24th sept,
WEDNESDAY

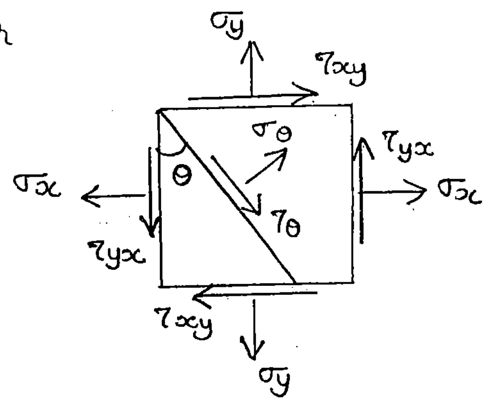
02 COMPLEX STRESS & STRAINS

→ 2D (or) Biaxial (or) Plane Stress system

All the stresses will be developing in one perpendicu plane only.

Eg: Beams, shafts, any thin member

2D Stress Tensor :- $\begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{bmatrix}_{2\times2}$



⊙ In a member (or element) normal stresses are balanced by force equilibrium, shear stresses are balanced by moment equilibrium.

For moment equilibrium, $\tau_{xy} = \tau_{yx}$.

∴ for a 2D stress tensor, there will be a total of 4 stress components available. Among them, 3 are independent components.

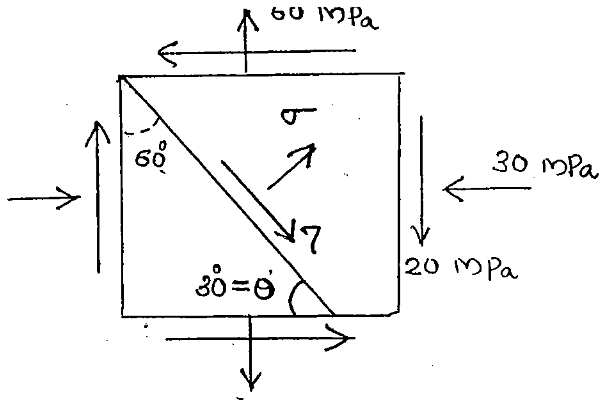
If horizontal shear stress is due to external loads, a vertical shear stress of opposite nature develops for balancing called complementary shear stress.

Stress components on Inclined Plane :

$$\boxed{\begin{aligned} \sigma_\theta &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \\ \tau_\theta &= \frac{\sigma_x - \sigma_y}{2}\sin 2\theta - \tau_{xy}\cos 2\theta \end{aligned}}$$

NOTE : Above formulas are valid only for the given basic element.

Q.



$\sigma_x = -30$ MPa

$\sigma_y = 60$ MPa

$\tau_{xy} = -20$ MPa.

$\theta = 60°$

$$\sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta.$$

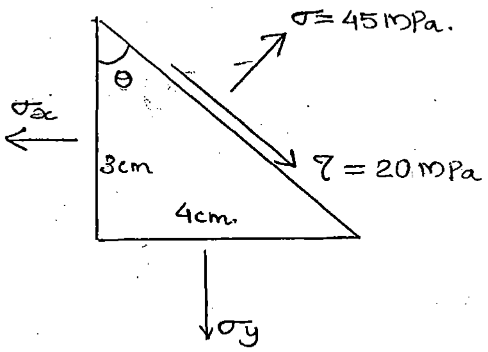
$$= \frac{-30+60}{2} + \frac{-30-60}{2}\cos 2(60°) + -20\sin 2(60) = \underline{\underline{20.18}} \text{ MPa}$$

$$\tau_\theta = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta - \tau_{xy}\cos 2\theta.$$

$$= \frac{-30-60}{2}\sin(2\times 60) - -20\cos 2(60).$$

$= \underline{\underline{-48.97}}$ MPa (−ve means shear should be opp. direction)

Q.



$\tan\theta = \frac{4}{3}$

$\theta = \underline{\underline{53.13°}}$

$\tau_{xy} = 0$

$$45 = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos(2\times 53.13) + 0.$$

$$90 = \sigma_x + \sigma_y + (\sigma_x - \sigma_y)\times -0.28$$

$$= 0.72\,\sigma_x + 1.28\,\sigma_y. \longrightarrow (1)$$

$$20 = \frac{\sigma_x - \sigma_y}{2}\sin(2\times 53.13) - 0.$$

$$40 = 0.96\,\sigma_x - 0.96\,\sigma_y. \longrightarrow (2)$$

$\sigma_x = 71.66$ MPa

$\sigma_y = \underline{\underline{30}}$ MPa

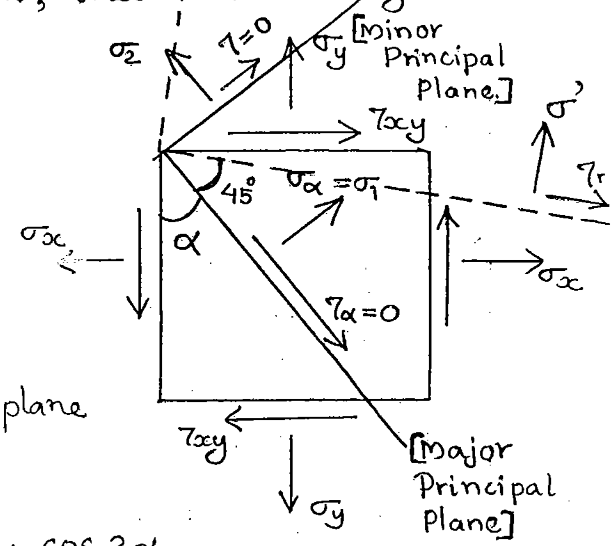
→ Principal Stresses.

