A QUICK REVIEW

OHM'S LAW

"Ohm's law states that the voltage *v* across a resistor is directly proportional to the current *i* flowing through the resistor, at constant temperature."

$$v \propto i$$

Ohm defined the constant of proportionality for a resistor to be the resistance, *R*.

$$v = iR$$

KIRCHHOFF’S LAWS

- Kirchhoff's circuit laws are two equalities that deal with the current and potential difference (commonly known as voltage) in the lumped element model of electrical circuits
- Applicable for both for DC circuits, and for AC circuits

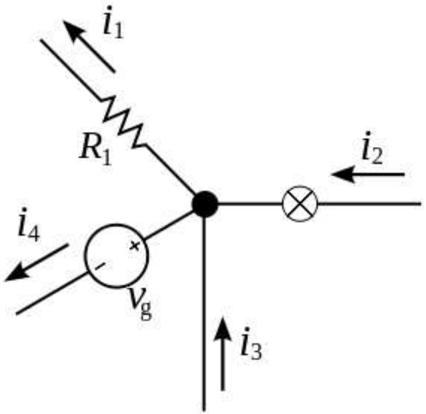
KIRCHHOFF'S CURRENT LAW

- Also called Kirchhoff's first law, Kirchhoff's point rule, or Kirchhoff's junction rule (or nodal rule)
- ***At any node (junction) in an electrical circuit, the sum of currents flowing into that node is equal to the sum of currents flowing out of that node,***

(Or)

- ***The algebraic sum of currents in a network of conductors meeting at a point is zero.***

- Based on principle of conservation of electric charge



$$\sum_{k=1}^{n} I_k = 0$$

The current entering any junction is equal to the current leaving that junction. $i_1 + i_4 = i_2 + i_3$

KIRCHHOFF'S VOLTAGE LAW

- Also called Kirchhoff's second law, Kirchhoff's loop (or mesh) rule, and Kirchhoff's second rule
- ***The directed sum of the electrical potential differences (voltage) around any closed network is zero.***

(Or)

- ***The sum of the emfs in any closed loop is equal to the sum of the potential drops in that loop.***

(Or)

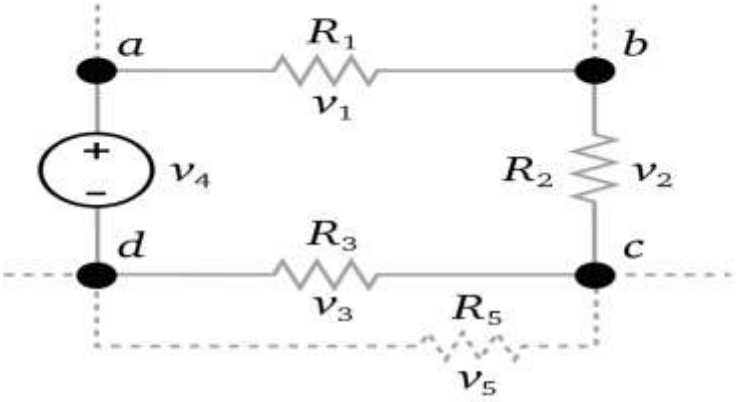
- ***The algebraic sum of the products of the resistances of the conductors and the currents in them in a closed loop is equal to the total emf available in that loop.***

$$\sum_{k=1}^{n} V_k = 0$$

- Based on the conservation of energy whereby voltage is defined as the energy per unit charge.

KIRCHHOFF'S voltage law

- The sum of all the voltages around the loop is equal to zero. - v_1 - v_2 - v_3 + v_4 = 0



Limitations of Kirchhoff's laws

- Applicable only to lumped network models
- KCL is valid only if the total electric charge, Q , remains constant in the region being considered
- KVL is based on the assumption that there is no fluctuating magnetic field linking the closed loop.
- KCL and KVL only apply to circuits with steady currents (DC). However, for AC circuits having dimensions much smaller than a wavelength, KCL, KVL are also approximately applicable.

VOLTAGE DIVIDER RULE

- Series circuit – Voltage divider
- Same current flows.
- Voltage drops proportional to value of resistors/impedance; Different voltage from single source; So called voltage divider.

CURRENT DIVIDER RULE

- Parallel circuit- Current divider rule.
- Current from source divides in all branches of parallel circuit; Hence it is called current divider rule.

NETWORK THEOREMS

- Important fundamental theorems of network analysis. They are the
  - Superposition theorem
  - Thévenin’s theorem
  - Norton’s theorem
  - Maximum power transfer theorem
  - Millman’s theorem
  - Reciprocity theorem

THEVENIN’S THEOREM

For DC Circuits

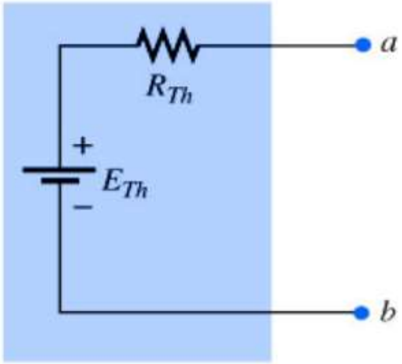
- Any two-terminal, linear, bilateral, active dc network can be replaced by an equivalent circuit consisting of an equivalent voltage source(Thévenin’s Voltage Source) and an equivalent series resistor (Thévenin’s Resistance)

For AC Circuits

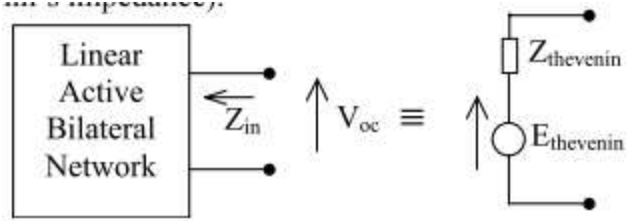
- Any two-terminal, linear, bilateral, active ac network can be replaced by an equivalent circuit consisting of an equivalent voltage source(Thévenin’s Voltage Source) and an equivalent series impedance (Thévenin’s Impedance)

THEVENIN’S THEOREM …continued

- DC Circuits



- AC Circuits



THEVENIN'S THEOREM …continued

- Thevenin's theorem can be used to:
  - Analyze networks with sources that are not in series or parallel.
  - Reduce the number of components required to establish the same characteristics at the output terminals.
  - Investigate the effect of changing a particular component on the behaviour of a network without having to analyze the entire network after each change.

THEVENIN'S THEOREM …continued

Procedure to determine the proper values of R_{Th} and E_{th}

1. Disconnect the load resistance.

2. Calculate R_{Th} by first setting all sources to zero (voltage sources are replaced by short circuits, and current sources by open circuits) and then finding the resultant resistance between the two marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.)

3. Calculate E_{Th} by first returning all sources to their original position and finding the open-circuit voltage between the marked terminals.

4. Draw the Thévenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.

NORTON’S THEOREM

- Dual of Thevenin's theorem

For DC Networks

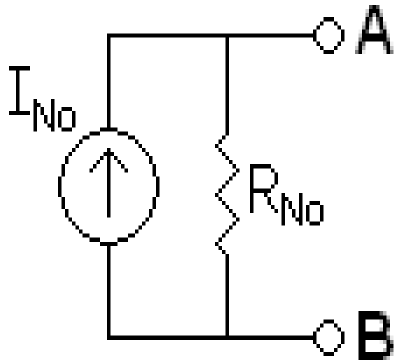
- Any two-terminal, linear, bilateral, active dc network can be replaced by an equivalent circuit consisting of an equivalent current source(Norton’s Current Source) and an equivalent parallel resistor (Norton’s Conductance)

For AC Circuits

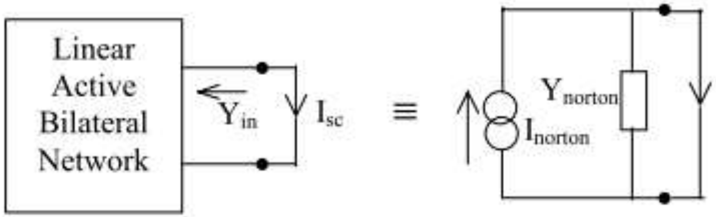
- Any two-terminal, linear, bilateral, active ac network can be replaced by an equivalent circuit consisting of an equivalent current source(Norton’s Current Source) and an equivalent shunt admittance (Norton’s Admittance)

NORTON'S THEOREM …continued

- DC Circuits



- AC Circuits



NORTON'S THEOREM …continued

Procedure to determine the proper values of R_N and I_N

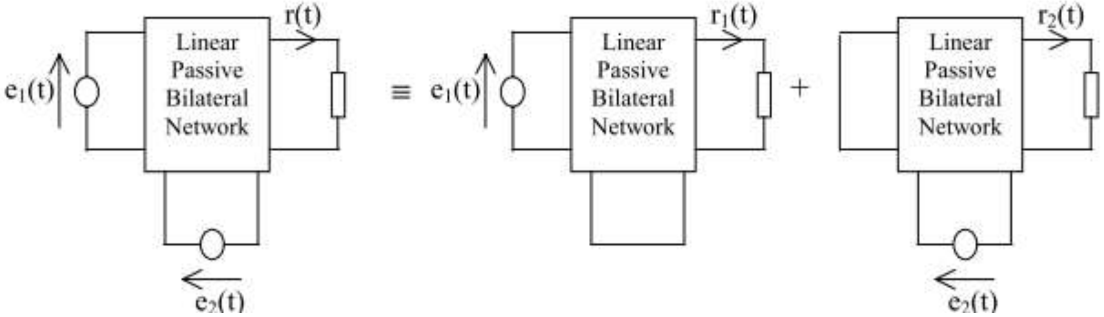
1. Disconnect the load resistance.
2. Calculate R_N by first setting all sources to zero (voltage sources are replaced with short circuits, and current sources with open circuits) and then finding the resultant resistance between the two marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.) Since $R_N = R_{Th}$, the procedure and value obtained using the approach described for Thevenin's theorem will determine the same value as R_N.
3. Calculate I_N by first returning all the sources to their original position and then finding the short-circuit current between the marked terminals
4. Draw the Norton equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.

Thévenin – NORTON EQUIVALENT

- Possible to find Norton equivalent circuit from Thévenin equivalent circuit
  - Use source transformation method
- $\mathbf{Z}_{\mathrm{N}} = \mathbf{Z}_{\mathrm{Th}}$
- $\mathbf{I}_{\mathrm{N}} = \mathbf{E}_{\mathrm{Th}}/\mathbf{Z}_{\mathrm{Th}}$

SUPERPOSITION THEOREM

- The current through, or voltage across, an element in a linear bilateral network is equal to the algebraic sum of the currents or voltages produced independently by each source.



SUPERPOSITION THEOREM… continued

- Used to find the solution to networks with two or more sources that are not in series or parallel.
- The current through, or voltage across, an element in a network is equal to the algebraic sum of the currents or voltages produced independently by each source.
- Since the effect of each source will be determined independently, the number of networks to be analyzed will equal the number of sources.
- Superposition theorem is applicable to linear and homogeneous networks.

SUPERPOSITION THEOREM… continued

- For a two-source network, if the current produced by one source is in one direction, while that produced by the other is in the opposite direction through the same resistor, the resulting current is the difference of the two and has the direction of the larger
- If the individual currents are in the same direction, the resulting current is the sum of two in the direction of either current

SUPERPOSITION THEOREM… continued

- Superposition theorem can be applied only to voltage and current
- It cannot be used to solve for total power dissipated by an element
- Power is not a linear quantity
  - Follows a square-law relationship

SUPERPOSITION THEOREM… continued

For applying Superposition theorem:-

- Replace all other independent voltage sources with a short circuit (thereby eliminating difference of potential. i.e. V=0, internal impedance of ideal voltage source is ZERO (short circuit)).
- Replace all other independent current sources with an open circuit (thereby eliminating current. i.e. I=0, internal impedance of ideal current source is infinite (open circuit).
- When this theorem is applied to an ac circuit, it has to be remembered that the voltage and current sources are in the phasor form and the passive elements are impedances.

MAXIMUM POWER TRANSFER THEORM

- **DC Circuits**

  A load will receive maximum power from a linear bilateral dc network when its load resistive value is exactly equal to the Thevenin’s resistance.

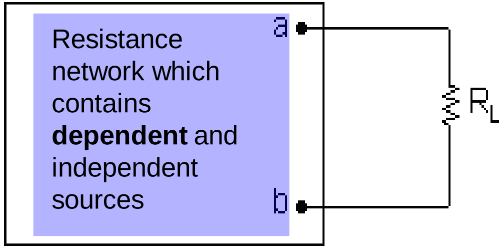
  $$R_{L} = R_{Th}$$

- **AC Circuits**

  A load will receive maximum power from a linear bilateral ac network when its load impedance is complex conjugate of the Thevenin’s impedance

  $$\mathbf{Z}_{L} = \mathbf{Z}_{\mathrm{Th}}^{*}$$

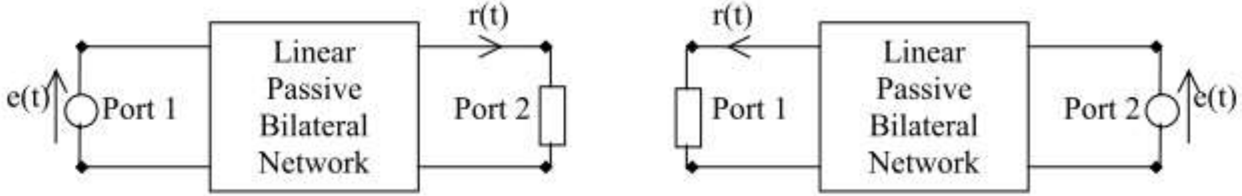
MAXIMUM Power transfer theorem



$$p_{max} = \frac{V_{Th}{}^2 R_L}{(2R_L)^2} = \frac{V_{Th}{}^2}{4R_L}$$

Reciprocity theorem

- In a linear bilateral single source network, the ratio of response to excitation remains the same even when the positions of response and excitation are interchanged.



  - The location of the voltage source and the resulting current may be interchanged without a change in current.

Mesh Analysis - Applicable to planar circuits.

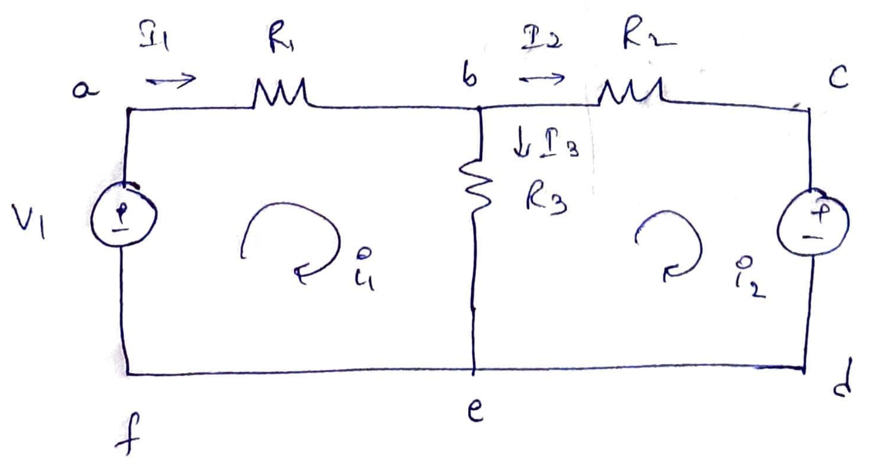
- Mesh analysis also provides another general procedure for analyzing circuits, using mesh currents as the circuit variables.

- Using mesh currents instead of element currents as circuit variables is convenient as it reduces the number of equations that must be solved simultaneously

Loop - It is a closed path with no node passed more than once.

Mesh - It is a loop that does not contain any other loop in it.

- Mesh analysis applies KVL to find the unknown currents.



1) Mesh Analysis to planar circuits that do not contain current sources.

2) Mesh analysis to planar circuits that contain current sources.

Steps to determine mesh currents:

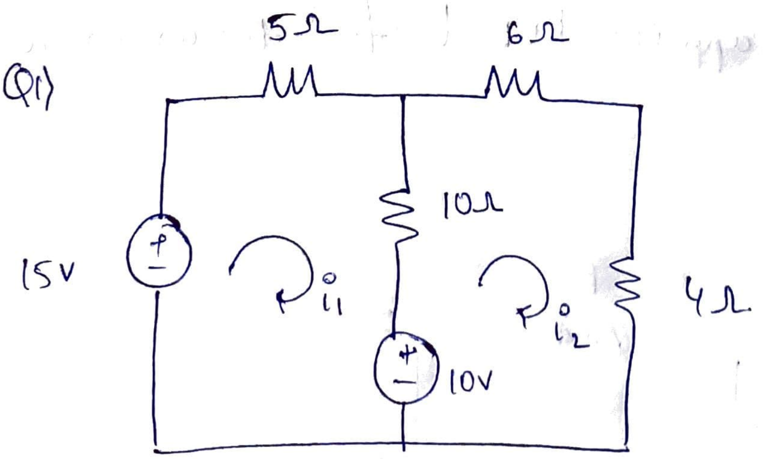
1) Assign mesh currents $i_1, i_2, \ldots i_n$ to the 'n' meshes.

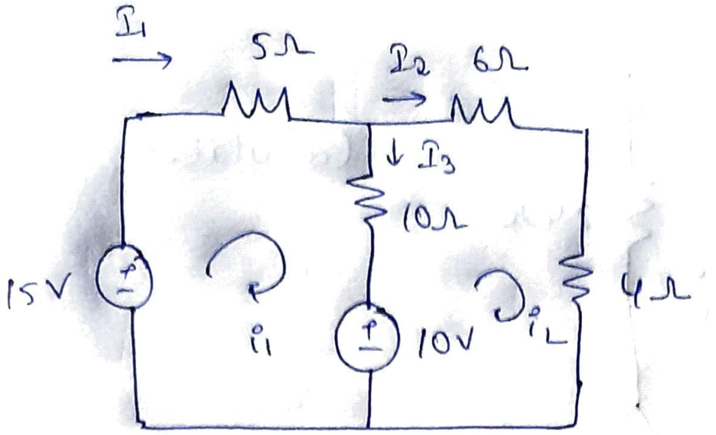
2) Apply KVL to each of the n meshes.

Use Ohm's law to express the voltages in terms of the mesh currents.

3) Solve the resulting n simultaneous equations to get the mesh currents.

Q1)





Apply KVL in first loop,

$$15 = 5(i_1) + 10(i_1 - i_2) + 10 \longrightarrow (1)$$

Apply KVL in second loop,

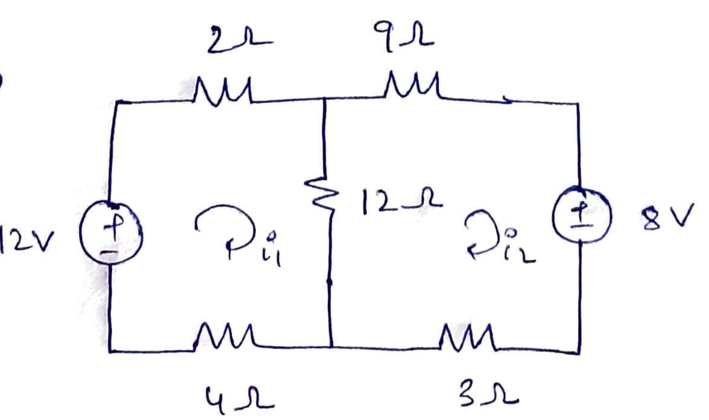
$$6i_2 + 4i_2 = 10 + 10(i_1 - i_2) \longrightarrow (2)$$

Solving (1), (2)

$$\Rightarrow \quad i_1 = 1A \qquad i_2 = 1A$$

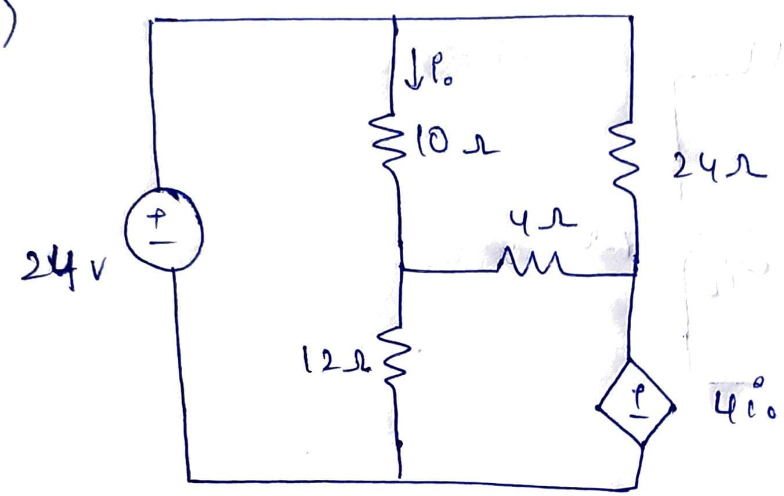
$$\Rightarrow I_1 = i_1 = 1A, \quad I_2 = i_2 = 1A, \quad I_3 = i_1 - i_2 = 0.$$

Q2)



Calculate mesh currents i_1, i_2.

