me: V. Akanksha STD: E-2 Sec.: CE-02 Roll No: 02

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Structural analysis

UNIT 1

Indeterminate structural analysis

Structural Analysis :-

Structural analysis is the application of solid mechanics to predict the response (in terms of force and displacements) of a given structure (existing or proposed) subjected to specified loads.

structure → stability (how much time it can withstand)
→ strength - Resistance of physical loads
→ stiffness -
→ economy
→ Aesthetics

Input → system → output
↓ ↓ ↓
load → structure → Response ?

[force & displacement]
↓
must satisfy
Compactibility
[Kinematic admissible]

Must satisfy
equilibrium
[statically admissible]

Loads :- 1) Direct loading
2) Indirect loading

structure :- 1) structural system
2) structural idealisation
3) structural elements

2) Joints → Responsible for internal stability
supports → Responsible for external stability

static indeterminancy
Kinematic Indeterminancy

* A structure may be defined as an assembly of load-bearing elements in a construction.

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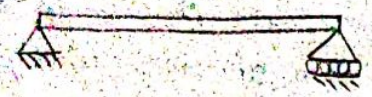
* By Making appropriate idealization, we reduce the structural system to separate sub-system. Each of which is made of structural element.

Beams :-

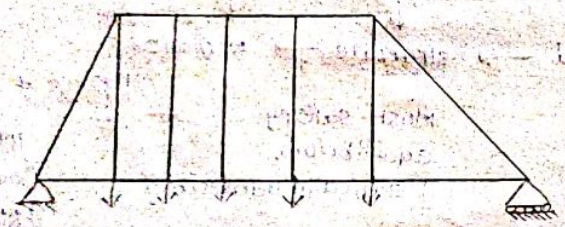


Simply supported

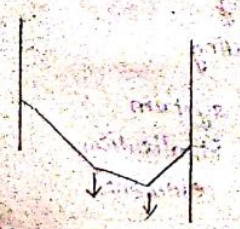
Fixed beam

Continuous beam

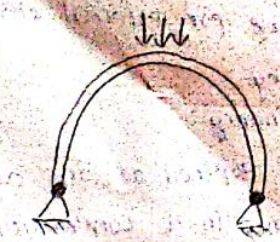
plane truss :-



Cable :-



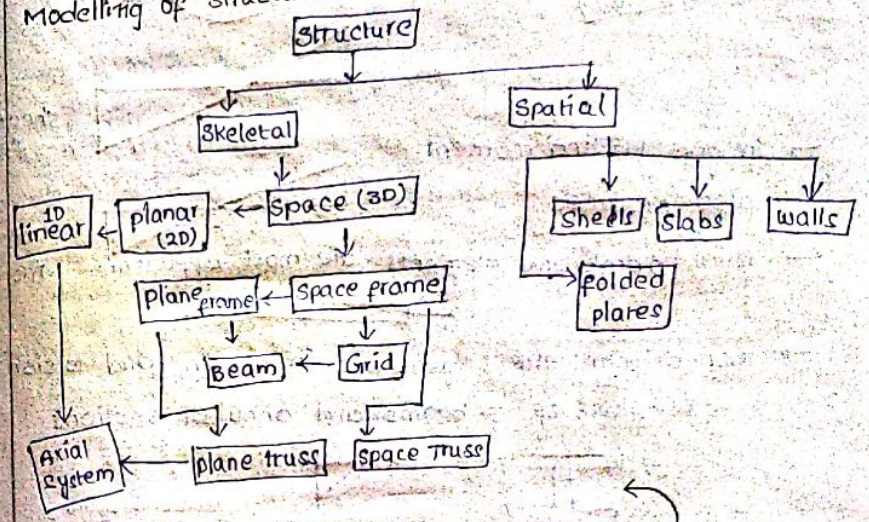
Arch :-

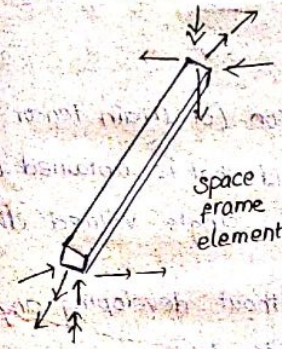


Response :-

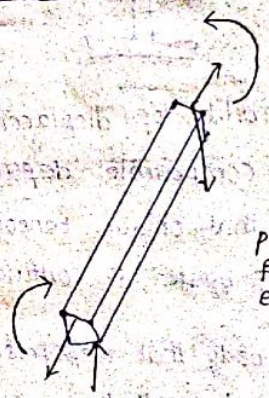
- equilibrium of forces
- compatibility of displacement
- force - displacement relations
- levels of structural analysis

Modelling of structures :-



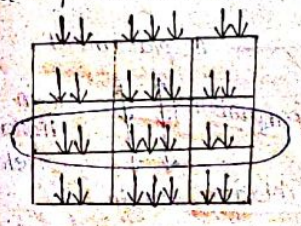


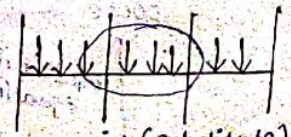
Space frame element



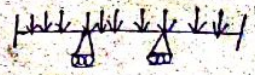
plane frame element

plane frame and Beams :-





partial (substitute) frame



continuous beam

Beam element

Bending moment

shear force

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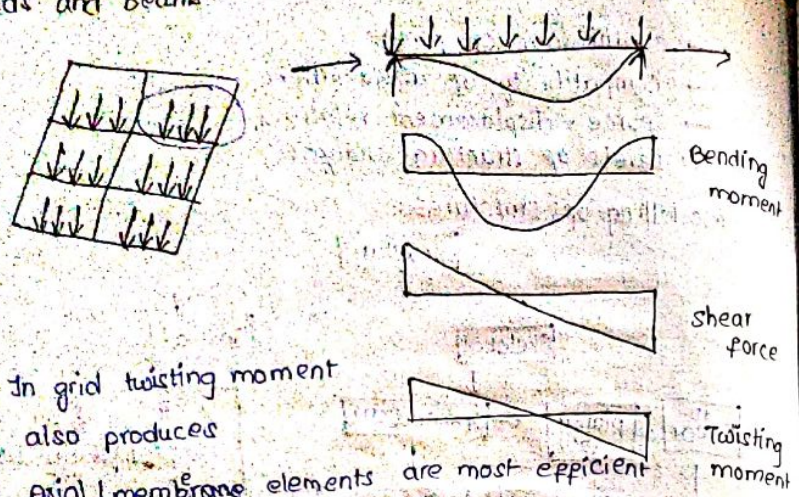
Grids and Beams :-



→ In grid twisting moment also produces

→ Axial / membrane elements are most efficient

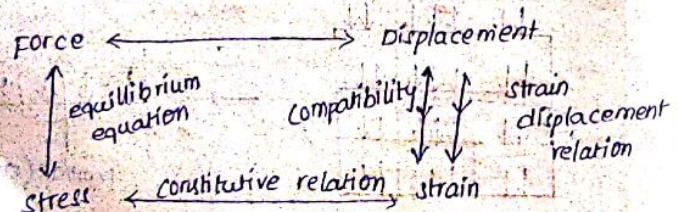
→ Non-uniform stress generated in bending and torsion

Ex :- use of I-beams and annular sections.

Compatibility of displacement :-

A compatible deformation (or strain tensor field in a body is that unique tensor field that is obtained when the body is subjected to continuous, single valued displacement field.

A body that deforms without developing any gaps/overlaps is called a compatible body.



Static Indeterminancy :-

A structure is statically indeterminate (or hypostatic) when the static equillibrium eqns are insufficient for determining the internal forces and relations on that structure.

Kinematic Indeterminancy :-

Independently of all other displacements. The number of independent joint displacement in a structure is known as the degree of kinematic indeterminancy or the number of degree of freedom.

Internally static Indeterminancy :-

It refers to the geometric stability of the structure after knowing the external reactions if it is not possible to determine the all internal forces or reaction using static equillibrium eqns alone. Then the structure is said to be internally indeterminate.

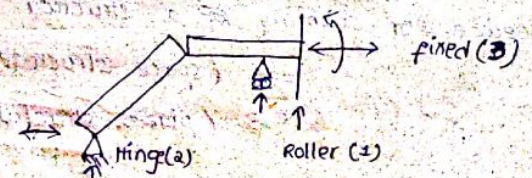
Joints and supports :-

Kinematic function
control relative displacement

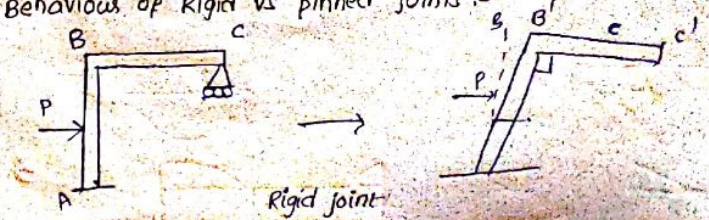
static function
provide for transmission of force

joint →
- pinned (free relative motion)
- semi rigid
- Rigid (No relative rotation)

support reactions :-



Behaviour of Rigid vs pinned joints :-



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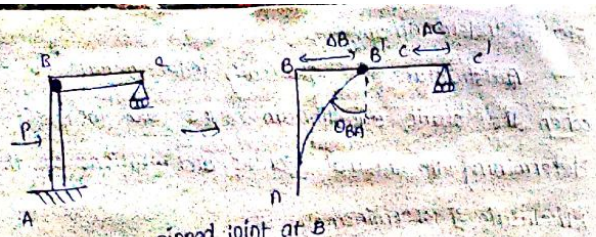


pinned joint at B

Supports and external Stability :-

When there are just adequate external restrains (supports) to render a structural system then it is externally stable we may use the term 'just rigid' to define the state of external stability.

→ If we give additional (extra or redulant) external (restrain) then we use the term "over rigid" to define the state of external stability.

→ we use the term under-rigid to define the external instability arising out of insufficient external restrains.

Note :-

Internal under-rigidity can sometimes be compensated by external over-rigidity to provide for overall stability.

→ minimum no. of independent support restrains needed for stability of a structure

= 6 (space structure)

= 3 (planar structure)

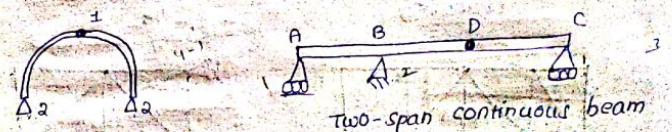
Just rigid structure with internal hinges :-

when internal hinge present → substract it from reactions

internally unstable

= 2+2 = 4 externally over rigid

= 2+2-1 = 3 Just rigid

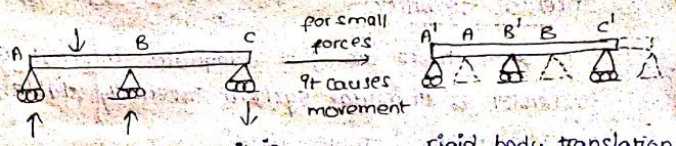


= stable Three hinged arch

Two-span continuous beam with internal hinge

Externally unstable systems :-

The support reactions should be located and aligned such that their line of action do not always converge at a point and are also not mutually parallel.



Here reactions = 3 but it is not stable because loads reactions are parallel

rigid body translation under horizontal peturbation.

⇒ equillibrium (3) possible but parallel so not externally stable

a) multiple support with parallel lines of reactions.

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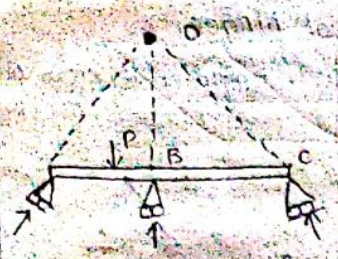
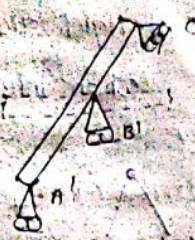


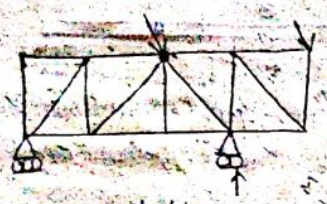
when reactions converge at a point it cause rotation



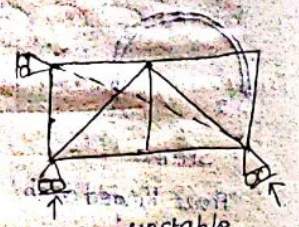
equillibrium possible only if loads acts along OB.

Rigid body rotation under any general pertubation.

b) Multiple supports with concurrent lines of reactions.



stable



unstable

since reactions → 3
There are not all parallel and not converging at one point

Converging to a point

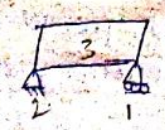
Internal stability :-

when there are just adequate internal constraints to render a structural system internally stable, we may use the term "Just - rigid"

additional → over-rigid
less → under-rigid

★ For Box from the total support reactions are "6"

∴ $n_s = 3$



6 - 3
= 3

Examples :-

unstable
= 6 - 4
= 2

stable (just rigid)
= 6 - 3
= 3 $n_s = 0$

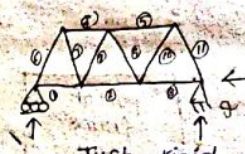
Stable (just rigid)
= 6 - 4 + 1
= 3

stable (over rigid)
= 6 - 4 + 2
= 4
N_{si} = 1

stable (over rigid)
= 6 - 2
= 4
N_{si} = 1

Stable (support reactions)
= 6
N_{si} = 6 - 3
= 3

Overall stability and rigidity, degree of static indeterminacy (n_s) :-



Just - rigid

$N_R = 1$ (only axial load) No of BM

$(am + r) - (Bj + r)$

$(2(11) + 3) - (2(7) + 0)$

$= 0$

statically determinacy

$\Sigma H = 0$ just rigid
$\Sigma V = 0$

simple plane truss

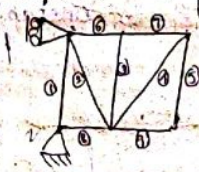
stable if $m + r = 2j$

m = member force
j = joints number
r = equillibrium eqns

Because here
$m = 11$, $r = 3$, $j = 7$

$m = 2j - r$
$11 = 2(7) - 3$
$11 = 11$

Examples :-

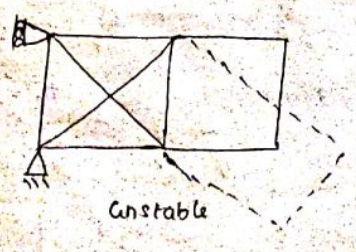


Just rigid

$(9+3) - (2(6)+0)$
$= 0$

internally stable (i.e., $n_s = 0$)
external stable (i.e reaction = 3, $n_s = 0$)

$m = 9$, $r = 3$, $j = 6$
$m + r = 2j$
$12 = 12$



Unstable

externally stable ($r = 3$)
$m = 9$, $r = 3$, $j = 6$
$12 = 12$

But it is internally unstable

∴ Totally unstable

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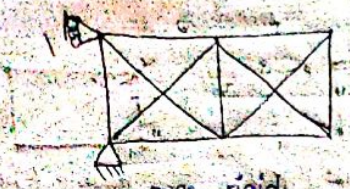


$m = 11,\ r = 3,\ j = 6$ $\quad (11+3) = (2)(6)$

externally stable

internally stable

$n_s = m + r - 2j$

$= 11 + 3 - 2(6)$

$n_s = 2$

over - rigid

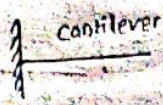
$(2m + r) - (2j) - (3)$

$(6+3) - (3(2) - 1)$

$9 - 5$

$= 4$

Stability and Rigidity :-

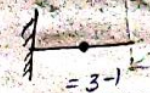


cantilever

just rigid $\quad m = 0,\ j = 1$

$n_s = 0 + 0$
$= 0$

$(0+3) - (3(1))$
$3 - 3$
$= 0$



$= 3 - 1$
$= 2$

∴ internal hinge

externally just rigid

internally under rigid

∴ unstable

propped cantilever

stable (over rigid)

$n_s = 1$

∴ $3 + 1 = 4$

extra = 1

Just rigid

$\Rightarrow 3 + 1 - 1$

$\Rightarrow 3$

$n_s = 2$

Fixed

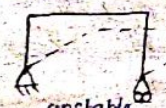
$n_s = 3$

portal frame

just rigid. $(n_s = 0)$

just rigid

$= 2 + 2 - 1$
$= 3$



unstable

$= 2 + 1 - 1$
$= 2$ (internally unstable)

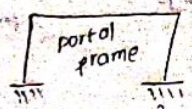
unstable

$n_s = 1$

(∵ $2 + 2 = 4$

$4 - 3 = 1$

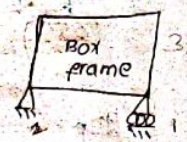
$n_s = 1$)



portal frame

3 rxns

$n_s = 3$



Box frame

$n_s = 3$

3 × no. of boxes

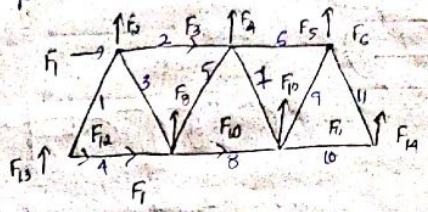
$r = 3 -$

I_s "

E_s "

$3 + 3 -$

forces and displacements :-



simple plane truss

$m = 11,\ r = 3,\ j = 7$

$m + r = 2j$

$= 14$

iii. Kinematic indeterminacy :-

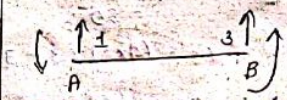
Kinematic indeterminacy is equal to unknown displacements

for planes → 2 unknowns at every support

for space → 3 unknowns at every support

→ The degree of kinematic indeterminancy may be defined as the total no. of degree of freedom (independent displacement coordinates) at the various joints in a skeletal structure.

n_k :-

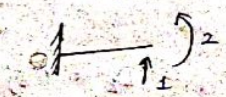


$n_k = 4$ (horizontal neglected)

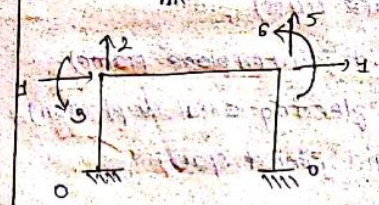
fixed beam

$n_k = 0$

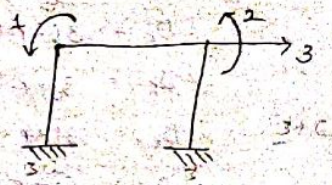
$n_k = 6$



$n_k = 2$



$n_k = 6$



Ignoring axial deformation

vertical → neglected

But horizontal it may ~~sway~~ sway. so consider total horizontal 1

∴ K indeterminacy - no. of free rxns of displacement

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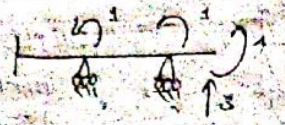
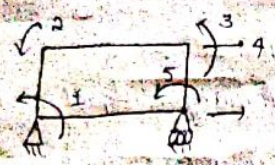
1) Continuous beam :-



$n_k = 7$

2) Box frame



Ignoring axial deformation

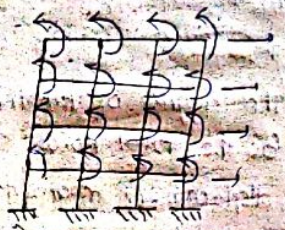
$n_k = 6$

$n_s = n_{inter} + n_{external}$

$n_{ext} = 3 \times 4 - 3$
$= 12 - 3$
$= 9$

$n_{inter} = 3 \times 9 = 27$

$n_s = 9 + 27 = 36$



Ignoring axial deformation

$n_k = 16 + 4$
$= 20$

$\therefore n_s = (4 \times 3) \times 3 = 36$

$n_k = (4+4) \times 3$
$= 48$

$D_k = 2j - r$ (plane truss)

$D_k = 3j - r$ (space truss)

$D_k = 3j - r$ (rigid jointed plane frame)

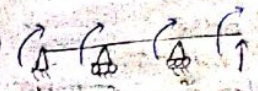
$D_k = 3j - r - m$ (neglecting axial deflection)

$D_k = 6j - r$ (rigid jointed space)

$D_k = 6j - r - m$ (space neglecting axial deformation)

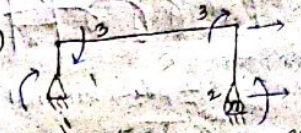
Examples :-

1)



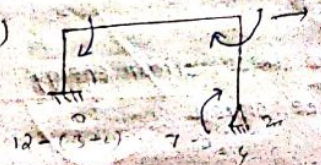
$n_k = 3$
$= 3(4) - 4$
$= 8 - 3 = 5$ axial

2)



$n_k = 6$ axial
$= 3(4) - 3 = 9$
$= 9 - \text{restriction}$
$= 9 - 3$
$= 6$

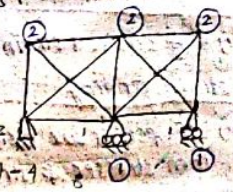
3)



$n_k = 4$
$\Rightarrow 3(4) - 5 = 7$
$\Rightarrow 7 - 3 = 4$

axial
$3 + 3 + 0 + 1 - 3$
$= 4$

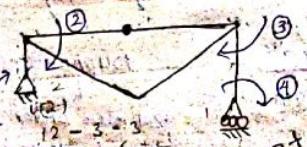
Truss :-



$n_k = 2 + 2 + 2 + 1 + 1$
$= 8$

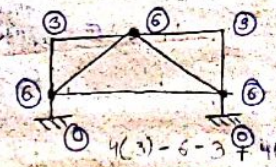
$aj - b + r$
$2(6) - 4 + 0$
$12 - 4 = 8$

(or)
$n_k = 2(6) - 4$
$= 8$

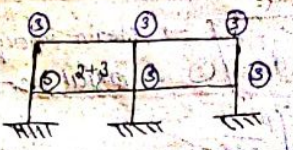


If internal hinge, present use need to find extra equations

$\Rightarrow 3(4) - 3 = 9$
$\Rightarrow 9 - 3 = 6$
$\Rightarrow 6 + 2 = 8$



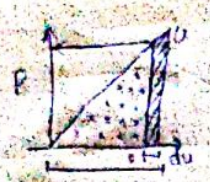
$n_k = (6 \times 4) - 9$
$= 15$



$= 20 - m$
$= 20 - 10$
$= 10$

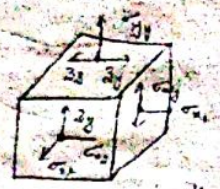
Strain energy:-

Strain energy due to axial force

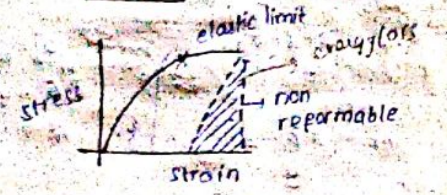


two apply force gradually $\frac{1}{2}Pdu$ → external work → energy

$$W = \int_0^u P\,du$$



→ Due to stress, strain energy will be stored



$$\boxed{U = \frac{1}{2}\,\text{stress} \times \text{strain} \times \text{volume}}$$

↓ strain energy

→ If forces are acted gradually then stresses & strain are also acted gradually

→ Strain = $\frac{\text{Change in length}}{\text{original length}}$

toughness = $\frac{\text{strain energy}}{\text{volume}}$

↓ total area under curve

$\frac{1}{2} \times \text{stress} \times \text{strain}$

$= \frac{du}{dx} = e$

$E = \frac{\text{stress}}{\text{strain}} = \frac{P/A}{e}$

$E = \frac{P}{Ae}$

$e = \frac{P}{AE}$

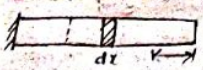
external work $= \frac{1}{2}Pu$ $\quad du = \left(\frac{P}{AE}\right)dx$

Strain energy $U = \int_0^L \frac{1}{2} P \left[\frac{P}{AE}\right] dx$

$$\boxed{U = \int_0^L \frac{1}{2}\left(P^2/AE\right) dx}$$

→ strain energy due to axial force

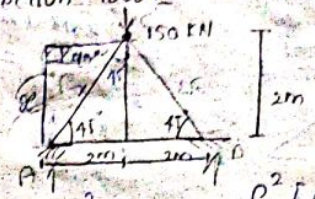
$$U = \int \frac{P^2\,dx}{2AE}$$



for total length

u → strain

1) Determine the total strain energy stored in a cross section of each section 1000mm²

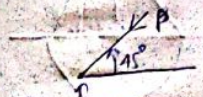


$E = 2\times10^5$ N/mm²

$U = \int_0^L \frac{P^2 dx}{2AE}$

$= \frac{P^2L}{2AE}$

$U = \frac{P^2L}{2AE} = \frac{P^2\,[2\sqrt{2}]\times1000}{2[1200][2\times10^5]}$



$AC = P = \sqrt{2}\,[150\cos45]$

$= 106.06$ kN [compress]

ΣF_y → $\sin45° = \frac{2}{x}$

$\frac{1}{\sqrt{2}} = \frac{2}{x}$

$\boxed{x = 2\sqrt{2}\,m}$

$R_A + R_B = 150$

$\Sigma M_A = 0$

$150(2) = R_B(4) = 0$

$R_B = 75$ kN

$R_A = 75$ kN

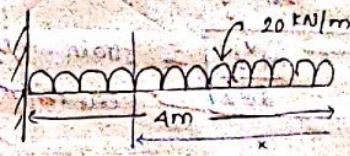
$U = \frac{(106.06)^2(2\sqrt{2})\times1000}{2(1200)(2\times10^5)} = 0.0661$ kN·mm

$= 66.1$ kN·m

for one side

For two sides $= 2\times66.1 = 133.69$ kN·m

2) Determine the strain energy of cantilever



$I = 25\times10^{-4}$ m⁴

$E = 2\times10^{11}$ N/mm²

$M^2 = \left(20(x)\times\frac{x}{2}\right)^2$

$= 100x^4$

$U = \int_0^L \frac{M^2}{2EI}\,dx$

$U = \int_0^L \frac{100x^4}{2EI}\,dx$

$= \frac{100}{2EI}\left[\frac{x^5}{5}\right]_0^L$

$= \frac{100}{2EI}\left[(4\times1000)^5 - (0\times1000)^5\right]$

$= \frac{10}{EI}\left[(4\times1000)^5\right] = 2.048\times10^{-4}$ N·mm

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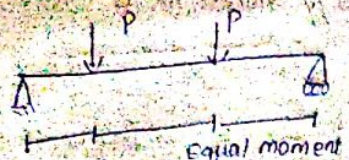
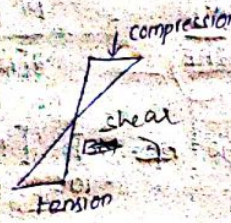


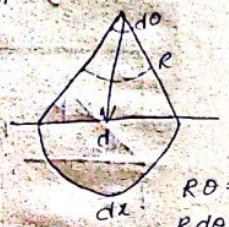
work done = force × displacement

$= \frac{1}{2}$ moment × ω

Equal moment

Sagging





$R\theta = S$

$R\,d\theta = dx$

$d\theta = \frac{1}{R}dx$

External work done $U = \frac{1}{2} M\, d\theta$

$$U = \int_0^L \frac{1}{2} M \left(\frac{1}{R}dx\right)$$

$$\frac{M}{I} = \frac{E}{R} \Rightarrow \frac{M}{EI} = \frac{1}{R}$$

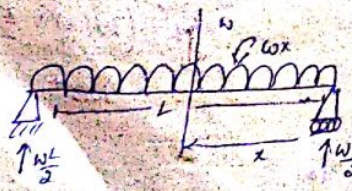
$$U = \int_0^L \frac{1}{2} M \left(\frac{M}{EI}\right) dx$$

$$= \int_0^L \frac{1}{2} \frac{M^2}{EI} dx$$

$$= \frac{M^2}{2EI} \int_0^L dx$$

$$\boxed{U = \frac{M^2 L}{2EI}} \rightarrow \text{strain energy due to bending}$$

⇒



$R_A = R_B = \omega L$

$-R_B(L) + \omega L\left(\frac{L}{2}\right) = 0$

$+R_B L = +\frac{\omega L^2}{2}$

$R_B = \frac{\omega L}{2}$

$$M = +\left(\frac{\omega L}{2} \times x\right) - \omega x\left(\frac{x}{2}\right)$$

$$U = \int_0^L \frac{M^2 dx}{2EI}$$

$$U = \int_0^L \frac{\left(\frac{\omega L x}{2} - \frac{\omega x^2}{2}\right)^2}{2EI} dx$$

$$U = \frac{1}{8EI} \int_0^L (\omega L x - \omega x^2)^2 dx$$

$$= \frac{1}{8EI} \int_0^L (\omega L x)^2 + (\omega x^2)^2 - 2(\omega^2 L x^3)\, dx$$

$$= \frac{1}{8EI}\left[\omega^2 L^2 \left(\frac{x^3}{3}\right)_0^L + \omega^2 \left(\frac{x^5}{5}\right)_0^L - 2\omega^2 L \left(\frac{x^4}{4}\right)_0^L\right]$$

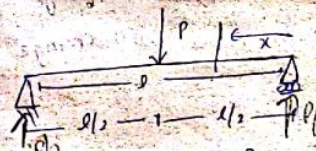
$$= \frac{1}{8EI}\left[\frac{\omega^2 L^2}{3} L^3 + \frac{\omega^2}{5} L^5 - 2\omega^2 L \frac{L^4}{4}\right]$$

$$= \frac{1}{8EI}\left[\frac{\omega^2 L^5}{3} + \frac{\omega^2 L^5}{5} - \frac{\omega^2 L^4}{2}\right]$$

$$= \frac{\omega^2 L^5}{8EI}\left[\frac{10 + 6 - 15}{30}\right]$$

$$= \frac{\omega^2 L^5}{8EI}\left[\frac{1}{30}\right] = \frac{\omega^2 L^5}{240EI}$$

⇒



$BM = \frac{P}{2}x$

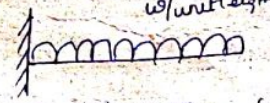
$$U = \int_0^{l/2} \frac{P^2 x^2}{4 \times 2EI} dx$$

$$= \frac{P^2}{8EI}\left(\frac{x^3}{3}\right)_0^{l/2} = \frac{P^2}{24EI}\left(\frac{l^3}{8}\right)$$

$$= \frac{P^2}{24EI} \times \frac{l^3}{8}$$

$$U = \frac{P^2 l^3}{96EI}$$

⇒



w/unit length

$\frac{P^2}{8EI}\left[\frac{l^3}{3EI}\right] = \frac{P^2 l^3}{24EI}$

$$= \frac{P^2 L^3}{96EI}$$

$$m = \frac{\omega x^2}{2}$$

$$U = \int_0^L \frac{\frac{\omega^2 x^4}{4} dx}{2EI}$$

$$= \frac{\omega^2}{8EI}\left(\frac{x^5}{5}\right)_0^L$$

$$= \frac{\omega^2 L^5}{40EI}$$

$$U = \frac{\omega^2 L^5}{40EI}$$

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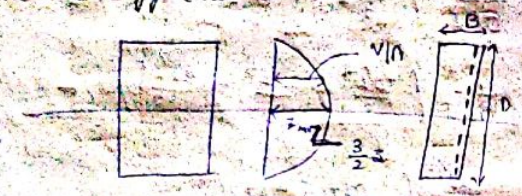
$m = -Px$

$$U = \int_0^L \frac{P^2 x^2 dx}{2EI}$$

$$= \frac{P^2}{2EI}\left(\frac{x^3}{3}\right)_0^L$$

$$= \frac{P^2 L^3}{6EI}$$

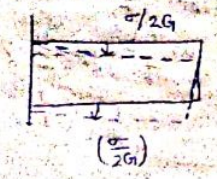
Strain energy due to shear :-



$$U = \int_0^L \frac{P^2 dx}{2AG}$$

A - shear Area P - shear force

After applied shear stress shear Area will change



Strain energy due to torsion :-

$$U = \int_0^L \frac{T^2 dx}{2GI_p}$$

$I_p = J$ = polar moment of inertia

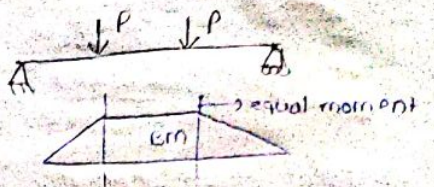
Strain energy due to axial force :-

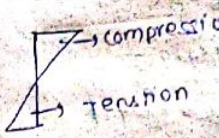
$$U = \int_0^L \frac{P^2}{2EA} dx$$

Strain energy due to Bending moment

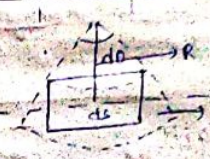
$$U = \int_0^L \frac{m^2}{2EI} dx$$

Q) Find strain energy

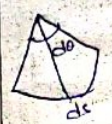




stress diagram strain diagram



Due to addition strain Bending rotation occurs in addition to bending stress of Bending strain.



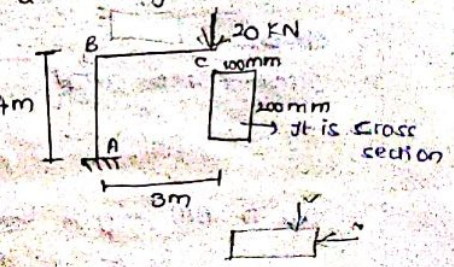
$R d\theta = ds$

$d\theta = \frac{1}{R} ds$

$d\theta = \frac{m}{EI} ds$ (already done)

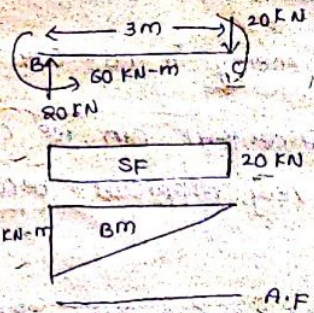
$$U = \int_0^L \frac{m^2}{2EI} ds$$

3) The strain energy stored in a rectangular bent as shown in fig. $E = 2\times10^5$ MPa, $G = 0.8\times10^5$ GPa



the energy will stored due to Bending moment, shear and axial force.

In BC :-



Area = 100 x 200

= 20000 mm²

= 0.02 m²

$I = \frac{bd^3}{12}$

$= \frac{100\times200^3}{12}$

$= 6.65\times10^{-5}$ m⁴

Strain energy Due to shear $U = \int_0^L \frac{V^2}{2AG} dx$

$$= \int_0^3 \frac{(20)^2}{2\times0.02\times0.8\times10^5} dx$$

$= 4.8$ J

Axial force $U = 0$

BM $\Rightarrow$ $U = \int_0^L \frac{M^2}{2EI} dx$

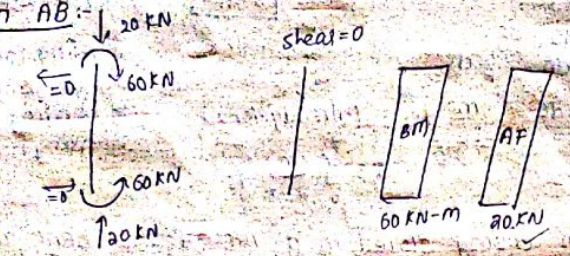
$= \int_0^L \frac{(20x)^2}{2EI} dx = \frac{400}{2EI} \left(\frac{x^3}{3}\right)_0^3$

$= 134.9\ J$

$\therefore$ Total strain energy in BC $= 4.8 + 0 + 134.9$

$= 139.7\ J$

In AB :-



~~shear~~

strain due to axial $= U = \frac{P^2 L}{2EA}$

$= \frac{(20)^2 \times 4}{2EA} = 0.8\ J$

Due to BM $= U = \frac{(60)^2 \times 4}{2EI} = 539.7\ J$

Due to shear $= 0$

$\therefore$ Total strain energy in AB $= 539.7\ J$

$\Rightarrow$ From this we notice that BM we can't rotate the beam, it is very difficult because it has more energy

$\rightarrow$ But from axial & shear we can easily rotate or move (low energy)

$\rightarrow$ strain energy is maximum due to BM.

shear & axial are negligible w.r.t BM

Castigliano's Theorem :-

Castigliano's theorem for linearly elastic structures the partial derivative of strain energy w.r.t. to applied force (or couple) is equal to the displacement (or rotation) of the force (or couple) along its line of action

deflection $(\delta) = \left[\frac{\partial U}{\partial P}\right] \rightarrow$ P's direction is δ's direction

slope - rotation $\theta = \left[\frac{\partial U}{\partial M}\right] \rightarrow$ moment direction is rotation rotation

$U = \int_0^L \frac{M^2 dx}{2EI}$

$\frac{\partial U}{\partial P} = \int_0^L \frac{\partial M}{\partial P} \cdot \frac{M}{EI} dx$

deflection

Internal energy stored $U = \frac{P^2 L^3}{6EI}$

external work done $= \frac{1}{2} P\delta$

$\frac{1}{2} P\delta = \frac{P^2 L^3}{6EI}$

$\delta = \frac{2PL^3}{6EI} = \frac{PL^3}{3EI}$

(or)

partial derivative of internal energy stored

$U = \frac{P^2 L^3}{6EI}$

$\frac{dU}{dP} = \frac{2PL^3}{6EI}$

$\delta = \frac{PL^3}{3EI}$

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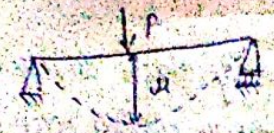


internal $= \frac{P^2L^3}{96EI}$ external $\frac{1}{2}Pu$

$\frac{1}{2}Pu = \frac{P^2L^3}{96EI}$ $\frac{P^2L^3}{96EI}$

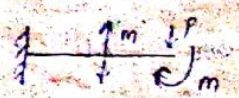
$u = \frac{2L^3P}{96EI}$ $\frac{2PL^3}{96EI}$

$= \frac{PL^3}{48EI}$ $\frac{PL^3}{48EI}$

$U = \frac{m^2L}{2EI}$

rotation $\frac{\partial U}{\partial m} = \frac{2mL}{2EI} = \frac{mL}{EI} = \theta$

For displacement $\frac{\partial U}{\partial P}$ but there no P so we should take imaginary load



$U = \frac{m^2L}{2EI} + \frac{P^2L^3}{6EI}$

moment $= (m+Px)$ so $U = \frac{(m+Px)^2\,dx}{2EI}$

$= \frac{m^2+P^2x^2+2mPx}{2EI}$

$= \frac{m^2}{2EI} + \frac{P^2L^3}{6EI} + \frac{mPL^2}{2EI}$

Eternal work $= \frac{1}{2}Pu + \frac{1}{2}M\theta$

$= \frac{1}{2}P(u+du) + \frac{1}{2}m(\theta+d\theta)$

$= \frac{1}{2}Pu + \frac{1}{2}M\theta + \boxed{\frac{1}{2}Pdu + \frac{1}{2}m\,d\theta}$ Neglected

$U = \frac{M^2L}{2EI} + \frac{P^2L^3}{6EI} + \frac{mPL^2}{2EI}$

$\delta = \frac{\partial U}{\partial P} = \frac{PL^3}{3EI} + \frac{mL^2}{2EI}$

when $P=0$ $\delta = \frac{mL^2}{2EI}$

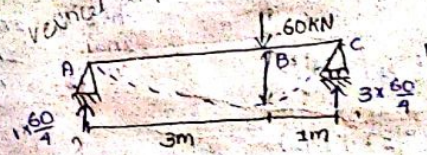
$\theta = \frac{\partial U}{\partial m} = \frac{2mL}{2EI} + \frac{PL^2}{2EI}$

$= \frac{mL}{EI} + \frac{PL^2}{2EI}$

@ $m=0 \Rightarrow \frac{PL^2}{2EI}$

4)

vertical displacement



$EI = 2.2\times10^6\ N\text{-}m^2$

$P = 60\ KN$
$= 60\times10^3\ N$

$\frac{\partial U}{\partial P} = \delta = \int_0^L \frac{\partial m}{\partial P}\cdot\frac{m}{EI}\,dx$

B/w B and C, $m = 3\times\frac{P}{4}x$

$\frac{\partial m}{\partial P} = \frac{3x}{4}$

B/w A and B, $M = 3\times\frac{P}{4}x - P(x-1)$ $1<x<3$

(or)

$M = \frac{P}{4}x$ $0<x<3$

$\frac{\partial m}{\partial P} = \frac{x}{4}$

$\delta = \int_0^1 \frac{3x}{4}\cdot\frac{3Px}{4EI}\,dx + \int_0^3 \frac{x}{4}\left(\frac{Px}{4}\right)\frac{1}{EI}\,dx$

$= \frac{9P}{16EI}\left[\frac{x^3}{3}\right]_0^1 + \frac{P}{16EI}\left[\frac{x^3}{3}\right]_0^3$

$= \frac{3P}{16EI} + \frac{P}{48EI}[27]$

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5)

$\delta = \frac{\partial U}{\partial P}$

$R d\theta = ds$

$M = PR\sin\theta$

$\frac{\partial m}{\partial P} = R\sin\theta$

$$\delta = \int_0^{\pi/2} \frac{\partial m}{\partial P} \cdot \frac{m}{EI}\, dx$$

$$= \int_0^{\pi/2} R\sin\theta \cdot \frac{PR\sin\theta}{EI} (R\, d\theta)$$

$$= \frac{PR^3}{EI} \int_0^{\pi/2} \sin^2\theta \, d\theta$$

$2n-1=0$ $\quad 2m-1=2$

$2n=1$ $\quad 2m=3$

$n=1/2$ $\quad m=3/2$

$$= \frac{PR^3}{EI} \frac{1}{2}\left[\Gamma(3/2, 1/2)\right]$$

$$= \frac{PR^3}{2EI} \frac{\Gamma(3/2)\,\Gamma(1/2)}{\Gamma(\frac{3}{2}+\frac{1}{2})} \qquad \frac{4}{2}=2! \qquad \Gamma(3/2) \quad \frac{1}{2}\Gamma 1/2$$

$$= \frac{PR^3}{2EI} \cdot \frac{\frac{1}{2}\sqrt{\pi} \cdot \sqrt{\pi}}{1} \qquad \frac{1}{2}\sqrt{\pi}$$

$$= \frac{PR^3\pi}{4EI}$$

6) Same 5th problem but find horizontal deflection

$R-R\cos\theta$

$M = PR\sin\theta + QR(1-\cos\theta)$

$\frac{\partial m}{\partial Q} = R[1-\cos\theta]$

$$\delta = \int_0^{\pi/2} R(1-\cos\theta) \frac{PR\sin\theta + QR(1-\cos\theta)}{EI} (R\, d\theta)$$

$$= \frac{1}{EI}\int_0^{\pi/2} \left(PR^3\sin\theta - PR^3\sin\theta\cos\theta + QR^3(1-\cos\theta) - QR^3(1-\cos\theta)\cos\theta\right) d\theta$$

$$= \frac{1}{EI}\int_0^{\pi/2} PR^3(\sin\theta - \sin\theta\cos\theta)\, d\theta + \int_0^{\pi/2} QR^3(1-\cos\theta)^2\cos\theta\, d\theta$$

$\cos\frac{\pi}{2} = 0$

$$= \frac{PR^3}{EI}\int_0^{\pi/2} \sin\theta(1-\cos\theta)\, d\theta$$

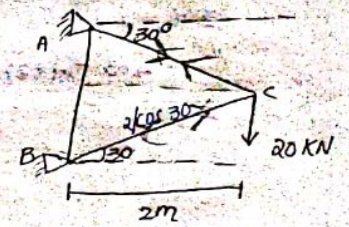
$\int f(x) f'(x) = \frac{(f(x))^2}{2}$

$$= \frac{PR^3}{EI}\left[\frac{(1-\cos\theta)^2}{2}\right]_0^{\pi/2}$$

$$= \frac{PR^3}{2EI}\left[(1-0)^2 - (1-1)\right]$$

$$= \frac{PR^3}{2EI}$$

7)

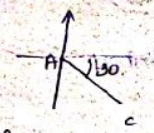


$A = 400\,mm^2$

$E = 2\times10^5\,MPa$

$$U = \int_0^L \frac{P^2\,dx}{2AE}$$

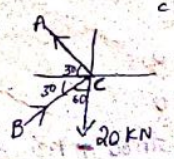
At A ⇒



$AC\cos 30 = H_1$

$AC\sin 30 = V_2$ } all unknowns

At C ⇒



$AC\sin 30 + BC\sin 30 = 20$

$AC = BC$

$x\left(\frac{1}{2}\right) + x\left(\frac{1}{2}\right) = 20$

$x = 20\,KN$

$\cos = \frac{2}{}$

$$U = \frac{P^2\left[\frac{2}{\cos 30}\right]}{2EA} \times 2$$

$$\frac{(20)\times1000)^2 \times \frac{2}{\cos 30} \times 1000}{400 \times 2\times10^5 \times 10^6}$$

$= 0.0115\,J$

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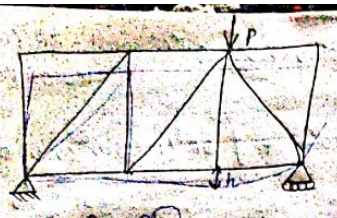
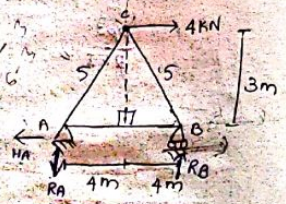
virtual work method
↓
unit load method



$\delta = \frac{PL}{AE}$ (each member's deflection)
stiffness

$\delta = \Sigma \eta \left[\frac{PL}{AE}\right]$

8) Determine the vertical displacement of joint C if 4 KN of force is applied at C. Area of each member is 400 mm² and E = 200 GPa



$\Sigma x = 0$

$H_A = +4$ KN

$\Sigma y = 0$

$R_A + R_B$

$R_A + R_B = 0$

$\Sigma M_A = 0$

$4 \times 3 - (4+4) R_B + 4(3) = 0$

$8R_B = +12$

$R_B = \frac{12}{8} = 1.5$ KN

$R_A = -1.5$ KN

$4^2 + 3^2 = 16 + 9 = \sqrt{25} = 5$

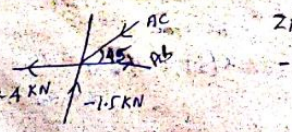
$\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.8$

+ tension

− compression

Finding forces (P) by using method of joints.

FBD at A :-



$\Sigma F_x = 0$

$\Sigma F_y = 0$

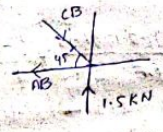
$-AC \sin 60 + 1.5 = 0$

$-AC \sin 45 = 1.5$

$-AC = \frac{1.5}{\sin 45}$

$AC = 2.12$ KN (T)

FBD at B :-



$\Sigma F_y = 0$

$CB \sin 45 = 1.5$

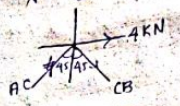
$CB = \frac{1.5}{\sin 45}$

$-AB + CB \cos 45 = 0$

$AB = CB \cos 45$

$= 2.12 \cos 45 = 1.5$ (T)

FBD at C :- not required



$\Sigma F_y = 0$

$-AB + CB \cos 45 = 0$

$-CB \sin 45 + 1.5 = 0$

$CB = 2.12$ (C)

$4 + CB \cos 45 + AC \sin 45 = 0$

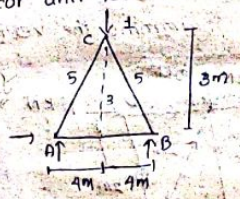
$4 + \cos 45\, CB + 2.12 \sin 45 = 0$

∴ $F_{AB} = 1.5$ (T)

$F_{BC} = 2.12$ (C)

$F_{AC} = 2.12$ (T)

For unit load at C :-



$\Sigma F_x = 0$

$\Sigma F_y = 0$

$R_A + R_B = 1$

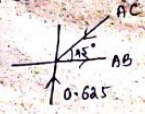
$\Sigma M_A = 0$

$-R_B(8) + 3(1) = 0$

$R_B = \frac{3}{8} = 0.375$ KN

$R_A = 0.625$ KN

FBD at A :-



$-AC \cos 45 + AB = 0$

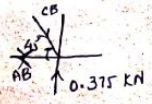
$0.625 - AC \sin 45 = 0$

$AC = 0.883$ (C)

$+0.883 \cos 45 = +AB$

$AB = 0.625$ KN (T)

FBD at B :-



$AB + CB \cos 45 = 0$

$-AB = -CB \cos 45$

$CB = +0.883$ (T)

$F_{AB} = 0.625$ KN (T)

$F_{BC} = 0.883$ KN (T)

$F_{AC} = 0.883$ KN (C)

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$$\delta = \Sigma \eta \frac{PL}{AE}$$

$$= \frac{(0.625)(1.5)(8)}{AE} + \frac{(0.883)(-2.12)(5)}{AE} + \frac{(-0.883)(2.12)}{AE}$$

$A = 400\ mm^2$

$E = 200\ GPa = 200 \times 10^9\ N/m^2$

$= 200 \times 10^9 \times \frac{N}{(10^3)^2}$

$= 200 \times 10^3\ N/mm^2$

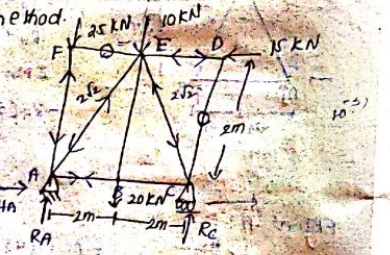
$$\delta = \frac{0.625 \times 1.5 \times 8 \times 1000}{400 \times 200 \times 10^3 \times 10^{-3}}$$

$= 0.093\ mm$

9) A pin jointed truss shown in fig. Determine the vertical joint of E by unit load method.

