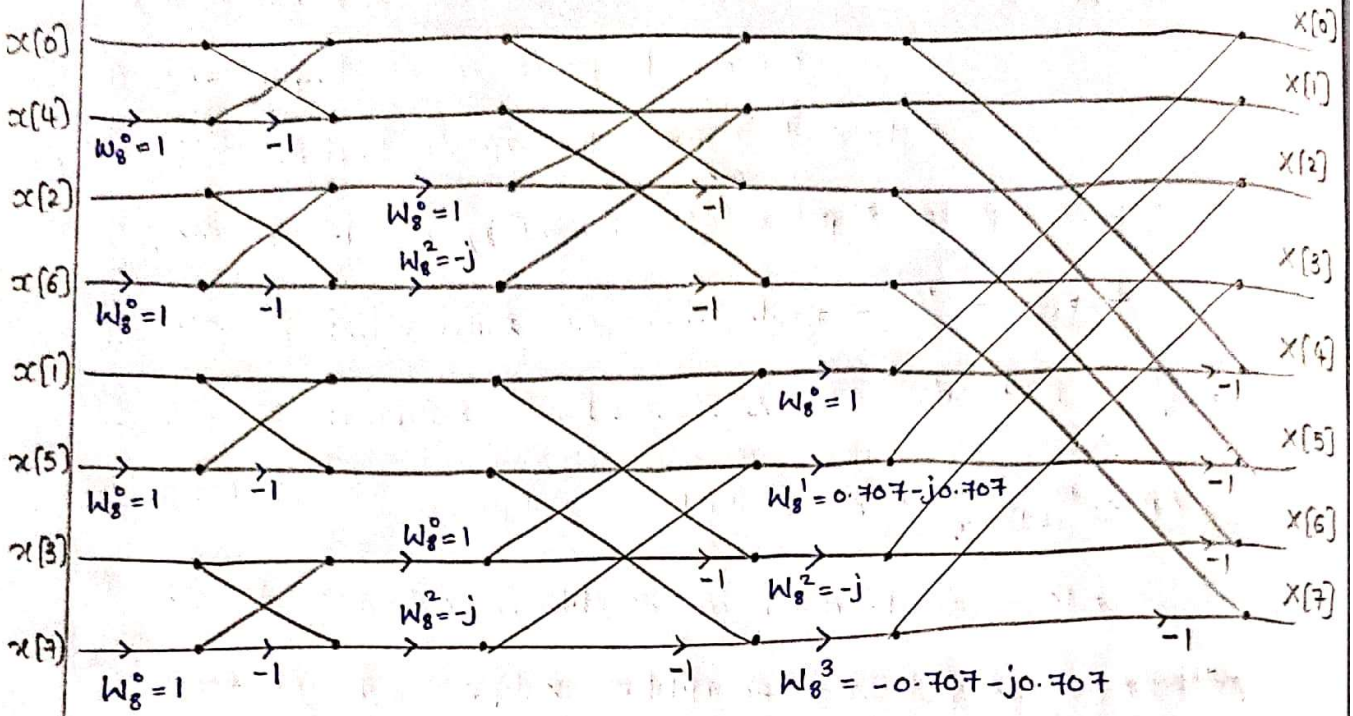
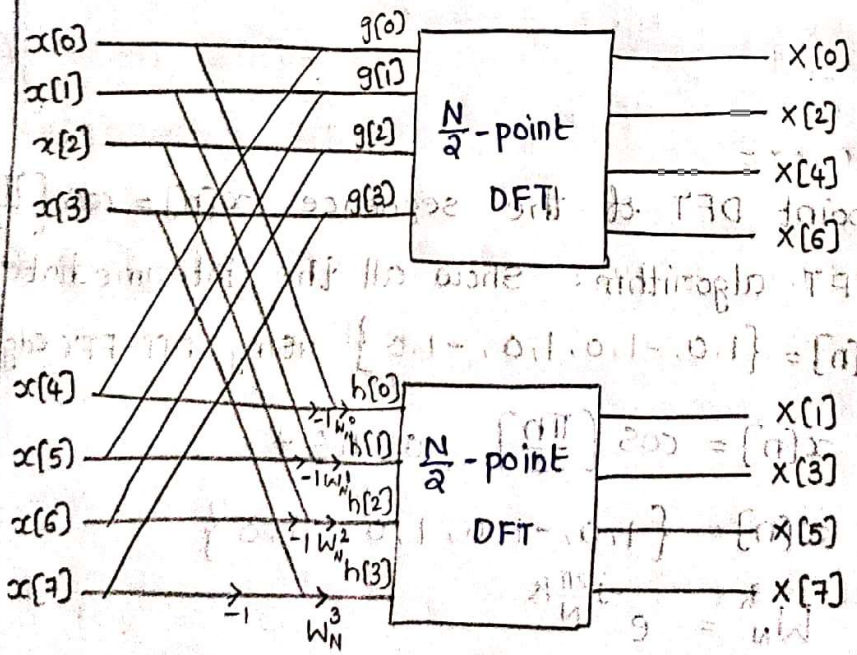
