Mechanics of Materials-II

COMBINED STRESSES

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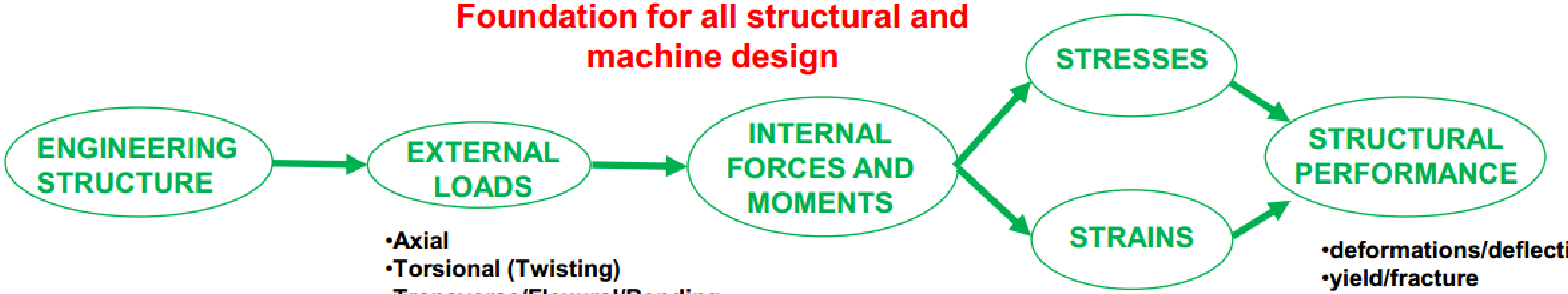
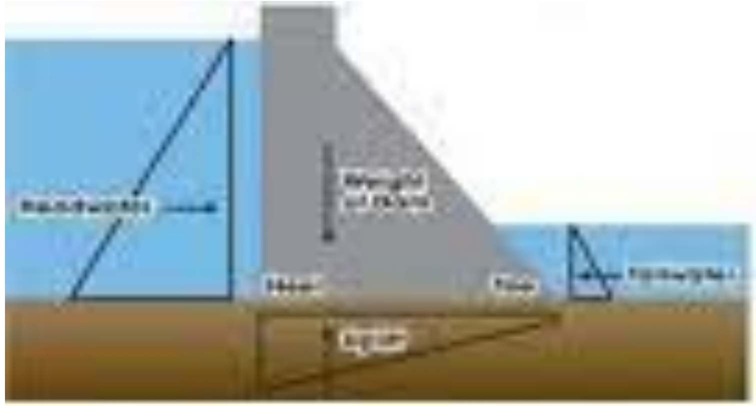
MOM (Course Outcomes)

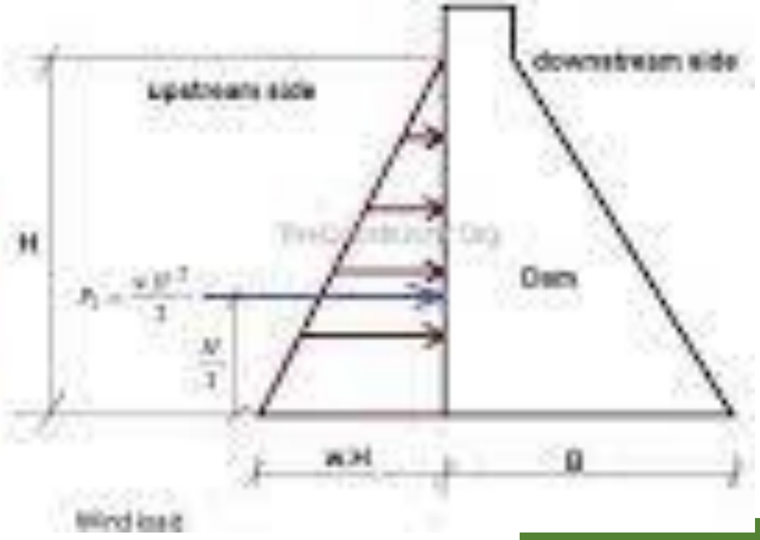
Foundation for all structural and machine design



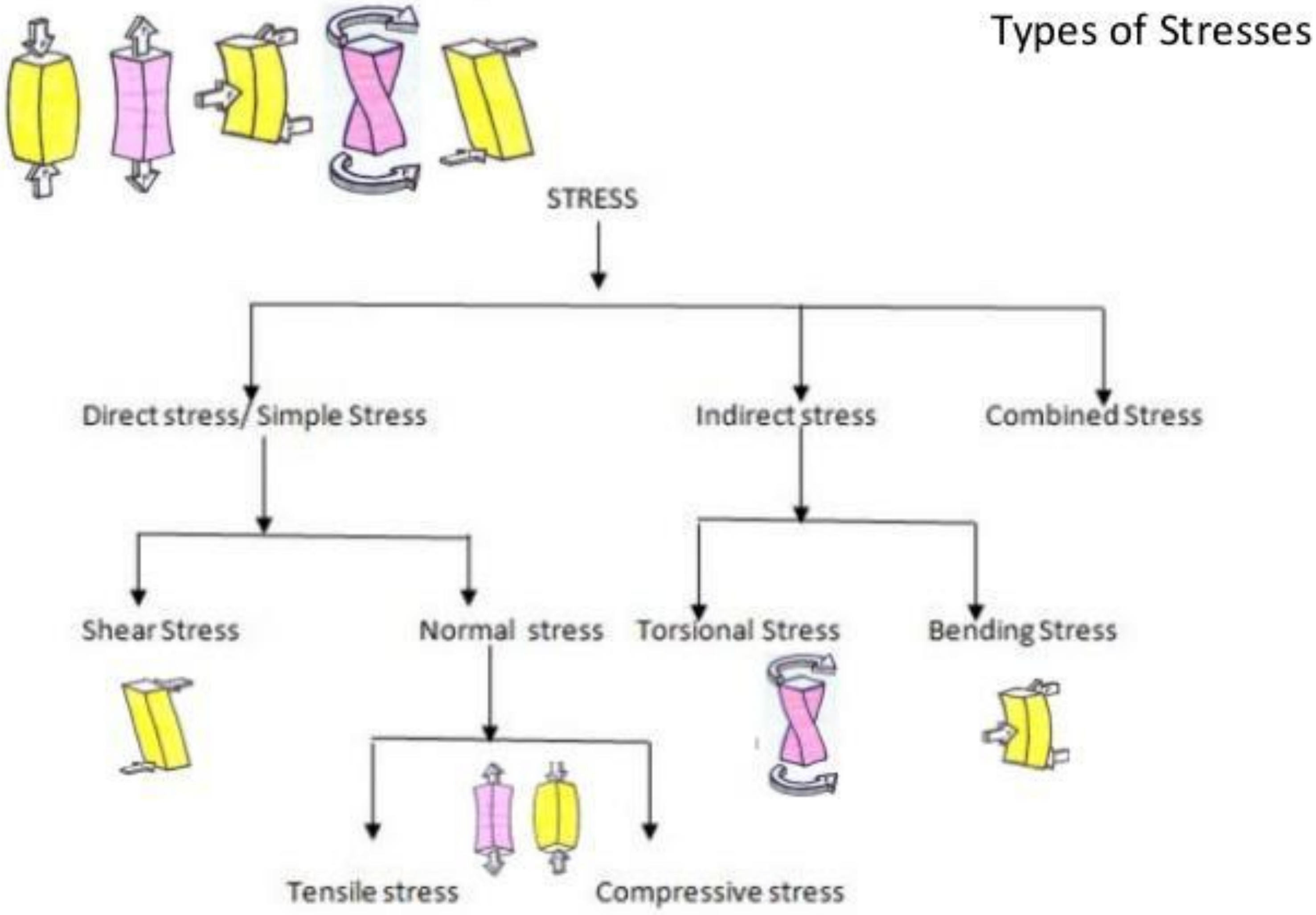
- Axial
- Torsional (Twisting)
- Transverse/Flexural/Bending
- Pressure
- Combined Loading

- deformations/deflections
- yield/fracture
- success/failure?

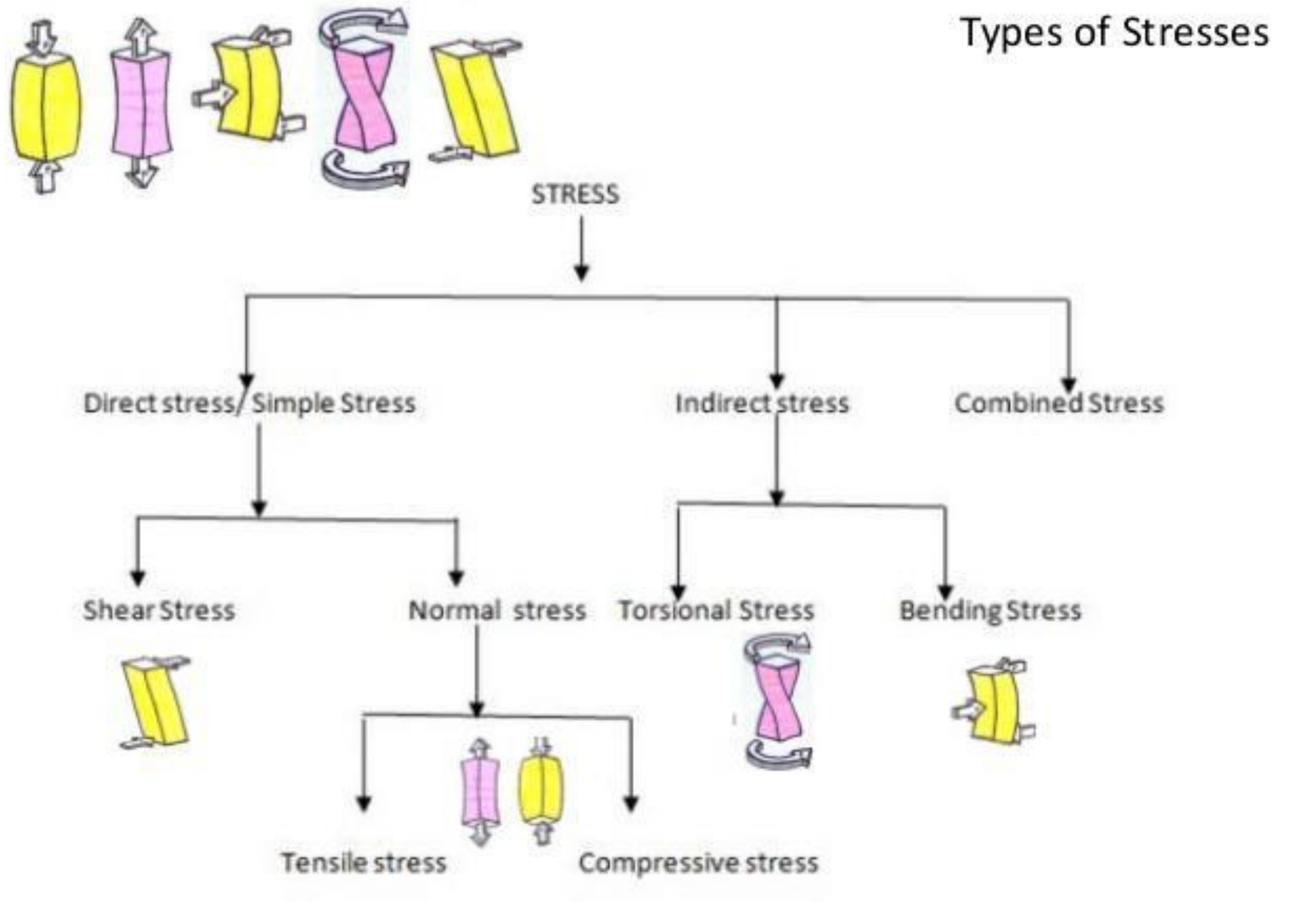




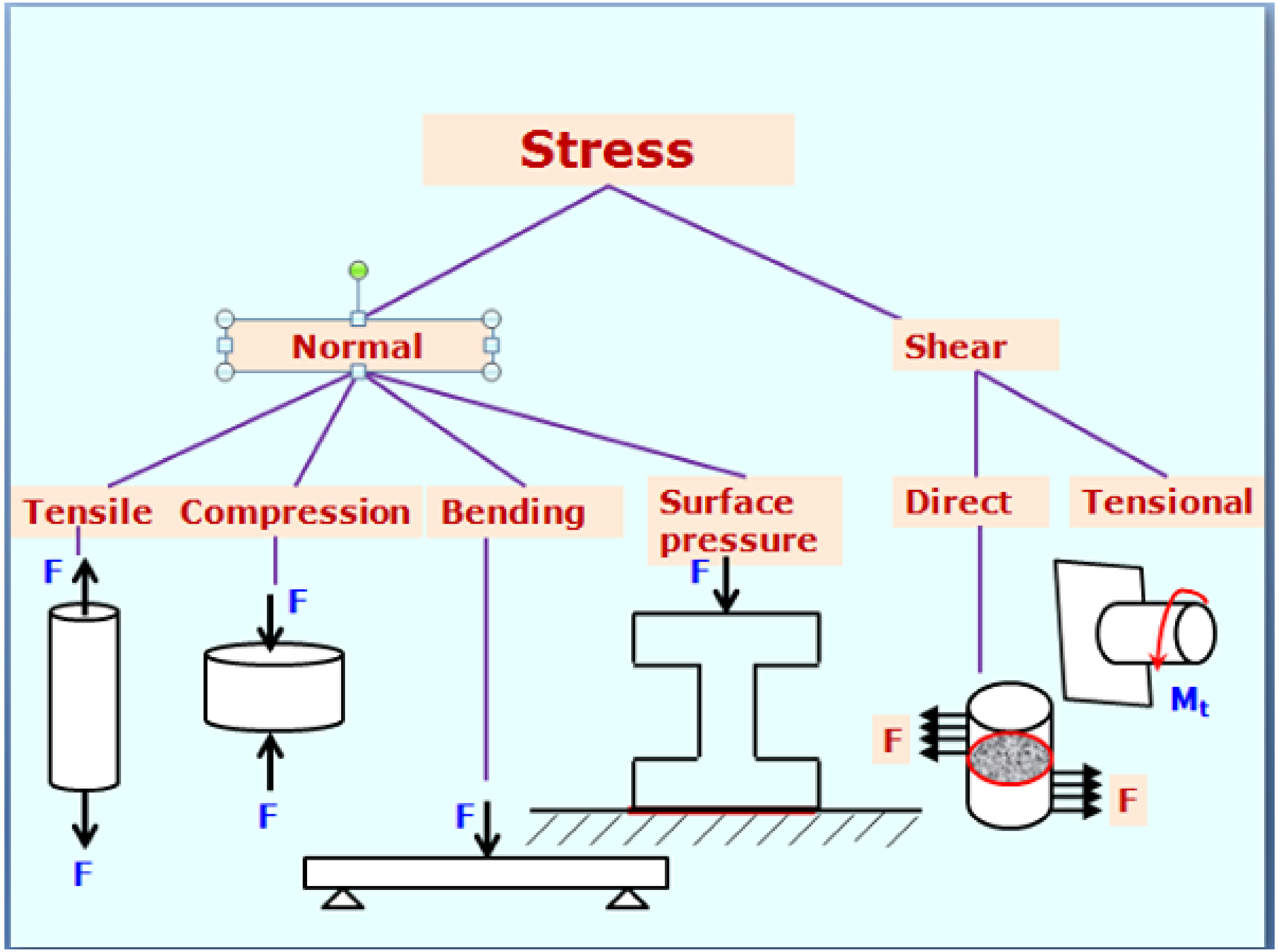
Combined Stress



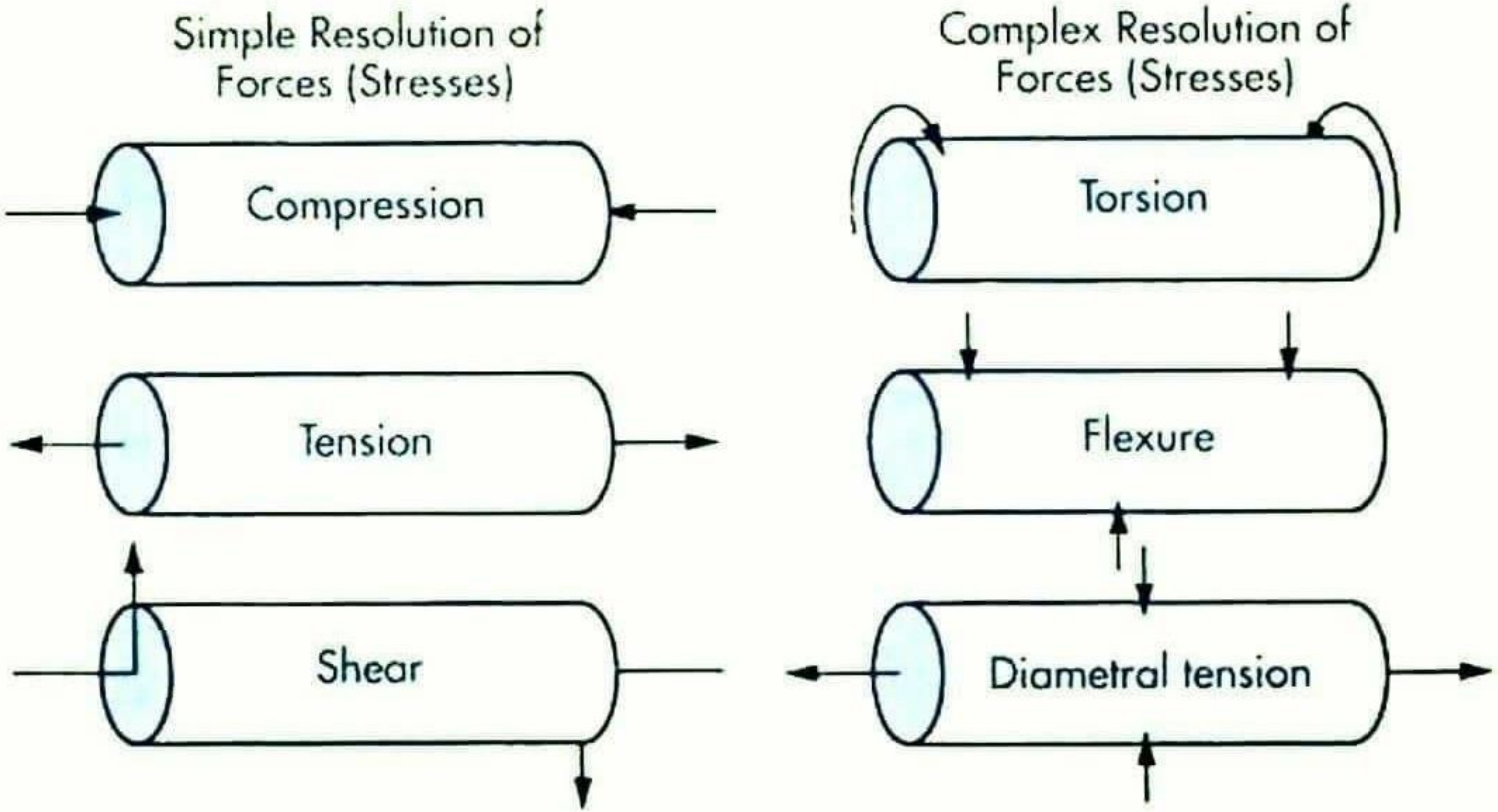
Combined Stress



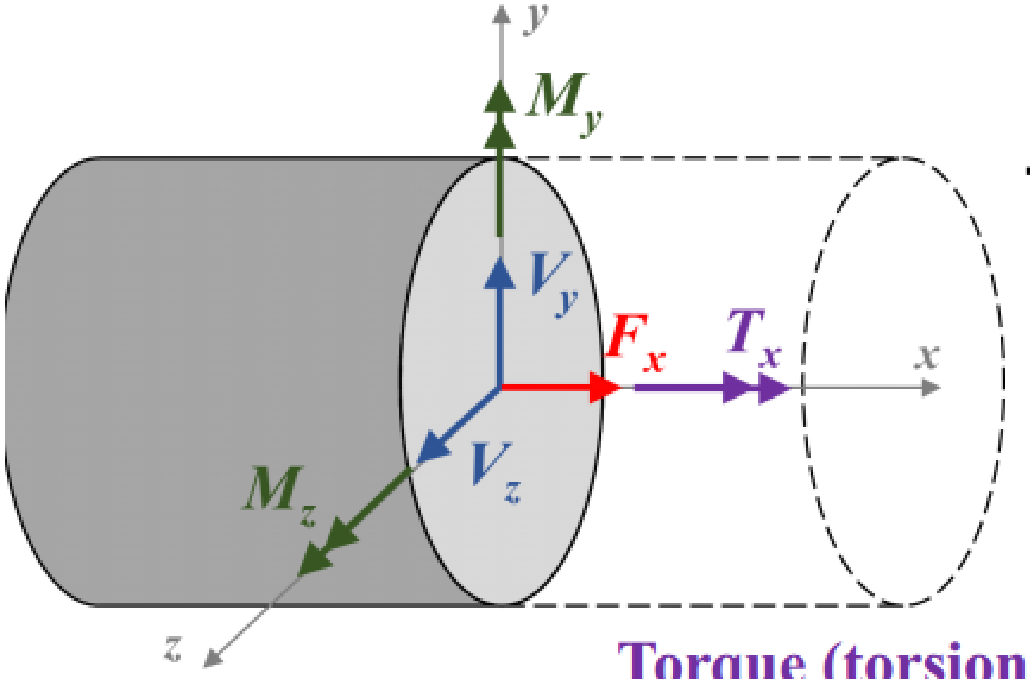
Combined Stress



Combined Stress

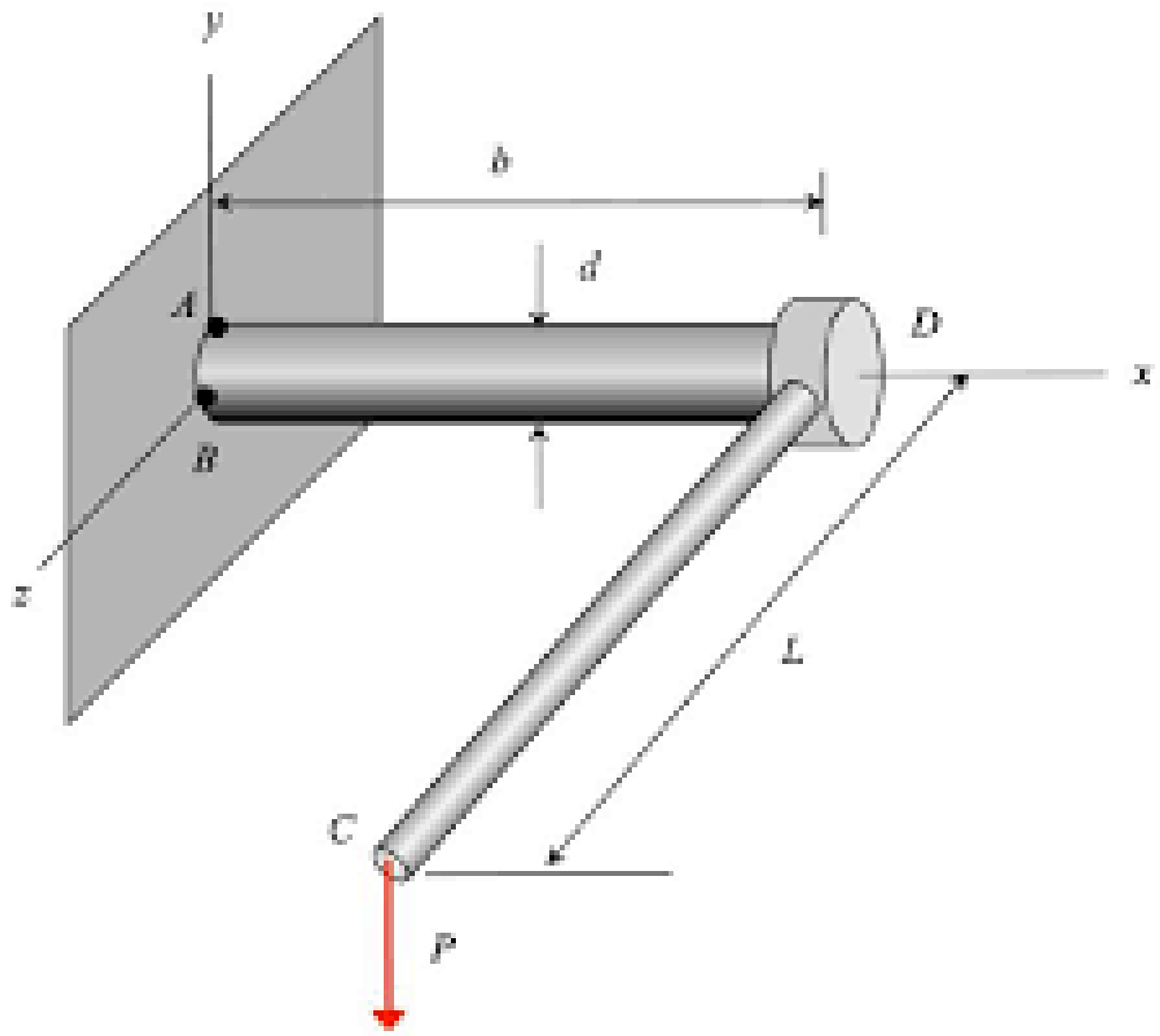


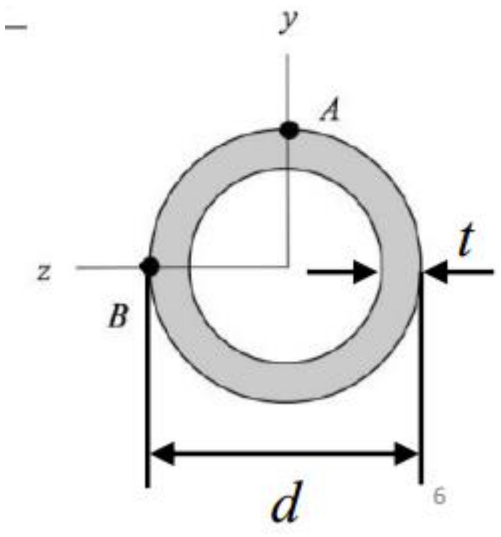
Combined Stress



Load	Type of stress	Stress distribution
Axial force F_x	Normal	$\sigma_x = F_x / A$
Shear force V_y	Shear	$\tau_{xy} = \frac{V_y Q}{I_{zz} t}$
Shear force V_z	Shear	$\tau_{xz} = \frac{V_z Q}{I_{yy} t}$
Torque (torsional moment) T_x	Shear	$\tau = T\rho / I_p$
Bending moment M_y	Normal	$\sigma_x = M_y z / I_{yy}$
Bending moment M_z	Normal	$\sigma_x = -M_z y / I_{zz}$

