Rajiv Gandhi University of Knowledge Technology-Nuzvid

DEPARTMENT OF CIVIL ENGINEERING

# SOIL MECHANICS

BY

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# SHEAR STRENGTH OF SOIL

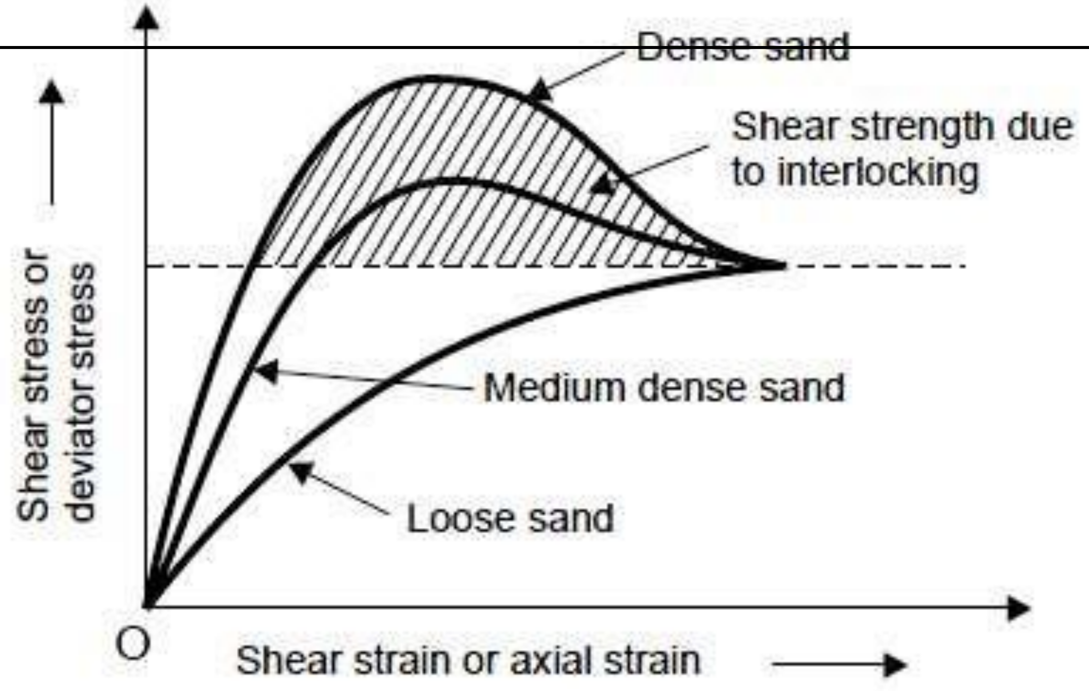
# PART-2

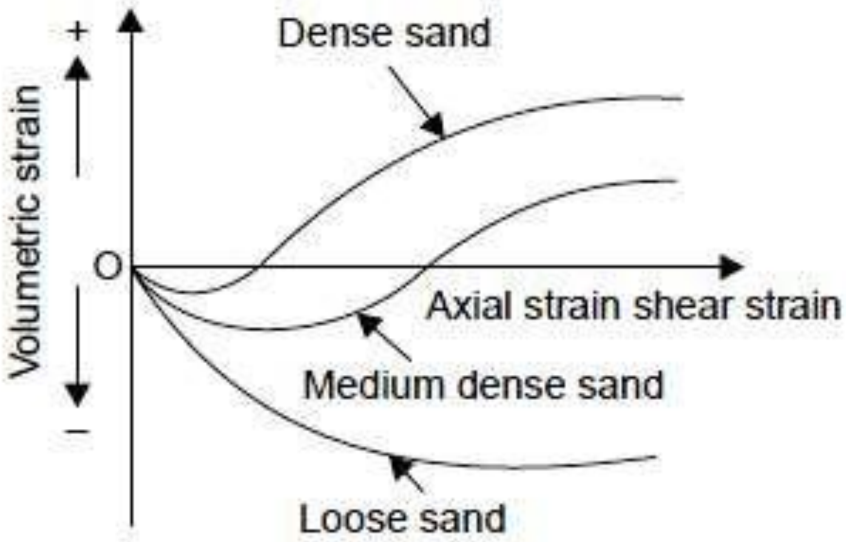
# Syllabus of Shear strength of soil

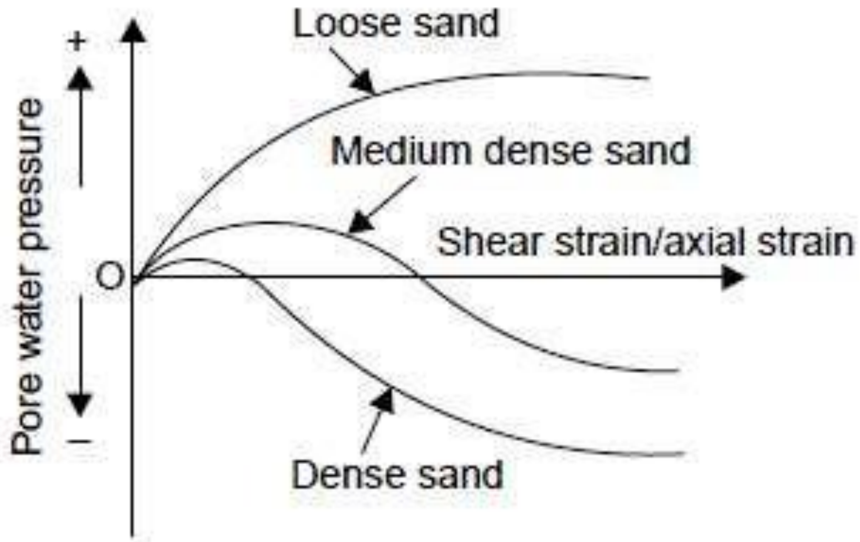
- Basic mechanism of shear strength
- Mohr – Coulomb Failure theories
- Stress-Strain behaviour of Sands – Critical Void Ratio
- Stress-Strain behaviour of clays
- Shear Strength determination- various drainage conditions.

