Maxwell's electromagnetic equations:

Maxwell's e.m. equations are based upon the well-known fundamental equations such as (i) Gauss law of electricity (ii) Gauss law of magnetism (iii) Faraday's law of e.m induction and (iv) Ampere's law.

When electric and magnetic fields are changing very rapidly with time, then varying magnetic fields give electric field and vice-versa. We therefore, ∴ consider e.m fields by a set of equations known as Maxwell's e.m equations.

(a) Maxwell's equations in integral form:

① Maxwell first equation is nothing but Gauss law in electricity. i.e $\int \vec{E}\cdot\vec{ds} = q/\epsilon_0$

$$\therefore \int \vec{E}\cdot\vec{ds} = \frac{1}{\epsilon_0}\rho\iiint dv \quad \left(\because q = \rho\iiint dv\right) \text{ (or) } \rho = \frac{q}{\iiint dv}$$

$$\Rightarrow \int \epsilon_0 E\, ds = \rho\iiint dv$$

$$\Rightarrow \boxed{\iint_S \vec{D}\cdot ds = \rho\iiint dv} \quad (\because D = \epsilon_0 E)$$

This is 1st equation in integral form.

② Maxwell's Second equation:

From Gauss law in magnetism, we have

$$\int \vec{B}\cdot ds = 0 \Rightarrow \boxed{\iint_S B\cdot ds = 0}$$

This is Second Maxwell's equation.

③ Maxwell's third equation:

From Faraday's law of e.m induction we have

$$\int E\cdot dl = -\frac{d\phi}{dt}$$

ie $$\boxed{\int E\cdot dl = -\frac{d}{dt}\iint_s B\cdot ds} \quad \left(\because \phi = \iint \vec{B}\cdot\vec{ds}\right)$$

This is third maxwell's equation.

④ Maxwell's fourth equation:

From modified form of Ampere's law, we have

$$\int B\cdot dl = \mu_0\left[J + \frac{\partial D}{\partial t}\right]$$

ie, it can be written as $$\boxed{\int_l B\cdot dl = \mu_0 \iint_s \left(J + \epsilon_0\frac{\partial E}{\partial t}\right)ds}$$

This is Maxwell's Fourth equation in integral form

(b) In Differential form:

① We have $$\iint_s D\cdot ds = \rho\iiint_V dv$$

From Gauss - Divergence theorem, $\iint_s D\cdot ds = \iiint_V (\nabla\cdot D)\,dv.$

then the above equation becomes,

$$\iiint_V (\nabla \cdot D)\, dv = \rho \iiint_V dv$$

$\therefore$ $\boxed{\nabla \cdot D = \rho}$ (or) $\boxed{\operatorname{div} D = \rho}$

This is 1st equation in differential form.

(2) We know, that $\iint_S B \cdot ds = 0$

From Gauss-Divergence theorem, $\iint_S B \cdot ds = \iiint_V (\nabla \cdot B)\, dv$

$\therefore$ The above equation becomes, i.e

$$\iint_S B \cdot ds = \iiint_V (\nabla \cdot B)\, dv = 0$$

ie $(\nabla \cdot B) = 0 \implies \boxed{\nabla \cdot B = 0}$ (or) $\operatorname{div} B = 0$.

This is Second equation in differential form.

(3) We have $\int_l E \cdot dl = -\frac{d}{dt} \iint_S B \cdot ds$

From stoke's theorem LHS Can be written as

$$\int_l E \cdot dl = \iint_S (\nabla \times E)\, ds,$$

then the above equation becomes, $\iint_S (\nabla \times E)\, ds = -\frac{d}{dt} \iint_S B \cdot ds$

$$\therefore \quad \nabla \times E = -\frac{dB}{dt} \quad \text{(or)} \quad \nabla \times E = -\frac{d}{dt}(\mu_0 H)$$

$$\implies \boxed{\nabla \times E = -\mu_0 \frac{\partial H}{\partial t}}$$

This is 3rd equation in differential form.

(4) We have $\int B \cdot dl = \mu_0 \iint_S \left(J + \epsilon_0 \frac{\partial E}{\partial t}\right) ds$.

the LHS Can be written as using Stoke's theorem ie $\int B \cdot dl = \iint_S (\nabla \times B)\, ds$.