Combined Stress affect at Point- problem-4





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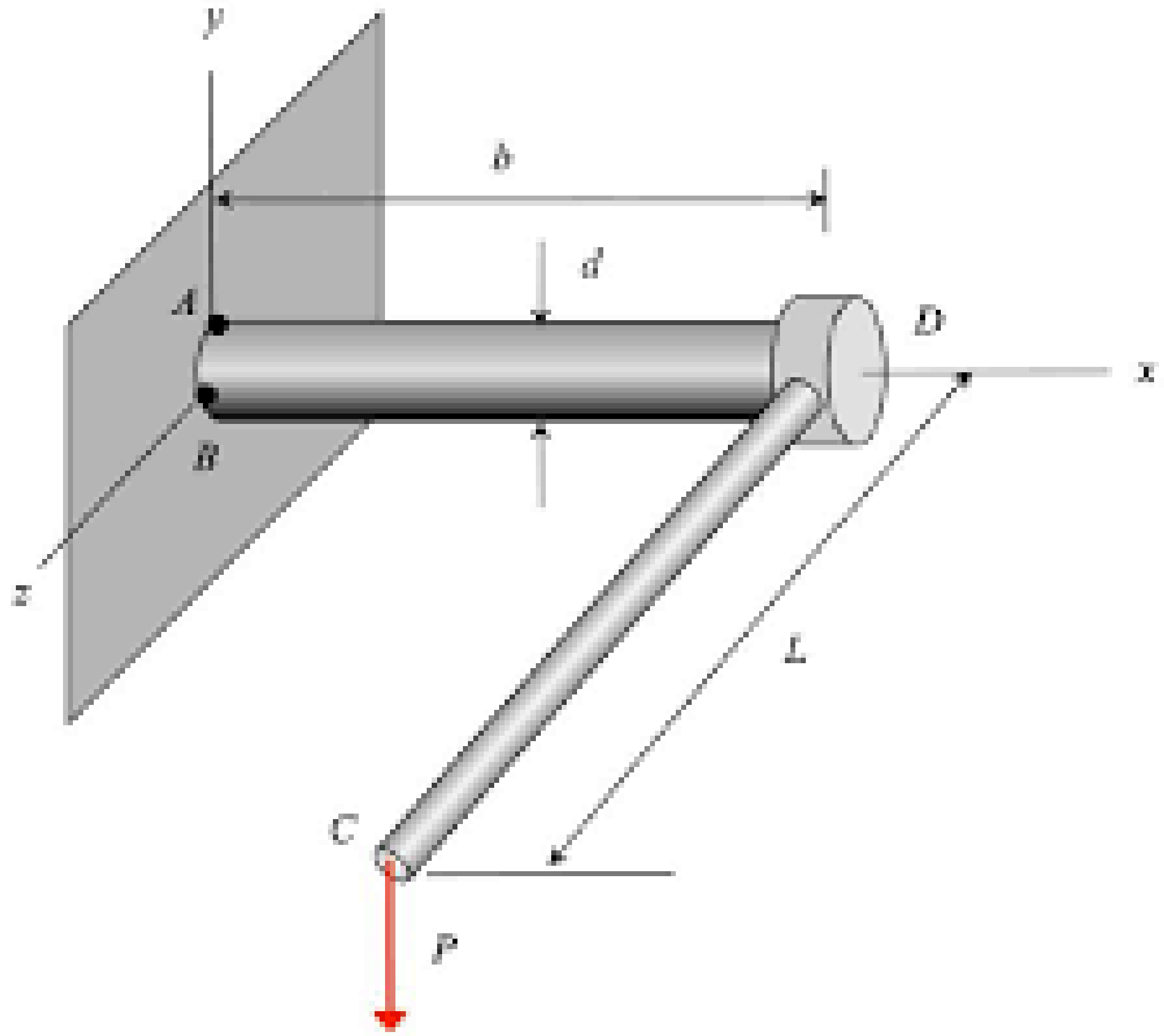
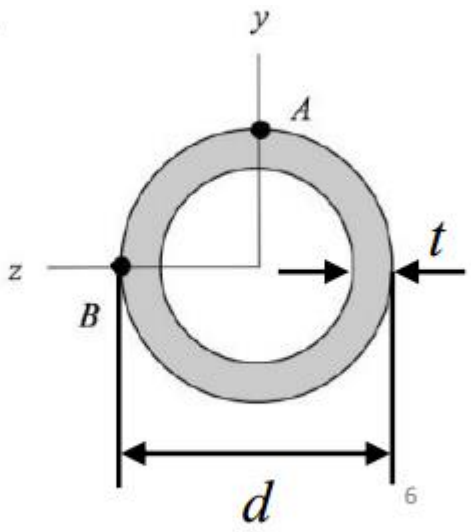
Combined Stress affect at Point- problem-4





Combined Stress (Torsion +Bending)

The principal stresses thus induced are

$$\sigma_{1,2} = \left\{\frac{\sigma_x + \sigma_y}{2}\right\} \pm \sqrt{\left\{\frac{\sigma_x - \sigma_y}{2}\right\}^2 + \{\tau\}^2}$$

$$\sigma_1 = \left\{\frac{\frac{32M}{\pi d^3}}{2}\right\} \pm \sqrt{\left\{\frac{\frac{32M}{\pi d^3}}{2}\right\}^2 + \left\{\frac{16T}{\pi d^3}\right\}^2}$$

$$\sigma_1 = \frac{16}{\pi d^3}\{M + \sqrt{\{M\}^2 + \{T\}^2}$$

$$\sigma_2 = \frac{16}{\pi d^3}\{M - \sqrt{\{M\}^2 + \{T\}^2}\}$$

Maximum shear stress

$$\tau_{max} = \sqrt{\left\{\frac{\sigma_x - \sigma_y}{2}\right\}^2 + \{\tau\}^2} \quad \text{or} \quad \frac{\sigma_1 - \sigma_2}{2}$$

$$\tau_{max} = \frac{16}{\pi d^3}\{\sqrt{\{M\}^2 + \{T\}^2}\}.$$

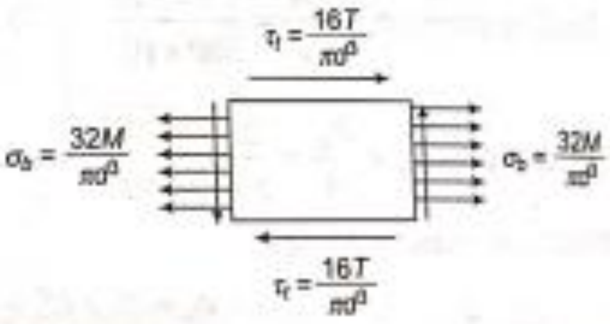


FIGURE 13.17

$$\sigma = \frac{32M_{eq}}{\pi d^3}$$

al stress due to combined bending and tw

$$\sigma_1 = \frac{16}{\pi d^3}\{M + \sqrt{\{M\}^2 + \{T\}^2}\}$$

equivalent BM $\sigma = \sigma_1$.

$$\frac{32M_{eq}}{\pi d^3} = \frac{16}{\pi d^3}\{M + \sqrt{\{M\}^2 + \{T\}^2}\}$$

$$M_{eq} = \frac{1}{2}\{M + \sqrt{\{M\}^2 + \{T\}^2}\}.$$

$$\tau_{max} = \frac{16}{\pi d^3}\{\sqrt{\{M\}^2 + \{T\}^2}\}$$

of equivalent BM $\tau = \tau_{max}$.

$$\frac{16T_{eq}}{\pi d^3} = \frac{16}{\pi d^3}\{\sqrt{\{M\}^2 + \{T\}^2}\}$$

Combined Stress (Torsion +Bending)

... stress d...

$$\sigma = \frac{32M_{eq}}{\pi d^3}$$

...al stress due to combined bending and tw...

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Combined Stress (Torsion +Bending)- Problem-5

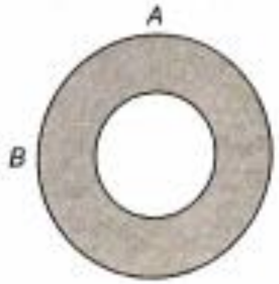
A Hollow shaft of outer diameter 10 mm and inner diameter 50 mm is subjected to a BM of 10 kN.m and Twisting moment of 6 kN.m. Determine the maximum normal and shear stress induced in the shaft at A and B shown in Figure

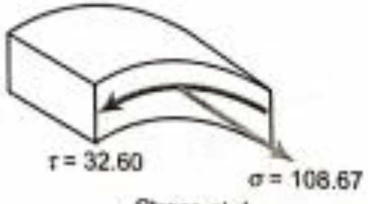
Maximum bending normal stress at the extreme fibers

$$\sigma_{\text{bending}} = \sigma = \frac{M}{I} y_{\max} = \frac{10\times10^6}{4.601\times10^6} \times 50 = 108.67\,\text{MPa}$$

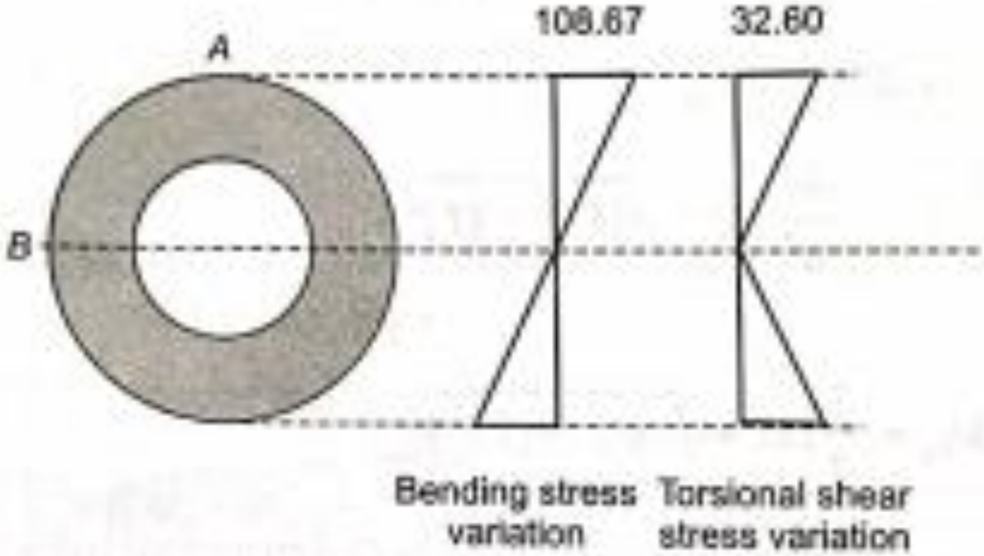
Maximum torsional shear stress at the extreme fibers

$$\tau_{\text{torsional}} = \tau = \frac{T}{J} r_{\max} = \frac{6\times10^6}{9.202\times10^6} \times 50 = 32.60\,\text{MPa}$$





Stress at A



Combined Stress (Torsion +Bending)- Problem-5

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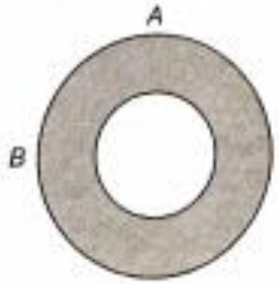
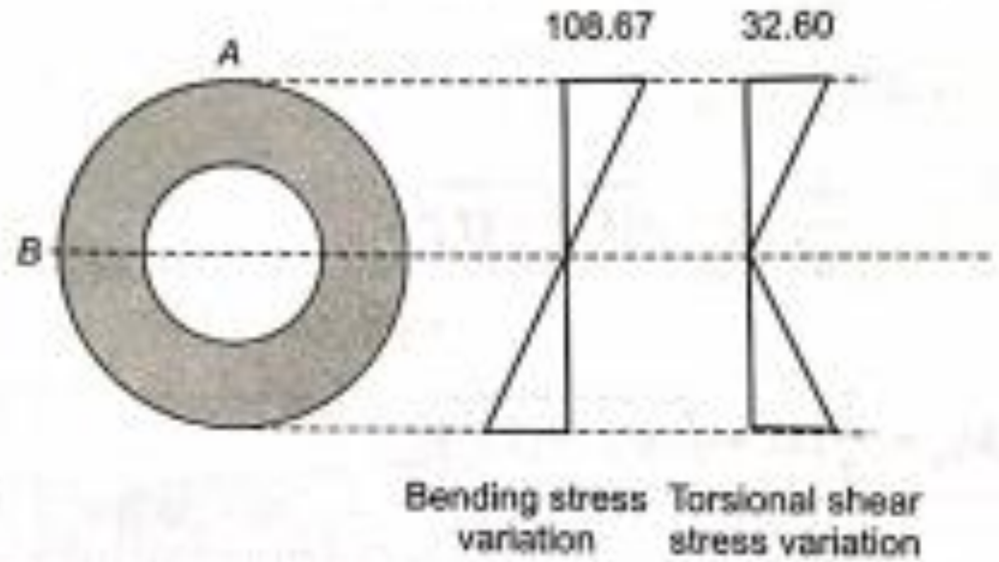
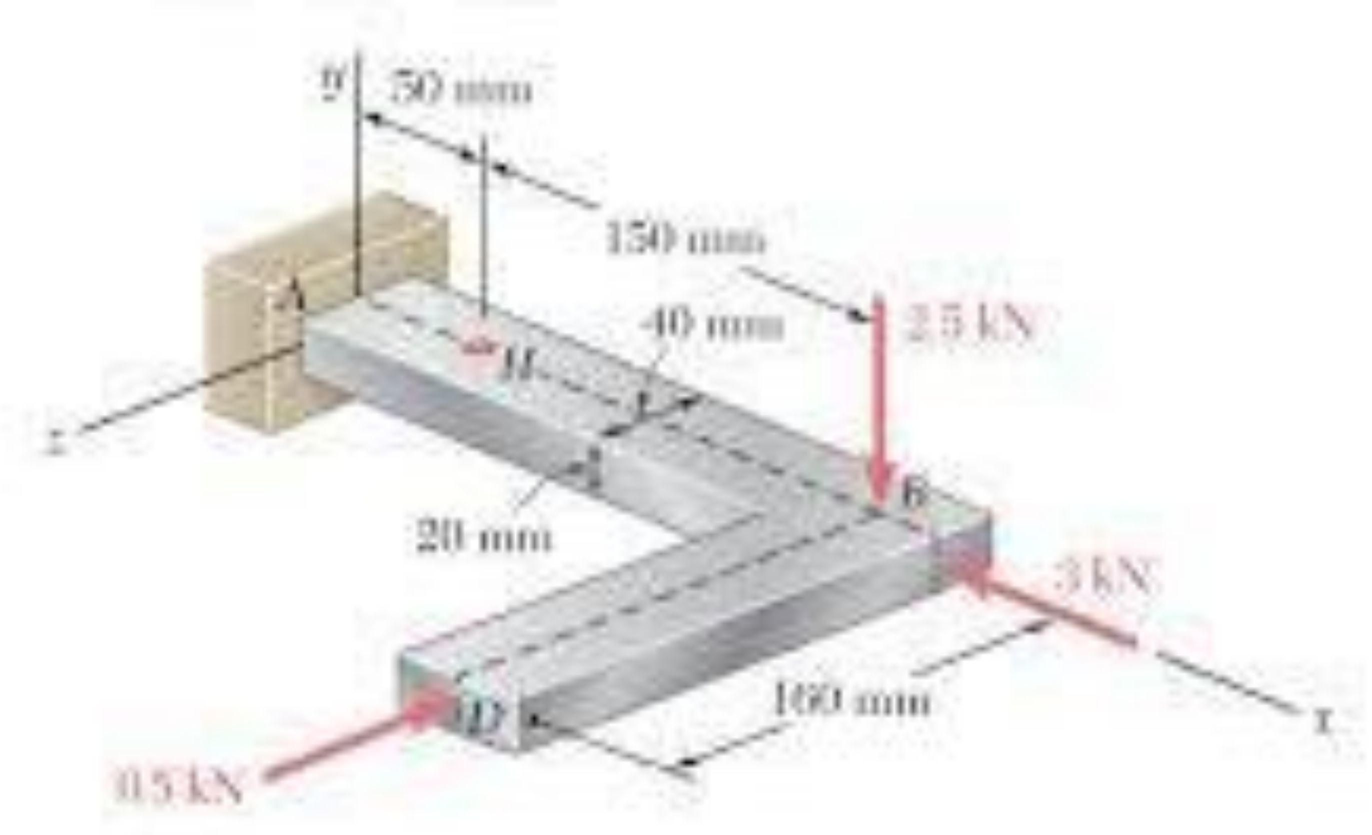