$A = 250\ mm^2$

$E = 200\ GPa$



$\Sigma F_x = 0$

$H_A - 15 = 0$

$H_A = 15\ KN$

$\Sigma F_y = 0$

$R_A + R_C - 10 - 25 - 20 = 0$

$R_A + R_C = 55\ KN$

$\Sigma M_A = 0$

$20(2) - R_C(4) - 15(2) + 10(2) = 0$

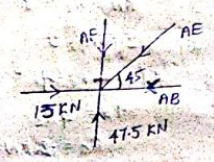
$30 = 4R_C$

$R_C = 7.5\ KN$

$R_A = 47.5\ KN$

By Method of joints :-

At A FBD :-



$\Sigma F_x = 0$

$15 - AE \cos 45 = AB$

$AE = 21.21$

$\Sigma F_y = 0$

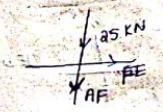
$AF + 47.5 - AE \sin 45 = 0$

$-25 + 47.5 = AE \sin 45$

$AE = 31.81\ KN\ (C)$

$AB = -7.5\ KN = 7.5\ KN\ (T)$

At F :-



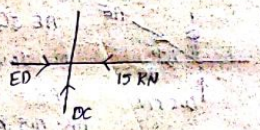
$AE = 0$

$-AF - 25 = 0$

$F_{AF} = -25\ KN$

$= 25\ KN\ (C)$

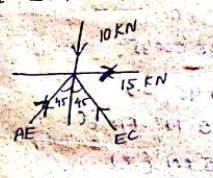
At D :-



$ED = 15\ KN\ (C)$

$DC = 0$

At E :-



$\Sigma F_x = 0$

$15 - AE \sin 45 + EC \sin 45 = 0$

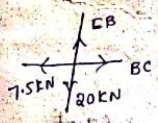
$15 - 22.49 = -EC \sin 45$

$EC = +10.6\ KN\ (C)$

$\Sigma F_y = 0$

$EC \cos 45 - AE \cos 45 - 10 = 0$

At B :-



$EB = 20\ KN\ (T)$

$BC = 7.5\ KN\ (T)$

$\therefore$

$F_{AB} = 7.5\ KN\ (T)$

$F_{BC} = 7.5\ KN\ (T)$

$F_{AF} = 25\ KN\ (C)$

$F_{FE} = 0\ KN$

$F_{BE} = 20\ KN\ (T)$

$F_{DC} = 0$

$F_{ED} = 15\ KN\ (C)$

$F_{EC} = 10.6\ KN\ (C)$

$F_{AE} = 31.81\ KN\ (C)$

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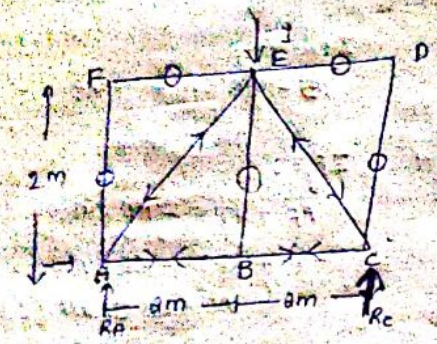
For unit load at E



$R_A + R_C = 1$

$\Sigma M_A = 0$

$1(3) + R_C(9) = 0$

$+3 = 9R_C$

$R_C = 0.5$ kN

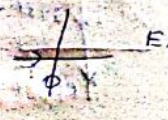
$R_A + 0.5 = 1$

$R_A = 0.5$ kN

FBD at F :-

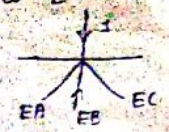
$F_A = 0$

FBD at D :-

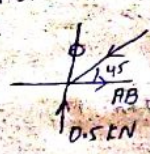


$E_D = 0$

FBD at E



FBD at A :-



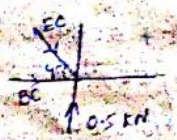
$AE \cos 45 = AE$

$AE \sin 45 = 0.5$

$AE = 0.707$ kN (C)

$AB = 0.5$ kN (T)

FBD at C :-



$BC + EC \cos 45 = 0$

$BC = -EC \cos 45$

$+0.5 + EC \sin 45 = 0$

$EC = 0.707$ kN (C)

$BC = -0.5$ kN (T)

$= 0.5$ kN

$\therefore F_{AB} = 0.5$ kN (T)

$F_{BC} = 0.5$ kN (T)

$F_{AF} = 0$

F_{AE} 0.707 kN (C)

$F_{EC} = 0.707$ kN (C)

$\delta = \Sigma \eta \frac{PL}{AE}$

$= \frac{1}{AE}\left[(0.5)(7.5)(2) + (0.5)(7.5)(2) + (-0.707)(-31.81) + (-0.707)(-10.6)(2\sqrt{2})\right]$

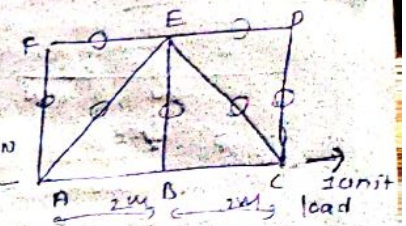
$= \frac{1}{250\times10^{-6}\times200\times10^{9}\times10^{-3}}\,[97.80]$

$= 1.99\times10^{-3}$ m

$= 1.996$ mm

At 'c' horizontal deflection :-

only members AB & BC contribute horizontal deflection



$\delta = \Sigma \eta \frac{PL}{AE}$

$= \frac{(0.409)(7.5)10^3\times2}{AE} +$

$\frac{0.409\times7.5\times10^3\times2}{AE}$

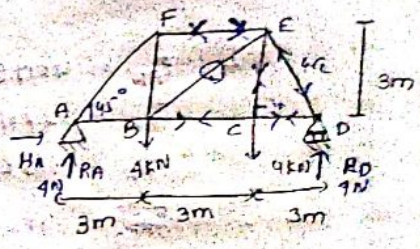
$= 2.454\times10^{-4}$ m

$= 0.24$ mm

10) Determine the vertical displacement of joint C.

A = 400 mm²

E = 200 GPa



$R_A + R_D = 8$ kN

$H_A = 0$

$\Sigma M_A = 0$

$4(3) + 4(6) - R_D(9) = 0$

$R_D(9) = 36$

$R_D = 4$ kN

$R_A = 4$ kN

$3^2 + 3^2 = x^2$

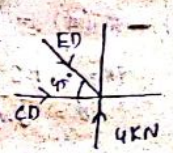
$9 + 9 = x^2$

$18 = x^2$

$x = \sqrt{18}$

$= 4\sqrt{2}$

at D :-



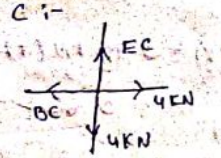
$CD + ED \cos 45 = 0$

$4 = ED \sin 45$

$ED = 5.6$ (C)

$CD = 4$ kN (T)

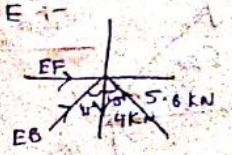
at C :-



$BC = 4$ kN (T)

$EC = 4$ kN (T)

at E :-



$EF = 5.6 \sin 45 - EB \sin 45$

$EB \cos 45 + 5.6 \cos 45 = 4$

$EB = 0$

$EF = 4$ kN (T)

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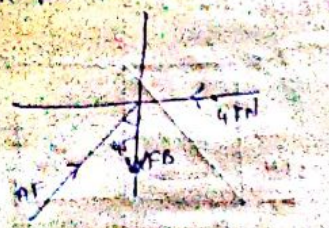
At F :-



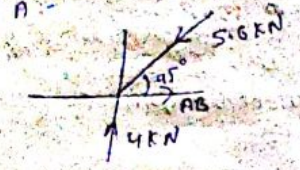
$FB = AF \cos 45$

$AF \sin 45 = 4$

$AF = 5.6\ KN\ (C)$

$FB = 4\ KN\ (T)$

At A :-



$AB = 5.6 \cos 45$

$= 4KN\ (T)$

$F_{AB} = 4KN\ (T)$

$F_{FB} = 4\ KN\ (T)$

$F_{AF} = 5.6\ KN\ (C)$

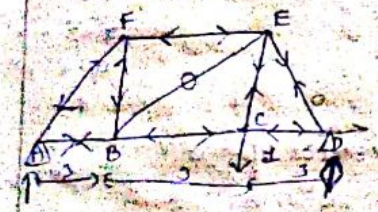
$F_{EF} = 4KN\ (C)$

$F_{BC} = 4KN\ (T)$

$F_{EC} = 4KN\ (T)$

$F_{ED} = 5.6\ (C)$

$F_{CD} = 4\ KN\ (T)$



$R_A + R_D = 1$

$\Sigma M_A = 0$

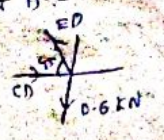
$1(6) - R_D(9) = 0$

$9R_D = +6$

$R_D = 0.66KN$

$R_A = 0.4\ KN$

at D :-



$0.6 = ED \sin 45$

$ED = 0.84\ KN\ (T) = 0.94\ KN\ (T)$

$CD = ED \cos 45$

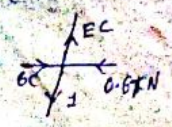
$= 0.6\ KN\ (C)$

$= 0.67\ KN\ (C)$

$0.66 = ED \times 1/\sqrt{2}$

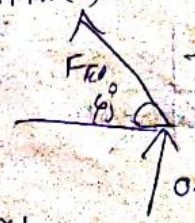
$ED = 0.6\sqrt{2}$

at C :-



$\Rightarrow EC = 1\ KN\ (T)$

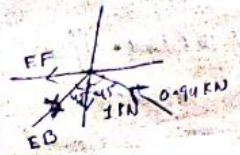
$BC = 0.67\ (C)$



$F_{CD} \times 1/\sqrt{2} + 0.66 = 0$

$F_{CD} = 0.66 \times$

at E :-



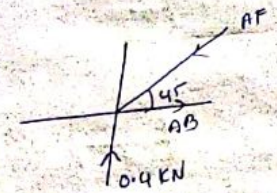
$1 + EB \cos 45 = 1$

$EB = 0.471\ (C)$

$EF + 0.94 \cos 45 = EB \sin 45$

$EF = 0.3316\ (C)$

At A :-



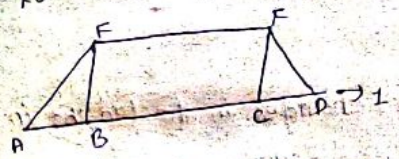
$AF \sin 45 = 0.4$

$AF = 0.56\ (C)$

$AB = AF \cos 45$

$= 0.4\ (T)$

For horizontal displacement at D :-



$CD = 1$

$BC = 1$

$AB = 1$

Member	P	vertical η	L	horizontal η
AB	4 (T)	0.333	3	1
BC	4 (T)	0.67	3	1
CD	4 (T)	0.67	3	1
AF	$4\sqrt{2}$ (C)	0.471 (C)	$3\sqrt{2}$	0
FE	4 (C)	0.333 (C)	3	0
ED	$4\sqrt{2}$ (C)	0.971 (C)	$3\sqrt{2}$	0
BF	4 (T)	0.333	3	0
CE	4 (T)	1	3	0
BE	0	0.471 (C)	$3\sqrt{2}$	0

$$\delta = \Sigma \eta \frac{PL}{AE}$$

$$= \frac{1}{AE}\left[0.333(4)(3) + (0.67 \times 4 \times 3) + (0.67 \times 4 \times 3) + (0.471)(4\sqrt{2})(3\sqrt{2}) + (-4 \times -0.33 \times 3) + (4\sqrt{2} \times 0.971 \times 3\sqrt{2}) + 4(0.33 \times 3) + (4 \times 3) + (0)\right]$$

$$= \frac{1}{AE}(61.076)$$

$$= \frac{10^3 \times 1000}{100 \times 200 \times 10^3}(74.6064) = 0.932\ mm$$

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For horizontal displacement at D:-

$$\delta = \frac{1}{AE}\left[1(4)(3) + (1\times4\times3) + (1\times4\times3)\right]$$

$$= \frac{10^3\times10^3}{200\times400\times10^3}\left[36\right]$$

$$= 0.45 \text{ mm}$$

11) Due to temperature changes, changes in the lengths in members like this AB = +4mm BE = −2mm CD = +2mm BF = −1mm

Find vertical displacement ① of joint C? Horizontal displacement ② at joint D?

For above 10th sum $\Delta l = \frac{PL}{AE}$

$$\delta_c = \sum \eta \frac{PL}{AE}$$

η	members
0.333	AB
−0.471	BE
0.67	CD
0.333	BF

① $\delta_c =$

$= 0.333(4) + (-0.471\times-2) + (0.67\times2) + (0.333\times-1)$

$= \frac{3.281 \text{ mm}}{200\times10^3\times400} = 4.10\times10^{-8}\text{ m} = 4.10\times10^{-5}\text{ mm}$

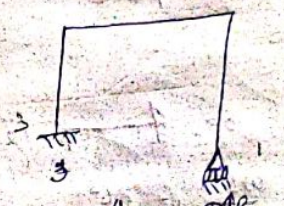
② $\delta = \frac{1}{}\left[(1\times4) + (1\times2)\right]$

$= 6\text{ mm}$

η	members
1	AB
2	BE
1	CD
0	BF

Minimum potential energy theory :-

In a statistically indeterminate structures if there are no support settlements and no change of temperature, the redundant to make strain energy minimum



3+1 = 4

4−3 = ①

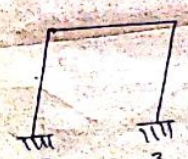
one extra so strain energy

R should be minimum

$$\left\{\frac{\partial U}{\partial R} = 0\right\}$$

To find R value we write all other values in terms of R then differential strain energy w.r.to "R"

Force method :-



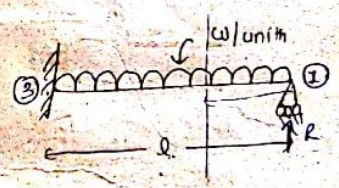
$\frac{\partial U}{\partial H} = 0$

$\frac{\partial U}{\partial V} = 0$

$\frac{\partial U}{\partial M} = 0$

3+3 = 6

6−3 = ③ unknowns

12) 

3+1 = 4

4−3 = ① unknown

Bending moment $M = Rx - \frac{wx^2}{2}$

$$U = \int_0^L \frac{M^2\,dx}{2EI}$$

$$\frac{\partial M}{\partial R} = x$$

$$\frac{\partial U}{\partial R} = \int_0^L \frac{M\cdot\frac{\partial M}{\partial R}\cdot dx}{EI} = 0$$

$$= \int_0^L \frac{Rx^2 - \frac{wx^3}{2}\,dx}{EI} = 0$$

$$= \frac{1}{EI}\left[R\left(\frac{x^3}{3}\right) - \frac{w}{2}\frac{x^4}{4}\right]_0^L = 0$$

$$R\frac{x^3}{3} = \frac{wx^4}{8} \quad \frac{R}{3} = \frac{wL}{8} \Rightarrow R = \frac{3}{8}wL$$

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At fixed beam we've 3 rxns

at that $5 \times \frac{1}{8} wL$ → vertical

Horizontal ⇒ 0

moment = $RL - \frac{wL^2}{2} = \frac{3}{8}wL^2 - \frac{wL^2}{2} = -\frac{wL^2}{8}$

(or) If Δ not there displacement take's place

$$\delta_c = \frac{wL^4}{8EI}$$

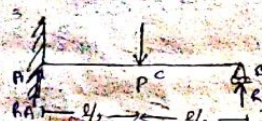
Remove ⌒⌒⌒ then δ_c of ⊥ is

$$\delta_c = \frac{wL^3}{3EI} = \frac{RL^3}{3EI}$$

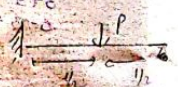
$$\frac{wL^4}{8EI} = \frac{RL^3}{3EI}$$

$$R = \frac{3}{8}wL$$

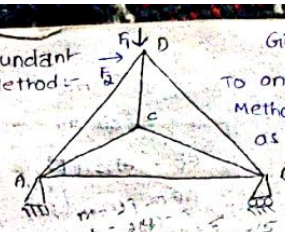
13)



indeterminate



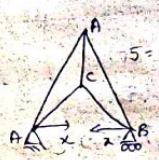
14) Redundant Method :-



Given truss as $n_s = 1$

To analyse the truss we use redundant Method. Assume any one of member as redundant (i.e., remove that member) and then apply 'x' unit load tensiley and find out each member forces. Before to calculate 'x' unit load, first calculate for 1 unit load and multiply it with 'x' and add the redundant load before redundant the load it.

θ - force after 1 unit applied in member (just like x)



$5 = 2(4) - 3$
$= 8 - 3$
$5 = 5$
$n_s = 0$

Assume AB is not present then it is determinate structure as $n_s = 0$

$P_1 + \theta_1 x$
$P_2 + \theta_2 x$
$P_3 + \theta_3 x$
⋮
$P_n + \theta_n x$

After applying "x" unit load of tensile The force in all member then,

$$U = \sum \frac{P^2}{2AE} dx$$

$$\frac{\partial U}{\partial x} = 0 \Rightarrow \sum \frac{(P+\theta x)\theta L}{AE} = 0$$

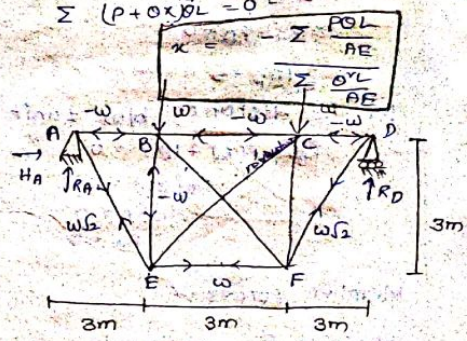
$$\sum (P + \theta x)\theta L = 0$$

$$x = -\frac{\sum \frac{PθL}{AE}}{\sum \frac{θ^2 L}{AE}}$$

15) Analyse the truss

Analyse whether it is determinate or not

$m = 2j - 3$
$10 = 2(6) - 3$
$10 = 12 - 3$
$10 \neq 9 \quad n_s = 1$



Taking 3m = l

consider EC as a redundant member.

$R_A + R_D = 2w$

$\Sigma M_A = 0 \Rightarrow w(l) + w(2l) = R_D(3l)$

$\Rightarrow 3wl = 3l R_D$

$R_D = w$

$R_A = w$

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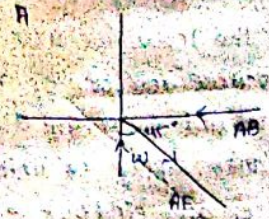
At A



$AE\cos 45 = AB$

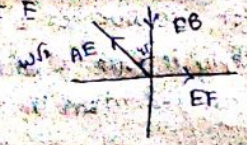
$\omega = AE\sin 45$

$AE = \omega\sqrt{2}\ (T)$

$AB = \omega\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)$

$= \omega\ (C)$

At E



$AE\cos 45 = EF$

$\omega\sqrt{2}\left(\frac{1}{\sqrt{2}}\right) = EF$

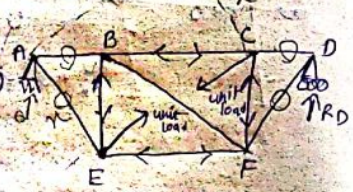
$EF = \omega\ (T)$

$EB = AE\cos 45$

$EB = \omega\sqrt{2}\left(\frac{1}{\sqrt{2}}\right) = \omega\ (C)$

Now apply 1 unit load of tensile in the redundant member direction and then analyse the truss.

support rxns always becomes zero. (as acting in opp. direction)



$\Sigma F_y = 0$

$R_A + R_B = 0$

$\Sigma M_A = 0$

let the distance b/w 1 unit applied and A be 'x'

$R_D(3L) - 1(x) + 1(x) = 0$

$R_D(3L) = 0$

$R_D = 0$

$R_A = 0$

Member forces :-

at joint E

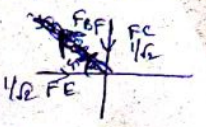
$1\cos 45 = EF$

$EF = \frac{1}{\sqrt{2}}\ (C)$

$EB = 1\sin 45$

$EB = \frac{1}{\sqrt{2}}\ (C)$

at joint F :-



$F_{BF}\cos 45 = \frac{1}{\sqrt{2}}$

$F_{BF}\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}$

$F_{BF} = 1\ (T)$

Member	P	θ	L	PθL	θ²L
1) AB	$-\omega$	0	L	0	0
BC	$-\omega$	$-\frac{1}{\sqrt{2}}$	L	$\frac{\omega L}{\sqrt{2}}$	$L/2$
CD	$-\omega$	0	L	0	0
AE	$\omega\sqrt{2}$	0	$L\sqrt{2}$	0	0
EF	ω	$-\frac{1}{\sqrt{2}}$	L	$-\frac{\omega L}{\sqrt{2}}$	$\frac{L}{2}$
DF	$\omega\sqrt{2}$	0	$L\sqrt{2}$	0	0
BE	$-\omega$	$-\frac{1}{\sqrt{2}}$	L	$\frac{\omega L}{\sqrt{2}}$	$\frac{L}{2}$
CF	$-\omega$	$-\frac{1}{\sqrt{2}}$	L	$\frac{\omega L}{\sqrt{2}}$	$L/2$
BF	0	1	$L\sqrt{2}$	0	$L\sqrt{2}$
10) EC	0	1	$L\sqrt{2}$	0	$L\sqrt{2}$

$\Sigma P\theta L = \frac{2\omega L}{\sqrt{2}}$ $\quad \Sigma\theta^2 L = 2L + 2\sqrt{2}$

$= \sqrt{2}\omega L$

$x = -\frac{\Sigma \frac{P\theta L}{AE}}{\Sigma \frac{\theta^2 L}{AE}} = -\frac{\sqrt{2}\omega L}{2L + 2\sqrt{2}L}$

$= -\frac{\sqrt{2}\omega(3)}{2(3) + 2\sqrt{2}(3)}$

$x = -0.292\omega$

Final forces :-

$P_1 + \theta_1 x \Rightarrow -\omega + 0(x) = -\omega$

$P_2 + \theta_2 x \Rightarrow -\omega - \frac{1}{\sqrt{2}}x = -\omega - \frac{1}{\sqrt{2}}(-0.292\omega)$

$= -\omega + 0.2\omega = -0.79\omega$

$P_3 + \theta_3 x \Rightarrow -\omega + 0(x) = -\omega$

$P_4 + \theta_4 x \Rightarrow \omega\sqrt{2} + 0(x) = \omega\sqrt{2}$

⋮

$P_{10} + \theta_{10} x \Rightarrow 0 + 1(x) = x$

$= -0.292\omega$

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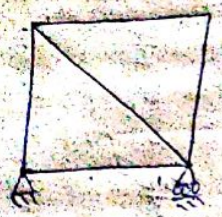
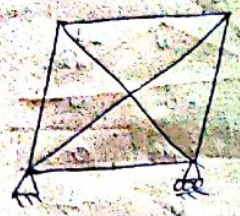
Lack of fit :-

$5 = 2(4) - 3$
$5 = 8 - 3$
$= 5$



$n_s = 0$

Determinate



$6 = 2(4) - 3$
$= 8 - 3$
$= 5$

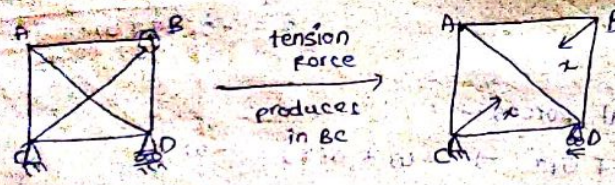
$n_s = 1$

indeterminate

→ In Determinate structures because of fabrication (i.e., short or long in length in one member) will not generate extra forces in members but in Indeterminate structures extra forces will generate

→ Here every member shares the force
if length is short → Tension produces
length is long → compression force

→ Let, consider the below structure in which BC length is short, due to this strain forces generates

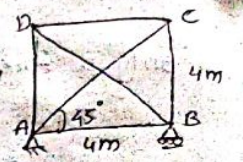


$$U = \sum \frac{P^2 dx}{2AE}$$

$$\frac{\partial U}{\partial x} = \Delta \text{ (deflection)}$$

Here x is force

16) A square truss is assembled in the field if the length of a diagonal member AC is defective short by 0.1% its length. Axial rigidity of all members is same. AC diagonal fitted forcefully, find the force developed in remaining members.



square

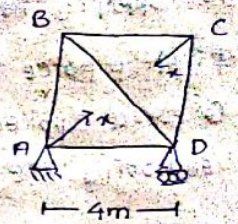
$L_{AC} = 4\sqrt{2} = 5.65\,m$

$5.65 \rightarrow 100\%$

$x \rightarrow 0.1\% \Rightarrow \frac{5.65}{x} = \frac{100}{0.1}$

$\Rightarrow x = 5.65\,mm$

Total strain energy stored in system :-



$$U = \frac{\sum P^2 L}{2AE}$$

Member	force	length	P^2L
AB	$-\frac{x}{\sqrt{2}}$	L	$x^2L/2$
BC	$-\frac{x}{\sqrt{2}}$	L	$x^2L/2$
CD	$-\frac{x}{\sqrt{2}}$	L	$x^2L/2$
AD	$-\frac{x}{\sqrt{2}}$	L	$x^2L/2$
BD	x	$L\sqrt{2}$	$x^2L\sqrt{2}$
AC	x	$L\sqrt{2}$	$x^2L\sqrt{2}$

$$\sum P^2 L = 2x^2L + 2\sqrt{2}x^2L$$
$$= 2x^2L[1+\sqrt{2}]$$

$$\therefore U = \frac{\sum P^2 L}{2AE}$$

$$U = \frac{2x^2L(1+\sqrt{2})}{2AE}$$

$$\frac{\partial U}{\partial x} = \frac{2xL(1+\sqrt{2})}{AE}$$

$$5.6 = \frac{2xL(1+\sqrt{2})}{AE} \Rightarrow x = \frac{5.6 \times 10 \times 2 \times 10^6}{2 \times 4 \times 10^2 (1+\sqrt{2})}$$

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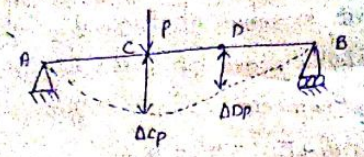
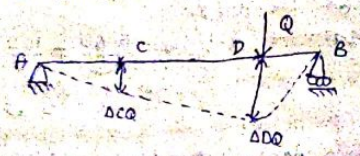
Betti's Law :-

only P force applied at C ⇒

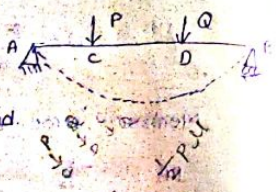


only Q force applied at D ⇒



Now applying both forces :-

First P force applied and then Q force applied without removing the first force (or) load.



1) P force followed by Q force

$$\omega_1 = \frac{1}{2} P\Delta_{CP} + \frac{1}{2} Q\Delta_{DQ} + P\Delta_{CQ}$$ → virtual work done (not gradual already applied) so no ½

2) Q force followed by P force

$$\omega_2 = \frac{1}{2} Q\Delta_{DQ} + \frac{1}{2} P\Delta_{CP} + Q\Delta_{DP}$$

Both are equal

$$\omega_1 = \omega_2$$

$$\Rightarrow \boxed{P\Delta_{CQ} = Q\Delta_{DP}}$$

Theorem :-

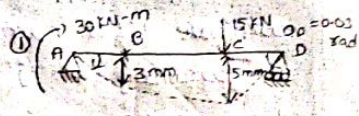
For linearly elastic structures subjected to two sets of forces P and Q the workdone by the set 'P' through the displacement produced by the set Q is equal to the workdone by the set Q through the displacement produced by the set P.

$$P\Delta_{CQ} = Q\Delta_{DP}$$

16) We've shown a system of forces and displacements for a simple beam. Calculate for unknown forces.

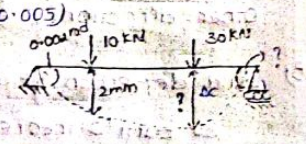
By Betti's law

$$P\Delta_{CQ} = Q\Delta_{DP}$$



B/w ① & ②

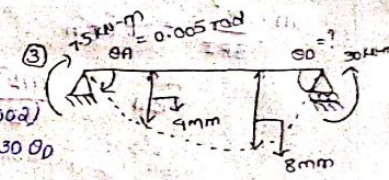
$$(30)(0.002) + 15\Delta_c = 10(0.003) + 30(0.005)$$



$$0.06 + 15\Delta_c = 0.03 + 0.15$$

$$\Delta_c = 8\times10^{-3}\ m$$

$$\Delta_c = 8\ mm$$



B/w ② & ③

$$10(0.004) + 30(0.008) = 7.5(0.002) + 30\,\theta_D$$

$$\theta_D = 0.0088\ rad$$

B/w ① & ③

$$30(0.0015) + 15(0.008) = 7.5(\theta_A) + 30(0.003)$$

$$\theta_A = 0.01\ rad$$

→ If P = Q ⇒ deflection at C and deflection at D are equal

$$P\Delta_{CQ} = Q\Delta_{DP}$$

$$\Delta_{CQ} = \Delta_{DP} \quad (\because P = Q)$$

⇒ It is Betti's reciprocal theorem

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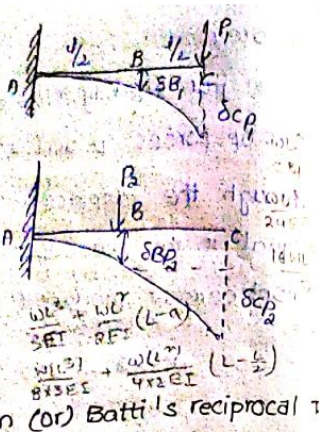
17) Find deflection at B :-



By Batti's eqn

$P_1 \delta C_{P_2} = P_2 \delta B_1$

$P_1 = P_2$ then

$\delta C_{P_2} = \delta B_2$

w.k.T $\delta_{CP} = \frac{5PL^3}{48EI}$

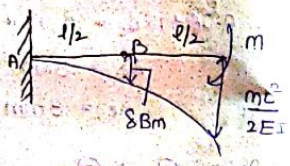
$\frac{wL^4}{8EI} + \frac{wL^2}{8EI}(L-a)$

$\frac{wL^3}{8\times3EI} + \frac{wL^2}{4\times2EI}(L-\frac{L}{2})$

Maxwell Reciprocal theorem (or) Batti's reciprocal Th

The displacement in any direction at coordinate due to a force applied at some other direction at coordinate 'j' is numerically equal to the displacem at coordinate ~~at~~ 'j' in the corresponding direction due to force applied at coordinate 'i' in some other direction.

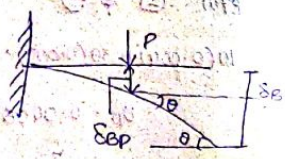
18) Find deflection at B



By Batti's theorem

$m \cdot \frac{PL^2}{8EI} = P\delta_{Bm}$

$\delta_{Bm} = \frac{mL^2}{8EI}$



Otherwise :-

By parabolic eqn

$\theta = \frac{PL^2}{8EI}$

$\frac{x^2}{y} = c$

$\frac{L^2}{\left(\frac{mL^2}{2EI}\right)} = \frac{\frac{L^2}{4}}{\delta_{Bm}}$

$= \frac{wL^3}{3EI} + \frac{wL^2}{2EI}(L-$

$= PL^3$

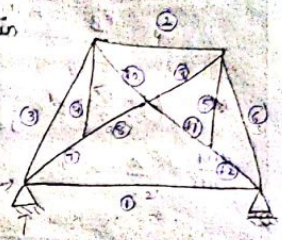
$\delta_{Bm} = \frac{mL^2}{8EI}$

MID - 1

Exam

Section - A

1. For a given truss structure explain



a) External stability

b) External inderminancy, internal indeterminacy

c) Total static indeterminacy, Kinamatic indeterminacy

a) A structure is externally unstable if its rxns are concurrent or parallel.

Here the structure is externally stable as $\gamma = 3 = a$

$\gamma \rightarrow$ No of ~~forces~~ rxns $= 3$

~~$\gamma \rightarrow$ no of required unknowns~~ forces

[Each 1 page] $\boxed{\text{stable}}$

b) External indeterminacy $= \gamma - 3$ [2D] externally determinate $= 3-3 = 0$

Internal indeterminacy $\Rightarrow (m+3) - (2j+0) \Rightarrow m - 2j + 3$

$a \rightarrow$ At each joint 2 equilibrium rxns are available in truss

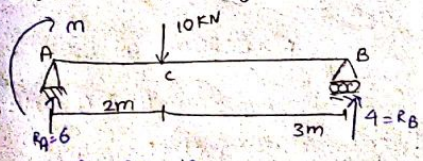
$\Sigma F_x = 0 \quad \Sigma F_y = 0$

$\therefore I.I = 12+3-2(7) = 1.$

c) $SI = EI + II = 0+1 = 1$ statically indeterminate

d) $KI = aj - \beta + \gamma \Rightarrow 2(7) - 3 = 11$

2. Calculate rotation at support A of the simple supported beam shown in fig. $EI = 100000\ KNm^2$



$R_A + R_B = 10$

$\Sigma m_A = 0$

$(10\times2) - R_B(5) = 0$

$R_B = 4$

$R_A = 6$

To get rotation at A

$R_A + R_B = 0$

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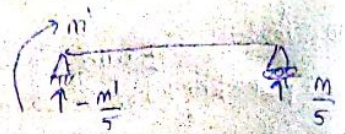
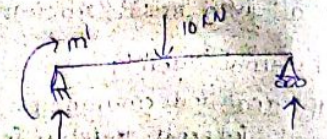
$\Sigma M_B = 0$



$5R_A + m' = 0$

$R_A = -\frac{m'}{5}$

$R_B = +\frac{m'}{5}$



$M_1 = \left(4+\frac{m'}{5}\right)x \qquad 0<x<3$

$\frac{\partial M_1}{\partial m} = \frac{x}{5}$

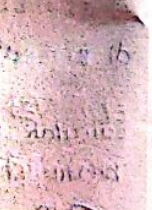
$M_2 = \left(4+\frac{m'}{5}\right)x - 10(x-3)$

$\frac{\partial M_2}{\partial m} = \frac{x}{5}$

$$\frac{\partial v}{\partial m'} = \int_0^l \frac{M}{EI} \cdot \frac{\partial M}{\partial m'}\, dx$$

$$= \frac{1}{EI}\int_0^3 \left(4+\frac{m'}{5}\right)\frac{x}{5}\,dx + \int_0^5 \left[\left(4+\frac{m'}{5}\right)x - 10(x-3)\right]\frac{x}{5}\,dx$$

$$= \frac{1}{EI}\int_0^3 \left(\frac{4x}{5}+\frac{m'x}{25}\right)dx + \int_0^5 \left(\frac{4x^2}{5}+\frac{m'x^2}{25}-\frac{10x^2}{5}-\frac{30x}{5}\right)dx$$

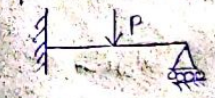


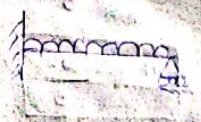
UNIT - II

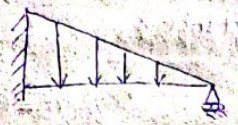
MID-02

PROPPED CANTILEVERS & FIXED BEAMS

propped cantilevers :-

1) 

2) 

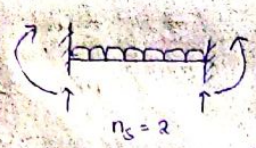
3) 

4)

5)

6)

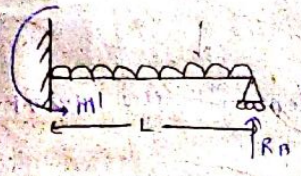
Here $n_s = 3$

7) 

$n_s = 2$

Horizontal not considered

Based on deflection :-

19) 

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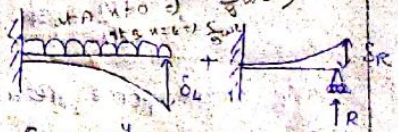


$\delta_L = \frac{wL^4}{8EI}$ $\delta_R = \frac{RL^3}{3EI}$

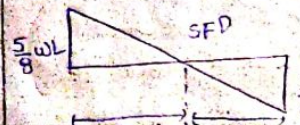
Both are equal

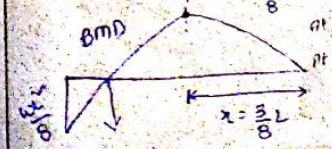
$S_L = \delta_R$

$\frac{RL^3}{3EI} = \frac{wL^4}{8EI}$

$R_A = \frac{3wL}{8}$

$R_B = \frac{5}{8}wL$





at this point curvature changes is called point of contraflexure

max BM at $x = \frac{3L}{8}$

$BM = R_A x - \frac{wx^2}{2}$

at $x = 0$ = 0

at $x = L$ $= \frac{3wL^2}{8} - \frac{wL^2}{2}$

$= \frac{3wL^2 - 4wL^2}{8}$

$= \frac{-wL^2}{8}$

$\frac{5}{8}wL \cdot x - \frac{w}{2}x^2 = \frac{3}{8}wL(L-x)$

$\frac{5x}{8} = \frac{3L}{8} - \frac{3x}{8}$

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For BMD :-

$$BM = \left(\frac{3}{8}wL\right)L - \frac{wL^2}{2} - m' = 0$$

$$m' = \frac{3}{8}wL^2 - \frac{wL^2}{2}$$

$$\boxed{m' = \frac{wL^2}{8}}$$

Let maximum BM occurs at distance x

By similar triangles $x = \frac{3}{8}L$

maximum BM :-

$$B.M = \left(\frac{3}{8}wL\right)\frac{3}{8}L - \left(\frac{3}{8}wL\right) \times \frac{3}{8} \times \frac{L}{2}$$

$$= \frac{9}{64}wL^2 - \frac{9}{128}wL^2$$

$$= \frac{9}{128}wL^2$$

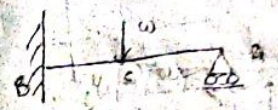
* moment at every point in a structure is zero. But BM will be there

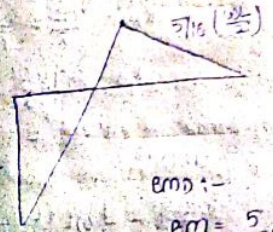
$$\frac{dM}{dx} = SF$$

* If shear force present slope will also present

$$\frac{M}{EI} = \frac{1}{R} = \text{curvature}$$

20)





$$\delta_L = \frac{wL^3}{3EI} + \frac{wL^2}{2EI}(L-L)$$

$$\delta_R = \frac{RL^3}{3EI}$$

$$\frac{wL^3}{3EI} + \frac{wL^2}{2EI}(L-L) = \frac{wL^3}{8EI}$$

$$\frac{wL^3}{3EI} - \frac{wL^3}{8EI} = \frac{5wL^3}{24EI}$$

$$\delta L = \delta R$$

$$\frac{5wL^3}{24EI} = \frac{RL^3}{3EI}$$

$$R = \frac{5w}{8} = \frac{5w}{16}$$

$$R_B = \frac{11}{16}w$$

$$SF = -\frac{5w}{16}$$

$$SF\ BC = -\frac{5w}{16} - w = -\frac{11w}{16}$$

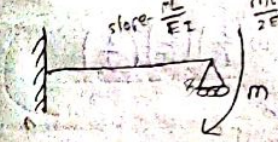
BMD :-

$$BM = \frac{5}{16}wL - \frac{wL}{2}$$

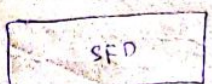
$$m' = \frac{5wL - 8wL}{16}$$

$$m' = -\frac{3wL}{16}$$

21) Concentrated moment m :-



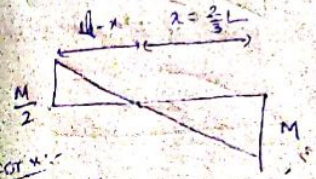
$$\delta_L = \frac{mL^2}{2EI} \qquad \delta_R = \frac{RL^3}{3EI}$$



$$R_A = \frac{3}{2}\frac{m}{L}$$

$$R_A + R_B = 0$$

$$R_B = -\frac{3}{2}\frac{m}{L}$$



BMD :-

$$BM = \frac{3}{2}\frac{m}{L} \times L - m$$

$$m' = \frac{3}{2}m - m$$

$$m' = \frac{m}{2}$$

For x :-

$$\frac{M}{EI} = \frac{x}{L-x}$$

$$2 = \frac{x}{L-x}$$

$$x = \frac{2}{3}L$$

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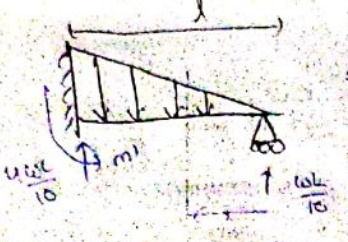
22)



$\delta_L = \frac{wL^4}{30EI}$ $\qquad S_R =$

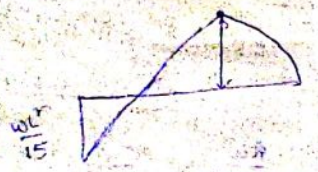
$R_A = \frac{wL^4}{30EI} \times \frac{3EI}{L^3}$

$R_A = \frac{wL}{10}$

$R_A + R_B = \frac{1}{2}wL$

$R_B = \frac{wL}{2} - \frac{wL}{10}$

$R_B = \frac{4wL}{10}$



SF zero at

$-\frac{wL}{10}x + \frac{1}{2}\left(\frac{w}{L}\right)x \times x = 0$

$-\frac{wL}{10}x + \frac{wx^2}{2L} = 0$

$x = \frac{L}{\sqrt{5}}$

BMD :-

$\frac{wL}{10}x - \frac{1}{2}wL \times \frac{L}{3} + m' = 0$

$m' = \frac{mL^2}{15}$

max BM :-

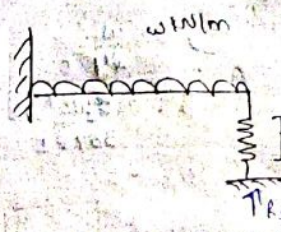
Occurs at $x = \frac{L}{\sqrt{5}}$ $\qquad \frac{wL}{10}x - \left(\frac{1}{2}wx \times x\right)\frac{x}{3}$

$BM = \frac{wL}{10}\left(\frac{L}{\sqrt{5}}\right) - \frac{1}{2}w\left(\frac{L}{\sqrt{5}}\right)\left(\frac{L}{\sqrt{5}}\right)\left(\frac{L}{\sqrt{5}}\right)$

$= \frac{wL^2}{10\sqrt{5}} - \frac{wL^2}{30\sqrt{5}}$

$= \frac{wL^2}{15\sqrt{5}}$

23)



$k = \frac{EI}{L^3}$

Because of spring it allows some deflection because of that $\delta_L \neq \delta_R$

$\frac{wL^4}{8EI} = \frac{R_s L^3}{3EI} + y$

$y = \frac{wL^4}{8EI} - \frac{R_s L^3}{3EI}$

For unit displacement how much force required is called stiffness

$\text{stiffness} = \frac{\text{force}}{\text{deflection}}$

$\frac{EI}{L^3} = \frac{R_s}{y}$

$y = \frac{wL^4}{8EI} - \frac{R_s L^3}{3EI}$

$y = \frac{R_s L^3}{EI}$

equal

$\therefore \frac{wL^4}{8EI} - \frac{R_s L^3}{3EI} = \frac{R_s L^3}{EI}$

$R_s = \frac{3wL}{32}$

* If stiffness increases it allows less deflection so R_s value increases

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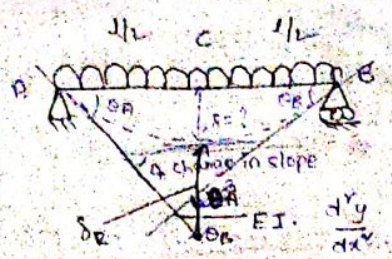
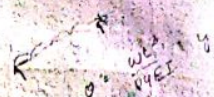
24)



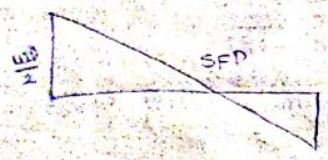
$\theta = \frac{wL^3}{24EI}$

$\delta = \frac{5wL^4}{384EI}$

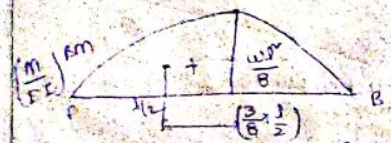


By mohr theorem 1 :-

$\theta_A - \theta_B = \frac{\text{Area b/w A and B BMD}}{EI}$



The change in slope b/w A and in deflection diagram = area under the curvature curve b/w the two points



$\theta_A = \frac{\frac{2}{3} \times \frac{wL^2}{8} \times \frac{l}{2}}{EI}$

$\boxed{\theta_A = \frac{wL^3}{24EI}}$

$\boxed{\theta_B = -\frac{wL^3}{24EI}}$

By mohr theorem 2 :-

The vertical deviation of point C on elastic curve w.r.to tangent at another point of A at elastic curve is equal to the

$\delta_R = \frac{\text{Area} \times \text{Centroid}}{EI}$

$= \frac{\left[\frac{2}{3} \times \frac{wl^2}{8} \times \frac{l}{2}\right]\left(\frac{3}{8}\frac{l}{2}\right)}{EI}$

$= \frac{3wl^3}{384EI}$

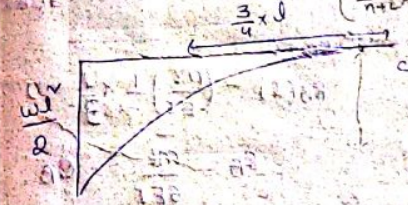
$\delta_D = \theta_A \times \frac{l}{2}$

$\delta_D = \frac{wl^3}{24EI} \times \frac{l}{2}$

$\delta_C = \delta_D - \delta_R$

$\boxed{\delta_C = \frac{5wL^4}{384EI}}$

25)



$\delta_C = \frac{\frac{1}{3} \times \frac{wl^2}{2} \times l \times \frac{3}{4}l}{EI}$

$= \frac{1}{3} \times \frac{wl^2}{2} \times l = \frac{wl^4}{8EI}$

$\theta_C - \theta_A = \frac{wl^2}{2} \times \frac{1}{3} l / EI$

$\boxed{\theta_C = \frac{wl^3}{6EI}}$

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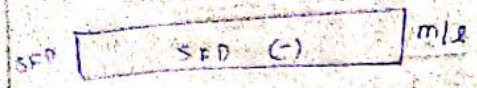
26)

$\theta_A - \theta_B = -$

unknows



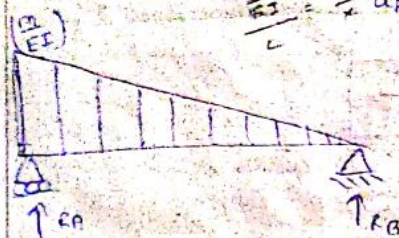
so conjugate beam

hinge place → ~~[illegible]~~

Roller place → hinge

fixed → free

free → fixed.