$\therefore$ The above equation becomes, $\iint_S (\nabla \times B)\, ds = \mu_0 \iint_S \left(J + \frac{\partial D}{\partial t}\right)$

ie $\nabla \times B = \mu_0 \left[J + \frac{\partial D}{\partial t}\right]$ (or) $\boxed{\nabla \times B = \mu_0 J + \epsilon_0 \frac{\partial E}{\partial t}}$

This is Maxwell's fourth equation in differential form.

(I) Maxwell's equations in free Space:

For free Space $\rho = 0$ and $J = 0$

Now the integral and differential forms of Maxwell's equations are as follow.

Integral form: (i) $\iint_S D \cdot ds = 0$ (ii) $\iint_S B \cdot ds = 0$

(iii) $\int_l E \cdot dl = -\frac{d}{dt} \iint_S B \cdot ds$ (iv) $\int B \cdot dl = \mu_0 \iint_S \frac{\partial D}{\partial t}\, ds$.

Differential form:

(i) $\nabla \cdot D = 0$ (ii) $\nabla \cdot B = 0$ (iii) $\nabla \times E = -\frac{dB}{dt}$ (iv) $\nabla \times B = \mu_0 \frac{\partial D}{\partial t}$

(II) Maxwell's equations in Dielectric medium

In di-electric medium, $\rho = 0$, $J = 0$, $B = \mu H$ and $D = \epsilon E$

Integral form:

(i) $\iint_S D \cdot ds = 0$ (ii) $\iint_S B \cdot ds = 0$. (iii) $\int E \cdot dl = -\frac{d}{dt} \iint_S B \cdot ds$

(iv) $\int B \cdot dl = \mu \iint \frac{\partial D}{\partial t}\, ds$ (or) $\int B \cdot dl = \mu \epsilon \iint \frac{\partial E}{\partial t}\, ds$.

Differential form: $\rho = 0$, $J = 0$

(i) $\nabla \cdot D = 0$ (ii) $\nabla \cdot B = 0$ (iii) $\nabla \times E = -\frac{dB}{dt}$ (iv) $\nabla \times B = \mu \epsilon \frac{\partial E}{\partial t}$

(III) Maxwell's equations in constant field with time.

Since the field is constant, then $\frac{\partial B}{\partial t} = 0$, $\frac{\partial E}{\partial t} = 0$

Integral form: (i) $\int D \cdot ds = \int \rho \, dv$ $\quad (\because D = \epsilon E)$

(ii) $\int B \cdot ds = 0$ (iii) $\int E \cdot dl = 0$ (iv) $\int B \cdot dl = \mu \int J \, ds$

Differential form: (i) $\nabla \cdot D = \rho$ (ii) $\nabla \cdot B = 0$ (iii) $\nabla \times E = 0$

(iv) $\nabla \times B = \mu J$

Maxwell's equations in conducting medium:

For good conducting medium, $J \neq 0$ and $\rho = 0$.

Since the amount of '+ve' charge and '−ve' charges are equal. ($\rho = 0$, since charge resides only on surface)

Integral form: (i) $\int \vec{D} \cdot ds = 0$, (ii) $\int B \cdot ds = 0$

(iii) $\int E \cdot dl = -\frac{dB}{dt} \int ds$ (iv) $\int B \cdot dl = \mu_0 \int \vec{J} \cdot ds$

Differential form:

(i) $\nabla \cdot D = 0$ (ii) $\nabla \cdot B = 0$ (iii) $\nabla \times E = -\frac{\partial B}{\partial t}$

(iv) $\nabla \times B = \mu_0 J$.

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Physical Significance of Maxwell's Equations:

① **Maxwell's first equation**: This represents the Gauss law in electrostatics, which states that the total electric flux over a closed surface is $\frac{1}{\epsilon_0}$ times the total charge enclosed with in the surface.

② **Maxwell's Second equation**: This represents the Gauss law in magneto statics, which states that the net magnetic flux through any closed surface is zero. It is known that a magnetic mono pole does not exists, therefore any closed volume will always contain equal and opposite magnetic poles. Thus the magnetic flux entering the region is equal to magnetic flux leaving it.

③ **Maxwell's third equation**: This is the Faraday's law of e.m induction, which signifies that an electric field is produced by a changing magnetic flux $(d\phi/dt)$.

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(4) Maxwell's fourth equation: It is the modified form of Ampere's law. It is valid for both steady and time varying electric fields. This equation states that the amount of work done to move a unit pole around a closed path is equal to sum of conduction current and displacement current.

This signifies that a conduction current as well as displacement current (time varying field) produces magnetic field.