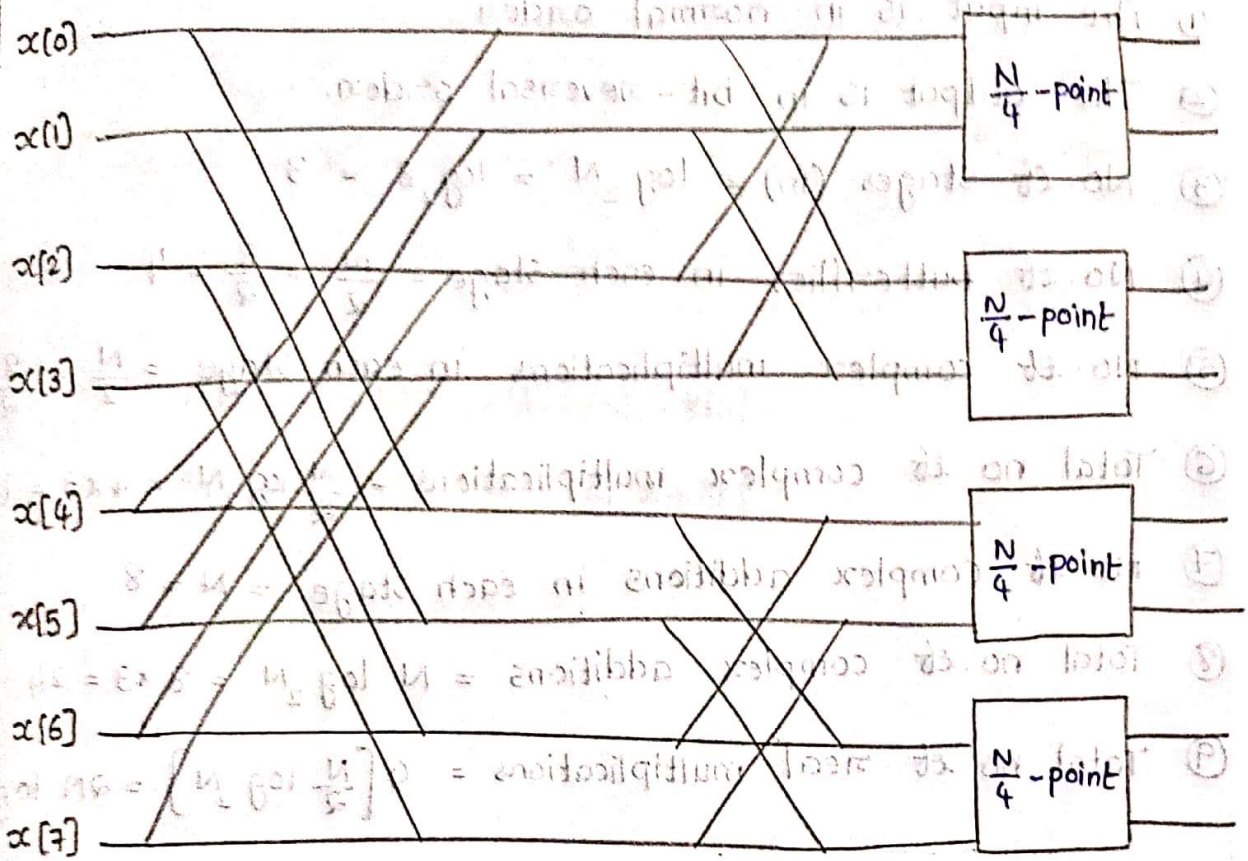