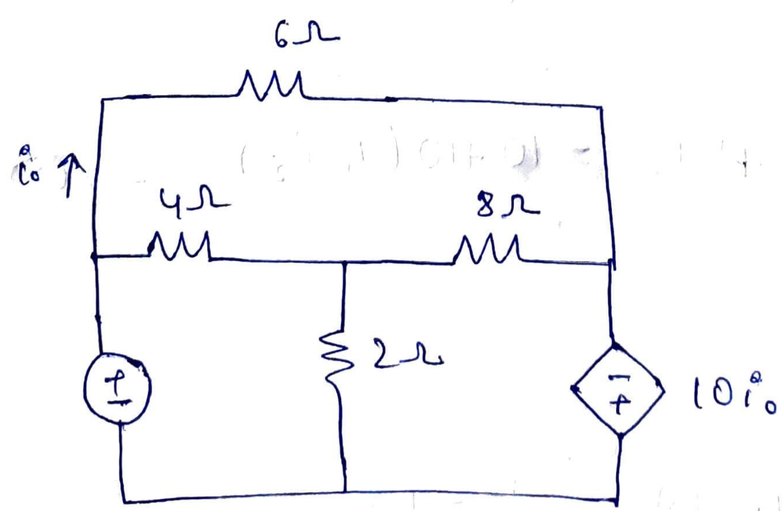
3Q)



Calculate i_o.

Ans: $i_o = i_1 - i_2 = 1.5\,A$

4Q)

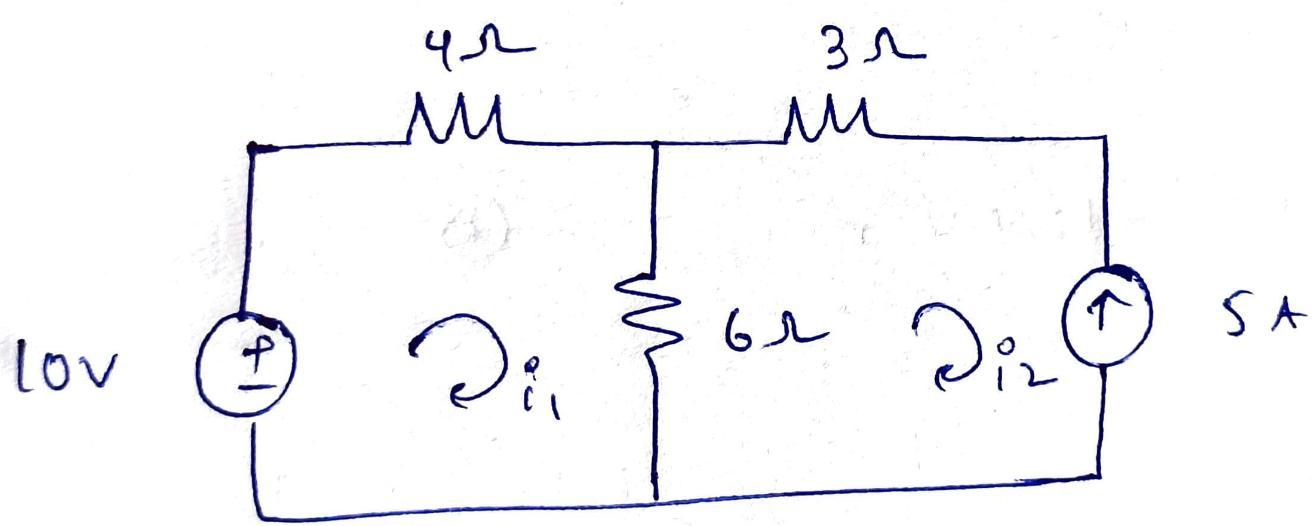


Calculate i_o.

Mesh analysis with current sources:

Case (i):

When current source exists only in one mesh

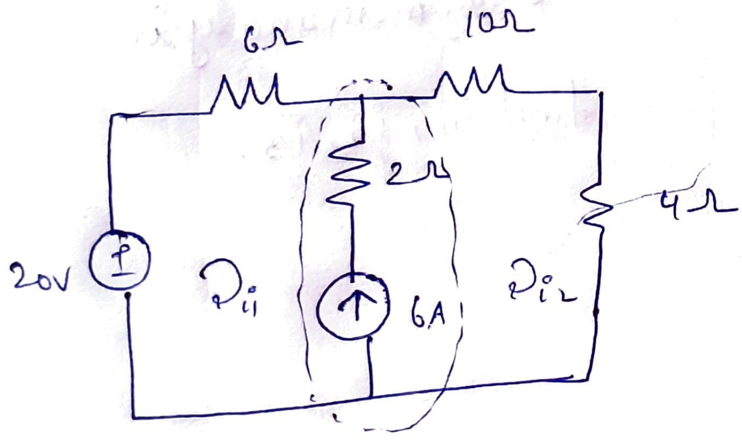


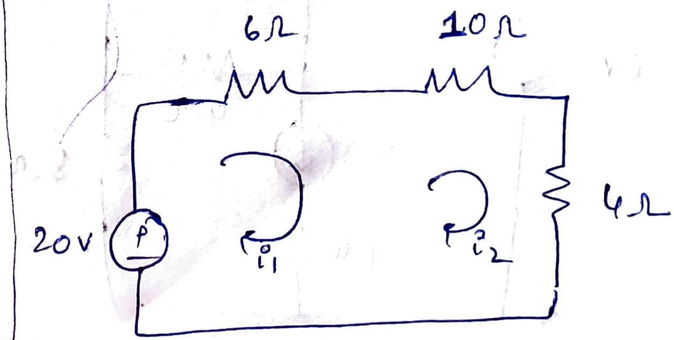
Case (ii):

When a current source exists between two meshes.

- we create a supermesh.

" A supermesh results when two meshes have a (dependent or independent) current source in common. "





Supermesh circuit

Supermesh $\rightarrow$ $i_2 - i_1 = 6$

$$\Rightarrow i_2 = 6 + i_1 \longrightarrow (1)$$

Apply KVL in the circuit where supermesh is create

$$20 = 6i_1 + 10i_2 + 4i_2 \longrightarrow (2)$$

Sub (1) in (2),

$$20 = 6(i_1) + 10(6 + i_1) + 4(6 + i_1)$$

$$= 20i_1 + 84$$

$$\Rightarrow 20i_1 = 20 - 84$$

$$= -64$$

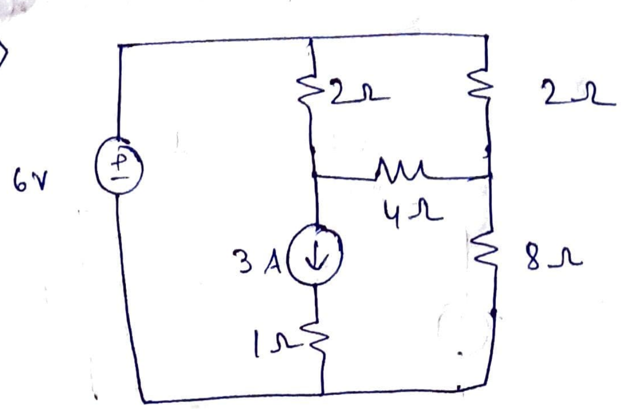
$$\Rightarrow \boxed{i_1 = -3.2A}$$

from (1)

$$\Rightarrow i_2 = 6 + i_1$$

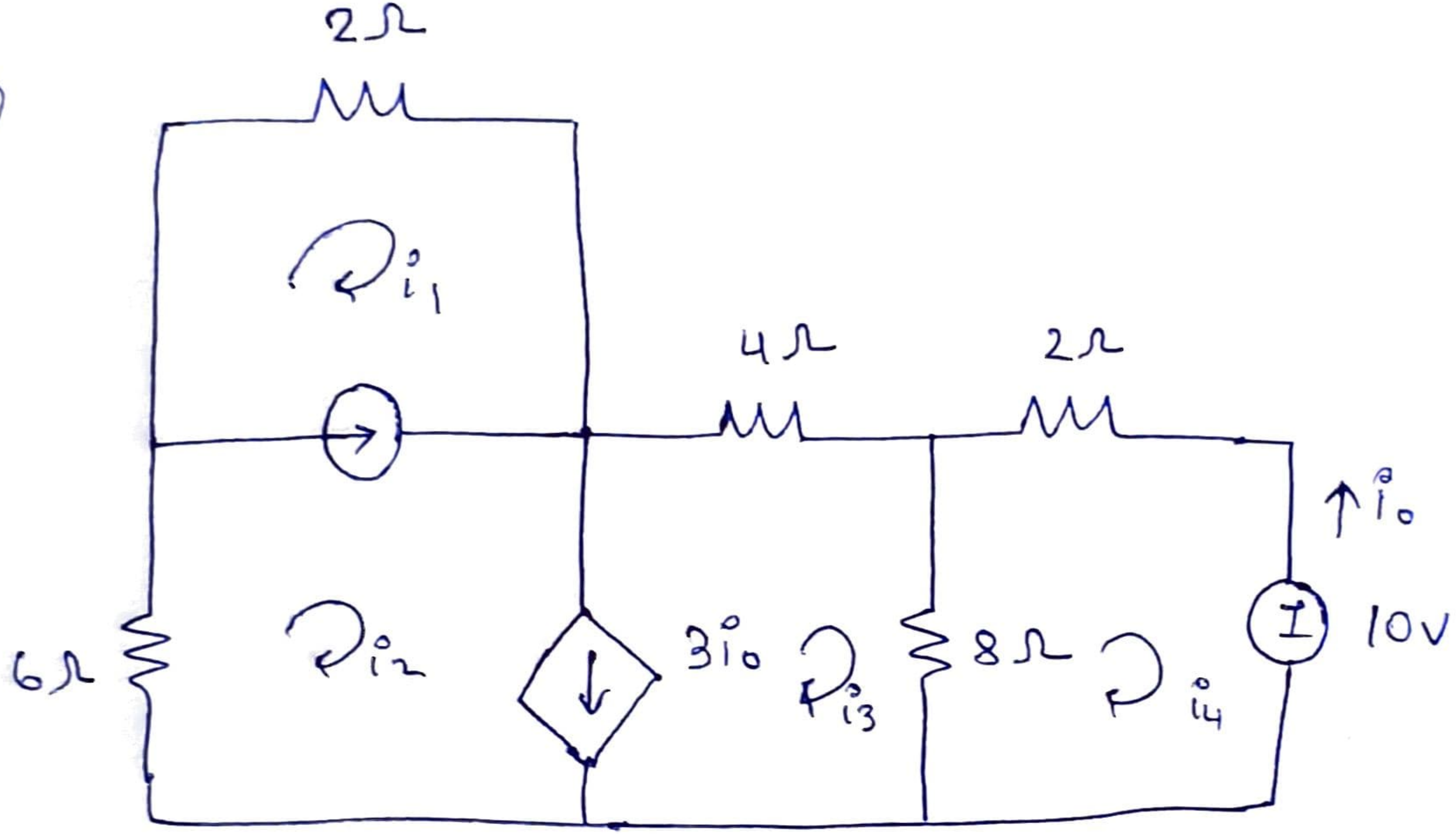
$$= 6 - 3.2 = 2.8\ A.$$

1Q)



Using mesh analysis find i_1, i_2, i_3.

2 Q)



Nodal Analysis: - It is used to analyze almost any circuit.

- It is an application of KCL.

{ Node: A point where two or more elements are joined together is called node.

Junction: A point where three or more branches meet is called a junction. }

- Nodal analysis also called as the node-voltage method.
- In nodal analysis, we will be finding nodal voltages.

~~Two conditions~~

1) Nodal analysis without voltage sources in the network
2) Nodal analysis with voltage sources in the network.

Steps to determine node voltages:

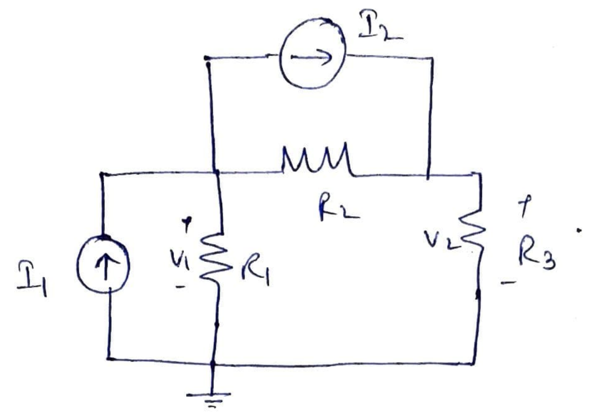
1) Select a node as reference node. It's voltage is taken as zero.
2) Assign voltage to rest of the nodes (If there are n nodes in the network, 'n-1' are the non-reference nodes).
3) Apply KCL at 'n-1' nodes (non-reference)

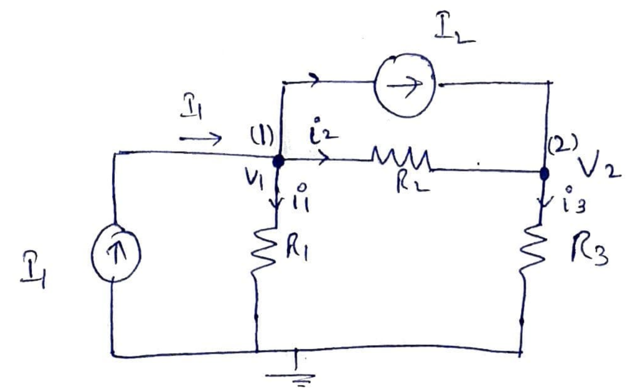
4) Using Ohm's law, express branch currents in terms of node voltages.

5) Solve the resulting equations to obtain unknown node voltages.

[" Current flows from higher potential to lower potential."]

- Nodal analysis without voltage sources in the network:





I_1, I_2 - known values of currents,

i_1, i_2, i_3 - unknown values of current.

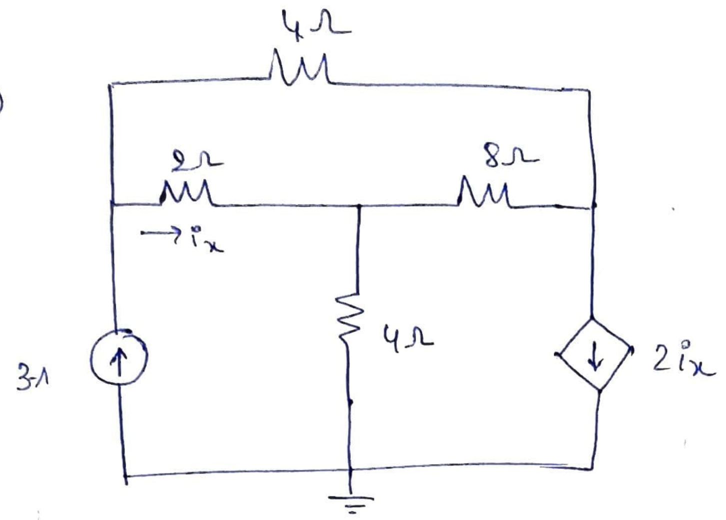
$$i = \frac{V_{higher} - V_{lower}}{R}$$

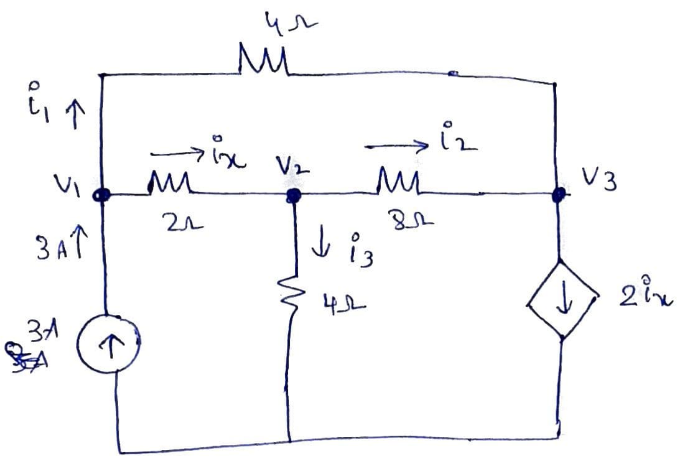
At node '1', $I_1 = I_2 + i_2 + i_1$

$$I_1 = I_2 + \frac{V_1 - V_2}{R_2} + \frac{V_1 - 0}{R_1} \rightarrow (1)$$

At node '2', $I_2 + i_2 = i_3 \Rightarrow I_2 + \frac{V_1 - V_2}{R_2} = \frac{V_2 - 0}{R_3} \rightarrow (2)$

2Q)

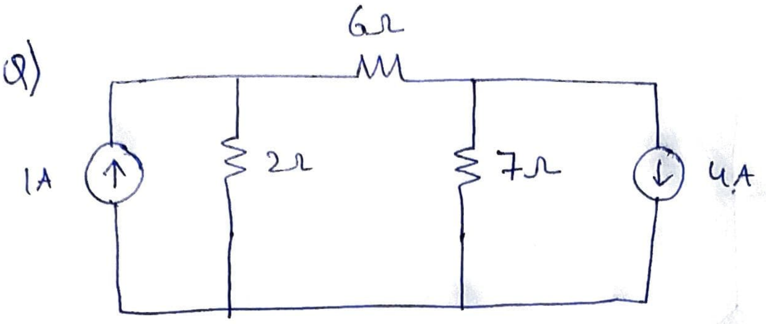




$3 = i_1 + i_x$

$V_1 = 4.8V$, $V_2 = 2.4V$, $V_3 = -2.4V$

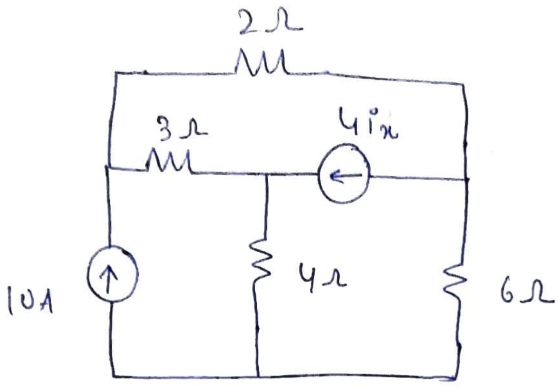
Q)



Obtain the node voltages for the above circuit.

Ans: $V_1 = -2V$, $V_2 = -14V$

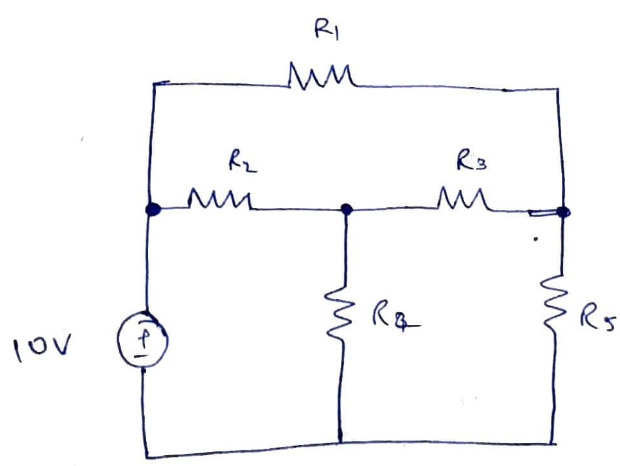
3Q)



Ans: $v_1 = 80V$, $v_2 = \cancel{6V} \cdot -64V$, $V3 = 156V$

Nodal Analysis with voltage sources:

Case (i): A voltage source connected between the reference node and a non-reference node.

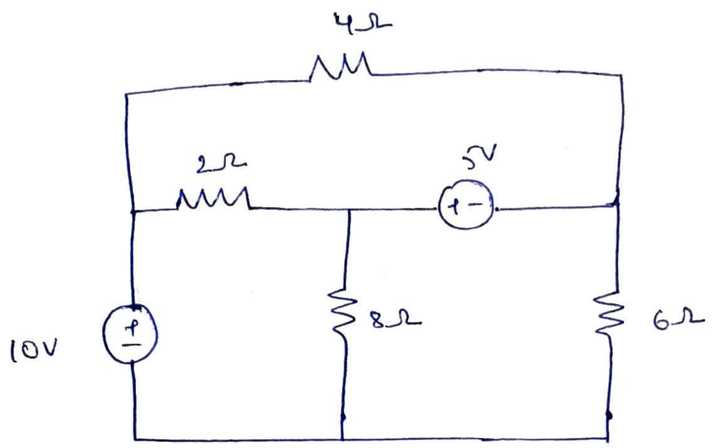


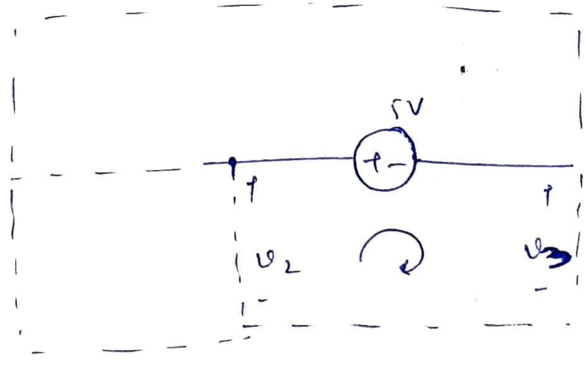
Case (ii):

A voltage source (dependent or independent) is connected between two nonreference nodes.

These two nonreference nodes form a generalized node (or) supernode.