# Stress-Strain behaviour of Sands

- The stress-strain behaviour of sands is dependent to a large extent on the initial density of packing, as characterised by the density index.
- It can be observed from Fig.(a), the shear stress (in the case of direct shear tests) or deviator stress (in the case of triaxial compression tests) builds up gradually for an initially loose sand, while for an initially dense sand, it reaches a peak value and decreases at greater values of shear/axial strain to an ultimate value comparable to that for an initially loose specimen.
- The volume change characteristics of sands is another interesting feature, as depicted in Fig. (b). An initially dense specimen tends to increase in volume and become loose with increasing values of strain, while an initially loose specimen tends to decrease in volume and become dense. This is explained in terms of the rearrangement of particles during s ear.
- The changes in pore water pressure during undrained shear, which is rather not very common owing to high permeability of sands, are depicted in Fig. (c). Positive pore pressures develop in the case of an initially loose specimen and negative pore pressures develop in the case of an initially dense specimen.

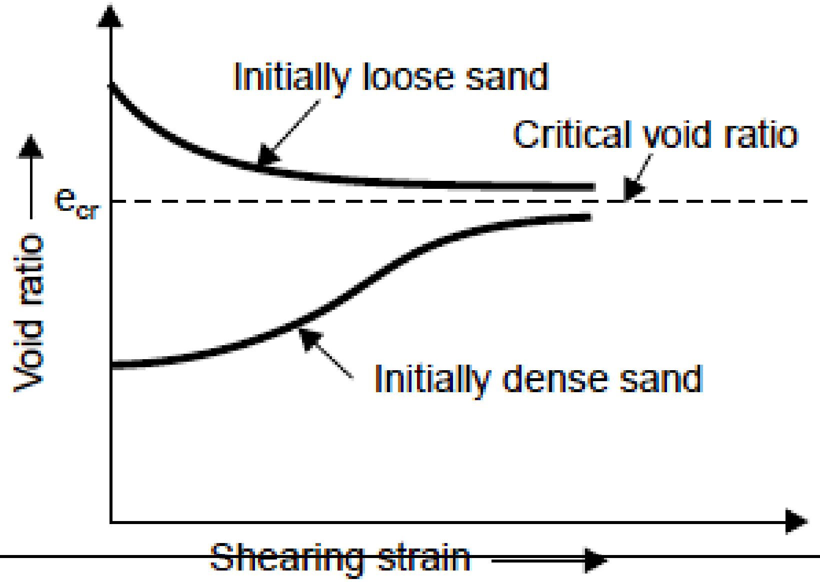

Dense sand
Shear strength due
to interlocking
Shear stress or
deviator stress
Medium dense sand
Loose sand
O
Shear strain or axial strain
(a)


+
Dense sand
Volumetric strain
O
Axial strain shear strain
Medium dense sand
−
Loose sand
(b)


+
Loose sand
Medium dense sand
Pore water pressure
Shear strain/axial strain
O
−
Dense sand
(c)

# Critical Void Ratio

- Volume change characteristics depend upon various factors such as the particle size, particle shape and distribution, principal stresses, previous stress history and significantly on density index. Volume changes, expressed in terms of the void ratio versus shear strain are typically as shown in Fig.
- At large strains both initially loose and initially dense specimens attain nearly the same void ratio, at which further strain will not produce any volume changes. Such a void ratio is usually referred to as the 'Critical Void Ratio'.
- Sands with initial void ratio greater than the critical value will tend to decrease in volume during shearing, while sands with initial void ratio less than the critical with tend to increase in volume.



# Shearing Strength of Clays

# Unconsolidated Undrained Tests

## Saturated Clay

- Tests on saturated clay may be carried out either on undisturbed or on remolded soil samples. The procedure of the test is the same in both cases.
- A series of samples (at least a minimum of three) having the same initial conditions are tested under undrained conditions. With $\sigma_3$ the all-round pressure, acting on a sample under conditions of no drainage, the axial pressure is increased until failure occurs at a deviator stress ($\sigma_1$ - $\sigma_3$).
- From the deviator stress, the major principal stress $\sigma_1$, is determined. If the other samples are tested in the same way but with different values of $\sigma_3$, it is found that for all types of saturated clay, the deviator stress at failure (compressive strength) is entirely independent of the magnitude of $\sigma_3$ as shown in Fig.
- The diameters of all the Mohr circles are equal and the Mohr envelope is parallel to the cr-axis indicating that the angle of shearing resistance $\phi_u = 0$. The symbol $\phi_u$ represents the angle of shearing resistance under undrained conditions.

- Thus saturated clays behave as purely cohesive materials with the following properties:

$$\phi_u = 0, \text{ and } c_u = \frac{1}{2}\ (\sigma_1 - \sigma_3)$$

- where $c_u$ is the symbol used for cohesion under undrained conditions.
- This equation holds true for the particular case of an unconfined compression test in which $\sigma_3 = 0$. Since this test requires a very simple apparatus, it is often used, especially for field work, as a ready means of measuring the shearing strength of saturated clay, in this case

$$c_u = \frac{q_u}{2}, \quad \text{where} \quad q_u = (\sigma_1 - \sigma_3)_f = (\sigma_1)_f$$