Poynting theorem: This theorem is used to find the power of electromagnetic wave using Maxwell's equations known as Poynting theorem. This theorem is analogous to work energy theorem in classical mechanics and mathematically similar to the continuity equation.

Suppose let us have some charge at a time 't' in the electric field E and magnetic field 'B'. Let the charge be moved a bit in a time 'dt'

The work done on the element of charge 'dq' in time 'dt' is $dw = \vec{F} \cdot \vec{dl} = [\vec{E} + (\vec{V} \times \vec{B})] dq \times \vec{V} . dt$

$(\because dl = \vec{V} . dt)$

$\therefore dw = \vec{E} . dq \, \vec{V} dt + (\vec{V} \times \vec{B}) \vec{V} \, dq \times dt$

but $(\vec{V} \times \vec{B}) . \vec{V} = 0$, then $\boxed{dw = \vec{E} . \vec{V} \, dq \, dt}$ — ①

Let $\rho \rightarrow$ volume charge density, $J \rightarrow$ current density

then $\boxed{dq = \rho \, dv}$ — ② $(\because q = \rho V)$

put ② in ①, we get

$dw = \vec{E} . \vec{V} . \rho \, dv \, dt$

The total work done on all charges per second is

$\frac{dw}{dt} = \int (\vec{E} . \vec{V}) . \rho \, dv$ — ③

put $\vec{V}.\rho = J$, $J \rightarrow$ current density

$\left(\because \frac{1}{t} \times \frac{q}{A \times l} = \vec{V} \times \rho = \frac{i}{A} = J \right)$

Then equation ③ becomes, $\boxed{\frac{dw}{dt} = \int (\vec{E} . \vec{J}) \, dv}$ — ④

Consider Maxwell's 4th equation,

$$\nabla \times \vec{B} = \mu \left[\vec{J} + \frac{\partial D}{\partial t} \right] = \left(\mu \vec{J} + \mu \epsilon \frac{\partial E}{\partial t} \right)$$

$\epsilon \rightarrow$ permittivity of a medium

$\mu \rightarrow$ permeability.

$$\Rightarrow \nabla \times \vec{B} = \mu \vec{J} + \mu \epsilon \frac{\partial E}{\partial t}$$

$$\vec{J} = \frac{\nabla \times B}{\mu} - \epsilon \frac{\partial E}{\partial t}$$

Now $\vec{E}.\vec{J}$ can be written as,

$$\vec{E}.\vec{J} = \frac{\vec{E}.(\nabla \times \vec{B})}{\mu} - \epsilon \vec{E} \frac{\partial E}{\partial t} \quad — ⑤$$

To find $E.(\nabla \times \vec{B})$ in equation ⑤ Consider

$$\vec{\nabla}.(\vec{E} \times \vec{B}) = \vec{B}(\nabla \times \vec{E}) - \vec{E}(\nabla \times \vec{B})$$

$$\Rightarrow \vec{E}(\nabla \times \vec{B}) = \vec{B}(\nabla \times E) - \nabla(\vec{E} \times \vec{B}) \quad — ⑥$$

put ⑥ in ⑤, we get

$$\vec{E}.\vec{J} = \frac{\vec{B}(\nabla \times \vec{E})}{\mu} - \frac{\nabla(\vec{E} \times \vec{B})}{\mu} - \epsilon \vec{E} \frac{\partial E}{\partial t} \quad — ⑦$$

but $\nabla \times E = -\frac{\partial B}{\partial t}$, from Maxwell's 3rd equation

then ⑦ becomes, $\vec{E}.\vec{J} = \frac{\vec{B}}{\mu}\left(-\frac{\partial B}{\partial t}\right) - \frac{\nabla(\vec{E} \times \vec{B})}{\mu} - \epsilon E \frac{\partial E}{\partial t}$

but $\vec{B}.\frac{\partial \vec{B}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}(B^2)$ and $\vec{E}\frac{\partial \vec{E}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}(\vec{E}^2)$

then ⑦ becomes, $\vec{E}.\vec{J} = -\frac{1}{2\mu}\frac{\partial}{\partial t}(B^2) - \frac{\epsilon}{2}\frac{\partial}{\partial t}(E^2) - \frac{\nabla(\vec{E} \times \vec{B})}{\mu}$

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$$\therefore \frac{dw}{dt} = \int (\vec{E}\cdot\vec{J})\, dv$$

$$= -\frac{1}{2}\frac{d}{dt}\int_V \left(\epsilon E^2 + \frac{B^2}{\mu}\right) dv - \frac{1}{\mu}\int_V \nabla\cdot(\vec{E}\times\vec{B})\, dv.$$

$$\therefore \boxed{\frac{dw}{dt} = P = -\frac{d}{dt}\int_V \frac{1}{2}\left(\epsilon E^2 + \frac{B^2}{\mu}\right) dv - \frac{1}{\mu}\int_S (\vec{E}\times\vec{B})\, ds}$$

($\because$ From Gauss - Divergence theorem)

This equation is called Poynting's theorem.

