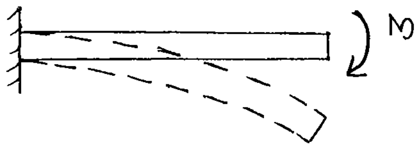
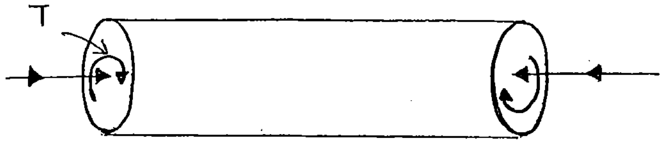
24th Oct,
FRIDAY

07. TORSION

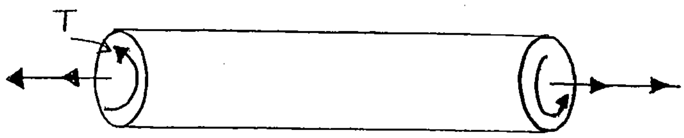


BM : along axis



Torsion : about axis
Clockwise : +ve

Torsion also called as
Twisting moment (or).
Axial couple (or).
Torque.



Anticlockwise : −ve

* Pure Torsion (impossible)

T = non zero const. & max

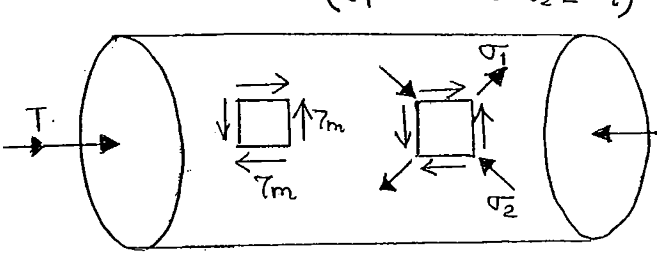
SF = 0 ; BM = 0 ; AF = 0

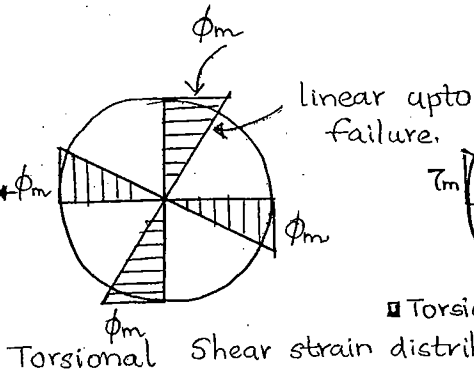
→ Assumptions :

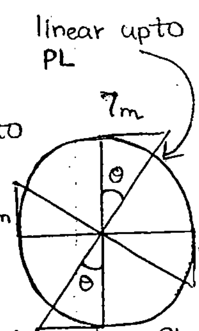
1. Euler – Bernoullie

As per Bernoullie, there is no distortion in the shape of c/s after the torsion (no warping and no bending in the c/s)

($\sigma_1 = +\tau$ & $\sigma_2 = -\tau$)







▫ Torsional Shear Str. Dist

▫ Torsional Shear strain distribution

As per Bernoulli, shear strain is linear in the c/s with zero at centre of shaft and max. at all extreme points on the surface of shaft.

* Limitations:

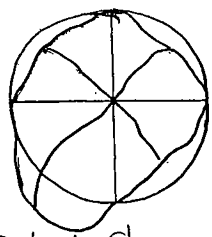
(i) Applicable for gradually applied torsion. (invalid for torsion with impact).

(ii) Applicable only for circular (solid or hollow), shafts. only

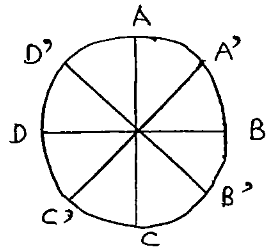
2. Torsion is constant along length of shaft.

3. Material is isotropic, homogenous and follows Hooke's Law

4. Radii remain straight after torsion (no distortion in c/s)



Distorted Shape
(Bernoulli's Assumption not valid).



5. Torsion applied must be within proportionality limit.

→ Torsion Equation.

$$\boxed{\frac{T}{J} = \frac{G\theta}{l} = \frac{\tau}{r}}$$

$J \rightarrow$ Polar MI $= I_z = I_p = I_x + I_y$.

$\theta \rightarrow$ angle of twist (in rad)

$\tau \rightarrow$ Torsional shear stress (indirect shear stress)

$r \rightarrow$ radial distance from centre of shaft.

⊙ Equation is valid only for circular shafts (both solid & hollow)

⊙ Not valid for composite shafts made of different materials

$$\boxed{\tau \propto r}$$

⊙ Due to torsion, shear stress is developing b/w the layers. The max. torsional shear stress is b/w the outermost thin layer and the layer below it.