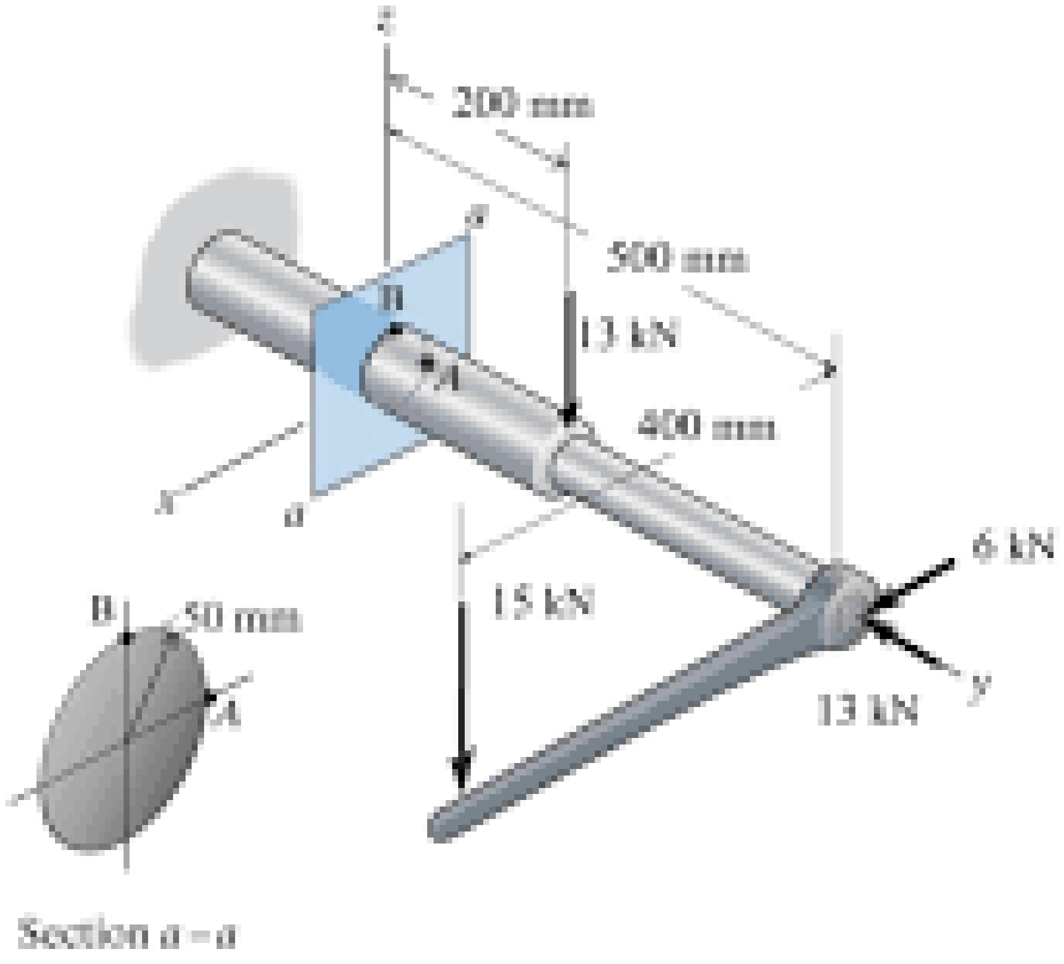
Stress at A



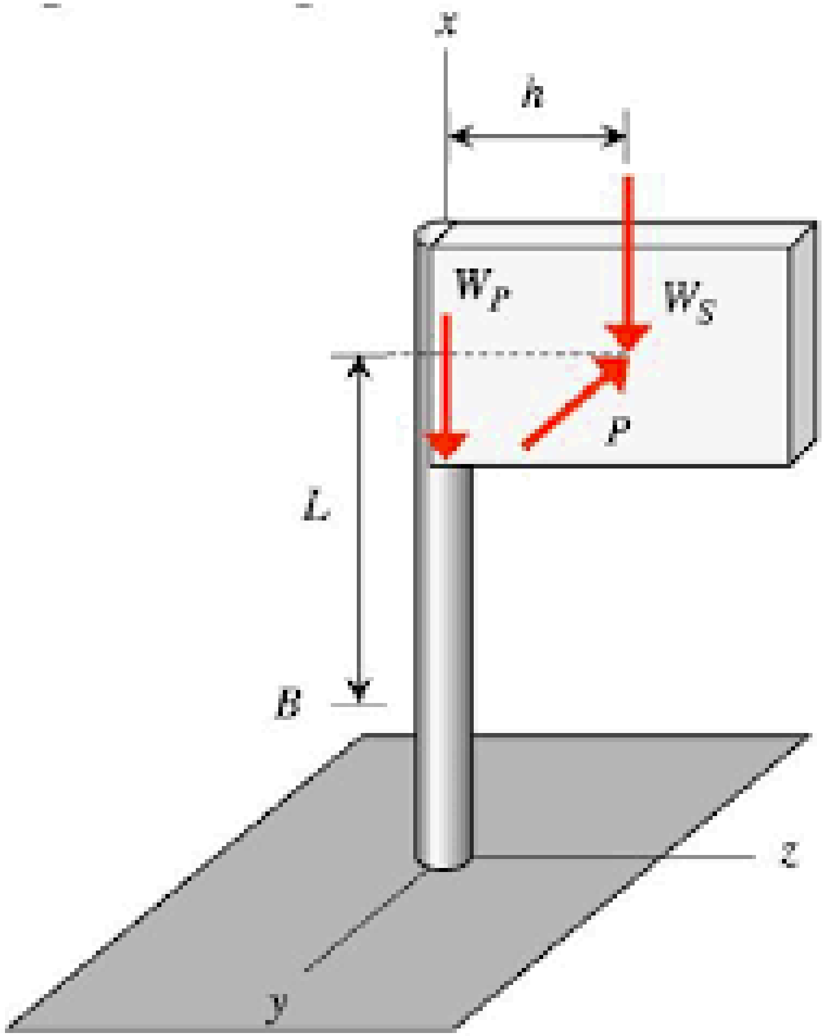
Combined Stress

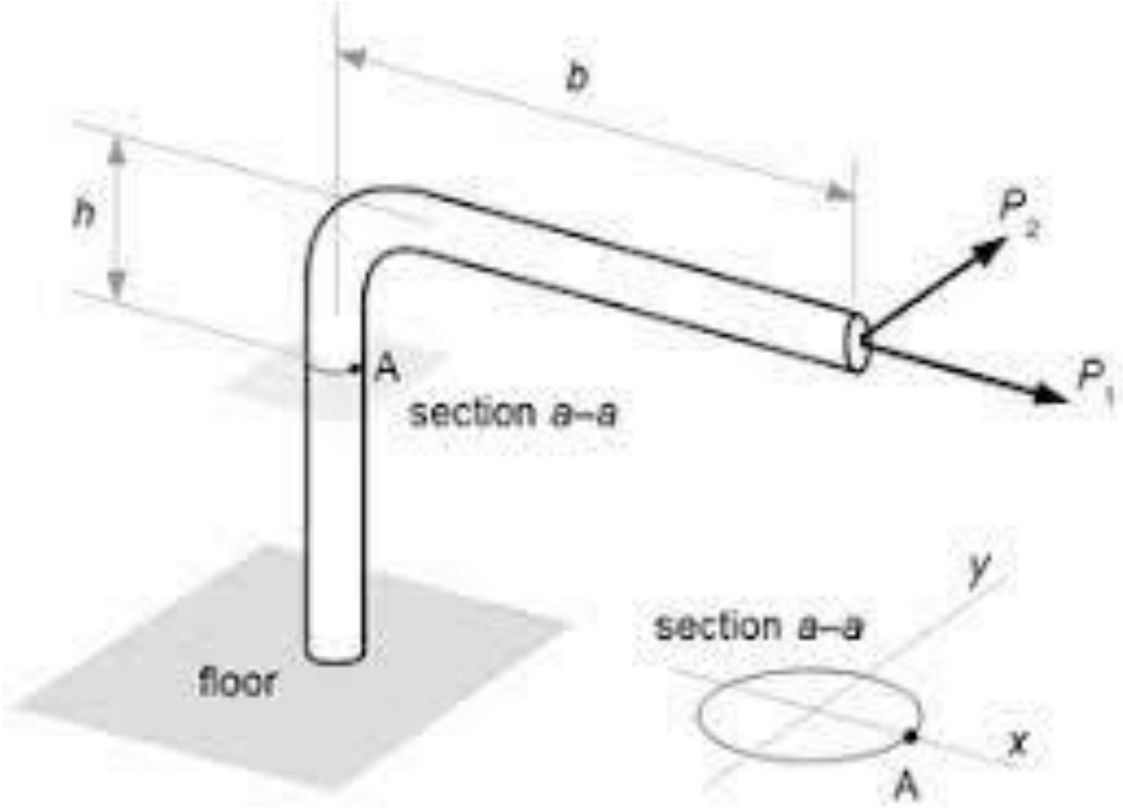


Combined Stress



Combined Stress





Combined Stress

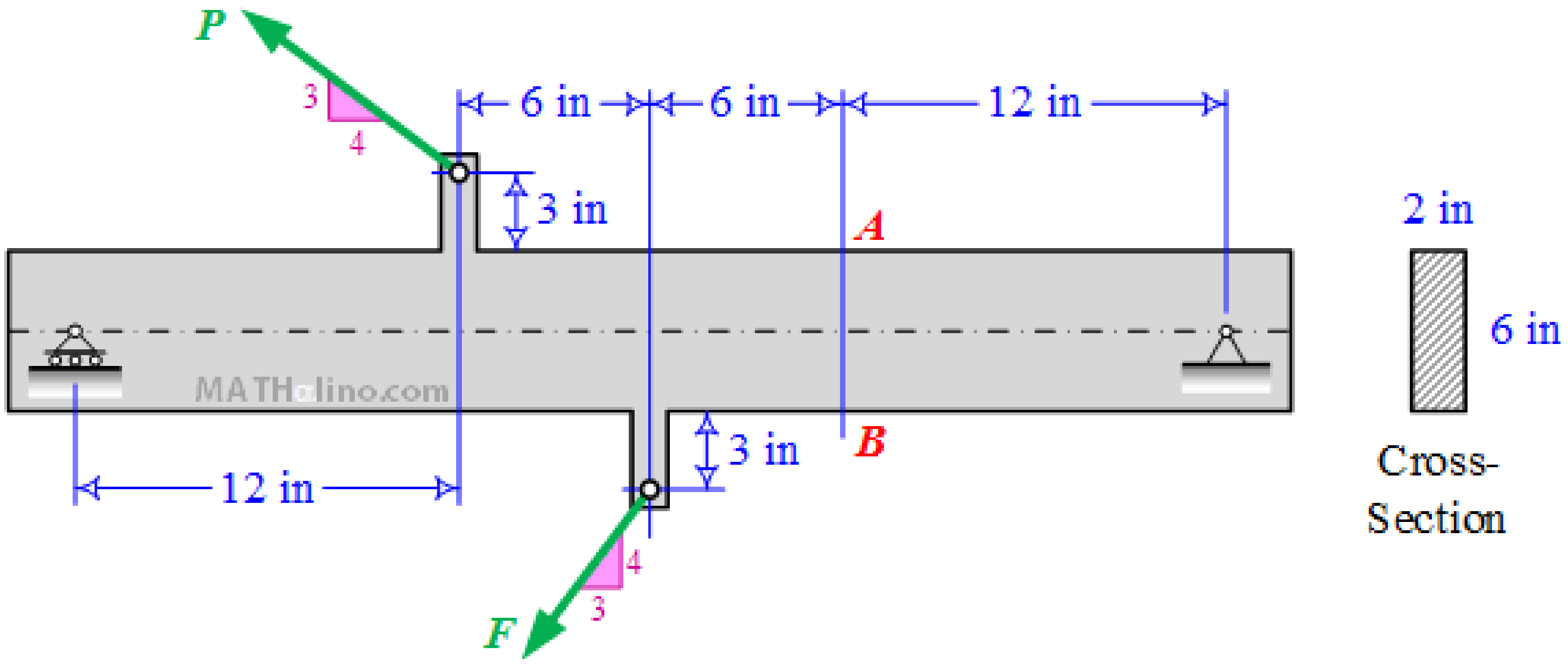


Figure P-912

Combined Stress

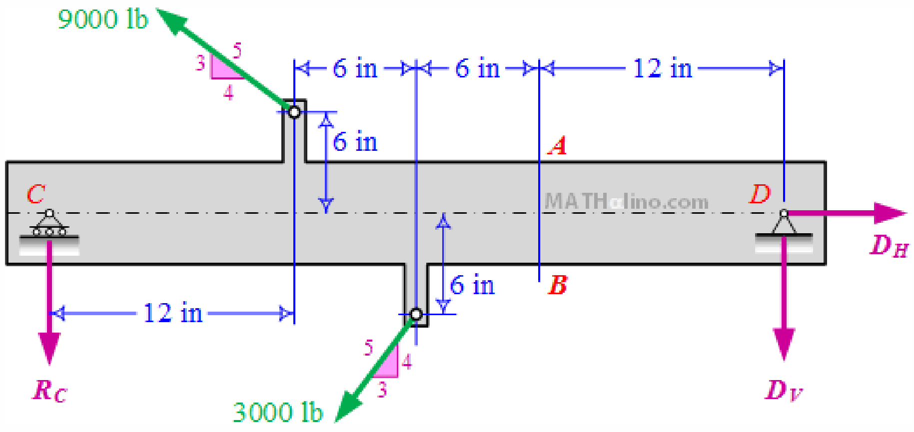
$$\Sigma M_C = 0$$

$$36D_V + 18(\tfrac{4}{5} \times 3000) + 6(\tfrac{3}{5} \times 3000) = 12(\tfrac{3}{5} \times 9000) + 6(\tfrac{4}{5} \times 9000)$$

$$36D_V + 43,200 + 10,800 = 64,800 + 43,200$$

$$36DV = 54,000$$

$$D_V = 1500 \text{ lb}$$



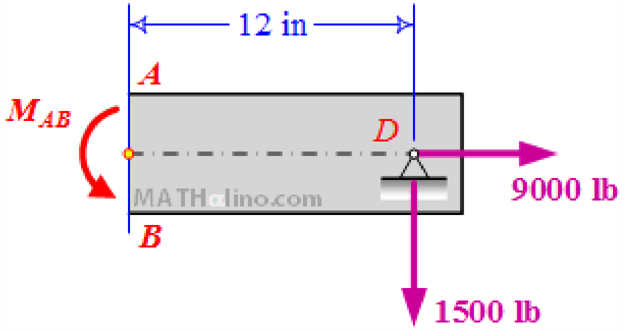
$$\Sigma F_H = 0$$

$$D_H = \tfrac{4}{5}(9000) + \tfrac{3}{5}(3000)$$

$$D_H = 9000 \text{ lb}$$

$$M_{AB} = \Sigma M_{\text{to the right of }AB}$$

$$M_{AB} = 12 \times 1500 = 18,000 \text{ lb} \cdot \text{in}$$



$$\sigma_a = \frac{D_H}{A_{AB}} = \frac{9000}{2(6)}$$

$$\sigma_a = 750 \text{ psi}$$

$$\sigma_f = \frac{6M_{AB}}{bd^2} = \frac{6(18,000)}{2(6^2)}$$

$$\sigma_f = 1500 \text{ psi}$$

$$\sigma_A = \sigma_a + \sigma_f = 750 + 1500$$

$$\sigma_A = 2250 \text{ psi} \quad \textit{answer}$$

$$\sigma_B = \sigma_a - \sigma_f = 750 - 1500$$

$$\sigma_B = -750 \text{ psi} \quad \textit{answer}$$

Combined Stress- (Problem-3)

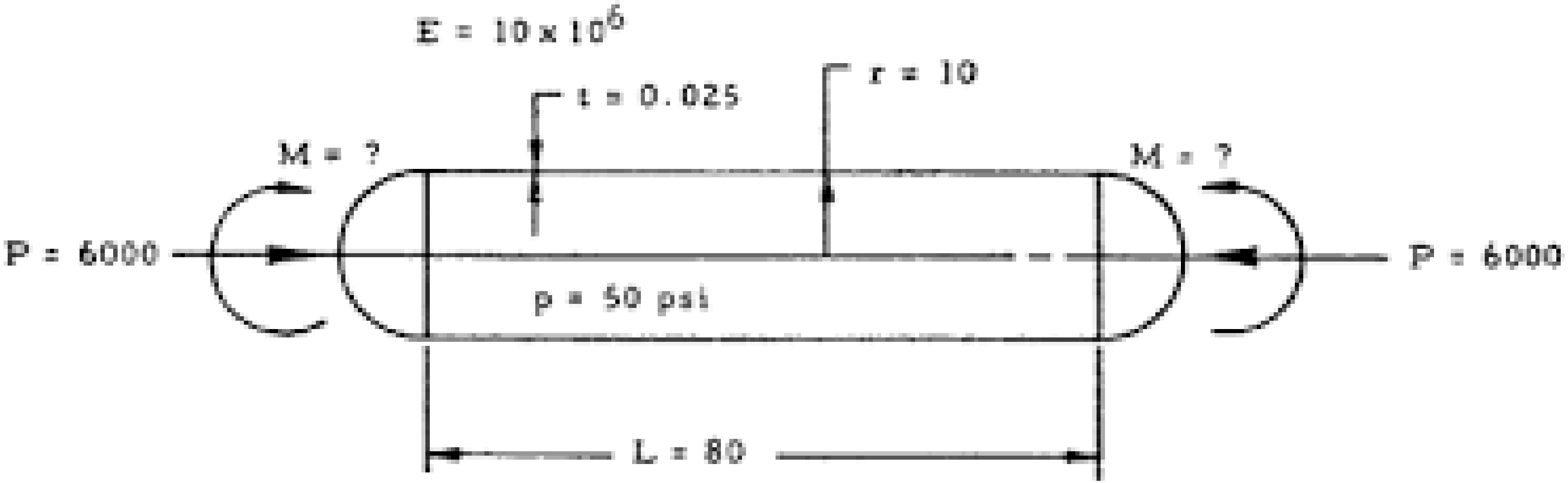
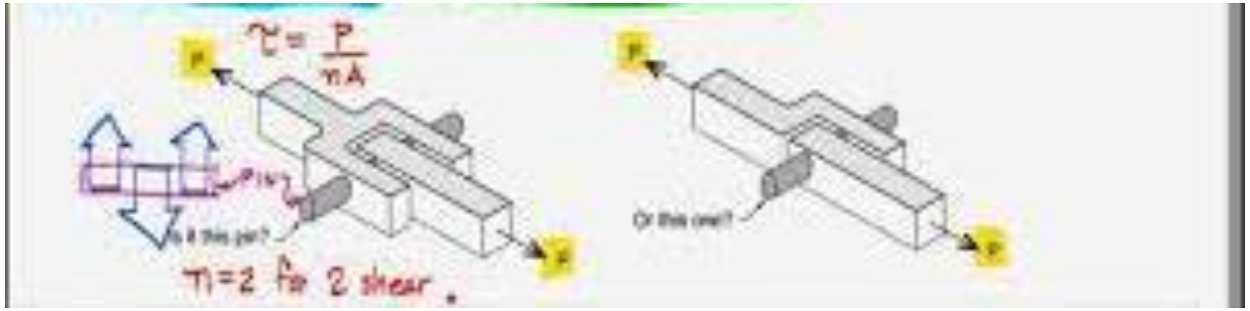
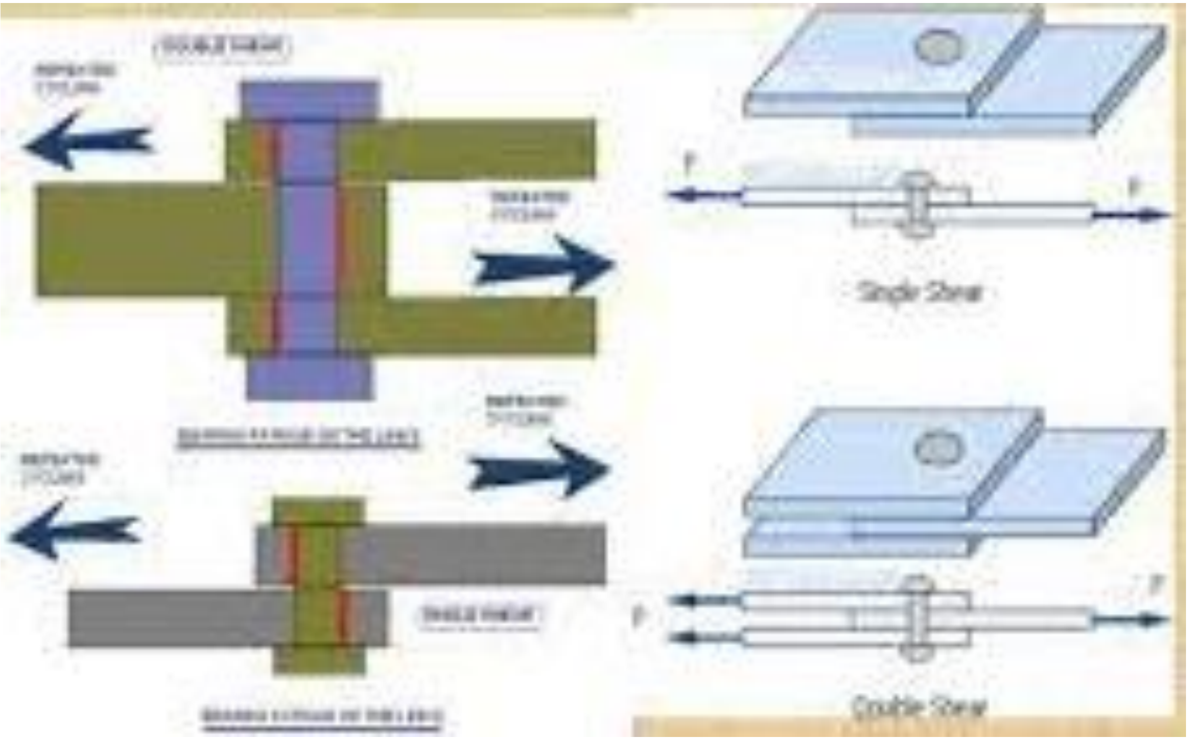
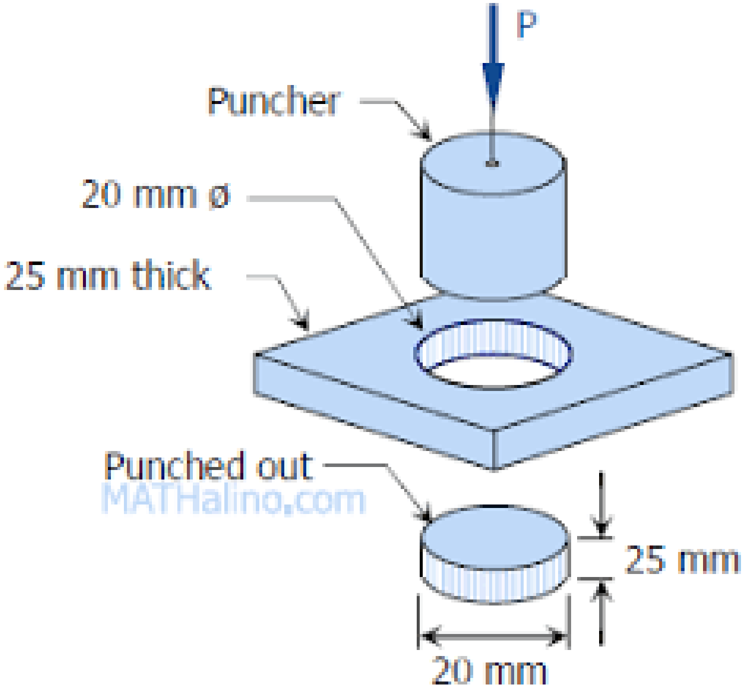


