac{R\sin\theta}{R\cos\theta} = \dfrac{A_2\sin\phi}{A_1 + A_2\cos\phi}$.

$\Rightarrow \boxed{\theta = \tan^{-1}\left(\dfrac{A_2\sin\phi}{A_1 + A_2\cos\phi}\right)} \longrightarrow (4)$

* Intensity :-

$$\boxed{\text{Intensity} \propto (\text{Amplitude})^2}$$

$$\boxed{I \propto R^2}$$

$$\therefore \boxed{I = R^2 = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\cos\phi}$$

* case-(i) :- constructive Interference :-

Here R_{max} when $\cos\phi = +1$ $[\phi = 0, 2\pi, 4\pi, \ldots 2n\pi]$

$\therefore \boxed{\phi = 2n\pi} \rightarrow$ phase difference

$$\boxed{R_{max} = A_1 + A_2}$$

$$\boxed{\text{Path difference, } \Delta = \frac{\lambda}{2\pi}\,[\text{phase difference}]}$$

$\therefore \Delta = \frac{\lambda}{2\pi}(2n\pi) \rightarrow \boxed{\Delta = n\lambda}$

if $A_1 = A_2 = A$

$\therefore \boxed{R_{max} = 2A}$

$\therefore \boxed{I_{max} = 4A^2}$

* case-(ii) :- Destructive Interference :-

Here R_{min} when $\cos\phi = -1$ $[\phi = \pi, 3\pi, 5\pi, \ldots (2n+1)\pi]$

$\therefore \boxed{\phi = (2n+1)\pi}$ phase diff $\therefore \Delta = \frac{\lambda}{2\pi}(2n+1)\pi$

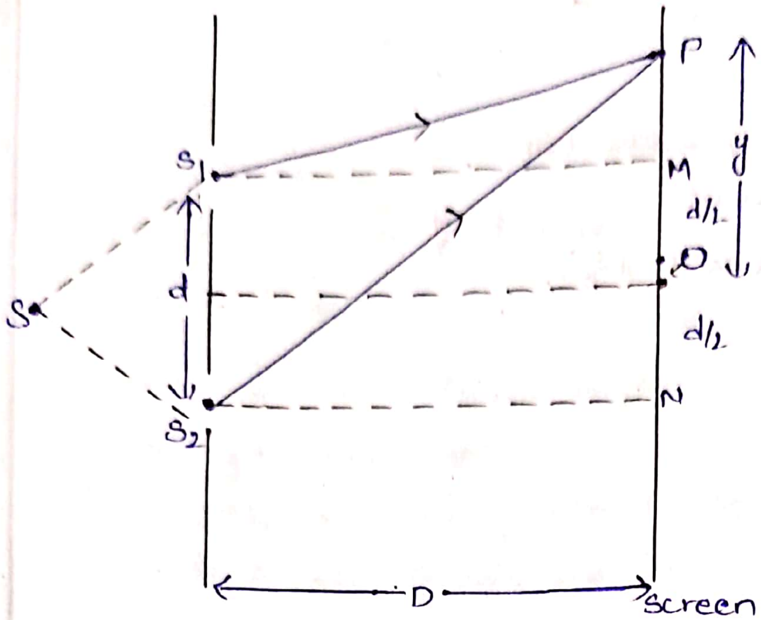
$$\boxed{\Delta = \frac{(2n+1)\lambda}{2}}$$ Path diff

$R_{min} = \sqrt{A_1^2 + A_2^2 - 2A_1A_2}$ $\therefore \boxed{R_{min} = |A_1 - A_2|}$

If $A_1 = A_2 = A$

$\boxed{R_{min} = 0}$, $\boxed{I_{min} = 0}$

* Young's Double Slit Experiment :- [Division of Wavefront]