⊙ Any element on the surface of shaft will be under pure shear (if normal stress on τ_{max} plane is zero, then it is called pure shear).

$$\sigma' = \frac{\sigma_x + \sigma_y}{2} = \frac{\sigma_1 + \sigma_2}{2} = 0.$$

where σ_1 & σ_2 are principal stresses.

⊙ Due to torsion, all the stresses are b/w the layers only, there is no stress developed in the plane of c/s.

$$\boxed{\frac{T}{J} = \frac{G}{(l/\theta)} = \frac{\tau}{r}}$$

$$\boxed{\frac{M}{I} = \frac{E}{R} = \frac{f}{y}}$$

→ Polar Section Modulus

$$Z_p = \frac{J}{r_{max}} \qquad \left(Z = \frac{I}{y_{max}}\right).$$

Unit : m^3, mm^3

$\uparrow Z_p \Rightarrow \uparrow$ strength in torsion

→ Torsional Rigidity (GJ)

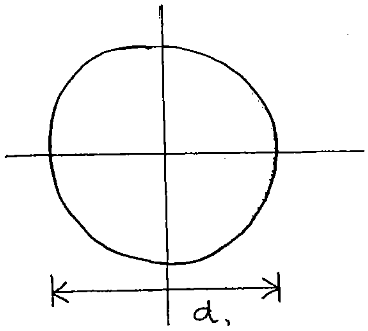
Unit : Nm^2.

$\uparrow GJ \Rightarrow \uparrow$ rigid shaft.

$\uparrow$ stiffness

$\downarrow \theta$

→ Solid Shaft.



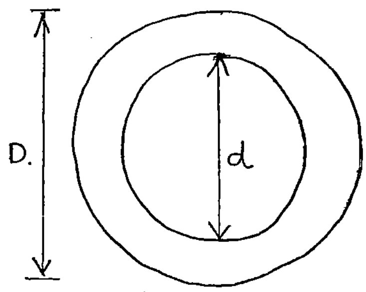
$$I_x = I_y = \frac{\pi}{64} d^4$$

$$I_z = I_x + I_y = \frac{\pi}{32} d^4 . .$$

$$Z_p = \frac{J}{d/2} = \frac{\frac{\pi}{32} d^4}{d/2}.$$

$$\Rightarrow \boxed{Z_p = \frac{\pi d^3}{16.}} \qquad \left\{ Z = \frac{\pi d^3}{32.} \right\}$$

→ Hollow Shaft.



$$\boxed{Z_p = \frac{\pi (D^4 - d^4)}{16 D.}}$$

→ Power Transmission.

$$P = \omega T.$$

$$\boxed{P = 2\pi N T.}$$

$T \rightarrow$ average torque (after losses). (Nm or J)

$N \rightarrow$ rps (or) Hz (or) cycles/sec.

$P \rightarrow$ average power $=$ Nm/s
$= J/s = W$

⊙ 1 watt (w) = 1 Nm/s = 1 J/s

kW = kNm/s.

⊙ HP = 746 W = 746 Nm/s.
= 0.746 kW = 0.746 kNm/s

- If N is given in rpm,

$$P = \frac{2\pi NT}{60}$$

(Theoretical) Max. Torque (without losses) $\rightarrow$

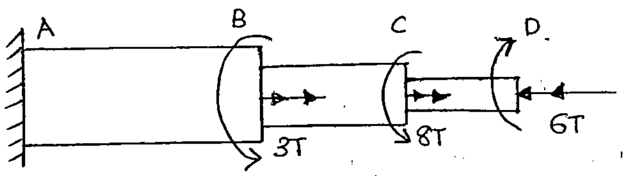
$$\frac{T}{J} = \frac{G\theta}{l} = \frac{\tau}{r}.$$

- If losses are not given in a problem, consider

$$T_{max} = T_{avg}$$

→ Arrangement of Shafts.

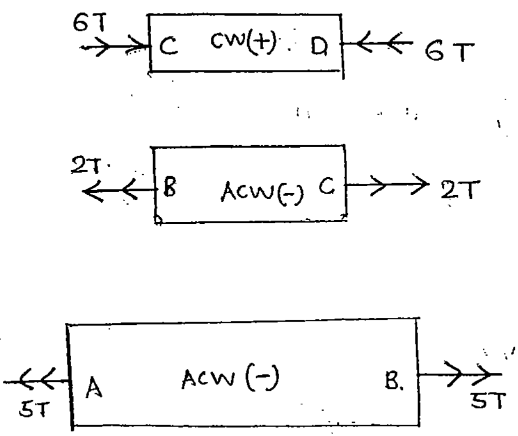
1. Series.