Figure 8-36. Pressurized Cylinder in Compression and Bending

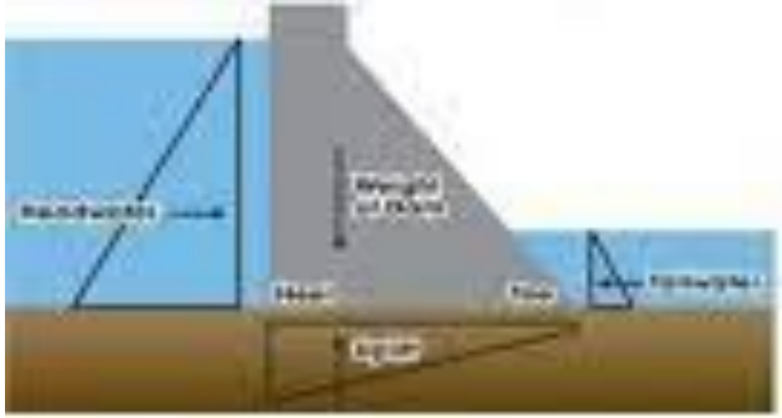
Direct Shear Stress

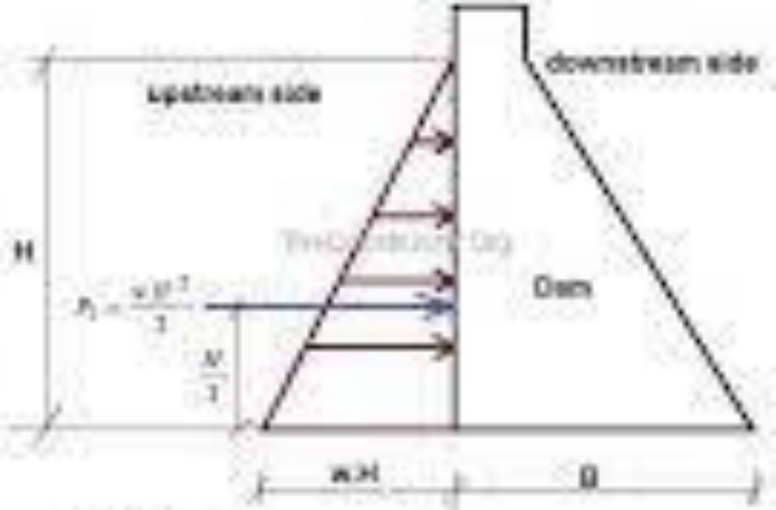


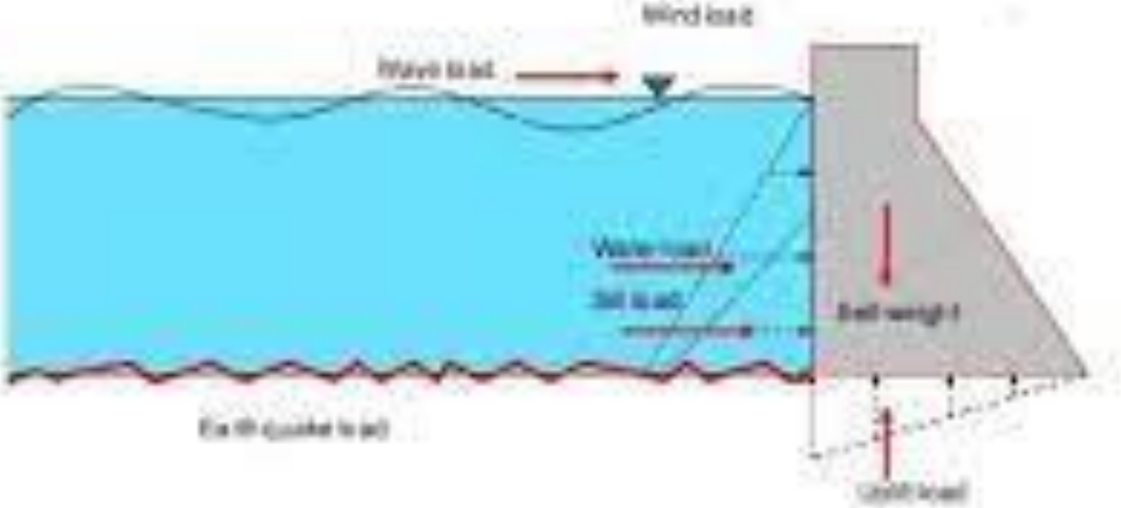




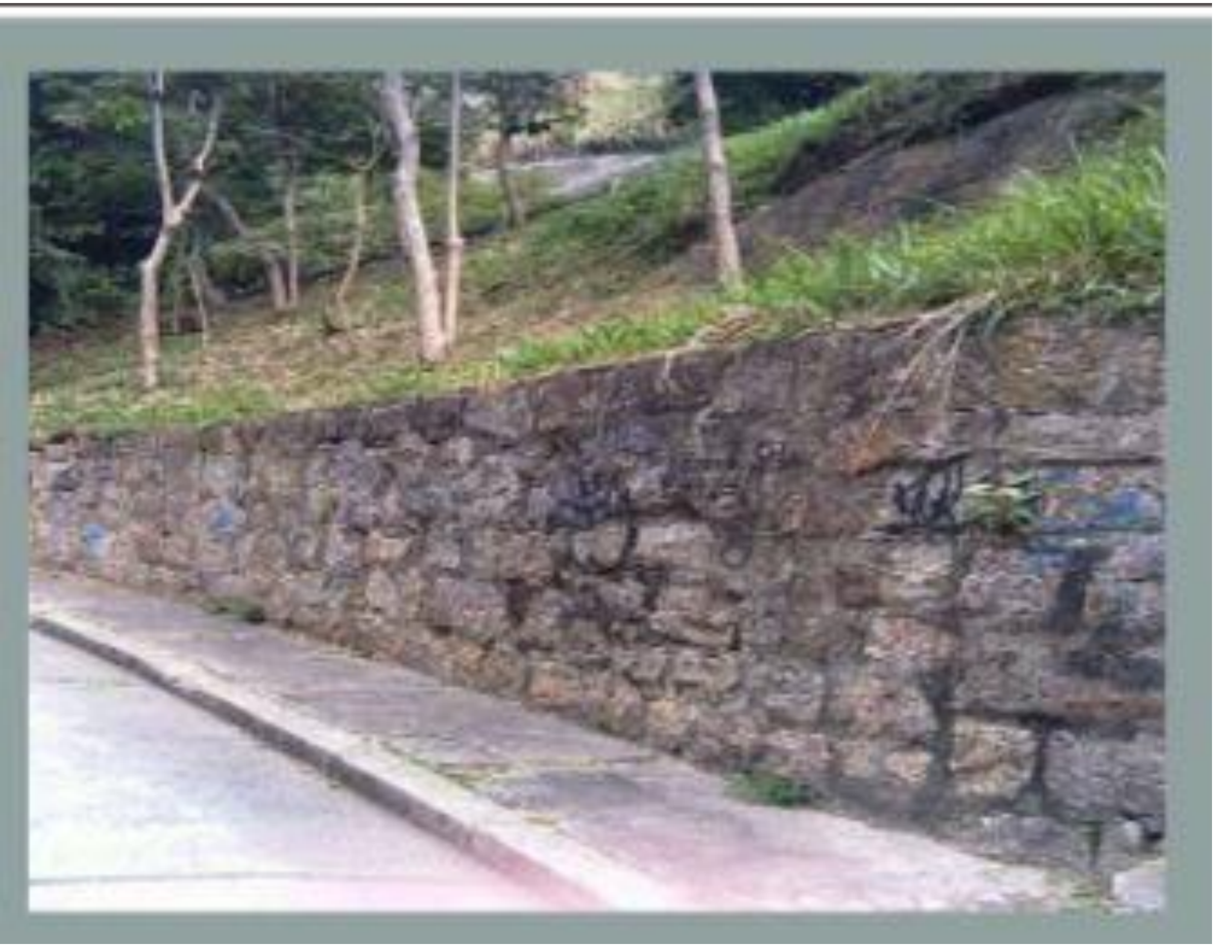
Combined Stress- Dam

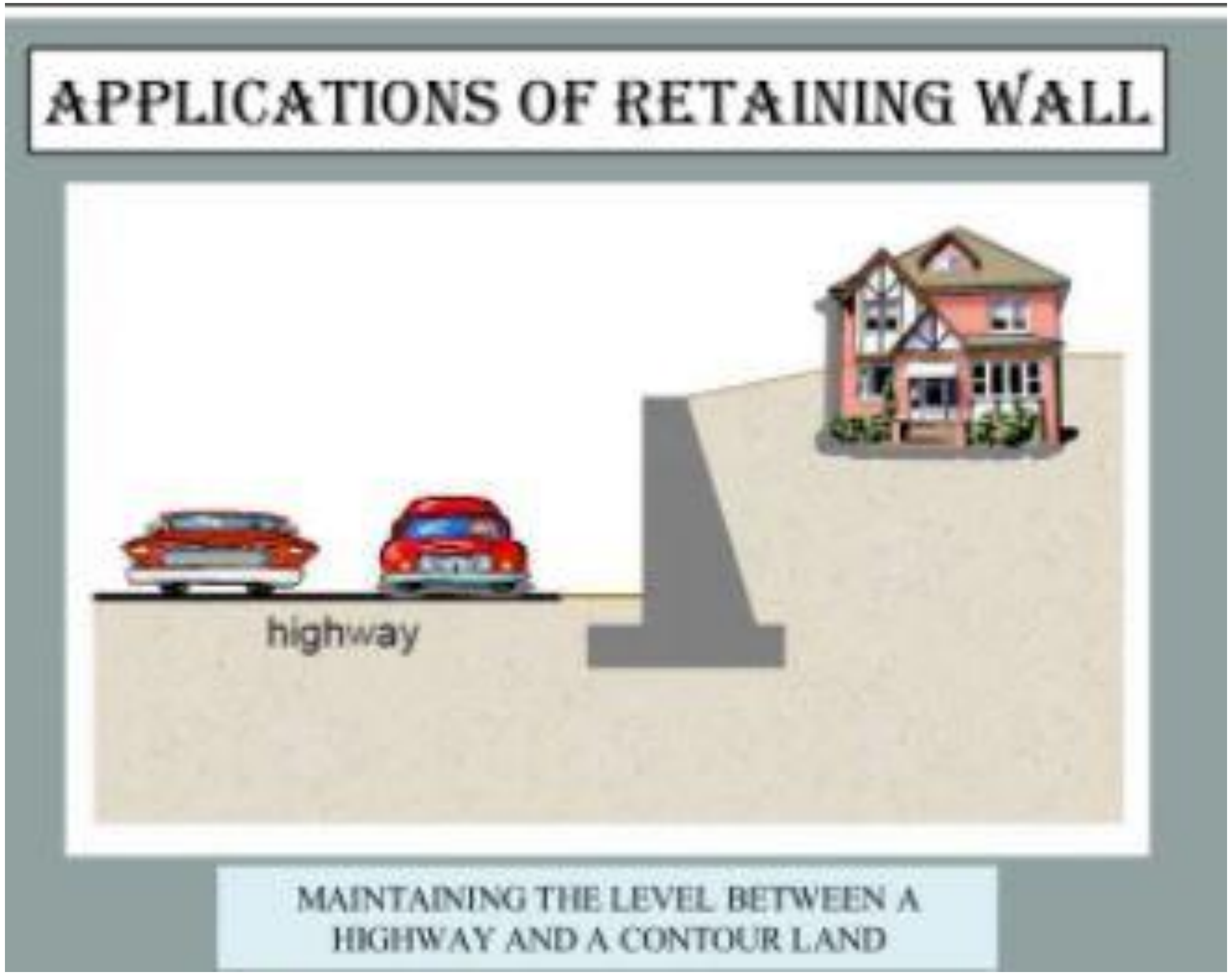




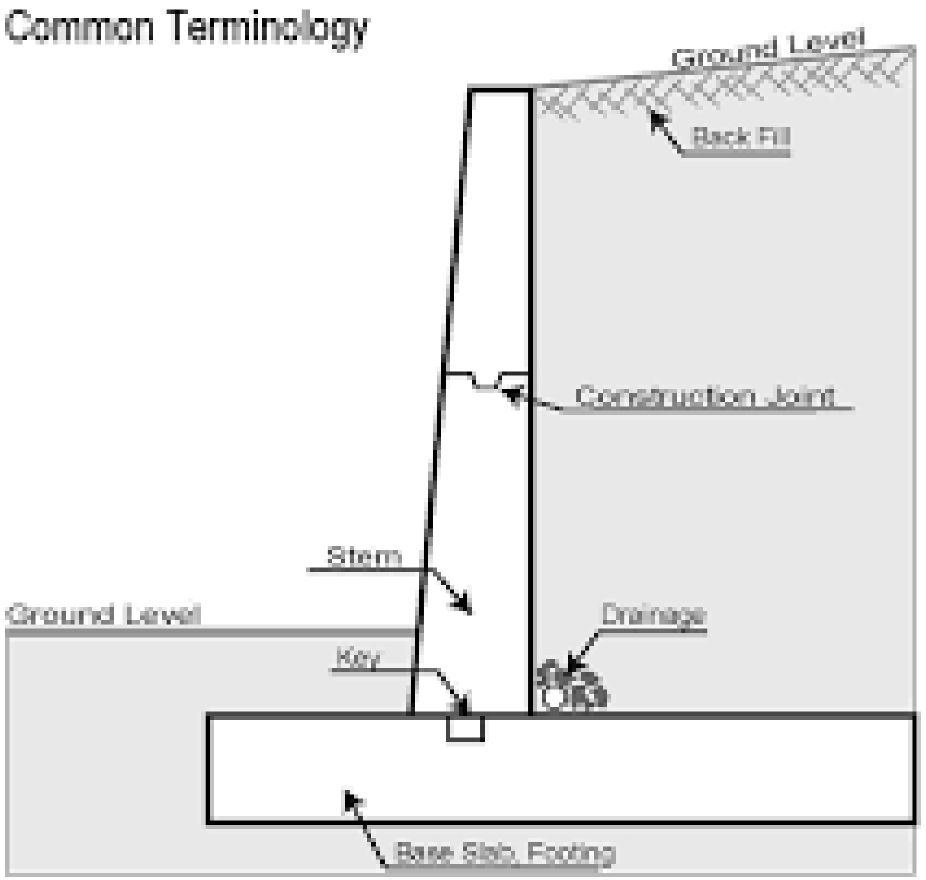


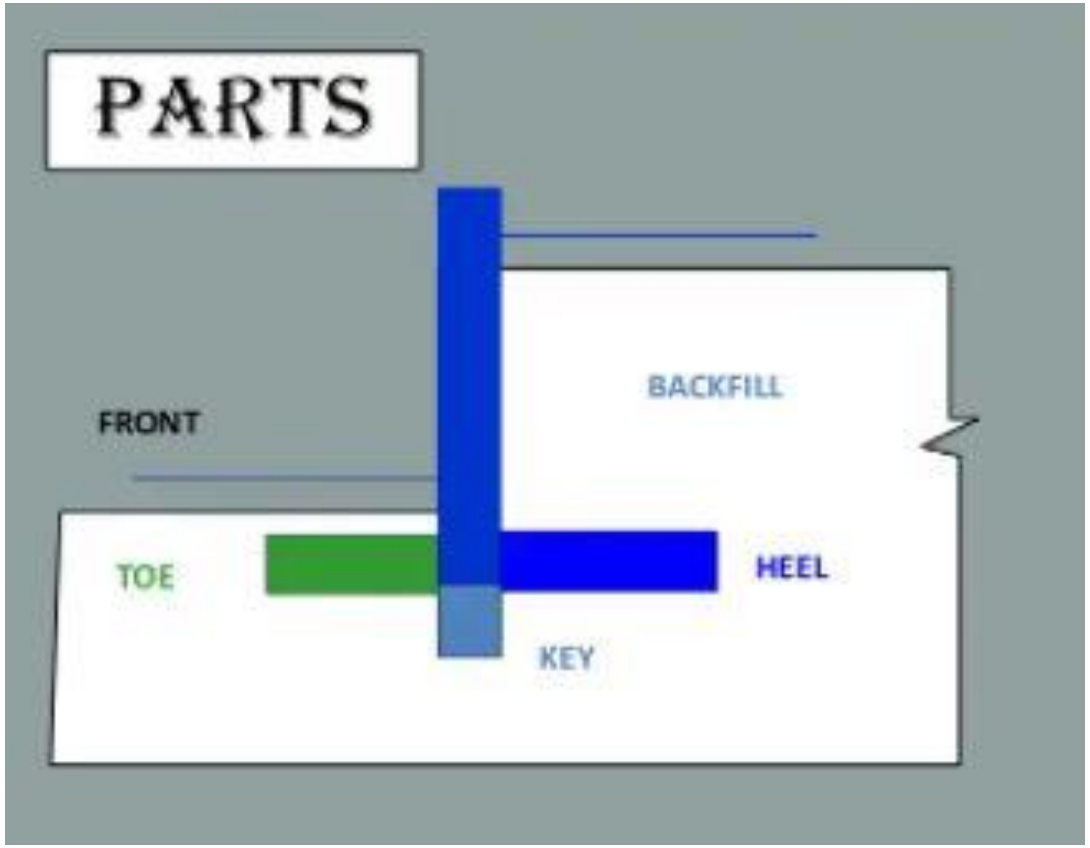
Combined Stress –Retaining Wall



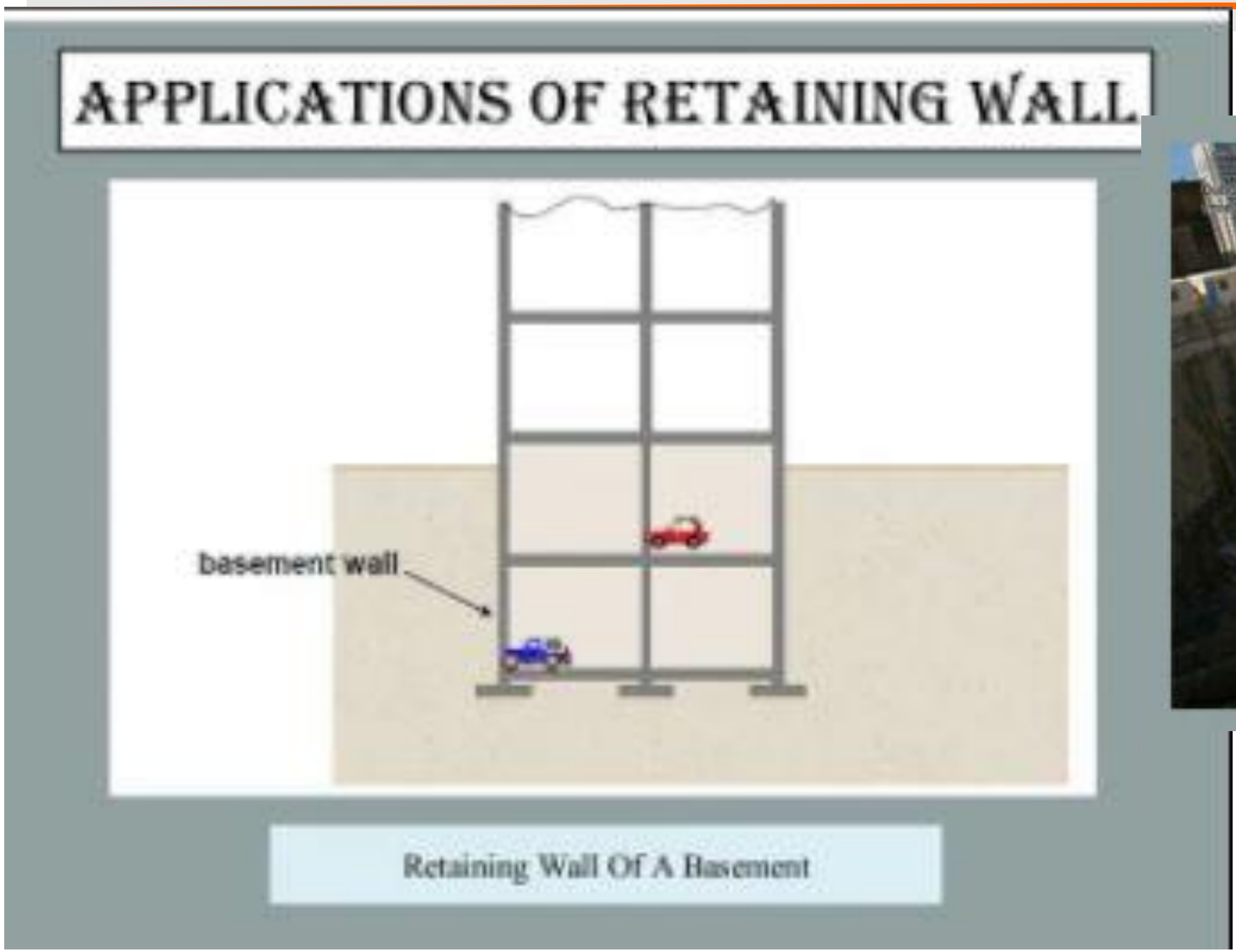


Combined Stress





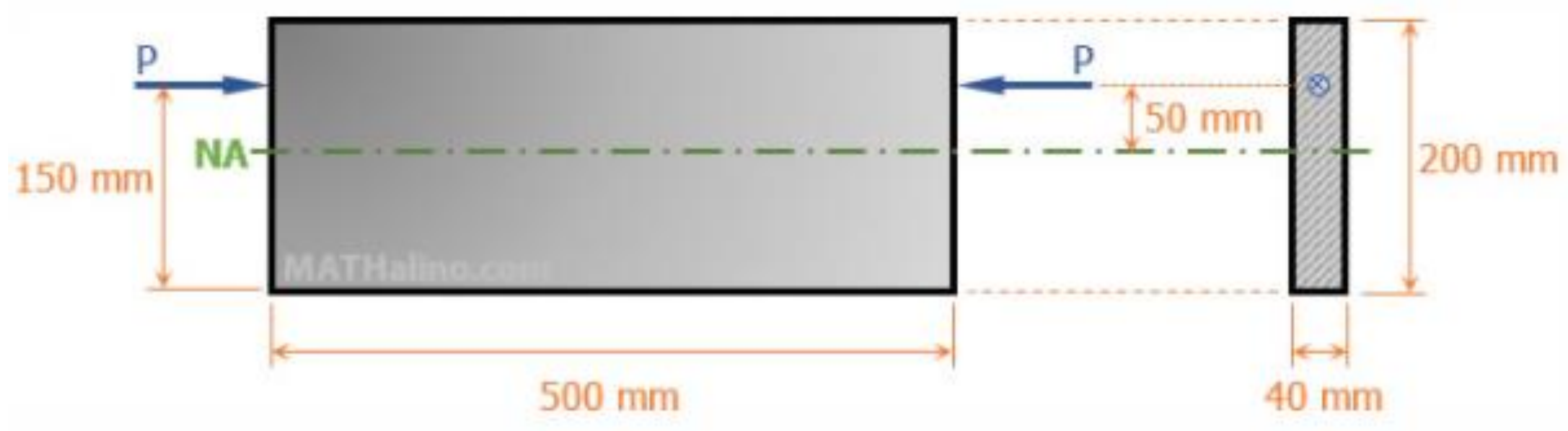
Combined Stress



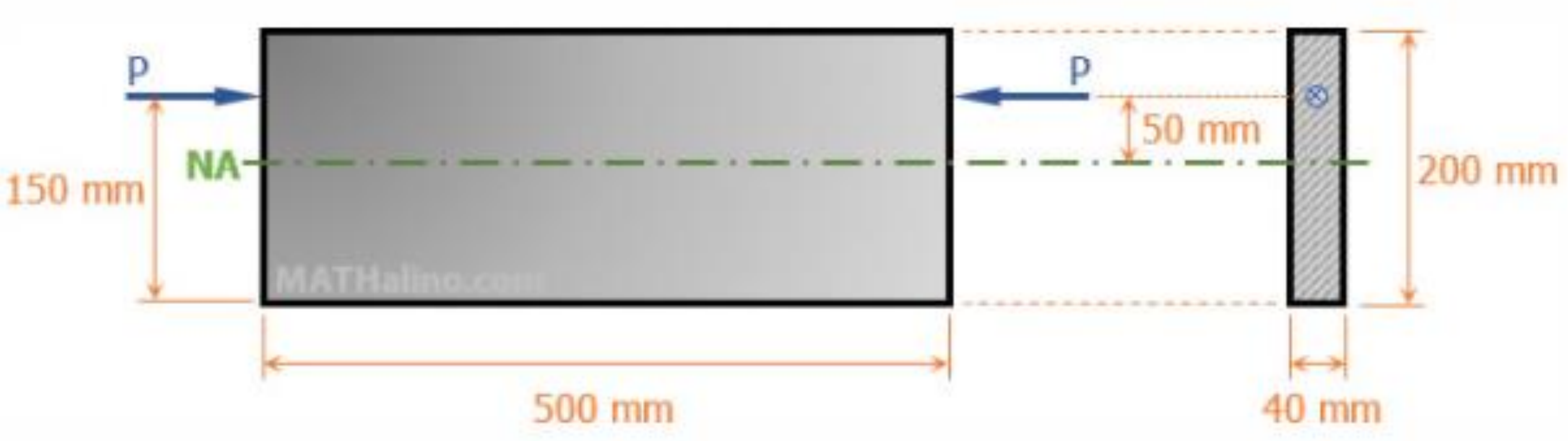
Combined Stress

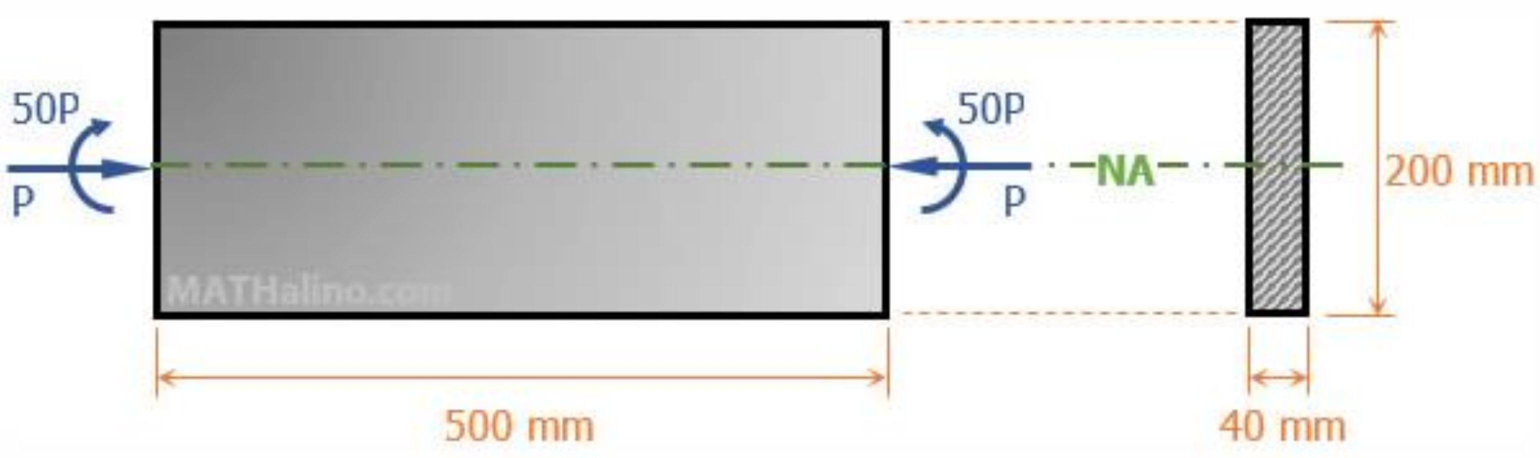
Combined Stress (Axial +Bending)

A cast iron link is 40 mm wide by 200 mm high by 500 mm long. The allowable stresses are 40 MPa in tension and 80 MPa in compression. Compute the largest compressive load P that can be applied to the ends of the link along a longitudinal axis that is located 150 mm above the bottom of the link.