$$\left.\begin{matrix}\text{Major, } \sigma_1 \\ \text{Minor, } \sigma_2\end{matrix}\right\} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

The normal stress across the principal plane is principal str

→ Principal Planes.

- The plane on which only principal (normal) stress will be acting.
- On principal plane, shear stress is zero.
- If shear stress is zero on a plane, on the perpendicu' plane also shear stress is zero.
- In 2D system, there will be two mutually perpendicul principal planes. On both the planes, shear / stress is zero.



* To locate principal plane :

Assume principal plane is making an angle α as shown. Shear stress on that plane must be zero if its a principal plane

$$\tau_\alpha = 0 = \frac{\sigma_x - \sigma_y}{2} \sin 2\alpha - \tau_{xy} \cos 2\alpha.$$

$$\boxed{\tan 2\alpha = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}}$$

α → angle of major principal plane

$(\alpha + 90)$ → angle of minor principal plane.

* Max Shear Stress :

$$\boxed{\tau_{max} = \pm\left[\frac{\sigma_1 - \sigma_2}{2}\right] = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

– In 2D system, there'll be two max. shear stresses of equal magnitude but opposite in nature

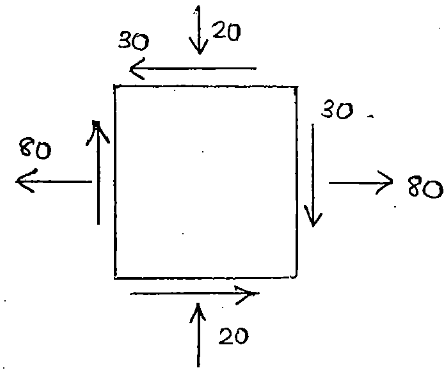
* Maximum Shear Stress Planes.

– The plane on which maximum shear stress is acting. In 2D system, there will be ~~be~~ two τ_{max} planes seperated by 90°.

– The angle b/w any principal plane and the nearest τ_{max} plane is 45°.

– On the τ_{max} plane, there may be normal stress which is equal to σ' or $\sigma_{avg} = \dfrac{\sigma_1+\sigma_2}{2} = \dfrac{\sigma_x+\sigma_y}{2}$

– If $\sigma' = 0$, then its called 'Pure shear Stress'. (On τ_{max} plane, only shear stress alone will be acting.)



Q. Calculate σ_1, σ_2, τ_m, σ'

$\tau_{xy} = -30$

$\sigma_x = +80$

$\sigma_y = -20$

$$\sigma_1 = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{80+(-20)}{2} + \sqrt{\left(\frac{80+20}{2}\right)^2 + (-30)^2}$$

$$= 30 + 58.309 = 88.31 \text{ kPa.}$$

$$\sigma_2 = 30 - 58.309 = -28.309 \text{ kPa}$$

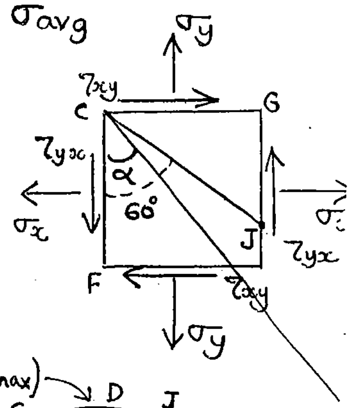
$$\tau_m = \frac{\sigma_1-\sigma_2}{2} = \frac{88.31-(-28.309)}{2} = 58.309$$

$$\sigma' = \frac{\sigma_1+\sigma_2}{2} = \frac{88.31+-28.31}{2} = \underline{\underline{30}}$$

3rd Oct, Friday

→ Mohr's Circle

- Graphical method given by Otto Mohr
- Basically developed for 2D (plane) stress system.
- Centre of Mohr Circle lies on x-axis where normal stress is represented. The distance of centre of Mohr circle from origin is OC = σ' or σ_{avg}