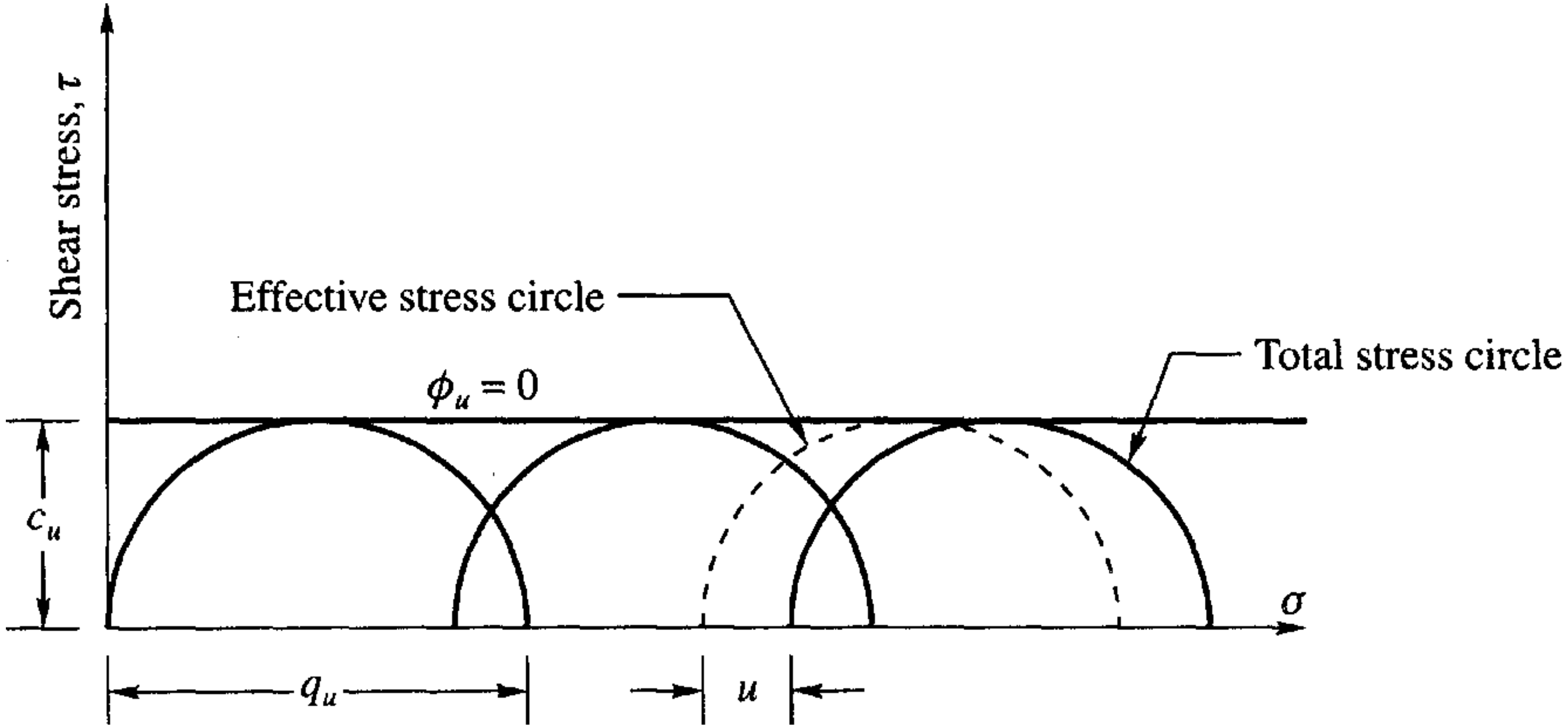
- **Effective Stresses:**
- If during the test, pore-pressures are measured, the effective principal stresses may be written as

$$\sigma_1' = \sigma_1 - u$$

$$\sigma_3' = \sigma_3 - u$$

- where u is the pore water pressure measured during the test. The effective deviator stress at failure may be written as $(\sigma_1' - \sigma_3')_f = (\sigma_1 - u)_f - (\sigma_3 - u)_f = (\sigma_1 - \sigma_3)_f$
- Eq. shows that the deviator stress is not affected by the pore water pressure. As such the effective stress circle is only shifted from the position of the total stress circle as shown in Fig.

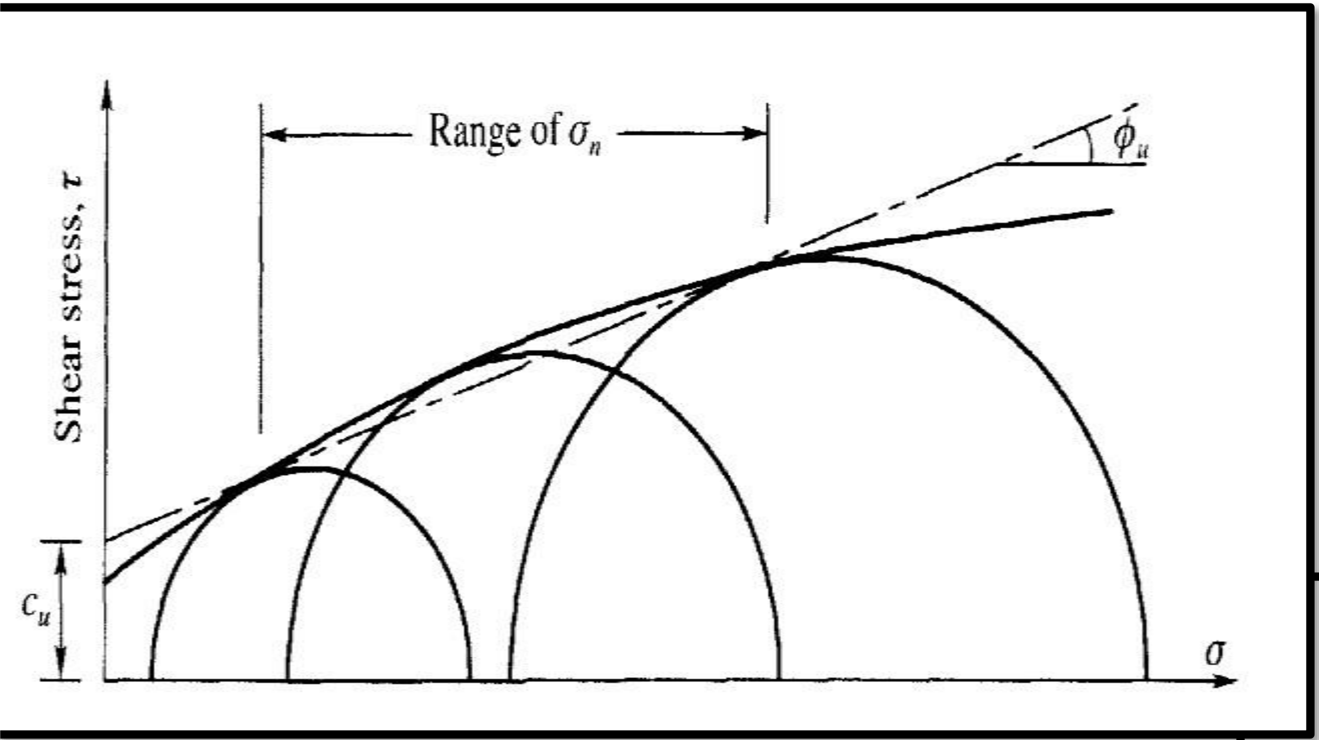
# Mohr circle for undrained shear test on saturated clay