$\frac{\frac{m}{EI}}{L} = \frac{q}{x}$ after that place the Bending curve on the beam as a load

slope at required point = SF at that point

θ_A = SF at A

deflection at required point = BM at that point

~~SF b/w A and B~~ $R_B l = \frac{1}{2} \times \frac{ml}{EI} \times l$

$= \frac{ml^2}{2EI}$

$\Sigma M_A = 0$

$R_B(l) - \left(\frac{ml}{EI}\right)\frac{1}{2} \times \frac{l}{3} = 0$

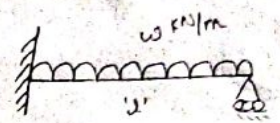
$R_B = \frac{ml}{6EI} = \theta_B$

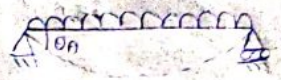
$R_A = \frac{ml}{3EI} = \theta_A$

Based on slope :-

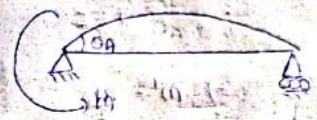
	slope
cantilever (couple)	$\frac{ml}{EI}$
simply supported (point load)	$\frac{ml}{3EI}$

27)





$\theta_A = \frac{wl^3}{24EI}$



$\theta_A = \frac{ml}{3EI}$ [simply supported couple]

$\frac{ml}{3EI} = \frac{wl^3}{24EI}$ $\left[\frac{ml}{3EI}\right]$

$m = \frac{wl^2}{8}$

$\theta_B = ?$ $R_A + R_B = wl$

$\Sigma M_B = 0$ $-R_A l + \frac{wl^2}{2} - m = 0$

$R_A l = m - \frac{wl^2}{2}$

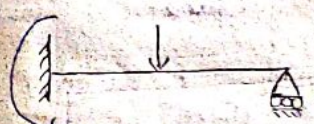
$R_B = \frac{5}{8}wl$ $R_B l = \frac{wl^2}{2} - \frac{wl^2}{8}$

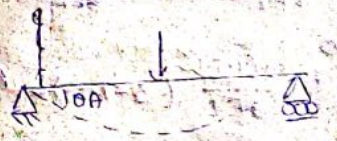
$R_A = \frac{3}{8}wl$ $R_B l = \frac{wl^2}{8} + \frac{wl^2}{2}$

$= \frac{wl^2 + 4}{8}$

$= \frac{5wl}{8}$

28)





$\theta_A = \frac{wl^3}{16EI}$

$\theta_A = \frac{ml}{3EI}$

$\frac{ml}{3EI} = \frac{wl^3}{16EI}$

$m = \frac{3}{16}wl$

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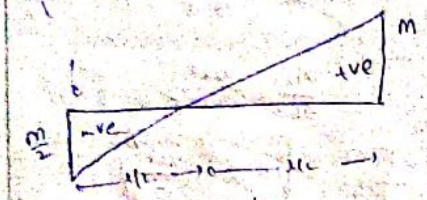
29)

$\Rightarrow \theta_A = \frac{ML}{4EI} \Rightarrow M =$

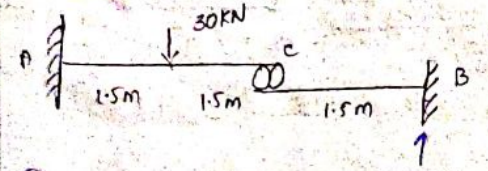
$\theta_A = \frac{ML}{6EI}$

$\theta_A = \frac{ml}{3EI}$

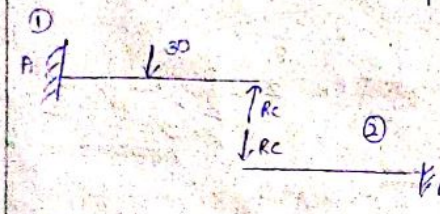
$\Rightarrow m' = \frac{m}{2}$



30)



Because of roller at c it allows some deflection because of that δ_L & δ_R R_c in both cases are same.



δ due to ① $\Rightarrow \frac{wa^3}{3EI} + \frac{wa^2}{2EI}(L-a) - \frac{R_c(3)^3}{3EI}$

$\Rightarrow \frac{w(1.5)^3}{3EI} + \frac{w(1.5)^2}{2EI}(3-1.5) - \frac{R_c(\frac{L}{3})^3}{3EI}$

$\Rightarrow$

δ due to ② $\Rightarrow \frac{R_cL^3}{3EI} \Rightarrow \frac{R_c(1.5)^3}{3EI}$

$$\frac{R_c(1.5)^3}{3EI} = \frac{w(1.5)^3}{3EI} + \frac{w(1.5)^2}{2EI}(1.5) - \frac{R_c(3)^3}{3EI}$$

$$\frac{9}{8}R_c = \frac{135}{1} + \frac{405}{8} - \frac{1}{8}R_c$$

$$\frac{81}{8}R_c = \frac{675}{8}$$

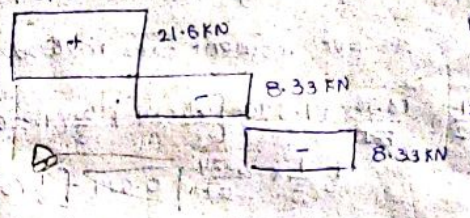
$$R_c = \frac{25}{3} = 8.33 \text{ KN}$$

$R_A + R_c = 30$

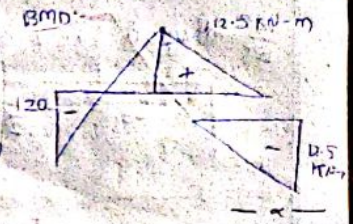
$R_A = 21.6$ KN $\quad R_c = 8.33$ KN

$R_B = R_c$

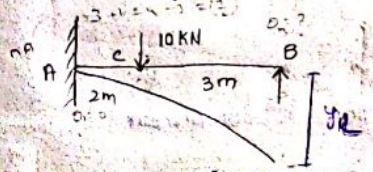
SFD:-



BMD:-



31) Determine the deflection at point c of the propped cantilever shown in fig.



$\Rightarrow \frac{wa^3}{3EI} + \frac{wa^2}{2EI}(L-a)$

$y_{PL} = \frac{10\times2^3}{3EI} + \frac{10\times2^2}{2EI}\times3$

$= \frac{86.67}{EI}$

$y_R = \frac{R_B\times3^3}{3EI}$

$y_L = y_R$

$\frac{R_B 3^3}{3EI} = \frac{86.67}{EI}$

$R_B = 2.08$ KN

$R_A + R_B = 10$

$R_A = 7.92$ KN

SFD

BM at C:-

$M_c = R_B\times3 = 2.08\times3 = 6.24$ KN·m

BM at A:-

$m_A = y_{R_B} - (10\times2) = -13.76$ KNm

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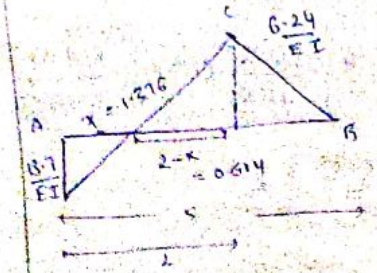


From similar triangles:-

$$\left(\frac{13.76}{EI}\right) \times \frac{1}{x} = \frac{6.24}{EI} \times \frac{1}{(2-x)}$$

$$x = 1.376 \text{ m}$$

The deflection at C is given by the moment of the $\frac{m}{EI}$ diagram about C as per moment area method.

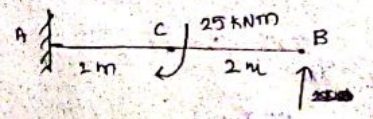
$$\delta_c = -\frac{1}{2} \times \frac{13.76}{EI} \times 1.376 \left[0.624 + \frac{2}{3}(1.376)\right] + \frac{1}{2} \times \frac{6.24}{EI} \times 0.624 \left\{\frac{1}{3}(0.624)\right\}$$

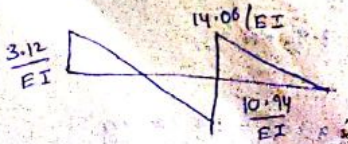
$$= \frac{-14.59}{EI} + \frac{0.405}{EI}$$

$$= \frac{-14.19}{EI}$$

'-ve' means deflection is downward.

32) Determine slope of the beam at the point where moment is applied. Take $EI = 11000$ KNm^2.





δ_L due to moment

$$\delta_L = \frac{mb^2}{2EI} + \frac{ma}{EI}(L-a)$$

$$\delta_L = \frac{25 \times 2^2}{2EI} + \frac{25 \times 2}{EI}(2)$$

$$= \frac{150}{EI}$$

$$\delta_R = \frac{R_B l^3}{3EI} = \frac{21.33 R_B}{EI}$$

$$\delta_L = \delta_R$$

$R_B = 7.03$ KN

$R_A = -7.03$ KN

BM at C:-

$$M_{C+} = R_B(2)$$

$$= 14.06 \text{ KN-m}$$

BM at A:-

$$M_A = 4R_B - 25$$

$$= 3.12 \text{ KNm}$$

Negative BM at C

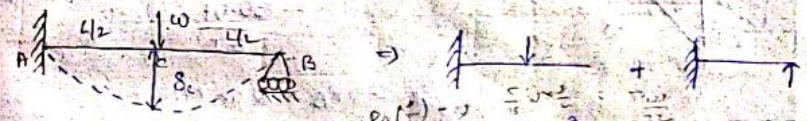
$$M_{C-} = R_B \times 2 - 25$$

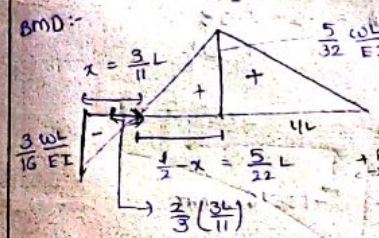
$$= -10.94$$

By moment area method, slope at C is

$$\theta_c = \frac{1}{2} \times \frac{14.06}{EI} \times 2 = 1.28 \times 10^{-3} \text{ rad}$$

33) Deflection of propped cantilever under load point at mid-span





$$\delta_L = \frac{5wL^3}{48EI} \qquad \delta_A = \frac{RL^3}{3EI}$$

BM b/w AC

$$R_B = \frac{5}{16}w$$

$$R_A = \frac{11}{16}w$$

$$+R_A(x) - w\left(x - \frac{L}{2}\right) = 0$$

$$\frac{5}{16}wx - wx + \frac{wL}{2} = 0$$

$$x\left\{\frac{5}{16}w - 1\right\} = -\frac{wL}{2}$$

$$x = -\frac{wL}{2} \div \frac{(5w-16)}{16}$$

The distance x from A where the BM becomes zero is obtained from similar Δle's.

$$\frac{\frac{5wL}{32EI}}{\frac{3}{16}\frac{wL}{EI}} = \frac{\frac{L}{2} - x}{x}$$

$$\Rightarrow x = \frac{3}{11}L$$

As the tangent drawn to the deflection curve at A is horizontal, the deflection at the load point C is obtained as a vertical distance b/w this horizontal and the tangent to the elastic curve at C. As per moment area method this is given by the moment of the $\frac{m}{EI}$ diagram b/w A and C about C.

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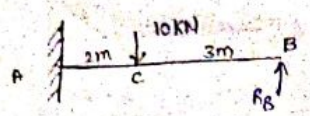
$$y_c = \delta_c = -\frac{1}{2}\left[\frac{3wL}{16EI} \times \frac{3}{11}L\right] \times \left(\frac{2}{3}\left(\frac{3L}{11}\right) + \frac{5L}{22}\right) + \left[\frac{1}{2} \times \frac{5wL}{32EI}\right]$$

$$= -\frac{81wL^3}{7744EI} + \frac{125}{92928}\frac{wL^3}{EI}$$

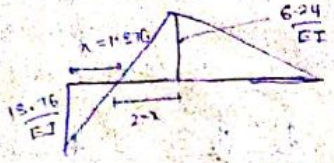
$$= -9.71 \times 10^{-3}\frac{wL^3}{EI}$$

34) Determine the deflection at point C of the propped can. shown in fig. EI is constant



$\Rightarrow$

$+$



$$y_L = \frac{10 \times 2^3}{9EI} + \frac{10 \times 2^2}{2EI} \times 3 \qquad y_R = \frac{R_B}{}$$

$$= \frac{86.67}{EI}$$

$$y_L = y_R \Rightarrow R_B = 2.08\ KN$$

$$R_A = 10 - 2.08$$

$$= 7.92\ KN$$

Bending moment at C:

$$M_c = 2.08 \times 3 = 6.24\ KN\text{-}m$$

BM at A:

$$M_A = 2.08 \times 3 - (10 \times 2) = -13.76\ KNm$$

From similar triangles:-

$$\frac{\frac{6.24}{EI}}{\frac{13.76}{EI}} = \frac{2-x}{x} \Rightarrow x = 1.376$$

$\therefore$ The deflection at C is given by the moment of the $\frac{M}{EI}$ diagram about C as per moment-area method.

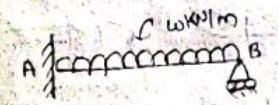
$$y_c = -\frac{1}{2} \times \frac{13.76}{EI} \times 1.376\left[0.624 + \frac{2}{3}(1.376)\right] + \frac{1}{2}\left(\frac{6.24}{EI} \times 0.624\right)\left(\frac{1}{3} \times 0.624\right)$$

$$= \frac{-14.59}{EI} + \frac{0.405}{EI}$$

$$= \frac{-14.19}{EI}$$

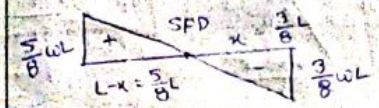
FIXED BEAMS WITH

35) Deflection of propped cantilever with UDL.



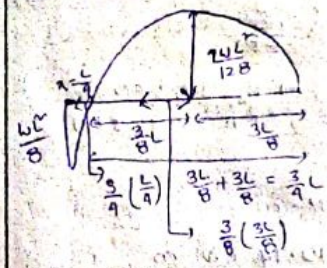
$\Rightarrow$

$$y_L = \frac{wL^4}{8EI} \qquad y_R = \frac{R_B L^3}{3EI}$$



$$\Rightarrow R_B = \frac{3}{8}wL$$

$$R_A = \frac{5}{8}wL$$



The distance of C from point B is given by similar Δs

$$\frac{\frac{3}{8}wL}{\frac{5}{8}wL} = \frac{x}{L-x}$$

$$\Rightarrow x = \frac{3}{8}L$$

The maximum BM where SF is zero.

$$M_c = \frac{3wL}{8} \times \frac{3L}{8} - \frac{3wL}{8}\left[\frac{3L}{8} \times \frac{L}{2}\right]$$

$$= \frac{9wL^2}{128}$$

BM at A:-

$$M_A = \frac{3wL}{8} \times L - wL \cdot \frac{L}{2}$$

$$= -\frac{wL^2}{8}$$

The BM at any section x from A is M_A which is zero.

$$\left(\frac{5}{8}wL\right)x - \frac{wL^2}{8} - \frac{wx^2}{2} = 0$$

$$\frac{5}{8}wLx - \frac{wL^2}{8} - \frac{wx^2}{2} = 0$$

$$\frac{wx^2}{2} - \frac{5}{8}wLx + \frac{wL^2}{8} = 0$$

$$x^2 - Lx + L^2 = 0$$

$$x = \frac{L}{4}$$

$$\therefore y = \delta_c = \left(-\frac{1}{3} \times \frac{wL^2}{8EI} \times \frac{L}{4}\right) \times \left(\frac{3}{4} \times \frac{L}{4} + \frac{3L}{8}\right) + \left(\frac{2}{3} \times \frac{9wL^2}{128EI} \times \frac{3L}{8}\right) \times \left(\frac{3}{8} \times \frac{3L}{8}\right)$$

$$= -\frac{3wL^4}{512EI} + \frac{81}{32768}\frac{wL^3}{EI}$$

$$= -1.08 \times 10^{-3}\frac{wL^4}{EI}$$

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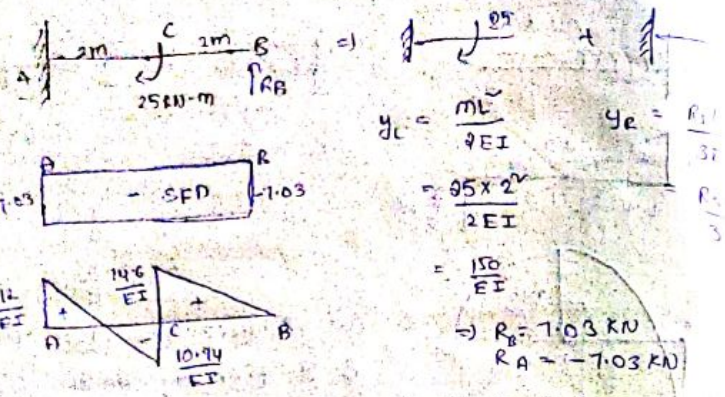
36) Find the slope of the beam as shown in fig at point where moment is applied. Take EI = 110



$y_C = \frac{ML^2}{2EI}$

$= \frac{25 \times 2^2}{2EI}$

$= \frac{150}{EI}$

$\Rightarrow R_B = 7.03$ kN

$R_A = -7.03$ kN

BM at C :-

$M_{C_+} = 7.03 \times 2 = 14.06$ kN-m

$M_{C_-} = 7.03 \times 2 - 25 = -10.94$ kN-m

BM at A :-

$M_A = 7.03 \times 4 - 25 = 3.12$ kN-m

From the moment area method, the slope at C is given by the area of the $\frac{M}{EI}$ diagram b/w C and B

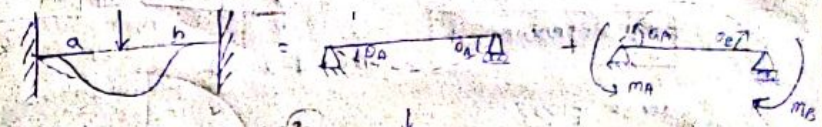
$\theta_C = \frac{1}{2} \times \frac{14.6}{EI} \times 2$

$= 1.28 \times 10^{-3}$ rad.

FIXED BEAMS

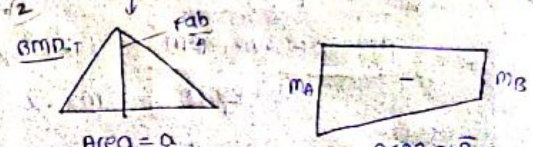
$n_c = 3$ $\quad$ $n_s = 2$

1) Eccentric point load :-

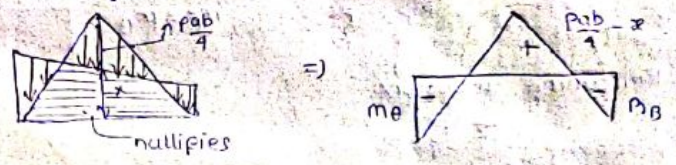


Let b > a, so

$M_a > M_b$



Combining both



For 1st diagram $\theta_A = 0$, $\theta_B = 0$

From mohr's Ist method:

$\theta_A - (-\theta_B)$ = Area of curvature diagram

$0 - 0$ = Area of +ve BM + Area of -ve BM

$\boxed{\therefore \text{Area of +ve curvature} = \text{Area of -ve curvature}}$

$\Rightarrow$ Area of free BMD = Area of fixed BMD

$a_1 = \bar{a}$

It gives one eqn

From mohr's IInd method :-

$y_B = ax - \bar{a}\bar{x}$ $\quad$ ($\because y_B = 0$ fixed)

$0 = ax - \bar{a}\bar{x}$ $\quad$ ($a = \bar{a}$ above prove a)

$ax = \bar{a}\bar{x}$

$x = \bar{x}$

$\therefore$ centroidal distance of free BMD = centroidal distance of fixed BMD.

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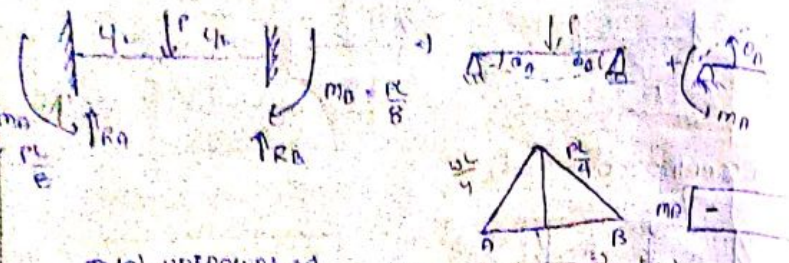
2) central point load or concentrated load



Total unknowns = 4

$m_B = 4 - 2 = 2$

By above proof:

Area of +ve BM = Area of -ve BM

$\frac{1}{2} \times \frac{PL}{4} \times L = m_A \times L$

$m_A = \frac{PL}{8}$

$m_B = \frac{PL}{8}$ $\quad m_A = m_B$

Remaining unknowns

$\Sigma F_y = 0$

$R_A + R_B = P$

$\Sigma m_B = 0$

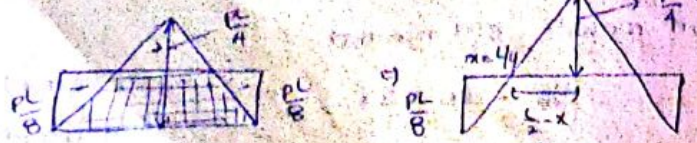
$R_A L + \frac{PL}{8} - \frac{PL}{2} - \frac{PL}{8} = 0$

$R_A L = \frac{PL}{2}$

$R_A = \frac{P}{2}$, $R_B = \frac{P}{2}$

SFD:-

BMD:-



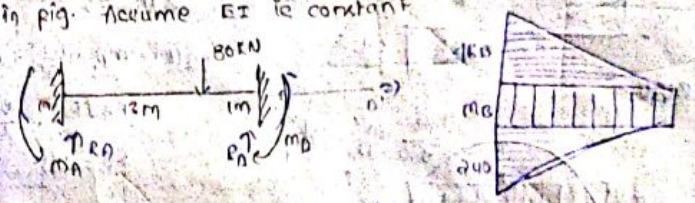
point of contra practure:

from similar Δ'les

$\frac{\frac{PL}{4}}{\frac{PL}{8}} = \frac{\frac{L}{2} - x}{x} \Rightarrow x = \frac{L}{4}$

∴ point of contra practure occurs at a point at $x = \frac{L}{4}$ $x = \frac{3L}{4}$ from A

38) Determine the fixed end moments in the beam shown in fig. Assume EI is constant



$0 = \frac{1}{2} \times 4R_B \times 4 - m_B \times 4 - \frac{1}{2} \times 240 \times 3$

$0 = 2R_B - m_B - 90$

$2R_B - m_B = 90$ — ①

The deviation of B from the tangent drawn at A is zero. so, from the second theorem

$\frac{1}{2} \times 4R_B \times 4 \times \frac{2}{3} \times 4 - m_B \times 4 \times 2 - \frac{1}{2} \times 240 \times 3 \times \left(1 + \frac{2}{3} \times 3\right) = 0$

$21.33 R_B - 8m_B = 1086$ — ②

solving ① & ②

$R_B = 67.54$ KN, $m_B = 45.08$ KN-m

Now, $\Sigma F_y = 0$

$R_A + R_B = 80$

$R_A = 12.46$ KN

$\Sigma m_A = 0$

$M_A + (67.54 \times 4) - (80 \times 3) - 45.08 = 0$

$M_A = -15.08$ KN-m

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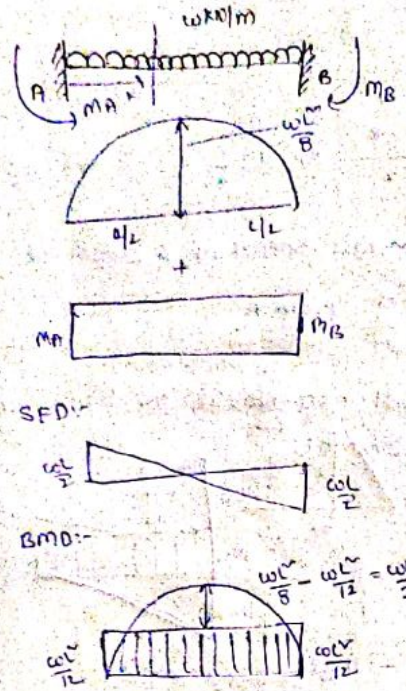
39)



$A_1 = A_2$

$\frac{2}{3} \times \frac{wL^2}{8} \times L = M_A \times L$

$M_A = \frac{wL^2}{12}$

$M_B = \frac{wL^2}{12}$

Remaining unknown

$\Sigma F_y = 0$

$R_A + R_B = wL$

$\Sigma M_A = 0$

$R_B(L) - M_B - \frac{wL^2}{2} + M_A$

$R_B(L) = \frac{wL^2}{2}$

$R_B = \frac{wL}{2}, R_A$

point of contraflexure:-

SF eqn = 0

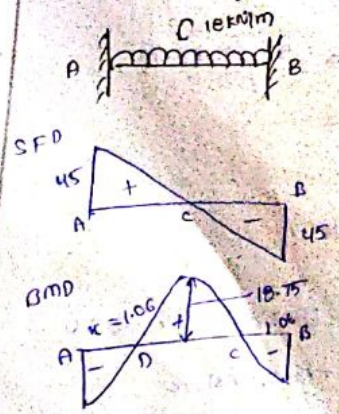
$\frac{wL^2}{2} - \frac{wx^2}{2} = 0$

$\frac{wx^2}{2} = \frac{wL^2}{12}$

$x^2 = L^2/6$

$x = L/\sqrt{6}$

40) Draw SFD & BMD



The distance of the point D where BM is zero from A is given by writing BM eqn and equating the same to zero.

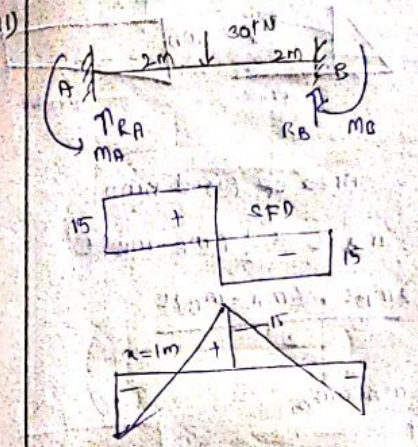
$M_x = 45x - 18\left(\frac{x^2}{2}\right) - 37.5 = 0$

$9x^2 - 15x + 37.5 = 0$

$x = 1.06\,m$ (or) $3.96\,m$

The points D & E are points of contraflexure in the beam because of symmetry, the point of contraflexure are equidistance from the ends.

41)



$R_A + R_B = 30$

COFT $R_A - R_B = \frac{w}{2}$

$R_A = R_B = 15$

$M_A = M_B = \frac{wL}{8} = \frac{30 \times 4}{8} = 15\,kN\text{-}m$

$M_E = \frac{wL}{8} = \frac{30 \times 4}{8} = 15\,kN\text{-}m$

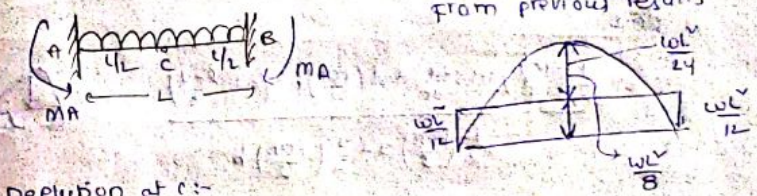
Point of contraflexure

$M_x = 0$

$15x - 15 = 0$

$x = 1m$

42) Derive the expression for the deflection at the centre of a fixed beam on which UDL of w KN/m is applied.



From previous results

Deflection at C:-

$y_c = \frac{1}{EI}\left[\frac{2}{3} \times \frac{wL^2}{8} \times \frac{L}{2} \times \frac{3}{8} \times \frac{L}{2} - \left(\frac{wL^2}{12} \times \frac{L}{2} \times \frac{L}{4}\right)\right]$

$= \frac{-wL^4}{384EI}$

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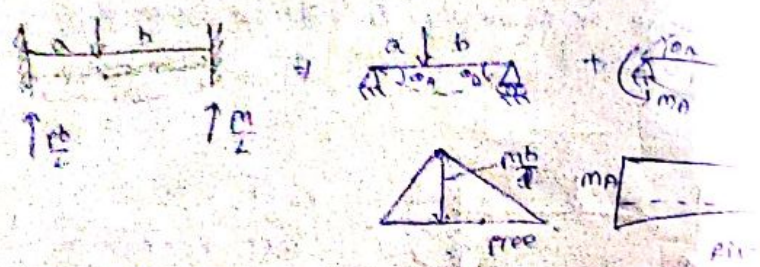

From Ist theorem:-

Area of free BMD = Area of fixed BMD

$$\frac{1}{2} \times \frac{Pab}{l} \times l = m_B \times L + \frac{1}{2}(m_A - m_B)L$$

$$\frac{Pab}{2} = \frac{2m_B L + (m_A - m_B)L}{2}$$

$$\frac{Pab}{L} = m_A + m_B$$

From IInd theorem

Centroid of free BMD = centroid of fixed BMD

$$\bar{x} = \bar{\bar{x}}$$

Now

$$\bar{x} = \frac{\frac{1}{2}\left(\frac{Pab}{l}\right) \times a \times \frac{2a}{3} + \frac{1}{2}\left(\frac{Pab}{l}\right) b \times \left(a + \frac{l-a}{3}\right)}{\frac{1}{2}\left(\frac{Pab}{l}\right) a + \frac{1}{2}\left(\frac{Pab}{l}\right) b}$$

$$\bar{x} = \frac{\frac{2a^2}{3} + (l-a)\left(\frac{3a+l-a}{3}\right)}{a+l-a}$$

$$= \frac{1}{3}\frac{(2a^2 + 2al - l^2 - 2a^2 - al)}{l}$$

$$= \frac{1}{3}\left(\frac{l^2 + al}{l}\right)$$

$$= \frac{l+a}{3}$$

$$\bar{\bar{x}} = \frac{(m_B \times L) \times \frac{L}{2} + \frac{1}{2}(m_A - m_B)L \times \frac{L}{3}}{m_B L + \frac{1}{2}(m_A - m_B)L}$$

$$= \frac{\frac{m_B L}{2} + \left(\frac{m_A - m_B}{6}\right)L}{\frac{2m_B + (m_A - m_B)}{2}}$$

$$= \frac{l}{3}\left[\frac{3m_B + (m_A - m_B)}{2m_B + (m_A - m_B)}\right]$$

$$= \left[\frac{m_A + 2m_B}{m_A + m_B}\right]\frac{l}{3}$$

$$\bar{x} = \bar{\bar{x}}$$

$$\frac{l+a}{3} = \left(\frac{m_A + m_B + m_B}{m_A + m_B}\right)\frac{l}{3}$$

$$l + a = \left[\frac{\frac{Pab}{l} + m_B}{\frac{Pab}{l}}\right] l$$

$$\frac{l+a}{l} = \frac{Pab + m_B l}{Pab}$$

$$\left(\frac{l+a}{l}\right) Pab = Pab + m_B L$$

$$m_B L = Pab - Pab\left(\frac{l+a}{l}\right)$$

$$m_B L = -\frac{Pa^2 b}{l}$$

$$m_B = \frac{Pa^2 b}{l^2}$$

$$m_A = \frac{Pab}{l} - m_B = \frac{Pab^2}{l^2}$$

m_A, m_B are calculated so we can easily calculate remaining $\bar{x}$.

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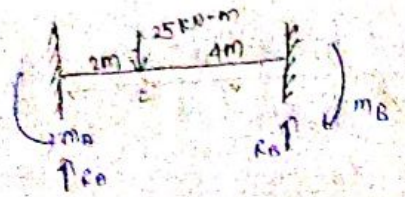
44)



$$M_A = \frac{Pab^2}{l^2} = \frac{25 \times 2 \times 4^2}{6^2} = 22.22$$

$$M_B = \frac{Pa^2b}{l^2} = \frac{25 \times 2^2 \times 4}{6^2} = 11.11$$

Remaining unknowns :-

R_A & $\Sigma F_y = 0$

$R_A + R_B = 25$

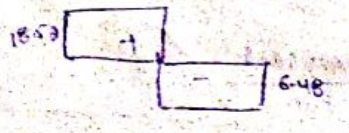
$\Sigma M_A = 0$

$-R_B(6) + M_B + 25 \times 2 - M_A = 0$

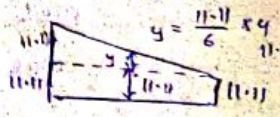
$R_B = 6.48$

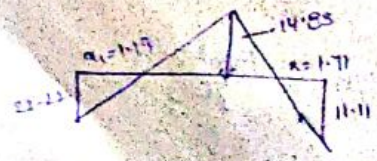
$R_A = 18.52$

SFD :-



BMD :-



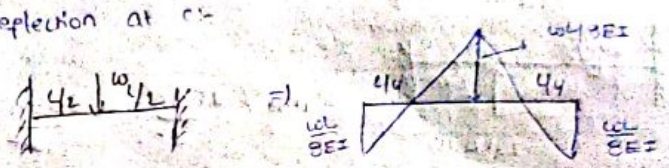


From similar Δles

$$\frac{14.83}{4-x_1} = \frac{22.22}{x_1}, \quad \frac{14.83}{4-x_2} = \frac{11.11}{x_2}$$

$x_1 = 1.19$ $x_2 = 1.71$

Deflection at C :-



$$y = \left[-\frac{1}{2}\left(\frac{wl}{8EI}\right) \times \frac{l}{4} \times \left(\frac{l}{4} + \frac{2}{3}\frac{l}{4}\right)\right] + \left[\frac{1}{2}\left(\frac{wl}{8EI}\right)\frac{l}{4} \times \frac{1}{3}\frac{l}{4}\right]$$

$$= \frac{1}{2}\frac{wl}{8EI} \times \frac{l}{4}\left[\left[-\left(\frac{l}{4} + \frac{l}{6}\right)\right] + \frac{l}{12}\right]\left[-\frac{10}{24} + \frac{1}{12}\right]$$

$$= \frac{wl^2}{64EI} \times \left(-\frac{1}{3}l\right)$$

$$= \frac{wl^3}{192EI}$$

↳ If fixed support then deflection decreases 4 times when compared to simple supported

∴ Deflection in s.p for point load $= \frac{wl^3}{48EI}$ — ①

Deflection in fixed for " $= \frac{wl^3}{192EI}$ — ②

$\frac{①}{②} = 4$

⇒ similarly,

For uniform load it decreases to 5 times in fixed beam when compared to simply support

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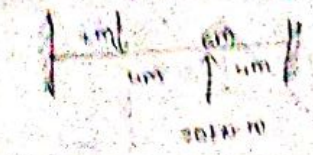
45)



$M_A = \frac{25 \times 12 \times 6^2}{8^2} - \frac{20 \times 4}{8^2}$

$= \frac{25 \times 12 \times 6^2}{8^2} - \frac{20 \times 4}{8^2}$

$= 8.12$

$M_B = \frac{25 \times 6 \times 12^2}{8^2} - \frac{20 \times 4}{8^2}$

$= -10.63$

Now,

$\Sigma F_y = 0$

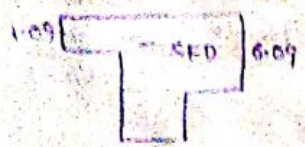
$R_A + R_B + 20 = 25$

$\Sigma M_A = 0$

$-R_B(8) + 10.63 - (20 \times 4) + (25 \times 12) - 8.12 = 0$

$R_A = 6.04, \quad R_B = -1.09$

SFD



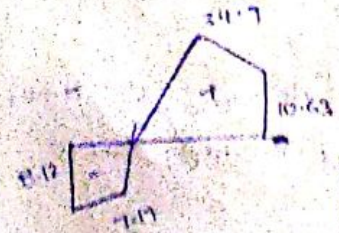
BM at D:

$M_D = 6.09 \times 4 + 10.63$

$= 34.7$

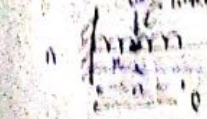
$M_C = 6.09 \times 6 + 10.63 - 8$

$= 7.17$



$\boxed{\text{Extra } M_A = M_{max} \cdot BM}$

46)



consider more of load

$M_A = \frac{20 \times (x^2 l^2 - x^3)}{l^2}$

$M_A = \int_0^a \frac{w}{l^2} x^2 (l - x)^2 \, dx$

$M_A = \int_0^a \frac{(w/l)x \, dx \, x^2 (l-x)}{l^2}$

$= \frac{w}{l^2} \int_0^a x(l^2 + x^2 - 2lx) \, dx$

$= \frac{w}{l^2} \int_0^a (l^2 x^2 - x^3) \, dx$

$= \frac{w}{l^2} \left[\frac{l^2 x^3}{3} - \frac{x^4}{4} \right]_0^a$

$= \frac{w}{l^2} \left[\frac{l^2 a^3}{3} - \frac{a^4}{4} \right]$

$= \frac{w}{l^2} \left[\frac{4a^2 l - 3a^4}{12} \right]$

$= \frac{wa^2}{12 l^2} [4l - 3a]$

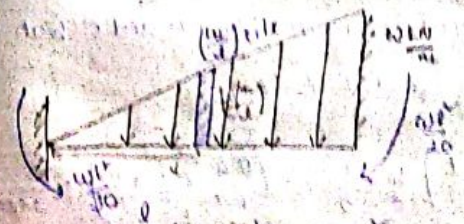
$= \frac{w}{l^2} \left[\frac{l^2 x^3}{3} + \frac{x^4}{4} - 2lx^3 \right]_0^a$

$= \frac{w}{l^2} \left[\frac{l^2 a^3}{3} + \frac{a^4}{4} - 2a^3 l \right]$

$= \frac{w}{l^2} \left[\frac{4l^2 a^3 + 3a^4 - 8a^3 l}{12} \right]$

$= \frac{wa^2}{12 l^2} [4l^2 + 3a^2 - 8la]$

47)



$M_A = \int_0^l \frac{(\frac{w}{l} x \, dx)(x)(l-x)^2}{l^2}$

$= \frac{w}{l^3} \int_0^l x^2 (l-x)^2 \, dx$

$= \frac{w}{l^3} \int_0^l (x^2 l^2 + x^4 - 2lx^3) \, dx$

$= \frac{w}{l^3} \left[\frac{l^2 x^3}{3} + \frac{x^5}{5} - 2l \frac{x^4}{4} \right]_0^l$

$= \frac{w}{l^3} \left[\frac{l^5}{3} + \frac{l^5}{5} - \frac{2l^5}{2} \right]$

$= \frac{w}{l^3} \left[\frac{10l^5 + 3l^5 - 30l^5}{60} \right]$

$= \frac{w}{l^3} \left(\frac{2l^5}{60} \right) = \frac{wl^2}{30}$

$M_B = \int \frac{(\frac{w}{l} x \, dx) x^2 (l-x)}{l^2}$

$= \frac{w}{l^3} \int x^3 \, dx \, (l-x)$

$= \frac{w}{l^3} \int x^3 l - x^4 \, dx$

$= \frac{w}{l^3} \left[\frac{x^4 l}{4} - \frac{x^5}{5} \right]_0^l$

$= \frac{w}{l^3} \left(\frac{l^5}{4} - \frac{l^5}{5} \right)$

$= \frac{wl^2}{20}$

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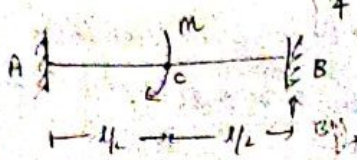
48)



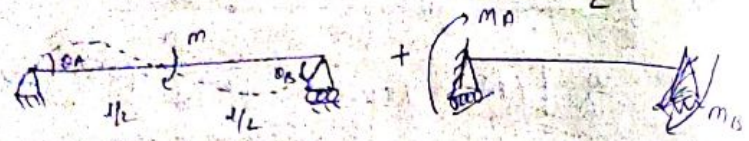
$\Sigma M_A = 0$

$V_B(l) - M = 0$

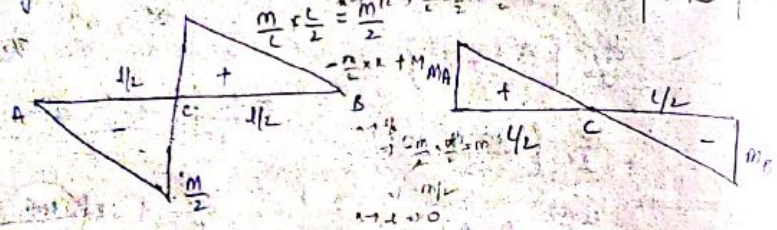
$V_B = \frac{M}{l}$

$V_A + V_B = 0$

$V_A = -\frac{M}{l}$



BM diagrams



Couple acting at centre so $M_A = M_B$

By applying Mohr's IInd theorem b/w A and C but we know that deflection at 'C' is zero.

$$y_C = -\left(\frac{M}{2EI} \times \frac{l}{2} \times \frac{l}{2}\right) \frac{l}{2} \times \frac{1}{3} + \left(\frac{M_A}{EI} \times \frac{l}{2} \times \frac{1}{2}\right) \times \frac{2l}{3 \times 2}$$

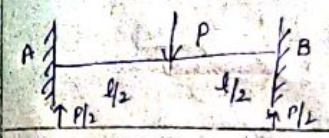
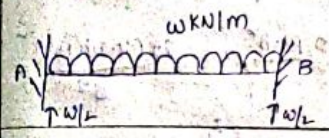
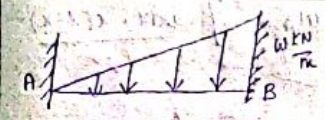
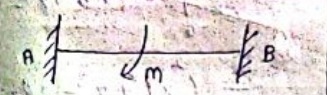
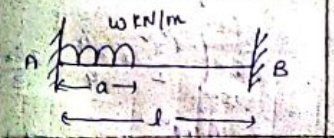
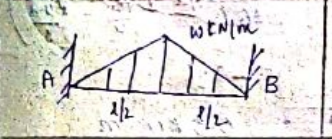
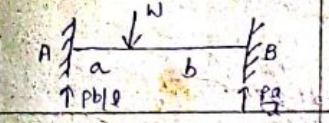
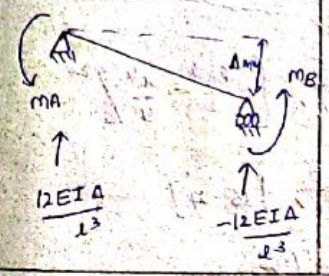
$$0 = -\frac{M}{2EI} \times \frac{l^2}{4} + \frac{M_A}{EI} \times \frac{l^2}{3}$$

$$\frac{M_A}{EI} \times \frac{l}{3} = \frac{M}{2EI} \times \frac{l}{6}$$

$$\times \frac{3}{4} = M_A$$

$$M_A = \frac{M}{4} = M_B$$

Bending Moments for fixed beams

Diagram	M_A	M_B
	$-\frac{PL}{8}$	$-\frac{PL}{8}$
	$-\frac{wl^2}{12}$	$-\frac{wl^2}{12}$
	$-\frac{wl^2}{30}$	$-\frac{wl^2}{20}$
	$+\frac{M}{4}$	$-\frac{M}{4}$
	$-\left[\frac{wa^2}{12l^2}(6l^2 - 8la + 3a^2)\right]$	$\frac{wa^3}{12l^2}(4l - 3a)$
	$-\frac{5wl^2}{96}$	$-\frac{5wl^2}{96}$
	$-\frac{Pab^2}{l^2}$	$-\frac{Pba^2}{l^2}$
	$\frac{6EI\Delta}{l^2}$	$\frac{6EI\Delta}{l^2}$

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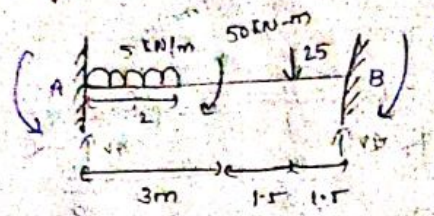
49) Analyse the fixed beam



m_A due to Couple $= +\frac{m}{4} = \frac{50}{4} = +12.5$

m_A due to point load $= \frac{25\,pab^2}{l^2} = -\frac{25(4.5)(1.5)^2}{6^2}$

m_A due to uniform load $= m_A = \int_0^2 \frac{w\,dx \cdot x(l-x)^2}{l^2}$

$= \frac{5}{6^2}\int_0^2 x^2(l^2 - 2lx + x^2)$

$= \frac{5}{36}\int_0^2 xl^2 + x^3 - 2lx^2$

$= \frac{5}{36}\left(\frac{l^2x^2}{2} + \frac{x^4}{4} - \frac{2lx^3}{3}\right)$

$= \frac{5}{36}\left(\frac{4l^2}{2} + \frac{16}{4} - \frac{16l}{3}\right)$

$= -6.1$

$m_A = -6.1$

Final $m_A = 12.5 - 7.08 - 6.1 = -0.6$ kN-m

$M_B = -\frac{m}{4} = -12.5$

$m_B = -\frac{25 \times 4.5^2 \times 1.5}{6^2} = -21.09$

$M_B = \int_0^2 \frac{(w\,dx)x^2(l-x)}{l^2} = \frac{5}{6^2}\left[\frac{lx^3}{3} - \frac{x^4}{4}\right]_0^2$

$= \frac{5}{6^2}[16-4]$

$= 1.66$

$\therefore m_B = -12.5 - 21.09 - 1.66$

$= -35.102$

Now support reactions

$V_A + V_B = 25 + 10$

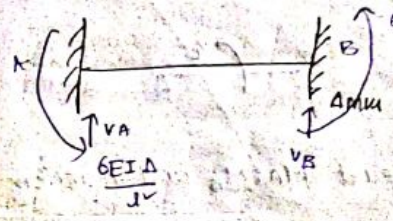
$-V_B(6) + 35.102 + 25(4.5) + 50 + (5\times2)\times1 = 0$

$V_B = 22.79$

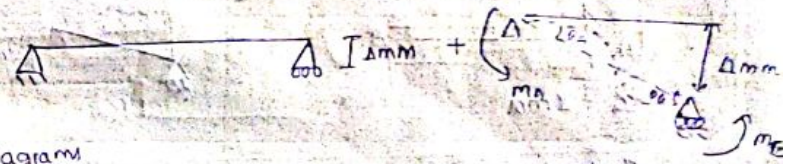
$V_A = 35 - 22.79$

$= 12.20$ kN

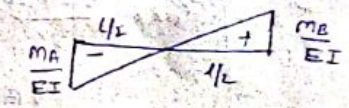
50)



one end shrinks or deflects then



BM diagram



By apply Mohr's second theorem b/w A and B we get

$$y_{BA} = \frac{-\left(m_A \times \frac{l}{2} \times \frac{l}{2}\right)\left(\frac{2}{3}\frac{l}{2} + \frac{l}{2}\right) + \left(m_B \times \frac{l}{2} \times \frac{l}{2}\right)\left(\frac{1}{3} \times \frac{l}{2}\right)}{EI}$$

$$-\Delta mm = \frac{-\left(m_A \frac{l}{4}\right)\left(\frac{5}{6}l\right) + \left(m_B \frac{l}{4}\right)\left(\frac{l}{6}\right)}{EI}$$

$$-\Delta mm = \frac{m_A l}{4EI}\left[-\frac{5}{6}l + \frac{l}{6}\right]$$

$$-\Delta = \frac{m_A l}{4EI}\left(\frac{1}{6}l\right)$$

$$m_A = \frac{6EI\Delta}{l^2} = m_B$$

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Now reaction

$$V_A + V_B = 0$$

$$V_A(l) + \frac{6EI\Delta}{l^2} + \frac{6EI\Delta}{l^2} = 0$$

$$V_B = -\frac{12EI\Delta}{l^3}$$

$$V_A = \frac{12EI\Delta}{l^3}$$

5) Support A sink by 1mm find M_A, M_B and reactions

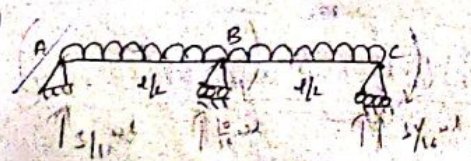
$\Delta = 1mm$

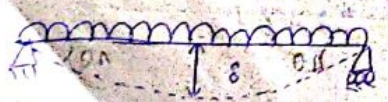
$$M_A = \frac{6EI\Delta}{l^2} = \frac{6EI}{l^2}$$

$$M_B = \frac{6EI(\Delta)}{l^2} = \frac{6EI}{l^2}$$

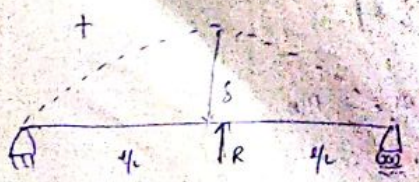
$$V_A = \frac{12EI}{l^3} \qquad V_B = -\frac{12EI}{l^3}$$

5a)





$$\frac{5wl^4}{384EI}$$



$$\frac{Rl^3}{48EI}$$

$$R = \frac{5wl^4}{384EI} \times \frac{48EI}{l^3}$$

$$= \frac{3}{16}wl$$

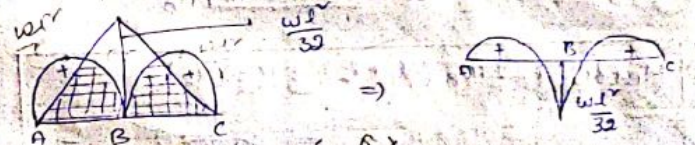
RXNS ⇒ $M_{RA} = \frac{3}{16}wl$ $\quad M_{RB} = \frac{10}{16}wl$ $\quad M_{RC} = \frac{3}{16}wl$

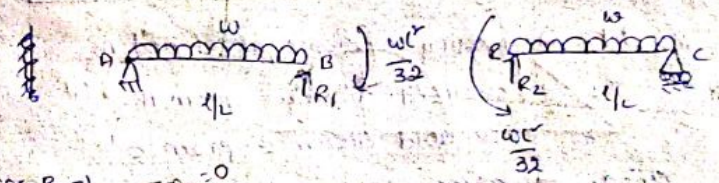
Bending moment at B = ? (NS)

$$\Rightarrow \frac{3}{16}wl \times \frac{l}{2} - \frac{wl}{2} \times \frac{l}{4}$$

$$\Rightarrow \frac{3wl^2 - 4wl^2}{32}$$

$$\Rightarrow \left[\frac{-wl^2}{32}\right]$$





For R_1 ⇒ $\Sigma M_A = 0$

$$R\left(\frac{l}{2}\right) - \frac{wl^2}{32} - \frac{wl}{2}\,\frac{l}{4} = 0$$

$$R\left(\frac{l}{2}\right) + \left[\frac{-wl^2 - 4wl^2}{32}\right] = 0$$

$$R\left(\frac{l}{2}\right) = \frac{5wl^2}{32}$$

$$R = \frac{10wl}{32}$$

$$\boxed{R = \frac{5}{16}wl}$$

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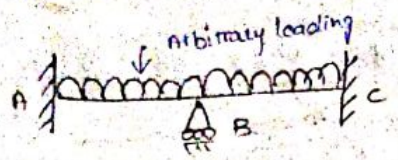
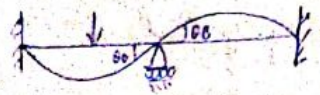
Three Moments theorem -

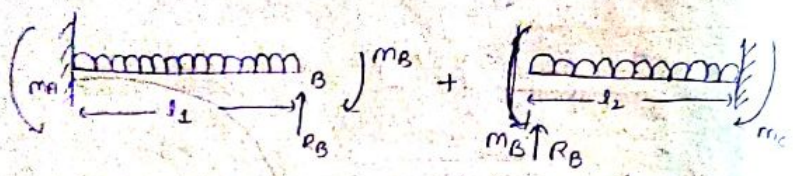
It is also called "clapeyron's theorem"

To find M_B it will be used.