$\theta_A = 0$

θ @ free end = ?

$\theta_C = ?$



$$\theta = \frac{TL}{GJ}.$$

$$\theta_{AD} = \theta_{AB} + \theta_{BC} + \theta_{CD}.$$

$$\theta_D - \theta_A = \theta_{AB} + \theta_{BC} + \theta_{CD}.$$

$$\theta_D - 0 = \frac{-5T \times 2L}{3GJ} + \frac{-2T \times L}{2GJ} + \frac{6TL}{GJ}.$$

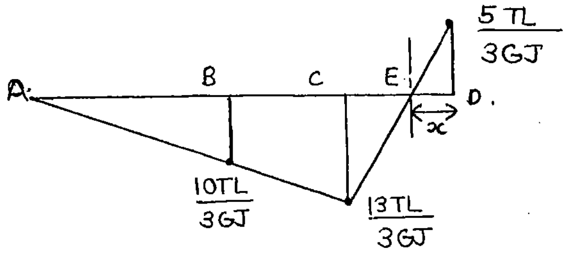
$$\theta_D = \theta_{max} \text{ @ free end} = \underline{\underline{\frac{5TL}{3GJ}}} \text{ (CW)}.$$

$$\theta_{AC} = \theta_{AB} + \theta_{BC},$$

$$\theta_C - \theta_A = \frac{-5T \times 2L}{3GJ} + \frac{-2T \times L}{2GJ}.$$

$$\therefore \theta_C = \underline{\underline{\frac{-13TL}{3GJ}}} \text{ (ACW)}$$

OR



$$\theta_{CD} = \frac{(6T)L}{GJ}$$

$$\theta_D - \theta_C = \frac{6TL}{GJ}.$$

$$\frac{5TL}{3GJ} - \theta_C = \frac{6TL}{GJ} \Rightarrow \theta_C = \frac{-13TL}{3GJ}$$

$$\theta_{AB} = \theta_B - \theta_A.$$

$$\frac{-10TL}{3GJ} = \theta_B \quad (ACW).$$

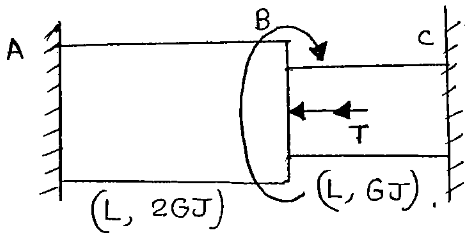
$$\frac{ED}{5/3} = \frac{CE}{13/3}.$$

$$\frac{x}{5} = \frac{l-x}{13}.$$

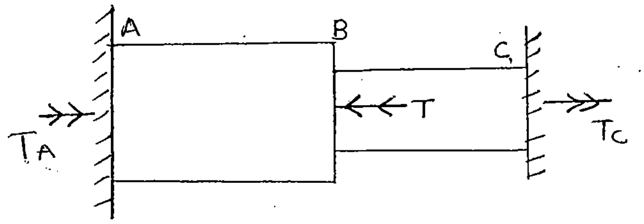
$$\Rightarrow x = \frac{5l}{18} \ \{\text{from free end D}\}$$

2. Parallel.

Q.



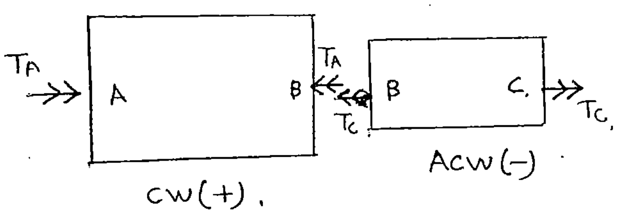
$T_A = ?$; $T_C = ?$; $\theta_B = ?$



Equilibrium equation;

$$T_A + T_C = T.$$

Compatibility condition,



$$\theta_{AC} = \theta_{AB} + \theta_{BC}$$

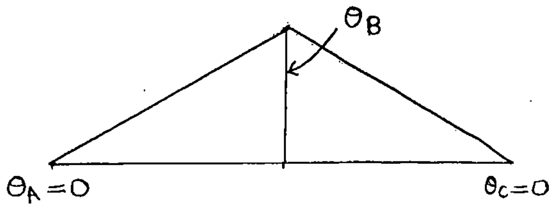
$$\cancel{\theta_C}^{0} - \cancel{\theta_A}^{0} = \theta_{AB} + \theta_{BC}.$$