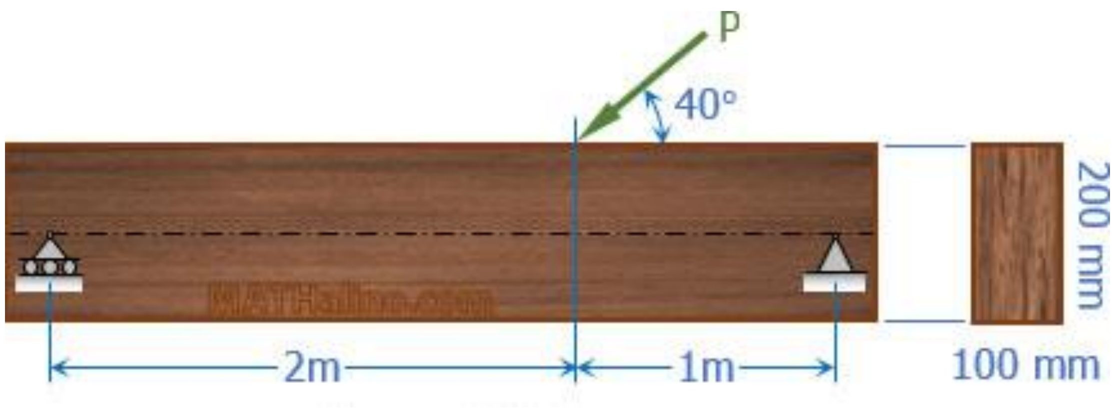
Combined Stress (Axial +Bending)



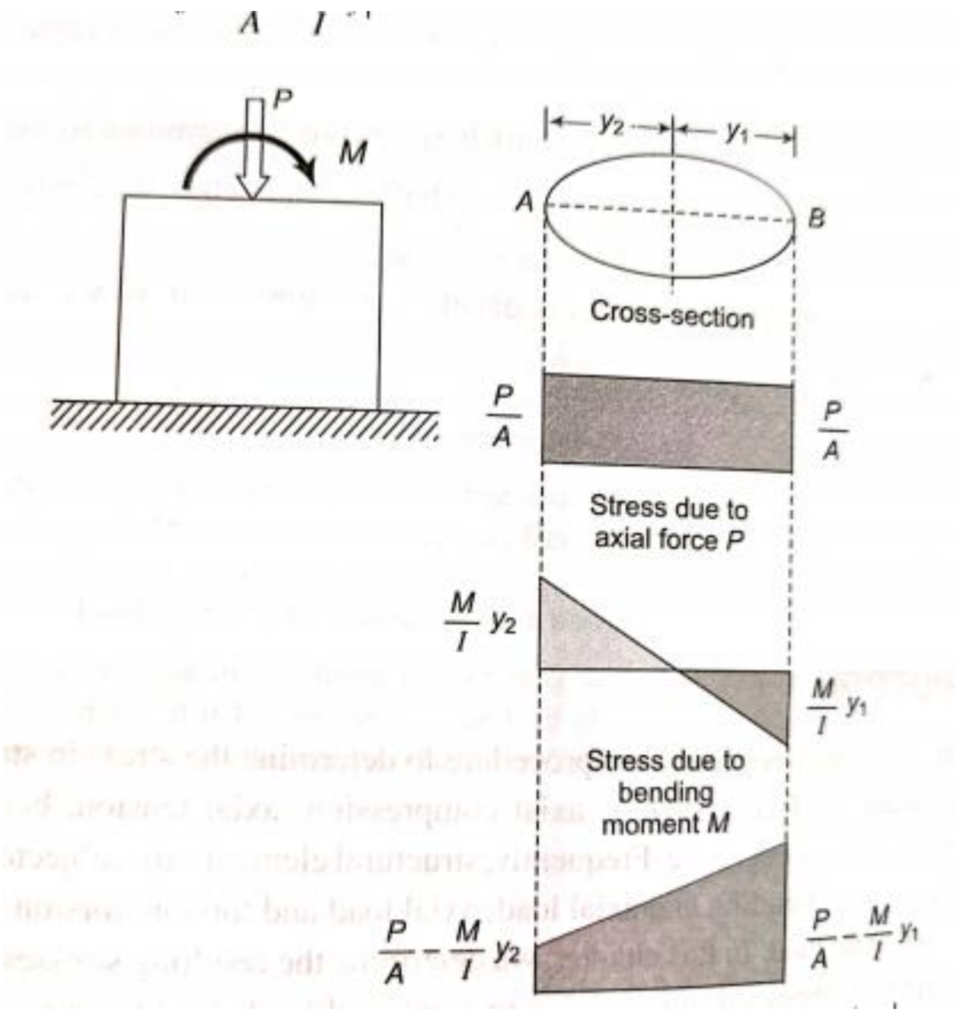


Combined Stress (Axial +Bending)

A wooden beam 100 mm by 200 mm, supported as shown in Figure P-905, carries a load P. What is the largest safe value of P is the maximum stress is not to exceed 10 MPa?



Combined Stress: Columns with Eccentric Load



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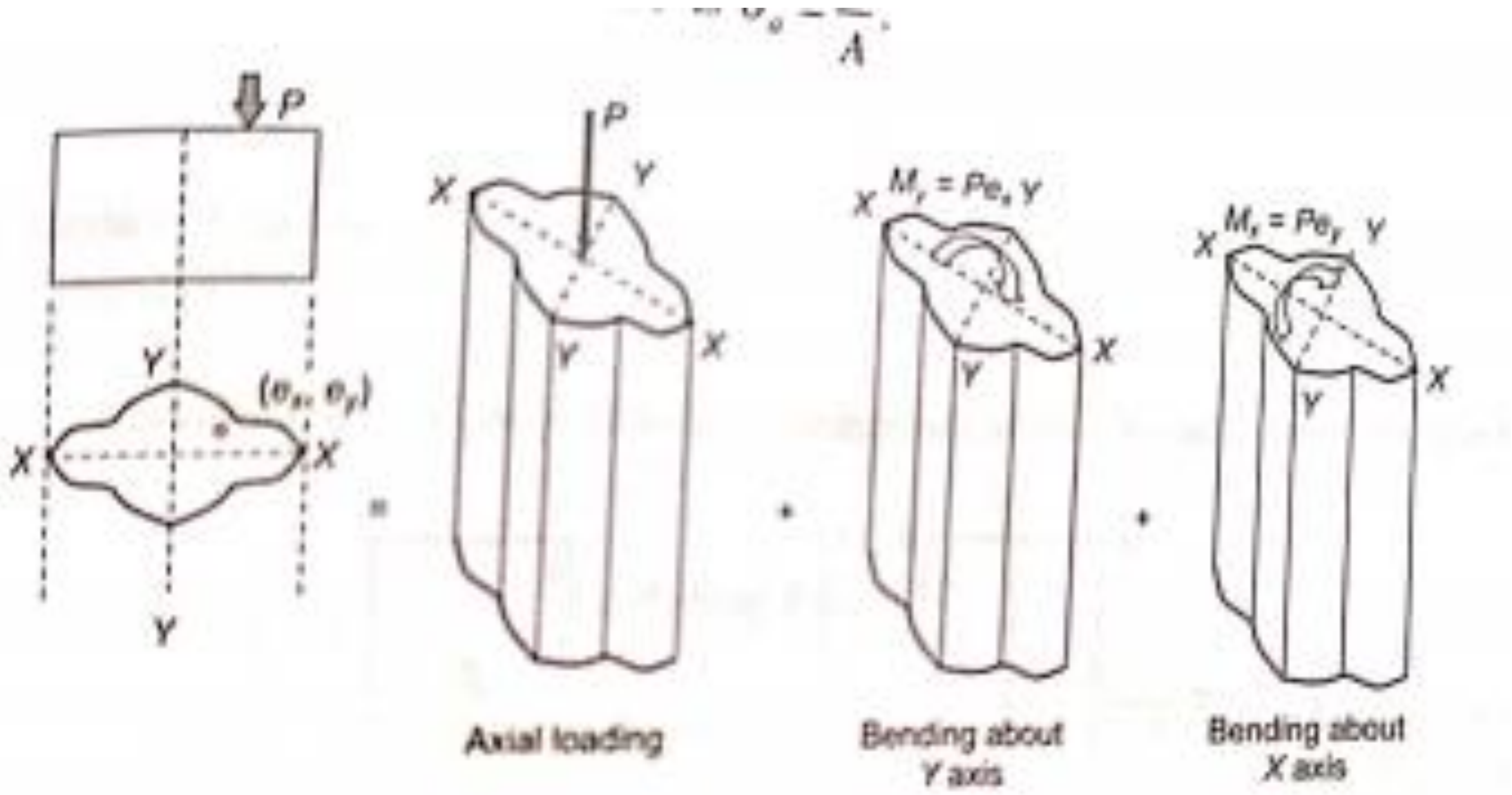
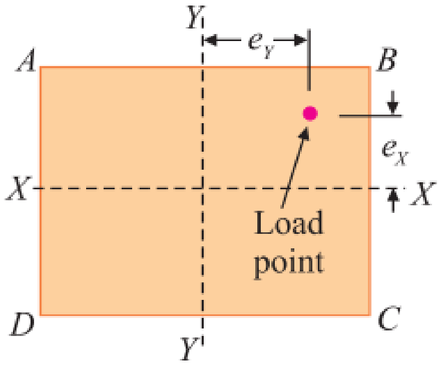
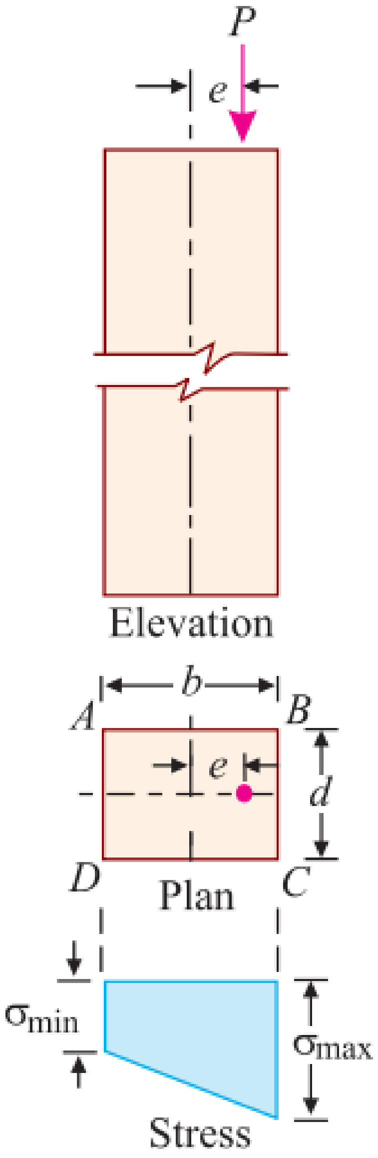


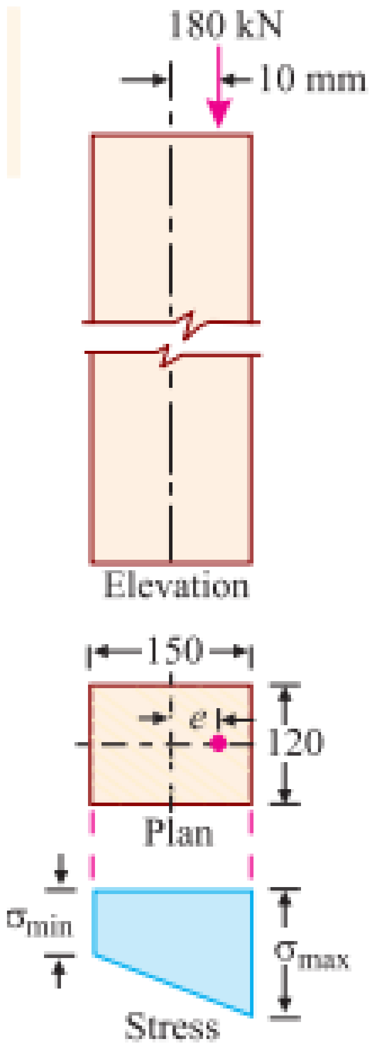
Combined Stress: Columns with Eccentric Load





Combined Stress: Columns with Eccentric Load

- A rectangular strut is 150 mm and 120 mm thick. It carries a load of 180 kN at an eccentricity of 10 mm in a plane bisecting the thickness. Find the maximum and minimum intensities of stress in the section.

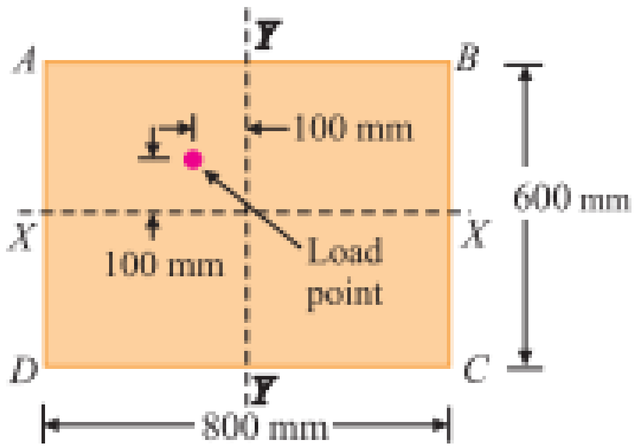


Combined Stress: Columns with Eccentric Load

- A rectangular strut is 200 mm and 150 mm thick is carrying a vertical load of 120 kN at an eccentricity of 50 mm in a plane bisecting the thickness. Determine the maximum and minimum intensities of stress in the section.

Combined Stress: Columns with Eccentric Load

- A column 800 mm x 600 mm is subjected to an eccentric load of 60 kN as shown in figure. What are the maximum and minimum intensities of stresses in the column?



Combined Stress: Columns with Eccentric Load

- A compressive load P = 100 kN is applied, as shown in Fig. 9-8a, at a point 70 mm to the left and 30 mm above the centroid of a rectangular section for which h = 300 mm and b = 250 mm. What additional load, acting normal to the cross section at its centroid, will eliminate tensile stress anywhere over the cross section?

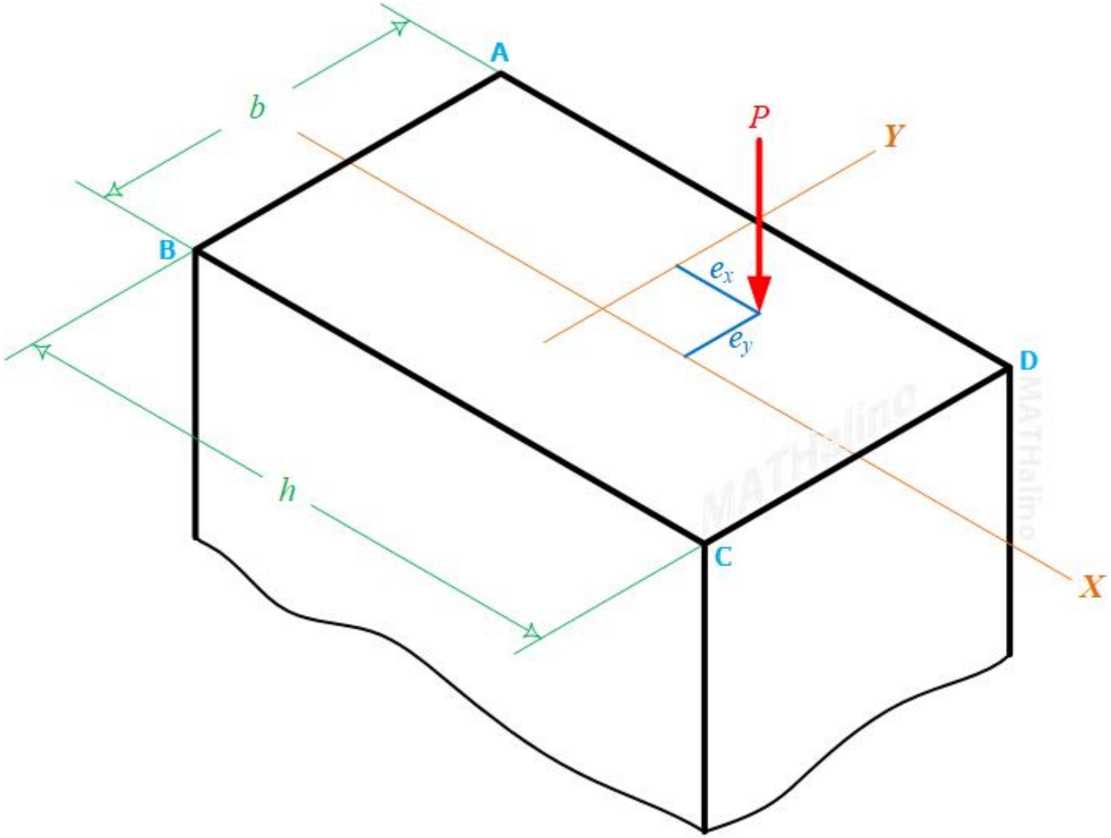
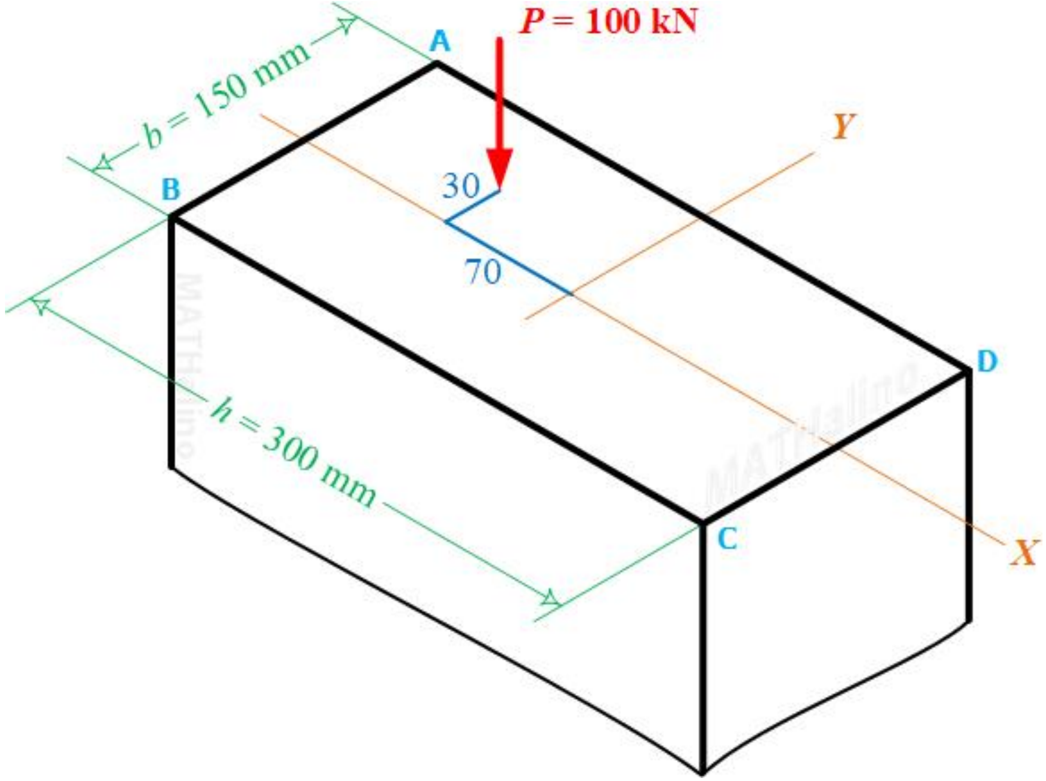
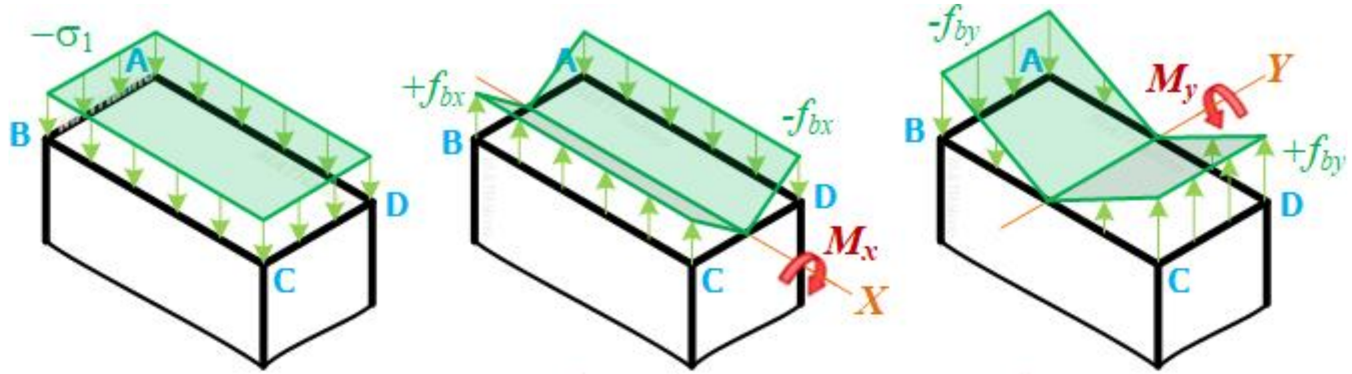