$\theta_B = \theta_B$ extra RRD we use



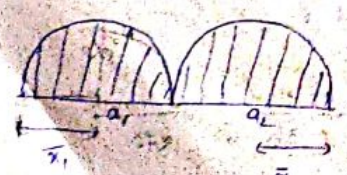
$$M_A(l_1) + 2M_B(l_1+l_2) + M_C(l_2) = \frac{6a_1\bar{x}_1}{l_1} + \frac{6a_2\bar{x}_2}{l_2}$$

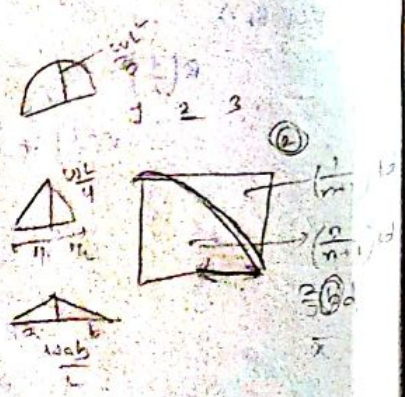
M_B, M_A → hogging moments at B and A

x_1 → centroid distance from A

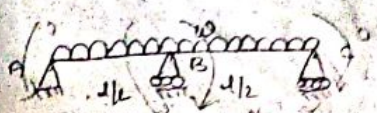
x_2 → centroid distance from C

a_1, a_2 → area of BM



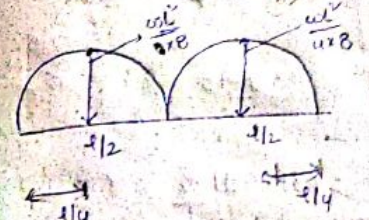


53)



$M_A = 0$

$M_C = 0$



$$M_A(l_1) + 2M_B(l_1+l_2) = M_C(l_2)$$

$$= \frac{6a_1\bar{x}_1}{l_1} + \frac{6a_2\bar{x}_2}{l_2}$$

$$\Rightarrow 0 + 2M_B\left(\frac{l}{2}+\frac{l}{2}\right) + 0$$

$$= \frac{6(2)\left[\frac{l}{2}\cdot\frac{wl^2}{32}\cdot\frac{l}{2}\right]\frac{2}{3}}{l/2}$$

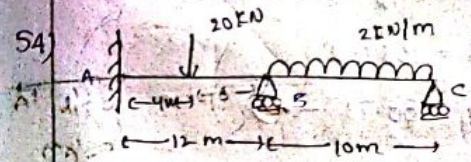
$$\Rightarrow 2M_B(l) = \frac{1}{32}\frac{wl^4}{l}\times 2$$

$$= \frac{wl^3}{16}$$

$$\boxed{M_B = \frac{wl^2}{32}}$$

52nd sum ↑

54)



$M_A = 0$

$l_0 = 0$

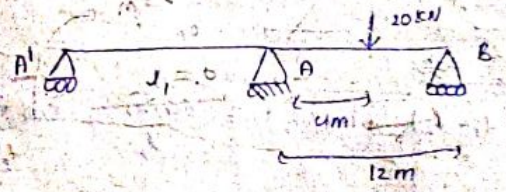
$M_C = 0$

A is fixed end (M_A)

B has some sa (M_B)

C has no moment ($M_C = 0$)

Assume an imaginary beam A' with length l_1



Apply eqn on A', A, B

$$M_{A'}(l_1) + 2M_A(l_1+l_2) + M_B(l_2) = \frac{6a_1\bar{x}_1}{l_1} + \frac{6a_2\bar{x}_2}{l_2}$$

$$0 + 2M_A(0+12) + M_B(12) = 0 + \frac{6a_2\bar{x}_2}{l_2}$$

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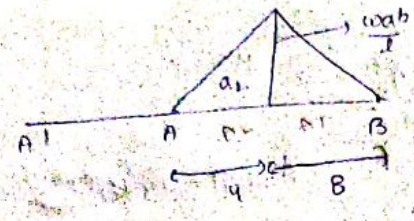


$\frac{wab}{l} = \frac{20\times4\times8}{12} = 53.33$

$a_1 = \frac{1}{2}\times53.33\times12 = 319.98$

$\therefore\ 2M_A(12) + M_B(12) = 6(319.98)\frac{\bar{x}_2}{l_2}$

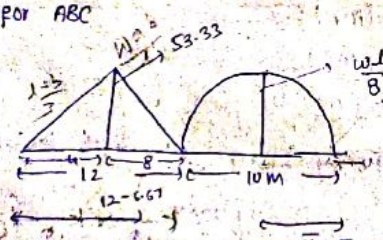
$\left[\bar{x}_2 = \frac{(\frac{1}{2}\times53.33\times8)(\frac{2}{3}\times8) + (\frac{1}{2}\times53.33\times4)(8+\ldots)}{\frac{1}{2}\times53.33\times12} = \frac{113.7+995.49}{330} = 6.67\right]$

$24M_A + 12M_B = \frac{6\times320\times6.67}{12}$

$24M_A + 12M_B = 1067.2$

$2M_A + M_B = 88.93$ — (1)

Apply for ABC



$\frac{wl^2}{8} = \frac{20\times10^2}{8} = 250$ kN

centroid $\frac{l_1+l_2}{3}$ $\bar{x} = 5.33$ $\bar{x} = 5$

$M_A(l_1) + 2M_B(l_1+l_2) + M_C(l_2) = \frac{6a_1\bar{x}_1}{l_1} + \frac{6a_2\bar{x}_2}{l_2}$

$M_A(12) + 2M_B(12+10) = \frac{6(320)(5.33)}{12} + \frac{6[\frac{2}{3}\times25\times10](5)}{10}$

$12M_A + 44M_B = 852.8 + 500$

$12M_A + 44M_B = 1352.8$

$M_A + 3.67M_B = 112.73$ — (2)

$M_A = 112.73 - 3.67M_B$

substitute in (1)

$2(112.73 - 3.67M_B) + 1M_B = 88.93$

$225.46 - 7.34M_B + M_B = 88.93$

$225.46 - 6.34M_B = 88.93$

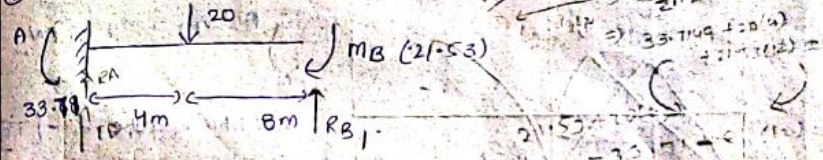
$6.34M_B = 136.53$

$M_B = 21.53$ kNm

$M_A = 112.73 - 3.67(21.53)$

$= 33.7149$ kNm

① Finding rxns :-



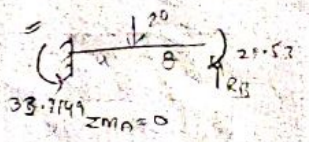
$\Sigma M_A = 0$

$33.71 - 21.53 + R_B(12) - 20(4) = 0$

$R_{B_1} = 5.65$ kN

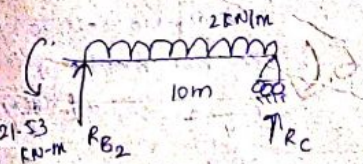
$R_A + R_B = 20$

$R_A = 14.35$ kN



$\Sigma M_A = 0$

$-33.7149 + 21.53 + 20(4) = 0$

② 

$R_B + R_C = 20$

$\Sigma M_C = 0$

$21.53 + 20(5) - R_{B_2}(10) = 0$

$R_{B_2} = 12.16$ kN

$R_C = 7.8$ kN

$R_B = R_{B_1} + R_{B_2} = 5.65 + 12.16$

$= 17.81$ kN

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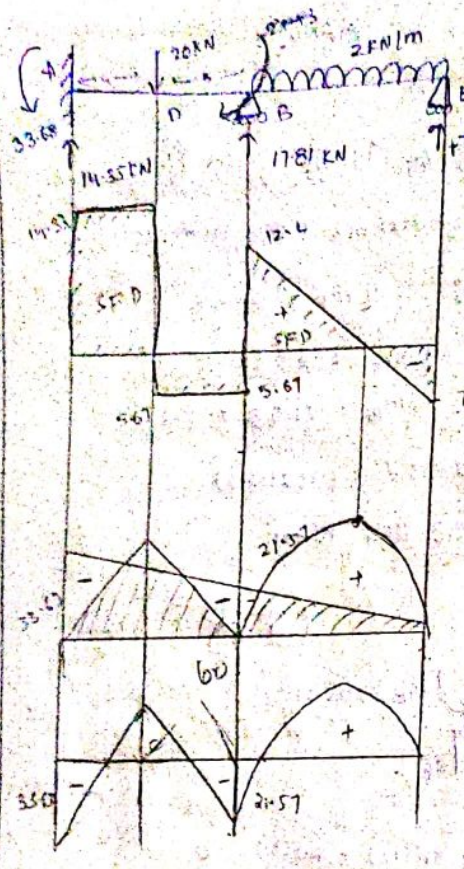


SF B/W B and C

$= -7.8 + 20$

at C $x \to 0$ = -7.8 kN

at B $x \to 10 \Rightarrow 20 - 7.8$
$= 12.2$

SF B/W B and D

$\Rightarrow -7.8 + 20 - 17.81$

at B $x \to 10 \Rightarrow 12.2$ kN

at D $x \to 18 \Rightarrow 12.21 - 17.81$
$\Rightarrow -5.6$ kN

SF b/w D and A $= -7.8 - 17.81 + 20 \times 10$

at D $x \to 18 \Rightarrow 14.39$ kN

at A $x \to 22 \Rightarrow 14.39$ kN

BM :-

BM b/w B and C $= -\left[7.8(x) - 2x^2\right]$

at C $x \to 0 \Rightarrow 0$

at B $x \to 10 \Rightarrow -\left[78 - \frac{200}{2}\right]$
$= 22$ kN-m

BM b/w B and D $\Rightarrow -\left[7.8x - 20(x-5) + 17.81(x-10)\right]$

at B $x \to 10 \Rightarrow -22$ kN-m

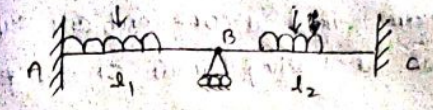
at D $x \to 18 \Rightarrow -22.88$ kN-m

BM b/w A and D $\Rightarrow$
$-\left[7.8(x) - 20(x-5) + 17.81(x-10) - 20(x-18)\right]$

at D $x \to 18 \Rightarrow -22.88$ kN-m

at A $x \to 22 \Rightarrow 57.12$ kN-m

5)



E_1I_1 E_2I_2

flexural rigidity EI is different for AB and BC

then

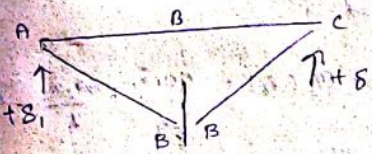
$$\frac{M_A(l_1)}{E_1I_1} + 2M_B\left[\frac{l_1}{E_1I_1} + \frac{l_2}{E_2I_2}\right] + M_C\left[\frac{l_2}{E_2I_2}\right] = \frac{6a_1\bar{x}_1}{l_1E_1I_1} + \frac{6a_2\bar{x}_2}{l_2E_2I_2}$$

when assumed n' EI should be assumed as 1

→ If B is sink, δ occurs for AB span and BC span

moment $= \frac{6EI\delta}{l^2}$ then

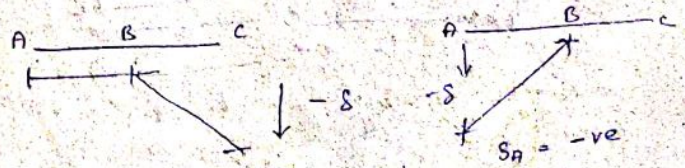
$$\frac{M_A(l_1)}{E_1I_1} + 2M_B\left[\frac{l_1}{E_1I_1} + \frac{l_2}{E_2I_2}\right] + M_C\left[\frac{l_2}{E_2I_2}\right] = \frac{6a_2\bar{x}_1}{l_1E_1I_1} + \frac{6a_2\bar{x}_2}{l_2E_2I_2} - \frac{6E_1I_1S_1}{l_1E_1I_1} - \frac{6E_2I_2S_2}{l_2E_2I_2}$$



$S_1 \Rightarrow$ relative displacement of joint A w.r.to point B (upward so +)

$S_2 \Rightarrow$ relative displacement of joint C w.r.to point B (upward so +)

If



S_B relative B w.r.to C $\Rightarrow -\delta$ (−ve)

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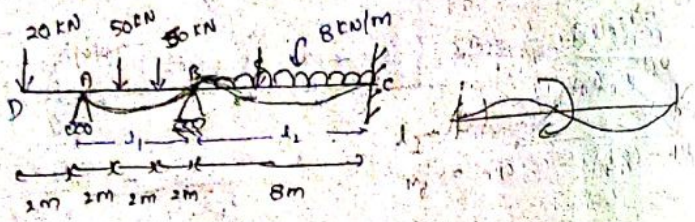
55) A figure shows a beam fixed at one end and supp. B settles by 10 mm on the loading draw SF and BM

$I = 100\times10^{-6}\ m^4$

$E = 2\times10^8\ kN/m^2$

EI is common for all



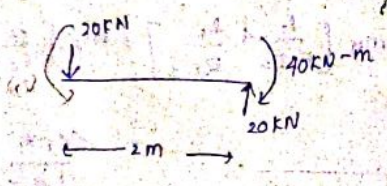
Draw imaginary beam

As there is a overhang at A there is moment M_A

$M_A = 20\times2 = 40\ kN\text{-}m$

Hogging

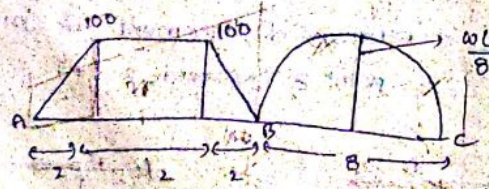
Draw AD ⇒



equilibrium

$$M_A(l_1) + 2M_B(l_1+l_2) + M_C(l_2) = \frac{6a_1\bar{x}_1}{l_1} + \frac{6a_2\bar{x}_2}{l_2} - \frac{6EI\delta}{l_1^2} - \frac{6EI\delta}{l_2^2}$$

Now AB and BC ⇒



$\frac{wL^2}{8} \Rightarrow 64\ kN\text{-}m$

$$40(6) + 2M_B(6+8) + M_C(8) = \frac{6(400)(3)}{6} + \frac{6(341.33)(4)}{8} - \frac{6(2\times10^8)(0.01)}{6^2} - \frac{6(2\times10^8)(0.01)}{8^2}$$

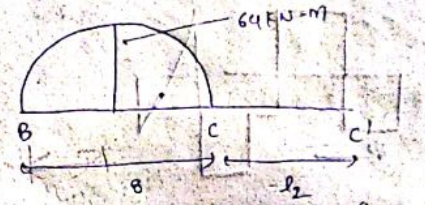
(100×10^{-6}) (100×10^{-6})

$$240 + 28M_B + 8M_C = 1200 + 1023.99 - 333.33 - 150$$

$$28M_B + 8M_C = -1873.99 - 240$$

$$3.5M_B + M_C = 204.24 \quad (1)$$

Now B, C, C'



$$M_B(l_1) + 2M_C(l_1+l_2) + M_C(l_2) = \frac{6a_1\bar{x}_1}{l_1} + \frac{6a_2\bar{x}_2}{l_2} - \frac{6EI\delta}{l_1^2} - \frac{6EI\delta}{l_2^2}$$

$$M_B(8) + 2(M_C)(8) + 0 = \frac{6(341.33)(4)}{8} + 0 - \frac{6(100\times10^{-6})(2\times10^8)}{8^2}$$

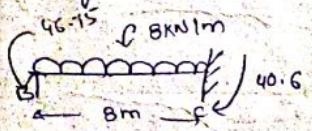
$$8M_B + 16M_C = 1023.99$$

$$0.5M_B + M_C = 63.99 \quad (2)$$

By solving (1) & (2)

$M_B = 46.75$ $M_C = 40.6$

Finding reactions :-



$$R_B(8) + 40.6 = 8(8)[4]$$

$R_B = 26.9\ kN$

$R_B + R_C = 64$

$R_C = 37.1\ kN$

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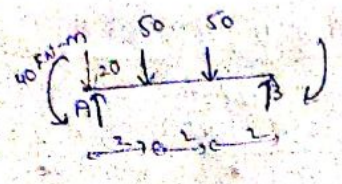


$R_A + R_B = 100$

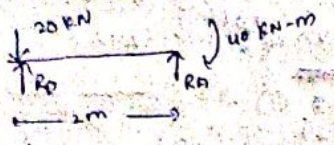
$\Sigma M_A = 0$

$50(2) + 50(4) + 46.75 = R_B$

$R_B = 51.125$ KN

$= 31.125$

$R_B = 78.025$ KN



$R_A + R_D = 20$

$\Sigma R_A = 0$

$40 + R_D(2) = 20(2)$

$R_D = -0$ KN

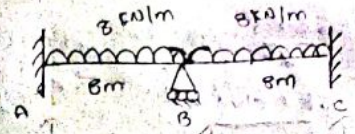
$R_A = 20$ KN

$R_D = 0$ KN

$R_B = 78.025$ KN

$R_C = 37.1$ KN

56)



consider A', A, B $\quad M_{A'} = 0 \quad q_1 = 0$

$\ell_1 = 0$

$2M_A(\ell) + M_B \ell = \dfrac{6 a \bar{x}_2}{\ell_2} = \dfrac{6 \times \left(\frac{2}{3} \times 64 \times 8\right) 4}{8}$

$2M_A \ell + M_B \ell = 1024$

$16 M_A + 8 M_B = 1024$ —①

consider A, B, C

$M_A(8) + 2M_B(8+8) + M_C(8) = \dfrac{2 \times 6 \times \left[\frac{2}{3} \times 64 \times 8\right] \times 4}{8}$

$8M_A + 32 M_B + 8 M_C = 2048$ —②

consider B, C, C'

$M_B(8) + 2M_C(8+0) = \dfrac{6 \times \left[\frac{2}{3} \times 64 \times 8\right] \times 4}{8}$

$8M_B + 16 M_C = 1024$ —③

By solving ①, ② & ③ $\quad M_A = 42.6$ KN-m

$M_B = 42.6$ KN-m

$M_C = 42.6$ KN-m

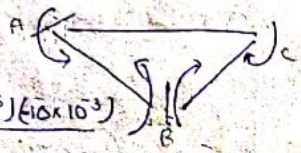
$M_A = 42.6$

$M_B = 42.6$

$M_C = 42.6$

B is sink by 10 mm $\quad EI = 2 \times 100 \times 10^8 \times 10^{-6} \ \frac{KN}{m^2} \cdot m^4$

$16 M_A + 8 M_B = 1024 - \dfrac{6EI\delta}{\ell^2}$



$= 1024 - \dfrac{6(2 \times 100 \times 10^8 \times 10^{-6})(10 \times 10^{-3})}{8^2}$

$= 874$ — ⓐ $\quad 1024 + 150 = 1174$

$8M_A + 32 M_B + 8 M_C = 2048 - \dfrac{12 EI \delta}{\ell^2}$

$= 2048 - \dfrac{12 EI (10 \times 10^{-3})}{8^2}$

$= 1748$ — ⓑ

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$$8M_B + 16M_C = 1024 - \frac{6EI\delta}{l^2}$$

$$= 1024 - \frac{6EI(10\times10^{-3})}{8}$$

$$= 1024 - 1.74$$

$M_A = 29.4$

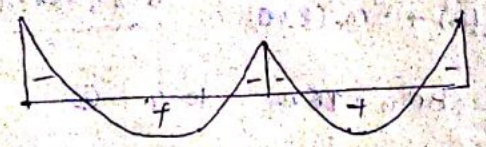
$M_B = 30.9$

$M_C = 67.3$

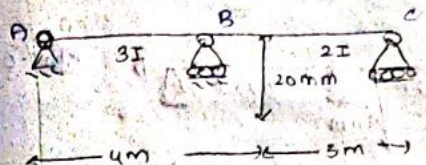
$M_A = 61.4$ kN·m

$M_B = 25.9$ kN·m

$M_C = 61.4$ kN·m



a) Determine the support moments and the rxns in the continuous beams if the support B sinks by 20mm. Take EI = 10000 kNm²



$M_A = M_C = 0$

$a_1 = 0$

$a_2 = 0$

$$M_B\left(\frac{l_1}{EI}\right) + 2M_B\left(\frac{l_1}{EI} + \frac{l_2}{EI}\right) + M_C(l_2) = \frac{6a_1\bar{x}_1}{l_1} + \frac{6a_2\bar{x}_2}{l_2} - \frac{6EI\delta_1}{l_1 EI} - \frac{6EI\delta_2}{l_2 EI}$$

$$2M_B\left(\frac{4}{3I} + \frac{3}{2I}\right) = +\frac{6E(0.02)}{4} + \frac{6E(0.01)}{3}$$

∴ Clapeyron's three-moment eqn for continuous beam including the effect of settlement of support is

$$M_A\left[\frac{L_1}{I_1}\right] + 2M_B\left[\frac{L_1}{I_1} + \frac{L_2}{I_2}\right] + M_C\left[\frac{L_2}{I_2}\right] = -\frac{6a_1}{I_1L_1} - \frac{6a_2}{I_2L_2} + \frac{6E\delta_1}{L_1} + \frac{6E\delta_2}{L_2}$$

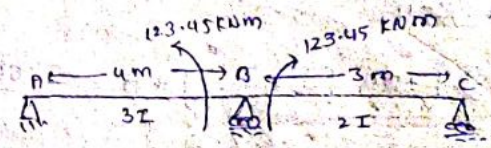
$$2M_B\left[\frac{8+9}{6I}\right] = (0.03 + 0.04)E$$

$$5.67M_B = 0.07EI$$

$$M_B = \frac{0.07}{5.67}(10000)$$

$$M_B = 123.45 \text{ kN.m}$$

Support rxns



Due to support moment: ↑ 30.86 kN, 80.87 kN ↓, 41.15 kN ↓, ↑ 41.15 kN

Total rxns: $R_A = 30.86$ kN ↑, $R_B = 72.02$ kN ↓, $R_C = 41.15$ kN ↑

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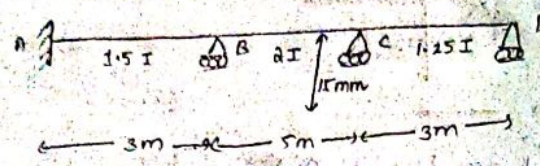
b) Use three-moment equation to determine the support rx in the beam shown below. If support c sinks by 15mm. Assume EI = 9000 kN m²



$M_D = 0$

consider A' having length = 0 and $M_{A'} = 0$, $I = \infty$

$$M_{A'}(L_0) + 2M_A\left(L_0 + \frac{L_1}{EI}\right) + M_B\left(\frac{L_1}{EI}\right) = \frac{6a_1\bar{x}_1}{L_0 EI} + \frac{6a_2\bar{x}_2}{L_1} - \frac{6E\delta}{L_0}$$

$$2M_A\left(\frac{3}{1.5I}\right) + M_B\left(\frac{3}{1.5I}\right) = 0$$

$$4M_A + 2M_B = 0 \quad -①$$

Consider ABC

$$M_A\left(\frac{L_0}{EI_1}\right) + 2M_B\left(\frac{L_1}{EI_1} + \frac{L_2}{EI_2}\right) + M_C\left(\frac{L_2}{EI_2}\right) = -\frac{6E(0.015)}{5}$$

$$M_A\left(\frac{3}{1.5I}\right) + 2M_B\left(\frac{3}{1.5I} + \frac{5}{2I}\right) + M_C\left(\frac{5}{2I}\right) = -\frac{6E(0.015)}{5}$$

$$\frac{2M_A}{I} + \frac{9M_B}{I} + \frac{2.5M_C}{I} = -0.018E$$

$$2M_A + 9M_B + 2.5M_C = -0.018(9000)$$

$$= -162 \quad -②$$

Consider BCD

$$M_B\left(\frac{5}{2I}\right) + 2M_C\left[\frac{5}{2I} + \frac{3}{1.25I}\right] + 0 = -\frac{6E(0.015)}{5} - \frac{6E(0.015)}{3}$$

$$2.5M_B + 9.8M_C = -0.048EI$$

$$2.5M_B + 9.8M_C = -432 \quad -③$$

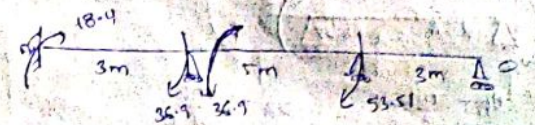
By solving ①②&③

$M_A = 18.4$ kN-m

$M_B = -36.9$ kN-m

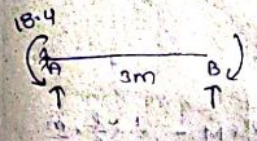
$M_C = 53.51$ kN-m

Finding rxns (not required from Qn)



Rxns due to Moment	↑ 6.133 kN + 12.3 = 18.43 kN ↓	6.16 ↑ ↓ 7.38 − 10.72 ↓ 3.32 kN	↑ ↓ 11.77 3.32	↕ 17.77 kN
Total	18.43 ↑↓	2.84 ↑	14.45 ↓	↑ 17.77

(or)



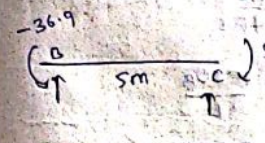
$R_A + R_B = 0$

$\Sigma M_A = 0$

$R_B(3) - 36.9 - 18.4 = 0$

$R_{B_1} = 18.4$

$R_A = -18.4$



$\Sigma M_B = 0$

$R_C(5) + 53.51 + 36.9 = 0$

$R_C = -18$ kN

$R_{B_2} = 18$ kN

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57)

$\theta_B = \frac{mL}{3EI}$ $\theta_A = \frac{mL}{6EI}$

$M = \frac{3EI\theta_B}{L}$ $m = \frac{6EI\theta_A}{L}$

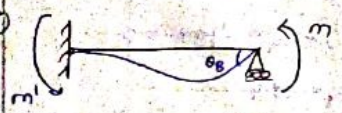
$\theta_A = \frac{\theta_B}{2}$

$m' = \frac{3EI\theta_A}{4}$

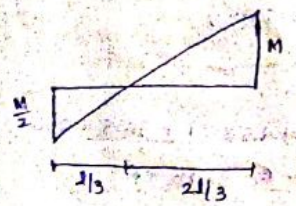
$m' = \frac{3EI\theta_B}{2L}$

$\boxed{m' = \frac{1.5\,EI\theta_B}{L}}$

58)



$\theta_B = \frac{\frac{1}{2}\times\frac{2l}{3}\times M - \frac{1}{2}\times\frac{l}{3}\times\frac{m}{2}}{EI}$

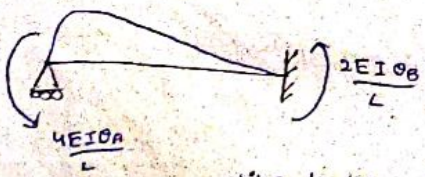


$\theta_B = \frac{mL}{4EI}$

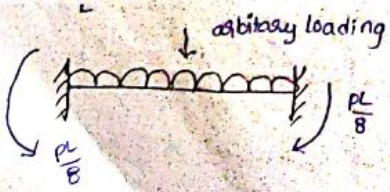
$\therefore M = \frac{4EI\theta_B}{L}$

$m' = \frac{2EI\theta_B}{L}$

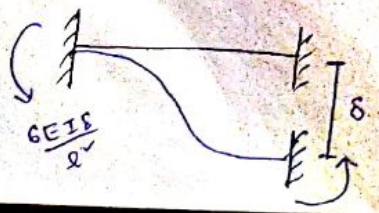
59)



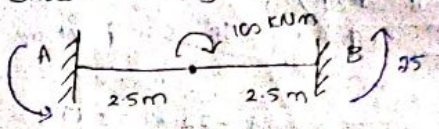
60)



$\frac{pL}{8}$ FEM (fixed end moment)



61) find the fixed end moments and the vertical rxn in the beam shown in fig.



$m_A = \frac{m}{4}$ $m_B = -\frac{m}{4}$

$= \frac{100}{4} = 25$ $= -25$

$R_A + R_B = 0$

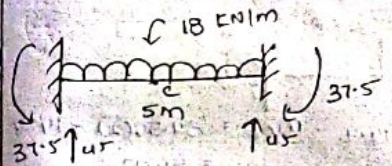
$\Sigma M_A = 0 \Rightarrow 5R_B + 25 = 100 + 25$

$5R_B = 100$

$R_B = 100/5 = 20$ kN

$R_A = -20$ kN

62)



$m_A = m_B = 37.5$

$m_C = \frac{18\times 2.5^2}{2} - 37.5 = 18.75$

point of contraflexture

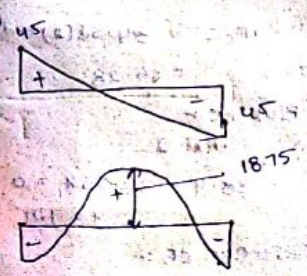
$m_x = 0$

$-45(x) + \frac{18x^2}{2} + 37.5 = 0$

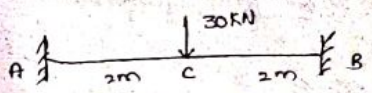
$\boxed{9x^2 - 45x + 37.5 = 0}$

$x = 1.056$ m

SFD

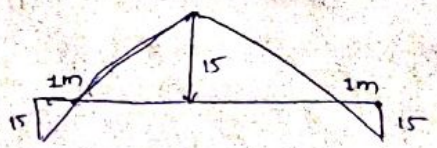


63) Draw SFD and BMD



$R_A + R_B = 30$

$R_A = R_B = \frac{W}{2} = 15$



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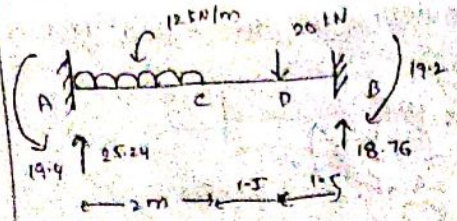
6(a)



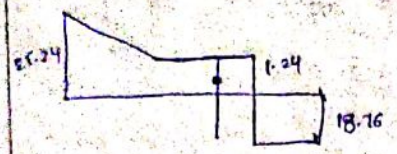
In AC :- point of contraflexure

$m_x = 0$

$-25.24x - 19.4 = 12\left(\frac{x^2}{2}\right)$

$6x^2 + 25.24x + 19.4 = 0$

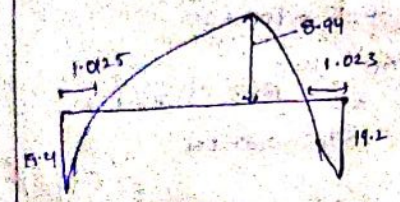
$x = 1.0125$ m from A



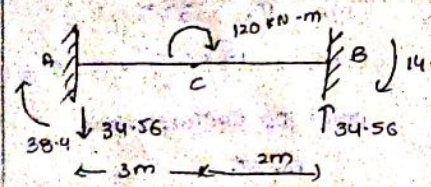
In BD :-

$-18.76x = 19.2$

$x = 1.023$ m from B



6(b)



$M_{C+} = 34.56(2) - 14.4$
$= 54.72$

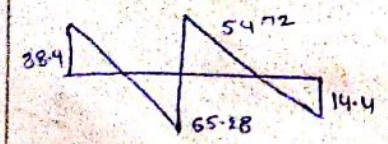
$M_{C-} = 34.56(2) - 14.4 - 120$
$= 65.28$

B/w AC :-

$m_x = 0$

$38.4 - 34.56x = 0$

$x = 1.11$



Between CB :-

$34.56x - 14.4 = 0$

$x = 0.417$

Slope deflection method :-

Step 1 - Treat each span as fixed beam and calculate fixed end moments (FEM), m.

Step 2 - write slope deflection eqns for each span considering FEM, rotations and settlement (slope deflection eqn)

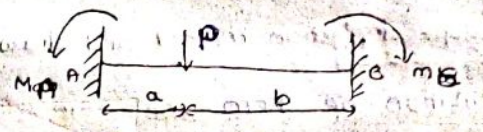
Step 3 - write equillibrium eqn for each joint

Step 4 - Solve the SD eqn and equillibrium eqn for rotation and settlement

Step 5 - put the value of θ and δ, find FEM (m). Also draw BMD.

Calculation of fixed end moment (FEM) :-

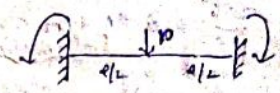
case-1



$M_A = -\frac{Pab^2}{l^2}$ we take anticlockwise negative (−)

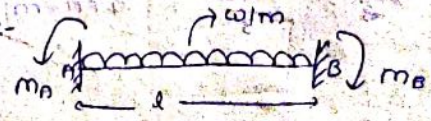
$M_B = \frac{Pa^2b}{l^2}$

case - 2 -



$m_a = -\frac{PL}{8}$ $m_b = \frac{PL}{8}$

Case - 3 :-



$m_A = -\frac{wl^2}{12}$ $m_B = \frac{wl^2}{12}$

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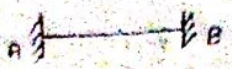
slope deflection eqn:-



$$M_A = FEM + \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\delta}{l}\right]$$

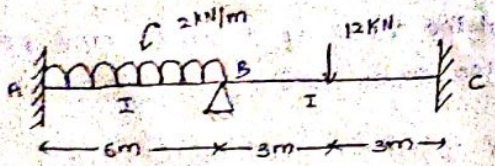
$$M_B = FEM + \frac{2EI}{l}\left[2\theta_B + \theta_A - \frac{3\delta}{l}\right]$$

Sign convention -

clockwise = +ve

Anticlockwise = -ve

6)



Solve by slope deflection method and draw BMD?

Step 1 - Calculation of FEM

~~FEM~~ consider AB as fixed span then

FEM $m_{AB} = -\frac{wl^2}{12} = -\frac{2\times(6)^2}{12} = -6$ kN-m

$m_{BA} = \frac{wl^2}{12} = \frac{2\times6^2}{12} = 6$ kN-m

BC span ⇒

FEM $m_{BC} = -\frac{PL}{8} = \frac{-12(6)}{8} = -9$ kN-m

$m_{CB} = \frac{PL}{8} = \frac{12(6)}{8} = 9$ kN-m

Step 2:- SD eqn

$$m_{AB} = FEM + \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\delta}{l}\right]$$

$\therefore \theta_A = 0$, $\theta_C = 0$ (due to fixed end), $\theta_B = ?$

$$= (-6) + \frac{2EI}{6}\left[2(0) + \theta_B - 0\right]$$

$$= -6 + \frac{EI\theta_B}{3}$$

$$M_{BA} = (6) + \frac{2EI}{6}(2\theta_B + 0 - 0)$$

$$= 6 + \frac{2EI\theta_B}{3}$$

$$m_{BC} = -9 + \frac{2EI}{6}\left[2\theta_B + \theta_C - \frac{3\delta}{J}\right]$$

$$= -9 + \frac{2EI\theta_B}{3}$$

$$m_{CB} = 9 + \frac{2EI}{J}\left[2\theta_C + \theta_B - \frac{3\delta}{J}\right]$$

$$= 9 + \frac{2EI\theta_B}{6}$$

Step 3:- equillibrium eqn:-

at joint B ⇒ $m_{BA} + m_{BC} = 0$

$$6 + \frac{2EI\theta_B}{3} - 9 + \frac{2EI\theta_B}{3} = 0$$

$$-3 + 0.67EI\theta_B + 0.67EI\theta_B = 0$$

$$1.33EI\theta_B = 3$$

$$EI\theta_B = \frac{3}{1.33} = 2.25$$

Step 4:-

Solve SD eqns

substitute these value " $EI\theta_B = 2.25$ "

$m_{AB} = -5.25$ kN-m

$m_{BA} = 7.5$ kN-m

$m_{BC} = -7.5$ kN-m

$m_{CB} = 9.74$ kN-m

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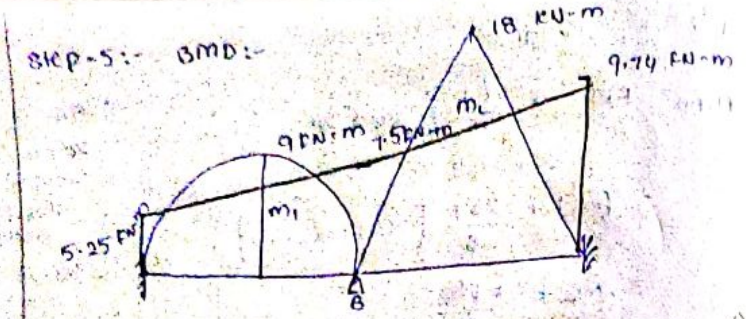
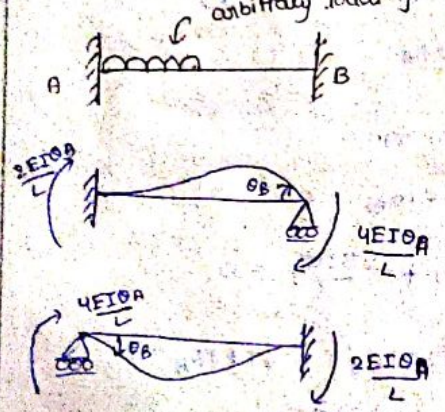
Step-5:- BMD:-



$m_1 = \frac{wl^2}{8} = \frac{2\times6^2}{8} = 9$ kN-m

$m_2 = \frac{wL}{4} = \frac{12\times6}{4} = 18$ kN-m

a) arbitrary loading



$m_A = FEM + \frac{2EI}{l}\theta_B + 4EI\theta_A$

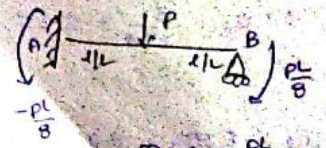
$m_A = FEM + \frac{2EI}{l}[2\theta_A + \theta_B]$

$m_B = FEM + \frac{2EI}{l}[2\theta_B + \theta_A]$

For fixed beam $\theta_A = \theta_B = 0$

$M_B = FEM$

b)



$m_B = \frac{PL}{8} + \frac{2EI}{l}[2\theta_B + 0]$

$0 = \frac{PL}{8} + \frac{4EI\theta_B}{l}$

$\theta_B = -\frac{PL^2}{32EI}$

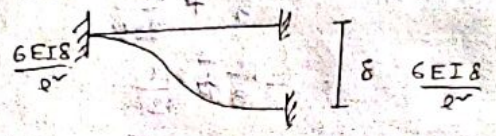
$M_A = -\frac{PL}{8} + \frac{2EI}{l}[\cancel{2\theta_A}^{0} + \theta_B]$

$m_A = -\frac{PL}{8} + \frac{2EI}{l}\left[\frac{-PL^2}{32EI}\right]$

$= -\frac{PL}{8} - \frac{PL}{16}$

$m_A = -\frac{3}{16}PL$ $\left[\frac{PL}{8} + \frac{PL}{8}\times\frac{1}{2}\right]$

Final Moments:-

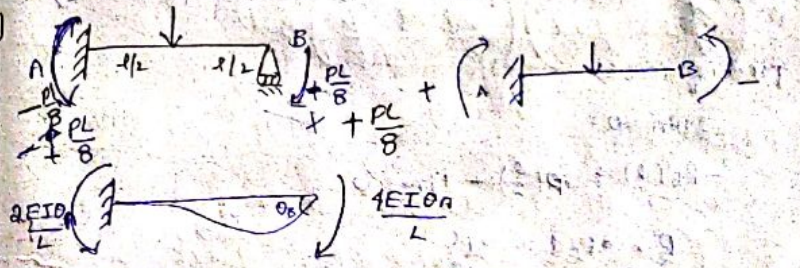


$m_A = FEM + \frac{2EI}{l}\theta_B + \frac{4EI\theta_A}{l} - \frac{6EI\delta}{l^2}$

$m_A = (FEM)_A + \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\delta}{l}\right]$

$m_B = FEM_B + \frac{2EI}{l}\left[2\theta_B + \theta_A - \frac{3\delta}{l}\right]$

b)



$M_{AB} = FEM_{AB} + \frac{2EI}{l}\left[\cancel{2\theta_A}^{0} + \theta_B - \cancel{\frac{3\delta}{l}}^{0}\right]$

$= -\frac{PL}{8} + \frac{2EI\theta_B}{l} \Rightarrow -PL/8 + \frac{2EI}{l}\left(\frac{-PL^2}{32EI}\right)$

$M_{BA} = FEM_{BA} + \frac{2EI}{l}\left[2\theta_B + \theta_A - \frac{3\delta}{l}\right]$ $-\frac{4PL}{32}$

$= \frac{PL}{8} + \frac{2EI}{l}[2\theta_B]$

$= \frac{PL}{8} + \frac{4EI\theta_B}{l}$ $-\frac{2PL + PL}{16}$

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To solve for θ_A we use the moment equillibrium condition at joint B $\Sigma M_{BA} = 0$

$\Sigma M_{BA} = 0$

$$\frac{PL}{8} = \frac{4EI\theta_B}{L}$$

$$\frac{PL^2}{8} = 4EI\theta_B \qquad \left(\theta_A = \frac{-PL^2}{32EI}\right)$$

$$-\frac{PL^2}{32} = EI\theta_B$$

In eqn (1) $\Rightarrow$ $M_{AB} = -\frac{PL}{8} + \frac{2PL^2}{32L}$

$$= -\frac{PL}{8} - \frac{2PL}{32}$$

$$= \frac{-4PL - 2PL}{32} = \frac{-6PL}{32}$$

$$= -\frac{3PL}{16}$$

in eqn (2) $\Rightarrow$ $M_{BA} = -\frac{PL}{8} + \frac{KPL^2}{32L}$

$= 0.$ (as derived from this)

Finding RRs $\Rightarrow$

$\Sigma M_A = 0$

$$-R_B(l) + P\left(\frac{l}{2}\right) - M_{AB} = 0$$

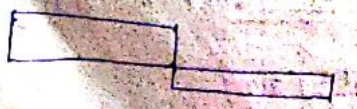
$$\frac{Pl}{2} = R_B l + \frac{3PL}{16}$$

$$R_B l = \frac{Pl}{2} - \frac{3PL}{6}$$

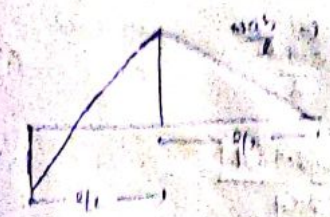
$$= \frac{3Pl - 3PL}{6} \Rightarrow \frac{3L}{6}(l - L) = 0.$$

$R_B = 0$

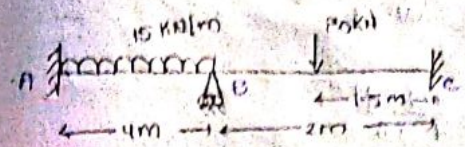
SFD



BMD



68)



$$M_{AB} = FEM + \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3R}{l}\right]$$

$$= -\frac{wL^2}{12} + \frac{2EI}{l}\left[2\theta_B\right]$$

$$= -\frac{(15)(4)^2}{12} + \frac{2EI}{4}\left[\theta_B\right] \Rightarrow -20 + \frac{2EI\theta_B}{4}$$

$$M_{BA} = \frac{wL^2}{12} + \frac{2EI}{l}\left[2\theta_B\right]$$

$$= \frac{15(4)^2}{12} + \frac{4EI\theta_B}{l}$$

$$M_{BC} = -\frac{Pab^2}{l^2} + \frac{2EI}{l}\left[2\theta_B\right] \qquad \frac{Pab^2}{l^2}$$

$$= -\frac{80(0.5)(1.5)^2}{4^2} + \frac{4EI\theta_B}{l} = -22.5 + \frac{4EI\theta_B}{l}$$

$$M_{CB} = \frac{Pab^2}{l^2} + \frac{2EI}{l}\left[\theta_B\right]$$

$$= \frac{80(0.5)^2(1.5)}{4^2} + \frac{2EI\theta_B}{l} = 22.5 + \frac{2EI\theta_B}{l} \qquad \frac{Pa^2b}{l^2}$$

We know moment at B is 0

$$M_{BA} + M_{BC} = 0$$

$$20 + \frac{4EI\theta_B}{l} - 22.5 + \frac{4EI\theta_B}{l} = 0$$

$$\frac{8EI\theta_B}{l} = 2.5$$

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$$EI\theta_B = \frac{2.5\,l}{8}$$

① ⇒ $m_{AB} = -20 + \frac{2(2.5\times4)}{8\times4} =$

$m_{AB} = -19.375$ kN-m

② ⇒ $m_{BA} = 20 + \frac{4(2.5(4))}{4\times8}$

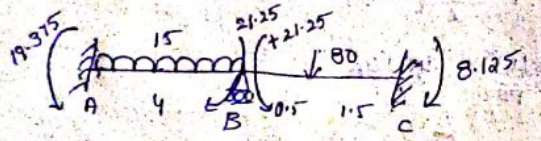
$= 21.25$

③ ⇒ $m_{BC} = -22.5 + \frac{4(2.5)(2)}{(2)\times8}$

$= -21.25$

④ ⇒ $m_{CB} = 2.5 + \frac{2(2.5\times2)}{8(2)}$

$= 8.125$



finding rxns

consider AB ⇒ $R_A + R_B = 60 + 21.25$

$\Sigma m_A = 0 \Rightarrow -R_B(4) + 60(2) - 19.375 = 0$

$R_{B_1} = 3.85$ kN

$R_A = 56.15$ kN

consider BC ⇒ $R_{B_2} + R_C = 80$

$\Sigma m_C = 0 \Rightarrow -R_{B(2)}(2) - 80(1.5) - 21.25 + 8.125 = 0$

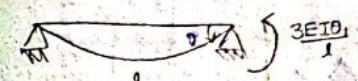
$R_{B(2)} = 45.31$

$R_B = 49.16$ kN

$R_C = 34.69$ kN

6

69)

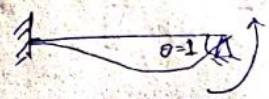


$\theta = 1$ radian $\boxed{K = \frac{3EI}{l}}$

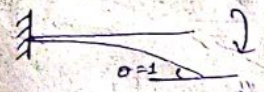
$K = \frac{F}{\delta} = \frac{m}{\theta}$ EI flexural rigidity

$\delta = \frac{FL}{AE}$

$\frac{AE\delta}{L} = F$

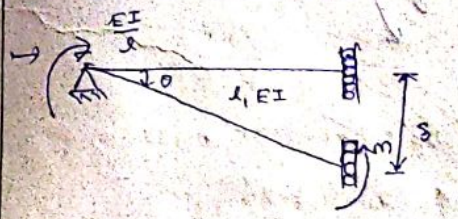


$\frac{4EI}{l} = k$



slope $= \frac{ml}{EI}$

$m = K = \frac{EI}{l}$

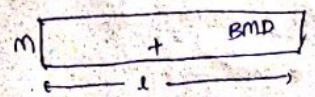


By area moment method

$\boxed{\frac{ml}{EI} = \theta}$

$\frac{ml}{EI} \times \frac{l}{2} = \delta$

$\delta = \frac{ml^2}{2EI}$



$\boxed{m = \frac{2EI\delta}{l^2}}$

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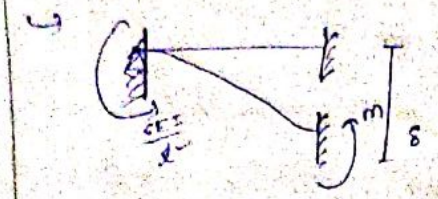
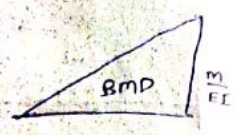
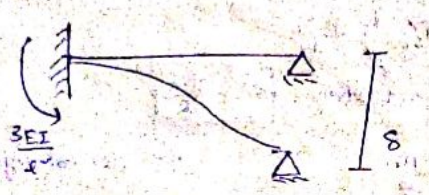
for $\theta = 1$ radian i.e. should apply that moment



$M = \delta = \frac{6EI}{l^2}$



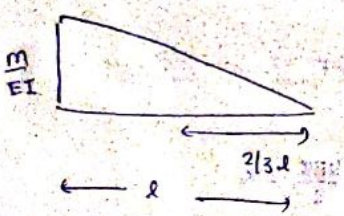


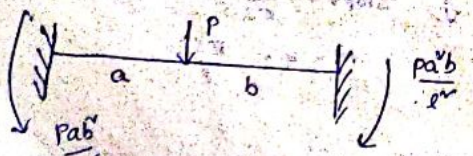
$\delta = \frac{1}{2} \times \frac{ml}{EI} \times \frac{2}{3} l$