∴ A supernode is formed by enclosing a (dependent or independant) voltage source connected between two nonreference nodes

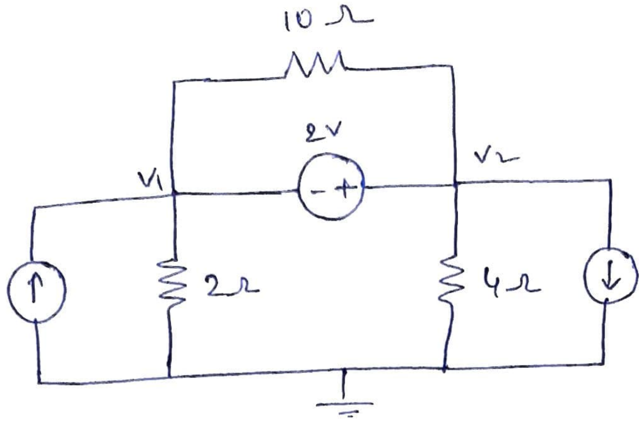




$$5 + v_3 - v_2 = 0$$

$$\boxed{5 = v_2 - v_3}$$

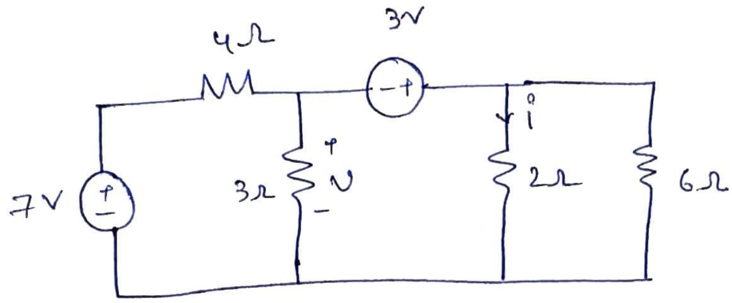
3Q)



Ans: $V_1 = -7.33V$

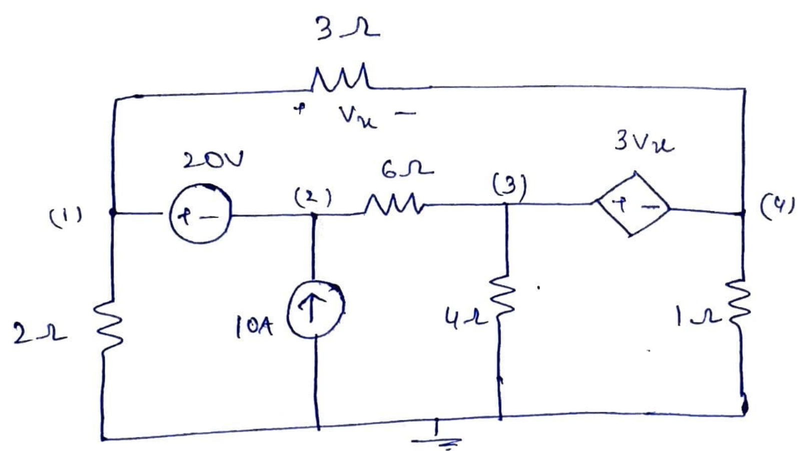
$V_2 = -5.33V$

4Q)



find v, i and all node voltages.

5Q)



Ans: $V_1 = 26.667 \text{ V}$

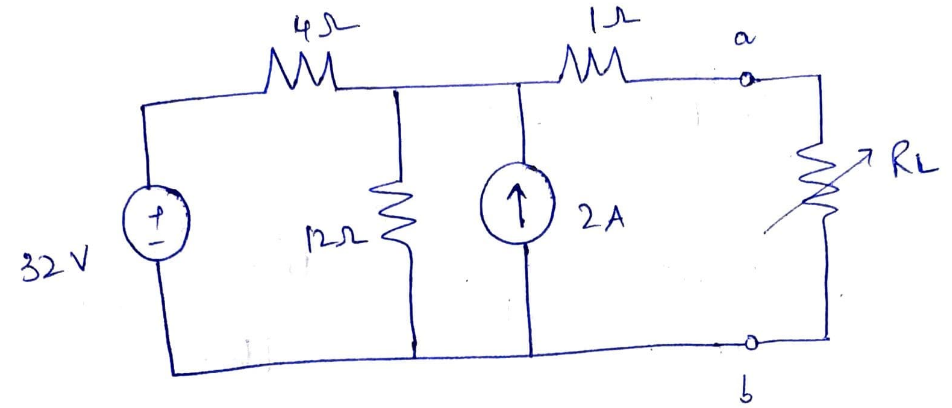
$V_2 = 6.667 \text{ V}$

$V_3 = 173.33V$

$V_4 = -46.667V$

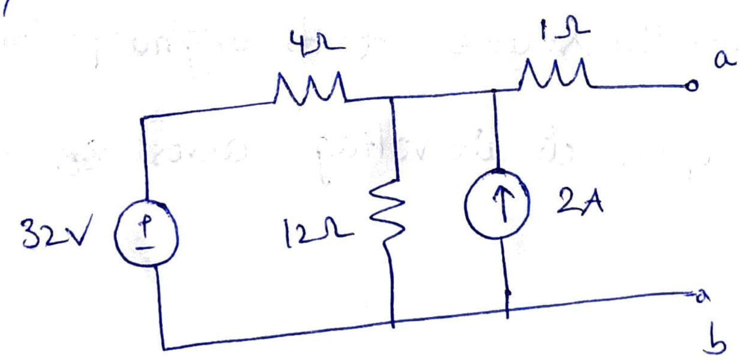
Thevenin's theorem.

Q) find the thevenin equivalent circuit with respect to the load R_L shown.



Then find the current through R_L, when $R_L = 1\Omega$, 8Ω and 32Ω.

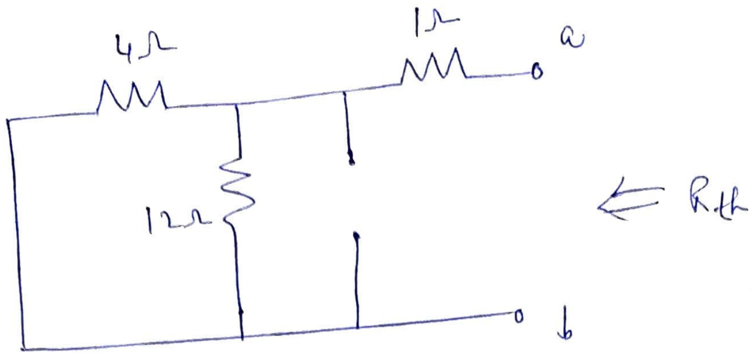
Sol) 1) Remove load



2) find R_{th}:

for finding R_{th}, network must be made passive

⇒ Replace all voltage & current sources by their internal impedances.



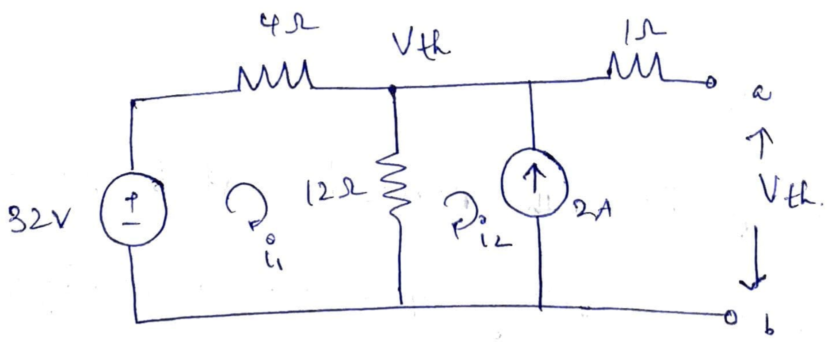
$R_{th} = (4 \| 12) + 1$

$= \dfrac{4 \times 12}{4+12} + 1 = \dfrac{\cancel{4} \times \cancel{12}^{3}}{\cancel{16}_{4}} + 1$

$\boxed{R_{th} = 4\Omega}$

3) Find Vth :

- Return all the sources to its original position
- find the open circuit voltage across ~~Rth~~ the terminals a and b



mesh 1 $\Rightarrow$ $32 = 4(i_1) + 12(i_1 - i_2)$

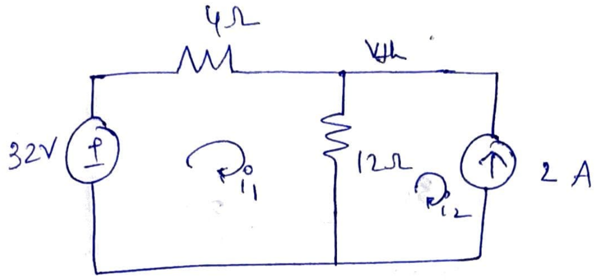
$= 4i_1 + 12i_1 - 12i_2$

$= 16i_1 - 12i_2$

$$\Rightarrow \quad 8 = 4i_1 - 3i_2 \quad \rightarrow ①$$

from mesh 2,

$$i_2 = -2A \quad \rightarrow ②$$



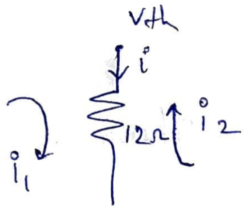
i_2 is the mesh current in mesh 2

Sub ② in ①,

$$8 = 4i_1 - 3(-2) = 4i_1 + 6$$

$$\Rightarrow \quad 4i_1 = 2 \Rightarrow i_1 = 0.5\,A$$

$$V_{th} = 12(i_1 - i_2)$$



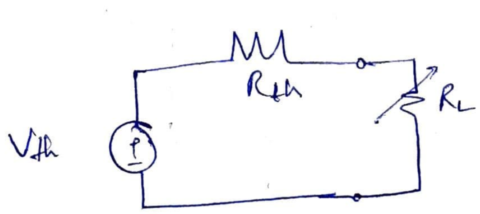
$$= 12(0.5 - (-2))$$

$i = i_1 - i_2$

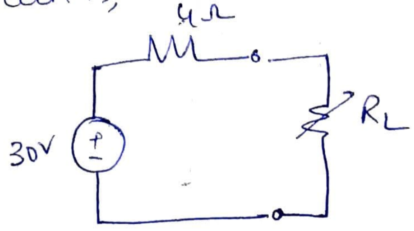
$$= 12(2.5)$$

$$\boxed{V_{th} = 30\,V}$$

∴ Thevenin's equivalent circuit is,

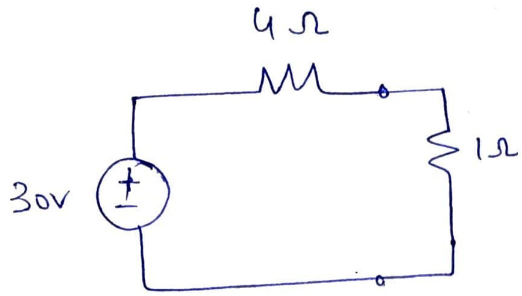


⇒



Now, when $R_L = 1\Omega, 8\Omega, 32\Omega$, we have to find the current flowing through load. R_L.

$R_L = 1\Omega$

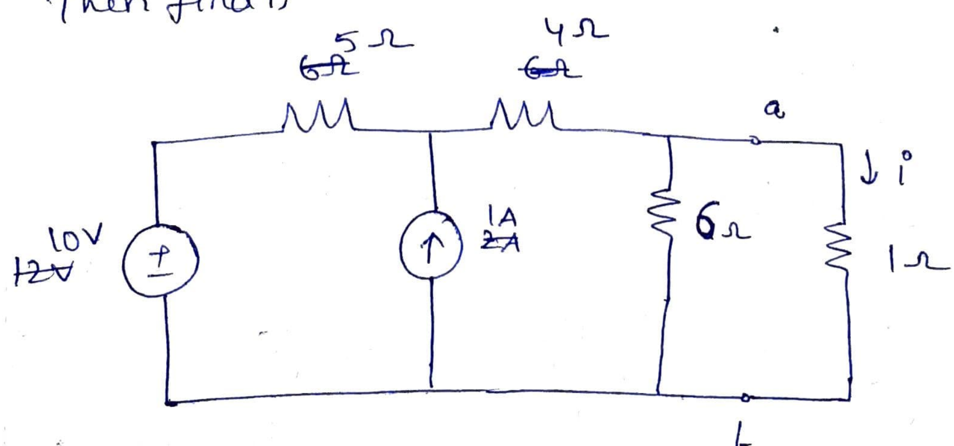


$$I_L = \frac{V_{th}}{R_{th} + R_L} = \frac{30}{4+1} = \frac{30}{5} = 6A$$

$R_L = 8\Omega \Rightarrow I_L = \frac{30}{4+8} = \frac{30}{12} = 2.5A$

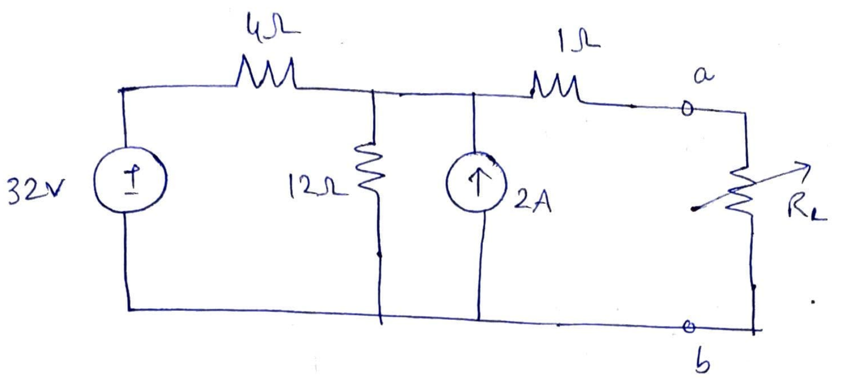
$R_L = 32\Omega \Rightarrow I_L = \frac{30}{4+32} = \frac{30}{36} = 0.833A$

2Q) Using Thevenin's theorem, find the equivalent circuit to the left of the terminals in the circuit shown. Then find i



Norton's theorem :

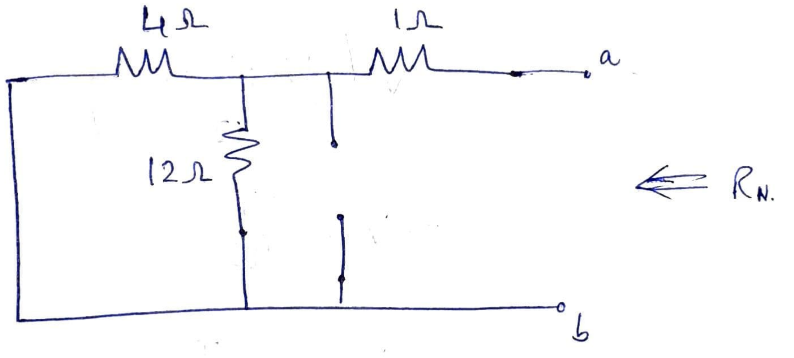
Q) find the norton's equivalent circuit with respect to the load R_L shown. find current through R_L, when $R_L = 5\Omega$



Sol) 1) Disconnect R_L.

2) find ~~Rth~~. R_N

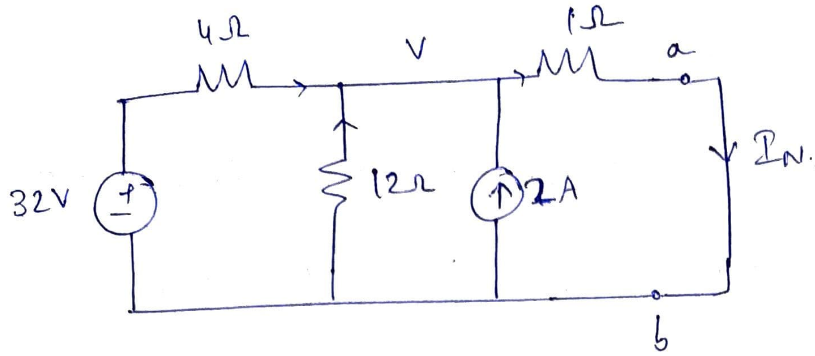
Make the network passive to find R_N.



$$R_N = R_{th} = \left(\frac{4 \times 12}{4+12}\right) + 1 = \boxed{4\Omega = R_N}$$

3) - Return all the sources to their original positions.

- Short the load terminals a b & find current flowing through that short circuited path.



One node is available.

Let the voltage of that node be V.

at node, apply KCL,

$$\frac{32-V}{4} + \frac{0-V}{12} + 2 = \frac{V-0}{1}$$

$$\Rightarrow \frac{\overset{8}{\cancel{32}}}{\cancel{4}} - \frac{V}{4} - \frac{V}{12} + 2 = V$$

$$\Rightarrow 8+2 = V + \frac{V}{4} + \frac{V}{12}$$

$$\Rightarrow 10 = V\left(\frac{12+3+1}{12}\right)$$

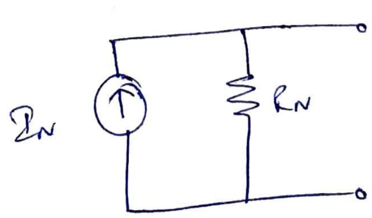
$$= V \times \frac{\cancel{16}\,4}{\cancel{12}\,3}$$

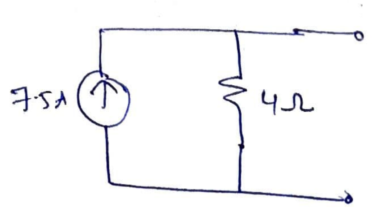
$$\Rightarrow V = \frac{30}{4} = \frac{15}{2} = 7.5V.$$

$$I_N = \frac{V-0}{1} = \frac{7.5-0}{1}$$

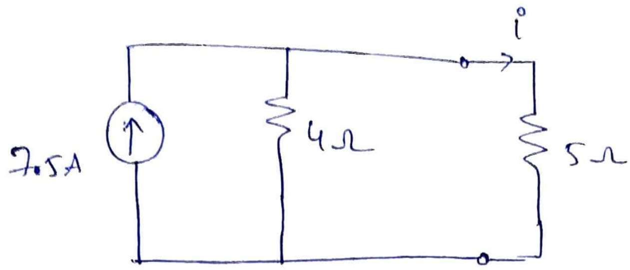
$$\boxed{I_N = 7.5A}$$

4) Draw norton's equivalent circuit,



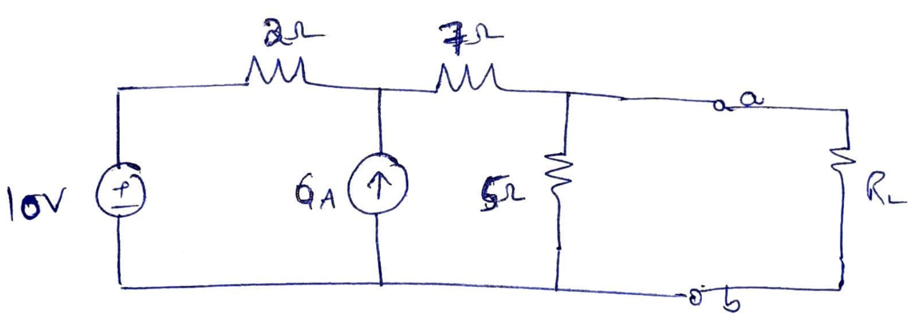


find i when $R_L = 5\Omega$



$$i = \frac{4}{4+5} \times 7.5 = \frac{4}{9} \times 7.5 = \boxed{3.33A. = i}$$

2Q) find the norton's equivalent circuit parameters & find the voltage drop across R_L when $R_L = 1\Omega$



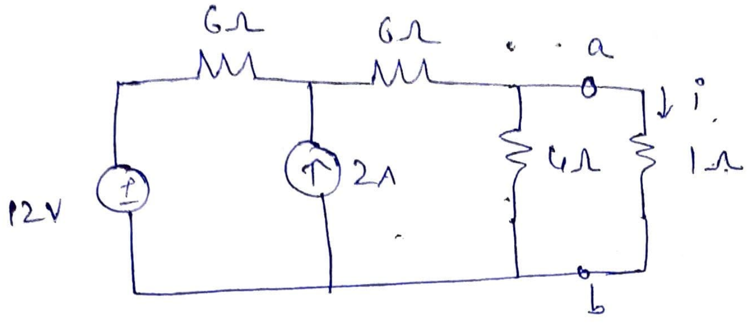
Ans: $I_N = 2.44A$

$$R_N = \frac{45}{14} = 3.2\Omega$$

$I = 2.37A$

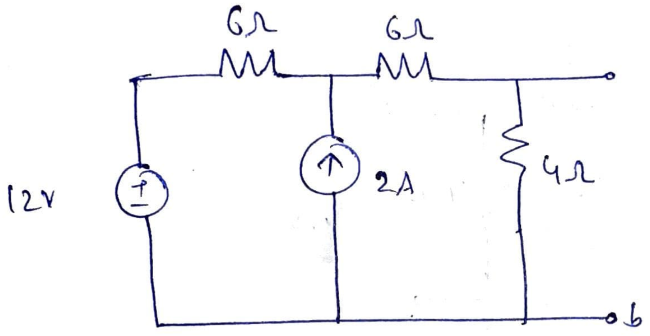
$V = 2.37V.$

3Q) Draw the Thevenin's, Norton's equivalent circuit. find i,



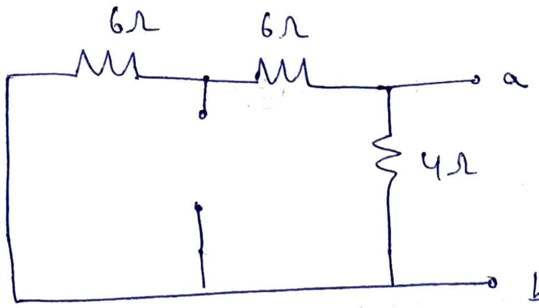
Sol) Thevenin's circuit.

1) Disconnect R_L.