Figure 9-8a

Combined Stress: Columns with Eccentric Load



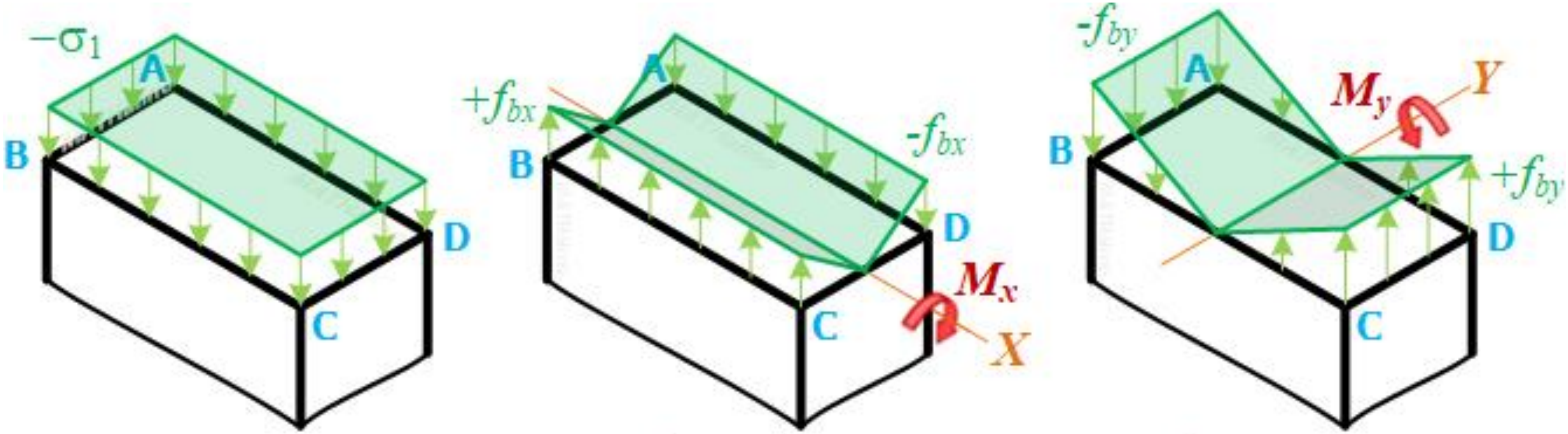


$\sigma_1 = 20/9$ MPa

$f_{bx} = 8/3$ MPa

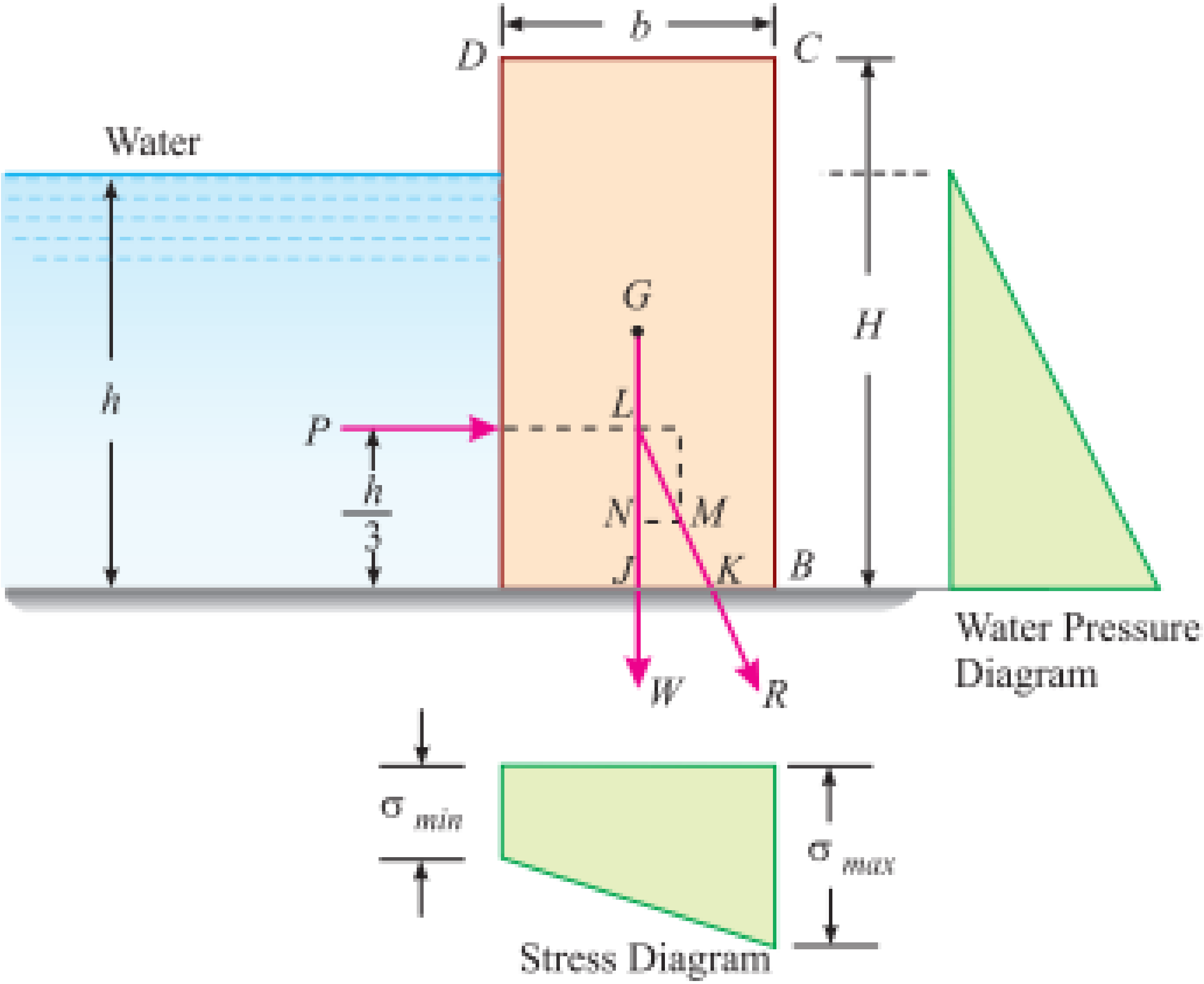
$f_{by} = 28/9$ MPa

Combined Stress: Columns with Eccentric Load



$\sigma_1 = 20/9$ MPa $\quad f_{bx} = 8/3$ MPa $\quad f_{by} = 28/9$ MPa

Combined Stress-Dams



∴ Weight of dam per unit length,

$$W = \rho \cdot b \cdot H$$

This weight will act through centre of gravity of the dam

We know that the intensity of water pressure will be zero by a straight line law to *wh* at the bottom. Thus the average in the dam

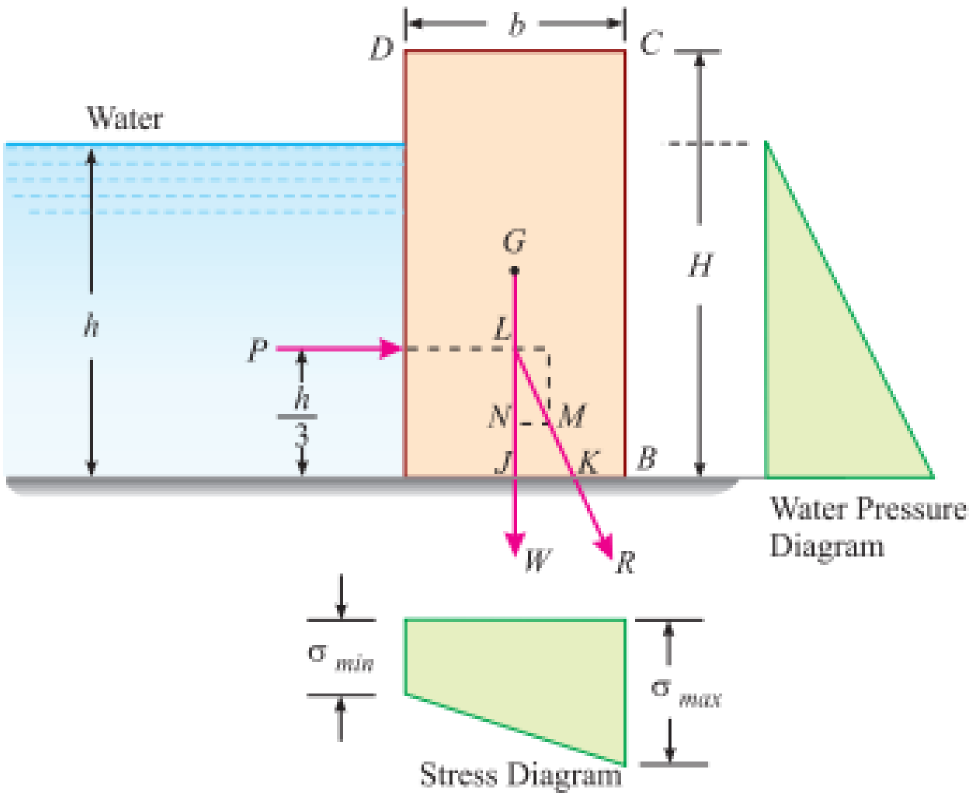
$$= \frac{wh}{2}$$

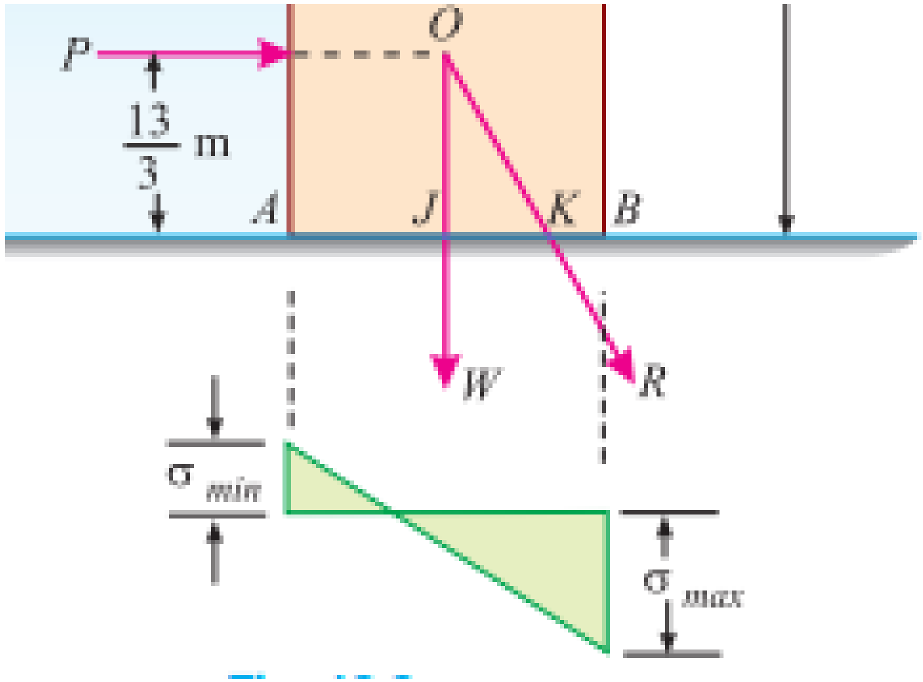
∴ Total pressure per unit length of the dam,

$$P = h \times \frac{wh}{2} = \frac{wh^2}{2}$$

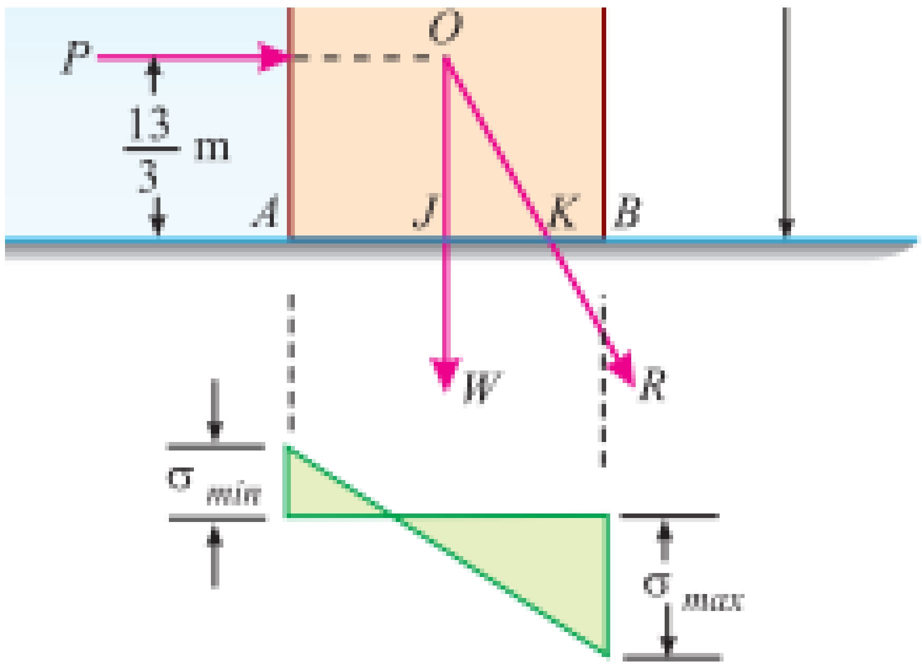
Combined Stress-Dams

A concrete dam of rectangular section 15 m high and 6 m wide contains water up to a height of 13 m. Find **(a) total pressure per meter length of the dam, (b) point, where the resultant cuts the base and (c) maximum and minimum intensities of stress at the base**. Assume weight of water and concrete as 10 and 25 kN/m^3





Combined Stress-Dams



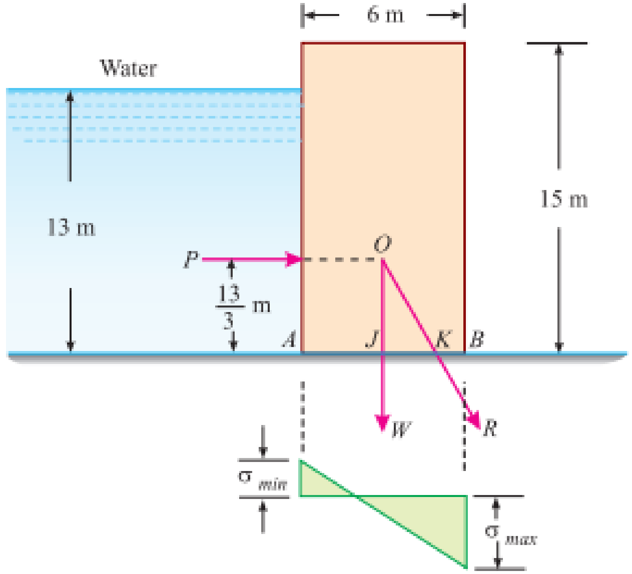


Fig. 18.3

(c) ***Maximum and minimum intensities of stress at the base***

We know that

*eccentricity of the resultant,

$$e = x = 1.63 \text{ m}$$

∴ Maximum intensity of stress at the base,

$$\sigma_{max} = \frac{W}{b}\left(1+\frac{6e}{b}\right) = \frac{2250}{6}\left(1+\frac{6\times1.63}{6}\right) \text{ kN/m}^2$$

$$= 986.25 \text{ kN/m}^2 = \mathbf{986.25\ kPa\ (Compression)}\quad \textbf{Ans.}$$

and minimum intensity of stress at the base,

$$\sigma_{min} = \frac{W}{b}\left(1-\frac{6e}{b}\right) = \frac{2250}{6}\left(1-\frac{6\times1.63}{6}\right) \text{ kN/m}^2$$

$$= -236.25 \text{ kN/m}^2 = \mathbf{236.25\ kPa\ (Tension)}\quad \textbf{Ans.}$$

Combined Stress-Dams

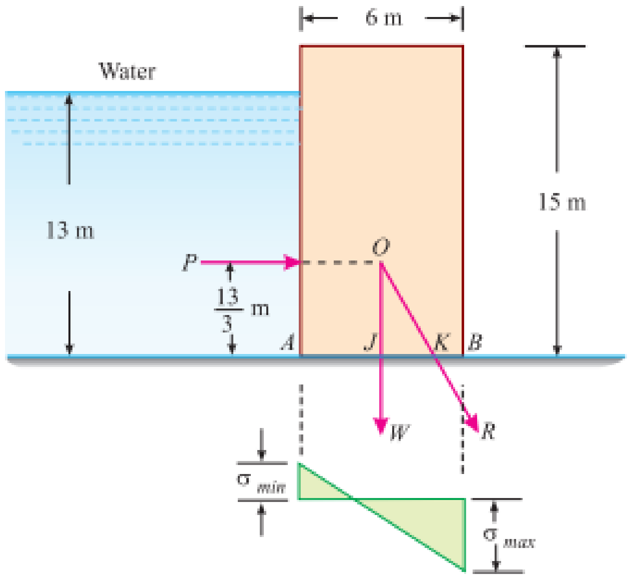


Fig. 18.3

(c) ***Maximum and minimum intensities of stress at the base***

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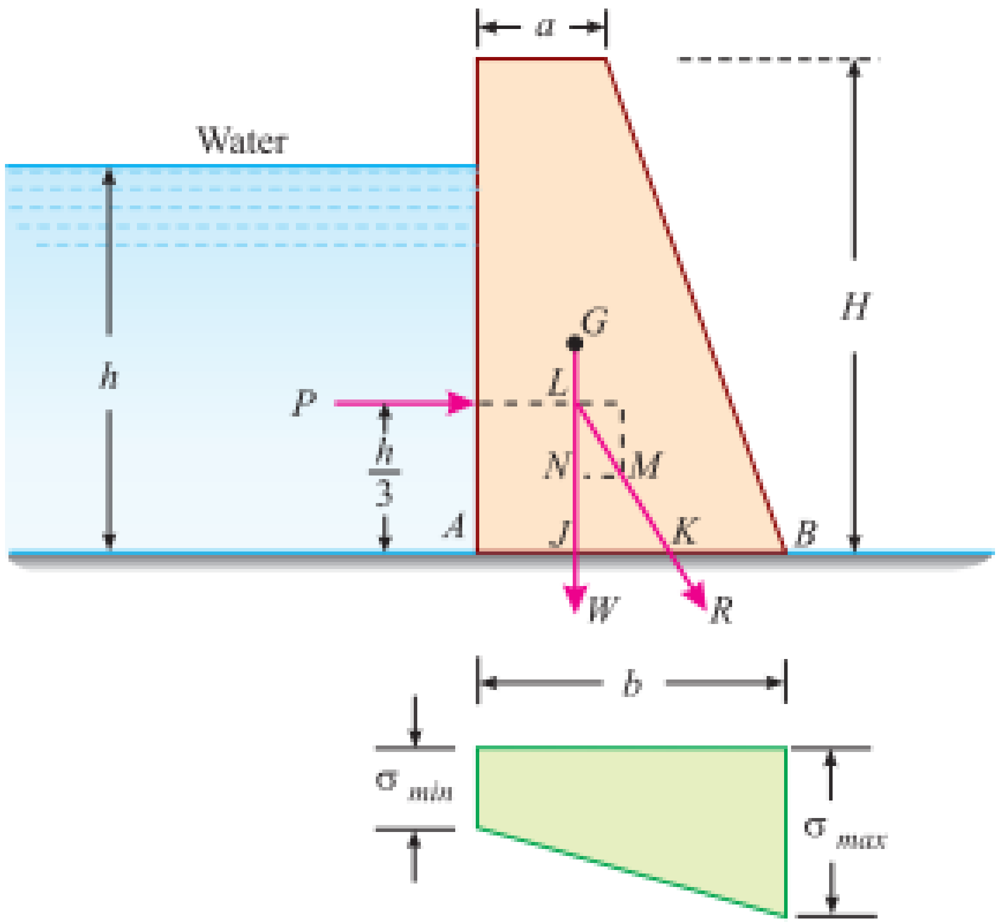
and minimum intensity of stress at the base,

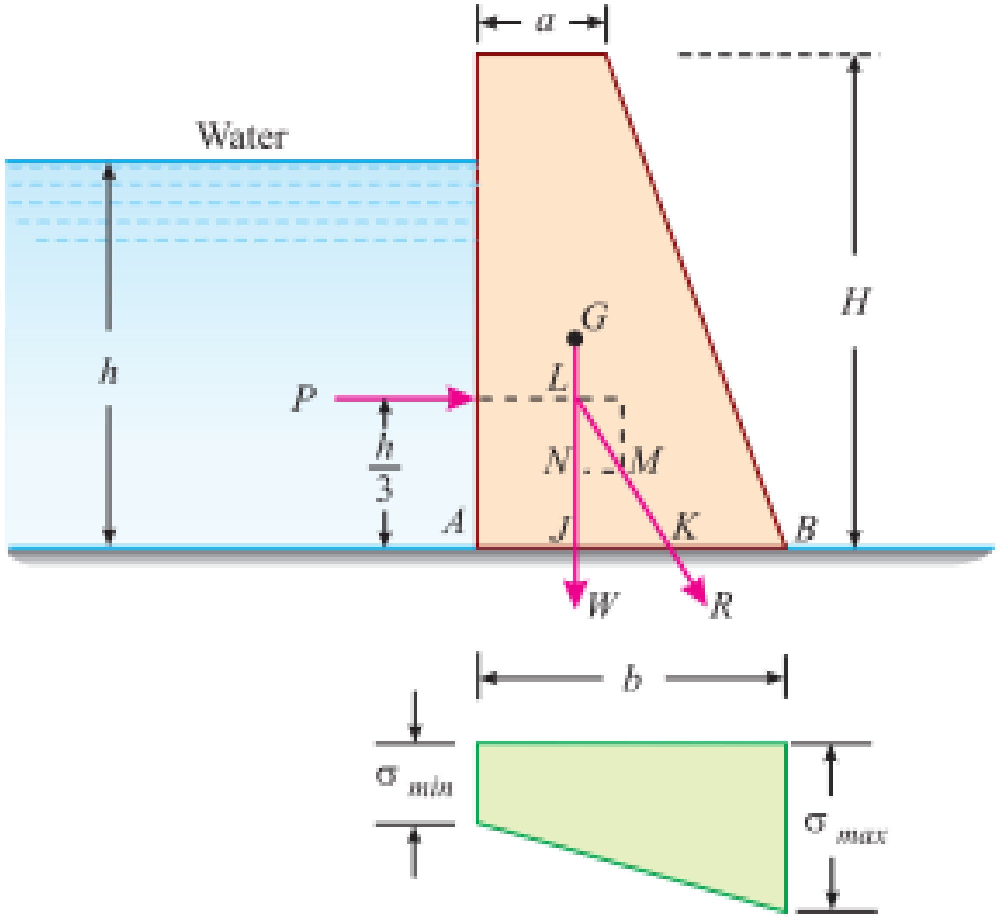
$$\sigma_{min} = \frac{W}{b}\left(1-\frac{6e}{b}\right) = \frac{2250}{6}\left(1-\frac{6\times 1.63}{6}\right) \text{ kN/m}^2$$

$$= -236.25 \text{ kN/m}^2 = \mathbf{236.25 \text{ kPa (Tension)}} \quad \textbf{Ans.}$$

Combined Stress-Trapezoidal Dams



Combined Stress-Trapezoidal Dams



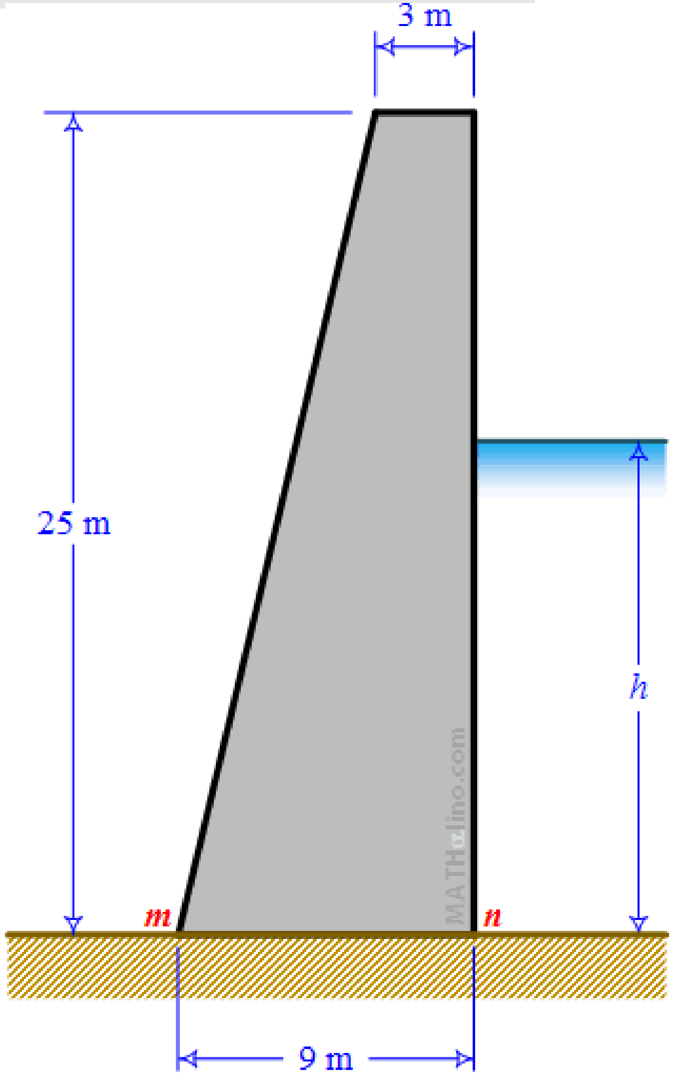
Assignment:

A concrete dam 8 m high, 1.5 m wide at top, and 4m wide at the base has its front face vertical and retains water to a depth of 6 m. Find the maximum and minimum stress intensities at the base. The density of water is 10 kN/m^3 and that of masonry is 24 kN/m^3

Check for stability of dam against tension at base, overturning and sliding (coefficient of friction is 0.6)

Combined Stress (Axial +Bending)

A concrete dam has the profile shown in Figure P-911. If the density of concrete is 2400 kg/m^3 and that of water is 1000 kg/m^3, determine the maximum compressive stress at section *m-n* if the depth of the water behind the dam is *h* = 15 m.