Note :- $\iint_S (\vec{E}\times\vec{B})\, ds = \iiint_V \nabla\cdot(\vec{E}\times\vec{B})\, dv$

Gauss – Divergence theorem.

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E.M wave equation

E.M wave consists of oscillating electric and magnetic fields at right angles to each other and inturn these are ⊥r to the direction of propagation of wave. Such waves require no medium for its propagation and can travel through vaccum as well.

Wave equation interms of electric field: (Conducting medium)

E.M wave equation can be derived from Maxwell's equations.

Consider Maxwell's third law.

$$\nabla \times E = -\frac{\partial B}{\partial t} \Rightarrow \boxed{\nabla \times E = -\mu \frac{\partial H}{\partial t}} \text{ — (1)} \quad (\because B = \mu H)$$

Taking curl on both sides for (1)

$$\therefore \nabla \times \nabla \times E = -\mu \left(\nabla \times \frac{\partial H}{\partial t}\right) \text{ — (2)}$$

Consider LHS, $\nabla \times \nabla \times E = \nabla(\nabla E) - (\nabla \nabla) E$

Here $\nabla \cdot E = 0$, since E inside a perfector is zero.

$$\therefore \boxed{\nabla \times \nabla \times E = -\nabla^2 E} \text{ — (3)}$$

Consider Maxwell's fourth equation

$$\nabla \times H = \left[J + \epsilon \frac{\partial E}{\partial t}\right] = \sigma E + \epsilon \frac{\partial E}{\partial t} \quad (\because J = \sigma E)$$

$$\therefore \boxed{\nabla \times H = \sigma E + \epsilon \frac{\partial E}{\partial t}} \text{ — (4)}$$

Diff. w.r.to 't',

$$\nabla \times \frac{\partial H}{\partial t} = \frac{\partial}{\partial t}\left[\sigma E + \epsilon \frac{\partial E}{\partial t}\right]$$

$$\boxed{\nabla \times \frac{\partial H}{\partial t} = \sigma \frac{\partial E}{\partial t} + \epsilon \frac{\partial^2 E}{\partial t^2}} \quad -⑤$$

put ③ and ⑤ in ② we get

$$+\nabla^2 E = +\mu\left[\sigma\, \partial E/\partial t + \epsilon \frac{\partial^2 E}{\partial t^2}\right]$$

$$\therefore \boxed{\nabla^2 E - \mu\sigma \frac{\partial E}{\partial t} - \mu\epsilon \frac{\partial^2 E}{\partial t^2} = 0}$$

Wave equation in magnetic field:

Consider 4th Maxwell's equation

$$\nabla \times H = \sigma E + \epsilon \frac{\partial E}{\partial t} \quad (\because J = \sigma E) \quad -①$$

Taking 'curl' on both sides

$$\nabla \times \nabla \times H = \sigma(\nabla \times E) + \epsilon\left(\nabla \times \frac{\partial E}{\partial t}\right) \quad -②$$

The LHS can be written as by using Vector identity, $\nabla \times \nabla \times H = \nabla(\nabla \cdot H) - (\nabla\cdot\nabla)H$

$$0 - \nabla^2 H \quad \left(\because \oint B\cdot ds = 0,\ \nabla\cdot B = 0,\ \nabla\cdot H = 0\right)$$

$$\therefore -\nabla^2 H = \nabla \times \nabla \times H \quad -③$$

From Maxwell's 3rd equation we have

$$\nabla \times E = -\mu\, \partial H/\partial t \quad -④$$

$$\Rightarrow \boxed{\nabla \times \frac{\partial E}{\partial t} = -\mu \frac{\partial^2 H}{\partial t^2}} \quad (\because \text{on differentiation})$$