2) Calculate R_{th}

- Make network passive

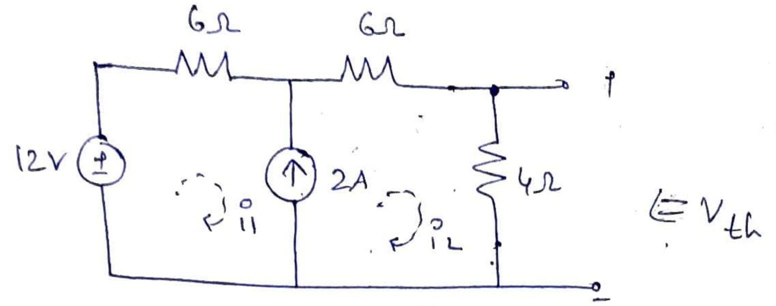


$R_{th} = (6+6) \parallel 4$

$= 12 \parallel 4 = \frac{12 \times 4}{12+4} = \frac{12 \times \cancel{4}}{\cancel{16}_4} = 3\,\Omega$

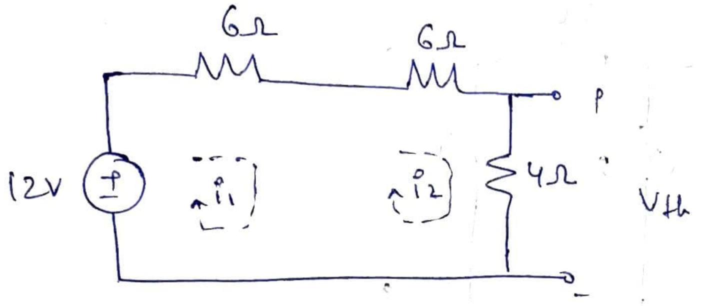
$\therefore R_{th} = 3\,\Omega$

3) Calculate V_{th}:



Apply supermesh,

$$i_2 - i_1 = 2 \rightarrow (1)$$



$$6i_1 + 6i_2 + 4i_2 - 12 = 0 \Rightarrow 6i_1 + 10i_2 = 12$$

$$\Rightarrow 3i_1 + 5i_2 = 6 \rightarrow (2)$$

from (1), $i_2 = 2 + i_1 \rightarrow (3)$

Sub (3) in (2)

$$\Rightarrow 3i_1 + 5(2 + i_1) = 6$$

$$\Rightarrow 3i_1 + 10 + 5i_1 = 6$$

$$\Rightarrow 8i_1 = -4 \Rightarrow i_1 = \frac{-4}{8} = -\frac{1}{2} = \boxed{-0.5A = i_1}$$

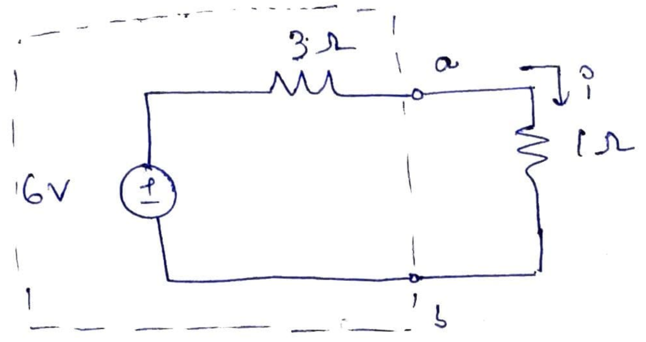
from (3), $i_2 = 2 + i_1 = 2 - 0.5 = \boxed{1.5A = i_2}$

Now,

$$V_{th} = 4i_2 = 4(1.5)$$

$$\boxed{V_{th} = 6V.}$$

∴ Thevenin's circuit with load,



$$6 = (3+1)i = 4i$$

$$\Rightarrow i = \frac{6}{4} = \frac{3}{2} = \boxed{1.5A = i}$$

Norton's equivalent circuit,

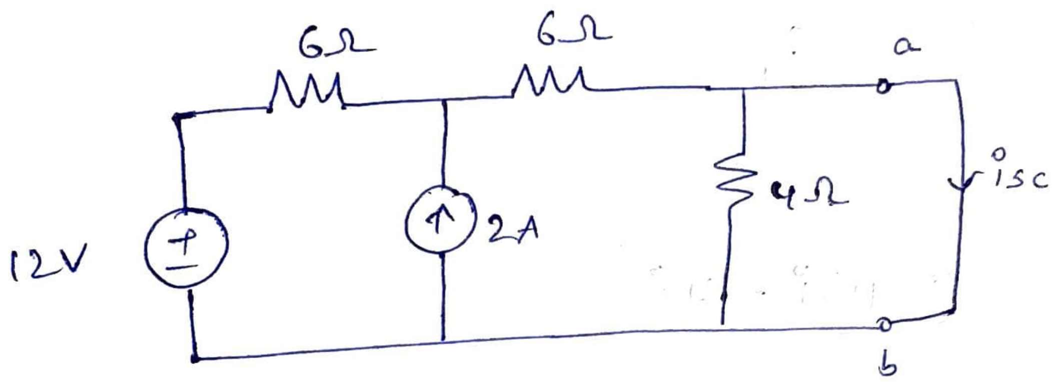
1) Disconnect R_L or load

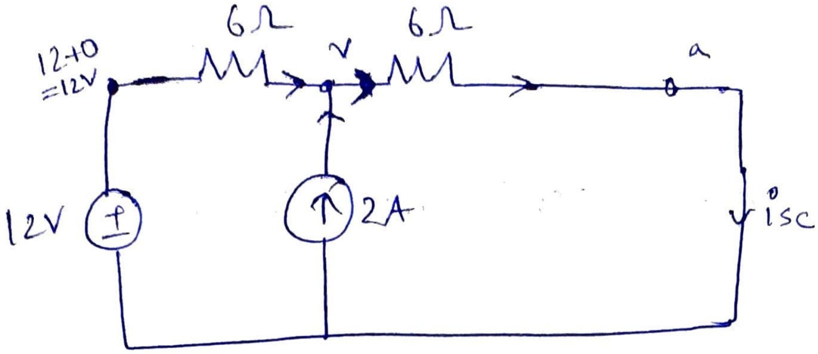
2) Calculate ~~Rth~~, R_N:

{ Same as R_{th}, from previous problem }.

$\therefore R_N = 3\,\Omega$

3) Calculate i_{sc}.





4Ω resistor is in parallel with a shoret circuited line,

So, 4Ω is neglected.

So, Only one node is present.

$$\frac{12-V}{6} + 2 = i_{sc} \Rightarrow \frac{12-V}{6} + 2 = \frac{V-0}{6}$$

$$\Rightarrow 2 - \frac{V}{6} + 2 = \frac{V}{6}$$

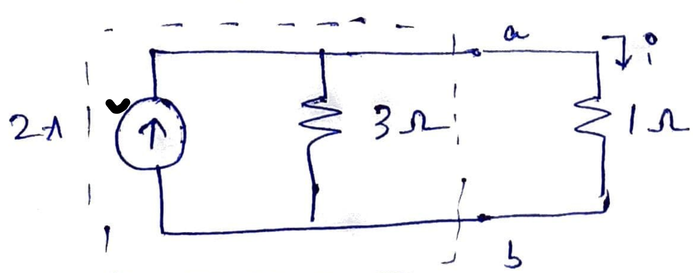
$$\Rightarrow 4 = \frac{V}{6} + \frac{V}{6}$$

$$= \frac{2V}{6} = \frac{V}{3}$$

$$\Rightarrow V = 12V$$

$$\therefore i_{sc} = \frac{V}{6} = \frac{12}{6} = 2A$$

4) Norton's equivalent circuit with load,



from, current divider rule,

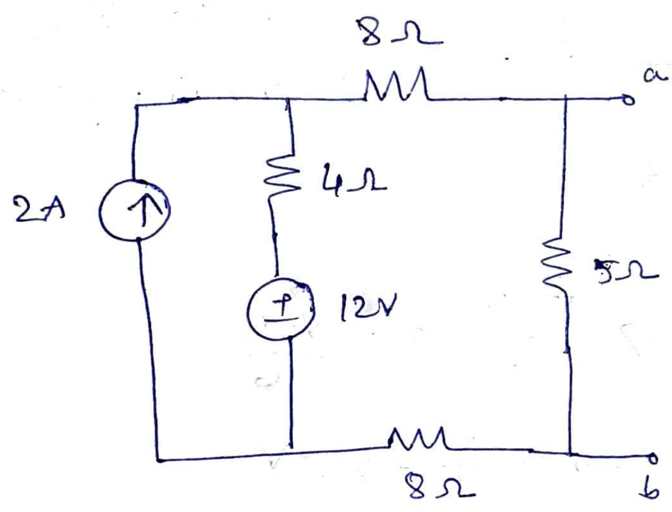
$$i = \frac{3}{3+1} \times 2A$$

$$= \frac{3}{4} \times 2$$

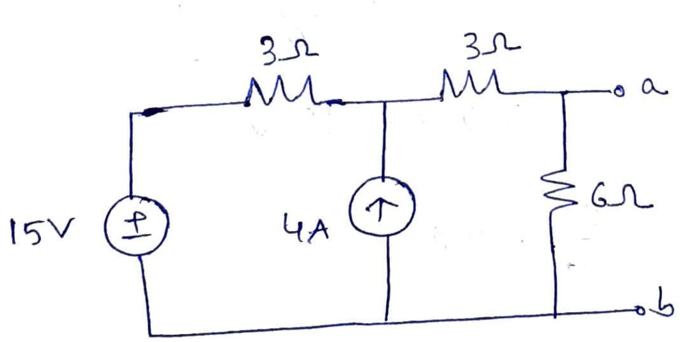
$$= 1.5\ A$$

$$\Rightarrow \boxed{i = 1.5\ A}$$

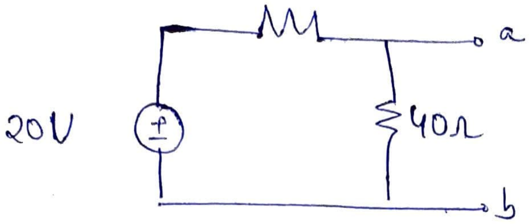
4Q) Draw Thevenin's, Norton's equivalent circuits.

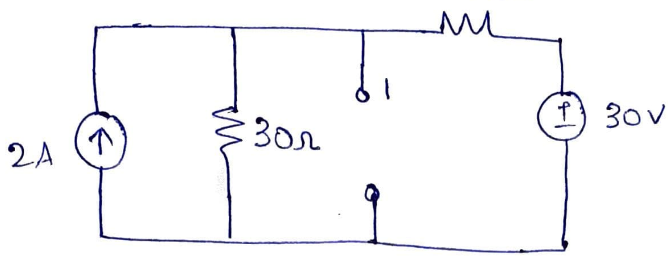


5Q) Draw Thevenin's & Norton's equivalent circuits.

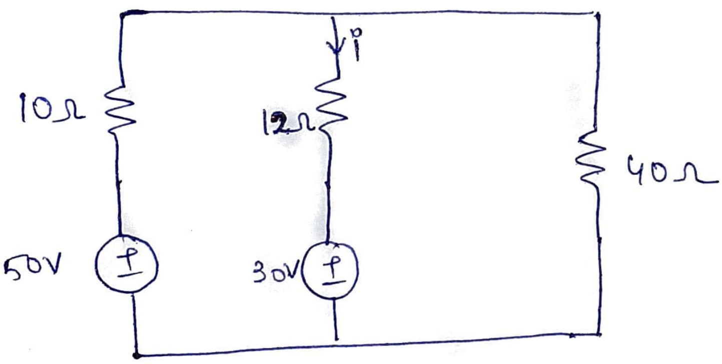


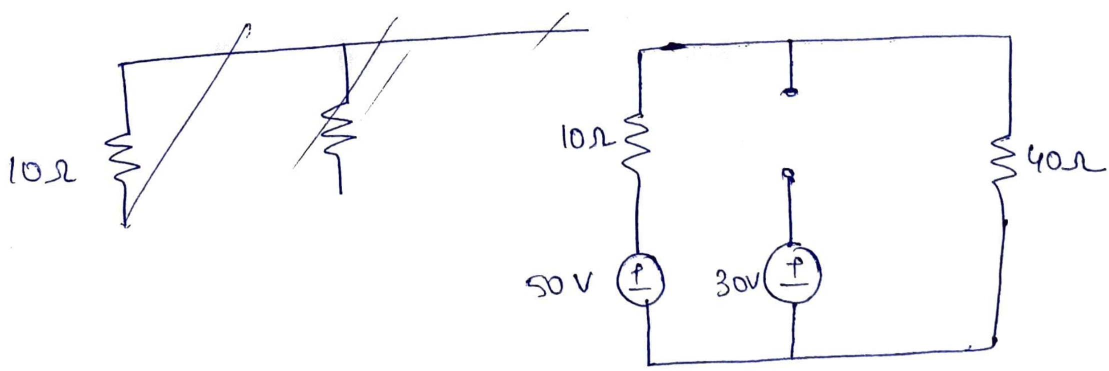
6.Q) Draw Thevenin's, Norton's equivalent circuits.





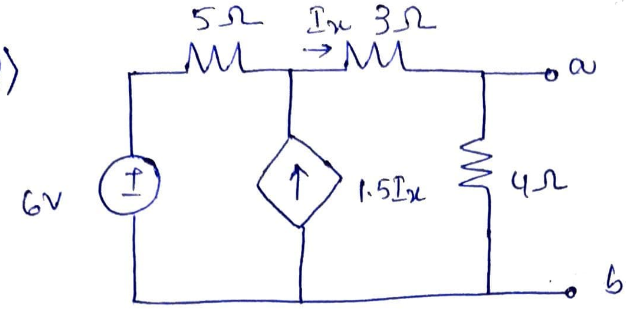
7Q) find 'i' using Thevenin's theorem, i is current through 12Ω





Problem on dependent sources:

Q)



find the thevenin equivalent circuit of the above network to the left of the terminals a, b

Sol) 1) Disconnect R_L (here however there is no R_L)

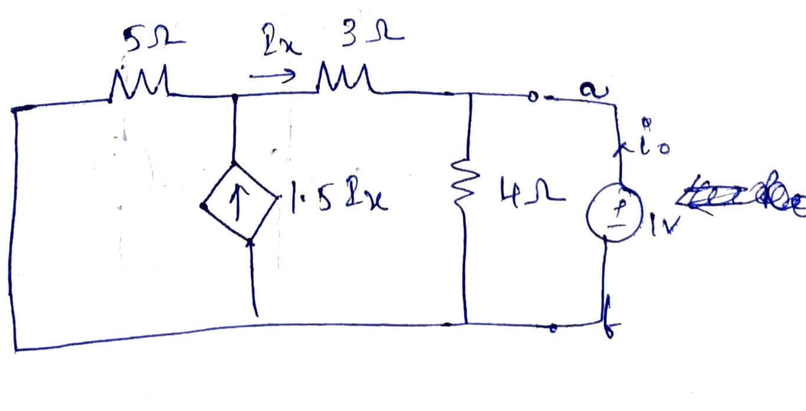
2) Calculate R_{th}.

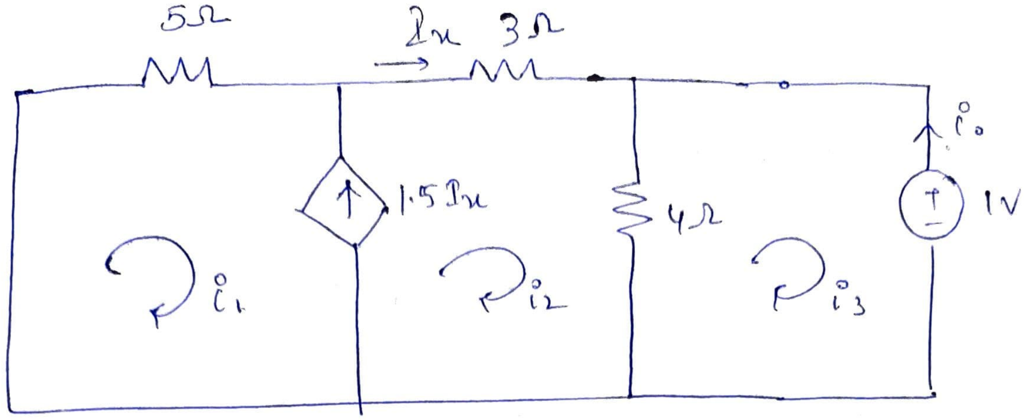
- Replace all independent sources by their internal impedances.

- Dependent sources must remain the same

- Connect a known voltage source $v_o = 1V$ & find i_o supplied by it.

- $R_{th} = \frac{v_o}{i_o}$





from the circuit, $i_2 = I_x \rightarrow (1)$

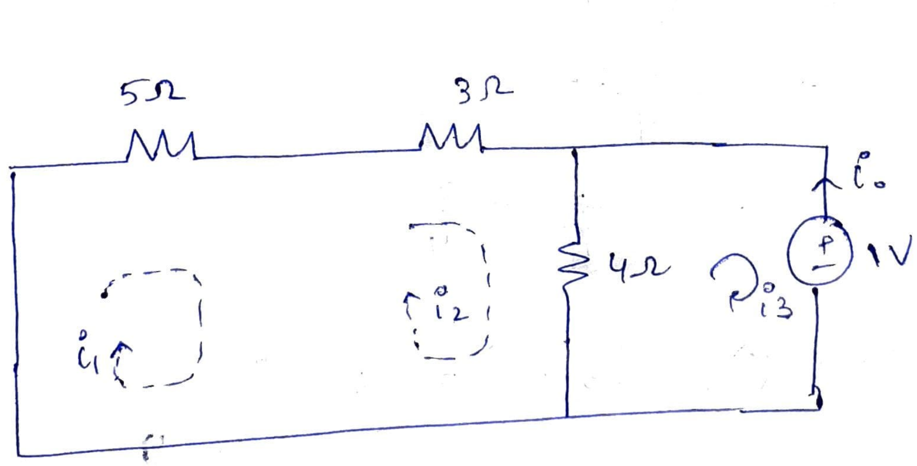
$i_3 = -i_0 \rightarrow (2)$

Apply supermesh, for mesh 1, 2

$i_2 - i_1 = 1.5\, I_x \rightarrow (3)$

$\Rightarrow i_2 - i_1 = 1.5(i_2)$

$\Rightarrow i_1 = -0.5 i_2 \rightarrow (4)$



KVL for supermesh, $5i_1 + 3i_2 + 4(i_2 - i_3) = 0$

$5i_1 + 7i_2 - 4i_3 = 0 \rightarrow (5)$

for mesh 3, $4(i_3 - i_2) + 1 = 0$

$\Rightarrow 4(i_3 - i_2) = -1$

$\Rightarrow i_3 - i_2 = -\frac{1}{4} \Rightarrow i_3 = -\frac{1}{4} + i_2$

$\rightarrow (6)$

Sub (4), (6) in (5).

$$5i_1 + 7i_2 - 4i_3 = 0$$

$$\Rightarrow 5(-0.5i_2) + 7i_2 - 4\left(-\frac{1}{4} + i_2\right) = 0$$

$$\Rightarrow -2.5i_2 + 7i_2 - 4\left(\frac{-1 + 4i_2}{4}\right) = 0$$

$$\Rightarrow -2.5i_2 + 7i_2 + 1 - 4i_2 = 0$$

$$\Rightarrow 0.5i_2 + 1 = 0$$

$$\Rightarrow i_2 = \frac{-1}{0.5} = \frac{-1}{\frac{5}{10}} = \frac{-10}{5} = -2$$

$$\Rightarrow \boxed{i_2 = -2}$$

from (6)

$$i_3 = -\frac{1}{4} + i_2$$

$$i_3 = -\frac{1}{4} - 2$$

$$= \cancel{-\frac{9}{4}} \; -\frac{9}{4}$$

$\therefore$ from (2), $i_3 = -i_0 \Rightarrow i_0 = -i_3$

$$= -\left(-\frac{9}{4}\right)$$

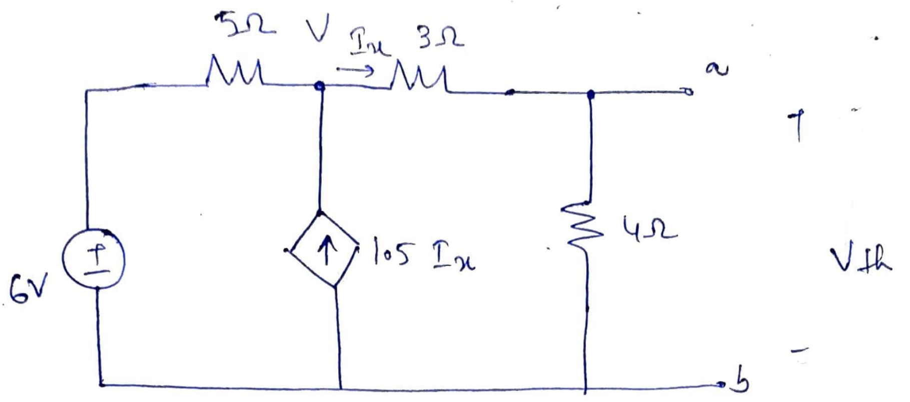
$$= \frac{9}{4}$$

$V_0 = 1V$. (known).

$$\therefore R_{th} = \frac{V_0}{i_0} = \frac{1}{9/4}$$

$$= \frac{4}{9} = \boxed{0.44\,\Omega = R_{th}}$$

3) Calculate V_{th}:



$$I_x = \frac{V - V_{th}}{3} \quad \text{or} \quad \frac{V-0}{3+4} = \frac{V}{7}$$

Apply nodal analysis;

$$\frac{6-V}{5} + 1.5 I_x = \frac{V-0}{3+4}$$

$$\frac{6-V}{5} + 1.5 \times \frac{V}{7} = \frac{V}{7}$$

$$\Rightarrow \frac{6-V}{5} + \frac{1.5V}{7} - \frac{V}{7} = 0$$

$$\Rightarrow \frac{6}{5} - \frac{V}{5} + \frac{0.5V}{7} = 0$$

$$\Rightarrow \frac{6}{5} = \frac{V}{5} - \frac{1}{2} \times \frac{V}{7}$$

$$\Rightarrow \frac{6}{5} = \frac{9V}{14 \times 5}$$

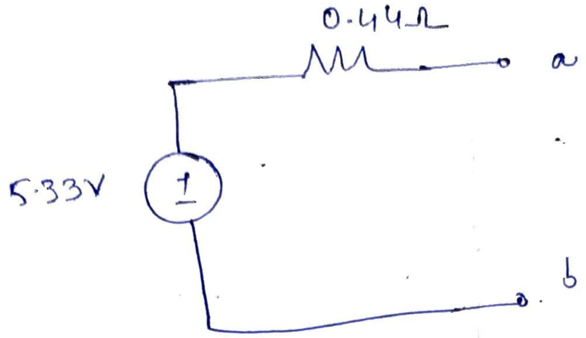
$$\Rightarrow V = \frac{14 \times 2}{3} = 9.33\,V$$

Now, V_{th} = voltage across 4Ω

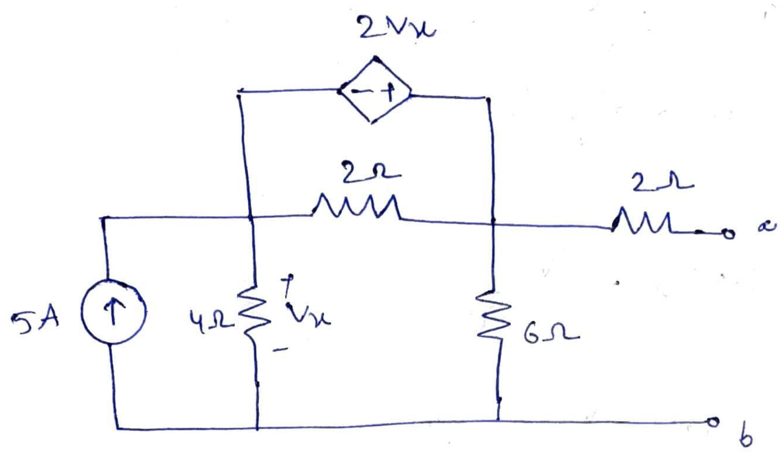
$$= V \times \frac{4}{3+4} = 9.33 \times \frac{4}{7}$$

$$\boxed{V_{th} = 5.33V}$$

∴ Thevenins equivalent circuit is.