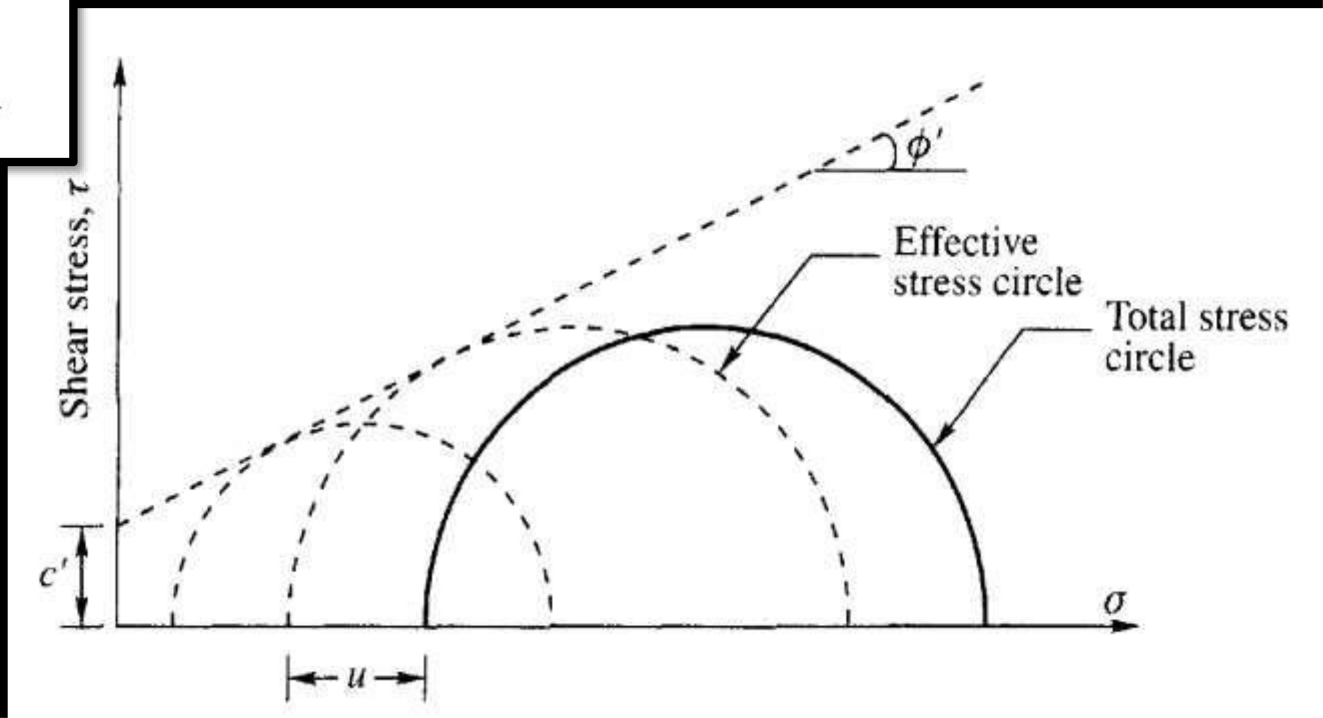
## Partially Saturated Clay

- Tests on partially saturated clay may be carried out either on undisturbed or on remolded soil samples. All the samples shall have the same initial conditions before the test, i.e., they should possess the same water content and dry density. The tests are conducted in the same way as for saturated samples. Each sample is tested under undrained conditions with different all-round pressures $\sigma_3$.
- Mohr circles for three soil samples and the Mohr envelope are shown in Fig. Though all the samples had the same initial conditions, the deviator stress increases with the increase in the all-round pressure $\sigma_3$ as shown in the figure. This indicates that the strength of the soil increases with increasing values of $\sigma_3$. The degree of saturation also increases with the increase in $\sigma_3$. The Mohr envelope which is curved at lower values of $\sigma_3$ becomes almost parallel to the $\sigma$-axis as full saturation is reached.
- Thus it is not strictly possible to quote single values for the parameters $c_u$ and $\phi_u$ for partially saturated clays, but over any range of normal pressure $\sigma_n$; encountered in a practical example, the envelope can be approximated by a straight line and the approximate values of $c_u$ and $\phi_u$ can be used in the analysis.
- **Effective Stresses**
- If the pore pressures are measured during the test, the effective circles can be plotted as shown in Fig. and the parameters c' and $\phi'$ obtained. The envelope to the Mohr circles, when plotted in terms of effective stresses, is linear.

Mohr circle for undrained shear tests on partially saturated clay soils




Effective stress circles for undrained shear tests on partially saturated clay soils