$\delta = \frac{ml^2}{3EI}$

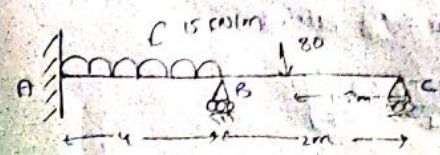
$K = \frac{m}{\delta} = \frac{3EI}{l^2}$

$m = \frac{3EI\delta}{l^2}$





70)



$M_{AB} = -20 + \frac{2EI}{4}\theta_B$

$M_{BA} = 20 + \frac{2EI}{4}[2\theta_B]$

$M_{BC} = -22.5 + \frac{2EI}{2}[2\theta_B + \theta_C]$

$M_{CB} = 7.5 + \frac{2EI}{2}[\theta_B + 2\theta_C]$

equillibrium eqns

$M_{CB} = 0$ — (1)

$M_{BA} + M_{BC} = 0$ — (2)

$7.5 + EI[\theta_B + 2\theta_C] = 0$

$\theta_B + 2\theta_C = -\frac{7.5}{EI}$ — (a)

(2) →

$20 - 22.5 + \frac{2EI}{2}\theta_B + EI(2\theta_B + \theta_C) = 0$

$-2.5 + EI\theta_B + 2EI\theta_B + EI\theta_C = 0$

$EI[\theta_B + 2\theta_B + \theta_C] = +2.5$

$3\theta_B + \theta_C = +\frac{2.5}{EI}$ — (b)

$\theta_B = +0.5$

$\theta_C = -1.5$

(a) & (b)

$\Rightarrow \theta_B + 2\theta_C = -\frac{7.5}{EI}$

$6\theta_B + 2\theta_C = +\frac{5.0}{EI}$

(−) (−) (+)

$+\frac{2.5}{EI} = -\frac{7.5}{EI} - 2\theta_C$

$\frac{10}{EI} = -2\theta_C$

$\theta_C = -\frac{5}{EI}$

$-5\theta_B = -\frac{12.5}{EI} + \frac{7}{EI}$

$+5\theta_B = +\frac{12.5}{EI}$

$\theta_B = +\frac{12.5}{5EI}$

$= +2.5/EI$

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Now substituting θ_A & θ_B values

$$M_{AB} = -20 + \frac{2EI}{4}\left[\frac{2.5}{EI}\right]$$

$$= -18.75 \text{ kN-m}$$

$$M_{BA} = 20 + \frac{2EI}{4}\left[1 \times \frac{2.5}{EI}\right]$$

$$= 22.5 \text{ kN-m}$$

$$M_{BC} = -22.5 + \frac{2EI}{2}\left[2\left(\frac{2.5}{EI}\right) + \left(-\frac{5}{EI}\right)\right]$$

$$= -22.5 \text{ kN-m}$$

$$M_{CB} = 7.5 + \frac{2EI}{2}\left[\frac{2.5}{EI} + 2\left(\frac{-5}{EI}\right)\right]$$

$$= 0$$

(01)

$$M_{BC} = -22.5 + \frac{3EI}{l}[2\theta_B]$$

$$= -22.5 + \frac{3EI}{2}(2\theta_B)$$

$$= -22.5 + 3EI\theta_B$$

$$M_{BA} = 20 + \frac{2EI}{4}[2\theta_B]$$

$$= 20 + EI\theta_B$$

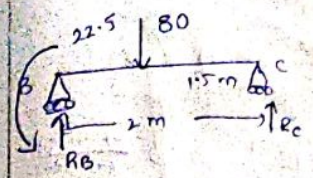
$$M_{BA} + M_{BC} = 0$$

$$-22.5 + 3EI\theta_B + 20 + EI\theta_B = 0$$

$$4EI\theta_B = 2.5$$

$$EI\theta_B = \cancel{0.625}\ 2.5$$

$$\theta_B = \frac{2.5}{EI}$$



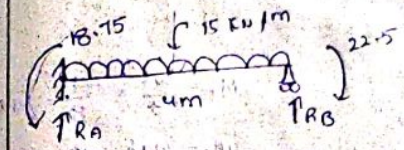
$$R_B + R_C = 80$$

$$\Sigma M_B = 0$$

$$-R_C(2) + 80(0.5) - 22.5 = 0$$

$$R_C = 8.75 \text{ kN}$$

$$R_B = 71.25 \text{ kN}$$



$$R_A + R_B = 60$$

$$\Sigma M_A = 0$$

$$22.5 - R_B(4) + 60(2) - 18.75 = 0$$

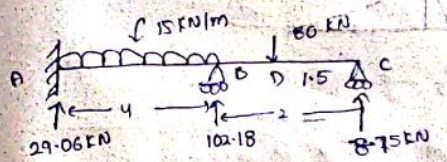
$$R_B = 30.93 \text{ kN}$$

$$R_A = 29.06 \text{ kN}$$

$\therefore R_A = 29.06$ kN

$R_B = 30.931 + 71.25 = 102.18$ kN

$R_C = \cancel{71.25}$ kN 8.75 kN



SF B/w BC $\Rightarrow -8.75$

at $C = -8.75$

$D = -8.75$

SF B/w B and D $\Rightarrow -8.75 + 80$

$\Rightarrow 71.25$

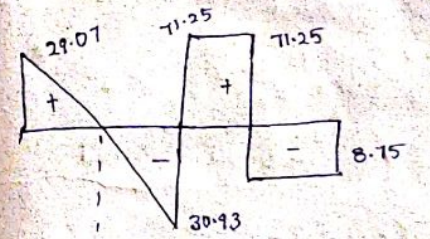
at $D = 71.25$

$B = 71.25$

SF B/w A and B $\Rightarrow -8.75 + 80 + 15(x-2) - 102.18$

at B $x \to 2 \Rightarrow 71.25 - 102.18 \Rightarrow -30.93$ kN

at A $x \to 6 \Rightarrow 131.25$ kN $- 102.18 \Rightarrow 29.07$ kN



BM B/w D and C $\Rightarrow -(8.75x)$

at $C = 0$

at $D = 13.125$ kN-m

BM B/w B and D $\Rightarrow 8.75x - 80(x-1.5)$

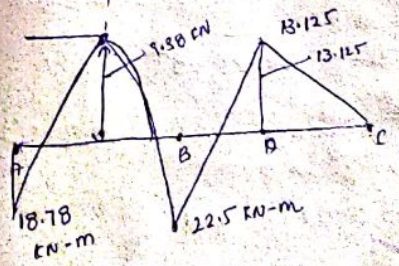
at D $x \to 1.5 = 13.125$ kN-m

at B $x \to 2 = -22.5$ kN-m

BM b/w A and B $\Rightarrow 8.75x - 80(x-1.5) - 15\frac{(x-2)^2}{2} + 102.18(x-2)$

at B $x \to 2 \Rightarrow -22.5$ kN-m

at A $x \to 6 \Rightarrow -18.78$ kN-m



$$-8.75 + 80 + 15(x-2) - 102.18 = 0$$

$$x - 2 = 2.062$$

$$x = 4.062$$

max BM $\Rightarrow$ 9.38 kN-m

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Free moment diagram

$M_{AB} = -18.75$ kN-m

$M_{BA} = 22.5$ kN-m

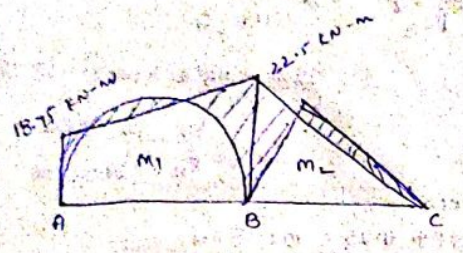
$M_{BC} = -22.5$ kN-m

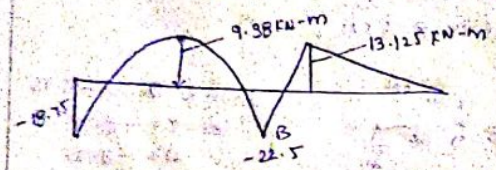
$M_{CB} = 0$

$M_1 = \frac{wL^2}{8} = \frac{15\times 5^2}{8}$

$M_2 = \frac{wab}{l} = \frac{30(0.5)(1.5)}{2}$

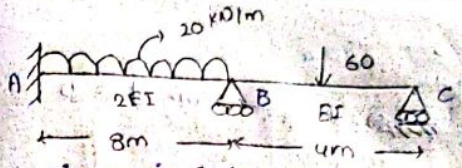
$= 30$ kN-m





— x —

71)



By slope deflection method.

B sink by 10 mm

Kinematic indeterminacy = 2

$EI = 40000$ kNm²

$M_{AB} = -\frac{wl^2}{12} + \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\delta}{l}\right]$

$= -\frac{20(8)^2}{12} + \frac{2(2EI)}{8}\left[\theta_B - \frac{3(0.01)}{8}\right]$

$= -106.67 + \frac{EI}{2}\left[\theta_B - 3.75\times10^{-3}\right]$

$M_{BA} = \frac{wl^2}{12} + \frac{2EI}{l}\left[2\theta_B + \theta_A - \frac{3\delta}{l}\right]$

$= \frac{20(8)^2}{12} + \frac{2(2EI)}{8}\left[2\theta_B - 3.75\times10^{-3}\right]$

$M_{BC} = -\frac{PL}{8} + \frac{2EI}{l}\left[2\theta_B + \theta_C + \frac{3\delta}{l}\right]$

$= -\frac{60(4)}{8} + \frac{2EI}{4}\left[2\theta_B + \theta_C + \frac{3(0.01)}{4}\right]$

$= -30 + \frac{EI}{2}\left[2\theta_B + \theta_C + 7.5\times10^{-3}\right]$

$M_{CB} = \frac{60(4)}{8} + \frac{2EI}{4}\left[2\theta_C + \theta_B + \frac{3\delta}{l}\right]$

$= 30 + \frac{EI}{2}\left[\theta_B + 2\theta_C + 7.5\times10^{-3}\right]$

$M_{BA} + M_{BC} = 0$

$106.67 + \frac{EI}{2}(2\theta_B - 3.75\times10^{-3}) - 30 + \frac{EI}{2}\left[2\theta_B + \theta_C + 7.5\times10^{-3}\right] = 0$

$76.67 + \frac{EI}{2}\left[2\theta_B - 3.75\times10^{-3} + 2\theta_B + \theta_C + 7.5\times10^{-3}\right] = 0$

$76.67 + \frac{EI}{2}\left[4\theta_B + \theta_C + 3.75\times10^{-3}\right] = 0$

$EI\cancel{(4\theta_B + \theta_C)} \cancel{(-\frac{76.67\times2}{EI})} - 3.75\times10^{-3}$

$4\theta_B + \theta_C = \cancel{\frac{38.3}{EI}}$

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$M_{CB} = 0$

$$30 + \frac{EI}{2}(2\theta_C + \theta_B + 7.5\times10^{-3}) = 0$$

$$\frac{EI}{2}\left[2\theta_C + \theta_B + 7.5\times10^{-3}\right] = -30$$

$$EI\left[2\theta_C + \theta_B + 7.5\times10^{-3}\right] = -60$$

$$2\theta_C + \theta_B + 7.5\times10^{-3} = \frac{-60}{EI} \quad -②$$

① + ②

$$4\theta_B + \theta_C + 3.75\times10^{-3} = \frac{-153.34}{EI} \quad -①$$

$$4\theta_B + 8\theta_C + 30\times10^{-3} = \frac{-240}{EI} \quad -②$$

(−) (−) (−) (+)

$$-7\theta_C - 0.02625 = \frac{86.66}{EI}$$

$$-7\theta_C = \frac{86.66}{EI} + 0.02625$$

$$-\theta_C = \frac{12.38}{EI} + 3.75\times10^{-3}$$

$EI = 4000$

$$\theta_C = -6.84\times10^{-3} \text{ rad}$$

~~In ②~~ $\Rightarrow$ $2(-6.84\times10^{-3}) + \theta_B + 7.5\times10^{-3} = \frac{-60}{4000}$

$$0.01368 + \theta_B + 7.5\times10^{-3} = -0.015$$

$$\theta_B = -0.03648$$

In ① $\Rightarrow$ $4\theta_B = \frac{-153.34}{4000} - 3.75\times10^{-3} + 6.84\times10^{-3}$

$$\theta_B = -8.8\times10^{-3} \text{ rad.}$$

Substituting θ_B and θ_C values

$M_{AB} = -131.77$

$M_{BA} = 63.97$

$M_{BC} = -63.88$

$M_{CB} = 0.04 \sim 0$

Consider AB $\Rightarrow$

$R_A + R_B = 160$

$\Sigma M_A = 0$

$-R_B(8) + 160(4) + 63.97 - 131.77 = 0$

$R_B = 71.52$ kN

$R_A = 88.47$ kN

consider BC $\Rightarrow$

$R_B + R_C = 60$

$\Sigma M_B = 0$

$-R_C(4) + 0 + 60(2) - 63.88 = 0$

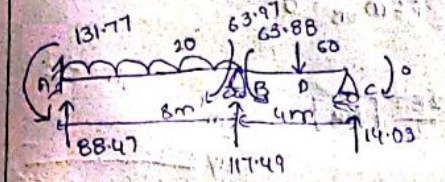
$R_C = 14.03$ kN

$R_B = 45.97$

$\therefore R_A = 88.47$ kN

$R_B = 117.49$ kN

$R_C = 14.03$ kN



SF b/w DC $\Rightarrow$ -14.03

at C, D = -14.03 kN

SF b/w DB $\Rightarrow$ $-14.03 + 60$

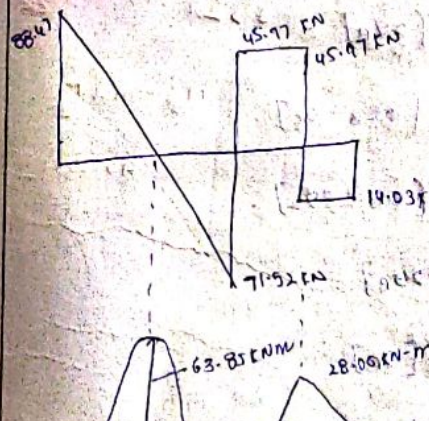
$= 45.97$ kN

SF b/w BA $\Rightarrow$ $-14.03 + 60 + 20(4)$

-117.49

at B $\Rightarrow$ ~~45.97 kN~~ -71.52 kN

at A $\Rightarrow$ 88.48 kN



BM b/w D and C $\Rightarrow$ $14.03x$

at C = 0

at D = 28.06 kN

BM b/w B and D $\Rightarrow$ $14.03x - 60(x-2)$

at $B_{x\to4}$ $\Rightarrow$ 28.06 kN m

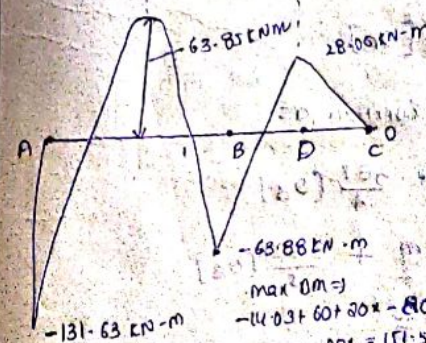
at $B_{x\to4}$ $\Rightarrow$ -63.88 kN m

BM b/w A and B $\Rightarrow$

$14.03(x) - 60(x-2) + 117.49(x-4) + 63.88 - 63.97 - 20\frac{(x-4)^2}{2}$

at $B_{x\to4}$ $\Rightarrow$ -63.97 kNm

at $A_{x\to12}$ $\Rightarrow$ -131.63 kNm



max BM $\Rightarrow$

$-14.03 + 60 + 20x - 80 - 117.49 = 0$ right

$20x = 151.52$

$x = 7.5$ m

$14.03(7.5) - 60(7.5-2) + 117.49(7.5-4) + 63.88 - 63.97 - 20\frac{(7.5-4)^2}{2}$

$= 63.85$ kN-m

$\int V$ −ve

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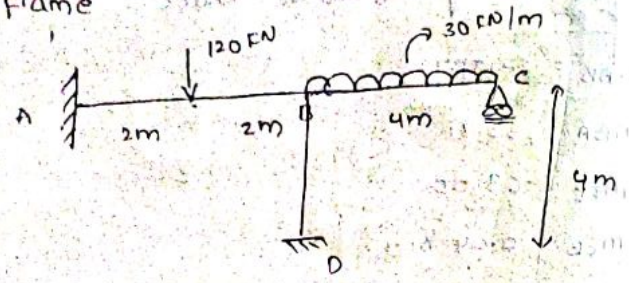
72) Frame



$FEM_{AB} = -\frac{PL}{8} = -\frac{120 \times 4}{8} = -60$

$FEM_{BA} = 60$

$FEM_{BC} = -\frac{wl^2}{12} = -\frac{30(4)^2}{12} = -40$

$FEM_{CB} = 40$

$FEM_{BD} = 0 \quad FEM_{DB} = 0$

$m_{AB} = -60 + \frac{2EI}{4}[\theta_B]$

$m_{BA} = 60 + \frac{2EI}{4}(2\theta_B)$

$m_{BC} = -40 + \frac{2EI}{4}[2\theta_B + \theta_C]$

$m_{CB} = 40 + \frac{2EI}{4}[\theta_B + 2\theta_C]$

$m_{BD} = 0 + \frac{2EI}{4}[2\theta_B]$

$m_{DB} = 0 + \frac{2EI}{4}[\theta_B]$

m_{BC} can also be written as

$m_{BC} = -60 + \frac{3EI}{4}[\theta_B]$

$m + \frac{m}{2} + \frac{3EI}{4}[\theta_B]$

$-40 + \frac{40}{2}$

$M_{BA} + M_{BC} + M_{BD} = 0$

$60 + \frac{2EI}{4}(2\theta_B) + \frac{3EI}{4}(\theta_B) - 60 + \frac{2EI}{4}(2\theta_B) = 0$

$2EI\theta_B + \frac{3EI}{4}\theta_B = 0$

$\theta_B\left[2EI + 3\frac{EI}{4}\right] = 0$

$\theta_B EI\left[\frac{4+3}{4}\right] = 0$

$\theta_B = 0$

$M_{CB} = 0$

$40 + \frac{2EI}{4}[0 + 2\theta_C] = 0$

$\frac{2EI}{4}(2\theta_C) = -40$

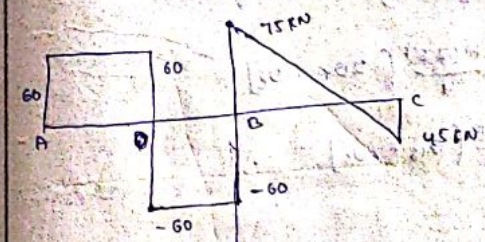
$EI\theta_C = -40$

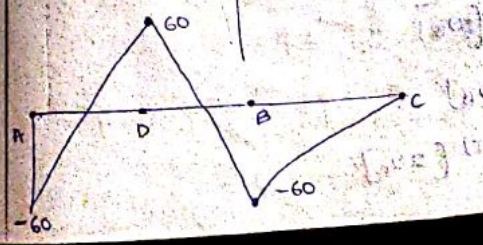
$m_{AB} = -60$
$m_{BA} = 60$
$m_{BC} = -60$
$m_{CB} = 0$
$m_{BD} = 0$
$m_{DB} = 0$

$R_A + R_B = 120$ KN
$\Sigma M_A = 0$
$-R_B(4) + 120(2) = 0$
$R_B = 60$ KN
$R_A = 60$ KN

$R_B + R_C = 120$
$\Sigma M_B = 0$
$-R_C(4) + 120(2) - 60 = 0$
$R_C = 45$ KN
$R_B = 75$ KN
$R_B = 135$ KN

SF b/w BC => $-45 + 30(x)$
at C = -45 KN
at B = 75 KN
SF b/w DB => $-45 + 120 - 135$
at B => -60 KN
at D => -60 KN
SF b/w AD => $-45 + 120 - 135 + 120$
at D(A) 60 KN
BM b/w BC => $45x - 30\left(\frac{x^2}{2}\right)$
at C = 0
at B = -60 KN-m
BM b/w DB => $45x - 120(x-2) + 60 - 60 + 135(x-4)$
at B x→4 => -60 KN-m
at D x→6 => 60 KN-m
BM b/w AD => $45x - 120(x-2) + 135(x-4) - 120(x-6)$
at D = 60 KN-m
at A = -60 KN-m





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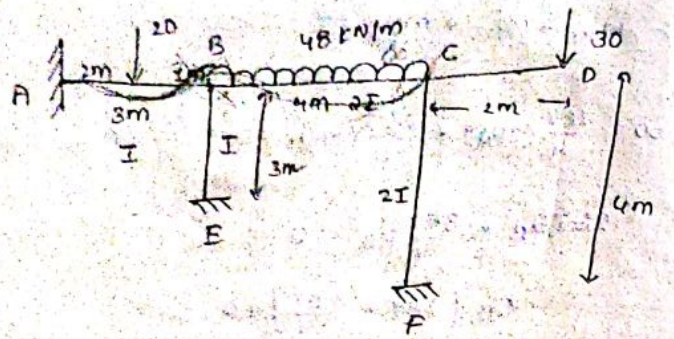
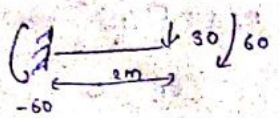
13)



CD ⇒



$FEM_{AB} = -\frac{Pab^2}{l^2} = -4.44$

$FEM_{BA} = +\frac{Pa^2b}{l^2} = 8.9$

$FEM_{BC} = -\frac{wl^2}{12} = -64$

$FEM_{CB} = 64$

$FEM_{BE} = 0 \quad FEM_{EB} = 0$

$FEM_{CF} = 0 \quad FEM_{FC} = 0$

$M_{AB} = -4.44 + \frac{2EI}{3}[\theta_B]$

$m_{BA} = 8.9 + \frac{2EI}{3}[2\theta_B]$

$m_{BC} = -64 + \frac{2E(2I)}{4}[2\theta_B + \theta_C]$

$m_{CB} = 64 + \frac{4EI}{4}[\theta_B + 2\theta_C]$

$m_{CD} = -60$

$m_{BE} = 0 + \frac{2EI}{3}[2\theta_B]$

$m_{EB} = 0 + \frac{2EI}{3}[\theta_B]$

$m_{CF} = 0 + \frac{2E(2I)}{4}[2\theta_C]$

$m_{FC} = 0 + \frac{2E(2I)}{4}(\theta_C)$

$M_{BA} + M_{BC} + M_{BE} = 0$

$M_{CB} + M_{CD} + m_{CF} = 0$

$8.9 + \frac{2EI}{3}\theta_B - 64 + 2EI\theta_B + EI\theta_C + \frac{4EI\theta_B}{3} = 0$

$EI\theta_B\left[\frac{4}{3} + 2 + \frac{4}{3}\right] + EI\theta_C = 55.1$

$EI\theta_B(4.7) + EI\theta_C = 55.1$

$4.7\theta_B + \theta_C = \frac{55.1}{EI}$ — ①

$64 + EI\theta_B + 2EI\theta_C - 60 + 2EI\theta_C = 0$

$EI\theta_B + 4EI\theta_C = -4$ — ②

$\theta_B + 4\theta_C = \frac{-4}{EI}$ — ②

$4.7\theta_B + \theta_C = 55.1/EI$

$4.7\theta_B +$

$\theta_B = +12.7/EI$

$\theta_C = -4.15/EI$

$m_{AB} = 4.02$ kN-m

$m_{BA} = 25.827$

$m_{BC} = -42.77$

$m_{CB} = +68.35$

$m_{CD} = -60$

$m_{BE} = 16.93$

$m_{EB} = 8.46$

$m_{CF} = -8.35$

$m_{FC} = -4.176$

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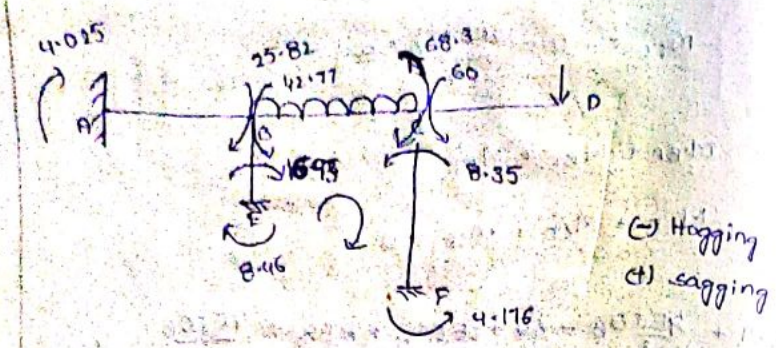
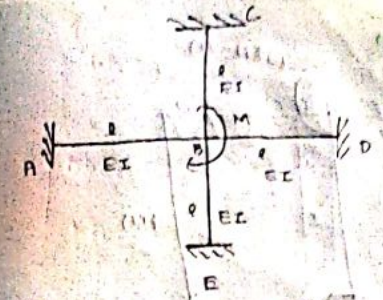


7A)

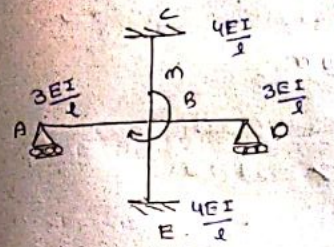


$$4\left[\frac{4EI}{l}\theta_B\right] = M$$

$$16\frac{EI}{l}\theta_B = M$$

$$\theta_B = \frac{Ml}{16EI}$$

7B)



$$2\left[\frac{4EI\theta_B}{l}\right] + 2\left[\frac{3EI\theta_B}{l}\right] = M$$

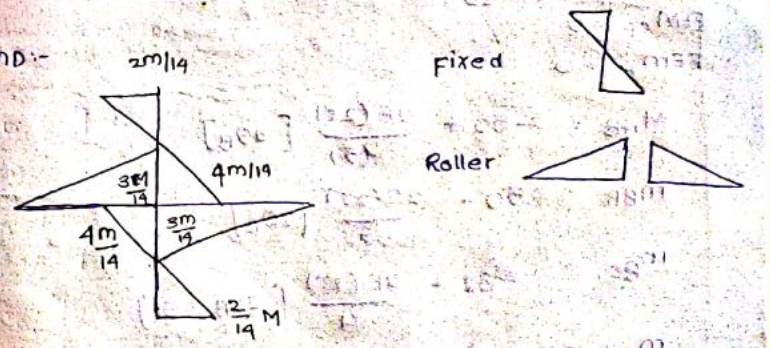
$$\frac{14EI}{l}\theta_B = M$$

$$\theta_B = \frac{ML}{14EI}$$

$$\theta_E = \frac{4}{14} \times M$$

$$= \frac{4}{14} \times \frac{14EI}{l}\theta_B = \frac{4EI\theta_B}{l}$$

BMD:-



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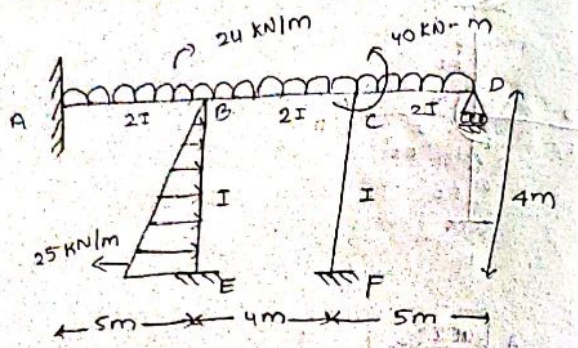
76)



Kinematic indeterminacy = 3 (B, C, D)

$FEM_{AB} = -\frac{wL^2}{12} = -\frac{24(5)^2}{12} = -50$ kN m

$FEM_{BA} = 50$ kN m

$FEM_{BC} = -\frac{24(4)^2}{12} = -32$ kN m

$FEM_{CB} = 32$ kN m

$FEM_{CD} = \frac{wL^2}{12} = -50$ kNm

$FEM_{DC} = 50$ kNm

$FEM_{BE} = +\frac{wL^2}{30} = +13.33$ kNm

$FEM_{EB} = -\frac{wL^2}{20} = -20$ kNm

$FEM_{CF} = 0$

$FEM_{FC} = 0$

$M_{AB} = -50 + \frac{2E(2I)}{(5)}[2\theta_B]$

$m_{BA} = 50 + \frac{2E(2I)}{5}[2\theta_B]$

$m_{BC} = -32 + \frac{2E(2I)}{4}[2\theta_B + \theta_C]$

$m_{CB} = 32 + \frac{2E(2I)}{4}[\theta_B + 2\theta_C]$

$m_{CD} = (-50) - \frac{50}{2} + \frac{3E(2I)}{5}[\theta_C]$

$M_{DC} = -50 + \frac{50}{2} + \frac{3E(2I)}{5}[\theta_C]$

$M_{BE} = +13.33 + \frac{2E(I)}{4}[2\theta_B]$

$m_{EB} = -20 + \frac{2E(I)}{4}[2\theta_B]$

$m_{CF} = 0 + \frac{2EI}{4}[2\theta_C]$

$m_{FC} = 0 + \frac{2EI}{4}[2\theta_C]$

$M_{BA} + M_{BE} + m_{BC} = 0$

$m_{CB} + m_{CD} + m_{CF} = -40$

$EI\theta_B = x$

$EI\theta_C = y$

$50 + \frac{8x}{5} + 13.33 + x - 32 + 2x + y = 0$

$4.6x + y = 31.33$

$32 + x + 2y - 50 - 25 + \frac{6}{5}y + y = -40$

$x + 4.2y = 43 - 40 = 3$

$EI\theta_B = -7.34$

$EI\theta_C = +2.4$

$x = 13.05 \quad 3.4M - 7.34$

$y = 2.46$

$M_{AB} = -55.87$

$m_{BA} = 38.256$

$m_{BC} = -44.22$

$m_{CB} = 29.58$

$m_{CD} = -72.048$

$m_{BE} = 5.99$

$m_{EB} = -23.67$

$m_{CF} = 2.46$

sway → (sway moment)

1) eccentricity load

2) Material property change

↓ (I ↓ and length ↑ percent)

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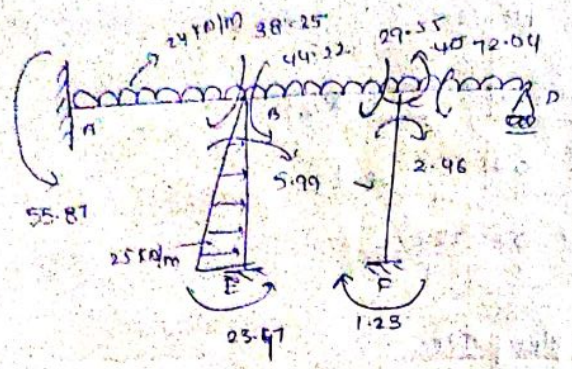


Finding rxnc :-

AB ⇒ $R_A + R_B = 24 \times 5 = 120$

$\Sigma m_A = 0$

$38.25 + 24(5)(2.5) - R_B(5) - 55.87 = 0$

$R_B(5) = 282.38$

$R_B = 56.476$ kN

$R_A = 63.524$ kN

BC ⇒ $R_B + R_C = 24 \times 4 = 96$

$\Sigma m_B = 0$

$29.55 + 24(4)(2) - 44.22 - R_C(4) = 0$

$R_C(4) = 177.33$

$R_C = 44.33$ kN

$R_B = 51.66$ kN

CD ⇒ $R_C + R_D = 120$

$\Sigma m_C = 0$

$-R_D(5) + 120(2.5) - 72.04 = 0$

$R_D(5) = 227.96$

$R_D = 45.5$

$R_C = 74.408$

BE ⇒ $R_B + R_E = \frac{1}{2} \times 25 \times 4 = 50$

$\Sigma m_E = 0$

$5.99 - R_B(4) + \frac{1}{2}(25)(4)\left(\frac{4}{3}\right) - 23.47 = 0$

$(4) R_B = 49.38$

$R_B = 12.25$ $R_E = 37.70$

CF ⇒ $R_C + R_F = 0$

$\Sigma m_F = 0$

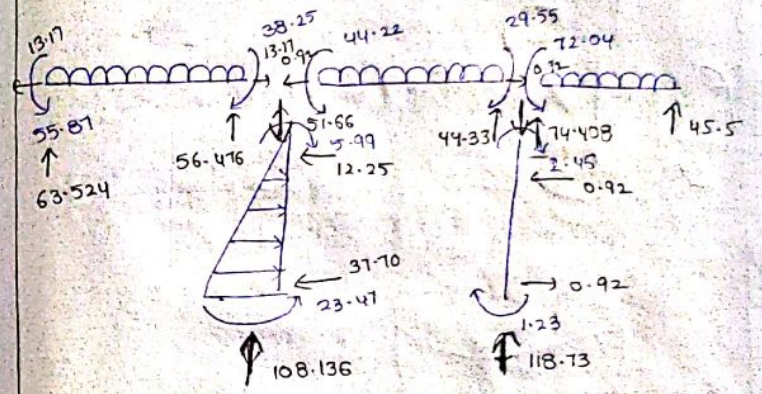
$-R_C(4) + 2.46 + 1.23 = 0$

$R_C = 0.92$

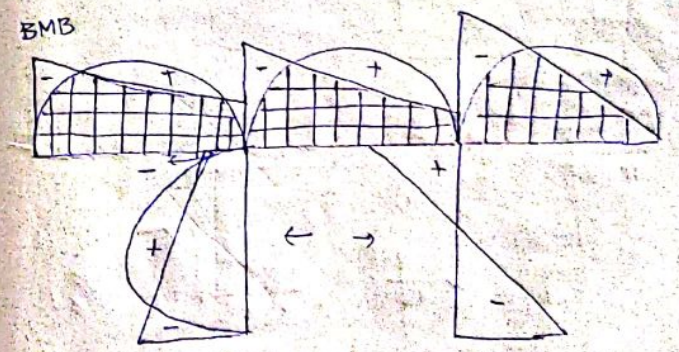
$R_F = -0.92$

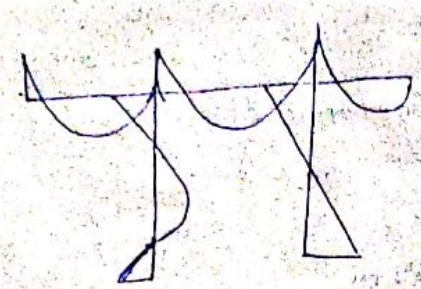
$R_A = 63.524$ kN

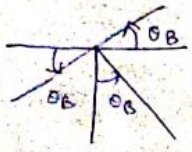
$R_D =$

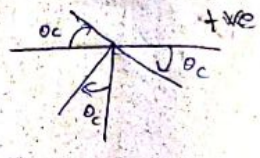


BMB



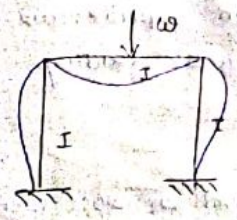




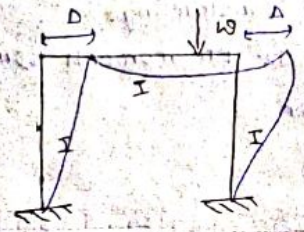


Sway Analysis :-

Its not because of axial, shear deformations only cos of BM or rotations

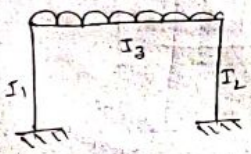


no sway



As the load is unsymmetrical

sway

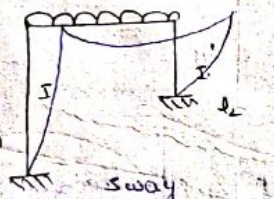


sway

E - material

I - dimensional property

even the load is symmetrical, the dimensional load is varying so chance of sway



sway

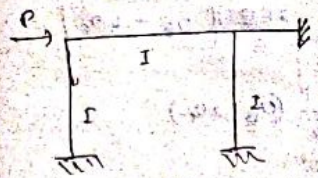
length - geometric property

Due to geometrical change there is chance of sway.

very minute difference in lengths.

horizontal load

sway



fixed beam is not allowing the rotation or movement.

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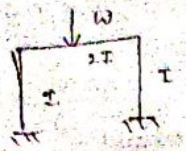


column properties are same

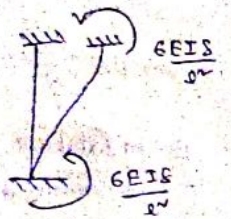
symmetric loading

No sway

↳ Body moment also considered so cross sectional area can be reduced while constructing.

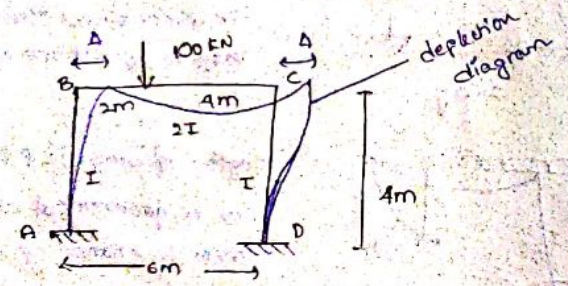
If sway considered the BM moments will reduce

Δ is also an unknown



If it is hinge $\frac{3EI\delta}{l^2}$

7)



$$M_{AB} = +\frac{2EI}{4}\left[\theta_B - \frac{3\Delta}{l}\right]$$ (−) because look at upper diagram anticlockwise

$$M_{BA} = \frac{2EI}{4}\left[2\theta_B - \frac{3\Delta}{l}\right]$$

$$M_{BC} = -\frac{100(2)(4)^2}{36} + \frac{2EI}{l}\left[2\theta_B + \theta_C - \frac{3\delta}{l}\right]$$

$$= -88.89 + \frac{2E(2I)}{6}\left[2\theta_B + \theta_C - \frac{3\delta}{6}\right]$$

$$M_{CB} = \frac{(100)(2^2)(4)}{36} + \frac{2E(2I)}{6}\left[\theta_B + 2\theta_C\right]$$

$$= 44.44 + \frac{4EI}{6}(\theta_B + 2\theta_C)$$

$$M_{CD} = \frac{2EI}{4}\left[2\theta_C - \frac{3\delta}{4}\right]$$

$$M_{DC} = \frac{2EI}{4}\left[\theta_C - \frac{3\delta}{4}\right]$$

$$M_{BA} + M_{BC} = 0$$

$$M_{CB} + M_{CD} = 0$$

$EI\theta_B = x$

$EI\theta_C = y$

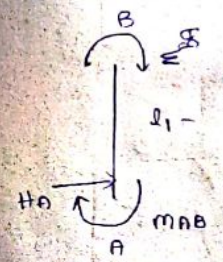
$$x - \frac{6}{4}\frac{EI\Delta}{(4)} + \frac{8}{6}x + \frac{4}{6}y - 88.89 = 0$$

$$2.33x - 0.375\frac{EI}{\Delta} + 0.667y - 88.89 = 0 \quad \text{—(1)}$$

$$44.44 + \frac{4}{6}x + \frac{8}{6}y + y - \frac{6EI\Delta}{16} = 0$$

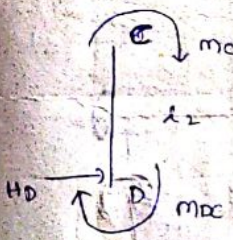
$$0.667x + 2.33y - 0.375EI\Delta - 44.44 = 0 \quad \text{—(2)}$$

Consider the columns for another $x(n)$ =



$$-H_A(l_1) + M_{AB} + M_{BA} = 0$$

$$H_A = \frac{M_{AB} + M_{BA}}{4}$$



$$H_D = \frac{M_{CD} + M_{DC}}{4}$$

$H_A + H_D = 0$ [total horizontal forces = 0]

shear at horizontal direction

If shear at vertical direction we write $V_A + V_D = 0$.

$$\frac{M_{AB} + M_{BA}}{4} + \frac{M_{CD} + M_{DC}}{4} = 0$$

$$M_{AB} + M_{BA} + M_{CD} + M_{DC} = 0.$$

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$$\frac{2}{4}x - \frac{6EI\Delta}{16} + x - \frac{6EI\Delta}{16} + y - \frac{6EI\delta}{16} + \frac{2}{4}y - \frac{6}{16}EI\delta = 0$$

$EI\delta = z$

$$0.5x + 1.5y - 1.125z = 0 \quad -(3)$$

$x = 46.74$

$y = 20.01$

$z = 89.02$

$x = 49.878$

$y = -30.125$

$z = 19.753$

$$M_{AB} = \frac{2}{4}\left[49.878 - \frac{3(19.753)}{4}\right]$$

$= 17.53$

$$m_{BA} = \frac{2}{4}\left[2(49.878) - \frac{3(19.753)}{4}\right]$$

$= 42.47$

~~= 49.47~~

$m_{BC} = -42.47$

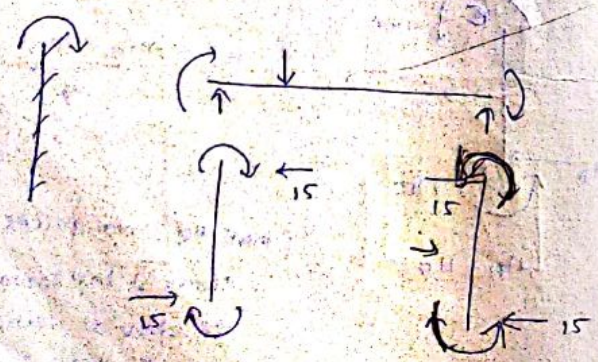
$m_{CB} = +37.53$

$m_{CD} = -37.53$

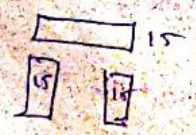
$m_{DC} = -22.47$

$H_A = 15$

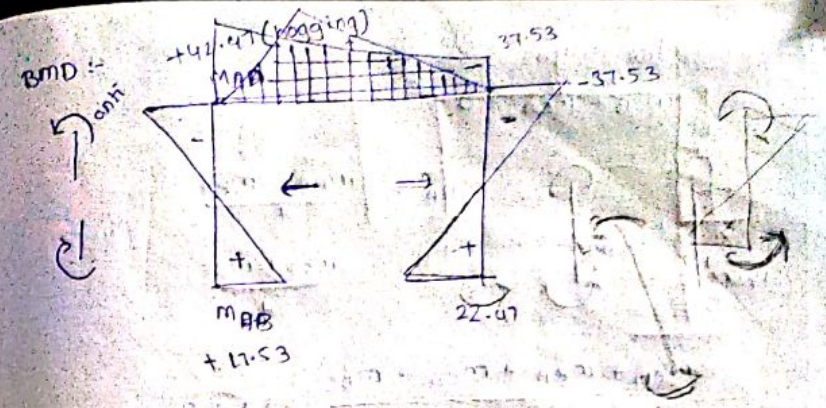
$H_B = -15$



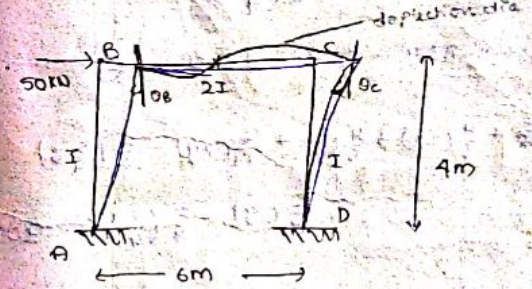
Axial diagrams :-



BMD :-



78)



at joint 50kN is acting, we'll consider it at shear equillibrium

$FEM_{AB} = 0 \quad FEM_{BA} = FEM_{CD} - FEM_{DC} = 0$

$$M_{AB} = \frac{2EI}{4}\left[\theta_B - \frac{3\delta}{4}\right] \qquad M_{CD} = \frac{2EI}{4}\left[2\theta_C - \frac{3\delta}{4}\right]$$

$$m_{BA} = \frac{2EI}{4}\left[2\theta_B - \frac{3\delta}{4}\right] \qquad m_{DC} = \frac{2EI}{4}\left[\theta_C - \frac{3\delta}{4}\right]$$

$$m_{BC} = \frac{4EI}{6}\left[2\theta_B + \theta_C\right]$$

$$m_{CB} = \frac{4EI}{6}\left[2\theta_C + \theta_B\right]$$

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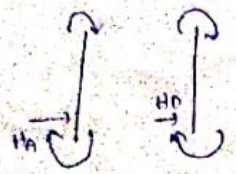
$M_{BA} + M_{BC} = 0$

$M_{CB} + M_{CD} = 0$



$$H_A = \frac{M_{AB} + M_{BA}}{4}$$

$$H_D = \frac{M_{CD} + M_{DC}}{4}$$

$$\frac{M_{AB} + M_{BA} + M_{CD} + M_{DC}}{4} + 50 = 0$$

$$M_{AB} + M_{BC} + M_{CD} + M_{DC} = -200$$

$EI\theta_B = x$

$EI\theta_C = y$

$EI\delta = z$

$$\frac{2}{4}(2x) - \frac{2}{4}\left(\frac{3z}{4}\right) + \frac{4}{6}(2x) + \frac{4}{6}y = 0$$

$$2.33x + 0.66y - 0.375z = 0 \quad \text{—(1)}$$

$$\frac{4}{6}(2y) + \frac{4}{6}(x) + \frac{2}{4}(2y) - \frac{2}{4}\left(\frac{3}{4}z\right) = 0$$

$$0.66x + 2.33y - 0.375z = 0 \quad \text{—(2)}$$

$$\frac{2}{4}x - \frac{2}{4}\frac{3}{4}z + \frac{4}{6}(2x) + \frac{4}{6}(y) + \frac{2}{4}(2y) - \frac{2}{4}\left(\frac{3}{4}z\right) + \frac{2}{4}(y) - \frac{2}{4}\left(\frac{3}{4}\right)z = -200$$

$$1.833x + 2.166y - 1.125z + 200 = 0 \quad \text{(3)}$$

$x = 22.22$

$y = 22.22$

$z = 177.77$

$M_{AB} = -55.55$

$M_{BA} = -44.44$

$M_{BC} = +44.44$

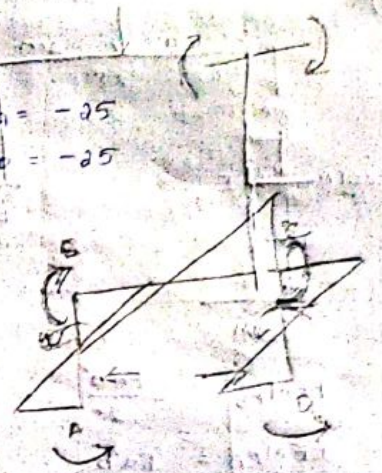
$M_{CB} = +44.44$

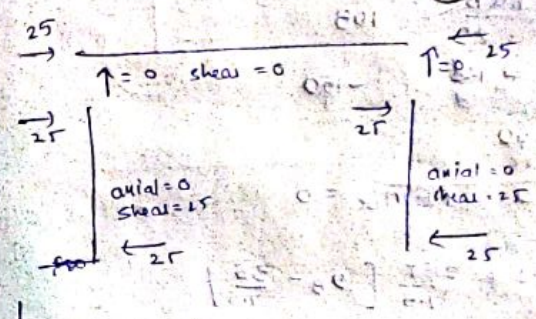
$M_{CD} = -44.44$

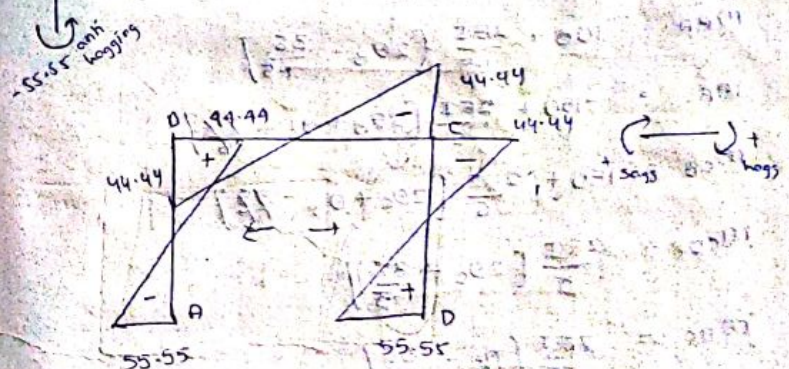
$M_{DC} = -55.55$

$H_A = -25$

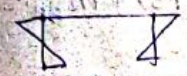
$H_D = -25$



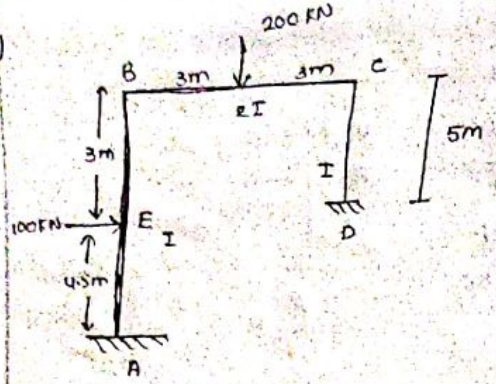




centre of load



79)



$FEM_{AB} = -\frac{Pab^2}{l^2} = -72$

$FEM_{BA} = \frac{Pa^2b}{l^2} = 108$

$FEM_{BC} = -\frac{Pl}{8} = -150$

$FEM_{CB} = 150$

$FEM_{CD} = 0 \quad FEM_{DC} = 0$

$m_{AB} = -72 + \frac{2EI}{7.5}\left[\theta_B - \frac{3\delta}{7.5}\right]$

$m_{BA} = 108 + \frac{2EI}{7.5}\left(2\theta_B - \frac{3\delta}{7.5}\right]$

$m_{BC} = -150 + \frac{2EI}{6}\left[2\theta_B + \theta_C - \frac{3\delta}{6}\right]$

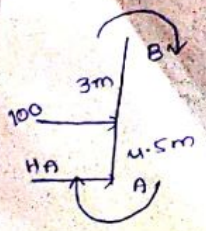
$m_{CB} = 150 + \frac{2EI}{6}\left[2\theta_C + \theta_B - \frac{3\delta}{6}\right]$

$m_{CD} = \frac{2EI}{5}\left[2\theta_C - \frac{3\delta}{5}\right]$

$m_{DC} = \frac{2EI}{5}\left[\theta_C - \frac{3\delta}{5}\right]$

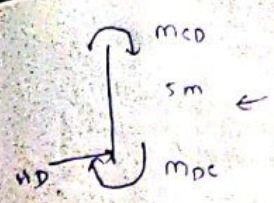
$M_{BA} + m_{BC} = 0$

$m_{CB} + m_{CD} = 0$



$-H_A(7.5) + m_{AB} - 100(3) + m_{BA} = 0$

$H_A = \frac{m_{AB} + m_{BA} - 300}{7.5}$



$-H_D(5) + m_{DC} + m_{CD} = 0$

$H_D = \frac{m_{DC} + m_{CD}}{5}$

$H_A + H_D + 100 = 0$

$5m_{AB} + 5m_{BA} - 5(300) + m_{DC}(7.5) + 7.5m_{CD} = -100(3)(7.5)$

$5m_{AB} + 5m_{BA} + 7.5m_{DC} + 7.5m_{CD} = -2250$

$5\left[-72 + \frac{2}{7.5}x - \frac{6}{7.5^2}z\right] + 5\left[108 + \frac{4}{7.5}x - \frac{6z}{7.5^2}\right] + 7.5\left[\frac{2}{5}y - \frac{6}{25}z\right]$

$+ 7.5\left[\frac{4}{5}y - \frac{6}{25}z\right] = -2250$

$-360 + 1.33x - 0.533z + 540 + 2.667x - 0.533z + 3y - 1.8z$

$+ 6y - 1.8z = -2250$

$3.997x + 9y - 4.667z = -2430$ —①

$108 + \frac{2}{7.5}\left(2x - \frac{3}{7.5}z\right) + \frac{2}{3}x + \frac{y}{3} = 150$

$0.533x - 0.106z + 0.67x + 0.33y = 42$

$1.203x + 0.33y - 0.106z = 42$ —②

$150 + \frac{1}{3}(2y + x) + \frac{2}{5}(2y - 3z) = 0$

$0.66y + 0.33x + 0.8y - 1.2z = -150$

$0.33x + 1.46y - 1.2z = -150$ —③

$x = 63.37$

$y = -32.46$

$z = 512.44$

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Final moments -

$M_{AB} = -109.76$

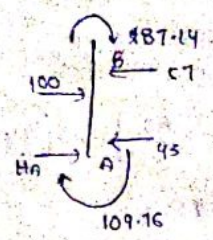
$M_{BA} = 87.14$

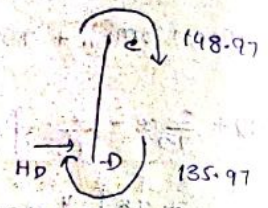
$M_{BC} = -87.14$

$M_{CB} = +148.97$

$M_{CD} = -148.97$

$M_{DC} = -135.97$





$H_A = \frac{M_{AB} + M_{BA} - 300}{7.5}$

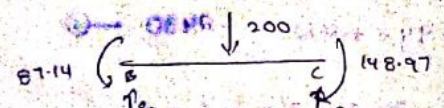
$H_A = -43$

$H_B = -57$

$H_D = -57$

$H_C + H_D = 0$

$H_C = 57$



$R_B = 89.7$

$R_C = 110.3$

$R_C + R_D = 200$

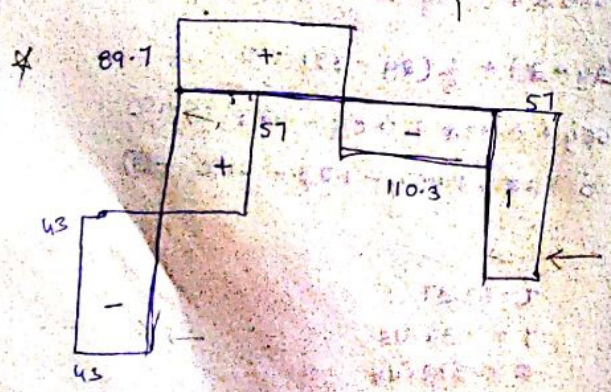
$\Sigma R_B = 0$

$148.97 + R_C(6) + 200(3) - 87.14 = 0$

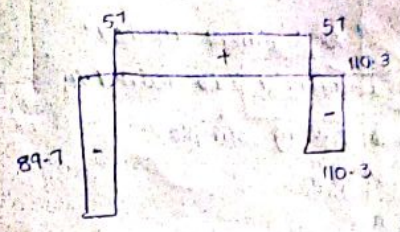
$R_C = 110.30$

$R_B = 89.69$

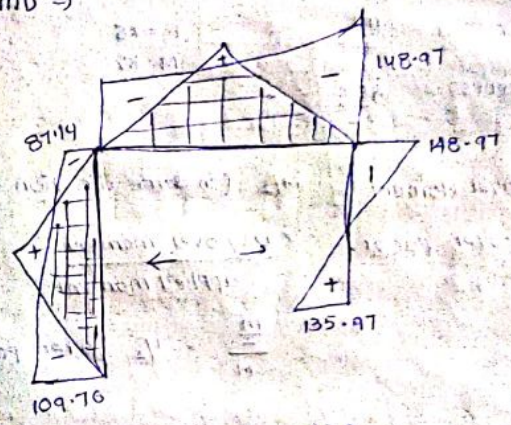
SFD :-

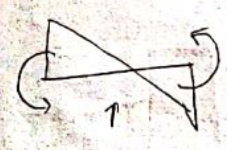


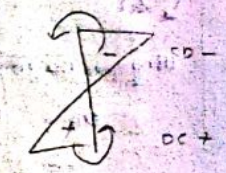
Axial force =>

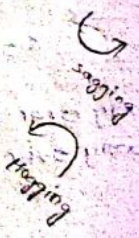


BMD =>









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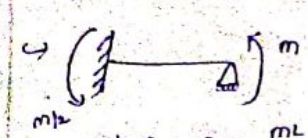
Moment Distribution Method:-

Also called as "Displacement method"

Hardy cross is a scientist who derived it.