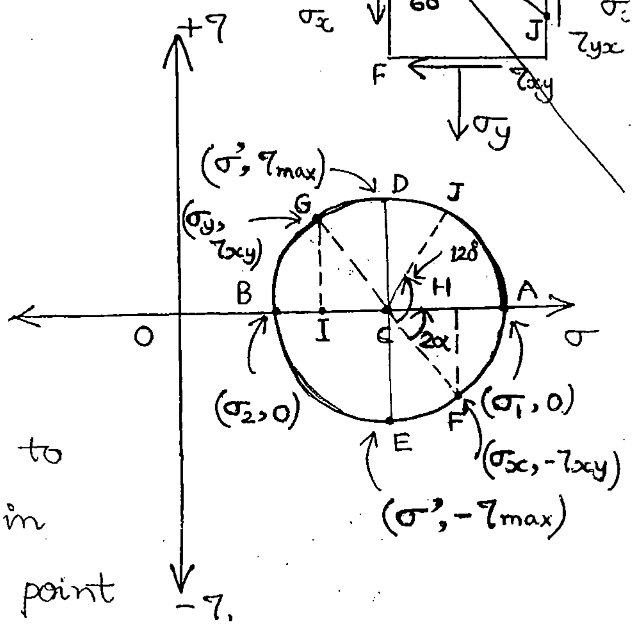
$$OC = \sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{\sigma_1 + \sigma_2}{2}$$

- Radius of Mohr circle,

$$R = \tau_{max}$$

$$= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

$$= \frac{\sigma_1 - \sigma_2}{2}$$



- Each radial line drawn to the Mohr circle is a plane in the material or element. The point on the circle corresponding to the radial line gives the co-ordinates of normal and shear stresses on the plane

CA : Major Principal Plane

CB : Minor Principal Plane

CD & CE : τ_{max} Plane.

- All the angles at the centre of Mohr Circle are twice of actual

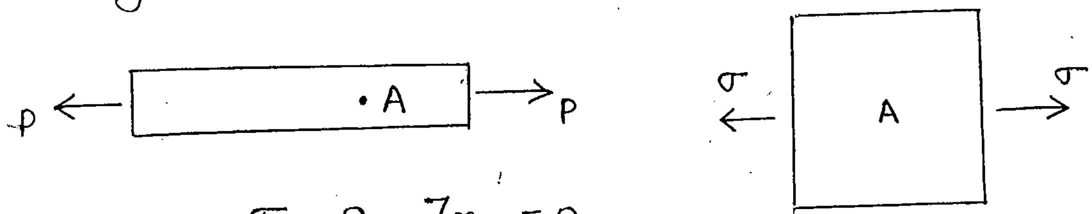
OH = σ_x & HF = $-\tau_{xy}$ (anti-cw)

OI = σ_y & IG = $+\tau_{xy}$ (clock-wise).

* Special Cases :

(i) 1 D

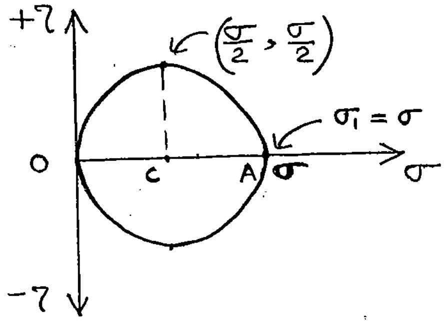
Eg: Tie, strut.



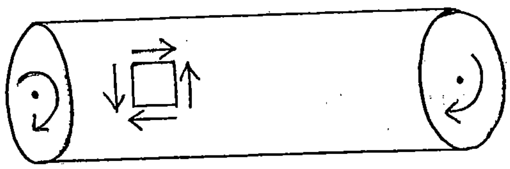
$\sigma_x = \sigma$, $\sigma_y = 0$, $\tau_{xy} = 0$.

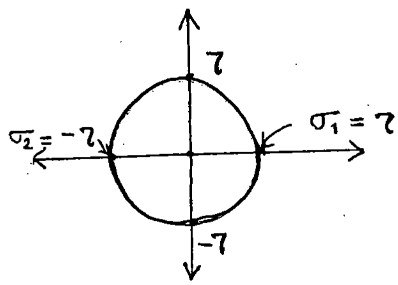
$OC = \frac{\sigma_x + \sigma_y}{2} = \underline{\underline{\frac{\sigma}{2}}}$; Radius $= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \underline{\underline{\frac{\sigma}{2}}}$



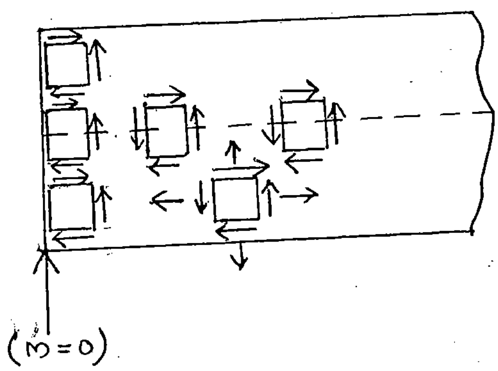
(ii) Pure Shear





If $\sigma = 0$ on τ_{max} plane, it is Pure Shear condition.

- any element on the axis of a beam
- element on surface of shaft.
- any element at the support of a beam.