Path difference,

$$\Delta = S_2P - S_1P = \sqrt{D^2 + \left(y+\frac{d}{2}\right)^2} - \sqrt{D^2 + \left(y-\frac{d}{2}\right)^2}.$$

$$\Rightarrow \Delta = D\left[\sqrt{1 + \frac{\left(y+\frac{d}{2}\right)^2}{D^2}} - \sqrt{1 + \frac{\left(y-\frac{d}{2}\right)^2}{D^2}}\right]$$

$$\Rightarrow \Delta = D\left[\cancel{1} + \frac{\left(y+\frac{d}{2}\right)^2}{2D^2} - \cancel{1} - \frac{\left(y-\frac{d}{2}\right)^2}{2D^2}\right].$$

$$\Rightarrow \Delta = \frac{\cancel{D}}{2\cancel{D}^2}\left[\left(y+\frac{d}{2}\right)^2 - \left(y-\frac{d}{2}\right)^2\right].$$

$$\Rightarrow \Delta = \frac{1}{2D}\left[\left(y+\frac{d}{2}\right)^2 - \left(y-\frac{d}{2}\right)^2\right]. \quad [\because (a+b)^2 - (a-b)^2 = 4ab]$$

$$\Rightarrow \Delta = \frac{1}{\cancel{2}D}\left[\cancel{4}y\left(\frac{d}{\cancel{2}}\right)\right].$$

$$\Rightarrow \Delta = \frac{yd}{D}.$$

$\therefore$ Path difference, $\boxed{\Delta = \frac{yd}{D}.}$

For constructive interference [Bright Fringe] :-

$\Delta = n\lambda \quad \Rightarrow \frac{yd}{D} = n\lambda \quad \therefore \boxed{y_n = \frac{n\lambda D}{d}}$ (n^{th} band)

$\boxed{\beta = \frac{\lambda D}{d}}$ Fringe width

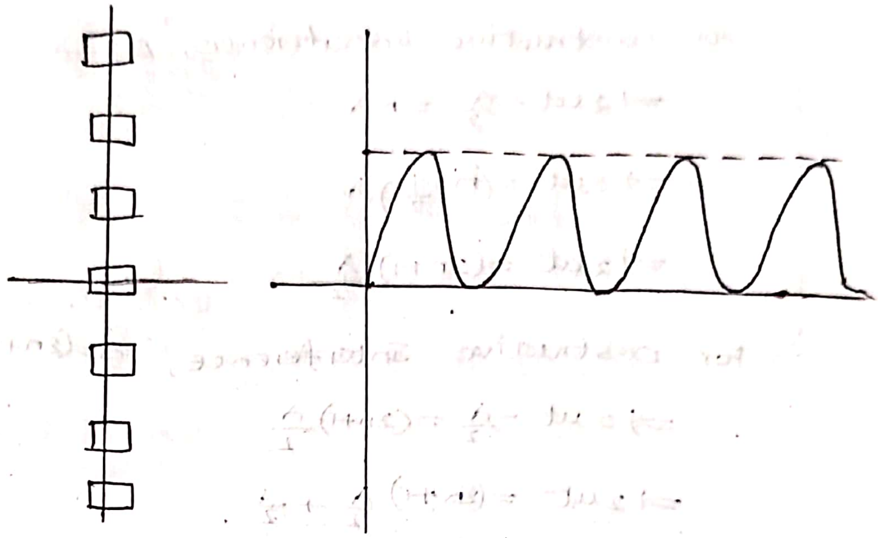
For Destructive Interference

$\Delta = (2n+1)\frac{\lambda}{2} \quad \Rightarrow \frac{yd}{D} = (2n+1)\frac{\lambda}{2}. \quad \therefore \boxed{y_n = (2n+1)\frac{\lambda D}{2d}}$

* conclusions :-

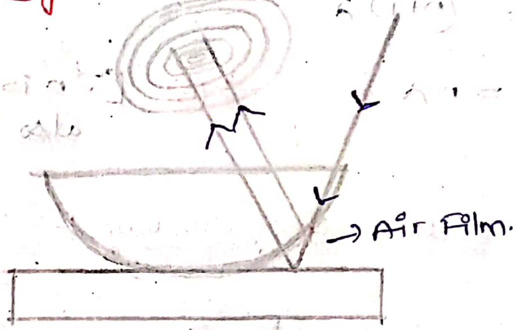
① 'Fringe width' is same for 'Bright' and 'Dark Fringes'.