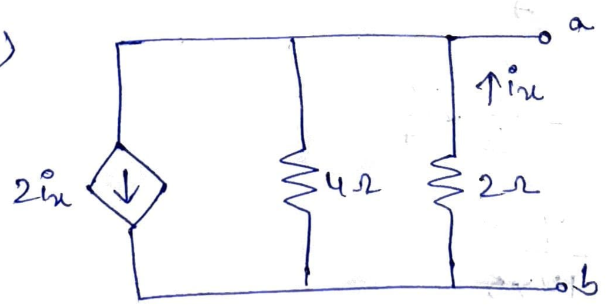


2Q)



ans: $V_{th} = 20V$, $R_{th} = 6\Omega$

3Q)

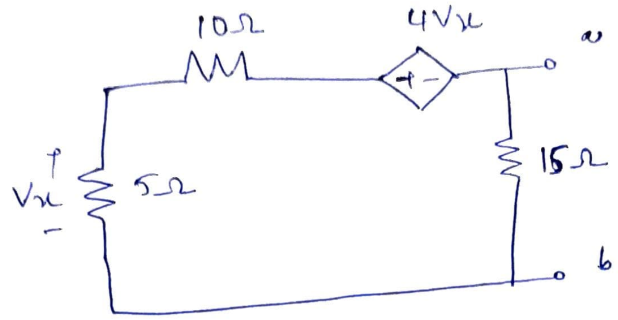


$R_{th} = -4\Omega$

$(V_{th} = 0)$

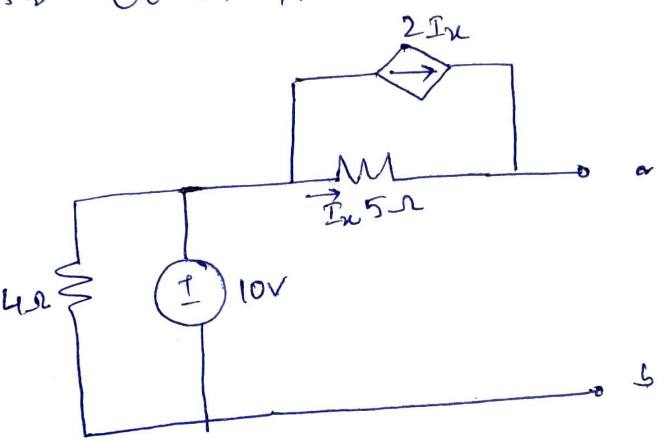
$V_{th} = 0$ as there is no ~~dep~~ independent source in the network.

4Q) Obtain Thevenin's equivalent circuit,

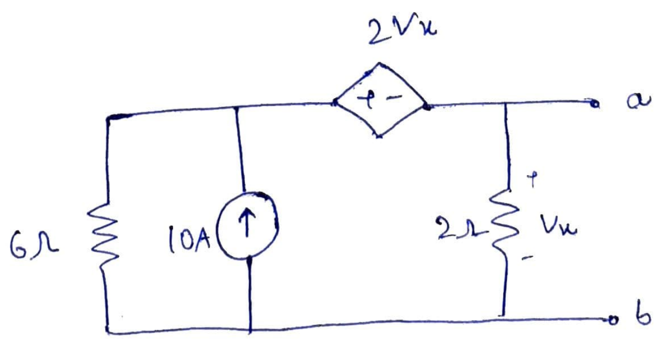


ans: $V_{th} = 0V$, $R_{th} = -7.5\Omega$.

5Q) Obtain, norton's, thevenin's equivalent ckt.



6Q) Draw, thevenin's, norton's equivalent ckts.



~~ans~~

Superposition theorem

It is applicable to linear circuits.

" The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

Note:

- while applying superposition theorem, we consider one independent source at a time, while all other independent sources are turned off

{ Turned off ⇒ the sources are replaced by their internal impedences

⇒ VS $\xRightarrow{by}$ short circuit

⇒ CS replaced by open circuit. }.

Thus, the network becomes simpler.

- Dependent sources are left intact because they are controlled by circuit variables.

Steps to apply Superposition theorem:

1) Turn off all independent sources except one source. find the output (voltage or current) due to that one active source using nodal or mesh analysis

2) Repeat step 1 for all other independent sources, each taken at a time.

3) find the total contribution by adding algebraically all the cond contributions due to independent sources.

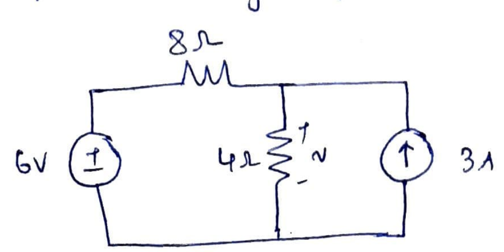
Disadvantages:

- This process becomes time consuming when there are more number of sources.

- Because, superposition theorem is based on linearity, this is not applicable for power calculations.

Example:

Q1) find v using superposition theorem.



Sol) - we have to find v.

- Since there are two sources in the network, we have to analyze two circuits.

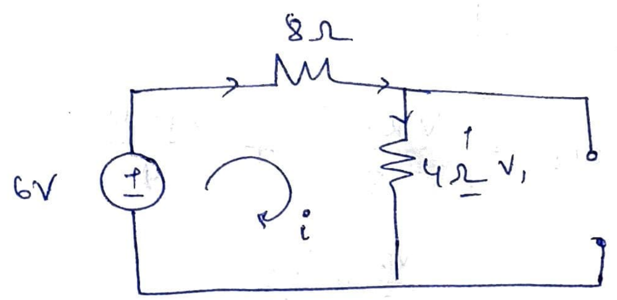
- The value of v obtained in each case be v_1, v_2 respectively

- According to superposition theorem,

$$\boxed{v = v_1 + v_2}$$

Case (i): v_1:

Let v_1 be the voltage at 4Ω when the 6V voltage source alone is acting. ⇒ Current source is turned off.



$6 = (8+4)i = 12i \Rightarrow i = 0.5A$ $\quad \therefore v_1 = i(4)$

$= 0.5 \times 4$

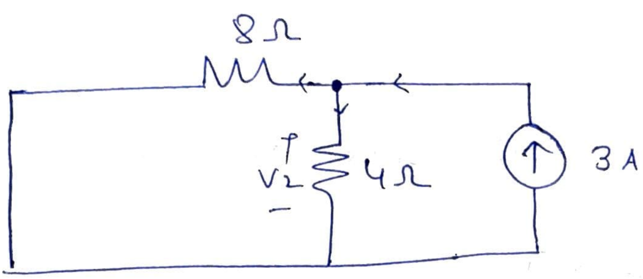
$= 2V.$

(or) apply voltage divider rule,

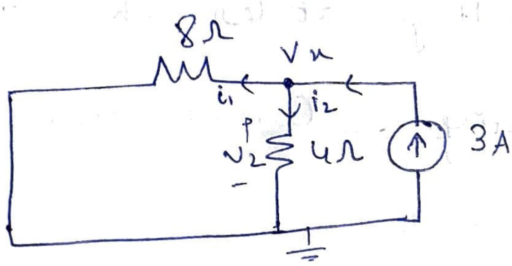
$$v_1 = \frac{4}{8+4} \times 6 = \frac{4}{12} \times 6 = 2V.$$

Case (ii): v_2:

Let v_2 be the voltage at 4Ω when current source of 3A is acting alone. $\Rightarrow$ the voltage source is turned off



Apply nodal analysis, let the node voltage be V_x



$$\Rightarrow \quad 3 = i_1 + i_2 = \frac{V_x - 0}{8} + \frac{V_x - 0}{4}$$

$$= \frac{V_x}{8} + \frac{V_x}{4}$$

$$= \frac{3V_x}{8}$$

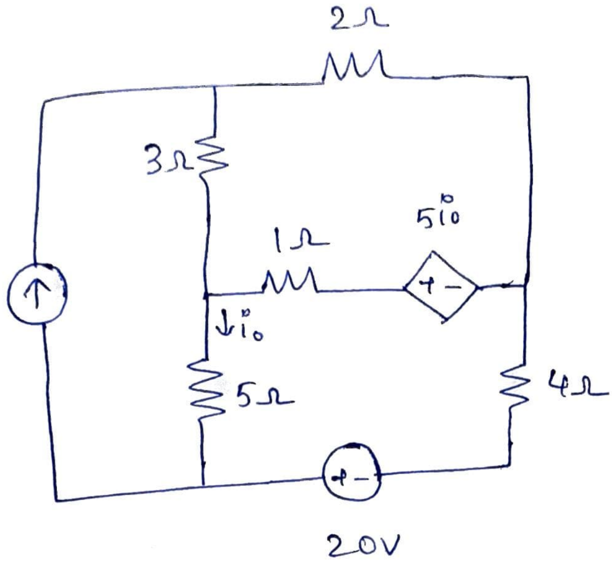
$$\Rightarrow \boxed{V_x = 8V}$$

V_x is same as v_2 here, $\Rightarrow \boxed{V_2 = 8V}$

According to superposition theorem.

$$v = v_1 + v_2 = 2 + 8 = \boxed{10V = v}$$

2Q)



Maximum power transfer theorem

- Practically, a circuit / network is designed to provide power to the load.

- In areas such as communications, it is desirable to maximize power to the load.

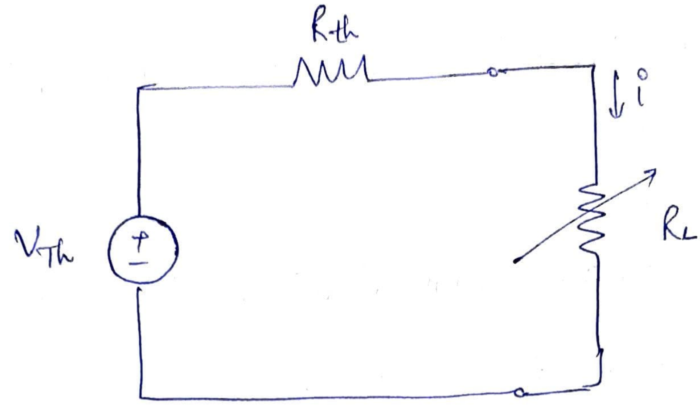
{ While for electric circuits, minimizing power losses in the process of transmission & distribution is critical for ~~f~~ efficiency and economic reasons }.

- When we deliver maximum power to the load, it must be noted that, this will result in significant internal losses, which will be greater than or equal to the power delivered to the load.

- The thevenin's equivalent is useful in finding the maximum power a linear circuit can deliver to a load.

- We assume that load is adjustable.

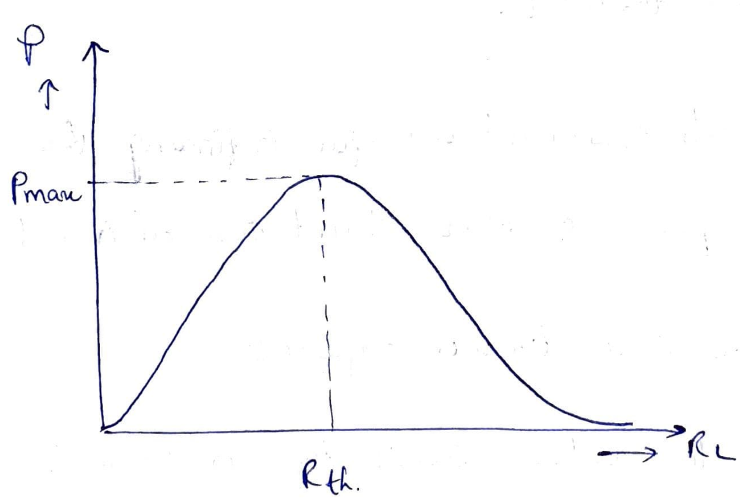
- Replace the entire circuit by its Thevenin's equivalent except for the load R_L. Then, the circuit will appear as shown.



From the above circuit, power delivered to the load is, $P = i^2 R_L = \left(\frac{V_{th}}{R_{th} + R_L}\right)^2 \times R_L \rightarrow (1)$

- for a given circuit, V_{th}, R_{th} are fixed. Hence, the only variable here is R_L.

Graph drawn for Power delivered to the load as a function of R_L :



- Power is small for small or large values of R_L
- It's maximum occurs at some point between 0 to ∞ of R_L.

- Proof that maximum power occurs at $R_L = R_{th}$:

To prove this, we differentiate P (i.e eq ①) with respect to R_L and set the result equal to zero.

$$\frac{dP}{dR_L} = V_{th}^2 \left[\frac{d}{dR_L} \left(\frac{R_L}{(R_{th}+R_L)^2} \right) \right] = 0$$

$$\Rightarrow V_{th}^2 \left[\frac{1 \cdot (R_{th}+R_L)^2 - R_L[2(R_{th}+R_L)]}{((R_{th}+R_L)^2)^2} \right] = 0$$

$$\Rightarrow V_{th}^2 \left[(R_{th}+R_L)^2 - R_L(2(R_{th}+R_L)) \right] = 0$$

$$\Rightarrow V_{th}^2 \left[(R_{th}+R_L) - R_L(2) \right] = 0$$

$$\Rightarrow (R_{th}+R_L) - R_L(2) = 0$$

$$\Rightarrow R_{th}+R_L = 2R_L$$

$$\Rightarrow \boxed{R_L = R_{th}}$$

* $\Rightarrow$ Maximum power transfer takes place when the load resistance R_L equals the Thevenin's resistance R_{th}.

This is known as the maximum power theorem

" Maximum power is transferred to the load ~~res~~ when the load resistance equals the thevenin's resistance as seen from the load ($R_L = R_{th}$) "

∴ The maximum power transferred is given by

$$P_{max} = P\Big|_{at\, R_L = R_{th}} \quad (\text{from eq } 1)$$

$$= \left(\frac{V}{R_{th} + R_L}\right)^2 R_L \Bigg|_{at\, R_L = R_{th}}$$

$$= \left(\frac{V}{R_{th} + R_{th}}\right)^2 R_{th}$$

$$= \frac{V^2}{(2R_{th})^2} \times R_{th}$$

$$= \frac{V^2}{4R_{th}^2} \times R_{th}$$

$$\boxed{P_{max} = \frac{V^2}{4R_{th}}}$$

This equation holds good only when $R_L = R_{th}$.

If $R_L \neq R_{th}$, we have to go with eq 1.

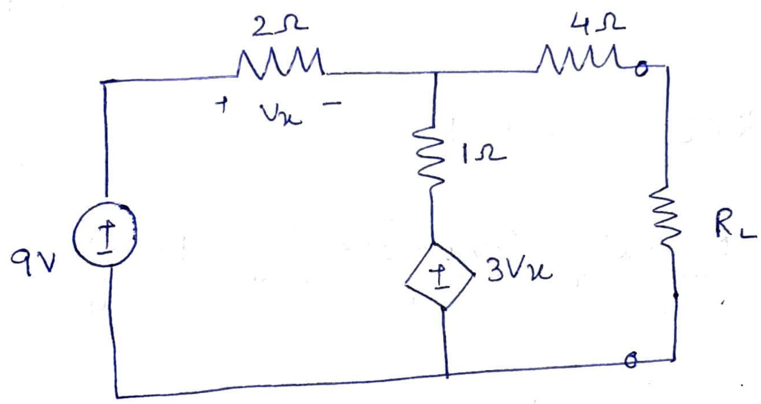
* For AC circuits, maximum power is transferred

when $Z_L = Z_S^*$

where, Z_L = Load impedance

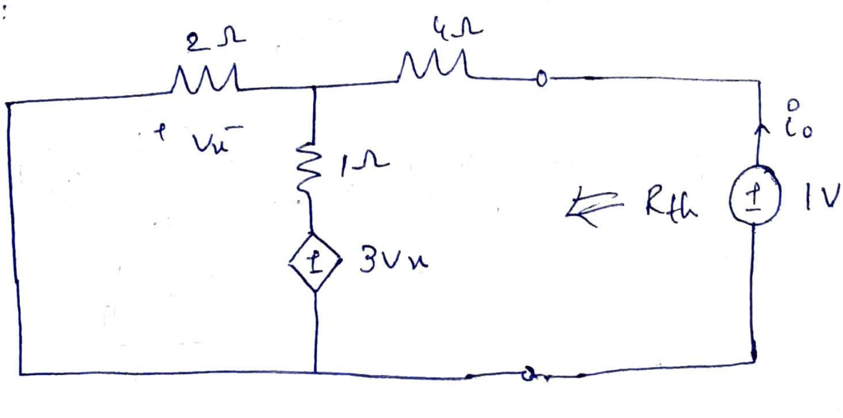
& Z_S = Source impedance

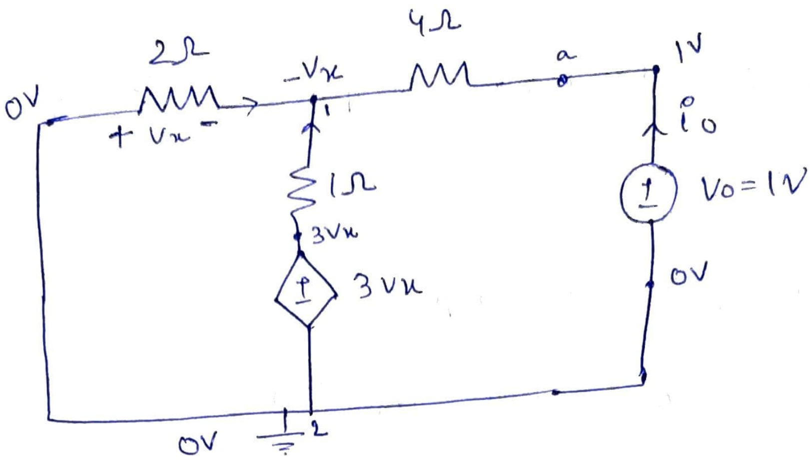
Q1) Determine the value of R_L that will draw maximum power from the circuit. Calculate maximum power.

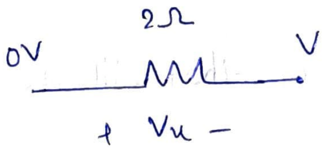


Sol) To find the value of R_L for maximum power transfer, firstly, the thevenin's equivalent circuit across the load terminals must be drawn.

R_{th}:







$$\Rightarrow 0 - V = V_x$$
$$\Rightarrow V = -V_x$$

Let all the currents be incoming currents at node 1.