Most widely used as it is very simple.



slope = $\theta = \frac{mL}{4EI}$

$F = K\delta$

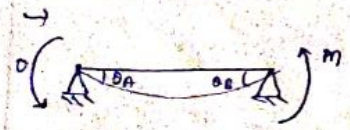
$M = K\theta$

Stiffness = $\frac{4EI}{L}$

1) carry over Moment – $M/2$ [in same direction of m]

2) carry over factor – $\frac{\text{carry over moment}}{\text{applied moment}}$

$= \frac{\frac{m}{2}}{M} = 1/2$ [when far end ...



Hinge → no moment

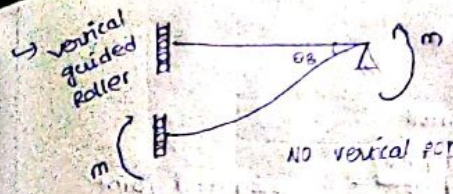
$\theta_A = \frac{mL}{6EI}$ $\theta_B = \frac{mL}{3EI}$

$= \theta_B/2$

$K = \frac{3EI}{L}$

carry over Moment = 0

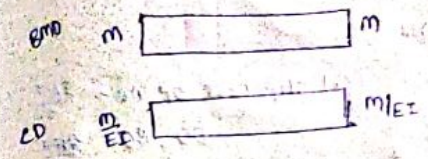
carry over factor = 0 [when far end is hinge] $\frac{0}{m}$



NO vertical forces

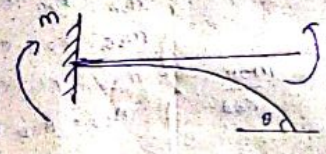
$\theta_B = \frac{mL}{EI}$

$K = \frac{EI}{L}$



carry over moment = −m

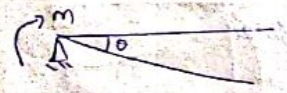
carry over factor = $\frac{-m}{m} = -1$

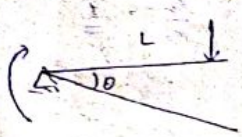


$\frac{mL^2}{2EI} = \left(\theta \times \frac{L}{2}\right)$

$\theta = \frac{mL}{EI}$

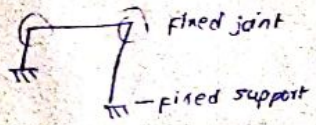
$K = \frac{EI}{L}$





(θL) if its st. line

intermediate hinge support



Hinge support has no moment but intermediate hinge support has moment.

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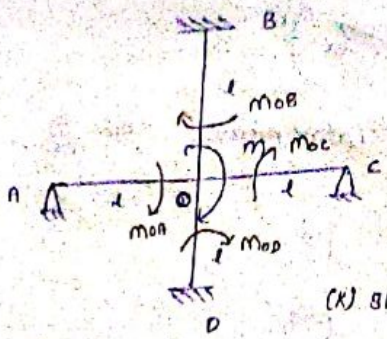


O is fixed joint

(K) stiffness of AO - $\frac{3EI}{l}$

DO, BO - $\frac{4EI}{l}$

CO - $\frac{3EI}{l}$

$M = K\theta_0$

total stiffness of joint 'O' = K_0

$K_0 = K_{OA} + K_{OB} + K_{OC} + K_{OD}$

Distribution factor $OA = \frac{K_{OA}}{K_0}$

$\theta_0 = \frac{M_{OA}}{K_{OA}} = \frac{M_{OB}}{K_{OB}} = \frac{M_{OC}}{K_{OC}} = \frac{M_{OD}}{K_{OD}} = \frac{M}{\Sigma K}$

$M_{OA} = K_{OA} \cdot \theta_{OA}$

$M_{OB} = K_{OB} \cdot \theta_{OB}$

$M_{OC} = K_{OC} \cdot \theta_{OC}$

$M_{OD} = K_{OD} \cdot \theta_{OD}$

all θ's equal

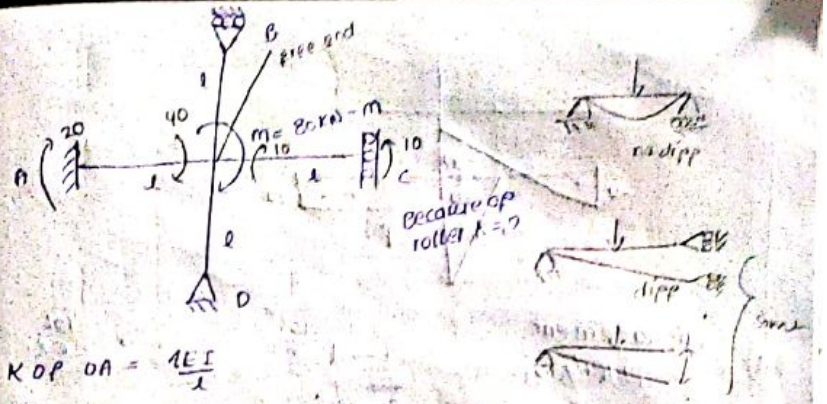
$\Rightarrow$ M_{OA} = distribution factor of OA × total Moment

$= \frac{K_{OA}}{K_0} \times M$

Rotation is same at every joint

$\Rightarrow \frac{K_{OA}}{K_0} + \frac{K_{OB}}{K_0} + \frac{K_{OC}}{K_0} + \frac{K_{OD}}{K_0} = \frac{K_0}{K_0} = 1$

80)



K of OA = $\frac{4EI}{l}$

K of OD = $\frac{3EI}{l}$

K of OB = 0

K of OC = $\frac{EI}{l}$

Total K = K_0 = $\frac{8EI}{l}$

distribution factor for OA = $\frac{4EI}{l} \times \frac{1}{8EI} = \frac{1}{2}$

OD = $\frac{3}{8}$

OC = 1/8

OB = 0

Total $\frac{1}{2} + \frac{3}{8} + \frac{1}{8} = \frac{4+3+1}{8} = \frac{8}{8} = 1$ check

$M_{OA} = \frac{1}{2} \times 80 = 40$ kN-m

$M_{OB} = 0$

$M_{OC} = \frac{1}{8} \times 80 = 10$ kN-m

$M_{OD} = \frac{3}{8} \times 80 = 30$ kN-m

moment at A $\Rightarrow \frac{40}{2} = 20$ carry over moment

carry over factor = $\frac{20}{40} = 1/2$

moment at C $\Rightarrow -10$ (opp. direction)

carry over factor = $\frac{-10}{10} = -1$

Moment at D $\Rightarrow$ 0

at B = 0

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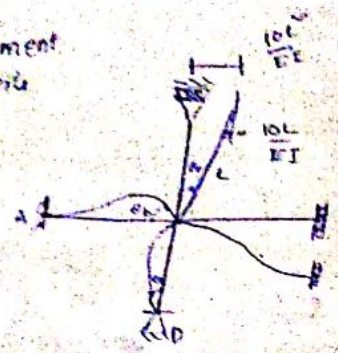
BMD -

displacement profile



$\theta_1 = \frac{10L}{EI}$

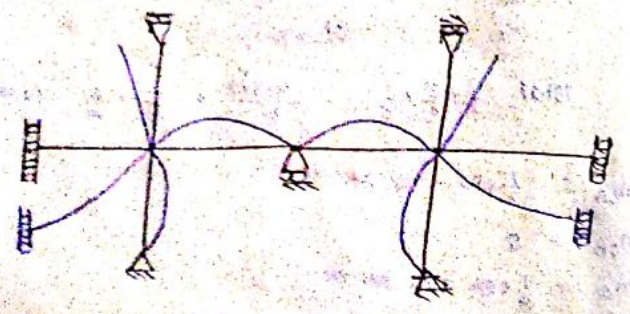
$\theta = \frac{10L}{EI}$

$\theta = 0$

$\delta = \frac{10L}{EI} \times \frac{L}{2}$

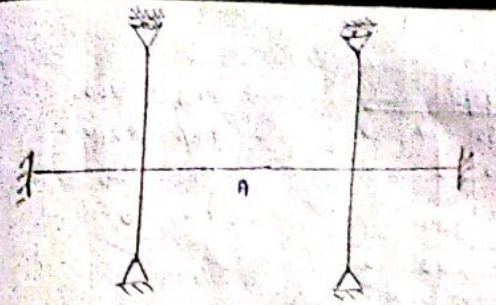
$\delta = \frac{5L^2}{EI}$

Q1)

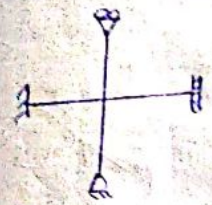


Mirror Image

Q2)

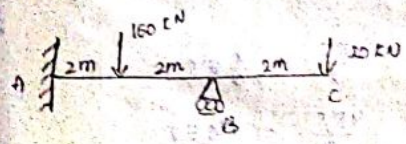


at 'A' place slope should be 0
displacement should be there
so it suits for roller.



solve it and multiply with 2 then problem is solved.

Q3)

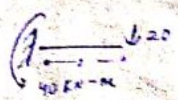


$FEM_{AB} = \frac{PL}{8} = -80$ kN-m

$FEM_{BA} = +80$ kN-m

$FEM_{BC} = -40$ kN-m

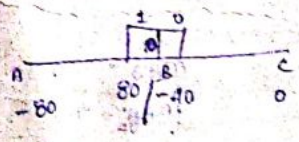
$FEM_{CB} = 0$



Distribution factors →

$BA = \frac{4EI}{L} = 1$

$BC = 0$



unbalanced moment = +40

stiffness at $BA = \frac{4EI}{4} = EI$

$BC = 0$

Distribution ratio of BA and BC = 1:0

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A	B		C
	1/2	1/2	
-80	80	-40	0
	-40	0	
-20			
-100	40	-40	0

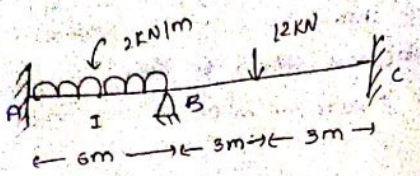
$M_{AB} = -100$

$M_{BA} = 40$

$M_{BC} = -40$

$M_{CB} = 0$

84)



FEM $M_{AB} = -\frac{wl^2}{12} = -6$

$M_{BA} = 6$

$M_{BC} = -\frac{Pl}{8} = -9$

$M_{CB} = 9.$

Joint B ⇒ $BA = \frac{4EI}{l} = \frac{4}{6}EI$

$BC = \frac{4EI}{6} = 4/6\,EI$

$\Sigma K = 8/6\,EI$

So $BA = 1/2$

$BC = 1/2$

A	B		C
	1/2	1/2	
-6	6	-9	9
	1.5	1.5	
0.75			0.75
-6	7.5	-7.5	9.75

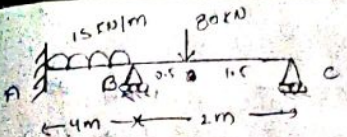
$M_{AB} = -6$

$M_{BA} = 7.5$

$M_{BC} = 7.5$

$M_{CB} = 9.75$

85)



FEM :-

$AB = -20$

$BA = 20$

$BC = -22.5 - \frac{Pab^2}{l^2}$

$CB = +7.5 + \frac{Pa^2b}{l^2}$

Stiffness ⇒ at joint B ⇒

$BA = \frac{4EI}{4} = EI$

$BC = \frac{3EI}{2} = 3/2\,EI$

$\Sigma K = \left(1+\frac{3}{2}\right)EI \Rightarrow \frac{5}{2}EI$

So distribution factors ⇒

$BA = 1/5/2 = 2/5$

$BC = 3/2 / 5/2 = 3/5$

A	B		C
	2/5	3/5	
-20	20	-22.5	7.5
			-7.5
		-3.75	
	+2.5	3.75	
1.25			
-18.75	+22.5	-22.5	0

hinge so moment should be 0

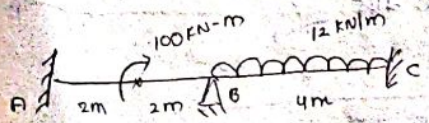
$M_{AB} = -18.75$

$M_{BA} = 22.5$

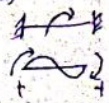
$M_{BC} = -22.5$

$M_{CD} = 0$

86)



To know $\pm\frac{m}{4}$ ⇒



FEM ⇒ $AB = +\frac{m}{4} = 25$

$BA = +\frac{m}{4} = 25$

$BC = -\frac{wl^2}{12} = -16$

$CB = +16$

at Joint B ⇒

$BA = \frac{4EI}{4} = EI$

$BC = \frac{4EI}{4} = EI$

distribution

1/2 1/2

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A	B		C
	1/2	1/2	
25	25	-16	16
	-4.5	-4.5	
-2.25			-2.25
22.75	20.5	-20.5	13.75

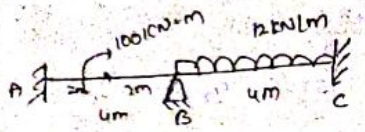
$M_{AB} = 22$

$M_{BA} = 20$

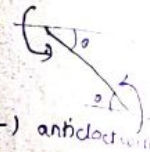
$M_{BC} = -20$

$M_{CB} = 13.75$

87) Same sum when joint B sinks by 10mm $EI = 48000$



AB ⇒



(−) anticlockwise

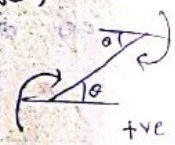
$AB \Rightarrow +\frac{m}{4} - \left[\frac{6EI(1)}{1000 \times 4^2}\right] = 7$

$BA \Rightarrow +\frac{m}{4} - \left[\frac{6EI(1)}{1000 \times 4^2}\right] = 25 - 18 = 7$

$BC \Rightarrow -16 + \left[\frac{6EI(1)}{1000 \times 4^2}\right] = 2$

$CB \Rightarrow +16 + 18 = 34$

BC ⇒



+ve clock

Stiffness ⇒ ~~AB~~ joint B ⇒

$BA = \frac{4EI}{4} = EI$

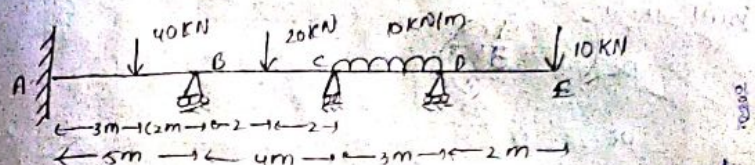
$BC = \frac{4EI}{4} = EI$

(1/2, 1/2)

A	B		C
	1/2	1/2	
7	7	2	34
	-4.5	-4.5	
-2.25			-2.25
4.75	2.5	-2.5	31.75

-8

88)



Fixed end moments

hogging −ve

$M_{AB} = -\frac{Pab^2}{l^2} = -19.2$

$M_{BA} = +\frac{Pa^2b}{l^2} = +28.80$

$M_{BC} = -\frac{PL}{8} = -10$

$M_{CB} = +10$

$M_{CD} = -\frac{wL^2}{12} = -7.5$

$M_{DC} = 7.5$

$M_{DE} = -20$

$-(10 \times 2) = -20$ (N)

joint B ⇒

$BA = \frac{4EI}{5} = 4/5\, EI$

$BC = \frac{4EI}{4} = EI$

$\Sigma K = 9/5\, EI$

so $BA = \frac{4/5}{9/5} = 4/9$ ✓

$BC = 1/9/5 = 5/9$ ✓

joint C ⇒

$CB = \frac{4EI}{4} = EI$

$CD = \frac{3EI}{3} = EI$

$\Sigma K = 2EI$

so $CB = 1/2$ ✓

$CD = 1/2$ ✓

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joint D ⇒

$DC = \frac{3EI}{3}$; $DC = \frac{4EI}{3}$

$DE = 0$; $DE = 0$

$\Sigma K = \frac{3EI}{3} = 1$; $\Sigma K = \frac{4EI}{3}$

so $DC = 1$ ✓ ; $DC = 1$

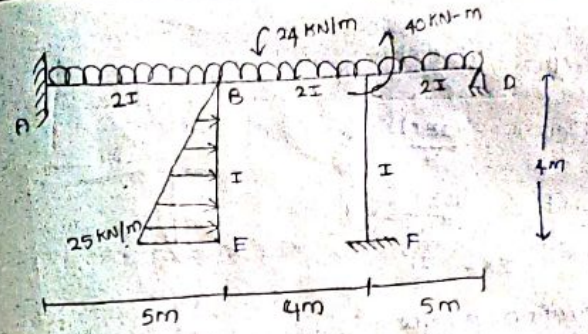
$DE = 0$ ✓ ; $DE = 0$

A	B		C		D		E
	4/9	5/9	1/2	1/2	1	0	
-19.2	28.8	-10	10	-7.5	7.5	-20	0
				+12.5			
				6.25			
	-8.35	-10.4	-4.37	-4.37			
-4.17		-2.185	-5.2				
	+0.971	1.213	2.61	2.61			
+0.4855		+1.305	+0.61				
	-0.58	-0.725	-0.305	-0.305			
0.29		-0.1525	0.3625				
-22.59	20.84	-20.94	3.68	-3.325	20	-20	

-12.5
30
+12.5

10 - 1.25
8.75
So - 8.75

89)



$M_{AB} = -50 \Rightarrow -\frac{wl^2}{12} \Rightarrow -\frac{24(25)}{12}$

$M_{BA} = 50$

$M_{BC} = -32 \Rightarrow \frac{wl^2}{12} \Rightarrow -\frac{(24)(16)}{12}$

$M_{BE} = 13.33 \Rightarrow \frac{wl^2}{30}$

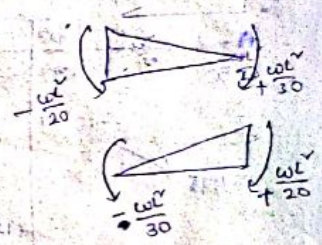
$M_{CB} = 32$

$M_{CD} = -50$

$M_{DC} = 50$

$M_{CF} = M_{FC} = 0$

$M_{EB} = -20$



At joint B :-

$BA = \frac{4(2EI)}{5} = \frac{8}{5}$

$BC = \frac{4(2EI)}{4} = \frac{8}{4} = 2$

$BE = \frac{4(EI)}{4} = 1$

$\Sigma K = \frac{8}{5} + 2 + 1 = \frac{23}{5}$

$\frac{8}{5} + 2 + 1$

$\frac{8+10+5}{5} = \frac{23}{5}$

Distributions -

$\theta_{BA} = \frac{8/5}{23/5} = \frac{8}{23}$

$\theta_{BC} = 10/23$

$\theta_{BE} = 5/23$

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At joint C :-

$$r_{CB} = \frac{4(2EI)}{4} = 2$$

$$r_{CD} = \frac{3(2EI)}{5} = \frac{6}{5}$$

$$r_{CF} = \frac{4(EI)}{4} = 1$$

$$\Sigma K = \frac{21}{5}$$

Distribution factors :-

$$\theta_{CB} = \frac{10}{21} \qquad \theta_{CF} = \frac{5}{21}$$

$$\theta_{CD} = \frac{6}{21}$$

A	B		C	D
	8/23	10/23	19/21	6/21
			−40	
−50	+50	−32	+32	−50
	−10.89	−13.62		−25
−5.445			1.5	1.5
			+39.52	+23.714
		+19.76	−6.81	
	−6.87	−8.59	+3.24	+1.94
−3.44				
		+1.62	−4.295	
−0.2815	−0.563	−0.704	+2.045	+1.227

E	B	F	C
	5/21		5/21
−20	+13.33	0	0
	−6.8		
−3.4			+19.76
	−4.29		+9.88
−2.145			+1.62
	−0.352		+0.81
−0.176			+1.0226
			0.5113

$M_{CB} + M_{CF} + M_{CD} = -40$

A	B		C	D
	8/23	10/23	19/21	6/21
			−40	
−50	50	−32	32	−50
				−25
	−10.89	−13.62	1.42	0.857
−5.448		0.704	−6.81	
	−0.278	−0.310	3.24	1.945
−0.124				
		1.64	−0.155	
	−0.57	−0.710	0.073	0.044
−55.57	39.432	−44.28	27.81	−72.154

E	B	F	C
	5/23		5/6
−20.11	13.33	0	0
	−6.81	0.357	0.714
−3.455			
	−0.155	0.810	0.621
0.077			
	−0.356	0.018	0.036
−0.178			
−23.506	6.009	1.185	2.371

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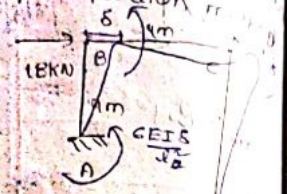
Mit-2

Q) section B.

(a) Analysis the loaded frame shown in fig having equal rigidity throughout the frame by slope deflection method consider side sway.



FEM ⇒

$M_{AB} = -\frac{6EI\delta}{l_1^2}$ $M_{BA} = -\frac{6EI\delta}{l_1^2}$

$M_{BC} = M_{CB} = 0$

$M_{CD} = -\frac{6EI\delta}{l_2^2}$ $M_{DC} = -\frac{6EI\delta}{l_2^2}$

$l_1 = 4$ $l_2 = 6$

$M_{AB} = -\frac{6EI\delta}{16}$ $M_{BA} = -\frac{6EI\delta}{16}$

$M_{CD} = -\frac{6EI\delta}{36}$ $M_{DC} = -\frac{6EI\delta}{36}$

$-\frac{6EI\delta}{4^2}$

$\frac{1}{16} : \frac{1}{36}$

36 : 16

Distribution factor :-

at joint B →

$K_{BA} = \frac{4EI}{4} = EI$ $\theta_{BA} = 1/2$

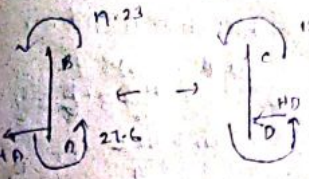
$K_{BC} = \frac{4EI}{4} = EI$ $\theta_{BC} = 1/2$

at joint C →

$K_{CD} = \frac{4EI}{6} = 4/6\,EI$ $\theta_{CD} = 4/10 = 2/5$

$K_{CB} = \frac{4EI}{4} = EI$ $\theta_{CB} = 6/10 = 3/5$

A	B		C		D
-36	-36	0	0	-16	-16
9	+18	+18	+9.6	6.4	3.2
		4.8	9		
-1.8	-2.4	-2.4	-5.4	-3.6	-1.8
		-2.7	-1.2		
0.675	1.35	1.35	0.72	0.48	0.24
		0.56	0.675		
-0.09	-0.18	-0.18	-0.405	-0.27	-0.135
-27.6	-19.23	19.2	+12.99	-12.99	-14.495



$\Sigma M_B = 0$

$H_A(4) - 27.6 - 19.23 = 0$

$H_A = 11.7$

$\Sigma M_C = 0$

$H_D(6) - 12.99 - 14.495 = 0$

$H_D = 4.58$

$F_x = 16.28$ KN

But we need for 18 KN (given true force)

Here all values obtained for 16.28 KN shear force

$M_{AB} \rightarrow \frac{18}{16.28} \times -27.6 = -30.515$

$M_{BA} = \frac{18}{16.28} \times -19.23 = -21.26$

$M_{BC} = 1.105 \times 19.2 = 21.22$

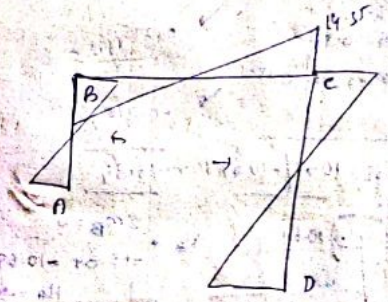
$M_{CB} = 1.105 \times 12.99 = 14.35$

$M_{CD} = 1.105 \times -12.99$
$= -14.35$

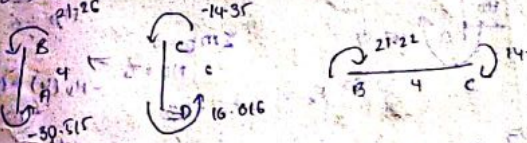
$M_{DC} = 1.105 \times -14.495$
$= -16.016$

$16.29 \rightarrow -27.6$
$18 \rightarrow ?$
$M_{AB} = \frac{18}{16.29}(-27.6)$

BMB



SFD ⇒



$\Sigma M_A = 0$

$-30.515 + 21.26 - R_B(4) = 0$

$R_B = -2.31$

$R_A = 2.31$

$\Sigma M_D = 0$

$-16.016 - 14.35 + R_C(6) = 0$

$R_C = 5.061$

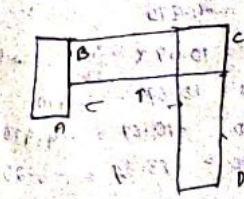
$R_D = -5.061$

$\Sigma M_B = 0$

$-21.22 + 14.35 - R_C(4) = 0$

$R_C = -1.71$

$R_B = 1.71$



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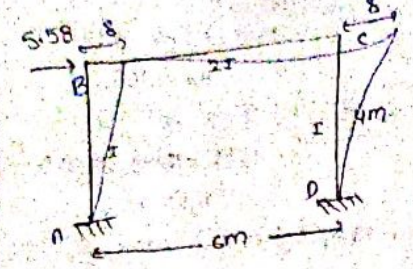
91)



Distribution factor

at B →

$BC = \frac{4EI}{6} = \frac{8}{6}$

$BA = \frac{4EI}{4} = EI$

$K = 7/3$

at C →

$CB = \frac{4EI}{6} = \frac{4}{3}$

$CD = \frac{4EI}{4} = EI$

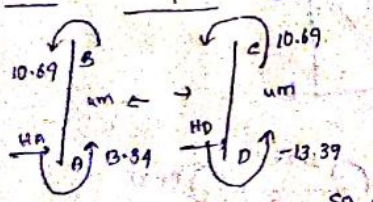
A	B		C		D
	3/7	4/7	4/7	3/7	
-16	-16	0	0	-16	-16
	6.857	9.142	9.142	6.857	
3.4285		4.571	4.57		3.4285
	-1.95	-2.61	-2.61	-1.95	
-0.975		-1.30	-1.30		-0.975
	0.55	0.74	0.737	0.55	
0.275		0.36	0.36		0.275
	-0.153	-0.204	-0.204	-0.153	
-0.076					-0.076
-13.34	-10.69	10.69	10.69	-10.69	-13.34

FEM :-

$-\frac{6EI\delta}{16} = -\frac{6EI\delta}{16}$

1 : 1

-16 = -16



$\Sigma m_B = 0$

$H_A = -13.34 + 10.69 - H_A(4) = 0$

$H_A = -5.995 \sim 6$

$H_A = 6$ ←

$\Sigma m_C = 0$

$H_D = 6$ ⇒ $-H_D(4) + 13.39 - 10.69 = 0$

$H_D = -6$

$= 6$ ←

so $F_x = 6 + 6 = 12$

⇒ $A \rightarrow \frac{5.58}{12} \times -13.34 = -6.203$

$B \rightarrow \frac{5.58}{12} \times -10.69 = -4.970$

Final Moments ⇒

$m_{AB} = -6.203$

$m_{BA} = -4.970$

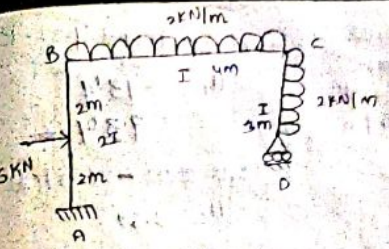
$m_{BC} = 10.69 \times \frac{5.58}{12} = +4.970$

$m_{CB} = 10.69 = 4.970$

$m_{CD} = -10.69 = -4.970$

$m_{DC} = -13.34 = -6.203$

92)



Fixed end Moments :-

$M_{AB} = -\frac{PL}{8} = -8$ $\quad M_{BA} = \frac{PL}{8} = 8$

$m_{BC} = -\frac{wL^2}{12} = -2.67$ $\quad m_{CB} = +2.67$

$m_{CD} = -\frac{wL^2}{12} = -1.5$ $\quad m_{DC} = +1.5$

At Joint B ⇒

$K_{BA} = \frac{4E(2I)}{4} = \frac{8}{4} = \frac{2}{3}$ θ_{BA}

$K_{BC} = \frac{4E(I)}{4} = EI = 1 \Rightarrow \frac{1}{3}$

$K = \frac{2}{3} + 1 =$

$\theta_{BA} = 2/3$

$\theta_{BC} = 1/3$

at Joint C ⇒

$K_{CB} = \frac{4E(I)}{4} = EI$

$K_{CD} = \frac{3EI}{3} = EI$

$\theta_{CB} = 1/2$

$\theta_{CD} = 1/2$

Non sway Analysis

A	B		C		D
	2/3	1/3	1/2	1/2	
-8	8	-2.67	2.67	-1.5	1.5
				-0.75	-1.5
	-3.55	-1.776	-0.21	-0.21	
-1.775		-0.105	-0.105		
	0.067	0.033	0.444	0.444	
0.0336		0.022	0.0165		
	-0.147	-0.073			
0.0736					
-9.6618	4.37	-4.37	2.02	-2.02	0

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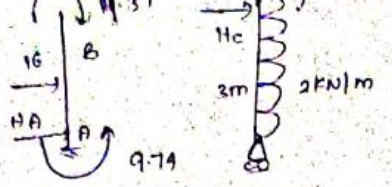


$H_A = -9.34$

$= 9.34$

$H_c(3) - 2.02 - 2(1.5)$

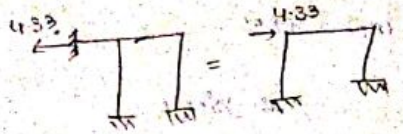
$H_c = 3.67$

For total structure

$\Sigma F_x = 0$

$16 - 9.34 + 8.67 - 6 = 0$

$\rightarrow 4.33 \neq 0$ so we have extra 4.33 in the structure



Sway analysis :-

$\frac{-6(2EI)\delta}{16} : \frac{-6(EI)\delta}{9}$

$\frac{2}{16} : \frac{1}{9}$

$1/8 : 1/9$

$9 : 8 \rightarrow$ From Lingo also take -8

The far end is hinge

(or) $\frac{-6(2EI)\delta}{10} : \frac{-3(EI)\delta}{9}$

$\frac{12}{16} : \frac{3}{9}$

$3/4 : 1/3$

$9 : 4$

From lingo take 9

A	B		C		D
	3/4	1/4	1/2	1/2	
$-\frac{6(EI)\delta}{4^2}$	$-\frac{6(EI)\delta}{4^2}$	0	0	$-\frac{6EI\delta}{3^2}$	$-\frac{6EI\delta}{3^2}$
-9	-9	0	0	-8	0 -8 0
2	6	3	+4	+4	0
	2		1.5		
-0.664	1.33 -0.667	-0.666	-0.75	-0.75	
		-0.375	0.333 -0.665		
0.125	0.25	0.125	0.166 -0.083	0.166 -0.083	
		0.0625	0.0625		
-7.205	-3.417	3.458	2.7	1.92	
-7.539	-4.58	4.16	5.31	4.58	

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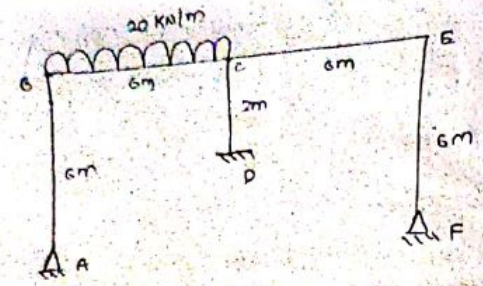
93) Analyse the Frame



Fixed end moments :-

$M_{AB} = M_{BA} = M_{CD} = M_{DC} = M_{CE} = M_{EC} = M_{EF} = M_{FE} = 0$

$M_{BC} = -\frac{wl^2}{12} = -60 \qquad M_{CB} = 60$

at joint B ⇒

$BA = \frac{3EI}{6} = \frac{1}{2}$

$BC = \frac{4EI}{6} = \frac{2}{3}$

$K = \frac{1}{2} + \frac{2}{3} = \frac{3+4}{6} = \frac{7}{6}$

$\theta_{BA} = \frac{1}{2} \times \frac{6}{7} = 3/7$

$\theta_{BC} = \frac{2}{3} \times \frac{6}{7} = 4/7$

at joint C ⇒

$CB = \frac{4EI}{6} = \frac{2}{3}$

$CD = \frac{4EI}{3} = \frac{4}{3}$

$CE = \frac{4EI}{6} = \frac{2}{3}$

$K = \frac{2+4+2}{3} = \frac{8}{3}$

$\theta_{CB} = 2/8 = 1/4$

$\theta_{CD} = 4/8 = 1/2$

$\theta_{CE} = 2/8 = 1/4$

at joint E ⇒

$EC = \frac{4EI}{6}$

$EF = \frac{3EI}{6}$

$K = \frac{2}{3} + \frac{1}{2}$

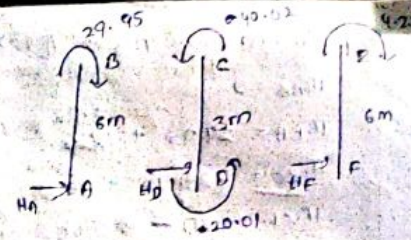
$\theta_{EC} = 4/7$

$\theta_{EF} = 3/7$

Non sway Analysis

A	B		C		E		F
	3/7	4/7	1/4	1/4	4/7	3/7	
0	0	-60	60	0	0		0
	25.71	34.28	-15	-15			
		-7.5	17.14		-7.5		
	3.214	4.285	-4.28	-4.28	4.28	3.214	
		-2.14	2.142		-2.14		
	0.917	1.22	-0.53	-0.53	1.2	0.91	
		-0.265	0.61		-0.265		
	0.113	0.151	-0.15	-0.15	0.15	0.113	
		0.075	0.075		0.075		
0	29.95	-29.89	60	-19.96	-4.2	4.237	0

D	C
0	0
	-30
-15	
	-8.57
-4.285	
	-1.07
-0.535	
	-0.305
-0.1525	
	-0.075
-0.0375	
-20.01	-40.02



$\Sigma M_B = 0$

$-H_A(6) + 29.95 = 0$

$H_A = 4.99$

$\Sigma M_C = 0$

$-H_D(3) - 20.01 - 40.02 = 0$

$H_D = -20.01$

$H_D = 20.01 \leftarrow$

$\Sigma F_x = 0$

$4.99 - 20.01 + 0.706 = 0$

-14.3

$\Sigma M_E = 0$

$-H_F(6) + 4.237 = 0$

$H_F = 0.706$

$\leftarrow 14.3$ (sway force)

sway Analysis :-

$\frac{3EI\delta}{36} : \frac{6EI\delta}{9}$

$1/12 : \frac{2}{9} \Rightarrow \frac{1}{12} : \frac{2}{3}$

$\Rightarrow 3 : 24$

$\Rightarrow 1 : 8 \Rightarrow 10 : 80$

A	B		C		E		F
	3/7	4/7	1/4	1/4	4/7	3/7	
0	10	0	0	0	0	10	0
	-4.26	-5.7	-20	20	-5.7	-4.26	
		-10	-2.85	-2.85	-10		
	4.28	5.714	1.425	1.425	5.71	4.28	
		0.71	2.857	2.857	0.71		
	-0.304	0.405	-1.428	-1.428	-0.40	-0.30	
	9.716	-9.68	-20	-20	-9.68	9.72	

D	C
30	-80
	-40
-20	
	+2.85
1.427	
	-2.855
-4.427	
60	40

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(viii) Final End moments :- (last step)

$m_{AB} = 0$

$m_{BA} = 29.95 + 3.72 = 33.62$

$m_{BC} = -29.95 - 3.72 = -33.65$

$m_{CB} = 60 - 7.68 = 52.25$

$m_{CE} = -19.96 - 7.68 = -27.54$

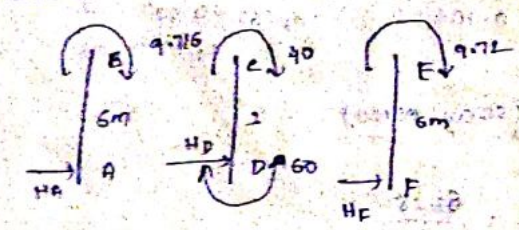
$m_{EC} = -4.2 - 3.73 = -8.08$

$m_{EF} = 4.237 + 3.73 = 8.08$

$m_{FE} = 0 + 0 = 0$

$m_{CD} = -40.02 + 15.36 = -24.58$

$m_{DC} = -20.01 + 23.6 = 3.6$



$\Sigma M_B = 0$

$-H_A(6) + 9.716 = 0$

$H_A = 1.61$

$\Sigma M_E = 0$

$-H_F(6) + 9.72 = 0$

$H_F = 1.62$

$\Sigma M_C = 0$

$-H_D(3) + 40 + 60 = 0$

$H_D = +33.33$

$\Sigma F_x = 0 \Rightarrow 1.61 + 1.62 + 33.33 = 0$

$+36.56 \leftarrow$

The moments we got are for +36.56

$36.56 \rightarrow 9.716$

$14.3 \rightarrow \frac{14.3}{36.56} \times 9.716$

$= 3.8$

$36.56 \rightarrow 20$

$14.3 \rightarrow \frac{14.3}{36.56} \times 20$

$= 7.822$

$36.56 \rightarrow 9.72$

$14.3 \rightarrow \frac{14.3}{36.56} \times 9.72$

$= 3.8$

$36.56 \rightarrow 60$

$14.3 \rightarrow \frac{14.3}{36.56} \times 60 = 23.48$

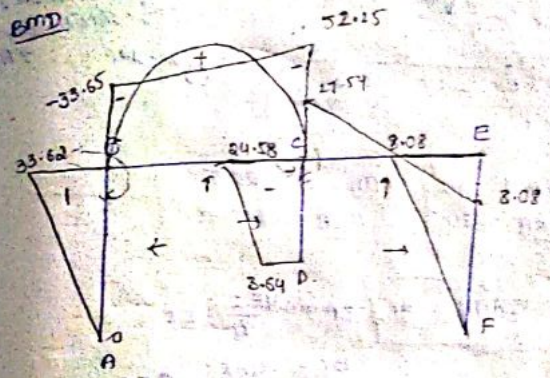
$36.56 \rightarrow 40$

$14.3 \rightarrow \frac{14.3}{36.56} \times 40 = 15.64$

Final fixed moments :-

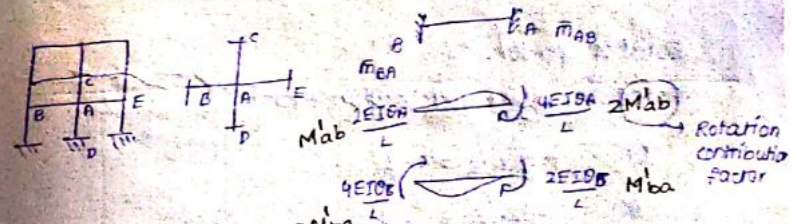
A	B	C	E	F	D	C
	29.93	-29.93	-19.96	4.35		-19.96	39.94
	3.70	-3.72	-7.68	-7.68	3.73	23.6	15.36
						3.64	-24.58
0	33.62	-33.65	52.25	-27.54	-8.08	8.08	0

BMD



"Kani's Method"

Rotation contribution Method



$$\boxed{\begin{aligned} M_{AB} &= \bar{m}_{AB} + 2M'_{ab} + M'_{ba} \\ M_{AC} &= \bar{m}_{AC} + 2m'_{ac} + m'_{ca} \\ M_{AD} &= \bar{m}_{AD} + 2m'_{ad} + m'_{da} \end{aligned}}$$

rotation contribution from far end w.r.to A

$\Sigma M_{AB} = \Sigma \bar{M}_{AB} + 2\Sigma M'_{ab} + \Sigma M'_{ba} = 0$

rotation contribution at joint A w.r.to joint B

$2M'_{ab} = -\frac{1}{2}\left[\Sigma \bar{M}_{AB} + \Sigma m'_{ba}\right]$

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Equilibrium eqn: $\Sigma M_{AB} = 0$ ⇒ $\Sigma M_{AB} = \Sigma \bar{M}_{AB} + 2\Sigma m'_{ab}$

$2M'_{ab} = K_{AB}\theta_A$

$\Sigma m'_{ab} = -\frac{1}{2}[\ldots]$

$2\Sigma m'_{ab} = \Sigma K_{AB}\theta_A$

$m'_{ab} = \frac{K_{AB}}{\Sigma K_{AB}} \Sigma M'_{ab}$

$$\boxed{M'_{ab} = \frac{K_{AB}}{\Sigma K_{AB}} \Sigma M'_{ab}}$$

$2m'_{ab} = K_{AB}\theta_A$

$2\Sigma m'_{ab} = \Sigma K_{AB}\theta_A$

$$\boxed{M'_{ab} = -\frac{1}{2}\frac{K_{AB}}{\Sigma K_{AB}}\left[\Sigma \bar{M}_{AB} + \Sigma M'_{ba}\right]}$$

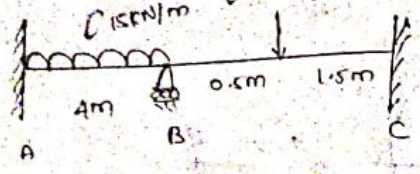
← from ① & ②

$[U_{AB}]$ → rotation factor at 'A' for member AB

→ rotation contribution at joint

$-\frac{1}{2}$ × distribution factor of AB

9A)



68th scan

FEM ⇒ $m_{AB} = -20$ $\quad m_{BC} = -22.5$

$m_{BA} = 20$ $\quad m_{CB} = 7.5$

at joint B ⇒

distribution factor ⇒ $BA = \frac{4EI}{4} = EI$

$BC = \frac{4EI}{2} = 2EI$

$\Sigma K = 1 + 2 = 3$

$\theta_{BA} = 1/3$

$\theta_{BC} = 2/3$

Rotation factors ⇒

Rotation factor $= -\frac{1}{2} \times$ distribution factor

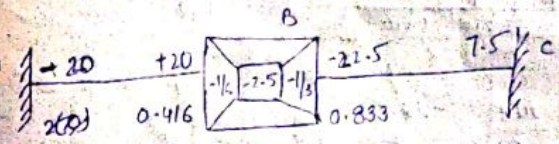
$BA = -\frac{1}{2} \times \frac{1}{3} = -\frac{1}{6}$