② In Interference pattern 'Amplitude' is same.

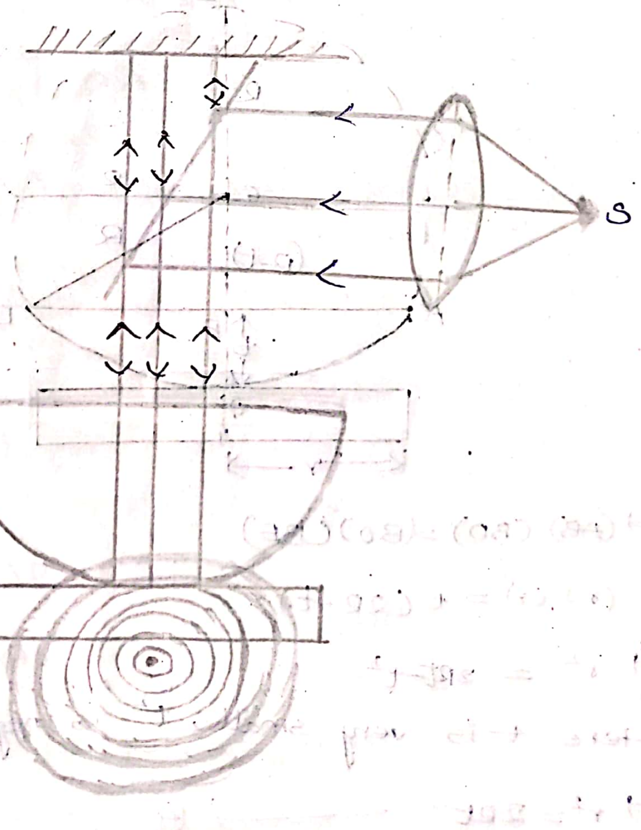


* Newton's Rings :- [Division of Amplitude].

① Basic :-



② Experimental :-



$$\Rightarrow \boxed{\Delta = 2\mu t \cos(r+\theta) - \frac{\lambda}{2}}$$

For Normal Incidence, $r+\theta = 0$.

$$\Rightarrow \cos(r+\theta) \simeq 1$$

$$\Rightarrow \Delta = 2\mu t - \frac{\lambda}{2} \longrightarrow (1)$$

For constructive interference, $\Delta = n\lambda$.

$$\Rightarrow 2\mu t - \frac{\lambda}{2} = n\lambda$$

$$\Rightarrow 2\mu t = \left(n+\frac{1}{2}\right)\lambda.$$

$$\Rightarrow 2\mu t = (2n+1)\frac{\lambda}{2} \longrightarrow (2)$$

For Destructive Interference, $\Delta = (2n+1)\frac{\lambda}{2}$.

$$\Rightarrow 2\mu t - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

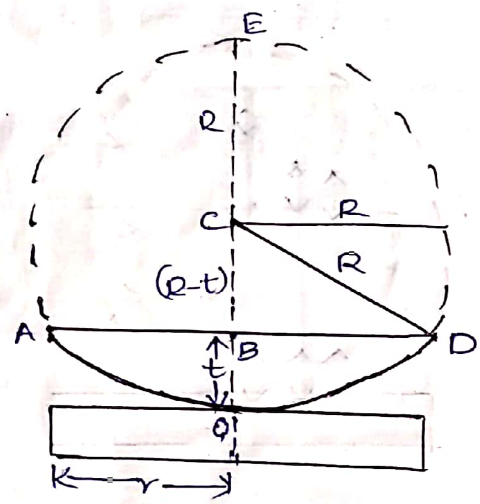
$$\Rightarrow 2\mu t = (2n+1)\frac{\lambda}{2} + \frac{\lambda}{2}$$

$$\Rightarrow 2\mu t = [(2n+1)+1]\frac{\lambda}{2}$$

$$\Rightarrow 2\mu t = (n+1)\lambda.$$

$$\Rightarrow 2\mu t = n\lambda \longrightarrow (3)$$ [$\because$ 'n' is integer, 'n+1' is also integer].