# Consolidated undrained tests

## Normally Consolidated Saturated Clay

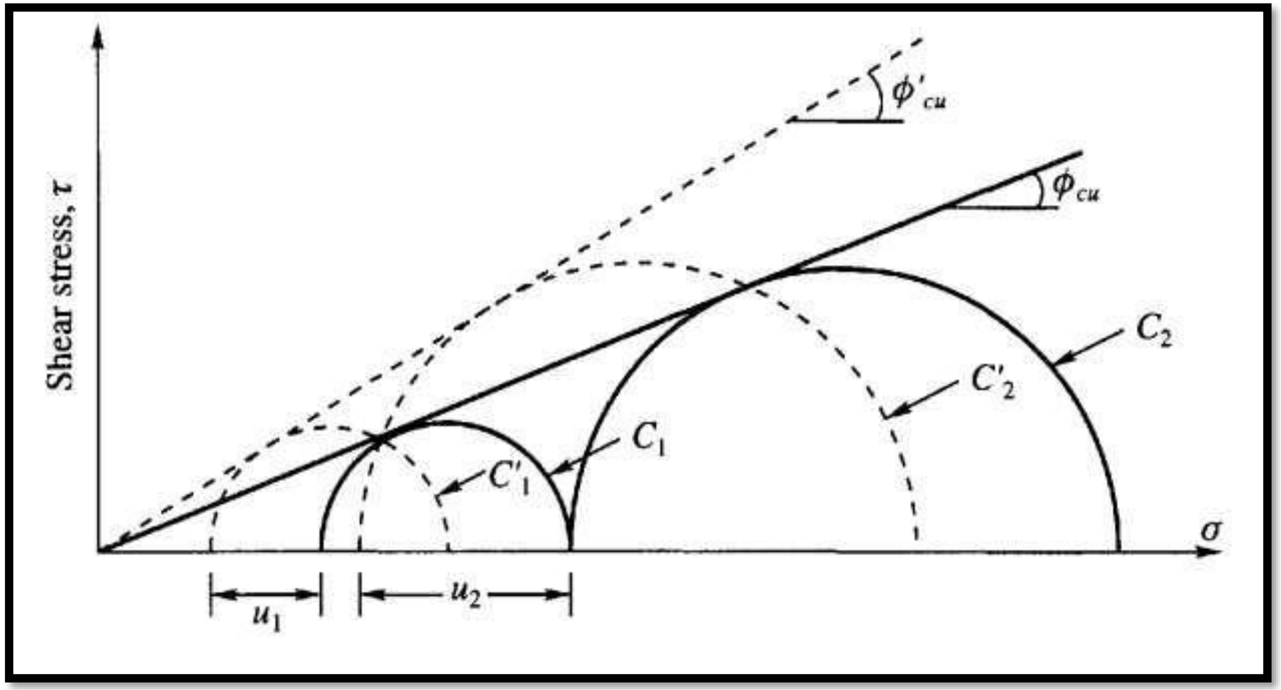
- If two clay samples 1 and 2 are.consolidated under ambient pressures of $p_l$ and $p_2$ and are then subjected to undrained triaxial tests without further change in cell pressure, the results may be expressed by the two Mohr circles $C_1$ and $C_2$ respectively as shown in Fig.
- The failure envelope tangential to these circles passes through the origin and its slope is defined by $\phi_{cu}$, the angle of shearing resistance in consolidated undrained tests. If the pore pressures are measured the effective stress Mohr circles $C'_1$ and $C'_2$ can also be plotted and the slope of this envelope is $\phi'_{cu}$.
- The effective principal stresses are:

$$\sigma'_{11} = \sigma_{11} - u_1; \ \sigma'_{12} = \sigma_{12} - u_2$$

$$\sigma'_{31} = \sigma_{31} - u_1; \ \sigma'_{32} = \sigma_{32} - u_2$$

where $u_l$ and $u_2$ are the pore water pressures for the samples 1 and 2 respectively.

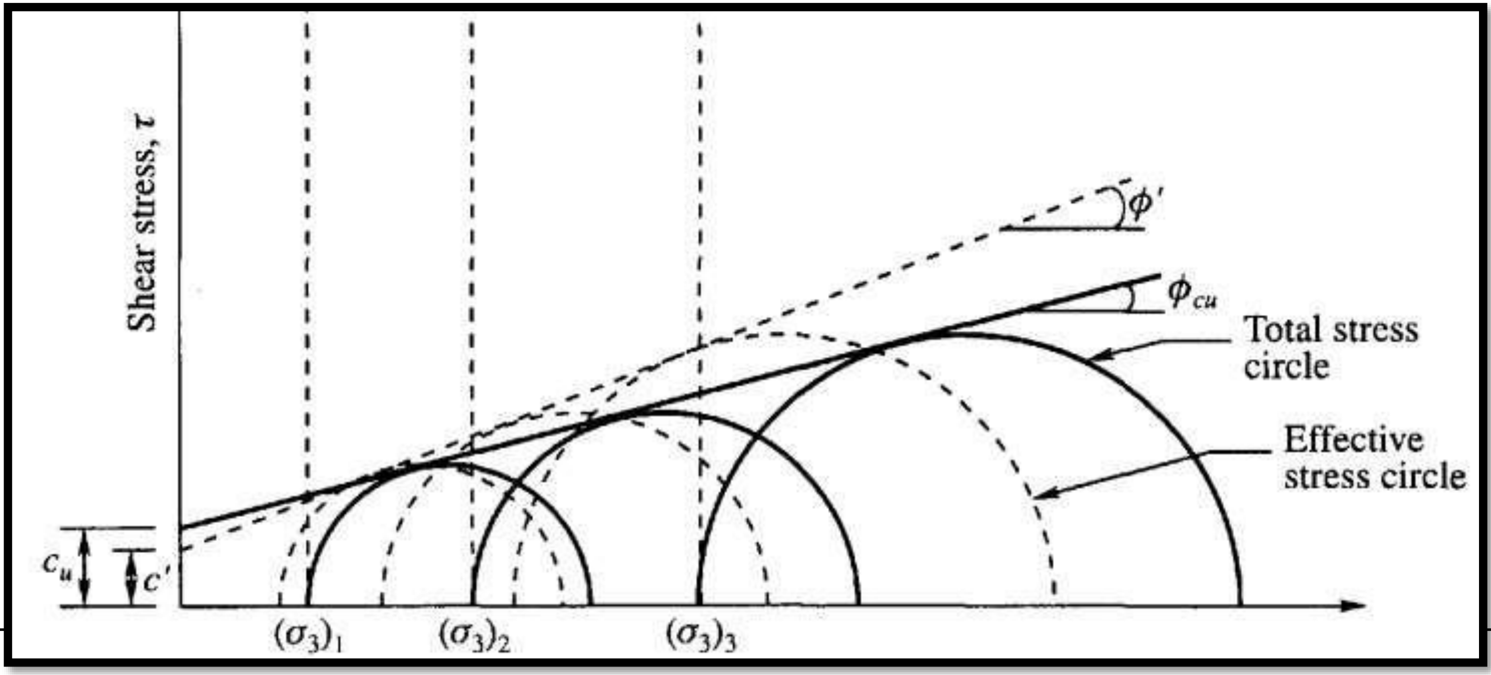
## Normally consolidated clay under undrained triaxial test