$$X[K] = \{0, 0.4, 0, 0, 0, 0.4, 0\}$$

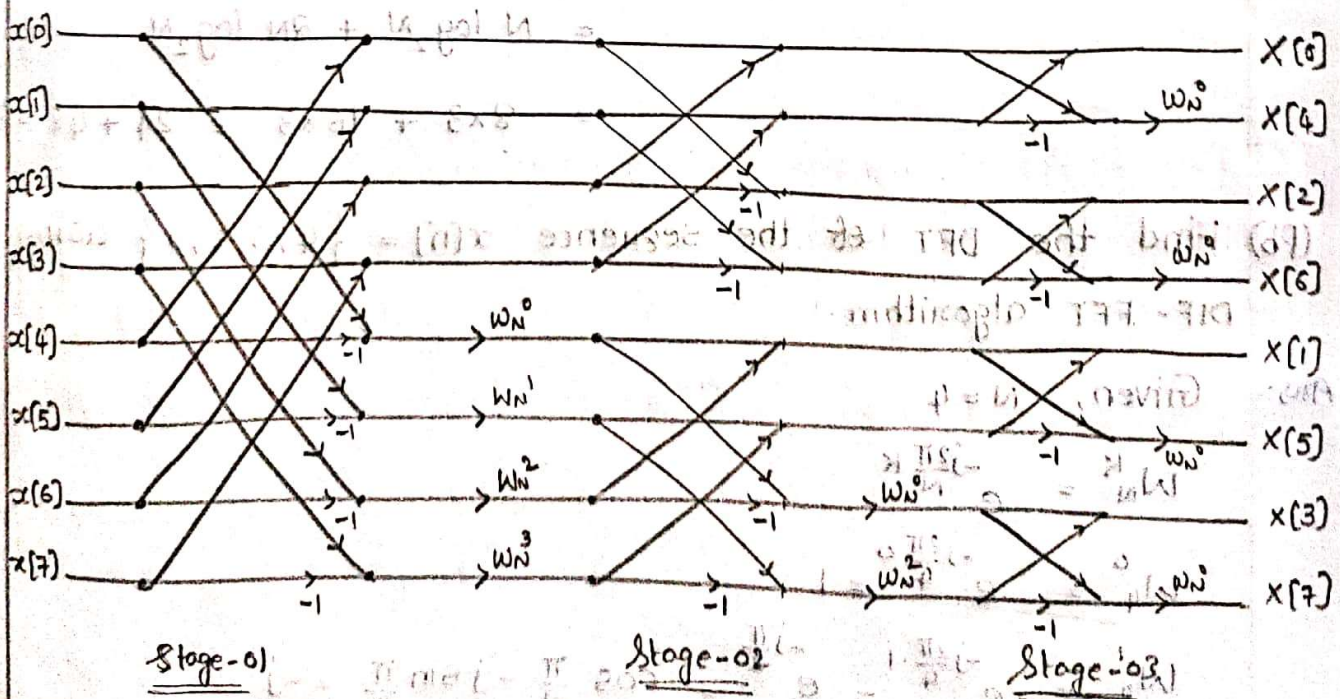
Decimation in Frequency - FFT Algorithm (DIF-FFT)



2nd stage of DIF-FFT algo:



3rd stage of DIF-FFT algo:



Observations:-

① The input is in normal order.

② The output is in bit-reversal order.

③ No. of stages (m) = $\log_2 N = \log_2 8 = 3$

④ No. of butterflies in each stage = $\frac{N}{2} = \frac{8}{2} = 4$

⑤ No. of complex multiplications in each stage = $\frac{N}{2} = \frac{8}{2} = 4$

⑥ Total no. of complex multiplications = $\frac{N}{2}\log_2 N = 4 \times 3 = 12$

⑦ No. of complex additions in each stage = $N = 8$

⑧ Total no. of complex additions = $N \log_2 N = 8 \times 3 = 24$

⑨ Total no. of real multiplications = $4\left[\frac{N}{2}\log_2 N\right] = 2N \log_2 N$

$= 16 \times 3 = 48$

⑩ Total no. of real additions = $2\left[\frac{N}{2}\log_2 N\right] + 2\left[N \log_2 N\right]$