⑤

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put ③, ④ and ⑤ in ②, we get

$$-\nabla^2 H = -\mu\sigma \frac{\partial H}{\partial t} + \epsilon\left(-\mu \frac{\partial^2 H}{\partial t^2}\right)$$

$$\therefore \nabla^2 H = \mu\sigma \frac{\partial H}{\partial t} + \mu\epsilon\, \partial^2 H/\partial t^2$$

$$\therefore \boxed{\nabla^2 H - \mu\sigma \frac{\partial H}{\partial t} - \mu\epsilon \frac{\partial^2 H}{\partial t^2} = 0}$$

$$\therefore \nabla^2 E - \mu\sigma \frac{\partial E}{\partial t} - \mu\epsilon \frac{\partial^2 E}{\partial t^2} = 0 \quad \text{and}$$

$$\nabla^2 H - \mu\sigma \frac{\partial H}{\partial t} - \mu\epsilon \frac{\partial^2 H}{\partial t^2} = 0 \quad \text{are Called}$$

wave equations of e.m. wave. in Conducting medium

Equation of e.m wave in free Space:

In free space $\rho = 0, \sigma = 0, J = 0, \epsilon = \epsilon_0$, $\mu = \mu_0$.

Now the above equations become

$$\nabla^2 E - \mu_0 \times 0 \times \frac{\partial E}{\partial t} - \mu_0\epsilon_0 \frac{\partial^2 E}{\partial t^2} = 0$$

$$\therefore \boxed{\nabla^2 E - \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} = 0}$$

and

$$\boxed{\nabla^2 H - \mu_0 \epsilon_0 \frac{\partial^2 H}{\partial t^2} = 0} \quad (\because \sigma = 0)$$

Velocity of e.m. wave in free Space

Consider the Maxwell's equations in free Space

ie $\rho = 0$, $J = 0$, hence these equations become

$$\therefore \nabla \cdot E = \rho / \epsilon_0 \Rightarrow \boxed{\nabla \cdot E = 0} \quad -(1)$$

$$\boxed{\nabla \cdot B = 0} \quad -(2)$$

$$\boxed{\nabla \times E = -\frac{\partial B}{\partial t}} \quad -(3) \text{ and } \boxed{\nabla \times B = \mu_0 \epsilon_0 \frac{\partial E}{\partial t}} \quad -(4)$$

Consider $\nabla \times B = \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$

Taking curl on both sides

$$\nabla \times \nabla \times B = \mu_0 \epsilon_0 \frac{\partial}{\partial t} (\nabla \times E) = \mu_0 \epsilon_0 \frac{\partial}{\partial t} \left(-\frac{\partial B}{\partial t}\right)$$

$$\nabla \times \nabla \times B = -\mu_0 \epsilon_0 \left(\frac{\partial^2 B}{\partial t^2}\right) \quad -(5)$$

from Vector identity $\nabla \times \nabla \times B = \nabla (\nabla \cdot B) - (\nabla \cdot \nabla) B$

$$\nabla \times \nabla \times B = 0 - \nabla^2 B \quad -(6)$$

From (5) & (6),

$$-\mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2} = -\nabla^2 B$$

$$\therefore \nabla^2 B = \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2} \quad -(7)$$

Compare the equation (7) with the general wave equation. ie

$$\nabla^2 y = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} \quad -(8)$$

$$\therefore \frac{1}{v^2} = \mu_0 \epsilon_0 \Rightarrow v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.85 \times 10^{-12}}}$$

$\therefore$ Velocity of e.m. wave $= 3 \times 10^8$ m/sec.

It is equal to the velocity of light.

Boundary Conditions for Electric field:

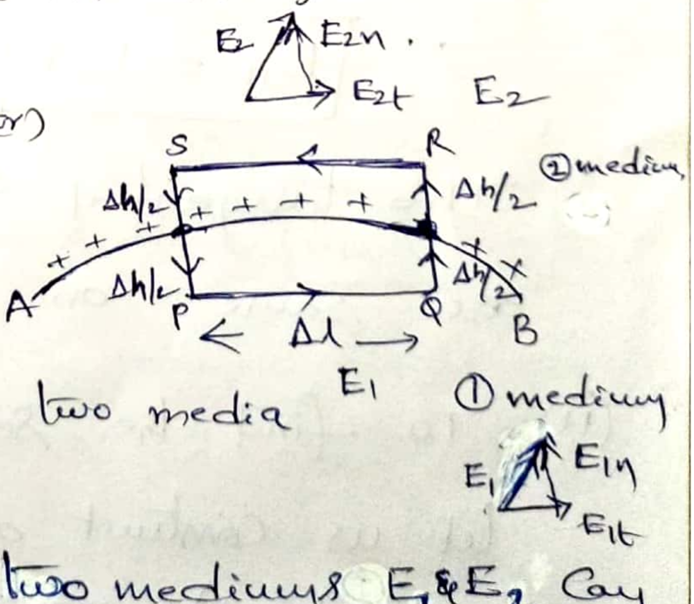
Boundary conditions for electric field are calculated to understand the continuity and discontinuity of electric field when it passes from one medium to another medium through a surface (or) boundary.