Combined Stress (Axial +Bending)

Consider 1-m length perpendicular to the drawing

$$W_1 = 2400 \times \tfrac{1}{2}(6)(25)(1)$$

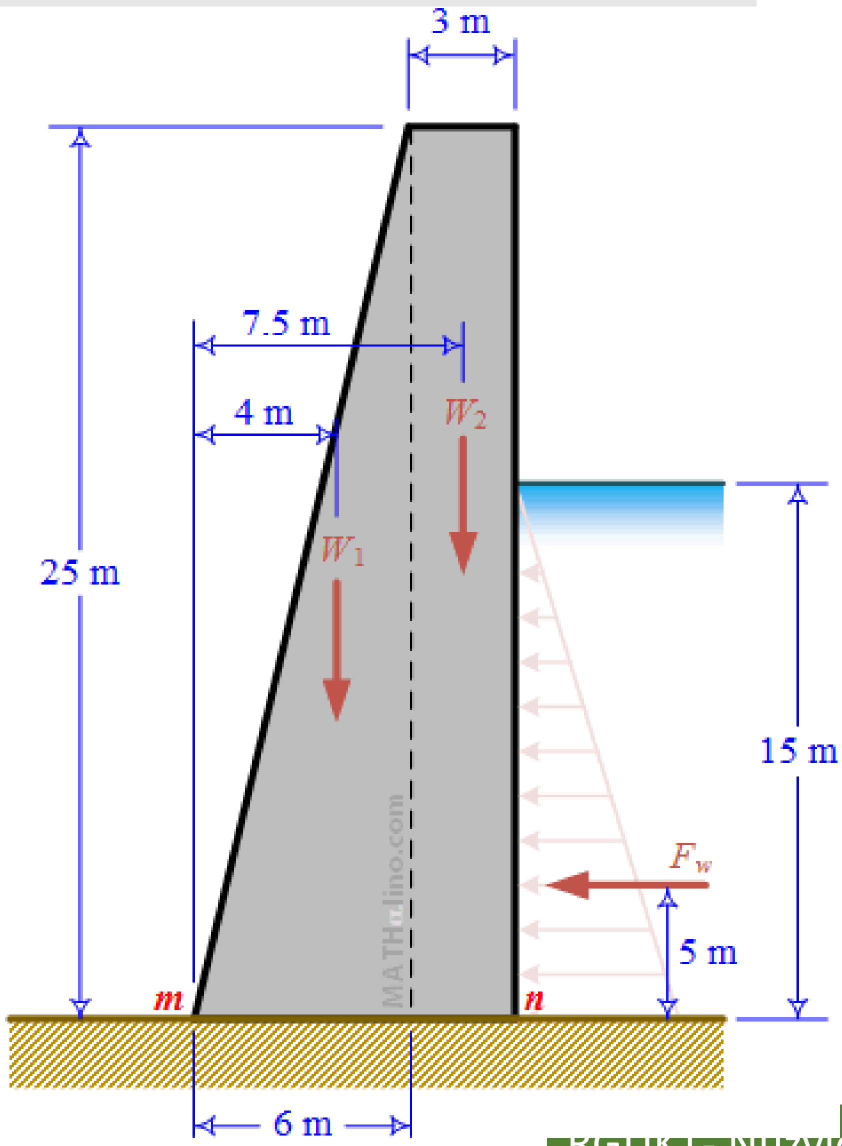
$$W_1 = 180,000 \text{ kg}$$

$$W_2 = 2400 \times 3(25)(1)$$

$$W_2 = 180,000 \text{ kg}$$

$$F_w = 1000(7.5) \times 15(1)$$

$$F_w = 112,500 \text{ kg}$$



Combined Stress (Axial +Bending)

Moment About m

Righting Moment, RM

$$RM = 4W_1 + 7.5W_2 = 4(180,000) + 7.5(180,000)$$

$$RM = 2,070,000 \text{ kg} \cdot \text{m}$$

Overturning Moment, OM

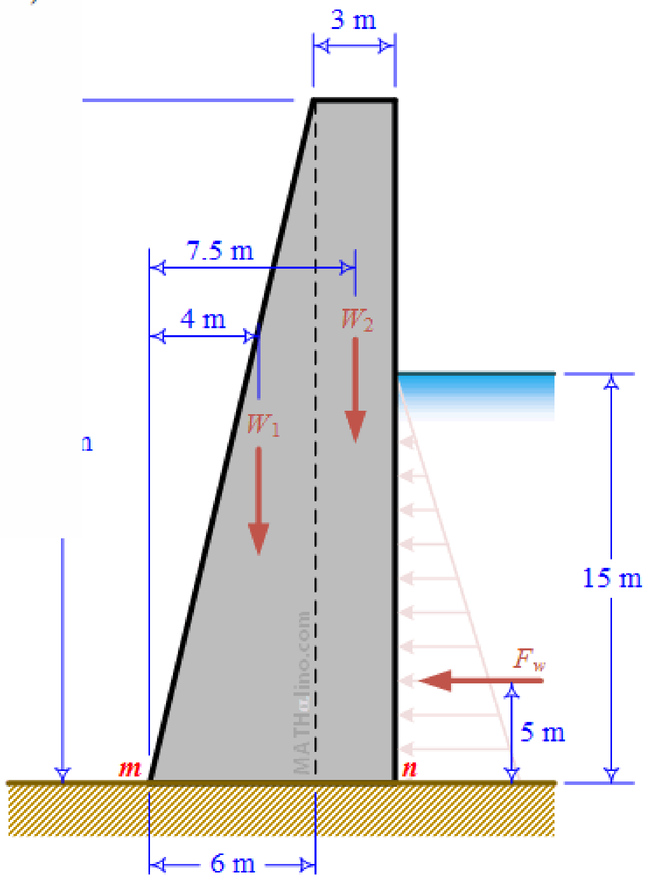
$$OM = 5F_w = 5(112,500)$$

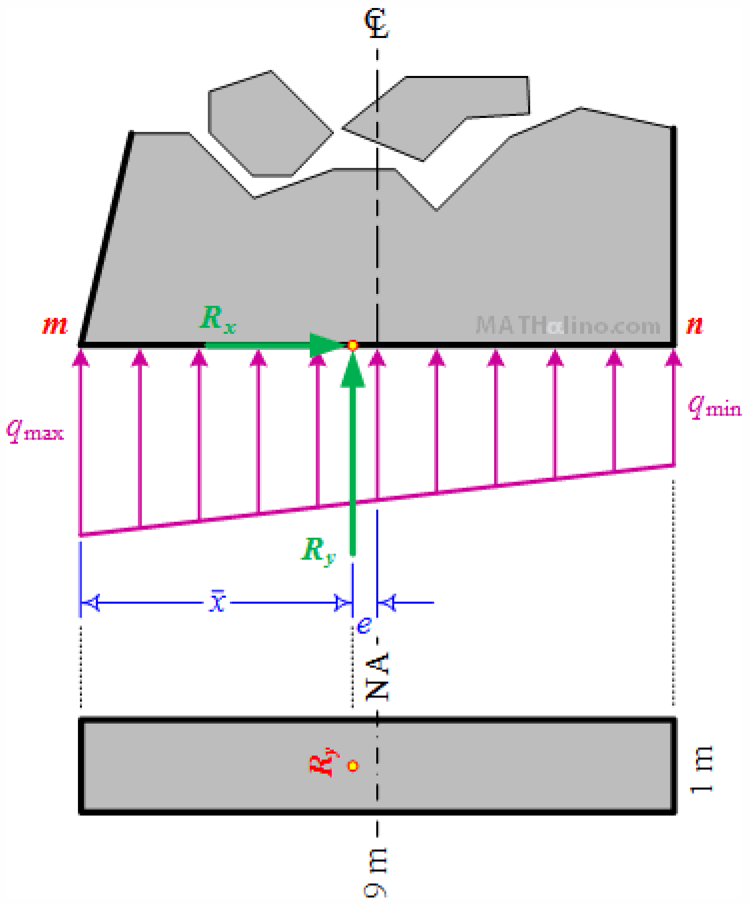
$$OM = 562,500 \text{ kg} \cdot \text{m}$$

Reactions at the Base

$$R_y = W_1 + W_2 = 360,000 \text{ kg}$$

$$R_x = F_w = 112,500 \text{ kg}$$





Combined Stress (Axial +Bending)

Location of R_y

$$\bar{x}R_y = RM - OM$$

$$\bar{x}(360,000) = 2,070,000 - 562,500$$

$$\bar{x} = 4.1875 \text{ m}$$

Eccentricity

$$e = 4.5 - \bar{x} = 4.5 - 4.1875$$

$$e = 0.3125 \text{ m}$$

$$M = R_y e = 360,000(0.3125)$$

$$M = 112,500 \text{ kg} \cdot \text{m}$$

$$M = R_y e = 360,000(0.3125)$$

$$M = 112,500 \text{ kg} \cdot \text{m}$$

$$\sigma_a = \frac{P}{A} = \frac{360,000}{1(9)}$$

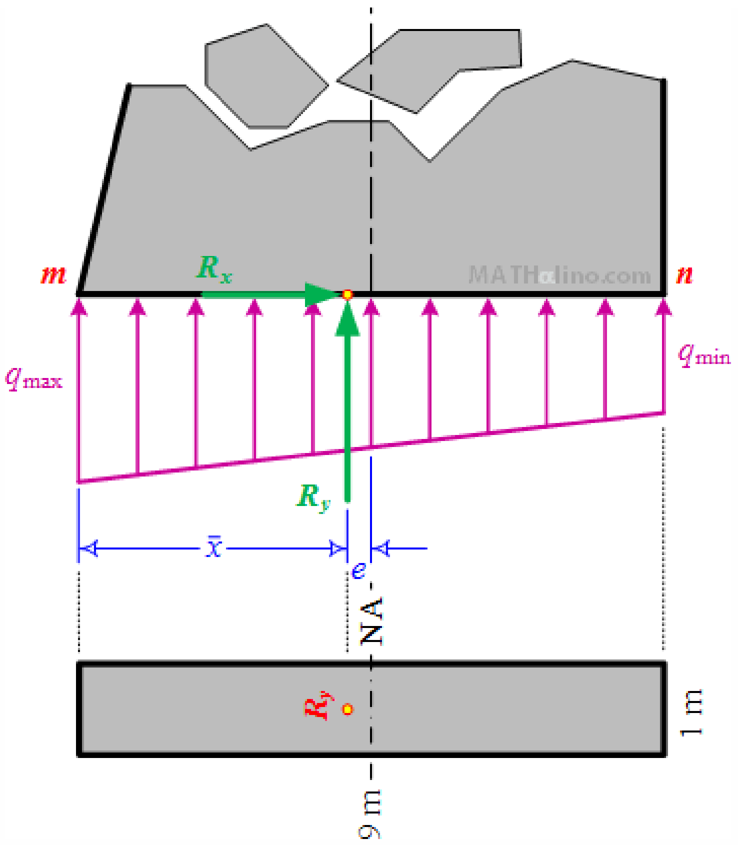
$$\sigma_a = 40,000 \text{ kg/m}^2$$

$$\sigma_f = \frac{6M}{bd^2} = \frac{6(112,500)}{1(9^2)}$$

$$\sigma_f = 8,333.33 \text{ kg/m}^2$$

$$q_{max} = \sigma_a + \sigma_f = 40,000 + 8,333.33$$

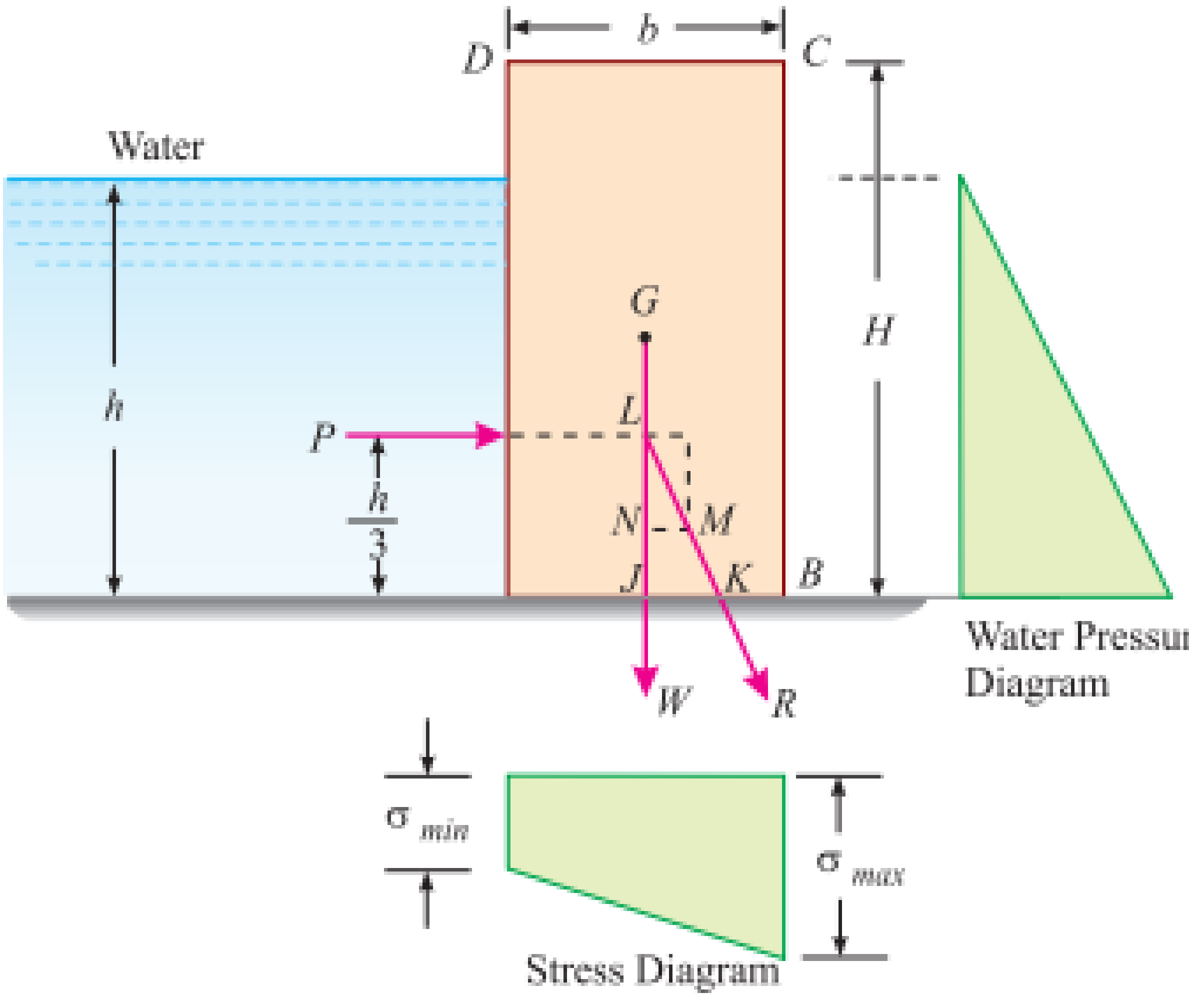
$$q_{max} = 48,333.33 \text{ kg/m}^2 \qquad \textit{answer}$$



Conditions for Stability of a Dam

1. To avoid tension in the masonry at the base of the dam,

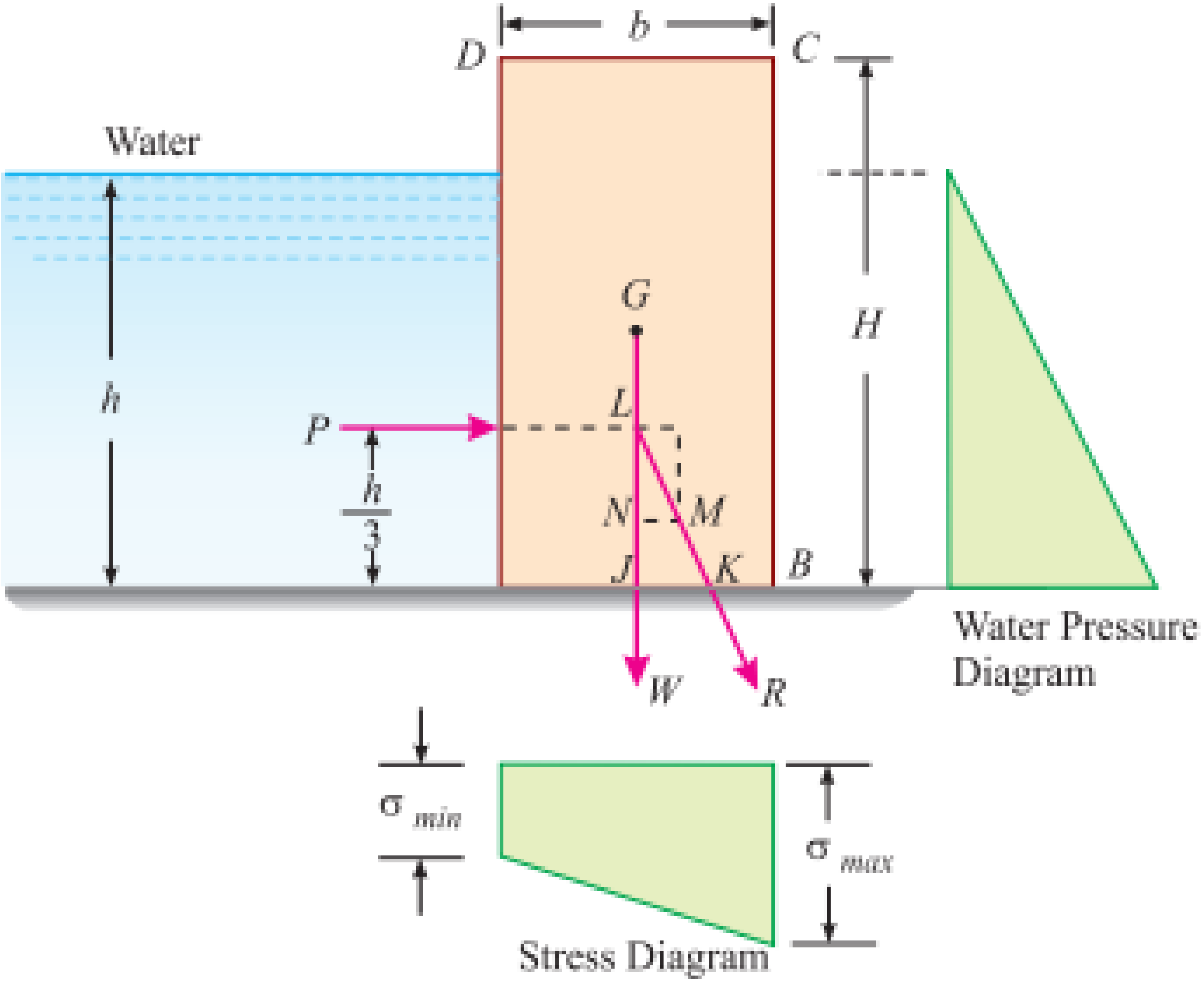
2. To safeguard the dam from overturning,



Conditions for Stability of a Dam

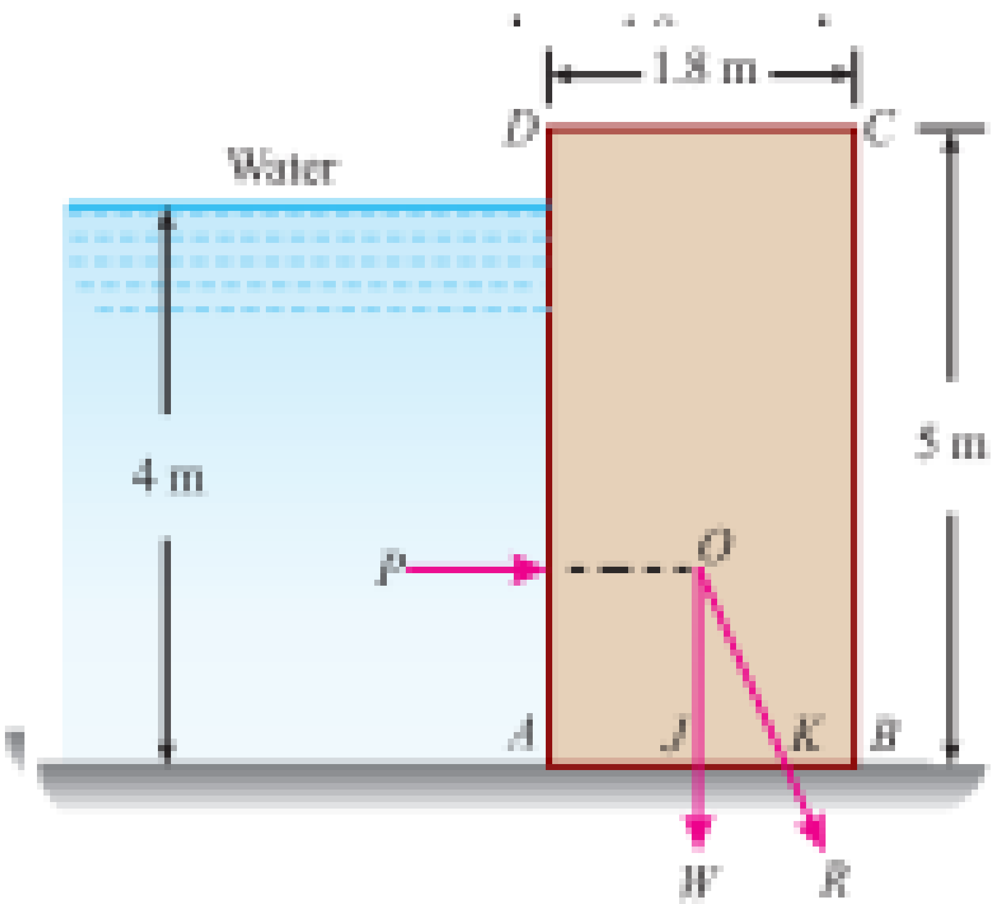
3. To prevent the sliding of dam and

4. To prevent the crushing of masonry at the base of the dam.



Conditions for Stability of a Dam

A masonry wall 5 metres high and 1.8 metre wide is containing water up to a height of 4 metres. If the coefficient of friction between the wall and the soil is 0.6, check the stability of the wall. Take weight of the masonry and water as 22 kN/m^3 and 9.81 kN/m^3