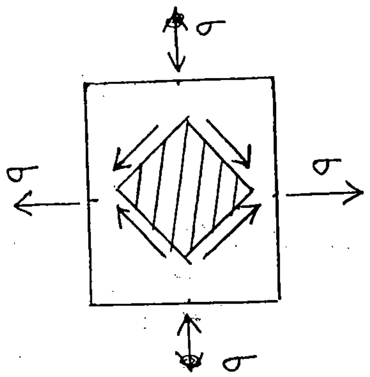
$\sigma_x = 0$, $\sigma_y = 0$, $\tau_{xy} = \tau$

$OC = \sigma' = 0$.

Radius, $\tau_{max} = \tau$.

*If centre of Mohr circle coincides with origin, it is a Pure Shear condition

(iii)



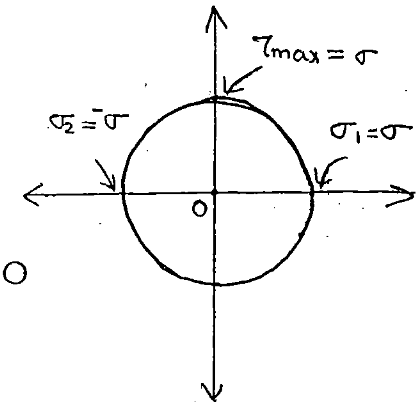
$\sigma_x = \sigma$

$\sigma_y = -\sigma$

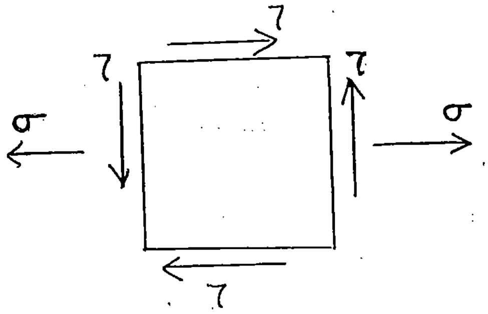
$\tau_{xy} = 0$

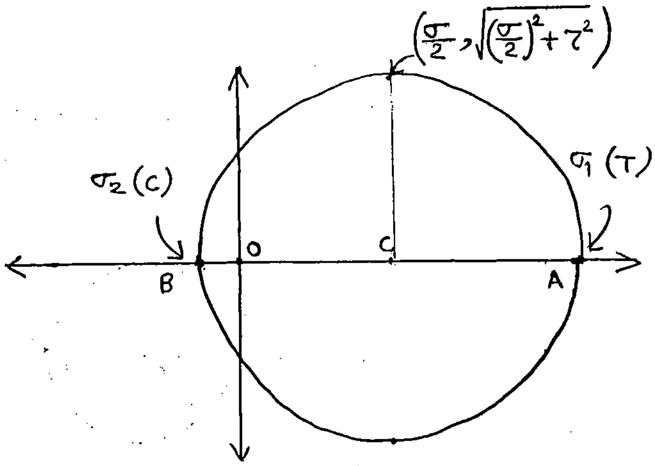
$OC = \sigma' = 0$

$\tau_{max} = \sigma$



(iv) Beams.





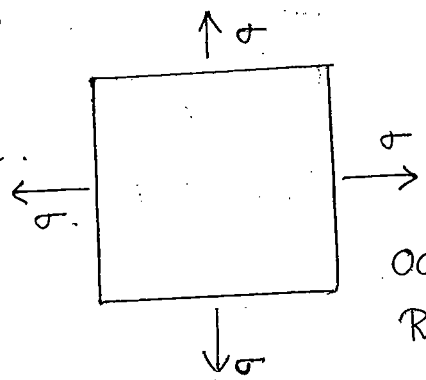
Even though transverse load is applied on the beam, which is normal to the axis of beams, the shear stress will develop b/w layers and tension or compression will act along the axis of the beam. The normal stress in the direction of load is always zero in beams.

$\sigma_x = \sigma$, $\sigma_y = 0$, $\tau_{xy} = \tau$.

$OC = \sigma' = \frac{\sigma}{2}$ & Radius, $\tau_{max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$

* In beams, Principal stress will be opposite in nature. because of bending, one face of beam is under tension and the other face is under compression

(V). Isotropic Condition.



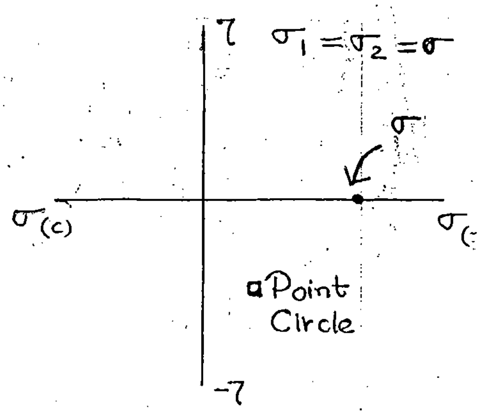
$\sigma_x = +\sigma$

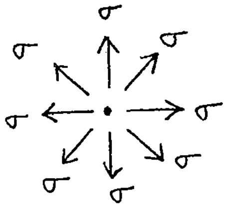
$\sigma_y = +\sigma$

$\tau_{xy} = 0$.