The boundary conditions of electric field are

(i) The tangential components of electric fields are same (continuous) on both sides of the boundary.

(ii) The normal components of electric fields are not same (discontinuous) at the charged surface (or) continuous across the charged free surface.

Consider an interface (or) boundary seperating two media. Let E_1, E_2 be the electric fields in the two media ① and ②, respectively.



The electric field in two mediums E_1 & E_2 can be taken as the sum of normal component and tangential component. $\therefore E_1 = E_{1t} + E_{1n}$

Similarly, $E_2 = E_{2t} + E_{2n}$.

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Consider a rectangular box PQRS which contains both the media. Let Δl be the length of the box and Δh be the height of the box.

The electric field passing through the box can be written as follows.

W.K.T in electrostatic field, the voltage around the closed path is zero.

$$\therefore V = -\int E \cdot dl = 0 \qquad \left(\because E = -\frac{dv}{dl} \text{ (or) } V = -\int E\,dl\right)$$

$\therefore$ The voltage around the box becomes,

$$\left(E_{1t}dl - E_{1n}\frac{\Delta h}{2} - E_{2n}\frac{\Delta h}{2}\right) - \left(E_{2t}dl - E_{2n}\frac{\Delta h}{2} - E_{1n}\frac{\Delta h}{2}\right)$$

$$E_{1t}\,dl - E_2\,dl = 0 \Rightarrow \boxed{E_1t - E_2t = 0} \quad (\because dl \neq 0)$$

$$\therefore \boxed{E_{1t} = E_2t}$$

$\therefore$ The tangential components of electric field are same on both sides of the boundary.

(ii) To find the second boundary condition,

Let us construct a small pill box shaped surface (cylindrical). The height of the pill box is assumed to be negligibly small in comparision with base diameter.

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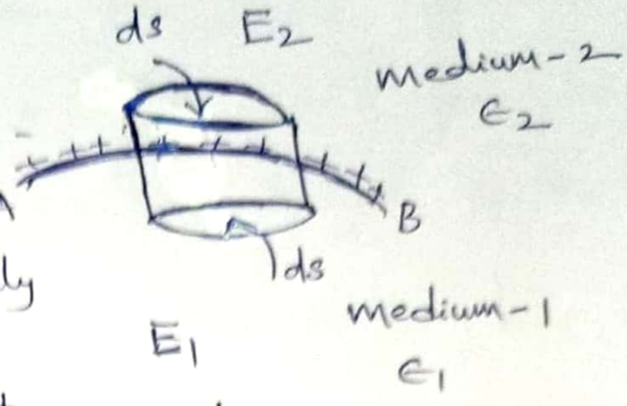
Let E_{n1}, E_{n2} be the normal components of electric-fields in medium-1 and medium-2 respectively



Apply Gauss law to the pill box,

$$\int E \cdot dl = q/\epsilon_0$$

The total flux $= q/\epsilon_0$

The total flux around the pill box $= E_{1n} ds - E_{2n} ds$

$\therefore$ From Gauss law, this electric flux $= q/\epsilon_0$

$$\therefore (E_{1n} - E_{2n}) ds = q.$$

$$\Rightarrow E_{1n} - E_{2n} = \frac{q}{ds}$$

let $\frac{q}{ds} = \sigma$; $\sigma \rightarrow$ surface charge density

$$\therefore E_{1n} - E_{2n} = \sigma \Rightarrow E_{1n} \neq E_{2n} \text{ (or)}$$

$E_{1n} - E_{2n} \neq 0$ and hence $\boxed{E_{1n} \neq E_{2n}}$

$\therefore$ The normal components of electric fields are not same on both sides of a charged boundary surface.

If the boundary surface is charge free.

ie $q = 0$ and hence $\sigma = 0$

$\therefore$ The equation becomes $E_{1n} - E_{2n} = 0 \Rightarrow \boxed{E_{1n} = E_{2n}}$

The normal components of electric fields are same over a charge free surface.