$$\Rightarrow \theta_{AB} + \theta_{BC} = 0.$$

$$0 = \frac{T_A L}{2GJ} + \frac{-T_C L}{GJ}.$$

$$T_A = 2T_C,$$

$$\Rightarrow \quad T_C = \frac{T}{3} \quad \& \quad T_A = \frac{2T}{3}$$

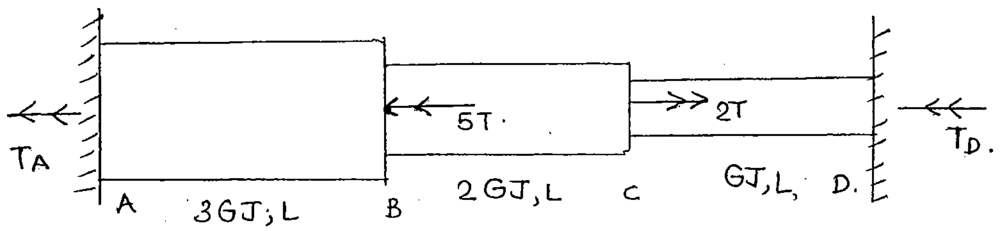


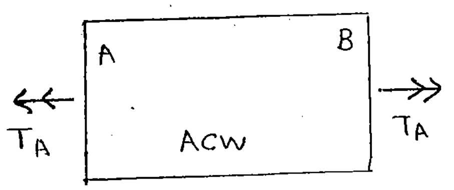
$$\theta_{AB} = \theta_B - \theta_A.$$

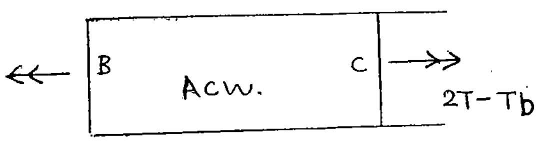
$$\frac{T_A L}{2GJ} = \theta_B - 0.$$

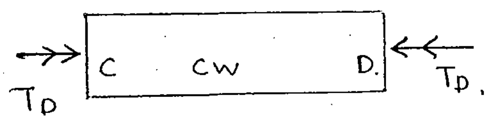
$$\Rightarrow \quad \theta_B = \frac{TL}{3GJ} \quad (CW).$$

Q.









Compatibility condition :

$$\theta_{AD} = \theta_{AB} + \theta_{BC} + \theta_{CD}$$

$$0 = -\frac{T_A L}{3GJ} - \frac{(2T - T_D)}{2GJ} + \frac{T_D L}{GJ}.$$

$$-\frac{T_A}{3} - T + \frac{3T_D}{2} = 0.$$

$$-2T_A + 9T_D = 6T.$$

Equilibrium condition :

$$T_A + T_D + 5T = 2T.$$

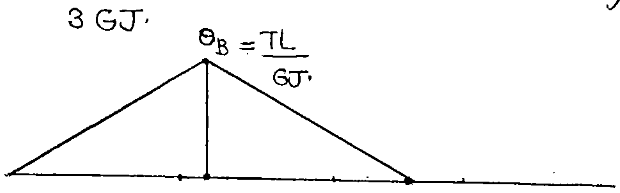
$$T_A + T_D = -3T.$$

$$T_A = -3T \quad \& \quad T_D = 0.$$

$$\therefore \quad T_A = 3T \ (CW) \quad \& \quad T_D = 0$$

$$\theta_{AB} = \theta_B - \cancel{\theta_A}^{\,0} = \frac{T_A . L}{3\,GJ} = \frac{3TL}{3\,GJ}$$

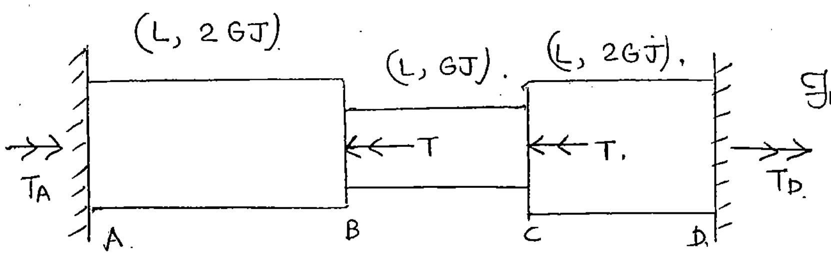
$$\underline{\underline{\theta_B = \frac{TL}{GJ}}}$$



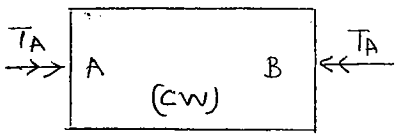
$$\theta_{CD} = \cancel{\theta_D}^{\,0} - \theta_C = \frac{-T_D L}{GJ} = \underline{\underline{0}}$$

$$\therefore \ \theta_C = 0$$

Q.