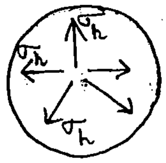
$OC = \sigma$

Radius $= 0$



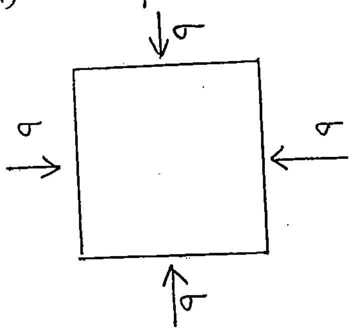


Isotropic condition.



- On the surface of a thin sphere, at a point in all the directions, only hoop tension will be acting without shear stress. called Isotropic condition.

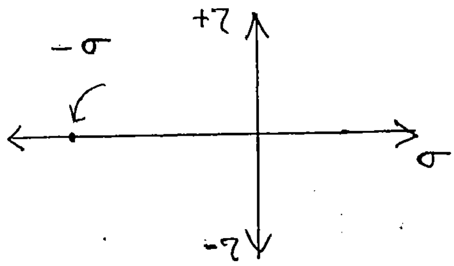
(vi) Isotropic condition.

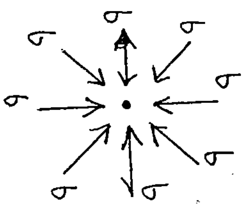


$\sigma_x = -\sigma$

$\sigma_y = -\sigma$

$\tau_{xy} = 0.$

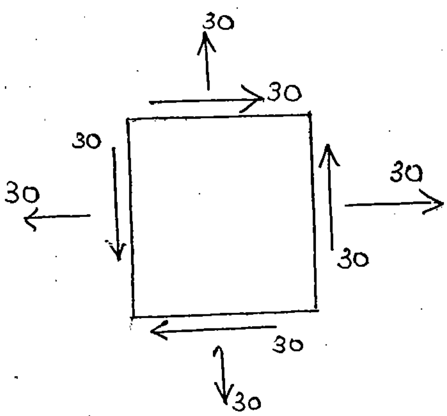




Hydrostatic pressure condition

- On a submerged body under hydrostatic pressure condition shear stress is zero. There will be only change in volume without distortion in shape.

(vii)



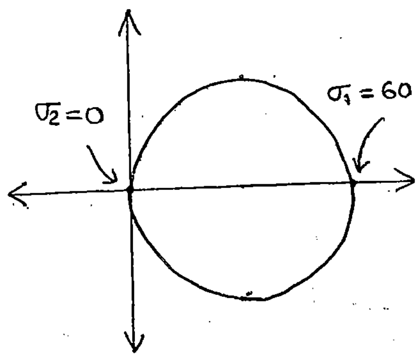
$\sigma_x = 30$

$\sigma_y = 30$

$\tau_{xy} = 30$

$\sigma = 30$

Radius $= 30.$

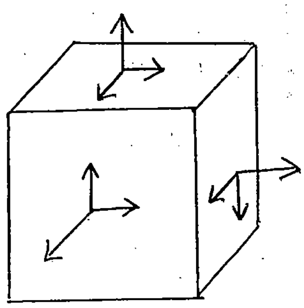


THURSDAY

October

→ 3D Stress System

$$\begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}_{3\times3}$$



For symmetry of stress tensor:

$\tau_{xy} = \tau_{yx}$

$\tau_{xz} = \tau_{zx}$

$\tau_{zy} = \tau_{yz}$

	3D	2D	1D
Total stress components	9	4	1
Independent components.	6	3	1

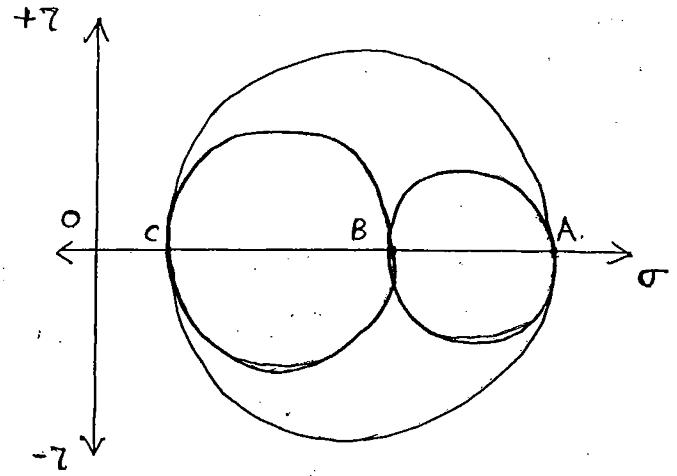
2D (Plane Stress)

$$\begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{bmatrix}_{2\times2}$$

1D (uni-axial)

$$\begin{bmatrix} \sigma \end{bmatrix}_{1\times1}$$

* 3D Mohr Circle:



$\sigma_1 \rightarrow$ major (OA)

$\sigma_2 \rightarrow$ intermediate (OB).