**Mohr envelope**

# Overconsolidated Clay

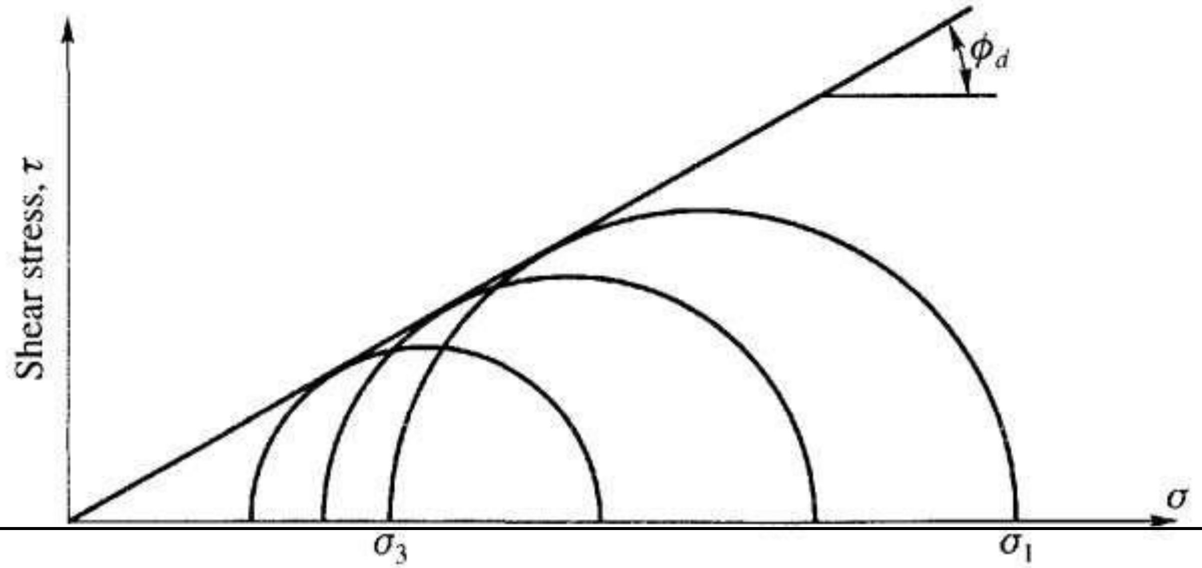
- Let a saturated sample 1 be consolidated under an ambient pressure $p_a$ and then allowed to swell under the pressure $p_1$. An undrained triaxial test is carried out on this sample under the all-round pressure $p_1 (= \sigma_{31})$.
- Another sample 2 is also consolidated under the same ambient pressure $p_a$ and allowed to swell under the pressure $p_2 (= \sigma_{32})$. An undrained triaxial test is carried out on this sample under the same all-round pressure $p_2$.
- The two Mohr circles are plotted and the Mohr envelope tangential to the circles is drawn as shown in Fig. The shear strength parameters are $c_u$ and $\phi_{CU}$.
- If pore water pressure is measured, effective stress Mohr circles may be plotted as shown in the figure. The strength parameters for effective stresses are represented by c'and $\phi'$.



# CONSOLIDATED-DRAINED TEST

- In drained triaxial tests the soil is first consolidated under an ambient pressure *pa and then subjected to* an increasing deviator stress until failure occurs, the rate of strain being controlled in such a way that at no time is there any appreciable pore-pressure in the soil.
- Thus at all times the applied stresses are effective, and when the stresses at failure are plotted in the usual manner, the failure envelope is directly expressed in terms of effective stresses.
- For normally consolidated clays and for sands the envelope is linear for normal working stresses and passes through the origin as shown in Fig. The failure criterion for such soils is therefore the angle of shearing resistance in the drained condition $\phi_d$. The drained strength is

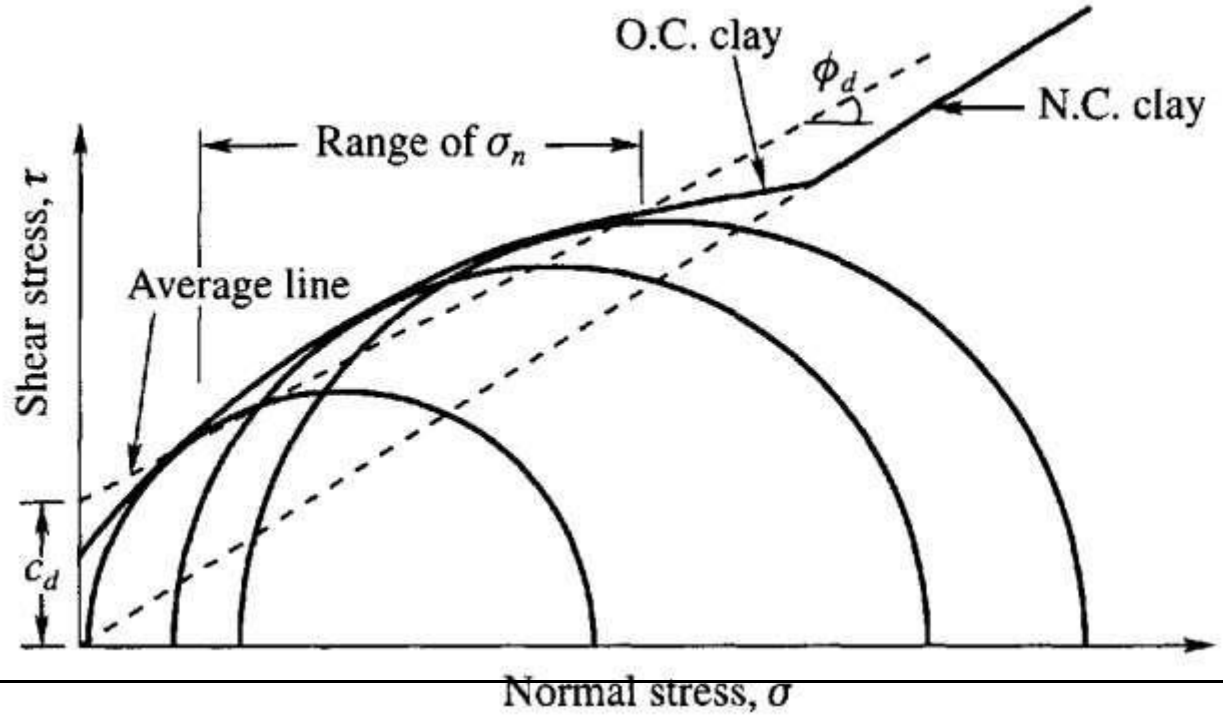
$$\frac{1}{2}(\sigma_1 - \sigma_3)_f = \frac{p \sin\phi_d}{1 - \sin\phi_d}$$