$= N \log_2 N + 2N \log_2 N$

$= 8 \times 3 + 16 \times 3 = 24 + 48 = 72$

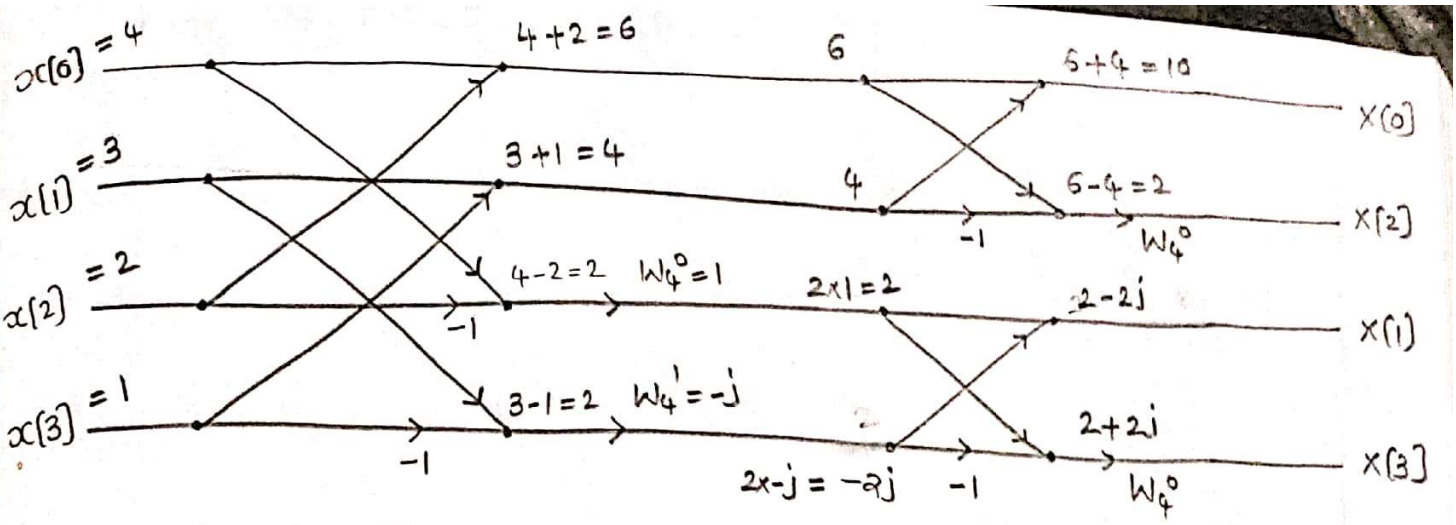
(Pb) Find the DFT of the sequence $x(n) = \{4, 3, 2, 1\}$ using DIF-FFT algorithm.

Ans:- Given, $N = 4$

$W_N^K = e^{-j\frac{2\pi}{N}K}$

$W_4^0 = e^{-j\frac{2\pi}{4}0} = 1$

$W_4^1 = e^{-j\frac{2\pi}{4}\cdot 1} = e^{-j\frac{\pi}{2}} = \cos\frac{\pi}{2} - j\sin\frac{\pi}{2} = -j$



$\therefore X[k] = \{X[0], X[1], X[2], X[3]\}$

$X[k] = \{10, 2-2j, 2, 2+2j\}$

2-point

W_2^0

4-point

W_4^0, W_4^1

8-point

$W_8^0, W_8^1, W_8^2, W_8^3$

16-point

$W_{16}^0, W_{16}^1, W_{16}^2 \ldots W_{16}^7$

(Pb) Compute the 8-point DFT of the sequence

$$x[n] = \begin{cases} 1 & 0 \le n \le 7 \\ 0 & \text{otherwise} \end{cases}$$

by using DIT-FFT algo & also show all the intermediate results.