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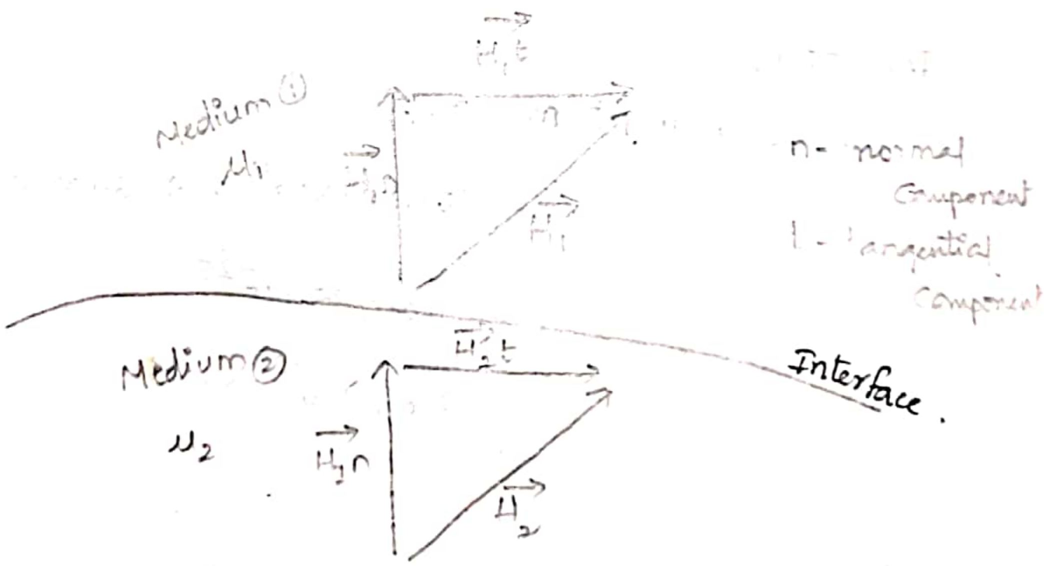
Boundary conditions for B and H at the surface of a Magnetic material:

The aim is consider that how the field vectors B and H change in passing an interface between two media. The two media may be two materials with different magnetic properties, or a material medium and vaccum.

Consider the interface between two different materials with dissimilar permeabilities.

Let us consider both magnetic field & magnetic ~~lines~~ flux density are present in both regions.

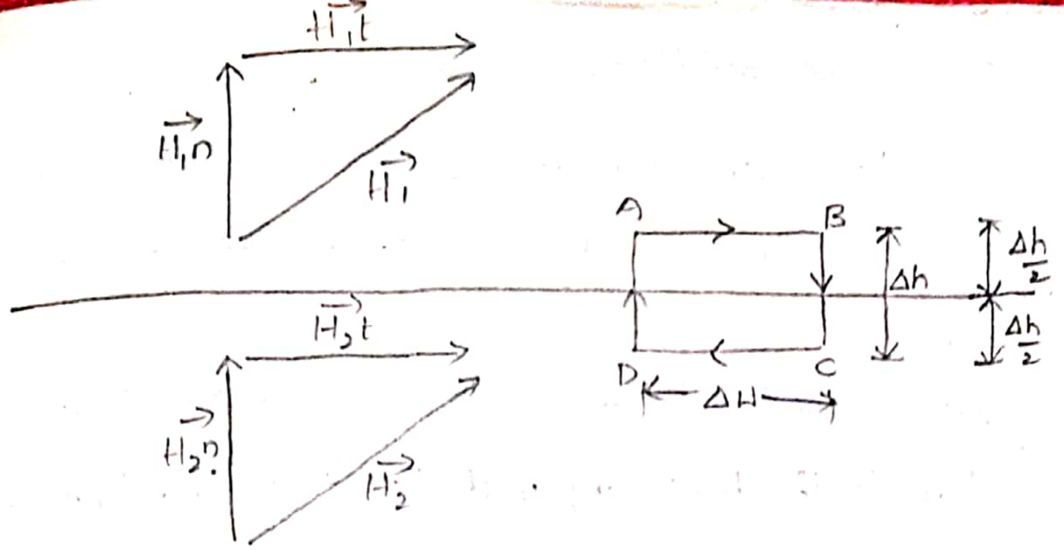
The magnetic fields at the interface in terms of their normal and tangential components as shown in figure.



Boundary condition 1:

It state that the tangential component of the magnetic field intensity is continuous across a boundary.

Let us consider a closed loop ABCD at the boundary between two media of different permeabilities μ_1 & μ_2 as shown in adjacent figure. The height and width of closed loop are Δh and Δw.