- For overconsolidated clays, the envelope intersects the axis of zero pressure at a value $c_d$. The apparent cohesion in the drained test and the strength are given by the expression.

$$\frac{1}{2}(\sigma_1 - \sigma_3)_f = \frac{c_d \cos\phi_d + p\sin\phi_d}{1-\sin\phi_d}$$

- The Mohr envelope for overconsolidated clays is not linear as may be seen in Fig. An average line is to be drawn within the range of normal pressure. The shear strength parameters $c_d$ and $\phi_d$ are referred to this line.
- Since the stresses in a drained test are effective, it might be expected that a given $\phi_d$ would be equal to $\phi'$ as obtained from undrained tests with pore-pressure measurement.



# PORE PRESSURE PARAMETERS

- Pore water pressures play an important role in determining the strength of soil. The change in pore water pressure due to change in applied stress is characterised by dimensionless coefficients, called ‘Pore pressure coefficients’ or ‘Pore pressure parameters’ A and B.
- These parameters have been proposed by Prof. A.W. Skempton (Skempton, 1954) and are now universally accepted.
- The difference between the total and effective stresses is simply the pore water pressure **u.** Consequently, the total and effective stress Mohr circles have the same diameter and are only separated along the **σ - axis** by the magnitude of the pore water pressure.
- It is easy to construct a series of total stress Mohr Circles but the inferred total stress parameters have no relevance to actual soil behaviour. In principle, the effective strength parameters are necessary to check the stability against failure for any soil construction in the field. To do this, the pore water pressure in the ground under the changed loading conditions must be known and in general they are not.
- In an undrained triaxial test with pore pressure measurement, this is possible and the effective stresses can then be determined. Alternatively, in drained tests, the loading rate can be made sufficiently slow so as to allow the dissipation of all excess pore water pressure. For low permeability soils, the drainage will require longer times.

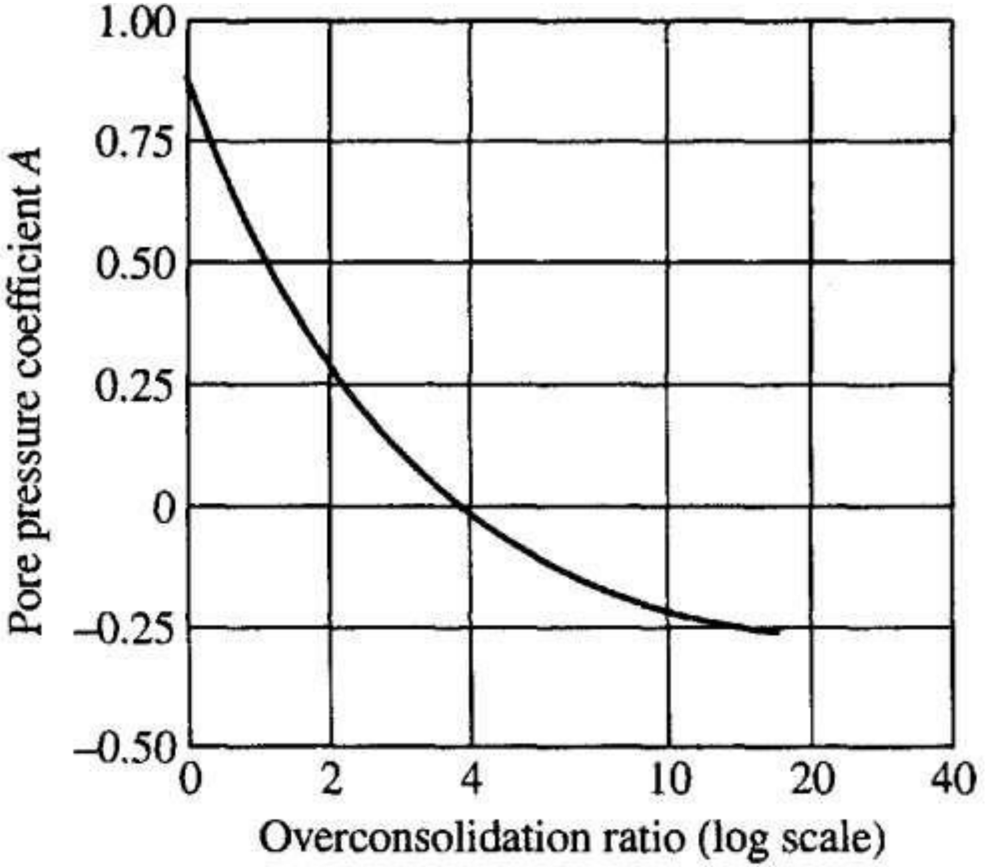
- In undrained tests, the general expression relating total pore water pressure developed and changes in applied stresses for both the stages is:

$$\Delta u = \Delta u_1 + \Delta u_2$$

$$= B.\Delta\sigma_3 + B.A.(\Delta\sigma_1 - \Delta\sigma_3)$$

- where $\Delta u_1$ = pore water pressure developed in the first stage during application of confining stress $\Delta\sigma_3$, $\Delta u_2$ = pore water pressure developed in the second stage during application of deviator stress $(\Delta\sigma_1 - \Delta\sigma_3)$, and
- **B** and **A** are Skempton's pore water pressure parameters.
- Parameter **B** is a function of the degree of saturation of the soil (= 1 for saturated soils, and = 0 for dry soils). Parameter **A** is also not constant, and it varies with the over-consolidaton ratio of the soil and also with the magnitude of deviator stress. The value of **A** at failure is necessary in plotting the effective stress Mohr circles.
- Consider the behaviour of saturated soil samples in undrained triaxial tests. In the first stage, increasing the cell pressure without allowing drainage has the effect of increasing the pore water pressure by the same amount. Thus, there is no change in the effective stress. During the second shearing stage, the change in pore water pressure can be either positive or negative.
- For **UU tests** on saturated soils, pore water pressure is not dissipated in both the stages (i.e., $\Delta u = \Delta u_1 + \Delta u_2$).
- For **CU tests** on saturated soils, pore water pressure is not dissipated in the second stage only (i.e., $\Delta u = \Delta u_2$).