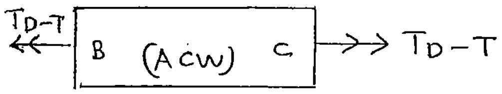


Find torsion in BC ?

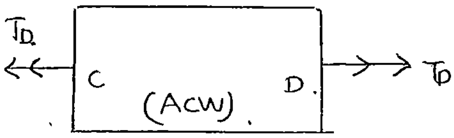


$$T_A + T_D = 2T.$$

$$0 = \frac{T_A \times L}{2GJ} - \frac{(T_D - T)\,L}{GJ} + \frac{-T_D \times L}{2GJ}$$



$$\frac{T_A}{2} - \frac{3T_D}{2} = -T.$$



$$\Rightarrow T_A = T$$

$$\underline{\underline{T_D = T}}$$

Tension in BC, $T_{BC} = T_D - T = T - T = \underline{\underline{0}}$

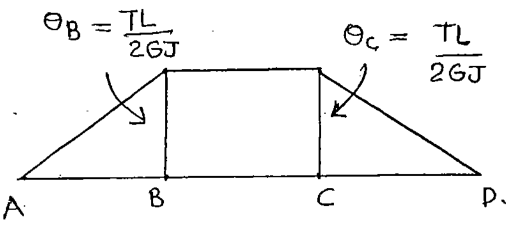
$$\theta_{AB} = \theta_B - \theta_A.$$

$$\theta_B = \frac{T_A L}{2GJ} = \underline{\underline{\frac{TL}{2GJ}}}$$

$$\theta_{CD} = \theta_D - \theta_C.$$

$$\frac{-T_D \times L}{2\,GJ} = -\theta_C.$$

$$\Rightarrow \theta_C = \underline{\underline{\frac{T.L}{2GJ}}}$$



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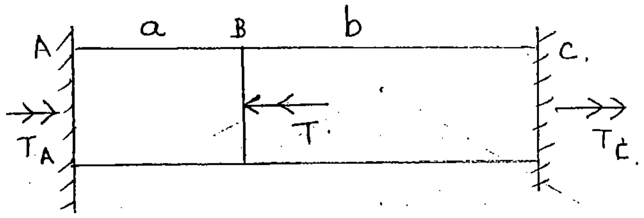


$$T_A + T_C = T.$$

$$\theta_{AC} = \theta_{AB} + \theta_{BC}.$$

$$0 = \frac{T_A a}{GJ} + \frac{T_B \cdot b}{GJ.}$$

$$a T_A = -b T_B.$$

$$T_A = \frac{Tb}{l.}$$

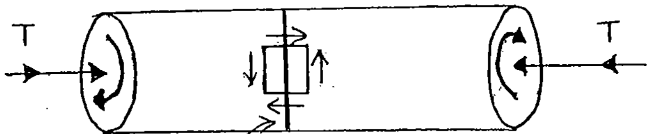
$$T_C = \frac{Ta}{l}$$

→ Failure Criteria.

1. Ductile Shaft.

Weak in shear.

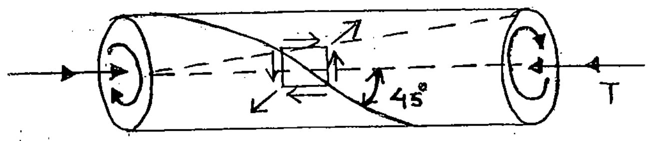
No failure in horizontal direction due to large area to resist the shear (length * diameter). So failure occurs as a vertical cut.



vertical cut (normal to axis). {for cw & ACw T}

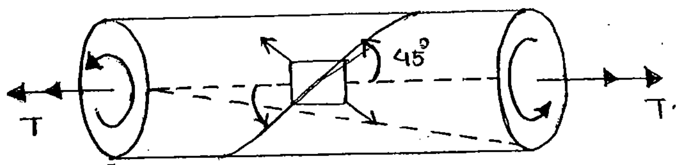
2. Brittle Shaft. (CI, glass).

Weak in tension.