Let us suppose that some current is flowing at the boundary.

$\vec{K}$ is the current density at the boundary.

Apply Ampere's law, to this closed loop ABCD

$$\oint \vec{H} \cdot \vec{dl} = I_{enc} \longrightarrow (1).$$

$$\Rightarrow H_1 t \, \Delta W - \cancel{H_1 n \frac{\Delta h}{2}} - \cancel{H_2 n \frac{\Delta h}{2}} - H_2 t \, \Delta W + \cancel{H_2 n \frac{\Delta h}{2}} + \cancel{H_1 n \frac{\Delta h}{2}} = \vec{K} \Delta W$$

$$\therefore H_1 t \, \Delta W - H_2 t \, \Delta W = \vec{K} \Delta W.$$

$$\Delta W [H_1 t - H_2 t] = \vec{K} \Delta W.$$

$$\Rightarrow \boxed{H_1 t - H_2 t = \vec{K}} \longrightarrow (2).$$

If the boundary is free of current then eq'n (2) becomes

$$H_1 t - H_2 t = 0$$

$$\boxed{H_1 t = H_2 t} \longrightarrow (3)$$

$\therefore$ Tangential component of $\vec{H}$ is continuous at the boundary.

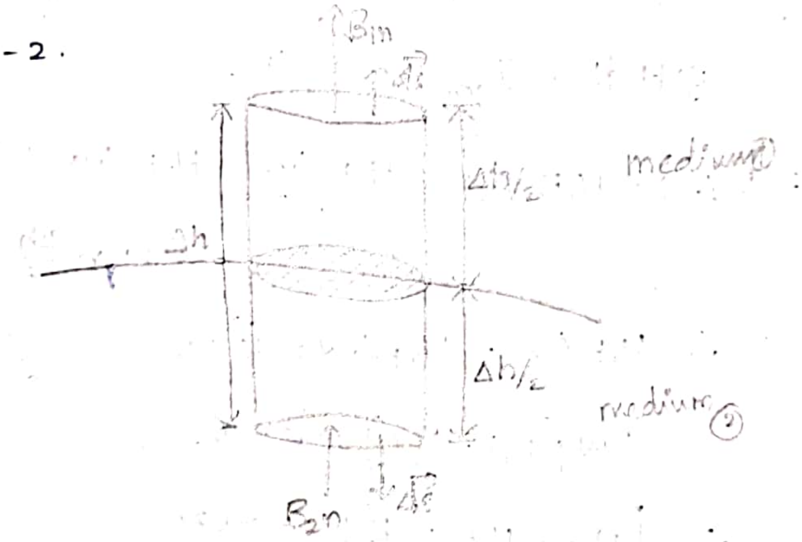
From eq'n (3) $\boxed{\frac{B_1 t}{\mu_1} = \frac{B_2 t}{\mu_2}} \longrightarrow (4)$ $\quad \because \boxed{B = \mu H}$

$\therefore$ The Tangential component of $\vec{B}$ is discontinuous of Boundary.

Boundary condition 2:

It states that the normal vector component of magnetic flux density is continuous across the magnetic boundary.

Let us consider a small pill box shaped surface which intersects the boundary with its end faces parallel to it Δh is the height of the pill box is assumed to be very small ($\Delta h \to 0$). Half of the pill box is present in the medium(1) and half of the pill box is present in medium-2.



We know that $\oint_S \vec{B}\cdot\vec{ds} = 0 \longrightarrow (5)$

The flux through the pill box is given by

$$\int_{top} \vec{B}\cdot\vec{ds} + \int_{Bottom} \vec{B}\cdot ds + \int_{Curve} \vec{B}\cdot\vec{ds} = 0 \longrightarrow (6)$$

Since the height of the pill box negligibly small ($\Delta h \to 0$) The only contribution of flux will come through top & bottom face.

$\therefore\ B_{1n}\Delta s - B_{2n}\Delta s = 0$

$$\boxed{B_{1n} = B_{2n}} \longrightarrow (7)$$

∴ The normal component of magnetic flux density is continuous at the boundary.

we know that, $\vec{B} = \mu \vec{H}$

∴ from eq'n 7) $\mu_1 H_{1n} = \mu_2 H_{2n}$

$$\therefore \boxed{\frac{H_{1n}}{H_{2n}} = \frac{\mu_2}{\mu_1}} \longrightarrow (8)$$

The normal component of magnetic field intensity is discontinuous at the boundary seperating two mediums.