Ans:- Given, $N = 8$

$$x[n] = \{1, 1, 1, 1, 1, 1, 1, 1\}$$

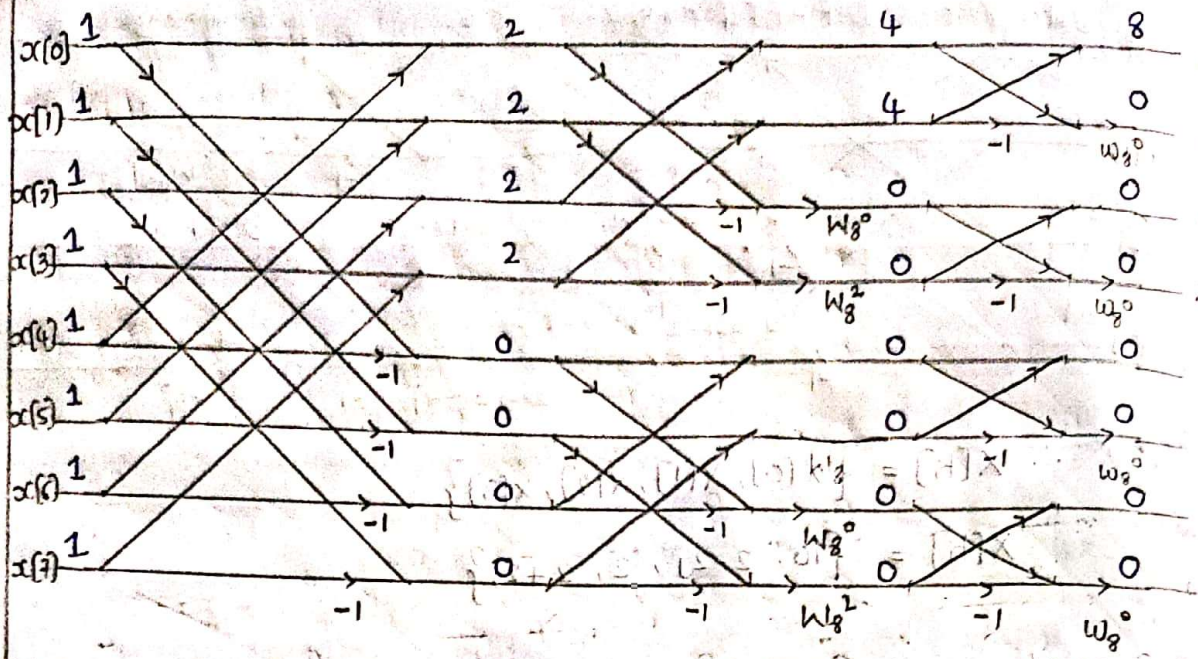
$$W_N^K = e^{-j\frac{2\pi}{N}.K}$$

$$W_8^0 = e^{-j\frac{2\pi}{8}.0} = 1$$

$$W_8^1 = e^{-j\frac{2\pi}{8}.1} = \frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}$$