To find the Radius of curvature :-



$$\Rightarrow (AB)(BD) = (BO)(BE)$$

$$\Rightarrow (r)(r) = t(2R-t)$$

$$\Rightarrow r^2 = 2Rt - t^2$$

Here t is very small, 't^2' is neglected

$$\Rightarrow r^2 = 2Rt. \longrightarrow (4)$$

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$$\Rightarrow t = \frac{r^2}{2R} \longrightarrow (5)$$

For Bright Fringe / Bright Ring :-

Substitute 't' in eqn (2)

$$\Rightarrow 2\mu\left[\frac{r^2}{2R}\right] = (2n+1)\frac{\lambda}{2}$$

$$\Rightarrow \boxed{r_n^2 = \frac{(2n+1)\lambda R}{2\mu}} \longrightarrow (6)$$

Now, $r_n = \frac{D_n}{2}$ $[\because D_n$ is Diameter$]$

$$\Rightarrow \frac{D_n^2}{4} = \frac{(2n+1)\lambda R}{2\mu}$$

$$\Rightarrow D_n^2 = \frac{2(2n+1)\lambda R}{\mu}$$

For Air, $\mu = 1$

$$\Rightarrow D_n^2 = 2(2n+1)\lambda R$$

$$\Rightarrow D_n = \sqrt{2\lambda R} \cdot \sqrt{2n+1}$$

$$\therefore \boxed{D_n \propto \sqrt{2n+1}} \quad [\text{For Bright Ring}]$$

For Dark Ring :-

Substitute 't' in eqn (3)

$$\Rightarrow 2\mu\left[\frac{r^2}{2R}\right] = n\lambda$$

$$\Rightarrow \boxed{r_n^2 = \frac{n\lambda R}{\mu}}$$

$$\Rightarrow \frac{D_n^2}{4} = \frac{n\lambda R}{\mu} \quad \left[\because r = \frac{D}{2}\right]$$

$$\Rightarrow D_n^2 = \frac{4n\lambda R}{\mu}$$

For Air, $\mu = 1$

$$\Rightarrow D_n^2 = 4n\lambda R$$

$$\Rightarrow D_n = \sqrt{4\lambda R} \cdot \sqrt{n}$$

$$\therefore \boxed{D_n \propto \sqrt{n}} \quad [\text{For Dark Ring}]$$

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Radius of curvature :-

We know, $D_n^2 = 4n\lambda R$

For p^{th} Ring, $D_{n+p}^2 = 4(n+p)\lambda R.$

$\Rightarrow D_{n+p}^2 - D_n^2 = 4(n+p)\lambda R - 4n\lambda R$

$= 4n\lambda R + 4p\lambda R - 4n\lambda R$

$= 4p\lambda R.$

$$\therefore \boxed{R = \frac{D_{n+p}^2 - D_n^2}{4p\lambda}}.$$

For Na, $\lambda = 5893 A^\circ.$

For wave length :-

$$\boxed{\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}}$$

For Refractive Index of Liquid :-

We have, $(D_n^2)_{air} = 4n\lambda R.$

$\Rightarrow (D_n^2)_{liquid} = \frac{4n\lambda R}{\mu}.$

$(D_{n+p}^2 - D_n^2)_{air} = 4p\lambda R.$

$(D_{n+p}^2 - D_n^2)_{Liquid} = \frac{4p\lambda R}{\mu}.$

By Dividing the above two eq^{n}s.