∴ Apply KCL at node 1,

$$\Rightarrow \frac{0-(-V_x)}{2} + \frac{3V_x-(-V_x)}{1} + \frac{1-(-V_x)}{4} = 0$$

$$\Rightarrow \frac{V_x}{2} + \frac{3V_x+V_x}{1} + \frac{1+V_x}{4} = 0$$

$$\Rightarrow \frac{V_x}{2} + 4V_x + \frac{1}{4} + \frac{V_x}{4} = 0$$

$$\Rightarrow \frac{19}{4} V_x = -\frac{1}{4}$$

$$\Rightarrow V_x = -\frac{1}{19}\ V$$

$$\therefore i_o = \frac{1-(-V_x)}{4} = \frac{1-\left(-\left(-\frac{1}{19}\right)\right)}{4}$$

$$= \frac{1-\frac{1}{19}}{4}$$

$$= \frac{18}{4 \times 19}$$

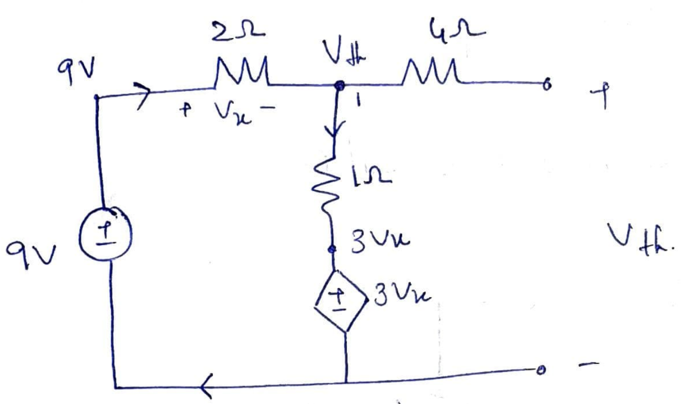
$$= 0.237 \, A$$

$$\therefore R_{th} = \frac{V_o}{i_o} = \frac{1}{0.237}$$

$$= 4.219$$

$$\approx 4.22 \, \Omega$$

V_{th}:



$$\left\{ 9 - V_{th} = V_x \right\}$$

Apply KCL at node 1,

$$\frac{9 - V_{th}}{2} = \frac{V_{th} - 3V_x}{1}$$

$$\frac{9}{2} - \frac{V_{th}}{2} = V_{th} - 3(9 - V_{th})$$

$$\frac{9}{2} - \frac{V_{th}}{2} = V_{th} - 27 + 3V_{th}$$

$$\Rightarrow \frac{9}{2} + 27 = 4V_{th} + \frac{V_{th}}{2}$$

$$\Rightarrow \frac{9+27(2)}{2} = \frac{8V_{th}+V_{th}}{2}$$

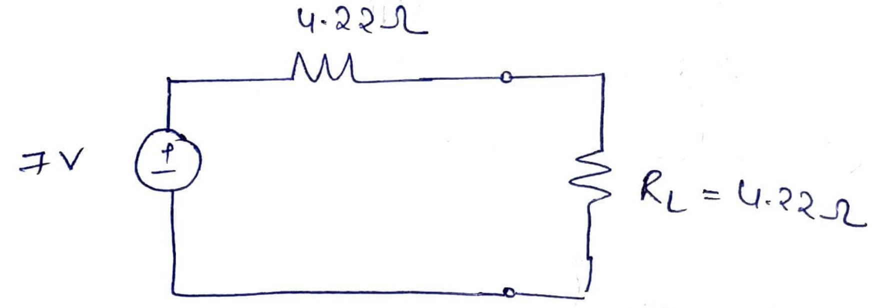
$$\Rightarrow 9+27(2) = 9V_{th}$$

$$\Rightarrow 9(1+3(2)) = 9V_{th}$$

$$\Rightarrow V_{th} = 1+6 = 7V$$

$$\Rightarrow \boxed{V_{th} = 7V}$$

Now, thevenin's equivalent circuit is,



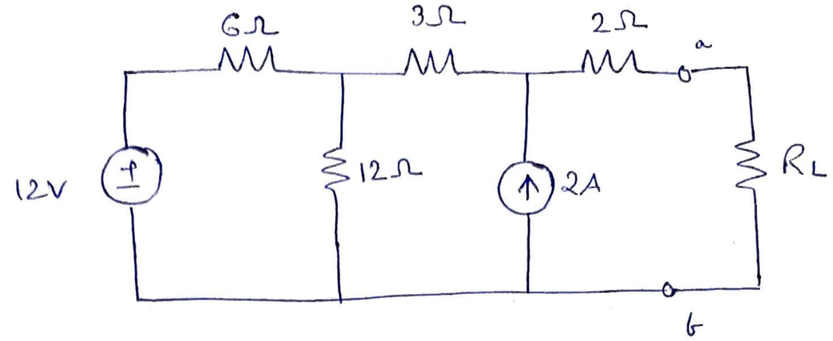
maximum power delivered, $P_{max} = \frac{V_{th}^2}{4R_{th}}$.

$$= \frac{7^2}{4 \times 4.22}$$

$$= \frac{49}{4 \times 4.22}$$

$$\therefore \boxed{P_{max} = 2.9\,W}$$

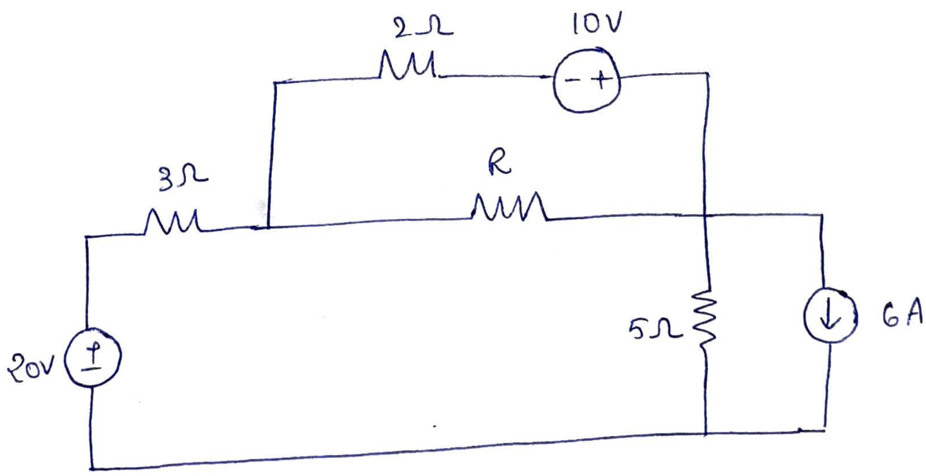
Q2)



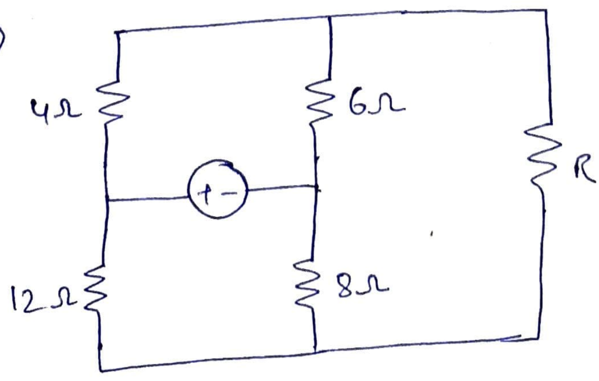
find the value of R_L for maximum power transfer in the above circuit. find the maximum power

Ans: $R_L = 9\Omega$, $P_{max} = 13.44\,W$.

Q3) find the maximum power that can be delivered to the resistor R in the circuit, shown.

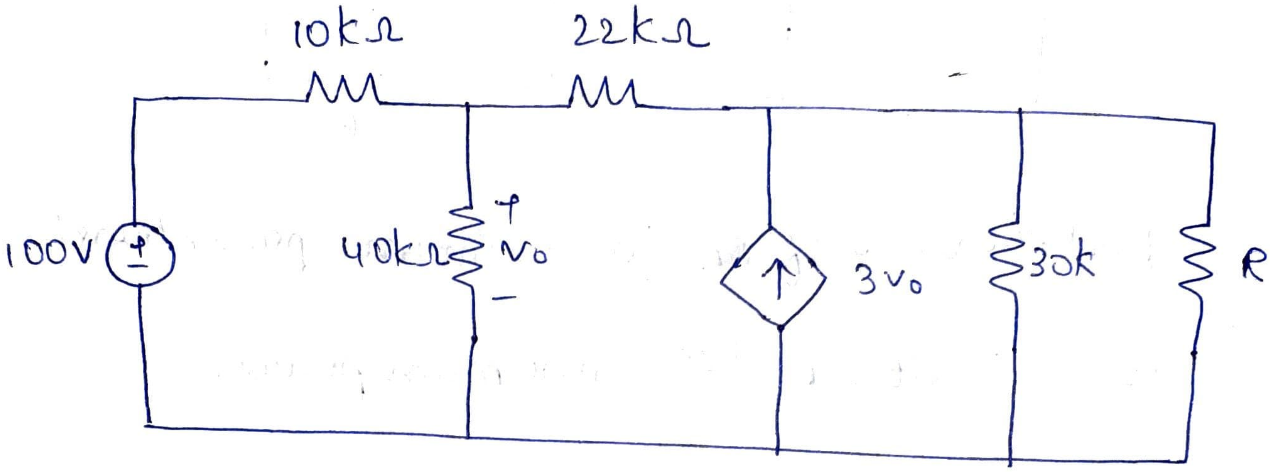


Q4)



find R for maximum power dissipation. Also calculate that power.

5Q) find the maximum power transferred to the resistor R in the circuit



Millman's theorem

Statement:

"If n no. of voltage sources $V_1, V_2, V_3, \ldots V_n$ having internal resistance $r_1, r_2, \ldots r_n$ are connected in parallel across the load R_L, then this arrangement may be replaced by a single voltage source V_{eq} in series with equivalent resistance R_{eq}.

Consider the below circuit

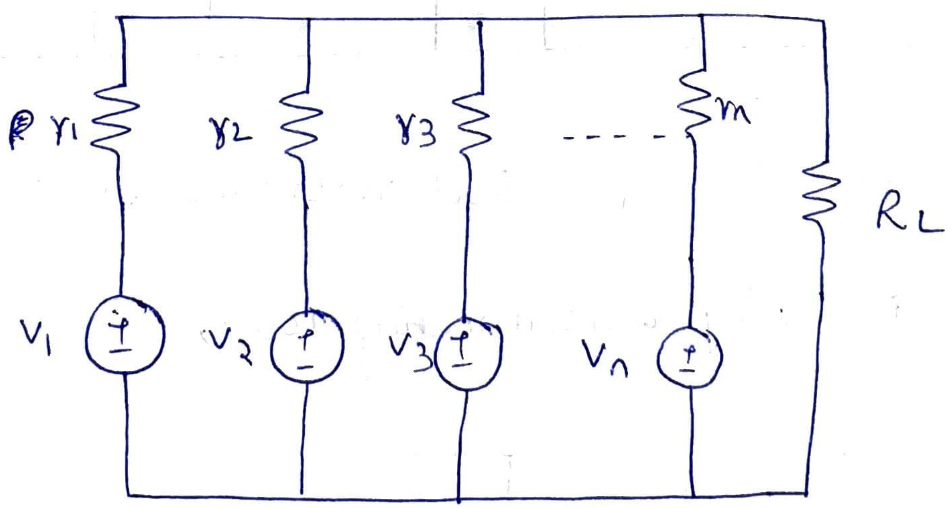


fig 1

then according to Millman's theorem, it can be represented as,

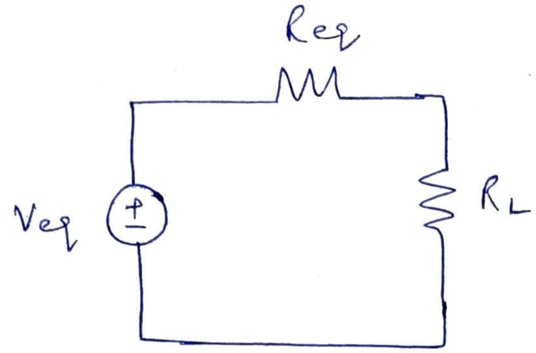
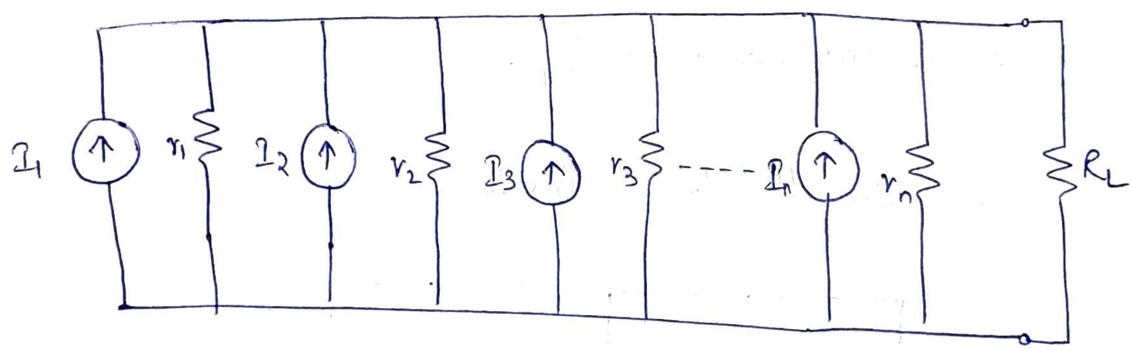


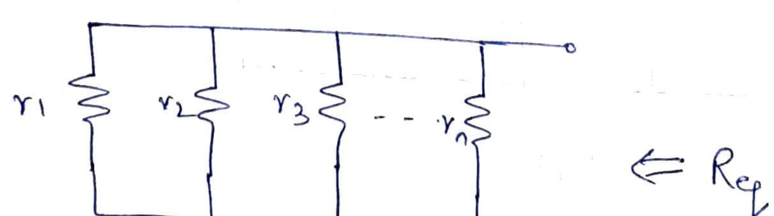
fig 2

Calculation of V_{eq}, R_{eq}

from fig 1, convert all the voltage sources into their respective current sources using source transform-ation.



Now, calculate equivalent resistance across the load terminals, by disconnecting load,



$$\frac{1}{Req} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \dots + \frac{1}{r_n}$$

$$\frac{1}{Req} = G_1 + G_2 + G_3 + \dots + G_n$$

$$\Rightarrow \boxed{Req = \frac{1}{G_1 + G_2 + G_3 + \dots + G_n.}} \longrightarrow (1)$$

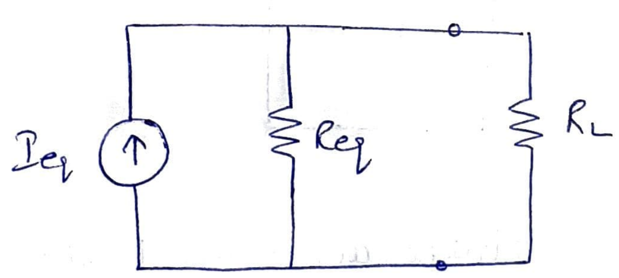
Let 'I' be the resultant current of the parallel current sources.

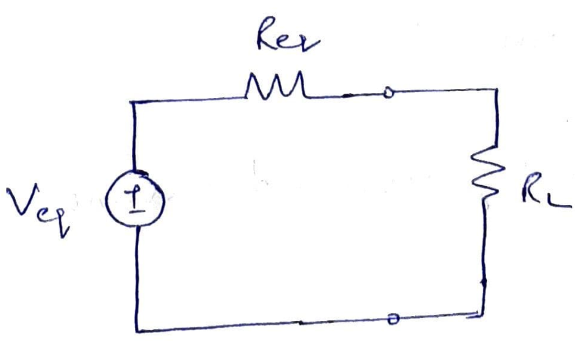
$$I_{eq} = I_1 + I_2 + I_3 + \dots + I_n$$

$$I_{eq} = \frac{V_1}{r_1} + \frac{V_2}{r_2} + \frac{V_3}{r_3} + \dots + \frac{V_n}{r_n}$$

$$I_{eq} = V_1 G_1 + V_2 G_2 + V_3 G_3 + \dots + V_n G_n \longrightarrow (2)$$

Now, equivalent circuit is,





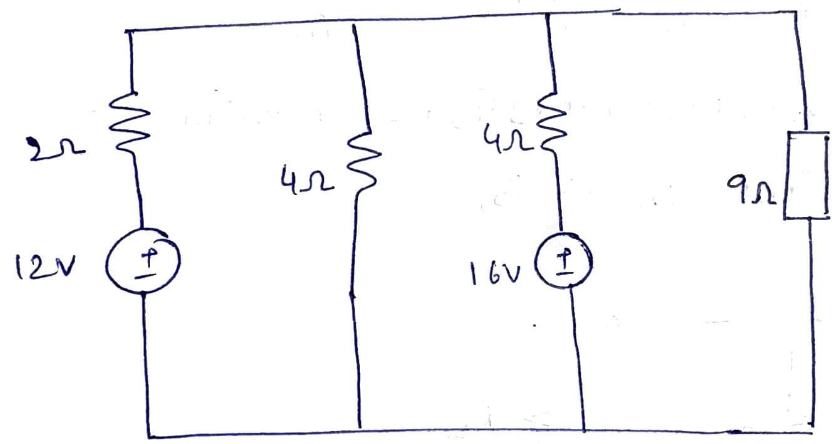
$$\therefore V_{eq} = I_{eq} \cdot R_{eq}.$$

$$\Rightarrow V_{eq} = \left(V_1 G_1 + V_2 G_2 + V_3 G_3 + \dots + V_n G_n\right) \cdot \left(\frac{1}{G_1 + G_2 + G_3 + \dots + G_n}\right)$$

(from ①, ②)

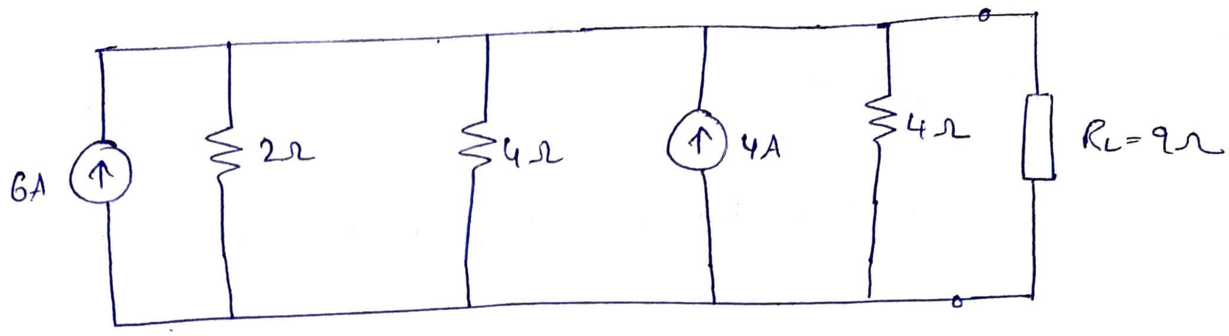
$$\boxed{V_{eq} = \frac{V_1 G_1 + V_2 G_2 + V_3 G_3 + \dots + V_n G_n}{G_1 + G_2 + G_3 + \dots + G_n}}$$

Example:



find current and voltage across the load terminal using millman's theorem.

Sol) Convert all the voltage sources to current sources:



$$R_{eq} = \frac{1}{G_1 + G_2 + G_3}$$

$$= \frac{1}{\frac{1}{2} + \frac{1}{4} + \frac{1}{4}}$$

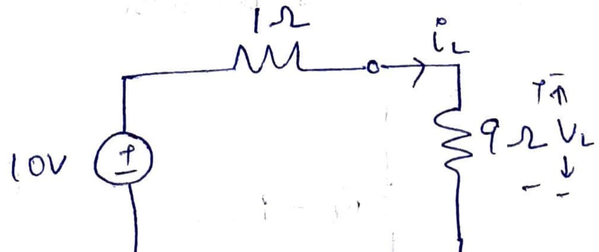
$$= \frac{1}{1} = 1\,\Omega$$

$$V_{eq} = \frac{12 \times \frac{1}{2} + 0 \times \frac{1}{4} + 16 \times \frac{1}{4}}{\frac{1}{2} + \frac{1}{4} + \frac{1}{4}}$$

$$= \frac{6 + 0 + 4}{1}$$

$$V_{eq} = 10\,V$$

Equivalent circuit is,



$$i_L = \frac{10}{1+9} = \frac{10}{10} = \boxed{1A = i_L}$$

$$V_L = i_L \times 9$$

$$= 1 \times 9 = \boxed{9V = V_L}$$

Applications of Millaman's theorem:

- ~~Adc~~ This theorem is very convenient for determining the voltage across a set of parallel branches.

- It is easy to apply.

- The Millman's theorem is mostly applied for circuits with several operational amplifiers representing complex circuit topology

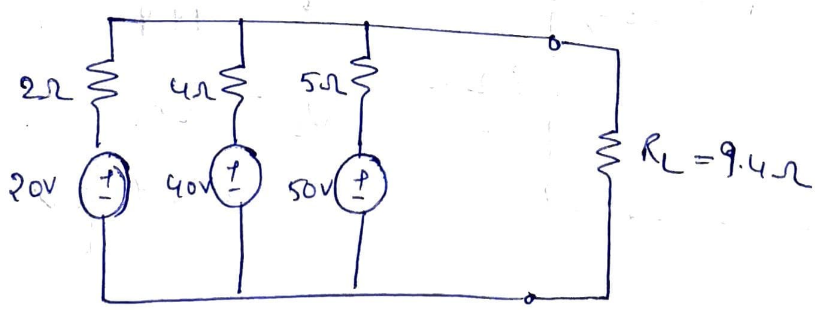
Disadvantages of Millman's theorem:

- Not applicable to circuits consisting of impedances between independent sources

- Not applicable to the circuits consisting of dependent source between the independent source

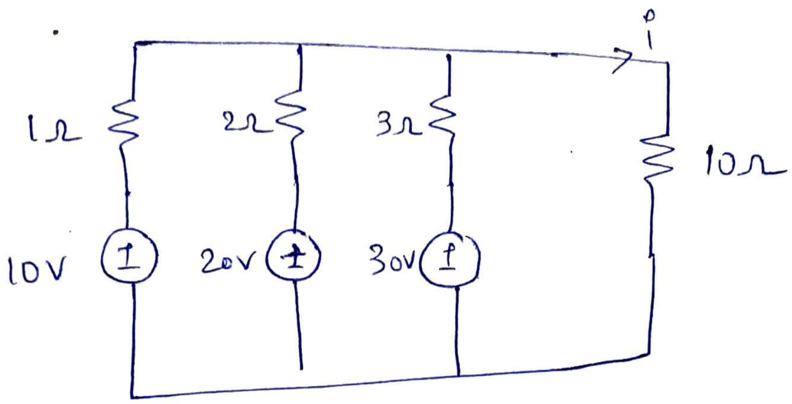
- Not applicable when the sources are connected in series.

Q1) find 'i' through R_L using Millman's theorem.