$$W_8^2 = e^{-j\frac{2\pi}{8}.2} = -j$$

$$W_8^3 = e^{-j\frac{2\pi}{8}.3} = \frac{-1}{\sqrt{2}} - \frac{j}{\sqrt{2}}$$



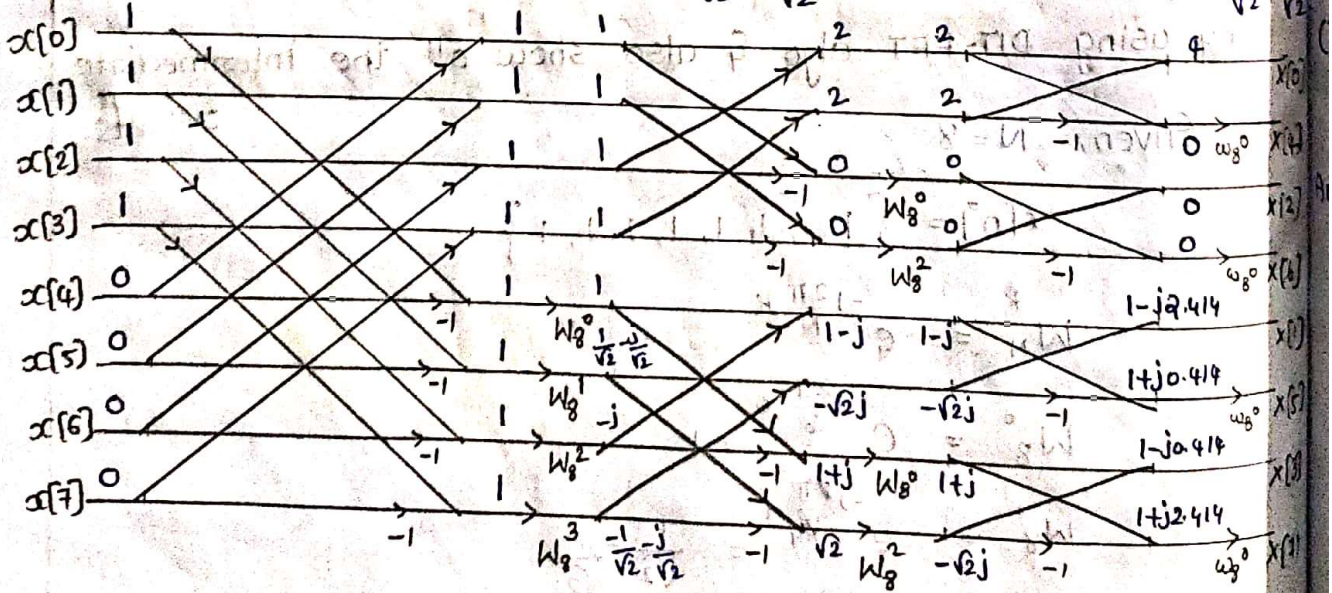
$$X[K] = \{8, 0, 0, 0, 0, 0, 0, 0\}$$

(Pb) Using the signal flow graph method DIF-FFT, determine the 8-point DFT of the sequence $x[n] = \{1, 1, 1, 1, 0, 0, 0, 0\}$. Show the intermediate values on the graph.

Ans:- Given $N = 8$, $x[n] = \{1, 1, 1, 1, 0, 0, 0, 0\}$

$$W_N^K = e^{-j\frac{2\pi}{N}.K}$$

$W_8^0 = 1$, $W_8^1 = e^{-j\frac{\pi}{4}} = \frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}$, $W_8^2 = -j$, $W_8^3 = \frac{-1}{\sqrt{2}} - \frac{j}{\sqrt{2}}$



$$X[K] = \{4, 1-j2.414, 0, 1-j0.414, 0, 1+j0.414, 0, 1+j2.414\}$$

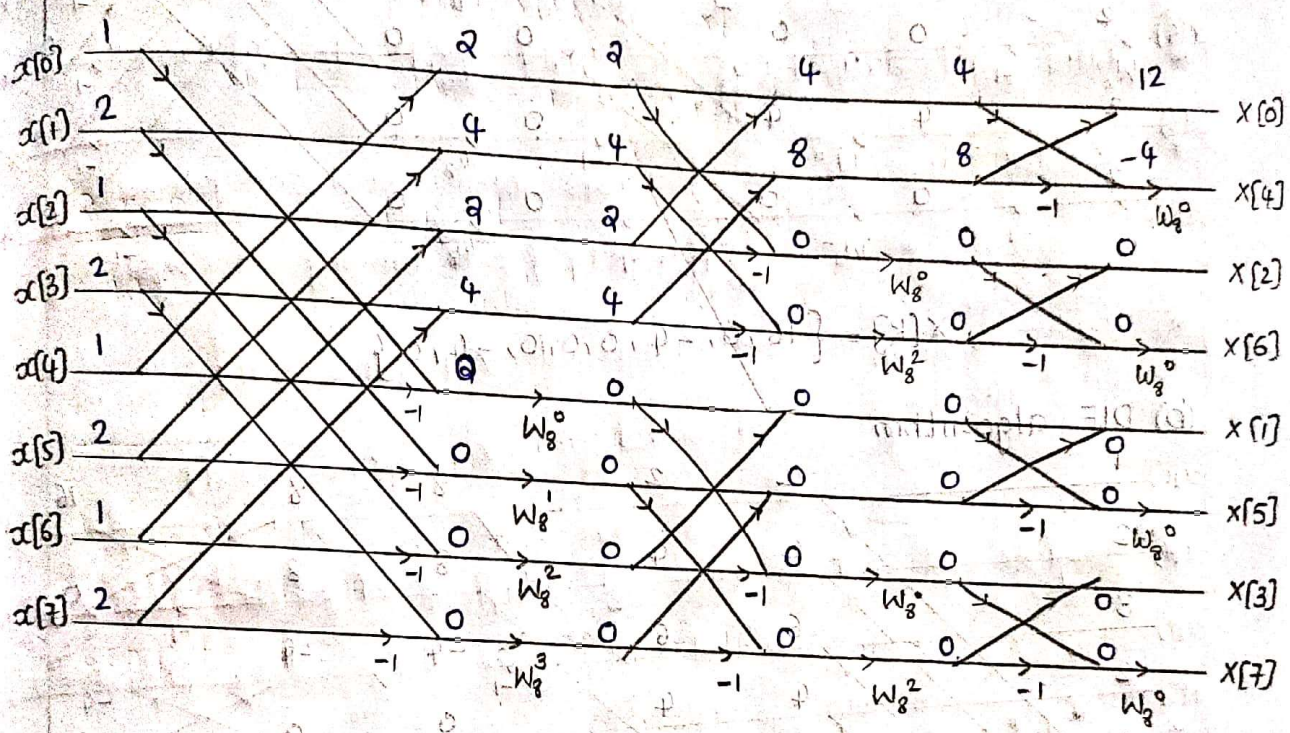
(Pb) Compute the 8-point DFT of the given sequence $x[n] = \{1,2,1,2,1,2,1,2\}$ using DIF-FFT algorithm.