$$\Rightarrow \frac{(D_{n+p}^2 - D_n^2)_{air}}{(D_{n+p}^2 - D_n^2)_{Liquid}} = \frac{\cancel{4p\lambda R}}{\frac{\cancel{4p\lambda R}}{\mu}}$$

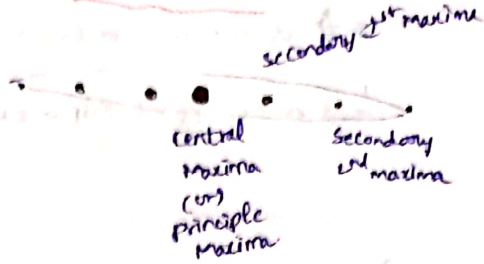
$\therefore$ Refractive Index of Liquid,

$$\boxed{\mu = \frac{(D_{n+p}^2 - D_n^2)_{air}}{(D_{n+p}^2 - D_n^2)_{Liquid}}}$$

* Diffraction :

→ The phenomenon of Bending of light into its Geometrical shape (or) shadow is called Diffraction.

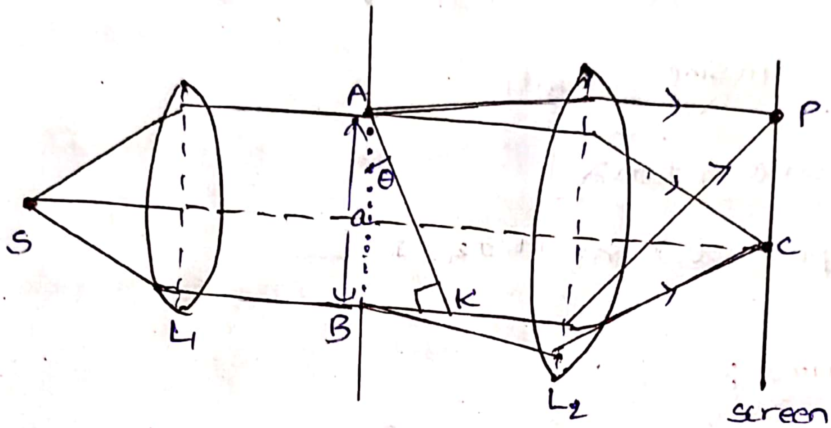
* Types of Diffraction :



① Fraenne l Diffraction.

② Fraun Hoffer Diffraction.

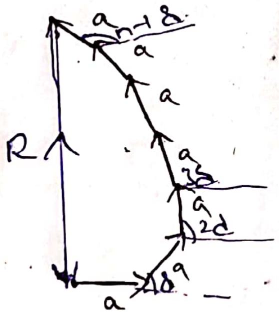
* Single slit [Fraun Hoffer] Diffraction :



Here path difference, 'BK = a sinθ'

phase difference = $\frac{2\pi}{\lambda}(a\sin\theta)$.

Vector Theorem :



$$\boxed{R = A \cdot \frac{\sin\alpha}{\alpha}}$$

Here, $A = na$

$\alpha = \frac{n\delta}{2}$.

Now, $\delta = \frac{2\pi a \sin\theta}{n\lambda}$ [∵ Two successive sources].

$R = A\frac{\sin\alpha}{\alpha}$

$\Rightarrow I = R^2 = A^2\frac{\sin^2\alpha}{\alpha^2}$

For maxima & minima intensity $\frac{dI}{d\alpha} = 0$.

Now, $\frac{dI}{d\alpha} = A^2\left[\frac{\alpha^2\, 2\sin\alpha\cos\alpha - 2\alpha\sin^2\alpha}{\alpha^4}\right] = 0.$

$\Rightarrow 2\alpha\sin\alpha\left[\alpha\cos\alpha - \sin\alpha\right] = (0)A^2\alpha^4.$

$\Rightarrow \alpha\sin\alpha\left[\alpha\cos\alpha - \sin\alpha\right] = 0.$

$\Rightarrow \alpha = 0$ (or) $\sin\alpha = 0$ (or) $\alpha\cos\alpha = \sin\alpha.$

$\alpha = \tan\alpha.$

For Minima :-

$\sin\alpha = 0$

$\Rightarrow \alpha = \pm m\pi.$

Here $m = 1, 2, 3, \ldots$ (except $m=0$).