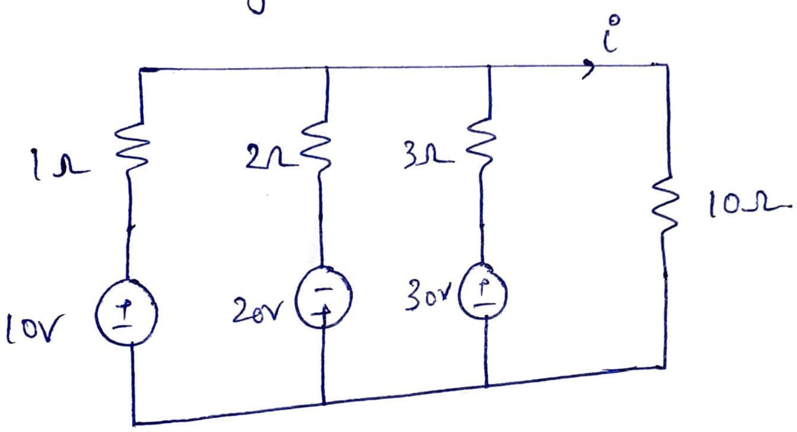


Ans: $i = 3.02A$

2) find i using Millman's theorem.



3) find i using Millman's theorem,



Ans: $i = 0.5172\,A$

Source transformation

- It is another used to simplify a given network

"A source transformation is the process of replacing a voltage source Vs in series with a resistor R by a current source is in parallel with a resistor R, (or) vice versa."

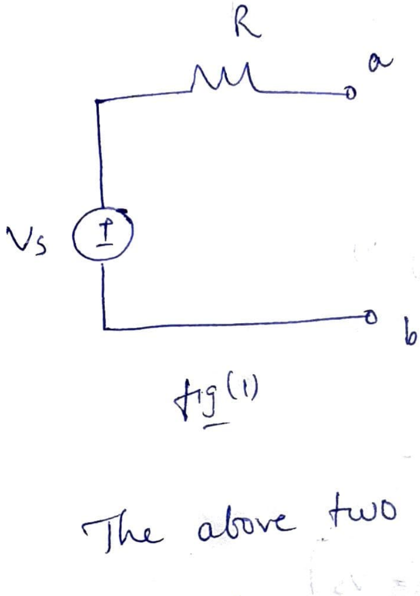
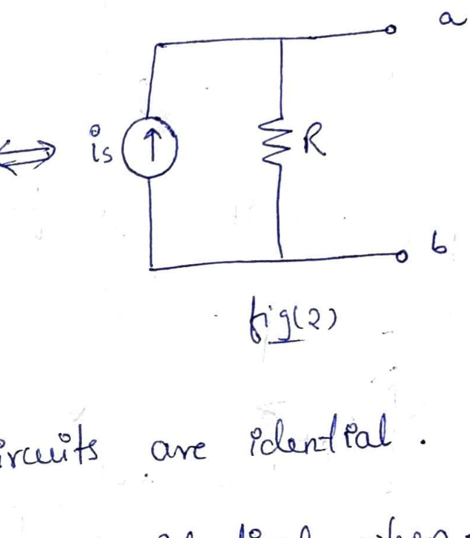


fig (1)



fig(2)

The above two circuits are identical.

Two circuits are said to be identical, when their v-i characteristics is identical

So, here, v, i at a, b terminals must be same for both circuit.

→ Equivalent resistance is same for both circuits.

fig (1).

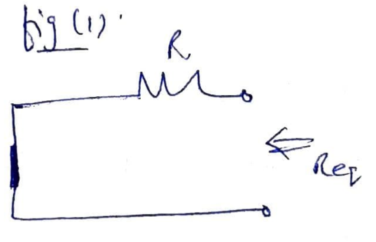
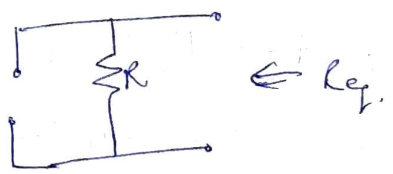
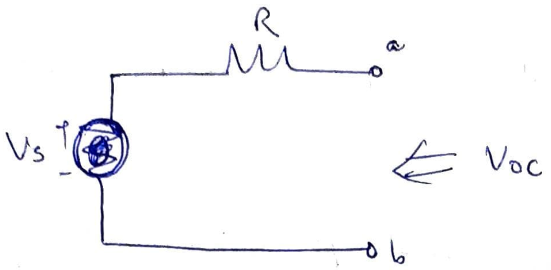


fig 2

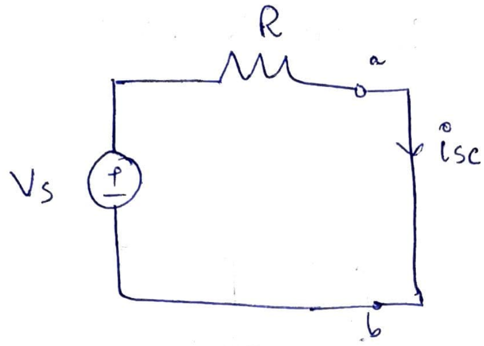


→ Open circuit voltage for fig (1)



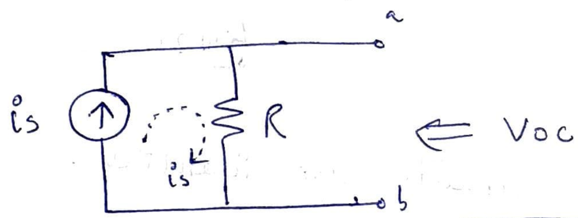
$\Rightarrow \boxed{V_{oc} = V_s}$

Short circuit current for fig (1)



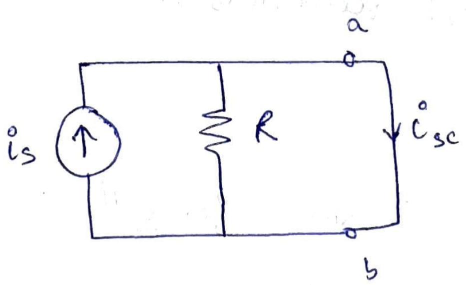
$\Rightarrow \boxed{i_{sc} = \frac{V_s}{R}} = i_s$

→ Open circuit voltage for fig (2),



$\boxed{V_{oc} = i_s \cdot R = V_s}$

Short circuit voltage for fig (2).



$\Rightarrow \boxed{i_{sc} = i_s}$

i.e resistance R is neglected.

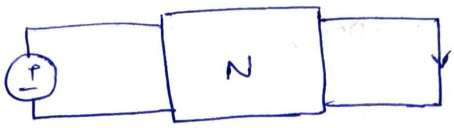
∴ fig 1, fig 2 circuits are identical.

- Source transformation can be applied to dependent sources as well.

Reciprocity Theorem:

" In a linear bilateral single source, the ratio of response to excitation remains the same even when the positions of response & excitation are interchanged."

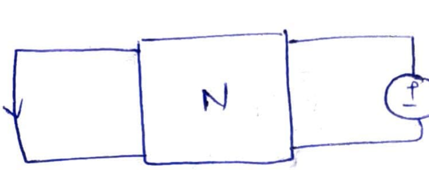
Vx (exct)



Iy (resp)

" The ratio of response to excitation is constant for reciprocal networks"

Iy (resp)



Vx (exct)

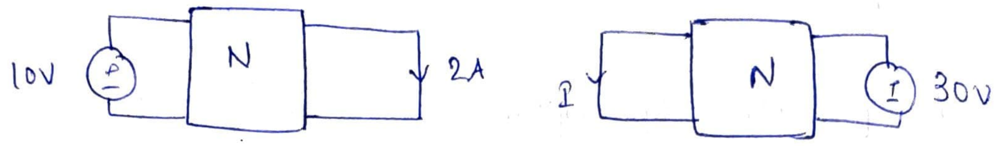
$$\boxed{\frac{\text{Resp}}{\text{exct}} = \text{constant}}$$

" If an emf E in one branch produces a current I in a second branch, then if the same emf E is moved from the first branch to the second branch, it will produce the same current in the first branch, when the emf E in the first branch is replaced by a short circuit."

Reciprocity theorem is applicable only if,

- The circuit is bilateral, linear.
- The ratio of response to excitation is either ohm or mho
- One independent source is only present in the network.
- No dependent sources are present.

Q1) find the value of current I when the network satisfies the reciprocity conditions.



Sol: According to reciprocity theorem,

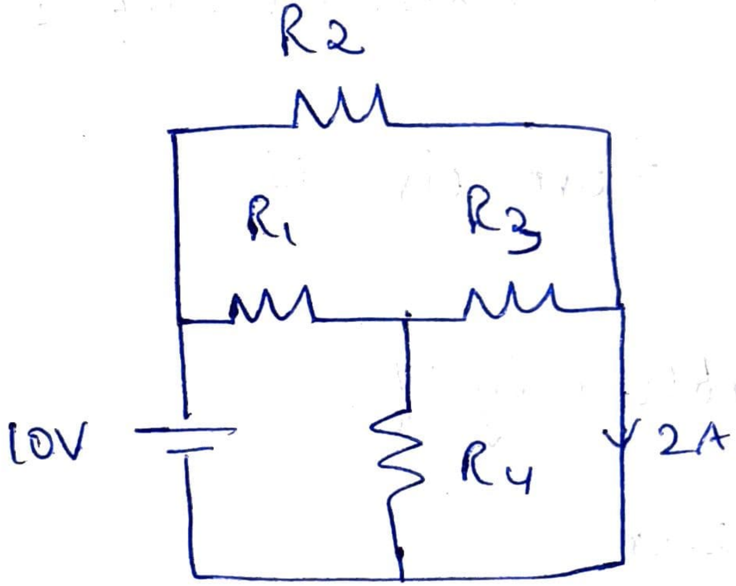
response to ~~excition~~ excitation ratio is a constant for both circuits.

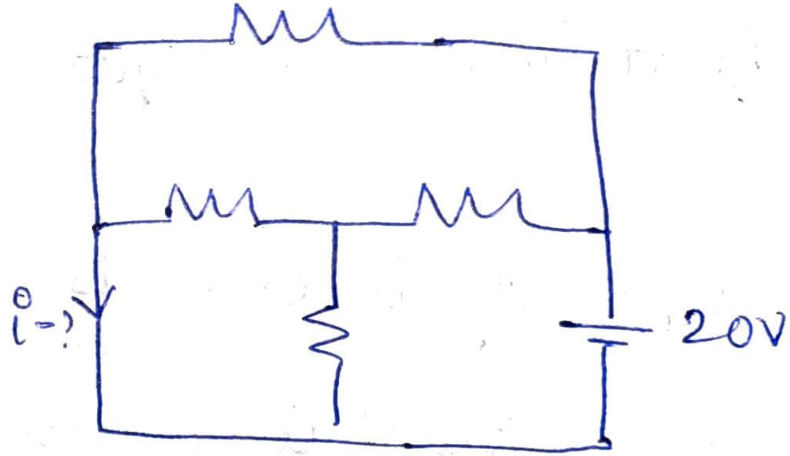
$$\therefore \left(\frac{Resp}{exct}\right)_1 = \left(\frac{Resp}{exct}\right)_2$$

$$\Rightarrow \frac{2}{10} = \frac{I}{30}$$

$$\Rightarrow \boxed{I = 6A}$$

Q2) find the current i in the circuit of the figure.





(a) −2A (b) 2A (c) −4A (d) +4A

Sol) Check whether reciprocity theorem is applicable or not,

1) All the elements are linear

2) $\frac{Resp}{Exct}$ = ohm (or) mho

3) One independent source must be present

4) No dependent source must be present

$$\frac{Resp}{Exct} = constant \Rightarrow \left(\frac{Resp}{exct}\right)_1 = \left(\frac{Resp}{exct}\right)_2$$

$$\Rightarrow \frac{2}{10} = \frac{i}{20}$$

$$\Rightarrow \boxed{i = 4A}$$

02) Mesh Analysis:

1Q) solved

2Q) $i_1 = \frac{2}{3} A$, $i_2 = 0A$

3Q) $i_o = 1.5 A$

4Q) $-5A$ (missing voltage source value is 20V)

(with current sources)

1Q) $i_1 = 3.474 A$

$i_2 = 0.4737 A$

$i_3 = 1.1052 A$

2Q) $i_1 = -7.5A$

$i_2 = -2.5A$

$i_3 = 3.93A$

$i_4 = 2.143A$

(Missing current source value is 5A)

03) Nodal Analysis:

2Q) $V_1 = 4.8V$, $V_2 = 24V$, $V_3 = -2.4V$

Q) $V_1 = -2V$, $V_2 = -14V$

3Q) $V_1 = 80V$, $V_2 = -64V$, $V_3 = 156V$

(i_x is current flowing through 4Ω downwards).

(with voltage sources)

3Q) $V_1 = -7.33V$

$V_2 = -5.33V$

(missing values ↑ 2A, ↓ 7A)

4Q) $v = -0.2V$

$i = 1.4A$

5Q) $V_1 = 26.667V$

$V_2 = 6.667V$

$V_3 = 173.33V$

$V_4 = -46.667V$

04) Thevenin's & Norton's Th

(Thevenin's)

2Q) $\{V_{th} = 6V,\ R_{th} = \frac{18}{5}\Omega\}$

$i = \frac{30}{23} A.$

05) Numericals on Th & N theorems :

4Q) $R_{th} = 4\Omega$

$V_{th} = 4V$

$I_N = 1A$

5Q) $R_{th} = 3\Omega$

$V_{th} = \frac{27}{2} V$

$I_N = \frac{9}{2} A$

6Q) – $R_{th} = 8\Omega$

$V_{th} = 16V$

$I_N = 2A$

– $R_{th} = 20\Omega$

$V_{th} = 50V$

$I_N = 2.5A$

7Q) $V_{th} = 10V$

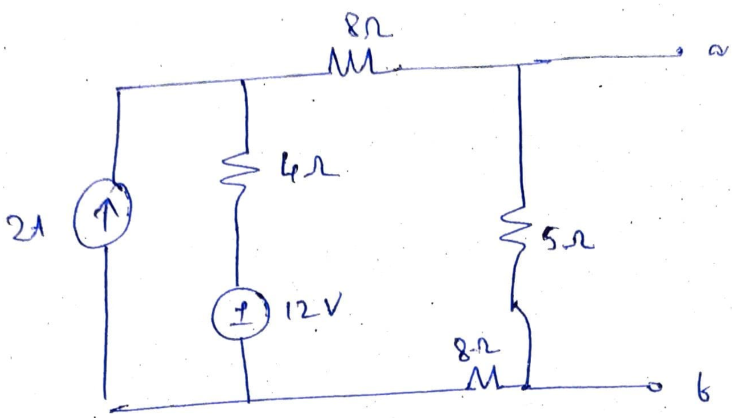
$R_{th} = 8\Omega$

~~i = ...~~

$i = 0.5A.$

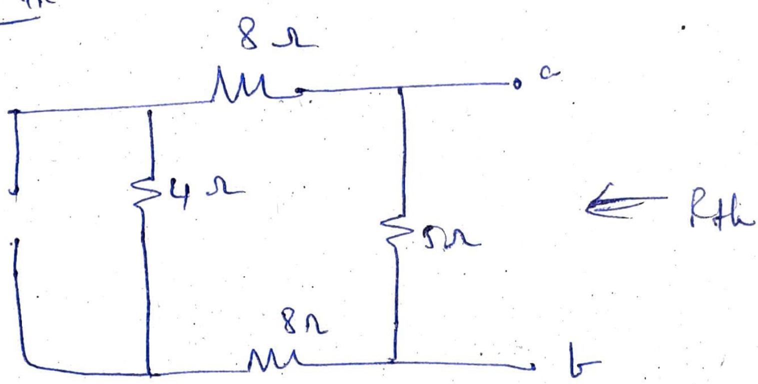
" * Solutions to (pdf 5) Numericals on Th & N Theorems * "

4Q) Draw Thevenin's & Norton's equivalent ckt



Sol)

Rth:



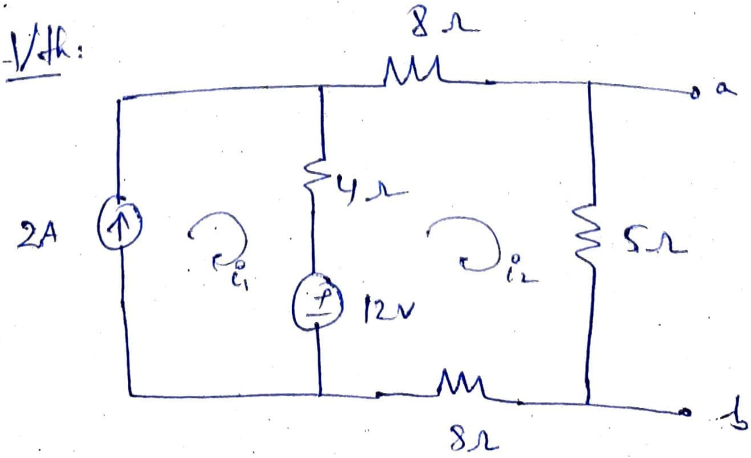
$$R_{th} = (8+4+8) \,||\, 5$$

$$= 20 \,||\, 5$$

$$= \frac{20 \times 5}{20+5}$$

$$= 4\Omega$$

Vth:



Mesh current for loop1 is $i_1 = 2A$

loop2: $8i_2 + 5i_2 + 8i_2 - 12 + 4(i_2 - i_1) = 0$

$25i_2 - 12 + (-4i_1) = 0$

$25i_2 - 12 - 8 = 0$

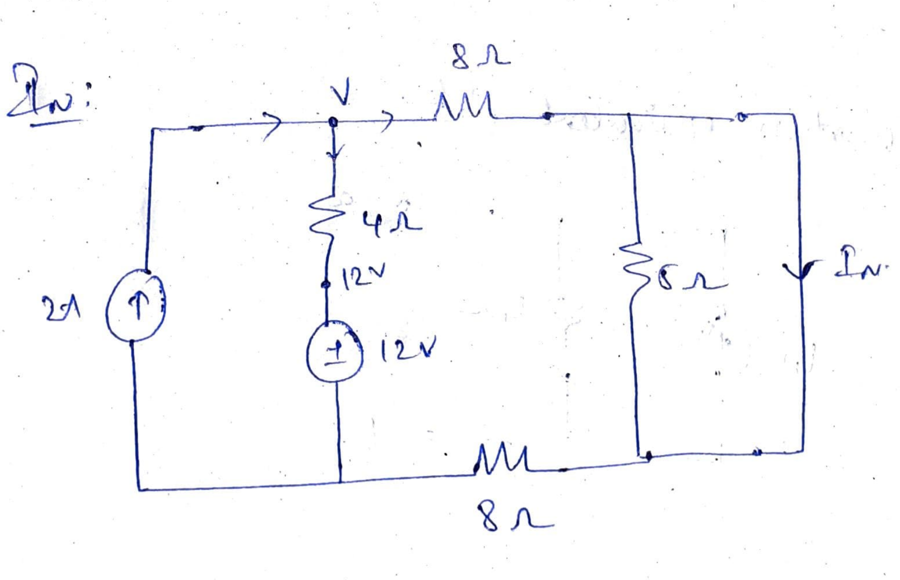
$25 i_2 = 20$

$i_2 = \frac{20}{25}$

$i_2 = \frac{4}{5} A$

$V_{th} = 5 i_2 = 5 \times \frac{4}{5} = \boxed{4V}$

I_N:



5Ω can be neglected as it is paralleled by a short circuit.

So, there will be one node, let its voltage be V.

Other node is reference node.