$\sigma_3 \rightarrow$ minor (OC)

τ_{max} in 3D = max. radius

$$= \frac{AC}{2}$$

$$= \frac{OA - OC}{2}$$

$$\boxed{\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}}$$

Eg 1: Principal stresses 40, 20, 10 MPa.

$$\tau_{max\,(3D)} = \frac{40-10}{2} = \underline{\underline{15 \text{ MPa}}}$$

Eg 2: Principal stresses 30 MPa, 50 MPa.

τ_{max} in 2D (Plane stress system) $= \frac{50-30}{2} = 10$ MPa.

$\tau_{max} = \frac{50-0}{2} = 25$ MPa

- In a problem, if only τ_{max} is asked to calculate, it should be based on 3D only. If only two principal stresses are given in the problem consider the third principal stress (σ_3) as zero

Eg: 3 Principal stresses :- 50 MPa & -20 MPa.

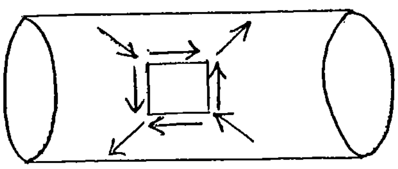
τ_{max} in 2D $= \frac{50-(-20)}{2} = 35$ MPa.

$\tau_{max} = \frac{50+20}{2} = \underline{\underline{35 \text{ MPa}}}$

$\sigma_1 = 50$ MPa
$\sigma_2 = 0$
$\sigma_3 = -20$ MPa.

- If principal stresses are opposite in nature (one tensile & the other compressive), $\tau_{max (2D)} = \tau_{max (3D)}$

Such a case will arise in beams, shafts or any member subjected to bending except thin cylinders and spheres.



$\left.\begin{matrix} \sigma_1 = +\tau \\ \sigma_3 = -\tau \end{matrix}\right\} 2D$ $\left.\begin{matrix} \sigma_1 = +\tau \\ \sigma_2 = 0 \\ \sigma_3 = -\tau \end{matrix}\right\} 3D$

Eg 4: Principal stresses -30 MPa, -80 MPa.

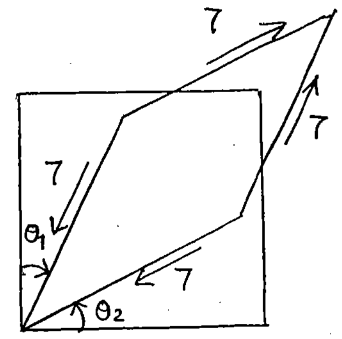
τ_{max} in 2D $= \frac{-30-(-80)}{2} = \underline{\underline{25 \text{ MPa}}}$

$\tau_{max} = \frac{0-(-80)}{2} = \underline{\underline{40 \text{ MPa}}}$

$\left.\begin{matrix} \sigma_1 = 0 \\ \sigma_2 = -30 \\ \sigma_3 = -80 \end{matrix}\right\} 3D.$

→ Strain Analysis (2D)

Stresses	σ_x	σ_y	τ_{xy}
Strain	ϵ_x	ϵ_y	$\phi_{xy}/2$



Shear strain is the angular deformation b/w two mutually ⊥r planes in radians.

$$\phi = \theta_1 + \theta_2$$

For square elements (due to symmetry)

$$\theta_1 = \theta_2$$

$$\Rightarrow \phi = \theta_1 + \theta_1 = 2\theta_1 = 2\theta_2$$

$$\therefore \boxed{\theta_1 = \frac{\phi}{2} \quad \& \quad \theta_2 = \frac{\phi}{2}}$$

→ Strain on Inclined Plane :

$$\epsilon_\theta = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\phi_{xy}}{2}\sin 2\theta .$$

$$\frac{\phi_\theta}{2} = \frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta - \frac{\phi_{xy}}{2}\cos 2\theta .$$

→ Principal Strains :

$$\left.\begin{matrix}\epsilon_1 \\ \epsilon_2\end{matrix}\right\} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\phi_{xy}}{2}\right)^2}$$

- The Plane on which shear stress and the corresponding shear strain is zero. On the same planes, both Principal stresses and corresponding Principal strains will be acting.

* $\tan(2\alpha) = \dfrac{2\left(\frac{\phi_{xy}}{2}\right)}{\epsilon_x - \epsilon_y}$

* Maximum shear strain (ϕ_{max})

$$\frac{\phi_{max}}{2} = \frac{\epsilon_1 - \epsilon_2}{2}$$

$$\Rightarrow \boxed{\phi_{max} = \epsilon_1 - \epsilon_2}$$

→ Strain Gauges

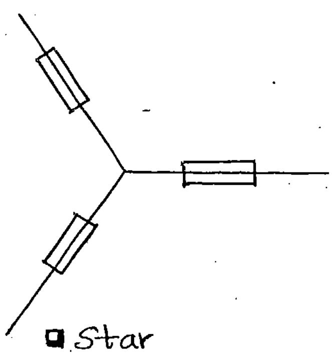
No: of strain gauges required:

$$\left.\begin{array}{l} 1D \rightarrow 1 \text{ no.} \\ 2D \rightarrow 3 \text{ no.} \\ 3D \rightarrow 6 \text{ no.} \end{array}\right\} \text{no. of independent stress components.}$$

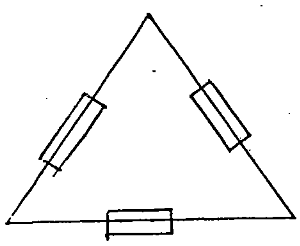
* Types:
(i) Mechanical.
(ii) Electrical.
(iii) Digital.