[FIX] CW (+)

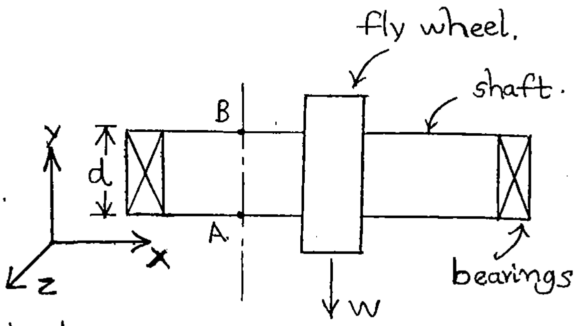
(45° cw cracks with axis)



[FIX] ACW torsion is applied

(45° ACW crack with axis)

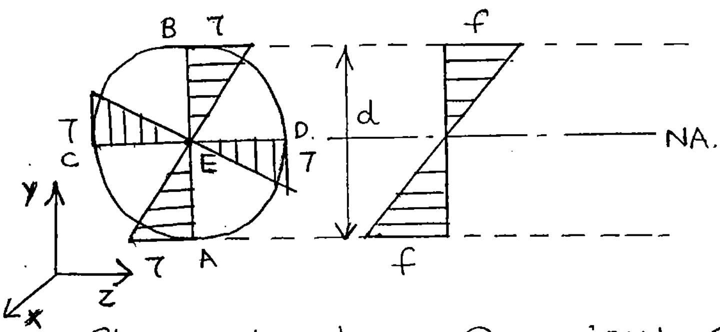
→ Combined Stresses.



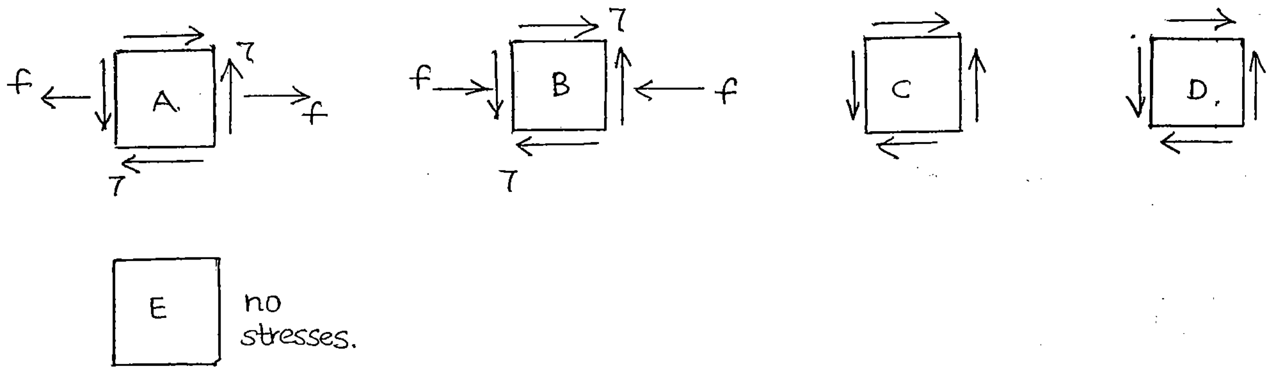
- Usually rotating shafts are subjected to torsion, BM & SF.

At the point of max. BM, SF is zero. ∴ the shaft must be designed for the combined effect of bending and torsion.

Assume diameter of shaft is **d**.



State of stress @ various points :-



The critical elements for the design of shaft are A and B

Now consider element A.

$$\sigma_x = f = \frac{M}{Z} \quad ; \quad \tau_{xy} = \tau = \frac{T}{Z_p}.$$

$$\sigma_y = 0$$

$$\sigma_x = \frac{M}{\frac{\pi d^3}{32}} = \underline{\underline{\frac{32M}{\pi d^3}}}.$$

$$\tau_{xy} = \frac{T}{\frac{\pi d^3}{16}} = \underline{\underline{\frac{16T}{\pi d^3}}}$$

⊙ Design is based on Principal Stresses:

$$\left.\begin{matrix}\sigma_1 \\ \sigma_3\end{matrix}\right\} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

$$= \frac{f}{2} \pm \sqrt{\left(\frac{f}{2}\right)^2 + \tau^2}$$

$$= \frac{16M}{\pi d^3} \pm \sqrt{\left(\frac{16M}{\pi d^3}\right)^2 + \left(\frac{16T}{\pi d^3}\right)^2}$$

$$\left.\begin{matrix}\sigma_1 \\ \sigma_3\end{matrix}\right\} = \frac{16}{\pi d^3}\left[M \pm \sqrt{M^2 + T^2}\right]$$

$$\boxed{\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{16}{\pi d^3}\sqrt{M^2 + T^2}}$$

In any member subjected to bending action, major and minor principal stresses will be opposite in nature. Intermediate principal stress $= 0$ $(\sigma_2 = 0)$.