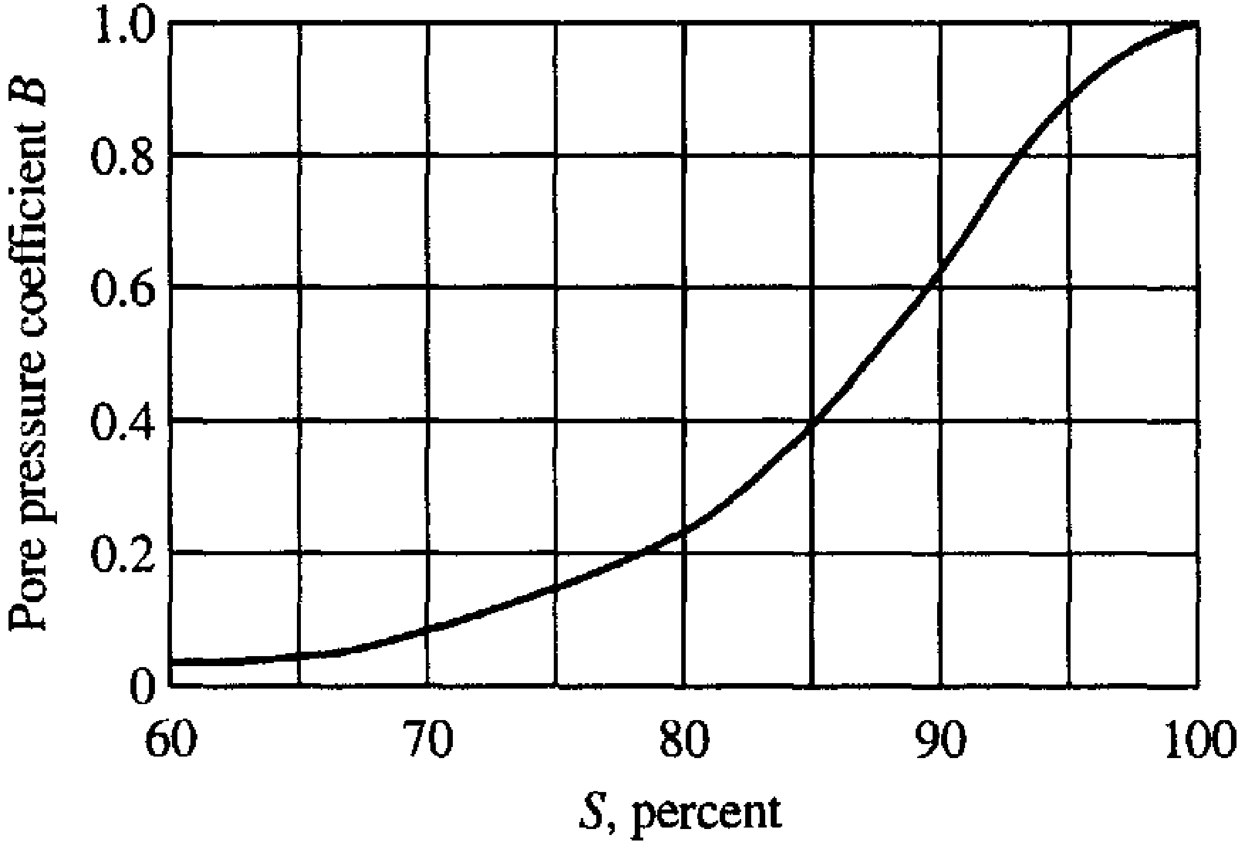
The variation of A under a failure condition (A) with the Overconsolidation ratio, OCR, is given in Fig.




Some typical values of A, are given in Table

| Type of Soil | Volume change | $A_f$ |
|---|---|---|
| Highly sensitive clay | large contraction | + 0.75 to + 1.5 |
| Normally consolidated clay | contraction | + 0.5 to + 1.0 |
| Compacted sandy clay | slight contraction | + 0.25 to + 0.75 |
| Lightly overconsolidated clay | none | + 0.00 to + 0.5 |
| Compacted clay gravel | expansion | – 0.25 to + 0.25 |
| Heavily overconsolidated clay | expansion | – 0.5 to 0 |

The value of B varies with the degree of saturation as shown in Fig



# Stress-strain behaviour of clays

- The stress-strain behaviour of clays is primarily dependent upon whether the clay is in a normally consolidated state or in an over-consolidated state.
- The behaviour of a normally consolidated clay is somewhat similar to that of a loose sand and that of an over-consolidated clay is similar to that of a dense sand.
- In the case of plastic nature of stress-strain relationship with no specific failure point, an arbitrary strain of 15 to 20% is considered to be representative of failure condition.
- In normally consolidated clays and in loose sands the two angles of shearing resistance are in fact closely equal since the rate of volume change in such materials at failure in the drained test is approximately zero and there is no volume change throughout an undrained test on saturated soils.
- But in dense sands and heavily over-consolidated clays there is typically a considerable rate of positive volume change at failure in drained tests, and work has to be done not only in overcoming the shearing resistance of the soils, but also in increasing the volume of the specimen against the ambient pressure.
- Yet in undrained tests on the same soils, the volume change is zero and consequently $\phi_d$ for dense sands and heavily over-consolidated clays is greater than $\phi'$.

# Stress-strain relationships for a normally consolidated and overconsolidated clay