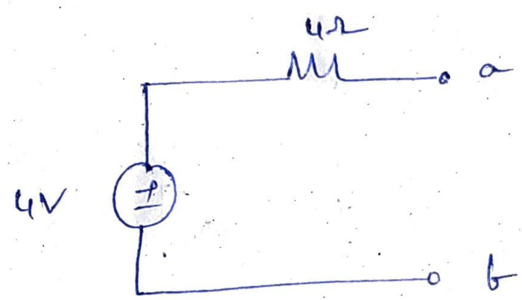
$2 = \frac{V-12}{4} + \frac{V-0}{8+8} \Rightarrow \boxed{V = 16V}$

$$I_N = \frac{V-0}{8+8}$$

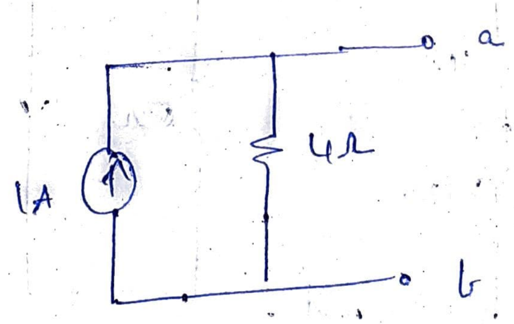
$$= \frac{16}{16} = 1A$$

$I_N = 1A$

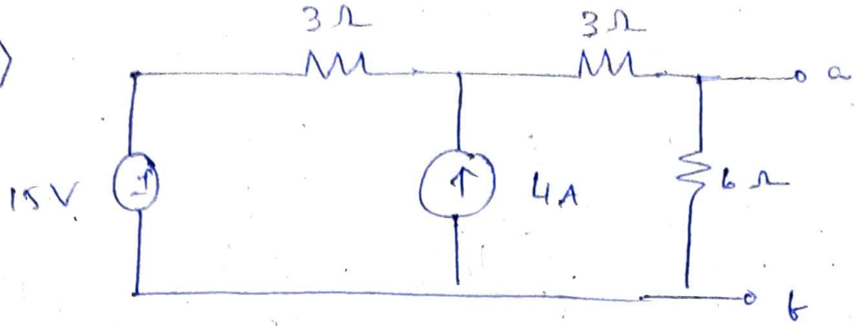
∴ Thevenin's equivalent ckt



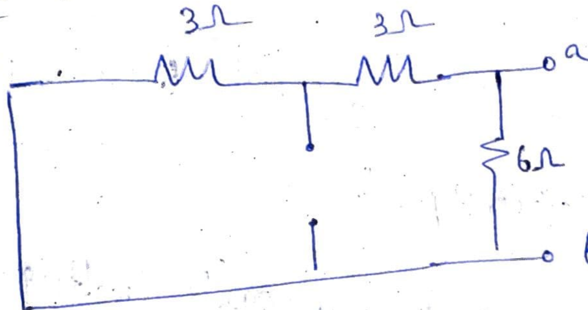
Norton's equivalent ckt



5Q) Th, N eq ckt:



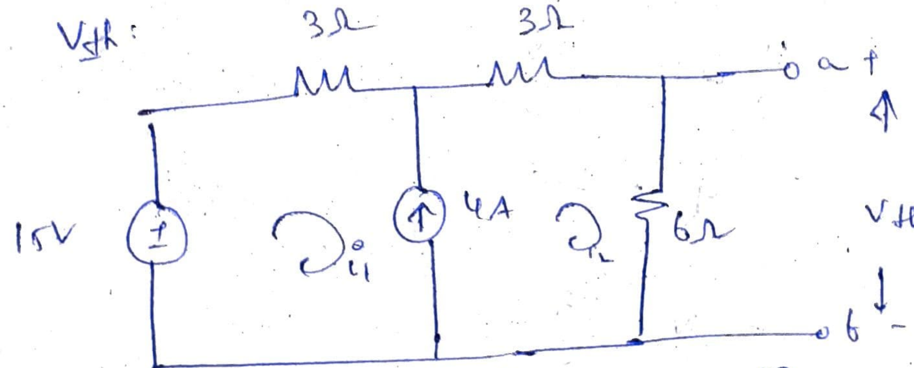
Sol) Rth:



$R_{th} = (3+3) \parallel 6$

$= 6 \parallel 6 = \boxed{3\Omega}$

Vth:



$i_2 - i_1 = 4A \longrightarrow (1)$

Supermesh:

$3i_1 + 3i_2 + 6i_2 = 15$

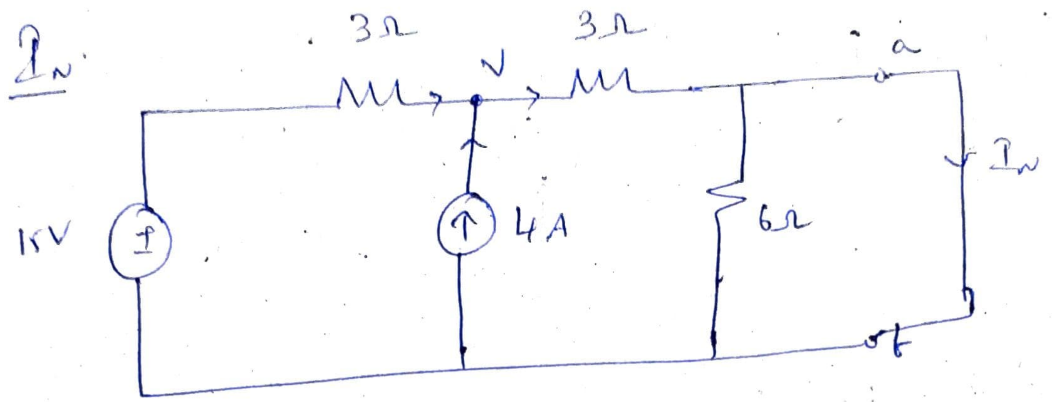
$\Rightarrow 3i_1 + 9i_2 = 15$

$\Rightarrow i_1 + 3i_2 = 5 \longrightarrow (2)$

Adding (1), (2), $4i_2 = 9 \Rightarrow i_2 = \frac{9}{4}$.

$\therefore V_{th} = 6 \times \frac{9}{4} = \boxed{\frac{27}{2} V}$

I_N:



6Ω can be neglected because it is paralleled by a short circuit path

Then, the unknown node voltage be V

$$\frac{15-V}{3} + 4 = \frac{V-0}{3}$$

$$\frac{15}{3} + 4 = \frac{2V}{3}$$

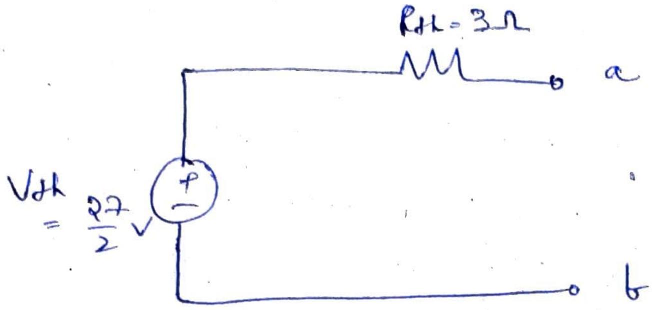
$$9 = \frac{2V}{3}$$

$$\Rightarrow V = \frac{27}{2}$$

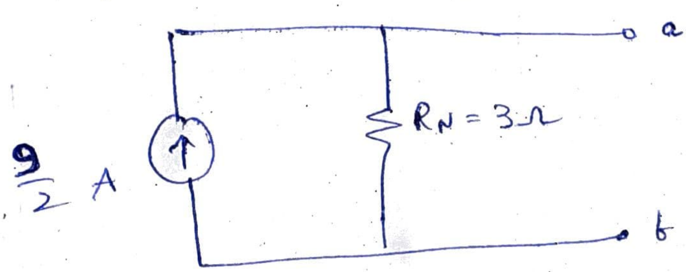
$$\therefore I_N = \frac{V-0}{3} = \frac{\frac{27}{2} - 0}{3}$$

$$\boxed{I_N = \frac{9}{2}\ A}$$

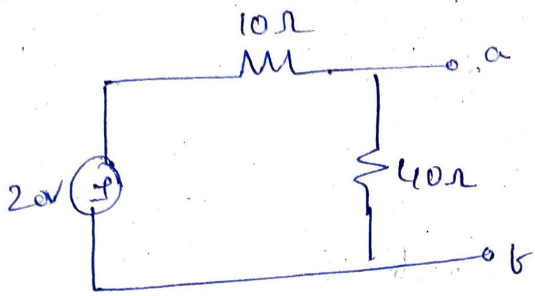
∴ Thevenin's equivalent circuit:



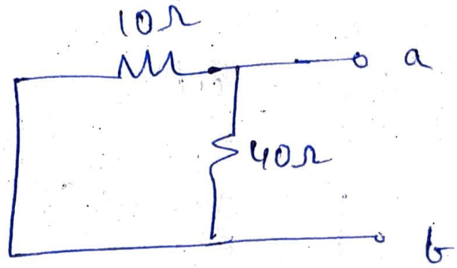
∴ Norton's equivalent ckt:



6Q) Th fnW Equ ckt :



Rth :

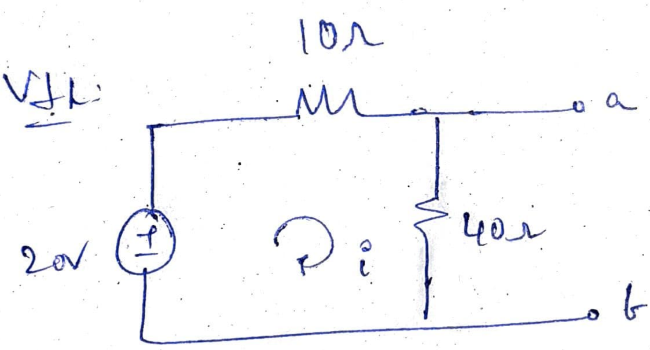


$R_{th} = 10 \parallel 40$

$= \frac{10 \times 40}{50}$

$= \boxed{8\Omega}$

Vth :



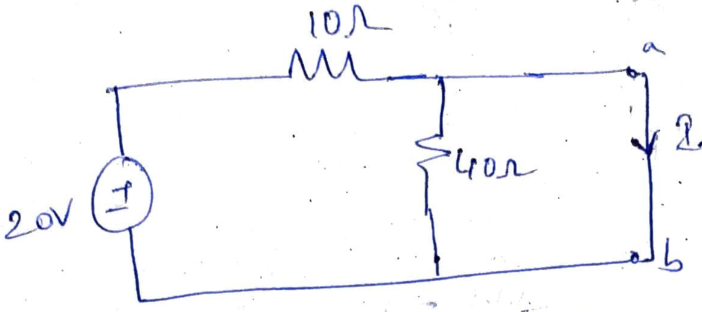
$V_{th} = i \times 40$

$= \frac{20}{(10+40)} \times 40$

$= \frac{20}{50} \times 40$

$\boxed{V_{th} = 16V}$

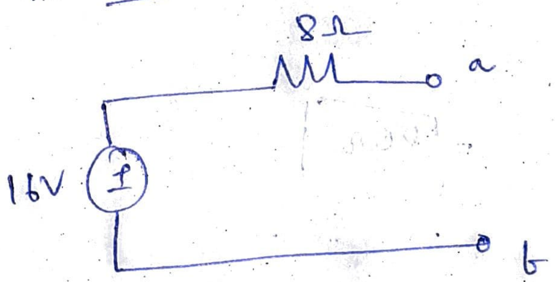
I_N:



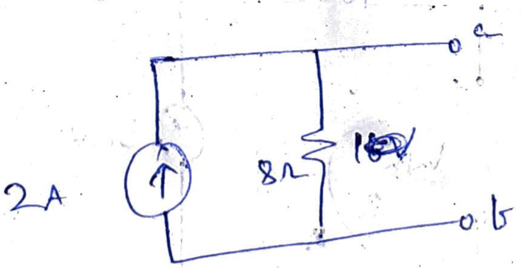
40Ω can be neglected.

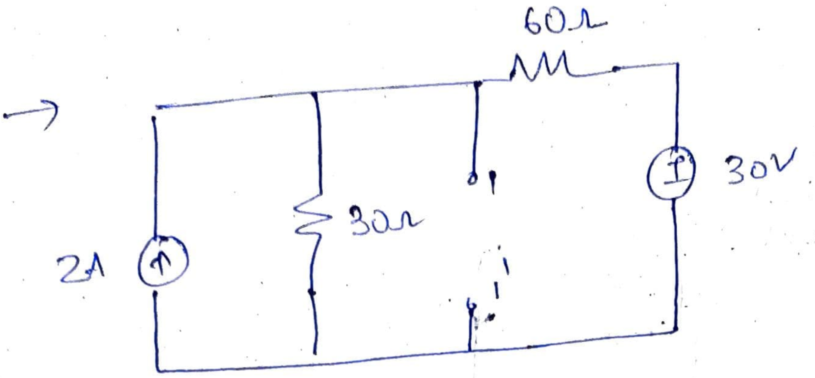
$$I_N = \frac{20}{10} = 2A$$

Thevenin's eq. ckt:

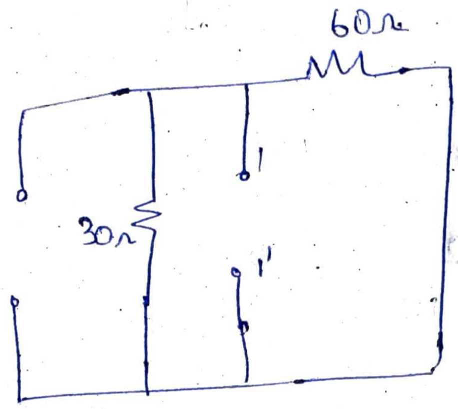


Norton's eq ckt:





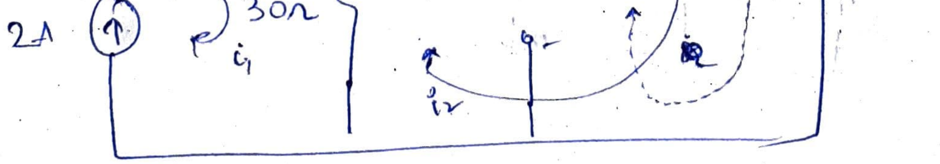
R_{th} =



$R_{th} = 30 \| 60$

$= \frac{30 \times 60}{90} = \boxed{20\Omega}$

V_{th}:



$i_1 = 2A$

Apply KVL in second loop.

$60 i_2 + 30 + 30(i_2 - i_1) = 0$

$$60i_2 + 30 + 30(i_2 - 2) = 0$$

$$90 i_2 - 30 = 0$$

$$90 i_2 = 30$$

$$i_2 = \frac{1}{3} A$$

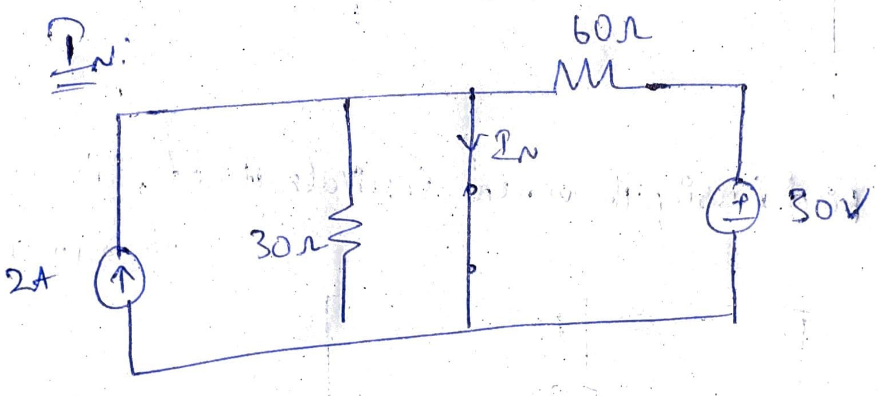
for Vth, apply kvl along the dotted lines,

$$60 i_2 + 30 = V_{th}$$

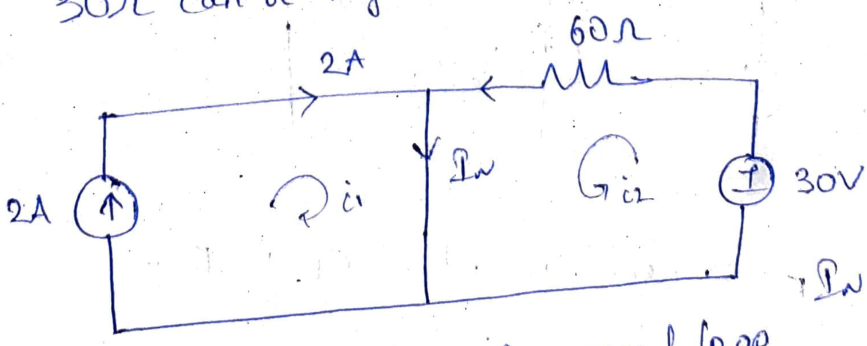
$$60\left(\frac{1}{3}\right) + 30 = V_{th}$$

$$\Rightarrow \boxed{V_{th} = 50V}$$

I_N:



30Ω can be neglected



$\therefore I_N = i_1 + i_2$

Apply KVL in second loop

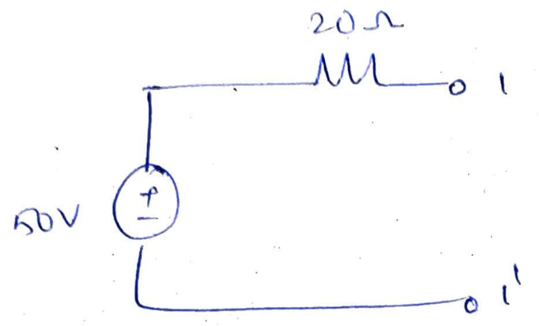
$$60 i_2 = 30 \Rightarrow i_2 = \frac{30}{60} = \frac{1}{2}$$

$$= 0.5A$$

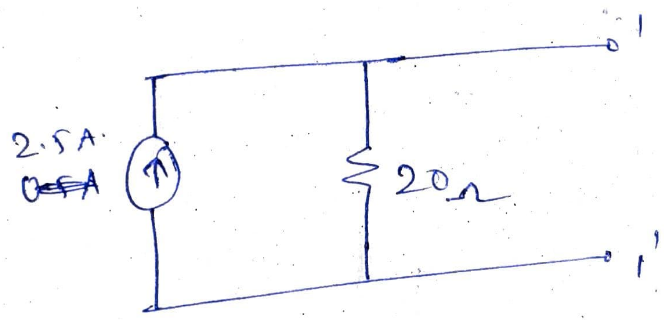
& $i_1 = 2A$

$$\therefore I_N = i_1 + i_2 = 2 + 0.5 = \boxed{2.5A}$$

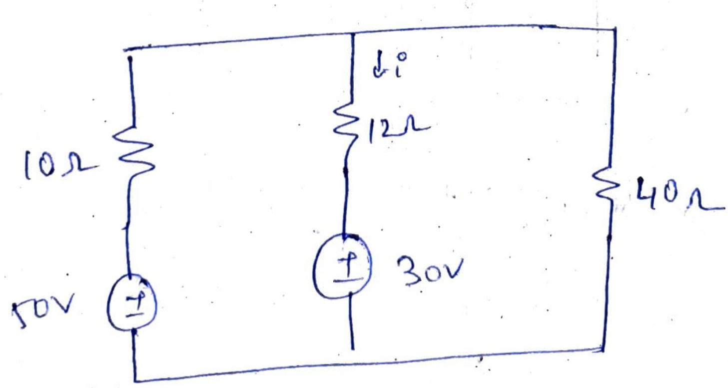
∴ Thevenin's equivalent ckt



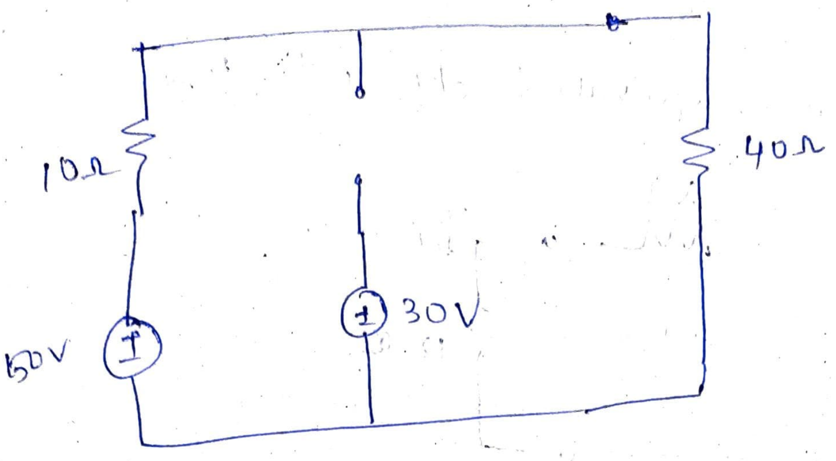
Norton's equivalent ckt:



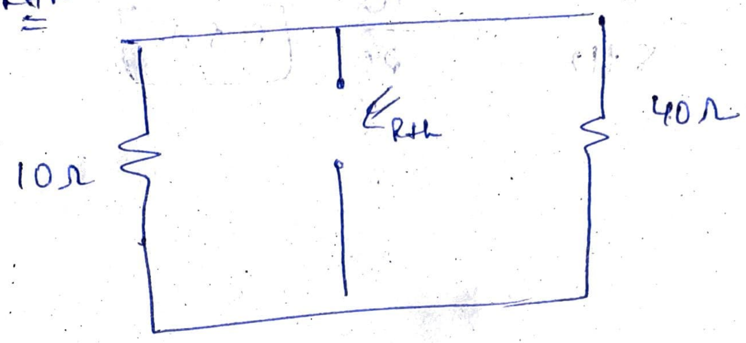
7Q) Find i using Thevenin's equivalent ckt, i is current through 12Ω



Sol) Disconnect R_L, here R_L is 12Ω



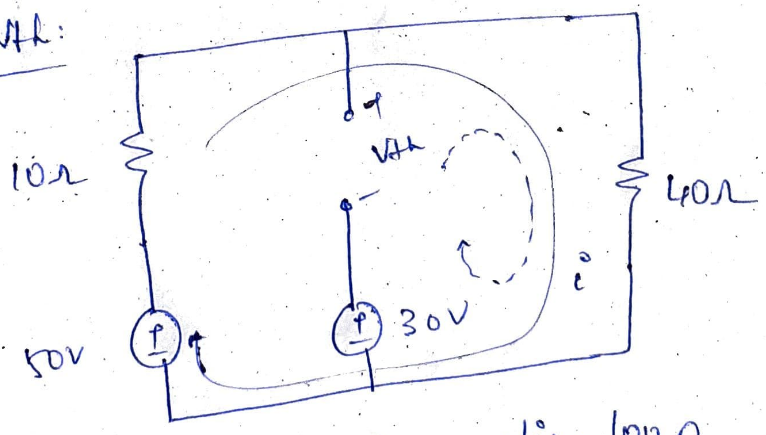
$\underline{R_{th}}$:



$R_{th} = 10 \parallel 40$

$= 8\,\Omega$

$\underline{V_{th}}$:



Apply KVL in the entire loop

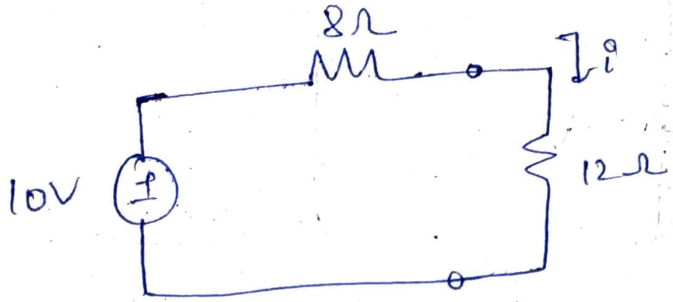
$10i + 40i = 50 \Rightarrow i = 1A$

For V_{th}, apply KVL along the dotted line

$40i - 30 - V_{th} = 0$

$40(1) - 30 = V_{th} \Rightarrow \boxed{V_{th} = 10V}$

∴ Thevenin's equivalent ckt, with 12Ω,



$$i = \frac{10}{8+12} = \frac{10}{20} = \boxed{0.5\,A}$$