* Equivalent BM $= M_e = \dfrac{M + \sqrt{M^2 + T^2}}{2}$

* Equivalent torsion, $T_e = \sqrt{M^2 + T^2}$

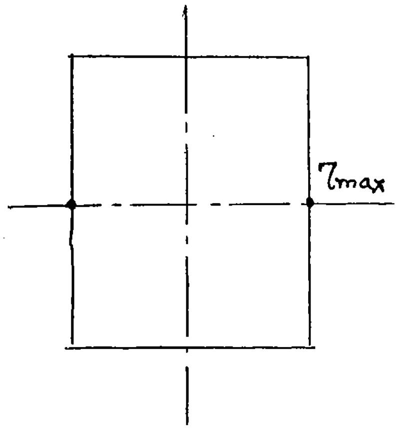
⊙ For a shaft, M & T act together to produce principal stress σ_1. But the equivalent moment, M_e, alone can produce the same value of σ_1 on the shaft.

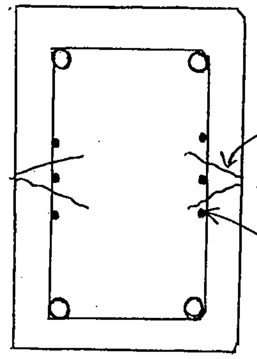
⊙ Similarly, M & T act together to produce max. shear stress, τ_{max}. But the equivalent torsion, T_e, alone can produce the same value of τ_{max} on the shaft.

25th Oct,
SATURDAY

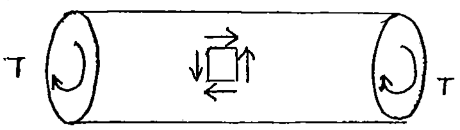
58.

9.





Deep beam (D > 750 mm).
[Torsion develops.]

11. 

For element, $\sigma_x = 0$, $\sigma_y = 0$, $\tau_{xy} = \tau$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \underline{\underline{+\tau}}$$

For element on surface subjected to pure shear, $\sigma_1 = +\tau$
$\sigma_3 = -\tau$

14. $\sigma_1 = \tau = \dfrac{16T}{\pi d^3}$

13. In the c/s, no stresses.

8. $P = 2\pi N T$.

$452.8 \times 0.746 = 2\pi \times 2\ T$

$T = 26.89$ kNm.

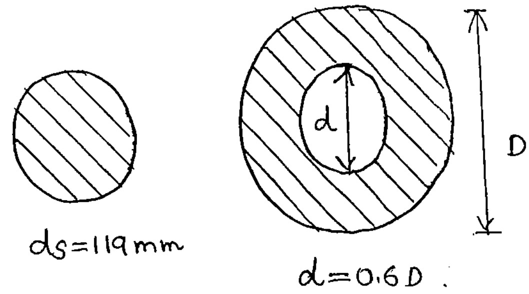
$$\frac{T}{J} = \frac{\tau}{r}$$

$$\tau = \frac{T}{Z_p} = \frac{T}{\frac{\pi}{16} d^3}$$

$$80 = \frac{16T}{\pi d^3} = \frac{16(26.89 \times 10^3)}{\pi d^3}$$

$d = \underline{\underline{119 \text{ mm}}}$

Replaced hollow shaft should tranfer same torsion.



$$\tau_s = \tau_h .$$

$$\left(\frac{T}{Z_P}\right)_s = \left(\frac{T}{Z_P}\right)_h .$$

$$(Z_P)_h = (Z_P)_s .$$

$$\frac{\pi}{16D}(D^4 - d^4) = \frac{\pi}{16} d_s^3 .$$

$$\frac{D^4 - (0.6D)^4}{D} = 119^3 .$$

Outer diameter of hollow shaft, $D = \underline{\underline{124.635}}$ mm

Weight, $W = \gamma A l$.

For both the shafts, 'l' & 'γ' must be same.

$$\Rightarrow \quad W \propto A .$$

$$\frac{W_h}{W_s} = \frac{\frac{\pi}{4}(D^2 - d^2)}{\frac{\pi}{4} \times d_s^2} = \frac{D^2(1 - 0.6^2)}{119^2} = 0.702$$

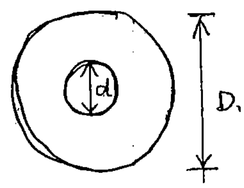
$$W_h = 0.702\ W_s .$$

$\Rightarrow$ 30% savings in weight when solid shaft replaced by hollow shaft.