$BC = -\frac{1}{2} \times \frac{2}{3} = -\frac{1}{3}$

$$M'_{ba} = -\frac{1}{2}\frac{K_{BA}}{\Sigma K_{BA}}\left[\Sigma \bar{m}_{BA} + \Sigma m'_{ab}\right]$$

$U_{BA}\left[\Sigma M_{BA} + \Sigma m'_{ab}\right]$

$M_{BA} + M_{BC}$ $\quad M'_{ab} + M'_{cb}$



cycle 1 ⇒

$m'_{bc} = -\frac{1}{3}[+2.5+0+0] = 0.833$

$M'_{ba} = -\frac{1}{6}[-2.5+0+0] = 0.416$

∵ rotation contribution m_{ab}, m_{cb} are '0' due to fixed end

$M_{AB} = \bar{M}_{AB} + 2m'_{ab} + m'_{ba}$

$= -20 + 2(0) + 0.416$

$= -19.584$

$M_{BA} = \bar{M}_{BA} + 2M'_{ba} + m'_{ab}$

$= +20 + 2(0.416) + 0$

$= 20.832$

$M_{BC} = \bar{m}_{BC} + 2M'_{bc} + m'_{cb}$

$= -22.5 + 2(0.833) + 0$

$= -20.832$

$m_{CB} = \bar{m}_{CB} + 2m'_{cb} + m'_{bc}$

$= 7.5 + 2(0) + 0.833$

$= 8.33$

BM, SF

-20	+20	-1/6 [-2.5] -1/3	-22.5	7.5
2(0)	2(0.416)		(0.833)2	2(0)
0.416	0		0	0.833
-19.58	20.53		-20.83	8.33

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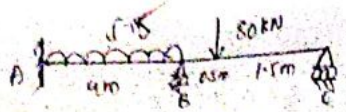
95)



(-10th sum)

$m_{AB} = -20$ $\quad m_{BC} = -22.5$

$m_{BA} = +20$ $\quad m_{CB} = 7.5$

Distribution factors =>

$BA = \frac{4EI}{4} = 1$ $\quad \theta_{BA} = 2/5$

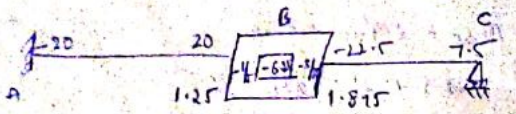
$BC = \frac{3EI}{2} = 3/2$ $\quad \theta_{BC} = 3/5$

$K = 1 + \frac{3}{2} = \frac{5}{2}$

Rotation factors =>

$BA = -\frac{1}{2} \times \frac{2}{5} = -1/5$

$BC = -\frac{1}{2} \times \frac{3}{5} = -\frac{3}{10}$



-7.5

$-\frac{7.5}{2} = -3.75$

$-20 \qquad 20 \qquad -26.25 \qquad 0$

$m'_{ba} = -\frac{1}{5}[-6.25 + 0 + 0]$ as we add $\frac{7.5}{2}$ value already

$= 1.25$

$m'_{bc} = -\frac{3}{10}[-6.25 + 0 + 0]$

$= 1.875$

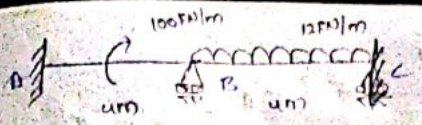
$M_{AB} = \bar{m}_{AB} + 2m'_{ab} + m'_{ba} = -20 + 2(0) + (1.25) = -18.75$

$M_{BA} = \bar{m}_{BA} + 2\bar{m}'_{ba} + \bar{m}'_{ab} = 20 + 2(1.25) + 0 = 22.5$

$M_{BC} = \bar{m}_{BC} + 2\bar{m}'_{bc} + m'_{cb} = -22.5 + 2(1.875) + 0 = -22.5$

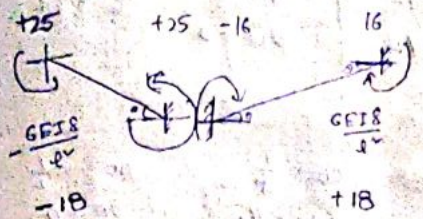
$M_{CB} = 0.$

96)



B sink by 1mm

$EI = 48000$ kN-m²



(8th sum)

FEM =>

$M_{AB} = +25 - 18 = 7$

$M_{BA} = 25 - 18 = 7$

$M_{BC} = -16 + 18 = 2$

$M_{CB} = 16 + 18 = 34$

distribution factor =>

$BA = \frac{4EI}{4} = EI$ $\quad \theta_{BA} = 1/2$

$BC = \frac{4EI}{4} = EI$ $\quad \theta_{BC} = 1/2$

$m'_{bc} = -\frac{1}{4}(9 + 0 - 0)$

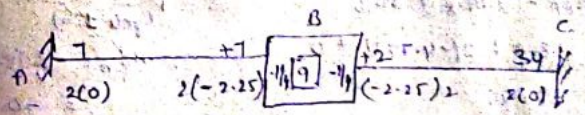
$= -2.25$

$m'_{bc} = -\frac{1}{4}(9 - 0 + 0)$

Rotation factor =>

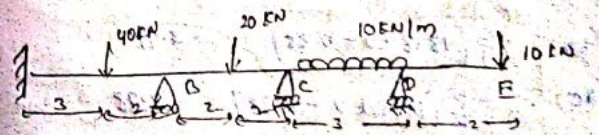
$BA = -1/4$

$BC = -1/4$



-2.25	0	0	-2.25
$M_{AB} = 4.75$	M_{BA} 2.5	$M_{BC} = -2.5$	$M_{CB} = 31.75$

97)



38th sum

$M_{AB} = -19.2$

$M_{BA} = +28.8$

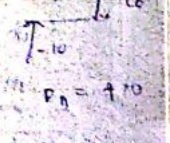
$M_{BC} = -10$

$M_{CB} = +10$

$M_{CD} = -7.5$

$M_{DC} = 7.5$

$M_{DE} = -20$



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Distribution factors =>

at B: $\theta_{BA} = 4/9$, $\theta_{BC} = 5/9$

at C: $\theta_{CB} = 1/2$, $\theta_{CD} = 1/2$

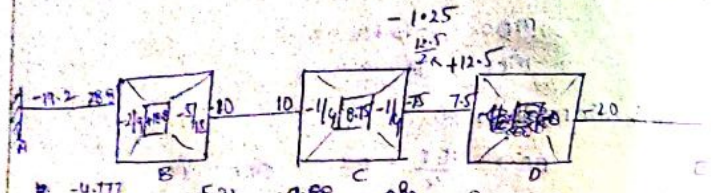
at D: $\theta_{DC} = 1$, $\theta_{DE} = 0$

Rotation factors =>

$BA = -\frac{1}{2} \times \frac{4}{9} = -\frac{2}{9}$ $\quad CB = \frac{1}{4}$ $\quad DC = -$

$BC = -\frac{1}{2} \times \frac{5}{9} = -\frac{5}{18}$ $\quad CD = -1/4$ $\quad DE = 0$

Not [illegible] we only [illegible]



-4.177	-5.22	-0.88	-0.88	
-3.98	-4.98	-0.94	-0.94	
-5.16	-4.96	-0.947	-0.947	

$M_{AB} = \bar{m}_{AB} + 2m'_{ab} + m'_{ba}$

$= -19.2 + 2(0) + (-4.777) = -23.97$

$m_{BA} = \bar{m}_{BA} + 2m'_{ba} + m'_{ab}$

$= 28.8 + 2(-4.777) + 0$

$= 19.246$

$M_{BC} = \bar{m}_{BC} + 2m'_{bc} + m'_{cb}$

$= -10/4 \; 4(-5.22) + 2$

Cycle 1 =>

$m'_{ba} = -\frac{2}{9}[18.8 \ldots$

$= -4.177$

$m'_{bc} = -\frac{5}{18}(18.8 \ldots$

$= -5.2$

Cycle 1 =>

$m'_{cb} = -\frac{1}{4}(8.75 \ldots$

$= -0.88$

$m'_{cd} = -\frac{1}{4}(8.75 - 5 \ldots$

$= -0.88$

Cycle 2 =>

$m'_{ba} = -\frac{2}{9}[18.8 + 0 - 0.88] = -3.98$

$m'_{BE} = -\frac{5}{18}[18.8 + 0 - 0.88] = -4.98$

$m'_{cb} = -\frac{1}{4}[8.75 - 4.98 + 0] = -0.94$

$m'_{cd} = -\frac{1}{4}[8.75 - 4.98 + 0] = -0.94$

Cycle 3 =>

$m'_{ba} = -\frac{2}{9}(18.8 + 0 - 0.94) = -3.96$

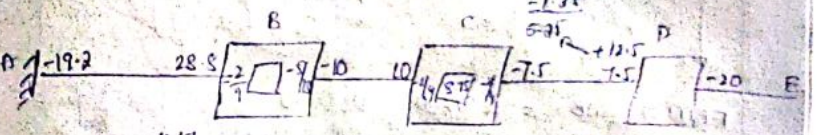
$m'_{bc} = -\frac{5}{18}(18.8 + 0 - 0.94) = -4.96$

$m'_{cb} = -\frac{1}{4}(8.75 - 4.96 + 0) = -0.947$

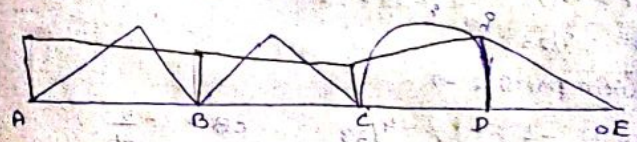
$m'_{bc} = -\frac{1}{4}[8.75 - 4.96 + 0] = -0.947$

Cycle 4 =>

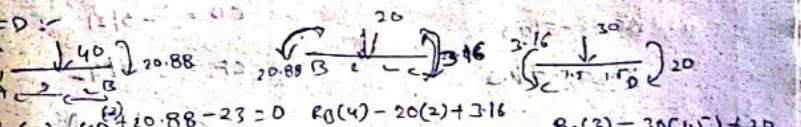
$m'_{bd} = -\frac{2}{9}(18.8 + 0 - 0.947) =$



-4.17	-5.22	-0.88	-0.88	
-3.98	-4.98	-0.94	-0.94	
-3.96	-4.96	-0.94	-0.94	
2(0)	2(-3.96)	2(-4.96)	2(-0.94)	2(-0.94)
-3.96	0	-0.94	-4.96	0
-23.16	+20.88	-20.88	3.16	-3.16	20	-20



SFD :-



$+R_A(5) - 40 + 20.88 - 23 = 0$

$R_A = 16.44$

$R_B = 23.576$

$R_B(4) - 20(2) + 3.16 - 20.88 = 0$

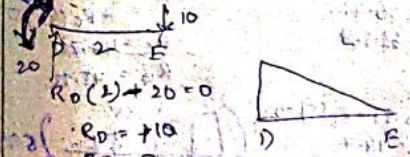
$R_B = 14.43$

$R_C = 5.57$

$R_C(3) - 30(1.5) + 20 - 3.16 = 0$

$R_C = 5.38$

$R_D = 20.6$

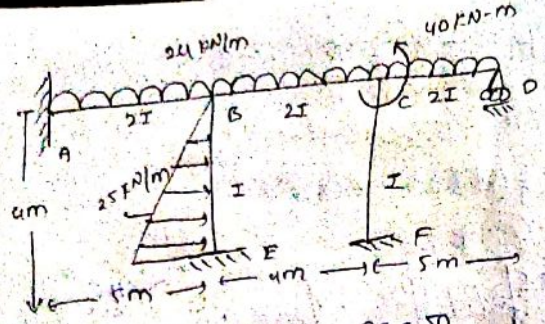


$R_D(2) + 20 = 0$

$R_D = +10$

$R_E = 0$

98.)



FEM ⇒ AB = −50 kNm
BA = +50
BC = −32
CB = 32
CD = −50
DC = 50
BE = 13.33
EB = −20
CF = 0
FC = 0

Distribution factors ⇒

at joint B ⇒

BA ← 4E(2I) — B

5 5

$\theta_{BA} = 8/23$

$\theta_{BC} = 10/23$

$\theta_{BE} = 5/23$

at joint C ⇒

CB = 10/21

CD = 6/21

CF = 5/21

Rotation factors ⇒

$BA = -\frac{1}{2} \times \frac{8}{23} = -4/23$

$BC = -5/23$

$BE = -5/46$

$CB = -\frac{5}{21}$

$CD = -3/21$

$CF = -5/42$

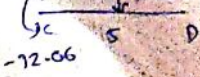
Rxns ⇒

$R_A + R_B = 120$

$-55.87 + 88.26 + 120(2.5) - R_B(5) = 0$

$R_B = 56.47$

$R_A = 63.53$

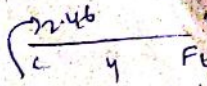


$R_C + R_D = 120$

$72.06 + 120(2.5) - R_D(5) = 0$

$R_D = 74.4$

$R_C = 45.5$

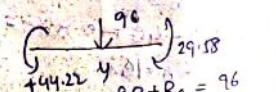


$R_C + R_F = 0$

$2.46 + 1.23 + R_F(4) = 0$

$R_F = -0.92$

$R_C = +0.92$

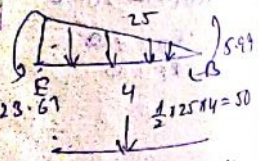


$R_B + R_C = 96$

$-44.22 + 29.88 + 96(2) - R_C(4) = 0$

$R_C = 44.26$

$R_B = 45.73$



Bar ← 1/3

Apex → 1/3

$-23.67 + 5.99 + 50(\frac{1}{3}(4)) - R_B(4) = 0$

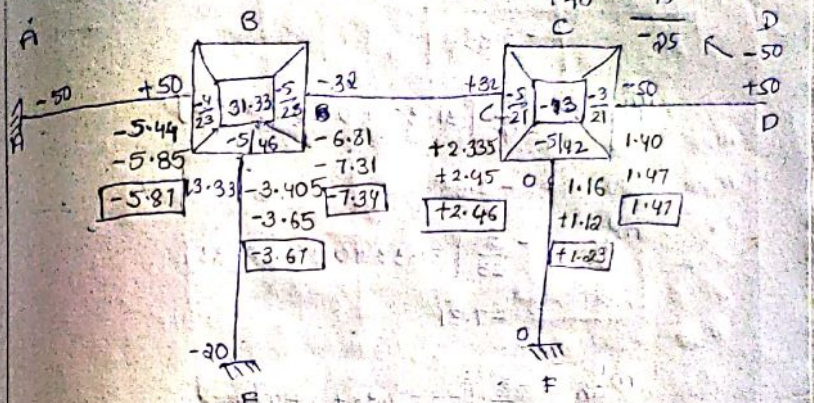
$R_B = 12.9$

$R_E = 31.08$

$M_{CB} + M_{CF} + M_{CD} = -40$

$M_{CB} + M_{CF} + M_{CD} + 40 = 0$

+40 −75 −25



cycle 1 →

$m'_{ba} = -\frac{4}{23}[31.33 + 0 + 0 + 0]$

$= -5.448$

$m'_{bc} = -\frac{5}{23}[31.33 + 0 + 0 + 0]$

$= -6.81$

$m'_{be} = -\frac{5}{46}[31.33 + 0 + 0 + 0]$

$= -3.405$

$m'_{cb} = -\frac{5}{21}[-93 - 6.81 + 0 + 0]$

$= 2.335$

$m'_{cd} = -\frac{3}{21}[-3 - 6.81 + 0 + 0]$

$= 1.40$

$m'_{cf} = -\frac{5}{42}[-3 - 6.81 + 0 + 0]$

$= +1.16$

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cycle 2 =)

$m'_{ba} = -\frac{4}{23}[31.33 + 0 + 0 + 2.33]$

$= -5.85$

$m'_{be} = -\frac{5}{46}[31.33 + 0 + 0 + 2.33]$

$= -3.65$

$m'_{bc} = -\frac{5}{23}[31.33 + 0 + 0 + 2.33]$

$= -7.31$

$m'_{cb} = -\frac{5}{21}[-3 - 7.31 + 0 + 0]$

$= 2.45$

$m'_{cf} = -\frac{5}{42}[-3 - 7.31]$

$= 1.22$

$m'_{cd} = -\frac{3}{21}[-3 - 7.31]$

$= 1.472$

cycle 3 =)

$m'_{ba} = -\frac{4}{23}[31.33 + 2.45 + 0 + 0]$

$= -5.87$

$m'_{bc} = -\frac{5}{23}[31.33 + 2.45]$

$= -7.343$

$m'_{be} = -\frac{5}{46}[31.33 + 2.45]$

$= -3.67$

$m'_{cb} = -\frac{5}{21}[-3 - 7.34 + 0 + 0]$

$= 2.46$

$m'_{cf} = -\frac{5}{42}[-3 - 7.34]$

$= 1.23$

$m'_{cd} = -3/21(-3 - 7.34) = 1.477$

final moments =)

$M_{AB} = \overline{M}_{AB} + 2m'_{ab} + m'_{ba}$

$M_{AB} = -50 + 2(0) + (-5.87)$

$= -55.87$

$M_{BA} = 50 + 2(-5.87) + 0$

$= 38.26$

$M_{BC} = -32 + 2(-7.34) + 2.46$

$= -44.22$

$M_{BE} = +13.33 + 2(-3.67) + 0$

$= 5.99$

$M_{CB} = 32 + 2(2.46) - 7.34$

$= 29.58$

$M_{CD} = -75 + 2(1.47)$

$= -72.06$

$M_{CF} = 0 - \frac{5}{42} + 2(1.23) + 0$

$= 2.46$

$M_{FC} = 0 + 2(0) + 1.23$

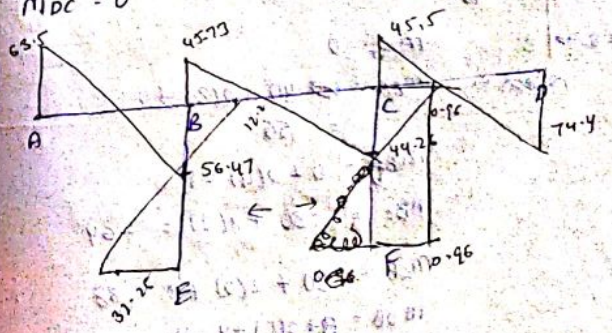
$= 1.23$

$M_{EB} = -20 + 2(0) - 3.67$

$= -23.67$

$M_{DC} = 0$

SFD



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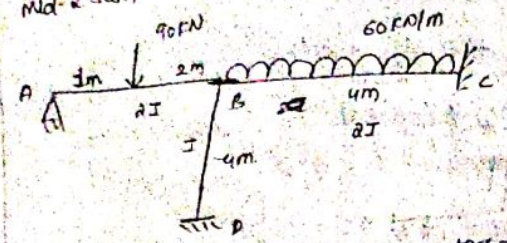
99) Mid-2nd Sum



$m_{AB} = -40$

$m_{BA} = +20$

$m_{BC} = -80$

$m_{CB} = +80$

$m_{BD} = 0$

$m_{DB} = 0$

Distribution factors →

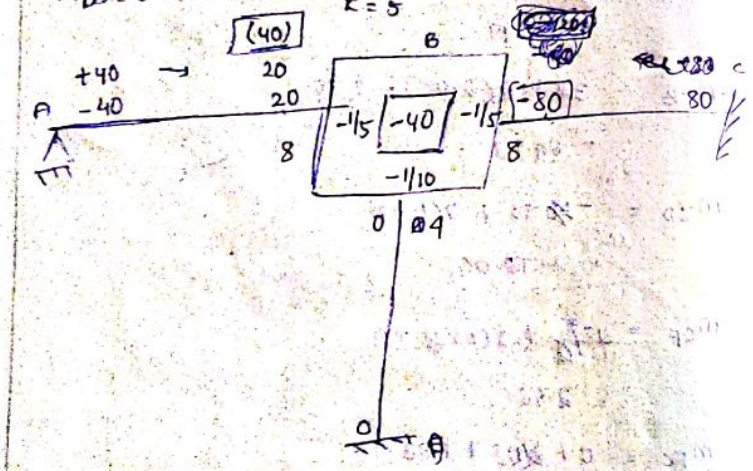
$BA = \frac{4(3EI)}{3} = 2 \Rightarrow 2/5 \Rightarrow -1/5$

$BC = \frac{4(2EI)}{4} = 2 \Rightarrow 2/5 \Rightarrow -1/5$

$BD = \frac{4EI}{4} = 1 \Rightarrow 1/5 \Rightarrow -1/10$

Rotation Factor

$K = 5$



Cycle 1 →

$m'_{ba} = -\frac{1}{5}[-40 + 0 + 0 + 0] = 8$

$m'_{bc} = -\frac{1}{5}[-40] = 8$

$m'_{be} = -\frac{1}{10}(-40) = 4$

Cycle 2 →

$m_{AB} = 0$

$m_{BA} = 40 + 2(8) + 0$

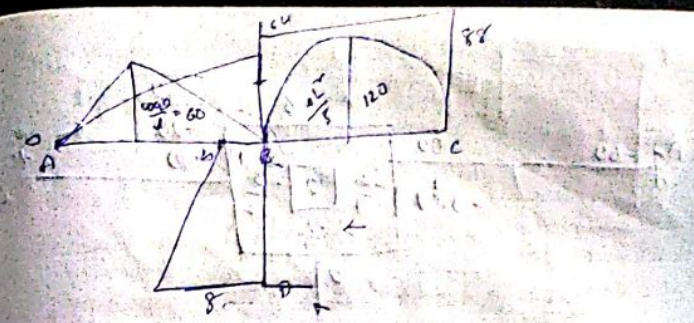
$= 56$

$m_{BD} = 0 + 2(4) = 8$

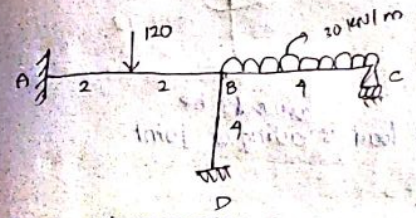
$m_{BC} = -80 + 2(8) = -64$

$m_{CB} = 80 + 2(0) + 8 = 88$

$m_{DB} = 0 + 2(0) + 4 = 4$



100)



72nd Sum

Fixed end moments :-

$m_{AB} = -\frac{PL}{8} = -60$

$m_{BA} = +60$

$m_{BC} = -\frac{wL^2}{12} = -40$

$m_{CB} = 40$

$m_{BD} = 0$

$m_{DB} = 0$

Distribution factor ⇒

$BA = \frac{4EI}{4} = 1 \Rightarrow 4/11$

$BC = \frac{3EI}{4} = \frac{3}{4} \Rightarrow 3/11$

$BD = \frac{4EI}{4} = 1 \Rightarrow 4/11$

$K = 1 + \frac{3}{4} + 1 \Rightarrow \frac{4+3+4}{4}$

$= 11/4$

Rotation factor ⇒

$\Rightarrow -\frac{1}{2} \times \frac{4}{11} = -\frac{2}{11}$

$\Rightarrow -\frac{1}{2} \times \frac{3}{11} = -\frac{3}{22}$

$\Rightarrow -\frac{2}{11}$

Cycle 1 :-

$m'_{ba} = -\frac{2}{11}[0 + 0] = 0$

$m'_{bc} = -\frac{3}{11}[0] = 0$

$m'_{bd} = -\frac{2}{11}(0) = 0$

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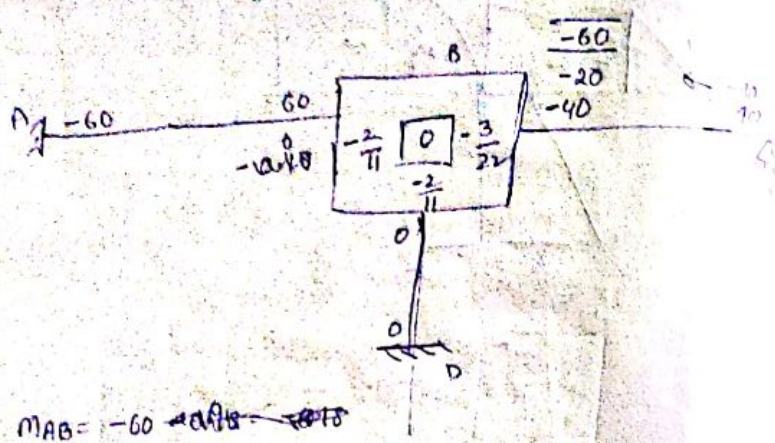


$m_{AB} = -60$

$m_{BA} = 60$ $\qquad m_{BD} = 0$

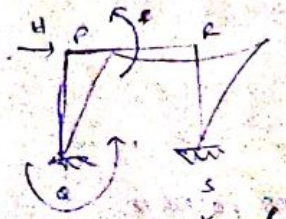
$m_{BC} = -60$ $\qquad m_{DB} = 0$

$m_{CB} = 0$

Sway Analysis :—

mostly horizontal load is assumed to be acting at joint

Wind winds only

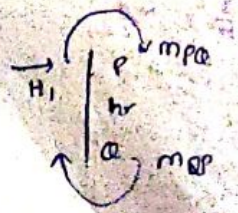


$m_{PQ} = \bar{m}_{PQ} + 2(m'_{PQ}) + m'_{QP} + m''_{PQ}$

$m_{PR} = \bar{m}_{PR} + 2m'_{PR} + m'_{RP} + 0$

$= \Sigma \bar{m}_{PR} + 2\Sigma m'_{PR} + \Sigma m'_{RP} + \Sigma m''_{PR}$

$m'_{PQ} = -\frac{1}{2}\frac{K_{PQ}}{\Sigma K_{PQ}}\left[\Sigma \bar{m}_{PQ} + \Sigma m'_{QP} + m''_{PQ}\right]$ displacement factor.



$H_1 h_r + m_{PQ} + m_{QP} = 0$

$H_1 = -\left[\frac{m_{PQ} + m_{QP}}{h_r}\right]$

$H_2 = -\left[\frac{m_{RS} + m_{SR}}{h_r}\right]$

$\cancel{H = H_1 + H_2}$

$\cancel{H = -\frac{m_{PQ} + m_{QP} - m_{RS} - m_{SR}}{h_r}}$

$H = -\left[\frac{\overset{\text{top}}{\Sigma m_{PQ}} + \overset{\text{bottom}}{\Sigma m_{QP}}}{h_r}\right]$

$m_{PQ} = \bar{m}_{PQ} + 2(m'_{PQ}) + m'_{QP} + m''_{PQ}$

$m_{QP} = \bar{m}_{QP} + 2(m'_{QP}) + m'_{PQ} + m''_{QP}$

$m_{PQ} + m_{QP} = 3m'_{PQ} + 3m'_{QP} + 2m''_{PQ}$

$m_{RS} + m_{SR} = \cancel{3m'_{SR} + 3m'_{RS} + 2m''_{RS}}$

$= 3m'_{RS} + 3m'_{SR} + 2m''_{RS}$

$H h_r = -\left[\Sigma 3m'_{PQ} + 3\Sigma m'_{QP} + 2\Sigma m''_{PQ}\right]$

$2\Sigma m''_{PQ} = -3\Sigma m'_{PQ} - 3\Sigma m'_{QP} - H h_r$

$\Sigma m''_{PQ} = -\left[\frac{H h_r}{2} + \frac{3}{2}\Sigma m'_{PQ} + \frac{3}{2}\Sigma m'_{QP}\right]$

$$\boxed{\Sigma m''_{PQ} = -\frac{3}{2}\left[\frac{H h_r}{3} + \Sigma m'_{PQ} + \Sigma m'_{QP}\right]}$$

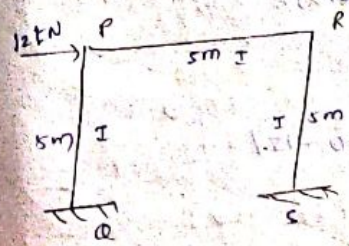
H - shear force in frame (total)

h_r = height of story

$\frac{H h_r}{3}$ - story moment

$m''_{PQ} = -\frac{3}{2} \times \frac{K_{PQ}}{\Sigma K_{PQ}}\left[\frac{h_r H}{2} + \Sigma m'_{PQ} + \Sigma m'_{QP}\right]$

1)
10)



NO FEM

Distribution factors ⇒

Rotation factor ⇒ Displacement

P ⇒ $PQ = \frac{4EI}{5} = \frac{4}{5} \Rightarrow \frac{1}{2}$ $\qquad \Rightarrow -\frac{1}{4}$ $\qquad \frac{1}{2} \times -\frac{1}{2} = -1/4$

$PR = \frac{4EI}{5} = \frac{4}{5} \Rightarrow \frac{1}{2}$ $\qquad \Rightarrow -\frac{1}{4}$

R ⇒ $RP = \frac{4EI}{5} \Rightarrow 1/2$ $\qquad \Rightarrow -1/4$ $\qquad -1/4$

$RS = 4EI/5 \Rightarrow 1/2$ $\qquad \Rightarrow -1/4$

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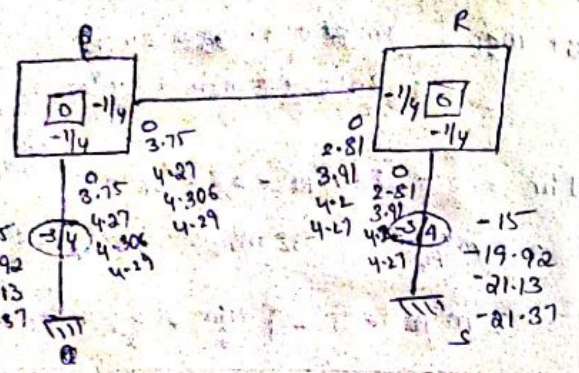
$K_{PQ} = \frac{6EI}{5}$

$K_{QP} = \frac{6EI}{5}$

$$\frac{K_{PQ}}{\Sigma K_{PQ}} = \frac{\frac{6EI}{5}}{\frac{12EI}{5}} = \frac{1}{2}$$



1st cycle →

$m'_{PQ} = -\frac{1}{4}[0+0+0+0]$ ↑ m''_{PQ}

$m''_{PQ} = -\frac{3}{2}\frac{K_{QP}}{\Sigma K_{PQ}}\left[\frac{hrH}{3} + \Sigma m'_{PQ} + \Sigma m'_{QP}\right]$ ↑ top ↑ bottom

$= -\frac{3}{4}\left[\frac{5(12)}{3} + 0+0+0+0\right]$

$= -15$

2nd cycle →

$m'_{PQ} = -\frac{1}{4}[0+0+0+15]$

$= 3.75$

$m'_{PR} = 3.75$

$m'_{QP} = -\frac{1}{4}[0+3.75+0-15]$

$= 2.81$

$m''_{PQ} = -\frac{3}{4}\left[\frac{5(12)}{3} + 3.75 + 2.81\right]$

$= -19.92$

3rd cycle ⇒

$m'_{PQ} = -\frac{1}{4}[0+0+2.81-19.92]$

$= 4.27$

$m'_{PR} = 4.27$

$m'_{RP} = -\frac{1}{4}[0+0+4.27-19.92]$

$= 3.91$

$m'_{RS} = 3.91$

$m''_{PQ} = -\frac{3}{4}[20+4.27+3.91]$

$= -21.13$

$m''_{RS} = -21.13$

4th cycle ⇒

$m'_{PQ} = -\frac{1}{4}[3.91-21.13]$

$= 4.306$

$m'_{PR} = 4.306$

$m'_{RP} = -\frac{1}{4}[4.306-21.13]$

$= 4.251$

$m''_{PQ} = -\frac{3}{4}[20+4.306+4.27]$

$= -21.37$

5th cycle ⇒

$m'_{PQ} = -\frac{1}{4}[4.2-21.37]$

$= 4.29$

$m'_{PR} = 4.29$

$m'_{RP} = -\frac{1}{4}[4.29-21.37]$

$= 4.27$

Final moments ⇒

$m_{QP} = 0+0+4.29-21.37 = -17.08$

$m_{PQ} = 0+2(4.29)+0-21.37 = -12.79$

$m_{RP} = 0+2(4.27)+4.29 = -12.83$

$m_{PR} = 0+2(4.29)+4.27 = -12.72$

$m_{RS} = 0+2(4.27)+0-21.37 = -12.7$

$m_{SR} = 0+0+4.27-21.37 = -17.1$

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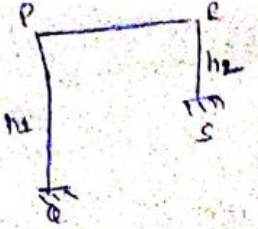


$$\frac{h_1}{h_1} = C_{PQ}$$
$$\frac{h_2}{h_2} = C_{RS}$$
} height factor

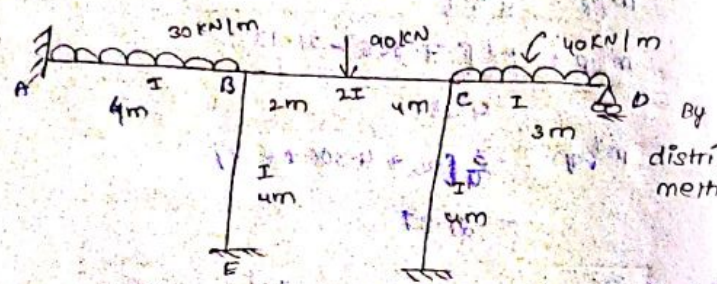
$$\frac{m''_{PQ}}{\Sigma m''_{PQ}} = -\frac{3}{2}\,\frac{K_{PQ}}{\Sigma K_{PQ}}$$ Displacement factor changes into

$$= -\frac{3}{2}\left[\frac{K_{CPQ}}{\Sigma C_{PQ}^2 K}\right]$$

$$= -\frac{3}{2}\left[\frac{K_{PQ}\,C_{PQ}}{C_{PQ}^2 K_{PQ} + C_{RS}^2 K_{RS}}\right]$$

$$M''_{PQ} = \left[-\frac{3}{2}\,\frac{K_{CPQ}}{\Sigma K_{PQ} C_{PQ}^2}\right]\left[\frac{h_{r1}}{3} + \left[\Sigma m'_{PQ} + \Sigma m'_{QP}\right] C_{PQ}\right]$$

2)



By distribution method.

FEM ⇒

$AB = -\frac{wL^2}{12} = -40$ $\qquad$ $CD = -\frac{wL^2}{12} = -30$

$BA = +40$ $\qquad$ $DC = +30$

$BC = -\frac{Pab^2}{l^2} = -80$ $\qquad$ $CF = 0$

$CB = +\frac{Pa^2b}{l^2} = +40$ $\qquad$ $FC = 0$

$BE = EB = 0$

Distribution factors ⇒

B ⇒ $BA = \frac{4EI}{4} = 1 \Rightarrow 3/10 \Rightarrow 0.3$

$BE = \frac{4EI}{4} = 1 \Rightarrow 3/10 \Rightarrow 0.3$

$BC = \frac{4E(2I)}{6} = \frac{8}{6} \Rightarrow 4/10 \Rightarrow 0.4$

$K = 2 + \frac{8}{6} = \frac{12+8}{6} = \frac{20}{6} = \frac{10}{3} = 3.33$

C ⇒ $CB = \frac{4E(2I)}{6} = \frac{8}{6} \Rightarrow 0.36 \Rightarrow \frac{8}{20}$

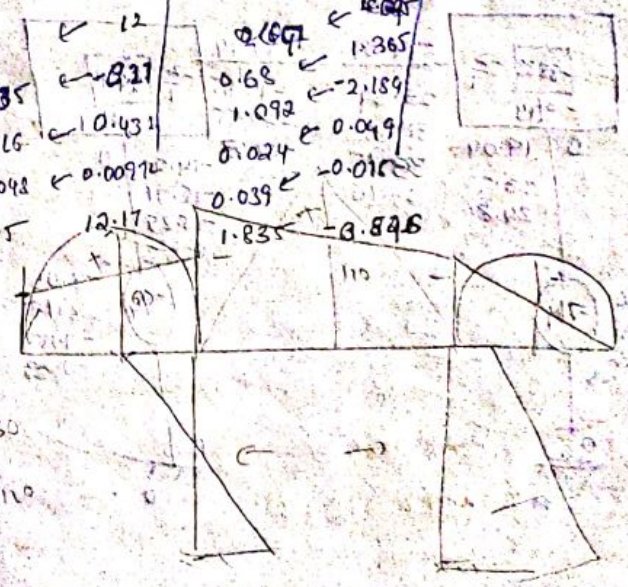
$CF = \frac{4E(I)}{4} = 1 \Rightarrow 0.213 \Rightarrow \frac{6}{20}$

$CD = \frac{3EI}{3} = 1 \Rightarrow 0.213 \Rightarrow \frac{6}{20}$

$K = 3.66$, $\frac{8}{6} \Rightarrow \frac{12+8}{6} = \frac{20}{6}$

A	B		C		D
	0.3	0.4	0.36	0.36	
-40	40	-80	40	-30	30
				-15	-30
	12	16		1.8	
6		0.9	8		
-0.135	-0.27	-0.36	-2.88	-2.88	
		-1.44	-0.18		
0.216	0.432	0.576	0.0648	0.0648	
		0.0324	0.288		
0.0486	0.00972	-0.0129	-0.10	-0.10	
	52.17	-64.30	46.99	-46.115	0

E		B	F		C
-33.91	0.3			0.20	
0	0	0	0	0	
6	12	0.667	1.335		
-0.135	-0.27	0.68	-2.159		
0.216	0.431	1.092	0.049		
0.0048	0.00972	0.024	-0.015		
		0.039			
6.085	12.17	1.835	-3.846		



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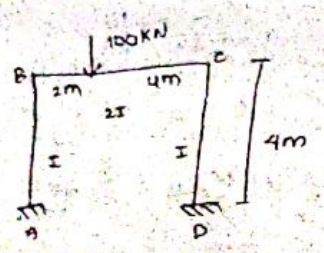
3)



FEM :-

$BC = -\frac{Pab^2}{l^2} = -88.88$

$CB = -\frac{Pa^2b}{l^2} = +44.44$

Distribution factor | Rotation factors | Displacement factors

at B ⇒

$BA = \frac{4EI}{4} = 1 \Rightarrow 3/7 \Rightarrow -\frac{1}{2}\times\frac{3}{7} = -\frac{3}{14} \Rightarrow -\frac{3}{2}\times\frac{3}{7} = -\frac{9}{14}$

$BC = \frac{4E(2I)}{6} = \frac{8}{6} \Rightarrow \frac{4}{7} \Rightarrow -\frac{1}{2}\times\frac{4}{7} = -\frac{2}{7} \Rightarrow -\frac{3}{2}\times\frac{4}{7} = -\frac{6}{7}$

$1+\frac{8}{6} = \frac{14}{6} = \frac{7}{3}$

at C ⇒

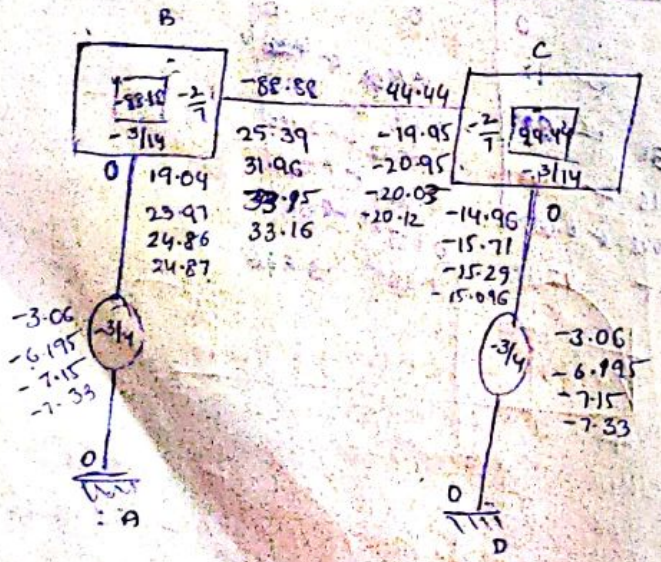
$CB = 4E(2I)/6 = 8/6 = \frac{4}{7} \Rightarrow -\frac{1}{2}\times\frac{4}{7} = -\frac{2}{7} \Rightarrow -\frac{3}{2}\times\frac{4}{7} = -\frac{6}{7}$

$CD = 1 = 3/7 \Rightarrow -\frac{3}{14} \Rightarrow -\frac{9}{14}$

Displacement factors ⇒

$BA = \frac{6EI}{4} = \frac{6EI}{4} = \frac{6}{4} = \frac{3}{2} \Rightarrow 1/2 \times -\frac{3}{2} = -\frac{3}{4}$

$CD = \frac{6EI}{4} = \frac{6}{4} = \frac{3}{2} \Rightarrow 1/2 \times -\frac{3}{2} = -\frac{3}{4}$



cycle 1 →

$m'_{ba} = \frac{-3}{14}(-88.88 + 0 + 0 + 0)$

$= 19.04$

$m'_{bc} = -\frac{2}{7}[-88.88]$

$= 25.39$

$m'_{cb} = -\frac{2}{7}[44.44 + 25.39 + 0 + 0]$

$= -19.95$

$m'_{cd} = -\frac{3}{14}[44.44 + 0 + 25.39 + 0]$

$= -14.96$

$m''_{ba} = -\frac{3}{4}[\frac{4(0)}{3} + 19.04 - 19.95]$

$= -3.06$

$m''_{cd} = -3.06$

cycle 2 ⇒

$m'_{ba} = -\frac{3}{14}[-88.88 + 0 - 19.95 - 3.06]$

$= 23.97$

$m'_{bc} = -\frac{2}{7}[-88.88 + 0 - 19.95 - 3.06]$

$= 31.96$

$m'_{cb} = -\frac{2}{7}[44.44 + 0 + 31.96 - 3.06]$

$= -20.95$

$m'_{cd} = -\frac{3}{14}(73.34) = -15.71$

$m''_{ba} = -\frac{3}{4}[0 + 23.97 - 15.71]$

$= -6.195$

$m''_{cd} = -6.195$

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cycle 3 ⇒

$m'_{ba} = -\frac{3}{14}[-88.88 + 0 - 20.95 - 6.195]$

$= 24.86$

$m'_{bc} = -\frac{2}{7}[-88.88 + 0 - 20.95 - 6.195]$

$= 33.15$

$m'_{cb} = -\frac{2}{7}[44.44 + 0 + 33.15 - 6.195]$

$= -20.03$

$m'_{cd} = -\frac{3}{14}[44.44 + 0 + 33.15 - 6.195]$

$= -15.29$

$m''_{ba} = -\frac{3}{4}[0 + 24.8 - 15.29]$

$= -7.15$

cycle 4 ⇒

$m'_{ba} = -\frac{3}{14}(-88.88 - 20.03 - 7.15)$

$= 24.87$

$m'_{bc} = -\frac{2}{7}(-88.88 - 20.03 - 7.15) = 33.16$

$m'_{cb} = -\frac{2}{7}(44.44 + 33.16 - 7.15] = -20.12$

$m'_{cd} = -\frac{3}{14}(44.44 + 33.16 - 7.15) = -15.096$

$m''_{ba} = -\frac{3}{4}[0 + 24.87 - 15.096] = -7.33$

$M_{AB} = [0 + 2(0) + 24.87 - 7.33] = 17.54$

$m_{BA} = [0 + 2(24.87) + 0 - 7.33] = 42.41$

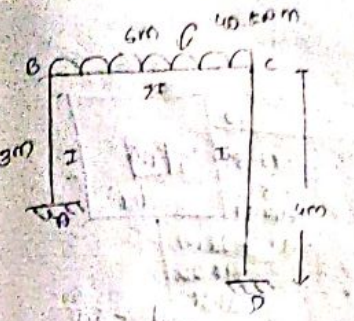
$m_{BC} = [-88.88 + 2(33.16) + 0] = -42.68$

$m_{CB} = (44.44 + 2(-20.12) + 33.16) = -37.36$

$m_{CD} = (0 + 2(-15.096) + 0 - 7.33) = -37.522$

$m_{DC} = (0 + 2(0) - 15.096 - 7.33) = -22.426$

A)



$C_{PQ} = \frac{h_r}{height}$

$h_r = 3m$ (Assume)

$C_{BA} = \frac{3}{3} = 1$

$C_{CD} = \frac{3}{4} = 0.75$

Displacement factors ⇒ $-\frac{3}{2}\left[\frac{K C_{PQ}}{\Sigma C_{PQ}^2 K}\right]$

$m = K\theta$

$m = \frac{6EI(\delta)}{2h^2}$

$K = \frac{6EI}{\delta}$

$K_{BA} = \frac{6EI}{3} = 2$

$K_{CD} = \frac{6EI}{4} = \frac{3}{2}$

$DF_{BA} = -\frac{3}{2}\left[\frac{2 \times 1}{1^2(2) + (0.75^2 \times \frac{3}{2})}\right]$

$= -1.055$

$DF_{CD} = -\frac{3}{2}\left[\frac{\frac{3}{2} \times 0.75}{1^2(2) + (0.75^2 \times \frac{3}{2})}\right]$

$= -0.594$

Rotation factors

Displace

Rotation factors ⇒

$BA = \frac{4EI}{3} = \frac{4}{3} \Rightarrow \frac{2}{3}$

$BC = \frac{4EI}{6} = \frac{8}{6} \Rightarrow \frac{2}{3}$

$\frac{2}{3} + \frac{2}{6} \Rightarrow \frac{8+8}{6} = \frac{16}{6} = 2$

at B ⇒ $BA = \frac{4EI}{3} = \frac{4}{3} = \frac{1}{2} \times -\frac{1}{3} = -1/4$

$BC = \frac{4E(2I)}{6} = \frac{4}{3} = \frac{1}{2} \times -\frac{1}{2} = -1/4$

at C ⇒ $CB = \frac{4E(2I)}{6} = \frac{4}{3} \Rightarrow \frac{4}{7} \times -\frac{1}{2} = -\frac{2}{7}$

$CD = \frac{4E(I)}{4} = 1 \Rightarrow 3/7 \times -\frac{1}{2} = \frac{-3}{14}$

$1 + \frac{4}{3} = \frac{3+4}{3} = \frac{7}{3}$

FEM

$BC = -120$

$CB = 120$

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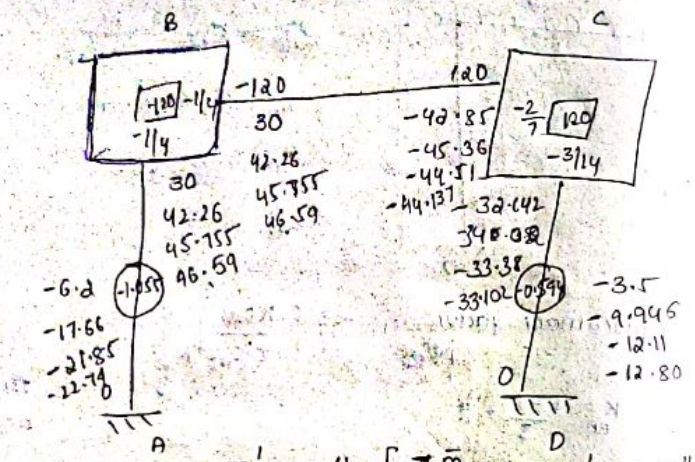

$$m'_{ab} = U_F\left[\bar{m}_{AB} + \Sigma m'_{ba} + m''_{ab}\right]$$

$$m''_{ab} = \theta_F\left[\frac{M_{AB}}{3} + \Sigma\left(m'_{ab} + m'_{ba}\right)c_{ab}\right]$$

cycle 1 →

$m'_{ba} =$

$m'_{ba} = -\frac{1}{4}\left[-120 + 0 + 0 + 0\right] = 30$

$m'_{bc} = -\frac{1}{4}(-120) = 30$

$m'_{cb} = -\frac{2}{7}\left(120 + 30 + 0\right) = -42.85$

$m'_{cd} = -\frac{3}{14}(120 + 30) = -32.142$

$m''_{ba} = -1.055\left[\frac{(0)(3)}{3} + \left[(30\times1) + (0\times1) + (-32.142\times0.75) + (0\times0.75)\right]\right]$

$= -6.2$

$m''_{cd} = -0.594\left[0 + (30\times1) + (-32.142\times0.75)\right]$

$= -3.5$

cycle 2 →

$m'_{ba} = -\frac{1}{4}(120 + 0 - 42.85 - 6.2)$

$= 42.26$

$m'_{bc} = 42.26$

$m'_{cb} = -\frac{2}{7}(120 + 0 + 42.26 - 3.5)$

$= -45.36$

$m'_{cd} = -\frac{3}{14}(120 + 0 + 42.26 - 3.5)$

$= -34.02$

$m''_{ba} = -1.055\left[0 + 42.26(1) + (-34.02\times0.75)\right]$

$= -17.66$

$m''_{cd} = -0.594\left[42.26(1) + (-34.02\times0.75)\right]$

$= -9.946$

cycle 3 ⇒

$m'_{ba} = -\frac{1}{4}\left[-120 + 0 - 45.36 - 17.66\right]$

$= 45.755$

$m'_{bc} = 45.755$

$m'_{cb} = -\frac{2}{7}\left(120 + 45.755 + 0 - 9.946\right)$

$= -44.851$

$m'_{cd} = -\frac{3}{14}\left(120 + 45.755 - 9.946\right)$

$= -33.38$

$m''_{ba} = -1.055\left[45.755 - 33.38(0.75)\right]$

$= -21.85$

$m''_{cd} = -0.594\left[0 + 45.755(1) - 33.38(0.75)\right]$

$= -12.11$

cycle 4 ⇒

$m'_{ba} = -\frac{1}{4}\left[-120 - 44.51 - 21.85\right]$

$= 46.59$

$m'_{bc} = 46.59$

$m'_{cb} = -\frac{2}{7}\left[120 + 46.59 - 12.11\right]$

$= -44.137$

$m'_{cd} = -\frac{3}{14}\left(120 + 46.59 - 12.11\right]$

$= -33.102$

$m''_{ba} = -1.055\left[0 + 46.59(1) - 33.102(0.75)\right]$

$= -22.74$

$m'_{bc} = -0.594\left[0 + 46.59 - 33.102(0.75)\right]$

$= -12.80$

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$M_{AB} = 0 + 2(0) + 46.59 - 22.74 = 23.85$