* Strain Rosette.

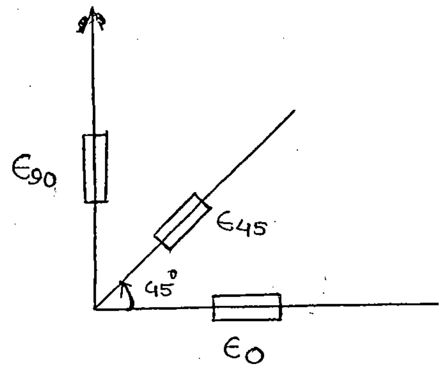
The arrangement of strain gauges to obtain relevant strain values is called Strain rosette.



▫ Star



▫ Delta



▫ Rectangular

Step 1: Read 3 strain gauge values

Step 2: Calculate ϵ_x, ϵ_y, ϕ_{xy}

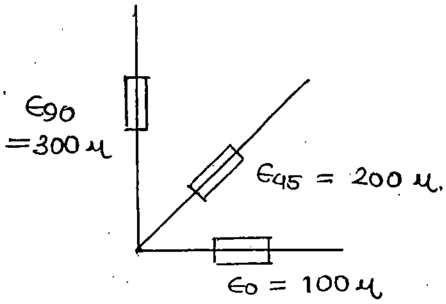
Step 3: P- strains ϵ_1 & ϵ_2

Step 4: P- stresses using E & μ

Step 5: $\sigma_1 \not> $ permissible stress.

Strain values on a rectangular strain rosette are shown in fig. Determine principal stresses, if $E = 2 \times 10^5$ MPa and $\mu = 0.3$. Also check the safety of the member if permissible stress in the material is 200 MPa.

$$\epsilon_\theta = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\phi_{xy}}{2}\sin 2\theta.$$



Use $\theta = 0$, $\epsilon_0 = 100\mu$

$$100\mu = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} + 0.$$

$$\boxed{\because\ \epsilon_{45} = \frac{\epsilon_0 + \epsilon_{90}}{2} \quad \Rightarrow \epsilon_1 = \epsilon_{90} \quad \epsilon_2 = \epsilon_0}$$

Use $\theta = 90$, $\epsilon_{90} = 300\mu$.

$$300\mu = \frac{\epsilon_x + \epsilon_y}{2} + \left(\frac{\epsilon_x - \epsilon_y}{2}\right) \times -1 + 0.$$

$$\Rightarrow \epsilon_x = 100\mu \quad \& \quad \epsilon_y = 300\mu.$$

Use $\theta = 45$, $\epsilon_{45} = 200\mu$.

$$\epsilon,\ 200\mu = \frac{\epsilon_x + \epsilon_y}{2} + 0 + \frac{\phi_{xy}}{2}$$

$$\Rightarrow \phi_{xy} = 0$$

$$\epsilon_1 = \frac{\epsilon_x + \epsilon_y}{2} + \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\phi_{xy}}{2}\right)^2}$$

$$= 200\mu + \frac{100 - 300}{2}$$

$$= 100\mu.$$

$$\epsilon_2 = 200\mu - \frac{100 - 300}{2} = 300\mu.$$

$$\therefore \epsilon_1 = 300\mu \quad \& \quad \epsilon_2 = 100\mu$$

- If $\phi_{xy} = 0$, then ϵ_x & ϵ_y are directly the values of ϵ_1 & ϵ_2.

$$\epsilon_1 = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} \Rightarrow 300\mu = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}$$

$$\epsilon_2 = \frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E} \Rightarrow 100\mu = \frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E}$$

$$\epsilon_1 + \mu\,\epsilon_2 = \frac{\sigma_1}{E}(1-\mu^2).$$

$$\therefore \sigma_1 = \frac{E(\epsilon_1 + \mu\epsilon_2)}{1-\mu^2} = \frac{2\times10^5(300\mu + 0.3\times100\mu)}{1-0.3^2}$$

$$= \underline{\underline{72.527}} \text{ MPa.}$$

$$\sigma_2 = \frac{E(\epsilon_2 + \mu\epsilon_1)}{1-\mu^2} = \frac{2\times10^5(100\mu + 0.3\times300\mu)}{1-0.3^2}$$

$$= \underline{\underline{41.76}} \text{ MPa}$$

$\Rightarrow \sigma_1 \not> $ Permissible stress ($= 200$ MPa)