P-56

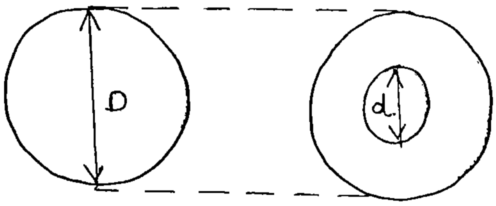
→ Comparison of Hollow & Solid shaft:

1. Areas are equal.



$$A_s = A_h \Rightarrow W_s = W_h .$$

$$\frac{T_h}{T_s} = \frac{P_h}{P_s} = \frac{(\text{Strength})_h}{(\text{Strength})_s} = \frac{(Z_P)_h}{(Z_P)_s} = \frac{1+K^2}{\sqrt{1-K^2}} \qquad K = \frac{d}{D}$$

2. 

$$\frac{T_h}{T_s} = \frac{P_h}{P_s} = \frac{(\text{strength})_h}{(\text{strength})_s} = \frac{(Z_p)_h}{(Z_p)_s} = \underline{\underline{1 - K^4}}$$

3. Solid and hollow shaft of equal strength

$$T_h = T_s$$

$$P_h = P_s$$

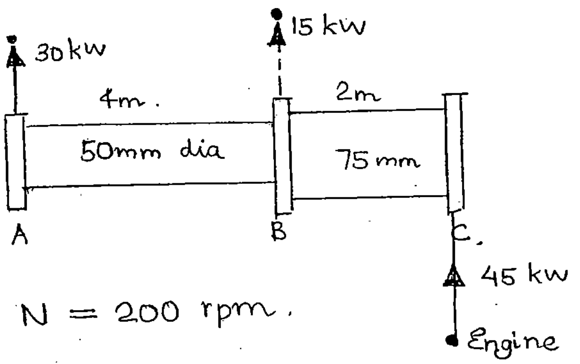
$$(\text{Str})_h = (\text{Str})_s$$

$$(Z_p)_h = (Z_p)_s$$

$$\Rightarrow \boxed{\frac{W_h}{W_s} = \frac{A_h}{A_s} = \frac{1-K^2}{(1-K^4)^{2/3}}}$$

8. $\frac{W_h}{W_s} = ?$ $\quad K = \frac{d}{D} = 0.6.$

$$\frac{W_h}{W_s} = \frac{1-0.6^2}{(1-(0.6)^4)^{2/3}} = \underline{\underline{0.702}}$$

09. 

$N = 200$ rpm.

$P_{AB} = 30$ kw. $\quad P_{BC} = 45$ kw.

Shaft AB :

$$P = \frac{2\pi N T}{60} \Rightarrow 30 \times 1000 = \frac{2\pi \times 200\,(T)}{60}$$

$$T_{AB} = 1.43 \text{ kNm.}$$

Illy $T_{BC} = 2.15$ kNm.

$$\tau_{AB} = \frac{16\, T_{AB}}{\pi d_{AB}^3} = \frac{16 \times 1.43 \times 10^6}{\pi \times 50^3} = \underline{\underline{58.3 \text{ MPa}}}$$

$$\tau_{BC} = \frac{16\, T_{BC}}{\pi d_{BC}^3} = \frac{16 \times 2.15}{\pi \times 75^3} = \underline{\underline{25.9 \text{ MPa}}}$$

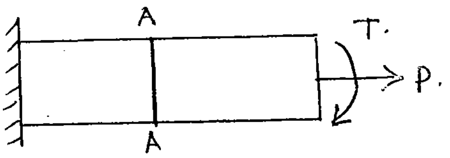
$$\tau_{max} = \tau_{AB} = \underline{\underline{58.2}} \text{ MPa. (max).}$$

10. $\theta_{AC} = \theta_{AB} + \theta_{AC}.$

$$= \frac{1.43 \times 10^6 \times 4000}{8.5 \times 10^4 \times \frac{\pi}{32}(50^4)} + \frac{2.15 \times 10^6 \times 2000}{8.5 \times 10^4 \times \frac{\pi}{32} \times 75^4} = \underline{\underline{0.126}} \text{ rad}$$

$$= \underline{\underline{7.14^\circ}}$$

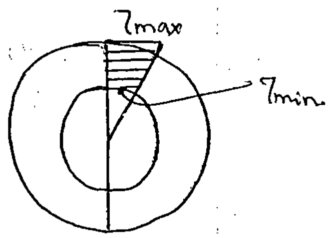
11.



$$\sigma = \frac{P}{A} = \text{const.}$$

$$\tau = \frac{16T}{\pi d^3} = \text{const.}$$

∴ Both normal and shear stress are continuous. at every section.



13. $$\tau_{max} = \frac{T}{J} r_{max} = \frac{100 \times 10^3}{\frac{\pi}{32}(30^4 - 26^4)} \times \frac{30}{2}$$

$$= \underline{\underline{43.27}} \text{ MPa}$$

$$\tau_{min} = \frac{T}{J} r_{min} = \frac{100 \times 10^3}{\frac{\pi}{32}(30^4 - 26^4)} \times \frac{26}{2} = \underline{\underline{37.5}} \text{ MPa}$$