$M_{BA} = 0 + 2(46.59) + 0 - 22.74 = 70.44$

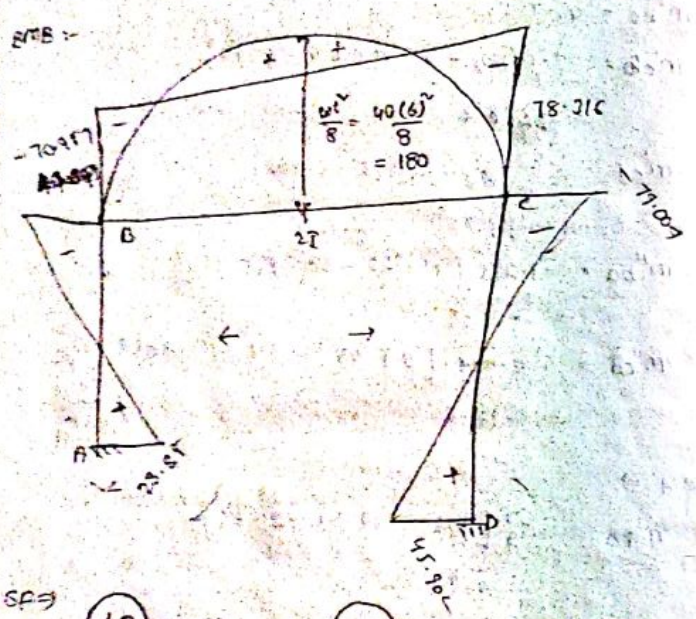
$M_{BC} = -120 + 2(46.59) - 44.137 + 0 = 49.094 - 70.$

$M_{CD} = 120 + 2(-44.137) + 46.59 = 78.316$

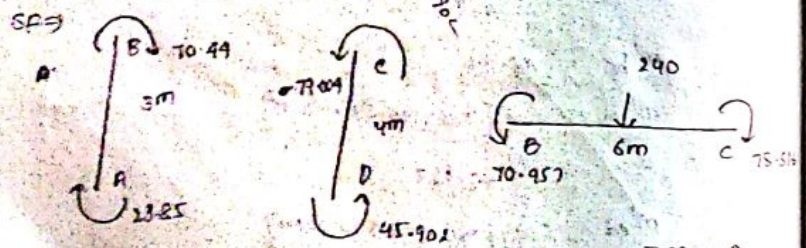
$M_{CB} = 0 + 2(-33.102) + 0 - 12.80 = -79.004$

$M_{DC} = 0 + 0 + (-33.102) - 12.80 = -45.902$

BMD :-



SFD



$\Sigma M_A = 0$

$-R_B(3) + 70.44 + 23.85 = 0$

$R_B = 31.43$

$R_A = -31.43$

$\Sigma M_D = 0$

$-R_C(4) - 45.902 - 79.004 = 0$

$R_C = -31.22$

$R_D = 31.22$

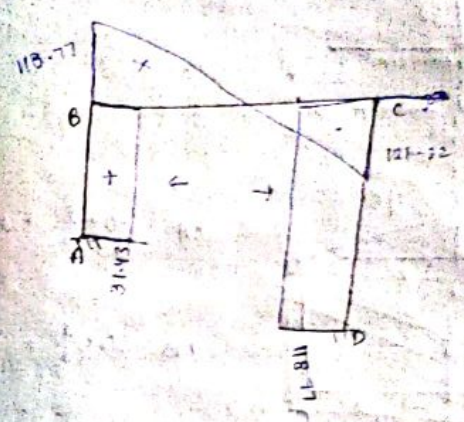
$\Sigma M_B = 0$

$240(3) + 78.316 - R_C$

$- 70.957 = 0$

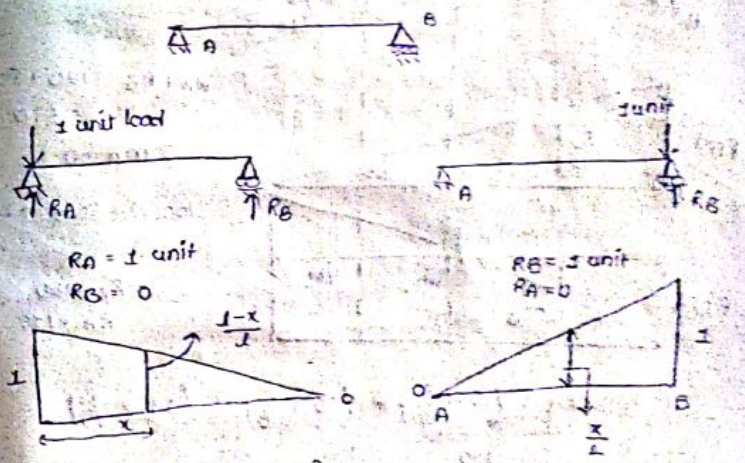
$R_C = 121.22$

$R_B = 113.77$



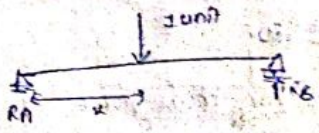
Rolling loads and Influence lines

1) Influence lines for support reactions -



Influence line for R_A

Influence line for R_B



$L(R_A) - 1(L - x) = 0,$

$R_A = \frac{L-x}{L}$

$\Sigma M_A = 0$

$LR_B - (1 \times x) = 0$

$R_B = \frac{x}{L}$

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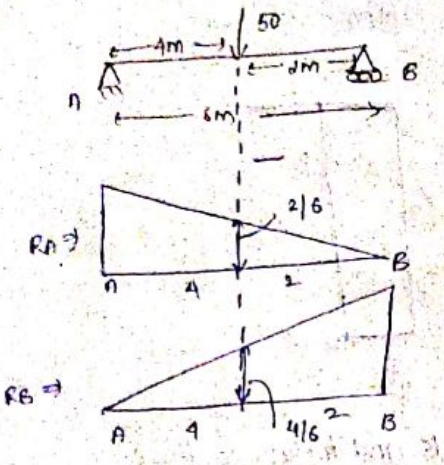
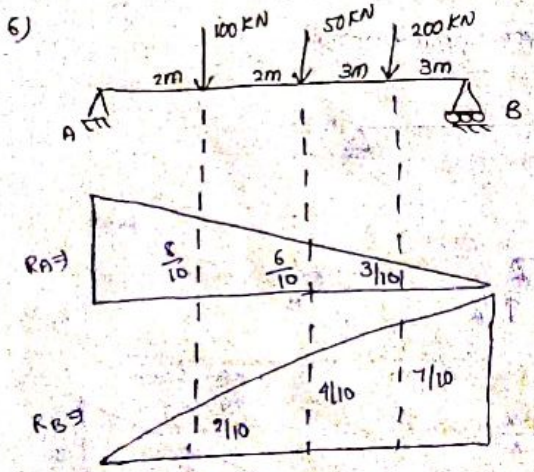
5)



$R_A = 50 \times \frac{2}{6} = 16.67$

$R_B = 50 \times \frac{4}{6} = 33.33$

6)



$R_A + R_B = 100 + 50 + 200$
$= 350$

$\Sigma M_A = 0$

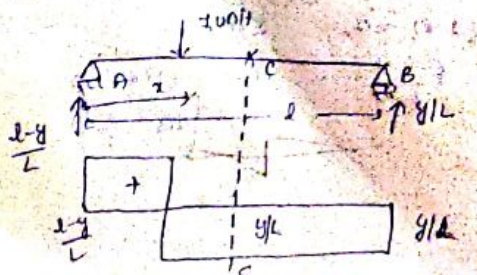
$100(2) + 50(4) + 200(7) = 10(R_B)$

$R_B = 180$
$R_A = 170$

$R_A = \left(100 \times \frac{8}{10}\right) + \left(50 \times \frac{6}{10}\right) + \left(200 \times \frac{3}{10}\right)$
$= 170$

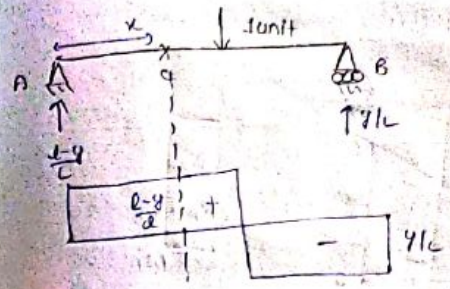
$R_B = \left(100 \times \frac{2}{10}\right) + \left(50 \times \frac{4}{10}\right) + \left(200 \times \frac{7}{10}\right)$
$= 180$

7) Influence lines for shear force at particular section



From this we observe that the load travelling from A to C the shear force is approaching to $R_B = y/L$

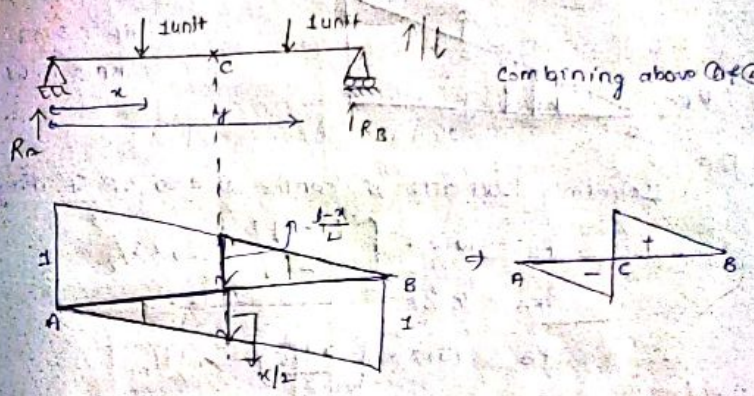
8)



From load travelling from B to A shear force is approaching to R_A

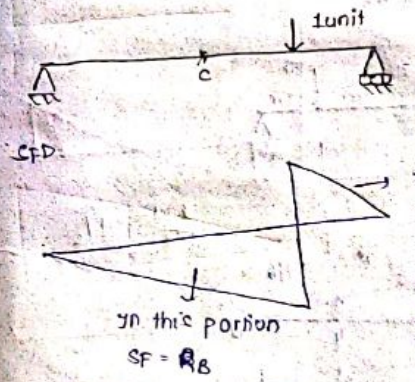
$R_A = \frac{L-y}{L}$ SF at C

9)

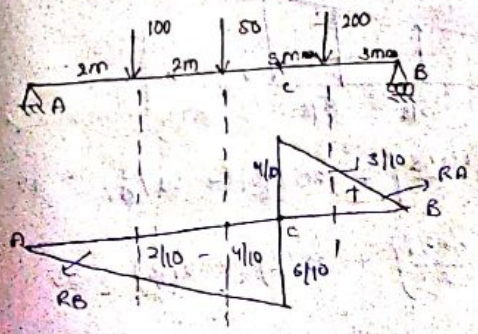


Combining above ② & ①

10)



11)



SF at C ⇒

$S_C = -100\left(\frac{2}{10}\right) - 50\left(\frac{4}{10}\right) + 200\left(\frac{3}{10}\right)$

$= 20$

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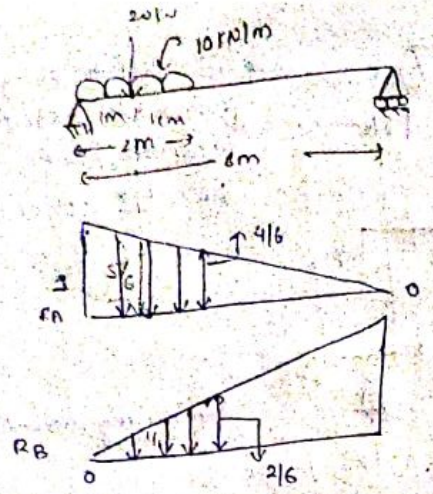
12)



$R_A + R_B = 20$

$\Sigma M_A = 0$

$6R_B = 10 \times 2 \times 1$

$R_B = 3.33$

$R_A = 16.67$

Uniformly load acts at centre at area of SF diagram

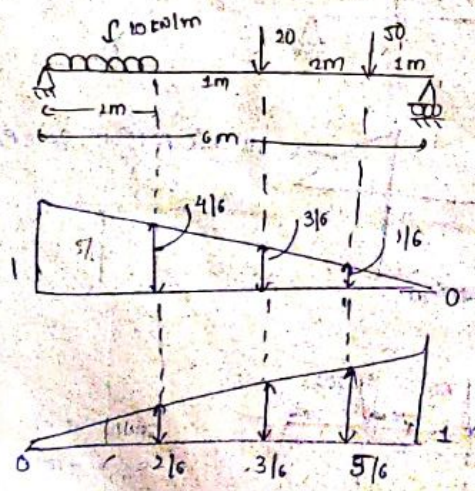
$R_A = 10 \times 2 \times \left[\frac{(1+\frac{4}{6})}{2}\right] \Rightarrow 20 \times 5/6$

$R_A = 16.67$ $\rightarrow 16.67$

$R_B = 10 \times 2 \times \left[\frac{0+\frac{2}{6}}{2}\right] \Rightarrow 20 \times 1/6$

$= 3.33$ $= 3.33$

13)

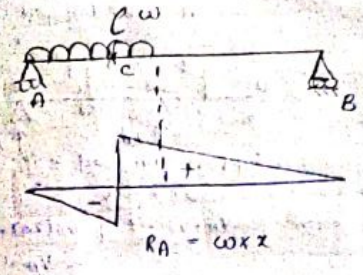


$\frac{l-x}{l}$, x/l

$R_A = 10(2)\left[\frac{1+4/6}{2}\right] + 20\left(\frac{3}{6}\right) + 50\left(\frac{1}{6}\right) = 35$

$R_B = 10(2)\left[\frac{0+2/6}{2}\right] + 20\left(\frac{3}{6}\right) + 50\left(\frac{5}{6}\right) = 55$

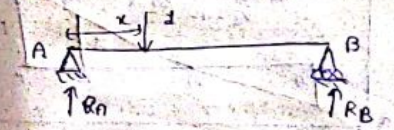
14)



These are all for determinent structures and it is varying linearly.

Definition:- ILD can be defined as a graphical presentation of the function (Rxn at supports, SF, BM & axial force at a particular section) for a unit load moving over the span.

ILD for support rxns:-



$0 \le x \le l$

$\Sigma F_x = 0 \Rightarrow H_A = 0$

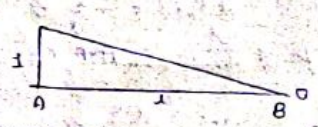
$\Sigma F_y = 0 \Rightarrow R_A + R_B = 1$

$\Sigma M_A = 0 \Rightarrow -R_B(l) + 1(x) = 0$

$\Rightarrow R_B = \frac{x}{l}$

$R_A = 1 - \frac{x}{l} = \frac{l-x}{l}$

ILD for $R_A \rightarrow$



$R_A = \frac{l-x}{l}$

at $x = 0$, $R_A = 1$

at $x = l$, $R_A = 0$

ILD for $R_B \rightarrow$

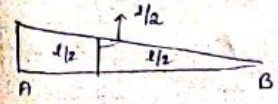
$R_B = x/l$

at $x = 0$, $R_B = 0$

at $x = l$, $R_B = 1$

Suppose if $x = l/2$

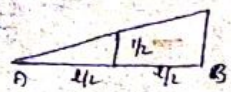
$R_A \Rightarrow$



$V_A = 1/2$

if load is at $x = l/2$

$R_B \Rightarrow$



$V_B = 1/2$

if load is at $x = l/2$

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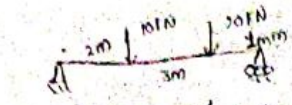
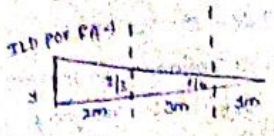


ILD for R_A →

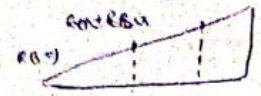


$R_A = 10\left[\frac{2}{3}\right] + 20\left(\frac{1}{6}\right)$

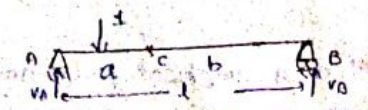
$= \frac{20}{3} + \frac{20}{6}$

$= \frac{60}{6} = 10$ kN

Cont R_B →

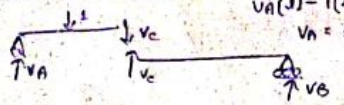


ILD for shear force at a section ⇒



ILD for shear force at c →

① Load is in section AC



$\Sigma M_B = 0$

$V_A(l) - 1(l-x) = 0$

$V_A = \frac{l-x}{l}$

$\Sigma F_y = 0$

$V_A - 1 - V_c = 0$

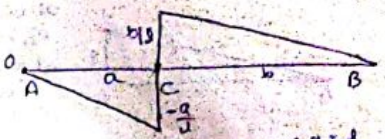
$V_c = V_A - 1$

$= \frac{l-x}{l} - 1 = -\frac{x}{l}$ ✓ $0 < x < a$

② Load is in section CB

$V_c = V_A$

$V_c = \frac{l-x}{l}$ ✓ $a \le x \le a+b$



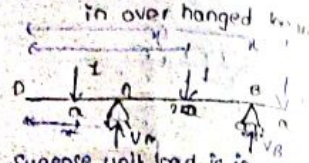
at $x = l$

$V_c = 0$

at $x = a$

$V_c = \frac{l-a}{l} = \frac{b}{l}$

ILD for support reac in over hanged beam



Suppose unit load is in section AB

$0 \le x \le a$

$V_A + V_B = 1$ | $V_A(2a) - 1(3a - x) = 0$

$V_A = \frac{3a - x}{2a}$

Suppose unit load is in section CB

$0 \le x \le 3a$ $V_A = \frac{3a - x}{2a}$

Suppose unit load is in section BC

$3a \le x \le 4a$ $V_A = \frac{3a - x}{2a}$

at D → $x = 0$

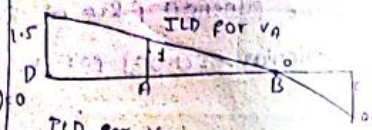
$V_A = \frac{3a}{2a} = 1.5$

at A → $x = a$, $V_A = \frac{2a}{2a} = 1$

at B → $x = 3a$, $V_A = 0$

at C → $x = 4a$, $\frac{-a}{2a} = -1/2$

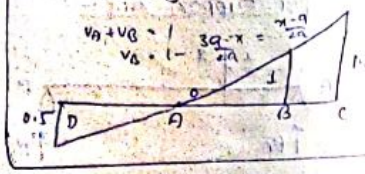
ILD for V_A

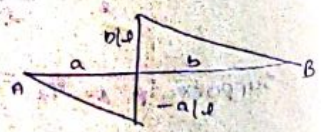


ILD for V_B

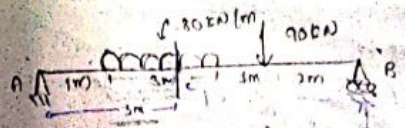
$V_A + V_B = 1$

$V_B = 1 - \frac{3a - x}{2a} = \frac{x - a}{2a}$

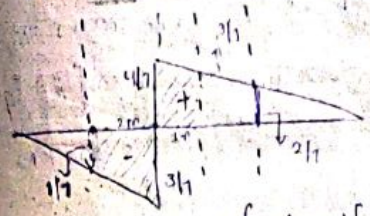




15)



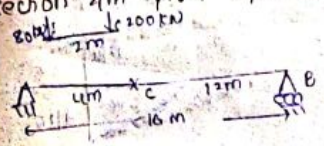
Find shear force at section at distance of 3m



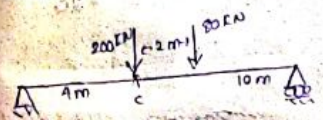
SF at C $= -80\left\{-(80\times2)\left[\frac{\frac{1}{7}+\frac{3}{7}}{2}\right] + (80\times1)\left[\frac{\frac{4}{7}+\frac{3}{7}}{2}\right] + 90\left[\frac{2}{7}\right]\right.$

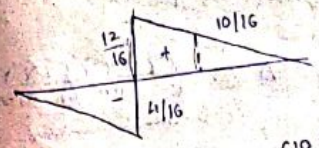
$= 20$ kN

16) Two wheel loads 80 kN & 200 kN placed 2m apart move on girder on span 16m. Find out max of +ve and -ve shear force at a section 4m from left end.



maximum +ve shear force occurs if 200 kN acts at 'C'

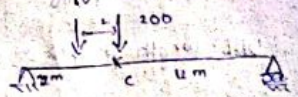


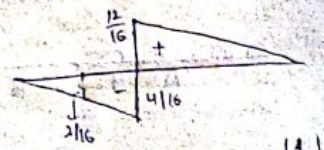


$+SF = +200\left[\frac{12}{16}\right] + 80\left[\frac{10}{16}\right]$

$= 200$ kN

maximum -ve SF

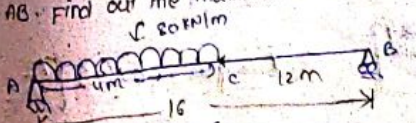


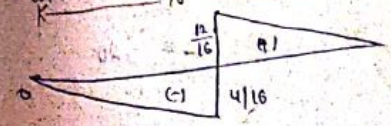


$-SF = -\left[80\left(\frac{2}{16}\right) + 200\left(\frac{4}{16}\right)\right]$

$= -60$ kN

17) A UDL of 80 kN/m of length 4m moving from one end other end of span AB. Find out the maximum shear. ~~and max positive & negative shear.~~





-ve shear ⇒

$-80(4)\left[\frac{0 + \frac{2}{16}}{2}\right]$

$= -20$ kN

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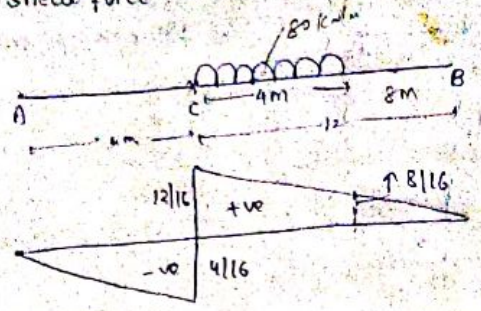
+ shear force maximum ⇒



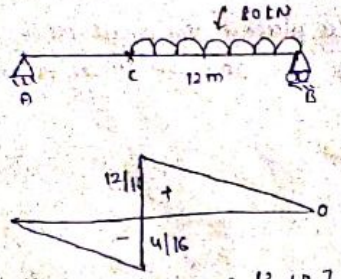
+ve shear F ⇒

$80 \times 4 \left[\frac{\frac{12}{16} + \frac{8}{16}}{2}\right]$

$= 200$ kN

UDL of length 12m ⇒ UDL shorter than span

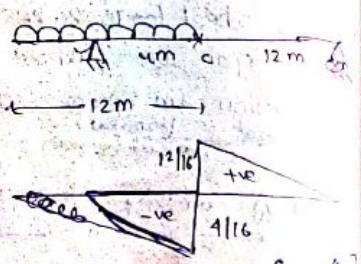
+ve shear max (Bus)



+ shear $= 80(12)\left[\frac{\frac{12}{16}+0}{2}\right]$

$= +360$

-ve shear max (train)

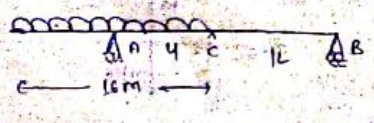


-ve shear $= (-80 \times 4)\left[\frac{0+\frac{4}{16}}{2}\right]$

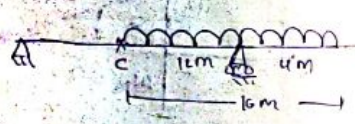
$= -20$

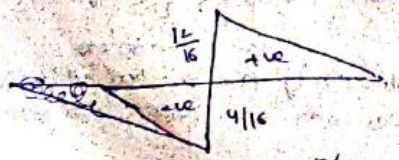
UDL length of 16m ⇒

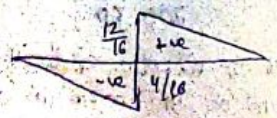
Max ~~+ve~~



Max +ve







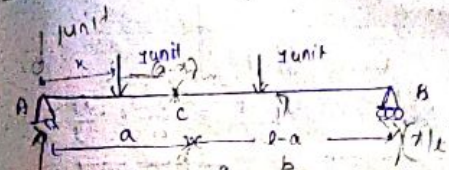
~~+ve~~ CF $= 80 \times (4)\left[\frac{0+\frac{4}{16}}{2}\right]$

$=$

+ shear $= 80(12)\left[\frac{\frac{12}{16}+0}{2}\right]$

$= -20$

Influence line diagram of BM for simply supported beam



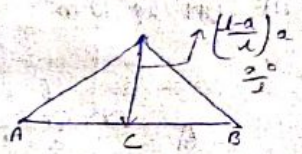
$\left[\frac{l-1}{l}\right]$ load at support A ⇒ $M_c = 0$ $\left((1 \times a) - (1 \times a) = 0\right)$

load at x distance from A $\left[\frac{l-x}{l}\right]a - 1(a-x)$

$M_c = \frac{x}{l} \times (l-a)$

$= \frac{(l-a)}{l} x$

at c, $x = a \Rightarrow M_c = \frac{(l-a)a}{l}$

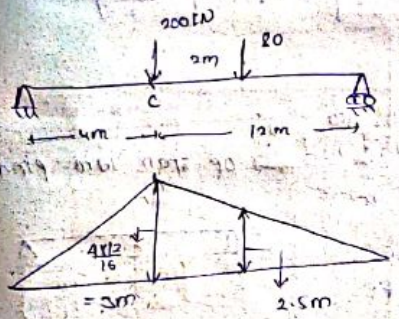


+ load at D,

$M_c = \left(\frac{l-x}{l}\right) \times a$

at $x = L$ $M_c = 0$

18) Two wheel loads 80 kN & 200 kN placed 3m apart move on girder of span 16m. Find out BM.

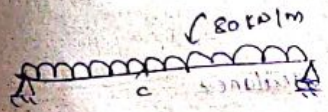


80 is placed in right side because the slope of BM in right is less than left so we get max BM

$12 \rightarrow 3$
$10 \rightarrow ? \Rightarrow \frac{10 \times 3}{12} = 2.5$

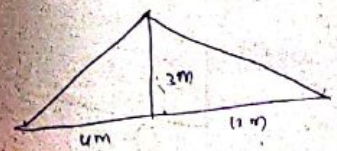
max BM $= 200(3) + 80(2.5) = 800$ kN-m

19) UDL of load 80 kN/m of length 16m moving from one end to another. Find out max BM at section 'c'



max BM $= (80 \times 4)\left[\frac{0+3}{2}\right] + (80 \times 12)\left[\frac{3}{2}\right]$

$= 1920$ kN-m



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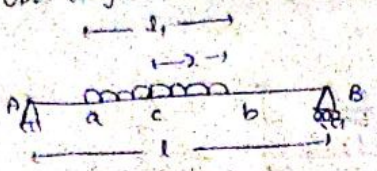
20) UDL length 18 m



$\omega = 1$

$\frac{l_1 - z}{a} = \frac{z}{b} = \frac{l_1}{l}$

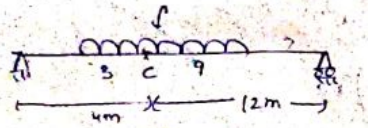
$\boxed{z = \frac{b l_1}{l}}$

Average load AC $= \frac{\omega(l_1 - z)}{a}$

Avg load BC $= \frac{\omega z}{b}$

$\frac{z}{b} = \frac{l_1 - z + z}{a + b} \Rightarrow \frac{l_1}{l}$

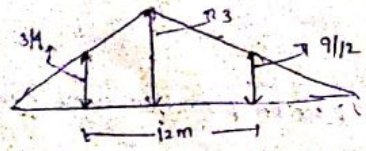
21) UDL of 18 m



max Bending moment

at $z = \frac{18 \times 12}{16}$

$= 9m$

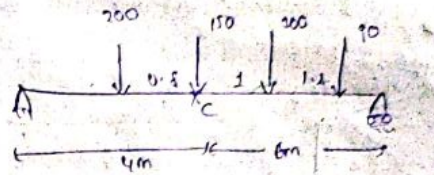


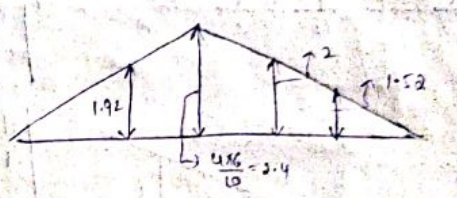
Max BM $= (80 \times 3)\left[\frac{\frac{3}{4} + 3}{2}\right] + (80 \times 9)\left[\frac{3 + \frac{9}{12}}{2}\right]$

$=$

22) Set of loads 200 150 300 90 d 0.8d 1.0d 1.2d of span 18m find max BM at C at 4m from A

Avg load AC	BC
$\frac{740}{4}$	0
$\frac{650}{4}$	$\frac{90}{6}$
$\frac{350}{4}$	$\frac{390}{6}$
$\frac{50}{4}$	$\frac{540}{6}$ ⇒ unbalance

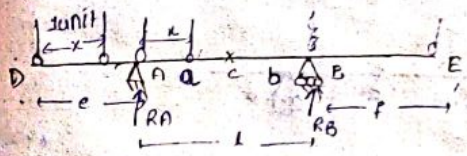




max $BM_C = 200(1.92) + 150(2.4) + 300(2) + 90(1.52)$

$= 1480$ KN-m

over-hanging beam-



$\Sigma M_B = 0$

at $x = 0$; $-1(l + e) + R_A(l) = 0$

$R_A = \frac{l + e}{l} = 1 + \frac{e}{l}$

$-1(l + e - x) + R_A(l) = 0$

$R_A = \frac{l + e - x}{l}$

at $x = e$; $-1(l) + R_A(l) = 0$

$R_A = \frac{l}{l} = 1$

$R_A(l) - 1(l - x) = 0$

$R_A = \frac{l - x}{l}$

$\Sigma M_B = 0$

$R_A(l) - 1(f) = 0$

$R_A = -\frac{f}{l}$

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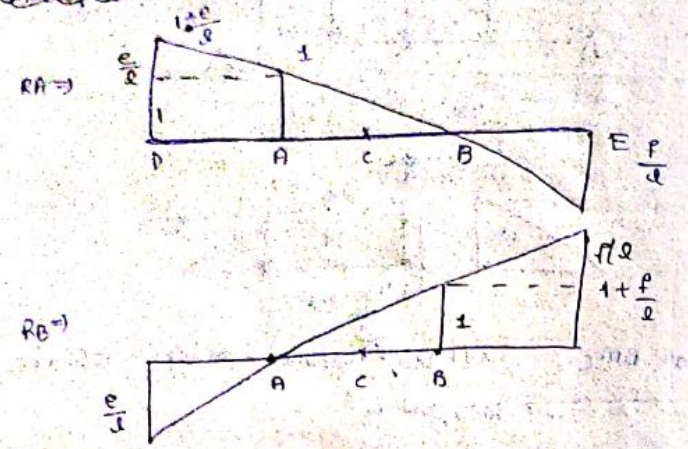
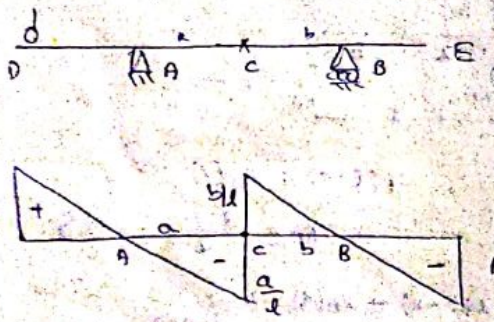
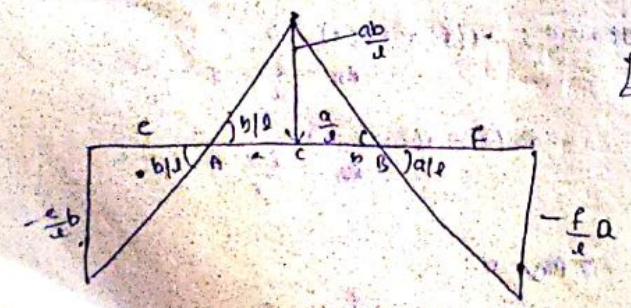
~~Shear force~~



Shear force at C :-



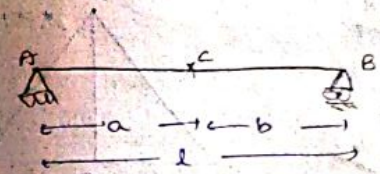
BM at C :-



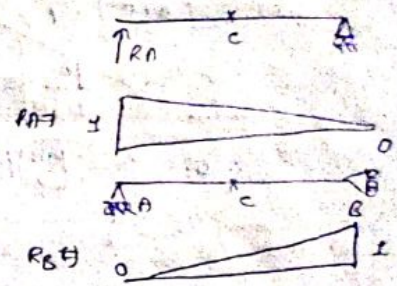
$\boxed{S = r\theta}$

Muller Breslou's method :-

The influence line for any force response function in a structure given by the deflected shape of the structure resulting from a unit displacement corresponding to force.

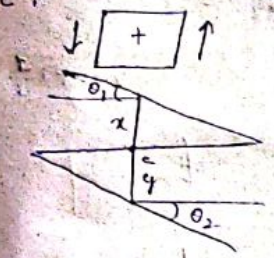


apply unit displacement in the direction of force



shear force ⇒

At C :-

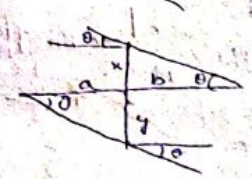


Give unit displacement at C

$x + y = 1$

θ_1 must be equal to θ_2 then only

⇒



From figure

$x = \theta b$

$y = \theta a$

$\theta(a+b) = 1$

$\theta = \frac{1}{a+b}$

$x = \left[\frac{b}{a+b}\right] = \frac{b}{l}$

$y = \left(\frac{a}{a+b}\right) = \frac{a}{l}$

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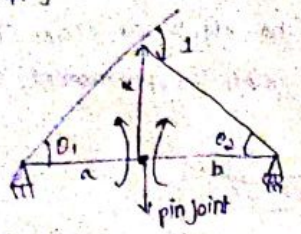
Bending Moment ⇒

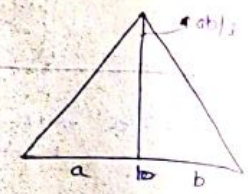
Apply unit rotation



$\theta_1 a = x$

$\theta_2 b = x$

w.k.T $\theta_1 + \theta_2 = 1$



$\Rightarrow \frac{x}{a} + \frac{x}{b} = 1$

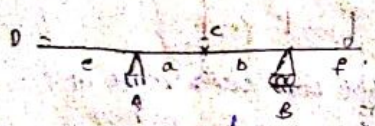
$x\left[\frac{a+b}{ab}\right] = 1$

$x = \frac{ab}{a+b} = \frac{ab}{l}$

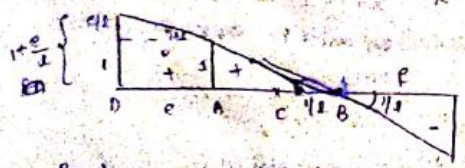
$\therefore \theta_1 = \frac{x}{a} = \left(\frac{ab}{l}\right)\left(\frac{1}{a}\right) = \frac{b}{l}$

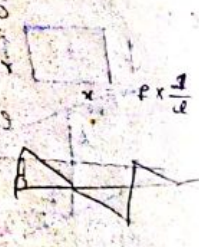
$\theta_2 = \frac{a}{l}$

23)

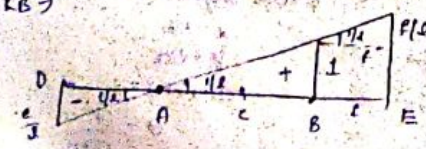


RA ⇒





RB ⇒



$1 + \frac{f}{l}$ $1 + \frac{e}{l}$

RA = ↑ +ve

$= \frac{e}{l}$

Positive

So high

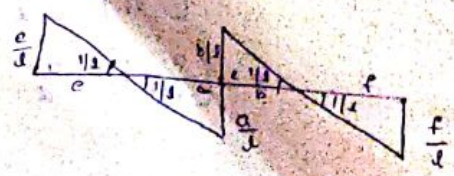
$S = \tau\theta$

$\frac{b}{l} = b\theta$

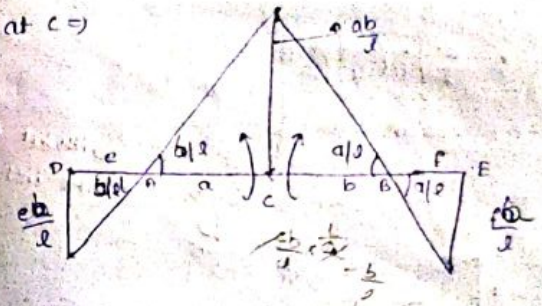
$\theta = b \times \frac{1}{b}$

$\theta = \frac{1}{l}$

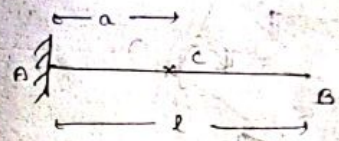
shear force at c ⇒



BM at c ⇒

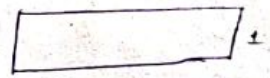


24)



RXNS ⇒ Here 'A' is fixed joint after applying unit load there will be no rotation allowed at A.

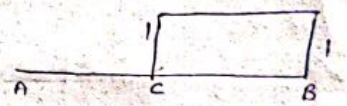
RA ⇒



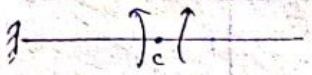
RB ⇒

(NO support)

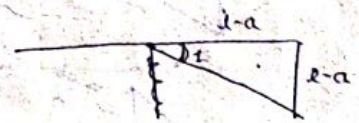
shear force at c ⇒



BM at c ⇒

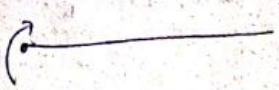


M_c ⇒

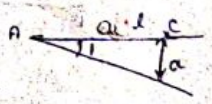


at A ⇒

MA



⇒



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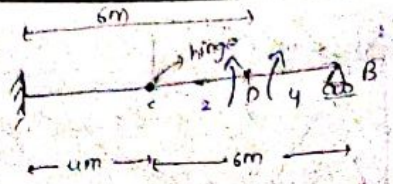
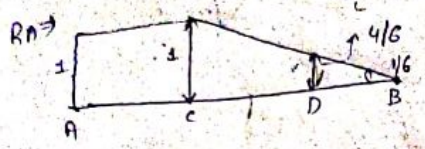
25)

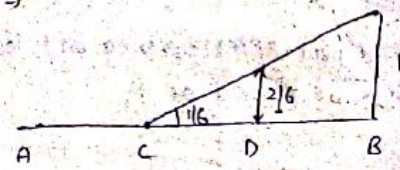


without hinge

Rins ⇒

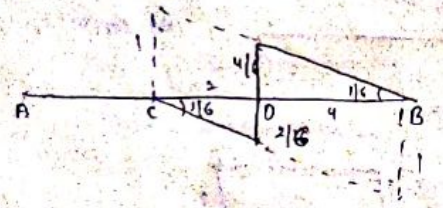


RB ⇒



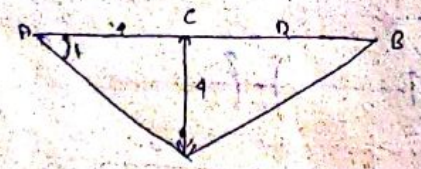
shearforce

at D ⇒

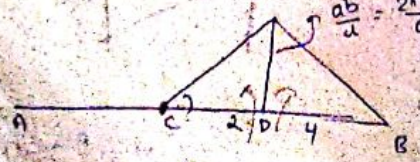


Bending Moment

MA ⇒

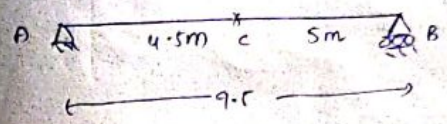


MD ⇒

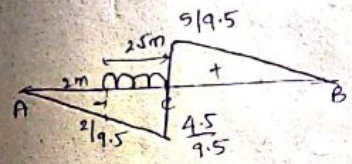


26) A UDL of length 2.5 m and intensity 25 KN/m rolls across glider of span 9.5 m shown in fig.

a) calculate the maximum -ve and +ve SF and max BM at section 4.5 m from left support



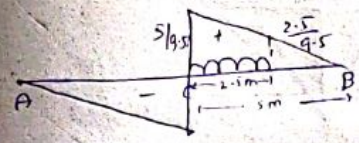
maximum -ve shear



max -ve SF ⇒

$$-25(2.5)\left[\frac{\frac{2}{9.5}+\frac{4.5}{9.5}}{2}\right] = -21.38 \text{ KN}$$

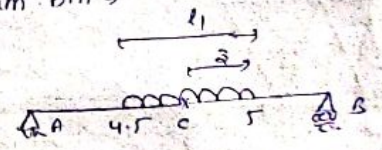
maximum +ve shear



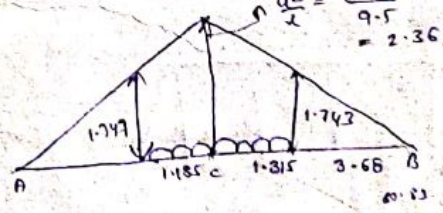
max -ve SF ⇒

$$+25(2.5)\left[\frac{\frac{5}{9.5}+\frac{2.5}{9.5}}{2}\right] = 24.67$$

maximum BM ⇒



$$a = \frac{b l_1}{l_0} = \frac{5(2.5)}{9.5} = 1.315$$



2.36 → 5
x → 3.68
3.68/5 × 2.36
1.743

Then $BM_{max} = (25 \times 1.185)\left[\frac{1.747+2.36}{2}\right] + (25 \times 1.315)\left[\frac{2.36+3.68}{2}\right]$

$= 128.27$ KN-m

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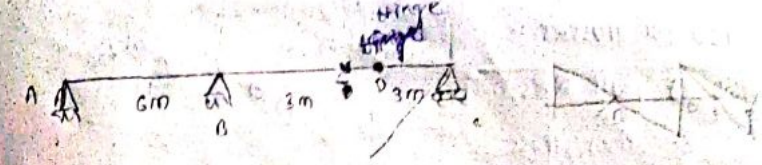
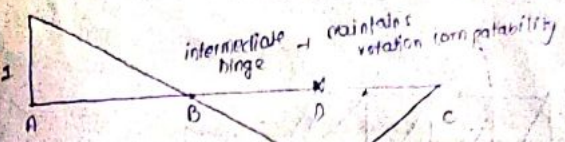
27) A simply supported beam of span 18 m carries a series loads is shown in fig. Determine the absolute maximum moment using the influence line.

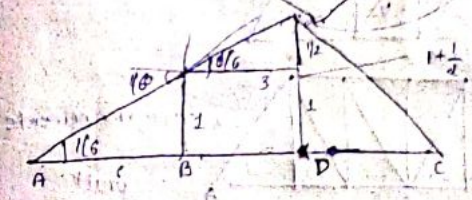
28)





Rms ⇒ Remove support put unit load

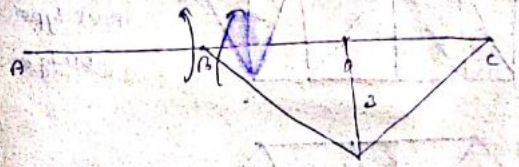
R_B ⇒



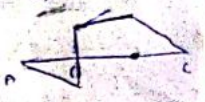
shear force ⇒

BM ⇒

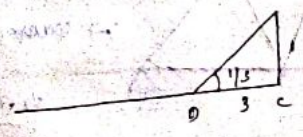
at B



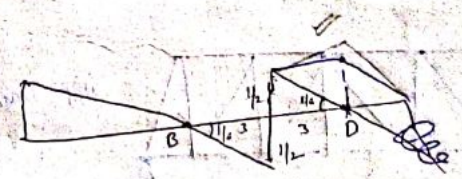
SF at B



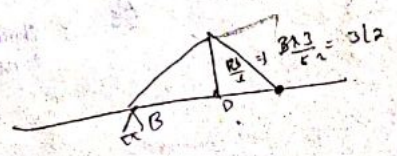
R_C ⇒



SF ⇒



M_D ⇒



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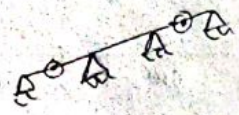
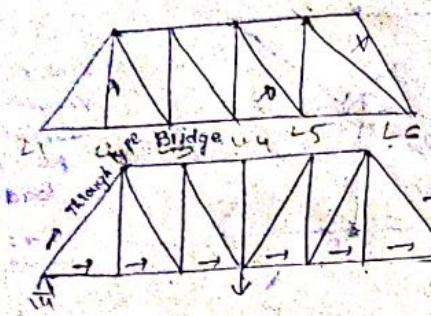
ILD for trusses :-

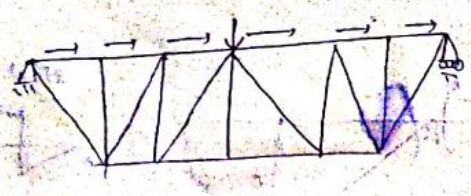


1. pratt truss
2. Howe truss
3. warren pattern

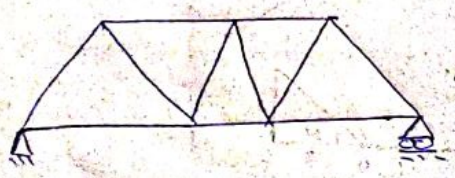


→ slope downwards

pratt



Deck type Bridge



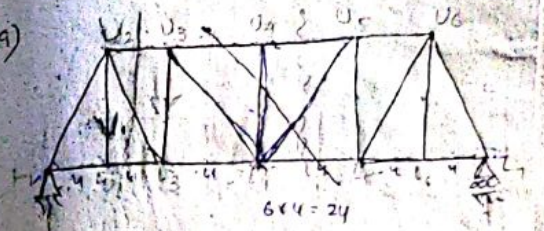
→ warren ~~pratt~~ pattern

No vertical members

slope upwards

Should make it as tensile not compression if it will buckle. To reduce buckling increase area

29)



unit load at L_1 so remaining all not responded ↓1 ↑1

upper cord values :-

U_2U_3

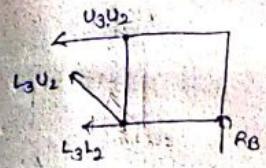
unit load at L_2 $R_A = \frac{20}{24}$ $\frac{4}{24} = R_B$

consider a section

at left side → no.of unknowns (2)

unit load, R_A

Right " → (1) R_B ✓



$\Sigma M_{L_3} = 0$

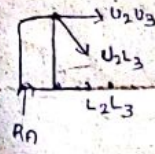
$-U_3U_2(4) - R_B(16) = 0$

$R_B(16) = -4\,U_3U_2$

$\frac{4}{24}(8) = -4(U_3U_2)$

$U_3U_2 = -\frac{2}{3}$ compression $U_3U_2 = -\frac{2}{3} = -0.66$

unit load at L_3 ⇒ $R_A = \frac{16}{24}$ $R_B = \frac{8}{24}$



$\Sigma M_{L_3} = 0$

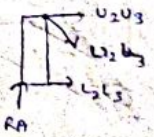
$R_A(8) + U_2U_3(4) = 0$

$\frac{16}{24}(8) + 4\,U_2U_3 = 0$

$\frac{2.66}{4} = -U_2U_3$ ⇒ $U_2U_3 = -1.33$ $-\frac{4}{3}$

unit load at L_4 ⇒

$R_A = \frac{12}{24}$ $R_B = \frac{12}{24}$ $\Sigma M_{L_3} = 0$



⇒ $R_A(8) + U_2U_3(4) = 0$

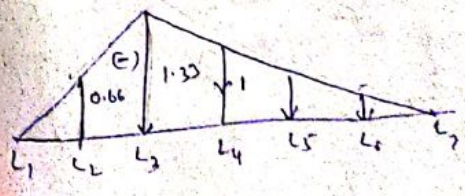
$\frac{12}{24}(8) + U_2U_3(4) = 0$

$U_2U_3 = -1$

First increase, decrease, increase

through out the span it is in compression

$F_{U_2U_3} = \frac{16 \times 8}{24 \times 4} = \frac{(l-a)(a)}{l \times h}$



Scanned by CamScanner