Conditions for Stability of a Dam

$$W = 22 \times 5 \times 1.8 = 198 \text{ kN}$$

1. *Check for tension in the masonary at the base*

We know that horizontal distance between the centre of gravity of resultant thrust (*R*) cuts the base,

$$x = \frac{P}{W} \times \frac{h}{3} = \frac{78.48}{198} \times \frac{4}{3} = 0.53 \text{ m}$$

$$\therefore \quad AK = AJ + x = 0.9 + 0.53 = 1.43 \text{ m}$$

Since the resultant thrust lies beyond the middle third of the base therefore the wall shall fail due to tension in its base. **Ans.**

2. *Check for overturning.*

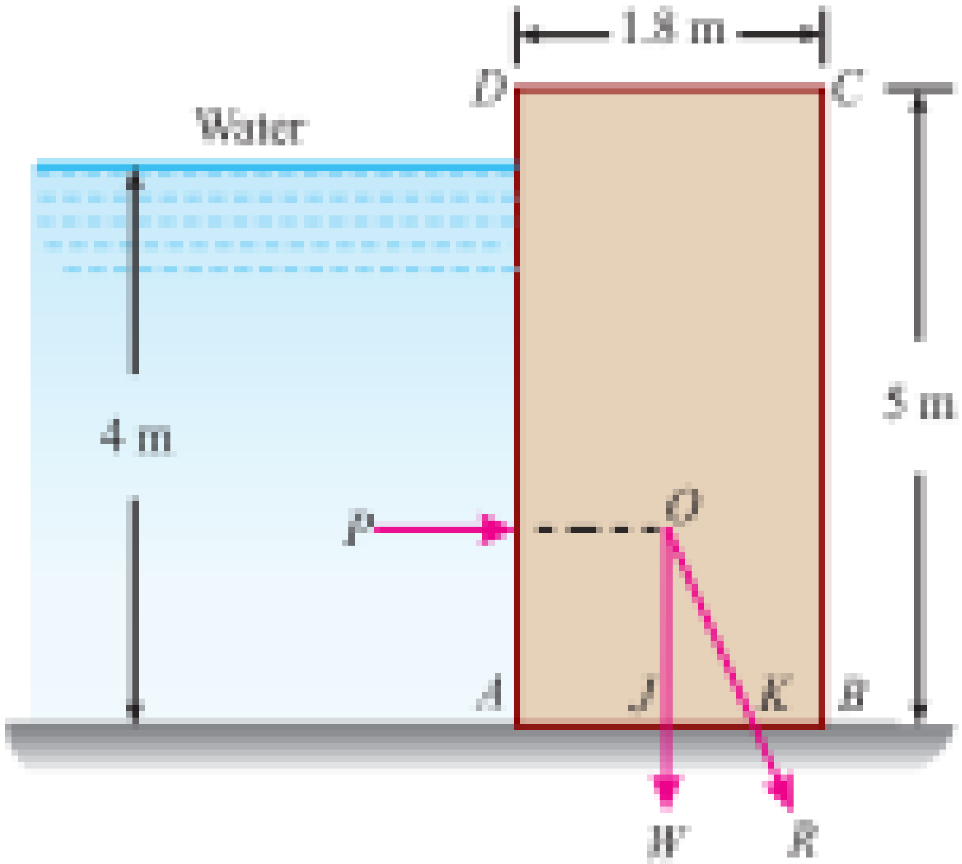
Since the resultant thrust is passing within the base as obtained ab against overturning. **Ans.**

3. *Check for sliding the wall.*

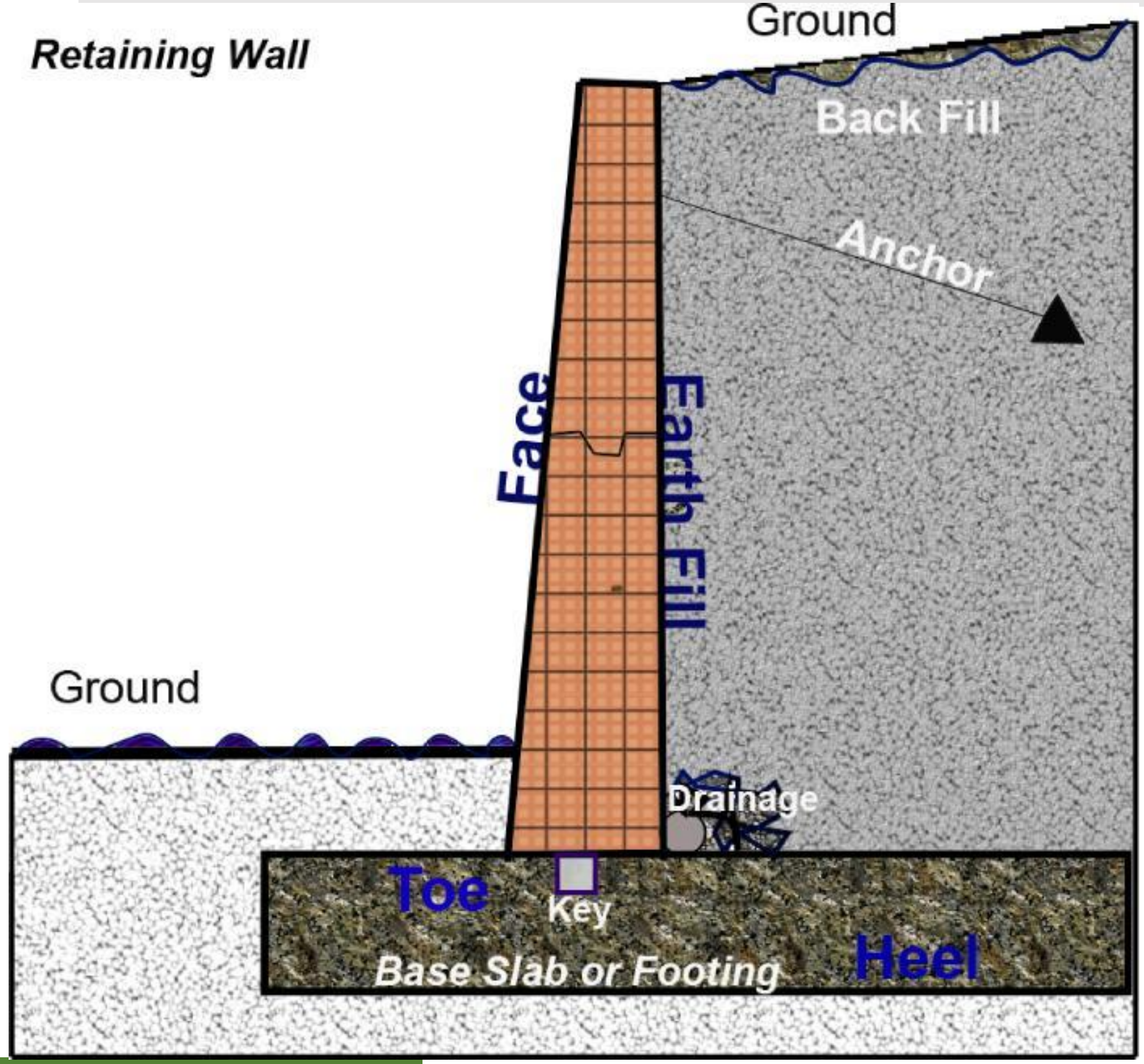
We know that horizontal pressure due to water, $(P) = 78.48$ kN.

And the frictional force $= \mu W = 0.6 \times 198 = 118.8$ kN

Since the frictional force (118.8 kN) is *more than the horizontal the wall is safe against sliding. **Ans.**



Combined Stress-Retaining Wall



Retaining wall is generally, constructed to retain earth in hilly areas.

The analysis of a retaining wall is, somewhat like a dam.

The retaining wall is subjected to pressure, produced by the retained earth in a similar manner, as the dam is subjected to water pressure.

Combined Stress-Retaining Wall

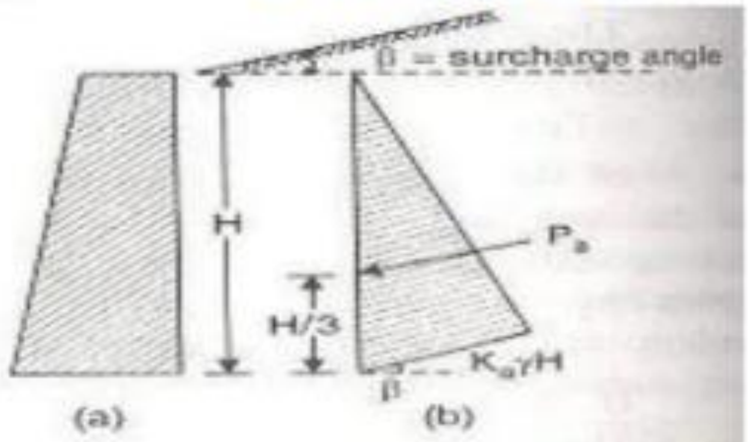


FIG. 18.6. LATERAL PRESSURE DISTRIBUTION FOR SLOPING SURCHARGE

β=inclination of sloping surface behind the wall with the horizontal
=Surcharge Angle

$$P = \frac{wh^2}{2}\cos\alpha \cdot \frac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi}}{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi}}$$

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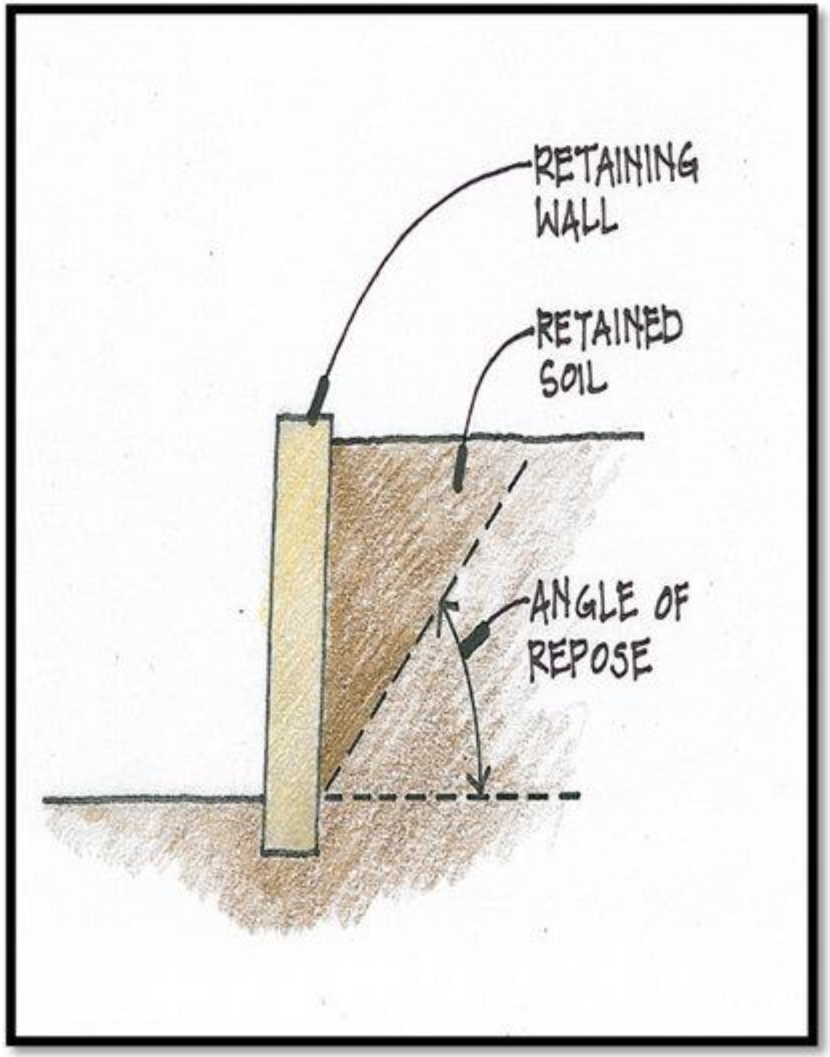
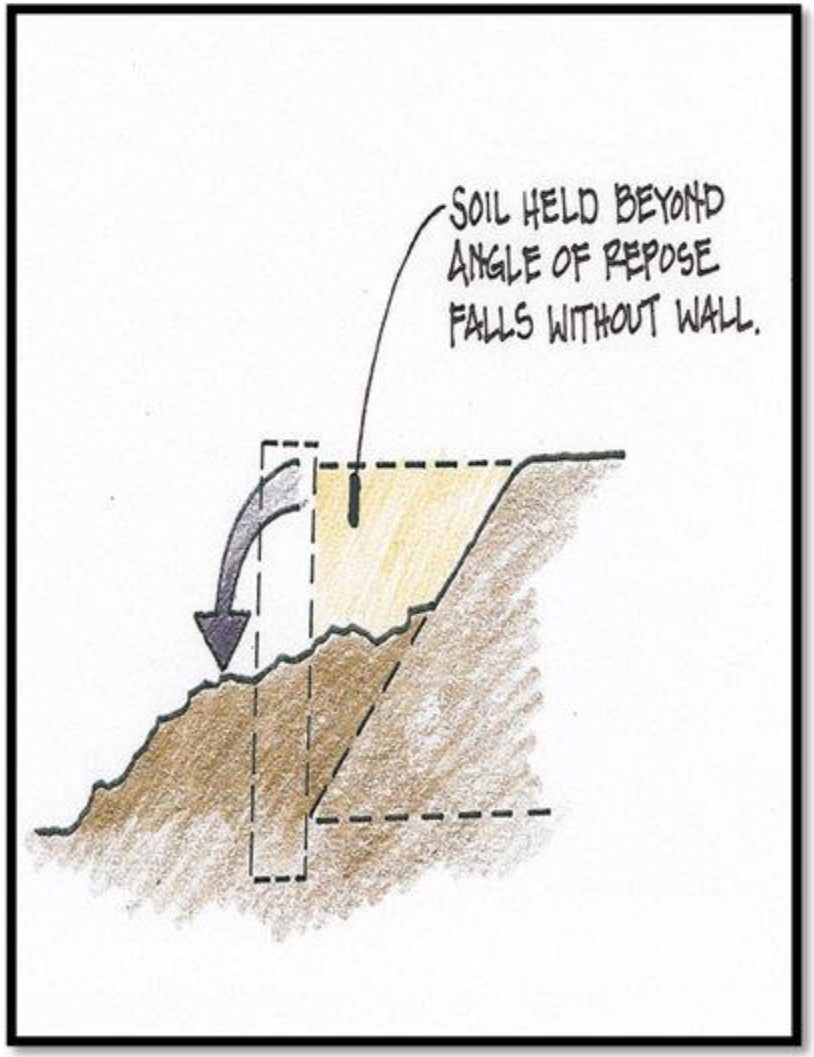
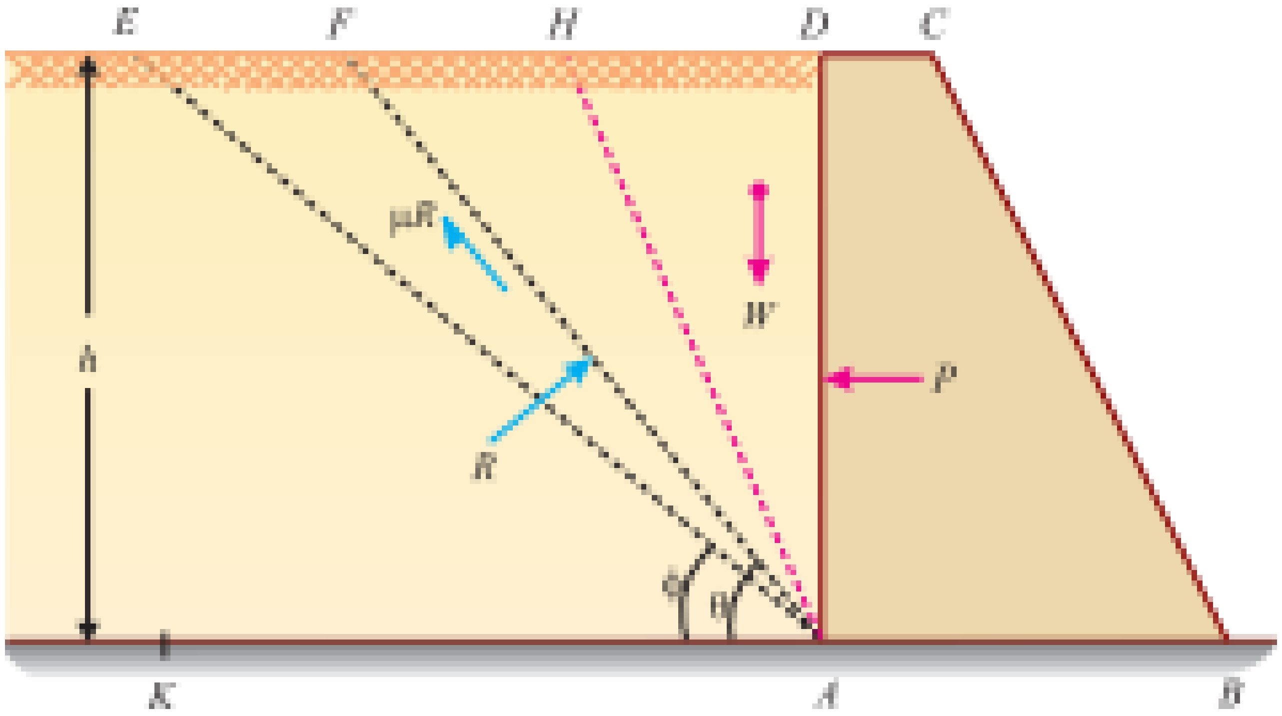
Combined Stress-Retaining Wall





Combined Stress-Retaining Wall



1. Active Earth pressure:
   i. *The pressure, exerted by the retained material called backfill, on the retaining wall is known as active earth pressure.*
   ii. *As a result of the active pressure, the retaining wall tends to slide away from the retained earth.*

1. Passive Earth Pressure
   i. Sometimes, the retaining wall moves laterally against the retained earth, which gets compressed.
   ii. As a result of the movement of the retaining wall, the compressed earth is subjected to a pressure

Combined Stress-Retaining Wall

1. A masonry retaining wall is 10 m high and retains earth weighting 2000 kg/m^{3}. The top width of the retaining wall is 2m and bottom width of retaining wall 6m. The angle of repose is 30^{o}. Weight of masonry is 2400 kg/m^{3}. Determine the maximum and minimum stresses in the wall at base

Combined Stress-Retaining Wall

1. A masonry retaining wall is 10 m high and retains earth weighting 2000 kg/m^{3}. The top width of the retaining wall is 2m and bottom width of retaining wall 6m. The angle of repose is 30^{o}. Weight of masonry is 2400 kg/m^{3}. Determine the maximum and minimum stresses in the wall at base

Combined Stress-Retaining Wall

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Combined Stress-Retaining Wall

1. A masonry retaining wall of trapezoidal section with a vertical face on the earth side is 1 m wide at the top, 3 m wide at the bottom and 6 m high. It retains sand over the entire height with an angle of surcharge of 20°. Determine the distribution of pressure at the base of the wall. The sand weighs 18 kN/m³ and has an angle of repose of 30°. The masonry weighs 24 kN/m³

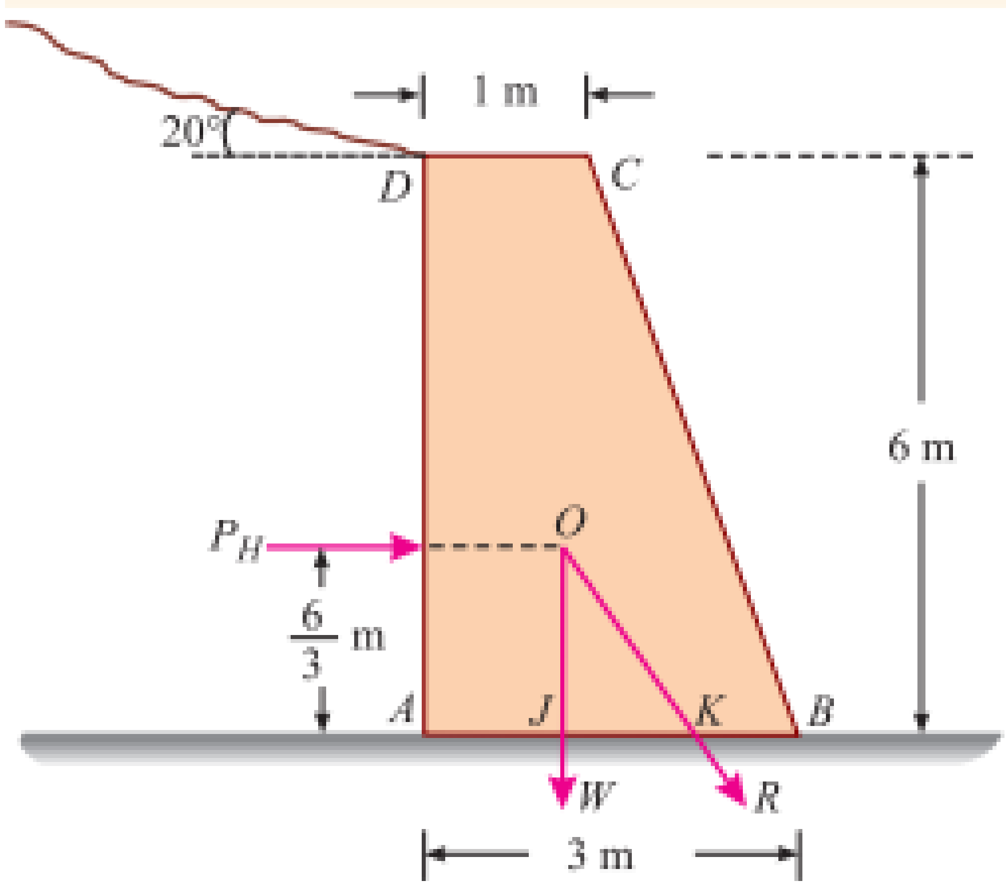

Fig. 18.24

Combined Stress-Retaining Wall

$$P = \frac{wh^2}{2} \cos\alpha \times \frac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\alpha}}{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\alpha}}$$

$$= \frac{18 \times (6)^2}{2} \cos 20° \times \frac{\cos 20° - \sqrt{\cos^2 20° - \cos^2 30°}}{\cos 20° + \sqrt{\cos^2 20° - \cos^2 30°}} \text{ kN}$$

$$= 324 \times 0.9397 \times \frac{0.9397 - \sqrt{(0.9397)^2 - (0.866)2}}{0.9397 + \sqrt{(0.9397)^2 - (0.866)^2}} \text{ kN}$$

$$= 304.5 \times \frac{0.575}{1.3044} = 134.2 \text{ kN}$$

∴ Horizontal component of the pressure,

$$P_H = 134.2 \cos 20° = 134.2 \times 0.9397 = 126.1 \text{ kN}$$

and vertical component of the pressure,

$$P_V = 134.2 \sin 20° = 134.2 \times 0.3420 = 45.9 \text{ kN}$$

We also know that weight of the retaining wall

$$= 24 \times \frac{(1+3)}{2} \times 6 = 288 \text{ kN}$$

∴ Total weight acting vertically down,

$$W = 45.9 + 288 = 333.9 \text{ kN}$$

Combined Stress

Queries?