$\Rightarrow \alpha = \frac{\pi a\sin\theta}{\lambda} = \pm m\pi.$

$\therefore \boxed{a\sin\theta = \pm m\lambda.}$

(except $m=0$,) $m = \pm 1, \pm 2, \pm 3$ ——

For Maxima :-

(i) For principle maxima, $\alpha = 0.$

We know intensity, $I = A^2\frac{\sin^2\alpha}{\alpha^2}.$

$\Rightarrow I = A^2 \lim_{\alpha\to 0}\frac{\sin^2\alpha}{\alpha^2}$

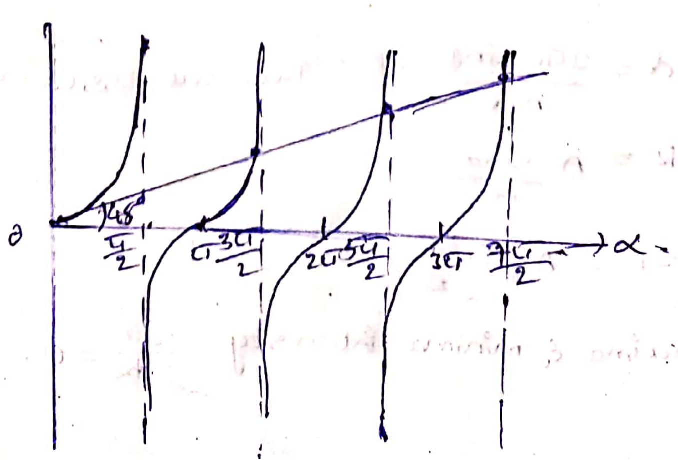
$\therefore \boxed{I = A^2}$ $[\because = 1]$

$\therefore$ It is maximum instant.

(ii) secondary maxima :-

$\alpha = \tan\alpha$

Let $y = \alpha = \tan\alpha$, $y = \alpha$, $y = \tan\alpha$.



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$$\boxed{\alpha = \frac{\pi a \sin\theta}{\lambda} = \pm (2m+1)\frac{\pi}{2}}$$

Here $m = \pm 1, \pm 2, \pm 3, \ldots$

$$\Rightarrow \boxed{a \sin\theta = \pm (2m+1)\frac{\lambda}{2}}$$ secondary maxima's.

if $m=1$, $\alpha = \frac{3\pi}{2}$ [secondary first maxima]

$m=2$, $\alpha = \frac{5\pi}{2}$ [secondary second maxima]

$m=3$, $\alpha = \frac{7\pi}{2}$ [secondary Third maxima].

$$I_1 = A^2 \cdot \frac{\sin^2\alpha}{\alpha^2} = \frac{A^2 \sin^2 \frac{3\pi}{2}}{\left(\frac{3\pi}{2}\right)^2} = A^2 \frac{4}{9\pi^2} = \left(\frac{4}{9\pi^2}\right) I_0$$

$\therefore I_1 = 4.5\%$ of I_0.

$$I_2 = A^2 \frac{\sin^2\alpha}{\alpha^2} = A^2 \frac{\sin^2\left(\frac{5\pi}{2}\right)}{\left(\frac{5\pi}{2}\right)^2} = \left(\frac{4}{25\pi^2}\right) I_0 = 1.5\% \text{ of } I_0$$

$$I_3 = A^2 \cdot \frac{\sin^2\left(\frac{7\pi}{2}\right)}{\left(\frac{7\pi}{2}\right)^2} = \frac{4}{49\pi^2} I_0$$

$$\therefore I_0 : I_1 : I_2 : I_3 : I_4 \ldots = 1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \frac{4}{49\pi^2} : \frac{4}{81\pi^2}$$

Graph :-