Ans: Given, $N = 8$ $x[n] = \{1,2,1,2,1,2,1,2\}$, $W_N^k = e^{-j\frac{2\pi}{N}k}$

$W_8^0 = 1$ $\quad W_8^3 = -\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}$

$W_8^1 = \frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}$

$W_8^2 = -1$



$X[k] = \{12, 0, 0, 0, -4, 0, 0, 0\}$

(Pb) Compute the 8-point DFT of the following sequence $x[n] = \{1,2,3,2,1,2,3,2\}$ using (a) DIT algo (b) DIF algo.

Ans:- Given $N = 8$

$x[n] = \{1,2,3,2,1,2,3,2\}$

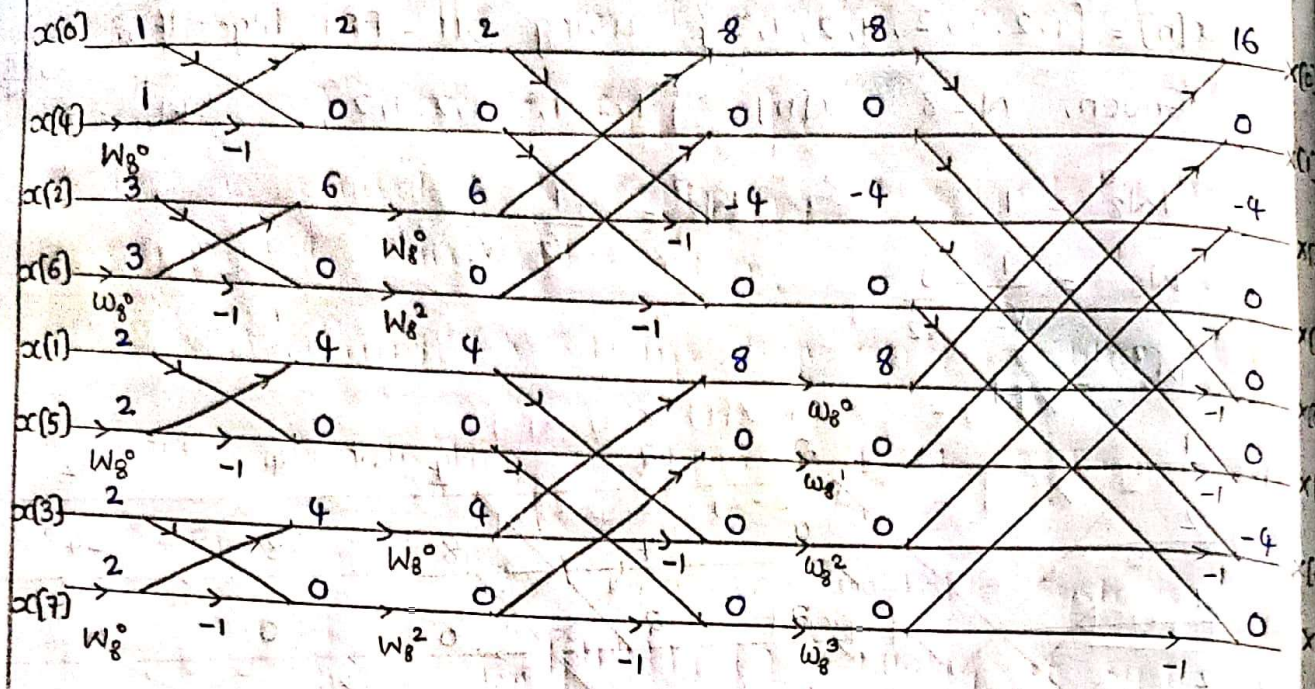
$W_N^k = e^{-j\frac{2\pi}{N}k}$

$W_8^0 = 1$ $\quad W_8^3 = -\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}$