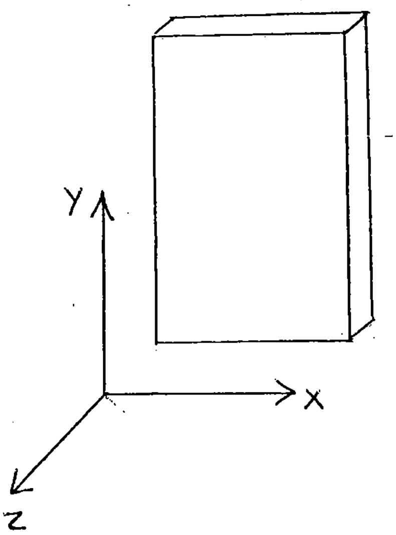
$\therefore$ <u>Safe</u>

→ Plane Stress System (2D)

Eg: Beams, shafts, thin members

$\sigma_x \neq 0$	$\boxed{\sigma_z = 0 \quad \epsilon_z \neq 0}$
$\sigma_y \neq 0$	$\tau_{xz} = 0$
$\tau_{xy} \neq 0$	$\tau_{yz} = 0$.

$z \rightarrow$ direction along thickness.



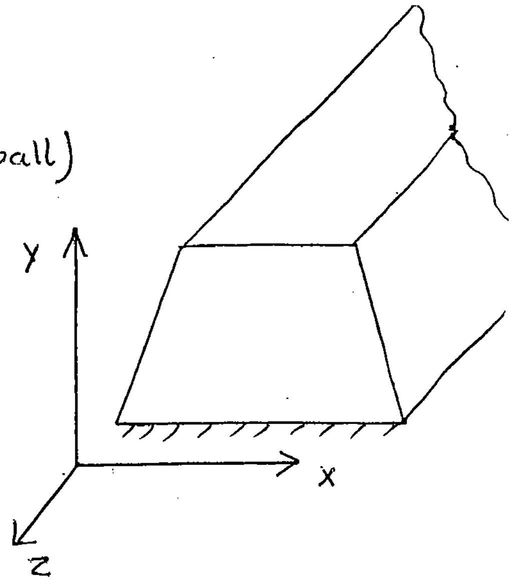
→ Plane Strain System.

Eg: Long members (dam, retaining wall)

$\epsilon_x \neq 0$ $\boxed{\epsilon_z = 0 \qquad \sigma_z \neq 0}$

$\epsilon_y \neq 0$ $\phi_{xz} = 0$

$\phi_{xy} \neq 0$ $\phi_{yz} = 0$



P-25

Q.08. $\epsilon_z = \frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E}$

$$0 = \sigma_z - 0.3 \times 150 - 0.3 \times -300.$$

$$\therefore \sigma_z = \underline{\underline{-45}} \text{ MPa}$$

09. $\sigma_x = 65 \text{ N/mm}^2$, $\sigma_y = -13 \text{ N/mm}^2$, $\tau_{xy} = 20 \text{ N/mm}^2$.

$$\sigma_1 = \frac{65-13}{2} + \sqrt{\left(\frac{65+13}{2}\right)^2 + 20^2}$$

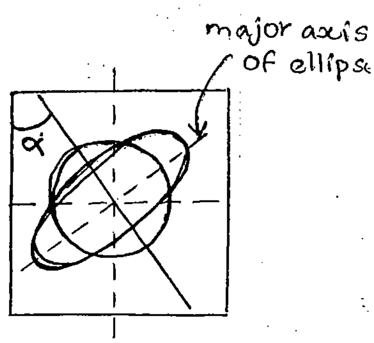
$$= 26 + 43.83 = 69.83 \text{ N/mm}^2.$$

$$\sigma_2 = 26 - 43.83 = \underline{\underline{-17.83}} \text{ N/mm}^2$$

10. Major axis of ellipse will develop in the direction of σ_1 which will be $\perp^r$ to major principal plane.

$$\tan 2\alpha = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2 \times 20}{65 - (-13)}.$$

$$\alpha = 13.57^\circ \text{ (with vertical).}$$



Angle of major axis of ellipse (along which σ_1 is acting)

$$= \alpha + 90 = \underline{\underline{103.5^\circ}}$$

11. Length of major axis:

$$\epsilon_1 = \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E}$$

$$\frac{\partial D}{D} = \frac{70}{2\times10^5} - 0.3\frac{(-18)}{2\times10^5}$$

$$\delta D = 0.113 \text{ mm.}$$

Major axis length $= 300 + 0.113 = \underline{\underline{300.113}}$ mm

Length of minor axis:

$$\epsilon_2 = \frac{\sigma_2}{E} - \mu \frac{\sigma_1}{E}$$

$$\frac{\partial D}{D} = \frac{-18}{2\times10^5} - 0.3\times\frac{70}{2\times10^5}$$

$$\partial D = -0.0585 \text{ mm.}$$

Minor axis length $= 300 - 0.0585 = \underline{\underline{299.94}}$ mm