$W_8^1 = \frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}$

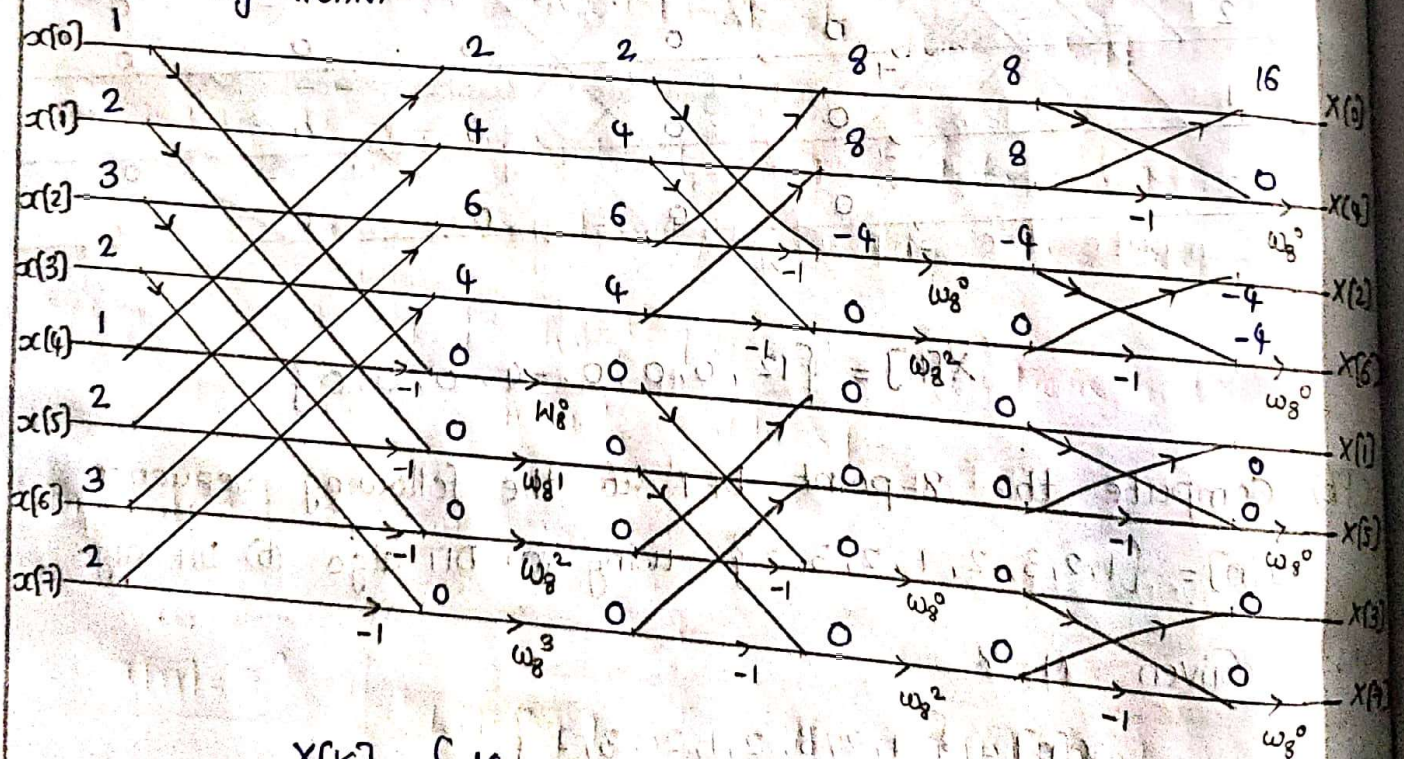
$W_8^2 = -j$

(a) DIT algorithm



$$X[K] = \{16, 0, -4, 0, 0, 0, -4, 0\}$$

(b) DIF algorithm



$$X[K] = \{16, 0, -4, 0, 0, 0, -4, 0\}$$